Rank-65863 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(16) is 303108629

General information

Number of lines	3
Number of points	289
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	47
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 ³
Type of lines on points	$2^2, 1^{47}, 0^{240}$

Singular Points

The surface has 1 singular points:

$$0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{33}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{49}$$

Rank of lines: (0, 69904, 69921)

Rank of points on Klein quadric: (0, 33, 49)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points: The double points on the surface are:

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_3 = (0, 0, 0, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 47 single points: The single points on the surface are:

 $0: P_0 = (1, 0, 0, 0)$ lies on line ℓ_0 1: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 2 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 $3: P_7 = (3,1,0,0)$ lies on line ℓ_0 4: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 5: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 6: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 7: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 8: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 9: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 10: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 11: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 12: $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 13: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 14: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 15 : $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 16: $P_{35} = (0, 1, 1, 0)$ lies on line ℓ_2 17: $P_{290} = (0, 1, 0, 1)$ lies on line ℓ_1 18: $P_{306} = (0, 2, 0, 1)$ lies on line ℓ_1 19: $P_{322} = (0, 3, 0, 1)$ lies on line ℓ_1 20: $P_{338} = (0, 4, 0, 1)$ lies on line ℓ_1 21: $P_{354} = (0, 5, 0, 1)$ lies on line ℓ_1 22: $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_1 23: $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_1

24: $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_1 25: $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_1 26: $P_{434} = (0, 10, 0, 1)$ lies on line ℓ_1 27: $P_{450} = (0, 11, 0, 1)$ lies on line ℓ_1 28: $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_1 29: $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_1 $30: P_{498} = (0, 14, 0, 1)$ lies on line ℓ_1 $31: P_{514} = (0, 15, 0, 1)$ lies on line ℓ_1 $32: P_{546} = (0, 1, 1, 1)$ lies on line ℓ_2 33: $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_2 $34: P_{1089} = (0,3,3,1)$ lies on line ℓ_2 $35: P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_2 $36: P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_2 $37: P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_2 38: $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_2 $39: P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_2 40: $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_2 41: $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_2 $42: P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_2 43: $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_2 44: $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_2 45: $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_2 46 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line: The points on the surface but not on lines are:

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0: P_4 = (1, 1, 1, 1)
                                                                  48: P_{1122} = (1, 5, 3, 1)
1: P_{65} = (14, 2, 1, 0)
                                                                  49: P_{1135} = (14, 5, 3, 1)
2: P_{78} = (11, 3, 1, 0)
                                                                  50: P_{1142} = (5, 6, 3, 1)
                                                                  51: P_{1152} = (15, 6, 3, 1)
3: P_{85} = (2,4,1,0)
4: P_{109} = (10, 5, 1, 0)
                                                                  52: P_{1177} = (8, 8, 3, 1)
5: P_{117} = (2, 6, 1, 0)
                                                                  53: P_{1178} = (9, 8, 3, 1)
6: P_{140} = (9,7,1,0)
                                                                  54: P_{1198} = (13, 9, 3, 1)
7: P_{158} = (11, 8, 1, 0)
                                                                  55: P_{1200} = (15, 9, 3, 1)
8: P_{167} = (4, 9, 1, 0)
                                                                  56: P_{1227} = (10, 11, 3, 1)
9: P_{180} = (1, 10, 1, 0)
                                                                  57: P_{1231} = (14, 11, 3, 1)
                                                                  58: P_{1305} = (8, 0, 4, 1)
10: P_{196} = (1, 11, 1, 0)
11: P_{225} = (14, 12, 1, 0)
                                                                  59: P_{1336} = (7, 2, 4, 1)
12: P_{231} = (4, 13, 1, 0)
                                                                  60: P_{1344} = (15, 2, 4, 1)
13: P_{252} = (9, 14, 1, 0)
                                                                  61: P_{1370} = (9, 4, 4, 1)
                                                                  62: P_{1384} = (7, 5, 4, 1)
14: P_{269} = (10, 15, 1, 0)
15: P_{531} = (1, 0, 1, 1)
                                                                  63: P_{1387} = (10, 5, 4, 1)
16: P_{570} = (9, 2, 1, 1)
                                                                  64: P_{1428} = (3, 8, 4, 1)
17: P_{572} = (11, 2, 1, 1)
                                                                  65: P_{1433} = (8, 8, 4, 1)
                                                                  66: P_{1476} = (3, 11, 4, 1)
18: P_{603} = (10, 4, 1, 1)
                                                                  67: P_{1477} = (4, 11, 4, 1)
19: P_{607} = (14, 4, 1, 1)
20: P_{675} = (2, 9, 1, 1)
                                                                  68: P_{1514} = (9, 13, 4, 1)
21: P_{684} = (11, 9, 1, 1)
                                                                  69: P_{1520} = (15, 13, 4, 1)
22: P_{693} = (4, 10, 1, 1)
                                                                  70: P_{1541} = (4, 15, 4, 1)
23: P_{703} = (14, 10, 1, 1)
                                                                  71: P_{1547} = (10, 15, 4, 1)
24: P_{707} = (2, 11, 1, 1)
                                                                  72: P_{1568} = (15, 0, 5, 1)
25: P_{714} = (9, 11, 1, 1)
                                                                  73: P_{1580} = (11, 1, 5, 1)
26: P_{757} = (4, 14, 1, 1)
                                                                  74: P_{1583} = (14, 1, 5, 1)
27: P_{763} = (10, 14, 1, 1)
                                                                  75: P_{1641} = (8, 5, 5, 1)
28: P_{790} = (5,0,2,1)
                                                                  76: P_{1682} = (1, 8, 5, 1)
29: P_{821} = (4, 2, 2, 1)
                                                                  77: P_{1683} = (2, 8, 5, 1)
30: P_{844} = (11, 3, 2, 1)
                                                                  78: P_{1698} = (1, 9, 5, 1)
31: P_{846} = (13, 3, 2, 1)
                                                                  79: P_{1704} = (7, 9, 5, 1)
32: P_{870} = (5, 5, 2, 1)
                                                                  80: P_{1715} = (2, 10, 5, 1)
33: P_{880} = (15, 5, 2, 1)
                                                                  81: P_{1724} = (11, 10, 5, 1)
34: P_{885} = (4, 6, 2, 1)
                                                                  82: P_{1764} = (3, 13, 5, 1)
                                                                  83: P_{1769} = (8, 13, 5, 1)
35: P_{889} = (8, 6, 2, 1)
                                                                  84: P_{1780} = (3, 14, 5, 1)
36: P_{915} = (2, 8, 2, 1)
37: P_{924} = (11, 8, 2, 1)
                                                                  85: P_{1784} = (7, 14, 5, 1)
38: P_{947} = (2, 10, 2, 1)
                                                                  86: P_{1807} = (14, 15, 5, 1)
39: P_{960} = (15, 10, 2, 1)
                                                                  87: P_{1808} = (15, 15, 5, 1)
40: P_{1017} = (8, 14, 2, 1)
