Notations

We use the following Hamiltonian with aperiodic boundary condition (ABC):

$$\hat{H}_K = -\sum_{x=1}^{L}\left(\hat{c}_x^\dagger \hat{c}_{x+1} + \delta \hat{c}_x^\dagger c_{x+1}^\dagger + \text{h.c.}\right) - \mu \sum_{j=1}^{L} \hat{c}_x^\dagger \hat{c}_x \ ; \tag{1}$$

where we consider $\hat{c}_{L+1} = -\hat{c}_1$.
In the scaling functions of this model, we fix these scaling variables:

$$\Upsilon = \tau_s L^{-(y_\mu + z)} \ , \tag{2}$$
$$\kappa = w_i \tau_s^{y_\mu/(y_\mu+z)} \ , \tag{3}$$
$$\Omega = w(t) \tau_s^{y_\mu/(y_\mu+z)} \ , \tag{4}$$
$$\tag{5}$$

with $w(t) = \mu(t) - \mu_c$ and $w(t) = t/\tau_s$.
Moreover, to analyze the system behavior, we introduce a set of observables:

- the time-dependent correlations functions:

$$C_x = \langle \psi(t) \big| \hat{c}_y^\dagger \hat{c}_{y+x} + \text{h.c.} \big| \psi(t) \rangle \ , \tag{6}$$
$$P_x = \langle \psi(t) \big| \hat{c}_y^\dagger \hat{c}_{y+x}^\dagger + \text{h.c.} \big| \psi(t) \rangle \ ; \tag{7}$$
$$\tag{8}$$

- the equilibrium correlations functions:

$$C_x^o = \langle GS(t) \big| \hat{c}_y^\dagger \hat{c}_{y+x} + \text{h.c.} \big| GS(t) \rangle \ , \tag{9}$$
$$P_x^o = \langle GS(t) \big| \hat{c}_y^\dagger \hat{c}_{y+x}^\dagger + \text{h.c.} \big| GS(t) \rangle \ ; \tag{10}$$
$$\tag{11}$$

- the adiabacity:

$$A = -\ln \left| \langle GS(t) \big| \psi(t) \rangle \right| \ ; \tag{12}$$

- the energy difference:

$$E - E_o = \langle \psi(t) \big| \hat{H}(t) \big| \psi(t) \rangle - \langle GS(t) \big| \hat{H}(t) \big| GS(t) \rangle \ ; \tag{13}$$

- the particle density:

$$\delta\rho(t) = \rho - \rho_c = \frac{\mathrm{N} - \mathrm{N}_c}{\mathrm{L}} \ . \tag{14}$$