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A general overview on Lepton Flavour Universality violation in B mesons physics

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Introduction

In this work we try to explore the huge world of Beyond Standard Model (BSM) theories using as gate some recent experimental observations indicating *Lepton Flavour Universality* (LFU) violation.

In the Standard Model (SM) every fermion field emerges as a triplet of an approximate symmetry called *Flavour Symmetry*, broken only by the Higgs couplings to fermions that acts with different intensities to the different components of these triplets named *families* or *generations*.
Since the different coupling to the Higgs boson means also a different mass between the different families, there is always a kinematic flavour sensitive effect enclosed in the phase space factor which affects the rate or the cross section of a process; for instance the decay of charged π produces mostly μ for this reasons, even if the coupling is the same for e and τ. Nevertheless, in the SM, the only dynamical effect which is not mass blind is the coupling of the fermions to the Higgs boson that is proportional to the mass of the corresponding fermion. Since leptons are very light (the τ, which is the heaviest, weighs less than 100 times the Higgs vacuum expectation value) we can normally neglect the Higgs coupling compared to gauge interactions and say that approximately SM exhibit a flavour universality in the interaction with leptons, while interaction of quark are sensitively different for different families mostly because of the non negligible coupling of the t quark with Higgs boson. Hence the flavour symmetry is explicitly broken by the Higgs coupling but the universality of the leptons interactions, known as LFU, is a well approximated accidental symmetry of the SM.

If we have accepted that LFU is a good symmetry we can also understand that if we observe consistently big effects of a coupling to leptons that depends on the generation, that effect has to emerge in a BSM framework.

Recently in different experiments were found several hints of LFU violation in semileptonic decays of the B mesons (mesons that contain a b quark). All the deviations from the SM appearing in these decays go under the name of *B-Physics Anomalies*.
The purpose of this work is, neglecting the theoretical details of every possible scenario that can provide the violation of LFU we observe, to see what are the fundamental features of the New Physics (NP) needed to accommodate these phenomena.

Since, as we will discuss, the B-Physics Anomalies are observed in processes at the hadronic scale (order few GeV), we will implement a model independent approach based on the Effective Field Theory (EFT), as usually done with the Fermi Theory to study low energy weak interactions. In this framework we parameterize the NP effects as deviations from the Lagrangian's coefficients of the operators, named Wilson coefficients, obtained from the matching to SM.

After having defined the EFT, the next step is to find the right heavy mediators which, once integrated out from the Lagrangian (as we do with W and Z bosons to get the Fermi Theory), give us the appropriate contributions to accommodate the B-Physics Anomalies.

When we generate the contributions needed to accommodate the semileptonic observables showing deviations from the SM, we need to pay attention to the effect of these contributions to the observables that do not show any anomaly.
In this sense a pleasant option is to include in the UV Lagrangian coloured heavy bosons named *Leptoquarks* that transform under the fundamental (or antifundamental) representation of $SU(3)_c$. In fact, as we will see, this option considerably limits the generation of non semileptonic contributions, avoiding possible tensions with the processes that involve only quarks or only leptons.

B-Physics Anomalies and heavy bosons are widely discussed in literature. The main purpose of this work is to offer a catalogue of solutions as general and complete as possible obtained adding a NP mediator or combinations of them to the SM spectrum, according to the low energies contributions needed to fit the experimental data.

In this thesis we will not go beyond simplified models that extend the SM by adding (scalar or vector) bosons.
More complete UV models, where particles and processes are accommodated - with few assumptions and input values - in terms of some underlying symmetry are nevertheless the first goal. We will mention some of the most interesting UV completions available in the literature.

1

B-Physics Anomalies

B-physics Anomalies one of the most important and coherent deviation from SM seen in last years, with a constantly growing set of data coming from different processes that seems to confirm them.
These anomalies are given by some semileptonic decays of B mesons that seem to treat differently different flavours of leptons, i.e. suggesting LFU violation in processes that involve b quark.

The clues of LFU violation in b-sector come from different and independent observables, nevertheless before to present them is possible to summarize some general characteristics:

- The fundamental transitions affected are always $b \to sl^+l^-$ and $b \to cl\nu$ (or crossing transformated) to which we will refer respectively as *Neutral Current* anomalies (NC) and *Charged Current* anomalies (CC). Hence the first family of quark is not involved.

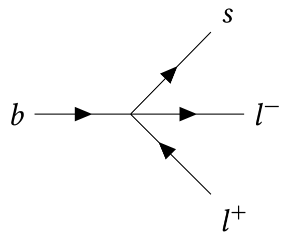
- In NC anomalies we see an excess of electrons in some decays that can be given by either a NP contribution of electron channel or a contribution for muons that interferes destructively with the SM.

- In CC anomalies NP seems to be universal for first two families but to couple more with τ, giving a preference for $b \to c\tau\nu$ compared to other lepton flavours.

In the anomaly concerning the LFU violation between the first two family of leptons we will assume that NP couples only with muons interfering destructively with SM contributions.

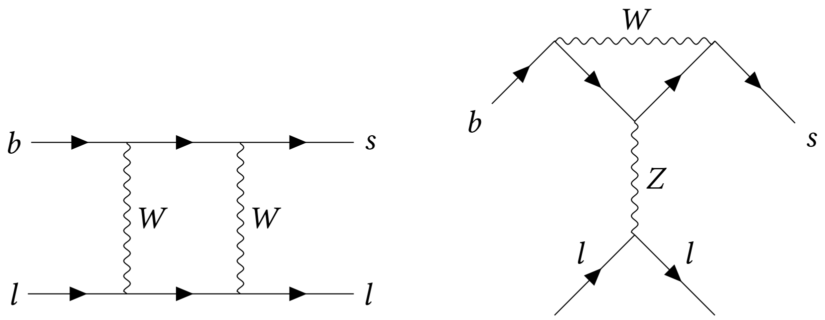
1.1 Main Processes for NC transitions

The transition $b \to sl^+l^-$ in which the leading contribution in Fermi theory is represented by the vertex



is, as every FCNC (see section 2.2), one-loop induced in the SM. The leading contributions are represented in the diagrams in Figure 1.1 in which all the couplings to the leptons are flavour universal.

Figure 1.1: Standard Model leading contributions to the transition $b \to sl^+l^-$



The first process we consider is the decay $B \to K^{(*)}l^+l^-$ where K, K^* are respectively the pseudoscalar and the vector state of the kaon and $l^{\pm}$ are charged leptons belonging to the same family.
In the SM that decay gets only lepton flavour universal contribution, in fact when the leptons pair takes an enough big part of the energy available such that the phase space difference between a electron pair and a muon pair is negligible, the decay widths corresponding to $B \to K^{(*)}\mu^+\mu^-$ and $B \to K^{(*)}e^+e^-$ are predicted to be equal.
The anomalous observables we consider are the ratios R_K which are defined as:

$$R_{K^+} \equiv \frac{\mathcal{B}(B^+ \to K^+\mu^-\mu^+)}{\mathcal{B}(B^+ \to K^+e^-e^+)} \tag{1.1}$$

$$R_{K_S} \equiv \frac{\mathcal{B}(B^0 \to K_S\mu^-\mu^+)}{\mathcal{B}(B^0 \to K_Se^-e^+)} \tag{1.2}$$

$$R_{K^*} \equiv \frac{\mathcal{B}(B^0 \to K^*\mu^-\mu^+)}{\mathcal{B}(B^0 \to K^*e^-e^+)} \tag{1.3}$$

$$R_{K^{*+}} \equiv \frac{\mathcal{B}(B^+ \to K^{*+}\mu^-\mu^+)}{\mathcal{B}(B^+ \to K^{*+}e^-e^+)} \tag{1.4}$$

where the K^*, K^{*+} are vector mesons and K^+, K_S are pseudoscalars and the S stays for the short-living neutral kaon which is a mixture of K_0 and $\overline{K}_0$ because of the CP Violation.

Table 1.1: Some anomalous observables characterized by $b \to sl^+l^-$ transition. $R_H^{[q_1^2,q_2^2]}$ means the ratio R_H in which the momenta of the pair lepton-antilepton has energy at rest q^2 included beetween q_1^2 and q_2^2.

Observable	*Experiment*
$R_{K^+}^{[1.1,6.0]}$	$0.846^{+0.042+0.013}_{-0.039-0.012}$ *[1]*
$R_{K_S^0}^{[1.1,6.0]}$	$0.846^{+0.020+0.013}_{-0.039-0.012}$ *[1]*
$R_{K^*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07} \pm 0.03$ *[2]*
$R_{K^*}^{[1.1,6.0]}$	$0.69^{+0.11}_{-0.07} \pm 0.05$ *[2]*
$R_{K^{*+}}^{[0.045,6.0]}$	$0.70^{+0.18+0.03}_{-0.13-0.04}$ *[1]*
$R_{pK}^{[0.1,6.0]}$	$0.86^{+0.14}_{-0.11} \pm 0.05$*[3]*

Analogous LFU ratios can be defined and measured for baryon decays:

$$R_{pK} \equiv \frac{\mathcal{B}(\Lambda_b \to pK^-\mu^-\mu^+)}{\mathcal{B}(\Lambda_b \to pK^-J/\psi(\to \mu^-\mu^+)}\Big/\frac{\mathcal{B}(\Lambda_b \to pK^-\mu^-\mu^+)}{\mathcal{B}(\Lambda_b \to pK^-J/\psi(\to \mu^-\mu^+)} \tag{1.5}$$

The reasons why we use ratios of Branching Ratios are mainly three:

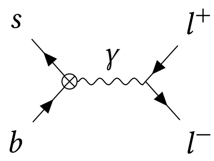
- To remove the dependence from hadronic form factors
- To remove the dependence from CKM matrix elements
- To reduce the systematic error in general

In Table 1.1 we report the experimental data from the LHCb collaboration [1–3] that seem to point to a LFU violation.
In the notation used $R_{K^*}^{[q_1^2,q_2^2]}$ is:

$$R_H^{[q_1^2,q_2^2]} = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to H\mu^+\mu^-)}{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to He^+e^-)}$$

where H is the corresponding hadronic state and $q^2 = (p_{l^+} + p_{l^-})^2$ is the rest frame energy of the leptons pair.

According to the momentum of the lepton pair the SM prediction is different. In fact, when the energy of leptons is high enough to neglect the phase space different between μ and e, the ratios are equal to 1 in SM because of LFU.
When we include in the measure events in which the energy of leptons q^2 is comparable with m_μ^2 we would need to include the phase space correction.
Plus, the contribution of the electromagnetic term - which is as well one-loop induced in the SM- given by the diagram

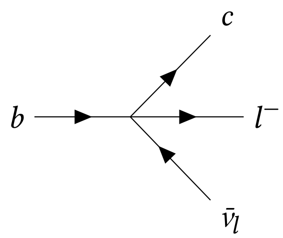


represents a flavour universal contribution proportional to $\frac{1}{q^2}$ because the photon propagator, so it becomes also important for low energies of the leptons pair.

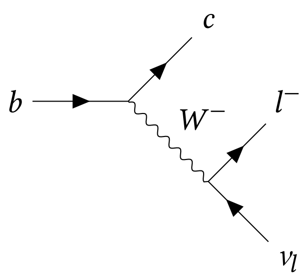
When the range of q^2 is $[0.045, 1.1]\ GeV^2$ the photon contribution is big and we would need include it in the expressions of the observables together with the phase space factors ratio.
On the other hand when the range is wide enough (for instance $[0.045, 6.0]\ GeV^2$) we neglect that contributions that would give only a small contribution to the whole integration on that range because of the relative smallness of the low energy bin.
Hence, in that case we will neglect both the photon contribution and the phase space effect.

1.2 Main Processes for CC transitions

The CC transition $b \to cl\bar{\nu}$ gets its tree level contribution in Fermi Theory from



which in this case is tree level generated in the UV theory through an exchange of W boson



and that means that whatever NP could be a candidate to accomodate the anomaly, it has to affect the transition $b \to cl\bar{\nu}$ with a bigger magnitude respect to the magnitude of NP contribution deed to accommodate NC anomalies.

Table 1.2: Table with the some anomalous ratios anomalies in $b \to cl\nu$ transition. ρ stays for the correlation between the ratio in D and D^*.

Observable	*Experiment*	*SM*
$\{R_D, R_{D^*}\}$	$\{0.337(30), 0.298(14)\}$	$\{0.299(3), 0.258(5)\}$
ρ	-0.42	$-$
$\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)$	$1.09(24) \cdot 10^{-4}$	$0.812(54) \cdot 10^{-4}$

The ratios we will consider this time are

$$R_{D^*} \equiv \frac{\mathcal{B}(B \to D^* \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathcal{B}(B \to D^* l \nu_l)} \tag{1.6}$$

$$R_D \equiv \frac{\mathcal{B}(B \to D \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathcal{B}(B \to D l \nu_l)} \tag{1.7}$$

In Table 1.2 we see the ratios measured by [4] compared to SM predictions provided by [4].
As we see the ratios are not predicted to be 1 in SM as in the previous case and this is due to the phase space suppression of the τ channel, the mass of which is not negligible compared to the meson masses involved. In other words at this energy scale the different mass of τ compared to μ and e brings a non negligible kinematic factor that makes the prediction for the ratio to be

$$R_D^{SM} = \frac{\Phi_\tau^D}{\frac{1}{2}[\Phi_e^D + \Phi_\mu^D]} \equiv \eta_\tau^D \tag{1.8}$$

that will be used as phenomenological parameter when we will compare anomalies with a BSM scenario. Analogous is the definition of $\eta_\tau^{D^*}$ that is smaller because of the bigger mass of D^* compared to D.

Plus it seems that, in CC processes, NP does not affect first two families. In fact as reported by [5]

$$R_{D^*}^{\mu e} = \left.\frac{\mathcal{B}(B \to D^* \mu \nu)}{\mathcal{B}(B \to D^* e \nu)}\right|_{exp} = 1.00 \pm 0.02 \tag{1.9}$$

that this time is equal to 1 even in SM prediction because $\frac{\Phi_\mu}{\Phi_e} \simeq 1$ both for decay in D and in D^*.

Since both the fundamental transitions of interest happen to be flavour breaking, we will describe the main features of the flavour phenomena of the SM to see how they affect four fermion operators of the Fermi theory and then we will see how possible NP effects affect the anomalous observables.

2

Flavour Physics

When we look for clues of New Physics one possible way is to look for processes that are not allowed in the known model either to measure NP effects or to test the available theory.
For similar reasons a good places to look for NP clues are those processes that are predicted to be rare by the SM, to bring either hints for deviations or an improvement on the measure of the SM parameters.

An environment in which lots of rare processes are treated is the *flavour sector* of the SM. The Flavour Physics is focused on the fact that the fermion fields of the SM

$$l_L,\ q_L,\ e_R,\ d_R,\ u_R$$

appear in three copies each, named *families* (or *generations*). These copies differ only in their mass, and hence in their coupling to the Higgs boson. Turning off the Higgs-fermions couplings there would be a big global accidental symmetry that would not allow us to distinguish the different generations, named *Flavour symmetry.*
It is interesting to see how the Higgs couplings to fermions (known as *Yukawa couplings*) affect the Flavour symmetry.

2.1 Introduction to Standard Model

The Standard Model of Fundamental Interactions is nowadays the most predicting and accurately tested theory available to treat a wide set of processes.

The founding principles of the SM can be summarized as follows:

1. A local *gauge* symmetry $\mathcal{G}_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ spontaneously broken at low enegies trough an Higgs mechanism with the pattern $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{em}$.

2. The theory has to be a local *renormalizable* Quantum Field Theory (QFT) , i.e. the Lagrangian has to be a sum of operators Lorentz invariant with dimension, in mass unit, ≤ 4.

3. Matter fields are defined to be irreducible representations of $\mathcal{G}_{SM}$ and are chiral fermions: $q_L^i \sim (3,2)_{1/6}$, $l_L^i \sim (3,2)_{-1/2}$,$e_R^i \sim (1,1)_{-1}$, $d_R^i \sim (3,1)_{-1/3}$, $u_R^i \sim (3,1)_{2/3}$ where $i = 1,2,3$ is the family index, and the scalar responsible for the *Electroweak Symmetry Breaking* (EWSB) $H \sim (1,2)_{1/2}$.
The notation is $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ and indicates the dimension of the representation for $SU(3)_c$ and $SU(2)_L$ and the eigenvalue of the only one generator of $U(1)_Y$.

Once stated the guidelines, we are ready to write the Lagrangian, which has to contain all the possible terms allowed by our assumptions. For simplicity we split it in pieces:

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_\theta \tag{2.1}$$

$\mathcal{L}_{kin}$ contains the kinetic term of fermions and gauge fields:

$$\mathcal{L}_{kin} = \sum_\psi i\bar{\psi}\not{D}\psi - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_j^{\mu\nu}W^j_{\mu\nu} - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} \tag{2.2}$$

where $D_\mu = \partial_\mu - ig_1\frac{Y}{2}B_\mu - ig_2T_L^aW_{\mu a} - ig_3T_c^aG_{\mu a}$ with $T_L^a = \frac{\sigma^a}{2}$ and $T_c^a = \frac{\lambda^a}{2}$ (σ are the Pauli matrices and λ Gell-Mann matrices), while $B_\mu \sim (1,1)_0$ $W_\mu \sim (1,3)_0$ and $G_\mu \sim (8,1)_0$ are the vector bosons arising from the local symmetry $\mathcal{G}_{SM}$.
$W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - \frac{i}{g_2}f^{ijk}W^j_\mu W^k_\nu$ is the gauge invariant tensor for $SU(2)_L$ bosons and f^{ijk} are the structure constants of $SU(2)$ and, analogously for the tensor $G^a_{\mu\nu}$ which inherit the structure constants from $SU(3)$ and for $B_{\mu\nu}$ that has vanishing structure constants (since $U(1)$ is an Abelian group).

$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H + m_H^2 H^\dagger H - \lambda(H^\dagger H)^2 \tag{2.3}$$

is the Higgs term that presents himself as a scalar field Lagrangian with a quartic coupling. The sign of the Higgs mass term gives us the vacuum expectation value (vev) $\langle 0|\, H |0\rangle \equiv \frac{v}{\sqrt{2}} = \sqrt{\frac{m_H^2}{2\lambda}}$ responsible for the EWSB.

Finally, the *Yukawa interactions*:

$$\mathcal{L}_Y = Y_u^{ij}\,(H\bar{q}_L^i)u_R^j + Y_d^{ij}(H^c\bar{q}_L^i)d_R^j + Y_l^{ij}(H\bar{l}_L^i)e_R^j + h.c. \tag{2.4}$$

Where the charge conjugated $H^c \equiv \varepsilon H$ with $\varepsilon = i\sigma_2$ acting on $SU(2)_L$ space and $Y_{u/d/e}$ are generic complex 3x3 matrices in flavour space.

In the end

$$\mathcal{L}_\theta = \frac{\theta_3}{32\pi}\varepsilon_{\mu\nu\rho\sigma}G_a^{\mu\nu}G^{\rho\sigma a} \tag{2.5}$$

is known as the $\theta - term$ which would imply CP violation in strong interaction. Nevertheless θ_3 is strongly bounded from the experiments and anyway this term would be negligible for our purposes, so we will just ignore it[1].

[1] Analogous theta terms arise for $SU(2)_L \otimes U(1)_Y$ gauge bosons, but they can be set to 0 through a redefinition of the gauge fields

It is clear that, without assuming renormalizability of the theory, we could have added infinit more terms to the Lagrangian. Since we have stopped at dimension 4, some more symmetry of the Lagrangian emerged accidentally and they are known as *accidental symmetries*.

Definition of Flavour basis

In the definition of $\mathscr{L}_{kin}$ we have set the flavour structure of the kinetic term equal to the identity δ_{ij}. Even if it seems a special choice it is consequence of the choice of the basis we use to describe fermion fields.
In fact, if we had included a generic hermitian matrix in the flavour space for every fermion field H_ψ, we could have diagonalized it through a transformation of fields defined as:

$$\psi^i \to \psi'^i = V_{\psi\,j}^{\;i}\psi^j \tag{2.6}$$

where i, j are flavour indexes and $V_\psi \in SU(3)$ is such that $H_\psi \to H_\psi^{(d)} = V_\psi{}^\dagger H^\psi V_\psi$ is diagonal, which is always possible since H_ψ has to be hermitian and, including the condition given by the canonical commutation rule, we find $H_\psi^{(d)} = \mathbf{1}$.
Once we have diagonalized H_ψ through the transformation 2.6 we notice that, if we act on the fields again with the a generic transformation of the type 2.6 the flavour matrices of the kinetic term become $1 \to V_\psi{}^\dagger V_\psi = \mathbf{1}$.
Hence having $H^\psi = \mathbf{1},\ \forall\psi$ is a consequence of the choice we made for the basis of the fermion fields.
A basis that satisfies this condition is called *flavour basis* and, since the transformation 2.6 is a symmetry of $\mathscr{L}_{kin}$ there are infinite flavour basis linked by the relation 2.6.

Let's see how the rotation 2.6 affects $\mathscr{L}_Y$. Considering only the flavour structure of the fields, the Yukawa terms transform as

$$\begin{aligned}(Hl_L)^\dagger Y_e e_R &= H\,e_L^\dagger Y_e e_R \to H\,e_L^\dagger (V_{l_L}{}^\dagger Y_e V_{e_R}) u_R\\ (Hd_L)^\dagger Y_u u_R &= H\,d_L^\dagger Y_u u_R \to H\,q_L^\dagger (V_{q_L}^\dagger Y_u V_{u_R}) u_R\\ (H^c d_L)^\dagger Y_u u_R &= H^*\,u_L^\dagger Y_d d_R \to H^*\,q_L^\dagger (V_{q_L}{}^\dagger Y_d V_{d_R}) d_R\end{aligned}$$

Now we use that for a generic 3x3 complex matrix two matrices exist in $SU(3)$ such that $A \to U_1^\dagger A U_2 = A^{(d)}$ where $A^{(d)}$ is diagonal. Since we have freedom to rotate the fields through a *gauge diagonal* transformation (2.6 form) leaving $\mathscr{L}_{kin}$ invariant, we can choose V_{l_L}, V_{e_R} such that $V_{l_L}{}^\dagger Y_e V_{e_R} = Y_e^{(d)}$ that means that leptons flavour eigenstates can diagonalize the Yukawa matrix, i.e. they are mass eigenstates as well.

Different is the case of quarks fields, in fact there we have two terms that cannot be rotated independently. We may choose $V_{q_L} \equiv V_{d_L}$ and V_{d_R} such that $V_{d_L}^\dagger Y_d V_{d_R} = Y_d^{(d)}$ is diagonal but then we have no more freedom to diagonalize the up-quark Yukawa. In other words the gauge eigenstates are not mass eigenstates in the quark sector.

We can always define V_{u_L}, V_{u_R} such that $V_{u_L}^{\dagger} Y_u V_{u_R} = Y_u^{(d)}$ then, after diagonalizing the down-quark Yukawa matrix, the one corresponding to up quarks results (fixed $V_{q_L} = V_{d_L}$):

$$H\, q_L^{\dagger} (V_{d_L}{}^{\dagger} Y_u V_{u_R}) u_R = H\, u_L^{\dagger} (V_{d_L}^{\dagger} V_{u_L}) Y_u^{(d)} u_R \equiv H (V_{CKM} u)^{\dagger} Y_u^{(d)} u_R$$

where $V_{CKM} \equiv V_{u_L}^{\dagger} V_{d_L}$ is the *Cabibbo-Kobayashi-Maskawa* matrix that tells us that if we call u_L^i the mass eigenstates, the flavour eigenstates will be $V_{ij}^* u_L^j$.

Interactions of leptons are, instead, always flavour diagonal in the SM and this fact is known as the first *Flavour Theorem*:

Flavour theorem 1. *Flavour breaking processes are absent in the lepton sector.*

Nevertheless if we consider an extension of the SM in agreement with the *neutrino flavour oscillation* we need to include in the Lagrangian $\nu_R \sim (1,1)_0$ among the SM fermion fields.
This would imply an analogous mechanism in which ν_L's interaction eigenstates would be given by $\nu_L^{Mi} = (U_{PMNS})^i_j\, \nu_L^j$, where $U_{PMNS} = V_{\nu_L}^{\dagger}\, V_{e_L}$ is the *Pontecorvo-Maki-Nakagawa-Sakata* (PMNS) matrix introduced to explain neutrino flavour oscillations.
Due to the smallness of neutrino masses ($m_\nu \lesssim 0.1\ eV$ as found by [6]) we will neglect every effect related to PMNS matrix assuming then Lepton Flavour (LF) conservation in the SM.