                                                                  88: P_{1812} = (3, 0, 6, 1)
41: P_{1022} = (13, 14, 2, 1)
                                                                  89: P_{1860} = (3, 3, 6, 1)
42: P_{1049} = (8,0,3,1)
                                                                  90: P_{1866} = (9, 3, 6, 1)
43: P_{1066} = (9, 1, 3, 1)
                                                                  91: P_{1883} = (10, 4, 6, 1)
44: P_{1067} = (10, 1, 3, 1)
                                                                  92: P_{1884} = (11, 4, 6, 1)
45: P_{1094} = (5, 3, 3, 1)
                                                                  93: P_{1918} = (13, 6, 6, 1)
46: P_{1106} = (1, 4, 3, 1)
                                                                  94: P_{1976} = (7, 10, 6, 1)
47: P_{1118} = (13, 4, 3, 1)
                                                                  95: P_{1978} = (9, 10, 6, 1)
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150: P_{2834} = (1, 0, 10, 1)
96: P_{1990} = (5, 11, 6, 1)
97: P_{1998} = (13, 11, 6, 1)
                                                                 151: P_{2850} = (1, 1, 10, 1)
98: P_{2019} = (2, 13, 6, 1)
                                                                 152: P_{2860} = (11, 1, 10, 1)
99: P_{2024} = (7, 13, 6, 1)
                                                                 153: P_{2916} = (3, 5, 10, 1)
100: P_{2038} = (5, 14, 6, 1)
                                                                 154: P_{2923} = (10, 5, 10, 1)
101: P_{2043} = (10, 14, 6, 1)
                                                                 155: P_{2932} = (3, 6, 10, 1)
102: P_{2051} = (2, 15, 6, 1)
                                                                 156: P_{2942} = (13, 6, 10, 1)
103: P_{2060} = (11, 15, 6, 1)
                                                                 157: P_{2953} = (8, 7, 10, 1)
104: P_{2073} = (8, 0, 7, 1)
                                                                 158: P_{2957} = (12, 7, 10, 1)
105: P_{2139} = (10, 4, 7, 1)
                                                                 159: P_{3004} = (11, 10, 10, 1)
106: P_{2144} = (15, 4, 7, 1)
                                                                 160: P_{3021} = (12, 11, 10, 1)
107: P_{2154} = (9, 5, 7, 1)
                                                                 161: P_{3022} = (13, 11, 10, 1)
108: P_{2156} = (11, 5, 7, 1)
                                                                 162: P_{3081} = (8, 15, 10, 1)
                                                                 163: P_{3083} = (10, 15, 10, 1)
109: P_{2189} = (12, 7, 7, 1)
110: P_{2195} = (2, 8, 7, 1)
                                                                 164: P_{3090} = (1, 0, 11, 1)
111: P_{2201} = (8, 8, 7, 1)
                                                                 165: P_{3106} = (1, 1, 11, 1)
112: P_{2227} = (2, 10, 7, 1)
                                                                 166: P_{3115} = (10, 1, 11, 1)
113: P_{2231} = (6, 10, 7, 1)
                                                                 167: P_{3148} = (11, 3, 11, 1)
114: P_{2253} = (12, 11, 7, 1)
                                                                 168: P_{3152} = (15, 3, 11, 1)
115: P_{2256} = (15, 11, 7, 1)
                                                                 169: P_{3222} = (5, 8, 11, 1)
116: P_{2263} = (6, 12, 7, 1)
                                                                 170: P_{3228} = (11, 8, 11, 1)
117: P_{2266} = (9, 12, 7, 1)
                                                                 171: P_{3255} = (6, 10, 11, 1)
118: P_{2299} = (10, 14, 7, 1)
                                                                 172: P_{3256} = (7, 10, 11, 1)
119: P_{2300} = (11, 14, 7, 1)
                                                                 173: P_{3275} = (10, 11, 11, 1)
120: P_{2324} = (3, 0, 8, 1)
                                                                 174: P_{3287} = (6, 12, 11, 1)
121: P_{2339} = (2, 1, 8, 1)
                                                                 175: P_{3296} = (15, 12, 11, 1)
122: P_{2347} = (10, 1, 8, 1)
                                                                 176: P_{3302} = (5, 13, 11, 1)
123: P_{2358} = (5, 2, 8, 1)
                                                                 177: P_{3304} = (7, 13, 11, 1)
124: P_{2365} = (12, 2, 8, 1)
                                                                 178: P_{3360} = (15, 0, 12, 1)
125: P_{2371} = (2, 3, 8, 1)
                                                                 179: P_{3387} = (10, 2, 12, 1)
126: P_{2372} = (3, 3, 8, 1)
                                                                 180: P_{3388} = (11, 2, 12, 1)
127: P_{2438} = (5, 7, 8, 1)
                                                                 181: P_{3454} = (13, 6, 12, 1)
128: P_{2448} = (15, 7, 8, 1)
                                                                 182: P_{3455} = (14, 6, 12, 1)
