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A general overview on Lepton Flavour Universality Violation in B mesons physics

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Contents

Introductıon

In this work we are trying to explore the huge world of Beyond Standard Model (BSM) theories using as gate some exmerimental observations seen recently, that bring clues of the breaking of an accidental symmetry.

In the Standard Model (SM) as is known today we have an accidental symmetry which is called Lepton Flavour Universality (LFU) which tells us that all the gauge interactions are flavour blind; in other words the Electro-Weak (EW) processes have the same strenght for the electron, the muon, the tau and same for the different types of neutrinos.
Since the only difference beetween the different lepton families is the mass, there is always a kinematic flavour sensitive effect i.e. the phase space factor which affect the rate or the cross section of a process, for instance the decay of charged π produces mostly muons for this reasons.
Nevertheless, in the SM, there is also a dynamical effect which is not mass-blind, which is the coupling with the Higgs Boson but since leptons are very light (the τ, which is the heaviest, weighs less than 50 times the Higgs'vacuum expectation value) we neglect the Higgs'coupling and say that LFU is a well approximated accidental symmetry of the SM.

If we have accepted that LFU is a good symmetry we can also understand why testing it is a good place to find clues for BSM theories. Recently in different experiments were found differrent hints of LFU Violation (LFUV) in semileptonic decays of the B mesons (mesons with non vacuum difference of b and $\bar{b}$ as valence quark). All the deviations from the SM appearing in these decays go under the name of *B-Physics Anomalies.*

Since, as we will discuss, the B-Physics Anomalies appear mostly at the hadronic scale (order few GeV), we will implement a model independent approach named the Effective Field Theory (EFT) approach, as is usually done with the Fermi Theory to study low energy weak interactions.
In this framework finding New Physics (NP) basically means to find deviation from the Lagrangian's coefficients of the operators (or Wilson coefficients).

So our purpose is to find the right heavy mediators which, once integrated out from the Lagrangian (again as we do with W and Z bosons to get the Fermi theory), give us the appropriate contribute to accomodate the B-Physics Anomalies. We also have to be very careful about the processes that are already tested, because introducing NP can affect also processes that don't concern B or hadrons at all.

It will be clear that introducing SM-like heavy degrees of freedom the risk to affect the phenomenological constraints is high.
This is one of the reason because we like to introduce Leptoquarks (LQs): coloured bosons which can be absorbed from a quark to become a lepton (+h.c.) and try in this way to affect only semileptonic processes without disturbing others SM constraints. We will take count also of the possible colour-less bosons (heavier version of W and Z) and in both cases we will try to find the best flavour structure to accomodate the Anomalies.

A lot of papers are already written about B-Anomalies and Leptoquarks; the main purpose of this work is to offer an updated catalogue as much general and complete we can do of solution with single NP mediator or combination of them according to the low energy contributions we need.

In the end we have to mention that adding bosons to SM is not enough to say that we have a BSM theory.
We would like to have a theory in which with few assumptions and few input values all the particles and processes come out naturally (as in SM). These complete theories are known as UV Completions and we will mention some of the most interesting in the available literature.

Hope you readers will find the work interesting and light to read.
Before to introduce the Anomalies we need to introduce tha Flavour sector of the SM.

1

Flavour Physics

When we look for clues of New Physics one possible way is to look for processes that aren't allowed from the known model either to measure NP effects or to test the available theory.
For similar reasons a good places to look for NP clues are those processes that are predicted to be rare by the SM to bring either hints for deviations or an improvement on the measure of the SM parameters.

An enviroment in which lot of rare processes arise is the *Flavour sector* of the SM. The Flavour Physics is focused on the fact that the fermion fields of the SM

$$l_L \; q_L \; e_R \; d_R \; u_R$$

appear in phenomenology in three copies each, named *families* (or *generations*), differing only from the mass and hence for the coupling to the Higgs boson. Turning off the Higgs-fermions coupling there would be a big global accidental symmetry that wouldn't allow us to distinguish the different generations named *Flavour symmetry*.
It is interesting to see how the Higgs coupling to fermions (known as *Yukawa coupling*) affect the Flavour symmetry.

1.1 Introduction to Standard Model

The Standard Model of Fundamental Interactions is nowaday the most predicting and accurately tested theory available to treat a wide set of processes.

The assumption done by the SM can be summarized as follows:

1. A local *gauge* symmetry $\mathcal{G}_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ spontaneously broken at low enegies trough an Higgs mechanism with the pattern $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{em}$.

2. The theory has to be a local QFT *renormalizable*, i.e. the lagrangian has to contain operators Lorentz invariant with dimension, in mass unit, ≤ 4.

3. Matter fields are defined to be irreducible representation of $\mathcal{G}_{SM}$ and are the fermions; $q_L^i \sim (3,2)_{1/6}$, $l_L^i \sim (3,2)_{-1/2}$,$e_R^i \sim (1,1)_{-1}$, $d_R^i \sim (3,1)_{-1/3}$, $u_R^i \sim (3,1)_{2/3}$ where

$i = 1, 2, 3$ is the family index, and the scalar responsible for the *Electro-Weak Symmetry Breaking* (EWSB) $H \sim (1, 2)_{1/2}$.
The notation is $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ and indicates the dimension of the representation for $SU(3)_c$ and $SU(2)_L$ and the eigenvalue of the only generator of $U(1)_Y$.

Once stated the guidelines we are ready to write the lagrangian, which has to contain all the possible terms allowed by our assumptions. For simplicity we split it in pieces:

$$\mathscr{L}_{SM} = \mathscr{L}_{kin} + \mathscr{L}_H + \mathscr{L}_\theta + \mathscr{L}_Y \tag{1.1}$$

Where $\mathscr{L}_{kin}$ contains the kinetik term of fermions and gauge fields:

$$\mathscr{L}_{kin} = \sum_\psi i\overline{\psi}^i H^\psi_{ij} \not{D} \psi^j - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_j^{\mu\nu} W^j_{\mu\nu} - \frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} \tag{1.2}$$

where $D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 T^a_L W_{\mu a} - ig_3 T^a_c G_{\mu a}$ with $T^a_L = \frac{\sigma^a}{2}$ and $T^a_c = \frac{\lambda^a}{2}$ (σ are the Pauli matrices and λ Gell-Mann matrices), while $B_\mu \sim (1, 1)_0$ $W_\mu \sim (1, 3)_0$ and $G_\mu \sim (8, 1)_0$ are the vector bosons arising from the local symmetry $\mathscr{G}_{SM}$.
H_{ij} a 3x3 hermitian matrix in flavour space that have to satisfy the canonical quantization condition to have all eigenvalues equal to 1.
In the end $W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - \frac{i}{g_2} f_{abc} W^b_\mu W^c_\nu$ is the gauge invariant tensor for $SU(2)_L$ bosons and f_{abc} are the structure's constants of $SU(2)$. Analogous for the tensor $G^a_{\mu\nu}$ which inherit the structure's constants from $SU(3)$ and for $B_{\mu\nu}$ that has vacuum structure's constants (since $U(1)$ is an Abelian group).

$$\mathscr{L}_H = (D_\mu H)^\dagger D^\mu H + m_H^2 H^\dagger H - \lambda (H^\dagger H)^2 \tag{1.3}$$

is the Higgs term that presents himself as a scalar field Lagrangian with a quartic coupling. The sign of the quartic coupling gives us the vacuum expectation value $\langle 0| H |0\rangle = v \neq 0$ responsible for the EWSB.

$$\mathscr{L}_Y = Y_u^{ij} (H\overline{q}^i_L) u^j_R + Y_d^{ij} (H^c \overline{q}^i_L) d^j_R + Y_l^{ij} (H\overline{l}^i_L) e^j_R + h.c. \tag{1.4}$$

Where the charge's conjugated $H^c \equiv \varepsilon H$ with $\varepsilon = i\sigma_2$ acting on $SU(2)_L$ space and Y are generic complex 3x3 matrices in flavour space.

In the end

$$\mathscr{L}_\theta = \frac{\theta_3}{32\pi} \varepsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G^{\rho\sigma a} \tag{1.5}$$

is known as the $\theta - term$ which would imply CP violation in strong interaction. Nevertheless θ_3 is strongly bounded from the experiments and anyway this term would be neglectable for our purposes, so we will just ignore it[1].

[1]Analogous theta terms arise for $SU(2)_L \otimes U(1)_Y$ gauge bosons, but them can be set to 0 through a redefinition of the gauge fields

It's clear that, without assuming renormalizability of the theory, we could have added infinity more terms to the lagrangian. Since we have stopped at dimension 4, some more symmetry arised accidentally and they are known as *accidental symmetries.*

1.1.1 *Definition of Flavour eigenstates*

As we have seen, to be completely general in equation 1.2 we have introduced an hermitian matrix which acts on flavour triplets H^{ψ}_{ij} which is in general different for every fermion. Nevertheless is always possible to redefine the fermion field trough an unitay trasformation:

$$\psi^i \to \psi'^i = V_{\psi}{}^i_j \psi^j \tag{1.6}$$

choosing V_ψ such that $H \to H^{\psi'} = V_\psi{}^\dagger H^\psi V_\psi = 1_{3x3}$ which is always possible since H is hermitian and its eigenvalues are equal to 1 to respect canonical commutation rule.

Thus, doing the transformation 1.6 for every fermion field we define a basis $\{\psi^{i'}\} \equiv \{q^i_L; l^i_L; e^i_R, d^i_R, u^i_R\}$ of flavour triplets that we know as the previously mentioned fermions of SM. In other words the fermion field triplets are defined as the ones that diagonalize the kinetik term, i.e. the interaction with gauge bosons. We call the basis of interaction eigenstates the *Flavour basis.*

Now we could do any flavour rotation like the 1.6 and we would always obtain $1 \to V_\psi{}^\dagger V_\psi = 1$, hence we find that H had to be the identity from beginning and the relation 1.6 defines a set of flavour basis that diagonalize the gauge interaction.

Let's see how the rotation 1.6 affects $\mathscr{L}_Y$. Considering only the flavour structure of the fields, the Yukawa terms transform as

$$(Hl_L)^\dagger Y_e e_R = H\, e_L^\dagger Y_e e_R \to H\, e_L^\dagger (V_{l_L}{}^\dagger Y_e V_{e_R}) u_R$$
$$(Hd_L)^\dagger Y_u u_R = H\, d_L^\dagger Y_u u_R \to H\, q_L^\dagger (V_{q_L}^\dagger Y_u V_{u_R}) u_R$$
$$(H^c d_L)^\dagger Y_u u_R = H^*\, u_L^\dagger Y_d d_R \to H^*\, q_L^\dagger (V_{q_L}{}^\dagger Y_d V_{d_R}) d_R$$

Now we use that for a generic 3x3 complex matrix exist two matrices in $SU(3)$ such that $A \to U_1^\dagger A U_2 = A^{(d)}$ where $A^{(d)}$ is diagonal. Since we have freedom to rotate the fields through a *gauge diagonal* trasformation (1.6 form) we can choose V_{l_L}, V_{e_R} such that $V_{l_L}{}^\dagger Y_e V_{e_R} = Y_e^{(d)}$ that means that leptons flavour eigenstates can diagonalize the Yukawa matrix, i.e. they are mass'eigenstates as well. This can be seen as the proof of the theorem 1 that will bw shown later.
Different is the story of quarks, in fact there we have two terms that can't be rotated independently. We may choose $V_{q_L} \equiv V_{d_L}, V_{d_R}$ such that $V_{d_L}^\dagger Y_d V_{d_R} = Y_d^{(d)}$ is diagonal but then we no more freedom to diagonalize the up-quark Yukawa. In other words the gauge eigenstates are not mass'eigenstates in the quark sector.

We can always define V_{u_L}, V_{u_R} such that $V_{u_L}^\dagger Y_u V_{u_R} = Y_u^{(d)}$, then the Yukawa coupling term of up quarks results (fixed $V_{q_L} = V_{d_L}$):

$$H\, q_L^\dagger (V_{d_L}{}^\dagger Y_u V_{u_R}) u_R = H\, u_L^\dagger (V_{d_L}^\dagger V_{u_L}) Y_u^{(d)} u_R \equiv H (V_{CKM}^\dagger u)^\dagger Y_u^{(d)} u_R$$

where $V_{CKM} \equiv V_{u_L}^\dagger V_{d_L}$ is the *Cabibbo-Kobayashi-Maskawa*, hence LH mass'eigenstates are (naming CKM matrix with V):

$$q_L^i = \begin{pmatrix} V^{*ij} u_{LJ} \\ d_L^i \end{pmatrix} \quad , l_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

That means that a quark of the i-family can couple at tree level with a j-family quark with intensity according to V_{ij}.

Interactions of leptons are, instead, always flavour diagonal, neglecting effects due to neutrino's mass that, if introduced in the lagrangian trough the insertion of $\nu_R \sim (1,1)_0$ among the SM fields, would imply an analogous mechanism in which ν_L's mass eigenstate would be $\nu_L^{Mi} = (U_{PMNS})^i_j \nu_L^j$, where $U_{PMNS} = V_{\nu_L}^\dagger V_{e_L}$ is the *Pontecorvo-Maki-Nakagawa-Sakata* (PMNS) matrix introduced to explain *neutrinos flavour oscillations.*
We will neglect every effect related to PMNS matrix assuming then Lepton Flavour (LF) conservation for the SM.

1.2 The Flavour sector

1.2.1 *Flavour symmetries of SM Lagrangian*

As previously mentioned, allowing terms with dimension in mass at most equal to four, the lagrangian exhibit many additional symmetries to the ones assumed by the theory, being them exact or approximated.

Simmetries concerning the flavour has to appear in either $\mathscr{L}_{kin}$ or $\mathscr{L}_Y$.
Since $\mathscr{L}_{kin}$ is proportional to identity in flavour space and even in gauge interactions space as already mentioned every field can be rotated by $\psi \to V_\psi \psi$ with $V \in U(3)$ and it would be a symmetry of $\mathscr{L}_{kin}$. That is true for any independent V_ψ associated with the fermion ψ i.e. the whole symmetry arising is $\mathscr{G}_F = [U(3)]^5 = U(3)_q \otimes U(3)_l \otimes U(3)_u \otimes U(3)_d \otimes U(3)_e$ which is called the *Flavour Group.*

Since every matrix in $U(3)$ can be obtained from the product of two matrix in $U(3)$, one proportional to identity and the other with determinant equal to 1, is instructive to decompose every $U(3)$ as $U(3) \sim SU(3) \otimes U(1)$.
All the $U(1)$ groups are given by the gauge diagonality of kinetik term and it represents the conservation of the number of a given fermion (seen as flavour eigenstate) in every gauge interaction.
$SU(3)$, instead, are due to the fact that flavour matrix is equal to identity, and they guarantee that every different flavour component of a given fermion interacts with gauge fields with the same intensity.

These symmetries of $\mathscr{L}_{kin}$ are partially broken by Yukawa coupling in fact, since Higgs couples different gauge representations of fields (in other worlds Yukawa coupling isn't gauge diagonal). Nevertheless, being the Higgs field a colour-singlet, it can couple quark with quark and lepton with lepton, hence the $[U(1)]^5$ symmetry is broken

only partially, the symmetry left is $[U(1)]^2 = U(1)_B \otimes U(1)_L$ which represent the conservation of *Barion Number* and *Lepton Number* that are realized exactly in SM.
In the lepton sector, since Yukawa matrix is diagonal, we have an additional $[U(1)]^3$ under the which the i-th family of leptons has the same charge being l_L^i or e_R^i; this symmetry represents the already mentioned Lepton Flavour conservation, which is exact in SM apart from effects derived from neutrino's mass.

Then, since Yukawa matrices are far to be proportional to identity, a generic $V \in SU(3)$ would change Yukawa matrices, even if it was the same for the two fields coupling the Higgs inside a given operator.
In $\mathscr{L}_{kin}$ we were guaranteed to have just flavour blind interactions by $SU(3)$. When we include Yukawa couplings the quark sector, Yukawa matrices of quark break the $[SU(3)]^3$ of quark fields, so the interactions with Higgs breaks the flavour universality of quark interactions. It is important to clarify that this is not due to the fact that we can't diagonalize Yukawa matrices with a flavour basis, because even if we could do it (i.e. in the limit $V_{CKM} = 1$) the Yukawa matrices of quark, being not proportional to identity, aren't invariant under $Y_{u/d} \to V^\dagger Y_{u/d} V$, then $[SU(3)]^3$ of the quark sector is explicitly broken.
Same argument can be brought for the lepton sector to say that $[SU(3)]^2$ is explicitly broken because of the different mass of charged leptons. Nevertheless the situation of lepton numerically is way different from quark, in fact if the heaviest quark, the top-quark, has a mass of 173 GeV comparable with Higgs'vev ($v \simeq 246$ GeV), charged leptons are way lighter; the heaviest lepton, the τ, weighs $\simeq 1.7$ GeV which means that the biggest matrix element of Y_e is $Y_e^{33} \simeq \frac{1.7}{246} \simeq 7 \cdot 10^{-3}$. If we neglect terms of order $\frac{m_\tau}{v}$ the flavour group $[SU(3)]^2$ of lepton sector remains unbroken making arise the so known *Lepton Flavour Universality* (LFU), which is realized approximately in the SM, that tells us the interactions of SM are almost flavour-blind in the lepton sector, apart from terms of τ-Higgs coupling.

Phenomenologically LFU can be tested in every process in which the phase space factor of the final state (i.e. when the mass of the final state products is neglectable compared to the total energy in the rest frame) is neglectable, the rate of events is the same for every flavour of leptons involved.
Some examples of ratios that test LFU are:

$$\left.\frac{\mathscr{B}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\mathscr{B}(\tau \to e \nu_\tau \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests electron-muon universality,

$$\left.\frac{\mathscr{B}(\tau \to e \nu_\tau \bar{\nu}_e)}{\mathscr{B}(\mu \to e \nu_\mu \bar{\nu}_e)}\right|_{SM} = 1 \quad ,$$

that tests muon-tau universality, and many others.

A measure of sensitive deviations of this ratios would mean beyond SM effects.

1.2.2 *Flavour Theorems*

We have seen so far that mass'eigenstates of quarks aren't gauge interaction eigenstates as well.
Nevertheless we define particles we observe as eigenstates of the time evolution i.e. states with a well-defined mass.
The fact that gauge interactions aren't flavour diagonal in mass'eigenstates basis makes arise processes that link different families of quarks, known as *flavour breaking* processes. These processes happen to be suppressed because of the values of CKM-matrix.

They main features of all breaking flavour processes are summarized in three statements that, because of their fundamental importance, are known as *Flavour theorems*:

Flavour theorem 1. *Flavour breaking processes are absent in lepton sector.*

in which the proof is shown above in section 1.1.1.

Flavour theorem 2. *Flavour breaking processes in the quark sector happen just in charge current processes (exchanging $W^{\pm}$).*

The proof of this theorem is reported in section 1.3.

Flavour theorem 3. *Applying CP transformation to the whole lagrangian we find that $\mathscr{L}^{CP} \equiv CP(\mathscr{L})$ is equal to $\mathscr{L}$ if and only if V_{CKM} is real.*

The proof of which 3 is reported in B.

The second flavour theorem tells us that flavour breaking processes can arise at three level just through exchange of W boson. Also, at every order they involve only Left Handed fermions apart from corrections due to mass insertions.

The third flavour theorem tells us that CP violation (CPV) can happen just if there are at least three family of quark.
Historically when in 1964 Christenson, Cronin, Fitch and Turley observed the first evidence of CPV in neutral Kaons system [1], the third family of quark wasn't observed yet. We have seen previously that CKM matrix has $\frac{N^2-3N+2}{2}$ irreducible phases, hence if only the two families known at the time existed CKM matrix would have been real and wouldn't have allowed CPV for the theorem 3.
In the years later, discovering the b quark and later t quark, was good test to the newborn Flavour Sector of the SM.

1.2.3 *CKM matrix*

Diagonalizing the quark mass matrices we found that up-quark (by convention could be down as well) interaction eigenstates are rotated respect to the mass eigenstates through:

$$V \equiv V_{u_L}^{\dagger} V_{d_L} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.7}$$

Where every elements is generally complex. Neverteheless it has to satisfy unitarity condition $V^\dagger V = 1$, plus it hasn't a unique form because, for what said so far in section 1.2.1, is possible to redefine CKM through a phase trasformation of the single flavour components that leaves invariant $\mathscr{L}_{kin}$, since the $[SU(3)]^5$ symmetry was already used to diagonalize Yukawa matrices. There is actually a global phase that has to remain free due to the Barion Number conservation $U(1)_B$, for the rest, for N families we have $2N-1$ (N for down quarks, N for up quarks minus 1 for the global barion number) phases that we can define as we prefer to write CKM matrix.

