

# SCHEMA REFINEMENT and NORMAL FORMS

Session 2:FUNCTIONAL  
DEPENDENCIES

# Functional Dependencies

- A *functional dependency* (*FD*) is a type of integrity constraint (IC) that plays a crucial role in the process of database schema design, particularly in the normalization process.
- Functional dependencies generalize the concept of a key by establishing relationships between different sets of attributes within a relation (table).
- Understanding FDs is essential for ensuring that the database schema is well-structured and free from redundancy and anomalies.

# Functional Dependencies

- **Definition of Functional Dependency:**
- Let  $R$  be a relation schema, and let  $X$  and  $Y$  be nonempty sets of attributes in  $R$ . We say that an instance  $r$  of  $R$  satisfies the Functional Dependency  $X \rightarrow Y$  if the following condition holds for every pair of tuples  $t_1$  and  $t_2$  in  $r$ :
  - If  $t_1:X = t_2:X$ , then  $t_1:Y = t_2:Y$ .
  - This means that if two tuples in the relation have the same values for the attributes in  $X$ , then they must also have the same values for the attributes in  $Y$ .
- **Notation:**
  - $t_1:X$  : This notation refers to the projection of the tuple  $t_1$  onto the attributes in  $X$ . In simpler terms, it means we are considering the values of the attributes in  $X$  for the tuple  $t_1$ .
  - $X \rightarrow Y$  : This notation means that  $X$  functionally determines  $Y$ . In other words, if the values of  $X$  are known, the values of  $Y$  are uniquely determined.

- Example:
- Consider the functional dependency  $AB \rightarrow C$ . This means that if two tuples in the relation agree on the values of attributes  $A$  and  $B$ , they must also agree on the value of attribute  $C$ .

- Example:

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
a1	b1	c1	d1
a1	b1	c1	d2
a1	b2	c2	d1
a2	b1	c3	d1

An Instance that Satisfies  $AB \rightarrow C$

- In this table:
  - The first two rows show that  $AB$  is not a key because even though  $AB$  determines  $C$ , it does not uniquely determine all the attributes in the table (specifically, the attribute  $D$  varies while  $AB$  remains the same).
  - The third and fourth rows show that if  $A$  or  $B$  differs, then  $C$  can also differ without violating the FD.
  - If we add a new tuple  $(a1, b1, c2, d1)$ , the resulting instance would violate the FD :  $AB \rightarrow C$ , because for the pair of tuples  $(a1, b1)$ ,  $(C)$  would have different values  $c1$  and  $c2$ , which contradicts the functional dependency.

# Legal Instances and Integrity Constraints:

- A legal instance of a relation must satisfy all specified integrity constraints, including all specified functional dependencies.
- Integrity constraints are derived from the semantics of the real-world scenario being modeled by the database.
- By looking at an instance of a relation, we might be able to tell that a certain FD does *not* hold.
- However, we can never deduce that an FD *does* hold by looking at one or more instances of the relation because an FD, like other ICs, is a statement about *all* possible legal instances of the relation.
- It is important to understand that functional dependencies cannot be deduced by just looking at a few instances of the data.
- An FD, like other integrity constraints, applies to all possible legal instances of the relation.
  - For instance, just because a set of data currently satisfies  $A \rightarrow B$ , it doesn't mean  $A$  always determines  $B$  unless it's explicitly defined as an FD based on the business rules or logic.

# Functional Dependencies and Keys

- A primary key is a special case of a functional dependency.
  - In a primary key constraint, the attributes in the key play the role of  $X$ , and the set of all attributes in the relation plays the role of  $Y$ . This means the key uniquely determines all other attributes in the table.
- The definition of a functional dependency does not require that the set  $X$  be minimal. For  $X$  to be a key, the following conditions must be met:
  - Minimality: There should not be a subset  $V$  of  $X$  such that  $V \rightarrow Y$ . In other words, no proper subset of the key should functionally determine all attributes.
  - Superkey vs. Key: If  $X \rightarrow Y$  holds for all attributes, but there exists a subset  $V$  of  $X$  such that  $V \rightarrow Y$ , then  $X$  is a superkey, but it is not a minimal key.