$$
\begin{aligned}
H_I &= \sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+ \\
H_{DST} &= H_I \otimes \mathbb{1} + \mathbb{1} \otimes H_I \\
|\psi_T\rangle &= cos(\theta/2)\,|0\rangle + e^{i\phi} sin(\theta/2)\,|1\rangle \\
\left|\dot{\psi}_T\right\rangle &= -sin(\theta/2)\dot{\theta}/2\,|0\rangle + e^{i\phi}(cos(\theta/2)\ \dot{\theta}/2 + i\dot{\phi}\ sin(\theta/2))\,|1\rangle \\
\frac{1}{2}\frac{dW}{dt} &= -\operatorname{Im}\left\langle \dot{\psi}_T \middle| \psi_T^{\lightning} \right\rangle = \operatorname{Im}\left\langle \dot{\psi}_T \middle| iH_T\,|\psi_T\rangle = \operatorname{Im}\langle\psi_D|\,\langle\psi_S|\left\langle \dot{\psi}_T \middle| iH_{DST}\,|\psi_D\rangle\,|\psi_S\rangle\,|\psi_T\rangle \\
&= \operatorname{Im}\langle\psi_D|\,\langle\psi_S|\left\langle \dot{\psi}_T \middle| iH_{DS} \otimes \mathbb{1}\,|\psi_D\rangle\,|\psi_S\rangle\,|\psi_T\rangle + \operatorname{Im}\langle\psi_D|\,\langle\psi_S|\left\langle \dot{\psi}_T \middle| i\mathbb{1} \otimes H_{ST}\,|\psi_D\rangle\,|\psi_S\rangle\,|\psi_T\rangle \\
&= \operatorname{Im} i\,\langle H_{DS}\rangle \left\langle \dot{\psi}_T \middle| \psi_T \right\rangle + \operatorname{Im}\langle\psi_S|\left\langle \dot{\psi}_T \middle| iH_{ST}\,|\psi_S\rangle\,|\psi_T\rangle \\
&= 0\ \ + \operatorname{Im}\langle\psi_S|\left\langle \dot{\psi}_T \middle| i\sigma_+ \otimes \sigma_-\,|\psi_S\rangle\,|\psi_T\rangle + \operatorname{Im}\langle\psi_S|\left\langle \dot{\psi}_T \middle| i\sigma_- \otimes \sigma_+\,|\psi_S\rangle\,|\psi_T\rangle \\
&= -\operatorname{Im} i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi} sin^2(\theta/2)\dot{\theta}/2 + \operatorname{Im} i\,\langle\psi_S|\,\sigma_-\,|\psi_S\rangle\, e^{-i\phi}(cos^2(\theta/2)\dot{\theta}/2 - i\dot{\phi}\ sin(\theta/2)) \\
&= -\operatorname{Im} i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi} sin^2(\theta/2)\dot{\theta}/2 + \operatorname{Im}[-i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi}(cos^2(\theta/2)\dot{\theta}/2 + i\dot{\phi}\ sin(\theta/2))]^* \\
&= -\operatorname{Im} i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi} sin^2(\theta/2)\dot{\theta}/2 + \operatorname{Im} i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi}(cos^2(\theta/2)\dot{\theta}/2 + i\dot{\phi}\ sin(\theta/2)) \\
&= \operatorname{Im} i\,\langle\psi_S|\,\sigma_+\,|\psi_S\rangle\, e^{i\phi}(cos(\theta)\dot{\theta}/2 + i\dot{\phi}\ sin(\theta/2))
\end{aligned}
$$

$$H_S(\theta_D(t),\phi_D(t),\theta_T(t),\phi_T(t)) = \frac{1}{2}\left[sin(\theta_D(t))e^{i\phi_D(t)} + sin(\theta_T(t))e^{i\phi_T(t)}\right]\sigma_+ + h.c.$$
$$= \alpha\sigma_+ + h.c.$$
$$\rho_S(t+1) = U\rho_S(t)U^\dagger, U = e^{-iH_S(t)\Delta t}$$
$$dW(t) = \mathrm{Tr}\ \rho(t)dH$$
$$dH = H_S(\theta_D(t),\phi_D(t),\theta_T(t+1),\phi_T(t+1)) - H_S(\theta_D(t),\phi_D(t),\theta_T(t),\phi_T(t))$$
$$= \frac{1}{2}(sin(\theta_T(t+1))e^{i\phi_T(t+1)} - sin(\theta_T(t))e^{i\phi_T(t)})\sigma_+ + h.c. =: (\tau' - \tau)\sigma_+ + h.c.$$

For N=2 only one jump, analytical solution is possible:

$$U = e^{-iH_S\Delta t} = exp\begin{pmatrix} 0 & -i\alpha^*\Delta t \\ -i\alpha\Delta t & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \frac{\alpha^*}{|\alpha|} & -\frac{\alpha^*}{|\alpha|} \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{-i|\alpha|\Delta t} & 0 \\ 0 & e^{i|\alpha|\Delta t} \end{pmatrix}\begin{pmatrix} \frac{|\alpha|}{\alpha^*} & 1 \\ -\frac{|\alpha|}{\alpha^*} & 1 \end{pmatrix}$$
$$= \begin{pmatrix} cos(|\alpha|\Delta t) & -i\frac{\alpha^*}{|\alpha|}sin(|\alpha|\Delta t) \\ -i\frac{|\alpha|}{\alpha^*}sin(|\alpha|\Delta t) & cos(|\alpha|\Delta t) \end{pmatrix}$$

With $\rho_0 = |0\rangle\langle 0|$, the time evolved state $\rho = U\rho_0 U^\dagger$is

$$\rho = \begin{pmatrix} cos^2(|\alpha|\Delta t) & i\frac{|\alpha|}{2\alpha}sin(2|\alpha|\Delta t) \\ -i\frac{|\alpha|}{2\alpha^*}sin(2|\alpha|\Delta t) & sin^2(|\alpha|\Delta t) \end{pmatrix}$$
$$W = \mathrm{Tr}\begin{pmatrix} 0 & \tau'^* - \tau^* \\ \tau' - \tau & 0 \end{pmatrix}\begin{pmatrix} cos^2(|\alpha|\Delta t) & i\frac{|\alpha|}{2\alpha}sin(2|\alpha|\Delta t) \\ -i\frac{|\alpha|}{2\alpha^*}sin(2|\alpha|\Delta t) & sin^2(|\alpha|\Delta t) \end{pmatrix}$$
$$= -sin(2|\alpha|\Delta t)\,\mathrm{Im}\left\{(\tau' - \tau)\frac{|\alpha|}{\alpha}\right\}$$