Vector Databases

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Machine Learning Seminar

Department of Scientific Computing, FSU

September 28, 2023

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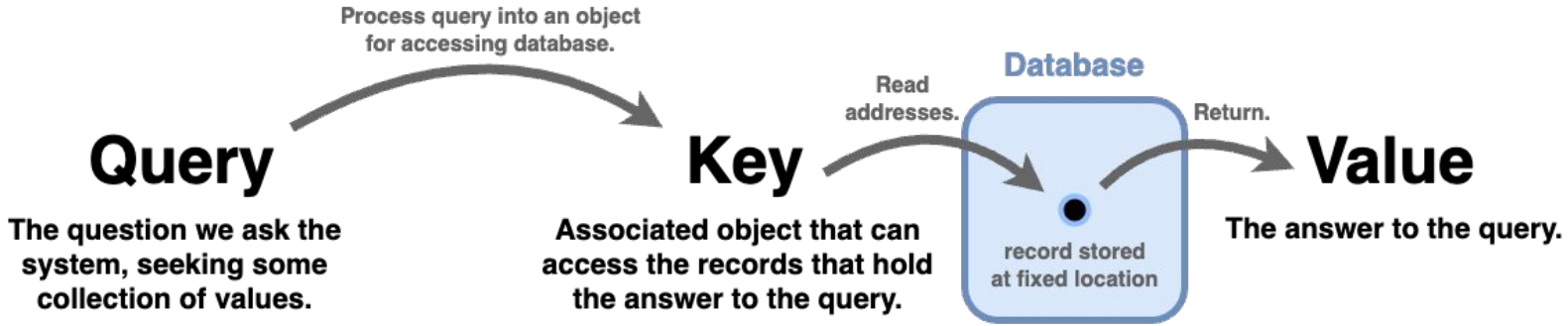

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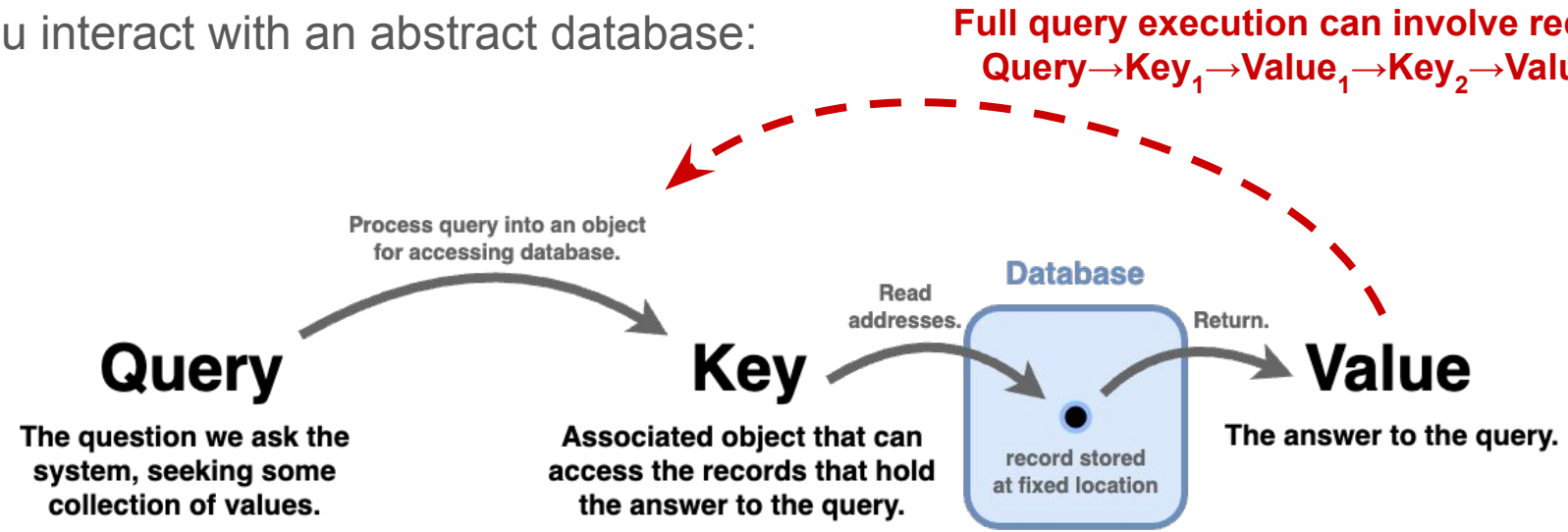

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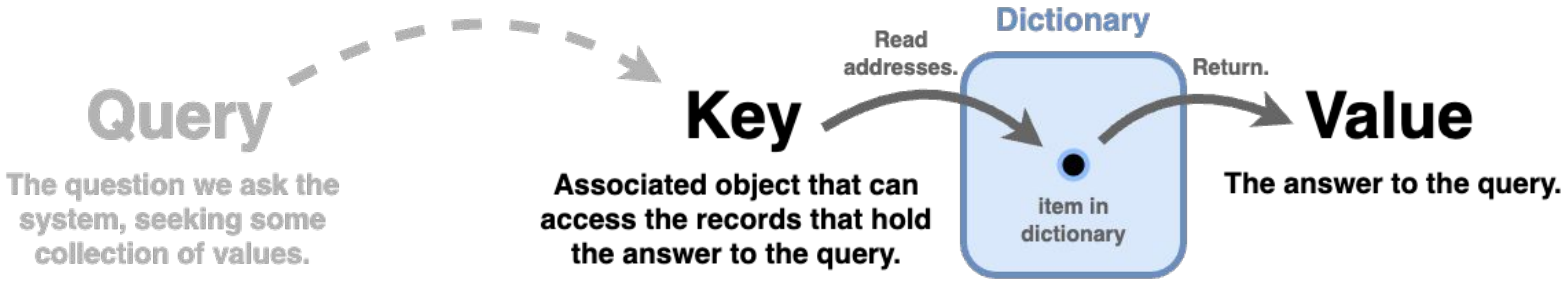

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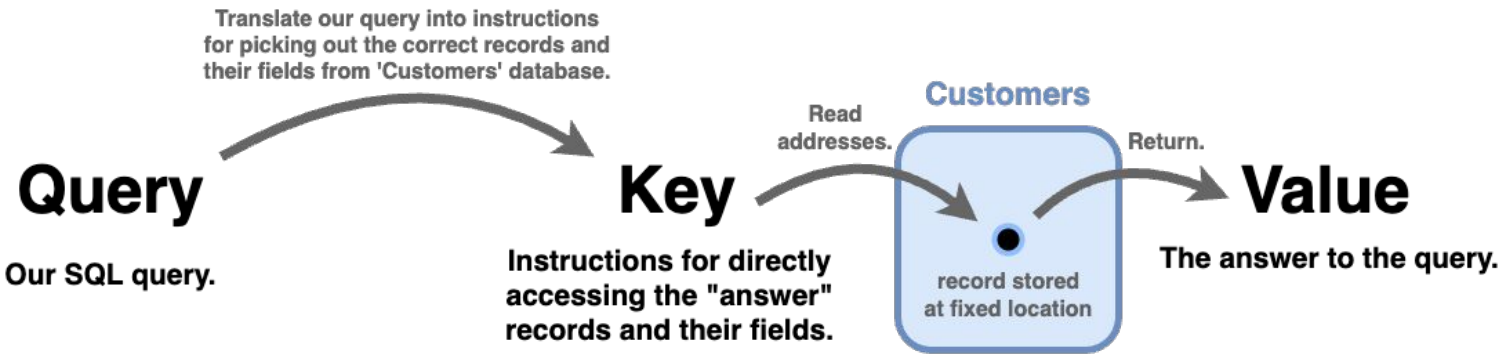

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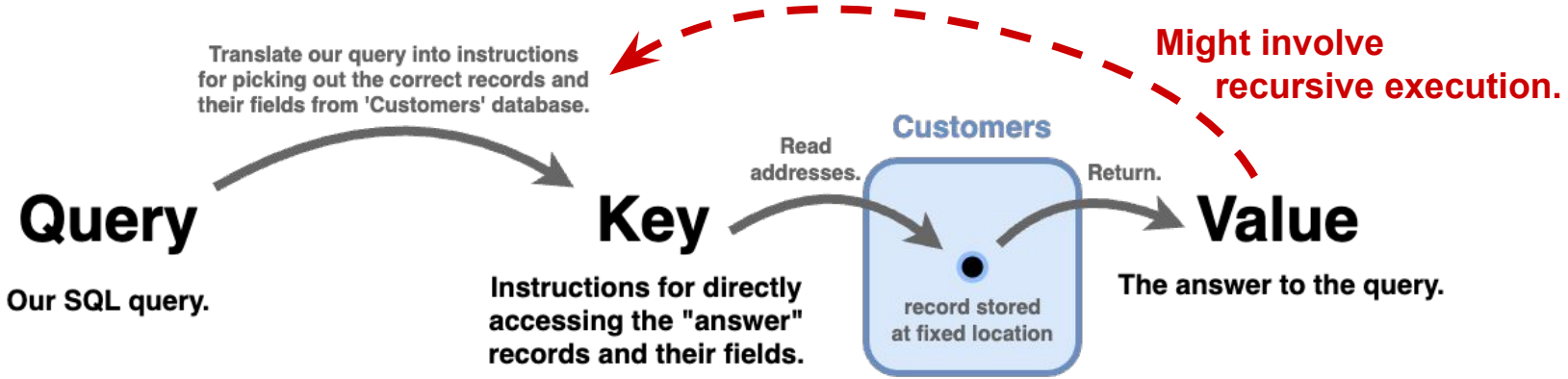

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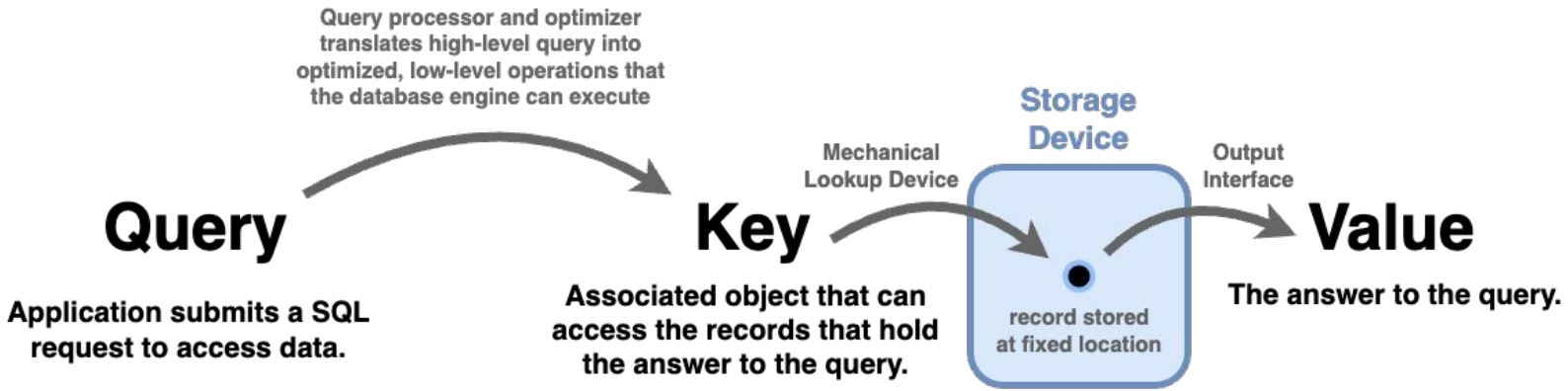

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…But how well can we really describe the world with this language?

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The SQL query format and its specific query-to-key processing cannot support “realistic” querying.

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Any part of query processing or address reading that Involves linear algebra in $\mathbf{R}^3$ (including navigation via $\mathbf{SO}(3) \subset \mathbf{GL}(3)$) has fast implementation on GPUs.

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We’ve used the geometric interpretation of linear-algebraic computations to endow our query, our query processor, and our address reader with the ability execute queries that are more physically realistic.

RECALL:

High-level idea for a solution.

Database records should include extra fields that encode more of the real-life-meanings attached to records.

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Query-to-key processing and the key's address-reads should utilize the extra fields to enable more realistic querying.

