Lab 5

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Group 16

Pattern Recognition

FMNS • RUG

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1 Assignment 1

1.1 Exercise 1

As seen on the code below (Listing 1), we load and plot the two sample data we were provided, we plot feature 2 vs feature 1, and we can see in Figure 1 that the class A is split in two clusters on the left and on the right of the class B, this means that we have to use at least two prototypes for class A and one for class B. We can also see that the distribution of data points is almost equally distributed among the two features, therefore we can use Euclidean Distance to calculate the distance measure.

Listing 1: False acceptance, hit point

```
function LVQ1ex1()
2
        %Load data class_a and class_b
        load data_lvq_A . mat
3
        load data_lvq_B.mat
4
       \%Plot\ feature 2\ vs\ feature 1\ of\ both\ classes
6
        scatter(matA(:,1), matA(:,2),20);
        title('Class distribution scatter plot')
        xlabel('Feature 1')
        ylabel ('Feature 2')
10
        hold on;
11
12
        scatter(matB(:,1),matB(:,2),20);
13
        legend('Class A', 'Class B');
14
        hold off;
15
   end
16
```

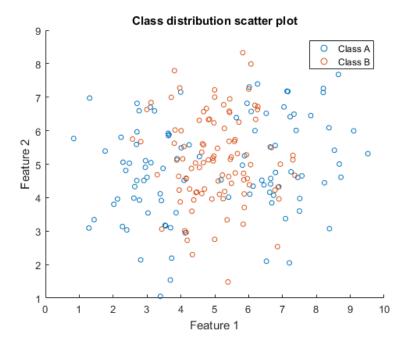


Figure 1: 2 dimension feature map of two different classes

1.2 Exercise 2

For class A we are goint to use two prototypes, since the data in class A is splitted in two clusters, one easy way to choose the starting prototypes is to choose two random points of class A, the first one with value lower than 4 and the second with value bigger than 6 on feature 1; for feature 2, we use a random initial value between the whole space of values (0 10). For class B we use a random value between 0 and 10 for both features.

The complete implementation of this algorithm is included on appendix 4.1, for the training phase we used a $\eta = 0.01$. We also introduced two control thresholds for the training of the algorithm, one for the maximum error rate we would like to have (errorThreshold=0.25) and the second to stop the training when the error rate converges after n epochs (variationThreshold=0.0001).

The code on appendix 4.1 also includes an animation to view the evolution of the prototypes after each epoch.

At the end of the training we obtain the following prototypes positions (Figure 2). We could observe through different η values, that if they are high values, sometimes the training converges after few epochs, but sometimes it never converges, furthermore the prototypes position is not too precise; when we use a small η value, the training takes more epochs to converge, but the result is more precise, the cost of this small value is the computational resources needed to compute the training prototypes.

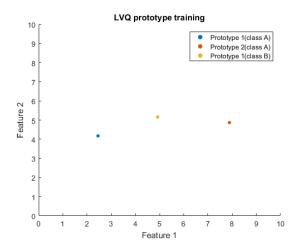


Figure 2: LVQ trained prototypes (2-1)

We also plotted the missclassification training error rate (Figure 3) to visualize how the training evolves after each epoch, due to the small training data, the error rate typically stays on 25%, that is why we added the errorThreshold control variable to this amount, and also because if we only used the variationThreshold between epochs, there were some scenarios where the two error rates were around 50% but since there was no variation in respect to the previous epoch value, the training stopped after few epochs and the prototypes were not fully trained.

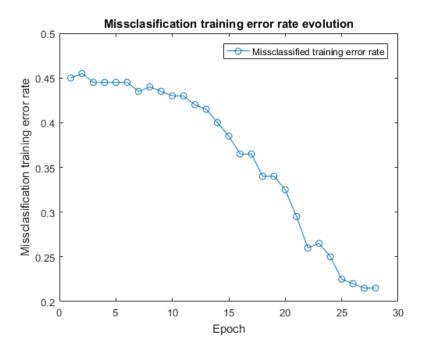


Figure 3: Missclassification error rate during training epochs

1.3 Exercise 3 a)

In this exercise we use one prototype for class A and one for class B, we had to adjust the errorThreshold control variable because the missclassification training error rate, is typically above 0.47, that is because, as we analyzed on figure 1, the class A is divided in two different clusters, therefore the center of mass of the cluster is similar to the center of mass of the class B cluster, which leads to classification error, and basically we have 0.47 probability of assigning the correct class to the incoming points.

This solution is not optimal because we cannot classify correctly the classes, we have to use at least two prototypes for class A.

These are the resulting prototypes after the training (Figure 4) and missclassification error through epochs (Figure 5).

We observe that when using (1-1) prototypes for class A prototype, it depends on the initial position of the prototype, and the prototype can never cross to the other cluster because class B is pushing it away, so it always stays on the cluster where it began, on the case of class B prototype, the prototype is also affected with this selection because when a new point from class A, and from the left cluster is presented, if the closest prototype is the prototype of class B, it gets pushed away, that is why is out of the center of mass of its own class data.

