Rank-76308 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_0^2 X_1 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is 1107562566

General information

Number of lines	4
Number of points	4161
Number of singular points	2
Number of Eckardt points	1
Number of double points	1
Number of single points	255
Number of points off lines	3904
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{4}
Type of lines on points	$3, 2, 1^{255}, 0^{3904}$

Singular Points

The surface has 2 singular points:

$$\begin{array}{l} 0: \ P_{233538} = \mathbf{P}(1,0,\epsilon^{42},1) = \mathbf{P}(1,0,56,1) \\ 1: \ P_{237634} = \mathbf{P}(1,0,\epsilon^{21},1) = \mathbf{P}(1,0,57,1) \end{array}$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{17047616} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{66}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{274562} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{274562} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{544642}$$

Rank of lines: (17047616, 270400, 8258, 274562)

Rank of points on Klein quadric: (1, 66, 544578, 544642)

Eckardt Points

The surface has 1 Eckardt points: $0: P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1).$

Double Points

The surface has 1 Double points: The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

Single Points

The surface has 255 single points: The single points on the surface are:

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0: P_3 = (0,0,0,1) lies on line \ell_0
                                                                     23: P_{9427} = (18, 18, 1, 1) lies on line \ell_2
1: P_4 = (1, 1, 1, 1) lies on line \ell_2
                                                                     24: P_{9492} = (19, 19, 1, 1) lies on line \ell_2
2: P_5 = (1, 1, 0, 0) lies on line \ell_2
                                                                     25: P_{9557} = (20, 20, 1, 1) lies on line \ell_2
3: P_{132} = (1, 1, 1, 0) lies on line \ell_3
                                                                     26: P_{9622} = (21, 21, 1, 1) lies on line \ell_2
4: P_{4163} = (1, 0, 0, 1) lies on line \ell_1
                                                                     27: P_{9687} = (22, 22, 1, 1) lies on line \ell_2
                                                                     28: P_{9752} = (23, 23, 1, 1) lies on line \ell_2
5: P_{4227} = (1, 1, 0, 1) lies on line \ell_3
                                                                     29: P_{9817} = (24, 24, 1, 1) lies on line \ell_2
6: P_{8259} = (1, 0, 1, 1) lies on line \ell_1
7: P_{8387} = (2, 2, 1, 1) lies on line \ell_2
                                                                     30: P_{9882} = (25, 25, 1, 1) lies on line \ell_2
8: P_{8452} = (3, 3, 1, 1) lies on line \ell_2
                                                                     31: P_{9947} = (26, 26, 1, 1) lies on line \ell_2
9: P_{8517} = (4, 4, 1, 1) lies on line \ell_2
                                                                     32: P_{10012} = (27, 27, 1, 1) lies on line \ell_2
10: P_{8582} = (5, 5, 1, 1) lies on line \ell_2
                                                                     33: P_{10077} = (28, 28, 1, 1) lies on line \ell_2
11 : P_{8647} = (6, 6, 1, 1) lies on line \ell_2
                                                                     34: P_{10142} = (29, 29, 1, 1) lies on line \ell_2
12: P_{8712} = (7,7,1,1) lies on line \ell_2
                                                                     35: P_{10207} = (30, 30, 1, 1) lies on line \ell_2
                                                                     36 : P_{10272} = (31, 31, 1, 1) lies on line \ell_2
13: P_{8777} = (8, 8, 1, 1) lies on line \ell_2
14: P_{8842} = (9, 9, 1, 1) lies on line \ell_2
                                                                     37: P_{10337} = (32, 32, 1, 1) lies on line \ell_2
15: P_{8907} = (10, 10, 1, 1) lies on line \ell_2
                                                                     38: P_{10402} = (33, 33, 1, 1) lies on line \ell_2
16: P_{8972} = (11, 11, 1, 1) lies on line \ell_2
                                                                     39: P_{10467} = (34, 34, 1, 1) lies on line \ell_2
17: P_{9037} = (12, 12, 1, 1) lies on line \ell_2
                                                                     40: P_{10532} = (35, 35, 1, 1) lies on line \ell_2
18: P_{9102} = (13, 13, 1, 1) lies on line \ell_2
                                                                      41: P_{10597} = (36, 36, 1, 1) lies on line \ell_2
19: P_{9167} = (14, 14, 1, 1) lies on line \ell_2
                                                                     42: P_{10662} = (37, 37, 1, 1) lies on line \ell_2
