# Rank-66764 over GF(64)

January 15, 2021

## The equation

The equation of the surface is:

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0 ) The point rank of the equation over  $\mathrm{GF}(64)$  is 1091047494

#### General information

Number of lines	7
Number of points	4289
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	441
Number of points off lines	3843
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 <sup>7</sup>
Type of lines on points	$4^2, 2^3, 1^{441}, 0^{3843}$

## Singular Points

The surface has 2 singular points:

$$\begin{array}{l} 0: \ P_{3651} = \mathbf{P}(0, \epsilon^{42}, 1, 0) = \mathbf{P}(0, 56, 1, 0) \\ 1: \ P_{3715} = \mathbf{P}(0, \epsilon^{21}, 1, 0) = \mathbf{P}(0, 57, 1, 0) \end{array}$$

### The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{270401}$$

$$\ell_{1} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{9} \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{12516345} = \begin{bmatrix} 1 & 0 & 0 & 47 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{12516345} = \mathbf{Pl}(56, 57, 62, 1, 0, 0)_{4154}$$

$$\ell_{2} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{18} \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{2929401} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{2929401} = \mathbf{Pl}(56, 57, 8, 1, 0, 0)_{752}$$

$$\ell_{3} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{36} \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{9587001} = \begin{bmatrix} 1 & 0 & 0 & 36 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{9587001} = \mathbf{Pl}(56, 57, 15, 1, 0, 0)_{1193}$$

$$\ell_{4} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{9} \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{12516344} = \begin{bmatrix} 1 & 0 & 0 & 47 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{12516344} = \mathbf{Pl}(57, 56, 52, 1, 0, 0)_{3525}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{18} \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{2929400} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{2929400} = \mathbf{Pl}(57, 56, 45, 1, 0, 0)_{3084}$$

$$\ell_{6} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{36} \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{9587000} = \begin{bmatrix} 1 & 0 & 0 & 36 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{9587000} = \mathbf{Pl}(57, 56, 33, 1, 0, 0)_{2328}$$

Rank of lines: ( 17043456, 12516345, 2929401, 9587001, 12516344, 2929400, 9587000) Rank of points on Klein quadric: ( 270401, 4154, 752, 1193, 3525, 3084, 2328)

#### **Eckardt Points**

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 3 Double points: The double points on the surface are:

$$P_{4172} = (10, 0, 0, 1) = \ell_1 \cap \ell_4$$
  

$$P_{4199} = (37, 0, 0, 1) = \ell_2 \cap \ell_5$$

 $P_{4208} = (46, 0, 0, 1) = \ell_3 \cap \ell_6$ 

#### Single Points

The surface has 441 single points: The single points on the surface are:

 $0: P_1 = (0, 1, 0, 0)$  lies on line  $\ell_0$ 15:  $P_{963} = (0, 14, 1, 0)$  lies on line  $\ell_0$ 1:  $P_2 = (0, 0, 1, 0)$  lies on line  $\ell_0$ 16:  $P_{1027} = (0, 15, 1, 0)$  lies on line  $\ell_0$ 2:  $P_{131} = (0, 1, 1, 0)$  lies on line  $\ell_0$ 17:  $P_{1091} = (0, 16, 1, 0)$  lies on line  $\ell_0$ 3 :  $P_{195} = (0,2,1,0)$  lies on line  $\ell_0$ 18:  $P_{1155} = (0, 17, 1, 0)$  lies on line  $\ell_0$ 4:  $P_{259} = (0, 3, 1, 0)$  lies on line  $\ell_0$ 19:  $P_{1219} = (0, 18, 1, 0)$  lies on line  $\ell_0$ 5:  $P_{323} = (0, 4, 1, 0)$  lies on line  $\ell_0$ 20:  $P_{1283} = (0, 19, 1, 0)$  lies on line  $\ell_0$ 6:  $P_{387} = (0, 5, 1, 0)$  lies on line  $\ell_0$ 21:  $P_{1347} = (0, 20, 1, 0)$  lies on line  $\ell_0$ 22:  $P_{1411} = (0, 21, 1, 0)$  lies on line  $\ell_0$ 7:  $P_{451} = (0, 6, 1, 0)$  lies on line  $\ell_0$ 8:  $P_{515} = (0, 7, 1, 0)$  lies on line  $\ell_0$ 23:  $P_{1475} = (0, 22, 1, 0)$  lies on line  $\ell_0$ 9:  $P_{579} = (0, 8, 1, 0)$  lies on line  $\ell_0$ 24:  $P_{1539} = (0, 23, 1, 0)$  lies on line  $\ell_0$ 10:  $P_{643} = (0, 9, 1, 0)$  lies on line  $\ell_0$ 25:  $P_{1603} = (0, 24, 1, 0)$  lies on line  $\ell_0$ 11:  $P_{707} = (0, 10, 1, 0)$  lies on line  $\ell_0$ 26:  $P_{1667} = (0, 25, 1, 0)$  lies on line  $\ell_0$ 12:  $P_{771} = (0, 11, 1, 0)$  lies on line  $\ell_0$ 27:  $P_{1731} = (0, 26, 1, 0)$  lies on line  $\ell_0$ 13:  $P_{835} = (0, 12, 1, 0)$  lies on line  $\ell_0$ 28:  $P_{1795} = (0, 27, 1, 0)$  lies on line  $\ell_0$ 14:  $P_{899} = (0, 13, 1, 0)$  lies on line  $\ell_0$ 29:  $P_{1859} = (0, 28, 1, 0)$  lies on line  $\ell_0$ 

