Rank-65839 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(64)$ is -2130173883

General information

Number of lines	7
Number of points	4289
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	441
Number of points off lines	3843
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 ⁷
Type of lines on points	$4^2, 2^3, 1^{441}, 0^{3843}$

Singular Points

The surface has 2 singular points:

$$0: P_{60} = \mathbf{P}(\epsilon^{42}, 1, 0, 0) = \mathbf{P}(56, 1, 0, 0) 1: P_{61} = \mathbf{P}(\epsilon^{21}, 1, 0, 0) = \mathbf{P}(57, 1, 0, 0)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_{1} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \mathbf{Pl}(0,0,1,1,56,1)_{14958978}$$

$$\ell_{2} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{241330} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{241330} = \mathbf{Pl}(0,0,56,57,1,1)_{551563}$$

$$\ell_{3} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{241329} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{241329} = \mathbf{Pl}(0,0,57,56,57,1)_{15228170}$$

$$\ell_{4} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \mathbf{Pl}(0,0,1,1,57,1)_{15221058}$$

$$\ell_{5} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{237169} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{237169} = \mathbf{Pl}(0,0,56,57,56,1)_{14965963}$$

$$\ell_{6} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{237168} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{237168} = \mathbf{Pl}(0,0,57,56,1,1)_{551690}$$

Rank of lines: (0, 241274, 241330, 241329, 237113, 237169, 237168)
Rank of points on Klein quadric: (0, 14958978, 551563, 15228170, 15221058, 14965963, 551690)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points: The double points on the surface are:

$$P_{8258} = (0, 0, 1, 1) = \ell_1 \cap \ell_4$$

$$P_{233537} = (0, 0, 56, 1) = \ell_2 \cap \ell_5$$

 $P_{237633} = (0, 0, 57, 1) = \ell_3 \cap \ell_6$

Single Points

The surface has 441 single points: The single points on the surface are:

 $0: P_0 = (1,0,0,0)$ lies on line ℓ_0 15: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0 16: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 2: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 17: $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0 3 : $P_6=(2,1,0,0)$ lies on line ℓ_0 18: $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0 4: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 19: $P_{22} = (18, 1, 0, 0)$ lies on line ℓ_0 5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 20: $P_{23} = (19, 1, 0, 0)$ lies on line ℓ_0 6: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 21: $P_{24} = (20, 1, 0, 0)$ lies on line ℓ_0 22: $P_{25} = (21, 1, 0, 0)$ lies on line ℓ_0 7: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 8: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 23: $P_{26} = (22, 1, 0, 0)$ lies on line ℓ_0 9: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 24: $P_{27} = (23, 1, 0, 0)$ lies on line ℓ_0 10: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 25: $P_{28} = (24, 1, 0, 0)$ lies on line ℓ_0 11: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 26: $P_{29} = (25, 1, 0, 0)$ lies on line ℓ_0 12: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 27: $P_{30} = (26, 1, 0, 0)$ lies on line ℓ_0 13 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0 28 : $P_{31} = (27,1,0,0)$ lies on line ℓ_0 14 : $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 29 : $P_{32} = (28, 1, 0, 0)$ lies on line ℓ_0

