Rank-65903 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65903

General information

Number of lines	4
Number of points	11
Number of singular points	0
Number of Eckardt points	0
Number of double points	4
Number of single points	4
Number of points off lines	3
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^4
Type of lines on points	2^4, 1^4, 0^3

Singular Points

The surface has 0 singular points:

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

Rank of lines: (0, 5, 14, 12)
Rank of points on Klein quadric: (0, 12, 6, 32)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$
$P_{12} = (0,0,1,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0:\ P_4 = (1,1,1,1)$ lies on line ℓ_3
$1:\ P_9 = (1,0,0,1)$ lies on line ℓ_2
$2:\ P_{11} = (1,1,0,1)$ lies on line ℓ_2
$3:\ P_{13} = (1,0,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 3 points not on any line:
The points on the surface but not on lines are:

$0:\ P_6 = (1,0,1,0)$
$1:\ P_7 = (0,1,1,0)$
$2:\ P_8 = (1,1,1,0)$

Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	0	1
2	1	0	0	0
3	1	1	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_{12}

Line 2 intersects

Line	ℓ_0
in point	P_1

Line 3 intersects

Line	ℓ_0	ℓ_1
in point	P_5	P_{12}

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (1,0,1,0)$
$5: P_7 = (0,1,1,0)$
$6: P_8 = (1,1,1,0)$
$7: P_9 = (1,0,0,1)$
$8: P_{11} = (1,1,0,1)$
$9: P_{12} = (0,0,1,1)$
$10: P_{13} = (1,0,1,1)$