Modelling Complex Systems

Self-propelled particles I

This lecture is adapted from Vicsek, T. & Zafeiris, A. (2012) Collective Motion. And slides of David Sumpter

See: arXiv:1010.5017v2

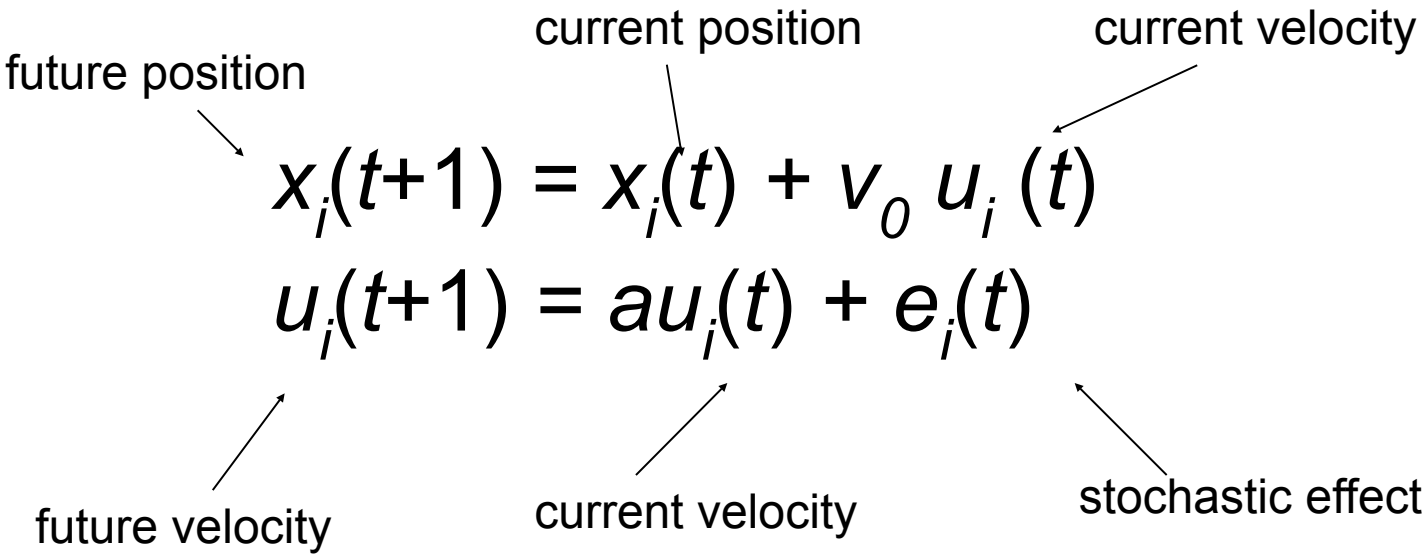
Why do animals move together?

- Increased accuracy (many estimates)
- Increased awareness (many eyes)
- Confuse predators and reduce encounters

How do animals move together?

- Group formation usually seems to be *spontaneous.*
- Based on local interactions
- Phenomenological models
- Can ignore 'first principles' physics!
  e.g. Conservation of momentum
- Use biological principles and limits instead.

Random walk in one dimension



$e_i(t)$ *is a* random number selected uniformly at random from a range $[-\eta/2, \eta/2]$

Attraction in one dimension

- Run ‘Aggregate1D’

future position

current position

current velocity

$$x_i(t+1) = x_i(t) + v_0\, u_i(t)$$

$$u_i(t+1) = a u_i(t) + (1-a)\, s_i(t) + e_i(t)$$

future velocity

current velocity

Direction to most neighbours

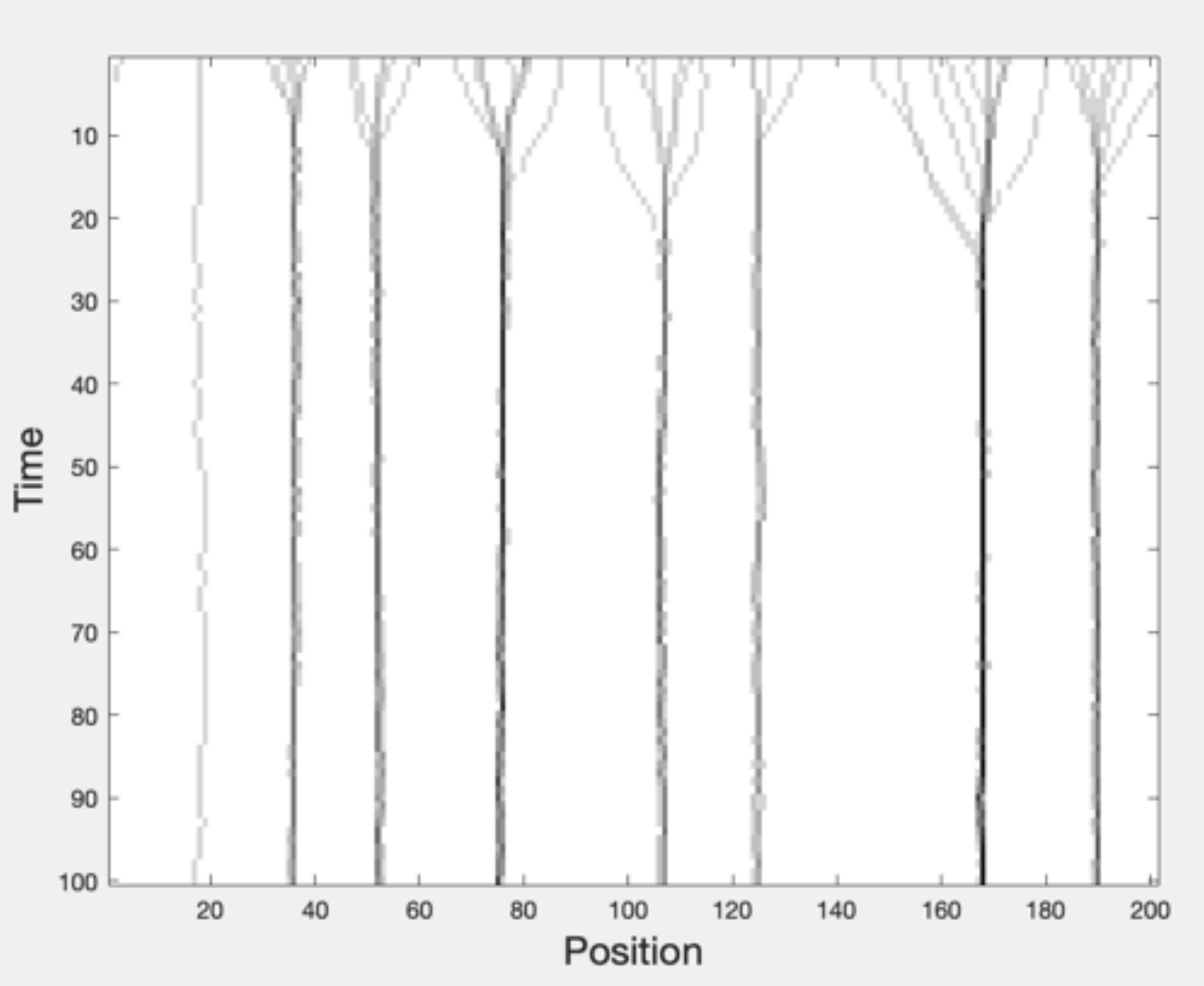
stochastic effect

$$s_i(t) = \frac{1}{|R_i|} \sum_{j \in R_i} \text{sign}\left(x_i(t) - x_j(t) \right)$$

$e_i(t)$ *is a* random number selected uniformly at random from a range $[-\eta/2, \eta/2]$

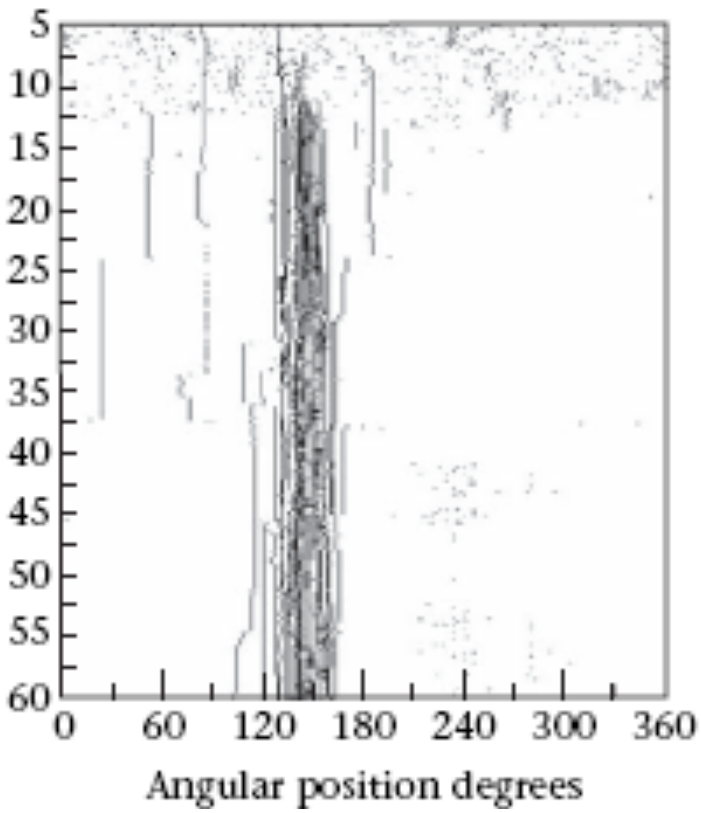
Attraction in one dimension

- Run ‘Aggregate1D’

