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Do U.S. factors impact the Brazilian yield curve? Evidence from a dynamic factor model

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ABSTRACT

This paper contributes to the literature on the relationship between the yield curve, macroeconomic variables, and the unexplored interactions with tae U.S. yield curve movements by focusing on an emerging market: Brazil. We incorporate factors for the U.S. yield curve and domestic macroeconomic variables into the Dynamic Nelson Siegel Model to explore comovements with the Brazilian yield curve. As noted here, foreign macroeconomic factors contain a lot of information about the domestic term structure of yields. The empirical results suggest that both American and macroeconomic components may explain the latent factors of the term structure; in particular, the U.S. factors influence the Brazilian yield curve, since almost half of the variance in the level factor was caused by movements in the U.S. curve. Furthermore, we find evidence that a specification with U.S. yield factors is better for short maturities and long forecasts horizons.

1. Introduction

Macroeconomic factors and yield curve factors are characterized by a strong interaction. The yield curve often contains useful information about real economic activity, interest rates, and inflation. In addition, yield curve fluctuations across different countries are highly correlated. A growing body of literature has investigated the interaction between the macro-economy and term structure. This paper employs the Dynamic Nelson-Siegel (DNS) model, augmented with macroeconomic factors in order to study the dynamic interactions between the yield curves in Brazil and the United States. The objective is to determine the relative importance of the domestic macroeconomy and the U.S. yield curve in driving movement in the term structure of interest rates in Brazil.

Many studies have analyzed the term structure of interest rates and macroeconomic variables jointly, considering both unidirectional and bidirectional linkages. The interest rate channel is an important transmission channel to better understand linkages in other financial assets and economic activity (Aguiar-Conraria, Martins, & Soares, 2012) Ang et al. (2003) and Diebold, Rudebusch, and Aruoba (2006) introduce inflation and the output gap to augment the term structure model and to show that macro factors can explain large variations in bond yields. Rudebusch and Wu (2008) found that the slope of the yield curve is related to cyclical variations in inflation and output gaps as the central bank moves short-term rates in order to achieve its macroeconomic goals. Moreover, Moench (2008) shows that a term structure model augmented with a broad macro-finance information set can provide superior forecasts. Joslin, Priebsch, and Singleton (2014) employed principal components analysis to decompose the term structure into three orthogonal latent yield curve factors within a no-arbitrage affine term structure model (ATSM) framework to investigate macroeconomic phenomena through the term structure. Also, Chen et al. (2013) applied the Dynamic Nelson-Siegel (DNS) factor

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model framework of Diebold et al. (2006) and found that yield curve factors can explain exchange rate movements and excess currency returns.

Another group of literature explores potential interactions among yield curves across different countries. Bernanke (2007) provided evidence that, in the integrated financial market, global factors play a role in determining long-term interest rates. Mehl (2009) reviews the literature on the predictive ability of the yield curve in emerging market economies (EMEs) and shows that, depending on the extent of market liquidity, it has informational value for future inflation and growth. It also examines international financial linkages and how the U.S. and the Euro Area yield curves help to predict yield curves and have in- and out-of-sample information content for futures inflation and growths in EMEs. Wong, Lucia, Price, and Startz (2011) found that the strong correlations between yields dynamics in the U.S. and Canada dissipated after Canadian monetary policy reforms in the early 1990s, while Lange (2014) demonstrated that the U.S. yield curve accounts for variations in Canadian term structure factors. A number of recent studies (Kulish & Rees, 2011; Wright, 2011, Dahlquist & Hasseltoft, 2013, Swanson & Williams, 2014) have highlighted the strong comovement between long-term interest rates in a number of inflation targeting open economies (such as Australia, Canada, New Zealand and the UK) and long-term interest rates in the U.S. An important issue addressed by these papers is that macro fundamentals drive common movements in international yields. It is this latter research line that this paper is most closely related.

In the same research filed, Bowman, Londono, and Sapriza (2015) explored the effect of U.S. monetary policy shocks on sovereign bond yields in emerging market economies (including Brazil), and determined that these bonds respond strongly to U.S. monetary policy announcements. Aside from this, there are few studies analyzing the connectedness between developing and developed markets yield curves. Sowmya, Prasanna, and Bhaduri (2016) investigated linkages in sovereign bond yields across different maturity spectrums among developed and Asian countries. Jotikasthira, Le, and Lundblad (2015) studied the comovement of foreign bond yields through the lens of an affine term structure model, and found that global inflation and U.S. yield level factors explained over 70% of the Germany and UK yields variations at all maturity spectrums.

Fig. (1) shows the pattern of interest rate correlations at different maturities on the yield curve for Brazil compared to the U.S. Long-term rates are highly correlated with their U.S. counterparts and are usually higher than rates at shorter maturities. The high interest rate correlations between the yield curve for Brazil with the U.S., and the recent research evidence about the impact of global factors on the domestic yield curve, motivates the present study.

In light of the discussions about potential interactions between international and domestic factors impacting the yield curve, we investigate the relationship among factors from DNS models with the observed macroeconomic variables and the U.S. yield curve factors. The main purpose of this paper is to evaluate the impact that the U.S. yield curve and macroeconomic variables have on the Brazilian curve, extending the Diebold et al. (2006) framework. We follow Lange (2014) by assuming that term structure of a small open economy will be affect by the U.S. curve, but not the other way around. We contribute to the literature of a yield curve cross-country iteration, such as Diebold, Li, and Yue (2008), Lange (2014) and Jotikasthira et al. (2015). Furthermore, we shed lights on an emerging economy, observing its financial market openness, and also expanding an underdeveloped literature.

In light of the discussions about potential interactions between international and domestic factors and their impact on the yield curve, we investigate the relationship among factors from DNS models with observed macroeconomic variables and the U.S. yield curve. The purpose of this paper is to evaluate the impact that the U.S. yield curve and macroeconomic variables have on the Brazilian curve, extending the Diebold et al. (2006) framework. We follow Lange (2014) by assuming that the term structure of a small open economy will be affected by the U.S. curve, but not the other way around. We contribute to the literature in this field, such as Diebold et al. (2008), Lange (2014) and Jotikasthira et al. (2015). Furthermore, we shed light on an emerging economy, observing its financial market openness, and also expanding an underdeveloped base of literature.

We empirically examine Brazilian term structure dynamics using monthly observations from 2004 to 2016. We identify the bidirectional linkage between the macroeconomy and the yield curve, with a stronger market capability to foresee economic results. We highlight the comovement between the level factor of Brazil and U.S. curves, as well as the important effect of the U.S. level factor on the slope of the Brazilian curve. In particular, we find that U.S. factors are responsible for nearly half of the variance in the level factor, and have almost the same capability to explain the slope and curvature variance in the long term as macroeconomic factors. These findings expand the literature of comovements between the U.S. yield curve and other countries, showing the influence of the United States over developing economies.

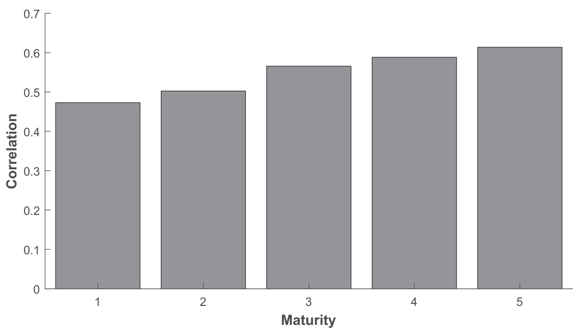

Fig. 1. Cross correlations with U.S. interest rates.

This paper is structured as follows: Section 2 lays out the model with U.S. yield curve states and macroeconomic variables. Section 3 presents the empirical results and compares the effects of foreign variable in the model, presenting impulse response function, forecast error variance decomposition and out-of-sample forecast. Section 5 concludes the paper.

