Climate-HANK: Carbon Shocks, Fiscal Policy, and Double Dividends

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Abstract

Carbon pricing introduces price fluctuations at business cycle frequency. I develop a heterogeneous-agent New Keynesian model with a carbon market to analyze the effects of these fluctuations on the business cycle and their distributional impact. An adverse carbon shock increases inflation and decreases economic activity with the costs borne relatively evenly across the income and wealth distribution. Fiscal policy can mitigate the macroeconomic and welfare effects of higher carbon prices in a revenue-neutral way using the revenue from higher carbon prices. Paying it back as lump-sum transfers, higher carbon prices have a welfare-enhancing effect that goes beyond welfare gains from lower emissions—a double dividend result.

Keywords: Carbon shocks, Fiscal policy, HANK, Distributional impact of climate change heterogeneity, inequality, households, Double dividends

JEL-Codes: D31, E62, E64, Q58

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1 Introduction

Climate change is one of the biggest challenges of our times. As carbon dioxide emissions are one of the main drivers for climate change, policymakers in many countries introduce costs of emitting carbon dioxide with the aim to reduce its demand by firms and households. The European Union (EU), for example, has introduced the European Union Emission Trading System (EU ETS) as a cap and trade system to manage the amount of carbon dioxide emitted in the EU. If regulated by the EU ETS, agents need to buy sufficient carbon dioxide certificates to cover their carbon dioxide emissions putting a price on carbon dioxide issuance. Figure 1 shows the price per ton of carbon dioxide use over the last 4 years in the EU. It demonstrates that the price for carbon dioxide use is highly volatile and that it fluctuates at business cycle frequency. This raises the questions: How does carbon price volatility affect the business cycle? Who bears the cost of carbon pricing? And which fiscal policy should be implemented to mitigate the costs of carbon price fluctuations?

To answer these questions, I build a *Climate-HANK* model: I extend a heterogeneous-agent New-Keynesian (HANK) model as in Bayer, Born, and Luetticke (2020) by a carbon market. Carbon dioxide enters the model two-fold: first, it is used as an input in the production function of firms alongside capital and labor. Second, households directly consume carbon dioxide and I allow for heterogeneity within the intensity of carbon dioxide in households' consumption baskets which I match to German micro data. I start by simulating the business cycle effects of a carbon shock, that is a surprise drop in the amount of carbon allowances. This increases the price per unit of carbon dioxide use. I find that this affects the economy similar to a cost-push shock: output, investment, and consumption decrease while inflation increases and average welfare in the economy decreases. I also show that the distributional effects are rather small: both along the income and along the wealth distribution, the costs of the carbon shock are borne relatively uniformly. While poorer households' welfare are hit harder by the recession, the consumption of richer households has a higher carbon dioxide intensity.

The adverse carbon shock increases the revenue of the government from selling carbon certificates. In the baseline, I assume it to be absorbed by lower debt in the short-run reflecting the lack of earmarking of carbon tax revenues for certain fiscal instruments in most countries. I show that earmarking these revenues for certain fiscal instruments could mitigate the aggregate effects of the carbon shock and eases its welfare burden. However, there is a trade-off between macroeconomic stabilization and welfare improvement depending on the fiscal instrument for which the extra revenue is earmarked: while paying a capital subsidy is most effective in curbing the recession, average welfare of households is lower than in the

Figure 1: The price of emissions allowance in the EU

Notes: Cost per ton of carbon dioxide use in €. Source: EMBER.

no-earmark case. In contrast, repaying the extra revenue directly to households in form of a lump-sum transfer mitigates the recession only slightly but it even increases average welfare above the status quo of no carbon shocks. In that sense, repayment by transfers is the HANK-version of the seminal double dividend result (see e.g. Goulder, 1995) as in this case, higher carbon pricing is even welfare-increasing. However, this comes at the cost of introducing much larger distributional consequences than the carbon shock itself as in this case, higher carbon prices redistribute from the income- and wealth-rich to the poor.

More in detail, in this paper I extend the medium-sized HANK model in Bayer, Born, and Luetticke (2020) by explicitly modelling the market for carbon dioxide, accounting for the energy consumption of households and firms. Based on the EU-ETS, I assume that the total supply of carbon dioxide is exogenously set by the fiscal authority. I allow for heterogeneity in the carbon dioxide share of households' consumption baskets, in line with German micro data. The other model features are by now fairly standard. Financial markets are incomplete and households face idiosyncratic risk which is why they self-insure through savings in a liquid and illiquid asset. As a result, there is a non-degenerate distribution of income and wealth. Wages and prices are sticky through the standard way in New Keynesian models. I calibrate the economy based on data for Germany—the largest country in the EU—including the degree of heterogeneity in the carbon dioxide intensity in the consumption baskets found for German households. My modelling choices allow me to capture the heterogeneity in carbon dioxide intensity conditional on income for which I find that the carbon intensity tends to be lower for lower income households. At the same time, my model can capture the large heterogeneity even when conditioning on income which I find to be large in the data.

I simulate a surprise and temporary reduction in the carbon dioxide allowances. This increases the price of carbon dioxide and, thus, the costs in firms' production and the price of the consumption basket of households. I find that in response to this cost-push shock, output, investment, and consumption fall, while inflation increases. The consumption Gini only moves slightly, which shows the small distributional impact of the carbon shock. This is also reflected in the welfare impact of a carbon shock along the wealth and income distribution. The average welfare impact for all ten deciles of the income as well as for the wealth distribution is negative. Yet, I find the welfare impact of the carbon shock to be mitigated by its positive impact on the government budget. While this leads to lower government debt in the short run, lower taxes bring government debt eventually back to its steady state level. These lower taxes in the long-run, themselves, have a positive effect on welfare thereby mitigating the overall loss in welfare from higher carbon prices.

I then use the model to compute counterfactual analyses in which fiscal policy directly spends its extra revenue. More specifically, I consider three different fiscal instruments through which the government directly repays its extra revenue from the carbon system: lump-sum transfers to households, lower income taxes, and higher capital subsidies. Repayment through transfers mitigates the fall in output through carbon taxes as it stimulates private consumption. However, the dampening effects on output are quantitatively small as the higher transfers further decrease investment. I find that average welfare now increases rather than decreases in response to an adverse carbon shock. This uncovers a novel, HANK-type, double-dividend result: using the extra revenue from higher carbon prices can be welfare-beneficial if the extra revenue is used to finance higher transfers. The reason is that higher transfers decrease consumption inequality and provide partial insurance against households' idiosyncratic income risk. Hence, in addition to the unmodelled welfare gain of lower carbon emissions, the restructuring of government expenditures induced by higher carbon prices adds an additional welfare gain. Yet, I show that the higher average welfare comes at the cost of higher distributional impact of carbon shocks: both along the income and the wealth distribution, it is mainly the bottom 50% who benefit from higher transfers while the richest deciles are slightly worse off compared to the baseline model.

I then show that repayment through a direct decrease in income taxes substantially mitigates the recession, especially in terms of output and consumption. In addition, it increases average welfare, albeit at a smaller magnitude than repayment through transfers. Thus, I show that the classical double-dividend result—using the revenue from the carbon system to reduce distortionary taxes—holds in my business cycle context. Yet, again, the double-dividend result comes at the cost of increasing the distributional impact of carbon shocks, but to a much smaller degree than repayment through transfers would.

Finally, I show that repayment through capital subsidies mitigates the recession the most and even increases output on impact. They are also the most effective in mitigating the fall in investment. Yet, with repayment through capital subsidies, average welfare falls even stronger than in the baseline. At the same time, it amplifies the distributional impact of carbon shocks as it is the wealthy households that directly benefit from the capital subsidy.

The paper is structured as follows. In the remainder of this section, I discuss the related literature. Section 2 provides a summary of the model. Most of the details are relegated to the appendix. Instead, the exposition focuses on the carbon market. Section 3 presents details of the calibration of the model and Section 4 the results. The final section offers some conclusions.

Related literature. My paper relates closely to two strands of the literature. First, there is the recent surge of HANK models which are used to revisit the transmission of traditional business cycle shocks and business cycle policies starting with the influential study of Kaplan, Moll, and Violante (2018), but also, for instance, Auclert (2019) and Bayer, Luetticke, Pham-Dao, and Tjaden (2019). This framework lends itself naturally to the analysis of fiscal policy more broadly (Auclert, Rognlie, and Straub, 2018; Bayer, Born, Luetticke, and Müller, 2023; Pfäuti, Seyrich, and Zinman, 2024) and in particular to the analysis of tax policies (Le Grand, Martin-Baillon, and Ragot, 2021; Bhandari, Evans, Golosov, and Sargent, 2021; Seidl and Seyrich, 2023).

Second, my paper relates to the strand of literature that analyzes the macroeconomic impact of climate change and, in particular, to its sub-strand that studies its business cycle implications (Känzig, 2023; Metcalf, 2019; Metcalf and Stock, 2023; Bernard and Kichian, 2021; Konradt and Weder, 2021; McKibbin, Morris, Panton, and Wilcoxen, 2017; Goulder and Hafstead, 2017). In contrast to these papers, I build a general-equilibrium business-cycle model with heterogeneous agents and rich portfolio choices which allows me to jointly analyze the macroeconomic impact of carbon pricing and its distributional impact as well as to run counterfactuals analyzing alternative repayment structures of the extra revenue from higher carbon prices. The parallel work of Langot, Malmberg, Tripier, and Hairault, 2023 is closest to mine. While I focus on German data and resort to a model with rich portfolio choice, they focus on France and on a HANK model without portfolio choice and without investment. Also, they do not compute the welfare impact of carbon pricing shocks. In addition, while they find in French data that the consumption baskets of lower-income households is more energy-intensive, I find in German data, that the consumption basket of lower-income households is less carbon dioxide intensive which makes the partial equilibrium effect of higher carbon prices progressive instead of regressive.

2 A Climate-HANK model

I evaluate the effects of temporary fluctuations in carbon pricing in a medium-scaled HANK model. In particular, I extend the HANK model in Bayer, Born, and Luetticke (2020) by a carbon market. The carbon market resembles the structure of the energy market in Bayer, Kriwoluzky, Müller, and Seyrich (2023). The model accounts for carbon dioxide emissions in production and in household consumption. The latter represents direct carbon dioxide emissions by households which may be due to heating, gasoline or transport services. The model features incomplete financial markets and assets with different liquidity (bonds and capital). The model captures household heterogeneity with respect to income, wealth, portfolios, and carbon dioxide intensity of consumption. The following is a brief summary of the model, with a particular focus on the carbon market. A full description of the model can be found in the Appendix.

2.1 Summary of the model

Markets are incomplete and households face idiosyncratic, that is, household-specific, risks but are able to self-insure. They can do so using a liquid asset that can be traded every period and an illiquid asset (physical capital) which can only be traded subject to a friction. As a result, households are heterogeneous in terms of income and wealth. Households with little wealth or households whose wealth consists mainly of illiquid assets (e.g. houses) have a high propensity to consume out of disposable income and transfers.

Prices and wages are sticky, as is common in the New Keynesian literature. The model consists of a firm sector, a household sector, and a government sector. The firm sector consists of (a) perfectly competitive intermediate goods firms that produce intermediate goods using capital and labor thereby emitting carbon dioxide; (b) final goods firms that operate under monopolistic competition and produce differentiated final goods from homogeneous intermediate goods; (c) capital goods producers that transform consumer goods into capital; (d) labor intermediaries that produce labor services by combining differentiated labor from (e) unions that differentiate the raw labor provided by households. Pricing by final goods producers goods and wage setting by unions is subject to frictions à la Calvo (1983).

There is a continuum of households that consume final goods and emit carbon dioxide. Households earn income from supplying (raw) labor and capital to the labor and capital markets and from owning their firm sector, absorbing any rents arising from the market power of unions and final goods producers and from diminishing returns to scale in capital goods production.

The government sector comprises a monetary policy and a fiscal authority. The fiscal authority levies taxes on labor income and distributed profits, issues government bonds and sells carbon certificates. The fiscal authority also operates a targeted transfer system. Monetary policy sets the nominal interest rate in the economy using a Taylor rule, that is, it adjusts the interest rate to inflation.

2.2 Modelling carbon dioxide emissions

A distinct and novel feature of my analysis is to model carbon dioxide emissions in a HANK framework. The way I model carbon dioxide emissions in the economy is closely related to how Bayer, Kriwoluzky, Müller, and Seyrich (2023) model energy usage in the economy. In the following, I provide details in this regard, first discussing sources of carbon dioxide issuance and then turning to the market for carbon certificates.

