PSYC40005 - 2018
ADVANCED DESIGN AND DATA ANALYSIS

Lecture 5:
Structural equation modelling 1:
Confirmatory factor analysis

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The agenda for this lecture

1. What is structural equation modeling?
2. Confirmatory vs exploratory factor analysis
3. Issues for CFA
4. CFA in SPSS AMOS

GOALS OF THIS LECTURE

- To introduce the basic concept behind of structural equation modeling
- To show how SEM can combine a number of different analyses
- To distinguish between manifest and latent variables and between measurement and structural models
- To illustrate how to conduct a confirmatory factor analysis
- To show how to do this in SPSS AMOS

Section 1

STRUCTURAL EQUATION MODELING (SEM)

SEM in a nutshell

(Tabachnik & Fidell, 2007)

- A collection of statistical techniques that permit analysis of relationships between one or more IVs and DVs, in possibly complex ways
  - Also known as causal modelling, causal analysis, simultaneous equation modelling, analysis of covariance structures.
  - Special types of SEM include confirmatory factor analysis and path analysis.
- SEM enables a combined analysis that otherwise requires multiple techniques
  - For instance, factor analysis and regression analysis

Combining factor and regression analysis

- For example, a 5 item Satisfaction with Life (SWL) scale measured at 2 time points.
- Research question: Does SWL at time point 1 predict SWL at time point 2?

SWL2.sav [DataSet1] - IBM SPSS Statistics Data Editor

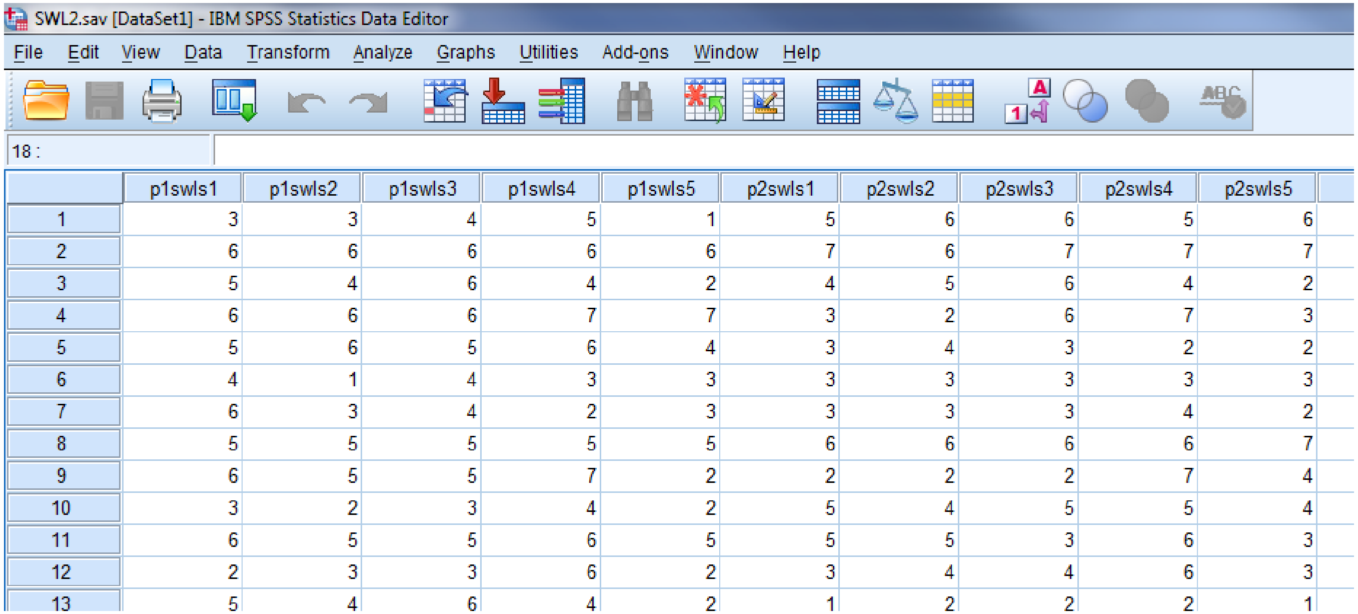
File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

18 :

	p1swls1	p1swls2	p1swls3	p1swls4	p1swls5	p2swls1	p2swls2	p2swls3	p2swls4	p2swls5
1	3	3	4	5	1	5	6	6	5	6
2	6	6	6	6	6	7	6	7	7	7
3	5	4	6	4	2	4	5	6	4	2
4	6	6	6	7	7	3	2	6	7	3
5	5	6	5	6	4	3	4	3	2	2
6	4	1	4	3	3	3	3	3	3	3
7	6	3	4	2	3	3	3	3	4	2
8	5	5	5	5	5	6	6	6	6	7
9	6	5	5	7	2	2	2	2	7	4
10	3	2	3	4	2	5	4	5	5	4
11	6	5	5	6	5	5	5	3	6	3
12	2	3	3	6	2	3	4	4	6	3
13	5	4	6	4	2	1	2	2	2	1

Combining factor and regression analysis

- Given the techniques we have learnt to date, we could perform a factor analysis of the scale at each time point, save the factor scores, and then regress the time 2 factor on the time 1 factor.



	p1swls1	p1swls2	p1swls3	p1swls4	p1swls5	p2swls1	p2swls2	p2swls3	p2swls4	p2swls5
1	3	3	4	5	1	5	6	6	5	6
2	6	6	6	6	6	7	6	7	7	7
3	5	4	6	4	2	4	5	6	4	2
4	6	6	6	7	7	3	2	6	7	3
5	5	6	5	6	4	3	4	3	2	2
6	4	1	4	3	3	3	3	3	3	3
7	6	3	4	2	3	3	3	3	4	2
8	5	5	5	5	5	6	6	6	6	7
9	6	5	5	7	2	2	2	2	7	4
10	3	2	3	4	2	5	4	5	5	4
11	6	5	5	6	5	5	5	3	6	3
12	2	3	3	6	2	3	4	4	6	3
13	5	4	6	4	2	1	2	2	2	1

Combining factor and regression analysis

- Given the techniques we have learnt to date, we could perform a factor analysis of the scale at each time point, save the factor scores, and then regress the time 2 factor on the time 1 factor.

Time 1

Factor Matrix[a]

	Factor
	1
P1SWLS1	.860
P1SWLS2	.879
P1SWLS3	.885
P1SWLS4	.714
P1SWLS5	.698

Extraction Method: Maximum Likelihood.

a. 1 factors extracted. 3 iterations required.

Time 2

Factor Matrix[a]

	Factor
	1
P2SWLS1	.918
P2SWLS2	.898
P2SWLS3	.878
P2SWLS4	.742
P2SWLS5	.760

Extraction Method: Maximum Likelihood.

a. 1 factors extracted. 4 iterations required.

Combining factor and regression analysis

- Given the techniques we have learnt to date, we could perform a factor analysis of the scale at each time point, save the factor scores, and then regress the time 2 factor on the time 1 factor.

The Regression

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.616[a]	.380	.376	.81521476

a. Predictors: (Constant), SWL1

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	6.009E-017	.062		.000	1.000
	SWL1	.610	.060	.616	10.197	.000

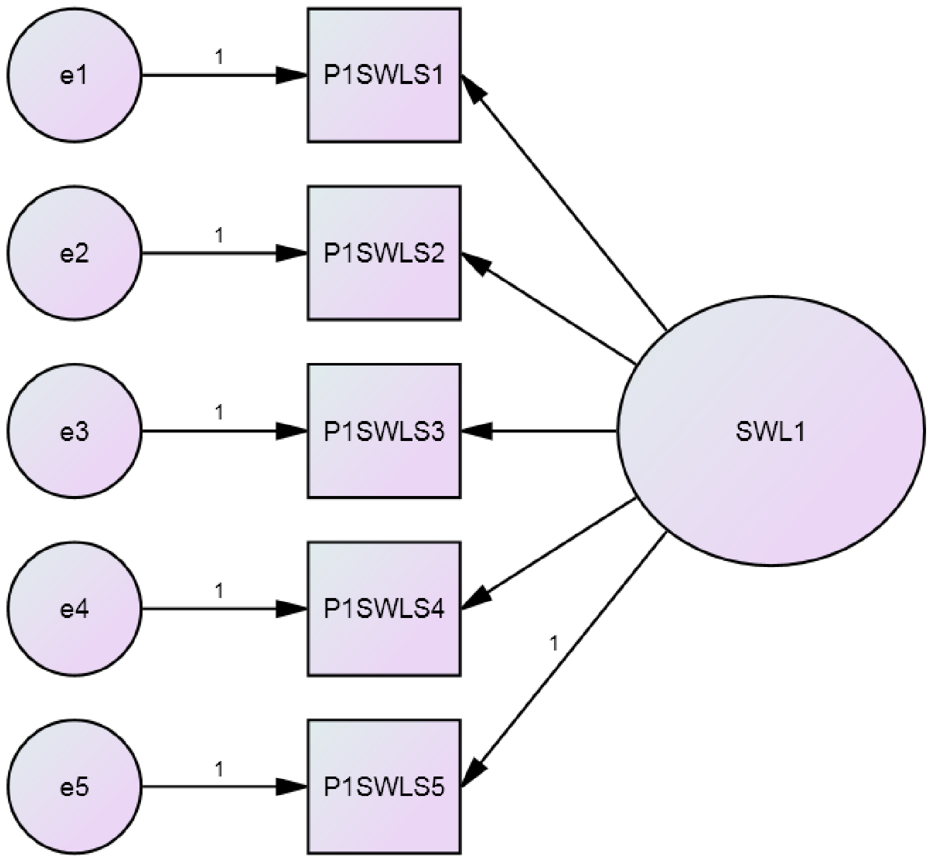
a. Dependent Variable: SWL2

Combining factor and regression analysis

What have we done here?

FIRST, (for each time point)
We have combined five measures into one underlying factor.

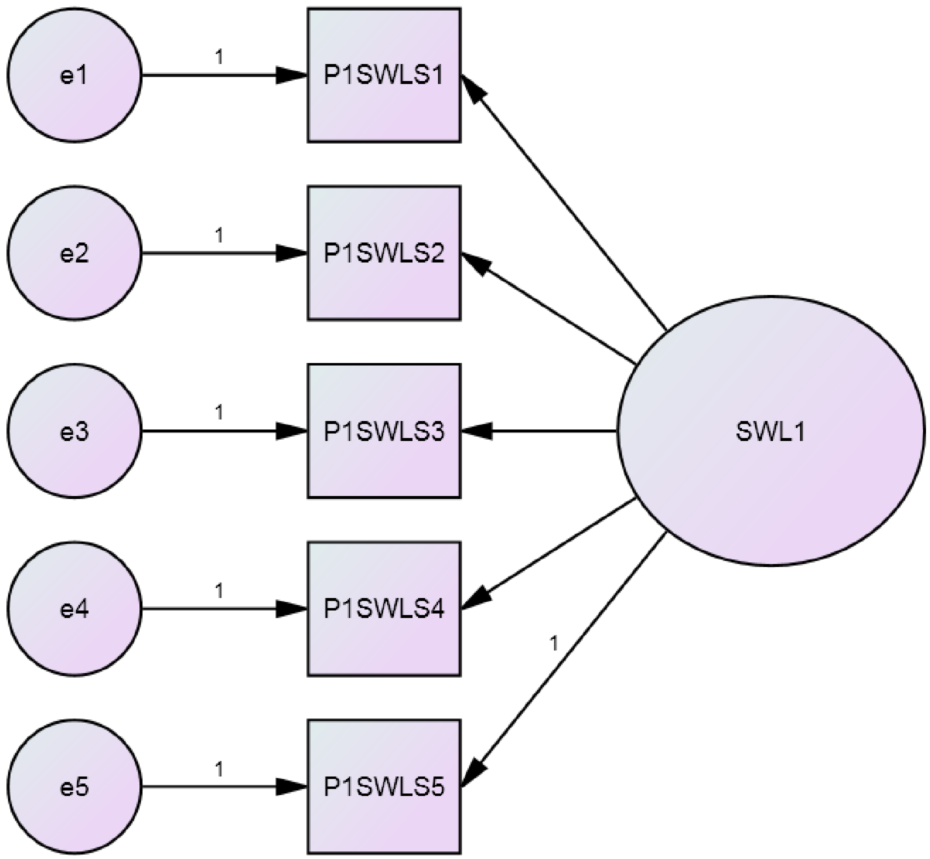
We are supposing that the one factor (the one construct), which we don't directly observe, influences each of the measures .



Combining factor and regression analysis

What have we done here?

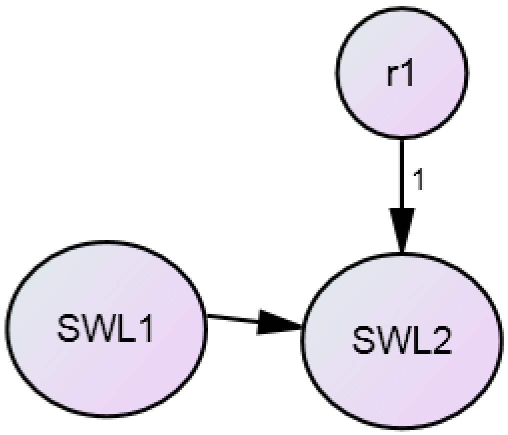
The observed measures (**manifest** variables) are represented in boxes. The **latent factor** is represented in an oval. Each observed measure also has some residual variance (the e's) because it is not fully explained by the factor.



Combining factor and regression analysis

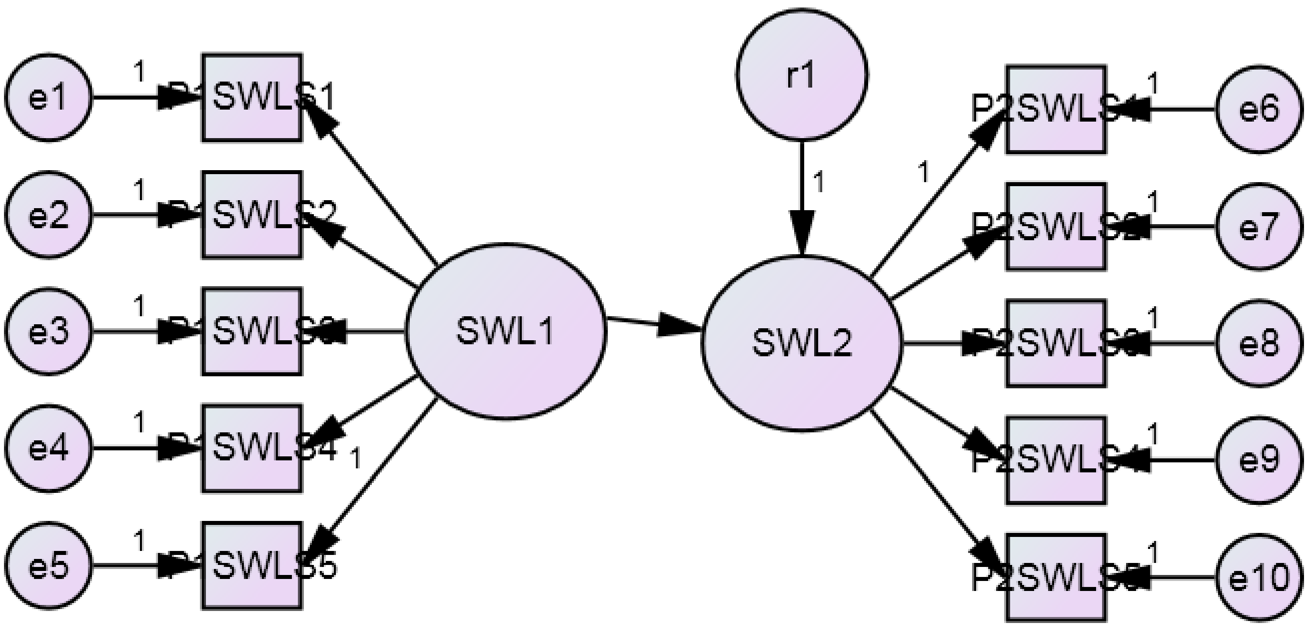
What have we done here?