In conclusion the gauge interaction eigenstates, in the limit $U_{PMNS} = \mathbf{1}$ are given by:

$$q_L^i = \begin{pmatrix} V_{CKM}^{*}{}^{ij} u_{LJ} \\ d_L^i \end{pmatrix} \quad , l_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}. \tag{2.7}$$

From that we can conclude that, due to the unitarity of V_{CKM} the interaction terms given by the diagonal generators of $SU(2)_L$, known as *Neutral Current* (NC) interactions (placing $V_{CKM} = V$)

$$\begin{aligned} \bar{q}_L^j \gamma^\mu q_L^i =& \bar{u}_L^i V_{jk} \gamma^\mu V_{kl}^* u_L^l + d_L^i \gamma^\mu d_L^i = \\ =& \bar{u}_L^i \gamma^\mu u_L^i + d_L^i \gamma^\mu d_L^i \end{aligned}$$

and hence NC interactions are diagonal in mass eigenstates basis.

Instead, in the interactions $SU(2)_L$ matrix is off diagonal, i.e. for *Charged Current* (CC) interactions we have:

$$\bar{q}_L^j \gamma^\mu \sigma_- q_L^i + h.c. = \bar{u}_L^i V_{jk} \gamma^\mu d_L + h.c.$$

that links different families of mass eigenstates giving rise to the so called *flavour breaking* processes.
This fact is known as the second *Flavour theorem*:

Flavour theorem 2. *Flavour breaking processes in the quark sector appear at tree-level just in charge current processes (exchanging $W^\pm$).*

nevertheless we will see that flavour breaking NC processes can emerge in the SM at one-loop level.

Flavour symmetries of SM Lagrangian

As previously mentioned, allowing terms with dimension in mass at most equal to four, the Lagrangian exhibit many additional symmetries to the ones assumed by the theory, being them exact or approximated.

Symmetries concerning the flavour has to involve fermions and hence they appear in either $\mathscr{L}_{kin}$ or $\mathscr{L}_Y$.
In the limit in which the Yukawa couplings are vanishing ($Y_{u,d,e} = 0$) the SM Lagrangian exhibit an accidental symmetry described by $\mathscr{G}_F = [U(3)]^5 = U(3)_q \otimes U(3)_l \otimes U(3)_u \otimes U(3)_d \otimes U(3)_e$ which is called the *Flavour Group*.

Since every matrix in $U(3)$ can be obtained from the product of two matrix in $U(3)$, one proportional to identity and the other with determinant equal to 1, it is instructive to decompose every $U(3)$ as $U(3) \sim SU(3) \otimes U(1)$.
All the $U(1)$ groups are given by the gauge diagonality of kinetic term and they represent the conservation of the number of a given fermion (seen as flavour eigenstate) in every gauge interaction.
$SU(3)$ symmetries, instead, are due to the fact that flavour matrices in $\mathscr{L}_{kin}$ are equal to the identity, and they guarantee that every different flavour component of a given fermion interacts with gauge fields with the same intensity.

When we turn on the Yukawa couplings transforming the fields with a generic $U(3)$ matrix in the flavour space as 2.6 does not leave the SM Lagrangian invariant because the Higgs couples different flavour eigenstates each other breaking explicitly $\mathscr{G}_F$.
Nevertheless, since the Higgs is a $SU(3)_c$-singlet it is allowed by $\mathscr{G}_{SM}$ to couple only quarks with quarks and this fact gives rise to an accidental $U(1)_B$ symmetry under the which every quark has the same charge. This symmetry, through the Noether theorem, implies the *Baryon Number Conservation* that consists in the conservation of the difference between the total number of quark and anti-quark in every process described by the SM.
The lepton number is conserved as well but, being both Yukawa and gauge couplings of leptons simultaneously diagonal, we have an independent $U(1)_i$ symmetry that acts on the i-th component of both the flavour triplets e^i_R and l^i_L, namely $[U(1)]^3 = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$ that implies the conservation of the difference between leptons and antileptons belonging to the same generation in every process described by the SM apart from effects due to the PMNS matrix.

Then, since Yukawa matrices are far to be proportional to identity, a generic $V \in SU(3)$ would change Yukawa matrices, even if it was the same for the two fields coupling the Higgs inside a given operator.
In $\mathscr{L}_{kin}$ we were guaranteed to have just flavour blind interactions by $SU(3)$. When we include Yukawa couplings of quarks, Yukawa matrices break the $[SU(3)]^3$ of quark fields, so the interactions with Higgs breaks the flavour universality of quark interactions. It is important to clarify that this is not due to the fact that we cannot diagonalize Yukawa matrices with a flavour basis, because even if we could do it (i.e. in the limit $V_{CKM} = 1$) the Yukawa matrices of quark, being not proportional to identity, are not invariant under

$Y_{u/d} \to V^\dagger Y_{u/d} V$, then $[SU(3)]^3$ of the quark sector is explicitly broken by the Yukawa couplings.

The same argument can be brought for the lepton sector to say that $[SU(3)]^2$ is explicitly broken because of the different mass of charged leptons. Nevertheless the situation of leptons is numerically different from quarks, in fact if the heaviest quark, the top-quark, has a mass $m_t = 173\ GeV$ comparable with Higgs vev ($v = 246\ GeV$), charged leptons are way lighter; the heaviest lepton, the τ, weighs $\simeq 1.7\ GeV$ which means that the biggest matrix element of Y_e is $Y_e^{33} \simeq \frac{1.7}{246} \simeq 7 \cdot 10^{-3}$. If we neglect terms of order $\frac{m_\tau}{v}$ the flavour group $[SU(3)]^2$ of lepton sector breaks itself leaving unbroken the subgroup $SU(3)_{LFU}$ constituted by the subgroup in which the $SU(3)$ matrix acting on the flavour indexes of fermion fields is the same for l_L^i and e_R^i triplets. This fact gives rise to the so known *Lepton Flavour Universality* (LFU) -which is realized approximately in the SM- that tells us the interactions of SM are almost flavour-blind in the lepton sector, within terms of order $\frac{m_\tau}{v}$.

Phenomenologically LFU can be tested in every process in which we can compare the dynamical effects that appear in the calculation of decay rates or cross sections for every different flavour of leptons involved separate from the phase space factor.
Some examples of ratios testing LFU in which the ratio is given by the ratio of transition matrix elements (the values of the masses involved allow to neglect kinematic mass effects) are:

$$\left.\frac{\mathcal{B}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\mathcal{B}(\tau \to e \nu_\tau \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests electron-muon universality,

$$\left.\frac{\mathcal{B}(\tau \to e \nu_\tau \bar{\nu}_e)}{\mathcal{B}(\mu \to e \nu_\mu \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests muon-tau universality, and many others.
Since LFU is much tested in processes involving different generations of leptons, it is often assumed to be preserved in the model building BSM, except when the building is deed to accommodate phenomena that break remarkably the LFU.

CKM matrix

Diagonalizing the quark mass matrices we found that LH up-quark (by convention could be down as well) interaction eigenstates are rotated respect to the mass eigenstates through the matrix:

$$V \equiv V_{u_L}^\dagger V_{d_L} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{2.8}$$

where every element is generally complex. Nevertheless it has to satisfy unitarity condition $V^\dagger V = 1$, plus it has not a unique form because, for what said so far in section 2.1, is possible to redefine CKM through a phase trasformation of the single flavour components that leaves invariant $\mathscr{L}_{kin}$, since the freedom given by the $[SU(3)]^5$ symmetry was already used to diagonalize Yukawa matrices. There is actually a global phase that has to remain free due to the Baryon Number conservation $U(1)_B$, for the rest, for N families we have $2N-1$ (N for down quarks, N for up quarks minus 1 for the global baryon number) phases that we can define as we prefer to write CKM matrix.

These conditions limit the number of free parameters of V.
A generic NxN unitary matrix has $2N^2 - N^2 = N^2$ parameters because of the N^2 conditions in $V^\dagger V = 1$. Among these N^2 parameters we have the $\frac{N(N-1)}{2}$ angles describing a orthogonal matrix plus the $\frac{N(N+1)}{2}$ phases of which $2N-1$ can be chosen by redefinition of quark fields. Thus we remain with $\frac{N(N-1)}{2}$ angles and $\frac{N(N+1)}{2} - (2N-1) = \frac{N^2-3N+2}{2}$ irreducible phases.

With three families we count three angles and one phase that define our matrix. The standard parameterization is given by the product of three real rotations, where the phase is included in the $1-3$ rotation, i.e.

$$V = R_{12}(\theta_{12}) \cdot R_{13}(\theta_{13}, e^{i\delta}) \cdot R_{23}(\theta_{23}) \tag{2.9}$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \end{aligned} \tag{2.10}$$

where $c_{ij} \equiv cos(\theta_{ij})$ and $s_{ij} \equiv sin(\theta_{ij})$.
From this parameterization the hierarchy of the flavour breaking transitions is not clear, so we introduce the *Wolfenstain parameterization* defining:

$$\lambda \equiv s_{12} \quad , A\lambda^2 \equiv s_{23} \quad , A\lambda^3(\rho - i\eta) \equiv s_{13}e^{-i\delta}.$$

We obtain, up to order λ^3

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{2.11}$$

Since $\lambda^{exp} \simeq 0.22$ we find that, $V = 1 + O(\lambda)$, thus flavour diagonal transitions are favoured. Flavour breaking transitions are suppressed at order λ if they link first and second family, at order λ^2 if they link first and second ones and at order λ^3 if they link first and third family.

The presence if an irreducible phase in the CKM matrix is fundamental because of the third flavour theorem:

Flavour theorem 3. *Applying CP transformation to the whole Lagrangian we find that $\mathscr{L}^{CP} \equiv CP(\mathscr{L})$ is equal to $\mathscr{L}$ if and only if V_{CKM} is real.*

the proof of which is reported in appendix B.

This theorem tells us that CP violation (CPV) is predicted by the SM only if there is at least one irreducible phase in the CKM matrix. Historically when in 1964 Christenson, Cronin, Fitch and Turley observed the first evidence of CPV in neutral Kaons system [7], the third family of quark was not observed yet. We have seen previously that CKM matrix has $\frac{N^2-3N+2}{2}$ irreducible phases, hence if only the two families known at the time existed CKM matrix would have been real and would not have allowed CPV for the theorem 3.
The observation of CP violation implied the existence of a third generation. In the years later, discovering the b quark, and later t quark, was a striking test to the newborn Flavour Sector of the SM.

2.2 Weak Effective Lagrangian

When we need to describe processes induced by the exchange of particles much heavier than the energy scale of the processes considered is recommended to study them in an EFT that does not contain all the particles too heavy to be produced on shell simplifying considerably the calculations.

The EFT used to describe the weak processes of the SM when the energy scale is much below $M_W \simeq 80$ GeV is known as the *Fermi Theory* and it is historically the first theory used to describe the weak interaction before W and Z bosons were observed.
This theory describes the weak processes at low energies ignoring the details of the UV Physics like the coupling between fermions and vectors that mediate the interaction, the mass of these mediators nor the theoretical nature of all the particles heavier than $\Lambda \equiv M_W = 80$ GeV, which we call the *matching scale*.

In an EFT all the physics beyond the matching scale can be ignored, nevertheless the condition for the EFT to be a low energy version of an UV theory is for the coefficients of the operators belonging to the effective Lagrangian is to satisfy the *matching condition* that consists in imposing the coefficients of the EFT, to recreate the same transition amplitudes as the UV theory when the energy is equal to the matching scale.

Hence the coefficients of the effective operators, known as *Wilson coefficients*, contain all the information on the UV theory, being that the known SM or a BSM theory.

Derivation of the Fermi Theory from the SM

When the energy available in a given process is not enough to produce on-shell some of the particles belonging to the spectrum of the theory, is possible to *integrate them out* replacing the fields that describe these heavy particles with the solution of their motion equations, in that way we would automatically satisfy the matching condition for all tree level amplitudes.

We here show how to generate operators in the Fermi Theory through the integration of the W and Z bosons from the SM Lagrangian. Since from 2.3 can be shown that Higgs boson couples only to a linear combination of B_μ and W^3_μ we define the mass eigenstates:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} cos\, \theta_W & -sin\, \theta_W \\ sin\, \theta_W & cos\, \theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} W^3_\mu cos\, \theta_W - B_\mu sin\, \theta_W \\ W^3_\mu sin\, \theta_W + B_\mu cos\, \theta_W \end{pmatrix},$$

where A_μ is the field that describes the photon which, being massless, does not need to be integrated out at low energy, Z_μ is the field that describes the Z boson, responsible for neutral current weak interactions and θ_W is the *Weinberg angle* defined as

$$\frac{g_1^2}{g_1^2 + g_2^2} \equiv sin^2\, \theta_W.$$

Since at low energy the symmetry is broken to $U(1)_{em}$ we write $W^{1,2}_\mu$ in the electric charge eigenstates basis:

$$W^\pm_\mu \equiv \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}}, \quad \sigma_\pm \equiv \frac{\sigma_1 \pm i\sigma_2}{2}$$

hence $W_1\sigma_1 + W_2\sigma_2 = \sqrt{2}(W^+\sigma_- + W^-\sigma_+)$.
Now we can write the covariant derivative D_μ in terms of the mass eigenstates as:

$$D_\mu = \partial_\mu - i\frac{g_2}{\sqrt{2}}[W^-_\mu \sigma_- + W^+_\mu \sigma_+] - i\frac{g_2}{cos\, \theta_W} Z_\mu (T_3 - Q sin^2\, \theta_W) - i\frac{g_2}{sin\, \theta_W} Q\, A_\mu - i g_3 T^a_c G_{a\mu},$$

where $Q \equiv Y - T_3$ is the electric charge operator and $g_2 sin\theta_W = g_1 cos_W \equiv e$ is the absolute value of the electron charge.
The Lagrangian that contains the W and the Z is given by:

$$\begin{aligned} \mathscr{L} \supset & -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} + \frac{M_Z^2}{2} Z_\mu Z^\mu + M_W^2 W^+_\mu W^{\mu-} \\ & + Z_\mu J^\mu_0 + W^-_\mu J^\mu_- + W^+_\mu J^\mu_+, \end{aligned}$$

where

$$J^\mu_0 = \sum_\psi g_Z^\psi \bar{\psi} \gamma^\mu \psi,$$

where the sum is done of every SM fermion after EWSB, i.e. $\{\psi\} = \{e_L, \nu_L, e_R, u'_L, d_L, u_R, d_R\}$ and

$$g_Z^\psi = \frac{g_2}{cos\, \theta_W}(T_3^\psi - Q^\psi sin^2\, \theta_W). \tag{2.12}$$

The charged currents are:

$$J^\mu_+ \equiv \frac{g_2}{\sqrt{2}}[\bar{d}_L \gamma^\mu u'_L + \bar{e}_L \gamma^\mu \nu_L], \quad J^\mu_- \equiv (J^\mu_+)^\dagger \tag{2.13}$$

where $u'_L \equiv V^\dagger_{CKM} u_L$ is the positive T_3 component of the flavour eigenstate q_L defined in section 2.1.

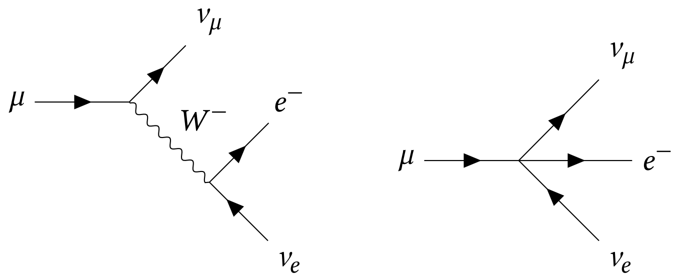


Figure 2.1: Left: Feynman diagram for muon decay in SM, the Lagrangian terms responsible for this interaction is of the form $\sim g_2\overline{\psi}_{1L}\gamma^\mu\psi_{2L}W_\mu$.
Right: Feynman diagram for muon decay in Fermi Theory, the Lagrangian terms responsible for this interaction is of the form $\sim G_F\overline{\psi}_{1L}\gamma^\mu\psi_{2L}\overline{\psi}_{3L}\gamma_\mu\psi_{4L}$

.

Then, the equations of motion result:

$$\frac{\delta\mathscr{L}}{\delta W^+_\mu} = 0 = M_W^2 W^{\mu-} + J_+^\mu \to W^{\mu-} = -\frac{1}{M_W^2}J_+^\mu; \quad W^{\mu+} = (W^{\mu-})^\dagger = -\frac{1}{M_W^2}J_-^\mu$$
$$\frac{\delta\mathscr{L}}{\delta Z_\mu} = 0 = M_Z^2 Z^\mu + J_0^\mu \to Z^\mu = -\frac{1}{M_Z^2}J_0^\mu;$$

Replacing in the Lagrangian the solution of equations of motion we find the *Fermi Lagrangian*

$$\mathscr{L}_F = -\frac{1}{M_W^2}J_+^\mu J_{\mu_-} - \frac{1}{2M_Z^2}J_0^\mu J_{0\mu},$$

then we use the relation $M_W^2\, cos^2\theta_W = M_Z^2$ and the definition of the *Fermi constant* $\frac{g_2^2}{4\sqrt{2}M_W^2} \equiv G_F$ to write:

$$\begin{aligned}\mathscr{L}_F = -\frac{4G_F}{\sqrt{2}}&\left[\overline{u'_L}\gamma^\mu d_L\, \overline{d_L}\gamma_\mu u'_L \;+\; \overline{u'_L}\gamma^\mu d_L\, \overline{\nu_L}\gamma_\mu e_L \;+\; \overline{e_L}\gamma^\mu \nu_L\, \overline{d_L}\gamma_\mu u'_L \;+\; \overline{e_L}\gamma^\mu \nu_L \overline{\nu_L}\gamma_\mu e_L\right] \\ &\frac{-4G_F}{\sqrt{2}}[\sum_\psi (T_3^\psi - Q^\psi sin^2\theta_W)\overline{\psi}\gamma^\mu\psi][\sum_\chi (T_3^\chi - Q^\chi sin^2\theta_W)\overline{\chi}\gamma_\mu\chi].\end{aligned} \tag{2.14}$$

Since $(sin^2\theta_W)^{exp} \simeq 0.22$ we notice that the weak interaction for RH fermions ($T_3 = 0$) exist just in the neutral current form, and it gets a suppression due to the accidental smallness of θ_W.

The Lagrangian 2.14 satisfies at tree level the matching condition for every tree level amplitude of the SM, to explicit the coefficients of the flavour breaking terms we need to write $u'^i = (V^*)^{ij}u_j$.

The Fermi Theory was historically the first theory that described the weak processes as the beta decay or the muon decay (Feynman diagrams in Figure 2.1), whose leading contributions are tree level induced in the SM without any flavour suppression.

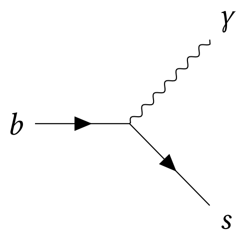
Adding all the Flavour breaking terms we get a lot of terms with a small coupling due to CKM suppression. Also it is possible to get some non trivial loop induced operators, like the neutral current flavour breaking processes, also known as *Flavour Changing Neutral Currents* (FCNC).

Flavour Changing Neutral Currents

As we have seen, flavour breaking processes arise at tree level just through exchange of W bosons. Nevertheless, at low energy we could have flavour breaking transitions in neutral current processes due to the matching to one-loop amplitudes in the SM.

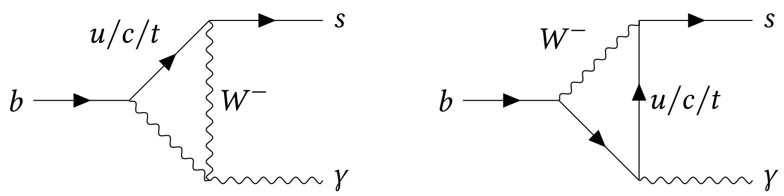
These processes are named Flavour Changing Neutral Currents (FCNC) and, because of the theorem 2, they get a double suppression for the additional vertex and the loop factor.

Theorem 2 tells us that the vertex:

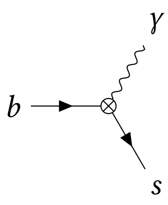


is not allowed at tree level in the SM, but $b \to s\gamma$ transition can arise at one-loop level through a loop of W as shown in figure 2.2.

Figure 2.2: Standard model leading contributions to the transition $b \to s\gamma$



Now if we integrate out W and Z bosons we get the effective vertex:



The FCNC $b \to s\gamma$ in the UV theory can be matched in the EFT introducing the effective operators defined as

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2}\bar{s}\sigma_{\mu\nu}P_L b\, F^{\mu\nu}, \quad \mathcal{O}_7' = \frac{e}{(4\pi)^2}\bar{s}\sigma_{\mu\nu}P_R b\, F^{\mu\nu},$$

where we have factorized the factor $\frac{e}{(4\pi)^2}$ that is common to all the contributions to $b \to s\gamma$ in the SM.

Since we will study the deviations from the effective contributions to $b \to sl^+l^-$ we list the operators deed to satisfy the matching condition to the one-loop amplitudes described in figure 1.1.
Following the convention adopted by [8–10] we define:

$$\begin{aligned}
\mathcal{O}_9^l &= \frac{e^2}{(4\pi)^2}\bar{b}\gamma_\mu P_L s\ \bar{l}\gamma^\mu l, & \mathcal{O}_9^{l\prime} &= \frac{e^2}{(4\pi)^2}\bar{b}\gamma_\mu P_R s\ \bar{l}\gamma^\mu l,\\
\mathcal{O}_{10}^l &= \frac{e^2}{(4\pi)^2}\bar{b}\gamma_\mu P_L s\ \bar{l}\gamma^\mu\gamma^5 l, & \mathcal{O}_{10}^{l\ \prime} &= \frac{e^2}{(4\pi)^2}\bar{b}\gamma_\mu P_R s\ \bar{l}\gamma^\mu\gamma^5 l,\\
\mathcal{O}_S^l &= \frac{e^2}{(4\pi)^2}\bar{b}P_R s\ \bar{l}l, & \mathcal{O}_S^{l\prime} &= \frac{e^2}{(4\pi)^2}\bar{b}P_L s\ \bar{l}l\\
\mathcal{O}_P^l &= \frac{e^2}{(4\pi)^2}\bar{b}P_R s\ \bar{l}\gamma^5 l, & \mathcal{O}_P^{l\prime} &= \frac{e^2}{(4\pi)^2}\bar{b}P_L s\ \bar{l}\gamma^5 l,
\end{aligned} \tag{2.15}$$

where $l = e, \mu, \tau$ and $P_{L/R} = \frac{1\mp\gamma^5}{2}$ are the projectors on the irreducible representations of the Lorentz group contained in a Dirac field (details in appendix A).

Including operators generated at one-loop that can contribute to the transition $b \to sl^+l^-$, the effective Lagrangian becomes:

$$\mathcal{L}_{EFT} = \mathcal{L}_F - \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*[C_7\mathcal{O}_7 + C_7{}'\mathcal{O}_7{}' + \sum_{i=9,10,S,P}\ \sum_{l=e,\mu,\tau}(C_i^l\mathcal{O}_i^l + C_i^{l\prime}\mathcal{O}_i^{l\prime})]$$

where we factorized the common factor $\frac{4G_F}{\sqrt{2}} = \frac{2}{v^2}$ and the CKM factor due $V_{tb}V_{ts}^*$ given by the leading contribution among the ones described by the diagrams in figures 1.1 and 2.2 i.e. the one with a t quark in the loop.
As explained in chapter 1 the electromagnetic contribution to $b \to sl^+l^-$, which is proportional to C_7 (or $C_7{}'$), decreases with the energy of the leptons pair as $\frac{1}{q^2}$ because of the photon propagator and hence it is important only when the energy of the leptons pair is low.
The values of the coefficients $C_i, C_i{}'$ suitable to satisfy the matching condition to the SM are reported by [10].

The Wilson coefficients C_i include the features of the UV theory relevant to the processes described such as loop factors, couplings, flavour structure etc.
So,if the measured Wilson coefficient is different from the one that allows the low energy theory to match the SM, it means that to build the EFT we need to integrate out BSM heavy particles which we do not already know.

3

New Physics Effective Field Theory

We want to explore scenarios able to accommodate the anomalies described so far by means of NP contributions originating above the EW scale. This is motivated by the small size of the observed effects ($\sim 20\%$ of the SM), which point to a NP scale in the few *TeV* range, as explained below, as well as the many experimental constraints on lighter particles [11–14].
The first step is to study these effects in a model-independent framework, i.e. an Effective Field theory act to parameterize the NP contributions.

First we will categorize the different possible four fermions operators that can be generated integrating out heavy degrees of freedom at the *TeV* scal, then we will focus on semileptonic operators that give the leading contribution on the processes of our interest.

