CS 486 Assignment 1

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June 11, 2021

Changes

- Q1: Changed the descriptions to use wolf and sheep instead of missionaries and cannibals. Changed the formulation to use S_j instead of M_j for a sheep and W_j instead of C_j for a wolf. This change does not affect the problem solution at all.

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1 The Wolves and Sheep Problem (42 marks)

There are three wolves, three sheep, and a boat on the left side of the river. Let's assume that the wolves and sheep know how to row the boat. The boat can hold one or two animals. Every time the boat crosses the river, we count it as one boat trip. Our goal is to find a way to move all the animals from the left side to the right side of the river using the smallest number of boat trips subject to the constraints below.

- The boat can only cross the river if there is at least one animal in it.
- The sheep must stay alive.

  At any moment in time, on either side of the river, if there is at least one sheep and the number of wolves is greater than the number of sheep, the wolves will eat the sheep.

You will apply uninformed and informed search algorithms to solve this problem.

Please complete the following tasks.

(a) Formulate this problem as a search problem. Make sure you define the states, the initial state, the goal state, the successor function, and the cost function.

In parts b and c, we will provide you with some other formulations. After that, you will have a chance to revise and improve your formulation in part d. Come up with your own formulation first before looking at the next two parts. Don't miss this valuable learning opportunity.

Marking Scheme: (2 marks) We will mark this part for completion only. As long as you provide a state definition and a successor function, you will get the 2 marks.

- (1 mark) for stating a state definition.
- (1 mark) for stating a successor function.

Solutions:
States: Each state is given by $S_L W_L S_R W_R B$,
S_L, W_L represents the number of sheep/wolf in left side.
S_R, W_R represents the number of sheep/wolf in right side.
B = d if the boat is on the left bank (departure side) and B = a if the boat is on the right bank (arrival side).
Start State: 3300d

Goal State: 0033a
Successor Function: Assume A is a state, if $B = d$, we have to decrease 1 or 2 for $S_L W_L$ and increase $S_R W_R$ correspondingly.
Cost Function: cost of each boat trip is 1.

(b) Your friend Taylor came up with the following formulation. Taylor's formulation is correct, which means applying a (optimal) search algorithm on this formulation will allow us to find a (optimal) solution. Unfortunately, Taylor's formulation has a problem, which significantly affects the performance of a search algorithm on this problem.

Describe the problem with Taylor's formulation in no more than 3 sentences. Then, discuss how this problem affects the performance of a search algorithm in no more than a paragraph.

Hint: If you have trouble identifying the problem, draw the search graph and look at the states in the search graph. How many states are there? Why are these states in the graph?

Taylor's formulation:

- **State:** Each state is given by $S_1 S_2 S_3 W_1 W_2 W_3 B$. S_i and W_j are 1 if the sheep/wolf is on the left bank and 0 otherwise. $B = d$ if the boat is on the left bank (departure side) and $B = a$ if the boat is on the right bank (arrival side).

  A state is valid if and only if it satisfies the following two constraints.

  If there is at least one sheep on the left bank, then the number of wolves on the left bank must be less than or equal to the number of sheep on the left bank.
  If there is at least one sheep on the right bank, then the number of wolves on the right bank must be less than or equal to the number of sheep on the right bank.
- **Initial state:** 111111d.
- **Goal states:** Any state where every animal is on the right bank is a goal state. 000000x where x can be a or d.
- **Successor function:** Assume that A and B are two states satisfying the state definition. B is a successor of A if and only if we can start from A, move the boat with one or two animals from one side of the river to the other side of the river, and arrive at state B.
- **Cost function:** The cost of each boat trip is one.

Marking Scheme: (4 marks)

- (2 mark) Describe the problem with Taylor's formulation.

- (2 marks) Describe how the problem affects the performance of a search algorithm.

Solutions:

- For Taylor's formulation, she put the constraints in the definition of state instead of successor function.
- For this problem, the performance is affected because the successor function will produce much more states to frontier. Since we put the constraint in the state, for each iteration, we have to check the validity of current state. This will affect the performance of search algorithm.

