A Distributed Robust Adaptive Control for a class of Nonlinearly Coupled Hierarchical Systems with Actuator Faults

Sina Ameli

FAMU-FSU College of Engineering

RAS lab Physics-Data-Driven

Abstract

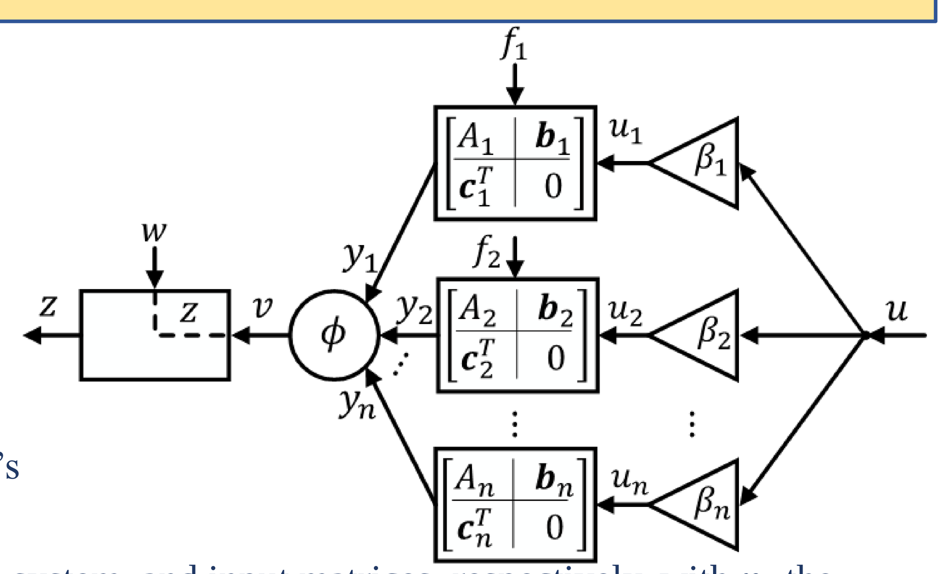
This talk proposes an approach to address the control challenges posed by a fault induced uncertainty in both dynamics and control input effectiveness for a class of hierarchical nonlinear system. The nonlinear system has two layers in which the high-level dynamics is coupled with low-level subsystems through a nonlinear function. Such nonlinearity in the coupling makes the problem challenging as it is not an invertible function. In other words, if the nonlinear function is designed as the control input for the high-level dynamics, the individual low-level outputs cannot be obtained from it. In addition, both the high-level dynamics and the low-level subsystems have multiplicative uncertainty in their control input effectiveness, but the former is due to an exogenous signal and the latter is due to the time-varying actuator faults.

To address this problem, a distributed robust-adaptive controller is designed. For the low-level subsystems, an adaptive mechanism estimates fast time-varying parameters, and then a splitting mechanism is designed to distribute the control input automatically among subsystems in response to the faults. The splitter works in a way that faulty subsystems receive less control inputs and healthy ones receive more inputs such that the overall control input effectiveness is preserved, and then a distributed state-feedback controller is designed to recover the system from the faults. In the high-level dynamics, a nonlinear L_2-gain based controller is designed to reject the exogenous disturbance. The resulting analysis guarantees a robust tracking of the high-level reference command signal.

Problem Formulation

System Model:

$$\mathcal{H}:\begin{cases}\dot{z} = f(z,w) - g(z,w)\phi(y_1,\cdots,y_n)\\ y_i = \begin{bmatrix} A_i & b_i\\ c_i^T & 0\end{bmatrix} u_i, i = 1,\ldots,n\end{cases}$$



$f:\mathbb{R}\times\mathbb{R}\mapsto\mathbb{R}$, $g:\mathbb{R}\times\mathbb{R}\mapsto\mathbb{R}$ are unknown smooth nonlinear functions, $\phi:\mathbb{R}\times\cdots\times\mathbb{R}\longmapsto\mathbb{R}$ is a smooth nonlinear function coupling the high-level to the low-level dynamics, $y_i\in\mathbb{R}$, $u_i\in\mathbb{R}$ are the *i-th* agent's low-level output and control input, respectively.

