CS 251, LE 2 **Fall 2016**

Midterm 1 - KEY

Tuesday, September 27, 2016

The exam has 2+24 questions to bubble in. Maximum is 44 points.

2 pts except 1 pt for questions 1, 5, 8, 11, 15, 20,

Correct answers are either in bold or indicated by **

1. I have spelled out and bubbled in **correctly** my first name, last name, and Purdue ID.

   A True **B** False

2. Your version is Version A. Bubble in the version below. If you do not bubble in the version, your score will be 0/44.

   A Version A **B** Version B

3. Algorithm X has a running time of $2n^2 - 4n\log n + 16n$. Which big-O expressions hold for algorithm X's time?
   1. $O(n)$
   2. $O(n\log n)$
   3. $O(n^2)$
   4. $O(n^3)$
   5. $O(2^n)$

   A 1, 2, and 3

   B 3, 4, and 5

   C 2 and 3

   D 3 and 4

   E 3

4. The asymptotic ordering of the four running times from smallest to largest corresponds to what order? Running times: $2\sqrt{n}$, $\frac{6n}{\log n}$, $8n+\sqrt{n}$, $16\log n$

   A $16\log n$, $\frac{6n}{\log n}$, $2\sqrt{n}$, $8n+\sqrt{n}$

   B $2\sqrt{n}$, $16\log n$, $\frac{6n}{\log n}$, $8n+\sqrt{n}$

   C $16\log n$, $2\sqrt{n}$, $8n+\sqrt{n}$, $\frac{6n}{\log n}$

   D **$16\log n$**, **$2\sqrt{n}$**, **$\frac{6n}{\log n}$**, $8n+\sqrt{n}$

5. The running time of your algorithm is $4n^{3/2} + \frac{12n^2}{\log n} + 14n^2\log n + 1024$. Which bound below is the tightest asymptotic bound?

   A $n\log n$ **B** n^2 **C** **$n^2\log n$ **D** $n^{3/2}$

6. Consider the code segment below. How many times is function help called? Mark the tightest bound.

```
for (int i = 1; i < n; i++)
    j = i+1
    while (j < n) {
            help()
            j = j + 4
    }
```

A $O(n)$ **B** $O(n \log n)$ **C** **$O(n^2)$ **D** $O(n^3)$

7. Consider the code segment below. How many times is function help called? Mark the tightest bound.

```
for (int a = 1; a < n; a++)
    b = n
    while (b > 1) {
            help()
            b = (b - 2)/2
    }
```

A $O(\log n)$ **B** **$O(n \log n)$ **C** $O(n)$ **D** $O(n^2)$

8. Suppose an **intermixed** sequence of push and pop operations are performed on a stack. The pushes operations push the six integers 0 through 5 in that order. The pops happen at arbitrary times (when the stack is not empty) and each pop prints out the return value.

   Which of the following printed sequences could **not** occur?

   A 3 2 1 0 5 4

   B 4 3 2 1 5 0

   C 2 1 0 3 4 5

   D 0 1 2 3 4 5

   E 3 4 5 2 0 1

9. An infix expression can be evaluated by using two stacks, one for operators and one for operands. For the expression ((8/2) + (5 * (6-3))), what is on the operand stack **after 3** has been read, but before) is read? Top of stack is rightmost entry.

   A / + * -

   B + -

   C **+ * -**

   D - * +

10. The prefix expression + + A * B C D is equivalent to what **postfix** notation?

A A B + C D + *

B A B * C D * +

C A B C * + D +

D A + B * C + D

11. Consider a sorted circular doubly-linked list where the head element points to the smallest element in the list. One can find the largest element in what time? Mark the tightest bound.

A **$O(1)$ **B** $O(n)$ **C** $O(\log n)$

12. Consider a sorted circular doubly-linked list where the head element points to the smallest element in the list. Given an entry x, we can determine whether x is in the list in what time? Mark the tightest bound.

A $O(\log n)$ **B** **$O(n)$ **C** $O(1)$

13. A queue contains the elements A, B, C, D, E with A being at the front of the queue. What is the minimum number of dequeue and enqueue operations needed to generate a queue containing D, E, C, A, B (in that order, with D being at the front of the queue)?

A 2 dequeues and 2 enqueues

B 3 dequeues and 3 enqueues

C 4 dequeues and 4 enqueues

D 2 dequeues and 3 enqueues

14. A 180 student implemented a queue with a linked list as shown below. The lab assignments asks the students to achieve constant time per operation. Using the implementation illustrated, what performance is achieved (on a queue containing n elements)?

