1. Classification

    (a) K-Nearest Neighbor (Algorithm 1)
    (b) Support Vector Machines (Algorithm 32)
    (c) Iterative Dichotomiser 3 (Algorithm 6)
    (d) C4.5 (Algorithm 16)
    (e) Naive Bayes Multinomial Model (Algorithm 18)
    (f) Random Forest (Algorithm 5)
    (g) Winnow Algorithm(Algorithm 30)

2. Neural Networks

    (a) Perceptron (Algorithm 7,8)
    (b) Back-Propagation (Algorithm 9,10)
    (c) Learning Vector Quantization (Algorithm 11)
    (d) Self Organizing Map (Algorithm 12,13)

3. Hidden Markov Models

    (a) The forward algorithm (Algorithm 31)
    (b) Viterbi algorithm (Algorithm 32)
    (c) The forward-backward algorithm(Algorithm 33)

4. Clustering Algorithms

    (a) Hierarchial Agglomerative Clustering (Algorithm 14 and Algorithm 15)
    (b) Hierarchial Divisive Clustering (Algorithm 16 and Algorithm 17)
    (c) DBSCAN (Algorithm 28)
    (d) K means(Algorithm 26)
    (e) K medoids(Algorithm 27)

5. Regression Algorithms

    (a) Lasso Regression (Algorithm 19)
    (b) Logistic Regression (Algorithm 30)

6. Deep Learning Algorithms

    (a) Deep Q-Learning (Algorithm 23)

7. Ensemble Learning

    (a) Adaboost (Algorithm 2, 3, 4)
    (b) Bagging (Algorithm 22)
    (c) Stacking (Algorithm 33)

8. Dimensionality Reduction Algorithms

    (a) Principal Components Analysis(Algorithm 29)

9. Other Methods

    (a) Gradient Descent (Algorithm 19)
    (b) PageRank(Algorithm 24)
    (c) Gaussian Process(Algorithm 31)

Algorithm 1 k-Nearest Neighbor   [Tay et al., 2014] link:16,31

Input: X: training data, Y:Class labels of X, x: unknown sample
Output: Class label of unknown sample
1: **function** CLASSIFY(X, Y, x)
2:     **for** $i = 1$ to m **do**
3:         Compute distance $d(X_i, x)$
4:     **end for**
5:     Compute set I containing indices for the k smallest distances $d(X_i, x)$
6:     Return majority label $\{Y_i \text{ where } i \in I\}$
7: **end function**

Algorithm 2 Adaboost   [Schapire, 2014]

Input:
    Training data $\{(x_i, y_i)_{i=1}^N$ where $x_i \in \mathbb{R}^k$ and $y_i \in \{-1, 1\}\}$
    Large number of classifiers denoted by $f_m(x) \in \{-1, 1\}$
    0-1 loss function I defined as

$$I(f_m(x, y)) = \begin{cases} 0, & \text{if } f_m(x_i) = y_i \\ 1, & \text{if } f_m(x_i) \neq y_i \end{cases} \quad\quad (1) \\ (2)$$

Output: The final classifier
1: **for** $i = 1$ to N **do**
2:     **for** $i = 1$ to M **do**
3:         Fit weak classifier m to minimize the objective function:
4:         $\epsilon_m = \frac{\sum_{i=1}^N w_i^m I(f_m(x_i)) \neq y_i}{x^2 + 2x + 1}$
5:         where $I(f_m(x_i) \neq y_i) = 1$ if $f_m(x_i) \neq y_i$ and 0 otherwise
6:         $\alpha_m = \ln \frac{1-\epsilon_m}{\epsilon_m}$
7:     **end for**
8:     **for** all i **do**
9:         $w_i^{m+1} = w_i^{(m)} e^{\alpha_{mI(f_m(x_i) \neq y_i)}}$
10:    **end for**
11: **end for**

Algorithm 3 Adaboost   [Hertzmann et al., 2015]

Input:
    Training data $\{(x_i, y_i)_{i=1}^N$ where $x_i \in \mathbb{R}^k$ and $y_i \in \{-1, 1\}\}$
Output: Weighted sum that represents the final output of the boosted classifier
1: Given Training data $\{(x_i, y_i)$ where $y_i \in \{-1, 1\}\}$
2: initialize D_1 = uniform distribution on training examples
3: **for** $t = 1$ to T **do**
4:     Train weak classifier h_t on D_t
5:     choose $\alpha_t > 0$
6:     compute new distribution D_{t+1}:
7:     **for** all i **do**
8:         multiply $D_t(x)$ by

$$\begin{cases} e^{-\alpha_t}, & (< 1) \text{ if } y_i = h_t(x_i) \\ e^{\alpha_t}, & (> 1) \text{ if } y_i \neq h_t(x_i) \end{cases} \quad\quad (3) \\ (4)$$

9:         renormalize
10:    **end for**
11:    output final classifier $H_{final(x)} = sign(\sum \alpha_t h_t(x))$
12: **end for**

Algorithm 4 Adaboost4 [Hertzmann et al., 2015] Link:23,35,93,95

Input:

Training data $\{(x_i, y_i)_{i=1}^{N}$ where $x_i \in \mathbb{R}^k$ and $y_i \in \{-1, 1\}\}$

Output: Weighted sum that represents the final output of the boosted classifier

1: Set uniform example weights.
2: **for** each base learner do **do**
3:     Train base learner with weighted sample.
4:     Test base learner on all data.
5:     Set learner weight with weighted error.
6:     Set example weights based on ensemble predictions.
7: **end for**

Algorithm 5 Random forest [Bernstein, 2016] Link:78

Input: S: training set, F:Features and number of trees in forest B

Output: Constructed tree

1: **function** RANDOMFOREST(S, F)
2:     $H \leftarrow \emptyset$
3:     **for** $i \in 1, ... B$ **do**
4:         $S^{(i)} \leftarrow$ A bootstrap sample from S
5:         $h_i \leftarrow RANDOMIZEDTREELEARN(S^i, F)$
6:         $H \leftarrow H \bigcup \{h_i\}$
7:     **end for**
8:     return H
9: **end function**
10: **function** RANDOMIZEDTREELEARN(S, F)
11:     At each node:
12:     $f \leftarrow$ a very small subset of F
13:     Split on best feature in f
14:     return The learned tree
15: **end function**

Algorithm 6 Iterative Dichotomiser 3 [., 2015a] Link:12,14,41, 69,95,98,114,134,164,177,186,193

Input: D : Training Data, X : Set of Input Attributes

Output: A decision tree

1: **function** ID3(D, X)
2:     Let T be a new tree
3:     **if** all instances in D have the same class c **then**
4:         Label $(T) = c$; Return T
5:     **end if**
6:     **if** $X = \emptyset$ or no attribute has positive information gain **then**
7:         Label $(T) =$ most common class in D; Return T
8:     **end if**
9:     $X \leftarrow$ attribute with highest information gain
10:     Label(T) = X
11:     **for** each value x of X **do**
12:         $D_x \leftarrow$ instances in D with $X = x$
13:         **if** D_x is empty **then**
14:             Let T_x be a new tree
15:             Label(T_x) = most common class in D
16:         **else**
17:             $T_x = $ ID3($D_x, X - \{x\}$)
18:         **end if**
19:         Add a branch from T to T_x labeled by x
20:     **end for**
21:     return T
22: **end function**