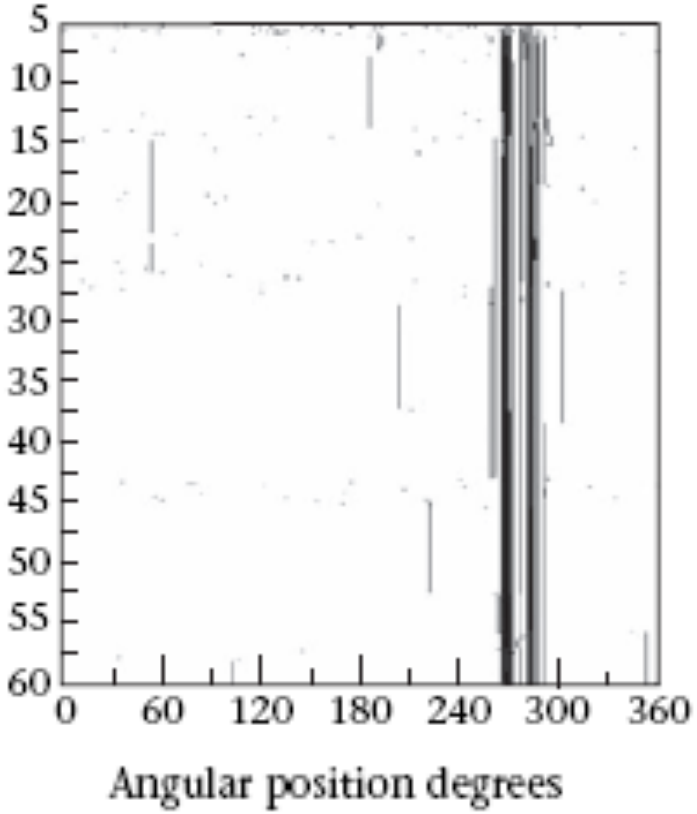


Cockroach aggregation

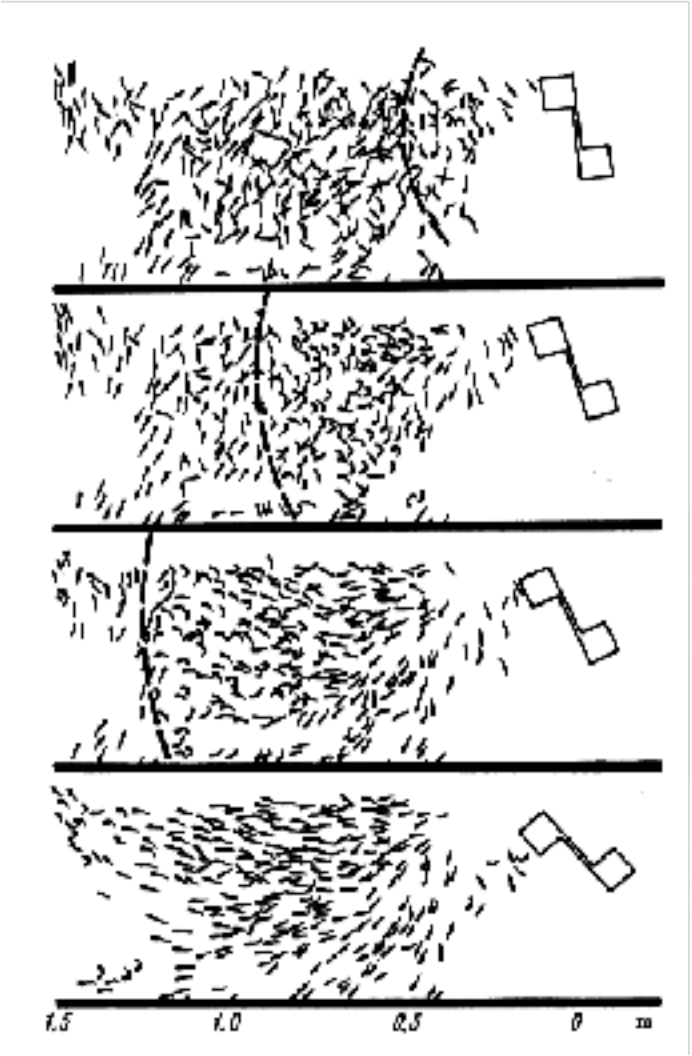
Cockroaches



Model



Radakov's fish



Alignment model in one dimension

- Run 'Align1D'

future position

current position

current velocity

$$x_i(t+1) = x_i(t) + v_0\, u_i(t)$$

$$u_i(t+1) = a u_i(t) + (1-a)\, s_i(t) + e_i(t)$$

future velocity

current velocity

velocity of neighbours

stochastic effect

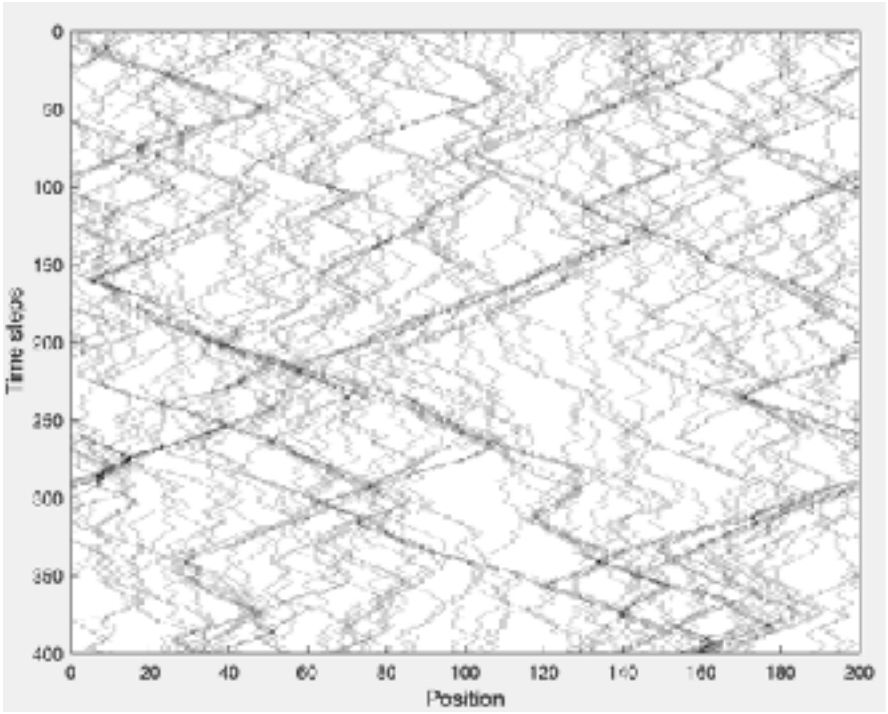
$$s_i = G\left(\frac{1}{|R_i|} \sum_{j \in R_i} u_j(t) \right)$$

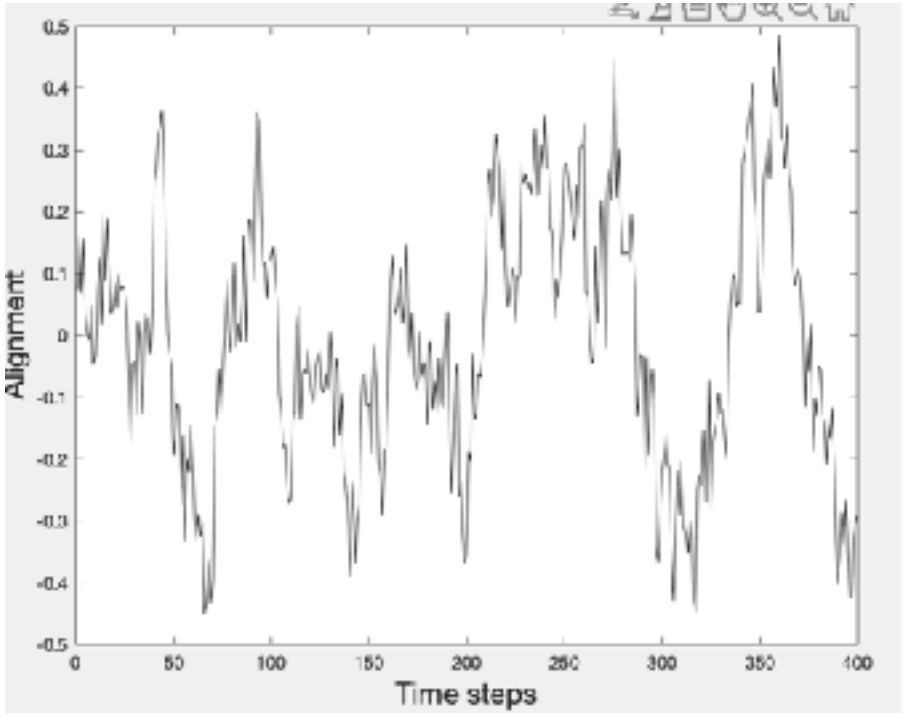
$$G(u) = \begin{cases} (u+1)/2 & for \quad u > 0 \\ (u-1)/2 & for \quad u < 0 \end{cases}$$

e is a random number selected uniformly at random from a range $\left[-\eta/2, \eta/2\right]$

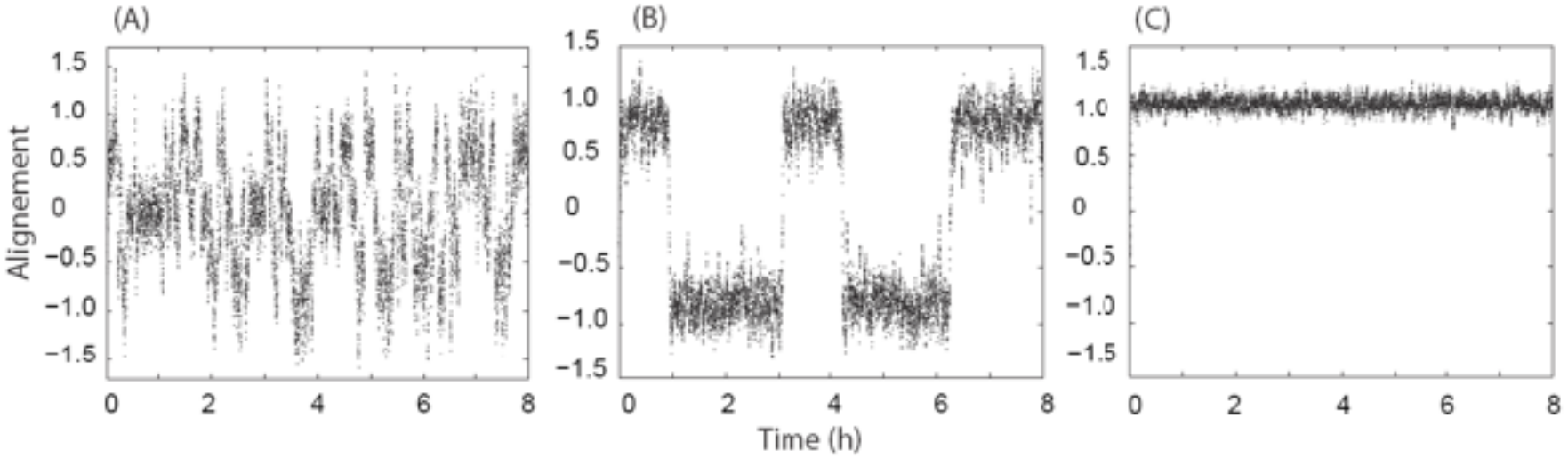
Alignment model in one dimension

- Run ‘Align1D’





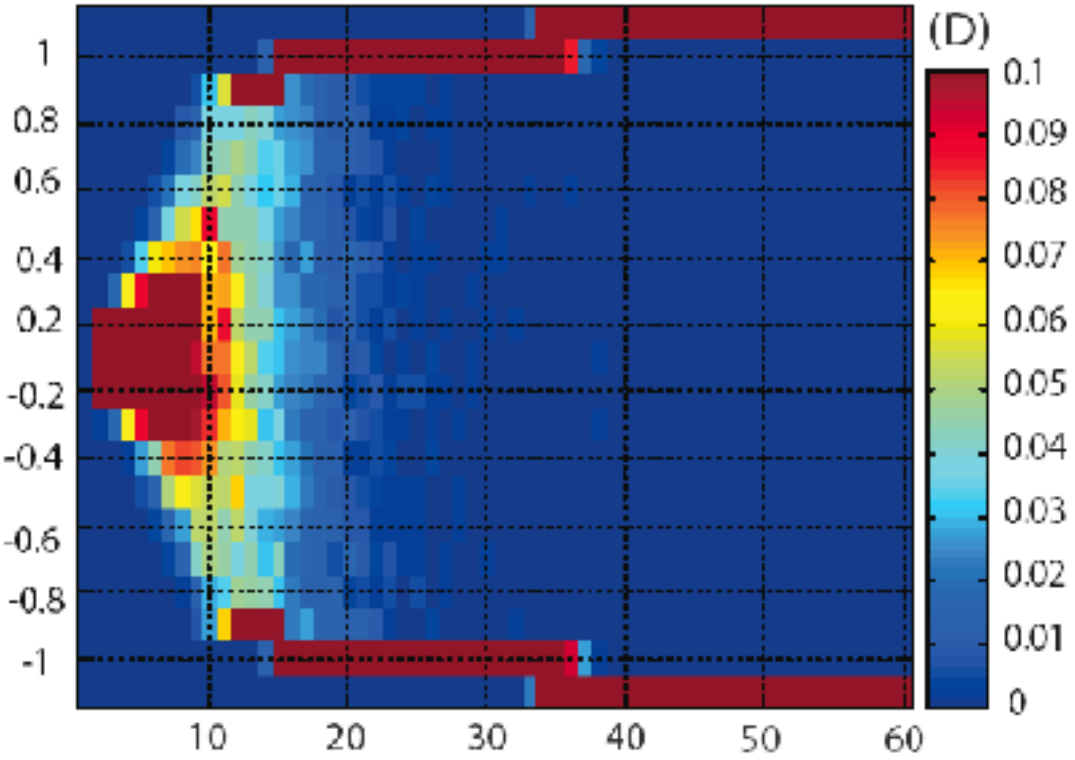
Alignment



$$\phi = \frac{1}{n}\sum_{i=1}^{n} \underline{u}_i(t)$$ measures order in the system.

