

* Disadvantages of array →

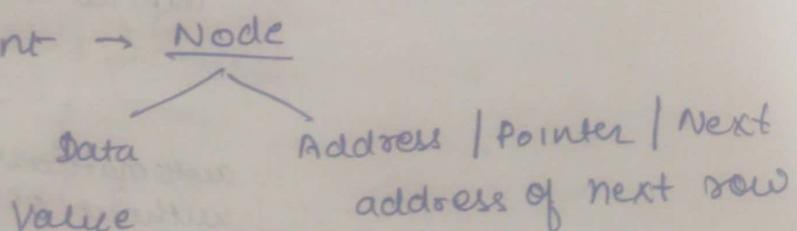
① static memory allocation

→ at compile time specify no. of elements
 so if $A[100]$ & we use only 10 place
 memory for 80 places is wasted.

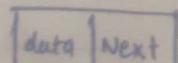
② Insertion & deletion → complex

Linked List →

Element → Node



Representation →



10	2000
head	

10	3000
2000	

30	null
3000	tail

declaring Node ...

```
struct node  
{  
    int data;  
    struct node *next;  
};
```

self-referential structure
because we can't store address in pts variable

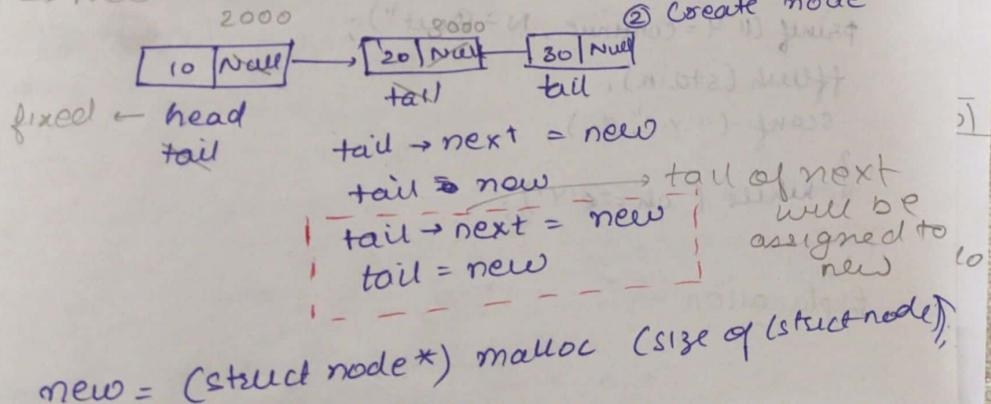
allocate memory dynamically

```
new = (struct node *) malloc (sizeof (struct node));
```

dynamic memory location
datatype (typecast) *to allocate memory*
size

creating list →

10 → New
2000 20 → New
800 30 → New



Program to input

do

```
{ new = (struct node*) malloc (size of (struct node));  
    Memory allocation  
    scanf (" %d ", & value); → value intake  
    new → next vali location me  
    new → data = value; → data me 20  
    new → next = NULL; → next me null  
if (head == NULL)  
{ head = new;           Head or tail bnaog  
    tail = new;  
}  
else  
{ tail → next = new;      tail of next = new  
    tail = new;           tail ki location me to next of alh,  
};                                new ki location add  
printf (" Y=continue, N=Quit ");  
fflush (stdin);  
scanf (" %c ", ch );  
while ( ch != 'Y' );
```

Explanation -

20 → 2000

30 → 3000

50 → 5000

value = 20

new [20 | Null]
data Next

As not initialised automatically

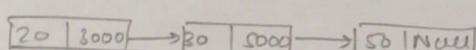
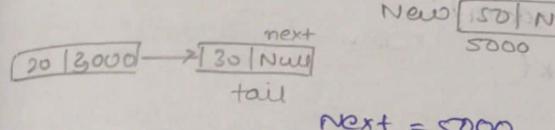
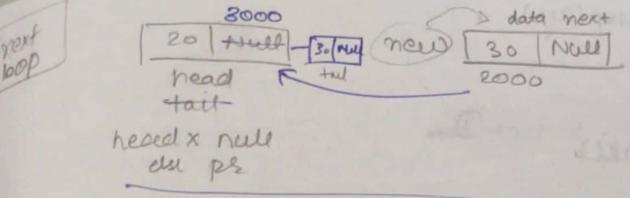
Head [20 | Null]

tail [20 | Null]

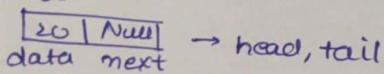
If head == null

Head [20 | Null] ← tail

If pressed y

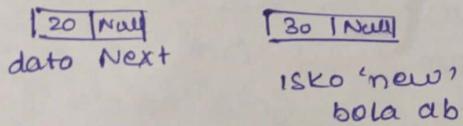


① New

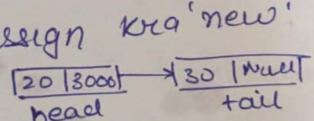


②

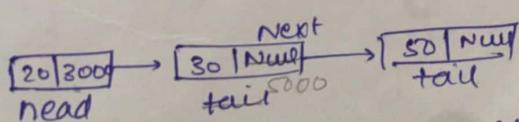
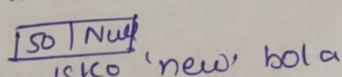
~~Next~~ agla value lia



- (i) tail of 'next' ko assign kro 'new'
ka address
- (ii) tail ab 'new'
bna



③ agla value lia



tail of Next is assigned to new
tail ab new ko bna dia

program output (display)

$\text{temp} = \text{head};$

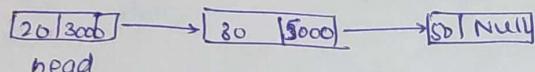
while ($\text{temp} \neq \text{NULL}$)

 4 $\text{printf}("%d", \text{temp} \rightarrow \text{data});$ access the address of

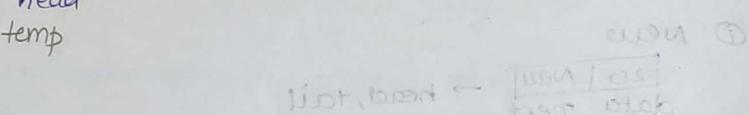
 temp = temp \rightarrow next;

 3 'next' k pointer ko
 access kora = agar dat
 access k

explain →



① temp



② temp \rightarrow ① Node (head), ko assign kia

③ temp is not null

④ data print (20) + xahi otak

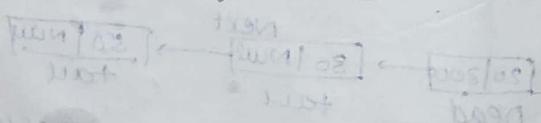
⑤ temp assigned 'next' address
 ki value = (30)

⑥ 30 print hoga

⑦ 50 print hoga

⑧ temp = 'next' address per value
(NULL)

⑨ condition false exit.