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84: P_{22987} = (10, 38, 4, 1) lies on line \ell_4
30: P_{1923} = (0, 29, 1, 0) lies on line \ell_0
                                                                     85: P_{23014} = (37, 38, 4, 1) lies on line \ell_5
31: P_{1987} = (0, 30, 1, 0) lies on line \ell_0
                                                                     86: P_{23023} = (46, 38, 4, 1) lies on line \ell_6
32: P_{2051} = (0, 31, 1, 0) lies on line \ell_0
33: P_{2115} = (0, 32, 1, 0) lies on line \ell_0
                                                                     87: P_{26315} = (10, 26, 5, 1) lies on line \ell_1
34: P_{2179} = (0, 33, 1, 0) lies on line \ell_0
                                                                     88: P_{26342} = (37, 26, 5, 1) lies on line \ell_2
                                                                     89: P_{26351} = (46, 26, 5, 1) lies on line \ell_3
35: P_{2243} = (0, 34, 1, 0) lies on line \ell_0
36: P_{2307} = (0, 35, 1, 0) lies on line \ell_0
                                                                     90: P_{26635} = (10, 31, 5, 1) lies on line \ell_4
37: P_{2371} = (0, 36, 1, 0) lies on line \ell_0
                                                                     91: P_{26662} = (37, 31, 5, 1) lies on line \ell_5
38: P_{2435} = (0, 37, 1, 0) lies on line \ell_0
                                                                     92: P_{26671} = (46, 31, 5, 1) lies on line \ell_6
39: P_{2499} = (0, 38, 1, 0) lies on line \ell_0
                                                                     93: P_{32011} = (10, 51, 6, 1) lies on line \ell_1
40: P_{2563} = (0, 39, 1, 0) lies on line \ell_0
                                                                     94: P_{32038} = (37, 51, 6, 1) lies on line \ell_2
41: P_{2627} = (0, 40, 1, 0) lies on line \ell_0
                                                                     95: P_{32047} = (46, 51, 6, 1) lies on line \ell_3
42: P_{2691} = (0, 41, 1, 0) lies on line \ell_0
                                                                     96: P_{32139} = (10, 53, 6, 1) lies on line \ell_4
43: P_{2755} = (0, 42, 1, 0) lies on line \ell_0
                                                                     97: P_{32166} = (37, 53, 6, 1) lies on line \ell_5
44: P_{2819} = (0, 43, 1, 0) lies on line \ell_0
                                                                     98: P_{32175} = (46, 53, 6, 1) lies on line \ell_6
45: P_{2883} = (0, 44, 1, 0) lies on line \ell_0
                                                                     99: P_{33547} = (10, 11, 7, 1) lies on line \ell_1
46: P_{2947} = (0, 45, 1, 0) lies on line \ell_0
                                                                     100: P_{33574} = (37, 11, 7, 1) lies on line \ell_2
47: P_{3011} = (0, 46, 1, 0) lies on line \ell_0
                                                                     101: P_{33583} = (46, 11, 7, 1) lies on line \ell_3
48: P_{3075} = (0, 47, 1, 0) lies on line \ell_0
                                                                     102: P_{33611} = (10, 12, 7, 1) lies on line \ell_4
49: P_{3139} = (0, 48, 1, 0) lies on line \ell_0
                                                                     103: P_{33638} = (37, 12, 7, 1) lies on line \ell_5
50: P_{3203} = (0, 49, 1, 0) lies on line \ell_0
                                                                     104: P_{33647} = (46, 12, 7, 1) lies on line \ell_6
51: P_{3267} = (0, 50, 1, 0) lies on line \ell_0
                                                                     105: P_{39307} = (10, 37, 8, 1) lies on line \ell_1
                                                                     106: P_{39334} = (37, 37, 8, 1) lies on line \ell_2
52: P_{3331} = (0, 51, 1, 0) lies on line \ell_0
                                                                     107 : P_{39343} = (46, 37, 8, 1) lies on line \ell_3
53: P_{3395} = (0, 52, 1, 0) lies on line \ell_0
54: P_{3459} = (0, 53, 1, 0) lies on line \ell_0
                                                                     108: P_{39819} = (10, 45, 8, 1) lies on line \ell_4
55: P_{3523} = (0, 54, 1, 0) lies on line \ell_0
                                                                     109: P_{39846} = (37, 45, 8, 1) lies on line \ell_5
56: P_{3587} = (0, 55, 1, 0) lies on line \ell_0
                                                                     110: P_{39855} = (46, 45, 8, 1) lies on line \ell_6
57: P_{3779} = (0, 58, 1, 0) lies on line \ell_0
                                                                     111: P_{42315} = (10, 20, 9, 1) lies on line \ell_4
                                                                     112: P_{42342} = (37, 20, 9, 1) lies on line \ell_5
58: P_{3843} = (0, 59, 1, 0) lies on line \ell_0
59: P_{3907} = (0, 60, 1, 0) lies on line \ell_0
                                                                     113: P_{42351} = (46, 20, 9, 1) lies on line \ell_6
60: P_{3971} = (0, 61, 1, 0) lies on line \ell_0
                                                                     114: P_{42891} = (10, 29, 9, 1) lies on line \ell_1
61: P_{4035} = (0, 62, 1, 0) lies on line \ell_0
