A SURVEY ON SKELETONS IN DIGITAL IMAGE PROCESSING.

1.Mrs.J.Komala lakshmi,Lecturer,Department of Computer science,SNR SONS college,Coimbatore.E.mail:ashwathraju@yahoo.com

2. Dr. Mrs. M. Punithavalli, Director, Department of computer scince,SriRamakrishna college of arts and science for women,Coimbatore.Email:mpunitha_srcw@yahoo.co.in

Abstract:

An image is digitized to convert it to a form which can be stored in a computer's memory or on some form of storage media such as a hard disk or CD-ROM. Once the image has been digitized, it can be operated upon by various image processing operations like enhancement, restoration, reconstruction, compression. An image defined in the "real world" is considered to be a function of two real variables, for example, $a(x,y)$ with a as the amplitude (e.g. brightness) of the image at the *real* coordinate position (x,y). An image may be considered to contain sub-images sometimes referred to as *regions-of-interest*, *ROIs*, or simply *regions*. This concept reflects the fact that images frequently contain collections of objects each of which can be the basis for a region. In a sophisticated image processing system it should be possible to apply specific image processing operations to selected regions. Thus one part of an image (region) might be processed to suppress motion blur while another part might be processed to improve color rendition. For performing image processing operations ,the basic structure called skeleton is much more essential and highly adaptive tool. Skeletons are important shape descriptors in object representation and recognition. A skeleton that captures essential topology and shape information of the object in a simple form is extremely useful in solving various problems such as character recognition, 3D model matching and retrieval, and medical image analysis. Medical imaging systems. Due to its compact shape representation, image skeleton has been studied for a long time in computer vision, pattern recognition, and optical character recognition. It is a powerful tool for intermediate representation for a number of geometric operations on solid models. Many image processing applications depend on the skeletons.

key words: Skeleton, Shape representation, Skeleton quality, Skeletonization, Shape decomposition.

1.Introduction

Representing and understanding shapes play central roles in many of today's graphics and vision applications. These applications often benefit from some form of shape descriptors, one of which is known as skeletons. A skeleton is a compact, medial structure that lies within a solid object [1]. A skeleton of a 2D object consists of 1D (e.g., curve) elements, whereas the skeleton of a 3D object may consist of both 1D and 2D (e.g., surface) elements. A skeleton is a lower dimensional object that essentially represents the shape of its target object. Because a skeleton is simpler than the original object, many operations, e.g., shape recognition and deformation, can be performed more efficiently on the skeleton than on the full object. The process of generating such a skeleton is called skeleton extraction or skeletonization. Examples of automatic skeleton extraction include the Medial Axis Transform (MAT) [2] and skeletonization into a one dimensional poly-line skeleton (or simply 1D skeleton) [3] [4] [5].Skeletons have been extracted from different sources, such as voxel (image) based data [6] [7] [8], boundary represented models [9] [10] [11], and scattered points [12], and for different purposes, such as shape description [13] [14], shape approximation [15][16], similarity estimation [17], collision detection [18],[19], biological applications [20], navigation in virtual environments [21], and animation [22] [5]. Although it has been noted before that a good shape decomposition can be used to extract a high quality skeleton [4],[5] and that a high quality skeleton can be used to produce a good decomposition [23], this relation- ship between shape decomposition and skeleton extraction is a relatively unexplored concept, especially in 3D. Instead, when a relationship is noted, the skeletons are usually

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treated as an intermediate result or a by-product of the shape decomposition. In the technical literature, the concepts of skeleton and medial axis are used interchangeably by some authors[24][25[26][27][28], while some other authors[29][30][31] regard them as related, but not the same.

2.Categories of methods.

All the existing skeletons are categorized into classes based upon the shape reduction operators used to extract the skeletons .The operators work on the data that are captured from the objects. Based on the data, it is classified into two cases.

2.1The Continuous case

The *skeleton* is defined as the locus of centers of maximal inscribed (open) balls (or disks in 2D) [32]. More formally, let *X C ¼ R3* be a 3D shape. An (open) ball of radius *r* centered at *x C¸ X* is defined as *Sr(x) = {y C¸ R3, d(x, y) < r}*, where *d(x, y)* is the distance between two points *x* and *y* in *R3*. A ball *Sr(x) C ¼ X* is maximal if it is not completely included in any other ball included in *X* [33]. The skeleton is then the set of centers of all maximal balls included in *X*. The process of obtaining a skeleton is called *skeletonization*. In 2D, the medial axis [34] of a shape is a set of curves defined as the locus of points that have at least two closest points on the boundary of the shape [32]. In the 3D case, the corresponding object is called the *medial surface* [35] because in addition to curves, it can also contain surface patches.