2. The model

The popular yield curve model developed by (henceforth NS Nelson & Siegel (1987)) provides a flexible and parsimonious approach to fit the cross-section of yields. Diebold and Li (2006) henceforth DL reinterpreted the NS model as a dynamic factor model

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon_t(\tau), \tag{1}$$

where $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are time-varying parameters referred as level (L_t), slope (S_t) and curvature (C_t) factors of the yield curve.

The DL generalization of the NS model can be expressed in state space form

$$\mathbf{y}_t = \Lambda \mathbf{X}_t + \epsilon_t, \quad \epsilon_t \sim N(0, \Omega), t = 1, ..., T, \tag{2}$$

$$(\mathbf{X}_t - \mu_{\mathbf{X}}) = \Upsilon(\mathbf{X}_{t-1} - \mu_{\mathbf{X}}) + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta), \tag{3}$$

where (2) and (3) are the measurement and transition equations, respectively. In (2), $y_t$is a vector of Nyields with maturities τ, Λ is the loading matrix, $\mathbf{X}_t$ is the vector of latent dynamic factors, where we can include the yield curve factors level, slope and curvature. Our purpose is to also include macroeconomic variables and the U.S. yield curve factors in the $\mathbf{X}_t$vector, aiming to evaluate its impacts on the Brazil yield curve. $\mu_{\mathbf{X}}$ is the mean vector, Υ is the autoregressive matrix, which we will take a closer look soon. The disturbance vector, ϵ_t, is normally distributed with diagonal variance matrix Ω.

The structure for the loading matrix Λ is inherited from the NS model and is identified by setting the predetermined loadings, so

$$\Lambda = \left[1, \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right), \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{\lambda\tau}\right)\right], \tag{4}$$

where λ is a decay parameter and the only parameter to be estimated. The dimension of Λ could be generalized as a $N \times k$ matrix, of which the first three columns include the loadings on the three yield factors and the $k-3$ columns on the right contain only zeros, so that the yields still load only on the yield curve factors, as the original DL model.

A model with macroeconomic variables and/or foreign yield curve factors would increases the $\mathbf{X}_t$ and, consequently, most of elements in the Eqs. (2) and (3). Dealing with the curse of dimensionality, some restrictions will be imposed, so we can estimate four models specifications, which are briefly described in Section 3.2.

3. Empirical application

3.1. Data

The Brazilian yields for the estimations consists of end-of-month yields of Brazilian interbank deposit future contracts (DI-futuro) collected on a monthly basis. The data set were obtained from the Brazilian Mercantile and Futures Exchange (BM&FBovespa), which is the entity that offers DI-futuro contracts and determines the maturities with authorized contracts. Our sample range from January 30th, 2004 to November 30th, 2016, with a monthly frequency, and consider the following 14 maturities: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 36, 42, 48, and 60 months. This choice provides us with a panel of 165 monthly observations on 14 different yields.[1] At the same time, we took advantage of the Gurkaynak, Sack, and Wright (1961) resulting estimates to compute the U.S. yield curve for the same fixed maturities used by Diebold et al. (2006).[2]

For the macroeconomic variables, we used the industrial capacity utilization (uci_t), the 12-month percentage change in the Broad National Consumer Price Index ($ipca_t$), the measure used as the inflation target in Brazil, the Selic interest rate ($selic_t$), which is equivalent to the U.S. federal funds rate, as the instrument of monetary policy, and the percentage change in the exchange rate (exr_t), defined as the price of foreign exchange. All three variables are public available on the Brazil Central Bank and the Institute for Applied Economic Research (IPEA) databases.

3.2. Empirical specifications

We estimated four models specifications, named: Yield-Macro, Yield-UScurve, Yield-USM and Yield-USMX. The main difference among them is the composition and size of the Υ matrix. The Yield-Macro is the baseline model. It has the same specification introduced by Diebold et al. (2006), with $\mathbf{X}_t = (L_t, S_t, C_t, uci_t, selic_t, ipca_t)$, where Υ is a full 6×6matrix.

In the Yield-UScurve, the $\mathbf{X}_t$ vector is composed by the three unobserved Brazilian yield curve factors and the empirical factors of

[1] Additional details about this data set and the DI-futuro contract can be found in Caldeira, Moura, and Santos (2016).

[2] The authors follow the framework developed by Svensson (1994), an extension of the form proposed by Nelson and Siegel (1987), using yearly maturities.

the U.S. curve[3], so $\mathbf{X}_t = (L_t, S_t, C_t, USL_t, USS_t, USC_t)$. We can summarize the matrix Υ into four 3×3 blocks:

$$\Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ \Upsilon_3 & \Upsilon_4 \end{bmatrix}. \tag{5}$$

The effects of lagged U.S. factors on the Brazilian curve are incorporated in the sub-matrix Υ_2. At the same time, as the opposite effect is very unlikely, the Υ_3 block is composed of zeros, in a way that lagged Brazilian factors have no effect on the U.S. curve.

The Yield-USM specification includes both U.S. curve empirical factors and macroeconomic variables, in that order. This time, the Υ matrix can be summarized into six 3×3 blocks:

$$\Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3 \\ \Upsilon_4 & \Upsilon_5 & \Upsilon_6 \\ \Upsilon_7 & \Upsilon_8 & \Upsilon_9 \end{bmatrix}. \tag{6}$$

The sub-matrix Υ_4 is equivalent to the Υ_3 block in (5), and so is composed of zeros. Similarly, Υ_8 and Υ_6 are also formed of zeros, assuming that lagged U.S. factors have no direct effect on Brazilian macroeconomic variables, and the other way around. Beyond the economic belief behind this restriction, that no significant changes in the macroeconomy of a country could be caused by changes in the yield curve of another country and that macroeconomic results of a developing country could move the yield country of a leading economy, we also carry out Wald tests to confirm this hypothesis, which confirm it for nearly all parameters.[4] For those parameters which demonstrates some minor impact, we capture most of its effect through the covariance matrix in the transition equation.

The last specification includes the percentage change in the exchange rate, with the following vector of latent factors $\mathbf{X}_t = (L_t, S_t, C_t, USL_t, USS_t, USC_t, uci_t, selic_t, ipca_t, exr_t)$. Since this is an extension of the previous specification, we maintain the same effects on the Υmatrix, including the notion that exchange rate did not affect factors of the U.S. yield curve, while it can be affected by them. Computational procedures can be found on Appendix B.

3.3. Estimation procedures

Eqs. (2) and (3) characterize a linear and Gaussian state space model, thus the Kalman filter can be used to obtain estimates of the unobserved factors, as well as to construct the log- likelihood function. However, the computational burden associated with the Kalman filter recursions depends crucially on the dimension of both the state and observation vectors. Moreover, in yield curve models the dimension of the observation vector $(N \times 1)$ is often much larger than that of the state vector $(K \times 1)$. In these circumstances, Jungbacker and Koopman (2015) have shown that significant computational gains can be achieved by a simple transformation. First, define the $N \times N$ and the $K \times N$ matrices:

$$J = \begin{bmatrix} J^L \\ J^H \end{bmatrix}, \qquad J^L = C\Lambda(\lambda)'\Sigma^{-1},$$

respectively, where C can be any $K \times K$ invertible matrix, and J^H is chosen to guarantee that J is full rank. Selecting $C = (\Lambda(\lambda)'\Sigma^{-1}\Lambda(\lambda))^{-1}$ implies:

$$J(\mathbf{y}_t - \Gamma) = \begin{bmatrix} J^L(\mathbf{y}_t - \Gamma) \\ J^H(\mathbf{y}_t - \Gamma) \end{bmatrix} = \begin{bmatrix} \mathbf{y}_t^L \\ \mathbf{y}_t^H \end{bmatrix} = \begin{bmatrix} \mathbf{X}_t \\ 0 \end{bmatrix} + \begin{bmatrix} A^L\varepsilon_t \\ A^H\varepsilon_t \end{bmatrix}, \begin{bmatrix} A^L\varepsilon_t \\ A^H\varepsilon_t \end{bmatrix} \sim N\left(0, \begin{bmatrix} C & 0 \\ 0 & \Sigma^H \end{bmatrix}\right).$$

