

Few-shot Learning by Statistical Methods (Part 2)

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CVPR 2023 Tutorial: Few-shot Learning from Meta-Learning, Statistical Understanding to Applications

<https://fsl-fudan.github.io>

Few-Shot Learning Revisited



Machine Learning by Cost

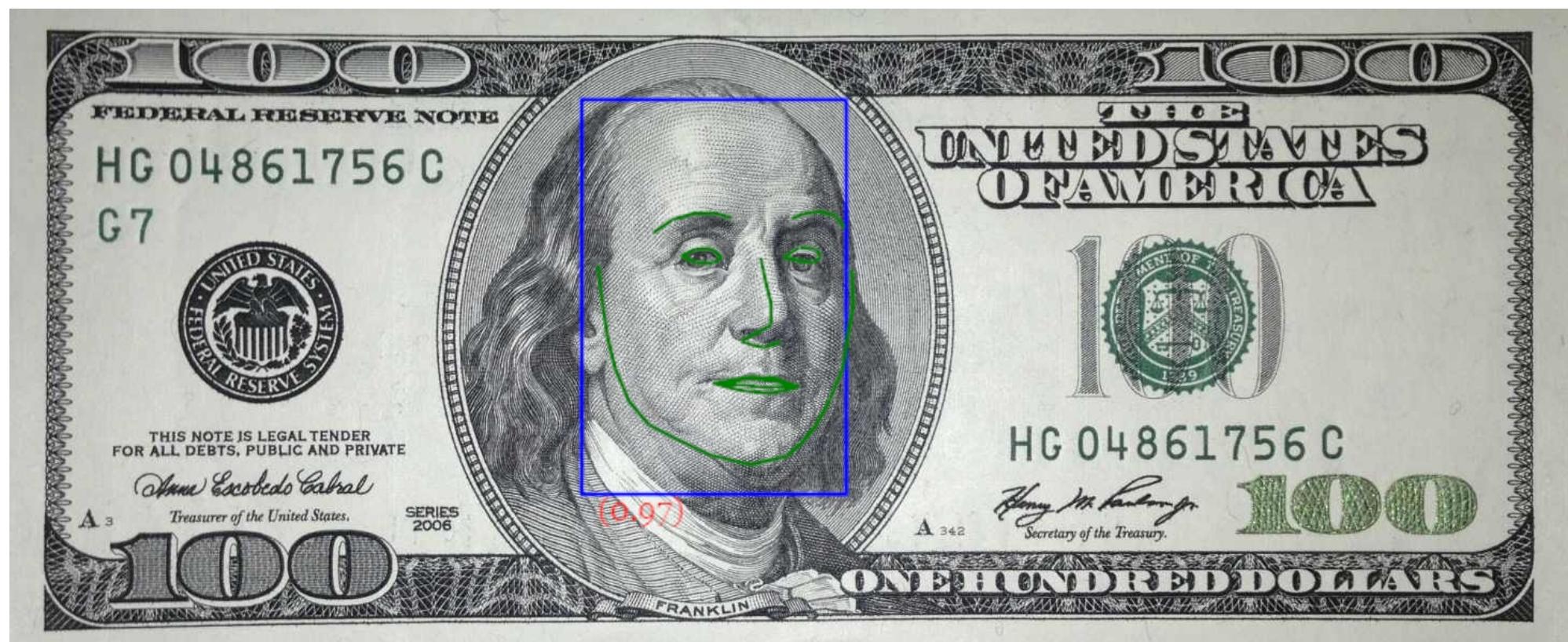
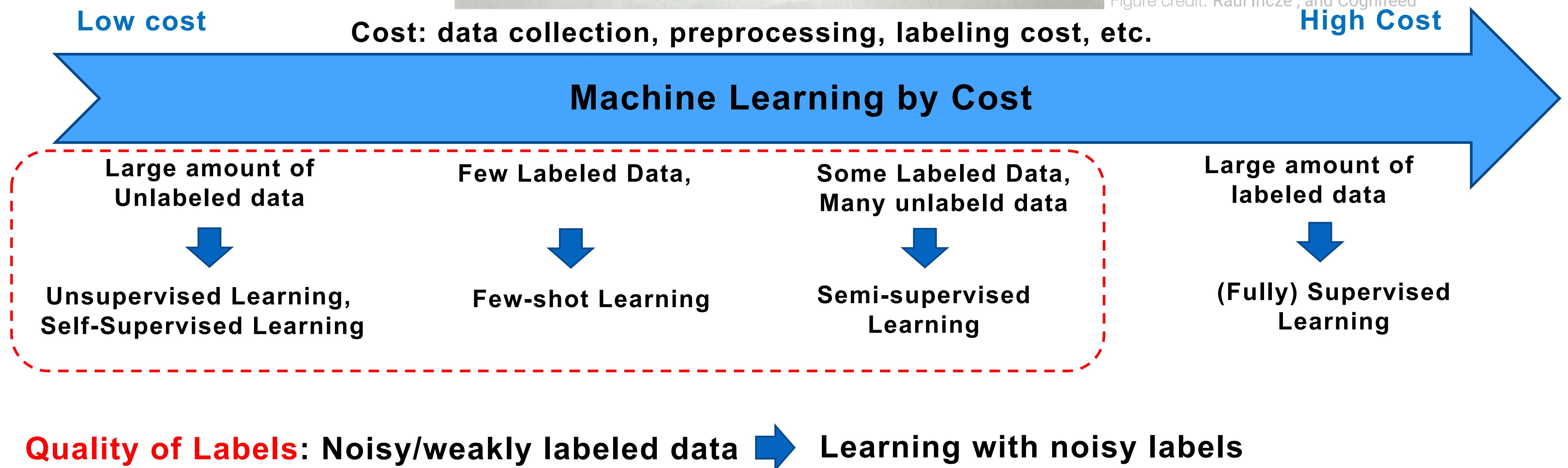
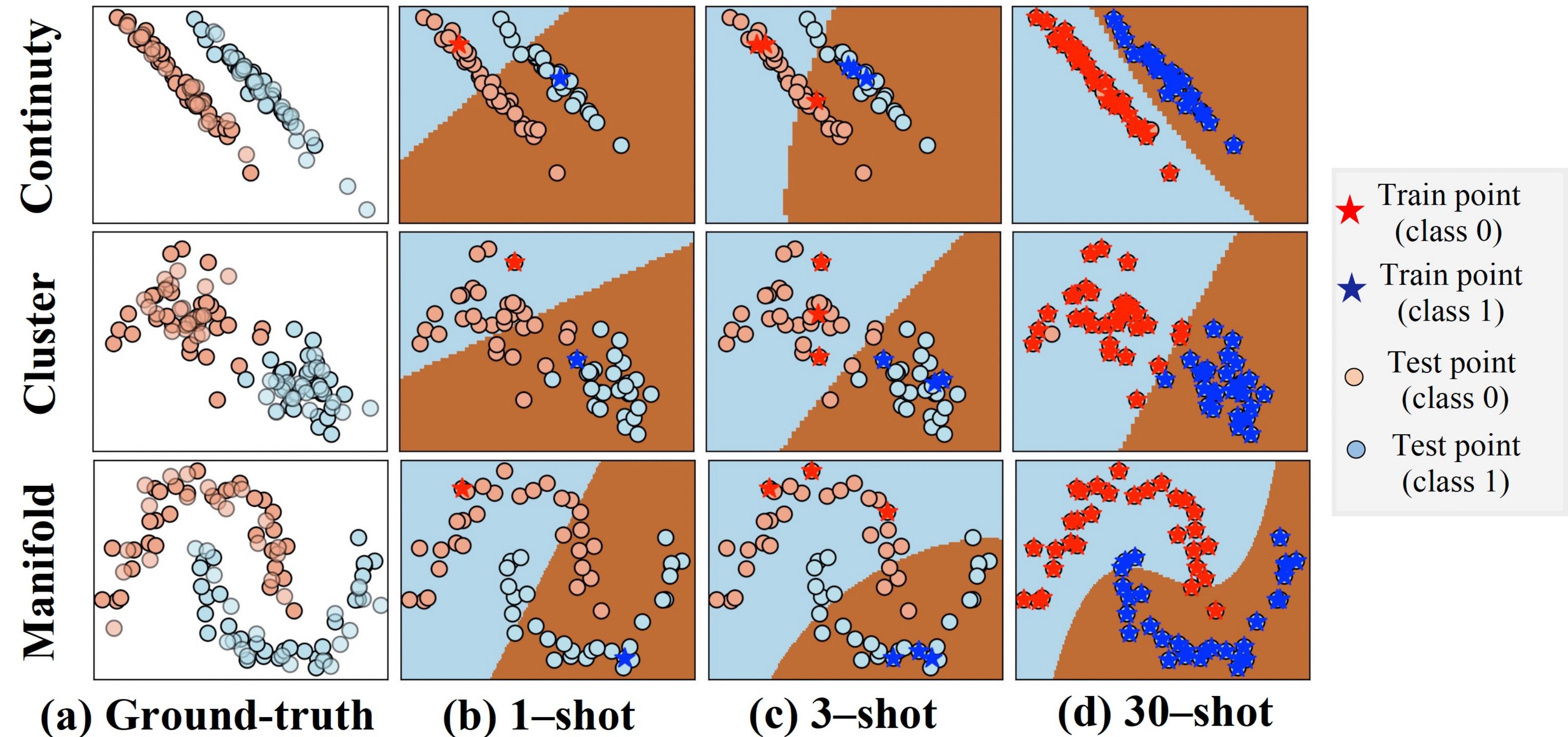


Figure credit: Raul Incze , and Cognifeed



From Few-shot Learning to Many-shot Learning



The continuity, cluster, and manifold assumptions of underlying data distribution.

Few-Shot Learning Setup

Task formulation



Classes with many samples

Base Data

- N-way K-shot meta-learning setting (N random categories)
 - K samples for each category in support set $\mathcal{S} = \{(I_i^{supp}, y_i^{supp})\}$
 - Q samples for each category in query set $\mathcal{Q} = \{I_i^q, y_i^q\}$
- Goal: transfer knowledge from $\mathcal{D}_S = \{(I_i, y_i), y_i \in \mathcal{C}_S\}$ to $\mathcal{D}_t = \{(I_i, y_i), y_i \in \mathcal{C}_t\}$ ($\mathcal{C}_S \cap \mathcal{C}_t = \emptyset$)

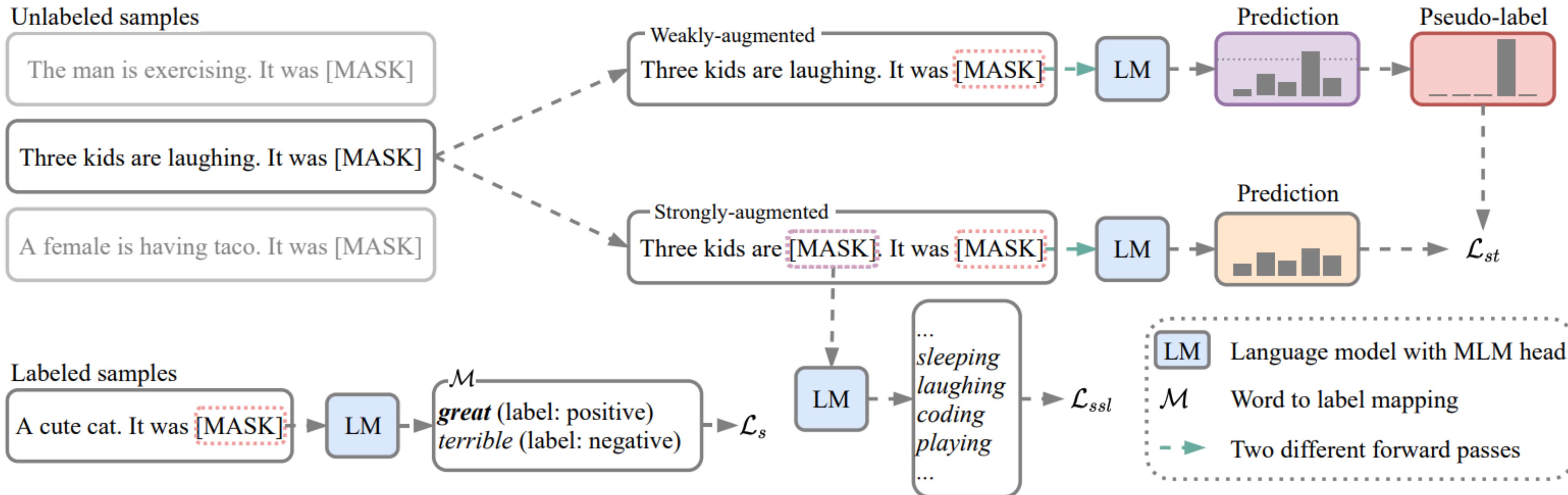


Classes with few samples

Novel Data

Few-Shot Learning by Unlabeled Data

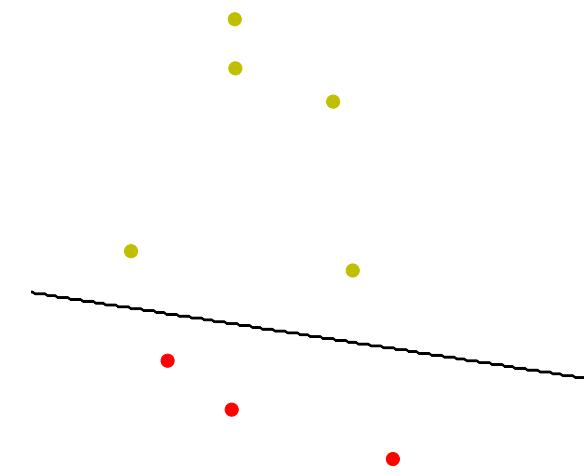
Task formulation



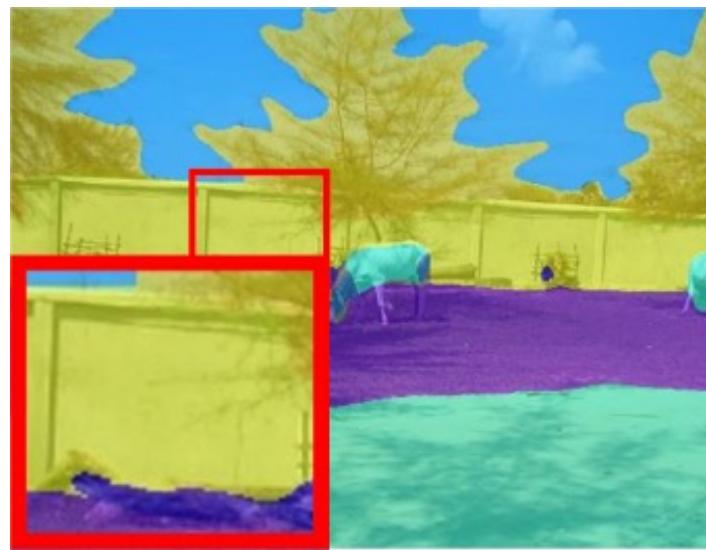
*Few-Shot Learning with unlabeled data
as Semi-supervised Learning?*

Few-Shot Learning Tasks

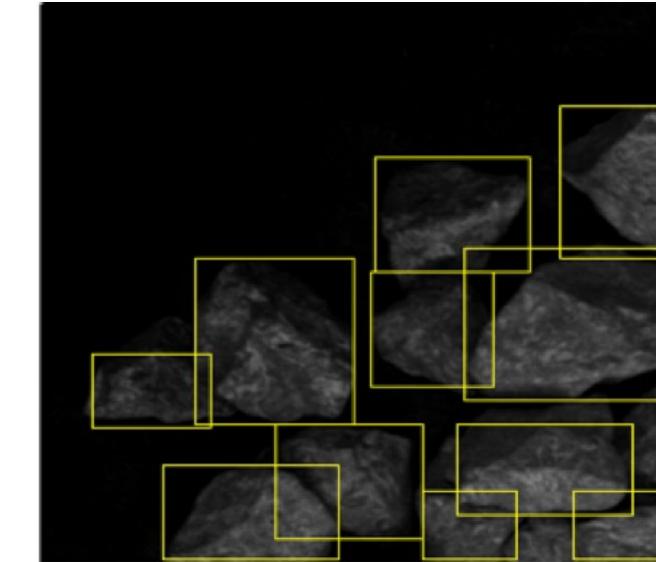
Task formulation



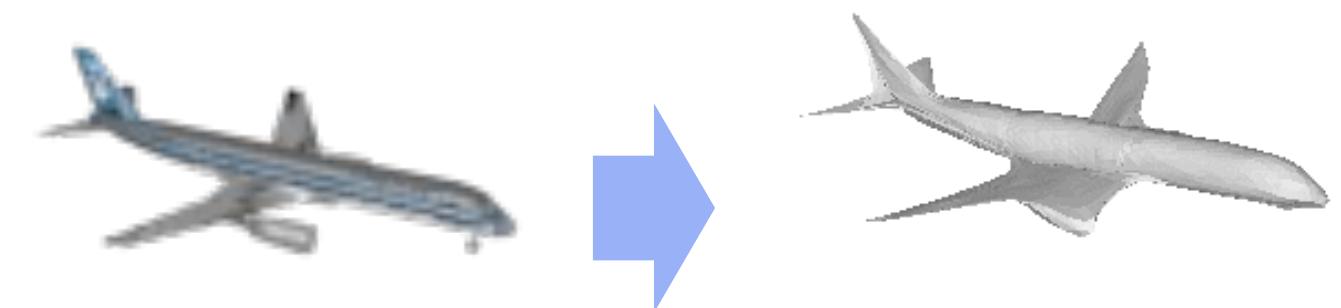
Recognition



Segmentation

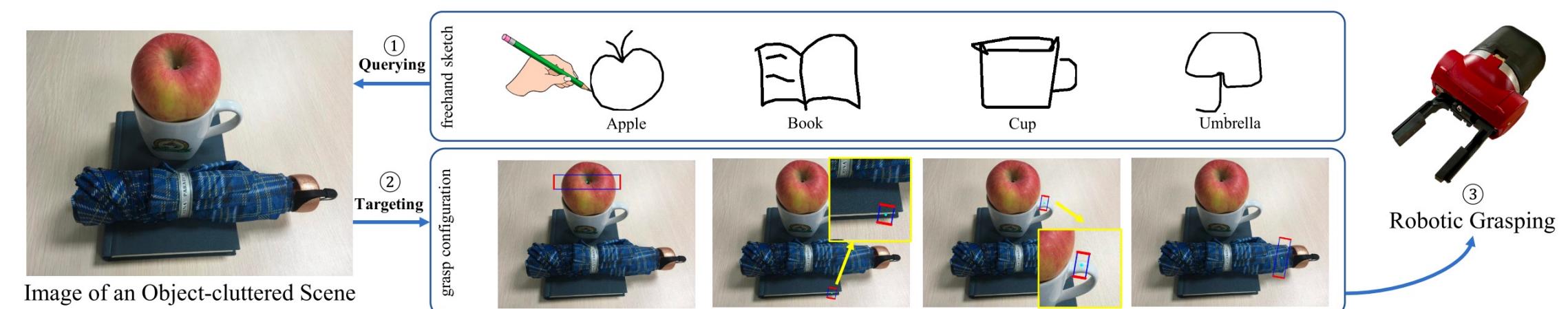


Object Detection



Reconstruction

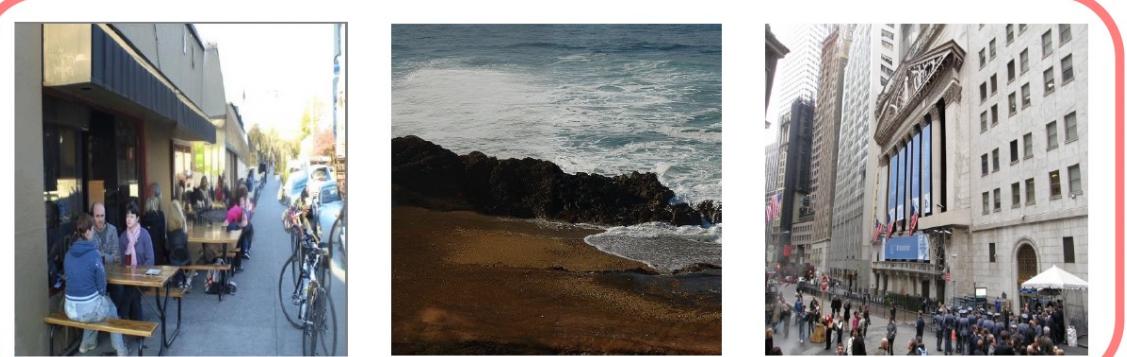
Robotic Grasping



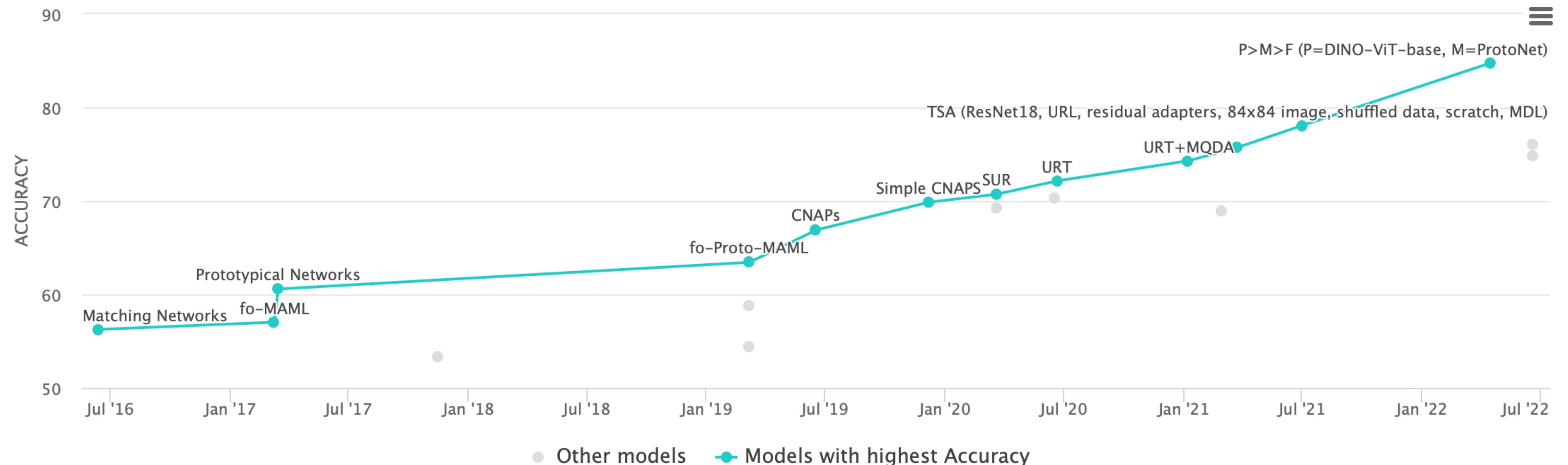
In-context Learning



LDM
(a) Frozen LDM for
inpainting-based generation



Few-shot Learning: Current State



Leaderboard on Meta-Dataset. Image credit: paperswithcode.com

What is Required for Few-Shot Learning?