Then we run a regression on two latent factors (remembering there is also a residual *r* in the regression equation)



Combining factor and regression analysis

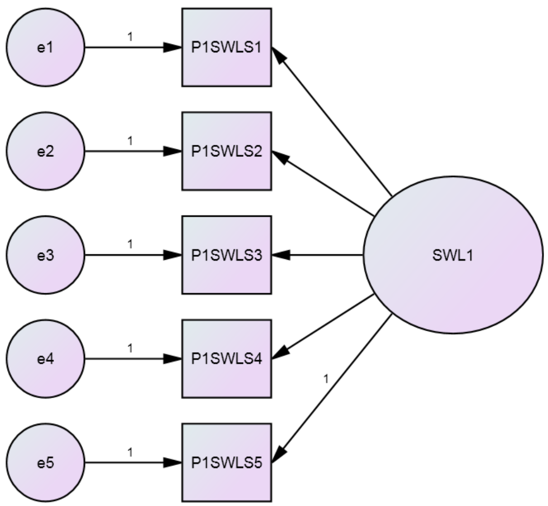
SEM lets you do it altogether

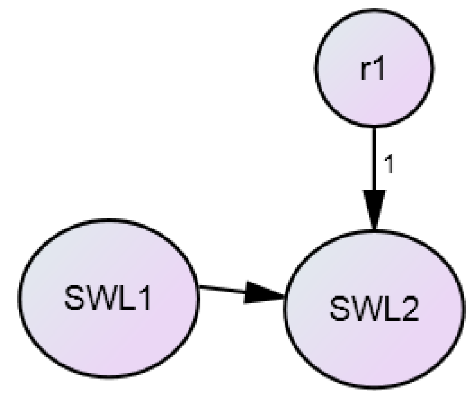


Combining factor and regression analysis

The measurement model

The structural model





Confirmatory factor analysis is a measurement model.

Section 2

CONFIRMATORY VS EXPLORATORY FACTOR ANALYSIS

CFA vs EFA

- Exploratory factor analysis can impose two kinds of restrictions.
  - restrict the number of factors
  - constrain the factor loadings to be uncorrelated with an orthogonal rotation.

- Confirmatory factor analysis can restrict factor loadings (or factor correlations or variances) to take certain values.
  - A common value: zero
  - If a factor loading was set to zero, the hypothesis is that the observed variable score was not due to the factor

CFA vs EFA

- MOREOVER:
  - using maximum likelihood and generalized least squares estimation, CFA has a test of fit
  - SO it is possible to test the hypothesis that the factor loading is zero.
  - If data fit the model, hypothesis supported.
- Hence ***confirmatory*** factor analysis.

Example: 11 subtests of the WISC in a sample of learning disabled students (Tabachnik & Fidell)

Two factors: Verbal IQ and Performance IQ

Subtests loading on Verbal:

Information
Comprehension
Arithmetic
Similarities
Vocabulary
Digit span

Subtests loading on Performance:

Picture completion
Picture arrangement
Block design
Object assembly
Coding

Exploratory factor analysis of the WISC data

Pattern Matrix[a]

	Factor	
	1	2
info	.815	
comp	.465	
arith	.575	
simil	.542	
vocab	.733	
digit	.466	
pictcomp		.607
parang		.403
block		.623
object		.650
coding		

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 5 iterations.

But this is with low factor loadings suppressed – the complete solution is:

Pattern Matrix[a]

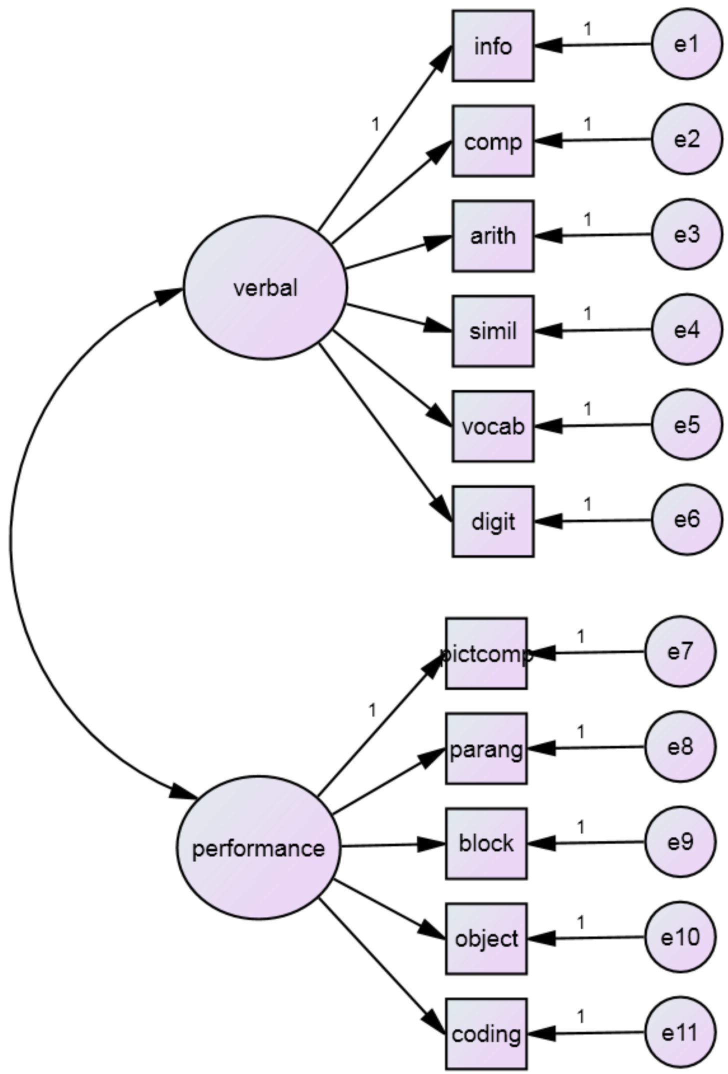
	Factor	
	1	2
info	.815	-.029
comp	.465	.366
arith	.575	.012
simil	.542	.260
vocab	.733	.077
digit	.466	-.095
pictcomp	.040	.607
parang	.057	.403
block	.029	.623
object	-.094	.650
coding	.071	.003

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.

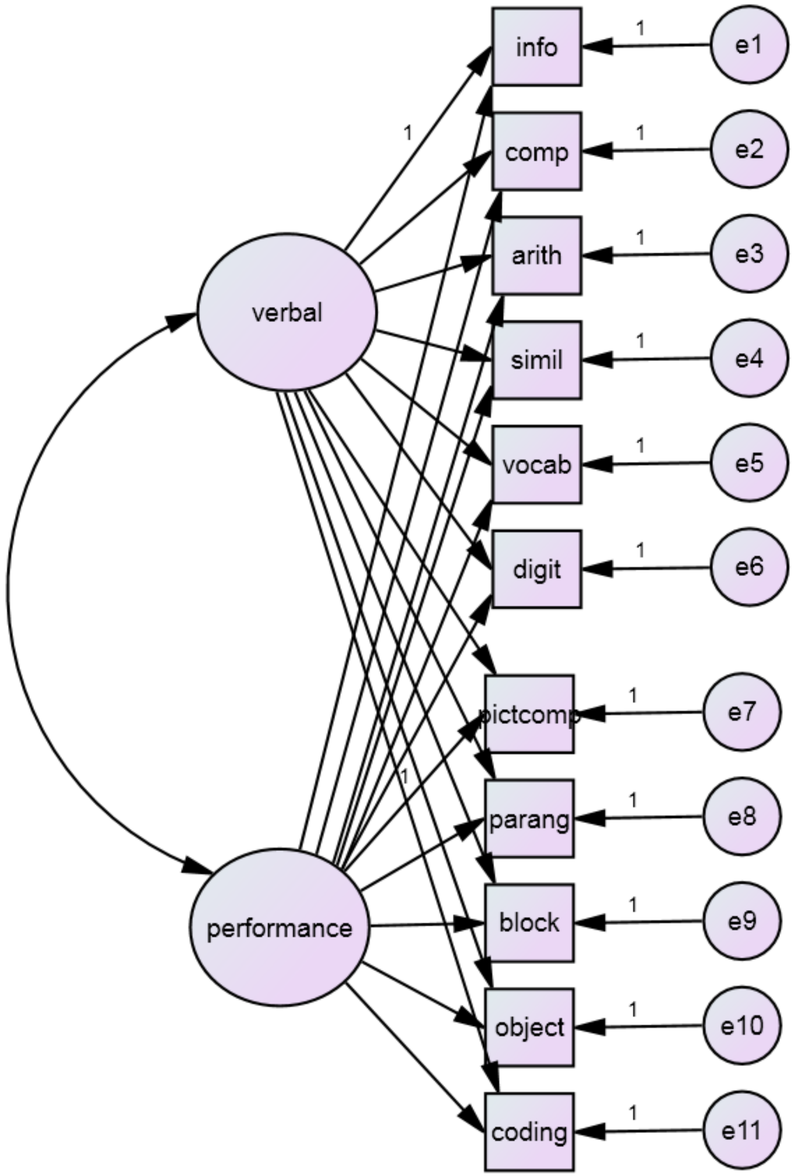
a. Rotation converged in 5 iterations.

Except for “Coding”, the factor structure seems to be reasonably reproduced.

The hypothesized model



The EFA model



The estimated CFA loadings

	Verbal	Performance
Info	0.76	0
Comp	0.69	0
Arith	0.57	0
Simil	0.70	0
Vocab	0.77	0
Digit	0.39	0
Pictcomp	0	0.60
Parang	0	0.47
Block	0	0.68
Object	0	0.57
Coding	0	0.07

Verbal – Performance correlation 0.59

Pattern Matrix[a]

	Factor	
	1	2
info	.815	
comp	.465	
arith	.575	
simil	.542	
vocab	.733	
digit	.466	
pictcomp		.607
parang		.403
block		.623
object		.650
coding		

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 5 iterations.

Factor Correlation Matrix

Factor	1	2
1	1.000	.458
2	.458	1.000

Extraction Method: Maximum Likelihood.
Rotation Method: Oblimin with Kaiser Normalization.

Section 3

SEVEN ISSUES FOR CFA

Issues for CFA: 1. Sample size

- Wolf et al (2013) show "one size fits all" rules work poorly in this context.
- Jackson (2003) provides support for the N:q rule.
  - Ratio of cases (N) to parameters being estimated (q)
  - > 20:1 recommended. < 10:1 likely to cause problems.
- Absolute sample size harder to assess
  - $N = 200$ is common, but may be too small.
  - Barrett (2007) suggests journal editors routinely reject any CFA with $N < 200$.

Issues for CFA:
2. Significance testing

- Kline (2016) reports a diminished emphasis on significance testing, because:
  - We emphasise testing the whole model rather than individual effects
  - Large-sample requirement means even trivial effects may be statistically significant
  - p-value estimates could change we if we used a different method to estimate model parameters
  - Greater general awareness of issues with significance testing.

Issues for CFA:
3. Distributional assumptions

- The default estimation technique (maximum likelihood) assumes multivariate normality.
  - Possible to transform variables to obtain normality
  - Widaman (2012): maximum likelihood estimation appears relatively robust to moderate violations of distributional assumptions.
  - Some robust methods of estimation are available (Tabachnik & Fidell, 2013)
- CFA generally assumes continuous variables
  - Some programs allow for ordered categorical data

Issues for CFA:
4. Identification

- Necessary but insufficient requirements for identification
  1. Model degrees of freedom must be ≥ 0
  2. All latent variables must be assigned a scale
- Estimation is based on the solving of a number of complex equations
- Constraints need to be placed on the model (not the data) in order for these equations to be solved unambiguously
- Model is ***identified*** if it's theoretically possible for a unique estimate of every model parameter to be derived

Issues for CFA:
4. Identification

- CFA seeks parameter estimates that can best reproduce the variance-covariance matrix of the data.
  - Parameters include factor loadings, factor correlations and unique variances (residuals)
- If the number of variables in the data is n than the number of observations (cells in the variance-covariance matrix) is $n(n+1)/2$
- A model in which the number of free parameters to be estimated is equal to this number is said to be a ***just-identified*** model

Issues for CFA:
4. Identification (Loehlin, 1992)

Underidentified

Not identified. Not possible to uniquely estimate all the model's free parameters (usually because there are more free parameters than observations, and thus model $df < 0$)

You'll need to respecify your model

Just-identified

Identified and has the same number of observations as free parameters (model $df = 0$)

Model will reproduce your data exactly, but won't test your theory

Overidentified

Identified and has more observations than free parameters (model $df > 1$)

Permits discrepancies between model and data, permits tests of model fit, and of theory

$$10 = 2x + y$$

Unidentified [x = 5, y = 0]

$$10 = 2 \times 5 + 0$$

Unidentified [x = 3.50, y = 3]

$$10 = 2 \times 3.50 + 3$$

Unidentified

$$10 = 2x + y$$

$$10 = 2x + y$$

$$2 = x - y$$

Just-identified [$x = 4$, $y = 2$]

$$10 = 2x + y$$

$$2 = x - y$$

$$10 = 2x + y$$
$$2 = x - y$$
$$5 = x + 2y$$

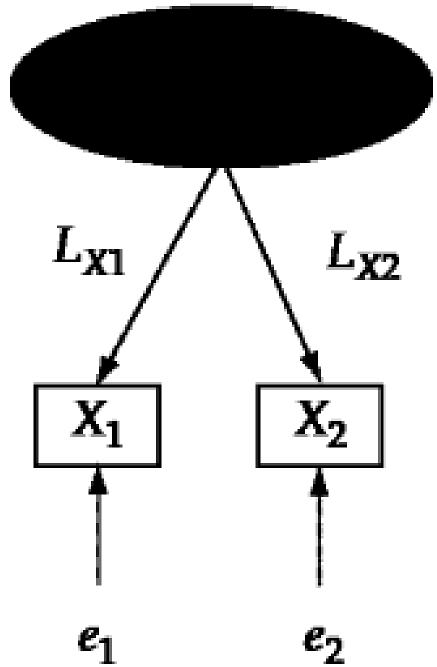
Overidentified

$$10 = 2x + y$$

$$2 = x - y$$

$$5 = x + 2y$$

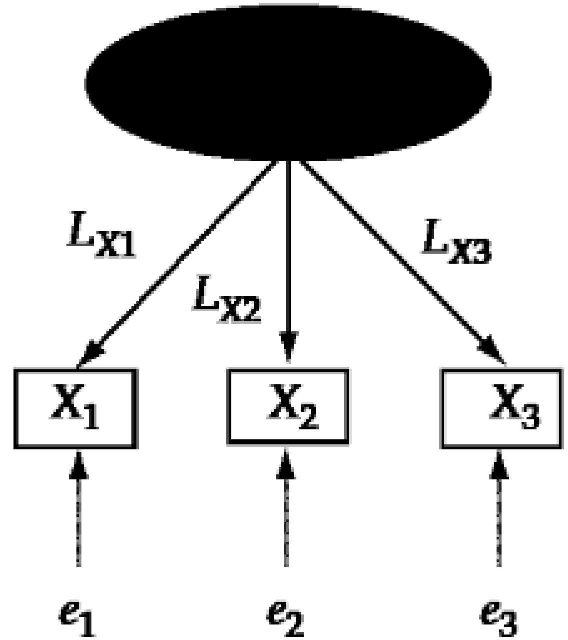
Underidentified



Four parameters to estimate
($L_{X1}, L_{X2}, e_{11}, e_{22}$)

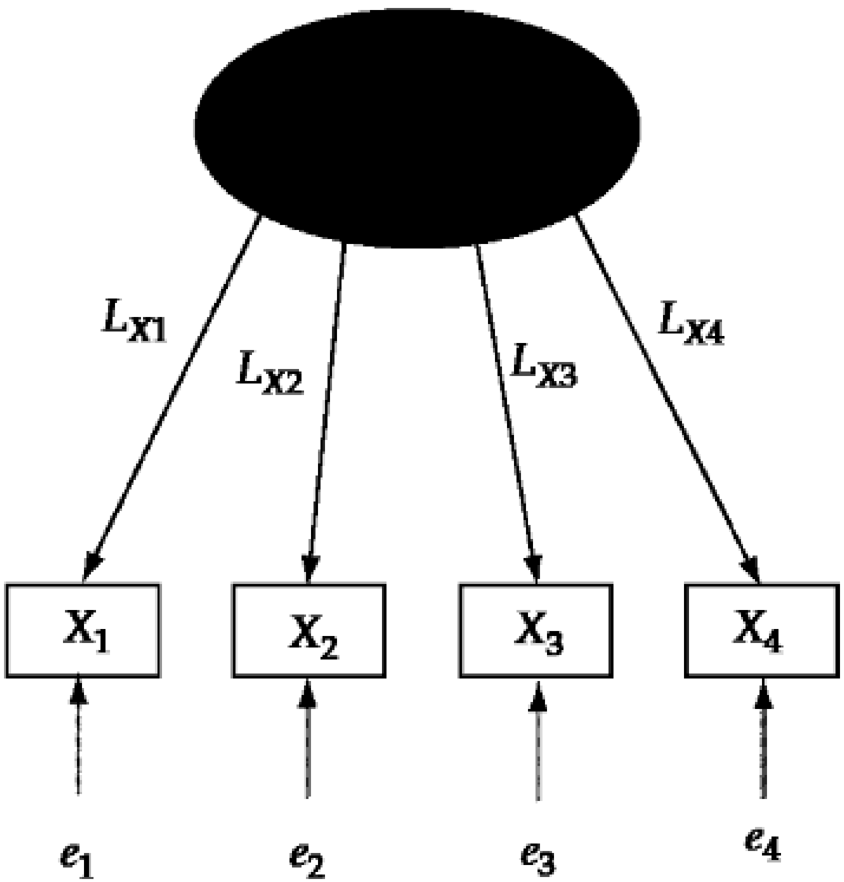
$$
\begin{array}{c|cc|}
\mathbf{S} & X_1 & X_2 \\
\hline
X_1 & \text{var}(1) & \text{cov}(1,2) \\
X_2 & \text{cov}(1,2) & \text{var}(2)
\end{array}
$$

