Robust Control for a Class of Nonlinearly Coupled Hierarchical Systems with Actuator Faults

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Road Map

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  - Controller design
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Problem Formulation

System model

$$\mathcal{H}: \begin{cases} \dot{z} = f(z,w) - g(z,w)\phi(y_1, \cdots, y_n) \\ y_i = \left[\begin{array}{c|c} A_i & b_i \\ \hline c_i^T & 0 \end{array}\right] u_i, i = 1, \dots, n \end{cases}$$

The problem: To stabilize the high-level system with uncertainty and disturbance (w) and improve the performance of the controller for the low-level subsystems while some are subjected to actuator faults.

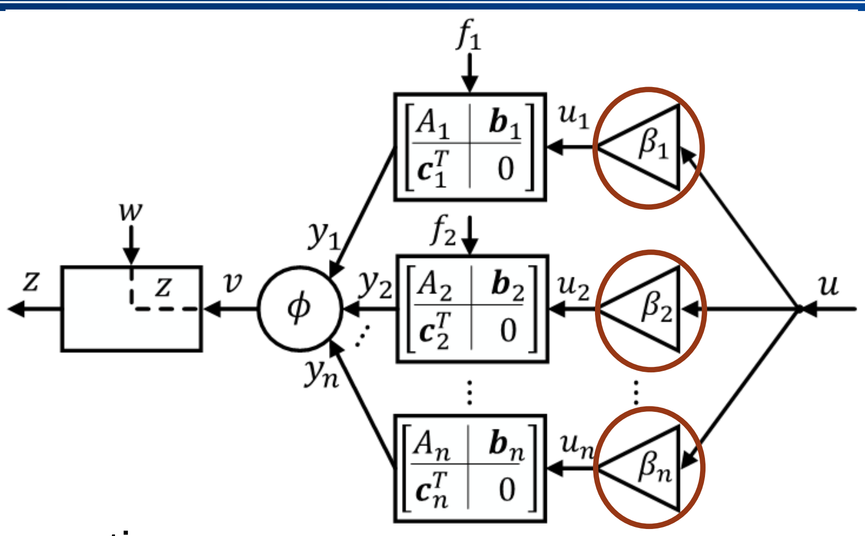
Major challenges:

1) The high-level system and low-level subsystems are coupled through a nonlinear function $\phi(y_1, \cdots, y_n)$, which is not invertible, i.e, we cannot obtain individual $y_i s$ by designing that function as the control input.
2) Faults in the low-level subsystems which degrade the overall performance of the system.

