Rank-65665 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 304226853

General information

Number of lines	4
Number of points	289
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	60
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^4
Type of lines on points	2^4, 1^{60}, 0^{225}

Singular Points

The surface has 2 singular points:

$0: \ P_{291} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
$1: \ P_{531} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(1,0,1,0,0,1)_{4657}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \mathbf{Pl}(1,0,0,0,1,0)_{290}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \mathbf{Pl}(0,0,1,0,1,0)_{320}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1,1,1,1,0,1)_{5586}$$

Rank of lines: (274, 16, 257, 4642)
Rank of points on Klein quadric: (4657, 290, 320, 5586)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_{35} = (0,1,1,0) = \ell_0 \cap \ell_3$
$P_0 = (1,0,0,0) = \ell_1 \cap \ell_2$
$P_{291} = (1,1,0,1) = \ell_1 \cap \ell_3$
$P_{531} = (1,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 60 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_{20} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{54} = (3,2,1,0)$ lies on line ℓ_0
3 : $P_{69} = (2,3,1,0)$ lies on line ℓ_0
4 : $P_{88} = (5,4,1,0)$ lies on line ℓ_0
5 : $P_{103} = (4,5,1,0)$ lies on line ℓ_0
6 : $P_{122} = (7,6,1,0)$ lies on line ℓ_0
7 : $P_{137} = (6,7,1,0)$ lies on line ℓ_0
8 : $P_{156} = (9,8,1,0)$ lies on line ℓ_0
9 : $P_{171} = (8,9,1,0)$ lies on line ℓ_0
10 : $P_{190} = (11,10,1,0)$ lies on line ℓ_0
11 : $P_{205} = (10,11,1,0)$ lies on line ℓ_0
12 : $P_{224} = (13,12,1,0)$ lies on line ℓ_0
13 : $P_{239} = (12,13,1,0)$ lies on line ℓ_0
14 : $P_{258} = (15,14,1,0)$ lies on line ℓ_0
15 : $P_{273} = (14,15,1,0)$ lies on line ℓ_0
16 : $P_{290} = (0,1,0,1)$ lies on line ℓ_1
17 : $P_{292} = (2,1,0,1)$ lies on line ℓ_1
18 : $P_{293} = (3,1,0,1)$ lies on line ℓ_1
19 : $P_{294} = (4,1,0,1)$ lies on line ℓ_1
20 : $P_{295} = (5,1,0,1)$ lies on line ℓ_1
21 : $P_{296} = (6,1,0,1)$ lies on line ℓ_1
22 : $P_{297} = (7,1,0,1)$ lies on line ℓ_1
23 : $P_{298} = (8,1,0,1)$ lies on line ℓ_1
24 : $P_{299} = (9,1,0,1)$ lies on line ℓ_1
25 : $P_{300} = (10,1,0,1)$ lies on line ℓ_1
26 : $P_{301} = (11,1,0,1)$ lies on line ℓ_1
27 : $P_{302} = (12,1,0,1)$ lies on line ℓ_1
28 : $P_{303} = (13,1,0,1)$ lies on line ℓ_1
29 : $P_{304} = (14,1,0,1)$ lies on line ℓ_1
30 : $P_{305} = (15,1,0,1)$ lies on line ℓ_1
31 : $P_{530} = (0,0,1,1)$ lies on line ℓ_2
32 : $P_{532} = (2,0,1,1)$ lies on line ℓ_2
33 : $P_{533} = (3,0,1,1)$ lies on line ℓ_2
34 : $P_{534} = (4,0,1,1)$ lies on line ℓ_2
35 : $P_{535} = (5,0,1,1)$ lies on line ℓ_2
36 : $P_{536} = (6,0,1,1)$ lies on line ℓ_2
37 : $P_{537} = (7,0,1,1)$ lies on line ℓ_2
38 : $P_{538} = (8,0,1,1)$ lies on line ℓ_2
39 : $P_{539} = (9,0,1,1)$ lies on line ℓ_2
40 : $P_{540} = (10,0,1,1)$ lies on line ℓ_2
41 : $P_{541} = (11,0,1,1)$ lies on line ℓ_2
42 : $P_{542} = (12,0,1,1)$ lies on line ℓ_2
43 : $P_{543} = (13,0,1,1)$ lies on line ℓ_2