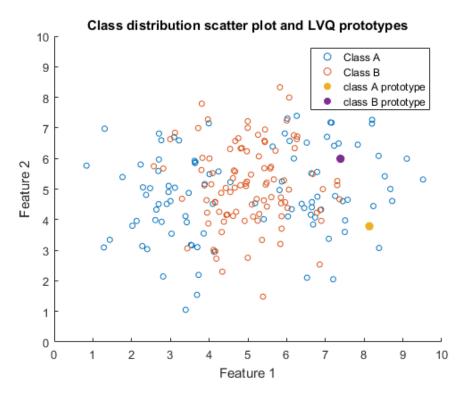


Figure 4: LVQ trained prototypes (1-1) with $\eta = 0.01$

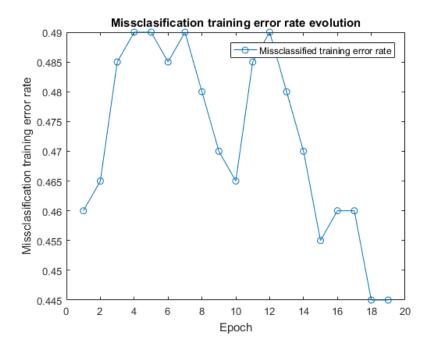


Figure 5: Missclasification error rate during training epochs with $\eta = 0.01$ (1-1)

As we see on figure 5 the missclassification error is too high, and the only way to stop the training is by accepting a big error rate and just waiting for two similar results to converge, but clearly one can see from this plot that along the epochs the missclassification training error rate does not seem to converge.

1.4 Exercise 3 b)

If we take one prototype from class A and two prototypes of class B, we can see that the behaviour is similar to the one in the previous exercise, the prototype of class A is pushed away of the class B prototypes (figure 6), causing that even after 20 epoches, the missclassification error is still high (figure 7), close to 0.5, the main difference now is that with more prototypes of the wrong class, you give more importance to this class and two prototypes push further away the protype of class A.

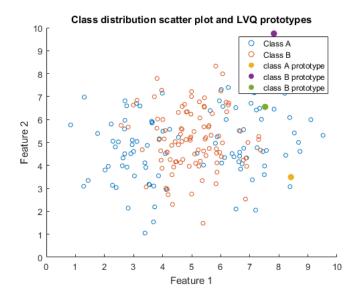


Figure 6: LVQ trained prototypes (1-2) with $\eta = 0.01$

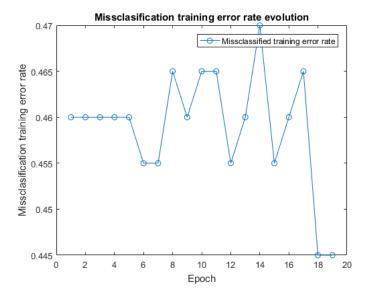


Figure 7: Missclasification error rate during training epochs with $\eta=0.01$ (1-2)

We still see no traces of convergence (Figure 7).

1.5 Exercise 3 c)

For this exercise we use two class A prototypes and one class B prototype (Figure 8), we are able to see that finally the system is able to converge (Figure 9), it even converges with a smaller missclassification error rate (clse to 0.3). We can conclude that this is a working solution for the sample data.

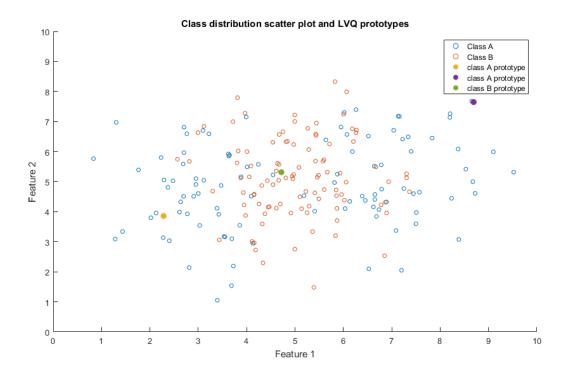


Figure 8: LVQ trained prototypes (2-1) with $\eta = 0.01$

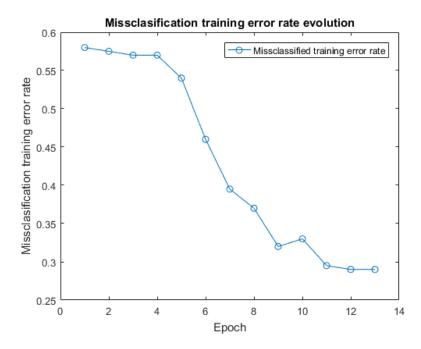


Figure 9: Missclasification error rate during training epochs with $\eta = 0.01$ (2-1)

Of course this solution only works if the two initial prototypes for the class A begin on a position close to each one of the clusters, otherwise they would be pushed away from the class B center and the prototype for class B would have been also moved away of its right position.

1.6 Exercise 3 d)

When using two and two prototypes of each class, we see that the general method still works, because the missclassification error rate tends to decrease and converge at lower error rate values (Figure 11), but not as low as when we used the minimum necessary prototypes number, this is due to the fact that the extra prototype of class A, pushes the class B prototypes a little further from itself while, "steals" some of the corrections that the other prototype would have gotten (Figure 10).

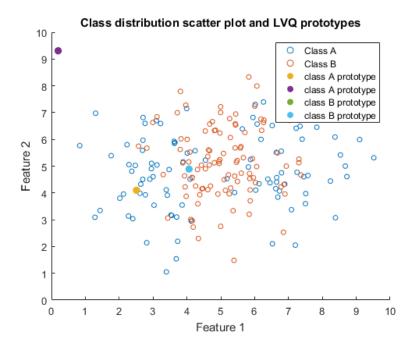


Figure 10: LVQ trained prototypes (2-2) with $\eta=0.01$

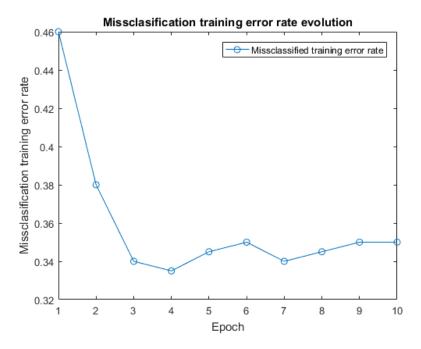


Figure 11: Missclasification error rate during training epochs with $\eta=0.01$ (2-2)

After all of the tests we can conclude that one has to use the minimal number of prototypes necessary for each class, that way the missclassification error decreases, and the final position of the prototypes is more precise.

