Rank-65554 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1091051654

General information

Number of lines	3
Number of points	4161
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	3969
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^3, 1^{189}, 0^{3969}

Singular Points

The surface has 3 singular points:

$0: P_{132} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
$1: P_{3708} = \mathbf{P}(\epsilon^{21}, \epsilon^{42}, 1, 0) = \mathbf{P}(57, 56, 1, 0)$
$2: P_{3771} = \mathbf{P}(\epsilon^{42}, \epsilon^{21}, 1, 0) = \mathbf{P}(56, 57, 1, 0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4162} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4162} = \mathbf{Pl}(1,0,1,0,0,1)_{270529}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & \epsilon^{42} & 0 \\ 0 & 1 & \epsilon^{21} & 0 \end{array}\right]_{233073} = \left[\begin{array}{cccc} 1 & 0 & 56 & 0 \\ 0 & 1 & 57 & 0 \end{array}\right]_{233073} = \mathbf{Pl}(57,0,56,0,0,1)_{277570}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & \epsilon^{21} & 0 \\ 0 & 1 & \epsilon^{42} & 0 \end{array}\right]_{237233} = \left[\begin{array}{cccc} 1 & 0 & 57 & 0 \\ 0 & 1 & 56 & 0 \end{array}\right]_{237233} = \mathbf{Pl}(56,0,57,0,0,1)_{277696}$$

Rank of lines: (4162, 233073, 237233)
Rank of points on Klein quadric: (270529, 277570, 277696)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{3771} = (56,57,1,0) = \ell_0 \cap \ell_1$
$P_{3708} = (57,56,1,0) = \ell_0 \cap \ell_2$
$P_{132} = (1,1,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 189 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_{60} = (56,1,0,0)$ lies on line ℓ_1
2 : $P_{61} = (57,1,0,0)$ lies on line ℓ_2
3 : $P_{68} = (1,0,1,0)$ lies on line ℓ_0
4 : $P_{123} = (56,0,1,0)$ lies on line ℓ_2
5 : $P_{124} = (57,0,1,0)$ lies on line ℓ_1
6 : $P_{131} = (0,1,1,0)$ lies on line ℓ_0
7 : $P_{198} = (3,2,1,0)$ lies on line ℓ_0
8 : $P_{235} = (40,2,1,0)$ lies on line ℓ_1
9 : $P_{238} = (43,2,1,0)$ lies on line ℓ_2
10 : $P_{261} = (2,3,1,0)$ lies on line ℓ_0
11 : $P_{275} = (16,3,1,0)$ lies on line ℓ_1
12 : $P_{277} = (18,3,1,0)$ lies on line ℓ_2
13 : $P_{328} = (5,4,1,0)$ lies on line ℓ_0
14 : $P_{350} = (27,4,1,0)$ lies on line ℓ_1
15 : $P_{353} = (30,4,1,0)$ lies on line ℓ_2
16 : $P_{391} = (4,5,1,0)$ lies on line ℓ_0
17 : $P_{422} = (35,5,1,0)$ lies on line ℓ_1
18 : $P_{426} = (39,5,1,0)$ lies on line ℓ_2
19 : $P_{458} = (7,6,1,0)$ lies on line ℓ_0
20 : $P_{461} = (10,6,1,0)$ lies on line ℓ_1
21 : $P_{464} = (13,6,1,0)$ lies on line ℓ_2
22 : $P_{521} = (6,7,1,0)$ lies on line ℓ_0
23 : $P_{565} = (50,7,1,0)$ lies on line ℓ_1
24 : $P_{567} = (52,7,1,0)$ lies on line ℓ_2
25 : $P_{588} = (9,8,1,0)$ lies on line ℓ_0
26 : $P_{600} = (21,8,1,0)$ lies on line ℓ_2
27 : $P_{607} = (28,8,1,0)$ lies on line ℓ_1
28 : $P_{651} = (8,9,1,0)$ lies on line ℓ_0
29 : $P_{679} = (36,9,1,0)$ lies on line ℓ_1
30 : $P_{687} = (44,9,1,0)$ lies on line ℓ_2
31 : $P_{713} = (6,10,1,0)$ lies on line ℓ_2
32 : $P_{718} = (11,10,1,0)$ lies on line ℓ_0
33 : $P_{720} = (13,10,1,0)$ lies on line ℓ_1
34 : $P_{781} = (10,11,1,0)$ lies on line ℓ_0
35 : $P_{824} = (53,11,1,0)$ lies on line ℓ_1
36 : $P_{834} = (63,11,1,0)$ lies on line ℓ_2
37 : $P_{848} = (13,12,1,0)$ lies on line ℓ_0
38 : $P_{886} = (51,12,1,0)$ lies on line ℓ_2
39 : $P_{897} = (62,12,1,0)$ lies on line ℓ_1
40 : $P_{905} = (6,13,1,0)$ lies on line ℓ_1
41 : $P_{909} = (10,13,1,0)$ lies on line ℓ_2
42 : $P_{911} = (12,13,1,0)$ lies on line ℓ_0
43 : $P_{978} = (15,14,1,0)$ lies on line ℓ_0
44 : $P_{995} = (32,14,1,0)$ lies on line ℓ_2
45 : $P_{1010} = (47,14,1,0)$ lies on line ℓ_1
46 : $P_{1041} = (14,15,1,0)$ lies on line ℓ_0
47 : $P_{1050} = (23,15,1,0)$ lies on line ℓ_1
48 : $P_{1052} = (25,15,1,0)$ lies on line ℓ_2
49 : $P_{1094} = (3,16,1,0)$ lies on line ℓ_2
50 : $P_{1108} = (17,16,1,0)$ lies on line ℓ_0
51 : $P_{1109} = (18,16,1,0)$ lies on line ℓ_1

