

EID305: Design and Analysis of Algorithms

Module I: Part B: Finding Maximum and Minimum

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Finding Maximum and Minimum: Straight method

1. **Algorithm Straightmaxmin**(a, n, max, min)

2. //let max be the maximum and min be the minimum of a[1:n]

3. { max:= min := a[1];

4. for i:=2 to n do

5. { if(a[i] > max) then max :=a[i];

6. if (a[i] < min) then min := a[i];

7. }

8. }

Index	Element
1	22
2	25
3	20
4	47
5	37
6	25
7	10
8	45
9	66
10	55

Max : 66

Min : 10



Case1: if ($a[i] > \max$) then $\max := a[i]$;
 if ($a[i] < \min$) then $\min := a[i]$;

- This requires $2(n-1)$ elements comparisons in the best , average and worst case.
- The comparison $a[i] < \min$ is necessary only when $a[i] > \max$ is false.
 if ($a[i] > \max$) then $\max := a[i]$;
 else if ($a[i] < \min$) then $\min := a[i]$;
- The **best case** occurs when the elements are in increasing order.
- Number of elements comparisons : $(n-1)$.
- The **worst case** occurs when the elements are in decreasing order.
- The no. of elements comparisons is $2(n-1)$.

Index	Element
1	22
2	32
3	34
4	47
5	57
6	65
7	70
8	85
9	86
10	95



Index	Element
1	92
2	82
3	64
4	57
5	47
6	45
7	40
8	35
9	26
10	15



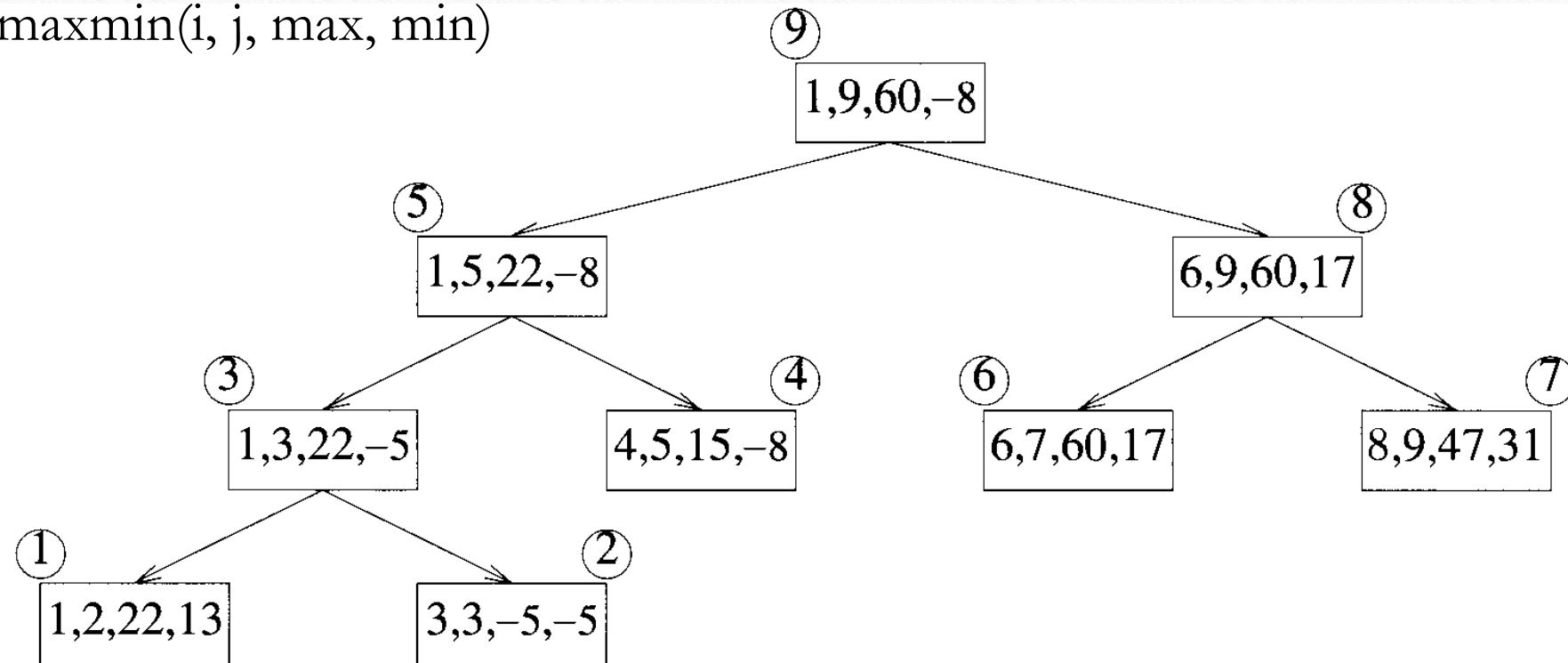
Finding Maximum and Minimum: DAndC

```
1. Algorithm maxmin(i, j, max, min)
2. // a[1:n] is a global array, i and j integers  $1 \leq i \leq j \leq n$ 
3. { if(i=j) then max := min := a[i]; //Small(P)
4.   else if (i = j-1) then // Another case for small(P)
5.     { if (a[i] < a[j]) then { min :=a[i]; max :=a[j];
6.       else {min := a[j]; max:=a[i]}
7.     }
8.   else { // if P is not small divide P into sub problems. Find where to split the set.
9.     mid= (i+j)/2;
10.    // solve the sub problem/
11.    maxmin (i, mid, max, min);
12.    maxmin (mid+1, j, max1, min1);
13.  }
14.  // combine the solutions
15.  if (max< max1) then max:=max1;
16.  if(min>min1)the min := min1;
17.  } // end of if-else
18. } // end of algorithm
```




Index	1	2	3	4	5	6	7	8	9
Value	22	13	-5	-8	15	60	17	31	47

maxmin(i, j, max, min)





Finding Maximum and Minimum: Introduction

No . of elements comparisons needed for max min is:

- $$T(n) = \begin{cases} T(n/2) + T([n/2]) + 2 & n > 2 \\ 1 & n = 2 \\ 0 & n = 1 \end{cases}$$

When n is a power of 2. I.e., $n = 2^k$ where k is positive integer.

$$T(n) = 2T(n/2) + 2$$

$$2(2T(n/4) + 2) + 2$$

$$4T(n/4) + 2^2 + 2$$

$$4(2T(n/8) + 2) + 2^2 + 2$$

$$8T(n/8) + 2^3 + 2^2 + 2..$$

$$2^{k-1}T(2) + \sum_{1 \leq i \leq k-1} 2^i$$

$$2^{k-1} + 2^k - 2$$

$3n/2 - 2$ number of comparisons for best, average and worst case comparisons when n is a power of w .



- **Recursive calls of max min**
- In terms of storage maxmin is worse than the straight forward algorithm.. Because it requires stack space for $i, j, \max, \min, \max1, \min1$.
- For n elements there will be $\lceil \log_2 n \rceil + 1$ levels of recursion need to save 'n' values for recursive call
- If comparisons among the elements of $a[]$ are much more costly than comparisons of integers variables, then the divide and conquer technique has given a more efficient algorithm. If not, it yields a less-efficient algorithm.
- DAndC strategy is only a guide to better algorithm design which may not always succeed. Both maxmin & straightmaxmin are $O(n)$