Rank-65548 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431655834

General information

Number of lines	7
Number of points	29
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	3
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	4^2, 2^3, 1^{21}, 0^3

Singular Points

The surface has 2 singular points:

$0: \ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \mathbf{Pl}(2,0,0,1,0,0)_{11}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \mathbf{Pl}(3,0,0,1,0,0)_{12}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_{6}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \mathbf{Pl}(0,3,1,0,0,0)_{8}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \mathbf{Pl}(0,2,1,0,0,0)_{7}$$

Rank of lines: (336, 84, 252, 168, 100, 268, 184)
Rank of points on Klein quadric: (101, 10, 11, 12, 6, 8, 7)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{23} = (1,0,0,1) = \ell_1 \cap \ell_4$
$P_{24} = (2,0,0,1) = \ell_2 \cap \ell_5$
$P_{25} = (3,0,0,1) = \ell_3 \cap \ell_6$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_{11} = (0,1,1,0)$ lies on line ℓ_0
1 : $P_{15} = (0,2,1,0)$ lies on line ℓ_0
2 : $P_{19} = (0,3,1,0)$ lies on line ℓ_0
3 : $P_{27} = (1,1,0,1)$ lies on line ℓ_1
4 : $P_{28} = (2,1,0,1)$ lies on line ℓ_2
5 : $P_{29} = (3,1,0,1)$ lies on line ℓ_3
6 : $P_{31} = (1,2,0,1)$ lies on line ℓ_1
7 : $P_{32} = (2,2,0,1)$ lies on line ℓ_2
8 : $P_{33} = (3,2,0,1)$ lies on line ℓ_3
9 : $P_{35} = (1,3,0,1)$ lies on line ℓ_1
10 : $P_{36} = (2,3,0,1)$ lies on line ℓ_2
11 : $P_{37} = (3,3,0,1)$ lies on line ℓ_3
12 : $P_{39} = (1,0,1,1)$ lies on line ℓ_4
13 : $P_{40} = (2,0,1,1)$ lies on line ℓ_5
14 : $P_{41} = (3,0,1,1)$ lies on line ℓ_6
15 : $P_{54} = (1,0,2,1)$ lies on line ℓ_4
16 : $P_{55} = (2,0,2,1)$ lies on line ℓ_5
17 : $P_{56} = (3,0,2,1)$ lies on line ℓ_6
18 : $P_{70} = (1,0,3,1)$ lies on line ℓ_4
19 : $P_{71} = (2,0,3,1)$ lies on line ℓ_5
20 : $P_{72} = (3,0,3,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 3 points not on any line:
The points on the surface but not on lines are:

$0: P_{12} = (1, 1, 1, 0)$
$1: P_{18} = (3, 2, 1, 0)$
$2: P_{21} = (2, 3, 1, 0)$

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 1 1 1 1
1	1 0 1 1 1 0 0
2	1 1 0 1 0 1 0
3	1 1 1 0 0 0 1
4	1 1 0 0 0 1 1
5	1 0 1 0 1 0 1
6	1 0 0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_1	P_2	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_1	P_1	P_1	P_{23}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_1	P_1	P_1	P_{24}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_1	P_1	P_1	P_{25}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_2	P_{23}	P_2	P_2

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_{24}	P_2	P_2

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_2	P_{25}	P_2	P_2

The surface has 29 points:
The points on the surface are:

$0: P_1 = (0, 1, 0, 0)$
$1: P_2 = (0, 0, 1, 0)$
$2: P_{11} = (0, 1, 1, 0)$
$3: P_{12} = (1, 1, 1, 0)$
$4: P_{15} = (0, 2, 1, 0)$
$5: P_{18} = (3, 2, 1, 0)$
$6: P_{19} = (0, 3, 1, 0)$
$7: P_{21} = (2, 3, 1, 0)$
$8: P_{23} = (1, 0, 0, 1)$
$9: P_{24} = (2, 0, 0, 1)$
$10: P_{25} = (3, 0, 0, 1)$
$11: P_{27} = (1, 1, 0, 1)$
$12: P_{28} = (2, 1, 0, 1)$
$13: P_{29} = (3, 1, 0, 1)$
$14: P_{31} = (1, 2, 0, 1)$
$15: P_{32} = (2, 2, 0, 1)$
$16: P_{33} = (3, 2, 0, 1)$
$17: P_{35} = (1, 3, 0, 1)$
$18: P_{36} = (2, 3, 0, 1)$
$19: P_{37} = (3, 3, 0, 1)$
$20: P_{39} = (1, 0, 1, 1)$

$21: P_{40} = (2, 0, 1, 1)$
$22: P_{41} = (3, 0, 1, 1)$
$23: P_{54} = (1, 0, 2, 1)$
$24: P_{55} = (2, 0, 2, 1)$
$25: P_{56} = (3, 0, 2, 1)$
$26: P_{70} = (1, 0, 3, 1)$
$27: P_{71} = (2, 0, 3, 1)$
$28: P_{72} = (3, 0, 3, 1)$