Bag-of-Words models

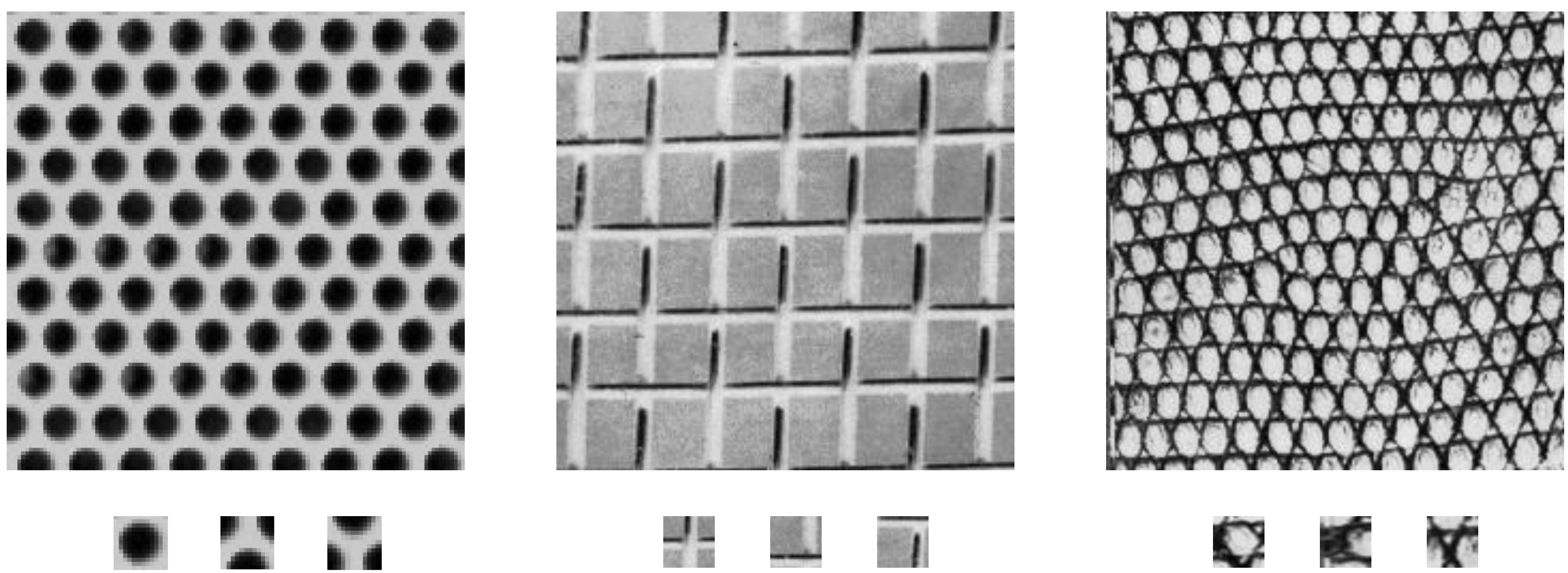
Lecture 9

Slides from: S. Lazebnik, A. Torralba, L. Fei-Fei, D. Lowe, C. Szurka

Bag-of-features models

Origin 1: Texture recognition

- Texture is characterized by the repetition of basic elements or *textons*
- For stochastic textures, it is the identity of the textons, not their spatial arrangement, that matters



Julesz, 1981; Cula & Dana, 2001; Leung & Malik 2001; Mori, Belongie & Malik, 2001; Schmid 2001; Varma & Zisserman, 2002, 2003; Lazebnik, Schmid & Ponce, 2003

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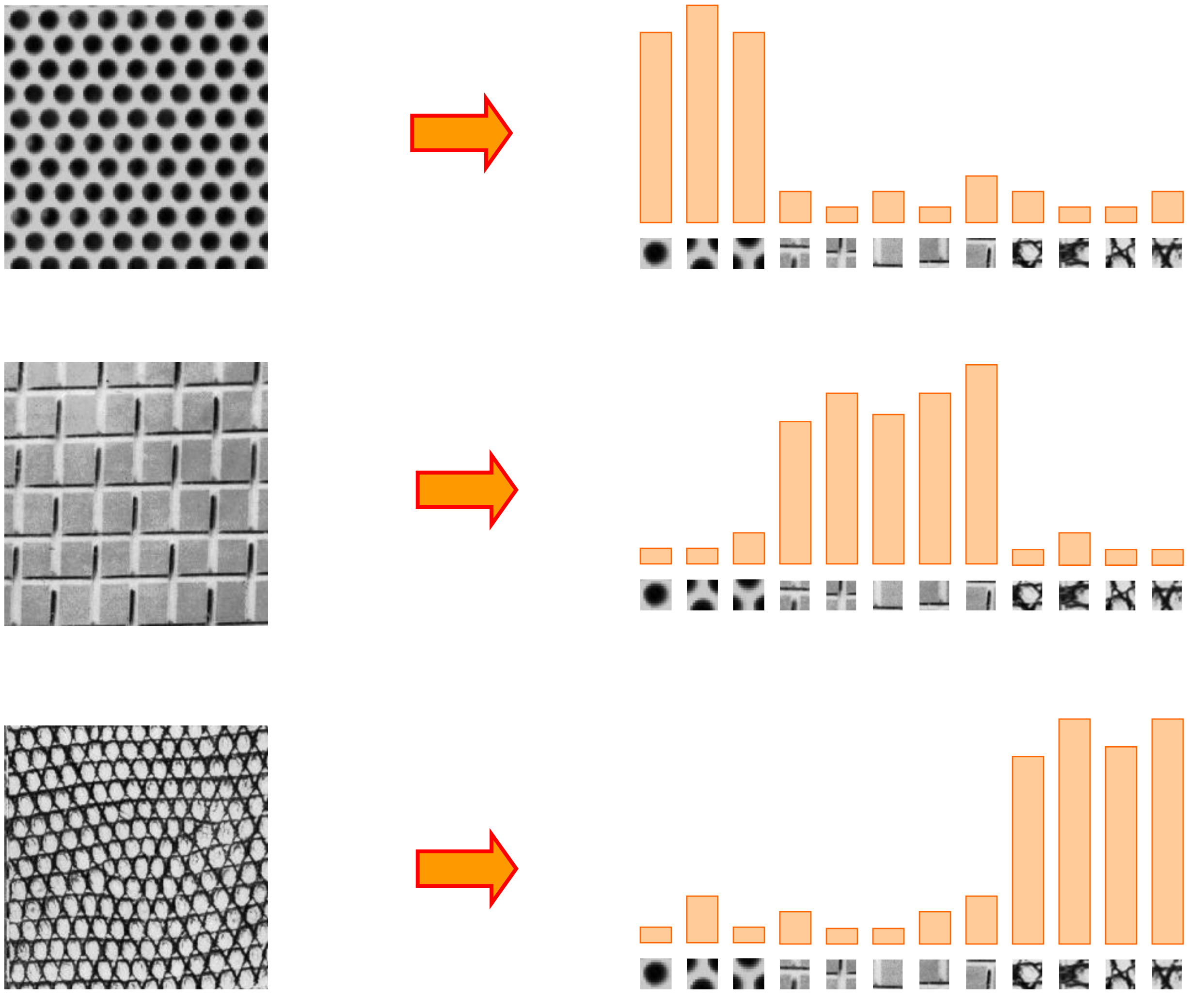

Julesz, 1981; Cula & Dana, 2001; Leung & Malik 2001; Mori, Belongie & Malik, 2001; Schmid 2001; Varma & Zisserman, 2002, 2003; Lazebnik, Schmid & Ponce, 2003

Origin 2: Bag-of-words models

- Orderless document representation: frequencies of words from a dictionary Salton & McGill (1983)

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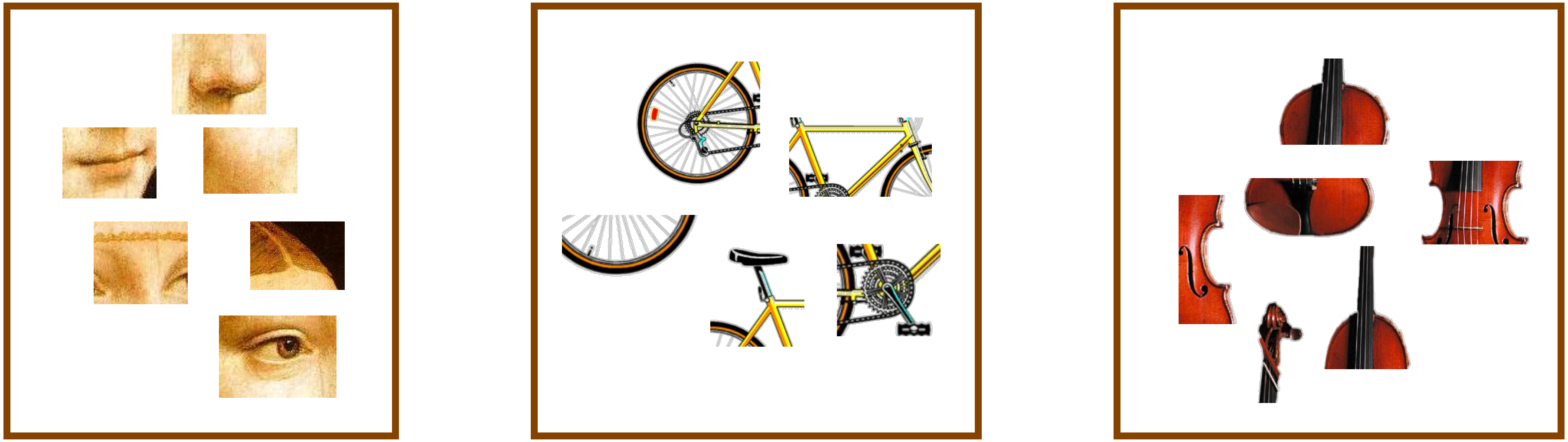
Bags of features for image classification

1. Extract features



Bags of features for image classification

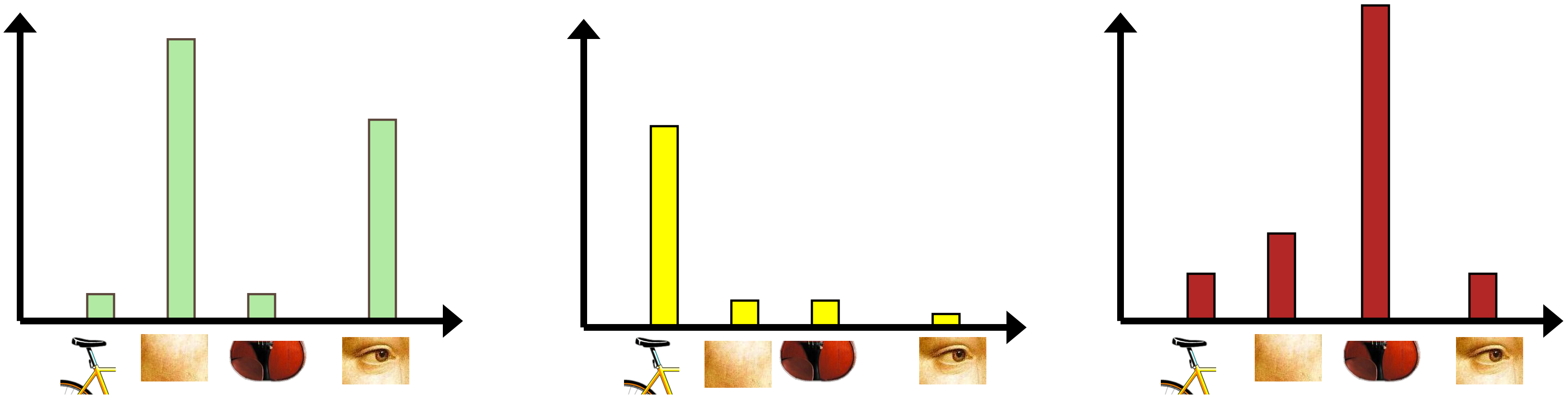
1. Extract features
2. Learn “visual vocabulary”

