B. Tech Computer Science & Engineering,ECE (School of Technology)

SEMESTER –V

MATH2381: PROBABITY AND STATISTCS
(ENGINEERING MATHEMATICS)

UNIT-I
Data Science and Probability

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Learning Objectives

At the end of the module students able to learn:

- Probability is the probability an event will occur based on an analysis in which each measure is based on a recorded observation or a long history of collected data.
- Define event, outcome, trial, simple event, sample space and calculate the probability that an event will occur.
- Calculate the probability of events for more complex outcomes.
- Solve applications involving probabilities.

Learning Outcomes

At the end of the unit on probability, students should be able to:

- Know the definitions of Probability
- Understand event, outcome, trial, simple event, sample space and
- Calculate the probability that an event will occur by laws of Probabilities .
- Calculate the probability of events for more complex outcomes.
- Solve applications involving probabilities by Bayes theorem.

Contents

Why Probability ?

SNo	Statement	Sure/Impossible/ May or May not Happen
1	Sun rises in the West	Impossible
2	Increase in the length or breadth of a Rectangle results its Area of a Rectangle Increases	Sure
3	Tomorrow will be a sunny day	May or may not
4	Mother is younger than her daughter	Impossible
5	A leap Year consists of 366 days	Sure
6	I am flipping a fair coin I get a Head	May or may not
7	I am taking GAT I will get 1st rank	May or may not

Introduction

- In most areas of human endeavor, there is always an element of uncertainty (doubt)
- Consider the following

Weather report, a sporting event, stock transaction, working condition of a machine, Efficiency of Civil Engineer or a machine operator, an election result or a matter relating to health etc…

- There always facing with a certain degree of risk in nature
- Therefore it must be able to assess the degree of uncertainty in any given situation or event and this is done mathematically by using a measure Probability

General Definition Probability

The mathematical measurement of occurrence or non occurrence of an event is known as Probability

- The measure probability deal with the trial or experiment
- Experiment: The work done under certain homogeneous identically Conditions is known as trial or experiment
- Random and Deterministic Experiment
- Random: Result is not unique or not same for repetitions
- Deterministic: Result is unique or same for repetitions
- Sample space: The set of all possible outcomes is a sample space
- Sample space finite or Infinite

Sample Examples to Follow

- Single coin Tossing exp: S={ H,T }
- Die rolling experiment: S={ 1, 2, 3 ,4, 5, 6 }
- Two coins tossing : S={HH, HT, TH, TT}
- A family consists two kids S={ BB, BG, GB, GG }
- Machine Working Condition S={ good, not good }
- Today's Weather Report S={ Cloudy, Sunny, Rainy }
- Arrival time the interval [0, T], or $[0, T] = \{t: 0 \leq y \leq T\}$
- Human height. The experiment is to randomly select a human and measure his or her length

Classification of Events

- Sure Event: An event which is definitely occurs
- Impossible Event : An event which do not occurs
- Mutually Exclusive : No two events simultaneously occurs
- Dependent and Independents : Occurrence of an event affect on other else independent
- Equally Likely : proportion of happening of events same
- Favourable and Exhaustive : An event which is favour to count is favourable and events which are counted as total from the sample space (i.e all outcomes)

Mutually Exclusive Events

- Two events A,B are said be mutually exclusive events if they do not occurs simultaneously

Examples

- If student takes a test : either he may get pass or fail but not both at a time
- Tossing (Flip) a coin : either head appears or tail appears but not both at a time
- Machining working condition: Good or Not or in Repair
- Efficiency of a Machine operator : Trained or Not
- Efficiency of a Civil Engineer : Good or Not

Equally likely Events

Two or more events which have an equal probability of occurrence are said to be equally likely, Equally likely events may be elementary or compound events

Example: In the experiment of tossing a coin:

A : the event of getting a "HEAD" and B : the event of a "TAIL"

Events "A" and "B" are said to be equally likely events

[Both the events have the same chance of occurrence].

In the experiment of throwing a die: Events "A", "B", "C", "D", "E", "F" are said to be equally likely events

[All these events have the same chance of Occurrence]

Mathematical or Classical Definition

- The mathematical or classical definition probability of an event E (mutually exclusive, equally likely) is defined as
- P(E)= Favourable No. of cases /Total or Exhaustive No. of cases i.e

$$P(E) = \frac{m}{n}$$

$$P(\overline{E}) = 1 - \frac{m}{n} = 1 - P(E)$$

$$\Rightarrow P(E) + P(\overline{E}) = 1 \quad or$$

$$P(S) + P(F) = 1 \quad or \quad p + q = 1$$

- Here E is occurrence event and E bar is its compliment treated as success(s) and failure(F) with probability p and q respectively

