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An up to date overview on Lepton Flavour Universality Violation in B mesons physics

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Introduction

In this work we are trying to explore the huge world of Beyond Standard Model (BSM) theories using as gate some exmerimental observations seen recently, that bring clues of the breaking of an accidental symmetry.

In the Standard Model (SM) as is known today we have an accidental symmetry which is called Lepton Flavour Universality (LFU) which tells us that all the gauge interactions are flavour blind; in other words the Electro-Weak (EW) processes have the same strenght for the electron, the muon, the tau and same for the different types of neutrinos.
Since the only difference beetween the different lepton families is the mass, there is always a kinematic flavour sensitive effect i.e. the phase space factor which affect the rate or the cross section of a process, for instance the decay of charged π produces mostly muons for this reasons.
Nevertheless, in the SM, there is also a dynamical effect which is not mass-blind, which is the coupling with the Higgs Boson but since leptons are very light (the τ, which is the heaviest, weighs less than 50 times the Higgs'vacuum expectation value) we neglect the Higgs'coupling and say that LFU is a well approximated accidental symmetry of the SM.

If we have accepted that LFU is a good symmetry we can also understand why testing it is a good place to find clues for BSM theories. Recently in different experiments were found differrent hints of LFU Violation (LFUV) in semileptonic decays of the B mesons (mesons with non vacuum difference of b and $\bar{b}$ as valence quark). All the deviations from the SM appearing in these decays go under the name of *B-Physics Anomalies.*

Since, as we will discuss, the B-Physics Anomalies appear mostly at the hadronic scale (order few GeV), we will implement a model independent approach named the Effective Field Theory (EFT) approach, as is usually done with the Fermi Theory to study low energy weak interactions.
In this framework finding New Physics (NP) basically means to find deviation from the Lagrangian's coefficients of the operators (or Wilson coefficients).

So our purpose is to find the right heavy mediators which, once integrated out from the Lagrangian (again as we do with W and Z bosons to get the Fermi theory), give us the appropriate contribute to accomodate the B-Physics Anomalies. We also have to be very careful about the processes that are already tested, because introducing NP can affect also processes that don't concern B or hadrons at all.

It will be clear that introducing SM-like heavy degrees of freedom the risk to affect the phenomenological constraints is high.
This is one of the reason because we like to introduce Leptoquarks (LQs): coloured bosons which can be absorbed from a quark to become a lepton (+h.c.) and try in this way to affect only semileptonic processes without disturbing others SM constraints. We will take count also of the possible colour-less bosons (heavier version of W and Z) and in both cases we will try to find the best flavour structure to accomodate the Anomalies.

A lot of papers are already written about B-Anomalies and Leptoquarks; the main purpose of this work is to offer an updated catalogue as much general and complete we can do of solution with single NP mediator or combination of them according to the low energy contributions we need.

In the end we have to mention that adding bosons to SM is not enough to say that we have a BSM theory.
We would like to have a theory in which with few assumptions and few input values all the particles and processes come out naturally (as in SM). These complete theories are known as UV Completions and we will mention some of the most interesting in the available literature.

Hope you readers will find the work interesting and light to read.
Before to introduce the Anomalies we need to introduce tha Flavour sector of the SM.

1

Flavour Physics

When we look for clues of New Physics one possible way is to look for processes that aren't allowed from the known model either to measure NP effects or to test the available theory.
For similar reasons a good places to look for NP clues are those processes that are predicted to be rare by the SM to bring either hints for deviations or an improvement on the measure of the SM parameters.

An enviroment in which lot of rare processes arise is the *Flavour sector* of the SM. The Flavour Physics is focused on the fact that the fermion fields of the SM

$$l_L \; q_L \; e_R \; d_R \; u_R$$

appear in phenomenology in three copies each, named *families* (or *generations*), differing only from the mass and hence for the coupling to the Higgs boson. Turning off the Higgs-fermions coupling there would be a big global accidental symmetry that wouldn't allow us to distinguish the different generations named *Flavour symmetry.*
It is interesting to see how the Higgs coupling to fermions (known as *Yukawa coupling*) affect the Flavour symmetry.

1.1 Introduction to Standard Model

The Standard Model of Fundamental Interactions is nowaday the most predicting and accurately tested theory available to treat a wide set of processes.

The assumption done by the SM can be summarized as follows:

1. A local *gauge* symmetry $\mathcal{G}_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ spontaneously broken at low enegies trough an Higgs mechanism with the pattern $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{em}$.

2. Matter fields are defined to be irreducible representation of $\mathcal{G}_{SM}$ and are the fermions; $q_L^i \sim (3,2)_{1/6}$, $l_L^i \sim (3,2)_{-1/2}$,$e_R^i \sim (1,1)_{-1}$, $d_R^i \sim (3,1)_{-1/3}$, $u_R^i \sim (3,1)_{2/3}$ where $i = 1, 2, 3$ is the family index, and the scalar responsible for the *Electro-Weak Symmetry Breaking* (EWSB) $H \sim (1,2)_{1/2}$.

The notation is $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ and indicates the dimension of the representation for $SU(3)_c$ and $SU(2)_L$ and the eigenvalue of the only generator of $U(1)_Y$.

3. The theory has to be a local QFT *renormalizable*, i.e. the lagrangian has to contain operators Lorentz invariant with dimension, in mass unit, ≤ 4.

Once stated the guidelines we are ready to write the lagrangian, which has to contain all the possible terms allowed by our assumptions. For simplicity we split it in pieces:

$$\mathscr{L}_{SM} = \mathscr{L}_{kin} + \mathscr{L}_H + \mathscr{L}_\theta + \mathscr{L}_Y \tag{1.1}$$

Where $\mathscr{L}_{kin}$ contains the kinetik term of fermions and gauge fields:

$$\mathscr{L}_{kin} = \sum_\psi i\overline{\psi}^i H^\psi_{ij} \not{D}\psi^j - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^{\mu\nu}_j W^j_{\mu\nu} - \frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} \tag{1.2}$$

where $D_\mu = \partial - ig_1\frac{Y}{2}B_\mu - ig_2T^a_LW_{\mu a} - ig_3T^a_cG_{\mu a}$ with $T^a_L = \frac{\sigma^a}{2}$ and $T^a_c = \frac{\lambda^a}{2}$ (σ are the Pauli matrices and λ Gell-Mann matrices), while $B_\mu \sim (1,1)_0$ $W_\mu \sim (1,3)_0$ and $G_\mu \sim (8,1)_0$ are the vector bosons arising from the local symmetry $\mathscr{G}_{SM}$.
H_{ij} a 3x3 hermitian matrix in flavour space that have to satisfy the canonical quantization condition to have all eigenvalues equal to 1.
In the end $W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - \frac{i}{g_2}f_{abc}W^b_\mu W^c_\nu$ is the gauge invariant tensor for $SU(2)_L$ bosons and f_{abc} are the structure's constants of $SU(2)$. Analogous for the tensor $G^a_{\mu\nu}$ which inherit the structure's constants from $SU(3)$ and for $B_{\mu\nu}$ that has vacuum structure's constants (since $U(1)$ is an Abelian group).

$$\mathscr{L}_H = (D_\mu H)^\dagger D^\mu H + m_H^2 H^\dagger H - \lambda(H^\dagger H)^2 \tag{1.3}$$

is the Higgs term that presents himself as a scalar field Lagrangian with a quartic coupling. The sign of the quartic coupling gives us the vacuum expectation value $\langle 0|H|0\rangle = v \neq 0$ responsible for the EWSB.

$$\mathscr{L}_Y = Y^{ij}_u\,(H\overline{q}^i_L)u^j_R + Y^{ij}_d(H^c\overline{q}^i_L)d^j_R + Y^{ij}_l(H\overline{l}^i_L)e^j_R + h.c. \tag{1.4}$$

Where the charge's conjugated $H^c \equiv \varepsilon H$ with $\varepsilon = i\sigma_2$ acting on $SU(2)_L$ space and Y are generic complex 3x3 matrices in flavour space.

In the end

$$\mathscr{L}_\theta = \frac{\theta_3}{32\pi}\varepsilon_{\mu\nu\rho\sigma}G^{\mu\nu}_a G^{\rho\sigma a} \tag{1.5}$$

is known as the $\theta - term$ which would imply CP violation in strong interaction. Nevertheless θ_3 is strongly bounded from the experiments and anyway this term would be neglectable for our purposes, so we will just ignore it[1].

[1]Analogous theta terms arise for $SU(2)_L \otimes U(1)_Y$ gauge bosons, but them can be set to 0 through a redefinition of the gauge fields

It's clear that, without assuming renormalizability of the theory, we could have added infinity more terms to the lagrangian. Since we have stopped at dimension 4, some more symmetry arised accidentally and they are known as *accidental symmetries.*

1.1.1 *Definition of Flavour eigenstates*

As we have seen, to be completely general in equation 1.2 we have introduced an hermitian matrix which acts on flavour triplets H^{ψ}_{ij} which is in general different for every fermion. Nevertheless is always possible to redefine the fermion field trough an unitay trasformation:

$$\psi^i \rightarrow \psi'^i = V_{\psi}{}^i_j \psi^j \tag{1.6}$$

choosing V_ψ such that $H \rightarrow H^{\psi'} = V_\psi{}^\dagger H^\psi V_\psi = 1_{3x3}$ which is always possible since H is hermitian and its eigenvalues are equal to 1 to respect canonical commutation rule.

Thus, doing the transformation 1.6 for every fermion field we define a basis $\{\psi^{i'}\} \equiv \{q^i_L; l^i_L; e^i_R, d^i_R, u^i_R\}$ of flavour triplets that we know as the previously mentioned fermions of SM. In other words the fermion field triplets are defined as the ones that diagonalize the kinetik term, i.e. the interaction with gauge bosons. We call the basis of interaction eigenstates the *Flavour basis.*

Now we could do any flavour rotation like the 1.6 and we would always obtain $1 \rightarrow V_\psi{}^\dagger V_\psi = 1$, hence we find that H had to be the identity from beginning and the relation 1.6 defines a set of flavour basis that diagonalize the gauge interaction.

Let's see how the rotation 1.6 affects $\mathscr{L}_Y$. Considering only the flavour structure of the fields, the Yukawa terms transform as

$$\begin{aligned}
(Hl_L)^\dagger Y_e e_R = H\, e_L^\dagger Y_e e_R \rightarrow H\, e_L^\dagger (V_{l_L}{}^\dagger Y_e V_{e_R}) u_R \\
(Hd_L)^\dagger Y_u u_R = H\, d_L^\dagger Y_u u_R \rightarrow H\, q_L^\dagger (V_{q_L}^\dagger Y_u V_{u_R}) u_R \\
(H^c d_L)^\dagger Y_u u_R = H^*\, u_L^\dagger Y_d d_R \rightarrow H^*\, q_L^\dagger (V_{q_L}{}^\dagger Y_d V_{d_R}) d_R
\end{aligned}$$

Now we use that for a generic 3x3 complex matrix exist two matrices in $SU(3)$ such that $A \rightarrow U_1^\dagger A U_2 = A^{(d)}$ where $A^{(d)}$ is diagonal. Since we have freedom to rotate the fields through a *gauge diagonal* trasformation (1.6 form) we can choose V_{l_L}, V_{e_R} such that $V_{l_L}{}^\dagger Y_e V_{e_R} = Y_e^{(d)}$ that means that leptons flavour eigenstates can diagonalize the Yukawa matrix, i.e. they are mass'eigenstates as well. This can be seen as the proof of the theorem 1 that will bw shown later.

Different is the story of quarks, in fact there we have two terms that can't be rotated independently. We may choose $V_{q_L} \equiv V_{d_L}, V_{d_R}$ such that $V_{d_L}^\dagger Y_d V_{d_R} = Y_d^{(d)}$ is diagonal but then we no more freedom to diagonalize the up-quark Yukawa. In other words the gauge eigenstates are not mass'eigenstates in the quark sector.

We can always define V_{u_L}, V_{u_R} such that $V_{u_L}^\dagger Y_u V_{u_R} = Y_u^{(d)}$, then the Yukawa coupling term of up quarks results (fixed $V_{q_L} = V_{d_L}$):

$$H\, q_L^\dagger (V_{d_L}{}^\dagger Y_u V_{u_R}) u_R = H\, u_L^\dagger (V_{d_L}^\dagger V_{u_L}) Y_u^{(d)} u_R \equiv H(V_{CKM}^\dagger u)^\dagger Y_u^{(d)} u_R$$

where $V_{CKM} \equiv V^{\dagger}_{u_L} V_{d_L}$ is the *Cabibbo-Kobayashi-Maskawa*, hence LH mass'eigenstates are (naming CKM matrix with V):

$$q^i_L = \begin{pmatrix} V^{*ij} u_{LJ} \\ d^i_L \end{pmatrix} \quad , l^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}$$

That means that a quark of the i-family can couple at tree level with a j-family quark with intensity according to V_{ij}.

Interactions of leptons are, instead, always flavour diagonal, neglecting effects due to neutrino's mass that, if introduced in the lagrangian trough the insertion of $\nu_R \sim (1,1)_0$ among the SM fields, would imply an analogous mechanism in which ν_L's mass eigenstate would be $\nu^{Mi}_L = (U_{PMNS})^i_j \nu^j_L$, where $U_{PMNS} = V^{\dagger}_{\nu_L} V_{e_L}$ is the *Pontecorvo-Maki-Nakagawa-Sakata* (PMNS) matrix introduced to explain *neutrinos flavour oscillations.*
We will neglect every effect related to PMNS matrix assuming then Lepton Flavour (LF) conservation for the SM.

1.2 The Flavour sector

1.2.1 *Flavour symmetries of SM Lagrangian*

As previously mentioned, allowing terms with dimension in mass at most equal to four, the lagrangian exhibit many additional symmetries to the ones assumed by the theory, being them exact or approximated.

Simmetries concerning the flavour has to appear in either $\mathscr{L}_{kin}$ or $\mathscr{L}_Y$.
Since $\mathscr{L}_{kin}$ is proportional to identity in flavour space and even in gauge interactions space as already mentioned every field can be rotated by $\psi \to V_\psi \psi$ with $V \in U(3)$ and it would be a symmetry of $\mathscr{L}_{kin}$. That is true for any independent V_ψ associated with the fermion ψ i.e. the whole symmetry arising is $\mathscr{G}_F = [U(3)]^5 = U(3)_q \otimes U(3)_l \otimes U(3)_u \otimes U(3)_d \otimes U(3)_e$ which is called the *Flavour Group.*

Since every matrix in $U(3)$ can be obtained from the product of two matrix in $U(3)$, one proportional to identity and the other with determinant equal to 1, is instructive to decompose every $U(3)$ as $U(3) \sim SU(3) \otimes U(1)$.
All the $U(1)$ groups are given by the gauge diagonality of kinetik term and it represents the conservation of the number of a given fermion (seen as flavour eigenstate) in every gauge interaction.
$SU(3)$, instead, are due to the fact that flavour matrix is equal to identity, and they guarantee that every different flavour component of a given fermion interacts with gauge fields with the same intensity.