1D self-propelled particles

Alignment

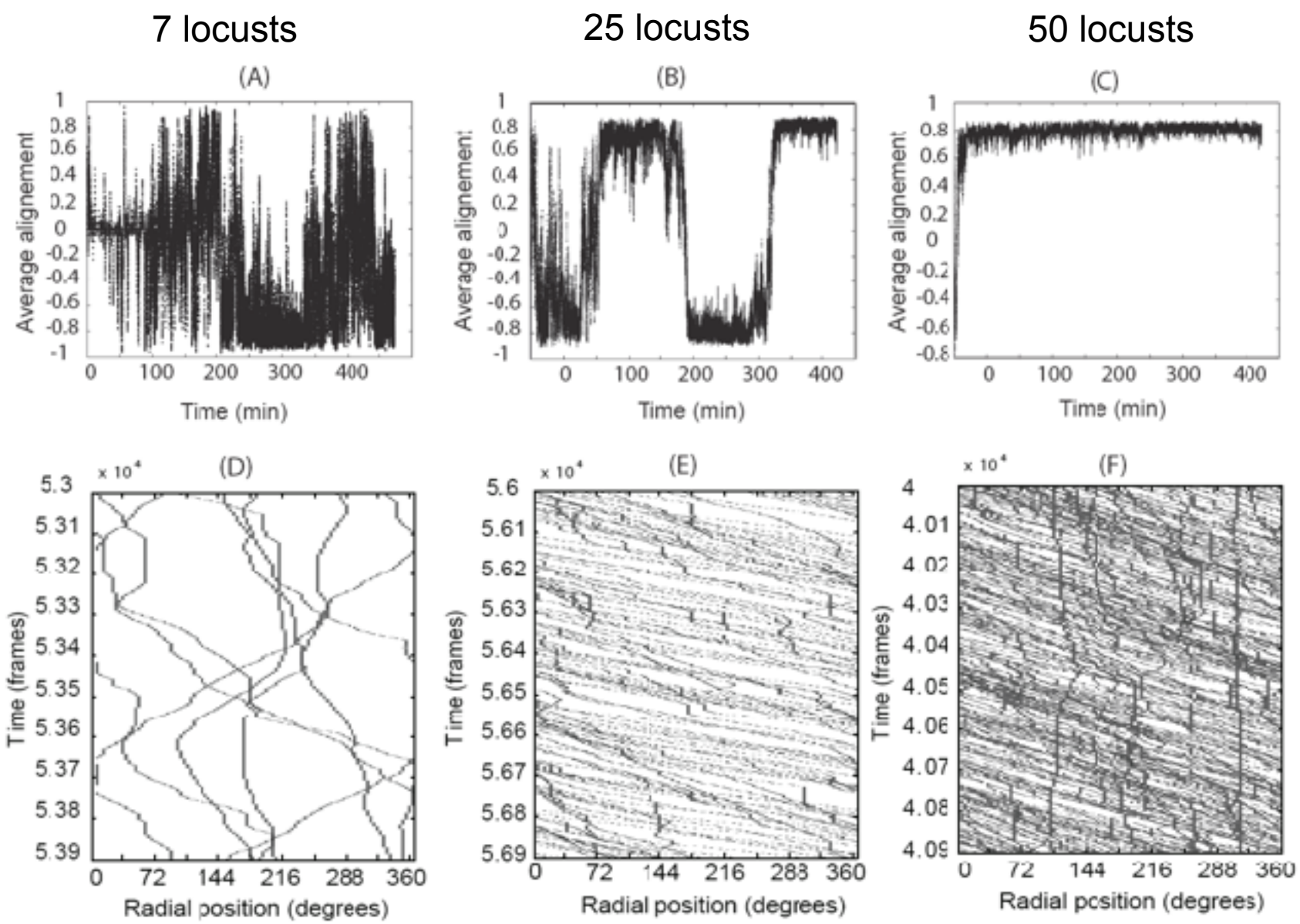


Number of particles

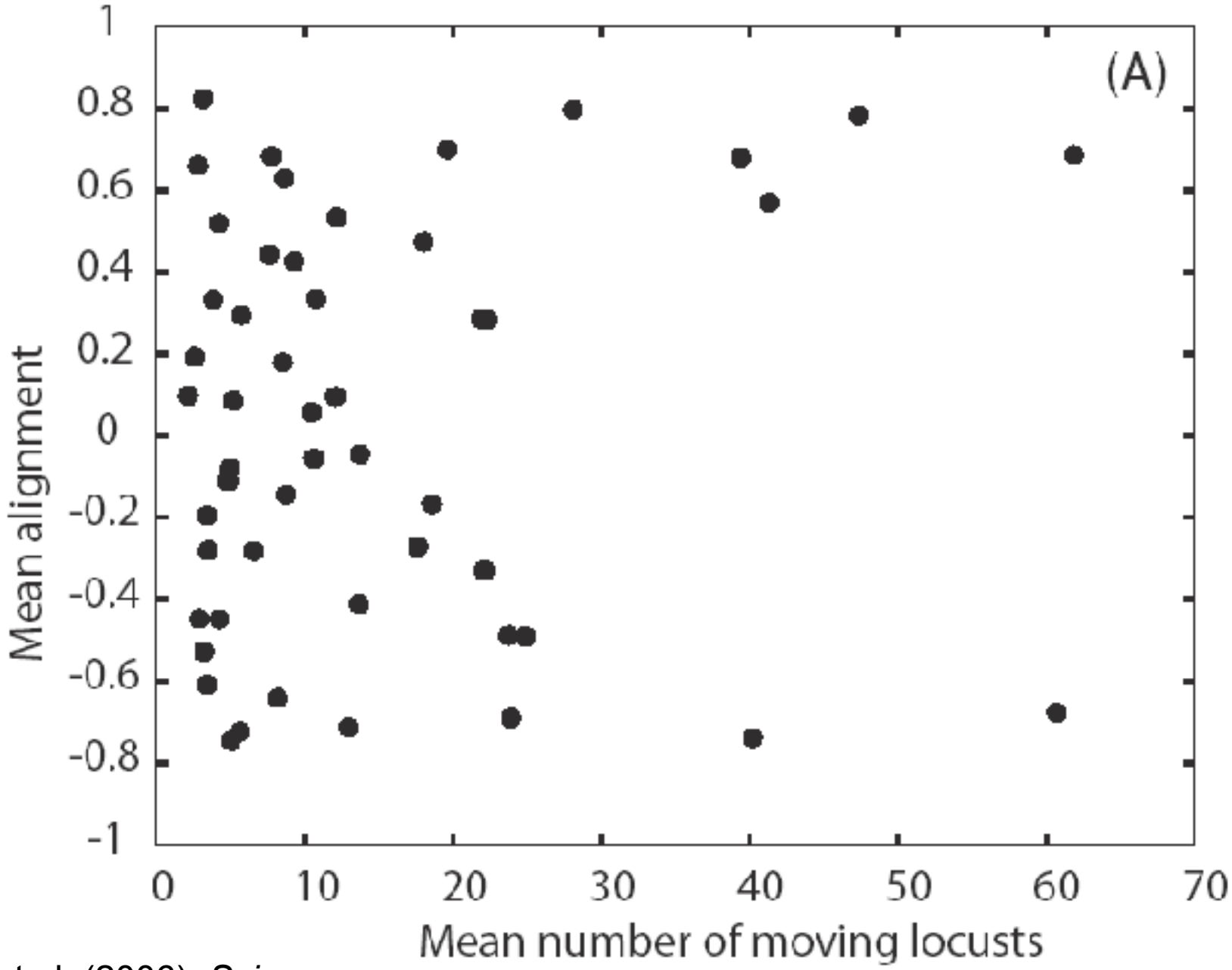
$\phi = \frac{1}{n}\sum_{i=1}^{n} \underline{u}_i(t)$ measures order in the system (alignment).