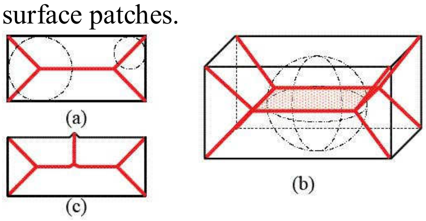


fig [2.1.1]

A major disadvantage of the medial surface (axis)/skeleton is its intrinsic sensitivity to small changes in the object's boundary due to the way it is defined [36][37].

2.2 The Discrete Case

In discrete space, the definitions are analogous to the continuous case, but problems may occur because of discretization. For example, a maximal ball may touch the discrete boundary of an object in a single point in cases other than the limit case, such as the case of an object whose width is an even number of voxels. Since the diameter of the ball is always an odd number of voxels (assuming the center of the ball must be one of the voxels), the ball will be maximal when it touches the boundary on only one side. As a result, in order to include all centers of maximal balls, the discrete skeleton may be more than one image element (pixel or voxel) thick. Furthermore, resolution can cause a loss of detail for certain objects such as merging or even disappearance of small features.

2.3 Conversions

Conversion between these two representations can be performed using well-known algorithms: continuous geometric data can be transformed into a discrete representation by voxelization [38], while voxelized data can be converted into a geometric representation using a surface extraction algorithm [39]. [72] A. Lieutier. Any open bounded subset of Rn has the same homotopy type than its medial axis, Proc. ACM SMI, 2003.

3.Skeleton Representation

The Medial Axis (MA), Voronoi diagram, Shock graph and Reeb graph are common skeleton representations. Although the MA can represent a lossless shape descriptor [2], it is di_cult and expensive to compute accurately in high (> 2) dimensional space [40]. Several ideas for approximating the MA have been proposed, e.g., using Voronoi diagram, and its dual Delaunay triangulation [10] [41] [42], of densely sampled points from the object boundary. Shock graphs [43] [44], another representation of the MA, encode the formation order and, therefore, the importance of each part of the MA. Reeb graphs, a type of 1D skeleton, extracted from various Morse functions, are a powerful tool for shape matching [12][45][46][17]. Since Morse functions are defined on mesh vertices, re-meshing [17][46] is usually needed to generate a good (accurate) skeleton. Several methods have been proposed to extract a skeleton from the components of a decomposition [4][5]. Skeletons can also be constructed by simplifying (contracting) a polygonal mesh to line segments [23].

3.1 Geometric methods

Geometric methods apply to objects represented by polygonal meshes or scattered point sets.A popular approach is to use the **Voronoi diagram** [47][48][49][50] generated by the vertices of the 3D polygonal representation or directly by a set of unorganized points [51][52][53].

3.2Cores and M-reps

All [54][55][56][57] are also medialaxis/ surface approaches. A core is a locus in a space whose coordinates are position, radius, and associated orientations. The location of the core represents the middle of the figure and the spread of the core represents the width of the figure. M-reps are a generalization of the Core concept. The M-rep models the medial surface using a “web” of connected atoms. Each atom has a position and additional information escribing the shape locally, such as: width, a local figural frame (which implies the figural directions) and an object angle between opposing corresponding positions on the implied boundary.

3.3The shock graph

It is based on the concept of contact spheres [58][59] and represents the medial axis/surface by a set of shock curves, defined as the intersection of medial surface sheets.

3.4The Reeb graph

The Reeb graph based shape descriptors, with roots in Morse theory, are 1D structures encoding the topology and geometry of the original shape. The Reeb graph captures the topology of a compact manifold by following the evolution of the level sets of a real-valued function defined on the respective manifold.

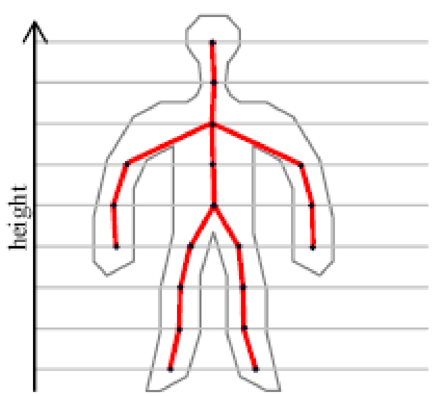


fig [3.4.1]

An embedding of the Reeb graph into the original image space. Each node is taken to be the centroid of its corresponding contour.