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30: P_{33} = (29, 1, 0, 0) lies on line \ell_0
                                                                     84: P_{8973} = (12, 11, 1, 1) lies on line \ell_1
31: P_{34} = (30, 1, 0, 0) lies on line \ell_0
                                                                     85: P_{9032} = (7, 12, 1, 1) lies on line \ell_1
32: P_{35} = (31, 1, 0, 0) lies on line \ell_0
                                                                     86: P_{9036} = (11, 12, 1, 1) lies on line \ell_4
33: P_{36} = (32, 1, 0, 0) lies on line \ell_0
                                                                     87: P_{9139} = (50, 13, 1, 1) lies on line \ell_4
34: P_{37} = (33, 1, 0, 0) lies on line \ell_0
                                                                     88: P_{9152} = (63, 13, 1, 1) lies on line \ell_1
35: P_{38} = (34, 1, 0, 0) lies on line \ell_0
                                                                     89: P_{9175} = (22, 14, 1, 1) lies on line \ell_1
36: P_{39} = (35, 1, 0, 0) lies on line \ell_0
                                                                     90: P_{9177} = (24, 14, 1, 1) lies on line \ell_4
37: P_{40} = (36, 1, 0, 0) lies on line \ell_0
                                                                     91: P_{9250} = (33, 15, 1, 1) lies on line \ell_4
38: P_{41} = (37, 1, 0, 0) lies on line \ell_0
                                                                     92: P_{9263} = (46, 15, 1, 1) lies on line \ell_1
39: P_{42} = (38, 1, 0, 0) lies on line \ell_0
                                                                     93: P_{9324} = (43, 16, 1, 1) lies on line \ell_1
40: P_{43} = (39, 1, 0, 0) lies on line \ell_0
                                                                     94: P_{9340} = (59, 16, 1, 1) lies on line \ell_4
41: P_{44} = (40, 1, 0, 0) lies on line \ell_0
                                                                     95: P_{9347} = (2, 17, 1, 1) lies on line \ell_4
42: P_{45} = (41, 1, 0, 0) lies on line \ell_0
                                                                     96: P_{9364} = (19, 17, 1, 1) lies on line \ell_1
43: P_{46} = (42, 1, 0, 0) lies on line \ell_0
                                                                     97: P_{9449} = (40, 18, 1, 1) lies on line \ell_4
44: P_{47} = (43, 1, 0, 0) lies on line \ell_0
                                                                     98: P_{9467} = (58, 18, 1, 1) lies on line \ell_1
45: P_{48} = (44, 1, 0, 0) lies on line \ell_0
                                                                     99: P_{9475} = (2, 19, 1, 1) lies on line \ell_1
46: P_{49} = (45, 1, 0, 0) lies on line \ell_0
                                                                     100: P_{9490} = (17, 19, 1, 1) lies on line \ell_4
47: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                     101: P_{9546} = (9, 20, 1, 1) lies on line \ell_1
48: P_{51} = (47, 1, 0, 0) lies on line \ell_0
                                                                     102: P_{9566} = (29, 20, 1, 1) lies on line \ell_4
49: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                     103: P_{9637} = (36, 21, 1, 1) lies on line \ell_4
50: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                     104: P_{9650} = (49, 21, 1, 1) lies on line \ell_1
51: P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                     105: P_{9679} = (14, 22, 1, 1) lies on line \ell_4
52: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                     106: P_{9689} = (24, 22, 1, 1) lies on line \ell_1
                                                                     107 : P_{9761} = (32, 23, 1, 1) lies on line \ell_1
53: P_{56} = (52, 1, 0, 0) lies on line \ell_0
54: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                     108: P_{9784} = (55, 23, 1, 1) lies on line \ell_4
55: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                     109: P_{9807} = (14, 24, 1, 1) lies on line \ell_1
56: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                     110: P_{9815} = (22, 24, 1, 1) lies on line \ell_4
57: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                     111: P_{9904} = (47, 25, 1, 1) lies on line \ell_4
58: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                     112: P_{9911} = (54, 25, 1, 1) lies on line \ell_1
59: P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                     113: P_{9926} = (5, 26, 1, 1) lies on line \ell_4
60: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                     114: P_{9952} = (31, 26, 1, 1) lies on line \ell_1
61: P_{66} = (62, 1, 0, 0) lies on line \ell_0
                                                                     115: P_{10024} = (39, 27, 1, 1) lies on line \ell_1
62: P_{67} = (63, 1, 0, 0) lies on line \ell_0
                                                                     116: P_{10045} = (60, 27, 1, 1) lies on line \ell_4
                                                                     117 : P_{10093} = (44, 28, 1, 1) lies on line \ell_1