44 : $P_{544} = (14, 0, 1, 1)$ lies on line ℓ_2
45 : $P_{545} = (15, 0, 1, 1)$ lies on line ℓ_2
46 : $P_{834} = (1, 3, 2, 1)$ lies on line ℓ_3
47 : $P_{1074} = (1, 2, 3, 1)$ lies on line ℓ_3
48 : $P_{1378} = (1, 5, 4, 1)$ lies on line ℓ_3
49 : $P_{1618} = (1, 4, 5, 1)$ lies on line ℓ_3
50 : $P_{1922} = (1, 7, 6, 1)$ lies on line ℓ_3
51 : $P_{2162} = (1, 6, 7, 1)$ lies on line ℓ_3
52 : $P_{2466} = (1, 9, 8, 1)$ lies on line ℓ_3
53 : $P_{2706} = (1, 8, 9, 1)$ lies on line ℓ_3
54 : $P_{3010} = (1, 11, 10, 1)$ lies on line ℓ_3
55 : $P_{3250} = (1, 10, 11, 1)$ lies on line ℓ_3
56 : $P_{3554} = (1, 13, 12, 1)$ lies on line ℓ_3
57 : $P_{3794} = (1, 12, 13, 1)$ lies on line ℓ_3
58 : $P_{4098} = (1, 15, 14, 1)$ lies on line ℓ_3
59 : $P_{4338} = (1, 14, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

0 : $P_{61} = (10, 2, 1, 0)$
1 : $P_{82} = (15, 3, 1, 0)$
2 : $P_{94} = (11, 4, 1, 0)$
3 : $P_{102} = (3, 5, 1, 0)$
4 : $P_{123} = (8, 6, 1, 0)$
5 : $P_{134} = (3, 7, 1, 0)$
6 : $P_{152} = (5, 8, 1, 0)$
7 : $P_{173} = (10, 9, 1, 0)$
8 : $P_{179} = (0, 10, 1, 0)$
9 : $P_{195} = (0, 11, 1, 0)$
10 : $P_{216} = (5, 12, 1, 0)$
11 : $P_{242} = (15, 13, 1, 0)$
12 : $P_{254} = (11, 14, 1, 0)$
13 : $P_{267} = (8, 15, 1, 0)$
14 : $P_{275} = (1, 0, 0, 1)$
15 : $P_{319} = (13, 2, 0, 1)$
16 : $P_{335} = (13, 3, 0, 1)$
17 : $P_{345} = (7, 4, 0, 1)$
18 : $P_{361} = (7, 5, 0, 1)$
19 : $P_{381} = (11, 6, 0, 1)$
20 : $P_{397} = (11, 7, 0, 1)$
21 : $P_{414} = (12, 8, 0, 1)$
22 : $P_{430} = (12, 9, 0, 1)$
23 : $P_{434} = (0, 10, 0, 1)$
24 : $P_{450} = (0, 11, 0, 1)$
25 : $P_{476} = (10, 12, 0, 1)$
26 : $P_{492} = (10, 13, 0, 1)$
27 : $P_{504} = (6, 14, 0, 1)$
28 : $P_{520} = (6, 15, 0, 1)$
29 : $P_{555} = (10, 1, 1, 1)$
30 : $P_{556} = (11, 1, 1, 1)$
31 : $P_{629} = (4, 6, 1, 1)$
32 : $P_{630} = (5, 6, 1, 1)$
33 : $P_{655} = (14, 7, 1, 1)$
34 : $P_{656} = (15, 7, 1, 1)$
35 : $P_{695} = (6, 10, 1, 1)$
36 : $P_{696} = (7, 10, 1, 1)$
37 : $P_{717} = (12, 11, 1, 1)$
38 : $P_{718} = (13, 11, 1, 1)$
39 : $P_{723} = (2, 12, 1, 1)$
40 : $P_{724} = (3, 12, 1, 1)$
41 : $P_{745} = (8, 13, 1, 1)$
42 : $P_{746} = (9, 13, 1, 1)$
43 : $P_{798} = (13, 0, 2, 1)$
44 : $P_{826} = (9, 2, 2, 1)$
45 : $P_{830} = (13, 2, 2, 1)$
46 : $P_{853} = (4, 4, 2, 1)$
47 : $P_{858} = (9, 4, 2, 1)$
48 : $P_{883} = (2, 6, 2, 1)$
49 : $P_{890} = (9, 6, 2, 1)$
50 : $P_{900} = (3, 7, 2, 1)$
51 : $P_{909} = (12, 7, 2, 1)$
52 : $P_{915} = (2, 8, 2, 1)$
53 : $P_{927} = (14, 8, 2, 1)$
54 : $P_{934} = (5, 9, 2, 1)$
55 : $P_{944} = (15, 9, 2, 1)$
56 : $P_{948} = (3, 10, 2, 1)$
57 : $P_{949} = (4, 10, 2, 1)$
58 : $P_{996} = (3, 13, 2, 1)$
59 : $P_{1003} = (10, 13, 2, 1)$
60 : $P_{1011} = (2, 14, 2, 1)$
61 : $P_{1013} = (4, 14, 2, 1)$
62 : $P_{1031} = (6, 15, 2, 1)$
63 : $P_{1033} = (8, 15, 2, 1)$
64 : $P_{1054} = (13, 0, 3, 1)$
65 : $P_{1118} = (13, 4, 3, 1)$
66 : $P_{1120} = (15, 4, 3, 1)$
67 : $P_{1170} = (1, 8, 3, 1)$
68 : $P_{1179} = (10, 8, 3, 1)$
69 : $P_{1189} = (4, 9, 3, 1)$

