1 PLAN

1. $\mathcal{L}_{EFT}$ semileptonic.
2. Matching UV.
3. $\mathcal{A} = f(C_i)$.
4. $\mathcal{A} = f(g, M)$

2 EFT

2.1 $\mathcal{L}_{EFT}$ semileptonic

Set of relevant 2q2l operators hopefully complete:

$$\mathcal{O}_S = q_L^\dagger \overline{\sigma}^\mu q_L \; l_L^\dagger \overline{\sigma}_\mu l_L \quad , \mathcal{O}_T = q_L^\dagger \overline{\sigma}^\mu \sigma_a q_L \; l_L^\dagger \overline{\sigma}_\mu \sigma^a l_L$$

$$\mathcal{O}_{LR1} = q_L^\dagger \overline{\sigma}^\mu q_L \; e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LR2}^{u/d} = q_R^\dagger \sigma^\mu q_R \; l_L^\dagger \overline{\sigma}_\mu l_L$$

$$\mathcal{O}_R^{u/d} = q_R^\dagger \sigma^\mu q_R \; e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LQ} = e_R^\dagger q_L \; \varepsilon \; u_R^\dagger l_L$$

$$\mathcal{O}_S^u = q_L^\dagger u_R \; \varepsilon \; l_L^\dagger e_R \quad , \mathcal{O}_S^d = q_L^\dagger d_R \; \varepsilon \; e_R^\dagger l_L$$

2.2 $\mathcal{L}_{EFT}$ proton decay

I wouldn't concern too much about proton decay because it would be anyway flavour suppressed but the relevant operators are : $d_R^\dagger u_R^c \; u_R^\dagger e_R^c \quad d_R^\dagger u_R^c \; q_L^\dagger l_L^c$ $d_R^\dagger d_R^c \; d_R^\dagger e_R$

2.3 $\mathcal{L}_{EFT}$ $\Delta F = 2$

To catalogate the operators which contribute to meson mixing we just consider that the operators are alrady present in SM and are obtained contracting the neutral quark currents in all the possible ways. Since the meson mixing concerns just neutral mesons without flipping of the third component of $SU(2)_L$ i.e. without including the triplet current $q_L^\dagger \overline{\sigma}^\mu \sigma_a q_L$. Plus with 4 quark we have two different ways to contract colour index ($SU(3)_c$ is not that easy to Fierz as $SU(2)_L$) so we neglect the colour index in the case the single current is an $SU(3)_c$ singlet.

$$\mathcal{O}_{4q} = q_L^\dagger \overline{\sigma}^\mu q_L \; q_L^\dagger \overline{\sigma}_\mu q_L \quad , \mathcal{O'}_{4q} = q_L^{\alpha\dagger} \overline{\sigma}^\mu q_{L\beta} \; q_L^{\beta\dagger} \overline{\sigma}_\mu q_{L\alpha}$$

$$\mathcal{O}_{2q2u/d} = q_R^\dagger \sigma^\mu q_R \; q_L^\dagger \overline{\sigma}_\mu q_L \quad , \mathcal{O'}_{2q2u/d} = q_R^{\alpha\dagger} \sigma^\mu q_{R\beta} \; q_L^{\beta\dagger} \overline{\sigma}_\mu q_{L\alpha}$$

$$\mathcal{O}_{4u/d} = q_R^\dagger \sigma^\mu q_R \; q_R^\dagger \sigma_\mu q_R \quad , \mathcal{O'}_{4u/d} = q_R^{\alpha\dagger} \sigma^\mu q_{R\beta} \; q_R^{\beta\dagger} \sigma_\mu q_{R\alpha}$$

$$\mathcal{O}_{2d2u} = d_R^\dagger \sigma^\mu d_R \; u_R^\dagger \sigma_\mu u_R \quad , \mathcal{O'}_{2d2u} = d_R^{\alpha\dagger} \sigma^\mu d_{R\beta} \; u_R^{\beta\dagger} \sigma_\mu u_{R\alpha}$$

3 Matching UV

3.1 W' B'

$$\mathcal{L}_{UV} \supset \lambda^l_W g_{W'} \overline{l_L}\gamma^\mu \sigma_a l_L W^a_\mu \ + \ \lambda^q_W g_{W'} \overline{q_L}\gamma^\mu \sigma_a q_L W^a_\mu \ +$$

$$\lambda^l_B g_{B'} \overline{l_L}\gamma^\mu l_L B_\mu + \lambda^q_B g_{B'} \overline{q_L}\gamma^\mu q_L B_\mu + \lambda^e_B g_{B'} \overline{e_R}\gamma^\mu e_R B_\mu + \lambda^u_B g_{B'} \overline{u_R}\gamma^\mu u_R B_\mu + \lambda^d_B g_{B'} \overline{d_R}\gamma^\mu d_R B_\mu +$$