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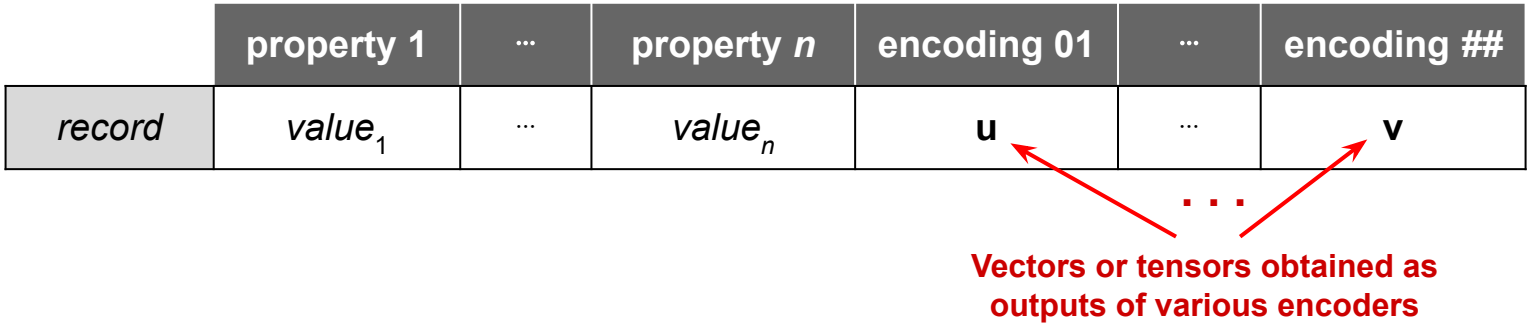

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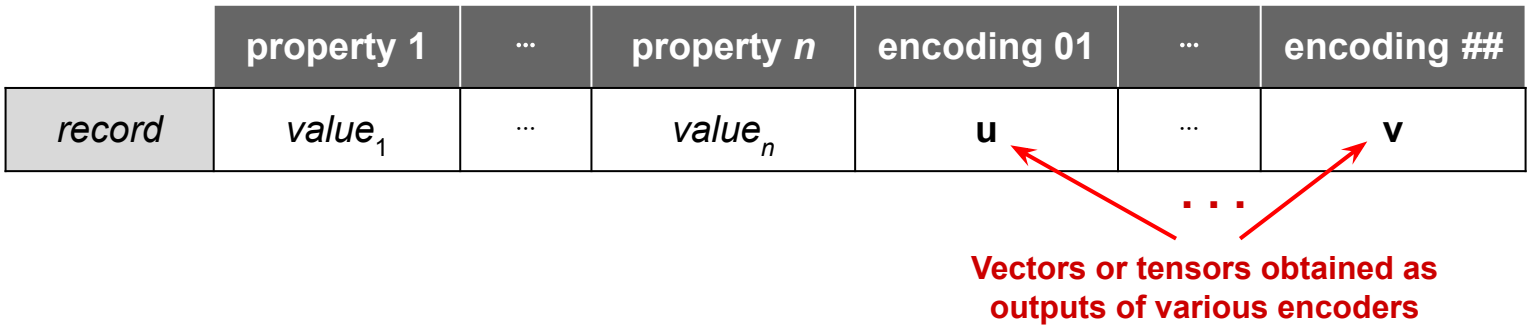
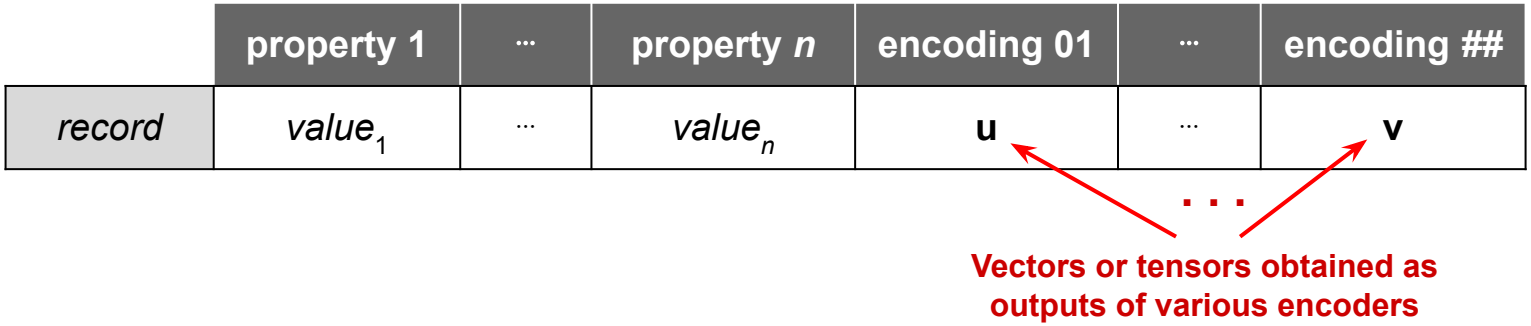
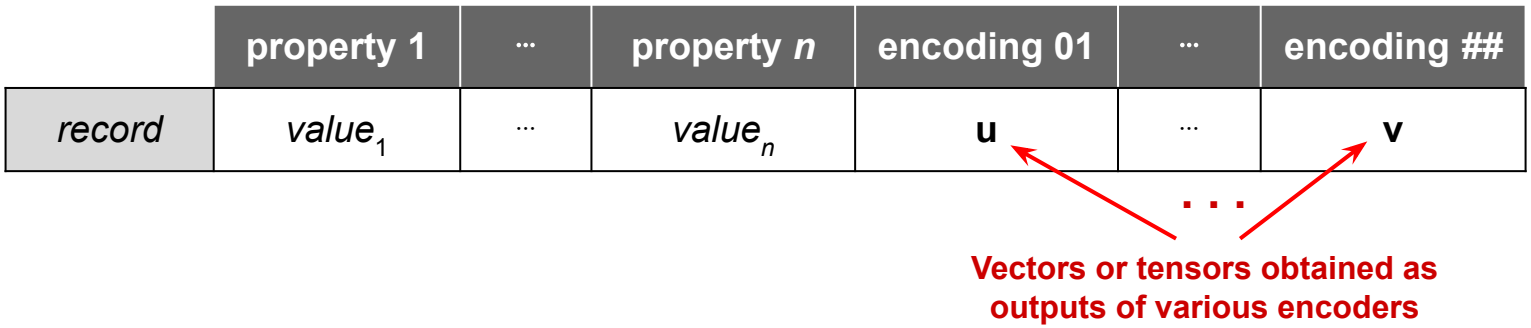


More-or-less obvious to ML Engineers:

Any trainable PyTorch model with “tensor → tensor” forward pass is a reasonable candidate for an operator we might use as part of executing queries in a database with records that look like this.

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Vectors or tensors obtained as
outputs of various encoders

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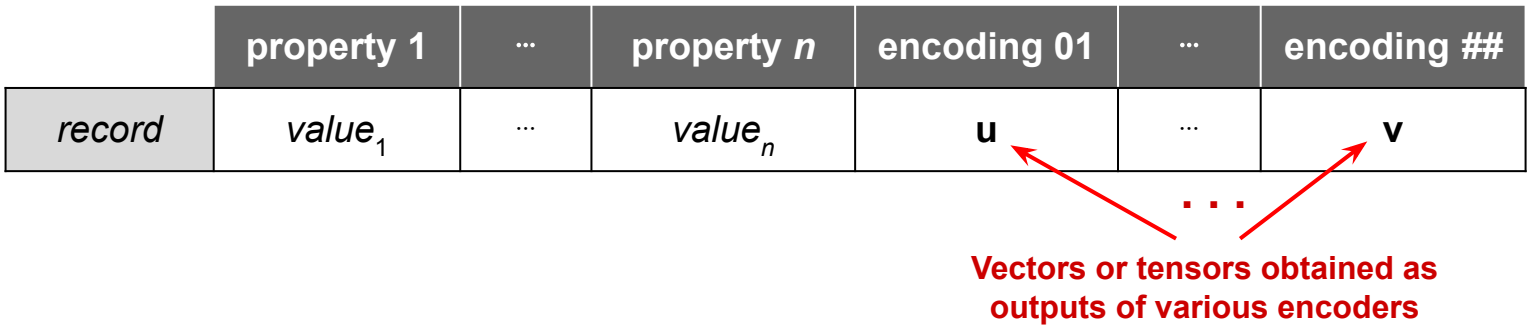

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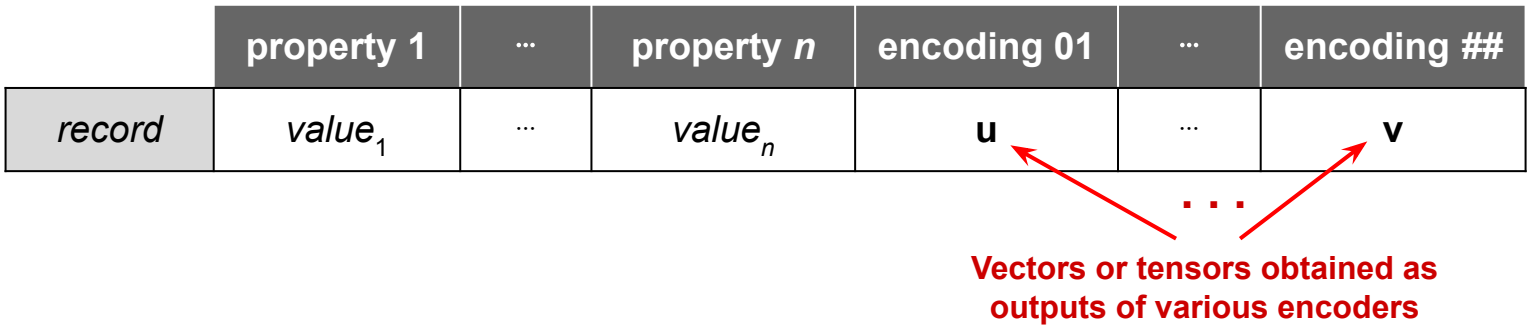

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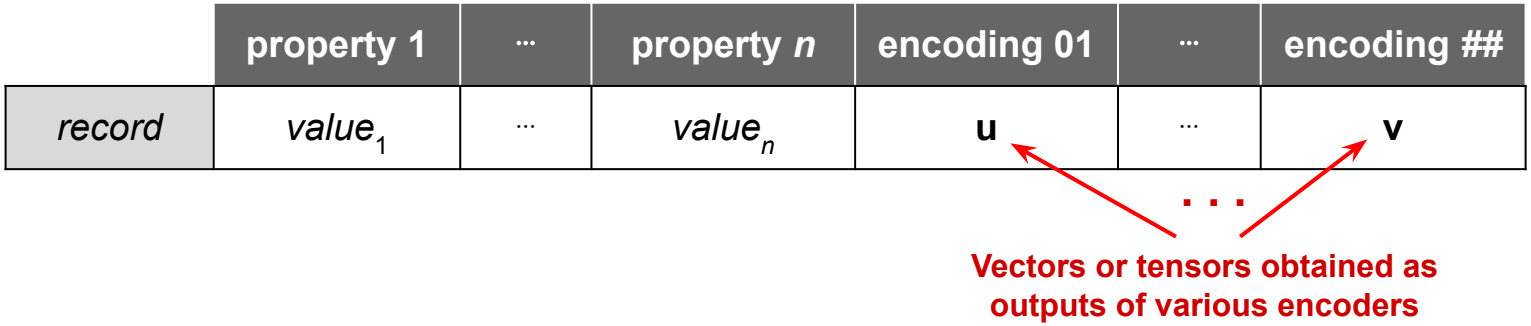
2. Query execution exploits modern tensor-based computational tools (GPUs, ML, etc.) when working with vectors/tensors in these columns.



Time/good place for Jupyter notebook example?

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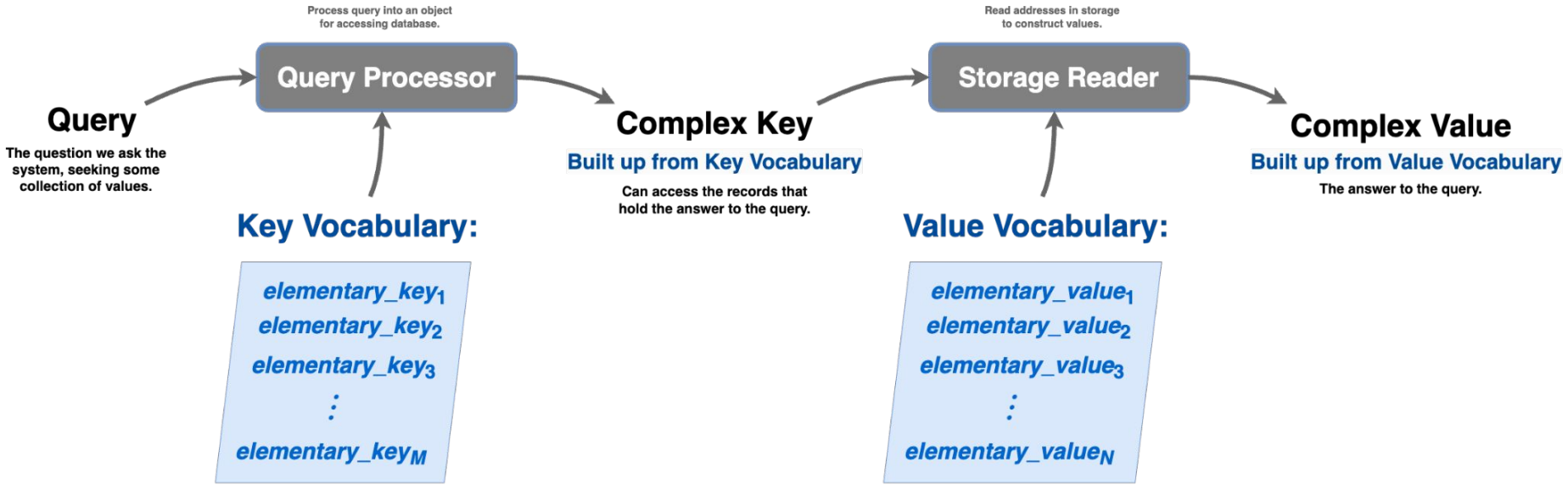

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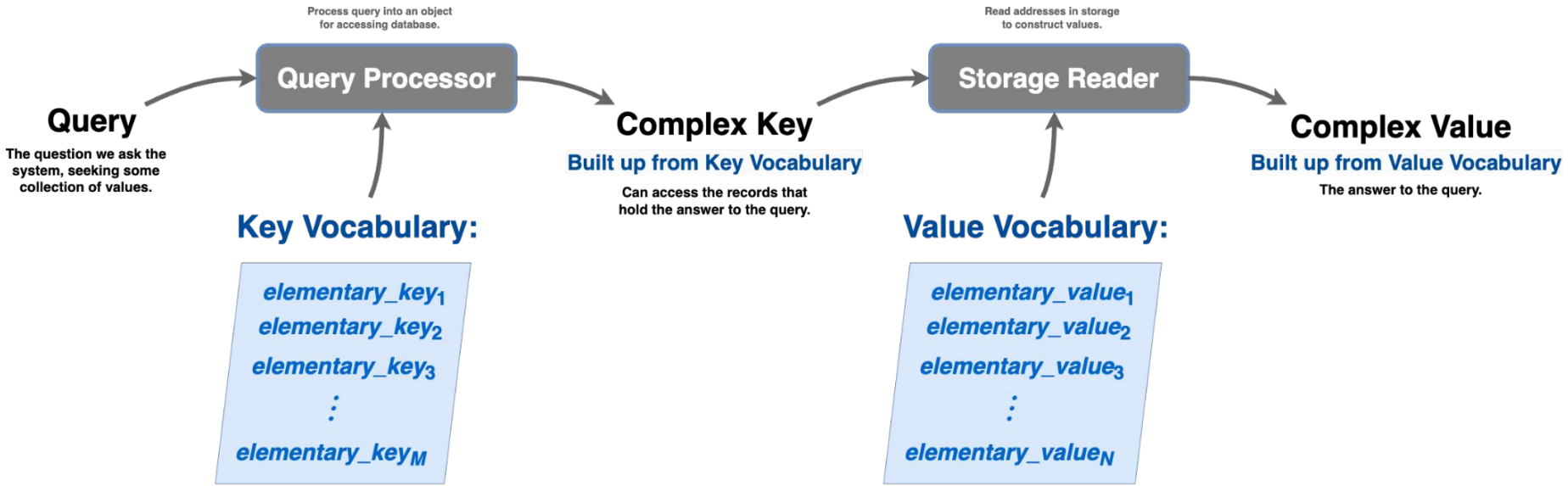