These conditions limit the number of free parameters of V. A generic NxN unitary matrix has $2N^2 - N^2 = N^2$ because of the N^2 conditions in $V^\dagger V = 1$. Among these N^2 parameters we have the angles describing a ortogonal matrix $\frac{N(N-1)}{2}$ plus the $\frac{N(N+1)}{2}$ phases in which $2N-1$ can be chosen by redefinition of quark fields. Thus we remain with $\frac{N(N-1)}{2}$ angles and $\frac{N(N+1)}{2} - (2N-1) = \frac{N^2-3N+2}{2}$ phases.

With three families we count three angles and one phase that define our matrix. The standard parametrization is given by the product of three real rotations, and the phase included in the $1-3$ rotation, i.e.

$$V = R_{12}(\theta_{12}) \cdot R_{13}(\theta_{13}, e^{i\delta}) \cdot R_{23}(\theta_{23}) \tag{1.8}$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \end{aligned} \tag{1.9}$$

where $c_{ij} \equiv cos(\theta_{ij})$ and $s_{ij} \equiv sin(\theta_{ij})$.
From this parametrization is indeed not clear the hierarchy of the flavour breaking transition, so we like to introduce *Wolfenstain's parametrization.* We define

$$\lambda \equiv s_{12} \quad , A\lambda^2 \equiv s_{23} \quad , A\lambda^3(\rho - i\eta) \equiv s_{13}e^{-i\delta}$$

We obtain, up to order λ^3

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \tag{1.10}$$

Since $\lambda^{exp} \simeq 0.22$ we find that, as mentioned previously, $V = 1 + O(\lambda)$, thus flavour diagonal transitions are encouraged. Flavour breaking transitions are suppressed at order λ if they link first and second family, at order λ^2 if they link first and second ones and at order λ^3 if they link first and third family.

1.3 Fermi Theory

The Fermi Theory is the QFT used to describe the weak processes of the SM when the energy is way below $M_W = 80\ GeV$.
This theory describes the weak processes at low energies ignoring the details of the UV Physics like the coupling beetween fermions and vectors that mediate the force, the mass of these mediators nor the theoretical nature of all the particles heavier than $\Lambda = 80\ GeV$, which we call the *matching scale*.

In an EFT all the physics beyond the matching scale is contained in the numerical coefficient in front of the Lagrangian's operators. In fact the condition for the EFT to be a low energy version of an UV theory is for those coefficient to satisfy the *matching condition* that consists in impose the coefficients of the two theories, to recreate the same transition amplitudes at an energy equal to the matching scale.

1.3.1 *Derivation of the Fermi Theory from the SM*

When the energy available in a given process is not enough to produce some of the particles among the spectrum of the theory, is possible to *integrate them out* replacing the fields that describe these heavy particles with the solution of their motion equations.
Doing in that way we would automatically satisfy the matching condition for all tree level amplitudes.

We here show how to generate operators in the Fermi Theory through the integration of the W and Z bosons.
Since in 1.2 we have B_μ and W_μ^3 phenomenologically happen to have a not defined mass, the first step is to define:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} cos\,\theta_W & -sin\,\theta_W \\ sin\,\theta_W & cos\,\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} W_\mu^3 cos\,\theta_W - B_\mu sin\,\theta_W \\ W_\mu^3 sin\,\theta_W + B_\mu cos\,\theta_W \end{pmatrix}$$

Where A_μ is the field that describes the photon that, being massless doesn't need to be integrated out, and Z_μ is the field describing the Z boson, responsible for neutral current weak interactions. Instead θ_W is the *Weinberg angle* defined as

$$\frac{g_1^2}{g_1^2 + g_2^2} \equiv sin^2\,\theta_W$$

Since at low energy the symmetry is broken to $U(1)_{em}$ we would like to write $W_\mu^{1,2}$ in electric charge eigenstates basis:

$$W_\mu^\pm \equiv \frac{W_\mu^1 \pm iW_\mu^2}{\sqrt{2}}, \quad \sigma_\pm \equiv \frac{\sigma_1 \pm i\sigma_2}{2}$$

hence $W_1\sigma_1 + W_2\sigma_2 = \sqrt{2}(W^+\sigma_- + W^-\sigma_+)$.
Now we can write the *covariantderivative* in term of mass eigenstates as:

$$D_\mu = \partial_\mu - i\frac{g_2}{\sqrt{2}}[W_\mu^+\sigma_- + W_\mu^-\sigma_+] - i\frac{g_2}{cos\,\theta_W}Z_\mu(T_3 - Qsin^2\,\theta_W) - i\frac{g_2}{sin\,\theta_W}A_\mu - ig_3T_c^aG_{a\mu}$$

where $Q \equiv Y - T_3$ is the electric charge operator Then the lagrangian that contains the W and the Z is given by:

$$\begin{aligned}\mathcal{L} \supset &-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}W_{\mu\nu}^+W_{\mu\nu}^- + \frac{M_Z^2}{2}Z_\mu Z^\mu + M_W^2W_\mu^+W^{\mu-}\\ &+ Z_\mu J_0^\mu + W_\mu^+J_-^\mu + W_\mu^-J_+^\mu\end{aligned}$$

where

$$J_0^\mu = \sum_\psi g_Z^\psi\bar{\psi}\gamma^\mu\psi$$

where the sum is done of every SM fermion after EWSB, i.e. $\{\psi\} = \{e_L, \nu_L, e_R, u_L', d_L, u_R', d_R\}$ and

$$g_Z^\psi = \frac{g_2}{cos\,\theta_W}(T_3^\psi - Q^\psi sin^2\,\theta_W) \tag{1.11}$$

while the charged currents are:

$$J_+^\mu \equiv \frac{g_2}{\sqrt{2}}[\bar{d}_L\gamma^\mu u_L' + \bar{e}_L\gamma^\mu\nu_L], \quad J_-^\mu \equiv (J_+^\mu)^\dagger \tag{1.12}$$

where $u' \equiv V_{CKM}^\dagger u$. Here we can see that the neutral current interaction uf LH up quarks is flavour diagonal because of unitarity of CKM matrix $V_{ij}^*V_{jk} = \delta_{ik} \rightarrow$

$$\bar{u}_L^i V_{jk}\gamma^\mu V_{kl}^* u_L^l = \bar{u}_L^i\gamma^\mu u_L^i$$

in the Charged current instead we have:

$$\bar{u}_L^i V_{jk}\gamma^\mu d_L$$

that breaks the flavour explicitly. This fact can be seen as the proof of the theorem 2.

Then, the equation of motion result:

$$\begin{aligned}&\frac{\delta\mathcal{L}}{\delta W_\mu^+} = 0 = M_W^2W^{\mu-} + J_-^\mu \rightarrow\\ &W^{\mu-} = -\frac{1}{M_W^2}J_-^\mu; \quad W^{\mu+} = (W^{\mu-})^\dagger = -\frac{1}{M_W^2}J_+^\mu\\ &\frac{\delta\mathcal{L}}{\delta Z_\mu} = 0 = M_Z^2Z^\mu + J_0^\mu \rightarrow\\ &Z^\mu = -\frac{1}{M_Z^2}J_0^\mu;\end{aligned}$$

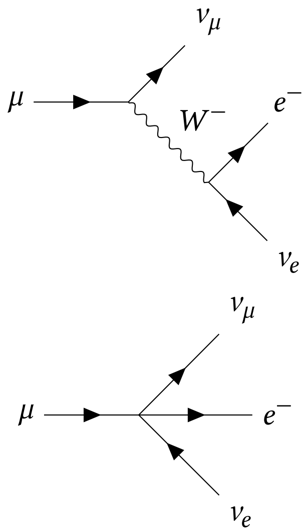


Figure 1.1: Above: Feynman diagram for muon decay in SM, the lagrangian terms responsible for this interaction is of the form $\sim g_2\overline{\psi}_{1L}\gamma^\mu\psi_{2L}W_\mu$.
Below: Feynman diagram for muon decay in Fermi Theory, the lagrangian terms responsible for this interaction is of the form $\sim G_F\overline{\psi}_{1L}\gamma^\mu\psi_{2L}\overline{\psi}_{3L}\gamma_\mu\psi_{4L}$
.

Replacing in the lagrangian the solution of equations of motion we find

$$\mathscr{L}_{EFT} = -\frac{1}{M_W^2}J_+^\mu J_{\mu_-} - \frac{1}{2M_Z^2}J_0^\mu J_{0\,\mu}$$

then we use the relation $M_W^2\, cos^2\theta_W = M_Z^2$ and the definition of the *Fermi constant* $\frac{g_2^2}{4\sqrt{2}M_W^2} \equiv G_F$ to write:

$$\begin{aligned}\mathscr{L}_{EFT} \supset &-\frac{4G_F}{\sqrt{2}}\left[\overline{u'_L}\gamma^\mu d_L\,\overline{d_L}\gamma_\mu u'_L \;+\; \overline{u'_L}\gamma^\mu d_L\,\overline{\nu_L}\gamma_\mu e_L \;+\; \overline{e_L}\gamma^\mu\nu_L\,\overline{d_L}\gamma_\mu u'_L \;+\; \overline{e_L}\gamma^\mu\nu_L\overline{\nu_L}\gamma_\mu e_L + h.c.\right]\\ &\frac{-4G_F}{\sqrt{2}}\Big[\sum_\psi(T_3^\psi - Q^\psi sin^2\theta_W)\overline{\psi}\gamma^\mu\psi\Big]\Big[\sum_\chi(T_3^\chi - Q^\chi sin^2\theta_W)\overline{\chi}\gamma_\mu\chi\Big]\end{aligned} \tag{1.13}$$

up to $O(\lambda)$ corrections.
Since $(sin^2\theta_W)^{exp} \simeq 0.22$ we notice that the weak interaction for RH fermions ($T_3 = 0$) exist just in the neutral current form, and it gets a suppression due to the accidental smallness of θ_W.

The lagrangian 1.13 satisfies the matching condition for every tree level amplitude of the SM.
The intensity of flavour breaking terms becomes clear writing $u'^i = (V^*)^{ij}u_j$.

The Fermi Theory was historically the first theory that described the weak processes as the beta decay or the muon decay (Feynman diagram in Figure 1.1), whose leading contributions are three level induced in the SM without any flavour suppression.

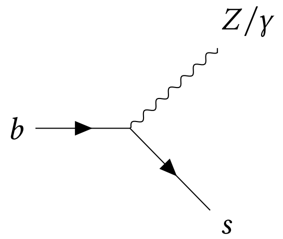
Adding all the Flavour breaking terms we get a lot of terms with a small coupling due to CKM suppression. Also it is possible to get some non trivial loop induced operators, like the neutral current flavour breaking processes, also known as *Flavour Changing Neutral Currents* (FCNC).

1.3.2 *Flavour Changing Neutral Currents*

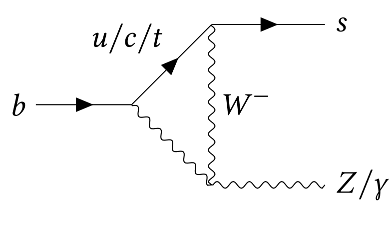
As we have seen flavour breaking processes arise at tree level just through exchange of W bosons. Nevertheless, at low energy we could have flavour breaking transitions in neutral current processes due to the matching to a one loop amplitude in the high energy theory.

These processes are named Flavour Changing Neutral Currents and, because of the theorem 2, they gets a double suppression for the additional vertex and the loop factor.

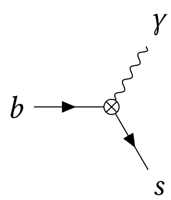
For example the vertex:



is not allowed in the SM, but the $b \to sZ/\gamma$ transition can arise at one loop level through a loop of W:



Now if we integrate out W's and Z bosons we get:



In the previous diagram we neglected the Z boson because in Fermi Theory it doesn't belong to the spectrum. Indeed if the Z were virtual in the SM diagram, perhaps producing a pair l^+l^-, we would have exactly the effective vertex mentioned in section 2 that allows the decay $b \to sl^-l^+$ and so the decay $B \to K^{(*)}l^-l^+$ which will be of the most interested for us.

In SM we have CKM, loop factor, and gauge coupling that give me a prediction for the coefficient in front of the operator that mediate this decay: the Wilson coefficent. So if the mesaured Wilson coefficient is different from the theoretical one, it means that probably to build the EFT we need to integrate out other heavy particles which we don't already know.

2

B-Physics Anomalies

One of the most important and coherent deviation from SM seen in last years, with a constantly growing set of data that corroborating them, are the *B-physics Anomalies*. That anomalies are given by some semileptonic decays of B mesons that seem to treat differently different flavours of leptons, i.e. suggesting LFU violation in processes that involves b quark.

The clues of LFU violation in b-sector come from different and independent observables but anyway before to list them is possible to summarize some characteristics:

- The fundamental transitions affected are always $b \to sl^+l^-$ and $b \to cl\nu$ (or crossing transformated) to which we will refer respectively as *Neutral Current* anomalies (NC) and *Charged Current* anomalies (CC). Hence the first family of quark is not involved.

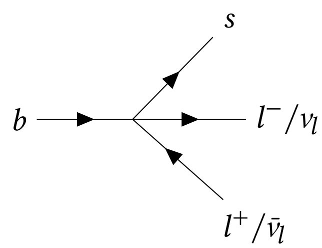
- In NC anomalies we see an excess of electrons in some decays that can be given by either a NP contribution of electron channel or a contribution for muons that interferes destructively with the SM.

- In CC anomalies NP seems to be universal for first two families but to couple more with τ, giving a preference for $b \to c\tau\nu$ compared to other lepton flavours.

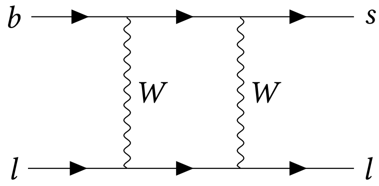
In the anomaly concerning the LFU violation between the first two family of leptons we will assume that NP couples only with muons interfering destructively with SM contributions.

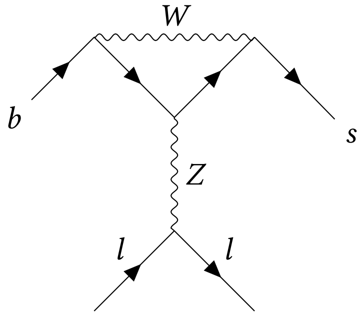
2.1 Main Processes for NC transitions

The transition $b \to sl^+l^-$ in which the leading contribution in Fermi theory from the vertex



is, as every FCNC (see section 1.3.2), one loop induced in the SM from the diagrams:





The first anomalous observables sensitive to the transition $b \to sl^+l^-$ are the ratios R_K for which we have four independent observables according to the different spin, parity or electric charge states.
Thus we define

$$R_{K^+} \equiv \frac{\mathscr{B}(B^+ \to K^+\mu^-\mu^+)}{\mathscr{B}(B^+ \to K^+e^-e^+)} \tag{2.1}$$

$$R_{K_S} \equiv \frac{\mathscr{B}(B^+ \to K^+\mu^-\mu^+)}{\mathscr{B}(B^+ \to K^+e^-e^+)} \tag{2.2}$$

$$R_{K^*} \equiv \frac{\mathscr{B}(B^+ \to K^+\mu^-\mu^+)}{\mathscr{B}(B^+ \to K^+e^-e^+)} \tag{2.3}$$

$$R_{K^{*+}} \equiv \frac{\mathscr{B}(B^+ \to K^+\mu^-\mu^+)}{\mathscr{B}(B^+ \to K^+e^-e^+)} \tag{2.4}$$

where the K^*, K^{*+} are vector mesons and K^+, K_S are pseudoscalars and the S stays for the short-living neutral kaon which is a mixture of K_0 and $\overline{K}_0$ because of CP Violation.

We have another anomalous observable given by the ratio R_{pK} defined as:

$$R_{K^{*+}} \equiv \frac{\mathscr{B}(\Lambda_b \to pK^-\mu^-\mu^+)}{\mathscr{B}(\Lambda_b \to pK^-J/\psi(\to \mu^-\mu^+)} \Big/ \frac{\mathscr{B}(\Lambda_b \to pK^-\mu^-\mu^+)}{\mathscr{B}(\Lambda_b \to pK^-J/\psi(\to \mu^-\mu^+)} \tag{2.5}$$

The reasons why we use ratios of Branching Ratios are mainly three:

Table 2.1: Some anomalous observables characterized by $b \to s l^+ l^-$ transition. $R_H^{[q_1^2, q_2^2]}$ means the ratio R_H in which the momenta of the pair lepton-antilepton has energy at rest q^2 included beetween q_1^2 and q_2^2.

Observable	*Experiment*
$R_{K^+}^{[1.1,6.0]}$	$0.846^{+0.042+0.013}_{-0.039-0.012}$ *[2]*
$R_{K_S^0}^{[1.1,6.0]}$	$0.846^{+0.020+0.013}_{-0.039-0.012}$ *[3]*
$R_{K^*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07} \pm 0.03$ *[2]*
$R_{K^*}^{[1.1,6.0]}$	$0.69^{+0.11}_{-0.07} \pm 0.05$ *[2]*
$R_{K^{*+}}^{[0.045,6.0]}$	$0.70^{+0.18+0.03}_{-0.13-0.04}$ *[3]*
$R_{pK}^{[0.1,6.0]}$	$0.86^{+0.14}_{-0.11} \pm 0.05$*[4]*

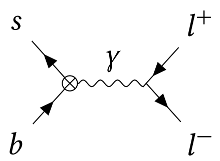
- To reduce the dependence from hadronic form factors
- To reduce the dependence from CKM matrix elements
- To reduce the systematic error in general

In the Table 2.1 we finally see the experimental data that seem to point to a LFU violation.
In the notation used $R_{K^*}^{[q_1^2, q_2^2]}$ is:

$$R_H^{[q_1^2, q_2^2]} = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to H\mu^+\mu^-)}{\int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathcal{B}}{dq^2}(B \to He^+e^-)}$$

where H is the corresponding hadronic state.

According to the momentum of the lepton pair the prediction SM is different. In fact, when the energy of leptons is high enough to neglect the phase space different beetween μ and e, the ratios are equal to 1 in SM because of LFU.
When we include in the measure events in which the energy of leptons q^2 is comparable with m_μ^2 we would need to include the phase space correction and also the contribution of the electromagnetic term given by the diagram

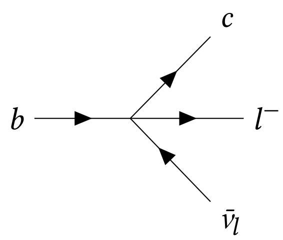


that represents a flavour universal contribution proportional to $\frac{1}{q^2}$ due to the photon propagator.

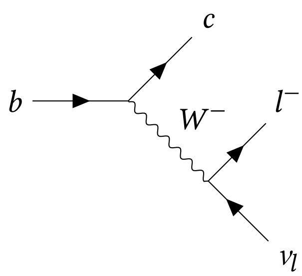
When the range of q^2 is $[0.045, 1.1]$ GeV^2 that contribution is big and we would need to keep it in the computation of NP observables together with the phase space factrors ratio.
On the other hand when the range is wide enough (for instance $[0.045, 6.0]$ GeV^2) we can neglect that contributions that would give only a small contribution to the whole integration on that range because if the relative smallness of the low energy bin.
Hence, in that case we will neglect both the photon contribution and the phase space effect.

2.2 Main Processes for CC transitions

The CC transition $b \to cl\bar{\nu}$ gets its tree level contribution in Fermi Theory from



which is tree level generated in the UV theory through an exchange of W



and that means that whatever NP could be a candidate to accomodate the anomaly, it has to affect the transition $b \to cl\bar{\nu}$ with a biggest magnitude respect to the magnitude of NP contribution deed to accomodate NC anomalies, that instead is one loop induced in SM.

The ratios we will consider this time are

$$R_{D^*}^{\tau l} \equiv \frac{\mathscr{B}(B \to D^* \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathscr{B}(B \to D^* l \nu_l)} \tag{2.6}$$

$$R_D \equiv \frac{\mathscr{B}(B \to D \tau \nu_\tau)}{\frac{1}{2}\sum_{l=e,\mu} \mathscr{B}(B \to D l \nu_l)} \tag{2.7}$$

Table 2.2: Table with the some anomalous ratios anomalies in $b \to cl\nu$ transition. ρ stays for the correlation beetween the ratio in D and D^*.

