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Inventory management in distribution systems – case of an Indian FMCG company

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Abstract

Managing inventory in distribution systems involves taking decisions on the quantity of inventory to be placed at different stages so that a desired customer service level can be achieved at minimum cost. In this paper, we present the case of the distribution system of an Indian FMCG company, which despite having centralized control takes replenishment decisions based on local stock information. The objective of the study is to develop single period representative models based on installation stock and echelon stock, and show by simulation that had the company taken decisions based on system stock information, it would have saved money in terms of investment in inventory.

Keywords: Case study; Inventory; Distribution system; Installation stock; Echelon stock

1. Introduction

Managing inventory in distribution systems involves taking decisions on the quantity of inventory to be placed at different stages so that a desired customer service level can be achieved at minimum cost. If most of the inventory is placed at the lowest stage or the stage facing external demand, the customer service level improves; however, this is accompanied by an increase in the inventory carrying cost due to value addition at the lower stages. On the other hand, if most of the inventory is placed away from the lowest stage, the inventory carrying cost decreases, but at the same time the delivery lead time increases, leading to the deterioration of the customer service level. A trade-off between these two counteracting issues has to be made while taking decisions on the positioning and quantity of inventory in a distribution system.

There are two types of control in a distribution system – centralized and decentralized. In a centralized control system, the decision maker at the highest stage decides on how much to order and how to allocate the available inventory among the downstream locations based on an echelon stock policy, where echelon stock at a location is the stock at that location plus the stock

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(including in-transit) at all of its downstream locations [6]. The entire available inventory can be allocated [7, 9, 10] or some inventory can be held back after shipping out most of it at the start of each order cycle, and shipped later in the cycle to balance the inventories at the downstream locations [15, 16]. A virtual allocation system was studied by Graves [12], where stock at the higher stage was reserved for certain downstream locations as demand occurred. One of the primary findings of the study was that a certain amount of inventory should be held at the higher stages, but most of the inventory should be located downstream. In fact, holding inventory at higher stages is meaningful if the inventory carrying cost is very low and/or the delivery lead times are negligible. Heijden et al. [14] discussed several allocation rules in multi-stage distribution systems.

In a decentralized control system, every location takes replenishment decisions on its own. The decisions can be taken based on either an echelon stock or an installation stock policy. Installation stock at a location refers to the stock at that location only. It has been mentioned in the literature [2, 3, 4] that in serial and assembly systems one can always find an echelon stock policy which is at least as good as an installation stock policy, but in distribution systems the policies can outperform each other under different operating conditions. One of the problems with an installation stock policy is that the demand distribution at every higher stage is to be derived. This can be avoided with an echelon stock policy, which needs only the end-item demand distribution. To overcome the problem of deriving the demand distributions in an installation stock policy, several assumptions and approximations have been made in the literature [22, 26]. The problem can also be overcome by making available the end-item demand information at every location. This is done in the base stock policy where every location takes replenishment decisions based on the actual end-item demand, rather than the demand generated by downstream locations [13, 25]. The availability of the end-item demand information also reduces the so-called “bullwhip effect” [17]. Several authors have quantified the benefits of information sharing in decentralized multi-stage supply chains [11, 18, 19]. For a review of the centralized and decentralized planning models, refer to [8] and [1] respectively. More references can be found in [24].

Most of the literatures discussed so far deal with stationary demand. When demand becomes non-stationary, the analytical process becomes quite complicated in terms of the derivation of the policy parameters. Distribution Requirement Planning (DRP) followed in practice to manage inventory is essentially a deterministic process where uncertainty is taken care of by a periodic revision of dispatch schedules. Most of the DRP systems are decentralized and based on an echelon stock policy. Very few DRP systems are centralized [24]. In this paper, we present the case of the dairy division of an Indian FMCG company whose distribution system is centralized, but replenishment decisions are taken based on local stock information only. The objective of the study is to develop single period models based on an installation stock policy (representing the present distribution system) and an echelon stock policy (representing the ideal distribution system), and show by simulation that the company would have saved money in terms of investment in inventory had it taken decisions based on an echelon stock policy.

The organization of the paper is as follows. Section 2 describes the distribution system of the dairy division. The current ordering/inventory control policy is presented in Section 3, followed by a description of the system under study in Section 4. In Section 5, the simulation procedure is discussed and models used for simulation are developed. A discussion on the simulation results is presented in Section 6. Section 7 concludes the paper.

2. Distribution system of the dairy division

Currently the company does not have a production facility for its dairy whitener and cheese products; instead it outsources its requirements from another dairy, and sells them through conventional distribution channels. The dairy has a contract with the company such that the company's current requirements can be met within a short span of time. The various Stock Keeping Units (SKUs) are shipped from the factory to the Carrying and Forwarding Agents (C&FAs), and from the C&FAs to the Authorized Wholesalers (A/Ws), which constitutes the primary sales of the company. The Authorized Wholesalers in turn sell the SKUs to retailers, independent wholesalers and institutions such as hotels and restaurants, which constitutes the secondary sales. The company takes care of the primary freight between the factory and the C&FAs as well as the secondary freight between the C&FAs and the A/Ws. The transportation systems for dairy whitener and cheese are different since cheese requires refrigerated transportation and needs cold storage at the C&FAs and A/Ws. Fig. 1 shows the distribution system of the dairy products.