Buhl et al. (2006), *Science*
Yates et al. (2009), *PNAS*

Buhl et al. (2006), *Science*
Yates et al. (2009), *PNAS*

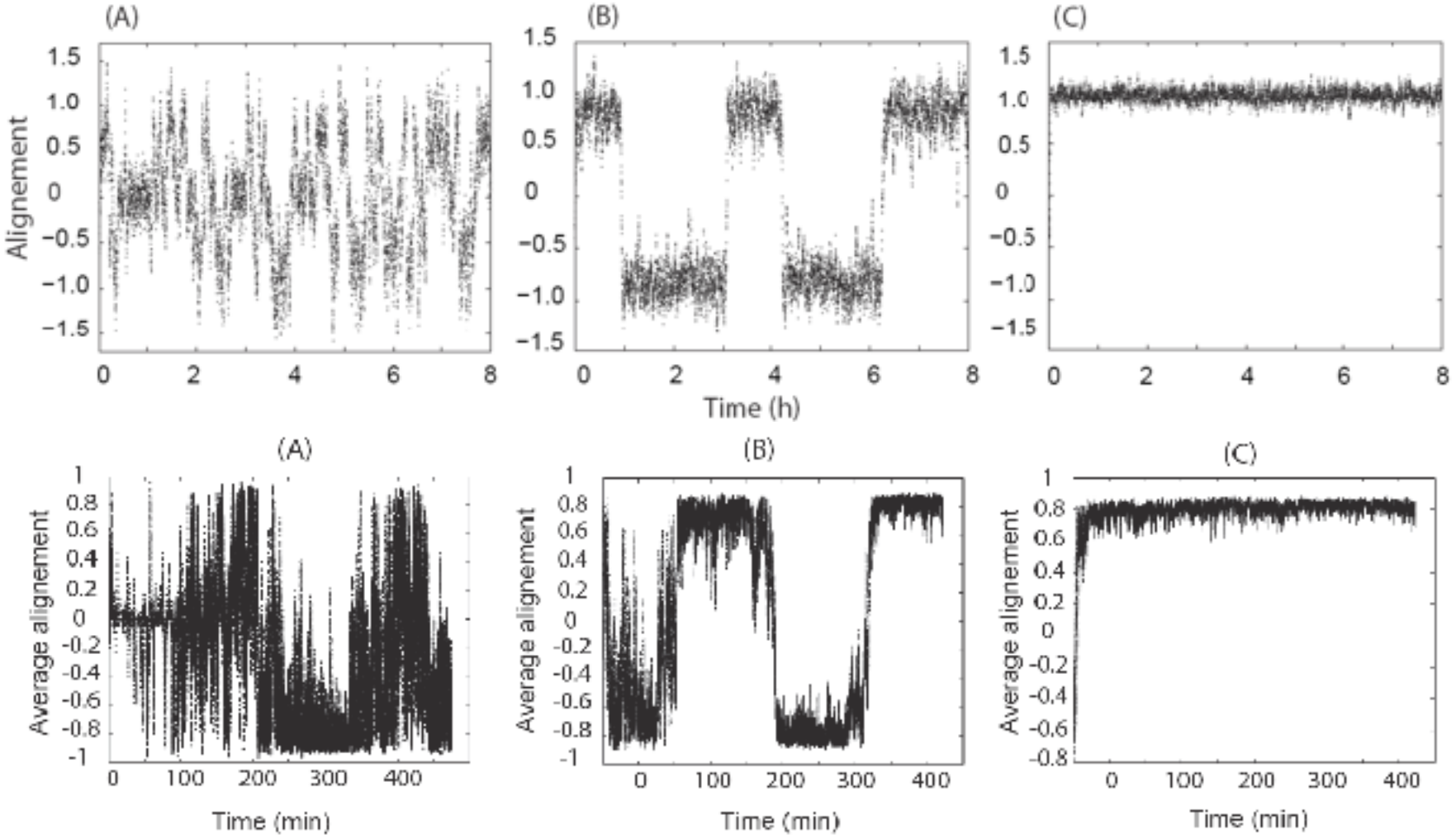


Buhl et al. (2006), *Science*

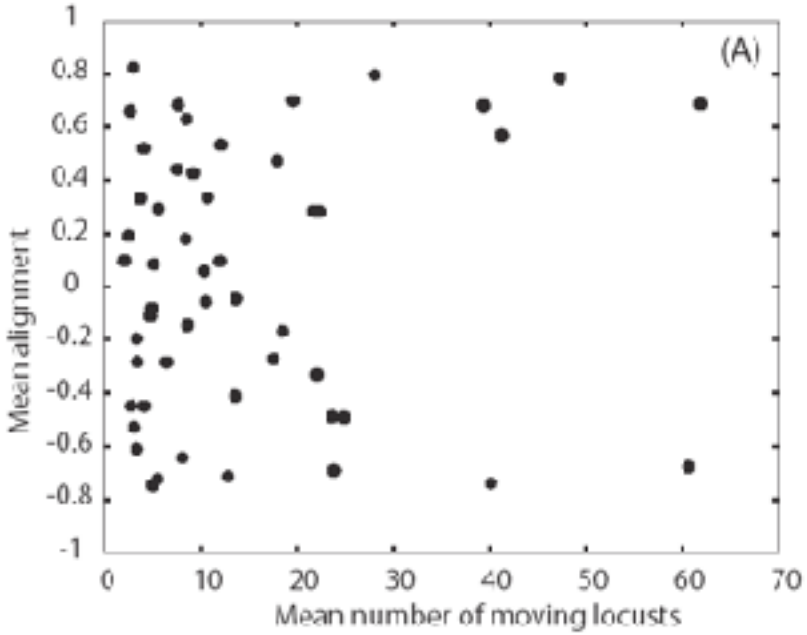


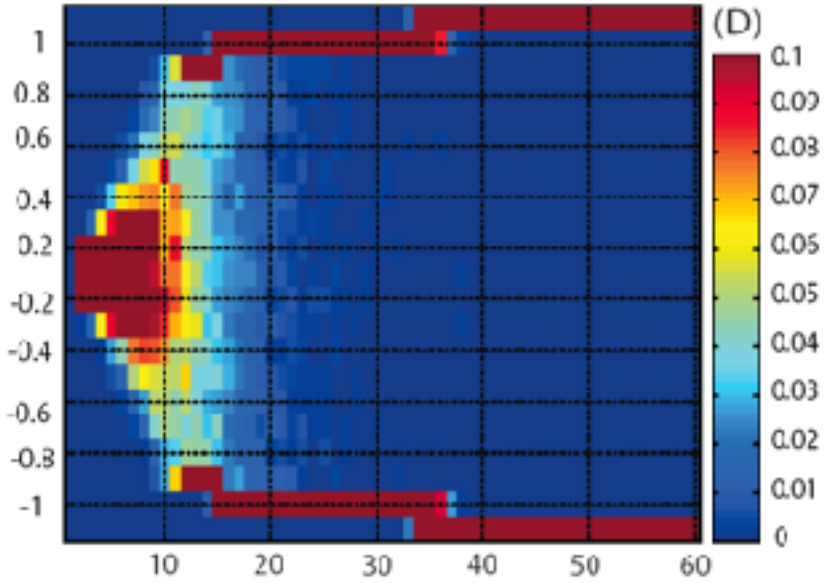
Buhl et al. (2006), *Science*

Model vs Experiment



Model vs Experiment





Vicsek Model

- Introduced in Lab 5, we now discuss the theory.

- Code: ‘Align2D.m’ or the python implementation on the course webpage.

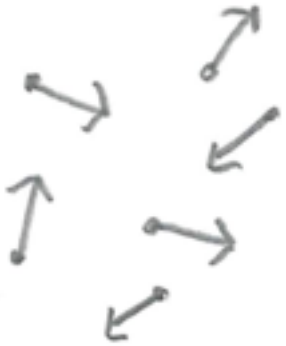
Measure of Alignment: Polarisation

High polarisation

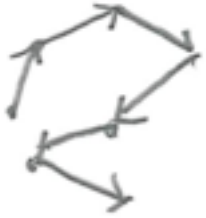
Low Polarisation

Measure of Alignment: Polarisation

High polarisation

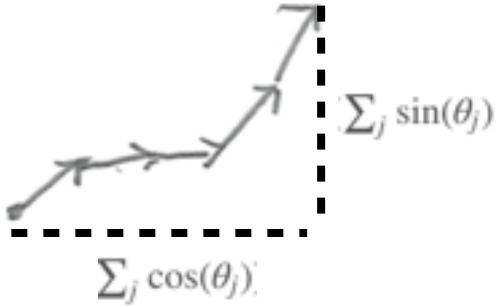


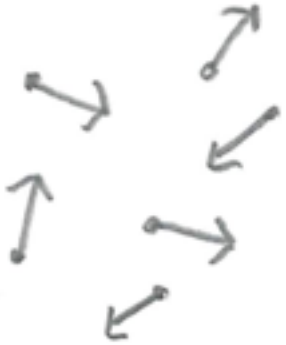
Low Polarisation



Measure of Alignment: Polarisation

High polarisation

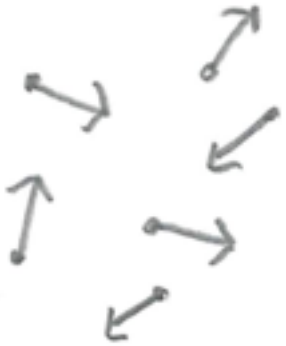




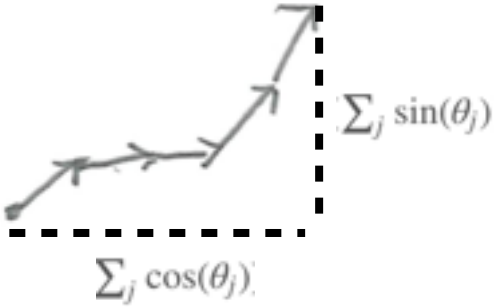
Low Polarisation

Measure of Alignment: Polarisation

High polarisation



Low Polarisation



Polarisation of:

$\theta_1, \theta_2, \ldots, \theta_N$

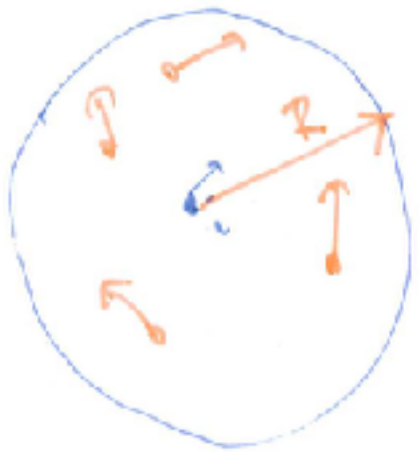
$$= \frac{1}{N}\sqrt{\left(\sum_j \sin(\theta_j)\right)^2 + \left(\sum_j \cos(\theta_j)\right)^2}$$

Measure of Aggregation?

Definition -

a cluster of things that have come or been brought together

Vicsek Model



N: number of particles
η: noise parameter
L: size of domain
R: radius of interaction
v: speed

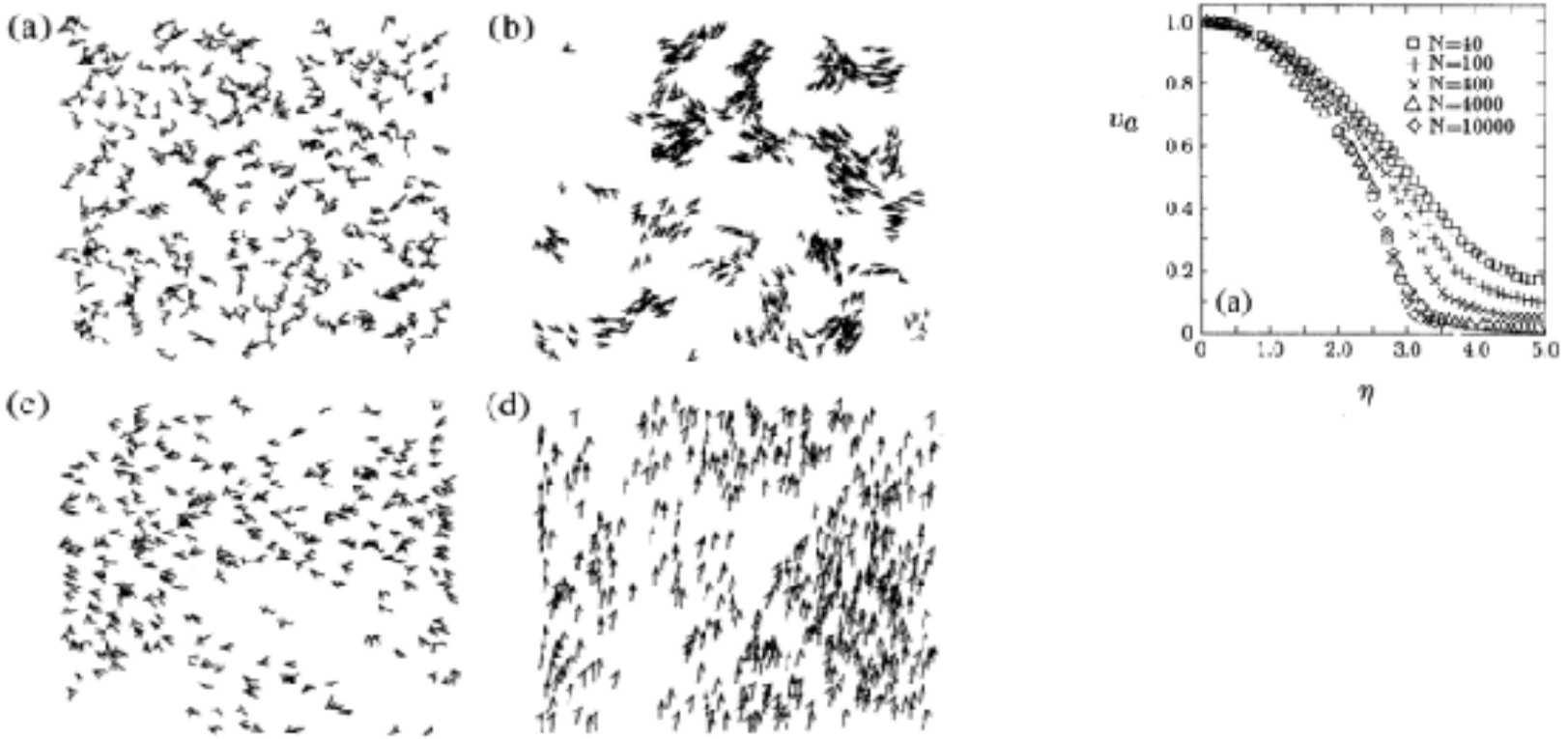
Angular update rule:

$$\theta_i(t+1) = \tan^{-1}\left(\frac{\sum_{j\in R_i}\sin(\theta_j(t))}{\sum_{j\in R_i}\cos(\theta_j(t))}\right) + e(t)$$

e(t) is a random number selected uniformly at random from a range $[-\eta/2, \eta/2]$

