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Pisa, a.a. 2021-2022

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UNIVERSITÀ DI PISA

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Path along different solutions for the B-Physics Anomalies

Introduction of one or two mediators Beyond Standard Model to accomodate those Anomalies

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Sessione autunnale
Anno Accademico 2021/2022

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Introduction

In this work I am trying to explore the huge world of Beyond Standard Model (BSM) theories using as gate some exmerimental observations seen recently. In the Standard Model (SM) as is known today we have an accidental symmetry which is called Lepton Flavour Universality (LFU) which tells us that all the gauge interactions are flavour blind; in other words the Electro-Weak (EW) processes have the same strenght for the electron, the muon, the tau and same for neutrinos. Since the only difference beetween the different lepton families is the mass, there is the phase space factor which affect the rate or the cross section of a process (for example the decay of charged π produces just muons for this reasons). Nevertheless there is also a dynamical effect which is not mass-blind, which is the coupling with the Higgs Boson, but since leptons are very light (the τ, which is the heaviest, weighs less than 50 times the Higgs'vacuum expectation value) we decide to neglect the Higgs's coupling when we speak about LFU.

If we have accepted that LFU is a good symmetry we can also understand why testing it is a good place to find clues for BSM theories. Recently in different experiments were found differrent hints of LFU Violation (LFUV) in semileptonic decays of the B mesons (mesons with non vacuum difference of b and $\bar{b}$ as valence quark). All the deviations from the SM appearing in these decays go under the name of $B-$*Physics Anomalies.*

Since, as we will discuss, the B-Physics Anomalies appear mostly at the hadronic scale (order few GeV) the most natural approach is the Effective Field Theory (EFT) approach, as is usually done with the Fermi Theory. In this approach finding New Physics (NP) basically means to find deviation from the Lagrangian's coefficients of the operators (or Wilson's coefficients).

So our purpose is to find the right heavy mediators which, once integrated out from the Lagrangian (again as we do with W's and Z bosons to get the Fermi theory), give us the appropriate contribute to accomodate the B-Physics Anomalies. We also have to be very careful about the processes that are already tested, because introducing NP can affect also processes that don't concern B or hadrons at all.

This is one of the reason because we like to introduce LeptoQuarks (LQs): coloured bosons which can be absorbed from a quark to become a lepton (+h.c.) and try in this way to affect just semileptonic processes without disturbing others SM constraints. We will take count also of the possible colour-less bosons (heavier version of W and Z) and in both cases we will try to find the best flavour structure to accomodate the Anomalies.

We also would like to see those bosons, that's why apart from the low energy processes we want to take in consideration the direct searches in which we try to see if this bosons are produced at LHC smashing proton against proton. In this point we will find interesting constraints on the masses and some reasons to improve the colliders we already have.

A lot of papers are already written about B-Anomalies and LeptoQuarks; the main purpose of this work is to offer a catalogue as much clear and complete we can do of solution with single NP mediator and with a pair of them.

In the end we have to mention that adding bosons to SM is not enough to say that we have a NP theory, we need a theory in which with few assumptions and few input values all the particles and processes come out naturally (as in SM). These complete theories are known as UV Completions and we will mention some of them. Most of them belong to the family of Great Unification Theories (GUTs) which are the natural enviroment to get LQs.

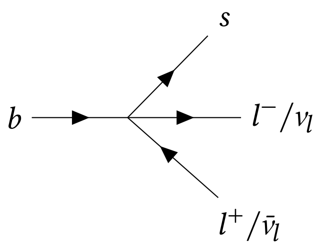
Hope you readers will find the work interesting and light to read, for now we have done enough of introductions; let's begin with B-Physics Anomalies.

1

B-Physics Anomalies

The two fundamental processes in which we are interested in are: the neutral current (NC) decay

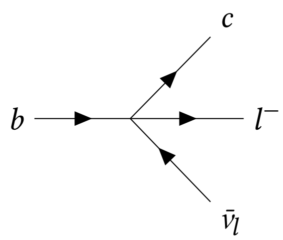
$$b \to s l^+ l^- \text{ or } b \to s \nu \bar{\nu}$$



and the charged current (CC) decay

$$b \to c l \bar{\nu}$$

.