The distribution network is divided into four regions. The headquarters of the eastern region is located in Calcutta. There are 8 C&FAs and around

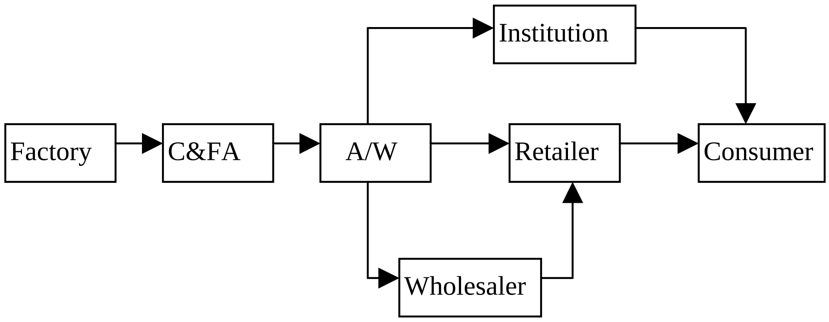

Figure 1 Distribution System of the Dairy Products

185 A/Ws in this region. The biggest C&FA is located in Calcutta, which also acts as the “mother depot” in this region feeding other C&FAs having low sales volumes. The SKUs are shipped from the dairy to the Calcutta C&FA from where they are dispatched to other C&FAs in the region where the sales volumes of those SKUs do not justify direct shipment from the dairy.

From now on, the C&FAs and A/Ws will be referred to as “depots” and “distributors” respectively, as they are commonly called by the company personnel.

3. Present ordering/inventory control policy

At the beginning of every month, the company makes a primary sales plan for each distributor, based on its secondary sales in the previous month and the current stock availability. The primary sales plans for all the distributors are then translated into overall requirements for the month, and orders are placed with the dairy with dispatch schedules for various depots. The dairy operates with a three months’ rolling plan, and is capable of meeting the current requirements of the company. The average transportation time from the dairy to the Calcutta depot is 20 days. The depot ordering policy is as follows. Based on the primary sales plan and the availability of stock at the depot, the order quantity is calculated as follows: order quantity = primary sales plan (including inter-depot transfers) – availability + desired closing balance. For the Calcutta depot, the desired closing balance is 25 days’ planned sales volume. The distributors are also stocked in such a way that at the beginning of every month they have around 30 days’ inventory. The movement of stock closely follows the consumers’ buying patterns, which peak during the first and last weeks of every month. For both primary and secondary sales, the last 10 days' sales accounts for 40-45% of total sales in the month.

From the above discussion it is readily visible that the combined stock of the Calcutta depot and its distributors at the beginning of every month is equivalent to 55 days' secondary sales. This brings out the fact that the distributors are always carrying one month's stock in excess. The company's policy is to push stock to expedite secondary sales under inventory pressure. Though this fulfills the company's target for primary sales for the month, it results in the distributors carrying excess stock.

4. Description of the system under study

As a prototype, the Calcutta depot and three of its distributors, which account for most of the sales from the depot, are considered for the study. This is equivalent to a two-echelon distribution system, shown in Fig. 2, with the depot at the higher echelon and the distributors at the lower echelon.

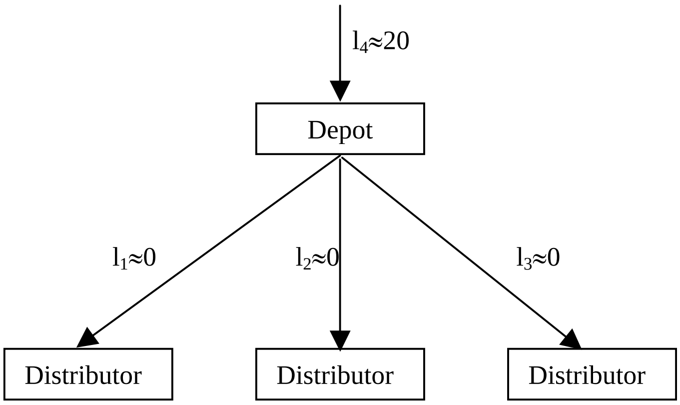

Figure 2 Distribution System for the Case Study
(l_i = lead time in days at stage i)

The system can be generalized across all the depots and distributors. The dairy has excess production capacity, and has been excluded from the study. It is assumed that the depot outsources its requirements from a location with infinite capacity. As there are no resource constraints anywhere in the distribution system, and no interactions among the various SKUs, the SKUs can be treated independently. In this study, a specific SKU of dairy whitener, namely 500g poly pouch, which sells most in this product category, is considered. The average transportation time for dairy whitener from the dairy to the depot, as mentioned earlier, is 20 days. The transportation time from the depot to the distributors is taken to be zero as the distributors are located in and around Calcutta, and the shipping time is a few hours.

The following assumptions have been made for modelling the system.

