A HANK^2 model of monetary unions

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Abstract

How does a monetary union alter the impact of business cycle shocks at the household level? We develop a Heterogeneous Agent New Keynesian model of two countries (HANK^2) and show in closed form that a monetary union shifts the adjustment to a shock horizontally—across countries—within the brackets of the union-wide wealth distribution rather than vertically—that is, across the brackets of the union-wide wealth distribution. Calibrating the model to the euro area reveals that a monetary union alters the impact of shocks most strongly in the tails of the wealth distribution but leaves the middle class almost unaffected.

Keywords: HANK^2, OCA theory, Two-country model, monetary union, spillovers, monetary policy, heterogeneity, inequality, households

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1 Introduction

Following the seminal work of Mundell (1961), Optimum Currency Area theory analyzes the costs and benefits of monetary unions at the level of regions or countries. Likewise, the policy debate is framed in such terms, as the 20-year plus history of the euro illustrates: discussions of whether specific *countries* would have been better off without the euro abound. Heterogeneity is at the heart of the issue: if countries differ, say, because of country-specific shocks, one (monetary) policy doesn't fit all. However, heterogeneity across households—in terms of income, wealth and shocks—dwarfs the heterogeneity across countries. Hence, we offer a change of perspective. We focus on households rather than countries and ask: How does a monetary union alter the impact of business cycle shocks at the household level?

To answer this question, we propose a Heterogeneous Agent, New Keynesian model of two countries: HANK^2. The model features incomplete markets, idiosyncratic risk, and self-insurance in a standard New Keynesian two-country setup. It is therefore able to capture key features of both, the business cycle and the wealth distribution and allows us to contrast the effects of country-specific shocks in monetary unions and under independent monetary policies—at three levels of aggregation. First, we show that when aggregated across both countries and households, macroeconomic dynamics are independent of whether there is a monetary union or not. Second, monetary union is also irrelevant for the effect of the shock on households in specific brackets of the wealth (and income) distribution aggregated across countries of residence. In other words, the monetary union itself does not redistribute the impact of the shock *vertically* across wealth classes. Instead, and this is our third and main result, it shifts the impact of the shock *horizontally* across borders within the brackets of the wealth distribution. Quantitatively, this burden-shifting is strongest in the tails of the distribution.

We first study an analytically tractable small-scale version of the model similar to Auclert et al. (2021b) where we make a number of simplifying assumptions. These assumptions allow us to obtain strong analytical knife-edge results. First, we restrict the two countries in the model to be symmetric, except for the occurrence of country-specific shocks. What is more, we abstract from capital accumulation and allow for trade in liquid, one-period debt only. Further, we abstract from price stickiness and maintain wage rigidities as the only nominal friction. A key difference to Auclert et al. (2021b) is that they consider a small-open economy rather than a two-country model. Relying on a sequence-space representation as in Auclert et al. (2018) or McKay and Wolf (2022), we then show that it is possible to cast the union-wide dynamics of the simplified model into the canonical form which is familiar from the textbook-version of the New Keynesian model (Galí 2015).

To assess the quantitative relevance of the analytical results, we consider a richer version of the model. Specifically, we introduce capital formation, portfolio choice, and price rigidities building on the medium-scale HANK model in Bayer et al. (2020). We calibrate this version of the model to capture key features and asymmetries between the Italian economy and the German one at the micro level. In terms of microdata, Italy and Germany are very different with respect to wealth inequality. According to a number of indicators, wealth inequality is significantly higher in Germany. We show that the model is able to account for these structural differences through an appropriate choice of model parameters. At the macro level, we maintain the assumption of identical frictions. These have been the subject of earlier research which we review below. Still, we verify that the model is able to capture key aspects of the business cycle, including its co-movement across Italy and Germany. We find that our results for the simplified model carry over to the medium-scale HANK model with asymmetries.

In more detail, we present our main results as we put forward three propositions: First, we show that whether countries operate a monetary union or independent monetary policies makes no difference for how country-specific shocks play out at the union level: Monetary union is irrelevant for union-wide dynamics. This result holds exactly for a first-order approximation in the aggregate states and under the assumption that countries are perfectly symmetric (except for the incidence of shocks). Moreover, we require—in line with actual practice in the EA—that the common monetary policy adjusts the policy rate to the average inflation rate (and possibly output gap) in both countries, to which we refer as "Home" and "Foreign." Against this background the irrelevance result is intuitive. Under a monetary union, monetary policy does not fit all: relative to a benchmark with independent monetary policies, the common policy responds too much in one country and too little in the other. It follows that macro dynamics at the country level do very much depend on whether countries operate a monetary union or not. But when countries are symmetric the changes induced by the monetary union in both countries offset each other such that union-wide dynamics do not change with monetary union.

Our second proposition concerns the household level. Taking a union-wide perspective and aggregating households across countries of residence within the brackets of the wealth distribution, we find that the impact of a business cycle shock for specific wealth classes does not depend on whether there is a monetary union or not. Put differently, just like with union-wide aggregate dynamics, monetary union is also irrelevant to the impact of shocks along the union-wide wealth distribution. It does not, say, shift the adjustment vertically from the rich to the poor or vice versa. Intuitively, how saving and consumption at the household level change in response to shocks within one country does depend on whether there is a monetary union in place or not because these depend on the price adjustments within that country. But given that a monetary union is irrelevant for union-wide price paths, it follows that the changes that a monetary union induces for the response of a generic household in Home are perfectly offset by the changes of its "twin" in Foreign—where a twin is defined in terms of a location in the income and wealth space. Aggregating across countries of residence we thus find the overall adjustment unchanged within the brackets of the wealth distribution.

Our third proposition is implicit in the argument above. It establishes that monetary union is potentially very relevant for the impact of country-specific shocks along the wealth distribution within a country. More specifically, comparing the outcome under a monetary union to the outcome under independent monetary policies, we observe that the union shifts the impact of shocks horizontally across borders within the brackets of the wealth distribution. Put differently, the impact of shocks changes for specific households at the expense of their twins in the other country: in the face of specific shocks, the poor (rich) in one country benefit from union membership at the expense of the poor (rich) in the other country.

We simulate the calibrated model and verify that our main results hold approximately once we allow for asymmetries in terms of household-level heterogeneity across countries. In particular, as we study the adjustment to country-specific shocks we find—consistent with Proposition 1—that union-wide aggregate dynamics are basically independent of whether there is a monetary union in place or not. In contrast, country-level dynamics change fundamentally due to the monetary union. In this regard, our model simulation confirms the classic notion that one size doesn't fit all. We also provide suggestive evidence that Proposition 2 holds approximately in the asymmetric model analysis by comparing the response of Gini coefficients to country-specific shocks across monetary regimes.

Last, we perform a quantitative analysis that relates to Proposition 3. We compute the consumption equivalent welfare variation of a shock as a comprehensive (ex-post) measure of its impact and find that a monetary union induces strong changes in this measure in the tails of the wealth distribution, both in Home and Foreign. These changes can be traced back to how a monetary union changes the interest-rate dynamics to which households in the tails of the wealth distribution—rich and poor—are more exposed than the middle class which neither borrows nor saves much (in excess of what it implicitly owes through government debt). We find accordingly, that monetary union does not change the impact of shocks for the middle class. This result provides a potential rationale for why the EA did not break up after all: During its 20-year-plus history, the EA witnessed political movements that campaigned against the euro, yet their appeal to the larger electorate turned out to be limited. Our analysis offers an explanation for why this is.

Related literature. Our analysis builds on two earlier generations of OCA theory. The first generation stresses that countries should be sufficiently homogeneous to qualify as OCA. The original contribution of Mundell (1961) emphasizes that economic regions as opposed to nation-states or countries are the relevant category when it comes to operating a common currency. We thus follow Mundell's lead as we attempt to shift the focus away from countries (and towards households). Other contributions to the first generation of OCA theory stress the role of trade openness and the asymmetry of shocks (McKinnon 1963; Kenen 1981; Bayoumi and Eichengreen 1992; Krugman 1993). Lastly, influential work has emphasized the potential endogeneity of the OCA criteria (Frankel and Rose 1998; Rose 2000).

The second generation of OCA theory zooms in on specific aspects, notably on the trade-offs faced by monetary and fiscal policy in monetary unions as well as on the conduct of optimal policy, relying on explicit welfare criteria (Beetsma and Uhlig 1999; Alesina and Barro 2002). These criteria are typically micro-founded within New Keynesian models featuring representative agents (see, for instance, Benigno 2004; Kollmann 2004; Benigno and López-Salido 2006; Beetsma and Jensen 2005; Corsetti 2008; Gali and Monacelli 2008; Galí and Monacelli 2016; Farhi and Werning 2017; Hettig and Müller 2018; Groll and Monacelli 2020).

The present paper belongs to a new set of studies that explicitly accounts for within-country heterogeneity when revisiting open-economy issues. In particular, several studies rely on small open-economy HANK models to reassess the merits of alternative exchange-rate policies. Ferra et al. (2021) find that household heterogeneity rationalizes "fear of floating" in the face of sudden stops. Auclert et al. (2021b), in turn, stress that household heterogeneity can amplify the real income channel of exchange rates, potentially giving rise to contractionary depreciations. Guo et al. (2020) find that fixing the exchange rate leads to larger spillovers of foreign shocks but dampens their distributional impact, in contrast to what we find for HANK^2. Oskolkov (2021) and Zhou (2021) also study the distributional impact of foreign shocks and exchange-rate policies in small open-economy HANK models. Aggarwal et al. (2022) study the implications of fiscal deficits through the lens of a multi-country HANK model. Bellifemine et al. (2023) develop a HANK model of a monetary union composed of small open economies. What sets our paper apart is the two-country structure of HANK^2: it allows us to study how a monetary union alters the impact of shocks along the wealth distribution— both, vertically and horizontally across borders. Bayer et al. (2022) and Chen et al. (2023) also develop a two-country HANK model and calibrate to the EA, focusing on fiscal frameworks rather than on monetary union as such. In Bayer et al. (2022), in particular, we develop the notion that "attitudes" towards fiscal policy may be traced back to how differences in income and wealth interact with different social security systems.

2 The Model

We develop a two-country New Keynesian model with incomplete markets, idiosyncratic risk, and heterogeneous agents (HANK^2). In this section, we first introduce a smaller model, a one-asset-HANK^2 model, for which we are able to establish a number of closed-form results in Section 3. We extend the model in Section 4 to a two-asset, medium-scale-HANK^2 model and calibrate it to data for the EA in order to assess the quantitative relevance of our results.

We borrow our two-country framework from Corsetti et al. (2012), while the specification of the household problem follows the small-open economy setup of Auclert et al. (2021b). Their setup, in turn, extends Galí and Monacelli (2005) by allowing for household heterogeneity. Countries are isomorphic and our exposition focuses on the domestic economy or "Home". "Foreign" looks the same, but may differ in terms of size: We normalize the total population to unity, a fraction n of which resides in Home. In what follows, we denote foreign variables with the superscript * and use subscripts H and F to distinguish between domestic and foreign variables within a country. To benchmark the case of a monetary union against a scenario of independent monetary policies, we allow Home and Foreign to operate different currencies. In case there is a monetary union there will be an irrevocable conversion rate. We further assume that all households have perfect foresight and focus on a first-order approximation around the stationary equilibrium.

2.1 Households

There is a continuum of households, each of which faces idiosyncratic income risk. This, in turn, is due to idiosyncratic productivity, $e_{i,t}$, which is determined exogenously by a first-order Markov chain with mean $\mathbb{E}e_{i,t} = 1$. Households save via a riskless bond which is denominated in domestic currency and issued by a mutual fund. To keep the household problem simple, we assume below that foreign currency bonds are traded via that mutual fund, too. This yields the familiar UIP condition but is otherwise inconsequential for the household savings decisions given perfect foresight.

Household labor supply, N_t, is determined by a labor union as described below and we assume that the labor union allocates hours worked uniformly across households. A generic household with bond holdings a and productivity level e at time t optimally chooses consumption, c, and savings a', by solving the dynamic program

$$V_t(a,e) = \max_{c,a'} u(c,N_t) + \xi_t \beta E_t[V_{t+1}(a',e')] \tag{1}$$

$$\text{s.t. } c + a' = (1+r_t^b)a + e\frac{W_t}{P_t}N_t - \tilde{\tau}e \tag{2}$$

$$a' \geq \underline{a'},$$

where P_t is the consumption price index specified below, r_t^b is the return on the bond, W_t the nominal wage, $0 < \beta < 1$ is the time discount factor, and $\tilde{\tau}$ is a non-distortionary tax on households.[1] In addition, ξ_t is an impatience shock which we use to showcase how country-specific demand shocks will enter the IS relation later.

For now, we assume the functional form

$$u(c,N_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \psi\frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\gamma, \varphi > 0$ are the inverse of the intertemporal elasticity of substitution and the inverse of the Frisch elasticity of labor supply respectively.

We state the solution to the household's problem in sequence-space form as in Auclert et al. (2021a), which we will use later to characterize the dynamics of the model. In particular, the solution to household's i consumption-savings problem described by (1) and (2) maps the paths of wages $\boldsymbol{W}$, hours worked $\boldsymbol{N}$, real returns $\boldsymbol{r^b}$, taxes $\boldsymbol{\tau}$, prices $\boldsymbol{P}$, and shocks $\boldsymbol{\xi}$ to that household's consumption $\boldsymbol{c_i}$:

$$\boldsymbol{c}_i = \mathcal{C}_i\Big(\boldsymbol{W}/\boldsymbol{P}, \boldsymbol{N}, \boldsymbol{r^b}, \boldsymbol{\tilde{\tau}}, \boldsymbol{\xi}\Big). \tag{3}$$

Aggregating across all domestic households, we obtain an aggregate domestic consumption function $\mathcal{C}(\cdot)$, similar as in Auclert et al. (2018) or McKay and Wolf (2022):

$$\boldsymbol{c} = \mathcal{C}\Big(\boldsymbol{W}/\boldsymbol{P}, \boldsymbol{N}, \boldsymbol{r^b}, \boldsymbol{\tilde{\tau}}, \boldsymbol{\xi}\Big). \tag{4}$$

In each period, households allocate their consumption expenditures, c, across a domestically produced good c_H and an imported good c_F so as to enjoy an overall consumption level

$$c_t = \left\{[1-(1-n)\alpha_H]^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + [(1-n)\alpha_H]^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{1-\eta}}. \tag{5}$$

[1]Non-distortionary taxes simplify our arguments below but they are not necessary: our result also holds if taxes are standard distortionary labor taxes.

Here $\alpha_H \in [0,1]$ indicates a home bias in consumption: The weight of the domestic good in total consumption is larger than what the size of the domestic economy would imply. If $\alpha_H = 1$ there is no home bias. η is the elasticity of substitution between the domestic and the imported good. Letting $P_{H,t}$ and $P_{F,t}$ denote the price of these goods, both expressed in domestic currency, expenditure minimization implies for the consumer price index:

$$P_t = \left\{ [1-(1-n)\alpha_H] P_{H,t}^{1-\sigma} + [(1-n)\alpha_H] P_{F,t}^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}. \tag{6}$$

The optimal intratemporal allocation of expenditures implies the demand functions:

$$c_H = (1-(1-n)\alpha_H)\left(\frac{P_{H,t}}{P_t}\right)^{-\sigma} c, \quad c_F = (1-n)\alpha_H \left(\frac{P_{F,t}}{P_t}\right)^{-\sigma} c.$$

Let $\mathcal{E}_t$ denote the nominal exchange rate, that is, the price of foreign currency expressed in terms of the domestic currency. We assume that the law of one price holds, that is, the foreign currency price of the domestically produced good is given by $P^*_{H,t} = \mathcal{E}_t P_{H,t}$ and likewise for the foreign-currency price of the imported good. For future reference, it is also useful to define the terms of trade as the relative price of foreign goods to domestic goods $s_t = P_{F,t}/P_{H,t}$ and the real exchange rate $Q_t = P_t \mathcal{E}_t / P^*_t$.