If we neglect the possibility to violate Baryon Number conservation, which is tested to very high accuracy, we are left with three types of possible operators:

- Four quarks operators, which can mediate for instance the Kaon decay in pion channels. These operators are quite hard to match with the SM because we can have QCD interactions beetween the two SM currents which both couple with gluons. Since at the meson scale of energies the strong interaction is non-perturbative, precises predictions for these amplitudes are difficult.

- Four leptons operators, which can for instance mediate the muon decay. Those operators are used for processes that have just leptons both in the initial and final state avoiding all the QCD troubles for both of them.

- Semileptonic operators, which can mediate the charged pion decay, but also all the processes that can be used to address the B-Physics Anomalies. Of course the predictions are not clean as in the purely leptonic case, nevertheless we have no gluon exchange between the two SM currents and this tells us that we can see how the quark current renormalize just using global symmetries of QCD. In fact, in QCD, the vector current is conserved and so the quark current in a semileptonic decay is not affected by renormalization group of QCD. This fact reduces the theoretical uncertainty to the computation of the matrix element of a quark current between the initial and the final hadronic states.

If we include to the SM some new particles with a mass of few *TeV* order, at the EW scale (order of the Higgs vev v) we can integrate out these heavy particles creating an *high energy EFT* BSM. The Lagrangian that describe will include $\mathscr{L}_{SM}$ and some additional operators in order to match the UV theory at the matching scale.

The path to identify the proper NP begins with looking at what NP contributions are needed in the low energy EFT (i.e. at energy below M_W) to solve the discrepancy between theory and data at the hadronic scale.
Once we have included them we are going to see what are the proper BSM heavy particles to match those contributions at the *TeV* scale.

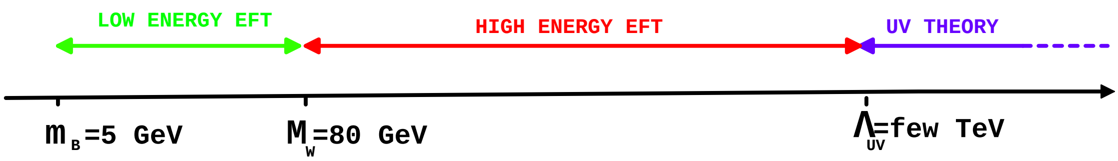


Figure 3.1: Representation of the energy ranges of validity of the UV *complete* theory (blue), the high energy EFT (red) and the low energy EFT (green)

To compare the contributions generated in the different UV frameworks we need to choose a basis of semileptonic operators that includes all possible terms that is possible to generate through a tree level matching to an UV theory that respects the assumptions of the SM. In particular, since the UV matching scale will be above the EW scale, the new EFT has to respect the gauge symmetry of the SM.

3.1 Effective Semileptonic Lagrangian

As we mentioned many times so far we want to find the coefficients that parametrize the B-Physics Anomalies which appear just in semileptonic B mesons decays.
In this chapter we choose to write fermion fields in Weyl picture described in appendix A.

There are many possible bases to describe all possible semileptonic operators, nevertheless to respect the Lorentz symmetry all of them have to be made of two currents both being scalars, vectors or 2-rank tensors.
The choice we make is to avoid the explicit action of charge conjugation operator and, when it is possible, to have every current being a colour singlet (we will see later why this choice is useful in the calculation of the observables).

Our first step is to collect all the effective operators that can contribute to those processes. First we will list five operators that can be written as contraction of two vector currents. The classification follows the one of [15] and [16].
If we assume for NP to be coupled just to LH fermions, as done by [17], we would need

just two operators:

$$\begin{aligned}\mathcal{O}_S =&(q_L^\dagger\bar\sigma^\mu q_L)\,(l_L^\dagger\bar\sigma_\mu l_L) = \big((V^\dagger u_L)^\dagger\bar\sigma^\mu(V^\dagger u_L)\big)\,(e_L^\dagger\bar\sigma_\mu e_L) + (d_L^\dagger\bar\sigma^\mu d_L)\,(e_L^\dagger\bar\sigma_\mu e_L)+\\ &\big((V^\dagger u_L)^\dagger\bar\sigma^\mu(V^\dagger u_L)\big)\,(\nu_L^\dagger\bar\sigma_\mu\nu_L) + (d_L^\dagger\bar\sigma^\mu d_L)\,(\nu_L^\dagger\bar\sigma_\mu\nu_L)\\ \mathcal{O}_T =&(q_L^\dagger\bar\sigma^\mu\sigma_a q_L)\,(l_L^\dagger\bar\sigma_\mu\sigma^a l_L) = 2\big((V^\dagger u_L)^\dagger\bar\sigma^\mu d_L\big)\,(l_L^\dagger\bar\sigma_\mu\nu_L) + 2\big(d_L^\dagger\bar\sigma^\mu(V^\dagger u_L)\big)\,(\nu_L^\dagger\bar\sigma_\mu l_L)+\\ &(u_L^\dagger\bar\sigma^\mu u_L)\,(\nu_L^\dagger\bar\sigma_\mu\nu_L) + (d_L^\dagger\bar\sigma^\mu d_L)\,(e_L^\dagger\bar\sigma_\mu e_L) - (d_L^\dagger\bar\sigma^\mu d_L)\,(\nu_L^\dagger\bar\sigma_\mu\nu_L) - (u_L^\dagger\bar\sigma^\mu u_L)\,(l_L^\dagger\bar\sigma_\mu l_L)\end{aligned} \tag{3.1}$$

where S, T, say that the two currents contracting each other are in the two case a singlet and a triplet of $SU(2)_L$.
We neglected the flavour indexes, keeping them explicit we have, for instance, the singlet operator equal to $\mathcal{O} = \mathcal{O}^{ijkl} = q_L^{i\,\dagger}\bar\sigma^\mu q_L^j\, l_L^{k\,\dagger}\bar\sigma_\mu l_L^l$ with

$$q_L^i = \begin{pmatrix} V^{*ij}u_{L\,J} \\ d_L^i\end{pmatrix} \quad , l_L^i = \begin{pmatrix}\nu_L^i \\ e_L^i\end{pmatrix}.$$

In other words we are keeping the flavour structure free at this level, and it is defined by Wilson coefficients.

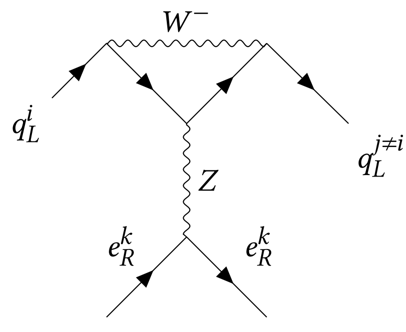
Any time we keep EW indexes implicit it means that both the vector currents composing the operator are the same irreducible representation of $SU(2)_L$.

Besides, looking at the electric charge eigenstates form, we notice that $\mathcal{O}_S$ contributes only to NC processes, while $\mathcal{O}_T$ can contribute to CC transitions as well as NC ones.

Including the RH fermions, we find three more operators involving LH vector currents compatible with the gauge symmetry $\mathcal{G}_{SM}$ that we define:

$$\begin{aligned}\mathcal{O}_{LR1} =&(q_L^\dagger\bar\sigma^\mu q_L)\,(e_R^\dagger\sigma_\mu e_R) = \big((V^\dagger u_L)^\dagger\bar\sigma^\mu(V^\dagger u_L)\big)\,(e_R^\dagger\sigma_\mu e_R) + (d_L^\dagger\bar\sigma^\mu d_L)\,(e_R^\dagger\sigma_\mu e_R)\\ \mathcal{O}_{LR2}^{u/d} =&(q_R^\dagger\sigma^\mu q_R)\,(l_L^\dagger\bar\sigma_\mu l_L) = (q_R^\dagger\sigma^\mu q_R)\,(\nu_L^\dagger\bar\sigma_\mu\nu_L) + (q_R^\dagger\sigma^\mu q_R)\,(e_L^\dagger\bar\sigma_\mu e_L)\end{aligned} \tag{3.2}$$

when the u/d means that we have two independent versions of the $\mathcal{O}_{LR2}$ for q_R equal to the u_R or d_R flavour triplets. These two operators can describe just NC transitions and so the quark flavour diagonal part, which gets tree level contributions in SM, is not much interesting for us.
The flavour breaking contributions are as well generated in SM but suppressed for different reasons. The FCNC contribution given by $\mathcal{O}_{LR1}$ described by the SM diagram:



is accidentally suppressed because of the smallness of the Weinberg angle θ_W that suppresses the coupling between the Z boson and RH fermions.
$\mathcal{O}_{LR2}$ is even more suppressed because, even if the Z this time couple to LH fermions with larger coupling, to have the FCNC for RH quarks we need to flip the chirality twice (once per quark) because of the Flavour theorem 2 which states that flavour breaking couplings of the SM are allowed only in LH sector, and so the contribution is suppressed due to the light mass of the quarks involved.

The last vector-vector operators are the ones that, in the SM, take both the suppressions described above:

$$\mathcal{O}_R^{u/d} = (q_R^\dagger \sigma^\mu q_R)\,(e_R^\dagger \sigma_\mu e_R) \tag{3.3}$$

where again we have the u/d versions according to the flavour triplet being in the place pf q_R.

Now we have three operators made from the contraction of two scalar currents.
First:

$$\begin{aligned}\mathcal{O}_S^u =&(q_L^\dagger u_R)\,\varepsilon\,(l_L^\dagger e_R) = \big((V^\dagger u)_L^\dagger u_R\big)\,(e_L^\dagger e_R) - (d_L^\dagger u_R)\,(\nu_L^\dagger e_R)\\ \mathcal{O}_S^d =&(q_L^\dagger d_R)\,(e_R^\dagger l_L) = \big((V^\dagger u)_L^\dagger d_R\big)\,(e_R^\dagger \nu_L) + (d_L^\dagger d_R)\,(e_R^\dagger e_L)\end{aligned} \tag{3.4}$$

in which $\mathcal{O}_S^d$ could indeed be written as vector currents contraction through Fierz identities, but renouncing to the request of having currents transforming as $SU(3)_c$ singlets. The ε tensor is the one introduced in section 3.2, a part that acts on $SU(2)_L$ to select the singlet in the product $\mathbf{2} \otimes \mathbf{2}$.

The last possible scalar-scalar operator is the one we call the *Leptoquark* operator:

$$\mathcal{O}_{LQ} = e_R^\dagger q_L\,\varepsilon\,u_R^\dagger l_L = e_R^\dagger (V^\dagger u)_L\,u_R^\dagger e_L - e_R^\dagger d_L\,u_R^\dagger \nu_L \tag{3.5}$$

that cannot be written as contraction of two colour singlet currents but only of two quark-lepton scalar currents, unless we involve explicitly the charge conjugation operator, in a way such that

$$\varepsilon_{ij}\varepsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} \rightarrow q^{c\dagger}_L \varepsilon l_L\; e_R^\dagger u_R^c = \mathcal{O}_{LQ} - \mathcal{O}_S^u$$

that anyway would not help the simplification of hadronic matrix elements since we would obtain anyway a current which appears only in this operator.
It is straightforward to see, from the electric charge eigenstates form, that all the scalar operators can contribute both to CC and NC transitions.

In the end we mention for completeness two operators that cannot be generated in an EFT through a tree level matching: the *tensor* operators

$$\begin{aligned}\mathcal{O}_T^u =&(q_L^\dagger \sigma^{\mu\nu} u_R)\,\varepsilon(l_L^\dagger \sigma_{\mu\nu} e_R) = (u_L^\dagger \sigma^{\mu\nu} u_R)\,(e_L^\dagger \sigma_{\mu\nu} e_R) - (d_L^\dagger \sigma^{\mu\nu} u_R)\,(\nu_L^\dagger \sigma_{\mu\nu} e_R)\\ \mathcal{O}_T^d =&(q_L^\dagger \sigma^{\mu\nu} d_R)\,(e_R^\dagger \bar{\sigma}_{\mu\nu} l_L) = (u_L^\dagger \sigma^{\mu\nu} d_R)\,(e_R^\dagger \bar{\sigma}_{\mu\nu} \nu_L) + (d_L^\dagger \sigma^{\mu\nu} d_R)\,(e_R^\dagger \bar{\sigma}_{\mu\nu} e_L)\end{aligned} \tag{3.6}$$

where $\sigma^{\mu\nu} \equiv \sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}$ and $\bar{\sigma}^{\mu\nu} \equiv \bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}$. Nevertheless, as already mentioned, these operators can be generated through the matching to amplitudes at one or more loop level in SM. Since we will implement in the next chapter a tree level matching with NP contributions at high energy we are going to neglect these last two operators.

The basis of operators defined above is complete, which means that any other semileptonic operator can be expressed as a linear combination of those above by means of field ridefinitions or Fierz identities described in section 3.2.
The effective high energy Lagrangian including the NP contributions is given by:

$$\begin{aligned}\mathscr{L}_{EFT} =& \mathscr{L}_{SM} + \Big[C_S\mathcal{O}_S + C_T\mathcal{O}_T + (C_{LQ}\mathcal{O}_{LQ} + h.c.) + C_{LR1}\mathcal{O}_{LR1} + \\ & \sum_{q=u,d} \left[C_R^q\mathcal{O}_R^q + C_{LR2}^q\mathcal{O}_{LR2}^q + (C_S^q\mathcal{O}_S^q + C_T^q\mathcal{O}_T^q + h.c.) \right] \Big]\end{aligned} \tag{3.7}$$

in which, expilciting flavour indexes $C\mathcal{O} = C_{q_1q_2l_1l_2}\mathcal{O}^{q_1q_2l_1l_2}$ where, in our convention, q_1 and l_1 are the ones appearing in the operator *daggered*.

The only assumption we do on the Wilson coefficients is to neglect the coupling to first families of quarks and leptons, i.e. every $C_{q_1q_2l_1l_2}$ will be $\neq 0$ only for $(l_1, l_2) \in \{\mu, \tau\}$ and $(q_1, q_2) \in \{s, b\}$ where we have labelled every family with the name of its negative charged component.

3.2 Fierz identities

When we generate operators in an EFT through a tree level matching they could be different from the ones listed above, nevertheless they will always be equal to a linear combination of them.

To project the operators we generate in another basis we need to use some relations that derive from $SU(N)$ structure known as *Fierz Identities.*

It is possible to prove that, since any hermitian matrix NxN can be written as

$$H = c_0 1 \; + \sum_{i=1}^{N^2-1} c_i T_i$$

where T_i are the generators of the fundamental representation of $SU(N)$, these generators satisfy the completeness condition:

$$\sum_{a=1}^{N^2-1} T^a_{ij} T_{a\,kl} = \frac{1}{2}\big(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\big).$$

In the case $N = 2$ since $\sigma_a = 2T_a$ we find (keeping the sum on a implicit):

$$\sigma_{a\;ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl} \tag{3.8}$$

this relation can help us to write the EW structure to have all the currents contracted in the operators listed in 3.1, i.e. to have every single current transforming with an irreducible representation of $SU(2)_L$.
If we need to compare two terms with the indexes arranged in the same way we can invert the relation 3.8 and obtain:

$$\sigma^a_{ij}\sigma_{a\ kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\ kj}). \tag{3.9}$$

Similar relations hold for the Lorentz group, since the group is isomorphic to $SU(2)xSU(2)^*$. Having written fermions as Weyl spinors we have explicited the relation between the Lorentz algebra, used to define the currents in appendix and the Pauli matrices as shown in appendix A.

The structure of the vector and the scalar chiral currents is described in appendix A Taking back 3.8 and, mindful that $\sigma^0_{\alpha\dot{\alpha}} = \overline{\sigma}^0_{\alpha\dot{\alpha}} = \delta_{\alpha\dot{\alpha}}$[1], and including the minus sign to space indexes due to Minkowski metric we write:

$$\sigma_{\mu\ \alpha\dot{\alpha}}\sigma^{\mu}_{\beta\dot{\beta}} = \delta_{\alpha\dot{\alpha}}\delta_{\beta\dot{\beta}} - \sigma_{i\ \alpha\dot{\alpha}}\sigma^{i}_{\beta\dot{\beta}} = 2(\delta_{\alpha\dot{\alpha}}\delta_{\beta\dot{\beta}} - \delta_{\alpha\dot{\beta}}\delta_{\beta\dot{\alpha}}) \tag{3.10}$$

now using that $\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}$ we find:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\,\beta\dot{\beta}} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} = -\sigma^{\mu}_{\alpha\dot{\beta}}\sigma_{\mu\,\beta\dot{\alpha}} \tag{3.11}$$

And then, generalizing to the overlined matrices, we write:

$$\overline{\sigma}^{\dot{\alpha}\alpha}_{\mu}\overline{\sigma}^{\mu\ \dot{\beta}\beta} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}} \tag{3.12}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\sigma_{\mu}}^{\dot{\beta}\beta} = 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}}. \tag{3.13}$$

In some cases we have to use both the relations for $SU(2)_L$ and Lorentz to write the operators generated in the basis presented in section 3.1. In these cases the tensor structure could be complicated and eventually confusing.
We are going to derive some useful results in the appendix C, so that we will have them ready when we will handle the physics.

In the next chapter we will use these relations to project the operators we generate with the *tree level matching* onto the basis to which we refer, in order to catalogue and compare the contributions arising in the different NP scenarios.
The purpose of doing this is that, once we find the numerical values of the Wilson coefficients that parameterize NP effects referring to the basis chosen in a model-independent EFT, it will be easy to combine them either to enhance the accommodation of the anomalies or to cancel contributions that are constrained from the experiments.

[1]The dotted indexes indicate the indexes corresponding to $SU(2)^*$ following the notation adopted by [18].

4

Some of the possible heavy bosons

As we previously said one possible way to modify the Wilson coefficients in an EFT is to introduce heavy particles to the UV theory which couple with the SM fermions involved in the processes we want to accommodate.

When we introduce new particles interacting with the SM we need to look carefully at the experimental constraints. In particular two conditions have to be satisfied:

- The contribution to the observable given by the diagrams in which the new particles appear as virtual particles has to show agreement with the experiments, both the ones that show deviations and the ones that agree with the SM;
- If the new particle can be produced at colliders, its production cross-section has to agree with the limits from direct searches for BSM states.

To begin, we will see how we can get contributions to the anomalous observables introducing different type of heavy vector and scalar bosons.
We will begin showing the effect of the inclusion in the spectrum of the theory of two colour-less vectors charged under $SU(2)_L$: $B' \sim (\mathbf{1}, \mathbf{1})_0$ and $W' \sim (\mathbf{1}, \mathbf{3})_0$, where the name reminds to their lighter version W and B of the SM. Then we will describe the vector Leptoquarks $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$ and scalar Leptoquarks $S_1 \sim (\mathbf{3}^*, \mathbf{1})_{1/3}$, $S_3 \sim (\mathbf{3}^*, \mathbf{3})_{1/3}$, $R_2 \sim (\mathbf{3}, \mathbf{2})_{7/6}$ and $\widetilde{R}_2 \sim (\mathbf{3}, \mathbf{2})_{1/6}$.

Once we have acknowledged what contributions are generated from the different bosons we will be ready to see what mediator, or what combination of mediators, is able to accommodate the B-Physics anomalies without contradicting the other Flavour tests done so far.

4.1 Colour-less bosons

$B' \sim (\mathbf{1}, \mathbf{1})_0$

The first candidate is a heavier version of the EW singlet of the SM: $B' \sim (\mathbf{1}, \mathbf{1})_0$. Apart from the bigger mass the main difference between these two bosons is the flavour

structure of the coupling, which is diagonal in SM and instead is kept free a priori for the BSM version.
Once we have introduced this boson the most general UV Lagrangian contains:

$$\begin{aligned}\mathscr{L}_{UV} \supset \frac{M_{B'}^2}{2} B_\mu B^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + g_{B'}(l_L^\dagger \overline{\sigma}^\mu \lambda_B^l l_L) B_\mu + g_{B'}(q_L^\dagger \overline{\sigma}^\mu \lambda_B^q q_L) B_\mu + \\ g_{B'}(e_R^\dagger \sigma^\mu \lambda_B^e e_R) B_\mu + g_{B'}(u_R^\dagger \sigma^\mu \lambda_B^u u_R) B_\mu + g_{B'}(d_R^\dagger \sigma^\mu \lambda_B^d d_R) B_\mu\end{aligned} \tag{4.1}$$

where the λ matrices are flavour matrices that are real since B' transforms under a real representation of the gauge group. We have also factorized out an overall coupling constant $g_{B'}$.

When we go down to energies $E \ll M_{B'}$ B' can not be produced on shell anymore, and so it can't appear in a process but as virtual particle. In this condition it is possible to *integrate it out* from the Lagrangian, i.e. replacing B'^μ with the solution of the equation of motion:

$$\frac{\delta \mathscr{L}}{\delta B_\mu} = 0 =$$

$$= \frac{M_B^2}{2} B^\mu + g_{B'}(l_L^\dagger \overline{\sigma}^\mu \lambda_B^l l_L) + g_{B'}(q_L^\dagger \overline{\sigma}^\mu \lambda_B^q q_L) + g_{B'}(e_R^\dagger \sigma^\mu \lambda_B^e e_R) + g_{B'}(u_R^\dagger \sigma^\mu \lambda_B^u u_R) + g_{B'}(d_R^\dagger \sigma^\mu \lambda_B^d d_R)$$

$$\to B^\mu = -\frac{2 g_{B'}}{M_{B'}^2} \big[(l_L^\dagger \overline{\sigma}^\mu \lambda_B^l l_L) + (q_L^\dagger \overline{\sigma}^\mu \lambda_B^q q_L) + (e_R^\dagger \sigma^\mu \lambda_B^e e_R) + (u_R^\dagger \sigma^\mu \lambda_B^u u_R) + (d_R^\dagger \sigma^\mu \lambda_B^d d_R) \big].$$

obtaining an effective Lagrangian:

$$\mathscr{L}_{EFT} \supset -2 G_{B'} \big[\big(\sum_\psi \psi^\dagger \lambda_B^\psi \sigma_\mu \psi \big) \big(\sum_\chi \chi^\dagger \lambda_B^\chi \sigma_\mu \chi \big) \big]$$

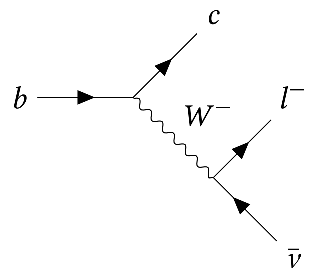
where $G_{B'} \equiv \frac{g_{B'}^2}{M_{B'}^2}$ and the sums on ψ and χ are done over every SM fermion that couples to B'.
Among the 25 possible combinations of these five neutral currents we recognize some semileptonic operators, precisely all the ones constructed with two $SU(2)_L$-singlet vector currents inside the list shown in section 3.1. The coefficients of these operators are equal to:

$$C_S = -2 G_{B'} \lambda_B^q \lambda_B^l, \quad C_R^{u/d} = -2 G_{B'} \lambda_B^e \lambda_B^{u/d},$$
$$C_{LR1} = -2 G_{B'} \lambda_B^q \lambda_B^e, \quad C_{LR2}^{u/d} = -2 G_{B'} \lambda_B^{u/d} \lambda_B^l.$$

Integrating out B' we also generate lots of four-quarks and four-leptons operators that, according to the parameters, could generate contributions to observables tested by EW Precision Tests (EWPT) at LEP [19] or to the meson mixing [20].
In particular, since to accommodate the anomaly of the $b \to s l^- l^+$ transition we need a non zero $(\lambda_B^{q/u/d})^{bs} = (\lambda_B^{q/u/d})^{sb}$ (where $q/u/d$ in this case means that has to be non zero at least in one of those three flavour matrices), we produce a tree-level contribution to B_s mixing, an observable which is one-loop induced in the SM.
Also we have to highlight that we have not generate any operator which contributes to CC transitions, i.e. B' cannot be used to accommodate the anomaly of the $b \to c l \nu$ decays.

$W' \sim (\mathbf{1}, \mathbf{3})_0$

Since B' cannot contribute to CC anomalies, we expect that to be done by an $SU(2)_L$ triplet as happens in SM, where $b \to cl\bar{\nu}$ is mediated by $W^{\pm}$ bosons which are given by the non diagonal components of the $SU(2)_L$ matrix $W^a_\mu \sigma^a$. The amplitude for the fundamental process in the SM takes the leading contribution from



which takes a flavour suppression from the quark vertex equal to V_{cb} and is diagonal and flavour universal in the lepton vertex (in one words the flavour matrix is the $\mathbf{1}_{3x3}$ for what concern the lepton current).
Since the CC anomaly consists in a preference for the decay channel with the τ as charged lepton, an obvious way to accommodate it is to add another $SU(2)_L$ triplet (much heavier than the $W^{\pm}$ to pass collider constraints) with a bigger coupling for the third flavour component of the lepton fields, and similarly for the third generation of quarks.