- The unit of the SKU under consideration (500g poly pouch) is taken as one Card Board Box (CBB) containing 24 such SKUs, which is the form in which it is transported from the factory to the depot and then to the distributors before bulk breaking at the latter's site.
- Since the dairy division of the company began operation in the middle of 1999, the monthly secondary sales data of the years 2000, 2001 and 2002 (January and February) are only available with the author, which are shown in Appendix 1. Since the available data are not adequate in number, sophisticated forecasting techniques such as Univariate Box-Jenkins (UBJ) or Auto Regressive Integrated Moving Average (ARIMA) could not be used, which require at least fifty observations for reliable forecasting [5, 21]. For the purpose of the study, an exponential smoothing forecasting model with trend correction [20, 23] is used. The forecast errors are assumed to be stationary and normally distributed with mean zero and standard deviation equal to the Root Mean Square Error (RMSE). Hence the forecasted demand for any month can be assumed to be normally distributed with mean equal to the forecasted value, and standard deviation as the RMSE.
- The safety factor at the depot is assumed to be 2 (corresponding to 98% service level for normally distributed demand) in concurrence with the inventory norms followed by the company. Holding costs at the depot stage have not been considered; instead the performances of the models are compared with respect to the month-end residual inventories at the depot. On the other hand, positive holding and shortage costs are considered at the distributor stage, and the value of the safety factor is computed from the well-known single-period newsboy model formulation. The holding cost is the cost of capital tied up plus the cost of one unsold unit. Cost of one unsold unit is the cost per unit minus the salvage value, and is equal to zero since it is assumed that the inventory left over at the end of the month can be salvaged at least at its cost price. The shortage cost is the loss of profit. The holding and shortage costs are assessed based on the month-

end inventories. Calculations for holding and shortage costs are shown in Appendix 2.

- There is no fixed cost anywhere in the system. The cost of ordering is not significant.
- Distributors' demands for any month occur at the beginning of the month. In practice, there may be multiple deliveries. But since it is assumed that holding and shortage costs are assessed based on the month-end inventories, this will not affect the models' validity. Demand unsatisfied at any stage is backordered.
- The transportation time from the depot to the distributors, as mentioned before, is assumed to be zero.

For each of the two models developed for the distribution system, the performance is examined by simulation for the period January 2001 to February 2002. The forecast for demand is made in each month based on the data available till that time. The monthly forecast for secondary sales from January 2001 to February 2002 is shown in Appendix 3. The performance of the models is measured in terms of the month-end depot inventories.

5. Simulation procedure and models used for simulation

The depot places order with the dairy at the beginning of every month, after knowing the actual secondary sales in the previous month and current availability of stock at the distributors' ends. The deliveries are received around 20th of the month. So, to cater to the requirements of the distributors before the deliveries are realized, the depot plans in such a way that it has 25-30 days' stock at the beginning of every month. In the beginning of January 2001, when the simulation starts, the initial stock levels at the distributors are assumed to be equal to their corresponding safety values. The initial depot stock is set at a level equivalent to 20 days' combined demand forecasts for the distributors. For each month, the order quantities for the distributors and the depot are computed depending on the model being used. The primary demand for the month is equal to the combined order quantity of the distributors. For secondary demands two cases are considered. In the first case, secondary demands are normal random variables with parameters obtained as per the assumption made in Section 4, and 1000 replications are performed to arrive at the expected month-end depot and distributor inventories. The second case is a specific instance where secondary demands are denoted by the actual secondary sales figures for the corresponding month. During the simulation, it is assumed that whenever a distributor faces shortage, it is replenished with the shortage quantity by the depot instantly, provided that the depot has sufficient stock for the month (opening balance plus receivables). Finally, the primary/secondary sales figures, and the month-end inventories for the depot and the distributors are computed. The flowchart of the simulation is shown in Figure 3.

