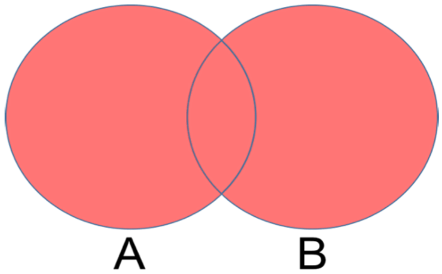
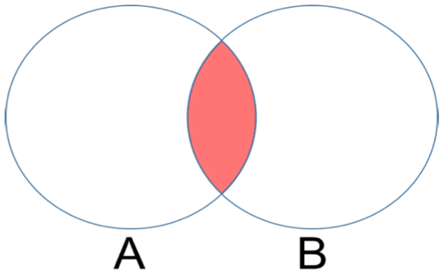
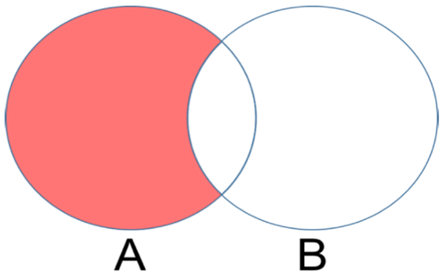
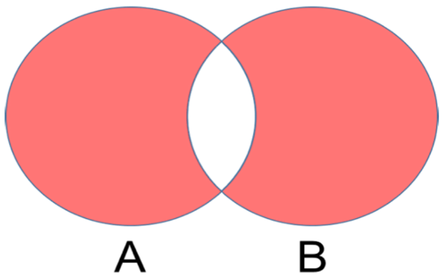
WEEK-1

Set Operation	Venn Diagram	Interpretation
Union		$A \cup B$, is the set of all values that are a member of A, or B, or both.
Intersection		$A \cap B$, is the set of all values that are members of both A and B.
Difference		$A \setminus B$, is the set of all values of A that are not members of B
Symmetric Difference		$A \triangle B$, is the set of all values which are in one of the sets, but not both.

SET/VENN DIAGRAM NOTATION TABLE			
SYMBOL	DEFINITION	EXAMPLE	EXPLANATION
$\in$	element of	$A \in B$	A is an element of B
$\notin$	not an element of	$A \notin B$	A is not an element of B
$\subseteq$	subset of	$A \subseteq B$	A is a subset of B
$\not\subset$	not a subset of	$A \not\subset B$	A is not a subset of B
$\cap$	intersection	$A \cap B$	A intersect B (in both sets)
$\cup$	union	$A \cup B$	A union B (in set A and/or B)
'	complement	A'	A complement (not A)

Set Notations

Symbol	Set
N	A set of all the natural numbers
R	A set of all the real numbers
R+	A set of all the positive real numbers
Q	A set of all the rational numbers
Z	A set of all the integers
Z+	A set of all the positive numbers

The Number of Subsets of a Set

- If a finite set has n elements, then the set has 2^n subsets and $2^n - 1$ proper subsets.

OPERATIONS ON SETS

$$A \cup A = A \qquad A \cap A = A$$

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$U' = \emptyset \qquad \emptyset' = U$$

$$A \cup U = U \qquad A \cap U = A$$

$$A \cup \emptyset = A \qquad A \cap \emptyset = \emptyset$$

$$(\mathrm{A}')' = \mathrm{A}$$

$$A \cup A' = U \qquad A \cap A' = \emptyset$$

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

1. If A and B are overlapping set, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. If A and B are disjoint set, $n(A \cup B) = n(A) + n(B)$