20: P_{9232} = (15, 15, 1, 1) lies on line \ell_2
                                                                     43: P_{10727} = (38, 38, 1, 1) lies on line \ell_2
21: P_{9297} = (16, 16, 1, 1) lies on line \ell_2
                                                                     44: P_{10792} = (39, 39, 1, 1) lies on line \ell_2
22: P_{9362} = (17, 17, 1, 1) lies on line \ell_2
                                                                      45: P_{10857} = (40, 40, 1, 1) lies on line \ell_2
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100 : P_{53314} = (1, 0, 12, 1) lies on line \ell_1
46: P_{10922} = (41, 41, 1, 1) lies on line \ell_2
47: P_{10987} = (42, 42, 1, 1) lies on line \ell_2
                                                                     101: P_{54158} = (13, 13, 12, 1) lies on line \ell_3
48: P_{11052} = (43, 43, 1, 1) lies on line \ell_2
                                                                     102: P_{57409} = (0, 0, 13, 1) lies on line \ell_0
49: P_{11117} = (44, 44, 1, 1) lies on line \ell_2
                                                                     103: P_{57410} = (1, 0, 13, 1) lies on line \ell_1
50: P_{11182} = (45, 45, 1, 1) lies on line \ell_2
                                                                     104: P_{58189} = (12, 12, 13, 1) lies on line \ell_3
51: P_{11247} = (46, 46, 1, 1) lies on line \ell_2
                                                                     105: P_{61505} = (0, 0, 14, 1) lies on line \ell_0
52: P_{11312} = (47, 47, 1, 1) lies on line \ell_2
                                                                     106: P_{61506} = (1, 0, 14, 1) lies on line \ell_1
53: P_{11377} = (48, 48, 1, 1) lies on line \ell_2
                                                                     107: P_{62480} = (15, 15, 14, 1) lies on line \ell_3
54: P_{11442} = (49, 49, 1, 1) lies on line \ell_2
                                                                     108: P_{65601} = (0, 0, 15, 1) lies on line \ell_0
55: P_{11507} = (50, 50, 1, 1) lies on line \ell_2
                                                                     109: P_{65602} = (1, 0, 15, 1) lies on line \ell_1
56: P_{11572} = (51, 51, 1, 1) lies on line \ell_2
                                                                     110: P_{66511} = (14, 14, 15, 1) lies on line \ell_3
57: P_{11637} = (52, 52, 1, 1) lies on line \ell_2
                                                                     111: P_{69697} = (0, 0, 16, 1) lies on line \ell_0
58: P_{11702} = (53, 53, 1, 1) lies on line \ell_2
                                                                     112: P_{69698} = (1, 0, 16, 1) lies on line \ell_1
59: P_{11767} = (54, 54, 1, 1) lies on line \ell_2
                                                                     113: P_{70802} = (17, 17, 16, 1) lies on line \ell_3
                                                                     114: P_{73793} = (0, 0, 17, 1) lies on line \ell_0
60: P_{11832} = (55, 55, 1, 1) lies on line \ell_2
61: P_{11897} = (56, 56, 1, 1) lies on line \ell_2
                                                                     115: P_{73794} = (1, 0, 17, 1) lies on line \ell_1
62: P_{11962} = (57, 57, 1, 1) lies on line \ell_2
                                                                     116: P_{74833} = (16, 16, 17, 1) lies on line \ell_3
63: P_{12027} = (58, 58, 1, 1) lies on line \ell_2
                                                                     117: P_{77889} = (0, 0, 18, 1) lies on line \ell_0
64: P_{12092} = (59, 59, 1, 1) lies on line \ell_2
                                                                     118: P_{77890} = (1, 0, 18, 1) lies on line \ell_1
65: P_{12157} = (60, 60, 1, 1) lies on line \ell_2
                                                                     119: P_{79124} = (19, 19, 18, 1) lies on line \ell_3
66: P_{12222} = (61, 61, 1, 1) lies on line \ell_2
                                                                     120: P_{81985} = (0, 0, 19, 1) lies on line \ell_0
67: P_{12287} = (62, 62, 1, 1) lies on line \ell_2
                                                                     121: P_{81986} = (1, 0, 19, 1) lies on line \ell_1
                                                                     122: P_{83155} = (18, 18, 19, 1) lies on line \ell_3
68: P_{12352} = (63, 63, 1, 1) lies on line \ell_2
69 : P_{12353} = (0, 0, 2, 1) lies on line \ell_0
                                                                     123: P_{86081} = (0, 0, 20, 1) lies on line \ell_0
70: P_{12354} = (1,0,2,1) lies on line \ell_1
                                                                     124: P_{86082} = (1, 0, 20, 1) lies on line \ell_1
