**Exercise 1.** An *Eulerian circuit* of $G$ is a sequence of vertices $v_1v_2 \ldots v_\ell$ (a vertex may appear more than once) so that every edge $uw \in E(G)$ appears as $v_iv_{i+1}$ for some $i$ in the sequence, and so that $v_1 = v_\ell$. A Eulerian graph is one which has a Eulerian circuit.

A *Hamiltonian cycle* of graph $G$ on atleast three vertices is an sequence $v_1v_2 \ldots v_n$ such that each $u \in V(G)$ appears exactly once, $v_1 = v_2$ and each $v_iv_{i+1} \in E(G)$. A graph is Hamiltonian if it has a Hamiltonian cycle.

(a) Let $\mathcal{A}$ be the set of Eulerian graphs. Show that $\mathcal{A}$ is not monotone.

(b) Let $\mathcal{B}$ be the set of Hamiltonian graphs. Is $\mathcal{B}$ monotone?

**Exercise 2.** A graph $G$ with $n \geq 3$ vertices, denoted $C_n$, is a cycle if its vertices can be (re)-labelled $v_1, \ldots, v_n$ such that $E(G) = \{v_iv_{i+1} : i \in [n]\}$ where the subscript addition is taken modulo $n$. For example a cycle on 3 vertices is  and there are three cycles on four vertices , , .

A *connected graph* is one in which any two vertices $uv$ are connected by a sequence of vertices $v_1 \ldots v_\ell$ so that $u = v_1$, $v = v_\ell$ and each $v_iv_{i+1}$ is an edge. For example  is connected but  is not connected.

(a) A graph with $n$ vertices and $n$ edges must contain a cycle as a subgraph.

(b) A connected graph with $n$ vertices and $n$ edges must contain exactly one cycle.

(c) Give an example to show that the assumption of connectivity is needed for part b.

**Exercise 3.** Let $\mathcal{A}_\triangle$ be the set of all graphs which contain  as a subgraph.

(a) Show that $\mathbb{P}(G(n, 1/2) \in \mathcal{A}_\triangle) \to 1$.

(b) (optional) Fix a constant $0 < p \leq 1$, and show that $\mathbb{P}(G(n,p) \in \mathcal{A}_\triangle) \to 1$.

**Exercise 4.** Prove the following:

Let $X_1, X_2, \ldots$ be a sequence of random variables each taking non-negative values. If $\mathbb{E}[X_n] \to 0$ then

$$\mathbb{P}(X_n = 0) \to 1,$$

and if $\mathbb{E}[X_n] > 0$ for each $n$, and $\mathbb{V}[X_n]/\mathbb{E}[X_n] \to 0$ then

$$\mathbb{P}(X_n = 0) \to 0.$$