Adding to SM $W' \sim (\mathbf{1}, \mathbf{3})_0$ the general additional terms to high energy Lagrangian are:

$$\mathscr{L}_{UV} \supset \frac{M^2_{W'}}{2} W'^a_\mu W'^\mu_a - \frac{1}{4} W'^a_{\mu\nu} W'^{\mu\nu}_a + g_{W'} (l^\dagger_L \bar{\sigma}^\mu \lambda^l_W \sigma_a l_L) W^a_\mu + g_{W'} (q^\dagger_L \bar{\sigma}^\mu \lambda^q_W \sigma_a q_L) W^a_\mu \quad (4.2)$$

To see how this new terms contribute to processes at the EW scale we need again to integrate out W' to see what effective semileptonic operators are generated in the high energy EFT.

$$\frac{\delta \mathscr{L}}{\delta W^a_\mu} = 0 = \frac{M^2_W}{2} W^\mu_a + g_{W'} l^\dagger_L \bar{\sigma}^\mu \sigma_a \lambda^l_W l_L + g_{W'} q^\dagger_L \bar{\sigma}^\mu \sigma_a \lambda^q_W q_L \rightarrow$$

$$W^\mu_a = -\frac{2 g_{W'}}{M^2_{W'}} \left[l^\dagger_L \bar{\sigma}^\mu \sigma_a \lambda^l_W l_L + q^\dagger_L \bar{\sigma}^\mu \sigma_a \lambda^q_W q_L \right].$$

Then inserting it in the Lagrangian we obtain:

$$\begin{aligned} \mathscr{L}_{EFT} \supset & -2G_{W'} \big[(l^\dagger_L \lambda^l_W \bar{\sigma}^\mu \sigma_a l_L)\, (l^\dagger_L \lambda^l_W \bar{\sigma}_\mu \sigma^a l_L) \; + \; (q^\dagger_L \lambda^q_W \bar{\sigma}^\mu \sigma_a q_L)\, (q^\dagger_L \lambda^q_W \bar{\sigma}_\mu \sigma^a q_L) + \\ & 2 (q^\dagger_L \lambda^q_W \bar{\sigma}^\mu \sigma_a q_L)\, (l^\dagger_L \lambda^l_W \bar{\sigma}_\mu \sigma^a l_L) \big] \end{aligned} \quad (4.3)$$

in which we recognize the triplet operator $\mathcal{O}_T$ in the last term implying

$$C_T = -4 G_{W'} \lambda^l_W \lambda^q_W$$

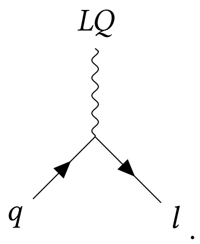
with $G_{W'} \equiv \frac{g_{W'}^2}{M_{W'}^2}$. Notice that this time we generate $C_T \neq 0$ that could contribute to CC anomalous observables.

Again we generate in addition four-leptons and four-quarks operators that possibly create some tensions between the anomalies solution and EWPT or meson mixing.

In conclusion colour-less vectors could together, a priori, accommodate both CC and NC anomalies but we can already see, without numerical analysis, that they generate a lot of unpleasant terms that don't easily pass the constraints once we included non semileptonic observables.
We notice that so far we did not need to use Fierz identities, this is due to the choice we did to write operators as contractions of $SU(3)_c$-singlet currents, and they can couple directly to $SU(3)_c$-singlet bosons.

4.2 Leptoquarks

A possible way to avoid the tension between the anomalies and the precisely tested flavour sector of the SM is to introduce bosons with quantum numbers different from the Higgs and gauge bosons.
A good way to proceed is to include bosons that couple leptons to quarks in order to generate only semileptonic contributions at tree level.
Hence we introduce colour-triplet bosons that connect quark and lepton sector at tree level known as *Leptoquark*, allowing the vertex



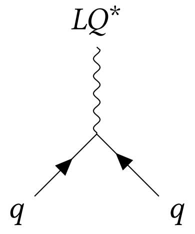
There could be different types of Leptoquarks differing in the representation under the which their fields transform under Lorentz group and under $\mathcal{G}_{SM}$.

The only one feature common to all the Leptoquarks is that, to link a colour triplet (or antitriplet) and a colour singlet, they have to transform under the fundamental (or antifundamental) representation of $SU(3)_c$, hence they are *coloured*.

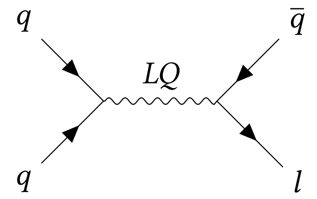
In the limit in which the only coupling allowed for Leptoquarks is the quark-lepton coupling, we would generate at tree level just semileptonic operators.
Nevertheless, because of $SU(3)$ structure we have that the product of two fundamental representations is:

$$\mathbf{3} \otimes \mathbf{3} \sim \mathbf{6} \oplus \mathbf{3}^*$$

and hence, if the Lorentz and $SU(2)_L \otimes U(1)_Y$ symmetries allow, it is possible to have the quark-quark vertex as well:



The presence of this vertex would generate the already mentioned meson mixing because of four-quarks tree level amplitudes generated. Also with both of the vertexes above allowed is possible to generate at tree level the amplitude



and that would violate the Baryion Number consrvation.

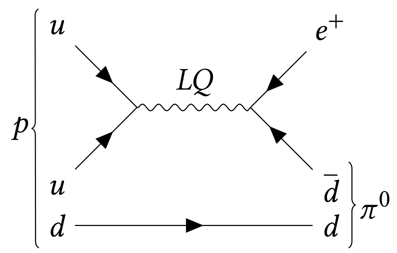
The easiest solution that can be adopted in a *bottom-up* approach[1] would be just to switch off the quark-quark vertex see what values for the other couplings are suitable to describe the phenomenology.
Instead if we would implement a properly said UV completion, setting that coupling to zero has to be in some way justified.

In this work we won't aim to formulate a complete theory that justifies the proton stability. On the other hand we will to discuss briefly this possible issue.

Proton decay

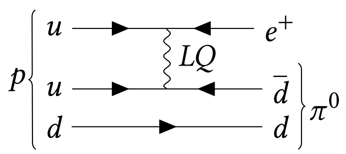
Leptoquarks emerge easily as heavy gauge bosons when we try to extend the gauge sector of the SM.
The most stringeny contraint on the existence of Leptoquarks has always been that some of them could mediate proton decay $p \to e^+\pi^0$

The diagrams corresponding to the leading order amplitudes would be:



or even in t-channel

[1] An approach in which we worry only about the values of the parameters that explain the phenomenology without wondering about the theoretical nature of the NP.



Now the available experimental limit for the proton lifetime is $\tau_p > 10^{33} ys$ and so if LQs arise the contribution sketched above should be very suppressed.
This constraint can be satisfied in two cases:

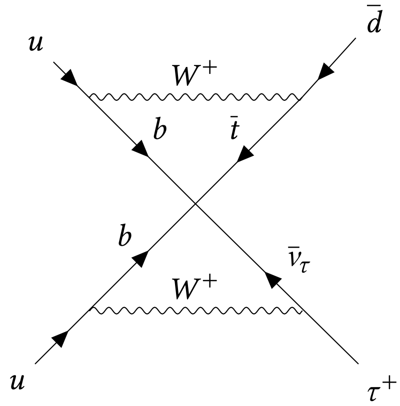
- $M_{LQ} \simeq \Lambda_{GUT} \simeq 10^{16}\ GeV$ to have a decoupling at low energies as shown in D,
- some symmetry among the assumptions of the theory set automatically to zero the quark-quark vertex. For instance if among the assumptions we can ask explicitly to preserve Baryon Number or Lepton Number. That happens for some Leptoquarks as consequence of the gauge symmetry $\mathcal{G}_{SM}$.

In the framework we have chosen we will not concern about the fundamental reason to suppress the quark-quark but we will point out when that suppression emerges automatically from the request to respect SM symmetries.

Nevertheless, the flavour structure we have described so far helps the purpose of suppressing the proton decay in $\pi^0\ e^+$ at tree level. Even if we consider the flavour component with the biggest coupling in the NP we are considering:

$$(\bar{t}\Gamma b)\ (\bar{\nu}_\tau \Gamma b)$$

where Γ is some generic Lorentz structure, the decay $p \to \tau^+ \pi^0 \to \bar{\nu}_\tau + hadrons$ the four fermion vertex would be suppressed the with CKM matrix elements and loop factors coming from the insertion of two W loops needed to switch the flavour of quarks involved to the third family.



and we get an additional suppression from the propagator of the τ^+ off-shell. In any case we cannot state that our flavour structure is enough to pass the experimental test, but for sure it points in the right direction.

4.3 Vector Leptoquarks

First we are going to analyze Leptoquark vectors known as $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$. The values of the $SU(2)_L \otimes U(1)_Y$ quantum numbers are chosen such that a coupling to a LH semileptonic current $(q_L^\dagger \bar{\sigma}^\mu l_L)$ is possible. We will see that they have some pleasant features; for instance, since they couple to vector currents the value of the hypercharge $Y = 2/3$ doesn't allow them to couple with a quark-quark current and hence, in general, they can never mediate proton decay.

$U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$

The first LQ we take in consideration is the $SU(2)_L$ singlet U_1 which is one of the most popular in the available literature [8, 9, 17] for reasons that will be clear later.

Including U_1 in our theory we get an UV Lagrangian that reads:

$$\mathscr{L}_{UV} \supset -\frac{1}{2} U_{1\mu\nu}^\dagger U_1^{\mu\nu} + M_{U_1}^2 U_{1\mu}^\dagger U_1^\mu + g_{U_1} \left[U_{1\mu} \left(q_L^\dagger \beta_{U_1}^L \bar{\sigma}^\mu l_L + d_R^\dagger \beta_{U_1}^R \sigma^\mu e_R \right) + h.c. \right] \tag{4.4}$$

One difference that can be seen immediately is that belonging to the fundamental representation of $SU(3)_c$, which is complex, the current coupled to it has not to be hermitian, hence the flavour matrices β are complex in general.

We can integrate out U_1 from the Lagrangian through the equation of motion:

$$\begin{aligned} \frac{\delta \mathscr{L}}{\delta U_{1\mu}^\dagger} =& 0 = M_{U_1}^2 U_1^\mu + g_{U_1} \left[l_L^\dagger \beta_{U_1}^{\dagger L} \bar{\sigma}^\mu q_L + e_R^\dagger \beta_{U_1}^{\dagger R} \sigma^\mu d_R \right] \rightarrow \\ U_1^\mu =& - \frac{g_{U_1}}{M_{U_1}^2} \left[l_L^\dagger \beta_{U_1}^{\dagger L} \bar{\sigma}^\mu q_L + e_R^\dagger \beta_{U_1}^{\dagger R} \sigma^\mu d_R \right] \end{aligned} \tag{4.5}$$

obtaining

$$\mathscr{L}_{EFT} \supset -G_{U_1} \left[(l_L^\dagger \beta_{U_1}^L \bar{\sigma}^\mu q_L)(q_L^\dagger \beta_{U_1}^{\dagger L} \bar{\sigma}_\mu l_L) + [(q_L^\dagger \beta_{U_1}^L \bar{\sigma}^\mu l_L)(e_R^\dagger \beta_{U_1}^{\dagger R} \sigma_\mu d_R) + h.c.] + (e_R^\dagger \beta_{U_1}^R \sigma^\mu d_R)(d_R^\dagger \beta_{U_1}^{\dagger R} \sigma^\mu e_R) \right] \tag{4.6}$$

where, $G_{U_1} \equiv \frac{g_{U_1}^2}{M_{U_1}^2}$.

The effective Lagrangian is made of a sum of semileptonic operators, now we need to project them on the basis listed in section 3.1.

The second term is proportional to $\mathcal{O}_S^d$ because of the relation $\bar{\sigma}^\mu_{\dot{\alpha}\alpha} \sigma_\mu^{\beta\dot{\beta}} = 2\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}}$ it's precisely equal to $-2\mathcal{O}_S^d$, where the minus comes from the anticommutation rule for two fermion fields $\{\psi, \chi\} = 0$ used three times to sort the fields in $q^* l e^* d \rightarrow q^* d e^* l$.

The Lorentz structure of the first and the third get us a minus sign because of the relations $\sigma^\mu_{\alpha\dot{\alpha}} \sigma_{\mu\, \beta\dot{\beta}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon_{\alpha\beta} = -\sigma^\mu_{\alpha\dot{\beta}} \sigma_{\mu\beta\dot{\alpha}}$ and $\bar{\sigma}^\mu_{\dot{\alpha}\alpha} \bar{\sigma}_{\mu\, \dot{\beta}\beta} = 2\varepsilon_{\dot{\alpha}\dot{\beta}} \varepsilon_{\alpha\beta} = -\bar{\sigma}^\mu_{\alpha\dot{\beta}} \bar{\sigma}_{\mu\beta\dot{\alpha}}$.

Using these relations and anticommutation of fermions we see that the third term is equal to $\mathcal{O}_R$ while the first term seems equal to $\mathcal{O}_S$ apart from the fact that the quark and the lepton current aren't $SU(2)_L$ singlets, in fact both quarks contract the $SU(2)_L$ index with a lepton.
To project that operator on our basis we need to recall 3.8

$$\sigma^i_{ad}\sigma_{cbi} = 2\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} \rightarrow \delta_{ab}\delta_{cd} = \frac{\sigma^i_{ad}\sigma_{cbi} + \delta_{ad}\delta_{cb}}{2}$$

to rewrite the first term as $-\frac{1}{2}(\mathcal{O}_S + \mathcal{O}_T)$. Adding up everything we obtain:

$$\mathcal{L}_{EFT} \supset -G_{U_1}\Big[\frac{1}{2}\beta^L_{U_1}\beta^{\dagger L}_{U_1}(\mathcal{O}_S + \mathcal{O}_T) + \beta^R_{U_1}\beta^{\dagger R}_{U_1}\mathcal{O}^d_R - 2\beta^R_{U_1}\beta^{\dagger L}_{U_1}\mathcal{O}^d_S + \Big] \tag{4.7}$$

that means in other words

$$C_S = C_T \equiv -\frac{1}{2}G_{U_1}\beta^{L\dagger}_{U_1}\beta^L_{U_1} \quad C^d_R \equiv -G_{U_1}\beta^{R\dagger}_{U_1}\beta^R_{U_1} \quad C^d_S \equiv -2G_{U_1}\beta^{R\dagger}_{U_1}\beta^L_{U_1}$$

The first feature we notice is that, even though we take all the possible terms in the UV Lagrangian, at tree-level just semileptonic contributions are generated. This means that when we accommodate the anomalies we need to be mindful of constraints that come just from semileptonic observables.

Without considering the details of the computation we can also see how one of this constraints is avoided from the fact that $C_S = C_T$. In fact $C_S = C_T \neq 0$ gives us a way to accomodate the anomaly on $b \rightarrow s l^+ l^-$ where the amplitude due to NP effects is (considering just purely LH contributions) $\propto (C_S + C_T)$ as can be seen from the electric charge eigenstates form of $\mathcal{O}_S$ and $\mathcal{O}_T$ shown in section 3.1.
From that form we can see also that the transition $b \rightarrow s\nu\bar{\nu}$ is linked to $b \rightarrow s l^+ l^-$ by the gauge symmetry and results to be $\propto (C_S - C_T)$ which means no contribution coming from the integration of U_1 as shown by [17]. This is pleasant since $\mathcal{B}(B \rightarrow K^{\dagger}\nu\bar{\nu})$ is a rare decay suppressed in SM and much constrained by experimental data and U_1 doesn't contribute to it at tree-level.

$U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$

The vector $SU(2)_L$ triplet LQ couples just to the triplet LQ current $J^a_T = q^{\dagger}_L\bar{\sigma}^{\mu}\sigma^a\beta_{U_3}l_L$ i.e. just to LH fermions so, for what said in section 3.1, we expect to generate at low energy a combination of $\mathcal{O}_S$ and $\mathcal{O}_T$.

The UV Lagrangian gains the following terms:

$$\mathcal{L}_{UV} \supset -\frac{1}{2}U^{\dagger a}_{3\mu\nu}U^{\mu\nu}_{3a} + M^2_{U_3}U^{\dagger\mu}_{3a}U^a_{3\mu} + g_{U_3}\big[U^a_{3\mu}q^{\dagger}_L\beta_{U_3}\bar{\sigma}^{\mu}\sigma_a l_L + h.c\big] \tag{4.8}$$

To build the effective Lagrangian at $E < M_{U_3}$ scale we ask as always

$$\frac{\delta\mathcal{L}}{\delta U^a_{3\mu}} = 0 = M^2_{U_3}U^{mu}_{3a} + g_{U_3}l^{\dagger}_L\beta^{\dagger}_{U_3}\bar{\sigma}^{\mu}\sigma_a q_L \rightarrow U^{\mu}_{3a} = -\frac{g_{U_3}}{M^2_{U_3}}l^{\dagger}_L\beta^{\dagger}_{U_3}\bar{\sigma}^{\mu}\sigma_a q_L \tag{4.9}$$

generating

$$\mathscr{L}_{EFT} \supset -G_{U_3}\left[q_L^\dagger \beta_{U_3}\overline{\sigma}^\mu\sigma_a l_L \; l_L^\dagger \beta_{U_3}^\dagger \overline{\sigma}_\mu \sigma^a q_L \;\right] \tag{4.10}$$

Now we get a minus sign from the Lorentz structure and one more minus from exchanging the positions of fermion fields as in the U_1 case, so that we get the fields in the operators written as contractions of colour singlet currents.

Meanwhile the EW structure recalls 3.9

$$\begin{gathered} \sigma^a_{ij}\sigma_{a\;kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\;kj}) \rightarrow \\ \mathscr{L}_{EFT} \supset -\frac{1}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger[3\mathcal{O}_S - \mathcal{O}_T] \end{gathered} \tag{4.11}$$

The Wilson coefficients are equal to

$$C_S = -3C_T \equiv -\frac{3}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$$

that give tree level contributions on both CC and NC processes.

To summarize the features of vector Leptoquarks we would say that they are a good possibility since they affect at tree level just semileptonic processes even if we include all possible terms allowed by SM symmetries.
Particularly U_1 is mentioned many times in the literature as the only one that can accommodate both anomalies alone passing most of the constraints, while U_3 is not that appreciated as solution of the anomalies but possibly it can be combined with other mediators to build a relevant model.

4.4 Scalar Leptoquarks

After having shown vectors we introduce some scalar Leptoquarks that can be helpful for our purposes.

By convention the ones belonging to real representations of $SU(2)_L$ are defined in the antifundamental of $SU(3)_c$ i.e. we have $S_1 \sim (3^*, 1)_{1/3}$, $S_3 \sim (3^*, 3)_{1/3}$.
While the $SU(2)_L$-doublets we consider are two as well differing for the value of hypercharge: $R_2 \sim (3, 2)_{7/6}$ and $\widetilde{R}_2 \sim (3, 2)_{1/6}$. As before the values of the hypercharge are chosen such that they can couple the corresponding semileptonic currents.

In this case the values of the hypercharge allow S_1 and S_3 to couple a quark-quark current, this is not possible instead for the doublets because of $SU(2)_L$ symmetry[2].

In addition, differently from vector case, the scalar current has to be made by $(\psi_{L/R})^\dagger \chi_{R/L}$ and it is possible to use the charge conjugation to flip the chirality since $(\psi_L^c) \sim (0, \frac{1}{2})$ and viceversa.

[2]A doublet LQ has to couple with a left-right scalar current that has always only one conjugated field. That makes forbidden quark-quark coupling because of $SU(3)_c$ $1 \not\supset 3 \otimes 3^* \otimes 3$.

That was possible also for vector Leptoquark but there the hypercharge did not allow to couple a current $(\psi^c_{L/R})^\dagger \sigma^\mu \chi_{R/L}$ for any $\{\psi_L, \chi_R\} \in \{q_L, l_L, u_R, d_R, e_R\}$.

So, to see what currents couple to a given scalar LQ, we have to include all the currents allowed by $\mathscr{G}_{SM}$, mindful that the charge conjugation acts as complex conjugation on the complex representations.

$S_1 \sim (\mathbf{3}^*, \mathbf{1})_{1/3}$

A scalar current which couples with a $SU(2)_L$ singlet must involve charge's conjugation, because we can't make a $SU(2)_L$ singlet with only one LH field.
Mindful of this point we find that, this time, the $\mathscr{G}_{SM}$ symmetry allows a quark-quark coupling. Precisely the UV Lagrangian that includes S_1 is:

$$\begin{aligned}\mathscr{L}_{UV} \supset (D_\mu S_1)^\dagger D^\mu S_1 + M^2 S_1^\dagger S_1 + \\ + g_{S_1} S_1^i \big[q_{Li}^{\dagger c} \beta^L_{S_1} \varepsilon l_L + u_{Ri}^{\dagger c} \beta^R_{S_1} e_R + {q_L^j}^\dagger \lambda_{S_1} \varepsilon {q_L^c}^k \varepsilon_{ijk}\big] + h.c.\end{aligned} \tag{4.12}$$

where the ε with the indexes kept implicit is the $SU(2)_L$ tensor that guarantee the current to be a EW singlet, while ε_{ijk} is the completely antisymmetric tensor in three dimensions deed to make $SU(3)_c$ singlet.
The $+h.c.$ term of these type of scalar currents its obtained as usual, but we have to be careful to include a minus sign that arises from $\varepsilon^\dagger = -\varepsilon$, i.e.

$$[(a^c)^\dagger \varepsilon b]^\dagger = -b^\dagger \varepsilon a^c$$

where a and b are LH fields.

Integrating out S_1 we need to explicit the colour index, since this time it is not contracted trivially in all the terms:

$$\begin{aligned}\frac{\delta \mathscr{L}_{UV}}{\delta S_1^{i\dagger}} =& 0 = M^2_{S_1} S_{1i} - g_{S_1} \big[l_L^\dagger \varepsilon \beta^{L\dagger}_{S_1} q^c_{Li} + + e_R^\dagger \beta^{R\dagger}_{S_1} u^c_{Ri} + q_L^{\dagger c j} \lambda^\dagger_{S_1} \varepsilon q^k_L \varepsilon_{ijk}\big] \to \\ S_{1i} =& \frac{g_{S_1}}{M^2_{S_1}} \big[l_L^\dagger \beta^\dagger_{S_1} \varepsilon q^c_{Li} + e_R^\dagger \beta^{R\dagger}_{S_1} u^c_{Ri} + {q^{cj}_L}^\dagger \lambda^\dagger_{S_1} \varepsilon q^k_L \varepsilon_{ijk}\big]\end{aligned} \tag{4.13}$$

Inserting this term in the UV Lagrangian we would obtain nine terms, for simplicity we here keep implicit all the terms that violate Baryon Number

$$\begin{aligned}\mathscr{L}_{EFT} \supset & G_{S_1} \big[l^\dagger \beta^\dagger_{S_1} \varepsilon q^c_{Li} + e_R^\dagger \beta^{R\dagger}_{S_1} \varepsilon u^c_{Ri} + {q^{cj}_L}^\dagger \lambda^\dagger_{S_1} \varepsilon q^k_L \varepsilon_{ijk}\big] \big[q^{\dagger c}_{Li} \beta^L_{S_1} \varepsilon l_L + u^{\dagger c}_{Ri} \beta^R_{S_1} e_R + {q^j_L}^\dagger \lambda_{S_1} \varepsilon {q^c_L}^k \varepsilon_{ijk}\big] = \\ & G_{S_1} \big[(l_L^\dagger \beta^{L\dagger}_{S_1} \varepsilon q^c_{Li})\, (q^{\dagger c}_{Li} \beta^L_{S_1} \varepsilon l_L) + (e^\dagger_R \beta^{R\dagger}_{S_1} u^c_{Ri})\, (q^{c\dagger}_{Li} \beta^L_{S_1} \varepsilon l_L) + (q_L^{c\dagger j} \lambda^\dagger_{S_1} \varepsilon q^k_L) \varepsilon_{ijk}\, ({q^l_L}^\dagger \lambda_{S_1} \varepsilon {q^c_L}^m) \varepsilon_{ilm} + \\ & (l_L^\dagger \beta^{L\dagger}_{S_1} \varepsilon q^c_{Li})\, (u^{\dagger c}_{Ri} \beta^R_{S_1} e_R) + (e^\dagger_R \beta^{R\dagger}_{S_1} u^c_{Ri})\, (u^{\dagger c}_{Ri} \beta^R_{S_1} e_R) + (BNV\, terms)\big]\end{aligned} \tag{4.14}$$

Here we can see four terms describing semileptonic contributions and a four-quarks term that can generate tree level amplitudes contributing to the meson mixing plus four more terms kept implicit that can violate Baryon Number conservation.