A dequeue = O(1), enqueue = O(1)

B dequeue = O(n), enqueue = O(1)

C dequeue = O(1), enqueue = O(n)

D dequeue = O(n), enqueue = O(n)

15. What is the most appropriate data structure for modeling a supermarket checkout lane? Customers enter at the back and are checked out at the front. Sometimes, the last customer in line leaves.

   A stack **B** queue **C** heap **D deque**

16. Given are two arrays A and B, each of size n and each containing integers **sorted in increasing order**. For a given target value T, one can determine whether there exist an entry a in array A and an entry b in array B such that $a + b = T$ in what time? Mark the tightest bound.

   A **$O(n)$

   B $O(\log n)$

   C $O(n \log n)$

   D $O(n^2)$

   E none of the above holds

17. Consider the number of **array exchanges** made when sorting n elements. What is the maximum number of array exchanges made by Selection sort? Mark the tightest bound.

   A **$O(n)$

   B $O(n \log n)$

   C $O(n^2)$

   D $O(n^3)$

18. A heap contains 36 entries. How many nodes are on the lowest (last) level?

   A 2 **B** 4 **C 5** **D** 6 **E** 8

19. Randomized Quicksort with 3-way partition happens to run on input data consisting of three distinct elements (e.g., 5, 22, and 38). How many times is the partition routine invoked?

   A 1 **B** 2 **C 3** **D** 4 **E** $\log n$

20. Assume Quicksort sorts an array of size n containing n distinct elements and always uses the median element as the pivot. There is no additional cost in obtaining the median. The resulting running time is $O(n \log n)$ because

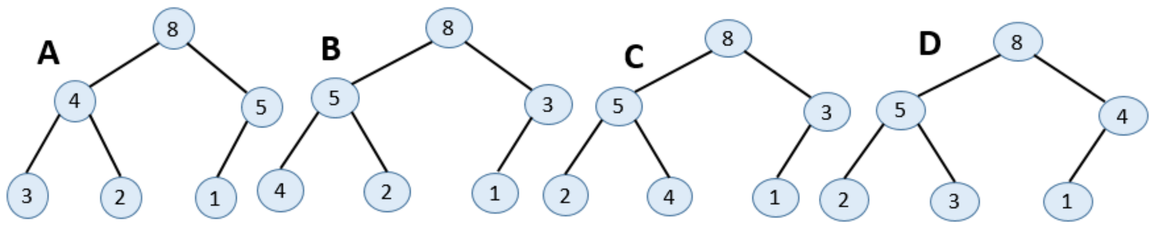
   A we choose a good pivot.

   B we generate two subproblems whose size differs by at most 1.

   C the number of levels of recursion is now constant.

   D It is not $O(n \log n)$. The worst case remains $O(n^2)$.

21. What does the max-heap look like when the six element sequence 5 2 3 4 8 1 is inserted (one by one) into an empty heap?



C is the correct answer

22. In a general binary tree, a node can have 0, 1 or 2 children. The **level** of a node is the number of links on the path from the root to the node. The root is at level 0. In a general binary tree consisting of n nodes, the maximum level of a node is

A 1

B $\lfloor \log n \rfloor$

C $n/2$

D **$n-1$

E n

23. Given is a flow grid of size n by n. Assume the flow factors are 0, 1, 2, 3, and 4. If the earliest flow path contains k cells, what is the **latest time** the flow on this path **can leave** row $n-1$?

A k **B** $4(k-1)$ **C** $n-1$ **D** $3k$ **E** **$4k$

24. An array of size n contains records with keys -15, 1, 5.5, 25, 125. The records can be sorted in $O(n)$ time using what algorithm?

1. Heapsort
2. Insertion Sort
3. **Counting Sort**
4. Standard Merge Sort

A 1

B 2

C 3

D 4

E 2 and 3

25. It is 2026 and you are the Director of Software Engineering at MacroHard. You happen to interview a senior from Purdue who has taken ECE 368 and who claims to know a lot about Quicksort. He makes the following claims:

1. There exist input sequences having all distinct elements on which Quicksort achieves $O(n)$ time.
2. Quicksort has an $O(n \log n)$ worst-case performance.
3. Quicksort is a stable sort.
4. Partitioning always uses $O(n)$ additional space.
5. Randomized Quicksort always beats deterministic Quicksort.

You remember what you learned in CS 251 and realize the correct claims are:

A 1 and 2

B 3 and 4

C 5

D 1

E **None**

26. You are asked to sort N inventory IDs. These IDs consist of 8 characters: six digits, followed by two characters chosen from a set of size K. For example, for K=2 and the set containing A and B, 485567AB is a valid ID while A123466B is not. Using Radix sort, the N IDs can be sorted in time proportional to (check the most accurate bound)

A $10N \log N$

B $600N$

C **$8N + 2K + 60$

D $14N + 2K + 160$

E $2N + 10K + 60$