These symmetries of $\mathscr{L}_{kin}$ are partially broken by Yukawa coupling in fact, since Higgs couples different gauge representations of fields (in other worlds Yukawa coupling isn't gauge diagonal). Nevertheless, being the Higgs field a colour-singlet, it can couple quark with quark and lepton with lepton, hence the $[U(1)]^5$ symmetry is broken

only partially, the symmetry left is $[U(1)]^2 = U(1)_B \otimes U(1)_L$ which represent the conservation of *Barion Number* and *Lepton Number* that are realized exactly in SM.
In the lepton sector, since Yukawa matrix is diagonal, we have an additional $[U(1)]^3$ under the which the i-th family of leptons has the same charge being l^i_L or e^i_R; this symmetry represents the already mentioned Lepton Flavour conservation, which is exact in SM apart from effects derived from neutrino's mass.

Then, since Yukawa matrices are far to be proportional to identity, a generic $V \in SU(3)$ would change Yukawa matrices, even if it was the same for the two fields coupling the Higgs inside a given operator.
In $\mathscr{L}_{kin}$ we were guaranteed to have just flavour blind interactions by $SU(3)$. When we include Yukawa couplings the quark sector, Yukawa matrices of quark break the $[SU(3)]^3$ of quark fields, so the interactions with Higgs breaks the flavour universality of quark interactions. It is important to clarify that this is not due to the fact that we can't diagonalize Yukawa matrices with a flavour basis, because even if we could do it (i.e. in the limit $V_{CKM} = 1$) the Yukawa matrices of quark, being not proportional to identity, aren't invariant under $Y_{u/d} \to V^\dagger Y_{u/d} V$, then $[SU(3)]^3$ of the quark sector is explicitly broken. Same argument can be brought for the lepton sector to say that $[SU(3)]^2$ is explicitly broken because of the different mass of charged leptons. Nevertheless the situation of lepton numerically is way different from quark, in fact if the heaviest quark, the top-quark, has a mass of 173 GeV comparable with Higgs'vev ($v \simeq 246\ GeV$), charged leptons are way lighter; the heaviest lepton, the τ, weighs $\simeq 1.7\ GeV$ which means that the biggest matrix element of Y_e is $Y_e^{33} \simeq \frac{1.7}{246} \simeq 7 \cdot 10^{-3}$. If we neglect terms of order $\frac{m_\tau}{v}$ the flavour group $[SU(3)]^2$ of lepton sector remains unbroken making arise the so known *Lepton Flavour Universality* (LFU), which is realized approximately in the SM, that tells us the interactions of SM are almost flavour-blind in the lepton sector, apart from terms of τ-Higgs coupling.

Phenomenologically LFU can be tested in every process in which the phase space factor of the final state (i.e. when the mass of the final state products is neglectable compared to the total energy in the rest frame) is neglectable, the rate of events is the same for every flavour of leptons involved.
Some examples of ratios that test LFU are:

$$\left.\frac{\mathscr{B}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\mathscr{B}(\tau \to e \nu_\tau \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests electron-muon universality,

$$\left.\frac{\mathscr{B}(\tau \to e \nu_\tau \bar{\nu}_e)}{\mathscr{B}(\mu \to e \nu_\mu \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests muon-tau universality, and many others.

A measure of sensitive deviations of this ratios would mean beyond SM effects.

1.2.2 *Flavour Theorems*

We have seen so far that mass'eigenstates of quarks aren't gauge interaction eigenstates as well.
Nevertheless we define particles we observe as eigenstates of the time evolution i.e. states with a well-defined mass.
The fact that gauge interactions aren't flavour diagonal in mass'eigenstates basis makes arise processes that link different families of quarks, known as *flavour breaking* processes. These processes happen to be suppressed because of the values of CKM-matrix.

They main features of all breaking flavour processes are summarized in three statements that, because of their fundamental importance, are known as *Flavour theorems*:

Flavour theorem 1. *Flavour breaking processes are absent in lepton sector.*

in which the proof is shown above in section 1.1.1.

Flavour theorem 2. *Flavour breaking processes in the quark sector happen just in charge current processes (exchanging $W^{\pm}$).*

Flavour theorem 3. *Applying CP transformation to the whole lagrangian we find that $\mathscr{L}^{CP} \equiv CP(\mathscr{L})$ is equal to $\mathscr{L}$ if and only if V_{CKM} is real.*

The proof of theorems 2 and 3 are reported in A.

The second flavour theorem tells us that flavour breaking processes can arise at three level just through exchange of W boson. Also, at every order they involve only Left Handed fermions apart from corrections due to mass insertions.

The third flavour theorem tells us that CP violation (CPV) can happen just if there are at least three family of quark.
Historically when in 1964 Christenson, Cronin, Fitch and Turley observed the first evidence of CPV in neutral Kaons system [1], the third family of quark wasn't observed yet. We have seen previously that CKM matrix has $\frac{N^2-3N+2}{2}$ irreducible phases, hence if only the two families known at the time existed CKM matrix would have been real and wouldn't have allowed CPV for the theorem 3.
In the years later, discovering the b quark and later t quark, was good test to the newborn Flavour Sector of the SM.

1.2.3 *CKM matrix*

Diagonalizing the quark mass matrices we found that up-quark (by convention could be down as well) interaction eigenstates are rotated respect to the mass eigenstates through:

$$V \equiv V_{u_L}^{\dagger} V_{d_L} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.7}$$

Where every elements is generally complex. Neverteheless it has to satisfy unitarity condition $V^{\dagger}V = 1$, plus it hasn't a unique form because, for what said so far in section

1.2.1, is possible to redefine CKM through a phase trasformation of the single flavour components that leaves invariant $\mathscr{L}_{kin}$, since the $[SU(3)]^5$ symmetry was already used to diagonalize Yukawa matrices. There is actually a global phase that has to remain free due to the Barion Number conservation $U(1)_B$, for the rest, for N families we have $2N-1$ (N for down quarks, N for up quarks minus 1 for the global barion number) phases that we can define as we prefer to write CKM matrix.

These conditions limit the number of free parameters of V. A generic NxN unitary matrix has $2N^2 - N^2 = N^2$ because of the N^2 conditions in $V^\dagger V = 1$. Among these N^2 parameters we have the angles describing a ortogonal matrix $\frac{N(N-1)}{2}$ plus the $\frac{N(N+1)}{2}$ phases in which $2N-1$ can be chosen by redefinition of quark fields. Thus we remain with $\frac{N(N-1)}{2}$ angles and $\frac{N(N+1)}{2} - (2N-1) = \frac{N^2-3N+2}{2}$ phases.

With three families we count three angles and one phase that define our matrix. The standard parametrization is given by the product of three real rotations, and the phase included in the $1-3$ rotation, i.e.

$$V = R_{12}(\theta_{12}) \cdot R_{13}(\theta_{13}, e^{i\delta}) \cdot R_{23}(\theta_{23}) \tag{1.8}$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \end{aligned} \tag{1.9}$$

where $c_{ij} \equiv cos(\theta_{ij})$ and $s_{ij} \equiv sin(\theta_{ij})$.
From this parametrization is indeed not clear the hierarchy of the flavour breaking transition, so we like to introduce *Wolfenstain's parametrization*. We define

$$\lambda \equiv s_{12} \quad , A\lambda^2 \equiv s_{23} \quad , A\lambda^3(\rho - i\eta) \equiv s_{13}e^{-i\delta}$$

We obtain, up to order λ^3

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{1.10}$$

Since $\lambda^{exp} \simeq 0.22$ we find that, as mentioned previously, $V = 1 + O(\lambda)$, thus flavour diagonal transitions are encouraged. Flavour breaking transitions are suppressed at order λ if they link first and second family, at order λ^2 if they link first and second ones and at order λ^3 if they link first and third family.

1.3 Fermi Theory

The Fermi Theory is the QFT used to describe the weak processes of the SM when the energy is way below $M_W = 80\ GeV$.

This theory describes the weak processes at low energies ignoring the details of the UV Physics like the coupling beetween fermions and vectors that mediate the force, the mass of these mediators nor the theoretical nature of all the particles heavier than $\Lambda = 80\, GeV$, which we call the *matching scale.*

In an EFT all the physics beyond the matching scale is contained in the numerical coefficient in front of the Lagrangian's operators. In fact the condition for the EFT to be a low energy version of an UV theory is for those coefficient to satisfy the *matching condition* that consists in impose the coefficients of the two theories, to recreate the same transition amplitudes at an energy equal to the matching scale.

1.3.1 *Derivation of the Fermi Theory from the SM*

When the energy available in a given process is not enough to produce some of the particles among the spectrum of the theory, is possible to *integrate them out* replacing the fields that describe these heavy particles with the solution of their motion equations.
Doing in that way we would automatically satisfy the matching condition for all tree level amplitudes.

We here show how to generate operators in the Fermi Theory through the integration of the W and Z bosons.
Since in 1.2 we have B_μ and W^3_μ phenomenologically happen to have a not defined mass, the first step is to define:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} cos\, \theta_W & -sin\, \theta_W \\ sin\, \theta_W & cos\, \theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} W^3_\mu cos\, \theta_W - B_\mu sin\, \theta_W \\ W^3_\mu sin\, \theta_W + B_\mu cos\, \theta_W \end{pmatrix}$$

Where A_μ is the field that describes the photon that, being massless doesn't need to be integrated out, and Z_μ is the field describing the Z boson, responsible for neutral current weak interactions. Instead θ_W is the *Weinberg angle* defined as

$$\frac{g_1^2}{g_1^2 + g_2^2} \equiv sin^2\, \theta_W$$

Since at low energy the symmetry is broken to $U(1)_{em}$ we would like to write $W_\mu^{1,2}$ in electric charge eigenstates basis:

$$W^\pm_\mu \equiv \frac{W^1_\mu \pm iW^2_\mu}{\sqrt{2}}, \quad \sigma_\pm \equiv \frac{\sigma_1 \pm i\sigma_2}{2}$$

hence $W_1\sigma_1 + W_2\sigma_2 = \sqrt{2}(W^+\sigma_- + W^-\sigma_+)$.
Now we can write the *covariantderivative* in term of mass eigenstates as:

$$D_\mu = \partial_\mu - i\frac{g_2}{\sqrt{2}}[W^+_\mu\sigma_- + W^-_\mu\sigma_+] - i\frac{g_2}{cos\, \theta_W} Z_\mu (T_3 - Q sin^2\, \theta_W) - i\frac{g_2}{sin\, \theta_W} A_\mu - ig_3 T^a_c G_{a\mu}$$

where $Q \equiv Y - T_3$ is the electric charge operator Then the lagrangian that contains the W and the Z is given by:

$$\begin{aligned}\mathscr{L} \supset &-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}W^{+}_{\mu\nu}W^{-}_{\mu\nu} + \frac{M_Z^2}{2}Z_\mu Z^\mu + M_W^2 W^+_\mu W^{\mu-} \\ &+ Z_\mu J_0^\mu + W^+_\mu J^\mu_- + W^-_\mu J^\mu_+\end{aligned}$$

where

$$J_0^\mu = \sum_\psi g_Z^\psi \bar{\psi}\gamma^\mu\psi$$

where the sum is done of every SM fermion after EWSB, i.e. $\{\psi\} = \{e_L, \nu_L, e_R, u'_L, d_L, u'_R, d_R\}$ and

$$g_Z^\psi = \frac{g_2}{\cos\theta_W}(T_3^\psi - Q^{\psi^2} sin^2\,\theta_W) \tag{1.11}$$

while the charged currents are:

$$J^\mu_+ \equiv \frac{g_2}{\sqrt{2}}[\bar{d}_L\gamma^\mu u'_L + \bar{e}_L\gamma^\mu\nu_L], \quad J^\mu_- \equiv (J^\mu_+)^\dagger \tag{1.12}$$

where $u' \equiv V^\dagger_{CKM}u$. Here we can see that the neutral current interaction uf LH up quarks is flavour diagonal because of unitarity of CKM matrix $V^*_{ij}V_{jk} = \delta_{ik} \rightarrow$

$$\bar{u}^i_L V_{jk}\gamma^\mu V^*_{kl}u^l_L = \bar{u}^i_L\gamma^\mu u^i_L$$

in the Charged current instead we have:

$$\bar{u}^i_L V_{jk}\gamma^\mu d_L$$

that breaks the flavour explicitly. This fact can be seen as the proof of the theorem 2.

Then, the equation of motion result:

$$\begin{aligned}&\frac{\delta\mathscr{L}}{\delta W^+_\mu} = 0 = M_W^2 W^{\mu-} + J^\mu_- \rightarrow \\ &W^{\mu-} = -\frac{1}{M_W^2}J^\mu_-; \quad W^{\mu+} = (W^{\mu-})^\dagger = -\frac{1}{M_W^2}J^\mu_+ \\ &\frac{\delta\mathscr{L}}{\delta Z_\mu} = 0 = M_Z^2 Z^\mu + J_0^\mu \rightarrow \\ &Z^\mu = -\frac{1}{M_Z^2}J_0^\mu;\end{aligned}$$

Replacing in the interation term the solution of equations of motion we find

$$\mathscr{L}_{EFT} = -\frac{1}{M_W^2}[J^\mu_+ J_{\mu_-} + J^\mu_- J_{\mu_+}] - \frac{1}{M_Z^2}J_0^\mu J_{0\,\mu}$$

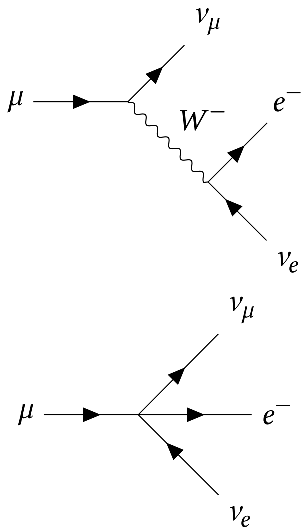


Figure 1.1: Above: Feynman diagram for muon decay in SM, the lagrangian terms responsible for this interaction is of the form $\sim g_2\overline{\psi}_{1L}\gamma^\mu\psi_{2L}W_\mu$.
Below: Feynman diagram for muon decay in Fermi Theory, the lagrangian terms responsible for this interaction is of the form $\sim G_F\overline{\psi}_{1L}\gamma^\mu\psi_{2L}\overline{\psi}_{3L}\gamma_\mu\psi_{4L}$
.

then we use the relation $M_W^2 cos^2\theta_W = M_Z^2$ and the definition of the *Fermi constant* $\frac{g_2^2}{2\sqrt{2}} \equiv G_F$ to write:

$$\begin{aligned}\mathscr{L}_{EFT} \supset &- \frac{4G_F}{\sqrt{2}}\big[\overline{u'_L}\gamma^\mu d_L\ \overline{d_L}\gamma_\mu u'_L\ +\ \overline{u'_L}\gamma^\mu d_L\ \overline{\nu_L}\gamma_\mu e_L\ +\ \overline{e_L}\gamma^\mu\nu_L\ \overline{d_L}\gamma_\mu u'_L\ +\ \overline{e_L}\gamma^\mu\nu_L\overline{\nu_L}\gamma_\mu e_L + h.c.\big] \\ &- 4\sqrt{2}G_F[\sum_\psi(T_3^\psi - Q^\psi sin^2\theta_W)\overline{\psi}\gamma^\mu\psi][\sum_\chi(T_3^\chi - Q^\chi sin^2\theta_W)\overline{\chi}\gamma_\mu\chi]\end{aligned} \tag{1.13}$$

up to $O(\lambda)$ corrections.
Since $(sin^2\theta_W)^{exp} \simeq 0.22$ we notice that the weak interaction for RH fermions ($T_3 = 0$) exist just in the neutral current form, and it is suppressed due to the accidental smallness of θ_W.