2.2 Production

The production function is linear in labor:

$$Y_t = N_t, \tag{7}$$

where N_t is the aggregate labor input. For now, we assume perfect competition in the domestic goods market such that the price of domestic goods is equal to marginal costs given by the nominal wage: $P_{Ht} = W_t$. It is convenient to rewrite the real wage as a function of the terms of trade:

$$w_t = \frac{W_t}{P_t} = \frac{P_{Ht}}{P_t} = [(1-(1-n)\alpha_H) + ((1-n)\alpha_H) s_t^{1-\sigma}]^{-\frac{1}{1-\sigma}}. \tag{8}$$

Aggregate labor is composed of differentiated types:

$$N_t = \left(\int_k N_{k,t}^{\frac{\epsilon_t - 1}{\epsilon_t}} \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}, \tag{9}$$

where ϵ_t is the elasticity of substitution between labor types and may vary over time. We use this "cost-push shock" to showcase how country-specific shocks to the Philips curve affect the dynamics of the model. Labor types, in turn, are efficiency units of work: $N_{k,t} = \int e_{i,t} n_{i,k,t} di$, where i indexes a household and $k \in [0,1]$ indexes the labor type. As in the recent literature, we assume that a household's number of hours worked as type k, $n_{k,t}$ is determined by a union which also determines the wage for each type $W_{k,t}$ (Erceg et al. 2000; Auclert et al. 2018; McKay and Wolf 2022). A union can reset the wage with a constant probability θ.

The solution to the union problem gives rise to a standard linearized open-economy New Keynesian Philips curve at Home:

$$\hat{\pi}_{H,t} = \kappa((1-n)\alpha_H \hat{s}_t + \phi \hat{y}_t + \gamma \hat{c}_t) + \beta \hat{\pi}_{H,t+1} + \psi \hat{\epsilon}_t, \tag{10}$$

where $\pi_{H,t} := \frac{p_{H,t}}{p_{H,t-1}}$ is gross domestic producer price inflation, $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\phi \equiv \frac{v_{ll}(\bar{l})\bar{l}}{v_l(\bar{l})}$, and $\psi \equiv -\frac{\kappa}{(\bar{\epsilon}-1)}$.[2]

2.3 Financial markets

There are two bonds, a home bond and a foreign bond each denoted in their own currency (unless there is monetary union). In the absence of arbitrage, the expected returns on both bonds are equal which implies the standard uncovered interest parity (UIP) condition:

$$1 + i_t = (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}. \tag{11}$$

In order to abstract from potentially heterogeneous household portfolios we assume that bond trading takes place via the mutual fund. Generally, up to the first order, the fund's portfolio is indeterminate. We assume that, in the steady state, the fund only holds domestic-currency debt. In this way, we rule out valuation effects that may arise in response to shocks under flexible exchange rates. Off steady-state, without loss of generality, we assume that cross-border trade is restricted to the foreign bond.

The domestic real interest rate is then pinned down by the Fisher equation and given by:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}, \tag{12}$$

where $\pi_{t+1} := \frac{P_t}{P_{t-1}}$ is domestic CPI inflation. Given our assumptions, $r_t^b = r_t$ holds up to first-order.

[2] We assume that the union neglects the impatience shock of households when setting wages in order to study the role of distinct demand and supply shocks.

2.4 Monetary and fiscal policy

In case there is a monetary union the common central bank adjusts interest rates based on the following simple rule:

$$i_t = \theta_\pi \Big(n\pi_{Ht} + (1-n)\pi^*_{Ft} \Big). \tag{13}$$

where π_{Ht} is the producer price inflation at Home while π^*_{Ft} is producer price inflation in Foreign. The coefficient $\theta_\pi \geq 0$ governs the extent to which the central bank adjusts the policy rate in response to average price inflation in the monetary union which we state as producer price inflation but may equivalently be expressed as consumer price inflation. Our results below also extend to the case where interest rates are adjusted to the (average) output gap. Note also that UIP (11) implies that $i_t = i^*_t$ once the nominal exchange rate is irrevocably fixed.

Alternatively, we consider a case with flexible exchange rates, assuming the following rules for monetary policy in Home:

$$i_t = \theta_\pi \pi_{Ht}, \tag{14}$$

and symmetrically for Foreign:

$$i^*_t = \theta_\pi \pi^*_{Ft}. \tag{15}$$

The conduct of fiscal policy is independent of whether there is a monetary union or not. It is set at the national level. Each government issues government bonds B_t to finance deficits and sets the tax rate. The budget constraint of the national fiscal policy reads as follows:

$$\frac{1+i_t}{\pi_t} B_t = B_{t+1} + \tau_t w_t N_t, \tag{16}$$

with $\tau_t = \frac{\tilde{\tau}_t}{w_t N_t}$. We assume that tax rates adjust to stabilize the level of government debt:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{B_{t+1}}{\bar{B}} \right)^{\gamma^\tau_B}, \tag{17}$$

where γ^τ_B governs the speed with which debt returns to its target.

2.5 Market Clearing

Bond market clearing requires:

$$A_{t+1} = B_{t+1} + \frac{B_{F,t+1}}{Q_t}, \tag{18}$$

that is, the total amount of domestic savings, A_t, equals the domestic bonds plus the net foreign asset position, $B_{F,t+1}$, which is held in foreign bonds. Analogously bond market clearing requires for Foreign:

$$A^*_{t+1} = B^*_{t+1} - \frac{n}{1-n} B_{F,t+1}. \tag{19}$$

Aggregating over the domestic households' budget constraints gives the net amount of domestic holdings of foreign bonds, $B_{F,t}$:

$$w_t Y_t - T_t + (1+r_t)B_t + \frac{(1+r^*_t)}{Q_t} B_{F,t} = C_t + B_{t+1} + \frac{B_{F,t+1}}{Q_t}. \tag{20}$$

Finally, goods markets clearing requires:

$$Y_t = \left(w_t\right)^{-\sigma}\left[(1-(1-n)\alpha_H)C_t + (1-n)\alpha_H Q_t^{-\sigma} C^*_t\right] \tag{21}$$

$$Y^*_t = \left(w^*_t\right)^{-\sigma}\left[n\alpha_H Q_t^{\sigma} C_t + (1-n\alpha_H)C^*_t\right]. \tag{22}$$

2.6 Equilibrium

We now define a linearized perfect-foresight transition economy of the model. That is, we always refer to the linearized versions of the relevant model equations.

Given exogenous paths for the supply and the demand shocks, $\{\epsilon_t, \epsilon^*_t, \xi_t, \xi^*_t\}_{t=0}^{\infty}$, a linearized perfect-foresight equilibrium is a set of aggregates $\{\pi_t, \pi^*_t, \pi_{H,t}, \pi^*_{H,t}, \pi_{F,t}, \pi^*_{F,t}, i_t, i^*_t, r_t, r^*_t,$ $\tau_t, \tau^*_t, w_t, w^*_t, c_t, c^*_t, Y_t, Y^*_t, N_t, N^*_t, \Delta\mathcal{E}_t, s_t, Q_t, A_t, A^*_t, B_t, B^*_t, B_{F,t}\}_{t=0}^{\infty}$ such that:

1. The paths of aggregate consumption at Home and at Foreign $\{c_t, c^*_t\}_{t=0}^{\infty}$ are consistent with the linearized aggregate consumption functions (4), and the path of household asset holdings in Home and in Foreign $\{A_t, A^*_t\}_{t=0}^{\infty}$ are consistent with the budget constraints (2), aggregated across households in Home and in Foreign.

2. The real wages $\{w_t, w^*_t\}_{t=0}^{\infty}$ are consistent with (8) and the counterpart in Foreign.

3. The paths of $\{N_t, N^*_t, Y_t, Y^*_t\}_{t=0}^{\infty}$ satisfy the aggregate production functions in Home and in Foreign (7).

4. The paths of $\{\pi_{H,t}, \pi^*_{F,t}, Y_t, Y^*_t, s_t\}_{t=0}^{\infty}$ are consistent with the national Philips curves (10).

5. The paths of $\{\pi^*_{H,t}, \pi_{F,t}\}_{t=0}^{\infty}$ are consistent with the law of one price stated in the main text.

6. Nominal interest rates and the change in the nominal exchange rate satisfy the interest rate rules given above and the UIP (11) condition holds.

7. The evolution of the government debt levels and taxes $\{B_t, B^*_t, \tau_t, \tau^*_t\}_{t=0}^{\infty}$ are consistent with the government budget constraints (16) and the feedback function for taxes (17).

8. CPI rates in both countries $\{\pi_t, \pi^*_t\}_{t=0}^{\infty}$ are consistent with the definition of the CPI given by (6).

9. The net foreign asset position $\{B_{F,t}\}_{t=0}^{\infty}$ evolves according to the home budget constraint (20).

10. Terms of trade $\{s_t\}_{t=0}^{\infty}$ and the real exchange rate $\{Q_t\}_{t=0}^{\infty}$ evolve as defined in the main text.

11. The bond markets (19), and the goods markets clear (21).

3 Closed-Form Results

In this section, we derive our main results in closed form. In particular, we show that a monetary union shifts the impact of country-specific shocks at the household level horizontally, that is, across borders within the brackets of the wealth distribution. To set the stage, we first derive two propositions that show that a monetary union makes no difference for union-wide outcomes, both in terms of how aggregate variables respond to country-specific shocks and how the impact of the shock spreads vertically across the brackets of the union-wide wealth distribution. Put differently, it is irrelevant to union-wide outcomes whether countries form a monetary union or not. At the same time, a monetary union alters the adjustment to country-specific shocks across borders—both, at the aggregate level and at the household level.

For what follows, we define union-wide variables as a weighted average of the realizations in Home and Foreign, $X^W_t = nX_t + (1-n)X^*_t$, and write the canonical form for union-wide dynamics using the sequence-space representation (see Appendix A.1 for details).[3] As with

[3]For lack of a better term, we also refer to these variables as "union-wide" variables even if the two countries operate independent monetary policies and let their exchange rates float.

the textbook representative agent version of the New Keynesian model, the canonical form is sufficient to describe the aggregate dynamics of the economy. Specifically, we summarize inflation dynamics with a union-wide New Keynesian Phillips curve:

$$\widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} = \kappa \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \beta \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}_{\boldsymbol{+1}}} + \psi \widehat{\boldsymbol{\eta}^{\boldsymbol{W}}}, \tag{23}$$

where η^W is a sequence of cost shocks. The union-wide IS relation, in turn, is given by:

$$\widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} + \tilde{\mathcal{C}}_\xi \widehat{\boldsymbol{\xi}^{\boldsymbol{W}}}. \tag{24}$$

Importantly, (23) and (24), hold independently of whether there is a monetary union or not. To close the model, we need to specify a rule that pins down the union-wide interest rate $\widehat{\boldsymbol{i}^{\boldsymbol{W}}}$. This is where the monetary union comes into play. However, it turns out to be irrelevant for dynamics of the union-wide interest rate whether or not the two countries operate a monetary union.

Proposition 1. *The union-wide aggregate dynamics are characterized by* (23) *and* (24) *and a mapping from aggregate union-wide inflation to aggregate union-wide policy rates. As this mapping is the same in a monetary union and with independent monetary policies, so are aggregate dynamics.*

Proof. With a monetary union, we have $i_t = i_t^* = i_t^W$ and given equation (13), we have:

$$i_t^W = \theta_\pi (n\pi_{H,t} + (1-n)\pi^*_{F,t}) = \theta_\pi \pi_t^W.$$

With independent monetary policies, given by equations (14) and (15), the union-wide interest rate is:

$$i_t^W = ni_t + (1-n)i_t^* = n(\theta_\pi \pi_{H,t}) + (1-n)(\theta_\pi \pi^*_{F,t}) = \theta_\pi \pi_t^W \tag{25}$$

and, hence, *exactly the same as with a monetary union.* □

Proposition 1 implies that if a monetary union experiences country-specific shocks, union-wide aggregates like output, consumption, and inflation behave exactly the same independently of whether countries form a monetary union or not. To see why, consider a shock originating in Foreign. In a monetary union, the response of monetary policy is a response to the weighted average of the dynamics in both countries. This implies, for instance, that monetary policy reacts "too much" from the perspective of Home and "too little" from the perspective of Foreign, compared to what would happen under independent policies. But given that the countries are isomorphic—in particular given $(\kappa, \mathcal{C}, \theta_\pi) = (\kappa^*, \mathcal{C}^*, \theta_\pi^*)$—"too little" and "too much" means the same in absolute value and, thus, the contribution of each country to union-wide dynamics exactly offsets each other. Note that this holds even if the countries are not of the same size.

From a union-wide perspective, monetary union is also irrelevant to the impact of shocks along the wealth distribution. To see this, consider a generic household j in Home. Given symmetry, there are $\frac{n}{1-n}$ times identical households in Foreign, that is, households with the same idiosyncratic productivity and the same wealth. We label these twin households j^* and define $c_J = nc_j + (1-n)c_{j^*}$ as aggregate consumption of household j and its twins. Note that in linearized form, we have

$$\widehat{\boldsymbol{c}}_j = \mathcal{C}_{w,j}\widehat{\boldsymbol{w}} + \mathcal{C}_{N,j}\widehat{\boldsymbol{N}} + \mathcal{C}_{i,j}\widehat{\boldsymbol{i}} + \mathcal{C}_{\pi,j}\widehat{\boldsymbol{\pi}} + \mathcal{C}_{\tau,j}\widehat{\boldsymbol{\tau}} + \mathcal{C}_{\xi,j}\widehat{\boldsymbol{\xi}} \tag{26}$$

$$\widehat{\boldsymbol{c}^*}_{j^*} = \mathcal{C}^*_{w^*,j^*}\widehat{\boldsymbol{w}^*} + \mathcal{C}^*_{N^*,j^*}\widehat{\boldsymbol{N}^*} + \mathcal{C}^*_{i^*,j^*}\widehat{\boldsymbol{i}^*} + \mathcal{C}^*_{\pi^*,j^*}\widehat{\boldsymbol{\pi}^*} + \mathcal{C}^*_{\tau^*,j^*}\widehat{\boldsymbol{\tau}^*} + \mathcal{C}^*_{\xi^*,j^*}\widehat{\boldsymbol{\xi}^*}. \tag{27}$$

Given symmetry, $\mathcal{C}_{x,j} = \mathcal{C}^*_{x^*,j^*}$ and, thus:

$$\widehat{c}_J = \mathcal{C}_{w,j}\widehat{\boldsymbol{w}^{\boldsymbol{W}}} + \mathcal{C}_{N,j}\widehat{\boldsymbol{N}^{\boldsymbol{W}}} + \mathcal{C}_{i,j}\widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \mathcal{C}_{j,\pi}\widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} + \mathcal{C}_{j,\tau}\widehat{\boldsymbol{\tau}^{\boldsymbol{W}}} + \mathcal{C}_{\xi,j}\widehat{\boldsymbol{\xi}^{\boldsymbol{W}}}. \tag{28}$$

The same logic applies to all policy functions of the household. Given Proposition 1, the inputs of the aggregate policy functions of the twin households do not depend on whether there is a monetary union or not. Hence, the weighted average (or union-wide aggregate) of a choice variable of household j in Home and its $\frac{n}{1-n}$ twins in Foreign does therefore not depend on whether there is a monetary union or not. Our next irrelevance result follows directly:

Proposition 2. *The impact of country-specific shocks along the union-wide wealth and income distribution is independent of whether two countries form a monetary union or not. In other words, the monetary union does not shift the vertical impact of the shock.*

Against this background, the next proposition follows directly. It summarizes our main result.