Observable	*Experiment*	*SM*
$\{R_D, R_{D^*}\}$	$\{0.337(30), 0.298(14)\}$	$\{0.299(3), 0.258(5)\}$
ρ	-0.42	$-$
$BR(B^- \to \tau^- \bar{\nu}_\tau)$	$1.09(24) \cdot 10^{-4}$	$0.812(54) \cdot 10^{-4}$

In Table 2.2 we see the measured ratios compared to SM predictions provided by [2]. As we see the ratios are not predicted to be 1 in SM as in the previous case and this is due to the phase's space suppression of the τ channel which. In other words at this energy scale the different mass of τ compared to μ and e brings a non neglectable kinematic factor that makes the prediction for the ratio to be

$$R_D^{SM} = \frac{\Phi_\tau^D}{\frac{1}{2}[\Phi_e^D + \Phi_\mu^D]} \equiv \eta_\tau^D \tag{2.8}$$

that will be used as phenomenological parameter when we will compare anomalies with a BSM scenario. Analogoud is the definition of $\eta_\tau^{D^*}$ that is smaller because of the bigger mass of D^* compared to D.

Plus it seems that, in CC processes, NP doesn't affect first two families. In fact as reported by [5]

$$R_D^{\mu e} = \frac{\mathscr{B}(B \to D\mu\nu)}{\mathscr{B}(B \to De\nu}\bigg|_{exp} = 1.00 \pm 0.02 \tag{2.9}$$

that this time is equal to 1 even in SM prediction because $\frac{\Phi_\mu}{\Phi_e} \simeq 1$ both for decay in D and in D^*.

To understand what type of NP framework can fit the data better than SM, the first step is to implement an EFT with BSM contributions to some Wilson coefficients.
At this level the only assumption we will do is for NP to not interact with first families of quark and leptons.

3

New Physics Effective Field Theory

First step to implement a scenario that accomodate the anomalies described so far is to study these effects in a model independent framework, i.e. an Effective Field theory deed to parametize eventual NP contributions.

First we will categorize the different possible four fermion operators that can be generated integrating out NP degrees of freedom, then we will focus on semileptonic operators whose give the leading contribution on processes of interest.

Keeping in mind that we don't want to break the conservation of Barion number, which is very tested, we have just three types of operators:

- Purely quark operators, which can mediate for instance the Kaon's decay in pions'channels. Nevertheless those operators are quite hard to match with the SM because we can have loop of gluons beetween the two currents which both couple with gluons. Since at the meson scale of energies the strong interaction is non-perturbative there are a lot of next to leading contributions that we have to consider.

- Purely leptonic operators, which can for instance mediate the muon's decay. Those operators are used for processes that have just leptons both in the initial and final state avoiding all the QCD's mess for both of them. In fact the cited muon decay (shown previously as example in 1.1) is predicted so much accurately that it was the main process used to measure the Fermi's constant G_F.

- Semileptonic operators, which can mediate the charge pion's decay, but also all the processes to which we can address the B-Physics Anomalies. Of course the prediction are not clean as the purely leptonic case, nevertheless we have no gluon loop beetween the two currents and this tells us that we can see how the quark current renormalize just using global symmetries of QCD. In fact, in QCD, the vector current is conserved and so the quark current in a semileptonic decay is not affected by renormalization group of QCD. This fact reducts the theoretical uncertainity to the computation of the matrix element of a quark current beetween the initial and the final hadron states.

The path to identify the proper NP is, first to see what NP contributions in the EFT, that with those contributions wouldn't match with SM at matching scale, and then find what UV theories BSM could match these NP contributions without contraddicting other tested observables.

To compare the different UV frameworks we need to choose a basis of semileptonic operators that includes all possible contributions that is possible to generate through a tree level matching.

3.1 Effective Semileptonic Lagrangian

As we mentioned many times so far we want to find the coefficients that parametrize the B-Physics Anomalies which appear just in semileptonic B mesons decays.
In this chapter we choose to write fermion fields in Weyl representation A

Since every four fermion operator has to be made of two spinor bilinears both being scalar, vector or tensor.
There are many possible basis to describe all possible semileptonic operators, the choice we make is to avoid the explicit conjugation of charge and, if possible, to have every bilinear being a colour singlet. This choice is due to the fact that in hadronic transitions is confortable to have few shapes of quark currents in order to simplify them, when it's possible, in the computation of experimental observables.

Our first step is to collect all the effective operators that can contribute to those processes. For first we will list five operators that can be written as contraction of two vector currents.
If we assume for NP to be coupled just to LH fermions, as done by [6], we would need just two operators:

$$\begin{aligned}\mathcal{O}_S =& q_L^\dagger \bar{\sigma}^\mu q_L\ l_L^\dagger \bar{\sigma}_\mu l_L = (V^\dagger u_L)^\dagger \bar{\sigma}^\mu (V^\dagger u_L)\ e_L^\dagger \bar{\sigma}_\mu e_L + d_L^\dagger \bar{\sigma}^\mu d_L\ e_L^\dagger \bar{\sigma}_\mu e_L + \\ & (V^\dagger u_L)^\dagger \bar{\sigma}^\mu (V^\dagger u_L)\ \nu_L^\dagger \bar{\sigma}_\mu \nu_L + d_L^\dagger \bar{\sigma}^\mu d_L\ \nu_L^\dagger \bar{\sigma}_\mu \nu_L\end{aligned} \tag{3.1}$$

$$\begin{aligned}\mathcal{O}_T = q_L^\dagger \bar{\sigma}^\mu \sigma_a q_L\ l_L^\dagger \bar{\sigma}_\mu \sigma^a l_L =& 2(V^\dagger u_L)^\dagger \bar{\sigma}^\mu d_L\ l_L^\dagger \bar{\sigma}_\mu \nu_L + 2 d_L^\dagger \bar{\sigma}^\mu (V^\dagger u_L)\ \nu_L^\dagger \bar{\sigma}_\mu l_L + \\ & u_L^\dagger \bar{\sigma}^\mu u_L\ \nu_L^\dagger \bar{\sigma}_\mu \nu_L + d_L^\dagger \bar{\sigma}^\mu d_L\ l_L^\dagger \bar{\sigma}_\mu l_L - \\ & d_L^\dagger \bar{\sigma}^\mu d_L\ \nu_L^\dagger \bar{\sigma}_\mu \nu_L - u_L^\dagger \bar{\sigma}^\mu u_L\ l_L^\dagger \bar{\sigma}_\mu l_L\end{aligned} \tag{3.2}$$

where S, T, say that the two currents contracting each other are in the two case a singlet and a triplet of $SU(2)_L$.
We neglected the flavour indexes, keeping them explicit we have, for instance, the singlet operator equal to $\mathcal{O} = \mathcal{O}^{abcd} = {q_L^a}^\dagger \bar{\sigma}^\mu q_L^b\ {l_L^c}^\dagger \bar{\sigma}_\mu l_L^d$ with

$$q_L^i = \begin{pmatrix} V^{*ij} u_{L\,J} \\ d_L^i \end{pmatrix} \quad , l_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$$

in other words we are keeping the flavour structure free at this level, that will be *ruled* by Wilson coefficients.

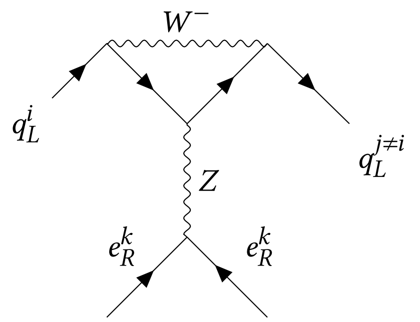
Any time we keep EW indexes inplicit it means that both the vector currents composing the operator are irreducible representation of $SU(2)_L$.

Besides, looking at the electric charge eigenstates form, we notice that $\mathcal{O}_S$ contributes only to NC processes, while $\mathcal{O}_T$ can contribute to CC transitions as well as NC ones.

Including the RH fermions, we find other two operators involving LH currents compatible with the gauge symmetry $\mathcal{G}_{SM}$:

$$\begin{aligned}
\mathcal{O}_{LR1} =& q_L^\dagger \bar{\sigma}^\mu q_L \, e_R^\dagger \sigma_\mu e_R = (V^\dagger u_L)^\dagger \bar{\sigma}^\mu (V^\dagger u_L) \, e_R^\dagger \sigma_\mu e_R + d_L^\dagger \bar{\sigma}^\mu d_L \, e_R^\dagger \sigma_\mu e_R \\
\mathcal{O}_{LR2}^{u/d} =& q_R^\dagger \sigma^\mu q_R \, l_L^\dagger \bar{\sigma}_\mu l_L = q_R^\dagger \sigma^\mu q_R \, \nu_L^\dagger \bar{\sigma}_\mu \nu_L + q_R^\dagger \sigma^\mu q_R \, e_L^\dagger \bar{\sigma}_\mu e_L
\end{aligned}$$

when the u/d means that we have two independent versions of the $\mathcal{O}_{LR2}$ for q_R equal to the up or down quark's flavour triplet. These two operators can describe just NC transitions and so the quark flavour diagonal part, which gets tree level contributions in SM, is not much interesting for us.
The flavour breaking contributions are as well generated in SM but suppressed for different reasons: the FCNC contribute given by $\mathcal{O}_{LR1}$ described by the SM diagram:



is accidentally suppressed because of the smallness of the Weinberg's angle θ_W that suppresses the coupling beetween the Z boson and RH fermions.
$\mathcal{O}_{LR2}$ is even more suppressed because, even if the Z this time couple to its favourites LH fermions, to have the FCNC for RH quarks we need to flip the chirality twice (once per quark) because of the Flavour theorem 2 which states that flavour breaking couplings of the SM are allowed only in LH sector, and so the contribute is suppressed due to the light mass of the quarks involved.

The last vector-vector operator is the one that, in SM, takes both the suppressions described above:

$$\mathcal{O}_R^{u/d} = q_R^\dagger \sigma^\mu q_R \, e_R^\dagger \sigma_\mu e_R$$

again in the u/d versions according to the flavour triplet involved.

Now we have three operators made up the contraction of two scalar currents. For first:

$$\begin{aligned}
\mathcal{O}_S^u &= q_L^\dagger u_R \, \varepsilon \, l_L^\dagger e_R = (V^\dagger u)_L^\dagger u_R \, e_L^\dagger e_R - d_L^\dagger u_R \, \nu_L^\dagger e_R \\
\mathcal{O}_S^d &= q_L^\dagger d_R \, e_R^\dagger l_L = (V^\dagger u)_L^\dagger d_R \, e_R^\dagger \nu_L + d_L^\dagger d_R \, e_R^\dagger e_L
\end{aligned}$$

in which $\mathcal{O}_S^d$ could indeed be written as vector currents contraction through Fierz identities, but renouncing to the request of having currents trasforming as $SU(3)_c$ singlets.

The ε tensor is the one introduced in section 3.2, a part that acts on $SU(2)_L$ to select the singlet.

In the end we have the *Leptoquark* operator:

$$\mathcal{O}_{LQ} = e_R^\dagger q_L \, \varepsilon \, u_R^\dagger l_L = e_R^\dagger (V^\dagger u)_L \; u_R^\dagger e_L - e_R^\dagger d_L \; u_R^\dagger \nu_L \tag{3.3}$$

that can't be written as contraction of two colour singlets but just of two LQs scalar currents, unless we involve charge's conjugate fields, in fact

$$\varepsilon_{ij}\varepsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} \to q^{c\dagger}_L \varepsilon l_L \; e_R^\dagger u_R^c = \mathcal{O}_{LQ} - \mathcal{O}_{\mathcal{S}}^u$$

that anyway wouldn't help the simplification of hadronic form factors, since we would have anyway a current which appears just in this operator.
It is straightforward to see, from the electric charge's eigenstates form, that all the scalar operators contribute both to CC and NC transitions.

In the end we mention for completeness two operators that can't be generated in an EFT through a three level matching: the *tensor* operators

$$\begin{aligned} \mathcal{O}_T^u &= q_L^\dagger \sigma^{\mu\nu} u_R \; \varepsilon l_L^\dagger \sigma_{\mu\nu} e_R \\ \mathcal{O}_T^d &= q_L^\dagger \sigma^{\mu\nu} u_R \; e_R^\dagger \bar{\sigma}_{\mu\nu} l_L \end{aligned} \tag{3.4}$$

where $\sigma^{\mu\nu} \equiv \sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu$ and $\bar{\sigma}^{\mu\nu} \equiv \bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu$ nevertheless, as already mentioned, these operators can be generate through the matching to amplitudes at one or more loop level in SM. Since we will implement in the next chapter a tree level matching with eventual NP contributions at high energy we are going to neglect these last two operators.

We choose to separate the SM Lagrangian from the operators defined above, i.e. to use a Lagrangian that recreate the SM predictions in the limit $C_S = C_T = C_{LR1} = C_{LR2}^{u/d} = C_R^{u/d} = C_{LQ} = C_S^u = C_S^d = C_T^u = C_T^d = 0$:

$$\begin{aligned} \mathcal{L}_{EFT} =& \mathcal{L}_{SM} + \Bigg[C_S \mathcal{O}_S + C_T \mathcal{O}_T + + C_{LQ} \mathcal{O}_{LQ} + C_{LR1} \mathcal{O}_{LR1} + \\ & \sum_{q=u,d} \left[C_R^q \mathcal{O}_R^q + C_{LR2}^q \mathcal{O}_{LR2}^q + C_S^q \mathcal{O}_S^q + C_T^q \mathcal{O}_S^q \right] \Bigg] \end{aligned} \tag{3.5}$$

in which, expilciting flavour indexes $C\mathcal{O} = C_{abcd}\mathcal{O}^{abcd}$ and hence we define c_i as

$$C_i^{q_1 q_2 l_1 l_2} \equiv c_i \Lambda_i^{q_1 q_2 l_1 l_2}$$

that seem to be redundant but will be useful to describe separately the flavour structure separately from the rest. In our convention q_1 and l_1 are the ones appearing in the operator *daggered.*

From the experiments we acknowledge what are the observables of interest to explore the clues of NP. Plus they suggest us that NP, at leading level, doesn't concern the lightest families of quark and leptons and couples preferly to the heaviest.

3.2 Fierz identities

When we generate operators in an EFT through a tree level matching they could be different from the ones listed above, nevertheless they will always be equal to a linear combination of them.

To project the operators we generate in another basis we need to use some relations that derive from $SU(N)$ structure known as *Fierz Identities.*

Is possible to prove that, since any hermitian matrix NxN can be written as

$$H = c_0 1 \; + \; \sum_{i=1}^{N^2-1} c_i T_i$$

where T_i are the generators of the fundamental representation of $SU(N)$, these generators satisfy the completeness condition:

$$\sum_{a=1}^{N^2-1} T^a_{ij} T_{a\ kl} = \frac{1}{2}\big(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\big).$$

In the case $N = 2$ since $\sigma_a = 2T_a$ we find (neglecting the sum on a):

$$\sigma_{a\ ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl} \tag{3.6}$$

this relation can help us to write the EW structure to have all the currents contracted in the operators listed in 3.1, i.e. to have every single current trasforming with an irreducible representation of $SU(2)_L$.
Comparing two terms with the indexes sorted at the same way we can rewrite the relation 3.6:

$$\sigma^a_{ij}\sigma_{a\ kl} = \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\ kj}). \tag{3.7}$$

For what concern the Lorentz group, since the group is $SU(2)xSU(2)^*$, the situation is analogous.
Having written fermions as Weyl spinors we have explicited the relation beetween the Lorentz structure of the currents and Pauli matrices:

$$\sigma^\mu = (\mathbf{I}, \vec{\sigma}) \qquad \overline{\sigma}^\mu = (\mathbf{I}, -\vec{\sigma})$$

These matrices allow us to select the Lorentz representation made up a triplet and a singlet respectively of $SU(2)$ and $SU(2)^*$ or viceversa.
Using them we can write a current which transforms as a vector under Lorentz group, combining two spinors transforming both as the $(\frac{1}{2}, 0)$ or the $(0, \frac{1}{2})$, and so the current with explicit lorentz index is written with the *dotted notation*[1]

$$J^\mu_R = (\psi^\dagger_R)^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\chi^{\dot{\alpha}}_R \sim (0,1) \quad , J^\mu_L = (\psi^\dagger_L)^{\dot{\alpha}}(\overline{\sigma}^\mu)_{\dot{\alpha}\alpha}\chi^\alpha_L \sim (1,0)$$

[1]The dotted indexes indicate $SU(2)^*$, i.e. the RH fermions.

In which the dots distinguish the two fundamental representation of the Lorentz group. The scalar current instead is given by the product of two spinors trasforming one as $(\frac{1}{2},0)$ and the other as $(0,\frac{1}{2})$:

$$S = (\psi_R^\dagger)^\alpha \chi_{L\alpha} \ + \ (\chi_L^\dagger)^{\dot\alpha} \psi_{R\dot\alpha} \sim (0,0)$$

that for $\chi = \psi$ correspond to the fermions'mass term, indicated in Dirac representation as $S = \overline{\psi}\psi$ (see appendix A).

It is useful to introduce the charge coniugation operator:

$$C(\psi_{R(L)_\alpha}) = (\psi_{R(L)})^c_\alpha \equiv \varepsilon_{\alpha\beta}(\psi^*_{R(L)})^\beta \sim (\frac{1}{2},0)\Big(\sim (0,\frac{1}{2})\Big)$$

that links the right and the left representations.

The ε is the completely antisymmetric tensor in two dimensions (fixed to the value $\varepsilon^{12} = -\varepsilon^{21} = -\varepsilon_{12} = \varepsilon_{21} = 1$) that guarantees the antisymmetry typical of the singlet. This allow us to build a scalar with two spinors belonging to the same representation of the Lorentz group, eventually even two copies of the same spinor that would introduce a mass term built fermions that belong to only one representation of lorentz group. The term just described $(\psi^c_{L(R)})^\dagger \psi_{L(R)} \sim (0,0)$ is known as *Majorana mass term* and it can't appear in SM lagrangian because there aren't fermions belonging to real (or pseudoreal) representation of $\mathcal{G}_{SM}$, in other words $(\psi^c_{L(R)})^\dagger \psi_{L(R)}$ can never be a singlet of $\mathcal{G}_{SM}$ if $\psi_{L(R)}$ is a SM fermion (except for the case in which we include ν_R described in 1.1.1 in the spectrum).

Now that we described the structure of the vector and the scalar currents we can take back 3.6 and, mindful that $\sigma^0_{\alpha\dot\alpha} = \overline{\sigma}^0_{\alpha\dot\alpha} = \delta_{\alpha\dot\alpha}$, and including the minus sign to space indexes due to Minkowski's metric we write:

$$\sigma_{\mu\ \alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = \delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} - \sigma_{i\ \alpha\dot\alpha}\sigma^i_{\beta\dot\beta} = 2(\delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} - \delta_{\alpha\dot\beta}\delta_{\beta\dot\alpha}) \tag{3.8}$$

now using that $\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}$ we find:

$$\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} = -\sigma^\mu_{\alpha\dot\beta}\sigma_{\mu\beta\dot\alpha} \tag{3.9}$$

Now, generalizing at the overlined matrices:

$$\overline{\sigma}^{\dot\alpha\alpha}_\mu \overline{\sigma}^{\mu\ \dot\beta\beta} = 2\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta} \tag{3.10}$$

$$\sigma^\mu_{\alpha\dot\alpha}\overline{\sigma_\mu}^{\dot\beta\beta} = 2\delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha}. \tag{3.11}$$

Despite to the similar aspect of the equations it is clear that the relations for $SU(2)_L$ and for $SU(2)xSU(2)^*$ has to be used indipendently.

In some cases we have to use both to write the operators generated in the basis presented in section 3.1. In these cases the tensor structire could be complicated and eventually confusing.
So we are going to derive some useful results now to have them ready when we will handle the physics.

3.2.1 *Fierzing in the scalar lagrangian*

When we introduce to the theory scalar Leptoquarks, integrating them out from the lagrangian we could generate four-fermion operators of this shape:

$$\overline{l^{1c}}\varepsilon q^1\ \overline{q^2}\varepsilon l^{2c} = l^1_{\alpha a} q^1_{\beta b} q^{2*}_{\gamma c} l^{2*}_{\delta d}\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

where all the fields are LH fermions, i, j, k, l are EW indexes and $\alpha, \beta, \gamma, \delta$ are Lorentz indexes and the ε of the Lorentz's group comes from the definition of charge's coniugation operator.