71: P_{12548} = (3, 3, 2, 1) lies on line \ell_3
                                                                     125: P_{87446} = (21, 21, 20, 1) lies on line \ell_3
72: P_{16449} = (0,0,3,1) lies on line \ell_0
                                                                     126: P_{90177} = (0, 0, 21, 1) lies on line \ell_0
73: P_{16450} = (1,0,3,1) lies on line \ell_1
                                                                     127: P_{90178} = (1, 0, 21, 1) lies on line \ell_1
74: P_{16579} = (2, 2, 3, 1) lies on line \ell_3
                                                                     128: P_{91477} = (20, 20, 21, 1) lies on line \ell_3
75: P_{20545} = (0, 0, 4, 1) lies on line \ell_0
                                                                     129: P_{94273} = (0, 0, 22, 1) lies on line \ell_0
76: P_{20546} = (1, 0, 4, 1) lies on line \ell_1
                                                                     130: P_{94274} = (1, 0, 22, 1) lies on line \ell_1
77: P_{20870} = (5, 5, 4, 1) lies on line \ell_3
                                                                     131: P_{95768} = (23, 23, 22, 1) lies on line \ell_3
78: P_{24641} = (0, 0, 5, 1) lies on line \ell_0
                                                                     132 : P_{98369} = (0, 0, 23, 1) lies on line \ell_0
79: P_{24642} = (1, 0, 5, 1) lies on line \ell_1
                                                                     133: P_{98370} = (1, 0, 23, 1) lies on line \ell_1
80: P_{24901} = (4, 4, 5, 1) lies on line \ell_3
                                                                     134: P_{99799} = (22, 22, 23, 1) lies on line \ell_3
                                                                     135: P_{102465} = (0, 0, 24, 1) lies on line \ell_0
81: P_{28737} = (0, 0, 6, 1) lies on line \ell_0
82: P_{28738} = (1, 0, 6, 1) lies on line \ell_1
                                                                     136: P_{102466} = (1, 0, 24, 1) lies on line \ell_1
83: P_{29192} = (7,7,6,1) lies on line \ell_3
                                                                     137: P_{104090} = (25, 25, 24, 1) lies on line \ell_3
84: P_{32833} = (0, 0, 7, 1) lies on line \ell_0
                                                                     138: P_{106561} = (0, 0, 25, 1) lies on line \ell_0
85: P_{32834} = (1, 0, 7, 1) lies on line \ell_1
                                                                     139: P_{106562} = (1, 0, 25, 1) lies on line \ell_1
                                                                     140: P_{108121} = (24, 24, 25, 1) lies on line \ell_3
86: P_{33223} = (6, 6, 7, 1) lies on line \ell_3
87: P_{36929} = (0,0,8,1) lies on line \ell_0
                                                                     141: P_{110657} = (0, 0, 26, 1) lies on line \ell_0
88: P_{36930} = (1,0,8,1) lies on line \ell_1
                                                                     142: P_{110658} = (1, 0, 26, 1) lies on line \ell_1
89: P_{37514} = (9, 9, 8, 1) lies on line \ell_3
                                                                     143: P_{112412} = (27, 27, 26, 1) lies on line \ell_3
90: P_{41025} = (0,0,9,1) lies on line \ell_0
                                                                     144: P_{114753} = (0, 0, 27, 1) lies on line \ell_0
                                                                     145 : P_{114754} = (1, 0, 27, 1) lies on line \ell_1
91: P_{41026} = (1, 0, 9, 1) lies on line \ell_1
92: P_{41545} = (8, 8, 9, 1) lies on line \ell_3
                                                                     146: P_{116443} = (26, 26, 27, 1) lies on line \ell_3
93: P_{45121} = (0, 0, 10, 1) lies on line \ell_0
                                                                     147: P_{118849} = (0, 0, 28, 1) lies on line \ell_0
94: P_{45122} = (1, 0, 10, 1) lies on line \ell_1
                                                                     148: P_{118850} = (1, 0, 28, 1) lies on line \ell_1
95: P_{45836} = (11, 11, 10, 1) lies on line \ell_3
                                                                     149: P_{120734} = (29, 29, 28, 1) lies on line \ell_3
                                                                     150: P_{122945} = (0, 0, 29, 1) lies on line \ell_0
96: P_{49217} = (0, 0, 11, 1) lies on line \ell_0
97: P_{49218} = (1, 0, 11, 1) lies on line \ell_1
                                                                     151: P_{122946} = (1, 0, 29, 1) lies on line \ell_1
98 : P_{49867} = (10, 10, 11, 1) lies on line \ell_3
                                                                     152: P_{124765} = (28, 28, 29, 1) lies on line \ell_3
