# Rank-65612 over GF(64)

January 15, 2021

## The equation

The equation of the surface is:

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

 $(\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0)$ 

The point rank of the equation over GF(64) is 1091047494

## General information

Number of lines	7
Number of points	4289
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	441
Number of points off lines	3843
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 <sup>7</sup>
Type of lines on points	$4^2, 2^3, 1^{441}, 0^{3843}$

## Singular Points

The surface has 2 singular points:

$$0: P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0) 1: P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

### The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{270401}$$

$$\ell_{1} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{9} \\ 0 & 1 & 0 & 0 \end{bmatrix}_{12516288} = \begin{bmatrix} 1 & 0 & 0 & 47 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{12516288} = \mathbf{Pl}(10, 0, 0, 1, 0, 0)_{139}$$

$$\ell_{2} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{18} \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2929344} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2929344} = \mathbf{Pl}(37, 0, 0, 1, 0, 0)_{166}$$

$$\ell_{3} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{36} \\ 0 & 1 & 0 & 0 \end{bmatrix}_{9586944} = \begin{bmatrix} 1 & 0 & 0 & 36 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{9586944} = \mathbf{Pl}(46, 0, 0, 1, 0, 0)_{175}$$

$$\ell_{4} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{9} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{12520384} = \begin{bmatrix} 1 & 0 & 0 & 47 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{12520384} = \mathbf{Pl}(0, 47, 1, 0, 0, 0)_{112}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{18} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2933440} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2933440} = \mathbf{Pl}(0, 11, 1, 0, 0, 0)_{76}$$

$$\ell_{6} = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{36} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{9591040} = \begin{bmatrix} 1 & 0 & 0 & 36 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{9591040} = \mathbf{Pl}(0, 36, 1, 0, 0, 0)_{101}$$

Rank of lines: ( 17043456, 12516288, 2929344, 9586944, 12520384, 2933440, 9591040) Rank of points on Klein quadric: ( 270401, 139, 166, 175, 112, 76, 101)

#### **Eckardt Points**

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 3 Double points: The double points on the surface are:

$$P_{4172} = (10, 0, 0, 1) = \ell_1 \cap \ell_4 P_{4199} = (37, 0, 0, 1) = \ell_2 \cap \ell_5$$

 $P_{4208} = (46, 0, 0, 1) = \ell_3 \cap \ell_6$ 

## Single Points

The surface has 441 single points: The single points on the surface are:

$0: P_{131} = (0, 1, 1, 0)$ lies on line $\ell_0$	15: $P_{1091} = (0, 16, 1, 0)$ lies on line $\ell_0$
1: $P_{195} = (0, 2, 1, 0)$ lies on line $\ell_0$	16: $P_{1155} = (0, 17, 1, 0)$ lies on line $\ell_0$
$2: P_{259} = (0, 3, 1, 0)$ lies on line $\ell_0$	17: $P_{1219} = (0, 18, 1, 0)$ lies on line $\ell_0$
$3: P_{323} = (0, 4, 1, 0)$ lies on line $\ell_0$	18: $P_{1283} = (0, 19, 1, 0)$ lies on line $\ell_0$
4: $P_{387} = (0, 5, 1, 0)$ lies on line $\ell_0$	19: $P_{1347} = (0, 20, 1, 0)$ lies on line $\ell_0$
$5: P_{451} = (0, 6, 1, 0)$ lies on line $\ell_0$	20: $P_{1411} = (0, 21, 1, 0)$ lies on line $\ell_0$
6: $P_{515} = (0, 7, 1, 0)$ lies on line $\ell_0$	21: $P_{1475} = (0, 22, 1, 0)$ lies on line $\ell_0$
7: $P_{579} = (0, 8, 1, 0)$ lies on line $\ell_0$	22: $P_{1539} = (0, 23, 1, 0)$ lies on line $\ell_0$
8: $P_{643} = (0, 9, 1, 0)$ lies on line $\ell_0$	23: $P_{1603} = (0, 24, 1, 0)$ lies on line $\ell_0$
9: $P_{707} = (0, 10, 1, 0)$ lies on line $\ell_0$	24 : $P_{1667} = (0, 25, 1, 0)$ lies on line $\ell_0$
10: $P_{771} = (0, 11, 1, 0)$ lies on line $\ell_0$	25: $P_{1731} = (0, 26, 1, 0)$ lies on line $\ell_0$
11: $P_{835} = (0, 12, 1, 0)$ lies on line $\ell_0$	26: $P_{1795} = (0, 27, 1, 0)$ lies on line $\ell_0$
12: $P_{899} = (0, 13, 1, 0)$ lies on line $\ell_0$	27: $P_{1859} = (0, 28, 1, 0)$ lies on line $\ell_0$
13: $P_{963} = (0, 14, 1, 0)$ lies on line $\ell_0$	28: $P_{1923} = (0, 29, 1, 0)$ lies on line $\ell_0$
14: $P_{1027} = (0, 15, 1, 0)$ lies on line $\ell_0$	29: $P_{1987} = (0, 30, 1, 0)$ lies on line $\ell_0$