52 : $P_{1171} = (16, 17, 1, 0)$ lies on line ℓ_0
53 : $P_{1197} = (42, 17, 1, 0)$ lies on line ℓ_1
54 : $P_{1213} = (58, 17, 1, 0)$ lies on line ℓ_2
55 : $P_{1222} = (3, 18, 1, 0)$ lies on line ℓ_1
56 : $P_{1235} = (16, 18, 1, 0)$ lies on line ℓ_2
57 : $P_{1238} = (19, 18, 1, 0)$ lies on line ℓ_0
58 : $P_{1301} = (18, 19, 1, 0)$ lies on line ℓ_0
59 : $P_{1324} = (41, 19, 1, 0)$ lies on line ℓ_2
60 : $P_{1342} = (59, 19, 1, 0)$ lies on line ℓ_1
61 : $P_{1368} = (21, 20, 1, 0)$ lies on line ℓ_0
62 : $P_{1384} = (37, 20, 1, 0)$ lies on line ℓ_2
63 : $P_{1395} = (48, 20, 1, 0)$ lies on line ℓ_1
64 : $P_{1419} = (8, 21, 1, 0)$ lies on line ℓ_1
65 : $P_{1431} = (20, 21, 1, 0)$ lies on line ℓ_0
66 : $P_{1439} = (28, 21, 1, 0)$ lies on line ℓ_2
67 : $P_{1498} = (23, 22, 1, 0)$ lies on line ℓ_0
68 : $P_{1508} = (33, 22, 1, 0)$ lies on line ℓ_1
69 : $P_{1529} = (54, 22, 1, 0)$ lies on line ℓ_2
70 : $P_{1554} = (15, 23, 1, 0)$ lies on line ℓ_2
71 : $P_{1561} = (22, 23, 1, 0)$ lies on line ℓ_0
72 : $P_{1564} = (25, 23, 1, 0)$ lies on line ℓ_1
73 : $P_{1628} = (25, 24, 1, 0)$ lies on line ℓ_0
74 : $P_{1649} = (46, 24, 1, 0)$ lies on line ℓ_2
75 : $P_{1658} = (55, 24, 1, 0)$ lies on line ℓ_1
76 : $P_{1682} = (15, 25, 1, 0)$ lies on line ℓ_1
77 : $P_{1690} = (23, 25, 1, 0)$ lies on line ℓ_2
78 : $P_{1691} = (24, 25, 1, 0)$ lies on line ℓ_0
79 : $P_{1758} = (27, 26, 1, 0)$ lies on line ℓ_0
80 : $P_{1769} = (38, 26, 1, 0)$ lies on line ℓ_1
81 : $P_{1792} = (61, 26, 1, 0)$ lies on line ℓ_2
82 : $P_{1799} = (4, 27, 1, 0)$ lies on line ℓ_2
83 : $P_{1821} = (26, 27, 1, 0)$ lies on line ℓ_0
84 : $P_{1825} = (30, 27, 1, 0)$ lies on line ℓ_1
85 : $P_{1867} = (8, 28, 1, 0)$ lies on line ℓ_2
86 : $P_{1880} = (21, 28, 1, 0)$ lies on line ℓ_1
87 : $P_{1888} = (29, 28, 1, 0)$ lies on line ℓ_0
88 : $P_{1951} = (28, 29, 1, 0)$ lies on line ℓ_0
89 : $P_{1968} = (45, 29, 1, 0)$ lies on line ℓ_1
90 : $P_{1972} = (49, 29, 1, 0)$ lies on line ℓ_2
91 : $P_{1991} = (4, 30, 1, 0)$ lies on line ℓ_1
92 : $P_{2014} = (27, 30, 1, 0)$ lies on line ℓ_2
93 : $P_{2018} = (31, 30, 1, 0)$ lies on line ℓ_0
94 : $P_{2081} = (30, 31, 1, 0)$ lies on line ℓ_0
95 : $P_{2085} = (34, 31, 1, 0)$ lies on line ℓ_2
96 : $P_{2111} = (60, 31, 1, 0)$ lies on line ℓ_1
97 : $P_{2129} = (14, 32, 1, 0)$ lies on line ℓ_1
98 : $P_{2148} = (33, 32, 1, 0)$ lies on line ℓ_0
99 : $P_{2162} = (47, 32, 1, 0)$ lies on line ℓ_2
