Rank-73802 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498768750

General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	21
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	$3^4, 2^6, 1^{21}, 0^2$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{45} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{45} = \mathbf{Pl}(3,0,2,0,0,1)_{118}$$

1

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{65} = \mathbf{Pl}(2,0,3,0,0,1)_{124}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{129} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{129} = \mathbf{Pl}(3,2,2,3,0,1)_{173}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{149} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{149} = \mathbf{Pl}(2,3,3,2,0,1)_{166}$$

Rank of lines: (22, 45, 65, 345, 106, 355, 129, 350, 149)
Rank of points on Klein quadric: (109, 118, 124, 13, 150, 15, 173, 14, 166)

Eckardt Points

The surface has 4 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_{11} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$2 : P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0)$,
$3 : P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{21} = (2,3,1,0) = \ell_0 \cap \ell_1$ $P_{66} = (1,3,2,1) = \ell_4 \cap \ell_8$
$P_{18} = (3,2,1,0) = \ell_0 \cap \ell_2$ $P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$
$P_{12} = (1,1,1,0) = \ell_1 \cap \ell_2$
$P_{78} = (1,2,3,1) = \ell_4 \cap \ell_6$

Single Points

The surface has 21 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_0 $7 : P_{31} = (1,2,0,1)$ lies on line ℓ_8
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_1 $8 : P_{35} = (1,3,0,1)$ lies on line ℓ_6
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_2 $9 : P_{39} = (1,0,1,1)$ lies on line ℓ_4
$3 : P_8 = (1,0,1,0)$ lies on line ℓ_0 $10 : P_{42} = (0,1,1,1)$ lies on line ℓ_3
$4 : P_9 = (2,0,1,0)$ lies on line ℓ_2 $11 : P_{45} = (0,2,1,1)$ lies on line ℓ_5
$5 : P_{10} = (3,0,1,0)$ lies on line ℓ_1 $12 : P_{49} = (0,3,1,1)$ lies on line ℓ_7
$6 : P_{27} = (1,1,0,1)$ lies on line ℓ_4 $13 : P_{54} = (1,0,2,1)$ lies on line ℓ_6

14 : $P_{57} = (0,1,2,1)$ lies on line ℓ_7 18 : $P_{73} = (0,1,3,1)$ lies on line ℓ_5
15 : $P_{61} = (0,2,2,1)$ lies on line ℓ_3 19 : $P_{77} = (0,2,3,1)$ lies on line ℓ_7
16 : $P_{65} = (0,3,2,1)$ lies on line ℓ_5 20 : $P_{81} = (0,3,3,1)$ lies on line ℓ_3
17 : $P_{70} = (1,0,3,1)$ lies on line ℓ_8

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

0 : $P_{24} = (2,0,0,1)$
1 : $P_{25} = (3,0,0,1)$

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7 8
--+-----------------
0 |0 1 1 1 1 0 0 0 0
1 |1 0 1 0 0 1 1 0 0
2 |1 1 0 0 0 0 0 1 1
3 |1 0 0 0 1 1 0 1 0
4 |1 0 0 1 0 0 1 0 1
5 |0 1 0 1 0 0 1 1 0
6 |0 1 0 0 1 1 0 0 1
7 |0 0 1 1 0 1 0 0 1
8 |0 0 1 0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_{21}	P_{18}	P_{11}	P_{11}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_{21}	P_{12}	P_{15}	P_{15}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_{18}	P_{12}	P_{19}	P_{19}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_{11}	P_{11}	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_8
in point	P_{11}	P_{11}	P_{78}	P_{66}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_{15}	P_3	P_{15}	P_3

Line 6 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_8
in point	P_{15}	P_{78}	P_{15}	P_4

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_{19}	P_3	P_3	P_{19}

Line 8 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_7
in point	P_{19}	P_{66}	P_4	P_{19}

The surface has **33** points:
The points on the surface are:

$0: P_3 = (0, 0, 0, 1)$

$1: P_4 = (1, 1, 1, 1)$

$2: P_5 = (1, 1, 0, 0)$

$3: P_6 = (2, 1, 0, 0)$

$4: P_7 = (3, 1, 0, 0)$

$5: P_8 = (1, 0, 1, 0)$

$6: P_9 = (2, 0, 1, 0)$

$7: P_{10} = (3, 0, 1, 0)$

$8: P_{11} = (0, 1, 1, 0)$

$9: P_{12} = (1, 1, 1, 0)$

$10: P_{15} = (0, 2, 1, 0)$

$11: P_{18} = (3, 2, 1, 0)$

$12: P_{19} = (0, 3, 1, 0)$

$13: P_{21} = (2, 3, 1, 0)$

$14: P_{24} = (2, 0, 0, 1)$

$15: P_{25} = (3, 0, 0, 1)$

$16: P_{27} = (1, 1, 0, 1)$

$17: P_{31} = (1, 2, 0, 1)$

$18: P_{35} = (1, 3, 0, 1)$

$19: P_{39} = (1, 0, 1, 1)$

$20: P_{42} = (0, 1, 1, 1)$

$21: P_{45} = (0, 2, 1, 1)$

$22: P_{49} = (0, 3, 1, 1)$

$23: P_{54} = (1, 0, 2, 1)$

$24: P_{57} = (0, 1, 2, 1)$

$25: P_{61} = (0, 2, 2, 1)$

$26: P_{65} = (0, 3, 2, 1)$

$27: P_{66} = (1, 3, 2, 1)$

$28: P_{70} = (1, 0, 3, 1)$

$29: P_{73} = (0, 1, 3, 1)$

$30: P_{77} = (0, 2, 3, 1)$

$31: P_{78} = (1, 2, 3, 1)$

$32: P_{81} = (0, 3, 3, 1)$