1.1 Main Processes for NC transitions

For what concern the NC decays the first process we want to discuss is the decay:

$$B \to K^* l^+ l^-.$$

Table 1.1: Table with the most important anomalies in $b \to sl^+l^-$ transition. $R_{K^*}^{[q_1^2,q_2^2]}$ means the ratio R_{K^*} in which the momenta of the pair lepton-antilepton has energy at rest q^2 included beetween q_1^2 and q_2^2. Fonte: [1]

Observable	*Experiment*	*SM*
$R_{K^*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07} \pm 0.03$	0.906 ± 0.028
$R_{K^*}^{[1.1,6.0]}$	$0.69^{+0.11}_{-0.07} \pm 0.05$	1.00 ± 0.01
$R_{K}^{[1.1,6.0]}$	$0.846^{+0.042+0.013}_{-0.039-0.012}$	1.00 ± 0.01
$BR(B_s \to \mu^-\mu^+)$	$2.85^{+0.32}_{-0.31} \cdot 10^{-9}$	$(3.66 \pm 0.14) \cdot 10^{-9}$

We know that in the SM this process is loop-induced (as all the Flavour Changing Neutral Current (FCNC)) and should have the same rate for the charged lepton being electron or muon because of LFU. Given the masses of the particle involved:

$$m_B \simeq 5.3GeV \;\; m_K \simeq 430MeV \;\; m_\mu \simeq 106MeV \;\; m_e \simeq 500keV$$

the phase space factor is neglectable; and the ratio

$$R_{K^*}^{\mu e} \equiv \frac{\mathscr{B}(B \to K^*\mu^-\mu^+)}{\mathscr{B}(B \to K^*e^-e^+)}$$

is equal to 1 in the SM.

The reasons why we use ratios of Branching Ratios are mainly three:

- To reduce the dependence from hadronic form factors
- To reduce the dependence from CKM matrix elements
- To reduce the systematic error in general

In the Table 1.1 we can finally see the first B-Physics Anomaly; in fact the lack of muons among the decay's products is a clear hint of LFU violation. The formal definition for the ratio $R_{K^*}^{[q_1^2,q_2^2]}$ is:

$$R_{K^*}^{[q_1^2,q_2^2]} = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathscr{B}}{dq^2}(B \to K^*\mu^+\mu^-)}{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathscr{B}}{dq^2}(B \to K^*e^+e^-)}$$

and same for the pseudoscalar K.

1.1.1 $B_s \to \mu\mu$:

Also in Table 1.1 we can see the data referred to the $B_s \to \mu\mu$ decay we can easily see that the 4-fermion vertex factor involved is exactly the same of the $B \to K^*\mu\mu$ decay because of the so called *crossing symmetry.*

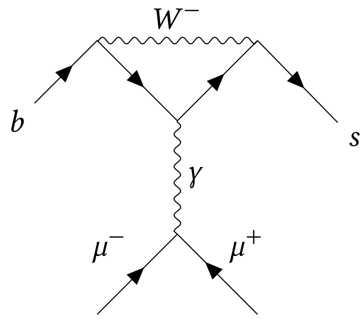


Figure 1.1: Diagram of the fundamental transition $b \to s\mu^-\mu^+$ in the SM.

With this decay indeed we need to be careful to the possibility that perturbativity condition is not satisfied because of $c\bar{c}$ resonances. The diagram describing this decay in SM is the one in Figure 1.1.1 which is one of the so called *penguin diagrams.*

If the energy of the virtual photon is few GeV, there is a no-neglectable contribution due to the fact that the photon can produce a pair cc nearby of the known resonances that have those quark as valence quark : J/ψ, $\psi(1S)$, $\psi(2S)$,...
So we can have issues due to QCD non-perturbativity but on the other hand we have a final state which is *clean* theoretically, since is a two lepton state with no hadronic form factor. That's why we will treat this kind of process separately.

1.2 Main Processes for CC transitions

The main CC decay in which we are interested in is:

$$B \to D^* l \nu_l$$

We define the ratio analogously:

$$R_{D^*}^{\tau l} \equiv \frac{\mathcal{B}(B \to D^* \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathcal{B}(B \to D^* l \nu_l)}.$$

In Table 1.2 we see the most important CC anomalies.
The main differences with the NC can be summarized as follows:

- From a first look to the Table 1.2 we can already notice that the significance of the anomaly is less important than the NC case;
- In SM this process is tree level generated (no penguins):

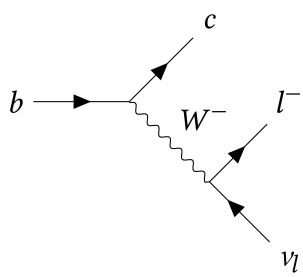


Table 1.2: Table with the most important anomalies in $b \to sl\nu$ transition. ρ stays for the correlation beetween the ratio with D and D^*. Fonte:[1]

Observable	*Experiment*	*SM*
$\{R_D, R_{D^*}\}$	$\{0.337(30), 0.298(14)\}$	$\{0.299(3), 0.258(5)\}$
ρ	-0.42	$-$
$BR(B^- \to \tau^- \bar{\nu}_\tau)$	$1.09(24) \cdot 10^{-4}$	$0.812(54) \cdot 10^{-4}$

and that's why the SM prediction is more accurated than the FCNC process.

