Smoothing out the Noise

Fred Kaesmann

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```
library(FredsVietorisRips)
Load_FVR_Dependencies()
Source_FVR_py(notebook = TRUE)
```

Data

We begin with a set of points, X, and we would like to determine the number of holes present via persistent homology.

```
##### Import Data ####
df_original <- read.csv("../data/Clark_Sample_data.csv", col.names = c("x", "y"))</pre>
```

Initial Condiditions

```
##### Trials and Samples ####
# How many points are we sampling
# and how many times are we sampling
sample_size <- 1000</pre>
ntrials <- 7
##### Size and Overlap of Pullback Covers ####
# What is the length of our pullback cover
     and what is the overlap between them?
resolution <- 0.05
gain <- 0.75
##### Connection Distance and Cluster Requirements ####
# Points within distance epsilon are connected
# and only points with a minimum number of
  connections are considered valid clusters
epsilon <- 0.1
min_connections <- 3</pre>
##### Number of Components per Cover per Trial ####
ncomponents <- vector("list", length = ntrials)</pre>
```

We run 7 iterations where we sample 1000 points from our original point cloud. Each cover beginning at 0 has resolution, or length, 0.05 and they overlap 75%. As we move up the y-axis we allocate to each cover

the points

Cover_j =
$$\{(x_i, y_i) | y_i \in [y_j, y_j + r)\}$$

where y_j is the lower bound of our cover, and $y_j + r$ is its upper bound.

Run Trials

When we run each trial we process the data as normal, the only difference is that we create a list of list which contains the number of components in each cover for each of our trials. We use this list of lists to determine the average number of components in each cover and plot that.

```
for (trial in c(1:ntrials)) {
    ##### Sample Original Data ####
    df <- df_original %>%
        sample_n(sample_size)

# Make it compatible with Pyton, start with zero
    rownames(df) <- as.numeric(rownames(df)) - 1

##### Filter ###

list_of_covers <- PullBackCovers(df$x, df$y, resolution, gain)

##### How many components do we have? #####

components <- FindComponents(list_of_covers, epsilon, min_connections, notebook = TRUE)

