Fundamental Properties of Dynamic Occupancy Grid Systems for Vehicle Environment Perception

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***Abstract*—Using probabilistic reasoning to model and perceive driving environments is a challenging problem due to both geometric and dynamic random natures. The occupancy grid system, providing an intermediate representation for complicated environments and having many beneficial implications (such as avoiding direct data association and more freedom in data fusion), has been increasingly becoming a popular paradigm for vehicle environment perception. The conventional static occupancy grid (SOG) system only describes static environments; to incorporate dynamic information into the conventional occupancy grid is naturally desirable. The corresponding system is called dynamic occupancy grid (DOG). The DOG systems are still developing and some fundamental questions remain unanswered. In a statistical sense, the DOG extends the SOG in generalizing its grid as a random field defined over a parameter space in not only geometric space but also time domain. In this paper, under a formal statistical definition of the DOG, we carry out a Bayesian analysis and examine commonly-used assumptions and approximations in the literature. In view of works having been done, we present a particle-based multiple model approach for our DOG system and the corresponding results are given in a typical vehicle driving scenario.**

***Index Terms*—Dynamic Occupancy Grid, Binary Bayesian analysis, Particle Filter, Multiple Model, Hybrid System**

I. Introduction

Autonomous driving poses unique challenges for vehicle environment perception due to the complex driving environment the autonomous vehicle finds itself in. To detect, classify and track the external objects, an autonomous vehicle uses a variety of sensors, such as radars, cameras and Lidars. In order to overcome limitations of the individual sensors and to increase the likelihood of correct detections and classifications, the sensor data needs to undergo a fusion step to obtain more accurate estimation results. This sensor fusion either happens on an object level or, as described in more detail in this paper, the raw data level using occupancy grid maps.

Occupancy grid systems provide a popular yet reliable framework for vehicle environment perception. Compared to object-based approaches, the systems avoid object concept, handle directly grid cells and related information, and therefore have many beneficial characteristics (such as no direct data association and more freedom in data fusion). In the literature, there are two distinct systems: 1) static occupancy grid (SOG), assuming stationary obstacles, is a random field defined over geometric grid; 2) dynamic occupancy grid (DOG), which incorporates kinematic information into geometric grid cells, is a random field defined over not only geometric space but also dynamic space. Due to the inherently dynamic nature of perceiving environments and ever-increasing computational power, the DOG systems become more and more desirable.

In this paper, we examine some representative DOG works, including the popular 4-D Bayesian occupancy filter (BOF) approach described in [5], the 2-D histogram BOF in [4], the particle-based BOF approach in [9] and the particle-based hypotheses DOG approach in [6]. In view of their advantages and disadvantages, we hereby try to answer some fundamental questions about the DOG systems from a Bayesian analysis and aim to develop our own DOG environment perception system. We first, with a formal statistical definition for DOG, identify the assumptions and approximations for processing grid on a single-cell basis — an additional assumption is proposed about cell independence in prediction stage. We use the single-cell basis processing to avoid complicated parameter design and data validation/association in a multi-target tracking approach. We stay clear from multiple hypothesis tracker (MHT) due to unaffordable computational complexity. We deal with dynamic state only based on (already) occupied grid cells, rather than arguing cell-associated dynamic is moving-then-observed (e.g., track before detection). Some compromises between computational complexity and estimation accuracy are made: the cell occupancy state is obtained based on fast binary Bayesian analysis (BBA) [12] or DempsterShafer theory (DST); we explain the occupancy state variations (at a same cell) are caused by dynamic information "flow" among the grids over time. Finally, based on a typical use case in vehicle tracking, the effectiveness and efficiency of our DOG system using a particle-based multiple model (MM) approach is examined and verified. Noticeably, the MM approach used is a simple naive one — a fixed weight is given to a uniform probability density on velocity space (i.e., a certain amount of particles are representing possible opportunities of moving targets). What we show here is what we have in current running system; the more desirable switching multiple model approach [2][13] is an ongoing work.

The paper is organized as follows. In Section II, we examine DOG systems with a Bayesian analysis. In Section III, we present a particle-based MM BOF. In Section IV, a scenario study is carried out and in Section V summaries are presented.

II. The Problem

Conventional SOG is a random field defined over a discrete geometric grid. Accordingly, the DOG can be considered as

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a random field, with occupancy state in specified geometric grid and associated dynamic state taking values in a kinematic space, defined over a parameter space spanning in *both geometric manifold and time domain*.