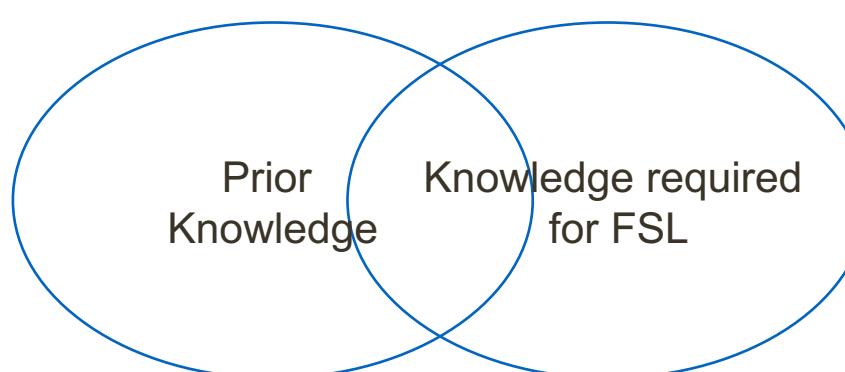
- Prior knowledge

- A Large and relevant dataset



credit: GluonCV

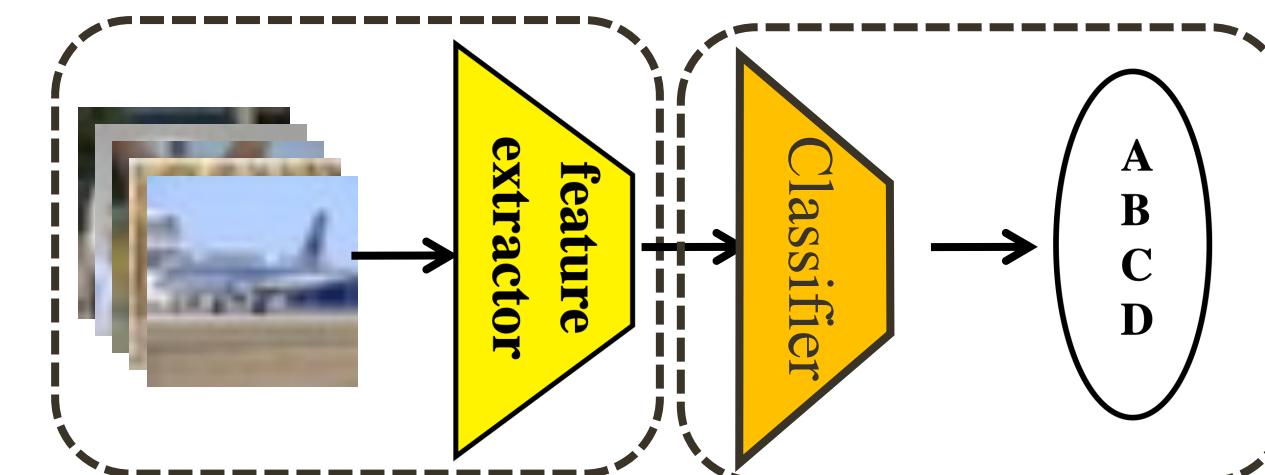
- Adaptable Knowledge



- Fast Adaptation Capacity

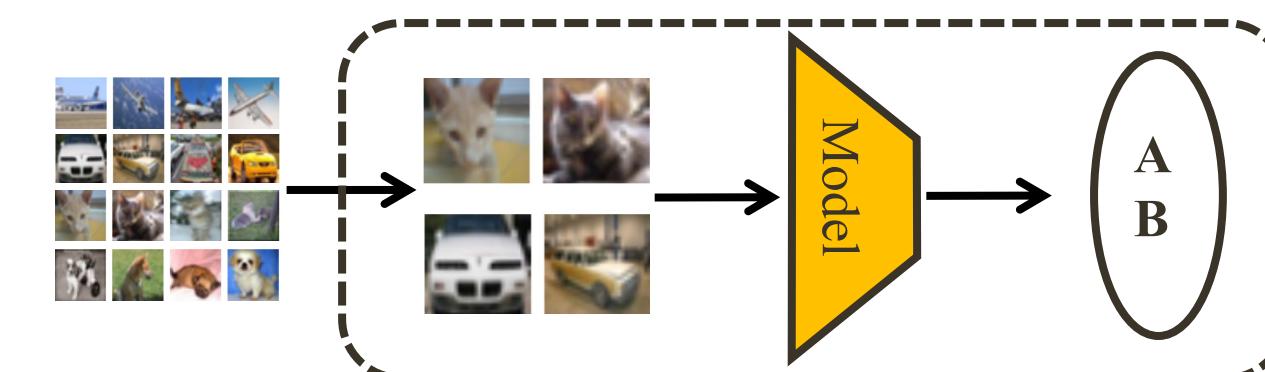
- Pre-train and fine-tune:

Reducing learning difficulty

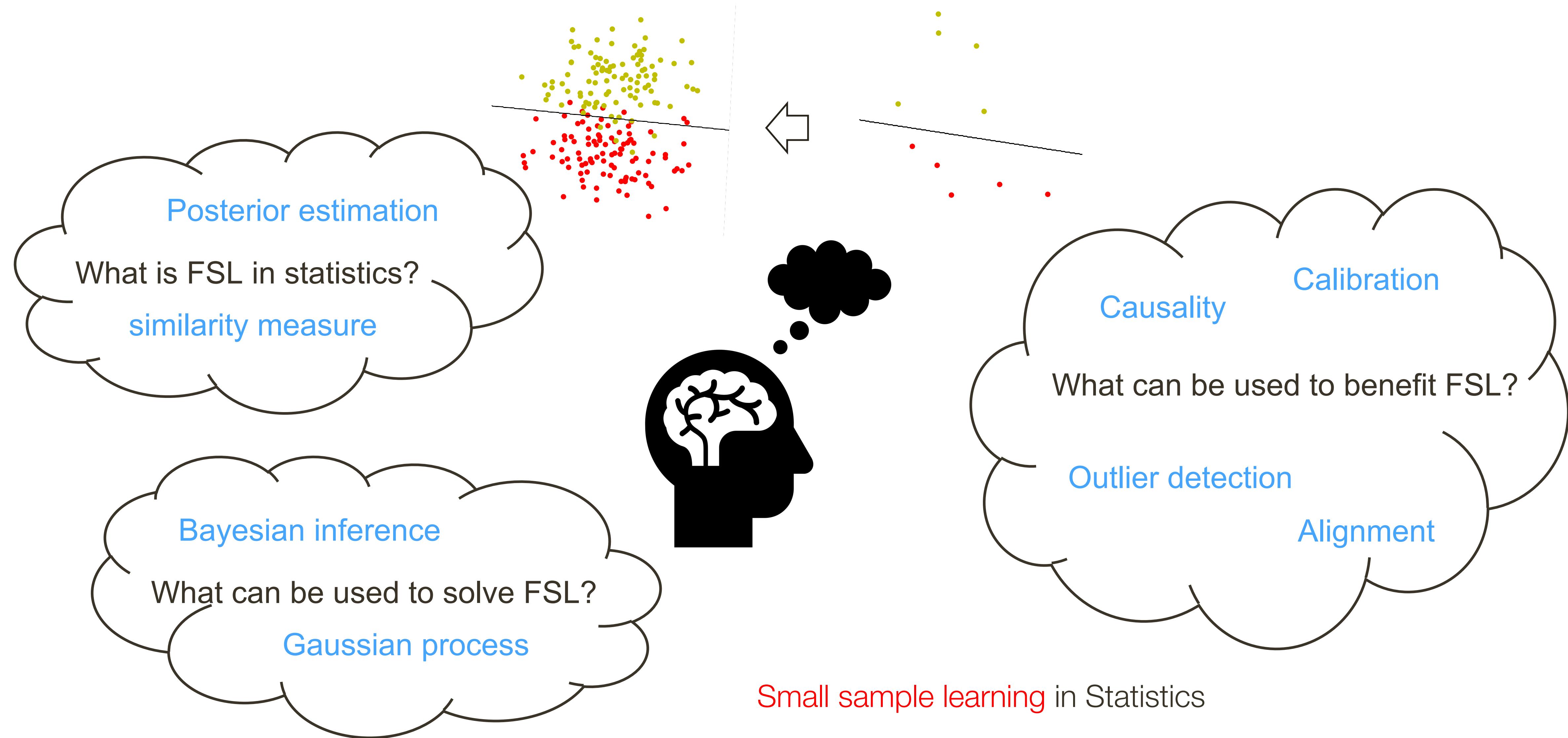


- Meta-learning:

Learning to learn from few examples



When Statistics Meets Few-Shot Learning



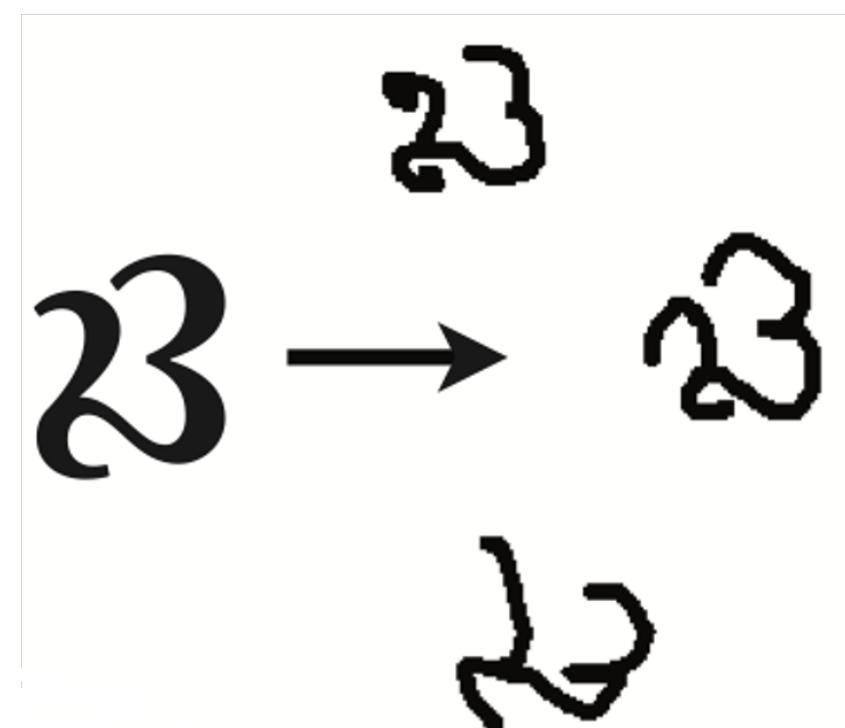
Few-Shot Learning

- Learning from base data
 - Causality
 - Similarity Measurement
 - Neural Collapse
- Adaptation on novel data
- FSL in 2020s

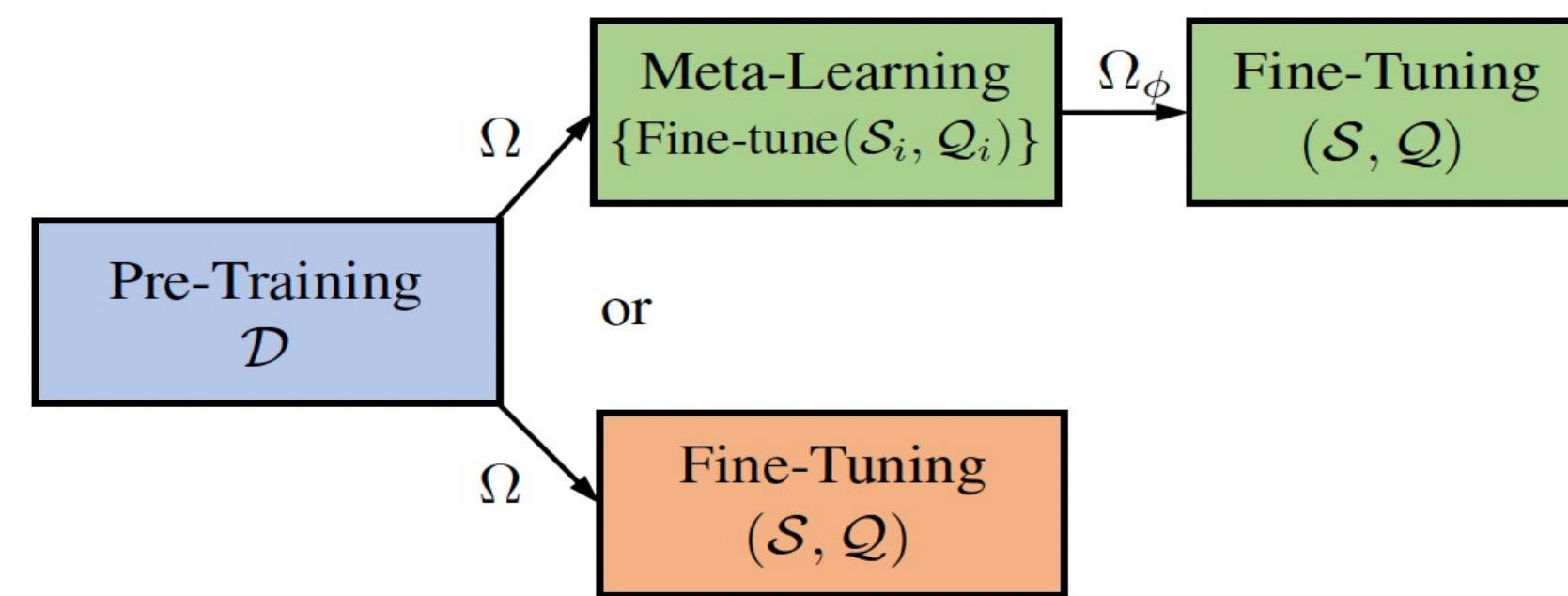
The Key Idea of “Causality”

Causality v.s. Correlation:

Find features that *determines* the class instead of *correlated* with the class.

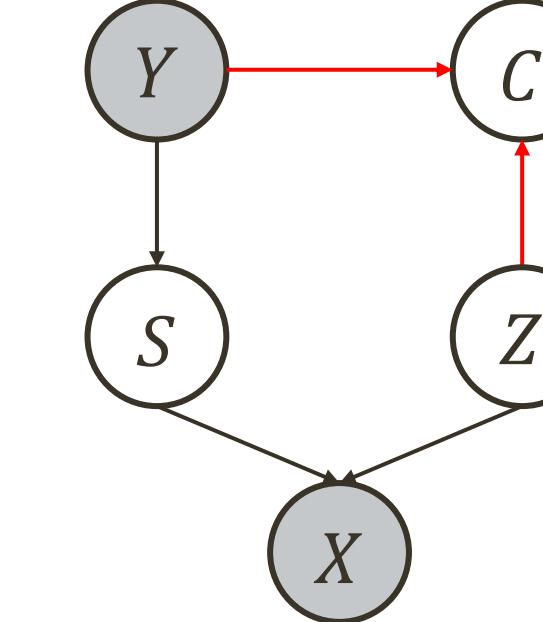


Causality as part of probabilistic
program induction for FSL

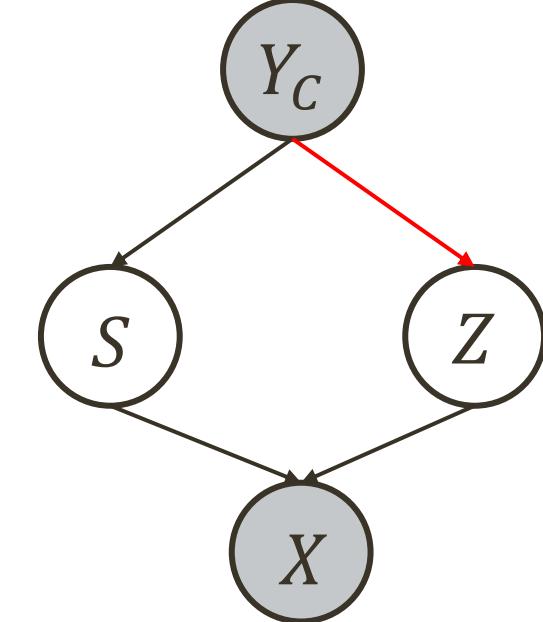


credit: [Brenden-CS2015]

Causality to remove the dependency of
pre-knowledge from pre-trained dataset

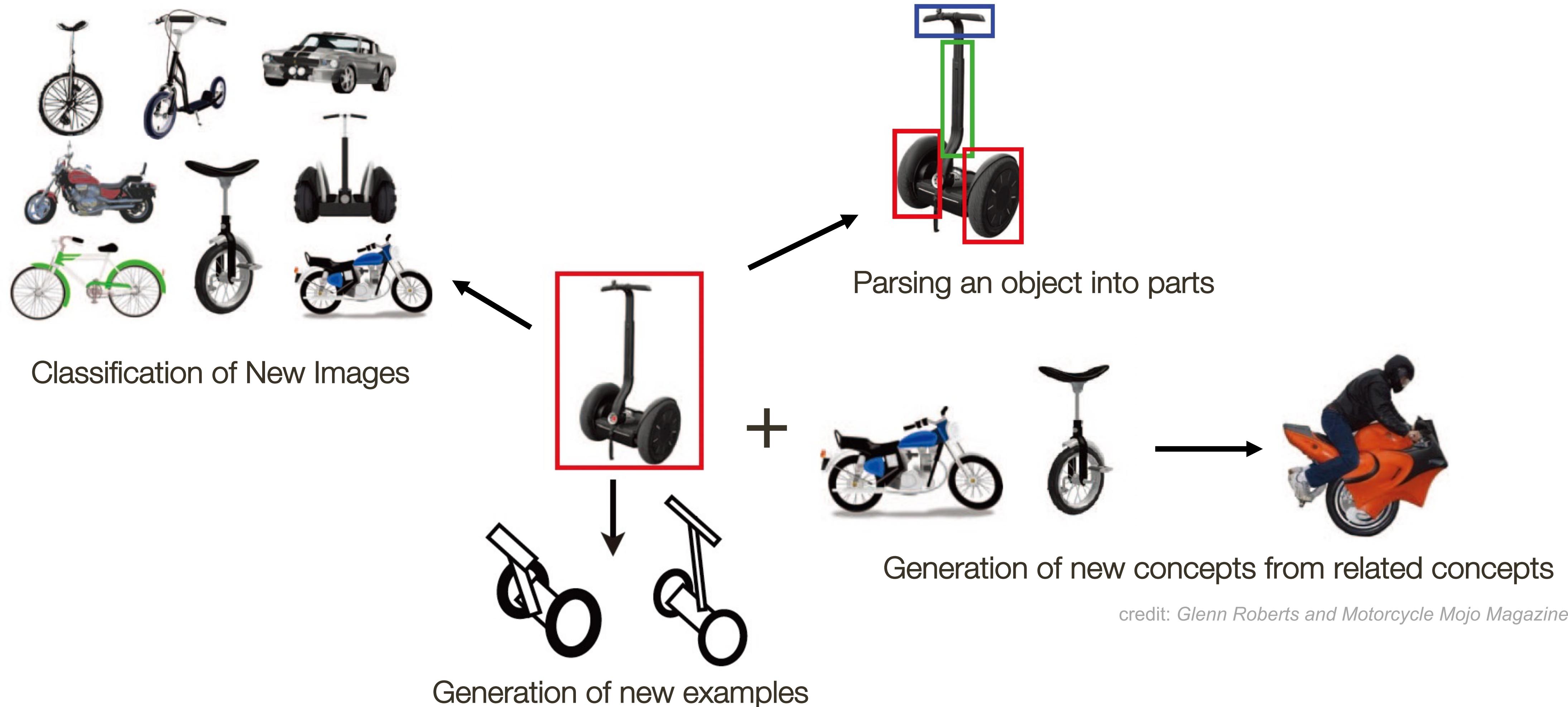


Causality to remove spurious feature

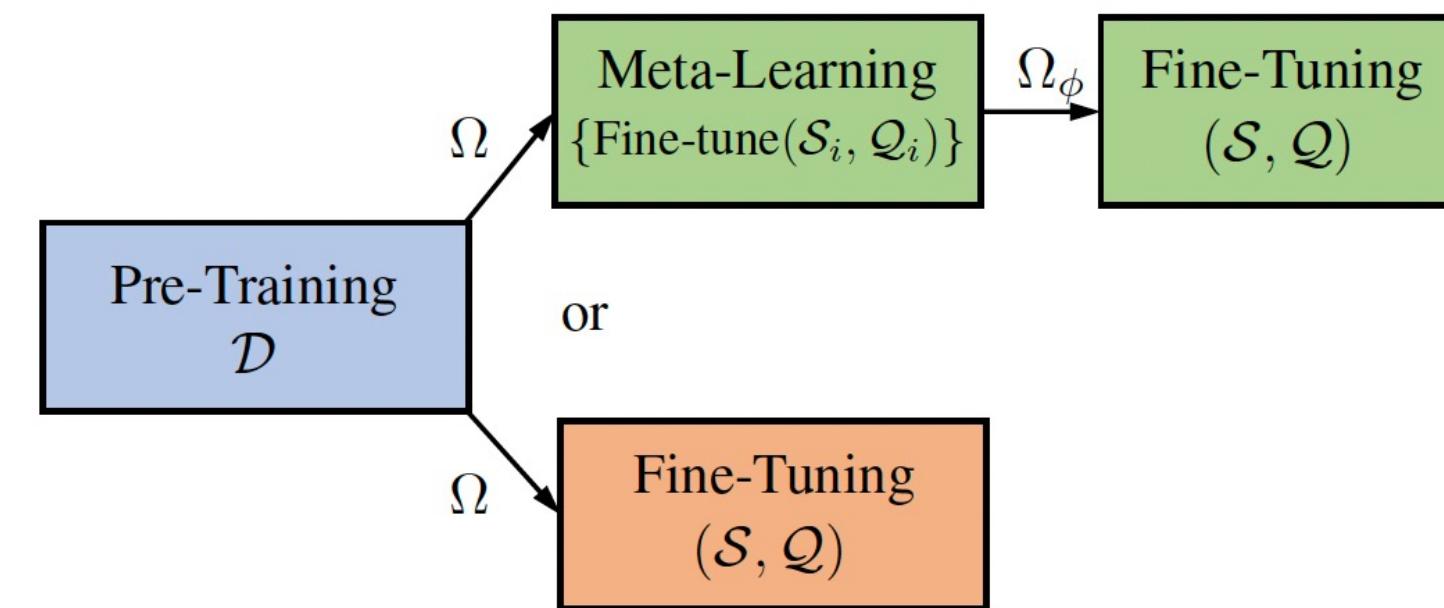


Causality as part of probabilistic program induction

Three key ideas: *compositionality*, **causality**, *learning to learn*, *in learning new concepts from few examples*.



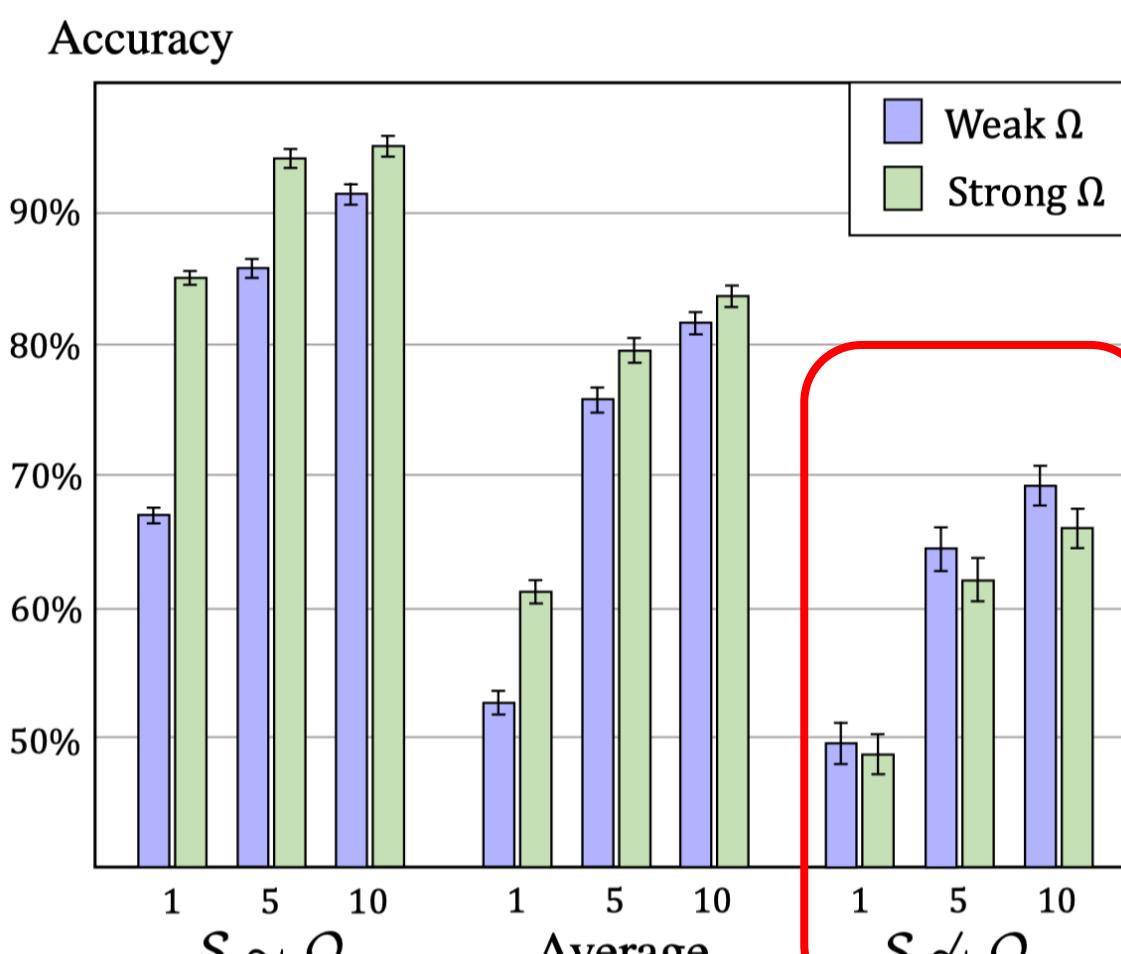
Influence of Pre-trained Knowledge



D = ImageNet, Ω = ResNet

D = Wikipedia, Ω = BERT in natural language processing

credit: [Yue-NeurIPS2020]



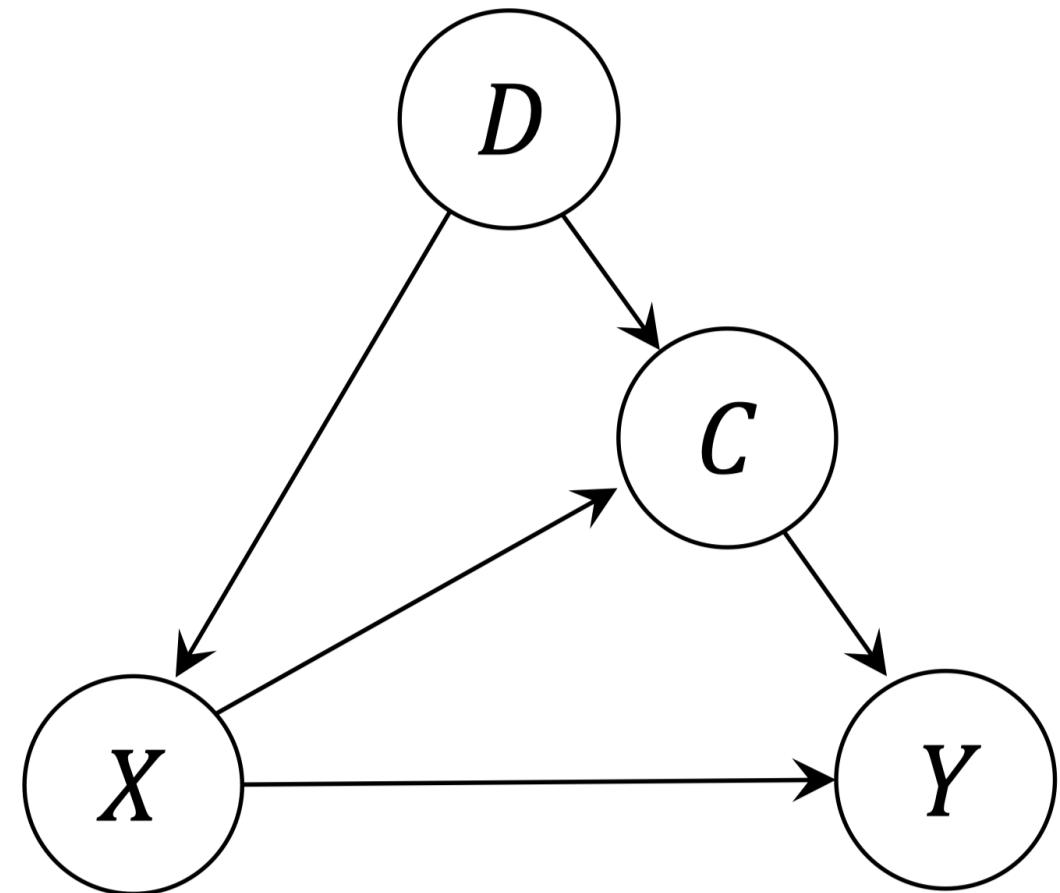
credit: [Yue-NeurIPS2020]



credit: [Yue-NeurIPS2020]

the pre-trained knowledge can do evil in FSL.