The code for generating all the tests for exercise 3, is included in the appendix (Script 4.2)

1.7 Exercise 4 (bonus)

We could observe through different η values, that if they are high values, sometimes the training converges after few epochs, but sometimes it never converges, furthermore the prototypes position is not too precise; when we use a small η value, the training takes more epochs to converge, but the result is more precise, the cost of this small value is the computational resources needed to compute the training prototypes.

Below we can see an example using $\eta = 0.1$

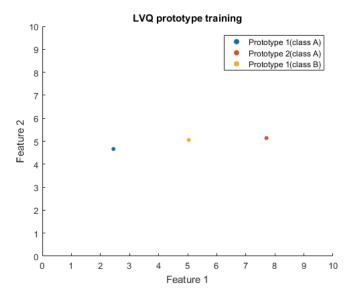


Figure 12: LVQ trained prototypes (2-1) with $\eta = 0.1$

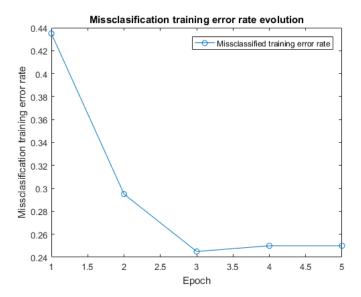


Figure 13: Missclasification error rate during training epochs with $\eta=0.1$

And an example using $\eta = 0.001$

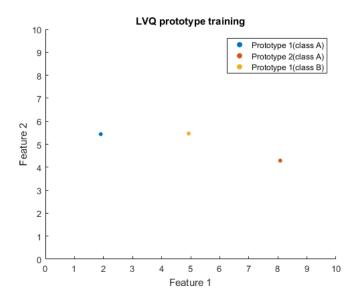


Figure 14: LVQ trained prototypes (2-1) with $\eta = 0.001$

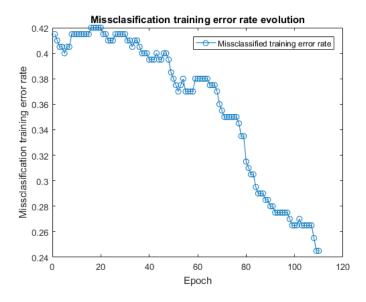


Figure 15: Missclassification error rate during training epochs with $\eta = 0.001$

2 Assignment 2

2.1 Exercise 1

We modified the script from assignment 1 exercise 3, to use ten folding validation method, this means, we divided the sample data into 10 groups and during ten iterations we left one different group for testing the LVQ after training the prototypes with the other 9 groups, we used 2 prototypes for class A and 1 prototype for class B. We obtained the same prototypes positions (Figure 16), and the following classification error for each fold (Figure 17).

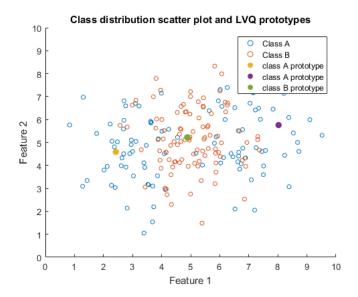


Figure 16: LVQ trained prototypes (2-1) with $\eta = 0.01$, using ten folding technique

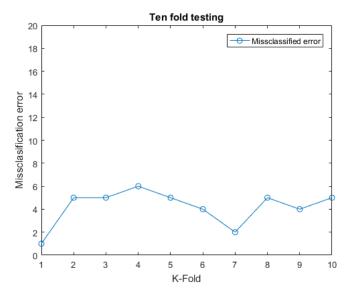


Figure 17: Testing error using ten folding technique, for prototypes (2-1) and $\eta=0.01$

We used the matlab function:

```
indices = crossvalind('Kfold', data(:,3),10);
```

To randomly split the data into the ten folding samples.

The complete code used is on the appendix section under script 4.3.

2.2 Exercise 2

In order to compute the total test error, we calculated the average of the test errors of each fold (Figure 17) and simply computed the mean with the following instruction, obtaining a result of 4.2

```
%The test error is the mean of the classification errors
mean(testingError);
```

3 Assignment 3

3.1 Exercise 1

We implemented the GRLVQ algorithm using initial lambda values of 0.5, the complete code can be seen on appendix 4.4.

For calculating the new lambda, we used the following formula, using the Euclidean distance.

```
lambda1=lambda1-etha*phi*pdist2(point, prototype);
```

We can see that the distance the prototype is adjusted get determined by how far the prototype is of its final location.

3.2 Exercise 2

The final relevances after each epoch can be seen on the following plot (Figure 18). We can see that they start separating as the prototypes converge.

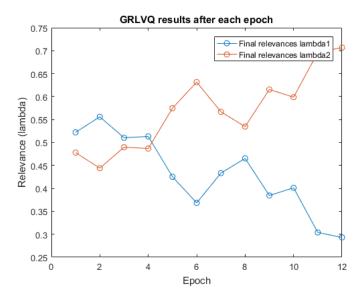


Figure 18: Final relevances (lambda) values after each epoch for prototypes (2-1) and $\eta = 0.01$

3.3 Exercise 3

The final LVQ prototypes are the following (Figure 19), the prototypes always converge, even if the initial prototypes are far from its ideal location.