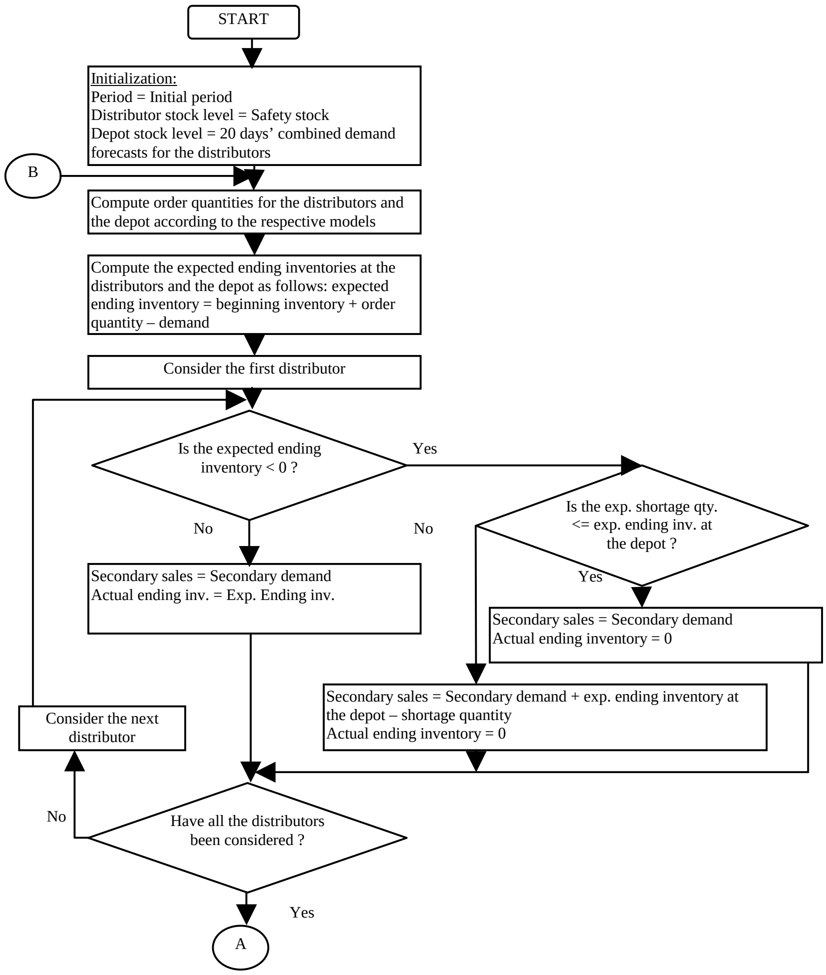

Figure 3 Flowchart of the Simulation

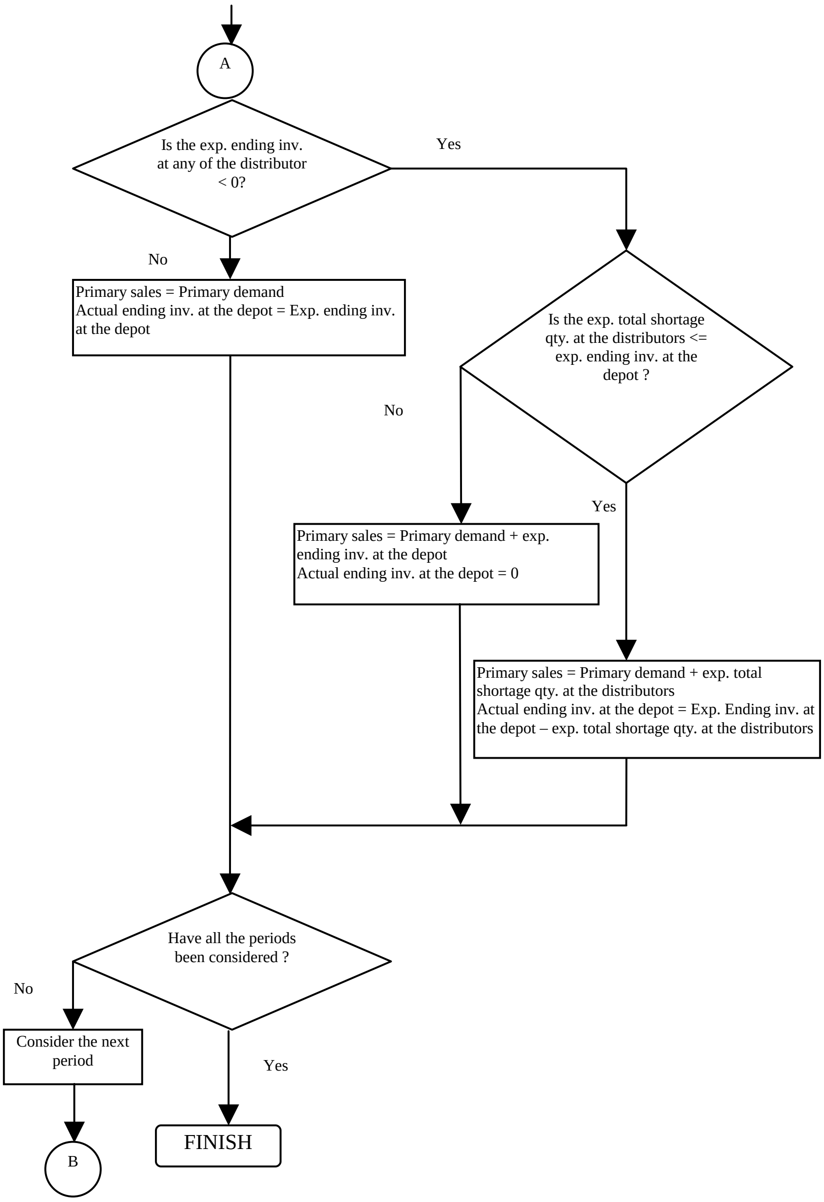

Figure 3 Flowchart of the Simulation (Continued)

The three distributors and the depot are represented by stage 1, stage 2, stage 3 and stage 4 respectively. Before discussing the models, the following symbols are noted.

μ_j Average demand in a period at stage *j*

σ_j Standard deviation of demand in a period at stage *j*

h_j Holding cost per unit per period at stage *j*

p_j Shortage cost per unit at stage *j*

l_j Lead time at stage *j*

S_j Order-up-to level at stage *j*

k_j Safety factor at stage *j*

x_j Inventory position at stage *j*

T Review period (one month in this case)

$\Phi(\cdot)$ Cumulative distribution function of the standard normal distribution

5.1 Single period myopic model based on installation inventory

In this model, the decision is taken for the current month without giving any weight to the expected sales patterns for the coming months. Also, the depot's ordering decision is based on its own inventory, without taking into account the distributors' inventory positions. The sequence of decisions that are taken at the depot and the distributor levels is as follows.