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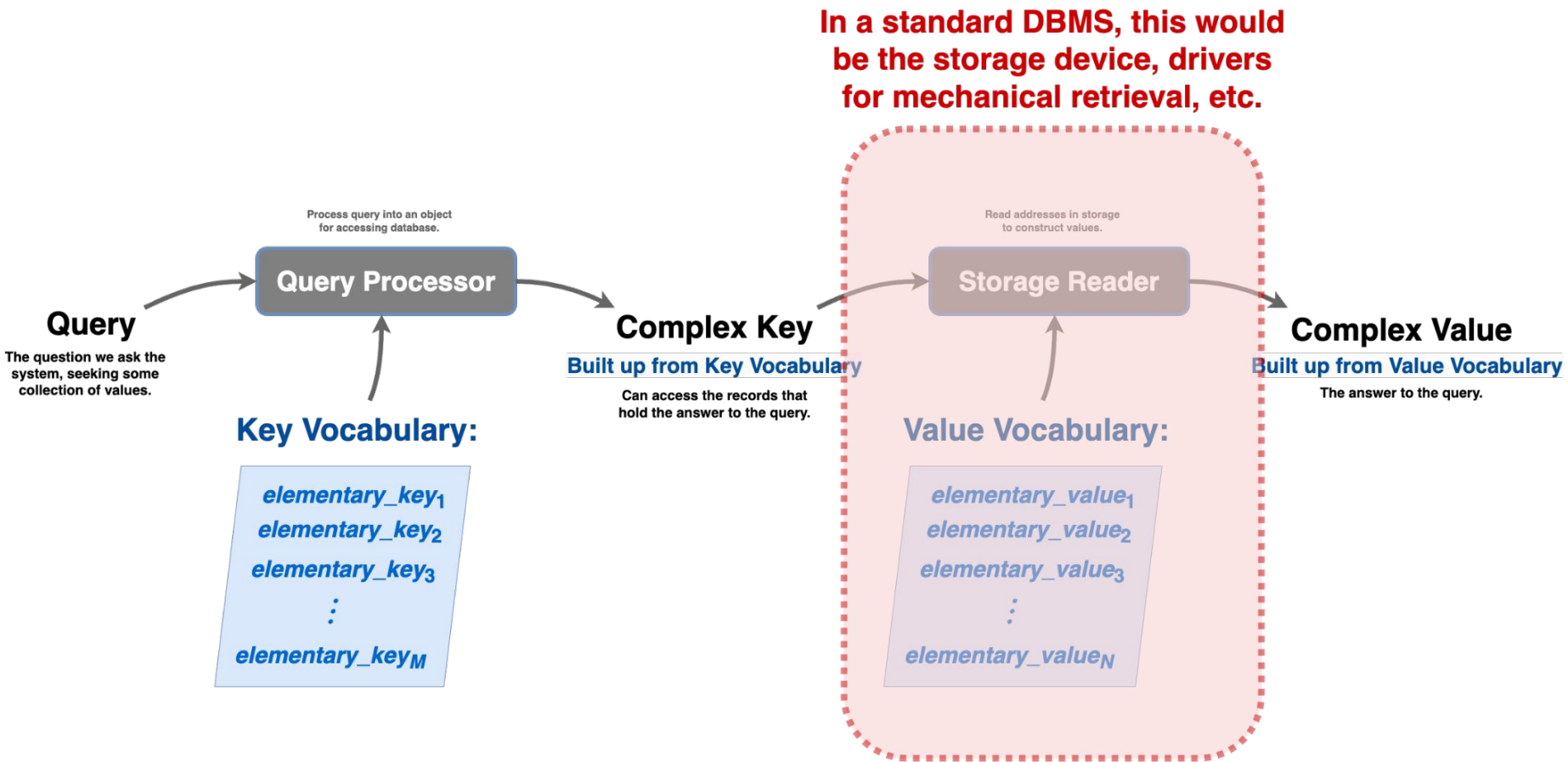

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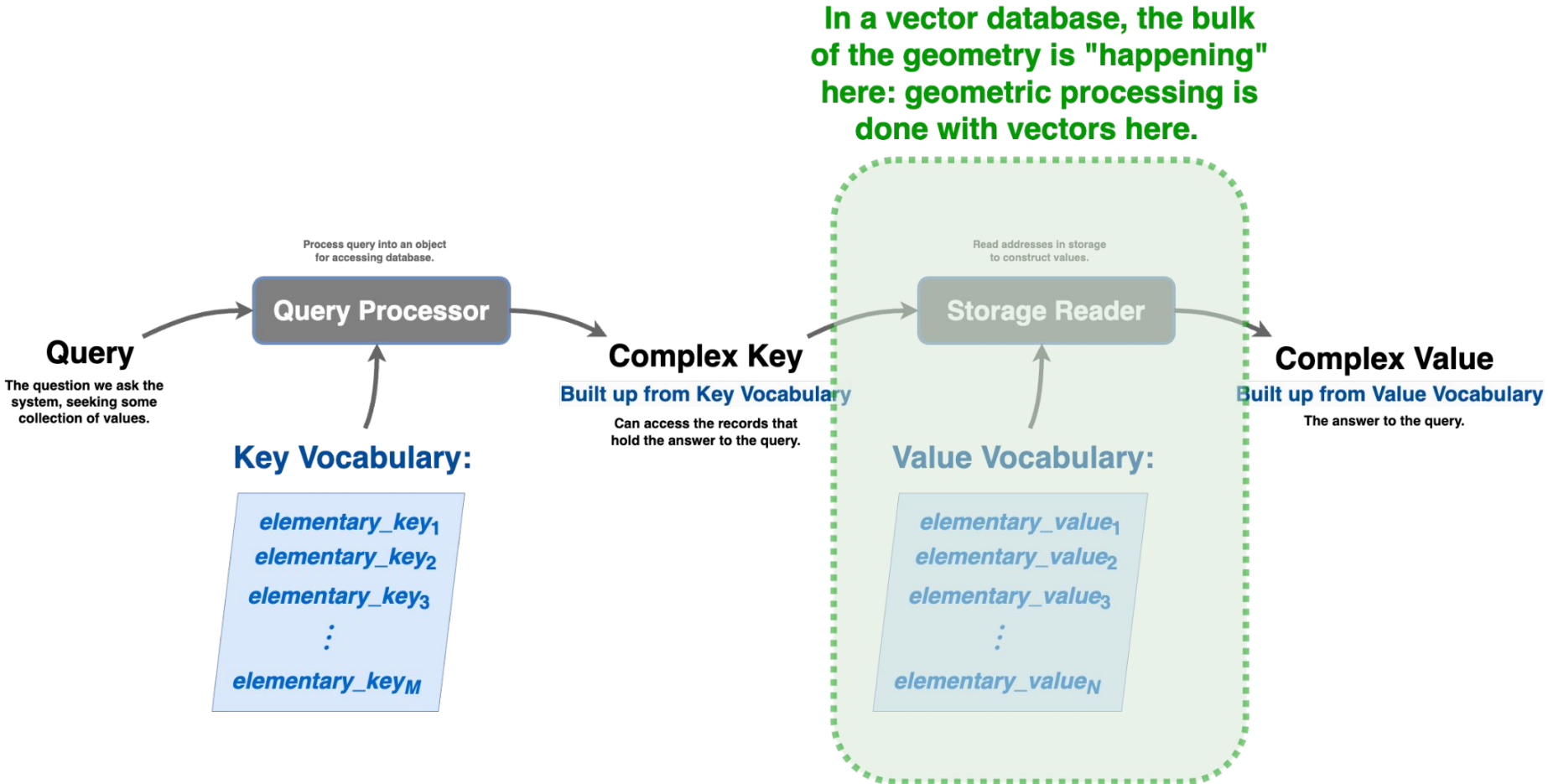

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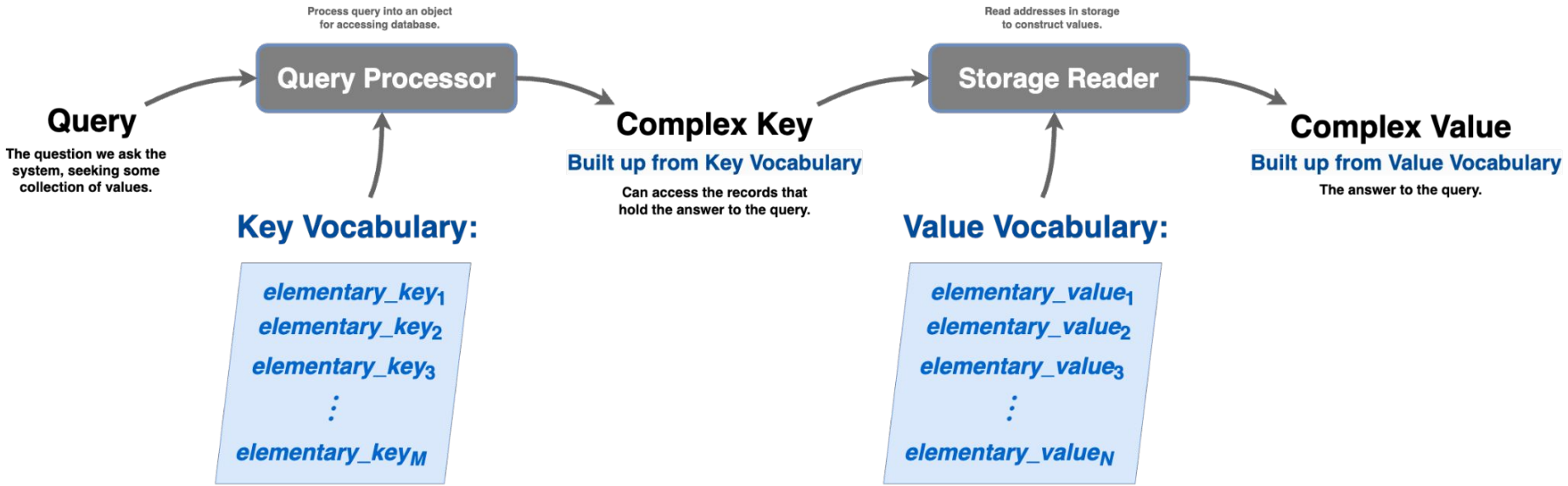

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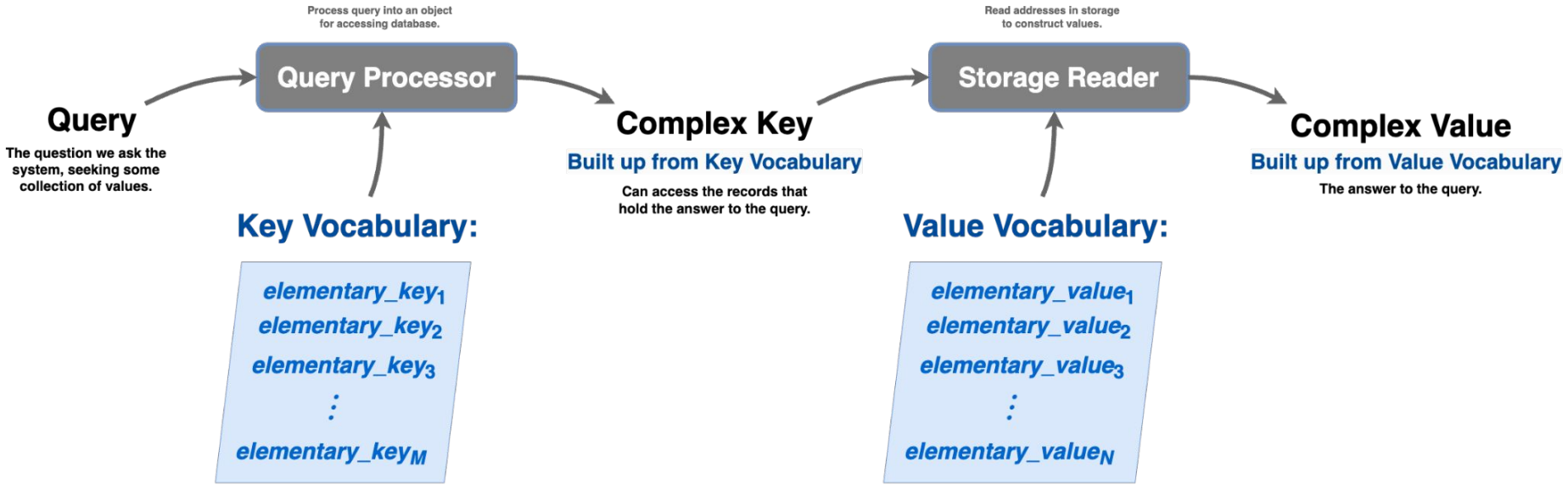