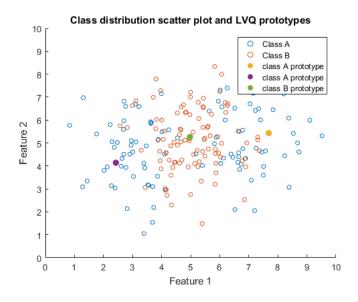


Figure 19: Final prototypes using GRLVQ for (2-1) prototypes and $\eta = 0.01$

3.4 Exercise 4

Below we can see the training error E after each epoch validation (Figure 20) and the relevances as a function of the epochs (Figure 21). We can observer that at the beginning the training started with two initial values well located, that is why only after 12 epochs the training phase finished.

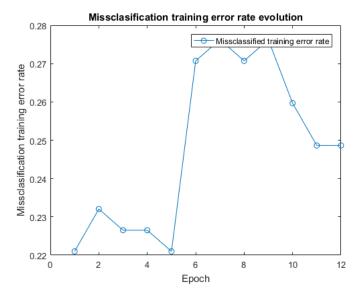


Figure 20: Training error after each epoch using (2-1) prototypes and $\eta = 0.01$

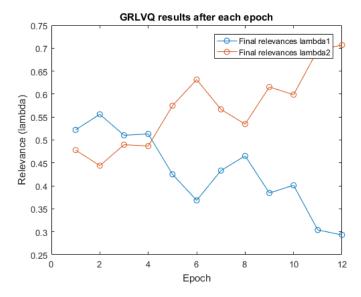


Figure 21: Final relevances (lambda) values after each epoch for prototypes (2-1) and $\eta = 0.01$

3.5 Exercise 5

Using the ten fold cross validation we obtain the following results (Figure 22), with a testing group of 20 samples, we can observe that the error always remain low.

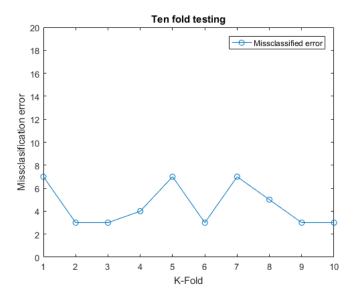


Figure 22: Errors using ten fold validation for (2-1) and $\eta = 0.01$

3.6 Exercise 6

We compute the test error using the mean of the ten fold cross validation errors, and the result is 4.5, the complete code can be seen on the appendix (script 4.4), this means that 4.5 out of 20 test were correct, is a hit rate around 77.5%.

4 Appendix

4.1 Script for assignment 1, exercise 2

```
function LVQ1ex2(eta)
%Load data class_a and class_b
load data_lvq_A.mat
load data_lvq_B.mat

%Add category label 0=class A, 1=class B
matA(:,3)=0;
matB(:,3)=1;

%Concatenate the two matrices
data=vertcat(matA,matB);
```

```
12
        %Randomly permute the rows, so we have an unbiased training data
13
        data2=data(randperm(length(data(:,1))),:);
14
15
       \mbox{\ensuremath{\mbox{\tiny WW}}}\mbox{\ensuremath{\mbox{\tiny obtain}}} the final position of the three prototypes after N
16
        %epochs, where the variation between epochs becomes smaller than the
17
        %threshold
18
        [prototype1A, prototype2A, prototype1B, epochNumber, epochError]=LVQeval(data2, eta);
19
20
        x=1:1:length(epochError);
21
22
        %Plot of the missclasification training error rate
23
        figure
24
25
        plot(x,epochError, '-o');
            \%axis([0 10 0 10]);
26
        title ('Missclasification training error rate evolution')
27
        xlabel('Epoch')
28
        ylabel ('Missclasification training error rate')
29
        legend('Missclassified training error rate');
30
   end
31
32
   function [prototype1A, prototype2A, prototype1B, epochNumber, epochError]=LVQeval(data, eta)
33
        %Randomly generate two prototypes for class A
34
        prototype1A = [rand()*4, rand()*9];
35
        prototype2A = [6 + rand() *4, rand() *9];
36
37
        %Randomly generate one prototype for class B
38
39
        prototype1B = [rand()*10, rand()*9];
40
        %Missclasiffied training error
41
42
        newMTE=1:
        MTEdiff=1;
43
        variationThreshold = 0.0001;
44
        errorThreshold=0.25;
45
46
        epochNumber=0;
47
48
        %Run epochs until the eror difference becomes smaller than threshold
49
        while (MTEdiff>variationThreshold | | newMTE>errorThreshold)
50
51
            epochNumber=epochNumber+1;
52
            oldMTE=newMTE;
53
54
             [newMTE, prototype1A, prototype2A, prototype1B]=epoch(data, prototype1A, prototype2A,
55
                 prototype1B, eta);
56
            MTEdiff=abs (oldMTE-newMTE);
57
            We save the missclassified training error for each epoch
58
            epochError(epochNumber)=newMTE;
59
60
            %Animated plotting
61
             scatter(prototype1A(1),prototype1A(2),20,'filled');
62
             axis([0 10 0 10]);
63
             title('LVQ prototype training')
64
             xlabel('Feature 1')
65
             ylabel ('Feature 2')
66
             hold on;
67
68
             scatter(prototype2A(1),prototype2A(2),20,'filled');
scatter(prototype1B(1),prototype1B(2),20,'filled');
69
70
             legend('Prototype 1(class A)', 'Prototype 2(class A)', 'Prototype 1(class B)');
71
```

```
hold off
72
73
             pause (0.10);
74
         \quad \text{end} \quad
75
    end
76
77
    function [trainingErrorRate, prototype1A, prototype2A, prototype1B]=epoch(data, prototype1A,
78
         prototype2A, prototype1B, eta)
    %Learning phase
79
         mte=0;
80
81
         for i=1:1: size (data(:,1))
82
83
              point=data(i,1:2);
84
85
              %Find the class of the closest prototype
86
              closestPrototype=WinnerEuc(point, prototype1A, prototype2A, prototype1B);
87
88
89
              switch closestPrototype
                   case '1A'
90
91
                        %Compare if they belong to the same class
                        if(data(i,3)==0)
92
                             prototype1A=newPosition(prototype1A, point, eta, 1);
93
94
                             prototype1A=newPosition(prototype1A, point, eta, -1);
95
                             mte=mte+1;
96
                        end
97
                   case '2A'
98
                        %Compare if they belong to the same class
99
                        if(data(i,3) == 0)
100
101
                             prototype2A=newPosition(prototype2A, point, eta, 1);
102
                             prototype2A = newPosition(prototype2A, point, eta, -1);
103
104
                             mte=mte+1;
                        end
105
                   case '1B'
106
                        %Compare if they belong to the same class
107
                        if(data(i,3)==1)
108
                             prototype1B=newPosition(prototype1B, point, eta, 1);
109
110
                             prototype1B = newPosition \, (\, prototype1B \, , \ point \, , \ eta \, , \ -1) \, ;
111
                             mte=mte+1;
112
                        end
113
              end
114
115
116
         trainingErrorRate=mte/length(data(:,1));
117
118
    %Return the closest prototype to the point
119
    function minD=WinnerEuc(point, prototype1A, prototype2A, prototype1B)
120
         d1A \hspace{-0.05cm}=\hspace{-0.05cm} pdist2 \hspace{0.05cm} (\hspace{0.05cm} point\hspace{0.1cm} , \hspace{0.1cm} prototype1\hspace{0.1cm} A\hspace{0.1cm} ) \hspace{0.1cm} ;
121
         minD='1A';
122
123
         d2A=pdist2(point, prototype2A);
124
125
         if(d2A < d1A)
              minD='2A';
126
127
128
         d1B=pdist2(point, prototype1B);
129
130
         if (d1B<d1A & d1B<d2A)
              minD='1B';
131
```

```
end
end

function w=newPosition(prototype, point, eta, phi)
w=prototype+(eta*phi*(point-prototype));
end

end
```