Just Identified



Six parameters to estimate

$$
\begin{array}{c|ccc|}
\mathbf{S} & X_1 & X_2 & X_3 \\
\hline
X_1 & \text{var}(1) & \text{cov}(1,2) & \text{cov}(1,3) \\
X_2 & \text{cov}(1,2) & \text{var}(2) & \text{cov}(2,3) \\
X_3 & \text{cov}(1,3) & \text{cov}(2,3) & \text{var}(3)
\end{array}
$$



Eight paths to estimate

Symmetric Covariance Matrix

	X_1	X_2	X_3	X_4
X_1	2.01			
X_2	1.43	2.01		
X_3	1.31	1.56	2.24	
X_4	1.36	1.54	1.57	2.00

10 unique variance–covariance terms

Model Fit:

$\chi^2 = 14.9$

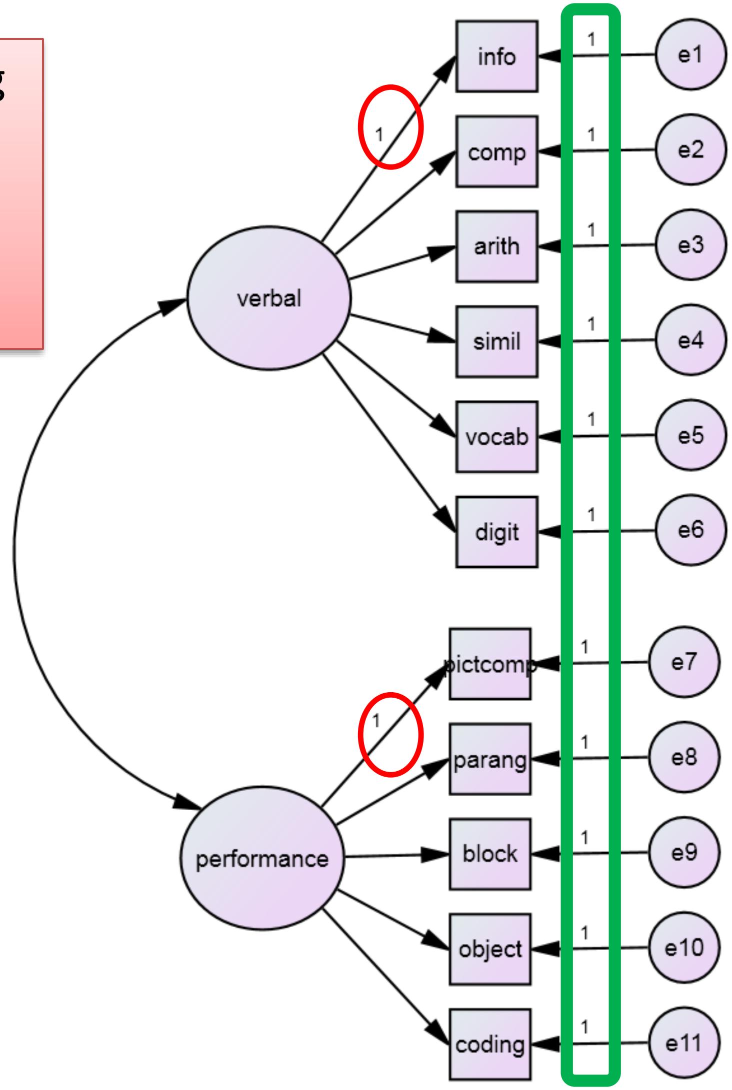
$df = 2$

$p = .001$

CFI = .99

Issues for CFA:
4. Identification

Setting one loading per factor to a coefficient of 1 determines a scale for the factor



Setting the residual path coefficient from each error term to its observed variable determines a scale for variance of the error terms.

Issues for CFA: 4. Identification

How do we get identified CFA models?

- Method used in this subject to scale the factors: set a reference variable for each factor.
- Factor loading (unstandardized pattern coefficient) for one of the variables is fixed at a non-zero value [usually 1.0].
  - In our example we set verbal -> info = 1.0, and performance -> pictcomp = 1.0.
  - Fixing the factor loading of one variable to 1.0, scales the factor in a metric related to the explained variance of the reference variable.

Issues for CFA:
4. Identification

How do we get identified CFA models?

- If there are at least two factors, then the model will be identified for only two variables loading per factor,
  - if the factor is correlated with another factor
  - and one of the variable loadings is fixed to a non-zero value.

For some more complex models these last two rules may not apply.
Computer programs are often able to detect a model that is not identified, and you will get an error message
Identification heuristics exist for humans to identify some of the other cases – not part of this course but see Kline (2016) for further reference.

Issues for CFA:
5. Methods of estimation

- As for EFA, the most commonly used are
  - Unweighted Least Squares,
  - Generalized Least Squares, and
  - Maximum Likelihood
- ML is often preferred, but assumes normality
- If you're picking between two methods and they yield substantially different results, report both

Issues for CFA:
6. Assessment of fit

- Model Test statistics (discrepancy between model covariance matrix and sample covariance matrix can be reasonably attributed to sampling error?)
  - Chi Square
- Approximate Fit indexes
  - Absolute (proportions of covariances in sample data matrix explained by model)
    - Standardized Root Mean square Residual [SRMR]
  - Comparative (relative improvement in fit compared to a baseline)
    - Comparative Fit Index [CFI]
  - Parsimony (model-sample discrepancy adjusted for sample size and number of parameters)
    - Root Mean Square Error of Approximation [RMSEA]
- May be best to cite one of each kind

Issues for CFA:
7. Setting up a CFA

How to specify a CFA model?

- use equation $\Sigma = \Lambda\Phi\Lambda' + \Psi$,
  - specify values for the elements of the matrices Λ, (factor loadings); Φ, (factor correlations) and Ψ (unique variances). LISREL
- specify regression equations.
  - EQS and MPLUS, also AMOS but we won't use it
- draw diagram (AMOS default)

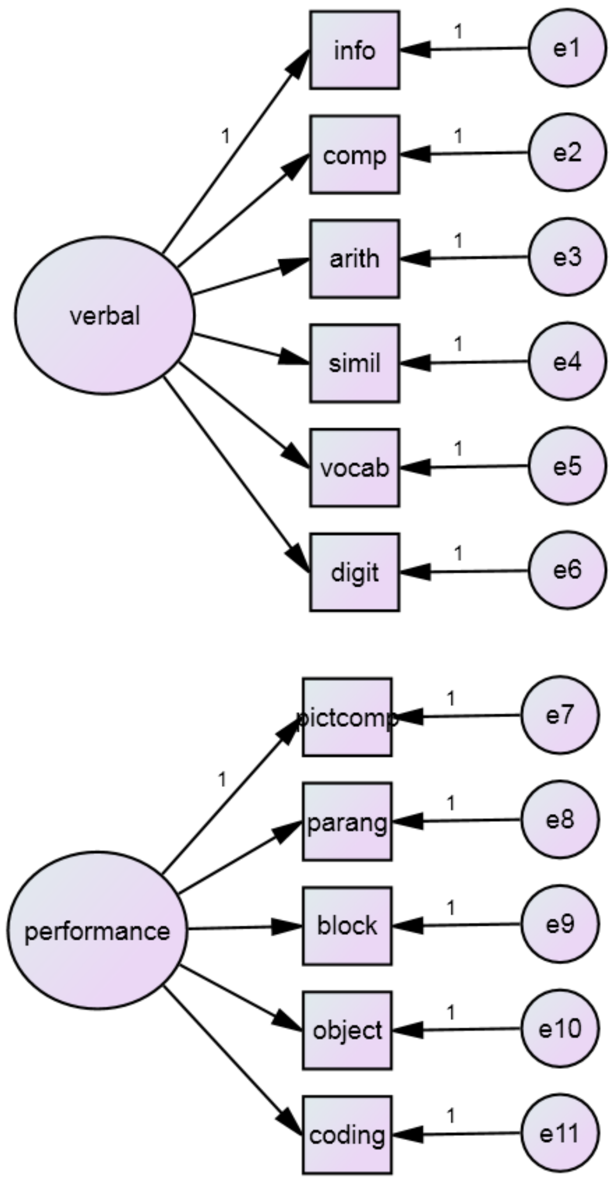
Section 4

CFA IN AMOS

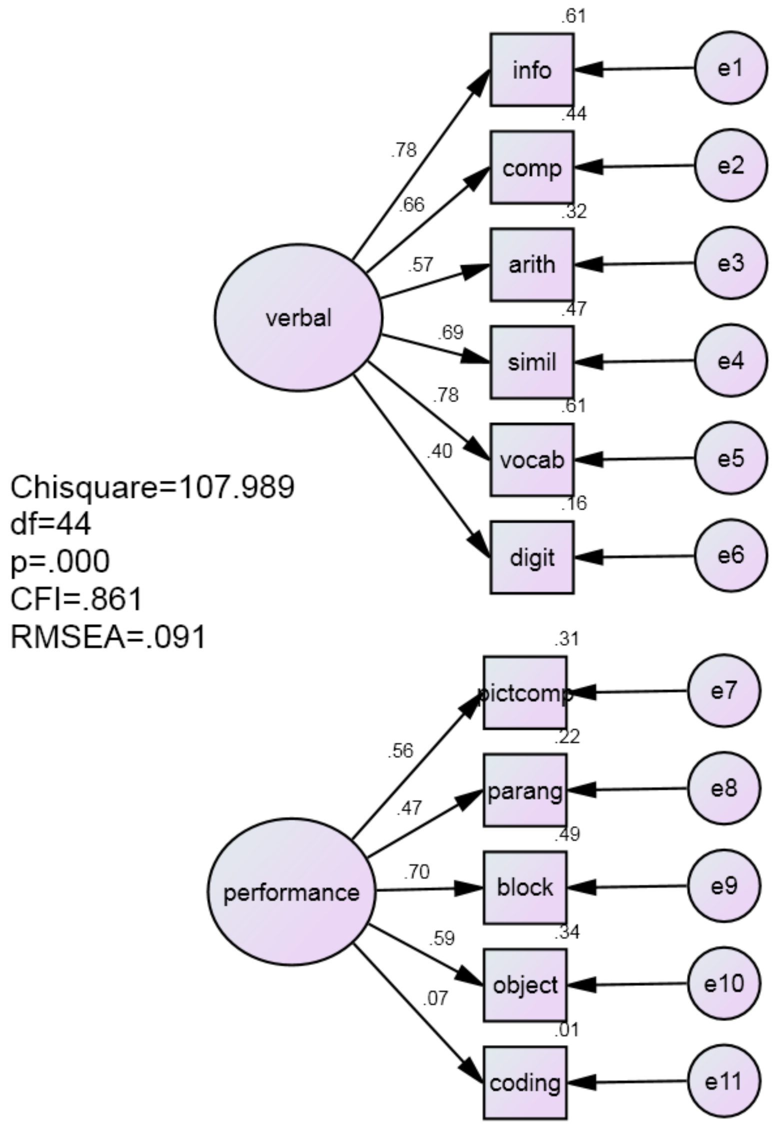
Drawing conventions

- observed variable: *rectangle*
- unobserved common variable: *ellipse*
- unobserved unique or residual component: *circle*
- relationship
  - **correlation:** *curved, double-headed arrow*;
  - **regression:** *straight single-headed arrow* aligned with direction of prediction.