##### nComponents per Cover per Trial ####

ncomponents[[trial]] <- components %>%
    lapply(length) %>%
    unlist()
}
```

Below we turn our list of lists into a table to show the minor variations in the number of components in each cover.

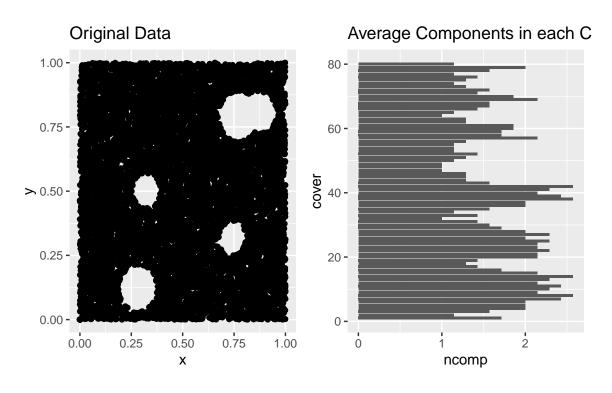
Table 1: Components in each Cover in each Trial

| | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 | Trial 7 | averageComponents |
|---------|---------|---------|---------|---------|---------|---------|---------|-------------------|
| Cover 1 | 1 | 1 | 3 | 1 | 2 | 2 | 2 | 1.714286 |
| Cover 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1.142857 |
| Cover 3 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1.571429 |

| Cover 4 2 1 2 2 2 2 2 2 2 2 </th <th></th> <th>Trial 1</th> <th>Trial 2</th> <th>Trial 3</th> <th>Trial 4</th> <th>Trial 5</th> <th>Trial 6</th> <th>Trial 7</th> <th>averageComponents</th> | | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 | Trial 7 | averageComponents |
|--|----------|---------|---------|---------|---------|---------|---------|---------|-------------------|
| Cover 6 | Cover 4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2.000000 |
| Cover 7 3 2 2 3 2 3 2 2.428571 Cover 9 2 2 3 2 2 2 2.1212857 Cover 10 2 2 2 2 2 2 2 2.142857 Cover 11 2 2 2 2 3 2 3 2.285714 Cover 13 2 2 2 2 2 3 3 2 2.285714 Cover 14 2 3 2 2 3 3 2 2.255714 Cover 16 2 1 3 2 2 3 2.142857 Cover 16 2 1 1 2 1 1 2 1 1 2 1.428571 Cover 18 2 1 1 2 1 1 1.228571 1 1 2.142857 Cover 20 3 1 2 2 3 </td <td>Cover 5</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2.000000</td> | Cover 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2.000000 |
| Cover 8 3 2 3 3 2 2 2 2.571429 Cover 9 2 2 2 2 2 2 2 2 2.285714 Cover 11 2 3 2 2 3 2 2 3 2.285714 Cover 13 2 2 2 2 2 3 2 2.285714 Cover 14 2 3 2 2 4 3 2 2.571429 Cover 16 2 1 3 2 2 2 3 2.142857 Cover 16 2 1 2 2 1 1 2 1.42857 Cover 17 2 1 1 2 1 1 2.142857 Cover 19 2 1 1 2 2 1 1 1.428571 Cover 20 3 1 2 2 3 2 2 1.142857 <td></td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> <td>2.000000</td> | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2.000000 |
| Cover 9 2 2 2 2 2 2 2 2 1 2 2 2 1 2 1 2 2 1 2 2 1 2 2 1 1 2 2 1 1 2 1 4 3 2 2 2 1 4 3 2 2 1 4 2 2 3 3 2 1 1 2 3 3 2 1 1 2 2 3 2 </td <td>Cover 7</td> <td>3</td> <td>2</td> <td>2</td> <td>3</td> <td>2</td> <td>3</td> <td>2</td> <td>2.428571</td> | Cover 7 | 3 | 2 | 2 | 3 | 2 | 3 | 2 | 2.428571 |
| Cover 10 2 2 2 2 3 2.285714 Cover 11 2 3 2.428571 Cover 12 3 2 2 2 3 2 1 2 2 2 3 2 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 2 2 1 | Cover 8 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 2.571429 |
| Cover 11 2 3 2 2 3 2.428571 Cover 13 2 3 2 2 2 3 2 2 2 3 2 1 1 2 2 1 1 2 2 1 1 2 1 1 2 1 1 1 2 1 1 1 1 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 1 2 2 1 1 2 2 2 | Cover 9 | 2 | 2 | 3 | 2 | 2 | 2 | 2 | 2.142857 |
| Cover 12 2 2 2 2 2 2 2.142857 Cover 13 2 2 2 2 2 3 3 2 2.255714 Cover 15 2 1 3 2 2 2 3 2.142857 Cover 16 2 1 2 2 1 1 3 1.714286 Cover 18 2 1 1 2 1 1 2 1.428571 Cover 19 2 1 1 2 2 1 1 1.428571 Cover 20 3 3 1 2 2 1 1 1.42857 Cover 21 3 1 2 2 3 2 2 2 2.142857 Cover 22 2 2 2 2 2 2 2 2 2.142857 Cover 24 2 2 2 2 2 2 3 | Cover 10 | 2 | 2 | 2 | 2 | 3 | 2 | 3 | 2.285714 |
| Cover 13 2 2 2 2 3 3 2 2.2857142 Cover 15 2 1 3 2 2 2 3 2.142857 Cover 16 2 1 2 2 1 1 3 1.714286 Cover 17 2 1 1 2 1 1 2 1.428571 Cover 19 2 1 1 2 1 1 1.285714 Cover 19 2 1 1 2 2 1 1 1.428571 Cover 20 3 1 2 2 3 2 1 1.428577 Cover 21 3 1 2 2 3 2 <t< td=""><td>Cover 11</td><td>2</td><td>3</td><td>2</td><td>2</td><td>3</td><td>2</td><td>3</td><td>2.428571</td></t<> | Cover 11 | 2 | 3 | 2 | 2 | 3 | 2 | 3 | 2.428571 |
| Cover 14 | Cover 12 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2.142857 |
| Cover 15 | Cover 13 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2.285714 |