Causality to remove the dependency of pre-knowledge

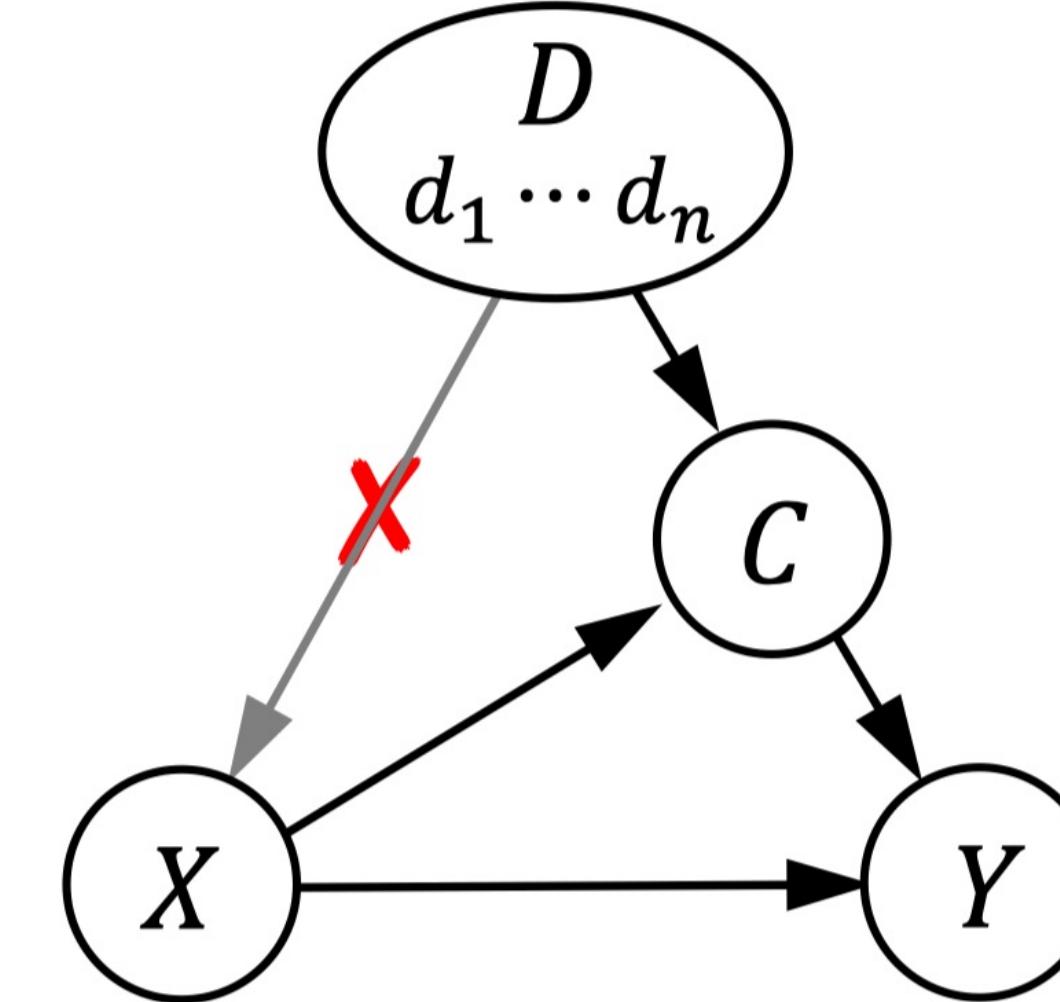


credit: [Yue-NeurIPS2020]

$$D \rightarrow X$$

$$D \rightarrow C \leftarrow X$$

$$X \rightarrow Y \leftarrow C$$



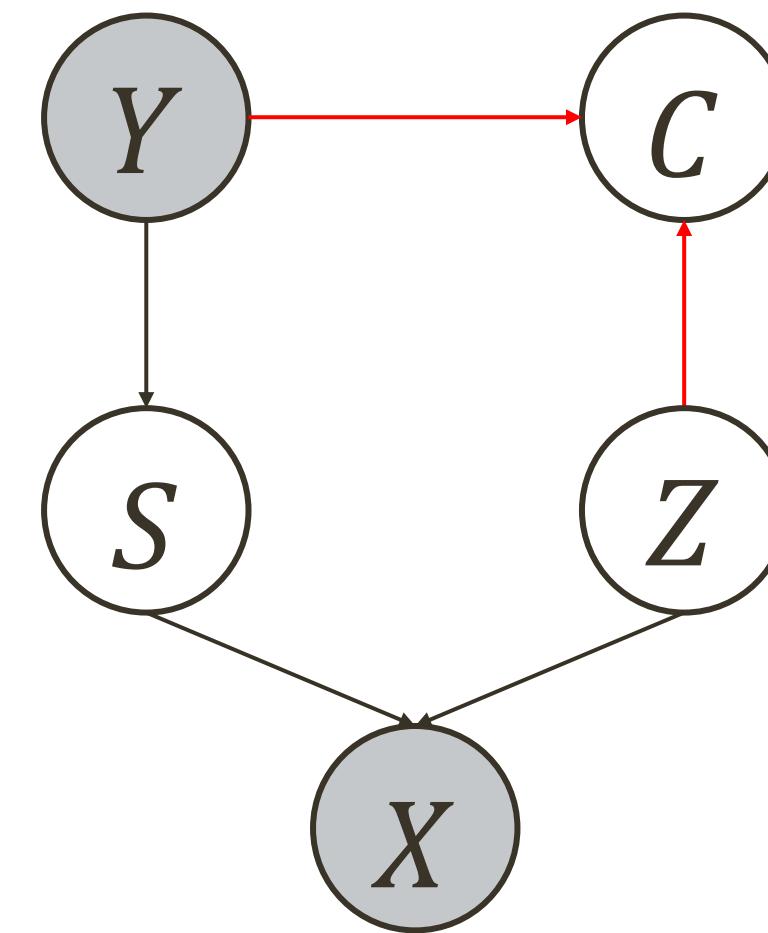
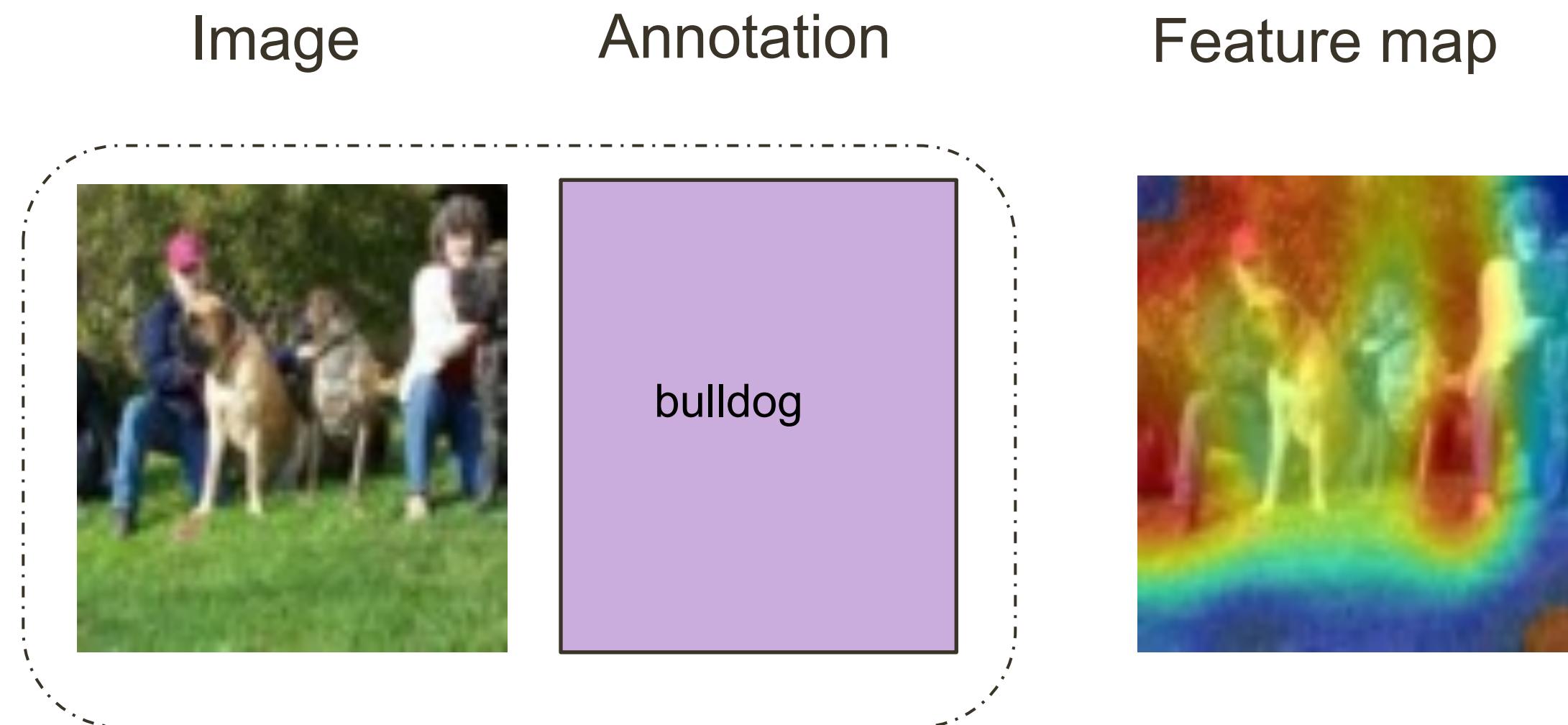
credit: [Yue-NeurIPS2020]

Causal Intervention by backdoor adjustment

$$P(Y|do(X = \mathbf{x})) = \sum_d P(Y|X = \mathbf{x}, D = d, C = g(\mathbf{x}, d)) P(D = d)$$

Learn $P(Y|do(X))$ instead of $P(Y|X)$.

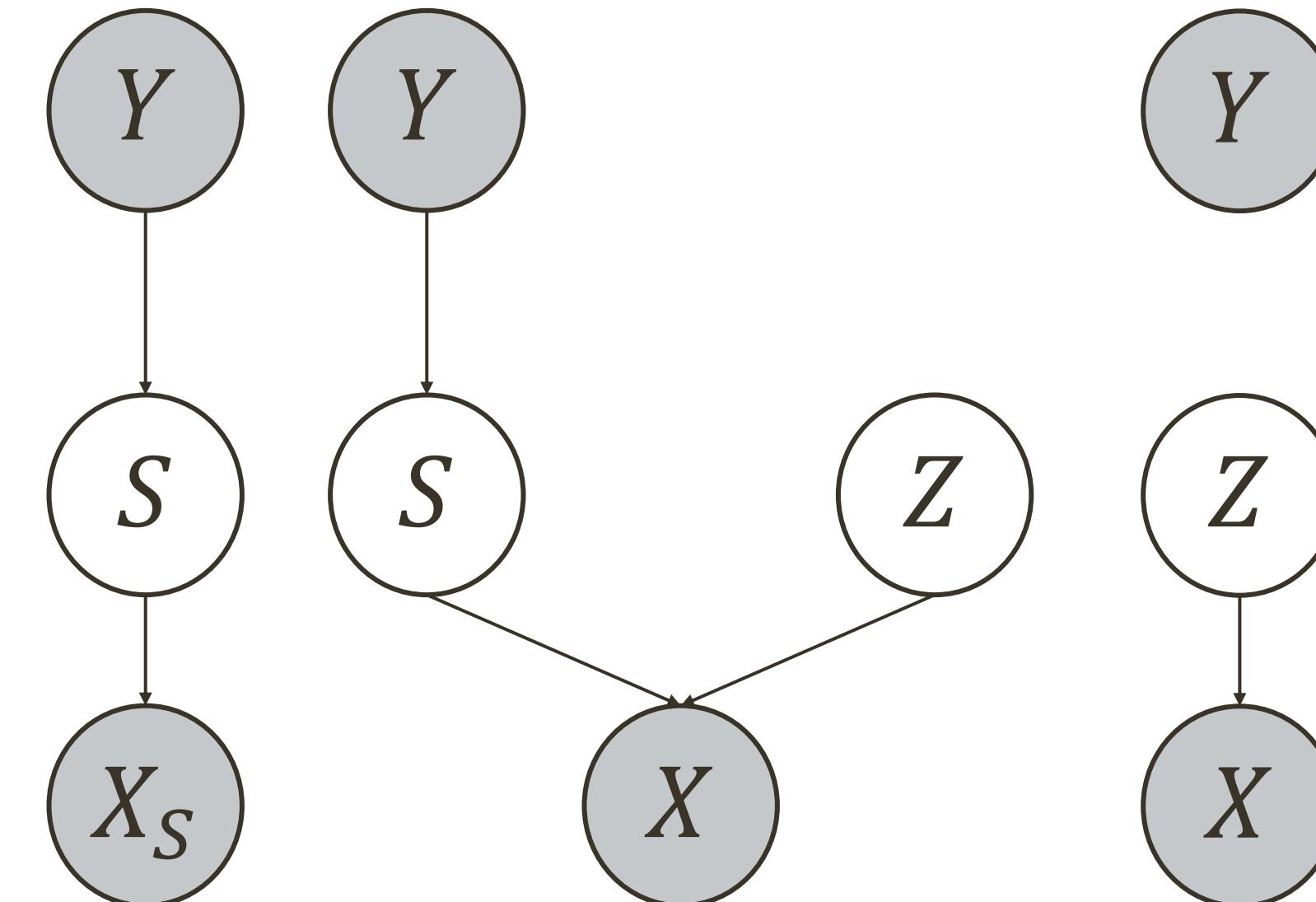
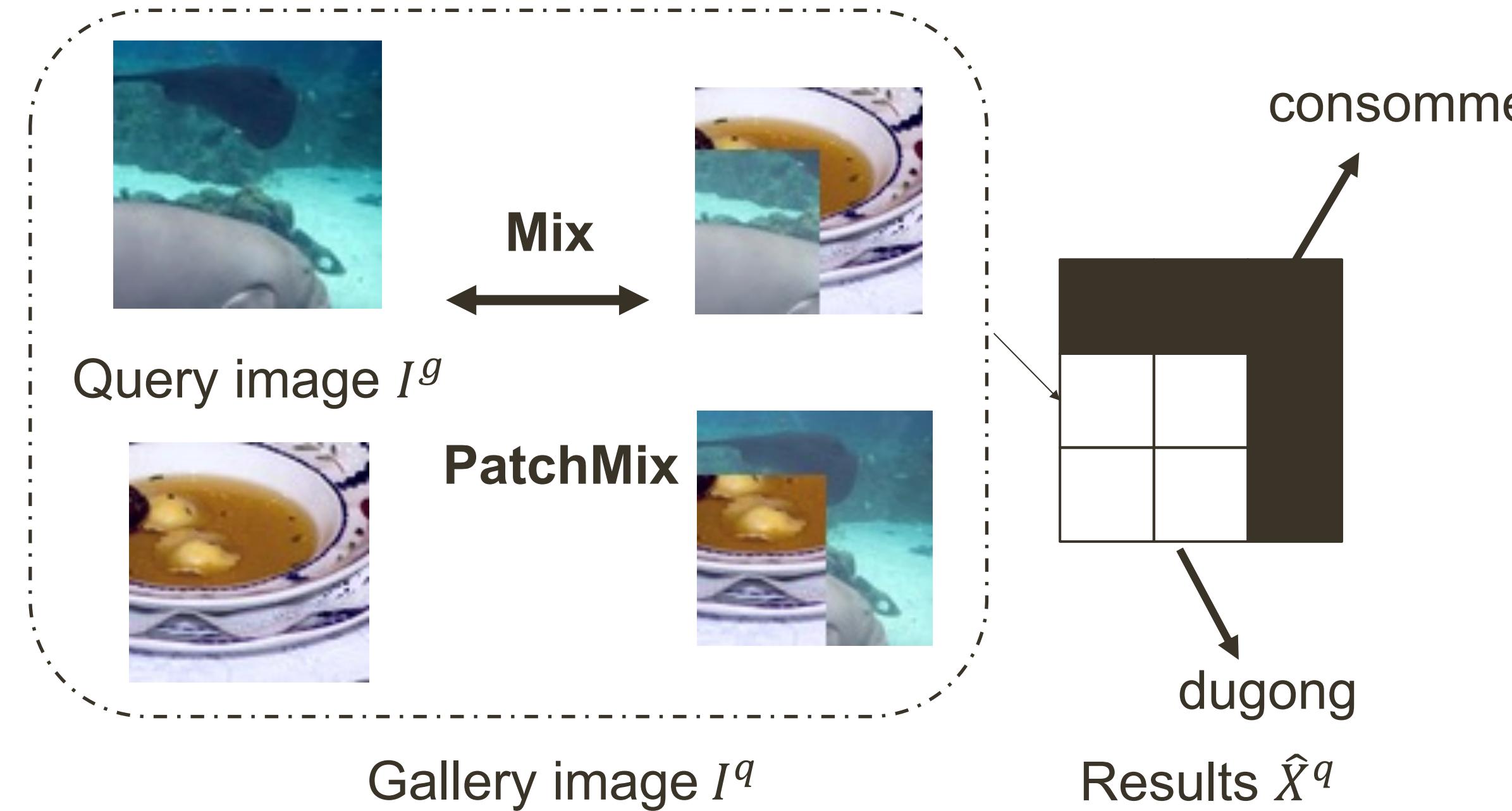
Structural Causal Model in FSL: data selection perspective



Class Y , Causal feature S and non-causal feature Z ,
 $C=1$ (training set)/0 (testing set)

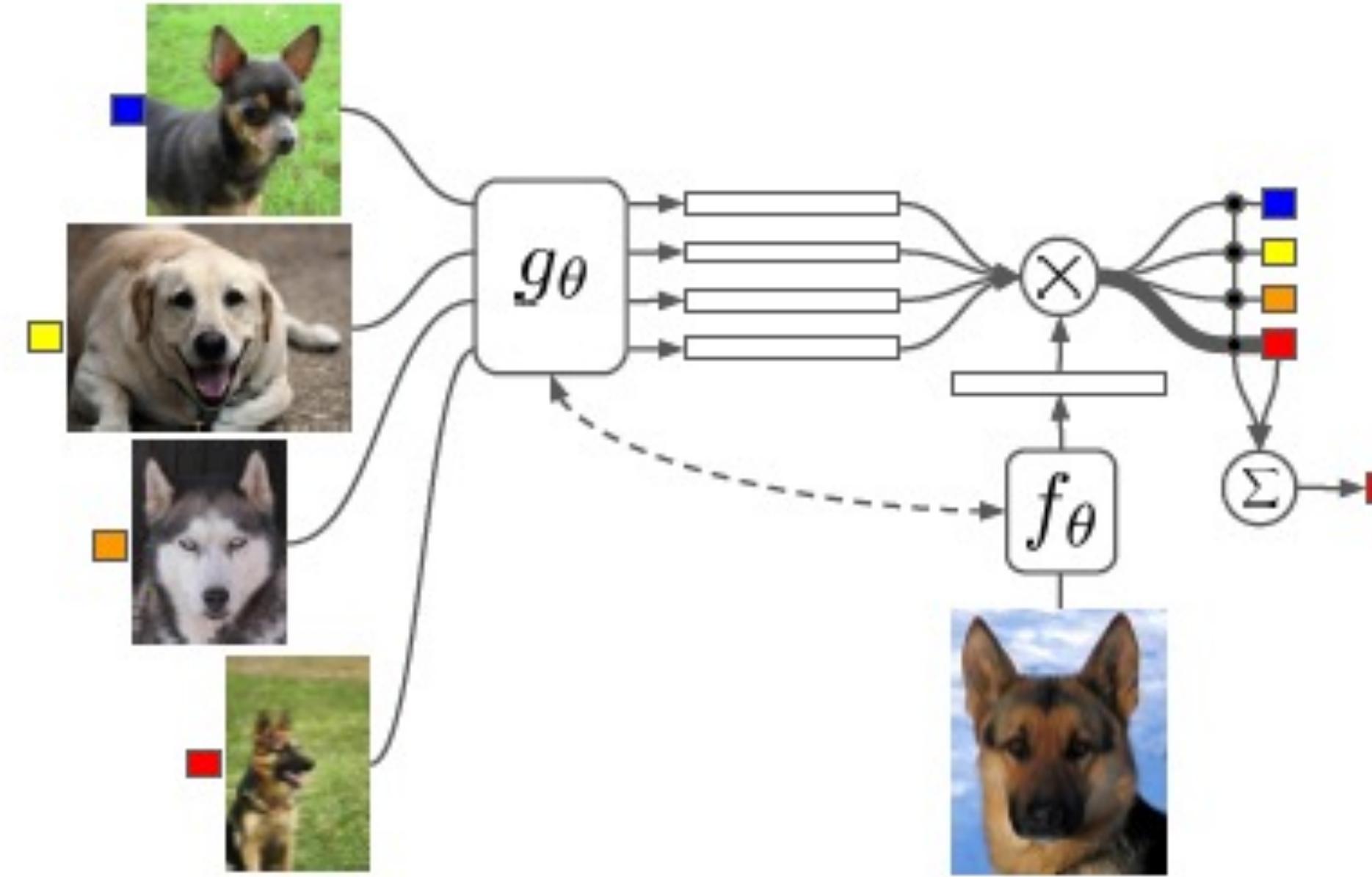
The data selection bias can induce spurious correlation between causal and non-causal features.

Structural Causal Model in FSL: data selection perspective

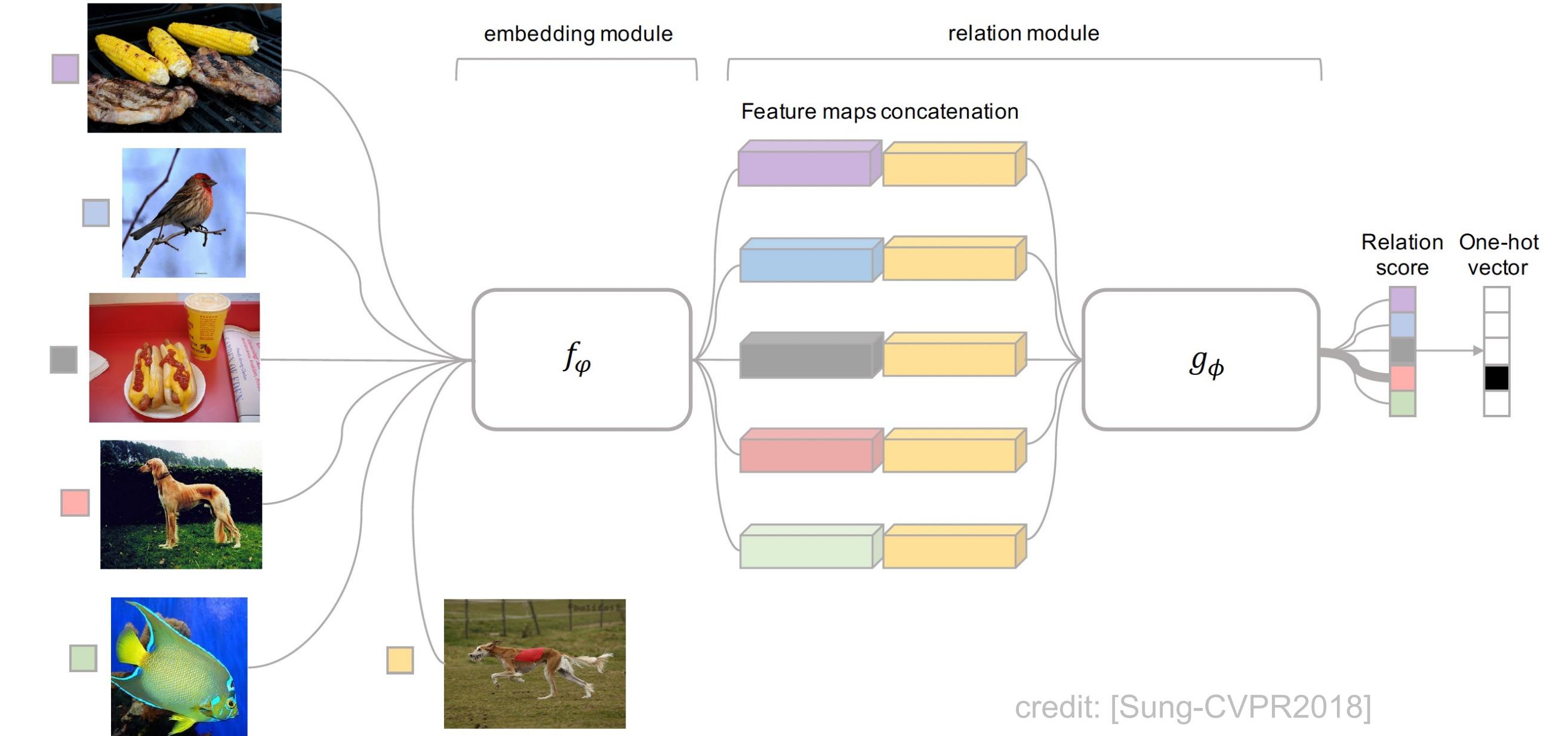


Solving spurious correlation problem by exchanging visual & label information between images.

The Key Idea of “Learning better metrics”



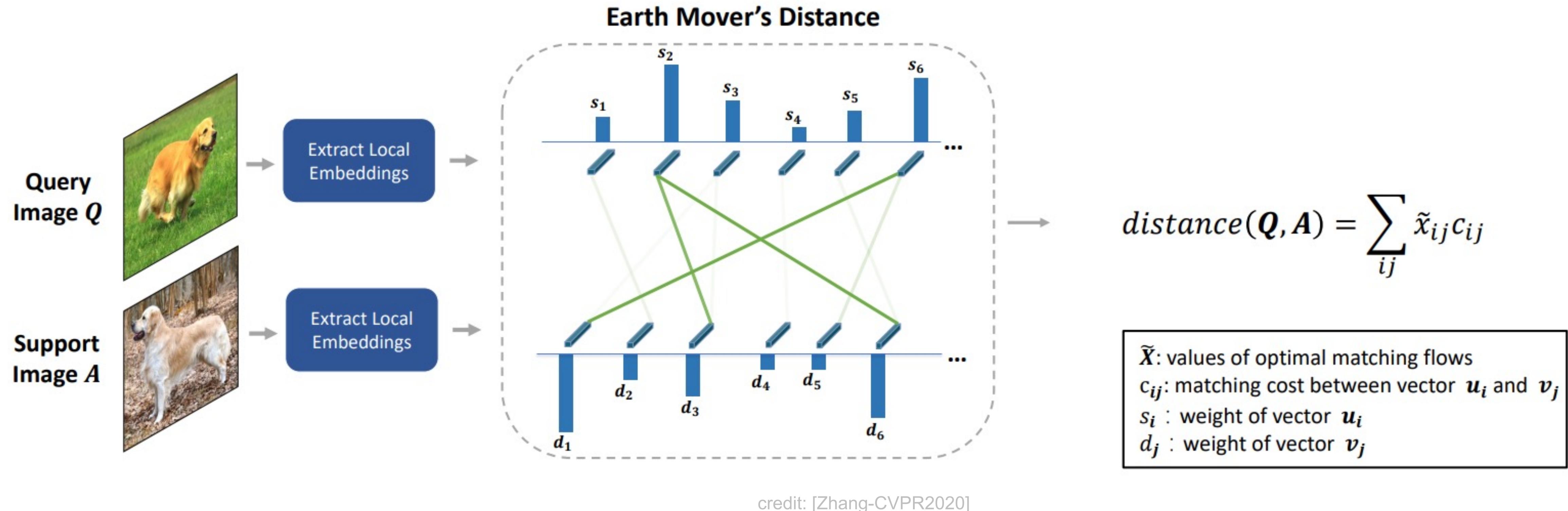
Matching Network



Relation Network

Can we learn **the better metrics** by probability distribution?