(c) Your other friend Avery proposed a formulation that is similar to Taylor's. However, Taylor included the constraints in the state definition, whereas Avery put the constraints in the successor function. See the state and the successor function in the two formulations below.

Taylor and Avery argued for a while but couldn't figure out which choice is better. Could you help them settle the argument? *Which of the two formulations is better? State your answer and justify your answer in no more than a paragraph.*

Taylor's formulation:

- **State:** Each state consists of $(S_1, S_2, S_3, W_1, W_2, W_3, B)$. S_i and W_j are 1 if the sheep/wolf is on the left bank and 0 otherwise. $B = d$ if the boat is on the left bank (departure side) and $B = a$ if the boat is on the right bank (arrival side).

  A state is invalid if it violates either constraint below.

  - If there is at least one sheep on the left bank, then the number of wolves on the left bank must be less than or equal to the number of sheep on the left bank.
  - If there is at least one sheep on the right bank, then the number of wolves on the right bank must be less than or equal to the number of sheep on the right bank.
- **Successor function:** Assume that A and B are two states satisfying the state definition. B is a successor of A if and only if we can start from A, move the boat with one or two animals from one side of the river to the other side of the river, and arrive at state B.

Avery's formulation:

- **State:** Each state consists of $(S_1, S_2, S_3, W_1, W_2, W_3, B)$. S_i and W_j are 1 if the sheep/wolf is on the left bank and 0 otherwise. $B = d$ if the boat is on the left bank (departure side) and $B = a$ if the boat is on the right bank (arrival side).
- **Successor function:** Consider a state A that satisfies the state definition. A does not have any successor if it violates either constraint below.
  - If there is at least one sheep on the left bank, then the number of wolves on the left bank must be less than or equal to the number of sheep on the left bank.
  - If there is at least one sheep on the right bank, then the number of wolves on the right bank must be less than or equal to the number of sheep on the right bank.

  If state A satisfies both constraints above, then we will determine A's successor states as follows. Assume that B is a state satisfying the state definition. B is a successor of A if and only if we can start from A, move the boat with one or two animals from one side of the river to the other side of the river, and arrive at state B.

Marking Scheme: (4 marks)

- (2 marks) Correct answer
- (2 marks) A reasonable justification

Solutions:

- Avery's formulation is better.
- For Taylor's formulation, once we call the successor function of current state, it will add more states to frontier than Avery's formulation (since it has no constraints). For each iteration we have to check the validity of state in order to meet constraints. These steps will affect the performance of search algorithm.

(d) Based on the previous parts, come up with the best formulation for this problem. Make sure you define the states, the initial state, the goal state, and the successor function. Assume the cost function is the same as the one in Taylor's formulation.

We will mark your formulation on its correctness and its quality.

Marking Scheme: (10 marks)

- (4 marks) The quality of your formulation.

- (2 marks) State definition.
- (2 marks) Constraints.
- (1 mark) Initial state and goal states.
- (1 mark) Successor function.

Solutions:

- **State:** $S_L W_L S_R W_R B$
  S_L, W_L represents the number of sheep/wolf in left side.
  S_R, W_R represents the number of sheep/wolf in right side.
  $B =$ d if the boat is on the left bank (departure side) and B = a if the boat is on the right bank (arrival side).
- **Start State:** 3300d
- **Goal State:** 0033a
- **Successor Function:** Consider a state A satisfies the state definition. A does not have any successor if it violates either constraint below.
  - if $S_L \neq 0$, then $W_L \leq S_L$
  - if $S_R \neq 0$, then $W_R \leq S_R$

  If state A satisfies both constraints above, then we will determine A's successor states as follows. Assume that B is a state satisfying the state definition. B is a successor of A if and only if we can start from A, move the boat with one or two animals from one side of the river to the other side of the river, and arrive at state B.

(e) Draw the search graph. The search graph should contain the states and the arcs in your problem formulation in part d. Highlight the start state and the goal states in your graph.

If you search graph has more than 35 nodes in it, go back to part d and rethink your formulation.

Marking Scheme: (6 marks)

- (4 marks) includes all the states (and handles the constraints correctly).
- (2 marks) draws the arcs correctly (and handles the constraints correctly).

