

Rank-65871 over GF(4)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431725481

General information

Number of lines	4
Number of points	21
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	12
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	$2^4, 1^{12}, 0^5$

Singular Points

The surface has 2 singular points:

$$0 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$1 : P_{27} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{10} \\ \ell_2 &= \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \mathbf{Pl}(0, 0, 1, 1, 2, 1)_{258} \\ \ell_3 &= \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \mathbf{Pl}(0, 0, 1, 1, 3, 1)_{318}\end{aligned}$$

Rank of lines: (0, 84, 80, 59)

Rank of points on Klein quadric: (0, 10, 258, 318)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_6 = (2, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_7 = (3, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{38} = (0, 0, 1, 1) = \ell_2 \cap \ell_3$$

Single Points

The surface has 12 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_{23} = (1, 0, 0, 1) \text{ lies on line } \ell_1$$

$$3 : P_{27} = (1, 1, 0, 1) \text{ lies on line } \ell_1$$

$$4 : P_{31} = (1, 2, 0, 1) \text{ lies on line } \ell_1$$

$$5 : P_{35} = (1, 3, 0, 1) \text{ lies on line } \ell_1$$

$$6 : P_{43} = (2, 1, 1, 1) \text{ lies on line } \ell_2$$

$$7 : P_{44} = (3, 1, 1, 1) \text{ lies on line } \ell_3$$

$$8 : P_{46} = (1, 2, 1, 1) \text{ lies on line } \ell_3$$

$$9 : P_{48} = (3, 2, 1, 1) \text{ lies on line } \ell_2$$

$$10 : P_{50} = (1, 3, 1, 1) \text{ lies on line } \ell_2$$

$$11 : P_{51} = (2, 3, 1, 1) \text{ lies on line } \ell_3$$

The single points on the surface are:

Points on surface but on no line

The surface has 5 points not on any line:

The points on the surface but not on lines are:

$$0 : P_{11} = (0, 1, 1, 0)$$

$$1 : P_{16} = (1, 2, 1, 0)$$

$$2 : P_{20} = (1, 3, 1, 0)$$

$$3 : P_{53} = (0, 0, 2, 1)$$

$$4 : P_{69} = (0, 0, 3, 1)$$

Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	0	0
2	1	0	0	1
3	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_1	P_6	P_7

Line 1 intersects

Line	ℓ_0
in point	P_1

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_7	P_{38}

The surface has 21 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_5 = (1, 1, 0, 0)$$

$$3 : P_6 = (2, 1, 0, 0)$$

$$4 : P_7 = (3, 1, 0, 0)$$

$$5 : P_{11} = (0, 1, 1, 0)$$

$$6 : P_{16} = (1, 2, 1, 0)$$

$$7 : P_{20} = (1, 3, 1, 0)$$

$$8 : P_{23} = (1, 0, 0, 1)$$

$$9 : P_{27} = (1, 1, 0, 1)$$

$$10 : P_{31} = (1, 2, 0, 1)$$

$$11 : P_{35} = (1, 3, 0, 1)$$

$$12 : P_{38} = (0, 0, 1, 1)$$

$$13 : P_{43} = (2, 1, 1, 1)$$

$$14 : P_{44} = (3, 1, 1, 1)$$

$$15 : P_{46} = (1, 2, 1, 1)$$

$$16 : P_{48} = (3, 2, 1, 1)$$

$$17 : P_{50} = (1, 3, 1, 1)$$

$$18 : P_{51} = (2, 3, 1, 1)$$

$$19 : P_{53} = (0, 0, 2, 1)$$

$$20 : P_{69} = (0, 0, 3, 1)$$