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30: P_{2051} = (0, 31, 1, 0) lies on line \ell_0
                                                                     84: P_{4684} = (10, 8, 0, 1) lies on line \ell_1
31: P_{2115} = (0, 32, 1, 0) lies on line \ell_0
                                                                     85: P_{4711} = (37, 8, 0, 1) lies on line \ell_2
                                                                     86: P_{4720} = (46, 8, 0, 1) lies on line \ell_3
32: P_{2179} = (0, 33, 1, 0) lies on line \ell_0
33: P_{2243} = (0, 34, 1, 0) lies on line \ell_0
                                                                     87: P_{4748} = (10, 9, 0, 1) lies on line \ell_1
34: P_{2307} = (0, 35, 1, 0) lies on line \ell_0
                                                                     88: P_{4775} = (37, 9, 0, 1) lies on line \ell_2
                                                                     89: P_{4784} = (46, 9, 0, 1) lies on line \ell_3
35: P_{2371} = (0, 36, 1, 0) lies on line \ell_0
36: P_{2435} = (0, 37, 1, 0) lies on line \ell_0
                                                                     90: P_{4812} = (10, 10, 0, 1) lies on line \ell_1
37: P_{2499} = (0, 38, 1, 0) lies on line \ell_0
                                                                     91: P_{4839} = (37, 10, 0, 1) lies on line \ell_2
38: P_{2563} = (0, 39, 1, 0) lies on line \ell_0
                                                                     92: P_{4848} = (46, 10, 0, 1) lies on line \ell_3
39: P_{2627} = (0, 40, 1, 0) lies on line \ell_0
                                                                     93: P_{4876} = (10, 11, 0, 1) lies on line \ell_1
40: P_{2691} = (0, 41, 1, 0) lies on line \ell_0
                                                                     94: P_{4903} = (37, 11, 0, 1) lies on line \ell_2
                                                                     95: P_{4912} = (46, 11, 0, 1) lies on line \ell_3
41: P_{2755} = (0, 42, 1, 0) lies on line \ell_0
42: P_{2819} = (0, 43, 1, 0) lies on line \ell_0
                                                                     96: P_{4940} = (10, 12, 0, 1) lies on line \ell_1
43: P_{2883} = (0, 44, 1, 0) lies on line \ell_0
                                                                     97: P_{4967} = (37, 12, 0, 1) lies on line \ell_2
                                                                     98: P_{4976} = (46, 12, 0, 1) lies on line \ell_3
44: P_{2947} = (0, 45, 1, 0) lies on line \ell_0
45: P_{3011} = (0, 46, 1, 0) lies on line \ell_0
                                                                     99: P_{5004} = (10, 13, 0, 1) lies on line \ell_1
46: P_{3075} = (0, 47, 1, 0) lies on line \ell_0
                                                                     100: P_{5031} = (37, 13, 0, 1) lies on line \ell_2
47: P_{3139} = (0, 48, 1, 0) lies on line \ell_0
                                                                     101: P_{5040} = (46, 13, 0, 1) lies on line \ell_3
48: P_{3203} = (0, 49, 1, 0) lies on line \ell_0
                                                                     102: P_{5068} = (10, 14, 0, 1) lies on line \ell_1
49: P_{3267} = (0, 50, 1, 0) lies on line \ell_0
                                                                     103: P_{5095} = (37, 14, 0, 1) lies on line \ell_2
50: P_{3331} = (0, 51, 1, 0) lies on line \ell_0
                                                                     104: P_{5104} = (46, 14, 0, 1) lies on line \ell_3
51: P_{3395} = (0, 52, 1, 0) lies on line \ell_0
                                                                     105: P_{5132} = (10, 15, 0, 1) lies on line \ell_1
52: P_{3459} = (0, 53, 1, 0) lies on line \ell_0
                                                                     106: P_{5159} = (37, 15, 0, 1) lies on line \ell_2
53: P_{3523} = (0, 54, 1, 0) lies on line \ell_0
                                                                     107: P_{5168} = (46, 15, 0, 1) lies on line \ell_3
54: P_{3587} = (0, 55, 1, 0) lies on line \ell_0
                                                                     108: P_{5196} = (10, 16, 0, 1) lies on line \ell_1
55: P_{3651} = (0, 56, 1, 0) lies on line \ell_0
                                                                     109: P_{5223} = (37, 16, 0, 1) lies on line \ell_2
56: P_{3715} = (0, 57, 1, 0) lies on line \ell_0
                                                                     110: P_{5232} = (46, 16, 0, 1) lies on line \ell_3
57: P_{3779} = (0, 58, 1, 0) lies on line \ell_0
                                                                     111: P_{5260} = (10, 17, 0, 1) lies on line \ell_1
                                                                     112: P_{5287} = (37, 17, 0, 1) lies on line \ell_2
58: P_{3843} = (0, 59, 1, 0) lies on line \ell_0
59: P_{3907} = (0, 60, 1, 0) lies on line \ell_0
                                                                     113: P_{5296} = (46, 17, 0, 1) lies on line \ell_3
60: P_{3971} = (0, 61, 1, 0) lies on line \ell_0
                                                                     114: P_{5324} = (10, 18, 0, 1) lies on line \ell_1