$$\frac{M_B^2}{2} B_\mu B^\mu + \frac{M_W^2}{2} W^a_\mu W^\mu_a - \frac{1}{4}(W^a_{\mu\nu} W^{\mu\nu}_a + B_{\mu\nu} B^{\mu\nu})$$

$$\frac{\delta \mathcal{L}}{\delta B_\mu} = 0 = \frac{M_B^2}{2} B^\mu + \lambda^l_B g_{B'} \overline{l_L}\gamma^\mu l_L \ + \ \lambda^q_B g_{B'} \overline{q_L}\gamma^\mu q_L \ +$$

$$\lambda^e_B g_{B'} \overline{e_R}\gamma^\mu e_R \ + \ \lambda^u_B g_{B'} \overline{u_R}\gamma^\mu u_R \ + \ \lambda^d_B g_{B'} \overline{d_R}\gamma^\mu d_R \rightarrow$$

$$B^\mu = -\frac{2 g_{B'}}{M_{B'}^2} \big[\lambda^l_B \overline{l_L}\gamma^\mu l_L + \lambda^q_B \overline{q_L}\gamma^\mu q_L + \lambda^e_B \overline{e_R}\gamma^\mu e_R + \lambda^u_B \overline{u_R}\gamma^\mu u_R + \lambda^d_B \overline{d_R}\gamma^\mu d_R\big].$$

$$\frac{\delta \mathcal{L}}{\delta W^a_\mu} = 0 = \frac{M_W^2}{2} W^\mu_a + \lambda^l_W g_{W'} \overline{l_L}\gamma^\mu \sigma_a l_L \ + \ \lambda^q_W g_{W'} \overline{q_L}\gamma^\mu \sigma_a q_L \rightarrow$$

$$W^\mu_a = -\frac{2 g_{W'}}{M_{W'}^2} \big[\lambda^l_W \overline{l_L}\gamma^\mu \sigma_a l_L \ + \ \lambda^q_W \overline{q_L}\gamma^\mu \sigma_a q_L\big].$$

$$\mathcal{L}_{EFT} \supset -2G_{W'} \big[l_L^\dagger \lambda^l \overline{\sigma}^\mu \sigma_a l_L \ l_L^\dagger \lambda^l \overline{\sigma}_\mu \sigma^a l_L + q_L^\dagger \lambda^q \overline{\sigma}^\mu \sigma_a q_L \ q_L^\dagger \lambda^q \overline{\sigma}_\mu \sigma^a q_L + 2 q_L^\dagger \lambda^q \overline{\sigma}^\mu \sigma_a q_L \ l_L^\dagger \lambda^l \overline{\sigma}_\mu \sigma^a l_L\big]$$

$$-2G_{B'} \big[\big(\sum_\psi \overline{\psi} \lambda^\psi \sigma_\mu \psi\big) \ \big(\sum_\chi \overline{\chi} \lambda^\chi \sigma_\mu \chi\big)\big]$$

where

$$G_W \equiv \frac{g_{W'}^2}{M_{W'}^2} \quad , G_B \equiv \frac{g_{B'}^2}{M_{B'}^2}.$$

So we generate:

$$C_S \equiv 2G_{B'} \lambda^q_{B'} \lambda^l_{B'} \quad C_T \equiv 2G_{W'} \lambda^q_{W'} \lambda^l_{W'} \quad C_{LR1} \equiv 2G_{B'} \lambda^q_{B'} \lambda^e_{B'} \quad C^{u/d}_{LR2} \equiv 2G_{B'} \lambda^{u/d}_{B'} \lambda^l_{B'}$$

3.2 $U_1 \sim (3,1)_{2/3}$

$$\mathcal{L}_{UV} \supset -\frac{1}{2} U^\dagger_{1\mu\nu} U_1^{\mu\nu} \ + \ M^2_{U_1} U^\dagger_{1\mu} U_1^\mu \ + g_U U_{1\mu} \big[q_L^\dagger \beta^L_{U_1} \overline{\sigma}^\mu l_L \ + \ d_R^\dagger \beta^R_{U_1} \sigma^\mu e_R\big] \ + \ h.c.$$

$$\frac{\delta \mathcal{L}}{\delta U^\dagger_{1\mu}} = 0 = M^2_{U_1} U_1^\mu \ + \ g_{U_1} \big[l_L^\dagger \beta^{*L}_{U_1} \overline{\sigma}^\mu q_L \ + \ e_R^\dagger \beta^{*R}_{U_1} \sigma^\mu d_R\big] \rightarrow$$

