Rank-76291 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361351245

General information

Number of lines	8
Number of points	89
Number of singular points	2
Number of Eckardt points	3
Number of double points	7
Number of single points	49
Number of points off lines	30
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^8
Type of lines on points	3^3, 2^7, 1^{49}, 0^{30}

Singular Points

The surface has 2 singular points:

$0:\ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1:\ P_{12} = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \mathbf{Pl}(1,0,0,0,0,1)_{650}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{9} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{9} = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_5 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \mathbf{Pl}(1,1,1,0,1,1)_{1224}$$

$$\ell_7 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \mathbf{Pl}(0,1,1,1,1,1)_{1330}$$

Rank of lines: (0, 64, 73, 9, 4744, 138, 82, 722)
Rank of points on Klein quadric: (0, 2, 650, 97, 1, 1322, 1224, 1330)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_{12} = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$,
$2 : P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$.

Double Points

The surface has 7 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_5$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_4$
$P_{20} = (1,1,1,0) = \ell_2 \cap \ell_7$
$P_4 = (1,1,1,1) = \ell_3 \cap \ell_5$
$P_{146} = (0,1,1,1) = \ell_3 \cap \ell_6$
$P_{83} = (1,1,0,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 49 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_4
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$3 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$4 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$5 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$6 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$7 : P_{13} = (2,0,1,0)$ lies on line ℓ_1
$8 : P_{14} = (3,0,1,0)$ lies on line ℓ_1
$9 : P_{15} = (4,0,1,0)$ lies on line ℓ_1
$10 : P_{16} = (5,0,1,0)$ lies on line ℓ_1
$11 : P_{17} = (6,0,1,0)$ lies on line ℓ_1
$12 : P_{18} = (7,0,1,0)$ lies on line ℓ_1
$13 : P_{28} = (1,2,1,0)$ lies on line ℓ_2
$14 : P_{36} = (1,3,1,0)$ lies on line ℓ_2
$15 : P_{44} = (1,4,1,0)$ lies on line ℓ_2

16 : $P_{52} = (1, 5, 1, 0)$ lies on line ℓ_2
17 : $P_{60} = (1, 6, 1, 0)$ lies on line ℓ_2
18 : $P_{68} = (1, 7, 1, 0)$ lies on line ℓ_2
19 : $P_{147} = (2, 1, 1, 1)$ lies on line ℓ_3
20 : $P_{148} = (3, 1, 1, 1)$ lies on line ℓ_3
21 : $P_{149} = (4, 1, 1, 1)$ lies on line ℓ_3
22 : $P_{150} = (5, 1, 1, 1)$ lies on line ℓ_3
23 : $P_{151} = (6, 1, 1, 1)$ lies on line ℓ_3
24 : $P_{152} = (7, 1, 1, 1)$ lies on line ℓ_3
25 : $P_{155} = (2, 2, 1, 1)$ lies on line ℓ_5
26 : $P_{164} = (3, 3, 1, 1)$ lies on line ℓ_5
27 : $P_{173} = (4, 4, 1, 1)$ lies on line ℓ_5
28 : $P_{182} = (5, 5, 1, 1)$ lies on line ℓ_5
29 : $P_{191} = (6, 6, 1, 1)$ lies on line ℓ_5
30 : $P_{200} = (7, 7, 1, 1)$ lies on line ℓ_5
31 : $P_{201} = (0, 0, 2, 1)$ lies on line ℓ_4
32 : $P_{212} = (3, 1, 2, 1)$ lies on line ℓ_6
33 : $P_{228} = (3, 3, 2, 1)$ lies on line ℓ_7
34 : $P_{265} = (0, 0, 3, 1)$ lies on line ℓ_4
35 : $P_{275} = (2, 1, 3, 1)$ lies on line ℓ_6
36 : $P_{283} = (2, 2, 3, 1)$ lies on line ℓ_7
37 : $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_4
38 : $P_{342} = (5, 1, 4, 1)$ lies on line ℓ_6
39 : $P_{374} = (5, 5, 4, 1)$ lies on line ℓ_7
40 : $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_4
41 : $P_{405} = (4, 1, 5, 1)$ lies on line ℓ_6
42 : $P_{429} = (4, 4, 5, 1)$ lies on line ℓ_7
43 : $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_4
44 : $P_{472} = (7, 1, 6, 1)$ lies on line ℓ_6
45 : $P_{520} = (7, 7, 6, 1)$ lies on line ℓ_7
46 : $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_4
47 : $P_{535} = (6, 1, 7, 1)$ lies on line ℓ_6
48 : $P_{575} = (6, 6, 7, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 30 points not on any line:
The points on the surface but not on lines are:

0 : $P_{94} = (4, 2, 0, 1)$
1 : $P_{103} = (5, 3, 0, 1)$
2 : $P_{113} = (7, 4, 0, 1)$
3 : $P_{120} = (6, 5, 0, 1)$
4 : $P_{125} = (3, 6, 0, 1)$
5 : $P_{132} = (2, 7, 0, 1)$
6 : $P_{224} = (7, 2, 2, 1)$
7 : $P_{233} = (0, 4, 2, 1)$
8 : $P_{248} = (7, 5, 2, 1)$
9 : $P_{259} = (2, 7, 2, 1)$
10 : $P_{293} = (4, 3, 3, 1)$
11 : $P_{305} = (0, 5, 3, 1)$
12 : $P_{316} = (3, 6, 3, 1)$
13 : $P_{325} = (4, 7, 3, 1)$
14 : $P_{349} = (4, 2, 4, 1)$
15 : $P_{363} = (2, 4, 4, 1)$
16 : $P_{379} = (2, 6, 4, 1)$
17 : $P_{385} = (0, 7, 4, 1)$
18 : $P_{416} = (7, 2, 5, 1)$
19 : $P_{422} = (5, 3, 5, 1)$
20 : $P_{440} = (7, 5, 5, 1)$
21 : $P_{441} = (0, 6, 5, 1)$
22 : $P_{481} = (0, 3, 6, 1)$
23 : $P_{491} = (2, 4, 6, 1)$
24 : $P_{503} = (6, 5, 6, 1)$
25 : $P_{507} = (2, 6, 6, 1)$
26 : $P_{537} = (0, 2, 7, 1)$
27 : $P_{549} = (4, 3, 7, 1)$
28 : $P_{560} = (7, 4, 7, 1)$
29 : $P_{581} = (4, 7, 7, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	0	1	0	0
1	1	0	1	1	1	0	1	0
2	1	1	0	0	0	0	1	1
3	1	1	0	0	0	1	1	0
4	0	1	0	0	0	1	0	1
5	1	0	0	1	1	0	0	1
6	0	1	1	1	0	0	0	1
7	0	0	1	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_0	P_1	P_0	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_0	P_{12}	P_0	P_2	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7
in point	P_1	P_{12}	P_{12}	P_{20}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_0	P_0	P_4	P_{146}

Line 4 intersects

Line	ℓ_1	ℓ_5	ℓ_7
in point	P_2	P_{138}	P_{138}

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_7
in point	P_5	P_4	P_{138}	P_{138}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_7
in point	P_{12}	P_{12}	P_{146}	P_{83}

Line 7 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_{20}	P_{138}	P_{138}	P_{83}