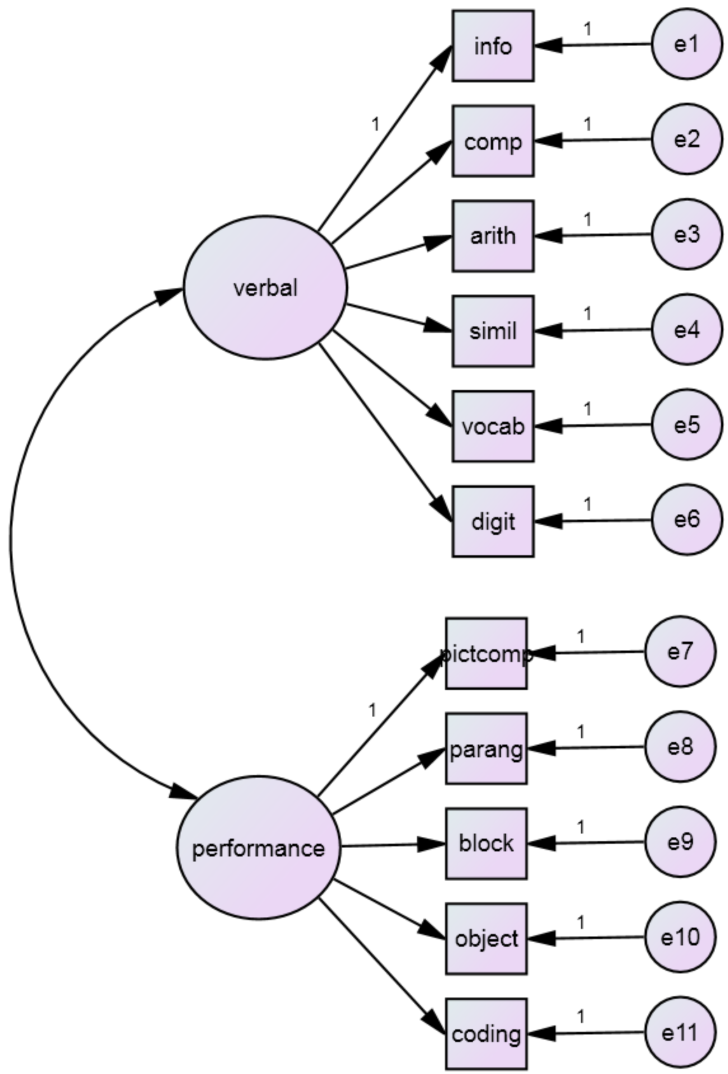
Uncorrelated factors



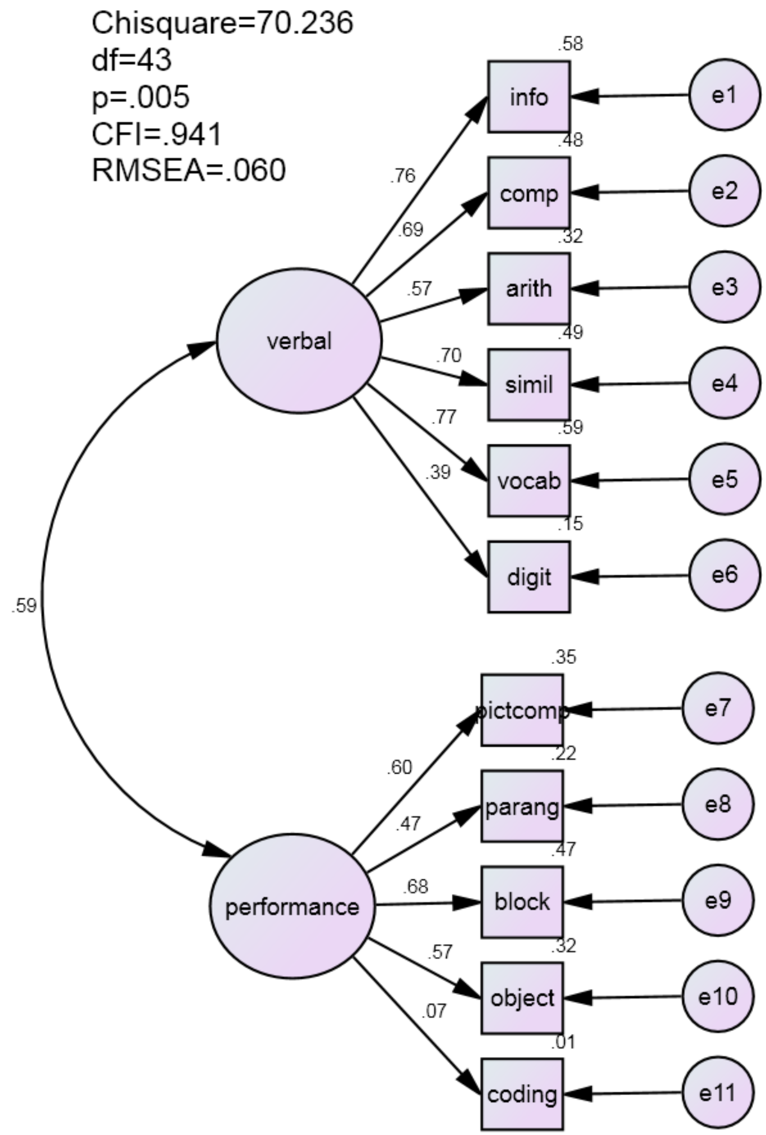
Uncorrelated factors



Correlated factors

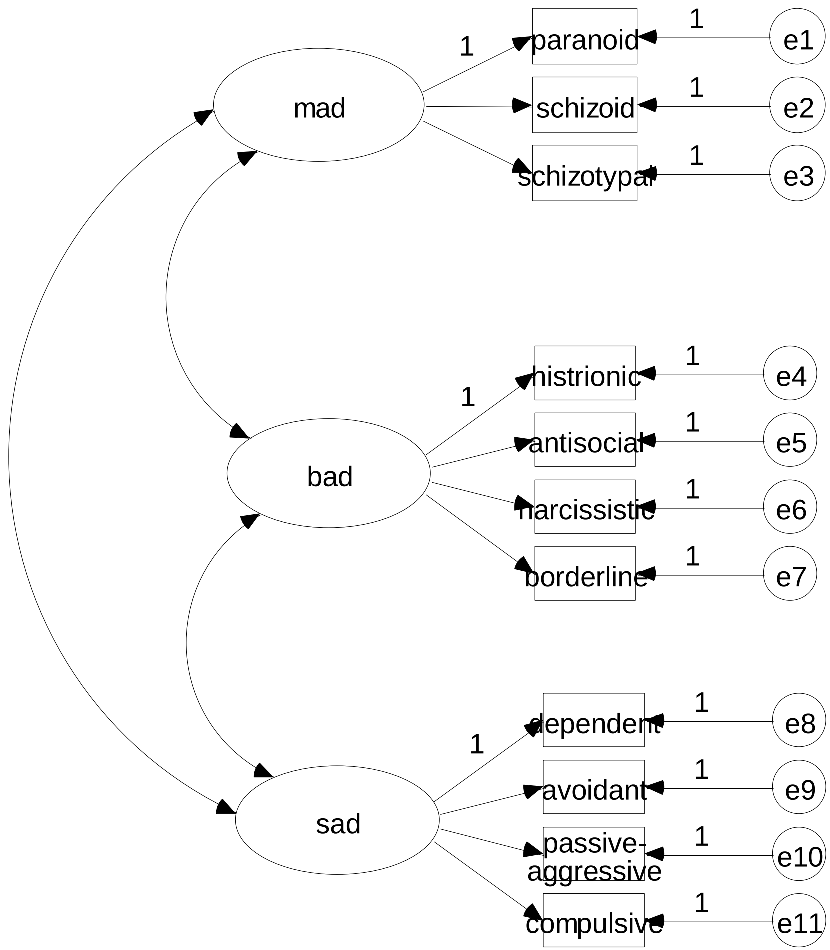


Correlated factors



Correlated factors

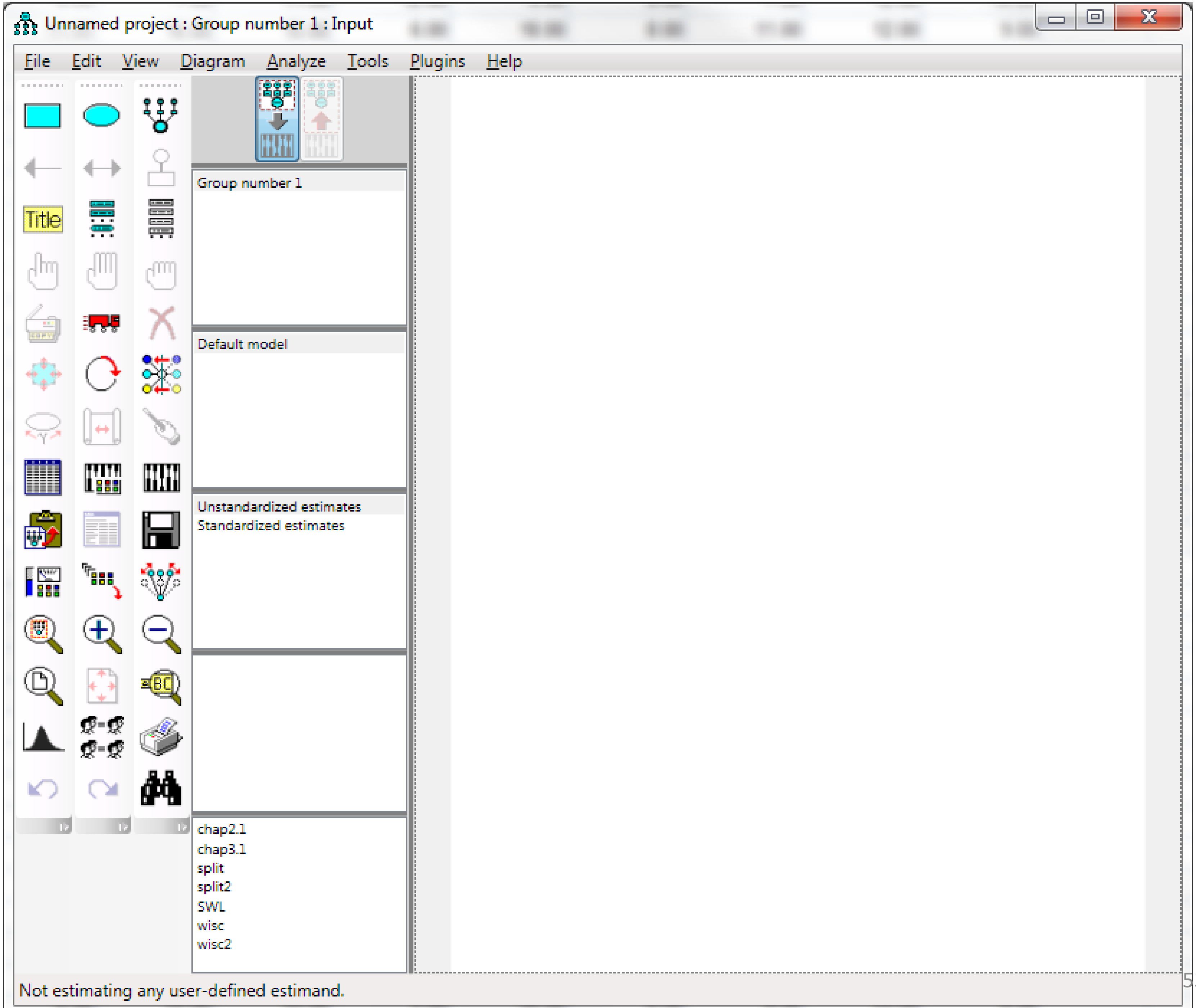
If you've got three factors, don't forget to put correlations between all of them!



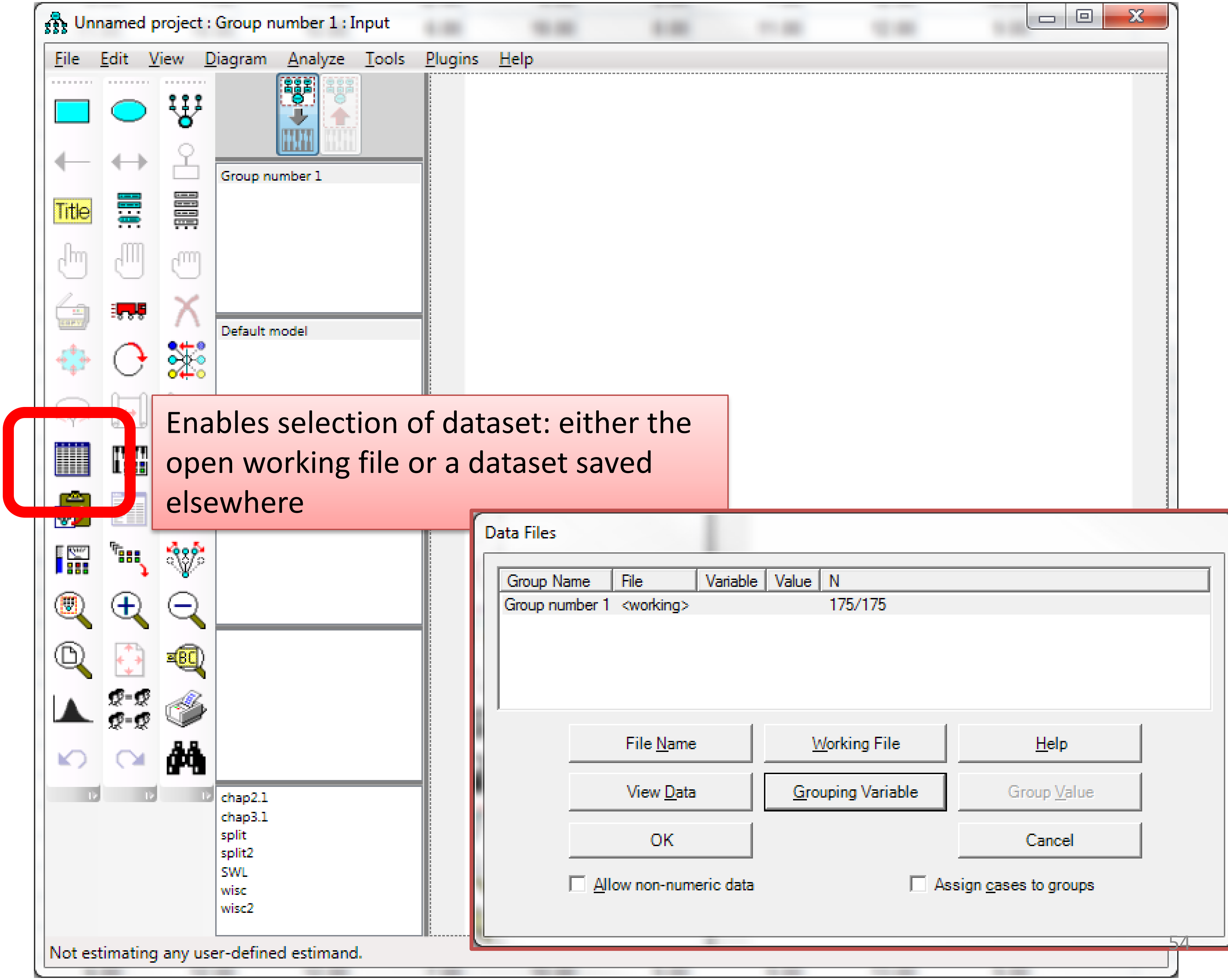
Using AMOS

- **AMOS** (**A**nalysis of **MO**ment **S**tructures)
- separate from SPSS
- but linked
- models defined by drawing them

Running AMOS
- Open datafile in SPSS
- Find AMOS on the Analysis pull-down list
- Wait – Amos is a separate program and has to start up



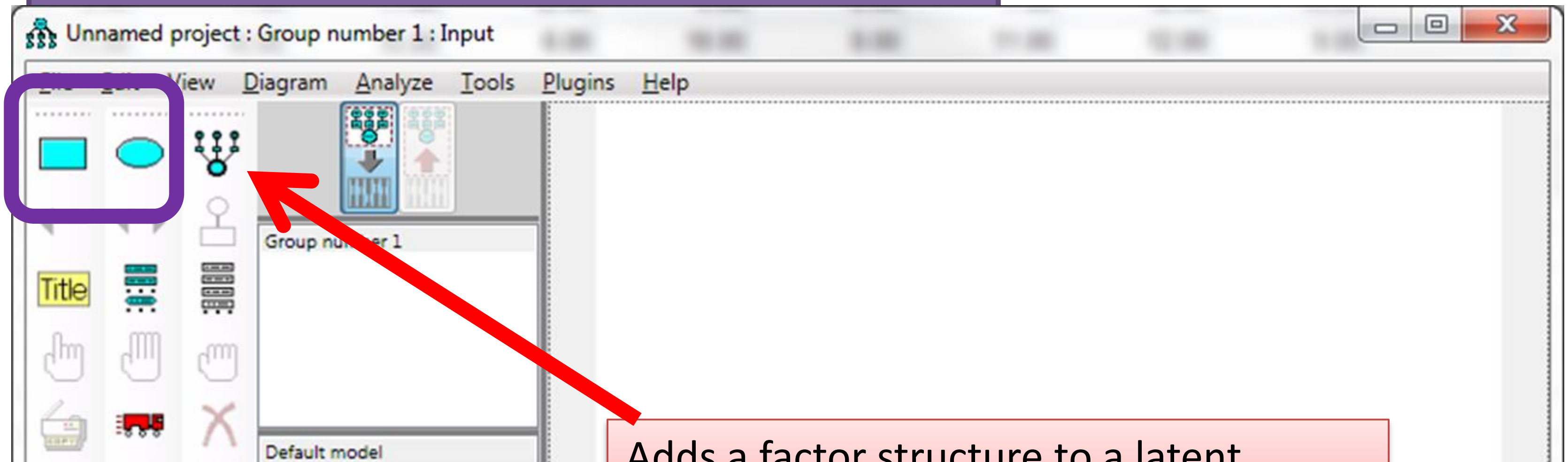
Unnamed project : Group number 1 : Input
File Edit View Diagram Analyze Tools Plugins Help
Title
Group number 1
Default model
Unstandardized estimates
Standardized estimates
chap2.1
chap3.1
split
split2
SWL
wisc
wisc2
Not estimating any user-defined estimand.


Unnamed project : Group number 1 : Input
File Edit View Diagram Analyze Tools Plugins Help
Title
Group number 1
Default model
Enables selection of dataset: either the open working file or a dataset saved elsewhere
Data Files
Group Name | File | Variable | Value | N
Group number 1 <working> 175/175
File Name
Working File
Help
View Data
Grouping Variable
Group Value
OK
Cancel
Allow non-numeric data
Assign cases to groups
chap2.1
chap3.1
split
split2
SWL
wisc
wisc2
Not estimating any user-defined estimand.

Instructions to draw a diagram.

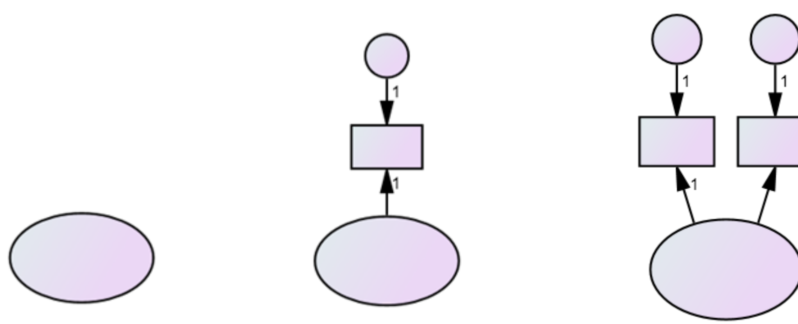
- About eighty drawing and modelling operations.
- four different ways to pick the operation you want to perform:
  - Using the mouse to press a button in a toolbox *(this method will be our focus here)*
  - Using the mouse or the keyboard to select an item from a pull-down menu
  - Pressing a "hot key" on the keyboard (for some operations)
  - Using the second mouse button to select an item from a pop-up menu (for some operations).

