

Understanding Wasserstein t-SNE

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1.2 Hierarchical Data

1.2.1 Loosing Information by taking the mean

Chapter 2

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Analysis: Is there structure in Covariance?

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Outlook and Discussion

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4.1.1 Improving the runtime of Exact Wasserstein

4.2 Finding more Use-Cases

4.2.1 Medical Data