Bags of features for image classification

1. Extract features
2. Learn “visual vocabulary”
3. Quantize features using visual vocabulary

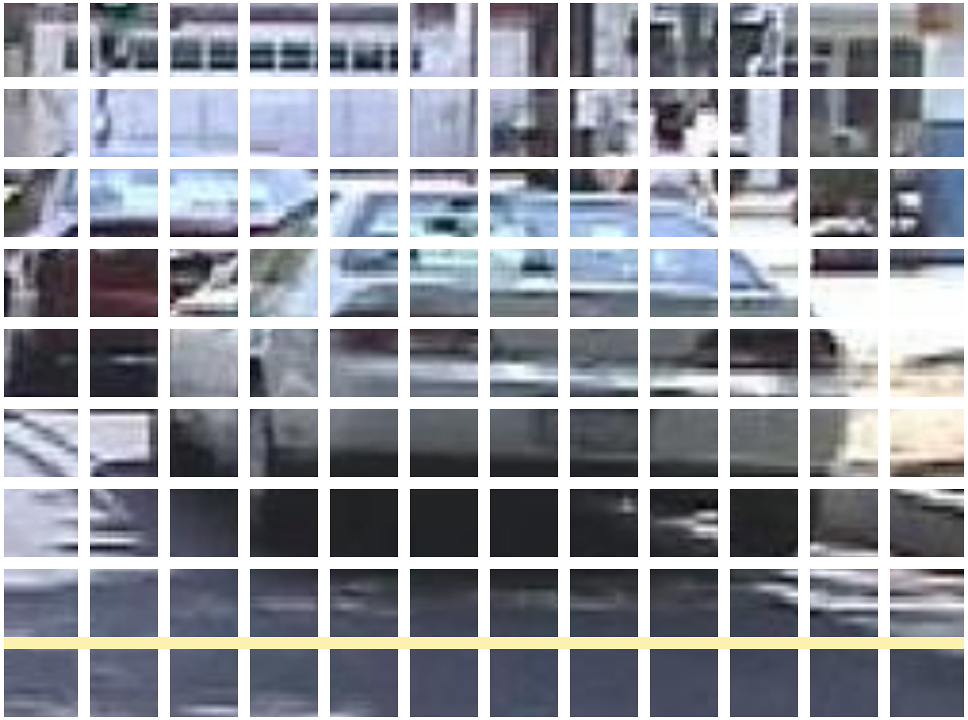
Bags of features for image classification

1. Extract features
2. Learn "visual vocabulary"
3. Quantize features using visual vocabulary
4. Represent images by frequencies of "visual words"



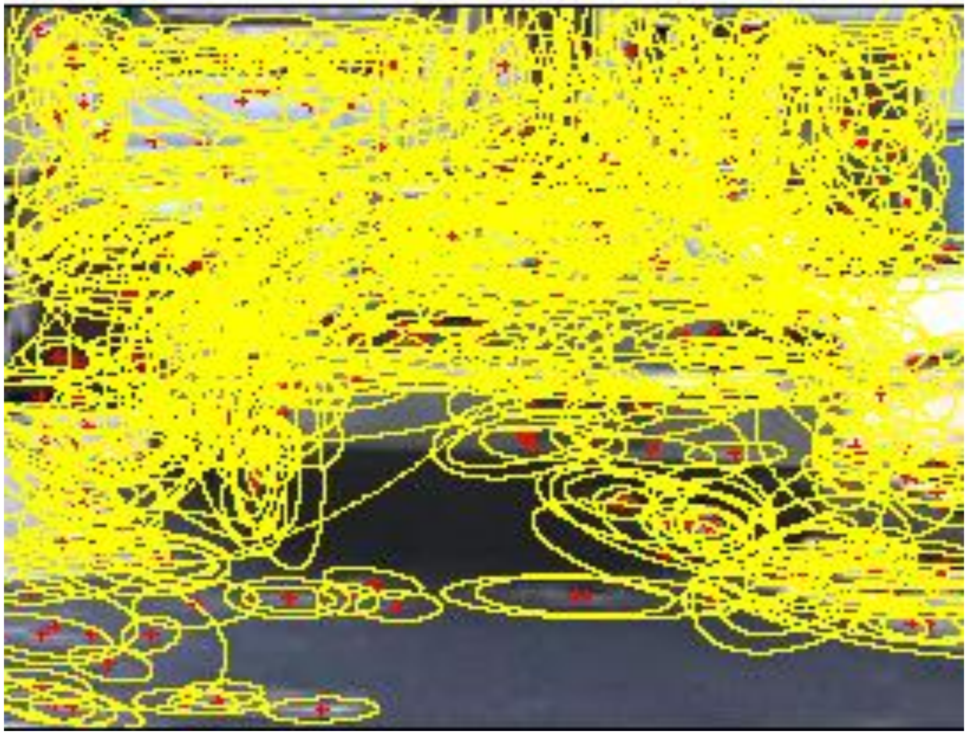
1. Feature extraction

- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005



1. Feature extraction

- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005
- Interest point detector
  - Csurka et al. 2004
  - Fei-Fei & Perona, 2005
  - Sivic et al. 2005



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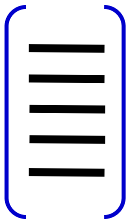
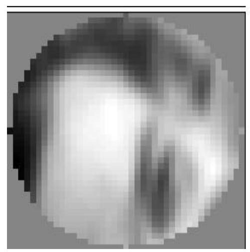
  - Sivic et al. 2005
- Other methods
  - Random sampling (Vidal-Naquet & Ullman, 2002)
  - Segmentation-based patches (Barnard et al. 2003)

1. Feature extraction

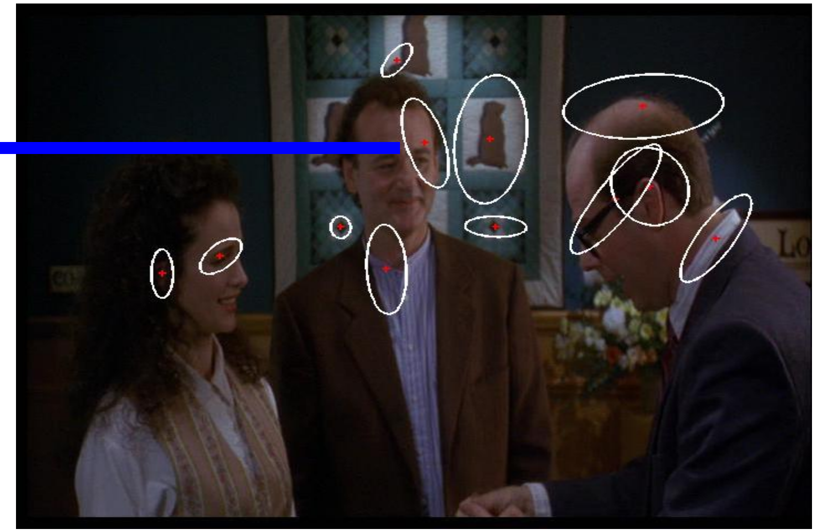


Compute SIFT descriptor

[Lowe'99]



Normalize patch



Detect patches

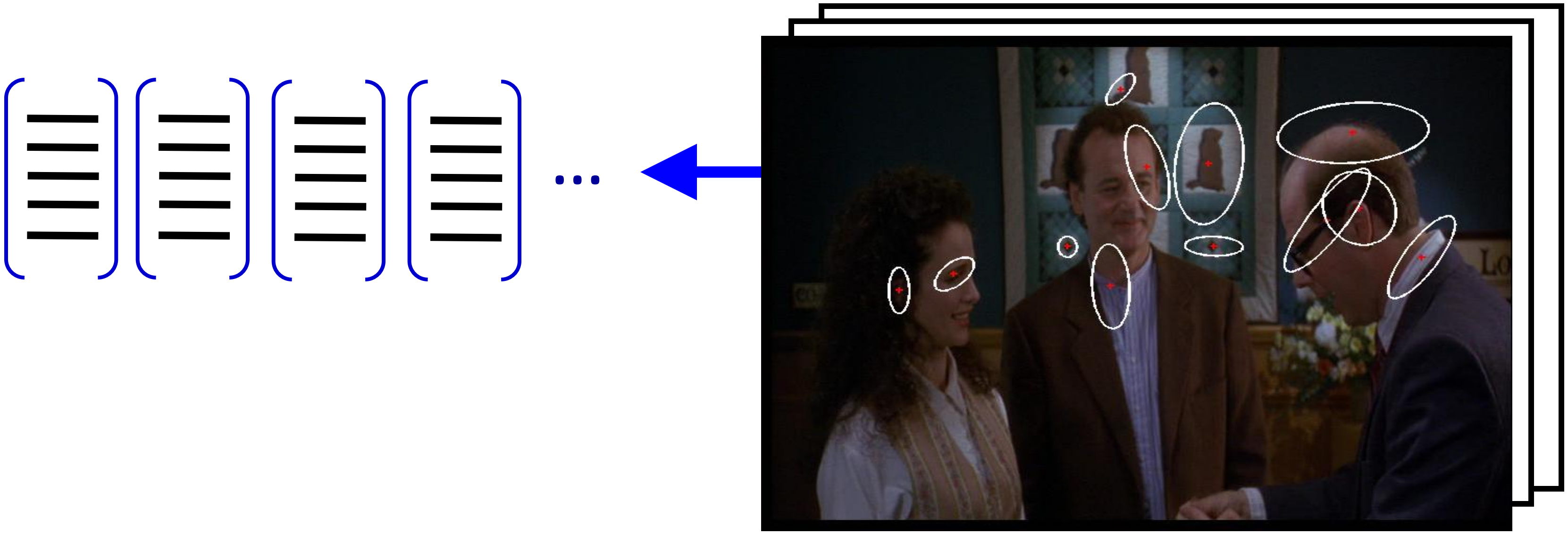
[Mikojaczyk and Schmid '02]

[Mata, Chum, Urban & Pajdla, '02]