Algorithm 7 Perceptron [Brownlee, 2015d] Link:65

Input: ProblemSize,InputPatterns,Max number of iterations,Learning rate
Output: WeightsClassification of the entire training set)
1: **for** $i = 1$ to Max number of iterations **do**
2:     $Pattern_i \leftarrow$ SelectInputPattern(InputPatterns)
3:     $Activation_i \leftarrow$ ActivateNetwork($Pattern_i$,Weights)
4:     $Output_i \leftarrow$ TransferActivation($Activation_i$)
5:     UpdateWeights($Pattern_i$,$Output_i$,Learning rate)
6: **end for**
7: Return Weights

Algorithm 8 Perceptron [Amini, 2015] Link:65

Input:
    Training set S = $((x_1, y_1),....(x_m, y_m))$
    Learning rate $\eta > 0$
    Maximum number of iterations T
Output: Parameters of the linear model $w^{(t)}$
    Initialize weights $w^{(0)} = (\bar{w}^{(0)}, w_0^{(0)})$
    $t \leftarrow 0$
1: **while** $t \leq T$ **do**
2:     Choose an example at random $(x, y) \in S$
3:     **if** $y \times (\langle \bar{w}^{(t)}, x \rangle + w_0^{(t)}) \leq 0$ **then**
4:         $w_0^{(t+1)} \leftarrow w_0^{(t)} + \eta \times y$
5:         $\bar{w}^{(t+1)} \leftarrow \bar{w}^{(t)} + \eta \times y \times x$
6:     **else**
7:         $w^{(t+1)} \leftarrow w^{(t)}$
8:     **end if**
9: **end while**
10: t $\leftarrow$ t +1

Algorithm 9 Back-propagation [Brownlee, 2015a] Link:15,53,82,95

Input: ProblemSize,InputPatterns,Max number of iterations,Learning rate
Output: Network
1: Network $\leftarrow$ ConstructNetworkLayers()
2: $Network_{weights} \leftarrow$ InitializeWeights(Network,ProblemSize)
3: **for** $i = 1$ to Max number of iterations **do**
4:     $Pattern_i \leftarrow$ SelectInputPattern(InputPatterns)
5:     $Output_i \leftarrow$ ForwardPropagate($Pattern_i$,Network)
6:     BackwardPropagateError($Pattern_i$,$Output_i$,Network)
7:     UpdateWeights($Pattern_i$,$Output_i$,Network,Learning rate)
8: **end for**
9: Return Network

Algorithm 10 Back-propagation2 [Ng,]

Input:
  Training Set $x^{(1)}, y^{(1)}, , ..., (x^{(m)}, y^{(m)})$
Output:
  Gradient of the cost function
1: $\Delta_{ij}^{(l)} = 0$(for all l,i,j)
2: **for** $i = 1$ to m **do**
3:    Set $a^{(1)} = x^{(i)}$
4:    Perform forward propagation to compute $a^{(l)}$ for $l = 2, 3, ...L$
5:    Using $y^{(}i)$, compute $\delta^{(}L) = a^{(}L) - y^{(}i)$
6:    Compute $\delta^{(}L-1), \delta^{(}L-2), ...\delta^{(}2)$
7:    $\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)}$
8: **end for**
9: $D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij} + \lambda\theta_{ij}^{(l)}$ if $j \neq 0$
10: $D_{ij}^{(l)} := \frac{1}{m}\Delta_{ij}$ if $j = 0$
11: $\frac{\partial}{\partial\theta_{ij}^{(l)}}J(\theta) = D_{ij}^{(}l)$

Algorithm 11 Learning Vector Quantization [Brownlee, 2015c] Link : 97 and 105

Input: ProblemSize,InputPatterns,Max number of iterations,$CodebookVectors_{num}$,Learning rate
Output: CodebookVectors
1: $CodebookVectors \leftarrow InitializeCodebookVectors(CodebookVectors_{num}, ProblemSize)$
2: **for** $i = 1$ to Max number of iterations **do**
3:    $Pattern_i \leftarrow$ SelectInputPattern(InputPatterns)
4:    $Bmu_i \leftarrow$ SelectBestMatchingUnit($Pattern_i$,CodebookVectors)
5:    **for** $Bmu_i^{attribute} \in Bmu_i$ **do**
6:       **if** $Bmu_i^{class} \equiv Pattern_i^{class}$ **then**
7:          $Bmu_i^{attribute} \leftarrow Bmu_i^{attribute} + learn_{rate} \times (Pattern_i^{attribute} - Bmu_i^{attribute})$
8:       **else**
9:          $Bmu_i^{attribute} \leftarrow Bmu_i^{attribute} - learn_{rate} \times (Pattern_i^{attribute} - Bmu_i^{attribute})$
10:      **end if**
11:   **end for**
end for
Return CodebookVectors

Algorithm 12 Self Organizing Map [Brownlee, 2015b] Link:63,71,85,86,98,105

Input: InputPatterns,Max number of iterations,Learning rate,$Grid_{width}, Grid_{height}$
Output: CodebookVectors
1: $CodebookVectors \leftarrow InitializeCodebookVectors(Grid_{width}, Grid_{height}, InputPatterns)$
2: **for** $i = 1$ to Max number of iterations **do**
3:    $Learn_{rate}^i \leftarrow$ CalculateLearningRate($i, learn_{rate}^{init}$)
4:    $neighborhood_{size}^i \leftarrow$ CalculateNeighborhoodSize($i, neighborhood_{init}^{size}$)
5:    $Pattern_i \leftarrow$ SelectInputPattern(InputPatterns)
6:    $Bmu_i \leftarrow$ SelectBestMatchingUnit($Pattern_i$,CodebookVectors)
7:    Neighborhood $\leftarrow Bmu_i$
8:    Neighborhood $\leftarrow$ SelectNeighbors(Bmu_i,CodebookVectors,$neighborhood_{size}^i$)
9:    **for** $Vector_i \in$ Neighborhood **do**
10:      **for** $Vector_i^{attribute} \in Vector_i$ **do**
11:         $Vector_i^{attribute} \leftarrow Vector_i^{attribute} + learn_{rate} \times (Pattern_i^{attribute} - Vector_i^{attribute})$
12:      **end for**
13:   **end for**
14: **end for**
15: Return CodebookVectors

Algorithm 13 Self Organizing Map [., g] Link:69

Input:

    Training data: Vectors of length n

1: $(x_{1,1}, x_{1,2}, ...x_{1,n})$

2: ....