Carbon dioxide is issued in two different ways: First, producing the consumption good issues carbon dioxide such that carbon dioxide—along with labor and capital—is an input to the production of intermediate goods. Carbon dioxide can be substituted away in the production but this leads to efficiency losses in production. This allows me to capture how carbon prices affect the supply side of the economy through its effect on industrial production. Specifically, I assume intermediate goods Y_t are produced with the (nested) CES production function:

$$Y_t = \left((1-a_P)^{\frac{1}{\sigma_P}} \; {Y_t^P}^{\frac{\sigma_P - 1}{\sigma_P}} + {a_P}^{\frac{1}{\sigma_P}} \; \left(E_t^Y\right)^{\frac{\sigma_P - 1}{\sigma_P}} \right)^{\frac{\sigma_P}{\sigma_P - 1}}, \text{ where } Y_t^P = (u_t K_t^s)^{\alpha} \; {N_t}^{1-\alpha}. \quad (1)$$

As this expression shows, the intermediate good is made of a physical input, Y_t^P, which, in turn, combines capital, K_t, with capacity utilization u_t, and labor, N_t, on the one hand, and carbon dioxide, E_t^Y, on the other hand. The coefficient α is the capital share, the coefficient σ_P captures the (short-run) substitutability of carbon dioxide in the production process, and a_P is the carbon dioxide issuance during production in normal times.

Second, households directly emit carbon dioxide as part of their consumption basket. This can be thought of burning gasoline while driving a car, the carbon dioxide emissions caused by transport services, or carbon dioxide issued by heating or other energy sources.[1] In addition, I allow households to differ in their carbon dioxide intensity in consumption. Total consumption c_{it} of household i at time t consists of the physical consumption good c_{it}^P and

[1]In the terminology of Kuhn and Schlattmann (2024), households emit carbon dioxide both *directly* as well as *indirectly* with the latter being the carbon dioxide emission coming from the physical consumption good that households consume.

carbon dioxide, E^C_{it}, again combined in a CES aggregator:

$$c_{it} = \left(\left(1 - a^C_{it}\right)^{\frac{1}{\sigma_C}} \; c^P_{it}{}^{\frac{\sigma_C - 1}{\sigma_C}} + a^C_{it}{}^{\frac{1}{\sigma_C}} \; \left(E^C_{it}\right)^{\frac{\sigma_C - 1}{\sigma_C}} \right)^{\frac{\sigma_C}{\sigma_C - 1}}. \tag{2}$$

Here σ_C represents the elasticity of substitution in consumption, that is, it measures the extent to which carbon dioxide can be substituted for physical consumption goods as relative prices fluctuate.

Households differ in the carbon dioxide intensity of their consumption baskets, captured by a^C_{it}. I assume that the long-run share of carbon dioxide in consumption varies exogenously across households and over time. The transitions from low to high and from high to low carbon dioxide intensity are random but related to the income state of the household. Concretely, I assume for the probability $\rho(h, a^C)$ to switch from one carbon dioxide type to the other the following functional form:

$$\rho(h, a^C) = \bar{\rho} + (\mathbb{I}_{a^C = a^C_H} - \mathbb{I}_{a^C = a^C_L}) A(h) + \mathbb{I}_{a^C = a^C_L} B, \tag{3}$$

where A is a linear function of the human capital quintile and B is a constant that captures that it is in general more likely to remain a low carbon dioxide type.

This allows me to capture two key dimensions of heterogeneity in households' carbon dioxide share in the data: First, there is a strong positive correlation between the carbon dioxide emissions and household income. Second, there is a large dispersion in the carbon dioxide emissions even conditional on income. I will further discuss the large heterogeneity in Section 3. However, while I allow for transitions in carbon dioxide-intensity types, I model type transitions as infrequent, so that the carbon dioxide intensity of the household is very persistent—in line with the fact that households preferences probably change very slowly.

The differences in carbon dioxide intensity also imply heterogeneity in inflation rates across households when carbon prices move. Since carbon dioxide is a component of household consumption, an increase in carbon prices raises the household price index and, all else equal, leads to a reduction in real income and, potentially, consumption. This effect is more pronounced for households with high carbon dioxide intensity than for households with low carbon dioxide intensity.

2.3 Demand for carbon certificates and market clearing

I assume that whenever firms or households issue carbon dioxide, they need to be in possession of a carbon certificate. In the spirit of the EU ETS, I assume that policy sets the amount of carbon certificates, E_t. For simplicity, I assume that the EU ETS covers all economic activity. This implies that there is a price on issuing carbon dioxide, p_t^C. As there are no other costs involved when issuing carbon dioxide, the price of issuance one unit of carbon dioxide equals the carbon price $p_t^E = p_t^C$. I further assume that the government sells the certificates such that the government receives all revenue from the carbon certificate system. The demand for carbon dioxide from the production sector satisfies the following first-order condition:

$$p_t^C = mc_t(1 - a_P)^{\frac{1}{\sigma_P}} \left(\frac{Y_t}{E_t^Y} \right)^{\frac{1}{\sigma_P}}, \tag{4}$$

and the demand for carbon dioxide from households satisfy the type-specific first-order conditions:

$$E_{it}^C = \frac{1 - a_{it}^C}{a_{it}^C} (p_t^C)^{-\sigma^C} c_{it}^P. \tag{5}$$

Total carbon dioxide issuance in the economy then equals the amount of carbon certificates:

$$E_t = E_t^C + E_t^Y. \tag{6}$$

I model a carbon shock by an exogenous and temporary decrease in carbon certificates. More precisely, I assume E_t to follow the following AR(1)-process:

$$E_t = (1 - \rho_E)\bar{E} + \rho_E E_{t-1} + \varepsilon_t^E, \tag{7}$$

2.4 Carbon shocks and the role of fiscal policy

An increase in carbon pricing increases the fiscal surplus. There is a wide debate how to repay this fiscal surplus. In my baseline, I assume that there is no direct repayment reflecting the fact that in most countries, revenue through carbon pricing is not earmarked for certain expenditures or tax cuts. To analyze the potential usefulness of such earmarking, I then also analyze three counterfactual scenarios in which the extra revenue is earmarked for i) repayment through lump-sum transfers, ii) repayment through lower income taxes, and iii) repayment through capital subsidies.

Baseline. In the no repayment case, I assume that the extra tax income generated by higher carbon taxes enters the budget of the government without being earmarked for certain expenditures or tax cuts. Hence, in the short-run, higher revenue from higher carbon prices decreases government debt. In the long-run, lower income taxes bring back government debt to its steady state level resembling the idea that the carbon dioxide emissions revenue will eventually reduce the tax revenue that the government needs to collect.

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{\tau_{t-1}}{\bar{\tau}}\right)^{\rho_\tau} \left(\frac{B_{t+1}}{\bar{B}}\right)^{(1-\rho_\tau)\gamma_B^\tau}. \tag{8}$$

Repayment through lump-sum transfers. In this case, I assume that the extra revenue generated by the increase in higher carbon prices is directly redistributed to households via uniform lump-sum transfers. This implies:

$$Tr_t = p_{E,t}E_t - \bar{p_E}\bar{E}. \tag{9}$$

Repayment through income taxes. In this case, I assume that the extra revenue generated by the increase in higher carbon prices is earmarked for lower income taxes. In particular, I assume that income taxes are reduced by τ_t^C, with:

$$\tau_t^C = \frac{(p_{E,t}E_t - \bar{p_E}\bar{E})}{TB}, \tag{10}$$

where TB is the income tax base.

Repayment through capital subsidies. In this case, I assume that the extra revenue generated by the increase in higher carbon prices is earmarked for capital subsidies. In particular, I assume that capital subsides s_t^C are directly paid to firms per unit of capital such that:

$$s_t^C = \frac{(p_{E,t}E_t - \bar{p_E}\bar{E})}{K}. \tag{11}$$

In all three counterfactuals, I assume that all effects on the government budget which are not due to changes in carbon revenues are absorbed by debt in the short-run with debt being brought back to steady state in the long-run by taxes.

3 Calibration

As for the model exposition, I mainly focus here on my calibration strategy for the carbon market. For the rest of the calibration, I only briefly sketch the calibration strategy leaving the details for Appendix B.

I calibrate the model economy to Germany for two reasons: first, it is the largest economy in the EU and, second, there is detailed data on households' consumption which allows

Table 1: Calibration of the carbon sector

	Description	Value
σ_P	Elasticity of substitution in production	0.100
σ_C	Elasticity of substitution in consumption	0.100
a_P	Share of energy in production	0.017
a_{CH}	Proportion of carbon in consumption: Type "high"	0.018
a_{CN}	Proportion of carbon in consumption: Type "low"	0.010
$\bar{\rho}$	Persistence of high carbon state at median income	0.966
A	Slope of probability to stay in low carbon state	0.005
B	Shift in probability to remain in low carbon state	0.010

me to determine the degree of heterogeneity in the carbon dioxide intensity of households' consumption baskets. When matching the income and wealth distribution in Germany, I follow the strategy employed in Bayer, Kriwoluzky, Müller, and Seyrich (2023): A targeted transfer system mimics the German minimum benefits system which pays transfers to households with income below a certain threshold. I then set key parameter values in order to match the debt ratio, the capital ratio, the wealth Gini, the share of the 10% richest in total wealth, the share of the 50% poorest in total wealth, and the share of indebted households in Germany. I set the remaining parameters to values that have been established in business cycle analyses based on New Keynesian models. Appendix B provides details on the calibration.

Given the focus on this paper, I report the key parameters related to the carbon dioxide sector in some detail, see Table 1. Specifically, I choose the carbon dioxide share, a^P, for the firm sector to match the steady-state carbon dioxide expenditure shares of 1.7% of production costs.[2] I set the elasticity of substitution in production σ_P to 0.2. This captures the limited substitutability of carbon dioxide especially in the short-run reflecting the energy substitutability found in (Bachmann et al., 2022). For the household sector, I set the elasticity of substitution to $\sigma_C = 0.2$ which is an intermediate value between Bachmann et al. (2022)'s value for the energy elasticity of households and the value in Känzig (2023). To compute the share of carbon dioxide in households consumption baskets, I take expenditures on the three consumption categories that have by the far the highest carbon intensity:[3] fuel, transportation services, and energy consumption.[4] In addition, I follow Bayer, Kriwoluzky,

[2] In 2022, German production emitted 573 millions tons of carbon dioxide. Assuming a carbon price of 100€—which amounts to the highest value in the EU ETS so far—this equals 57 billions € which amounts to 1.7% of the production costs.

[3] This calibration strategy resembles the calibration strategy in Kuhn and Schlattmann (2024).

[4] More precisely, I compute how much an individual household spend for these three consumption categories. I then compute the carbon dioxide per € ratio for these three sectors indicating how much carbon dioxide is emitted in these sectors per € and multiply this with households expenditures in these sectors. I then aggregate households carbon dioxide emission over these three sectors.

Table 2: Emission of carbon dioxide of households

Emission of CO2 Data (Model) in tons per quartal									
Income quintiles				Emission quartiles					
		Mean		p25		p50		p75	
		D	M	D	M	D	M	D	M
I:	0-20%	0.72	0.79	0.39	0.47	0.59	0.58	0.93	0.86
II:	20-40%	1.17	1.03	0.65	0.56	1.04	0.79	1.51	1.22
III:	40-60%	1.54	1.33	0.92	0.74	1.39	1.13	1.95	1.63
VI:	60-80%	1.97	1.77	1.25	1.04	1.80	1.70	2.48	2.14
V:	80-100%	2.39	2.75	1.50	1.93	2.15	2.70	2.97	3.24
Targets: relative moment by income quintile									
Mean(I)/Mean(V)		**0.30**	**0.29**	p25(III)/p75(III)		**0.47**	**0.46**		
p25(I)/p75(V)		**0.13**	**0.14**						

Source: German Einkommens- und Verbrauchsstichprobe (EVS) 2018, own calculations. Income quintiles refer to household net incomes. Expenditure quartiles refer to the within-income-quintile carbon dioxide emission. Columns *D* refer to the data, *M* to the model. Targeted moments in bold.