Since all the fermions involved are LH we have to write them as a linear comnbination of O_S and O_T. To do that we need to write

$$\varepsilon^{ab}\varepsilon^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = c_1\ \delta^{ad}\delta^{bc}\overline{\sigma}_\mu^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}\ +\ c_2\ \sigma^{ad}\sigma^{bc}\overline{\sigma}_\mu^{\alpha\delta}\overline{\sigma}^{\gamma\beta\mu}$$

basically switching the position of $q^1 \longleftrightarrow l^2$.
Now we use that

$$\varepsilon^{ab}\varepsilon^{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} = \frac{1}{2}\sigma^{ad}_a\sigma^{bc^a} - \frac{1}{2}\delta^{ad}\delta^{bc}$$

Where we used 3.6 in the second equality.
Then, acting on the 3.10,

$$\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta} = \frac{1}{2}\overline{\sigma}_\mu^{\alpha\gamma}\overline{\sigma}^{\mu\ \beta\delta} = -\frac{1}{2}\overline{\sigma}_\mu^{\delta\alpha}\overline{\sigma}^{\mu\ \gamma\beta}$$

And multiplicating is straightforward to obtain:

$$\overline{l^{1c}}\varepsilon q^1\ \overline{q^2}\varepsilon l^{2c} = \frac{1}{4}\left[q^{2\dagger}\overline{\sigma}^\mu q^1\ l^{2\dagger}\overline{\sigma}_\mu l^1 -\ q^{2\dagger}\overline{\sigma}^\mu\sigma^a q^1\ l^{2\dagger}\overline{\sigma}_\mu\sigma_a l^1\right] \tag{3.12}$$

Once obtained that result is easy to derive the same in the case of the contraction of two triplet scalar currents:

$$l^{1c^\dagger}\varepsilon\sigma_a q^1\ q^{2^\dagger}\sigma^a\varepsilon l^{2c} = l^1_{\alpha a} q^1_{\beta b} q^{2*}_{\gamma c} l^{2*}_{\delta d}(\sigma^a\varepsilon)^{ab}(\varepsilon\sigma_a)^{cd}\varepsilon^{\alpha\beta}\varepsilon^{\gamma\delta}$$

We already know how to treat the lorentz structure that will bring a factor $-\frac{1}{2}\overline{\sigma}_\mu^{\alpha\delta}\overline{\sigma}^{\mu\ \beta\gamma}$.
The EW structure instead is different:

$$\begin{aligned}\varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cea} &= (\delta^{af}\delta^{ed} - \delta^{ad}\delta^{fe})(2\delta^{e}\delta^{kj} - \delta^{mj}\delta^{kn}) = \\ 2\delta^{ad}\delta^{bc} - 4\delta^{ad}\delta^{bc} &+ \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd} = -(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd})\end{aligned}$$

then recalling the 3.6

$$\begin{aligned}\sigma^a_{da}\sigma_{cba} &= 2\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc} \rightarrow \\ \varepsilon^{ae}\varepsilon^{fd}\sigma^{eb}_a\sigma^{cf^a} &= -\frac{1}{2}(\sigma^{ad}_a\sigma^{bc^a} + 3\delta^{da}\delta^{cb})\end{aligned}$$

multiplying the factors coming from EW and Lorentz group:

$$\overline{l^{1c}}\varepsilon\sigma_a q^1\ \overline{q^2}\sigma^a\varepsilon l^{2c} = \frac{1}{4}\left[3q^{2\dagger}\overline{\sigma}^\mu q^1\ l^{2\dagger}\overline{\sigma}_\mu l^1 +\ q^{2\dagger}\overline{\sigma}^\mu\sigma^a q^1\ l^{2\dagger}\overline{\sigma}_\mu\sigma_a l^1\right] \tag{3.13}$$

that again is a combination of terms that reminds to $\mathcal{O}_S$ and $\mathcal{O}_T$.

In the next section we will use these relations and some more to project the operators we generate on the basis to which we refer to catalogue and compare the contributions arising in different NP scenarios.
The purpose of doing that is that, once we find the numerical values of the Wilson coefficients that parametrize NP effects referring to the basis chosen in a model independent EFT, if we had these contributions referred to the same form of operators it is easy to combine them either to enhance the accomodation of the anomalies or to cancel constrained contributions.

4

Some of the possible heavy bosons

As we previously said one possible way to modify the Wilson coefficients in an EFT (or analogously to get non vacuum values for NP Wilson coefficients) is to introduce heavy particles to the UV theory which couple with the fermions involved in the processes we want to accomodate.

When we introduce new particle interacting with the SM particles we need to look carefully at the experimental constraints. In particular the particle has to satisfy two bounds:

- The contribution to the observable given by the diagrams in which the new particles appears as virtual particles has to show agreement with the experiments, both the ones who seem anomalous to the SM and the ones that has tested the SM.
- If the new particles interact with the particles that are smashing at the colliders, the mass range allowed for these particles has to not belong to the energy range explored at the colliders so far.

To begin we will see how we can accomodate the B-Physics Anomalies introducing different type of heavy vector and scalar bosons.
We will begin with the most familiar case to the ones who know the SM: colour-less vector charged under $SU(2)_L$. Indicating the quantum number as $(SU(3)_c, SU(2)_L)_Y$ we can address to these particles $B' \sim (\mathbf{1}, \mathbf{1})_0$ and $W' \sim (\mathbf{1}, \mathbf{3})_0$, where the names already suggest the connection with their SM's lighter sisters W,B[1].

Then we will describe the vector Leptoquarks $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$.

In the end we will descibe the behavior of some scalar Leptoquarks $S_1 \sim (\mathbf{3}^\dagger, \mathbf{1})_{1/3}$, $S_3 \sim (\mathbf{3}^\dagger, \mathbf{3})_{1/3}$ and $R_2 \sim (\mathbf{3}, \mathbf{2})_{7/6}$.

Once we have aknowledged what contributions are taken from the different bosons we will be ready to see what mediator, or what combination of mediators, are proper to accomodate the B-Physics anomalies without contradicting the Flavour tests done so far.

[1]That phenomenological are better known as $W^\pm$, Z and the photon γ because of the so known EWSB.

4.1 Colour-less bosons

4.1.1 $B' \sim (\mathbf{1}, \mathbf{1})_0$

The first candidate is the heaviest version of the EW singlet of the SM B: $B' \sim (\mathbf{1}, \mathbf{1})_0$. Apart from the bigger mass the main difference beetween these two bosons is the flavour structure, which is diagonal in SM and is free a priori instead for the BSM version.
Once introduced this boson the most general UV Lagrangian contains:

$$\begin{aligned}\mathscr{L}_{UV} \supset \frac{M_{B'}^2}{2} B_\mu B^\mu - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + g_{B'} l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L B_\mu + g_{B'} q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L B_\mu + \\ g_{B'} e_R^\dagger \sigma^\mu \lambda_B^e e_R B_\mu + g_{B'} u_R^\dagger \sigma^\mu \lambda_B^u u_R B_\mu + g_{B'} d_R^\dagger \sigma^\mu \lambda_B^d d_R B_\mu\end{aligned} \tag{4.1}$$

where the λ matrices are flavour matrices that are real since B' belongs to a real representation of the gauge group.

When we go down to energies $\ll M_{B'}$ B' can't be produced on shell anymore, and so it can't appear in a process but as virtual particle. In this condition is possible to *integrate it out* from the lagrangian, i.e. replacing B'^μ with the solution of the equation of motion:

$$\frac{\delta \mathscr{L}}{\delta B_\mu} = 0 = \frac{M_B^2}{2} B^\mu + g_{B'} l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L + g_{B'} q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L$$

$$g_{B'} e_R^\dagger \sigma^\mu \lambda_B^e e_R + g_{B'} u_R^\dagger \sigma^\mu \lambda_B^u u_R + g_{B'} d_R^\dagger \sigma^\mu \lambda_B^d d_R \rightarrow$$

$$B^\mu = -\frac{2 g_{B'}}{M_{B'}^2} \left[l_L^\dagger \bar{\sigma}^\mu \lambda_B^l l_L + q_L^\dagger \bar{\sigma}^\mu \lambda_B^q q_L + e_R^\dagger \sigma^\mu \lambda_B^e e_R B_\mu + u_R^\dagger \sigma^\mu \lambda_B^u u_R B_\mu + d_R^\dagger \sigma^\mu \lambda_B^d d_R B_\mu \right].$$

obtaining an effective lagrangian:

$$\mathscr{L}_{EFT} \supset -2 G_{B'} \left[\Big(\sum_\psi \psi^\dagger \lambda_B^\psi \sigma_\mu \psi \Big) \Big(\sum_\chi \chi^\dagger \lambda_B^\chi \sigma_\mu \chi \Big) \right]$$

where $G_{B'} \equiv \frac{g_{B'}^2}{M_{B'}^2}$ and the sums on ψ and χ are done over every SM fermion.
Among the 25 four-fermion operators, made of all the possible combination of these five neutral currents we recognize some semileptonic operators, precisely all the ones constructed with two vectorcurrents $SU(2)_L$-singlet inside the list shown in section 3.1. The coefficients of these operators are equal to:

$$C_S = -2 G_{B'} \lambda_B^q \lambda_B^l \quad , C_R^{u/d} = -2 G_{B'} \lambda_B^e \lambda_B^{u/d}$$

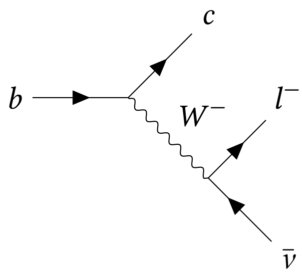
$$C_{LR1} = -2 G_{B'} \lambda_B^q \lambda_B^e \quad , C_{LR2}^{u/d} = -2 G_{B'} \lambda_B^{u/d} \lambda_B^l$$

Plus integrating out B' we generate lots of four-quarks and four-leptons operators that, according to the parameters, could generate uncorfortable contributes to observables tested by EW Precision Tests (EWPT) or to the meson mixing. In particular, since to accomodate the anomaly of the $b \rightarrow s l^- l^+$ transition i need a non vacuum $(\lambda_B^{q/u/d})^{bs} =$

$(\lambda_B^{q/u/d})^{sb}$ (where the / in this case means that has to be non vacuum at least in one of those three flavour matrices), we could produce an unpleasant tree-level contribute to $B_s \longleftrightarrow \overline{B_s}$ mixing which is one-loop induced in the SM.
Also we have to point out that we haven't generate any operator which contributes to CC transitions, i.e. B' can't be used to accomodate the anomaly of the $b \to cl\nu$ decays.

4.1.2 $W' \sim (\mathbf{1}, \mathbf{3})_0$

Since B' can't accomodate CC anomalies, we expect that to be done by an $SU(2)_L$ triplet as happens in SM, where $b \to cl\bar{\nu}$ is mediated by $W^{\pm}$ bosons which are the non $SU(2)_L$ diagonal components of the gauge boson W_a. The amplitude for the fondamental process in the SM takes the leading contribute from



which takes a flavour suppression from the quark vertex equal to V_{cb} and is diagonal and flavour universal (in one word the flavour matrix is the $\mathbf{1}_{3x3}$) for what concern the lepton current.
Since the anomaly consists in a preference for the decay channel with the τ as charged lepton, the most easy try to accomodate it is another $SU(2)_L$ triplet (much heavier than the $W^{\pm}$ to avoid collider constraints) with a non universal flavour structure in the lepton sector, that gets involved just when b quark belongs to the process.

Adding to SM $W' \sim (\mathbf{1}, \mathbf{3})_0$ the most general additional terms to high energy Lagrangian are:

$$\mathscr{L}_{UV} \supset \frac{M_{W'}^2}{2} W_\mu'^a W_a'^\mu - \frac{1}{4} W_{\mu\nu}'^a W_a'^{\mu\nu} + g_{W'} l_L^\dagger \bar{\sigma}^\mu \lambda_W^l \sigma_a l_L W_\mu^a + g_{W'} q_L^\dagger \bar{\sigma}^\mu \lambda_W^q \sigma_a q_L W_\mu^a \tag{4.2}$$

To see how this new terms contribute to processes at hadronic scale we need again to integrate out W' to see what effective semileptonic operators are generated in the EFT.

$$\frac{\delta \mathscr{L}}{\delta W_\mu^a} = 0 = \frac{M_W^2}{2} W_a^\mu + g_{W'} l_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^l l_L + g_{W'} q_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^q q_L \to$$

$$W_a^\mu = -\frac{2 g_{W'}}{M_{W'}^2} \left[l_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^l l_L + q_L^\dagger \bar{\sigma}^\mu \sigma_a \lambda_W^q q_L \right].$$

Then inserting it in the lagrangian we obtain:

$$\mathscr{L}_{EFT} \supset -2G_{W'} \left[l_L^\dagger \lambda_W^l \bar{\sigma}^\mu \sigma_a l_L \, l_L^\dagger \lambda_W^l \bar{\sigma}_\mu \sigma^a l_L + q_L^\dagger \lambda_W^q \bar{\sigma}^\mu \sigma_a q_L \, q_L^\dagger \lambda_W^q \bar{\sigma}_\mu \sigma^a q_L + 2 q_L^\dagger \lambda_W^q \bar{\sigma}^\mu \sigma_a q_L \, l_L^\dagger \lambda_W^l \bar{\sigma}_\mu \sigma^a l_L \right] \tag{4.3}$$

in which we recognize the triplet operator $\mathcal{O}_T$ in the last term implying

$$C_T = -4G_{W'}\lambda_W^l\lambda_W^q$$

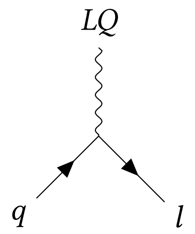
with $G_{W'} \equiv \frac{g_{W'}^2}{M_{W'}^2}$. Notice that $C_T \neq 0$ could accomodate CC anomalies.

Again we generate in addition four-leptons and four-quark operators that are unpleasant because of the tension they create beetween the anomalies and EWPT or meson mixing.

In conclusion colour-less vectors could together, a priori, accomodate both CC and NC anomalies but we can already see, without numerical analysis, that they generate a lot of unpleasant terms that can't be tuned easily once we included precision observables. We noticed that so far we didn't need to use Fierz identities, this is due to the choice we did to write operators as contractions of $SU(3)_c$-singlet currents that can couple directly to $SU(3)_c$-singlet bosons.

4.2 Leptoquarks

A possible way to avoid the tension beetween the anomalies and the precisely tested sector of the SM is to introduce bosons with quantum numbers different from the Higgs and gauge's bosons.
Since our purpose is to generate semileptonic operators, a good guess is to introduce bosons that connect quark and lepton sector at classical level, allowing the vertex



Bosons like that are known as *Leptoquarks* and there could be different shape of them differing from the mass and the representation under the which their fields trasform under Lorentz group and SM gauge group.

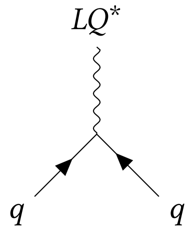
The only features common to all the LQs is that, to link a colour triplet (or antitriplet) and a colour singlet, they have to trasform under the fundamental (or antifundamental) representation of $SU(3)_c$, hence they are *coloured*.

Being the LQs coloured they have to be confined and that protects the tests on Barion Number conservation at a phenomenological level but that's true in the limit in which the only vertex allowed is a quark-lepton vertex.
Plus, in that limit, we would generate at tree level just semileptonic operators.

Nevertheless, because of $SU(3)$ structure we have that:

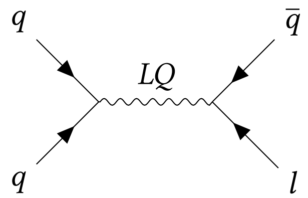
$$\mathbf{3}^* \otimes \mathbf{3}^* \sim \mathbf{6}^* \oplus \mathbf{3}$$

and hence, if the lorentz and $SU(2)_L x U(1)_Y$ symmetries allow, is possible to have



vertex as well.

The presence of this vertex would generate the already mentioned meson mixing because of four-quarks tree level contributions.
Also with both of the vertexes above allowed is possible to generate at tree level the amplitude



where the lepton is conjugated because of the $U(1)_Y$ symmetry and that would violate the Barion Number at every level.
The easiest solution that can be adopted in a *bottom-up* approach[2] would be just to switch off the quark-quark vertex.
Instead if we would implement a properly said UV completion, kill that contribute has to be in some way justified.

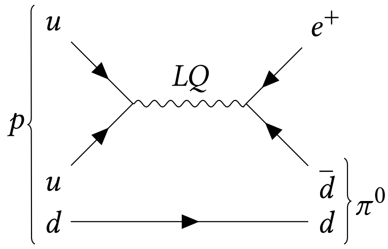
In this work we won't aim to formulate a complete theory that justify the proton stability. On the other hand since this has always been one of the reasons that have made scentists scared of Leptoquarks is worthy to discuss briefly this possible issue.

4.2.1 *Proton decay*

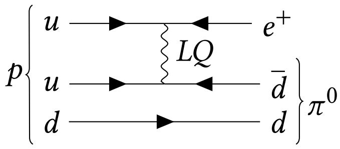
Before B-Physics Anomalies came up there where different attempt to go BSM making LQs arise. This is mostly because LQs arise naturally as gauge bosons of some GUT groups.
The most common reason to not believe in LQs (and as consequence in GUT as well) has always been that some of them could mediate proton decay $p \to e^+\pi^0$

The diagrams corresponding to the leading order amplitudes would be:

[2] An approach in which we worry only about the values of the parameters that explain the phenomenology without wondering about the theoretical nature of the NP.



or even in t-channel

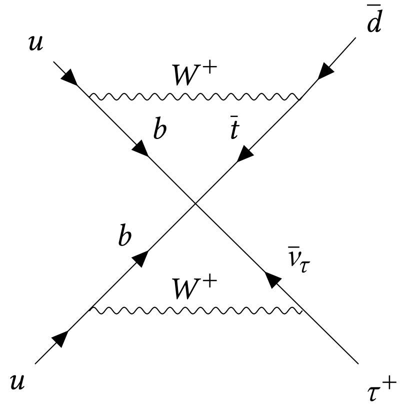


Now the available experimental limit for the proton lifetime is $\tau_p > 10^{33} ys$ and so if LQs arise the contribute sketched above should be very suppressed. This happens in two cases:

- $M_{LQ} \simeq \Lambda_{GUT} \simeq 10^{16}$ GeV to have a decoupling at low energies as shown in C,
- some symmetry among the assumptions of the theory set automatically ro zero the quark-quark vertex. For instance if among the assumptions we can ask explictly to conserv Barion Number or Lepton Number. That happens for some LQs as consequence of the gauge symmetry of the SM.

In the framework we have chosen we will not concern about the fundamental reason to suppress the quark-quark but we will point out when that arises automatically from the request to respect $\mathscr{G}_{SM}$ gauge symmetry.

Nevertheless, the flavour structure we have described so far helps the purpose forbidding the proton decay in π^0 e^+. Even if we consider the favourite flavour of the NP we are going to describe, i.e. the decay $p \to \tau^+ \pi^0 \to \bar{\nu}_{tau} + hadrons$ the four fermion vertex would be suppressed the with CKM matrix elements and loop factors coming from the insertion of two W loops deed to switch the flavour of quarks involved to the third family.



In which we get an additional suppression from the propagator of the τ^+ off-shell. In any case we can't state that our flavour structure is always enough to pass the experimental test, but for sure it points in the right direction.

4.3 Vector Leptoquarks

First we are going to analyze Leptoquark vectors known as $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$ and $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$. We will see that they are a pleasant options because, among the rest, since they couple to vector currents the value of the hypercharge $Y = 2/3$ doesn't allow them to couple with a quark-quark current, hence then they can never mediate proton decay.

4.3.1 $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$

The first LQ we take in consideration is the $SU(2)_L$ singlet U_1 which is one of the most popular in the available literature for reasons that will be clear at the end.

Including U_1 in our theory we get an UV lagrangian that reads:

$$\mathscr{L}_{UV} \supset -\frac{1}{2}U_{1\mu\nu}^{\dagger}U_1^{\mu\nu} + M_{U_1}^2 U_{1\mu}^{\dagger}U_1^{\mu} + g_{U_1}\left[U_{1\mu}\left(q_L^{\dagger}\beta_{U_1}^L\bar{\sigma}^{\mu}l_L + d_R^{\dagger}\beta_{U_1}^R\sigma^{\mu}e_R\right) + h.c.\right] \tag{4.4}$$

One difference that can be seen immediately is that belonging to the fundamental representation of $SU(3)_c$, which is complex, the current coupled to it is complex too and so, in general, the flavour matrices β are complex in general.