The law of motion of the factors in (3) is not affected by the transformation. Note that $\mathbf{y}_t^H$ is neither dependent on $\mathbf{X}_t$, nor correlated with $\mathbf{y}_t^L$ and, therefore, does not need to be considered for the estimation of the factors.[5] This implies that the Kalman filter only needs to be applied to the low dimensional subvector $\mathbf{y}_t^L$ for signal extraction, generating large computational gains when $N \gg K$ (see Table 1 of Jungbacker and Koopman (2015))

Denote $l(y)$ the log-likelihood function of the untransformed model in (2) and (3), where $y = (y_1', ..., y_T')'$. Evaluation of $l(y)$ can also take advantage of the transformations presented above. Jungbacker and Koopman (2015) show that the log-likelihood of the untransformed model can be represented as

$$l(\mathbf{y}) = c + l(\mathbf{y}^L) - \frac{T}{2}\log\frac{|\Sigma|}{|C|} - \frac{1}{2}\sum_{t=1}^{T} e_t'\Sigma^{-1}e_t, \tag{7}$$

where c is a constant independent of both y and the parameters, $l(\mathbf{y}^L)$ is the log-likelihood function of the reduced system, and $e_t = \mathbf{y}_t - \Gamma - \Lambda(\lambda)\mathbf{X}_t$. Note that computation of matrix J^H is not required at any point, as proved in Lemma 2 of Jungbacker and Koopman (2015).

[3] The empirical proxy for level, slope and curvature are $USL_t = (y_t(3) + y_t(24) + y_t(120))/3$, $USS_t = y_t(3) - y_t(120)$ and $USC_t = 2y_t(24) - y_t(3) - y_t(120)$, respectively.

[4] The restricted hypothesis is rejected for the relation between USL_t and USC_t with Brazil's inflation index and USC_t with Brazil's industrial capacity utilization. However, it is unexpected that lagged macroeconomic variables of a small open economy would affect the curvature and level of the U.S. yield curve. Since there is no economic reasoning to explore these effects, we decided to mute these channels.

[5] Γ is a vector of constants, which is fixed to zero in the DNS specification

Table 1
Correlation between empirical and estimate factors.

	L_t	S_t	C_t
Yield-Macro	0.819	0.966	0.897
Yield-UScurve	0.825	0.966	0.869
Yield-USM	0.824	0.965	0.869
Yield-USMX	0.824	0.966	0.867

Note: Correlation between estimated and empirical factors for level, slope and curvature considering each of the four specifications.

4. Results

We now discuss the correlation between empirical and estimated factors, comparison of decay parameters (λ) and in-sample fitting of the models. In general, the models fit the term structures in all four specifications well, with a high correlation between the empirical and the estimate factors, as showed in Table 1, and small measurement errors in the parameters estimates. The $\lambda$for the Yield-Macro, Yield-UScurve, Yield-USM, and Yield-USMX model is 0.0928, 0.0913, 0.0899, and 0.0912, respectively. This implies that the loading on the curvature factor is maximized at approximately 18 and 21 months, a result consistent with the literature and the fixed maturities used.

Table A.7 in Appendix A shows in-sample fitting statistics, with model predictions regarding mean and standard deviations of the 14 maturities included as observables in the estimation. We also calculate the Root Mean Squared Error of maturities predicted in each of the specifications. Furthermore, as a benchmark, we estimate a Dynamic Nelson-Siegel (DNS) for purpose of comparison. Overall, this table demonstrates that all four specifications capture the first and second moment of data quite well, and the inclusion of U.S. factors slightly improved the RMSE for medium- and long-term maturities. However, since this paper is not intended to forecast, these results serve as guide to emphasize the adequacy of estimation.

4.1. Estimation results

Table 2 presents the Υ matrix of estimated parameters for the Yield-Macro model. All variables show significant persistence. As already pointed out by Sekkel and Alves (2010), lagged macroeconomic variables have important effects on Brazilian yield curve factors. The bidirectional link between term structure factors and macroeconomic variables is presented, with latent factors more significant for future macroeconomic factors, indicating the market?s capability to anticipate economic movements.

The lagged monetary instrument, the $selic_{t-1}$interest rate, has a negative effect on all yield factors. Even though the level parameter is not significant, the negative effect on level and slope factors is quite unusual when compared to the literature analyzing developed countries, and is inconsistent with the Expectations Hypothesis, since an increase in interest rates gives the yield curve a positive slope. Regressing each maturity on the policy interest rate, we observe that its effects on the slope factor are mainly related to changes in the shorter end of the term structure. This result could be related to historical episodes of high inflation in Brazil. In this sense, higher interest rates are mainly associated with the focus on inflation control by Brazil?s monetary authorities. Since the central bank seems to be accurate on this objective, as expressed by the significant parameter for lagged $selic_t$ on inflation, the markets react through the yield curve in an opposite direction.

We show the Yield-UScurve parameter estimates in Table 3. As the Yield-Macro model, the persistence parameters are significant, with results similar to latent factors. The parameters for the U.S. curve closely resemble those in the literature, where the main

Table 2
Yield-macro model parameter estimates.

	L_{t-1}	S_{t-1}	C_{t-1}	uci_{t-1}	$selic_{t-1}$	$ipca_{t-1}$	μ
L_t	**0.97**	**0.13**	− 0.01	**0.02**	−0.08	0.01	− 0.59
	(0.52)	(0.03)	(0.02)	(0.01)	(0.04)	(0.04)	(0.36)
S_t	**0.20**	**0.96**	**0.08**	**− 0. 02**	**− 0. 10**	0.05	0.12
	(0.04)	(0.41)	(0.03)	(0.01)	(0.04)	(0.05)	(0.43)
C_t	**0. 47**	**0.38**	**0.96**	0.00	**− 0. 47**	**− 0. 20**	**1.19**
	(0.14)	(0.15)	(0.23)	(0.01)	(0.17)	(0.07)	(0.57)
uci_t	0.02	− 0.03	−0.02	**0.99**	0.01	**− 0. 05**	**0.34**
	(0.05)	(0.04)	(0.01)	(0.77)	(0.05)	(0.02)	(0.15)
$selic_t$	**0.36**	**0.31**	**0.07**	**0.01**	**0.62**	0.02	**− 0. 32**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)	(0.12)
$ipca_t$	**0.15**	**0.12**	**0.05**	**0.01**	**− 0. 17**	**1.00**	**− 0. 37**
	(0.03)	(0.03)	(0.01)	(0.00)	(0.03)	(0.86)	(0.12)

Note: Bold and underline entries denote parameter estimates significant at the 5% and 10% level, respectively. Standard errors appear in parentheses.

Table 3
Yield-UScurve model parameter estimates.

	L_{t-1}	S_{t-1}	C_{t-1}	USL_{t-1}	USS_{t-1}	USC_{t-1}	μ
L_t	**0.90**	**0.12**	**−0.02**	**0.18**	**−0.35**	−0.07	0.30
	(0.23)	(0.03)	(0.02)	(0.08)	(0.13)	(0.16)	(0.28)
S_t	**0.15**	**0.85**	**0.10**	**−0.30**	**0.32**	0.20	−0.56
	(0.04)	(0.10)	(0.03)	(0.10)	(0.16)	(0.19)	(0.34)
C_t	**−0.18**	−0.11	**0.82**	**0.38**	**−0.61**	**0.62**	**1.13**
	(0.07)	(0.06)	(0.10)	(0.16)	(0.25)	(0.28)	(0.52)
USL_t				**0.99**	**−0.06**	**0.12**	**0.07**
				(0.84)	(0.01)	(0.02)	(0.03)
USS_t				−0.01	**0.93**	**0.14**	**0.10**
				(0.01)	(0.22)	(0.03)	(0.03)
USC_t				−0.02	0.00	**0.98**	0.01
				(0.01)	(0.01)	(0.81)	(0.03)

Note: Bold and underline entries denote parameter estimates significant at the 5% and 10% level, respectively. Standard errors appear in parentheses.