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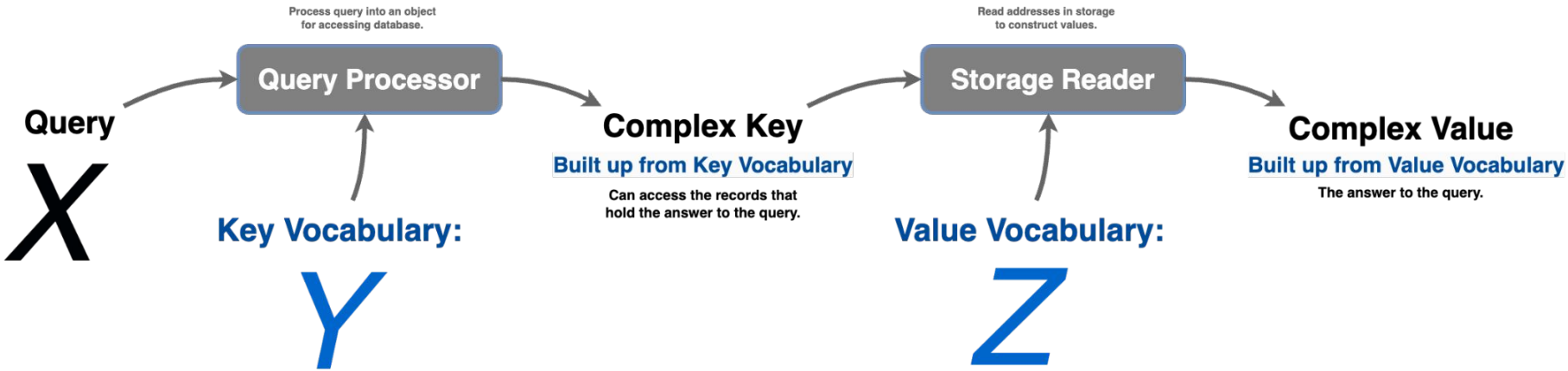

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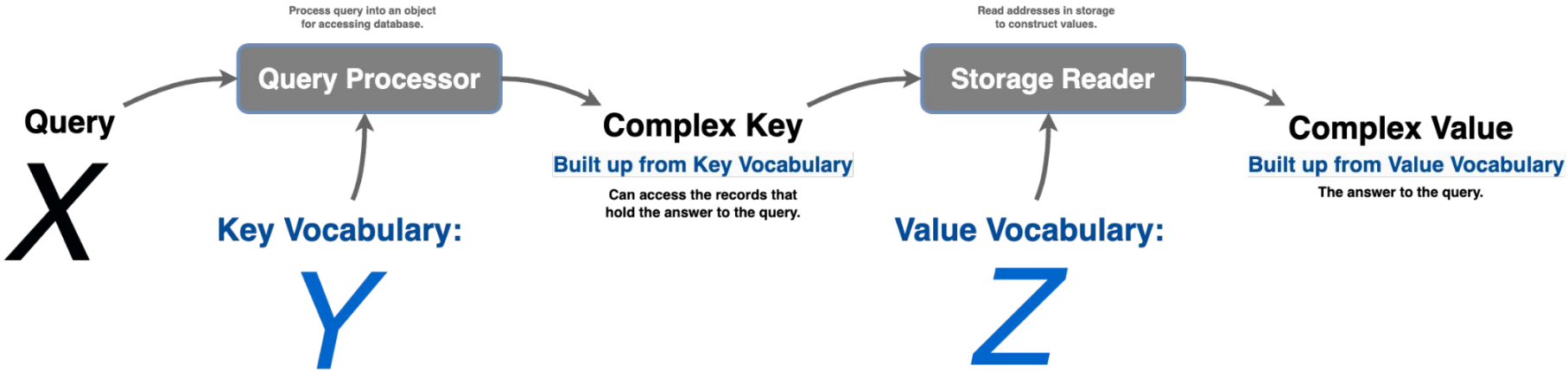
A cross-attention head is like a DBMS ***so robust*** that you can load it with any (correctly formatted) key and value vocabulary you like, and it will be able to do "closest match" query execution:



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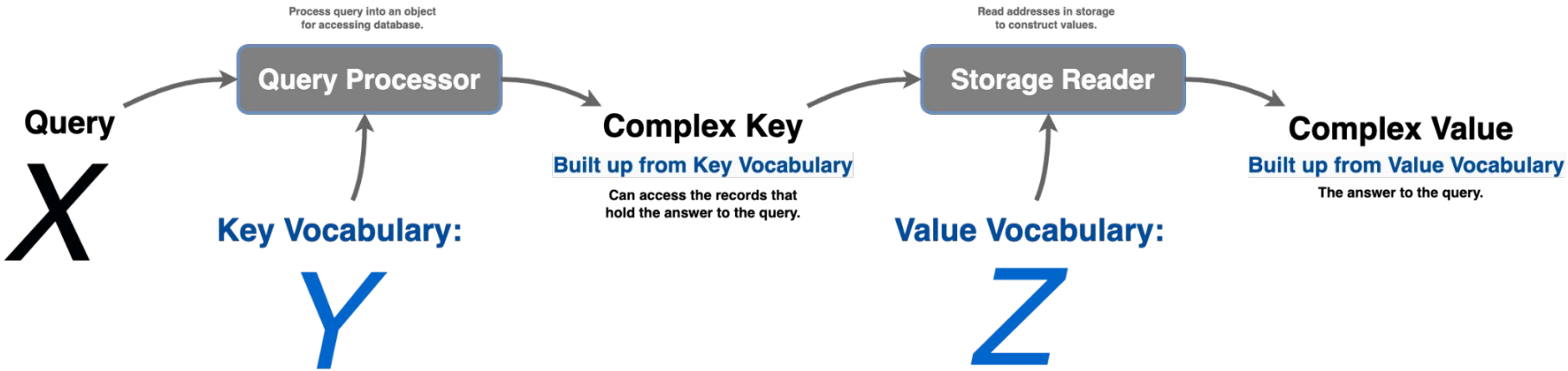

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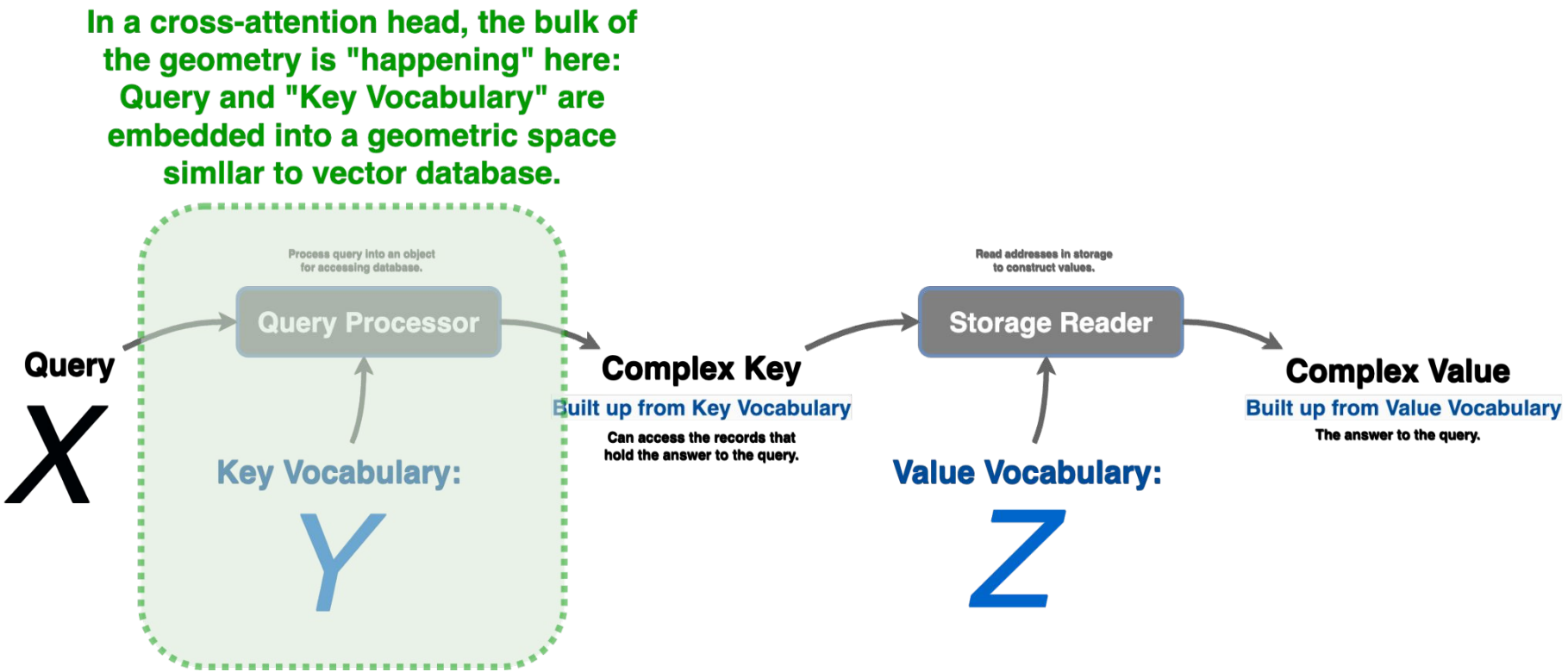

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How can we execute queries more efficiently in a vector database?

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$$\text{continuous space } \mathbf{R}^d \quad \longmapsto \quad \text{finite set } \{C_1, C_2, \ldots, C_k\}$$

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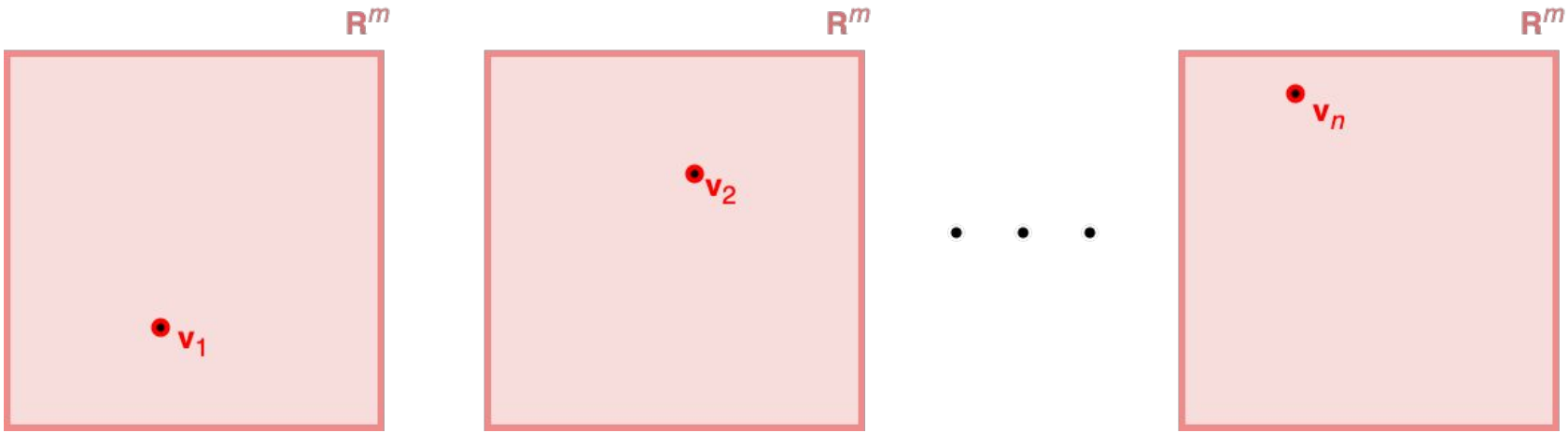

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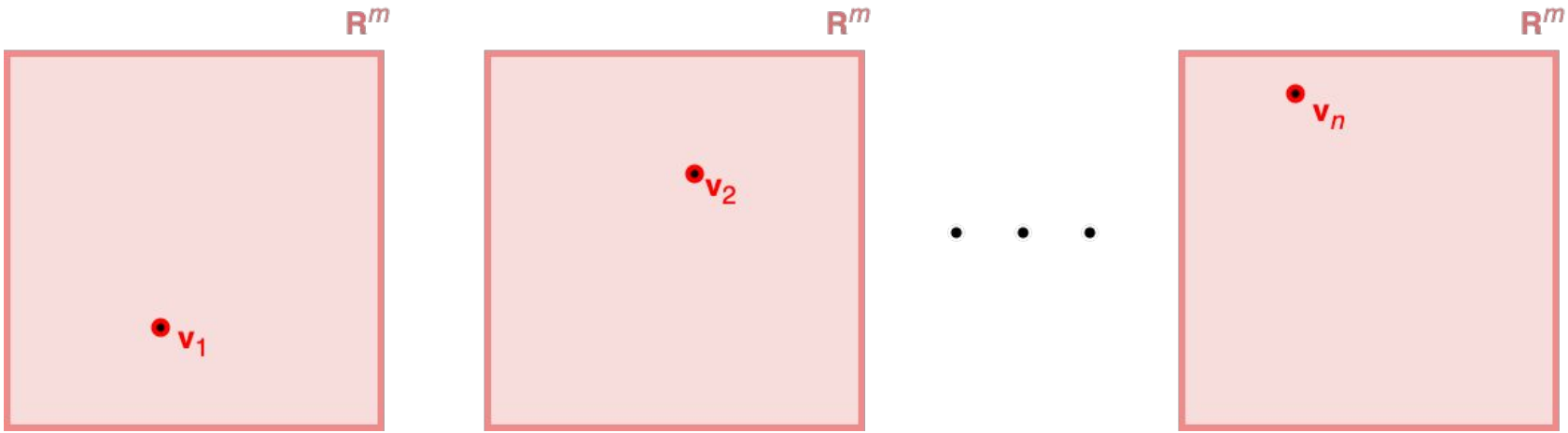