70 : $P_{1194} = (9, 9, 3, 1)$
71 : $P_{1202} = (1, 10, 3, 1)$
72 : $P_{1209} = (8, 10, 3, 1)$
73 : $P_{1304} = (7, 0, 4, 1)$
74 : $P_{1333} = (4, 2, 4, 1)$
75 : $P_{1338} = (9, 2, 4, 1)$
76 : $P_{1358} = (13, 3, 4, 1)$
77 : $P_{1360} = (15, 3, 4, 1)$
78 : $P_{1368} = (7, 4, 4, 1)$
79 : $P_{1375} = (14, 4, 4, 1)$
80 : $P_{1414} = (5, 7, 4, 1)$
81 : $P_{1420} = (11, 7, 4, 1)$
82 : $P_{1450} = (9, 9, 4, 1)$
83 : $P_{1455} = (14, 9, 4, 1)$
84 : $P_{1478} = (5, 11, 4, 1)$
85 : $P_{1482} = (9, 11, 4, 1)$
86 : $P_{1494} = (5, 12, 4, 1)$
87 : $P_{1495} = (6, 12, 4, 1)$
88 : $P_{1509} = (4, 13, 4, 1)$
89 : $P_{1519} = (14, 13, 4, 1)$
90 : $P_{1524} = (3, 14, 4, 1)$
91 : $P_{1529} = (8, 14, 4, 1)$
92 : $P_{1539} = (2, 15, 4, 1)$
93 : $P_{1541} = (4, 15, 4, 1)$
94 : $P_{1560} = (7, 0, 5, 1)$
95 : $P_{1700} = (3, 9, 5, 1)$
96 : $P_{1704} = (7, 9, 5, 1)$
97 : $P_{1730} = (1, 11, 5, 1)$
98 : $P_{1744} = (15, 11, 5, 1)$
99 : $P_{1786} = (9, 14, 5, 1)$
100 : $P_{1791} = (14, 14, 5, 1)$
101 : $P_{1794} = (1, 15, 5, 1)$
102 : $P_{1804} = (11, 15, 5, 1)$
103 : $P_{1820} = (11, 0, 6, 1)$
104 : $P_{1829} = (4, 1, 6, 1)$
105 : $P_{1830} = (5, 1, 6, 1)$
106 : $P_{1843} = (2, 2, 6, 1)$
107 : $P_{1850} = (9, 2, 6, 1)$
108 : $P_{1961} = (8, 9, 6, 1)$
109 : $P_{1966} = (13, 9, 6, 1)$
110 : $P_{2044} = (11, 14, 6, 1)$
111 : $P_{2048} = (15, 14, 6, 1)$
112 : $P_{2076} = (11, 0, 7, 1)$
113 : $P_{2095} = (14, 1, 7, 1)$
114 : $P_{2096} = (15, 1, 7, 1)$
115 : $P_{2100} = (3, 2, 7, 1)$
116 : $P_{2109} = (12, 2, 7, 1)$
117 : $P_{2134} = (5, 4, 7, 1)$
118 : $P_{2140} = (11, 4, 7, 1)$
119 : $P_{2211} = (2, 9, 7, 1)$
120 : $P_{2218} = (9, 9, 7, 1)$
121 : $P_{2333} = (12, 0, 8, 1)$
122 : $P_{2355} = (2, 2, 8, 1)$
123 : $P_{2367} = (14, 2, 8, 1)$
124 : $P_{2370} = (1, 3, 8, 1)$
125 : $P_{2379} = (10, 3, 8, 1)$
126 : $P_{2482} = (1, 10, 8, 1)$
127 : $P_{2484} = (3, 10, 8, 1)$
128 : $P_{2550} = (5, 14, 8, 1)$
129 : $P_{2557} = (12, 14, 8, 1)$
130 : $P_{2589} = (12, 0, 9, 1)$
131 : $P_{2614} = (5, 2, 9, 1)$
132 : $P_{2624} = (15, 2, 9, 1)$
133 : $P_{2629} = (4, 3, 9, 1)$
134 : $P_{2634} = (9, 3, 9, 1)$
135 : $P_{2650} = (9, 4, 9, 1)$
136 : $P_{2655} = (14, 4, 9, 1)$
137 : $P_{2660} = (3, 5, 9, 1)$
138 : $P_{2664} = (7, 5, 9, 1)$
139 : $P_{2681} = (8, 6, 9, 1)$
140 : $P_{2686} = (13, 6, 9, 1)$
141 : $P_{2691} = (2, 7, 9, 1)$
142 : $P_{2698} = (9, 7, 9, 1)$
143 : $P_{2723} = (2, 9, 9, 1)$
144 : $P_{2733} = (12, 9, 9, 1)$
145 : $P_{2745} = (8, 10, 9, 1)$
146 : $P_{2751} = (14, 10, 9, 1)$
147 : $P_{2777} = (8, 12, 9, 1)$
148 : $P_{2779} = (10, 12, 9, 1)$
149 : $P_{2803} = (2, 14, 9, 1)$
150 : $P_{2815} = (14, 14, 9, 1)$
151 : $P_{2833} = (0, 0, 10, 1)$
152 : $P_{2855} = (6, 1, 10, 1)$
153 : $P_{2856} = (7, 1, 10, 1)$
154 : $P_{2868} = (3, 2, 10, 1)$
155 : $P_{2869} = (4, 2, 10, 1)$
156 : $P_{2882} = (1, 3, 10, 1)$
157 : $P_{2889} = (8, 3, 10, 1)$
158 : $P_{2962} = (1, 8, 10, 1)$
159 : $P_{2964} = (3, 8, 10, 1)$
160 : $P_{2985} = (8, 9, 10, 1)$
161 : $P_{2991} = (14, 9, 10, 1)$
162 : $P_{2996} = (3, 10, 10, 1)$
163 : $P_{3001} = (8, 10, 10, 1)$
164 : $P_{3089} = (0, 0, 11, 1)$
165 : $P_{3117} = (12, 1, 11, 1)$
166 : $P_{3118} = (13, 1, 11, 1)$
167 : $P_{3158} = (5, 4, 11, 1)$
168 : $P_{3162} = (9, 4, 11, 1)$
169 : $P_{3170} = (1, 5, 11, 1)$
170 : $P_{3184} = (15, 5, 11, 1)$
171 : $P_{3270} = (5, 11, 11, 1)$
172 : $P_{3280} = (15, 11, 11, 1)$
173 : $P_{3315} = (2, 14, 11, 1)$
174 : $P_{3328} = (15, 14, 11, 1)$
175 : $P_{3330} = (1, 15, 11, 1)$
176 : $P_{3334} = (5, 15, 11, 1)$
177 : $P_{3355} = (10, 0, 12, 1)$