3: ....

4: $(x_{p,1}, x_{p,2}, ...x_{p,n})$ ▷ p distinct training vectors

Output:

    A vector Y of length m :$(y_1, y_2, ...y_m)$

5:                                   ▷ Sometimes $m < n$, sometimes $m > n$, sometimes $m = n$

6: Select output layer network topology   ▷ Initialize current neighborhood distance,$D(0)$, to a positive value

7: Initialize weights from inputs to outputs to small random values

8: Let $t = 1$

9: **while** computational bounds are not exceeded **do**

10:     Select an input sample i_l

11:     Compute the square of the Euclidean distance of i_l

12: from weight vectors(w_j) associated with each output node

$$\sum_{k=1}^{n} ((i_{l,k} - w_{j,k}(t)))^2$$

13:     Select output node j^* that has weight vector with minimum value from step 2)

14:     Update weights to all nodes within a topological distance given by $D(t)$ from j^* using the weight update rule: $w_j(t+1) = w_j(t) + \eta(t)(i_l - w_j(t))$

15:     increment t

16: **end while**

Algorithm 14 Hierarchial Agglomerative Algorithm [Stein, 2016a] Link:46,101

Input:

    $\langle V, E, w \rangle$.

    d_c.Distance measure for two clusters

Output: $\langle V_T, E_T \rangle$.Cluster hierarchy or dendogram

1: $C = \{\{v \mid v \in V\}\}$ ▷ Initial Clustering

2: $V_t = \{v_C \mid C \in C\}, E_T = \emptyset$ ▷ Initial Dendogram

3: **while** $|C| > 1$ **do**

4:     $update\_distance\_matrix(C, G, d_c)$

5:     $\{C, C'\} = \underset{\{C_i, C_j\} \in C : C_i \neq C_j}{argmin} \ d_c(C_i, C_j)$

6:     $C = (C \backslash \{C, C'\}) \cup \{C \cup C'\}$ ▷ Merging

7:     $V_T = V_T \cup \{v_{C,C'}\}, E_T = E_T \cup \{\{v_{C,C'}, v_C\}, \{v_{C,C'}, v_C\}\}$ ▷ Dendogram

8: **end while**

9: Return T

Algorithm 15 Hierarchial Agglomerative Algorithm 2 [., 2015c] Link:46,101,158

Input:

    N number of Data points

    d_c.Distance measure for two clusters

Output: Cluster hierarchy or dendogram of size N

1: **while** More than one cluster remains **do**

2:     Compute the proximity graph if necessary

3:     repeat

4:     Merge the closest two clusters.

5:     Update the proximity matrix to reflect the proximity between the new cluster and the original clusters.

6: **end while**

Algorithm 16 Hierarchial Divisive Algorithm [Stein, 2016b]

Input:
$\langle V, E, w \rangle$.Weighted graph
d_c.Distance measure for two clusters

Output: $\langle V_T, E_T \rangle$.Cluster hierarchy or dendogram

1: $C = \{V\}$ ▷ Initial Clustering
2: $V_t = \{v_C \mid C \in C\}, E_T = \emptyset$ ▷ Initial Dendogram
3: **while** $\exists C_x : (C_x \in C \wedge |C| > 1)$ **do**
4:      $update\_distance\_matrix(C, G, d_c)$
5:      $\{C, C'\} = \underset{\{C_i, C_j\}: C_i \cup C_j = C_x \wedge \ C_i \cap C_j = \emptyset}{argmax} d_c(C_i, C_j)$
6:      $C = (C \backslash \{C, C'\}) \cup \{C \cup C'\}$ ▷ Merging
7:      $V_T = V_T \cup \{v_{C,C'}\}, E_T = E_T \cup \{\{v_{C,C'}, v_C\}, \{v_{C,C'}, v_C\}\}$ ▷ Dendogram
8: **end while**
9: Return T

Algorithm 17 Hierarchial Divisive Algorithm [Stein, 2016b]

Input:
$\langle V, E, w \rangle$.Weighted graph
d_c.Distance measure for two clusters

Output: $\langle V_T, E_T \rangle$.Cluster hierarchy or dendogram

1: **while** More than one cluster remains **do**
2:      Create a new cluster by breaking the link corresponding to the largest distance (smallest similarity).
3: **end while**

Algorithm 18 C4.5 [Dai and Ji, 2014] Link :18,72,87,102,130,131,132,133,134,135 ,178

Input:
T : Training dataset
S : Attributes

Output: decision tree $Tree$

1: **function** C4.5(T)
2:      **if** T is $NULL$ **then**
3:          return failure
4:      **end if**
5:      **if** S is $NULL$ **then**
6:          return $Tree$ as a single node with most frequent class label in T
7:      **end if**
8:      **if** $|S| = 1$ **then**
9:          return $Tree$ as a single node S
10:      **end if**
11:      set $Tree = \{\}$
12:      **for** $a \in S$ **do**
13:          set $Info(a, T) = 0$ and $SplitInfo(a, T) = 0$
14:          compute $Entropy(a)$
15:          **for** $v \in values(a, T)$ **do**
16:              set $T_{a,v}$ as the subset of T with attribute $a = v$
17:              $Info(a, T) += \frac{|T_{a,v}|}{|T_a|} Entropy(a)$
18:              $SplitInfo(a, T) += -\frac{|T_{a,v}|}{|T_a|} \log \frac{|T_{a,v}|}{|T_a|}$
19:          **end for**
20:          $Gain(a, T) = Entropy(a) - Info(a, T)$
21:          $GainRatio(a, T) = \frac{Gain(a, T)}{SplitInfo(a, T)}$
22:      **end for**
23:      set $a_{best} = argmax\{GainRatio(a, T)\}$
24:      a_{best}into $Tree$
25:      **for** $v \in values(a_{best}, T)$ **do** call $C4.5(T_{a,v})$
26:      **end for**
27:      return $Tree$
28: **end function**

Algorithm 19 Gradient Descent [., 2015b] Link :35,106,142, 210

Input:
  f
  starting value x_1
  termination tolerances
Output: $x_{maxIters}$
1: **for** $i = 1$ to $maxIters$ **do**
2:     Compute the search direction $d_t = -\delta f(x_t)$
3:     **if** $|d_T| < \epsilon_g$ **then**
4:         return "Converged to critical point", output x_t
5:         Find α_t so that $f(x_t + \alpha_t d_t) < f(x_t)$
6:     **end if**
7:     **if** $|\alpha_t d_T| < \epsilon_x$ **then**
8:         return "Converged in x", output x_t
9:         Find α_t so that $f(x_t + \alpha_t d_t) < f(x_t)$
10:     **end if**
11:     Let $x_{t+1} = x_t + \alpha_t d_t$
12: **end for**
13: Return "Max number of iterations reached", output $x_{maxIters}$

Algorithm 20 Naive Bayes [., e] Link:6,7,8, 9, 57,88,89,90,102,157,166,167,168,178,179,185,187