Step 1: For each distributor, find the order-up-to level from the following.

$S_j = \mu_j\left(1+\frac{l_j}{T}\right)+k_j\sigma_j\sqrt{1+\frac{l_j}{T}}$, where k_j is obtained from the well-known newsboy problem, $\Phi(k_j) = \frac{p_j}{p_j+h_j}$ for j = 1,2,3.

Step 2: For each distributor, find the order quantity, S_j-x_j for j = 1,2,3.

Step 3: The depot order quantity is calculated as follows. Depot order quantity = total issues + desired closing balance – available inventory. Total issue quantity is the sum of the order quantities of the distributors. The desired closing balance is the average l_4 days' sales plus safety stock for l_4 days.

For calculation of the safety stock, it is assumed that the variance of the combined order quantities of the distributors is equal to the sum of the variances of the secondary sales forecasts. Hence the depot order quantity can be written as $\sum_{j=1}^{3}\left(S_j - x_j\right) + \frac{l_4}{T}\sum_{j=1}^{3}\left(S_j - x_j\right) + k_4\sqrt{\frac{l_4}{T}\sum_{j=1}^{3}\sigma_j^2} - x_4$, where k_4 is assumed to be 2.

It is to be noted here that the depot replenishments are based on primary demands from the distributors. This is exactly equivalent to the present ordering policy followed by the company, where primary demands are substituted by primary sales plans. The only difference is that in this model, the distributors are not carrying excess inventory, and the ordering decisions are made theoretically rather than intuitively.

5.2 Single period myopic model based on echelon inventory

This model differs from the previous model in that the replenishment decision for the depot is taken based on the secondary sales forecast and the depot echelon inventory level. The order-up-to level for the depot is set to cover ($T+l_4$) days' secondary sales. Hence the depot order quantity becomes $\left(1+\frac{l_4}{T}\right)\sum_{j=1}^{3}\mu_j + k_4\sqrt{1+\frac{l_4}{T}}\sqrt{\sum_{j=1}^{3}\sigma_j{}^2} - x_4$, where x_4 is the depot echelon inventory. The order quantities for the distributors are calculated as before.

6. Discussion of the simulation results

Table 1 shows the expected month-end depot installation inventories obtained by applying the two models described in the previous section when secondary demands are normal random variables. Fig. 4 shows the monthwise differences between the expected depot inventories obtained from the model based on echelon inventory and the same obtained from the model based on installation inventory.

For the specific instance where secondary demands are equal to the actual secondary sales, the simulation results are shown in Appendix 4 and Appendix 5. All the inventory figures in Appendix 5 are based on echelon stock except the figures in the last column, i.e., "Difference in depot inventory", which are in terms of installation stock.

Table 1 Expected Month-end Depot Inventories: Simulation Results
(Secondary Demands are Normal Random Variables)

Month	Expected depot inventory from the model based on	
	Installation inventory	echelon inventory
Jan-01	237	226
Feb-01	276	240
Mar-01	286	253
Apr-01	266	254
May-01	309	278
Jun-01	209	267
Jul-01	272	271
Aug-01	291	279
Sep-01	290	283
Oct-01	263	275
Nov-01	289	278
Dec-01	240	267
Jan-02	313	284
Feb-02	334	313

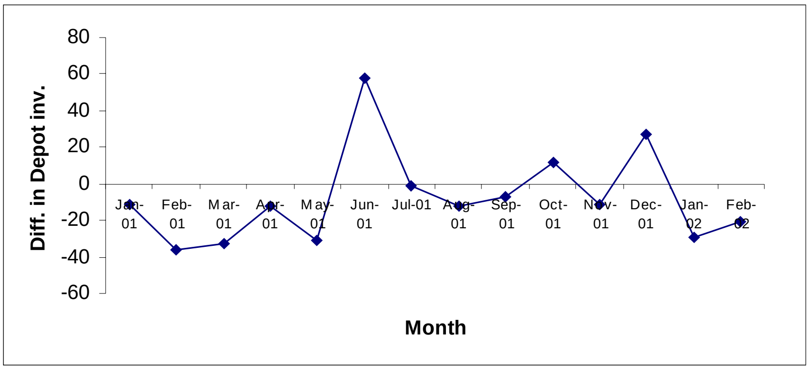

Figure 4 Month-wise Differences in Expected Depot Inventories
(Secondary Demands are Normal Random Variables)

The column "Difference in depot inventory" in Appendix 5 shows the differences between the month-end depot inventories obtained from the model based on echelon inventory (In this case, the month-end depot installation inventories are obtained by subtracting the total month-end distributor inventories from the month-end depot echelon inventories) and the same obtained from the model based on installation inventory. Fig. 5 shows graphically the month-wise differences in depot inventories.