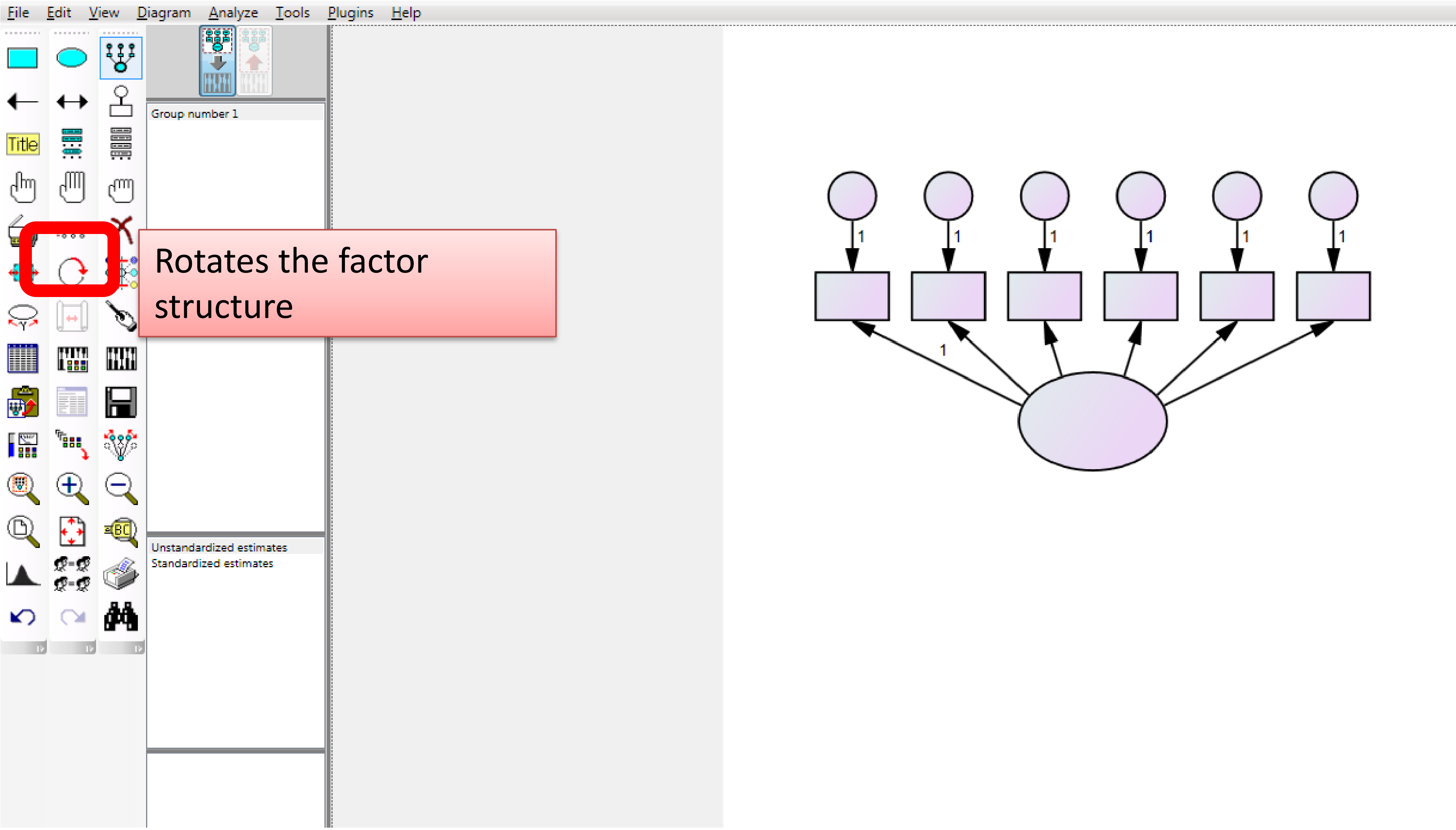
To draw observed and latent variables

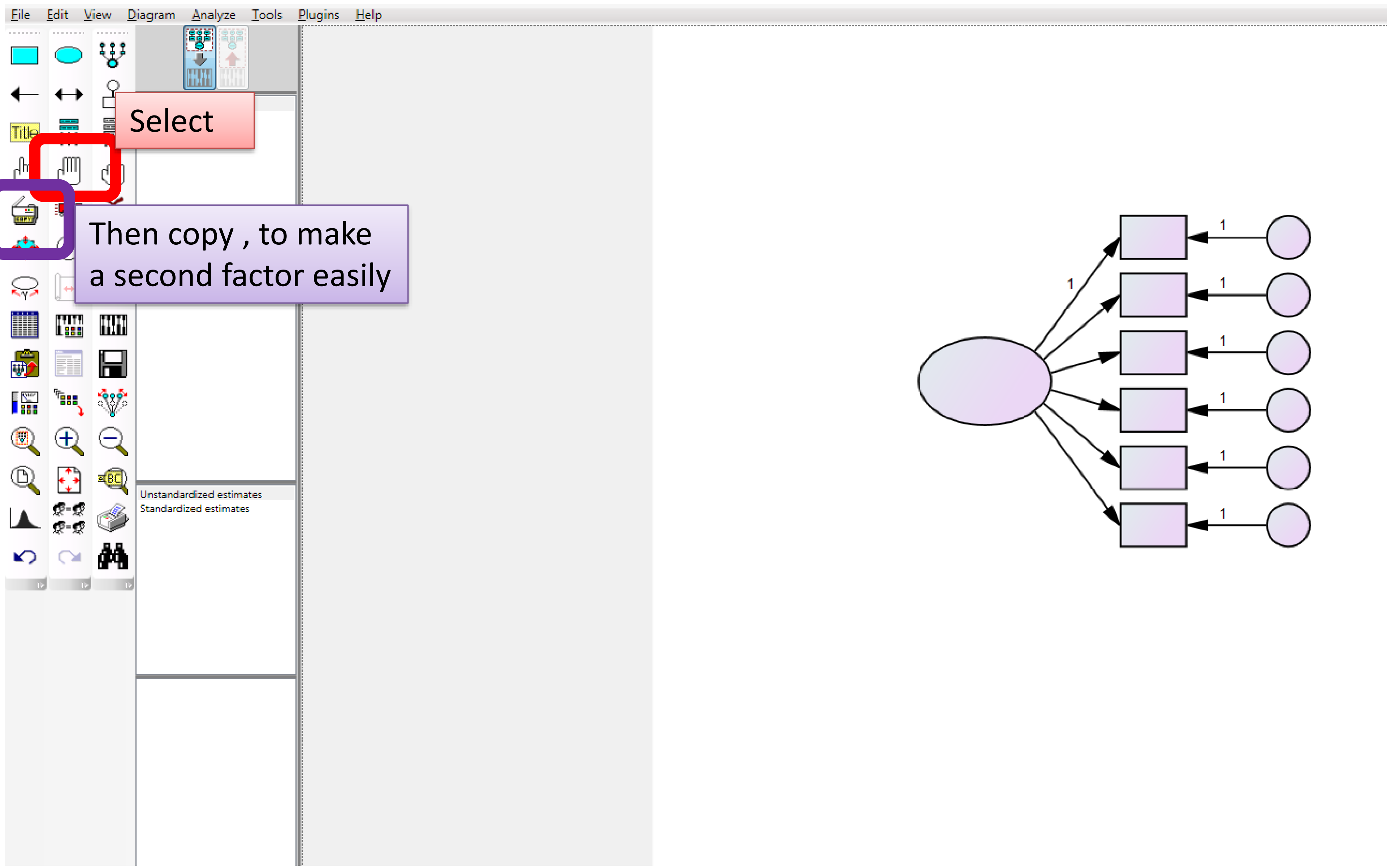


Adds a factor structure to a latent variable

Draw the central ellipse to represent an factor – then click on the 'factor structure' button – then click on the ellipse, repeatedly unit you have enough observed variables.



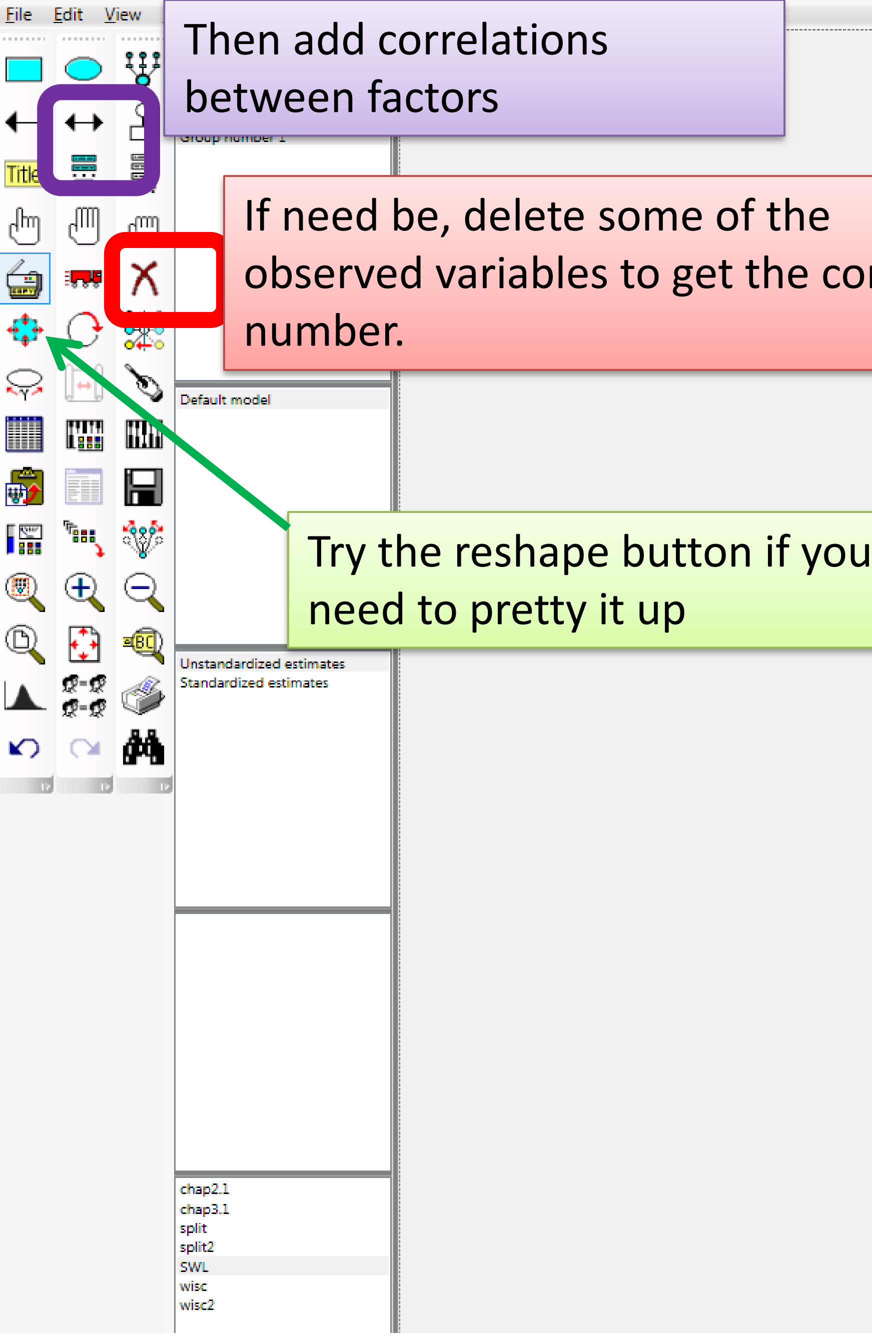

File Edit View Diagram Analyze Tools Plugins Help
Title
Group number 1
Rotates the factor structure
Unstandardized estimates
Standardized estimates
1
1
1
1
1
1
1

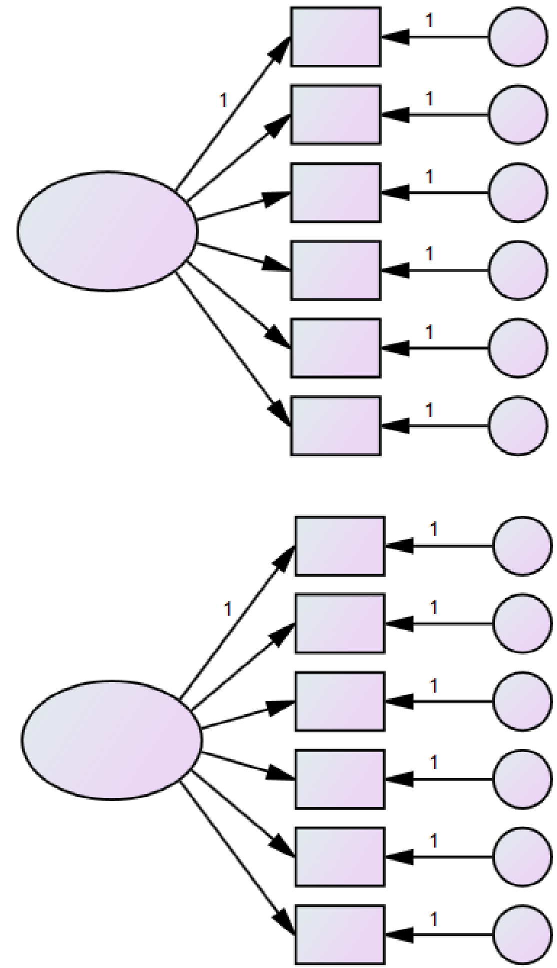

File Edit View Diagram Analyze Tools Plugins Help
Title
Select
Then copy , to make
a second factor easily
Unstandardized estimates
Standardized estimates
1
1
1
1
1
1
1

Then add correlations between factors

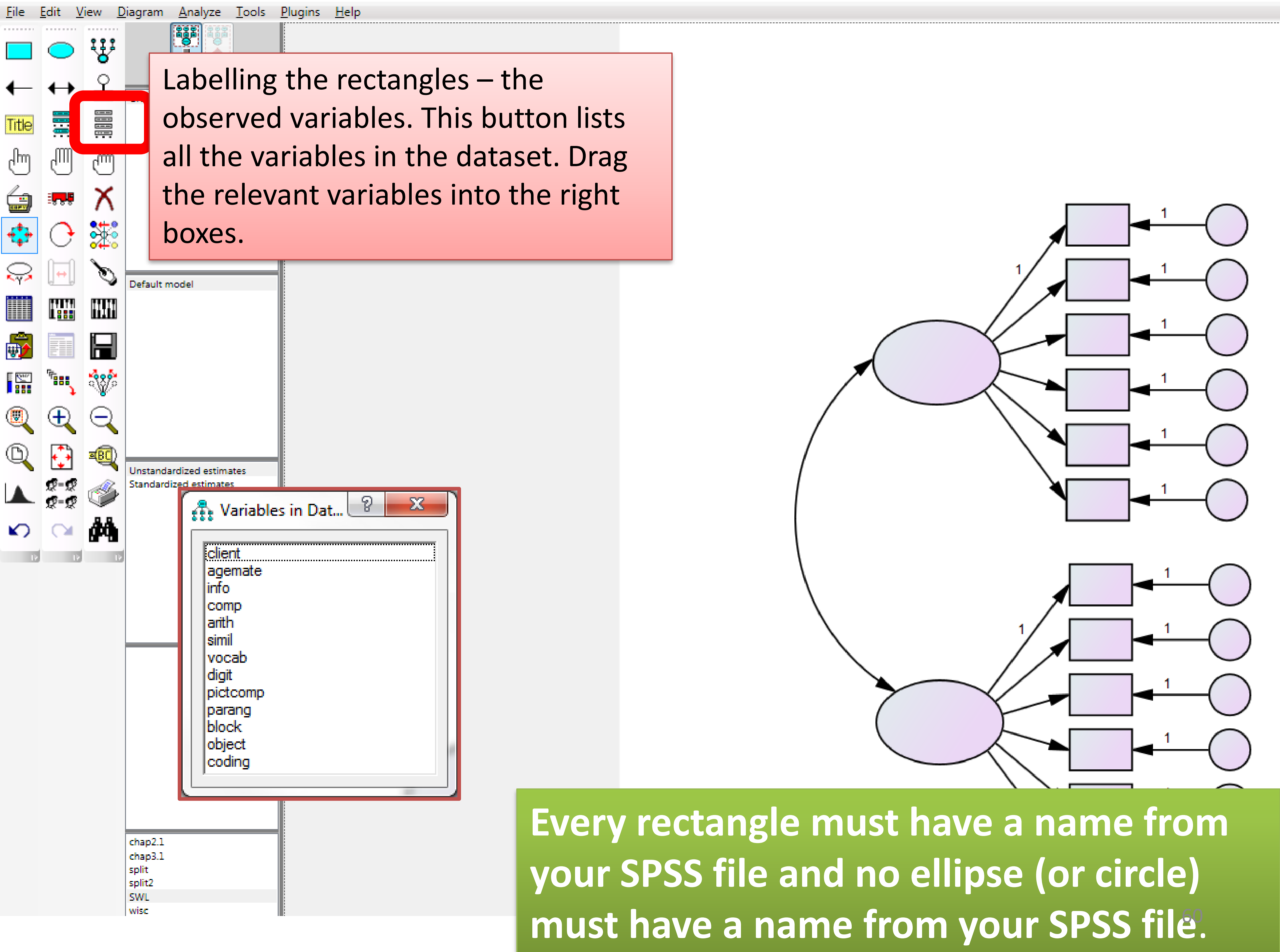
If need be, delete some of the observed variables to get the correct number.