63: P_{8377} = (56, 1, 1, 1) lies on line \ell_1
64: P_{8378} = (57, 1, 1, 1) lies on line \ell_4
                                                                     118: P_{10097} = (48, 28, 1, 1) lies on line \ell_4
                                                                     119: P_{10122} = (9, 29, 1, 1) lies on line \ell_4
65: P_{8402} = (17, 2, 1, 1) lies on line \ell_1
66: P_{8404} = (19, 2, 1, 1) lies on line \ell_4
                                                                     120: P_{10133} = (20, 29, 1, 1) lies on line \ell_1
67: P_{8490} = (41, 3, 1, 1) lies on line \ell_1
                                                                     121: P_{10212} = (35, 30, 1, 1) lies on line \ell_4
68: P_{8491} = (42, 3, 1, 1) lies on line \ell_4
                                                                     122: P_{10238} = (61, 30, 1, 1) lies on line \ell_1
69: P_{8547} = (34, 4, 1, 1) lies on line \ell_1
                                                                     123: P_{10246} = (5, 31, 1, 1) lies on line \ell_1
70 : P_{8551} = (38, 4, 1, 1) lies on line \ell_4
                                                                     124: P_{10267} = (26, 31, 1, 1) lies on line \ell_4
71: P_{8603} = (26, 5, 1, 1) lies on line \ell_1
                                                                     125: P_{10328} = (23, 32, 1, 1) lies on line \ell_4
72: P_{8608} = (31, 5, 1, 1) lies on line \ell_4
                                                                     126: P_{10360} = (55, 32, 1, 1) lies on line \ell_1
73: P_{8692} = (51, 6, 1, 1) lies on line \ell_1
                                                                     127: P_{10384} = (15, 33, 1, 1) lies on line \ell_1
74: P_{8694} = (53, 6, 1, 1) lies on line \ell_4
                                                                     128: P_{10415} = (46, 33, 1, 1) lies on line \ell_4
75: P_{8716} = (11, 7, 1, 1) lies on line \ell_1
                                                                     129: P_{10437} = (4, 34, 1, 1) lies on line \ell_4
76: P_{8717} = (12, 7, 1, 1) lies on line \ell_4
                                                                     130: P_{10471} = (38, 34, 1, 1) lies on line \ell_1
77: P_{8806} = (37, 8, 1, 1) lies on line \ell_1
                                                                     131: P_{10527} = (30, 35, 1, 1) lies on line \ell_1
78: P_{8814} = (45, 8, 1, 1) lies on line \ell_4
                                                                     132: P_{10558} = (61, 35, 1, 1) lies on line \ell_4
79: P_{8853} = (20, 9, 1, 1) lies on line \ell_4
                                                                     133: P_{10582} = (21, 36, 1, 1) lies on line \ell_1
                                                                     134: P_{10610} = (49, 36, 1, 1) lies on line \ell_4
80: P_{8862} = (29, 9, 1, 1) lies on line \ell_1
81: P_{8949} = (52, 10, 1, 1) lies on line \ell_1
                                                                     135: P_{10633} = (8, 37, 1, 1) lies on line \ell_4
                                                                     136 : P_{10670} = (45, 37, 1, 1) lies on line \ell_1
82: P_{8959} = (62, 10, 1, 1) lies on line \ell_4
83: P_{8968} = (7, 11, 1, 1) lies on line \ell_4
                                                                      137: P_{10693} = (4, 38, 1, 1) lies on line \ell_1
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192 : P_{233684} = (19, 2, 56, 1) lies on line \ell_5
138: P_{10723} = (34, 38, 1, 1) lies on line \ell_4
139 : P_{10780} = (27, 39, 1, 1) lies on line \ell_4
                                                                    193: P_{233770} = (41, 3, 56, 1) lies on line \ell_2
140: P_{10813} = (60, 39, 1, 1) lies on line \ell_1
                                                                     194: P_{233771} = (42, 3, 56, 1) lies on line \ell_5
141: P_{10835} = (18, 40, 1, 1) lies on line \ell_1
                                                                    195: P_{233827} = (34, 4, 56, 1) lies on line \ell_2
                                                                    196: P_{233831} = (38, 4, 56, 1) lies on line \ell_5
142: P_{10875} = (58, 40, 1, 1) lies on line \ell_4
143: P_{10884} = (3, 41, 1, 1) lies on line \ell_4
                                                                    197: P_{233883} = (26, 5, 56, 1) lies on line \ell_2
144: P_{10923} = (42, 41, 1, 1) lies on line \ell_1
                                                                     198: P_{233888} = (31, 5, 56, 1) lies on line \ell_5
145: P_{10948} = (3, 42, 1, 1) lies on line \ell_1
                                                                     199: P_{233972} = (51, 6, 56, 1) lies on line \ell_2
146: P_{10986} = (41, 42, 1, 1) lies on line \ell_4
                                                                    200: P_{233974} = (53, 6, 56, 1) lies on line \ell_5
147: P_{11025} = (16, 43, 1, 1) lies on line \ell_4
