# In-band Network Telemetry Optimization Problem, Introduction and Solution

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## 1 Introduction

In-band telemetry is a technique which allows Network Administrators to gather information regarding the state in which the network is currently in. There are multiple information that could help improve and decipher what is happening in the network, for instance, one could get the average size of the output queue on each of the router interfaces, how many packets are passing by at a given interval, etc.

To gather this information, a flow passing by a router can create what we call telemetry headers and fill the information from a given router in such packets and eventually send them to an external controller. The problem is: such operations can be costly in a network and instead of helping the network, it will damage it even further.

Our aim in this work is to present and solve the In-band Network Telemetry Optimization Problem. Whose goal is to minimize the collection of such information.

In summary, our goal is to minimize the amount of creations and delivery of telemetry headers, while still gathering information from all router interfaces which are being used by some given flow and respecting the capacity of routers that a packet can gather without delivering it.

Some restrictions: a flow which has an open telemetry header and passes by a router, has to gather it's information, a telemetry header can only carry information of Q routers in sequence, this value is passed as a parameter to our problem, a router interface can only be covered by a flow which passes by it and, lastly, a router interface has to be covered by 1 flow, not more, not less.

# 2 Overall

$$
\begin{aligned}
&\mathbf{min} \sum_{k\in K}\sum_{u\in V} Y_{ku} && \\
&\mathbf{s.t} \sum_{k\in K} X_{ak} = 1 && \forall a \in A \\
&X_{af} = 0 && \forall a \in A - f, \forall f \in F \\
&X_{ak} \geq Y_{ku} && \forall k \in K, \forall a = (u,v) \in A \\
&X_{ak} \geq C_{ku} && \forall k \in K, \forall a = (u,v) \in A \\
&Y_{ku} - X_{ak} \geq -C_{kv} && \forall k \in K, \forall a = (u,v) \in A \\
&(-Y_{ku} + X_{ak}) * Q \geq C_{kv} && \forall k \in K, \forall a = (u,v) \in A \\
&C_{kv} \leq C_{ku} + 1 + (1 - X_{ak}) * Q + Y_{ku} * Q && \forall k \in K, \forall a = (u,v) \in A \\
&C_{kv} \geq C_{ku} + 1 - (1 - X_{ak}) * Q - Y_{ku} * Q && \forall k \in K, \forall a = (u,v) \in A \\
&Y_{ku} \geq X_{ak} && \forall k \in K, \forall a = (u,v) \in L \\
&C_{ku} \in \mathcal{Z}^+ && \\
&X_{ak} \in \{0,1\} && \\
&Y_{ku} \in \{0,1\} &&
\end{aligned}
$$

# 3 Constant and Variable Definition

## 3.1 Constants

$F$ = Set of flows present in a network.
$K$ = Set of solution flows.
$A$ = Set of links (or archs) in a given network.
$L$ = Set that represents the last links on all flows.
$V$ = Sets of routers in a network.
$Q$ = Value that defines the max amount any given route can carry.

## 3.2 Variables

$C_{ku}$ = Array variable that represents the charge of flow k when arriving at node u.
$X_{ak}$ = Binary array variable that controls whether a link a is covered by route k.
$Y_{ku}$ = Binary array variable that defines whether route k dispatches on node u.

## 4 Objective

We want to minimize the subgroups where the item fetching is done in a network environment. To do so, we want to minimize the values in Y.

$$\mathbf{min} \sum_{k \in K} \sum_{u \in V} Y_{ku}$$

## 5 Restrictions

Make sure that all nodes are covered.

$$\sum_{k \in K} X_{ak} = 1 \qquad \forall a \in A$$

Make sure that a route only covers an arch if the flow with the same index passes by it.

$$X_{af} = 0 \qquad \forall a \in A - f, \forall f \in F$$

Make sure that a group only dispatches if it collects in that arch.

$$X_{ak} \geq Y_{ku} \qquad \forall k \in K, \forall a = (u, v) \in A$$

Make sure that if a flow is not collecting a given node, the weight when it arrives there, its 0.

