Formal Modeling of Communicating System - Homework 2

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1 Exercise 3.2

1.1 Question 1

Let **P** $\underline{\underline{def}}$ **(CA|CTM)** $\backslash$**{coin,coffee,tea}** and **Q** $\underline{\underline{def}}$ **(CA|CTM')** $\backslash$**{coin,coffee,tea}** which are obtained from the following processes:

- CA $\underline{\underline{def}}$ $\overline{coin}$.coffee.CA
- CTM $\underline{\underline{def}}$ coin.($\overline{coffee}$.CTM + $\overline{tea}.CTM$)
- CTM' $\underline{\underline{def}}$ coin.$\overline{coffee}$.CTM' + coin.$\overline{tea}.CTM'$

If we draw the transition graph of both processes P and Q, the results should be similar to the following figures:

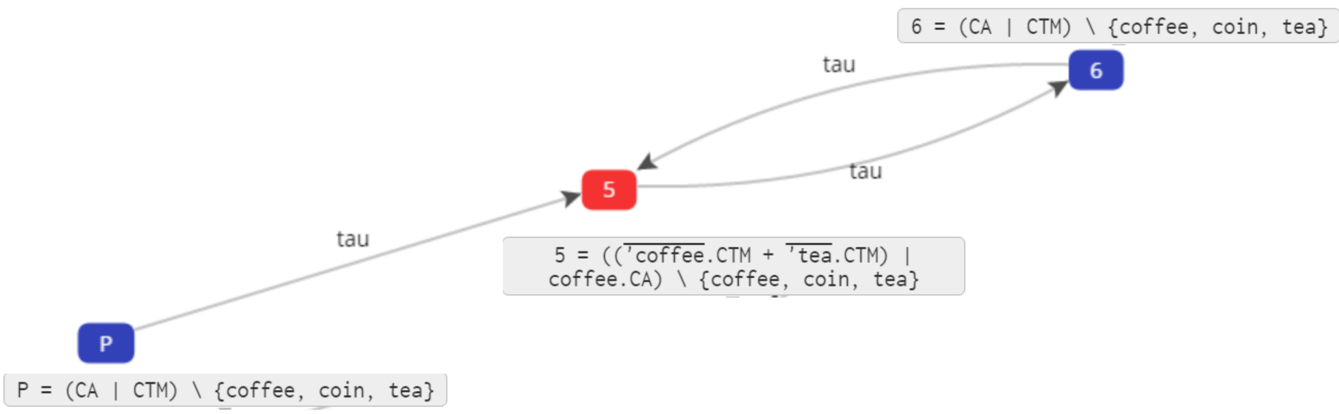

Figure 1: Transition graph of process P.

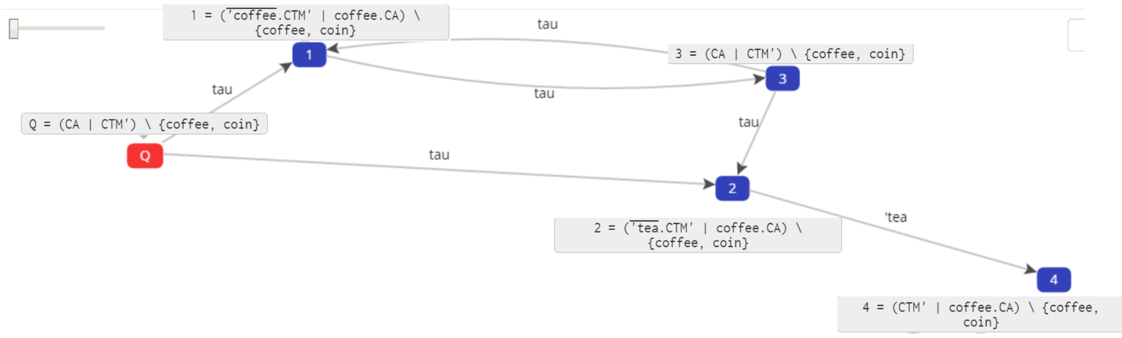

Figure 2: Transition graph of process Q.

The question is, *do the above processes have the same completed traces?* The answer is yes because the transition graphs of both processes depict the same behaviors. Below are the traces they produced:

- (($\overline{coffee}$.CTM | $\overline{tea}$.CTM) | coffee.CA) can be matched by ($\overline{coffee}$.CTM' | coffee.CA) or ($\overline{tea}$.CTM' | coffee.CA).
- (CA|CTM) can be matched by (CA|CTM').

The traces of these process can also be verified using trace equivalence on caal[1] and the result shows that they are behaviorally equivalence.

1.2 Question 2

Of course both process P and Q will have the same completed traces if they are affording the same traces and the same set of labels (L). As long as they refer to the same labels, the traces will remain the same. This can be verified using processes defined in the first question. If we apply relabeling function with the same set of labels on process (CA|CTM) and (CA|CTM'), the results would always be the same. However, if the labels on both processes are different, lets say, one process has extra label than the other, the results would of course be different since one process will create additional traces in which the other process won't be able to perform the relabeling function corresponding to specific label.

[1]http://caal.cs.aau.dk/

2 Exercise 3.3

Figure 3 depicts the transition graph for both process P and Q.