Treating the physics at the hadronic scale we can integrate out U_1 from the Lagrangian through the equation of motion:

$$\begin{aligned} \frac{\delta\mathscr{L}}{\delta U_{1\mu}^{\dagger}} =&0 = M_{U_1}^2 U_1^{\mu} + g_{U_1}\left[l_L^{\dagger}\beta_{U_1}^{\dagger L}\bar{\sigma}^{\mu}q_L + e_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}d_R\right] \rightarrow \\ U_1^{\mu} =& -\frac{g_{U_1}}{M_{U_1}^2}\left[l_L^{\dagger}\beta_{U_1}^{\dagger L}\bar{\sigma}^{\mu}q_L + e_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}d_R\right] \end{aligned} \tag{4.5}$$

Obtaining

$$\mathscr{L}_{EFT} \supset -G_{U_1}\left[l_L^{\dagger}\beta_{U_1}^L\bar{\sigma}^{\mu}q_L\ q_L^{\dagger}\beta_{U_1}^{\dagger L}\bar{\sigma}_{\mu}l_L\ +\ 2l_L^{\dagger}\beta_{U_1}^L\bar{\sigma}^{\mu}q_L\ d_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma_{\mu}e_R\ +\ e_R^{\dagger}\beta_{U_1}^R\sigma^{\mu}d_R\ d_R^{\dagger}\beta_{U_1}^{\dagger R}\sigma^{\mu}e_R\right] \tag{4.6}$$

where, $G_{U_1} \equiv \frac{g_{U_1}^2}{M_{U_1}^2}$.

The effective lagrangian is made of a sum of semileptonic operators, now we need to project them on the basis listed in section 3.1.

The second term is proportional to $\mathcal{O}_S^d$ because of the relation $\bar{\sigma}_{\dot{\alpha}\alpha}^{\mu}\sigma_{\mu}^{\beta\dot{\beta}} = 2\delta_{\alpha}^{\beta}\delta_{\dot{\alpha}}^{\dot{\beta}}$ it's precisely equal to $2\mathcal{O}_S^d$.

The lorentz structure of the first and the third get us a minus sign because of the relations $\sigma_{\alpha\dot{\alpha}}^{\mu}\sigma_{\mu\ \beta\dot{\beta}} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta} = -\sigma_{\alpha\dot{\beta}}^{\mu}\sigma_{\mu\beta\dot{\alpha}}$ and $\bar{\sigma}_{\dot{\alpha}\alpha}^{\mu}\bar{\sigma}_{\mu\ \dot{\beta}\beta} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta} = -\bar{\sigma}_{\alpha\dot{\beta}}^{\mu}\bar{\sigma}_{\mu\beta\dot{\alpha}}$.

Using these relations we see that the third term is equal to $-\mathcal{O}_R$ while the first term seems equal to $-\mathcal{O}_S$ apart from the fact that the quark and the lepton current aren't

$SU(2)_L$ singlets. In fact both quarks contract the $SU(2)_L$ index with a lepton, to project that operator on our basis we need to recall 3.6

$$\sigma^i_{ad}\sigma_{cbi} = 2\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} \rightarrow \delta_{ab}\delta_{cd} = \frac{\sigma^i_{ad}\sigma_{cbi} + \delta_{ad}\delta_{cb}}{2}$$

to rewrite the first term as $-\frac{1}{2}(\mathcal{O}_S + \mathcal{O}_T)$. Adding up everything we obtain:

$$\mathcal{L}_{EFT} \supset G_{U_1}\big[\frac{1}{2}\beta^L_{U_1}\beta^{\dagger L}_{U_1}(\mathcal{O}_S + \mathcal{O}_T) + \beta^R_{U_1}\beta^{\dagger R}_{U_1}\mathcal{O}^d_R - 4\beta^R_{U_1}\beta^{\dagger L}_{U_1}\mathcal{O}^d_S + \big] \tag{4.7}$$

that means in other words

$$C_S = C_T \equiv \frac{1}{2}G_{U_1}\beta^{L\dagger}_{U_1}\beta^L_{U_1} \quad C^d_R \equiv G_{U_1}\beta^{R\dagger}_{U_1}\beta^R_{U_1} \quad C^d_S \equiv -4G_{U_1}\beta^{R\dagger}_{U_1}\beta^L_{U_1}$$

The first feature we notice is that, even though we take all the possible terms in the UV lagrangian, just semileptonic contribute are generated. This means that when we accomodate the anomalies we need to be mindful of constraints that come just from semileptonic observables.

Without considering the details of the computation we can also see how one of this constraints is avoided from the fact that $C_S = C_T$. In fact $C_S = C_T \neq 0$ gives us a way to accomodate the anomaly on $b \rightarrow sl^+l^-$ where the amplitude due to NP effects is (considering just purely LH contributions) $\propto (C_S + C_T)$ as can be seen from the electric charge eigenstates form of $\mathcal{O}_S$ and $\mathcal{O}_T$ shown in section 3.1.
From that form we can see also that the transition $b \rightarrow s\nu\bar{\nu}$ is linked to $b \rightarrow sl^+l^-$ by the gauge symmetry and results to be $\propto C_S - C_T$ which means no contribution coming from the integration of U_1 as shown by [6]. This is very pleasant since $\mathcal{B}(B \rightarrow K^\dagger\nu\bar{\nu})$ is a rare decay extremely suppressed in SM and very constrained by experimental data. Nevertheless U_1 doesn't contribute to that decay at leading order even taking count of the coupling to RH fermions which can as well help to accomodate the anomalies.

Another reason to support U_1 is that it comes naturally as gauge boson in the UV completion proposed by Pati and Salam [7] that will be briefly shown later.

4.3.2 $U_3 \sim (\mathbf{3}, \mathbf{3})_{2/3}$

The vector $SU(2)_L$ triplet LQ couples just to the triplet LQ current $J^a_T = q^\dagger_L\bar{\sigma}^\mu\sigma^a\beta_{U_3}l_L$ i.e. just to LH fermions so, for what said in section 3.1, we expect to generate at low energy a combination of $\mathcal{O}_S$ and $\mathcal{O}_T$.

The UV lagrangian gains the following terms:

$$\mathcal{L}_{UV} \supset -\frac{1}{2}U^{\dagger a}_{3\mu\nu}U^{\mu\nu}_{3a} + M^2_{U_3}U^{\dagger\mu}_{3a}U^a_{3\mu} + g_{U_3}\big[U^a_{3\mu}q^\dagger_L\beta_{U_3}\bar{\sigma}^\mu\sigma_a l_L + h.c\big] \tag{4.8}$$

To build the effective lagrangian at m_b scale we set as always

$$\begin{aligned} \frac{\delta\mathscr{L}}{\delta U^a_{3\mu}} =& 0 = M^2_{U_3} U^{mu}_{3a} + g_{U_3} l^\dagger_L \beta^\dagger_{U_3} \bar{\sigma}^\mu \sigma_a q_L \rightarrow \\ U^\mu_{3a} =& -\frac{g_{U_3}}{M^2_{U_3}} l^\dagger_L \beta^\dagger_{U_3} \bar{\sigma}^\mu \sigma_a q_L \end{aligned} \tag{4.9}$$

generating

$$\mathscr{L}_{EFT} \supset -G_{U_3} \big[q^\dagger_L \beta_{U_3} \bar{\sigma}^\mu \sigma_a l_L \; l^\dagger_L \beta^\dagger_{U_3} \bar{\sigma}_\mu \sigma^a q_L \; \big] \tag{4.10}$$

Now we get a minus sign as in the U_1 case to make colour singlet currents, meanwhile the EW structure recalls 3.7

$$\begin{aligned} \sigma^a_{ij} \sigma_{a\;kl} =& \frac{1}{2}(3\delta_{il}\delta_{jk} - \sigma^a_{il}\sigma_{a\;kj}) \rightarrow \\ \mathscr{L}_{EFT} \supset & \frac{1}{2} G_{U_3} \beta_{U_3} \beta^\dagger_{U_3} [3\mathscr{O}_S - \mathscr{O}_T] \end{aligned} \tag{4.11}$$

The Wilson coefficients are equal to

$$C_S = -3C_T \equiv \frac{3}{2} G_{U_3} \beta_{U_3} \beta^\dagger_{U_3}$$

that give tree level contributions on both CC and NC processes.

To summarize the features of vector LQs we would say that they are a good possibility since they affect at tree level just semileptonic processes even if we include all possible terms allowed by SM symmetries.
Particularly U_1 is mentioned many times in the literature as the only one that can accomodate both anomalies alone passing most of the constraints, while U_3 is not that appreciated as solution of the anomalies but possibly it can be combined with other mmediators to build a relevant model.

4.4 Scalar Leptoquarks

In the end of this list we will show what are the scalar LQs that can be helpful for our purposes.

By convention the ones belonging to real represententation of $SU(2)_L$ are defined in the antifundamental of $SU(3)_c$ i.e. we have $S_1 \sim (3^\dagger, 1)_{1/3}$, $S_3 \sim (3^*, 3)_{1/3}$.
While the doublets are also two differing for the value of hypercharge: $R_2 \sim (3, 2)_{7/6}$ and $\widetilde{R}_2 \sim (3, 2)_{1/6}$.

In this case the values of the hypercharge allow S_1 and S_3 to couple a quark-quark current, this is not possible instead for the doublets because of $SU(2)_L$ symmetry[3].

[3]A doublet LQ has to couple with a left-right scalar current that has always only one conjugated field. That makes forbidden quark-quark coupling because of $SU(3)_c$ $1 \not\supset 3 \otimes 3^* \otimes 3$.

In addition, differently from vector case, the scalar current has to be made by $(\psi_{L/R})^\dagger \chi_{R/L}$ and it is possible to use the charge conjugation to flip the chirality since $(\psi_L^c) \sim (0, \frac{1}{2})$ and viceversa.
That was possible also for vector LQ but there the hypercharge wasn't the proper to a current $(\psi_{L/R})^c \sigma^\mu \chi_{R/L}$ for any $\{\psi_L, \chi_R\} \in \{q_L, l_L, u_R, d_R, e_R\}$.

So, to see what current couples to a given LQ, we have to see all the bilinears, according to $SU(2)_L$, with the right hypercharge, mindful that the charge conjugation changes its sign.

4.4.1 $S_1 \sim (3^*, 1)_{1/3}$

A scalar current which couples with a $SU(2)_L$ singlet has to involve charge's conjugation, because we can't make a $SU(2)_L$ singlet with just one LH field.
Mindful of this point we find that, this time, the $\mathcal{G}_{SM}$ symmetry allows a quark-quark coupling
Precisely the UV lagrangian that includes S_1 is:

$$\begin{aligned} \mathscr{L}_{UV} \supset (D_\mu S_1)^\dagger D^\mu S_1 + M^2 S_1^\dagger S_1 + \\ + g_{S_1} S_1^i \big[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + q_L^{j\,\dagger} \lambda_{S_1} \varepsilon q_L^{c\,k} \varepsilon_{ijk} \big] + h.c. \end{aligned} \tag{4.12}$$

where the ε with indexes implicit is the $SU(2)_L$ tensor that guarantee the current to be a EW singlet, while ε_{ijk} is the completelly antisymmetric tensor in three dimensions deed to make $SU(3)_c$ singlet.

Integrating out S_1 we need to explicit the colour index, since this time is not contracted trivially in all the terms:

$$\begin{aligned} \frac{\delta \mathscr{L}_{UV}}{\delta S_1^{i\dagger}} =0 &= M_{S_1}^2 S_{1a} + g_{S_1} \big[l_L^\dagger \varepsilon \beta_{S_1}^{L\dagger} q_{Li}^c + + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c + q_L^{\dagger c\,j} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \big] \rightarrow \\ S_{1i} &= -\frac{g_{S_1}}{M_{S_1}^2} \big[l_L^\dagger \beta_{S_1}^\dagger \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c + q_L^{cj\,\dagger} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \big] \end{aligned} \tag{4.13}$$

Inserting this term in the UV lagrangian we would obtain nine terms, for simplicity we here neglect all the BNV terms,

$$\begin{aligned} \mathscr{L}_{EFT} \supset -G_{S_1} \big[l^\dagger \beta_{S_1}^\dagger \varepsilon q_{Li}^c + e_R^\dagger \beta_{S_1}^{R\dagger} \varepsilon u_{Ri}^c + q_L^{cj\,\dagger} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \big] \big[q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + q_L^{j\,\dagger} \lambda_{S_1} \varepsilon q_L^{c\,k} \varepsilon_{ijk} \big] = \\ -G_{S_1} \big[l_L^\dagger \beta_{S_1}^{L\dagger} \varepsilon q_{Li}^c \, q_{Li}^{\dagger c} \beta_{S_1}^L \varepsilon l_L + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c \, q_{Li}^{c\dagger} \beta_{S_1}^L \varepsilon l_L + q_L^{c\dagger j} \lambda_{S_1}^\dagger \varepsilon q_L^k \varepsilon_{ijk} \, q_L^{l\,\dagger} \lambda_{S_1} \varepsilon q_L^{c\,m} \varepsilon_{ilm} + \\ l_L^\dagger \beta_{S_1}^{L\dagger} \varepsilon q_{Li}^c \, u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + e_R^\dagger \beta_{S_1}^{R\dagger} u_{Ri}^c \, u_{Ri}^{\dagger c} \beta_{S_1}^R e_R + (BNV\,terms) \big] \end{aligned} \tag{4.14}$$

Here we can see four terms describing semileptonic contributes and a four-quark term that can generate tree level amplitudes contributing to meson mixing plus four more terms kept implicit that could violate barion number conservation.

To project these terms on our operators basis we need to recall the relations obtained using Fierz Identities of EW group together with the ones of the Lorentz group, in particular we need 3.12:

$$l^{1c\dagger}\varepsilon q^1\, q^{2\dagger}\varepsilon l^{2c} = \frac{1}{4}\left[q^{2\dagger}\bar{\sigma}^\mu q^1\, l^{2\dagger}\bar{\sigma}_\mu l^1 - q^{2\dagger}\bar{\sigma}^\mu\sigma^a q^1\, l^{2\dagger}\bar{\sigma}_\mu\sigma_a l^1\right]$$

that projects the semileptonic term with LH fermions. Then to project the left-right semileptonic term we need to use that:

$$\varepsilon_{ij}\varepsilon_{kl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$$

obtaining two scalar operators that we recognize to be $\mathcal{O}_{LQ}$ and $-\mathcal{O}^u_{S^\dagger}$.
In the end, the completely RH term turns to a vector-vector operator using

$$\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} = -\frac{1}{2}\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta}$$

obtaining $-\frac{1}{2}\mathcal{O}_R$.
Adding up all the terms we obtain the following effective lagrangian:

$$\begin{aligned}\mathscr{L}_{EFT} \supset -G_{S_1}\Big[\frac{1}{4}\beta^L_{S_1}\beta^{L\dagger}_{S_1}\mathcal{O}_S - \frac{1}{4}\beta^L_{S_1}\beta^{L\dagger}_{S_1}\mathcal{O}_T + [\beta^{R\dagger}_{S_1}\beta^L_{S_1}\mathcal{O}_{LQ} - \beta^{R\dagger}_{S_1}\beta^L_{S_1}\mathcal{O}^{u\dagger}_S + h.c.] - \frac{1}{2}\beta^{R\dagger}_{S_1}\beta^R_{S_1}\mathcal{O}^u_R \\ + {q^{cj}_L}^\dagger\lambda^\dagger_{S_1}\varepsilon q^k_L\varepsilon_{ijk}\, {q_{Ll}}^\dagger\lambda_{S_1}\varepsilon q^c_{Lm}\varepsilon^{ilm} + (BNVterms)\Big]\end{aligned} \tag{4.15}$$

Here we have generated different semileptonic contributes that contribute both to CC and NC processes and a term that contributes to tree level amplitude of meson mixing, plus some BNV terms that we neglected.
There is one substantial difference with the colour-less case, in fact here we can set $\lambda_{S_1} = 0$ without affecting semileptonic contributes and turn off all four-quark and BNV terms.
The Wilson coefficients of semileptonic operators result to be

$$C_S = -C_T \equiv -\frac{G_{S_1}}{4}\beta^{L\dagger}_{S_1}\beta^L_{S_1} \quad , C_{LQ} = -C^{u\dagger}_S = G_{S_1}\beta^{R\dagger}_{S_1}\beta^L_{S1} \quad , C_R = -\frac{1}{2}G_{S_1}\beta^{R\dagger}_{S_1}\beta^R_{S_1}$$

For what we said about U_1 is easy to see that the purely LH tree level contribute to $b \to s l^+ l^-$, which is proportional to $C_S + C_T$, turns out to be null and that takes out from our toolbox a possible weapon to accomodate NC anomalies.
Nevertheless as we will see S_1 is a good candidate which appears in some UV scenarios.

4.4.2 $R_2 \sim (3,2)_{7/6}$

The scalar doublet with $Y = 7/6$ couples to a left-right current to contract the $SU(2)_L$ index.
That means that it can't generate the purely LH operators $\mathcal{O}_S, \mathcal{O}_T$ and this is the reason

why [6] neglects every LQ EW doublet.

The lagrangian terms allowed are:

$$\begin{aligned} \mathscr{L}_{UV} \supset &(D_\mu R_2)^\dagger D^\mu R_2 + M^2_{R_2} R_2^\dagger R_2 + \\ & g_{R_2} R_2 \big[u_R^\dagger \beta^l_{R_2} l_L \varepsilon + q_L^\dagger \beta^q_{R_2} e_R\big] + h.c. \end{aligned} \tag{4.16}$$

where replacing at low energies the equation of motion

$$\begin{aligned} \frac{\delta \mathscr{L}_{UV}}{\delta R_2^\dagger} &= 0 = M^2_{R_2} R_2 + g_{R_2}\big[\varepsilon l_L^\dagger \beta^{l\dagger}_{R_2} u_R + e_R^\dagger \beta^{q\dagger}_{R_2} q_L\big] \to \\ R_2 &= -\frac{g_{R_2}}{M^2_{R_2}}\big[\varepsilon l_L^\dagger \beta^{l\dagger}_{R_2} u_R + e_R^\dagger \beta^{q\dagger}_{R_2} q_L\big] \end{aligned} \tag{4.17}$$

we find

$$\mathscr{L}_{EFT} \supset -G_{R_2}\big[-u_R^\dagger \beta^l_{R_2} l_L\, l_L^\dagger \beta^{l\dagger}_{R_2} u_R + u_R^\dagger \beta^l_{R_2} l_L\, e_R^\dagger \beta^{q\dagger}_{R_2} q_L + q_L^\dagger \beta^{q\dagger}_{R_2} e_R\, l_L^\dagger \beta^{l\dagger}_{R_2} u_R + q_L^\dagger \beta^q_{R_2} e_R\, e_R^\dagger \beta^{q\dagger}_{R_2} q_L\big] \tag{4.18}$$

where we used $\varepsilon \cdot \varepsilon = -\mathbf{1}$. Apart from the flavour structure, the second and the third term are $\mathcal{O}_{LQ}$ and its hermitian conjugate, then using the (+h.c.) we can write the sum of them as $2Re[\beta^{l\dagger}_{R_2}\beta^q_{R_2}]\mathcal{O}_{LQ}$.
The first and the fourth terms can be fierzed through

$$\delta^b_a \delta^d_c = \frac{1}{2}\sigma^\mu_{ac}\overline{\sigma}_\mu^{bc}$$

to obtain $\frac{1}{2}\mathcal{O}_{LR1}$ and $\frac{1}{2}\mathcal{O}^u_{LR2}$.
Adding up we obtain

$$\mathscr{L}_{EFT} \supset -\frac{G_{R_2}}{2}\big[-\beta^l_{R_2}\beta^{l\dagger}_{R_2}\mathcal{O}^u_{LR2} + [\beta^l_{R_2}\beta^{q\dagger}_{R_2}\mathcal{O}^u_{LQ} + h.c.] + \beta^q_{R_2}\beta^{q\dagger}_{R_2}\mathcal{O}_{LR1}\big] \tag{4.19}$$

Again we generated just semileptonic terms even including all possible terms.
The Wilson coefficients result to be

$$C^u_{LR2} = \frac{1}{2}G_{R_2}\beta^l_{R_2}\beta^{l\dagger}_{R_2} \quad , C_{LQ} = -\frac{1}{2}G_{R_2}\beta^q_{R_2}\beta^{q\dagger}_{R_2} \quad , C_{LR1} = -\frac{1}{2}G_{R_2}\beta^l_{R_2}\beta^{q\dagger}_{R_2}$$

4.4.3 $\widetilde{R}_2 \sim (3,2)_{1/6}$

We want to consider another possible value for the hypercharge of a scalar LQ EW-doublet: $\widetilde{R}_2 \sim (3,2)_{1/6}$.
In this case the Lagrangian becomes:

$$\mathscr{L}_{UV} \supset M^2_{\widetilde{R}_2}\widetilde{R}_2^\dagger \widetilde{R}_2 + (D_\mu \widetilde{R}_2)^\dagger D^\mu \widetilde{R}_2 + g_{\widetilde{R}_2}\widetilde{R}_2(d_R^\dagger \beta_{\widetilde{R}_2} l_L + h.c.) \tag{4.20}$$

