

Rank-46 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(8) is 1186

General information

Number of lines	3
Number of points	73
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$3, 1^{24}, 0^{48}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{503} = \left[\begin{array}{cccc} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{503} = \mathbf{Pl}(0, 0, 1, 1, 2, 1)_{1826} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{284} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{284} = \mathbf{Pl}(0, 0, 1, 1, 4, 1)_{2834}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{430} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{430} = \mathbf{PI}(0, 0, 1, 1, 7, 1)_{4346}$$

Rank of lines: (503, 284, 430)

Rank of points on Klein quadric: (1826, 2834, 4346)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_{138} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 24 single points:

The single points on the surface are:

- | | |
|---|---|
| 0 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 | 13 : $P_{175} = (6, 4, 1, 1)$ lies on line ℓ_2 |
| 1 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_1 | 14 : $P_{176} = (7, 4, 1, 1)$ lies on line ℓ_1 |
| 2 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_2 | 15 : $P_{178} = (1, 5, 1, 1)$ lies on line ℓ_2 |
| 3 : $P_{147} = (2, 1, 1, 1)$ lies on line ℓ_0 | 16 : $P_{180} = (3, 5, 1, 1)$ lies on line ℓ_1 |
| 4 : $P_{149} = (4, 1, 1, 1)$ lies on line ℓ_1 | 17 : $P_{184} = (7, 5, 1, 1)$ lies on line ℓ_0 |
| 5 : $P_{152} = (7, 1, 1, 1)$ lies on line ℓ_2 | 18 : $P_{186} = (1, 6, 1, 1)$ lies on line ℓ_0 |
| 6 : $P_{156} = (3, 2, 1, 1)$ lies on line ℓ_2 | 19 : $P_{187} = (2, 6, 1, 1)$ lies on line ℓ_1 |
| 7 : $P_{157} = (4, 2, 1, 1)$ lies on line ℓ_0 | 20 : $P_{190} = (5, 6, 1, 1)$ lies on line ℓ_2 |
| 8 : $P_{158} = (5, 2, 1, 1)$ lies on line ℓ_1 | 21 : $P_{195} = (2, 7, 1, 1)$ lies on line ℓ_2 |
| 9 : $P_{162} = (1, 3, 1, 1)$ lies on line ℓ_1 | 22 : $P_{196} = (3, 7, 1, 1)$ lies on line ℓ_0 |
| 10 : $P_{165} = (4, 3, 1, 1)$ lies on line ℓ_2 | 23 : $P_{199} = (6, 7, 1, 1)$ lies on line ℓ_1 |
| 11 : $P_{167} = (6, 3, 1, 1)$ lies on line ℓ_0 | |
| 12 : $P_{174} = (5, 4, 1, 1)$ lies on line ℓ_0 | |

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

- | | |
|-----------------------------|-------------------------------|
| 0 : $P_{12} = (1, 0, 1, 0)$ | 6 : $P_{75} = (1, 0, 0, 1)$ |
| 1 : $P_{19} = (0, 1, 1, 0)$ | 7 : $P_{82} = (0, 1, 0, 1)$ |
| 2 : $P_{20} = (1, 1, 1, 0)$ | 8 : $P_{83} = (1, 1, 0, 1)$ |
| 3 : $P_{40} = (5, 3, 1, 0)$ | 9 : $P_{103} = (5, 3, 0, 1)$ |
| 4 : $P_{57} = (6, 5, 1, 0)$ | 10 : $P_{120} = (6, 5, 0, 1)$ |
| 5 : $P_{62} = (3, 6, 1, 0)$ | 11 : $P_{125} = (3, 6, 0, 1)$ |

12 : $P_{206} = (5, 0, 2, 1)$	31 : $P_{407} = (6, 1, 5, 1)$
13 : $P_{223} = (6, 2, 2, 1)$	32 : $P_{409} = (0, 2, 5, 1)$
14 : $P_{235} = (2, 4, 2, 1)$	33 : $P_{411} = (2, 2, 5, 1)$
15 : $P_{241} = (0, 5, 2, 1)$	34 : $P_{448} = (7, 6, 5, 1)$
16 : $P_{246} = (5, 5, 2, 1)$	35 : $P_{450} = (1, 7, 5, 1)$
17 : $P_{253} = (4, 6, 2, 1)$	36 : $P_{461} = (4, 0, 6, 1)$
18 : $P_{272} = (7, 0, 3, 1)$	37 : $P_{468} = (3, 1, 6, 1)$
19 : $P_{278} = (5, 1, 3, 1)$	38 : $P_{474} = (1, 2, 6, 1)$
20 : $P_{298} = (1, 4, 3, 1)$	39 : $P_{483} = (2, 3, 6, 1)$
21 : $P_{309} = (4, 5, 3, 1)$	40 : $P_{489} = (0, 4, 6, 1)$
22 : $P_{321} = (0, 7, 3, 1)$	41 : $P_{493} = (4, 4, 6, 1)$
23 : $P_{328} = (7, 7, 3, 1)$	42 : $P_{524} = (3, 0, 7, 1)$
24 : $P_{335} = (6, 0, 4, 1)$	43 : $P_{544} = (7, 2, 7, 1)$
25 : $P_{360} = (7, 3, 4, 1)$	44 : $P_{545} = (0, 3, 7, 1)$
26 : $P_{364} = (3, 4, 4, 1)$	45 : $P_{548} = (3, 3, 7, 1)$
27 : $P_{377} = (0, 6, 4, 1)$	46 : $P_{563} = (2, 5, 7, 1)$
28 : $P_{383} = (6, 6, 4, 1)$	47 : $P_{582} = (5, 7, 7, 1)$
29 : $P_{389} = (4, 7, 4, 1)$	
30 : $P_{395} = (2, 0, 5, 1)$	