In our discussion, we separate the cell geometric occupancy state $\mathbf{o}$ (with support $\{_{free}\ \bar{\mathbf{O}},_{occupied}\ \mathbf{O}\}$, denoted as $\mathcal{O}$) and its associated dynamic state $\mathbf{x}$ (with support $\mathcal{R}_d$; without loss of generality we only consider *velocity* components). The probability density function of the "composite" (occupancy and dynamic) information state for a specific cell c (the cell index $c\in\{1,\ldots,C\}$) at time k is

$$p(\mathbf{y}_k^c) \triangleq p(\mathbf{o}_k^c,\mathbf{x}_k^c) \tag{1}$$

Note that $\mathbf{y}_k^c$ and $(\mathbf{o}_k^c,\mathbf{x}_k^c)$ will be used interchangeably.

Given a sequence of observations (compensated for ego-vehicle motions already) $\mathbf{z}_{1:k} \triangleq \{\mathbf{z}_i, i=1,\ldots,k\}$ up to time k, we face following *joint* conditional (posterior) density probability over the whole grid

$$p\left(\mathbf{o}_k^{1:C},\mathbf{x}_k^{1:C}|\mathbf{z}_{1:k}\right) \tag{2}$$

Assuming Markov processes, we have

$$p\left(\mathbf{y}_k^{1:C}|\mathbf{z}_{1:k}\right)=\frac{1}{p(\mathbf{z}_k|\mathbf{z}_{1:\,k-1})}\,p\left(\mathbf{y}_k^{1:C}|\mathbf{z}_{1:\,k-1}\right)p\left(\mathbf{z}_k|\mathbf{y}_k^{1:C}\right) \tag{3}$$

This consists of the following two stages [2]

- *Prediction stage* — Chapman-Kologorov (CK) equation.

$$p\left(\mathbf{y}_k^{1:C}|\mathbf{z}_{1:\,k-1}\right)=\int p\left(\mathbf{y}_k^{1:C}|\mathbf{y}_{k-1}^{1:C}\right)p\left(\mathbf{y}_{k-1}^{1:C}|\mathbf{z}_{1:\,k-1}\right)d\mathbf{y}_{k-1}^{1:C} \tag{4}$$

- *Update stage* — Bayes measurement update

$$p\left(\mathbf{z}_k|\mathbf{y}_k^{1:C}\right)=\prod_{j=1}^{N_k}p\left(z_{k,j}|\mathbf{y}_k^{1:C}\right) \tag{5}$$

A. The Challenges

To obtain the exact solution from (4) and (5) is nearly impossible. The desire to handle the DOG on a single-cell basis puts even more strict requirements.

In general, we face three challenges ("||" loosely indicates different kinds of dependence):

$\mathbb{C}$1. At a specific time, the information of (neighboring) grid cells are geometrically dependent in general [12] — *"cell-spatial dependence"*, e.g., $(\mathbf{y}_k^{\xi}||\mathbf{y}_k^{\zeta})$ for different cells ξ and ζ.

$\mathbb{C}$2. At a specific grid cell, the dynamic information of different kinematic components (i.e., position, velocity, acceleration, etc.) are generally dependent [1] — *"dynamic component-dependence"*, e.g., the velocity-acceleration correlation $(\mathbf{v}_k^c||\mathbf{a}_k^c)$.

$\mathbb{C}$3. The dynamic processes (patterns) of the grids over consecutive time instances are dependent [1] — *"grid-temporal-dependence"*, e.g., $(\mathbf{y}_k^{1:C}||\mathbf{y}_{k-1}^{1:C})$.

B. Assumptions and Approximations

In the literature, many attempts have been made on systems. We examine the following popular works by focusing on how the dynamic (velocity) information is handled: the 4-D BOF (Bayesian Occupancy filter) approach in [5], the 2-D histogram BOF in [4], the particle-based BOF approach in [9] and the particle-based hypotheses DOG approach in [6].