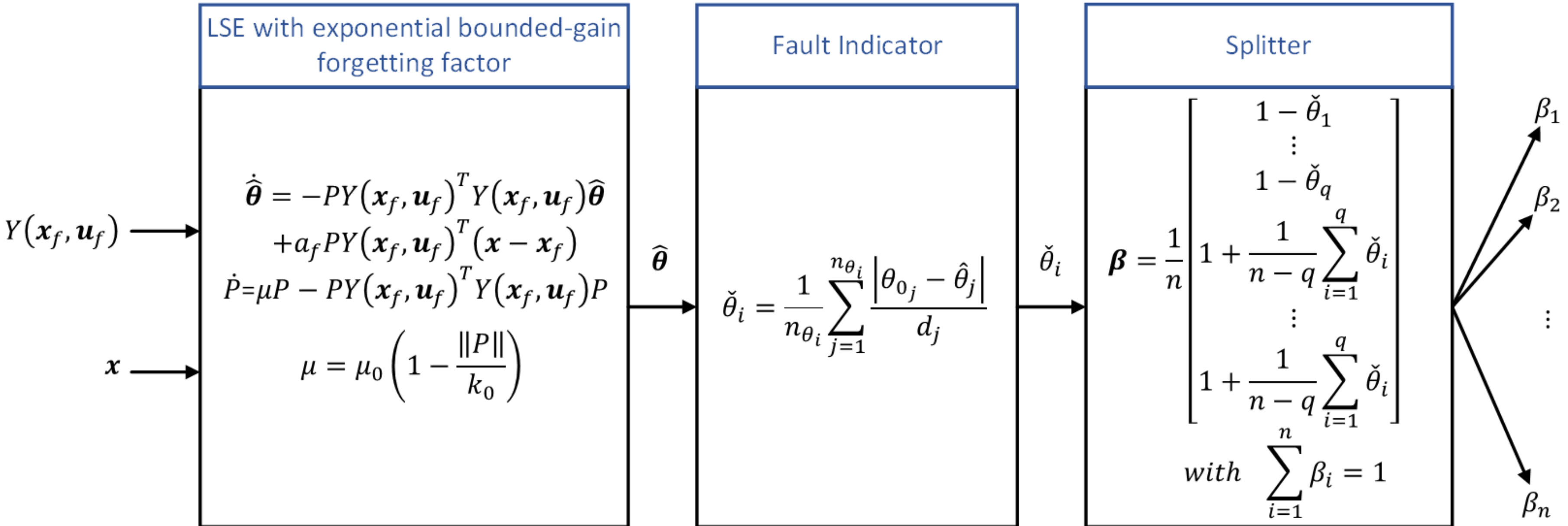
Control Development

Control allocation

$$\mathcal{H}:\begin{cases}\dot{z} = f(z,w) - g(z,w)\phi(y_1,\cdots,y_n)\\ y_i = \begin{bmatrix} A_i & b_i \\ c_i^T & 0\end{bmatrix} u_i, i = 1,\ldots,n\end{cases}$$



Control allocation schematic



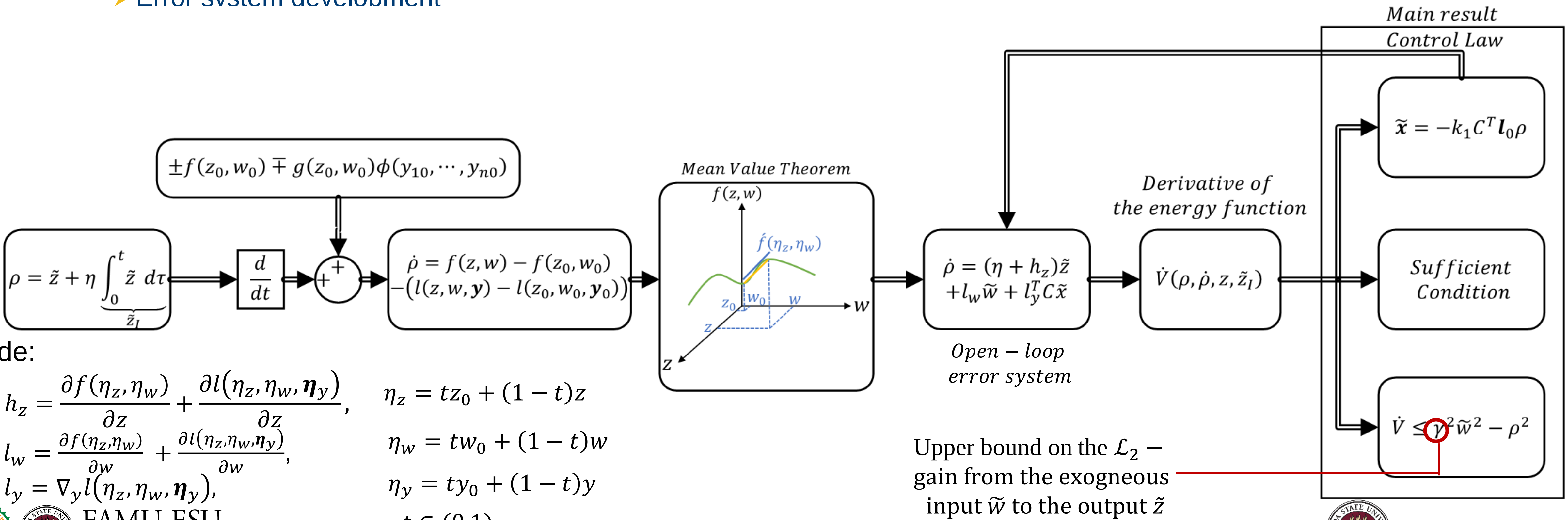
Assumptions:

