

Algorithms

Date: / /

$O(n)$ \rightarrow space
 $O(n^2)$ \rightarrow time

① Insertion sort \rightarrow like sorting a hand of cards. Pick a card from right & give it a place in left hand.

time = $O(n^2)$

pseudo code \rightarrow

```

for (i = 1; i < n; i++)
{
    t = a[i];
    for (j = i; j > 0 & t < a[j-1]; j--)
    {
        a[j] = a[j-1];
    }
    a[j] = t;
}
    
```

best case $\rightarrow O(n)$ when already sorted

No. of

[5 | 2 | 4 | 6 | 1]

$t = 2$
 $t < a[j-1]$ swap
 $a[j] = t$

[2 | 5 | 4 | 6 | 1]

$t = 4$
 $a[j-1] > t$ swap
 $t < a[j-1]$ x
 $a[j] = t$

[2 | 4 | 5 | 6 | 1]

$t = 6$
 $a[j-1] > t$ x
 $a[j] = t$

[2 | 4 | 5 | 6 | 1]

$t = 1$

2 4 5 6 1

2 4 5 5 6

2 4 4 5 6

2 2 4 5 6

1 2 4 5 6

J/O x

Preferred where doesn't matter
 → complexity
 → short code needed
 ② BUBBLE SORT $O(n^2)$ → time / $O(1)$ → space

Total
 Max swaps
 needed
 → $n(n-1)/2$

iteration
 ↓
 $(n-1)$

In every pass 1st element takes its posⁿ → eg in 3 elements - in 1st iteration 1st element takes 3rd posⁿ, and it → 2nd highest take 3rd posⁿ & ultimately last highest gets its last posⁿ ∴ $n-1$ iterations are required.

swaps → 1st (it) → 1st highest - needs 1 swap to reach 3rd posⁿ max.

to reach 2nd posⁿ and highest needs 1 swap max

swap max

pseudo code →

for ($i=1; i < n; i++$) → $n-1$ iteration
 for ($j=0; j < n-i; j++$) → $n-i$ swaps

if $a[j] > a[j+1]$ for each iteration
 swap($a[j], a[j+1]$)

$i \rightarrow 1, 2$

70	40	50
----	----	----

 $i=1, j=0$ $j=1$ $j=2$ $j=3$ - 1 value = 0, 1

40	70	50
----	----	----

 $j=0$ $j=1$ $j=2$ $j=3$ $j=4$

40	50	70
----	----	----

40	50	70
----	----	----

 $i=2, j=2$ $j=3$ - 2 value = 0
 No swap needed

40	50	70
----	----	----

40	50	70
----	----	----

 → done

③ SELECTION SORT

$O(1)$ space

$O(n^2)$ time
No. of comparisons $\rightarrow \frac{n(n-1)}{2}$

min element is placed at its posn compar by each element next in the series

pseudocode \rightarrow for $(i=0; i < n; i++)$

$\{$ min = $a[i]$

for $(j=i+1; j < n; j++)$

$\{$ if $(a[j] < a[min])$

$\{$ min = j

temp = swap $[a[i], a[min]]$

10 | 30 | 20 | 50 | 40

$i=0$ $j=1, 2, 3, 4$

all sorted steps

10 | 30 | 20 | 50 | 40

$i=1$ $j=2, 3, 4$

10 | 30 | 20 | 50 | 40

$i=2$

10 | 20 | 30 | 50 | 40

$j=3, 4$

10 | 20 | 30 | 50 | 40

10 | 20 | 30 | 50 | 40

$i=3$ $j=4$

10 | 20 | 30 | 50 | 40

10 | 20 | 30 | 40 | 50

done

④

DIVIDE AND CONQUER APPROACH

Date: ② /

DIVIDE divide into smaller subprob
CONQUER solving sub divided recursively
COMBINE combine sol of sub into the sol of original prob