Solutions:

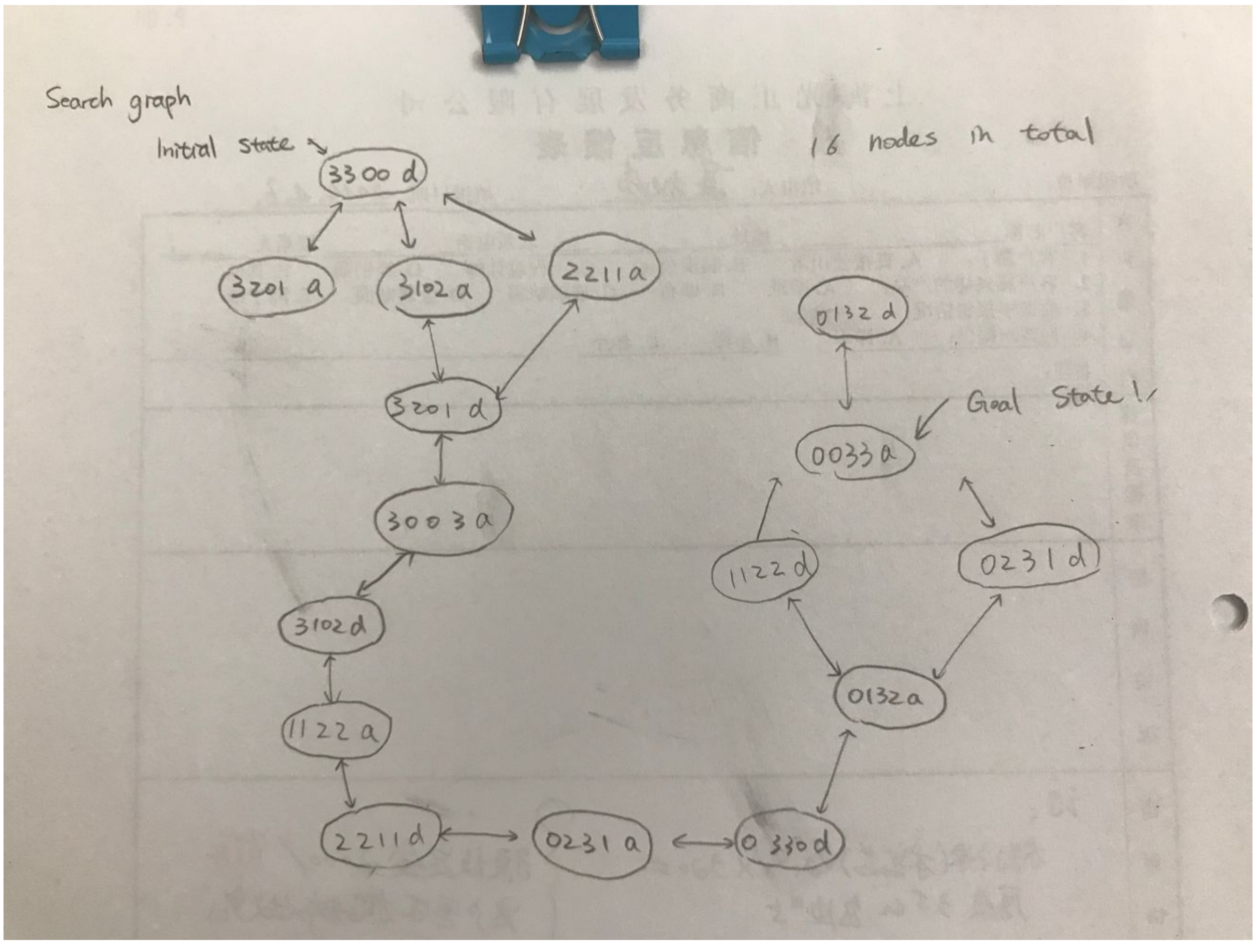

Figure 1: Search graph

(f) If your goal is to find a solution quickly (while minimizing the number of nodes expanded), which **uninformed search algorithm** would you use to solve this problem?