$$U_1^\mu = -\frac{g_{U_1}}{M^2_{U_1}} \big[l_L^\dagger \beta^{*L}_{U_1} \overline{\sigma}^\mu q_L + e_R^\dagger \beta^{*R}_{U_1} \sigma^\mu d_R\big] \quad , U_1^{\mu\dagger} = -\frac{g_{U_1}}{M^2_{U_1}} \big[q_L^\dagger \beta^L_{U_1} \overline{\sigma}^\mu l_L + d_R^\dagger \beta^R_{U_1} \sigma^\mu e_R\big]$$

$$\mathcal{L}_{EFT} \supset -G_{U_1} \big[l_L^\dagger \beta^L_{U_1} \overline{\sigma}^\mu q_L \ q_L^\dagger \beta^{*L}_{U_1} \overline{\sigma}_\mu l_L + 2 l_L^\dagger \beta^L_{U_1} \overline{\sigma}^\mu q_L \ d_R^\dagger \beta^{*R}_{U_1} \sigma_\mu e_R + e_R^\dagger \beta^R_{U_1} \sigma^\mu d_R \ d_R^\dagger \beta^{*R}_{U_1} \sigma^\mu e_R + h.c\big]$$

Where, $G_{U_1} \equiv \frac{g_{U_1}^2}{M_{U_1}^2}$, now we easily recognize $\mathcal{O}_{LQ}^d$; also using that $\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta} = -\sigma^{\mu}_{\alpha\dot{\beta}}\sigma_{\mu\beta\dot{\alpha}}$.
Plus for what concern the $SU(2)_L$ structure we have to fix just the first term, since for the others the singlet is realized trivially.

$$\sigma^i_{ad}\sigma_{cbi} = 2\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} \to \delta_{ab}\delta_{cd} = \frac{\sigma^i_{ad}\sigma_{cbi} + \delta_{ad}\delta_{cb}}{2}$$

$$\mathcal{L}_{EFT} \supset G_{U_1}\,[\frac{\beta^L_{U_1}\beta^{*L}_{U_1}}{2}(\mathcal{O}_S + \mathcal{O}_T) + \beta^R_{U_1}\beta^{*R}_{U_1}\mathcal{O}^d_R - 4\beta^R_{U_1}\beta^{*L}_{U_1}\mathcal{O}_{S1} + h.c.]$$

Generating so:

$$C_S = C_T \equiv \frac{G_{U_1}}{2}\beta^{L*}_{U_1}\beta^L_{U_1} \quad C^d_R \equiv G_{U_1}\beta^{R*}_{U_1}\beta^R_{U_1} \quad C_{S2} \equiv -4G_{U_1}\beta^{R*}_{U_1}\beta^L_{U_1}$$

and nothing else!!!

3.3 $U_3 \sim (3,3)_{2/3}$

$$\mathcal{L}_{UV} \supset -\frac{1}{2}U^{\dagger a}_{3\mu\nu}U^{\mu\nu}_{3a} + M^2_{U_3}U^{\dagger\mu}_{3a}U^a_{3\mu} + g_{U_3}\left[U^a_{3\mu}q^\dagger_L\beta_{U_3}\overline{\sigma}^\mu\sigma_a l_L + h.c\right]$$

$$\frac{\delta\mathcal{L}}{\delta U^a_{3\mu}} = 0 = M^2_{U_3}U^{mu}_{3a} + g_{U_3}l^\dagger_L\beta^*_{U_3}\overline{\sigma}^\mu\sigma_a q_L \to$$

$$U^\mu_{3a} = -\frac{g_{U_3}}{M^2_{U_3}}l^\dagger_L\beta^*_{U_3}\overline{\sigma}^\mu\sigma_a q_L$$

$$\mathcal{L}_{EFT} \supset -G_{U_3}\left[q^\dagger_L\beta_{U_3}\overline{\sigma}^\mu\sigma_a l_L\ l^\dagger_L\beta^*_{U_3}\overline{\sigma}_\mu\sigma^a q_L\ \ + h.c\right]$$

Using the same identity I find a minus sign, but this time i have to make also the $SU(2)_L$ structure according with the basis defined. To do this I have to write tensors which combine the first and the fourth field (and second with third) starting from first with second and third with fourth, trough Fierz's identity i find:

$$\sigma^a_{ij}\sigma_{a\ kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\ kj}) \to$$

$$\mathcal{L}_{EFT} \supset \frac{G_{U_3}}{2}\beta_{U_3}\beta^*_{U_3}[3\mathcal{O}_S - \mathcal{O}_T]$$

which in term of Wilson coefficients:

$$C_S = -3C_T \equiv \frac{3G_{U_3}}{2M^2_{U_3}}\beta_{U_3}\beta^*_{U_3}$$

3.4 $S_1 \sim (3^*, 1)_{1/3}$

$$\mathcal{L}_{UV} \supset (D_\mu S_1)^\dagger D^\mu S_1 + M^2 S_1^\dagger S_1 +$$

$$+ g_{S_1} S_1^i \left[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + \overline{q_L^j} \lambda_{S_1} \varepsilon q_L^{c\,k} \varepsilon_{ijk} \right] + h.c.$$

where ε act on $SU(2)_L$ and ε_{abc} on $SU(3)_c$.