$A_i\in\mathbb{R}^{n_i\times n_i}$, $b_i\in\mathbb{R}^{n_i}$ are the corresponding uncertain system, and input matrices, respectively, with n_i the length of the associated state vector.

Problem: To stabilize the high-level system with uncertainty and disturbance (w) and improve the performance of the controller for the low-level subsystems while some are subjected to actuator faults.

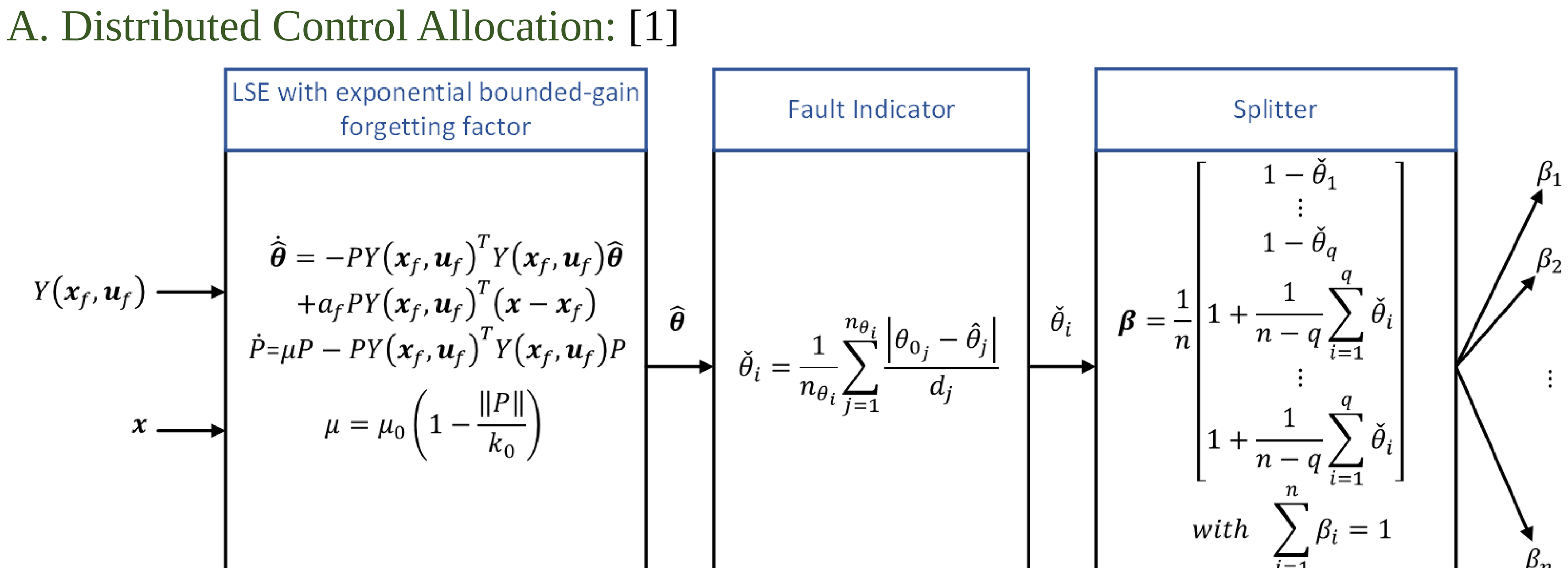
Major Challenges:

1) The high-level system and low-level subsystems are coupled through a nonlinear function $\phi(y_1,\cdots,y_n)$, which is not invertible, i.e, we cannot obtain individual $y_i s$ by designing that function as the control input.

2) Faults in the low-level subsystems which degrade the overall performance of the system.