The surface has 89 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_4 = (1,1,1,1)$
5 : $P_5 = (1,1,0,0)$
6 : $P_6 = (2,1,0,0)$
7 : $P_7 = (3,1,0,0)$
8 : $P_8 = (4,1,0,0)$
9 : $P_9 = (5,1,0,0)$
10 : $P_{10} = (6,1,0,0)$
11 : $P_{11} = (7,1,0,0)$
12 : $P_{12} = (1,0,1,0)$
13 : $P_{13} = (2,0,1,0)$
14 : $P_{14} = (3,0,1,0)$
15 : $P_{15} = (4,0,1,0)$
16 : $P_{16} = (5,0,1,0)$
17 : $P_{17} = (6,0,1,0)$
18 : $P_{18} = (7,0,1,0)$
19 : $P_{20} = (1,1,1,0)$
20 : $P_{28} = (1,2,1,0)$
21 : $P_{36} = (1,3,1,0)$
22 : $P_{44} = (1,4,1,0)$
23 : $P_{52} = (1,5,1,0)$
24 : $P_{60} = (1,6,1,0)$
25 : $P_{68} = (1,7,1,0)$
26 : $P_{83} = (1,1,0,1)$
27 : $P_{94} = (4,2,0,1)$
28 : $P_{103} = (5,3,0,1)$
29 : $P_{113} = (7,4,0,1)$
30 : $P_{120} = (6,5,0,1)$
31 : $P_{125} = (3,6,0,1)$
32 : $P_{132} = (2,7,0,1)$
33 : $P_{138} = (0,0,1,1)$
34 : $P_{146} = (0,1,1,1)$
35 : $P_{147} = (2,1,1,1)$
36 : $P_{148} = (3,1,1,1)$
37 : $P_{149} = (4,1,1,1)$
38 : $P_{150} = (5,1,1,1)$
39 : $P_{151} = (6,1,1,1)$
40 : $P_{152} = (7,1,1,1)$
41 : $P_{155} = (2,2,1,1)$
42 : $P_{164} = (3,3,1,1)$
43 : $P_{173} = (4,4,1,1)$
44 : $P_{182} = (5,5,1,1)$
45 : $P_{191} = (6,6,1,1)$
46 : $P_{200} = (7,7,1,1)$
47 : $P_{201} = (0,0,2,1)$
48 : $P_{212} = (3,1,2,1)$
49 : $P_{224} = (7,2,2,1)$
50 : $P_{228} = (3,3,2,1)$
51 : $P_{233} = (0,4,2,1)$
52 : $P_{248} = (7,5,2,1)$
53 : $P_{259} = (2,7,2,1)$
54 : $P_{265} = (0,0,3,1)$
55 : $P_{275} = (2,1,3,1)$
56 : $P_{283} = (2,2,3,1)$
57 : $P_{293} = (4,3,3,1)$
58 : $P_{305} = (0,5,3,1)$
59 : $P_{316} = (3,6,3,1)$
60 : $P_{325} = (4,7,3,1)$
61 : $P_{329} = (0,0,4,1)$
62 : $P_{342} = (5,1,4,1)$
63 : $P_{349} = (4,2,4,1)$
64 : $P_{363} = (2,4,4,1)$
65 : $P_{374} = (5,5,4,1)$

66 : $P_{379} = (2, 6, 4, 1)$
67 : $P_{385} = (0, 7, 4, 1)$
68 : $P_{393} = (0, 0, 5, 1)$
69 : $P_{405} = (4, 1, 5, 1)$
70 : $P_{416} = (7, 2, 5, 1)$
71 : $P_{422} = (5, 3, 5, 1)$
72 : $P_{429} = (4, 4, 5, 1)$
73 : $P_{440} = (7, 5, 5, 1)$
74 : $P_{441} = (0, 6, 5, 1)$
75 : $P_{457} = (0, 0, 6, 1)$
76 : $P_{472} = (7, 1, 6, 1)$
77 : $P_{481} = (0, 3, 6, 1)$
78 : $P_{491} = (2, 4, 6, 1)$
79 : $P_{503} = (6, 5, 6, 1)$
80 : $P_{507} = (2, 6, 6, 1)$
81 : $P_{520} = (7, 7, 6, 1)$
82 : $P_{521} = (0, 0, 7, 1)$
83 : $P_{535} = (6, 1, 7, 1)$
84 : $P_{537} = (0, 2, 7, 1)$
85 : $P_{549} = (4, 3, 7, 1)$
86 : $P_{560} = (7, 4, 7, 1)$
87 : $P_{575} = (6, 6, 7, 1)$
88 : $P_{581} = (4, 7, 7, 1)$