129: P_{2464} = (15, 8, 8, 1)
                                                                 183: P_{3483} = (10, 8, 12, 1)
                                                                 184: P_{3487} = (14, 8, 12, 1)
130: P_{2501} = (4, 11, 8, 1)
                                                                 185: P_{3492} = (3, 9, 12, 1)
131: P_{2507} = (10, 11, 8, 1)
132: P_{2546} = (1, 14, 8, 1)
                                                                 186: P_{3500} = (11, 9, 12, 1)
                                                                 187: P_{3508} = (3, 10, 12, 1)
133: P_{2557} = (12, 14, 8, 1)
134: P_{2562} = (1, 15, 8, 1)
                                                                 188: P_{3511} = (6, 10, 12, 1)
135: P_{2565} = (4, 15, 8, 1)
                                                                 189: P_{3525} = (4, 11, 12, 1)
136: P_{2592} = (15, 0, 9, 1)
                                                                 190: P_{3534} = (13, 11, 12, 1)
137: P_{2634} = (9, 3, 9, 1)
                                                                 191: P_{3543} = (6, 12, 12, 1)
138: P_{2636} = (11, 3, 9, 1)
                                                                 192: P_{3589} = (4, 15, 12, 1)
139: P_{2644} = (3, 4, 9, 1)
                                                                 193: P_{3600} = (15, 15, 12, 1)
140: P_{2653} = (12, 4, 9, 1)
                                                                 194: P_{3606} = (5, 0, 13, 1)
                                                                 195: P_{3641} = (8, 2, 13, 1)
141: P_{2692} = (3,7,9,1)
142: P_{2703} = (14, 7, 9, 1)
                                                                 196: P_{3644} = (11, 2, 13, 1)
143: P_{2716} = (11, 8, 9, 1)
                                                                 197: P_{3653} = (4, 3, 13, 1)
144: P_{2717} = (12, 8, 9, 1)
                                                                 198: P_{3659} = (10, 3, 13, 1)
145: P_{2735} = (14, 9, 9, 1)
                                                                 199: P_{3686} = (5, 5, 13, 1)
146: P_{2742} = (5, 10, 9, 1)
                                                                 200: P_{3695} = (14, 5, 13, 1)
147: P_{2746} = (9, 10, 9, 1)
                                                                 201: P_{3717} = (4,7,13,1)
148: P_{2822} = (5, 15, 9, 1)
                                                                 202: P_{3725} = (12, 7, 13, 1)
149: P_{2832} = (15, 15, 9, 1)
                                                                 203: P_{3755} = (10, 9, 13, 1)
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204: P_{3756} = (11, 9, 13, 1)
                                                                223: P_{4107} = (10, 15, 14, 1)
                                                                224: P_{4118} = (5, 0, 15, 1)
205: P_{3768} = (7, 10, 13, 1)
                                                                225: P_{4133} = (4, 1, 15, 1)
206: P_{3769} = (8, 10, 13, 1)
                                                                226: P_{4140} = (11, 1, 15, 1)
207: P_{3789} = (12, 11, 13, 1)
208: P_{3791} = (14, 11, 13, 1)
                                                                227: P_{4146} = (1, 2, 15, 1)
209: P_{3816} = (7, 13, 13, 1)
                                                                228: P_{4151} = (6, 2, 15, 1)
210: P_{3860} = (3, 0, 14, 1)
                                                                229: P_{4162} = (1, 3, 15, 1)
211: P_{3908} = (3, 3, 14, 1)
                                                                230: P_{4170} = (9, 3, 15, 1)
                                                                231: P_{4183} = (6, 4, 15, 1)
212: P_{3913} = (8, 3, 14, 1)
                                                                232: P_{4185} = (8, 4, 15, 1)
213: P_{3947} = (10, 5, 14, 1)
214: P_{3951} = (14, 5, 14, 1)
                                                                233: P_{4197} = (4, 5, 15, 1)
                                                                234: P_{4198} = (5, 5, 15, 1)
215: P_{4006} = (5, 9, 14, 1)
                                                                235: P_{4282} = (9, 10, 15, 1)
216: P_{4007} = (6, 9, 14, 1)
217: P_{4041} = (8, 11, 14, 1)
                                                                236: P_{4284} = (11, 10, 15, 1)
218: P_{4047} = (14, 11, 14, 1)
                                                                237: P_{4308} = (3, 12, 15, 1)
219: P_{4051} = (2, 12, 14, 1)
                                                                238: P_{4313} = (8, 12, 15, 1)
220: P_{4054} = (5, 12, 14, 1)
                                                                239: P_{4356} = (3, 15, 15, 1)
221: P_{4083} = (2, 14, 14, 1)
222: P_{4103} = (6, 15, 14, 1)
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Line Intersection Graph

