

# Rank-65859 over GF(32)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(32) is -2112846811

## General information

Number of lines	35
Number of points	1089
Number of singular points	33
Number of Eckardt points	16
Number of double points	34
Number of single points	1039
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$33^{35}$
Type of lines on points	$3^{16}, 2^{34}, 1^{1039}$

## Singular Points

The surface has 33 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{2082} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{3105} = \mathbf{P}(0, 0, \eta, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{4129} = \mathbf{P}(0, 0, \eta^{18}, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{5153} = \mathbf{P}(0, 0, \eta^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{6177} = \mathbf{P}(0, 0, \eta^5, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{7201} = \mathbf{P}(0, 0, \eta^{19}, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{8225} = \mathbf{P}(0, 0, \eta^{11}, 1) = \mathbf{P}(0, 0, 7, 1)$$

$$9 : P_{9249} = \mathbf{P}(0, 0, \eta^3, 1) = \mathbf{P}(0, 0, 8, 1)$$

$$10 : P_{10273} = \mathbf{P}(0, 0, \eta^{29}, 1) = \mathbf{P}(0, 0, 9, 1)$$

$$11 : P_{11297} = \mathbf{P}(0, 0, \eta^6, 1) = \mathbf{P}(0, 0, 10, 1)$$

$$12 : P_{12321} = \mathbf{P}(0, 0, \eta^{27}, 1) = \mathbf{P}(0, 0, 11, 1)$$

$$13 : P_{13345} = \mathbf{P}(0, 0, \eta^{20}, 1) = \mathbf{P}(0, 0, 12, 1)$$

$$14 : P_{14369} = \mathbf{P}(0, 0, \eta^8, 1) = \mathbf{P}(0, 0, 13, 1)$$

$$15 : P_{15393} = \mathbf{P}(0, 0, \eta^{12}, 1) = \mathbf{P}(0, 0, 14, 1)$$

$$16 : P_{16417} = \mathbf{P}(0, 0, \eta^{23}, 1) = \mathbf{P}(0, 0, 15, 1)$$

$$17 : P_{17441} = \mathbf{P}(0, 0, \eta^4, 1) = \mathbf{P}(0, 0, 16, 1)$$

$$\begin{aligned}
18 : P_{18465} &= \mathbf{P}(0, 0, \eta^{10}, 1) = \mathbf{P}(0, 0, 17, 1) \\
19 : P_{19489} &= \mathbf{P}(0, 0, \eta^{30}, 1) = \mathbf{P}(0, 0, 18, 1) \\
20 : P_{20513} &= \mathbf{P}(0, 0, \eta^{17}, 1) = \mathbf{P}(0, 0, 19, 1) \\
21 : P_{21537} &= \mathbf{P}(0, 0, \eta^7, 1) = \mathbf{P}(0, 0, 20, 1) \\
22 : P_{22561} &= \mathbf{P}(0, 0, \eta^{22}, 1) = \mathbf{P}(0, 0, 21, 1) \\
23 : P_{23585} &= \mathbf{P}(0, 0, \eta^{28}, 1) = \mathbf{P}(0, 0, 22, 1) \\
24 : P_{24609} &= \mathbf{P}(0, 0, \eta^{26}, 1) = \mathbf{P}(0, 0, 23, 1) \\
25 : P_{25633} &= \mathbf{P}(0, 0, \eta^{21}, 1) = \mathbf{P}(0, 0, 24, 1)
\end{aligned}$$

$$\begin{aligned}
26 : P_{26657} &= \mathbf{P}(0, 0, \eta^{25}, 1) = \mathbf{P}(0, 0, 25, 1) \\
27 : P_{27681} &= \mathbf{P}(0, 0, \eta^9, 1) = \mathbf{P}(0, 0, 26, 1) \\
28 : P_{28705} &= \mathbf{P}(0, 0, \eta^{16}, 1) = \mathbf{P}(0, 0, 27, 1) \\
29 : P_{29729} &= \mathbf{P}(0, 0, \eta^{13}, 1) = \mathbf{P}(0, 0, 28, 1) \\
30 : P_{30753} &= \mathbf{P}(0, 0, \eta^{14}, 1) = \mathbf{P}(0, 0, 29, 1) \\
31 : P_{31777} &= \mathbf{P}(0, 0, \eta^{24}, 1) = \mathbf{P}(0, 0, 30, 1) \\
32 : P_{32801} &= \mathbf{P}(0, 0, \eta^{15}, 1) = \mathbf{P}(0, 0, 31, 1)
\end{aligned}$$