- in the $B^- \to \tau\nu$ decay we have no issues linked to the charmonic resonances but the final state is not clean as the $\mu\mu$ pair (the neutrino is invisible and the τ decays briefly in hadrons). Also the rate is way bigger than the $B_s \to \mu\mu$.

If we begin to wonder about the NP's shape we notice that the NP couple mostly at the third generation for what concern quarks. A naive approach would be to guess a coupling growing with the mass (*Higgs-like*) but the NC decays suggest a smaller coupling for muons that the one for electrons. In the CC case, instead, we could imagine that the new physics couples more to the heaviest lepton: the τ.
All this kind of considerations (and way more of them) will be the ones that allow us to build a low energy model independent Field Theory: the Effective Field Theory; which later will give us the shape of the BSM particles we need.

2

Effective Field Theory approach

The first step to see how the low energy deviations from the SM is to parametrize these effects in a Quantum Field Theory which is independent from the physics at higher energies.

The usual example of an EFT is the Fermi Theory which is used to describe the weak processes of the SM when the energy is way below $M_W = 80\ GeV$. This theory can describe the weak processes at low energy ignoring the details of the UV Physics like the coupling beetween fermions and vectors that mediate the force, the mass of these mediators nor the theoretical nature of all the particles heavier than 80 GeV, which is called the *matching scale*.

In an EFT all the physics beyond the matching scale is contained in the numerical coefficient in front of the Lagrangian's operators. In fact the condition for the EFT to be a low energy version of an UV theory is for those coefficient to satisfy the *matching condition* that consists in impose the coefficients of the two theories, which normally run with the energy scale of the process via Renormalization Group Equations (RGE), to recreate the same transition amplitudes at an energy equal to the matching scale.

In the Fermi Theory, as is normally intended, we find the desciption of the 4-fermion weak processes that are tree-level generated in SM without flavour suppression. So, for first, we introduce the *Extended Fermi Theory* which takes in consideration basically all the allowed 4-fermion process.
Once this is done we can take just the semileptonic operators which concern the b quark and then try to find the proper flavour structure to explain the Anomalies without bothering the SM.

2.1 Extended Fermi Theory

The Fermi Theory was historically the first theory that described the weak processes as the beta decay or the muon decay.
When the EW theory was formulated from Glashow-Weinberg-Salam they of course needed to predict the low energy decays with same rates tested by the Fermi Theory. Basically the EW theory had to satisfy the matching condition with the Fermi Theory.

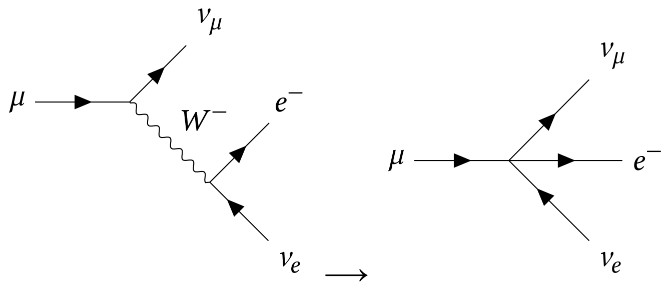


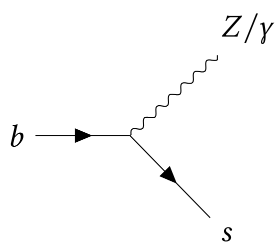
Figure 2.1: Left: Feynman diagram for muon decay in SM, the lagrangian terms responsible for this interaction is of the form $\sim g_2\overline{\psi}_{1L}\gamma^\mu\psi_{2L}W_\mu$.
Right: Feynman diagram for muon decay in Fermi Theory, the lagrangian terms responsible for this interaction is of the form $\sim G_F\overline{\psi}_{1L}\gamma^\mu\psi_{2L}\overline{\psi}_{3L}\gamma_\mu\psi_{4L}$
.

In Figure 2.1 we can see how Feynman diagrams change passing from the SM to the Fermi Theory or, in other words, integrating out all the particles heavier than 80 GeV. Basically, if we study processes in which the energy available to produce particles is less than the rest energy of the W boson, we can ignore it in the spectrum of our theory and take count of its presence as virtual particle trough a form factor which happens to be constant at low energy: the Fermi's constant G_F[1] .