                                                                     115: P_{42918} = (37, 29, 9, 1) lies on line \ell_2
62: P_{4099} = (0, 63, 1, 0) lies on line \ell_0
                                                                     116: P_{42927} = (46, 29, 9, 1) lies on line \ell_3
                                                                     117: P_{48459} = (10, 52, 10, 1) lies on line \ell_1
63: P_{11851} = (10, 56, 1, 1) lies on line \ell_1
64: P_{11878} = (37, 56, 1, 1) lies on line \ell_2
                                                                     118: P_{48486} = (37, 52, 10, 1) lies on line \ell_2
                                                                     119: P_{48495} = (46, 52, 10, 1) lies on line \ell_3
65: P_{11887} = (46, 56, 1, 1) lies on line \ell_3
66: P_{11915} = (10, 57, 1, 1) lies on line \ell_4
                                                                     120: P_{49099} = (10, 62, 10, 1) lies on line \ell_4
67: P_{11942} = (37, 57, 1, 1) lies on line \ell_5
                                                                     121: P_{49126} = (37, 62, 10, 1) lies on line \ell_5
68: P_{11951} = (46, 57, 1, 1) lies on line \ell_6
                                                                     122: P_{49135} = (46, 62, 10, 1) lies on line \ell_6
69: P_{13451} = (10, 17, 2, 1) lies on line \ell_1
                                                                     123: P_{49675} = (10, 7, 11, 1) lies on line \ell_4
70 : P_{13478} = (37, 17, 2, 1) lies on line \ell_2
                                                                     124: P_{49702} = (37, 7, 11, 1) lies on line \ell_5
71: P_{13487} = (46, 17, 2, 1) lies on line \ell_3
                                                                     125: P_{49711} = (46, 7, 11, 1) lies on line \ell_6
                                                                     126 : P_{49995} = (10, 12, 11, 1) lies on line \ell_1
72: P_{13579} = (10, 19, 2, 1) lies on line \ell_4
73: P_{13606} = (37, 19, 2, 1) lies on line \ell_5
                                                                     127: P_{50022} = (37, 12, 11, 1) lies on line \ell_2
74: P_{13615} = (46, 19, 2, 1) lies on line \ell_6
                                                                     128: P_{50031} = (46, 12, 11, 1) lies on line \ell_3
75: P_{19083} = (10, 41, 3, 1) lies on line \ell_1
                                                                     129: P_{53771} = (10, 7, 12, 1) lies on line \ell_1
76: P_{19110} = (37, 41, 3, 1) lies on line \ell_2
                                                                     130: P_{53798} = (37, 7, 12, 1) lies on line \ell_2
77: P_{19119} = (46, 41, 3, 1) lies on line \ell_3
                                                                     131: P_{53807} = (46, 7, 12, 1) lies on line \ell_3
78: P_{19147} = (10, 42, 3, 1) lies on line \ell_4
                                                                     132: P_{54027} = (10, 11, 12, 1) lies on line \ell_4
79: P_{19174} = (37, 42, 3, 1) lies on line \ell_5
                                                                     133: P_{54054} = (37, 11, 12, 1) lies on line \ell_5
80: P_{19183} = (46, 42, 3, 1) lies on line \ell_6
                                                                     134: P_{54063} = (46, 11, 12, 1) lies on line \ell_6
81: P_{22731} = (10, 34, 4, 1) lies on line \ell_1
                                                                     135: P_{60619} = (10, 50, 13, 1) lies on line \ell_4
82 : P_{22758} = (37, 34, 4, 1) lies on line \ell_2
                                                                     136 : P_{60646} = (37, 50, 13, 1) lies on line \ell_5
83: P_{22767} = (46, 34, 4, 1) lies on line \ell_3
                                                                     137: P_{60655} = (46, 50, 13, 1) lies on line \ell_6
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138 : P_{61451} = (10, 63, 13, 1) lies on line \ell_1
                                                                    192 : P_{95819} = (10, 24, 22, 1) lies on line \ell_1
                                                                    193 : P_{95846} = (37, 24, 22, 1) lies on line \ell_2
139: P_{61478} = (37, 63, 13, 1) lies on line \ell_2
                                                                    194: P_{95855} = (46, 24, 22, 1) lies on line \ell_3
140: P_{61487} = (46, 63, 13, 1) lies on line \ell_3
141: P_{62923} = (10, 22, 14, 1) lies on line \ell_1
                                                                    195: P_{100427} = (10, 32, 23, 1) lies on line \ell_1
142: P_{62950} = (37, 22, 14, 1) lies on line \ell_2
                                                                    196: P_{100454} = (37, 32, 23, 1) lies on line \ell_2
143 : P_{62959} = (46, 22, 14, 1) lies on line \ell_3
                                                                    197: P_{100463} = (46, 32, 23, 1) lies on line \ell_3
144: P_{63051} = (10, 24, 14, 1) lies on line \ell_4
                                                                    198: P_{101899} = (10, 55, 23, 1) lies on line \ell_4
145: P_{63078} = (37, 24, 14, 1) lies on line \ell_5
                                                                    199: P_{101926} = (37, 55, 23, 1) lies on line \ell_5
