UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Assignment 2
Spring 2022

Lab 2: Networks.

The deadline for this sheet is midnight Wednesday 11th of May.

Please submit hand-ins on studium. All code should be included. Please feel free to submit videos illustrating your results where appropriate, via studium or uploaded elsewhere such as vimeo or youtube. You may work in groups of size 1-5, and only one group member needs to submit the assignment. State clearly the members of the group. This exercise will be covered in lab session on 28th of April.

5a. Robustness of networks

For this exercise all sections are to be done both for an example real world network and a random networks model of your choice. For this question it is enough to simulate the network and/or random deletion process multiple times (≥ 10) and take the average value.

What is the value of the (global) clustering coefficient after α proportion of edges have been deleted. By repeatedly simulating the model while systematically changing α plot the average size of the clustering coefficient in the graph as a function of α.

1. If edges deleted are chosen randomly
2. If edges are deleted by choosing the vertex of max degree then choosing a random edge incident with that vertex (breaking ties somehow - explain how).
3. Another node or edge deletion method of your choosing. If you delete vertices then set α to be proportion of the edges which have been deleted. Describe your chosen algorithm clearly.

(4 points)

5b. Random graphs as null models for networks

For this exercise you may consider either the (global) clustering coefficient, the maximum modularity or any other non-trivial measure of the network as the statistic of interest.

Find a real network examples and calculate the statistic (clustering coefficient, max modularity etc) of those networks. In the case of maximum modularity you can use a heuristic method such as Louvain algorithm or Leiden algorithm which outputs a modularity and a score, but is not necessarily the maximum modularity (it would be too slow to test all possible partitions).

Find a real network example and calculate the statistic of that network.

1. **Erdos-Renyi as a null model** Simulate an Erdős-Rényi random graph with the same expected number of edges as your example network. By repeated simulations, ($\geq$ 30), record the values of your statistic on the random graph. Illustrate this using a box and whisker plot and show on the same plot the value of your statistic on the real network.

   Discuss your findings in terms of the Monte Carlo method. Was your test statistic on the real-network greater then 95% of the values for the test statistic which you recorded for the random graphs? **(3 points)**

2. **Configuration Model as a null model.** Simulate a graph with the configuration model which has the same degree sequence as your example network. (You may choose to generate a multigraph with same degree sequence, or sample repeatedly keeping only simple graphs). By repeated simulations, ($\geq$ 30), record the values of your statistic on the random graph. Illustrate this using a box and whisker plot and show on the same plot the value of your statistic on the real network.

   Discuss your findings in terms of the Monte Carlo method. Was your test statistic on the real-network greater then 95% of the values for the test statistic which you recorded for the random graphs? **(3 points)**