99: P_{53313} = (0, 0, 12, 1) lies on line \ell_0
                                                                     153: P_{127041} = (0, 0, 30, 1) lies on line \ell_0
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154 : P_{127042} = (1, 0, 30, 1) lies on line \ell_1
                                                                     205 : P_{196674} = (1, 0, 47, 1) lies on line \ell_1
155 : P_{129056} = (31, 31, 30, 1) lies on line \ell_3
                                                                     206: P_{199663} = (46, 46, 47, 1) lies on line \ell_3
156: P_{131137} = (0, 0, 31, 1) lies on line \ell_0
                                                                     207: P_{200769} = (0, 0, 48, 1) lies on line \ell_0
157: P_{131138} = (1, 0, 31, 1) lies on line \ell_1
                                                                     208: P_{200770} = (1, 0, 48, 1) lies on line \ell_1
                                                                     209 : P_{203954} = (49, 49, 48, 1) lies on line \ell_3
158: P_{133087} = (30, 30, 31, 1) lies on line \ell_3
159: P_{135233} = (0, 0, 32, 1) lies on line \ell_0
                                                                     210 : P_{204865} = (0, 0, 49, 1) lies on line \ell_0
                                                                     211: P_{204866} = (1, 0, 49, 1) lies on line \ell_1
160: P_{135234} = (1, 0, 32, 1) lies on line \ell_1
161: P_{137378} = (33, 33, 32, 1) lies on line \ell_3
                                                                     212: P_{207985} = (48, 48, 49, 1) lies on line \ell_3
162: P_{139329} = (0, 0, 33, 1) lies on line \ell_0
                                                                     213: P_{208961} = (0, 0, 50, 1) lies on line \ell_0
163: P_{139330} = (1, 0, 33, 1) lies on line \ell_1
                                                                     214: P_{208962} = (1, 0, 50, 1) lies on line \ell_1
164: P_{141409} = (32, 32, 33, 1) lies on line \ell_3
                                                                     215: P_{212276} = (51, 51, 50, 1) lies on line \ell_3
165: P_{143425} = (0, 0, 34, 1) lies on line \ell_0
                                                                     216: P_{213057} = (0, 0, 51, 1) lies on line \ell_0
166: P_{143426} = (1, 0, 34, 1) lies on line \ell_1
                                                                     217: P_{213058} = (1, 0, 51, 1) lies on line \ell_1
167: P_{145700} = (35, 35, 34, 1) lies on line \ell_3
                                                                     218: P_{216307} = (50, 50, 51, 1) lies on line \ell_3
168: P_{147521} = (0, 0, 35, 1) lies on line \ell_0
                                                                     219: P_{217153} = (0, 0, 52, 1) lies on line \ell_0
169: P_{147522} = (1, 0, 35, 1) lies on line \ell_1
                                                                     220: P_{217154} = (1, 0, 52, 1) lies on line \ell_1
170: P_{149731} = (34, 34, 35, 1) lies on line \ell_3
                                                                     221: P_{220598} = (53, 53, 52, 1) lies on line \ell_3
171: P_{151617} = (0, 0, 36, 1) lies on line \ell_0
                                                                     222: P_{221249} = (0, 0, 53, 1) lies on line \ell_0
172: P_{151618} = (1, 0, 36, 1) lies on line \ell_1
                                                                     223: P_{221250} = (1, 0, 53, 1) lies on line \ell_1
173: P_{154022} = (37, 37, 36, 1) lies on line \ell_3
                                                                     224: P_{224629} = (52, 52, 53, 1) lies on line \ell_3
174: P_{155713} = (0, 0, 37, 1) lies on line \ell_0
                                                                     225: P_{225345} = (0, 0, 54, 1) lies on line \ell_0
175: P_{155714} = (1, 0, 37, 1) lies on line \ell_1
                                                                     226: P_{225346} = (1, 0, 54, 1) lies on line \ell_1
                                                                     227: P_{228920} = (55, 55, 54, 1) lies on line \ell_3
176: P_{158053} = (36, 36, 37, 1) lies on line \ell_3
177: P_{159809} = (0, 0, 38, 1) lies on line \ell_0
                                                                     228: P_{229441} = (0, 0, 55, 1) lies on line \ell_0
178: P_{159810} = (1, 0, 38, 1) lies on line \ell_1
                                                                     229: P_{229442} = (1, 0, 55, 1) lies on line \ell_1
179: P_{162344} = (39, 39, 38, 1) lies on line \ell_3
                                                                     230: P_{232951} = (54, 54, 55, 1) lies on line \ell_3
180: P_{163905} = (0, 0, 39, 1) lies on line \ell_0