4.2 Script for assignment 1, exercise 3

```
function LVQ1ex3a(etha, noProtA, noProtB)
1
    Load\ data\ class_a\ and\ class_b
       load data_lvq_A.mat
3
       load data_lvq_B.mat
4
5
       %Add category label 0=class A, 1=class B
6
       matA(:,3) = 0;
       matB(:,3)=1;
8
       %Concatenate the two matrices
10
       data=vertcat (matA, matB);
11
12
       %Randomly permute the rows, so we have an unbiased training data
13
       data2=data(randperm(length(data(:,1))),:);
14
15
       We obtain the final position of the three prototypes after N
16
17
       %epochs, where the variation between epochs becomes smaller than the
       %threshold
18
19
        [prototype,epochNumber,epochError]=LVQeval(data2, etha,noProtA,noProtB);
20
21
       %Plot feature2 vs feature1 of both classes
22
       figure
        scatter(matA(:,1),matA(:,2),20,'DisplayName','Class A');
23
        axis([0 10 0 10]);
24
        title ('Class distribution scatter plot and LVQ prototypes')
25
        xlabel('Feature 1')
26
        ylabel ('Feature 2')
27
       hold on;
28
29
        scatter(matB(:,1),matB(:,2),20,'DisplayName','Class B');
30
31
        for(i=1:length(prototype(:,1)))
32
33
            if(prototype(i,3)==0)
34
                strLeg='class A prototype';
35
36
                strLeg='class B prototype';
37
38
            scatter(prototype(i,1),prototype(i,2),50,'filled','DisplayName',strLeg);
39
40
41
       legend('show');
42
43
       hold off;
44
       x=1:1:length (epochError);
45
46
       %Plot of the missclasification training error rate
47
48
       plot(x,epochError, '-o');
49
           %axis([0 10 0 10]);
50
        title ('Missclasification training error rate evolution')
51
```

```
xlabel('Epoch')
ylabel('Missclasification training error rate')
52
53
        legend('Missclassified training error rate');
54
55
56
    end
57
    function [prototype,epochNumber,epochError]=LVQeval(data, etha, noProtA, noProtB)
58
        %Randomly generate the prototype matrix, column 3 identifies the class
59
        \%(A=0, B=1)
60
        for ( i = 1:(noProtA+noProtB))
61
             if (i<=noProtA)
62
                 prototype(i,:) = [rand()*10, rand()*10, 0];
63
64
                 prototype(i,:) = [rand()*10, rand()*10,1];
65
             end
66
67
68
        %Missclasiffied training error
69
70
        newMTE=1;
        MTEdiff=1;
71
72
         variationThreshold = 0.0001;
        errorThreshold = 0.45;
73
74
        epochNumber=0;
75
76
        %Run epochs until the eror difference becomes smaller than threshold
77
        %while (MTEdiff>variationThreshold)
78
79
        while (MTEdiff>variationThreshold | | newMTE>errorThreshold)
80
             epochNumber=epochNumber+1
81
82
             oldMTE=newMTE;
83
             [newMTE, prototype] = epoch (data, prototype, etha);
84
             disp (newMTE)
85
             MTEdiff=abs (oldMTE-newMTE)
86
             We save the missclassified training error for each epoch
87
             epochError (epochNumber)=newMTE;
88
89
             strLeg='';
90
91
            %Animated plotting
92
             for ( i = 1: length ( prototype (:,1) ))
93
94
                 if(prototype(i,3)==0)
95
                      strLeg='class A';
96
97
                 else
                      strLeg='class B';
98
99
                 end
100
                 scatter(prototype(i,1), prototype(i,2),50, 'filled', 'DisplayName', strLeg);
101
                 axis([0 10 0 10]);
102
                  title ('LVQ prototype training')
103
                 xlabel('Feature 1')
104
                  ylabel ('Feature 2')
105
                 hold on;
106
107
             end
108
109
             legend('show');
110
             hold off
111
112
```

```
pause (0.10);
113
         end
114
    end
115
116
    function [trainingErrorRate, prototype] = epoch(data, prototype, etha)
117
    %Learning phase
118
         mte=0;
119
120
         for i=1:1: size (data(:,1))
121
122
              point=data(i,1:2);
123
124
             %Find the class of the closest prototype
125
126
             closestPrototype=WinnerEuc(point, prototype);
127
             %Compare if they belong to the same class
128
              if (data(i,3)=prototype(closestPrototype,3))
129
                  prototype \, (\, closestPrototype \,\, , 1:2 \, ) = newPosition \, (\, prototype \, (\, closestPrototype \,\, , 1:2 \, ) \,\, ,
130
                       point, etha, 1);
              else
131
132
                  prototype (closestPrototype, 1:2) = newPosition (prototype (closestPrototype, 1:2),
                       point, etha, -1);
133
                  mte=mte+1;
134
             end
         end
135
         trainingErrorRate=mte/length(data(:,1));
136
    end
137
138
    Return the closest prototype to the point
139
    function closestPrototypeIndex=WinnerEuc(point, prototype)
140
141
         oldD = 100;
         for (i=1:length(prototype(:,1)))
142
             d=pdist2(point, prototype(i,1:2));
143
144
              if (d<oldD)</pre>
145
                  closestPrototypeIndex=i;
146
                  oldD=d;
147
             end
148
         end
149
150
    end
151
    function w=newPosition(prototype, point, etha, phi)
152
         w=prototype+(etha*phi*(point-prototype));
153
    end
154
```