Once integrated out $\widetilde{R}_2$

$$\begin{aligned} \frac{\delta \mathscr{L}_{UV}}{\delta R_2^\dagger} =& M^2_{\widetilde{R}_2}\widetilde{R}_2 + g_{\widetilde{R}_2} l_L \beta^\dagger_{\widetilde{R}_2} d_R \to \\ \widetilde{R}_2 =& -\frac{g_{\widetilde{R}_2}}{M^2_{\widetilde{R}_2}} d_R^\dagger \beta_{\widetilde{R}_2} l_L \end{aligned}$$

which gives us the effective Lagrangian:

$$\mathscr{L}_{EFT} \supset -G_{\widetilde{R}_2} l_L^\dagger \beta_{\widetilde{R}_2} d_R d_R^\dagger \beta_{\widetilde{R}_2} l_L$$

Then using again

$$\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}} = \frac{1}{2}\sigma^\mu{}_{\alpha\dot{\alpha}} \bar{\sigma}_\mu^{\dot{\beta}\beta}$$

obtaining

$$\mathscr{L}_{EFT} \supset -\frac{G_{\widetilde{R}_2}}{2}\mathcal{O}_{LR2}^d \rightarrow C_{LR2}^d \equiv -\frac{1}{2} G_{\widetilde{R}_2} \beta_{\widetilde{R}_2}^\dagger \beta_{\widetilde{R}_2}$$

4.4.4 $S_3 \sim (3^*, 3)_{1/3}$

The last heavy boson we want to compute the affection at low energy is the scalar EW triplet.
As U_3 it couples with a triplet current, hence it doesn't interact with RH fermions. The main difference with U_3 is that it allows quark-quark coupling, in fact the UV lagrangian turns out to be:

$$\begin{aligned} \mathscr{L}_{UV} \supset & (D_\mu S_{3a})^\dagger D^\mu S_3^a + M^2 S_{3a}^\dagger S_3^a + \\ & + g_{S_3} S_3^{ia} ({q_L^j}^\dagger \lambda_{S_3} \varepsilon \sigma_a {q_L^c}^k \varepsilon_{ijk} + {q_{Li}^c}^\dagger \beta_{S_3} \varepsilon \sigma_a l_L) + h.c. \end{aligned} \tag{4.21}$$

in which $i = 1, 2, 3$ is the colour index and $a = 1, 2, 3$ is the EW index. Again the annoying part could be turned off without affecting semileptonic operators, as for S_1.
Integrating it out

$$\frac{\delta \mathscr{L}_{UV}}{\delta {S_3^{ia}}^\dagger} = 0 = M_{S_3}^2 S_{3ia} + g_{S_3}\big[{q_{Lj}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lk} \varepsilon^{ijk} + l_L^\dagger \beta_{S_3}^\dagger \sigma_a \varepsilon {q_L^c}^i\big] \rightarrow$$

$$S_{3ia} = -\frac{g_{S_3}}{M_{S_3}^2}\big[{q_{Lj}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lk} \varepsilon^{ijk} + l_L^\dagger \beta_{S_3}^\dagger \sigma_a \varepsilon {q_L^c}^i\big]$$

hence the EFT lagrangian results

$$\begin{aligned} \mathscr{L}_{EFT} \supset -G_{S_3}\big[& {q_{Li}^c}^\dagger \beta_{S_3} \varepsilon \sigma_a l_L \, \bar{l}_L \beta_{S_3}^\dagger \sigma_a \varepsilon {q_L^c}^i \; + \; {q_{Li}^c}^\dagger \beta_{S_3} \varepsilon \sigma_a l_L \, {q_{Lj}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lk} \varepsilon^{ijk} + \\ & {q_L^j}^\dagger \lambda_{S_3} \varepsilon \sigma_a {q_L^c}^k \varepsilon_{ijk} \, l_L^\dagger \beta_{S_3}^\dagger \sigma_a \varepsilon {q_L^c}^i + \; {q_L^j}^\dagger \lambda_{S_3} \varepsilon \sigma_a {q_L^c}^k \varepsilon_{ijk} {q_{Ll}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lm} \varepsilon^{ilm}\big] \end{aligned}$$

The first term is the only one affecting semileptonic processes at tree level. To project it on our basis we remember that we obtained 3.13

$$ {l^{1c}}^\dagger \varepsilon \sigma_a q^1 \, {q^2}^\dagger \sigma^a \varepsilon l^{2c} = \frac{1}{4}\big[3 q^{2\dagger} \bar{\sigma}^\mu q^1 \, l^{2\dagger} \bar{\sigma}_\mu l^1 \; + \; q^{2\dagger} \bar{\sigma}^\mu \sigma^a q^1 \, l^{2\dagger} \bar{\sigma}_\mu \sigma_a l^1\big]$$

which means

$$\begin{aligned} \mathscr{L}_{EFT} \supset -G_{S_3}\big[& \frac{3}{4}\beta_{S_3}^\dagger \beta_{S_3} \mathcal{O}_S + \frac{1}{4}\beta_{S_3}^\dagger \beta_{S_3} \mathcal{O}_T + \; {q_{Li}^c}^\dagger \beta_{S_3} \varepsilon \sigma_a l_L \, {q_{Lj}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lk} \varepsilon^{ijk} + \\ & {q_L^j}^\dagger \lambda_{S_3} \varepsilon \sigma_a {q_L^c}^k \varepsilon_{ijk} \, l_L^\dagger \beta_{S_3}^\dagger \varepsilon \sigma_a {q_L^c}^i + \; {q_L^j}^\dagger \lambda_{S_3} \varepsilon \sigma_a {q_L^c}^k \varepsilon_{ijk} {q_{Ll}^c}^\dagger \lambda_{S_3}^\dagger \sigma_a \varepsilon q_{Lm} \varepsilon^{ilm}\big] \end{aligned} \tag{4.22}$$

As already mentioned, meson mixing and BNV amplitudes would arise at tree level just for non vacuum values of the flavour matrix λ_{S_3} that we can turn off for our purposes. The Wilson coefficients of semileptonic terms are

$$C_S = 3C_T \equiv -\frac{3}{4} G_{S_3} \beta^\dagger_{S_3} \beta_{S_3}$$

that means, a priori, contributions to both NC and CC processes.

4.5 Summary

The last thing we need to decide what recipes are suitable to accomodate the anomalies is to decide what Wilson coefficients we need to accomodate the B-Physics anomalies and compute what values of them. Once we have done it we will be ready to check if those values pass the constraint given by the tests to Flavour Physics done so far.

Before to implement the numerical analysis we summarize what contributions are generated in the different scenarios in tables 4.1.

4.5.1 *Comparison beetween 4-quark contributions generated*

As we mentioned along the listing of heavy bosons some of them are, without furthermore assumptions on the theory, generating 4-quark operators that could affect tested phenomena. In particular, since the flavour structure must be non vacuum in third-second family transitions in quark sector, we need to be careful to the constraints on B_s meson mixing.

As we have seen, for Leptoquarks, is possible to turn to zero the quark quark coupling without affecting semileptonic contributions. On the other hand that is not true for colour-less vectors that generate 4-quark contributions of order $\simeq \lambda^{q^2}$ and semileptonic operators of order $\simeq \lambda^q \lambda^l$ where λ_l can't be big as will because that would affect 4 leptons operators that have coefficients $\simeq \lambda^{l^2}$.

If we want to use colour-less vectors to generate the desired semileptonic contributions we need to cancel the 4-quark contributions and we want to see whether that can be done by contributions generated by LQ.
To do that, we need for the contributions to cancel each other, because even if we keep contribution small at UV scale, the tree-mathing level wouldn't be worthy at B mesons scale because the loop induced contributions are in general of the same order of the tree level induced.

The LQs that are allowed to couple two quarks by $\mathcal{G}_{SM}$ are S_1 and S_3, and these couplings involve only LH quarks, hence we would like to see if que can write them in terms of the left left operators that are generated by W' and B':

$$\mathcal{O}_S^{4q} = q_L^\dagger \bar{\sigma}_\mu q_L \, q_L^\dagger \bar{\sigma}^\mu q_L, \quad \mathcal{O}_T^{4q} = q_L^\dagger \bar{\sigma}_\mu \sigma_a q_L \, q_L^\dagger \bar{\sigma}^\mu \sigma^a q_L. \tag{4.23}$$

Figure 4.1: Wilson coefficients corresponding to effective generators generated by the integration of heavy vector bosons (above) and scalar bosons (below).

	B'	W'	U_1	U_3
C_S	$-2G_{B'}\lambda_B^q\lambda_B^l$	$\varnothing$	$\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$\frac{3}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_T	$\varnothing$	$-4G_{W'}\lambda_W^q\lambda_W^l$	$\frac{1}{2}G_{U_1}\beta_{U_1}^{L\dagger}\beta_{U_1}^L$	$-\frac{1}{2}G_{U_3}\beta_{U_3}\beta_{U_3}^\dagger$
C_{LR1}	$-2G_{B'}\lambda_B^q\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^u	$-2G_{B'}\lambda_B^u\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LR2}^d	$-2G_{B'}\lambda_B^d\lambda_B^l$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^u	$-2G_{B'}\lambda_B^u\lambda_B^e$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$-2G_{B'}\lambda_B^d\lambda_B^e$	$\varnothing$	$G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^R$	$\varnothing$
C_{LQ}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^u	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$-4G_{U_1}\beta_{U_1}^{R\dagger}\beta_{U_1}^{R\dagger}$	$\varnothing$

	S_1	R_2	$\widetilde{R}_2$	S_3
C_S	$-\frac{1}{4}G_{S_1}\beta_{S_1}^L\beta_{S_1}^{L\dagger}$	$\varnothing$	$\varnothing$	$-\frac{3}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_T	$\frac{1}{4}G_{S_1}\beta_{S_1}^{L\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$-\frac{1}{4}G_{S_3}\beta_{S_3}^\dagger\beta_{S_3}$
C_{LR1}	$\varnothing$	$-G_{R_2}\beta_{R_2}^l\beta_{R_2}^{q\dagger}$	$\varnothing$	$\varnothing$
C_{LR2}^u	$\varnothing$	$\frac{1}{2}G_{R_2}\beta_{R_2}^{l\dagger}\beta_{R_2}^l$	$\varnothing$	$\varnothing$
C_{LR2}^d	$\varnothing$	$\varnothing$	$-\frac{1}{2}G_{\widetilde{R}_2}\beta_{\widetilde{R}_2}^\dagger\beta_{\widetilde{R}_2}$	$\varnothing$
C_R^u	$-\frac{1}{2}G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^R$	$\varnothing$	$\varnothing$	$\varnothing$
C_R^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
C_{LQ}	$G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$-\frac{1}{2}G_{R_2}\beta_{R_2}^{q\dagger}\beta_{R_2}^q$	$\varnothing$	$\varnothing$
C_S^u	$-G_{S_1}\beta_{S_1}^{R\dagger}\beta_{S_1}^L$	$\varnothing$	$\varnothing$	$\varnothing$
C_S^d	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

That are respectevely generated by B' with coefficent $-2G_B\lambda_B^{q\,\dagger}\lambda_B^q$ and W' with coefficient $-2G_W\lambda_W^{q\,\dagger}\lambda_W^q$, where we keep in mind that the only flavour component of $\lambda_{B/W}^q$ that needs to be non vacuum to affect anomalous obrevables is $(\lambda_{B/W}^q)^{bs}$.
Instead, four quarks operator generated from S_1 and S_3 are

$$\mathcal{O}_{S_1}^{4q} = (q^{cj}{}_L^{\dagger}\, \varepsilon q_L^k\, q_{Ll}^{\dagger}\, \varepsilon q_{Lm}^c)\varepsilon_{ijk}\varepsilon^{ilm}, \quad \mathcal{O}_{S_3}^{4q} = (q_L^{\dagger j}\, \varepsilon\sigma_a q_L^{c\,k}\, \overline{q_{Ll}^c}\sigma_a\varepsilon q_{Lm})\varepsilon_{ijk}\varepsilon^{ilm} \tag{4.24}$$

with coefficients respectively $-G_{S_1}\lambda_{S_1}^{\dagger}\lambda_{S_1}$ and $-G_{S_3}\lambda_{S_3}^{\dagger}\lambda_{S_3}$.
The first thing we notice is that in LQ generated operators the fòavou structure is non trivial while in $\mathcal{O}_S$ and $\mathcal{O}_T$ is left implicit because both currents are colour singlets.

We can use the relations derived in 3.2 to write

$$\begin{aligned}
\mathcal{O}_{S_1}^{4q} &= \frac{1}{4}(q_L^{l\,\dagger}\bar{\sigma}^\mu q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu q_{Lm}^c \;-\; q_L^{l\,\dagger}\bar{\sigma}^\mu\sigma^a q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu\sigma_a q_{Lm}^c)\varepsilon_{ijk}\varepsilon^{ilm}\\
\mathcal{O}_{S_3}^{4q} &= \frac{1}{4}(3q_L^{l\,\dagger}\bar{\sigma}^\mu q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu q_{Lm}^c \;+\; q_L^{l\,\dagger}\bar{\sigma}^\mu\sigma^a q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu\sigma_a q_{Lm}^c)\varepsilon_{ijk}\varepsilon^{ilm}
\end{aligned}$$

Now using the property of *Levi-Civita* tensor:

$$\varepsilon_{ijk}\varepsilon^{ilm} = (\delta_j^l\delta_k^m - \delta_j^m\delta_k^l) \tag{4.25}$$

results

$$\begin{aligned}
\mathcal{O}_{S_1}^{4q} &= \frac{1}{4}(q_L^{l\,\dagger}\bar{\sigma}^\mu q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu q_{Lm}^c \;-\; q_L^{l\,\dagger}\bar{\sigma}^\mu\sigma^a q_L^k\, q_{Lj}^{\dagger}\bar{\sigma}_\mu\sigma_a q_{Lm}^c)\varepsilon_{ijk}\varepsilon^{ilm}\\
\mathcal{O}_{S_3}^{4q} &= \frac{1}{4}(3\mathcal{O}_S'^{4q} + \mathcal{O}_T'^{4q} - 3\mathcal{O}_S^{4q} - \mathcal{O}_T^{4q})
\end{aligned}$$

where we have defined

$$\mathcal{O}_S'^{4q} = q_L^{i\,\dagger}\bar{\sigma}_\mu q_L^j\, q_{L\,j}^{\dagger}\bar{\sigma}^\mu q_{Li}, \quad \mathcal{O}_T'^{4q} = q_L^{i\,\dagger}\bar{\sigma}_\mu\sigma_a q_L^j\, q_{L\,j}^{\dagger}\bar{\sigma}^\mu\sigma^a q_{Li}. \tag{4.26}$$

that are *independent* from $\mathcal{O}_S$ and $\mathcal{O}_T$.
In conclusion the four-quarks contribution of LQ is independent from the colour-less one, hence they cannot cancel each other.

This leaves us with only one possibility in which we can include colourless vectors without affect B_s mixing but that happens to be impossible. In fact including both B' and W' the contribution to $b\bar{s} \to \bar{b}s$ would be proportional to $C_S^{4q^{bbss}} + C_T^{4q^{bbss}}$ that can't be cancelled because they are both negative in that flavour component because $\lambda^\dagger = \lambda \to (\lambda_{B/W}^{q\;\;bs})^2 > 0$.

5

Phenomenological Analysis

To understand what is the proper way to set the analysis of what values of NP parameters can accomodate the anomalies we need first to analyze the clues given by the data.

The main issue of computing the Branching ratios of the semileptonic processes we are interested in is that we need, apart from the EW nature of the processes, to consider that the quark involved are *confined* in hadrons.
From now we will use as example the NC anomalous decay $B \to K^* l^+ l^-$ which is the one with the biggest significant anomaly, nevertheless the discussion can be easily applied on every $M \to M' l_1 l_2$ process, where M, M' are mesons and $l_{1,2}$ are leptons.

The decay width of $B \to K^* l^+ l^-$ is given by:

$$\Gamma(B \to K^* l^+ l^-) = \frac{1}{2m_B} \int |\mathscr{M}|^2 \frac{d^3 p_{K^*}}{(2\pi)^3\ 2E_{K^*}} \frac{d^3 p_{l^+}}{(2\pi)^3\ 2E_{l^+}} \frac{d^3 p_{l^-}}{(2\pi)^3\ 2E_{l^-}}$$

Where $\mathscr{M}$ is defined to be

$$\mathscr{M} \equiv (-i) \langle B| \mathscr{O} |K^* l^+ l^- \rangle$$

in which $\mathscr{O}$ is a given operator that at three level corresponds to the sum of lagrangian's operators that can annihilate a b quark and create s quark and a pair of charged lepton anti-lepton.
If that operator can be factorized in a piece that contain only lepton fields and another one containing only quark,hence

$$\mathscr{O} = \mathscr{O}_q(\{q\}) \cdot \mathscr{O}_l(\{l\})$$

the transition matrix element results:

$$\mathscr{M} = \langle B| \mathscr{O}_q |K^* \rangle \langle 0| \mathscr{O}_l |l^+ l^- \rangle$$

i.e. the transition matrix is the product of a leptonic perturbative term which depends only from the form of the operator and an *hadronic matrix element* which at B-meson mass'scale is non calculable perturbatively.

Those factors are generally a bad deal because the non perturbativity of QCD limits the accuracy of theoretical predictions for these decay widths. Nevertheless if we want to compare processes where the hadronic contribution to the amplitude is given by operators with the same $\mathcal{O}_{had}$ is possible to simplify the non perturbative contribution expressing observables in terms of ratios of branching ratios (now we see the reason for the choice done at the begin of section 3.1 to write the operators as contraction of colour-singlet currents).

Even if the QCD is non perturbative at B mesons mass scale sometimes is possible to link different form factors without compute them explictly but just using general symmetries of strong interactions. Since QCD conserves parity it is convenient to *express from now on fermion fields in Dirac representation* through the mapping shown in A.

5.1 Left Handed framework

In the SM, because of the theorem 2, flavour breaking processes get their leading contributions from LH quarks. Since any process addressed to B-Physics Anomalies is flavour breaking, a good guess is to assume that NP should involve only LH quark as well. For what concern leptons we have that for $q^2 = (p_{l^+} + p_{l^-})^2 \in [1.1; 6.0]GeV^2$ the vector like QED contribution is suppressed by the photon propagator $\simeq \frac{1}{q^2}$ and, as previously said in section 1.3, the weak contribution to RH fermions is subbressed because of to the accidental smallness of θ_W, and so every contribute given to the RH-RH lepton current is not interfering with SM requiring a big Wilson coefficient to imply sensitive deviations, that would be bounded from many semileptonic LFU ratios like

$$\frac{\mathcal{B}(J/\psi \to \mu^+\mu^-)}{\mathcal{B}(J/\psi \to e^+e^-)}$$

and many others.

If we keep only the operators with a LH lepton current, we are left with $\mathcal{O}_S$, $\mathcal{O}_T$, $\mathcal{O}^d_{LR2}$ where O_T is the only one that contributes to CC processes.
Another clues that pushes in the direction of a purely LH NP is given by the ratio R_{K^*}/R_K.

5.1.1 R_K *and* R_{K*}

From the point of view of the EW transition there is no difference beetween the decay channel in $\to Kl^+l^-$ and $\to K^*l^+l^-$ since the quark transition is always $b \to sl^+l^-$. In the limit in which leptons take the biggest part of the available energy (in the way we can neglect kinematic effects due to the mass difference beetween μ and e), the difference beetween the two ratios is hidden in the hadronic part of the matrix element:

$$\frac{\Gamma(B \to K^*l^+l^-)}{\Gamma(B \to Kl^+l^-)} = \frac{|\langle B|\, C_{SM} + \sum_i C_i\mathcal{O}_i\, |K^*\rangle\, |^2}{|\langle B|\, C_{SM} + \sum_i C_i\mathcal{O}_i\, |K\rangle\, |^2}$$

Where $\mathcal{O}_i$ are the different possible hadronic BSM operator that allow the $b \to s$ transition that products a $l^+_L l^-_L$ pair.

Since we want the leptons pair to be LH, the hadronic part of the operator has to be a vector under rotations and the most general form of $\sum_i C_i \mathcal{O}_i$ is given by $\bar{b}\gamma^\mu(\alpha_{NP} + \beta_{NP}\gamma^5)s \equiv \alpha_{NP}\mathcal{O}_V + \beta_{NP}\mathcal{O}_A$. Written as combination of two currents, the conservation of parity gives me that:

$$\begin{aligned} \langle B| \mathcal{O}_V |K^*\rangle = 0 \quad &, \langle B| \mathcal{O}_A |K^*\rangle \equiv A(q^2) \\ \langle B| \mathcal{O}_V |K\rangle \equiv V(q^2) \quad &, \langle B| \mathcal{O}_A |K\rangle = 0 \end{aligned}$$

where $A(q^2)$ and $V(q^2)$ are unknown functions of the exchanged momentum $q^2 = (p_B - p_K)^2$.

