A simple parallel 3D thinning algorithm

WeiXin GONG and Gilles BERTRAND

Laboratoire Intelligence Artificielle et Analyse d'Images ESIEE

BP.99 2, Boulevard Blaise Pascal 93162 Noisy-Le-Grand Cedex France

Abstract :

In this paper, we present a parallel 3D thinning algorithm, which conserves medial surfaces. A new characterization of simple points is proposed and some new topological predicates are given which are very simple to calculate. Some new geometrical predicates are also given. We prove that the thinning operation based on those new predicates does not deconnect a 3D object. The experiments show that the method gives a satisfactory result.

Key words: 3D binary image, parallel thinning operation, medial surface, simple point

1 Introduction

In the continuous space, the medial surface of a 3-dimensional (3D) object is defined as the set of centres of maximal euclidean balls included in the object. In the discrete space, one way to approach the continuous medial surface is to thin iteratively the object. Let S be an object in Z^3, ($S \subset Z^3$): S is represented by 1's, $\overline{S}$ by 0's.

Like a 2D thinning algorithm, a 3D thinning algorithm deletes iteratively the border points of S, until no more points , according to some predicates, can be deleted. The remaining points constitute the medial surface of S, an example is shown in fig.6. The 3D thinning algorithm should preserve the topological properties of the 3D object, i.e. the medial surface should have the same number of holes, cavities, and connected components as the object. There are two ways to implement thinning operations, one is sequential, the other is parallel. In this paper we will present a 3D parallel thinning algorithm.

In the discrete space Z^3, the neighborhood $N_{26}(p)$ of a point comprises 26 points adjacent to p, see fig.1. $p_{(r,s,t)}$ is said 6-adjacent (26-adjacent) to $p_{(i,j,k)}$, if $|i-r|+|j-s|+|k-t| = 1$ ($max(|i-r|, |j-s|, |k-t|) = 1$). $N_6(p)$ denotes the set of points 6-adjacent to p. $N_{26}(p)$ denotes the set of points 26-adjacent to p and $N'_{26}(p) = N_{26}(p) \cup p$. A point $p \in S$ is called a border point , if at least one of its 6-neighbor is in $\overline{S}$. A border point is called an U (B,E,S,W or N) type border point if it's up(bottom, east, south, west or north) 6-neighbor is in $\overline{S}$ (see fig.1). The definition of a path, connected component, arc, curve, cavity and hole can be found in [3,5,6,7].

In this work we present a parallel algorithm based on some deletion predicates which are very simple. In the second section, we propose a new characterization of simple points. In the 3th section, we present the deletion predicates. Then we present two thinning operations. We will compare our algorithm with other algorithms in the 7th section.

2 simple point

If the deletion of a point p from S does not change the topological properties of S, the point is called a simple point. In [1,8,10], path connectivity in $N'_{26}(p)$ is used to check the preservation of topological properties. In fact, it is not sufficient, because a hole may be created in $N'_{26}(p)$ even if path connectivity in $N'_{26}(p)$ is preserved, see fig.2.

a

x	x	x
x	p_U	x
x	x	x

b.window 1

x	p_N	x
p_W	p	p_E
x	p_S	x

c.window 2

x	x	x
x	p_D	x
x	x	x

d.window 3

Figure 1: a) the neighborhood $N'_{26}(p)$ of p: the up (medium, bottom) layer is referred to window 1 (2,3). p_U ($p_B, p_N, ..., p_W$) is the up(bottom,north,...,west) 6-neighbor of p.

Thus, thinning algorithms based on path connectivity may change the topological properties of S. In [10], Tsao et al. use other additional conditions to complete their algorithm. While in[8], no other conditions have been used, hence it seems that this algorithm can not obtain a satisfactory result.

A characterization of simple point is given in [4]. A point is simple if and only if it satisfies the following **conditions 1:**

$1a$	$NC(S \cap N'_{26}(p)) = NC(S \cap N_{26}(p))$
$1b$	$NC((\overline{S} \cap N_{26}(p)) \cup p) = NC(\overline{S} \cap N_{26}(p))$
$1c$	$NH(S \cap N'_{26}(p)) = NH(S \cap N_{26}(p))$
$1d$	$NH((\overline{S} \cap N_{26}(p)) \cup p) = NH(\overline{S} \cap N_{26}(p))$

where NC(A) denotes the number of connected components of A and NH(A) denotes the number of holes in A. According to this characterization, a straightforward way to check if a point is simple is to count the number of connected components and holes in $N_{26}(p)$ and in $N'_{26}(p)$. This calculation is not easy to implement in parallel.