## The 35 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2081} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2081} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{34912} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{65} \\
\ell_4 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_5 &= \begin{bmatrix} 1 & \eta^{30} & 0 & 0 \\ 0 & 0 & 1 & \eta^{16} \end{bmatrix}_{20077} = \begin{bmatrix} 1 & 18 & 0 & 0 \\ 0 & 0 & 1 & 27 \end{bmatrix}_{20077} = \mathbf{Pl}(0, 0, 31, 27, 19, 1)_{661700} \\
\ell_6 &= \begin{bmatrix} 1 & \eta^{13} & 0 & 0 \\ 0 & 0 & 1 & \eta^{27} \end{bmatrix}_{30631} = \begin{bmatrix} 1 & 28 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{30631} = \mathbf{Pl}(0, 0, 16, 11, 29, 1)_{988115} \\
\ell_7 &= \begin{bmatrix} 1 & \eta^{29} & 0 & 0 \\ 0 & 0 & 1 & \eta \end{bmatrix}_{10539} = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{10539} = \mathbf{Pl}(0, 0, 18, 2, 8, 1)_{300785} \\
\ell_8 &= \begin{bmatrix} 1 & \eta^{26} & 0 & 0 \\ 0 & 0 & 1 & \eta^{23} \end{bmatrix}_{25350} = \begin{bmatrix} 1 & 23 & 0 & 0 \\ 0 & 0 & 1 & 15 \end{bmatrix}_{25350} = \mathbf{Pl}(0, 0, 13, 15, 22, 1)_{758774} \\
\ell_9 &= \begin{bmatrix} 1 & \eta^{12} & 0 & 0 \\ 0 & 0 & 1 & \eta^4 \end{bmatrix}_{15838} = \begin{bmatrix} 1 & 14 & 0 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}_{15838} = \mathbf{Pl}(0, 0, 11, 16, 15, 1)_{529496} \\
\ell_{10} &= \begin{bmatrix} 1 & \eta^{20} & 0 & 0 \\ 0 & 0 & 1 & \eta^{28} \end{bmatrix}_{13730} = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 22 \end{bmatrix}_{13730} = \mathbf{Pl}(0, 0, 8, 22, 13, 1)_{463835} \\
\ell_{11} &= \begin{bmatrix} 1 & \eta^{28} & 0 & 0 \\ 0 & 0 & 1 & \eta^{23} \end{bmatrix}_{24293} = \begin{bmatrix} 1 & 22 & 0 & 0 \\ 0 & 0 & 1 & 15 \end{bmatrix}_{24293} = \mathbf{Pl}(0, 0, 13, 15, 23, 1)_{791510} \\
\ell_{12} &= \begin{bmatrix} 1 & \eta^2 & 0 & 0 \\ 0 & 0 & 1 & \eta^7 \end{bmatrix}_{5272} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{5272} = \mathbf{Pl}(0, 0, 30, 20, 5, 1)_{203333} \\
\ell_{13} &= \begin{bmatrix} 1 & \eta^{25} & 0 & 0 \\ 0 & 0 & 1 & \eta^{15} \end{bmatrix}_{27480} = \begin{bmatrix} 1 & 25 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{27480} = \mathbf{Pl}(0, 0, 27, 31, 24, 1)_{825128} \\
\ell_{14} &= \begin{bmatrix} 1 & \eta^4 & 0 & 0 \\ 0 & 0 & 1 & \eta^{14} \end{bmatrix}_{17965} = \begin{bmatrix} 1 & 16 & 0 & 0 \\ 0 & 0 & 1 & 29 \end{bmatrix}_{17965} = \mathbf{Pl}(0, 0, 19, 29, 17, 1)_{595472} \\
\ell_{15} &= \begin{bmatrix} 1 & \eta^{11} & 0 & 0 \\ 0 & 0 & 1 & \eta^{30} \end{bmatrix}_{8441} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{8441} = \mathbf{Pl}(0, 0, 2, 18, 6, 1)_{234305}
\end{aligned}$$