 $\begin{array}{c|c}
 & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
2 & 0 & 1 & 0
\end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_3

Line 2 intersects

Line	ℓ_1
in point	P_3

The surface has 289 points:

The points on the surface are:

$0: P_0 = (1,0,0,0)$	13: $P_{14} = (10, 1, 0, 0)$	$26: P_{158} = (11, 8, 1, 0)$
$1: P_1 = (0, 1, 0, 0)$	$14: P_{15} = (11, 1, 0, 0)$	$27: P_{167} = (4, 9, 1, 0)$
$2: P_3 = (0,0,0,1)$	15: $P_{16} = (12, 1, 0, 0)$	$28: P_{180} = (1, 10, 1, 0)$
$3: P_4 = (1, 1, 1, 1)$	16: $P_{17} = (13, 1, 0, 0)$	$29: P_{196} = (1, 11, 1, 0)$
$4: P_5 = (1, 1, 0, 0)$	17: $P_{18} = (14, 1, 0, 0)$	$30: P_{225} = (14, 12, 1, 0)$
$5: P_6 = (2, 1, 0, 0)$	18: $P_{19} = (15, 1, 0, 0)$	$31: P_{231} = (4, 13, 1, 0)$
$6: P_7 = (3, 1, 0, 0)$	$19: P_{35} = (0, 1, 1, 0)$	$32: P_{252} = (9, 14, 1, 0)$
$7: P_8 = (4, 1, 0, 0)$	$20: P_{65} = (14, 2, 1, 0)$	$33: P_{269} = (10, 15, 1, 0)$
$8: P_9 = (5, 1, 0, 0)$	$21: P_{78} = (11, 3, 1, 0)$	$34: P_{290} = (0, 1, 0, 1)$
$9: P_{10} = (6, 1, 0, 0)$	$22: P_{85} = (2,4,1,0)$	$35: P_{306} = (0, 2, 0, 1)$
$10: P_{11} = (7, 1, 0, 0)$	$23: P_{109} = (10, 5, 1, 0)$	$36: P_{322} = (0, 3, 0, 1)$
11: $P_{12} = (8, 1, 0, 0)$	$24: P_{117} = (2, 6, 1, 0)$	$37: P_{338} = (0,4,0,1)$
$12: P_{13} = (9, 1, 0, 0)$	$25: P_{140} = (9, 7, 1, 0)$	$38: P_{354} = (0, 5, 0, 1)$