| Cover 16 2 1 2 2 1 1 3 1.714286 Cover 17 2 1 1 2 1 1 2 1.428571 Cover 19 2 1 1 2 1 1 1.285714 Cover 20 3 1 2 3 3 2 1 1.1428571 Cover 20 3 1 2 3 3 2 1 1.1428571 Cover 20 3 1 2 2 3 2 2 1.428571 Cover 22 2 2 2 2 2 2 2 2.142857 Cover 23 2 2 2 2 2 2 3 2.142857 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 26 3 2 2 2 2 2 2 2 2 2 | Cover 14 | 2 | 3 | 2 | 2 | 4 | 3 | 2 | 2.571429 |
| Cover 17 2 1 1 2 1 1 2 1.428571 Cover 18 2 1 1 2 1 1 1.285714 Cover 19 2 1 1 2 2 1 1 1.24857 Cover 20 3 1 2 2 3 2 1 2.142857 Cover 21 3 1 2 2 3 2 2 2.2422 2 2.2425774 Cover 22 2 2 2 2 2 3 2.248577 2 2 2 2 3 2.142857 2 2 2 2 3 2.2485774 2 2 2 3 2.2485774 2 2 2 3 2.248577 2 <td>Cover 15</td> <td>2</td> <td>1</td> <td>3</td> <td>2</td> <td>2</td> <td>2</td> <td>3</td> <td>2.142857</td> | Cover 15 | 2 | 1 | 3 | 2 | 2 | 2 | 3 | 2.142857 |
| Cover 18 2 1 1 2 1 1 1.285714 Cover 19 2 1 1 2 2 1 1 1.428571 Cover 20 3 1 2 3 3 2 1 1.42857 Cover 21 3 1 2 2 3 2 2 2.142857 Cover 22 2 2 2 2 2 2 2.2425714 Cover 23 2 2 2 2 2 2 3 2.142857 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 26 3 2 <th< td=""><td>Cover 16</td><td>2</td><td>1</td><td>2</td><td>2</td><td>1</td><td>1</td><td>3</td><td>1.714286</td></th<> | Cover 16 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 1.714286 |
| Cover 19 2 1 1 2 2 1 1 1.428571 Cover 20 3 1 2 3 3 2 1 2.142857 Cover 21 3 1 2 2 3 2 2 2.2142857 Cover 23 2 2 2 2 2 2 3 2.142857 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.285714 Cover 26 3 2 2 2 2 2 2 2 2.2000000 Cover 27 3 3 2 1 2 2 2 2 2.2000000 Cover 29 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 | Cover 17 | 2 | 1 | 1 | 2 | 1 | 1 | 2 | 1.428571 |
| Cover 20 3 1 2 3 3 2 1 2.142857 Cover 21 3 1 2 2 3 2 2 2.142857 Cover 22 2 2 2 2 2 3 2.285714 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.142857 Cover 26 3 2 2 2 2 2 3 2.285714 Cover 26 3 2 2 2 2 2 2 2.285714 Cover 36 3 2 2 2 2 2 2.285714 Cover 37 3 3 2 <td>Cover 18</td> <td>2</td> <td>1</td> <td>1</td> <td>2</td> <td>1</td> <td>1</td> <td>1</td> <td>1.285714</td> | Cover 18 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1.285714 |
| Cover 21 3 1 2 2 3 2 2 2.142857 Cover 22 2 2 2 2 2 3 2.285714 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.142857 Cover 26 3 2 2 2 2 2 2 2 2000000 Cover 27 3 3 2 2 2 2 2 2 2.2000000 Cover 28 3 2 1 2 2 2 2 2.2000000 Cover 30 2 1 2 2 1 1 2 1.571429 Cover 31 2 1 1 2 1 1 1 1.00000 Cover 31 2 1 1 1 1 1 1 1.00000 </td <td>Cover 19</td> <td></td> <td>1</td> <td></td> <td></td> <td></td> <td></td> <td>1</td> <td>1.428571</td> | Cover 19 | | 1 | | | | | 1 | 1.428571 |
| Cover 22 2 2 2 2 3 2.285714 Cover 23 2 2 2 2 2 3 2.142857 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.285714 Cover 26 3 2 2 2 2 2 2 2 2.2000000 Cover 28 3 2 1 2 2 2 2 2 2.2000000 Cover 29 2 2 1 2 2 1 2 1.714286 Cover 30 2 1 1 2 1 1 2 1.757429 Cover 31 2 1 1 1 1 1 1 1.00000 Cover 32 1 1 1 1 1 1 1 1 1 1 | Cover 20 | 3 | 1 | 2 | 3 | 3 | 2 | 1 | 2.142857 |
| Cover 23 2 2 2 2 2 3 2.142857 Cover 24 2 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.285714 Cover 26 3 2 2 2 2 1 2 2.000000 Cover 28 3 2 1 2 2 2 2 2.285714 Cover 29 2 2 1 2 2 2 2 2.000000 Cover 30 2 1 2 2 1 2 1.714286 Cover 31 2 1 1 2 1 1 2 1.42857 Cover 32 1 1 1 1 1 1 1 1.00000 Cover 33 2 1 2 1 1 1 1.142857 Cover 34 1 < | Cover 21 | 3 | 1 | 2 | 2 | 3 | 2 | 2 | 2.142857 |
| Cover 24 2 2 2 2 2 3 2.142857 Cover 25 3 2 2 2 2 2 3 2.285714 Cover 26 3 2 2 2 2 2 2 2.20000000 Cover 27 3 3 2 2 2 2 2 2.20000000 Cover 28 3 2 1 2 2 2 2.20000000 Cover 30 2 1 2 2 1 2 1.714286 Cover 30 2 1 1 2 1 1 2 1.771429 Cover 31 2 1 1 1 1 1 1 1.000000 Cover 32 1 1 1 1 1 1 1.1000000 Cover 32 1 1 1 1 1 1 1.1142857 Cover 34 1 1 2 | Cover 22 | 2 | 2 | 2 | 2 | 3 | 2 | 3 | 2.285714 |
| Cover 25 | Cover 23 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2.142857 |
| Cover 26 | Cover 24 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2.142857 |
| Cover 27 3 3 2 2 2 2 2 2 2.000000 Cover 28 3 2 1 2 2 2 2 2.000000 Cover 30 2 1 2 2 1 2 1.714286 Cover 31 2 1 1 2 1 1 2 1.571429 Cover 31 2 1 1 2 1 1 2 1.428571 Cover 32 1 1 1 1 1 1 1 1.000000 Cover 33 2 1 2 1 1 1 1 1.1428571 Cover 34 1 1 2 1 1 1 1.1428571 Cover 35 1 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2< | Cover 25 | 3 | | 2 | 2 | 2 | 2 | 3 | 2.285714 |