4.Framework

Image skeleton is presumed to represent the shape of the object in a relatively small number of pixels, all of which are, in some sense, structural and therefore necessary. The skeleton of an object is conceptually defined as the locus of center pixels in the object. [60] Unfortunately, no generally agreed upon definition of a digital image skeleton exists. But for all definitions, at least 4 requirements must be satisfied for skeleton objects [60]:

1**. Centeredness**: The Skeleton must be centered within the object boundary.

2. **Preservation of Connectivity**: The output skeleton must have the same connectivity as the original object and should not contain any background elements.

3. **Consistency of Topology**: The topology must remain constant.

4. **Thinness**: The output skeleton must be as thin as possible: 1-pixel thin is the requirement for a 2D skeleton, and 1-voxel thin in 3D.

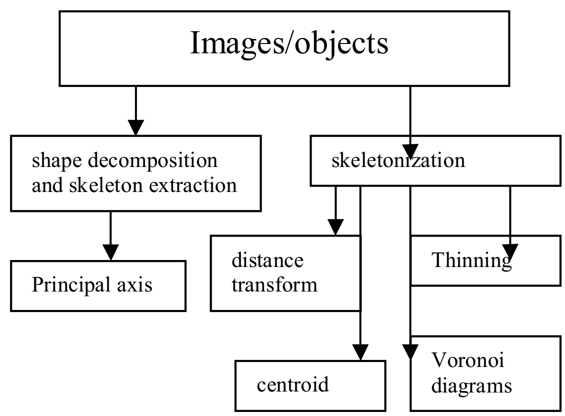


fig [4..1]

4.1 Shape Decomposition

A skeleton of a model is extracted from the components of its decomposition that is, both processes and the qualities of their results are interdependent. In particular, if the quality of the extracted skeleton does not meed some user specified criteria, then the model is decomposed into finner components and a new skeleton is extracted from these components. The process of simultaneous shape decomposition and skeletonization iterates until the quality of the skeleton becomes satisfactory.Shape decomposition partitions a model into (visually) meaningful components. Recently shape decomposition has been applied to texture mapping [61], shape manipulation [5], shape matching [62][63][64], and collision detection [23].

Early work on shape decomposition can be found in pattern recognition and computer vision; see surveys in [65],[66]. Simultaneous shape decomposition and skeleton extraction. The set fCig is a decomposition of the input model P and initially $\{C_I\} = \{P\}$.

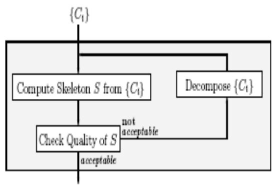


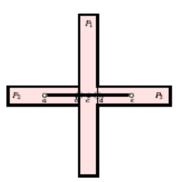
fig [4.1.1]

4.2 Extracting skeletons

we discuss two simple methods to extract a (local) skeleton from a component of a decomposition. These local skeletons can be connected to form a global skeleton of the input model. The centroid method, is very simplistic but can result in skeletons that do not represent the shape of the object. The second method, based on the principal axis of a component, is slightly more expensive to compute, but leads to improved skeletons in some cases.

4.2.1Using Centroids.

One of the easiest ways to construct a skeleton for a component C (in a decomposition) is to connect the centroids of the openings, called opening centroids, on @C to the centroid of C. These openings are generated when a component is split into sub-components during the decomposition process, Several similar methods for extracting skeletons have been proposed [4],[5]].

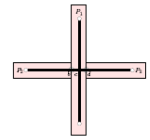


(using centroids)

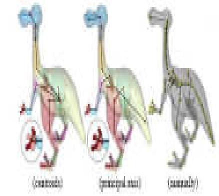
fig[4.2.1.1]

4.2.2Using the Principal Axis.

