

CSE 220: Fourier Transform Practice

Term: July 2025

Task 1

The purpose of this practice is to understand how Fourier Transform (FT) can approximate various functions and serve as a tool for differentiation. You will implement custom Fourier Transform and Inverse Fourier Transform (IFT) algorithms, then use these to approximate parabolic, triangular, sawtooth, and rectangular wave functions. Additionally, you will apply these transforms to compute and visualize the derivatives of the specified functions

Function creation

- **Parabolic Function:** Create a parabolic function $y = x^2$ within the interval $[-2, 2]$ and 0 elsewhere.
- **Triangular Function:** Create a triangular wave within the interval $[-2, 2]$ with height 1 and 0 elsewhere.
- **Sawtooth Function:** Create a sawtooth wave within the interval $[-2, 2]$ with a slope of 1 and 0 elsewhere.
- **Rectangular Function:** Create a rectangular pulse within the interval $[-2, 2]$ with a height of 1 and 0 elsewhere.

Fourier Transform

Go to `FT basic.py`. Complete the function `fourier_transform(signal, frequencies, sampled_times)` where `signal` is the y values of the function, `frequencies` are the frequency values for which you want to store the transform and `sampled_times` are the corresponding x values for y . Also complete the `inverse_fourier_transform(ft_signal, frequencies, sampled_times)` and `inverse_fourier_transform_for_derivative(ft_signal, frequencies, sampled_times)` where `ft_signal` is the Fourier transformed signal containing two parts - real and imaginary. Read the comments for better understanding and edit other parts according to the comments. Remember, you can only use `trapz` for integration. Any built-in libraries for FT, FFT, DFT, IFT are not allowed.

Experiment with Frequency Limits

Try different frequency ranges, starting with low frequencies and gradually increasing. Observe how the approximation improves as higher frequencies are included. But at some point higher frequencies will distort the results. Why?

Differentiation using Fourier Transform

Compute the derivatives of the functions (specifically the parabolic and triangular functions) using the Fourier Transform property. Recall the differentiation property in the frequency domain:

$$\mathcal{F}\{f'(t)\} = (2\pi if) \cdot \mathcal{F}\{f(t)\}$$

Apply this property by multiplying the Fourier transformed signal by $2\pi if$ (where f represents your frequency array) before applying the Inverse Fourier Transform. This will reconstruct the derivative of the original signal in the time domain.

Plotting

For each function:

- Plot the original function.
- Plot the frequency spectrum.
- Plot the Fourier approximated function.
- Plot the derivative obtained via Fourier Transform against the analytical (theoretical) derivative.