                                                                    201: P_{233996} = (11, 7, 56, 1) lies on line \ell_2
148: P_{11068} = (59, 43, 1, 1) lies on line \ell_1
                                                                    202: P_{233997} = (12, 7, 56, 1) lies on line \ell_5
149 : P_{11101} = (28, 44, 1, 1) lies on line \ell_4
                                                                    203: P_{234086} = (37, 8, 56, 1) lies on line \ell_2
150: P_{11121} = (48, 44, 1, 1) lies on line \ell_1
                                                                    204: P_{234094} = (45, 8, 56, 1) lies on line \ell_5
151: P_{11145} = (8, 45, 1, 1) lies on line \ell_1
                                                                    205: P_{234133} = (20, 9, 56, 1) lies on line \ell_5
152 : P_{11174} = (37, 45, 1, 1) lies on line \ell_4
                                                                     206: P_{234142} = (29, 9, 56, 1) lies on line \ell_2
153: P_{11216} = (15, 46, 1, 1) lies on line \ell_4
                                                                    207: P_{234229} = (52, 10, 56, 1) lies on line \ell_2
154: P_{11234} = (33, 46, 1, 1) lies on line \ell_1
                                                                     208: P_{234239} = (62, 10, 56, 1) lies on line \ell_5
155: P_{11290} = (25, 47, 1, 1) lies on line \ell_1
                                                                    209: P_{234248} = (7, 11, 56, 1) lies on line \ell_5
                                                                    210: P_{234253} = (12, 11, 56, 1) lies on line \ell_2
156: P_{11319} = (54, 47, 1, 1) lies on line \ell_4
157: P_{11357} = (28, 48, 1, 1) lies on line \ell_1
                                                                    211: P_{234312} = (7, 12, 56, 1) lies on line \ell_2
158: P_{11373} = (44, 48, 1, 1) lies on line \ell_4
                                                                    212: P_{234316} = (11, 12, 56, 1) lies on line \ell_5
159: P_{11414} = (21, 49, 1, 1) lies on line \ell_4
                                                                    213: P_{234419} = (50, 13, 56, 1) lies on line \ell_5
160: P_{11429} = (36, 49, 1, 1) lies on line \ell_1
                                                                    214: P_{234432} = (63, 13, 56, 1) lies on line \ell_2
161 : P_{11470} = (13, 50, 1, 1) lies on line \ell_1
                                                                    215 : P_{234455} = (22, 14, 56, 1) lies on line \ell_2
162: P_{11520} = (63, 50, 1, 1) lies on line \ell_4
                                                                    216: P_{234457} = (24, 14, 56, 1) lies on line \ell_5
163: P_{11527} = (6, 51, 1, 1) lies on line \ell_4
                                                                    217: P_{234530} = (33, 15, 56, 1) lies on line \ell_5
164: P_{11574} = (53, 51, 1, 1) lies on line \ell_1
                                                                    218: P_{234543} = (46, 15, 56, 1) lies on line \ell_2
165: P_{11595} = (10, 52, 1, 1) lies on line \ell_4
                                                                    219: P_{234604} = (43, 16, 56, 1) lies on line \ell_2
166: P_{11647} = (62, 52, 1, 1) lies on line \ell_1
                                                                    220: P_{234620} = (59, 16, 56, 1) lies on line \ell_5
167: P_{11655} = (6, 53, 1, 1) lies on line \ell_1
                                                                     221: P_{234627} = (2, 17, 56, 1) lies on line \ell_5
168: P_{11700} = (51, 53, 1, 1) lies on line \ell_4
                                                                    222: P_{234644} = (19, 17, 56, 1) lies on line \ell_2
169: P_{11738} = (25, 54, 1, 1) lies on line \ell_4
                                                                    223: P_{234729} = (40, 18, 56, 1) lies on line \ell_5
170: P_{11760} = (47, 54, 1, 1) lies on line \ell_1
                                                                    224: P_{234747} = (58, 18, 56, 1) lies on line \ell_2
                                                                     225 : P_{234755} = (2, 19, 56, 1) lies on line \ell_2
171: P_{11800} = (23, 55, 1, 1) lies on line \ell_1
172: P_{11809} = (32, 55, 1, 1) lies on line \ell_4
                                                                    226: P_{234770} = (17, 19, 56, 1) lies on line \ell_5
                                                                    227 : P_{234826} = (9, 20, 56, 1) lies on line \ell_2
173: P_{11842} = (1, 56, 1, 1) lies on line \ell_4
174: P_{11898} = (57, 56, 1, 1) lies on line \ell_1
                                                                    228: P_{234846} = (29, 20, 56, 1) lies on line \ell_5
175: P_{11906} = (1, 57, 1, 1) lies on line \ell_1
                                                                     229: P_{234917} = (36, 21, 56, 1) lies on line \ell_5
176: P_{11961} = (56, 57, 1, 1) lies on line \ell_4
                                                                    230: P_{234930} = (49, 21, 56, 1) lies on line \ell_2