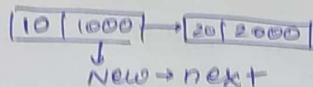


2nd loop
list base
data print 50 + xahi je list
base and on uski do list

Insertion operation →



① Insertion At Beginning → (Element is 10)



scanf ("<rd", &value);

new → data = value

new → next = head

head = new

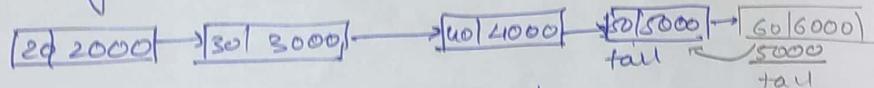
new ko head
bnaya

new ka data
me '10

new ka next

me head ko
address

② Inserting at Ending



scanf ("<rd", &value);

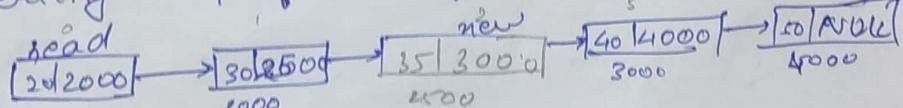
new → data = value; new ka data
me value dalo

tail → next = new; tail ka next me

new → next = NULL; new ko data

tail = new → tail ab new
ko bnaya

③ Inserting at specific pos → (pos = 2, value = 55)



scanf ("<rd", pos);

scanf ("<rd", value);

temp = head

for (i=0; i<pos-1; i++)

new → next
me temp ka next
bhabha

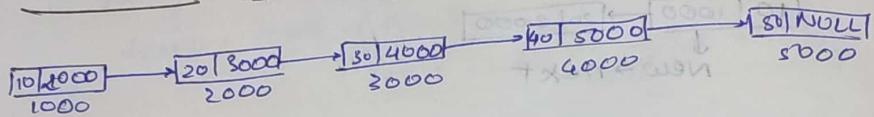
temp = temp → next → jab tak par
na pauchhe

new → data = value new my data value

new → next = temp → next

temp k temp → next = next
 next me new ka address
 bhe dia

Deletion Operation



① Delete At Beginning (\rightarrow says "bx")

head
 \downarrow
 [20|3000]
 2000

$\text{new} = \text{head} \leftarrow \text{curr}$
 $\text{temp} = \text{head};$
 $\text{temp head} = \text{head} \rightarrow \text{next};$
 $\text{temp} \rightarrow \text{next} = \text{NULL};$

② Delete At End (\rightarrow tail → [50|NULL])

temp = head; or s = "bx";
 while (temp → next != tail)

{
 $\text{temp} = \text{temp} \rightarrow \text{next};$ }

$\text{temp} \rightarrow \text{next} = \text{NULL};$

tail = temp;

③ Deletion anywhere

for (i=0; i < pos-1; i++)

{
 $\text{temp} = \text{temp} \rightarrow \text{next};$ }

$\text{temp} \rightarrow \text{next} = \text{temp} \rightarrow \text{next} \rightarrow \text{next}$

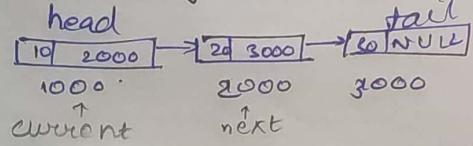
Counting no of nodes →

```
c = 0  
temp = head;  
while (temp != NULL)  
{  
    c++;  
    temp = temp -> next;  
}
```

Reversing →

```
struct node *current, *next,  
            *prev = NULL;  
current = head;  
while (current != NULL)  
{  
    next = current -> next;  
    current -> next = prev;  
    prev = current;  
    current = next;  
}  
head = prev;
```

Reversing →



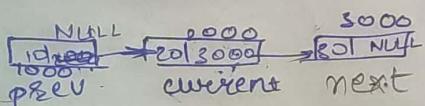
Iteration true

next = 2000

current -> next = NULL

prev = 10

current = 20



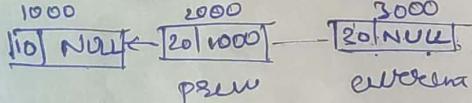
Iteration true

next = 3000

current -> next = 1000

prev = current

current = next

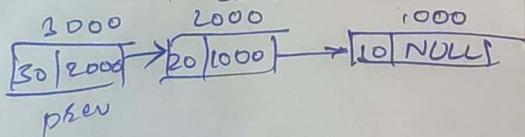


Iteration true

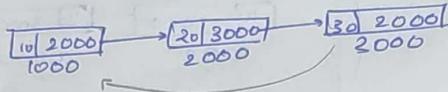
next = NULL

current -> next = 2000

prev = 3000 current = next NULL



CIRCULAR LINKED List



```
struct node
```

```
{ int data;
```

```
    struct node * next;
```

```
} * new, * head, * tail, * temp;
```

```
main()
```

```
{ new = (struct node*) malloc (size of (struct node));
```

```
scanf ("%d", & value);
```

```
new-> data = value;
```

```
new-> next = NULL;
```

```
if (head == NULL);
```

```
{ head = new;
```

```
tail = new;
```

```
}
```

```
else
```

```
{ tail
```

```
head-> next = new;
```

```
tail = new;
```

```
tail-> next = head;
```

```
while (ch != 'q');
```

```
// display
```

```
temp = head;
```

```
while (temp->next != head)
```

```
{ printf ("%d", temp->data);
```

```
temp = temp->next;
```

```
printf ("\n%d", temp->data);
```

last vala nhi hoga usse.

Insertion at Beginning

```

new → data = value;
new → next = head;
tail → next = new;
tail → head = new;
    
```

Insertion at End

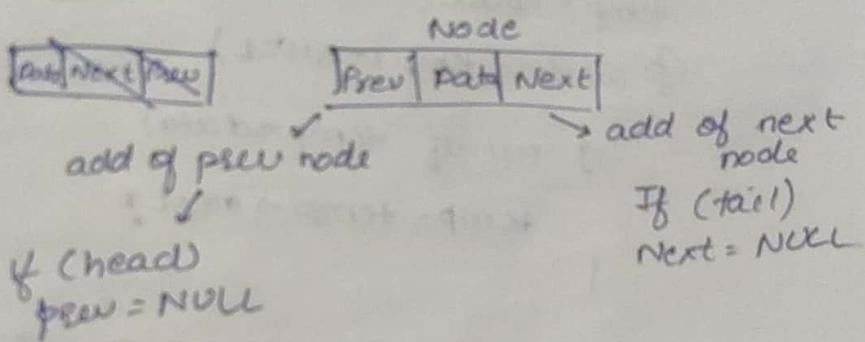
* Insertion at beginning
Prog written (simple)

Insertion

Deletion

① At Beginning, end, anywhere (Complex logic
see laptop)

Double Linked List



struct node

```

{ int data;
  struct node * prev;
  struct node * next;
}
new*, temp*, head*, tail*; 
```

```
}; main()
do
```

```

  new = (struct node*) malloc (sizeof (struct node));
  scanf ("%d", & value);
  new → data = value;
  new → prev = NULL;
  new → next = NULL; 
```

NULL → [initial]

```

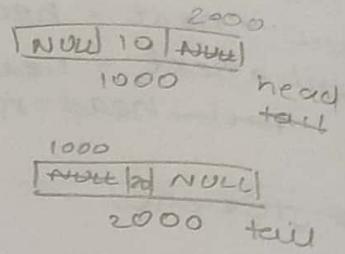
If (head == NULL)
{
    head = new;
    tail = new;
}
else
{
    tail->next = new;
    new->prev = tail
    tail = new;
}

```

```

while (ch == 'u')
{
    temp = head;
    while (temp != NULL)
    {
        printf("%d", temp->data);
        temp = temp->next;
    }
}

```



- ① Linear
- ② Pollo
- ③ push
- pop
- top

Impl

(1)

- over
- sta
- Ine
- ine
- to a sta
- having

Insertion

① At begin →
 $new \rightarrow prev = NULL;$
 $new \rightarrow next = head;$
 $head \rightarrow prev = new;$
 $head = new;$

② At end →
 $tail \rightarrow next = new;$
 $new \rightarrow prev = tail;$
 $new \rightarrow next = NULL;$
 $tail = new$

③ Anywhere →
 $new \rightarrow prev = temp$
 $new \rightarrow next = temp \rightarrow next$
 $temp \rightarrow next = new$

Deletion →

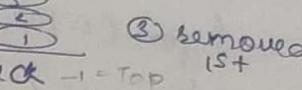
① At begin →
~~head = head->next~~
~~head->prev = null~~

② At end →
~~tail = prev~~
~~tail = tail->prev~~
~~tail->next = NULL~~

③ Anywhere →
 $temp \rightarrow next = temp \rightarrow next \rightarrow next$
 $temp \rightarrow next \rightarrow prev = temp$

STACKS

using ARRAYS

- ① Linear DS (sequential)
- ② Follows LIFO (last in first out) → 
- ③ push → inserting an element into stack → 
- ④ pop → deleting " " " " "
- Top → top of stack ← Push Pop
- * Top = -1 (initial value → stack is empty)

Implementing in two ways →

② LL

① Arrays

- overflow condn (Push)
 - Stack can only have 5 elements
 - Insert 6th element
 - inserting an element to a stack which is already having max elements.

