Notes

I. MODEL

We consider the following Kitaev Hamiltonian:

$$\hat{H}_K = -\mu_i \sum_{x=1}^{L} \hat{c}_x^\dagger \hat{c}_x - \sum_{x=1}^{L-1} \left[\hat{c}_x^\dagger \hat{c}_{x+1} \hat{c}_x \hat{c}_{x+1} + \text{h.c.} \right] . \quad (1)$$

To monitor the critical behavior close the critical point $\mu_c = -2$ we introduce the following scaling variables:

$$\kappa_i = (\mu_i - \mu_c) L^{y_\mu} \ , \quad (2)$$

$$\eta = T L^z \ ; \quad (3)$$

where in the last equation, we introduce the initial temperature T of the system and $z = 1$ is the dynamical critical exponent. Since we take the Kitaev chain in equilibrium with the temperature T, the initial density matrix corresponds with the Gibbs mixture:

$$\hat{\rho}_T = e^{-\beta \hat{H}_K} \ , \quad (4)$$

$$\beta = \frac{1}{T} \ . \quad (5)$$

To evaluate the correlations in the state ρ_T, we take the steady state obtained by the limit $t \to +\infty$ of the solution associated with the Lindblad equation. The dissipation term is given by a set of equal thermal bath homogeneous coupled with each site [?].

Thus, introducing the two-point correlations functions as:

$$C(x,y) = \text{Tr}\left[\rho_T \hat{c}_x^\dagger \hat{c}_y + \text{h.c.} \right] \ , \quad (6)$$

$$P(x,y) = \text{Tr}\left[\rho_T \hat{c}_x \hat{c}_y + \text{h.c.} \right] \ ; \quad (7)$$

in the initial thermal steady state, their behavior is described by the scaling laws:

$$C(x,y;\mu_i,T) = L^{-2y_c} \mathcal{C}(x/L, y/L, \kappa_i, \eta) \ ; \quad (8)$$

$$P(x,y;\mu_i,T) = L^{-2y_c} \mathcal{P}(x/L, y/L, \kappa_i, \eta) \ . \quad (9)$$

II. OUT-OF-EQUILIBRIUM DYNAMICS

A. Unitary dynamics

Starting from the state introduced above, at $t = 0$, we change suddenly the value of the chemical potential μ_i to a final value μ_f. In other words, we perform a quench protocol on the Hamiltonian (1) from an initial value to another one. This process lead to a reformulation of the Bogoliubov bases in which the time evolution operator acts.

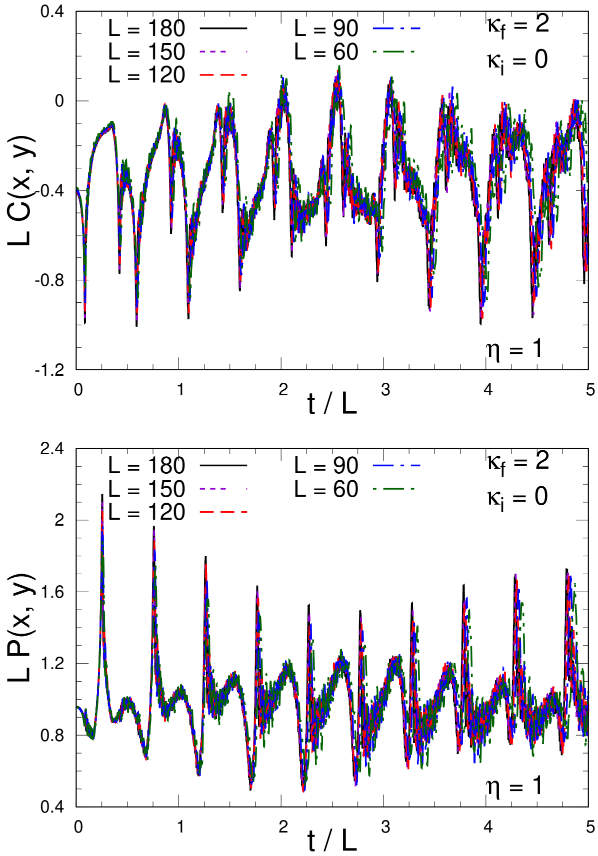

FIG. 1: Scaling behavior of the two-points correlations functions $C(x,y,t)$ (top) and $P(x,y,t)$ (bottom) for $x = L/3$, $y = 2L/3$ keeping the scaling variables $\kappa_i = 0$, $\kappa_f = 2$, $\eta = 1$ fixed in function of tL^{-z}, up to system size $L = 180$.

This operator is unitary because the state is not in contact with any thermal bath. The corresponding scaling law in Eqs. 8 become:

$$C(x,y,t;\mu_i,\mu_f,T) = L^{-2y_c} \mathcal{C}(x/L, y/L, tL^{-z}, \kappa_i, \kappa_f, \eta) \ , \quad (10)$$

$$P(x,y,t;\mu_i,\mu_f,T) = L^{-2y_c} \mathcal{P}(x/L, y/L, tL^{-z}, \kappa_i, \kappa_f, \eta) \ ; \quad (11)$$

where κ_f is the scaling variable associated the final chemical potential μ_f.