4.3 Script for assignment 2

```
function LVQ2(etha, noProtA, noProtB)
1
    %Load data class_a and class_b
2
       load data_lvq_A.mat
       load data_lvq_B.mat
4
       \% Add category label 0=class A, 1=class B
6
       matA(:,3) = 0;
       matB(:,3)=1;
9
       %Concatenate the two matrices
10
       data=vertcat (matA, matB);
11
12
       %Randomly permute the rows, so we have an unbiased training data
13
```

```
data2=data(randperm(length(data(:,1))),:);
14
15
16
        We obtain the final position of the three prototypes after N
17
        %epochs, where the variation between epochs becomes smaller than the
18
        %threshold
19
        [prototype, testingError]=LVQeval(data2, etha, noProtA, noProtB);
20
21
        %Plot feature2 vs feature1 of both classes
22
        figure
23
        scatter (matA(:,1), matA(:,2),20, 'DisplayName', 'Class A');
24
        axis([0 10 0 10]);
25
        title ('Class distribution scatter plot and LVQ prototypes')
26
        xlabel('Feature 1')
27
        ylabel ('Feature 2')
28
        hold on;
29
30
        scatter(matB(:,1),matB(:,2),20,'DisplayName','Class B');
31
32
        for (i=1:length (prototype (:,1)))
33
34
             if(prototype(i,3)==0)
35
                  strLeg='class A prototype';
36
37
                  strLeg='class B prototype';
38
39
             end
             scatter\left(\,prototype\left(\,i\,\,,1\right)\,,prototype\left(\,i\,\,,2\right)\,,50\,,\,'\,filled\,\,'\,,\,'DisplayName\,'\,,strLeg\,\right);
40
41
42
        legend('show');
43
44
        hold off;
45
        x=1:1:length (testingError);
46
47
        %Plot of the missclasification training error rate
48
49
        figure
        plot(x, testingError, '-o');
50
        axis([1 10 0 20]);
51
        title ('Ten fold testing')
52
        xlabel('K-Fold')
53
        ylabel ('Missclasification error')
54
        legend('Missclassified error');
55
56
        %The test error is the mean of the classification errors
57
        disp(mean(testingError));
58
59
   end
60
61
    function [prototype, testing Error] = LVQeval(data, etha, noProtA, noProtB)
62
        %Randomly generate the prototype matrix, column 3 identifies the class
63
        \%(A=0, B=1)
64
        for ( i = 1:(noProtA+noProtB) )
65
66
             if (i \le noProtA)
                  prototype\,(\,i\,\,,:\,)=\![\,\mathbf{rand}\,(\,)*10\,,\!\mathbf{rand}\,(\,)*10\,,\!0\,]\,;
67
             else
68
                  prototype(i,:) = [rand()*10, rand()*10,1];
69
             end
70
        end
71
72
        indices = crossvalind('Kfold', data(:,3),10);
73
74
        data(:,4)=indices;
```

```
75
        %Run ten-fold training validatino
76
        for (i=1:1:10)
77
             trainData = [0 \ 0 \ 0];
78
             testData = [0 \ 0 \ 0];
79
             for (j=1:1:200)
80
                 if (data(j,4)~=i)
81
                      trainData(length(trainData(:,1))+1,:)=data(j,1:3);
82
83
                      testData(length(testData(:,1))+1,:)=data(j,1:3);
84
                 end
85
             end
86
87
             [testingError(i), prototype] = epoch(trainData, testData, prototype, etha);
88
89
            %Animated plotting
90
             for(i=1:length(prototype(:,1)))
91
92
93
                  if(prototype(i,3)==0)
                      strLeg='class A';
94
95
                      strLeg='class B';
96
                 end
97
98
                 scatter(prototype(i,1), prototype(i,2),50, 'filled', 'DisplayName', strLeg);
99
                 axis([0 10 0 10]);
100
                  title ('LVQ prototype training')
101
102
                 xlabel ('Feature 1')
                 ylabel ('Feature 2')
103
                 hold on;
104
105
             end
106
107
             legend('show');
108
             hold off
109
110
            pause (0.10);
111
        end
112
    end
113
114
    function [testingError, prototype] = epoch(data, testData, prototype, etha)
115
    %Learning phase
116
        testingError = 0;
117
118
         for i=2:1: size (data(:,1))
119
120
             point=data(i,1:2);
121
122
             %Find the class of the closest prototype
123
             closestPrototype=WinnerEuc(point, prototype);
124
125
             %Compare if they belong to the same class
126
             if (data(i,3)=prototype(closestPrototype,3))
127
                 prototype (closestPrototype, 1:2)=newPosition(prototype(closestPrototype, 1:2),
128
                      point, etha, 1);
             else
129
                 prototype (closestPrototype, 1:2) = newPosition (prototype (closestPrototype, 1:2),
130
                      point, etha, -1);
             end
131
        end
132
133
```

```
for i = 2:1: size (testData(:,1))
134
135
              point=testData(i,1:2);
              %Find the class of the closest prototype
136
              closestPrototype=WinnerEuc(point, prototype);
137
138
              %Compare if they belong to the same class if (\text{testData}(i,3)^{\sim}=\text{prototype}(\text{closestPrototype},3))
139
140
                   testingError=testingError+1;
141
              end
142
         end
143
144
145
    %Return the closest prototype to the point
146
    function closestPrototypeIndex=WinnerEuc(point, prototype)
         oldD=100:
148
         for(i=1:length(prototype(:,1)))
149
              d=pdist2(point, prototype(i,1:2));
150
151
152
              if (d<oldD)
                   closestPrototypeIndex=i;
153
154
                   oldD=d;
              end
155
         end
156
157
    end
158
    function w=newPosition(prototype, point, etha, phi)
159
         w=prototype+(etha*phi*(point-prototype));
160
161
```