$$X_{ak} \geq C_{ku} \qquad \forall k \in K, \forall a = (u, v) \in A$$

The next two restrictions make sure of the following:

- If $X_{ak}$ equals 1 and $C_{kv}$ equals 0, it means that the content was dispatched, therefore $Y_{ku}$ equals 1.
- If $X_{ak}$ equals 1 and $C_{kv}$ is more than 0, it means that the content was not dispatched, therefore $Y_{ku}$ equals 0.
- If $X_{ak}$ equals 0, then $C_{ku}$ has to be 0 and $Y_{ku}$ also has to be 0.

$$Y_{ku} - X_{ak} \geq -C_{kv} \qquad \forall k \in K, \forall a = (u, v) \in A$$

$$(-Y_{ku} + X_{ak}) * Q \geq C_{kv} \qquad \forall k \in K, \forall a = (u, v) \in A$$

The next two restrictions make sure of the following:

- If $X_{ak}$ equals 1 and $Y_{ku}$ equals 0, $C_{kv}$ has to be $C_{ku}$ plus 1.
- If $X_{ak}$ equals 1 and $Y_{ku}$ equals 1, $C_{kv}$ has to be 0.
- If $X_{ak}$ equals 0, $C_{kv}$ has to be 0.

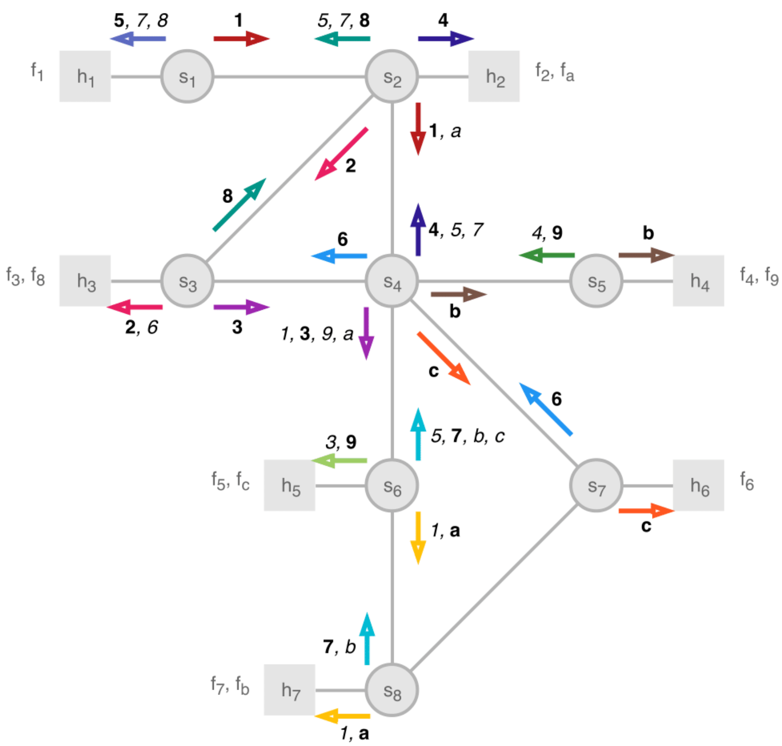
$$C_{kv} \leq C_{ku} + 1 + (1 - X_{ak}) * Q + Y_{ku} * Q \qquad \forall k \in K, \forall a = (u, v) \in A$$

$$C_{kv} \geq C_{ku} + 1 - (1 - X_{ak}) * Q - Y_{ku} * Q \qquad \forall k \in K, \forall a = (u, v) \in A$$

Make sure that if a link that belongs to a node which is last in its flow is collected, dispatch it.