Proposition 3. *Monetary union shifts the distributional impact of asymmetric shocks horizontally across borders within the brackets of the wealth distribution.*

To see what drives this result, recall that the monetary policy alters the dynamics of *country-specific* variables relative to what would be observed under independent monetary policies. This means that the arguments that feature in the consumption function of individual households in Home and Foreign, (26) and (27), generally differ compared to what would be the case with independent monetary policies. Also, the consumption choice of a household with a given wealth and productivity state in Home will generally differ from that of its twin in Foreign. Yet, as established in Proposition 2, how the *union-wide* wealth distribution changes in response to country-specific shocks do not depend on the monetary union as a monetary union does not redistribute vertically. Assuming countries are of the same size, this then requires that monetary union changes the effect of a shock on a household's consumption choice in Home in exactly the opposite way as it does for its Foreign twin. When countries differ in size, the differential impact of monetary union on the choice of a generic household in Home is of the opposite sign as that of its Foreign twin, weighted by the number of twins that a Home household has in Foreign. It follows that the distributional effect of monetary union operates horizontally across borders: It shifts the distributional impact of shocks (compared to a scenario of independent monetary policy) between households in Home and Foreign *within the same individual states* or within the same bracket of the wealth distribution. For instance, if the consumption of the poor at Home is higher with a monetary union than with independent monetary policies after a given shock, the consumption of the poor at Foreign must be lower by the (weighted) same amount.

4 Quantitative Analysis

Our analysis above has shown qualitatively that monetary union alters the impact of shocks at the household level. It does so, as it shifts the adjustment horizontally across countries within the brackets of the wealth distribution. We now perform a quantitative analysis in order to assess how strongly different types of households are affected by the monetary union. We perform the quantitative analysis in a version of the model that is extended in a number of dimensions and, importantly, no longer restricts Home and Foreign to be symmetric. Specifically, we calibrate the model to two countries of the euro area that represent polar cases in terms of the wealth distribution: Germany and Italy. For this version of the model, we also show that the results established by Propositions 1 and 2, which rely on symmetry, are still approximately satisfied.

4.1 Medium-sized HANK^2 model

Since our question at hand is a quantitative one, we enrich our model laid out in Section 2 by features that are frequently used in medium-sized business cycle models. In particular, we use a two-country version of the model developed in Bayer et al. (2020) which has been shown to be able to generate business-cycle dynamics that conform well with the data. We calibrate this medium-sized HANK^2 model to capture key aspects when it comes to asset holdings and wealth distributions in Germany and Italy. In what follows, we briefly sketch the main extensions of the model and delegate a full description of the extended model to Appendix B. As before the structural features are the same in Home and Foreign. Yet by assigning different parameter values below we make sure that Home and Foreign differ—in accordance with the data.

Households. We modify the household side in three ways in order to be better able to match the wealth distribution in the data. First, we assume that a group of households is employed by firms while others are self-employed entrepreneurs. The former group receives only labor income while entrepreneurs earn firm profits due to monopolistic competition in the goods market (see below). Yet, households may move from one group (or employment state) to the other according to some exogenous probability. Both labor income and profit income are subject to a proportional income tax. Second, we assume that households can hold two different types of assets, liquid government bonds, and illiquid capital. Capital holdings are illiquid because we assume that only a random share of households can trade capital in a given period. Third, we assume that in Foreign, which will be calibrated to Germany, households will receive a minimum income benefit which we model as a targeted transfer which those households receive whose income is below a certain threshold. As Bayer et al. (2022) show in detail, large differences in minimum income benefits across Germany and Italy can explain a large part of the differences in the wealth distribution,[4] and, as a result, requires large differences in government debt (high in Italy and low in Germany) in order to obtain the same real interest in both countries in steady state.

Firm sector. We also extend the firm sector by assuming that not only wages but also prices are adjusted infrequently. To this end, we assume a multi-layered production structure. Intermediate goods producers operate under perfect competition using both domestic capital and labor which we assume to be immobile across countries. We also assume that production is subject to national total factor productivity (TFP). Final good producers, in turn, differentiate domestic intermediate goods under monopolistic competition and subject to Calvo (1983)-type price setting frictions in Home and Foreign. Domestically and imported goods are then bundled into consumer goods as in Section 2. Capital producers also use intermediate goods and face quadratic investment adjustment costs.

Fiscal policy. Lastly, we also consider a somewhat richer set of fiscal policies. First, the government in Foreign has to fund the minimum income benefits. Second, we now also

[4] Pham-Dào (2016) shows that this is the case more general across Eurozone countries. However, she uses a single-asset incomplete markets model

consider government spending. This will allow us to analyze how a government spending shock plays out, both under flexible exchange rates and in the monetary union.

Shocks. In what follows we focus on TFP shocks and government spending shocks which may originate either in Home or Foreign. We assume all shocks to follow an exogenous AR(1)-process.

4.2 Symmetric Calibration: Propositions 1 - 3 still hold exactly

The medium-sized HANK^2 model outlined here features a richer structure in order to better capture key features of the data, both at the micro and the macro level. Before calibrating the model to the EA, we verify that the results established by Proposition 1 - 3 still hold exactly once we simulate a perfectly *symmetric* version of the model. For this purpose, we pick parameter values for both Home and Foreign in line with the "Italy calibration" below. We provide detailed results in Appendix D. We study, in particular, the transmission of TFP shocks at the country level and at the union level. We find that monetary union alters the effects of a country-specific shock at the country level very much. Yet, while there is "too little" adjustment in one country and "too much" in the other these effects also offset each other completely in the larger model—in line with Proposition 1: The response of union-wide prices and quantities to country-specific shocks is independent of whether the countries operate in a monetary union or not. Consequently, the arguments in the aggregate consumption function of union-wide twins are the same (Proposition 2), implying that redistribution takes place only horizontally within the brackets of the wealth and income distribution (Proposition 3). To the extent that our results below differ from those stated in Propositions 1 - 3, this thus reflects the asymmetric calibration which in turn captures the differences between the German and the Italian wealth distribution.

4.3 Asymmetric Calibration to the euro area

We outline how we calibrate the model to the EA and refer readers to Appendix C for more details. Importantly, we now allow countries to differ not only in terms of shocks but also in terms of heterogeneity at the household level, in line with the data for Germany and Italy. For this purpose, we set parameters to target the wealth distributions and asset holdings in both countries.

For most parameters, we use standard values (see Appendix C). We specify the parameters that determine the income process at the household level to match micro-level estimates for German and Italian data. In particular, we set the persistence of idiosyncratic income

Table 4.1: Calibrated Model v Data

			Model		Data	
			H	**F**	**ITA**	**GER**
Steady state	Assets	Debt (% of output)	132	71	132	71
		Capital-Output-Ratio	3.3	3.2	3.3	3.2
(targeted)	Distribution	Wealth gini	0.60	0.72	0.61	0.73
		Top-10% wealth share	0.43	0.55	0.44	0.52
		Bottom-50% wealth share	0.10	0.01	0.09	0.02
		Borrowers	0.08	0.18	0.08	0.18
Business Cycle	Volatility	Std(Y)*100 (targeted)	3.78	2.90	3.78	2.74
		Std(C)/Std(Y)	0.95	0.86	0.95	0.90
		Std(I)/Std(Y)	2.52	3.00	1.82	1.60
		Std(π)/Std(Y)	0.63	0.67	0.33	0.40
	Co-Movement	Corr(Y, Y^*) (targeted)	0.80		0.80	
		Corr(C, C^*)	0.95		0.49	
		Corr(I, I^*)	0.89		0.33	
		Corr(π, π^*)	0.97		0.77	

Notes: Model predictions based on baseline calibration, see Appendix C for details. Micro data based on the 2017 wave of the Household Finance and Consumption survey of the ECB. Macro data from Eurostat and Worldbank (Inflation). Quantities are measured in real per capita terms, yoy changes; sample: 1999Q1-2022Q2.

shocks, to a standard value found for the euro area, see for example Pham-Dào (2016); and set the respective standard deviations as to match income inequality in Italy and in Germany. Moreover, we model that Germany (Foreign) pays minimum income benefits in line with German data. The overall size of these transfers represents 1% of GDP. There are no minimum income benefits in Home which represents Italy. We then use six parameters to target key features of the wealth distributions and asset holdings in Germany and Italy. In particular, we use the discount factor, the portfolio adjustment probability, the probability which governs the transition of households to become entrepreneurs, and the borrowing penalty to match the level of government debt, the capital-to-output ratio, the wealth Gini, the top-10% wealth share, the bottom-50% wealth share, and the mass of borrowers.

Table 4.1 shows the implications of the model for the steady state in the top panel. It compares the model predictions and the empirical counterparts that serve as calibration targets. Note that the model is able to generate the observed large asymmetry between both countries: The wealth distribution is much more unequal in Germany compared to Italy, while government debt is considerably higher in Italy. As explained in detail in Bayer et al. (2022), from the perspective of the model these two aspects are interrelated and can be explained by the higher need for self-insurance in the absence of minimum income benefits. For what follows, we note that the degree of asymmetry between both countries along these dimensions is among the largest in Europe (see also Pham-Dào (2016) and Kindermann and Kohls (2017)).

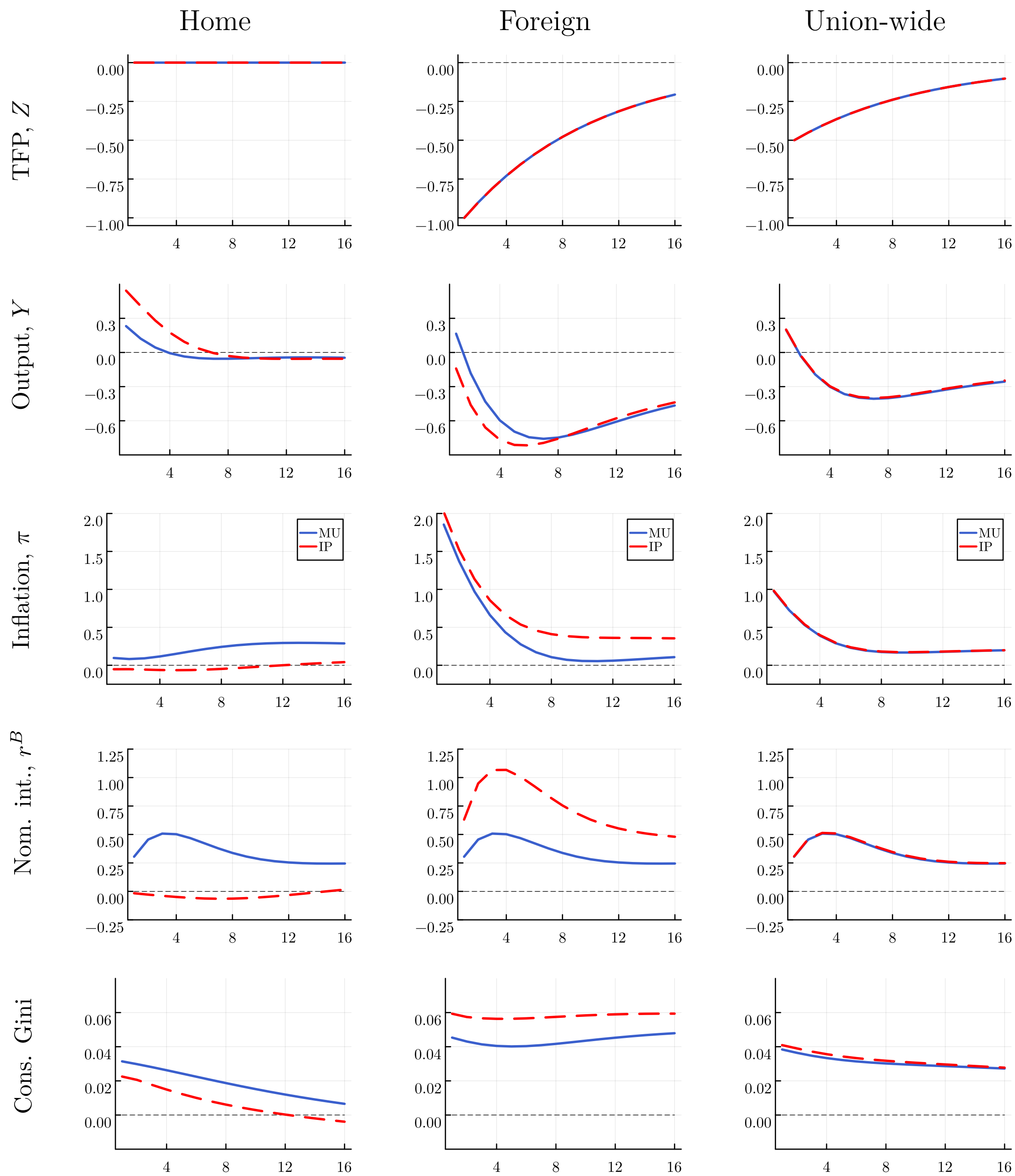
At the same time, the model is able to capture key features of the business cycle. For this purpose, we set parameters in line with estimates of Bayer et al. (2020). Note that for the baseline we assume a common monetary policy (monetary union) and assume that the exchange rate is permanently fixed. Monetary policy is described by an interest rate feedback rule with interest rate smoothing. For the open economy parameters, we rely on standard parameter values in the literature, see again Appendix C. We follow Enders et al. (2013) in specifying country-specific government spending shocks and TFP shocks. We also add a common TFP shock since, otherwise, the model cannot account for the high degree of business-cycle comovement across both countries. We specify TFP shocks and the common component as we target output volatility in both countries and the co-movement of output across countries. The lower part of Table 4.1 shows that our model then does a fairly good job in matching other key statistics of the Euro area business cycle. In particular, the relative volatility of investment, consumption, and inflation is the right ballpark, as is the co-movement across countries.

4.4 The macroeconomic adjustment to country-specific shocks

We use the calibrated model to analyze the macroeconomic adjustments to country-specific shocks. In particular, we consider a TFP shock and a government spending shock that originates in Foreign. Even though the countries are no longer symmetric, a very similar pattern obtains for shocks that originate in Home. Hence we do not report results for this case to economize on space.

Figure 4.1 shows the responses of the national macroeconomic aggregates to a contractionary TFP shock that originates in Foreign as well as the union-wide aggregates. Throughout, we contrast results for the monetary union with those for independent monetary policies in Home (left column), in Foreign (middle column), and in the entire union by displaying the aggregate responses (right column). Recall that in case of independent monetary policies, the interest rate feedback rule is the same as in the monetary-union case, except that monetary policy in each country responds to country-level rather than union-wide inflation rates. The top panels show the shock process which is independent of whether there is monetary union or not: TFP in Foreign contracts; it is unchanged in Home. The second row shows the

Figure 4.1: Adjustment to adverse TFP shock originating in Foreign



Notes: monetary union v independent monetary policies in Home (left), in Foreign (middle), and aggregate of Home and Foreign (right). Y-axis: Percentage deviation from steady state and percentage points in case of interest rates. X-axis: Quarters.

adjustment of output. Here monetary union makes a fundamental difference. Output in Foreign *increases* on impact with a monetary union in place (solid blue line), but *decreases* under independent policies (dashed red line). Likewise, output in Home also responds very differently across monetary regimes: it increases much more under independent monetary policies.

