CS 161

HW 5

1.

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q | P => ˥Q | Q => ˥P |
| F | F | F | F |
| F | T | T | T |
| T | F | T | T |
| T | T | F | F |

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q | P <=> ˥Q | ((P^ ˥Q) v (˥P ^ Q)) |
| F | F | F | F |
| F | T | T | T |
| T | F | T | T |
| T | T | F | F |

All truth tables (worlds) are the same for all combinations of P, Q, so they are equivalent.

2. Let Smoke = a, Fire = b, Heat = c

i) (Smoke => Fire) => (˥Smoke => ˥Fire)

(˥Smoke v Fire) => (Smoke v ˥Fire)

˥(˥Smoke v Fire) v (Smoke v ˥Fire)

(Smoke ^ ˥Fire) v (Smoke v ˥Fire)

(Smoke ^ ˥Fire) v (Smoke) v (˥Fire)

|  |  |  |
| --- | --- | --- |
| Smoke | Fire | (Smoke ^ ˥Fire) v (Smoke) v (˥Fire) |
| F | F | T |
| F | T | F |
| T | F | T |
| T | T | T |

Neither valid nor unsatisfiable, since some models, but not all models satisfy the sentence.

ii)

(Smoke => Fire) => ((Smoke v Heat) => Fire)

(˥Smoke v Fire) => ((˥Smoke ^ ˥Heat) v Fire)

(Smoke ^ ˥Fire) v ((˥Smoke v Fire) ^ (˥Heat v Fire))

((Smoke ^ ˥Fire) v (˥Smoke v Fire)) ^ ((Smoke ^ ˥Fire) v (˥Heat v Fire))

((Smoke v ˥Smoke v Fire) ^ (˥Fire v ˥Smoke ˥Fire)) ^ ((Smoke v ˥Heat v Fire) ^ (˥Fire v ˥Heat ˥Fire))

|  |  |  |  |
| --- | --- | --- | --- |
| Smoke | Fire | Heat | Smoke v Fire v ˥Heat |
| F | F | F | T |
| F | F | T | F |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

Neither valid nor unsatisfiable, since some models satisfy the sentence.

iii)

|  |  |  |  |
| --- | --- | --- | --- |
| Smoke | Fire | Heat | ((Smoke ^ Heat) => Fire) ⬄ ((Smoke => Fire) v (Heat => Fire)) |
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

This is valid, since all models are true for all sentences.

3)

a)

Let:

a = mythical

b = immortal

c = mammal

d = horned

e = magical

Knowledge Base:

a => b

˥a => ˥b ^ c

b v c => d

d => e

b) Convert to CNF

1. ˥a v b

2. a v ˥b

3. a v c

4. ˥b v d

5. ˥c v d

6. ˥d v e

c)

i) Prove a (mythical)?

1. ˥a v b

2. a v ˥b

3. a v c

4. ˥b v d

5. ˥c v d

6. ˥d v e

7. ˥a

8. ˥b 2,7

9. c 3,7

The Knowledge Base does not imply alpha (mythical) , since there is no contradiction.

ii) Prove e (magical)?

1. ˥a v b

2. a v ˥b

3. a v c

4. ˥b v d

5. ˥c v d

6. ˥d v e

7. ˥e

8. ˥d 6,7

9. ˥b 4,8

10. ˥a 1,9

11. ˥c 5,7

12. ˥b ^ c 9, 12

13. empty 10,12

The Knowledge Base implies alpha (magical), since there is a contradiction.

iii) Provide d (horned)?

1. ˥a v b

2. a v ˥b

3. a v c

4. ˥b v d

5. ˥c v d

6. ˥d v e

7. ˥d

8. ˥b 4,7

9. ˥c 5,7

10. ˥a 1,8

11. c 3,10

12. empty 9,11

The Knowledge Base implies alpha (horned), since there is a contradiction.