- underflow condn (Pop)
 - Deleting an element from an empty stack.

```

implementation → stack[10], size=10, top=-1
PUSH
push()
{
    if (top == -1)
        scanf("%d", element);
    if (top == size-1)
        printf("overflow");
    else
        {
            top++;
            stack[top] = element;
        }
}
POP
pop()
{
    if (top == -1)
        printf("Underflow");
    else
        {
            element = stack[top];
            top--;
        }
}

```

0	1	2
	30	
	20	
	10	
		top = -1

```

display()
{
    if (top == -1)
        for (i = top; i >= 0; i--)
            printf("%d", stack[i]);
    else
        printf("empty stack");
}

```

3	40
2	30
1	20
0	10

$i = 3 \quad i > 0$
 $\text{stack}[i] = 40$

using ARRAYS

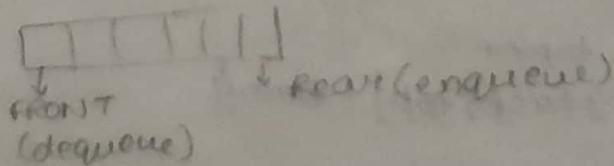
QUEUE

implement < arrays
 LL

- Linear DS (sequential)
- Follows FIFO (1st in last out)
- Operations →
 - ENQUEUE → inserting
 - DEQUEUE → deleting
- CONDITIONS →
 - (enqueue) → OVERFLOW → Insertion in a Queue that's full
 - (dequeue) → UNDERFLOW → deletion from empty Queue

- two ends →
 - FRONT → starting element
 - REAR → last "

- enqueue will be done at REAR end.
- dequeue " " " " FRONT "



- * Initially FRONT = REAR = -1

Implement "→

SIZE = 10

FRONT = REAR = -1

QUEUE [];

ENQUEUE()

{ if (REAR == SIZE-1)

{ printf ("overflow");

else

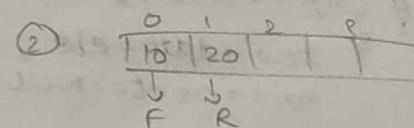
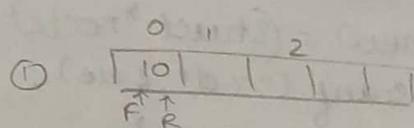
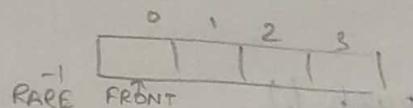
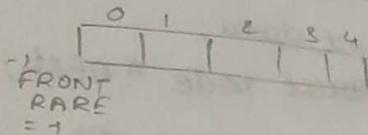
{ scanf ("%d", ele);

if (FRONT == -1)

{ FRONT = 0; }

REAR++;

{ QUEUE [REAR] = ele; }



DEQUEUE()

{ if (REAR == -1 || FRONT > REAR)

{ printf ("underflow"); }

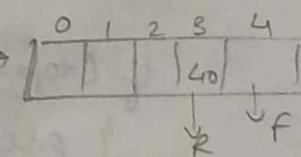
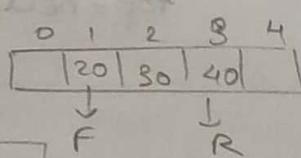
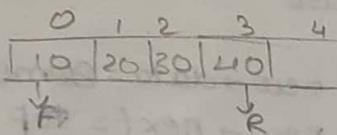
else

{ ele = QUEUE [FRONT];

printf ("element %d is deleted", ele);

FRONT++;

}



DISPLAY()

{ if (REAR == -1 || FRONT > REAR)

{ printf ("empty Queue"); }

else

{ for (i=FRONT; i<=REAR; i++)

{ printf ("%d", QUEUE [i]); }

}

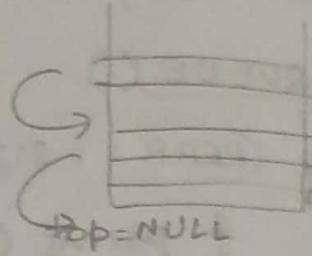
STACK using LL

struct node

{ int data;

struct node *next;

{*new, *temp, top



push()

{ new = (struct *node) malloc (sizeof (struct node));

scanf ("%d", &ele);

if (top == NULL),

{ new->data = ele;
new->next = NULL;

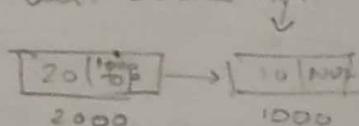
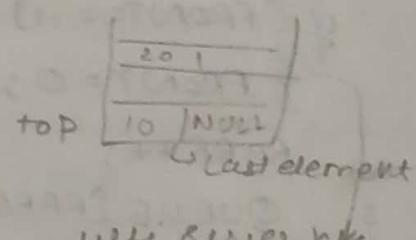
top = new;

else

new->data = element;

new->next = top; new & next make

top = new;



pop()

if (top == NULL)

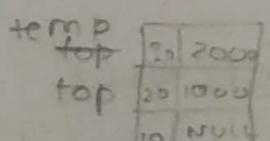
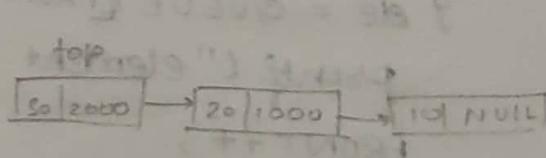
{ printf ("stack empty"); }

else

{ temp = top; ele = top->data; (element is deleted)
top = top->next;

or [temp->next = NULL;

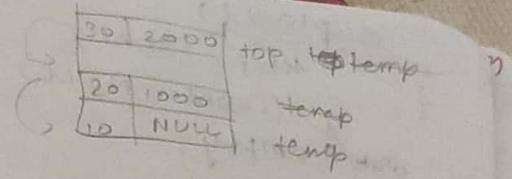
free (temp);



```

display()
{
    if (top == NULL)
        printf("Stack is empty");
    else
    {
        temp = top;
        while (temp != NULL)
        {
            printf("%d", temp->data);
            temp = temp->next;
        }
    }
}

```

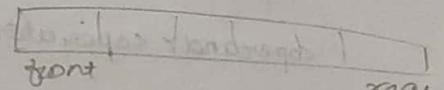


QUEUE USING LL

```

struct node
{
    int data;
    struct node *next;
} *new, *temp, *front, *rear;

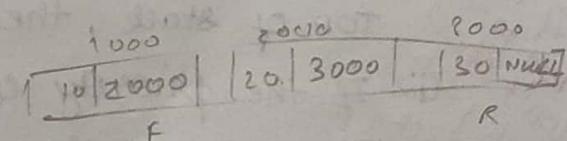
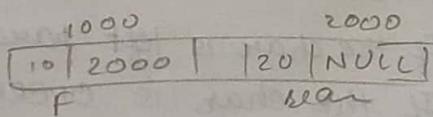
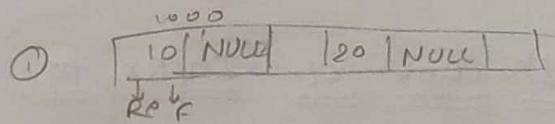
```



```

enqueue()
{
    new = (struct node *) malloc();
    scanf("%d", &ele);
    if (FRONT == rear)
        new->data = ele;
    new->next = NULL;
    if (rear == NULL)
        front = new;
    rear = new;
    else
        rear->next = new;
    rear = new;
}

```

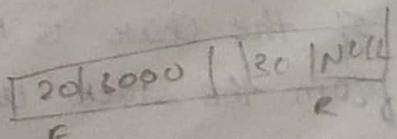


dequeue()

```

{
    if (FRONT == NULL)
        printf("empty");
    else
    {
        temp = front;
        front = front->next;
        free(temp);
    }
}

```



```

display()
{
    temp = FRONT;
    if (FRONT == NULL)
        write "empty";
    else
        while (temp != NULL)
            printf ("v.d", temp->data);
            temp = temp->next;
}