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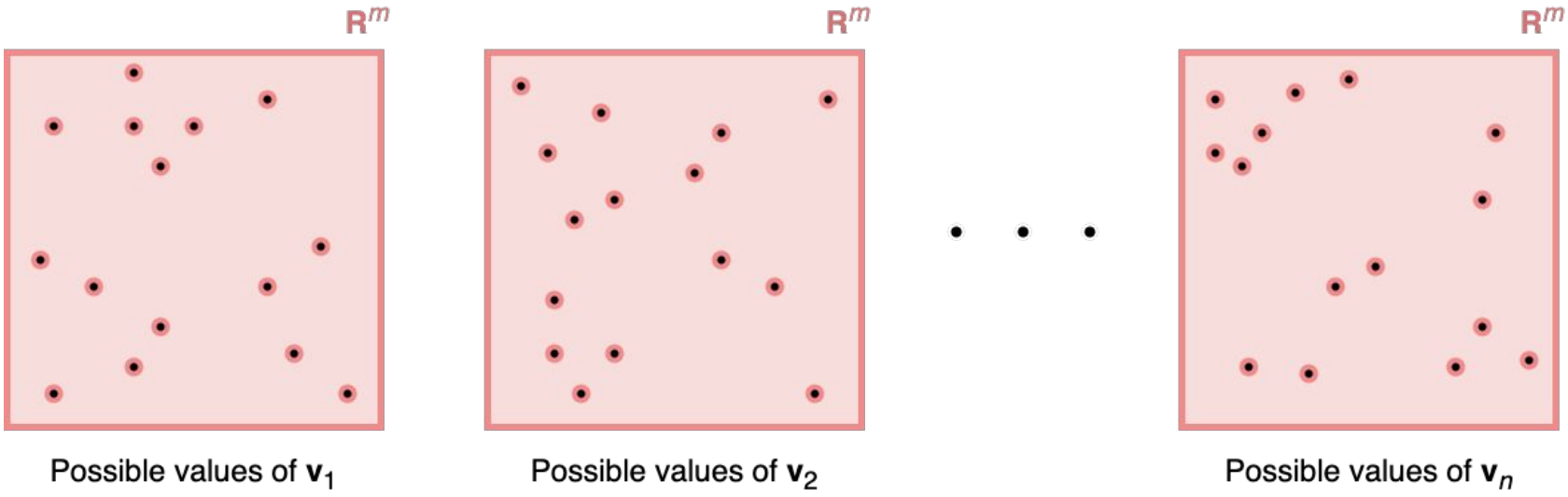

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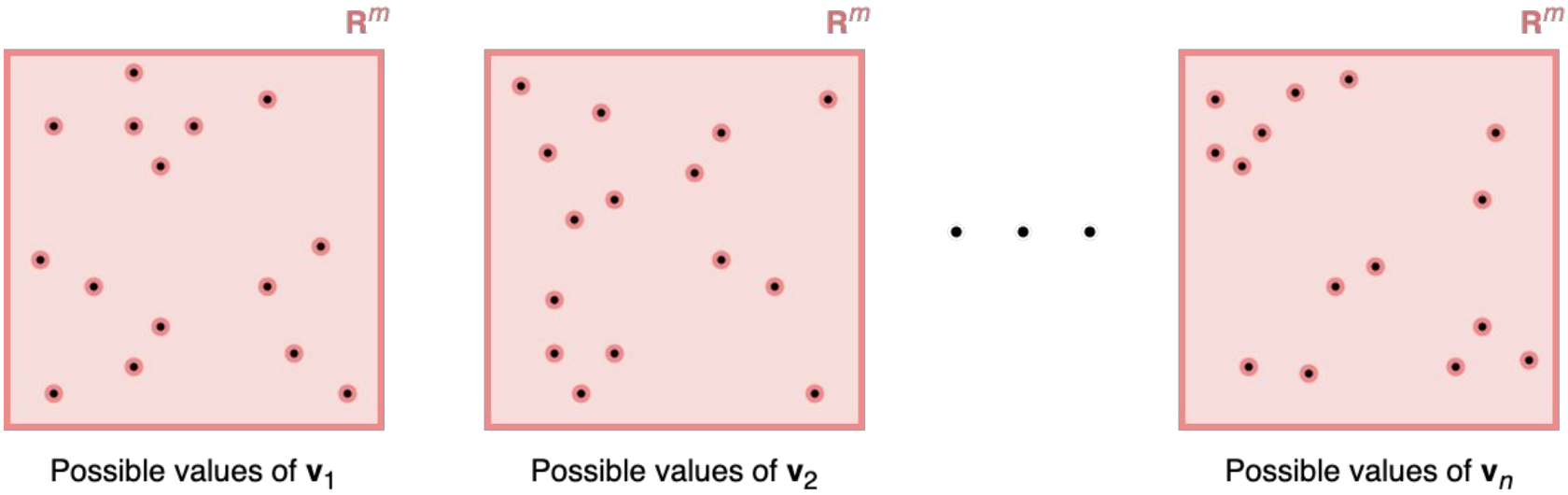

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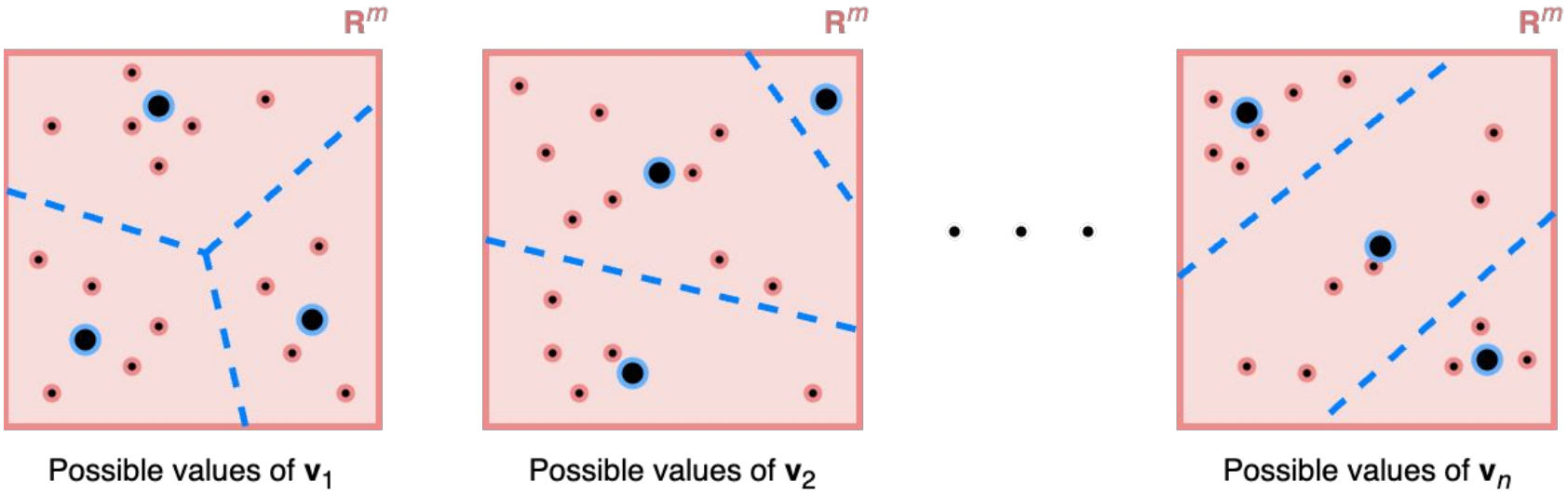

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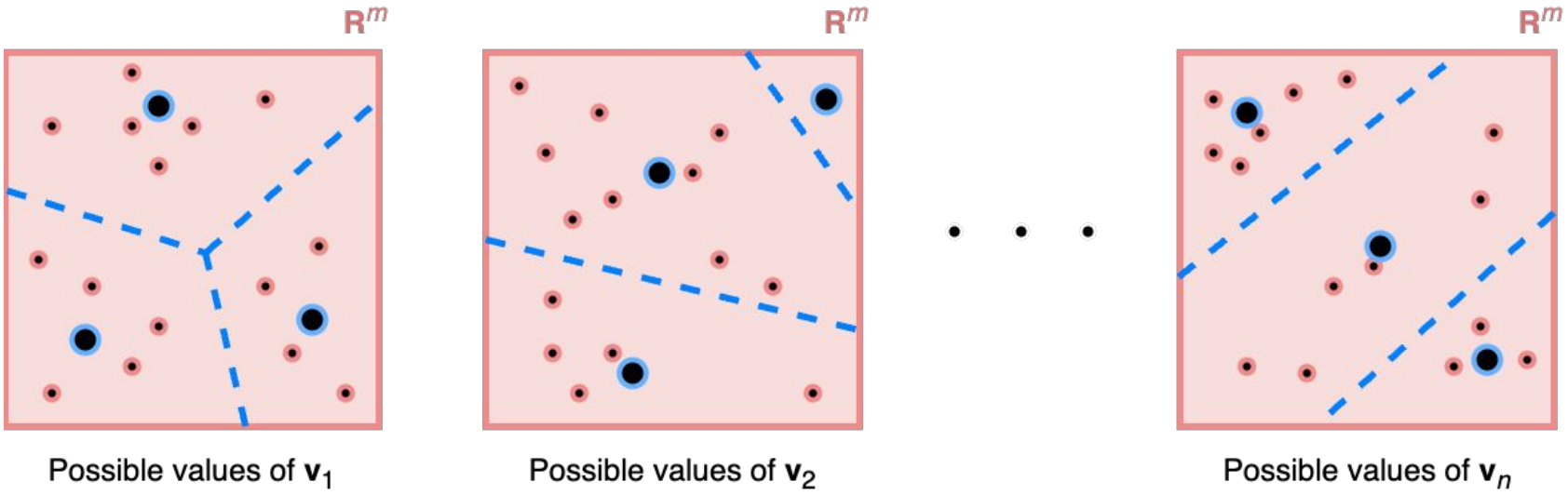

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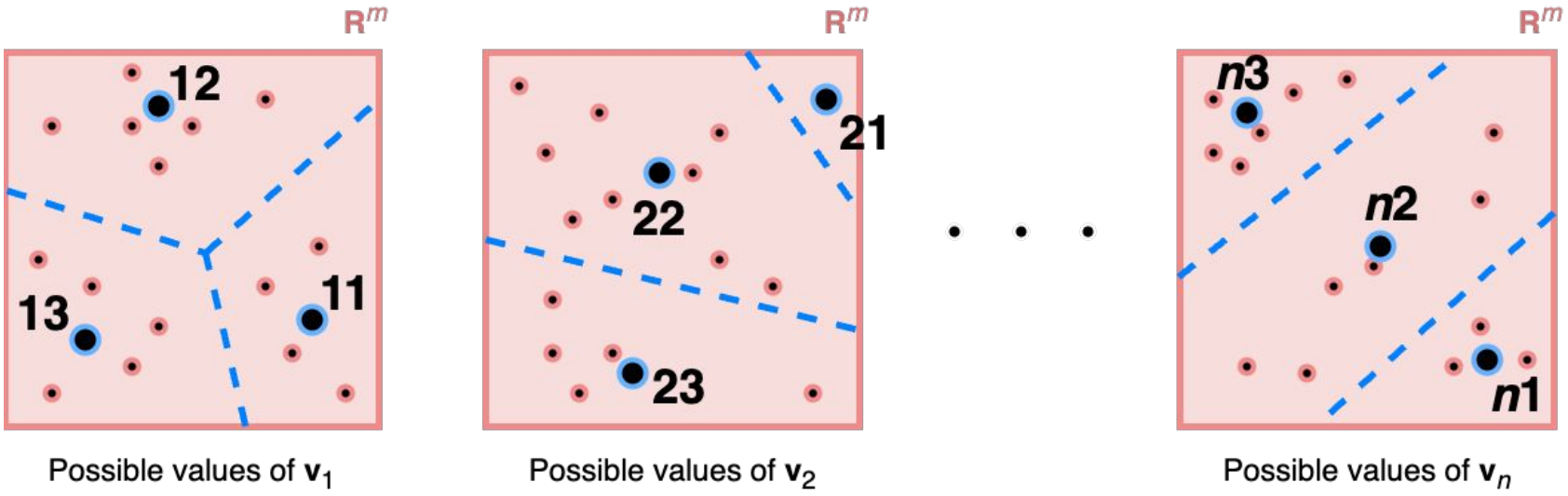

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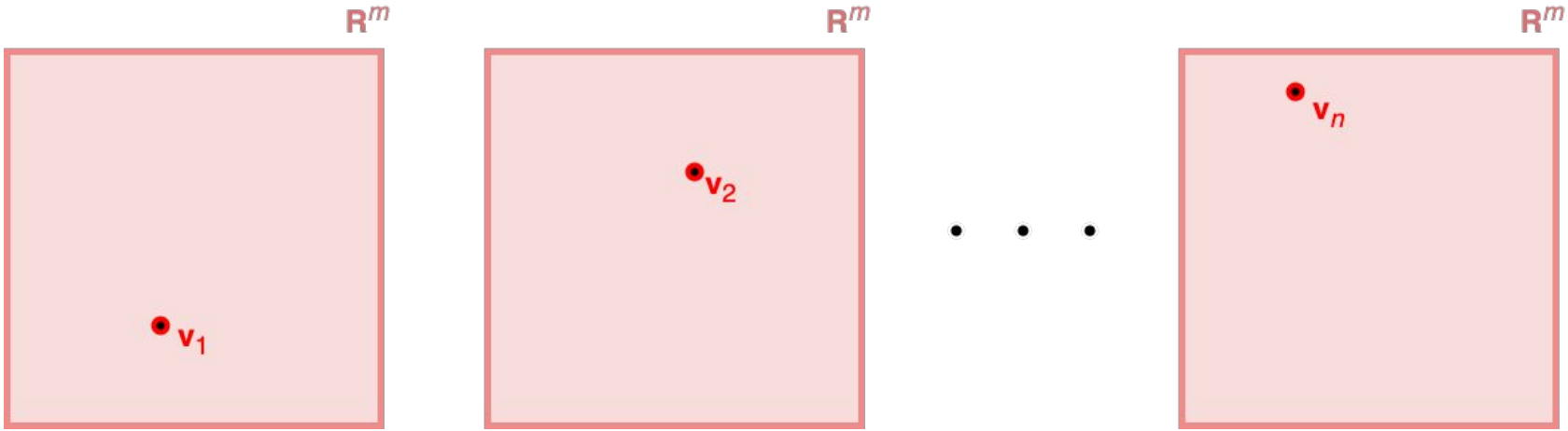