The lagrangian 1.13 satisfies the matching condition for every tree level amplitude of the SM.
To include the flavour breaking terms it is sufficient to replace u_L with $V^\dagger u_L$

The Fermi Theory was historically the first theory that described the weak processes as the beta decay or the muon decay (Feynman diagram in Figure 1.1), whose leading contributions are three level induced in the SM without any flavour suppression.

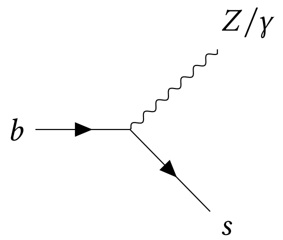
Adding all the Flavour breaking terms we get a lot of terms with a small coupling due to CKM suppression, and also is possible to get some non trivial loop induced operators, like the neutral current flavour breaking processes, also known as *Flavour Changing Neutral Currents* (FCNC).

1.3.2 *Flavour Changing Neutral Currents*

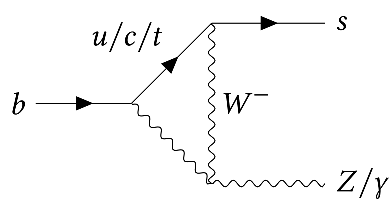
As we have seen flavour breaking processes arise at tree level just through exchange of W bosons. Nevertheless, at low energy we could have flavour breaking transitions in neutral current processes due to the matching to a one loop amplitude in the high energy theory.

These processes are named Flavour Changing Neutral Currents and, because of the theorem 2, they gets a double suppression for the additional vertex and the loop factor.

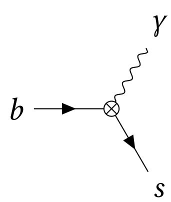
For example the vertex:



is not allowed in the SM, but the $b \to sZ/\gamma$ transition can arise at one loop level through a loop of W:



Now if we integrate out W's and Z bosons we get:



In the previous diagram we neglected the Z boson because in Fermi Theory it doesn't belong to the spectrum. Indeed if the Z were virtual in the SM diagram, perhaps producing a pair l^+l^-, we would have exactly the effective vertex mentioned in section 2 that allows the decay $b \to sl^-l^+$ and so the decay $B \to K^{(*)}l^-l^+$ which will be of the most interested for us.

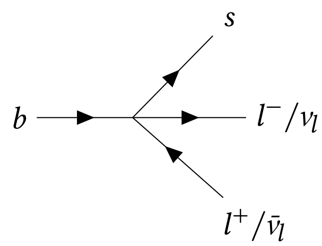
In SM we have CKM, loop factor, and gauge coupling that give me a prediction for the coefficient in front of the operator that mediate this decay: the Wilson coefficent. So if the mesaured Wilson coefficient is different from the theoretical one, it means that probably to build the EFT we need to integrate out other heavy particles which we don't already know.

2

B-Physics Anomalies

The two fundamental processes in which we are interested in are: the neutral current (NC) decay

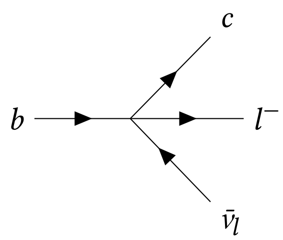
$$b \to s l^+ l^- \text{ or } b \to s \nu \bar{\nu}$$



and the charged current (CC) decay

$$b \to c l \bar{\nu}$$

.



2.1 Main Processes for NC transitions

For what concern the NC decays the first process we want to discuss is the decay:

$$B \to K^* l^+ l^-.$$

Table 2.1: Table with the most important anomalies in $b \to sl^+l^-$ transition. $R_{K^*}^{[q_1^2,q_2^2]}$ means the ratio R_{K^*} in which the momenta of the pair lepton-antilepton has energy at rest q^2 included beetween q_1^2 and q_2^2. Fonte: [2]

Observable	*Experiment*	*SM*
$R_{K^*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07} \pm 0.03$	0.906 ± 0.028
$R_{K^*}^{[1.1,6.0]}$	$0.69^{+0.11}_{-0.07} \pm 0.05$	1.00 ± 0.01
$R_{K}^{[1.1,6.0]}$	$0.846^{+0.042+0.013}_{-0.039-0.012}$	1.00 ± 0.01
$BR(B_s \to \mu^-\mu^+)$	$2.85^{+0.32}_{-0.31} \cdot 10^{-9}$	$(3.66 \pm 0.14) \cdot 10^{-9}$

We know that in the SM this process is loop-induced (as all the Flavour Changing Neutral Current (FCNC)) and should have the same rate for the charged lepton being electron or muon because of LFU. Given the masses of the particle involved:

$$m_B \simeq 5.3GeV \;\; m_K \simeq 430MeV \;\; m_\mu \simeq 106MeV \;\; m_e \simeq 500keV$$

the phase space factor is neglectable; and the ratio

$$R_{K^*}^{\mu e} \equiv \frac{\mathcal{B}(B \to K^*\mu^-\mu^+)}{\mathcal{B}(B \to K^*e^-e^+)}$$

is equal to 1 in the SM.

The reasons why we use ratios of Branching Ratios are mainly three:

- To reduce the dependence from hadronic form factors
- To reduce the dependence from CKM matrix elements
- To reduce the systematic error in general

In the Table 2.1 we can finally see the first B-Physics Anomaly; in fact the lack of muons among the decay's products is a clear hint of LFU violation. The formal definition for the ratio $R_{K^*}^{[q_1^2,q_2^2]}$ is:

$$R_{K^*}^{[q_1^2,q_2^2]} = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to K^*\mu^+\mu^-)}{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to K^*e^+e^-)}$$

and same for the pseudoscalar K.

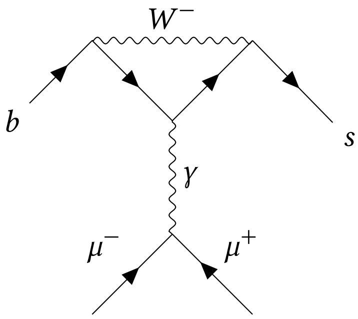


Figure 2.1: Diagram of the fundamental transition $b \to s\mu^-\mu^+$ in the SM.

2.1.1 $B_s \to \mu\mu$

:

Also in Table 2.1 we can see the data referred to the $B_s \to \mu\mu$ decay we can easily see that the 4-fermion vertex factor involved is exactly the same of the $B \to K^*\mu\mu$ decay because of the so called *crossing symmetry.*
With this decay indeed we need to be careful to the possibility that perturbativity condition is not satisfied because of $c\bar{c}$ resonances. The diagram describing this decay in SM is the one in Figure 2.1.1 which is one of the so called *penguin diagrams.*

If the energy of the virtual photon is few GeV, there is a no-neglectable contribution due to the fact that the photon can produce a pair cc nearby of the known resonances that have those quark as valence quark : J/ψ, $\psi(1S)$, $\psi(2S)$,...
So we can have issues due to QCD non-perturbativity but on the other hand we have a final state which is *clean* theoretically, since is a two lepton state with no hadronic form factor. That's why we will treat this kind of process separately.

2.2 Main Processes for CC transitions

The main CC decay in which we are interested in is:

$$B \to D^* l \nu_l$$

We define the ratio analogously:

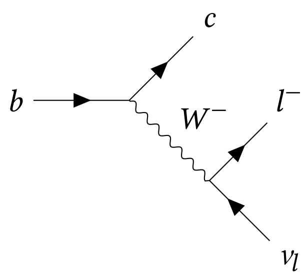
$$R_{D^*}^{\tau l} \equiv \frac{\mathcal{B}(B \to D^* \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathcal{B}(B \to D^* l \nu_l)}.$$

In Table 2.2 we see the most important CC anomalies.
The main differences with the NC can be summarized as follows:

- From a first look to the Table 2.2 we can already notice that the significance of the anomaly is less important than the NC case;
- In SM this process is tree level generated (no penguins):

Table 2.2: Table with the most important anomalies in $b \to sl\nu$ transition. ρ stays for the correlation beetween the ratio with D and D^*. Fonte:[2]

Observable	*Experiment*	*SM*
$\{R_D, R_{D^*}\}$	$\{0.337(30), 0.298(14)\}$	$\{0.299(3), 0.258(5)\}$
ρ	-0.42	$-$
$BR(B^- \to \tau^- \bar{\nu}_\tau)$	$1.09(24) \cdot 10^{-4}$	$0.812(54) \cdot 10^{-4}$



and that's why the SM prediction is more accurated than the FCNC process.

- in the $B^- \to \tau\nu$ decay we have no issues linked to the charmonic resonances but the final state is not clean as the $\mu\mu$ pair (the neutrino is invisible and the τ decays briefly in hadrons). Also the rate is way bigger than the $B_s \to \mu\mu$.

If we begin to wonder about the NP's shape we notice that the NP couple mostly at the third generation for what concern quarks. A naive approach would be to guess a coupling growing with the mass (*Higgs-like*) but the NC decays suggest a smaller coupling for muons that the one for electrons. In the CC case, instead, we could imagine that the new physics couples more to the heaviest lepton: the τ.
All this kind of considerations (and way more of them) will be the ones that allow us to build a low energy model independent Field Theory: the Effective Field Theory; which later will give us the shape of the BSM particles we need.

3

Effective Field Theory approach

3.0.1 *Semileptonic operators*

:

Once we have included all the 4-fermions operators in the effective lagrangian we can choose the ones we need to calculate the amplitudes of the processes of interest. Keeping in mind that we don't want to break the conservation of Barion number, which is very tested, we have just three types of operators:

- Purely quark operators, which can mediate for instance the Kaon's decay in pions'channels. Nevertheless those operators are quite hard to match with the SM because we can have loop of gluons beetween the two currents which both couple with gluon. Since at the meson scale of energies the strong interaction is non-perturbative we have to consider a lot of contributions that are not easy to parametrize.

- Purely leptonic operators, which can for instance mediate the muon's decay. Those operators are used to the processes that have just leptons in the initial and final state avoiding all the QCD's mess for both of them. In fact the cited muon decay is predicted so much accurately that it was the main process used to measure the Fermi's constant G_F.

- Semileptonic operators, which can mediate the charge pion's decay, but also all the processes to which we can address the B-Physics Anomalies. Of course the prediction are not clean as the purely leptonic case, nevertheless we have no gluon loop beetween the two currents and this tells us that we can see how the quark current renormalize just using global symmetries of QCD. In fact, in QCD, the vector current is conserved and so the quark current in a semileptonic decay is not affected by renormalization group of QCD. This fact reducts the theoretical uncertainity to the hadronic form factor.

3.1 Fierz identities

Is possible to prove that, since any hermitian matrix NxN can be written as

$$H = c_0\mathbf{I} + \sum_{i=1}^{N^2-1} c_i T_i$$

where T_i are the generators of the fundamental representation of $SU(N)$, these generators satisfy the completeness condition:

$$\sum_{a=1}^{N^2-1} T^a_{ij} T_{a\ kl} = \frac{1}{2}(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}).$$

In the case $N = 2$ since $\sigma_a = 2T_a$ we find (neglecting the sum on a):

$$\sigma_{a\ ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl} \tag{3.1}$$

this relation can help us to write the EW structure to have all the currents contracted in the operators listed in 3.2, i.e. to have every single current trasforming with an irreducible representation of $SU(2)_L$.
Comparing two terms with the indexes sorted at the way we can rewrite the relation 3.1:

$$\sigma^a_{ij}\sigma_{a\ kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\ kj}). \tag{3.2}$$

For what concern the Lorentz group, since the group is $SU(2)xSU(2)^*$, the situation is analogous.
Having written fermions as Weyl spinors we have explicited the relation beetween the Lorentz structure of the currents and Pauli matrices:

$$\sigma^\mu = (\mathbf{I}, \vec{\sigma}) \qquad \overline{\sigma}^\mu = (\mathbf{I}, -\vec{\sigma})$$

These matrices allow us to write a current which trasform as a vector under Lorentz group, combining two spinors trasforming both as the $(\frac{1}{2}, 0)$ or the $(0, \frac{1}{2})$, and so the current with explicit lorentz index is written with the *dotted notation*[1]

$$J^\mu_R = (\psi^\dagger_R)^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\chi^{\dot{\alpha}}_R \sim (0,1) \quad , J^\mu_L = (\psi^\dagger_L)^{\dot{\alpha}}(\overline{\sigma}^\mu)_{\dot{\alpha}\alpha}\chi^\alpha_L \sim (1,0)$$

In which the dots distinguish the two fundamental representation of the Lorentz group.
The scalar bilinear instead is the product of two spinors trasforming one as $(\frac{1}{2}, 0)$ and the other as $(0, \frac{1}{2})$:

$$S = (\psi^\dagger_R)^\alpha \chi_{L\alpha} \sim (0,0)$$

and same with all the indexes dotted.
It is useful to introduce the charge coniugated spinor:

$$(\psi_R)^c_\alpha \equiv \varepsilon_{\alpha\beta}(\psi^*_R)^\beta \sim (\frac{1}{2}, 0)$$

[1]The dotted indexes indicate $SU(2)^*$, i.e. the RH fermions.

where the ε is the totally antisymmetric tensor in two dimensions (fixed $\varepsilon^{12} = -\varepsilon^{21} = -\varepsilon_{12} = \varepsilon_{21} = 1$). This allow us to build a scalar with two spinors belonging to the same representation of the Lorentz group, eventually even two copies of the same spinor. In that case the ε tensor guarantees the antisymmetry typical of the singlet.