To project these terms on our operators basis we need to recall the relations obtained using Fierz Identities of EW group together with the ones of the Lorentz group, in particular we need C.2:

$$l^{1c\dagger}\varepsilon q^1\; q^{2\dagger}\varepsilon l^{2c} = \frac{1}{4}\big[(q^{2\dagger}\bar\sigma^\mu q^1)\,(l^{2\dagger}\bar\sigma_\mu l^1) - \;(q^{2\dagger}\bar\sigma^\mu\sigma^a q^1)\,(l^{2\dagger}\bar\sigma_\mu\sigma_a l^1)\big]$$

that projects the semileptonic term with LH fermions. Then to project the left-right semileptonic term we need to use that:

$$\varepsilon_{ij}\varepsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$$

obtaining two scalar operators that we recognize to be $\mathcal{O}_{LQ}$ and $-\mathcal{O}_S^{u\dagger}$.
In the end, the completely RH term turns to a vector-vector operator using

$$\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} = -\frac{1}{2}\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta}$$

obtaining $-\frac{1}{2}\mathcal{O}_R$.
Adding up all the terms we obtain the following effective Lagrangian:

$$\begin{aligned}\mathscr{L}_{EFT} \supset -G_{S_1}\Big[&\frac{1}{4}\beta^L_{S_1}\beta^{L\dagger}_{S_1}\mathcal{O}_S - \frac{1}{4}\beta^L_{S_1}\beta^{L\dagger}_{S_1}\mathcal{O}_T + [\beta^{R\dagger}_{S_1}\beta^L_{S_1}\mathcal{O}_{LQ} - \beta^{R\dagger}_{S_1}\beta^L_{S_1}\mathcal{O}^{u\dagger}_S + h.c.] - \frac{1}{2}\beta^{R\dagger}_{S_1}\beta^R_{S_1}\mathcal{O}^u_R \\ &+ (q^{cj\,\dagger}_{L}\lambda^\dagger_{S_1}\varepsilon q^k_L)\varepsilon_{ijk}\,(q_{Ll}^\dagger\lambda_{S_1}\varepsilon q^c_{Lm})\varepsilon^{ilm} + (BNVterms)\Big]\end{aligned} \tag{4.15}$$

Here we have generated different semileptonic contributions that contribute both to CC and NC processes and a term that contributes to tree level amplitude of meson mixing, plus some BNV terms that we neglected.
There is one substantial difference with the colour-less case, in fact here we can set $\lambda_{S_1} = 0$ without affecting semileptonic contributions and turn off both four-quarks and BNV terms.
The Wilson coefficients of semileptonic operators result to be

$$C_S = -C_T \equiv \frac{G_{S_1}}{4}\beta^{L\dagger}_{S_1}\beta^L_{S_1} \quad ,C_{LQ} = -C^{u\dagger}_S = G_{S_1}\beta^{R\dagger}_{S_1}\beta^L_{S1} \quad ,C_R = -\frac{1}{2}G_{S_1}\beta^{R\dagger}_{S_1}\beta^R_{S_1}$$

For what we said about U_1 is easy to see that the purely LH tree level contribution to $b \to sl^+l^-$, which is proportional to $C_S + C_T$, turns out to be null and that takes out from our toolbox a possible weapon to accommodate NC anomalies through a tree level matching.
Nevertheless as we will see S_1 is a good candidate for CC currents which appears in some UV scenarios.

$R_2 \sim (\mathbf{3}, \mathbf{2})_{7/6}$

The scalar doublet with $Y = 7/6$ couples to a left-right current to contract the $SU(2)_L$ index.
That means that it can't generate the purely LH operators $\mathcal{O}_S, \mathcal{O}_T$ and this is the reason why [17] neglects every LQ EW doublet.

The Lagrangian terms allowed are:

$$\mathscr{L}_{UV} \supset (D_\mu R_2)^\dagger D^\mu R_2 + M_{R_2}^2 R_2^\dagger R_2 + \left[g_{R_2} R_2 [u_R^\dagger \beta_{R_2}^l l_L \varepsilon + q_L^\dagger \beta_{R_2}^q e_R] + h.c.\right] \tag{4.16}$$

where replacing at low energies the equation of motion

$$\begin{aligned} \frac{\delta \mathscr{L}_{UV}}{\delta R_2^\dagger} &= 0 = M_{R_2}^2 R_2 - g_{R_2}\left[\varepsilon l_L^\dagger \beta_{R_2}^{l\dagger} u_R - e_R^\dagger \beta_{R_2}^{q\dagger} q_L\right] \rightarrow \\ R_2 &= \frac{g_{R_2}}{M_{R_2}^2}\left[\varepsilon l_L^\dagger \beta_{R_2}^{l\dagger} u_R - e_R^\dagger \beta_{R_2}^{q\dagger} q_L\right] \end{aligned} \tag{4.17}$$

we find

$$\mathscr{L}_{EFT} \supset -G_{R_2}\left[(u_R^\dagger \beta_{R_2}^l l_L)\,(l_L^\dagger \beta_{R_2}^{l\dagger} u_R) + (u_R^\dagger \beta_{R_2}^l l_L)\,\varepsilon\,(e_R^\dagger \beta_{R_2}^{q\dagger} q_L) - (q_L^\dagger \beta_{R_2}^{q\dagger} e_R)\,\varepsilon\,(l_L^\dagger \beta_{R_2}^{l\dagger} u_R) + (q_L^\dagger \beta_{R_2}^q e_R)\,(e_R^\dagger \beta_{R_2}^{q\dagger} q_L)\right] \tag{4.18}$$

where we used $\varepsilon \cdot \varepsilon = -\mathbf{1}$.
The second and the third terms are equal to $(\beta_{R_2}^l \beta_{R_2}^{q\;\dagger} \mathcal{O}_{LQ} + h.c.)$.
The first and the fourth terms can be transformed through

$$\delta_a^b \delta_c^d = \frac{1}{2} \sigma_{ac}^\mu \overline{\sigma}_\mu^{bc}$$

and, after 3 permutation of fermion fields that give us a factor $(-1)^3$, we obtain $-\frac{1}{2}\mathcal{O}_{LR1}$ and $-\frac{1}{2}\mathcal{O}_{LR2}^u$.
Adding up we obtain

$$\mathscr{L}_{EFT} \supset G_{R_2}\left[\frac{1}{2}\beta_{R_2}^l \beta_{R_2}^{l\dagger} \mathcal{O}_{LR2}^u - [\beta_{R_2}^l \beta_{R_2}^{q\dagger} \mathcal{O}_{LQ}^u + h.c.] + \frac{1}{2}\beta_{R_2}^q \beta_{R_2}^{q\dagger} \mathcal{O}_{LR1}\right] \tag{4.19}$$

Again we generated only semileptonic terms without further assumptions.
The Wilson coefficients result to be

$$C_{LR2}^u = \frac{1}{2} G_{R_2} \beta_{R_2}^l \beta_{R_2}^{l\dagger} \quad , C_{LQ} = -G_{R_2} \beta_{R_2}^q \beta_{R_2}^{q\dagger} \quad , C_{LR1} = \frac{1}{2} G_{R_2} \beta_{R_2}^l \beta_{R_2}^{q\dagger}$$

$\widetilde{R}_2 \sim (\mathbf{3}, \mathbf{2})_{1/6}$

We want to consider another possible value for the hypercharge of a scalar LQ EW-doublet: $\widetilde{R}_2 \sim (3, 2)_{1/6}$.
In this case the Lagrangian becomes:

$$\mathscr{L}_{UV} \supset M_{\widetilde{R}_2}^2 \widetilde{R}_2^\dagger \widetilde{R}_2 + (D_\mu \widetilde{R}_2)^\dagger D^\mu \widetilde{R}_2 + g_{\widetilde{R}_2} \widetilde{R}_2 (d_R^\dagger \beta_{\widetilde{R}_2} l_L + h.c.) \tag{4.20}$$

Once integrated out $\widetilde{R}_2$

$$\begin{aligned}\frac{\delta \mathscr{L}_{UV}}{\delta R_2^\dagger} =& M^2_{\widetilde{R}_2}\widetilde{R}_2 + g_{\widetilde{R}_2} l_L \beta^\dagger_{\widetilde{R}_2} d_R \rightarrow \\ \widetilde{R}_2 =& -\frac{g_{\widetilde{R}_2}}{M^2_{\widetilde{R}_2}} d_R^\dagger \beta_{\widetilde{R}_2} l_L\end{aligned}$$

which gives us the effective Lagrangian:

$$\mathscr{L}_{EFT} \supset -G_{\widetilde{R}_2}(l_L^\dagger \beta_{\widetilde{R}_2} d_R)(d_R^\dagger \beta_{\widetilde{R}_2} l_L)$$

Then using again

$$\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}} = \frac{1}{2}\sigma^\mu{}_{\alpha\dot{\alpha}}\bar{\sigma}_\mu^{\dot{\beta}\beta}$$

obtaining

$$\mathscr{L}_{EFT} \supset \frac{G_{\widetilde{R}_2}}{2}\mathcal{O}^d_{LR2} \rightarrow C^d_{LR2} \equiv \frac{1}{2}G_{\widetilde{R}_2}\beta^\dagger_{\widetilde{R}_2}\beta_{\widetilde{R}_2}$$

that, among the NP particles we consider, is generated only by $\widetilde{R}_2$.

$S_3 \sim (\boldsymbol{3}^*, \boldsymbol{3})_{1/3}$

The last heavy boson we want to compute the effect at low energy is the scalar EW triplet.
As U_3 it couples with a triplet current, hence it doesn't interact with RH fermions. The main difference with U_3 is that it allows quark-quark coupling, in fact the UV Lagrangian turns out to be:

$$\begin{aligned}\mathscr{L}_{UV} \supset& (D_\mu S_{3a})^\dagger D^\mu S_3^a + M^2 S_{3a}^\dagger S_3^a + \\ &+ g_{S_3} S_3^{ia}({q_L^j}^\dagger \lambda_{S_3}\varepsilon\sigma_a {q_L^c}^k \varepsilon_{ijk} + {q_{Li}^c}^\dagger \beta_{S_3}\varepsilon\sigma_a l_L) + h.c.\end{aligned} \tag{4.21}$$

in which $i = 1, 2, 3$ is the colour index and $a = 1, 2, 3$ is the EW index. Again the di-quark coupling could be turned off without affecting semileptonic operators, as for S_1.
Integrating it out

$$\frac{\delta \mathscr{L}_{UV}}{\delta {S_3^{ia}}^\dagger} = 0 = M^2_{S_3} S_{3ia} - g_{S_3}\big[{q_{Lj}^c}^\dagger \lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lk}\varepsilon^{ijk} + l_L^\dagger \beta^\dagger_{S_3}\sigma_a\varepsilon {q_L^c}^i\big] \rightarrow$$

$$S_{3ia} = \frac{g_{S_3}}{M^2_{S_3}}\big[{q_{Lj}^c}^\dagger \lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lk}\varepsilon^{ijk} + l_L^\dagger \beta^\dagger_{S_3}\sigma_a\varepsilon {q_L^c}^i\big]$$

hence the EFT Lagrangian results

$$\begin{aligned}\mathscr{L}_{EFT} \supset G_{S_3}\big[&({q_{Li}^c}^\dagger \beta_{S_3}\varepsilon\sigma_a l_L)\,(\bar{l}_L \beta^\dagger_{S_3}\sigma_a\varepsilon {q_L^c}^i) \;+\; ({q_{Li}^c}^\dagger \beta_{S_3}\varepsilon\sigma_a l_L)\,({q_{Lj}^c}^\dagger \lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lk})\varepsilon^{ijk} + \\ &({q_L^j}^\dagger \lambda_{S_3}\varepsilon\sigma_a {q_L^c}^k)\varepsilon_{ijk}\,(l_L^\dagger \beta^\dagger_{S_3}\sigma_a\varepsilon {q_L^c}^i) + \;({q_L^j}^\dagger \lambda_{S_3}\varepsilon\sigma_a {q_L^c}^k)\varepsilon_{ijk}({q_{Ll}^c}^\dagger \lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lm})\varepsilon^{ilm}\big]\end{aligned}$$

The first term is the only one affecting semileptonic processes at tree level. To project it on our basis we remember that we obtained C.3

$$l^{1c^\dagger} \varepsilon \sigma_a q^1 \; q^{2^\dagger} \sigma^a \varepsilon l^{2c} = \frac{1}{4}\left[3(q^{2\dagger}\bar{\sigma}^\mu q^1)\,(l^{2\dagger}\bar{\sigma}_\mu l^1) + \; (q^{2\dagger}\bar{\sigma}^\mu \sigma^a q^1)\,(l^{2\dagger}\bar{\sigma}_\mu \sigma_a l^1)\right]$$

which means

$$\begin{aligned}\mathscr{L}_{EFT} \supset G_{S_3}\Big[&\frac{3}{4}\beta^\dagger_{S_3}\beta_{S_3}\mathscr{O}_S + \frac{1}{4}\beta^\dagger_{S_3}\beta_{S_3}\mathscr{O}_T + \; q^{c\,\dagger}_{Li}\beta_{S_3}\varepsilon\sigma_a l_L \; q^{c\,\dagger}_{Lj}\lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lk}\varepsilon^{ijk} + \\ &(q^{j\,\dagger}_L \lambda_{S_3}\varepsilon\sigma_a q^{c\,k}_L)\varepsilon_{ijk}\,(l^\dagger_L\beta^\dagger_{S_3}\varepsilon\sigma_a q^{c\,i}_L) + \; (q^{j\,\dagger}_L \lambda_{S_3}\varepsilon\sigma_a q^{c\,k}_L\varepsilon_{ijk} q^{c\,\dagger}_{Ll}\lambda^\dagger_{S_3}\sigma_a\varepsilon q_{Lm})\varepsilon^{ilm}\Big]\end{aligned} \tag{4.22}$$

As already mentioned, meson mixing and BNV amplitudes would arise at tree level just for non zero values of the flavour matrix λ_{S_3}, that controls the di-quark couplings, that we can turn off for our purposes.
The Wilson coefficients of semileptonic terms are

$$C_S = 3C_T \equiv \frac{3}{4}G_{S_3}\beta^\dagger_{S_3}\beta_{S_3}$$

that means, a priori, contributions to both NC and CC processes.

4.5 Summary

We acknowledged the contributions to the high energy EFT given by the inclusion of different types of heavy bosons. The next step is to see what numerical values of the coefficients involved in the anomalous observables are needed to fit the anomalies.

Before implementing the numerical analysis we summarize what contributions are generated in the different scenarios in tables 4.1.

Comparison between 4-quark contributions generated

As we mentioned along the listing of heavy bosons some of them are, without furthermore assumptions on the theory, generating 4-quark operators that could affect the tested phenomena. In particular, since the flavour structure must allow third-second family transitions in quark sector, we need to be pay attention to the constraints on B_s meson mixing.

As we have seen, for Leptoquarks, is possible to turn to zero the quark quark coupling without affecting semileptonic contributions. On the other hand this is not true for colour-less vectors that generate 4-quark contributions of order $\simeq \lambda^{q^2}$ and semileptonic operators of order $\simeq \lambda^q\lambda^l$. Notice that λ_l cannot be arbitrarily large because that would affect 4 leptons operators that have coefficients $\simeq \lambda^{l^2}$.

We want to see if, using colour-less vectors to generate the desired semileptonic contributions, we can cancel the 4-quark contributions by means of the ones generated by

Table 4.1: Wilson coefficients corresponding to semileptonic effective operators generated by the integration of heavy vector bosons (above) and scalar bosons (below).

	B'	W'	U_1	U_3
C_S	$-2G_{B'}\lambda_B^q\lambda_B^l$	$\varnothing$	$-\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$-\frac{3}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_T	$\varnothing$	$-4G_{W'}\lambda_W^q\lambda_W^l$	$-\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$\frac{1}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_{LR1}	$-2G_{B'}\lambda_B^q\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^u	$-2G_{B'}\lambda_B^u\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^d	$-2G_{B'}\lambda_B^d\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^u	$-2G_{B'}\lambda_B^u\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$-2G_{B'}\lambda_B^d\lambda_B^e$	$\varnothing$	$-G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^R$	$\varnothing$
C_{LQ}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^u	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$2G_{U_1}\beta_{U_1}^R\beta_{U_1}^{L\dagger}$	$\varnothing$

	S_1	R_2	$\widetilde{R}_2$	S_3
C_S	$\frac{1}{4}G_{S_1}\beta_{S_1}^L\beta_{S_1}^{L\dagger}$	$\varnothing$	$\varnothing$	$\frac{3}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_T	$-\frac{1}{4}G_{S_1}\beta_{S_1}^{L\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$\frac{1}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_{LR1}	$\varnothing$	$-\frac{1}{2}G_{R_2}\beta_{R_2}^l\beta_{R_2}^{q\dagger}$	$\varnothing$	$\varnothing$
C_{LR2}^u	$\varnothing$	$\frac{1}{2}G_{R_2}\beta_{R_2}^{l\dagger}\beta_{R_2}^l$	$\varnothing$	$\varnothing$
C_{LR2}^d	$\varnothing$	$\varnothing$	$\frac{1}{2}G_{\widetilde{R}_2}\beta_{\widetilde{R}_2}^\dagger\beta_{\widetilde{R}_2}$	$\varnothing$
C_R^u	$\frac{1}{2}G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^R$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LQ}	$-G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$-G_{R_2}\beta_{R_2}^{q\dagger}\beta_{R_2}^q$	$\varnothing$	$\varnothing$
C_S^u	$G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

LQ.
To do that, we need the contributions to cancel each other precisely, because even if we keep contribution small at UV scale, RGE evolution down to the B meson scale would spoil the cancellation.

The Leptoquarks that are allowed to couple two quarks by $\mathcal{G}_{SM}$ are S_1 and S_3, and these couplings involve only LH quarks, hence we would like to see if we can write them in terms of the left left operators that are generated by W' and B':

$$\mathcal{O}_S^{4q} = (q_L^\dagger \bar{\sigma}_\mu q_L)\,(q_L^\dagger \bar{\sigma}^\mu q_L), \quad \mathcal{O}_T^{4q} = (q_L^\dagger \bar{\sigma}_\mu \sigma_a q_L)\,(q_L^\dagger \bar{\sigma}^\mu \sigma^a q_L). \tag{4.23}$$

That are respectively generated by B' with coefficent $-2G_{B'}\lambda_B^{q\,\dagger}\lambda_B^q$ and W' with coefficient $-2G_{W'}\lambda_W^{q\,\dagger}\lambda_W^q$, where we keep in mind that the only flavour component of $\lambda^q_{B/W}$ that needs to be non zero to affect anomalous observables is $(\lambda^q_{B/W})^{bs}$.
Instead, four-quarks operators generated by S_1 and S_3 are

$$\mathcal{O}_{S_1}^{4q} = (q^{cj}_L{}^\dagger \varepsilon q_L^k)\,(q_{Ll}^\dagger \varepsilon q^c_{Lm})\varepsilon_{ijk}\varepsilon^{ilm}, \quad \mathcal{O}_{S_3}^{4q} = (q_L^{\dagger j}\varepsilon\sigma_a q_L^{c\,k})(q^{c\,\dagger}_{Ll}\sigma_a \varepsilon q_{Lm})\varepsilon_{ijk}\varepsilon^{ilm} \tag{4.24}$$

with coefficients respectively $-G_{S_1}\lambda_{S_1}^\dagger\lambda_{S_1}$ and $-G_{S_3}\lambda_{S_3}^\dagger\lambda_{S_3}$.
The first thing we notice is that in Leptoquark case we generated operators the colour structure is non trivial while in $\mathcal{O}_S$ and $\mathcal{O}_T$ is left implicit because both currents are now colour singlets.

We can use the relations derived in C to write

$$\begin{aligned}
\mathcal{O}_{S_1}^{4q} &= \frac{1}{4}[(q_L^l{}^\dagger \bar{\sigma}^\mu q_L^k)\,(q_{Lj}^\dagger \bar{\sigma}_\mu q_{Lm}) \;-\; (q_L^l{}^\dagger \bar{\sigma}^\mu \sigma^a q_L^k)\,(q_{Lj}^\dagger \bar{\sigma}_\mu \sigma_a q_{Lm})]\varepsilon_{ijk}\varepsilon^{ilm} \\
\mathcal{O}_{S_3}^{4q} &= \frac{1}{4}[3(q_L^l{}^\dagger \bar{\sigma}^\mu q_L^k)\,(q_{Lj}^\dagger \bar{\sigma}_\mu q_{Lm}) \;+\; (q_L^l{}^\dagger \bar{\sigma}^\mu \sigma^a q_L^k)\,(q_{Lj}^\dagger \bar{\sigma}_\mu \sigma_a q_{Lm})]\varepsilon_{ijk}\varepsilon^{ilm}
\end{aligned}$$

Now using the property of *Levi-Civita* tensor:

$$\varepsilon_{ijk}\varepsilon^{ilm} = (\delta_j^l\delta_k^m - \delta_j^m\delta_k^l) \tag{4.25}$$

results

$$\begin{aligned}
\mathcal{O}_{S_1}^{4q} &= \frac{1}{4}(\mathcal{O}_S'^{4q} - \mathcal{O}_T'^{4q} - \mathcal{O}_S^{4q} + \mathcal{O}_T^{4q}) \\
\mathcal{O}_{S_3}^{4q} &= \frac{1}{4}(3\mathcal{O}_S'^{4q} + \mathcal{O}_T'^{4q} - 3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q})
\end{aligned} \tag{4.26}$$

where $\mathcal{O}_S'^{4q}$ and $\mathcal{O}_T'^{4q}$ are defined as:

$$\mathcal{O}_S'^{4q} = (q_L^i{}^\dagger \bar{\sigma}_\mu q_L^j)\,(q_{Lj}^\dagger \bar{\sigma}^\mu q_{Li}), \quad \mathcal{O}_T'^{4q} = (q_L^i{}^\dagger \bar{\sigma}_\mu \sigma_a q_L^j)\,(q_{Lj}^\dagger \bar{\sigma}^\mu \sigma^a q_{Li}). \tag{4.27}$$

To see if these two operators can be written as a linear combination of $\mathcal{O}_S^{4q}$ and $\mathcal{O}_T^{4q}$ the first step is to arrange them in a contraction of $SU(3)$-singlet currents using:

$$\bar{\sigma}^\mu{}_{\alpha\beta}\bar{\sigma}_{\mu_{\gamma\delta}} = -\bar{\sigma}^\mu{}_{\alpha\delta}\bar{\sigma}_{\mu_{\gamma\beta}}$$

and including the minus sign coming from the triple permutation of the fermion fields we get:

$$\mathcal{O}_S'^{4q} = [(q_{Li}^\dagger \bar{\sigma}_\mu q_{Lj})\,(q_{Lk}^\dagger \bar{\sigma}^\mu q_{Ll})]\delta_{il}\delta_{kj}, \quad \mathcal{O}_T'^{4q} = [(q_{La}^\dagger \bar{\sigma}_\mu \sigma_a q_{Lb})\,(q_{Lc}^\dagger \bar{\sigma}^\mu \sigma^a q_{Ld})]\sigma_{ad}\sigma_{bc}.$$

where this time we made explicit the $SU(2)_L$ structure, while the colour indexes are kept implicit because the two currents are $SU(3)_c$-singlets.

Then, we recall 3.8 to write

$$\begin{aligned}\delta_{il}\delta_{kj} &= \frac{1}{2}(\sigma_{aij}\sigma^a_{kl} + \delta_{ij}\delta_{kl}) \\ \sigma_{ail}\sigma^a_{kj} &= \frac{1}{2}(3\delta_{ij}\delta_{kl} - \sigma_{aij}\sigma^a_{kl}).\end{aligned}$$

Using these relations we obtain

$$\mathcal{O}_S'^{4q} = \frac{1}{2}[\mathcal{O}_S^{4q} + \mathcal{O}_T^{4q}], \quad \mathcal{O}_T'^{4q} = \frac{1}{2}[3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q}]. \tag{4.28}$$

and inserting them in 4.26 we obtain:

$$\begin{aligned}\mathcal{O}_{S_1}^{4q} &= \frac{1}{4}\Big[\frac{1}{2}(\mathcal{O}_S^{4q} + \mathcal{O}_T^{4q}) - \frac{1}{2}(3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q}) - \mathcal{O}_S^{4q} + \mathcal{O}_T^{4q}\Big] = \frac{1}{2}[\mathcal{O}_T^{4q} - \mathcal{O}_S^{4q}] \\ \mathcal{O}_{S_3}^{4q} &= \frac{1}{4}\Big[\frac{3}{2}(\mathcal{O}_S^{4q} + \mathcal{O}_T^{4q}) + \frac{1}{2}(3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q}) - 3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q}\Big] = 0.\end{aligned} \tag{4.29}$$

The result obtained means that S_3 does not generate any 4-quark operator.
Meanwhile the tree-level contributions generated by S_1 are a triplet operator and a singlet operator, the coefficients of which emerge with opposite sign. Hence these contributions do not affect $b\bar{s} \to s\bar{b}$ at tree-level, because the corresponding amplitude would be proportional to the combination of Wilson coefficients $C_S^{4q} + C_T^{4q}$.