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93: P_{1227} = (10, 11, 3, 1)
                                                                                        147: P_{2154} = (9, 5, 7, 1)
39: P_{370} = (0,6,0,1)
                                                                                        148: P_{2156} = (11, 5, 7, 1)
40: P_{386} = (0,7,0,1)
                                            94: P_{1231} = (14, 11, 3, 1)
41: P_{402} = (0, 8, 0, 1)
                                           95: P_{1305} = (8, 0, 4, 1)
                                                                                        149: P_{2177} = (0,7,7,1)
                                                                                        150: P_{2189} = (12, 7, 7, 1)
42: P_{418} = (0, 9, 0, 1)
                                           96: P_{1336} = (7, 2, 4, 1)
43: P_{434} = (0, 10, 0, 1)
                                           97: P_{1344} = (15, 2, 4, 1)
                                                                                        151: P_{2195} = (2, 8, 7, 1)
44: P_{450} = (0, 11, 0, 1)
                                           98: P_{1361} = (0, 4, 4, 1)
                                                                                        152: P_{2201} = (8, 8, 7, 1)
45: P_{466} = (0, 12, 0, 1)
                                            99: P_{1370} = (9, 4, 4, 1)
                                                                                        153: P_{2227} = (2, 10, 7, 1)
                                                                                        154: P_{2231} = (6, 10, 7, 1)
46: P_{482} = (0, 13, 0, 1)
                                            100: P_{1384} = (7, 5, 4, 1)
47: P_{498} = (0, 14, 0, 1)
                                           101: P_{1387} = (10, 5, 4, 1)
                                                                                        155: P_{2253} = (12, 11, 7, 1)
48: P_{514} = (0, 15, 0, 1)
                                           102: P_{1428} = (3, 8, 4, 1)
                                                                                        156: P_{2256} = (15, 11, 7, 1)
49: P_{531} = (1, 0, 1, 1)
                                           103: P_{1433} = (8, 8, 4, 1)
                                                                                        157: P_{2263} = (6, 12, 7, 1)
50: P_{546} = (0, 1, 1, 1)
                                           104: P_{1476} = (3, 11, 4, 1)
                                                                                        158: P_{2266} = (9, 12, 7, 1)
51: P_{570} = (9, 2, 1, 1)
                                           105: P_{1477} = (4, 11, 4, 1)
                                                                                        159: P_{2299} = (10, 14, 7, 1)
52: P_{572} = (11, 2, 1, 1)
                                           106: P_{1514} = (9, 13, 4, 1)
                                                                                        160: P_{2300} = (11, 14, 7, 1)
                                                                                        161: P_{2324} = (3, 0, 8, 1)
53: P_{603} = (10, 4, 1, 1)
                                           107: P_{1520} = (15, 13, 4, 1)
54: P_{607} = (14, 4, 1, 1)
                                            108: P_{1541} = (4, 15, 4, 1)
                                                                                        162: P_{2339} = (2, 1, 8, 1)
55: P_{675} = (2, 9, 1, 1)
                                           109: P_{1547} = (10, 15, 4, 1)
                                                                                        163: P_{2347} = (10, 1, 8, 1)
56: P_{684} = (11, 9, 1, 1)
                                                                                        164: P_{2358} = (5, 2, 8, 1)
                                           110: P_{1568} = (15, 0, 5, 1)
                                                                                        165: P_{2365} = (12, 2, 8, 1)
57: P_{693} = (4, 10, 1, 1)
                                           111: P_{1580} = (11, 1, 5, 1)
58: P_{703} = (14, 10, 1, 1)
                                           112: P_{1583} = (14, 1, 5, 1)
                                                                                        166: P_{2371} = (2, 3, 8, 1)
59: P_{707} = (2, 11, 1, 1)
                                           113: P_{1633} = (0, 5, 5, 1)
                                                                                        167: P_{2372} = (3, 3, 8, 1)
60: P_{714} = (9, 11, 1, 1)
                                                                                        168: P_{2438} = (5, 7, 8, 1)
                                           114: P_{1641} = (8, 5, 5, 1)
61: P_{757} = (4, 14, 1, 1)
                                                                                        169: P_{2448} = (15, 7, 8, 1)
                                           115: P_{1682} = (1, 8, 5, 1)
62: P_{763} = (10, 14, 1, 1)
                                           116: P_{1683} = (2, 8, 5, 1)
                                                                                        170: P_{2449} = (0, 8, 8, 1)
63: P_{790} = (5,0,2,1)
                                           117: P_{1698} = (1, 9, 5, 1)
                                                                                        171: P_{2464} = (15, 8, 8, 1)
64: P_{817} = (0, 2, 2, 1)
                                           118: P_{1704} = (7, 9, 5, 1)
                                                                                        172: P_{2501} = (4, 11, 8, 1)
65: P_{821} = (4, 2, 2, 1)
                                           119: P_{1715} = (2, 10, 5, 1)
                                                                                        173: P_{2507} = (10, 11, 8, 1)
66: P_{844} = (11, 3, 2, 1)
                                           120: P_{1724} = (11, 10, 5, 1)
                                                                                        174: P_{2546} = (1, 14, 8, 1)