Vicsek et al., PRL 75 (1995)

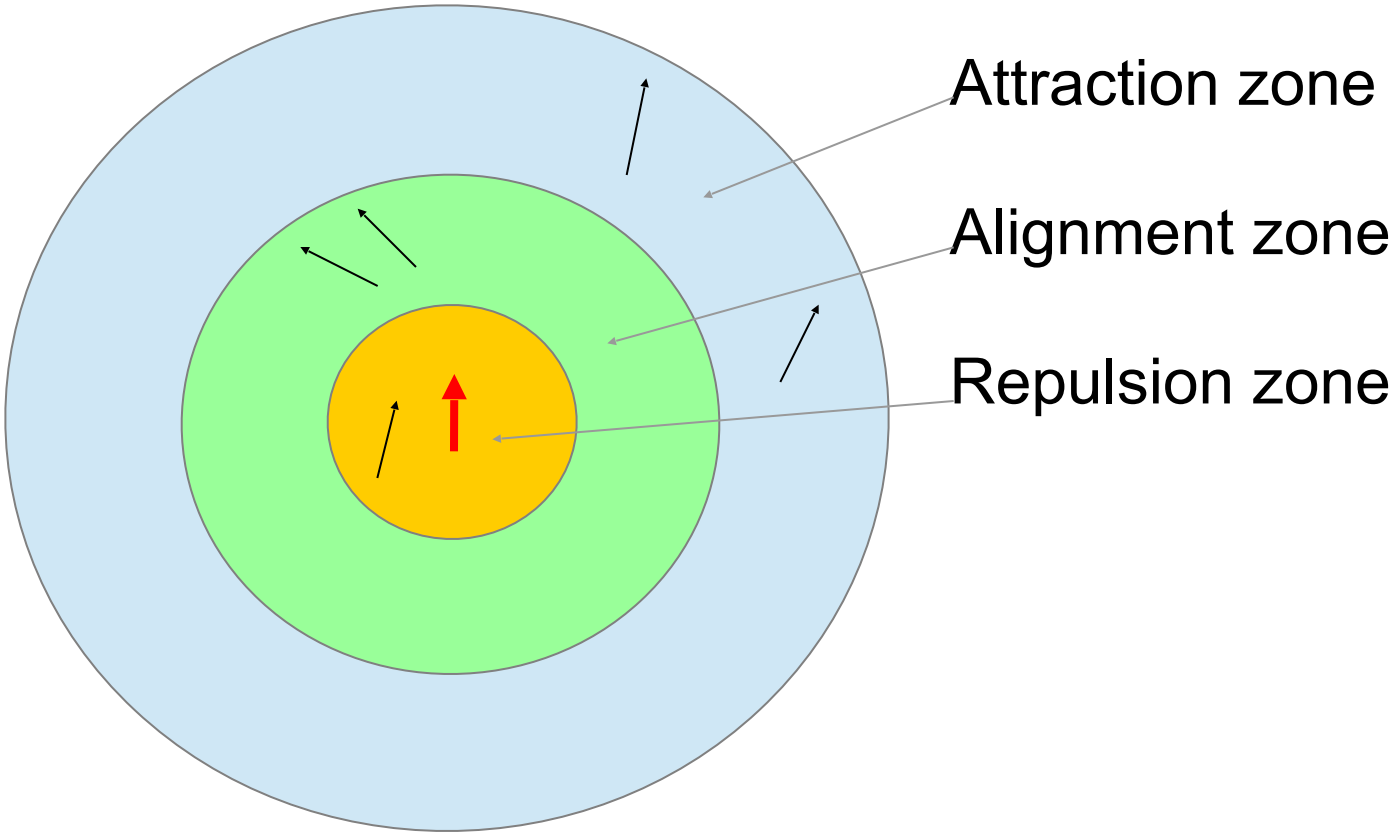
Vicsek Model



Vicsek et al., PRL 75 (1995)

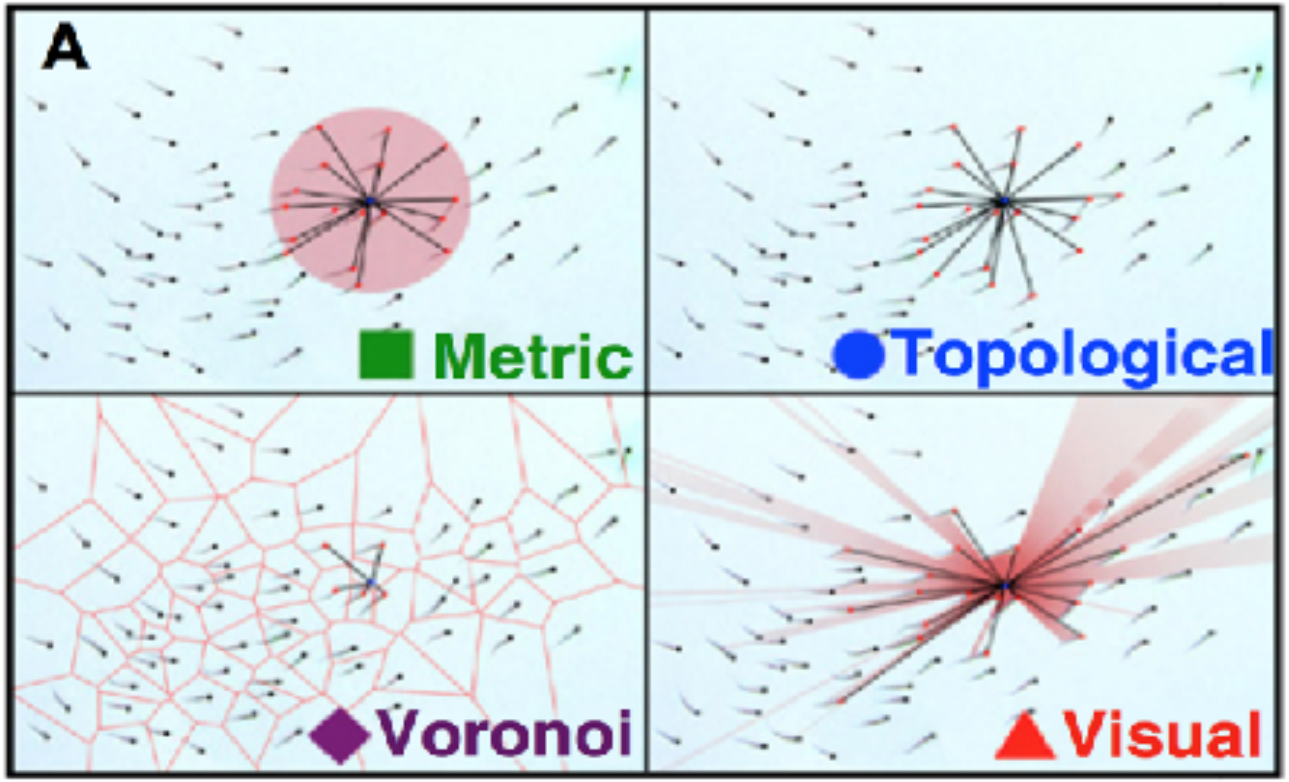
Attraction/Repulsion

"Boids" model



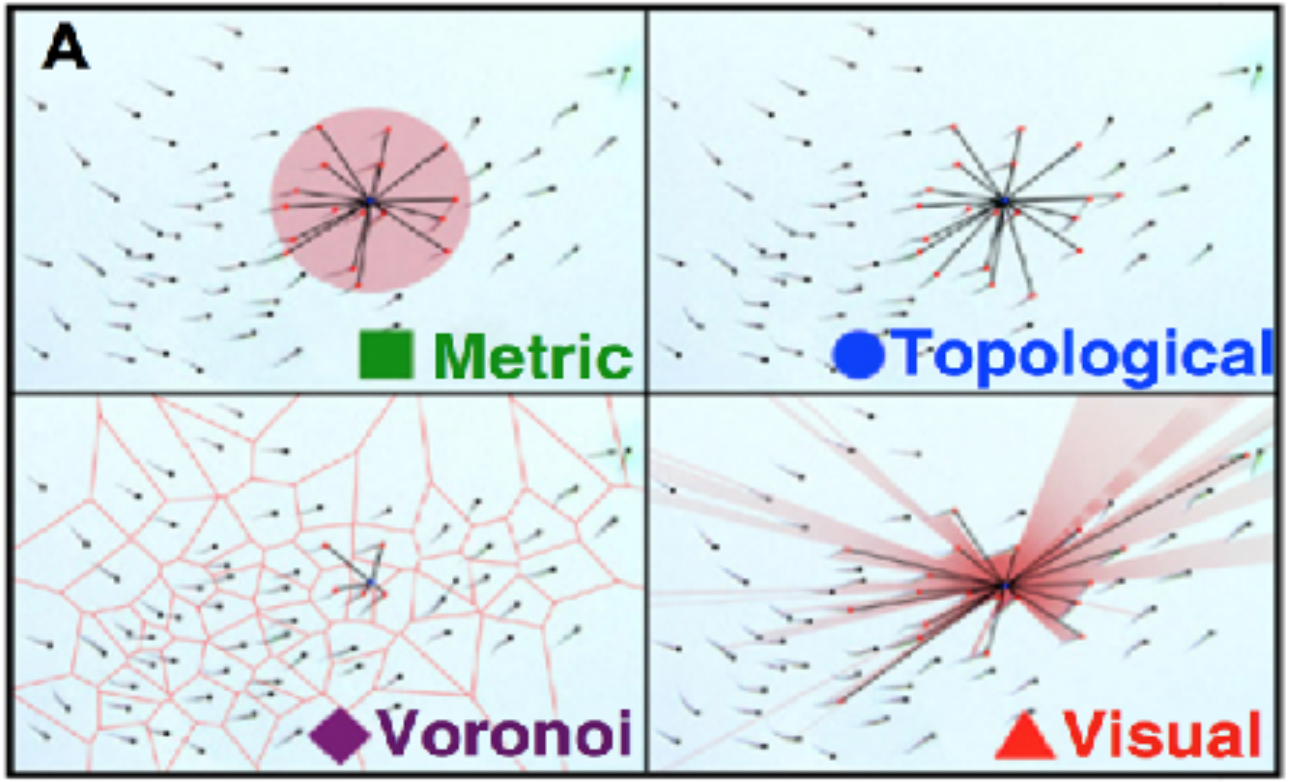
See: Couzin et al., J. Theor. Bio. (2002)

