

Rank-73796 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 73796

General information

Number of lines	7
Number of points	9
Number of singular points	3
Number of Eckardt points	7
Number of double points	0
Number of single points	0
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^7
Type of lines on points	$3^7, 0^2$

Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_1 &= \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0) \\ 1 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0) \end{aligned}$$

$$2 : P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{25} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{27}
\end{aligned}$$

Rank of lines: (28, 30, 29, 34, 31, 33, 32)

Rank of points on Klein quadric: (19, 5, 25, 1, 21, 7, 27)

Eckardt Points

The surface has 7 Eckardt points:

0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$, $T = -1$

1 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$, $T = -1$

2 : $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$, $T = 14$

3 : $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$, $T = 14$

4 : $P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$, $T = 14$

5 : $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$, $T = 14$

6 : $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$, $T = -1$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_8 = (1, 1, 1, 0)$$

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	0	1	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	1	0	1	1
5	1	1	1	1	1	0	1
6	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_2	P_2	P_7	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_3	P_{10}	P_3	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_{12}	P_{14}	P_{14}	P_{12}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_3	P_{12}	P_2	P_3	P_{12}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_{10}	P_{14}	P_2	P_{14}	P_{10}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_7	P_3	P_{14}	P_3	P_{14}	P_7

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_7	P_{10}	P_{12}	P_{12}	P_{10}	P_7

The surface has 9 points:

The points on the surface are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_7 = (0, 1, 1, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_{10} = (0, 1, 0, 1)$$

$$7 : P_{12} = (0, 0, 1, 1)$$

$$8 : P_{14} = (0, 1, 1, 1)$$