61: P_{4035} = (0, 62, 1, 0) lies on line \ell_0
                                                                     115: P_{5351} = (37, 18, 0, 1) lies on line \ell_2
62: P_{4099} = (0, 63, 1, 0) lies on line \ell_0
                                                                     116: P_{5360} = (46, 18, 0, 1) lies on line \ell_3
63: P_{4236} = (10, 1, 0, 1) lies on line \ell_1
                                                                     117: P_{5388} = (10, 19, 0, 1) lies on line \ell_1
64: P_{4263} = (37, 1, 0, 1) lies on line \ell_2
                                                                     118: P_{5415} = (37, 19, 0, 1) lies on line \ell_2
                                                                     119: P_{5424} = (46, 19, 0, 1) lies on line \ell_3
65: P_{4272} = (46, 1, 0, 1) lies on line \ell_3
66: P_{4300} = (10, 2, 0, 1) lies on line \ell_1
                                                                     120: P_{5452} = (10, 20, 0, 1) lies on line \ell_1
67: P_{4327} = (37, 2, 0, 1) lies on line \ell_2
                                                                     121: P_{5479} = (37, 20, 0, 1) lies on line \ell_2
68: P_{4336} = (46, 2, 0, 1) lies on line \ell_3
                                                                     122: P_{5488} = (46, 20, 0, 1) lies on line \ell_3
69: P_{4364} = (10, 3, 0, 1) lies on line \ell_1
                                                                     123: P_{5516} = (10, 21, 0, 1) lies on line \ell_1
70: P_{4391} = (37, 3, 0, 1) lies on line \ell_2
                                                                     124: P_{5543} = (37, 21, 0, 1) lies on line \ell_2
71: P_{4400} = (46, 3, 0, 1) lies on line \ell_3
                                                                     125: P_{5552} = (46, 21, 0, 1) lies on line \ell_3
72: P_{4428} = (10, 4, 0, 1) lies on line \ell_1
                                                                     126: P_{5580} = (10, 22, 0, 1) lies on line \ell_1
73: P_{4455} = (37, 4, 0, 1) lies on line \ell_2
                                                                     127: P_{5607} = (37, 22, 0, 1) lies on line \ell_2
74: P_{4464} = (46, 4, 0, 1) lies on line \ell_3
                                                                     128: P_{5616} = (46, 22, 0, 1) lies on line \ell_3
75: P_{4492} = (10, 5, 0, 1) lies on line \ell_1
                                                                     129: P_{5644} = (10, 23, 0, 1) lies on line \ell_1
76: P_{4519} = (37, 5, 0, 1) lies on line \ell_2
                                                                     130: P_{5671} = (37, 23, 0, 1) lies on line \ell_2
77: P_{4528} = (46, 5, 0, 1) lies on line \ell_3
                                                                     131: P_{5680} = (46, 23, 0, 1) lies on line \ell_3
78: P_{4556} = (10, 6, 0, 1) lies on line \ell_1
                                                                     132: P_{5708} = (10, 24, 0, 1) lies on line \ell_1
79: P_{4583} = (37, 6, 0, 1) lies on line \ell_2
                                                                     133: P_{5735} = (37, 24, 0, 1) lies on line \ell_2
                                                                     134 : P_{5744} = (46, 24, 0, 1) lies on line \ell_3
80: P_{4592} = (46, 6, 0, 1) lies on line \ell_3
81: P_{4620} = (10, 7, 0, 1) lies on line \ell_1
                                                                     135: P_{5772} = (10, 25, 0, 1) lies on line \ell_1
82: P_{4647} = (37, 7, 0, 1) lies on line \ell_2
                                                                     136: P_{5799} = (37, 25, 0, 1) lies on line \ell_2
83: P_{4656} = (46, 7, 0, 1) lies on line \ell_3
                                                                     137: P_{5808} = (46, 25, 0, 1) lies on line \ell_3
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138 : P_{5836} = (10, 26, 0, 1) lies on line \ell_1
                                                                     192: P_{6988} = (10, 44, 0, 1) lies on line \ell_1
139: P_{5863} = (37, 26, 0, 1) lies on line \ell_2
                                                                     193: P_{7015} = (37, 44, 0, 1) lies on line \ell_2
140: P_{5872} = (46, 26, 0, 1) lies on line \ell_3
                                                                     194: P_{7024} = (46, 44, 0, 1) lies on line \ell_3
141: P_{5900} = (10, 27, 0, 1) lies on line \ell_1
                                                                     195: P_{7052} = (10, 45, 0, 1) lies on line \ell_1
142: P_{5927} = (37, 27, 0, 1) lies on line \ell_2
                                                                     196: P_{7079} = (37, 45, 0, 1) lies on line \ell_2
143: P_{5936} = (46, 27, 0, 1) lies on line \ell_3
                                                                     197: P_{7088} = (46, 45, 0, 1) lies on line \ell_3
144: P_{5964} = (10, 28, 0, 1) lies on line \ell_1
                                                                     198: P_{7116} = (10, 46, 0, 1) lies on line \ell_1
145: P_{5991} = (37, 28, 0, 1) lies on line \ell_2
