

Rank-65633 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431660205

General information

Number of lines	2
Number of points	25
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	10
Number of points off lines	15
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^2
Type of lines on points	$1^{10}, 0^{15}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \mathbf{Pl}(1, 0, 0, 0, 1, 0)_{26} \\ \ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{198}\end{aligned}$$

Rank of lines: (4, 38)

Rank of points on Klein quadric: (26, 198)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 10 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
 1 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_1
 2 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_1
 3 : $P_{26} = (0, 1, 0, 1)$ lies on line ℓ_0
 4 : $P_{27} = (1, 1, 0, 1)$ lies on line ℓ_0
 5 : $P_{28} = (2, 1, 0, 1)$ lies on line ℓ_0

6 : $P_{29} = (3, 1, 0, 1)$ lies on line ℓ_0
 7 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_1
 8 : $P_{47} = (2, 2, 1, 1)$ lies on line ℓ_1
 9 : $P_{52} = (3, 3, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 15 points not on any line:

The points on the surface but not on lines are:

0 : $P_{11} = (0, 1, 1, 0)$
 1 : $P_{12} = (1, 1, 1, 0)$
 2 : $P_{15} = (0, 2, 1, 0)$
 3 : $P_{16} = (1, 2, 1, 0)$
 4 : $P_{19} = (0, 3, 1, 0)$
 5 : $P_{20} = (1, 3, 1, 0)$
 6 : $P_{23} = (1, 0, 0, 1)$
 7 : $P_{30} = (0, 2, 0, 1)$

8 : $P_{34} = (0, 3, 0, 1)$
 9 : $P_{46} = (1, 2, 1, 1)$
 10 : $P_{50} = (1, 3, 1, 1)$
 11 : $P_{53} = (0, 0, 2, 1)$
 12 : $P_{60} = (3, 1, 2, 1)$
 13 : $P_{69} = (0, 0, 3, 1)$
 14 : $P_{75} = (2, 1, 3, 1)$

Line Intersection Graph

	0	1
0	0	0
1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 25 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_4 = (1, 1, 1, 1)$
 2 : $P_5 = (1, 1, 0, 0)$
 3 : $P_{11} = (0, 1, 1, 0)$
 4 : $P_{12} = (1, 1, 1, 0)$
 5 : $P_{15} = (0, 2, 1, 0)$
 6 : $P_{16} = (1, 2, 1, 0)$
 7 : $P_{19} = (0, 3, 1, 0)$
 8 : $P_{20} = (1, 3, 1, 0)$

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 10 : $P_{26} = (0, 1, 0, 1)$
 11 : $P_{27} = (1, 1, 0, 1)$
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 13 : $P_{29} = (3, 1, 0, 1)$
 14 : $P_{30} = (0, 2, 0, 1)$
 15 : $P_{34} = (0, 3, 0, 1)$
 16 : $P_{38} = (0, 0, 1, 1)$
 17 : $P_{46} = (1, 2, 1, 1)$

18 : $P_{47} = (2, 2, 1, 1)$
 19 : $P_{50} = (1, 3, 1, 1)$
 20 : $P_{52} = (3, 3, 1, 1)$
 21 : $P_{53} = (0, 0, 2, 1)$
 22 : $P_{60} = (3, 1, 2, 1)$
 23 : $P_{69} = (0, 0, 3, 1)$
 24 : $P_{75} = (2, 1, 3, 1)$