- The low-level subsystems are linearly parametrized
  $$\dot{\boldsymbol{x}}_i = A_i\boldsymbol{x}_i + \boldsymbol{b}_i u_i = Y_i(\boldsymbol{x}_i,u_i)\boldsymbol{\theta}_i$$
  $$\boldsymbol{\theta}_i \in [\boldsymbol{\theta}_{0i} - \boldsymbol{d}_i, \boldsymbol{\theta}_{0i} + \boldsymbol{d}_i]$$
- The system is persistently excited
  $$\int_t^{t+T} Y^T Y \, d\sigma \geq \alpha_1 I$$

Control Development

■ High-level control

- Objective: Regulate the error dynamics $(\tilde{z} = z - z_0)$ against the disturbance and uncertainty in the model
- Methodology:
  - Filtered error $\rho = \tilde{z} + \eta \int_0^t \tilde{z}\ d\tau$
  - Open-loop error dynamics $\dot{\rho} = (\eta + h_z)\tilde{z} + l_w\tilde{w} + \boldsymbol{l}_y^T C\tilde{\boldsymbol{x}}$
  - Error system development



Aside:

$$h_z = \frac{\partial f(\eta_z, \eta_w)}{\partial z} + \frac{\partial l(\eta_z, \eta_w, \boldsymbol{\eta}_y)}{\partial z},$$

$$l_w = \frac{\partial f(\eta_z, \eta_w)}{\partial w} + \frac{\partial l(\eta_z, \eta_w, \boldsymbol{\eta}_y)}{\partial w},$$

$$l_y = \nabla_y l(\eta_z, \eta_w, \boldsymbol{\eta}_y),$$

$$\eta_z = tz_0 + (1 - t)z$$

$$\eta_w = tw_0 + (1 - t)w$$

$$\eta_y = ty_0 + (1 - t)y$$

$$t \in (0,1)$$

High-level Design

Main Result:

Consider the following high-level auxiliary control law

$$\tilde{\boldsymbol{x}} = -k_1 C^T \boldsymbol{l}_0 \rho$$

, where $k_1 > 0$ is a control gain and $\boldsymbol{l}_0$ satisfies $\boldsymbol{l}_y^T C C^T \boldsymbol{l}_0 \geq \alpha$. Given $\gamma > 0$, if the control gain is chosen to satisfy the sufficient condition

$$k_1 \geq \frac{(\bar{h}_z + 2\eta)^2}{4\alpha\eta} + \frac{\bar{l}_w^2}{4\alpha\gamma^2} + \frac{1}{\alpha},$$

then the corresponding closed-loop error system is $\mathcal{L}_2 - gain$ stable and the $\mathcal{L}_2 - gain$ from the disturbance $\tilde{w}$ to the regulation error $\tilde{z}$ is upper bounded by γ.

- Assumptions:
  - The high-level dynamics is sufficiently smooth; thus, the uncertain terms are bounded:

$$|h_z| \leq \bar{h}_z, \ |l_w| \leq \bar{l}_w$$

, moreover, there exists $\boldsymbol{l}_0$ and $\alpha > 0$ such that

$$\boldsymbol{l}_y^T C C^T \boldsymbol{l}_0 \geq \alpha$$

Proof sketch:

$$V = \frac{1}{2}\rho^2 + \frac{1}{2}\eta^2 \tilde{z}_I^2$$

$$\dot{V} = \rho\dot{\rho} + \eta^2 \tilde{z}_I \dot{\tilde{z}}_I$$

Inserting the open-loop error dynamics, then the control law, and then add and subtract $\gamma^2\tilde{w}^2 - \rho^2$ leads to