2.1.1 *Flavour Changing Neutral Currents* :

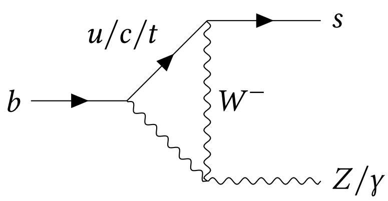
A complete and synthetic discussion on Flavour Physics can be found in Appendix. Here we report just the main parts that make us understand how we get tree level operators in the EFT that can't appear at tree level in SM.

The most natural example consists is the so called Flavour Changing Neutral Currents (FCNC) which are not allowed at a classical level in SM. For example the vertex:

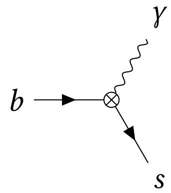


is not allowed from SM because the neutral interaction is diagonal in mass's basis (see Appendix). On the other hand that doesn't mean that we can't have the $b \to sZ/\gamma$ at all, in fact at one loop level we can have:

[1] $G_F = 1.166376(7) \cdot 10^{-5}\ GeV^{-2}$, [2]



Now if we integrate out W's and Z bosons we get:



Here we don't have G_F as vertex factor, in fact there is to multiplicate the loop factor and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and both those factors make the transition suppressed. This is why we say that the SM allows FCNCs but they are a suppressed effect and so a good place to look for NP.

In the previous diagram we neglected the Z boson because in Fermi Theory it doesn't belong to the spectrum. Indeed if the Z/γ were virtual in the SM diagram, perhaps producing a pair l^+l^-, we would have exactly the effective vertex mentioned in section 1 that allows the decay $b \to sl^-l^+$ and so the decay $B \to K^*l^-l^+$ which is one of the most interesting for us.

In SM we have CKM, loop factor, and gauge coupling that give me a prediction for the coefficient in front of the operator that mediate this decay: the Wilson coefficent. So if the mesaured Wilson coefficient is different from the theoretical one, it means that probably to build the EFT we need to integrate out other heavy particles which we don't know yet.

2.1.2 *4-fermions operators* :

Once we have included all the 4-fermions operators in the effective lagrangian we can choose the ones we need to calculate the amplitudes of the processes of interest. Keeping in mind that we don't want to break the conservation of Barion number, which is very tested, we have just three types of operators allowed by the gauge symmetry of the SM:

- Purely quark operators, which can mediate for instance the Kaon's decay in pions'channels. Nevertheless those operators are quite hard to match with the SM because we can have loop of gluons beetween the two currents which both couple with gluons. Since at the meson scale of energies the strong interaction is non-perturbative we have to consider a lot of contributions that are not easy to parametrize.

- Purely leptonic operators, which can for instance mediate the muon's decay. Those operators are used to the processes that have just leptons in the initial and final state avoiding all the QCD's mess for both of them. In fact the cited muon decay is predicted so much accurately that it is the main process used to measure the Fermi's constant G_F.

- Semileptonic operators, which can mediate the charged pion's decay, but also all the processes to which we can address the B-Physics Anomalies. Of course the prediction are not clean as the purely leptonic case, nevertheless we have no gluon loop beetween the two currents and this tells us that we can see how the quark current renormalize just using global symmetries of QCD. In fact, in QCD, the vector current is conserved in the massless limit and so the quark current in a semileptonic decay is not affected by renormalization group of QCD. This fact reducts the theoretical uncertainity to the hadronic form factor.

2.2 Effective Semileptonic Lagrangian

From now on we will write fermion's field as Weyl spinors.

As we mentioned many times so far we want to find the coefficients that parametrize the B-Physics Anomalies which appear just in semileptonic B mesons decays.

Our first step is to collect all the effective operators that can contribute to those processes. If we assume for NP to be coupled just to LH fermions, as done by [3], we would need just two operators:

$$\mathcal{O}_S = q_L^\dagger \bar{\sigma}^\mu q_L \, l_L^\dagger \bar{\sigma}_\mu l_L \quad , \mathcal{O}_T = q_L^\dagger \bar{\sigma}^\mu \sigma_a q_L \, l_L^\dagger \bar{\sigma}_\mu \sigma^a l_L$$