3. $n(A) = n(A \cup B) + n(A \cap B) - n(B)$

4. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

5. $n(B) = n(A \cup B) + n(A \cap B) - n(A)$

6. $n(U) = n(A) + n(B) - n(A \cap B) + n((A \cup B)^c)$

7. $n((A \cup B)^c) = n(U) + n(A \cap B) - n(A) - n(B)$

8. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

9. $n(A - B) = n(A \cup B) - n(B)$

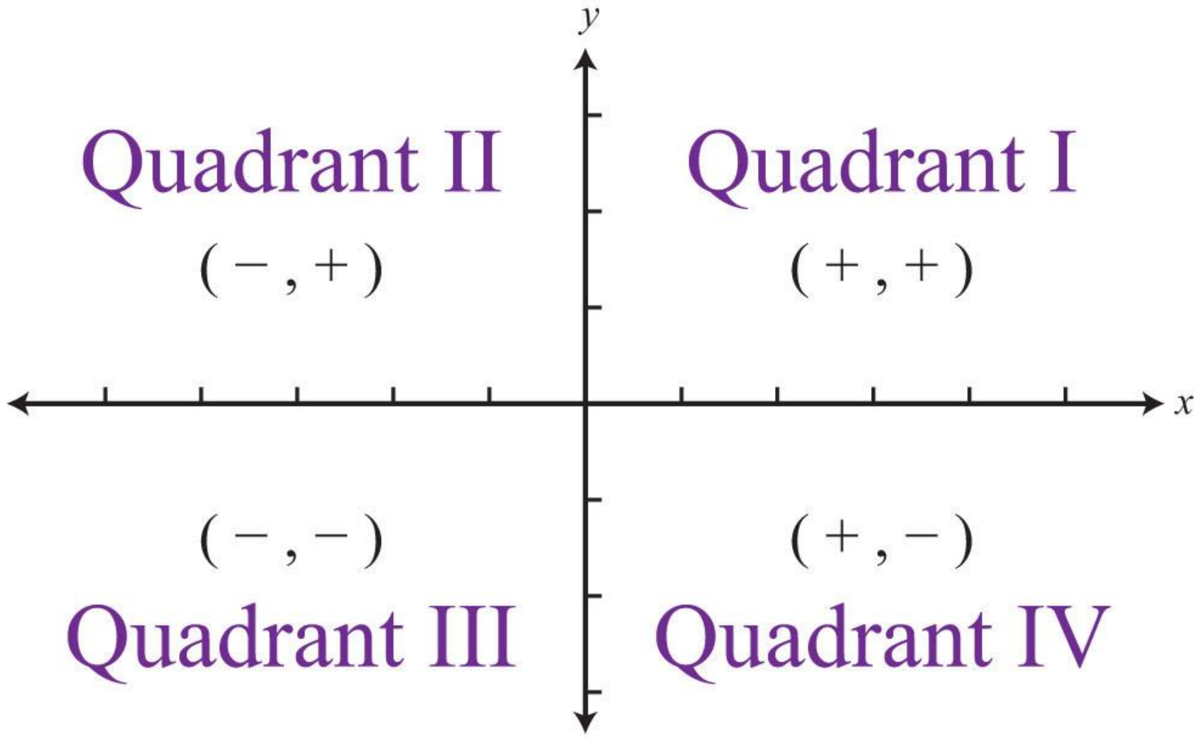
10. $n(A - B) = n(A) - n(A \cap B)$

11. $n(A^c) = n(U) - n(A)$

Some Important formulae: For any three sets A, B, C.

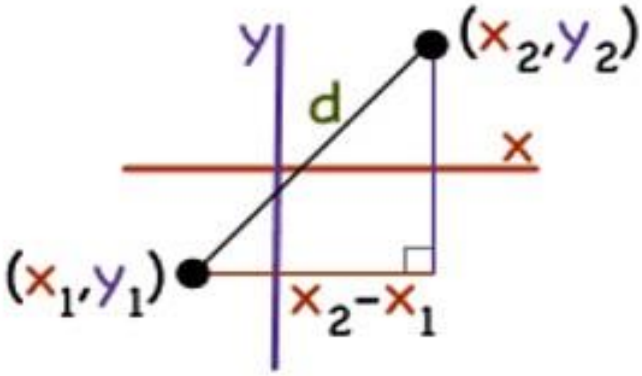
(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(ii) If $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$
(iii) $n(A - B) + n(A \cap B) = n(A)$
(iv) $n(B - A) + n(A \cap B) = n(B)$
(v) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$
(vi) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

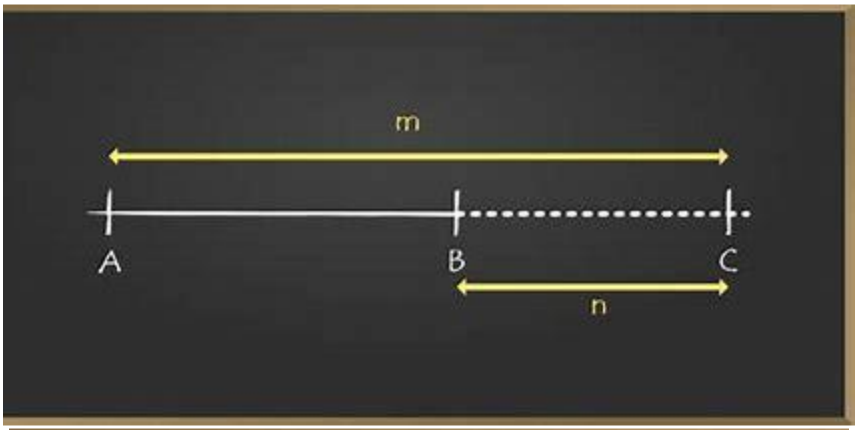
WEEK-2



Distance Formula

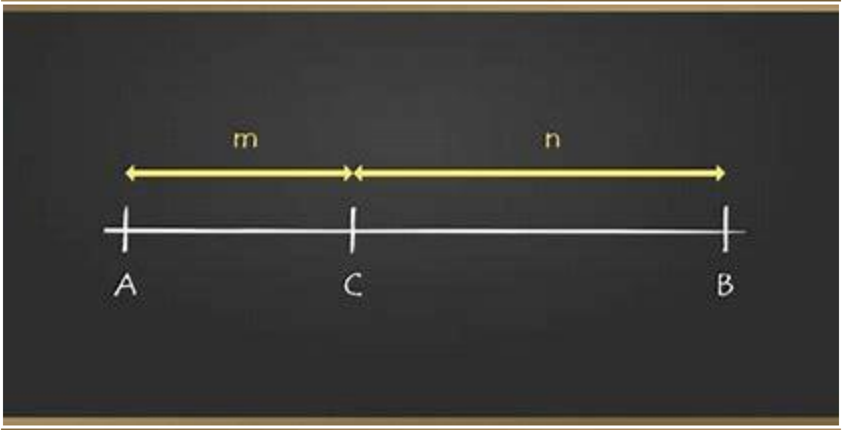
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$