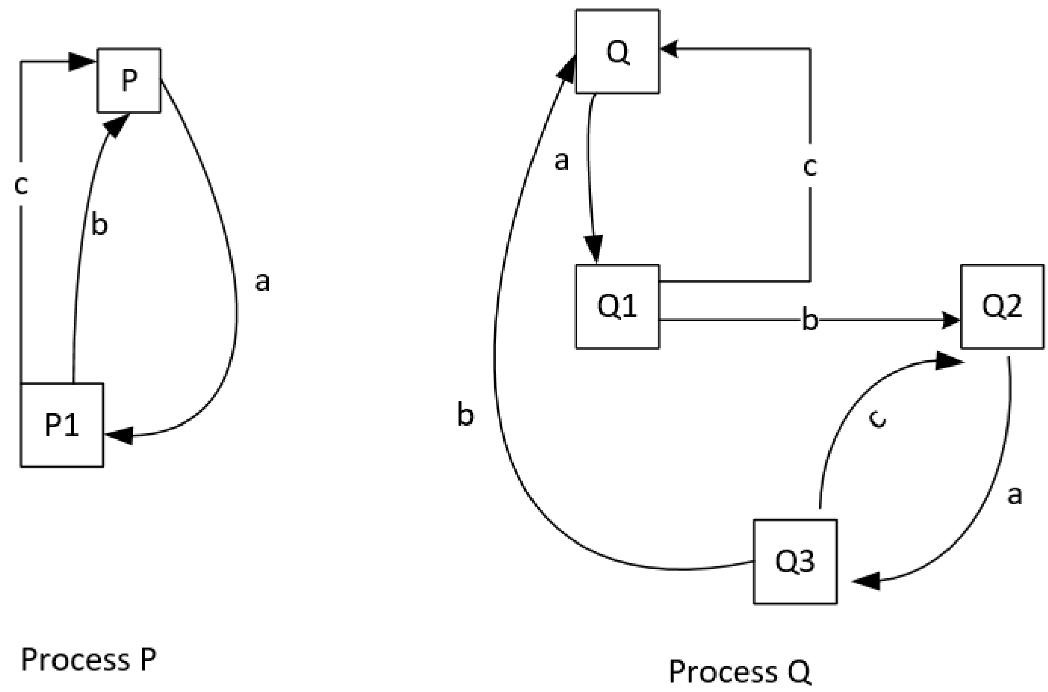

Figure 3: Transition graph of process P and Q.

In order to proof P $\sim$ Q exhibits an appropriate strong bisimulation, lets define relation: R = {(P,Q).(P_1,Q_1),(P_2,Q_2),(P_1,Q_3)}, then verify the transition of each pair as follows:

1. Consider the pair (P,Q)
   - Transition from P
     * P $\xrightarrow{a}$ P_1 can be matched by Q $\xrightarrow{a}$ Q_1.
   - Transition from Q
     * Q $\xrightarrow{a}$ Q_1 can be matched by P $\xrightarrow{a}$ P_1.
2. Consider the pair (P_1,Q_1)
   - Transition from P_1
     * P1 $\xrightarrow{b}$ P can be matched by Q_1 $\xrightarrow{b}$ Q_2.
     * P_1 $\xrightarrow{c}$ P can be matched by Q_1 $\xrightarrow{c}$ Q.
   - Transition from Q
     * Q_1 $\xrightarrow{b}$ Q can be matched by P_1 $\xrightarrow{b}$ P.
     * Q1 $\xrightarrow{c}$ Q can be matched by P_1 $\xrightarrow{c}$ P.
3. Consider the pair (P,Q_2)
   - Transition from P
     * P $\xrightarrow{a}$ P_1 can be matched by Q_2 $\xrightarrow{a}$ Q_3.
   - Transition from Q
     * Q_2 $\xrightarrow{a}$ Q_3 can be matched by P $\xrightarrow{a}$ P_1.

4. Consider the pair (P_1,Q_3)

    - Transition from P_1
      * $P_1 \xrightarrow{b} P$ can be matched by $Q_3 \xrightarrow{b} Q$.
      * $P_1 \xrightarrow{c} P$ can be matched by $Q_3 \xrightarrow{c} Q_2$.
    - Transition from Q_3
      * $Q_3 \xrightarrow{b} Q$ can be matched by $P_1 \xrightarrow{b} P$.
      * $Q_3 \xrightarrow{c} Q_2$ can be matched by $P_1 \xrightarrow{c} P$.

Since all transitions of process P can be matched by all transitions of process Q, therefore both processes exhibit strong bisimulation.

3 Exercise 3.5

In order to proof s $\sim$ t, lets perform bisimulation of relation R which contains the pair (s,t).

- Transition from s
  * $s \xrightarrow{a} s_1$ can be matched by $t \xrightarrow{a} t_1$.
  * $s \xrightarrow{a} s_2$ can be matched by $t \xrightarrow{a} t_3$.

- Transition from t
  * $t \xrightarrow{a} t_1$ can be matched by $s \xrightarrow{a} s_1$.
  * $t \xrightarrow{a} t_3$ can be matched by $s \xrightarrow{a} s_2$.

4 Exercise 3.13

4.1 Question 1

Is it true that, for all CCS processes P and Q, (P|Q) $\backslash\alpha \sim$ (P $\backslash\alpha$) | (Q $\backslash\alpha$)?
In order to proof these processes whether they are strongly bisimilar or not, lets define two processes which are able to simulate the bisimilarity as follows:

- A $\underline{\underline{def}}$ coin.$\overline{coffee}$.A;

- B $\underline{\underline{def}}$ $\overline{pub}$.$\overline{coin}$.coffee.B;

- P $\underline{\underline{def}}$ (A|B) $\backslash${coin,coffee};

- Q $\underline{\underline{def}}$ (A) $\backslash${coin,coffee } | B $\backslash${coin,coffee };

Figure 4 and Figure 5 depicts the transition graphs of process P and Q respectively.