In conclusion the 4-quark operators generated by the Leptoquarks considered cannot cancel the contribution to B_s meson mixing generated by B' and W'.

5

Phenomenological Analysis

To understand what is the proper way to set the analysis of what values of NP parameters can accommodate the anomalies we need first to analyze the clues given by the data.

The main issue of computing the Branching ratios of the semileptonic processes we are interested in is that we need, apart from the EW nature of the processes, to consider that the quark involved are *confined* in hadrons.
From now on we will use as example the NC anomalous decay $B \to K^* l^+ l^-$, nevertheless the discussion can be easily applied on every $M \to M' l_1 l_2$ process, where M, M' are mesons and $l_{1,2}$ are leptons.

The decay width of $B \to K^* l^+ l^-$ is given by:

$$\Gamma(B \to K^* l^+ l^-) = \frac{1}{2m_B} \int |\mathscr{M}|^2 \frac{d^3 p_{K^*}}{(2\pi)^3 \, 2E_{K^*}} \frac{d^3 p_{l^+}}{(2\pi)^3 \, 2E_{l^+}} \frac{d^3 p_{l^-}}{(2\pi)^3 \, 2E_{l^-}}$$

Where $\mathscr{M}$ is defined to be

$$\mathscr{M} \equiv (-i) \langle B | \mathscr{O} | K^* l^+ l^- \rangle$$

in which $\mathscr{O}$ is a given operator that at three level corresponds to the sum of Lagrangian operators that can annihilate a b quark and create a s quark and a pair of charged lepton anti-lepton.
If that operator can be factorized in a piece that contains only lepton fields and another one containing only quarks,

$$\mathscr{O} = \mathscr{O}_q(\{q\}) \cdot \mathscr{O}_l(\{l\})$$

the transition matrix element results:

$$\mathscr{M} = (-i) \langle B | \mathscr{O}_q | K^* \rangle \langle 0 | \mathscr{O}_l | l^+ l^- \rangle$$

i.e. the transition matrix is the product of a leptonic perturbative term which depends only from the on of the operator and an *hadronic matrix element* which at B-meson mass scale gets big QCD contributions and so is not calculable perturbatively.

These factors are generally a bad deal because the non perturbativity of QCD limits the accuracy of theoretical predictions for these decay widths. Nevertheless if we want to compare processes where the hadronic contribution to the amplitude is given by operators with the same $\mathcal{O}_{had}$ it is possible to simplify the non perturbative contribution expressing observables in terms of ratios of branching ratios as we previously mentioned in chapter 1.

Even if the QCD is non perturbative at B mesons mass scale sometimes it is possible to link different form factors without compute them explicitly but just using general symmetries of strong interactions. Since QCD describes vector-like interactions we will *express from now on fermion fields in Dirac representation* through the mapping shown in appendix A.

5.1 Left Handed framework

In the SM, because of the theorem 2, flavour breaking processes get their leading contributions from LH quarks. Since any process addressed to B-Physics Anomalies is flavour breaking, a good guess is to assume that NP should involve only LH quark as well.
For what concern leptons we have that for $q^2 = (p_{l^+} + p_{l^-})^2 \in [1.1; 6.0] GeV^2$ the vector-like QED contribution is suppressed by the photon propagator $\propto \frac{1}{q^2}$ and, as previously said in section 2.2, the weak contribution to RH fermions is suppressed again because the main contribution comes from $W^{\pm}$ exchange plus the accidental smallness of θ_W. As a result every contribution given to the RH-RH lepton current is not interfering with SM one in the limit of massless leptons and hence enters in the calculations of the observables only with a quadratic term. That would require a big Wilson coefficient for NP RH-RH contributions in order to fit the experimental anomalies creating possible tensions with the rest of semileptonic observables tested so far.

Even we keep all the operators containing a LH lepton current, we are left with $\mathcal{O}_S$, $\mathcal{O}_T$, $\mathcal{O}^d_{LR2}$ where O_T is the only one that can contribute to CC processes. We are going to show another clue that pushes in the direction of a purely LH NP -also for quarks- given by the ratio R_{K^*}/R_K.

R_K and R_{K*}

From the point of view of the EW transition there is no difference between the decay channel in $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ since the quark transition is always $b \to s l^+ l^-$. In the limit in which leptons take the biggest part of the available energy (in the way we can neglect kinematic effects due to the mass difference between μ and e), the difference between the two ratios is hidden in the hadronic part of the matrix element:

$$\frac{\mathcal{B}(B \to K^* l^+ l^-)}{\mathcal{B}(B \to K l^+ l^-)} = \frac{|\langle B| C^K_{SM} \mathcal{O}^q_{SM} + \sum_i C_i \mathcal{O}_i |K^*\rangle|^2}{|\langle B| C^K_{SM} \mathcal{O}^q_{SM} + \sum_i C_i \mathcal{O}_i |K\rangle|^2}$$

where $\mathcal{O}_i$ are the different possible hadronic part of BSM operators that allow the $b \to s l^+_L l^-_L$ transition and $\mathcal{O}^q_{SM}$ is the hadronic part of the operator that describes $b \to s l^+ l^-$ generated by the matching to the SM.

First, since we want the leptons pair to be LH due to the angular momentum conservation, the hadronic part of the operator in the limit of massless leptons has to be a vector under Lorentz group. Hence the most general form of $\sum_i C_i \mathcal{O}_i$ is given by $\bar{b}\gamma^\mu(\alpha_{NP} + \beta_{NP}\gamma^5)s \equiv \alpha_{NP}\mathcal{O}_V + \beta_{NP}\mathcal{O}_A$ where

$$\mathcal{O}_V \equiv \bar{b}\gamma^\mu s, \quad \mathcal{O}_A \equiv \bar{b}\gamma^\mu\gamma^5 s.$$

Second, we use the fact that K is a pseudoscalar meson, while $K*$ is a vector. Since the two operators $\mathcal{O}_V$ $\mathcal{O}_A$ have opposite parity, as shown appendix B, just the parity of the two operators tells us that in general the matrix elements are

$$\begin{aligned} \langle B| \mathcal{O}_A |K^*\rangle &\equiv A(q^2), \quad \langle B| \mathcal{O}_V |K^*\rangle = 0 \\ \langle B| \mathcal{O}_V |K\rangle &\equiv V(q^2), \quad\ \ \langle B| \mathcal{O}_A |K\rangle = 0 \end{aligned}$$

where we have used that the parities of the meson states are $J^P(B, K, K^*) = (0^-, 0^-, 1^-)$ and the form factors $A(q^2)$ and $V(q^2)$ are unknown functions of the exchanged momentum $q^2 = (p_B - p_{K^{(*)}})^2$.
Now if we consider the ratio between

$$R_{K^*} = \left.\frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)}\right|_{SM} = 1 \quad and \quad R_K = \left.\frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}\right|_{SM} = 1$$

that we call $\chi_K \equiv R_{K^*}/R_K$, it is clearly equal to 1 in the SM since both the ratios are 1. When we include NP effects that affect only the muon channel the expectation value is done on

$$C^K_{SM}[\mathcal{O}_V - \mathcal{O}_A] + \alpha_{NP}\mathcal{O}_V + \beta_{NP}\mathcal{O}_A$$

when we have used theorem 2 to write the SM leading contribution as purely LH.

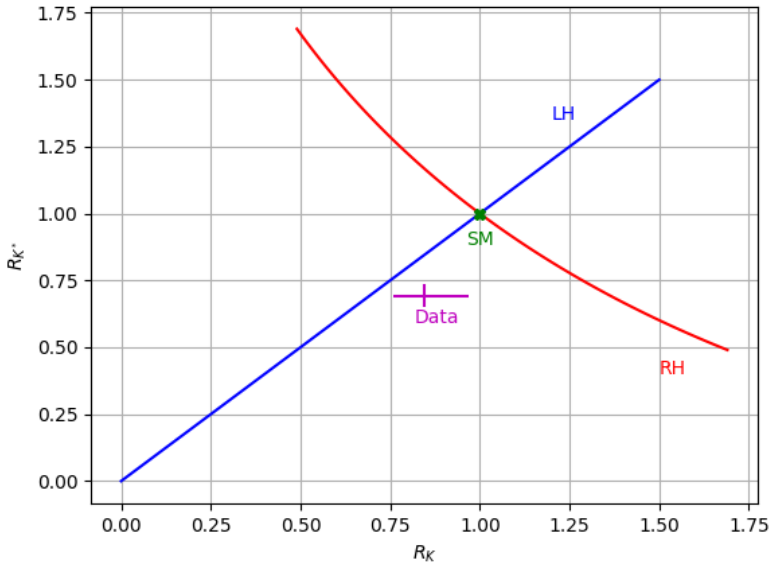
Generally, the ratio χ_K in a NP scenario realized as explained above is given by:

$$\begin{aligned} \chi_K = \frac{R_{K^*}}{R_K} &= \frac{|(-C^K_{SM} + \beta_{NP})A|^2}{|[C^K_{SM} + \alpha_{NP} + (\beta_{NP} - \beta_{NP})]V|^2}\frac{|C^K_{SM}V|^2}{|C^{K^*}_{SM}A|^2} \\ &= \left|\frac{\beta_{NP} - C^K_{SM}}{(\beta_{NP} - C^K_{SM}) + \alpha_{NP} + \beta_{NP}}\right|^2 \end{aligned} \tag{5.1}$$

where in the last line we made explicit that for LH NP, i.e. $\alpha_{NP} = -\beta_{NP}$, χ_K remains equal to 1 for any value of $\alpha_N P = -\beta_{NP}$, and the values of R_K, R_{K^*} are completely correlated. Otherwise if $\alpha_{NP}+\beta_{NP} \neq 0$ one of the two ratio could get bigger according to the relative sign between $\alpha_{NP} + \beta_{NP}$ and C^K_{SM}.

In Figure 5.1 we see how the data better align to a framework in which only LH quark are involved in NP interactions and so, from now on, we neglect $\mathcal{O}^d_{LR2}$ keeping only the contributions coming from $\mathcal{O}_S$ $\mathcal{O}_T$ limiting our analysis exclusively to LH interactions, significantly reducing the number of free parameters involved.

Figure 5.1: Plot of R_{K*} and R_K varying the vector and axial New Physics contributions α_{NP} and β_{NP} in the Left-Handed limit $\alpha_{NP} = -\beta_{NP}$ (blue) and in the Right-Handed limit $\alpha_{NP} = \beta_{NP}$ (red).



Anomalous observables in LH framework

The B-Physics anomalies we consider appear in the ratios listed in the tables 1.1-1.2. Since any Wilson coefficient in the Lagrangian 3.7 has dimension of $[Energy]^{-2}$ we choose to express them in unit of $v^{-2} = \sqrt{2}G_F = (246GeV)^{-2}$.

In the scenario in which NP couples only LH fermions it is easy to compute the predictions for the observables considered because, as should be clear now, non perturbative contributions are simplified in the ratio and we need only to add the proper Wilson coefficients to the SM ones.

The matching to SM tells us that the Wilson coefficient of the the operator $\bar{b}_L\gamma^\mu s_L\,\bar{l}_L\gamma_\mu l_L$ is given by

$$C^{NC}_{SM} = C^l_9 - C^l_{10} = -V_{tb}V^*_{ts}\frac{e^2}{8\pi^2}\cdot(8.314)\ v^{-2} \tag{5.2}$$

as reported e.g. by [10].
The ratio R_K, mindful of the electric charge eigenstates form of operators 3.1, results

$$\begin{aligned} R_K \equiv \frac{\mathcal{B}(B\to K\mu^+\mu^-)}{\mathcal{B}(B\to Ke^+e^-)} &= \frac{|C^K_{SM} + C^{bs\mu\mu}_S + C^{bs\mu\mu}_T|^2}{|C^K_{SM}|^2} \\ &= |1 + \frac{C^{bs\mu\mu}_S + C^{bs\mu\mu}_T}{C^K_{SM}}|^2 \end{aligned} \tag{5.3}$$

which depends only from the parameter $\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu}$.
Since we are allowing only LH contributions the simplification of the hadronic contribution in the ratio tells us that all the ratios listed in table 1.1 can be expressed as 5.3.

For what concerns CC the anomalous observables R_{D*} and R_D have also the same expression that gets NP contributions only from triplet operator O_T. Mindful of the relative coefficients of different component of LH doublets due to the gauge symmetry (see equation 3.1), we find:

$$\begin{aligned} R_D &\equiv \frac{\mathscr{B}(B \to D^* \tau\bar{\nu})}{\frac{1}{2}\left[\mathscr{B}(B \to D^* e\bar{\nu}) + \mathscr{B}(B \to D^* \mu\bar{\nu})\right]} = \\ &= \frac{[|C_{SM}^{CC} + 2C_T^{bs\tau\tau}V_{cs}^* + 2C_T^{bb\tau\tau}V_{cb}^*|^2 + |2C_T^{bs\tau\mu}V_{cs}^* + 2C_T^{bb\tau\mu}V_{cb}^*|^2\ ]\eta_\tau^D}{\frac{1}{2}\left[|C_{SM}^{CC}|^2 + |C_{SM}^{CC} + 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^*|^2 + |2C_T^{bs\mu\tau}V_{cs}^* + 2C_T^{bb\mu\tau}V_{cb}^*|^2\right]} \end{aligned} \tag{5.4}$$

where η_τ^D is defined in 1.8 and, apart from that which is fixed, we introduced several new free parameters.
Neglecting contributions that enter only at a quadratic level in the expression 5.4 we find

$$R_D = \frac{|C_{SM}^{CC} + 2C_T^{bs\tau\tau}V_{cs}^* + 2C_T^{bb\tau\tau}V_{cb}^*|^2\ \eta_\tau^D}{\frac{1}{2}\left[|C_{SM}^{CC}|^2 + |C_{SM}^{CC} + 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^*|^2\right]}. \tag{5.5}$$

In this case the SM matching gives us (see section 2.2)

$$C_{SM}^{CC} = -2V_{cb}^*\ v^{-2}. \tag{5.6}$$

When we are going to fit these anomalies we need to reduce the number of free parameters and we would like to do that without doing additional assumptions.

A possible way to reduce them is to look at non anomalous observables that would get an important contribution by the Wilson coefficients generated in the NP scenarios considered, and then use the experimental data to bound the corresponding NP coefficients.

5.2 Constraining non anomalous observables

Another powerful tool that can be used to reduce the free parameters of the NP model and to bound the NP contributions is to consider the constraints on already tested observables that do not show deviations from the SM predictions.

$\mathscr{B}(B \to K^*\bar{\nu}\nu)$

The $B \to K^*\bar{\nu}\nu$ decay is linked with the so mentioned $B \to K^* l^+ l^-$ by gauge symmetry. Besides $B \to K^*\bar{\nu}\nu$ differs for the fact that it has not been observed yet, and so only upper bounds are available from the experiments.

From [5] we get an upper limit at @95% CL and its relation with the SM prediction that is given by:

$$\mathscr{B}(B \to K^* \bar{\nu}\nu)_{exp} < 5.2\ \mathscr{B}(B \to K^* \bar{\nu}\nu)_{SM}. \tag{5.7}$$

On the other hand, in term of EFT Wilson coefficients, the branching ratio reads

$$\mathscr{B}(B \to K^* \bar{\nu}\nu)_{NP} = \mathrm{A}\left[|C^{\nu}_{SM}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\mu}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\tau}|^2\right] \tag{5.8}$$

where A is a given constant that includes the kinematic and the sum on Lorentz indexes and C^{ν}_{SM} which is flavour universal because include only SM interactions is equal to

$$C^{\nu}_{SM} = 12.8\ V^*_{ts} V_{tb} \frac{e^2}{8\pi^2}\ v^{-2} \tag{5.9}$$

as reported by [17], and $\delta C^{\nu_\mu(\nu_\tau)}$ include the NP coefficients for $b \to s\nu_\mu\bar{\nu}_\mu(\nu_\tau\bar{\nu}_\tau)$.
Using again the electric eigenstates form of 3.1 the NP contributions are $\delta C^{\nu_\mu} = C^{bs\mu\mu}_S - C^{bs\mu\mu}_T$ and $\delta C^{\nu_\tau} = C^{bs\tau\tau}_S - C^{bs\tau\tau}_T$.

The value of $\mathscr{B}(B \to K^* \bar{\nu}\nu)_{NP}$ has to not exceed the experimental limit, hence for 5.7

$$\begin{aligned} &\mathrm{A}\left[|C^{\nu}_{SM}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\mu}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\tau}|^2\right] < 5.2\mathscr{B}(B \to K^* \bar{\nu}\nu)_{SM} = 5.2 \cdot 3\mathrm{A}|C_{SM}|^2 \\ &\qquad \to \left|1 + \frac{\delta C^{\nu_\mu}}{C^{\nu}_{SM}}\right|^2 + \left|1 + \frac{\delta C^{\nu_\tau}}{C^{\nu}_{SM}}\right|^2 < 14.6 \end{aligned} \tag{5.10}$$

Here we could have different scenarios according to the relative intensity of the two contributions to μ and τ channels. But anyway we can do some general considerations. The biggest value that both the NP contributions can singularly assume is given by the case in which one of the two δC is equal to $-C_{SM}$. The other one has in any case to respect

$$|1 + \frac{\delta C}{C_{SM}}|^2 < 14.6 \to |1 + \frac{\delta C}{C_{SM}}| < \sqrt{14.6} \simeq 3.8 \tag{5.11}$$

and that is a limit for both coefficients which can never be exceeded. In fact it is easy to see that for any $\delta C^{\nu_\mu}(\delta C^{\nu_\tau}) \neq -C_{SM}$ the upper limit on $\delta C^{\nu_\tau}(\delta C^{\nu_\mu})$ would be lower.

That does not give direct constraint on Wilson coefficients, because there is a flat direction of δC given by the trasformation $(C_S, C_T) \to (C_S + x, C_T + x)$. Nevertheless if we manage to set a bound $\delta C^{NC} = C^{bs\mu\mu}_S + C^{bs\mu\mu}_T$ from the numerical analysis we can derive information on the magnitude of $C^{bs\mu\mu}_S, C^{bs\mu\mu}_T$.

$R^{\mu e}_{D^*}$

Here we look at a relative of R_{D^*} that compares the channel in muons with the channel in electrons.

The expression at leading order (neglecting again LF breaking contributions) is given by

$$\begin{aligned} R_{D^*}^{\mu e} =& \frac{\mathscr{B}(B \to D^* \mu \bar{\nu})}{\mathscr{B}(B \to D^* e \bar{\nu})} = \\ =& \frac{|C_{SM}^{CC} + \delta C^{CC\mu}|^2}{|C_{SM}^{CC}|^2} = \left|1 + \frac{\delta C^{CC\mu}}{C_{SM}^{CC}}\right|^2 \end{aligned} \tag{5.12}$$

where $\delta C^{CC\mu} = 2C_T^{bs\mu\mu} V_{cs}^* + 2C_T^{bb\mu\mu} V_{cb}^*$.
We know from [5] that $R_{D^*}^{\mu e}$ is a good test for LFU between the first two families.

$$R_{D^*}^{\mu e}|_{exp} = 1.000 \pm 0.021 \tag{5.13}$$

and that gives strong constraints on $\delta C^{CC\mu} = 2C_T^{bs\mu\mu} V_{cs}^* + 2C_T^{bb\mu\mu} V_{cb}^*$ that appears in the denominator of 5.5.
Neglecting the $\mu\mu$ terms compared to the $\tau\tau$ ones, allows us to approximate the ratio 5.5 to the simpler expression

$$\begin{aligned} R_D \simeq & \frac{|C_{SM}^{CC} + 2C_T^{bs\tau\tau} V_{cs}^* + 2C_T^{bb\tau\tau} V_{cb}^*|^2 \, \eta_\tau^D}{|C_{SM}^{CC}|^2} = \\ = & \left|1 + \frac{\delta C^{CC}}{C_{SM}^{CC}}\right|^2 \cdot \eta_\tau^D. \end{aligned} \tag{5.14}$$

that depends from the only parameter $\delta C^{CC} \equiv V_{cb}^* [2C_T^{bs\tau\tau} \frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}]$.

5.3 Numerical Analysis of the Anomalies

The next step is to find what values of the Wilson coefficients generated we need to accommodate the anomalies.

In Table 1.1 we have listed some anomalous observables belonging NC transitions. As we have already mentioned the expression 5.3 is valid only if the energy of the two leptons is greater than 1 GeV so that we can neglect phase space factor and electromagnetic contributions.

Our analysis is not a test of SM, because to do that we would need to include many other observables and the anomaly could probably being hidden by the values of other non anomalous observables.
If we use SM as starting point we can perform a likelihood-ratio test asking if the NP model considered fits the observables that we say to be anomalous better than SM itself does. The only degree of freedom of the fit is the free parameter $\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu}$, motivated by the discussion above.

Implementing a least squares fit we find that

$$
\begin{aligned}
&\delta C^{NC}|_{best} = 0.69\,\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\, v^{-2} \qquad \chi^2/ndof = 2.23 \\
&(0.60, 0.79)\,\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\, v^{-2} \quad whitin\ \ 1\sigma \\
&(0.51, 0.88)\,\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\, v^{-2} \quad whitin\ \ 2\sigma
\end{aligned}
\tag{5.15}
$$

hence the anomaly can be accommodated reducing C_{SM} reported in 5.2 of approximately 10%.

The point corresponding to SM ($\delta C^{NC} = 0$) compared to the minimum χ^2, is placed at $\sqrt{\Delta\chi^2} = \sqrt{\chi^2_{SM} - \chi^2_{min}} = 7.86$ showing that NP model can fit the considered observables way better than SM does.

The parameter δC^{CC} that appears in the expression 5.14 is constituted by different flavour components to the ones that constitute δC^{NC} and hence we are going to do an independent analysis to find the limits on that parameter.

Since $\eta_\tau^{D^{(*)}}$ are given by the SM predictions provided by [21] of $R_{D^{(*)}}$ we will fit the variables given by

$$
\frac{R_{D^{(*)}}}{\eta_\tau^{D^{(*)}}} = \left|1 + \frac{\delta C^{CC}}{C^{CC}_{SM}}\right|^2
\tag{5.16}
$$

such that we can include in the fit the theoretical uncertainty.
In both D and D^* cases clearly the SM prediction for the ratios is equal to 1 by definition.

Setting C^{CC}_{SM} at the value 5.6 we find that the least square is found for

$$
\begin{aligned}
&\delta C^{CC}|_{best} = -0.14\; V_{cb}^*\, v^{-2} \qquad \chi^2/ndof = 0.042 \\
&(-0.20, -0.08)\; V_{cb}^*\, v^{-2} \quad whitin\ \ 1\sigma \\
&(-0.26, -0.02)\; V_{cb}^*\, v^{-2} \quad whitin\ \ 2\sigma
\end{aligned}
\tag{5.17}
$$

where we find again that the anomaly is order 10% of C^{CC}_{SM}. This time the SM ($\delta C^{CC} = 0$) is placed at $\sqrt{\Delta\chi^2} = \sqrt{\chi^2_{SM} - \chi^2_{min}} = 2.31$ which means that this anomaly is less significant than the NC case, as we can see in figure 5.2.

Now that we have numerical values for NP coefficients we are ready to see what type of UV physics we need to generate these contributions.