Input:
  C : A fixed set of classes
  D : Documents
Output: Category(Class) of the Documents
1: **function** TRAINMULTINOMIALNB(C, D)
2:     $V \leftarrow EXTRACTVOCABULARY(D)$
3:     $N \leftarrow COUNTDOCS(D)$
4:     **for** each $c \in C$ **do**
5:         $N_c \leftarrow COUNTDOCSINCLASS(D, c)$
6:         $prior|c| \leftarrow N_c/N$
7:         $text_c \leftarrow CONCATENATETEXTOFALLDOCSINCLASS(D, C)$
8:         **for** each $t \in V$ **do**
9:             $condprob|t||c| \leftarrow \frac{T_{ct}+1}{\sum_{t'}(T_{ct'}+1)}$
10:         **end for**
11:     **end for**
12:     return $V, prior, condprob$
13: **end function**
14: **function** APPLYMULTINOMIALNB$(C, D, prior, condprob, d)$
15:     $W \leftarrow EXTRACTTOKENSFROMDOC(V, d)$
16:     **for** each $c \in C$ **do**
17:         $score|c| \leftarrow \log prior|c|$
18:         **for** each $t \in W$ **do**
19:             $score|c|+ = \log condprob|t||c|$
20:         **end for**
21:     **end for**
22:     return $arg\ max_{c \in C} score|c|$
23: **end function**

Algorithm 21 Lasso Regression [Wang et al.,] Link:52,61,209,214

Input:
    ipy : Inner product vector, $ipy_i = <y, X_{\cdot i}>$
    ipx : Inner product matrix, $ipx_{ij} = <X_{\cdot i}, X_{\cdot j}>$
    λ : Penalty parameter
    N : Number of samples
Output: $beta$: Regression parameter vector

1: **function** FASTLASSO(ipy, ipx, λ, N)
2:    stop_thr   ▷ Threshold for stopping iteration
3:    $p \leftarrow length(ipy)$
4:    $beta \leftarrow 0$ with length p
5:    $gc \leftarrow 0$ with length p
6:    **while** $difBeta_{max} \geq$ stop_thr **do**
7:        $difBeta_{max} \leftarrow 0$
8:        **for** $j = 1 \leftarrow p$ **do**
9:            $z \leftarrow (ipy|j| - gc|j|)/N + beta|j|$
10:           beta_tmp $\leftarrow max(0, z - \lambda) - max(0, -z - \lambda)$
11:           $difBeta \leftarrow$ beta_tmp $- beta|j|$
12:           $difabs \leftarrow abs(difBeta)$
13:           **if** $difabs > 0$ **then**
14:               $beta|j| \leftarrow$ beta_tmp
15:               $gc \leftarrow gc + ipx|j| \times difBeta$
16:               $difBeta_{max} = max(difBeta_{max}, difabs)$
17:           **end if**
18:        **end for**
19:    **end while**
20: **end function**

Algorithm 22 Bagging [., a] Link:43,105

Input:
    B: the number of bags or base hypotheses
    L: Base Learning Algorithm
Output: New Training Sets

1: **function** BAGGING($examples, B, L$)
2:    **for** $i = 1$ to B **do**
3:        $examples_i \leftarrow$ a bootstrap sample of $examples$
4:    **end for**
5:    Compute set I containing indices for the k smallest distances $d(X_i, x)$
6:    $h_i \leftarrow$ apply L to $examples_i$
7:    Return $h_1, h_2, ...h_B$
8: **end function**

Algorithm 23 Deep Q-Learning with Experience Replay [Mnih et al., 2013] Link: 42, 81, 155 ,156,177

Input:
    D: data set
    Q: Action-Value Function
Output: Actions that maximize the total reward
1: **for** $i = 1$ to M **do**
2:     Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi = \phi(s_1)$
3:     **for** $i = 1$ to T **do**
4:         With probability ϵ select a random action a_t otherwise select $a_t = max_a Q * (\phi(s_t).a : \theta)$
5:         Execute action a_t in emulator and observe reward r and image x_{t+1}
6:         Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
7:         Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
8:         Set $y_j =$

$$\begin{cases} r_j, & \text{for terminal } \phi_{j+1} \quad\quad\quad\quad (5) \\ r_j + \gamma max_{a'} Q(\phi_{j+1,a';\theta}), & \text{for terminal } \phi_{j+1} \quad\quad\quad\quad (6) \end{cases}$$

9:         Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to the following equation
10:

$$\Delta_\theta L_i(\theta_i) = \mathbb{E}_{s,a\sim\rho(.);s'\sim\epsilon}[(r+\gamma max_{a'} Q(s',a';\theta_{i-1})-Q(s,a;\theta_i))\Delta_{\theta_i} Q(s,a;\theta_i)]$$

11:     **end for**
12: **end for**

Algorithm 24 PageRank [., f] Link:20,109,203,204

Input:
    G: inlink file
    $iteration$: Number of iteration
Output: PageRank
1: **function** PAGERANK($G, iteration$)
2:     $d \leftarrow 0.85$ ▷ damping factor: 0.85
3:     $oh \leftarrow G$ ▷ get outlink hash from G
4:     $ih \leftarrow G$ ▷ get inlink hash from G
5:     $N \leftarrow G$ ▷ get number of pages from G
6:     **for** all p in the graph **do**
7:         $opg[p] \leftarrow \frac{1}{N}$
8:     **end for**
9:     **while** $iteration > 0$ **do**
10:         $dp \leftarrow 0$
11:         **for** all p that has no out-links **do**
12:             $dp \leftarrow dp + d * \frac{opg[p]}{N}$
13:         **end for**
14:         **for** all p in the graph **do**
15:             $npg[p] \leftarrow dp + \frac{[1-d]}{N}$
16:             **for** all ip in $ih[p]$ **do**
17:                 $npg[p] \leftarrow dp + \frac{d*opg[ip]}{oh[ip]}$
18:             **end for**
19:         **end for**
20:         $opg \leftarrow npg$
21:         $iteration \leftarrow iteration - 1$
22:     **end while**
23: **end function**

Algorithm 25 K means [., b] Link:48,59,139,187,205

Input:

    $E = \{e_1, e_2, ...e_k\}$

    k(number of clusters)

    $MaxIters$(limit of iterations)

Output:

    $C = \{c_1, c_2, ...c_k\}$

    $L = \{l(e)|e = 1, 2, ...n\}$ (set of cluster labels of E)

1: **for** each $c_i \in C$ **do**
2:     $c_i \leftarrow e_j \in E$(e.g random selection)
3: **end for**
4: **for** each $e_i \in E$ **do**
5:     $l(e_i) \leftarrow argminDistance(e_i, c_j)j \in \{1..k\}$
6: **end for**
7: $changed \leftarrow false$
8: $iter \leftarrow 0$
9: **while** $changed = true and iter <= MaxIters$ **do**
10:     **for** $c_i \in C$ **do**
11:         $UpdateCluster(c_i)$
12:     **end for**
13:     **for** $c_i \in E$ **do**
14:         $minDist \leftarrow argminDistance(e_i, e_j)j \in \{1..k\}$
15:         **if** $minDist \neq l(e_i)$ **then**
16:             $l(e_i) \leftarrow minDist$
17:             $changed \leftarrow true$
18:         **end if**
19:     **end for**
20:     $iter$**++**
21: **end while**