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| Cover 29 2 2 1 2 2 1 2 1.714286 Cover 30 2 1 2 2 1 1 2 1.571429 Cover 31 2 1 1 1 1 1 2 1.428571 Cover 32 1 1 1 1 1 1 1 1 1 1 1 1.000000 Cover 33 2 1 2 1 1 1 1 1 1.142857 Cover 34 1 1 2 1 1 1 1.142857 Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 2 2 2 2 2 2 2.000000 Cover 37 2 2 2 2 2 2 2 2.000000 Cover 38 2 2 3 2 3 | Cover 27 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2.285714 |
| Cover 30 2 1 2 2 1 1 2 1.571429 Cover 31 2 1 1 2 1 1 2 1.428571 Cover 32 1 1 1 1 1 1 1 1 1.000000 Cover 33 2 1 2 1 1 1 1 1 1.142857 Cover 34 1 1 2 1 1 1 1 1.142857 Cover 35 1 2 2 2 1 1 1 1.142857 Cover 36 2 | Cover 28 | 3 | | 1 | | | 2 | 2 | 2.000000 |
| Cover 31 2 1 1 2 1 1 1.000000 Cover 32 1 1 1 1 1 1 1.000000 Cover 33 2 1 2 1 1 1 1 1.000000 Cover 34 1 1 2 1 1 1 1 1.142857 Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 | Cover 29 | 2 | 2 | 1 | | 2 | 1 | 2 | |
| Cover 32 1 1 1 1 1 1 1 1 1000000 Cover 33 2 1 2 1 1 1 2 1.428571 Cover 34 1 1 2 1 1 1 1.142857 Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 <td>Cover 30</td> <td>2</td> <td>1</td> <td>2</td> <td></td> <td>1</td> <td>1</td> <td></td> <td>1.571429</td> | Cover 30 | 2 | 1 | 2 | | 1 | 1 | | 1.571429 |
| Cover 33 2 1 2 1 1 1 2 1.428571 Cover 34 1 1 2 1 1 1 1.142857 Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 <t< td=""><td>Cover 31</td><td>2</td><td>1</td><td>1</td><td>2</td><td>1</td><td>1</td><td>2</td><td>1.428571</td></t<> | Cover 31 | 2 | 1 | 1 | 2 | 1 | 1 | 2 | 1.428571 |
| Cover 34 1 1 2 1 1 1 1 1.142857 Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 2 2 2 2 2 2 2 2 2000000 Cover 37 2 | | | 1 | | 1 | 1 | 1 | | 1.000000 |
| Cover 35 1 2 2 1 2 2 1 1.571429 Cover 36 2 1 </td <td></td> <td>2</td> <td>1</td> <td></td> <td>1</td> <td>1</td> <td>1</td> <td>2</td> <td>1.428571</td> | | 2 | 1 | | 1 | 1 | 1 | 2 | 1.428571 |
| Cover 36 2 3 2 2 2 3 2 2 2 3 2 2 1< | | 1 | | | 1 | | | 1 | |
| Cover 37 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1< | | | | | | | | | |
| Cover 38 2 2 3 2 3 3 3 2.571429 Cover 39 2 2 3 2 3 3 2 2.428571 Cover 40 2 2 3 2 2 2 2 2 2.142857 Cover 41 3 2 3 2 2 2 2 2 2.285714 Cover 42 2 3 3 3 2 2 2 2 2.285714 Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1.571429 Cover 44 1 2 1 2 1 1 1.571429 Cover 45 1 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1.1000000 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> | | | | | | | | | |
| Cover 39 2 2 3 2 2.428571 Cover 40 2 2 3 2 2 2 2 2.142857 Cover 41 3 2 3 2 2 2 2 2.285714 Cover 42 2 3 3 3 2 2 3 2.571429 Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1 1.285714 Cover 45 1 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 1 1.000000 Cover 47 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1.000000 Cover 50 1 1 1 1 1 1 1 1.142857 Cover 52 3 | | | | | | | | | |
| Cover 40 2 2 3 2 2 2 2 2 2 2.142857 Cover 41 3 2 3 2 2 2 2 2.285714 Cover 42 2 3 3 3 2 2 3 2.571429 Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1 1.285714 Cover 45 1 2 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 1 1.285714 Cover 47 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1.142857 Cover 50 1 1 1 1 1 1 | | | | | | | | | |
| Cover 41 3 2 3 2 2 2 2 2.285714 Cover 42 2 3 3 3 2 2 3 2.571429 Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1 1 1.285714 Cover 45 1 2 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 1 1 1.285714 Cover 47 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1.142857 Cover 50 1 1 1 1 1 1 1 1.142857 Cover 52 3 2 1 1 1 1 | | | | | | | | | |
| Cover 42 2 3 3 2 2 3 2.571429 Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1 1 1.285714 Cover 45 1 2 2 1 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 2 1.285714 Cover 47 1 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1 1.142857 Cover 50 1 1 1 1 1 1 1 1.285714 Cover 51 2 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 | | | | | | | | | |