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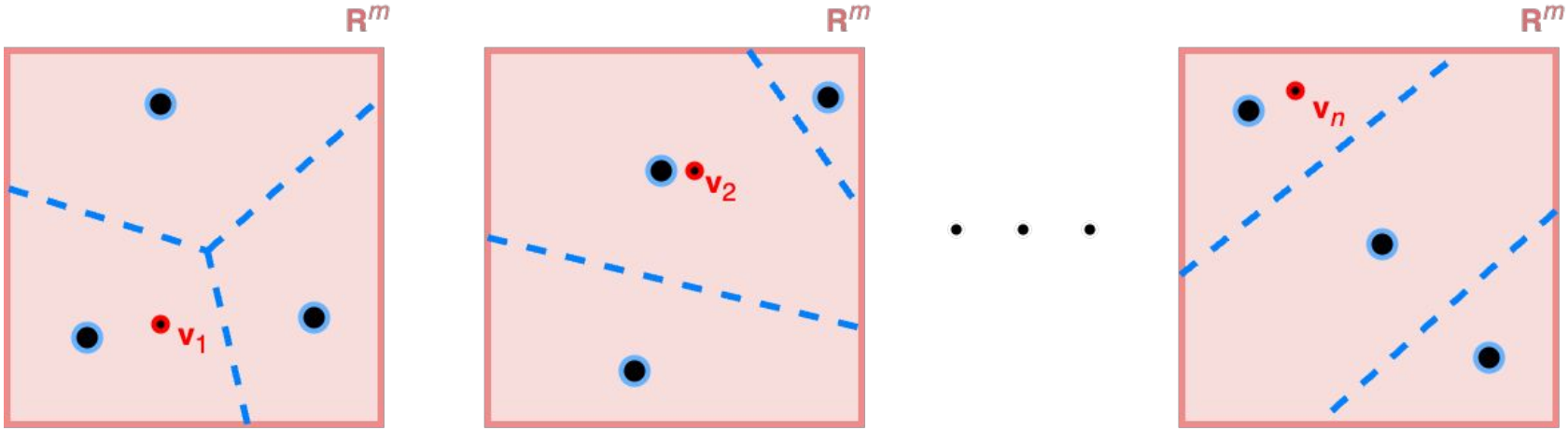

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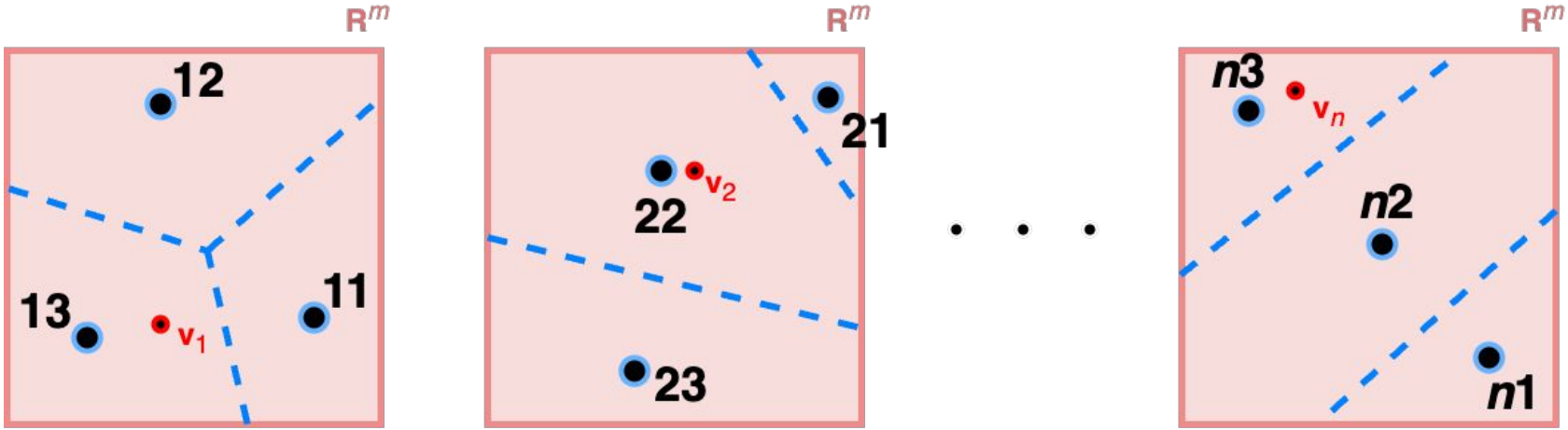

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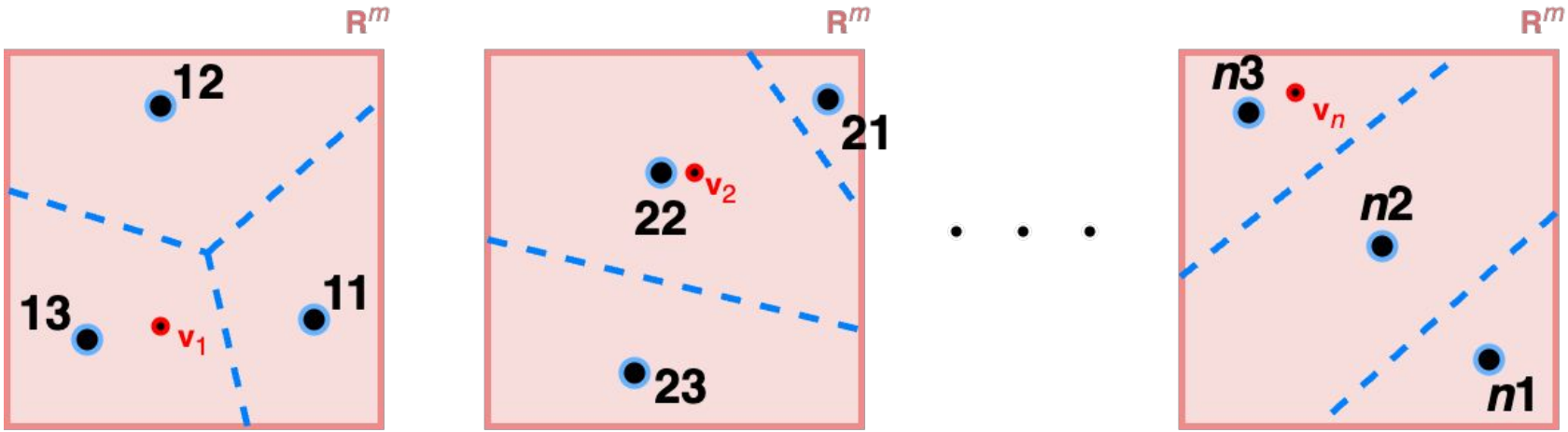

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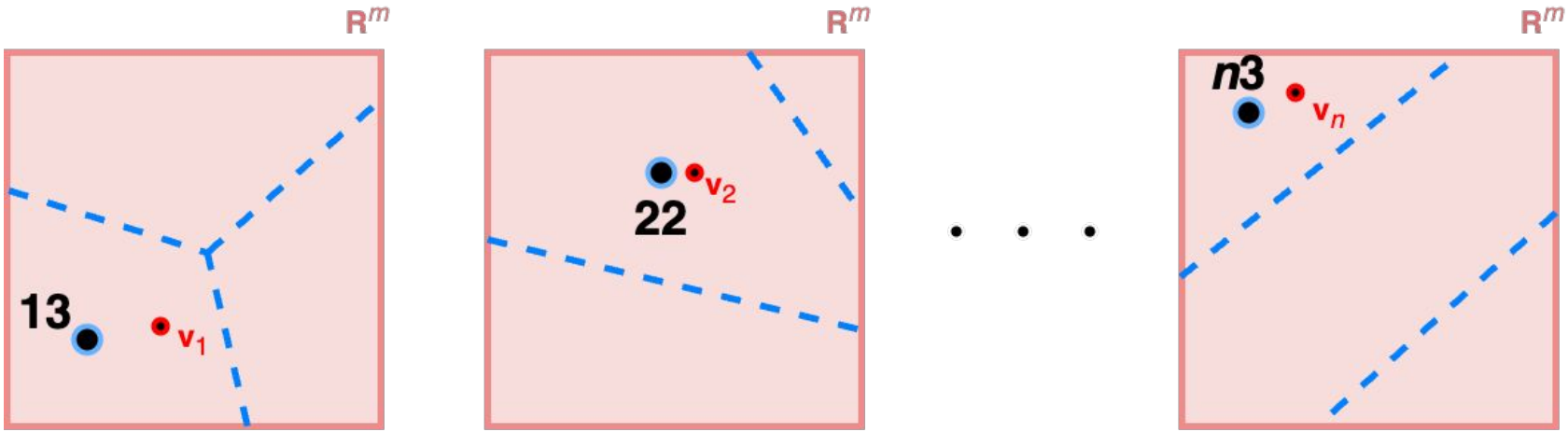

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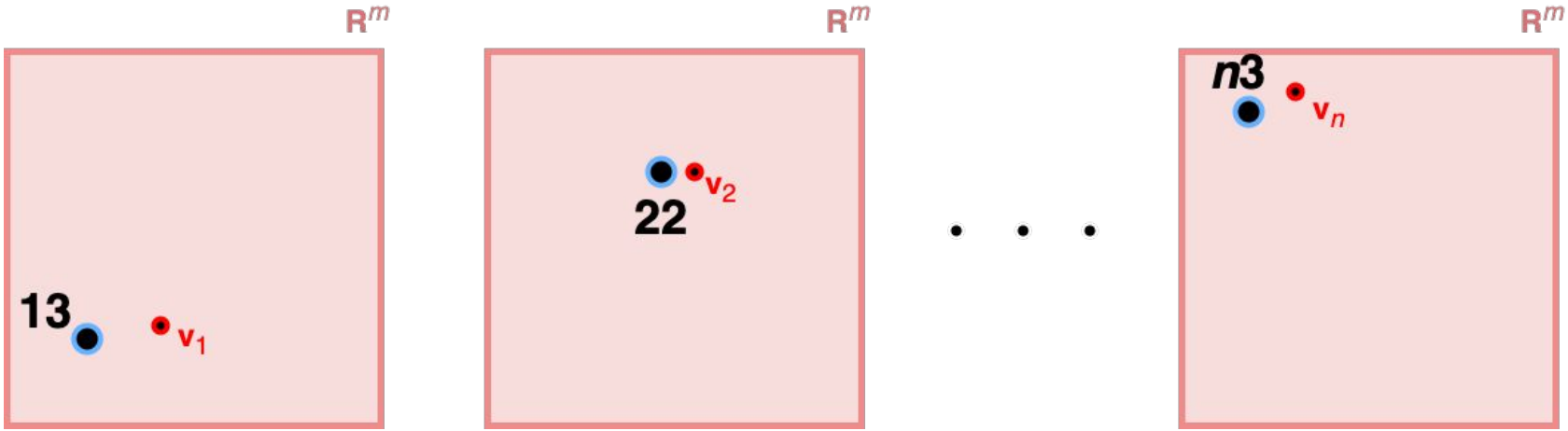

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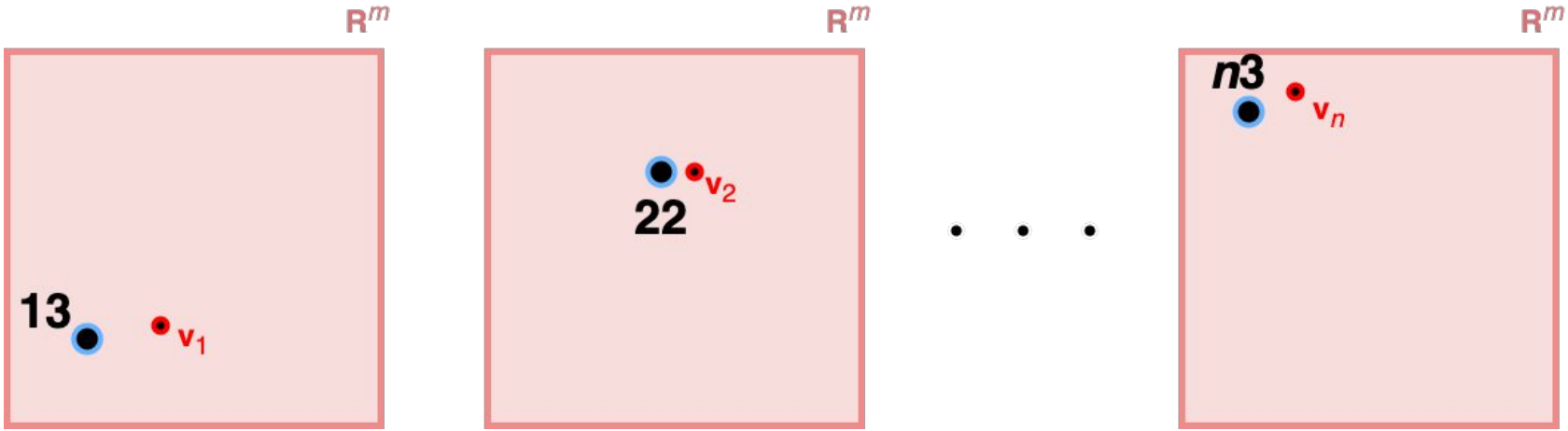

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How do we use this to speed up query execution?