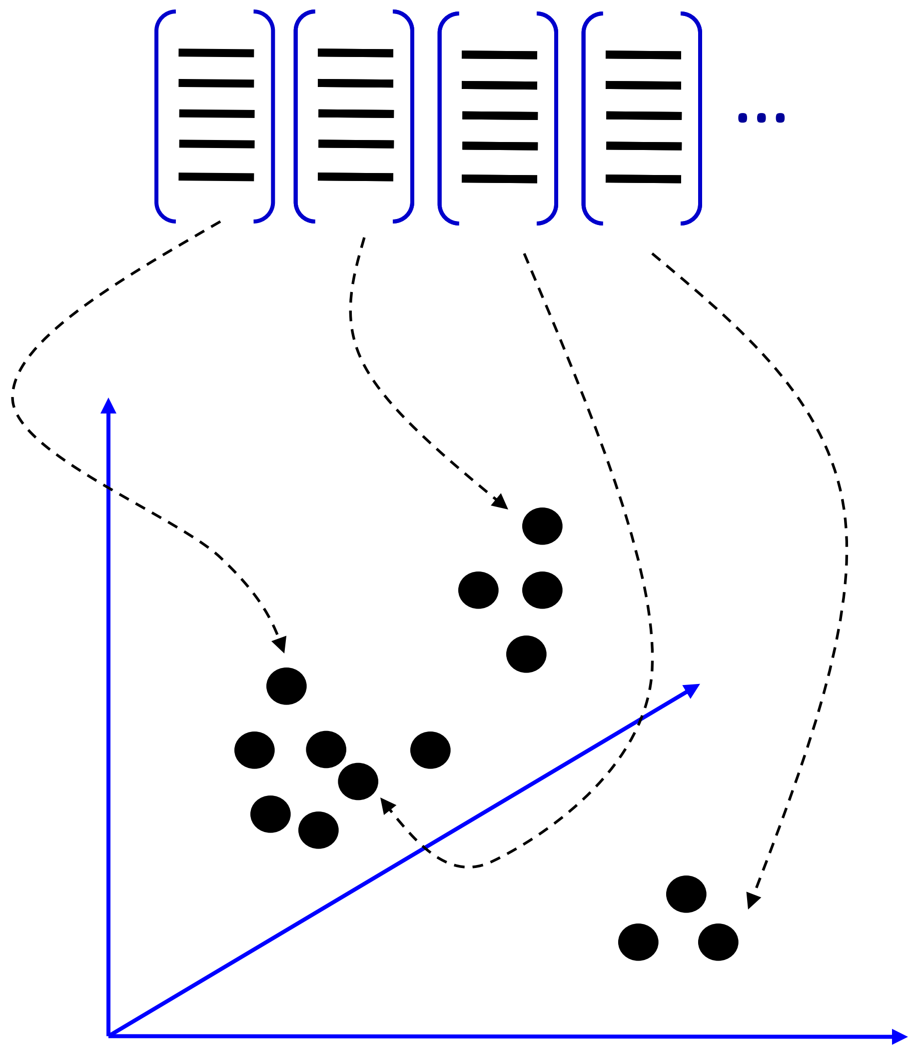
[Sivic & Zisserman, '03]

Slide credit: Josef Sivic

1. Feature extraction



2. Learning the visual vocabulary



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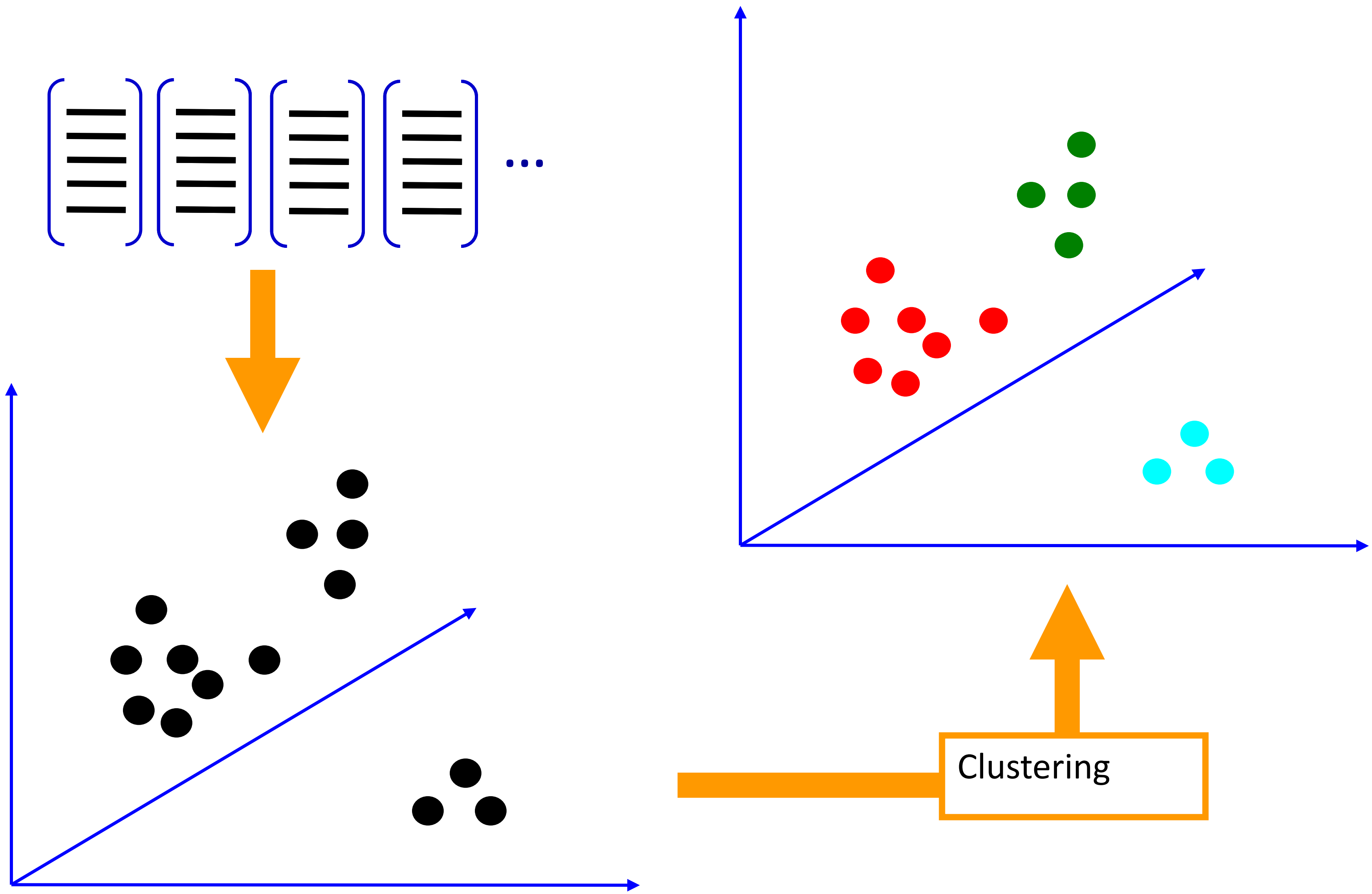

Slide credit: Josef Sivic

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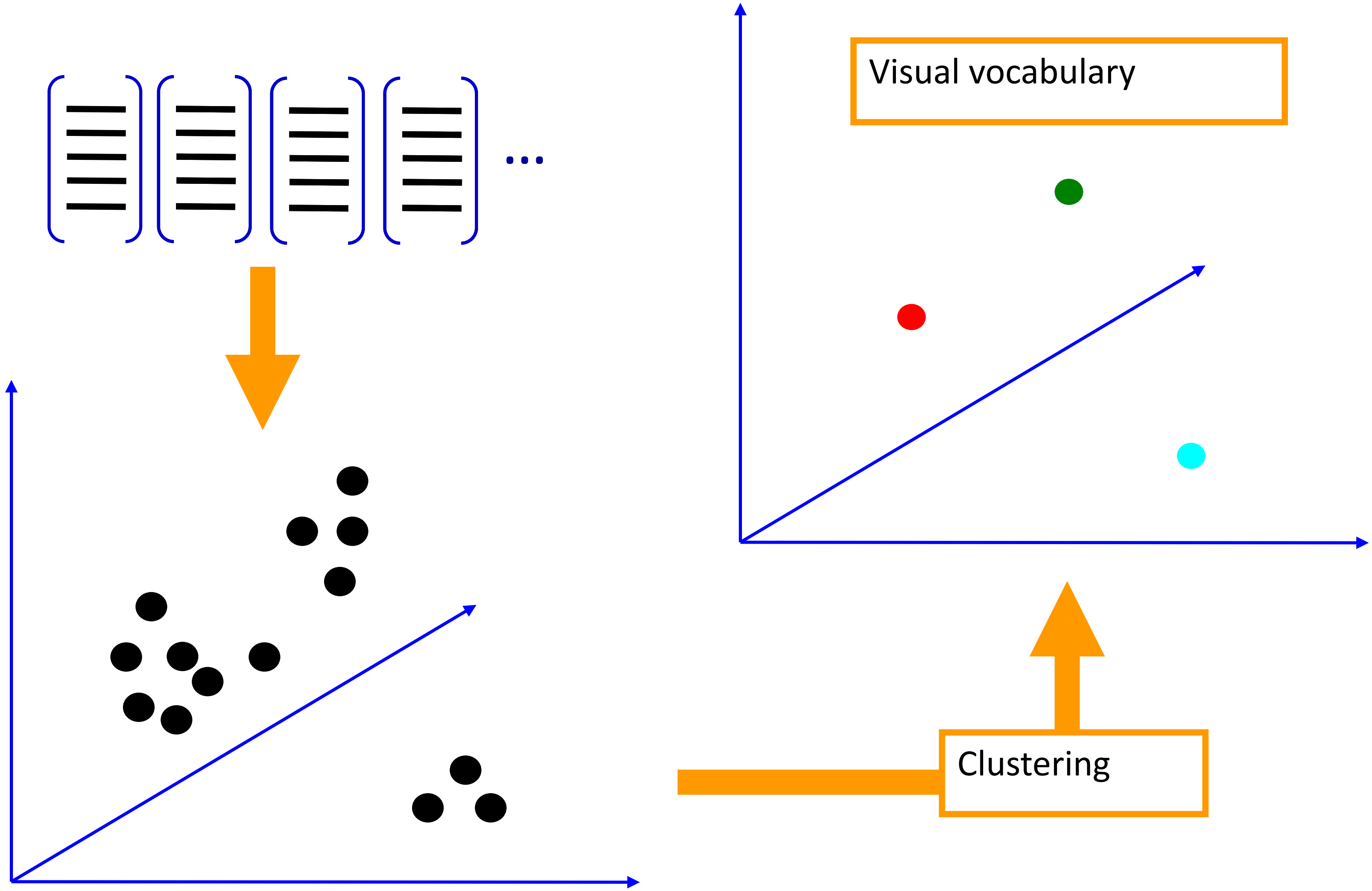


Slide credit: Josef Sivic

K-means clustering

- Want to minimize sum of squared Euclidean distances between points x_i and their nearest cluster centers m_k

$$D(X, M) = \sum_{\text{cluster } k} \sum_{\substack{\text{point } i \text{ in} \\ \text{cluster } k}} (x_i - m_k)^2$$