```

1000	2000	3000	NULL
F		R	

for eg
 A → char
 * → oper
 B → oper
 * → oper
 C → oper
 step

Applications of Stack

① Infix expression →
 operands → A, B, ... operators → + - * /
 operand < operator < operand → A + B

② Postfix expression
 (A + B) AB +

③ Prefix exp →
 +AB (A + B)

Conv. of Infix to Postfix exp (Algorithm) :-

STEPS →

- ① If the char is left parenthesis, push to the stack.
- ② If the char is OPERAND, add to the POSTFIX EXP.
- ③ If the char is OPERATOR, check whether stack is empty.
 - (i) If empty - push to stack. If not, check priority
 - (ii) If priority of OPERATOR > OPERATOR present at TOP of stack, then PUSH the operator in the stack.
 - (iii) If the priority of operator ≤ operator present at TOP of stack, then pop the operator from stack and add to POSTFIX EXP and goto step (i).
- ④ If the char is RIGHT Parenthesis, then pop all the operators from the stack until it reaches left par & add to postif
- ⑤ After reading all char, if stack is not empty then pop and add to postfix.

for eg. $A + B * C$

A → char (operand) by Step 2

* → operator Step 3

→ stack is empty - push to stack

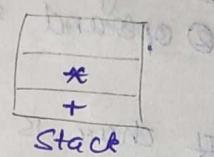
B → operand Step 2

* → operator Step 3 (i)

C → operand Step 2

Step 5

Postfix
A B C



Eg 2) $a - (b * c - d) / e \rightarrow \text{Infix}$

char	stack	postfix	
a		a	(→ push
-) → A → postfix
l			+ → empty - Push
b			not empty
*		ab	is of ↑ priority push
c		ab	pop before & postfix it check again
-		abc) → pop all till (add to postfix
d		abc*	last → pop, add to postfix
)		abc* d	
/		abc* d -	we don't insert / in postfix
e		abc* d - e	

* Create ()
int *
D newnode = (struct node *)
malloc ("Enter data");
printf ("Enter data", &x);

2) Evaluation of Postfix expression →

- ① only stack is used.
- ② operand stack

- ① If char is operator, push into stack.
- ② If char is operator, pop top 2 operands from the stack, perform calculation, push the result back into stack.

After reading all the char in from postfix exp.
STACK will be only having RESULT.

Eg →

$562 * +$

char

5

stack



6



2



*



+



$$5 + 12 = 17$$

Result = 17

Eg.

$4325 * - +$

char

4

stack



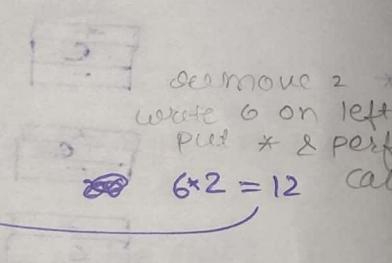
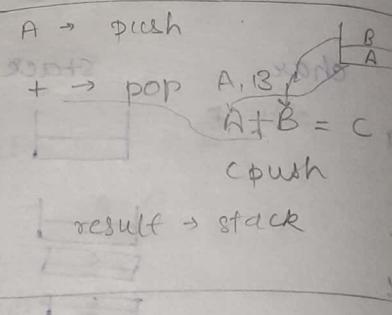
3



2



5



3) Balancing symbols

()
[]
{} -

Algorithm

- ① Read ch
- ② If char into stack
- ③ If ch stack is

④ After

Example

*

$$\begin{array}{|c|} \hline 10 \\ \hline 3 \\ \hline 4 \\ \hline \end{array}$$

$$2 * 5 = 10$$

-

$$\begin{array}{|c|} \hline -7 \\ \hline 4 \\ \hline \end{array}$$

$$3 - 10 = -7$$

+

$$\begin{array}{|c|} \hline -3 \\ \hline \end{array}$$

$$4 + -7 = -3$$

$$\text{Ans} = -3$$

3) Balancing Symbols →

Balancing symbols

{ } > exp.

[]

{ } → block of statements

→ Balancing every open symbol should have closed symbol

Algorithm →

- ① Read char from exp.
- ② If char is open symbol '{', '[', '{', check if stack is empty
 - If empty, exp is unbalanced
 - If not, pop the symbol from stack and compare with the symbol which is read.
 - = If matches, repeat the process
 - = If not, unbalanced.
- ③ After reading all the exp, stack \neq empty \rightarrow unbalanced.

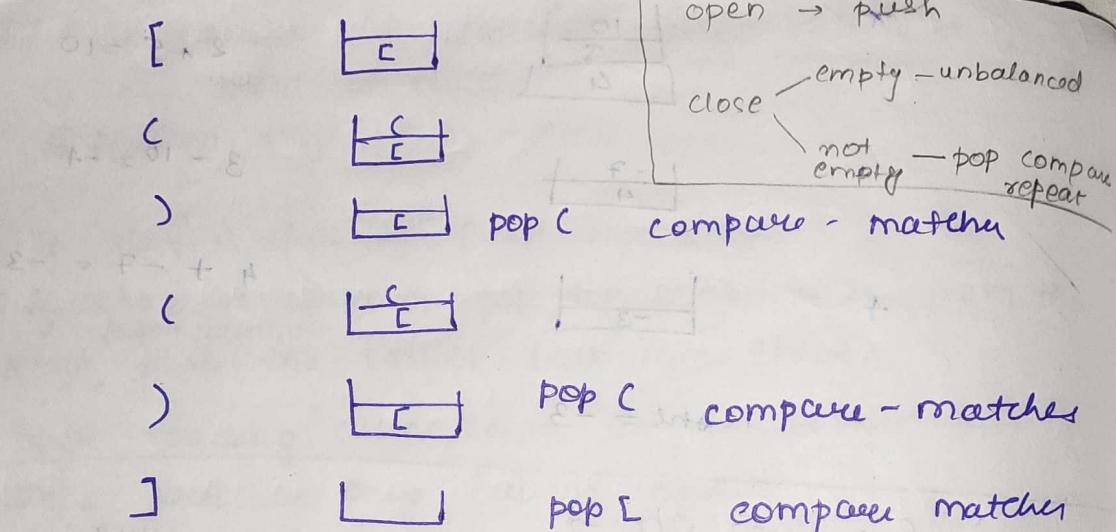
Example →

$$[(a+b)(a-b)]$$

```

    " create"
    create will
    return pointer
    to a node (to g(0))
    * Create()
    2 int x;
    3 D newnode = (struct node*) malloc(sizeof(struct node));
    printf("Enter data");
    scanf("%d", &x);

```



Node →

Root node

Edge →

Parent

Child

Siblings

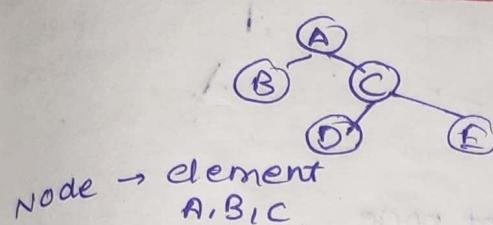
Leaf

Internal

Degree

TREE

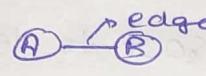
non linear
hierarchical fashion



Node → element
A, B, C

Root node → starting node

A is root node (only 1)

Edge →  - link/connection b/w 2 nodes.
N nodes - (N-1) edges.

Parent → A, C Node with branches

Child → Node with edge from bottom to top.
Branches of parent.

B, C, D, E

Siblings → child nodes of same parent.
B, C & D, E

Leaf → Node without child

B, D, E

Internal nodes → All nodes other than leaf
Node with child
A, B, C

Degree → no. of child nodes.

$$\text{degree}(A) = 2$$

$$\text{degree}(B) = 2$$

$$\text{degree}(C) = 2$$

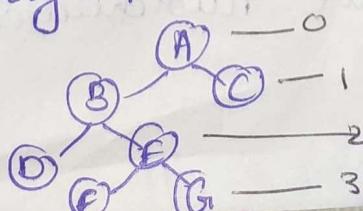
$$\text{Max degree} = \text{Degree of tree} = 2$$

Level → every step in tree

starts from 0.

