Rank-65839 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_2 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431722409

General information

Number of lines	7
Number of points	29
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	3
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	4^2, 2^3, 1^{21}, 0^3

Singular Points

The surface has 2 singular points:

$0:\ P_6 = \mathbf{P}(\omega, 1, 0, 0) = \mathbf{P}(2, 1, 0, 0)$
$1:\ P_7 = \mathbf{P}(\omega^2, 1, 0, 0) = \mathbf{P}(3, 1, 0, 0)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \mathbf{Pl}(0,0,1,1,2,1)_{258}$$

$$\ell_2 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{82} = \mathbf{Pl}(0,0,2,3,1,1)_{205}$$

$$\ell_3 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{81} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{81} = \mathbf{Pl}(0,0,3,2,3,1)_{332}$$

$$\ell_4 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \mathbf{Pl}(0,0,1,1,3,1)_{318}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{61} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{61} = \mathbf{Pl}(0,0,2,3,2,1)_{265}$$

$$\ell_6 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{60} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{60} = \mathbf{Pl}(0,0,3,2,1,1)_{212}$$

Rank of lines: (0, 80, 82, 81, 59, 61, 60)
Rank of points on Klein quadric: (0, 258, 205, 332, 318, 265, 212)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{38} = (0,0,1,1) = \ell_1 \cap \ell_4$
$P_{53} = (0,0,2,1) = \ell_2 \cap \ell_5$
$P_{69} = (0,0,3,1) = \ell_3 \cap \ell_6$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
2 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
3 : $P_{43} = (2,1,1,1)$ lies on line ℓ_1
4 : $P_{44} = (3,1,1,1)$ lies on line ℓ_4
5 : $P_{46} = (1,2,1,1)$ lies on line ℓ_4
6 : $P_{48} = (3,2,1,1)$ lies on line ℓ_1
7 : $P_{50} = (1,3,1,1)$ lies on line ℓ_1
8 : $P_{51} = (2,3,1,1)$ lies on line ℓ_4
9 : $P_{59} = (2,1,2,1)$ lies on line ℓ_2
10 : $P_{60} = (3,1,2,1)$ lies on line ℓ_5
11 : $P_{62} = (1,2,2,1)$ lies on line ℓ_5
12 : $P_{64} = (3,2,2,1)$ lies on line ℓ_2
13 : $P_{66} = (1,3,2,1)$ lies on line ℓ_2
14 : $P_{67} = (2,3,2,1)$ lies on line ℓ_5
15 : $P_{75} = (2,1,3,1)$ lies on line ℓ_3
16 : $P_{76} = (3,1,3,1)$ lies on line ℓ_6
17 : $P_{78} = (1,2,3,1)$ lies on line ℓ_6
18 : $P_{80} = (3,2,3,1)$ lies on line ℓ_3
19 : $P_{82} = (1,3,3,1)$ lies on line ℓ_3
20 : $P_{83} = (2,3,3,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 3 points not on any line:
The points on the surface but not on lines are:

$0: P_8 = (1,0,1,0)$
$1: P_{11} = (0,1,1,0)$
$2: P_{12} = (1,1,1,0)$

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 1 1 1 1
1	1 0 1 1 1 0 0
2	1 1 0 1 0 1 0
3	1 1 1 0 0 0 1
4	1 1 0 0 0 1 1
5	1 0 1 0 1 0 1
6	1 0 0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_6	P_6	P_6	P_7	P_7	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_6	P_6	P_6	P_{38}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_6	P_6	P_6	P_{53}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_6	P_6	P_6	P_{69}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_7	P_{38}	P_7	P_7

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_7	P_{53}	P_7	P_7

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_7	P_{69}	P_7	P_7

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_5 = (1,1,0,0)$
$3: P_6 = (2,1,0,0)$
$4: P_7 = (3,1,0,0)$
$5: P_8 = (1,0,1,0)$
$6: P_{11} = (0,1,1,0)$
$7: P_{12} = (1,1,1,0)$
$8: P_{38} = (0,0,1,1)$
$9: P_{43} = (2,1,1,1)$
$10: P_{44} = (3,1,1,1)$
$11: P_{46} = (1,2,1,1)$
$12: P_{48} = (3,2,1,1)$
$13: P_{50} = (1,3,1,1)$
$14: P_{51} = (2,3,1,1)$
$15: P_{53} = (0,0,2,1)$
$16: P_{59} = (2,1,2,1)$
$17: P_{60} = (3,1,2,1)$
$18: P_{62} = (1,2,2,1)$
$19: P_{64} = (3,2,2,1)$
$20: P_{66} = (1,3,2,1)$

$21: P_{67} = (2, 3, 2, 1)$
$22: P_{69} = (0, 0, 3, 1)$
$23: P_{75} = (2, 1, 3, 1)$
$24: P_{76} = (3, 1, 3, 1)$
$25: P_{78} = (1, 2, 3, 1)$
$26: P_{80} = (3, 2, 3, 1)$
$27: P_{82} = (1, 3, 3, 1)$
$28: P_{83} = (2, 3, 3, 1)$