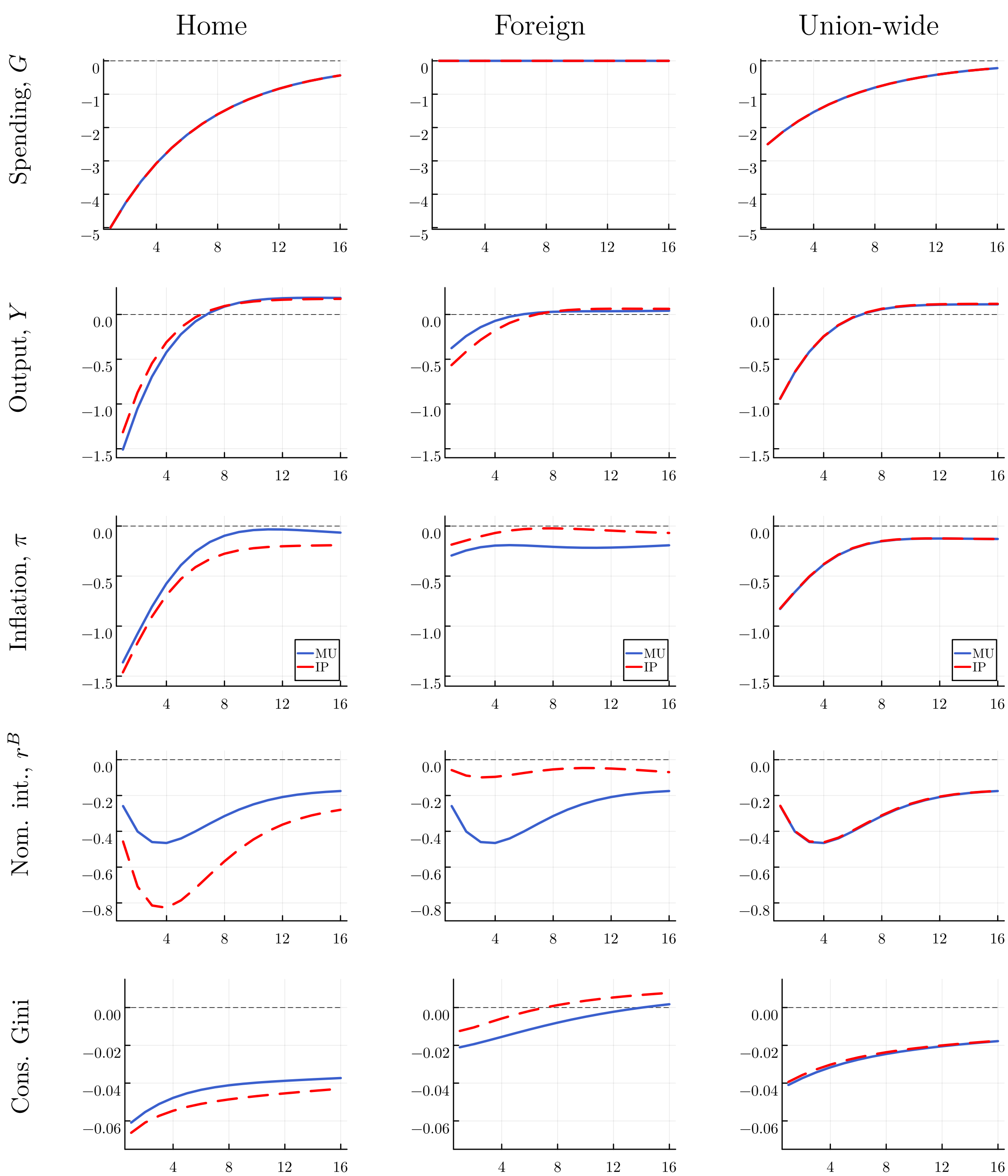
To rationalize these differences, it is instructive to study the adjustment of the policy rate, shown in the fourth row: in the monetary union it responds in the same way in both countries, while with independent monetary policies, we observe an increase in the short rate in Foreign and a decline in Home, reflecting, in turn, the differential impact of the shock on inflation in Foreign and Home which is shown in the third row: the contractionary TFP shock is strongly inflationary at Foreign, thus necessitating a monetary contraction, too. This effect is much weaker in Foreign once it operates in a monetary union. For Home it is the reverse: operating inside the monetary union implies a restrictive rather than an expansionary monetary policy. This is the one-size-doesn't-fit-all issue that is at the heart of the policy tensions in monetary unions. In our context, it even induces a change in the sign of the response of Foreign output in response to a TFP shock.

The right column of Figure 4.1 shows the aggregate response of Home and Foreign under monetary union and with independent monetary policies. For all variables, the two impulse responses lie almost perfectly on top of each other. This shows that even though the two countries now differ substantially in terms of household level heterogeneity, the result of Proposition 1 still holds approximately: monetary union does change the adjustments to country-specific shocks at the country level but it does so by shifting the adjustment between countries. The overall effect of a monetary union on the adjustment of union-wide aggregates is negligible.

Against this background, the bottom panels of the figure show the response of the consumption Gini. Consumption inequality increases after the shock in both countries and across both exchange rate regimes for reasons which become clear below. At this point it is important to point out that the response of aggregate consumption inequality (shown in the right column) is basically independent of the exchange rate regime—consistent with Proposition 2. Yet consumption inequality increases less at Foreign and more in Home in the monetary-union case, compared to what we observe under independent monetary policies.

Figure 4.2 depicts the adjustment to a government spending shock that originates in Foreign. It is structured in the same way as the previous figure. In particular, we show the shock process in the top panels. Spending goes up in Foreign only, reverting gradually to its steady state level. It is unchanged in Home. As with the transmission of the TPF shock, we observe that monetary union alters the dynamics within countries profoundly. In particular,

Figure 4.2: Adjustment to adverse government spending shock originating in Home



Notes: monetary union v independent monetary policies in Home (left), in Foreign (middle), and aggregate of Home and Foreign (right). Y-axis: Percentage deviation from steady state and percentage points in case of interest rates. X-axis: Quarters.

a monetary union amplifies the output and consumption response in Foreign but dampens it in Home. This again reflects the common monetary stance in the union, shown in the fourth row. Hence, a monetary union changes the macro adjustment at the country level by shifting the adjustment between countries. Consequently, the overall effect of monetary union on the adjustment of union-wide aggregates is close to zero which is again illustrated in the right column, in which the impulse responses lie almost perfectly on top of each other. We conclude that also for government spending shocks, the result of Proposition 1 still holds approximately even as well allow countries to be asymmetric.

4.5 Adjustment at the household level

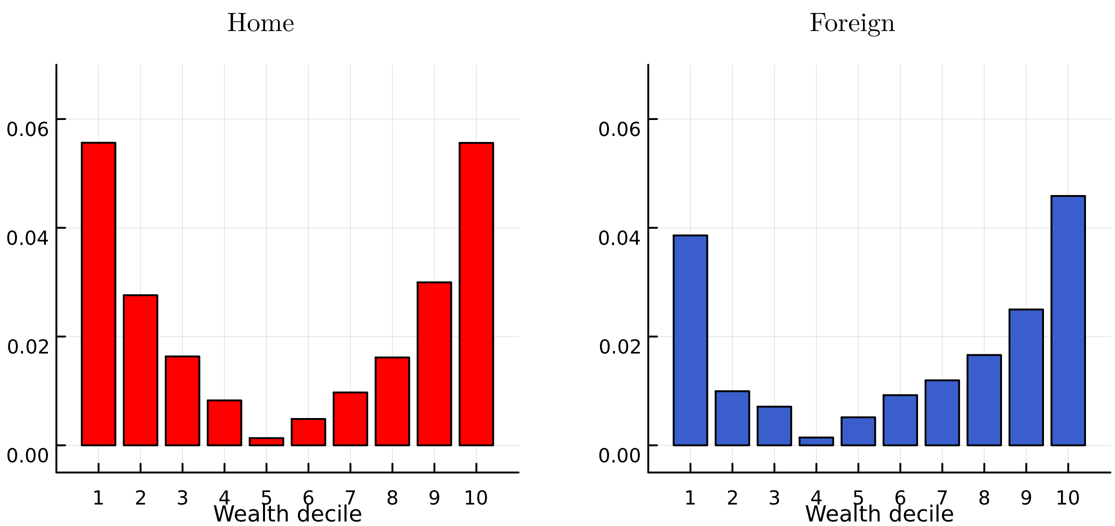
We are finally in a position to address the main question of the paper: How does monetary union alter the impact of country-specific shocks at the household level? Proposition 3 above establishes for the symmetric case that monetary union shifts the adjustment to country-specific shocks across borders. Specifically, monetary union shifts the adjustment at the household level horizontally across borders within the brackets of the wealth distribution. For the calibrated version of the model, we have shown that the results established in Propositions 1 and 2 approximately hold in the larger model even as it is calibrated to capture cross-country heterogeneity at the household level.

We now use this version of the model to quantify how monetary union alters the impact of shocks at the household level. Specifically, we compute the welfare impact of a shock for each household across the wealth distribution and contrast results for the monetary union with those under independent monetary policies. We measure the welfare impact with the consumption equivalent variation, that is, the permanent consumption change which makes an individual household just as well off as does the shock under consideration. We stress upfront that we take an ex-post perspective: We evaluate welfare conditional on specific shocks (that is, one-sided welfare) rather than providing an ex-ante welfare analysis based on a second-order approximation of the utility function.

We compute how the consumption equivalent variation changes for each decline of the wealth distribution as countries move from independent monetary policy to monetary union. We report the absolute value of the change in the consumption equivalent variation in order to remain agnostic about the sign of the shock. Rather our measure is meant to capture for which bracket of the wealth distribution the welfare impact of a shock changes most strongly as countries form a monetary union rather than running independent monetary policies.

Figure 4.3 shows the results. It shows the welfare differences between the monetary-union case and the independent-polices case for a TFP shock that originates in Foreign. Since it

Figure 4.3: How monetary union alters the welfare impact of shocks

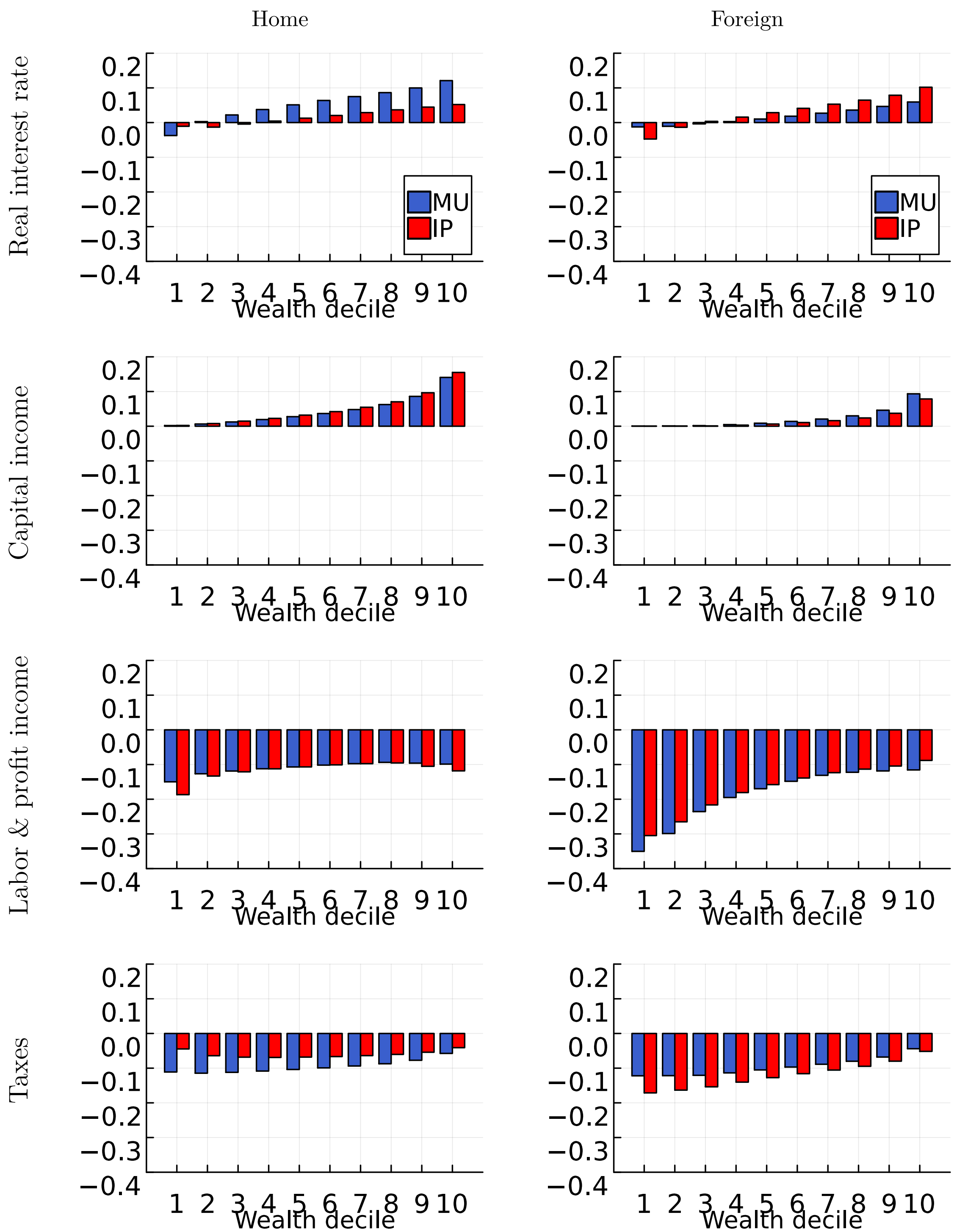


Notes: Difference of welfare impact of a Foreign TFP shock (in absolute value) between monetary union and independent monetary policies in Home (left) and Foreign (right). Y-axis: Difference in consumption compensation (in absolute values). X-axis: Wealth percentiles.

displays the absolute value of the change, it holds for both, positive and negative shocks. The left panel depicts the effect along the deciles of the wealth distribution in Home, while the right panel depicts the effects in Foreign. The emerging pattern is clear-cut and warrants two observations. First, the pattern is consistent with the result of Proposition 3 according to which monetary union shifts the impact of shocks across borders within the brackets of the wealth distribution. For the calibrated model this does not hold exactly because we relax the assumption of country symmetry. The patterns in Home and Foreign are not perfectly identical but they are quite similar: While the vertical redistribution induced by a monetary union is not zero anymore in the asymmetric model, it is still quantitatively small and, in particular, much smaller than the horizontal redistribution.[5] Second, we find that the change in the impact of shocks on welfare due to monetary union is very much concentrated in the tails of the wealth distribution. The middle class is much less affected. Put differently, whether there is a monetary union in place or not matters for the shock's welfare impact, but only for the poor and the rich. This holds—again consistent with Proposition 3—for both, Home and Foreign. A very similar pattern obtains for government spending shocks which we do not report here to economize on space.

[5]Figure D.2 in the appendix reports results for the symmetric calibration: in this case, the patterns in both countries are also perfectly symmetric.

Figure 4.4: Decomposition of welfare effect of TFP shock in Foreign



Notes: Decomposition of welfare effects of a contractionary TFP shock in Foreign across the deciles of the wealth distribution. Monetary union (blue) vs Independent monetary policies (red).