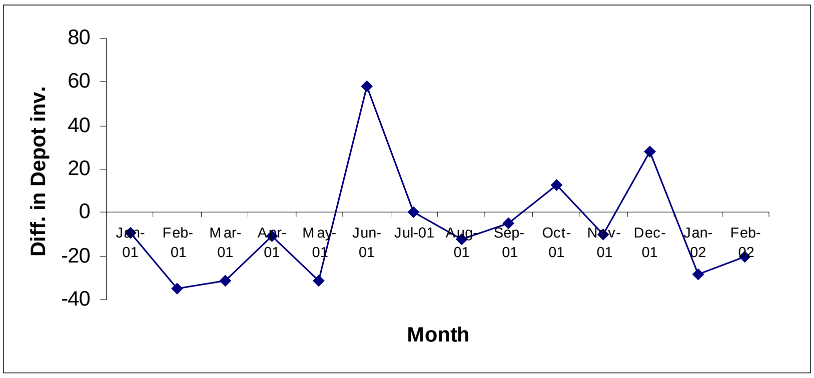

Figure 5 Month-wise Differences in Depot Inventories
(Secondary Demands are Equal to the Actual Secondary Sales)

It is seen from either Fig. 4 or Fig. 5 that in most of the months the model based on echelon inventory results in less depot inventory than the model based on installation inventory, the maximum reduction in investment in depot inventory being 12.41%. It is expected in real life when all the distributors and depots are taken into account, the total savings in monetary terms would be substantial. It is also observed that the less the combined distributor inventory at the beginning of a month, the more is the difference between the month-end depot inventories in favour of the model based on echelon inventory. The phenomenon is shown graphically in Fig. 6.

The phenomenon can be explained as follows. The opening stock levels at the distributors at the beginning of any month can be attributed to the gap between the forecast for demand and the actual secondary sales during the previous month. When the opening stock levels at the distributors are higher than their average levels due to overestimated demand in the previous month,

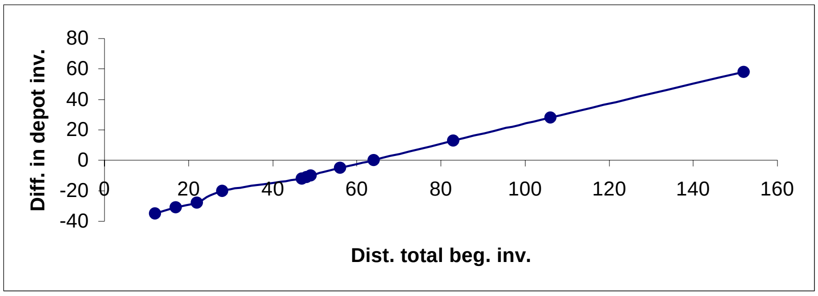

Figure 6 Differences in Depot Inventories against Distributors' Combined Inventory at the Beginning of the Month

the order quantities placed by the distributors with the depot to satisfy demand for the current month will be lower than their average values. In this situation the model based on installation inventory, where the depot order quantity depends on the combined order quantities for the distributors, may result in less month-end depot inventory than the model based on echelon inventory, where the depot order quantity is computed from the forecasted demand distributions. From Appendix 1, we can see that the secondary sales suddenly shot up in April 2001 due to a promotional campaign carried out by the company. In the next month sales plummeted, and subsequently at the beginning of June 2001, there are high stock levels at the distributors, which favours the model based on installation inventory as we can see from Appendix 5. In practice, the forecasts made by the sales personnel will be more accurate than the forecasts made by the exponential smoothing model used here, and it is expected that the more accurate the forecast, the better will be the performance of the model based on echelon inventory, as far as the month-end depot inventories are concerned. This was confirmed by a paired-sample signed-rank test at 5% level of significance excluding the data for June 2001 (Appendix 6).

7. Concluding remarks

In this paper, we have presented the case of an Indian FMCG company in the context of inventory management in its dairy products distribution system. Though the company has centralized control, i.e., it has full information on the actual secondary sales and stock positions of all the distributors, the ordering decisions taken by it are based on local stock information only. It has been shown in this paper through simulation that had the company taken ordering decisions based on echelon inventory, it would have saved money in terms of investment in depot inventory.

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Appendices

Appendix 1

Monthly Secondary Sales Data for the Distributors (Jan 2000 – Feb 2002)

Month	Distributor 1	Distributor 2	Distributor 3
Jan-00	85	111	-
Feb-00	118	100	-
Mar-00	102	104	-
Apr-00	95	94	-
May-00	79	121	12
Jun-00	38	53	15
Jul-00	103	76	35
Aug-00	142	106	31
Sep-00	86	100	27
Oct-00	85	107	36
Nov-00	109	111	33
Dec-00	105	129	66
Jan-01	155	129	58
Feb-01	130	144	52
Mar-01	131	123	57
Apr-01	181	135	108
May-01	57	130	55
Jun-01	98	140	76
Jul-01	111	151	76
Aug-01	118	163	59
Sep-01	125	139	56
Oct-01	123	151	68
Nov-01	100	136	53
Dec-01	127	180	67
Jan-02	112	229	74
Feb-02	116	211	69