$$\dot{V} \leq -\left(k_1\alpha - \frac{(h_z + 2\eta)^2}{4\eta} - \frac{\bar{l}_w^2}{4\gamma^2}\right)\rho^2 + (\gamma^2\tilde{w}^2 - \rho^2)$$

$$\leq \gamma^2\tilde{w}^2 - \rho^2$$

Upper bound on the $\mathcal{L}_2 -$ gain from the exogenous input $\tilde{w}$ to the output $\tilde{z}$

Low-level Design

Low-level control

- **Objective:** Improve tracking performance for faulty low-level subsystems using the splitter
  - Error: $\boldsymbol{e} = \widetilde{\boldsymbol{x}} + k_1 C^T \boldsymbol{l}_0$
  - Closed-loop error dynamics: $\dot{\boldsymbol{e}} = (A - B\boldsymbol{\beta}\boldsymbol{k}_2^T)\boldsymbol{e} + \boldsymbol{w}_\rho$, where $\boldsymbol{w}_\rho \triangleq k_1(C^T \boldsymbol{l}_0 \dot{\rho} - AC^T \boldsymbol{l}_0 \rho)$

Main result:

Consider the low-level control law

$$\boldsymbol{u} = -\varphi(\boldsymbol{x}_0) - \boldsymbol{\beta}\boldsymbol{k}_2^T \boldsymbol{e}.$$

Given $\alpha_l > 0$, if the control gain $\boldsymbol{k}_2$ is chosen to satisfy

$$\left(2\underline{a}_r + \alpha_l\right)I - B\boldsymbol{\beta}\boldsymbol{k}_2^T - \boldsymbol{k}_2\boldsymbol{\beta}^T B^T \leq 0,$$

where $\underline{a}_r = \max Re(eig\{A\})$, then the closed-loop error system is finite-gain $\mathcal{L}_2 - stable$ and the $\mathcal{L}_2 - gain$ with respect to the exogenous input $\boldsymbol{w}_\rho$ is upper bounded by $\frac{\lambda_1}{\lambda_2}$, where $\lambda_1 > 0$, and $\lambda_2 > 0$ satisfy

$$\frac{1}{\lambda_1} + \lambda_2 = \alpha_l$$

Proof sketch:

$$V = \frac{1}{2}\|\boldsymbol{e}\|^2$$

Taking the first time-derivative and substituting the closed-loop error system leads to

$$\dot{V} = \frac{1}{2}\boldsymbol{e}^T(A^T + A - B\beta\boldsymbol{k}_2^T - \boldsymbol{k}_2\beta^T B^T)\boldsymbol{e} + \boldsymbol{e}^T\boldsymbol{w}_\rho$$

After using Young's inequality and adding and subtracting the term $\frac{\lambda_2\|e\|^2}{2}$ becomes

$$\dot{V} \leq \frac{1}{2}\boldsymbol{e}^T(A^T + A - B\beta\boldsymbol{k}_2^T - \boldsymbol{k}_2\beta^T B^T + \alpha_l I)\boldsymbol{e} + \frac{\lambda_2}{2}\left(\frac{\lambda_1}{\lambda_2}\|w_\rho\|^2 - \|e\|^2\right)$$

Case Study (Wind Turbine)