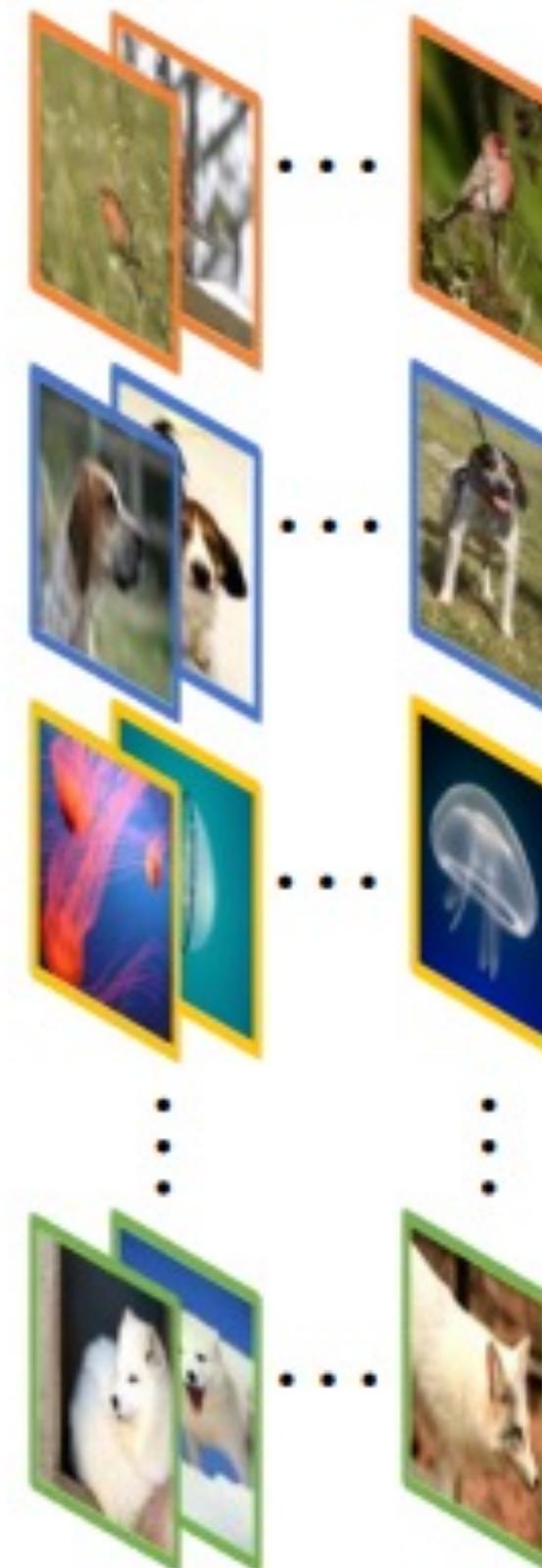
Earth Mover's Distance



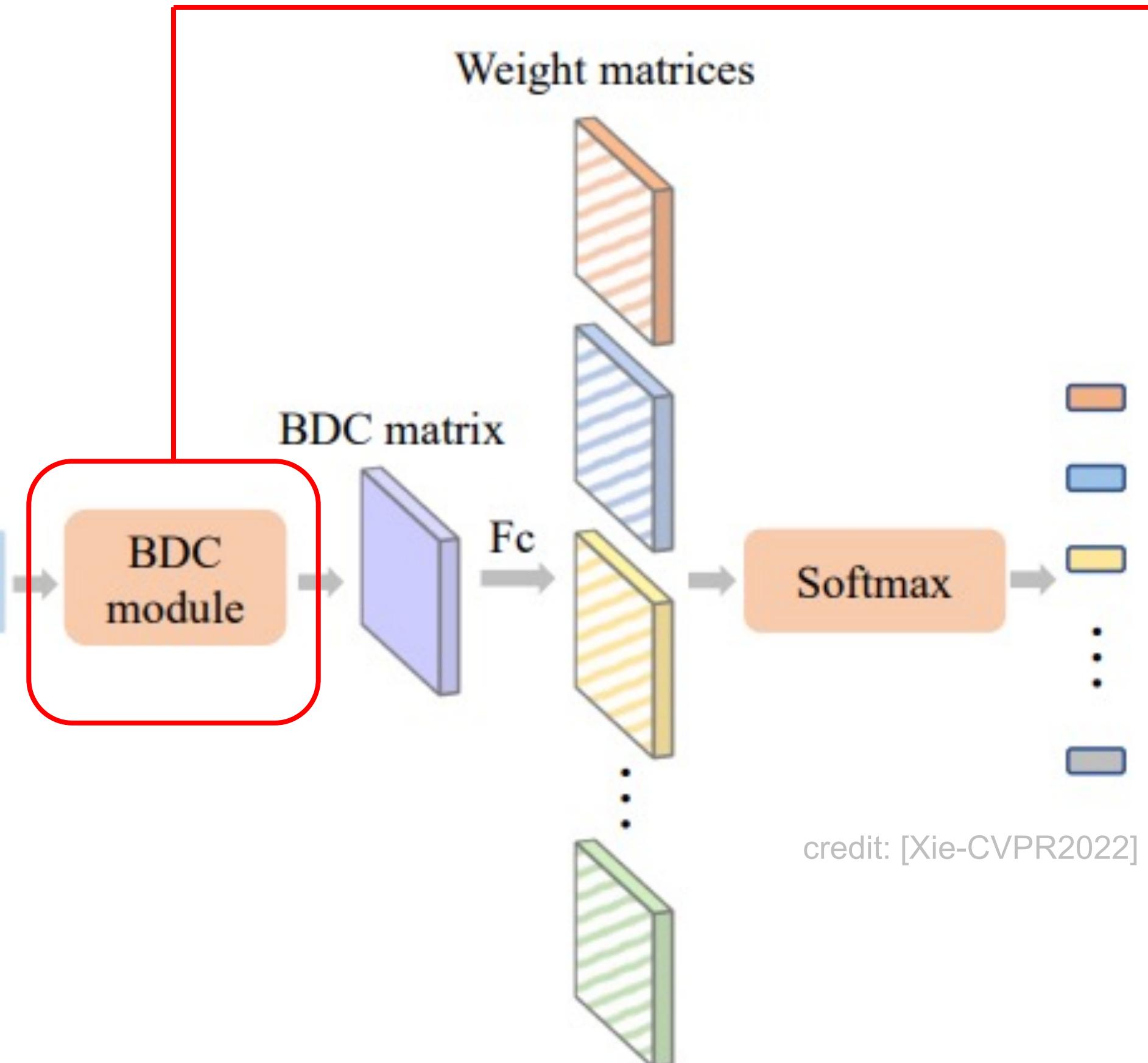
credit: [Zhang-CVPR2020]

Brownian Distance Covariance in FSL

Whole meta-training set of all classes



Backbone



BDC metric ρ between two sets
 $\{(x_1, y_1), \dots, (x_m, y_m)\}$:

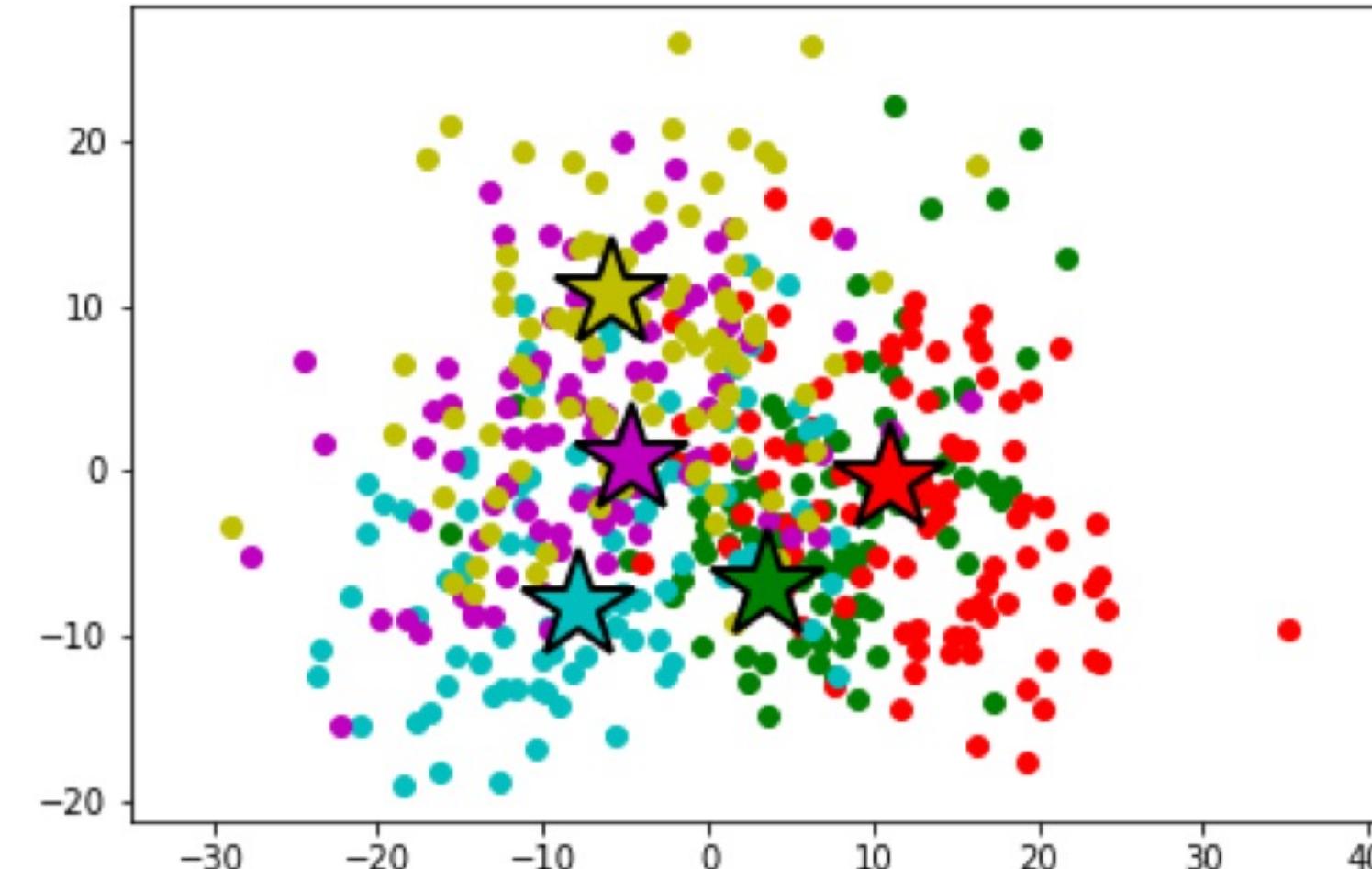
$$\hat{A} = (\hat{a}_{kl}), \hat{a}_{kl} = \|x_k - x_l\|$$
$$\hat{B} = (\hat{b}_{kl}), \hat{b}_{kl} = \|y_k - y_l\|$$

$$\rho(X, Y) = \text{tr}(A^T B) \text{ after normalization}$$

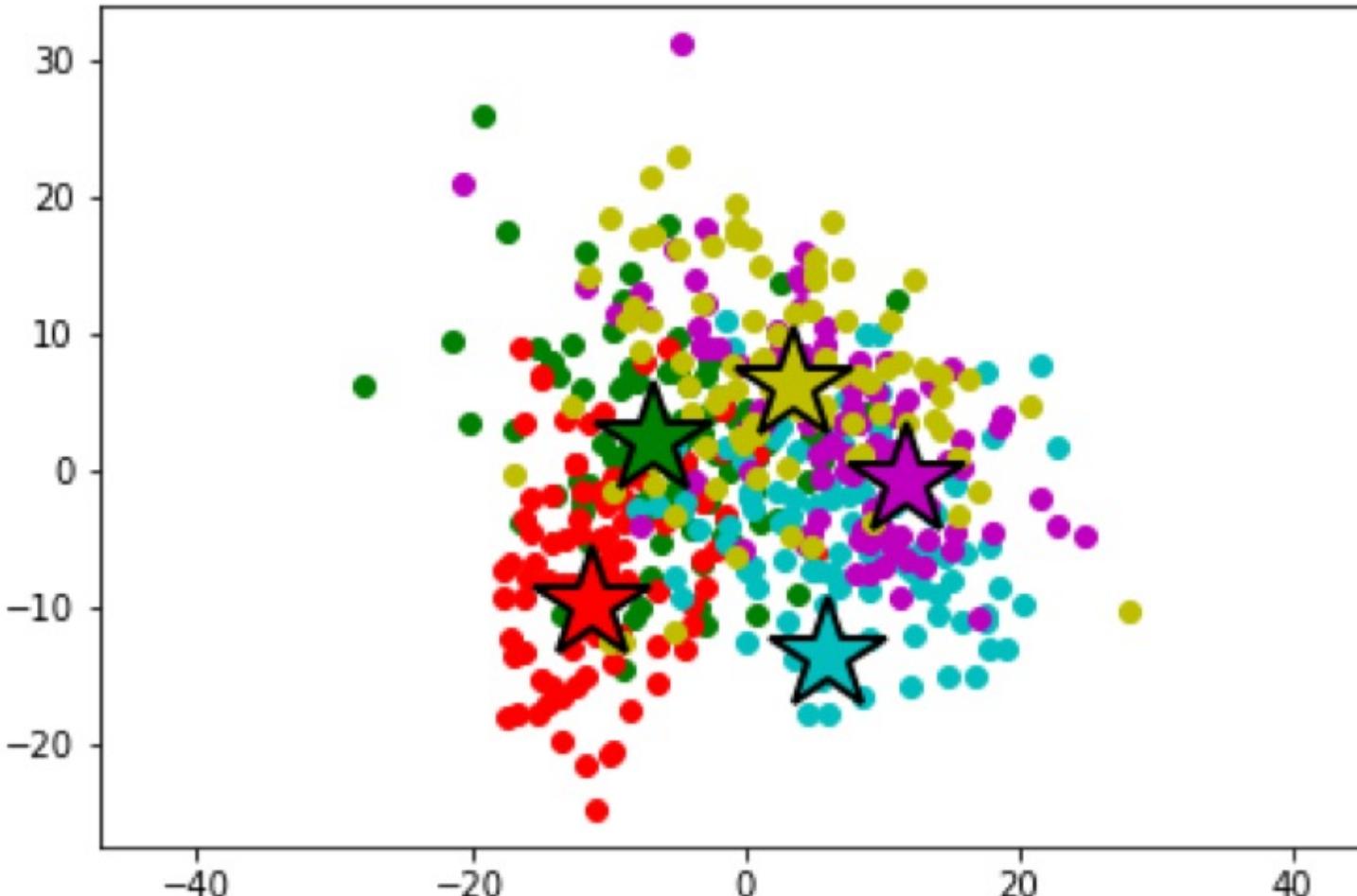
- Non-negative, 0 iff X, Y are independent
- Characterize linear and non-linear dependency
- Invariant to individual translations and orthonormal transformations

credit: [Xie-CVPR2022]

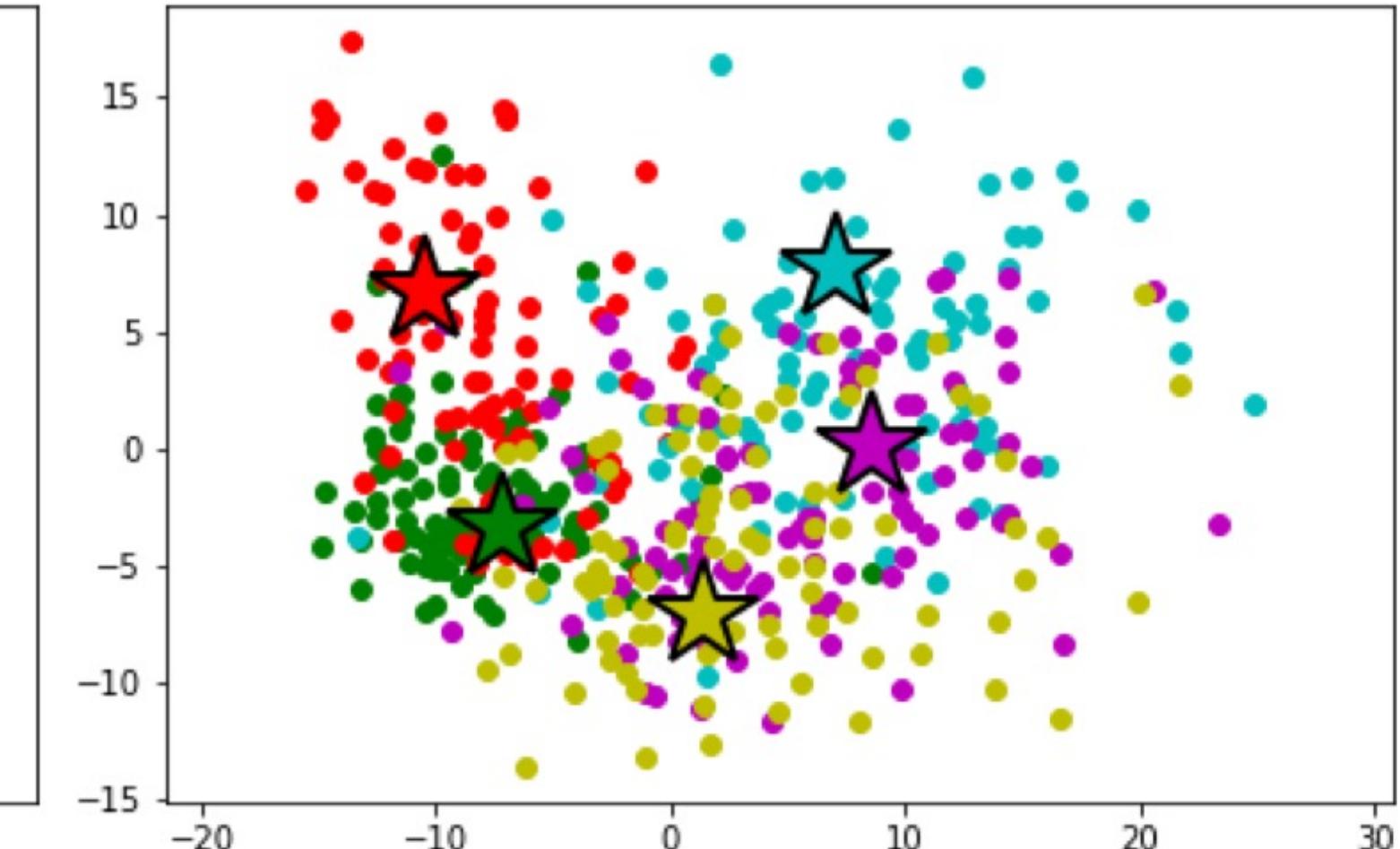
Squared root of the Euclidean distance and the Norm distance for dissimilarity measurement



Prototypical Network



Prototypical Network with ring loss



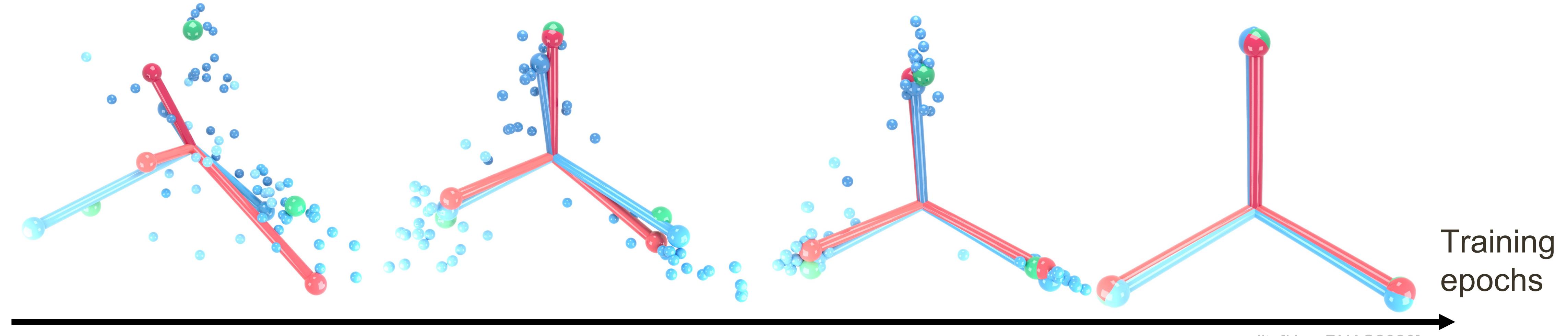
SEN

credit: [Nguyen-ECCV2020]

SEN dissimilarity between query feature z and prototype c :

$$d_s(z, c) = \sqrt{d_e(z, c) + \epsilon d_n(z, c)}, d_e(z, c) = \|z - c\|^2, d_n(z, c) = (\|z\| - \|c\|)^2$$

The Key Idea of “Neural Collapse”



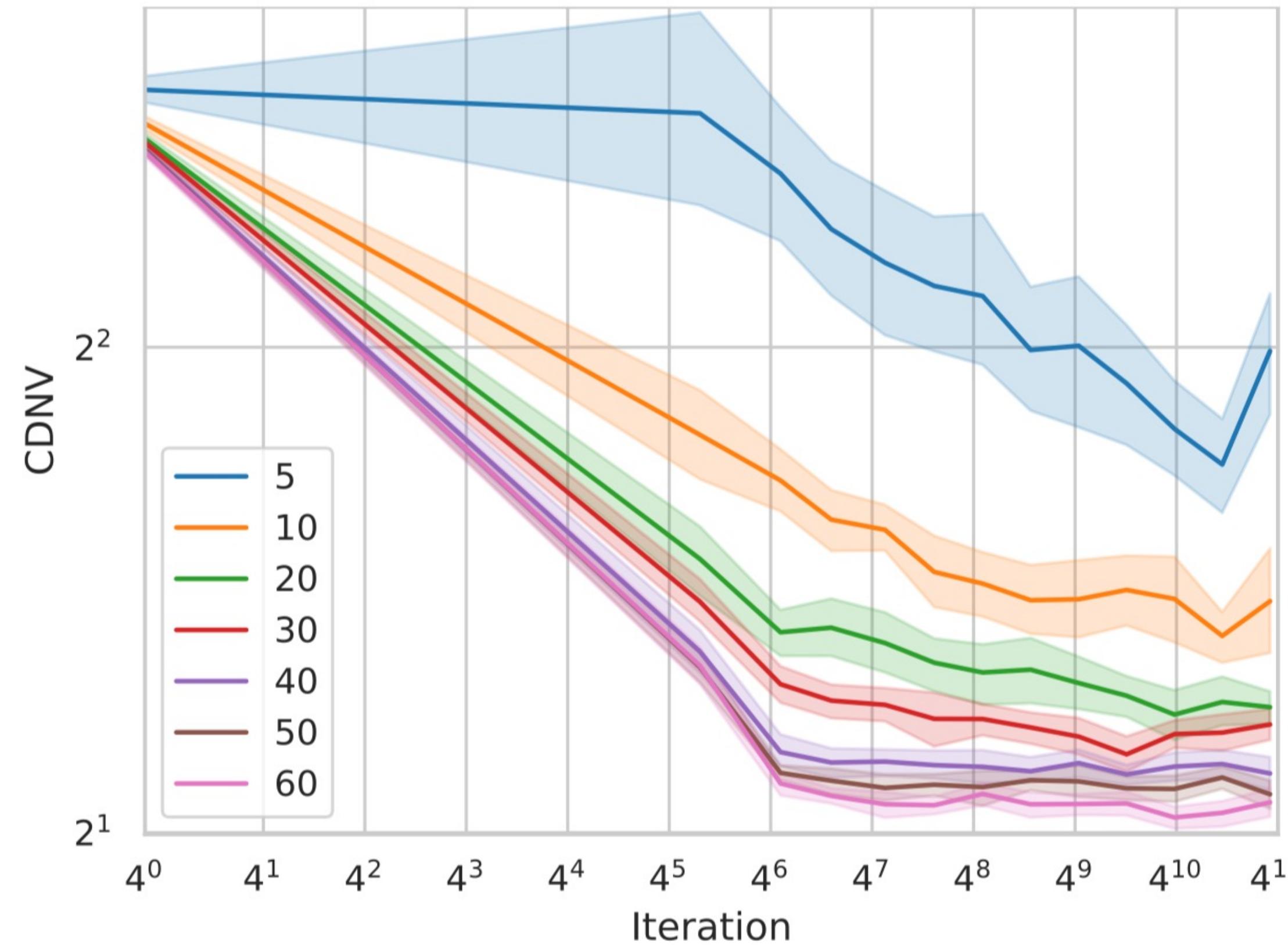
credit: [Han-PNAS2020]

Green spheres: vertices of Simplex ETF
Red ball-and-sticks: linear classifiers
Blue ball-and-sticks: class-means
Small blue spheres: last-layer features.