Control Development

A. Distributed Control Allocation: [1]



Assumptions:

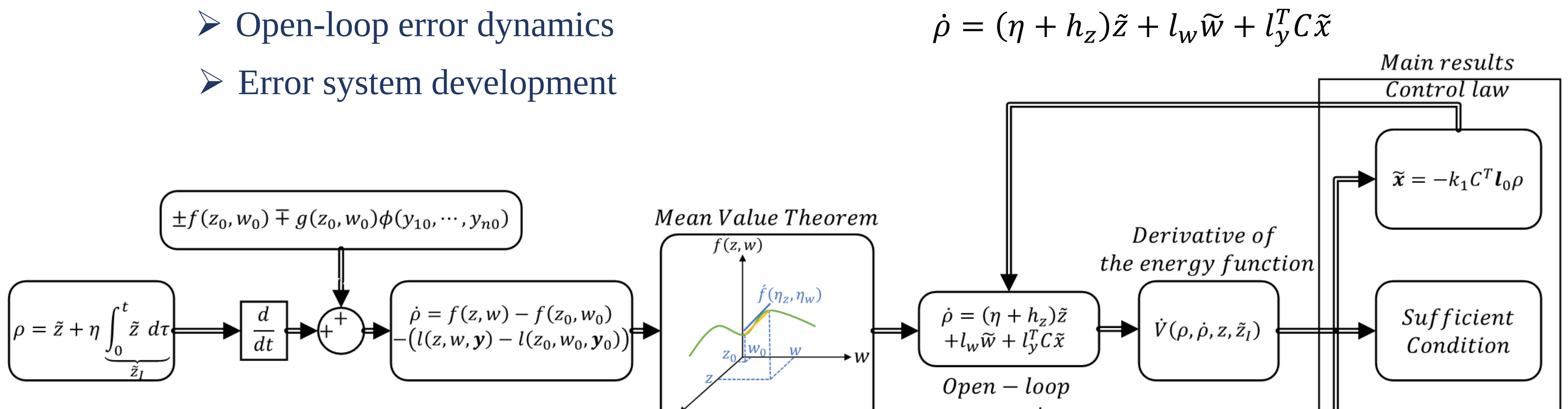
- The low-level subsystems are linearly parameterized:

$$\dot{\boldsymbol{x}}_i = A_i\boldsymbol{x}_i + \boldsymbol{b}_i u_i = Y_i(x_i,u_i)\boldsymbol{\theta}_i$$
$$\boldsymbol{\theta}_i\in[\boldsymbol{\theta}_{0i}-\boldsymbol{d}_i\ \boldsymbol{\theta}_{0i}+\boldsymbol{d}_i]$$

- The system is persistently excited. $\int_t^{t+T} Y^T Y\, d\sigma \geq \alpha_1 I$

B. High-level Control

- **Objective:** Regulate the error dynamics $\tilde{z} = z - z_0$ against the disturbance and uncertainty in the model.
- **Methodology:**
  - Filtered error $\rho = \tilde{z} + \eta\int_0^t \tilde{z}\ d\tau$
  - Open-loop error dynamics $\dot{\rho} = (\eta + h_z)\tilde{z} + l_w\tilde{w} + l_y^T C\tilde{x}$
  - Error system development



$$h_z = \frac{\partial f(\eta_z,\eta_w)}{\partial z} + \frac{\partial l(\eta_z,\eta_w,\boldsymbol{\eta}_y)}{\partial z},\quad \eta_z = tz_0 + (1-t)z$$
$$l_w = \frac{\partial f(\eta_z,\eta_w)}{\partial w} + \frac{\partial l(\eta_z,\eta_w,\boldsymbol{\eta}_y)}{\partial w},\quad \eta_w = tw_0 + (1-t)w$$
$$l_y = \nabla_y l(\eta_z,\eta_w,\boldsymbol{\eta}_y),\quad \eta_y = ty_0 + (1-t)y,\quad t\in(0,1)$$

Upper bound on the $\mathcal{L}_2-$gain from the exogeneous input $\tilde{w}$ to the output $\tilde{z}$

Main Result:

Consider the following high-level auxiliary control law [1], [2]

$$\tilde{\boldsymbol{x}} = -k_1 C^T\boldsymbol{l}_0\rho,$$

where $k_1 > 0$ is a control gain and $\boldsymbol{l}_0$ satisfies $\boldsymbol{l}_y^T CC^T\boldsymbol{l}_0 \geq \alpha$. Given $\gamma > 0$, if the control gain is chosen to satisfy the sufficient condition

$$k_1 \geq \frac{(\bar{h}_z+2\eta)^2}{4\alpha\eta} + \frac{\bar{l}_w^2}{4\alpha\gamma^2} + \frac{1}{\alpha},$$

then the corresponding closed-loop error system is $\mathcal{L}_2-gain$ stable and the $\mathcal{L}_2-gain$ from the disturbance $\tilde{w}$ to the regulation error $\tilde{z}$ is upper bounded by γ.