Algorithm 26 K means [Hamerly,] Link:48,59,139,187,205

Input:

    k the number of centers

    X the n data points in d dimensions

    initial location of the centers $C = \{c_j\}$

Output: C The final clusters

1: **while** the any c_j change location **do**
2:     **for** $i \in \{1...n\}$ **do**
3:         $class(x_i) \leftarrow arg\ min_j||x_i - c_j||$
4:     **end for**
5:     **for** $j \in \{1...k\}$ **do** $c_j \leftarrow \frac{\sum_i I(class(x_i)=j)x_i}{\sum_i I(class(x_i)=j)}$
6:     **end for**
7: **end while**
8: return C

Algorithm 27 K medoids [Hamerly,]

Input:

    D (Pairwise distance matrix which describe the pairwise distances for the datapoints being clustered)

    $n \times n$ matrix of distances between all pairs of datapoints

    the number of medoids desired k

    M The initial medoids

Output: M The final clusters

1: **while** any medoids m_j changes **do**
2:     **for** $i \in \{1...n\}$ **do**
3:         $class(x_i) \leftarrow arg\ min_j D_{ij}$
4:     **end for**
5:     **for** $j \in \{1...|M|\}$ **do** $m_j \leftarrow arg\ min_i I(class(i) = j)D_{ij}$
6:     **end for**
7: **end while**

Algorithm 28 DBSCAN link:13,156,161,182 [Ram et al., 2010]

Input:
    D: Data
    ϵ:Threshold distance
    $MinPts$: Minimum number of points required to form a cluster
Output: Clustered Data
1: **function** DBSCAN($D, \epsilon, minPts$)
2:    $C = 0$
3:    **for** each point P in dataset D **do**
4:        **if** P is visited **then**
5:            continue next point
6:        **end if**
7:        mark P as visited
8:        $NeighborPts = regionQuery(P, \epsilon)$
9:        **if** $sizeof(NeighborPts) < MinPts$ **then**
10:           mark P as NOISE
11:        **else**
12:           C = next cluster
13:           $expandCluster(P, NeighborPts, C, \epsilon, MinPts)$
14:        **end if**
15: **end for**
end function
function EXPANDCLUSTER($P, NeighborPts, C, \epsilon, MinPts$)
18: add P to Cluster C
19: **for** each point $P' in NeighborPts$ **do**
20:    **if** P' is not visited **then**
21:        mark P' as visited
22:        $NeighborPts' = regionQuery(P', eps)$
23:        **if** $sizeof(NeighborPts) >= MinPts$ **then**
24:           $NeighborPts = NeighborPts$ joined with $NeighborPts'$
25:        **end if**
26:    **end if**
27:    **if** P' is not yet member of any cluster **then**
28:        add P' to cluster C
29:    **end if**
30: **end for**
end function
function REGIONQUERY(P, ϵ)
33: return all points within $P's\ \epsilon\ neighborhood$
end function

Algorithm 29 Principle Component Analysis [., 2014] Link:41,69,72

Input:
    $x_1, ... x_n$ d length vector k
Output: Transform matrix R
1: $X \leftarrow n \times d$ data matrix with x_i in each row
2: $x \leftarrow \frac{1}{n} \sum_{i=1}^{n} x_i$
3: $X \leftarrow$ subtract x from each row $x_i in X$
4: $COV \leftarrow \frac{1}{n-1} X^T \times X$ Compute eigenvalue $e_1, ..., e_d$ of COV, and sort them
5: Compute matrix V which satisfy $V^{-1} \times COV \times V = D$, D is the diagonal matrix of eigenvalue of COV
6: $R \leftarrow$ the first k column of V

Algorithm 30 Logistic Regression [., d] link:10,11,109,110,115,180

Input:

Training data of the form $\{(x_1, 1), (x_2, 0), ..\}$

x: unknown sample

Output: The output is a probability that the given input point belongs to a certain class

1: $0 \leftarrow \beta$

2: Compute y by setting its elements to

$$y = \begin{cases} 1, & \text{if } g_i = 1 \\ 0, & \text{if } g_i = 2 \end{cases} \tag{7} \tag{8}$$

i = 1,2,..N

3: Compute p by setting its elements to

$$p(x_i, \beta) = \frac{e^{\beta^T x_i}}{1 + e^{\beta^T x_i}}$$

i = 1,2,..N

4: Compute the diagonal matrix W. The ith diagonal element is $p(x_i, \beta)(1 - p(x_i; \beta))$

5: $z \leftarrow X\beta + W^{-1}(y - p)$

6: $\beta \leftarrow (X^T W X)^{-1} X^T W z$

7: If the stopping criteria,stop;otherwise go back to step 3

Algorithm 31 Gaussian Process [., c] Link:78, 122

Input:

$X = \begin{bmatrix} x_1^T \\ \dots \\ x_n^T \end{bmatrix} \in \mathbb{R}^{n \times D}, \text{m training inputs}$

$y = \begin{bmatrix} y_1^T \\ .. \\ y_n^T \end{bmatrix} \in \mathbb{R}^n$

$k(.,.) : \mathbb{R}^{D \times D}$

x_*test input

σ^2noise level on the observations

$$[y(x) = f(x) + \epsilon, \epsilon \sim N(0, \sigma^2)]$$

Output:

f_*

$cov(f_*)$

1: $K \in \mathbb{R}^{n \times n}$Gram matrix.$K_{ij} = k(x_i, x_j)$

$$k(x_*) = k_* = k(X, x_*) = \begin{bmatrix} k(x_1, x_*) \\ \dots\dots\dots \\ k(x_n, x_*) \end{bmatrix} \in \mathbb{R}$$
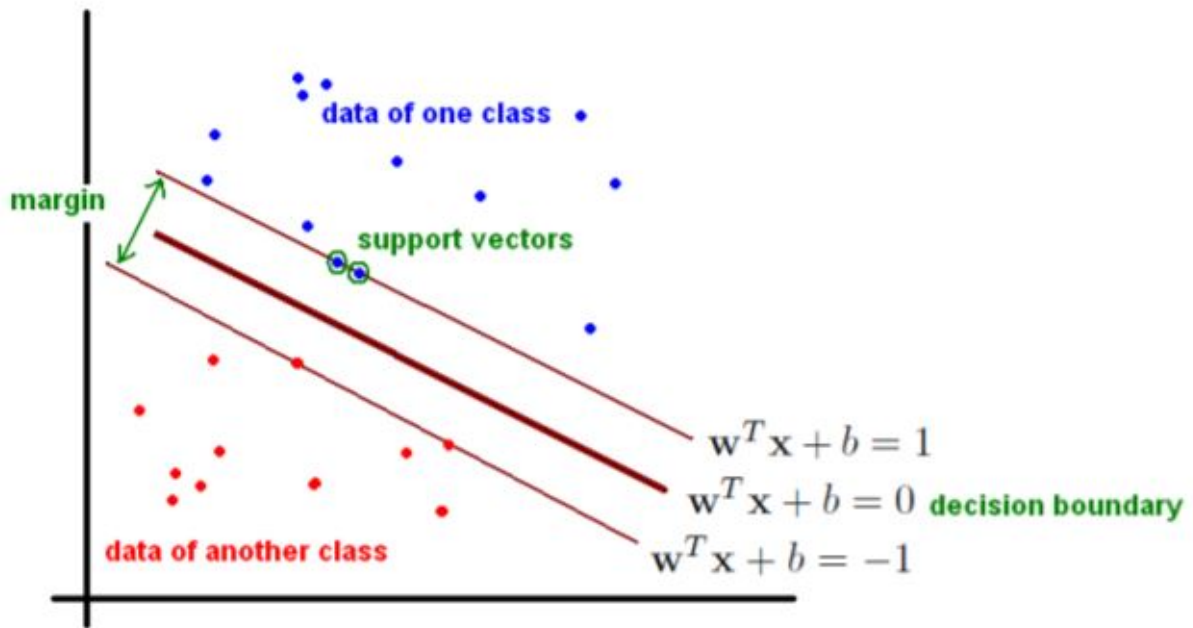
2: $\alpha = (K + \sigma_2 \mathbb{I}_n)^{-1} y$

3: $f_* = k_*^T \alpha \in \mathbb{R}$

4: $cov(f_*) = k(x_*, x_*) - k_*^T [K + \sigma_2 \mathbb{I}n]^{-1} k_*$

Algorithm 32 Support Vector Machines [Gavrilov,] Link : 39,70

Input:
Set of N input-output pairs $\{x, y\}^{N_1}$ x: input vectors of the same dimension and y : set of output target labels $y_i = \{0, 1\}$

1 Simple case : linearly-separable data,binary classification

Goal:we want to find the hyperplane(i.e. decision boundary)linearly separating our classes. Our boundary will have the equation: $\mathbf{w^T}\mathbf{x} + \mathbf{b} = 0$

Anything above the decision boundary should have label 1 i.e., $\mathbf{w^T}\mathbf{x}_i + b > 0$ will have corresponding $y_i = 1$

Similarly, anything below the decision boundary should have label -1 i.e. $\mathbf{w^T}\mathbf{x}_i + b < 0$ will have corresponding $y_i = -1$

The reason for this labeling scheme is that it lets us condense for the decision function to

$$f(x) = sign(\mathbf{w^T} + b)$$

since $f(x) = +1$ for all x above the boundary , and $f(x) = -1$ for all x below the boundary.

Thus, we can figure out if an instance has been classified properly by checking that $y(\mathbf{w^T} + b) \geq 1$ (which will be the case as long as either both $y, \mathbf{w^T} + b > 0$ or else $y, \mathbf{w^T} + b < 0$)

You'll notice that we will now have some space between our decision boundary and the nearest data points of either class. Thus,let's rescale the data such that anything on or above the boundary $\mathbf{w^T}\mathbf{x} + b = 1$ is of one class(with label 1), and anything on or below the boundary $\mathbf{w^T}\mathbf{x} + b = -1$ is of another class (with label -1)

What is the distance between these newly added boundaries?

First note that the two lines are parallel,and thus share their parameters w, b.Pick an arbitary point x_1 to lie on line $\mathbf{w^T}\mathbf{x} + b = -1$. Then the closest point on line $\mathbf{w^T}\mathbf{x} + b = 1$ is the point $\mathbf{x}_2 = \mathbf{x}_1 + \lambda\mathbf{w}$(since the closest point will always lie on the perpendicular;recall that the vector $\mathbf{w}$ is perpendicular to both lines). Using this formulation, $\lambda\mathbf{w}$ will be the line segment segment connecting $\mathbf{x}_1$ and $\mathbf{x}_2$, and thus, $\lambda \|\mathbf{w}\|$, the distance between $\mathbf{x}_1$ and $\mathbf{x}_2$ is the shortest distance between the two lines/boundaries.

References

[., a] . (.a). Bagging.

[., b] . (.b). Data mining algorithms in r/clustering/k-means.

[., c] . (.c). Gaussian process.

[., d] . (.d). Logistic regression.

[., e] . (.e). Naive bayes.

[., f] . (.f). Page rank.

[., g] . (.g). Self-organizing maps.

[., 2014] . (2014). Principal component analysis.

[., 2015a] . (2015a). Decision trees.

[., 2015b] . (2015b). Gradient descent.

[., 2015c] . (2015c). Hierarchical.

[., 2017] . (2017). Introduction to machine learning - online learning and mistake bounds.

[Amini, 2015] Amini, M. (2015). Perceptron.

[Bernstein, 2016] Bernstein, M. (2016). Random forests.

[Brownlee, 2015a] Brownlee, J. (2015a). Back-propagation.

[Brownlee, 2015b] Brownlee, J. (2015b). Clever algorithms: Nature-inspired programming recipes.

[Brownlee, 2015c] Brownlee, J. (2015c). Learning vector quantization.

[Brownlee, 2015d] Brownlee, J. (2015d). Perceptron.

[Dai and Ji, 2014] Dai, W. and Ji, W. (2014). A mapreduce implementation of c4.5 decision tree algorithm.

[Gavrilov,] Gavrilov, Z. (.). Support vector machines.

[Hamerly,] Hamerly, G. J. (.). Learning structure and concepts in data through data clustering.

[Hertzmann et al., 2015] Hertzmann, A., Fleet, D., and Brubaker, M. (2015). Adaboost.

[Jurafsky and Martin, 2008] Jurafsky, D. and Martin, J. H. (2008). *Hidden Markov Models*. Prentice Hall, 2nd edition.

[Mnih et al., 2013] Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., and Riedmiller, M. A. (2013). Playing atari with deep reinforcement learning. *CoRR*, abs/1312.5602.

[Ng,] Ng, A. Coursera machine learning.

[Ram et al., 2010] Ram, A., Jalal, S., Jalal, A. S., and Kumar, M. (2010). A density based algorithm for discovering density varied clusters in large spatial databases. *International Journal of Computer Applications*, 3(6):1–4.

[Russell and Norvig, 2016] Russell, S. J. and Norvig, P. (2016). *Artificial intelligence: a modern approach*. Pearson, 3rd edition.

[Schapire, 2014] Schapire, R. (2014). Machine learning algorithms for classification.

[Stein, 2016a] Stein, B. (2016a). Unit hierarchial cluster analysis.