67: P_{846} = (13, 3, 2, 1)
                                           121: P_{1764} = (3, 13, 5, 1)
                                                                                        175: P_{2557} = (12, 14, 8, 1)
68: P_{870} = (5, 5, 2, 1)
                                           122: P_{1769} = (8, 13, 5, 1)
                                                                                        176: P_{2562} = (1, 15, 8, 1)
69: P_{880} = (15, 5, 2, 1)
                                           123: P_{1780} = (3, 14, 5, 1)
                                                                                        177: P_{2565} = (4, 15, 8, 1)
70: P_{885} = (4, 6, 2, 1)
                                           124: P_{1784} = (7, 14, 5, 1)
                                                                                        178: P_{2592} = (15, 0, 9, 1)
71: P_{889} = (8, 6, 2, 1)
                                           125: P_{1807} = (14, 15, 5, 1)
                                                                                        179: P_{2634} = (9, 3, 9, 1)
72: P_{915} = (2, 8, 2, 1)
                                                                                        180: P_{2636} = (11, 3, 9, 1)
                                           126: P_{1808} = (15, 15, 5, 1)
73: P_{924} = (11, 8, 2, 1)
                                           127: P_{1812} = (3, 0, 6, 1)
                                                                                        181: P_{2644} = (3, 4, 9, 1)
                                           128: P_{1860} = (3, 3, 6, 1)
74: P_{947} = (2, 10, 2, 1)
                                                                                        182: P_{2653} = (12, 4, 9, 1)
75: P_{960} = (15, 10, 2, 1)
                                           129: P_{1866} = (9, 3, 6, 1)
                                                                                        183: P_{2692} = (3,7,9,1)
                                                                                        184: P_{2703} = (14, 7, 9, 1)
76: P_{1017} = (8, 14, 2, 1)
                                           130: P_{1883} = (10, 4, 6, 1)
77: P_{1022} = (13, 14, 2, 1)
                                           131: P_{1884} = (11, 4, 6, 1)
                                                                                        185: P_{2716} = (11, 8, 9, 1)
78: P_{1049} = (8, 0, 3, 1)
                                           132: P_{1905} = (0, 6, 6, 1)
                                                                                        186: P_{2717} = (12, 8, 9, 1)
79: P_{1066} = (9, 1, 3, 1)
                                           133: P_{1918} = (13, 6, 6, 1)
                                                                                        187: P_{2721} = (0, 9, 9, 1)
                                                                                        188: P_{2735} = (14, 9, 9, 1)
80: P_{1067} = (10, 1, 3, 1)
                                           134: P_{1976} = (7, 10, 6, 1)
81: P_{1089} = (0,3,3,1)
                                           135: P_{1978} = (9, 10, 6, 1)
                                                                                        189: P_{2742} = (5, 10, 9, 1)
82: P_{1094} = (5, 3, 3, 1)
                                           136: P_{1990} = (5, 11, 6, 1)
                                                                                        190: P_{2746} = (9, 10, 9, 1)
83: P_{1106} = (1, 4, 3, 1)
                                           137: P_{1998} = (13, 11, 6, 1)
                                                                                        191: P_{2822} = (5, 15, 9, 1)
84: P_{1118} = (13, 4, 3, 1)
                                           138: P_{2019} = (2, 13, 6, 1)
                                                                                        192: P_{2832} = (15, 15, 9, 1)
85: P_{1122} = (1, 5, 3, 1)
                                           139: P_{2024} = (7, 13, 6, 1)
                                                                                        193: P_{2834} = (1, 0, 10, 1)
86: P_{1135} = (14, 5, 3, 1)
                                           140: P_{2038} = (5, 14, 6, 1)
                                                                                        194: P_{2850} = (1, 1, 10, 1)
87: P_{1142} = (5, 6, 3, 1)
                                           141: P_{2043} = (10, 14, 6, 1)
                                                                                        195: P_{2860} = (11, 1, 10, 1)
88: P_{1152} = (15, 6, 3, 1)
                                           142: P_{2051} = (2, 15, 6, 1)
                                                                                        196: P_{2916} = (3, 5, 10, 1)
89: P_{1177} = (8, 8, 3, 1)
                                           143: P_{2060} = (11, 15, 6, 1)
                                                                                        197: P_{2923} = (10, 5, 10, 1)
90: P_{1178} = (9, 8, 3, 1)
                                           144: P_{2073} = (8, 0, 7, 1)
                                                                                        198: P_{2932} = (3, 6, 10, 1)
91: P_{1198} = (13, 9, 3, 1)
                                           145: P_{2139} = (10, 4, 7, 1)
                                                                                        199: P_{2942} = (13, 6, 10, 1)
92: P_{1200} = (15, 9, 3, 1)
                                           146: P_{2144} = (15, 4, 7, 1)
                                                                                        200: P_{2953} = (8, 7, 10, 1)
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261: P_{3951} = (14, 5, 14, 1)
201: P_{2957} = (12, 7, 10, 1)
                                           231: P_{3500} = (11, 9, 12, 1)
                                                                                      262: P_{4006} = (5, 9, 14, 1)
202: P_{2993} = (0, 10, 10, 1)