Assumptions:

- The high-level dynamics is sufficiently smooth; thus, the uncertain terms are bounded:

$$|h_z|\leq\bar{h}_z,\ |l_w|\leq\bar{l}_w,$$

moreover, there exists $\boldsymbol{l}_0$ and $\alpha > 0$ such that

$$\boldsymbol{l}_y^T CC^T\boldsymbol{l}_0 \geq \alpha$$

C. Low-level Control

- **Objective:** Improve the tracking performance of the faulty low-level subsystems using the splitter
  - Error $\boldsymbol{e} = \boldsymbol{a} + k_1 C^T\boldsymbol{l}_0$
  - Closed-loop error dynamics $\dot{\boldsymbol{e}} = (A - B\boldsymbol{\beta}\boldsymbol{k}_2^T)\boldsymbol{e} + \boldsymbol{w}_\rho,\ \boldsymbol{w}_\rho \triangleq k_1(C^T\boldsymbol{l}_0\dot{\rho} - AC^T\boldsymbol{l}_0\rho)$

Main Result:

Consider the low-level control law

$$\boldsymbol{u} = -\varphi(\boldsymbol{x}_0) - \boldsymbol{\beta}\boldsymbol{k}_2^T\boldsymbol{e}.$$

Given $\alpha_l > 0$, if the control gain $\boldsymbol{k}_2$ is chosen to satisfy

$$(2\underline{a}_r + \alpha_l)I - B\boldsymbol{\beta}\boldsymbol{k}_2^T - \boldsymbol{k}_2\boldsymbol{\beta}^T B^T \leq 0,$$

where $\underline{a}_r = \max Re(eig\{A\})$, then the closed-loop error system is finite-gain $\mathcal{L}_2 - stable$ and the $\mathcal{L}_2 - gain$ with respect to the exogenous input $\boldsymbol{w}_\rho$ is upper bounded by $\frac{\lambda_1}{\lambda_2}$, where $\lambda_1 > 0$, and $\lambda_2 > 0$ satisfy

$$\frac{1}{\lambda_1} + \lambda_2 = \alpha_l$$

Case Study: Wind Turbine

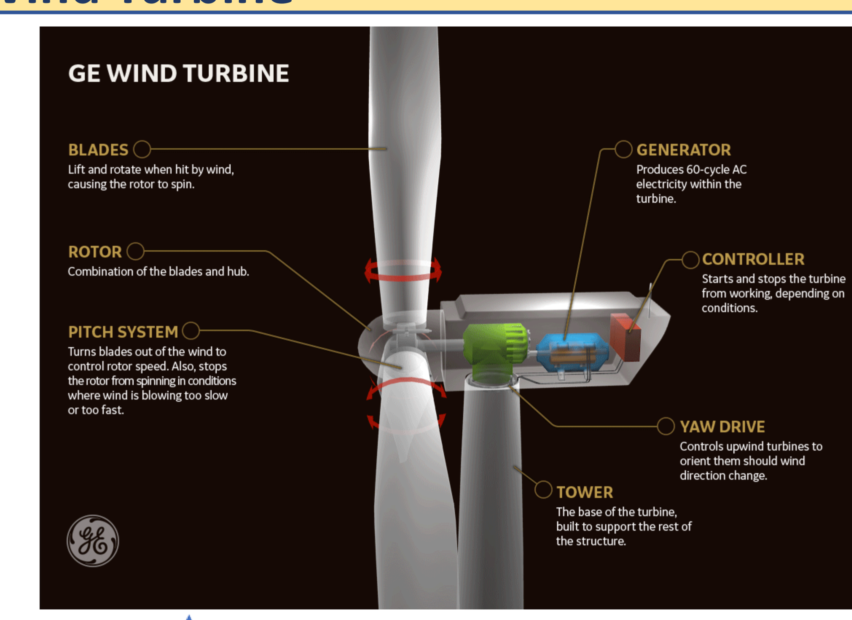
- Wind turbine model (WT) dynamics [2]
  - High-level dynamics:

$$f(z,w) = \frac{cw^3}{2Jz}\left(\frac{w}{z} - m_1\right)e^{\left(-m_2\frac{w}{z}\right)} - \frac{P_0}{Jz},$$
$$g(z,w) = \frac{cw^3}{6Jz}m_3 e^{\left(-m_2\frac{w}{z}\right)},$$
$$\phi(\boldsymbol{y}) = \|\boldsymbol{y}\|_2^2,\ \boldsymbol{y}\in\mathcal{R}^3$$

  - Low-level dynamics

$$\dot{x}_i = \begin{bmatrix} 0 & 1\\ -\omega_{ni}^2 & -2\xi\omega_{ni}\end{bmatrix}x_i + \begin{bmatrix}0\\ \omega_{ni}^2\end{bmatrix}u_i, i = 1,2,3$$

,where $\boldsymbol{x}_i = \begin{bmatrix}x_{1i}\\ x_{2i}\end{bmatrix}$

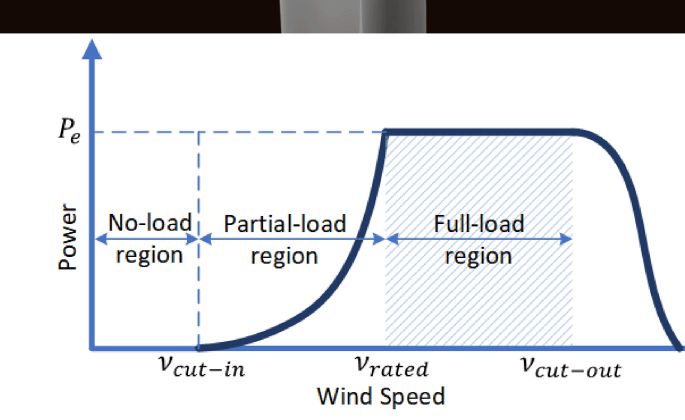


- Regression model

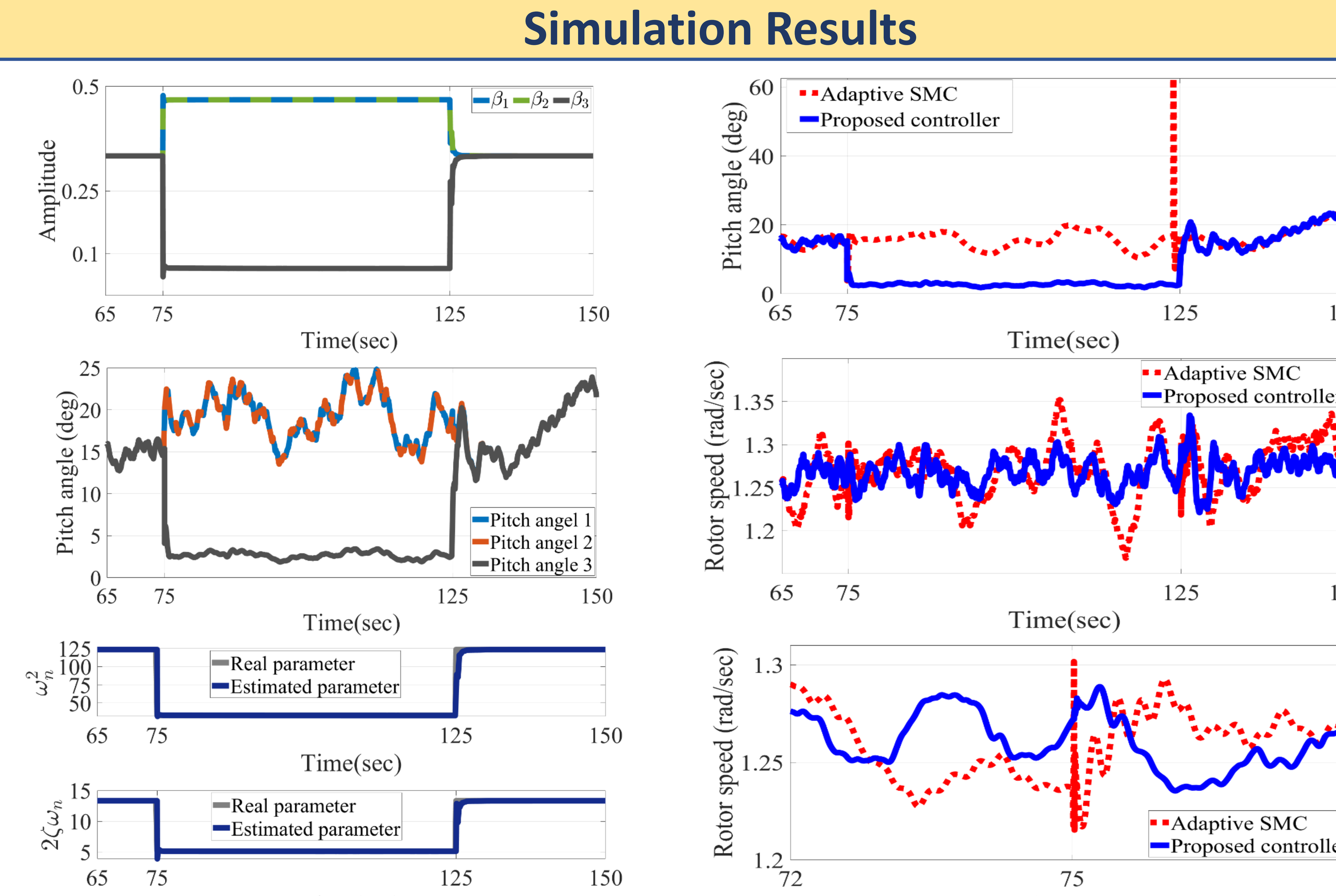
$$\check{x}_i = a_f x_{2i} - a_f x_{2fi},$$
$$\boldsymbol{\theta}_i = \begin{bmatrix}\omega_{ni}^2\\ 2\xi_i\omega_{ni}\end{bmatrix},\ Y_i = [u_{fi} - x_{1fi}\quad -x_{2fi}]$$

- Deviation indicator

$$\check{\vartheta}_i = \frac{1}{2}\left(\frac{|\omega_{n0}^2 - \widehat{\omega_{ni}^2}|}{d_\omega} + \frac{|(2\xi\omega_n)_0 - \widehat{2\xi\omega_{ni}}|}{d_\xi}\right)$$



Simulation Results



Conclusion

- A Splitter was designed to distribute control inputs automatically among low-level subsystems in the presence of faults