178 : $P_{3363} = (2, 1, 12, 1)$
179 : $P_{3364} = (3, 1, 12, 1)$
180 : $P_{3414} = (5, 4, 12, 1)$
181 : $P_{3415} = (6, 4, 12, 1)$
182 : $P_{3497} = (8, 9, 12, 1)$
183 : $P_{3499} = (10, 9, 12, 1)$
184 : $P_{3573} = (4, 14, 12, 1)$
185 : $P_{3583} = (14, 14, 12, 1)$
186 : $P_{3611} = (10, 0, 13, 1)$
187 : $P_{3625} = (8, 1, 13, 1)$
188 : $P_{3626} = (9, 1, 13, 1)$
189 : $P_{3636} = (3, 2, 13, 1)$
190 : $P_{3643} = (10, 2, 13, 1)$
191 : $P_{3669} = (4, 4, 13, 1)$
192 : $P_{3679} = (14, 4, 13, 1)$
193 : $P_{3832} = (7, 14, 13, 1)$
194 : $P_{3840} = (15, 14, 13, 1)$
195 : $P_{3863} = (6, 0, 14, 1)$
196 : $P_{3891} = (2, 2, 14, 1)$
197 : $P_{3893} = (4, 2, 14, 1)$
198 : $P_{3924} = (3, 4, 14, 1)$
199 : $P_{3929} = (8, 4, 14, 1)$
200 : $P_{3946} = (9, 5, 14, 1)$
201 : $P_{3951} = (14, 5, 14, 1)$
202 : $P_{3964} = (11, 6, 14, 1)$
203 : $P_{3968} = (15, 6, 14, 1)$
204 : $P_{3990} = (5, 8, 14, 1)$
205 : $P_{3997} = (12, 8, 14, 1)$
206 : $P_{4003} = (2, 9, 14, 1)$
207 : $P_{4015} = (14, 9, 14, 1)$
208 : $P_{4035} = (2, 11, 14, 1)$
209 : $P_{4048} = (15, 11, 14, 1)$
210 : $P_{4053} = (4, 12, 14, 1)$
211 : $P_{4063} = (14, 12, 14, 1)$
212 : $P_{4072} = (7, 13, 14, 1)$
213 : $P_{4080} = (15, 13, 14, 1)$
214 : $P_{4085} = (4, 14, 14, 1)$
215 : $P_{4087} = (6, 14, 14, 1)$
216 : $P_{4119} = (6, 0, 15, 1)$
217 : $P_{4151} = (6, 2, 15, 1)$
218 : $P_{4153} = (8, 2, 15, 1)$
219 : $P_{4179} = (2, 4, 15, 1)$
220 : $P_{4181} = (4, 4, 15, 1)$
221 : $P_{4194} = (1, 5, 15, 1)$
222 : $P_{4204} = (11, 5, 15, 1)$
223 : $P_{4290} = (1, 11, 15, 1)$
224 : $P_{4294} = (5, 11, 15, 1)$

