

Rank-76323 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1503222105

General information

Number of lines	2
Number of points	25
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	10
Number of points off lines	15
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^2
Type of lines on points	$1^{10}, 0^{15}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1\end{aligned}$$

Rank of lines: (0, 356)

Rank of points on Klein quadric: (0, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 10 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

2 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1

3 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1

4 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0

5 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0

6 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0

7 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_1

8 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_1

9 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 15 points not on any line:

The points on the surface but not on lines are:

0 : $P_{13} = (2, 1, 1, 0)$

1 : $P_{14} = (3, 1, 1, 0)$

2 : $P_{27} = (1, 1, 0, 1)$

3 : $P_{33} = (3, 2, 0, 1)$

4 : $P_{36} = (2, 3, 0, 1)$

5 : $P_{39} = (1, 0, 1, 1)$

6 : $P_{42} = (0, 1, 1, 1)$

7 : $P_{56} = (3, 0, 2, 1)$

8 : $P_{63} = (2, 2, 2, 1)$

9 : $P_{64} = (3, 2, 2, 1)$

10 : $P_{65} = (0, 3, 2, 1)$

11 : $P_{71} = (2, 0, 3, 1)$

12 : $P_{77} = (0, 2, 3, 1)$

13 : $P_{83} = (2, 3, 3, 1)$

14 : $P_{84} = (3, 3, 3, 1)$

Line Intersection Graph

	0	1
0	0	0
1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 25 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_5 = (1, 1, 0, 0)$
 5 : $P_6 = (2, 1, 0, 0)$
 6 : $P_7 = (3, 1, 0, 0)$
 7 : $P_{13} = (2, 1, 1, 0)$
 8 : $P_{14} = (3, 1, 1, 0)$

9 : $P_{27} = (1, 1, 0, 1)$
 10 : $P_{33} = (3, 2, 0, 1)$
 11 : $P_{36} = (2, 3, 0, 1)$
 12 : $P_{38} = (0, 0, 1, 1)$
 13 : $P_{39} = (1, 0, 1, 1)$
 14 : $P_{42} = (0, 1, 1, 1)$
 15 : $P_{53} = (0, 0, 2, 1)$
 16 : $P_{56} = (3, 0, 2, 1)$
 17 : $P_{63} = (2, 2, 2, 1)$

18 : $P_{64} = (3, 2, 2, 1)$
 19 : $P_{65} = (0, 3, 2, 1)$
 20 : $P_{69} = (0, 0, 3, 1)$
 21 : $P_{71} = (2, 0, 3, 1)$
 22 : $P_{77} = (0, 2, 3, 1)$
 23 : $P_{83} = (2, 3, 3, 1)$
 24 : $P_{84} = (3, 3, 3, 1)$