Now that we described the structure of the vector and the scalar identities we can take back 3.1 and, mindful that $\sigma^0_{\alpha\dot{\alpha}} = \overline{\sigma}^0_{\alpha\dot{\alpha}} = \delta_{\alpha\dot{\alpha}}$ and including the Minkowski's metric, write:

$$\sigma_{\mu\ \alpha\dot{\alpha}}\sigma^{\mu}_{\beta\dot{\beta}} = 2(\delta_{\alpha\dot{\alpha}}\delta_{\beta \dot{beta}} - \delta_{\alpha\dot{\beta}}\delta_{\beta\dot{\alpha}}) \tag{3.3}$$

now using that $\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}$ we find:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} = -\sigma^{\mu}_{\alpha\dot{\beta}}\sigma_{\mu\beta\dot{\alpha}} \tag{3.4}$$

Now, generalizing at the overlined matrices:

$$\overline{\sigma}^{\dot{\alpha}\alpha}_{\mu}\overline{\sigma}^{\mu\ \dot{\beta}\beta} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}} \tag{3.5}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\sigma_{\mu}}^{\dot{\beta}\beta} = 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}}. \tag{3.6}$$

Despite to the similar aspect of the equations it is clear that the relations for $SU(2)_L$ and for $SU(2)xSU(2)^*$ has to be used indipendently.
In some cases we have to use both to write the operators generated in the basis presented in section 3.2. In these cases the tensor structire could be complicated and eventually confusing, so we are going to present the useful results now to have them ready when we will handle the physics.

3.1.1 *Fierzing in the scalar lagrangian*

When we introduce to the theory scalar Leptoquarks, integrating them out from the lagrangian we could generate four-fermion operators of this shape:

$$\overline{l^{1c}}\varepsilon q^1\ \overline{q^2}\varepsilon l^{2c} = l^1_{\alpha a}q^1_{\beta b}q^{2*}_{\gamma c}l^{2*}_{\delta d}\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

where i, j, k, l are EW indexes and $\alpha, \beta, \gamma, \delta$ are Lorentz indexes and the ε of the Lorentz's group comes from the charge coniugation. Since all the fermions involved happens to be LH we have to write them as a linear comnbination of O_S and O_T. To do that we need to write

$$\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = c_1\ \delta^{ad}\delta^{bc}\overline{\sigma_{\mu}}^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}\ +\ c_2\ \sigma^{ad}\sigma^{bc}\overline{\sigma_{\mu}}^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}$$

basically switching the position of $q^1 \longleftrightarrow l^2$.
Now we use that

$$\varepsilon^{ab}\varepsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} = \frac{1}{2}\sigma^{ad}_{a}\sigma^{bc\,a} - \frac{1}{2}\delta^{ad}\delta^{bc}$$

Where we used 3.1 in the second equality.
Then, acting on the 3.5,

$$\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = \frac{1}{2}\overline{\sigma}^{\alpha\gamma}_{\mu}\overline{\sigma}^{\mu\ \beta\delta} = -\frac{1}{2}\overline{\sigma}^{\delta\alpha}_{\mu}\overline{\sigma}^{\mu\ \gamma\beta}$$

And multiplicating is straightforward to obtain:

$$\overline{l^{1c}}\varepsilon q^1\ \overline{q^2}\varepsilon l^{2c} = \frac{1}{4}\left[q^{2\dagger}\bar{\sigma}^\mu q^1\ l^{2\dagger}\bar{\sigma}_\mu l^1 -\ q^{2\dagger}\bar{\sigma}^\mu\sigma^a q^1\ l^{2\dagger}\bar{\sigma}_\mu\sigma_a l^1\right] \tag{3.7}$$

Once obtained that result is easy to derive the same in the case of the contraction of two triplet scalar currents:

$$\overline{l^{1c}}\varepsilon\sigma_a q^1\ \overline{q^2}\sigma^a\varepsilon l^{2c} = l^1_{\alpha a}q^1_{\beta b}q^{2*}_{\gamma c}l^{2*}_{\delta d}(\sigma^a\varepsilon)^{ab}(\varepsilon\sigma_a)^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

We already know how to treat the lorentz structure that will bring a factor $-\frac{1}{2}\bar{\sigma}^{\alpha\delta}_\mu\bar{\sigma}^{\mu\ \beta\gamma}$. The EW structure instead is different:

$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cea} = (\delta^{af}\delta^{ed} - \delta^{ad}\delta^{fe})(2\delta^{e}\delta^{kj} - \delta^{mj}\delta^{kn}) =$$

$$2\delta^{ad}\delta^{bc} - 4\delta^{ad}\delta^{bc} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} = -(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd})$$

then recalling the 3.1

$$\sigma^a_{da}\sigma_{cba} = 2\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \rightarrow$$

$$\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cea} = -\frac{1}{2}(\sigma^{ad}_a\sigma^{bc^a} + 3\delta^{da}\delta^{cb})$$

multiplying the factors coming from EW and Lorentz group:

$$\overline{l^{1c}}\varepsilon\sigma_a q^1\ \overline{q^2}\sigma^a\varepsilon l^{2c} = \frac{1}{4}\left[3q^{2\dagger}\bar{\sigma}^\mu q^1\ l^{2\dagger}\bar{\sigma}_\mu l^1 +\ q^{2\dagger}\bar{\sigma}^\mu\sigma^a q^1\ l^{2\dagger}\bar{\sigma}_\mu\sigma_a l^1\right] \tag{3.8}$$

3.2 Effective Semileptonic Lagrangian

As we mentioned many times so far we want to find the coefficients that parametrize the B-Physics Anomalies which appear just in semileptonic B mesons decays.

Since every four fermion operator has to be made of two spinor bilinears both being vector or scalar[2]. The choice we make is to avoid charge's conjugated spinors and, if possible, to have every bilinear being a colour singlet. This choice is due to the fact that in hadronic transitions is confortable to have few shapes of quark operators in order to simplify them, when it's possible, in the computation of experimental observables.

Our first step is to collect all the effective operators that can contribute to those processes. For first we will list five operators that can be written as contraction of two vector currents.

If we assume for NP to be coupled just to LH fermions, as done by [3], we would need just two operators:

$$\mathcal{O}_S = q^\dagger_L\bar{\sigma}^\mu q_L\ l^\dagger_L\bar{\sigma}_\mu l_L = (V^*u_L)^\dagger\bar{\sigma}^\mu(V^*u_L)\ e^\dagger_L\bar{\sigma}_\mu e_L + d^\dagger_L\bar{\sigma}^\mu d_L\ e^\dagger_L\bar{\sigma}_\mu e_L +$$

$$(V^*u_L)^\dagger\bar{\sigma}^\mu(V^*u_L)\ \nu^\dagger_L\bar{\sigma}_\mu\nu_L + d^\dagger_L\bar{\sigma}^\mu d_L\ \nu^\dagger_L\bar{\sigma}_\mu\nu_L$$

[2]we could have even 2 rank tensor if we used Dirac representation for the fields

$$\mathcal{O}_T = q_L^\dagger \bar\sigma^\mu \sigma_a q_L \, l_L^\dagger \bar\sigma_\mu \sigma^a l_L = 2(V^* u_L)^\dagger \bar\sigma^\mu d_L \, l_L^\dagger \bar\sigma_\mu \nu_L + 2 d_L^\dagger \bar\sigma^\mu (V^* u_L) \, \nu_L^\dagger \bar\sigma_\mu l_L +$$
$$(V^* u_L)^\dagger \bar\sigma^\mu (V^* u_L) \, \nu_L^\dagger \bar\sigma_\mu \nu_L + d_L^\dagger \bar\sigma^\mu d_L \, l_L^\dagger \bar\sigma_\mu l_L - d_L^\dagger \bar\sigma^\mu d_L \, \nu_L^\dagger \bar\sigma_\mu \nu_L - (V^* u_L)^\dagger \bar\sigma^\mu (V^* u)_L \, l_L^\dagger \bar\sigma_\mu l_L \tag{3.9}$$

where we neglect the flavour indexes. Writing them explicitly we have, for instance, the singlet operator equal to $\mathcal{O} = \mathcal{O}^{abcd} = q_L^{a\,\dagger} \bar\sigma^\mu q_L^b \, l_L^{c\,\dagger} \bar\sigma_\mu l_L^d$ with

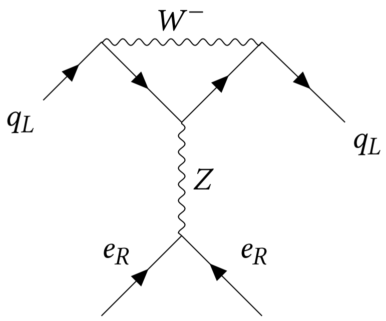
$$q_L^i = \begin{pmatrix} V^{*ij} u_{L\,J} \\ d_L^i \end{pmatrix} \quad , l_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

Also the EW indexes are neglected and here that means that both the vector currents composing the operator are irreducible representation of $SU(2)_L$ (two triplets for $\mathcal{O}_T$ and two singlets for any other). Besides we want to point out that $\mathcal{O}_T$ can mediate CC transitions as well as NC ones.

Including the RH fermions, we find other two operators compatible with the gauge symmetry $SU(3)_c x SU(2)_L x U(1)_Y$ already present in SM:

$$\mathcal{O}_{LR1} = q_L^\dagger \bar\sigma^\mu q_L \, e_R^\dagger \sigma_\mu e_R = (V^* u_L)^\dagger \bar\sigma^\mu (V^* u_L) \, e_R^\dagger \sigma_\mu e_R + d_L^\dagger \bar\sigma^\mu d_L \, e_R^\dagger \sigma_\mu e_R$$
$$\mathcal{O}_{LR2}^{u/d} = q_R^\dagger \sigma^\mu q_R \, l_L^\dagger \bar\sigma_\mu l_L = q_R^\dagger \sigma^\mu q_R \, \nu_L^\dagger \bar\sigma_\mu \nu_L + q_R^\dagger \sigma^\mu q_R \, e_L^\dagger \bar\sigma_\mu e_L$$

when the u/d means that we have two independent versions of the $\mathcal{O}_{LR2}$ for q_R equal to the up or down quark's flavour triplet. These two operators can describe just NC transitions and so the flavour diagonal part is not much interesting. The flavour breaking contributes are as well generated in SM but suppressed for different reasons: the FCNC contribute given by $\mathcal{O}_{LR1}$ described by the diagram:



is accidentally suppressed because of the smallness of the *Weinberg's angle* θ_W that suppresses the coupling beetween the Z boson and RH fermions; O_{LR2} is even more suppressed because, even if the Z this time couple to its favourite LH fermions, to have the FCNC for RH quarks we need to flip the chirality twice because of the Flavour theorem which states that flavour breaking couplings of the SM are allowed just for LH fermions, and so the contribute is suppressed due to the light mass of the quarks involved.
The last vector-vector operator is the one that, in SM, takes both the suppressions described above:

$$\mathcal{O}_R^{u/d} = q_R^\dagger \sigma^\mu q_R \, e_R^\dagger \sigma_\mu e_R$$

again in the u/d versions according to the flavour triplet involved.

Now we have three operators made up the contraction of two scalar currents. For first:

$$\mathcal{O}_S^u = q_L^\dagger u_R \, \varepsilon \, l_L^\dagger e_R = (V^* u)_L^\dagger u_R \, e_L^\dagger e_R - d_L^\dagger u_R \, \nu_L^\dagger e_R$$

$$\mathcal{O}_S^d = q_L^\dagger d_R \, e_R^\dagger l_L = (V^* u)_L^\dagger d_R \, e_R^\dagger \nu_L + d_L^\dagger d_R \, e_R^\dagger e_L$$

in which $\mathcal{O}_S^d$ could indeed be written as vector currents contraction through Fierz identities, but renouncing to the request of having currents trasforming as $SU(3)_c$ singlets. The ε tensor is the one introduced in section 3.1, a part that acts on $SU(2)_L$ to select the singlet.

In the end we have the *LeptoQuark* operator:

$$\mathcal{O}_{LQ} = e_R^\dagger q_L \, \varepsilon \, u_R^\dagger l_L = e_R^\dagger (V^* u)_L \, u_R^\dagger e_L - e_R^\dagger d_L \, u_R^\dagger \nu_L$$

that can't be written as contraction of two colour singlets but just of two LQs scalar currents, unless we involve charge's conjugate fields, in fact

$$\varepsilon_{ij}\varepsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} \rightarrow q^{c\dagger}_L \varepsilon l_L \, e_R^\dagger u_R^c = \mathcal{O}_{LQ} - \mathcal{O}_S^u$$

that anyway wouldn't help the simplification of hadronic form factors, since we would have anyway a current which appears just in this operator.
It is straightforward to see, from the electric charge's eigenstates form, that all the scalar operators contribute both to CC and NC transitions,

We choose to separate the SM Lagrangian from the operators defined above, i.e. to use a Lagrangian that recreate the SM predictions in the limit $C_S = C_T = C_{LR1} = C_{LR2}^{u/d} = C_R^{u/d} = C_{LQ} = C_S^u = C_S^d = 0$:

$$\begin{aligned} \mathscr{L}_{EFT} = & \mathscr{L}_{SM} + C_S \mathcal{O}_S + C_T \mathcal{O}_T + + C_{LQ}\mathcal{O}_{LQ} + C_{LR1}\mathcal{O}_{LR1} + \\ & \sum_{q=u,d} \left[C_R^q \mathcal{O}_R^q + C_{LR2}^q \mathcal{O}_{LR2}^q + C_S^q \mathcal{O}_S^q \right] \end{aligned} \tag{3.10}$$

in which, expilciting flavour indexes $C\mathcal{O} = C_{abcd}\mathcal{O}^{abcd}$ and hence we define c_i as

$$C_i^{q_1 q_2 l_1 l_2} \equiv c_i \Lambda_i^{q_1 q_2 l_1 l_2}$$

that seem to be redundant but will be useful to describe separately the flavour structure separately from the rest. In our convention q_1 and l_1 are the ones appearing in the operator *daggered.*

From the experiments we acknowledge what are the observables of interest to explore the clues of NP. Plus they suggest us that NP, at leading level, doesn't concern the lightest families of quark and leptons and couples preferly to the heaviest.

4

Some of the possible heavy bosons

As we previously said one possible way to modify the Wilson coefficients in an EFT is to introduce heavy particles to the theory which couple with the fermions involved in the processes we want to accomodate.
When we introduce new particle interacting with the SM particles we need to look carefully at the experimental constraints. In particular the particle has to satisfy two bounds:

- The contribution to the observable given by the diagrams in which the new particles appears as virtual particles has to show agreement with the experiments, both the ones who seem anomalous to the SM and the ones that has tested the SM.

- If the new particles interact with the particles that are smashing at the colliders, so the mass range allowed for those particles introduced has to not contain the energies explored at that colliders so far.

