# Rank-65687 over GF(32)

January 15, 2021

## The equation

The equation of the surface is:

$$X_2^3 + X_0^2 X_1 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(32) is 1109428261

### General information

Number of lines	34
Number of points	1089
Number of singular points	33
Number of Eckardt points	0
Number of double points	0
Number of single points	1088
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$33^{34}$
Type of lines on points	$34, 1^{1088}$

### Singular Points

The surface has 33 singular points:

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\begin{array}{lll} 0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1) & 8: P_{8456} = \mathbf{P}(\eta^{11},\eta^{11},\eta^{11},1) = \mathbf{P}(7,7,7,1) \\ 1: P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1) & 9: P_{9513} = \mathbf{P}(\eta^3,\eta^3,\eta^3,1) = \mathbf{P}(8,8,8,1) \\ 2: P_{68} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0) & 10: P_{10570} = \mathbf{P}(\eta^{29},\eta^{29},\eta^{29},1) = \mathbf{P}(9,9,9,1) \\ 3: P_{3171} = \mathbf{P}(\eta,\eta,\eta,1) = \mathbf{P}(2,2,2,1) & 11: P_{11627} = \mathbf{P}(\eta^6,\eta^6,\eta^6,1) = \mathbf{P}(10,10,10,1) \\ 4: P_{4228} = \mathbf{P}(\eta^{18},\eta^{18},\eta^{18},1) = \mathbf{P}(3,3,3,1) & 12: P_{12684} = \mathbf{P}(\eta^{27},\eta^{27},\eta^{27},1) = \mathbf{P}(11,11,11,1) \\ 5: P_{5285} = \mathbf{P}(\eta^2,\eta^2,\eta^2,1) = \mathbf{P}(4,4,4,1) & 13: P_{13741} = \mathbf{P}(\eta^{20},\eta^{20},\eta^{20},\eta^{20},1) = \mathbf{P}(12,12,12,1) \\ 6: P_{6342} = \mathbf{P}(\eta^{19},\eta^{19},\eta^{19},\eta^{19},1) = \mathbf{P}(6,6,6,1) & 15: P_{15855} = \mathbf{P}(\eta^{12},\eta^{12},\eta^{12},1) = \mathbf{P}(14,14,14,1) \end{array}
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\begin{array}{lll} 16: \ P_{16912} = \mathbf{P}(\eta^{23}, \eta^{23}, \eta^{23}, 1) = \mathbf{P}(15, 15, 15, 1) \\ 17: \ P_{17969} = \mathbf{P}(\eta^4, \eta^4, \eta^4, 1) = \mathbf{P}(16, 16, 16, 1) \\ 18: \ P_{19026} = \mathbf{P}(\eta^{10}, \eta^{10}, \eta^{10}, 1) = \mathbf{P}(17, 17, 17, 1) \\ 19: \ P_{20083} = \mathbf{P}(\eta^{30}, \eta^{30}, \eta^{30}, 1) = \mathbf{P}(18, 18, 18, 1) \\ 20: \ P_{21140} = \mathbf{P}(\eta^{17}, \eta^{17}, \eta^{17}, 1) = \mathbf{P}(19, 19, 19, 1) \\ 21: \ P_{22197} = \mathbf{P}(\eta^7, \eta^7, \eta^7, 1) = \mathbf{P}(20, 20, 20, 1) \\ 22: \ P_{23254} = \mathbf{P}(\eta^{22}, \eta^{22}, \eta^{22}, 1) = \mathbf{P}(21, 21, 21, 1) \\ 23: \ P_{24311} = \mathbf{P}(\eta^{28}, \eta^{28}, \eta^{28}, 1) = \mathbf{P}(22, 22, 22, 1) \\ 24: \ P_{25368} = \mathbf{P}(\eta^{26}, \eta^{26}, \eta^{26}, 1) = \mathbf{P}(23, 23, 23, 1) \end{array}
25: \ P_{26425} = \mathbf{P}(\eta^{21}, \eta^{21}, \eta^{21}, 1) = \mathbf{P}(24, 24, 24, 1) \\ 26: \ P_{27482} = \mathbf{P}(\eta^{25}, \eta^{25}, 1) = \mathbf{P}(25, 25, 25, 1) \\ 27: \ P_{28539} = \mathbf{P}(\eta^9, \eta^9, \eta^9, 1) = \mathbf{P}(26, 26, 26, 1) \\ 28: \ P_{29596} = \mathbf{P}(\eta^{16}, \eta^{16}, 1) = \mathbf{P}(27, 27, 27, 1) \\ 29: \ P_{30653} = \mathbf{P}(\eta^{13}, \eta^{13}, \eta^{13}, 1) = \mathbf{P}(28, 28, 28, 1) \\ 30: \ P_{31710} = \mathbf{P}(\eta^{14}, \eta^{14}, \eta^{14}, 1) = \mathbf{P}(29, 29, 29, 1) \\ 31: \ P_{32767} = \mathbf{P}(\eta^{24}, \eta^{24}, \eta^{24}, 1) = \mathbf{P}(30, 30, 30, 1) \\ 32: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(31, 31, 31, 1) \\ 29: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(31, 31, 31, 1) \\ 31: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(31, 31, 31, 1) \\ 31: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(31, 31, 31, 1) \\ 31: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) \\ 31: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) \\ 31: \ P_{33824} = \mathbf{P}(\eta^{15}, \eta^{15}, \eta^{15}, 1) = \mathbf{P}(\eta^{15}, \eta^{15}, \eta
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## The 34 Lines