Now if we consider the ratio beetween

$$R_{K^*} = \left.\frac{\Gamma(B \to K^*\mu^+\mu^-)}{\Gamma(B \to K^*e^+e^-)}\right|_{SM} = 1 \quad and \quad R_K = \left.\frac{\Gamma(B \to K\mu^+\mu^-)}{\Gamma(B \to Ke^+e^-)}\right|_{SM} = 1$$

that we define $\chi_K \equiv R_{K^*}/R_K$is of course equal to 1 in the SM since they are both 1.

When we include NP effects that affect just the muon channel things could change. In fact the expectation value is done on

$$\sum \mathcal{O} = C^K_{SM}[\mathcal{O}_V - \mathcal{O}_A] + \alpha_{NP}\mathcal{O}_V + \beta_{NP}\mathcal{O}_A$$

when we have used theorem 2 to write SM leading contribution as purely LH.

At this point is straightforward to notice that in a NP scenario

$$\begin{aligned} \chi_K = \frac{R_{K^*}}{R_K} &= \frac{|(-C^K_{SM} + \beta_{NP})A|^2}{|[C^K_{SM} + \alpha_{NP} + (\beta_{NP} - \beta_{NP})]V|^2} \frac{|C^K_{SM}V|^2}{|C^{K^*}_{SM}A|^2} \\ &= \left|\frac{\beta_{NP} - C^K_{SM}}{(\beta_{NP} - C^K_{SM}) + \alpha_{NP} + \beta_{NP}}\right|^2 \end{aligned} \tag{5.1}$$

where in the last line we explicited that for NP LH, i.e. $\alpha_{NP} = -\beta_{NP}$, χ_K remains equal to 1.

Otherwise if $\alpha_{NP} + \beta_{NP} \neq 0$ one of the two ratio could get bigger according to the relative sign beetween $\alpha_{NP} + \beta_{NP}$ and $C^{K^*}_{SM}$.

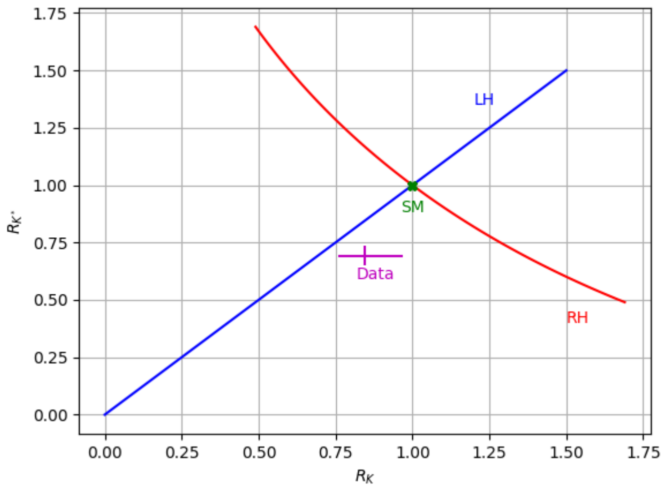
In Figure 5.1.1 we see how the data prefer a framework in which just LH quark are involved and so we assume that $\mathcal{O}^d_{LR2}$ doesn't serve our purpose.

For all these reasons we will consider the NP to involve only LH fermions, i.e. described by effective operators $\mathcal{O}_S$ and $\mathcal{O}_T$ reducing significantly the number of free parameters involved.

5.1.2 *Anomalous observables in LH framework*

The most significant B-Physics anomalies observed so far concern the ratios R_{K^*} and R_{D*}. Since any Wilson coefficient in the lagrangian 3.5 has dimension of $[Energy]^{-2}$ we choose to express them in unit of $v^{-2} = \sqrt{2}G_F = (246GeV)^{-2}$.

Figure 5.1: Plot of R_{K*} and R_K variating the vector and axial New Physics contributions α_{NP} and β_{NP} in the Left-Handed limit $\alpha_{NP} = -\beta_{NP}$ (blue) and in the Right-Handed limit $\alpha_{NP} = \beta_{NP}$ (red).



In the scenario in which NP couples only LH fermions it is easy to compute the previsions for those observables because, as should be clear now, non perturbative contributions are simplified in the ratio and we have just to add proper Wilson coefficients to the SM one.

The matching to SM tells us that the Wilson coefficient of the the operator $\bar{b}_L \gamma^\mu s_L \, \bar{l}_L \gamma_\mu l_L$ is given by

$$C^{NC}_{SM} = V_{tb} V^*_{ts} \frac{e^2}{8\pi^2} \cdot (8.314)\ v^{-2} \tag{5.2}$$

as calculated by [8]. The ratio R_{K^*}, mindful of the electric charge eigenstates form of operators 3.2, results

$$\begin{aligned} R_{K^*} &\equiv \frac{\mathcal{B}(B \to K^* \mu^+ \mu^-)}{\mathcal{B}(B \to K^* e^+ e^-)} = \frac{|C^K_{SM} + C^{bs\mu\mu}_S + C^{bs\mu\mu}_T|^2}{|C^K_{SM}|^2} \\ &= |1 + \frac{C^{bs\mu\mu}_S + C^{bs\mu\mu}_T}{C^K_{SM}}|^2 \end{aligned} \tag{5.3}$$

which depends only from the parameter $\delta C^{NC} = C^{bs\mu\mu}_S + C^{bs\mu\mu}_T$.
Since we are allowing only LH contributions, that are alligned with the SM, every other process $b \to sll$ expressed like a ratio beetween the BR of the muon channel and BR of the electronic channel can be expressed as 5.3, because of the factorization of hadronic contribution explained above.

For what concern CC the most significant anomalies appear in R_{D*}, R_D that take contribution only from triplet operator O_T. Mindful of the relative coefficients of different component of LH doublets given by gauge symmetry (see equation 3.2),

$$\begin{aligned} R_D &\equiv \frac{\mathscr{B}(B \to D^* \tau \bar{\nu})}{\frac{1}{2}\left[\mathscr{B}(B \to D^* e \bar{\nu}) + \mathscr{B}(B \to D^* \mu \bar{\nu})\right]} = \\ &= \frac{|C^{CC}_{SM} + 2C_T^{bs\tau\tau} V_{cs}^* + 2C_T^{bb\tau\tau} V_{cb}^*|^2 + |2C_T^{bs\tau\mu} V_{cs}^* + 2C_T^{bb\tau\mu} V_{cb}^*|^2 \, \eta_\tau}{\frac{1}{2}\left[|C^{CC}_{SM}|^2 + |C^{CC}_{SM} + 2C_T^{bs\mu\mu} V_{cs}^* + 2C_T^{bb\mu\mu} V_{cb}^*|^2 + |2C_T^{bs\mu\tau} V_{cs}^* + 2C_T^{bb\mu\tau} V_{cb}^*|^2\right]} \end{aligned} \tag{5.4}$$

where η_τ is defined in 2.8, and apart of that we introduced several new parameters. In this case the SM matching gives us (see section 1.3)

$$C^{CC}_{SM} = -2V_{cb}^* \, v^{-2} \tag{5.5}$$

When we are going to fit these anomalies we need to reduce the number of free parameters and we would like to do that without doing additional assumption.

A possible way to reduce them is to look at non anomalous observables that would get an important contribution from the Wilson coefficients of the NP and then use the experimental data to constraint those coefficients.

5.2 Constraining non anomalous observables

Another powerful tool that can be used to reduce the free parameters of the NP model is to use constraints derived by observables already tested that don't show deviations from the SM.

5.2.1 $B \to K^* \bar{\nu}\nu$

The $B \to K^* \bar{\nu}\nu$ decay is linked with the so mentioned $B \to K^* l^+ l^-$ by gauge symmetry. Besides phenomenologically has the huge difference to haven't been observed yet. From [5] we get an upper limit at @95% CL and its relation with the SM prevision that s given by:

$$\mathscr{B}(B \to K^* \bar{\nu}\nu)_{exp} < 5.2 \, \mathscr{B}(B \to K^* \bar{\nu}\nu)_{SM} \tag{5.6}$$

Where the If we include NP that interacts only with LH fermions apart for the first families the expression, neglecting LF breaking contributions that would enter at order C_{NP}^2

$$\mathscr{B}(B \to K^* \bar{\nu}\nu)_{NP} = \mathrm{A} \left[|C^\nu_{SM}|^2 + |C^\nu_{SM} + \delta C^{\nu_\mu}|^2 + |C^\nu_{SM} + \delta C^{\nu_\tau}|^2\right] \tag{5.7}$$

where A is a given constant that includes kinematik and sum on lorentz indexes and C^ν_{SM} which is flavour universal because include only SM interactions is equal to

$$C^\nu_{SM} = 12.8 \, V_{ts}^* V_{tb} \frac{e^2}{8\pi^2} \, v^{-2} \tag{5.8}$$

taken by [6].

The value of $\mathscr{B}(B \to K^* \bar{\nu}\nu)_{NP}$ has to not exceed the experimental limit hence for 5.6

$$
\begin{aligned}
\mathrm{A}\left[|C^{\nu}_{SM}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\mu}|^2 + |C^{\nu}_{SM} + \delta C^{\nu_\tau}|^2\right] &< 5.2\mathscr{B}(B \to K^* \bar{\nu}\nu)_{SM} = 5.2 \cdot 3\mathrm{A}|C_{SM}|^2 \\
\to \left|1 + \frac{\delta C^{\nu_\mu}}{C^{\nu}_{SM}}\right|^2 + \left|1 + \frac{\delta C^{\nu_\tau}}{C^{\nu}_{SM}}\right|^2 &< 14.6
\end{aligned}
\tag{5.9}
$$

Using again the electric eigenstates form of 3.2 the BSM terms are $\delta C^{\nu_\mu} = C_S^{bs\mu\mu} - C_T^{bs\mu\mu}$ and $\delta C^{\nu_\tau} = C_S^{bs\tau\tau} - C_T^{bs\tau\tau}$.

Here we could have different scenarios according to the relative intensity of the two contributions in μ and τ channels. But anyway we can do some general considerations. The biggest value that both the NP contributions can singularly assume is given by the case in which one of the two δC is equal to $-C_{SM}$. The other one has in any case to respect

$$
|1 + \frac{\delta C}{C_{SM}}|^2 < 14.6 \to |1 + \frac{\delta C}{C_{SM}}| < \sqrt{14.6} \simeq 3.8 \tag{5.10}
$$

and that is a limit for both coefficients which can never be exceeded. In fact it is easy to see that if $\delta C^{\nu_\mu}(\delta C^{\nu_\tau}) \neq -C_{SM}$ the upper limit on $\delta C^{\nu_\tau}(\delta C^{\nu_\mu})$ would be smaller.

That doesn't give direct constraint on Wilson coefficients, because there is a flat direction of δC given by the trasformation $(C_S, C_T) \to (C_S + x, C_T + x)$. Nevertheless if we manage to set a bound $\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu}$ from the phenomenology we can derive information on the magnitude of $C_S^{bs\mu\mu}, C_T^{bs\mu\mu}$.

5.2.2 $R_{D^*}^{\mu e}$

Here we look at a relative of R_{D^*} that compares the channel in muons with the channel in electrons.

The expression at leading order (neglecting again LF breaking contributions) is given by

$$
\begin{aligned}
R_{D^*}^{\mu e} =& \frac{\mathscr{B}(B \to D^* \mu\bar{\nu})}{\mathscr{B}(B \to D^* e\bar{\nu})} = \\
=& \frac{|C^{CC}_{SM} + \delta C^{CC\mu}|^2}{|C^{CC}_{SM}|^2} = 1 + |\frac{\delta C^{CC}}{C^{CC}_{SM}}|^2
\end{aligned}
\tag{5.11}
$$

where $\delta C^{CC\mu} = 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^*$.
We know from [5] that $R_{D^*}^{\mu e}$ is a good test for LFU beetween the first two families.

$$
R_{D^*}^{\mu e}|_{exp} = 1.000 \pm 0.021 \tag{5.12}
$$

that gives strong constraints on $\delta C^{CC\mu} = 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^*$ and, combined with bounds on $C_T^{bs\mu\mu}$ that we can derive from $\mathscr{B}(B \to K^* \bar{\nu}\nu)$ and anomalous NC observables we can set limits on $C_T^{bb\mu\mu}$.

5.3 Numerical Analysis of the Anomalies

5.3.1 *Fit to NC Anomalies*

In Table 2.1 we have listed some anomalous observables derived from NC transitions. As we have already mentioned the expression 5.3 is valid only if the energy of the two leptons is greater than 1 *GeV* so that we can neglect phase space factor and electromagnetic contributions.

Our analysis is not a test of SM, because for that we would need to include many other observables and the anomaly could probably being hidden by the other observable values.
If we use SM as starting point the question we can ask is if a NP scenario fits the observables we say to be anomalous, better than SM itself does. Hence the only degree of freedom of the fit is the free parameter $\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu}$.

Implementing a least squares fit we find that

$$\begin{aligned} &\delta C^{NC}|_{best} = -0.70\ \frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\ v^{-2} \qquad \chi^2/ndof = 2.97 \\ &(-0.82, -0.57)\ \frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\ v^{-2} \quad whitin\ \ 1\sigma \end{aligned} \tag{5.13}$$

hence the anomaly can be accomodated reducing C_{SM} in equation 5.2 of approximately 10%.

The point corresponding to SM ($\delta C^{NC} = 0$) compared to the minimum χ^2, is placed at $\sqrt{\Delta\chi^2} = \sqrt{\chi^2_{SM} - \chi^2_{min}} = 7.81$ showing that NP model can fit the considered observables way better than SM does.

Without doing any additional assumption we can take the limit coming from the decay in neutrinos:

$$\left| 1 - \frac{C_S^{bs\mu\mu} - C_T^{bs\mu\mu}}{C_{SM}^{\nu}} \right| < 3.8$$

where the value of C_{SM}^{ν} is given in 5.8,we have that

$$\begin{aligned} &-35.84\ v^{-2} < \frac{(C_S^{bs\mu\mu} - C_T^{bs\mu\mu})}{\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}} < 61.15\ v^{-2} \\ &-0.82 v^{-2} < \frac{(C_S^{bs\mu\mu} + C_T^{bs\mu\mu})}{\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}} < -0.57 v^{-2} \quad within\ 1\sigma \end{aligned} \tag{5.14}$$

We see that, having kept the flavour structure completely general, $B \to K^* \nu\nu$ doesn't give us acceptable range for the single Wilson coefficients $C_S^{bs\mu\mu}, C_T^{bs\mu\mu}$.

Anyway the only limit on $C_S^{bs\mu\mu} + C_T^{bs\mu\mu}$ would allow us to move as much as we want along the direction $(C_S^{bs\mu\mu}, C_T^{bs\mu\mu}) \to (C_S^{bs\mu\mu} + x, C_T^{bs\mu\mu} - x)$; But the range setted on $C_S^{bs\mu\mu} - C_T^{bs\mu\mu}$ limits that freedom and, in particular flavour scenarios when the bounds on δC^{ν_μ} keeping $(C_S^{bs\mu\mu}, C_T^{bs\mu\mu}) \simeq \mathcal{O}(\delta C^{NC})$.

Even without constraining $C_T^{bs\mu\mu}$ is possible to use the ratio $R_{D^*}^{\mu e}$ given by 5.11 to state from 5.12 that the whole coefficient $\delta C^{CC\mu} = 2C_T^{bs\mu\mu}V_{cs}^* + 2C_T^{bb\mu\mu}V_{cb}^* \ll C_{SM}^{CC}$, thus we remain with an expression of R_D equal to

$$
\begin{aligned}
R_D \equiv & \frac{\mathcal{B}(B \to D^* \tau\bar{\nu})}{\frac{1}{2}\left[\mathcal{B}(B \to D^* e\bar{\nu}) + \mathcal{B}(B \to D^* \mu\bar{\nu})\right]} \simeq \\
\simeq & \frac{|C_{SM}^{CC} + 2C_T^{bs\tau\tau}V_{cs}^* + 2C_T^{bb\tau\tau}V_{cb}^*|^2 \; \eta_\tau}{|C_{SM}^{CC}|^2} = \\
= & \left|1 + \frac{\delta C^{CC}}{C_{SM}^{CC}}\right|^2 \cdot \eta_\tau
\end{aligned}
\tag{5.15}
$$

where we have as well neglected LF violation contributions.
Then we have the same simple expression of the NC case with $\delta C^{CC} \equiv V_{cb}^*[2C_T^{bs\tau\tau}\frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}]$.

5.3.2 *Fit to CC Anomalies*

The expression 5.15 is not just easier to treat than 5.4, also the only free parameter $\delta C^{CC} \equiv V_{cb}^*[2C_T^{bs\tau\tau}\frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}]$ is completely independent from δC^{NC} and hence we are going to do an independent analysis to find limits on that parameter.

Since η_τ is given by the measure of $R_D^{(*)}$ we will consider the observables given by

$$
\frac{R_{D^{(*)}}}{\eta_\tau^{D^{(*)}}} = \left|1 + \frac{\delta C^{CC}}{C_{SM}^{CC}}\right|^2 \tag{5.16}
$$

that in both cases has SM prediction equal to 1 by definition.

Setting C_{SM}^{CC} at the value 5.5 we find that the least square is found for

$$
\begin{aligned}
&\delta C^{CC}|_{best} = -0.14\; V_{cb}^*\; v^{-2} \qquad \chi^2/ndof = 0.042 \\
&(-0.19, -0.08)\; V_{cb}^*\; v^{-2} \quad whitin\;\; 1\sigma
\end{aligned}
\tag{5.17}
$$

where we find again that the anomaly is order 10% of C_{SM}^{CC}. This time the SM ($\delta C^{CC} = 0$) is placed at $\sqrt{\Delta\chi^2} = \sqrt{\chi_{SM}^2 - \chi_{min}^2} = 2.31$ hence the anomaly is less significant than the NC case.

Now that we have numerical values for NP coefficients we are ready to see what type of UV physics we need to generate these contributions.

5.4 Selection of possible UV scenarios

In this section we are going to use numerical results found in the last section combined with UV matching done in chapter 4 summarized in table 4.1.

The single flavour components that enters in the anomalous observables are $\{C_S^{bs\mu\mu}, C_T^{bs\mu\mu}, C_T^{bb\tau\tau}, C_T^{bs\tau\tau}\}$ and we have to stay mindful of bounds on $C_S^{bs\tau\tau} - C_T^{bs\tau\tau}$ given by $B \to K^* \nu\bar{\nu}$.

At 1σ level the values we need are:

$$\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu} \in (-0.82, -0.57)\frac{e^2}{8\pi^2} V_{ts}^* V_{tb}\, v^{-2}$$

$$\delta C^{CC} = V_{cb}^*[2C_T^{bs\tau\tau}\frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}] \in (-0.050, -0.020) V_{cb}^*\, v^{-2}$$

$$\left|1+\frac{\delta C^{\nu_\mu}}{C_{SM}^{\nu}}\right|^2 + \left|1+\frac{\delta C^{\nu_\tau}}{C_{SM}^{\nu}}\right|^2 < 14.6$$

where the most significative deviation is the one accomodated by δC^{NC} and so could be worthy also to see how we could accomodate that one alone.

We are going to need those combinations bosons that generate $\mathcal{O}_S$ and $\mathcal{O}_T$ and vacuum sum of Wilson coefficients corresponding to other operators (that we assume constrained by observable we didn't take in count).