                                                                     199: P_{7143} = (37, 46, 0, 1) lies on line \ell_2
146: P_{6000} = (46, 28, 0, 1) lies on line \ell_3
                                                                     200: P_{7152} = (46, 46, 0, 1) lies on line \ell_3
147: P_{6028} = (10, 29, 0, 1) lies on line \ell_1
                                                                     201: P_{7180} = (10, 47, 0, 1) lies on line \ell_1
148: P_{6055} = (37, 29, 0, 1) lies on line \ell_2
                                                                     202: P_{7207} = (37, 47, 0, 1) lies on line \ell_2
                                                                     203: P_{7216} = (46, 47, 0, 1) lies on line \ell_3
149: P_{6064} = (46, 29, 0, 1) lies on line \ell_3
150: P_{6092} = (10, 30, 0, 1) lies on line \ell_1
                                                                     204: P_{7244} = (10, 48, 0, 1) lies on line \ell_1
151: P_{6119} = (37, 30, 0, 1) lies on line \ell_2
                                                                     205: P_{7271} = (37, 48, 0, 1) lies on line \ell_2
152: P_{6128} = (46, 30, 0, 1) lies on line \ell_3
                                                                     206: P_{7280} = (46, 48, 0, 1) lies on line \ell_3
153: P_{6156} = (10, 31, 0, 1) lies on line \ell_1
                                                                     207: P_{7308} = (10, 49, 0, 1) lies on line \ell_1
154: P_{6183} = (37, 31, 0, 1) lies on line \ell_2
                                                                     208: P_{7335} = (37, 49, 0, 1) lies on line \ell_2
155: P_{6192} = (46, 31, 0, 1) lies on line \ell_3
                                                                     209: P_{7344} = (46, 49, 0, 1) lies on line \ell_3
156: P_{6220} = (10, 32, 0, 1) lies on line \ell_1
                                                                     210: P_{7372} = (10, 50, 0, 1) lies on line \ell_1
157: P_{6247} = (37, 32, 0, 1) lies on line \ell_2
                                                                     211: P_{7399} = (37, 50, 0, 1) lies on line \ell_2
158: P_{6256} = (46, 32, 0, 1) lies on line \ell_3
                                                                     212: P_{7408} = (46, 50, 0, 1) lies on line \ell_3
159: P_{6284} = (10, 33, 0, 1) lies on line \ell_1
                                                                     213: P_{7436} = (10, 51, 0, 1) lies on line \ell_1
160: P_{6311} = (37, 33, 0, 1) lies on line \ell_2
                                                                     214: P_{7463} = (37, 51, 0, 1) lies on line \ell_2
161: P_{6320} = (46, 33, 0, 1) lies on line \ell_3
                                                                     215: P_{7472} = (46, 51, 0, 1) lies on line \ell_3
162: P_{6348} = (10, 34, 0, 1) lies on line \ell_1
                                                                     216: P_{7500} = (10, 52, 0, 1) lies on line \ell_1
163: P_{6375} = (37, 34, 0, 1) lies on line \ell_2
                                                                     217: P_{7527} = (37, 52, 0, 1) lies on line \ell_2
                                                                     218: P_{7536} = (46, 52, 0, 1) lies on line \ell_3
164: P_{6384} = (46, 34, 0, 1) lies on line \ell_3
165: P_{6412} = (10, 35, 0, 1) lies on line \ell_1
                                                                     219: P_{7564} = (10, 53, 0, 1) lies on line \ell_1
166: P_{6439} = (37, 35, 0, 1) lies on line \ell_2
                                                                     220: P_{7591} = (37, 53, 0, 1) lies on line \ell_2
167: P_{6448} = (46, 35, 0, 1) lies on line \ell_3
                                                                     221: P_{7600} = (46, 53, 0, 1) lies on line \ell_3
168: P_{6476} = (10, 36, 0, 1) lies on line \ell_1
                                                                     222: P_{7628} = (10, 54, 0, 1) lies on line \ell_1
169: P_{6503} = (37, 36, 0, 1) lies on line \ell_2
                                                                     223: P_{7655} = (37, 54, 0, 1) lies on line \ell_2
170: P_{6512} = (46, 36, 0, 1) lies on line \ell_3
                                                                     224: P_{7664} = (46, 54, 0, 1) lies on line \ell_3
171: P_{6540} = (10, 37, 0, 1) lies on line \ell_1
                                                                     225: P_{7692} = (10, 55, 0, 1) lies on line \ell_1
172: P_{6567} = (37, 37, 0, 1) lies on line \ell_2
                                                                     226: P_{7719} = (37, 55, 0, 1) lies on line \ell_2
173: P_{6576} = (46, 37, 0, 1) lies on line \ell_3
                                                                     227: P_{7728} = (46, 55, 0, 1) lies on line \ell_3
174: P_{6604} = (10, 38, 0, 1) lies on line \ell_1
                                                                     228: P_{7756} = (10, 56, 0, 1) lies on line \ell_1
175: P_{6631} = (37, 38, 0, 1) lies on line \ell_2
                                                                     229: P_{7783} = (37, 56, 0, 1) lies on line \ell_2