Müller, and Seyrich (2023) and set $\bar{\rho} = 0.97$ to match the average probability to switch carbon dioxide types to roughly 10%. This leaves me with four additional parameters to characterize the carbon dioxide intensity of household consumption: the carbon dioxide share in the consumption basket of high and low carbon intensive households (a_H^C, a_L^C) and the parameters (B, A) which govern the process that determines the carbon dioxide type of households, given in Equation (3) above. I set these parameters so that the average expenditure share on carbon dioxide amounts to 1.4%,[5] and to capture the dispersion of energy expenditures within and across incomes as shown in Table 2. Concretely, I match the following three targets: (i) the average increase in carbon dioxide emission across income quintiles, (ii) the interquartile range within the median income quintile, (iii) the bottom quartile of energy consumption in the bottom income quintile relative to the top quartile of energy consumption in the top income quintile.

In this way, I capture a high gradient of carbon dioxide emissions with income, that

[5]The average German household emits 1.55 ton carbon dioxide quarterly. Assuming again a current carbon price of 100€ per ton, this amounts to 155€ spent on carbon dioxide per household quarterly which represents 1.4% of average private consumption.

is, some non-homotheticity in carbon dioxide emissions on average, without resorting to non-homothetic preferences themselves. At the same time, I capture the large dispersion in carbon dioxide emissions even conditional on income. In fact, Table 2 also shows that the non-targeted carbon dioxide emissions (relative to the average) of the different groups in the carbon dioxide issuance and income distribution are relatively well matched, despite the very coarse parameterization. It shows the umbers implied by the model alongside the empirical distribution from German micro data (the German equivalent of the Consumption Expenditure Survey, CEX).

4 Results

In what follows, I study the effects of carbon shocks through the lens of the calibrated model. I compute a linearized state-space solution using the toolkit provided by Bayer, Born, and Luetticke (2020).

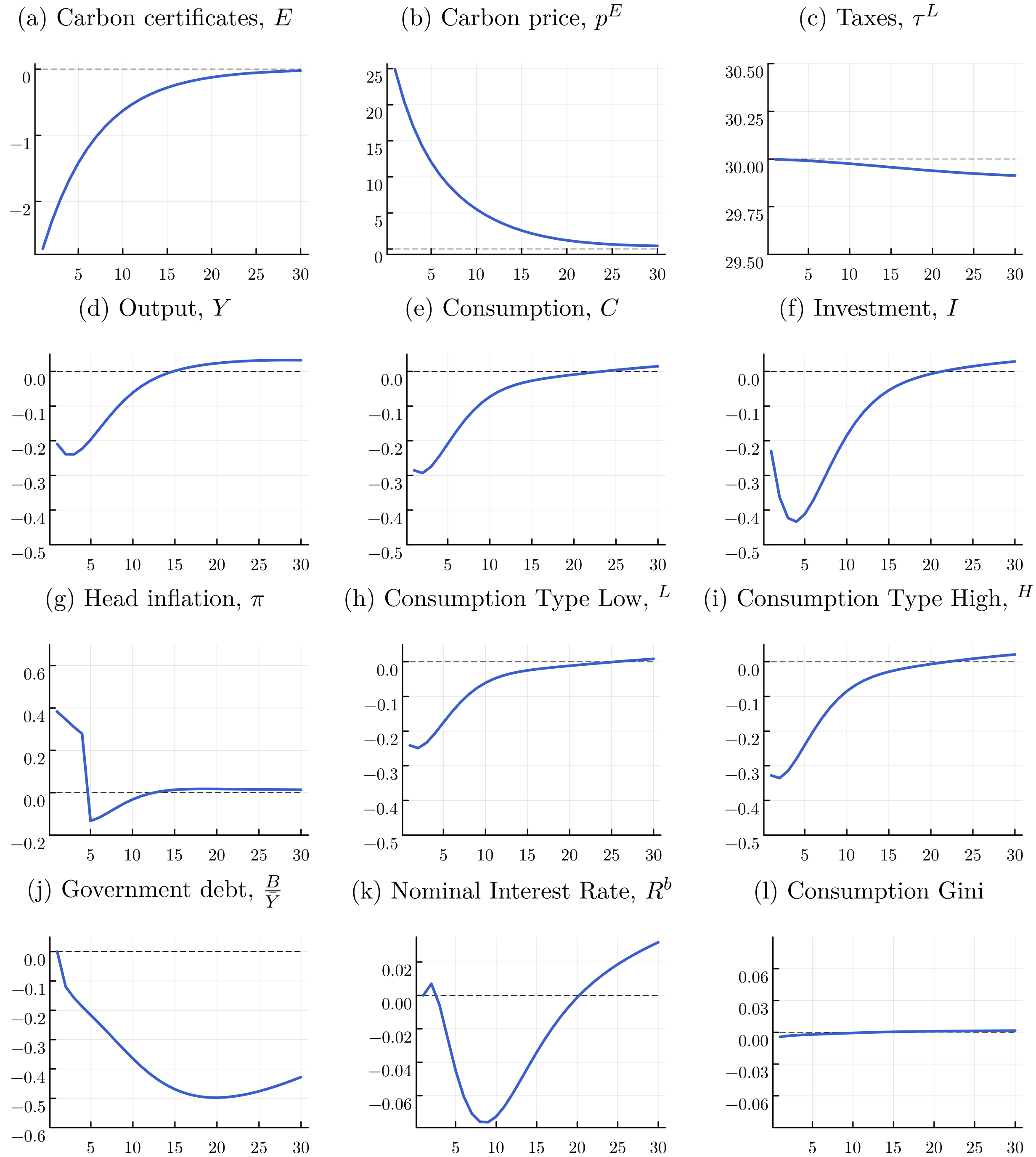
4.1 Baseline

Figure 2 shows the macroeconomic effects of an adverse carbon shock: the amount of carbon certificates decreases on impact by 2.5% and gradually converges back to steady state. This increases the carbon price by 25%. Consequently, output, consumption, and inflation decline by 0.22%, 0.29%, and 0.42% at their respective peaks. Year-on-year headline inflation increases by at most 0.4 percentage points. The average consumption of households with a high carbon dioxide intensity in their consumption basket falls by around 50% more than that of households with a low carbon dioxide intensity. Consumption inequality, measured by the consumption Gini, falls mildly. The higher carbon price increases the revenue from the carbon certificate trade which is absorbed by lower public debt in the short run. Taxes gradually decreases to bring back debt in the long-run to its steady state level.

Who bears the cost of higher carbon prices? To quantify the impact of carbon shocks at the household level, I measure the welfare impact using the consumption equivalent variation, which is the permanent consumption change that would make an individual household equally well off as the shock under consideration.[6] Figure 3 shows the impact of the adverse carbon shock on welfare along the income distribution (left panel) and along the wealth distribution (right panel). It depicts the average welfare of the respective income and wealth decile. The results document that higher carbon prices decrease average welfare for all deciles. There is

[6]I take an ex-post perspective, evaluating welfare based on the specific shock at hand (that is, one-sided welfare), rather than providing an ex-ante welfare analysis based on a second-order approximation of the utility function.

Figure 2: Response to an exogenous decrease in CO2 certificates, baseline

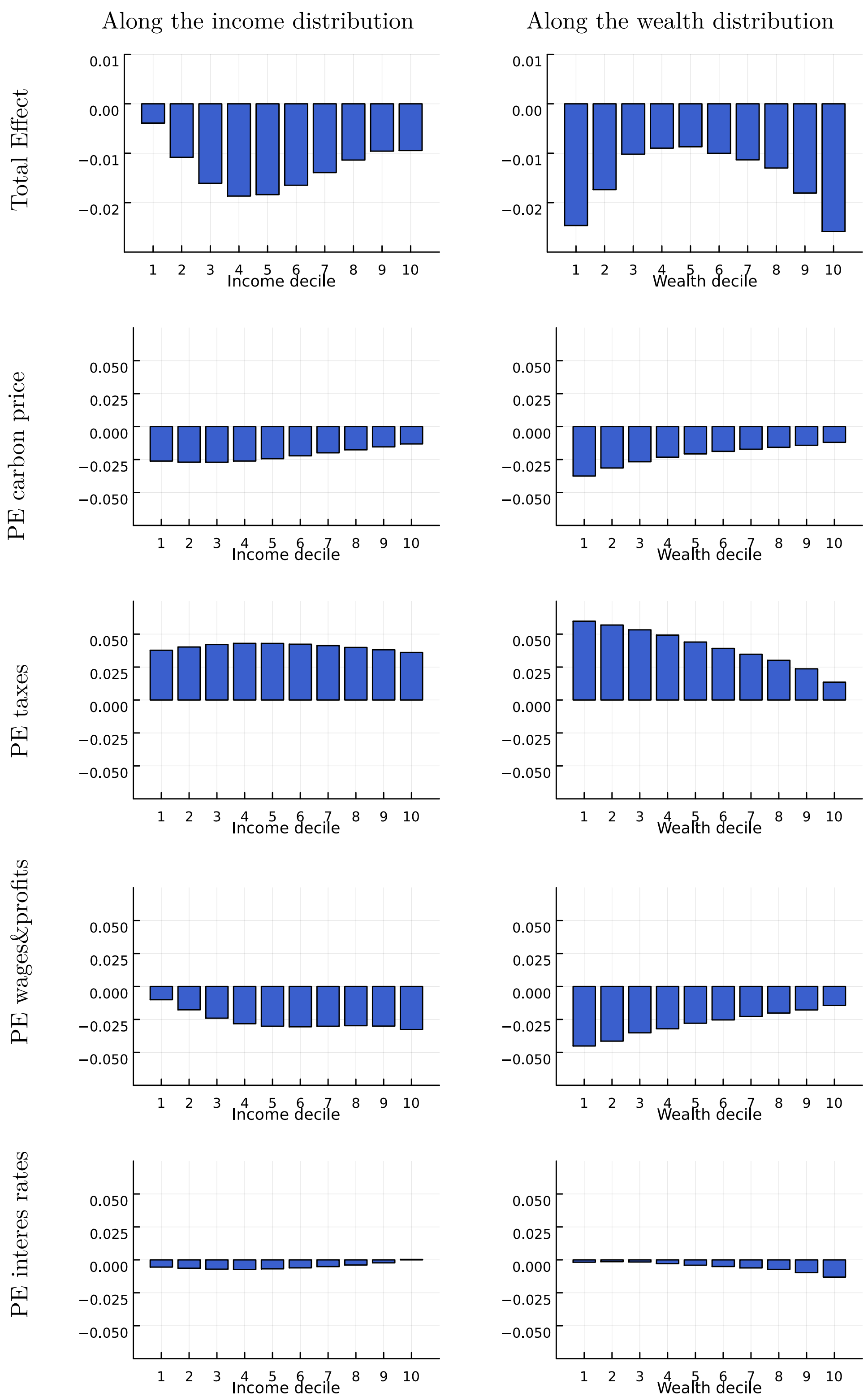


Impulse responses after an adverse carbon shock. Y-axis:Percentage deviation from steady state, percentage points in case of inflation and interest rate, and tax level in percentage in case of taxes. X-axis: Quarters.

some heterogeneity in the impact of higher carbon prices. Whether the general equilibrium effects of higher carbon prices affect the poor, the middle class or rich households depends on whether income or welfare is taken as a basis: along the income distribution, higher carbon prices hit the middle-class hardest, whereas along the wealth distribution, it affects both asset-poor and asset-rich households hardest. Overall, however, the costs of higher carbon prices is quite uniform reflected by the fact that the welfare impact has the same sign for all deciles.

To better understand the welfare impact of carbon shocks along the income and wealth distributions, I decompose the overall effect into its partial equilibrium effects. For this purpose, I exploit the fact that how the welfare of a given household is affected by a carbon shock depends on the arguments that enter its decision problem. I group the arguments into i) the carbon price itself, ii) changes in taxes, iii) changes in wage and profit income, and iv) changes in interest rates including the return on liquid bonds and the return on capital. These four partial equilibrium effects are reported in the lower four panels of Figure 3.

Figure 3: The welfare and distributional effects of carbon shocks



Notes: Welfare impact of a carbon shock along the income (left panels) and the wealth distribution. The upper panel shows the total impact whereas the lower panels decompose the total effects in its partial equilibrium components. Y-axis: Consumption equivalent compensating variations. X-axis: Income/wealth deciles.