146: P_{63087} = (46, 24, 14, 1) lies on line \ell_6
                                                                    200: P_{101935} = (46, 55, 23, 1) lies on line \ell_6
147: P_{67723} = (10, 33, 15, 1) lies on line \ell_4
                                                                    201: P_{103371} = (10, 14, 24, 1) lies on line \ell_1
148: P_{67750} = (37, 33, 15, 1) lies on line \ell_5
                                                                    202: P_{103398} = (37, 14, 24, 1) lies on line \ell_2
149: P_{67759} = (46, 33, 15, 1) lies on line \ell_6
                                                                    203: P_{103407} = (46, 14, 24, 1) lies on line \ell_3
150: P_{68555} = (10, 46, 15, 1) lies on line \ell_1
                                                                    204: P_{103883} = (10, 22, 24, 1) lies on line \ell_4
                                                                    205: P_{103910} = (37, 22, 24, 1) lies on line \ell_5
151: P_{68582} = (37, 46, 15, 1) lies on line \ell_2
152 : P_{68591} = (46, 46, 15, 1) lies on line \ell_3
                                                                    206: P_{103919} = (46, 22, 24, 1) lies on line \ell_6
153: P_{72459} = (10, 43, 16, 1) lies on line \ell_1
                                                                    207: P_{109579} = (10, 47, 25, 1) lies on line \ell_4
                                                                    208 : P_{109606} = (37, 47, 25, 1) lies on line \ell_5
154: P_{72486} = (37, 43, 16, 1) lies on line \ell_2
155: P_{72495} = (46, 43, 16, 1) lies on line \ell_3
                                                                    209: P_{109615} = (46, 47, 25, 1) lies on line \ell_6
156 : P_{73483} = (10, 59, 16, 1) lies on line \ell_4
                                                                    210 : P_{110027} = (10, 54, 25, 1) lies on line \ell_1
157: P_{73510} = (37, 59, 16, 1) lies on line \ell_5
                                                                    211: P_{110054} = (37, 54, 25, 1) lies on line \ell_2
158: P_{73519} = (46, 59, 16, 1) lies on line \ell_6
                                                                    212: P_{110063} = (46, 54, 25, 1) lies on line \ell_3
159: P_{73931} = (10, 2, 17, 1) lies on line \ell_4
                                                                    213: P_{110987} = (10, 5, 26, 1) lies on line \ell_4
160: P_{73958} = (37, 2, 17, 1) lies on line \ell_5
                                                                    214: P_{111014} = (37, 5, 26, 1) lies on line \ell_5
                                                                    215 : P_{111023} = (46, 5, 26, 1) lies on line \ell_6
161: P_{73967} = (46, 2, 17, 1) lies on line \ell_6
162: P_{75019} = (10, 19, 17, 1) lies on line \ell_1
                                                                    216: P_{112651} = (10, 31, 26, 1) lies on line \ell_1
163: P_{75046} = (37, 19, 17, 1) lies on line \ell_2
                                                                    217: P_{112678} = (37, 31, 26, 1) lies on line \ell_2
164: P_{75055} = (46, 19, 17, 1) lies on line \ell_3
                                                                    218: P_{112687} = (46, 31, 26, 1) lies on line \ell_3
165: P_{80459} = (10, 40, 18, 1) lies on line \ell_4
                                                                    219: P_{117259} = (10, 39, 27, 1) lies on line \ell_1
166: P_{80486} = (37, 40, 18, 1) lies on line \ell_5
                                                                    220: P_{117286} = (37, 39, 27, 1) lies on line \ell_2
167: P_{80495} = (46, 40, 18, 1) lies on line \ell_6
                                                                    221: P_{117295} = (46, 39, 27, 1) lies on line \ell_3
168: P_{81611} = (10, 58, 18, 1) lies on line \ell_1
                                                                    222: P_{118603} = (10, 60, 27, 1) lies on line \ell_4
169: P_{81638} = (37, 58, 18, 1) lies on line \ell_2
                                                                    223: P_{118630} = (37, 60, 27, 1) lies on line \ell_5
170: P_{81647} = (46, 58, 18, 1) lies on line \ell_3
                                                                    224: P_{118639} = (46, 60, 27, 1) lies on line \ell_6
171 : P_{82123} = (10, 2, 19, 1) lies on line \ell_1
                                                                    225 : P_{121675} = (10, 44, 28, 1) lies on line \ell_1
172: P_{82150} = (37, 2, 19, 1) lies on line \ell_2
                                                                    226: P_{121702} = (37, 44, 28, 1) lies on line \ell_2
                                                                    227 : P_{121711} = (46, 44, 28, 1) lies on line \ell_3
173: P_{82159} = (46, 2, 19, 1) lies on line \ell_3
174: P_{83083} = (10, 17, 19, 1) lies on line \ell_4
                                                                    228: P_{121931} = (10, 48, 28, 1) lies on line \ell_4
175: P_{83110} = (37, 17, 19, 1) lies on line \ell_5
                                                                    229: P_{121958} = (37, 48, 28, 1) lies on line \ell_5