176: P_{6640} = (46, 38, 0, 1) lies on line \ell_3
                                                                     230: P_{7792} = (46, 56, 0, 1) lies on line \ell_3
177: P_{6668} = (10, 39, 0, 1) lies on line \ell_1
                                                                     231: P_{7820} = (10, 57, 0, 1) lies on line \ell_1
178: P_{6695} = (37, 39, 0, 1) lies on line \ell_2
                                                                     232: P_{7847} = (37, 57, 0, 1) lies on line \ell_2
179: P_{6704} = (46, 39, 0, 1) lies on line \ell_3
                                                                     233: P_{7856} = (46, 57, 0, 1) lies on line \ell_3
180: P_{6732} = (10, 40, 0, 1) lies on line \ell_1
                                                                     234: P_{7884} = (10, 58, 0, 1) lies on line \ell_1
181: P_{6759} = (37, 40, 0, 1) lies on line \ell_2
                                                                     235: P_{7911} = (37, 58, 0, 1) lies on line \ell_2
182 : P_{6768} = (46, 40, 0, 1) lies on line \ell_3
                                                                     236: P_{7920} = (46, 58, 0, 1) lies on line \ell_3
183: P_{6796} = (10, 41, 0, 1) lies on line \ell_1
                                                                     237: P_{7948} = (10, 59, 0, 1) lies on line \ell_1
184: P_{6823} = (37, 41, 0, 1) lies on line \ell_2
                                                                     238: P_{7975} = (37, 59, 0, 1) lies on line \ell_2
185: P_{6832} = (46, 41, 0, 1) lies on line \ell_3
                                                                     239: P_{7984} = (46, 59, 0, 1) lies on line \ell_3
186: P_{6860} = (10, 42, 0, 1) lies on line \ell_1
                                                                     240: P_{8012} = (10, 60, 0, 1) lies on line \ell_1
187: P_{6887} = (37, 42, 0, 1) lies on line \ell_2
                                                                     241: P_{8039} = (37, 60, 0, 1) lies on line \ell_2
188: P_{6896} = (46, 42, 0, 1) lies on line \ell_3
                                                                     242: P_{8048} = (46, 60, 0, 1) lies on line \ell_3
189: P_{6924} = (10, 43, 0, 1) lies on line \ell_1
                                                                     243: P_{8076} = (10, 61, 0, 1) lies on line \ell_1
190 : P_{6951} = (37, 43, 0, 1) lies on line \ell_2
                                                                     244: P_{8103} = (37, 61, 0, 1) lies on line \ell_2
191: P_{6960} = (46, 43, 0, 1) lies on line \ell_3
                                                                     245: P_{8112} = (46, 61, 0, 1) lies on line \ell_3
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246 : P_{8140} = (10, 62, 0, 1) lies on line \ell_1
                                                                     300: P_{73803} = (10, 0, 17, 1) lies on line \ell_4
                                                                     301: P_{73830} = (37, 0, 17, 1) lies on line \ell_5
247: P_{8167} = (37, 62, 0, 1) lies on line \ell_2
248: P_{8176} = (46, 62, 0, 1) lies on line \ell_3
                                                                     302: P_{73839} = (46, 0, 17, 1) lies on line \ell_6
249 : P_{8204} = (10, 63, 0, 1) lies on line \ell_1
                                                                     303: P_{77899} = (10, 0, 18, 1) lies on line \ell_4
250: P_{8231} = (37, 63, 0, 1) lies on line \ell_2
                                                                     304: P_{77926} = (37, 0, 18, 1) lies on line \ell_5
251: P_{8240} = (46, 63, 0, 1) lies on line \ell_3
                                                                     305: P_{77935} = (46, 0, 18, 1) lies on line \ell_6
252: P_{8268} = (10, 0, 1, 1) lies on line \ell_4
                                                                     306: P_{81995} = (10, 0, 19, 1) lies on line \ell_4
253: P_{8295} = (37, 0, 1, 1) lies on line \ell_5
                                                                     307: P_{82022} = (37, 0, 19, 1) lies on line \ell_5
254 : P_{8304} = (46, 0, 1, 1) lies on line \ell_6
                                                                     308: P_{82031} = (46, 0, 19, 1) lies on line \ell_6
255: P_{12363} = (10, 0, 2, 1) lies on line \ell_4
                                                                     309: P_{86091} = (10, 0, 20, 1) lies on line \ell_4
256: P_{12390} = (37, 0, 2, 1) lies on line \ell_5
                                                                     310: P_{86118} = (37, 0, 20, 1) lies on line \ell_5
257: P_{12399} = (46, 0, 2, 1) lies on line \ell_6
                                                                     311: P_{86127} = (46, 0, 20, 1) lies on line \ell_6
258: P_{16459} = (10, 0, 3, 1) lies on line \ell_4
                                                                     312: P_{90187} = (10, 0, 21, 1) lies on line \ell_4
259: P_{16486} = (37, 0, 3, 1) lies on line \ell_5
                                                                     313: P_{90214} = (37, 0, 21, 1) lies on line \ell_5