In this method, we extract a skeleton from a component C (in a decomposition) using the principal axis of the convex hull HC of C. Instead of connecting the centroids of C's openings to the center of mass of C, we connect these centroids to the principal axis enclosed in HC. Figure 5 shows an example of skeletons constructed in this manner



(using the principal axis)



fig[4.2.1] fig [4.2.2]

4.2 Skeletonization methods

Skeletonization is a transformation of a component of a digital image into a subset of the original component. There are different categories of skeletonization methods: one category is based on distance transforms, and a specified subset of the transformed image is a distance skeleton. Another category is defined by thinning approaches; and the result of skeletonization using thinning algorithms should be a connected set of digital curves or arcs.Another category is defined by voronoi diagrams.

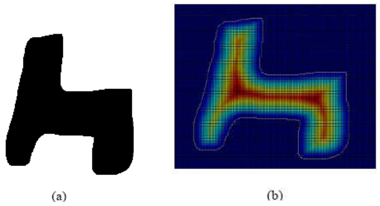
4.2.1Distance skeleton

The skeleton of a two-dimensional object is a transformation of the shape object into a onedimensionalline. Skeleton representation as introduced by Blum [1] meets most of these requirements.Map distance methods implement the idea of medial axis transformation in straightforward way [67] [68], but there seem to be serious problems finding a correct and connected set of discreteskeleton elements due to several problems when dealing with discrete metrics and balls.

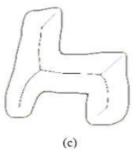
Calculating the Distance Map

Computing a distance map ( ) *DM* is not a very difficult task in discrete space. Given a suitable metric, one can compute the distance transform by propagating distance values from the boundary inwards [67] [68] [69][70].

Identifying Local Maxima in the Distance Map The set of axial points is derived from the Distance Map by applying the Medial Axes Transform (MAT). If the Distance Map is represented as a 3D surface, with the height corresponding to the distance values, the MAT is the set of local maxima M.



fig[4.2.1.1] Object fig[4.2.1.2] Distance map



fig[4.2.1.3] Skeleton by mean Crvature

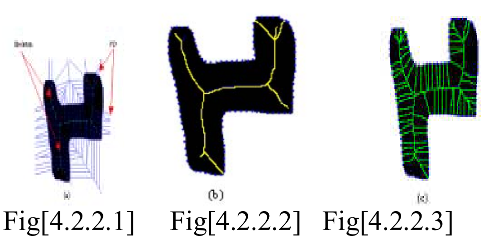
4.2.2VORONOI DIAGRAM SKELETON

Voronoi Diagram

Among the algorithms known for computing Voronoi diagrams of points in 2D, 3D and higher dimensions are the divide-and-conquer algorithm .

Skeleton from Voronoi Diagram

The 2D Voronoi skeleton of an object can be computed from set points from the objects boundary.The skeleton is obtained from the dual of the Delaunay triangulation of the sample point, a skeleton is a subgraph of the Voronoi tessellation of the point. As stated in [71] [72], the 2D Voronoi skeleton has been defined in many ways : Voronoi vertexes included in *S* Fig [4.2.2.1].a, Voronoi elements included in *S* Fig [4.2.2.2].b, intersection of Voronoi diagrams with *S* Fig [4.2.2.3].c. However, it seems most convenient to define the skeleton as the Voronoi vertexes and edges included in the object, since this is the smallest set which approximates the medial axis.



Fig[4.2.2.1] Fig[4.2.2.2] Fig[4.2.2.3]

4.2.3Thinning methods

Thinning or erosion of the image is a method that iteratively peels off the boundary layer by layer from outside to inside. The removal does not affect the topology of the image. This is a repetitive and time-consuming process of testing and deletion of each pixel. It is good for connectivity preservation. The problem with this approach is that the set of rules defining the removal of a pixel is highly dependent on the type of image and different sets of rules will be applied to different types of images. The phases are the thinning layers.are depicted as follows.

fig[4.2.3.1]originalimage

fig[4.2.3.2] intermediate step

fig[4.2.3.3]skeleton of the original image using thinning.

5.Measuring Skeleton Quality

Although several criteria exist for measuring the quality of a skeleton, the general principles we adopt are that the skeleton should reside in the interior of the model and it should encode the \topology" of the model's shape.Thus, using these general criteria, our strategy of computing the quality of a skeleton S is to compare S with its associated component C. In this section, we propose three methods for measuring quality. This first method checks whether S intersects ∂C and the second method checks the topological representation of S w.r.t. C. In the third method, we propose an adaptive measurement based on the volume of the component.An important property of these three methods is that, as the decomposition becomes _ner, the error of the skeleton becomes smaller. This property is justi_ed in Appendix C. Fig[5.1] shows an example of extracted skeletons based on these three quality measurements.