In the following, we assume that 26-connectedness is used for S, while 6-connectedness is used for $\overline{S}$. We introduce two conditions (1a') $\overline{C_6} = 1$ and (1b') $C_{26} = 1$, $\overline{C_6}$ is the number of connected components in $\overline{S} \cap N_{26}(p)$ adjacent (in the 6-connectedness sense) to p, C_{26} is the number of connected components in $S \cap N_{26}(p)$ adjacent (in the 26-connectedness sense) to p. It is obvious that C_{26} is equal to NC($S \cap N_{26}(p)$). With these new conditions, a new characterization of simple points can be proposed, the proof of this proposition is omitted:

Proposition 1.

A point is simple if and only if it satisfies the conditions $1a'$,$1b'$,$1c$ and $1d$.

3 new topological predicates

Though we have simplified the characterization of simple point, it is still not easy to check if a point is simple. In the 2D case (see [11]), let $\overline{C_4}$ be the number of connected components of $\overline{S} \cap N_8(p)$ 4-adjacent to p, let C_8 be the number of connected components of $S \cap N_8(p)$ 8-adjacent to p. $N_8(p)$ is the 8-neighborhood of p in 2D. It can be seen that if p is a border point and not isolated, $\overline{C_4} = C_8$ is true. Since $\overline{C_4}$ is easier to calculate than C_8, one calculates $\overline{C_4}$ to get C_8, if one wants C_8. Unfortunately in 3D, the relation $\overline{C_6} = C_{26}$ does not hold. Moreover,

o	o	o
o	o	o
o	•	•

window 1

•	•	•
•	•	•
o	o	•

window 2

•	•	•
•	•	•
•	•	•

window 3

Figure 2: • represents a point in S, o a point in $\overline{S}$.

even if we know the values of $\overline{C}_6$ and C_{26}, we have to compute the four numbers appearing in 1c and 1d to know whether a point is simple or not. So we try to find other simpler conditions.

Before giving the conditions, we introduce some notations. In 3D, a point can be viewed as an elementary cube. If a point p_i shares a common edge with another point p, we say that p_i is 12-adjacent to p. If a point p_i shares a common vertex with another point p, we say that p_i is 8-adjacent to p.

It is well known that, if all simple points are deleted simultaneously, an object may vanish. To avoid this, an iteration is divided into 6 subcycles; let d denote one of the 6 directions: up, bottom,..., north; in the d-subcycle, only points belonging to S having a neighbor in $\overline{S}$ in the d direction are considered. In the following, p will denote a point belonging to S, p_d the 6-neighbor of p in the d direction, $p_{\overline{d}}$ the 6-neighbor of p in the direction opposite to d. Let $N_d(p) = (N_6(p) - \{p_d\} - \{p_{\overline{d}}\})$. We present now the following conditions:

2a $\quad p_d = 0.$

2b $\quad p_{\overline{d}} = 1.$

2c $\quad if\ p_i = 0,\ then\ p_l = 0.$

$$for\ any\ p_i \in N_d(p),\ \ and\ for\ the\ point\ p_l\ 6-adjacent\ to\ p_i\ and\ to\ p_d.$$

2d $\quad if\ p_i, p_j\ and\ p_k = 0,\ then\ p_m = 0.$

$$for\ any\ p_i \in N_d(p),\ p_j \in N_d(p),\ p_j\ 12-adjacent\ to\ p_i,\ p_k$$

$$6-adjacent\ to\ p_i\ and\ p_j, p_m\ 12-adjacent\ to\ p_d\ and\ 6-adjacent\ to\ p_k.$$

It can be seen that if a point satisfies the above conditions 2, it satisfies the conditions 1. In the following we give some lemmas to verify it. We suppose that d = up (see fig.1).

Lemma 2. If a point satisfies condition 2, it satisfies condition $1a'$.
proof. If p_d is p_U, we have $p_B = 1$. Since 26-connectedness is used for S, any object point in the second window or in the third window is 26-connected to p_B. From 2c and 2d any object point in the first window is 26-connected to an object point in the second window. Thus in $N_{26}(p)$, we have one object component. Q.E.D.