| Cover 43 1 2 2 2 1 2 1 1.571429 Cover 44 1 2 1 2 1 1 1 1.285714 Cover 45 1 2 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 2 1.285714 Cover 47 1 | | | | | | | | | |
| Cover 44 1 2 1 2 1 1 1 1.285714 Cover 45 1 2 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 2 1.285714 Cover 47 1 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> | | | | | | | | | |
| Cover 45 1 2 2 1 1 1 1 1.285714 Cover 46 1 2 1 1 1 1 2 1.285714 Cover 47 1 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> | | | | | | | | | |
| Cover 46 1 2 1 1 1 2 1.285714 Cover 47 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1 1 1.000000 Cover 50 1 1 1 2 1 1 1 1 1.142857 Cover 51 2 2 1 1 1 1 1 1.285714 Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 47 1 1 1 1 1 1 1 1.000000 Cover 48 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1 1.000000 Cover 50 1 1 1 2 1 1 1 1 1.142857 Cover 51 2 2 1 1 1 1 1 1.428571 Cover 52 3 2 1 1 1 1 1 1.142857 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 48 1 1 1 1 1 1 1 1.000000 Cover 49 1 1 1 1 1 1 1 1.000000 Cover 50 1 1 1 2 1 1 1 1.142857 Cover 51 2 2 1 1 1 1 1 1.285714 Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 49 1 1 1 1 1 1 1 1.000000 Cover 50 1 1 1 2 1 1 1 1.142857 Cover 51 2 2 1 1 1 1 1 1.285714 Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 50 1 1 1 2 1 1 1 1.142857 Cover 51 2 2 1 1 1 1 1 1.285714 Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 51 2 2 1 1 1 1 1 1.285714 Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 52 3 2 1 1 1 1 1 1.428571 Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 53 2 1 1 1 1 1 1 1.142857 Cover 54 2 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| Cover 54 2 1 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| | | | | | | | | | |
| Cover 55 2 1 1 1 1 1 1 1 1.142857 | | | | | | | | | |
| | Cover 55 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1.142857 |

| | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Trial 6 | Trial 7 | averageComponents |
|----------|---------|---------|---------|---------|---------|---------|---------|-------------------|
| Cover 56 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 1.285714 |
| Cover 57 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 2.142857 |
| Cover 58 | 2 | 1 | 2 | 2 | 2 | 2 | 1 | 1.714286 |
| Cover 59 | 1 | 1 | 2 | 2 | 2 | 3 | 1 | 1.714286 |
| Cover 60 | 2 | 1 | 2 | 2 | 2 | 3 | 1 | 1.857143 |
| Cover 61 | 2 | 1 | 2 | 1 | 2 | 4 | 1 | 1.857143 |
| Cover 62 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 1.285714 |
| Cover 63 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 1.285714 |
| Cover 64 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1.000000 |
| Cover 65 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1.142857 |
| Cover 66 | 3 | 1 | 1 | 2 | 1 | 1 | 1 | 1.428571 |
| Cover 67 | 2 | 1 | 1 | 3 | 2 | 1 | 1 | 1.571429 |
| Cover 68 | 2 | 1 | 1 | 2 | 3 | 1 | 1 | 1.571429 |
| Cover 69 | 3 | 2 | 3 | 3 | 2 | 1 | 1 | 2.142857 |
| Cover 70 | 1 | 2 | 2 | 3 | 3 | 1 | 1 | 1.857143 |
| Cover 71 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1.428571 |
| Cover 72 | 3 | 2 | 1 | 2 | 1 | 1 | 1 | 1.571429 |
| Cover 73 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1.285714 |
| Cover 74 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1.142857 |
| Cover 75 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1.285714 |
| Cover 76 | 1 | 2 | 3 | 1 | 1 | 1 | 1 | 1.428571 |
| Cover 77 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1.142857 |
| Cover 78 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 1.571429 |
| Cover 79 | 3 | 2 | 1 | 3 | 1 | 2 | 2 | 2.000000 |
| Cover 80 | 1 | 0 | 2 | 3 | 0 | 2 | 0 | 1.142857 |

Visualization of our Data and our Components



Some Additional Thoughts

Our above bar chart shows that our initial conditions appear to be well chosen to represent our specific data. This is not the case for what appears to be even minor changes in those initial conditions.