B. Thermal dissipation case

Now, in the case in which we turn on a homogeneous thermal dissipation mechanism, the time evolution is not more unitary, but it is the interplay between two processes:

- the Quench protocol;
- the dissipation arising from equal thermal baths with temperature T_b.

To write the scaling laws, we have to define new scaling variable whose aim is to tune the dissipation with the critical behavior:

$$\eta_b = T_b L^z \ ; \tag{12}$$
$$\Gamma = \gamma L^z \ . \tag{13}$$

The γ factor is simple the dissipation coupling which enters in the following Lindblad equation:

$$\frac{\partial}{\partial t}\rho = -i\left[\hat{H}, \rho\right] + \mathbb{D}[\rho] \ , \tag{14}$$

$$\mathbb{D}[\rho] = \gamma \sum_k (1 - f(\omega_k)) \left[2\hat{b}_k \rho \hat{b}_k^\dagger - \left\{\rho, \, \hat{b}_k^\dagger \hat{b}_k\right\}\right] + \\ + \gamma \sum_k f(\omega_k) \left[2\hat{b}_k^\dagger \rho \hat{b}_k - \left\{\rho, \, \hat{b}_k \hat{b}_k^\dagger\right\}\right] \tag{15}$$

where the Bogoliubov operator $\hat{b}_k$ corresponds with the quasi-particle annihilation operator of the k^{th} mode. Moreover, they are associated with the Hamiltonian after the quench, i.e. when the chemical potential is equal to μ_f.

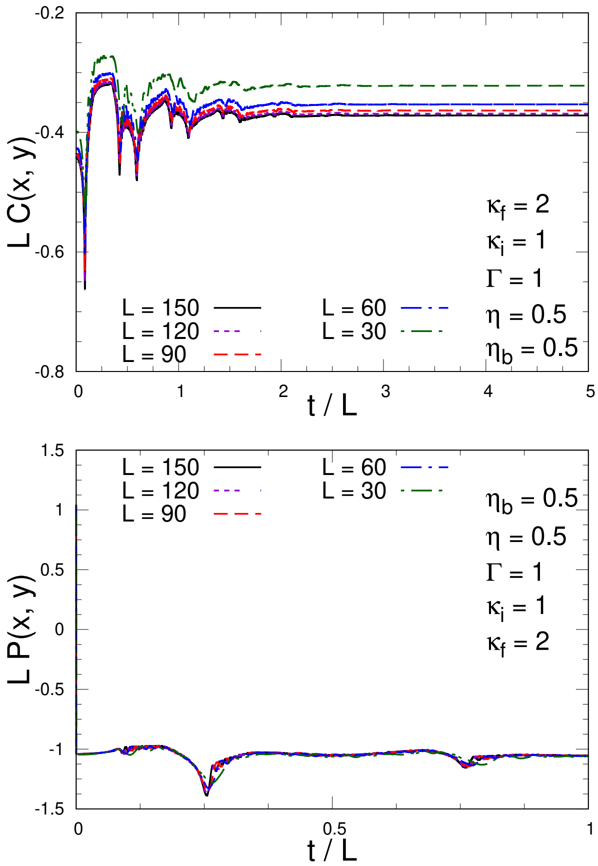

FIG. 2: Scaling behavior of the two-points correlations functions $C(x, y, t)$ (top) and $P(x, y, t)$ (bottom) for $x = L/3$, $y = 2L/3$ keeping the scaling variables $\kappa_i = 1$, $\kappa_f = 2$, $\eta = 0.5$, $\eta_b = 0.5$, $\Gamma = 1$ fixed in function of tL^{-z}, up to system size $L = 150$.