Assuming independence of cells in estimation, one has the following prior batch

$$p\left(\mathbf{o}_k^{1:C},\mathbf{x}_k^{1:C}|\mathbf{z}_{1:k}\right)\approx\prod_{c=1}^{C}p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{z}_{1:k}\right) \tag{6}$$

and the CK recursion is

$$\begin{aligned}p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{z}_{1:\,k-1}\right)=&\int\int p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{o}_{k-1}^{1:C},\mathbf{x}_{k-1}^{1:C}\right)\\&\cdot p\left(\mathbf{o}_{k-1}^{1:C},\mathbf{x}_{k-1}^{1:C}|\mathbf{z}_{1:\,k-1}\right)d\mathbf{o}_{k-1}^{1:C}d\mathbf{x}_{k-1}^{1:C}\end{aligned} \tag{7}$$

In order to handle DOG system on a single-cell basis, an additional assumption on cell independence in prediction is made — this is to relax the cross-correlation among estimation errors in nearby cells [1](e.g., common process noise effect [1]). Assuming independence of cell in prediction, we can have

$$\begin{aligned}p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{z}_{1:\,k-1}\right)\propto&\int\int\Big[\prod_{i=1}^{C}p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{o}_{k-1}^i,\mathbf{x}_{k-1}^i\right)\\&p\left(\mathbf{o}_{k-1}^i,\mathbf{x}_{k-1}^i|\mathbf{z}_{1:\,k-1}\right)\Big]d\mathbf{o}_{k-1}^{1:C}d\mathbf{x}_{k-1}^{1:C}\end{aligned} \tag{8}$$

Compared to (7), this apparently brings significant simplicity for the DOG system; just an additional normalization step is required. Now the DOG systems can be processed on a single-cell basis — this is very desirable in practical situations: i) it is essentially a single target tracking technique accommodating characteristic of DOG systems — complicated parameter design in a multi-target tracking approach (say, based on RFS) is avoided; ii) the procedure can be efficiently implemented by parallel processing.

In the 4-D BOF [5], the following modification is made

$$p\left(\mathbf{o}_k^{1:C},\mathbf{x}_k^{1:C}|\mathbf{z}_{1:k}\right)\approx\prod_{c_{\mathbf{x}}=1}^{C_{\mathbf{x}}}p\left(\boldsymbol{\varkappa}_k^{c_{\mathbf{x}}}|\mathbf{z}_{1:k}\right) \tag{9}$$

where $\boldsymbol{\varkappa}\triangleq[\mathbf{o}\ \ \mathbf{x}]'$ with $\mathbf{x}$ of only velocity components, i.e., the grid originally defined in geometric space (in a SOG) is directly extended to the geometry-velocity (stacked) joint space for the DOG;$c_{\mathbf{x}}$ is indexed of discretized joint space. However, components in geometry space and in velocity space are generally correlated in estimation, leading to aliasing issue.

In the 2-D BOF approach [4], the desired $\mathbf{x}_{k-1}^i$ take values in a L-case velocity space $\{\mathbf{h}_{k-1,l}^i, l=1,\ldots,L\}$

$$\begin{aligned}p(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{z}_{1:\,k-1})\approx&\int\sum_{l=1}^{L}\Big[p\left(\mathbf{o}_k^c|\mathbf{o}_{k-1}^{1:C},\mathbf{h}_{k-1,l}^{1:C}\right)\\&\cdot p\left(\mathbf{x}_k^c|\mathbf{o}_k^c,\mathbf{o}_{k-1}^{1:C},\mathbf{h}_{k-1,l}^{1:C}\right)\Big]\\&\cdot p\left(\mathbf{o}_{k-1}^{1:C},\mathbf{x}_{k-1}^{1:C}|\mathbf{z}_{1:\,k-1}\right)d\mathbf{o}_{k-1}^{1:C}\end{aligned} \tag{10}$$

This is a simple grid-based histogram approach [12].

In the particle-based BOF approach [9], using a set of weighted particles $\left\{\mathbf{x}_{k-1}^{1:C,s},w_{k-1}^{1:C,s};s\in\{1,\ldots,S\}\right\}$ for all possible *geometrically occupied cell*, one has

$$\begin{aligned}p(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{z}_{1:\,k-1})\approx&\int\sum_{s=1}^{S}p\left(\mathbf{o}_k^c,\mathbf{x}_k^c|\mathbf{o}_{k-1}^{1:C},\left\{\mathbf{x}_{k-1}^{1:C,s},w_{k-1}^{1:C,s}\right\}\right)\\&\cdot p\left(\mathbf{o}_{k-1}^{1:C}|\mathbf{z}_{1:\,k-1}\right)d\mathbf{o}_{k-1}^{1:C}\end{aligned} \tag{11}$$

This is a non-parametric particle-based approach [12].

Moreover, as in [6], a special kind of dual-nature particles that represent both velocity hypothesis and static blocks of the environment, the corresponding DOG system is described by "flows" of the particles. We therefore present the following particle-based multiple model approach.