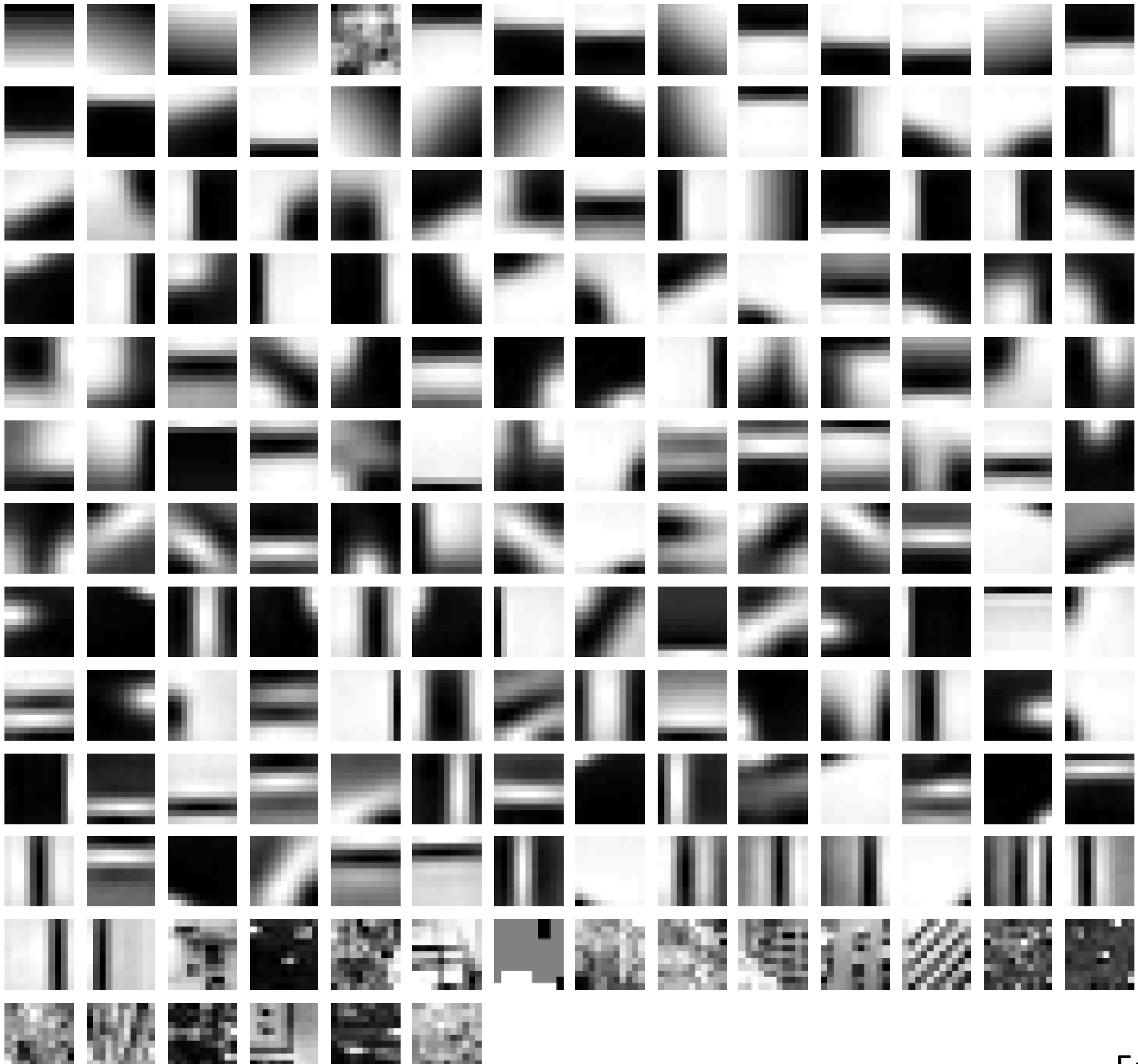
- Algorithm:
- Randomly initialize K cluster centers
- Iterate until convergence:
  - Assign each data point to the nearest center
  - Recompute each cluster center as the mean of all points assigned to it

From clustering to vector quantization

- Clustering is a common method for learning a visual vocabulary or codebook
  - Unsupervised learning process
  - Each cluster center produced by k-means becomes a codevector
  - Codebook can be learned on separate training set
  - Provided the training set is sufficiently representative, the codebook will be "universal"

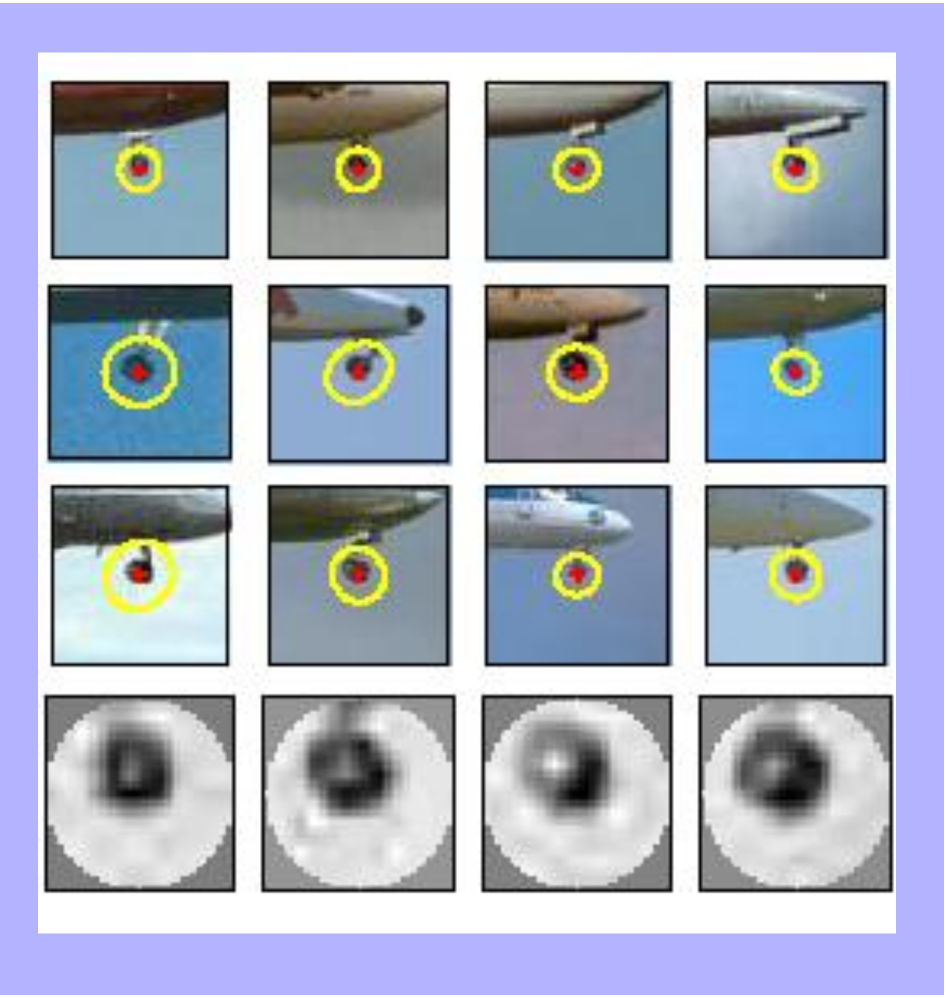
- The codebook is used for quantizing features
  - A *vector quantizer* takes a feature vector and maps it to the index of the nearest codevector in a codebook
  - Codebook = visual vocabulary
  - Codevector = visual word

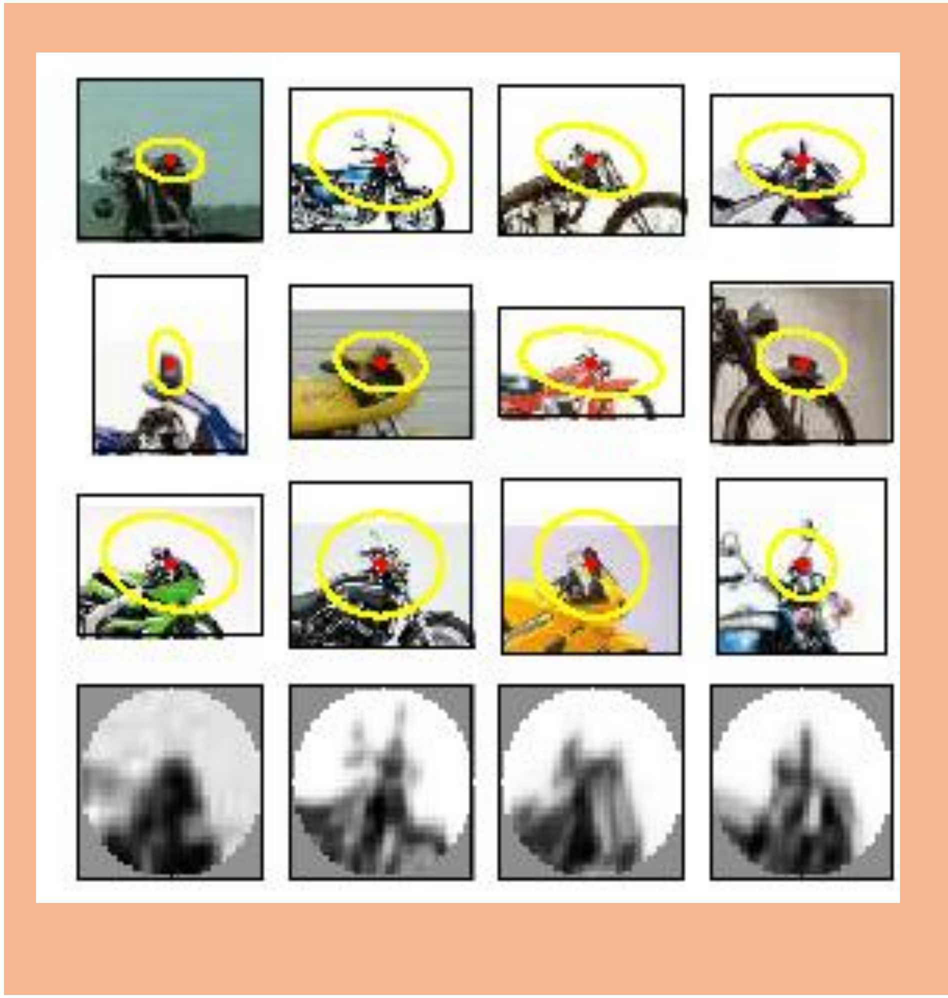
Example visual vocabulary



Fei-Fei et al. 2005

Image patch examples of visual words

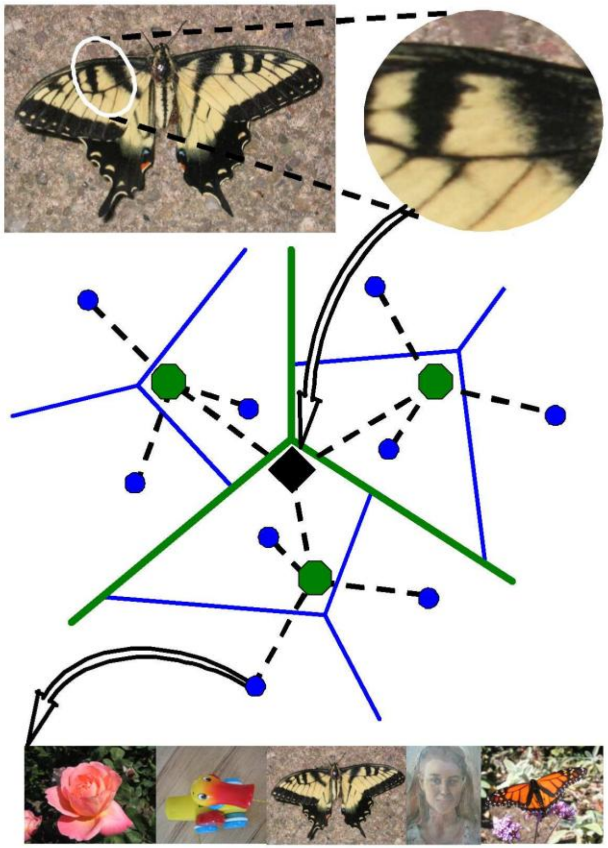




Sivic et al. 2005

Visual vocabularies: Issues

- How to choose vocabulary size?
  - Too small: visual words not representative of all patches
  - Too large: quantization arti
- Generative or discriminati
- Computational efficiency
  - Vocabulary trees
    (Nister & Stewenius, 2006)



