General Assembly

STUDY GUIDE

AXIOMS OF PROBABILITY AND BAYES THEOREM

Definitions:

Probability: The likelihood that an event will occur.
Experiment: A procedure that can be repeated an indefinite amount of times and has a well-defined set of outcomes.
Event: Any collection of outcomes of an experiment.
Sample space: The set of all possible outcomes of an experiment, denoted by S.
When discussing probability, we must also discuss sets and elements.
Set: A well-defined collection of distinct objects.
Element: An object that is a member of a set.

Set Operations
The union of two sets (A and B) is the set of elements that are in either A or B. We denote the union of A and B as $A \cup B$.
The intersection of two sets (A and B) is the set of elements that are in both A and B. We denote the intersection of A and B as $A \cap B$.

Probability

Probability relies on sets.

We define events within a sample space and then evaluate the probability of that event occurring within that sample space.

The likelihood of finding a parking spot is an example of how we follow this process in evaluating simple day-to-day probabilities.

A naive definition of probability would be to say that, for some event (A), the probability that A occurs is:
P(A) = the number of outcomes in (A) / the number of all possible outcomes.

However, this isn't always correct unless we make a series of assumptions, which further complicates the definition of probability.

In order to avoid errors and help us understand probability more accurately, mathematicians and logicians have defined a few rules called the axioms of probability, aka Kolmogorov's axioms.

Probability will always follow these axioms.

The First Axiom of Probability

A, $P(A) \geq 0$

The probability of an event cannot be negative.

The Second Axiom of Probability

S, $P(S) = 1$

The probability of an outcome occurring in the sample space is 1.

The Third Axiom of Probability

For mutually exclusive series of events, the probability of a union of those events is the sum of the probabilities of the individual events.

Marginal and Joint Probability

P(A) is the marginal probability of Event A (a single event).
$P(A \cap B)$ is the joint probability of A and B.
This can give us more information about how phenomena relate to one another.

Conditional Probability

$P(A \mid B)$
Conditional probabilities are used in cases when we know that an event has already occurred.

Bayes' Theorem

A powerful tool in statistics.
It is used as the basis for Bayesian statistics.
Also known as Bayes' rule, it allows us to connect two related conditional probabilities.
$P(A \mid B)$ and $P(B \mid A)$
Use Bayes' theorem to describe the probability of an event based on prior knowledge of conditions that might be related to it.
The difference between Bayes' statistics and frequentist statistics: A Bayesian statistician would use prior or historical data as an input to their model, whereas a frequentist would use new information to draw their conclusions.