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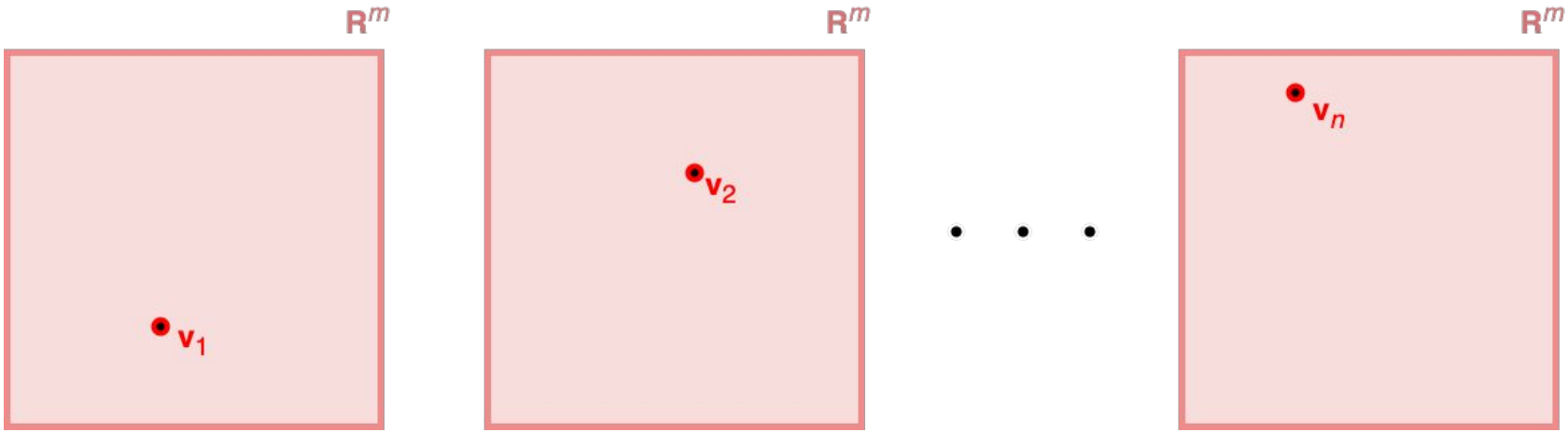
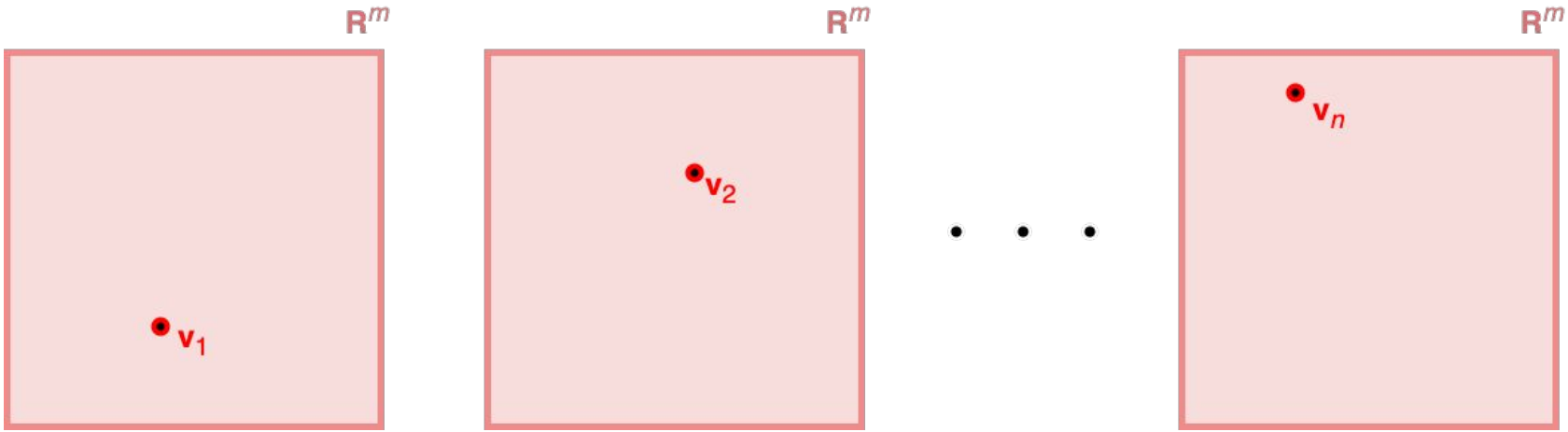
Transforming each component $\mathbf{v}_i$ separately means fewer arithmetic operations for matrix multiplication:



$$\begin{pmatrix} \Phi_1(\mathbf{v}_1) \\ \Phi_2(\mathbf{v}_2) \\ \vdots \\ \Phi_n(\mathbf{v}_n) \end{pmatrix} = \begin{pmatrix} \boxed{\Phi_1} & & 0 \\ & \boxed{\Phi_2} & \\ 0 & & \ddots \\ & & \boxed{\Phi_n} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_n \end{pmatrix}$$



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⇒ substantial speed-up for query execution

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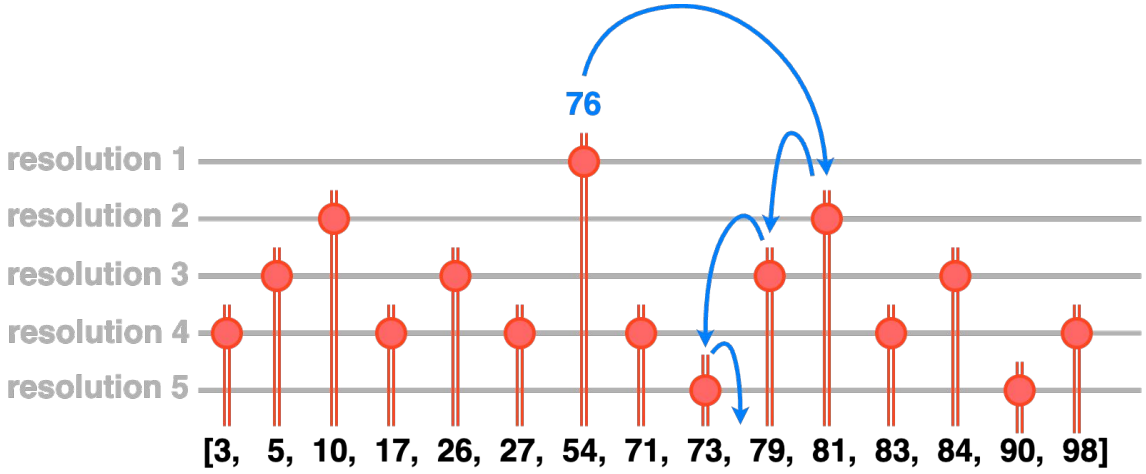

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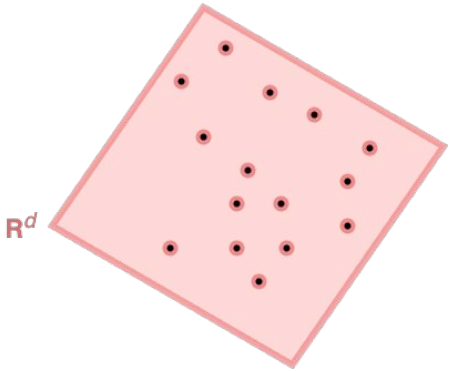

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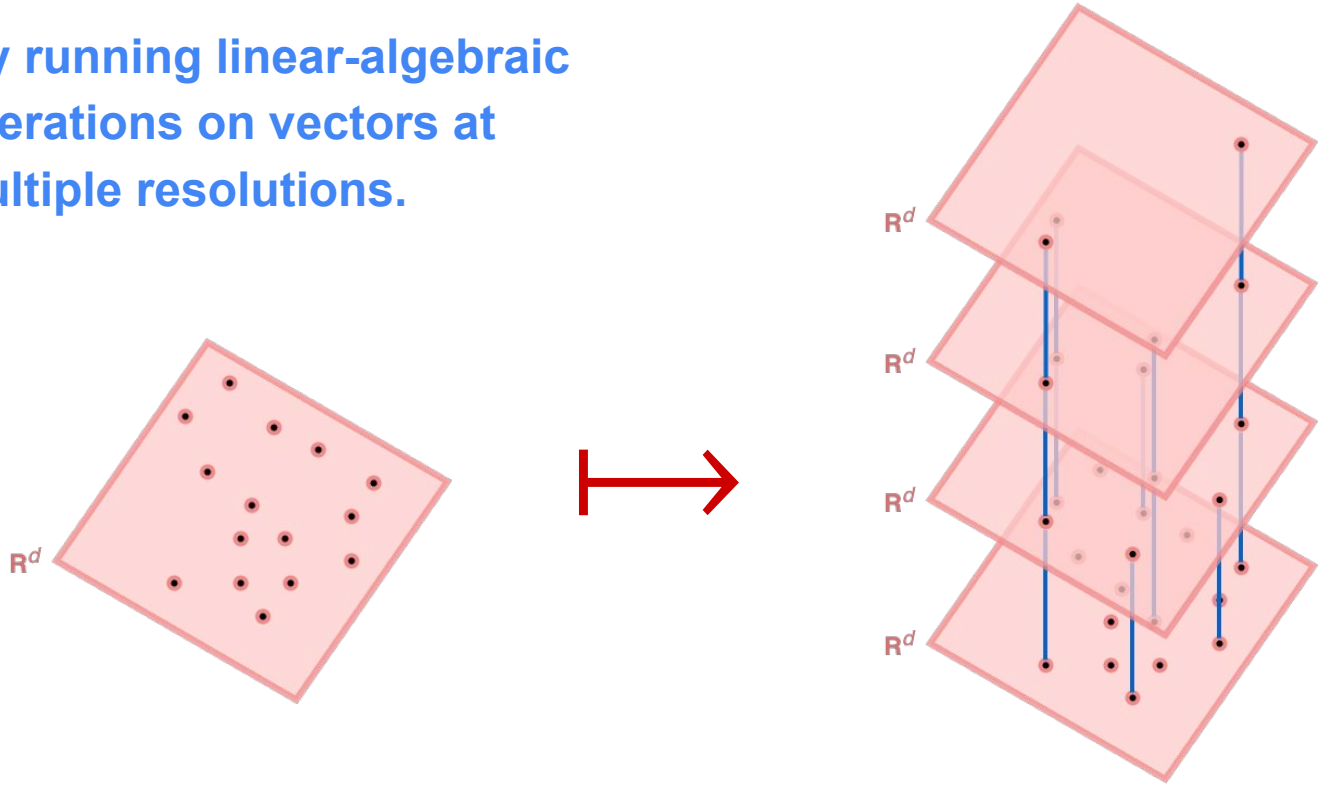

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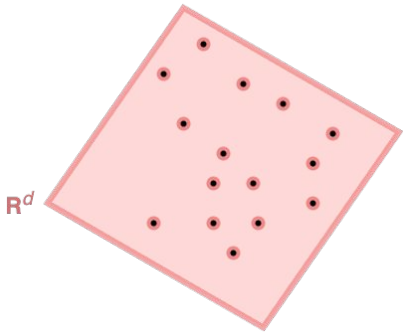

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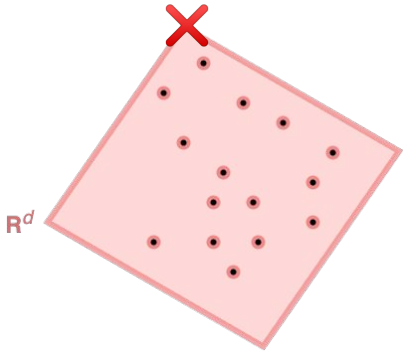

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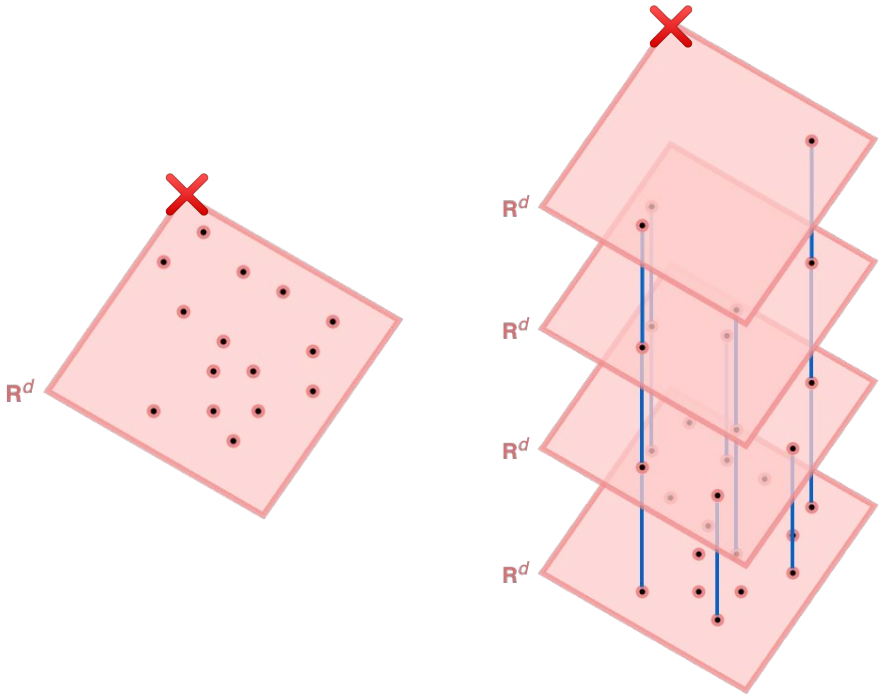