                                                                     231: P_{233537} = (0, 0, 56, 1) lies on line \ell_0
181: P_{163906} = (1, 0, 39, 1) lies on line \ell_1
                                                                     232: P_{233538} = (1, 0, 56, 1) lies on line \ell_1
182: P_{166375} = (38, 38, 39, 1) lies on line \ell_3
                                                                     233: P_{237242} = (57, 57, 56, 1) lies on line \ell_3
183 : P_{168001} = (0, 0, 40, 1) lies on line \ell_0
                                                                     234 : P_{237633} = (0, 0, 57, 1) lies on line \ell_0
184: P_{168002} = (1, 0, 40, 1) lies on line \ell_1
                                                                     235: P_{237634} = (1, 0, 57, 1) lies on line \ell_1
185: P_{170666} = (41, 41, 40, 1) lies on line \ell_3
                                                                     236: P_{241273} = (56, 56, 57, 1) lies on line \ell_3
186: P_{172097} = (0, 0, 41, 1) lies on line \ell_0
                                                                     237: P_{241729} = (0, 0, 58, 1) lies on line \ell_0
                                                                     238: P_{241730} = (1, 0, 58, 1) lies on line \ell_1
187: P_{172098} = (1, 0, 41, 1) lies on line \ell_1
188: P_{174697} = (40, 40, 41, 1) lies on line \ell_3
                                                                     239: P_{245564} = (59, 59, 58, 1) lies on line \ell_3
189 : P_{176193} = (0, 0, 42, 1) lies on line \ell_0
                                                                     240: P_{245825} = (0, 0, 59, 1) lies on line \ell_0
190: P_{176194} = (1, 0, 42, 1) lies on line \ell_1
                                                                     241: P_{245826} = (1, 0, 59, 1) lies on line \ell_1
191 : P_{178988} = (43, 43, 42, 1) lies on line \ell_3
                                                                     242 : P_{249595} = (58, 58, 59, 1) lies on line \ell_3
                                                                     243 : P_{249921} = (0, 0, 60, 1) lies on line \ell_0
192 : P_{180289} = (0, 0, 43, 1) lies on line \ell_0
                                                                     244: P_{249922} = (1, 0, 60, 1) lies on line \ell_1
193: P_{180290} = (1, 0, 43, 1) lies on line \ell_1
194: P_{183019} = (42, 42, 43, 1) lies on line \ell_3
                                                                     245: P_{253886} = (61, 61, 60, 1) lies on line \ell_3
195: P_{184385} = (0, 0, 44, 1) lies on line \ell_0
                                                                     246: P_{254017} = (0, 0, 61, 1) lies on line \ell_0
196: P_{184386} = (1, 0, 44, 1) lies on line \ell_1
                                                                     247: P_{254018} = (1, 0, 61, 1) lies on line \ell_1
197: P_{187310} = (45, 45, 44, 1) lies on line \ell_3
                                                                     248: P_{257917} = (60, 60, 61, 1) lies on line \ell_3
198: P_{188481} = (0, 0, 45, 1) lies on line \ell_0
                                                                     249: P_{258113} = (0, 0, 62, 1) lies on line \ell_0
199 : P_{188482} = (1, 0, 45, 1) lies on line \ell_1
                                                                     250: P_{258114} = (1, 0, 62, 1) lies on line \ell_1
200: P_{191341} = (44, 44, 45, 1) lies on line \ell_3
                                                                     251: P_{262208} = (63, 63, 62, 1) lies on line \ell_3
201: P_{192577} = (0, 0, 46, 1) lies on line \ell_0
                                                                     252: P_{262209} = (0, 0, 63, 1) lies on line \ell_0
202: P_{192578} = (1, 0, 46, 1) lies on line \ell_1
                                                                     253: P_{262210} = (1, 0, 63, 1) lies on line \ell_1
203: P_{195632} = (47, 47, 46, 1) lies on line \ell_3
                                                                     254: P_{266239} = (62, 62, 63, 1) lies on line \ell_3
204: P_{196673} = (0, 0, 47, 1) lies on line \ell_0
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The single points on the surface are:

Points on surface but on no line

The surface has 3904 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c|c} 0123 \\ \hline 0 & 0111 \\ 1 & 1000 \\ 2 & 1001 \\ 3 & 1010 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_{8258}	P_{8258}

 ${\bf Line~1~intersects}$

Line	ℓ_0
in point	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{8258}	P_{8258}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{8258}	P_{8258}

The surface has 4161 points:

Too many to print.