Try the reshape button if you need to pretty it up



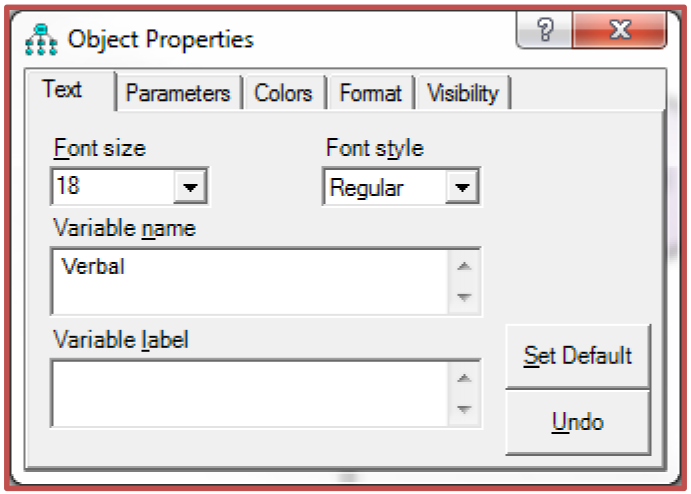


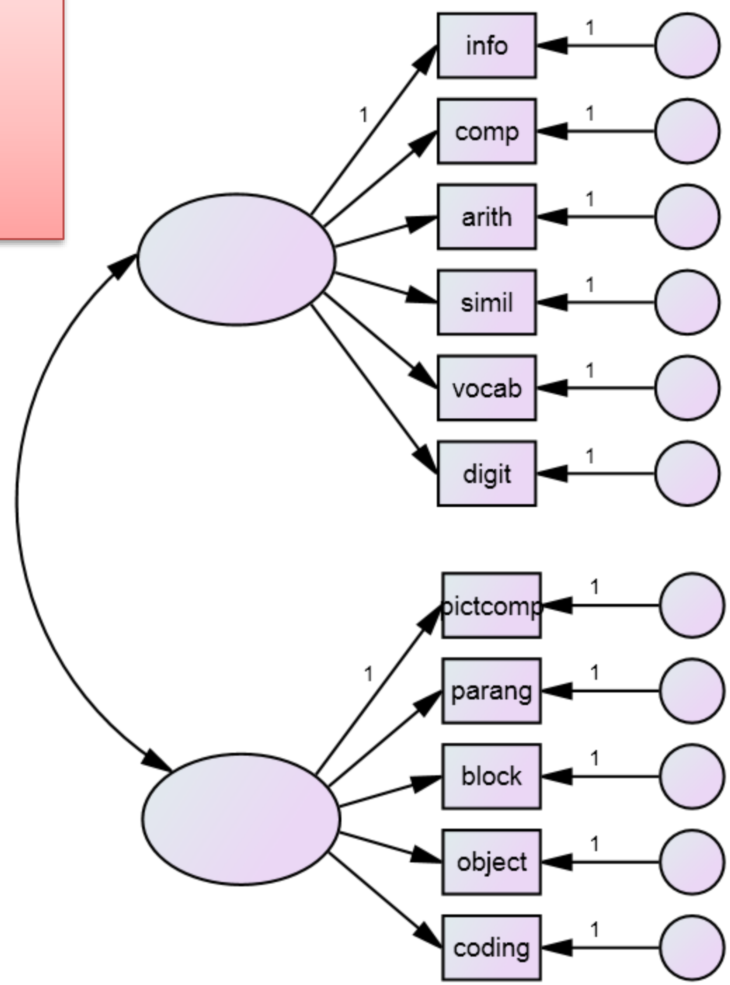
Labelling the rectangles – the observed variables. This button lists all the variables in the dataset. Drag the relevant variables into the right boxes.

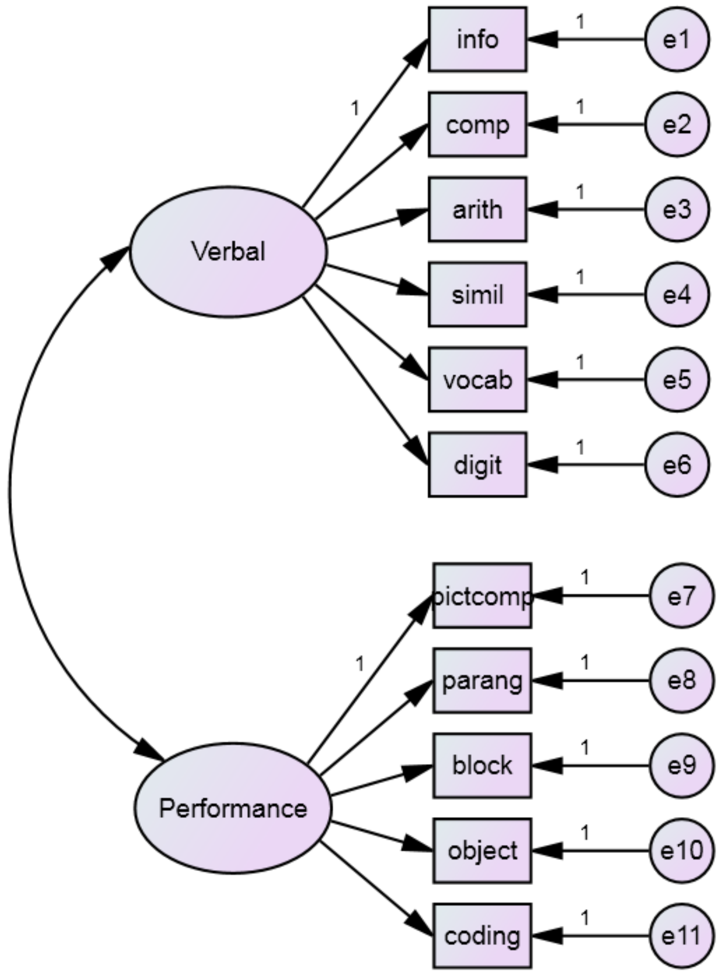


Every rectangle must have a name from your SPSS file and no ellipse (or circle) must have a name from your SPSS file.

Naming latent factors and variables. Right click on the ellipse to produce the Object Properties window. This enables you to give a name to the latent variable.

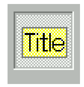


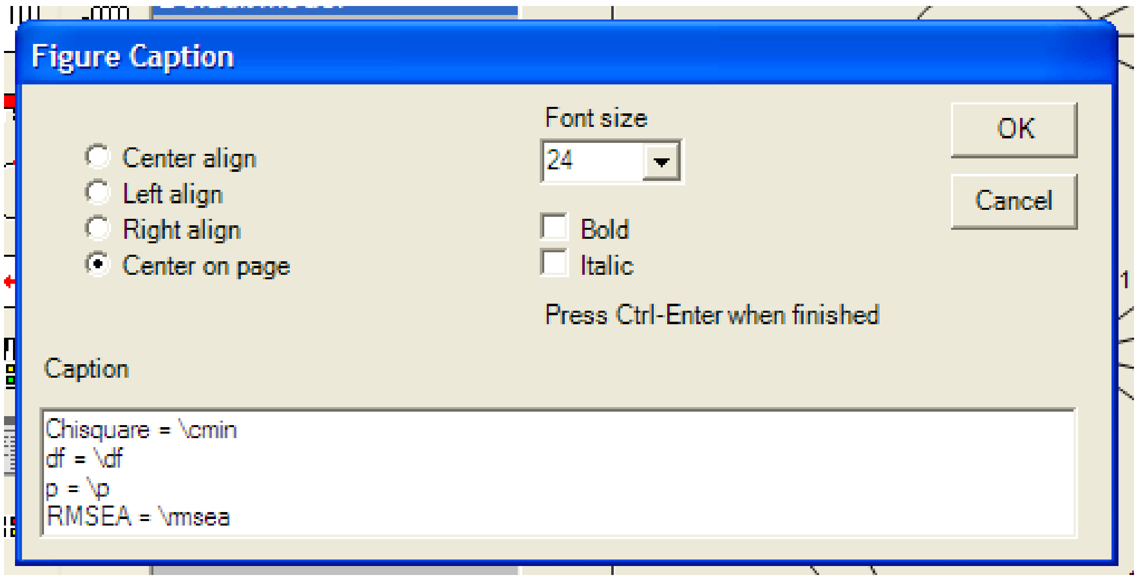



info
comp
arith
simil
vocab
digit
e1
e2
e3
e4
e5
e6
Verbal
1
pictcomp
parang
block
object
coding
e7
e8
e9
e10
e11
Performance

Another useful action is to insert the fit statistics onto the drawing.

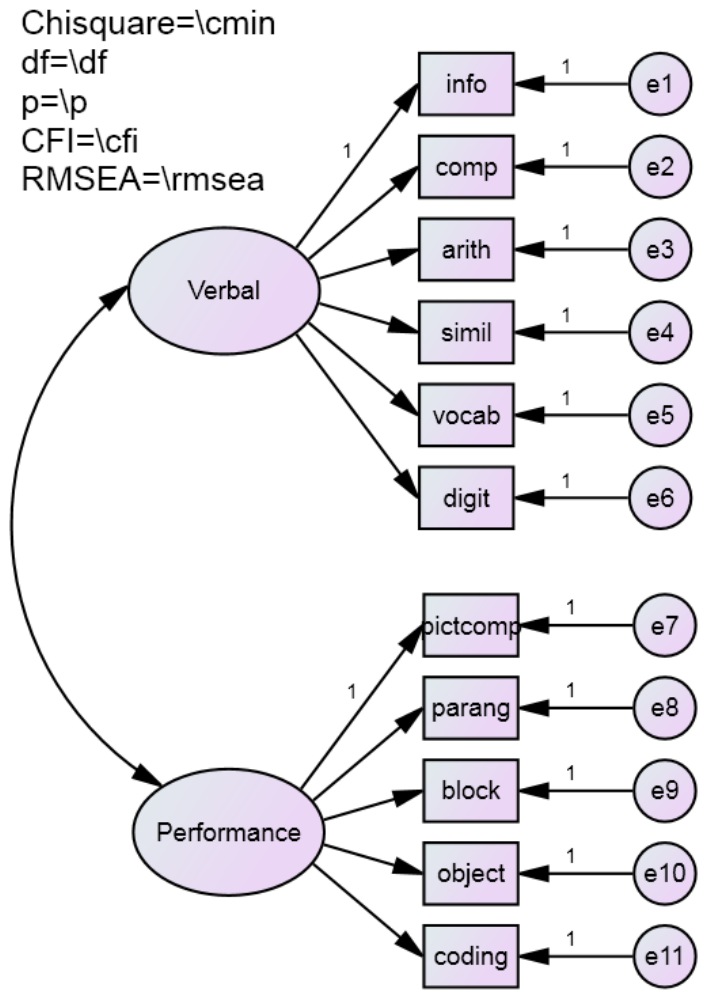
Do this by

clicking on the title button  which brings up a box



The text that is proceeded by a backslash ‘\’ is a keyword recognized by AMOS. Most of the keywords are self explanatory [cmin for chisquare is a little cryptic]

Another useful index of fit, the comparative fit index or CFI is represented by \cfi


Chisquare=\cmin
df=\df
p=\p
CFI=\cfi
RMSEA=\rmsea
Verbal
Performance
info
comp
arith
simil
vocab
digit
pictcomp
parang
block
object
coding
e1
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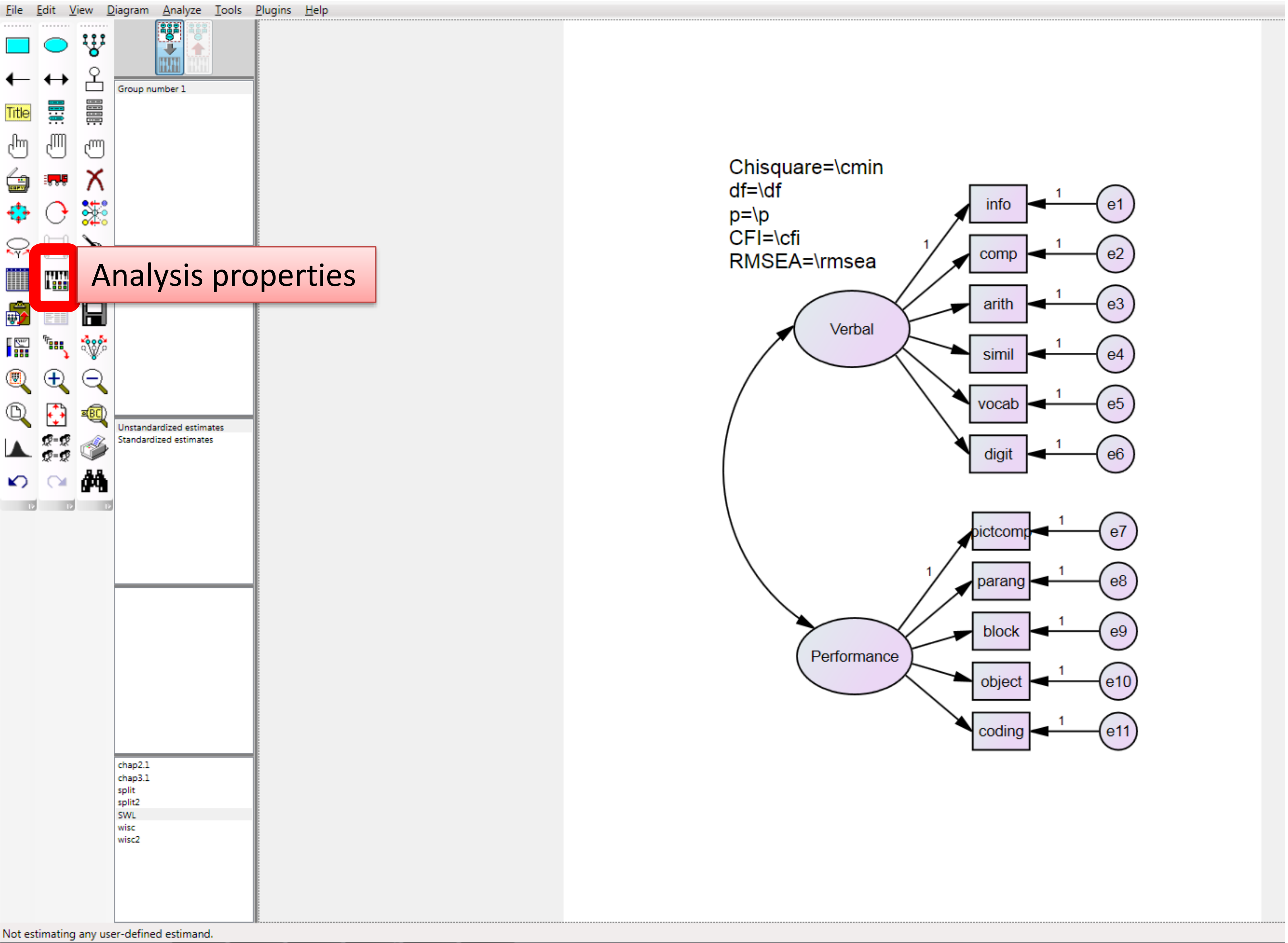
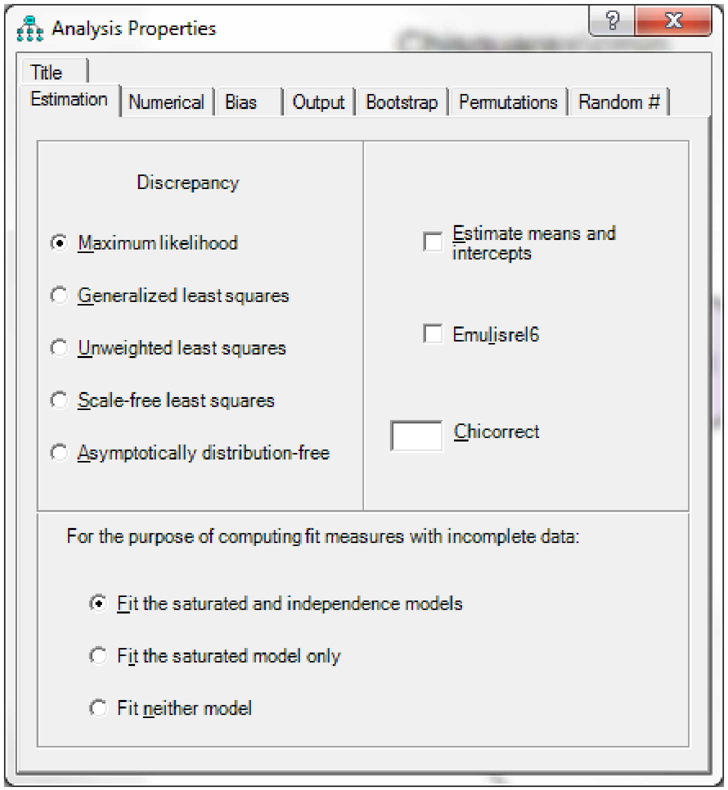
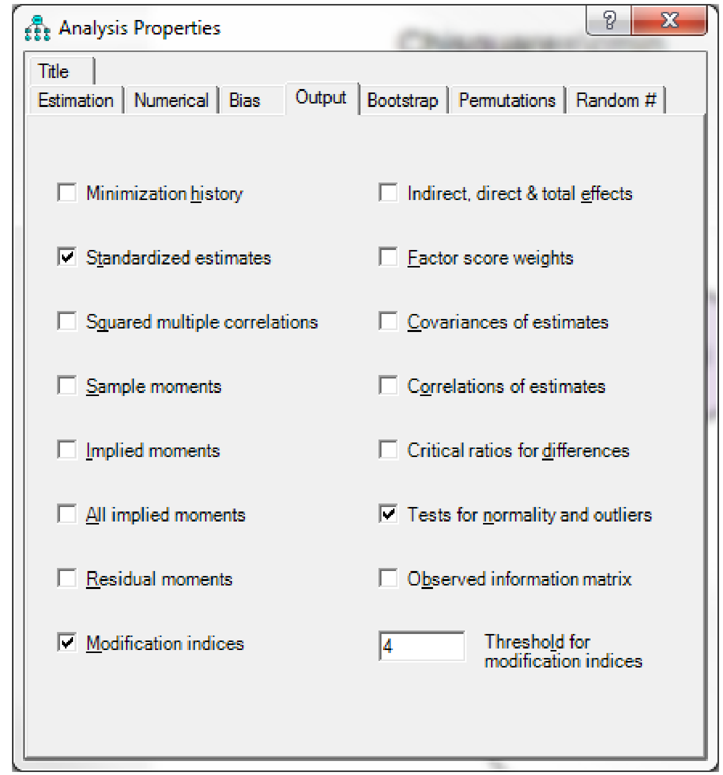
e9
e10
e11
1


File Edit View Diagram Analyze Tools Plugins Help
Title
Group number 1
Analysis properties
Unstandardized estimates
Standardized estimates
chap2.1
chap3.1
split
split2
SWL
wisc
wisc2
Chisquare=\cmin
df=\df
p=\p
CFI=\cfi
RMSEA=\rmsea
Verbal
Performance
info
comp
arith
simil
vocab
digit
pictcomp
parang
block
object
coding
e1
e2
e3
e4
e5
e6
e7
e8
e9
e10
e11
Not estimating any user-defined estimand.