Lemma 3. If a point satisfies condition 2, it satisfies condition $1b'$.
proof. According to 2a, $p_U = 0$. To verify $\overline{C}_6 = 1$, it suffices to verify that for any $p_i \in N_d(p)$, if $p_i = 0$, a 6-path can be found between p_i and p_U in $\overline{S} \cap N_{26}(p)$. According to 2c, it is true. Q.E.D.

Lemma 4. If a point satisfies condition 2, it satisfies condition 1c.
proof. When a point p changes from 1 to 0, there exists either the possibility to create a hole or the possibility to deconnect a 26-curve in $S \cap N'_{26}(p)$ so that a hole vanishes. Thus to verify the proposition, it suffices to verify that when a point satisfying condition 2 changes from 1 to 0, neither a hole is created nor a hole is suppressed.
(a) no hole can be created in $S \cap N_{26}(p)$.

In any one of the two following cases, a hole will be created if the point p is removed:

x	a	x
o	o	b
o	•	x

window 1

x	a	x
•	•	b
o	o	x

window 2

x	a	x
x	•	b
x	x	x

window 3

Figure 3: o's constitute a 6-arc. At least one a is 1 and one b, 26-adjacent to this a, is 1; two such points and two • constitute a 26-curve in $N_{26}(p)$. Other points(a's,b's and x's) should take such values that only one 6-component exists in $\overline{S}$ and one 26-component exists in S.

- a 26-curve in $S \cap N_{26}(p)$ exists and two components in $\overline{S} \cap N_{26}(p)$ 6-adjacent to p exist.
- a 26-curve γ_1 in $S \cap N_{26}(p)$ exists and a 6-arc α in $\overline{S} \cap N_{26}(p)$ exists such that $\alpha \cup p$ is a 6-curve γ_2 and the two curves γ_1 and γ_2 interleave.

It can be seen that if a point meets one of the above cases, it cannot be removed because it does not satisfy condition 2. For the first case, it is obvious that if the point p has two components in $\overline{S} \cap N_{26}(p)$ 6-adjacent to it, it does not satisfies condition 2. For the second case, the only possibility is shown in fig.3. It can be seen that the point p can not satisfy condition 2.
(b) no hole can vanish.

No 26-curve, which passes by p, can be constructed in $S \cap N'_{26}(p)$. Q.E.D.

Lemma 5. If a point satisfies condition 2, it satisfies condition 1d.
proof. When a point p changes from 1 to 0, a hole may vanish in $\overline{S} \cap N_{26}(p)$. In such a case, there should be a 6-curve in $\overline{S} \cap N_{26}(p)$ and two components in $S \cap N_{26}(p)$. It can be seen that such a point can not satisfy the conditions 2. Q.E.D.

According to lemmas 2-5, we have the following proposition:
Proposition 6.
If a point satisfies conditions 2, it must be a simple point.

The reciprocal of proposition 6 does not hold, i.e., a simple point may not satisfy conditions 2. One can find examples to show it. Thus in a subcycle, the number of removed points using conditions 2 will be less than those using condition1, but it is not important because such points are very rare and the complexity of the new conditions is considerably reduced.

4 a topological thinning operation

In this section, we present a parallel thinning operation T1. We will prove that T1 does not deconnect a 3D object.

The thinning operation T1 consists of removing simultaneously one type of border points which satisfy conditions 2 in a subcycle. Suppose that up border points are treated in the subcycle. Let R be the set of removed points, we have following proposition.

Proposition 7.
After an application of thinning operation T1 to S, points of $S - R$ connected in S are also connected in $S - R$.
proof. It suffices to prove that if $p_0, ..., p_n$ is a path with p_0, p_n in $S - R$ and $p_1, ..., p_{n-1}$ in R , then $p_1, ..., p_{n-1}$ can be replaced by a new path in $S - R$ such that p_0 and p_n are connected in $S - R$. According to the condition 2b, for any $p_i, p_{i+1} (1 \le i < n-1)$,one can find respectively in their neighborhood y_i, y_{i+1} such that $y_i, y_{i+1} \in S - R$ and y_i is adjacent to y_{i+1}(in fact, y_i is the bottom 6-neighbor of p_i and y_{i+1} is the bottom 6-neighbor of p_{i+1}.) . The remaining problem to resolve is to find a path $S - R$ which connects p_0 and y_1.