176: P_{83119} = (46, 17, 19, 1) lies on line \ell_6
                                                                    230 : P_{121967} = (46, 48, 28, 1) lies on line \ell_6
                                                                    231 : P_{123531} = (10, 9, 29, 1) lies on line \ell_4
177: P_{86667} = (10, 9, 20, 1) lies on line \ell_1
178 : P_{86694} = (37, 9, 20, 1) lies on line \ell_2
                                                                    232: P_{123558} = (37, 9, 29, 1) lies on line \ell_5
179: P_{86703} = (46, 9, 20, 1) lies on line \ell_3
                                                                    233: P_{123567} = (46, 9, 29, 1) lies on line \ell_6
                                                                    234 : P_{124235} = (10, 20, 29, 1) lies on line \ell_1
180: P_{87947} = (10, 29, 20, 1) lies on line \ell_4
181: P_{87974} = (37, 29, 20, 1) lies on line \ell_5
                                                                    235: P_{124262} = (37, 20, 29, 1) lies on line \ell_2
182: P_{87983} = (46, 29, 20, 1) lies on line \ell_6
                                                                    236: P_{124271} = (46, 20, 29, 1) lies on line \ell_3
183: P_{92491} = (10, 36, 21, 1) lies on line \ell_4
                                                                    237: P_{129291} = (10, 35, 30, 1) lies on line \ell_4
184: P_{92518} = (37, 36, 21, 1) lies on line \ell_5
                                                                    238: P_{129318} = (37, 35, 30, 1) lies on line \ell_5
185: P_{92527} = (46, 36, 21, 1) lies on line \ell_6
                                                                    239: P_{129327} = (46, 35, 30, 1) lies on line \ell_6
186: P_{93323} = (10, 49, 21, 1) lies on line \ell_1
                                                                    240: P_{130955} = (10, 61, 30, 1) lies on line \ell_1
187: P_{93350} = (37, 49, 21, 1) lies on line \ell_2
                                                                    241: P_{130982} = (37, 61, 30, 1) lies on line \ell_2
188: P_{93359} = (46, 49, 21, 1) lies on line \ell_3
                                                                    242: P_{130991} = (46, 61, 30, 1) lies on line \ell_3
189: P_{95179} = (10, 14, 22, 1) lies on line \ell_4
                                                                    243: P_{131467} = (10, 5, 31, 1) lies on line \ell_1
190: P_{95206} = (37, 14, 22, 1) lies on line \ell_5
                                                                    244: P_{131494} = (37, 5, 31, 1) lies on line \ell_2
191: P_{95215} = (46, 14, 22, 1) lies on line \ell_6
                                                                    245: P_{131503} = (46, 5, 31, 1) lies on line \ell_3
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300: P_{171723} = (10, 58, 40, 1) lies on line \ell_4
246: P_{132811} = (10, 26, 31, 1) lies on line \ell_4
247: P_{132838} = (37, 26, 31, 1) lies on line \ell_5
                                                                    301: P_{171750} = (37, 58, 40, 1) lies on line \ell_5
                                                                    302: P_{171759} = (46, 58, 40, 1) lies on line \ell_6
248: P_{132847} = (46, 26, 31, 1) lies on line \ell_6
249: P_{136715} = (10, 23, 32, 1) lies on line \ell_4
                                                                    303: P_{172299} = (10, 3, 41, 1) lies on line \ell_4
250: P_{136742} = (37, 23, 32, 1) lies on line \ell_5
                                                                    304: P_{172326} = (37, 3, 41, 1) lies on line \ell_5
251: P_{136751} = (46, 23, 32, 1) lies on line \ell_6
                                                                    305: P_{172335} = (46, 3, 41, 1) lies on line \ell_6
252: P_{138763} = (10, 55, 32, 1) lies on line \ell_1
                                                                    306: P_{174795} = (10, 42, 41, 1) lies on line \ell_1
253: P_{138790} = (37, 55, 32, 1) lies on line \ell_2
                                                                    307: P_{174822} = (37, 42, 41, 1) lies on line \ell_2
254 : P_{138799} = (46, 55, 32, 1) lies on line \ell_3
                                                                    308: P_{174831} = (46, 42, 41, 1) lies on line \ell_3
255: P_{140299} = (10, 15, 33, 1) lies on line \ell_1
                                                                    309: P_{176395} = (10, 3, 42, 1) lies on line \ell_1
256: P_{140326} = (37, 15, 33, 1) lies on line \ell_2
                                                                    310: P_{176422} = (37, 3, 42, 1) lies on line \ell_2
                                                                    311 : P_{176431} = (46, 3, 42, 1) lies on line \ell_3
257: P_{140335} = (46, 15, 33, 1) lies on line \ell_3
258: P_{142283} = (10, 46, 33, 1) lies on line \ell_4
                                                                    312: P_{178827} = (10, 41, 42, 1) lies on line \ell_4
259: P_{142310} = (37, 46, 33, 1) lies on line \ell_5
                                                                    313: P_{178854} = (37, 41, 42, 1) lies on line \ell_5
260: P_{142319} = (46, 46, 33, 1) lies on line \ell_6
