

July 2025 CSE220: Signals and Linear Systems Sessional

Offline Assignment on Continuous Fourier Transform

General Instructions

- All implementations must use **Object-Oriented Programming**.
- Use of any built-in Fourier Transform functions is **strictly prohibited**, including:
 - `np.fft`, `fft2`, `ifft`, `scipy.fft`
- All continuous integrals must be approximated numerically using `np.trapz()` or `np.trapezoid()` only.
- Complex exponentials must be handled by explicitly separating **real** and **imaginary** parts.
- Hard-coded Fourier transform pairs are forbidden.
- Skeleton codes (i.e., `task1_signal_framework.py` and `task2_image_cft.py`) will be provided with this assignment and must not be modified structurally.
- You are not allowed to initialize/declare any other new classes or new attributes/methods inside a given class. Follow the convention provided in the problem statement.

1 Task 1: Object-Oriented Continuous Signal Modeling and Spectral Analysis

1.1 Objective

The objective of this task is to design a reusable and extensible **object-oriented framework** for modeling continuous-time signals, computing their **Continuous Fourier Transform (CFT)**, and reconstructing them using the **Inverse Continuous Fourier Transform (ICFT)**.

Rather than analyzing a single predefined signal, students are required to build a general-purpose signal analysis system capable of generating, combining, and analyzing a wide variety of continuous-time signals.

1.2 Background

In practical signal processing systems, signals are often composed of multiple components including periodic oscillations, polynomial trends, and time-limited pulses. Such signals cannot be analyzed effectively using rigid, hard-coded formulations.

This task introduces abstraction through classes, allowing signals to be:

- Parameterized
- Composed dynamically
- Analyzed in a unified framework

1.3 Definition and Generation of Signal Types

In this assignment, continuous-time signals are generated and analyzed using an **object-oriented framework**. The `SignalGenerator` class defines various signal types as methods, each returning the signal values over a continuous time axis. The problem statement is to implement and combine these signals to form complex waveforms without using hard-coded expressions.

1. Periodic Signals

Periodic signals repeat after a fixed period $T = \frac{1}{f}$. They are defined mathematically as:

$$x(t) = \begin{cases} A \sin(2\pi ft) & \text{(sine wave)} \\ A \cos(2\pi ft) & \text{(cosine wave)} \end{cases}$$

2. Non-Sinusoidal Periodic Signals

These signals are periodic but non-sinusoidal. They can be constructed mathematically using functions like sign, floor, and absolute value:

$$\begin{aligned} \text{Square wave: } x(t) &= A \operatorname{sign}(\sin(2\pi ft)) \\ \text{Sawtooth wave: } x(t) &= A [2(ft - \lfloor 0.5 + ft \rfloor)] \\ \text{Triangle wave: } x(t) &= A [2|\operatorname{sawtooth}(1, f)| - 1] \end{aligned}$$

3. Polynomial Signals

Polynomial signals vary smoothly with time according to a polynomial function:

$$x(t) = ct^2 \quad (\text{parabolic}), \quad x(t) = ct^3 \quad (\text{cubic})$$

4. Time-Limited Signals

Time-limited signals exist only within a specific time interval and are zero elsewhere:

$$x(t) = \begin{cases} 1, & |t| \leq \frac{\text{width}}{2} \quad (\text{rectangular}) \\ 1, & \text{start} \leq t \leq \text{end} \quad (\text{pulse}) \\ 0, & \text{otherwise} \end{cases}$$

Each signal must be defined over a continuous time axis and evaluated numerically.

1.4 Composite Signal Requirement

Using the developed framework, construct the following composite signal:

$$x(t) = 2 \sin(2\pi t) + 0.5 \cos(6\pi t) + \text{Square}(t) + t^3 \cdot \text{Rect}(t)$$

Important: This expression must not be implemented directly. Each component must be created as a separate object and combined programmatically using the composite signal class.

1.5 Class Specifications

1. ContinuousSignal (Abstract Base Class)

Purpose: Defines a common interface for all continuous-time signals.

Attributes:

- `t`: Continuous time axis

Methods:

- `values()` – returns signal samples
- `plot()` – visualizes the signal in time domain

2. SignalGenerator (Inherits ContinuousSignal)

Purpose: Generates parameterized continuous-time signals defined in Section 1.3.

Methods:

- `sine(amplitude, frequency)`
- `cosine(amplitude, frequency)`
- `square(amplitude, frequency)`
- `sawtooth(amplitude, frequency)`
- `triangle(amplitude, frequency)`
- `cubic(coefficient)`
- `parabolic(coefficient)`
- `rectangular(width)`
- `pulse(start, end)`

3. CompositeSignal (Inherits ContinuousSignal)

Purpose: Combines multiple signal components into a single signal.