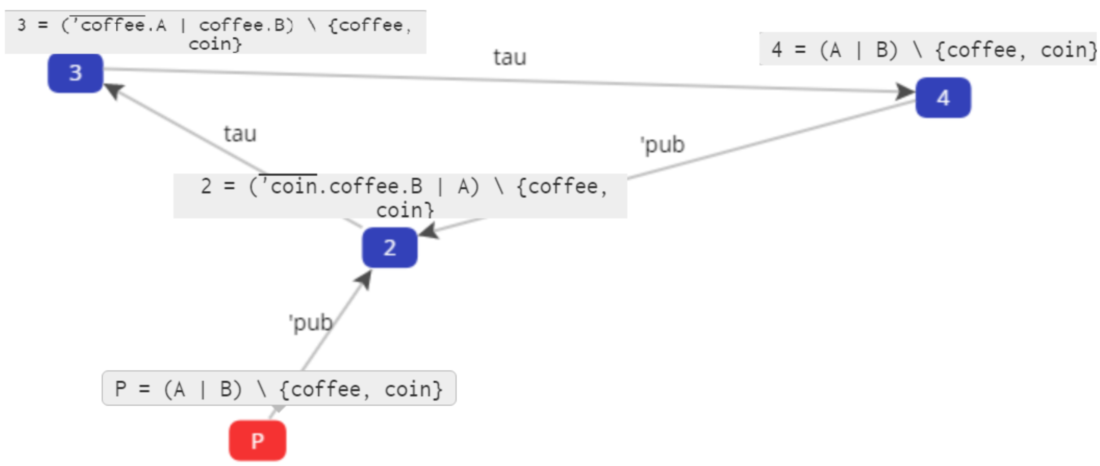

Figure 4: Transition graph of process P.

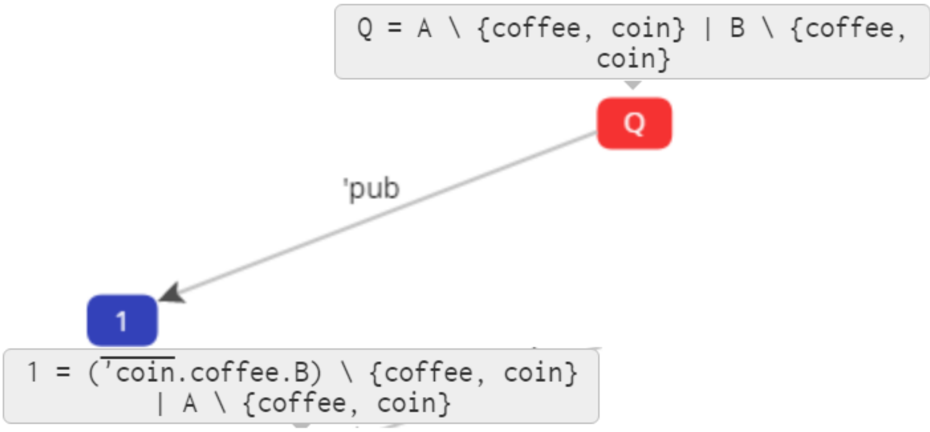

Figure 5: Transition graph of process Q.

Now, lets verify whether the process P and Q are strongly bisimilar by performing bisimulation. Given the relation R = {(P,Q)(P_1,Q_1)}.

- Consider the pair (P,Q)
  - Transition from P
    * P $\xrightarrow{\overline{pub}}$ P_1 can be matched by Q $\xrightarrow{\overline{pub}}$ Q_1.
  - Transition from Q
    * Q $\xrightarrow{\overline{pub}}$ Q_1 can be matched by P $\xrightarrow{\overline{pub}}$ P_1.
- Consider the pair (P_1,Q_1)
  - Transition from P_1
    * P1 $\xrightarrow{\tau}$ P_2 can't be matched by any Q_1 actions.

Since process Q_1 has a deadlock state and it can't proof the transition $P_1 \xrightarrow{\tau} Q_2$, therefore these two processes are not strongly bisimilar. In conclusion, for all CCS processes P and Q, (P|Q) $\backslash\alpha$ $\sim$ (P $\backslash\alpha$) | (Q $\backslash\alpha$) is not true.

4.2 Question 2

Does the equivalence (P/Q[f]) ∼ (P[f]) / (Q[f]) hold for all CCS processes P and Q, and relabeling function f?
Similar to what we did in the previous question, lets define two processes which are able to simulate the bisimilarity as follows:

- A $\underline{\underline{def}}$ alpha.A;
- B $\underline{\underline{def}}$ alpha.$\overline{betha}$.A;
- P $\underline{\underline{def}}$ (A|B)[f];
- Q $\underline{\underline{def}}$ (P[f]) | (Q[f]):