The lines and their Pluecker coordinates are:

$$\begin{split} \ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \mathbf{Pl}(0,0,0,1,0)_{1089} \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{Pl}(0,0,0,1,0,0)_{65} \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2113} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2113} = \mathbf{Pl}(0,0,0,1,1,0)_{3105} \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \mathbf{Pl}(0,1,0,1,1,0)_{3137} \\ \ell_4 &= \begin{bmatrix} 1 & \eta^{29} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{754697} = \begin{bmatrix} 1 & 9 & 22 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{754697} = \mathbf{Pl}(0,22,0,9,1,0)_{3662} \\ \ell_5 &= \begin{bmatrix} 1 & \eta^5 & \eta^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{547525} = \begin{bmatrix} 1 & 5 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{547525} = \mathbf{Pl}(0,16,0,5,1,0)_{3404} \\ \ell_6 &= \begin{bmatrix} 1 & \eta^{27} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{858283} = \begin{bmatrix} 1 & 11 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{858283} = \mathbf{Pl}(0,25,0,11,1,0)_{3791} \\ \ell_7 &= \begin{bmatrix} 1 & \eta^{10} & \eta^8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{458737} = \begin{bmatrix} 1 & 17 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{458737} = \mathbf{Pl}(0,13,0,17,1,0)_{4157} \\ \ell_8 &= \begin{bmatrix} 1 & \eta^7 & \eta^8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{563380} = \begin{bmatrix} 1 & 20 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{563380} = \mathbf{Pl}(0,18,0,4,1,0)_{3343} \\ \ell_{10} &= \begin{bmatrix} 1 & \eta^{12} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1079196} = \begin{bmatrix} 1 & 14 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{322384} = \mathbf{Pl}(0,31,0,28,1,0)_{4868} \\ \ell_{12} &= \begin{bmatrix} 1 & \eta^{13} & \eta^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{322384} = \begin{bmatrix} 1 & 16 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{322384} = \mathbf{Pl}(0,18,0,23,1,0)_{490} \\ \ell_{13} &= \begin{bmatrix} 1 & \eta^{14} & \eta^8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \begin{bmatrix} 1 & 29 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \mathbf{Pl}(0,18,0,23,1,0)_{490} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^{24} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \begin{bmatrix} 1 & 29 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \mathbf{Pl}(0,18,0,23,1,0)_{490} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^{24} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \begin{bmatrix} 1 & 29 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} = \mathbf{Pl}(0,18,0,23,1,0)_{490} \\ \ell_{15} &= \begin{bmatrix} 1 & \eta^{24} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{776894} = \begin{bmatrix} 1 & 30 & 22 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{776894} = \mathbf{Pl}(0,22,0,30,1,0)_{4905} \end{aligned}$$

