UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Assignment 5
Spring 2020

Lab 4: Network properties.

The deadline for this sheet is midnight Sunday 10th of May.

Please submit hand-ins on Studentportalen. All code should be included. Please feel free to submit videos illustrating your results where appropriate, via Studentportalen or uploaded elsewhere. You may work in groups of size 1-4, and only one group member needs to submit the assignment. State clearly the members of the group. This exercise will be covered in lab session on Friday 24th of April.

5a. Robustness of Networks

For this exercise all sections are to be done both for an example real world network and a random networks model of your choice. For this question it is enough to simulate the network and/or random deletion process multiple times (≥ 10) and take the average value.

What is the size of the maximum connected component after α proportion of nodes have been deleted. By repeatedly simulating the model while systematically changing α plot the probability of the disease spreading as a function of α.

1. If nodes deleted are chosen randomly
2. If nodes are deleted in order of highest degree to lowest degree (breaking ties somehow).
3. Another node or edge deletion method of your choosing. If you delete edges then set α to be proportion of the edges which have been deleted. Describe your chosen algorithm clearly.

(4 points)

5b. Random graphs as null models for networks

For this exercise you may consider either the (global) clustering coefficient, the maximum modularity or any other non-trivial measure of the network as the statistic of interest.

Find a real network examples and calculate the statistic (clustering coefficient, max modularity etc) of those networks. In the case of maximum modularity you can use a heuristic method such as Louvain algorithm or Leiden algorithm which outputs a modularity and a score, but is not necessarily the maximum modularity (it would be too slow to test all possible partitions).

1. **Erdos-Renyi as a null model** Simulate an Erdős-Rényi random graph with the same expected number of edges as your example network. By repeated simulations, ($\geq$ 30), record the values of your statistic on the random graph. Illustrate this using a box and whisker plot and show on the same plot the value of your statistic on the real network.

   Was your test statistic on the real-network greater then 95% of the values for the test statistic which you recorded for the random graphs? **(3 points)**

2. **Configuration Model as a null model.** Simulate a graph with the configuration model which has the same degree sequence as your example network. (You may choose to generate a multigraph with same degree sequence, or sample repeatedly keeping only simple graphs). By repeated simulations, ($\geq$ 30), record the values of your statistic on the random graph. Illustrate this using a box and whisker plot and show on the same plot the value of your statistic on the real network.

   Was your test statistic on the real-network greater then 95% of the values for the test statistic which you recorded for the random graphs? **(3 points)**