Rank-21 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 = 0$$

(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 21

General information

Number of lines	7
Number of points	7
Number of singular points	3
Number of Eckardt points	7
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^7
Type of lines on points	3^7

Singular Points

The surface has 3 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

$1: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

$2: P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{11} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{11} = \mathbf{Pl}(0,0,1,0,0,1)_{22}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{34} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{34} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$
$$\ell_2 = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{25} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{25} = \mathbf{Pl}(0,1,1,0,0,1)_{24}$$
$$\ell_3 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$
$$\ell_4 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{12} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$
$$\ell_5 = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{27} = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{27} = \mathbf{Pl}(0,1,0,1,1,0)_{17}$$
$$\ell_6 = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{26} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{26} = \mathbf{Pl}(0,1,1,1,1,1)_{34}$$

Rank of lines: (11, 34, 25, 13, 12, 27, 26)
Rank of points on Klein quadric: (22, 1, 24, 15, 32, 17, 34)

Eckardt Points

The surface has 7 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0),\ T = -1$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1),\ T = -1$
$2 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1),\ T = 13$
$3 : P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0),\ T = 13$
$4 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0),\ T = 13$
$5 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1),\ T = 13$
$6 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1).\ T = -1$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	0	1	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	1	0	1	1
5	1	1	1	1	1	0	1
6	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_5	P_5	P_8	P_8

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_3	P_{12}	P_3	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_{11}	P_4	P_4	P_{11}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_5	P_3	P_{11}	P_5	P_3	P_{11}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_5	P_{12}	P_4	P_5	P_4	P_{12}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_8	P_3	P_4	P_3	P_4	P_8

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_8	P_{12}	P_{11}	P_{11}	P_{12}	P_8

The surface has 7 points:
The points on the surface are:

$0:\ P_2 = (0,0,1,0)$
$1:\ P_3 = (0,0,0,1)$
$2:\ P_4 = (1,1,1,1)$
$3:\ P_5 = (1,1,0,0)$
$4:\ P_8 = (1,1,1,0)$
$5:\ P_{11} = (1,1,0,1)$
$6:\ P_{12} = (0,0,1,1)$