There are three important observations from this exercise. First, focusing on the income distribution, the middle-income households experience the strongest impact from three of the four partial equilibrium effects.[7] This explains why middle-income households are most affected by the carbon shock. Second, focusing on the wealth distribution, households with the lowest wealth are the most affected, as they have the smallest buffer stock and are thus hit hardest by rising prices and falling incomes. The exception is the partial equilibrium effect of interest rates, which impacts the wealthiest households the most, though this partial equilibrium effect is quantitatively small. Third, all partial equilibrium effects reduce welfare in each income or wealth decile, with the exception of the partial equilibrium effect of taxes. Taxes have a positive and a rather large impact on the welfare of households. This demonstrates the "double dividend" of higher carbon prices as they allow the government to decrease distortionary taxes. Similar to the classical double-dividend result (see, e.g., Goulder, 1995), the eventual reduction in distortionary taxes is insufficient to prevent higher carbon prices from being detrimental to welfare (excluding the welfare gains from reduced carbon dioxide emissions from which I abstract in this paper). However, it does account for why the overall impact on welfare is relatively moderate: the average consumption equivalent is only -0.015%.[8]

4.2 Counterfactual scenarios: immediate repayments of carbon revenues

So far, I have assumed that fiscal policy does not respond to changes in carbon revenues, reflecting the status quo in many countries of not earmarking carbon revenues for specific fiscal instruments. As shown in the previous section, an increase in carbon prices increases the revenue from selling carbon certificates and, thus, reduces government debt quite substantially. What if fiscal policy immediately repays this extra revenue? To answer this question, I simulate three counterfactual scenarios: I assume that all extra carbon revenues are immediately i) paid back by lump-sum transfers to households, ii) used to decrease income taxes, iii) used to pay a capital subsidy.

Figure 4 shows the macroeconomic effects of the adverse carbon shock in the baseline (left panels) and in the three counterfactuals. To economize on space, I only show impulse

[7] This is not the case for wage & profit income which affects the high-income households the most.

[8] This small loss in welfare heavily depends on the fact that the higher carbon revenue eventually leads to lower distortionary taxes. If instead, the higher revenue would be used for inefficient "rent-seeking", the welfare impact would be much higher. I approximate this scenario assuming that government debt will be brought back to steady state by higher wasteful government spending in the long-run. In this case, the average consumption equivalent is -0.060% and, thus, the welfare loss would be four times as high.

responses for output, consumption, investment, inflation, and the consumption Gini.[9] In all three counterfactual cases, the recession caused by an adverse carbon shock is smaller than in the baseline, but the extents of mitigation differ across the counterfactual cases: While in the transfer counterfactual, output is only slightly different from the baseline, it falls not even half as strong in the tax counterfactual. And in the capital subsidy counterfactual, output actually increases rather than decreases on impact. Non-surprisingly, capital subsides are also most effective in mitigating the fall in investment. In contrast, investment falls even more in the transfer counterfactual than it does in the baseline.

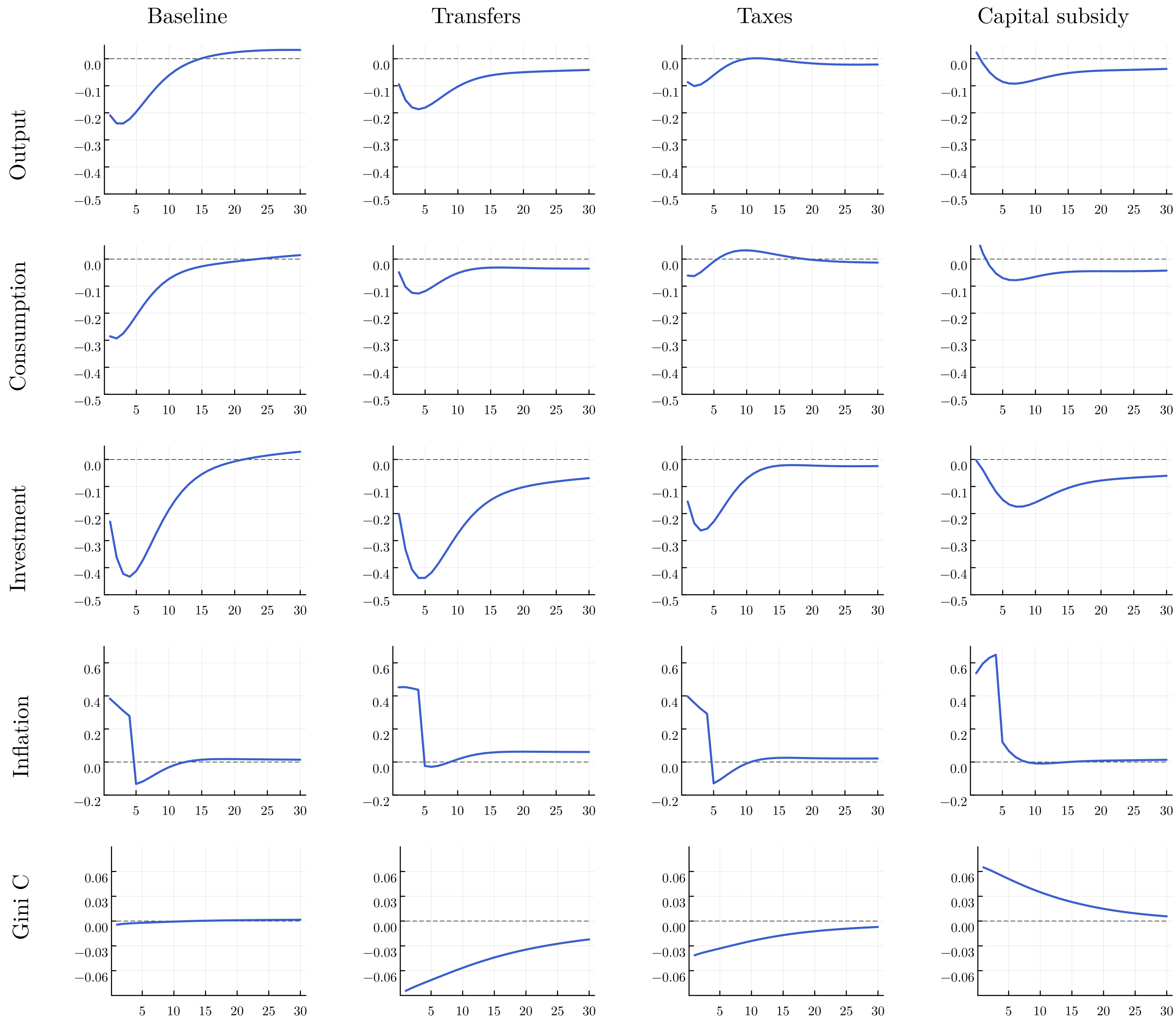
The carbon shock is slightly more inflationary in the transfer counterfactual than in the baseline and even much more inflationary in the capital subsidy counterfactual. In contrast, the carbon shock affects inflation the same in the tax counterfactual as it does in the baseline.[10] In all three counterfactuals, the consumption Gini responds much more strongly than in the baseline, in which it remains almost constant. The consumption Gini also differs strongly across the three different counterfactual scenarios: While consumption inequality falls in the tax case and even more strongly in the transfer case, it strongly increases in the capital subsidy case. The latter reflects the fact that mostly wealthy households benefit from capital subsidies.

[9] The full set of impulse responses for the counterfactuals can be found in Appendix C.

[10] In the HANK model, income taxes have both a supply side effect (through lower wages and hence lower marginal costs) as well as a demand side effect (through higher disposable income). These two opposite effects on inflation cancel out, such that they do not affect the inflation response of carbon shocks.

4.2.1 Macro effects

Figure 4: The macro effects of carbon shocks, alternative repayment methods



Notes: Impulse responses after an adverse carbon shock. Y-axis:Percentage deviation from steady state, percentage points in case of inflation. X-axis: Quarters.

4.2.2 Distributional effects

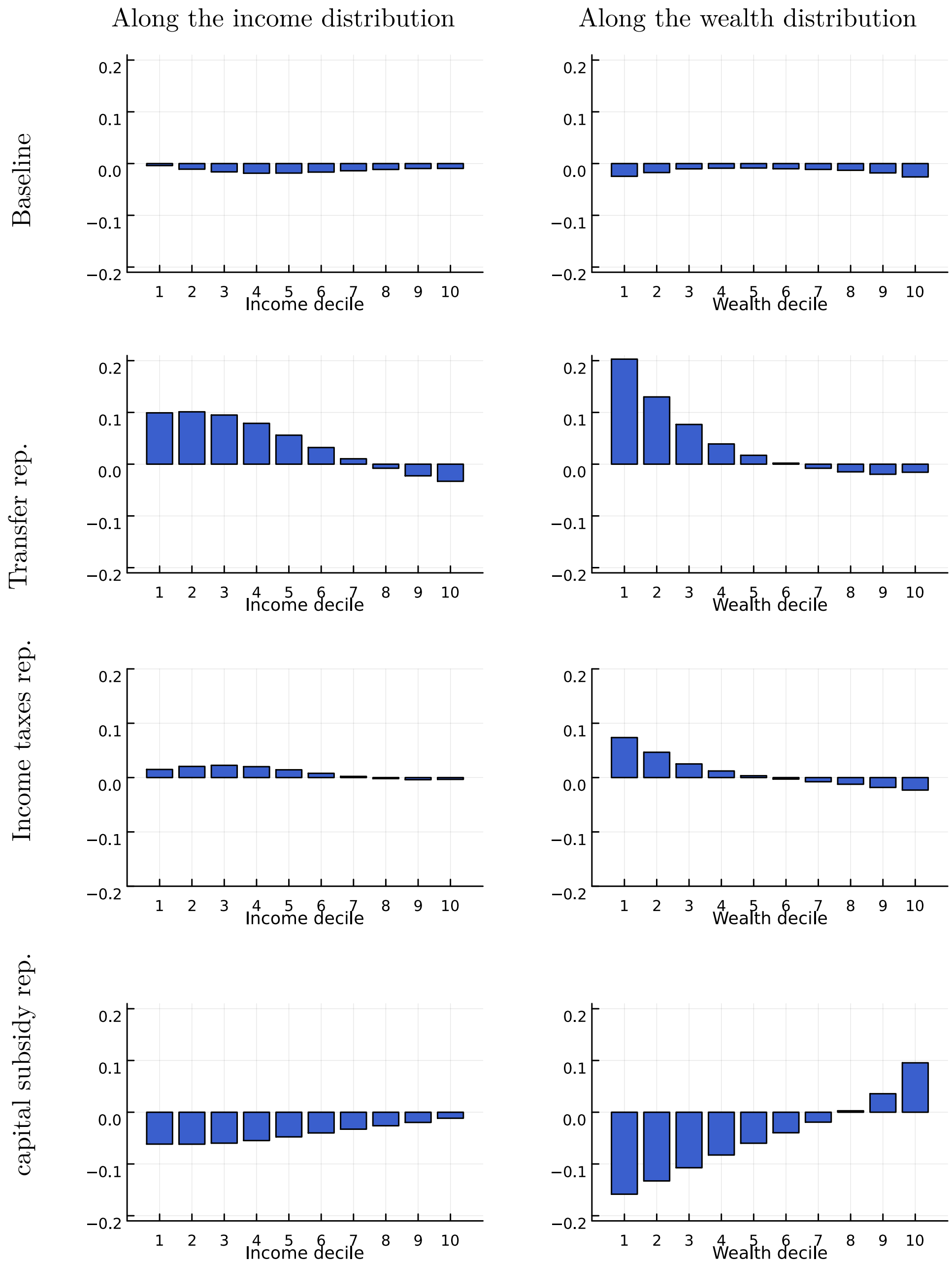
Figure 5 compares the welfare impact of the three counterfactual cases (lower panels) with the baseline case (upper panel). It does so again along the income distribution (left panel) and along the wealth distribution (right panel). All three counterfactuals changes the welfare impact of carbon shocks quite drastically. First, in all counterfactuals, it is no longer the case that the welfare impact of each income and wealth decile has the same sign. In other words, there are now winners and losers from the higher carbon prices amplifying the distributional impact of carbon pricing. In the transfer and in the tax case, the poorer households benefit from higher carbon prices whereas richer households suffer. This distributional differences are particularly strong in the transfer case as the induced higher lump-sum transfers redistribute more strongly towards relatively poorer households. In the capital subsidy case, most of the households suffer from carbon prices, with the exception of the wealthiest households which benefit from it. This reflects the fact that capital subsidies redistributes towards asset-rich households.