```
##### Constant Conditions ####
sample_size <- 750
ntrials <- 3
min_connections <- 3
epsilon <- 0.1

##### Varrying Condidtions
resolutionSet <- c(0.25, 0.2, 0.15, 0.1, 0.05, 0.025)
gainSet <- c(0.5, 0.75)

##### Number of Components per Cover per Trial ####
ncomponents <- vector("list", length = ntrials)</pre>
```

```
for (resolution in resolutionSet) {
  for (gain in gainSet) {
    for (trial in c(1:ntrials)) {
      ##### Sample Original Data ####
      df <- df_original %>%
        sample_n(sample_size)
      # Make it compatible with Pyton, start with zero
      rownames(df) <- as.numeric(rownames(df)) - 1</pre>
      ##### Filter ####
      list_of_covers <- PullBackCovers(df$x, df$y, resolution, gain)</pre>
      ##### How many components do we have? #####
      components <- FindComponents(list_of_covers, epsilon, min_connections, notebook = TRUE)</pre>
      ##### nComponents per Cover per Trial ####
      ncomponents[[trial]] <- components %>%
        lapply(length) %>%
        unlist()
    ##### Matrix of Trial Results ####
    trialMatrix <- matrix(unlist(ncomponents),</pre>
                   ncol = ntrials)
    ##### Average Components by Cover ####
    averageComponents <- trialMatrix %>%
    rowSums() / ntrials
    chart <- data.frame(</pre>
      cover = c(1:nrow(trialMatrix)),
      ncomp = averageComponents) %>%
      ggplot(aes(x = cover,
```