[Stein, 2016b] Stein, B. (2016b). Unit hierarchial cluster analysis.

[Tay et al., 2014] Tay, B., Hyun, J., and Sejong, O. (2014). A machine learning approach for specification of spinal cord injuries using fractional anisotropy values obtained from diffusion tensor images.

[Wang et al.,] Wang, T., Ren, Z., Ding, Y., Fang, Z., Sun, Z., MacDonald, M. L., Sweet, R. A., Wang, J., and Chen, W. Fastggm: An efficient algorithm for the inference of gaussian graphical model in biological networks.

[Wolpert, 1992] Wolpert, D. H. (1992). Stacked generalization. *Neural Networks*, 5:241–259.

Algorithm 32 Support Vector Machines (continued)

Solving for λ : $\mathbf{w^T}\mathbf{x}_2 + b = 1$ where $\mathbf{x}_2 = \mathbf{x}_1 + \lambda\mathbf{w}$

$\mathbf{w^T}(\mathbf{x}_1\lambda\mathbf{w}) + b = 1$ $\mathbf{w^T}(\mathbf{x}_1\lambda\mathbf{w}) + b = 1$

$\mathbf{w^T}\mathbf{x}_1 + b + \lambda\mathbf{w^T}\mathbf{w} = 1$ where $\mathbf{w^T}\mathbf{x}_1 + b = -1$

$-1 + \lambda\mathbf{w^T}\mathbf{w} = 1$

$\lambda\mathbf{w^T}\mathbf{w} = 2$

$\lambda = \frac{2}{\mathbf{w^T}\mathbf{w}} = \frac{2}{\|\mathbf{w}\|^2}$

And, so the distance $\lambda\|w\| is \frac{2}{\|\mathbf{w}\|^2}\|w\| = \frac{2}{\|w\|} = \frac{2}{\sqrt{\mathbf{w^T}\mathbf{w}}}$

It's intuitive that we would want to maximize the distance between the two boundaries demarcating the classes(Why?We want to be as sure that we are not making classification mistakes and thus we want our data points from the two classes to lie as far away from each other as possible).This distance is called the margin,so went to obtain the maximal margin.

Thus, we want to maximize $\frac{2}{\sqrt{\mathbf{w^T}\mathbf{w}}}$, which is equivalent to minimizing $\frac{\sqrt{\mathbf{w^T}\mathbf{w}}}{2}$ which is in turn equivalent to minimizing $\frac{\mathbf{w^T}\mathbf{w}}{2}$(since square root is a monotonic function)

This quadratic programming problem is expressed as :

$min_{w,b} \frac{\mathbf{w^T}\mathbf{w}}{2}$

subject to : $y_i(\mathbf{w}^T\mathbf{x} + b) \geq 1(\forall$ data points $\mathbf{x}_i)$

2 Soft-margin extension

Consider the case that your data isn't linearly separable.For instance, maybe you aren't guaranteed that all your data points are correctly labelled, so you want to allow some data points of one class to appear on the other side of the boundary.

We can introduce $slack variables$ an $- \epsilon_i \geq 0$. Our quadratic programming problem becomes:

$min_{\mathbf{w},b,\epsilon} \frac{\mathbf{w^T}\mathbf{w}}{2} + C\sum_i \epsilon_i$

subject to : $y_i(\mathbf{w^T}\mathbf{x}_i + b) \geq 1 - \epsilon$

3 Nonlinear decision boundary

Mapping your data vectors, x_i,into a higher-dimension(even infinite) feature space may make them linearly separable in that space (whereas they may not be linearly separable in the original space). The formation of the quadratic programmatic problem is as above,but with all $\mathbf{x}_i$ replaced with $\phi(\mathbf{x}_i)$, where ϕprovides the higher-dimensional mapping.So we have the standard SVM formulation:

$min_{\mathbf{w},b,\epsilon} \frac{\mathbf{w}^T\mathbf{w}}{2}$

subject to : $y_i(\mathbf{w}^T\phi(\mathbf{x}_i) + b) \geq 1 - \epsilon$ and $\epsilon_i \geq 0(\forall$ data points $\mathbf{x}_i)$

4 Reformulating as a Lagrangian

We can introduce Lagrange multipliers to represent the condition:

$y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b))$ must be as close to 1 as possible. This condition is captured by: $max_{ai\geq0}\alpha_i[1 - y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b))]$ This ensures that when $y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b)) \geq 1$, the expression above is maximal when $\alpha_i = 0$(since [1-$y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b)]$ ends up being negative). Otherwise, $y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b)) < 1$, so $[1 - y_i(\mathbf{w}^T\phi(\mathbf{x}_i + b))]$ is a positive value, and the expression is maximal when $a_i \rightarrow \infty$. This has the effect of penalizing any misclassified data points, while assigning 0 penalty to properly classified instances.

We thus have the following formulation:

$min_{w,b}[\frac{\mathbf{w}^T\mathbf{w}}{2} + \sum_i max_{\alpha\geq0}\alpha_i[1 - \mathbf{w}^T\phi(\mathbf{x}_i + b)]]$

To allow for slack(soft-margin), preventing the α variables from going to ∞, we can impose constraints on the Lagrange multipliers to lie within: $0 \leq \alpha_i \leq C$. We can define the dual problem by interchanging the max and min as follows (i.e minimize after fixing alpha):

$max_{alpha\geq zero}[min_{w,b}J(\mathbf{w}, b; \alpha)] where = \frac{\mathbf{w}^T\mathbf{w}}{2} + \sum_i \alpha_i[1 - y_i(\mathbf{w}^T\phi(x_i) + b)]$

Algorithm 32 Support Vector Machines (continued)

Since, we're solving an optimization problem, we set $\frac{\partial J}{\partial \mathbf{w}} = 0$ and discover that the optimal setting of $\mathbf{w}$ is $\sum_i \alpha_i y_i \phi(x_i)$, while seeing $\frac{\partial J}{\partial \mathbf{b}} = 0$ yields the constraint $\sum_i \alpha_i y_i = 0$

Thus, after substituting and simplifying, we get:
$min_{w,b} J(\mathbf{w}, b, \alpha) = \sum_i \alpha_i \alpha_j y_i y_j \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ And thus our dual is:
$max_{\alpha \geq 0} [\sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_j y_i y_j \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)]$
Subject to: $\sum_i \alpha_i y_i = 0$ and $0 \leq \alpha_i \leq C$