                                                                    231 : P_{234959} = (14, 22, 56, 1) lies on line \ell_5
177: P_{11987} = (18, 58, 1, 1) lies on line \ell_4
                                                                    232: P_{234969} = (24, 22, 56, 1) lies on line \ell_2
178: P_{12009} = (40, 58, 1, 1) lies on line \ell_1
179: P_{12049} = (16, 59, 1, 1) lies on line \ell_1
                                                                    233: P_{235041} = (32, 23, 56, 1) lies on line \ell_2
180: P_{12076} = (43, 59, 1, 1) lies on line \ell_4
                                                                    234: P_{235064} = (55, 23, 56, 1) lies on line \ell_5
181: P_{12124} = (27, 60, 1, 1) lies on line \ell_1
                                                                    235: P_{235087} = (14, 24, 56, 1) lies on line \ell_2
182: P_{12136} = (39, 60, 1, 1) lies on line \ell_4
                                                                    236: P_{235095} = (22, 24, 56, 1) lies on line \ell_5
                                                                     237 : P_{235184} = (47, 25, 56, 1) lies on line \ell_5
183: P_{12191} = (30, 61, 1, 1) lies on line \ell_4
184: P_{12196} = (35, 61, 1, 1) lies on line \ell_1
                                                                    238: P_{235191} = (54, 25, 56, 1) lies on line \ell_2
185: P_{12235} = (10, 62, 1, 1) lies on line \ell_1
                                                                    239: P_{235206} = (5, 26, 56, 1) lies on line \ell_5
186: P_{12277} = (52, 62, 1, 1) lies on line \ell_4
                                                                    240: P_{235232} = (31, 26, 56, 1) lies on line \ell_2
187: P_{12302} = (13, 63, 1, 1) lies on line \ell_4
                                                                    241: P_{235304} = (39, 27, 56, 1) lies on line \ell_2
188: P_{12339} = (50, 63, 1, 1) lies on line \ell_1
                                                                    242: P_{235325} = (60, 27, 56, 1) lies on line \ell_5
189: P_{233657} = (56, 1, 56, 1) lies on line \ell_2
                                                                    243: P_{235373} = (44, 28, 56, 1) lies on line \ell_2
190 : P_{233658} = (57, 1, 56, 1) lies on line \ell_5
                                                                    244: P_{235377} = (48, 28, 56, 1) lies on line \ell_5
191: P_{233682} = (17, 2, 56, 1) lies on line \ell_2
                                                                    245: P_{235402} = (9, 29, 56, 1) lies on line \ell_5
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300: P_{237178} = (57, 56, 56, 1) lies on line \ell_2
246: P_{235413} = (20, 29, 56, 1) lies on line \ell_2
247: P_{235492} = (35, 30, 56, 1) lies on line \ell_5
                                                                    301: P_{237186} = (1, 57, 56, 1) lies on line \ell_2
248: P_{235518} = (61, 30, 56, 1) lies on line \ell_2
                                                                    302: P_{237241} = (56, 57, 56, 1) lies on line \ell_5
249: P_{235526} = (5, 31, 56, 1) lies on line \ell_2
                                                                    303: P_{237267} = (18, 58, 56, 1) lies on line \ell_5
                                                                    304: P_{237289} = (40, 58, 56, 1) lies on line \ell_2
250: P_{235547} = (26, 31, 56, 1) lies on line \ell_5
251: P_{235608} = (23, 32, 56, 1) lies on line \ell_5
                                                                    305: P_{237329} = (16, 59, 56, 1) lies on line \ell_2
252: P_{235640} = (55, 32, 56, 1) lies on line \ell_2
                                                                    306: P_{237356} = (43, 59, 56, 1) lies on line \ell_5
253: P_{235664} = (15, 33, 56, 1) lies on line \ell_2
                                                                    307: P_{237404} = (27, 60, 56, 1) lies on line \ell_2
254 : P_{235695} = (46, 33, 56, 1) lies on line \ell_5
                                                                    308: P_{237416} = (39, 60, 56, 1) lies on line \ell_5
255: P_{235717} = (4, 34, 56, 1) lies on line \ell_5
                                                                    309: P_{237471} = (30, 61, 56, 1) lies on line \ell_5
256: P_{235751} = (38, 34, 56, 1) lies on line \ell_2
                                                                    310: P_{237476} = (35, 61, 56, 1) lies on line \ell_2
257 : P_{235807} = (30, 35, 56, 1) lies on line \ell_2
                                                                    311: P_{237515} = (10, 62, 56, 1) lies on line \ell_2
258: P_{235838} = (61, 35, 56, 1) lies on line \ell_5
                                                                    312: P_{237557} = (52, 62, 56, 1) lies on line \ell_5
259: P_{235862} = (21, 36, 56, 1) lies on line \ell_2
                                                                    313: P_{237582} = (13, 63, 56, 1) lies on line \ell_5
260: P_{235890} = (49, 36, 56, 1) lies on line \ell_5
                                                                    314: P_{237619} = (50, 63, 56, 1) lies on line \ell_2