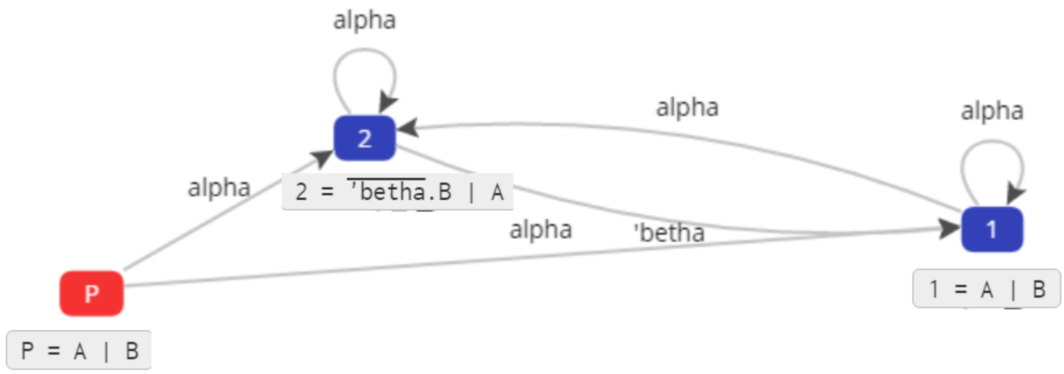

Figure 6: Transition graph of process P.

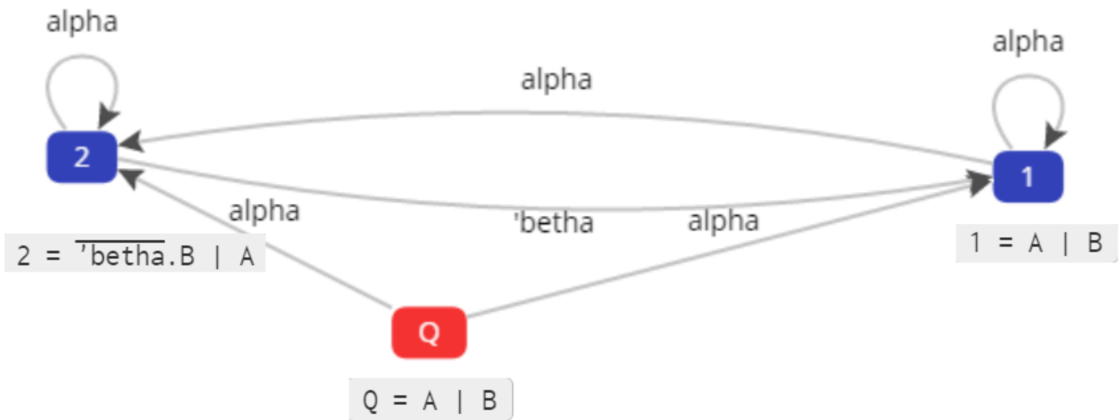

Figure 7: Transition graph of process Q.

Now, lets verify whether the process P and Q are strongly bisimilar by performing bisimulation. Given the relation R = {(P,Q)(P_1,Q_1),(P_2,Q_2)}.

- Consider the pair (P,Q)
  - Transition from P
    * P $\xrightarrow{\text{alpha}}$ P_1 can be matched by Q $\xrightarrow{\text{alpha}}$ Q_1.
    * P $\xrightarrow{\text{alpha}}$ P_2 can be matched by Q $\xrightarrow{\text{alpha}}$ Q_2.
  - Transition from Q
    * Q $\xrightarrow{\text{alpha}}$ Q_1 can be matched by P $\xrightarrow{\text{alpha}}$ P_1.
    * Q $\xrightarrow{\text{alpha}}$ Q_2 can be matched by P $\xrightarrow{\text{alpha}}$ P_2.
- Consider the pair (P_1,Q_1)
  - Transition from P_1
    * P_1 $\xrightarrow{\text{alpha}}$ P_2 can be matched by Q_1 $\xrightarrow{\text{alpha}}$ Q_2.
    * P_1 $\xrightarrow{\text{alpha}}$ P_1 can be matched by Q_1 $\xrightarrow{\text{alpha}}$ Q_1.
  - Transition from Q_1
    * Q_1 $\xrightarrow{\text{alpha}}$ Q_2 can be matched by P_1 $\xrightarrow{\text{alpha}}$ P_2.
    * Q_1 $\xrightarrow{\text{alpha}}$ Q_1 can be matched by P_1 $\xrightarrow{\text{alpha}}$ P_1.

- Consider the pair (P_2,Q_2)
  - Transition from P_2
    * $P_2 \xrightarrow{\text{betha}} P_1$ can be matched by $Q_2 \xrightarrow{\text{betha}} Q_1$.
    *$P_2 \xrightarrow{\text{alpha}} P_2$ can be matched by $Q_2 \xrightarrow{\text{alpha}} Q_2$.
  - Transition from Q_2
    * $Q_2 \xrightarrow{\text{betha}} Q_1$ can be matched by $P_2 \xrightarrow{\text{betha}} P_1$.
    * $Q_2 \xrightarrow{\text{alpha}} Q_2$ can be matched by $P_2 \xrightarrow{\text{alpha}} P_2$.

This completes the proof that R is strongly bisimilar therefore (P|Q[f]) $\sim$ (P[f]) | (Q[f]) holds for all CCS processes P and Q, and relabeling function [f].

5 Exercise 3.21

Prove that the behavioural equivalences claimed in Exercise 2.11 hold with respect to observational equivalence (weak bisimilarity)

Below are the processes defined in Exercise 2.11:

- User $\overset{def}{=}$ $\overline{p}$.enter.exit.$\overline{v}$.User;
- Sem $\overset{def}{=}$ p.v.Sem;
- FUser $\overset{def}{=}$ $\overline{p}$.enter.(exit.$\overline{v}$.FUser + exit.$\overline{v}$.0)
- Mutex2 $\overset{def}{=}$ ((User|Sem) | User) \{p,v}
- FMutex $\overset{def}{=}$ ((User | Sem) | Fuser) \{p,v}

The transition graph of process Mutex2 and FMutex are described in the Figure [] and Figure [] respectively.