3. Image representation

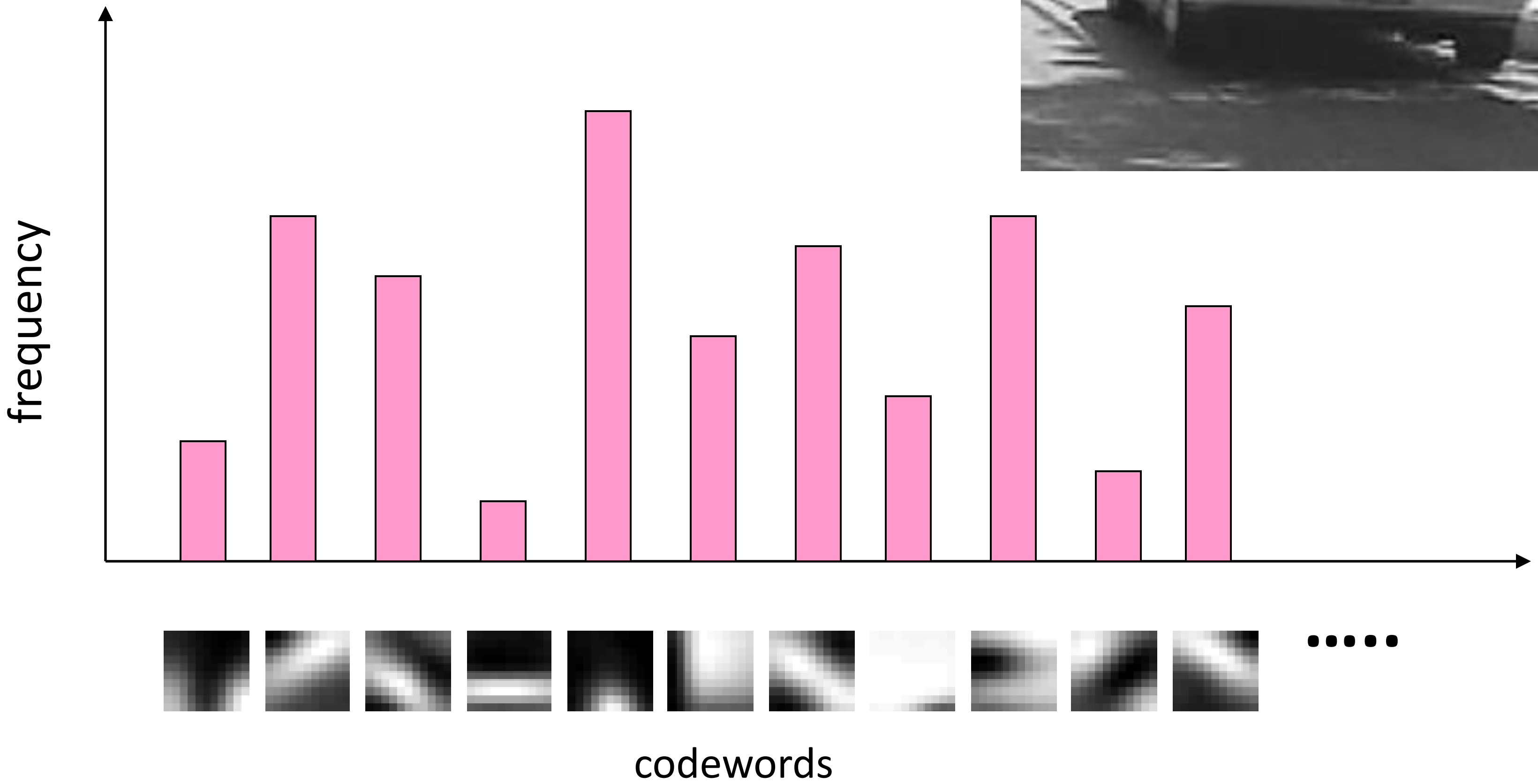
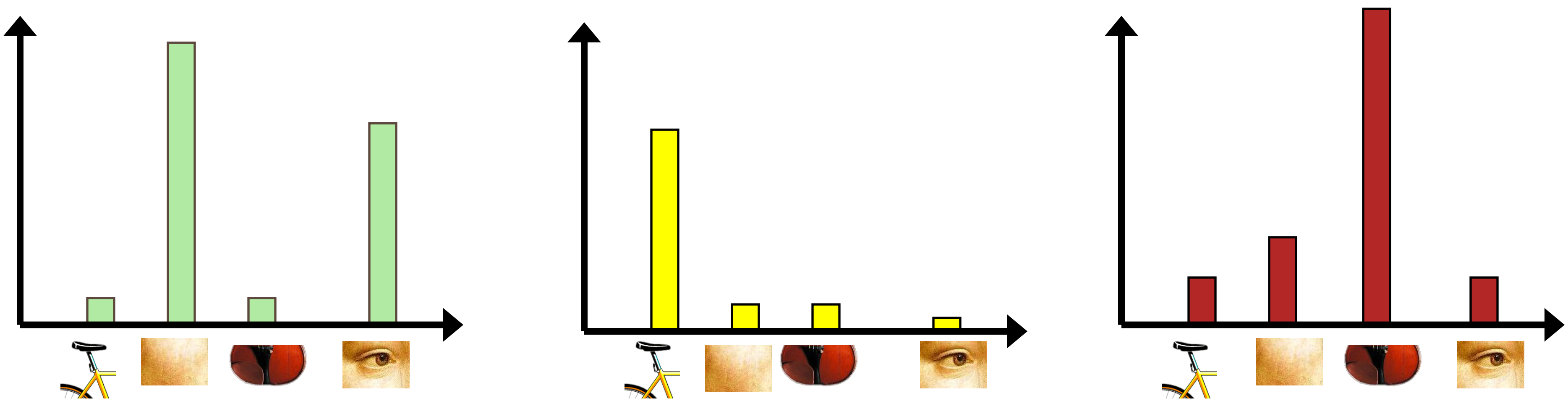
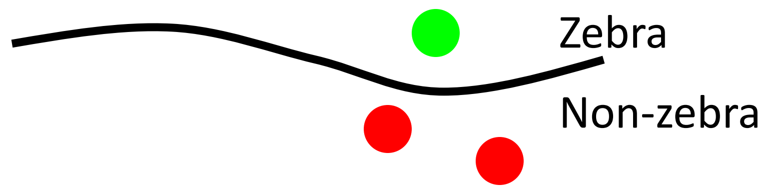


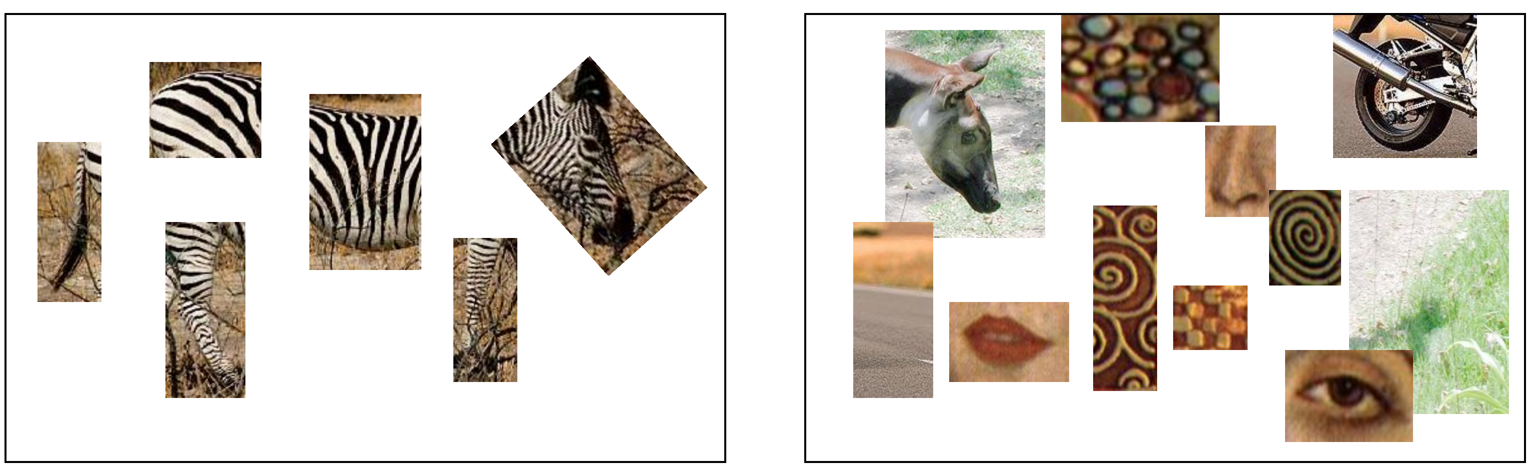
Image classification

- Given the bag-of-features representations of images from different classes, how do we learn a model for distinguishing them?