4.4 Script for assignment 3

```
function LVQ3(etha, noProtA, noProtB)
1
    %Load data class_a and class_b
        load data_lvq_A.mat
3
        load data_lvq_B.mat
5
        %Add category label 0=class A, 1=class B
6
        matA(:,3) = 0;
        matB(:,3) = 1;
10
        %Concatenate the two matrices
        data=vertcat (matA, matB);
11
12
        %Randomly permute the rows, so we have an unbiased training data
13
14
        data2=data(randperm(length(data(:,1))),:);
15
16
        We obtain the final position of the three prototypes after N
        %epochs, where the variation between epochs becomes smaller than the
17
        %threshold
18
        [\ prototype\ , epochNumber\ , epochError\ , testingError\ , lambda1\ , lambda2] = LVQeval(\ data2\ , \ etha\ , \ lambda2) = LVQeval(\ data2\ , \ etha\ , \ lambda2)
             noProtA, noProtB);
20
        %Plot feature2 vs feature1 of both classes
21
        figure
22
        scatter(matA(:,1), matA(:,2), 20, 'DisplayName', 'Class A');
23
        axis([0 10 0 10]);
24
        title ('Class distribution scatter plot and LVQ prototypes')
25
        xlabel ('Feature 1')
26
        ylabel ('Feature 2')
27
        hold on;
28
```

```
29
        scatter(matB(:,1),matB(:,2),20,'DisplayName','Class B');
30
31
32
        for(i=1:length(prototype(:,1)))
33
             if(prototype(i,3)==0)
34
                 strLeg='class A prototype';
35
36
                 strLeg='class B prototype';
37
            end
38
             scatter(prototype(i,1),prototype(i,2),50,'filled','DisplayName',strLeg);
39
        end
40
41
42
        legend('show');
        hold off;
43
44
        x=1:1:length (epochError);
45
46
       %Plot of the missclasification training error rate
47
        figure
48
        plot(x,epochError, '-o');
49
            %axis([0 10 0 10]);
50
        title ('Missclasification training error rate evolution')
51
        xlabel('Epoch')
52
        ylabel('Missclasification training error rate')
legend('Missclassified training error rate');
53
54
55
56
       %Plot of the clasification error
        x=1:1:length (testingError);
57
        figure
58
59
        plot(x, testingError, '-o');
        axis([1 10 0 20]);
60
        title ('Ten fold testing')
61
        xlabel('K-Fold')
62
        ylabel('Missclasification error')
legend('Missclassified error');
63
64
65
       %The test error is the mean of the classification errors
66
        disp(mean(testingError));
67
68
       %Plot of the final relevances after each epoch
69
        x=1:1:length(lambda1);
70
71
        figure
        plot(x,lambda1, '-o', 'DisplayName', 'Final relevances lambda1');
72
        title ('GRLVQ results after each epoch')
73
        xlabel('Epoch')
74
        ylabel ('Relevance (lambda)')
75
76
        hold on;
77
78
        plot(x,lambda2,'-o','DisplayName','Final relevances lambda2');
79
        legend('show');
80
        hold off;
81
82
   end
83
84
   function [prototype,epochNumber,epochError,testingError,lambda1E,lambda2E]=LVQeval(data,
        etha, noProtA, noProtB)
        %Randomly generate the prototype matrix, column 3 identifies the class
86
        \%(A=0, B=1)
87
       for ( i = 1:(noProtA+noProtB))
88
```

```
if (i<=noProtA)</pre>
89
                   prototype\,(\,i\,\,,:\,)=\![\,\mathbf{rand}\,(\,)*10\,,\!\mathbf{rand}\,(\,)*10\,,\!0\,]\,;
90
91
                   prototype(i,:) = [rand()*10, rand()*10,1];
92
              end
93
94
95
         indices = crossvalind('Kfold', data(:,3),10);
96
         data(:,4)=indices;
97
98
         %Run ten-fold training validatino
99
         for (i=1:1:10)
100
              trainData = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix};
101
102
              testData = [0 \ 0 \ 0];
              for (j=1:1:200)
103
                   if (data(j,4)~=i)
104
                        trainData(length(trainData(:,1))+1,:)=data(j,1:3);
105
106
                        testData(length(testData(:,1))+1,:)=data(j,1:3);
107
                   end
108
109
              end
110
              %Missclasiffied training error
111
112
              newMTE=1;
              MTEdiff=1;
113
              variationThreshold = 0.0001;
114
              errorThreshold = 0.25;
115
116
              epochNumber=0;
117
118
              %Run epochs until the eror difference becomes smaller than threshold
119
              %while (MTEdiff>variationThreshold)
120
              while (MTEdiff>variationThreshold | newMTE>errorThreshold)
121
122
                   epochNumber=epochNumber+1;
123
                   oldMTE=newMTE;
124
125
                   [newMTE, prototype, testingError(i), lambda1, lambda2]=epoch(trainData, testData,
126
                        prototype , etha);
127
                   \label{eq:mtediff}  \text{MTEdiff} = & abs \left( \text{oldMTE--newMTE} \right); 
128