261: P_{235913} = (8, 37, 56, 1) lies on line \ell_5
                                                                    315: P_{237753} = (56, 1, 57, 1) lies on line \ell_3
262: P_{235950} = (45, 37, 56, 1) lies on line \ell_2
                                                                    316: P_{237754} = (57, 1, 57, 1) lies on line \ell_6
263: P_{235973} = (4, 38, 56, 1) lies on line \ell_2
                                                                    317: P_{237778} = (17, 2, 57, 1) lies on line \ell_3
264: P_{236003} = (34, 38, 56, 1) lies on line \ell_5
                                                                    318: P_{237780} = (19, 2, 57, 1) lies on line \ell_6
                                                                    319 : P_{237866} = (41, 3, 57, 1) lies on line \ell_3
265: P_{236060} = (27, 39, 56, 1) lies on line \ell_5
266: P_{236093} = (60, 39, 56, 1) lies on line \ell_2
                                                                    320: P_{237867} = (42, 3, 57, 1) lies on line \ell_6
267: P_{236115} = (18, 40, 56, 1) lies on line \ell_2
                                                                    321: P_{237923} = (34, 4, 57, 1) lies on line \ell_3
268: P_{236155} = (58, 40, 56, 1) lies on line \ell_5
                                                                    322: P_{237927} = (38, 4, 57, 1) lies on line \ell_6
269 : P_{236164} = (3, 41, 56, 1) lies on line \ell_5
                                                                    323: P_{237979} = (26, 5, 57, 1) lies on line \ell_3
270: P_{236203} = (42, 41, 56, 1) lies on line \ell_2
                                                                    324: P_{237984} = (31, 5, 57, 1) lies on line \ell_6
271: P_{236228} = (3, 42, 56, 1) lies on line \ell_2
                                                                    325: P_{238068} = (51, 6, 57, 1) lies on line \ell_3
272: P_{236266} = (41, 42, 56, 1) lies on line \ell_5
                                                                    326: P_{238070} = (53, 6, 57, 1) lies on line \ell_6
273: P_{236305} = (16, 43, 56, 1) lies on line \ell_5
                                                                    327: P_{238092} = (11, 7, 57, 1) lies on line \ell_3
274: P_{236348} = (59, 43, 56, 1) lies on line \ell_2
                                                                    328: P_{238093} = (12, 7, 57, 1) lies on line \ell_6
275: P_{236381} = (28, 44, 56, 1) lies on line \ell_5
                                                                    329: P_{238182} = (37, 8, 57, 1) lies on line \ell_3
276: P_{236401} = (48, 44, 56, 1) lies on line \ell_2
                                                                    330: P_{238190} = (45, 8, 57, 1) lies on line \ell_6
277: P_{236425} = (8, 45, 56, 1) lies on line \ell_2
                                                                    331 : P_{238229} = (20, 9, 57, 1) lies on line \ell_6
278: P_{236454} = (37, 45, 56, 1) lies on line \ell_5
                                                                    332: P_{238238} = (29, 9, 57, 1) lies on line \ell_3
                                                                    333: P_{238325} = (52, 10, 57, 1) lies on line \ell_3
279: P_{236496} = (15, 46, 56, 1) lies on line \ell_5
280: P_{236514} = (33, 46, 56, 1) lies on line \ell_2
                                                                    334: P_{238335} = (62, 10, 57, 1) lies on line \ell_6
281: P_{236570} = (25, 47, 56, 1) lies on line \ell_2
                                                                    335: P_{238344} = (7, 11, 57, 1) lies on line \ell_6
282: P_{236599} = (54, 47, 56, 1) lies on line \ell_5
                                                                    336: P_{238349} = (12, 11, 57, 1) lies on line \ell_3
283: P_{236637} = (28, 48, 56, 1) lies on line \ell_2
                                                                    337: P_{238408} = (7, 12, 57, 1) lies on line \ell_3
284 : P_{236653} = (44, 48, 56, 1) lies on line \ell_5
                                                                    338: P_{238412} = (11, 12, 57, 1) lies on line \ell_6
                                                                    339: P_{238515} = (50, 13, 57, 1) lies on line \ell_6
285: P_{236694} = (21, 49, 56, 1) lies on line \ell_5
286: P_{236709} = (36, 49, 56, 1) lies on line \ell_2
                                                                    340: P_{238528} = (63, 13, 57, 1) lies on line \ell_3
287: P_{236750} = (13, 50, 56, 1) lies on line \ell_2
                                                                    341: P_{238551} = (22, 14, 57, 1) lies on line \ell_3
288 : P_{236800} = (63, 50, 56, 1) lies on line \ell_5
                                                                    342: P_{238553} = (24, 14, 57, 1) lies on line \ell_6
289: P_{236807} = (6, 51, 56, 1) lies on line \ell_5
                                                                    343: P_{238626} = (33, 15, 57, 1) lies on line \ell_6
290: P_{236854} = (53, 51, 56, 1) lies on line \ell_2