Analysis properties



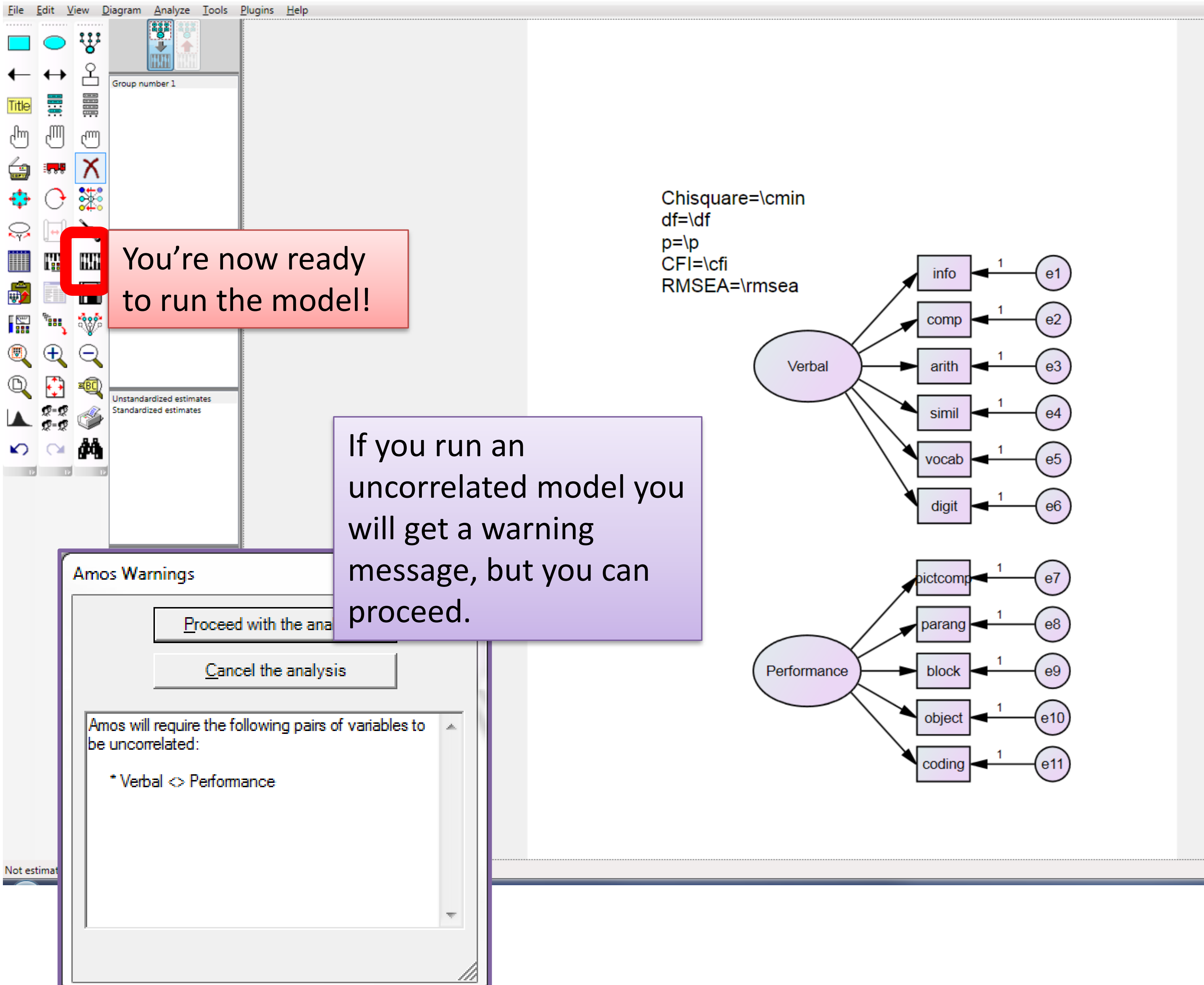


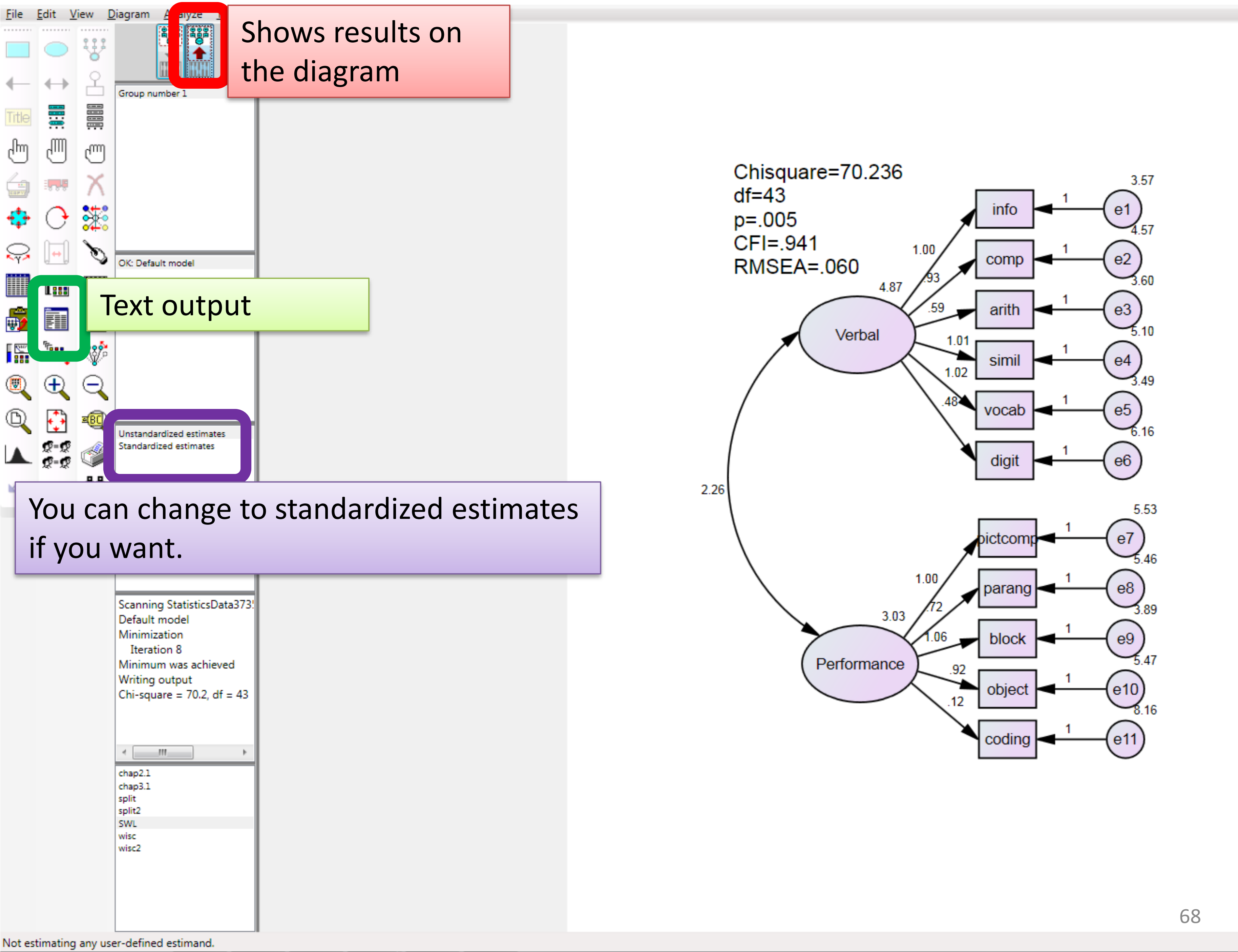
If you have missing data, you have to tick estimate means and intercepts, else usually use the defaults on Estimation

Some suggestions for Output – you can try others as well. Standardized estimates will enable scales in terms of correlations rather than covariances

You're now ready to run the model!

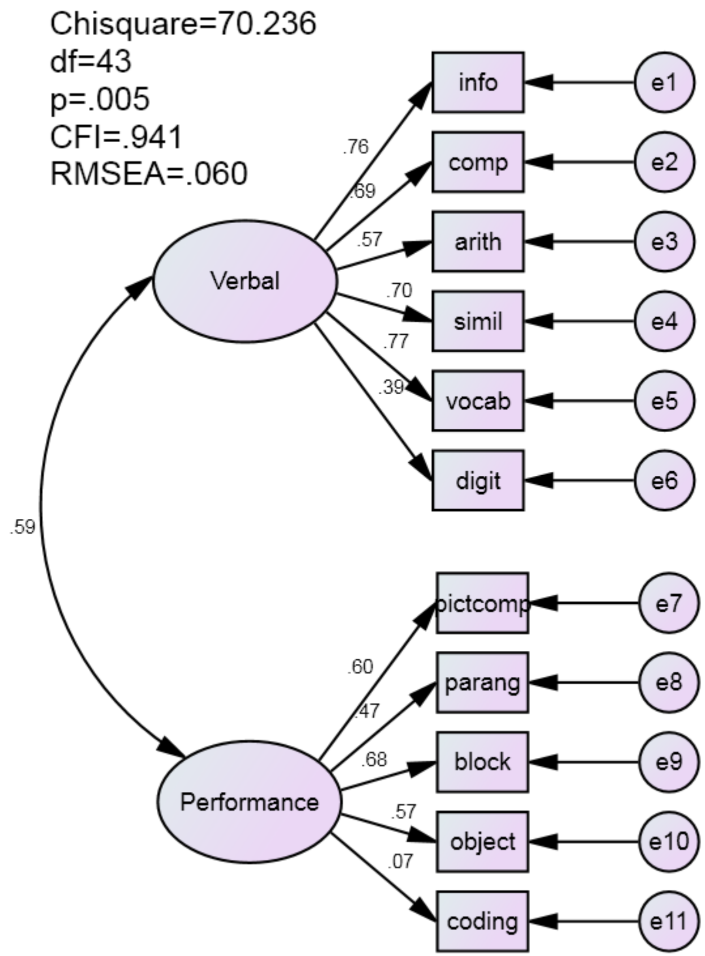
If you run an uncorrelated model you will get a warning message, but you can proceed.



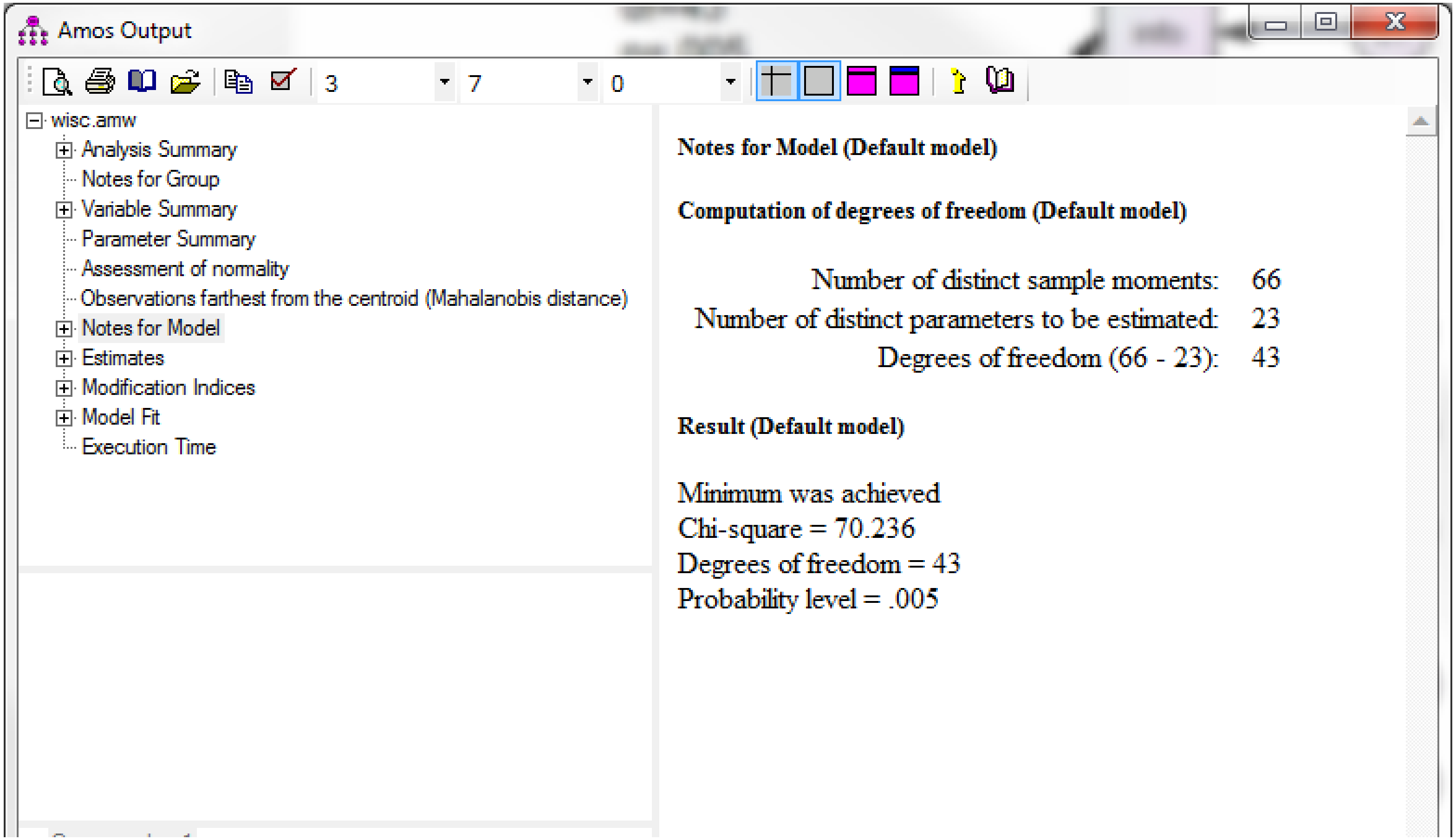


File Edit View Diagram Analyze
Shows results on the diagram
Group number 1
OK: Default model
Text output
Unstandardized estimates
Standardized estimates
You can change to standardized estimates if you want.
Scanning StatisticsData373!
Default model
Minimization
Iteration 8
Minimum was achieved
Writing output
Chi-square = 70.2, df = 43
chap2.1
chap3.1
split
split2
SWL
wisc
wisc2
Not estimating any user-defined estimand.
Chisquare=70.236
df=43
p=.005
CFI=.941
RMSEA=.060
Verbal
Performance
2.26
4.87
3.03
info
comp
arith
simil
vocab
digit
pictcomp
parang
block
object
coding
e1
e2
e3
e4
e5
e6
e7
e8
e9
e10
e11
1.00
.93
.59
1.01
1.02
.48
1.00
.72
1.06
.92
.12
3.57
4.57
3.60
5.10
3.49
6.16
5.53
5.46
3.89
5.47
8.16
1