- WT dynamics
  - High-level dynamics:

$$f(z,w)=\frac{cw^3}{2Jz}\left(\frac{w}{z}-m_1\right)e^{\left(-m_2\frac{w}{z}\right)}-\frac{P_0}{Jz},$$

$$g(z,w)=\frac{cw^3}{6Jz}m_3e^{\left(-m_2\frac{w}{z}\right)},$$

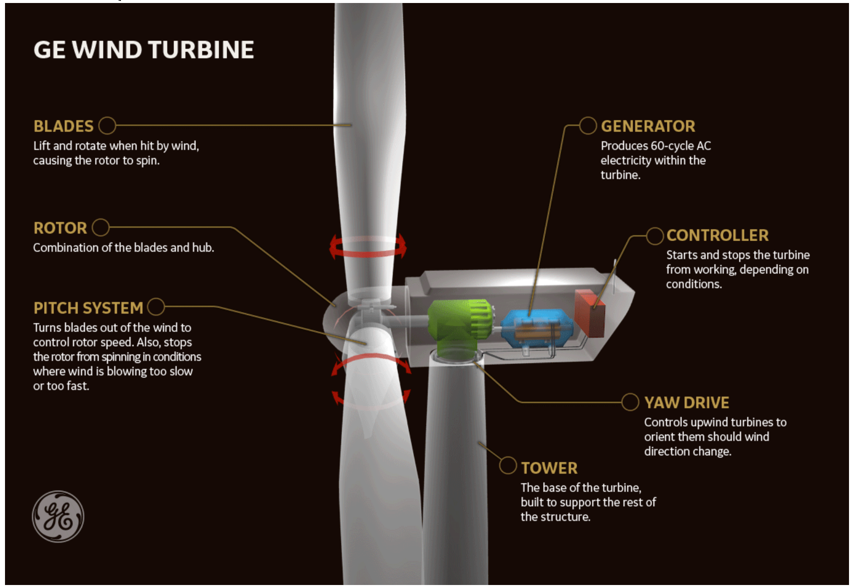
$$\phi(\boldsymbol{y})=\|\boldsymbol{y}\|_2^2,\ \mathbf{y}\in\mathcal{R}^3$$

  - Low-level dynamics

$$\dot{x}_i=\begin{bmatrix}0 & 1\\ -\omega_{ni}^2 & -2\xi\omega_{ni}\end{bmatrix}x_i+\begin{bmatrix}0\\ \omega_{ni}^2\end{bmatrix}u_i,i=1,2,3$$

where $\boldsymbol{x}_i=\begin{bmatrix}x_{1i}\\ x_{2i}\end{bmatrix}$

$$\mathcal{H}:\begin{cases}\dot{z}=f(z,w)-g(z,w)\phi(y_1,\cdots,y_n)\\ y_i=\begin{bmatrix}A_i & b_i\\ c_i^T & 0\end{bmatrix}u_i,i=1,\dots,n\end{cases}$$

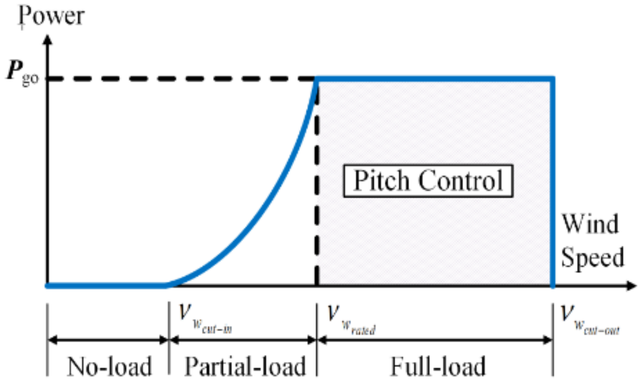


- Regression model

$$\check{x}_i=a_fx_{2i}-a_fx_{2fi},$$

$$\boldsymbol{\theta}_i=\begin{bmatrix}\omega_{ni}^2\\ 2\xi_i\omega_{ni}\end{bmatrix},$$