                                                                    314: P_{178863} = (46, 41, 42, 1) lies on line \ell_6
261: P_{143691} = (10, 4, 34, 1) lies on line \ell_4
                                                                    315: P_{181323} = (10, 16, 43, 1) lies on line \ell_4
262: P_{143718} = (37, 4, 34, 1) lies on line \ell_5
                                                                    316: P_{181350} = (37, 16, 43, 1) lies on line \ell_5
                                                                    317: P_{181359} = (46, 16, 43, 1) lies on line \ell_6
263: P_{143727} = (46, 4, 34, 1) lies on line \ell_6
264: P_{145867} = (10, 38, 34, 1) lies on line \ell_1
                                                                    318: P_{184075} = (10, 59, 43, 1) lies on line \ell_1
                                                                    319: P_{184102} = (37, 59, 43, 1) lies on line \ell_2
265: P_{145894} = (37, 38, 34, 1) lies on line \ell_2
266: P_{145903} = (46, 38, 34, 1) lies on line \ell_3
                                                                    320: P_{184111} = (46, 59, 43, 1) lies on line \ell_3
267: P_{149451} = (10, 30, 35, 1) lies on line \ell_1
                                                                    321: P_{186187} = (10, 28, 44, 1) lies on line \ell_4
268: P_{149478} = (37, 30, 35, 1) lies on line \ell_2
                                                                    322: P_{186214} = (37, 28, 44, 1) lies on line \ell_5
269 : P_{149487} = (46, 30, 35, 1) lies on line \ell_3
                                                                    323 : P_{186223} = (46, 28, 44, 1) lies on line \ell_6
270: P_{151435} = (10, 61, 35, 1) lies on line \ell_4
                                                                    324: P_{187467} = (10, 48, 44, 1) lies on line \ell_1
271: P_{151462} = (37, 61, 35, 1) lies on line \ell_5
                                                                    325: P_{187494} = (37, 48, 44, 1) lies on line \ell_2
272: P_{151471} = (46, 61, 35, 1) lies on line \ell_6
                                                                    326: P_{187503} = (46, 48, 44, 1) lies on line \ell_3
273: P_{152971} = (10, 21, 36, 1) lies on line \ell_1
                                                                    327: P_{189003} = (10, 8, 45, 1) lies on line \ell_1
274: P_{152998} = (37, 21, 36, 1) lies on line \ell_2
                                                                    328: P_{189030} = (37, 8, 45, 1) lies on line \ell_2
275: P_{153007} = (46, 21, 36, 1) lies on line \ell_3
                                                                    329: P_{189039} = (46, 8, 45, 1) lies on line \ell_3
276: P_{154763} = (10, 49, 36, 1) lies on line \ell_4
                                                                    330: P_{190859} = (10, 37, 45, 1) lies on line \ell_4
277: P_{154790} = (37, 49, 36, 1) lies on line \ell_5
                                                                    331: P_{190886} = (37, 37, 45, 1) lies on line \ell_5
278: P_{154799} = (46, 49, 36, 1) lies on line \ell_6
                                                                    332: P_{190895} = (46, 37, 45, 1) lies on line \ell_6
279: P_{156235} = (10, 8, 37, 1) lies on line \ell_4
                                                                    333: P_{193547} = (10, 15, 46, 1) lies on line \ell_4
280: P_{156262} = (37, 8, 37, 1) lies on line \ell_5
                                                                    334: P_{193574} = (37, 15, 46, 1) lies on line \ell_5
281 : P_{156271} = (46, 8, 37, 1) lies on line \ell_6
                                                                    335: P_{193583} = (46, 15, 46, 1) lies on line \ell_6
282: P_{158603} = (10, 45, 37, 1) lies on line \ell_1
                                                                    336: P_{194699} = (10, 33, 46, 1) lies on line \ell_1
283: P_{158630} = (37, 45, 37, 1) lies on line \ell_2
                                                                    337: P_{194726} = (37, 33, 46, 1) lies on line \ell_2
284 : P_{158639} = (46, 45, 37, 1) lies on line \ell_3
                                                                    338: P_{194735} = (46, 33, 46, 1) lies on line \ell_3
                                                                    339 : P_{198283} = (10, 25, 47, 1) lies on line \ell_1
285: P_{160075} = (10, 4, 38, 1) lies on line \ell_1
                                                                    340: P_{198310} = (37, 25, 47, 1) lies on line \ell_2
286: P_{160102} = (37, 4, 38, 1) lies on line \ell_2
287: P_{160111} = (46, 4, 38, 1) lies on line \ell_3
                                                                    341: P_{198319} = (46, 25, 47, 1) lies on line \ell_3
288 : P_{161995} = (10, 34, 38, 1) lies on line \ell_4
                                                                    342: P_{200139} = (10, 54, 47, 1) lies on line \ell_4
289: P_{162022} = (37, 34, 38, 1) lies on line \ell_5
                                                                    343: P_{200166} = (37, 54, 47, 1) lies on line \ell_5
290: P_{162031} = (46, 34, 38, 1) lies on line \ell_6