[1]Put simply, given $p(EF)=p(E)p(F)$, in general $p(G|EF)\neq\frac{1}{\text{const}}p(G|E)p(G|F)$.

III. Particle-Based BOF with Multiple Model Scheme

On a single-cell basis and using the fundamental assumptions:

- *Markov property for joint process* $\{\mathbf{o}_k^c, \mathbf{x}_k^c, k=1,2,\dots\}$.
- *Independence of cells in estimation.*
- *Independence of cells in prediction.*

we estimate occupancy and dynamic state in a two-stage manner

$$p(\mathbf{o}_k^c, \mathbf{x}_k^c | \mathbf{z}_{1:k}) = p(\mathbf{x}_k^c | \mathbf{o}_k^c, \mathbf{z}_{1:k}) p(\mathbf{o}_k^c | \mathbf{z}_{1:k}) \tag{12}$$

i.e., we obtain occupancy state first and then the dynamic state. This leads to the following CKB recursion

$$\begin{array}{ccc} \left\{p\left(\mathbf{o}_{k-1}^i, \mathbf{x}_{k-1}^i | \mathbf{z}_{1:k-1}\right)\right\} & \left\{ \begin{array}{l} \rightarrow \left\{p\left(\mathbf{o}_k^c | \mathbf{o}_{k-1}^i, \mathbf{x}_{k-1}^i\right)\right\} \\ \rightarrow \left\{p\left(\mathbf{x}_k^c | \mathbf{o}_k^c, \mathbf{o}_{k-1}^i, \mathbf{x}_{k-1}^i\right)\right\} \end{array} \right. & \\ p(\mathbf{o}_k^c, \mathbf{x}_k^c | \mathbf{z}_{1:k-1}) & \rightarrow & p(\mathbf{z}_k | \mathbf{o}_k^c, \mathbf{x}_k^c) \end{array}$$

There are some extremely important aspects in this

- Fundamentally different from object-based Bayesian estimation that generally only one prior is available, the DOG system requires considering every possible priors of the whole grid.
- The dynamic estimates from different cells "overlapping" together essentially provide a multi-modal estimate (e.g., a Gaussian mixture) at the grid cell. Therefore, a multiple-model approach is a natural choice.
- As there is no object concept in DOG, direct dynamic observations from the measurements can provide a reliable/fast *correction* for the dynamic estimation. In our present application, in additional to a Lidar system, we have range rate information from Radar system to provide significant performance boost.

A. Occupied Cell and Free Space

First, the uncertainty of occupancy state switching between *free* and *occupied* is accounted for by occupancy transition probability matrix (TPM):

$$\boldsymbol{\pi}_k^{\mathcal{O}} \triangleq \left[p(\mathbf{o}_k^c | \mathbf{o}_{k-1}^c)\right] \tag{13}$$

where $\mathbf{o}_k^c, \mathbf{o}_{k-1}^c \in \mathcal{O}$. This TPM should be application-oriented and state-dependent but a fixed state-independent time invariant transition matrix is used in scenario study.

Given a set of measurements, we obtain the occupancy state of a grid cell by BBA/DST first [12]. The choice of inverse sensor model is a reasonable compromise for alleviating the computational complexity, due to large set of Lidar data. Then CK equation is all about $p(\mathbf{O}_k^c, \mathbf{x}_k^c | \mathbf{z}_{1:k-1})$, instead of $p(\mathbf{o}_k^c, \mathbf{x}_k^c | \mathbf{z}_{1:k-1})$. In the following discussion, the total number of occupied cell is now $\bar{C}(k)$ out of C, i.e., $c \in \{1, \dots, \bar{C}(k)\}$. Also we drop the conditional side $\mathbf{z}_{1:k-1}$ and the time index of $\bar{C}(k)$ for conciseness.

B. Particle-based BOF for Dynamic State

- *Prior(s)*

Now starting with particles $\{\mathbf{x}_k^{c,s}, w_k^{c,s}, s=1,2,\dots N_k^c\}$, we can have

$$p(\mathbf{O}_k^c, \mathbf{x}_k^c) \approx \sum_{s=1}^{N_k^c} w_k^{c,s} \delta\left(\mathbf{x}_k^c - \mathbf{x}_k^{c,s}\right) \tag{14}$$

$$p(\mathbf{O}_k^c) = \int p(\mathbf{O}_k^c, \mathbf{x}_k^c) d\mathbf{x}_k^c \approx \sum_{s=1}^{N_k^c} w_k^{c,s} \tag{15}$$

$$p(\mathbf{x}_k^c | \mathbf{O}_k^c) \approx \sum_{s=1}^{N_k^c} \frac{w_k^{c,s}}{p(\mathbf{O}_k^c)} \delta\left(\mathbf{x}_k^c - \mathbf{x}_k^{c,s}\right) \triangleq \sum_{s=1}^{N_k^c} \bar{w}_k^{c,s} \delta\left(\mathbf{x}_k^c - \mathbf{x}_k^{c,s}\right) \tag{16}$$