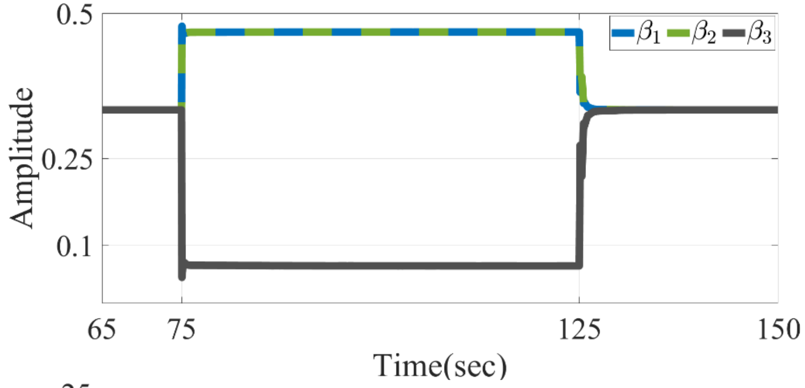
$$Y_i=\begin{bmatrix}u_{fi}-x_{1fi} & -x_{2fi}\end{bmatrix}$$

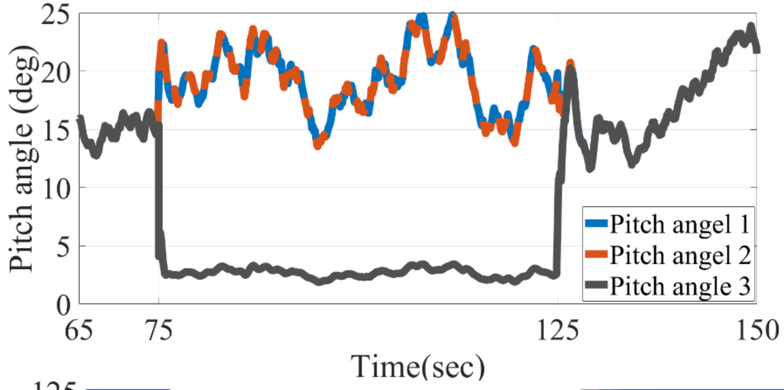


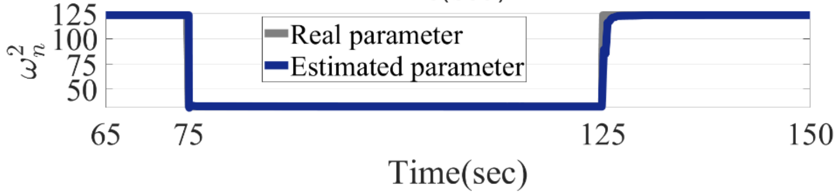
- Deviation indicator

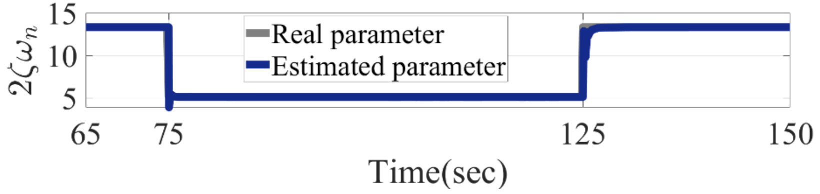
$$\check{\theta}_i=\frac{1}{2}\left(\frac{\left|\omega_{n0}^2-\widehat{\omega_{ni}^2}\right|}{d_\omega}+\frac{\left|(2\xi\omega_n)_0-\widehat{2\xi\omega_{ni}}\right|}{d_\xi}\right)$$

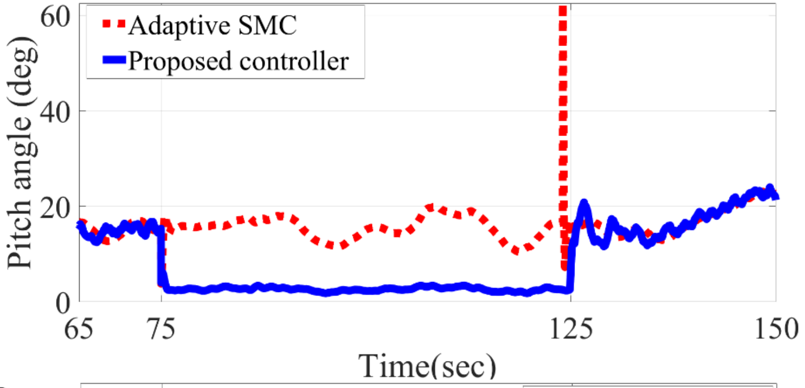
Case Study (Simulation results)