Alternative distance measures



Strandburg-Peshkin et al., Current Biology (2013)

Alternative distance measures



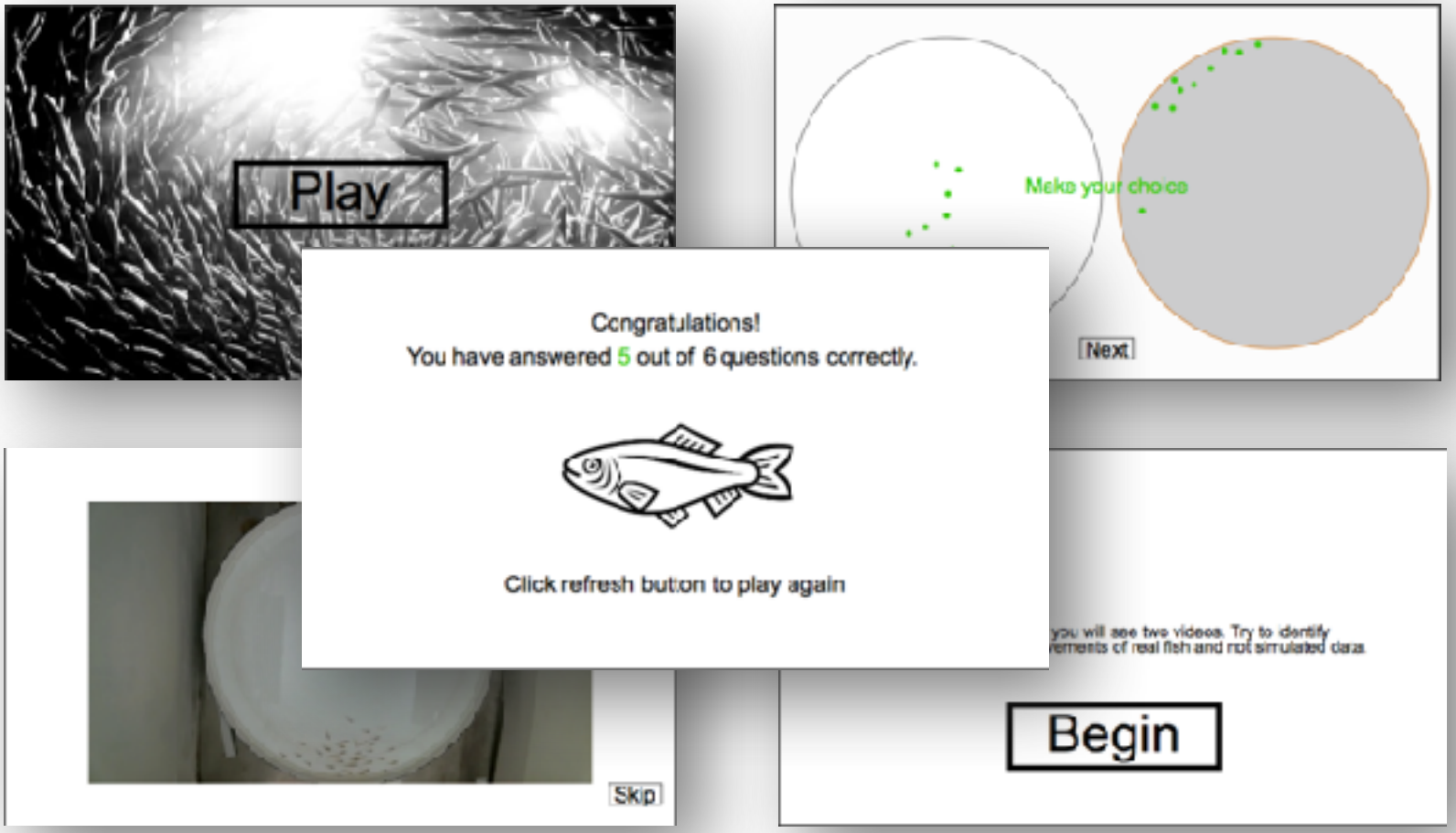
Metric: all individuals within a certain distance.
Topological: a fixed number of nearest neighbors.
Voronoi: those individuals sharing a boundary in a Voronoi tessellation of the group.
Visual: all individuals that occupy an angular area on the retina of the focal fish that is greater than a threshold value.

Strandburg-Peshkin et al., Current Biology (2013)

Even more options

- Maximum turning angles
- Blind angles
- Attraction/repulsion potentials
- Reaction times
- Wall interactions
- Variable speed
- Variation in individuals
- Pheromone trails
- Etc....

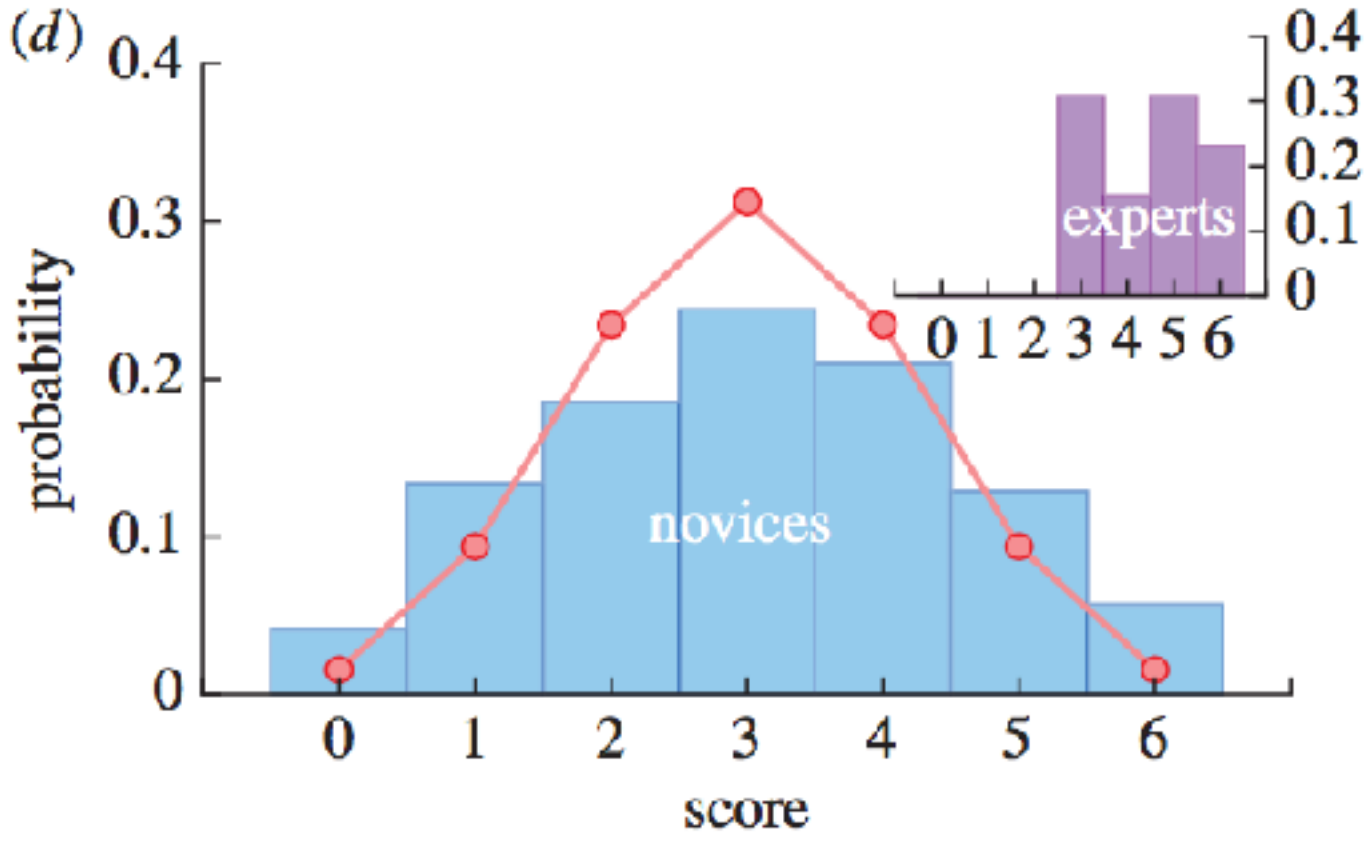
Can you tell the difference between real and simulated fish?



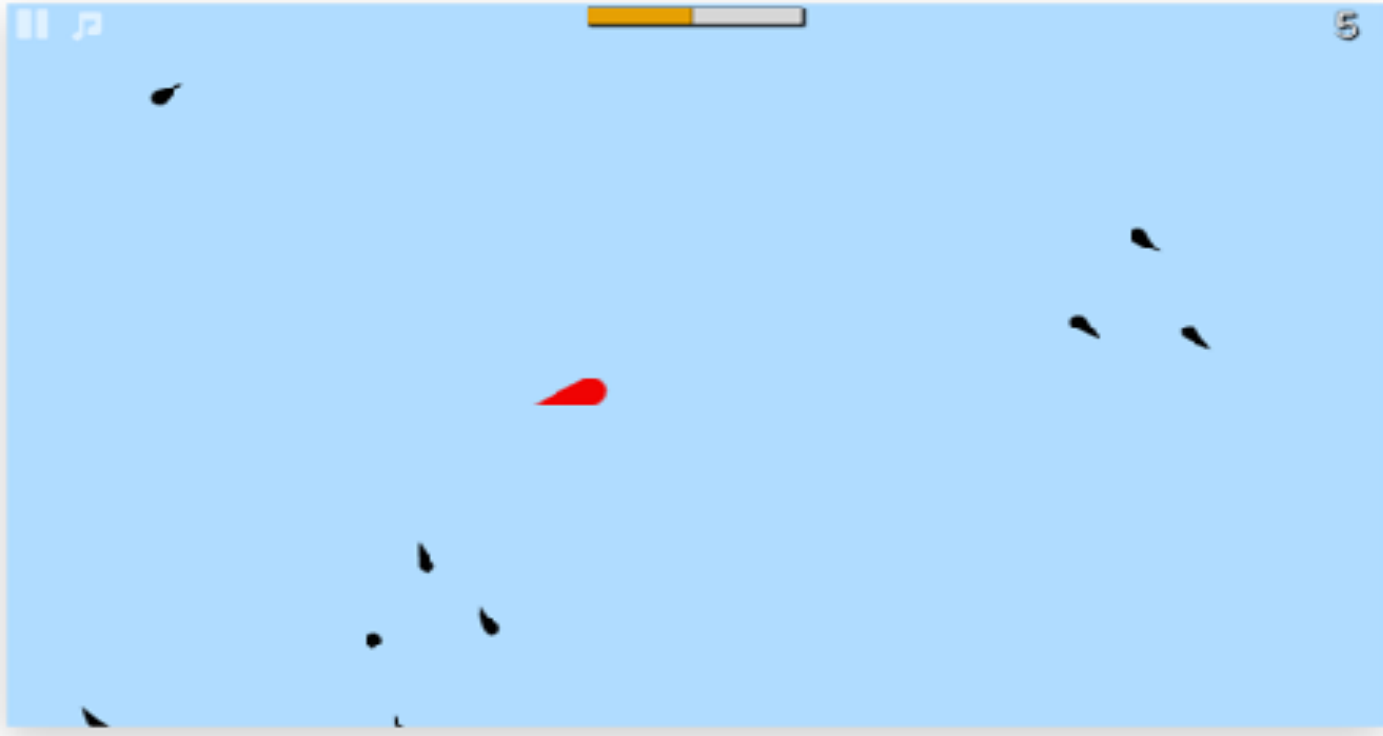
Get playing!

http://www.collective-behavior.com/apps/

Can people tell the difference between real and simulated fish?