where we neglect the flavour index. Writing them explicitly we have, for instance, the singlet operator equal to $\mathcal{O} = \mathcal{O}^{abcd} = q_L^{a\dagger} \bar{\sigma}^\mu q_L^b \, l_L^{c\dagger} \bar{\sigma}_\mu l_L^d$. Also the EW indexes are neglected and here that means that both the vector currents composing the operator are irreducible representation of $SU(2)_L$ (two triplets for $\mathcal{O}_T$ and two singlets for any other). With these two operators we can describe all semileptonic processes that involve just LH fermions. To keep the approach totally general, i.e. including the possible coupling of NP with RH fermions, we find other four operators compatible with the gauge symmetry $SU(3)_c x SU(2)_L x U(1)_Y$. Two of them are as well already generated in the SM:

$$\mathcal{O}_{LR1} = q_L^\dagger \bar{\sigma}^\mu q_L \, e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LR2}^{u/d} = q_R^\dagger \sigma^\mu q_R \, l_L^\dagger \bar{\sigma}_\mu l_L$$

that describe the already mentioned FCNCs. The u/d means that we have two independent versions of the $\mathcal{O}_{LR2}$ for q_R equal to the up or down quark's flavour triplet.
The last two operators that we include are

$$\mathcal{O}_R^{u/d} = q_R^\dagger \sigma^\mu q_R \, e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LQ}^{u/d} = q_L^\dagger \bar{\sigma}^\mu l_L \, e_R^\dagger \sigma_\mu q_R$$

Indeed $\mathcal{O}_R$ can describe weak processes which are generated from a contribute that includes at least two mass insertion and so widely neglectable. We can therefor say that in a good approximation $\mathcal{O}_R$ and $\mathcal{O}_{LQ}$ aren't generated in SM.

We choose to separate the SM Lagrangian from the operators defined above, i.e. to use a Lagrangian that recreate the SM predictions in the limit $C_S = C_T = C_{LR1} = C_{LR2}^{u/d} = C_R^{u/d} = C_{LQ}^{u/d} = 0$:

$$\mathscr{L}_{EFT} = \mathscr{L}_{SM} + C_S\mathscr{O}_S + C_T\mathscr{O}_T + \sum_{q=u,d}\left[C_R^q\mathscr{O}_R^q + C_{LQ}^q\mathscr{O}_{LQ}^q + C_{LR1}^q\mathscr{O}_{LR1}^q + C_{LR2}^q\mathscr{O}_{LR2}^q\right] \tag{2.1}$$

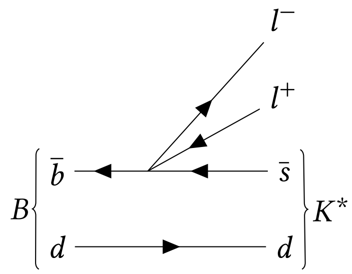
in which, expilciting flavour indexes $C\mathscr{O} = C_{abcd}\mathscr{O}^{abcd}$ and hence we define c_i as

$$C_i^{abcd} \equiv c_i\Lambda^{abcd}$$

that seem to be redundant but will be useful when we will match the EFT with UV theory in which the intensity of interactions is almost universal apart for the flavour structure.

From the experiments we acknowledge what are the observables of interest to explore the clues of NP. Plus they suggest us that NP, at leading level, doesn't concern the lightest families of quark and leptons and couple preferly to the heaviest. Now we can compute the anomalous observables in a framework independent from the high energy physics.

2.2.1 R_{K^*}



One of the main observables that bring clues of LFU violation is

$$R_{K^*} = \frac{\mathscr{B}(B \to K^*\mu\mu)}{\mathscr{B}(B \to K^*ee)}$$

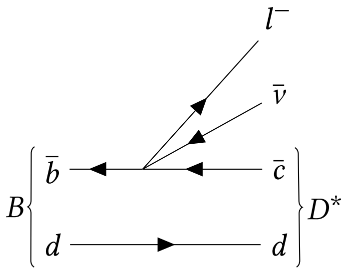
Since the current is neutral and doesn't involve neutrinos we have contribute from *all* the operators listed above.
To compute easily this observable we assume the center of mass frame energy of the lepton pairs way bigger than m_μ to simplify the phase space factor. Plus, using that electrons don't couple to NP, we have:

$$R_{K^*} = |1 + \frac{1}{C_{SM}}\sum_{i=all} C_i^{bs\mu\mu}|^2$$

where $C_{SM} \equiv V_{ts}^*V_{tb}\frac{e^2}{4\pi v^2}\mathscr{F}_{loop}(y_t)$ is roughly the amplitude for that process in SM. As previously pointed out in the limit $C_S = C_T = C_{LR1} = C_{LR2}^d = C_R^d = C_{LQ}^d = 0$ we find $R_{K^*} = 1$ which is the SM result because of LFU.

