

# Rank-74055 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1244172941

## General information

Number of lines	4
Number of points	81
Number of singular points	1
Number of Eckardt points	1
Number of double points	1
Number of single points	31
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^4$
Type of lines on points	$3, 2, 1^{31}, 0^{48}$

## Singular Points

The surface has 1 singular points:

$$0 : P_{146} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$$

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17} \\ \ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{18} \\ \ell_3 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25}\end{aligned}$$

Rank of lines: ( 0, 4680, 584, 4689 )

Rank of points on Klein quadric: ( 0, 17, 18, 25 )

### Eckardt Points

The surface has 1 Eckardt points:

0 :  $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$ .

### Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_3 = (0, 0, 0, 1) = \ell_1 \cap \ell_3$$

### Single Points

The surface has 31 single points:

The single points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$  lies on line  $\ell_0$   
1 :  $P_5 = (1, 1, 0, 0)$  lies on line  $\ell_0$   
2 :  $P_6 = (2, 1, 0, 0)$  lies on line  $\ell_0$   
3 :  $P_7 = (3, 1, 0, 0)$  lies on line  $\ell_0$   
4 :  $P_8 = (4, 1, 0, 0)$  lies on line  $\ell_0$   
5 :  $P_9 = (5, 1, 0, 0)$  lies on line  $\ell_0$   
6 :  $P_{10} = (6, 1, 0, 0)$  lies on line  $\ell_0$   
7 :  $P_{11} = (7, 1, 0, 0)$  lies on line  $\ell_0$   
8 :  $P_{19} = (0, 1, 1, 0)$  lies on line  $\ell_3$   
9 :  $P_{75} = (1, 0, 0, 1)$  lies on line  $\ell_2$   
10 :  $P_{82} = (0, 1, 0, 1)$  lies on line  $\ell_1$   
11 :  $P_{83} = (1, 1, 0, 1)$  lies on line  $\ell_2$   
12 :  $P_{90} = (0, 2, 0, 1)$  lies on line  $\ell_1$   
13 :  $P_{91} = (1, 2, 0, 1)$  lies on line  $\ell_2$   
14 :  $P_{98} = (0, 3, 0, 1)$  lies on line  $\ell_1$   
15 :  $P_{99} = (1, 3, 0, 1)$  lies on line  $\ell_2$

16 :  $P_{106} = (0, 4, 0, 1)$  lies on line  $\ell_1$   
17 :  $P_{107} = (1, 4, 0, 1)$  lies on line  $\ell_2$   
18 :  $P_{114} = (0, 5, 0, 1)$  lies on line  $\ell_1$   
19 :  $P_{115} = (1, 5, 0, 1)$  lies on line  $\ell_2$   
20 :  $P_{122} = (0, 6, 0, 1)$  lies on line  $\ell_1$   
21 :  $P_{123} = (1, 6, 0, 1)$  lies on line  $\ell_2$   
22 :  $P_{130} = (0, 7, 0, 1)$  lies on line  $\ell_1$   
23 :  $P_{131} = (1, 7, 0, 1)$  lies on line  $\ell_2$   
24 :  $P_{146} = (0, 1, 1, 1)$  lies on line  $\ell_3$   
25 :  $P_{217} = (0, 2, 2, 1)$  lies on line  $\ell_3$   
26 :  $P_{289} = (0, 3, 3, 1)$  lies on line  $\ell_3$   
27 :  $P_{361} = (0, 4, 4, 1)$  lies on line  $\ell_3$   
28 :  $P_{433} = (0, 5, 5, 1)$  lies on line  $\ell_3$   
29 :  $P_{505} = (0, 6, 6, 1)$  lies on line  $\ell_3$   
30 :  $P_{577} = (0, 7, 7, 1)$  lies on line  $\ell_3$

The single points on the surface are:

### Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

0 : $P_{31} = (4, 2, 1, 0)$	25 : $P_{367} = (6, 4, 4, 1)$
1 : $P_{42} = (7, 3, 1, 0)$	26 : $P_{403} = (2, 1, 5, 1)$
2 : $P_{50} = (7, 4, 1, 0)$	27 : $P_{407} = (6, 1, 5, 1)$
3 : $P_{53} = (2, 5, 1, 0)$	28 : $P_{410} = (1, 2, 5, 1)$
4 : $P_{63} = (4, 6, 1, 0)$	29 : $P_{416} = (7, 2, 5, 1)$
5 : $P_{69} = (2, 7, 1, 0)$	30 : $P_{418} = (1, 3, 5, 1)$
6 : $P_{207} = (6, 0, 2, 1)$	31 : $P_{419} = (2, 3, 5, 1)$
7 : $P_{208} = (7, 0, 2, 1)$	32 : $P_{440} = (7, 5, 5, 1)$
8 : $P_{222} = (5, 2, 2, 1)$	33 : $P_{455} = (6, 7, 5, 1)$
9 : $P_{255} = (6, 6, 2, 1)$	34 : $P_{468} = (3, 1, 6, 1)$
10 : $P_{262} = (5, 7, 2, 1)$	35 : $P_{469} = (4, 1, 6, 1)$
11 : $P_{264} = (7, 7, 2, 1)$	36 : $P_{476} = (3, 2, 6, 1)$
12 : $P_{278} = (5, 1, 3, 1)$	37 : $P_{490} = (1, 4, 6, 1)$
13 : $P_{280} = (7, 1, 3, 1)$	38 : $P_{491} = (2, 4, 6, 1)$
14 : $P_{293} = (4, 3, 3, 1)$	39 : $P_{498} = (1, 5, 6, 1)$
15 : $P_{302} = (5, 4, 3, 1)$	40 : $P_{501} = (4, 5, 6, 1)$
16 : $P_{314} = (1, 6, 3, 1)$	41 : $P_{507} = (2, 6, 6, 1)$
17 : $P_{320} = (7, 6, 3, 1)$	42 : $P_{525} = (4, 0, 7, 1)$
18 : $P_{322} = (1, 7, 3, 1)$	43 : $P_{526} = (5, 0, 7, 1)$
19 : $P_{325} = (4, 7, 3, 1)$	44 : $P_{556} = (3, 4, 7, 1)$
20 : $P_{331} = (2, 0, 4, 1)$	45 : $P_{557} = (4, 4, 7, 1)$
21 : $P_{332} = (3, 0, 4, 1)$	46 : $P_{566} = (5, 5, 7, 1)$
22 : $P_{347} = (2, 2, 4, 1)$	47 : $P_{580} = (3, 7, 7, 1)$
23 : $P_{351} = (6, 2, 4, 1)$	
24 : $P_{356} = (3, 3, 4, 1)$	

## Line Intersection Graph

	0 1 2 3
0	0 1 1 0
1	1 0 1 1
2	1 1 0 0
3	0 1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_1$	$P_1$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$
in point	$P_1$	$P_1$	$P_3$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_1$	$P_1$

Line 3 intersects

Line	$\ell_1$
in point	$P_3$

The surface has 81 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_1 = (0, 1, 0, 0)$   
 2 :  $P_3 = (0, 0, 0, 1)$   
 3 :  $P_5 = (1, 1, 0, 0)$   
 4 :  $P_6 = (2, 1, 0, 0)$   
 5 :  $P_7 = (3, 1, 0, 0)$   
 6 :  $P_8 = (4, 1, 0, 0)$   
 7 :  $P_9 = (5, 1, 0, 0)$   
 8 :  $P_{10} = (6, 1, 0, 0)$   
 9 :  $P_{11} = (7, 1, 0, 0)$   
 10 :  $P_{19} = (0, 1, 1, 0)$   
 11 :  $P_{31} = (4, 2, 1, 0)$   
 12 :  $P_{42} = (7, 3, 1, 0)$   
 13 :  $P_{50} = (7, 4, 1, 0)$   
 14 :  $P_{53} = (2, 5, 1, 0)$   
 15 :  $P_{63} = (4, 6, 1, 0)$   
 16 :  $P_{69} = (2, 7, 1, 0)$   
 17 :  $P_{75} = (1, 0, 0, 1)$   
 18 :  $P_{82} = (0, 1, 0, 1)$   
 19 :  $P_{83} = (1, 1, 0, 1)$   
 20 :  $P_{90} = (0, 2, 0, 1)$   
 21 :  $P_{91} = (1, 2, 0, 1)$   
 22 :  $P_{98} = (0, 3, 0, 1)$   
 23 :  $P_{99} = (1, 3, 0, 1)$   
 24 :  $P_{106} = (0, 4, 0, 1)$   
 25 :  $P_{107} = (1, 4, 0, 1)$   
 26 :  $P_{114} = (0, 5, 0, 1)$   
 27 :  $P_{115} = (1, 5, 0, 1)$

