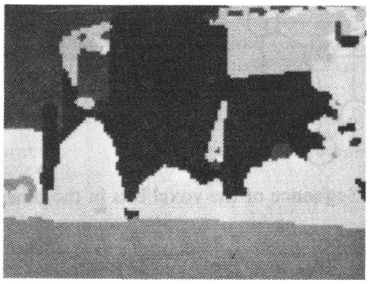

Fig. 6. Results of 19 iterations of relaxation.

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Three-Dimensional Skeletonization: Principle and Algorithm

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Abstract—**An algorithm is proposed for skeletonization of 3-D images. The criterion to preserve connectivity is given in two versions: global and local. The latter allows local decisions in the erosion process. A table of the decisions for all possible configurations is given in this paper. The algorithm using this table can be directly implemented both on general purpose computers and on dedicated machinery.**

Index Terms—**Connectivity, erosion, 3-D image processing, skeleton.**

Introduction

A special class of transformations are those applied to two-dimensional (2-D) images that consist of objects and background. Objects can be eroded or dilated, touching objects can be separated, or gaps can be filled in. These types of transformation are described by Hersant *et al.* [4], [5] and Rosenfeld and Kak [6].

A connectivity preserving way of erosion called skeletonization is described by Hilditch [3]. The resulting skeletons are one picture element (pixel) thick objects, which have the same connectivity as the original objects. Skeletons are of special interest because they reflect the structure of the original objects in their end pixels and vertices.

Three-dimensional (3-D) images consisting of volume elements (voxels) are present in tomography and echocardiography, for example. Analogous to the 2-D case, transformations like erosion and dilation can be applied to a voxel and its 3-D neighbors. Some of those 3-D transformations are easily generalized from the 2-D case. However, for a skeletonization algorithm one needs a connectivity preserving condition in 3-D. This paper deals with this topic.

The connectivity condition alone does not ensure stability of the skeleton. Line and leaf shaped parts of the objects, although one voxel thick would still further be eroded during skeletonization. To avoid this, one needs an additional criterion (end voxel condition). However, such an additional condition depends on the problem involved, and is not discussed here.

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Theoretical Background

Euler and Poincaré (see [2]) provided the means for checking the connectivity for objects in 3-D. They proved that for each single closed netted surface

$$n - e + f = 2 \tag{1}$$

where n denotes the number of nodal points of the net, e denotes the number of edges, and f denotes the number of faces. More generally, when an object or set of objects and closed cavities in objects are present in a 3-D space, it can be shown (Hilbert and Cohn-Vossen [2]) that for each netted surface S_i^* which encloses an object or a cavity

$$n_i - e_i + f_i = 2 - 2h_i \tag{2}$$

where h_i denotes the number of handles (i.e., tunnels, cf. Fig. 1) in S_i^*. So for the entire 3-D image one may define the connectivity number N,

$$N = \sum_i (2 - 2h_i). \tag{3}$$

The number of closed netted surfaces minus the number of handles in them equals $\frac{1}{2}N$. This results in the rule: in order to preserve the connectivities present, the value of N should not change during skeletonization. The algorithm to be presented will be based on a local version of this global criterion.

Consider the 3-D quantized space filled with cubic volume elements (voxels). Each voxel has several types of neighbors. One may consider the following types: those which are 6-adjacent to a voxel v and so have a face in common with v [Fig. 2(a)] those which are 26-adjacent to v and so have a face, an edge, or a point in common with v [Fig. 2(b)].

When the set of voxels belonging to the objects is called S and if voxels within an object are 6-connected, then the voxels not belonging to S (belonging to $\overline{S}$) should be 26-connected and vice versa. (Whether 6-connectivity is adopted for S and 26 for $\overline{S}$ or the other way around constitutes a free choice.) This is similar to the situation in 2-D where paradoxical situations are avoided if S is taken 4-connected when $\overline{S}$ is 8-connected and vice versa (Rosenfeld and Kak [6]).