$$\begin{aligned}
\ell_{16} &= \begin{bmatrix} 1 & \eta^{23} & 0 & 0 \\ 0 & 0 & 1 & \eta^4 \end{bmatrix}_{16895} = \begin{bmatrix} 1 & 15 & 0 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}_{16895} = \mathbf{Pl}(0, 0, 11, 16, 14, 1)_{496760} \\
\ell_{17} &= \begin{bmatrix} 1 & \eta^{19} & 0 & 0 \\ 0 & 0 & 1 & \eta^{30} \end{bmatrix}_{7384} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{7384} = \mathbf{Pl}(0, 0, 2, 18, 7, 1)_{267041} \\
\ell_{18} &= \begin{bmatrix} 1 & \eta^8 & 0 & 0 \\ 0 & 0 & 1 & \eta^{28} \end{bmatrix}_{14787} = \begin{bmatrix} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 22 \end{bmatrix}_{14787} = \mathbf{Pl}(0, 0, 8, 22, 12, 1)_{431099} \\
\ell_{19} &= \begin{bmatrix} 1 & \eta^{27} & 0 & 0 \\ 0 & 0 & 1 & \eta^2 \end{bmatrix}_{12655} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{12655} = \mathbf{Pl}(0, 0, 9, 4, 10, 1)_{365690} \\
\ell_{20} &= \begin{bmatrix} 1 & \eta^{21} & 0 & 0 \\ 0 & 0 & 1 & \eta^{15} \end{bmatrix}_{26423} = \begin{bmatrix} 1 & 24 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{26423} = \mathbf{Pl}(0, 0, 27, 31, 25, 1)_{857864} \\
\ell_{21} &= \begin{bmatrix} 1 & \eta & 0 & 0 \\ 0 & 0 & 1 & \eta^{19} \end{bmatrix}_{3144} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{3144} = \mathbf{Pl}(0, 0, 14, 6, 3, 1)_{136853} \\
\ell_{22} &= \begin{bmatrix} 1 & \eta^{14} & 0 & 0 \\ 0 & 0 & 1 & \eta^{27} \end{bmatrix}_{31688} = \begin{bmatrix} 1 & 29 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{31688} = \mathbf{Pl}(0, 0, 16, 11, 28, 1)_{955379} \\
\ell_{23} &= \begin{bmatrix} 1 & \eta^{24} & 0 & 0 \\ 0 & 0 & 1 & \eta^8 \end{bmatrix}_{32747} = \begin{bmatrix} 1 & 30 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{32747} = \mathbf{Pl}(0, 0, 15, 13, 31, 1)_{1053524} \\
\ell_{24} &= \begin{bmatrix} 1 & \eta^9 & 0 & 0 \\ 0 & 0 & 1 & \eta^{25} \end{bmatrix}_{28531} = \begin{bmatrix} 1 & 26 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{28531} = \mathbf{Pl}(0, 0, 10, 25, 27, 1)_{922265} \\