From Table 1.1 we see that, in the limit $E_{ll} \gg m_\mu$, $R_{K^*} < 1$ and so we can say at this point that to accomodate the R_{K*} anomaly we need for the sum of the Wilson coefficients to be negative.

2.2.2 R_{D^*}



The second process we want to predict trough our EFT is:

$$R_{D^*} \equiv \frac{\mathscr{B}(B \to D^*\tau\bar{\nu})}{\frac{1}{2}\left[\mathscr{B}(B \to D^*e\bar{\nu}) + \mathscr{B}(B \to D^*\mu\bar{\nu})\right]}$$

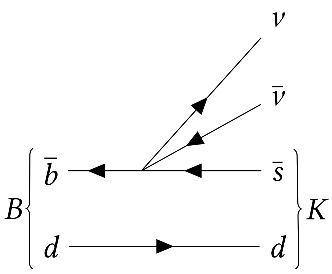
which is not supposed to be 1 in SM because of non-neglectable phase space factor and the anomaly consists in a larger value compared to SM prediction as shown in Table 1.2. Here we could have a contribute SM like from $\mathcal{O}_T$ and another from $\mathcal{O}_{LQ}$. Defining $C_{D^*}^{bcl_1l_2} \equiv \sum_d 2V_{cd}^* C_T^{cdl_1l_2} + C_{LQ}^{d\;\;bsl_1l_2}$ we have:

$$R_{D^*} = \frac{|1 + \frac{1}{C_{SM}}\left[C_{D^*}^{bc\tau\tau} + C_{D^*}^{bc\tau\mu}\right]|^2}{\frac{1}{2}\{1 + |1 + \frac{1}{C_{SM}}\left[C_{D^*}^{bc\mu\mu} + C_{D^*}^{bc\mu\tau}\right]^2|\}}$$

where in this case $C_{SM} = \frac{V_{cb}}{v^2}$.

Here we see why NP should prefer coupling to τ to the other two families.

2.2.3 $R_{B\to K\nu\nu}$



Another interesting process that we want to examinate is the semileptonic decay in neutrinos pair: $B \to K\nu\nu$, The reason why we wabt to predict this process is that it is a FCNC predicted to be suppressed in SM and precisely measured. Since for gauge invariance this process is tight linked to $B \to Kll$ we need to be very carefull when we accomodate R_{K^*} because if a non vacuum value for the NP Wilson coefficients could affect $B \to K\nu\nu$, eventually contraddicting the data.

Defining $C_{\nu\nu}^{bsl_1l_2} \equiv C_S^{bsl_1l_2} - C_T^{bsl_1l_2} + C_{LR2}^{d\;\;bsl_1l_2}$ and $C_{SM} \equiv V_{ts}^* V_{tb} \frac{1}{cos^2\theta_W\, v^2} \mathscr{F}_{loop}(y_t)$ we write:

$$R_{B\to K\nu\nu} = |1 + \frac{\sum_{l_1,l_2=\mu,\tau} C_{\nu\nu}^{bsl_1l_2}}{C_{SM}}|^2$$

Here we can point out some aspect. If NP, as in the approach followed by [3], coupled just with LH fermions we can accomodate R_{K^*} without disturbing $R_{B\to K\nu\nu}$ if $C_S^{bs\mu\mu} \sim C_T^{bs\mu\mu}$ which, as we will see, can be implemented in a particular UV scenario. In general we want $\sum_{l_1,l_2=\mu,\tau} C_{\nu\nu}^{bsl_1l_2} \ll C_{SM}$.

3

Some of the possible heavy bosons

As we previously said one possible way to modify the Wilson coefficients in an EFT is to introduce heavy particles to the theory which couple with the fermions involved in the processes we want to accomodate.
When we introduce new particle interacting with the SM particles we need to look carefully at the experimental constraints. In particular the particle has to satisfy two bounds:

- The contribution to the observable given by the diagrams in which the new particles appear as virtual particles has to show agreement with the experiments, both the ones who seem anomalous to the SM and the ones that has tested the SM.
- If the new particles interact with the particles that are smahing at the colliders, so the mass range allowed for those particles introduced has to not contain the energies explored at that colliders so far.

To begin we will see how we can accomodate the B-Physics Anomalies introducing different type of heavy vector and scalar bosons.
We will begin with the most familiar case to the ones who know the SM: colour-less vector charged under $SU(2)_L$. Indicating the quantum number as $(SU(3)_c, SU(2)_L)_Y$ we can address to these particles $B' \sim (\mathbf{1}, \mathbf{1})_0$ and $W' \sim (\mathbf{1}, \mathbf{3})_0$, where the names already suggest the connection with their SM's lighter sisters W,B[1].
Then we will describe the vector LeptoQuarks $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$.
In the end we will descibe the behavior of some scalar Leptoquarks $S_1 \sim (3^*, 1)_{1/3}$, $S_3 \sim (3^*, 3)_{1/3}$ and $R_2 \sim (3^*, 2)_{7/6}$.

