UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Assignment 3
Spring 2020

Lab 2: Automata.

The deadline for this sheet is midnight Tuesday 5th of May.

Please submit hand-ins on Studentportalen. All code should be included. Please feel free to submit videos illustrating your results where appropriate, via Studentportalen or uploaded elsewhere. This exercise will be covered in lab session on Tuesday 13th of April.

3a. A firing brain

In this exercise you implement a two-dimensional cellular automata model on an N by N grid. Use periodic boundary conditions, so that cells interact over the left and right and the top and bottom boundaries.

1. Consider a model of a brain made up of a two dimensional array of neurons. The neurons can have one of three states: ready, firing or resting. On each time step,

   (a) A *ready* neuron *fires* on the next time step if has exactly two neighbors that are *firing*

   (b) A *firing* neuron goes to the *resting* state on the next time step.

   (c) A *resting* neuron goes to the *ready* state on the next time step.

   Simulate this model for a $N = 40$ grid where each cell has a probability 0.3 of initially being in the firing state and all other neurons are ready. For an example simulation, plot how the cells typically look after 10 time steps, after 20 time steps, after 100 time steps and after 1000 time steps. Simulate your model 100 times for different initial conditions and for 1000 time steps and plot the average number of firing cells over time. Describe what typically happens as the system moves in to the equilibrium state. **(3 points)**.

2. Give examples of shapes that

   (a) Move forward at a rate of one cell per time step, while preserving the same shape.

   (b) Move forward at a rate of one cell per time step, launching other shapes behind them.

(c) Move forward at a rate of less than one cell per time step, while returning to the same shape after some period.

(d) Stay stationary but oscillate periodically.

(2 points).

3b. Spatial Epidemics

Consider the following simple model of an epidemics spread. An individual is either infected or susceptible to a disease.

1. An infected individual recovers with probability γ and continues to be infected with probability $1 - \gamma$
2. Those individuals that do not recover can infect others. Each susceptible individual coming in contact with an infected individual becomes infected.
3. Already infected neighbours remain infected (i.e. you can't be 'double infected').

You are going to model the epidemic and gauge the affect of the parameter γ and of the initial conditions.

1. First assume space is a one dimensional grid of N = 100 discrete points, each corresponding to a single person. Start with one infected individual placed in the middle of this line, with everyone else is susceptible. We say that all people next to each other on the grid are in contact. Implement the above model and show how the population changes through time and space for $\gamma = 0.6$,$\gamma = 0.5$, $\gamma = 0.4$ and $\gamma = 0.3$. By repeatedly simulating the model while systematically changing γ plot the probability of the disease spreading as a function of γ. **(1 point)**
2. Now assume that there is a population of N = 100 individuals and initially each member of the population is infected with probability p. By repeatedly simulating the model while systematically changing both p and γ plot the probability of the disease dying out. **(1 point)**

Design your own spatial epidemic model on an $N \times N$ grid with periodic boundary conditions. It should have a probabilistic cellular automata rule as above but may have more states. You may take inspiration from the previous project and include states, susceptible, infected, recovered etc. Explain the rule carefully. Feel free to fix some parameters arbitrarily. Points will be awarded depending for the explanation of the automata, and investigation its behaviour. **(3 points)**