For every step / hierarchy level + 1

level of tree = 3
level of root node = 0



```

    create null
    return pointer
    to a node (to g)
2. * Create()
3. D int x;
4. D newnode = (struct node*)
5. cout << "Enter data";
6. cin << "d", &x;

```

Height → longest path from leaf node to that node is height

$$\text{Height } (B) = 2$$

$$\text{Height } (A) = 3$$

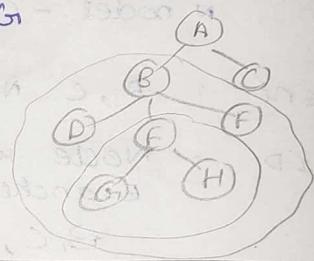
depth = longest path from root node to that node

$$\text{depth } (E) = 2$$

Path → sequence of nodes from root to leaf

$$\text{Path } (A \text{ to } G) = A - B - E - G$$

Subtree → node with child node



Binary Tree

→ every node in a tree should have atmost 2 children

~~If a newly connected node value is greater than root then it goes on right side~~

0, 1, 2 children

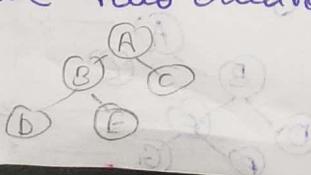


Types →

- Full binary tree / strictly binary tree
- Almost complete binary tree / incomplete binary tree
- Complete binary tree / Perfect BT
- Left skewed BT
- Right skewed BT

Full B / Strictly P →

every node must have two children except leaf



Incomplete
every node
except in

CBT →
every node
in all level
each level

LSBT
every
node
have only

RSBT

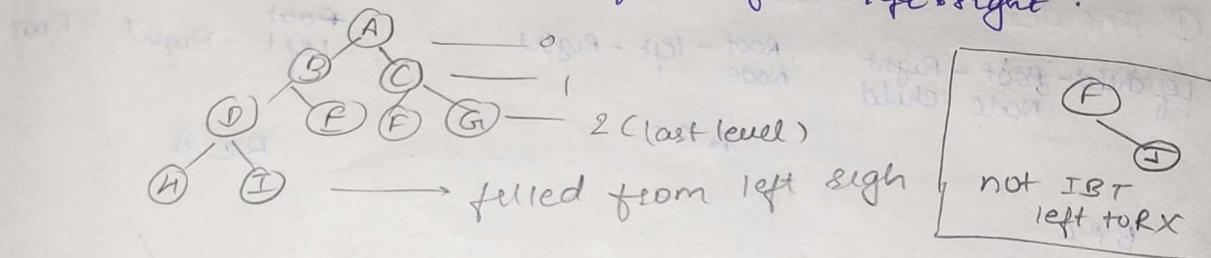
① Using

Algo:
① Consider
② left child is
i → parent
③ Right child
exit + 2

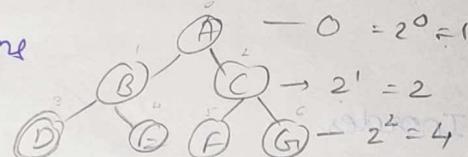
A	B	C	D
---	---	---	---

B → $2^{i+1} - 1$
F → $2^{i+1} - 1$
G → $2^{i+1} - 1$

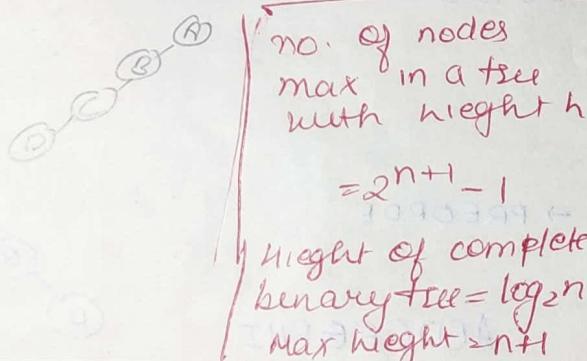
Incomplete BT →
every node must have 2 children in all levels
except in last level but filled from left-right.



CBT →
every node must have 2 children
in all levels.
each level 2^L nodes.



LSBT →
every node should
have only left chain.



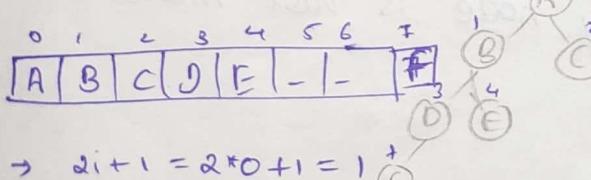
RSBT



① Using Array →

Algo:

- ① Consider root node at index 0
- ② Left child is placed at $2i + 1$
 $i \rightarrow$ parent node
- ③ Right child is placed at $2i + 2$

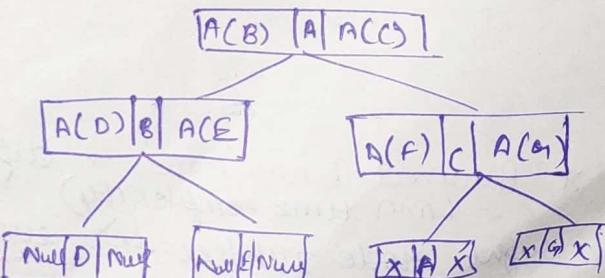


$$B \rightarrow 2i + 1 = 2 \cdot 0 + 1 = 1$$

$$F \rightarrow 2i + 1 = 2 \cdot 3 + 1 = 7$$

② Using LL →

Add of Left Add of Right



A(B) → add of B
X → Null

create will
 return you
 to a node
 1. * Create()
 2. int x;
 3. struct node * newnode = (struct node *
 x; if ("Enter data");
 d", &x);

TREE TRAVERSAL

① Inorder

Left child - Root - Right
Node child

B A C

② Preorder

Root - Left - Right
Node

A B C



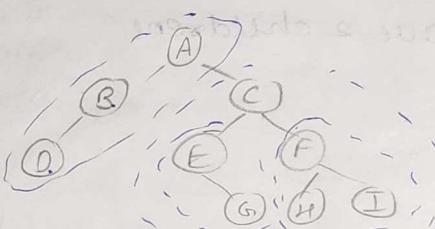
③ Post order

Root
left - Right - Root

B C A

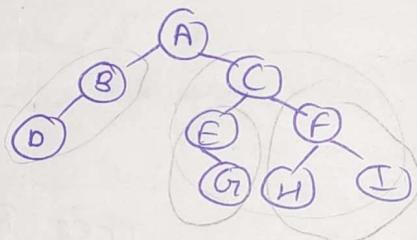
→ Inorder

D B A E G C H F I



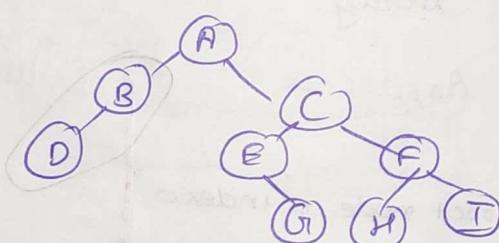
→ PREORDER

A B D C E G F H I



→ POSTORDER →

D B G E H I F C A

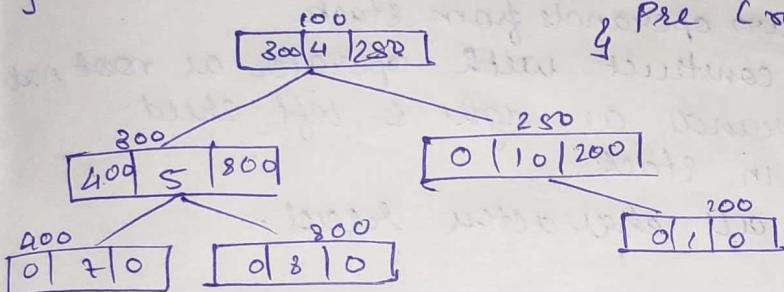


* Balanced binary tree → diff b/w height of left & right node subtree for every node is not more than
(min time complexity)