28 :  $P_{122} = (0, 6, 0, 1)$   
 29 :  $P_{123} = (1, 6, 0, 1)$   
 30 :  $P_{130} = (0, 7, 0, 1)$   
 31 :  $P_{131} = (1, 7, 0, 1)$   
 32 :  $P_{146} = (0, 1, 1, 1)$   
 33 :  $P_{207} = (6, 0, 2, 1)$   
 34 :  $P_{208} = (7, 0, 2, 1)$   
 35 :  $P_{217} = (0, 2, 2, 1)$   
 36 :  $P_{222} = (5, 2, 2, 1)$   
 37 :  $P_{255} = (6, 6, 2, 1)$   
 38 :  $P_{262} = (5, 7, 2, 1)$   
 39 :  $P_{264} = (7, 7, 2, 1)$   
 40 :  $P_{278} = (5, 1, 3, 1)$   
 41 :  $P_{280} = (7, 1, 3, 1)$   
 42 :  $P_{289} = (0, 3, 3, 1)$   
 43 :  $P_{293} = (4, 3, 3, 1)$   
 44 :  $P_{302} = (5, 4, 3, 1)$   
 45 :  $P_{314} = (1, 6, 3, 1)$   
 46 :  $P_{320} = (7, 6, 3, 1)$   
 47 :  $P_{322} = (1, 7, 3, 1)$   
 48 :  $P_{325} = (4, 7, 3, 1)$   
 49 :  $P_{331} = (2, 0, 4, 1)$   
 50 :  $P_{332} = (3, 0, 4, 1)$   
 51 :  $P_{347} = (2, 2, 4, 1)$   
 52 :  $P_{351} = (6, 2, 4, 1)$   
 53 :  $P_{356} = (3, 3, 4, 1)$   
 54 :  $P_{361} = (0, 4, 4, 1)$   
 55 :  $P_{367} = (6, 4, 4, 1)$

56 :  $P_{403} = (2, 1, 5, 1)$   
 57 :  $P_{407} = (6, 1, 5, 1)$   
 58 :  $P_{410} = (1, 2, 5, 1)$   
 59 :  $P_{416} = (7, 2, 5, 1)$   
 60 :  $P_{418} = (1, 3, 5, 1)$   
 61 :  $P_{419} = (2, 3, 5, 1)$   
 62 :  $P_{433} = (0, 5, 5, 1)$   
 63 :  $P_{440} = (7, 5, 5, 1)$   
 64 :  $P_{455} = (6, 7, 5, 1)$   
 65 :  $P_{468} = (3, 1, 6, 1)$   
 66 :  $P_{469} = (4, 1, 6, 1)$   
 67 :  $P_{476} = (3, 2, 6, 1)$   
 68 :  $P_{490} = (1, 4, 6, 1)$   
 69 :  $P_{491} = (2, 4, 6, 1)$   
 70 :  $P_{498} = (1, 5, 6, 1)$   
 71 :  $P_{501} = (4, 5, 6, 1)$   
 72 :  $P_{505} = (0, 6, 6, 1)$   
 73 :  $P_{507} = (2, 6, 6, 1)$   
 74 :  $P_{525} = (4, 0, 7, 1)$   
 75 :  $P_{526} = (5, 0, 7, 1)$   
 76 :  $P_{556} = (3, 4, 7, 1)$   
 77 :  $P_{557} = (4, 4, 7, 1)$   
 78 :  $P_{566} = (5, 5, 7, 1)$   
 79 :  $P_{577} = (0, 7, 7, 1)$   
 80 :  $P_{580} = (3, 7, 7, 1)$