I

MERGE SORT

Time: $O(n \log n)$, space: $O(n)$
 divides n subsequence into

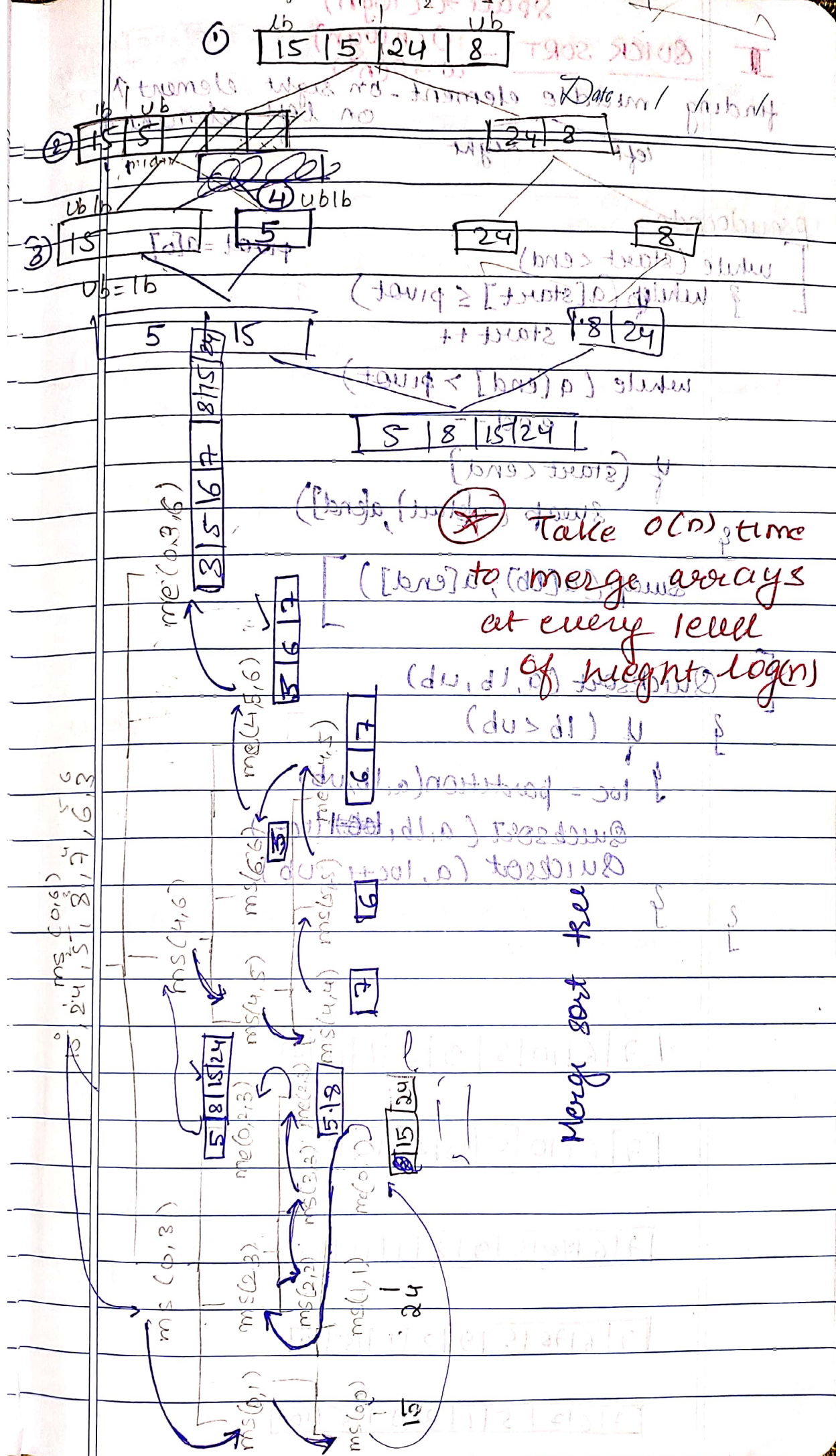
2 subarrays of $n/2$, sort these subarrays recursively
 Merge the sorted subarrays.

pseudocode \rightarrow Merge sort, (A, lb, ub)

```

if (lb < ub)
{
    mid = lb + ub / 2
    merge sort (A, lb, mid)
    merge sort (A, mid + 1, ub)
    merge (A, lb, mid, ub)
}

Merge (A, lb, mid, ub)
{
    i = lb    j = mid + 1    k = lb
    while (i <= mid && j <= ub)
    {
        if (a[i] <= a[j])
        {
            b[k] = a[i]
            i++
        }
        else
        {
            b[k] = a[j]
            j++
        }
        k++
    }
    if (i > mid)
    while (j <= ub)
    {
        b[k] = a[j]
        j++
        k++
    }
    else if (j > ub)
    while (i <= mid)
    {
        b[k] = a[i]
        i++
        k++
    }
}
  
```

Space $\rightarrow O(\log n)$
QUICK SORT $\rightarrow O(n \log n)$ \rightarrow Tree of $w = \log n$ with every step
 finding middle element - on right element $w \rightarrow O(n^2)$
 on left element

left right

pseudocode \rightarrow

while (start < end)
 { while (a[start] <= pivot) ①

start++

while (a[end] > pivot) ②

end--

if (start < end)

swap (a[start], a[end])

swap (a[lb], a[end])

Quicksort (a, lb, ub)

if (lb < ub)

loc = partition(a, lb, ub)

Quicksort (a, lb, loc-1)

Quicksort (a, loc+1, ub)

7 6 10 5 9 2 1 15 7

pivot = 7

checking ① & ②

7 6 10 5 9 2 1 15 7

swap

7 6 7 5 9 2 1 15 10

checking ① & ②

7 6 7 5 9 2 1 15 10

swap

7 6 7 5 1 2 9 15 10

checking ① & ②

7	6	7	5	1	2	9	15	10
---	---	---	---	---	---	---	----	----

25

end < start
 swap(a[lb], end),
 Date:

2	6	7	5	1	7	9	15	10
---	---	---	---	---	---	---	----	----

loc, e

repeat .

Choose pivot as middle element

SEARCHING

- ① **LINEAR SEARCH** → start at one end
 ↓
 good for $n \leq 100$ check every item until find key.

pseudocode. search(a, n, key)

```

  for (i=0, i<n, i++)
    if (a[i] = key)
      return i
  return -1
  
```

if -1 → not found

else → found at index i

$$T(n) = O(n)$$

- ② **BINARY SEARCH** → sort elements $O(\log n)$
 search at middle