To see what gives rise to the U-shaped welfare changes we decompose the welfare impact of a contractionary TFP shock across the deciles of the wealth distribution into partial equilibrium effects, contrasting what happens in the case of a monetary union and under independent monetary policies. How the welfare of a given household is affected by a TFP shock depends on the arguments that enter the household's policy functions. By changing the adjustment of these at the country level, a monetary union changes the effect of a shock on households' welfare.

Figure 4.4 shows the decomposition of the welfare effect into changes due to, in turn, the real rate on liquid bonds, capital income, labor and profit income, and taxes. The blue bars represent the case of a monetary union and the red bars the case of independent monetary policies. The left column shows the contribution to the overall welfare effect in Home and the right column for Foreign (the country where the shock originates). We observe that monetary union has a strong bearing on the welfare impact by changing the way the real interest rate (top panel) and taxes (bottom panel) respond to the shock, and more so than for labor and capital income (middle panels). This is intuitive because—as discussed above—monetary union changes the interest rate response to a country-specific TFP shock. Changes in the interest rate then impact governments' budgets by altering the interest rate burden on the outstanding debt which, ultimately, results in adjustments of the tax rate. The way in which the different adjustments of the real interest rate and the taxes affect households' welfare is highly heterogeneous along the wealth distribution. But we observe that the changes due to monetary union are largest at the tails of the wealth distribution—reflecting a different interest rate exposure. High-wealth households are directly exposed through their assets, low wealth agents through the wage and tax response. A negative TPF shock, for instance, raises interest rates and hence the return on the liquid asset.[6] At the same time, wages fall and labor income taxes rise. This benefits the asset-rich and hurts the asset-poor. The monetary union changes the size of these price responses. As a result, the impact of the shock depends a lot on whether there is a monetary union in place or not.

Figure 4.4 also illustrates why the middle class is largely unaffected by a monetary union. In contrast to the low-wealth or high-wealth households, for the middle class the real rate effects on the one hand, and tax and wage effects on the other hand, largely offset each other. This is intuitive. These households have a very balanced portfolio of human, financial, and real capital.

Our result might explain why the European Monetary Union did not break up in the

[6] The baseline version of the model features short-term debt only. To account for possible valuation effects, we consider an extended version of the model which allows for long-term debt, see Appendix E. We find our main results hold also in this case.

face of sizeable asymmetric shocks during its 20-year-plus history. In every country, those that benefit from the (union-wide) monetary response can always form a sufficiently large coalition with the middle class to support the union, as long as there is a small (and here non-modeled) cost of breaking up the union. Focusing on how monetary union alters the welfare impact of business cycle shocks thus offers new insights into the political economy of monetary unions, an issue that calls for further research.

5 Conclusion

Asymmetric shocks are a classic theme of OCA theory. They bring to the fore the one-size-doesn't-fit-all problem from which monetary unions are bound to suffer at times. We revisit the issue through the lens of a Heterogeneous Agent New Keynesian model with two countries: HANK^2. It belongs to a class of models that breaks with the representative agent paradigm and thus opens up new perspectives. In particular, in contrast to earlier generations of OCA theory, we are no longer confined to analyzing what membership in a monetary union means for countries or regions.

Instead, we investigate how monetary union alters the implications of shocks for individual households along the wealth distribution. In particular, the HANK^2 structure allows us to distinguish how monetary union alters the impact of shocks horizontally across borders within the brackets of the union-wide wealth distribution and vertically across the wealth brackets. A key result of our analysis is that a monetary union shifts the adjustment to shocks horizontally and not so much vertically. In fact, we can show in closed form that a monetary union neither changes the union-wide dynamics after a country-specific shock nor vertical impact across the brackets of the union-wide wealth distribution. Instead, it shifts the burden horizontally across borders within the brackets of the wealth distribution. Thus, for a given shock, being in a monetary union benefits the poor (rich) in one country at the expense of the poor (rich) in the other country.

Our quantitative analysis shows that this effect is particularly strong for the tails of the wealth distribution and weaker for the middle class. This brings to the fore questions about the political economy of currency unions which we take up in a companion paper (Bayer et al. 2022). Here we just note that our results may provide a rationale for why the EA did not break up during its 20-year-plus history despite several severe crises and calls for an exit of individual countries: in the face of a specific shock (or crisis), whether a country operates inside a monetary union or not does not matter so much for a large fraction of the population.

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A Derivations and Proofs

A.1 Deriving the Canonical Form

We now derive the canonical form of the model, that is the model in terms of union-wide variables. To this end, we define union-wide variables, as e.g., union-wide GDP, as $\widehat{X_t^W} = n\widehat{X_t} + (1-n)\widehat{X_t^*}$.

Using the goods market clearing conditions, we have,

$$\widehat{y_t^W} = n\widehat{C_{H,t}} + (1-n)\widehat{C_{F,t}} = \widehat{C_t^W}. \tag{29}$$

From national aggregate consumption functions to union-wide IS equation. The aggregate consumption functions of the countries are:

$$\begin{aligned} \boldsymbol{c} &= \mathcal{C}(\boldsymbol{w}, \boldsymbol{N}, \boldsymbol{i}, \boldsymbol{\pi}, \boldsymbol{\tau}) \\ \boldsymbol{c}^* &= \mathcal{C}^*(\boldsymbol{w}^*, \boldsymbol{N}^*, \boldsymbol{i}^*, \boldsymbol{\pi}^*, \boldsymbol{\tau}^*) \end{aligned} \tag{30}$$

Linearizing these consumption functions around the deterministic steady state:

$$\begin{aligned} \widehat{\boldsymbol{c}} &= \mathcal{C}_w \widehat{\boldsymbol{w}} + \mathcal{C}_N \widehat{\boldsymbol{N}} + \mathcal{C}_i \widehat{\boldsymbol{i}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}} \\ \widehat{\boldsymbol{c}^*} &= \mathcal{C}^*_{w^*} \widehat{\boldsymbol{w}^*} + \mathcal{C}^*_{N^*} \widehat{\boldsymbol{N}^*} + \mathcal{C}^*{}_{i^*} \widehat{\boldsymbol{i}^*} + \mathcal{C}^*{}_{\pi^*} \widehat{\boldsymbol{\pi}^*} + \mathcal{C}^*{}_{\tau^*} \widehat{\boldsymbol{\tau}^*}. \end{aligned} \tag{31}$$

Given symmetric countries, we have $\mathcal{C}(\boldsymbol{w}, \boldsymbol{N}, \boldsymbol{i}, \boldsymbol{\pi}, \boldsymbol{\tau}) = \mathcal{C}^*(\boldsymbol{w}^*, \boldsymbol{N}^*, \boldsymbol{i}^*, \boldsymbol{\pi}^*, \boldsymbol{\tau}^*)$ and thus, $(\mathcal{C}_w, \mathcal{C}_N, \mathcal{C}_i, \mathcal{C}_\pi, \mathcal{C}_\tau) = (\mathcal{C}^*{}_{w^*}, \mathcal{C}^*{}_{N^*}, \mathcal{C}^*{}_{i^*}, \mathcal{C}^*{}_{\pi^*}, \mathcal{C}^*{}_{\tau^*})$. Using this, we can write world consumption as:

$$\begin{aligned} \widehat{\boldsymbol{c}^{\boldsymbol{W}}} &= n\widehat{\boldsymbol{c}} + (1-n)\widehat{\boldsymbol{c}^*} \\ &= \mathcal{C}_w(n\widehat{\boldsymbol{w}} + (1-n)\widehat{\boldsymbol{w}^*}) + \mathcal{C}_N(n\widehat{\boldsymbol{N}} + (1-n)\widehat{\boldsymbol{N}^*}) \\ &\quad + \mathcal{C}_i(n\widehat{\boldsymbol{i}} + (1-n)\widehat{\boldsymbol{i}^*}) + \mathcal{C}_\pi(n\widehat{\boldsymbol{\pi}} + (1-n)\widehat{\boldsymbol{\pi}^*}) + \mathcal{C}_\tau(n\widehat{\boldsymbol{\tau}} + (1-n)\widehat{\boldsymbol{\tau}^*}) \\ &= \mathcal{C}_w \widehat{\boldsymbol{w}^{\boldsymbol{W}}} + \mathcal{C}_N \widehat{\boldsymbol{N}^{\boldsymbol{W}}} + \mathcal{C}_i \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}^{\boldsymbol{W}}} \end{aligned} \tag{32}$$

Using the linearized version of (8)

$$\hat{w}_t^H = -\alpha_H(1-n)\hat{s}_t^H$$

and its Foreign country counterpart

$$\hat{w}_t^F = -\alpha_H n \hat{s}_t^F$$

it follows that $\widehat{\boldsymbol{w}^{\boldsymbol{W}}} = n\widehat{\boldsymbol{w}^{\boldsymbol{H}}} + (1-n)\widehat{\boldsymbol{w}^{\boldsymbol{F}}} = 0$ because for terms of trade, s, $\widehat{\boldsymbol{s}}^H = -\widehat{\boldsymbol{s}}^F$ holds. Using this and $\widehat{\boldsymbol{c}^{\boldsymbol{W}}} = \widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \widehat{\boldsymbol{N}^{\boldsymbol{W}}}$ (see (29) and (7)) and writing $\mathcal{C}_y = \mathcal{C}_N$, we obtain:

$$\widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \mathcal{C}_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \mathcal{C}_i \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}^{\boldsymbol{W}}} \tag{33}$$

Furthermore, we can use national government budget constraints and tax feedback functions to solve for taxes:

$$\begin{aligned}
\widehat{\tau}_{t+1} &= ((1+\bar{i}) - \gamma\frac{\bar{\tau}}{b_y})\widehat{\tau}_t + \bar{i}\widehat{i}_t - (1+\bar{i})\widehat{\pi}_{t+1} - \gamma\frac{\bar{\tau}}{b_y}\widehat{y}_t \\
\widehat{\tau^*}_{t+1} &= ((1+\bar{i}) - \gamma\frac{\bar{\tau}}{b_y})\widehat{\tau}_t + \bar{i}\widehat{i^*}_t - (1+\bar{i})\widehat{\pi^*}_{t+1} - \gamma\frac{\bar{\tau}}{b_y}\widehat{y^*}_t,
\end{aligned} \tag{34}$$

and aggregating gives:

$$\widehat{\tau^W}_{t+1} = ((1+\bar{i}) - \gamma\frac{\bar{\tau}}{b_y})\widehat{\tau}_t + \bar{i}\widehat{i^W}_t - (1+\bar{i})\widehat{\pi^W}_{t+1} - \gamma\frac{\bar{\tau}}{b_y}\widehat{y^W}_t \tag{35}$$

Hence, we can stack taxes as:

$$\begin{aligned}
\widehat{\boldsymbol{\tau}} = \tau(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\pi}}),\ \widehat{\boldsymbol{\tau}^*} &= \tau(\widehat{\boldsymbol{y}^*}, \widehat{\boldsymbol{i}^*}, \widehat{\boldsymbol{\pi}^*}) \\
\widehat{\boldsymbol{\tau}^{\boldsymbol{W}}} &= \tau(\widehat{\boldsymbol{y}^{\boldsymbol{W}}}, \widehat{\boldsymbol{i}^{\boldsymbol{W}}}, \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}})
\end{aligned} \tag{36}$$

Using this, we can write:

$$\widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \mathcal{C}_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \mathcal{C}_i \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \mathcal{C}_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} + \mathcal{C}_\tau \widehat{\boldsymbol{\tau}^{\boldsymbol{W}}}(\widehat{\boldsymbol{y}^{\boldsymbol{W}}}, \widehat{\boldsymbol{i}^{\boldsymbol{W}}}, \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}}) \tag{37}$$

and:

$$\widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \underbrace{[\mathcal{C}_y + \mathcal{C}_\tau \mathcal{T}_y]}_{\tilde{\mathcal{C}}_y} \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \underbrace{[\mathcal{C}_i + \mathcal{C}_\tau \mathcal{T}_i]}_{\tilde{\mathcal{C}}_i} \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \underbrace{[\mathcal{C}_\pi + \mathcal{C}_\tau \mathcal{T}_\pi]}_{\tilde{\mathcal{C}}_\pi} \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} \tag{38}$$

where $\mathcal{T}_y$, $\mathcal{T}_r$ and $\mathcal{T}_\pi$ are derivative matrices for the maps $\widehat{\boldsymbol{\tau}}(\widehat{\boldsymbol{y}}, \widehat{\boldsymbol{i}}, \widehat{\boldsymbol{\pi}})$.

Importantly, the world IS equation (38) is potentially only affected by different exchange rate regimes through its effects on the world interest rate.

Philips Curve. Using the consumer price indexes and the law of one prices, one obtains:

$$\pi_t^W = n\pi_{H,t} + (1-n)\pi_{F,t} \tag{39}$$

Hence, world CPI inflation is just the weighted average of domestic producer price inflation. Using this and aggregating the two national Philips curves gives the union-wide Philips curve:

$$\pi_t^W = \beta \pi_{t+1}^W + \kappa^W \hat{y_t}^W, \tag{40}$$

with $\kappa^W = \kappa(\phi + \gamma)$.

Stacking the World Phillips Curve yields:

$$\Pi_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} = \Pi_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}}, \tag{41}$$

where

$$\Pi_\pi = \begin{pmatrix} 1 & -\beta & 0 & ... \\ 0 & 1 & -\beta & ... \\ 0 & 0 & 1 & ... \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \Pi_y = \kappa^W \omega I, \Pi_y = \kappa^W \sigma^{-1} I \tag{42}$$

Lemma 1. *In a two-country model of symmetric countries, we can describe the canonical system for world variables by the following two linear mappings:*

$$\Pi_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} = \Pi_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}}, \tag{43}$$

$$\widehat{\boldsymbol{y}^{\boldsymbol{W}}} = \tilde{\mathcal{C}}_y \widehat{\boldsymbol{y}^{\boldsymbol{W}}} + \tilde{\mathcal{C}}_i \widehat{\boldsymbol{i}^{\boldsymbol{W}}} + \tilde{\mathcal{C}}_\pi \widehat{\boldsymbol{\pi}^{\boldsymbol{W}}} \tag{44}$$

and a function for the nominal interest rate, i_t^W.

Note in particular that different exchange rate regimes can only potentially influence world variables through their effect on the world interest rates.

B The medium-sized HANK2

Our medium-sized HANK^2 model extends our baseline model in Section 2 such that its structure in each country mimics the closed-economy set up of Bayer et al. (2020), except for the fact that there is trade across the two countries, both in goods and financial markets. A brief overview of the extensions that we made can be found in Section 4. Here, we present the entire model from scratch.

Each country consists of a firm sector and a household sector. The firm sector of each country comprises (a) perfectly competitive intermediate goods producers who rent out labor services and capital on national labor and a national capital market, respectively; (b) final goods producers that face monopolistic competition when selling differentiated final goods, in turn, produced on the basis of homogeneous domestic intermediate inputs; (c) a representative consumption good bundler bundling domestic and imported foreign final goods to consumption goods; (d) producers of capital goods that turn consumption goods into capital subject to adjustment costs; (e) labor packers that produce labor services combining differentiated labor from (f) unions that differentiate raw labor rented out from households. Price setting for the final goods, as well as wage setting by unions, is subject to a pricing friction à la Calvo (1983). Only final goods are traded across countries.

