Rank-76308 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_1 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1503221338

General information

Number of lines	4
Number of points	21
Number of singular points	2
Number of Eckardt points	1
Number of double points	1
Number of single points	15
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	3, 2, 1^{15}, 0^4

Singular Points

The surface has 2 singular points:

$0:\ P_{54} = \mathbf{P}(1,0,\omega,1) = \mathbf{P}(1,0,2,1)$
$1:\ P_{70} = \mathbf{P}(1,0,\omega^2,1) = \mathbf{P}(1,0,3,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \mathbf{Pl}(0,1,1,1,1,1)_{202}$$

Rank of lines: (356, 100, 38, 122)
Rank of points on Klein quadric: (1, 6, 198, 202)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$.

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 15 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_0
$1 : P_4 = (1,1,1,1)$ lies on line ℓ_2
$2 : P_5 = (1,1,0,0)$ lies on line ℓ_2
$3 : P_{12} = (1,1,1,0)$ lies on line ℓ_3
$4 : P_{23} = (1,0,0,1)$ lies on line ℓ_1
$5 : P_{27} = (1,1,0,1)$ lies on line ℓ_3
$6 : P_{39} = (1,0,1,1)$ lies on line ℓ_1
$7 : P_{47} = (2,2,1,1)$ lies on line ℓ_2
$8 : P_{52} = (3,3,1,1)$ lies on line ℓ_2
$9 : P_{53} = (0,0,2,1)$ lies on line ℓ_0
$10 : P_{54} = (1,0,2,1)$ lies on line ℓ_1
$11 : P_{68} = (3,3,2,1)$ lies on line ℓ_3
$12 : P_{69} = (0,0,3,1)$ lies on line ℓ_0
$13 : P_{70} = (1,0,3,1)$ lies on line ℓ_1
$14 : P_{79} = (2,2,3,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

$0 : P_1 = (0,1,0,0)$
$1 : P_{42} = (0,1,1,1)$
$2 : P_{65} = (0,3,2,1)$
$3 : P_{77} = (0,2,3,1)$

Line Intersection Graph

	0 1 2 3
0	0 1 1 1
1	1 0 0 0
2	1 0 0 1
3	1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_{38}	P_{38}

Line 1 intersects

Line	ℓ_0
in point	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{38}	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{38}	P_{38}

The surface has 21 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_5 = (1,1,0,0)$
$5: P_{12} = (1,1,1,0)$
$6: P_{23} = (1,0,0,1)$
$7: P_{27} = (1,1,0,1)$
$8: P_{38} = (0,0,1,1)$
$9: P_{39} = (1,0,1,1)$
$10: P_{42} = (0,1,1,1)$
$11: P_{47} = (2,2,1,1)$
$12: P_{52} = (3,3,1,1)$
$13: P_{53} = (0,0,2,1)$
$14: P_{54} = (1,0,2,1)$
$15: P_{65} = (0,3,2,1)$
$16: P_{68} = (3,3,2,1)$
$17: P_{69} = (0,0,3,1)$
$18: P_{70} = (1,0,3,1)$
$19: P_{77} = (0,2,3,1)$
$20: P_{79} = (2,2,3,1)$