Rank-24 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 = 0$$

(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 36

General information

Number of lines	9
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	72
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^9
Type of lines on points	$9, 1^{72}$

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \mathbf{Pl}(0,0,0,1,1,0)_{201}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \mathbf{Pl}(0,1,0,0,1,0)_{89}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_3 = \begin{bmatrix} 1 & \gamma^5 & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4379} = \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4379} = \mathbf{Pl}(0,7,0,3,1,0)_{245}$$

$$\ell_4 = \begin{bmatrix} 1 & \gamma & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3138} = \begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3138} = \mathbf{Pl}(0,5,0,2,1,0)_{228}$$

$$\ell_5 = \begin{bmatrix} 1 & \gamma^3 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1605} = \begin{bmatrix} 1 & 5 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1605} = \mathbf{Pl}(0,2,0,5,1,0)_{270}$$

$$\ell_6 = \begin{bmatrix} 1 & \gamma^2 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3868} = \begin{bmatrix} 1 & 4 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3868} = \mathbf{Pl}(0,6,0,4,1,0)_{259}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^4 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2335} = \begin{bmatrix} 1 & 7 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2335} = \mathbf{Pl}(0,3,0,7,1,0)_{301}$$

$$\ell_8 = \begin{bmatrix} 1 & \gamma^6 & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2846} = \begin{bmatrix} 1 & 6 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2846} = \mathbf{Pl}(0,4,0,6,1,0)_{287}$$

Rank of lines: (145, 656, 4689, 4379, 3138, 1605, 3868, 2335, 2846)
Rank of points on Klein quadric: (201, 89, 25, 245, 228, 270, 259, 301, 287)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 72 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0