$$\frac{\delta \mathcal{L}_{UV}}{\delta S_1^{i\dagger}} = 0 = M_{S_1}^2 S_{1a} + g_{S_1} \left[l_L^\dagger \varepsilon \beta_{S_1}^{L*} q_{Li}^c + + e_R^\dagger \beta_{S_1}^{R*} u_{Ri}^c + q_L^{\dagger c\, j} \lambda_{S_1}^* \varepsilon q_L^k \varepsilon_{ijk} \right] \rightarrow$$

$$S_{1i} = -\frac{g_{S_1}}{M_{S_1}^2} \left[\overline{l_L} \beta_{S_1}^* \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R*} u_{Ri}^c + \overline{q^c}_L^j \lambda_{S_1}^* \varepsilon q_L^k \varepsilon_{ijk} \right]$$

$$\mathcal{L}_{EFT} \supset -G_{S_1} \left[\overline{l_L} \beta_{S_1}^* \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R*} \varepsilon u_{Ri}^c + \overline{q^c}_L^j \lambda_{S_1}^* \varepsilon q_L^k \varepsilon_{ijk} \right] \left[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + \overline{q_L^j} \lambda_{S_1} \varepsilon q_L^{c\,k} \varepsilon_{ijk} \right]$$

$$\mathcal{L}_{EFT} \supset -G_{S_1} \left[l_L^\dagger \beta_{S_1}^{L*} \varepsilon q_{Li}^c \; q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + e_R^\dagger \beta_{S_1}^{R*} u_{Ri}^c \; q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + q_L^{c\dagger j} \lambda_{S_1}^* \varepsilon q_L^k \varepsilon_{ijk} \; \overline{q_L^l} \lambda_{S_1} \varepsilon q_L^{c\,m} \varepsilon^{ilm} + \right.$$
$$\left. l_L^\dagger \beta_{S_1}^{L*} \varepsilon q_{Li}^c \; u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + e_R^\dagger \beta_{S_1}^{R*} u_{Ri}^c \; u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + (BNV\ terms) \right]$$

Then, (see appendix)

$$\overline{a^c} \varepsilon b \; \overline{c} \varepsilon d^c = \frac{1}{4} \left[a^\dagger \overline{\sigma}^\mu b \; c^\dagger \overline{\sigma}_\mu d \; - \; a^\dagger \overline{\sigma}^\mu \sigma_a b \; c^\dagger \overline{\sigma}_\mu \sigma^a d \right]$$

$$\varepsilon_{ij} \varepsilon_{kl} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}$$

$$\varepsilon_{ij} \varepsilon_{kl} = -\frac{1}{2} \sigma_{il}^\mu \sigma_{\mu jk}$$

$$\mathcal{L}_{EFT} \supset -G_{S_1} \left[\frac{1}{4} \beta_{S_1}^L \beta_{S_1}^{L*} \mathcal{O_S} - \frac{1}{4} \beta_{S_1}^L \beta_{S_1}^{L*} \mathcal{O}_T + 2 \beta_{S_1}^{R*} \beta_{S_1}^L \mathcal{O}_{LQ} - 2 \beta_{S_1}^{R*} \beta_{S_1}^L \mathcal{O}_S^{u\dagger} - \frac{1}{2} \beta_{S_1}^{R*} \beta_{S_1}^R \mathcal{O_R} \right.$$
$$\left. + \overline{q^c}_L^j \lambda_{S_1}^* \varepsilon q_L^k \varepsilon_{ijk} \; \overline{q_{Ll}} \lambda_{S_1} \varepsilon q_{Lm}^c \varepsilon^{ilm} + (BNV terms) \right] + h.c.$$

Terms that give problems to $\Delta F = 2$ and proton decay can be swtiched off setting the flavour matrix $\lambda_{S_1} = 0$. Last term can be written in term of known $\mathcal{O}_{1-2}$ using $\varepsilon_{ijk} \varepsilon^{ilm} = \delta_j^l \delta_k^m - \delta_j^m \delta_k^l$. In terms of Wilson's semileptonic coefficients:

$$C_S = -C_T \equiv -\frac{G_{S_1}}{4} \beta_{S_1}^* \beta_{S_1} \quad , C_{LQ} = -C_S^{u*} = 2 G_{S_1} \beta_{S_1}^{R*} \beta_{S1}^L \quad , C_R = G_{S1} = -\frac{1}{2} G_{S_1} \beta_{S_1}^{R*} \beta_{S1}^R$$