Table 4
Yield-USM model parameter estimates.

	L_{t-1}	S_{t-1}	C_{t-1}	USL_{t-1}	USS_{t-1}	USC_{t-1}	uci_{t-1}	$selic_{t-1}$	$ipca_{t-1}$	μ
L_t	**0.85**	**0.09**	−0.03	**0.37**	−0.28	−0.33	**−0.02**	**0.04**	**0.14**	0.68
	(0.01)	(0.02)	(0.03)	(0.08)	(0.15)	(0.27)	(0.01)	(0.01)	(0.05)	(0.53)
S_t	**0.21**	**0.88**	**0.13**	−0.33	0.25	0.41	**0.04**	**−0.11**	0.01	**−3.07**
	(0.04)	(0.02)	(0.04)	(0.15)	(0.28)	(0.31)	(0.01)	(0.04)	(0.11)	(0.6)
C_t	**−0.43**	−0.29	**0.87**	**−0.91**	−0.40	0.89	0.11	**0.68**	**−0.78**	−5.07
	(0.11)	(0.34)	(0.05)	(0.29)	(0.72)	(0.72)	(0.07)	(0.29)	(0.14)	(2.51)
USL_t				**0.98**	**−0.06**	0.12				0.09
				(0.09)	(0.02)	(0.07)				(0.05)
USS_t				**−0.02**	**0.92**	**0.14**				0.08
				(0.05)	(0.01)	(0.05)				(0.18)
USC_t				−0.03	−0.01	**0.97**				0.00
				(0.02)	(0.02)	(0.51)				(0.12)
uci_t	−0.23	**−0.26**	**−0.05**				**0.93**	0.28	−0.04	**5.28**
	(0.13)	(0.09)	(0.02)				(0.04)	(0.17)	(0.03)	(0.25)
$selic_t$	**0.26**	**0.23**	**0.06**				**0.02**	**0.72**	0.06	**−2.00**
	(0.06)	(0.05)	(0)				(0.01)	(0.1)	(0.01)	(0.63)
$ipca_t$	**0.11**	0.06	0.01				−0.01	−0.08	**0.94**	**0.67**
	(0.04)	(0.04)	(0.01)				(0)	(0.07)	(0.05)	(0.14)

Note: Bold and underline entries denote parameter estimates significant at the 5% and 10% level, respectively. Standard errors appear in parentheses.

distinguishing factor is slope persistence, which presents a higher result than other estimations.[6] The main result observed in this specification is the effect of lagged U.S. empirical factors on latent Brazilian factors, where most parameters are significant.

There is an indication of comovement between the U.S. and Brazilian yield curve, expressed by the same factor relations, in the second 3×3 block of the Υ matrix diagonal. The USC_{t-1} and C_t parameter, for example, highlights the predictive power of U.S. factors for the Brazilian curve. Moreover, cross-factors relations also demonstrate the importance of the U.S. curve in this specification, mainly the USL_{t-1} and USS_{t-1}. An increase in the U.S. level factor raises the curvature and reduces the slope of the yield curve in Brazil. At the same time, a positive change in the U.S. slope factor leads to a decrease in the level factor, due to a reduction of the yields on the shorter end of the curve and a loosening international risk perception within the market.

In the next specification, we include U.S. factors and Brazilian macroeconomic variables. The results in Table 4 are similar to previous cases with regard to persistence. Compared to the Yield-UScurve model, lagged U.S. factors have less relevance, with only the USL_{t-1} factor being significant for Brazilian factors. Conversely, the macroeconomic parameters are comparable in significance, in the same way that capacity utilization turns into a pertinent variable both predicting and being predicted by latent factors.

The comovement of level factors between U.S. and Brazilian curves is stronger than the one observed in the Yield-UScurve model. The impact of U.S. level factor on slope is constant, but it has an opposite sign and more substantial impact on curvature. The result of Yield-UScurve indicates that a rise in U.S. level factors decreases the slope and increases the curvature, with a stronger effect on short- and medium-term maturities. However, Table 6 give us a different perspective, suggesting a smaller reaction of mid-maturities in favor of a longer end of the term structure, since there is a negative relation between shocks on USL_{t-1} and the curvature factor.

All lagged macroeconomic factors are pertinent to the level factor. An increase in capacity utilization leads to a small negative

[6] Such as Diebold et al. (2006) and Laurini and Caldeira (2016).

Table 5
Yield-USMX model parameter estimates.

	teste testeL_{t-1}	S_{t-1}	C_{t-1}	USL_{t-1}	USS_{t-1}	USC_{t-1}	uci_{t-1}	$selic_{t-1}$	$ipca_{t-1}$	exr_{t-1}	μ
L_t	**0.93**	**0.19**	**−0.06**	**0.57**	**− 0. 46**	− 0.13	**− 0. 02**	**− 0. 20**	**0.24**	*0.02*	**1.69**
	(0.01)	(0.03)	(0.02)	(0.14)	(0.15)	(0.22)	(0.01)	(0.05)	(0.07)	(0.02)	(0.58)
S_t	**0.31**	**0.94**	**0.16**	**− 0. 63**	**0.49**	0.30	**0.04**	**− 0. 06**	−0.18	− 0.03	**− 3. 08**
	(0.04)	(0.03)	(0.03)	(0.17)	(0.18)	(0.252)	(0.01)	(0.04)	(0.08)	(0.02)	(0.62)
C_t	− 0.59	**− 0. 42**	**0.86**	− 0.21	− 0.69	0.85	**0.07**	**0.63**	**− 0. 34**	0.02	**− 3. 27**
	(0.11)	(0.07)	(0.03)	(0.24)	(0.28)	(0.43)	(0.02)	(0.09)	(0.13)	(0.03)	(1.10)
USL_t				**0.98**	**− 0. 06**	**0.12**					**0.08**
				(0.08)	(0.01)	(0.02)					(0.02)
USS_t				− 0.01	**0.94**	**0.11**					0.08
				(0.01)	(0.01)	(0.02)					(0.03)
USC_t				− 0.01	0.01	**0.96**					−0.01
				(0.01)	(0.01)	(0.02)					(0.03)
uci_t	**− 0. 05**	− 0.07	− 0.03				**0.97**	0.06	**− 0. 05**	**− 0. 02**	**2.64**
	(0.04)	(0.03)	(0.01)				(0.02)	(0.04)	(0.03)	(0.01)	(0.27)
$selic_t$	**0.32**	**0.28**	**0.07**				**0.02**	**0.66**	0.03	− 0.01	**− 1. 34**
	(0.02)	(0.02)	(0.01)				(0.00)	(0.00)	(0.01)	(0.00)	(0.32)
$ipca_t$	**0.18**	**0.15**	**0.04**				0.00	**− 0. 20**	**0.97**	0.00	0.21
	(0.03)	(0.03)	(0.01)				(0.00)	(0.03)	(0.06)	(0.01)	(0.29)
exr_t	0.50	**0.84**	**− 0. 02**	− 0.16	0.26	− 1.43	− 0.01	− 0.58	0.09	0.06	0.88
	(0.37)	(0.30)	(0.11)	(0.79)	(0.69)	(0.98)	(0.06)	(0.42)	(0.41)	(0.07)	(4.07)

Note: Bold and underline entries denote parameter estimates significant at the 5% and 10% level, respectively. Standard errors appear in parentheses.

change of level factor. The same is observed regarding decreases in inflation and Selic rate. The negative reaction of slope factor to positive changes in the Selic rate remains, with a greater effect on curvature. So, an increase in the monetary instrument has the same effect on level factor, mainly because of movements in short- and medium-term yield maturities. Inflation provides a comparable analysis, though it is more likely an increase in long-term rates instead of medium-term rates. When scrutinizing other directional effects, the main impact of lagged factors on macroeconomic variables is related to slope. Since the slope is negative on average, and more related to modifications on the shorter end of the term structure, we can say that a decrease of short-term yields has a predictive effect on the expansion of capacity utilization and on the decline of Selic rates. Moreover, we highlight the fact that the curvature of the Brazilian yield curve is linked to macroeconomic variables, converse to the U.S. economy finding in Diebold et al. (2006).