$$\begin{split} \ell_{16} &= \begin{bmatrix} 1 & \eta^9 & \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{96186} = \begin{bmatrix} 1 & 26 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{96186} = \mathbf{PI}(0, 2, 0, 26, 1, 0)_{4713} \\ \ell_{17} &= \begin{bmatrix} 1 & \eta^{15} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{710303} = \begin{bmatrix} 1 & 31 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{710303} = \mathbf{PI}(0, 20, 0, 31, 1, 0)_{5046} \\ \ell_{18} &= \begin{bmatrix} 1 & \eta^8 & \eta^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{386861} = \begin{bmatrix} 1 & 13 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{386861} = \mathbf{PI}(0, 11, 0, 13, 1, 0)_{3903} \\ \ell_{19} &= \begin{bmatrix} 1 & \eta^{28} & \eta^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{937558} = \begin{bmatrix} 1 & 22 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{330840} = \mathbf{PI}(0, 27, 0, 22, 1, 0)_{4486} \\ \ell_{20} &= \begin{bmatrix} 1 & \eta^{21} & \eta^{29} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{330840} = \begin{bmatrix} 1 & 24 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{330840} = \mathbf{PI}(0, 9, 0, 24, 1, 0)_{4594} \\ \ell_{21} &= \begin{bmatrix} 1 & \eta^{17} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{366739} = \begin{bmatrix} 1 & 19 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{366739} = \mathbf{PI}(0, 25, 0, 19, 1, 0)_{4295} \\ \ell_{22} &= \begin{bmatrix} 1 & \eta^{23} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{219855} = \begin{bmatrix} 1 & 15 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{219855} = \mathbf{PI}(0, 6, 0, 15, 1, 0)_{4024} \\ \ell_{23} &= \begin{bmatrix} 1 & \eta^{20} & \eta^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{926988} = \begin{bmatrix} 1 & 12 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{326938} = \mathbf{PI}(0, 27, 0, 12, 1, 0)_{3856} \\ \ell_{24} &= \begin{bmatrix} 1 & \eta^{22} & \eta^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{688106} = \begin{bmatrix} 1 & 10 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{688106} = \mathbf{PI}(0, 20, 0, 10, 1, 0)_{3723} \\ \ell_{26} &= \begin{bmatrix} 1 & \eta^{19} & \eta^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1051714} = \begin{bmatrix} 1 & 2 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13051714} = \mathbf{PI}(0, 4, 0, 3, 1, 0)_{3266} \\ \ell_{29} &= \begin{bmatrix} 1 & \eta^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{139523} = \begin{bmatrix} 1 & 18 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{130573} = \mathbf{PI}(0, 4, 0, 3, 1, 0)_{3266} \\ \ell_{29} &= \begin{bmatrix} 1 & \eta^{16} & \eta^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{36955} = \begin{bmatrix} 1 & 27 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{305075} = \mathbf{PI}(0, 15, 0, 27, 1, 0)_{4789} \\ \ell_{31} &= \begin{bmatrix} 1 & \eta^{25} & \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{36955} = \begin{bmatrix} 1 & 27 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{36955} = \mathbf{PI}(0, 2, 0, 25, 1, 0)_{456} \\ \ell_{32} &= \begin{bmatrix} 1 & \eta^{3} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{312456} = \begin{bmatrix} 1 & 25 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{38559} = \mathbf{PI}(0, 6, 0, 8, 1, 0)_{3583} \\ = \mathbf{PI}(0, 0, 0, 0, 1, 1, 0)_{3525}$$

Rank of lines: ( 1056, 1082400, 2113, 35937, 754697, 547525, 858283, 458737, 614116, 563380, 996750, 1079196, 322384, 471421, 634199, 776894, 96186, 710303, 386861, 937558, 330840, 866739, 219855, 926988, 530613, 688106, 142694, 1051714, 139523, 1000978, 536955, 95129, 212456, 380519)
Rank of points on Klein quadric: ( 1089, 65, 3105, 3137, 3662, 3404, 3791, 4157, 3343, 4349, 3984, 4868, 4090, 4913, 4540, 4985, 4713, 5046, 3903, 4486, 4594, 4295, 4024, 3856, 4411, 3723, 3455, 3230, 3266, 4236,

4789, 4650, 3583, 3525)

## **Eckardt Points**

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 0 Double points: The double points on the surface are:

## Single Points

The surface has 1088 single points: Too many to print.

### Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

# Line Intersection Graph

	0	1	2	3	1 :	5 (	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24:	25	26	27	28	29	30	31	32	33
0	0	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	0	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1 (	) [	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1 (	)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1 :	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1 :	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1 :	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1 :	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1 :	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1 :	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1 :	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1 :	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	Ι-	1	_	_		_	_	_	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20		1							1	_	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
21		1									1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
22		1									1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
23	-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
24	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
25	-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
27	-	1	_	_		_	_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
	Ι-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
30	-	1	_	_		_	_	_	1	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
31	-	1	_	_	-	-	_	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
32	-	1	_	_		_	_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
33	1	1	1	1	1 .	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$									