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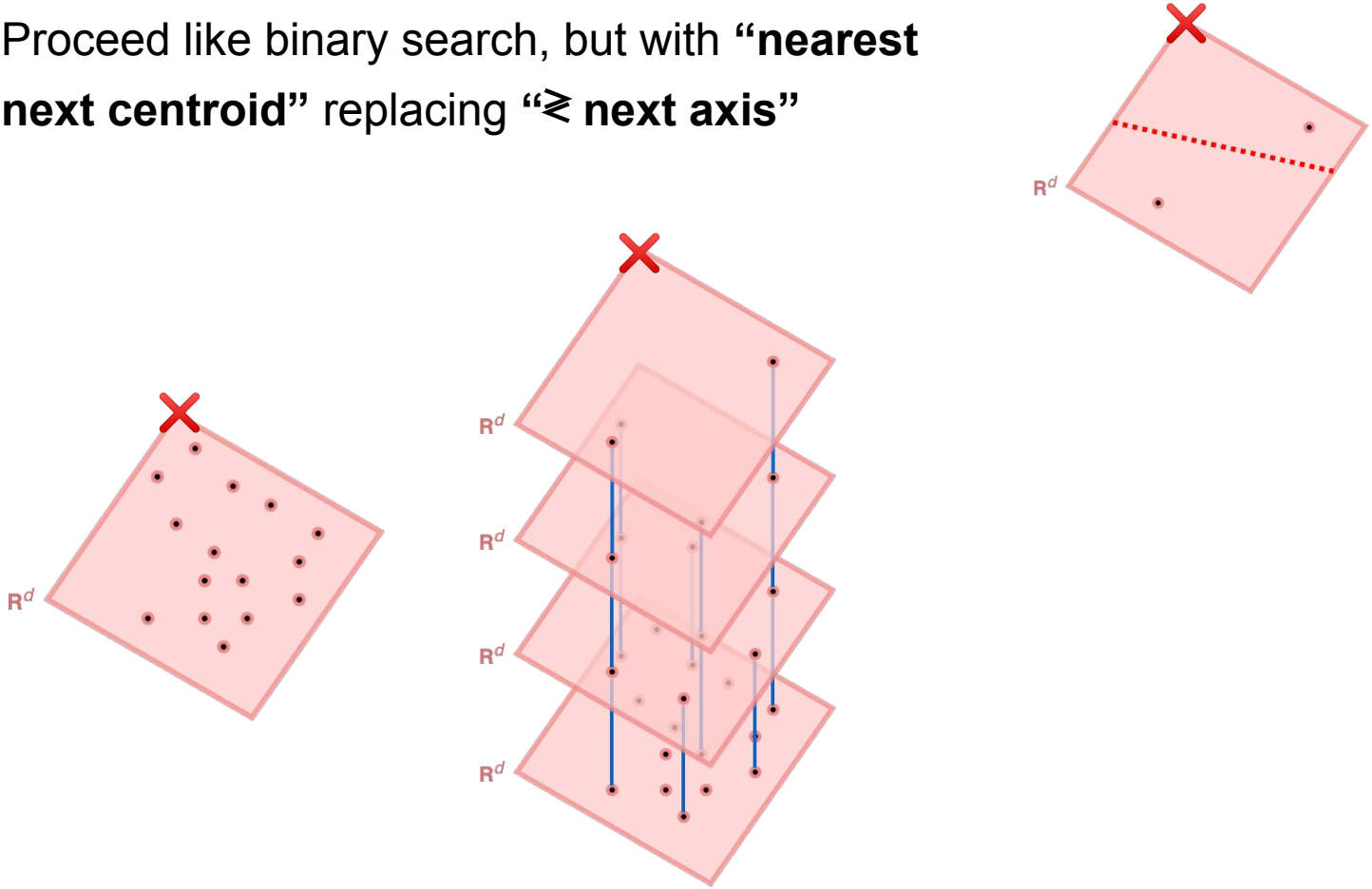

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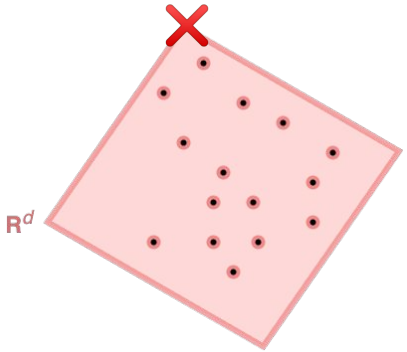

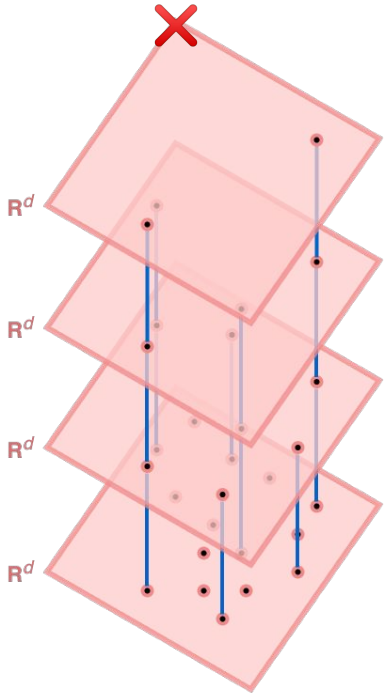

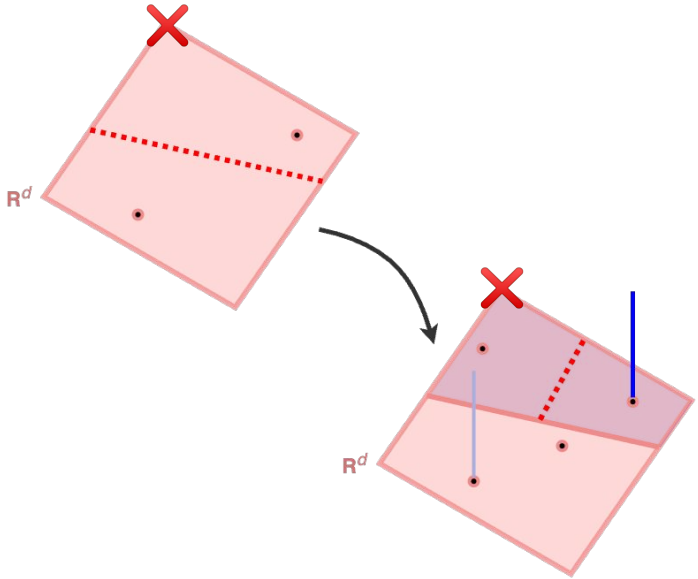

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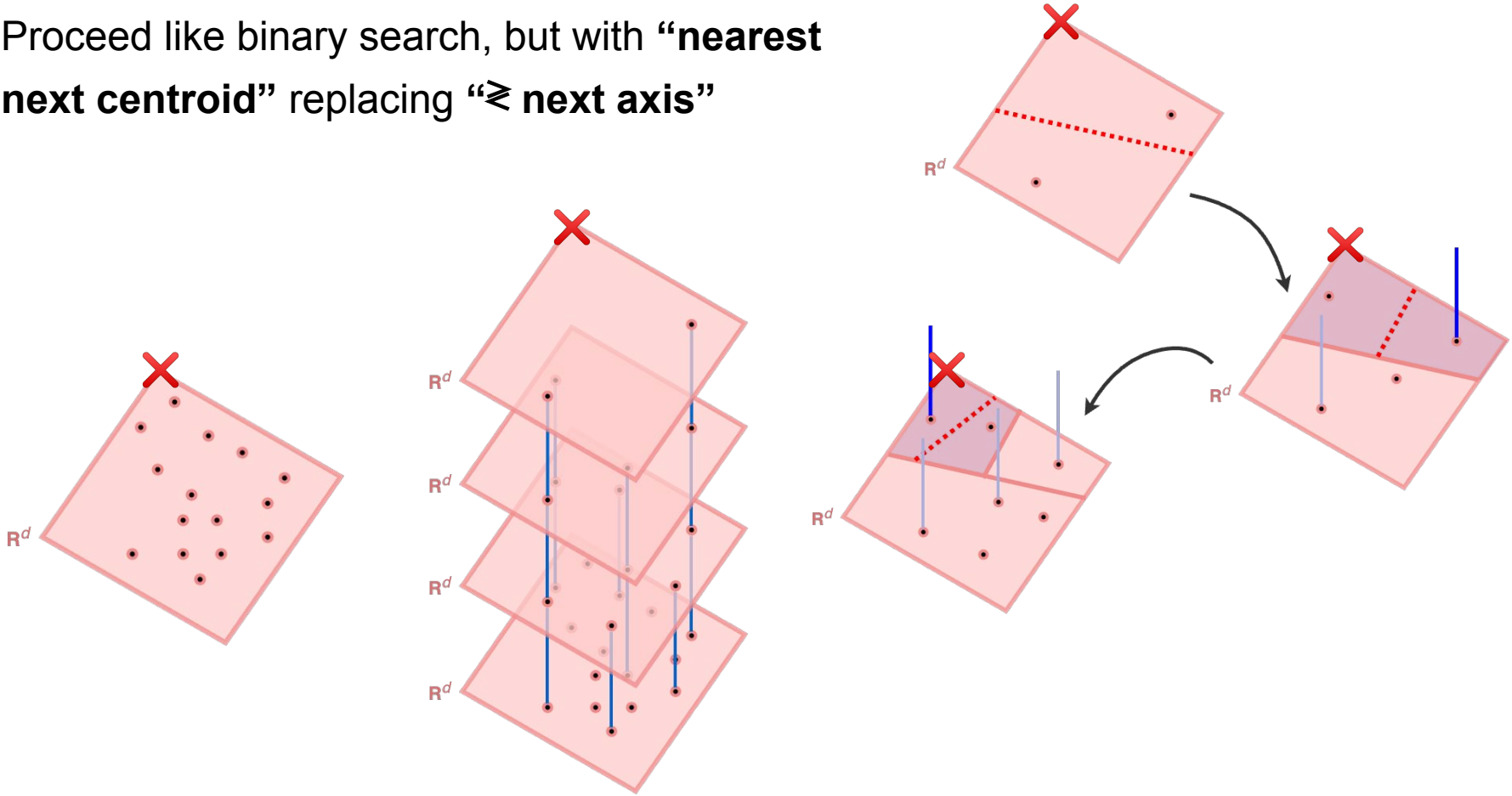
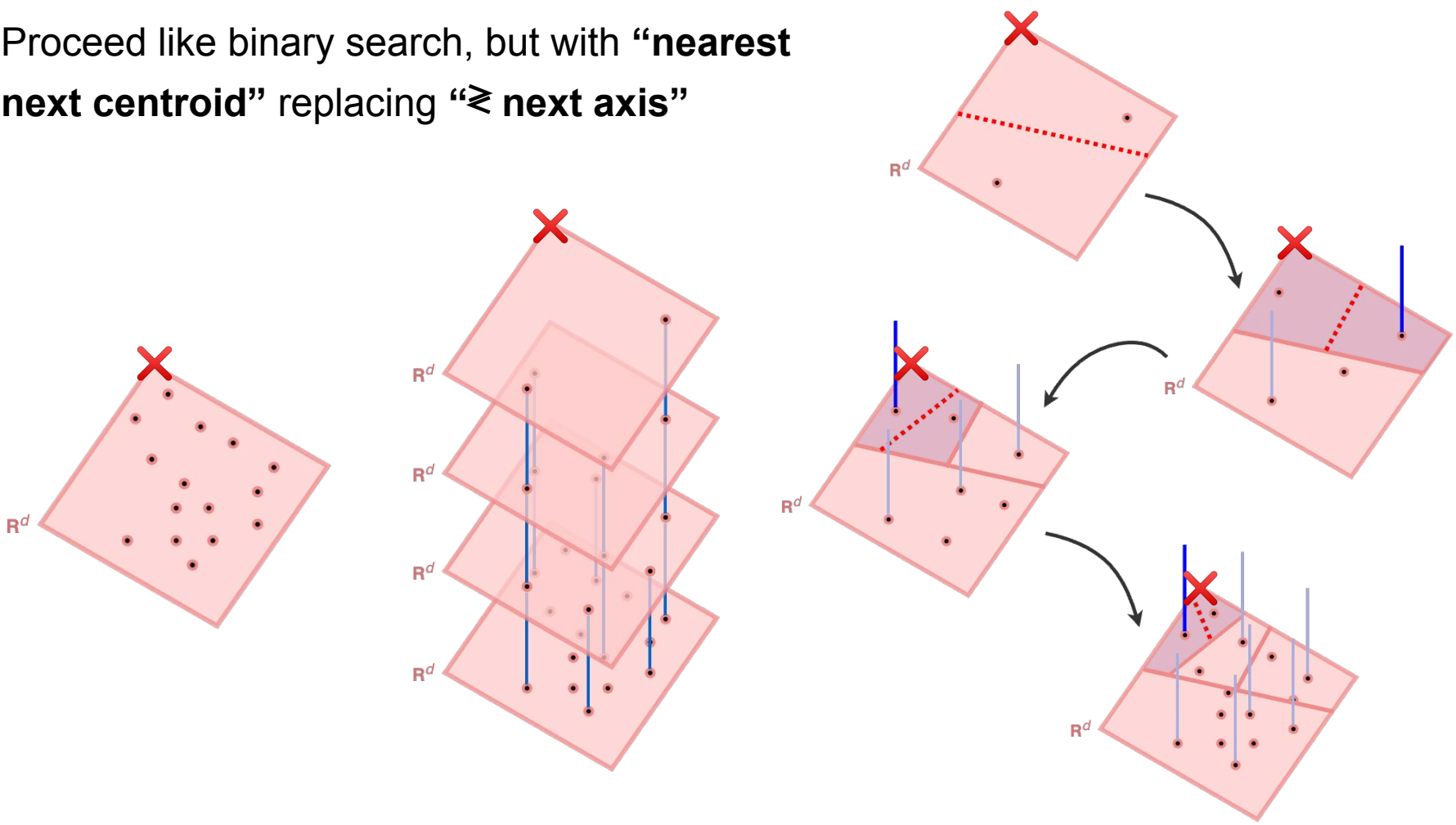


Proceed like binary search, but with **“nearest next centroid”** replacing **“≷ next axis”**



Thank you!