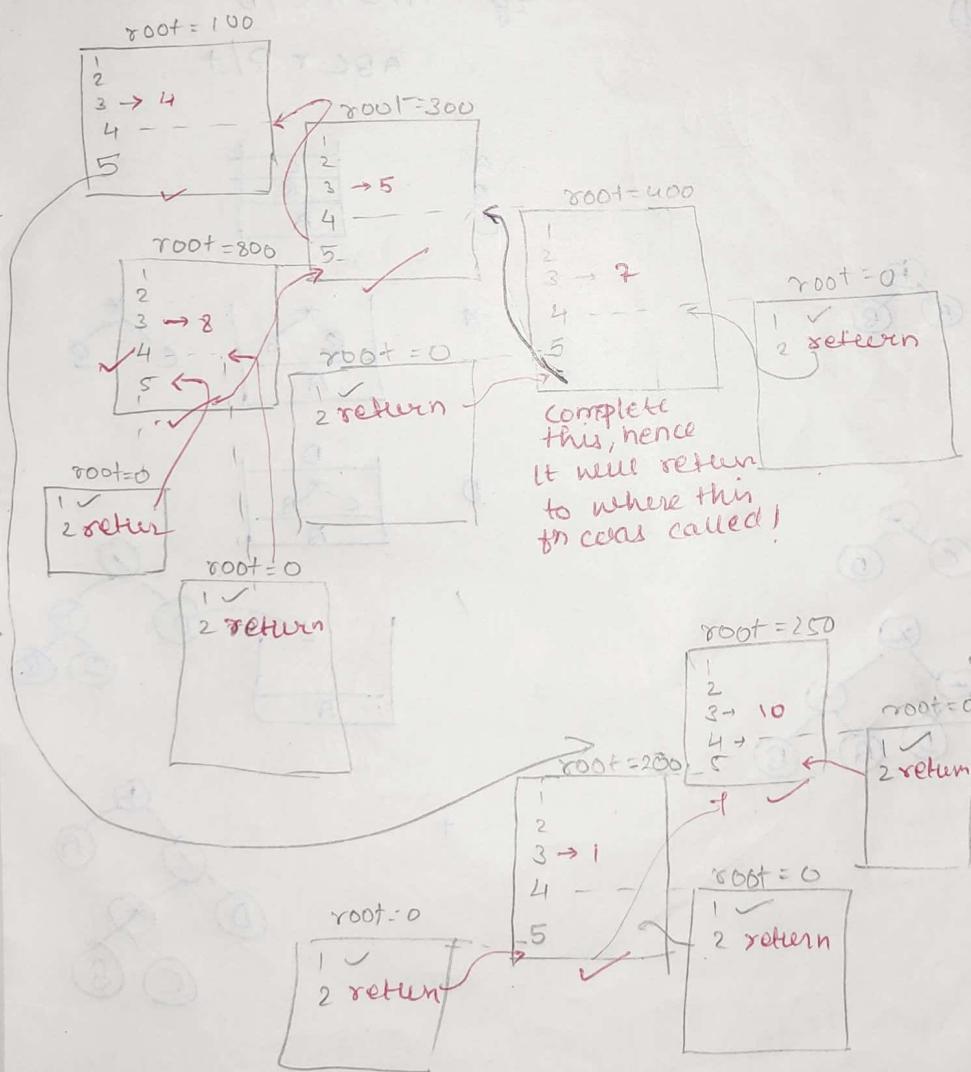
Preorder Code & Memory allocation

main()

```
{ struct node *root;
    write Preorder is;
    Pre (root);
}
```



```
void Pre (struct node *root)
{
    if (root == 0)
        return;
    printf ("%d", root->data);
    Pre (root->left);
    Pre (root->right);
}
```



```

    void main()
    {
        node *root = create();
        cout << "return pointer to a node (for eg. 100)" << endl;
        cout << "Enter data" << endl;
        int x;
        node *newnode = (struct node *) malloc(sizeof(node));
        typecast
        newnode->data = x;
        newnode->left = newnode->right = NULL;
        cout << "Enter data" << endl;
        cout << "Enter data" << endl;
    }

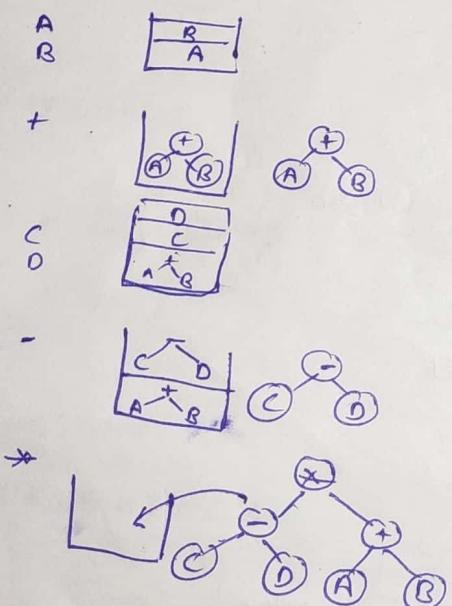
```

Expression Tree

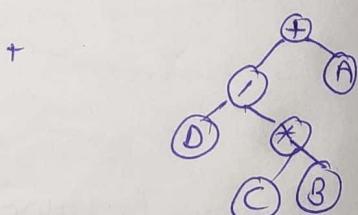
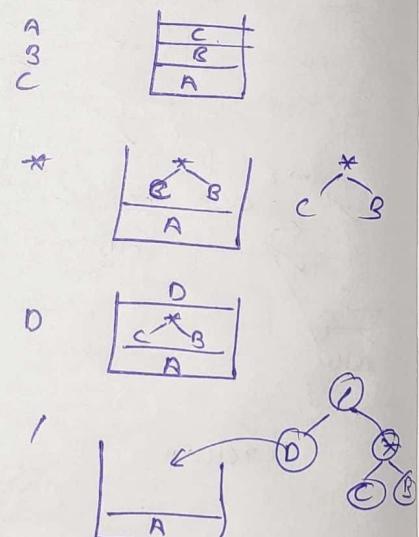
- ① Connect Infix to Postfix.
- ② Read all character
- ③ If character is operand, push in stack.
- ④ If " " operator
 - (i) POP two operands from stack
 - (ii) Tree construct with operator as root node & operands as right & left child
- ⑤ Push tree back in stack
- ⑥ Repeat until all characters read.

Eg. $(A+B)*(C+D)$

$AB+CD-*$

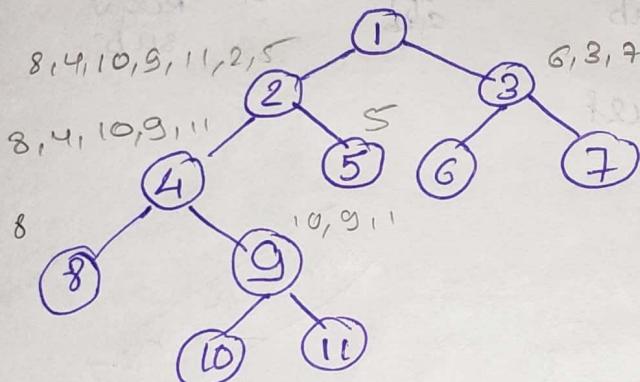


Eg. $A+B*C/D$
 $AB+C*D/+$



Constructing BT from Preorder & Inorder

Pre $\rightarrow 1, 2,$
First complete
left side



R LR Pre $\rightarrow \underline{1}, 2, 4, 8, 9, 10, 11, 5, 3, 6, 7$
L RR In $\rightarrow \underline{8}, \underline{4}, \underline{10}, \underline{9}, \underline{11}, \underline{2}, \underline{5}, \underline{1}, \underline{6}, \underline{3}, \underline{7}$