Since we want to see how the introduction in high energy theory affect the operators described in 2.2 we will soon find out that we will generate semileptonic operators that seem different to the one listed. That because the six operator listed are contraction of of some particular vector currents, meanwhile our bosons could couple with totally different currents like $q_L^\dagger \bar{\sigma}^\mu l_L$ which couples with U_1 or some other scalar currents coupled to the scalar bosons.
Indeed the operators we will find in this way are non independent from the six listed in 2.2 and we will project the operators generated on the operators basis described so far to compare on the same terms all the different scenarios.

[1]That phenomenological are better known as $W^\pm$, Z and the photon γ because of the so known EWSB.

To do this change of basis we will use some property of the algebra of Lorentz group $SO(3,1) \sim SU(2)xSU(2)^*$ and $SU(2)_L$ EW known as *Fierz identities* or completeness relations, that we will briefly describe before to list our candidates.

3.1 Fierz identities

Is possible to prove that, since any hermitian matrix NxN can be written as

$$H = c_0 \mathbf{I} + \sum_{i=1}^{N^2-1} c_i T_i$$

where T_i are the generators of the fundamental representation of $SU(N)$, these generators satisfy the completeness condition:

$$\sum_{a=1}^{N^2-1} T^a_{ij} T_{a\ kl} = \frac{1}{2}\big(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\big).$$

In the case $N = 2$ since $\sigma_a = 2T_a$ we find (neglecting the sum on a):

$$\sigma_{a\ ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl} \tag{3.1}$$

this relation can help us to write the EW structure to have all the currents contracted in the operators listed in 2.2, i.e. to have every single current trasforming with an irreducible representation of $SU(2)_L$.
Comparing two terms with the indexes sorted at the way we can rewrite the relation 3.1:

$$\sigma^a_{ij}\sigma_{a\ kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\ kj}). \tag{3.2}$$

For what concern the Lorentz group, since the group is $SU(2)xSU(2)^*$, the situation is analogous.
Having written fermions as Weyl spinors we have explicited the relation beetween the Lorentz structure of the currents and Pauli matrices:

$$\sigma^\mu = (\mathbf{I}, \vec{\sigma}) \qquad \overline{\sigma}^\mu = (\mathbf{I}, -\vec{\sigma})$$

These matrices allow us to write a current which trasform as a vector under Lorentz group, combining two spinors trasforming as the $(\frac{1}{2}, 0)$ and the $(0, \frac{1}{2})$, and so the current with explicit lorentz index is written with the *dotted notation*

$$J^\mu = \psi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\chi^{\dot{\alpha}} \sim (\frac{1}{2}, \frac{1}{2})$$

In which the dots distinguish the two fundamental representation of the Lorentz group. The scalar bilinear instead is the product of two spinors both from the same representation beetween $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$:

$$S = \psi^\alpha(\varepsilon)_{\alpha\beta}\chi^\beta \sim (0,0)$$

and same with all the indexes dotted. The ε is the total antisymmetric tensor in two dimensions (fixed $\varepsilon^{12} = -\varepsilon^{21} = -\varepsilon_{12} = \varepsilon_{21} = 1$), which guarantees the antisymmetry typical of the singlet.

Now that we described the structure of the vector and the scalar identities we can take back 3.1 and, mindful that $\sigma^0_{\alpha\dot{\alpha}} = \overline{\sigma}^0_{\alpha\dot{\alpha}} = \delta_{\alpha\dot{\alpha}}$ and including the Minkowski's metric we write:

$$\sigma_{\mu\ \alpha\dot{\alpha}}\sigma^{\mu}_{\beta b\dot{e}ta} = 2(\delta_{\alpha\dot{\alpha}}\delta_{\beta b\dot{e}ta} - \delta_{\alpha\dot{\beta}}\delta_{\beta\dot{\alpha}}) \tag{3.3}$$

now using that $\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}$ we find:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} = -\sigma^{\mu}_{\alpha\dot{\beta}}\sigma_{\mu\beta\dot{\alpha}} \tag{3.4}$$

Now, generalizing at the overlined matrices:

$$\overline{\sigma}^{\alpha\dot{\alpha}}_{\mu}\overline{\sigma}^{\mu\ \beta\dot{\beta}} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}} \tag{3.5}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\sigma_{\mu}}^{\beta\dot{\beta}} = 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}}. \tag{3.6}$$

Despite to the similar aspect of the equations it is clear that the relations for $SU(2)_L$ and for $SU(2)xSU(2)^*$ has to be used indipendently.
In some cases we have to use both to write the operators generated in the basis presented in section 2.2. In these cases the tensor structire could be complicated and eventually confusing, so we are going to present the useful results now to have them ready when we will handle the physics.