$$Y_{ku} \geq X_{ak} \qquad \forall k \in K, \forall a = (u, v) \in L$$

# 6 Example




$f_1 : h_1 > h_7 : (s_1, s_2, s_4, s_6, s_8)$
$f_2 : h_2 > h_3 : (s_2, s_3)$
$f_3 : h_3 > h_5 : (s_3, s4, s_6)$
$f_4 : h_4 > h_2 : (s_5, s_4, s_2)$
$f_5 : h_5 > h_1 : (s_6, s_4, s_2, s_1)$
$f_6 : h_6 > h_3 : (s_7, s_4, s_3)$
$f_7 : h_7 > h_1 : (s_8, s_6, s_4, s_2, s_1)$
$f_8 : h_3 > h_1 : (s_3, s_2, s_1)$
$f_9 : h_4 > h_5 : (s_5, s_4, s_6)$
$f_a : h_2 > h_7 : (s_2, s_4, s_6, s_8)$
$f_b : h_7 > h_4 : (s_8, s_6, s_4, s_5)$
$f_c : h_5 > h_6 : (s_6, s_4, s_7)$

$f_1 : \{(s_1, s_2)\ (s_2, s_4)\}$
$f_2 : \{(s_2, s_3)\ (s_3, h_3)\}$
$f_3 : \{(s_3, s_4)\ (s_4, s_6)\}$
$f_4 : \{(s_4, s_2)\ (s_2, h_2)\}$
$f_5 : \{(s_1, h_1)\}$
$f_6 : \{(s_7, s_4)\ (s_4, s_3)\}$
$f_7 : \{(s_8, s_6)\ (s_6, s_4)\}$
$f_8 : \{(s_3, s_2)\ (s_2, s_1)\}$
$f_9 : \{(s_5, s_4)\}\ \{(s_6, h_5)\}$
$f_a : \{(s_6, s_8)\ (s_8, h_7)\}$
$f_b : \{(s_4, s_5)\ (s_5, s_4)\}$
$f_c : \{(s_4, s_7)\ (s_7, h_6)\}$

To explain how our output would look like in this solution:

Note: We are only showing the values which are 1, as to not overcrowd the output.

## 6.1 Variable X

$$X[(s1, s2), 1] = 1$$
$$X[(s2, s4), 1] = 1$$
$$X[(s2, s3), 2] = 1$$
$$X[(s3, h3), 2] = 1$$
$$X[(s3, s4), 3] = 1$$
$$X[(s4, s6), 3] = 1$$
$$X[(s4, s2), 4] = 1$$
$$X[(s2, h2), 4] = 1$$
$$X[(s1, h1), 5] = 1$$
$$X[(s7, s4), 6] = 1$$
$$X[(s4, s3), 6] = 1$$
$$X[(s8, s6), 7] = 1$$
$$X[(s6, s4), 7] = 1$$
$$X[(s3, s2), 8] = 1$$
$$X[(s2, s1), 8] = 1$$
$$X[(s5, s4), 9] = 1$$
$$X[(s6, h5), 9] = 1$$
$$X[(s6, s8), a] = 1$$
$$X[(s8, h7), a] = 1$$
$$X[(s6, s6), b] = 1$$
$$X[(s5, s4), b] = 1$$
$$X[(s4, s7), c] = 1$$
$$X[(s7, h6), c] = 1$$

## 6.2 Variable Y

$$Y[1, s2] = 1$$
$$Y[2, s3] = 1$$
$$Y[3, s6] = 1$$
$$Y[4, s2] = 1$$
$$Y[5, s1] = 1$$
$$Y[6, s4] = 1$$
$$Y[7, s6] = 1$$
$$Y[8, s2] = 1$$
$$Y[9, s5] = 1$$
$$Y[9, s6] = 1$$
$$Y[a, s8] = 1$$
$$Y[b, s5] = 1$$
$$Y[c, s7] = 1$$

## 6.3 Variable C

Note: Since all groups have max size of 2, the weight can is maxed at 1, but if the groups were larger (because of a higher Q capacity), the values could increase.

$$C[1, s2] = 1$$
$$C[2, s3] = 1$$
$$C[3, s4] = 1$$
$$C[4, s2] = 1$$
$$C[6, s4] = 1$$
$$C[7, s6] = 1$$
$$C[8, s2] = 1$$
$$C[a, s8] = 1$$
$$C[b, s5] = 1$$
$$C[c, s7] = 1$$

## 6.4 Objective value

As we can count the number of 1's in array Y, we find that the objective value found is **13**.