                                                                    344: P_{238639} = (46, 15, 57, 1) lies on line \ell_3
291: P_{236875} = (10, 52, 56, 1) lies on line \ell_5
                                                                    345: P_{238700} = (43, 16, 57, 1) lies on line \ell_3
292: P_{236927} = (62, 52, 56, 1) lies on line \ell_2
                                                                    346: P_{238716} = (59, 16, 57, 1) lies on line \ell_6
293: P_{236935} = (6, 53, 56, 1) lies on line \ell_2
                                                                    347: P_{238723} = (2, 17, 57, 1) lies on line \ell_6
294: P_{236980} = (51, 53, 56, 1) lies on line \ell_5
                                                                    348: P_{238740} = (19, 17, 57, 1) lies on line \ell_3
295: P_{237018} = (25, 54, 56, 1) lies on line \ell_5
                                                                    349: P_{238825} = (40, 18, 57, 1) lies on line \ell_6
296: P_{237040} = (47, 54, 56, 1) lies on line \ell_2
                                                                    350: P_{238843} = (58, 18, 57, 1) lies on line \ell_3
297: P_{237080} = (23, 55, 56, 1) lies on line \ell_2
                                                                    351: P_{238851} = (2, 19, 57, 1) lies on line \ell_3
298 : P_{237089} = (32, 55, 56, 1) lies on line \ell_5
                                                                    352: P_{238866} = (17, 19, 57, 1) lies on line \ell_6
299: P_{237122} = (1, 56, 56, 1) lies on line \ell_5
                                                                    353: P_{238922} = (9, 20, 57, 1) lies on line \ell_3
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398 : P_{240362} = (41, 42, 57, 1) lies on line \ell_6
354: P_{238942} = (29, 20, 57, 1) lies on line \ell_6
                                                                    399 : P_{240401} = (16, 43, 57, 1) lies on line \ell_6
355: P_{239013} = (36, 21, 57, 1) lies on line \ell_6
                                                                    400: P_{240444} = (59, 43, 57, 1) lies on line \ell_3
356: P_{239026} = (49, 21, 57, 1) lies on line \ell_3
357: P_{239055} = (14, 22, 57, 1) lies on line \ell_6
                                                                    401 : P_{240477} = (28, 44, 57, 1) lies on line \ell_6
                                                                    402: P_{240497} = (48, 44, 57, 1) lies on line \ell_3
358: P_{239065} = (24, 22, 57, 1) lies on line \ell_3
359: P_{239137} = (32, 23, 57, 1) lies on line \ell_3
                                                                    403: P_{240521} = (8, 45, 57, 1) lies on line \ell_3
                                                                    404: P_{240550} = (37, 45, 57, 1) lies on line \ell_6
360: P_{239160} = (55, 23, 57, 1) lies on line \ell_6
361: P_{239183} = (14, 24, 57, 1) lies on line \ell_3
                                                                    405: P_{240592} = (15, 46, 57, 1) lies on line \ell_6
362: P_{239191} = (22, 24, 57, 1) lies on line \ell_6
                                                                    406: P_{240610} = (33, 46, 57, 1) lies on line \ell_3
363: P_{239280} = (47, 25, 57, 1) lies on line \ell_6
                                                                    407: P_{240666} = (25, 47, 57, 1) lies on line \ell_3
364: P_{239287} = (54, 25, 57, 1) lies on line \ell_3
                                                                    408: P_{240695} = (54, 47, 57, 1) lies on line \ell_6
365: P_{239302} = (5, 26, 57, 1) lies on line \ell_6
                                                                    409: P_{240733} = (28, 48, 57, 1) lies on line \ell_3
366: P_{239328} = (31, 26, 57, 1) lies on line \ell_3
                                                                    410: P_{240749} = (44, 48, 57, 1) lies on line \ell_6
367: P_{239400} = (39, 27, 57, 1) lies on line \ell_3
                                                                    411: P_{240790} = (21, 49, 57, 1) lies on line \ell_6
368: P_{239421} = (60, 27, 57, 1) lies on line \ell_6
                                                                    412: P_{240805} = (36, 49, 57, 1) lies on line \ell_3
369: P_{239469} = (44, 28, 57, 1) lies on line \ell_3
                                                                    413: P_{240846} = (13, 50, 57, 1) lies on line \ell_3
370: P_{239473} = (48, 28, 57, 1) lies on line \ell_6
                                                                    414: P_{240896} = (63, 50, 57, 1) lies on line \ell_6
371: P_{239498} = (9, 29, 57, 1) lies on line \ell_6
                                                                    415: P_{240903} = (6, 51, 57, 1) lies on line \ell_6
372 : P_{239509} = (20, 29, 57, 1) lies on line \ell_3
                                                                    416: P_{240950} = (53, 51, 57, 1) lies on line \ell_3
373: P_{239588} = (35, 30, 57, 1) lies on line \ell_6