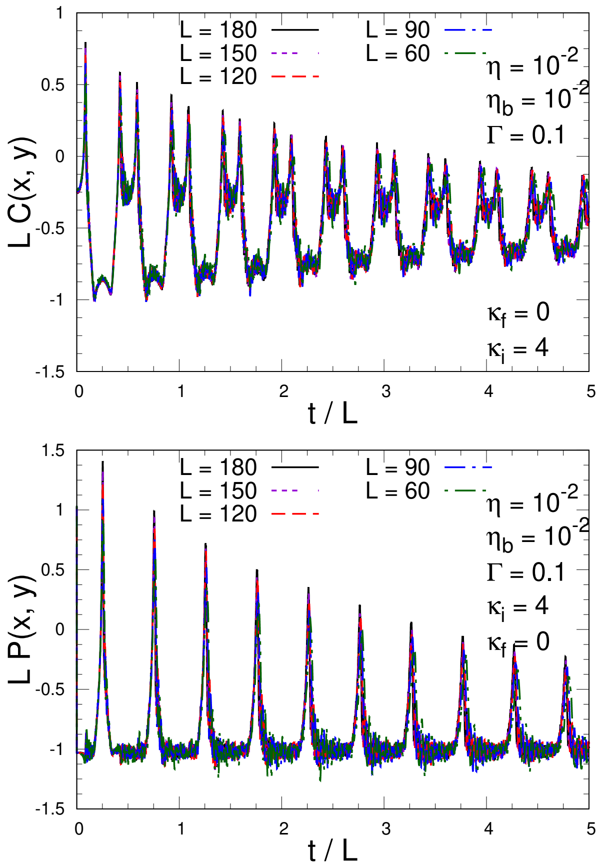

FIG. 3: Scaling behavior of the two-points correlations functions $C(x, y, t)$ (top) and $P(x, y, t)$ (bottom) for $x = L/3$, $y = 2L/3$ keeping the scaling variables $\kappa_i = 4$, $\kappa_f = 0$, $\eta = 0.01$, $\eta_b = 0.01$, $\Gamma = 0.1$ fixed in function of tL^{-z}, up to system size $L = 180$.

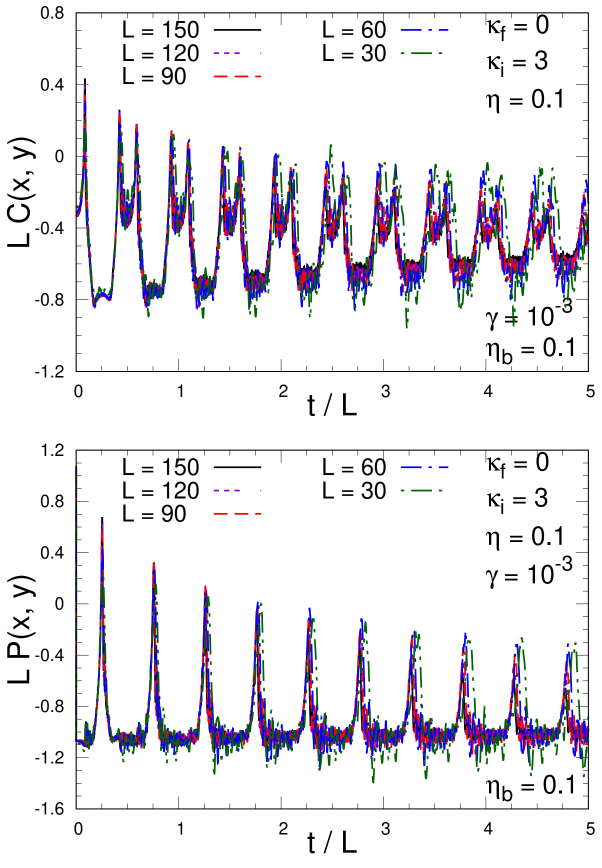

FIG. 4: Scaling behavior of the two-points correlations functions $C(x, y, t)$ (top) and $P(x, y, t)$ (bottom) for $x = L/3$, $y = 2L/3$ keeping the scaling variables $\kappa_i = 3$, $\kappa_f = 0$, $\eta = 0.1$, $\eta_b = 0.1$, $\gamma = 10^{-3}$ fixed in function of tL^{-z}, up to system size $L = 150$.

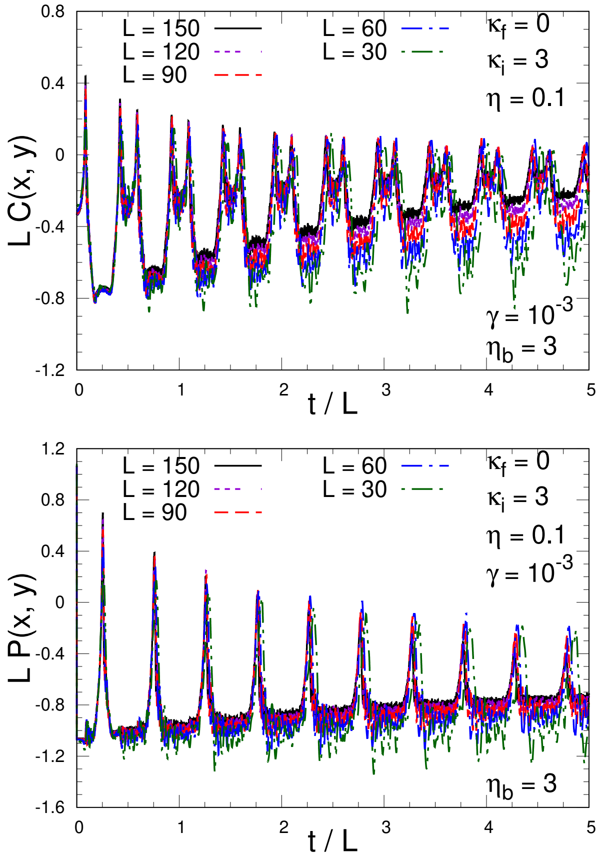

FIG. 5: Scaling behavior of the two-points correlations functions $C(x, y, t)$ (top) and $P(x, y, t)$ (bottom) for $x = L/3$, $y = 2L/3$ keeping the scaling variables $\kappa_i = 3$, $\kappa_f = 0$, $\eta = 0.1$, $\eta_b = 3$,$\gamma = 10^{-3}$ fixed in function of tL^{-z}, up to system size $L = 150$.