5 Kernel trick

Because we're working in a higher-dimension space(and potentially even an infinite-dimensional space), calculating $\phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ may be intractable. However, it turns out there are special *kernel* functions that operate on the lower dimension vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ to produce a value equivalent to the dot-product of the higher dimensional vectors. For instance, consider the function $\phi \colon \mathbb{R}^3 \longmapsto \mathbb{R}^{10}, where :$
$\phi(x) = (1, \sqrt{2}x^{(1)}, \sqrt{2}x^{(2)}, \sqrt{2}x^{(3)}, [\mathbf{x}^{(1)}]^2, [\mathbf{x}^{(2)}]^2, [\mathbf{x}^{(3)}]^2, \sqrt{2}x^{(1)(2)}, \sqrt{2}x^{(1)(3)}, \sqrt{2}x^{(2)(3)})$ Instead, we have the the kernel trick, which tells us that $K(\mathbf{x}_i, \mathbf{x}_j) = (1 + \mathbf{x}_i^T \mathbf{x}_j)^2 = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ for the given ϕ. Thus, we can simplify our calculations. Re-writing the dual in terms of the kernel yields: $max_{\alpha \geq 0} [\sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j]$

6 Decision function

To classify a novel instance $\mathbf{x}$ once you've learned the optimal α_i parameters, all you have to do is calculate $f(\mathbf{x}) = sign(\mathbf{w}^T \mathbf{x} + b) = \sum_i \alpha_i y_i \phi(\mathbf{x}_i$ and using the kernel trick). Note that α_i is only non-zero for instances $\phi(\mathbf{x}_i)$ on or near the boundary-those are called the *support vector* since they alone specify the decision boundary. We can toss out the other data points once training is complete. Thus, we only sum over the x_i which constitute the support vectors.

Algorithm 33 Stacking [Wolpert, 1992] link : 95,104

Input:
  A training set
Output: the final output of the combined learners
 1: Split the training set into two disjoint sets
 2: Train several base learners on the first part
 3: Test the base learners on the second part.
 4: Using the predictions from 3) as the inputs, and the correct responses as the outputs, train a higher level learner.

Algorithm 34 Winnow [., 2017] Link :96,97,153

Input:
  A training set
  d : The Number of attributes
Output: Weights(Classification of the entire training set)
 1: $\theta \leftarrow \frac{d}{2}$
 2: $w \leftarrow (1, 1, ...1)$
 3: **for** $i = 1, 2, ...$ **do**
 4:     **if** $w^T x^{(i)} > \theta$ **then**
 5:         $c_*(x^{(i)}) = 1$
 6:     **else**
 7:         $c_*(x^{(i)}) = 0$
 8:     **end if**
 9:     **if** $c_*(x^{(i)}) \neq c(x^{(i)})$ **then**
10:         $\forall : x_j^{(i)} = 1, w_j \leftarrow 2w_j$
11:     **else**
12:         $\forall : x_j^{(i)} = 1, w_j \leftarrow 0$
13:     **end if**
14: **end for**

Algorithm 35 Hidden Markov Models - The forward algorithm [Jurafsky and Martin, 2008]
Link:27,28,89,90,91,98

Input:
   observations of length T, state-graph of length N
Output: forward-probability
 1: create a probability matrix $forward[N+2,T]$
 2: **for** state s from 1 *to* N **do**
 3:     $forward[s,1] \leftarrow a_{o,s} \times b_s(o_1)$
 4: **end for**
 5: **for** time step t from 2 *to* T **do**
 6:     **for** state s from 1 *to* N **do**
 7:         $forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s,1] \times a_{s',s} \times b_s(o_t)$
 8:     **end for**
 9: **end for**
10: $forward[q_F,T] \leftarrow \sum_{s=1}^{N} forward[s,t] \times a_{s,qF}$
11: return $forward[q_F,T]$

Algorithm 36 Hidden Markov Models - Viterbi algorithm [Jurafsky and Martin, 2008] Link:26,27,90,91,98

Input:
   observations of length T, state-graph of length N
Output: best-path
 1: create a path probability matrix $viterbi[N+2,T]$
 2: create a path probability matrix $backpointer[N+2,T]$
 3: **for** state s from 1 *to* N **do**
 4:     $viterbi[s,1] \leftarrow a_{o,s} \times b_s(o_1)$
 5:     $backpointer[s,1] \leftarrow 0$
 6: **end for**
 7: **for** time step t from 2 *to* T **do**
 8:     **for** state s from 1 *to* N **do**
 9:         $viterbi[s,t] \leftarrow max_{s'=1}^{N} viterbi[s',1] \times a_{s',s} \times b_s(o_t)$
10:         $backpointer[s,t] \leftarrow argmax_{s'=1}^{N} viterbi[s',1] \times a_{s',s}$
11:     **end for**
12: **end for**
13: $viterbi[q_F,T] \leftarrow max_{s=1}^{N} viterbi[s,t] \times a_{s,qF}$
14: $backpointer[q_F,T] \leftarrow argmax_{s=1}^{N} viterbi[s,1] \times a_{s,qF}$
15: return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

Algorithm 37 Hidden Markov Models - The Forward-Backward algorithm [Jurafsky and Martin, 2008] Link : 26

Input:
   observations of len T
   output vocabulary V
   hidden state set Q
Output:
   The forward and backward probabilities A and B
 1: **initialize** A and B
 2: **iterate** until convergence **E-step**
 3: $\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{\alpha_t(q_F)} \ \forall$ t and j
 4: $\xi_t(i,j) = \frac{\alpha_t(i)a_{ij}b_j(o_{t+1})\beta_{t+1}(j)}{\alpha_T(N)} \ \forall$ t, i, and j **M-step**
 5: $\hat{\alpha_{ij}} = \frac{\sum_{t=1}^{|T-1|}\xi_t(i,j)}{\sum_{t=1}^{|T-1|}\sum_{j=1}^{N}\epsilon_t(i,j)}$
 6: $\hat{\beta_j}(v_k) = \frac{\sum_{t=1 s.t. O_t=v_k}^{|T|}\gamma_t(j)}{\sum_{t=1}^{T}\gamma_t(j)}$
 7: return A,B

Algorithm 38 Genetic Algorithm [Russell and Norvig, 2016] link :38,39,141

Input:
    population , a set of individuals
    Fitness-FN, a function that measures the fitness of an individual
Output: the best-individual in population, according to FITNESS-FN

 1: **function** GENETIC ALGORITHM(*population*, FITNESS-FN)
 2:    **while** some individual is not fit enough or enough time has not elapsed **do**
 3:        *new_population* ← empty set
 4:        **for** $i = 1$ to SIZE(*population*) **do**
 5:            x ← RANDOM-SELECTION (*population*,FITNESS-FN)
 6:            y ← RANDOM-SELECTION (*population*,FITNESS-FN)
 7:            $child$ ← REPRODUCE(x, y)
 8:            **if** small random propbability **then**
 9:                $child$ ← MUTATE($child$)
10:                add child to *new_population*
11:            **end if**
12:            $population$ ← $new\_population$
13:        **end for**
14:        return the best individual in *population* , according to FITNESS-FN
15:        **function** REPRODUCE(x, y)
16:            n ← LENGTH
17:            c ← random number from 1 to n
18:            APPEND(SUBSTRING$(x, 1, c)$,SUBSTRING$(y, c + 1, n)$)
19:        **end function**
20:    **end while**
21: **end function**