Since p_0 is adjacent to p_1, p_0 belongs $N_{26}(p_1)$. If p_0 is not in the first window, it is connected to y_1. If p_0 is in the first window, two cases should be discussed:

a. p_0 is 12-adjacent to p_1.
According to condition 2c, if p_1 can be removed, the point x must be 1(see fig.4). Since p_0 is 1, x can not be removed in this subcycle. Thus, the point x connects p_0 and y_1.

b. p_0 is 8-adjacent to p_1.
As $p_0 = 1$, according to condition 2d, at least one of the three points x, y, z is 1 (see fig.5). Two cases should be discussed. If y is 1, then y connects p_0 and y_1. If x is 1, (since y is 0 and p_0 is 1, according to condition 2c, x can not removed in the subcycle) then x connects p_0 and y_1. Q.E.D.

In the similar way, one can prove that T1 does not connect two components of the complement of a 3D object, i.e., after an application of thinning operation T1 to S, points of $\overline{S}$ connected in $\overline{S} \cup R$ are also connected in $\overline{S}$. One can also prove that T1 has another property : it does not create holes or cavities in S. Thus the proposed thinning operation T1 does not change the topological properties of S.

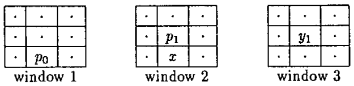

Figure 4: the neighborhood of p_1.

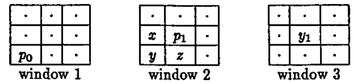

Figure 5: the neighborhood of p_1.

5 geometrical predicates

We have not yet discussed the preservation of the geometrical properties of the object. It is evident that if the thinning operation T1 is applied iteratively to an object S without holes and cavities, S may be thinned until a point.

In 2D thinning algorithms, the geometrical properties are preserved by keeping, at each iteration, the end points of arcs. In 3D, in order to preserve medial surfaces of S, border points of surfaces should not be removed. Let n be the number of 1's in $N_{26}(p)$. Let n_i be the number of 1's 6-adjacent to p in the ith octant of $N_{26}(p)$, an octant is a $2 \times 2 \times 2$ block of points, there are 8 octants in $N_{26}(p)$. We give now **condition 3**:

$$3 \qquad n \geq 8 \vee (4 \leq n \leq 7 \wedge (\exists i \in \{1, ..., 8\}, n_i = 3))$$

If a simple point satisfies condition 3, it is not considered as a border point of surfaces. We propose now the **thinning operation T2**. T2 consists of removing simultaneously one type of border points which satisfy conditions 2 and condition 3 in a subcycle, i.e., T2 removes points that are simple and are not border points of surfaces.

6 a geometrical thinning algorithm

The thinning algorithm consists of using iteratively the thinning operation T2. In each iteration, T2 is applied to one type of border points in a fixed sequence of direction: U,N,E,B,S,W. The thinning operation stops, when no point can be removed. An example is shown in fig.6.

7 discussion and conclusion

There exist several algorithms in the literature[1,2,4,8,10,11]; in this section, we cite the two most recent one's.

Tsao's algorithm[11] may be regarded as an improvement of Morgenthaler's algorithm. Several techniques have been used. Since the number of genus (see [2,5,10,11]) in $N'_{26}(p)$ is easier to calculate than the number of components in $N'_{26}(p)$, and since the simple point condition 1 can be presented with the number of genus(see [11]), Tsao used the number of genus in its algorithm. Tsao's algorithm deletes points which are 1) simple in the sense of genus, 2) simple in at least two checking windows (the details can be found in [10]), 3) are non-end points. In order to determine if a point is non-end point, each subcycle, has to be implemented in two steps. Thus the algorithm needs more time.

Hafford et al.[1] use a face-centered cubic grid, which is different from the one used in our algorithm. The advantage of using such a grid is that the determination of simple points becomes easier. The path connectivity can be used to check the preservation of topological properties. But such a grid is not widely used in 3D.

In this paper , we have presented a parallel 3D thinning algorithm, which conserves medial surfaces. We have also proved that the algorithm does not deconnect a 3D object and does not connect two components of the complement of an object. It has been pointed that the algorithm does not change the topological properties of the objects. The proposed topological predicates can be calculated by simple logical computations.

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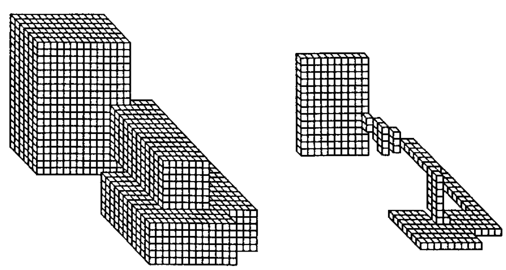

Figure 6: an example