Appendix 2

Calculations for holding and shortage costs

Cost per unit to the distributors for buying from the company = Rs. 1376
Revenue per unit for the distributors from selling to the retailers = Rs. 1431.
Estimated cost of capital = Rs. 0.15 per Re per year
Cost of capital per unit per month for the distributors = Rs. $0.15\times1376\times(1/12)$ = Rs. 17.2
Hence holding cost = 17.2 and shortage cost = 1431 - 1376 = 55

Appendix 3

Forecasts of Secondary Sales from January 2001 to February 2002

Forecast made in	Location	Forecast for the next month
Dec-00	Dist 1	97
	Dist 2	122
	Dist 3	49
Jan-01	Dist 1	105
	Dist 2	130
	Dist 3	55
Feb-01	Dist 1	109
	Dist 2	143
	Dist 3	57
Mar-01	Dist 1	113
	Dist 2	136
	Dist 3	60
Apr-01	Dist 1	123
	Dist 2	139
	Dist 3	83
May-01	Dist 1	115
	Dist 2	137
	Dist 3	77
Jun-01	Dist 1	114
	Dist 2	142
	Dist 3	80
Jul-01	Dist 1	114
	Dist 2	150
	Dist 3	83
Aug-01	Dist 1	115
	Dist 2	162
	Dist 3	77
Sep-01	Dist 1	118
	Dist 2	153
	Dist 3	71
Oct-01	Dist 1	119
	Dist 2	155
	Dist 3	72
Nov-01	Dist 1	117
	Dist 2	146
	Dist 3	66
Dec-01	Dist 1	119
	Dist 2	168
	Dist 3	68
Jan-02	Dist 1	119
	Dist 2	209
	Dist 3	71
Feb-02	Dist 1	120
	Dist 2	220
	Dist 3	72

Appendix 4

Single Period Myopic Model Based on Installation Inventory: Simulation Results (Secondary Demands are Equal to the Actual Secondary Sales)

Month	Location	Beginning inventory	Order quantity	Actual Primary/ Secondary demand	Expected ending inventory	Actual Primary/ Secondary sales	Actual ending inventory
Jan-01	Dist 1	21	97	155	-37	155	0
	Dist 2	16	122	129	9	129	9
	Dist 3	12	49	58	3	58	3
	Depot	179	336	268	246	305	209
Feb-01	Dist 1	0	126	130	-4	130	0
	Dist 2	9	137	144	2	144	2
	Dist 3	3	64	52	15	52	15
	Depot	209	405	328	286	332	282
Mar-01	Dist 1	0	130	131	-1	131	0
	Dist 2	2	157	123	36	123	36
	Dist 3	15	54	57	12	57	12
	Depot	282	355	342	295	343	294
Apr-01	Dist 1	0	134	181	-47	181	0
	Dist 2	36	116	135	17	135	17
	Dist 3	12	60	108	-36	108	0
	Depot	294	292	311	275	394	192
May-01	Dist 1	0	144	57	87	57	87
	Dist 2	17	138	130	25	130	25
	Dist 3	0	95	55	40	55	40
		192	505	378	319	378	319
Jun-01	Dist 1	87	49	98	38	98	38
	Dist 2	25	128	140	13	140	13
	Dist 3	40	49	76	13	76	13
	Depot	319	127	227	219	227	219
Jul-01	Dist 1	38	97	111	24	111	24
	Dist 2	13	145	151	7	151	7
	Dist 3	13	79	76	16	76	16
	Depot	219	385	322	282	322	282
Aug-01	Dist 1	24	111	118	17	118	17
	Dist 2	7	159	163	3	163	3
	Dist 3	16	79	59	36	59	36
	Depot	282	369	350	301	350	301
Sep-01	Dist 1	17	119	125	11	125	11
	Dist 2	3	175	139	39	139	39
	Dist 3	36	53	56	33	56	33
	Depot	301	346	348	299	348	299
Oct-01	Dist 1	11	128	123	16	123	16
	Dist 2	39	130	151	18	151	18
	Dist 3	33	50	68	15	68	15
	Depot	299	283	309	273	309	273
Nov-01	Dist 1	16	124	100	40	100	40
	Dist 2	18	153	136	35	136	35
	Dist 3	15	69	53	31	53	31
	Depot	273	373	347	299	347	299
Dec-01	Dist 1	40	98	127	11	127	11
	Dist 2	35	127	180	-18	180	0
	Dist 3	31	47	67	11	67	11
	Depot	299	223	273	249	291	231
Jan-02	Dist 1	11	129	112	28	112	28
	Dist 2	0	184	229	-45	229	0
	Dist 3	11	69	74	6	74	6
	Depot	231	475	383	323	428	278
Feb-02	Dist 1	28	112	116	24	116	24
	Dist 2	0	225	211	14	211	14
	Dist 3	6	77	69	14	69	14
	Depot	278	481	415	344	415	344

Appendix 5

Single Period Myopic Model Based on Echelon Inventory: Simulation Results (Secondary Demands are Equal to the Actual Secondary Sales)