3.5 $S_3 \sim (3^*, 3)_{1/3}$

$$\mathcal{L}_{UV} \supset (D_\mu S_{3a})^\dagger D^\mu S_3^a + M^2 S_{3a}^\dagger S_3^a +$$
$$+ g_{S_3} S_3^{ia} (\overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q_L^{c\,k} \varepsilon_{ijk} + \overline{q^c_L}_i \beta_{S_3} \varepsilon \sigma_a l_L) + h.c.$$
$$\frac{\delta \mathcal{L}_{UV}}{\delta S_3^{ia\,\dagger}} = 0 = M_{S_3}^2 S_{3ia} + g_{S_3} \big[\overline{q^c_L}_j \lambda^*_{S_3} \varepsilon \sigma_a q_{Lk} \varepsilon^{ijk} + \overline{l_L} \beta^*_{S_3} \varepsilon \sigma_a q_L^{c\,i}\big] \to$$
$$S_{3ia} = -\frac{g_{S_3}}{M_{S_3}^2} \big[\overline{q^c_L}_j \lambda^*_{S_3} \varepsilon \sigma_a q_{Lk} \varepsilon^{ijk} + \overline{l_L} \beta^*_{S_3} \varepsilon \sigma_a q_L^{c\,i}\big]$$

$$\mathcal{L}_{EFT} \supset -G_{S_3} \big[\overline{q^c_L}_i \beta_{S_3} \varepsilon \sigma_a l_L \; \overline{l_L} \beta^*_{S_3} \varepsilon \sigma_a q_L^{c\,i} \; + \; \overline{q^c_L}_i \beta_{S_3} \varepsilon \sigma_a l_L \; \overline{q^c_L}_j \lambda^*_{S_3} \varepsilon \sigma_a q_{Lk} \varepsilon^{ijk} \; +$$
$$\overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q_L^{c\,k} \varepsilon_{ijk} \; \overline{l_L} \beta^*_{S_3} \varepsilon \sigma_a q_L^{c\,i} + \; \overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q_L^{c\,k} \varepsilon_{ijk} \overline{q^c_L}_l \lambda^*_{S_3} \varepsilon \sigma_a q_{Lm} \varepsilon^{ilm}\big]$$

And then, (see appendix)

$$\overline{a^c} \varepsilon \sigma_a b \; \bar{c} \sigma^a \varepsilon d^c = \frac{1}{4} \big[3 a^\dagger \bar{\sigma}^\mu b \; c^\dagger \bar{\sigma}_\mu d \; + \; a^\dagger \bar{\sigma}^\mu \sigma_a b \; c^\dagger \bar{\sigma}_\mu \sigma^a d\big] \to$$

$$\mathcal{L}_{EFT} \supset -G_{S_3} \big[\frac{3}{4} \beta^*_{S_3} \beta_{S_3} \mathcal{O}_S + \frac{1}{4} \beta^*_{S_3} \beta_{S_3} \mathcal{O}_T + \; \overline{q^c_L}_i \beta_{S_3} \varepsilon \sigma_a l_L \; \overline{q^c_L}_j \lambda^*_{S_3} \varepsilon \sigma_a q_{Lk} \varepsilon^{ijk} \; +$$
$$\overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q_L^{c\,k} \varepsilon_{ijk} \; \overline{l_L} \beta^*_{S_3} \varepsilon \sigma_a q_L^{c\,i} + \; \overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q_L^{c\,k} \varepsilon_{ijk} \overline{q^c_L}_l \lambda^*_{S_3} \varepsilon \sigma_a q_{Lm} \varepsilon^{ilm}\big]$$

Terms that give problems to $\Delta F = 2$ and proton decay can be swtiched off setting the flavour matrix $\lambda_{S_3} = 0$. Last term can be written in term of known $\mathcal{O}_{1-2}$ using $\varepsilon_{ijk} \varepsilon^{ilm} = \delta^l_j \delta^m_k - \delta^m_j \delta^l_k$. And in terms of Wilson's coefficients

$$C_S = 3 C_T \equiv \frac{3}{4} G_{S_3} \beta^*_{S_3} \beta_{S_3}$$

3.6 $R_2 \sim (3, 2)_{7/6}$

$$\mathcal{L}_{UV} \supset (D_\mu R_2)^\dagger D^\mu R_2 + M_{R_2}^2 R_2^\dagger R_2 +$$
$$g_{R_2} R_2 \big[u_R^\dagger \beta^l_{R_2} l_L + q_L^\dagger \beta^q_{R_2} e_R\big]$$
$$\frac{\delta \mathcal{L}_{UV}}{\delta R_2^\dagger} = 0 = M_{R_2}^2 R_2 + g_{R_2} \big[l_L^\dagger \beta^{l*}_{R_2} u_R + e_R^\dagger \beta^{q*}_{R_2} q_L\big] \to$$
$$R_2 = -\frac{g_{R_2}}{M_{R_2}^2} \big[l_L^\dagger \beta^{l*}_{R_2} u_R + e_R^\dagger \beta^{q*}_{R_2} q_L\big]$$