To begin we will see how we can accomodate the B-Physics Anomalies introducing different type of heavy vector and scalar bosons.
We will begin with the most familiar case to the ones who know the SM: colour-less vector charged under $SU(2)_L$. Indicating the quantum number as $(SU(3)_c, SU(2)_L)_Y$ we can address to these particles $B' \sim (\mathbf{1}, \mathbf{1})_0$ and $W' \sim (\mathbf{1}, \mathbf{3})_0$, where the names already suggest the connection with their SM's lighter sisters W,B[1].
Then we will describe the vector Leptoquarks $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$.
In the end we will descibe the behavior of some scalar Leptoquarks $S_1 \sim (\mathbf{3}^\dagger, \mathbf{1})_{1/3}$, $S_3 \sim (\mathbf{3}^\dagger, \mathbf{3})_{1/3}$ and $R_2 \sim (\mathbf{3}, \mathbf{2})_{7/6}$.

Once we have aknowledged what contributions are taken from the different bosons we will be ready to see what mediator, or what combination of mediators, are proper to accomodate the B-Physics anomalies without contradicting the Flavour tests done so far.

[1]That phenomenological are better known as $W^\pm$, Z and the photon γ because of the so known EWSB.

4.1 Colour-less bosons

4.1.1 $B' \sim (\mathbf{1}, \mathbf{1})_0$

The first candidate is the heaviest version of the EW singlet of the SM B: $B' \sim (\mathbf{1}, \mathbf{1})_0$. Apart from the bigger mass the main difference beetween these two bosons is the flavour structure, which is diagonal in SM because of the gauge symmetry and is free a priori instead for the BSM version.
Once introduced this boson the most general UV Lagrangian contains:

$$\begin{aligned}\mathscr{L}_{UV} \supset \frac{M_{B'}^2}{2} B_\mu B^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + g_{B'} l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L B_\mu + g_{B'} q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L B_\mu + \\ g_{B'} e_R^\dagger \sigma^\mu \lambda_B^e e_R B_\mu + g_{B'} u_R^\dagger \sigma^\mu \lambda_B^u u_R B_\mu + g_{B'} d_R^\dagger \sigma^\mu \lambda_B^d d_R B_\mu\end{aligned} \tag{4.1}$$

where the λ matrices are flavour matrices that are real since B' belongs to a real representation of the gauge group.

When we go down to energies $\ll M_{B'}$ B' can't be produced on shell anymore, and so it can't appear in a process but as virtual particle. In this condition is possible to *integrate it out* from the lagrangian, i.e. replacing B'^μ with the solution of the equation of motion:

$$\frac{\delta \mathscr{L}}{\delta B_\mu} = 0 = \frac{M_B^2}{2} B^\mu + g_{B'} l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L + g_{B'} q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L$$

$$g_{B'} e_R^\dagger \sigma^\mu \lambda_B^e e_R + g_{B'} u_R^\dagger \sigma^\mu \lambda_B^u u_R + g_{B'} d_R^\dagger \sigma^\mu \lambda_B^d d_R \rightarrow$$

$$B^\mu = -\frac{2 g_{B'}}{M_{B'}^2} \left[l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L + q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L + e_R^\dagger \sigma^\mu \lambda_B^e e_R B_\mu + u_R^\dagger \sigma^\mu \lambda_B^u u_R B_\mu + d_R^\dagger \sigma^\mu \lambda_B^d d_R B_\mu \right].$$

obtaining an effective lagrangian:

$$\mathscr{L}_{EFT} \supset -2 G_{B'} \left[\Big(\sum_\psi \bar{\psi} \lambda_B^\psi \sigma_\mu \psi \Big) \Big(\sum_\chi \bar{\chi} \lambda_B^\chi \sigma_\mu \chi \Big) \right]$$

where $G_{B'} \equiv \frac{g_{B'}^2}{M_{B'}^2}$ and the sums on ψ and χ are done over every SM fermion.
Among the 25 four-fermion operators, made of all the possible combination of these five neutral currents we recognize some semileptonic operators, precisely all the ones constructed with two vectorcurrents $SU(2)_L$-singlet inside the list shown in section 3.2. The coefficients of these operators are equal to:

$$C_S = -2 G_{B'} \lambda_B^q \lambda_B^l \quad , C_R^{u/d} = -2 G_{B'} \lambda_B^e \lambda_B^{u/d}$$

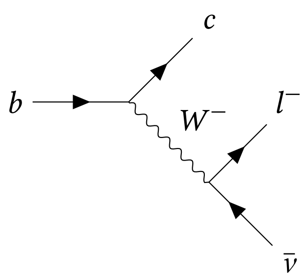
$$C_{LR1} = -2 G_{B'} \lambda_B^q \lambda_B^e \quad , C_{LR2}^{u/d} = -2 G_{B'} \lambda_B^{u/d} \lambda_B^l$$

Plus integrating out B' we generate lots of four-quarks and four-leptons operators that, according to the parameters, could generate uncorfortable contributes to observables tested by EW Precision Tests (EWPT) or to the meson mixing. In particular, since to accomodate the anomaly of the $b \rightarrow s l^- l^+$ transition i need a non vacuum $(\lambda_B^{q/u/d})^{bs} =$

$(\lambda_B^{q/u/d})^{sb}$ (where the / in this case means that has to be non vacuum at least in one of those three flavour matrices), we could produce an unpleasant tree-level contribute to $B_s \longleftrightarrow \overline{B_s}$ mixing which is one-loop induced in the SM. Also we need to point out that we haven't generate any operator which concerns CC transition, so B' can't be used to accomodate the anomaly of the $b \to cl\nu$ decays.

4.1.2 $W' \sim (\mathbf{1}, \mathbf{3})_0$

Since B' can't accomodate CC anomalies, we expect that to be done by an $SU(2)_L$ triplet as happens in SM, where $b \to cl\bar{\nu}$ is mediated by $W^{\pm}$ bosons which are the non $SU(2)_L$ diagonal components of the gauge boson W_a. The amplitude for the fondamental process in the SM takes the leading contribute from



which takes a flavour suppression from the quark vertex equal to V_{cb} and is diagonal and flavour universal (in one word the flavour matrix is the $\mathbf{1}_{3x3}$) for what concern the lepton current.
Since the anomaly consists in a preference for the decay channel with the τ as charged lepton, the most easy try to accomodate it is another $SU(2)_L$ triplet (much heavier than the $W^{\pm}$ to avoid collider constraints) with a non universal flavour structure in the lepton sector, that gets involved just when b quark belongs to the process.

Adding to SM $W' \sim (\mathbf{1}, \mathbf{3})_0$ the most general additional terms to high energy Lagrangian are:

$$\mathscr{L}_{UV} \supset \frac{M_{W'}^2}{2} W_\mu'^a W_a'^\mu - \frac{1}{4} W_{\mu\nu}'^a W_a'^{\mu\nu} + g_{W'} l_L^\dagger \bar{\sigma}^\mu \lambda_W^l \sigma_a l_L W_\mu^a + g_{W'} q_L^\dagger \bar{\sigma}^\mu \lambda_W^q \sigma_a q_L W_\mu^a \tag{4.2}$$

To see how this new terms contribute to processes at hadronic scale we need again to integrate out W' to see what effective semileptonic operators are generated in the EFT.

$$\frac{\delta \mathscr{L}}{\delta W_\mu^a} = 0 = \frac{M_W^2}{2} W_a^\mu + g_{W'} l_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^l l_L + g_{W'} q_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^q q_L \to$$

$$W_a^\mu = -\frac{2 g_{W'}}{M_{W'}^2} \left[l_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^l l_L + q_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^q q_L \right].$$

Then inserting it in the lagrangian we obtain:

$$\mathscr{L}_{EFT} \supset -2G_{W'} \left[l_L^\dagger \lambda_W^l \bar{\sigma}^\mu \sigma_a l_L \, l_L^\dagger \lambda_W^l \bar{\sigma}_\mu \sigma^a l_L + q_L^\dagger \lambda_W^q \bar{\sigma}^\mu \sigma_a q_L \, q_L^\dagger \lambda_W^q \bar{\sigma}_\mu \sigma^a q_L + 2 q_L^\dagger \lambda_W^q \bar{\sigma}^\mu \sigma_a q_L \, l_L^\dagger \lambda_W^l \bar{\sigma}_\mu \sigma^a l_L \right] \tag{4.3}$$

in which we recognize the triplet operator $\mathcal{O}_T$ in the last term implying

$$C_T = -4G_{W'}\lambda_W^l\lambda_W^q$$

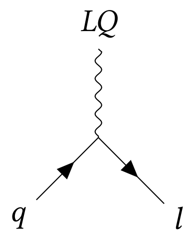
with $G_{W'} \equiv \frac{g_{W'}^2}{M_{W'}^2}$. Notice that $C_T \neq 0$ could accomodate CC anomalies.

Again we generate in addition four-leptons and four-quark operators that are unpleasant because of the tension they create beetween the anomalies and EWPT or meson mixing.

In conclusion colour-less vectors could together, a priori, accomodate both CC and NC anomalies but we can already see, without numerical analysis, that they generate a lot of unpleasant terms that can't be tuned easily once we included precision observables. In fact we will see, after a proper analysis, that colourless vectors don't costitute much more than an exercise to exemplify our *modus operandi*.

4.2 Leptoquarks

A possible way to avoid the tension beetween the anomalies and the precisely tested sector of the SM is to introduce bosons with quantum numbers different from the Higgs and gauge's bosons.
Since our purpose is to generate semileptonic operators, a good guess is to introduce bosons that connect quark and lepton sector at classical level, allowing the vertex



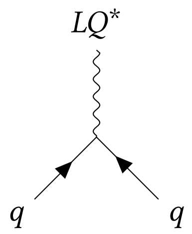
Bosons like that are known as *Leptoquark* and there could be different shape of them differing from the mass and the representation under their fields trasform under Lorentz group and SM gauge group.
The only features common to all the LQs is that, to link a colour triplet (or antitriplet) and a colour singlet, it has to trasform under the fundamental (or antifundamental) representation of $SU(3)_c$.

Being the LQs coloured they have to be confined and that protects the test on Barion Number conservation at a phenomenological level. That's true in the limit in which the only vertex allowed is a quark-lepton vertex.
Also, in that limit, we would generate at tree level just semileptonic operators.

Nevertheless, because of $SU(3)$ structure we have that:

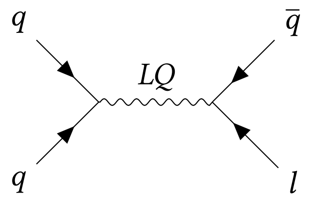
$$\mathbf{3}^* \otimes \mathbf{3}^* \sim \mathbf{6}^* \oplus \mathbf{3}$$

and so, if the lorentz and $SU(2)_L xU(1)_Y$ group allow, is possible to have



at tree level.

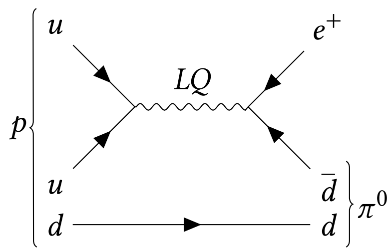
The presence of this vertex would generate the already mentioned meson mixing because of four-quarks tree level contributions. Also with both of the vertexes above allowed is possible to generate the amplitude



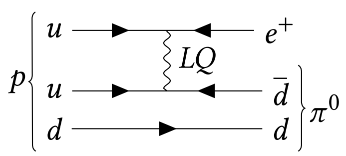
where the lepton is conjugated because of the $U(1)_Y$ symmetry and that would violate the Barion Number even at a phenomenological level.
The easiest solution at the level of simplified model would be just to switch off the quark-quark vertex. But when we want to implement a properly said UV completion, kill that contribute could need a non natural fine tuning.
We won't concern about naturalness in our analysis but it is still worthy to discuss the BNV issue in the Leptoquarks frame.

4.2.1 *Proton decay*

Before B-Physics Anomalies came up there where different attempt to go BSM making LQs arise. This is because LQs arise naturally as gauge bosons of most of the GUT groups.
The most common reason to not believe in LQs (and as consequence in GUT as well) has always been that they could mediate proton decay $p \to e^+\pi^0$



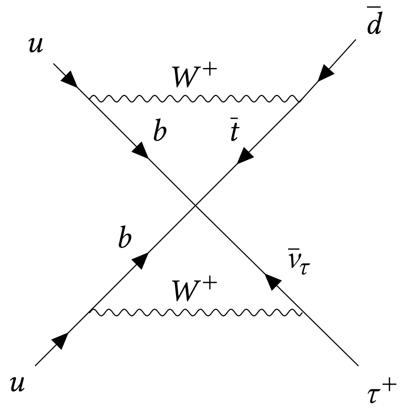
or even in t-channel



Now the available experimental limit for the proton lifetime is $\tau_p > 10^{34} ys$ and so if LQs arise the contribute sketched above should be very suppressed and that can happen if:

- $M_{LQ} \simeq \Lambda_{GUT} \simeq 10^{16}\ GeV$ to have a decoupling at low energies,
- some symmetry present in the theory doesn't allow the quark-quark vertex, for instance if among the assumptions we ask explictly to conserv Barion Number or Lepton Number. That can be realized in some cases as consequence of the gauge symmetry of the SM.

In the framework we have chosen we will not concern about the fundamental reason to suppress the quark-quark but we will point out when that arises as consequence of SM symmetry.
Nevertheless, the flavour structure we have described so far helps the purpose suppressing the proton decay. In fact if we consider $p \to \tau^+\pi^0 \to \bar{\nu}_{tau} + hadrons$ the four fermion vertex would be suppressed the with CKM matrix elements and loop factors coming from the insertion of two W's loops deed to switch the flavour of quark and leptons involved to the third family.



In which we get an additional suppression from the propagator of the τ^+ off-shell. Anyway we can't state that our flavour structure is always enough to get rid of that amplitude, but for sure it points in the right direction.

4.3 Vector Leptoquarks

For first we are going to analyze Leptoquark vectors named $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$. We will see that they are a pleasant option because, among the rest, since they couple to vector currents the value of the hypercharge $Y = 2/3$ doesn't allow them to couple with a quark-quark current and then they can never mediate proton decay.

4.3.1 $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$

The first LQ we take in consideration is the $SU(2)_L$ singlet U_1 which is one of the most popular in the available literature for reasons that will be clear at the end.

Including U_1 in our theory we get an UV lagrangian that reads:

$$\mathscr{L}_{UV} \supset -\frac{1}{2}U_{1\mu\nu}^{\dagger}U_1^{\mu\nu} + M_{U_1}^2 U_{1\mu}^{\dagger}U_1^{\mu} + \left[g_U U_{1\mu}\left(q_L^{\dagger}\beta_{U_1}^L\overline{\sigma}^{\mu}l_L + d_R^{\dagger}\beta_{U_1}^R\sigma^{\mu}e_R\right) + h.c.\right] \tag{4.4}$$

One difference that can be seen immediately is that belonging to the fundamental representation of $SU(3)_c$, which is complex, the current coupled to it is complex too and so, in general, the flavour matrices β are complex in general.