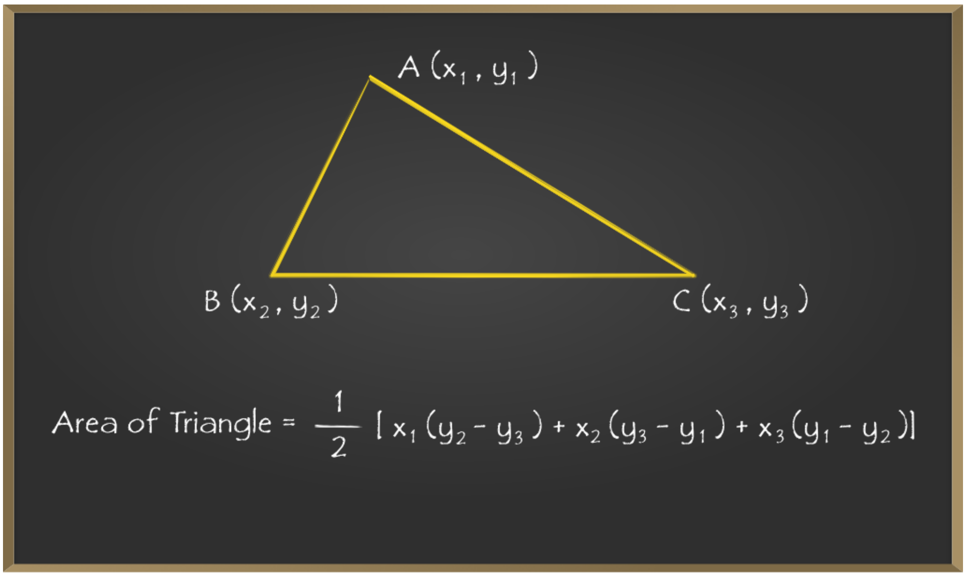
External Section Formula

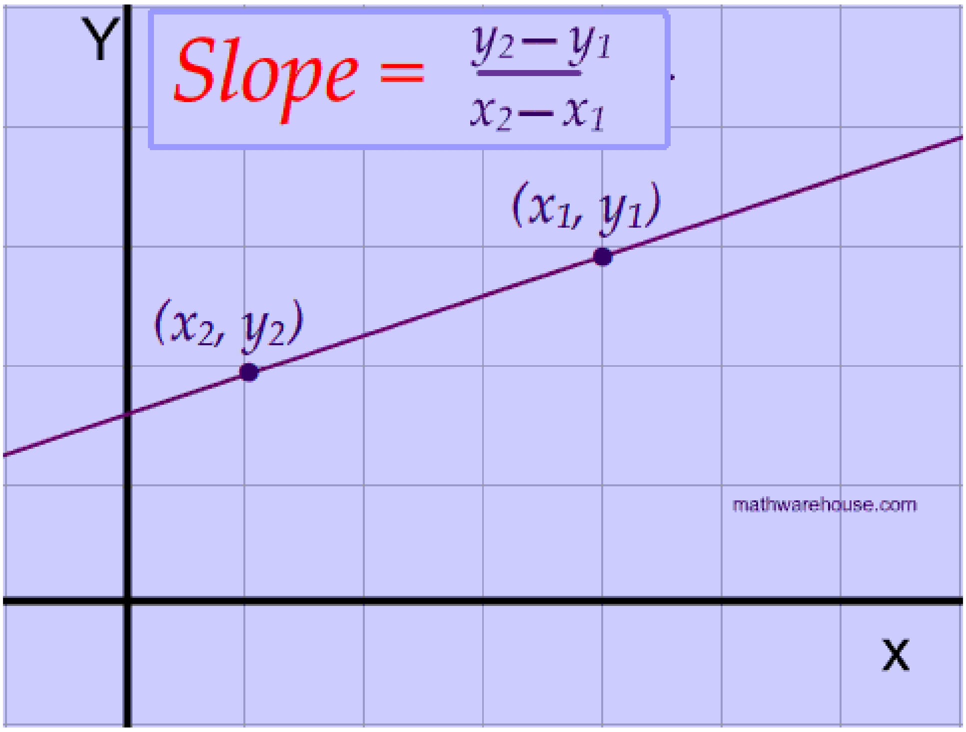
$$C(x, y) = \left(\frac{(m \times x_2 - n \times x_1)}{(m - n)}, \frac{(m \times y_2 - n \times y_1)}{(m - n)} \right)$$