Month	Location	Beginning inventory	Order quantity	Actual Primary/ Secondary demand	Expected ending inventory	Actual Primary/ Secondary sales	Actual ending inventory	Difference in depot inventory*
Jan-01	Dist 1	21	97	155	-37	155	0	
	Dist 2	16	122	129	9	129	9	
	Dist 3	12	49	58	3	58	3	
	Depot	228	326	269	212	306	212	-9
Feb-01	Dist 1	0	126	130	-4	130	0	
	Dist 2	9	137	144	2	144	2	
	Dist 3	3	64	52	15	52	15	
	Depot	212	378	328	264	332	264	-35
Mar-01	Dist 1	0	130	131	-1	131	0	
	Dist 2	2	157	123	36	123	36	
	Dist 3	15	54	57	12	57	12	
	Depot	264	358	342	311	343	311	-31
Apr-01	Dist 1	0	134	181	-47	181	0	
	Dist 2	36	116	135	17	135	17	
	Dist 3	12	60	108	-36	108	0	
	Depot	311	311	311	198	394	198	-11
May-01	Dist 1	0	144	57	87	57	87	
	Dist 2	17	138	130	25	130	25	
	Dist 3	0	95	55	40	55	40	
		198	484	378	440	378	440	-31
Jun-01	Dist 1	87	49	98	38	98	38	
	Dist 2	25	128	140	13	140	13	
	Dist 3	40	49	76	13	76	13	
	Depot	440	215	227	341	227	341	58
Jul-01	Dist 1	38	97	111	24	111	24	
	Dist 2	13	145	151	7	151	7	
	Dist 3	13	79	76	16	76	16	
	Depot	341	326	322	329	322	329	0
Aug-01	Dist 1	24	111	118	17	118	17	
	Dist 2	7	159	163	3	163	3	
	Dist 3	16	79	59	36	59	36	
	Depot	329	356	350	345	350	345	-12
Sep-01	Dist 1	17	119	125	11	125	11	
	Dist 2	3	175	139	39	139	39	
	Dist 3	36	53	56	33	56	33	
	Depot	345	352	348	377	348	377	-5
Oct-01	Dist 1	11	128	123	16	123	16	
	Dist 2	39	130	151	18	151	18	
	Dist 3	33	50	68	15	68	15	
	Depot	377	300	309	335	309	335	13
Nov-01	Dist 1	16	124	100	40	100	40	
	Dist 2	18	153	136	35	136	35	
	Dist 3	15	69	53	31	53	31	
	Depot	335	349	347	395	347	395	-10
Dec-01	Dist 1	40	98	127	11	127	11	
	Dist 2	35	127	180	-18	180	0	
	Dist 3	31	47	67	11	67	11	
	Depot	395	260	273	281	291	281	28
Jan-02	Dist 1	11	129	112	28	112	28	
	Dist 2	0	184	229	-45	229	0	
	Dist 3	11	69	74	6	74	6	
	Depot	281	418	383	284	428	284	-28
Feb-02	Dist 1	28	112	116	24	116	24	
	Dist 2	0	225	211	14	211	14	
	Dist 3	6	77	69	14	69	14	
	Depot	284	488	415	376	415	376	-20

*Difference in depot inventory = (depot inventory based on echelon stock – total distributors' inventory) – depot inventory based on installation stock

Appendix 6

Paired-sample Wilcoxon signed-rank test to ascertain if there is any significant difference between the simulation results of the models based on installation inventory and echelon inventory

As required by the test, it is assumed that the pair-wise differences come from continuous distributions (not necessarily the same) that are symmetric about zero.

As noted, we are excluding the data for June 2001. The data for July 2001 is also being excluded since the difference in depot inventory is zero. Hence we have 12 data points corresponding to 12 months. The differences in depot inventories and their signed ranks are shown in Table A6. DI_{inst} and DI_{ech} denote the month-end depot inventories based on installation stock and echelon stock respectively.

Table A6 Differences in Depot Inventories and Their Signed Ranks

Month	$DI_{inst} – DI_{ech}$	Signed rank
Jan-01	9	+2
Feb-01	35	+12
Mar-01	31	+10.5
Apr-01	11	+4
May-01	31	+10.5
Aug-01	12	+5
Sep-01	5	+1
Oct-01	-13	-6
Nov-01	10	+3
Dec-01	-28	-8.5
Jan-02	28	+8.5
Feb-02	20	+7

From the above table, it is found that $T^+ = 63.5$ and $T^- = 14.5$. Given n as the sample size, the expectation and variance of T, which is approximated by a normal distribution, are calculated from the following formula.

$$E(T) = \frac{n(n+1)}{4} = 39$$

$$Var(T) = \frac{n(n+1)(2n+1)}{24} = 162.5$$

The null hypothesis is stated as θ (median of the differences) = 0 against the alternative hypothesis θ > 0. The one-sided level of significance, α, is taken as 0.05. The criterion for rejecting the null hypothesis is that the value of T at 0.05 level of significance, $T_{0.05}$, must be greater than or equal to T^-. The value of $T_{0.05}$ is calculated as follows.

$$\frac{T_{0.05} - E(T)}{\sqrt{Var(T)}} = -1.645$$

$$or \ \frac{T_{0.05} - 39}{\sqrt{162.5}} = -1.645$$

$$or \ T_{0.05} = 18.03$$

Since $T_{0.05} > T^-$, the null hypothesis must be rejected, i.e., it can be concluded that there is a significant difference between the methods at 5% level of significance.