In this case the netted surface S^* which separates S from $\overline{S}$ consists of square faces. In general S^* consists of several separate surfaces S_i^*. Equations (2) and (3) applied to S^* provide the number N.

The Algorithm

The rule for conservation of connectivity in the skeletonization algorithm is: a voxel v belonging to an object can be deleted when this deletion does not cause a change in N. This implies determining N before and after possible deletion of v. One can restrict oneself to checking the contribution to N from the $3 \times 3 \times 3$ neighborhood of v.

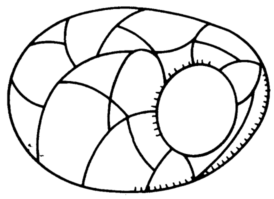


Fig. 1. Netted surface with one handle (tunnel).

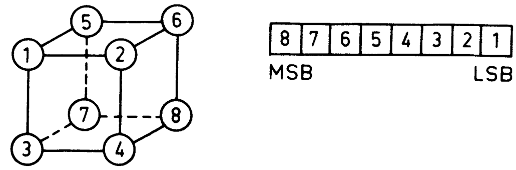
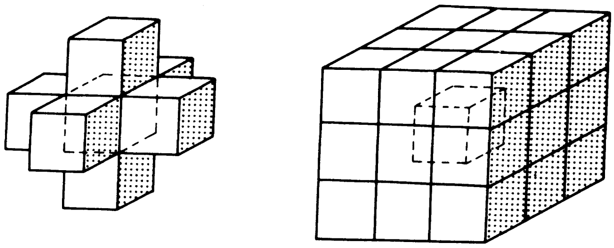


Fig. 3. Sequence of the voxel bits in the binary word.



(a) (b)

Fig. 2. A voxel surrounded by (a) its 6-connected neighbors and by (b) its 26-connected neighbors.

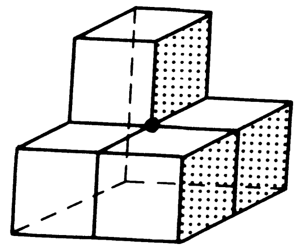


Fig. 4. Example of a $2 \times 2 \times 2$ configuration around a nodal point which contains one 6-connected surface, $N_6^{(220)} = N_{26}^{(220)} = -\frac{1}{4}$.

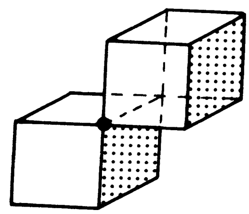


Fig. 5. Example of a $2 \times 2 \times 2$ configuration where two 6-connected surfaces touch and $N^{(c)}$ depends on the choice between 6- and 26-connectivity, $N_6^{(96)} = \frac{1}{2}, N_{26}^{(96)} = -\frac{1}{2}$.

A fast and simple algorithm can be obtained when the $3 \times 3 \times 3$ neighborhood of v is considered as eight partially overlapping $2 \times 2 \times 2$ cubes (centered around each nodal point j of v) for which the contributions N_j to N are computed separately.

There are $2^8 = 256$ possible configurations for a $2 \times 2 \times 2$ cube, as a voxel may belong to an object (denoted by 1) or may not (denoted by 0). Consequently, N_j can assume only 256 values $N^{(c)}$. (The 8 voxel bits constitute a binary word $c(j)$, which is attached to each configuration. The sequence of the voxel bits is given in Fig. 3.) The contribution $N^{(c)}$ of each cube configuration can easily be stored in a table, the index of which is the binary word $c(j)$ of voxel bits. So by addressing the table with and without the centre voxel v 8 times, the contribution to N of the $3 \times 3 \times 3$ neighborhood with and without v can be determined.

Let us restrict ourselves now to 6-connected objects and denote the 256 possible contributions by $N_6^{(c)}$. Two essentially different types of configuration can be distinguished.

