# Rank-74099 over GF(32)

January 15, 2021

# The equation

The equation of the surface is:

$$X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 ) The point rank of the equation over GF(32) is -2078243803

### General information

| Number of lines            | 2                     |
|----------------------------|-----------------------|
| Number of points           | 1089                  |
| Number of singular points  | 1                     |
| Number of Eckardt points   | 0                     |
| Number of double points    | 1                     |
| Number of single points    | 64                    |
| Number of points off lines | 1024                  |
| Number of Hesse planes     | 0                     |
| Number of axes             | 0                     |
| Type of points on lines    | $33^{2}$              |
| Type of lines on points    | $2, 1^{64}, 0^{1024}$ |

## Singular Points

The surface has 1 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

### The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{65}$$

$$\ell_1 = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{1083424} = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{1083424} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (1082400, 1083424)
Rank of points on Klein quadric: (65, 1)

#### Eckardt Points

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 1 Double points: The double points on the surface are:

$$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$$

#### Single Points

The surface has 64 single points: The single points on the surface are:

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0: P_1 = (0, 1, 0, 0) lies on line \ell_0
                                                                     29: P_{1954} = (0, 28, 0, 1) lies on line \ell_0
1: P_2 = (0, 0, 1, 0) lies on line \ell_1
                                                                      30: P_{1986} = (0, 29, 0, 1) lies on line \ell_0
2: P_{1090} = (0, 1, 0, 1) lies on line \ell_0
                                                                     31: P_{2018} = (0, 30, 0, 1) lies on line \ell_0
3: P_{1122} = (0, 2, 0, 1) lies on line \ell_0
                                                                     32: P_{2050} = (0, 31, 0, 1) lies on line \ell_0
                                                                     33 : P_{2082} = (0, 0, 1, 1) lies on line \ell_1
4: P_{1154} = (0, 3, 0, 1) lies on line \ell_0
5: P_{1186} = (0, 4, 0, 1) lies on line \ell_0
                                                                     34: P_{3105} = (0,0,2,1) lies on line \ell_1
6: P_{1218} = (0, 5, 0, 1) lies on line \ell_0
                                                                     35: P_{4129} = (0,0,3,1) lies on line \ell_1
7: P_{1250} = (0, 6, 0, 1) lies on line \ell_0
                                                                     36: P_{5153} = (0,0,4,1) lies on line \ell_1
8: P_{1282} = (0,7,0,1) lies on line \ell_0
                                                                      37: P_{6177} = (0,0,5,1) lies on line \ell_1
9: P_{1314} = (0, 8, 0, 1) lies on line \ell_0
                                                                     38: P_{7201} = (0,0,6,1) lies on line \ell_1
10: P_{1346} = (0, 9, 0, 1) lies on line \ell_0
                                                                     39: P_{8225} = (0,0,7,1) lies on line \ell_1
11: P_{1378} = (0, 10, 0, 1) lies on line \ell_0
                                                                      40: P_{9249} = (0,0,8,1) lies on line \ell_1
                                                                     41 : P_{10273} = (0, 0, 9, 1) lies on line \ell_1
12: P_{1410} = (0, 11, 0, 1) lies on line \ell_0
13: P_{1442} = (0, 12, 0, 1) lies on line \ell_0
                                                                     42: P_{11297} = (0, 0, 10, 1) lies on line \ell_1
14: P_{1474} = (0, 13, 0, 1) lies on line \ell_0
                                                                     43: P_{12321} = (0, 0, 11, 1) lies on line \ell_1
15: P_{1506} = (0, 14, 0, 1) lies on line \ell_0
                                                                      44: P_{13345} = (0, 0, 12, 1) lies on line \ell_1
16: P_{1538} = (0, 15, 0, 1) lies on line \ell_0
                                                                      45: P_{14369} = (0, 0, 13, 1) lies on line \ell_1
                                                                     46: P_{15393} = (0, 0, 14, 1) lies on line \ell_1
17: P_{1570} = (0, 16, 0, 1) lies on line \ell_0
18: P_{1602} = (0, 17, 0, 1) lies on line \ell_0
                                                                     47: P_{16417} = (0, 0, 15, 1) lies on line \ell_1
19: P_{1634} = (0, 18, 0, 1) lies on line \ell_0
                                                                      48: P_{17441} = (0, 0, 16, 1) lies on line \ell_1
                                                                      49: P_{18465} = (0, 0, 17, 1) lies on line \ell_1
20: P_{1666} = (0, 19, 0, 1) lies on line \ell_0
                                                                     50: P_{19489} = (0, 0, 18, 1) lies on line \ell_1
21: P_{1698} = (0, 20, 0, 1) lies on line \ell_0
22: P_{1730} = (0, 21, 0, 1) lies on line \ell_0
                                                                     51: P_{20513} = (0, 0, 19, 1) lies on line \ell_1
23: P_{1762} = (0, 22, 0, 1) lies on line \ell_0
                                                                     52: P_{21537} = (0, 0, 20, 1) lies on line \ell_1
24: P_{1794} = (0, 23, 0, 1) lies on line \ell_0
                                                                     53: P_{22561} = (0,0,21,1) lies on line \ell_1
25: P_{1826} = (0, 24, 0, 1) lies on line \ell_0
                                                                     54: P_{23585} = (0, 0, 22, 1) lies on line \ell_1
                                                                     55: P_{24609} = (0, 0, 23, 1) lies on line \ell_1
26: P_{1858} = (0, 25, 0, 1) lies on line \ell_0
27: P_{1890} = (0, 26, 0, 1) lies on line \ell_0
                                                                     56: P_{25633} = (0, 0, 24, 1) lies on line \ell_1
28: P_{1922} = (0, 27, 0, 1) lies on line \ell_0
                                                                     57: P_{26657} = (0, 0, 25, 1) lies on line \ell_1
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 $\begin{array}{lll} 58: \ P_{27681} = (0,0,26,1) \ \text{lies on line} \ \ell_1 \\ 59: \ P_{28705} = (0,0,27,1) \ \text{lies on line} \ \ell_1 \\ 60: \ P_{29729} = (0,0,28,1) \ \text{lies on line} \ \ell_1 \end{array}$ 

The single points on the surface are:

61 :  $P_{30753} = (0, 0, 29, 1)$  lies on line  $\ell_1$ 

### Points on surface but on no line

The surface has 1024 points not on any line: Too many to print.

# Line Intersection Graph

 $\begin{array}{c|c} 0 \ 1 \\ \hline 0 \ 0 \ 1 \\ 1 \ 1 \ 0 \end{array}$ 

Neighbor sets in the line intersection graph: Line 0 intersects

| Line     | $\ell_1$ |
|----------|----------|
| in point | $P_3$    |

Line 1 intersects

| Line     | $\ell_0$ |
|----------|----------|
| in point | $P_3$    |

The surface has 1089 points: Too many to print.