Rank-74243 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361351245

General information

Number of lines	4
Number of points	81
Number of singular points	2
Number of Eckardt points	1
Number of double points	2
Number of single points	29
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	3, 2^2, 1^{29}, 0^{49}

Singular Points

The surface has 2 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{64} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$
$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4672} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$
$$\ell_3 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 64, 4672, 4744)
Rank of points on Klein quadric: (0, 2, 649, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 29 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_3
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$5 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$6 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$7 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$8 : P_{12} = (1,0,1,0)$ lies on line ℓ_1
$9 : P_{13} = (2,0,1,0)$ lies on line ℓ_1
$10 : P_{14} = (3,0,1,0)$ lies on line ℓ_1
$11 : P_{15} = (4,0,1,0)$ lies on line ℓ_1
$12 : P_{16} = (5,0,1,0)$ lies on line ℓ_1
$13 : P_{17} = (6,0,1,0)$ lies on line ℓ_1
$14 : P_{18} = (7,0,1,0)$ lies on line ℓ_1
$15 : P_{19} = (0,1,1,0)$ lies on line ℓ_2
$16 : P_{27} = (0,2,1,0)$ lies on line ℓ_2
$17 : P_{35} = (0,3,1,0)$ lies on line ℓ_2
$18 : P_{43} = (0,4,1,0)$ lies on line ℓ_2
$19 : P_{51} = (0,5,1,0)$ lies on line ℓ_2
$20 : P_{59} = (0,6,1,0)$ lies on line ℓ_2
$21 : P_{67} = (0,7,1,0)$ lies on line ℓ_2
$22 : P_{138} = (0,0,1,1)$ lies on line ℓ_3
$23 : P_{201} = (0,0,2,1)$ lies on line ℓ_3
$24 : P_{265} = (0,0,3,1)$ lies on line ℓ_3
$25 : P_{329} = (0,0,4,1)$ lies on line ℓ_3
$26 : P_{393} = (0,0,5,1)$ lies on line ℓ_3
$27 : P_{457} = (0,0,6,1)$ lies on line ℓ_3
$28 : P_{521} = (0,0,7,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:
The points on the surface but not on lines are:

0 : $P_{83} = (1, 1, 0, 1)$
1 : $P_{94} = (4, 2, 0, 1)$
2 : $P_{103} = (5, 3, 0, 1)$
3 : $P_{113} = (7, 4, 0, 1)$
4 : $P_{120} = (6, 5, 0, 1)$
5 : $P_{125} = (3, 6, 0, 1)$
6 : $P_{132} = (2, 7, 0, 1)$
7 : $P_{160} = (7, 2, 1, 1)$
8 : $P_{165} = (4, 3, 1, 1)$
9 : $P_{171} = (2, 4, 1, 1)$
10 : $P_{184} = (7, 5, 1, 1)$
11 : $P_{187} = (2, 6, 1, 1)$
12 : $P_{197} = (4, 7, 1, 1)$
13 : $P_{213} = (4, 1, 2, 1)$
14 : $P_{223} = (6, 2, 2, 1)$
15 : $P_{231} = (6, 3, 2, 1)$
16 : $P_{237} = (4, 4, 2, 1)$
17 : $P_{242} = (1, 5, 2, 1)$
18 : $P_{258} = (1, 7, 2, 1)$
19 : $P_{279} = (6, 1, 3, 1)$
20 : $P_{284} = (3, 2, 3, 1)$
21 : $P_{291} = (2, 3, 3, 1)$
22 : $P_{307} = (2, 5, 3, 1)$
23 : $P_{319} = (6, 6, 3, 1)$
24 : $P_{324} = (3, 7, 3, 1)$
25 : $P_{344} = (7, 1, 4, 1)$
26 : $P_{346} = (1, 2, 4, 1)$
27 : $P_{364} = (3, 4, 4, 1)$
28 : $P_{372} = (3, 5, 4, 1)$
29 : $P_{378} = (1, 6, 4, 1)$
30 : $P_{392} = (7, 7, 4, 1)$
31 : $P_{404} = (3, 1, 5, 1)$
32 : $P_{414} = (5, 2, 5, 1)$
33 : $P_{420} = (3, 3, 5, 1)$
34 : $P_{430} = (5, 4, 5, 1)$
35 : $P_{437} = (4, 5, 5, 1)$
36 : $P_{445} = (4, 6, 5, 1)$
37 : $P_{470} = (5, 1, 6, 1)$
38 : $P_{488} = (7, 3, 6, 1)$
39 : $P_{495} = (6, 4, 6, 1)$
40 : $P_{502} = (5, 5, 6, 1)$
41 : $P_{512} = (7, 6, 6, 1)$
42 : $P_{519} = (6, 7, 6, 1)$
43 : $P_{531} = (2, 1, 7, 1)$
44 : $P_{539} = (2, 2, 7, 1)$
45 : $P_{546} = (1, 3, 7, 1)$
46 : $P_{554} = (1, 4, 7, 1)$
47 : $P_{574} = (5, 6, 7, 1)$
48 : $P_{582} = (5, 7, 7, 1)$

Line Intersection Graph

	0 1 2 3
0	0 1 1 0
1	1 0 1 1
2	1 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_1	P_2	P_2

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

The surface has 81 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (4,1,0,0)$
$8: P_9 = (5,1,0,0)$
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$22: P_{51} = (0,5,1,0)$
$23: P_{59} = (0,6,1,0)$
$24: P_{67} = (0,7,1,0)$
$25: P_{83} = (1,1,0,1)$
$26: P_{94} = (4,2,0,1)$
$27: P_{103} = (5,3,0,1)$
$28: P_{113} = (7,4,0,1)$
$29: P_{120} = (6,5,0,1)$
$30: P_{125} = (3,6,0,1)$
$31: P_{132} = (2,7,0,1)$
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