1) All object voxels in the $2 \times 2 \times 2$ cube are 6-adjacent to each other, so only one surface S_i^* occurs in the cube. In the center of each $2 \times 2 \times 2$ cube one nodal point may be present. At this position 6 edges may join and 12 faces may touch. Going through the $3 \times 3 \times 3$ neighborhood by the overlapping $2 \times 2 \times 2$ cubes each nodal point of S_i^* is encountered once, each edge of S_i^* is encountered twice and each face of S_i^* is encountered four times. This results in a contribution $N^{(c)}$ to N for each $2 \times 2 \times 2$ cube of

$$N^{(c)} = n^{(c)} - e^{(c)}/2 + f^{(c)}/4. \tag{4}$$

Here $n^{(c)}$ denotes the number of nodal points of S_i^* in the center of the $2 \times 2 \times 2$ cube, $e^{(c)}$ is the number of edges of S_i^* which join at the center, and $f^{(c)}$ is the number of faces of S_i^* which touch at the center. Obviously $n^{(c)}$ is equal to zero or one and when $n^{(c)} = 0$, then also $e^{(c)} = f^{(c)} = 0$. This means that any $2 \times 2 \times 2$ cubes centered around points in the bulk of an object or background (which hence are no part of S^*) do not contribute to N.

An example with one surface is given in Fig. 4. The binary representation of this configuration is 11011100, which is decimal 220. In this example, $n^{(c)} = 1$, $e^{(c)} = 5$, and $f^{(c)} = 5$ resulting in

$$N_6^{(220)} = 1 - \tfrac{5}{2} + \tfrac{5}{4} = -\tfrac{1}{4}. \tag{5}$$

2) Not all object voxels within the $2 \times 2 \times 2$ cube are 6-adjacent to each other.

When two voxels share only an edge (Fig. 5) they locally form at that edge two different surfaces, by definition of connectivity. As this edge is shared by two surfaces it must be counted twice.

When there are object voxels which only share an edge or a point with the other object voxels in the $2 \times 2 \times 2$ cube, two or more 6-connected surfaces are present in the $2 \times 2 \times 2$ cube. When k 6-connected surfaces in the cube are present the central nodal point is part of each and must be counted k times. When $e_s^{(c)}$ is the number of shared edges we obtain for the contribution $N^{(c)}$,

$$N^{(c)} = kn^{(c)} - \frac{e^{(c)} + e_s^{(c)}}{2} + \frac{f^{(c)}}{4}. \tag{6}$$

An example with two surfaces is given in Fig. 5. The binary representation is 01100000 which is decimal 96. So $k = 2$, $n^{(c)} = 1$, $e^{(c)} = 5$, $e_s^{(c)} = 1$, and $f^{(c)} = 6$, resulting in

$$N_6^{(96)} = 2 - \tfrac{6}{2} + \tfrac{6}{4} = \tfrac{1}{2}. \tag{7}$$

One may also study the complement of this situation binary 10011111 decimal 159. In this case one surface is present, and one edge is shared $e^{(c)} = 5, f^{(c)} = 6$

$$N_6^{(159)} = 1 - \tfrac{6}{2} + \tfrac{6}{4} = -\tfrac{1}{2}. \tag{8}$$

When the objects are 26-connected we obtain the complementary situation of the 6-connected configuration. The contribution $N_{26}^{(c)}$ for the 26-connected case, equals the contribution $N_6^{(c)}$ for the complementary 6-connected case.

For example, the configuration of Fig. 4 (binary 11011100 decimal 220) gives $N_6^{(220)} = -\frac{1}{4}$. The complementary configuration (binary 00100011, decimal 35) gives $N_{26}^{(35)} = -\frac{1}{4} = N_6^{(220)}$.