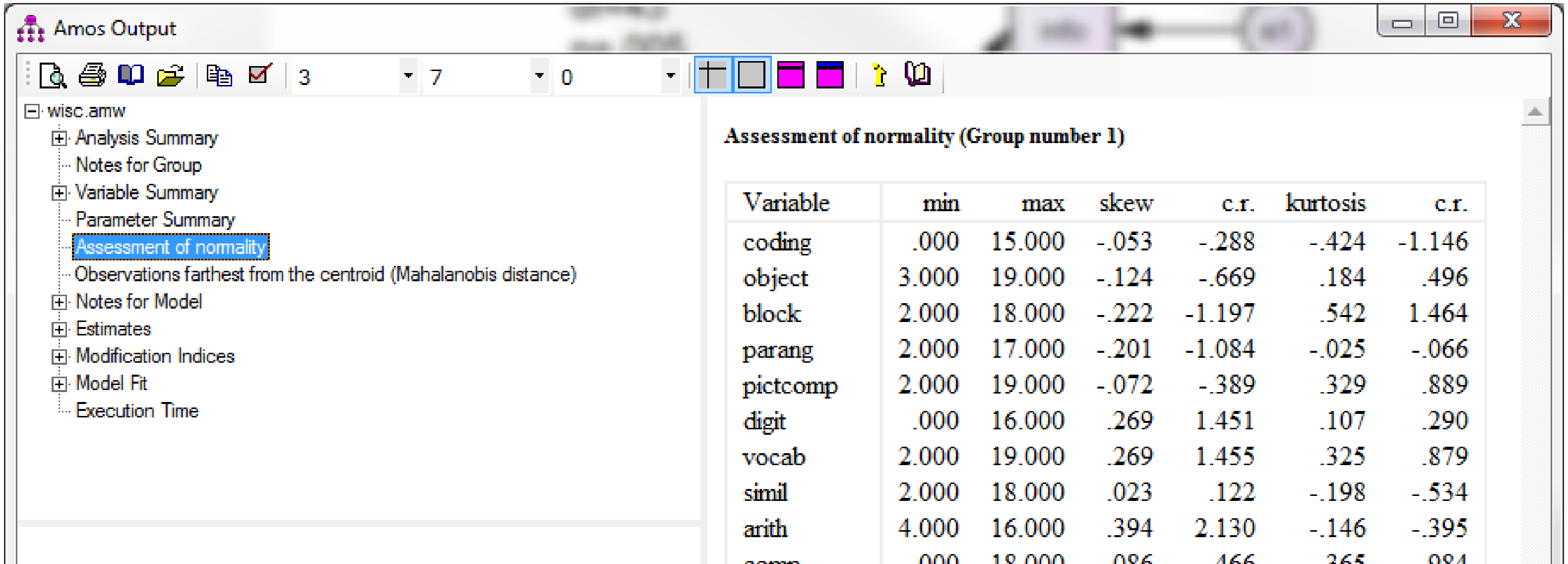
Standardized results



Text output

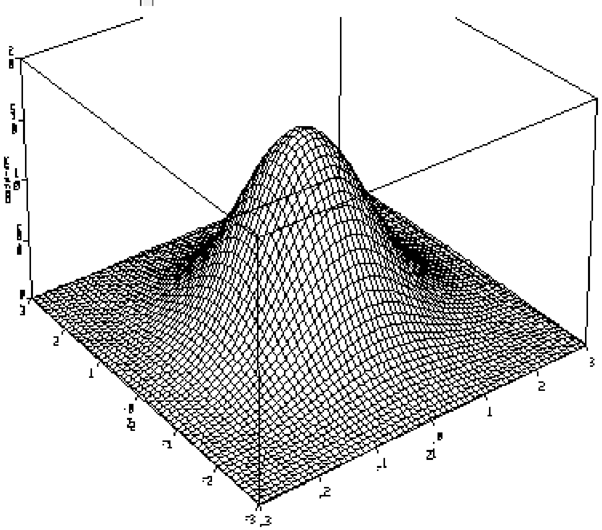

Amos Output
3
7
0
wisc.amw
Analysis Summary
Notes for Group
Variable Summary
Parameter Summary
Assessment of normality
Observations farthest from the centroid (Mahalanobis distance)
Notes for Model
Estimates
Modification Indices
Model Fit
Execution Time
Notes for Model (Default model)
Computation of degrees of freedom (Default model)
Number of distinct sample moments: 66
Number of distinct parameters to be estimated: 23
Degrees of freedom (66 - 23): 43
Result (Default model)
Minimum was achieved
Chi-square = 70.236
Degrees of freedom = 43
Probability level = .005

Assessment of normality output



Assessment of normality (Group number 1)

Variable	min	max	skew	c.r.	kurtosis	c.r.
coding	.000	15.000	-.053	-.288	-.424	-1.146
object	3.000	19.000	-.124	-.669	.184	.496
block	2.000	18.000	-.222	-1.197	.542	1.464
parang	2.000	17.000	-.201	-1.084	-.025	-.066
pictcomp	2.000	19.000	-.072	-.389	.329	.889
digit	.000	16.000	.269	1.451	.107	.290
vocab	2.000	19.000	.269	1.455	.325	.879
simil	2.000	18.000	.023	.122	-.198	-.534
arith	4.000	16.000	.394	2.130	-.146	-.395
comp	.000	18.000	.086	.466	.365	.984
info	3.000	19.000	.084	.454	-.048	-.129
Multivariate					7.526	2.943



Generally, we want the critical ratio (c.r.) to be less than 2, so this seems to be OK for most, although we may have a problem with multivariate kurtosis. We will proceed, relying on robustness of ML estimation (Widaman, 2012).

Parameter estimates: Unstandardized

			Estimate	S.E.	C.R.	P
info	<---	Verbal	1.000			
comp	<---	Verbal	.926	.108	8.585	***
arith	<---	Verbal	.589	.084	6.993	***
simil	<---	Verbal	1.012	.116	8.739	***
vocab	<---	Verbal	1.020	.107	9.521	***
digit	<---	Verbal	.477	.100	4.791	***
pictcomp	<---	Performance	1.000			
parang	<---	Performance	.719	.156	4.601	***
block	<---	Performance	1.060	.187	5.659	***
object	<---	Performance	.921	.177	5.200	***
coding	<---	Performance	.119	.147	.807	.419

Parameter estimates: Standardized

			Estimate
info	<---	Verbal	.760
comp	<---	Verbal	.691
arith	<---	Verbal	.565
simil	<---	Verbal	.703
vocab	<---	Verbal	.770
digit	<---	Verbal	.390
pictcomp	<---	Performance	.595
parang	<---	Performance	.473
block	<---	Performance	.683
object	<---	Performance	.566
coding	<---	Performance	.072

Covariances: (Group number 1 - Default model)

			Estimate	S.E.	C.R.	P
Verbal	<-->	Performance	2.263	.516	4.385	***

Correlations: (Group number 1 - Default model)

			Estimate
Verbal	<-->	Performance	.589

Measures of fit

Amos Output

- wisc.amw
  - Analysis Summary
  - Notes for Group
  - Variable Summary
  - Parameter Summary
  - Assessment of normality
  - Observations farthest from the centroid (Mahalanobis distance)
  - Notes for Model
  - Estimates
  - Modification Indices
  - Model Fit
  - Execution Time

Group number 1

Default model

Model Fit Summary

CMIN

Model	NPAR	CMIN	DF	P	CMIN/DF
Default model	23	70.236	43	.005	1.633
Saturated model	66	.000	0		
Independence model	11	516.237	55	.000	9.386

RMR, GFI

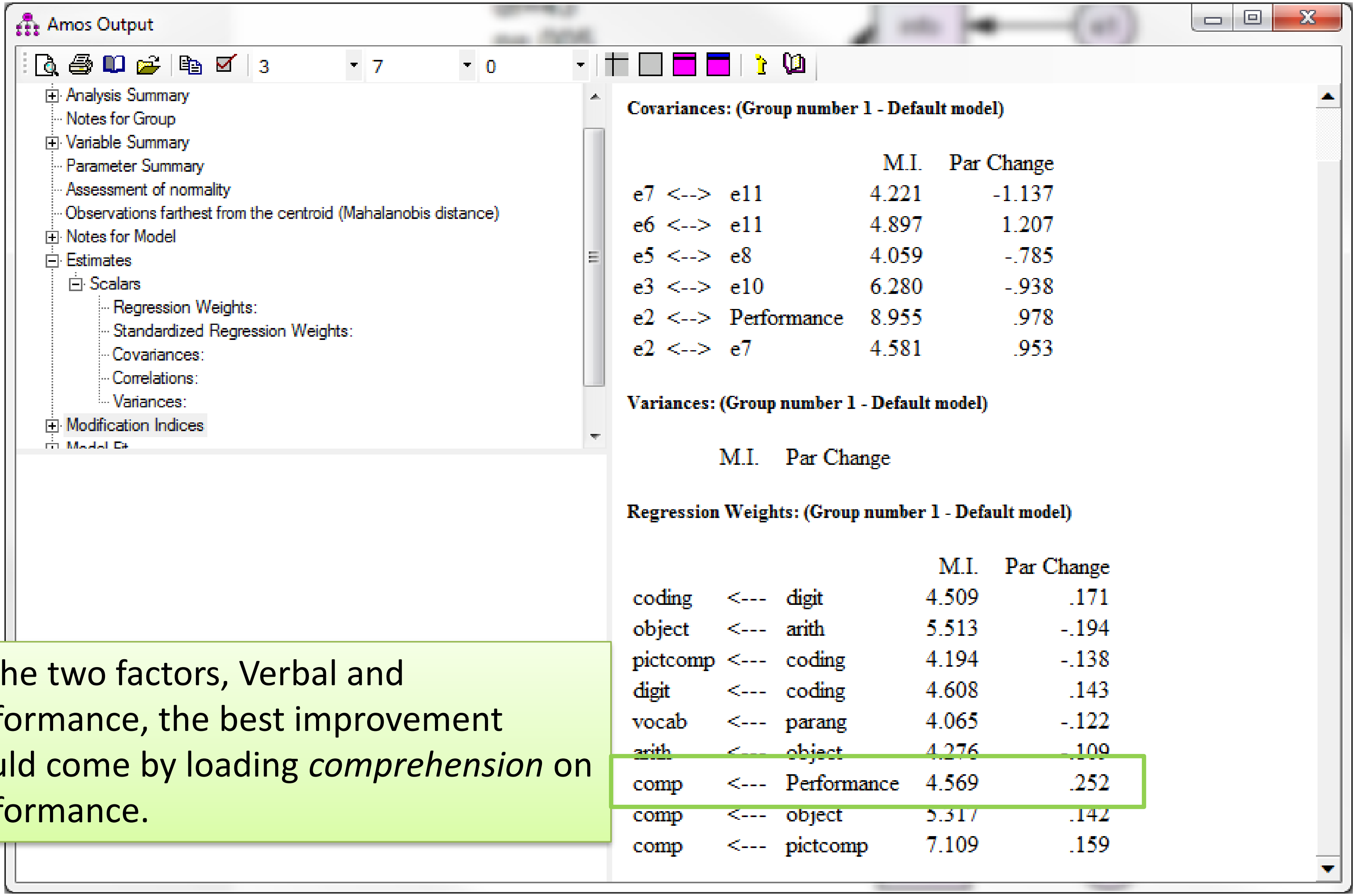
Model	RMR	GFI	AGFI	PGFI
Default model	.466	.931	.894	.606
Saturated model	.000	1.000		
Independence model	2.241	.527	.432	.439

Baseline Comparisons

Model	NFI Delta1	RFI rho1	IFI Delta2	TLI rho2	CFI
Default model	.864	.826	.942	.924	.941
Saturated model	1.000		1.000		1.000
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

AMOS has many measures of fit in addition to those we have studied so far. More on this next week.

Modification indices:
These tell us which additional covariances would most improve the model



Covariances: (Group number 1 - Default model)

			M.I.	Par Change
e7	<-->	e11	4.221	-1.137
e6	<-->	e11	4.897	1.207
e5	<-->	e8	4.059	-.785
e3	<-->	e10	6.280	-.938
e2	<-->	Performance	8.955	.978
e2	<-->	e7	4.581	.953

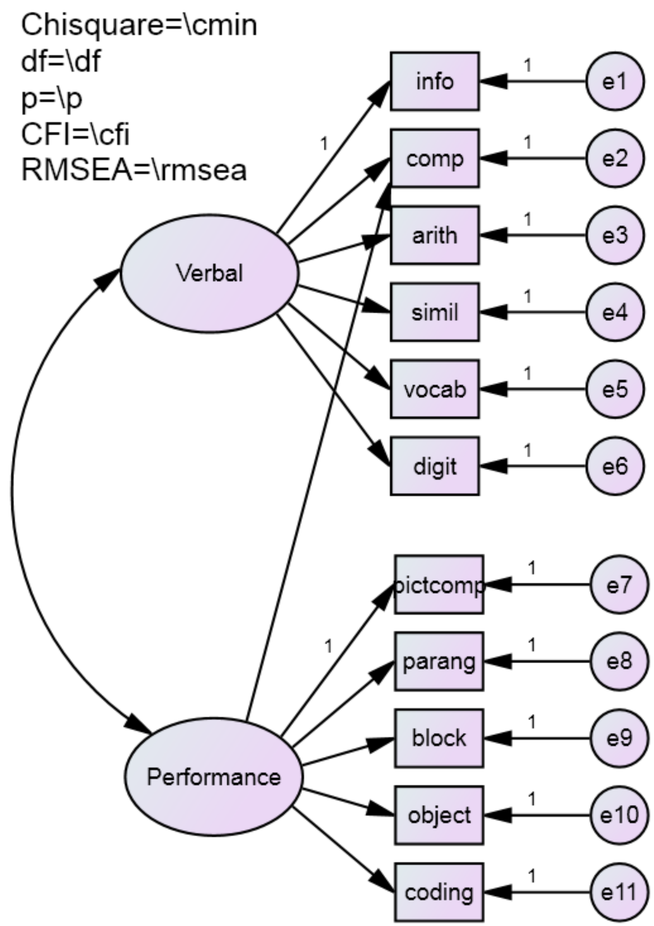
Variances: (Group number 1 - Default model)

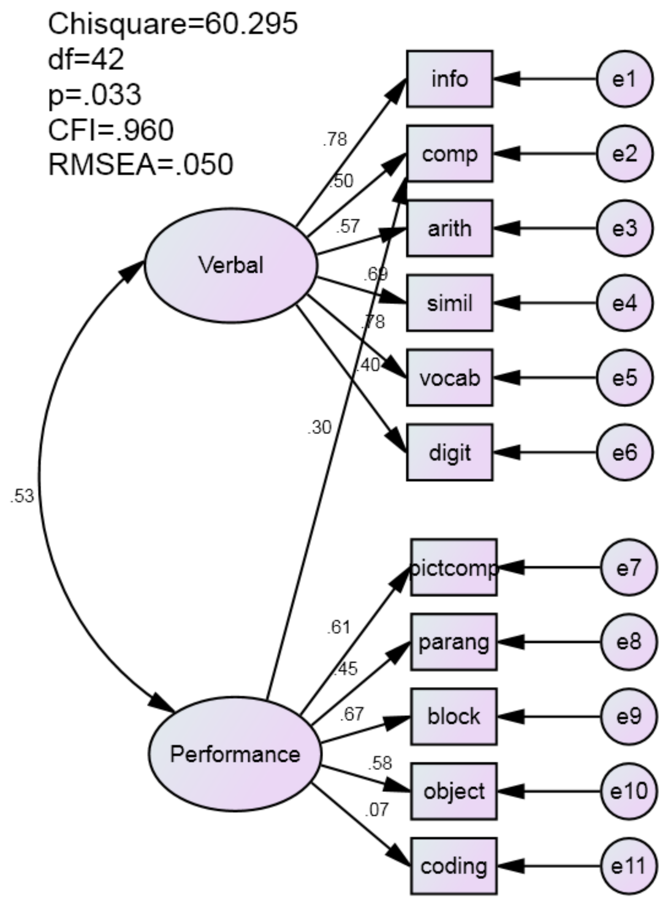
M.I.	Par Change

Regression Weights: (Group number 1 - Default model)

			M.I.	Par Change
coding	<---	digit	4.509	.171
object	<---	arith	5.513	-.194
pictcomp	<---	coding	4.194	-.138
digit	<---	coding	4.608	.143
vocab	<---	parang	4.065	-.122
arith	<---	object	4.276	-.109
comp	<---	Performance	4.569	.252
comp	<---	object	5.317	.142
comp	<---	pictcomp	7.109	.159

Of the two factors, Verbal and Performance, the best improvement would come by loading *comprehension* on Performance.


Chisquare=\cmin
df=\df
p=\p
CFI=\cfi
RMSEA=\rmsea
Verbal
Performance
info
comp
arith
simil
vocab
digit
pictcomp
parang
block
object
coding
e1
e2
e3
e4
e5
e6
e7
e8
e9
e10
e11
1


Chisquare=60.295
df=42
p=.033
CFI=.960
RMSEA=.050
info
comp
arith
simil
vocab
digit
pictcomp
parang
block
object
coding
e1
e2
e3
e4
e5
e6
e7
e8
e9
e10
e11
Verbal
Performance
.78
.50
.57
.69
.78
.40
.30
.53
.61
.45
.67
.58
.07

IN THIS LECTURE, you learnt

- the basic idea behind structural equation modeling
- that SEM can combine regression and factor analysis
- the differences between manifest and latent variables and between measurement and structural models
- how to conduct a confirmatory factor analysis in AMOS

References

Barrett, P. (2007). Structural equation modelling: Adjudging model fit. *Personality and Individual differences*, 42(5), 815-824.

Jackson, D. L. (2003). Revisiting sample size and number of parameter estimates: Some support for the N: q hypothesis. *Structural equation modeling*, 10(1), 128-141.

Kline, R. B. (2016). *Principles and practice of structural equation modeling*. Guilford Press.

Loehlin, J. C. (1992). *Latent variable models*. Erlbaum.

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics.* Pearson.

Widaman, K. (2012). Exploratory factor analysis and confirmatory factor analysis. In H. Cooper (Ed.) *APA Handbook of Research Methods in Psychology, Vol 3* (pp.361-389).

Wolf, E. J., Harrington, K. M., Clark, S. L., & Miller, M. W. (2013). Sample size requirements for structural equation models: An evaluation of power, bias, and solution propriety. *Educational and psychological measurement*, 73(6), 913-934.