Sample Output

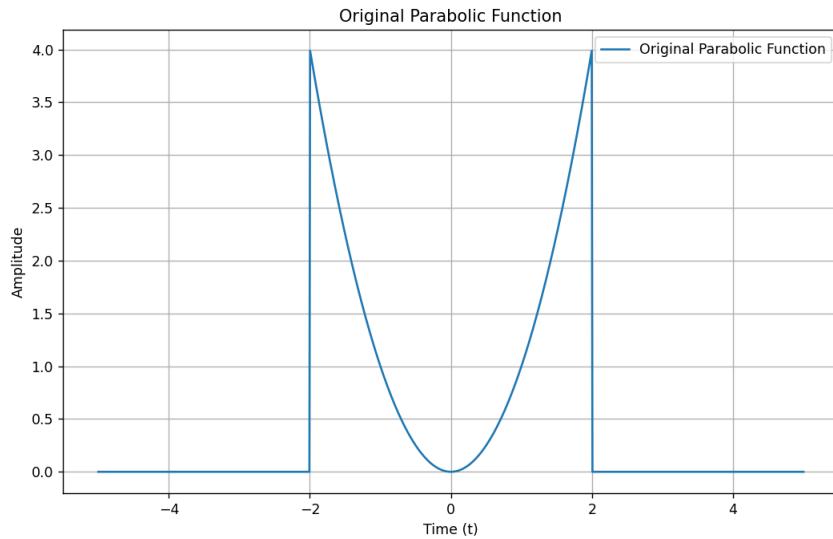


Figure 1: Original Parabolic Function

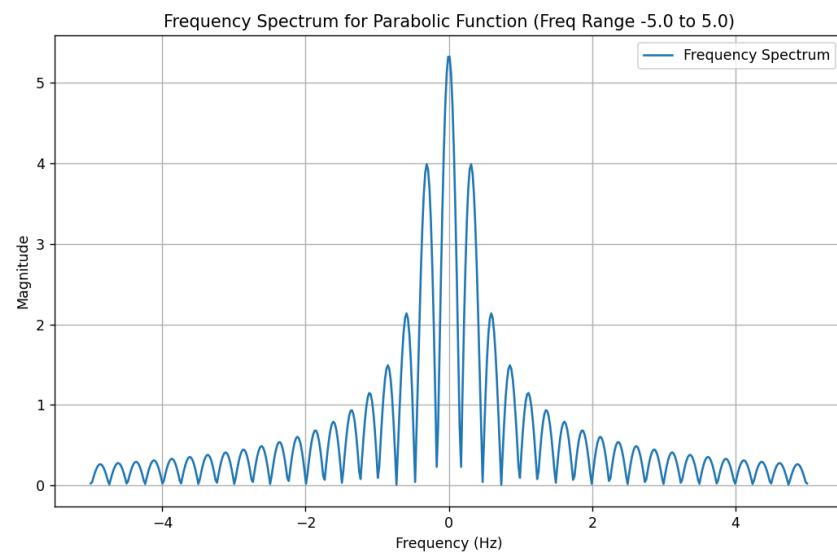


Figure 2: Frequency range from -5 to 5

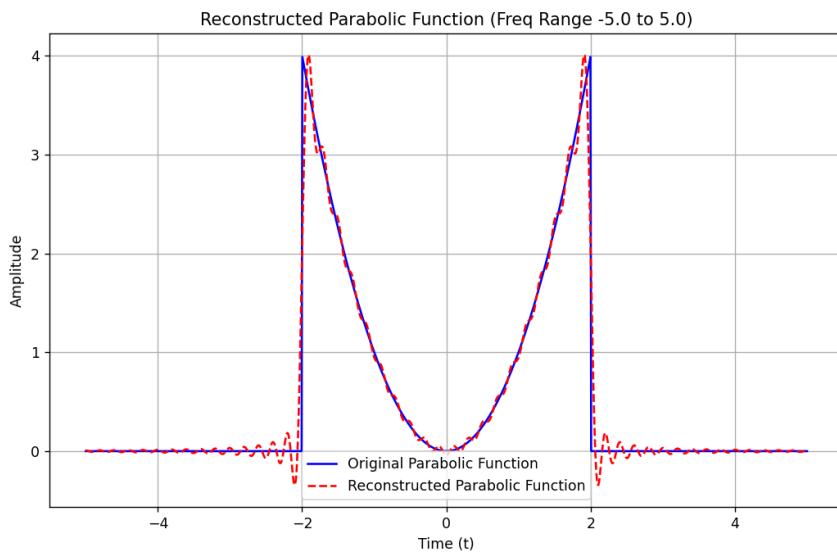


Figure 3: Reconstructed signal for frequency range -5 to 5

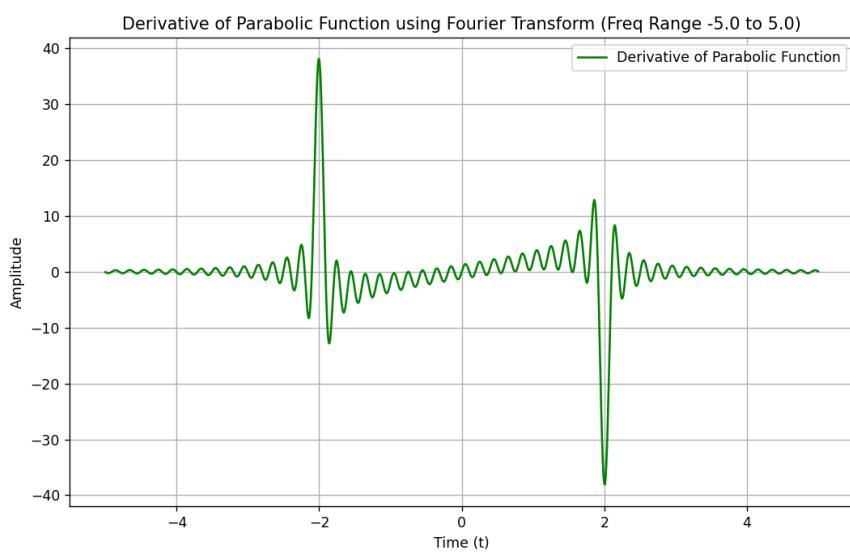


Figure 4: Derivative of parabolic function using Fourier Transform