1) $\underline{1}$ is 1st element in Pre (which is Root)
2) Mark $\underline{1}$ in Inorder and left $\leftarrow \underline{1} \rightarrow$ Right Sub

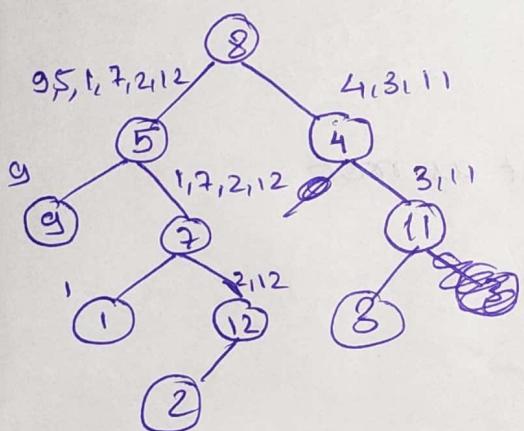
3) Out of left ele. the first one coming in Pre is child

Post & In

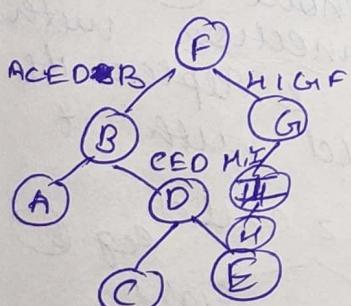
LRR Post $\rightarrow 9, 1, 2, 12, 7, 5, 3, 11, 4, 8$
R L R R In $\rightarrow \underline{9}, \underline{5}, \underline{1}, \underline{7}, \underline{2}, \underline{12}, \underline{8}, \underline{4}, \underline{3}, \underline{11}$

1) Last element in Post is root

2) Mark 8 in Inorder, 8 is left me left sub & right me right sub
3) In pre from right to left whatever comes 1st from left sub ele. is child
4) 1st complete left



Pre & Post (not possible to construct unique BT but possible to complete BT)



Pre $\rightarrow \underline{F}, \underline{B}, \underline{A}, \underline{C}, \underline{E}, \underline{D}, \underline{B}, \underline{G}, \underline{I}, \underline{H}$
Post $\rightarrow \underline{A}, \underline{C}, \underline{E}, \underline{D}, \underline{B}, \underline{F}, \underline{G}, \underline{I}, \underline{H}$

1) 1st element in Pre is root (F)
2) check next successor in Pre
3) mark it in Post

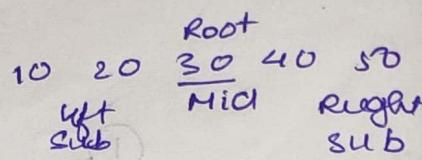
left + B $\leftarrow \underline{B} \rightarrow$ Right sub

Pre In $\rightarrow \underline{G}, \underline{I}, \underline{H}$
Post HI

3) Predecessor of F in Post
Mark \underline{G} in Pre
left $\leftarrow \underline{G} \rightarrow$ Right

Binary search tree

- ① Every node \rightarrow 0, 1, 2 child
- ② sort elements
- ③ Binary search
- ④ All elements of left sub must be \leq root.
- ⑤ All ele of right sub must be $>$ root.

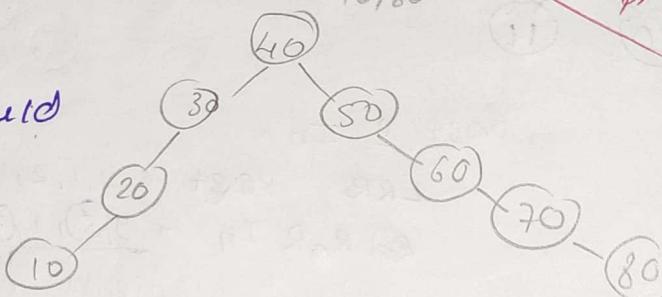


Operations \rightarrow

- ① Insertion
- ② Delete \leq
 \geq 1 child
- ③ Search
- ④ Min & Max

40, 30, 20, 50, 60, 70,
10, 80

Top + BST code



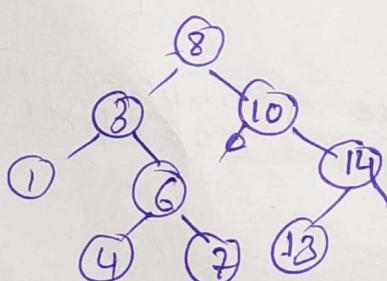
Insertion \rightarrow

- ① If tree is empty \rightarrow set element as root
- ② If element $<$ root \rightarrow Left sub
- ③ If " $>$ " \rightarrow Right sub.

Deletion \rightarrow

\rightarrow Deletion a node having no child:

8, 3, 10, 1, 6, 14, 4, 7, 13



② having 1 child
for eg ⑪
remove ⑭ node
connection with
⑩ & replace its
child with it

③ having 2 child
eg ⑧

left sub
replace
1 ele

right
replace
smallest
element

LEAF NODE \rightarrow 4, 7, 13

- ① Remove connection b/w leaf node & parent

Searching

- ① key == root (Mid ele is root)
return root;
- ② key < Root
key = NULL
return root;
- ③ key > Root
search space - left sub returns search (root → left, key)
search space - right sub
return root → right, key)

Min / Max element

min → left sub

max → right sub

AVL Tree

1) Type of Binary search tree.

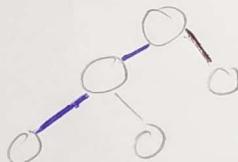
2) self balancing binary search tree

Balancing factor \leftarrow height of left sub - height of right sub

Balancing factor (-1) → Heavy Right
(0) → Balancing ~~AVL~~
(1) → Heavy Left.

$$\text{key. BF} = 2 - 1 = 1$$

left Heavy tree / heavy left.



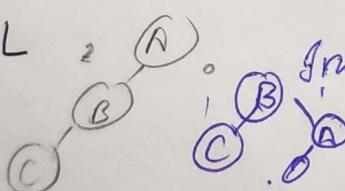
operations →

① Deletion

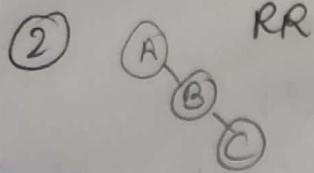
② Insertion

rotation →

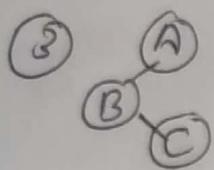
① LL ↗ (A)



Inserting node on left of left sub' to balance

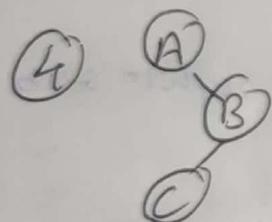
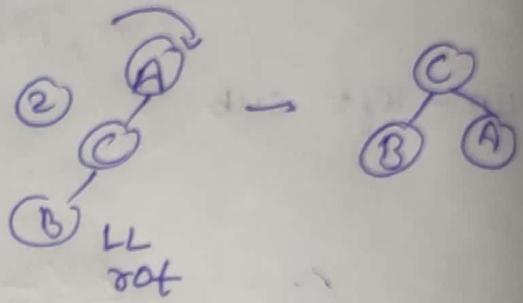
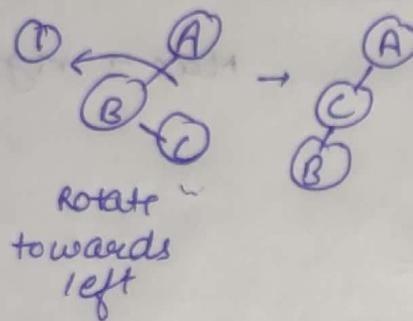


Inserting node on right of right sub
② A B C

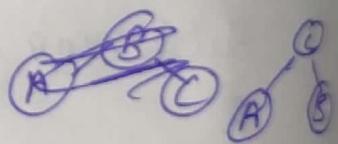
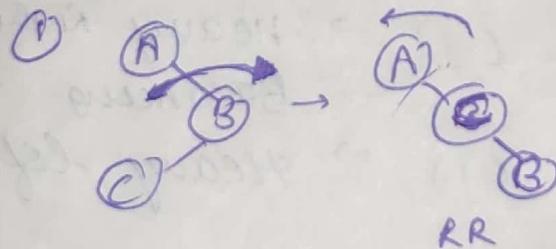