Axiomatic Approach to Probability

- The probability of an event (or Probability function) follows the **three axioms**
- Axiom(1) is called positivity
- Axiom(2) is called definitivity
- Axiom(3) is called additivity
- P(Ø)=0 where Ø (pie)-is an impossible event
- $P(A)+p(A^C)= 1$ where A, A^C two complementary events

(1) $0 \le P(E) \le 1$ where A be an event

(2) $P(S) = 1$ where S be sure event

(3) $P(AUB) = P(A) + P(B)$

where A, B are two mutually exclusive or disjoint events

Addition law of Probability

- Let A,B are any two non Disjoint events then
- $P(A \cup B) = P(A) + P(B) - P(A \& B)$
- The probability that at least one event occurs is the probability of one event plus the probability of the other
- But to avoid double counting, the probability of the intersection of the two events is subtracted
- $P(A \cup B) = P(A) + P(B)$ If A,B are Mutually Exclusive

Addition law of Probability Cont'd

- Two events are **mutually exclusive** if the events have no sample points in common
- If two events *A* and *B* are mutually exclusive, the probability of *A* and *B* is zero
- In this case, the probability of *A* or *B* is the sum of the probability of *A* and the probability of *B*. That is,
- P(*A*&*B*) = 0 if *A* and *B* are mutually exclusive
- In this case P(*A*U*B*) = P(*A*) + P(*B*)

Conditional Probabilities

- A **conditional probability** refers to the probability of an event *A* occurring, given that another event *B* has already occurred
- Notation: P(*A* | *B*), Read this as the "conditional probability of *A* given *B*" or the "probability of *A* given *B*"
- These are especially useful in analysis because probabilities of an event differ, depending on other events occurring

Notations : Conditional Probabilities

- The probability of *A* given *B* is $$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$
- The probability of *B* given *A* is $$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$
- Independent Events : Any two events A and B are said to be independent Occurrence of an event does not effect on occurrence of other event Mathematically, $P(A \cap B) = P(A)P(B)$
- The above expression is also known as multiplicative Law of Probability

Multiplicative law of Probabilities

- The multiplicative law of probability for two dependent events A and B is

$$P(A \cap B) = P(B)P(A|B)$$

(or)

$$P(A \cap B) = P(A)P(B|A)$$

- If the events A, B are independent

$$P(A \cap B) = P(A)P(B)$$

Bayes Theorem (Introduction)

- From the definition of conditional probability we have,

  $$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

  (or)

  $$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$

- Using the above P(A/B) can be written as

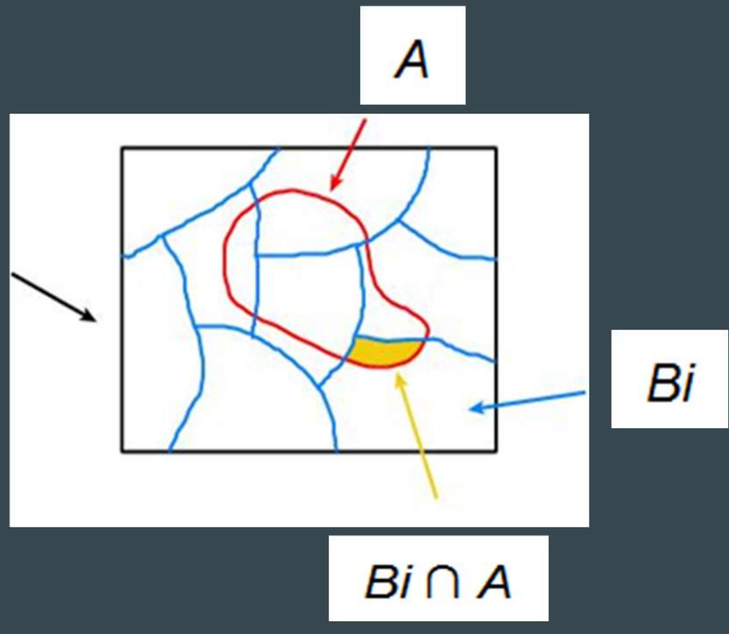
  $$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- First published (posthumously) by the Reverend Thomas Bayes (1702−1761)

Bayes Theorem(statement only)

- Let $B_1, B_2, \ldots B_n$ are n- mutually exclusive events
- Consider an event *A* which is subset to sample space S
- S is divided into n disjoint subsets B_i such that A is subset of any Bi(i=1,2,...,n) then, Baye's rule states that

$$P(B_i / A) = \frac{P(A / B_i)P(B_i)}{\sum_{i=1}^{n} P(A / B_i)P(B_i)}$$



- P(Bi/A) are called posterior probabilities and
- P(A/Bi) are called priori probabilities
- P(A/B)-likelihood events probabilities

Thank you

feed back to
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