Second, for the majorities of deciles, the welfare impact is stronger in the counterfactual cases than in the baseline case. This implies that the mitigated macroeconomic effects in the counterfactual cases depicted in Figure 4 actually hide stronger adjustments on the micro level.

The third observation regards the *average* sign and size of the welfare impact. In the tax and in the transfer case, average welfare actually increases rather than decreases after an adverse carbon shock. This adds a second dividend to the unmodelled gain in welfare due to lower carbon emissions. This effect is particularly pronounced for the transfer counterfactual in which case the average consumption equivalent is 0.040% (whereas it is 0.008% in the tax case). Paying the higher carbon revenue to households via lump-sum transfers decreases consumption inequality and partly insures households' idiosyncratic income risk. This is sufficiently welfare-enhancing to dominate the detrimental welfare costs of higher carbon prices which can be seen as a HANK-version of the double-dividend result.

In contrast, average welfare falls even more strongly in the capital subsidy case with an average consumption equivalent of -0.046% instead of -0.015% in the baseline. While capital subsidies are effective in preventing output from falling on impact, they prevent the eventual fall in distortionary taxes in the long-run which occurs in the baseline case. This highlights a trade-off between macroeconomic stabilization and welfare impact when it comes to the choice of fiscal instrument for repaying higher carbon revenues: On the one hand, the instrument which is most effective in stabilizing output on impact (capital subsidies), increases the welfare loss of higher carbon prices even further. And on the other hand, the instrument that is best for average welfare (lump-sum transfers) is not very effective in mitigating the recession.

Figure 5: The distributional effects of carbon shocks, alternative repayment methods



Notes: Welfare impact of an adverse carbon shocks along the income distribution (left panels) and along the wealth distribution (right panels). Y-axis:Consumption equivalent compensating variations. X-axis: Income/wealth deciles.

5 Conclusion

There is a consensus that combating climate change requires assigning a price to carbon dioxide emissions. However, implementing such pricing systems can be a source of business cycle fluctuations. This paper introduces a Climate-HANK model to study the business cycle impact and distributional effects of carbon shocks. The findings indicate that adverse carbon shocks influence the economy similarly to cost-push shocks, leading to reduced economic activity and increased inflation. Yet, the distributional impact of these shocks is found to be quantitatively small.

Although higher carbon prices can induce recessions, they can still increase overall welfare. The reason is that higher carbon prices increases the revenue of the government. I show that if the government pays this extra revenue directly back to households via lump-sum transfers—a repayment approach that has gained considerable support in recent years—higher carbon prices increases average welfare beyond the welfare gains from lower carbon emissions. This result represents a HANK adaptation of the classical double-dividend theory in the environmental taxation literature. However, lump-sum transfers are not as effective in mitigating the macroeconomic impact of carbon shocks as other fiscal instruments. Future research could therefore address the question of which revenue-neutral fiscal mix is best suited to counteract carbon shocks, considering their macroeconomic, welfare, and distributional impacts.

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A A Climate-HANK model

The model in the paper is based on the two-asset, medium-scale HANK model in Bayer, Born, and Luetticke (2020). I extend the model to cover carbon dioxide emissions in production and in household consumption.

The economy consists of a firm sector and a household sector. The firm sector comprises (a) perfectly competitive intermediate goods producers, who produce intermediate goods using capital, labor, and carbon dioxide; (b) final goods producers that face monopolistic competition when selling differentiated final goods, in turn, produced on the basis of homogeneous intermediate inputs; (c) producers of capital goods that turn consumption goods into capital subject to adjustment costs; (d) labor packers that produce labor services combining differentiated labor from (e) unions that differentiate raw labor rented out from households. Price setting for the final goods, as well as wage setting by unions, is subject to a pricing friction à la Calvo (1983).

Households consume a bundle that consists of produced goods and carbon dioxide directly. Households earn income from supplying (raw) labor and capital to the labor and the capital markets and from owning firms in their respective country. Households absorb all rents that stem from the market power of unions and final good producers, and decreasing returns to scale in capital goods production.

There is a monetary authority and a fiscal authority. The fiscal authority levies taxes on labor income and profits, issues bonds, pays transfers, sells carbon certificates, and adjusts taxes to stabilize the level of outstanding debt in the long run. Public debt is risk-free and, in turn, determined by monetary policy by means of a simple interest rate feedback rule.

A.1 Households

The household sector is subdivided into two types of agents: workers and entrepreneurs. The transition between both types is stochastic. Both rent out physical capital, but only workers supply labor. The efficiency of a worker's labor evolves randomly exposing households to labor-income risk. Entrepreneurs do not work but earn all pure rents in the economy except for the rents of unions which are equally distributed across workers.

All households self-insure against the income risks they face by saving in a liquid nominal asset (bonds) and a less liquid asset (capital). Trading illiquid assets is subject to random participation in the capital market. To be specific, there is a continuum of ex-ante identical households of measure 1, indexed by i. Households are infinitely lived, have time-separable preferences with time discount factor β, and derive felicity from consumption and leisure. Total consumption c_{it} consists of carbon dioxide, E^C_{it}, and the physical consumption good c^P_{it}.

Households obtain income from supplying labor, n_{it}, from renting out capital, k_{it}, and from earning interest on bonds, b_{it}, and potentially from profits or union transfers. Households pay taxes on labor and profit income and receive minimum income benefits as well as other transfers.

A.1.1 Productivity, labor supply, and labor income

A household's gross labor income $w_t n_{it} h_{it}$ is composed of the aggregate wage rate on raw labor, w_t, the household's hours worked, n_{it}, and its idiosyncratic labor productivity, h_{it}. I assume that productivity evolves according to a log-AR(1) process with time-varying volatility and a fixed probability of transition between the worker and the entrepreneur state:

$$\tilde{h}_{it} = \begin{cases} \exp(\rho_h \log \tilde{h}_{it-1} + \epsilon_{it}^h) & \text{with probability } 1-\zeta \text{ if } h_{it-1} \neq 0, \\ 1 & \text{with probability } \iota \text{ if } h_{it-1} = 0, \\ 0 & \text{else.} \end{cases} \tag{12}$$

with individual productivity $h_{it} = \frac{\tilde{h}_{it}}{\int \tilde{h}_{it} di}$ such that $\tilde{h}_{it}$ is scaled by its cross-sectional average, $\tilde{h}_{it} di$, to make sure that average worker productivity is constant. The shocks ϵ_{it}^h to productivity are normally distributed with variance $\sigma_{h,t}^2$. With probability ζ households become entrepreneurs ($h = 0$). With probability ι an entrepreneur returns to the labor force with median productivity. An entrepreneur obtains a share of the pure rents (aside from union rents), Π_t^F, in the economy (from monopolistic competition in the goods sector and the creation of capital). I assume that the claim to the pure rent cannot be traded as an asset. Union rents, Π_t^U are distributed lump sum across workers, leading to labor-income compression. For tractability, I assume union profits to be taxed at a fixed rate independent of the recipient's labor income.

With respect to leisure and consumption, households have Greenwood, Hercowitz, and Huffman (1988) (GHH) preferences and maximize the discounted sum of felicity:

$$E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u[c_{it} - G(h_{it}, n_{it})] \tag{13}$$

Total consumption c_{it} of household i at time t consists of carbon dioxide E_{it}^C and the physical consumption good c_{it}^P, again combined in a CES aggregator:

$$c_{it} = \left(\left(1 - a_{it}^C\right)^{\frac{1}{\sigma_C}} c_{it}^{P\,\frac{\sigma_C - 1}{\sigma_C}} + a_{it}^{C\,\frac{1}{\sigma_C}} \left(E_{it}^C\right)^{\frac{\sigma_C - 1}{\sigma_C}} \right)^{\frac{\sigma_C}{\sigma_C - 1}}. \tag{14}$$

Here σ_C represents the elasticity of substitution in consumption, which determines how much utility the household loses by substituting carbon dioxide for physical consumption goods. a^C_{it} determines the share of the carbon dioxide in the consumption good. The parameter follows a Markov chain to capture households with relatively high carbon dioxide intensity as well as households with relatively low carbon dioxide intensity. The switching probability $\rho(h, a^C)$ from one type to the other is a function of the current productivity level, h, and the current carbon dioxide intensity, a^C. I specify

$$\rho(h, a^C) = \bar{\rho} + (\mathbb{I}_{a^C = a^C_H} - \mathbb{I}_{a^C = a^C_L})A(h) + \mathbb{I}_{a^C = a^C_L}B,$$

where A is a linear function of the human capital quintile. With higher human capital the household is more likely to remain type low and more likely become type low. B is a constant that captures that it is in general more likely to remain type low.

The maximization is subject to the budget constraints described further below. The felicity function u exhibits a constant relative risk aversion (CRRA) with risk aversion parameter $\xi > 0$,

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \tag{15}$$

where $x_{it} = c_{it} - G(h_{it}, n_{it})$ is household i's composite demand for (carbon dioxide and physical composite) goods consumption c_{it} and leisure and G measures the dis-utility from work.

The household's labor income gets taxed at rate τ_t, such that its net labor income, expressed in physical consumption units (i.e. without carbon dioxide consumption), is given by

$$y_{it} := (1 - \tau_t) w_t h_{it} n_{it}, \tag{16}$$

where w_t is the aggregate real wage rate (in physical consumption units). Given net labor income, the first-order condition for labor supply is

$$\frac{\partial G(h_{it}, n_{it})}{\partial n_{it}} = (1 - \tau_t) \frac{w_t}{p^c_t(a^C_{it})} h_{it} = \frac{y_{it}}{n_{it}} / p^c_t(a^C_{it}). \tag{17}$$

Here $p^c_t(a^C_{it})$ is the cost in terms of physical goods at which household i buys its carbon dioxide-physical consumption bundle. This price depends on the carbon intensity of the household and is given by

$$p_t^c(a_{it}^C) = \left[(1 - a_{it}^C) + a_{it}^C(p_t^E - \tau_t^E)^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}}.$$

Assuming that G has a constant elasticity w.r.t. n, $\frac{\partial G(h_{it},n_{it})}{n_{it}} = (1+\gamma)\frac{G(h_{it},n_{it})}{n_{it}}$ with $\gamma > 0$, I can simplify the expression for the composite consumption good, x_{it}, making use of this first-order condition (17), and substitute $G(h_{it}, n_{it})$ out of the individual planning problem:

$$x_{it} = c_{it} - G(h_{it}, n_{it}) = c_{it} - \frac{1}{1+\gamma} y_{it}/p_t^c(a_{it}^C). \tag{18}$$

When the Frisch elasticity of labor supply is constant and the tax schedule has the form (16), the dis-utility of labor is always a fraction of labor income and constant across households. Therefore, in both the household's budget constraint and felicity function, only after-tax income enters and neither hours worked nor productivity appears separately.

What remains to be determined is individual and aggregate effective labor supply. Without further loss of generality, I assume $G(h_{it}, n_{it}) = h_{it}\frac{n_{it}^{1+\gamma}}{1+\gamma}$. This functional form simplifies the household problem in the stationary equilibrium as h_{it} drops out from the first-order condition and all households supply the same number of hours $n_{it} = N(w_t)$. Total effective labor input, $\int n_{it}h_{it}di$, is hence also equal to $N(w_t)$ because I normalized $\int h_{it}di = 1$.[11]

Households also receive profit income from union profits Π_t^U or firms profits Π_t^{fi} as workers or entrepreneurs, respectively. Both profits get taxed at rate τ_t. What is more, households may receive *non-distortionary* targeted transfer as minimum income benefits tr_{it} as well as lump-sum transfers, Tr_t. All together, after-tax non-capital income, plugging in the optimal supply of hours, is then:

$$y_{it} = \left[(1 - \tau_t)w_t/p_t^c(a_{it}^C)\right]^{\frac{1+\gamma}{\gamma}} h_{it} + \mathbb{I}_{h_{it}\neq 0}(1 - \tau_t)\Pi_t^U + \mathbb{I}_{h_{it}=0}(1 - \tau_t)\Pi_t^{fi} + tr_{it} + Tr_t. \tag{19}$$

A.1.2 Consumption, savings, and portfolio choice

Given this labor income, households optimize inter-temporally subject to their budget constraint expressed in terms of physical consumption goods:

$$p_t^c(a_{it}^C)c_{it} + b_{it+1} + q_t k_{it+1} = y_{it} + b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t^{core}} + (q_t + r_t)k_{it}, \quad k_{it+1} \geq 0,\, b_{it+1} \geq \underline{B} \tag{20}$$

[11]This means that I can read off average productivity risk from the estimated income risk series in the literature. Without scaling the labor dis-utility by productivity, I would need to translate productivity risk to income risk through the endogenous hour response.

b_{it} is real bond holdings, k_{it} is the amount of illiquid assets, q_t is the price of these assets, r_t is their dividend, $\pi_t^{core} = \frac{P_t}{P_{t-1}}$ is realized average core inflation (inflation of physical goods, i.e., without carbon dioxide), and R is the gross nominal interest rate on bonds, which depends on the portfolio position of the household and the central bank's interest rate R_t^b, which is set one period before.