3.1.1 *Fierzing in the scalar lagrangian*

When we introduce to the theory scalar Leptoquarks, integrating them out from the lagrangian we could generate four-fermion operators of this shape:

$$\overline{l^{1c}}\varepsilon q^1\ \overline{q^2}\varepsilon l^{2c} = l^1_{\alpha a}q^1_{\beta b}q^2_{\gamma c}l^2_{\delta d}\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

where i, j, k, l are EW indexes and $\alpha, \beta, \gamma, \delta$ are Lorentz indexes and the ε of the Lorentz's group comes from the charge coniugation. Since all the fermions involved happens to be LH we have to write them as a linear comnbination of O_S and O_T. To do that we need to write

$$\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = c_1\ \delta^{ad}\delta^{bc}\overline{\sigma_{\mu}}^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}\ +\ c_2\ \sigma^{ad}\sigma^{bc}\overline{\sigma_{\mu}}^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}$$

basically switching the position of $q^1 \longleftrightarrow l^2$.
Now we use that

$$\varepsilon^{ab}\varepsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} = \frac{1}{2}\sigma^{ad}_{a}\sigma^{bc^{a}} - \frac{1}{2}\delta^{ad}\delta^{bc}$$

Where we used 3.1 in the second equality.
Then, acting on the 3.5,

$$\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = \frac{1}{2}\overline{\sigma}^{\alpha\gamma}_{\mu}\overline{\sigma}^{\mu\ \beta\delta} = -\frac{1}{2}\overline{\sigma}^{\alpha\delta}_{\mu}\overline{\sigma}^{\mu\ \beta\gamma}$$

And multiplicating is straightforward to obtain:

$$\overline{l^{1c}}\varepsilon q^1 \, \overline{q^2}\varepsilon l^{2c} = \frac{1}{4}\big[q^{1\dagger}\overline{\sigma}^{\mu}q^2 \, l^{1\dagger}\overline{\sigma}_{\mu}l^2 \; - \; q^{1\dagger}\overline{\sigma}^{\mu}\sigma^a q^2 \, l^{1\dagger}\overline{\sigma}_{\mu}\sigma_a l^2\big] \tag{3.7}$$

Once obtained that result is easy to derive the same in the case of the contraction of two triplet scalar currents:

$$\overline{l^{1c}}\varepsilon\sigma_a q^1 \, \overline{q^2}\sigma^a\varepsilon l^{2c} = l^1_{\alpha a}q^1_{\beta b}q^2_{\gamma c}l^2_{\delta d}(\sigma^a\varepsilon)^{ab}(\varepsilon\sigma_a)^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

We already know how to treat the lorentz structure that will bring a factor $-\frac{1}{2}\overline{\sigma}^{\alpha\delta}_{\mu}\overline{\sigma}^{\mu\ \beta\gamma}$. The EW structure instead is different:

$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cea} = (\delta^{af}\delta^{ed} - \delta^{ad}\delta^{fe})(2\delta^{e}\delta^{kj} - \delta^{mj}\delta^{kn}) =$$

$$2\delta^{ad}\delta^{bc} - 4\delta^{ad}\delta^{bc} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} = -(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd})$$

then recalling the 3.1

$$\sigma^a_{ad}\sigma_{bca} = 2\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \rightarrow$$

$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cea} = -\frac{1}{2}(\sigma^{ad}_a\sigma^{bc^a} + 3\delta^{ad}\delta^{bc})$$

multiplying the factors coming from EW and Lorentz group:

$$\overline{a^c}\varepsilon\sigma_a b \, \overline{c}\sigma^a\varepsilon d^c = \frac{1}{4}\big[3a^{\dagger}\overline{\sigma}^{\mu}b \, c^{\dagger}\overline{\sigma}_{\mu}d \; + \; a^{\dagger}\overline{\sigma}^{\mu}\sigma_a b \, c^{\dagger}\overline{\sigma}_{\mu}\sigma^a d\big] \tag{3.8}$$

Bibliography

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[3]D. Buttazzo, A. Greljo, G. Isidori, and D. Marzocca, *B-physics anomalies: a guide to a combined explanation* (2017).