5.1Checking penetration.

Our first method measures the quality of S by checking whether S intersects the component boundary ∂C. If so, the function Error(C; S) returns a large number Otherwise, zero will be returned. The consequence is that C will be decomposed if $\partial C \cap S \neq 0$.

5.2Measuring centeredness.

In the second method, we measure the o_sets of S from the level sets of the geodesic distance map on ∂C. The value for each point in this map is the shortest distance to its closest opening of C. Ideally, a skeleton should pass through all connected components in all level sets. Therefore, this measurement method simply checks the number of times that S does not do so. Let LC be all the connected components in the level sets of C. We define the error of a skeleton S as:

$$Err(C; S) = \frac{\sum_{lc \in LC} f(lC,S)}{|L_C|}$$

where f(lc; S) returns 0 if S intersects component lc, and 1 otherwise, and jLCj is the total number of the connected components in C.

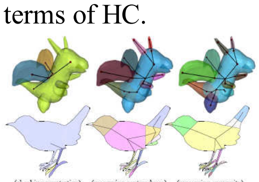
5.3Measuring convexity.

Our idea for the last quality measurement comes from the observation that in many cases the significance of a feature depends on its volumetric proportion to its “base".

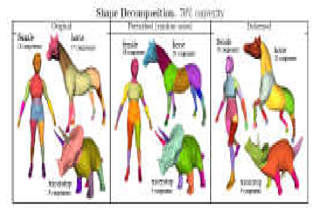
For example, a 5 cm stick attached to a ball with 5 cm radius is a more signi_cant feature than a 5 cm stick attached to a ball with 5 km radius. This intuition can be captured by the concept of the convexity of a component C defined as convexity(C) = vol(C) / vol(H_C) , where vol(X) is the volume of a set X. Thus, we can defin the error measurement as:

$$\boxed{Err(C, S) = 1\ \_\ convexity(C)}$$

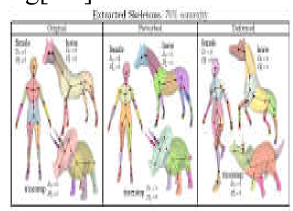
Assume that the skeleton S is a good representation of the convex hull HC. Then, a smaller difference between HC and C means that S is a better representation of C. Thus, although the skeleton S is not included in Equation 4, S is implicitly considered in terms of HC.



fig[5.1]



fig[5.2]



fig[5.3]

5.4 PROPERTIES OF SKELETON CLASSES.

Summary of properties achievable by the various skeleton classes

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	Thinning	Distance Field	Geometric	General Field
Homotopic	Y		Y	N
Transf. Invariance		Y		Y
Reconstruction	N		N	N
Thin				
Centered				
Reliable				
Junction Detection			Y	Y
Connected	Y			
Robust	N	N	N	Y
Smooth				Y
Hierarchic	N			Y
Efficiency	Y	Y	Y	N
Handle Point Sets	N		Y	Y

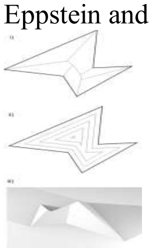
fig[5.4.1]

6.Skeleton types.

In Digital image processing ,for 3D objects there are many types known from the literature.Some of them are Straight skeletons, Morphological skeletons, Topological skeleton,Curve skeletons.

6.1Straight skeletons

Straight skeletons were first defined for simple polygons by Aichholzer et al.,[73] and generalized to planar straight line graphs by Aichholzer and Aurenhammer.[74] Cheng and Vigneron showed how to compute the straight skeleton of a simple polygon with n vertices, r of which have angles greater than Pi; in time O(n log2 n + r3/2 log r).[75] For more general polygonal inputs, the best known time bound is from a more complex and slower algorithm by Eppstein and Erickson.[76]



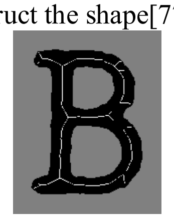
fig[6.1]

6.2Morphological skeleton

In digital image processing, **morphological skeleton** is a skeleton (or medial axis) representation of a shape or binary image, computed by means of morphological operators.[79]