All households that do not participate in the capital market ($k_{it+1} = k_{it}$) still obtain dividends and can adjust their bond holdings. Depreciated capital has to be replaced for maintenance, such that the dividend, r_t, is the net return on capital. Holdings of bonds have to be above an exogenous debt limit $\underline{B}$, and holdings of capital have to be non-negative.

Substituting the expression $c_{it} = x_{it} + \frac{1}{1+\gamma}\left[(1-\tau_t)w_t/p_t^c(a_{it}^C)\right]^{\frac{1+\gamma}{\gamma}} h_{it}$ for consumption, I obtain the budget constraint for the composite leisure-consumption good:

$$p_t^c(a_{it}^C)x_{it} + b_{it+1} + q_t k_{it+1} = b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t^{core}} + (q_t + r_t)k_{it} + z_{it}, \quad k_{it+1} \geq 0,\, b_{it+1} \geq \underline{B}, \tag{21}$$

where $z_{it} = \frac{\gamma}{1+\gamma}\left[(1-\tau_t)w_t/p_t^c(a_{it}^C)\right]^{\frac{1+\gamma}{\gamma}} h_{it} + \mathbb{I}_{h_{it}\neq 0}(1-\tau_t)\Pi_t^U + \mathbb{I}_{h_{it}=0}(1-\tau_t)\Pi_t^{fi} + tr_{it} + Tr_t$ is income corrected for the dis-utility of labor.

Households make their savings choices and their portfolio choice between liquid bonds and illiquid capital in light of a capital market friction that renders capital illiquid because participation in the capital market is random and i.i.d. in the sense that only a fraction, λ, of households are selected to be able to adjust their capital holdings in a given period. This means that I specify:

$$R(b_{it}, R_t^b) = \begin{cases} R_t^b & \text{if } b_{it} \geq 0 \\ R_t^b + \bar{R} & \text{if } b_{it} < 0 \end{cases}. \tag{22}$$

The extra wedge for unsecured borrowing, $\bar{R}$, creates a mass of households with zero unsecured credit but with the possibility to borrow, though at a penalty rate.

Since a household's saving decision—–(b'_a, k') for the case of adjustment and (b'_n, k') for non-adjustment—will be some non-linear function of that household's wealth and productivity, inflation and all other prices will be functions of the joint distribution, Θ_t, of (b, k, h) in t and the foreign joint distribution, Θ_t^*. This makes Θ and Θ^* state variables of the household's planning problem and these distributions evolve as a result of the economy's reaction to aggregate shocks. For simplicity, I summarize all effects of aggregate state variables, including the distributions of wealth and income, by writing the dynamic planning problem with time-dependent continuation values.

This leaves me with three functions that characterize the household's problem: value function V^a for the case where the household adjusts its capital holdings, the function V^n for the case in which it does not adjust, and the expected continuation value, $\mathbb{W}$, over both:

$$
\begin{aligned}
V_t^a(b,k,h,a^C) &= \max_{k',b'_a} u[x(b,b'_a,k,k',h,a^C)] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_a,k',h,a^C) \\
V_t^n(b,k,h,a^C) &= \max_{b'_n} u[x(b,b'_n,k,k,h,a^C)] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_n,k,h,a^C) \\
\mathbb{W}_{t+1}(b',k',h,a^C) &= \lambda V_{t+1}^a(b',k',h,a^C) + (1-\lambda) V_{t+1}^n(b',k,h,a^C).
\end{aligned}
\tag{23}
$$

Expectations about the continuation value are taken with respect to all stochastic processes conditional on the current states, i.e., over both human capital, h, and carbon intensity, a^C. Maximization is subject to the corresponding budget constraint.

A.2 Firm sector

The firm sector consists of four sub-sectors: (a) a labor sector composed of unions that differentiate raw labor and labor packers who buy differentiated labor and then sell labor services to intermediate goods producers, (b) intermediate goods producers who hire labor services and rent out capital and buy energy to produce goods, (c) final goods producers who differentiate intermediate goods and then sell them to households and to (d) capital goods producers, who turn bundled goods into capital goods.

When profit maximization decisions in the firm sector require inter-temporal decisions (i.e. in price and wage setting and in producing capital goods), I assume for tractability that they are delegated to a mass-zero group of households (managers) that are risk-neutral and compensated by a share in profits. They do not participate in any asset market and have the same discount factor as all other households. Since managers are a mass-zero group in the economy, their consumption does not show up in any resource constraint, and all but the unions' profits go to the entrepreneur households (whose $h = 0$). Union profits go lump-sum to worker households.

A.2.1 Labor packers and unions

Worker households sell their labor services to a mass-n_A continuum of unions indexed by j, each of whom offers a different variety of labor to labor packers who then provide labor services to intermediate goods producers. Labor packers produce final labor services according to the production function

$$
N_t = \left(\int_0^{n_A} \hat{n}_{jt}^{\frac{\eta_W - 1}{\eta_W}} \, dj \right)^{\frac{\eta_W}{\eta_W - 1}}. \tag{24}
$$

out of labor varieties $\hat{n}_{jt}$. Cost minimization by labor packers implies that each variety of labor, each union j, faces a downward-sloping demand curve

$$\hat{n}_{jt} = \left(\frac{W_{jt}}{W_t^{fi}}\right)^{-\eta_W} N_t \tag{25}$$

where W_{jt} is the nominal wage set by union j and W_t^{fi} is the nominal wage at which labor packers sell labor services to final goods producers. Since unions have market power, they pay the households a wage lower than the price at which they sell labor to labor packers. Given the nominal wage W_t at which they buy labor from households and given the nominal wage index W_t^{fi}, unions seek to maximize their discounted stream of profits. However, they face a Calvo (1983) type adjustment friction with indexation with the probability λ_w to keep wages constant. They therefore maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_w^t \frac{W_t^{fi}}{P_t} N_t \left\{ \left(\frac{W_{jt}(\bar{\pi}_W)^t}{W_t^{fi}} - \frac{W_t}{W_t^{fi}} \right) \left(\frac{W_{jt}(\bar{\pi}_W)^t}{W_t^{fi}} \right)^{-\eta_W} \right\}. \tag{26}$$

by setting W_{jt} in period t and keeping it constant except for indexation to π_W, the steady state wage inflation rate.

Since all unions are symmetric, I focus on a symmetric equilibrium and obtain the linearized wage Phillips curve from the corresponding first-order condition as follows, leaving out all terms irrelevant at a first-order approximation around the stationary equilibrium:

$$\log\left(\frac{\pi_t^W}{\bar{\pi}^W}\right) = \beta \mathbb{E}_t \log\left(\frac{\pi_{t+1}^W}{\bar{\pi}^W}\right) + \kappa_w \left(mc_t^w - \frac{1}{\mu^W} \right), \tag{27}$$

with $\pi_t^W := \frac{W_t^{fi}}{W_{t-1}^{fi}} = \frac{w_t^{fi}}{w_{t-1}^{fi}} \pi_t^{CPI}$ being wage inflation, w_t and w_t^{fi} being the respective *real* wages for households and firms, $mc_t^w = \frac{w_t}{w_t^{fi}}$ is the mark-down of wages the unions pay to households, W_t, relative to the wages charged to firms, W_t^{fi} and $\kappa_w = \frac{(1-\lambda_w)(1-\lambda_w \beta)}{\lambda_w}$. Union profits paid to workers therefore are $\Pi_t^U = (w_t^{fi} - w_t) N_t$.

A.2.2 Final goods producers

Similar to unions, final goods producers differentiate the homogeneous intermediate goods and set prices. They buy the intermediate good at the nominal price, MC_t. As I do for unions, I assume price adjustment frictions à la Calvo (1983) with indexation.

Under this assumption, the firms' managers maximize the present value of real profits given this price adjustment friction, i.e., they maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_Y^t (1-\tau_t) \left(\frac{p_{jt}(\bar{\pi})^t}{P_t} - \frac{MC_t}{P_t} \right) Y_t^d(j) \tag{28}$$

with a time-constant discount factor.

The corresponding first-order condition for price setting implies a Phillips curve

$$\log\left(\frac{\pi_t}{\bar{\pi}}\right) = \beta \mathbb{E}_t \log\left(\frac{\pi_{t+1}}{\bar{\pi}}\right) + \kappa_Y \left(mc_t - \frac{1}{\mu^Y} \right) \tag{29}$$

where I again dropped all terms irrelevant for a first-order approximation and have $\kappa_Y = \frac{(1-\lambda_Y)(1-\lambda_Y \beta)}{\lambda_Y}$. Here, $\pi_t := \frac{P_t}{P_{t-1}}$, is the gross producer price inflation rate, i.e., the gross inflation rate of the physical good, $mc_t := \frac{MC_t}{P_t}$ are the real marginal costs, $\bar{\pi}$ is steady-state inflation, and $\frac{1}{\mu^Y} = \frac{\eta-1}{\eta}$ is the target markup. Profits paid to entrepreneurs therefore are $\Pi_t^F = (1-mc_t)Y_t$.

A.2.3 Intermediate goods producers

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = \left((1-a_P)^{\frac{1}{\sigma_P}} \ {Y_t^P}^{\frac{\sigma_P - 1}{\sigma_P}} + a_P^{\frac{1}{\sigma_P}} \ \left(E_t^Y\right)^{\frac{\sigma_P - 1}{\sigma_P}} \right)^{\frac{\sigma_P}{\sigma_P - 1}}, \text{ where } Y_t^P = (u_t K_t^s)^{\alpha} \ N_t^{1-\alpha}. \tag{30}$$

Production combines physical production Y_t^P using capital K_t with capacity utilization u_t, labor N_t, and carbon dioxide E_t^Y. The coefficient α is the capital share, the coefficient σ_P captures the (short-run) substitutability of carbon dioxide in the production process, and a_P is the carbon dioxide share of production in normal times. Using capital with an intensity higher than normal increases depreciation of capital according to $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \delta_2/2(u_t - 1)^2$, which, assuming $\delta_1, \delta_2 > 0$, is an increasing and convex function of utilization. Without loss of generality, capital utilization in the steady state is normalized to 1, so that δ_0 denotes the steady-state depreciation rate of capital goods.

Let mc_t be the relative price at which the intermediate good is sold to final goods producers. The intermediate goods producer maximizes profits,

$$mc_t Y_t - w_t^{fi} N_t - [r_t^F + q_t \delta(u_t)] K_t - (p_t^E - \tau_t^E) E_t^Y, \tag{31}$$

where r_t^F and q_t are the rental rate of firms and the (producer) price of capital goods, respectively. The intermediate goods producer operates in perfectly competitive markets, such that the real wage and the user costs of capital are determined by the following equations:

$$MPK_t = mc_t \left(1 - a_P\right)^{\left(\frac{1}{\sigma_p}\right)} \alpha \left(\frac{K_t}{N_t}\right)^{(\alpha-1)} \left(\frac{Y_t}{Y_t^p}\right)^{\left(\frac{1}{\sigma_p}\right)}, \tag{32}$$

$$r_t = 1 + MPK_t u_t - q_t \delta(u_t), \tag{33}$$

$$w_t^{fi} = mc_t \left(1 - a_P\right)^{\left(\frac{1}{\sigma_p}\right)} (1-\alpha) \left(\frac{u_t K_t}{N_t}\right)^{\alpha} \left(\frac{Y_t}{Y_t^p}\right)^{\left(\frac{1}{\sigma_p}\right)}, \tag{34}$$

$$p_t^E - \tau_t^E = mc_t a_P^{\left(\frac{1}{\sigma_p}\right)} \left(\frac{Y_t}{E_t^Y}\right)^{\left(\frac{1}{\sigma_p}\right)}. \tag{35}$$

Here MPK is the marginal product of capital services. I assume that utilization is decided by the owners of the capital goods, taking the aggregate supply of capital services as given. The optimality condition for utilization is given by

$$MPK_t = q_t[\delta_1 + \delta_2(u_t - 1)] \tag{36}$$

i.e., capital owners increase utilization until the marginal maintenance costs equal the marginal product of capital services.