In each country, there is a continuum of households of size $n \in (0, 1)$ and $1-n$, respectively, such that the total population is 1. Households in both countries consume a bundle that consists of domestically produced and imported goods. Households earn income from supplying (raw) labor and capital to the national labor and the national capital markets and from owning firms in their respective country. Households absorb all rents that stem from the market power of unions and final good producers, and decreasing returns to scale in capital goods production.

In the baseline, there is a common monetary authority and the exchange rate is permanently fixed. Fiscal policy is run at the country level. It levies taxes on labor income and profits, issues bonds, and adjusts taxes to stabilize the level of outstanding debt in the long run. Public debt is risk-free and thus yields the same return in both countries, in turn, determined by monetary policy by means of a simple interest rate feedback rule. We assume that countries are perfectly symmetric and differ only because of asymmetric shocks and different parameterizations. In what follows, our exposition thus focuses on the domestic economy and uses an asterisk to denote foreign variables whenever they are relevant.

B.1 Households

The household sector is subdivided into two types of agents: workers and entrepreneurs. The transition between both types is stochastic. Both rent out physical capital, but only workers supply labor. The efficiency of a worker's labor evolves randomly exposing worker-households to labor-income risk. Entrepreneurs do not work but earn all pure rents in the economy except for the rents of unions which are equally distributed across workers. All households self-insure against the income risks they face by saving in a liquid nominal asset (bonds) and a less liquid asset (capital). Trading illiquid assets is subject to random participation in the capital market. To be specific, there is a continuum of ex-ante identical households of measure n, indexed by i. Households are infinitely lived, have time-separable preferences with time discount factor β, and derive felicity from consumption c_{it} and leisure. They obtain income from supplying labor, n_{it}, from renting out capital, k_{it}, and from earning interest on bonds, b_{it}, and potentially from profits or union transfers. Households pay taxes on labor and profit income.

B.1.1 Productivity, labor supply, and labor income

A household's gross labor income $w_t n_{it} h_{it}$ is composed of the aggregate wage rate on raw labor, w_t, the household's hours worked, n_{it}, and its idiosyncratic labor productivity, h_{it}. We assume that productivity evolves according to a log-AR(1) process with time-varying volatility and a fixed probability of transition between the worker and the entrepreneur state:

$$\tilde{h}_{it} = \begin{cases} \exp(\rho_h \log \tilde{h}_{it-1} + \epsilon^h_{it}) & \text{with probability } 1-\zeta \text{ if } h_{it-1} \neq 0, \\ 1 & \text{with probability } \iota \text{ if } h_{it-1} = 0, \\ 0 & \text{else.} \end{cases} \tag{45}$$

with individual productivity $h_{it} = \frac{\tilde{h}_{it}}{\int \tilde{h}_{it} di}$ such that $\tilde{h}_{it}$ is scaled by its cross-sectional average, $\tilde{h}_{it} di$, to make sure that average worker productivity is constant. The shocks ϵ^h_{it} to productivity are normally distributed with variance $\sigma^2_{h,t}$. With probability ζ households become entrepreneurs ($h = 0$). With probability ι an entrepreneur returns to the labor force with median productivity. In our baseline specification, an entrepreneur obtains a share of the pure rents (aside from union rents), Π^F_t, in the economy (from monopolistic competition in the goods sector and the creation of capital). We assume that the claim to the pure rent cannot be traded as an asset. Union rents, Π^U_t are distributed lump sum across workers, leading to labor-income compression. For tractability, we assume union profits to be taxed at a fixed rate independent of the recipient's labor income.[7]

With respect to leisure and consumption, households have Greenwood et al. (1988) (GHH) preferences and maximize the discounted sum of felicity:

$$E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u[c_{it} - G(h_{it}, n_{it})] \tag{46}$$

The maximization is subject to the budget constraints described further below. The felicity function u exhibits a constant relative risk aversion (CRRA) with risk aversion parameter $\xi > 0$,

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \tag{47}$$

where $x_{it} = c_{it} - G(h_{it}, n_{it})$ is household i's composite demand for goods consumption c_{it} and leisure and G measures the dis-utility from work. The consumption good c is a bundle of domestic and imported foreign final goods as described in Section B.2.2.

The household's labor income gets taxed at rate τ_t, such that its net labor income is given by

$$(1 - \tau_t) w_t h_{it} n_{it}, \tag{48}$$

where w_t is the aggregate wage rate. Given net labor income, the first-order condition for labor supply is

$$\frac{\partial G(h_{it}, n_{it})}{\partial n_{it}} = (1 - \tau_t) w_t h_{it} = \frac{y_{it}}{n_{it}}. \tag{49}$$

Assuming that G has a constant elasticity w.r.t. n, $\frac{\partial G(h_{it}, n_{it})}{n_{it}} = (1+\gamma)\frac{G(h_{it}, n_{it})}{n_{it}}$ with $\gamma > 0$, we can simplify the expression for the composite consumption good, x_{it}, making use of this first-order condition (49), and substitute $G(h_{it}, n_{it})$ out of the individual planning problem:

$$x_{it} = c_{it} - G(h_{it}, n_{it}) = c_{it} - \frac{1}{1+\gamma} y_{it}. \tag{50}$$

When the Frisch elasticity of labor supply is constant and the tax schedule has the form (48), the dis-utility of labor is always a fraction of labor income and constant across households.

[7] This modeling strategy serves two purposes. First and foremost, it generally solves the problem of the allocation of pure rents without distorting factor returns and without introducing another tradable asset. Second, we use the entrepreneur state in particular – a transitory state in which incomes are very high – to match the income and wealth distribution following the idea by Castaneda et al. (1998). The entrepreneur state does not change the asset returns or investment opportunities available to households.

Therefore, in both the household's budget constraint and felicity function, only after-tax income enters and neither hours worked nor productivity appears separately.

What remains to be determined is individual and aggregate effective labor supply. Without further loss of generality, we assume $G(h_{it}, n_{it}) = h_{it}\frac{n_{it}^{1+\gamma}}{1+\gamma}$. This functional form simplifies the household problem in the stationary equilibrium as h_{it} drops out from the first-order condition and all households supply the same number of hours $n_{it} = N(w_t)$. Total effective labor input, $\int n_{it}h_{it}di$, is hence also equal to $N(w_t)$ because we normalized $\int h_{it}di = 1$.[8]

Households also receive profit income from union profits Π_t^U or firms profits Π_t^{fi} as workers or entrepreneurs, respectively. Both profits get taxed at rate τ_t. What is more, households may receive *non-distortionary* targeted transfer as minimum income benefits tr_{it}. All together, after-tax non-capital income, plugging in the optimal supply of hours, is then:

$$y_{it} = [(1-\tau_t)w_t]^{\frac{1+\gamma}{\gamma}} h_{it} + \mathbb{I}_{h_{it}\neq 0}(1-\tau_t)\Pi_t^U + \mathbb{I}_{h_{it}=0}(1-\tau_t)\Pi_t^{fi} + tr_{it}. \tag{51}$$

B.1.2 Consumption, savings, and portfolio choice

Given this labor income, households optimize inter-temporally subject to their budget constraint:

$$c_{it} + b_{it+1} + q_t k_{it+1} = y_{it} + b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t^{CPI}} + (q_t + r_t)k_{it}k_{it+1} \geq 0,\ b_{it+1} \geq \underline{B} \tag{52}$$

b_{it} is real bond holdings, k_{it} is the amount of illiquid assets, q_t is the price of these assets, r_t is their dividend, $\pi_t^{CPI} = \frac{P_t}{P_{t-1}}$ is realized domestic CPI inflation, and R is the gross nominal interest rate on bonds, which depends on the portfolio position of the household and the central bank's interest rate R_t^b, which is set one period before.

All households that do not participate in the capital market ($k_{it+1} = k_{it}$) still obtain dividends and can adjust their bond holdings. Depreciated capital has to be replaced for maintenance, such that the dividend, r_t, is the net return on capital. Holdings of bonds have to be above an exogenous debt limit $\underline{B}$; and holdings of capital have to be non-negative.

Substituting the expression $c_{it} = x_{it} + \frac{1}{1+\gamma}[(1-\tau_t)w_t]^{\frac{1+\gamma}{\gamma}} h_{it}$ for consumption, we obtain the budget constraint for the composite leisure-consumption good:

$$x_{it} + b_{it+1} + q_t k_{it+1} = b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t} + (q_t + r_t)k_{it} + z_{it}, \quad k_{it+1} \geq 0,\ b_{it+1} \geq \underline{B}, \tag{53}$$

[8] This means that we can read off average productivity risk from the estimated income risk series in the literature. Without scaling the labor dis-utility by productivity, we would need to translate productivity risk to income risk through the endogenous hour response.

where $z_{it} = \frac{\gamma}{1+\gamma}\left[(1-\tau_t)w_t\right]^{\frac{1+\gamma}{\gamma}} h_{it} + \mathbb{I}_{h_{it}\neq 0}(1-\tau_t)\Pi_t^U + \mathbb{I}_{h_{it}=0}(1-\tau_t)\Pi_t^{fi} + tr_{it}$ is income corrected for the dis-utility of labor.

Households make their savings choices and their portfolio choice between liquid bonds and illiquid capital in light of a capital market friction that renders capital illiquid because participation in the capital market is random and i.i.d. in the sense that only a fraction, λ, of households are selected to be able to adjust their capital holdings in a given period. This means that we specify:

$$R(b_{it}, R_t^b) = \begin{cases} R_t^b & \text{if } b_{it} \geq 0 \\ R_t^b + \bar{R} & \text{if } b_{it} < 0 \end{cases}. \tag{54}$$

The extra wedge for unsecured borrowing, $\bar{R}$, creates a mass of households with zero unsecured credit but with the possibility to borrow, though at a penalty rate.

Since a household's saving decision—–(b'_a, k') for the case of adjustment and (b'_n, k') for non-adjustment—will be some non-linear function of that household's wealth and productivity, inflation and all other prices will be functions of the domestic joint distribution, Θ_t, of (b, k, h) in t and the foreign joint distribution, Θ_t^*. This makes Θ and Θ^* state variables of the household's planning problem and these distributions evolve as a result of the economy's reaction to aggregate shocks. For simplicity, we summarize all effects of aggregate state variables, including the distributions of wealth and income, by writing the dynamic planning problem with time-dependent continuation values.

This leaves us with three functions that characterize the household's problem: value function V^a for the case where the household adjusts its capital holdings, the function V^n for the case in which it does not adjust, and the expected continuation value, $\mathbb{W}$, over both:

$$\begin{aligned} V_t^a(b, k, h) &= \max_{k', b'_a} u[x(b, b'_a, k, k', h] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_a, k', h) \\ V_t^n(b, k, h) &= \max_{b'_n} u[x(b, b'_n, k, k, h] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_n, k, h) \\ \mathbb{W}_{t+1}(b', k', h) &= \lambda V_{t+1}^a(b', k', h) + (1-\lambda) V_{t+1}^n(b', k, h). \end{aligned} \tag{55}$$

Expectations about the continuation value are taken with respect to all stochastic processes conditional on the current states. Maximization is subject to the corresponding budget constraint.

B.2 Firm sector

The firm sector of each country consists of five sub-sectors: (a) a labor sector composed of unions that differentiate raw labor and labor packers who buy differentiated labor and then sell labor services to intermediate goods producers, (b) intermediate goods producers who hire labor services and rent out capital to produce goods, (c) final goods producers who differentiate intermediate goods and then sell them to (d) goods bundlers who bundle them with foreign final goods and finally sell them as consumption goods to households and to (e) capital goods producers, who turn bundled goods into capital goods. None of these products and goods can be traded between both countries, except for the differentiated final goods.

When profit maximization decisions in the firm sector require inter-temporal decisions (i.e. in price and wage setting and in producing capital goods), we assume for tractability that they are delegated to a mass-zero group of households (managers) that are risk-neutral and compensated by a share in profits. They do not participate in any asset market and have the same discount factor as all other households. Since managers are a mass-zero group in the economy, their consumption does not show up in any resource constraint, and all but the unions' profits go to the entrepreneur households (whose $h = 0$). Union profits go lump-sum to worker households.

B.2.1 Labor packers and unions

Worker households sell their labor services to a mass-n_A continuum of unions indexed by j, each of whom offers a different variety of labor to labor packers who then provide labor services to intermediate goods producers. Labor packers produce final labor services according to the production function

$$N_t = \left(\int_0^{n_A} \hat{n}_{jt}^{\frac{\eta_W - 1}{\eta_W}} \, dj \right)^{\frac{\eta_W}{\eta_W - 1}}. \tag{56}$$

out of labor varieties $\hat{n}_{jt}$. Cost minimization by labor packers implies that each variety of labor, each union j, faces a downward-sloping demand curve

$$\hat{n}_{jt} = \left(\frac{W_{jt}}{W_t^{fi}} \right)^{-\eta_W} N_t \tag{57}$$

where W_{jt} is the nominal wage set by union j and W_t^{fi} is the nominal wage at which labor packers sell labor services to final goods producers. Since unions have market power, they pay the households a wage lower than the price at which they sell labor to labor packers. Given the nominal wage W_t at which they buy labor from households and given the nominal wage index W_t^{fi}, unions seek to maximize their discounted stream of profits. However, they face a Calvo (1983) type adjustment friction with indexation with the probability λ_w to keep wages constant. They therefore maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_w^t \frac{W_t^{fi}}{P_t} N_t \left\{ \left(\frac{W_{jt}(\bar{\pi}_W)^t}{W_t^{fi}} - \frac{W_t}{W_t^{fi}} \right) \left(\frac{W_{jt}(\bar{\pi}_W)^t}{W_t^{fi}} \right)^{-\eta_W} \right\}. \tag{58}$$

by setting W_{jt} in period t and keeping it constant except for indexation to π_W, the steady state wage inflation rate.

Since all unions are symmetric, we focus on a symmetric equilibrium and obtain the linearized wage Phillips curve from the corresponding first-order condition as follows, leaving out all terms irrelevant at a first-order approximation around the stationary equilibrium:

$$\log\left(\frac{\pi_t^W}{\bar{\pi}^W}\right) = \beta \mathbb{E}_t \log\left(\frac{\pi_{t+1}^W}{\bar{\pi}^W}\right) + \kappa_w \left(mc_t^w - \frac{1}{\mu^W} \right), \tag{59}$$

with $\pi_t^W := \frac{W_t^{fi}}{W_{t-1}^{fi}} = \frac{w_t^{fi}}{w_{t-1}^{fi}} \pi_t^{CPI}$ being domestic wage inflation, w_t and w_t^{fi} being the respective *real* wages for households and firms, $mc_t^w = \frac{w_t}{w_t^{fi}}$ is the mark-down of wages the unions pay to households, W_t, relative to the wages charged to firms, W_t^{fi} and $\kappa_w = \frac{(1-\lambda_w)(1-\lambda_w \beta)}{\lambda_w}$. Union profits paid to workers therefore are $\Pi_t^U = (w_t^{fi} - w_t) N_t$.

