UPPSALA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Resit version
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Lab 2: Network Properties.

The deadline for this sheet is midnight Monday 22nd of August.

Please submit hand-ins on studium. All code should be submitted also or uploaded to github. Please feel free to submit videos illustrating your results where appropriate, via studium or uploaded elsewhere such as vimeo or youtube. You may work in groups of size 1-5, and only one group member needs to submit the assignment. State clearly the members of the group.

3 Network Epidemics

1. Create a random undirected social network with 5000 individuals and a link density of 0.0016. That is, every pair of individuals has a 0.16% chance of being linked to each other. Plot a histogram of the degree distribution. What kind of distribution is this? What is the average degree of this network? **(2 points)**

2. Simulate the following process on the network. Start with 100 random infected individuals. Every day, an individual connected to n infected individuals becomes infected with probability

$$P_{\text{infected}}(n) = 1 - e^{(-pn)}$$

per day, where p is a constant. An infected individual recovers with constant probability $r = 0.03$ per day. Recovered individuals can become infected again. Plot the number of infected individuals over the first 1000 days of the epidemic, using $p = 0.01$. Then run the simulation for $p = 0.001, 0.002, \ldots, 0.01$. Plot the number of infected individuals against time for each of these values on the same graph. Finally, plot the number of infected individuals on day 1000 against r/p for as you vary p. **(4 points)**

3. Now create a social network using preferential attachment. Start with $n = 2$ individuals linked together. Now add individuals, $n > 2$, to the network one after another. Every new person initially links to just one other randomly chosen individual, such that, the probability of the newly added individual linking to individual i is proportional to the degree (number of existing links) of individual i (denoted as d_i).

That is for the n-th node added, the probability of it linking to node i is equal to

$$\frac{d_i}{2(n-2)}$$

Try 100 people first to make sure your algorithm works, and then build a network of 5000 individuals. Plot a histogram of the degree distribution on a log-log scale. What kind of distribution is this? What is the average degree of the network? **(2 points)**

4. Repeat question 2 for this preferential attachment network. Make sure the initially infected individuals are chosen randomly. Describe how the equilibrium number of infected individuals depends on the infection probability p. How is this different to the random network? Discuss this result in the context of the spread of memes on the Internet. **(4 points)**