A.2.4 Capital goods producers

Capital goods producers transform the physical good, investment I_t, into capital. They take the relative price of capital goods, q_t, as given in deciding about their output, i.e., they maximize[12]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2}\left(\log \frac{I_t}{I_{t-1}}\right)^2\right] - 1 \right\}. \tag{37}$$

[12]As I use a first order approximation changes in the stochastic discount factor are irrelevant. So are changes in the relative price $p_t(a^C)$ of the physical to the final consumption good.

Optimality of the capital goods production requires (again dropping all terms irrelevant up to first order)

$$q_t \left[1 - \phi \log \frac{I_t}{I_{t-1}}\right] = 1 - \beta \mathbb{E}_t \left[q_{t+1} \psi \log \left(\frac{I_{t+1}}{I_t}\right)\right], \tag{38}$$

and each capital goods producer will adjust its production until (38) is fulfilled.

Since all capital goods producers are symmetric, I obtain the law for motion for aggregate capital as

$$K_t - (1 - \delta(u_t))K_{t-1} = \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}}\right)^2\right] I_t \tag{39}$$

The functional form assumption implies that investment adjustment costs are minimized and equal to 0 in the steady state.

A.3 Government Sector

There is a monetary authority and a fiscal authority. The monetary authority controls the nominal interest rate on liquid assets, while the fiscal authorities issue government bonds to finance deficits, choose the average tax rate, make expenditures for government consumption and their transfer system, and receive revenue from selling carbon certificates.

A.3.1 Monetary Union

I assume that monetary policy sets the nominal interest rate following a Taylor (1993)-type rule with interest rate smoothing:

$$\frac{R^b_{t+1}}{\bar{R}^b} = \left(\frac{R^b_t}{\bar{R}^b}\right)^{\rho_R} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\rho_R)\theta_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{(1-\rho_R)\theta_Y}. \tag{40}$$

The coefficient $\bar{R}^b \geq 0$ determines the nominal interest rate in the steady state. The coefficients $\theta_\pi, \theta_Y \geq 0$ govern the extent to which the central bank attempts to stabilize producer price inflation and output growth. $\rho_R \geq 0$ captures interest rate smoothing.

A.3.2 Fiscal Policy

The budget constraint of the fiscal policy reads

$$G_t + TR_t = B_{t+1} + T_t - \frac{R_t^b}{\pi_t^{CPI}} B_t + T_t^E. \tag{41}$$

Hence, the government has expenditure for government spending, G_t, aggregate spending on its transfer system specified below, TR_t, and repaying its debt, B_t. It finances its expenditures by issuing new debt, collecting tax revenue, T_t, and by collecting the revenue from selling carbon certificates, $T_t^E = p_t^E * E_t$. Tax revenue is

$$T_t = \tau_t (w_t N_t + \mathbb{I}_{h_{it}=0} \Pi_t^{fi} + \mathbb{I}_{h_{it}\neq 0} \Pi_t^U). \tag{42}$$

I assume that the average tax rate is a feedback function of government debt:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{\tau_{t-1}}{\bar{\tau}}\right)^{\rho_\tau} \left(\frac{B_{t+1}}{\bar{B}}\right)^{(1-\rho_\tau)\gamma_B^\tau}. \tag{43}$$

where γ_B^τ governs the speed with which debt returns to its target.

A.3.3 Targeted Transfer System

The targeted transfer system follows the design in Bayer, Kriwoluzky, Müller, and Seyrich (2023). It provides additional resources if net labor income $w_t n_t h_{it}$ falls short of some target level. For simplicity, I assume that these transfers are non-distortionary for the labor supply decision. In particular, I assume that transfers are paid to households according to the following scheme:

$$tr_{it} = \max\{0, a_1 \bar{y} - a_2 (1-\tau_t) w_t h_{it} n_{it}\}, \tag{44}$$

where $\bar{y}$ is the median income and $0 \leq a_1, a_2 \leq 1$. Thus, transfers decrease in individual income with a transfer withdrawal rate of a_2 and no transfers are paid to households whose net labor income $(1-\tau_t) w_t h_{it} n_{it} \geq \frac{a_1}{a_2} \bar{y}$. Total transfer payments are then

$$TR_t = \mathbb{E}_t tr_{it} + Tr_t, \tag{45}$$

where again, the expectation operator is the cross-sectional average.

A.4 Carbon dioxide, goods, bonds, capital, and labor market clearing

The market for carbon certificates clears, when total carbon dioxide emission, consisting of household and firm carbon dioxide emission, equals the exogenous supply of carbon certificates:

$$E_t = E_t^C + E_t^Y. \tag{46}$$

The labor market clears at the competitive wage given in (34). The bond markets clear whenever the following equation holds:

$$B_{t+1} = B^d(p_t^E, Tr_t, R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t, \tau_t, \Theta_t, \Theta_t^*, \mathbb{W}_{t+1}) \quad := \mathbb{E}_t[\lambda \mathbb{B}_{a,t} + (1-\lambda)\mathbb{B}_{n,t}], \tag{47}$$

where $\mathbb{B}_{a,t}$, $\mathbb{B}_{n,t}$ are functions of the states (b, k, h, a^c), and depend on how the households value asset holdings in the future, $\mathbb{W}_{t+1}$, and the current set of prices (and tax rates) $(p_t^E, Tr_t, R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t^{CPI}, \tau_t)$. Future prices do not show up because I can express the value functions such that they summarize all relevant information on the expected future price paths. Expectations in the right-hand-side expression are taken w.r.t. the distributions $\Theta_t(b, k, h, a^c)$. Equilibrium requires the total *net* amount of bonds the household sectors demand to equal the supply of government bonds. In gross terms, there are more liquid assets in circulation as some households borrow up to $\underline{B}$.

In addition, the market for capital has to clear:

$$\begin{aligned} K_{t+1} &= K^d(p_t^E, Tr_t, R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t^{CPI}, \tau_t, \Theta_t, \Theta_t^*, \mathbb{W}_{t+1}) \\ &:= \mathbb{E}_t[\lambda(\mathbb{K}_t) + (1-\lambda)(k)] \end{aligned} \tag{48}$$

where the first equation stems from competition in the production of capital goods, and the second equation defines the aggregate supply of funds from households - both those that trade capital, $\lambda(\mathbb{K}_t)$ and those that do not, $(1-\lambda)(k)$. Again $\mathbb{K}_t$ is a function of the current prices and continuation values.

Finally, goods market clearing requires:

$$Y_t = C_t + I_t + BD_t\bar{R} + G_t. \tag{49}$$

A.5 Equilibrium

A sequential equilibrium with recursive planning in my Climate-HANK model is a sequence of policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$, a sequence of value functions $\{V_t^a, V_t^n\}$, a sequence of prices $\{p_t^E, \tau_t^E, Tr_t, w_t, w_t^{fi}, \Pi_t^U, \Pi_t^{fi}, q_t, r_t, R_t^b, \pi_t^{CPI}, \pi_t^W, \tau_t\}$, a sequence of carbon certificates, $\{E_t\}$, aggregate capital, labor supply, distributions Θ_t over individual asset holdings and productivity, and expectations for the distribution of future prices, Γ, such that

1. Given the functionals $\mathbb{E}_t \mathbb{W}_{t+1}$ for the continuation value and period-t prices, policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$ solve the households' planning problem; and given the policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$ and prices, the value functions $\{V_t^a, V_t^n\}$ are a solution to the Bellman equation.

2. Distributions of wealth and income evolve according to households' policy functions.

3. All markets clear in every period, interest rates on bonds are set according to the central bank's Taylor rule, fiscal policies are set according to the fiscal rules, and stochastic processes evolve according to their law of motion.

4. Expectations are model consistent.

I solve the model by using the perturbation method in Bayer, Born, and Luetticke (2020).

B Calibration

I calibrate the economy to match German data. To this end, I match the wealth distributions. Table 3 shows the calibration choices required for my calibration strategy which is described in 3. The rest of the parameters are calibrated by matching long-run averages and using standard parameters from the literature. Table 4 summarizes my calibration of those parameters. I calibrate to quarterly frequency.

The labor share in production, $(1-\alpha)$, is 68% corresponding to a labor income share of 62%, given a markup of 10% due to an elasticity of substitution between differentiated goods of 11. The elasticity of substitution between labor varieties is also set to 11, yielding a wage markup of 10%. The parameter δ_1 that governs the cyclicality of utilization is set to 5.0. The investment adjustment cost parameter is set to 4.0. I set the Calvo parameters for price and wage adjustment probability both to 0.25. All these parameter choices are standard values in the literature.

I set relative risk aversion, ξ, to 4, following Kaplan and Violante (2014) and the Frisch elasticity, γ to 0.5 following Chetty, Guren, Manoli, and Weber (2011). The persistence of

Table 3: Calibration—Asymmetric Parameters

	Description	Germany	Source/Target
a_1	Transfer level	0.5	German MIB system
a_2	Transfer withdrawal rate	0.8	German MIB system
G/Y	Gov. cons. share	0.20	German data
σ_h	STD labor inc.	0.135	German income data
β	Discount factor	0.9823	Six wealth targets
λ	Portfolio adj. prob.	0.071	Six wealth targets
ζ	Trans. prob. from W to E	0.001	Six wealth targets
ι	Trans prob. E to W	0.0625	Six wealth targets
$\bar{R}$	Borrowing penalty	0.029	Six wealth targets
B_{min}/Y	Borrowing limit	1.7	Six wealth targets

idiosyncratic income shocks is set to $\rho_h = 0.9815$. The stationary equilibrium real rate(-growth difference) is set to a net rate of zero.

The steady-state tax level is set to 0.3. I assume that monetary policy only targets inflation, as this is the primary mandate of the ECB, and set the Taylor coefficient to 1.25 and the smoothing parameter to 0.85. The steady-state inflation is zero.

Table 4: Rest of Calibration

	Description	Value	Source/Target
Firms			
$1-\alpha$	Share of labor	0.68	62% lab. income
η	Elast. of substitution	11	10% Price markup
η_W	Elast. of substitution	11	10% Wage markup
κ	Price adj. prob.	0.25	1 year avg. price duration
κ_W	Wage adj. prob.	0.25	1 year avg. wage duration
ϕ	Inv. adj. cost	4.0	Bayer, Born, and Luetticke (2020)
δ_0	Depreciation rate	0.018	Bayer, Born, and Luetticke (2020)
δ_1	Depr. rate increase	5.0	Bayer, Born, and Luetticke (2020)
Households			
ξ	Risk aversion	4	Kaplan and Violante (2014)
γ	Inv. Frisch elast.	2	Chetty, Guren, Manoli, and Weber (2011)
Government			
$\bar{\tau}$	Tax rate	0.3	Standard value
ρ_R	Pers. in Tax rule	0.9	standard value
γ_B^τ	Reaction to debt.	0.85	standard value
$\bar{R}^b$	Gross interest rate	1.00	zero interest-growth difference
ρ_R	Pers. in Taylor rule	0.85	standard value
θ_π	Reaction to Infl.	1.25	standard value
θ_Y	Reaction to Output	0	ECB mandate