5.4 Selection of possible UV scenarios

In this section we are going to compare numerical results found in the last section with the effective contributions generated through the UV matching done in chapter 4

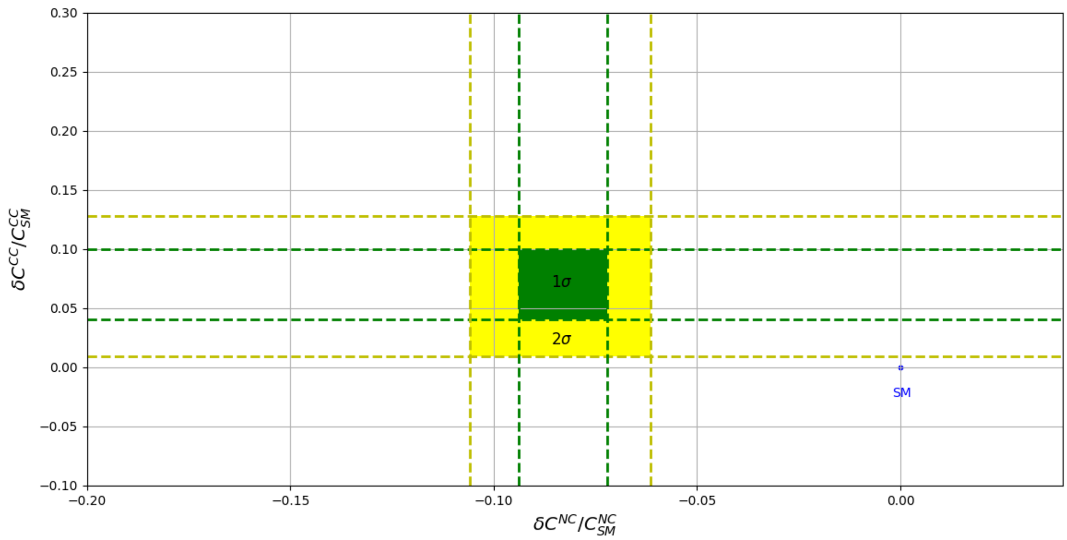


Figure 5.2: Values if the NP Wilson coefficients within 1σ (yellow) and 2σ (green).

and summarized in table 4.1.

The single flavour components that enters in the anomalous observables are $\{C_S^{bs\mu\mu}, C_T^{bs\mu\mu}, C_T^{bb\tau\tau}, C_T^{bs\tau\tau}\}$ and we have to stay mindful of bounds on $C_S^{bs\tau\tau} - C_T^{bs\tau\tau}$ given by $B \to K^* \nu\bar{\nu}$.

At 1σ level the values we need are:

$$\begin{aligned}
\delta C^{NC} &= C_S^{bs\mu\mu} + C_T^{bs\mu\mu} \in (0.60, 0.79)\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\, v^{-2} \\
\delta C^{CC} &= V_{cb}^*[2C_T^{bs\tau\tau}\frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}] \in (-0.20, -0.08) V_{cb}^*\, v^{-2} \\
&\left|1+\frac{\delta C^{\nu_\mu}}{C^{\nu}_{SM}}\right|^2 + \left|1 + \frac{\delta C^{\nu_\tau}}{C^{\nu}_{SM}}\right|^2 < 14.6
\end{aligned} \tag{5.18}$$

where the most statistically significant deviation is given by δC^{NC}.

In table 5.1 we have listed the contributions to the observables in terms of $\{\delta C^{NC}, \delta C^{CC}, \delta C^{\nu_\mu}, \delta C^{\nu_\tau}\}$ given by the heavy bosons described in chapter 4. We did not include the doublets R_2 and $\widetilde{R}_2$ because they are allowed to couple only L-R currents by $\mathcal{G}_{SM}$ and so they cannot generate $\mathcal{O}_{S,T}$.

Referring to tables 4.1 and 5.1 we can see what type of scenario, characterized by the insertion of one or more heavy bosons can accommodate the anomalous observables generating the contributions listed in equations 5.18.
We will also see if without setting to 0 UV parameters by hand -i.e. considering non zero values for all the entries in the tables 4.1- it is possible to cancel all contributions

	δC^{NC}	δC^{CC}	$\delta C^{\nu_\mu(\nu_\tau)}$
B'	$-2G_{B'}(\lambda_B^q)^{bs}(\lambda_B^l)^{\mu\mu}$	$\emptyset$	$-2G_{B'}(\lambda_B^q)^{bs}(\lambda_B^l)^{\mu\mu(\tau\tau)}$
W'	$-4G_{W'}(\lambda_W^q)^{bs}(\lambda_W^l)^{\mu\mu}$	$-8G_{W'}V_{cb}^*(\lambda_{W'}^l)^{\tau\tau}((\lambda_W^q)^{bb}\frac{V_{cs}^*}{V_{cb}^*}+(\lambda_W^q)^{ss})$	$4G_{W'}(\lambda_W^q)^{bs}(\lambda_W^l)^{\mu\mu(\tau\tau)}$
U_1	$-G_{U_1}(\beta_{U_1}^L)^{b\mu}(\beta_{U_1}^{L*})^{s\mu}$	$-G_{U_1}V_{cb}^*(\beta_{U_1}^L)^{b\tau}[(\beta_{U_1}^{L*})^{s\tau}\frac{V_{cs}^*}{V_{cb}^*}+(\beta_{U_1}^{L*})^{b\tau}]$	$\emptyset$
U_3	$-G_{U_3}(\beta_{U_3})^{b\mu}(\beta_{U_3}^*)^{s\mu}$	$G_{U_3}V_{cb}^*(\beta_{U_3})^{b\tau}[(\beta_{U_3}^*)^{s\tau}\frac{V_{cs}^*}{V_{cb}^*}+(\beta_{U_3}^*)^{b\tau}]$	$-2G_{U_3}(\beta_{U_3})^{b\mu}(\beta_{U_3}^*)^{s\mu}$
S_1	$\emptyset$	$-\frac{1}{2}G_{S_1}V_{cb}^*(\beta_{S_1}^L)^{b\tau}[(\beta_{S_1}^{L*})^{s\tau}\frac{V_{cs}^*}{V_{cb}^*}+(\beta_{S_1}^{L*})^{b\tau}]$	$G_{S_1}(\beta_{S_1})^{b\mu}(\beta_{S_1}^{L*})^{s\mu}$
S_3	$G_{S_3}(\beta_{S_3})^{b\mu}(\beta_{S_3}^*)^{s\mu}$	$G_{S_3}V_{cb}^*(\beta_{S_3})^{b\tau}[(\beta_{S_3}^*)^{s\tau}\frac{V_{cs}^*}{V_{cb}^*}+(\beta_{S_3}^*)^{b\tau}]$	$\frac{1}{2}G_{S_3}(\beta_{S_3})^{b\mu}(\beta_{S_3}^*)^{s\mu}$
	$(0.60, 0.79)\frac{e^2}{8\pi^2}V_{ts}^*V_{tb}\,v^{-2}$	$(-0.20;-0.08)V_{cb}^*v^{-2}$	$< 3.8C_{SM}^{\nu}$

Table 5.1: Parameters generated from the integration of heavy bosons, respect to table 4.1 we have selected the combinations involved in the observables considered.

to semileptonic operators different from $\mathcal{O}_S$ and $\mathcal{O}_T$, that we assume to be bounded by observables we aren't considering, and eventually to four-quarks operators, besides for what we said in section 4.5 is not possible to cancel them each other precisely with the heavy bosons we are considering.

The EW-singlets Leptoquarks: U_1 & S_1

The first possibility we consider is to accommodate the anomalies with the insertion of the only boson U_1.
We can assume the coupling to RH current β_R to be small (as it happens for the $SU(2)_L$-singlet of the SM) and hence neglect the coefficients of $\mathcal{O}_R^d$ and $\mathcal{O}_S^d$.
Thus we remain with δC^{NC} and δC^{CC} that can be set respectively positive and negative independently according to limits in 5.18 without creating tensions with $\mathcal{B}(B \to K^*\nu\nu)$ as highlighted in table 5.1.
Here the values of the β matrix has to be such that $\beta_{U_1}^{b\mu}\beta_{U_1}^{*\ s\mu} < 0$ and, since we have that for any value of $\beta_{U_1}^{b\tau}$ the addend $\beta_{U_1}^{b\tau}\beta_{U_1}^{*\ b\tau}$ turns to be positive and so we need a value for $\beta_{U_1}^{b\tau}\beta_{U_1}^{*\ s\tau}$ such that $\beta_{U_1}^{b\tau}[\frac{V_{cs}^*}{V_{cb}^*}\beta_{U_1}^{*\ s\tau} + \beta_{U_1}^{*\ b\tau}] > 0$.

The automatic cancellation of contributions to neutrinos decay is one of the reason that makes U_1 one of the most popular scenarios in literature. Plus, it avoids automatically some radiative constraints from EWPT and from the *high:p_t* searches considered in the analysis performed by [8, 9, 17]. Plus, it appears in many UV completions as shown by [9, 17, 22].

S_1, instead gives no tree level contributions to NC anomalies but it does generate a $\delta C^{CC} \neq 0$ (see table 5.1) and it also contributes to $b \to s\nu\nu$. A possibility is to include S_1 and U_1 together giving an additional contribution to $b \to c\tau\bar{\nu}_\tau$ that can help to accommodate $R_{D^{(*)}}$ within the values allowed by the limits on $\mathcal{B}(B \to K^*\nu\nu)$.

The EW-triplets Leptoquarks U_3 & S_3

Introducing a scalar and a vector with the same quantum numbers under $\mathcal{G}_{SM}$ (as we would do incluing the two EW-singlets U_1 & S_1) lends itself well to be implemented in an UV completion characterized by a strongly coupled sector at high energy with a global approximated chiral symmetry $SU(N)_L \otimes SU(N)_R$ spontaneusly broken at EW scale to the vectorial subgroup $SU(N)_V$.
In this scenario there are scalar and vector bosons emerging with the same quantum number, the first as pseudo Nambu-Goldstone Bosons (pNGB) and the second as composite state excited by the unbroken Noether's currents of $SU(2)_V$, in analogy with the π and ρ of the chiral symmetry of QCD [23]. In that case the contribution of the scalar would be generally bigger because of the lower mass protected by the unbroken symmetry.

Including U_3 or S_3 in the Lagrangian we would get the same tension with $\mathcal{B}(B \to K^*\nu\nu)$. In fact being in both cases $\delta C^{\nu_\mu} \propto \delta C^{NC}$ to accommodate NC anomalies we have also to contribute to $b \to s\nu_\mu\bar{\nu}_\mu$; nevertheless with a general flavour structure it is possible to include the CC deviations without exceeding limits on δC^{ν_μ} as shown by [8].
The biggest tension comes out when we try to accommodate CC anomalies. In fact both for U_3 and S_3, to compensate the $\beta^{b\tau}\beta^{*b\tau} > 0$ we need a big $\beta^{b\tau}\beta^{*s\tau}$ to get a negative δC^{CC} as requested by 5.18 and that corresponds to an important contribution to δC^{ν_τ}.

Including simultaneously both of them this tension can be avoided. In fact the total contributions result to be

$$\begin{aligned}
\delta C^{NC} &= -G_{U_3}\beta_{U_3}{}^{b\mu}\beta^{*}_{U_3}{}^{s\mu} + G_{S_3}\beta_{S_3}{}^{b\mu}\beta^{*}_{S_3}{}^{s\mu} \\
\delta C^{CC} &= V^*_{cb}[G_{U_3}(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}_{U_3}\beta^{*}_{U_3}{}^{s\tau} + \beta^{b\tau}_{U_3}\beta^{*}_{U_3}{}^{b\tau}) + G_{S_3}(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}_{S_3}\beta^{*}_{S_3}{}^{s\tau} + \beta^{b\tau}_{S_3}\beta^{*}_{S_3}{}^{b\tau})] \\
\delta C^{\nu_\mu(\nu_\tau)} &= -2G_{U_3}\beta_{U_3}{}^{b\mu(\tau)}\beta^{*}_{U_3}{}^{s\mu(\tau)} + \frac{1}{2}G_{S_3}\beta_{S_3}{}^{b\mu(\tau)}\beta^{*}_{S_3}{}^{s\mu(\tau)}
\end{aligned} \tag{5.19}$$

We consider the case in which the flavour structure is the same, i.e. $\beta_{U_3} = \beta_{S_3} \equiv \beta$ (which is reasonable if these two bosons derive from the same aspect of the theory as ρ and π).

The effective contributions in 5.19 become

$$\begin{aligned}
\delta C^{NC} &= (G_{S_3} - G_{U_3})\beta^{b\mu}\beta^{*s\mu} \\
\delta C^{CC} &= V_{cb}^{*}(G_{U_3} + G_{S_3})(\frac{V_{cs}^{*}}{V_{ts}^{*}}\beta^{b\tau}\beta^{*s\tau} + \beta^{b\tau}\beta^{*b\tau})] \\
\delta C^{\nu_\mu(\nu_\tau)} &= (\frac{1}{2}G_{S_3} - 2G_{U_3})\beta^{b\mu(\tau)}\beta^{*s\mu(\tau)}
\end{aligned} \tag{5.20}$$

In this framework we can accommodate the NC anomalies for instance with $\beta^{b\mu}\beta^{*s\mu} < 0$ for $G_{S_3} > G_{U_3}$ (that is normally the case if S_3 is a pNGB).
For $G_{S_3} \simeq 4G_{U_3}$ the limits on $B \to K^*\nu\nu$ are satisfied independently from the flavour structure (as it happens for U_1), hence there are different combinations of the β elements that can accommodate CC anomalies as well.

Otherwise we could have a problem coming from $\beta^{b\mu}\beta^{*s\mu}$ that needs to be negative to acccommodate NC anomalies and that contributes to $b \to s\nu_\mu\bar{\nu}_\mu$.
For what concerns CC anomalies accommodation, assuming $G_{S_3} > 2G_{U_3}$, we need $(\frac{V_{cs}^*}{V_{ts}^*}\beta^{b\tau}\beta^{*s\tau} + \beta^{b\tau}\beta^{*b\tau})$ to be positive and, since $\beta^{b\tau}\beta^{*b\tau} \geq 0$, we do not need a big value of $\beta^{b\tau}\beta^{*s\tau}$ and hence we can choose values such that $\delta C^{\nu_\tau} \simeq 0$.
With a non vanishing contribution to ν_μ channel and a vanishing one to ν_τ could be possible (according to the magnitude of $G_{S_3} - G_{U_3}$) to accommodate the anomalies, even if a mild tuning is needed.

Instead if $G_{U_3} < G_{S_3} < 2G_{U_3}$ we would need a big value of $\beta^{b\tau}\beta^{*s\tau}$ to flip the sign of δC^{CC} producing too big value of δC^{ν_τ}. Also we would have a reduction of $G_{U_3} - G_{S_3}$ requiring for a bigger $\beta^{b\mu}\beta^{*s\mu}$ which could exceed the constraint on $\mathscr{B}(B \to K^*\nu\nu)$.

In conclusion $U_3 + S_3$ can be a good candidate but it need for the NP parameters to assume special values.

$S_1 + U_3$

We have noticed that the LQ triplets cannot accommodate CC anomalies without creating tension with $\mathscr{B}(B \to K^*\nu\nu)$; on the other hand S_1 does not contribute to NC anomalies at tree level. Hence a possibility we want to consider is to include both of them dividing the tasks of accommodating CC and NC anomalies between the two of them.

The generic δC^ν generated are

$$\delta C^{\nu_\mu(\nu_\tau)} = G_{S_1}(\beta_{S_1})^{b\mu(\tau)}(\beta_{S_1}^*)^{s\mu(\tau)} - 2G_{U_3}(\beta_{U_3})^{b\mu(\tau)}(\beta_{U_3}^*)^{s\mu(\tau)} \tag{5.21}$$

where $(\beta_{S_1})^{b\mu}(\beta_{S_1}^*)^{s\mu}$ is *free* in the sense that does not need to have any particular value since it does not contribute to any other anomalous observable and so we can ask for it to have the right value to cancel the contribution to ν_μ channel. This fact allows $(\beta_{U_3})^{b\mu(\tau)}(\beta_{U_3}^*)^{s\mu(\tau)}$ to assume the value needed for δC^{NC} (that in this case is equal to the only contribution of U_3 in table 5.1) to accommodate NC anomalies.

The contribution to $R_{D^{(*)}}$ instead is given by

$$\begin{aligned}\delta C^{CC} =& V_{cb}^*\big[G_{U_3}((\beta_{U_3})^{b\tau}(\beta^*_{U_3})^{s\tau}\frac{V^*_{cs}}{V^*_{cb}} + (\beta_{U_3})^{b\tau}(\beta^*_{U_3})^{b\tau}) \\ & - \frac{1}{2}G_{S_1}((\beta^L_{S_1})^{b\tau}(\beta^{L*}_{S_1})^{s\tau}\frac{V^*_{cs}}{V^*_{cb}} + (\beta^L_{S_1})^{b\tau}(\beta^{L*}_{S_1})^{b\tau})\big]\end{aligned} \tag{5.22}$$

Here we see that there are different appreciable scenarios that could arise

- If $(\beta_{S_1})^{s\tau} \simeq (\beta_{U_3})^{s\tau} \simeq 0$ we would have a negative δC^{CC} if $G_{S_1}|\beta^L_{S_1}{}^{b\tau}|^2 > 2G_{U_3}|\beta^L_{U_3}{}^{b\tau}|^2$ with vanishing contribution to both neutrinos channels.
- If the $(\beta_{S_1})^{s\tau} \simeq (\beta_{U_3})^{s\tau} \neq 0$ is possible to cancel δC^{ν_τ} for $G_{S_1} \simeq 2G_{U_3}$ and then we can have a $\delta C^{CC} < 0$ for $G_{S_1}|\beta^L_{S_1}{}^{b\tau}|^2 > 2G_{U_3}|\beta^{b\tau}_{U_3}|^2$.
- Since the contribution to $b \to s\nu_\mu\bar{\nu}_\mu$ is independent from the others the two $\beta^{b\tau}\beta^{*s\tau}$ are as much free as δC^{ν_μ} is small.

In this scenario is possible to accommodate all the observables considered with many combination of numerical values of the parameters.

$S_1 + S_3$

The combination of S_1 and S_3 is also a popular LQ option to accommodate B-Physics Anomalies, so we want going to see what are in general the values that UV parameters can assume to do that.

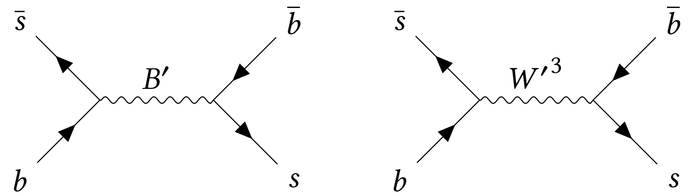
The whole contributions are given by

$$\begin{aligned}\delta C^{NC} &= G_{S_3}(\beta_{S_3})^{b\mu}(\beta^*_{S_3})^{s\mu} \\ \delta C^{CC} &= V^*_{cb}\big[-\frac{1}{2}G_{S_1}((\beta^L_{S_1})^{b\tau}(\beta^{L*}_{S_1})^{s\tau}\frac{V^*_{cs}}{V^*_{cb}} + (\beta^L_{S_1})^{b\tau}(\beta^{L*}_{S_1})^{b\tau}) + G_{S_3}((\beta_{S_3})^{b\tau}(\beta^*_{S_3})^{s\tau}\frac{V^*_{cs}}{V^*_{cb}} + (\beta_{S_3})^{b\tau}(\beta^*_{S_3})^{b\tau})\big] \\ \delta C^{\nu_\mu(\nu_\tau)} &= G_{S_1}(\beta_{S_1})^{b\mu(\tau)}(\beta^{L*}_{S_1})^{s\mu(\tau)} + \frac{1}{2}G_{S_3}(\beta_{S_3})^{b\mu(\tau)}(\beta^*_{S_3})^{s\mu(\tau)}\end{aligned} \tag{5.23}$$

where again we notice that the contrbution to $b \to s\nu_\mu\bar{\nu}_\mu$ can become small at will without affecting the other contributions and as consequence we will assume $G_{S_3}(\beta_{S_3})^{b\mu}(\beta^*_{S_3})^{s\mu}$ to have the right value to accommodate NC anomalies.

As we have seen in section 5.4 the contribution to δC^{CC} given by S_3 cannot become negative at will without contributing sentitively to $b \to s\nu_\tau\bar{\nu}_\tau$, nevertheless in this case that is as much less constrained as much δC^{ν_μ} is small as in the case of $S_1 + U_3$.
This time we have that the contribution generated by S_1 to δC^{CC} enters with a minus

Figure 5.3: Tree level contributions to $b\bar{s} \to s\bar{b}$; the combination of them grows with the growing of single amplitudes.



sign, hence for values of $(\beta_{S_1})^{b\tau}(\beta^*_{S_1})^{b\tau}$ not much big and negative the total contribution can results negative according to the relative magnitude of G_{S_1}/G_{S_3}.

This scenario has been theoretically studied in detail by [24] that brings S_1 and S_3 out as composite states of heavy fermions charged under a strongly coupled group $SU(N_{HC})$. In this work is shown how they can emerge together with other two colourless scalar fields with the quantum numbers of H and H^c.
Since all these states comes out as pNGBs of a spontaneously broken global symmetry that would explain the accidental lightness of Higgs boson solving at the same time the B-Physics Anomalies and the so-known *Electroweak Scale Naturalness Problem.*

Giving a try to colourless NP scenario

Another possible way to make two different bosons to fit independently NC and CC anomalies is to include in the lagrangian W' and B' making the effective contributions to LH operators equal to

$$\begin{aligned}
\delta C^{NC} &= -2G_{B'}(\lambda^q_B)^{bs}(\lambda^l_B)^{\mu\mu} - 4G_{W'}(\lambda^q_W)^{bs}(\lambda^l_W)^{\mu\mu} \\
\delta C^{CC} &= -8G_{W'}V^*_{cb}(\lambda^l_{W'})^{\tau\tau}[(\lambda^q_W)^{bb}\frac{V^*_{cs}}{V^*_{cb}} + (\lambda^q_W)^{ss}] \\
\delta C^{\nu_\mu(\nu_\tau)} &= -2G_{B'}(\lambda^q_B)^{bs}(\lambda^l_B)^{\mu\mu(\tau\tau)} + 4G_{W'}(\lambda^q_W)^{bs}(\lambda^l_W)^{\mu\mu(\tau\tau)}
\end{aligned} \tag{5.24}$$

where we notice right away that

$$\delta C^{NC} = \delta C^{\nu_\mu} - 8G_{W'}(\lambda^q_W)^{bs}(\lambda^l_W)^{\mu\mu}$$

which means that we can set the elements of $\lambda^{q/l}_B$ such that $\delta C^{\nu_\mu} \simeq 0$ and simultaneously ask a positive value for δC^{NC}.

The further semileptonic contributions generated by B' can be cancelled setting the RH couplings $\lambda^e_B = \lambda^u_B = \lambda^d_B = 0$. Nevertheless even having them small the contributions to $\mathcal{O}^{u/d}_R$ are proportional to the square of RH couplings and can be neglected, while contributions to RH-LH operators $\{\mathcal{O}_{LR1}, \mathcal{O}^u_{LR2}, \mathcal{O}^d_{LR2}\}$ can be cancelled introducing R_2 and $\widetilde{R}_2$ with appropriate parameters as we can see in table 4.1.
The contribution to $\delta C^{NC} > 0$ forces $(\lambda^q)^{bs}$ to be $\neq 0$ but it can be decreased at will increasing $(\lambda^l_W)^{\mu\mu}$.
Nevertheless if we want to fit CC anomalies as well we would need a non zero value for

$(\lambda_W^l)^{\tau\tau}$ combined to a non zero $(\lambda^l)^{\mu\mu}$ would contribute to $\tau \to \mu\nu_\tau\bar{\nu}_\mu$ tested by LEP [19]. Decreasing $(\lambda_W^l)^{\tau\tau}$ we could still get a negative contribution for δC^{CC} through $(\lambda_W^q)^{bb}$ (or even with $(\lambda_W^q)^{bs}$ which however is more unpleasant because of the B_s mixing discussed in 4.5).

The CC anomalies can be accommodated without disturbing much the constraints with a considerable value of $(\lambda_W^l)^{\tau\tau}(\lambda_W^q)^{bb}$.
On the other hand the NC anomalies to be accommodate need to include the parameter $(\lambda^q)^{bs} \neq 0$ that gives tree level contributions to B_s meson mixing through the amplitudes represented in figure 5.3 that are $\propto G_{B'}[(\lambda_B^q)^{bs}]^2 + G_{W'}[(\lambda_W^q)^{bs}]^2$, hence they always interfere constructively for any value of $(\lambda_{B/W}^q)^{bs}$.

Furthermore, direct searches fro these states at LHC require their masses to be large, which in turn implies also large couplings.
In conclusion $B' + W'$ is a good scenario only for a very special choice of the UV parameters and hence it is hard to have these contributions emerging naturally from general assumptions.

6

Summary and conclusions

Clues of LFU violation in the physics of B mesons have been observed in several channels by different experiments.
Many types of theories have been formulated and we do not know yet what is the exact form of the -if any- NP involved in these processes.
Nevertheless the guideline given by the available data is clear.