260: P_{16495} = (46, 0, 3, 1) lies on line \ell_6
                                                                     314 : P_{90223} = (46, 0, 21, 1) lies on line \ell_6
261: P_{20555} = (10, 0, 4, 1) lies on line \ell_4
                                                                     315: P_{94283} = (10, 0, 22, 1) lies on line \ell_4
262: P_{20582} = (37, 0, 4, 1) lies on line \ell_5
                                                                     316: P_{94310} = (37, 0, 22, 1) lies on line \ell_5
263: P_{20591} = (46, 0, 4, 1) lies on line \ell_6
                                                                     317: P_{94319} = (46, 0, 22, 1) lies on line \ell_6
                                                                     318 : P_{98379} = (10, 0, 23, 1) lies on line \ell_4
264: P_{24651} = (10, 0, 5, 1) lies on line \ell_4
265: P_{24678} = (37, 0, 5, 1) lies on line \ell_5
                                                                     319: P_{98406} = (37, 0, 23, 1) lies on line \ell_5
266: P_{24687} = (46, 0, 5, 1) lies on line \ell_6
                                                                     320 : P_{98415} = (46, 0, 23, 1) lies on line \ell_6
267: P_{28747} = (10, 0, 6, 1) lies on line \ell_4
                                                                     321: P_{102475} = (10, 0, 24, 1) lies on line \ell_4
268: P_{28774} = (37, 0, 6, 1) lies on line \ell_5
                                                                     322: P_{102502} = (37, 0, 24, 1) lies on line \ell_5
269 : P_{28783} = (46, 0, 6, 1) lies on line \ell_6
                                                                     323 : P_{102511} = (46, 0, 24, 1) lies on line \ell_6
270: P_{32843} = (10, 0, 7, 1) lies on line \ell_4
                                                                     324: P_{106571} = (10, 0, 25, 1) lies on line \ell_4
271: P_{32870} = (37, 0, 7, 1) lies on line \ell_5
                                                                     325: P_{106598} = (37, 0, 25, 1) lies on line \ell_5
272: P_{32879} = (46, 0, 7, 1) lies on line \ell_6
                                                                     326: P_{106607} = (46, 0, 25, 1) lies on line \ell_6
273: P_{36939} = (10, 0, 8, 1) lies on line \ell_4
                                                                     327: P_{110667} = (10, 0, 26, 1) lies on line \ell_4
274: P_{36966} = (37, 0, 8, 1) lies on line \ell_5
                                                                     328: P_{110694} = (37, 0, 26, 1) lies on line \ell_5
275: P_{36975} = (46, 0, 8, 1) lies on line \ell_6
                                                                     329: P_{110703} = (46, 0, 26, 1) lies on line \ell_6
276: P_{41035} = (10, 0, 9, 1) lies on line \ell_4
                                                                     330: P_{114763} = (10, 0, 27, 1) lies on line \ell_4
277: P_{41062} = (37, 0, 9, 1) lies on line \ell_5
                                                                     331: P_{114790} = (37, 0, 27, 1) lies on line \ell_5
278: P_{41071} = (46, 0, 9, 1) lies on line \ell_6
                                                                     332: P_{114799} = (46, 0, 27, 1) lies on line \ell_6
279 : P_{45131} = (10, 0, 10, 1) lies on line \ell_4
                                                                     333: P_{118859} = (10, 0, 28, 1) lies on line \ell_4
280: P_{45158} = (37, 0, 10, 1) lies on line \ell_5
                                                                     334: P_{118886} = (37, 0, 28, 1) lies on line \ell_5
                                                                     335 : P_{118895} = (46, 0, 28, 1) lies on line \ell_6
281 : P_{45167} = (46, 0, 10, 1) lies on line \ell_6
282: P_{49227} = (10, 0, 11, 1) lies on line \ell_4
                                                                     336: P_{122955} = (10, 0, 29, 1) lies on line \ell_4
283 : P_{49254} = (37, 0, 11, 1) lies on line \ell_5
                                                                     337: P_{122982} = (37, 0, 29, 1) lies on line \ell_5
284 : P_{49263} = (46, 0, 11, 1) lies on line \ell_6
                                                                     338: P_{122991} = (46, 0, 29, 1) lies on line \ell_6
285 : P_{53323} = (10, 0, 12, 1) lies on line \ell_4
                                                                     339: P_{127051} = (10, 0, 30, 1) lies on line \ell_4
286 : P_{53350} = (37, 0, 12, 1) lies on line \ell_5
                                                                     340: P_{127078} = (37, 0, 30, 1) lies on line \ell_5
287: P_{53359} = (46, 0, 12, 1) lies on line \ell_6
                                                                     341: P_{127087} = (46, 0, 30, 1) lies on line \ell_6
                                                                     342 : P_{131147} = (10, 0, 31, 1) lies on line \ell_4
288 : P_{57419} = (10, 0, 13, 1) lies on line \ell_4
289 : P_{57446} = (37, 0, 13, 1) lies on line \ell_5
                                                                     343: P_{131174} = (37, 0, 31, 1) lies on line \ell_5