In our last specification (Table 5), we add the percentage change in the exchange rate to the Yield-USM, calling it Yield-USMX. Overall, the signs of significant coefficients are equal in both specifications, with few changes in parameter significance and magnitude. In the Υ_1 sub-matrix, the main discrepancy between Yield-USM and Yield-USMX specifications is the significance of all parameters in the second. On the effects of lagged U.S. factors on the domestic curve, the Υ_2 sub-matrix shows the importance of U.S. slope factor on Brazilian level and slope factors, distinguishing it from Yield-USM. Furthermore, parameters that were significant in the previous specification now have larger parameters values. This reinforce our finding, i.e. there is a relevant comovement between U.S. and Brazilian slope and level factors, where Brazil?s yield curve responds to movements in the U.S. curve. We expand this analysis in the next subsection, by examining impulse response functions and forecast error variance decomposition.

In the relationship between level factor and lagged macroeconomic variables, capacity utilization lost its significant affect on level factor, showing small but significant effects on the other two factors, while the Selic interest rate and inflation hold significant at 5% with greater parameter values. In contrast to the previous specification, the Selic interest rate is negative in relation to the level factor; thus, an increase in the Selic rate has a negative effect on the level of the yield curve. On the other hand, the lagged capacity utilization is the only significant parameter for slope factor variations. Considering the effect of lagged yield factors on macroeconomic variables, we notice a decrease in size and significance of yield parameters? influence on capacity utilization. At the same time, all three latent factors have a positive and significant impact on inflation (in contrast to the Yield-USM model, where only the level factor has a relevant yet smaller impact on the price index). Even with these changes, the factors are still the main variables influencing the Selic interest rate, after inflation.

The central difference of our last specification is the inclusion of the exchange rate (exr_t). Regarding this variable, we observe that its lagged value has limited relevance to the rest of the model variables. However, our results highlight a strong relationship between the lagged domestic slope factor and the exchange rate. It is interesting to note that inflation and Selic interest rates are not related to exchange rates, which was expected, since the exchange rate is close to being a random walk. Also, this result is consistent with the exchange rate disconnect literature, in which exchange rates lack correlation with other macro variables (see Meese & Rogoff (1983)), piling up more empirical evidence on the broader exchange rate disconnect puzzle summarized by Itskhoki and Mukhin (2017).[7] In any case, the inclusion of exchange rate variations emphasizes the main results captured in previous specifications.

[7] We thanks one anonymous referee for the recommendation of this literature.

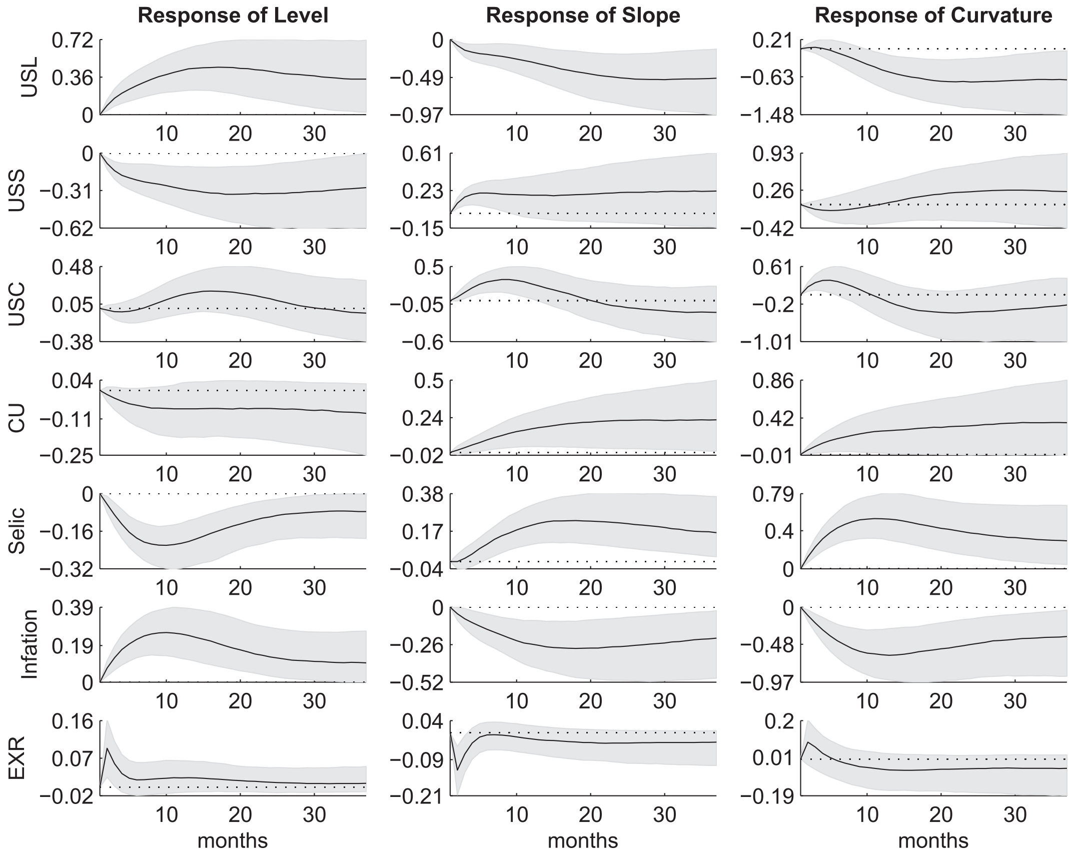

Fig. 2. Impulse-Response of yield curve factors. Note: Impulse response of Yield-USMX model. Grey area represents the one standard deviation confidence interval.

4.2. Impulse response functions

To analyze the interaction among variables in the measurement equation, we perform impulse response functions. Fig. (2) plots impulse response functions for the Yield-USMX model[8] at a horizon of 36 months, so that we can investigate the counter effect of latent factors to shocks on U.S. factors and macro variables. In Fig. (3) we explore the bidirectional linkage characteristic of this representation, plotting the Brazilian and U.S. yield curve factors on macroeconomic variables. In both cases, we present the response to one standard deviation shock on the variable in the vertical axis, and we show the median as well as the 16% and the 84% quantiles for the sample of impulse responses[9].

The response of level factor to shocks in U.S. levels is positive and significant up to 36 months. Conversely, slope and curvature respond negatively to increases in the U.S. level curve, which take more than 36 months to dissipate.[10] The response of all three factors to *USL*shocks is steady, as well as the level response to increases in the U.S. yield curve slope. This illustrates an inverse connection between them, and a lasting effect on Brazil?s level factor. While less important, there is also a comovement effect between slope and curvature factors, which lasts for around ten and six months, respectively. These results reinforce the analysis of Jotikasthira et al. (2015), that U.S. yield factors have power to explain movements in the curves of other countries. Moreover, since the authors look to Germany and the UK, we could say that this comovement is not only between developed economies, but also from advanced economies to emerging ones.

Regarding macro variables, an increase in capacity utilization has a not statistically significant effect on level factor, and a small but persistent effect on slope. Regarding shocks in the Selic interest rate and inflation, it is interesting to note that curvature factor has a higher response to macro variables than the other two factors, in contrast to the empirical results obtained in Diebold et al. (2006).