                                                                    344 : P_{200175} = (46, 54, 47, 1) lies on line \ell_6
291: P_{165643} = (10, 27, 39, 1) lies on line \ell_4
                                                                    345: P_{202571} = (10, 28, 48, 1) lies on line \ell_1
292: P_{165670} = (37, 27, 39, 1) lies on line \ell_5
                                                                    346: P_{202598} = (37, 28, 48, 1) lies on line \ell_2
293: P_{165679} = (46, 27, 39, 1) lies on line \ell_6
                                                                    347: P_{202607} = (46, 28, 48, 1) lies on line \ell_3
294: P_{167755} = (10, 60, 39, 1) lies on line \ell_1
                                                                    348: P_{203595} = (10, 44, 48, 1) lies on line \ell_4
295: P_{167782} = (37, 60, 39, 1) lies on line \ell_2
                                                                    349: P_{203622} = (37, 44, 48, 1) lies on line \ell_5
296: P_{167791} = (46, 60, 39, 1) lies on line \ell_3
                                                                    350: P_{203631} = (46, 44, 48, 1) lies on line \ell_6
297: P_{169163} = (10, 18, 40, 1) lies on line \ell_1
                                                                    351: P_{206219} = (10, 21, 49, 1) lies on line \ell_4
298 : P_{169190} = (37, 18, 40, 1) lies on line \ell_2
                                                                    352: P_{206246} = (37, 21, 49, 1) lies on line \ell_5
299: P_{169199} = (46, 18, 40, 1) lies on line \ell_3
                                                                    353: P_{206255} = (46, 21, 49, 1) lies on line \ell_6
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354: P_{207179} = (10, 36, 49, 1) lies on line \ell_1
                                                                    398 : P_{237231} = (46, 57, 56, 1) lies on line \ell_3
355: P_{207206} = (37, 36, 49, 1) lies on line \ell_2
                                                                    399: P_{237707} = (10, 1, 57, 1) lies on line \ell_1
356: P_{207215} = (46, 36, 49, 1) lies on line \ell_3
                                                                    400: P_{237734} = (37, 1, 57, 1) lies on line \ell_2
357: P_{209803} = (10, 13, 50, 1) lies on line \ell_1
                                                                    401: P_{237743} = (46, 1, 57, 1) lies on line \ell_3
                                                                    402: P_{241227} = (10, 56, 57, 1) lies on line \ell_4
358: P_{209830} = (37, 13, 50, 1) lies on line \ell_2
359: P_{209839} = (46, 13, 50, 1) lies on line \ell_3
                                                                    403: P_{241254} = (37, 56, 57, 1) lies on line \ell_5
360: P_{213003} = (10, 63, 50, 1) lies on line \ell_4
                                                                    404: P_{241263} = (46, 56, 57, 1) lies on line \ell_6
361: P_{213030} = (37, 63, 50, 1) lies on line \ell_5
                                                                    405: P_{242891} = (10, 18, 58, 1) lies on line \ell_4
362: P_{213039} = (46, 63, 50, 1) lies on line \ell_6
                                                                    406: P_{242918} = (37, 18, 58, 1) lies on line \ell_5
363: P_{213451} = (10, 6, 51, 1) lies on line \ell_4
                                                                    407: P_{242927} = (46, 18, 58, 1) lies on line \ell_6
364: P_{213478} = (37, 6, 51, 1) lies on line \ell_5
                                                                    408: P_{244299} = (10, 40, 58, 1) lies on line \ell_1
365: P_{213487} = (46, 6, 51, 1) lies on line \ell_6
                                                                    409: P_{244326} = (37, 40, 58, 1) lies on line \ell_2
366: P_{216459} = (10, 53, 51, 1) lies on line \ell_1
                                                                    410: P_{244335} = (46, 40, 58, 1) lies on line \ell_3
367: P_{216486} = (37, 53, 51, 1) lies on line \ell_2
                                                                    411: P_{246859} = (10, 16, 59, 1) lies on line \ell_1
368: P_{216495} = (46, 53, 51, 1) lies on line \ell_3
                                                                    412: P_{246886} = (37, 16, 59, 1) lies on line \ell_2
369: P_{217803} = (10, 10, 52, 1) lies on line \ell_4
                                                                    413: P_{246895} = (46, 16, 59, 1) lies on line \ell_3
                                                                    414: P_{248587} = (10, 43, 59, 1) lies on line \ell_4
370: P_{217830} = (37, 10, 52, 1) lies on line \ell_5
371: P_{217839} = (46, 10, 52, 1) lies on line \ell_6
                                                                    415: P_{248614} = (37, 43, 59, 1) lies on line \ell_5
372: P_{221131} = (10, 62, 52, 1) lies on line \ell_1
                                                                    416: P_{248623} = (46, 43, 59, 1) lies on line \ell_6
373: P_{221158} = (37, 62, 52, 1) lies on line \ell_2
                                                                    417: P_{251659} = (10, 27, 60, 1) lies on line \ell_1
374: P_{221167} = (46, 62, 52, 1) lies on line \ell_3