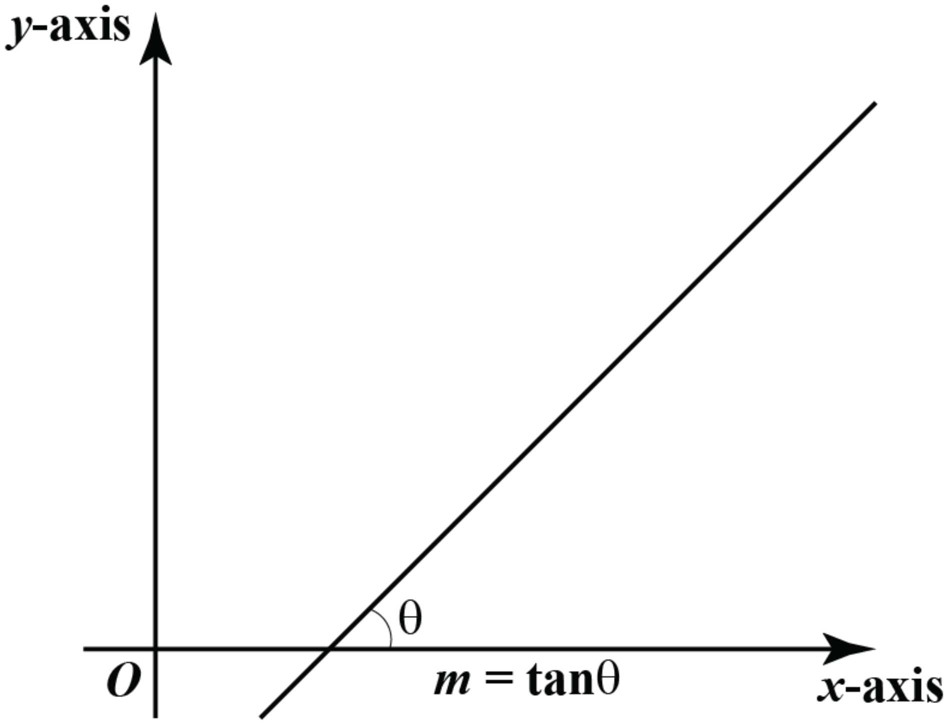


Internal Section Formula

$$C(x, y) = \left(\frac{(m \times x_2 + n \times x_1)}{(m + n)}, \frac{(m \times y_2 + n \times y_1)}{(m + n)} \right)$$

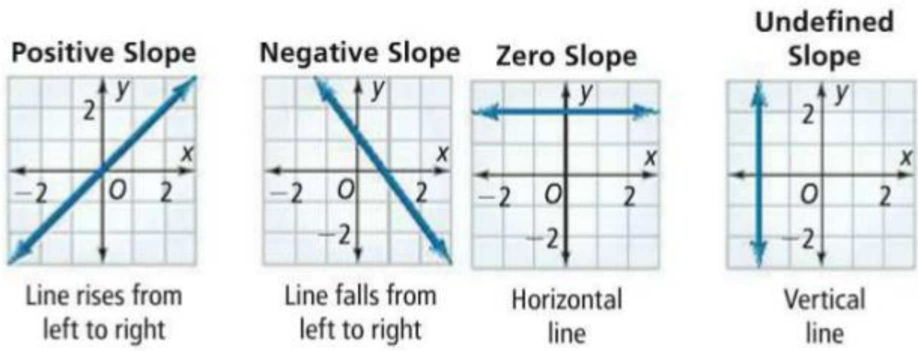

A (x_1, y_1)
B (x_2, y_2)
C (x_3, y_3)
Area of Triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$


Y
Slope = $\frac{y_2 - y_1}{x_2 - x_1}$
(x_1, y_1)
(x_2, y_2)
mathwarehouse.com
X



Slope

- How many different types of slopes are there?



Two non-vertical lines with slopes m_1 and m_2 are:

Parallel
if the lines have the same slope,
$m_1 = m_2$.

Perpendicular
if the slopes are negative reciprocals,
$$m_2 = -\frac{1}{m_1}$$
or equivalently, if $m_1 \cdot m_2 = -1$.

Question. Show that the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2=0$, $b_1,b_2 \neq 0$ are

- parallel if $a_1b_2= a_2b_1$, and
- perpendicular if $a_1a_2+ b_1b_2=0$.

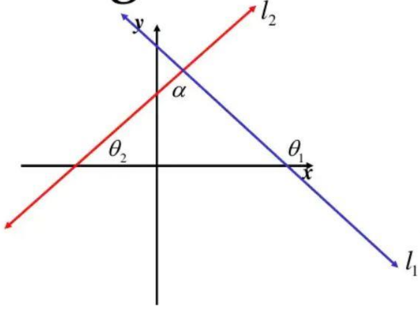
Using Slope-intercept form,

$$m_1 = -\frac{a_1}{b_1} \text{ and } m_2 = -\frac{a_2}{b_2}$$

If the lines are parallel, then $a_1b_2= a_2b_1$.

If the lines are perpendicular, then $a_1a_2+ b_1b_2=0$.