[8] Since the Yield-USMX is the most complete specification we estimate, we decide to present IRF and FEVD only for this specification, however, further results are available upon request.

[9] We build bias-corrected bootstrap confidence interval following Kilian (1998). Furthermore, if the distribution was normal, the selected quantiles would correspond to a one standard deviation band with 68% Confidence Interval.

[10] In turn, the relation to the Yield-USM specification is nearly the same, where the response of level factor to shocks in U.S. levels is positive and hump-shaped, turning negative after around 25 months, and the magnitude effect of shocks in the U.S. slope and curvature are overall smaller than those observed in the level.

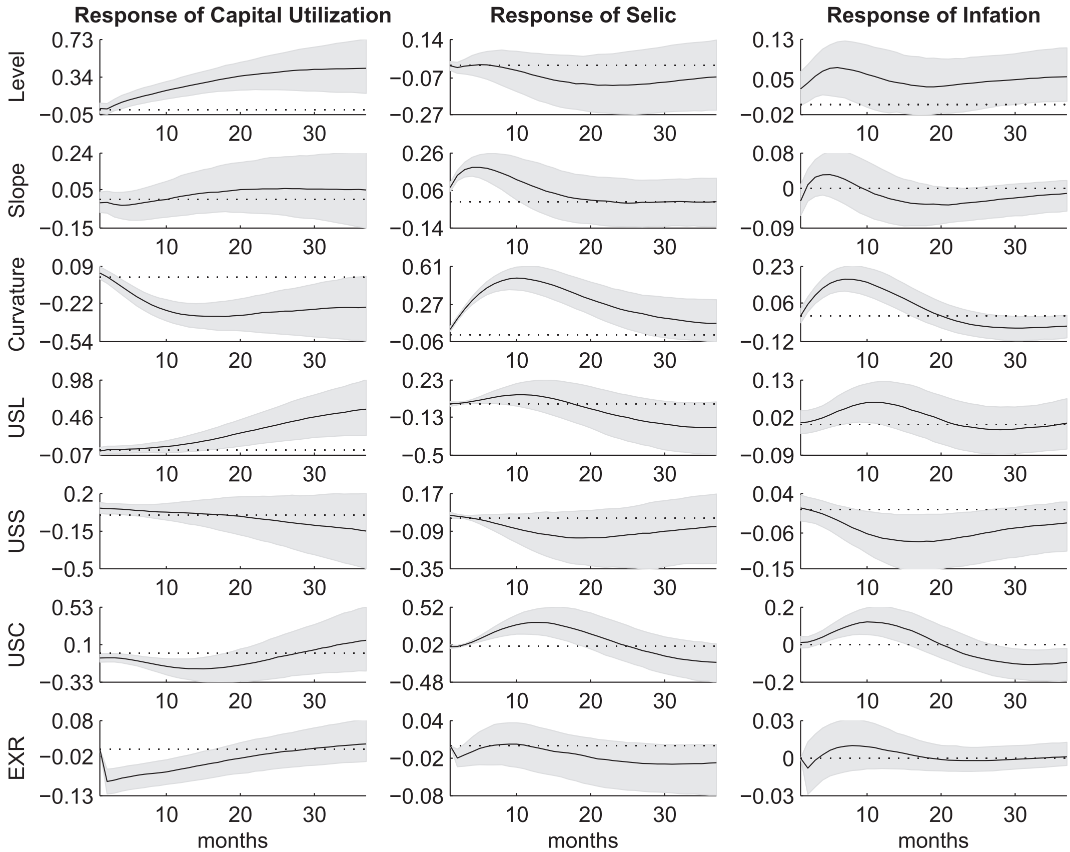

Fig. 3. Impulse-Response of Macroeconomic Variables. Note: Impulse response of Yield-USMX model. Grey area represents the one standard deviation confidence interval.

Moreover, a positive shock to inflation has the same impact on the level factor, which becomes non-significant after 30 months, demonstrating the risk perception of the market to an increase in inflation.

The other two macroeconomic variables have a significant impact on the Brazilian yield curve, while a monetary policy shock (such as the increase in the federal interest rate) has positive effects on both slope and curvature factor, and negative effect on level. While the slope and curvature reaction is no different from reports in the literature, the level reaction deserves closer attention. Compared to Sekkel and Alves (2010) results for Brazil and Diebold et al. (2006) for the U.S., the negative level response is an unusual effect, since the literature has typically reported a positive or null effect on the yield curve level for positive shocks to the federal interest rate. At the same time, looking to the same result in the Yield-USM specification, the response follows Sekkel and Alves (2010) findings, with a positive sign. Hence, although inconclusive, this response may pave the way for a debate on the impact of federal rates shocks on the yield curve in Brazil, and the importance of exchange rates, since this effect resulted from the inclusion of exchange rate in the specification.

Consider now the response of macroeconomic variables to yields curve shocks (see Fig. 3). At first, we notice a stronger response of capacity utilization to shocks on the level than the other way around, which might indicate an anticipatory behavior of the market. A similar but smaller effect can be seen in the response of inflation, once there is some evidence of economic channels linking the yield curve and price levels in the real economy. A positive shock on the slope, with a decrease of short compared to long maturities, indicates an increase in the Selic interest rate in the short term, which dissipates after about one year. Note that the effect of Selic indicates a higher response pattern in the curvature, and also shows that capacity utilization responds more intensely to shocks to this variable. It is interesting to note the significant impact of curvature on all three macroeconomic variables. The effects of level and slope shocks are the same as observed in the literature, such as Diebold et al. (2006).

A more challenging analysis would be the response of Brazilian macroeconomics to U.S. yield curve factors, since we already assume that lagged U.S. factors have no effect on these factors by the restrictions in the Υ matrix in the Yield-USMX specification. However, we impose no restriction on the Σ_η variance-covariance matrix in the transition equation of the state-space representation, which is why Fig. (3) shows this effect. These responses are related to a global market movement, which is worthy of further research, since this is not the aim of this paper. At the same time, it is worth to emphasize the negative response of inflation to a positive shock

Table 6
Forecast Error Variance Decomposition, Yield-USMX Model.

	Horizon	L_t	S_t	C_t	USL_t	USS_t	USC_t	uci_t	$selic_t$	inf_t	exr_t
Level Factor	12	0.5438	0.0685	0.0129	0.1301	0.095	0.005	0.0085	0.0624	0.0712	0.0026
	24	0.373	0.0349	0.0199	0.2303	0.1637	0.0203	0.0127	0.061	0.0825	0.0018
	36	0.3279	0.0224	0.0133	0.2614	0.2071	0.0144	0.0194	0.0524	0.0799	0.0017
Slope Factor	12	0.4557	0.0443	0.1993	0.0496	0.0573	0.0825	0.0201	0.0304	0.0578	0.0031
	24	0.3484	0.0209	0.1292	0.1275	0.0649	0.0474	0.0542	0.0792	0.1252	0.0031
	36	0.282	0.0115	0.0866	0.195	0.0727	0.032	0.0777	0.0897	0.1489	0.0038
Curvature Factor	12	0.1849	0.128	0.4078	0.015	0.0021	0.0155	0.0273	0.1139	0.1044	0.0014
	24	0.1992	0.0624	0.1977	0.1064	0.0104	0.0337	0.0557	0.1466	0.185	0.003
	36	0.1825	0.035	0.1407	0.1598	0.024	0.0391	0.0806	0.135	0.1996	0.0038
Capital Utilization	12	0.0827	0.004	0.1957	0.0049	0.0102	0.0545	0.6156	0.0032	0.0021	0.0269
	24	0.2281	0.012	0.2965	0.0587	0.0044	0.06	0.2369	0.0394	0.0528	0.0112
	36	0.2639	0.0126	0.2014	0.1639	0.0123	0.0272	0.1029	0.0858	0.1252	0.0047
Selic rate	12	0.0006	0.0923	0.6962	0.0216	0.0121	0.1403	0.0173	0.0167	0.0026	0.0003
	24	0.013	0.0364	0.5929	0.0177	0.041	0.145	0.0596	0.0417	0.0514	0.0013
	36	0.0239	0.0232	0.4483	0.0322	0.0433	0.1087	0.1067	0.0859	0.1242	0.0038
Inflation	12	0.0549	0.0038	0.2137	0.0282	0.0251	0.0788	0.0009	0.1234	0.4703	0.0008
	24	0.0682	0.0188	0.1893	0.0485	0.1016	0.0903	0.0006	0.1017	0.3801	0.0008
	36	0.0864	0.0246	0.171	0.0392	0.1297	0.1465	0.0032	0.0844	0.3144	0.0006
Exchange Rate	12	0.1642	0.0186	0.0109	0.038	0.0024	0.0104	0.0003	0.0032	0.0005	0.7514
	24	0.166	0.0189	0.0122	0.0478	0.0035	0.0114	0.0023	0.0058	0.0045	0.7274
	36	0.1656	0.0183	0.0118	0.0629	0.0066	0.0163	0.0055	0.009	0.0101	0.6939