Inserting a node on right of left sub
③ A B C

↓
2 steps



Inserting node on left of left sub
④ A B C



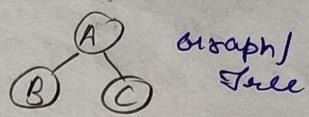
Red - Black Tree

① AVL type

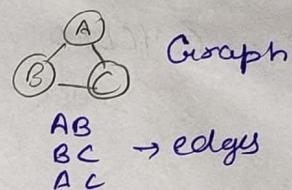
Graphs

, set of (V, E) pairs
 $V \rightarrow$ vertices set
 $E \rightarrow$ set of edges

- vertices \rightarrow represented as circles also known as nodes.
- edges \rightarrow represented as lines connecting 2 vertices/nodes



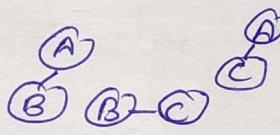
graph / tree
 $A, B, C \rightarrow$ vertices
 $\{A, B\}, \{A, C\} \rightarrow$ edges



Graph
 $AB, BC, AC \rightarrow$ edges

Terminology \rightarrow

① Adjacent nodes \rightarrow

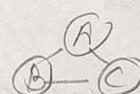


$A \& B$, $B \& C$, $C \& A$ which connect same edge

② Degree of Node \rightarrow No. of edges connected to that node

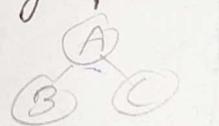


$A \rightarrow 2$
 $C \rightarrow 1$



$B \rightarrow 0$
 $C \rightarrow 2$

③ Size of graph \rightarrow Total no. of edges in graph

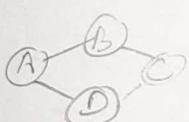


size = 2



size $\rightarrow 3$

④ path \rightarrow sequence of vertices from source node to destination node. (vertices & edges not repeat)



$A \rightarrow C$
 $A \rightarrow B$
 $B \rightarrow C$
 $B \rightarrow D$
 $C \rightarrow D$

$A \rightarrow B \rightarrow C$
 $A \rightarrow D \rightarrow C$

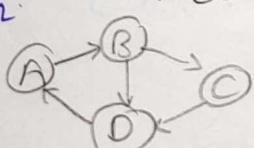
$\langle A, B, C \rangle$

⑤ Trail \rightarrow a walk in which no edge is repeated

Types of Graphs \rightarrow

① undirected

Directed or digraph



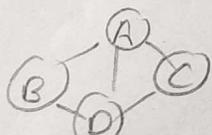
(B, C)

$(A, B) \neq (B, A)$

In 1 direction \Rightarrow

undirected

Bi-directional
 $\{B, C\}$

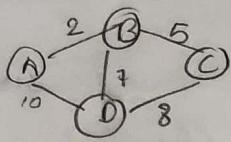


$(A, B) = (B, A)$

$(B, C) = (C, B)$

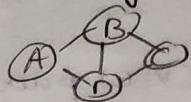
can go in any direction

② weighted

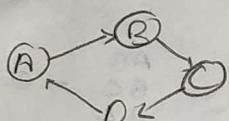


cost/weight is specified for every edge

unweighted



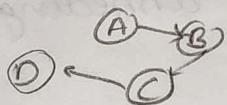
③ CYCLIC



starting node = end node

A → B → C → D → A

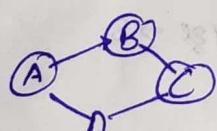
ACYCLIC



(A) self loop
A can have edge with itself
can be used to refresh a page

Representation -

① By using multidim. array → Adjacency Matrix

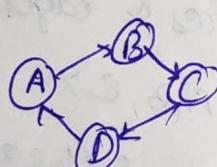


$$\begin{matrix} & \text{A} & \text{B} & \text{C} & \text{D} \\ \text{A} & 0 & 1 & 0 & 1 \\ \text{B} & 1 & 0 & 1 & 0 \\ \text{C} & 0 & 1 & 0 & 1 \\ \text{D} & 1 & 0 & 1 & 0 \end{matrix}$$

0 → if edge not
1 → if edge yes

A to A → 0

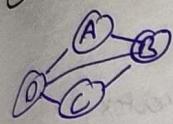
A to B → 1



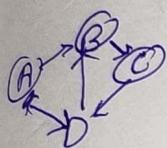
$$\begin{matrix} & \text{A} & \text{B} & \text{C} & \text{D} \\ \text{A} & 0 & 1 & 0 & 0 \\ \text{B} & 0 & 0 & 1 & 0 \\ \text{C} & 0 & 0 & 0 & 1 \\ \text{D} & 1 & 0 & 0 & 0 \end{matrix}$$

* V are represented as → {v₁, v₂, v₃}
as → {{v₁, v₂}, {v₃, v₄}}
as → {v₁, v₂, v₃, v₄}

② using list →



A:	<table border="1"><tr><td>B</td></tr><tr><td>→</td></tr><tr><td>D X</td></tr></table>	B	→	D X	→ A is having edge to B, D aerosuk		
B							
→							
D X							
B:	<table border="1"><tr><td>C</td></tr><tr><td>→</td></tr><tr><td>C X</td></tr></table>	C	→	C X	A: B,D B: A,C		
C							
→							
C X							
C:	<table border="1"><tr><td>B</td></tr><tr><td>→</td></tr><tr><td>D X</td></tr></table>	B	→	D X	C: D		
B							
→							
D X							
D:	<table border="1"><tr><td>A</td></tr><tr><td>→</td></tr><tr><td>B</td></tr><tr><td>→</td></tr><tr><td>C X</td></tr></table>	A	→	B	→	C X	A: A,B,C
A							
→							
B							
→							
C X							

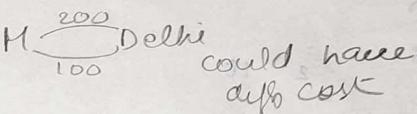


A:	<table border="1"><tr><td>B</td></tr><tr><td>→</td></tr><tr><td>D X</td></tr></table>	B	→	D X	A: B,D
B					
→					
D X					
B:	<table border="1"><tr><td>C</td></tr><tr><td>X</td></tr></table>	C	X	all O rep.	
C					
X					
C:	<table border="1"><tr><td>D</td></tr><tr><td>X</td></tr></table>	D	X	like this	
D					
X					
D:	<table border="1"><tr><td>B</td></tr><tr><td>X</td></tr></table>	B	X		
B					
X					

Properties →

① Multiedge / Parallel edge →

like in airlines



② No. of edges →

min → 0

$$\begin{matrix} v_0 & v_2 \\ v_3 & v_4 \end{matrix}$$

$$v = \{v_1, v_2, v_3, v_4\}$$

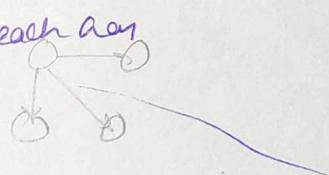
$$|V| = 4$$

$$|E| = \emptyset$$

max
in directed

→ nodes × edges each aay

$$\begin{matrix} 4 \times 3 \\ = 12 \end{matrix}$$



if $|V| = n$

then $0 \leq |E| \leq n(n-1)$ (directed)

$0 \leq |E| \leq \frac{n(n-1)}{2}$ (undirected)

Graph is called dense → too many edges
called sparse → less edges

Graph Representation

Vertex List
A
B
C
D
E
F

array can
be used to store
vertices

Edge List
BA
BA
RA
RA
AB
AB
AC
BC
CD
DE
EF
ED
CF
BF
AF

struct
can be
used to
store end
& starting vertex

struct edge

{ char *startvertex;
char *endvertex;

int weight;

};