- The nonlinear coupling is solved using the mean value theorem; that is $\tilde{\boldsymbol{x}}$ is the auxiliary control instead of $\phi(y_1,\cdots,y_n)$
- An $\mathcal{L}_2 - gain$ based controller regulates the error despite the disturbance and faults

Future Work

I. Obtain the minimum $\mathcal{L}_2 - gain$ of the system to improve the robustness of the controller
II. Make the robust controller to be less conservative
III. Consider sensor faults, and failure

References

[1] Ameli, S. and Anubi, O.M. "Robust Control for a Class of Nonlinearly Coupled Hierarchical Systems with Actuator Faults" IFAC, Modeling, Estimation, and Control Conference, Austin, USA. [to appear].

[2] Ameli, S. and Anubi, O.M. "Hierarchical robust adaptive control for wind turbines with actuator fault" ASME Letters in Dynamic Systems and Control, 2021. [to appear].

Contacts

✓ Sina Ameli, Ph.D candidate
Florida State University
Email: sa19bk@my.fsu.edu

Advisor:
✓ Olugbenga Moses Anubi
IEEE Senior Member
Florida State University
Email: oanubi@fsu.edu
Website:
https://web1.eng.famu.fsu.edu/~anubi/

Simulations are available online at: https://github.com/Sina-eng/Robust-control-and-estimator-for-Wind-Turbine-with-Actuator-Fault-Simulation-FAST-NREL-