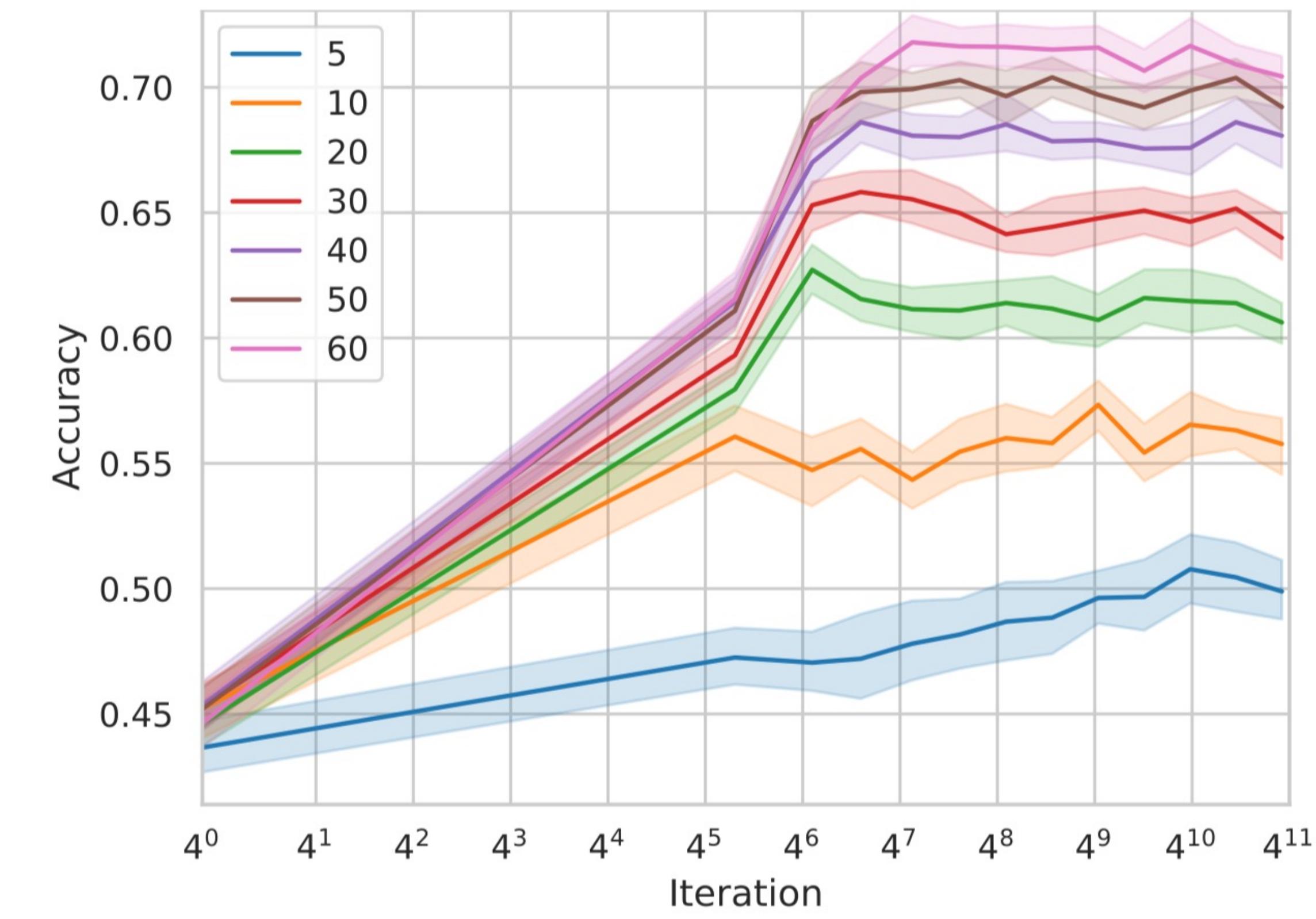
Neural Collapse is characterized by four manifestations in the classifier and last-layer activations

- NC1: Intra-class variation collapse to 0
- NC2: Class centers converge to simplex ETF
- NC3: Linear classifiers converge to class centers
- NC4: Classifier acts like nearest class center

NC in FSL: Within-class Variation Collapse



(c) target CNDV

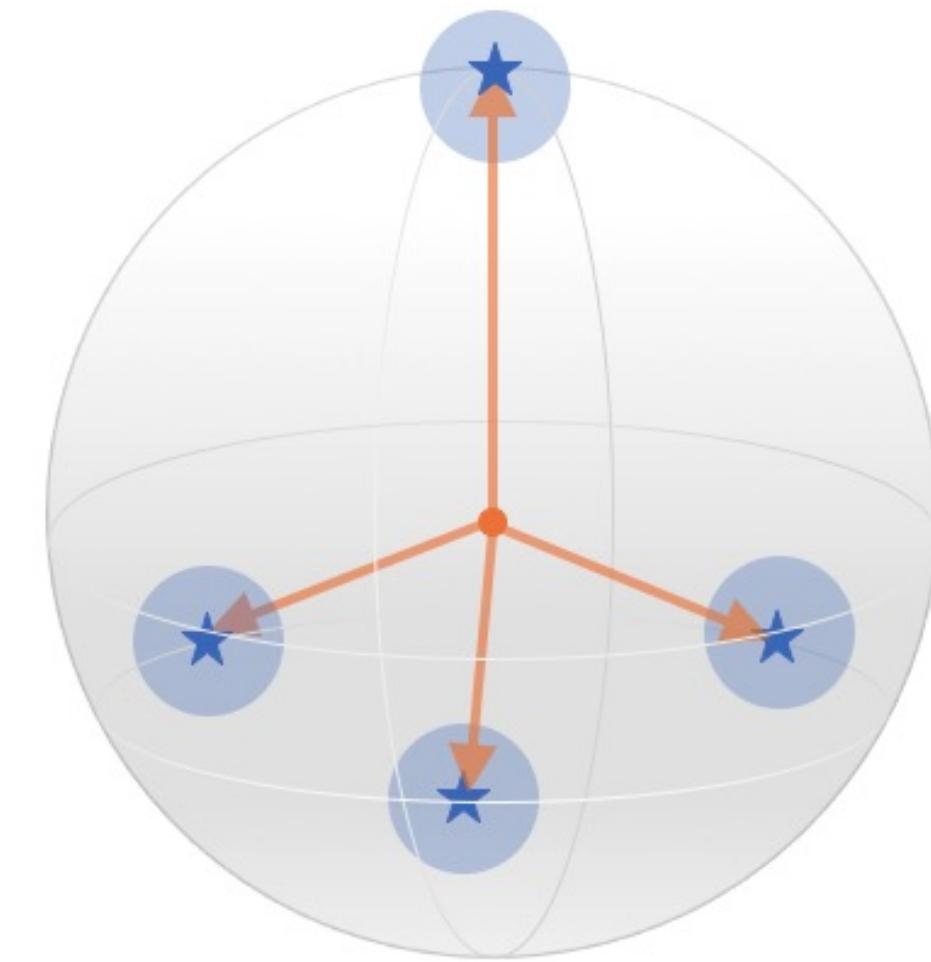


(d) target accuracy

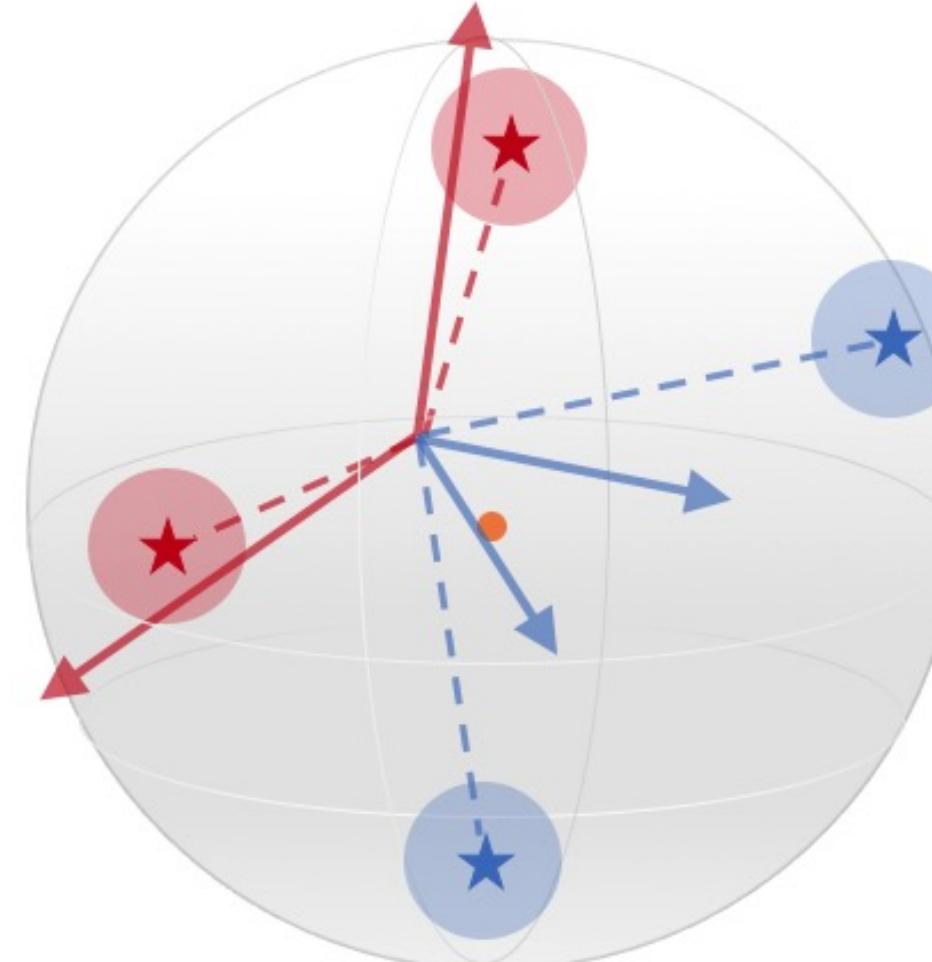
credit: [Galanti-ICLR2021]

$$\text{CDNV: } V_f(Q_1, Q_2) = \frac{\text{Var}_f(Q_1) + \text{Var}_f(Q_2)}{2\|\mu_f(Q_1) - \mu_f(Q_2)\|^2}$$

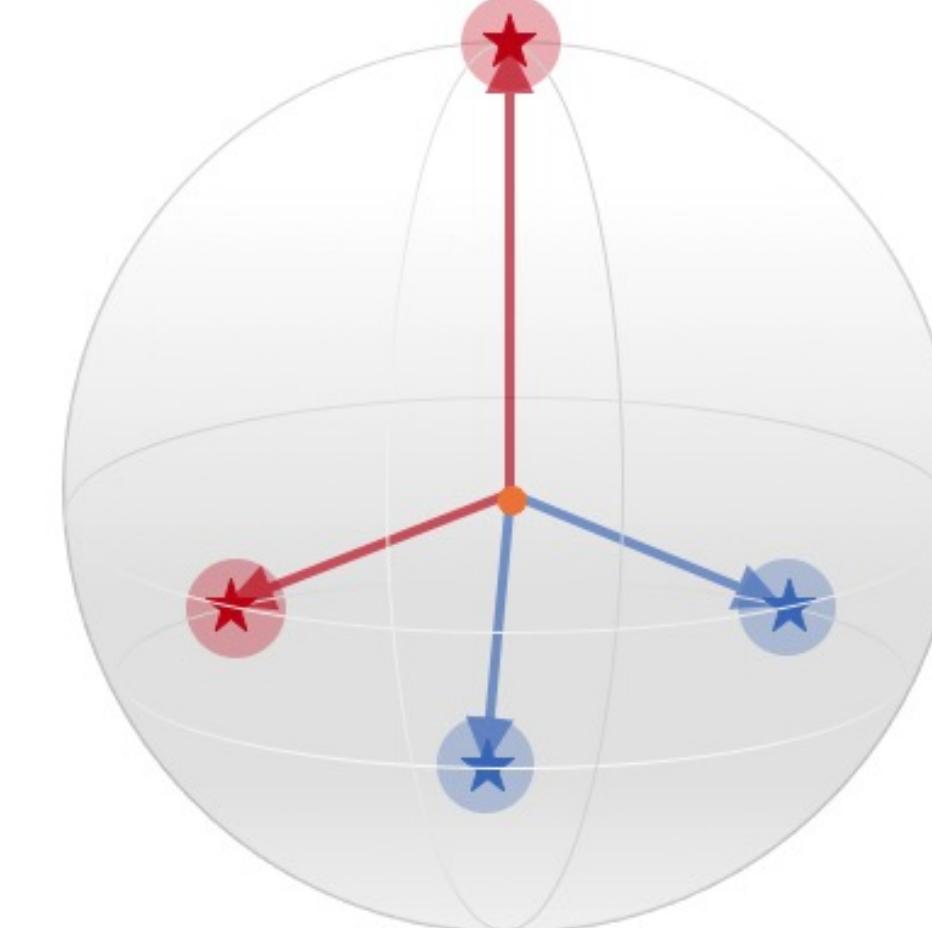
NC in imbalanced learning: stronger regularization



Balanced dataset



imbalanced dataset

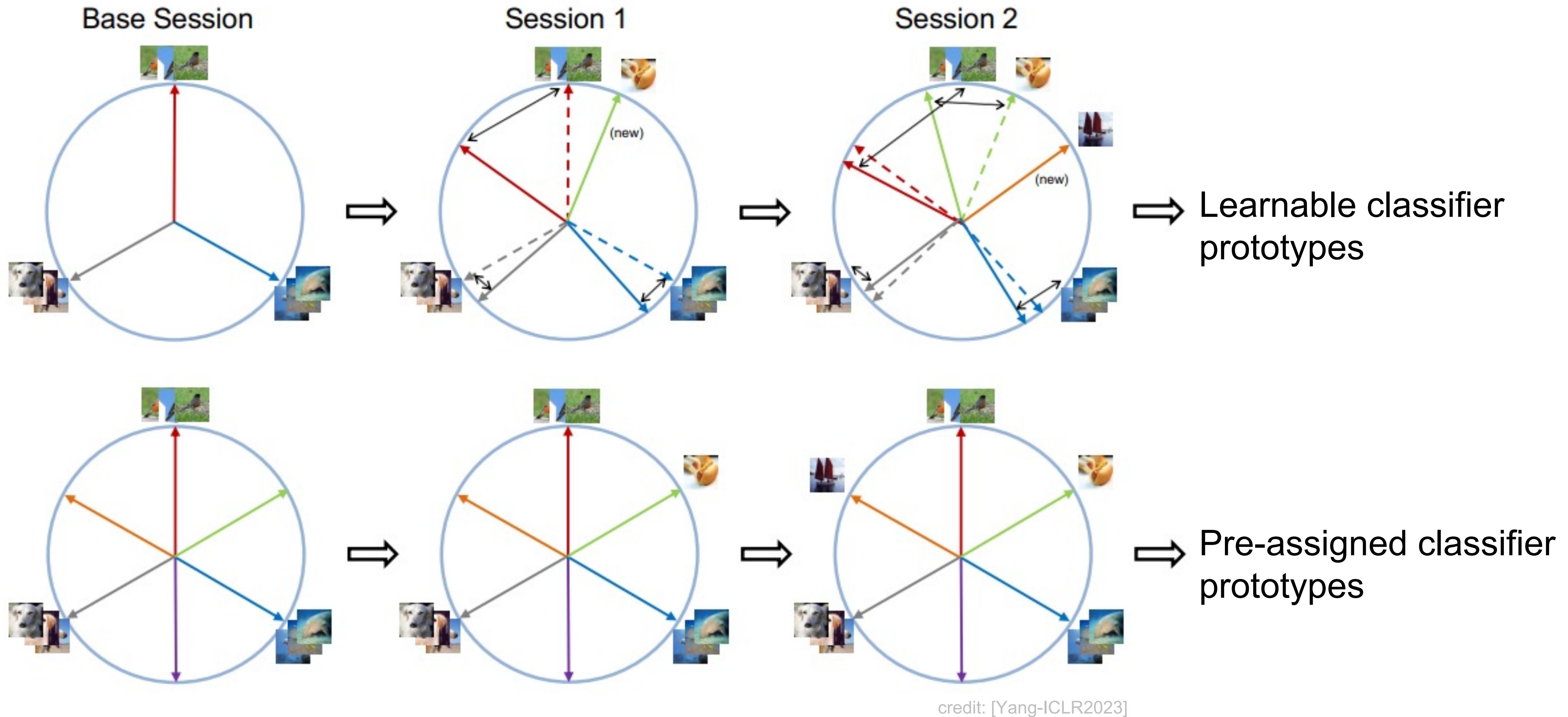


imbalanced dataset with NC

credit: [Liu-AISTATS2023]

- NC1: Intra-class variation collapse to 0
 - Compact within-class features: $L_W = \sum_{k=1}^K \sum_{y_k=k} \frac{1}{n_k} \|h_i - \mu_k\|_2^2$
- NC2: Class centers converge to simplex ETF
 - Distinct between-class features: $L_B = -\frac{1}{K} \sum_{k=1}^K \min_{k' \neq k} \arccos \frac{\langle \mu_k, \mu_{k'} \rangle}{\|\mu_k\| \cdot \|\mu_{k'}\|}$

NC in imbalanced learning: ETF classifier



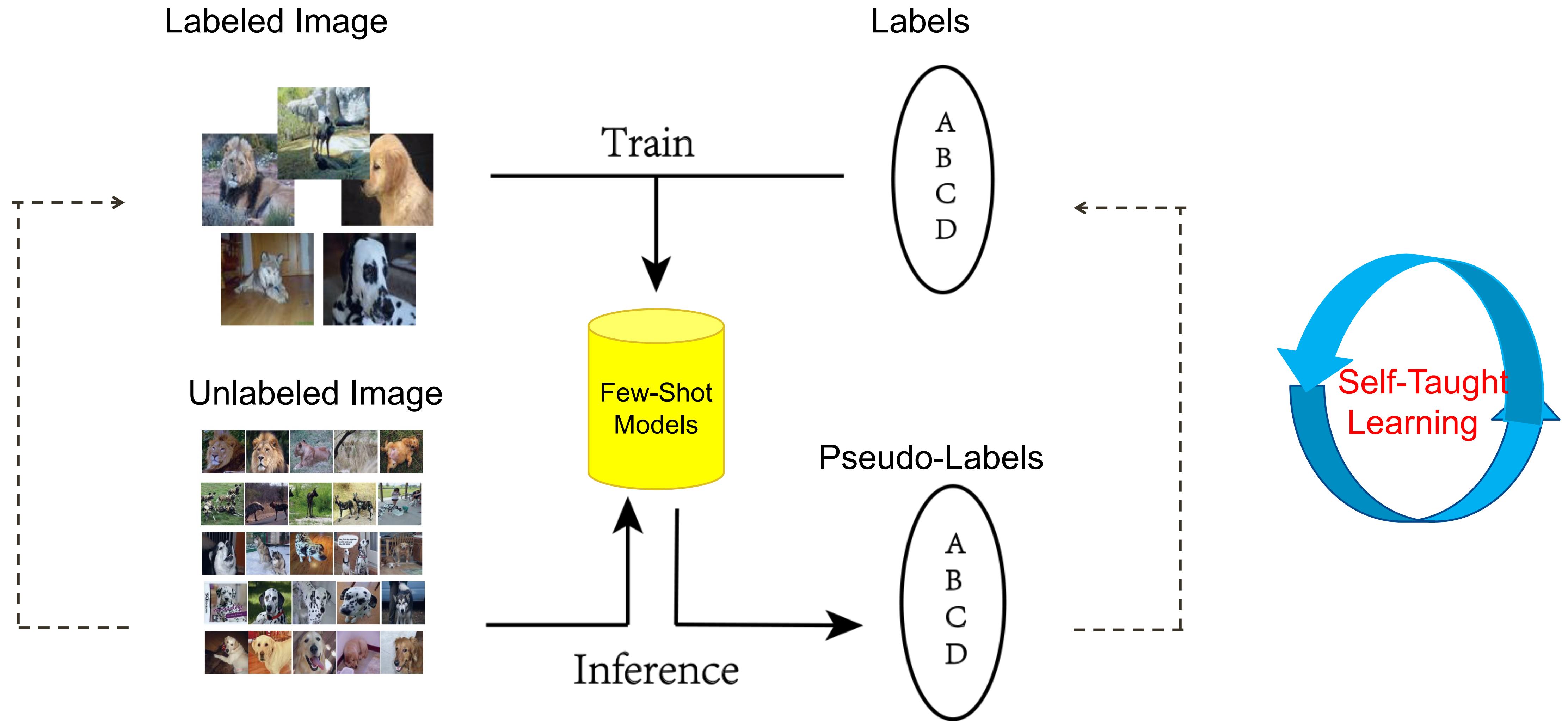
Few-Shot Learning

- Learning from base data
- **Adaptation on novel data**
 - FSL+ unlabeled data as SSL
 - Calibrating prototypes
 - Calibrating class distribution
- FSL in 2020s



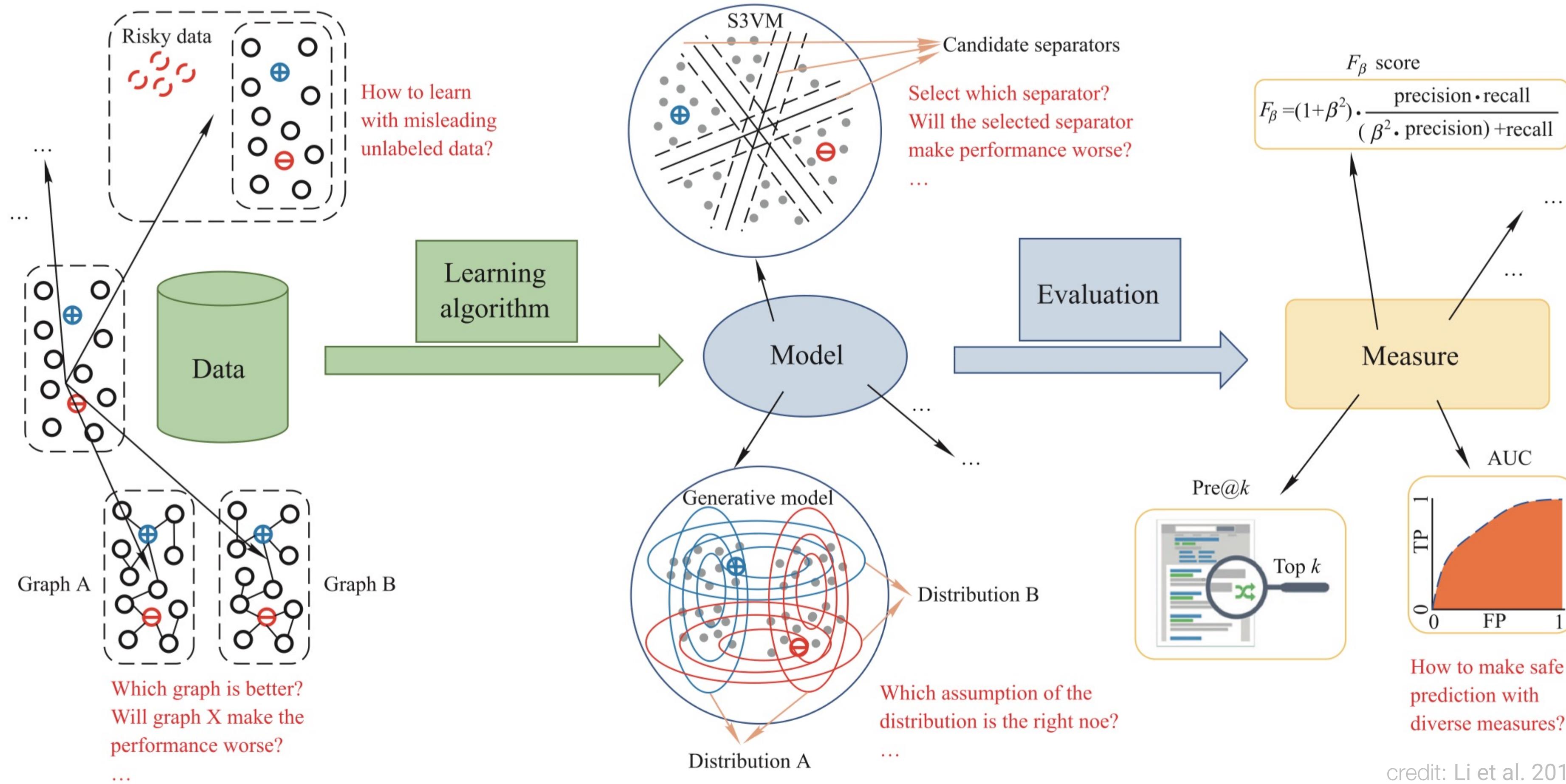
The Key Idea of “FSL with unlabeled data as SSL”

Fast Adaptation: Sample Selection



How to **select credible samples** to benefit learning?

Safe Semi-Supervised Learning Revisited



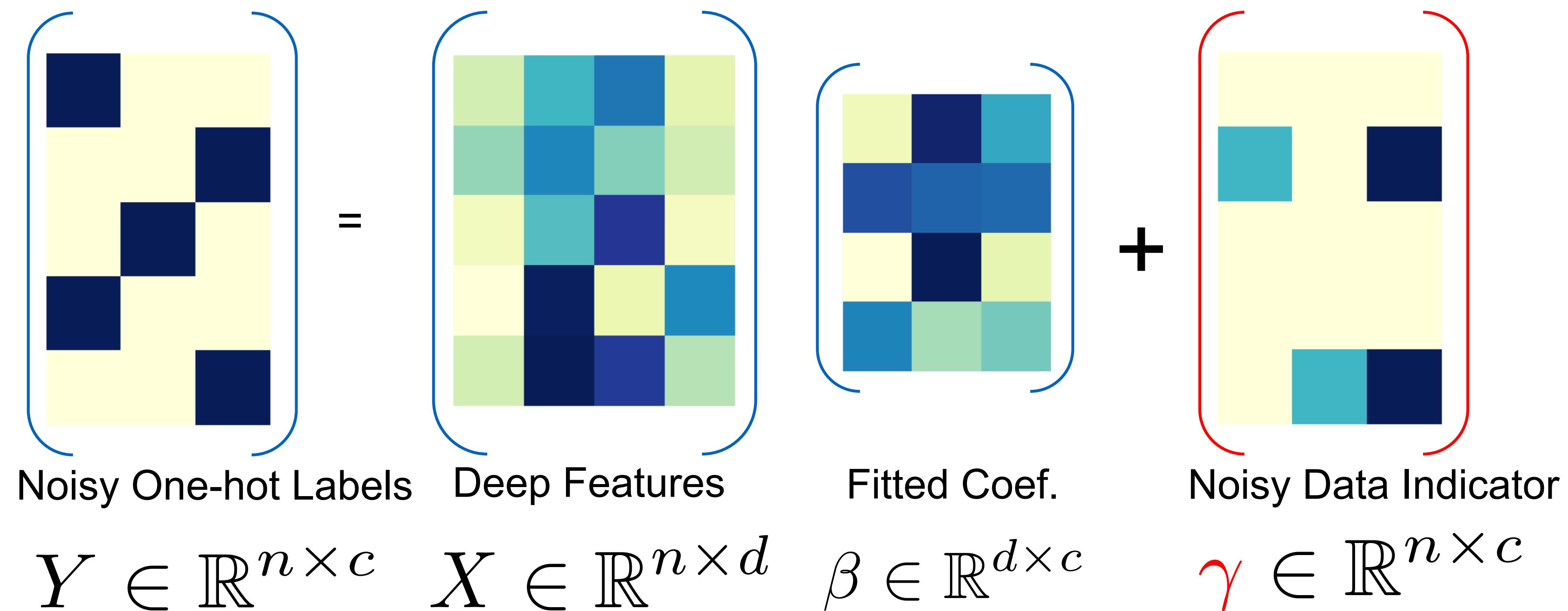
Li, Yu-Feng, and Zhi-Hua Zhou. "Towards making unlabeled data never hurt." TPAMI 2014.

