EXT 1: Calculus (Ext1), C2 Further Calculus Skills (Y12)

Harder Trig Calculus (Ext1)

Teacher: Alireza Razzaghi-Pour

Exam Equivalent Time: 16.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

Questions

1. Calculus, EXT1 C2 2016 HSC 5 MC

Which expression is equal to $\int \sin^2 2x \, dx$?

(A) $\frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c$

(B) $\frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) + c$

(C) $\frac{\sin^3 2x}{6} + c$

(D) $\frac{-\cos^3 2x}{6} + c$

---

2. Calculus, EXT1 C2 2012 HSC 7 MC

Which expression is equal to $\int \sin^2 3x \, dx$?

(A) $\frac{1}{2}\left(x - \frac{1}{3}\sin 3x\right) + C$

(B) $\frac{1}{2}\left(x + \frac{1}{3}\sin 3x\right) + C$

(C) $\frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$

(D) $\frac{1}{2}\left(x + \frac{1}{6}\sin 6x\right) + C$

---

3. Calculus, EXT1 C2 2019 HSC 11e

Find $\int 2\sin^2 4x \, dx$. *(2 marks)*

---

4. Calculus, EXT1 C2 2010 HSC 2a

The derivative of a function $f(x)$ is given by

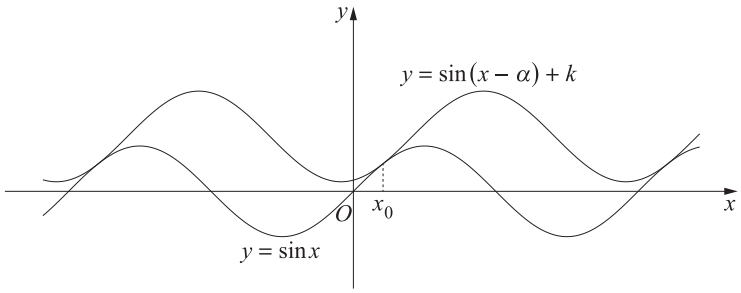
$$f'(x) = \sin^2 x.$$

Find $f(x)$, given that $f(0) = 2$. *(2 marks)*

---

5. Calculus, EXT1 C2 2019 HSC 14c

The diagram shows the two curves $y = \sin x$ and $y = \sin(x - \alpha) + k$, where $0 < \alpha < \pi$ and $k > 0$. The two curves have a common tangent at x_0 where $0 < x_0 < \frac{\pi}{2}$.



(i) Explain why $\cos x_0 = \cos(x_0 - \alpha)$. *(1 mark)*

(ii) Show that $\sin x_0 = -\sin(x_0 - \alpha)$. *(2 marks)*

(iii) Hence, or otherwise, find k in terms of α. *(2 marks)*

Worked Solutions

1. Calculus, EXT1 C2 2016 HSC 5 MC

$$\begin{aligned}
&\int \sin^2 2x \, dx \\
&\quad = \frac{1}{2}\int (1-\cos 4x)\, dx \\
&\quad = \frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c \\
&\Rightarrow \ A
\end{aligned}$$

---

2. Calculus, EXT1 C2 2012 HSC 7 MC

$$\begin{aligned}
&\text{Using: } \sin^2 a = \frac{1}{2}(1 - \cos 2a) \\
&\int \sin^2 3x \, dx = \frac{1}{2}\int (1 - \cos 6x)\, dx \\
&\qquad\qquad = \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C \\
&\Rightarrow \ C
\end{aligned}$$

---

3. Calculus, EXT1 C2 2019 HSC 11e

$$\begin{aligned}
&\text{Using } \sin^2\theta = \frac{1}{2}(1-\cos 2\theta): \\
&\int 2\sin^2 4x \, dx = \int 2 \times \frac{1}{2}(1-\cos 8x)\, dx \\
&\qquad\qquad = \int 1 - \cos 8x \, dx \\
&\qquad\qquad = x - \frac{1}{8}\sin x + C
\end{aligned}$$

Worked Solutions

4. Calculus, EXT1 C2 2010 HSC 2a

$$\begin{aligned}
&f'(x) = \sin^2 x \\
&f(x) = \int \sin^2 x \, dx \\
&\qquad = \int \frac{1}{2}(1-\cos 2x)\ dx \\
&\qquad = \int \frac{1}{2} - \frac{1}{2}\cos 2x \ dx \\
&\qquad = \frac{1}{2}x - \frac{1}{4}\sin 2x + c \\
&\text{Given } \ f(0) = 2, \\
&2 = \frac{1}{2} \times 0 - \frac{1}{4}\sin 0 + c \\
&\therefore c = 2 \\
&\therefore \ f(x) = \frac{1}{2}x - \frac{1}{4}\sin 2x + 2
\end{aligned}$$

---

5. Calculus, EXT1 C2 2019 HSC 14c

(i) $y_1 = \sin x$

$\dfrac{dy_1}{dx} = \cos x$

$y_2 = \sin(x - \alpha) + k$

$\dfrac{dy_2}{dx} = \cos(x - \alpha)$

At $x = x_0$, tangent is common

$\therefore \cos x_0 = \cos(x_0 - \alpha)$

♦ *Mean mark part (i) 47%.*

(ii) x_0 is in 1st quadrant (given)

Using part (i):

$\cos x_0 = \cos(x_0 - \alpha) > 0$

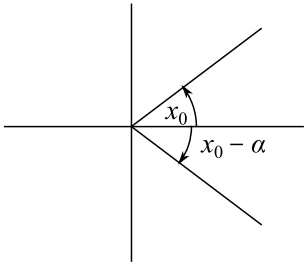
$\Rightarrow x_0 - \alpha$ is in 4th quadrant $(0 < \alpha < \pi)$

Since sin is positive in 1st quadrant and negative in 4th quadrant

$\Rightarrow \sin x_0 = -\sin(x_0 - \alpha)$

♦♦♦ *Mean mark part (ii) 19%.*

(iii)



When $x = x_0$,

$y_1 = \sin x_0$

$y_2 = \sin(x_0 - \alpha) + k$

$\sin x_0 = \sin(x_0 - \alpha) + k$

$= -\sin x_0 + k$

$k = = 2 \sin x_0$

Since $\cos x_0 = \cos(x_0 - \alpha)$

$x_0 = -(x_0 - \alpha)$

♦♦ *Mean mark part (iii) 21%.*

$2x_0 = \alpha$

$x_0 = \dfrac{\alpha}{2}$

$\therefore k = 2 \sin \dfrac{\alpha}{2}$