Line Intersection Graph

	0 1 2 3
0	0 0 0 1
1	0 0 1 1
2	0 1 0 1
3	1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_3
in point	P_{35}

Line 1 intersects

Line	ℓ_2	ℓ_3
in point	P_0	P_{291}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_0	P_{531}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_{35}	P_{291}	P_{531}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{20} = (1, 0, 1, 0)$
3 : $P_{35} = (0, 1, 1, 0)$
4 : $P_{54} = (3, 2, 1, 0)$
5 : $P_{61} = (10, 2, 1, 0)$
6 : $P_{69} = (2, 3, 1, 0)$
7 : $P_{82} = (15, 3, 1, 0)$
8 : $P_{88} = (5, 4, 1, 0)$

9 : $P_{94} = (11, 4, 1, 0)$
10 : $P_{102} = (3, 5, 1, 0)$
11 : $P_{103} = (4, 5, 1, 0)$
12 : $P_{122} = (7, 6, 1, 0)$
13 : $P_{123} = (8, 6, 1, 0)$
14 : $P_{134} = (3, 7, 1, 0)$
15 : $P_{137} = (6, 7, 1, 0)$
16 : $P_{152} = (5, 8, 1, 0)$
17 : $P_{156} = (9, 8, 1, 0)$
18 : $P_{171} = (8, 9, 1, 0)$
19 : $P_{173} = (10, 9, 1, 0)$
20 : $P_{179} = (0, 10, 1, 0)$
21 : $P_{190} = (11, 10, 1, 0)$
22 : $P_{195} = (0, 11, 1, 0)$
23 : $P_{205} = (10, 11, 1, 0)$
24 : $P_{216} = (5, 12, 1, 0)$
25 : $P_{224} = (13, 12, 1, 0)$
26 : $P_{239} = (12, 13, 1, 0)$
27 : $P_{242} = (15, 13, 1, 0)$
28 : $P_{254} = (11, 14, 1, 0)$
29 : $P_{258} = (15, 14, 1, 0)$
30 : $P_{267} = (8, 15, 1, 0)$
31 : $P_{273} = (14, 15, 1, 0)$
32 : $P_{275} = (1, 0, 0, 1)$
33 : $P_{290} = (0, 1, 0, 1)$
34 : $P_{291} = (1, 1, 0, 1)$
35 : $P_{292} = (2, 1, 0, 1)$
36 : $P_{293} = (3, 1, 0, 1)$
37 : $P_{294} = (4, 1, 0, 1)$
38 : $P_{295} = (5, 1, 0, 1)$
39 : $P_{296} = (6, 1, 0, 1)$
40 : $P_{297} = (7, 1, 0, 1)$
41 : $P_{298} = (8, 1, 0, 1)$
42 : $P_{299} = (9, 1, 0, 1)$
43 : $P_{300} = (10, 1, 0, 1)$
44 : $P_{301} = (11, 1, 0, 1)$
45 : $P_{302} = (12, 1, 0, 1)$
46 : $P_{303} = (13, 1, 0, 1)$
47 : $P_{304} = (14, 1, 0, 1)$
48 : $P_{305} = (15, 1, 0, 1)$
49 : $P_{319} = (13, 2, 0, 1)$
50 : $P_{335} = (13, 3, 0, 1)$
51 : $P_{345} = (7, 4, 0, 1)$
52 : $P_{361} = (7, 5, 0, 1)$
53 : $P_{381} = (11, 6, 0, 1)$
54 : $P_{397} = (11, 7, 0, 1)$
55 : $P_{414} = (12, 8, 0, 1)$
56 : $P_{430} = (12, 9, 0, 1)$
57 : $P_{434} = (0, 10, 0, 1)$
58 : $P_{450} = (0, 11, 0, 1)$
59 : $P_{476} = (10, 12, 0, 1)$
60 : $P_{492} = (10, 13, 0, 1)$
61 : $P_{504} = (6, 14, 0, 1)$
62 : $P_{520} = (6, 15, 0, 1)$
63 : $P_{530} = (0, 0, 1, 1)$
64 : $P_{531} = (1, 0, 1, 1)$
65 : $P_{532} = (2, 0, 1, 1)$
66 : $P_{533} = (3, 0, 1, 1)$
67 : $P_{534} = (4, 0, 1, 1)$
68 : $P_{535} = (5, 0, 1, 1)$
69 : $P_{536} = (6, 0, 1, 1)$
70 : $P_{537} = (7, 0, 1, 1)$
71 : $P_{538} = (8, 0, 1, 1)$
72 : $P_{539} = (9, 0, 1, 1)$
73 : $P_{540} = (10, 0, 1, 1)$
74 : $P_{541} = (11, 0, 1, 1)$
75 : $P_{542} = (12, 0, 1, 1)$
76 : $P_{543} = (13, 0, 1, 1)$
77 : $P_{544} = (14, 0, 1, 1)$
78 : $P_{545} = (15, 0, 1, 1)$
79 : $P_{555} = (10, 1, 1, 1)$
80 : $P_{556} = (11, 1, 1, 1)$
81 : $P_{629} = (4, 6, 1, 1)$
82 : $P_{630} = (5, 6, 1, 1)$