$$\mathcal{L}_{EFT} \supset -G_{R_2} \big[u_R^\dagger \beta^l_{R_2} l_L \; l_L^\dagger \beta^{l*}_{R_2} u_R + u_R^\dagger \beta^l_{R_2} l_L \; e_R^\dagger \beta^{q*}_{R_2} q_L + q_L^\dagger \beta^{q*}_{R_2} e_R \; l_L^\dagger \beta^{l*}_{R_2} u_R + q_L^\dagger \beta^q_{R_2} e_R \; e_R^\dagger \beta^{q*}_{R_2} q_L\big] + h.c$$

Now, since

$$\delta^b_a \delta^d_c = \frac{1}{2} \sigma^\mu_{ac} \overline{\sigma_\mu}^{bc}$$

$$\mathcal{L}_{EFT} \supset -\frac{G_{R_2}}{2} \big[\beta^l_{R_2} \beta^{l*}_{R_2} \mathcal{O}^u_{LR2} + \beta^l_{R_2} \beta^{q*}_{R_2} \mathcal{O}^u_{LQ} + \beta^q_{R_2} \beta^{l*}_{R_2} \mathcal{O}^{u\dagger}_{LQ} + \beta^q_{R_2} \beta^{q*}_{R_2} \mathcal{O}^u_{LR1}\big] + h.c.$$

4 Processes

$$\mathcal{O}_S = q_L^\dagger \overline{\sigma}^\mu q_L \; l_L^\dagger \overline{\sigma}_\mu l_L \quad , \mathcal{O}_T = q_L^\dagger \overline{\sigma}^\mu \sigma_a q_L \; l_L^\dagger \overline{\sigma}_\mu \sigma^a l_L$$

$$\mathcal{O}_R^{u/d} = q_R^\dagger \sigma^\mu q_R \; e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LQ}^{u/d} = q_L^\dagger \overline{\sigma}^\mu l_L \; e_R^\dagger \sigma_\mu q_R$$

$$\mathcal{O}_{LR1} = q_L^\dagger \overline{\sigma}^\mu q_L \; e_R^\dagger \sigma_\mu e_R \quad , \mathcal{O}_{LR2}^{u/d} = q_R^\dagger \sigma^\mu q_R \; l_L^\dagger \overline{\sigma}_\mu l_L$$

$$\mathcal{L}_{EFT} \supset C_S \mathcal{O}_S + C_T \mathcal{O}_T + \sum_{q=u,d} \left[C_R^q \mathcal{O}_R^q + C_{LQ}^q \mathcal{O}_{LQ}^q + C_{LR1}^q \mathcal{O}_{LR1}^q + C_{LR2}^q \mathcal{O}_{LR2}^q \right]$$

Where $C_i O_i \equiv C_i^{q_1 q_2 l_1 l_2} O_{i q_1 q_2 l_1 l_2}$ and

$$C_i^{q_1 q_2 l_1 l_2} \equiv c_i \Lambda_i^{q_1 q_2 l_1 l_2}$$

in which $q_{1-2} = d, s, b$ and $l_{1-2} = e, \mu, \tau$.
We will see in the different cases the SM contribute for those coefficients.

Before to compute some semileptonic observable we just write down somw guidelines:

- Processes corcening neutrinos get contribute from $\mathcal{O}_{LR1}, \mathcal{O}_R$.
- The non flavour diagonal contributes are suppresed in SM for what concern O_T and more suppressed for $\mathcal{O}_S, \mathcal{O}_{LR1-2}$
- In SM $C_R = C_{LQ} \simeq 0$
- NP in which we are interested in doesn't concern lightest families of quark and lepton.

4.1 R_{K^*}

$$R_{K^*} \equiv \frac{\mathcal{B}(B \to K^* \mu\mu)}{\mathcal{B}(B \to K^* ee)}$$

We know that in SM, because of LFU, $R_{K^*} = 1$. Both those processes are FCNC and the intensity is given by $C_{SM} \equiv V_{ts}^* V_{tb} \frac{e^2}{4\pi v^2} \mathcal{F}_{loop}(y_t)$.