Discriminative and generative methods for bags of features





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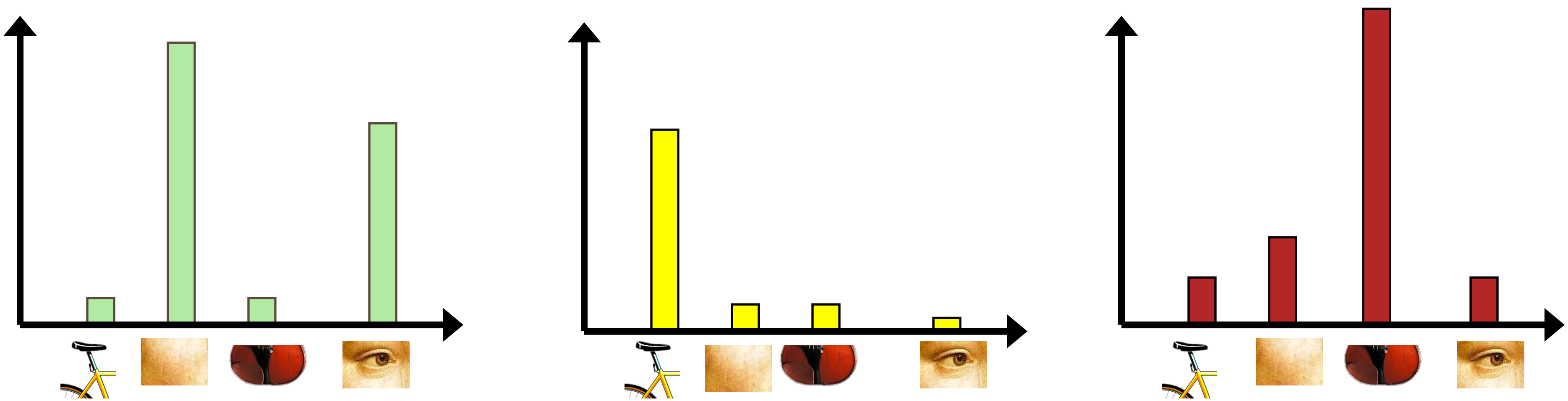
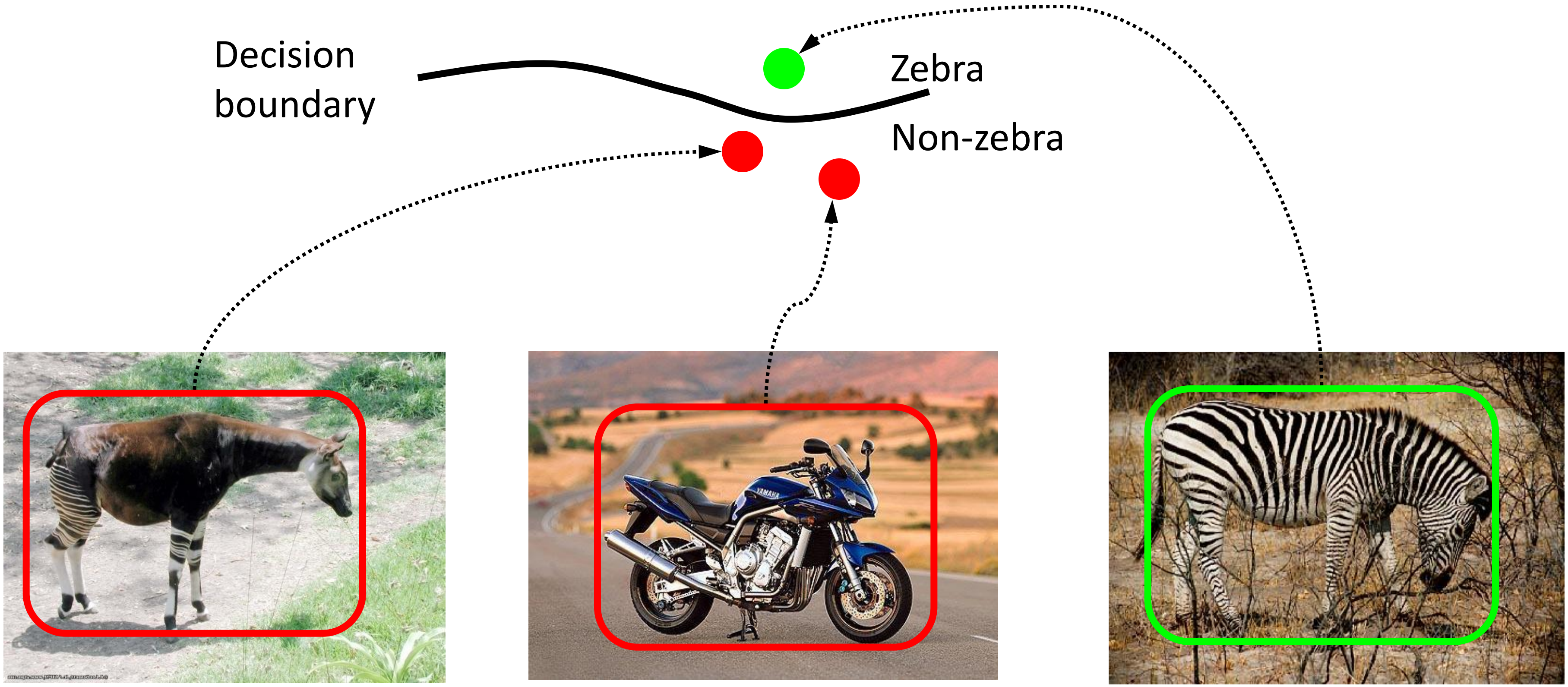


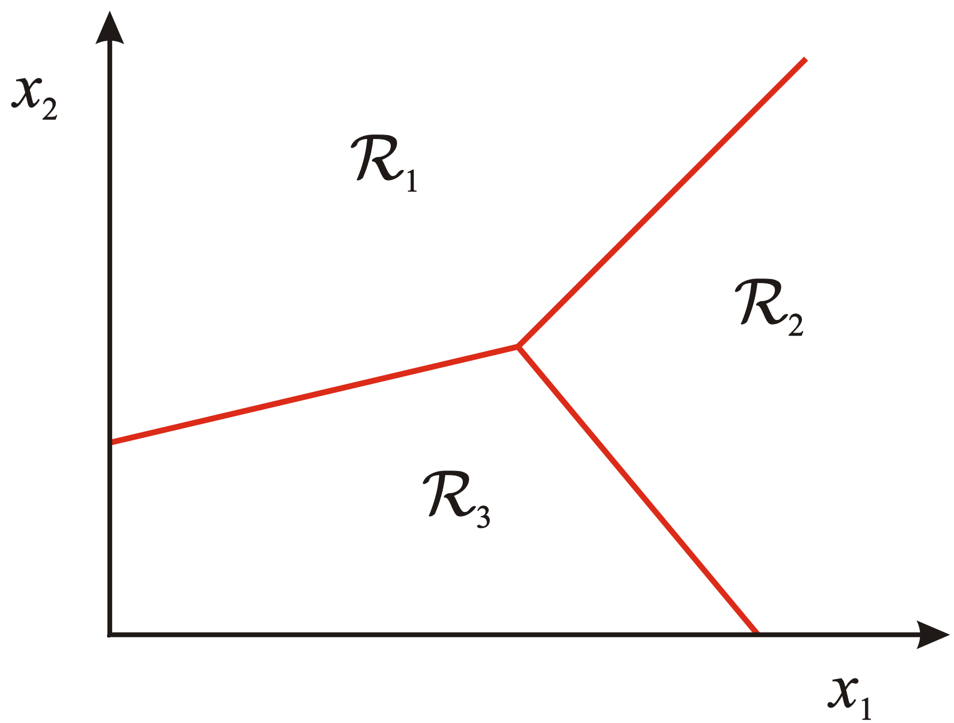
Discriminative methods

- Learn a decision rule (classifier) assigning bag-of-features representations of images to different classes



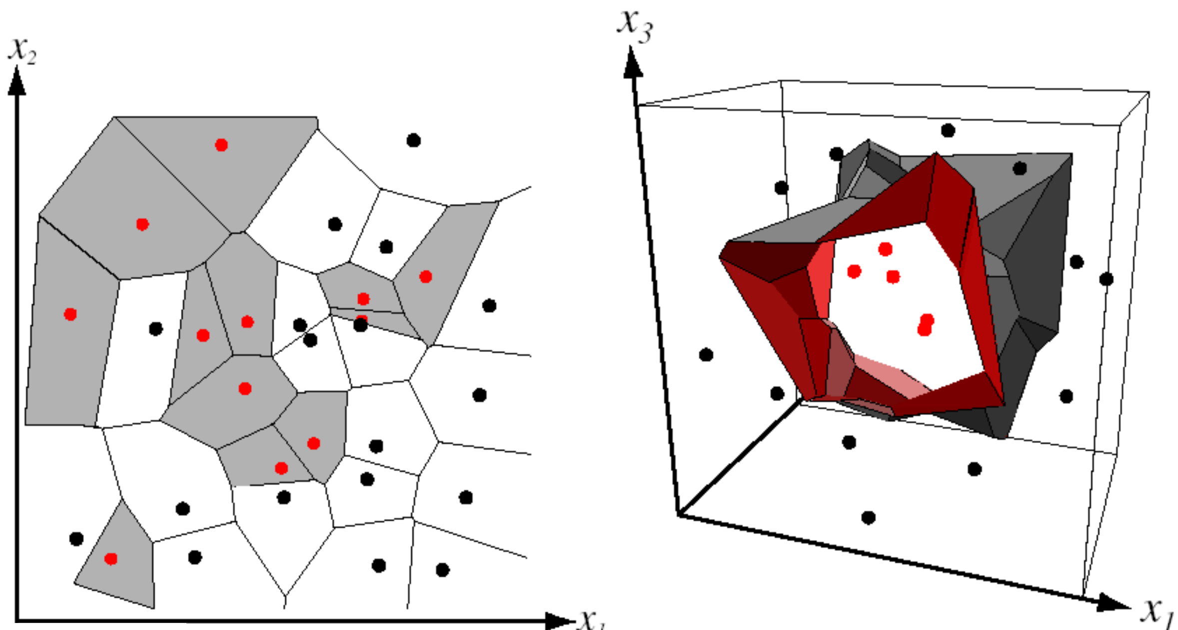
Classification

- Assign input vector to one of two or more classes
- Any decision rule divides input space into *decision regions* separated by *decision boundaries*



Nearest Neighbor Classifier

- Assign label of nearest training data point to each test data point



from Duda *et al.*

Voronoi partitioning of feature space for two-category 2D and 3D data

Source: D. Lowe

Functions for comparing histograms

- L1 distance

$$D(h_1, h_2) = \sum_{i=1}^{N} | h_1(i) - h_2(i) |$$

- χ^2 distance

$$D(h_1, h_2) = \sum_{i=1}^{N} \frac{\left(h_1(i) - h_2(i)\right)^2}{h_1(i) + h_2(i)}$$

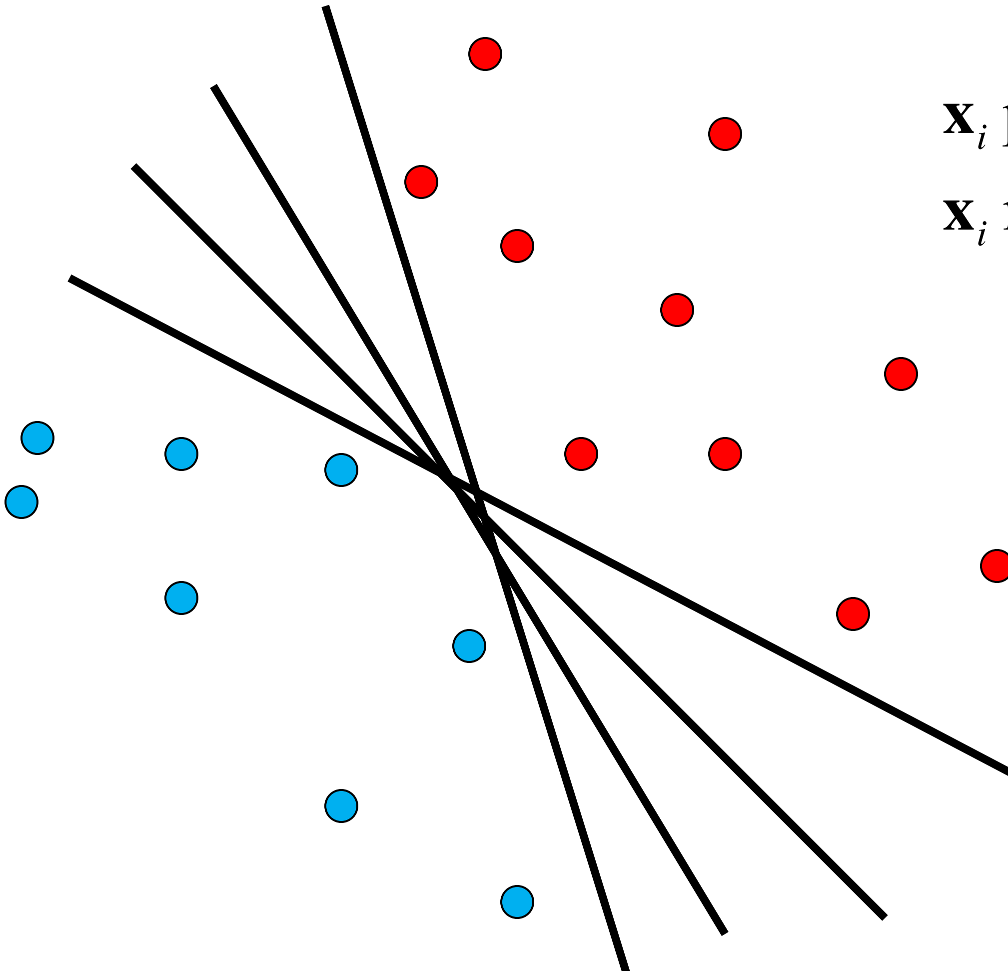
- Quadratic distance (*cross-bin*)

$$D(h_1, h_2) = \sum_{i,j} A_{ij} (h_1(i) - h_2(j))^2$$

Jan Puzicha, Yossi Rubner, Carlo Tomasi, Joachim M. Buhmann: Empirical Evaluation of Dissimilarity Measures for Color and Texture. ICCV 1999

Linear classifiers

- Find linear function (*hyperplane*) to separate positive and negative examples



$\mathbf{x}_i$ positive: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 0$

$\mathbf{x}_i$ negative: $\quad \mathbf{x}_i \cdot \mathbf{w} + b < 0$

Which hyperplane is best?

