Rank-65611 over GF(2)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65611

General information

Number of lines	5
Number of points	9
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	5
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	3^2, 2^2, 1^5

Singular Points

The surface has 3 singular points:

$0: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

$1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

$2: P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$
$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

Rank of lines: (0, 4, 28, 14, 18)
Rank of points on Klein quadric: (0, 2, 19, 6, 4)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),\ T = -1$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).\ T = -1$

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_9 = (1,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 5 single points:
The single points on the surface are:

$0:\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$1:\ P_6 = (1,0,1,0)$ lies on line ℓ_1
$2:\ P_7 = (0,1,1,0)$ lies on line ℓ_2
$3:\ P_{11} = (1,1,0,1)$ lies on line ℓ_3
$4:\ P_{13} = (1,0,1,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	1	1
3	1	0	1	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_1	P_2	P_1	P_2

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_1	P_1	P_9

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_9

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (1,0,1,0)$
$5: P_7 = (0,1,1,0)$
$6: P_9 = (1,0,0,1)$
$7: P_{11} = (1,1,0,1)$
$8: P_{13} = (1,0,1,1)$