Treating the physics at the hadronic scale we can integrate out U_1 from the Lagrangian through the equation of motion:

$$\frac{\delta\mathscr{L}}{\delta U_{1\mu}^{\dagger}} = 0 = M_{U_1}^2 U_1^{\mu} + g_{U_1}\left[l_L^{\dagger}\beta_{U_1}^{\dagger L}\overline{\sigma}^{\mu}q_L + e_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}d_R\right] \rightarrow$$

$$U_1^{\mu} = -\frac{g_{U_1}}{M_{U_1}^2}\left[l_L^{\dagger}\beta_{U_1}^{\dagger L}\overline{\sigma}^{\mu}q_L + e_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}d_R\right]$$

Obtaining

$$\mathscr{L}_{EFT} \supset -G_{U_1}\left[l_L^{\dagger}\beta_{U_1}^L\overline{\sigma}^{\mu}q_L\, q_L^{\dagger}\beta_{U_1}^{\dagger L}\overline{\sigma}_{\mu}l_L + 2l_L^{\dagger}\beta_{U_1}^L\overline{\sigma}^{\mu}q_L\, d_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma_{\mu}e_R + e_R^{\dagger}\beta_{U_1}^R\sigma^{\mu}d_R\, d_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}e_R + h.c\right]$$

where, $G_{U_1} \equiv \frac{g_{U_1}^2}{M_{U_1}^2}$.

The effective lagrangian is made of a sum of semileptonic operators, now we need to project them on the basis listed in section 3.2.

The second term is proportional to $\mathcal{O}_S^d$ because of the relation $\overline{\sigma}_{\dot{\alpha}\alpha}^{\mu}\sigma_{\mu}^{\beta\dot{\beta}} = 2\delta_{\alpha}^{\beta}\delta_{\dot{\alpha}}^{\dot{\beta}}$ it's precisely equal to $2\mathcal{O}_S^d$.

The lorentz structure of the first and the secon get us a minus sign in the first and the third term because of the relations $\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta} = -\sigma_{\alpha\dot{\beta}}^{\mu}\sigma_{\mu\beta\dot{\alpha}}$ and $\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\mu\beta\dot{\beta}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta} = -\sigma_{\alpha\dot{\beta}}^{\mu}\sigma_{\mu\beta\dot{\alpha}}$.

Using these relations we see that the third term is equal to $-\mathcal{O}_R$ while the first term seems equal to $-\mathcal{O}_S$ apart from the fact that the quark and the lepton current aren't $SU(2)_L$ singlets. In fact both quarks contract the $SU(2)_L$ index with a lepton, to project that operator on our basis we need to recall 3.1

$$\sigma_{ad}^{i}\sigma_{cbi} = 2\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} \rightarrow \delta_{ab}\delta_{cd} = \frac{\sigma_{ad}^{i}\sigma_{cbi} + \delta_{ad}\delta_{cb}}{2}$$

to rewrite the first term as $-\frac{1}{2}(\mathcal{O}_S + \mathcal{O}_T)$. Adding up everything we obtain:

$$\mathscr{L}_{EFT} \supset G_{U_1}\left[\frac{1}{2}\beta_{U_1}^L\beta_{U_1}^{\dagger L}(\mathcal{O}_S + \mathcal{O}_T) + \beta_{U_1}^R\beta_{U_1}^{\dagger R}\mathcal{O}_R^d - 4\beta_{U_1}^R\beta_{U_1}^{\dagger L}\mathcal{O}_S^d + \right] + h.c. \tag{4.5}$$

that means in other words

$$C_S = C_T \equiv \frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L \quad C_R^d \equiv G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^R \quad C_S^d \equiv -4G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^L$$

The first feature we notice is that, even though we take all the possible terms in the UV lagrangian, just semileptonic contribute are generated. This means that when we accomodate the anomalies we need to be mindful of constraints that come just from semileptonic observables.

Without considering the details of the computation we can also see how one of this constraints is avoided from the fact that $C_S = C_T$. In fact $C_S = C_T \neq 0$ gives us a way to accomodate the anomaly on $b \to sl^+l^-$ where the amplitude due to NP effects is (considering just purely LH contributions) $\propto (C_S + C_T)$ as can be seen from the electric charge eigenstates form of $\mathcal{O}_S$ and $\mathcal{O}_T$ shown in section 3.2.
From that form we can see also that the transition $b \to s\nu\bar{\nu}$ is linked to $b \to sl^+l^-$ by the gauge symmetry and results to be $\propto C_S - C_T$ which means no contribution coming from the integration of U_1 as shown by [3]. This is very pleasant since $\mathcal{B}(B \to K^\dagger \nu\bar{\nu})$ is a rare decay extremely suppressed in SM and very constrained by experimental data. Nevertheless U_1 doesn't contribute to that decay even taking count of the coupling to RH fermions which can as well help to accomodate the anomalies.

Another reason to support U_1 is that it comes naturally as gauge boson in the UV completion proposed by Pati and Salam [4] that will be briefly discussed later.

4.3.2 $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$

The vector $SU(2)_L$ triplet LQ couples just to the triplet LQ current $J_T^a = q_L^\dagger \bar{\sigma}^\mu \sigma^a \beta_{U_3} l_L$ i.e. just to LH fermions so, for what said in section 3.2, we expect to generate at low energy a combination of $\mathcal{O}_S$ and $\mathcal{O}_T$.

The UV lagrangian gains the following terms:

$$\mathscr{L}_{UV} \supset -\frac{1}{2} U_{3\mu\nu}^{\dagger a} U_{3a}^{\mu\nu} + M_{U_3}^2 U_{3a}^{\dagger\mu} U_{3\mu}^a + g_{U_3} \left[U_{3\mu}^a q_L^\dagger \beta_{U_3} \bar{\sigma}^\mu \sigma_a l_L + h.c \right] \tag{4.6}$$

To build the effective lagrangian at m_b scale we set as always

$$\frac{\delta \mathscr{L}}{\delta U_{3\mu}^a} = 0 = M_{U_3}^2 U_{3a}^{mu} + g_{U_3} l_L^\dagger \beta_{U_3}^\dagger \bar{\sigma}^\mu \sigma_a q_L \to$$

$$U_{3a}^\mu = -\frac{g_{U_3}}{M_{U_3}^2} l_L^\dagger \beta_{U_3}^\dagger \bar{\sigma}^\mu \sigma_a q_L$$

generating

$$\mathscr{L}_{EFT} \supset -G_{U_3} \left[q_L^\dagger \beta_{U_3} \bar{\sigma}^\mu \sigma_a l_L \; l_L^\dagger \beta_{U_3}^\dagger \bar{\sigma}_\mu \sigma^a q_L \; + h.c \right]$$

Now we get a minus sign as in the U_1 case to make colour singlet currents, meanwhile the EW structure recalls 3.2

$$\sigma_{ij}^a \sigma_{a\; kl} = \frac{1}{2} (3 \delta_{il} \delta_{jk} - \sigma_{il}^a \sigma_{a\; kj}) \to$$

$$\mathscr{L}_{EFT} \supset G_{U_3} \frac{1}{2} \beta_{U_3} \beta_{U_3}^{[} 3 \mathcal{O}_S - \mathcal{O}_T]$$

The Wilson coefficients are equal to

$$C_S = -3C_T \equiv \frac{3}{2} G_{U_3} \beta_{U_3} \beta^{\dagger}_{U_3}$$

that can accomodate both CC and NC anomalies, nevertheless since we mentioned for U_1 the constraint from $B \to K\nu\bar{\nu}$ we need to point out that this time the contraint is not passed automatically and could need a fine tuning.

To summarize the features of vector LQs we would say that they are a good possibility since they affect at tree level just semileptonic processes even if we include all possible terms allowed by SM symmetries.
Particularly U_1 is mentioned many times in the literature as the only one that can accomodate both anomalies alone passing most of the constraints, while U_3 is not that appreciated as solution of the anomalies but possibly it can be combined with other mmediators to build a relevant model.

4.4 Scalar Leptoquarks

In the end of this list we will show what are the scalar LQs that can be helpful for our purposes.
By convention the ones belonging to real represententation of $SU(2)_L$ are defined in the antifundamental of $SU(3)_c$ i.e. we have $S_1 \sim (3^{\dagger}, 1)_{1/3}$, $S_3 \sim (3^*, 3)_{1/3}$ and $R_2 \sim (3, 2)_{7/6}$. In literature sometimes another scalar LQ is mentioned : $\widetilde{R}_2 \sim (3, 2)_{1/6}$; nevertheless it doesn't serve the purpose properly while it has more than one quark-quark coupling, that's why we have chosen to neglect it.

We will see that the scalar currents that couple LQs can be of many shapes for two reasons. For first the values of the hypercharge allow S_1 and S_3 to couple a quark-quark current, secondly the scalar current has to be made by $(\psi_{L/R})^{\dagger} \chi_{R/L}$ and it is possible to use the charge conjugation to flip the chirality since $(\psi^c_L) \sim (0, \frac{1}{2})$ and viceversa. That was possible also for vector LQ but there the hypercharge wasn't the proper to a current $(\psi_{L/R})^c \sigma^{\mu} \chi_{R/L}$ for any $\psi_L, \chi_R \in \{q_L, l_L, u_R, d_R, e_R\}$.
So, to see what current couples to a given LQ, we have to see all the bilinears, according to $SU(2)_L$, with the right hypercharge, mindful that the charge conjugation brings a minus sign to it.

4.4.1 $S_1 \sim (3^*, 1)_{1/3}$

A scalar current which couples with a $SU(2)_L$ singlet has to involve charge's conjugation. Mindful of this point we find that, this time, the hypercharge allows a quark-quark coupling
Precisely the UV lagrangian that includes S_1 is:

$$\begin{aligned} \mathscr{L}_{UV} \supset (D_{\mu} S_1)^{\dagger} D^{\mu} S_1 + M^2 S_1^{\dagger} S_1 + \\ + g_{S_1} S_1^i \big[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + \overline{q_L^j} \lambda_{S_1} \varepsilon {q_L^c}^k \varepsilon_{ijk} \big] + h.c. \end{aligned} \tag{4.7}$$

Where the ε guarantees the current to be a EW singlet.
Integrating out S_1 we need to explicit the colour index, since this time is not contracted trivially in all the terms:

$$\frac{\delta \mathscr{L}_{UV}}{\delta S_1^{i\dagger}} = 0 = M_{S_1}^2 S_{1a} + g_{S_1}\left[l_L^\dagger \varepsilon \beta_{S_1}^{L\dagger} q_{Li}^c + +e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c + q_L^{\dagger c^j} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk}\right] \rightarrow$$

$$S_{1i} = -\frac{g_{S_1}}{M_{S_1}^2}\left[\overline{l_L} \beta_{S_1}^\dagger \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c + \overline{q^c}_L^j \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk}\right]$$

Inserting this term in the UV lagrangian we would obtain nine terms, for simplicity we here neglect all the BNV terms,

$$\mathscr{L}_{EFT} \supset -G_{S_1}\left[\overline{l_L} \beta_{S_1}^\dagger \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R\dagger} \varepsilon u_{Ri}^c + \overline{q^c}_L^j \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk}\right]\left[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + \overline{q_L^j} \lambda_{S_1} \varepsilon q_L^{c\,k} \varepsilon_{ijk}\right] + h.c. =$$

$$-G_{S_1}\Big[l_L^\dagger \beta_{S_1}^{L\dagger} \varepsilon q_{Li}^c \; q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c \; q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + q_L^{c\dagger j} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \; \overline{q_L^l} \lambda_{S_1} \varepsilon q_L^{c\,m} \varepsilon_{ilm} +$$
$$l_L^\dagger \beta_{S_1}^{L\dagger} \varepsilon q_{Li}^c \; u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c \; u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + (BNV\, terms)\Big]$$

Here we can see three terms describing semileptonic contributes and a four-quark term that can generate amplitudes contributing to meson mixing.
To project these terms on our operators basis we need to recall the relations obtained using Fierz Identities of EW group together with the ones of the Lorentz group, in particular we need 3.7:

$$\overline{l^{1c}} \varepsilon q^1 \; \overline{q^2} \varepsilon l^{2c} = \frac{1}{4}\left[q^{2\dagger} \overline{\sigma}^\mu q^1 \; l^{2\dagger} \overline{\sigma}_\mu l^1 \; - \; q^{2\dagger} \overline{\sigma}^\mu \sigma^a q^1 \; l^{2\dagger} \overline{\sigma}_\mu \sigma_a l^1\right]$$

that projects the semileptonic term with LH fermions. Then we notice that the left-right semileptonic term is generated in pair with his hermitian conjugate. Then, since we add ($+h.c.$) it gains a 2 factor in front. To project that term we need to use that:

$$\varepsilon_{ij} \varepsilon_{kl} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}$$

obtaining two scalar operators that we recognize to be $\mathscr{O}_{LQ}$ and $-\mathscr{O}_{S^\dagger}^u$.
In the end, the completely RH term turns to a vector-vector operator using

$$\varepsilon_{\alpha\beta} \varepsilon_{\dot{\alpha}\dot{\beta}} = -\frac{1}{2} \sigma_{\alpha\dot{\alpha}}^\mu \sigma_{\mu\beta\dot{\beta}}$$

obtaining $-\frac{1}{2}\mathscr{O}_R$.
Adding up all the terms we obtain the following effective lagrangian:

$$\begin{aligned}\mathscr{L}_{EFT} \supset -G_{S_1}\Big[&\frac{1}{4}\beta_{S_1}^L \beta_{S_1}^{L\dagger} \mathscr{O}_S - \frac{1}{4}\beta_{S_1}^L \beta_{S_1}^{L\dagger} \mathscr{O}_T + 2\beta_{S_1}^{R\dagger} \beta_{S_1}^L \mathscr{O}_{LQ} - 2\beta_{S_1}^{R\dagger} \beta_{S_1}^L \mathscr{O}_S^{u\dagger} - \frac{1}{2}\beta_{S_1}^{R\dagger} \beta_{S_1}^R \mathscr{O}_R^u \\ &+ \overline{q^c}_L^j \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \; \overline{q_{Ll}} \lambda_{S_1} \varepsilon q_{Lm}^c \varepsilon^{ilm} + (BNV terms)\Big] + h.c.\end{aligned} \tag{4.8}$$

Here we have generated different semileptonic contributes that contribute both to CC and NC processes and a term that contributes to tree level amplitude of meson mixing,

plus some BNV terms that we neglected.
There is one substantial difference with the colour-less case, in fact here we can set $\lambda_{S_1} = 0$ without affecting semileptonic contributes and turn off all four-quark and BNV terms.
The Wilson coefficients of semileptonic operators result to be

$$C_S = -C_T \equiv -\frac{G_{S_1}}{4}\beta_{S_1}^{L\dagger}\beta_{S_1}^{L} \quad ,C_{LQ} = -C_S^{u\dagger} = 2G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S1}^{L} \quad ,C_R = -\frac{1}{2}G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^{R}$$

For what we said about U_1 is easy to see that the purely LH tree level contribute to $b \to sl^+l^-$, which is proportional to $C_S + C_T$, turns out to be null and that takes out from our toolbox a possible weapon to accomodate NC anomalies.
Nevertheless as we will see S_1 is a good candidate which appears in many UV scenarios.