Angle Between Two Lines



l_1 has slope $m_1 = \tan\theta_1$

l_2 has slope $m_2 = \tan\theta_2$

$\theta_1 = \alpha + \theta_2$ (exterior $\angle\ \Delta$)

$$\alpha = \theta_1 - \theta_2$$

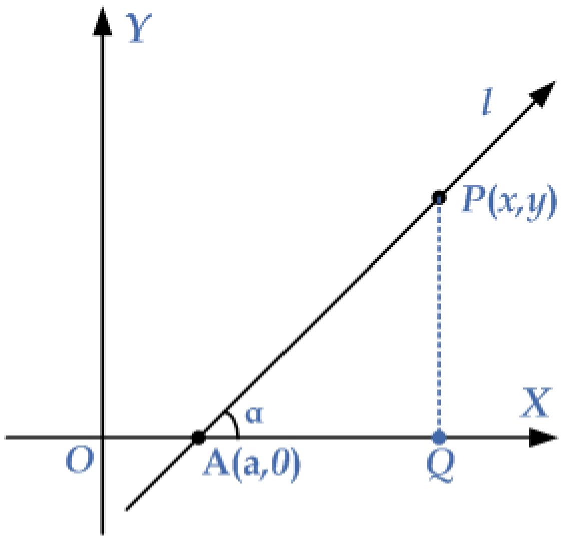
$$\tan\alpha = \tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1\tan\theta_2} = \frac{m_1 - m_2}{1+m_1 m_2}$$

The acute angle between two lines with slopes m_1 and m_2 can be found using;

Forms for the Equation of a Line		
Slope-Intercept	$y = mx + b$	*m* is the slope *b* is the *y*-intercept
Point-Slope	$y - y_1 = m(x - x_1)$	*m* is the slope (x_1, y_1) is a point on the line
Standard Form	$ax + by = c$	*a* is positive
Intercept Form	$\frac{x}{a} + \frac{y}{b} = 1$	*a* is the *x*-intercept *b* is the *y*-intercept
Vertical	$x = a$	Vertical line with *a* as the *x*-intercept
Horizontal	$y = b$	Horizontal line with *b* as the *y*-intercept

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

Two-Point Form



Now from the given diagram, consider the triangle ΔPAQ, i.e. $m\angle PAQ = \alpha$, and by the definition of slope we take

$$\tan\alpha = \frac{PQ}{AQ} = \frac{PQ}{OQ - OA}$$
$$\Rightarrow \tan\alpha = \frac{y}{x-a}$$

Now by the definition we can use m instead of $\tan\alpha$, and we get

$$\Rightarrow m = \frac{y}{x-a}$$
$$\Rightarrow m(x-a) = y$$

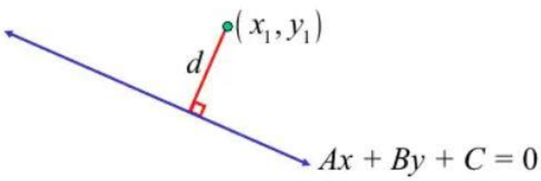
$$\boxed{y = m(x-a)}$$

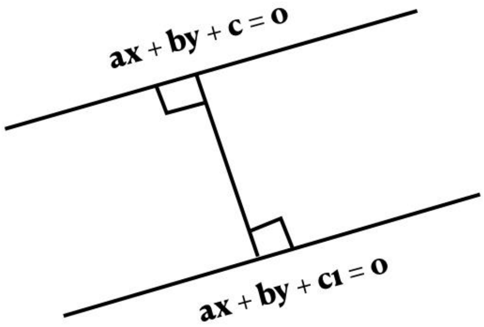
This is the equation of a straight line having the slope m and X-intercept a.

Line eq. With x-intercept

The shortest distance from a point to a line is the **perpendicular distance.**

$$\boxed{d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$





$$d = \frac{|c_1 - c|}{\sqrt{a^2 + b^2}}$$

$$SSE_{reg\ line} = \sum_{i=1}^{n} (y_i - (mx_i + b))^2$$

WEEK-3

$$ax^2 + bx + c = 0$$

A General Quadratic Equation

$$y = ax^2 + bx + c$$

Consider $x = 0$.