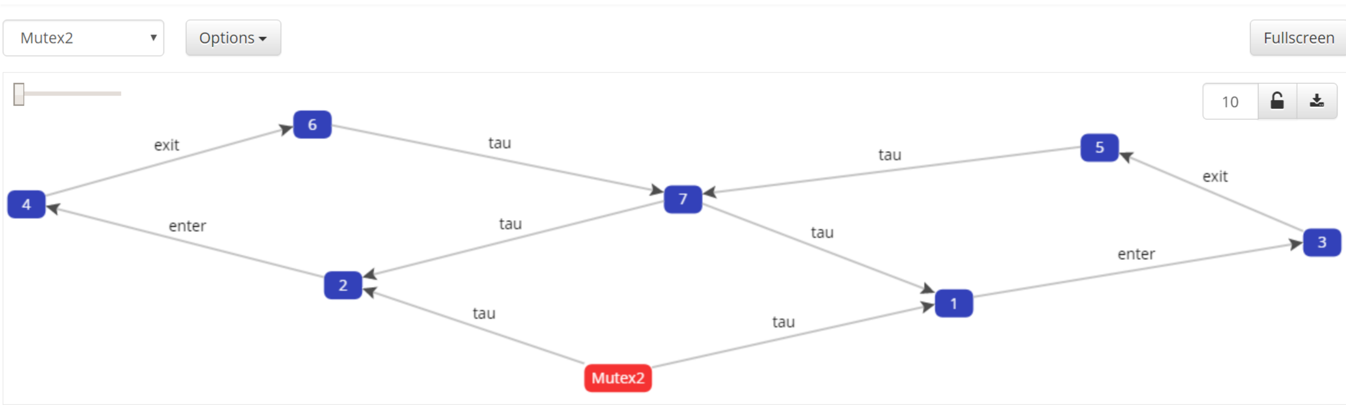

Figure 8: Transition graph of Mutex2.

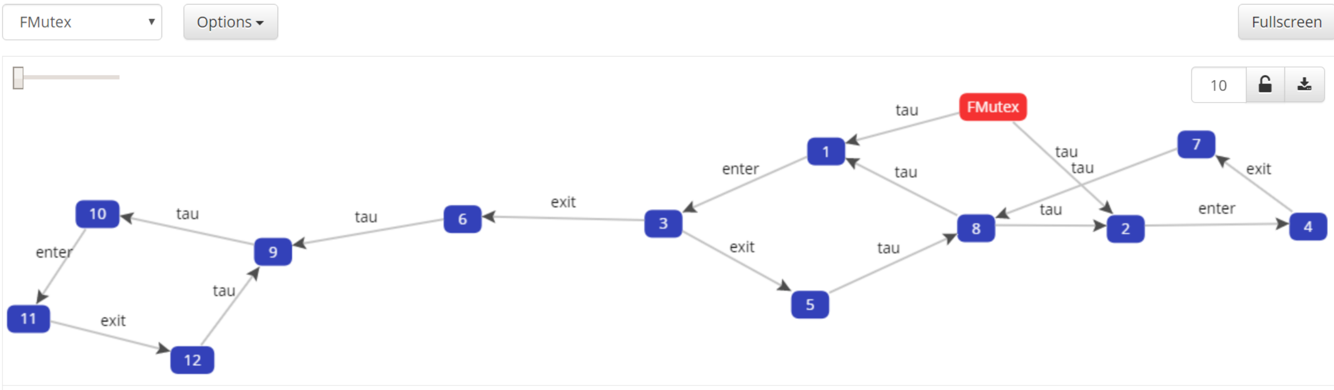

Figure 9: Transition graph of FMutex.

From the process Mutex2 and FMutex, we can define the relation
R = { (Mutex2,FMutex), ($Mutex2_1$,$FMutex_1$),($Mutex2_2$,$FMutex_2$), ($Mutex2_3$,$FMutex_3$), ($Mutex2_4$,$FMutex_4$), ($Mutex2_5$,$FMutex_5$), ($Mutex2_6$,$FMutex_6$), ($Mutex2_7$,$FMutex_8$)}

- From (Mutex2,FMutex) $\in$ R, both processes can do both τ action. We end up in either ($Mutex2_1$,$FMutex_1$) $\in$ R or ($Mutex2_2$,$FMutex_2$) $\in$ R.
- From ($Mutex2_1$,$FMutex_1$) $\in$ R, we end up in ($Mutex2_3$,$FMutex_3$) $\in$ R.
- From ($Mutex2_2$,$FMutex_2$) $\in$ R, we end up in ($Mutex2_4$,$FMutex_4$) $\in$ R.
- From ($Mutex2_3$,$FMutex_3$) $\in$ R, we end up in ($Mutex2_5$,$FMutex_5$) $\in$ R.
- From ($Mutex2_4$,$FMutex_4$) $\in$ R, we end up in ($Mutex2_6$,$FMutex_7$) $\in$ R.
- From ($Mutex2_5$,$FMutex_5$) $\in$ R, we end up in ($Mutex2_7$,$FMutex_8$) $\in$ R.
- From ($Mutex2_6$,$FMutex_6$) $\in$ R, we end up in ($Mutex2_7$,$FMutex_9$) $\in$ R.
- From ($Mutex2_7$,$FMutex_8$) $\in$ R, we end up in either ($Mutex2_1$,$FMutex_1$) $\in$ R or ($Mutex2_2$,$FMutex_2$) $\in$ R.