83 : $P_{655} = (14, 7, 1, 1)$
84 : $P_{656} = (15, 7, 1, 1)$
85 : $P_{695} = (6, 10, 1, 1)$
86 : $P_{696} = (7, 10, 1, 1)$
87 : $P_{717} = (12, 11, 1, 1)$
88 : $P_{718} = (13, 11, 1, 1)$
89 : $P_{723} = (2, 12, 1, 1)$
90 : $P_{724} = (3, 12, 1, 1)$
91 : $P_{745} = (8, 13, 1, 1)$
92 : $P_{746} = (9, 13, 1, 1)$
93 : $P_{798} = (13, 0, 2, 1)$
94 : $P_{826} = (9, 2, 2, 1)$
95 : $P_{830} = (13, 2, 2, 1)$
96 : $P_{834} = (1, 3, 2, 1)$
97 : $P_{853} = (4, 4, 2, 1)$
98 : $P_{858} = (9, 4, 2, 1)$
99 : $P_{883} = (2, 6, 2, 1)$
100 : $P_{890} = (9, 6, 2, 1)$
101 : $P_{900} = (3, 7, 2, 1)$
102 : $P_{909} = (12, 7, 2, 1)$
103 : $P_{915} = (2, 8, 2, 1)$
104 : $P_{927} = (14, 8, 2, 1)$
105 : $P_{934} = (5, 9, 2, 1)$
106 : $P_{944} = (15, 9, 2, 1)$
107 : $P_{948} = (3, 10, 2, 1)$
108 : $P_{949} = (4, 10, 2, 1)$
109 : $P_{996} = (3, 13, 2, 1)$
110 : $P_{1003} = (10, 13, 2, 1)$
111 : $P_{1011} = (2, 14, 2, 1)$
112 : $P_{1013} = (4, 14, 2, 1)$
113 : $P_{1031} = (6, 15, 2, 1)$
114 : $P_{1033} = (8, 15, 2, 1)$
115 : $P_{1054} = (13, 0, 3, 1)$
116 : $P_{1074} = (1, 2, 3, 1)$
117 : $P_{1118} = (13, 4, 3, 1)$
118 : $P_{1120} = (15, 4, 3, 1)$
119 : $P_{1170} = (1, 8, 3, 1)$
120 : $P_{1179} = (10, 8, 3, 1)$
121 : $P_{1189} = (4, 9, 3, 1)$
122 : $P_{1194} = (9, 9, 3, 1)$
123 : $P_{1202} = (1, 10, 3, 1)$
124 : $P_{1209} = (8, 10, 3, 1)$
125 : $P_{1304} = (7, 0, 4, 1)$
126 : $P_{1333} = (4, 2, 4, 1)$
127 : $P_{1338} = (9, 2, 4, 1)$
128 : $P_{1358} = (13, 3, 4, 1)$
129 : $P_{1360} = (15, 3, 4, 1)$
130 : $P_{1368} = (7, 4, 4, 1)$
131 : $P_{1375} = (14, 4, 4, 1)$
132 : $P_{1378} = (1, 5, 4, 1)$
133 : $P_{1414} = (5, 7, 4, 1)$
134 : $P_{1420} = (11, 7, 4, 1)$
135 : $P_{1450} = (9, 9, 4, 1)$
136 : $P_{1455} = (14, 9, 4, 1)$
137 : $P_{1478} = (5, 11, 4, 1)$
138 : $P_{1482} = (9, 11, 4, 1)$
139 : $P_{1494} = (5, 12, 4, 1)$
140 : $P_{1495} = (6, 12, 4, 1)$
141 : $P_{1509} = (4, 13, 4, 1)$
142 : $P_{1519} = (14, 13, 4, 1)$
143 : $P_{1524} = (3, 14, 4, 1)$
144 : $P_{1529} = (8, 14, 4, 1)$
145 : $P_{1539} = (2, 15, 4, 1)$
146 : $P_{1541} = (4, 15, 4, 1)$
147 : $P_{1560} = (7, 0, 5, 1)$
148 : $P_{1618} = (1, 4, 5, 1)$
149 : $P_{1700} = (3, 9, 5, 1)$
150 : $P_{1704} = (7, 9, 5, 1)$
151 : $P_{1730} = (1, 11, 5, 1)$
152 : $P_{1744} = (15, 11, 5, 1)$
153 : $P_{1786} = (9, 14, 5, 1)$
154 : $P_{1791} = (14, 14, 5, 1)$
155 : $P_{1794} = (1, 15, 5, 1)$
156 : $P_{1804} = (11, 15, 5, 1)$
157 : $P_{1820} = (11, 0, 6, 1)$
158 : $P_{1829} = (4, 1, 6, 1)$
159 : $P_{1830} = (5, 1, 6, 1)$
160 : $P_{1843} = (2, 2, 6, 1)$
161 : $P_{1850} = (9, 2, 6, 1)$
162 : $P_{1922} = (1, 7, 6, 1)$
163 : $P_{1961} = (8, 9, 6, 1)$
164 : $P_{1966} = (13, 9, 6, 1)$
165 : $P_{2044} = (11, 14, 6, 1)$
166 : $P_{2048} = (15, 14, 6, 1)$
167 : $P_{2076} = (11, 0, 7, 1)$
168 : $P_{2095} = (14, 1, 7, 1)$
169 : $P_{2096} = (15, 1, 7, 1)$
170 : $P_{2100} = (3, 2, 7, 1)$