Slide: S. Lazebnik

Support vector machines

- Find hyperplane that maximizes the *margin* between the positive and negative examples

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Support vector machines

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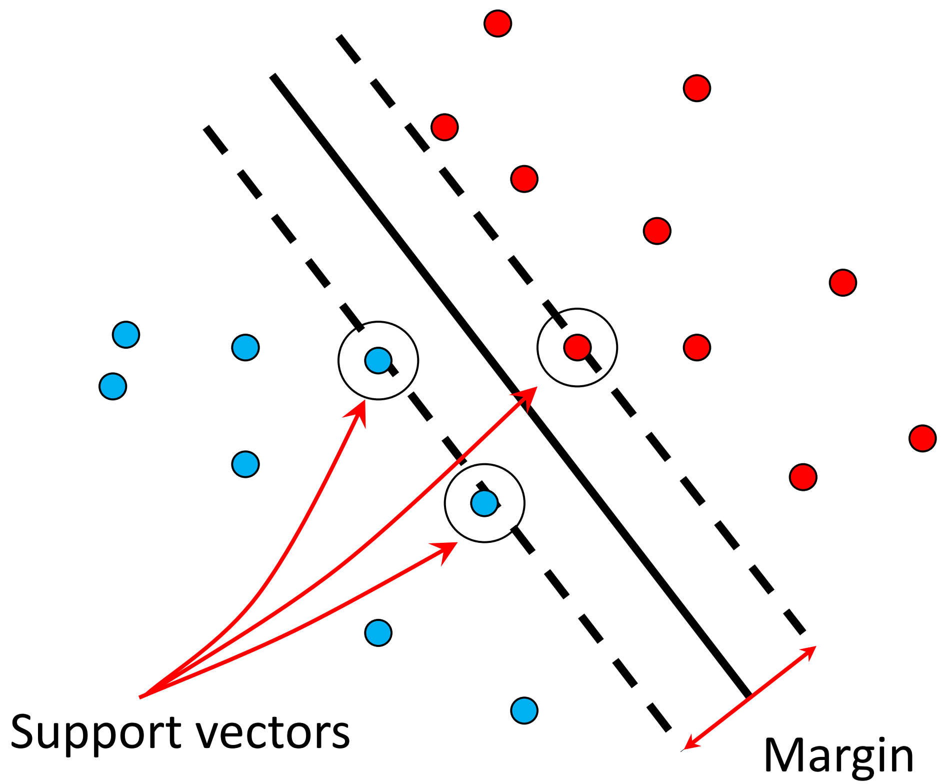


$\mathbf{x}_i$ positive $(y_i = 1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$

$\mathbf{x}_i$ negative $(y_i = -1)$: $\quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$

For support vectors, $\quad \mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and hyperplane: $\quad \dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{\|\mathbf{w}\|}$

Therefore, the margin is $2 / \|\mathbf{w}\|$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin hyperplane

1. Maximize margin $2/\|\mathbf{w}\|$
2. Correctly classify all training data:

$$\mathbf{x}_i \text{ positive } (y_i = 1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$$

$$\mathbf{x}_i \text{ negative } (y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$$

- *Quadratic optimization problem*:
- Minimize $\frac{1}{2}\mathbf{w}^T\mathbf{w}$

  Subject to $y_i(\mathbf{w}\cdot\boldsymbol{x}_i+b) \geq 1$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Finding the maximum margin hyperplane

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

learned weight

Support vector

C. Burges, [A Tutorial on Support Vector Machines for Pattern Recognition](), Data Mining and Knowledge Discovery, 1998

Finding the maximum margin hyperplane

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

  $b = y_i - \mathbf{w} \cdot \mathbf{x}_i$ for any support vector

- Classification function (decision boundary):

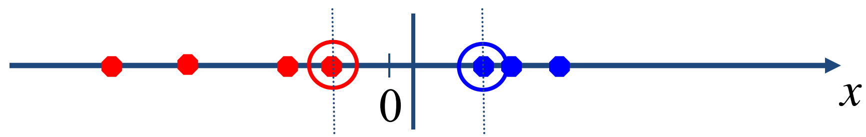
$$\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$$

- Notice that it relies on an *inner product* between the test point $\boldsymbol{x}$ and the support vectors $\boldsymbol{x}_i$
- Solving the optimization problem also involves computing the inner products $\boldsymbol{x}_i \cdot \boldsymbol{x}_j$ between all pairs of training points

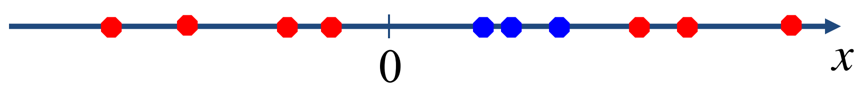
C. Burges, [A Tutorial on Support Vector Machines for Pattern Recognition], Data Mining and Knowledge Discovery, 1998

Nonlinear SVMs

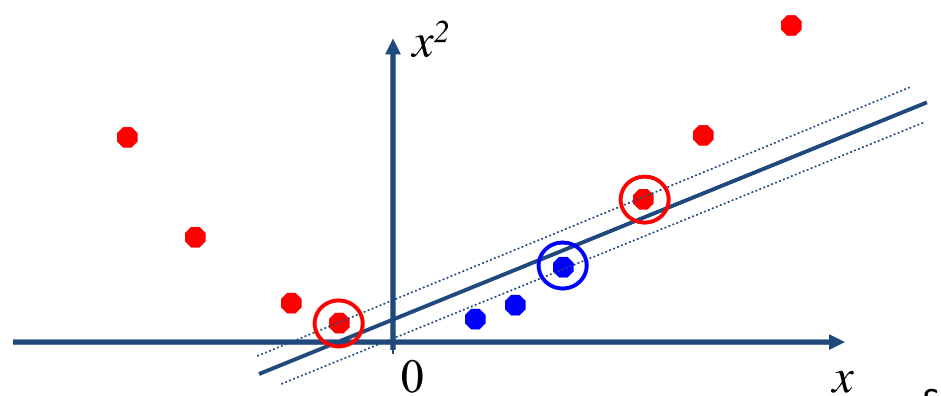
- Datasets that are linearly separable work out great:



- But what if the dataset is just too hard?



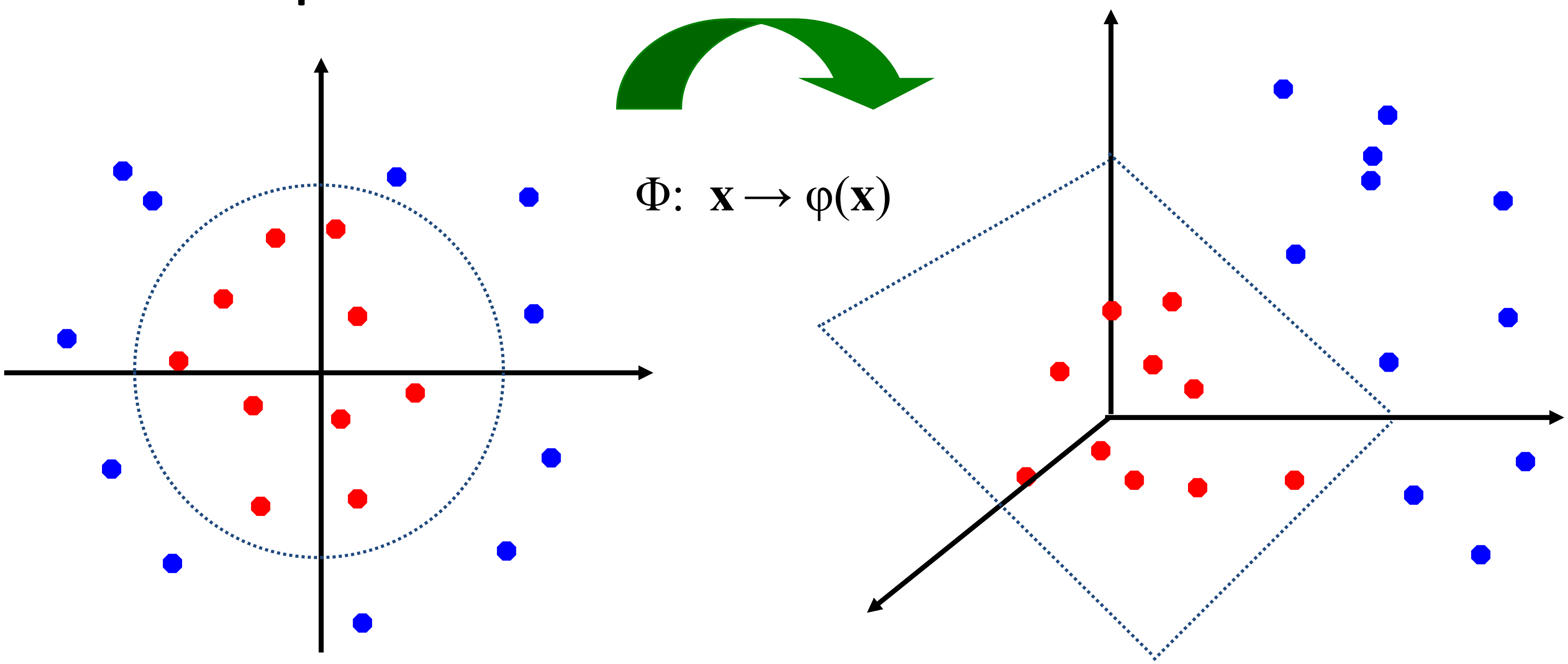
- We can map it to a higher-dimensional space:



Slide credit: Andrew Moore

Nonlinear SVMs

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable:



Slide credit: Andrew Moore

Nonlinear SVMs

- *The kernel trick*: instead of explicitly computing the lifting transformation $\boldsymbol{\varphi}(\mathbf{x})$, define a kernel function K such that

$$K(\mathbf{x}_i, \mathbf{x}_j) = \boldsymbol{\varphi}(\mathbf{x}_i) \cdot \boldsymbol{\varphi}(\mathbf{x}_j)$$

- (to be valid, the kernel function must satisfy *Mercer's condition*)
- This gives a nonlinear decision boundary in the original feature space:

$$\sum_i \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b$$

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Kernels for bags of features

- Histogram intersection kernel:

$$I(h_1, h_2) = \sum_{i=1}^{N} \min(h_1(i), h_2(i))$$

- Generalized Gaussian kernel:

$$K(h_1, h_2) = \exp\left(-\frac{1}{A} D(h_1, h_2)^2 \right)$$

- *D* can be Euclidean distance, χ^2 distance, Earth Mover's Distance, etc.