B.2.2 Consumption Good Bundler

The consumption goods are bundles of domestically produced and imported final goods and are not traded across countries. Letting F_t denote the consumption good and A_t and B_t bundles of domestically and imported final goods, we assume the following aggregation technology

$$F_t = \left\{ (1-(1-n)\omega_A)^{\frac{1}{\sigma}} A_t^{\frac{\sigma-1}{\sigma}} + ((1-n)\omega_A)^{\frac{1}{\sigma}} B_t^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{1-\sigma}}, \tag{60}$$

$$F_t^* = \left\{ (n\omega_B)^{\frac{1}{\sigma}} A_t^{\frac{\sigma-1}{\sigma}} + (1-n\omega_B)^{\frac{1}{\sigma}} B_t^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{1-\sigma}}. \tag{61}$$

Here σ measures the terms of trade elasticity of the relative demand for domestically produced goods. $\omega_A \in [0,1]$ provides a measure for the home bias, in the sense that with $\omega_A = 1$, the Country A has no home bias. The bundles of domestically and imported final goods are defined as follows:

$$A_t = \left[\left(\frac{1}{n_A}^{\frac{1}{\sigma}} \int_0^{n_A} A_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)\right]^{\frac{\epsilon}{\epsilon-1}},\ B_t = \left[\left(\frac{1}{1-n_A}^{\frac{1}{\sigma}} \int_{n_A}^1 B_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{62}$$

where $A_t(j)$ and $B_t(j)$ denote final goods produced in Home and Foreign, respectively, and ϵ measures the elasticity of substitution between final goods produced within the same country. Let $P(j)$ denote the price of a final good expressed in domestic currency. Then, letting $\mathcal{E}_t$ denote the nominal exchange rate (the price of domestic currency in terms of foreign currency) and assuming that the law of one price holds, we have

$$P_t^*(j) = \mathcal{E}_t P_t(j), \tag{63}$$

with $\mathcal{E}_t = 1\ \forall t$ since both countries form a monetary union.

The optimization problem of the good bundler is to minimize expenditures subject to $F_t = C_t + I_t$, and the aggregation technologies (60) and (62). Assuming that government consumption, G_t, is a bundle that is isomorphic to consumption goods, but consists of domestically produced goods only, global demand for a generic final good produced in Country A and B are given, respectively, by

$$Y_t^d(j) = \left(\frac{P_t(j)}{P_{At}}\right)^{-\epsilon} \left\{\left(\frac{P_{At}}{P_t}\right)^{\sigma} (1-(1-n)\omega_A)(C_t+I_t) + (1-n)\omega_B Q_t^{-\sigma}(I_t^* + C_t^*) + G_t\right\}, \tag{64}$$

$$Y_t^d(j)^* = \left(\frac{P_t(j)^*}{P_{Bt}^*}\right)^{-\epsilon} \left\{\left(\frac{P_{Bt}^*}{P_t^*}\right)^{\sigma} (n\omega_A)Q_t^{\sigma}(C_t+I_t) + (1-n\omega_B)(I_t^* + C_t^*) + G_t^*\right\}, \tag{65}$$

where the price indices are given by

$$P_{At} = \left[\frac{1}{n}\int_0^{n_A} P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}},\ P_{Bt} = \left[\frac{1}{1-n}\int_{n_A}^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{66}$$

and

$$P_t = [(1-(1-n)\omega_A)P_{At}^{1-\sigma} + ((1-n)\omega_A)P_{Bt}^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{67}$$

$$P_t^* = [(n\omega_B)(P_{At}^*)^{1-\sigma} + (1-n\omega_B)(P_{Bt}^*)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \tag{68}$$

The real exchange rate is given by

$$Q_t = \frac{P_t \mathcal{E}_t}{P_t^*}. \tag{69}$$

B.2.3 Final goods producers

Similar to unions, final goods producers in the home country differentiate the homogeneous home intermediate goods and set prices. They face the global demand (64) for each good $j \in [0, n]$ and buy the intermediate good at the national nominal price, MC_t. As we do for unions, we assume price adjustment frictions à la Calvo (1983) with indexation.

Under this assumption, the firms' managers maximize the present value of real profits given this price adjustment friction, i.e., they maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_Y^t (1-\tau_t) \left(\frac{p_{jt}(\bar{\pi})^t}{P_t} - \frac{MC_t}{P_t} \right) Y_t^d(j) \tag{70}$$

with a time-constant discount factor.

The corresponding first-order condition for price setting implies a domestic Phillips curve

$$\log\left(\frac{\pi_{At}}{\bar{\pi}}\right) = \beta \mathbb{E}_t \log\left(\frac{\pi_{At+1}}{\bar{\pi}}\right) + \kappa_Y \left(mc_t - \frac{1}{\mu^Y} \right) \tag{71}$$

where we again dropped all terms irrelevant for a first-order approximation and have $\kappa_Y = \frac{(1-\lambda_Y)(1-\lambda_Y \beta)}{\lambda_Y}$. Here, $\pi_{At} := \frac{P_{At}}{P_{At-1}}$, is the gross domestic producer price inflation rate, i.e., the gross inflation rate of domestic final goods, $mc_t := \frac{MC_t}{P_t}$ are the domestic real marginal costs, $\bar{\pi}$ is steady-state inflation, and $\frac{1}{\mu^Y} = \frac{\eta-1}{\eta}$ is the target markup. National profits paid to domestic entrepreneurs therefore are $\Pi_t^F = (1 - mc_t) Y_t$.

B.2.4 Intermediate goods producers

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = Z_t (N_t)^{\alpha} (u_t K_t)^{(1-\alpha)} \tag{72}$$

where Z_t is national total factor productivity and follows an autoregressive process in logs and $u_t K_t$ is the effective capital stock taking into account utilization, u_t, i.e., the intensity with which the existing capital stock is used. Using capital with an intensity higher than normal increases depreciation of capital according to $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \delta_2/2(u_t - 1)^2$, which, assuming $\delta_1, \delta_2 > 0$, is an increasing and convex function of utilization. Without loss of generality, capital utilization in the steady state is normalized to 1, so that δ_0 denotes the steady-state depreciation rate of capital goods.

Let mc_t be the relative price at which the intermediate good is sold to final goods producers. The intermediate goods producer maximizes profits,

$$mc_t Z_t Y_t - w_t^{fi} N_t - [r_t^F + q_t \delta(u_t)] K_t, \tag{73}$$

where r_t^F and q_t are the rental rate of firms and the (producer) price of capital goods, respectively. The intermediate goods producer operates in perfectly competitive national markets, such that the real wage and the user costs of capital are given by the marginal product of labor and effective capital:

$$w_t^{fi} = \alpha mc_t Z_t \left(\frac{u_t K_t}{N_t} \right)^{1-\alpha} \tag{74}$$

$$r_t^F + q_t \delta(u_t) = u_t (1-\alpha) mc_t Z_t \left(\frac{N_t}{u_t K_t} \right)^{\alpha} \tag{75}$$

We assume that utilization is decided by the owners of the capital goods, taking the aggregate national supply of capital services as given. The optimality condition for utilization is given by

$$q_t[\delta_1 + \delta_2(u_t - 1)] = (1-\alpha) mc_t Z_t \left(\frac{N_t}{u_t K_t} \right)^{\alpha}, \tag{76}$$

i.e., capital owners increase utilization until the marginal maintenance costs equal the marginal product of capital services.

B.2.5 Capital goods producers

Capital goods producers take the relative price of capital goods, q_t, as given in deciding about their output, i.e., they maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}} \right)^2 \right] - 1 \right\}. \tag{77}$$

Optimality of the capital goods production requires (again dropping all terms irrelevant up to first order)

$$q_t \left[1 - \phi \log \frac{I_t}{I_{t-1}}\right] = 1 - \beta \mathbb{E}_t \left[q_{t+1} \psi \log \left(\frac{I_{t+1}}{I_t}\right)\right], \tag{78}$$

and each capital goods producer will adjust its production until (78) is fulfilled.

Since all capital goods producers within a country are symmetric, we obtain the law for motion for domestic aggregate capital as

$$K_t - (1 - \delta(u_t))K_{t-1} = \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}}\right)^2\right] I_t \tag{79}$$

The functional form assumption implies that investment adjustment costs are minimized and equal to 0 in steady state.

B.3 Government Sector

The two countries form a monetary union such that they run a common monetary authority. In addition, each country runs a national fiscal authority. The monetary authority controls the nominal interest rate on liquid assets in both countries, while the national fiscal authorities issue government bonds in a union-wide bond market to finance deficits, choose both the average tax rate and the tax progressivity in their country, and make expenditures for government consumption and their national transfer system.

B.3.1 Monetary Union

We assume that monetary policy sets the nominal interest rate, which is the same in both countries, following a Taylor (1993)-type rule with interest rate smoothing:

$$\frac{R^b_{t+1}}{\bar{R}^b} = \left(\frac{R^b_t}{\bar{R}^b}\right)^{\rho_R} \left(\frac{n\pi_{At} + (1-n)(\pi_{Bt})}{\bar{\pi}}\right)^{(1-\rho_R)\theta_\pi} \left(n\frac{Y_t}{Y_{t-1}} + (1-n)\frac{Y^*_t}{Y^*_{t-1}}\right)^{(1-\rho_R)\theta_Y} \epsilon^R_t. \tag{80}$$

The coefficient $\bar{R}^b \geq 0$ determines the nominal interest rate in the steady state, Y^*_t determines output in Country B, and π_{Bt} is the producer price inflation in Country B. The coefficients $\theta_\pi, \theta_Y \geq 0$ govern the extent to which the central bank attempts to stabilize producer price inflation and the output growth in the monetary union. $\rho_R \geq 0$ captures interest rate smoothing and ϵ^R_t is an i.i.d. monetary policy shock.

B.3.2 Fiscal Policy

The budget constraint of the national fiscal policy reads

$$G_t + Tr_t = B_{t+1} + T_t - \frac{R_t^b}{\pi_t^{CPI}} B_t. \tag{81}$$

Hence, the government has expenditure for government spending, G_t, aggregate spending on its transfer system specified below, Tr_t, and repaying its debt, B_t. It finances its expenditures by issuing new debt and tax revenue, T_t. Tax revenue is

$$T_t = \tau_t (w_t N_t + \mathbb{I}_{h_{it}=0} \Pi_t^{fi} + \mathbb{I}_{h_{it} \neq 0} \Pi_t^U). \tag{82}$$

We assume that the average tax rate is a feedback function of government debt:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{B_{t+1}}{\bar{B}} \right)^{\gamma_B^\tau}, \tag{83}$$

where γ_B^τ governs the speed with which debt returns to its target.

B.3.3 Targeted Transfer System

The targeted transfer system provides additional resources if net labor income $w_t n_t h_{it}$ falls short of some target level. For simplicity, we assume that these transfers are non-distortionary for the labor supply decision. In particular, we assume that transfers are paid to households according to the following scheme:

$$tr_{it} = \max\{0, a_1 \bar{y} - a_2 (1 - \tau_t) w_t h_{it} n_{it}\}, \tag{84}$$

where $\bar{y}$ is the median income and $0 \leq a_1, a_2 \leq 1$. Thus, transfers decrease in individual income with a transfer withdrawal rate of a_2 and no transfers are paid to households whose net labor income $(1 - \tau_t) w_t h_{it} n_{it} \geq \frac{a_1}{a_2} \bar{y}$. Total transfer payments of the government in Country A are then

$$Tr_t = \mathbb{E}_t tr_{it}, \tag{85}$$

where again, the expectation operator is the cross-sectional average.

B.4 Goods, bonds, capital, and labor market clearing

The national labor market in Country A clears at the competitive wage given in (74). A symmetric labor market clearing condition is in place in Country B. The bond markets clear whenever the following equations hold:

$$\begin{aligned}
B_{t+1} &= B^d(R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t, \tau_t, \Theta_t, \Theta_t^*, \mathbb{W}_{t+1}) - \frac{B_{Bt+1}}{Q_t} \\
&:= \mathbb{E}_t[\lambda \mathbb{B}_{a,t} + (1-\lambda)\mathbb{B}_{n,t}] - \frac{B_{Bt+1}}{Q_t}, \\
B_{t+1}^* &= B^{d,*}(R_t^b, r_t^*, q_t^*, \Pi_t^{fi,*}, \Pi_t^{U,*} w_t^*, \pi_t^{CPI,*}, \tau_t^*, \Theta_t, \Theta_t^*, \mathbb{W}_{t+1}^*) + \frac{n_A}{1-n_A} B_{Bt+1} \\
&:= \mathbb{E}_t[\lambda \mathbb{B}_{a,t}^* + (1-\lambda)\mathbb{B}_{n,t}^*] + \frac{n_A}{1-n_A} B_{Bt+1}, \\
B_{t+1}^d + B_{t+1}^{d,*} &= B_{t+1} + B_{t+1}^* \qquad (86)
\end{aligned}$$

where $\mathbb{B}_{a,t}$, $\mathbb{B}_{n,t}$ are functions of the states (b, k, h), and depend on how the households in the Country A value asset holdings in the future, $\mathbb{W}_{t+1}$, and the current set of prices (and tax rates) $(R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t^{CPI}, \tau_t)$.[9] Future prices do not show up because we can express the value functions such that they summarize all relevant information on the expected future price paths. Expectations in the right-hand-side expression are taken w.r.t. the distributions in both countries $\Theta_t(b, k, h)$ and $\Theta_t^*(b, k, h)$. The total net amount of foreign bond holdings in Country A, B_{Bt}, is given by the aggregation over the households' budget constraint:

$$\begin{aligned}
(1-\tau_t)(w_t N_t + \Pi_t^U + \Pi_t^{fi}) + (P_{At} Y_t - w_t N_t - (\Pi_t^U + \Pi_t^{fi})) + Tr_t + B_t R_t^b / \pi_t^{CPI} \\
+ B_{Bt} R_t^b / (\pi_t^{CPI,*} Q_t)) = C_t + I_t + \bar{R} * BD_t + B_{t+1} + B_{Bt+1}/Q_t, \qquad (87)
\end{aligned}$$

where BD_t is the total amount of borrowing in Country A. Since both government bonds pay the same interest rate, we do not need to take track of the share of domestic vs. foreign bond holdings in each household's portfolio. Equilibrium requires the total *net* amount of bonds the household sectors in both countries demand to equal the supply of government bonds. In gross terms there are more liquid assets in circulation as some households borrow up to $\underline{B}$.

In addition, the national markets for capital have to clear. In Country A, we have:

$$\begin{aligned}
K_{t+1} &= K^d(R_t^b, r_t, q_t, \Pi_t^{fi}, \Pi_t^U, w_t, \pi_t^{CPI}, \tau_t, \Theta_t, \Theta_t^*, \mathbb{W}_{t+1}) \\
&:= \mathbb{E}_t[\lambda(\mathbb{K}_t) + (1-\lambda)(k)] \qquad (88)
\end{aligned}$$

where the first equation stems from competition in the production of capital goods, and the

[9]The same logic applies for $\mathbb{B}_{a,t}^*$, $\mathbb{B}_{n,t}^*$ in Country B.

second equation defines the aggregate supply of funds from households in Country A - both those that trade capital, $\lambda(\mathbb{K}_t)$ and those that do not, $(1-\lambda)(k)$. Again $\mathbb{K}_t$ is a function of the current prices and continuation values. In Country B, the capital market clearing condition is symmetric.