- Since B-Physics Anomalies appear only in semileptonic decays, the Leptoquark possibility seem absolutely proper and in most of the cases described in this work Baryon Number conservation is not threatened significantly.

- The deviations are observed only in flavour breaking processes i.e. in processes that in SM involve only LH fermions. The BSM physics could a priori couple also with RH fermions. Nevertheless the coupling to RH leptons is much suppressed in the SM and hence a NP kindred to RH leptons would create some tension with the available LFU tests. Instead, the RH quarks could be involved, even if the simultaneous measurement of R_K and R_{K^*} points toward a NP that involves only LH quarks.
  Furthermore the involvement of contributions radically different from SM ones would require the computation of hadronic matrix element with non perturbative methods. This was e.g. done by [8] in a general analysis and by [25] that presented a complete Great Unification Theory that makes S_3 and R_2 emerge with a suitable affection to B-Physics Anomalies.

- U_1 is a pleasant possibility that avoids tree level contributions to $b \to s\nu\bar{\nu}$, but that feature can also be implemented combining different Leptoquarks keeping a good level of generality for the UV parameters. $S_1 + S_3$ has been studied by the already mentioned [24] in a composite Higgs model but also by [17, 26].
  The other combinations of Leptoquarks like $U_1 + S_1$, $S_1 + U_3$ and $S_3 + U_3$ are not as much considered but they show as well many interesting features and all of them could be implemented in some UV completions.
  The colourless option, instead, accommodates the anomalies, without exceeding the experimental constraints, only for a very special choice of the parameters of the model; it has been considered as natural extension of the gauge sector of SM by [27, 28].

A

Mapping beetween Weyl and Dirac pictures

In this work fermion fields have been written in two different forms: as Dirac fields and as Weyl fields. Here we show that the two of them are equivalent and the choice of one rather than the other is done only to highlight different aspects of the Lorentz structure.

The Weyl fields are two-copmonent fermions that belong to an irreducible representation of Lorentz group, being that either $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$. It becomes useful when the dynamic treats differently fermions transforming under different representations. We indicate them with

$$\psi_L \sim (\frac{1}{2}, 0), \quad \psi_R \sim (0, \frac{1}{2})$$

that are linked by the charge conjugation explained in appendix B.

The two fundamental representations are labelled with L, R and this identifies a quantum number named *chirality* which by convention is set equal to -1 for fermions in $(\frac{1}{2}, 0)$ and $+1$ for fermions in $(0, \frac{1}{2})$.

When the interactions are *vector-like* (that means blind to chirality), can be useful to organize the two fundamental representations in a unique field:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A.1}$$

that has four components, two transforming under $(\frac{1}{2}, 0)$ and the other two under $(0, \frac{1}{2})$. A field defined in this way is clearly transforming under a reducible representation of Lorentz group (precisely $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$) it is named is *Dirac field*.

To link the two representations we define the *chirality matrix*

$$\gamma^5 = \begin{pmatrix} -\mathbf{1}_{2x2} & 0 \\ 0 & \mathbf{1}_{2x2} \end{pmatrix}$$

that acts on Dirac fields, where the name is due to the fact that eigenvalues of this matrix correspond to the chirality defined above.

Through this matrix we can define to projectors deed to get the access to the different irreducible representations:

$$P_L = \frac{1-\gamma^5}{2}, \quad P_R = \frac{1+\gamma^5}{2}.$$

Using them we can define a biunivoc mapping between Dirac and Weyl pictures:

$$\psi_L^{Weyl} \to \psi_L^{Dirac} \equiv P_L\psi^{Dirac} = \begin{pmatrix} \psi_L^{Weyl} \\ 0 \end{pmatrix}$$

$$\psi_R^{Weyl} \to \psi_R^{Dirac} \equiv P_R\psi^{Dirac} = \begin{pmatrix} 0 \\ \psi_R^{Weyl} \end{pmatrix}$$

and in the other way, using that $P_L + P_R = 1$

$$\psi^{Dirac} = (P_L + P_R)\psi^{Dirac} \equiv \psi_L^{Dirac} + \psi_R^{Dirac} \to \psi_L^{Weyl} + \psi_R^{Weyl}$$

A.1 Currents

The scalar and vector currents that can be constructed with Weyl fields using σ^μ and $\overline{\sigma}^\mu$ equal to

$$\sigma^\mu = (\mathbf{1}, \vec{\sigma}), \quad \overline{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices.

The chiral currents using these matrices are given by:

$$\{\psi_L^\dagger \overline{\sigma}^\mu \chi_L, \psi_R^\dagger \sigma^\mu \chi_R\} \sim \left(\frac{1}{2}, \frac{1}{2}\right)$$
$$\{\psi_L^\dagger \chi_R, \psi_R^\dagger \chi_L\} \sim (0, 0)$$

then is useful to write fermion currents in this picture when interactions of these fermions are *chiral*, i.e. different for fermions belonging to $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$.

To build currents in the Dirac picture we need to introduce a set of 4x4 matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad (\gamma^0)^\dagger = \gamma^0, \quad (\gamma^i)^\dagger = -\gamma^i \; for \, i = 1, 2, 3,$$

known as the Dirac matrices. The matrices so defined satisfy the anti commutation rule $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ from which follows that $(\gamma^0)^2 = \mathbf{1}$ and $(\gamma^i)^2 = -\mathbf{1}$. An useful consequence of these properties is

$$\gamma^0\gamma^\mu\gamma^0 = (\gamma^\mu)^\dagger. \tag{A.2}$$

In terms of these matrices, the γ^5 matrix is defined as

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3, \quad (\gamma^5)^\dagger = \gamma^5$$

and from $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ follows that $\{\gamma^\mu, \gamma^5\} = 0$ and hence $P_L\gamma^\mu = \gamma^\mu P_R$.

Once we have defined $\bar{\psi} \equiv \psi^\dagger\gamma^0$, the vector and scalar currents in Dirac picture, in terms of chiral currents, are given by:

$$\begin{aligned} &\psi_L^\dagger \chi_R + \psi_R^\dagger \chi_L \to \bar{\psi}_L\chi_R + \bar{\psi}_R\chi_L = \bar{\psi}\chi \sim (0,0) \\ &\psi_L^\dagger \bar{\sigma}^\mu \chi_L + \psi_R^\dagger \sigma^\mu \chi_R \to \bar{\psi}_L\gamma^\mu\chi_L + \bar{\psi}_R\gamma^\mu\chi_R = \bar{\psi}\gamma^\mu\chi \sim \left(\frac{1}{2}, \frac{1}{2}\right) \end{aligned} \tag{A.3}$$

where we have combined the currents belonging to the same irreducible representation of the Lorentz group between each other.

Since the special combination of the chiral currents in A.3 doesn't include all the possible combination of currents that belong to an irreducible representation of the Lorentz group, we define the pseudovector and the pseudoscalar currents through the γ^5 matrix as the orthogonal combination of chiral currents

$$\begin{aligned} &\psi_L^\dagger \chi_R - \psi_R^\dagger \chi_L \to \bar{\psi}_L\chi_R - \bar{\psi}_R\chi_L = \bar{\psi}\gamma^5\chi \sim (0,0) \\ &\psi_R^\dagger \sigma^\mu \chi_R - \psi_L^\dagger \bar{\sigma}^\mu \chi_L \to \bar{\psi}_R\gamma^\mu\chi_R - \bar{\psi}_L\gamma^\mu\chi_L = \bar{\psi}\gamma^\mu\gamma^5\chi \sim \left(\frac{1}{2}, \frac{1}{2}\right). \end{aligned}$$

In the opposite sense, the mapping from Dirac to Weyl picture consists in the combination of $\{\mathbf{1}, \gamma^5\}$ in $\{P_L, P_R\}$:

$$\begin{aligned} &\frac{1}{2}[\bar{\psi}\gamma^\mu\chi - \bar{\psi}\gamma^\mu\gamma^5\chi] = \bar{\psi}\gamma^\mu P_L\chi = \bar{\psi}_L\gamma^\mu\chi_L \to \psi_L^\dagger\bar{\sigma}_\mu\chi_L \\ &\frac{1}{2}[\bar{\psi}\gamma^\mu\chi + \bar{\psi}\gamma^\mu\gamma^5\chi] = \bar{\psi}\gamma^\mu P_R\chi = \bar{\psi}_R\gamma^\mu\chi_R \to \psi_R^\dagger\sigma_\mu\chi_R \end{aligned}$$

Here we have shown that the two pictures are equivalent and always interchangeable. One can prefer one rather than the other according that the interaction treated were chiral or vector-like.

We have written the effective high energy semileptonic operators in terms of Weyl fields (section 3.1). To compare their the Wilson coefficients to the ones of the low energy EFT introduced in 2.2 we need to combine the operators in order to get chiral currents using

$$1 = P_R + P_L, \quad \gamma^5 = P_R - P_L.$$

to obtain

$$\begin{aligned} &-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}[C_9^l\,\bar{b}\gamma^\mu P_L s\,\bar{e}\gamma_\mu e \;+\; C_{10}^l\,\bar{b}\gamma^\mu P_L s\, l\gamma_\mu\gamma^5 l] = \\ &-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}s[(C_9^l - C_{10}^l)\bar{b}_L\gamma^\mu s_L\,\bar{e}_L\gamma_\mu e_L + (C_9^l + C_{10}^l)\bar{b}_L\gamma^\mu s_L\,\bar{e}_R\gamma_\mu e_R] = \\ &-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}[(C_9^l - C_{10}^l)b_L^\dagger\bar{\sigma}^\mu s_L\,\bar{e}_L\bar{\sigma}_\mu e_L + (C_9^l + C_{10}^l)\bar{b}_L\bar{\sigma}^\mu s_L\,\bar{e}_R\sigma_\mu e_R] \end{aligned} \tag{A.4}$$

where the last term is written in terms the Weyl fields.
In conclusion when we compare the NP contributions to the amplitude $b \to sl^+l^-$ at low

energy with the SM ones in the low energy EFT the whole contributions is given by

$$\begin{aligned}
&-\frac{4G_F}{\sqrt{2}}V_{ts}^{*}V_{tb}\frac{e^2}{8\pi^2}(C_9^l - C_{10}^l) + C_S^{bsll} + C_T^{bsll};\\
&-\frac{4G_F}{\sqrt{2}}V_{ts}^{*}V_{tb}\frac{e^2}{8\pi^2}(C_9^l + C_{10}^l) + (C_{LR2})^{bsll}.
\end{aligned} \tag{A.5}$$

B

Parity and Charge conjugation

The*Parity P* act on Dirac fields as:

$$P\,:\,\psi=\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}\rightarrow\gamma_0\psi=\begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix}. \tag{B.1}$$

The action of the parity on the currents follows from B.1 and reads:

$$\begin{aligned}
&P(\overline{\psi}\chi)=\overline{\psi}\gamma^0\gamma^0\chi=\overline{\psi}\chi\\
&P(\overline{\psi}\gamma^5\chi)=\overline{\psi}\gamma^0\gamma^5\gamma^0\chi=-\overline{\psi}\gamma^5\chi\\
&P(\overline{\psi}\gamma^\mu\chi)=\overline{\psi}\gamma^0\gamma^\mu\gamma^0\chi=(\overline{\psi}\gamma^0\chi,-\overline{\psi}\vec{\gamma}\chi)\\
&P(\overline{\psi}\gamma^\mu\gamma^5\chi)=\overline{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\chi=(-\overline{\psi}\gamma^0\gamma^5\chi,\overline{\psi}\vec{\gamma}\gamma^5\chi)
\end{aligned} \tag{B.2}$$

where $\vec{\gamma}=(\gamma^1,\gamma^2,\gamma^3)$.

The *Charge conjugation C*, instead, acts on fermion fields as

$$C\,:\,\psi=\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}\rightarrow i\gamma_2\psi^*=\begin{pmatrix}-\varepsilon\psi_R^*\\ \varepsilon\psi_L^*\end{pmatrix} \tag{B.3}$$

where $\varepsilon=i\sigma_2$ is the two dimensions anti symmetric tensor

$$\varepsilon=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}.$$

that links the two fundamental representation of the Lorentz group as

$$\begin{aligned}
\varepsilon(\psi_L^*)&\sim\big(0,\frac{1}{2}\big)\\
-\varepsilon(\psi_R^*)&\sim\big(\frac{1}{2},0\big).
\end{aligned} \tag{B.4}$$

This operator allows us to build a scalar current with two Weyl fields belonging to the same representation of the Lorentz group.
If a scalar current is made of two copies of the same field that would introduce a $\mathcal{G}_{SM}$ invariant mass term for fermions $(\psi^c_{L(R)})^\dagger\psi_{L(R)}\sim(0,0)$ known as *Majorana mass term.* Nevertheless it is not allowed to be in the SM lagrangian because there aren't fermions

belonging to real (or pseudoreal) representation of $\mathscr{G}_{SM}$, in other words $(\psi^c_{L(R)})^\dagger\psi_{L(R)}$ can never be a singlet of $\mathscr{G}_{SM}$ if $\psi_{L(R)}$ is a SM fermion (except for the case in which we include the ν_R described in 2.1 in the spectrum of the theory).

The action of C on the currents can also be derived by B.3 mindful that in this currents all the Lorentz indexes are contracted we need to use the invariance under transposition of the currents.
For instance the scalar currents transforms as

$$C(\bar{\psi}\chi) = (i\gamma^2\psi^*)^\dagger\gamma^0(i\gamma^2\chi^*) = \psi^t(\gamma^2)^\dagger\gamma^0\gamma^2\chi^* = -\psi^t\gamma^0\chi^* = (-\psi^t\gamma^0\chi^*)^t = -\bar{\chi}\psi \tag{B.5}$$

Following this *iter* we find

$$\begin{aligned} C(\bar{\psi}\gamma^5\chi) &= (i\gamma^2\psi^*)^\dagger\gamma^0\gamma^5(i\gamma^2\chi^*) = \psi^t(\gamma^2)^\dagger\gamma^0\gamma^5\gamma^2\chi^* = \psi^t\gamma^0\gamma^5\chi^* = (-\psi^t\gamma^5\gamma^0\chi^*)^t \\ &= (\psi^t\gamma^5\gamma^0\chi^*)^t = \bar{\chi}\gamma^5\psi \end{aligned} \tag{B.6}$$

To see how it acts on vector and pseudovector currents we need to remind that, since $\sigma_i^* = -\sigma_2\sigma_i\sigma_2$, the γ^μ matrices enjoy the property

$$\gamma^2\gamma^\mu\gamma^2 = -(\gamma^\mu)^*, \tag{B.7}$$

that, used together with A.2, implies

$$\gamma^0\gamma^2\gamma^\mu\gamma^2\gamma^0 = (\gamma^\mu)^t. \tag{B.8}$$

$$\begin{aligned} C(\bar{\psi}\gamma^\mu\chi) &= (i\gamma^2\psi^*)^\dagger\gamma^0\gamma^\mu(i\gamma^2\chi^*) = \psi^t\gamma^2\gamma^0\gamma^\mu\gamma^2\chi^* = -\psi^t\gamma^0\gamma^2\gamma^\mu\gamma^2(\gamma^0\gamma^0)\chi^* = -\psi^t(\gamma^\mu)^t\gamma^0\chi \\ &= (-\psi^t(\gamma^\mu)^t\gamma^0\chi^*)^t = -\bar{\chi}\gamma^\mu\psi \\ C(\bar{\psi}\gamma^\mu\gamma^5\chi) &= (i\gamma^2\psi^*)^\dagger\gamma^0\gamma^\mu\gamma^5(i\gamma^2\chi^*) = \psi^t\gamma^2\gamma^0\gamma^\mu\gamma^5\gamma^2\chi^* = \psi^t\gamma^5\gamma^0\gamma^2\gamma^\mu\gamma^2(\gamma^0\gamma^0)\chi^* \\ &= \psi^t\gamma^5(\gamma^\mu)^t\gamma^0\chi^* = (\psi^t\gamma^5(\gamma^\mu)^t\gamma^0\chi^*)^t = \bar{\chi}\gamma^\mu\gamma^5\psi \end{aligned} \tag{B.9}$$

Under the action of the combined operator CP the transformation of the currents results

$$\begin{aligned} CP(\bar{\psi}\chi) &= -\bar{\chi}\psi \\ CP(\bar{\psi}\gamma^5\chi) &= -\bar{\chi}\gamma^5\psi \\ CP(\bar{\psi}\gamma^\mu\chi) &= (-\chi\gamma^0\psi, \bar{\chi}\vec{\gamma}\psi) \\ CP(\bar{\psi}\gamma^\mu\gamma^5\chi) &= (-\bar{\chi}\gamma^0\gamma^5\psi, \bar{\chi}\vec{\gamma}\gamma^5\psi) \end{aligned} \tag{B.10}$$

Since the bosons of the SM transform under CP as

$$\begin{aligned} CP(H) &= H^* \\ CP(A_\mu) &= (-A_0, \vec{A}) \\ CP(Z_\mu) &= (-Z_0, \vec{Z}) \\ CP(W^\pm_\mu) &= (-W^\mp_0, \vec{W}^\mp) \\ CP(G_\mu) &= (-G_0, \vec{G}) \end{aligned} \tag{B.11}$$

it follows that all the operators with a real coefficient in $\mathscr{L}_{SM}$ is CP-invariant, hence the only possible source of CPV in the SM comes from the CC interaction with W bosons that involves the CKM matrix.

$$\mathscr{L}_{CC} = -\frac{g_2}{\sqrt{2}}[(V^\dagger u_L)^\dagger \overline{\sigma}_\mu d_L W_\mu^+ + d_L^\dagger \overline{\sigma}_\mu (V^\dagger u_L) W_\mu^-] \tag{B.12}$$

That after the acting of CP becomes

$$CP(\mathscr{L}_{CC}) = -\frac{g_2}{\sqrt{2}}[(V^t u_L)^\dagger \overline{\sigma}_\mu d_L W_\mu^+ + d_L^\dagger \overline{\sigma}_\mu (V^t u_L) W_\mu^-] \tag{B.13}$$

from that we conclude that the SM Lagrangian allows CPV if and only if $V^* \neq V$, that proves the Theorem 3.

C

Fierz Identities in the Scalar Lagrangian

When we introduce to the theory scalar Leptoquarks, integrating them out from the lagrangian we could generate four-fermion operators of this shape:

$$\overline{l^{1c}}\varepsilon q^1 \ \overline{q^2}\varepsilon l^{2c} = l^1_{\alpha a} q^1_{\beta b} q^{2*}_{\gamma c} l^{2*}_{\delta d} \varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} \tag{C.1}$$

where all the fields are LH fermions, i, j, k, l are EW indexes and $\alpha, \beta, \gamma, \delta$ are Lorentz indexes and the ε of the Lorentz's group comes from the definition of charge's coniugation operator.
It is evident that that operator, as is written, doesn't belong to the list we have shown in 3.1

Since all the fermions involved are LH we expect to write them as a linear combinations of O_S and O_T. To do that we need to write

$$\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = c_1 \ \delta^{da}\delta^{bc}\overline{\sigma_\mu}^{\delta\alpha}\overline{\sigma}^{\gamma\beta^\mu} \ + \ c_2 \ \sigma^{da}\sigma^{bc}\overline{\sigma_\mu}^{\delta\alpha}\overline{\sigma}^{\gamma\beta^\mu}$$

and then switching the position of the fields using:

$$l^1 q^1 q^{2*} l^{2*} = (-1)^4 q^{2*} q^1 l^{2*} l^1$$

because the total number of permutations of the fields is four.

Now we use that

$$\varepsilon^{ab}\varepsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} = \frac{1}{2}\sigma^{da}_a\sigma^{cb^a} - \frac{1}{2}\delta^{da}\delta^{cb}$$

Where we used 3.8 in the second equality.
Then, acting on the 3.12,

$$\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = \frac{1}{2}\overline{\sigma}^{\alpha\gamma}_\mu\overline{\sigma}^{\mu \ \beta\delta} = -\frac{1}{2}\overline{\sigma}^{\delta\alpha}_\mu\overline{\sigma}^{\mu \ \gamma\beta}$$

Multiplicating we obtain:

$$\overline{l^{1c}}\varepsilon q^1 \ \overline{q^2}\varepsilon l^{2c} = \frac{1}{4}\big[(q^{2\dagger}\overline{\sigma}^\mu q^1)\,(l^{2\dagger}\overline{\sigma}_\mu l^1) - \ (q^{2\dagger}\overline{\sigma}^\mu\sigma^a q^1)\,(l^{2\dagger}\overline{\sigma}_\mu\sigma_a l^1)\big] \tag{C.2}$$

Once obtained that result it is easy to derive the same in the case of the contraction of two $SU(2)_L$-triplet scalar currents:

$$l^{1c\dagger}\varepsilon\sigma_a q^1\ q^{2\dagger}\sigma^a\varepsilon l^{2c} = l^1_{\alpha a}q^1_{\beta b}q^{2*}_{\gamma c}l^{2*}_{\delta d}(\sigma^a\varepsilon)^{ab}(\varepsilon\sigma_a)^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

We already know how to treat the lorentz structure that will bring a factor $-\frac{1}{2}\overline{\sigma}^{\alpha\delta}_{\mu}\overline{\sigma}^{\mu\ \beta\gamma}$. The EW structure instead is different:

$$\begin{aligned}&\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cf^a} = (\delta^{af}\delta^{ed} - \delta^{ad}\delta^{fe})(2\delta^{ef}\delta^{kj} - \delta^{eb}\delta^{ce}) = \\ &2\delta^{ad}\delta^{bc} - 4\delta^{ad}\delta^{bc} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} = -(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd})\end{aligned}$$

then recalling the 3.8

$$\begin{aligned}\sigma^a_{da}\sigma_{cba} &= 2\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \rightarrow \\ \varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cf^a} &= -\frac{1}{2}(\sigma^{ad}_a\sigma^{bc^a} + 3\delta^{da}\delta^{cb})\end{aligned}$$

multiplying the factors coming from EW and Lorentz group:

$$\overline{l^{1c}}\varepsilon\sigma_a q^1\ \overline{q^2}\sigma^a\varepsilon l^{2c} = \frac{1}{4}\big[3(q^{2\dagger}\overline{\sigma}^{\mu}q^1)\ (l^{2\dagger}\overline{\sigma}_{\mu}l^1) +\ (q^{2\dagger}\overline{\sigma}^{\mu}\sigma^a q^1)\ (l^{2\dagger}\overline{\sigma}_{\mu}\sigma_a l^1)\big] \tag{C.3}$$

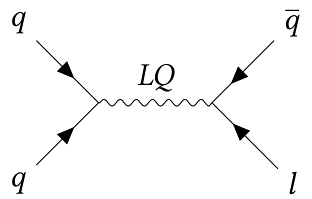
that again is a combination of terms that reminds to $\mathcal{O}_S$ and $\mathcal{O}_T$.

D

Rough estimation of Leptoquark mass from proton decay

Here we will show through a naive approach why $\tau_p > 10^{33} ys$ can be explained by a mediator with a mass of order $\Lambda_{GUT} \simeq 10^{16}\ GeV$ as we said in section 4.2.

The contribution given by the diagram to proton decay



is given by

$$\mathcal{M} \simeq g_{NP} \frac{1}{q^2 - M_{NP}^2} g_{NP} \tag{D.1}$$

that in the limit of $q^2 \ll M_{NP}^2$ becomes

$$\mathcal{M} \simeq -\frac{g_{NP}^2}{M_{NP}^2} \tag{D.2}$$

We know that the decay width is

$$\Gamma = (const.)|\mathcal{M}|^2$$

and that it has to have the dimension of a $[mass]^1$, hence the $(const.)$ has to have the dimension of a $[mass]^5$. Since the biggest mass in play is m_p, to get a rough estimation of the width we assume $(const.) = m_p^5$.

If we assume $g_{NP} \simeq 1$ -that would be justified in absence of flavour suppressions- the approximate width results:

$$\Gamma \simeq \frac{m_p^5}{M_{NP}^4} \rightarrow \tau \simeq \frac{M_{NP}^4}{m_p^5} > 10^{33}\ ys$$

To have the lifetime in unit of years we have first to write it in gaussian units, i.e.

$$\tau \simeq \frac{M_{NP}^4}{m_p^5} \cdot (\frac{\hbar}{c^2}) = (\frac{1}{0.938})^5 (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (6.58 \cdot 10^{-25} s)$$

hence

$$\tau \simeq (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (2.87 \cdot 10^{-32}\ ys) > 10^3 3ys \rightarrow$$
$$M_{NP}^4 > 10^{64-65}\ GeV^4 \rightarrow M_{NP} > 10^{16} GeV$$

that explain the estimation done in section 4.2.

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