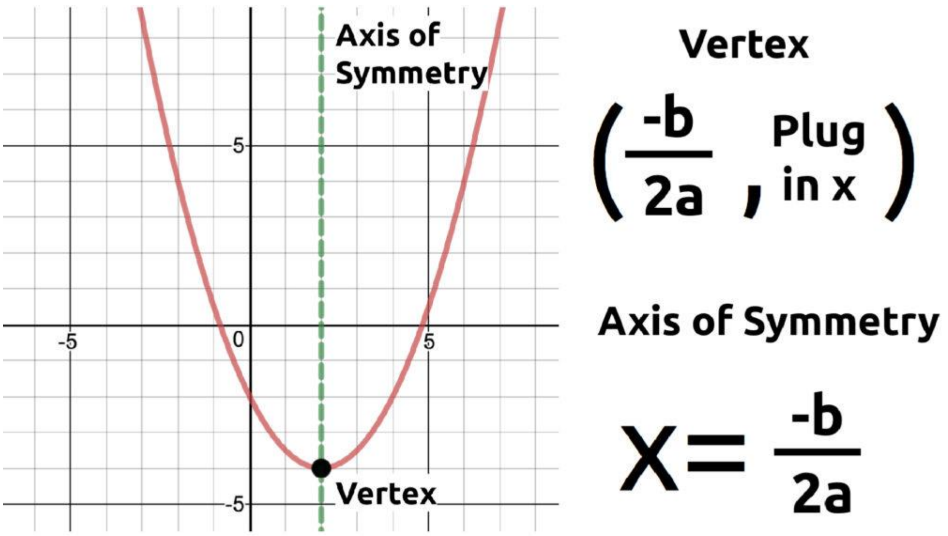
$$y = 0 + 0 + c$$

$$y = c$$

The graph of a quadratic function $f(x)= ax^2+bx+c$, where $a \neq 0$ is:

- Opens up and has minimum value, if $a > 0$.
- Opens down and has maximum value if $a < 0$.
- The range of a quadratic function is

$$\mathbf{R}\cap\{f(x)|f(x) \geq f_{min}\} \text{ or } \mathbf{R}\cap\{f(x)|f(x) \leq f_{max}\}.$$



a) The slope of the tangent to the graph of a function f is related to its first derivative. Let f be the quadratic function to find to be written as

$f(x) = a x^2 + b x + c$

The first derivative of f is given by

$f'(x) = 2 a x + b$

Equation	Parabola Form	Parabola Characteristics from this Form
$y=ax^2+bx+c$	**Standard Form**	c is the ***y*-intercept**
$y=a(x-h)^2+k$ or $y-k=a(x-h)^2$	**Vertex Form**	(h, k) is the **vertex**
$y=a(x-x_1)(x-x_2)$	**Factored Form** (also called **Intercept Form**)	x_1, x_2 (at the points $(x_1, 0)$ and $(x_2, 0)$) are the ***x*-intercepts** or **zeros** (if the parabola touches/cross the *x*-axis). **Zeros** are also called **values**, **solutions**, or **roots.**

Discriminant:

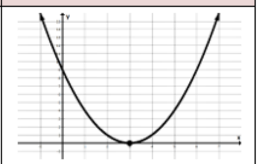
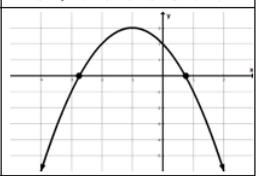
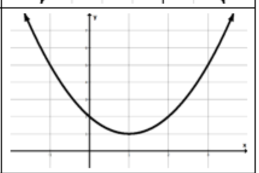
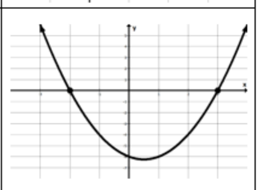
To find the number and types of roots of a quadratic equation use $b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{discriminant}$$

$b^2 - 4ac > 0$; positive — **2 real roots**

$b^2 - 4ac = 0$; — **1 real root**

$b^2 - 4ac < 0$; negative — **no real roots**

Discriminant	Number of roots	Example	Graph
$b^2-4ac\ = 0$	**1 real root** Touches x-axis once	$y=x^2-6x+9$ $b^2-4ac\ =$ $(-6)^2-4(1)(9)=$ $36-36=0$	
$b^2-4ac\ > 0$	**2 real roots** Touches x-axis twice	$y=-x^2-2x+2$ $b^2-4ac\ =$ $(-2)^2-4(-1)(2))=$ $4+8=12$	
$b^2-4ac\ < 0$	**No real roots** Doesn't touch x-axis – no x-intercepts (**Imaginary** roots)	$y=x^2-2x+2$ $b^2-4ac\ =$ $(-2)^2-4(1)(2)=$ $4-8=-4$	
And one more thing that's interesting: If $b^2-4ac=$ **a perfect square** $(0,1,4,9,16,25,36,\ldots)$	**2 real rational** ("easy") **roots** (1 root if discriminant = 0) (We'll see later that these quadratics can be **factored**)	$y=x^2-x-6$ $b^2-4ac\ =$ $(-1)^2-4(1)(-6)=$ $1+24=25$	

$$f(x) = ax^2 + bx + c$$

a > 0

Minimum point

a < 0

Maximum point

Maximum/minimum

$$= \left(-\frac{b}{2a}, f(-\frac{b}{2a}) \right)$$

WEEK-4

A **polynomial function** $of\ x\ with\ degree\ n\ is\ given\ by:$

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$where\ n\ is\ a\ nonnegative\ integer\ and\ a_n, a_{n-1}, \ldots a_2, a_1, a_0$
$with\ a_n \neq$

Types of Polynomials

Linear —— $ax + b = 0$

Quadratic —— $ax^2 + bx + c = 0$

Cubic —— $ax^3 + bx^2 + cx + d = 0$

Types Of Polynomial

Polynomial can be classified by number of non-zero term

Number of non-zero terms	Name	Example
0	Zero Polynomial	0
1	Monomial	X^2
2	Binomial	X^2+1
3	Trinomial	X^3+1