                                           232: P_{3508} = (3, 10, 12, 1)
203: P_{3004} = (11, 10, 10, 1)
                                           233: P_{3511} = (6, 10, 12, 1)
                                                                                      263: P_{4007} = (6, 9, 14, 1)
                                           234: P_{3525} = (4, 11, 12, 1)
                                                                                      264: P_{4041} = (8, 11, 14, 1)
204: P_{3021} = (12, 11, 10, 1)
205: P_{3022} = (13, 11, 10, 1)
                                           235: P_{3534} = (13, 11, 12, 1)
                                                                                      265: P_{4047} = (14, 11, 14, 1)
206: P_{3081} = (8, 15, 10, 1)
                                           236: P_{3537} = (0, 12, 12, 1)
                                                                                      266: P_{4051} = (2, 12, 14, 1)
207: P_{3083} = (10, 15, 10, 1)
                                           237: P_{3543} = (6, 12, 12, 1)
                                                                                      267: P_{4054} = (5, 12, 14, 1)
208: P_{3090} = (1, 0, 11, 1)
                                           238: P_{3589} = (4, 15, 12, 1)
                                                                                      268: P_{4081} = (0, 14, 14, 1)
209: P_{3106} = (1, 1, 11, 1)
                                           239: P_{3600} = (15, 15, 12, 1)
                                                                                      269: P_{4083} = (2, 14, 14, 1)
210: P_{3115} = (10, 1, 11, 1)
                                           240: P_{3606} = (5, 0, 13, 1)
                                                                                      270: P_{4103} = (6, 15, 14, 1)
211: P_{3148} = (11, 3, 11, 1)
                                           241: P_{3641} = (8, 2, 13, 1)
                                                                                      271: P_{4107} = (10, 15, 14, 1)
212: P_{3152} = (15, 3, 11, 1)
                                           242: P_{3644} = (11, 2, 13, 1)
                                                                                      272: P_{4118} = (5, 0, 15, 1)
213: P_{3222} = (5, 8, 11, 1)
                                           243: P_{3653} = (4, 3, 13, 1)
                                                                                      273: P_{4133} = (4, 1, 15, 1)
214: P_{3228} = (11, 8, 11, 1)
                                           244: P_{3659} = (10, 3, 13, 1)
                                                                                      274: P_{4140} = (11, 1, 15, 1)
                                           245: P_{3686} = (5, 5, 13, 1)
215: P_{3255} = (6, 10, 11, 1)
                                                                                      275: P_{4146} = (1, 2, 15, 1)
216: P_{3256} = (7, 10, 11, 1)
                                           246: P_{3695} = (14, 5, 13, 1)
                                                                                      276: P_{4151} = (6, 2, 15, 1)
                                           247: P_{3717} = (4, 7, 13, 1)
                                                                                      277: P_{4162} = (1, 3, 15, 1)
217: P_{3265} = (0, 11, 11, 1)
                                           248: P_{3725} = (12, 7, 13, 1)
                                                                                      278: P_{4170} = (9, 3, 15, 1)
218: P_{3275} = (10, 11, 11, 1)
219: P_{3287} = (6, 12, 11, 1)
                                           249: P_{3755} = (10, 9, 13, 1)
                                                                                      279: P_{4183} = (6, 4, 15, 1)
220: P_{3296} = (15, 12, 11, 1)
                                           250: P_{3756} = (11, 9, 13, 1)
                                                                                      280: P_{4185} = (8, 4, 15, 1)
221: P_{3302} = (5, 13, 11, 1)
                                           251: P_{3768} = (7, 10, 13, 1)
                                                                                      281: P_{4197} = (4, 5, 15, 1)
                                                                                      282: P_{4198} = (5, 5, 15, 1)
222: P_{3304} = (7, 13, 11, 1)
                                           252: P_{3769} = (8, 10, 13, 1)
223: P_{3360} = (15, 0, 12, 1)
                                           253: P_{3789} = (12, 11, 13, 1)
                                                                                      283: P_{4282} = (9, 10, 15, 1)
224: P_{3387} = (10, 2, 12, 1)
                                           254: P_{3791} = (14, 11, 13, 1)
                                                                                      284: P_{4284} = (11, 10, 15, 1)
225: P_{3388} = (11, 2, 12, 1)
                                           255: P_{3809} = (0, 13, 13, 1)
                                                                                      285: P_{4308} = (3, 12, 15, 1)
226: P_{3454} = (13, 6, 12, 1)
                                           256: P_{3816} = (7, 13, 13, 1)
                                                                                      286: P_{4313} = (8, 12, 15, 1)
227: P_{3455} = (14, 6, 12, 1)
                                           257: P_{3860} = (3, 0, 14, 1)
                                                                                      287: P_{4353} = (0, 15, 15, 1)
228: P_{3483} = (10, 8, 12, 1)
                                           258: P_{3908} = (3, 3, 14, 1)
                                                                                      288: P_{4356} = (3, 15, 15, 1)
                                           259: P_{3913} = (8, 3, 14, 1)
229: P_{3487} = (14, 8, 12, 1)
230: P_{3492} = (3, 9, 12, 1)
                                           260: P_{3947} = (10, 5, 14, 1)
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