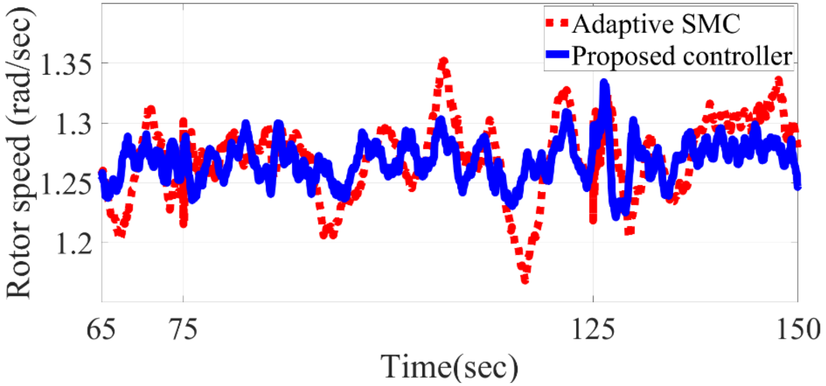


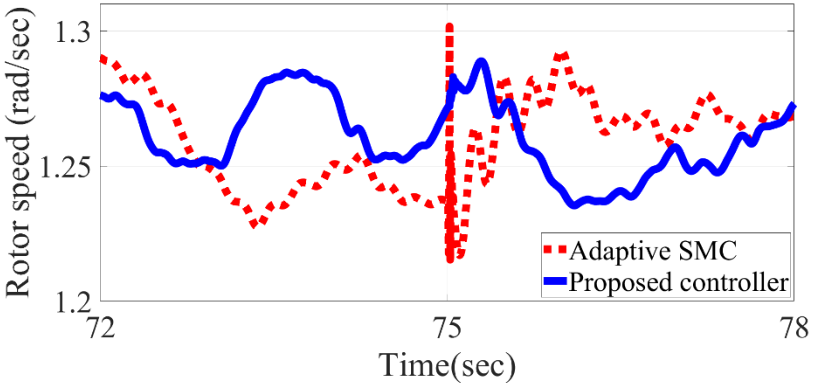












Simulations are available online at: https://github.com/Sina-eng/Robust-control-and-estimator-for-Wind-Turbine-with-Actuator-Fault-Simulation-FAST-NREL-

Conclusion and Future Works

- Conclusion:
  - A Splitter distributes control input automatically among low-level subsystems to deal with actuator faults
  - The problem of nonlinearly coupling between the low-level subsystems and the high-level dynamics is solved
  - An $\mathcal{L}_2 - gain$ based controller regulates the error despite an exogenous input and faults
- Future works for this class of nonlinearly coupled system:
  - I. Obtaining the minimum $\mathcal{L}_2 - gain$ of the system to improve the robustness of the controller
  - II. Making the robust controller less conservative
  - III. Sensor faults, and failure

THANK YOU

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[1] **S. Ameli**, O. M. Anubi, "**Hierarchical Robust Adaptive Control for Wind Turbines with Actuator Fault**", ASME Letters in Dynamic Systems and Control, 2021.

[2] **S. Ameli**, O. M. Anubi, "**Robust Control for a Class of Nonlinearly Coupled Hierarchical Systems with Actuator Faults**", Modeling, Estimation and Control Conference (MECC), Austin, 2021.

[3] **S. Ameli**, MJ. Morshed, A. Fekih "**Adaptive Integral Sliding Mode Design for the Pitch Control of a Variable Speed Wind Turbine**", IEEE Conference on Control Technology and application (CCTA), Hong Kong, 2019.