Rank-74264 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_2^3 + X_0^2 X_1 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over $\mathrm{GF}(8)$ is 1361355406

General information

Number of lines	1
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{48}$

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \mathbf{Pl}(1, 0, 0, 0, 0, 1)_{650}$$

Rank of lines: (73)

Rank of points on Klein quadric: (650)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

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\begin{array}{lll} 0: \ P_1 = (0,1,0,0) \ \mbox{lies on line} \ \ell_0 \\ 1: \ P_{12} = (1,0,1,0) \ \mbox{lies on line} \ \ell_0 \\ 2: \ P_{20} = (1,1,1,0) \ \mbox{lies on line} \ \ell_0 \\ 3: \ P_{28} = (1,2,1,0) \ \mbox{lies on line} \ \ell_0 \\ 4: \ P_{36} = (1,3,1,0) \ \mbox{lies on line} \ \ell_0 \\ \end{array}
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The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line: The points on the surface but not on lines are:

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0: P_3 = (0,0,0,1)
                                                                 22: P_{269} = (4,0,3,1)
1: P_4 = (1, 1, 1, 1)
                                                                 23: P_{298} = (1,4,3,1)
2: P_5 = (1, 1, 0, 0)
                                                                 24: P_{314} = (1, 6, 3, 1)
3: P_{40} = (5, 3, 1, 0)
                                                                 25: P_{316} = (3, 6, 3, 1)
4: P_{42} = (7,3,1,0)
                                                                 26: P_{317} = (4, 6, 3, 1)
5: P_{53} = (2, 5, 1, 0)
                                                                 27: P_{321} = (0,7,3,1)
6: P_{57} = (6, 5, 1, 0)
                                                                 28: P_{349} = (4, 2, 4, 1)
7: P_{62} = (3, 6, 1, 0)
                                                                 29: P_{358} = (5, 3, 4, 1)
8: P_{63} = (4, 6, 1, 0)
                                                                 30: P_{369} = (0, 5, 4, 1)
9: P_{75} = (1,0,0,1)
                                                                 31: P_{382} = (5, 6, 4, 1)
10: P_{83} = (1, 1, 0, 1)
                                                                 32: P_{400} = (7,0,5,1)
11: P_{141} = (3, 0, 1, 1)
                                                                 33: P_{409} = (0, 2, 5, 1)
12: P_{143} = (5, 0, 1, 1)
                                                                 34: P_{418} = (1,3,5,1)
13: P_{144} = (6, 0, 1, 1)
                                                                 35: P_{422} = (5, 3, 5, 1)
14: P_{146} = (0, 1, 1, 1)
                                                                 36: P_{424} = (7, 3, 5, 1)
                                                                 37: P_{450} = (1, 7, 5, 1)
15: P_{166} = (5, 3, 1, 1)
16: P_{183} = (6, 5, 1, 1)
                                                                 38: P_{459} = (2,0,6,1)
17: P_{188} = (3, 6, 1, 1)
                                                                 39: P_{474} = (1, 2, 6, 1)
18: P_{225} = (0, 3, 2, 1)
                                                                 40: P_{489} = (0, 4, 6, 1)
19: P_{244} = (3, 5, 2, 1)
                                                                 41: P_{498} = (1, 5, 6, 1)
20: P_{252} = (3, 6, 2, 1)
                                                                 42: P_{499} = (2, 5, 6, 1)
21: P_{259} = (2,7,2,1)
                                                                 43: P_{503} = (6,5,6,1)
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\begin{array}{l} 44:\ P_{551}=(6,3,7,1)\\ 45:\ P_{560}=(7,4,7,1)\\ 46:\ P_{567}=(6,5,7,1) \end{array}
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Line Intersection Graph

 $\frac{0}{0 \mid 0}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 57 points: The points on the surface are:

$0 \cdot D = (0.1, 0.0)$	00 · D (2 0 1 1)	40 · D (5 6 4 1)
$0: P_1 = (0, 1, 0, 0)$	$20: P_{141} = (3, 0, 1, 1)$	$40: P_{382} = (5, 6, 4, 1)$
$1: P_3 = (0,0,0,1)$	$21: P_{143} = (5, 0, 1, 1)$	$41: P_{400} = (7, 0, 5, 1)$
$2: P_4 = (1, 1, 1, 1)$	$22: P_{144} = (6, 0, 1, 1)$	$42: P_{409} = (0, 2, 5, 1)$
$3: P_5 = (1, 1, 0, 0)$	$23: P_{146} = (0, 1, 1, 1)$	$43: P_{418} = (1, 3, 5, 1)$
$4: P_{12} = (1,0,1,0)$	$24: P_{166} = (5, 3, 1, 1)$	$44: P_{422} = (5, 3, 5, 1)$
$5: P_{20} = (1, 1, 1, 0)$	$25: P_{183} = (6, 5, 1, 1)$	$45: P_{424} = (7, 3, 5, 1)$
$6: P_{28} = (1, 2, 1, 0)$	$26: P_{188} = (3, 6, 1, 1)$	$46: P_{450} = (1, 7, 5, 1)$
$7: P_{36} = (1, 3, 1, 0)$	$27: P_{225} = (0, 3, 2, 1)$	$47: P_{459} = (2, 0, 6, 1)$
$8: P_{40} = (5, 3, 1, 0)$	$28: P_{244} = (3, 5, 2, 1)$	$48: P_{474} = (1, 2, 6, 1)$
$9: P_{42} = (7,3,1,0)$	$29: P_{252} = (3, 6, 2, 1)$	$49: P_{489} = (0, 4, 6, 1)$
$10: P_{44} = (1, 4, 1, 0)$	$30: P_{259} = (2,7,2,1)$	$50: P_{498} = (1, 5, 6, 1)$
$11: P_{52} = (1, 5, 1, 0)$	$31: P_{269} = (4,0,3,1)$	$51: P_{499} = (2, 5, 6, 1)$
$12: P_{53} = (2, 5, 1, 0)$	$32: P_{298} = (1,4,3,1)$	$52: P_{503} = (6, 5, 6, 1)$
13: $P_{57} = (6, 5, 1, 0)$	$33: P_{314} = (1,6,3,1)$	$53: P_{551} = (6, 3, 7, 1)$
$14: P_{60} = (1, 6, 1, 0)$	$34: P_{316} = (3, 6, 3, 1)$	$54: P_{560} = (7, 4, 7, 1)$
15: $P_{62} = (3, 6, 1, 0)$	$35: P_{317} = (4, 6, 3, 1)$	$55: P_{567} = (6, 5, 7, 1)$
16: $P_{63} = (4, 6, 1, 0)$	$36: P_{321} = (0,7,3,1)$	$56: P_{569} = (0, 6, 7, 1)$
17: $P_{68} = (1, 7, 1, 0)$	$37: P_{349} = (4, 2, 4, 1)$	
18: $P_{75} = (1, 0, 0, 1)$	$38: P_{358} = (5, 3, 4, 1)$	
$19: P_{83} = (1, 1, 0, 1)$	$39: P_{369} = (0, 5, 4, 1)$	