Li, Yu-Feng, Lan-Zhe Guo, and Zhi-Hua Zhou. "Towards safe weakly supervised learning." TPAMI 2019.

Li, Yu-Feng, and De-Ming Liang. "Safe semi-supervised learning: a brief introduction." FCS, 2019.

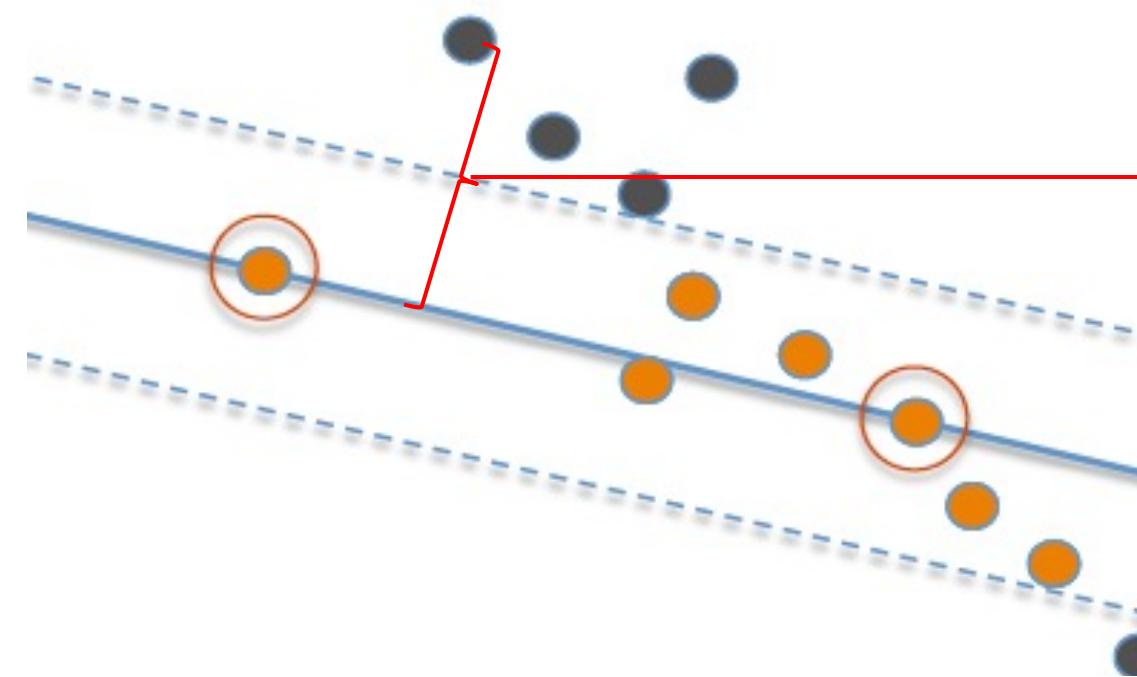
Instance Credibility Inference: Identify Noisy Data in Label Space

Linear system
with **Noisy** Data/Labels $Y = X\beta + \gamma$



Instance Credibility Inference: Understanding γ in Statistics

$$y = \mathbf{x}^\top \boldsymbol{\beta} + \varepsilon + \gamma$$



γ_i equals to the residual predict error $\gamma_i = y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}$



Leave-one-out externally studentized residual:

$$t_i = \frac{y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}_{(i)}}{\hat{\sigma}_{(i)} (1 + \mathbf{x}_i (\mathbf{X}_{(i)}^\top \mathbf{X}_{(i)})^{-1} \mathbf{x}_i)^{1/2}}$$

\Leftrightarrow test whether $\gamma = 0$ in $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \gamma \mathbf{1}_i + \varepsilon$.

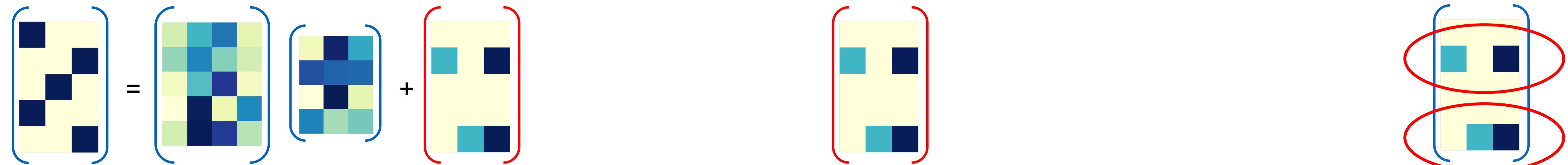
When there are multiple outliers:
masking and swamping



$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} + \gamma$$

Instance Credibility Inference: Statistical Outlier Detection

$$y_i = x_i^\top \beta + \varepsilon + \hat{\gamma}_i \longrightarrow \hat{\gamma}_i \longrightarrow O = \{i : \hat{\gamma}_i \neq 0\}$$



$$\underset{\boldsymbol{\beta}, \boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta}, \boldsymbol{\gamma}) := \|\mathbf{Y} - \mathbf{X}\boldsymbol{\beta} - \boldsymbol{\gamma}\|_{\text{F}}^2 + \lambda P(\boldsymbol{\gamma}) \longrightarrow \underset{\boldsymbol{\gamma}}{\operatorname{argmin}} \left\| \tilde{\mathbf{Y}} - \tilde{\mathbf{X}}\boldsymbol{\gamma} \right\|_{\text{F}}^2 + \lambda P(\boldsymbol{\gamma})$$

Instance Credibility Inference: Solving Gamma

$$\operatorname{argmin}_{\gamma} \left\| \tilde{\mathbf{Y}} - \tilde{\mathbf{X}}\gamma \right\|_F^2 + \lambda P(\gamma)$$

How to select λ ?

- heuristics rules $\lambda = 2.5\hat{\sigma}$?
- Cross-validation?
- Data adaptive techniques?
- AIC, BIC?

$$\hat{\gamma} = f(\lambda).$$

$$\lambda \rightarrow \infty, \hat{\gamma} \rightarrow 0.$$

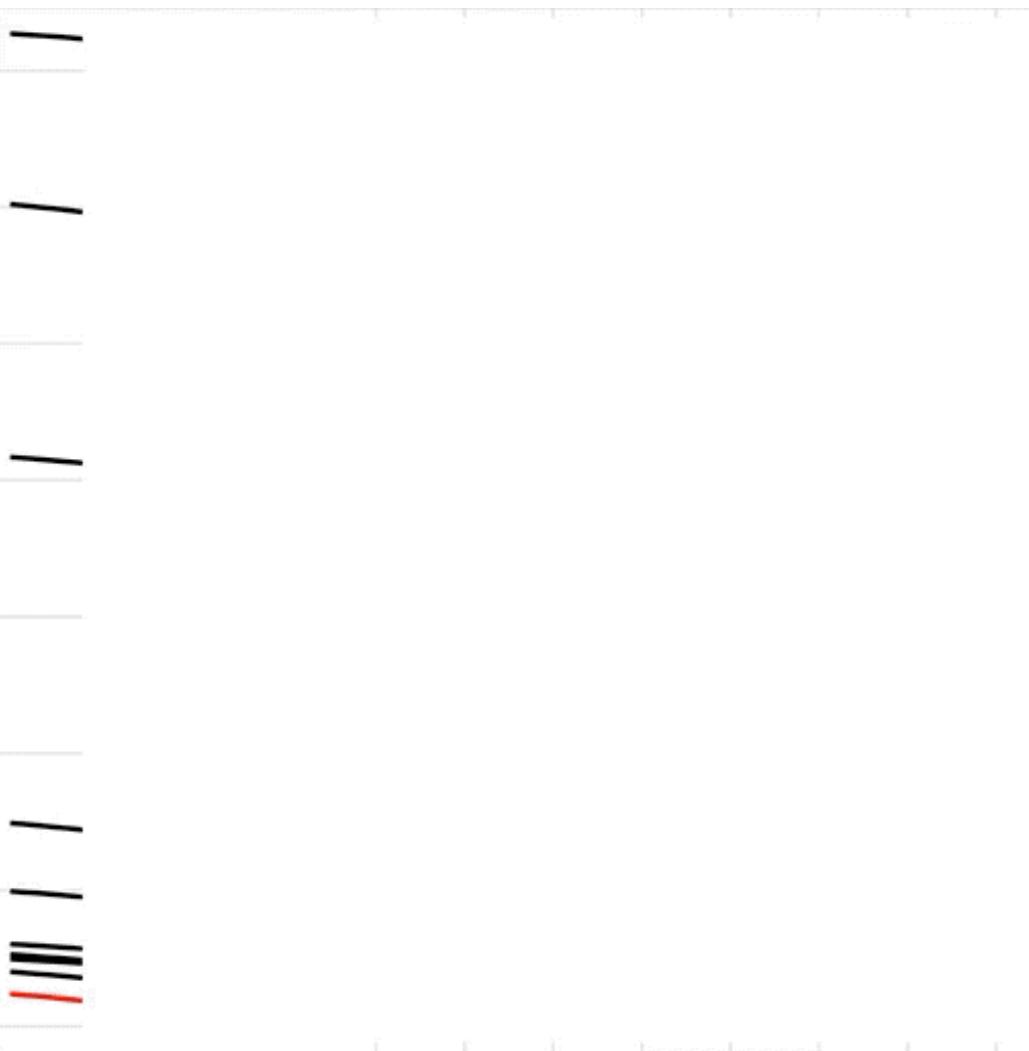
$$P(\gamma) = \sum_{i=1}^n \|\gamma_i\|_2,$$

$$C_i = \sup\{\lambda : \|\hat{\gamma}_i(\lambda)\| \neq 0\}$$

It is hard to select a proper λ .

This can be solved by GLM-Net.

Instance Credibility Inference: Theoretical Guarantees



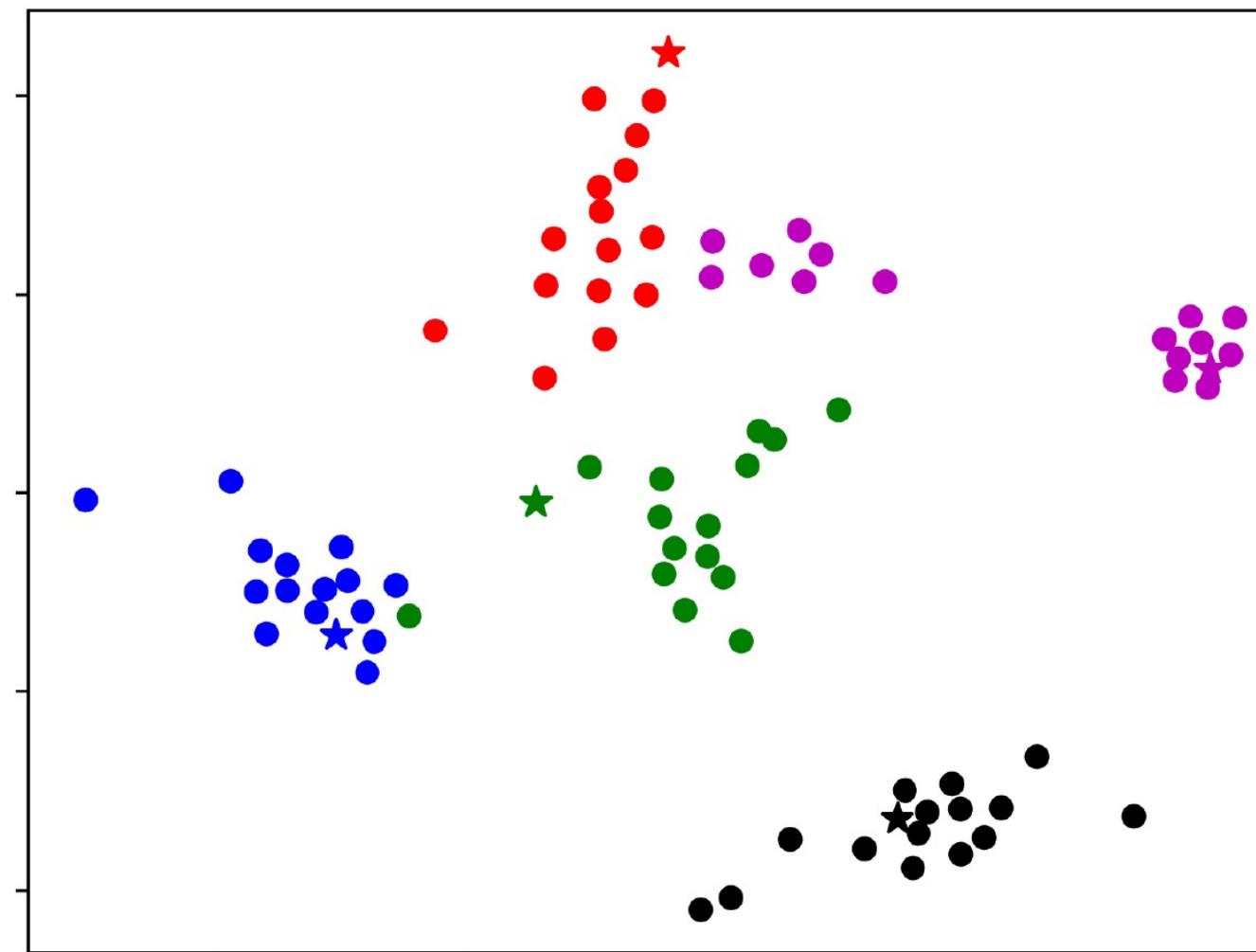
Theoretical guarantees:

- 1) Under the restricted eigenvalue and irrepresentability conditions, the noisy data identified by ICI is the subset of the ground-truth noisy data;
- 2) With further satisfied large error condition, ICI will identify all the noisy data.

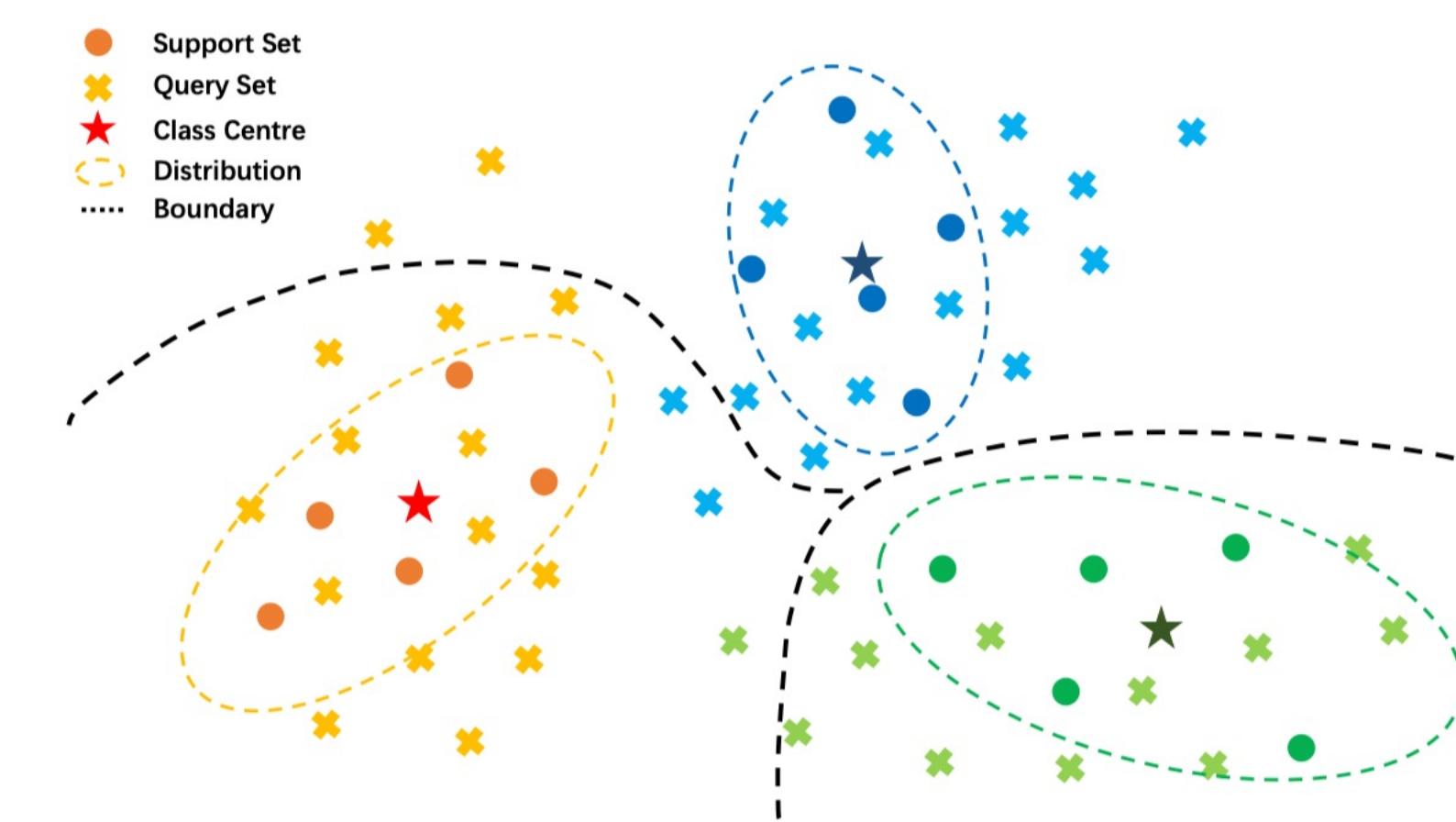
See our CVPR 2022 tutorial for details.
<https://sparse-learning.github.io>

The Key Idea of “Calibrating prototypes and class distribution”

- Bias towards base tasks

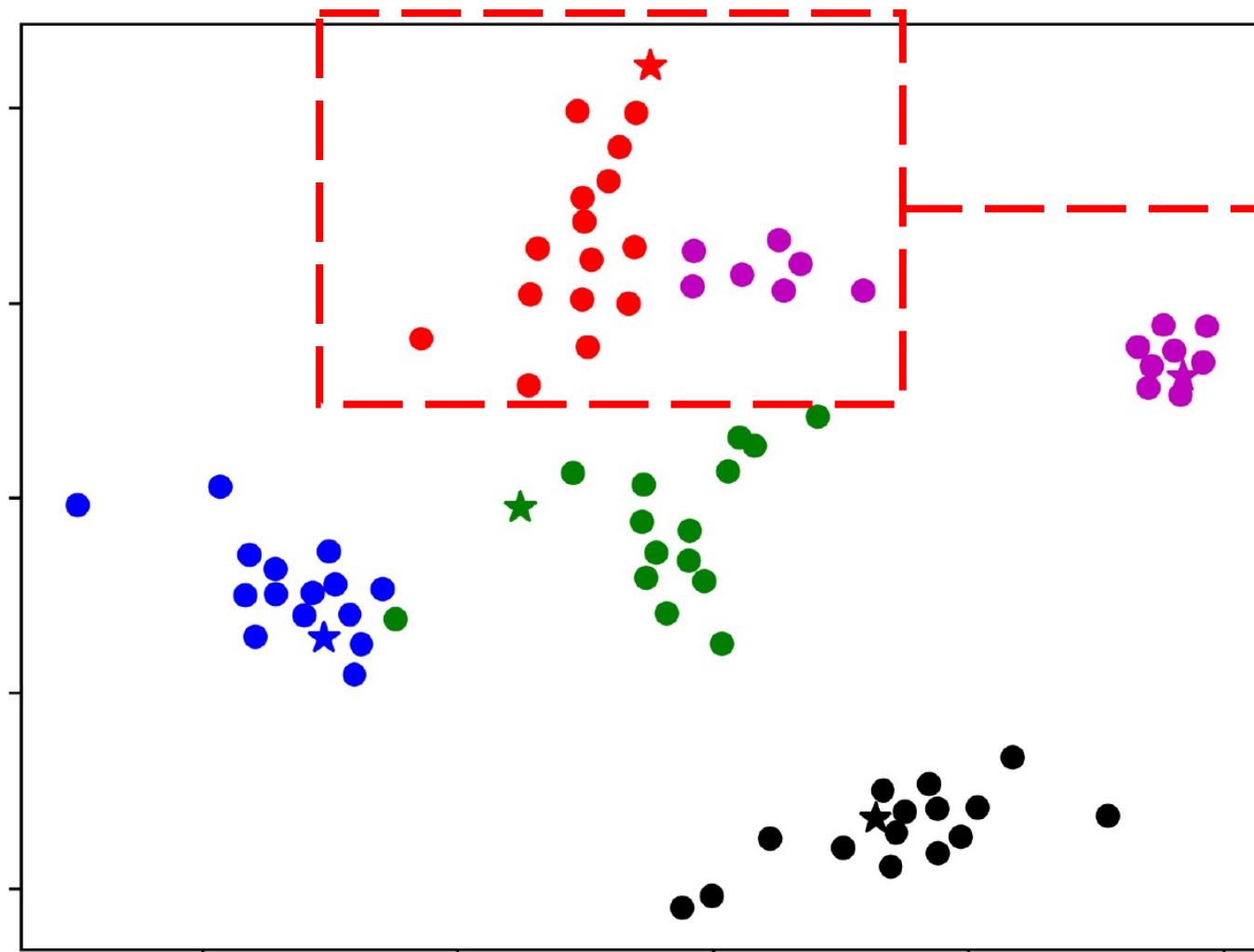


- Overfitting on few-shot samples

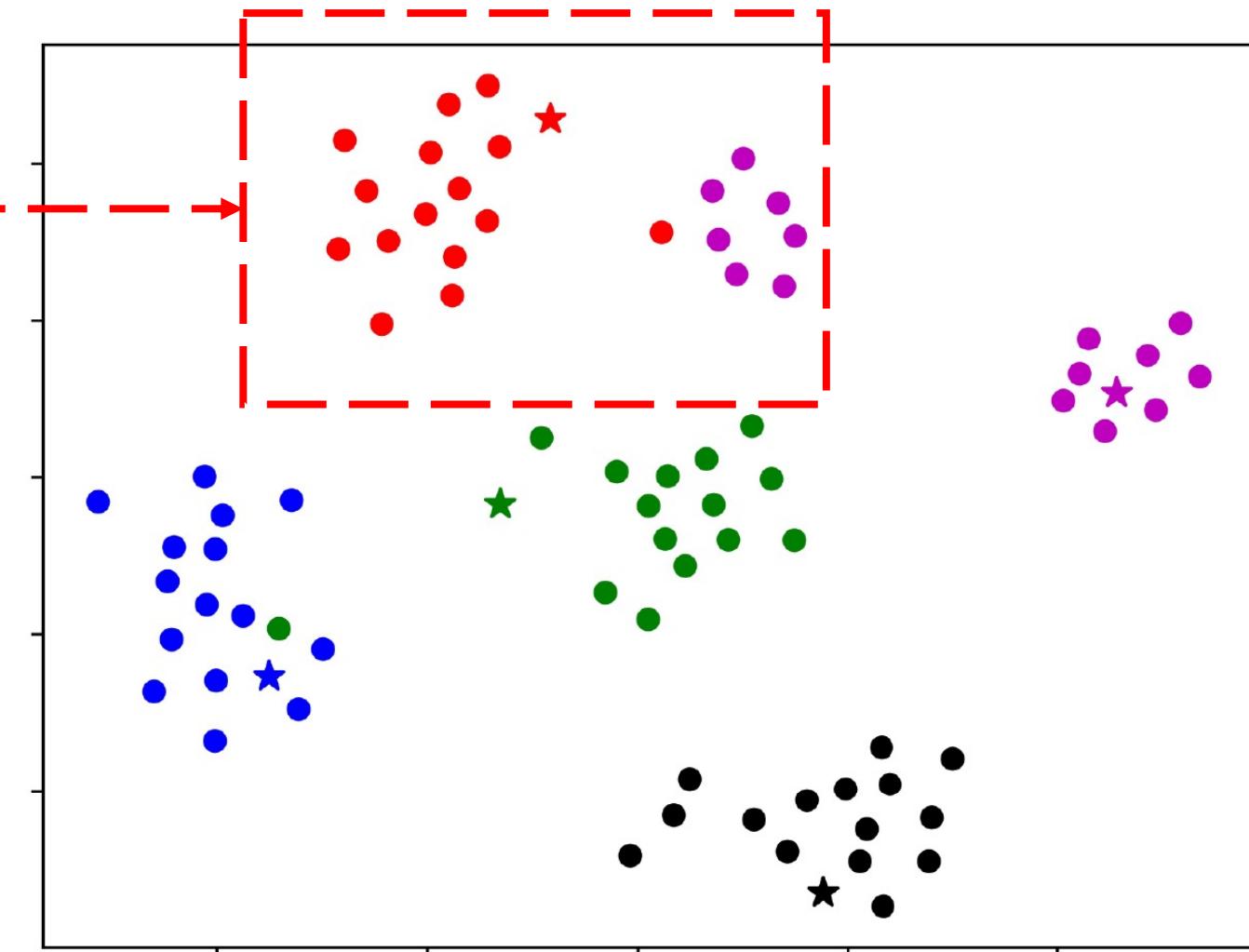


How to **calibrate novel task** to make robust estimation?