J. Zhang, M. Marszalek, S. Lazebnik, and C. Schmid, Local Features and Kernels for Classifcation of Texture and Object Categories: A Comprehensive Study, IJCV 2007

Summary: SVMs for image classification

1. Pick an image representation (in our case, bag of features)
2. Pick a kernel function for that representation
3. Compute the matrix of kernel values between every pair of training examples
4. Feed the kernel matrix into your favorite SVM solver to obtain support vectors and weights
5. At test time: compute kernel values for your test example and each support vector, and combine them with the learned weights to get the value of the decision function

What about multi-class SVMs?

- Unfortunately, there is no “definitive” multi-class SVM formulation
- In practice, we have to obtain a multi-class SVM by combining multiple two-class SVMs
- One vs. others
  - Traning: learn an SVM for each class vs. the others
  - Testing: apply each SVM to test example and assign to it the class of the SVM that returns the highest decision value
- One vs. one
  - Training: learn an SVM for each pair of classes
  - Testing: each learned SVM “votes” for a class to assign to the test example

Slide: S. Lazebnik

SVMs: Pros and cons

- Pros
  - Many publicly available SVM packages: http://www.kernel-machines.org/software
  - Kernel-based framework is very powerful, flexible
  - SVMs work very well in practice, even with very small training sample sizes

- Cons
  - No “direct” multi-class SVM, must combine two-class SVMs
  - Computation, memory
    - During training time, must compute matrix of kernel values for every pair of examples
    - Learning can take a very long time for large-scale problems

Slide: S. Lazebnik

Summary: Discriminative methods

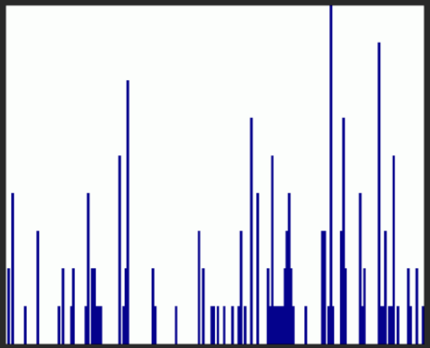
- Nearest-neighbor and k-nearest-neighbor classifiers
  - L1 distance, χ^2 distance, quadratic distance, Earth Mover's Distance
- Support vector machines
  - Linear classifiers
  - Margin maximization
  - The kernel trick
  - Kernel functions: histogram intersection, generalized Gaussian, pyramid match
  - Multi-class
- Of course, there are many other classifiers out there
  - Neural networks, boosting, decision trees, …

Adding spatial information

- Computing bags of features on sub-windows of the whole image
- Using codebooks to vote for object position
- Generative part-based models

Spatial pyramid representation

- Extension of a bag of features
- Locally orderless representation at several levels of resolution



level 0

Lazebnik, Schmid & Ponce (CVPR 2006)

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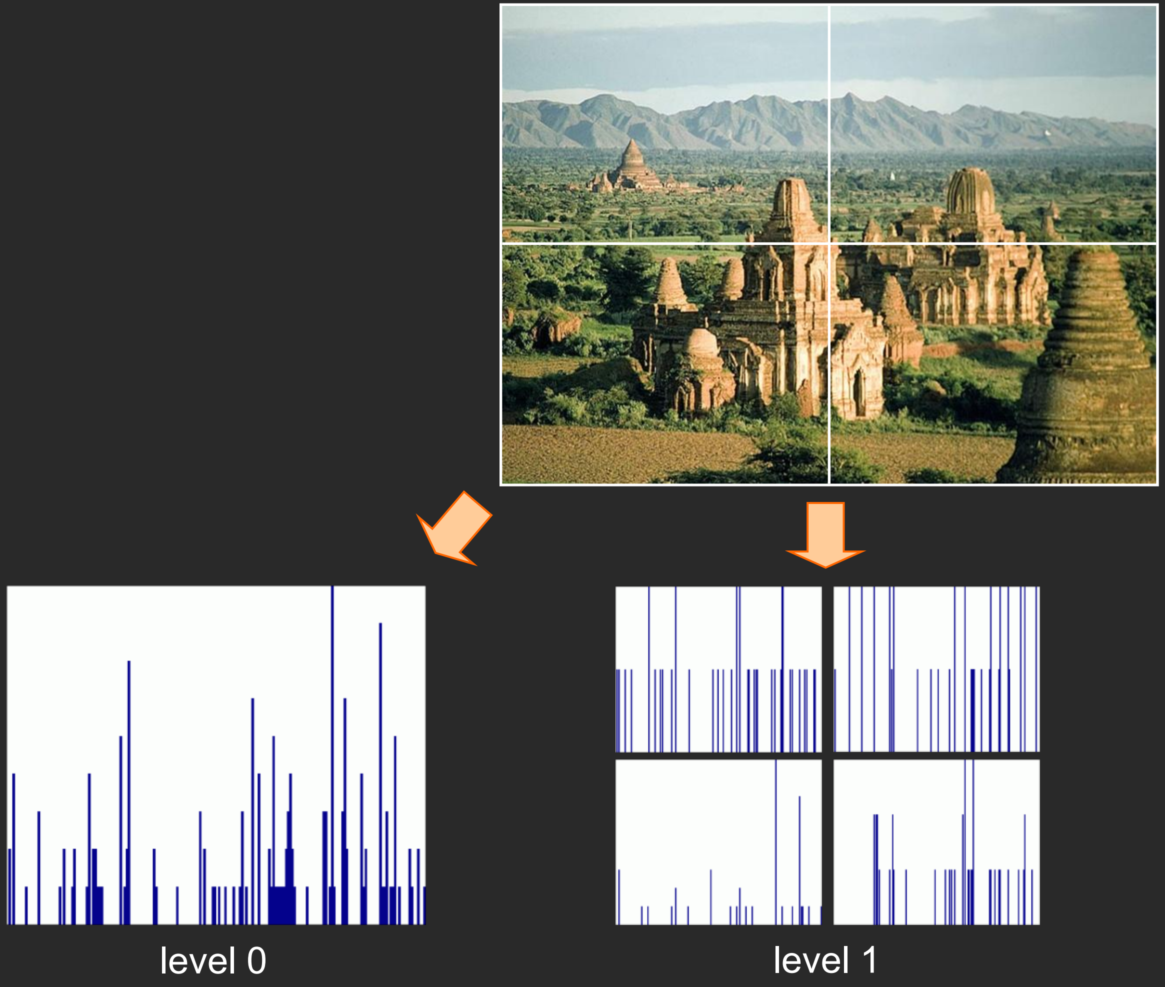

Lazebnik, Schmid & Ponce (CVPR 2006)

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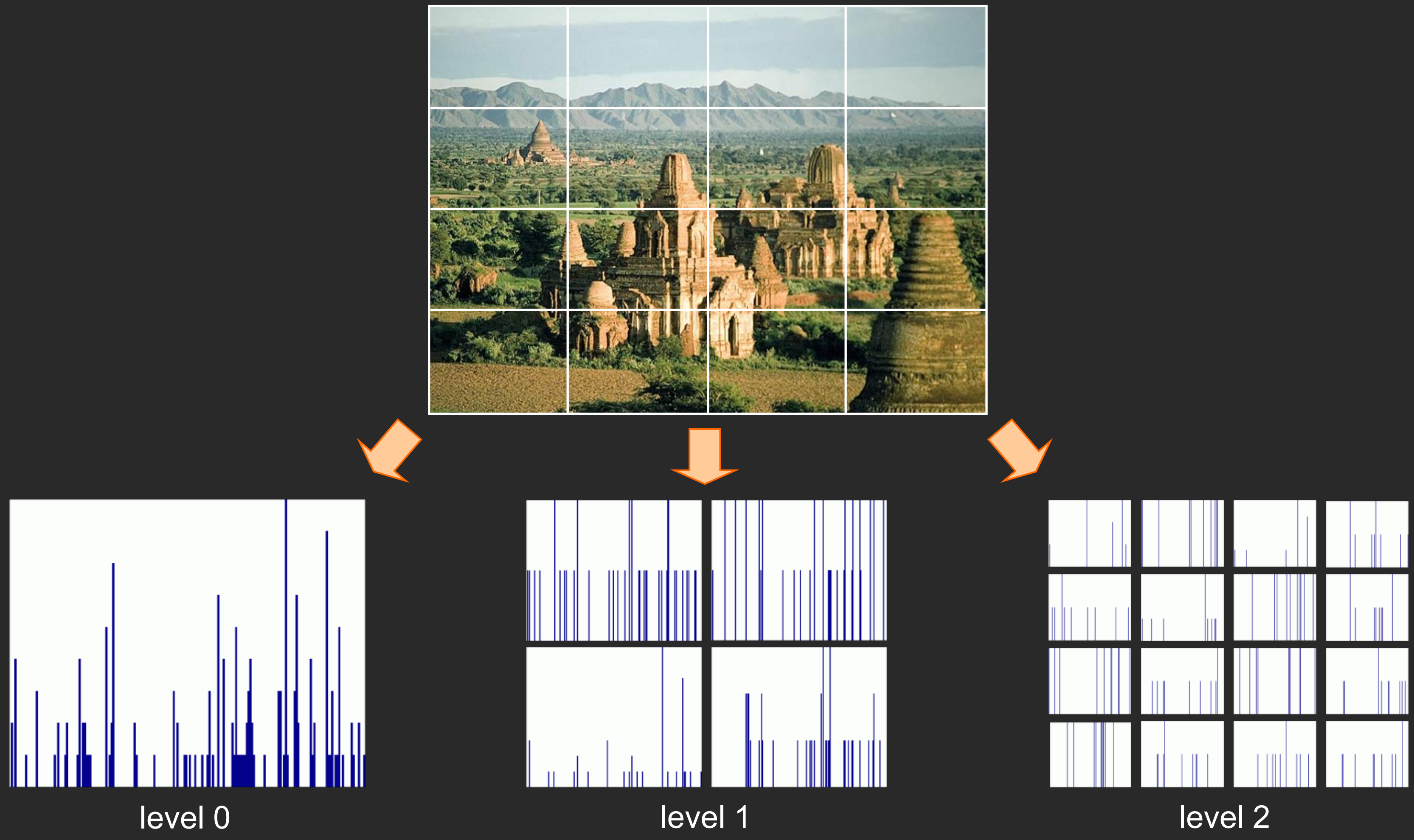

Lazebnik, Schmid & Ponce (CVPR 2006)

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