Note that the set $\{\mathbf{x}_k^{c,s}, w_k^{c,s}\}$ contains normalized particles for the joint distribution $p(\mathbf{O}_k^c, \mathbf{x}_k^c)$ whilst the set $\{\mathbf{x}_k^{c,s}, \bar{w}_k^{c,s}\}$, *incorporating the occupancy information*, contains normalized particles for the dynamic distribution $p(\mathbf{x}_k^c | \mathbf{O}_k^c)$.

- *Prediction*

we have the dynamic transition event for each particle

$$\mathbf{x}_{k+1}^i = f\left(\mathbf{x}_k^i\right) + \mathbf{v}_k \tag{17}$$

A nearly constant velocity is used in our scenario study.

According to the dynamic transition event model

$$\begin{aligned} p\left(\mathbf{x}_{k+1}^c, \mathbf{O}_{k+1}^c | \mathbf{O}_k^{1:\bar{C}}\right) &\approx \sum_i^{\bar{C}} \sum_{s=0}^{N_{k+1}^{c|i}} \frac{\bar{w}_k^{i,s}}{\eta_{k+1|k}^c} \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c|i,s}\right) \\ &\triangleq \sum_i^{\bar{C}} \sum_{s=0}^{N_{k+1}^{c|i}} \tilde{w}_k^{i,s} \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c|i,s}\right) \end{aligned} \tag{18}$$

where the normalization term is

$$\eta_{k+1|k}^c = 1 - \prod_{j=1}^{C} \left(1 - \sum_i^{\bar{C}} \sum_s \bar{w}_k^{i,s}\right) \tag{19}$$

with the production part indicates all the weighted particles that not jump to the cell c. Further, we can have

$$\begin{aligned} p\left(\mathbf{x}_{k+1|k}^c, \mathbf{O}_{k+1|k}^c\right) &\approx \sum_i^{\bar{C}} \sum_{s=0}^{N_{k+1}^{c|i}} \tilde{w}_k^{i,s} p\left(\mathbf{O}_k^i\right) \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c|i,s}\right) \\ &\triangleq \sum_i^{\bar{C}} \sum_{s=0}^{N_{k+1}^{c|i}} \breve{w}_k^{i,s} \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c|i,s}\right) \end{aligned} \tag{20}$$

Reindexing sets in (18) and (20) to accommodate in cell c:

$$\left\{\tilde{w}_k^{i,s}, \mathbf{x}_{k+1|k}^{c|i,s}\right\} \longleftrightarrow \left\{\bar{w}_{k+1|k}^{c,p}, \mathbf{x}_{k+1|k}^{c,p}\right\} \tag{21}$$

$$\left\{\breve{w}_k^{i,s}, \mathbf{x}_{k+1|k}^{c|i,s}\right\} \longleftrightarrow \left\{w_{k+1|k}^{c,p}, \mathbf{x}_{k+1|k}^{c,p}\right\} \tag{22}$$

where $i=1,\dots,\bar{C}$, $s=0,\dots,N_{k+1}^{c|i}$ and $p=1,\dots,\hat{N}_k^c$ with $\hat{N}_k^c$ is the total number of predicted particles reaching at cell c. That is,

$$p\left(\mathbf{x}_{k+1}^c, \mathbf{O}_{k+1}^c | \mathbf{O}_k^{1:\bar{C}}\right) \approx \sum_{p=0}^{\hat{N}_k^c} \bar{w}_k^{c,p} \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c,p}\right) \tag{23}$$

$$p\left(\mathbf{x}_{k+1|k}^c, \mathbf{O}_{k+1|k}^c\right) \approx \sum_{p=0}^{\hat{N}_k^c} w_k^{c,p} \delta\left(\mathbf{x}_{k+1|k}^c - \mathbf{x}_{k+1|k}^{c,p}\right) \tag{24}$$

Just as weight difference shown between the particle sets in (14) and (16), the $\left\{w_{k+1|k}^{c,p}, \mathbf{x}_{k+1|k}^{c,p}\right\}$ and $\left\{\bar{w}_{k+1|k}^{c,p}, \mathbf{x}_{k+1|k}^{c,p}\right\}$ are also particle sets with and without incorporating the occupancy information.