\ell_{25} &= \begin{bmatrix} 1 & \eta^3 & 0 & 0 \\ 0 & 0 & 1 & \eta \end{bmatrix}_{9482} = \begin{bmatrix} 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{9482} = \mathbf{Pl}(0, 0, 18, 2, 9, 1)_{333521} \\
\ell_{26} &= \begin{bmatrix} 1 & \eta^5 & 0 & 0 \\ 0 & 0 & 1 & \eta^7 \end{bmatrix}_{6329} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{6329} = \mathbf{Pl}(0, 0, 30, 20, 4, 1)_{170597} \\
\ell_{27} &= \begin{bmatrix} 1 & \eta^{10} & 0 & 0 \\ 0 & 0 & 1 & \eta^{14} \end{bmatrix}_{19022} = \begin{bmatrix} 1 & 17 & 0 & 0 \\ 0 & 0 & 1 & 29 \end{bmatrix}_{19022} = \mathbf{Pl}(0, 0, 19, 29, 16, 1)_{562736} \\
\ell_{28} &= \begin{bmatrix} 1 & \eta^6 & 0 & 0 \\ 0 & 0 & 1 & \eta^2 \end{bmatrix}_{11598} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{11598} = \mathbf{Pl}(0, 0, 9, 4, 11, 1)_{398426} \\
\ell_{29} &= \begin{bmatrix} 1 & \eta^{22} & 0 & 0 \\ 0 & 0 & 1 & \eta^{29} \end{bmatrix}_{23230} = \begin{bmatrix} 1 & 21 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{bmatrix}_{23230} = \mathbf{Pl}(0, 0, 4, 9, 20, 1)_{692735} \\
\ell_{30} &= \begin{bmatrix} 1 & \eta^{15} & 0 & 0 \\ 0 & 0 & 1 & \eta^8 \end{bmatrix}_{33804} = \begin{bmatrix} 1 & 31 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{33804} = \mathbf{Pl}(0, 0, 15, 13, 30, 1)_{1020788} \\
\ell_{31} &= \begin{bmatrix} 1 & \eta^{18} & 0 & 0 \\ 0 & 0 & 1 & \eta^{19} \end{bmatrix}_{4201} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{4201} = \mathbf{Pl}(0, 0, 14, 6, 2, 1)_{104117} \\
\ell_{32} &= \begin{bmatrix} 1 & \eta^{17} & 0 & 0 \\ 0 & 0 & 1 & \eta^{16} \end{bmatrix}_{21134} = \begin{bmatrix} 1 & 19 & 0 & 0 \\ 0 & 0 & 1 & 27 \end{bmatrix}_{21134} = \mathbf{Pl}(0, 0, 31, 27, 18, 1)_{628964} \\
\ell_{33} &= \begin{bmatrix} 1 & \eta^7 & 0 & 0 \\ 0 & 0 & 1 & \eta^{29} \end{bmatrix}_{22173} = \begin{bmatrix} 1 & 20 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{bmatrix}_{22173} = \mathbf{Pl}(0, 0, 4, 9, 21, 1)_{725471} \\
\ell_{34} &= \begin{bmatrix} 1 & \eta^{16} & 0 & 0 \\ 0 & 0 & 1 & \eta^{25} \end{bmatrix}_{29588} = \begin{bmatrix} 1 & 27 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{29588} = \mathbf{Pl}(0, 0, 10, 25, 26, 1)_{889529}
\end{aligned}$$