290: P_{57455} = (46, 0, 13, 1) lies on line \ell_6
                                                                     344: P_{131183} = (46, 0, 31, 1) lies on line \ell_6
                                                                     345: P_{135243} = (10, 0, 32, 1) lies on line \ell_4
291: P_{61515} = (10, 0, 14, 1) lies on line \ell_4
292: P_{61542} = (37, 0, 14, 1) lies on line \ell_5
                                                                     346: P_{135270} = (37, 0, 32, 1) lies on line \ell_5
293: P_{61551} = (46, 0, 14, 1) lies on line \ell_6
                                                                     347: P_{135279} = (46, 0, 32, 1) lies on line \ell_6
294: P_{65611} = (10, 0, 15, 1) lies on line \ell_4
                                                                     348: P_{139339} = (10, 0, 33, 1) lies on line \ell_4
295: P_{65638} = (37, 0, 15, 1) lies on line \ell_5
                                                                     349: P_{139366} = (37, 0, 33, 1) lies on line \ell_5
296 : P_{65647} = (46, 0, 15, 1) lies on line \ell_6
                                                                     350: P_{139375} = (46, 0, 33, 1) lies on line \ell_6
297: P_{69707} = (10, 0, 16, 1) lies on line \ell_4
                                                                     351: P_{143435} = (10, 0, 34, 1) lies on line \ell_4
298 : P_{69734} = (37, 0, 16, 1) lies on line \ell_5
                                                                     352 : P_{143462} = (37, 0, 34, 1) lies on line \ell_5
299: P_{69743} = (46, 0, 16, 1) lies on line \ell_6
                                                                     353: P_{143471} = (46, 0, 34, 1) lies on line \ell_6
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354: P_{147531} = (10, 0, 35, 1) lies on line \ell_4
                                                                    398: P_{204911} = (46, 0, 49, 1) lies on line \ell_6
                                                                    399: P_{208971} = (10, 0, 50, 1) lies on line \ell_4
355: P_{147558} = (37, 0, 35, 1) lies on line \ell_5
                                                                    400: P_{208998} = (37, 0, 50, 1) lies on line \ell_5
356: P_{147567} = (46, 0, 35, 1) lies on line \ell_6
357: P_{151627} = (10, 0, 36, 1) lies on line \ell_4
                                                                    401: P_{209007} = (46, 0, 50, 1) lies on line \ell_6
                                                                    402: P_{213067} = (10, 0, 51, 1) lies on line \ell_4
358: P_{151654} = (37, 0, 36, 1) lies on line \ell_5
359: P_{151663} = (46, 0, 36, 1) lies on line \ell_6
                                                                    403: P_{213094} = (37, 0, 51, 1) lies on line \ell_5
360: P_{155723} = (10, 0, 37, 1) lies on line \ell_4
                                                                    404: P_{213103} = (46, 0, 51, 1) lies on line \ell_6
361: P_{155750} = (37, 0, 37, 1) lies on line \ell_5
                                                                    405: P_{217163} = (10, 0, 52, 1) lies on line \ell_4
362: P_{155759} = (46, 0, 37, 1) lies on line \ell_6
                                                                    406: P_{217190} = (37, 0, 52, 1) lies on line \ell_5
363: P_{159819} = (10, 0, 38, 1) lies on line \ell_4
                                                                    407: P_{217199} = (46, 0, 52, 1) lies on line \ell_6
364: P_{159846} = (37, 0, 38, 1) lies on line \ell_5
                                                                    408: P_{221259} = (10, 0, 53, 1) lies on line \ell_4
365: P_{159855} = (46, 0, 38, 1) lies on line \ell_6
                                                                    409: P_{221286} = (37, 0, 53, 1) lies on line \ell_5
366: P_{163915} = (10, 0, 39, 1) lies on line \ell_4
                                                                    410: P_{221295} = (46, 0, 53, 1) lies on line \ell_6
367: P_{163942} = (37, 0, 39, 1) lies on line \ell_5
                                                                    411: P_{225355} = (10, 0, 54, 1) lies on line \ell_4
368: P_{163951} = (46, 0, 39, 1) lies on line \ell_6
                                                                    412: P_{225382} = (37, 0, 54, 1) lies on line \ell_5
369: P_{168011} = (10, 0, 40, 1) lies on line \ell_4
                                                                    413: P_{225391} = (46, 0, 54, 1) lies on line \ell_6
370: P_{168038} = (37, 0, 40, 1) lies on line \ell_5
                                                                    414: P_{229451} = (10, 0, 55, 1) lies on line \ell_4
371: P_{168047} = (46, 0, 40, 1) lies on line \ell_6
                                                                    415: P_{229478} = (37, 0, 55, 1) lies on line \ell_5
372: P_{172107} = (10, 0, 41, 1) lies on line \ell_4
                                                                    416: P_{229487} = (46, 0, 55, 1) lies on line \ell_6
                                                                    417 : P_{233547} = (10, 0, 56, 1) lies on line \ell_4