$Let\ p(x) = \sum_{k=0}^{n} a_k x^k,\ and\ q(x) = \sum_{j=0}^{m} b_j x^j. Then$
$p(x) + q(x) = \sum_{k=0}^{m \vee n} (a_k + b_k) x^k.$

$Let\ p(x) = \sum_{k=0}^{n} a_k x^k,\ and\ q(x) = \sum_{j=0}^{m} b_j x^j. Then$
$p(x) - q(x) = \sum_{k=0}^{m \vee n} (a_k - b_k) x^k.$

$$\text{Let } p(x) = \sum_{k=0}^{n} a_k x^k, \text{ and } q(x) = \sum_{j=0}^{m} b_j x^j. \text{ Then}$$
$$p(x)q(x) = \sum_{k=0}^{m+n} \sum_{j=0}^{k} (a_j b_{k-j}) x^k.$$

Degree of a Multivariate Polynomial

Highest sum of the exponents

Give the degree of the polynomial.

12 4

$-4u^{10} + 5x^4u^5w^3 - w^1x^3 - 6$

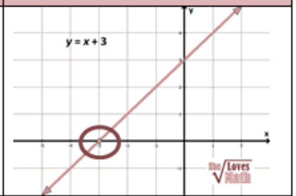
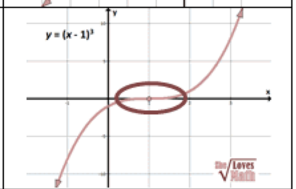
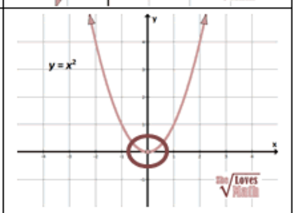
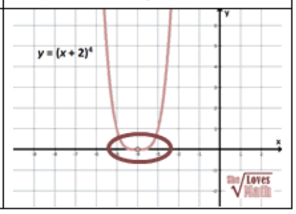
$\boxed{\text{Degree: } 12}$

Degree of polynomial

The degree of polynomial is the largest exponent occuring in its terms.

Eg : $8x^2 + 3x + 4$

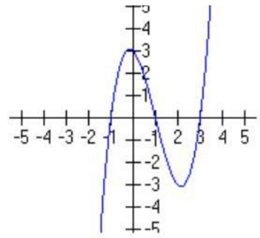
Degree of polynomial = 2

Factor(s)	Root	Multiplicity	Behavior	Example Graph
$(x+3)$	-3	**Odd:** 1	**Pass Through**	
$(x-1)^3$	1	**Odd:** 3, 5, … (the higher the odd degree, the flatter the "squiggle")	**"Squiggle" Pass Through**	
x^2	0	**Even:** 2	**Bounce** or **Touch**	
$(x+2)^4$	-2	**Even:** 4, 6, … (the higher the even degree, the flatter the bounce)	**"Flatter" Bounce**	

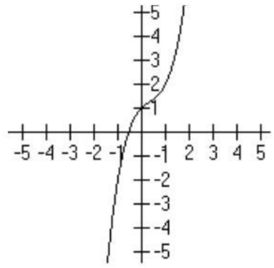
Turning Points of Polynomial Functions

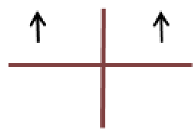
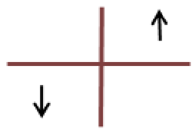
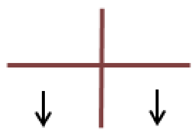
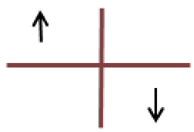
The graph of every polynomial function of degree *n* has at most *n* – 1 turning points. Moreover, if a polynomial function has *n* distinct real zeros, then its graph has exactly *n* -1 turning points.

$f(x) = x^3 - 3x^2 - 1x + 3$



$f(x) = x^3 - x^2 + x + 1$



Degree → / Sign of Leading Coefficient ↓	**Even**	**Odd**
	Examples: $-x^4+3x^2+1$, $-(x+3)^3(x+2)$, $x(3-x)^3(x+2)^2$	**Examples:** x^5+3x^2+1, $(x+3)^3(x+2)^2$, $x(3-x)^3(x+2)$
Positive (+) **Examples:** x^4+3x^2+1, $(x+3)^3(x+2)$, $(3-x)^4(x+2)$	 $x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to \infty$ **Example:** $y = x^2$	 $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$ **Examples:** $y = x,\ y = x^3$
Negative (–) **Examples:** $-x^4+3x^2+1$, $-(x+3)^3(x+2)$, $(3-x)^3(x+2)$	 $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to -\infty$ **Example:** $y = -x^2$	 $x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to -\infty$ **Examples:** $y = -x,\ y = -x^3$