                   We save the missclassified training error for each epoch
129
                   epochError (epochNumber)=newMTE;
130
                   lambda1E(epochNumber)=lambda1;
131
                   lambda2E(epochNumber)=lambda2;
132
133
                   strLeg='';
134
135
                   %Animated plotting
                   for ( i = 1: length ( prototype (:,1) ))
136
137
                        if(prototype(i,3)==0)
138
                             strLeg='class A';
139
140
                        else
                             strLeg='class B';
141
142
143
                        scatter(prototype(i,1),prototype(i,2),50,'filled','DisplayName',strLeg);
144
                        axis([0 10 0 10]);
145
                        title('LVQ prototype training')
146
                        xlabel('Feature 1')
147
                        ylabel ('Feature 2')
148
```

```
hold on;
149
150
                                        end
151
152
                                        legend('show');
153
                                        hold off
154
155
                                      pause (0.10);
156
                             end
157
                   end
158
         end
159
160
         function [trainingErrorRate, prototype, testingError, lambda1, lambda2] = epoch(data, testData,
161
                   prototype, etha)
         %Learning phase
162
                   mte=0;
163
                   lambda1 = 0.5;
164
                   lambda2 = 0.5;
165
166
                    testingError=0;
                    for i=1:1: size (data(:,1))
167
168
                             point=data(i,1:2);
169
170
                             %Find the class of the closest prototype
171
                             closestPrototype=WinnerEuc(point, prototype);
172
173
                             %Compare if they belong to the same class
174
175
                              if (data(i,3)=prototype(closestPrototype,3))
                                        [prototype(closestPrototype, 1:2), lambda1, lambda2]=newPosition(prototype(
176
                                                  closestPrototype\ , 1:2)\ ,\ point\ ,\ etha\ ,\ 1\ , lambda1\ , lambda2\ , data\left(i\ , 3\right))\ ;
177
                              else
                                         [\;prototype\,(\;closestPrototype\;,1:2)\;,lambda1\;,lambda2] = newPosition\,(\;prototype\,(\;archive archive 
178
                                                  closestPrototype, 1:2), point, etha, -1,lambda1,lambda2,data(i,3));
179
                                        mte=mte+1;
                             end
180
                   end
181
                    trainingErrorRate=mte/length(data(:,1));
182
183
         %Testing phase
184
                    for i=2:1: size (testData(:,1))
185
186
                             point=testData(i,1:2);
                             %Find the class of the closest prototype
187
                             closestPrototype=WinnerEuc(point, prototype);
188
189
                             %Compare if they belong to the same class
190
                              if(testData(i,3)~=prototype(closestPrototype,3))
191
                                        testingError=testingError+1;
192
193
                             end
                   end
194
195
196
         %Return the closest prototype to the point
197
         function closestPrototypeIndex=WinnerEuc(point, prototype)
198
                   oldD = 100;
199
                    for(i=1:length(prototype(:,1)))
200
                             d=pdist2(point, prototype(i,1:2));
201
202
                              if (d<oldD)
203
                                        {\tt closestPrototypeIndex=} i \; ; \\
204
                                        oldD=d;
205
206
                             end
```

```
\quad \text{end} \quad
207
      end
208
209
       function \ [w, lambda1\,, lambda2] = new Position (\,prototype\,,\ point\,,\ etha\,,\ phi\,,\ lambda1\,,\ lambda2\,,
210
            class)
            if(class==0)
211
                  w=prototype+(etha*phi*lambda1*(point-prototype));
212
                  %Calculate the new lambda
213
                  lambda1 \!\!=\! lambda1 \!\!-\! etha \! *\! phi \! *\! pdist2 \, (\, point \, , prototype \, ) \, ;
214
                  %Enforce
215
                  lambda2\!\!=\!\!1\!\!-\!lambda1\,;
216
            else
^{217}
                  w\!\!=\!\!\operatorname{prototype}\!+\!\!\left(\operatorname{etha*phi*lambda2*}(\operatorname{point-prototype})\right);
218
                  %Calculate the new lambda
219
                  lambda2\!\!=\!\!lambda2\!-\!etha*phi*pdist2\,(\,point\,,prototype\,)\;;
220
221
                  \% Enforce
                  lambda1\!\!=\!\!1\!\!-\!lambda2\,;
222
            end
223
     end
224
```