Rank-65609 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431659885

General information

Number of lines	6
Number of points	29
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	15
Number of points off lines	7
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^6
Type of lines on points	3, 2^6, 1^{15}, 0^7

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{129} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{129} = \mathbf{Pl}(3,2,2,3,0,1)_{173}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{149} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{149} = \mathbf{Pl}(2,3,3,2,0,1)_{166}$$

Rank of lines: (345, 106, 355, 129, 350, 149)
Rank of points on Klein quadric: (13, 150, 15, 173, 14, 166)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{11} = (0,1,1,0) = \ell_0 \cap \ell_1$
$P_{78} = (1,2,3,1) = \ell_1 \cap \ell_3$
$P_{66} = (1,3,2,1) = \ell_1 \cap \ell_5$
$P_{15} = (0,2,1,0) = \ell_2 \cap \ell_3$
$P_4 = (1,1,1,1) = \ell_3 \cap \ell_5$
$P_{19} = (0,3,1,0) = \ell_4 \cap \ell_5$

Single Points

The surface has 15 single points:
The single points on the surface are:

$0 : P_{27} = (1,1,0,1)$ lies on line ℓ_1
$1 : P_{31} = (1,2,0,1)$ lies on line ℓ_5
$2 : P_{35} = (1,3,0,1)$ lies on line ℓ_3
$3 : P_{39} = (1,0,1,1)$ lies on line ℓ_1
$4 : P_{42} = (0,1,1,1)$ lies on line ℓ_0
$5 : P_{45} = (0,2,1,1)$ lies on line ℓ_2
$6 : P_{49} = (0,3,1,1)$ lies on line ℓ_4
$7 : P_{54} = (1,0,2,1)$ lies on line ℓ_3
$8 : P_{57} = (0,1,2,1)$ lies on line ℓ_4
$9 : P_{61} = (0,2,2,1)$ lies on line ℓ_0
$10 : P_{65} = (0,3,2,1)$ lies on line ℓ_2
$11 : P_{70} = (1,0,3,1)$ lies on line ℓ_5
$12 : P_{73} = (0,1,3,1)$ lies on line ℓ_2
$13 : P_{77} = (0,2,3,1)$ lies on line ℓ_4
$14 : P_{81} = (0,3,3,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 7 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{47} = (2, 2, 1, 1)$
2 : $P_{52} = (3, 3, 1, 1)$
3 : $P_{59} = (2, 1, 2, 1)$
4 : $P_{64} = (3, 2, 2, 1)$
5 : $P_{76} = (3, 1, 3, 1)$
6 : $P_{83} = (2, 3, 3, 1)$

Line Intersection Graph

	0 1 2 3 4 5
0	0 1 1 0 1 0
1	1 0 0 1 0 1
2	1 0 0 1 1 0
3	0 1 1 0 0 1
4	1 0 1 0 0 1
5	0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{11}	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_{11}	P_{78}	P_{66}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_3	P_{15}	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_{78}	P_{15}	P_4

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_3	P_3	P_{19}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{66}	P_4	P_{19}

The surface has 29 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_{11} = (0, 1, 1, 0)$
4 : $P_{15} = (0, 2, 1, 0)$
5 : $P_{19} = (0, 3, 1, 0)$
6 : $P_{27} = (1, 1, 0, 1)$
7 : $P_{31} = (1, 2, 0, 1)$
8 : $P_{35} = (1, 3, 0, 1)$
9 : $P_{39} = (1, 0, 1, 1)$
10 : $P_{42} = (0, 1, 1, 1)$
11 : $P_{45} = (0, 2, 1, 1)$
12 : $P_{47} = (2, 2, 1, 1)$
13 : $P_{49} = (0, 3, 1, 1)$
14 : $P_{52} = (3, 3, 1, 1)$
15 : $P_{54} = (1, 0, 2, 1)$
16 : $P_{57} = (0, 1, 2, 1)$
17 : $P_{59} = (2, 1, 2, 1)$
18 : $P_{61} = (0, 2, 2, 1)$
19 : $P_{64} = (3, 2, 2, 1)$
20 : $P_{65} = (0, 3, 2, 1)$
21 : $P_{66} = (1, 3, 2, 1)$
22 : $P_{70} = (1, 0, 3, 1)$
23 : $P_{73} = (0, 1, 3, 1)$
24 : $P_{76} = (3, 1, 3, 1)$
25 : $P_{77} = (0, 2, 3, 1)$
26 : $P_{78} = (1, 2, 3, 1)$
27 : $P_{81} = (0, 3, 3, 1)$
28 : $P_{83} = (2, 3, 3, 1)$