Note: Forecast error variance decomposition for the Yield-USMX Model, comparing three forecast horizons, 12, 24 and 36 months.

on the U.S. yield curve slope. Mehl (2009) also finds that a steepening of the yield curve is expected to be associated with higher inflation. In particular, he finds that on average, a 100 basis point steepening of the U.S. or Euro yield curves observed a year earlier implies an expected acceleration of inflation of around 60 basis points a year ahead. Finally, we highlight the importance of exchange rate shocks on capacity utilization, showing a relevant bond between the Brazil economy and international trade.

4.3. Variance decomposition and out-of-sample forecast

The same effects can be observed in the variance decomposition of macro variables. Table 6 shows variance decomposition of yield factors and macro variables at forecast horizons of 12, 24 and 36 months.[11] It is interesting to note the importance of U.S. level factor on the variance of capacity utilization and slope and curvature on the inflation at longer horizons. As we previously pointed out, term structure interest rates carry information about macroeconomics and expectations. Thus, considering the global importance of the U.S., it is not surprising to observe an indirect effect through covariance matrix of our model, reflecting consequences on economic activity and inflation in Brazil. This is consistent with results in earlier studies, such as Mehl (2009), who noted that the U.S. slope adds more information than the Brazil domestic slope in forecasting inflation. We also highlight the influence of domestic factors on macro variables, such as level and curvature on capacity utilization, or curvature on Selic interest rates and inflation. Contrary to the literature for other countries, slope factor is not well-represented in this decomposition exercise.

We can also interpret the decomposition of domestic yield curve factors. Level factor has a relevant explanatory power over slope and curvature factor. At the same time, the variance decomposition of level factor for the long term is mainly explained by its own movements. However, USL_t and USC_t play an important role in domestic level factor movements. For a 36-month horizon, around 26% of level factor variance is due to U.S. level factor, and 20% to U.S. slope. Together, we can say that almost half of the variance in the level of the Brazilian yield curve is caused by changes in the U.S. term structure. Furthermore, the U.S. curve is also responsible to for 19–30% of the variance in the slope of the Brazilian yield curve. This equates to the importance of macroeconomic factors in the longer horizon. Although relevant, the role that U.S. factors play in curvature is smaller, and macroeconomic fundamentals reveal their importance in this domestic factor. These results reinforce the existence of a comovement between Brazilian and U.S. yield curves, since almost half of domestic level movements are related to variations in the U.S. yield curve.

Finally, we also carry out a forecast out of the sample. Since the largest specifications present more than 150 parameters to be estimated, it is unfeasible to re-estimate the whole model after each step of the forecast procedure. Furthermore, the main goal of this paper is not to compare the forecast performance of this model with existing competitors in the literature. At the same time, an out-of-

[11] We do not analyze shorter horizons because their results tend to only show persistence effects.

sample forecast highlights the importance of U.S.yield curve factors to the Brazilian yields. To achieve this task, we assume that parameters on the measurement equation are constants for each specification, i.e. λ and Ω, we take estimated states on the transition equation as given and forecast its VAR(1) process. Using 70% of the sample for the first estimation of the transition equation, where $t = 1, 2, \ldots, j$, we forecast $\mathbf{X_{j+h}}$ vector and use its results to forecast the vector of yields $\mathbf{y_{j+h}}$ till $j + h = T$, where $h = 1, 6, 12$ are the forecast horizons. We repeat this procedure for the four specifications re-estimating after each run.

In the Table C.8 of the Appendix C we compare the root mean squared forecasting error (RMSFE) $h-$months-ahead results to random walk (RW) and AR(1) for maturities of 3, 12, 24, 36 and 60 months. The best performers are the Yield-Macro and Yield-US specifications, even though all four present a reasonable error in comparison with the benchmarks. For the one month ahead forecast, the specification only with macroeconomic variables presents lower error statistics for the majority of selected maturities. The exception is the shortest maturity of three months, in which the Yield-US specification performs better. In fact, it is worth emphasizing that the Yield-US specification performs better for all three forecast horizons considering the three months? maturity. For the six months ahead forecast, this is the best specification too, losing to the Yield-Macro only for the longer maturity. In the end, the Yield-US specification is better than the others for all selected maturities on the 12-month horizon.

Summing up the RMSFE for each forecast horizon, as presented in the C, the Yield-US presents the lowest error for the three horizons. These results not only highlight the relevance of the U.S. yield curve for the forecast of Brazil's yields, but it also gives evidence of a clear pattern between the specification only with macroeconomic variables and the one with U.S. factors, providing a path for future research.

5. Concluding remarks

This paper compares four specifications of the DNS model, considering the effect of macroeconomic variables and U.S. yield curve factors. The results reinforce the bidirectional macro-finance linkage of the term structure of interest rates. We show that the market tends to anticipate economic movements through changes in the Brazilian yield curve. This can be observed through significant effects of level and slope on inflation and the Selic interest rate. We also observe controversial effects of lagged monetary instruments on yield factors—mainly level but also slope—which indicates a space for further research. At the same time, the effects of inflation and monetary policy on curvature hold for all specifications.

We prove U.S. factors have a definite impact on the Brazilian yield curve, and recognize that we should take this analysis further in order to have a clear understanding of the impact of movements in U.S. factors on Brazilian term structure forecasting. There is clear evidence of comovement between U.S. and Brazilian yield curve factors, where the lagged U.S. factors are highly predicative of movements in Brazilian term structures. Moreover, almost half of domestic level variances are related to the U.S. yield curve, which has more than twice the relevance of macroeconomic variables. Yet the U.S. curve level also impacts the Brazilian slope and curvature, given that around 20% of curvature and 30% of slope variance in the long term is due to U.S. yield curve movements.

On an out-of-sample forecast, we evidence a clear distinction between the specifications with the best performance. A specification only with macroeconomic variables seems better to forecast longer maturities for short horizons, while a specification only with U.S. yield factors is better for short maturities and long horizons of forecast. Since we do not explore extensively this forecast exercise, this is a path for future research, comparing these specifications with several well-known competitors in the literature.

These findings reinforce findings in the literature regarding comovements between term structures of different countries and bring an unprecedented result linking the term structure of emerging markets with developed markets. Our results highlight the importance of the U.S. curve in understanding movements in the Brazilian curve, which may assist fund managers and policymakers in better understanding the magnitude and direction of these activities.

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Appendix A. Model fit

Table A.7.

Table A.7
Fitting Statistics.