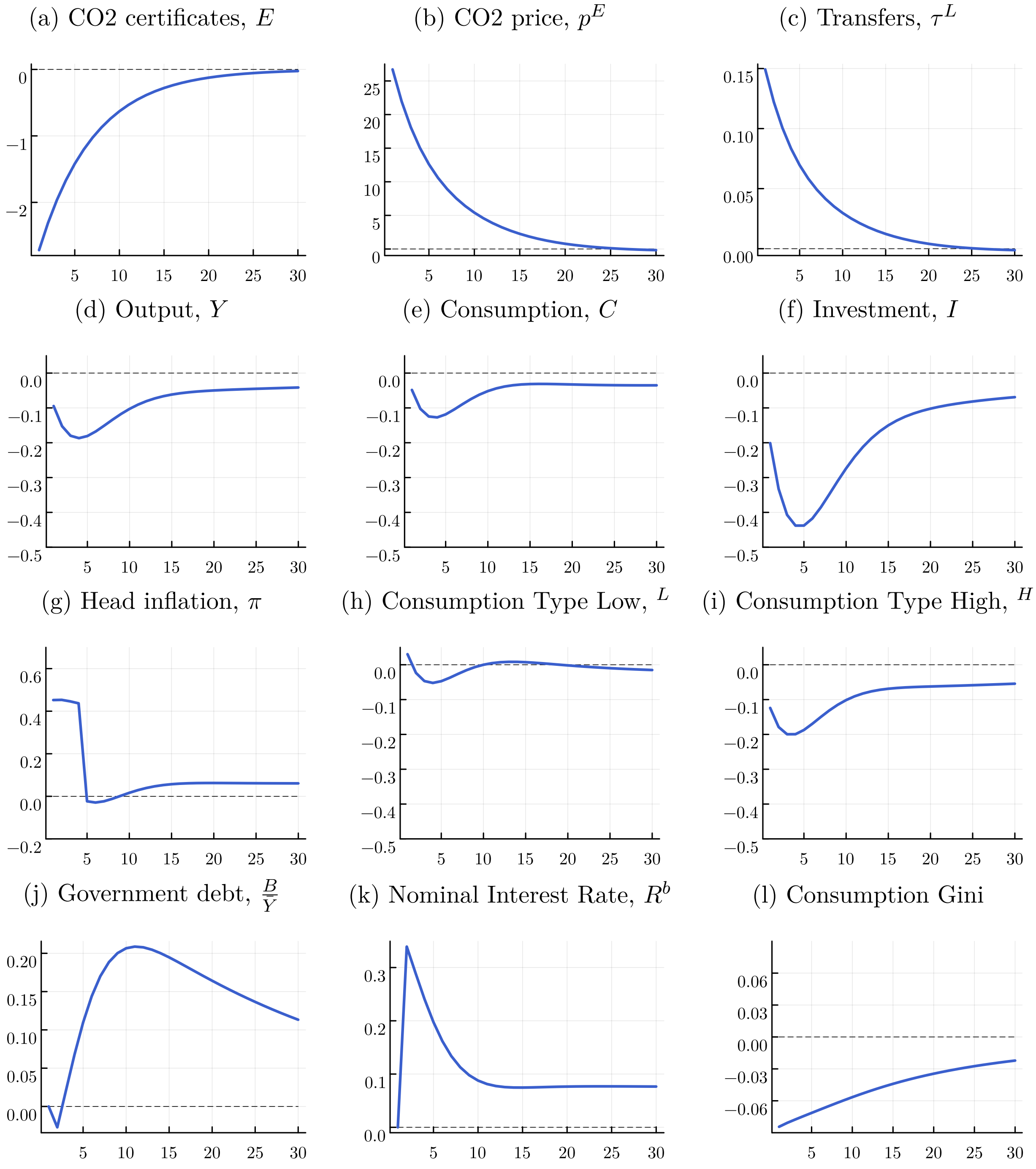
Table 5: Calibrated Model v Data

			Model		Data	
			F	**H**	**ITA**	**GER**
Steady state	Assets	Debt (% of output)	132	71	132	71
		Capital-Output-Ratio	3.3	3.2	3.3	3.2
(targeted)	Distribution	Wealth gini	0.60	0.72	0.61	0.73
		Top-10% wealth share	0.43	0.55	0.44	0.52
		Bottom-50% wealth share	0.10	0.01	0.09	0.02
		Borrowers	0.08	0.18	0.08	0.18

Notes: Model predictions based on baseline calibration, see Appendix B for details. Microdata based on the 2017 wave of the Household Finance and Consumption survey of the ECB. Macro data from Eurostat. Quantities are measured in real per capita terms, yoy changes; sample: 1999Q1-2022Q2.

C Further results

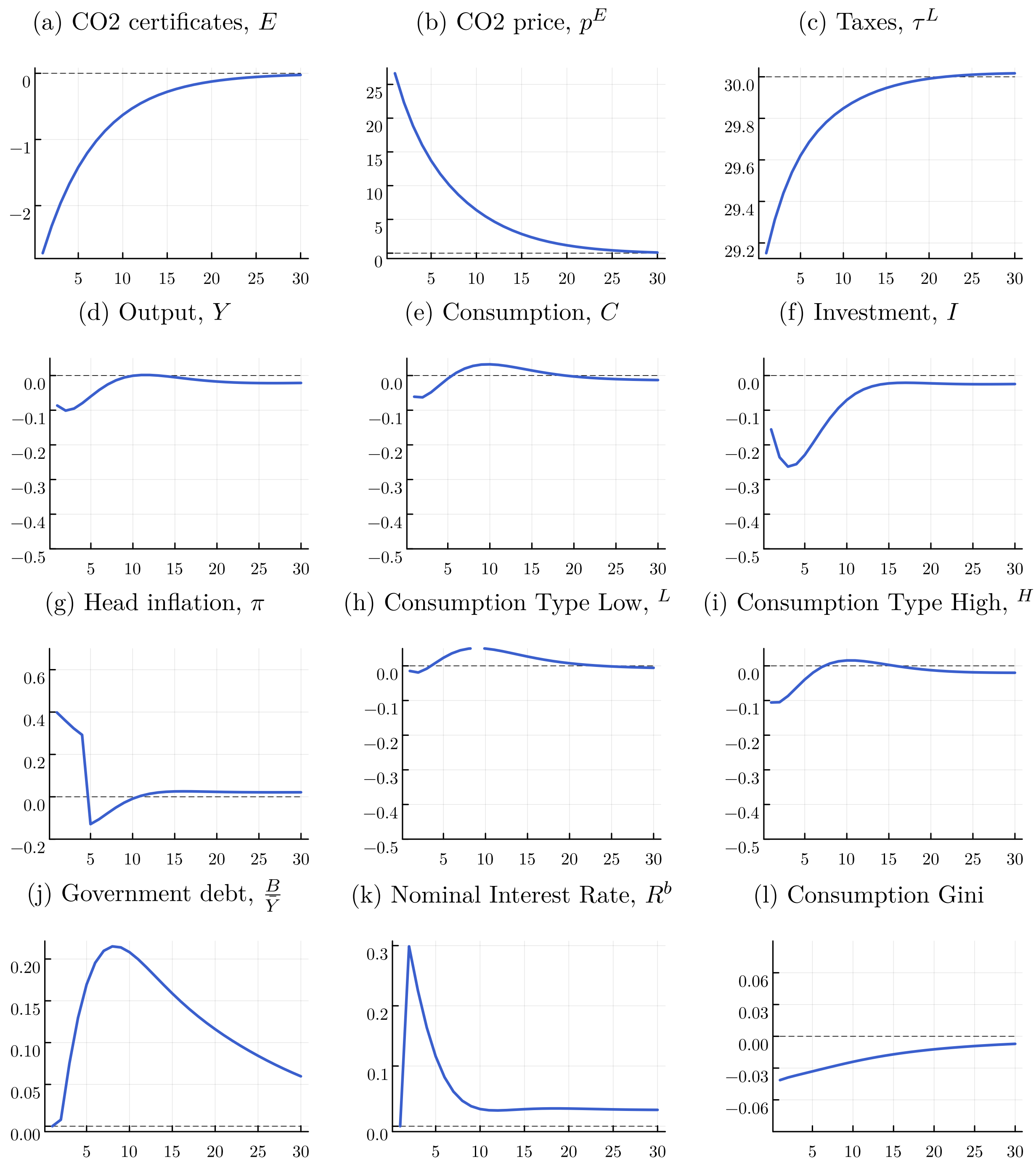
Figure 6: Response to carbon shock, transfer repayment



(a) CO2 certificates, E
(b) CO2 price, p^E
(c) Transfers, τ^L
(d) Output, Y
(e) Consumption, C
(f) Investment, I
(g) Head inflation, π
(h) Consumption Type Low, ^{L}
(i) Consumption Type High, ^{H}
(j) Government debt, $\frac{B}{Y}$
(k) Nominal Interest Rate, R^b
(l) Consumption Gini

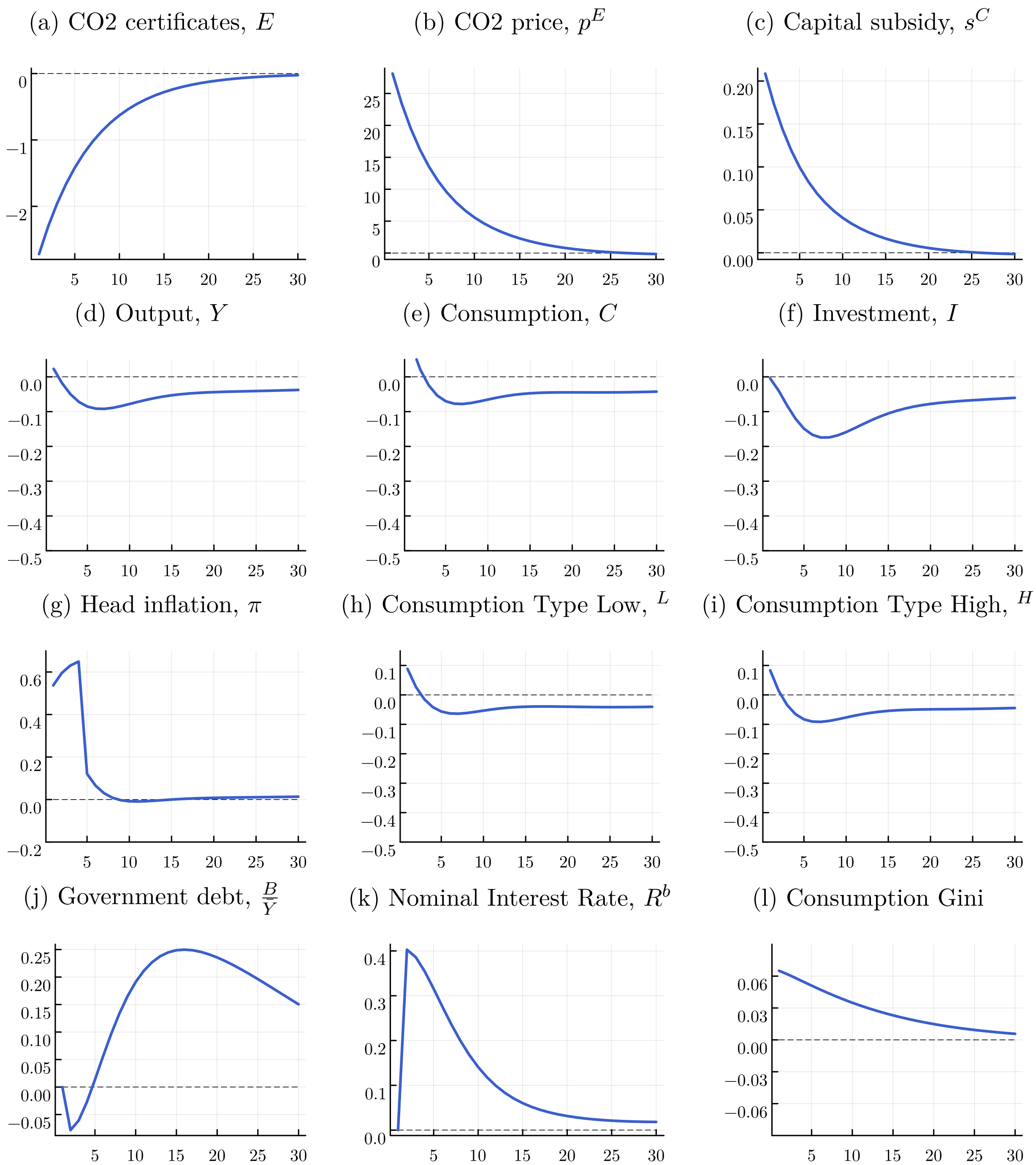
Impulse responses after an adverse carbon shock. Y-axis:Percentage deviation from steady state, percentage points in case of inflation and interest rate, and tax level in percentage in case of taxes. X-axis: Quarters.

Figure 7: Response to carbon shock, tax repayment



Impulse responses after an adverse carbon shock. Y-axis:Percentage deviation from steady state, percentage points in case of inflation and interest rate, and tax level in percentage in case of taxes. X-axis: Quarters.

Figure 8: Response to carbon shock, capital subsidy repayment



Impulse responses after an adverse carbon shock. Y-axis:Percentage deviation from steady state, percentage points in case of inflation, interest rate, and capital subsides. X-axis: Quarters.