Hence,we have shown that each pair in relation R satisfies the condition Mutex2 $\overset{def}{=}$ ((User|Sem) | User) \{p,v} and FMutex $\overset{def}{=}$ ((User | Sem) | Fuser) \{p,v} which means, these two processes are observational equivalence (weak bisimilarity) or can be considered as strongly bisimilar too.

6 Exercise 3.25

From the transition graph of process s and t, we can obtain the relation R = {(s,t),(s_1,t),(s_3,t),(s_3,t_2),(s_3,t_3),(s_5,t_1)}.

- Consider the pair (s,t)
  - Transition from s
    * s $\xrightarrow{\tau}$ s_1 can be matched by t $\xrightarrow{\tau}$ t_1 and (s_1,t_1) $\in$ R.
    * s $\xrightarrow{a}$ s_3 can be matched by t $\xrightarrow{a}$ t_2 and (s_3,t_2) $\in$ R.
  - Transition from t
    * t $\xrightarrow{\tau}$ t_1 can be matched by s $\xrightarrow{\tau}$ s_5 and (t_1,s_5) $\in$ R.
    * t $\xrightarrow{a}$ t_2 can be matched by s $\xrightarrow{a}$ s_3 and (t_2,s_3) $\in$ R.
    * t $\xrightarrow{b}$ t_3 can be matched by s_1 $\xrightarrow{b}$ s_4 and (t_3,s_4) $\in$ R.

Since both process can satisfy all the transitions of pair (s,t), therefore **s** $\approx$ **t**.

7 Exercise 3.27

In order to proof, *for all* P, Q, *if* $P \xrightarrow{\tau} Q$ *and* $Q \xrightarrow{\tau} P$, *then* $P \approx Q$, let's take an example of process s and t in the exercise 3.25 as shown in Figure 10.

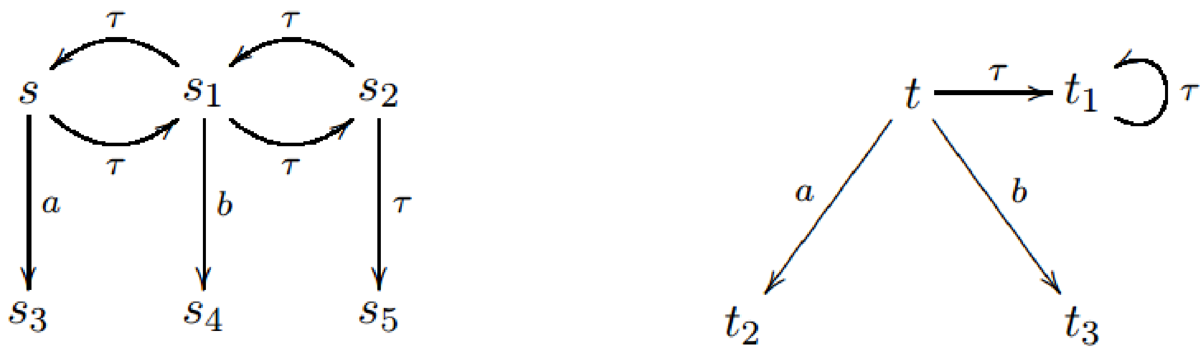

Figure 10: Labelled transition system in exercise 3.25.

From the above transition graphs, we can define relation R = {(s,t),(s_1,t),(s_2,t)} in which each pair must perform τ action. Let's break it down one by one.

- Consider the pair (s,t)
  - Transition from s
    * s $\xrightarrow{\tau}$ s_1 can be matched by t $\xrightarrow{\tau}$ t_1 $\in$ R.
  - Transition from t
    * t $\xrightarrow{\tau}$ t_1 can be matched by s $\xrightarrow{\tau}$ s_1 $\in$ R.
- Consider the pair (s_1,t)
  - Transition from s_1
    * s_1 $\xrightarrow{\tau}$ s_2 can be matched by t $\xrightarrow{\tau}$ t_1 $\in$ R.
    * s_1 $\xrightarrow{\tau}$ s can be matched by t $\xrightarrow{\tau}$ t_1 $\in$ R.
  - Transition from t * t $\xrightarrow{\tau}$ t_1 can be matched by either s_1 $\xrightarrow{\tau}$ s_2 or s_1 $\xrightarrow{\tau}$ s $\in$ R.
- Consider the pair (s_2,t)
  - Transition from s_2
    * s_2 $\xrightarrow{\tau}$ s_5 can be matched by t $\xrightarrow{\tau}$ t_1 $\in$ R.
    * s_2 $\xrightarrow{\tau}$ s_1 can be matched by t $\xrightarrow{\tau}$ t_1 $\in$ R.
  - Transition from t
    * t $\xrightarrow{\tau}$ t_1 can be matched by either s_2 $\xrightarrow{\tau}$ s_5 or s_2 $\xrightarrow{\tau}$ s_1 $\in$ R.