4.4.2 $R_2 \sim (3, 2)_{7/6}$

The scalar doublet with $Y = 7/6$ couples to a left-right current to contract the $SU(2)_L$ index.
That means that it can't generate the purely LH operators $\mathcal{O}_S$, $\mathcal{O}_T$ and this is the reason why [3] neglects every LQ EW doublet.

The lagrangian terms allowed are:

$$\begin{aligned}\mathcal{L}_{UV} \supset (D_\mu R_2)^\dagger D^\mu R_2 + M_{R_2}^2 R_2^\dagger R_2 + \\ g_{R_2} R_2\big[u_R^\dagger \beta_{R_2}^l l_L \varepsilon + q_L^\dagger \beta_{R_2}^q e_R\big] + h.c.\end{aligned} \tag{4.9}$$

where replacing at low energies the equation of motion

$$\frac{\delta \mathcal{L}_{UV}}{\delta R_2^\dagger} = 0 = M_{R_2}^2 R_2 + g_{R_2}\big[\varepsilon l_L^\dagger \beta_{R_2}^{l\dagger} u_R + e_R^\dagger \beta_{R_2}^{q\dagger} q_L\big] \to$$

$$R_2 = -\frac{g_{R_2}}{M_{R_2}^2}\big[\varepsilon l_L^\dagger \beta_{R_2}^{l\dagger} u_R + e_R^\dagger \beta_{R_2}^{q\dagger} q_L\big]$$

we find

$$\mathcal{L}_{EFT} \supset -G_{R_2}\big[-u_R^\dagger \beta_{R_2}^l l_L\, l_L^\dagger \beta_{R_2}^{l\dagger} u_R + u_R^\dagger \beta_{R_2}^l l_L\, e_R^\dagger \beta_{R_2}^{q\dagger} q_L + q_L^\dagger \beta_{R_2}^{q\dagger} e_R\, l_L^\dagger \beta_{R_2}^{l\dagger} u_R + q_L^\dagger \beta_{R_2}^q e_R\, e_R^\dagger \beta_{R_2}^{q\dagger} q_L\big] + h.c$$

where we used $\varepsilon \cdot \varepsilon = -1$. Apart from the flavour structure, the second and the third term are $\mathcal{O}_{LQ}$ and its hermitian conjugate, then using the (+h.c.) we can write the sum of them as $2Re[\beta_{R_2}^{l\dagger}\beta_{R_2}^q]\mathcal{O}_{LQ}$.
The first and the fourth terms can be fierzed through

$$\delta_a^b \delta_c^d = \frac{1}{2}\sigma_{ac}^\mu \overline{\sigma}_\mu^{bc}$$

to obtain $\frac{1}{2}\mathcal{O}_{LR1}$ and $\frac{1}{2}\mathcal{O}_{LR2}^u$.
Adding up we obtain

$$\mathcal{L}_{EFT} \supset -\frac{G_{R_2}}{2}\big[-\beta_{R_2}^l \beta_{R_2}^{l\dagger}\mathcal{O}_{LR2}^u + 2\beta_{R_2}^l \beta_{R_2}^{q\dagger}\mathcal{O}_{LQ}^u + \beta_{R_2}^q \beta_{R_2}^{q\dagger}\mathcal{O}_{LR1}\big] + h.c. \tag{4.10}$$

Again we generated just semileptonic terms even including all possible terms.
The Wilson coefficients result to be

$$C^u_{LR2} = \frac{1}{2} G_{R_2} \beta^l_{R_2} \beta^{l\dagger}_{R_2} \quad , C_{LQ} = -\frac{1}{2} G_{R_2} \beta^q_{R_2} \beta^{q\dagger}_{R_2} \quad , C_{LR1} = -2 G_{R_2} \beta^l_{R_2} \beta^{q\dagger}_{R_2}$$

4.4.3 $\widetilde{R}_2 \sim (3,2)_{1/6}$

We want to consider another possible value for the hypercharge of a scalar LQ EW-doublet: $\widetilde{R}_2 \sim (3,2)_{1/6}$.
In this case the Lagrangian becomes:

$$\mathscr{L}_{UV} \supset M^2_{\widetilde{R}_2} \widetilde{R}^\dagger_2 \widetilde{R}_2 + (D_\mu \widetilde{R}_2)^\dagger D^\mu \widetilde{R}_2 + g_{\widetilde{R}_2} (d^\dagger_R \beta_{\widetilde{R}_2} l_L + h.c.) \tag{4.11}$$

Once integrated out $\widetilde{R}_2$

$$\begin{aligned} \frac{\delta \mathscr{L}_{UV}}{\delta R^\dagger_2} =& M^2_{\widetilde{R}_2} \widetilde{R}_2 + g_{\widetilde{R}_2} l_L \beta^\dagger_{\widetilde{R}_2} d_R \rightarrow \\ \widetilde{R}_2 =& -\frac{g_{\widetilde{R}_2}}{M^2_{\widetilde{R}_2}} d^\dagger_R \beta_{\widetilde{R}_2} l_L \end{aligned}$$

which gives us the effective Lagrangian:

$$\mathscr{L}_{EFT} \supset -G_{\widetilde{R}_2} l^\dagger_L \beta_{\widetilde{R}_2} d_R d^\dagger_R \beta_{\widetilde{R}_2} l_L$$

Then using again

$$\delta^\beta_\alpha \delta_{\dot\alpha\dot\beta} = \frac{1}{2} \sigma^\mu{}_{\alpha\dot\alpha} \overline{\sigma}_\mu^{\dot\beta\beta}$$

obtaining

$$\mathscr{L}_{EFT} \supset -\frac{G_{\widetilde{R}_2}}{2} \mathscr{O}^d_{LR2} \rightarrow C^d_{LR2} \equiv -\frac{1}{2} G_{\widetilde{R}_2} \beta^\dagger_{\widetilde{R}_2} \beta_{\widetilde{R}_2}$$

4.4.4 $S_3 \sim (3^*,3)_{1/3}$

The last heavy boson we want to compute the affection at low energy is the scalar EW triplet.
As U_3 it couples with a triplet current, hence it doesn't interact with RH fermions. The main difference with U_3 is that it allows quark-quark coupling, in fact the UV lagrangian turns out to be:

$$\begin{aligned} \mathscr{L}_{UV} \supset& (D_\mu S_{3a})^\dagger D^\mu S^a_3 + M^2 S^\dagger_{3a} S^a_3 + \\ &+ g_{S_3} S^{ia}_3 (\overline{q_L}^j \lambda_{S_3} \varepsilon \sigma_a q^{c\,k}_L \varepsilon_{ijk} + \overline{q^c_L}_i \beta_{S_3} \varepsilon \sigma_a l_L) + h.c. \end{aligned} \tag{4.12}$$

in which $i = 1,2,3$ is the colour index and $a = 1,2,3$ is the EW index. Again the annoying part could be turned off without affecting semileptonic operators, as for S_1.
Integrating it out

$$\frac{\delta \mathscr{L}_{UV}}{\delta S^{ia\,\dagger}_3} = 0 = M^2_{S_3} S_{3ia} + g_{S_3} \left[\overline{q^c_L}_j \lambda^\dagger_{S_3} \varepsilon \sigma_a q_{Lk} \varepsilon^{ijk} + \overline{l_L} \beta^\dagger_{S_3} \varepsilon \sigma_a q^{c\,i}_L \right] \rightarrow$$

$$S_{3ia} = -\frac{g_{S_3}}{M_{S_3}^2}\big[\overline{q^c_{Lj}}\lambda^\dagger_{S_3}\varepsilon\sigma_a q_{Lk}\varepsilon^{ijk} + \overline{l_L}\beta^\dagger_{S_3}\varepsilon\sigma_a {q^c_L}^i\big]$$

hence the EFT lagrangian results

$$\mathscr{L}_{EFT} \supset -G_{S_3}\big[\overline{q^c_{Li}}\beta_{S_3}\varepsilon\sigma_a l_L\ \overline{l_L}\beta^\dagger_{S_3}\varepsilon\sigma_a {q^c_L}^i\ +\ \overline{q^c_{Li}}\beta_{S_3}\varepsilon\sigma_a l_L\ \overline{q^c_{Lj}}\lambda^\dagger_{S_3}\varepsilon\sigma_a q_{Lk}\varepsilon^{ijk} +$$

$$\overline{q_L}^j\lambda_{S_3}\varepsilon\sigma_a {q^c_L}^k\varepsilon_{ijk}\ \overline{l_L}\beta^\dagger_{S_3}\varepsilon\sigma_a {q^c_L}^i\ +\ \overline{q_L}^j\lambda_{S_3}\varepsilon\sigma_a {q^c_L}^k\varepsilon_{ijk}\overline{q^c_{Ll}}\lambda^\dagger_{S_3}\varepsilon\sigma_a q_{Lm}\varepsilon^{ilm}\big]$$

The first term is the only one affecting semileptonic processes at tree level. To project it on our basis we remember that we obtained 3.8

$$\overline{l^{1c}}\varepsilon\sigma_a q^1\ \overline{q^2}\sigma^a\varepsilon l^{2c} = \frac{1}{4}\big[3q^{2\dagger}\bar{\sigma}^\mu q^1\ l^{2\dagger}\bar{\sigma}_\mu l^1\ +\ q^{2\dagger}\bar{\sigma}^\mu\sigma^a q^1\ l^{2\dagger}\bar{\sigma}_\mu\sigma_a l^1\big]$$

which means

$$\begin{aligned}\mathscr{L}_{EFT} \supset -G_{S_3}\big[&\frac{3}{4}\beta^\dagger_{S_3}\beta_{S_3}\mathscr{O}_S + \frac{1}{4}\beta^\dagger_{S_3}\beta_{S_3}\mathscr{O}_T + \overline{q^c_{Li}}\beta_{S_3}\varepsilon\sigma_a l_L\ \overline{q^c_{Lj}}\lambda^\dagger_{S_3}\varepsilon\sigma_a q_{Lk}\varepsilon^{ijk} +\\ &\overline{q_L}^j\lambda_{S_3}\varepsilon\sigma_a {q^c_L}^k\varepsilon_{ijk}\ \overline{l_L}\beta^\dagger_{S_3}\varepsilon\sigma_a {q^c_L}^i\ +\ \overline{q_L}^j\lambda_{S_3}\varepsilon\sigma_a {q^c_L}^k\varepsilon_{ijk}\overline{q^c_{Ll}}\lambda^\dagger_{S_3}\varepsilon\sigma_a q_{Lm}\varepsilon^{ilm}\big] + h.c.\end{aligned} \tag{4.13}$$

As already mentioned, meson mixing and BNV amplitudes would arise at tree level just for non vacuum values of the flavour matrix λ_{S_3} that we can turn off for our purposes. The Wilson coefficients of semileptonic terms are

$$C_S = 3C_T \equiv -\frac{3}{4}G_{S_3}\beta^\dagger_{S_3}\beta_{S_3}$$

that means, a priori, contributions to both NC and CC processes.

4.5 Summary

The last thing we need to decide what recipes are suitable to accomodate the anomalies is to decide what Wilson coefficients we need to accomodate the B-Physics anomalies and compute what values of them. Once we have done it we will be ready to check if those values pass the constraint given by the tests to Flavour Physics done so far.

Before to implement the numerical analysis we summarize what contributions are generated in the different scenarios in tables 4.1.

Figure 4.1: Wilson coefficients corresponding to effective generators generated by the integration of heavy vector bosons (above) and scalar bosons (below).

	B'	W'	U_1	U_3
C_S	$-2G_{B'}\lambda_B^q\lambda_B^l$	$\varnothing$	$\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$\frac{3}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_T	$\varnothing$	$-4G_{W'}\lambda_W^q\lambda_W^l$	$\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$-\frac{1}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_{LR1}	$-2G_{B'}\lambda_B^q\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^u	$-2G_{B'}\lambda_B^u\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^d	$-2G_{B'}\lambda_B^d\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^u	$-2G_{B'}\lambda_B^u\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$-2G_{B'}\lambda_B^d\lambda_B^e$	$\varnothing$	$G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^R$	$\varnothing$
C_{LQ}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^u	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$-4G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^{R\dagger}$	$\varnothing$

	S_1	R_2	$\widetilde{R}_2$	S_3
C_S	$-G_{S_1}\beta_{S_1}^L\beta_{S_1}^{L\dagger}$	$\varnothing$	$\varnothing$	$-\frac{3}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_T	$G_{S_1}\beta_{S_1}^{L\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$-\frac{1}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_{LR1}	$\varnothing$	$-G_{R_2}\beta_{R_2}^l\beta_{R_2}^{q\dagger}$	$\varnothing$	$\varnothing$
C_{LR2}^u	$\varnothing$	$\frac{1}{2}G_{R_2}\beta_{R_2}^{l\dagger}\beta_{R_2}^l$	$\varnothing$	$\varnothing$
C_{LR2}^d	$\varnothing$	$\varnothing$	$-\frac{1}{2}G_{\widetilde{R}_2}\beta_{\widetilde{R}_2}^\dagger\beta_{\widetilde{R}_2}$	$\varnothing$
C_R^u	$-\frac{1}{2}G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^R$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LQ}	$2G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$-\frac{1}{2}G_{R_2}\beta_{R_2}^{q\dagger}\beta_{R_2}^q$	$\varnothing$	$\varnothing$
C_S^u	$-2G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

5

Phenomenological Analysis

To understand what is the proper way to set the analysis of what values of NP parameters can accomodate the anomalies we need first to analyze the clues given by the data.

The main issue of computing the Branching ratios of the semileptonic processes we are interested in is that we need, apart from the EW nature of the processes, to consider that the quark involved are *confined* in hadrons.
From now we will use as example the NC anomalous decay $B \to K^* l^+ l^-$ which is the one with the biggest significant anomaly, nevertheless the discussion can be easily applied on every $M \to M' l_1 l_2$ process, where M, M' are mesons and $l_{1,2}$ are leptons.

The decay width of $B \to K^* l^+ l^-$ is given by:

$$\Gamma(B \to K^* l^+ l^-) = \frac{1}{2m_B} \int |\mathscr{M}|^2 \frac{d^3 p_{K^*}}{(2\pi)^3 \ 2E_{K^*}} \frac{d^3 p_{l^+}}{(2\pi)^3 \ 2E_{l^+}} \frac{d^3 p_{l^-}}{(2\pi)^3 \ 2E_{l^-}}$$

Where $\mathscr{M}$ is defined to be

$$\mathscr{M} \equiv (-i) \langle B| \mathscr{O} |K^* l^+ l^- \rangle$$

in which $\mathscr{O}$ is a given operator that at three level corresponds to the sum of lagrangian's operators that can annihilate a b quark and create s quark and a pair of charged lepton anti-lepton.
If that operator can be factorized in a piece that contain only lepton fields and another one containing only quark results:

$$\mathscr{M} = \langle B| \mathscr{O}_q |K^* \rangle \langle 0| \mathscr{O}_l |l^+ l^- \rangle$$

i.e. the transition matrix is the product of a leptonic perturbative term which depends only from the form of the operator and an *hadronic form factor* which at B-meson mass'scale is non calculable perturbatively.