373: P_{172134} = (37, 0, 41, 1) lies on line \ell_5
374: P_{172143} = (46, 0, 41, 1) lies on line \ell_6
                                                                    418: P_{233574} = (37, 0, 56, 1) lies on line \ell_5
375: P_{176203} = (10, 0, 42, 1) lies on line \ell_4
                                                                    419: P_{233583} = (46, 0, 56, 1) lies on line \ell_6
376: P_{176230} = (37, 0, 42, 1) lies on line \ell_5
                                                                    420: P_{237643} = (10, 0, 57, 1) lies on line \ell_4
377 : P_{176239} = (46, 0, 42, 1) lies on line \ell_6
                                                                    421 : P_{237670} = (37, 0, 57, 1) lies on line \ell_5
378: P_{180299} = (10, 0, 43, 1) lies on line \ell_4
                                                                    422: P_{237679} = (46, 0, 57, 1) lies on line \ell_6
379: P_{180326} = (37, 0, 43, 1) lies on line \ell_5
                                                                    423: P_{241739} = (10, 0, 58, 1) lies on line \ell_4
380: P_{180335} = (46, 0, 43, 1) lies on line \ell_6
                                                                    424: P_{241766} = (37, 0, 58, 1) lies on line \ell_5
381: P_{184395} = (10, 0, 44, 1) lies on line \ell_4
                                                                    425: P_{241775} = (46, 0, 58, 1) lies on line \ell_6
382: P_{184422} = (37, 0, 44, 1) lies on line \ell_5
                                                                    426: P_{245835} = (10, 0, 59, 1) lies on line \ell_4
383: P_{184431} = (46, 0, 44, 1) lies on line \ell_6
                                                                    427: P_{245862} = (37, 0, 59, 1) lies on line \ell_5
384: P_{188491} = (10, 0, 45, 1) lies on line \ell_4
                                                                    428: P_{245871} = (46, 0, 59, 1) lies on line \ell_6
385: P_{188518} = (37, 0, 45, 1) lies on line \ell_5
                                                                    429: P_{249931} = (10, 0, 60, 1) lies on line \ell_4
386: P_{188527} = (46, 0, 45, 1) lies on line \ell_6
                                                                    430: P_{249958} = (37, 0, 60, 1) lies on line \ell_5
387: P_{192587} = (10, 0, 46, 1) lies on line \ell_4
                                                                    431: P_{249967} = (46, 0, 60, 1) lies on line \ell_6
388: P_{192614} = (37, 0, 46, 1) lies on line \ell_5
                                                                    432: P_{254027} = (10, 0, 61, 1) lies on line \ell_4
                                                                    433 : P_{254054} = (37, 0, 61, 1) lies on line \ell_5
389: P_{192623} = (46, 0, 46, 1) lies on line \ell_6
390: P_{196683} = (10, 0, 47, 1) lies on line \ell_4
                                                                    434: P_{254063} = (46, 0, 61, 1) lies on line \ell_6
391: P_{196710} = (37, 0, 47, 1) lies on line \ell_5
                                                                    435: P_{258123} = (10, 0, 62, 1) lies on line \ell_4
                                                                    436: P_{258150} = (37, 0, 62, 1) lies on line \ell_5
392 : P_{196719} = (46, 0, 47, 1) lies on line \ell_6
393: P_{200779} = (10, 0, 48, 1) lies on line \ell_4
                                                                    437: P_{258159} = (46, 0, 62, 1) lies on line \ell_6
394: P_{200806} = (37, 0, 48, 1) lies on line \ell_5
                                                                    438: P_{262219} = (10, 0, 63, 1) lies on line \ell_4
395: P_{200815} = (46, 0, 48, 1) lies on line \ell_6
                                                                    439: P_{262246} = (37, 0, 63, 1) lies on line \ell_5
396: P_{204875} = (10, 0, 49, 1) lies on line \ell_4
                                                                    440: P_{262255} = (46, 0, 63, 1) lies on line \ell_6
397: P_{204902} = (37, 0, 49, 1) lies on line \ell_5
```

The single points on the surface are:

#### Points on surface but on no line

The surface has 3843 points not on any line: Too many to print.

## Line Intersection Graph

 $\begin{array}{c} 0123456 \\ \hline 0 & 0111111 \\ 1 & 1011100 \\ 2 & 1101010 \\ 3 & 1110001 \\ 4 & 1100011 \\ 5 & 1010101 \\ 6 & 1001110 \end{array}$ 

Neighbor sets in the line intersection graph:

Line 0 intersects

$\operatorname{Line}$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_1$	$P_1$	$P_1$	$P_2$	$P_2$	$P_2$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_1$	$P_1$	$P_1$	$P_{4172}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_5$
in point	$P_1$	$P_1$	$P_1$	$P_{4199}$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_6$
in point	$P_1$	$P_1$	$P_1$	$P_{4208}$

Line 4 intersects

Line	$\ell_0$	$\ell_1$	$\ell_5$	$\ell_6$
in point	$P_2$	$P_{4172}$	$P_2$	$P_2$

Line 5 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_6$
in point	$P_2$	$P_{4199}$	$P_2$	$P_2$

Line 6 intersects

I	Line	$\ell_0$	$\ell_3$	$\ell_4$	$\ell_5$
	in point	$P_2$	$P_{4208}$	$P_2$	$P_2$

The surface has 4289 points:

Too many to print.