You can choose among breadth first search, depth first search, iterative-deepening search, and lowest-cost-first search. You can also use any pruning strategy discussed in lectures. Since you already drew the search graph, you can assume that it is given.

State the algorithm of your choice and justify your answer in at most 5 sentences.

Marking Scheme:

(3 marks) Provide a good reason for choosing a particular uninformed algorithm.

Solutions: From my perspective, the uninformed algorithm I would like to select is depth first search. Since we can use prune strategy, we do not need to worry about infinite loops. Using breadth first search, iterative-deepening search,and lowest-cost-first search will go through all nodes with lower costs that some of nodes may not be a part of path to goal state (for example, 3201a). However, when we use DFS, the ideal situation is that we do not need to add all nodes with lower cost into frontier and we can get the solution with less nodes. So I would like to use DFS.

(g) Propose an admissible heuristic function h. Describe the heuristic function in detail. Then, explain why the heuristic function is admissible. Some marks will be assigned to the quality of the heuristic function.

Hint: A heuristic function must specify a value for every state in the search graph.

Hint: If you relax the problem by removing a constraint, then the optimal solution to the relaxed problem is guaranteed to be an admissible heuristic function. One way to justify why your heuristic function is admissible is to explain how you relaxed the problem and solved the relaxed problem optimally.

Marking Scheme: (10 marks)

- (6 marks) Describe the heuristic function.
- (2 marks) Describe why the heuristic function is admissible.
- (2 marks) The quality of your heuristic function.

Solutions:

- My heuristic function is $h(x)$ = number of sheep and wolves on left side -1
- it is admissible because all pair of boat trips from left side to right side and

from right side to left side can take at most 1 animal from left side to right side(take 2 from left to right, then we need at least 1 animal back from right to left except last trip). So we need at least (2× number of sheep and wolves on left side - 1) trips.

(h) Your friend is considering using **the A* search algorithm** instead of using an uninformed search algorithm to solve the sheep and wolves problem. Would the A* search algorithm be a better choice than an uninformed search algorithm? Why or why not? State your answer and justify it in at most 5 sentences.

Tip: Do not trace the algorithm on the problem. We are not looking for the exact number of states expanded for either algorithm. Instead, formulate your answer by looking at the search graph and reasoning about the behaviour of the search algorithms.

Marking Scheme: (3 marks) A good justification for your answer.

Solutions: I agree that A* is a better choice than an uninformed search algorithm based on following reasons:

- A* with admissible heuristic function guarantees to get the optimal solution with less cost.
- A* will expand less nodes than uninformed search algorithm.

2 The Rush Hour Sliding Puzzle (98 marks)

In this programming question, you will solve the Rush Hour puzzle using the A* search and the depth-first search algorithms.

Take a look at an example of a Rush Hour puzzle below. The puzzle is on a 6 by 6 grid. We will number the rows as 0 to 5 from the top, and the columns as 0 to 5 from the left. In row 2, a horizontal car of length 2, called the goal car, is trying to escape through the exit on the right. There are horizontal and vertical cars of various lengths in the grid. A horizontal car can only move horizontally, whereas a vertical car can only move vertically. Each car may move more than one square in one step, but it cannot move over or through other cars. The goal is to move the cars around until the goal car reaches the exit, i.e. until the goal car is in the columns 4 and 5 in row 2.

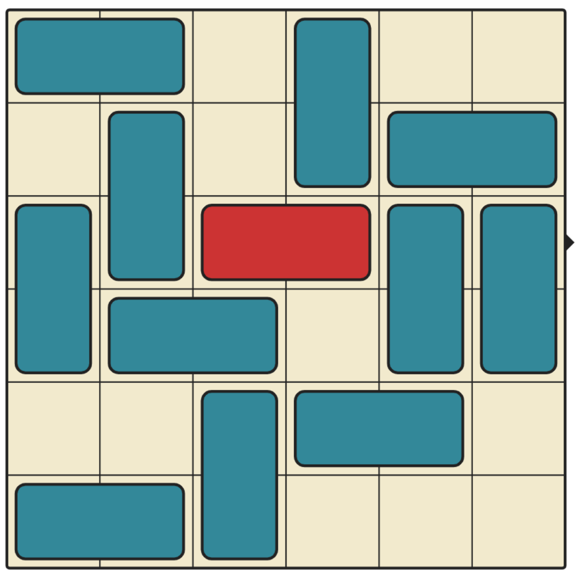

Figure 2: An example of a Rush Hour puzzle from https://www.michaelfogleman.com/rush/

You can make the following assumptions for this question.