Extra Formula

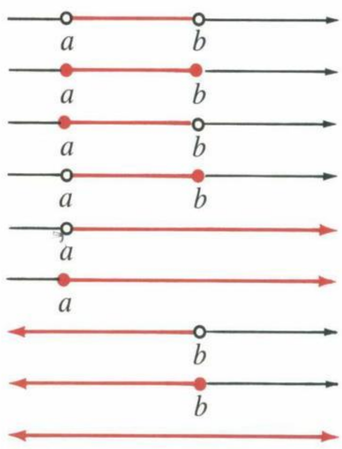
Summary of Rules for Exponents

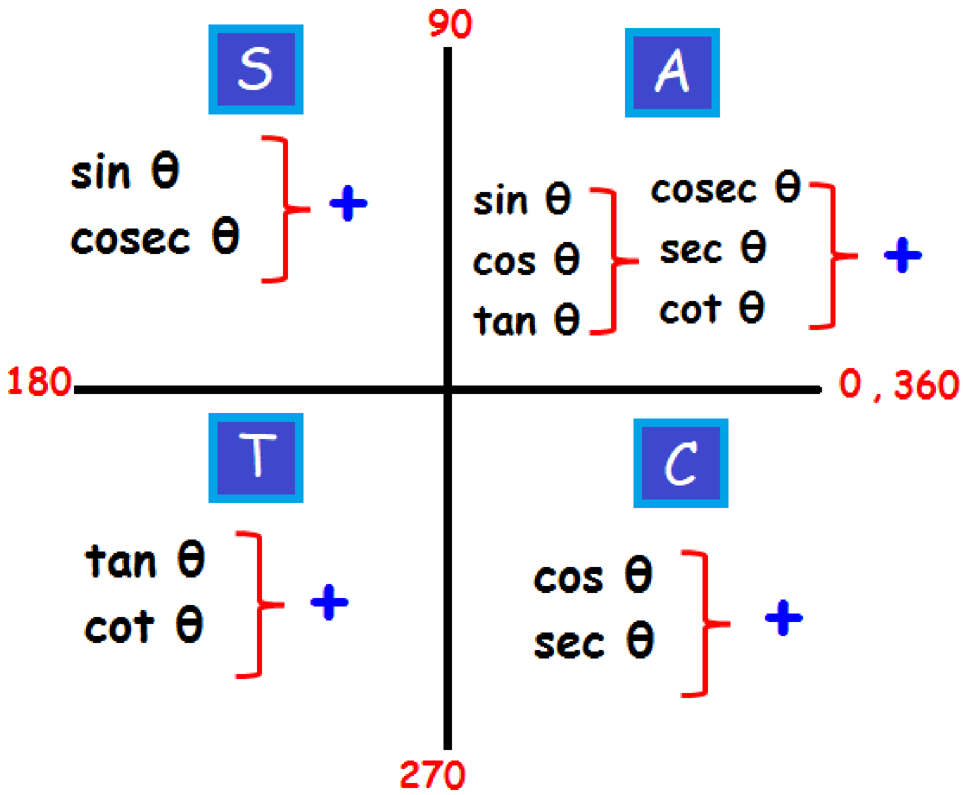
If a and b are real numbers and m and n are integers, then

Product rule	$a^m \cdot a^n = a^{m+n}$	
Zero exponent	$a^0 = 1$	$(a \neq 0)$
Negative exponent	$a^{-n} = \dfrac{1}{a^n}$	$(a \neq 0)$
Quotient rule	$\dfrac{a^m}{a^n} = a^{m-n}$	$(a \neq 0)$
Power rule	$(a^m)^n = a^{m \cdot n}$	
Power of a product	$(ab)^m = a^m \cdot b^m$	
Power of a quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$(b \neq 0)$

INTERVALS & INEQUALITIES

Interval Notation	Inequality Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \geq a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	R (set of all real numbers)	





Trigonometric Values for some Common angles

Degrees	Radians	sinθ	cosθ	tanθ	cotθ	secθ	cosecθ
0º	0	0	1	0	Undefined	1	Undefined
30º	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45º	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60º	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90º	$\frac{\pi}{2}$	1	0	Undefined	0	Undefined	1
180º	π	0	-1	0	Undefined	-1	Undefined
360º	2π	0	1	0	Undefined	1	Undefined

Trigonometric Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\cot\theta = \frac{1}{\tan\theta}$$
$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Sum Identities
Addition Formulas

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

Difference Identities
Subtraction Formulas

$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

Double Angle Formulas

$$\sin 2a = 2\sin a\cos a$$
$$\cos 2a = \cos^2 a - \sin^2 a$$
$$= 2\cos^2 a - 1$$
$$= 1 - 2\sin^2 a$$
$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

Co-function Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

Even-Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta$$
$$\sec(-\theta) = \sec\theta$$
$$\cot(-\theta) = -\cot\theta$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta}{1+\cos\theta}$$
$$= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Sum-to-Product Formulas

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$
$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Product-to-Sum Formulas

$$\sin a\sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$
$$\cos a\cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$
$$\sin a\cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$
$$\cos a\sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$n^2 \longrightarrow \text{sum of the first } n \text{ odd numbers}$$