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Squeezing the Efimov Effect in Real Space

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Contents

1 Methods

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\hbar^2}{2m_i}\right) \frac{\partial^2}{\partial \vec{r}_i} + \sum_{i=1}^{N} V_i(\vec{r}_i) + \sum_{i<j}^{N} V_{ij}(\vec{r}_i - \vec{r}_j) \tag{1}$$

1.1 Correlated Gaussian Method

$$\langle \boldsymbol{r}|g\rangle \equiv \exp\left(-\sum_{i,j=1}^{3\cdot N} A_{ij} r_i \cdot r_j + \sum_{i=1}^{3\cdot N} s_i \cdot r_i\right) = e^{-\boldsymbol{r}^T A \boldsymbol{r} + \boldsymbol{s}^T \boldsymbol{r}} \tag{2}$$

$$|\psi\rangle = \sum_{i=1}^{K} c_i \, |g_i\rangle \tag{3}$$

$$\hat{H} \, |\psi\rangle = \epsilon \, |\psi\rangle \tag{4}$$

$$\hat{H} \sum_{i=1}^{K} c_i \, |g_i\rangle = \epsilon \sum_{i=1}^{K} c_i \, |g_i\rangle \tag{5}$$

$$\sum_{i=1}^{K} c_i \, \langle g_j| \, \hat{H} \, |g_i\rangle = \epsilon \sum_{i=1}^{K} c_i \, \langle g_j|g_i\rangle \tag{6}$$

$$\mathcal{H}_{j,i} \equiv \langle g_j| \, \hat{H} \, |g_i\rangle \quad , \quad \mathcal{B}_{j,i} \equiv \langle g_j|g_i\rangle \tag{7}$$

$$\mathcal{H}\boldsymbol{c} = \epsilon \mathcal{B}\boldsymbol{c} \tag{8}$$

$$\boldsymbol{x} = \mathcal{U}\boldsymbol{r} \quad , \quad \hat{\boldsymbol{\nabla}}_x = \mathcal{U}\hat{\boldsymbol{\nabla}} \tag{9}$$

$$\mathcal{U} \;=\; \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ \frac{m_1}{m_1+m_2} & \frac{m_2}{m_1+m_2} & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{m_1}{m_1+\ldots+m_N} & \frac{m_2}{m_1+\ldots+m_N} & \cdots & \cdots & \frac{m_N}{m_1+\ldots+m_N} \end{pmatrix} . \tag{10}$$

$$\Psi\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N\right) = \Phi\left(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_{N-1}\right) \phi(\vec{x}_N) \tag{11}$$

$$\hat{T} = \hat{T}^{\mathrm{int}} + \hat{T}^{\mathrm{CM}} \quad , \quad \hat{V}_{\mathrm{HO}} = \hat{V}_{\mathrm{HO}}^{\mathrm{int}} + \hat{V}_{\mathrm{HO}}^{\mathrm{CM}} \tag{12}$$

$$\hat{H} = -\frac{\hbar^2}{2} \hat{\boldsymbol{\nabla}}_x^T \Lambda \hat{\boldsymbol{\nabla}}_x + \boldsymbol{x}^T \Omega \boldsymbol{x} + \sum_{i<j}^{N} V_{ij} + \hat{H}^{\mathrm{CM}} \tag{13}$$

$$\Lambda_{kj} = \sum_{i=1}^{N} \frac{\mathcal{U}_{ki}\mathcal{U}_{ji}}{m_i} \tag{14}$$

$$\Lambda_{kj} = \mu_k^{-1} \delta_{kj} \tag{15}$$

$$\mu_k = \frac{m_{k+1}\left(\sum_{i=1}^{k} m_i\right)}{\sum_{i=1}^{k+1} m_i} \tag{16}$$

$$\Omega_{kj} = \frac{\hbar^2 \Lambda_{kj}}{2\, b^4} \tag{17}$$