

Rank-65614 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227396182

General information

Number of lines	7
Number of points	89
Number of singular points	1
Number of Eckardt points	0
Number of double points	6
Number of single points	47
Number of points off lines	35
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^7
Type of lines on points	$4, 2^6, 1^{47}, 0^{35}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \mathbf{Pl}(0, 6, 1, 0, 0, 0)_{15}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \mathbf{Pl}(0, 3, 1, 0, 0, 0)_{12} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \mathbf{Pl}(0, 5, 1, 0, 0, 0)_{14} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{657} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^5 \\ 0 & 1 & \gamma^5 & 1 \end{bmatrix}_{2055} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{2055} = \mathbf{Pl}(4, 3, 7, 5, 3, 1)_{2690} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^3 \\ 0 & 1 & \gamma^3 & 1 \end{bmatrix}_{3444} = \begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 5 & 1 \end{bmatrix}_{3444} = \mathbf{Pl}(7, 5, 2, 6, 5, 1)_{3505} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^6 \\ 0 & 1 & \gamma^6 & 1 \end{bmatrix}_{3664} = \begin{bmatrix} 1 & 0 & 2 & 6 \\ 0 & 1 & 6 & 1 \end{bmatrix}_{3664} = \mathbf{Pl}(2, 6, 4, 3, 6, 1)_{4109}
\end{aligned}$$

Rank of lines: (3568, 1816, 2984, 4681, 2055, 3444, 3664)

Rank of points on Klein quadric: (15, 12, 14, 657, 2690, 3505, 4109)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_{331} = (2, 0, 4, 1) = \ell_0 \cap \ell_6$$

$$P_{525} = (4, 0, 7, 1) = \ell_1 \cap \ell_4$$

$$P_{208} = (7, 0, 2, 1) = \ell_2 \cap \ell_5$$

$$P_{273} = (0, 1, 3, 1) = \ell_3 \cap \ell_4$$

$$P_{401} = (0, 1, 5, 1) = \ell_3 \cap \ell_5$$

$$P_{465} = (0, 1, 6, 1) = \ell_3 \cap \ell_6$$

Single Points

The surface has 47 single points:

The single points on the surface are:

$$0 : P_{36} = (1, 3, 1, 0) \text{ lies on line } \ell_4$$

$$1 : P_{52} = (1, 5, 1, 0) \text{ lies on line } \ell_5$$

$$2 : P_{60} = (1, 6, 1, 0) \text{ lies on line } \ell_6$$

$$3 : P_{76} = (2, 0, 0, 1) \text{ lies on line } \ell_0$$

$$4 : P_{78} = (4, 0, 0, 1) \text{ lies on line } \ell_1$$

$$5 : P_{81} = (7, 0, 0, 1) \text{ lies on line } \ell_2$$

$$6 : P_{82} = (0, 1, 0, 1) \text{ lies on line } \ell_3$$

$$7 : P_{96} = (6, 2, 0, 1) \text{ lies on line } \ell_6$$

$$8 : P_{109} = (3, 4, 0, 1) \text{ lies on line } \ell_4$$

$$9 : P_{135} = (5, 7, 0, 1) \text{ lies on line } \ell_5$$

$$10 : P_{140} = (2, 0, 1, 1) \text{ lies on line } \ell_0$$

$$11 : P_{142} = (4, 0, 1, 1) \text{ lies on line } \ell_1$$

$$12 : P_{145} = (7, 0, 1, 1) \text{ lies on line } \ell_2$$

$$13 : P_{146} = (0, 1, 1, 1) \text{ lies on line } \ell_3$$

$$14 : P_{157} = (4, 2, 1, 1) \text{ lies on line } \ell_5$$

$$15 : P_{176} = (7, 4, 1, 1) \text{ lies on line } \ell_6$$

$$16 : P_{195} = (2, 7, 1, 1) \text{ lies on line } \ell_4$$

$$17 : P_{203} = (2, 0, 2, 1) \text{ lies on line } \ell_0$$

$$18 : P_{205} = (4, 0, 2, 1) \text{ lies on line } \ell_1$$

$$19 : P_{209} = (0, 1, 2, 1) \text{ lies on line } \ell_3$$

$$20 : P_{218} = (1, 2, 2, 1) \text{ lies on line } \ell_4$$

$$21 : P_{229} = (4, 3, 2, 1) \text{ lies on line } \ell_6$$

$$22 : P_{267} = (2, 0, 3, 1) \text{ lies on line } \ell_0$$

$$23 : P_{269} = (4, 0, 3, 1) \text{ lies on line } \ell_1$$

$$24 : P_{272} = (7, 0, 3, 1) \text{ lies on line } \ell_2$$

$$25 : P_{310} = (5, 5, 3, 1) \text{ lies on line } \ell_6$$

26 : $P_{311} = (6, 5, 3, 1)$ lies on line ℓ_5
 27 : $P_{333} = (4, 0, 4, 1)$ lies on line ℓ_1
 28 : $P_{336} = (7, 0, 4, 1)$ lies on line ℓ_2
 29 : $P_{337} = (0, 1, 4, 1)$ lies on line ℓ_3
 30 : $P_{362} = (1, 4, 4, 1)$ lies on line ℓ_5
 31 : $P_{376} = (7, 5, 4, 1)$ lies on line ℓ_4
 32 : $P_{395} = (2, 0, 5, 1)$ lies on line ℓ_0
 33 : $P_{397} = (4, 0, 5, 1)$ lies on line ℓ_1
 34 : $P_{400} = (7, 0, 5, 1)$ lies on line ℓ_2
 35 : $P_{444} = (3, 6, 5, 1)$ lies on line ℓ_6
 36 : $P_{447} = (6, 6, 5, 1)$ lies on line ℓ_4

37 : $P_{459} = (2, 0, 6, 1)$ lies on line ℓ_0
 38 : $P_{461} = (4, 0, 6, 1)$ lies on line ℓ_1
 39 : $P_{464} = (7, 0, 6, 1)$ lies on line ℓ_2
 40 : $P_{484} = (3, 3, 6, 1)$ lies on line ℓ_5
 41 : $P_{486} = (5, 3, 6, 1)$ lies on line ℓ_4
 42 : $P_{523} = (2, 0, 7, 1)$ lies on line ℓ_0
 43 : $P_{528} = (7, 0, 7, 1)$ lies on line ℓ_2
 44 : $P_{529} = (0, 1, 7, 1)$ lies on line ℓ_3
 45 : $P_{571} = (2, 6, 7, 1)$ lies on line ℓ_5
 46 : $P_{578} = (1, 7, 7, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 35 points not on any line:

The points on the surface but not on lines are:

0 : $P_5 = (1, 1, 0, 0)$	18 : $P_{307} = (2, 5, 3, 1)$
1 : $P_{22} = (3, 1, 1, 0)$	19 : $P_{350} = (5, 2, 4, 1)$
2 : $P_{24} = (5, 1, 1, 0)$	20 : $P_{355} = (2, 3, 4, 1)$
3 : $P_{25} = (6, 1, 1, 0)$	21 : $P_{365} = (4, 4, 4, 1)$
4 : $P_{83} = (1, 1, 0, 1)$	22 : $P_{407} = (6, 1, 5, 1)$
5 : $P_{162} = (1, 3, 1, 1)$	23 : $P_{408} = (7, 1, 5, 1)$
6 : $P_{167} = (6, 3, 1, 1)$	24 : $P_{427} = (2, 4, 5, 1)$
7 : $P_{178} = (1, 5, 1, 1)$	25 : $P_{445} = (4, 6, 5, 1)$
8 : $P_{180} = (3, 5, 1, 1)$	26 : $P_{455} = (6, 7, 5, 1)$
9 : $P_{186} = (1, 6, 1, 1)$	27 : $P_{467} = (2, 1, 6, 1)$
10 : $P_{190} = (5, 6, 1, 1)$	28 : $P_{468} = (3, 1, 6, 1)$
11 : $P_{219} = (2, 2, 2, 1)$	29 : $P_{476} = (3, 2, 6, 1)$
12 : $P_{256} = (7, 6, 2, 1)$	30 : $P_{488} = (7, 3, 6, 1)$
13 : $P_{260} = (3, 7, 2, 1)$	31 : $P_{517} = (4, 7, 6, 1)$
14 : $P_{277} = (4, 1, 3, 1)$	32 : $P_{559} = (6, 4, 7, 1)$
15 : $P_{278} = (5, 1, 3, 1)$	33 : $P_{565} = (4, 5, 7, 1)$
16 : $P_{288} = (7, 2, 3, 1)$	34 : $P_{584} = (7, 7, 7, 1)$
17 : $P_{302} = (5, 4, 3, 1)$	

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	0	0	1
1	1	0	1	1	1	0	0
2	1	1	0	1	0	1	0
3	1	1	1	0	1	1	1
4	0	1	0	1	0	0	0
5	0	0	1	1	0	0	0
6	1	0	0	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6
in point	P_2	P_2	P_2	P_{331}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_2	P_2	P_2	P_{525}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{208}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_2	P_{273}	P_{401}	P_{465}

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_{525}	P_{273}

Line 5 intersects

Line	ℓ_2	ℓ_3
in point	P_{208}	P_{401}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_{331}	P_{465}

The surface has 89 points:

The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$	30 : $P_{205} = (4, 0, 2, 1)$	60 : $P_{400} = (7, 0, 5, 1)$
1 : $P_5 = (1, 1, 0, 0)$	31 : $P_{208} = (7, 0, 2, 1)$	61 : $P_{401} = (0, 1, 5, 1)$
2 : $P_{22} = (3, 1, 1, 0)$	32 : $P_{209} = (0, 1, 2, 1)$	62 : $P_{407} = (6, 1, 5, 1)$
3 : $P_{24} = (5, 1, 1, 0)$	33 : $P_{218} = (1, 2, 2, 1)$	63 : $P_{408} = (7, 1, 5, 1)$
4 : $P_{25} = (6, 1, 1, 0)$	34 : $P_{219} = (2, 2, 2, 1)$	64 : $P_{427} = (2, 4, 5, 1)$
5 : $P_{36} = (1, 3, 1, 0)$	35 : $P_{229} = (4, 3, 2, 1)$	65 : $P_{444} = (3, 6, 5, 1)$
6 : $P_{52} = (1, 5, 1, 0)$	36 : $P_{256} = (7, 6, 2, 1)$	66 : $P_{445} = (4, 6, 5, 1)$
7 : $P_{60} = (1, 6, 1, 0)$	37 : $P_{260} = (3, 7, 2, 1)$	67 : $P_{447} = (6, 6, 5, 1)$
8 : $P_{76} = (2, 0, 0, 1)$	38 : $P_{267} = (2, 0, 3, 1)$	68 : $P_{455} = (6, 7, 5, 1)$
9 : $P_{78} = (4, 0, 0, 1)$	39 : $P_{269} = (4, 0, 3, 1)$	69 : $P_{459} = (2, 0, 6, 1)$
10 : $P_{81} = (7, 0, 0, 1)$	40 : $P_{272} = (7, 0, 3, 1)$	70 : $P_{461} = (4, 0, 6, 1)$
11 : $P_{82} = (0, 1, 0, 1)$	41 : $P_{273} = (0, 1, 3, 1)$	71 : $P_{464} = (7, 0, 6, 1)$
12 : $P_{83} = (1, 1, 0, 1)$	42 : $P_{277} = (4, 1, 3, 1)$	72 : $P_{465} = (0, 1, 6, 1)$
13 : $P_{96} = (6, 2, 0, 1)$	43 : $P_{278} = (5, 1, 3, 1)$	73 : $P_{467} = (2, 1, 6, 1)$
14 : $P_{109} = (3, 4, 0, 1)$	44 : $P_{288} = (7, 2, 3, 1)$	74 : $P_{468} = (3, 1, 6, 1)$
15 : $P_{135} = (5, 7, 0, 1)$	45 : $P_{302} = (5, 4, 3, 1)$	75 : $P_{476} = (3, 2, 6, 1)$
16 : $P_{140} = (2, 0, 1, 1)$	46 : $P_{307} = (2, 5, 3, 1)$	76 : $P_{484} = (3, 3, 6, 1)$
17 : $P_{142} = (4, 0, 1, 1)$	47 : $P_{310} = (5, 5, 3, 1)$	77 : $P_{486} = (5, 3, 6, 1)$
18 : $P_{145} = (7, 0, 1, 1)$	48 : $P_{311} = (6, 5, 3, 1)$	78 : $P_{488} = (7, 3, 6, 1)$
19 : $P_{146} = (0, 1, 1, 1)$	49 : $P_{331} = (2, 0, 4, 1)$	79 : $P_{517} = (4, 7, 6, 1)$
20 : $P_{157} = (4, 2, 1, 1)$	50 : $P_{333} = (4, 0, 4, 1)$	80 : $P_{523} = (2, 0, 7, 1)$
21 : $P_{162} = (1, 3, 1, 1)$	51 : $P_{336} = (7, 0, 4, 1)$	81 : $P_{525} = (4, 0, 7, 1)$
22 : $P_{167} = (6, 3, 1, 1)$	52 : $P_{337} = (0, 1, 4, 1)$	82 : $P_{528} = (7, 0, 7, 1)$
23 : $P_{176} = (7, 4, 1, 1)$	53 : $P_{350} = (5, 2, 4, 1)$	83 : $P_{529} = (0, 1, 7, 1)$
24 : $P_{178} = (1, 5, 1, 1)$	54 : $P_{355} = (2, 3, 4, 1)$	84 : $P_{559} = (6, 4, 7, 1)$
25 : $P_{180} = (3, 5, 1, 1)$	55 : $P_{362} = (1, 4, 4, 1)$	85 : $P_{565} = (4, 5, 7, 1)$
26 : $P_{186} = (1, 6, 1, 1)$	56 : $P_{365} = (4, 4, 4, 1)$	86 : $P_{571} = (2, 6, 7, 1)$
27 : $P_{190} = (5, 6, 1, 1)$	57 : $P_{376} = (7, 5, 4, 1)$	87 : $P_{578} = (1, 7, 7, 1)$
28 : $P_{195} = (2, 7, 1, 1)$	58 : $P_{395} = (2, 0, 5, 1)$	88 : $P_{584} = (7, 7, 7, 1)$
29 : $P_{203} = (2, 0, 2, 1)$	59 : $P_{397} = (4, 0, 5, 1)$	