Keeping all the contributes:

$$R_{K^*} = |1 + \frac{1}{C_{SM}} \sum_{i=all} C_i^{bs\mu\mu}|^2$$

4.2 R_{D^*}

$$R_{D^*} \equiv \frac{\mathcal{B}(B \to D^* \tau \overline{\nu})}{\frac{1}{2}\left[\mathcal{B}(B \to D^* e\overline{\nu}) + \mathcal{B}(B \to D^* \mu\overline{\nu})\right]}$$

now, since

$$|\mathcal{A}(b \to c\tau\overline{\nu})|^2 = \sum_{l=\mu,\tau}\left[|\mathcal{A}(b_L \to \tau_L c_L \overline{\nu}_{l\,L})|^2 + |\mathcal{A}(b_L \to \tau_R c_R \overline{\nu}_{l\,L})|^2 + \right.$$

$$\left. |\mathcal{A}(b_R \to \tau_R c_L \overline{\nu}_{l\,L})|^2 + |\mathcal{A}(b_R \to \tau_L c_R \overline{\nu}_{l\,L})|^2\right] =$$

$$|C_{SM} + 2V_{ts}^* C_T^{bs\tau\tau}|^2 + |2V_{ts}^* C_T^{bs\mu\tau}|^2 + \sum_{l=\mu,\tau}\left[|C_{S1}^{bsl\tau}|^2 + |C_{S2}^{bsl\tau}|^2 + |C_{T1}^{bsl\tau}|^2 + |C_{T2}^{bsl\tau}|^2\right]$$

The muon channel is analogous:

$$|\mathcal{A}(b \to c\mu\overline{\nu})|^2 = |C_{SM} + 2V_{ts}^* C_T^{bs\mu\mu}|^2 + |2V_{ts}^* C_T^{bs\tau\mu}|^2 + \sum_{l=\mu,\tau}\left[|C_{Scal}^{bsl\mu}|^2 + |C_{LQ}^{bsl\mu}|^2\right]$$

Meanwhile

$$|\mathcal{A}(b \to ce\overline{\nu})|^2 = |C_{SM}|^2$$

Now since $\langle B|\, b_L^\dagger c_R \,|D^*\rangle = \langle B|\, b_R^\dagger c_L \,|D^*\rangle = 0$

$$R_{D^*} = \frac{|C_{SM} + 2V_{ts}^* C_T^{bs\tau\tau}|^2 + |2V_{ts}^* C_T^{bs\mu\tau}|^2}{\frac{1}{2}\left[|C_{SM}|^2 + |C_{SM} + 2V_{ts}^* C_T^{bs\mu\mu}|^2 + |2V_{ts}^* C_T^{bs\tau\mu}|^2\right]}$$

With $C_{SM} = \frac{2V_{cb}}{v^2}$. Remember that R_{D^*} tests LFU but it's not equal to 1, in fact $R_{D^*SM} = \Phi_\tau$ if we set $\Phi_e = \Phi_\mu = 1$.

$$R_D =$$

4.3 $R_{B\to K\nu\nu}$

$$R_{B\to K\nu\nu} \equiv \frac{\mathcal{B}(B \to K\nu\nu)}{\mathcal{B}(B \to K\nu\nu)_{SM}}$$

Again the $C_{SM} \equiv V_{ts}^* V_{tb} \frac{1}{cos^2\theta_W \;\, v^2} \mathcal{F}_{loop}(y_t)$.
Meanwhile $C_{\nu\nu}^{bsl_1l_2} \equiv C_S^{bsl_1l_2} - C_T^{bsl_1l_2} + {C_{LR2}^{d}}^{bsl_1l_2}$

$$R_{B\to K\nu\nu} = |1 + \frac{\sum_{l_1,l_2=\mu,\tau} C_{\nu\nu}^{bsl_1l_2}}{C_{SM}}|^2$$

4.4 $\mathcal{B}(B \to K\mu\tau)$

4.5 $\mathcal{B}(\tau \to \mu\phi)$

5 Appendix

5.1 Fierz identities and useful Lorentz group relations

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\sigma_{\mu}}^{\beta\dot{\beta}} = 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}}$$
$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}$$
$$\overline{\sigma}^{\alpha\dot{\alpha}}_{\mu}\overline{\sigma}^{\mu\beta\dot{\beta}} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}$$
$$\varepsilon_{\alpha\beta}\varepsilon^{\gamma\delta} = \delta^{\gamma}_{\alpha}\delta^{\delta}_{\beta} - \delta^{\gamma}_{\beta}\delta^{\delta}_{\alpha}$$

5.2 Fierzing in scalar lagrangian

Doppio problema con la coniugazione complessa: 1- ignorata nei fermioni , 2 ignorata su una delle $\overline{\sigma}_{\mu}$, σ_a! *non è che in qualche modo si compensa?*
Here the case in which we need to switch the position of the spinors from 1234 to 1432. indicating with i, j, k, l the Lorentz indeces and with a, b, c, d the EW indeces:

$$\overline{a^c}\varepsilon b\ \overline{c}\varepsilon d^c = a_{ia}b_{jb}c^*_{kc}d^*_{ld}\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{ij}\varepsilon^{kl}$$

where the Lorent's ε comes from the charge's coniugation.
Now for what concern the Lorentz's structure

$$\varepsilon^{ij}\varepsilon^{kl} = -\varepsilon^{ji}\varepsilon^{kl} = -\frac{1}{2}\overline{\sigma}^{\mu}_{kj}\overline{\sigma}_{\mu li}$$