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{138}	P_{138}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{138}	P_{138}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{138}	P_{138}

The surface has 73 points:

The points on the surface are:

0 : $P_6 = (2, 1, 0, 0)$	13 : $P_{120} = (6, 5, 0, 1)$	26 : $P_{175} = (6, 4, 1, 1)$
1 : $P_8 = (4, 1, 0, 0)$	14 : $P_{125} = (3, 6, 0, 1)$	27 : $P_{176} = (7, 4, 1, 1)$
2 : $P_{11} = (7, 1, 0, 0)$	15 : $P_{138} = (0, 0, 1, 1)$	28 : $P_{178} = (1, 5, 1, 1)$
3 : $P_{12} = (1, 0, 1, 0)$	16 : $P_{147} = (2, 1, 1, 1)$	29 : $P_{180} = (3, 5, 1, 1)$
4 : $P_{19} = (0, 1, 1, 0)$	17 : $P_{149} = (4, 1, 1, 1)$	30 : $P_{184} = (7, 5, 1, 1)$
5 : $P_{20} = (1, 1, 1, 0)$	18 : $P_{152} = (7, 1, 1, 1)$	31 : $P_{186} = (1, 6, 1, 1)$
6 : $P_{40} = (5, 3, 1, 0)$	19 : $P_{156} = (3, 2, 1, 1)$	32 : $P_{187} = (2, 6, 1, 1)$
7 : $P_{57} = (6, 5, 1, 0)$	20 : $P_{157} = (4, 2, 1, 1)$	33 : $P_{190} = (5, 6, 1, 1)$
8 : $P_{62} = (3, 6, 1, 0)$	21 : $P_{158} = (5, 2, 1, 1)$	34 : $P_{195} = (2, 7, 1, 1)$
9 : $P_{75} = (1, 0, 0, 1)$	22 : $P_{162} = (1, 3, 1, 1)$	35 : $P_{196} = (3, 7, 1, 1)$
10 : $P_{82} = (0, 1, 0, 1)$	23 : $P_{165} = (4, 3, 1, 1)$	36 : $P_{199} = (6, 7, 1, 1)$
11 : $P_{83} = (1, 1, 0, 1)$	24 : $P_{167} = (6, 3, 1, 1)$	37 : $P_{206} = (5, 0, 2, 1)$
12 : $P_{103} = (5, 3, 0, 1)$	25 : $P_{174} = (5, 4, 1, 1)$	38 : $P_{223} = (6, 2, 2, 1)$

39 : $P_{235} = (2, 4, 2, 1)$
 40 : $P_{241} = (0, 5, 2, 1)$
 41 : $P_{246} = (5, 5, 2, 1)$
 42 : $P_{253} = (4, 6, 2, 1)$
 43 : $P_{272} = (7, 0, 3, 1)$
 44 : $P_{278} = (5, 1, 3, 1)$
 45 : $P_{298} = (1, 4, 3, 1)$
 46 : $P_{309} = (4, 5, 3, 1)$
 47 : $P_{321} = (0, 7, 3, 1)$
 48 : $P_{328} = (7, 7, 3, 1)$
 49 : $P_{335} = (6, 0, 4, 1)$
 50 : $P_{360} = (7, 3, 4, 1)$

51 : $P_{364} = (3, 4, 4, 1)$
 52 : $P_{377} = (0, 6, 4, 1)$
 53 : $P_{383} = (6, 6, 4, 1)$
 54 : $P_{389} = (4, 7, 4, 1)$
 55 : $P_{395} = (2, 0, 5, 1)$
 56 : $P_{407} = (6, 1, 5, 1)$
 57 : $P_{409} = (0, 2, 5, 1)$
 58 : $P_{411} = (2, 2, 5, 1)$
 59 : $P_{448} = (7, 6, 5, 1)$
 60 : $P_{450} = (1, 7, 5, 1)$
 61 : $P_{461} = (4, 0, 6, 1)$
 62 : $P_{468} = (3, 1, 6, 1)$

63 : $P_{474} = (1, 2, 6, 1)$
 64 : $P_{483} = (2, 3, 6, 1)$
 65 : $P_{489} = (0, 4, 6, 1)$
 66 : $P_{493} = (4, 4, 6, 1)$
 67 : $P_{524} = (3, 0, 7, 1)$
 68 : $P_{544} = (7, 2, 7, 1)$
 69 : $P_{545} = (0, 3, 7, 1)$
 70 : $P_{548} = (3, 3, 7, 1)$
 71 : $P_{563} = (2, 5, 7, 1)$
 72 : $P_{582} = (5, 7, 7, 1)$