                                                                    417: P_{240971} = (10, 52, 57, 1) lies on line \ell_6
374: P_{239614} = (61, 30, 57, 1) lies on line \ell_3
                                                                    418: P_{241023} = (62, 52, 57, 1) lies on line \ell_3
375: P_{239622} = (5, 31, 57, 1) lies on line \ell_3
                                                                    419: P_{241031} = (6, 53, 57, 1) lies on line \ell_3
376: P_{239643} = (26, 31, 57, 1) lies on line \ell_6
                                                                    420: P_{241076} = (51, 53, 57, 1) lies on line \ell_6
377 : P_{239704} = (23, 32, 57, 1) lies on line \ell_6
                                                                    421 : P_{241114} = (25, 54, 57, 1) lies on line \ell_6
378: P_{239736} = (55, 32, 57, 1) lies on line \ell_3
                                                                    422: P_{241136} = (47, 54, 57, 1) lies on line \ell_3
379: P_{239760} = (15, 33, 57, 1) lies on line \ell_3
                                                                    423: P_{241176} = (23, 55, 57, 1) lies on line \ell_3
380: P_{239791} = (46, 33, 57, 1) lies on line \ell_6
                                                                    424: P_{241185} = (32, 55, 57, 1) lies on line \ell_6
381: P_{239813} = (4, 34, 57, 1) lies on line \ell_6
                                                                    425: P_{241218} = (1, 56, 57, 1) lies on line \ell_6
382: P_{239847} = (38, 34, 57, 1) lies on line \ell_3
                                                                    426: P_{241274} = (57, 56, 57, 1) lies on line \ell_3
383: P_{239903} = (30, 35, 57, 1) lies on line \ell_3
                                                                    427: P_{241282} = (1, 57, 57, 1) lies on line \ell_3
                                                                    428: P_{241337} = (56, 57, 57, 1) lies on line \ell_6
384: P_{239934} = (61, 35, 57, 1) lies on line \ell_6
385: P_{239958} = (21, 36, 57, 1) lies on line \ell_3
                                                                    429: P_{241363} = (18, 58, 57, 1) lies on line \ell_6
386: P_{239986} = (49, 36, 57, 1) lies on line \ell_6
                                                                    430: P_{241385} = (40, 58, 57, 1) lies on line \ell_3
                                                                    431 : P_{241425} = (16, 59, 57, 1) lies on line \ell_3
387: P_{240009} = (8, 37, 57, 1) lies on line \ell_6
388: P_{240046} = (45, 37, 57, 1) lies on line \ell_3
                                                                    432: P_{241452} = (43, 59, 57, 1) lies on line \ell_6
389 : P_{240069} = (4, 38, 57, 1) lies on line \ell_3
                                                                    433 : P_{241500} = (27, 60, 57, 1) lies on line \ell_3
390: P_{240099} = (34, 38, 57, 1) lies on line \ell_6
                                                                    434: P_{241512} = (39, 60, 57, 1) lies on line \ell_6
391 : P_{240156} = (27, 39, 57, 1) lies on line \ell_6
                                                                    435 : P_{241567} = (30, 61, 57, 1) lies on line \ell_6
392: P_{240189} = (60, 39, 57, 1) lies on line \ell_3
                                                                    436: P_{241572} = (35, 61, 57, 1) lies on line \ell_3
                                                                    437: P_{241611} = (10, 62, 57, 1) lies on line \ell_3
393: P_{240211} = (18, 40, 57, 1) lies on line \ell_3
394: P_{240251} = (58, 40, 57, 1) lies on line \ell_6
                                                                    438: P_{241653} = (52, 62, 57, 1) lies on line \ell_6
395: P_{240260} = (3,41,57,1) lies on line \ell_6
                                                                    439: P_{241678} = (13, 63, 57, 1) lies on line \ell_6
396: P_{240299} = (42, 41, 57, 1) lies on line \ell_3
                                                                    440: P_{241715} = (50, 63, 57, 1) lies on line \ell_3
397: P_{240324} = (3, 42, 57, 1) lies on line \ell_3
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The single points on the surface are:

Points on surface but on no line

The surface has 3843 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c} 0123456 \\ \hline 0 & 01111111 \\ 1 & 1011100 \\ 2 & 1101010 \\ 3 & 1110001 \\ 4 & 1100011 \\ 5 & 1010101 \\ 6 & 1001110 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{60}	P_{60}	P_{60}	P_{61}	P_{61}	P_{61}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{60}	P_{60}	P_{60}	P_{8258}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_{60}	P_{60}	P_{60}	P_{233537}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_{60}	P_{60}	P_{60}	P_{237633}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_{61}	P_{8258}	P_{61}	P_{61}

Line 5 intersects

	Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in p	ooint	P_{61}	P_{233537}	P_{61}	P_{61}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_{61}	P_{237633}	P_{61}	P_{61}

The surface has 4289 points: Too many to print.