## Line 1 intersects

$\operatorname{Line}$	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$									

## Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$									

## Line 3 intersects $\frac{1}{2}$

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$									

Line 5 intersects   Line   L	Line 4 intersects	
Line 5 intersects   Line   L	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point   P <sub>3</sub>   P <sub>3</sub>	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 5 intersects	
Line 6 intersects    Line   Li		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
		2
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 7 intersects	
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 8 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3
Line 10 intersects  Line   \$\lambda_0 \begin{array}{c c c c c c c c c c c c c c c c c c c	Line 9 intersects	
Line 10 intersects  Line $\begin{pmatrix} l_0 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & l_8 & l_9 & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{19} & l_{19} & l_{19} & l_{19} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{1$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
Line 11 intersects  Line $  l_0   l_1   l_2   l_3   l_4   l_5   l_6   l_7   l_8   l_9   l_{10}   l_{12}   l_{13}   l_{14}   l_{15}   l_{16}   l_{17}   l_{18}   l_{19}   l_{20}   l_{21}   l_{22}   l_{10}   l_{10}   l_{10}   l_{11}   l_{13}   l_{14}   l_{15}   l_{16}   l_{17}   l_{18}   l_{19}   l_{20}   l_{21}   l_{22}   l_{10}   l_{10}   l_{10}   l_{11}   l_{13}   l_{14}   l_{15}   l_{16}   l_{17}   l_{18}   l_{19}   l_{20}   l_{21}   l_{22}   l_{21}   l_{22}   l_{10}   l_{10}   l_{11}   l_{13}   l_{14}   l_{15}   l_{16}   l_{17}   l_{18}   l_{19}   l_{20}   l_{21}   l_{22}   l_{21}   l_{22}   l_{21}   l_{22}   l_{22}   l_{23}   l_{24}   l_{25}   l_{2$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2
	in point $ P_3 $ $ P_$	;
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		
Line 12 intersects  Line $  \ell_0   \ell_1   \ell_2   \ell_3   \ell_4   \ell_5   \ell_6   \ell_7   \ell_8   \ell_9   \ell_{10}   \ell_{11}   \ell_{13}   \ell_{14}   \ell_{15}   \ell_{16}   \ell_{17}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{10}   \ell_{10}   \ell_{11}   \ell_{13}   \ell_{14}   \ell_{15}   \ell_{16}   \ell_{17}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{10}   \ell_{10}   \ell_{11}   \ell_{13}   \ell_{14}   \ell_{15}   \ell_{16}   \ell_{17}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{22}   \ell_{23}   \ell_{24}   \ell_{2$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	;
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 12 intersects	
Line 13 intersects    Line   $\ell_0$   $\ell_1$   $\ell_2$   $\ell_3$   $\ell_4$   $\ell_5$   $\ell_6$   $\ell_7$   $\ell_8$   $\ell_9$   $\ell_{10}$   $\ell_{11}$   $\ell_{12}$   $\ell_{14}$   $\ell_{15}$   $\ell_{16}$   $\ell_{17}$   $\ell_{18}$   $\ell_{19}$   $\ell_{20}$   $\ell_{21}$   $\ell_{22}$   in point   $P_3$		_
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	<u>:</u>
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 13 intersects	
Line 14 intersects    Line   $\ell_0$   $\ell_1$   $\ell_2$   $\ell_3$   $\ell_4$   $\ell_5$   $\ell_6$   $\ell_7$   $\ell_8$   $\ell_9$   $\ell_{10}$   $\ell_{11}$   $\ell_{12}$   $\ell_{13}$   $\ell_{15}$   $\ell_{16}$   $\ell_{17}$   $\ell_{18}$   $\ell_{19}$   $\ell_{20}$   $\ell_{21}$   $\ell_{22}$   in point   $P_3$	V 2 2 3 V V V V V 2 2 2 2 2 2 2 2 2 2 2	_
	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	<u>.                                    </u>
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 14 intersects	
Line 15 intersects  Line $\ell_0$ $\ell_1$ $\ell_2$ $\ell_3$ $\ell_4$ $\ell_5$ $\ell_6$ $\ell_7$ $\ell_8$ $\ell_9$ $\ell_{10}$ $\ell_{11}$ $\ell_{12}$ $\ell_{13}$ $\ell_{14}$ $\ell_{16}$ $\ell_{17}$ $\ell_{18}$ $\ell_{19}$ $\ell_{20}$ $\ell_{21}$ $\ell_{22}$ in point $P_3$	0 1 2 0 1 0 0 1 0 1 1 12 10 10 11 12 12	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	;
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 15 intersects	
Line 16 intersects  Line $ \ell_0   \ell_1   \ell_2   \ell_3   \ell_4   \ell_5   \ell_6   \ell_7   \ell_8   \ell_9   \ell_{10}   \ell_{11}   \ell_{12}   \ell_{13}   \ell_{14}   \ell_{15}   \ell_{17}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{18}   \ell_{19}   \ell_{20}   \ell_{21}   \ell_{22}   \ell_{22}   \ell_{21}   \ell_{22}   \ell_{22$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	,
	Line 16 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	;

Line 17 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 18 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 19 int	ersec	cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 20 int	erse	cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 21 int	ersec	cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 22 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 23 int		ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 24 int	ersec	cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 25 int																						
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 26 int																						
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 27 int																						
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 28 int		cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 29 int	ersec	cts																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

## Line 30 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	,
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$											

## Line 31 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

## ${\bf Line~32~intersects}$

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	1
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$											

## Line 33 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$   $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

The surface has 1089 points:

Too many to print.