- The puzzle must be on a 6 x 6 grid.
- The goal car is in row 2 and it has a length of 2.
- Besides the goal car, there is no other horizontal car in row 2.

Information on the Provided Code

We have provided three files `board.py`, `solve.py`, and `jams_posted.txt` on Learn. `board.py` contains code for handling input/output, representing states, etc. Your main task is to fill in the empty functions in `solve.py`.

Submit the `solve.py` to Marmoset only. We will use our version of `board.py` to test your code. Do not modify any provided function signatures in `solve.py`. Doing so will cause you to fail our tests. Feel free to add any new code to `solve.py`.

Input Format

The file `jams_posted.txt` contains 40 puzzles. You can use these puzzles to test your program. We will test your program using a different set of puzzles.

Below is an example of an input describing a puzzle.

```
example
6
1 2 h 2
2 0 v 2
4 0 h 2
3 1 v 3
4 1 v 2
4 3 h 2
.
```

- The first line assigns a name to the puzzle. In this case, the name is "example."
- The next line specifies the size of the grid. We only use 6 by 6 grid. So this number is always 6.
- The next line "`1 2 h 2`" gives a description of the goal car. The first two numbers $(1, 2)$ gives the (x, y) coordinates of the upper left corner of the car. The next letter "h" indicates that the car is horizontal ("v" would indicate that the car is vertical). The last number "2" indicates that the car has size 2.
- Each subsequent line, except the last line, describe a car in the puzzle, using the same format.
- The last line consists of a single period, indicating the end of the puzzle.

You can include multiple puzzles consecutively in the same file using the above format.

The Heuristic Functions for A* Search:

We have provided the implementation of the zero Heuristic function, which assigns a heuristic value of 0 to every state.

You must implement two other heuristic functions for A* search.

- Blocking Heuristic: The heuristic value for any goal state is zero. For any non-goal state, the heuristic value is one plus the number of cars blocking the path to the exit. For example, for the state in Figure 2, the blocking heuristic value is 3.
- Advanced Heuristic: Implement a third advanced heuristic of your choosing and invention. Your advanced heuristic should be consistent and should dominate the blocking heuristic.

Testing Your Program

Debugging and testing are essential skills for computer scientists. For this question, debugging your program may be especially challenging because of ties. Among "correct" implementations, the number of nodes expanded may vary widely due to how we handle the nodes with the same heuristic value on the frontier.

Please test your code using **Python 3.8.5**.

We rely on Python's hashing to generate a state's ID for breaking ties (see the Breaking Ties section below). However, Python' hashing function is not deterministic across different sessions. For example, you may get different hashing values of the same object for running your program multiple times. **Please set the environment variable PYTHONHASHSEED to 486 BEFORE running the Python script.** Note that setting the variable in the code/program will not work.

Implement **multi-path pruning for both DFS and A***. When there are multiple paths to a state, multi-path pruning explores the first path to the state and discards any subsequent path to the same state. Use an explored set to keep track of the states that have been expanded by the algorithm. When you **remove** a state from the frontier, check whether the state is in the explored set or not. If the state is in the explored set, then do nothing. Otherwise, add the state to the explored set and continue with the algorithm. Note that we perform pruning after we **remove** a state from the frontier, not before we **add** a state to the frontier.

DFS's behaviour depends on the order of adding a state's successors to the frontier. We will break ties by using the states' ID values. At each step, DFS will add the successors to the frontier in **decreasing** order of their IDs. In other words, DFS will expand the state with the smallest ID value among the successors.

A* search will also break ties using the states' ID values. Among several states with the same f value, A* will expand the state with the smallest ID value. If two states have the same ID value, A* will break ties using the states' parents — expanding the state whose parent has the smaller ID value.