171 : $P_{2109} = (12, 2, 7, 1)$
172 : $P_{2134} = (5, 4, 7, 1)$
173 : $P_{2140} = (11, 4, 7, 1)$
174 : $P_{2162} = (1, 6, 7, 1)$
175 : $P_{2211} = (2, 9, 7, 1)$
176 : $P_{2218} = (9, 9, 7, 1)$
177 : $P_{2333} = (12, 0, 8, 1)$
178 : $P_{2355} = (2, 2, 8, 1)$
179 : $P_{2367} = (14, 2, 8, 1)$
180 : $P_{2370} = (1, 3, 8, 1)$
181 : $P_{2379} = (10, 3, 8, 1)$
182 : $P_{2466} = (1, 9, 8, 1)$
183 : $P_{2482} = (1, 10, 8, 1)$
184 : $P_{2484} = (3, 10, 8, 1)$
185 : $P_{2550} = (5, 14, 8, 1)$
186 : $P_{2557} = (12, 14, 8, 1)$
187 : $P_{2589} = (12, 0, 9, 1)$
188 : $P_{2614} = (5, 2, 9, 1)$
189 : $P_{2624} = (15, 2, 9, 1)$
190 : $P_{2629} = (4, 3, 9, 1)$
191 : $P_{2634} = (9, 3, 9, 1)$
192 : $P_{2650} = (9, 4, 9, 1)$
193 : $P_{2655} = (14, 4, 9, 1)$
194 : $P_{2660} = (3, 5, 9, 1)$
195 : $P_{2664} = (7, 5, 9, 1)$
196 : $P_{2681} = (8, 6, 9, 1)$
197 : $P_{2686} = (13, 6, 9, 1)$
198 : $P_{2691} = (2, 7, 9, 1)$
199 : $P_{2698} = (9, 7, 9, 1)$
200 : $P_{2706} = (1, 8, 9, 1)$
201 : $P_{2723} = (2, 9, 9, 1)$
202 : $P_{2733} = (12, 9, 9, 1)$
203 : $P_{2745} = (8, 10, 9, 1)$
204 : $P_{2751} = (14, 10, 9, 1)$
205 : $P_{2777} = (8, 12, 9, 1)$
206 : $P_{2779} = (10, 12, 9, 1)$
207 : $P_{2803} = (2, 14, 9, 1)$
208 : $P_{2815} = (14, 14, 9, 1)$
209 : $P_{2833} = (0, 0, 10, 1)$
210 : $P_{2855} = (6, 1, 10, 1)$
211 : $P_{2856} = (7, 1, 10, 1)$
212 : $P_{2868} = (3, 2, 10, 1)$
213 : $P_{2869} = (4, 2, 10, 1)$
214 : $P_{2882} = (1, 3, 10, 1)$
215 : $P_{2889} = (8, 3, 10, 1)$
216 : $P_{2962} = (1, 8, 10, 1)$
217 : $P_{2964} = (3, 8, 10, 1)$
218 : $P_{2985} = (8, 9, 10, 1)$
219 : $P_{2991} = (14, 9, 10, 1)$
220 : $P_{2996} = (3, 10, 10, 1)$
221 : $P_{3001} = (8, 10, 10, 1)$
222 : $P_{3010} = (1, 11, 10, 1)$
223 : $P_{3089} = (0, 0, 11, 1)$
224 : $P_{3117} = (12, 1, 11, 1)$
225 : $P_{3118} = (13, 1, 11, 1)$
226 : $P_{3158} = (5, 4, 11, 1)$
227 : $P_{3162} = (9, 4, 11, 1)$
228 : $P_{3170} = (1, 5, 11, 1)$
229 : $P_{3184} = (15, 5, 11, 1)$
230 : $P_{3250} = (1, 10, 11, 1)$