8 Exercise 3.37

This exercise, both proving the strong bisimulation or suggesting the winning strategy, can be simulated on caal. The transition of each process can be written on caal as follows:

S=a.S1;
S1=b.S2;
S2=b.S2+a.S;

T=a.T1;
T1=b.T1+b.T2;
T2=a.T;

U=a.U1;
U1=b.U3;
U3=b.U2+a.U;
U2=b.U2+a.U;

V=a.V1;
V1=b.V2+b.V3;
V3=b.V3+b.V2;
V2=a.V;

Since LTS of each process has been converted into call syntax, now we can verify whether they are strongly bisimilar or not. The results show that $\mathbf{s} \nsim \mathbf{t}$, $\mathbf{s} \sim \mathbf{u}$, and $\mathbf{s} \nsim \mathbf{v}$ as shown in Figure 11.

Figure 11: Verification results of strong bisimulation.

In order to obtain the winning strategy, I performed strong bisimulation game against each pair of processes. The details of each game and the winning strategy corresponding to each pair are described as follows:

- **(s,t)**
  * Role : Attacker
  * Total Rounds : 4
  * Winning Strategy:
    - *Round 1*
      Current configuration (s,t).There are two options for attacker either to play $s \xrightarrow{a} t$ or $t \xrightarrow{a} t_1$. Attacker played $t \xrightarrow{a} t_1$ and defender played $s \xrightarrow{a} s1$.
    - *Round 2*
      Current configuration is changed to (s_1,t_1). Now, there are three available options for attacker: $s_1 \xrightarrow{b} s_2$, $t_1 \xrightarrow{b} t_1$, $t_1 \xrightarrow{b} t_2$. Attacker played $t_1 \xrightarrow{b} t_1$ and defender played $s_1 \xrightarrow{b} s_2$.
    - *Round 3*
      Current configuration is changed to (s_2,t_1). Now, there are four available options for attacker: $s_2 \xrightarrow{a} s$, $s_2 \xrightarrow{b} s_2$, $t_1 \xrightarrow{b} t_1$, $t_1 \xrightarrow{b} t_2$. Attacker played $t_1 \xrightarrow{b} t_2$ and defender played $s_2 \xrightarrow{b} s_2$.

– *Round 3*
Current configuration is changed to (s_2,t_2). There are three available options: $s_2 \xrightarrow{a} s$, $s_2 \xrightarrow{b} s_2$, $t_2 \xrightarrow{a} t$. Attacker played $s_2 \xrightarrow{b} s_2$ and defender has no available transitions. **Attacker win!**

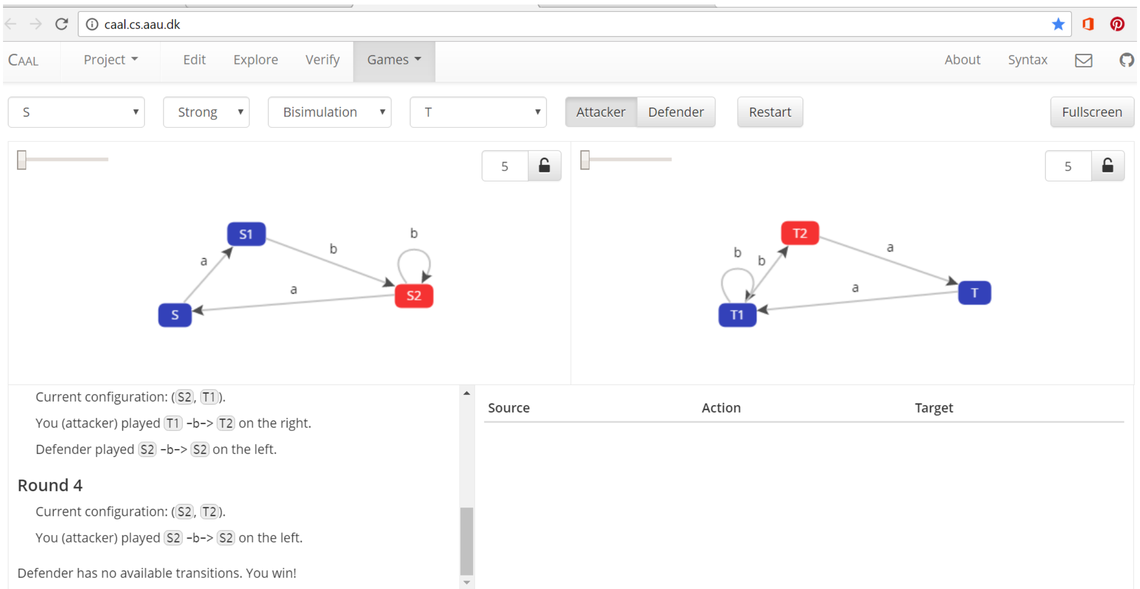

Figure 12: Strong bisimulation game (s,t).

- **(s,u)**
  * Role : Defender
  * Total Rounds : 4
  * Winning Strategy:

  – *Round 1*
  Current configuration (s,u). Attacker played $u \xrightarrow{a} u_1$ and defender played $s \xrightarrow{a} s_1$.

  – *Round 2*
  Current configuration is change to $u_1 \xrightarrow{b} u_3$. Attacker played $s_1 \xrightarrow{b} u_3$ and defender played u1 $\xrightarrow{b} u_3$.

  – *Round 3*
  Current configuration is change to $s_2 \xrightarrow{a} u_3$. Attacker played $u_3 \xrightarrow{a} u$ and defender played $s_2 \xrightarrow{a} s$.