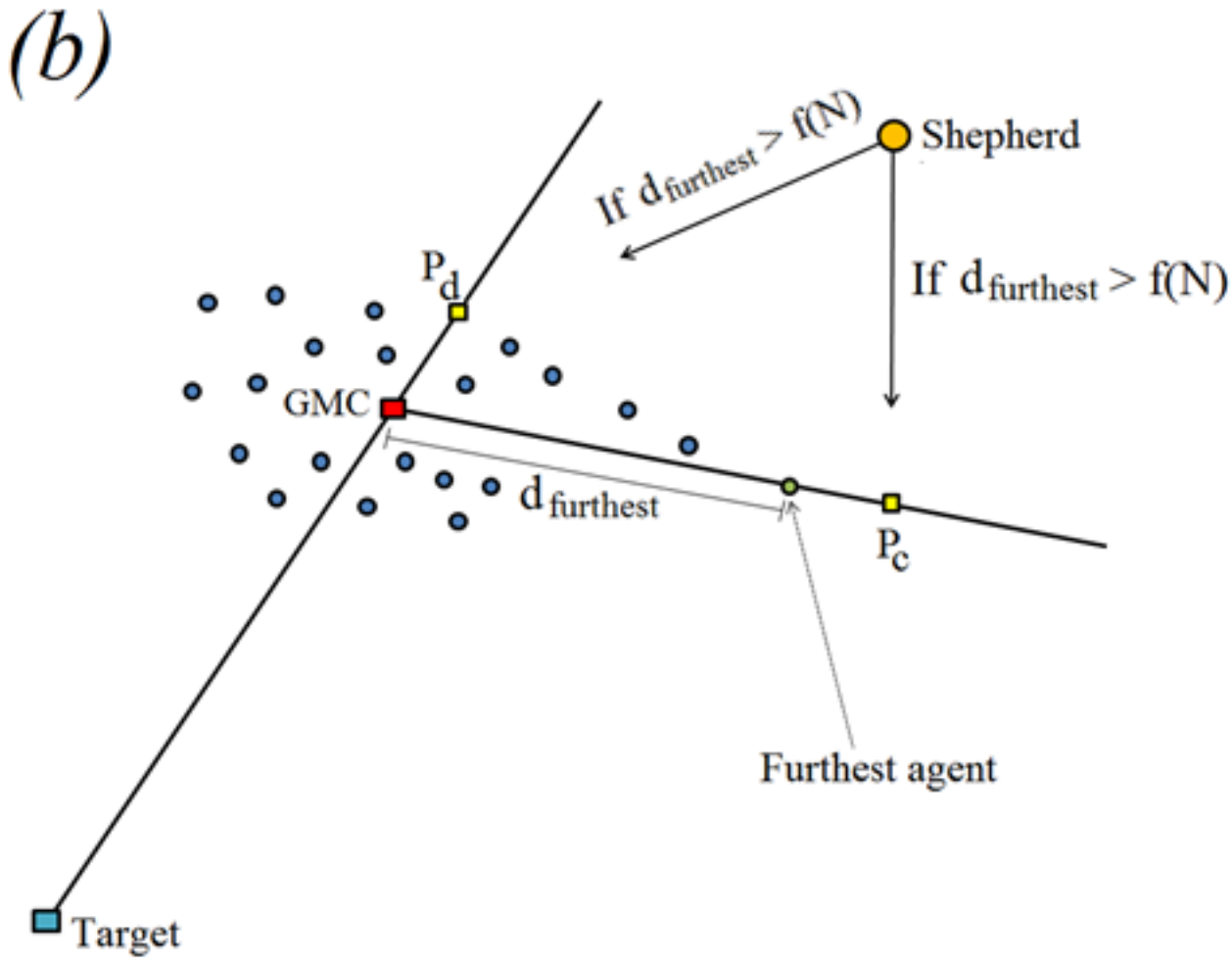
Evolving prey



http://collective-behavior.com/apps/fishindanger/webgl

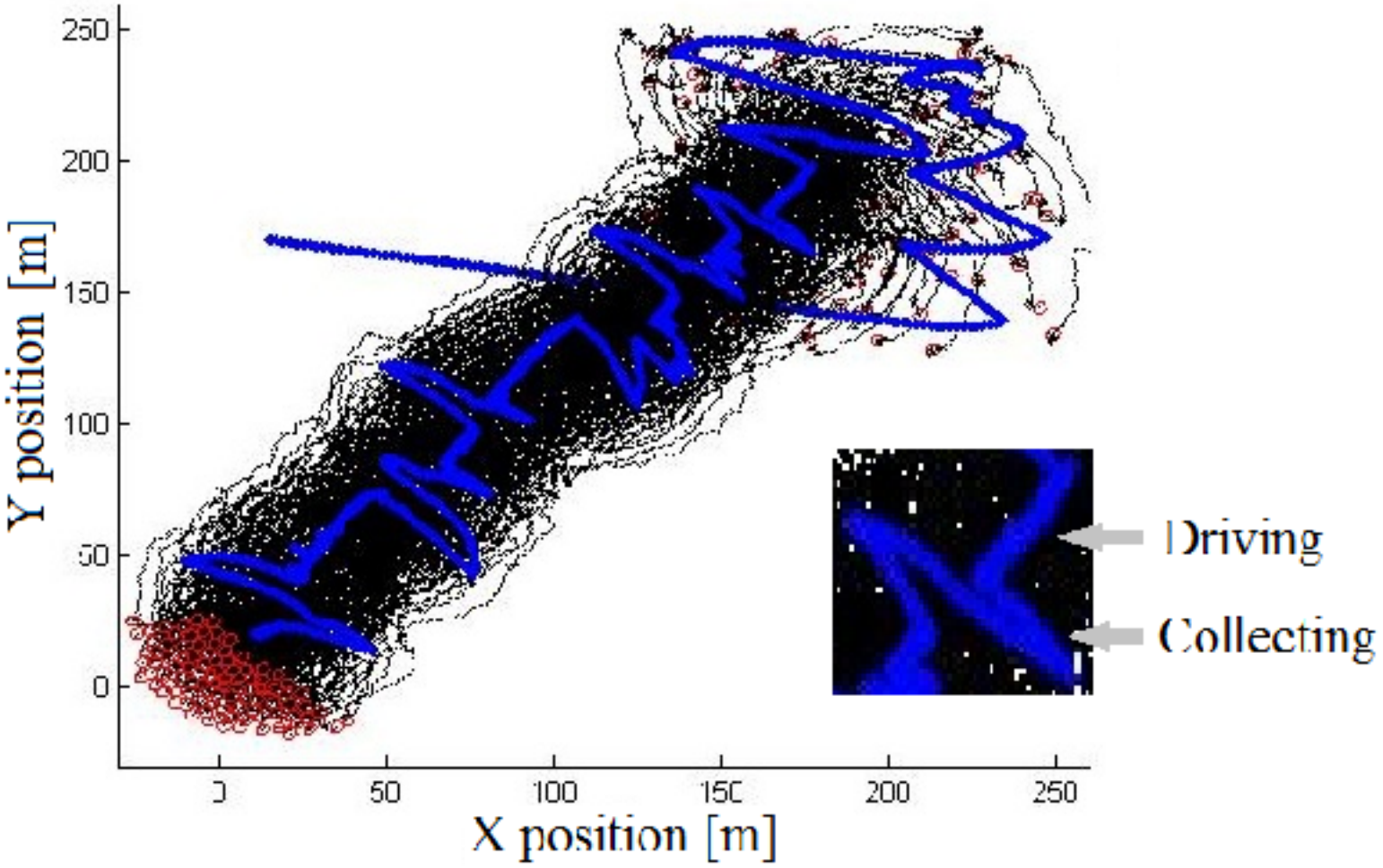
King et al. (2012) *Current Biology*

Sheepdog model



Strömbom et al., J. Royal Soc. Interface (2014)