231 : $P_{3270} = (5, 11, 11, 1)$
232 : $P_{3280} = (15, 11, 11, 1)$
233 : $P_{3315} = (2, 14, 11, 1)$
234 : $P_{3328} = (15, 14, 11, 1)$
235 : $P_{3330} = (1, 15, 11, 1)$
236 : $P_{3334} = (5, 15, 11, 1)$
237 : $P_{3355} = (10, 0, 12, 1)$
238 : $P_{3363} = (2, 1, 12, 1)$
239 : $P_{3364} = (3, 1, 12, 1)$
240 : $P_{3414} = (5, 4, 12, 1)$
241 : $P_{3415} = (6, 4, 12, 1)$
242 : $P_{3497} = (8, 9, 12, 1)$
243 : $P_{3499} = (10, 9, 12, 1)$
244 : $P_{3554} = (1, 13, 12, 1)$
245 : $P_{3573} = (4, 14, 12, 1)$
246 : $P_{3583} = (14, 14, 12, 1)$
247 : $P_{3611} = (10, 0, 13, 1)$
248 : $P_{3625} = (8, 1, 13, 1)$
249 : $P_{3626} = (9, 1, 13, 1)$
250 : $P_{3636} = (3, 2, 13, 1)$
251 : $P_{3643} = (10, 2, 13, 1)$
252 : $P_{3669} = (4, 4, 13, 1)$
253 : $P_{3679} = (14, 4, 13, 1)$
254 : $P_{3794} = (1, 12, 13, 1)$
255 : $P_{3832} = (7, 14, 13, 1)$
256 : $P_{3840} = (15, 14, 13, 1)$
257 : $P_{3863} = (6, 0, 14, 1)$
258 : $P_{3891} = (2, 2, 14, 1)$
259 : $P_{3893} = (4, 2, 14, 1)$
260 : $P_{3924} = (3, 4, 14, 1)$
261 : $P_{3929} = (8, 4, 14, 1)$
262 : $P_{3946} = (9, 5, 14, 1)$
263 : $P_{3951} = (14, 5, 14, 1)$
264 : $P_{3964} = (11, 6, 14, 1)$
265 : $P_{3968} = (15, 6, 14, 1)$
266 : $P_{3990} = (5, 8, 14, 1)$
267 : $P_{3997} = (12, 8, 14, 1)$
268 : $P_{4003} = (2, 9, 14, 1)$
269 : $P_{4015} = (14, 9, 14, 1)$
270 : $P_{4035} = (2, 11, 14, 1)$
271 : $P_{4048} = (15, 11, 14, 1)$
272 : $P_{4053} = (4, 12, 14, 1)$
273 : $P_{4063} = (14, 12, 14, 1)$
274 : $P_{4072} = (7, 13, 14, 1)$
275 : $P_{4080} = (15, 13, 14, 1)$
276 : $P_{4085} = (4, 14, 14, 1)$
277 : $P_{4087} = (6, 14, 14, 1)$
278 : $P_{4098} = (1, 15, 14, 1)$
279 : $P_{4119} = (6, 0, 15, 1)$
280 : $P_{4151} = (6, 2, 15, 1)$
281 : $P_{4153} = (8, 2, 15, 1)$
282 : $P_{4179} = (2, 4, 15, 1)$
283 : $P_{4181} = (4, 4, 15, 1)$
284 : $P_{4194} = (1, 5, 15, 1)$
285 : $P_{4204} = (11, 5, 15, 1)$
286 : $P_{4290} = (1, 11, 15, 1)$
287 : $P_{4294} = (5, 11, 15, 1)$
288 : $P_{4338} = (1, 14, 15, 1)$