Adaptation on novel data: Calibration

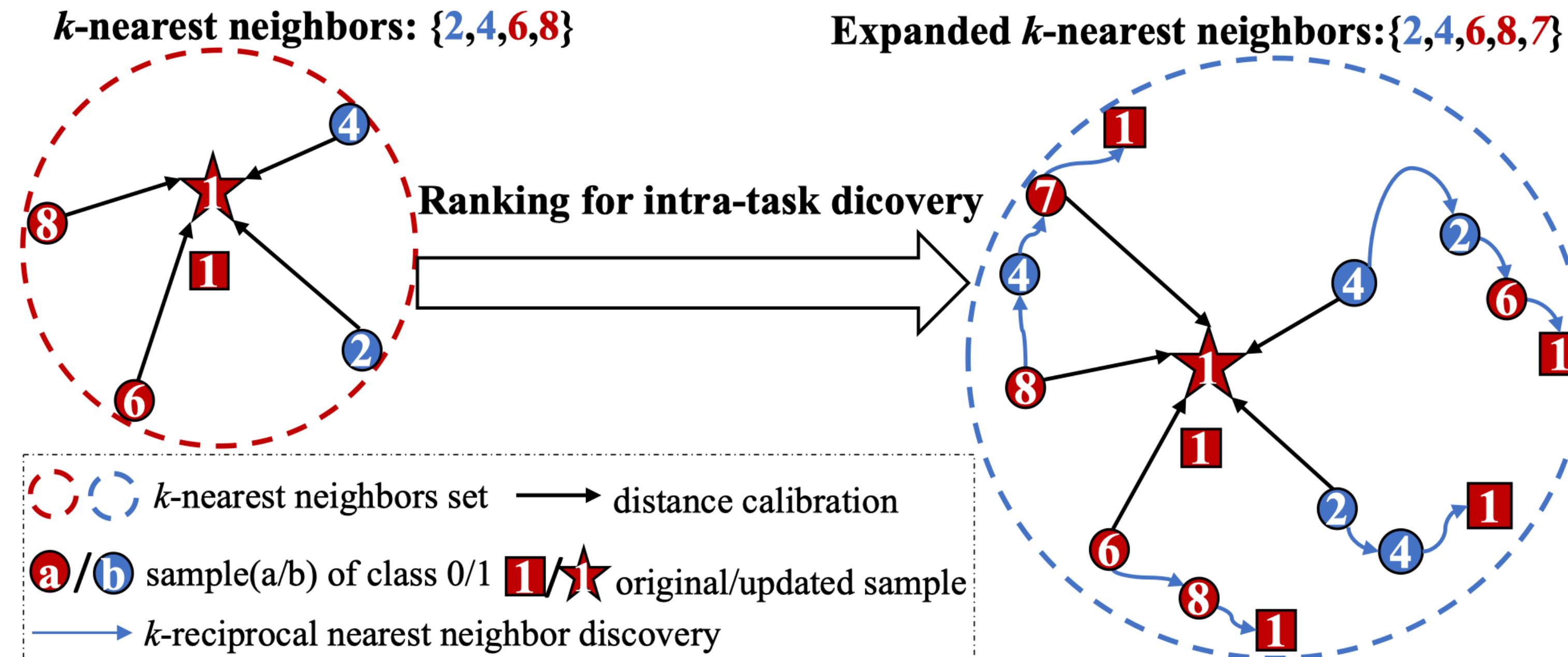


The representation learned on base dataset are biased.



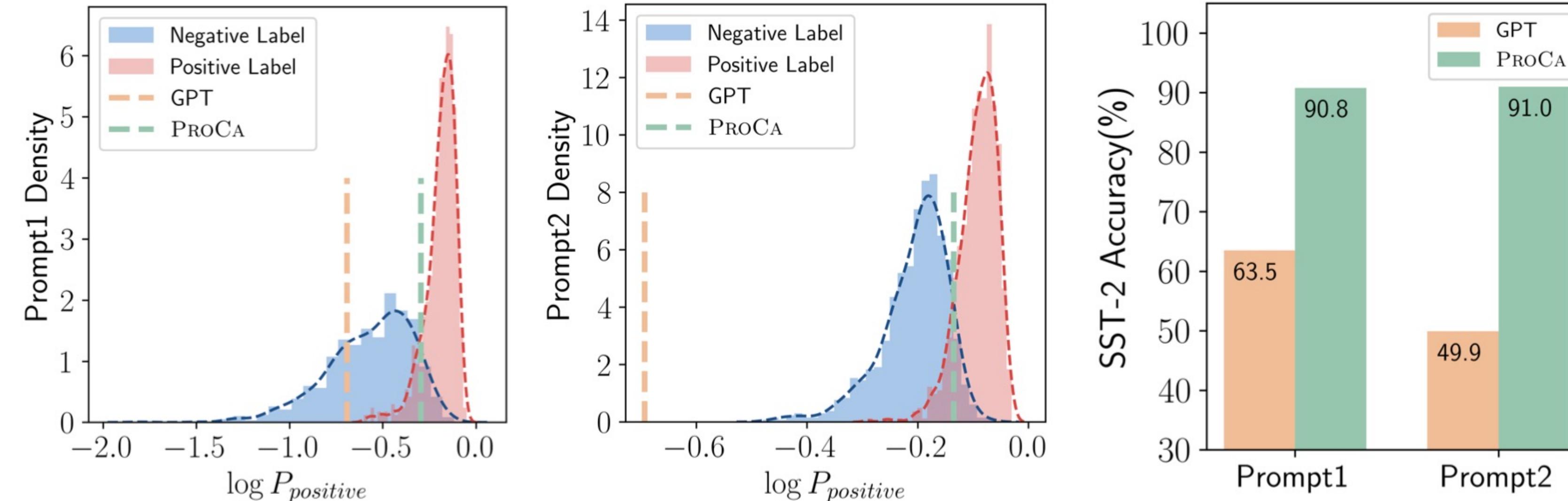
We can calibrate novel data to reduce the bias.

Ranking Distance Calibration



Discover likely positive samples and calibrate their pairwise distances

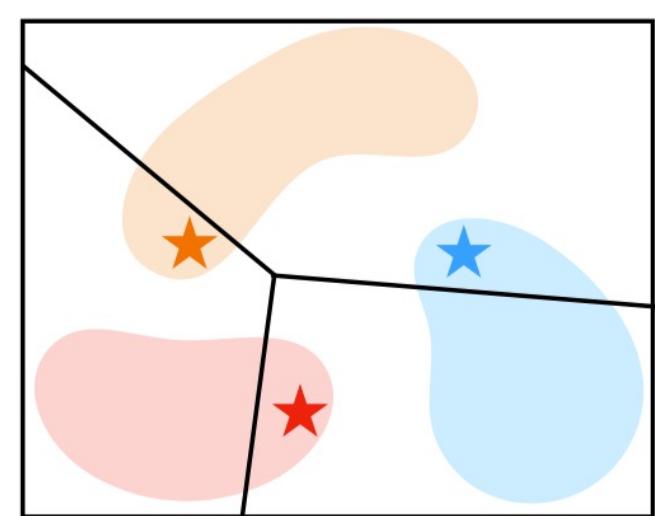
Novel Logit Distribution Calibration



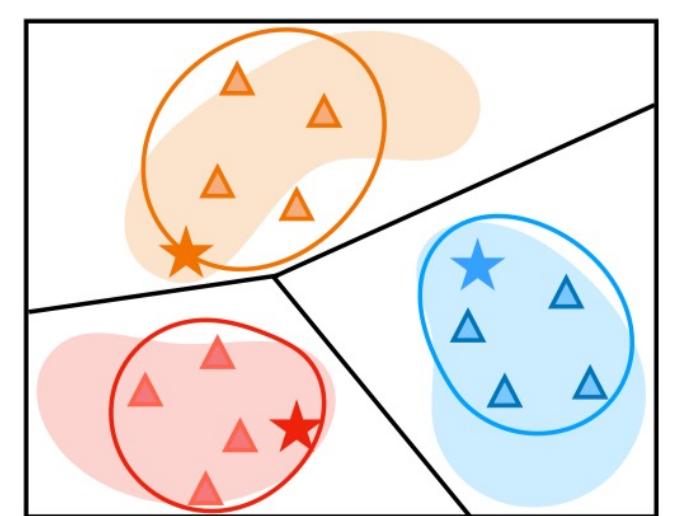
Learning a class-dependent threshold based on Gaussian mixture model of category logits distribution.

Novel Distribution Calibration

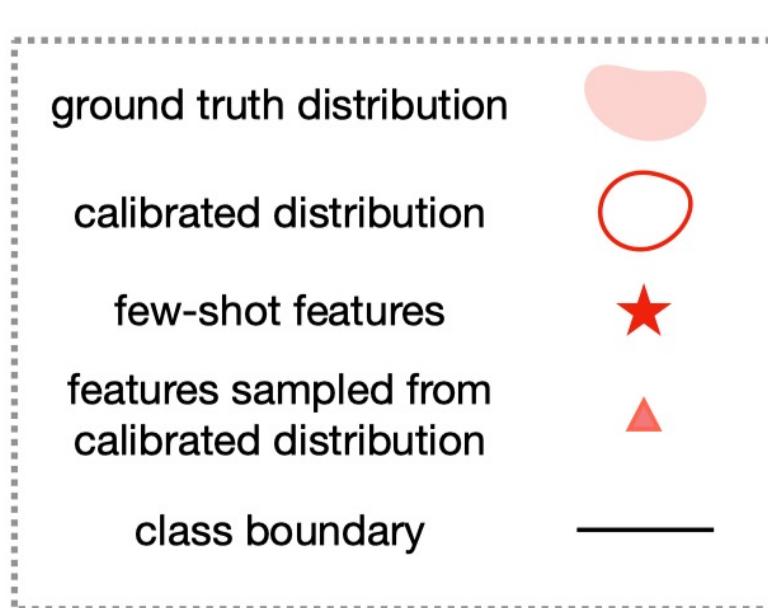
Transfer statistics from base classes to calibrate novel distribution.



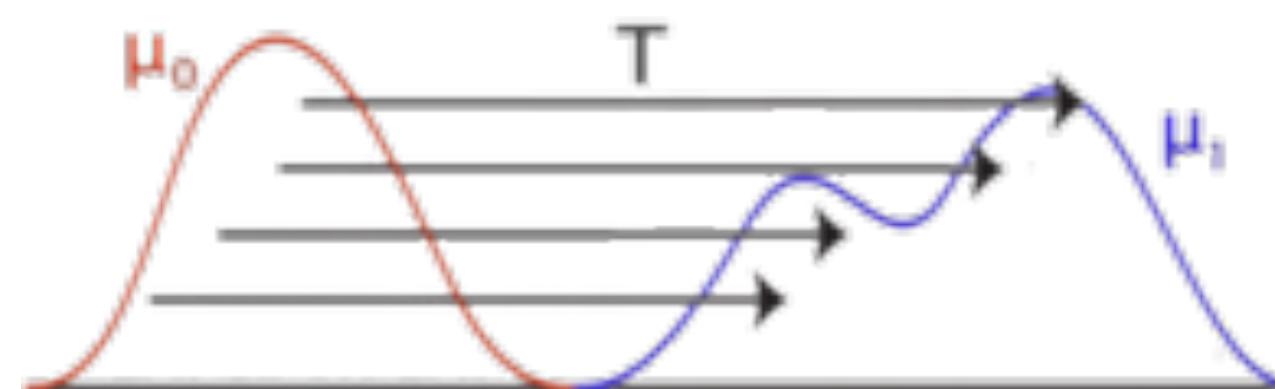
Classifier trained with
few-shot features



Classifier trained with features
sampled from calibrated distribution



credit: [Yang, et al. 2021]

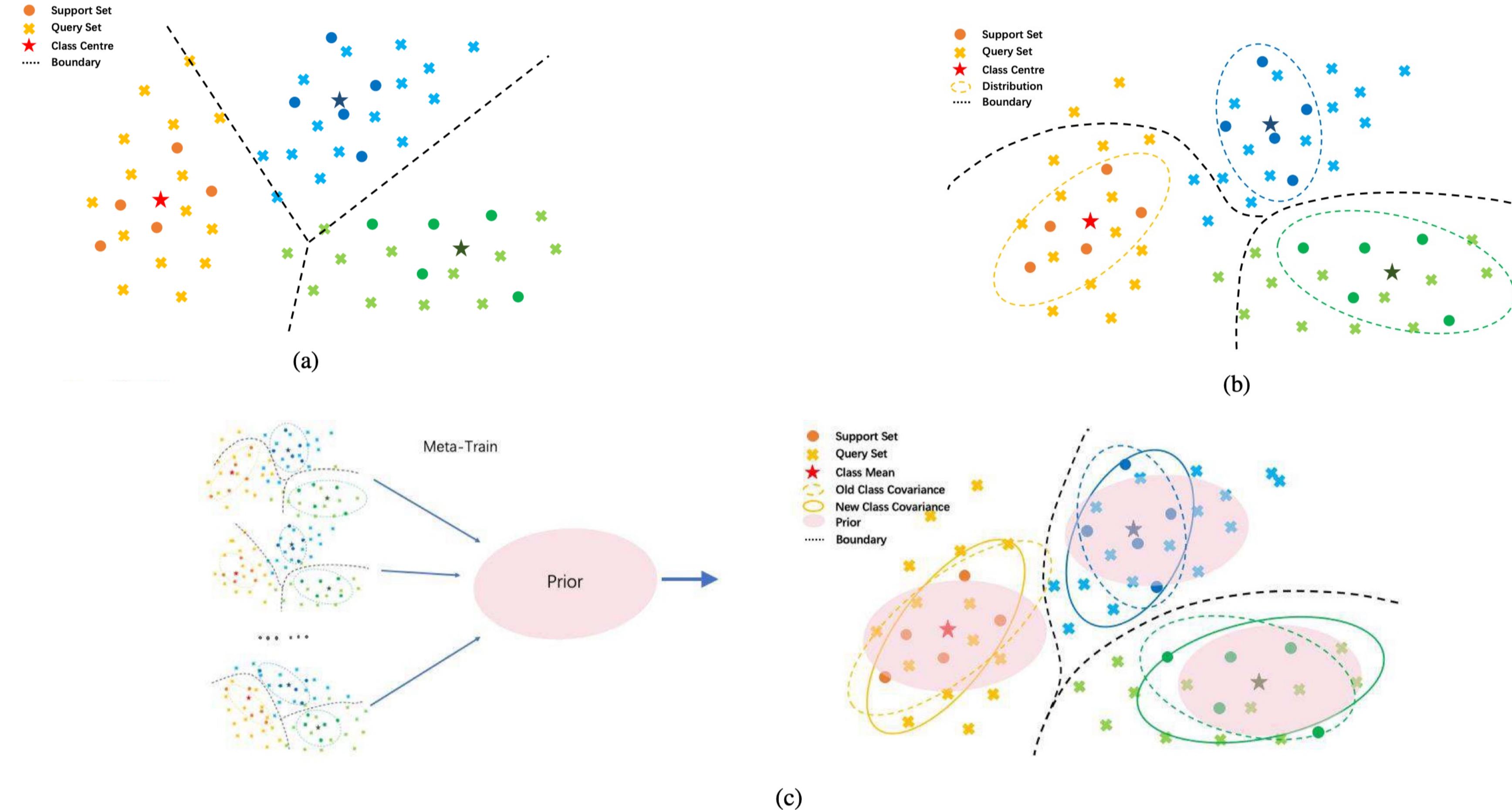


credit: wikipedia

The relationship between base classes can be modeled via Euclidean distance.

And can be modeled via more holistically, for example using optimal transport.

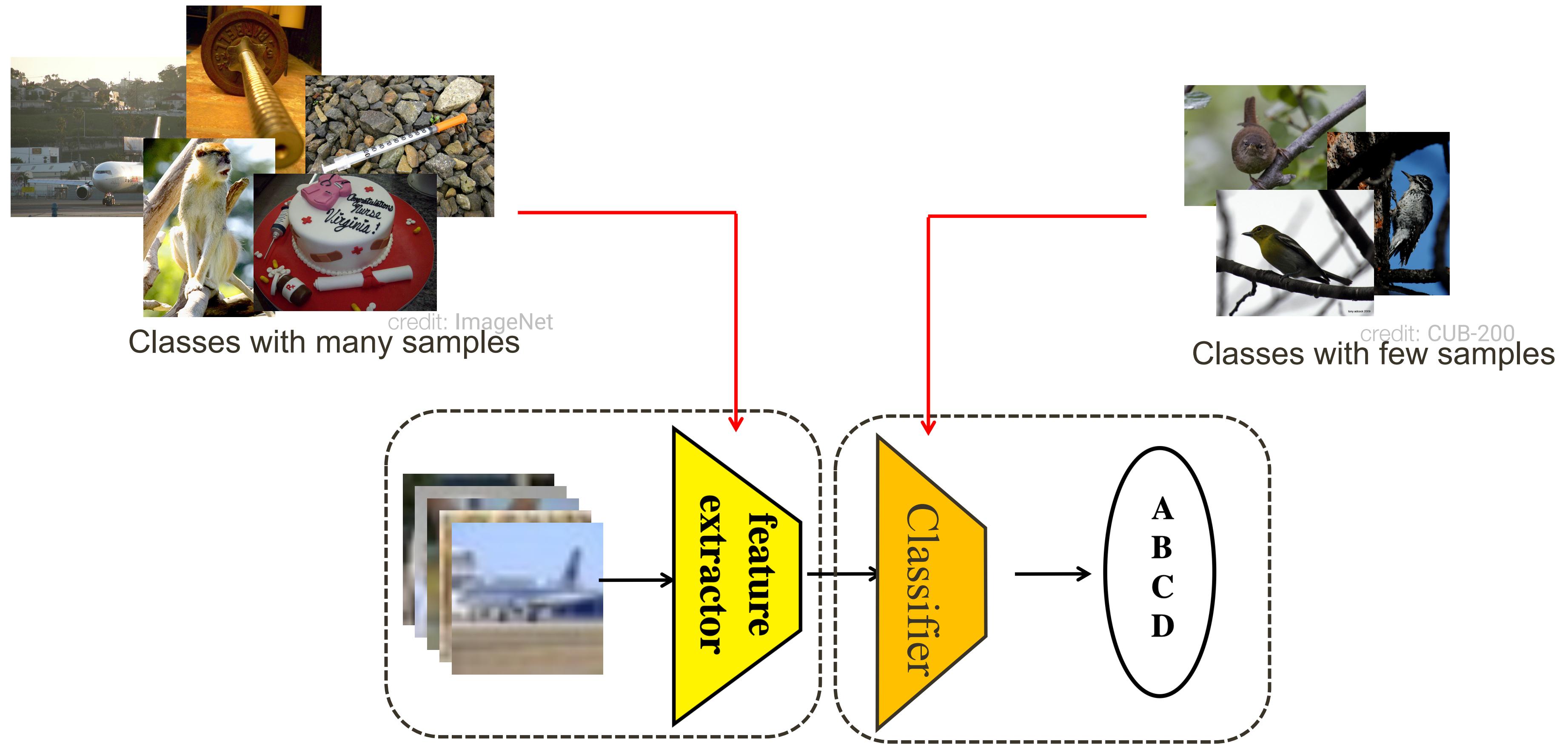
Bayesian Approach on Uncertainty Calibration



Few-Shot Learning

- Learning from base data
- Adaptation on novel data
- **FSL in 2020s**
 - Updating backbones
 - Cross-domain FSL
 - Foundation models as FSL learners

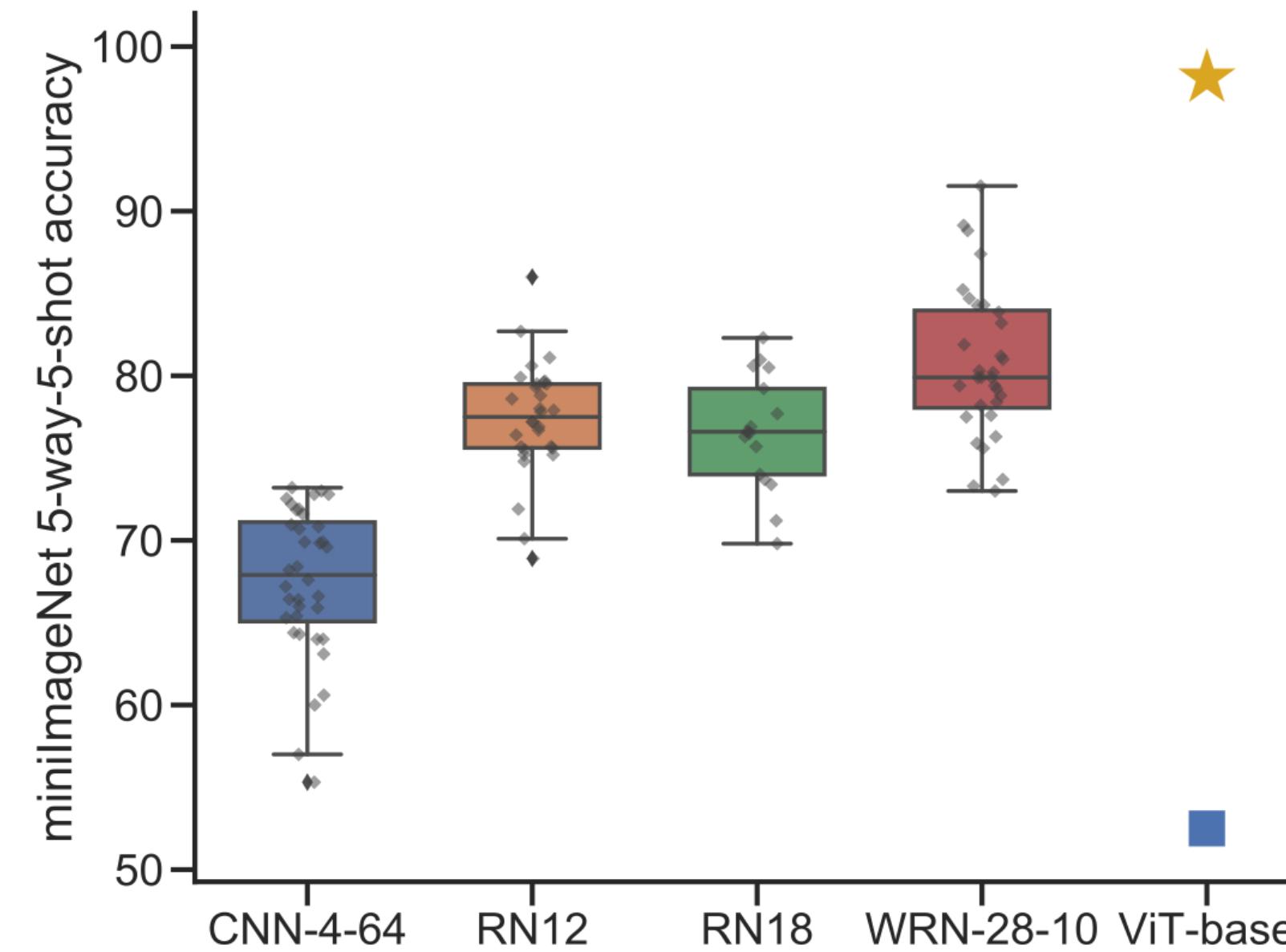
The Key Idea of “Updating backbones”



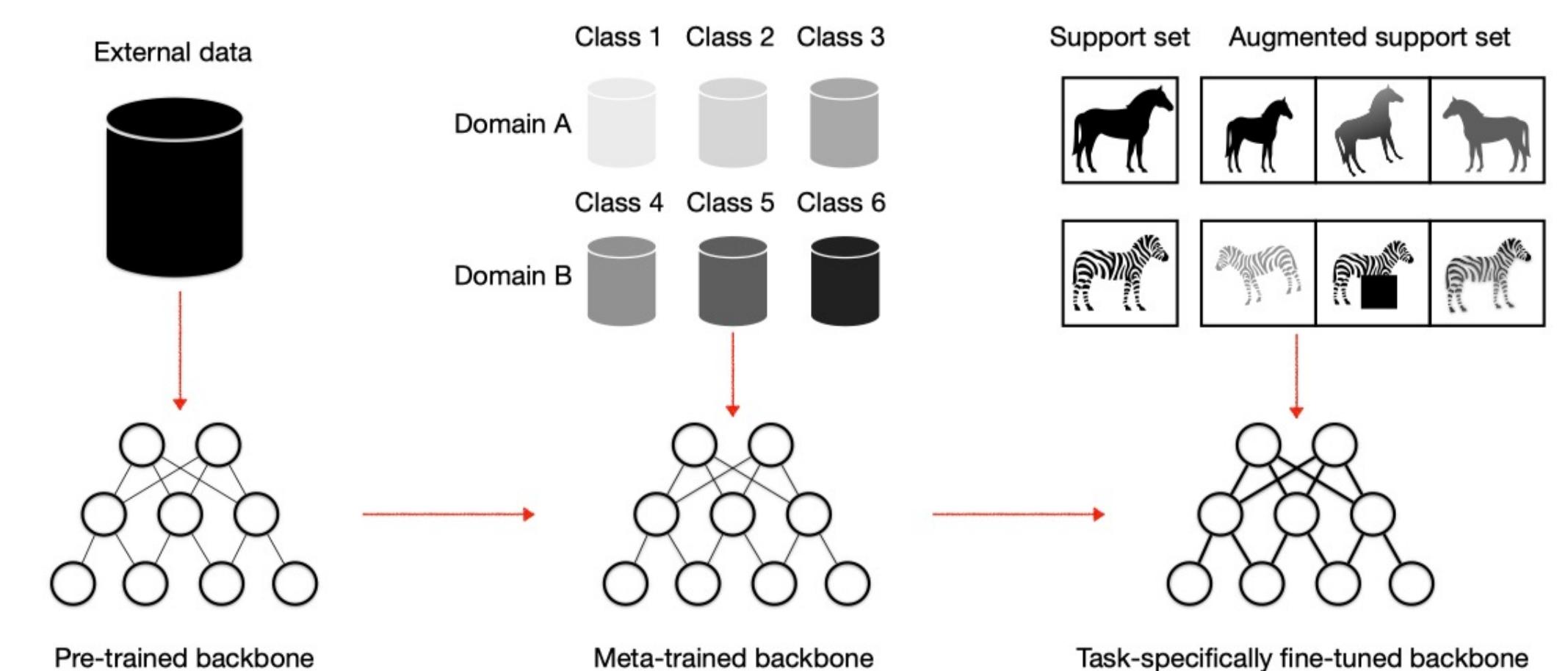
Can we enhance few-shot learning with recent development of deep learning?