Finally, goods market clearing requires:

$$Y_t = ((1-(1-n)\omega_A)\left(\frac{P_{At}}{P_t}\right)^{-\sigma}\left[C_t + I_t + BD_t\bar{R}\right] + (1-n_A)\omega_B Q_t^{-\sigma}\left[C_t^* + I_t^* + BD_t^*\bar{R})\right] + G_t$$
$$Y_t^* = n\omega_A Q_t^{\sigma}\left(\frac{P_{Bt}^*}{P_t^*}\right)^{-\sigma}\left[C_t + I_t + BD_t\bar{R}\right] + (1-n_A\omega_B)\left[C_t^* + I_t^* + BD_t^*\bar{R})\right] + G_t^*. \tag{89}$$

B.5 Equilibrium

A sequential equilibrium with recursive planning in our two-country model is a sequence of policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$ in Country A and $\{\mathbb{X}_{at}^*, \mathbb{X}_{nt}^*, \mathbb{B}_{at}^*, \mathbb{B}_{nt}^*, \mathbb{K}_t^*\}$ in Country B, a sequence of value functions $\{V_t^a, V_t^n\}$ in Country A and $\{V_t^{a,*}, V_t^{n,*}\}$ in Country B, a sequence of prices $\{w_t, w_t^{fi}, \Pi_t^U, \Pi_t^{fi}, q_t, r_t, R_t^b, \pi_t^{CPI}, \pi_{At}, \pi_t^W, \frac{P_{At}}{P_t}, \tau_t, Q_t\}$ in Country A and $\{w_t^*, w_t^{fi,*}, \Pi_t^{U,*}, \Pi_t^{fi,*}, q_t^*, r_t^*, \pi_t^{CPI,*}, \pi_{Bt}, \pi_t^{W,*}, \frac{P_{Bt}^*}{P_t^*}, \tau_t^*\}$ in Country B, a sequence of of the shock ϵ_t^R, aggregate capital, labor supply, and foreign bond holdings $\{K_t, N_t, B_{Bt}\}$ in Country A and $\{K_t^*, N_t^*\}$ in Country B, distributions Θ_t in Country A and Θ_t^* in Country B over individual asset holdings and productivity, and expectations for the distribution of future prices, Γ, such that

1. Given the functionals $\mathbb{E}_t\mathbb{W}_{t+1}$ and $\mathbb{E}_t\mathbb{W}_{t+1}^*$ for the continuation value and period-t prices, policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$ and $\{\mathbb{X}_{at}^*, \mathbb{X}_{nt}^*, \mathbb{B}_{at}^*, \mathbb{B}_{nt}^*, \mathbb{K}_t^*\}$ solve the households' planning problem; and given the policy functions $\{\mathbb{X}_{at}, \mathbb{X}_{nt}, \mathbb{B}_{at}, \mathbb{B}_{nt}, \mathbb{K}_t\}$ and $\{\mathbb{X}_{at}^*, \mathbb{X}_{nt}^*, \mathbb{B}_{at}^*, \mathbb{B}_{nt}^*, \mathbb{K}_t^*\}$ and prices, the value functions $\{V_t^a, V_t^n\}$ and $\{V_t^{a,*}, V_t^{n,*}\}$ are a solution to the Bellman equation.

2. Distributions of wealth and income evolve according to households' policy functions.

3. All markets clear in every period, interest rates on bonds are set according to the central bank's Taylor rule, fiscal policies are set according to the fiscal rules, and stochastic processes evolve according to their law of motion.

4. Expectations are model consistent.

We solve the model by using the perturbation method in Bayer and Luetticke (2020).

Table C.1: Calibration—Asymmetric Parameters

	Description	**Country A: Italy**	**Country B: Germany**
a_1	Transfer level	0	0.5
a_2	Transfer withdrawal rate	0	0.8
G/Y	Gov. cons. share	0.21	0.20
σ_h	STD labor inc.	0.123	0.135
β	Discount factor	0.9854	0.9823
λ	Portfolio adj. prob.	0.038	0.071
ζ	Trans. prob. from W to E	0.0007	0.001
ι	Trans prob. E to W	0.0625	0.0625
$\bar{R}$	Borrowing penalty	0.018	0.029

C Calibration

We calibrate the two countries in our model to match the wealth distributions in Germany and Italy. This requires asymmetric calibration choices regarding the households. Table C.1 shows the calibration choices required for our calibration strategy which is described in 4.

C.1 Calibration of asymmetric parameters

In order to match the data, the model requires German households to be slightly less patient, asset markets (this means housing markets for most households) to be less liquid, and borrowing penalties to be higher. Yet, the mass of entrepreneurs is larger such that pure profit incomes are smaller. The level of competition (in the sense of monopolistic competition) is higher.

C.2 Calibration of symmetric parameters

We keep the rest of the calibration symmetric. We calibrate the parameters by matching long-run averages and using standard parameters from the literature. Table C.2 summarizes our calibration of those parameters. We calibrate to quarterly frequency.

The labor share in production, α, is 68% corresponding to a labor income share of 62%, given a markup of 10% due to an elasticity of substitution between differentiated goods of 11. The elasticity of substitution between labor varieties is also set to 11, yielding a wage markup of 10%. The parameter δ_1 that governs the cyclicality of utilization is set to 5.0. The investment adjustment cost parameter is set to 4.0. We set the Calvo parameters for price and wage adjustment probability both to 0.25. All these parameter choices are standard values in the literature.

Table C.2: Calibration—Symmetric Parameters

	Description	Value	Source/Target
Firms			
α	Share of labor	0.68	62% lab. income
η	Elast. of substitution	11	10% Price markup
η_W	Elast. of substitution	11	10% Wage markup
κ	Price adj. prob.	0.25	1 year avg. price duration
κ_W	Wage adj. prob.	0.25	1 year avg. wage duration
ϕ	Inv. adj. cost	4.0	Bayer et al. (2020)
δ_0	Depreciation rate	0.018	Wealth Gini = 0.61
δ_1	Depr. rate increase	5.0	Bayer et al. (2020)
Households			
ξ	Risk aversion	4	Kaplan and Violante (2014)
γ	Inv. Frisch elast.	2	Chetty et al. (2011)
Open economy			
σ	Trade-price elasticity	0.66	Standard value
ω	Home bias	0.66	Standard value
n	Country size	0.5	Same size
Government			
$\bar{\tau}$	Tax rate	0.3	Standard value
$\bar{R}^b$	Gross interest rate	1.00	zero interest-growth difference
ρ_R	Pers. in Taylor rule	0.75	standard value
θ_π	Reaction to Infl.	1.25	standard value
θ_Y	Reaction to Output	0	ECB mandate

Notes: Parameter values for baseline calibration. Symmetric countries.

We set relative risk aversion, ξ, to 4, following Kaplan and Violante (2014) and the Frisch elasticity, γ to 0.5 following Chetty et al. (2011). The persistence of idiosyncratic income shocks is set to $\rho_h = 0.9815$. The stationary equilibrium real rate(-growth difference) is set to a net rate of zero.

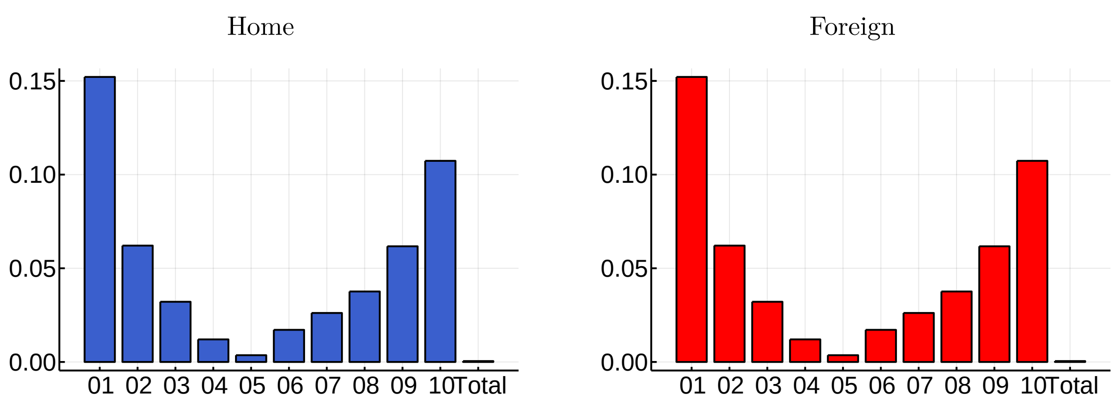
The steady state tax level is set to 0.3. We assume that monetary policy only targets inflation, as this is the primary mandate of the ECB, and set the Taylor coefficient to 1.5 and the smoothing parameter to 0.75. The steady state inflation is zero. We assume both countries are equally large and set $n = 0.5$. The home bias parameter, ω, and the terms of trade elasticity, σ are both set to 0.66—again standard values in the literature.

D The Effects of a Monetary Union with symmetric countries

This appendix shows the results using a symmetric calibration of our medium-sized HANK^2 model. Figure D.1 shows the IRFs after a contractionary TFP shock in Country A. Comparing the union-wide aggregates under a monetary union and with independent monetary policies reveals that our Proposition 1 holds exactly in that version of the model.

Figure D.2 shows that the same is true for Propositions 1 and 3: the bars of each wealth deciles in both countries are exactly the same size. If we do not depict absolute values, one would also see that they always have the opposite size. Hence, the welfare effect of a monetary union on a given union-wide wealth decile is always zero (2) implying that a monetary union only redistributes within wealth brackets across countries.

Figure D.2: Welfare impact of monetary union along the wealth distribution

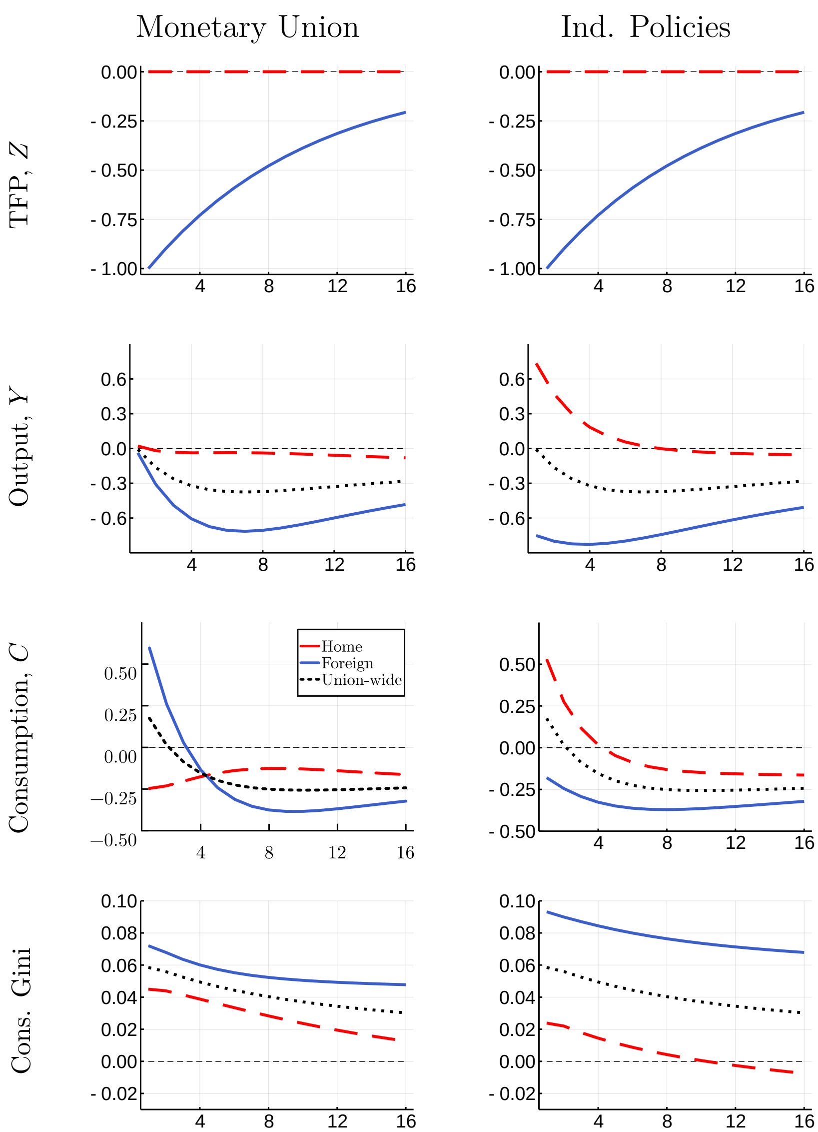


Notes: Welfare impact of monetary union measured as the difference to welfare under independent monetary policies in absolute value after TFP shocks in Country H (upper panels) and after TFP shocks in Country F (lower panel) in model with symmetric countries. Y-axis: Difference in consumption compensation (in absolute values). X-axis: Wealth percentiles.

E Long-term bonds

The baseline model features short-term debt and thus no valuation effects. With long-term debt a change in interest rates might induce valuation effects. To account for these effects, we extend the model by introducing long-term bonds. We follow the set-up in Bayer et al. (2023). The bonds issued by the government are long-term and we model their maturity as a geometric decay. Bonds are traded by perfectly competitive banks. Households save in

Figure D.1: Symmetric countries: Impulse responses to TFP shock in Country H

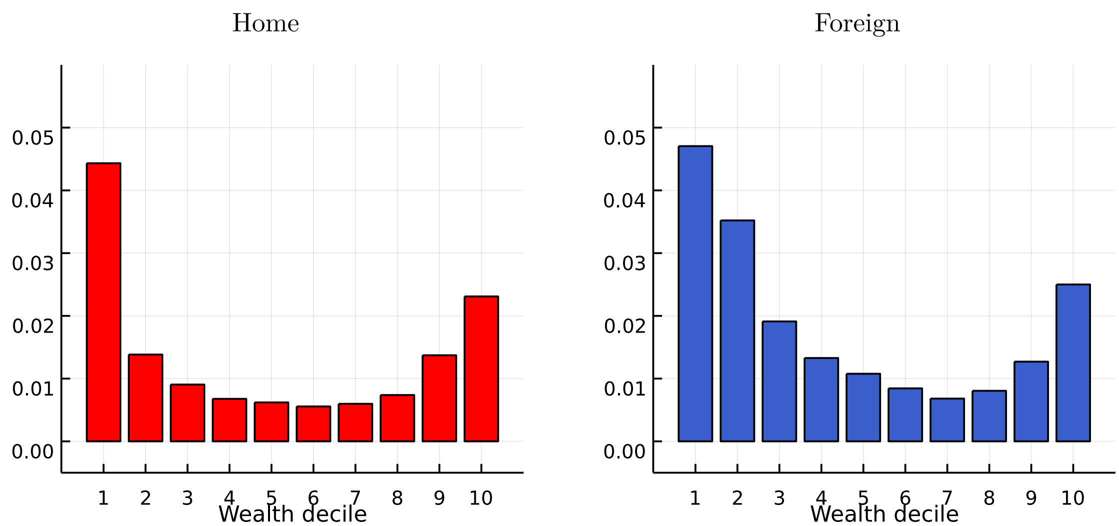


Notes: Effect of TFP shock in Country H in monetary union (left) and with independent monetary policies (right) in the model with symmetric countries. Y-axis: Percentage deviation from steady state. X-axis: Quarters.

deposits issued by the banks which pay the interest rate set by the monetary authority. In stationary equilibrium, banks make zero profits. Ex-post, banks make profits/losses when interest rates and therefore bond prices change. We assume that profits/losses of the banks are paid to households in proportion to their deposit holdings. This way, we account for the fact that wealthier households are more exposed to the re-valuation effects of assets.

Figure E.1 shows the welfare differences between the monetary-union case and the independent-policies case for (positive or negative) TFP shock that originates in Foreign. It shows that our main result is robust to the introduction of long-term bonds: A monetary union has the largest impact on the welfare at the tails of the wealth distribution. The middle class, on the other side, is almost unaffected by a monetary union.

Figure E.1: Welfare impact of monetary union along the wealth distribution with long-term bonds



Notes: Welfare impact of monetary union measured as the difference to welfare under independent monetary policies in absolute value after TFP shocks in Country H (upper panels) and after TFP shocks in Country F (lower panel) in model with long-term bonds. Y-axis: Difference in consumption compensation (in absolute values). X-axis: Wealth percentiles.