  – *Round 4*
  Current configuration is changed to (s,u). **A cycle has been detected and the defender win.**

- **(s,v)**
  * Role : Attacker
  * Total Rounds : 3
  * Winning Strategy:

  – *Round 1*
  Current configuration (s,v).Attacker has two available options, either $s \xrightarrow{a} s_1$ or $v \xrightarrow{a} v_1$. Attacker played $s \xrightarrow{a} s_1$ and defender played $v \xrightarrow{a} v_1$.

  – *Round 2*
  Current configuration is changed to (s_1,v_1). Now, there are three available options for attacker: $s_1 \xrightarrow{b} s_2$, $v_1 \xrightarrow{b} v_2$, $v_1 \xrightarrow{b} v_3$. Attacker played $v_1 \xrightarrow{b} v_2$ and defender played $s_1 \xrightarrow{b} s_2$.

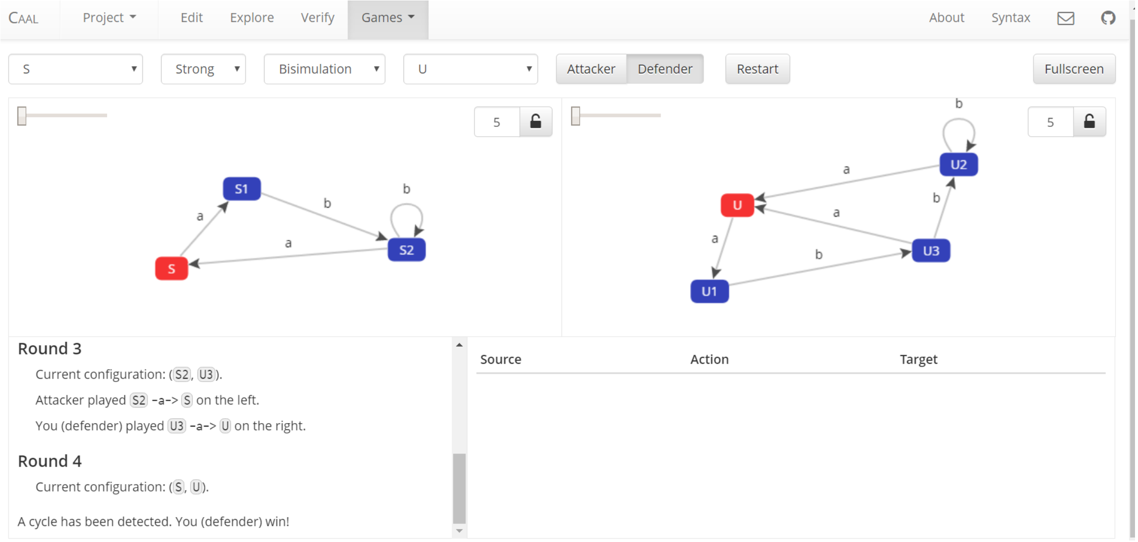

Figure 13: Strong bisimulation game (s,u).

– *Round 3*
Current configuration is changed to (s_2,v_2). Now, there are three available options for attacker: $s_2 \xrightarrow{a} s$, $s_2 \xrightarrow{b} s_2$, $v_2 \xrightarrow{a} v$. Attacker played $s_2 \xrightarrow{b} s_2$ and defender has no available transition. **Attacker win!**

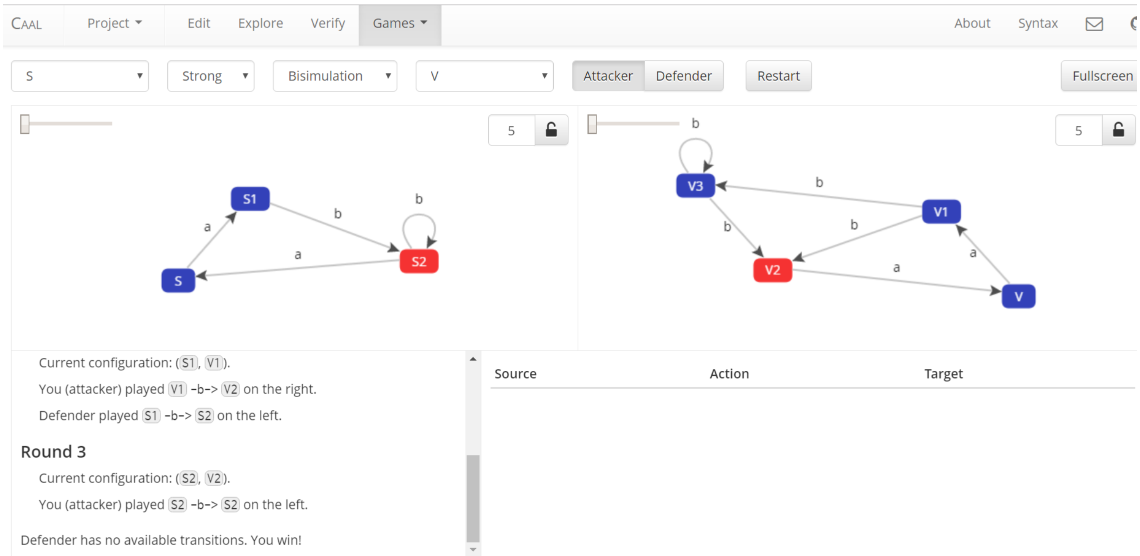

Figure 14: Strong bisimulation game (s,v).