Drive and collect



Next: when humans go ballistic

PRL 110, 228701 (2013) PHYSICAL REVIEW LETTERS week ending 31 MAY 2013

Collective Motion of Humans in Mosh and Circle Pits at Heavy Metal Concerts

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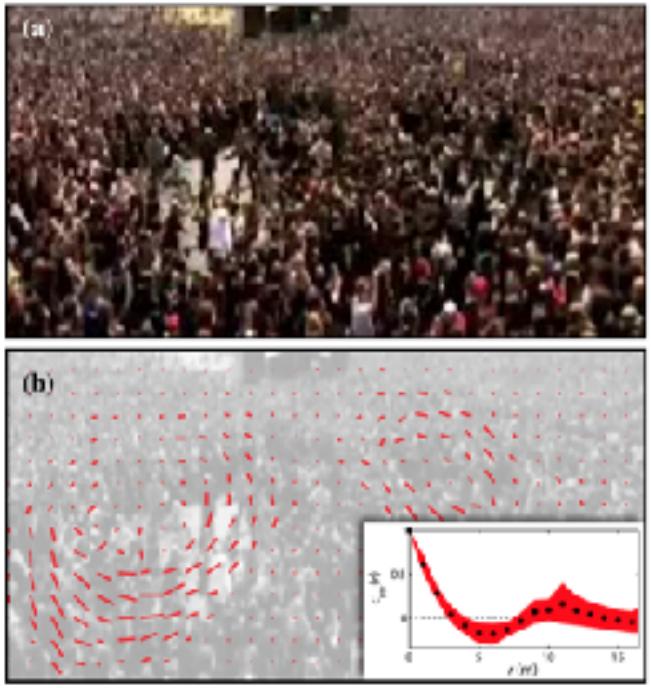
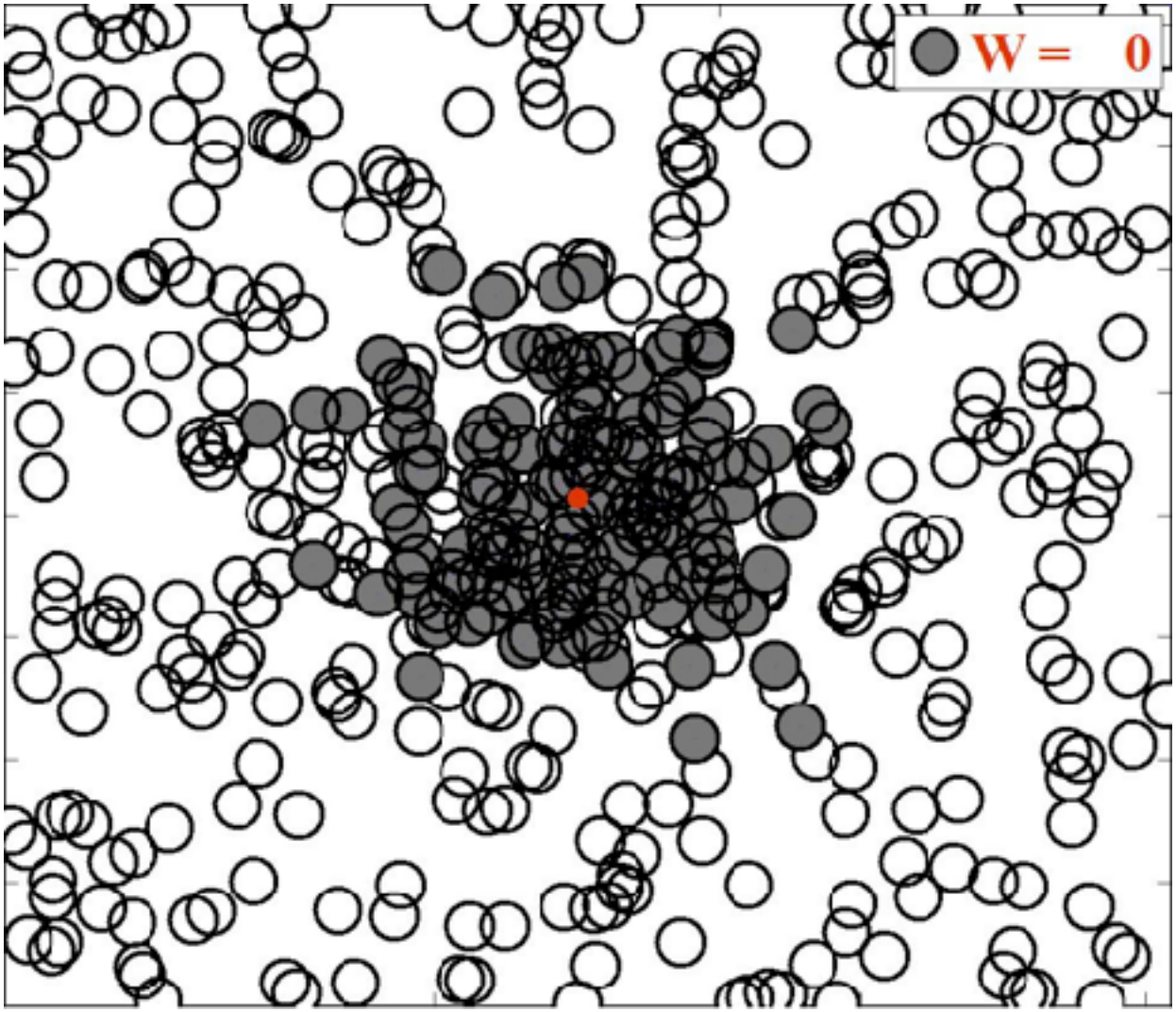
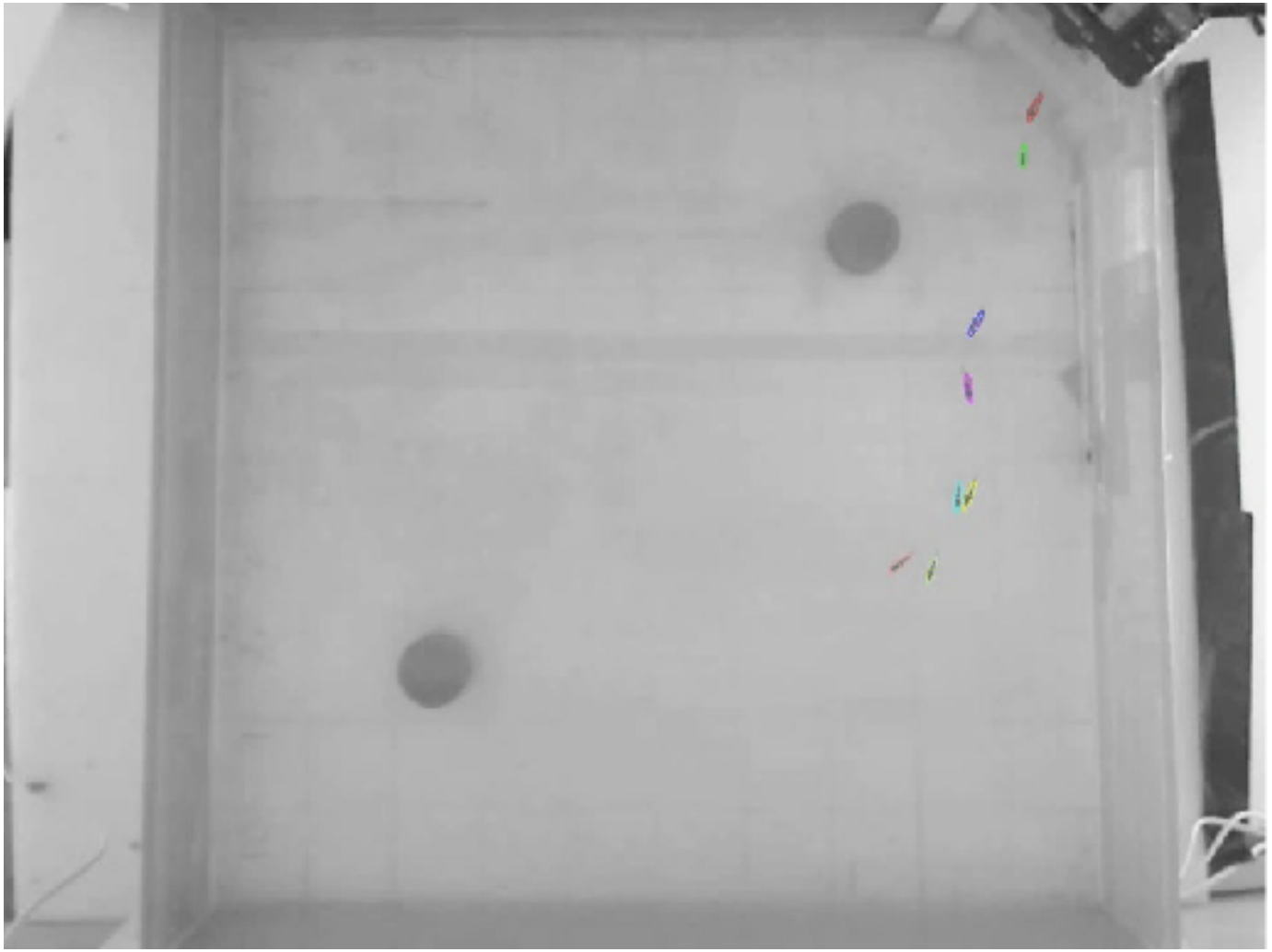


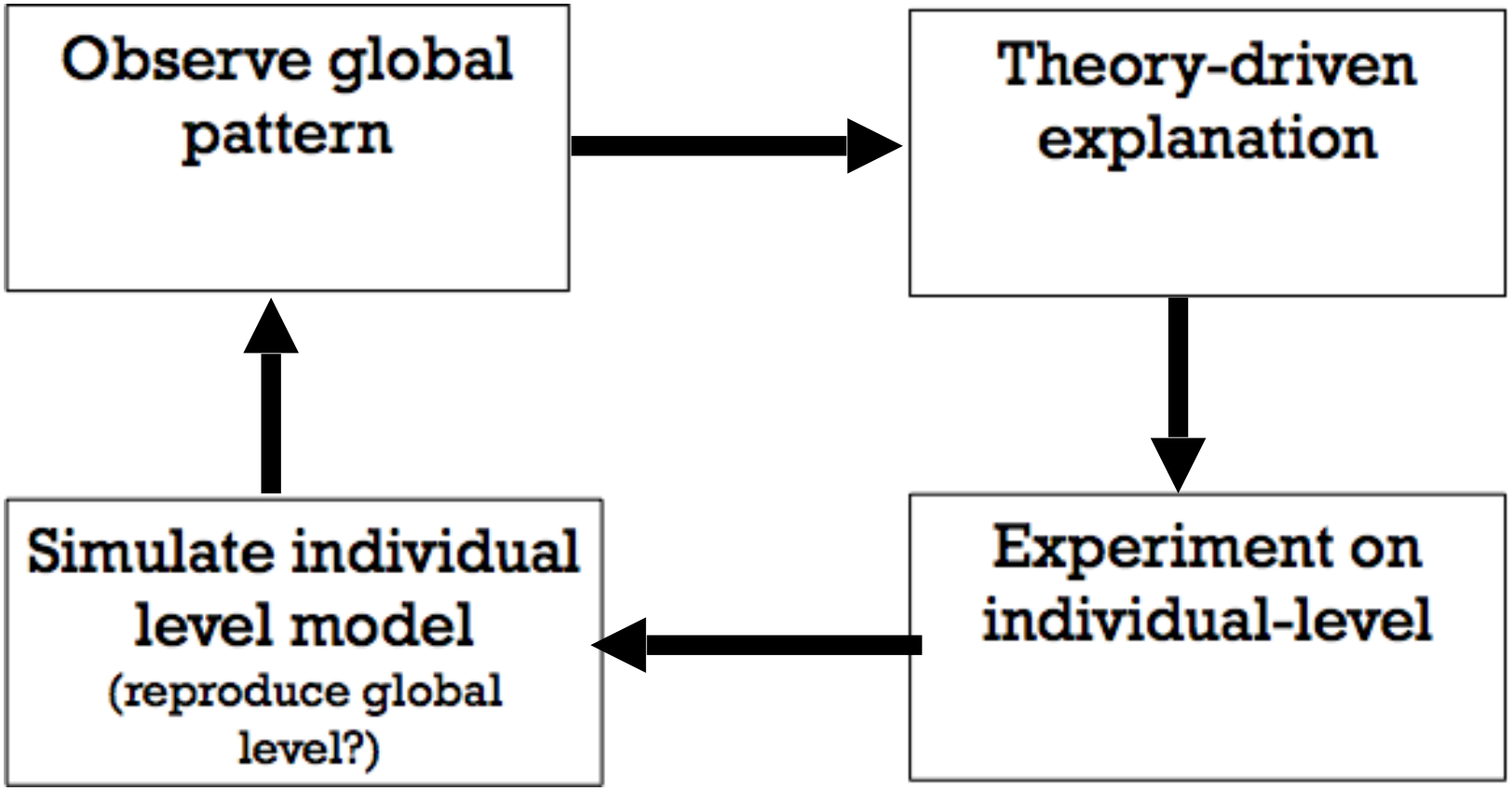
Moshpit model

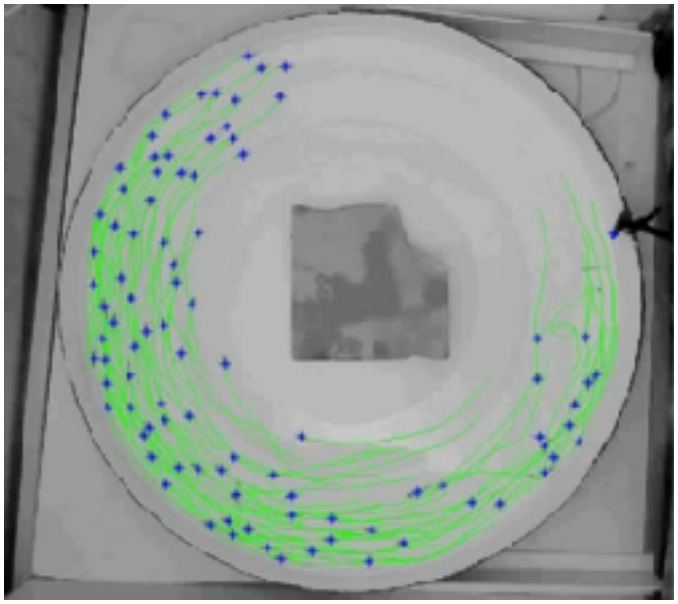


Rules of motion



The modelling cycle






C


COLLECTIVE BEHAVIOUR