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1. Is the value $13.333333\ldots$ rational or irrational?

Yes, it is rational. A rational number can be expressed as a fraction of integers. Let $x = 13.\overline{3}$.

Multiply both sides by 10: $10x = 133.\overline{3}$

Subtract: $10x - x = 133.\overline{3} - 13.\overline{3} \Rightarrow 9x = 120 \Rightarrow x = \frac{120}{9} = \frac{40}{3}$

Thus, $13.\overline{3} = \frac{40}{3}$, a *rational* number.

2. Evaluate $2 \times 10^{-2} \times 10^{\frac{3}{2}}$.

Combine exponents:

$10^{-2} \times 10^{\frac{3}{2}} = 10^{-2+\frac{3}{2}} = 10^{-\frac{1}{2}}$

Multiplying by 2:

$2 \times 10^{-\frac{1}{2}} = \frac{2}{\sqrt{10}} = \frac{2\sqrt{10}}{10} = \frac{\sqrt{10}}{5}$

3. Assuming the function $f(x) = 2x^4 - 3x^3 + 5x^2 - x + 7$ what is the slope at $x = 2$

First derivative:

$f'(x) = 8x^3 - 9x^2 + 10x - 1$

plug in $x = 2$:

$f'(2) = 8(2)^3 - 9(2)^2 + 10(2) - 1 = 64 - 36 + 20 - 1 = 47$

Slope at $x = 2$ is 47.

4. Assuming the function $f(x) = 2x^4 - 3x^3 + 5x^2 - x + 7$ What is the area under the curve between 2 and 6?

Integrate $f(x)$:

$\int f(x)\,dx = \frac{2}{5}x^5 - \frac{3}{4}x^4 + \frac{5}{3}x^3 - \frac{1}{2}x^2 + 7x$

Evaluate from 2 to 6:

$F(6) = \frac{2}{5}(7776) - \frac{3}{4}(1296) + \frac{5}{3}(216) - \frac{1}{2}(36) + 7(6) = 2522.4$

$F(2) = \frac{2}{5}(32) - \frac{3}{4}(16) + \frac{5}{3}(8) - \frac{1}{2}(4) + 7(2) = 26.133$

Area: $2522.4 - 26.133 = 2496.267 = \frac{37444}{15}$

5. True or False.

Determine whether each of the following statements is correct regarding Big-O notation and justify your answer.

1. $7n^2 + 3n\log n = O(n^2)$

- **Answer:** True
- **Explanation:** The dominant term is $7n^2$. Since n^2 grows faster than $n \log n$, the Big-O simplifies to $O(n^2)$.

2. $2^n + 5n^3 = O(2^n)$
   - **Answer:** True
   - **Explanation:** Exponential terms (2^n) dominate polynomial terms (n^3) for large n. Thus, 2^n dictates the growth rate.
3. $\log(n) + \log^2(n) = O(\log n)$
   - **Answer:** False
   - **Explanation:** $\log^2(n$ grows faster than $\log(n)$. The correct bound is $O(\log^2 n)$.
4. $n^{3/2} + 8 = O(n^2)$
   - **Answer:** True
   - **Explanation:** $n^{3/2} = \sqrt{n^3}$, which grows slower than n^2. Thus, $n^{3/2} + 8$ is bounded by $O(n^2)$.
5. $4n^3 + 2^n = O(n^3)$
   - **Answer:** False
   - **Explanation:** 2^n (exponential) cannot be bounded by n^3 (polynomial). The correct bound is $O(2^n)$.
6. $3^n + n^4 = O(3^n)$
   - **Answer:** True
   - **Explanation:** 3^n dominates n^4, so the entire expression is $O(3^n)$.

6. What is the time complexity of the function below? Explain.

Code:

```
int function(int n) {
    int i, j, m = 0;
    for (i = 1; i <= n; i *= 5) {
        for (j = 0; j <= i; j++) {
            m += 1;
        }
    }
    return m;
}
```

Analysis:

- **Outer Loop:** Runs $\log_5 n$ times because i grows as $1, 5, 25, \ldots, 5^k \leq n$.

- **Inner Loop:** For each i, it runs $i + 1$ times. The total iterations are:

$$\sum_{k=0}^{\log_5 n} (5^k + 1) \approx \sum_{k=0}^{\log_5 n} 5^k = \frac{5^{\log_5 n+1} - 1}{4} = O(n)$$

- **Total Time Complexity:** $O(n)$.

7. Given the array of positive integers, $\{$3, 2, 5, 10, 7$\}$, find the maximum product under the constraint that no two elements should be adjacent.

Array: $\{$3, 2, 5, 10, 7$\}$

Solution using Dynamic Programming:

- Define $dp[i]$ as the maximum product up to index i.
- Recurrence relation:

$$dp[i] = \max(dp[i-1],\ dp[i-2] \times \mathsf{arr}[i])$$

- Step-by-step calculation:

$$\begin{aligned}
&dp[0] = 3 \quad \text{(select 3)}\\
&dp[1] = \max(3, 2) = 3 \quad \text{(select 3)}\\
&dp[2] = \max(3, 3 \times 5) = 15 \quad \text{(select 3, 5)}\\
&dp[3] = \max(15, 3 \times 10) = 30 \quad \text{(select 3, 10)}\\
&dp[4] = \max(30, 15 \times 7) = 105 \quad \text{(select 3, 5, 7)}
\end{aligned}$$

- **Result:** Maximum product is $3 \times 5 \times 7 = 105$.