Maturities (in months)	3	6	9	12	15	18	21	24	27	30	36	42	48	60
								Mean						
Data	12.5751	12.6345	12.6890	12.7559	12.8244	12.8911	12.9483	13.0019	13.0461	13.0756	13.1247	13.1710	13.1965	13.2156
DNS	12.4709	12.5877	12.6862	12.7696	12.8405	12.9011	12.9530	12.9979	13.0367	13.0704	13.1259	13.1690	13.2031	13.2531
M	12.4716	12.5879	12.6861	12.7693	12.8401	12.9007	12.9527	12.9975	13.0364	13.0702	13.1258	13.1691	13.2034	13.2536
US	12.4790	12.5907	12.6856	12.7665	12.8357	12.8952	12.9465	12.9910	13.0297	13.0635	13.1193	13.1629	13.1976	13.2487
USM	12.4710	12.5877	12.6861	12.7695	12.8403	12.9009	12.9529	12.9977	13.0365	13.0703	13.1258	13.1690	13.2032	13.2533
USMX	12.4697	12.5868	12.6854	12.7689	12.8399	12.9006	12.9526	12.9975	13.0364	13.0702	13.1257	13.1689	13.2031	13.2532
								Standard Deviation						
Data	3.1844	3.1312	3.0573	2.9765	2.8914	2.8146	2.7487	2.6938	2.6542	2.6265	2.5945	2.5756	2.5596	2.5475
DNS	3.4202	3.2100	3.0605	2.9497	2.8647	2.7976	2.7438	2.7003	2.6650	2.6364	2.5950	2.5690	2.5535	2.5410
M	3.4168	3.2079	3.0597	2.9500	2.8655	2.7987	2.7449	2.7012	2.6656	2.6368	2.5948	2.5682	2.5522	2.5387
US	3.4100	3.1997	3.0507	2.9407	2.8563	2.7898	2.7364	2.6931	2.6579	2.6293	2.5877	2.5614	2.5456	2.5324
USM	3.4194	3.2103	3.0609	2.9501	2.8650	2.7979	2.7441	2.7005	2.6652	2.6365	2.5949	2.5688	2.5532	2.5407
USMX	3.4180	3.2095	3.0609	2.9506	2.8658	2.7988	2.7449	2.7012	2.6656	2.6368	2.5948	2.5683	2.5523	2.5391
								Root Mean Squared Error (RMSE)						
DNS	0.5015	0.1803	0.0198	0.0653	0.068	0.0504	0.0282	0.0248	0.0348	0.0408	0.0366	0.0299	0.0449	0.1345
M	0.4996	0.1783	0.0213	0.0651	0.0669	0.0487	0.0269	0.026	0.0359	0.0414	0.0363	0.0299	0.046	0.1354
US	0.5009	0.18	0.0205	0.0649	0.0675	0.0499	0.0279	0.0249	0.0348	0.0407	0.0366	0.0302	0.0446	0.1341
USM	0.4987	0.1788	0.0217	0.0649	0.0675	0.0498	0.0277	0.0252	0.0353	0.0411	0.0368	0.0299	0.0446	0.1351
USMX	0.4989	0.1782	0.0259	0.064	0.066	0.0483	0.0268	0.0262	0.036	0.0414	0.0365	0.0305	0.0449	0.1338

Appendix B. Computational procedures

We follow Diebold et al., 2006 estimation procedures, initializing the Kalman Filter[12] with the parameters obtained by the DL two-step approach computed with $\lambda = 0.077$. We maximize the likelihood by iterating Nelder-Mead and BFGS algorithms[13], with a convergence criterion of 10^{-6}. As a standard procedure, we optimize the Cholesky factor of matrix Σ_η, shrinking the number of parameters to be estimate from $k \times k$ to $k(1 + k)/2$, and ensuring that this matrix will be symmetric and positive definite.

We use a mixed optimization strategy due to the increasing complexity of the model's specifications; i.e. for the biggest specification we estimate 150 parameters. Considering the roughness of a likelihood function with such a great number of parameters, we avoid getting stuck on local maximum with the following algorithm:

1. Estimate initial parameters with the two-step approach;
2. Optimize for 1.000 iterations with the Nelder-Mead algorithm;
3. Maximize the likelihood function with the quasi-Newton method;
4. Evaluate if the main diagonal of matrix Υ, the parameter λ and the correlation with empirical factors are economically plausible. If so, store the parameters and function likelihood;
5. Re-optimize with the Nelder-Mead algorithm for 500 iterations from the point where step 3 terminate;
6. Loop from 3 to 5.

We run this procedure 40 times and pick the stored parameters associated with maximum likelihood. It takes 3.5 h for the specification with 150 parameters.[14] The criteria on step 4 are loose enough to avoid any kind of guided result. We expect that the diagonal of Υ matrix grater than 0.6 for the Brazil and U.S. yield curve factors and 0.5 for the macroeconomic factors, a bounded $\lambda \in [0.0009, 0.9]$ and correlation with empirical factors grater than 60%. Thus, we only exclude extremely implausible parameters.

Appendix C. Out-of-sample forecast

Table C.8.

Table C.8
Performance of out-of-sample forecasting results.

Maturity (τ)	Random-Walk	AR(1)	Yields-Macro	Yields-US	Yields-USM	Yields-USMX
			Horizon = 1-month ahead			
3 months	0.311	0.275	0.149	0.074	0.153	0.155
1 year	0.253	0.214	0.107	0.136	0.192	0.181
2 years	0.200	0.156	0.033	0.118	0.165	0.141
3 years	0.188	0.137	0.003	0.105	0.150	0.117
5 years	0.180	0.119	0.023	0.097	0.143	0.101
Trace-RMSE	0.261	0.208	0.069	0.136	0.197	0.172
			Horizon = 6-months ahead			
3 months	2.137	1.900	1.944	1.067	1.944	1.956
1 year	1.878	1.629	1.380	0.915	1.627	1.636
2 years	1.630	1.352	0.954	0.770	1.352	1.357
3 years	1.570	1.250	0.759	0.720	1.224	1.226
5 years	1.509	1.132	0.587	0.678	1.106	1.102
Trace-RMSE	2.017	1.684	1.283	0.962	1.680	1.686
			Horizon = 12-months ahead			
3 months	4.245	3.72	3.635	2.242	3.526	3.574
1 year	4.002	3.451	2.887	1.896	3.079	3.124
2 years	3.399	2.800	2.182	1.540	2.517	2.553
3 years	3.150	2.474	1.829	1.406	2.203	2.231

(*continued on next page*)

[12] We applied the procedures developed by Jungbacker and Koopman (2015) due to gains in efficiency.

[13] The second algorithm is a gradient quasi-Newton method with BFGS update of the estimated inverse Hessian, known as the csminwel.m routine by Christopher A. Sims. The interaction of both algorithms avoid the optimization stuck in a cliff edge, once we perturb the initial values of the parameter vector while not converged.

[14] For this specification we use a Windows machine with Intel Xeon E5-2630 v4 CPU 3.1 GHz processor, with 40 physical cores, and 32 GB of RAM.

Table C.8 (*continued*)

Maturity (τ)	Random-Walk	AR(1)	Yields-Macro	Yields-US	Yields-USM	Yields-USMX
5 years	2.915	2.133	1.466	1.273	1.842	1.861
Trace-RMSE	4.146	3.429	2.780	1.937	3.082	3.125

Note: The table reports root mean squared forecast errors (RMSFE) and trace RMSFE (TRMSFE) obtained by using the models considered, for the 1-month, 3-month, and 12- month forecast horizons. The evaluation sample is 2012:02 to 2016:09 (56 out-of-sample forecasts). The first line in each panel of the table reports the value of RMSFE and TRMSFE (expressed in basis points) for the random walk model (RW). The following model abbreviations are used in the table: AR(1) for the first-order univariate autoregressive model, Yields-Macro for dynamic Nelson-Siegel model with macro-factors, Yields-US for dynamic Nelson-Siegel model with US-factors, Yields-USM for dynamic Nelson-Siegel model with macro- and US-factors, and Yields-USMX for dynamic Nelson-Siegel model with US- and macro-factors, and exchange rate, respectively.

Appendix D. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.najef.2019.01.010.

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