Attributes:

- **components:** list of signal objects

Methods:

- `add_component(signal)`
- `values()`

4. CFTAnalyzer

Purpose: Computes the Continuous Fourier Transform.

Attributes:

- **signal:** CompositeSignal
- **t:** Continuous time axis
- **frequencies:** Continuous frequency axis

Methods:

- `compute_cft()`: Computes frequency domain components of the CFT
- `plot_spectrum()`

5. InverseCFT

Purpose: Reconstructs time-domain signals using ICFT.

Attributes:

- **spectrum:** Tuple consisting two numpy arrays indicating frequency-domain components
- **frequencies:** Continuous frequency axis
- **t:** Continuous time axis

Methods:

- `reconstruct()`

1.6 Analysis Requirements

Students must:

- Compare original and reconstructed signals
- Study reconstruction under different frequency truncation limits
- Comment on numerical errors and Gibbs phenomenon

2 Task 2: Two-Dimensional Continuous Fourier Transform for Image Processing

Objective

The objective of this task is to design and implement a complete object-oriented framework for performing the **Two-Dimensional Continuous Fourier Transform (2D-CFT)** and its inverse on grayscale images, using only numerical integration techniques. This task emphasizes the theoretical foundations of frequency-domain image analysis and demonstrates how spatial-domain image characteristics map into the frequency domain.

Students are strictly prohibited from using FFT, DFT, or any built-in frequency transform utilities. All transformations must be computed using numerical integration (e.g., trapezoidal rule).

Theoretical Background

Let $I(x, y)$ represent a continuous grayscale image defined over spatial coordinates (x, y) . The **2D Continuous Fourier Transform** of $I(x, y)$ is defined as:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y) e^{-j2\pi(ux+vy)} dx dy$$

Using Euler's identity, this can be expressed as:

$$\begin{aligned}\Re\{F(u, v)\} &= \int \int I(x, y) \cos(2\pi(ux + vy)) dx dy \\ \Im\{F(u, v)\} &= - \int \int I(x, y) \sin(2\pi(ux + vy)) dx dy\end{aligned}$$

The inverse 2D-CFT is defined as:

$$I(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

This operation reconstructs the spatial image from its frequency-domain representation.

Task Description

You are required to perform the following steps:

1. Load and normalize a grayscale image and interpret it as a continuous 2D signal. (**Done for you**)
2. Compute the 2D Continuous Fourier Transform using separable numerical integration.
3. Visualize the magnitude spectrum of the transformed image.
4. Design and apply a frequency-domain low-pass filter. (**Done for you**)
5. Reconstruct the filtered image using the inverse 2D-CFT.

Object-Oriented Design Specifications

Your implementation must follow the class structure described below.

Class: ContinuousImage

Purpose: Represents a grayscale image as a continuous spatial signal.

Attributes:

- **image:** A normalized 2D array representing image intensity.
- **x:** Continuous spatial coordinate vector along the horizontal axis.
- **y:** Continuous spatial coordinate vector along the vertical axis.

Methods:

- **show(title):** Displays the image with appropriate scaling and labels.

Class: CFT2D

Purpose: Computes the 2D Continuous Fourier Transform of a given image using numerical integration.

Attributes:

- **I:** Input image intensity matrix.
- **x, y:** Spatial coordinate grids.

Methods:

- **compute_cft():**
 - Computes real and imaginary components of the 2D-CFT
 - Uses separable trapezoidal integration
- **plot_magnitude():**
 - Visualizes the logarithmic magnitude spectrum

Class: FrequencyFilter (**Done for you**)

Purpose: Applies frequency-domain filtering operations.

Methods:

- **low_pass(real, imag, cutoff):**
 - Suppresses high-frequency components beyond a cutoff radius
 - Preserves low-frequency image information

Class: InverseCFT2D

Purpose: Reconstructs the spatial-domain image using inverse 2D-CFT.

Attributes:

- **real, imag:** Filtered frequency-domain components.
- **x, y:** Frequency coordinate vectors.

Methods:

- `reconstruct()`:
 - Performs inverse 2D numerical integration
 - Produces reconstructed image

Constraints and Rules

- Use of FFT, DFT, or frequency-domain libraries is strictly forbidden.
- All integrations must be implemented using numerical integration (e.g., trapezoidal rule).
- Object-oriented design must be strictly followed.
- Code must be well-documented and modular.

Marks Distribution

Class	Mark
SignalGenerator	20
CFTAnalyzer	20
InverseCFT	20
CFT2D	20
InverseCFT2D	20
Total	100

Submission

Create separate python files for the two parts. Rename them as `id_first.py` and `id_second.py`. Put the python files in a folder named by your student id. Zip the folder and submit the zip file. Do not include any images in the folder. **Deadline: Monday, 9 February, 11:59 PM.**