6.3. Topological Skeleton

In shape analysis, **skeleton** (or **topological skeleton**) of a shape is a thin version of that shape that is equidistant to its boundaries. The skeleton usually emphasizes geometrical and topological properties of the shape, such as its connectivity, topology, length, direction, and width. Together with the distance of its points to the shape boundary, the skeleton can also serve as a representation of the shape (they contain all the information necessary to reconstruct the shape[77]



fig[6.3.1]

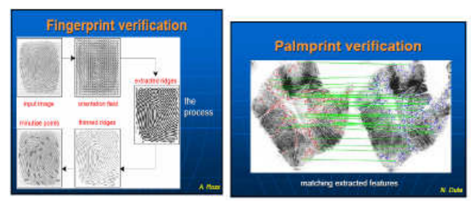
6.4 Curve skeleton

They are the thinned 1D representations of 3D objects useful for many visualization tasks including navigation,animation,etc.[78]

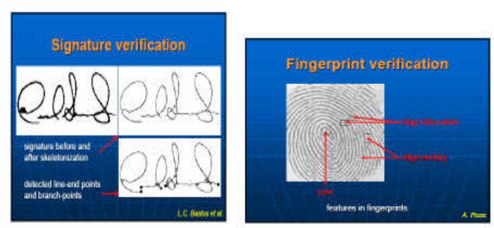
7.Applications.

We cannot predict the applications of skeletons in the cuurent digital world.Where ever computers are used , surely skeletons will occupy the first place in their developments.The extracted skeleton can be readily used to create animations. They also help to plan motion, e.g., for navigating in the human colon or removing a mechanical part from an airplane engine. Sampling-based motion planners have been shown to solve difficult motion planning problems.

fig[7.1]



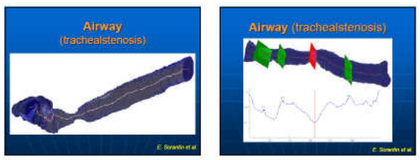
fig[7.2]



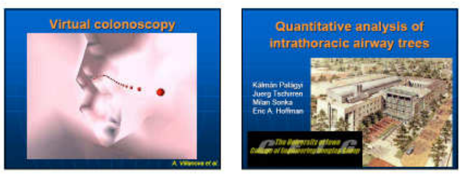
fig[7.3]

fig[7.4]

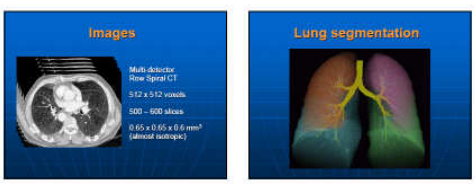
fig[7.5]



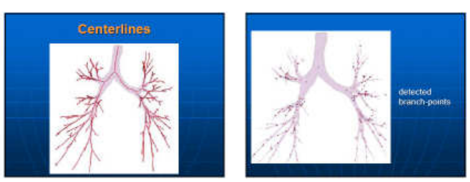
fig[7.6]



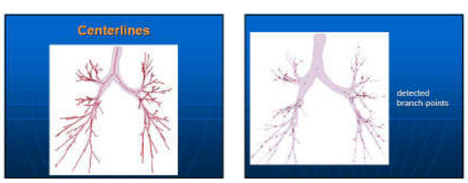
fig[7.7]



fig[7.8]

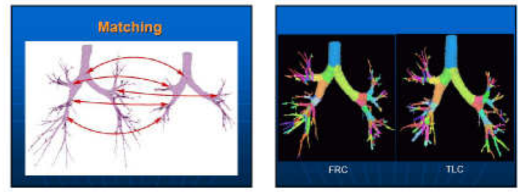


fig[7.9]

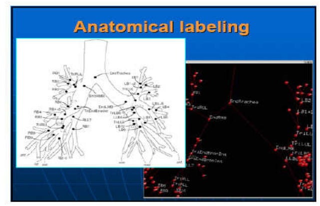


fig[7.10]

fig[7.11]



fig[7.12]



fig[7.13]

8. Conclusion

There are several ways to extend the current work. First, there is a need to establish a systematic framework for comparing qualities of shape decomposition and skeletons using more quantitative measuring methods and benchmarks. Although the proposed survey is based on a general idea of what a good skeleton should be, more studies are needed to investigate application -specific measurement criteria that should produce better and more “comparable" results. Any processing on the skeletons towards the enhancement of the properties of the skeleton described here will surely results in the development of the new digital image processing field.This survey on skeletons will help for that new step indigital image processing.

9.ACKNOWLEDGEMENT

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