                                                                    418: P_{251686} = (37, 27, 60, 1) lies on line \ell_2
375: P_{221643} = (10, 6, 53, 1) lies on line \ell_1
                                                                    419: P_{251695} = (46, 27, 60, 1) lies on line \ell_3
376: P_{221670} = (37, 6, 53, 1) lies on line \ell_2
                                                                    420: P_{252427} = (10, 39, 60, 1) lies on line \ell_4
377 : P_{221679} = (46, 6, 53, 1) lies on line \ell_3
                                                                    421: P_{252454} = (37, 39, 60, 1) lies on line \ell_5
                                                                    422: P_{252463} = (46, 39, 60, 1) lies on line \ell_6
378: P_{224523} = (10, 51, 53, 1) lies on line \ell_4
379: P_{224550} = (37, 51, 53, 1) lies on line \ell_5
                                                                    423: P_{255947} = (10, 30, 61, 1) lies on line \ell_4
380: P_{224559} = (46, 51, 53, 1) lies on line \ell_6
                                                                    424: P_{255974} = (37, 30, 61, 1) lies on line \ell_5
381: P_{226955} = (10, 25, 54, 1) lies on line \ell_4
                                                                    425: P_{255983} = (46, 30, 61, 1) lies on line \ell_6
382: P_{226982} = (37, 25, 54, 1) lies on line \ell_5
                                                                    426: P_{256267} = (10, 35, 61, 1) lies on line \ell_1
383: P_{226991} = (46, 25, 54, 1) lies on line \ell_6
                                                                    427: P_{256294} = (37, 35, 61, 1) lies on line \ell_2
384: P_{228363} = (10, 47, 54, 1) lies on line \ell_1
                                                                    428: P_{256303} = (46, 35, 61, 1) lies on line \ell_3
385: P_{228390} = (37, 47, 54, 1) lies on line \ell_2
                                                                    429: P_{258763} = (10, 10, 62, 1) lies on line \ell_1
386: P_{228399} = (46, 47, 54, 1) lies on line \ell_3
                                                                    430: P_{258790} = (37, 10, 62, 1) lies on line \ell_2
                                                                    431 : P_{258799} = (46, 10, 62, 1) lies on line \ell_3
387: P_{230923} = (10, 23, 55, 1) lies on line \ell_1
388: P_{230950} = (37, 23, 55, 1) lies on line \ell_2
                                                                    432: P_{261451} = (10, 52, 62, 1) lies on line \ell_4
                                                                    433: P_{261478} = (37, 52, 62, 1) lies on line \ell_5
389: P_{230959} = (46, 23, 55, 1) lies on line \ell_3
390: P_{231499} = (10, 32, 55, 1) lies on line \ell_4
                                                                    434: P_{261487} = (46, 52, 62, 1) lies on line \ell_6
391 : P_{231526} = (37, 32, 55, 1) lies on line \ell_5
                                                                    435 : P_{263051} = (10, 13, 63, 1) lies on line \ell_4
                                                                    436: P_{263078} = (37, 13, 63, 1) lies on line \ell_5
392: P_{231535} = (46, 32, 55, 1) lies on line \ell_6
                                                                    437: P_{263087} = (46, 13, 63, 1) lies on line \ell_6
393: P_{233611} = (10, 1, 56, 1) lies on line \ell_4
394 : P_{233638} = (37, 1, 56, 1) lies on line \ell_5
                                                                    438: P_{265419} = (10, 50, 63, 1) lies on line \ell_1
395: P_{233647} = (46, 1, 56, 1) lies on line \ell_6
                                                                    439: P_{265446} = (37, 50, 63, 1) lies on line \ell_2
396: P_{237195} = (10, 57, 56, 1) lies on line \ell_1
                                                                    440: P_{265455} = (46, 50, 63, 1) lies on line \ell_3
397: P_{237222} = (37, 57, 56, 1) lies on line \ell_2
```

The single points on the surface are:

#### Points on surface but on no line

The surface has 3843 points not on any line: Too many to print.

## Line Intersection Graph

 $\begin{array}{c} 0123456 \\ \hline 0 & 0111111 \\ 1 & 1011100 \\ 2 & 1101010 \\ 3 & 1110001 \\ 4 & 1100011 \\ 5 & 1010101 \\ 6 & 1001110 \end{array}$ 

Neighbor sets in the line intersection graph:

Line 0 intersects

$\operatorname{Line}$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_{3651}$	$P_{3651}$	$P_{3651}$	$P_{3715}$	$P_{3715}$	$P_{3715}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_{3651}$	$P_{3651}$	$P_{3651}$	$P_{4172}$

Line 2 intersects

	Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_5$
ſ	in point	$P_{3651}$	$P_{3651}$	$P_{3651}$	$P_{4199}$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_6$
in point	$P_{3651}$	$P_{3651}$	$P_{3651}$	$P_{4208}$

Line 4 intersects

Line	$\ell_0$	$\ell_1$	$\ell_5$	$\ell_6$
in point	$P_{3715}$	$P_{4172}$	$P_{3715}$	$P_{3715}$

 ${\bf Line~5~intersects}$ 

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_6$
in point	$P_{3715}$	$P_{4199}$	$P_{3715}$	$P_{3715}$

Line 6 intersects

Line	$\ell_0$	$\ell_3$	$\ell_4$	$\ell_5$
in point	$P_{3715}$	$P_{4208}$	$P_{3715}$	$P_{3715}$

The surface has 4289 points:

Too many to print.