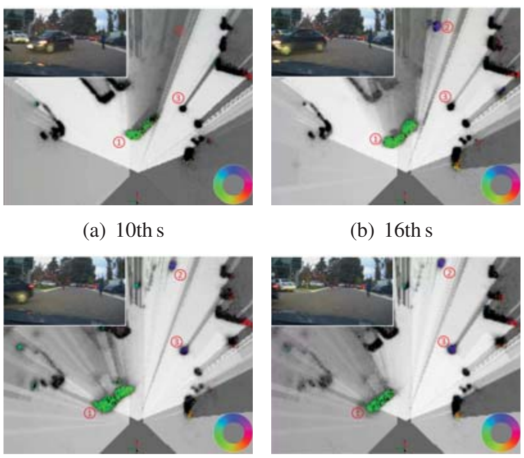
- *Measurements, Simple Multiple Model Scheme and Resampling*

The measurements in our current vehicle sensor system are the point cloud (range/angle) from a Lidar system and range/range rate from a Radar system. The information from point cloud are used for occupancy state updating only (via BBA). The range rate information from Radar system are affordable for likelihood evaluation, leading more accurate dynamic estimates and therefore guide the particles flow in a correct and accurate manner.

From the properties of DOG system, it is natural/desirable to represent the dynamic state by a multi-modal mixture and an MM approach is used. The MM approach is particularly important for objects of frequent move-stop-move behaviors. An IMM-PF estimator is possible but we choose to use a simple multiple model scheme in the particle resampling step — a small portion of particles in each cell is drawn from an uniform distribution over a specified velocity space. This can be considered as a fixed weight naive multiple model (or a birth model as that in some multi-target tracking approaches).

The particle resampling scheme we use is very similar to the one described in [6] but with one possible improvement: we use predicted particle weights to reflect expected occupancy, rather than the ratio of the number of arriving particles over a fixed number at each cell.

IV. Scenario Study



(a) 10th s (b) 16th s

(c) 23rd s (d) 24th s

Fig. 1: DOG using multiple-model Particle-based BOF

We use a test vehicle equipped with a Lidar and two Radars. For the details of sensor specification, sensor configuration, data collections, detailed measurement model and some other implementation issues, please refer to our companion work [8]. As shown in Fig. 1, the chosen scenario is a typical use case in vehicle tracking: T-cross road with three targets — a constant turn vehicle (marked by 1), an occluded-then-appears bicyclist (marked by 2) and a move-stop-move pedestrian (marked by 3).

The constantly moving vehicle is not an issue for our DOG system and the constant turn behavior is well-described by the associated particles: the dense green color particles indicates it moves towards lower left corner.

For the bicyclist, at the first 10 seconds or so, is occluded by the moving vehicle and appears at the time of about 16th second. Right after the first appearance, the small cluster of blue color particles well-indicates the bicyclist is moving away from the ego-vehicle; it is worth noting that using or not using range rate information make big difference: the former leads particles to picking up the dynamic information almost instantaneously; latter can still see the dynamic catch but not that dense.

For the pedestrian with move-stop-move behavior (in a sudden manner): at first, the marked (by 3) cluster of black particles indicates the static occupied cells associated with the pedestrian; then the pedestrian starts to move at about 23th second. The dynamic is first captured by a small portion of blue color particles and then becoming denser as time goes by. Without proposed MM scheme, the particles associated to the pedestrian will mostly die away if the pedestrian behaves in a move-stop-move manner, due to lack of good capture using single nearly constant velocity model on dynamic transition.

V. Summaries

In this paper, we present a feasible DOG vehicle environment perception system. An occupancy particle-based naive multiple mode approach is proposed based on a Bayesian analysis for "composite" (occupancy and dynamic) information state on a single-cell basis. The approximation and assumptions for existing representative works are examined. Several important fundamental properties of DOG system appear in our analysis. Aim to develop a real-time DOG system, some necessary compromises between estimation accuracy and computation complexity are made. A scenario study based on a typical vehicle driving use case validates the effectiveness and efficiency of the DOG system. In a statistical sense, the technique presented here is a single target tracking approach on a single cell basis accommodating fundamental properties of DOG systems.

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