Instead for what concern EW group in the singlet case, since

$$\sigma^{ab}_{a}\sigma^{a\,cd} = 2\delta^{ad}\delta^{bc} - \delta^{ab}\delta^{cd} \to$$
$$\varepsilon^{ab}\varepsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} = \frac{1}{2}\sigma^{ad}_{a}\sigma^{bc^{a}} - \frac{1}{2}\delta^{ad}\delta^{bc}$$

So

$$\overline{a^c}\varepsilon b\ \overline{c}\varepsilon d^c = \frac{1}{4}\left[d^{\dagger}\overline{\sigma}^{\mu}a\ c^{\dagger}\overline{\sigma}_{\mu}b\ -\ d^{\dagger}\overline{\sigma}^{\mu}\sigma_a a\ c^{\dagger}\overline{\sigma}_{\mu}\sigma^a b\right]$$

and in the triplet case we obtain

$$\overline{a^c}\varepsilon\sigma_a b\ \overline{c}\sigma^a\varepsilon d^c = a_{ia}b_{jb}c^*_{kc}d^*_{ld}\ \varepsilon^{ij}\varepsilon kl\ \varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_{a}\sigma^{a}$$
$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_{a}\sigma^{cea} = (\delta^{af}\delta^{ed} - \delta^{ad}\delta^{fe})(2\delta^{e}\delta^{kj} - \delta^{mj}\delta^{kn}) =$$
$$2\delta^{ad}\delta^{bc} - 4\delta^{ad}\delta^{bc} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} = -(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd})$$

then

$$\sigma^{a}_{ad}\sigma_{bca} = 2\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \to$$
$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_{a}\sigma^{cea} = -\frac{1}{2}(\sigma^{ad}_{a}\sigma^{bc^{a}} + 3\delta^{ad}\delta^{bc})$$

so

$$\overline{a^c}\varepsilon\sigma_a b\ \overline{c}\sigma^a\varepsilon d^c = \frac{1}{4}\left[3d^{\dagger}\overline{\sigma}^{\mu}a\ c^{\dagger}\overline{\sigma}_{\mu}b\ +\ d^{\dagger}\overline{\sigma}^{\mu}\sigma_a a\ c^{\dagger}\overline{\sigma}_{\mu}\sigma^a b\right]$$

5.3 From EW eigenstates to electric charge eigenstates

$$\sigma_\pm = \frac{\sigma_1 \pm i\sigma_2}{2} \to \sigma_1 = \sigma_+ + \sigma_- \quad , \sigma_2 = i(\sigma_- - \sigma_+)$$

$$\sigma_{a\,ij}\sigma^a{}_{kl} = \sigma_{1\,ij}\sigma_{1\,kl} + \sigma_{2\,ij}\sigma_{2\,kl} + \sigma_{3\,ij}\sigma_{3\,kl} =$$

$$(\sigma_{+\,ij} + \sigma_{-\,ij})(\sigma_{+\,kl} + \sigma_{-\,kl}) - (\sigma_{-\,ij} - \sigma_{+\,ij})(\sigma_{-\,kl} - \sigma_{+\,kl}) + \sigma_{3\,ij}\sigma_{3\,kl} =$$

$$2\sigma_{+\,ij}\sigma_{-\,kl} + 2\sigma_{+\,ij}\sigma_{-\,kl} + \sigma_{3\,ij}\sigma_{3\,kl}$$

So, in term of charge's eigenstates:

$$\mathcal{O}_T = q_L^\dagger \overline{\sigma}^\mu \sigma_a q_L \; l_L^\dagger \overline{\sigma}_\mu \sigma^a l_L = 2(V^* u_L)^\dagger \overline{\sigma}^\mu d_L \; l_L^\dagger \overline{\sigma}_\mu \nu_L \; + \; 2 d_L^\dagger \overline{\sigma}^\mu (V^* u_L) \; \nu_L^\dagger \overline{\sigma}_\mu l_L \; +$$

$$(V^* u_L)^\dagger \overline{\sigma}^\mu (V^* u_L) \; \nu_L^\dagger \overline{\sigma}_\mu \nu_L + d_L^\dagger \overline{\sigma}^\mu d_L \; l_L^\dagger \overline{\sigma}_\mu l_L - d_L^\dagger \overline{\sigma}^\mu d_L \; \nu_L^\dagger \overline{\sigma}_\mu \nu_L - (V^* u_L)^\dagger \overline{\sigma}^\mu (V^* u)_L \; l_L^\dagger \overline{\sigma}_\mu l_L$$