TABLE I
THE 22 BASIC CONFIGURATIONS: THEIR BINARY (cf. FIG. 2) AND DECIMAL REPRESENTATION AND THEIR CONTRIBUTIONS $N_6^{(c)}$ AND $N_{26}^{(c)}$ TO N IN CASE OF 6- AND 26-CONNECTIVITY

configuration			contribution x4	
k(j)	c(j) binary	c(j) decimal	$4N_6^{(c)}$	$4N_{26}^{(c)}$
1	00 00 00 00	0	0	0
2	00 00 00 01	1	1	1
3	00 00 00 11	3	0	0
4	00 00 10 01	9	2	-2
5	10 00 00 01	129	2	-6
6	00 00 01 11	7	-1	-1
7	01 00 00 11	67	1	-3
8	00 01 01 10	22	3	-1
9	00 00 11 11	15	0	0
10	00 10 01 11	39	-2	-2
11	00 01 01 11	23	-2	-2
12	10 00 01 11	135	0	0
13	11 00 00 11	195	0	0
14	01 10 10 01	105	4	4
15	11 10 10 01	233	-1	3
16	10 11 11 00	188	-3	1
17	11 11 10 00	248	-1	-1
18	01 11 11 10	126	-6	2
19	11 11 01 10	246	-2	2
20	11 11 11 00	252	0	0
21	11 11 11 10	254	1	1
22	11 11 11 11	255	0	0

THE TABLE

There are 22 basic different possibilities to fill the $2 \times 2 \times 2$ cube among the 256 configurations. All the other ones can be produced by the symmetry operations of the cube. In Table I the binary and the decimal representation of each of the 22 basic configurations are given, together with the contributions $N_6^{(c)}$ in case of 6-connectivity and $N_{26}^{(c)}$ in case of 26-connectivity. The decimal representation provides an index to the table in which all the 256 possibilities are stored.

The configurations $k(j)$ ($k(j) = 1$ to 8) are complementary to the configurations $23 - k(j)$; configurations 9 to 14 are self-complementary.

CONCLUSION

Using the skeletonization algorithm described one can erode three-dimensional binary images preserving connectivity. the algorithm is a fast one because of the use of tables. It can also be implemented on our dedicated image processor (Gerritsen *et al.* [1]) for which typically 16 operations of 250 ns per voxel would be needed. Through the use of special purpose logic based on the $3 \times 3 \times 3$ neighborhood this might be shortened by a factor 3.

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A Method for Automating the Visual Inspection of Printed Wiring Boards

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Abstract—**The application of pattern recognition techniques to manufacturing processes is a rapidly developing technology. Automatic verification of the quality of printed wiring boards (PWB's) using pattern recognition techniques is one potential application in this field. Qualitatively, this problem is finding small, irregular features in an environment of complicated, but larger and well-defined geometric features. In addition to the basic pattern recognition task, stringent performance requirements, both for throughput and accuracy, must be met if actual production usage is expected.**

The method employed in this study is based on characterizing 5×5 or 7×7 element binary patterns derived from the class of PWB's being inspected as good or defective. A database of 80 512×512 element images of PWB's was constructed and used to determine the number of unique patterns and their rates of occurrence. The major experimental result of this study is that *less* than 500 of the possible $(15/16)2^{24}$ 5×5 patterns are needed to describe *all* the border containing patterns in the 80 images. It is also apparent that more patterns would be required if the training database was larger.

The small number of patterns needed to represent virtually all of the normal border patterns suggests a two-stage inspection strategy. In the first stage, each border pattern from the PWB being inspected is compared to a previously prepared list. Those patterns not found in this list are passed to a second stage which employs a variety of techniques to determine if the pattern is indicative of a PWB flaw.

Index Terms—**Automated inspection, pattern recognition, printed wiring boards.**

I. INTRODUCTION

The application of pattern recognition techniques to manufacturing processes, particularly for visual inspection, is a rapidly developing technology. One such application, inspection of printed wiring board (PWB) conductor patterns for small flaws has been the subject of several studies [1]-[5]. This paper will propose another method for accomplishing this task.

Very qualitatively, the PWB inspection problem examined in this paper consists of looking for small, irregularly shaped conductor defects in the context of the larger and well-defined conductor patterns. In most cases, the border between the conductor and substrate is of primary interest while the surface condition of the conductor is only of marginal interest.

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