Pushing the Limits of Simple Pipelines for Few-Shot Learning

What pipeline do we need in 2020s?

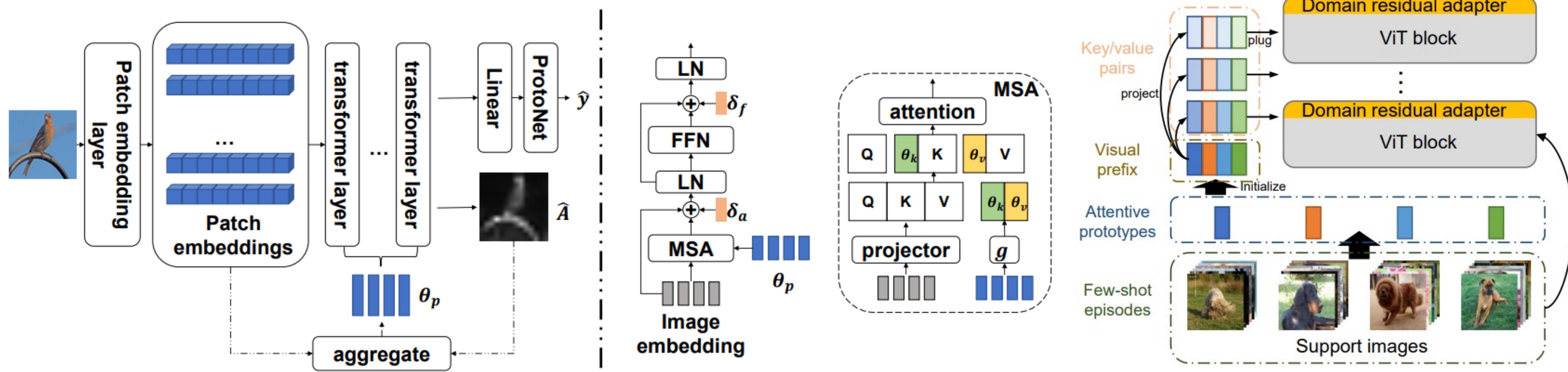


Larger backbone is better.



Both pre-train and fine-tuning are beneficial.

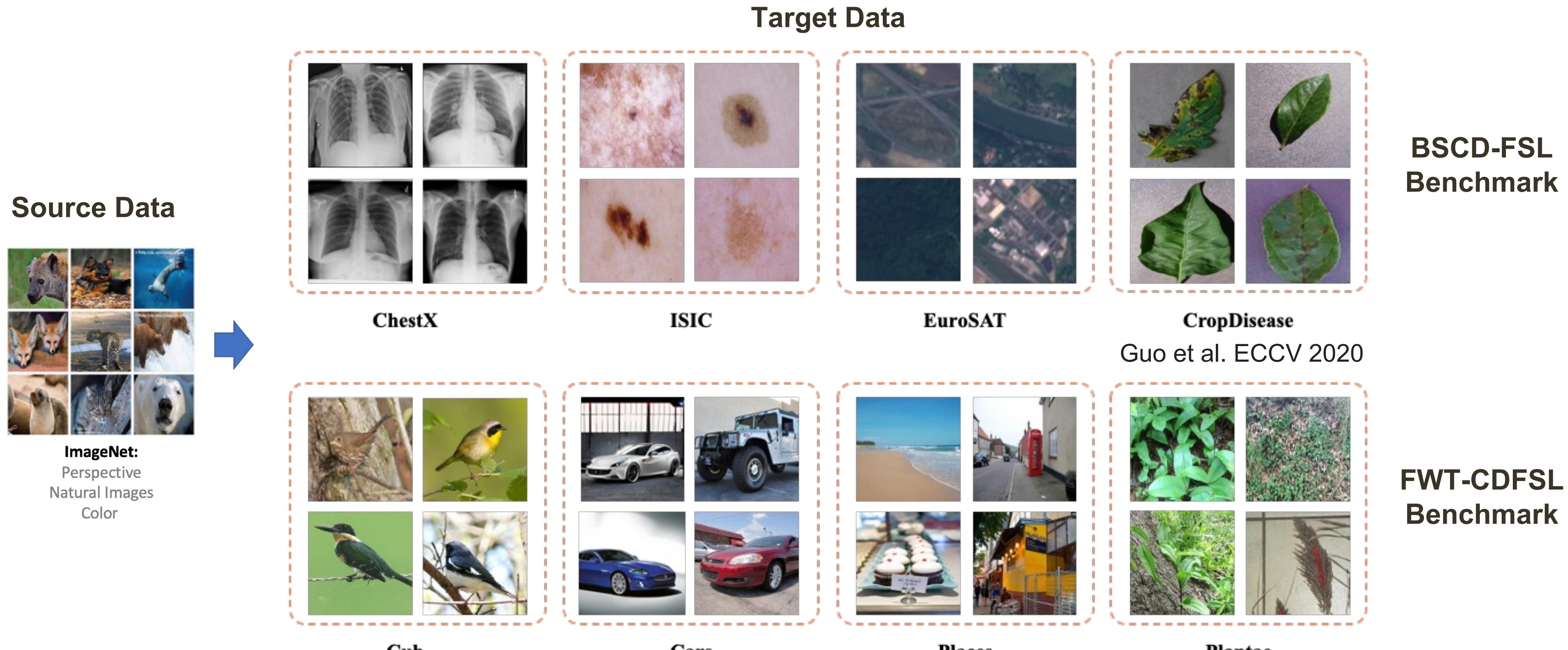
Efficient Transformer Tuning



Fewer learnable parameters, while being flexible and effective enough.

Cross-Domain Few-Shot Learning (CD-FSL)

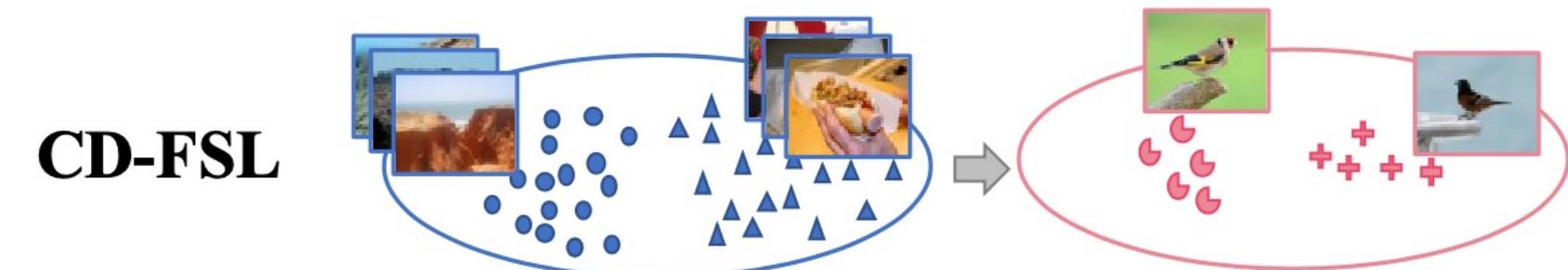
Improve FSL models across different domains.



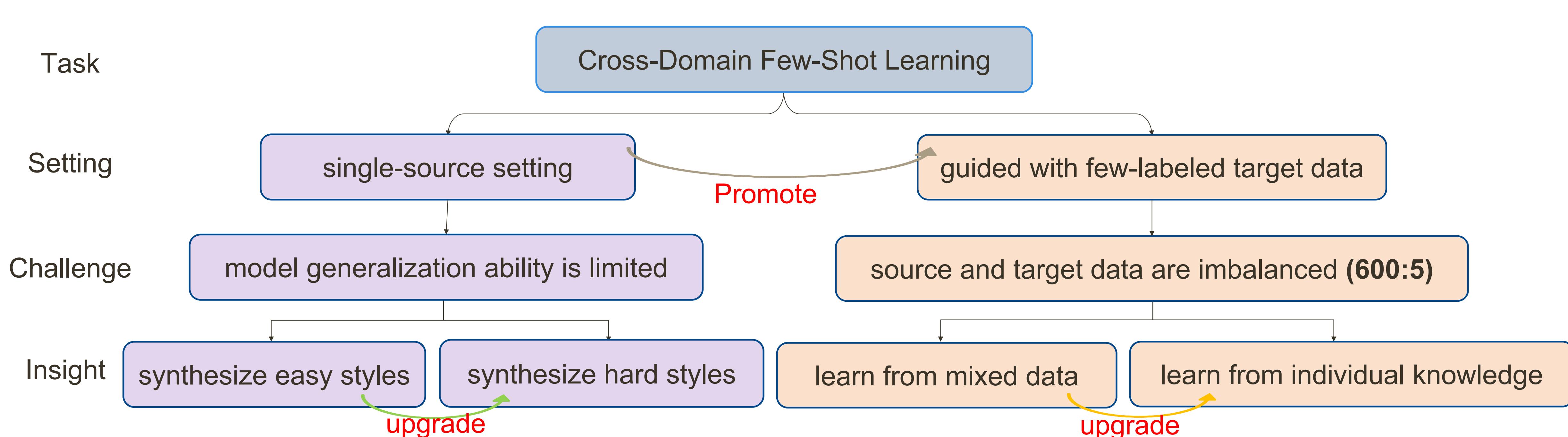
Cross-Domain Few-Shot Learning (CD-FSL)

Core Questions:

1. How to improve CD-FSL with only a source dataset as training data?
2. Could we utilize some target data for further promoting the CD-FSL?

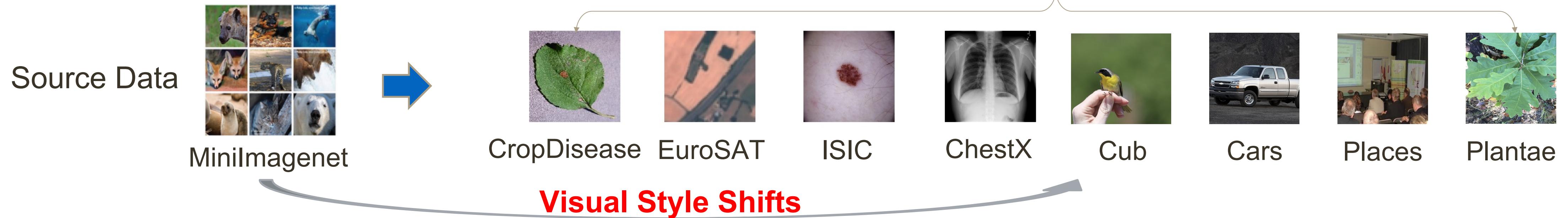


Overview:

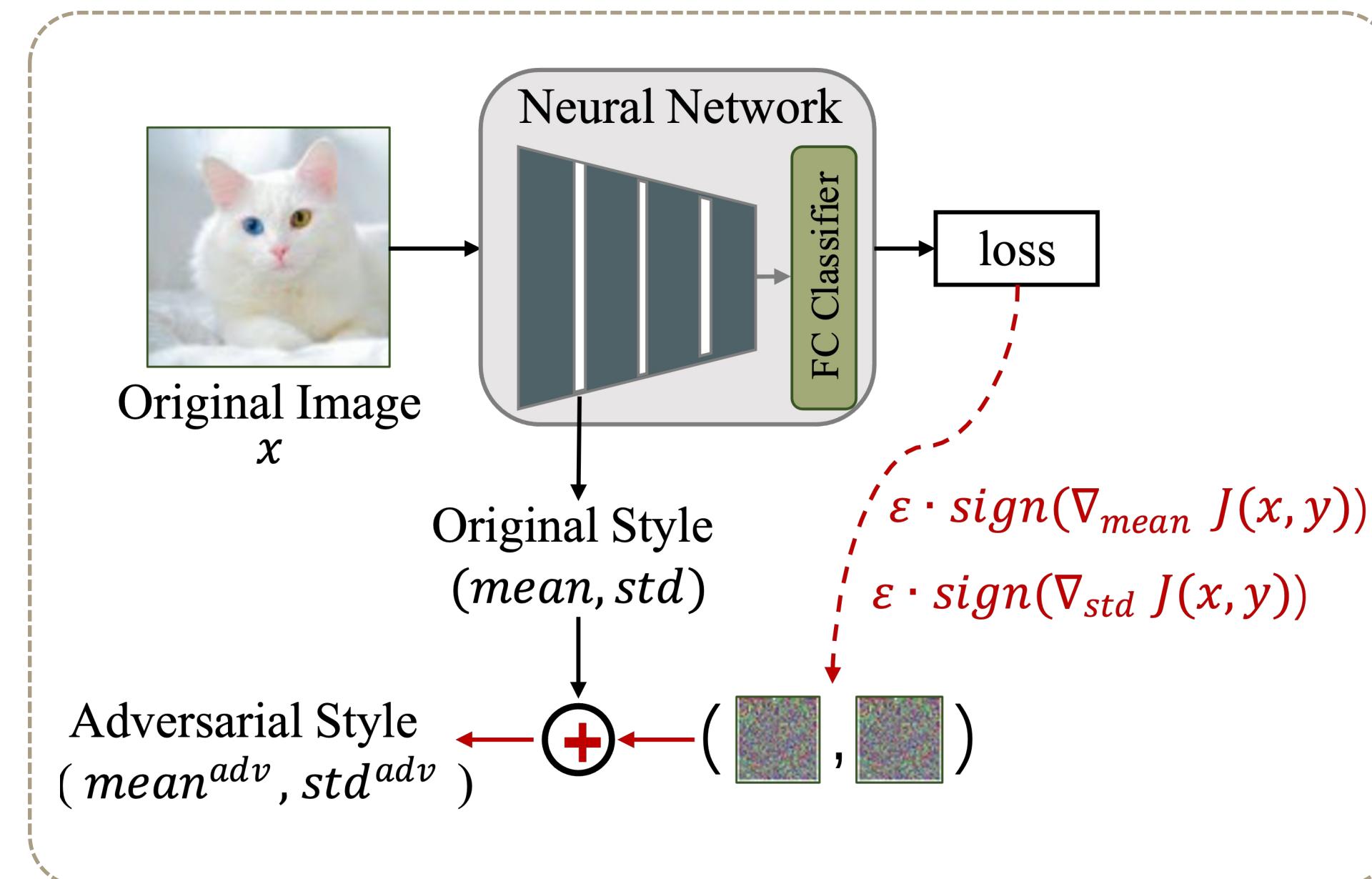


Cross-Domain Few-Shot Learning

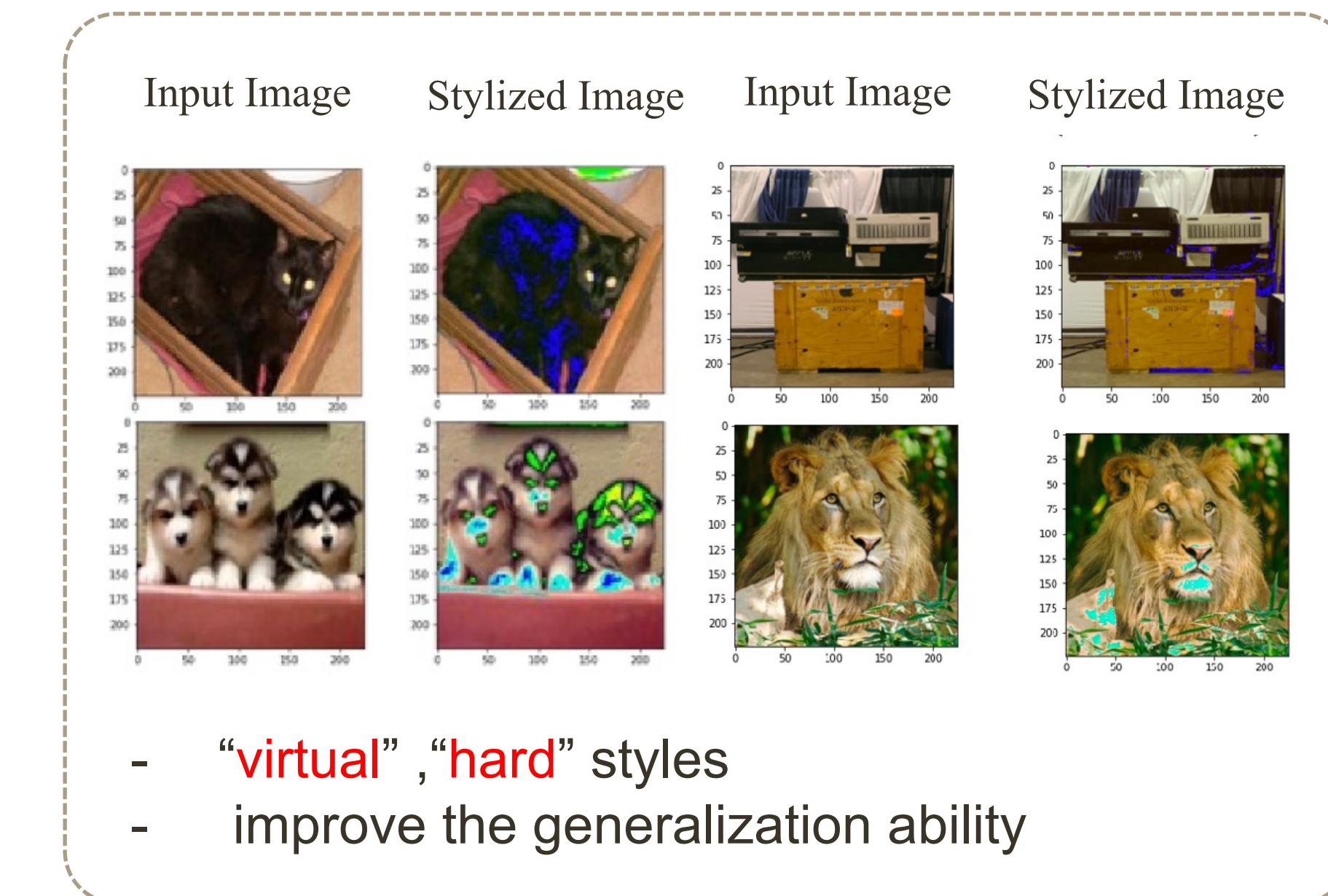
FSL across different Domains



StyleAdv: Adversarial style training



Novel Style Attack



Visualization Result

Cross-Domain Few-Shot Learning with Few-Labeled Target Data

Boost CD-FSL models with few labeled target data



Source Training Set Target Training Set Target Testing Set
(many examples) **(few examples)** **(few examples)**

disjoint

- more realistic
- good performance
- don't violate the CD-FSL setting

Challenges

- >> how to make use of the **imbalanced** source training set and target training set? (**600:5**)
- >> how **to narrow the domain gap** between the source domain and target domain?

Foundation Models as Few-Shot Learners

Quite a lot of works on foundation models, to name a few:

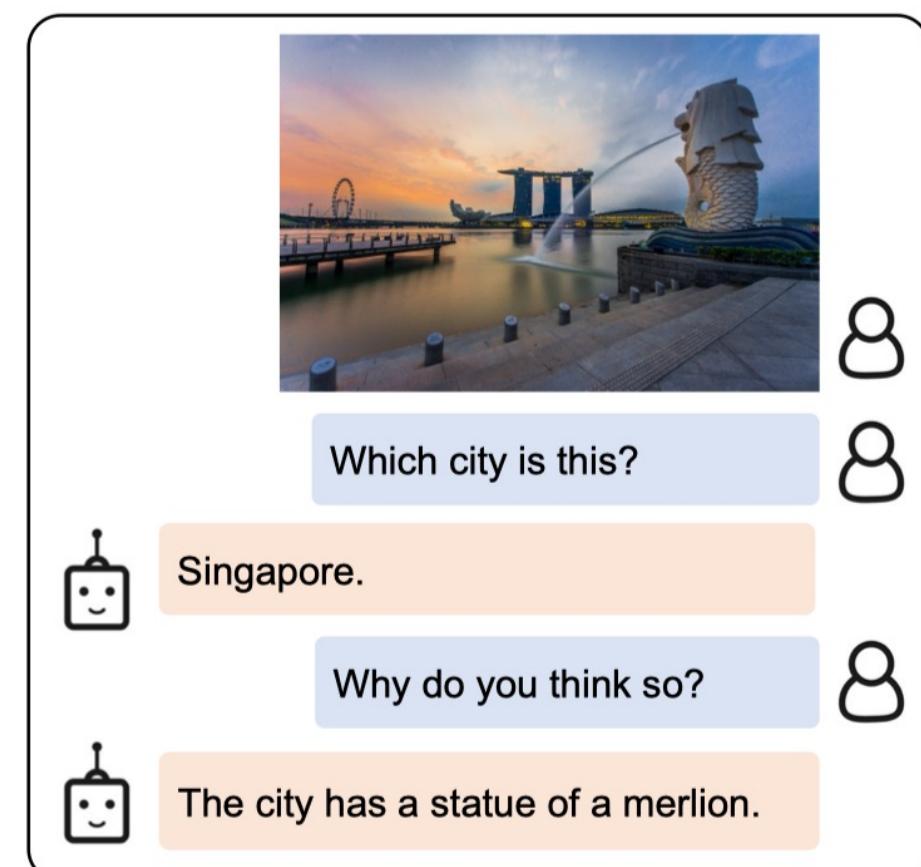
GPT-4

Q: How many prime numbers are there between 150 and 250?

A: There are 13 prime numbers between 150 and 250.

credit: [Bubeck, et al. 2023]

Text-to-Text



credit: [Li et al. 2023]

Image-to-Text



credit: [Rombach et al. 2022]

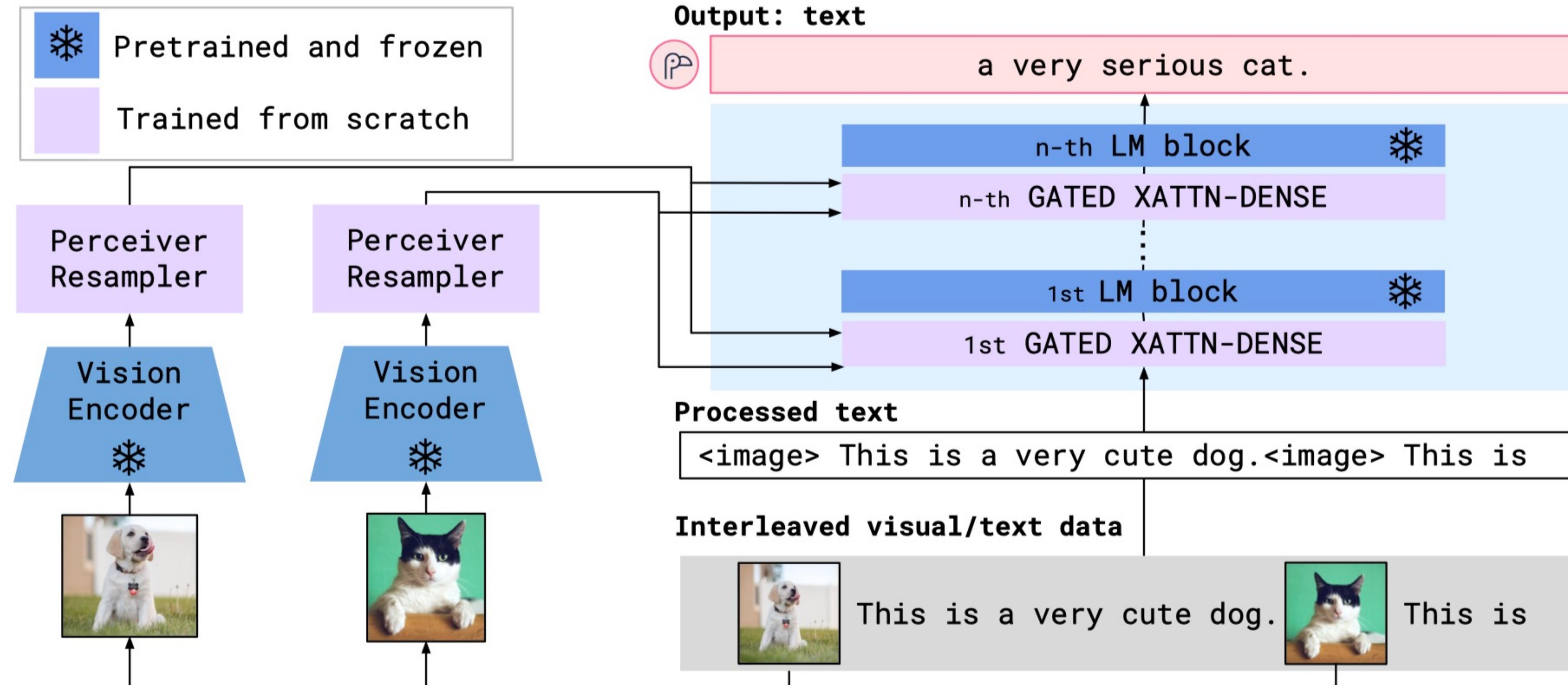
Text-to-Image



Image-to-Image



Foundation Models as Few-Shot Learners: Flamingo



Directly solving few-shot learning problems via the foundation model.

In-context Learning

Prompt:

This is awesome! // Negative
This is bad! // Positive
Wow that movie was rad! // Positive
What a horrible show! //

Output:

Negative

credit: promptingguide.ai



Take Home Message

To learn few-shot learning by statistical methods, we could,

- For learning from base data:
 - Learn causal-features that are truly transferable to novel tasks;
 - Model the similarity measurement in a more statistical way;
 - Utilize neural collapse to benefit few-shot learning.
- For adaptation on novel data:
 - Ensure the safety of learning with unlabeled data via statistical outlier detection;
 - Calibrate prototypes to reduce the bias;
 - Calibrate class distribution to prior knowledge;
- And for few-shot learning in 2020s:
 - Benefit few-shot learning with deeper architecture and more powerful pipeline;
 - Tackle more challenging cross-domain few-shot learning;
 - Adopt foundation models as few-shot learners.



Dr. Yanwei Fu



Yikai Wang



Chengming Xu



Yuqian Fu

Team to work on this talk in our group.



Yikai Wang

I'm looking for short-term visiting, or Potential Post-doc position (2024).

If there's any chance, please email me:
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THANKS

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