Please complete the following tasks:

Submit your solutions to part (a) on Marmoset and submit your solutions to parts (b) and (c) on Learn.

(a) Complete the empty functions in `solve.py` and submit `solve.py` on Marmoset. Marmoset will evaluate your program for its correctness and efficiency.

For correctness, we have written unit tests for these functions: `get_path`, `is_goal`, `blocking_heuristic`, `get_successors`, `dfs`, `a_star`.

For each function, Marmoset provides one public test, which tests the function in a trivial scenario. There are also several secret tests. Before the deadline, you can only view the results of the public tests. After the deadline, Marmoset will run all the tests and calculate your marks.

Each test runs the function up to a predefined time limit. If the test passes if and only if the function terminates within the time limit and returns the expected result. Each test is all or nothing — there are no partial marks available.

Marking Scheme: (88 marks)

Unit tests on `get_path`, `is_goal`, `blocking_heuristic`, `get_successors`, `dfs`, and `a_star`.

- `get_path`: (1 public test + 2 secret tests) * 1 mark = 3 marks.
- `is_goal`: (1 public test + 4 secret tests) * 1 mark = 5 marks.
- `blocking_heuristic`: (1 public test + 9 secret tests) * 2 marks = 20 marks.
- `get_successors`: (1 public test + 9 secret tests) * 2 marks = 20 marks.
- `dfs`: (1 public test + 9 secret tests) * 2 marks = 20 marks.
- `a_star`: (1 public test + 9 secret tests) * 2 marks = 20 marks.

(b) Prove that the blocking heuristic is consistent using the definition of a consistent heuristic.

Marking Scheme: (3 marks) Proof is correct and easy to understand.

Solutions:

- In order to prove this heuristic is a consistent heuristic.
- we want to prove $h(m) - h(n) \leq cost(m, n)$.

- $h(m) - h(n)$ means the difference of number of cars blocking goal car in state m and state n.
- Ideally, we need at least one move to increase/decrease one blocking car.
- so we need at least $h(m) - h(n)$ steps to move from a state m to a state n.
- so it is proved that blocking heuristic is consistent.

(c) Design and implement an advanced heuristic of your own invention. Your advanced heuristic should be consistent and dominate the blocking heuristic.

Prove that your advanced heuristic is consistent and dominates the blocking heuristic.

Implement your advanced heuristic. Show that A* search with the advanced heuristic expands fewer nodes than A* search with the blocking heuristic on all the 40 provided puzzles.

Marking Scheme: (7 marks)

- (3 marks) Prove that your advanced heuristic is consistent.
- (2 marks) Prove that your advanced heuristic dominates the blocking heuristic.
- (2 marks) Show program output to support that the advanced heuristic dominates the blocking heuristic.

Solutions:

- my advanced heuristic is that if the state is goal state, we return 0. Otherwise we return 1 + number of "blocking cars" described in blocking heuristic + for each "blocking car", min of (number of cars blocks "blocking car" upper side) and (number of cars blocks "blocking car" lower side)
- In order to prove this heuristic is a consistent heuristic. We want to prove $h(m) - h(n) \leq cost(m, n)$.
  - $cost(m, n)$ means the actual number of steps from state m to state n.
  - For cars blocks each "blocking car", since we only add minimum number of cars blocks "blocking car" on one side. So we need at least this number of move to make sure each "blocking car" are able to move.
  - the rest part is similar to blocking heuristic. Ideally we need at least 1 step to move "blocking cars" away from row 2.
  - so we can get that $h(m) - h(n) \leq cost(m, n)$
- To prove my advanced heuristic dominates the blocking heuristic.

  - it is obvious that $\forall a, h_a(a) = 1 +$ number of "blocking cars" described in blocking heuristic + min of (number of cars blocks "blocking car" upper side) and (number of cars blocks "blocking car" lower side) for each "blocking car" $\geq 1 +$ number of "blocking cars" $= h_0(a)$
  - $\exists b$, such that for one "blocking car" has one car blocks it both upper and lower side. $h_a(b) \geq 1+$number of "blocking cars" $+1 > h_0(b)$
  - so my advanced heuristic dominates the blocking heuristic.

- Attached as "output.txt"