Those factors are generally a bad deal because the non perturbativity limits the accuracy of theoretical predictions for these processes; nevertheless if we want to compare processes where the amplitude is given by operators with the same $\mathscr{O}_{had}$ is possible to simplify the non perturbative contribution expressing observables in terms of ratios of branching ratios (now we see the reason for the choice done at the begin of section 3.2 to write the operators as contraction of colour-singlet currents).

Since all the interaction involved in $\mathcal{O}_q$ are QCD interactions sometimes is possible to link different form factors without compute them explictly but just using general property of strong interactions. Since QCD conserves parity it is convenient to *express from now on fermion fields in Dirac representation*, i.e.:

$$q_L^{Weyl} \to q_L^{Dirac} = \frac{1-\gamma^5}{2} q^{Dirac} = \begin{pmatrix} q_L^{Weyl} \\ 0 \end{pmatrix}$$

$$q_R^{Weyl} \to q_R^{Dirac} = \frac{1+\gamma^5}{2} q^{Dirac} = \begin{pmatrix} 0 \\ q_R^{Weyl} \end{pmatrix}$$

5.1 Left Handed framework

In the SM, because of the theorem 2, flavour breaking processes get their leading contributions from LH quarks. Since any process addressed to B-Physics Anomalies is flavour breaking, a good guess is to assume that NP should involve only LH quark as well. For what concern leptons we have that for $q^2 = (p_{l^+} + p_{l^-})^2 \in [1.1; 6.0] GeV^2$ the vector like QED contribution is suppressed by the photon propagator $\simeq \frac{1}{q^2}$ and, as previously said, the *penguin* contribution of SM approximately LH due to the accidental smallness of θ_W, and so every contribute given to the RH-RH lepton current is not interfering with SM requiring a big Wilson coefficient to imply sensitive deviations, that would be bounded from many semileptonic LFU ratios like

$$\frac{\mathcal{B}(J/\psi \to \mu^+\mu^-)}{\mathcal{B}(J/\psi \to e^+e^-)}$$

If we keep only the operators with a left left lepton current, we are left with $\mathcal{O}_S$, $\mathcal{O}_T$, $\mathcal{O}_{LR2}^d$ where O_T is the only one that contributes to CC processes.
Another clues that pushes in the direction of a pyrely LH NP is given by the ratio R_{K^*}/R_K.

5.1.1 R_K *and* R_{K*}

From the point of view of the EW transition there is no difference beetween the decay channel in $\to Kl^+l^-$ and $\to K^*l^+l^-$ since the quark transition is always $b \to sl^+l^-$. In the limit in which leptons take the biggest part of the available energy (in the way we can neglect kinematic effects due to the mass difference beetween μ and e), the difference beetween the two ratios is hidden in the hadronic part of the matrix element:

$$\frac{\Gamma(B \to K^*l^+l^-)}{\Gamma(B \to Kl^+l^-)} = \frac{|\langle B| \sum_i C_i \mathcal{O}_i |K^*\rangle|^2}{|\langle B| \sum_i C_i \mathcal{O}_i |K\rangle|^2}$$

Where $\mathcal{O}_i$ are the different possible quark operator that allow the $b \to s$ transition that products a $l_L^+ l_L^-$ pair. If we consider just vector currents operators the most general form of $\sum_i C_i \mathcal{O}_i$ is $\bar{b}\gamma^\mu(\alpha + \beta\gamma^5)s \equiv \alpha\mathcal{O}_V + \beta\mathcal{O}_A$. Written as combination of two currents, the conservation of parity gives me that:

$$\begin{aligned} \langle B| \mathcal{O}_V |K^*\rangle = 0 \quad &, \langle B| \mathcal{O}_A |K^*\rangle \equiv A(q^2) \\ \langle B| \mathcal{O}_V |K\rangle \equiv V(q^2) \quad &, \langle B| \mathcal{O}_A |K\rangle = 0 \end{aligned}$$

where $A(q^2)$ and $V(q^2)$ are unknown functions of the exchanged momentum $q^2 = (p_B - p_K)^2$.
Now if we consider the ratio beetween

$$R_{K^*} = \left.\frac{\Gamma(B \to K^*\mu^+\mu^-)}{\Gamma(B \to K^*e^+e^-)}\right|_{SM} = 1 \quad and \quad R_K = \left.\frac{\Gamma(B \to K\mu^+\mu^-)}{\Gamma(B \to Ke^+e^-)}\right|_{SM} = 1$$

that we define $\chi_K \equiv R_{K^*}/R_K$is of course equal to 1 in the SM since they are both 1.
When we include NP effects that affect just the muon channel things could change. In fact the expectation value is done on

$$\sum \mathcal{O} = C^K_{SM}[\mathcal{O}_V - \mathcal{O}_A] + \alpha_{NP}\mathcal{O}_V + \beta_{NP}\mathcal{O}_A$$

since for what we previously said SM leading contribution is purely LH.

At this point is straightforward to notice that in a NP scenario

$$\begin{aligned} \chi_K = \frac{R_{K^*}}{R_K} &= \frac{|(-C^K_{SM} + \beta_{NP})A|^2}{|[C^K_{SM} + \alpha_{NP} + (\beta_{NP} - \beta_{NP})]V|^2} \frac{|C^K_{SM}V|^2}{|C^{K^*}_{SM}A|^2} \\ &= \left|\frac{\beta_{NP} - C^K_{SM}}{(\beta_{NP} - C^K_{SM}) + \alpha_{NP} + \beta_{NP}}\right|^2 \end{aligned} \tag{5.1}$$

where in the last line we explicited that for NP LH, i.e. $\alpha_{NP} = -\beta_{NP}$, χ_K remains equal to 1.
Otherwise if $\alpha_{NP} + \beta_{NP} \neq 0$ one of the two ratio could get bigger according to the relative sign beetween $\alpha_{NP} + \beta_{NP}$ and $C^{K^*}_{SM}$.

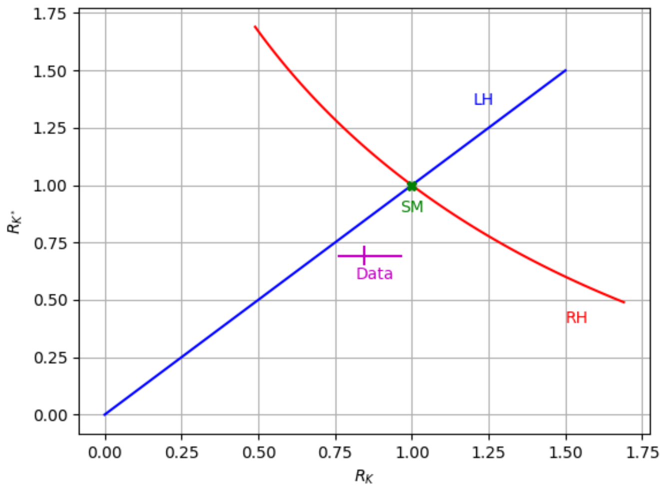
In Figure 5.1.1 we see how the data prefer a framework in which just LH quark are involved and so we assume that $\mathcal{O}^d_{LR2}$ doesn't serve our purpose.

For all these reasons we will consider the NP to involve only LH fermions, i.e. described by effective operators $\mathcal{O}_S$ and $\mathcal{O}_T$ reducing significantly the number of free parameters involved.

5.2 Anomalous observables in LH framework

The most significant B-Physics anomalies observed so far concern the ratios R_{K^*} and R_{D*}.
In the scenario in which NP couples only LH fermions it is easy to compute the previsions for those observables because, as should be clear now, non perturbative contributions

Figure 5.1: Plot of R_{K*} and R_K variating the vector and axial New Physics contributions α_{NP} and β_{NP} in the Left-Handed limit $\alpha_{NP} = -\beta_{NP}$ (blue) and in the Right-Handed limit $\alpha_{NP} = \beta_{NP}$ (red).



are simplified in the ratio and we have just to add proper Wilson coefficients to the SM one.

The matching to SM tells us that the Wilson coefficient of the the operator $\bar{b}_L\gamma^\mu s_L\,\bar{l}_L\gamma_\mu l_L$ is given by

$$C^K_{SM} = -\frac{V_{tb}V^*_{ts}}{8\pi^2 v^2}\mathcal{F}(y_t)$$

hence the ratio R_{K^*}, looking at the electric charge eigenstates form of equation 3.9, results

$$\begin{aligned} R_{K^*} \equiv \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)} &= \frac{|C^K_{SM} + C^{bs\mu\mu}_S + C^{bs\mu\mu}_T|^2}{|C^K_{SM}|^2} \\ &= |1 + \frac{C^{bs\mu\mu}_S + C^{bs\mu\mu}_T}{C^K_{SM}}|^2 \end{aligned} \tag{5.2}$$

which depends only from the parameter $C^{bs\mu\mu}_S + C^{bs\mu\mu}_T$. Since this observable shows the most significant anomaly it will be interesting to accomodate even that one alone to see which scenario is suggested.

The second most significant anomaly appears in R_{D*} that takes contribution only from triplet operator O_T. Mindful of the relative coefficients of different component of

LH doublets given by gauge symmetry (see equation 3.9),

$$
\begin{aligned}
R_{D^*} &\equiv \frac{\mathcal{B}(B \to D^* \tau\bar{\nu})}{\frac{1}{2}\big[\mathcal{B}(B \to D^* e\bar{\nu}) + \mathcal{B}(B \to D^* \mu\bar{\nu})\big]} = \\
&= \frac{|C_{SM}^D + 2C_T^{bs\tau\tau}V_{cs}^* + 2C_T^{bb\tau\tau}V_{cb}^*|^2 + |2C_T^{bs\tau\mu}V_{cs}^* + 2C_T^{bb\tau\mu}V_{cb}^*|^2}{\frac{1}{2}\big[|C_{SM}^D|^2 + |C_{SM}^D + 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^*|^2 + |2C_T^{bs\mu\tau}V_{cs}^* + 2C_T^{bb\mu\tau}V_{cb}^*|^2\big]}
\end{aligned}
\tag{5.3}
$$

that introduced several new parameters. In this case the SM matching gives us

$$
C_{SM}^D = -\frac{2V_{cb}^*}{v^2}
$$

5.2.1 *Fit to NC anomalies*

The first analysis we want to do is, neglecting the anomaly on $b \to cl\nu$ to see what values of Wilson coefficients we need to predict the measured value of R_{K^*}.

From the expression 5.2 we see that R_{K^*} depends of just one free parameter: $C_S^{bs\mu\mu} + C_T^{bs\mu\mu} \equiv C^K$.
Implementing a least squares fit we find that

$$
\begin{aligned}
C^K &\in ()GeV^{-2} \quad \textit{whitin } 1\sigma \\
&\in ()GeV^{-2} \quad \textit{whitin } 2\sigma
\end{aligned}
\tag{5.4}
$$

Referring to table 4.1 we want to understand which of the heavy bosons shown above can give us the contribution we need at low energies from a tree-level matching.

Since the collider constraints for the mass of those bosons is a lower limit of the order of few TeV we set, assuming a coupling of $O(1)$, $G_i = (2\ GeV)^{-2}$ allowing any value for flavour structures.

A

Flavour Theorems

A.0.1 *CP symmetry*

Among the consequences of flavour physics there is the arise of the CP symmetry violation.
In Dirac representation terms, defining

$$\psi_L \equiv \frac{1-\gamma_5}{2}\psi, \quad \psi_R \equiv \frac{1+\gamma_5}{2}\psi$$

charge conjugation, C, and spatial inversion, P, act on fermions as:

$$\begin{aligned} P \,:\, \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} &\to \gamma_0 \psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \\ C \,:\, \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} &\to i\gamma_2 \psi^* = \begin{pmatrix} -\varepsilon\psi_R^* \\ \varepsilon\psi_L^* \end{pmatrix} \\ CP \,:\, \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} &\to i\gamma_2\gamma_0 \psi^* = \begin{pmatrix} -\varepsilon\psi_L^* \\ \varepsilon\psi_R^* \end{pmatrix} \end{aligned}$$

where $\varepsilon = i\sigma_2$. Then we know that that the Higgs boson is transformed under CP as

$$CP \,:\, H \to CH \to H^c = \varepsilon H^*$$

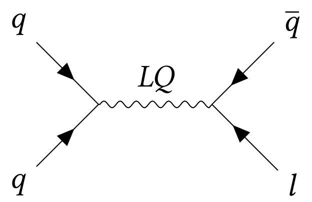
B

Notes on Great Unification Theories

B.0.1 *Rough esteemation of Λ_{GUT} from proton decay*

Here we will show through a naive approach why $\tau_p 10^{33} ys$ can be explained by a mediator with a mass of order $\Lambda_{GUT} \simeq 10^{16}\ GeV$ as we said in section 4.2.1.

The contribution given by the diagram to proton decay



if NP is flavour universal will be approximately

$$\mathscr{M} \simeq g_{NP} \frac{1}{q^2 - M_{NP}^2} g_{NP} \tag{B.1}$$

that in the limit of $q^2 \ll M_{NP}^2$ becomes

$$\mathscr{M} \simeq -\frac{g_{NP}^2}{M_{NP}^2} \tag{B.2}$$

We know that the decay width is

$$\Gamma = (const.)|\mathscr{M}|^2$$

and that it has to have the dimension of a $[mass]^1$, hence the $(const.)$ has to have the dimension of a $[mass]^5$. Since the biggest mass in play is m_p, to get a rough estimation of the width we assume $(const.) = m_p^5$.

If we assume $g_{NP} \simeq$ the approximate width results:

$$\Gamma \simeq \frac{m_p^5}{M_{NP}^5} \rightarrow \tau \simeq \frac{M_{NP}^4}{m_p^5} > 10^{33}\ ys$$

To have the prediction in unit of years we have first to get the lifetime in gaussian units, i.e.

$$\tau \simeq \frac{M_{NP}^4}{m_p^5} \cdot (\frac{\hbar}{c^2}) = (\frac{1}{0.938})^5 (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (6.58 \cdot 10^{-25} s)$$

hence

$$\tau \simeq (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (2.87 \cdot 10^{-32}\ ys) > 10^3 3 ys \rightarrow$$
$$M_{NP}^4 > 10^{64-65}\ GeV^4 \rightarrow M_{NP} > 10^{16} GeV$$

that explain the estimation done in section 4.2.1.

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