1 : $P_{12} = (1,0,1,0)$ lies on line ℓ_1

2 : $P_{19} = (0,1,1,0)$ lies on line ℓ_2

3 : $P_{32} = (5,2,1,0)$ lies on line ℓ_3

4 : $P_{42} = (7,3,1,0)$ lies on line ℓ_4

5 : $P_{49} = (6,4,1,0)$ lies on line ℓ_5

6 : $P_{53} = (2,5,1,0)$ lies on line ℓ_6

7 : $P_{63} = (4,6,1,0)$ lies on line ℓ_7

8 : $P_{70} = (3,7,1,0)$ lies on line ℓ_8

9 : $P_{83} = (1,1,0,1)$ lies on line ℓ_0

10 : $P_{92} = (2,2,0,1)$ lies on line ℓ_0

11 : $P_{101} = (3,3,0,1)$ lies on line ℓ_0

12 : $P_{110} = (4,4,0,1)$ lies on line ℓ_0

13 : $P_{119} = (5,5,0,1)$ lies on line ℓ_0

14 : $P_{128} = (6,6,0,1)$ lies on line ℓ_0

15 : $P_{137} = (7,7,0,1)$ lies on line ℓ_0

16 : $P_{139} = (1,0,1,1)$ lies on line ℓ_1

17 : $P_{146} = (0,1,1,1)$ lies on line ℓ_2

18 : $P_{158} = (5,2,1,1)$ lies on line ℓ_3

19 : $P_{168} = (7,3,1,1)$ lies on line ℓ_4

20 : $P_{175} = (6,4,1,1)$ lies on line ℓ_5

21 : $P_{179} = (2,5,1,1)$ lies on line ℓ_6

22 : $P_{189} = (4,6,1,1)$ lies on line ℓ_7

23 : $P_{196} = (3,7,1,1)$ lies on line ℓ_8

24 : $P_{203} = (2,0,2,1)$ lies on line ℓ_1

25 : $P_{214} = (5,1,2,1)$ lies on line ℓ_7

26 : $P_{217} = (0,2,2,1)$ lies on line ℓ_2

27 : $P_{231} = (6,3,2,1)$ lies on line ℓ_8

28 : $P_{240} = (7,4,2,1)$ lies on line ℓ_3
29 : $P_{242} = (1,5,2,1)$ lies on line ℓ_5
30 : $P_{252} = (3,6,2,1)$ lies on line ℓ_4
31 : $P_{261} = (4,7,2,1)$ lies on line ℓ_6
32 : $P_{268} = (3,0,3,1)$ lies on line ℓ_1
33 : $P_{280} = (7,1,3,1)$ lies on line ℓ_5
34 : $P_{287} = (6,2,3,1)$ lies on line ℓ_6
35 : $P_{289} = (0,3,3,1)$ lies on line ℓ_2
36 : $P_{302} = (5,4,3,1)$ lies on line ℓ_8
37 : $P_{309} = (4,5,3,1)$ lies on line ℓ_4
38 : $P_{315} = (2,6,3,1)$ lies on line ℓ_3
39 : $P_{322} = (1,7,3,1)$ lies on line ℓ_7
40 : $P_{333} = (4,0,4,1)$ lies on line ℓ_1
41 : $P_{343} = (6,1,4,1)$ lies on line ℓ_4
42 : $P_{352} = (7,2,4,1)$ lies on line ℓ_7
43 : $P_{358} = (5,3,4,1)$ lies on line ℓ_6
44 : $P_{361} = (0,4,4,1)$ lies on line ℓ_2
45 : $P_{372} = (3,5,4,1)$ lies on line ℓ_3
46 : $P_{378} = (1,6,4,1)$ lies on line ℓ_8
47 : $P_{387} = (2,7,4,1)$ lies on line ℓ_5
48 : $P_{398} = (5,0,5,1)$ lies on line ℓ_1
49 : $P_{403} = (2,1,5,1)$ lies on line ℓ_8
50 : $P_{410} = (1,2,5,1)$ lies on line ℓ_4

51 : $P_{421} = (4,3,5,1)$ lies on line ℓ_5
52 : $P_{428} = (3,4,5,1)$ lies on line ℓ_7
53 : $P_{433} = (0,5,5,1)$ lies on line ℓ_2
54 : $P_{448} = (7,6,5,1)$ lies on line ℓ_6
55 : $P_{455} = (6,7,5,1)$ lies on line ℓ_3
56 : $P_{463} = (6,0,6,1)$ lies on line ℓ_1
57 : $P_{469} = (4,1,6,1)$ lies on line ℓ_3
58 : $P_{476} = (3,2,6,1)$ lies on line ℓ_5
59 : $P_{483} = (2,3,6,1)$ lies on line ℓ_7
60 : $P_{490} = (1,4,6,1)$ lies on line ℓ_6
61 : $P_{504} = (7,5,6,1)$ lies on line ℓ_8
62 : $P_{505} = (0,6,6,1)$ lies on line ℓ_2
63 : $P_{518} = (5,7,6,1)$ lies on line ℓ_4
64 : $P_{528} = (7,0,7,1)$ lies on line ℓ_1
65 : $P_{532} = (3,1,7,1)$ lies on line ℓ_6
66 : $P_{541} = (4,2,7,1)$ lies on line ℓ_8
67 : $P_{546} = (1,3,7,1)$ lies on line ℓ_3
68 : $P_{555} = (2,4,7,1)$ lies on line ℓ_4
69 : $P_{567} = (6,5,7,1)$ lies on line ℓ_7
70 : $P_{574} = (5,6,7,1)$ lies on line ℓ_5
71 : $P_{577} = (0,7,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
 |0 1 2 3 4 5 6 7 8
0|0 1 1 1 1 1 1 1 1
1|1 0 1 1 1 1 1 1 1
2|1 1 0 1 1 1 1 1 1
3|1 1 1 0 1 1 1 1 1
4|1 1 1 1 0 1 1 1 1
5|1 1 1 1 1 0 1 1 1
6|1 1 1 1 1 1 0 1 1
7|1 1 1 1 1 1 1 0 1
8|1 1 1 1 1 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 73 points:
The points on the surface are:

$0: P_3 = (0,0,0,1)$
$1: P_5 = (1,1,0,0)$
$2: P_{12} = (1,0,1,0)$
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