Rank of lines: ( 0, 1024, 2081, 1082400, 1083424, 20077, 30631, 10539, 25350, 15838, 13730, 24293, 5272, 27480, 17965, 8441, 16895, 7384, 14787, 12655, 26423, 3144, 31688, 32747, 28531, 9482, 6329, 19022, 11598, 23230, 33804, 4201, 21134, 22173, 29588 )

Rank of points on Klein quadric: ( 0, 2, 34912, 65, 1, 661700, 988115, 300785, 758774, 529496, 463835, 791510, 203333, 825128, 595472, 234305, 496760, 267041, 431099, 365690, 857864, 136853, 955379, 1053524, 922265, 333521, 170597, 562736, 398426, 692735, 1020788, 104117, 628964, 725471, 889529 )

## Eckardt Points

The surface has 16 Eckardt points:

$$\begin{aligned}
0 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0), \\
1 : P_{3105} &= \mathbf{P}(0, 0, \eta, 1) = \mathbf{P}(0, 0, 2, 1), \\
2 : P_{5153} &= \mathbf{P}(0, 0, \eta^2, 1) = \mathbf{P}(0, 0, 4, 1), \\
3 : P_{9249} &= \mathbf{P}(0, 0, \eta^3, 1) = \mathbf{P}(0, 0, 8, 1), \\
4 : P_{10273} &= \mathbf{P}(0, 0, \eta^{29}, 1) = \mathbf{P}(0, 0, 9, 1), \\
5 : P_{11297} &= \mathbf{P}(0, 0, \eta^6, 1) = \mathbf{P}(0, 0, 10, 1), \\
6 : P_{12321} &= \mathbf{P}(0, 0, \eta^{27}, 1) = \mathbf{P}(0, 0, 11, 1), \\
7 : P_{14369} &= \mathbf{P}(0, 0, \eta^8, 1) = \mathbf{P}(0, 0, 13, 1), \\
8 : P_{15393} &= \mathbf{P}(0, 0, \eta^{12}, 1) = \mathbf{P}(0, 0, 14, 1), \\
9 : P_{16417} &= \mathbf{P}(0, 0, \eta^{23}, 1) = \mathbf{P}(0, 0, 15, 1), \\
10 : P_{17441} &= \mathbf{P}(0, 0, \eta^4, 1) = \mathbf{P}(0, 0, 16, 1), \\
11 : P_{19489} &= \mathbf{P}(0, 0, \eta^{30}, 1) = \mathbf{P}(0, 0, 18, 1), \\
12 : P_{20513} &= \mathbf{P}(0, 0, \eta^{17}, 1) = \mathbf{P}(0, 0, 19, 1), \\
13 : P_{28705} &= \mathbf{P}(0, 0, \eta^{16}, 1) = \mathbf{P}(0, 0, 27, 1), \\
14 : P_{31777} &= \mathbf{P}(0, 0, \eta^{24}, 1) = \mathbf{P}(0, 0, 30, 1), \\
15 : P_{32801} &= \mathbf{P}(0, 0, \eta^{15}, 1) = \mathbf{P}(0, 0, 31, 1).
\end{aligned}$$

## Double Points

The surface has 34 Double points:

The double points on the surface are:

$$\begin{aligned}
P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 & P_{21} &= (17, 1, 0, 0) = \ell_0 \cap \ell_{20} \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_2 & P_{22} &= (18, 1, 0, 0) = \ell_0 \cap \ell_{21} \\
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_3 & P_{23} &= (19, 1, 0, 0) = \ell_0 \cap \ell_{22} \\
P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_5 & P_{24} &= (20, 1, 0, 0) = \ell_0 \cap \ell_{23} \\
P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_6 & P_{25} &= (21, 1, 0, 0) = \ell_0 \cap \ell_{24} \\
P_8 &= (4, 1, 0, 0) = \ell_0 \cap \ell_7 & P_{26} &= (22, 1, 0, 0) = \ell_0 \cap \ell_{25} \\
P_9 &= (5, 1, 0, 0) = \ell_0 \cap \ell_8 & P_{27} &= (23, 1, 0, 0) = \ell_0 \cap \ell_{26} \\
P_{10} &= (6, 1, 0, 0) = \ell_0 \cap \ell_9 & P_{28} &= (24, 1, 0, 0) = \ell_0 \cap \ell_{27} \\
P_{11} &= (7, 1, 0, 0) = \ell_0 \cap \ell_{10} & P_{29} &= (25, 1, 0, 0) = \ell_0 \cap \ell_{28} \\
P_{12} &= (8, 1, 0, 0) = \ell_0 \cap \ell_{11} & P_{30} &= (26, 1, 0, 0) = \ell_0 \cap \ell_{29} \\
P_{13} &= (9, 1, 0, 0) = \ell_0 \cap \ell_{12} & P_{31} &= (27, 1, 0, 0) = \ell_0 \cap \ell_{30} \\
P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_{13} & P_{32} &= (28, 1, 0, 0) = \ell_0 \cap \ell_{31} \\
P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_{14} & P_{33} &= (29, 1, 0, 0) = \ell_0 \cap \ell_{32} \\
P_{16} &= (12, 1, 0, 0) = \ell_0 \cap \ell_{15} & P_{34} &= (30, 1, 0, 0) = \ell_0 \cap \ell_{33} \\
P_{17} &= (13, 1, 0, 0) = \ell_0 \cap \ell_{16} & P_{35} &= (31, 1, 0, 0) = \ell_0 \cap \ell_{34} \\
P_{18} &= (14, 1, 0, 0) = \ell_0 \cap \ell_{17} & P_3 &= (0, 0, 0, 1) = \ell_3 \cap \ell_4 \\
P_{19} &= (15, 1, 0, 0) = \ell_0 \cap \ell_{18} & & \\
P_{20} &= (16, 1, 0, 0) = \ell_0 \cap \ell_{19} & & 
\end{aligned}$$

## Single Points

The surface has 1039 single points:

Too many to print.

## Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

## Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
8	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_0$	$P_5$	$P_1$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{16}$	$P_{17}$	$P_{18}$	$P_{19}$	$P_{20}$	$P_{21}$	$P_{22}$	$P_{23}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$
in point	$P_0$	$P_2$	$P_2$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_4$
in point	$P_5$	$P_2$	$P_2$

Line 3 intersects

Line	$\ell_0$	$\ell_4$
in point	$P_1$	$P_3$

Line 4 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$
in point	$P_2$	$P_2$	$P_3$	$P_{32801}$	$P_{17441}$	$P_{19489}$	$P_{14369}$	$P_{12321}$	$P_{9249}$	$P_{14369}$	$P_{31777}$	$P_{28705}$	$P_{20513}$	$P_{3105}$	$P_{12321}$

Line 5 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{32}$
in point	$P_6$	$P_{32801}$	$P_{32801}$

Line 6 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{22}$
in point	$P_7$	$P_{17441}$	$P_{17441}$

Line 7 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{25}$
in point	$P_8$	$P_{19489}$	$P_{19489}$

Line 8 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{11}$
in point	$P_9$	$P_{14369}$	$P_{14369}$

Line 9 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{16}$
in point	$P_{10}$	$P_{12321}$	$P_{12321}$

Line 10 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{18}$
in point	$P_{11}$	$P_{9249}$	$P_{9249}$

Line 11 intersects

Line	$\ell_0$	$\ell_4$	$\ell_8$
in point	$P_{12}$	$P_{14369}$	$P_{14369}$

Line 12 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{26}$
in point	$P_{13}$	$P_{31777}$	$P_{31777}$

Line 13 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{20}$
in point	$P_{14}$	$P_{28705}$	$P_{28705}$

Line 14 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{27}$
in point	$P_{15}$	$P_{20513}$	$P_{20513}$

Line 15 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{17}$
in point	$P_{16}$	$P_{3105}$	$P_{3105}$

Line 16 intersects

Line	$\ell_0$	$\ell_4$	$\ell_9$
in point	$P_{17}$	$P_{12321}$	$P_{12321}$

Line 17 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{15}$
in point	$P_{18}$	$P_{3105}$	$P_{3105}$

Line 18 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{10}$
in point	$P_{19}$	$P_{9249}$	$P_{9249}$

Line 19 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{28}$
in point	$P_{20}$	$P_{10273}$	$P_{10273}$

Line 20 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{13}$
in point	$P_{21}$	$P_{28705}$	$P_{28705}$

Line 21 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{31}$
in point	$P_{22}$	$P_{15393}$	$P_{15393}$

Line 22 intersects

Line	$\ell_0$	$\ell_4$	$\ell_6$
in point	$P_{23}$	$P_{17441}$	$P_{17441}$

Line 23 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{30}$
in point	$P_{24}$	$P_{16417}$	$P_{16417}$

Line 24 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{34}$
in point	$P_{25}$	$P_{11297}$	$P_{11297}$

Line 25 intersects

Line	$\ell_0$	$\ell_4$	$\ell_7$
in point	$P_{26}$	$P_{19489}$	$P_{19489}$

Line 26 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{12}$
in point	$P_{27}$	$P_{31777}$	$P_{31777}$

Line 27 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{14}$
in point	$P_{28}$	$P_{20513}$	$P_{20513}$

Line 28 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{19}$
in point	$P_{29}$	$P_{10273}$	$P_{10273}$

Line 29 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{33}$
in point	$P_{30}$	$P_{5153}$	$P_{5153}$

Line 30 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{23}$
in point	$P_{31}$	$P_{16417}$	$P_{16417}$

Line 31 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{21}$
in point	$P_{32}$	$P_{15393}$	$P_{15393}$

Line 32 intersects

Line	$\ell_0$	$\ell_4$	$\ell_5$
in point	$P_{33}$	$P_{32801}$	$P_{32801}$

Line 33 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{29}$
in point	$P_{34}$	$P_{5153}$	$P_{5153}$

Line 34 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{24}$
in point	$P_{35}$	$P_{11297}$	$P_{11297}$

The surface has 1089 points:  
Too many to print.