The first possibility we consider is to accomodate the anomalies with the insertion of the only boson U_1.
To kill the contributions to the RH operators $\mathcal{O}_R^d$ and $\mathcal{O}_S^d$ we can just set $\beta_{U_1}^R = 0$ (i.e. annull the coupling with RH current).
Thus we remain with ($\beta_{U_1}^L \equiv \beta_{U_1}$)

$$\delta C^{NC} = C_S^{bs\mu\mu} + C_T^{bs\mu\mu} = G_{U1}\beta_{U_1}^{b\mu}\beta_{U_1}^{*\ s\mu}$$

$$\delta C^{CC} = V_{cb}^*[2C_T^{bs\tau\tau}\frac{V_{cs}^*}{V_{cb}^*} + 2C_T^{bb\tau\tau}] = 2V_{cb}^* G_{U_1}[\frac{V_{cs}^*}{V_{cb}^*}\beta_{U_1}^{b\tau}\beta_{U_1}^{*\ s\tau} + \beta_{U_1}^{b\tau}\beta_{U_1}^{*\ b\tau}]$$

$$\delta C^{\nu_\mu} = \delta C^{\nu_\tau} = 0$$

hence in general we have no contributions to neutrinos decay from the integration of U_1.
Here the values of beta matrix has to be such that $\beta_{U_1}^{s\mu}\beta_{U_1}^{*\ s\mu} < 0$ and, since we have for any value of $\beta_{U_1}^{b\tau}$ the addend $\beta_{U_1}^{n\tau}\beta_{U_1}^{*\ b\tau}$ has to be positive we need a value for $\beta_{U_1}^{b\tau}\beta_{U_1}^{*\ s\tau}$ such that $\beta_{U_1}^{b\tau}[\frac{V_{cs}^*}{V_{cb}^*}\beta_{U_1}^{*\ s\tau} + \beta_{U_1}^{*\ b\tau}] < 0$.

This is an interesting scenario because, since we have only one more field, we have few free parameters and the constraints coming from neutrinos decay are automatically passed. Plus it appears in many UV completions as reported by [2, 6, 7, 9].

Another possibility that includes U_1 is to include it together with S_1 that generates $\delta C^{NC} = 0$ but $\delta C^{CC} = 2G_{S_1} * V^*_{cb}\beta^{b\tau}[\beta^{*b\tau}\frac{V^*_{cs}}{V^*_{cb}} + \beta^{*s\tau}]$ that can help the accomodation of CC anomalies within the limits given by $B \to K^*\nu\bar{\nu}$.

Introducing a scalar and a vector with the same quantum numbers under $\mathcal{G}_{SM}$ lends itself well to be implemented in an UV completion characterized by a strongly coupled sector at high energy with a global chiral symmetry $SU(N)_L \otimes SU(N)_R$ spontaneusly broken at EW scale to the vectorial subgroup $SU(N)_V$.
In this scenario there are scalar and vector bosons arising with the same quantum number, the first as Pseudo Nambu-Goldston Bosons (pNGB) and the second as composite state excited by the unbroken Noether's currents of $SU(2)_V$, in analogy with the π and ρ of the chiral symmetry of QCD. In that case the contribution of the scalar would be generally bigger because its mass would be lower because of the protection given by the unbroken group.

Following the same line we can even think to include in the UV lagrangian U_3 and S_3. That would mean generate LH contributions equal to

$$\begin{aligned}
\delta C^{NC} &= G_{U_3}\beta_{U_3}{}^{b\mu}\beta^*_{U_3}{}^{s\mu} - G_{S_3}\beta_{S_3}{}^{b\mu}\beta^*_{S_3}{}^{s\mu} \\
\delta C^{CC} &= -V^*_{cb}[G_{U_3}(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}_{U_3}\beta^*_{U_3}{}^{s\tau} + \beta^{b\tau}_{U_3}\beta^*_{U_3}{}^{b\tau}) - \frac{1}{2}G_{S_3}(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}_{S_3}\beta^*_{S_3}{}^{s\tau} + \beta^{b\tau}_{S_3}\beta^*_{S_3}{}^{b\tau})] \\
\delta C^{\nu_\mu(\nu_\tau)} &= 2G_{U_3}\beta_{U_3}{}^{b\mu(\tau)}\beta^*_{U_3}{}^{s\mu(\tau)} - \frac{1}{2}G_{S_3}\beta_{S_3}{}^{b\mu(\tau)}\beta^*_{S_3}{}^{s\mu(\tau)}
\end{aligned} \tag{5.18}$$

suppose these two bosons derive from the same aspect of the theory as ρ and π, then it is reasonable to think they also have the same flavour structure, i.e. $\beta_{U_3} = \beta_{S_3} \equiv \beta$, then 5.18 becomes

$$\begin{aligned}
\delta C^{NC} &= (G_{U_3} - G_{S_3})\beta^{b\mu}\beta^{*s\mu} \\
\delta C^{CC} &= -V^*_{cb}(G_{U_3} - \frac{1}{2}G_{S_3})(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}\beta^{*s\tau} + \beta^{b\tau}\beta^{*b\tau})] \\
\delta C^{\nu_\mu(\nu_\tau)} &= (2G_{U_3} - \frac{1}{2}G_{S_3})\beta^{b\mu(\tau)}\beta^{*s\mu(\tau)}
\end{aligned} \tag{5.19}$$

In this framework we can accomodate the NC anomalies for instance with $\beta^{b\mu}\beta^{*s\mu} > 0$ for $G_{S_3} > G_{U_3}$ (that is normally the case if S_3 is a pNGB).
For $G_{S_3} \simeq 4G_{U_3}$ the limits on $B \to K^*\nu\nu$ are satisfied independently from the flavour structure, hence there are different comnbinations of the β elements that can accomodate CC anomalies as well.
Otherwise we could have two sources of problem, first $\beta^{b\mu}\beta^{*s\mu}$ needs to be positive to acccomodate NC anomalies, second, assuming $G_{S_3} > 2G_{U_3}$, we need $(\frac{V^*_{cs}}{V^*_{ts}}\beta^{b\tau}\beta^{*s\tau} + \beta^{b\tau}\beta^{*b\tau})$ to be negative and since $\beta^{b\tau}\beta^{*b\tau} \geq 0$ $\beta^{b\tau}\beta^{*s\tau}$ has be negative and bigger in absolute value than $\beta^{b\tau}\beta^{*b\tau}$, and both these contributions would increase the prediction for $\mathcal{B}(B \to K^*\nu\nu)$.
Otherwise if $G_{U_3} < G_{S_3} < 2G_{U_3}$ the last contribution described doesn't need to be that big, but we would have a decreasement of $G_{U_3} - G_{S_3}$ requiring for a bigger $\beta^{b\mu}\beta^{*s\mu}$ which

is again unpleasant for the constraints on $\mathcal{B}(B \to K^* \nu\nu)$.

A possibility that would cure only NC anomalies is the inclusion of B' that doesn't generate the triplet operator $\mathcal{O}_T$. Considering small the RH couplings $\lambda_d, \lambda u, \lambda_e$ (as happens in SM for the Z) is possible to neglect the contributions on $\mathcal{O}_{\mathcal{R}}^{u/d}$ which would be quadratic in RH couplings. Instead contributions to $\mathcal{O}_{LR1}, \mathcal{O}_{LR1}^{u/d}$ can be cancelled by the integration of the two LQ doublets R_2 and $\widetilde{R_2}$ that can kill those three contribution independently with a proper choice of the matrixes $\beta_{R_2}^{l/q}$ and $\beta_{\widetilde{R_2}}$.
In that framework we would have

$$\begin{aligned} \delta C^{NC} &= -2G_B \lambda_B^{q\,bs} \lambda_B^{l\;\mu\mu} \\ \delta C^{\nu_\mu} &= -2G_B \lambda_B^{q\,bs} \lambda_B^{l\;\mu\mu} \\ \delta C^{\nu_\tau} &= -2G_B \lambda_B^{q\,bs} \lambda_B^{l\;\tau\tau} \end{aligned} \tag{5.20}$$

where we could need to set a low $\lambda_B^{q\,bs}$ to protect B_s meson mixing and hence have an high $\lambda_B^{l\;\mu\mu}$.
The magnitude of $\lambda_B^{l\;\mu\mu}$ is not constrained if the rest of the matrix λ_B^l is null, hence we can set $\lambda_B^{l\;\tau\tau} = 0$ to protect simultaneously $B \to K^* \nu\bar{\nu}$ and $\tau \to \mu\nu\nu$.
The contribution to meson mixing could also be compensated by the integration of W' that would generate also contributions to CC anomalies toward a complete solution of the anomalies with four unobserved particles.

5.4.1 *τ dominance scenario*

The assumptions on flavour structure done so far a completely general in the limit NP doesn't interact with first two families

Nevertheless since both the deviation are $\sim$ 10% of SM, and the ratio SM Wilson coefficients is

$$\frac{C_{SM}^{NC}}{C_{SM}^{CC}} = \frac{e^2}{\pi^2} \frac{V_{ts}^* V_{tb}}{2V_{cb}^*} \cdot 8.31 \simeq \alpha_{em} \frac{8.31}{4\pi} \simeq 0.5 \cdot 10^{-2} \tag{5.21}$$

we always expect for δC^{CC} to be approximately two orders of magnitude bigger than δC^{NC}. Since δC^{CC} concerns only τ and δC^{NC} only μ that suggests a preference of NP to couple third family suggesting a sort of flavour symmetry involving the first two families $U(2)_q \otimes U(2)_l$ minimally broken to allow NC anomalies that is the followd by [6].

Staying general we have to say that we have flat directions in both coefficient δC^{NC} $((C_S^{bs\mu\mu}, C_T^{bs\mu\mu}) \to (C_S^{bs\mu\mu}+x, C_T^{bs\mu\mu}-x))$ and δC^{CC} $(C_S^{bs\mu\mu}, C_T^{bs\mu\mu}) \to (C_T^{bs\tau\tau}+x, C_T^{bb\tau\tau}-x\cdot\frac{V_{cb}^*}{V_{cs}^*}))$ that can make the addends big at will keeping $\frac{\delta C^{NC}}{\delta C^{CC}} = \mathcal{O}(10^{-2})$.

Nevertheless a possible assumption is to state that, if $(C_S^{bs\mu\mu}, C_T^{bs\mu\mu}) \sim \mathcal{O}(\delta C^{NC})$ and $(C_T^{bs\tau\tau}, C_T^{bb\tau\tau}) \sim \mathcal{O}(\delta C^{CC})$ within an order of magnitude all the couplings to τ heve to be approximately one hundred times bigger than couplings to μ.

That would mean that also the limits provided by $B \rightarrow K^* \nu\nu$ would be more constraining. In fact if the contribution due to $\delta C^{\nu_m u}$ is neglectable compared to the one coming from δC^{ν_τ}.
Then limits on δC^{ν_τ} would be similar to the ones written in 5.10 and limits on δC^{ν_μ} approximately one hundred times smaller for the assumptions done so far, pushing strongly toward the U_1 scenario that gives automatically null contribution to $B \rightarrow K^* \nu\nu$.

5.4.2 *Few examples of UV completions with Leptoquarks*

A

Mapping beetween Weyl and Dirac pictures

In this work fermion fields have been written in two different representations: the Dirac representation and the Weyl representation. Here we show that the two of them are equivalent and the choice of one rather than the other is done only to show different aspects of the lorentz structure.

The Weyl representation consists in writing fermion fields as irreducible representations of Lorentz group either $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$, it becomes useful when the dynamic treats differently fermions transforming under different representations. We indicate them with

$$\psi_L \sim (\frac{1}{2}, 0), \quad \psi_R \sim (0, \frac{1}{2})$$

that are linked by the charge conjugation. The two fundamental representations are labelled with L, R that identify a quantum number named *chirality* which by convention is set equal to -1 for fermions in $(\frac{1}{2}, 0)$ and $+1$ for fermions in $(0, \frac{1}{2})$.

The scalar and vector currents that can be constructed with Weyl fields using σ^μ and $\overline{\sigma}^\mu$ and, in this picture, they belong as well to irreducible representations of Lorentz group:

$$\begin{aligned} \psi_L^\dagger \overline{\sigma}^\mu \psi_L &\sim (1, 0) \\ \psi_R^\dagger \sigma^\mu \psi_R &\sim (0, 1) \\ \psi_L^\dagger \psi_R + h.c. &\sim (0, 0) \end{aligned}$$

then is useful to write fermion fields when interactions of these fermion are *chiral*, i.e. different for fermions belonging to $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$.

Instead, when the interactions are *vector-like* (that means blind to chirality), can be useful to organize the two fundamental representations in a unique field:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A.1}$$

that has four components, two trasforming under $(\frac{1}{2},0)$ and the other two under $(0,\frac{1}{2})$. A field defined in this way is clearly trasforming under a reducible representation of Lorentz group (precisely $(\frac{1}{2},0)\otimes(0,\frac{1}{2})$) and it is how we represent fermions in Dirac picture.

To build currents in this picture we need to introduce a set of 4x4 matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \overline{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}$$

known as the Dirac matrices.

The product of these matrices times i gives us the *chirality matrix*

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1}_{2x2} & 0 \\ 0 & \mathbf{1}_{2x2} \end{pmatrix}$$

where the name is due to the fact that eigenvalues of this matrix correspond to the chirality defined above.

Through this matrix we get the access to the different irreducible representations using the projectors

$$P_L = \frac{1-\gamma^5}{2}, \quad P_R = \frac{1+\gamma^5}{2}$$

Using them we can define a biunivoc mapping beetween Dirac and Weyl pictures

$$\psi_L^{Weyl} \to \psi_L^{Dirac} = P_L q^{Dirac} = \begin{pmatrix} \psi_L^{Weyl} \\ 0 \end{pmatrix}$$

$$\psi_R^{Weyl} \to \psi_R^{Dirac} = P_R \psi^{Dirac} = \begin{pmatrix} 0 \\ \psi_R^{Weyl} \end{pmatrix}$$

and in the other way, using that $P_L + P_R = 1$

$$\psi^{Dirac} = (P_L + P_R)\psi^{Dirac} \equiv \psi_L^{Dirac} + \psi_R^{Dirac} \to \psi_L^{Weyl} + \psi_R^{Weyl}$$

Here we have shown that the two pictures are equivalent and always interchangeable. One can prefer one rather than the other according that the interaction treated were chiral or vector-like.

A.0.1 *Currents*

To build currents with Dirac fermion fields we need to use the γ matrices as follows.

We define $\overline{\psi} \equiv \psi^\dagger\gamma^0$, hence the scalar current is given by:

$$\overline{\psi}\chi = \overline{\psi}_L\chi_R + \overline{\psi}_R\chi_L = \psi_L^\dagger\chi_R + \psi_R^\dagger\chi_L$$

where the last term is written in Weyl's representation.

The vector current, instead is given by

$$\bar{\psi}\gamma^\mu\chi = \bar{\psi}_L\gamma^\mu\chi_L - \bar{\psi}_R\gamma^\mu\chi_R = \psi_L^\dagger\bar{\sigma}^\mu\chi_L - \psi_R^\dagger\sigma^\mu\chi_R \sim (1,0)\oplus(0,1) \tag{A.2}$$

where we have used $\gamma^\mu, \gamma^5 = 0$ and $P_{L(R)}\psi_{R(L)} = 0$ to cancel L-R terms.
It is clear that it is not an irreducible representation, but it is an eigenstate of parity, in fact since parity acts like γ^0 on single fermion field

$$P(\psi) = \gamma^0\psi \to P(\bar{\psi}\gamma^\mu\chi) = \bar{\psi}\gamma^0\gamma^\mu\gamma^0\chi = -\bar{\psi}\gamma^\mu\chi \quad J^P(\bar{\psi}\gamma^\mu\chi) = 1^- \tag{A.3}$$

It is also possible to write a *pseudovector* current that has positive parity through insertion of γ^5 matrix

$$\begin{aligned}&\bar{\psi}\gamma^\mu\gamma^5\chi \\ &P(\bar{\psi}\gamma^\mu\gamma^5\chi) = \bar{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\chi = \bar{\psi}\gamma^\mu\gamma^5\chi \quad J^P(\bar{\psi}\gamma^\mu\gamma^5\chi) = 1^+\end{aligned} \tag{A.4}$$

Usual in works that use Dirac's picture semileptonic neutral current operator, for instance with quarks b and s as

$$C_9^l\,\bar{b}\gamma^\mu P_L s\,\bar{e}\gamma_\mu e + C_{10}^l\,\bar{b}\gamma^\mu P_L s\,\bar{e}\gamma_\mu\gamma^5 e \tag{A.5}$$

where the l corresponds to the family of e_L. The corrispondence with beetween these Wilson coefficients and the purely LH ones can be derived using

$$\begin{aligned}&1 = P_R + P_L, \quad \gamma^5 = P_R - P_L \to \\ &C_9^l\,\bar{b}\gamma^\mu P_L s\,\bar{e}\gamma_\mu e \;+\; C_{10}^l\,\bar{b}\gamma^\mu P_L s\, l\gamma_\mu\gamma^5 l \supset (C_9^l - C_{10}^l)\bar{b}_L\gamma^\mu s_L\,\bar{e}_L\gamma_\mu e_L = \\ &(C_9^l - C_{10}^l)b_L^\dagger\bar{\sigma}^\mu s_L\,e_L^\dagger\bar{\sigma}_\mu e_L\end{aligned} \tag{A.6}$$

where the last form in term of Weyl fields tells us that $C_9^l - C_{10}^l$ corresponds in our notation to $C_S^{bs\mu\mu} + C_T^{bsll}$.

B

Proof of the Flavour Theorem 3

B.0.1 *CP symmetry*

Among the consequences of flavour physics there is the arise of the CP symmetry violation.
In Dirac representation terms, defining

$$\psi_L \equiv \frac{1-\gamma_5}{2}\psi, \quad \psi_R \equiv \frac{1+\gamma_5}{2}\psi$$

charge conjugation, C, and spatial inversion, P, act on fermions as:

$$\begin{aligned}
P\,:\,\psi(\vec{x},t) = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix} &\to \gamma_0\psi(-\vec{x},t) = \begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix}(-\vec{x},t)\\
C\,:\,\psi = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}(\vec{x},t) &\to i\gamma_2\psi^*(\vec{x},t) = \begin{pmatrix}-\varepsilon\psi_R^*\\ \varepsilon\psi_L^*\end{pmatrix}(\vec{x},t)\\
CP\,:\,\psi(\vec{x},t) = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}(\vec{x},t) &\to i\gamma_2\gamma_0\psi^*(-\vec{x},t) = \begin{pmatrix}-\varepsilon\psi_L^*\\ \varepsilon\psi_R^*\end{pmatrix}(-\vec{x},t)
\end{aligned}$$

where $\varepsilon = i\sigma_2$. Then we know that that the Higgs boson is transformed under CP as

$$CP\,:\,H(\vec{x},t) \to CH(-\vec{x},t) \to H^c(-\vec{x},t) = \varepsilon H^*$$

Instead, on the gauge fields, CP acts as:

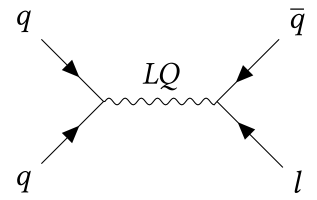
$$CP\,:\,(A_\mu(\vec{x},t), Z_\mu(\vec{x},t), W_\mu^\pm(\vec{x},t), G_\mu) \to (-A_\mu(-\vec{x},t), -Z_\mu(-\vec{x},t), W_\mu^\mp(-\vec{x},t), -G_\mu^a(-\vec{x},t))$$

C

Rough estimation of M_{LQ} from proton decay

Here we will show through a naive approach why $\tau_p 10^{33} ys$ can be explained by a mediator with a mass of order $\Lambda_{GUT} \simeq 10^{16}\ GeV$ as we said in section 4.2.1.

The contribution given by the diagram to proton decay



if NP is flavour universal will be approximately

$$\mathscr{M} \simeq g_{NP} \frac{1}{q^2 - M_{NP}^2} g_{NP} \tag{C.1}$$

that in the limit of $q^2 \ll M_{NP}^2$ becomes

$$\mathscr{M} \simeq -\frac{g_{NP}^2}{M_{NP}^2} \tag{C.2}$$

We know that the decay width is

$$\Gamma = (const.)|\mathscr{M}|^2$$

and that it has to have the dimension of a $[mass]^1$, hence the $(const.)$ has to have the dimension of a $[mass]^5$. Since the biggest mass in play is m_p, to get a rough estimation of the width we assume $(const.) = m_p^5$.

If we assume $g_{NP} \simeq$ the approximate width results:

$$\Gamma \simeq \frac{m_p^5}{M_{NP}^4} \rightarrow \tau \simeq \frac{M_{NP}^4}{m_p^5} > 10^{33}\ ys$$

To have the prediction in unit of years we have first to get the lifetime in gaussian units, i.e.

$$\tau \simeq \frac{M_{NP}^4}{m_p^5} \cdot (\frac{\hbar}{c^2}) = (\frac{1}{0.938})^5 (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (6.58 \cdot 10^{-25} s)$$

hence

$$\tau \simeq (\frac{M_{NP}c^2}{1\ GeV})^4 \cdot (2.87 \cdot 10^{-32}\ ys) > 10^3 3ys \rightarrow$$
$$M_{NP}^4 > 10^{64-65}\ GeV^4 \rightarrow M_{NP} > 10^{16} GeV$$

that explain the estimation done in section 4.2.1.

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