100 : $P_{2201} = (22, 33, 1, 0)$ lies on line ℓ_2
101 : $P_{2211} = (32, 33, 1, 0)$ lies on line ℓ_0
102 : $P_{2233} = (54, 33, 1, 0)$ lies on line ℓ_1
103 : $P_{2274} = (31, 34, 1, 0)$ lies on line ℓ_1
104 : $P_{2278} = (35, 34, 1, 0)$ lies on line ℓ_0
105 : $P_{2303} = (60, 34, 1, 0)$ lies on line ℓ_2
106 : $P_{2312} = (5, 35, 1, 0)$ lies on line ℓ_2
107 : $P_{2341} = (34, 35, 1, 0)$ lies on line ℓ_0
108 : $P_{2346} = (39, 35, 1, 0)$ lies on line ℓ_1
109 : $P_{2380} = (9, 36, 1, 0)$ lies on line ℓ_2
110 : $P_{2408} = (37, 36, 1, 0)$ lies on line ℓ_0
111 : $P_{2415} = (44, 36, 1, 0)$ lies on line ℓ_1
112 : $P_{2455} = (20, 37, 1, 0)$ lies on line ℓ_1
113 : $P_{2471} = (36, 37, 1, 0)$ lies on line ℓ_0
114 : $P_{2483} = (48, 37, 1, 0)$ lies on line ℓ_2
115 : $P_{2525} = (26, 38, 1, 0)$ lies on line ℓ_2
116 : $P_{2538} = (39, 38, 1, 0)$ lies on line ℓ_0
117 : $P_{2560} = (61, 38, 1, 0)$ lies on line ℓ_1
118 : $P_{2568} = (5, 39, 1, 0)$ lies on line ℓ_1
119 : $P_{2598} = (35, 39, 1, 0)$ lies on line ℓ_2
120 : $P_{2601} = (38, 39, 1, 0)$ lies on line ℓ_0
121 : $P_{2629} = (2, 40, 1, 0)$ lies on line ℓ_2
122 : $P_{2668} = (41, 40, 1, 0)$ lies on line ℓ_0
123 : $P_{2670} = (43, 40, 1, 0)$ lies on line ℓ_1
124 : $P_{2710} = (19, 41, 1, 0)$ lies on line ℓ_1
125 : $P_{2731} = (40, 41, 1, 0)$ lies on line ℓ_0
126 : $P_{2750} = (59, 41, 1, 0)$ lies on line ℓ_2
127 : $P_{2772} = (17, 42, 1, 0)$ lies on line ℓ_2
128 : $P_{2798} = (43, 42, 1, 0)$ lies on line ℓ_0
129 : $P_{2813} = (58, 42, 1, 0)$ lies on line ℓ_1
130 : $P_{2821} = (2, 43, 1, 0)$ lies on line ℓ_1
131 : $P_{2859} = (40, 43, 1, 0)$ lies on line ℓ_2
132 : $P_{2861} = (42, 43, 1, 0)$ lies on line ℓ_0
133 : $P_{2892} = (9, 44, 1, 0)$ lies on line ℓ_1
134 : $P_{2919} = (36, 44, 1, 0)$ lies on line ℓ_2
135 : $P_{2928} = (45, 44, 1, 0)$ lies on line ℓ_0
136 : $P_{2976} = (29, 45, 1, 0)$ lies on line ℓ_2
137 : $P_{2991} = (44, 45, 1, 0)$ lies on line ℓ_0
138 : $P_{2996} = (49, 45, 1, 0)$ lies on line ℓ_1
139 : $P_{3035} = (24, 46, 1, 0)$ lies on line ℓ_1
140 : $P_{3058} = (47, 46, 1, 0)$ lies on line ℓ_0
141 : $P_{3066} = (55, 46, 1, 0)$ lies on line ℓ_2
142 : $P_{3089} = (14, 47, 1, 0)$ lies on line ℓ_2
143 : $P_{3107} = (32, 47, 1, 0)$ lies on line ℓ_1
144 : $P_{3121} = (46, 47, 1, 0)$ lies on line ℓ_0
145 : $P_{3159} = (20, 48, 1, 0)$ lies on line ℓ_2
146 : $P_{3176} = (37, 48, 1, 0)$ lies on line ℓ_1
147 : $P_{3188} = (49, 48, 1, 0)$ lies on line ℓ_0
148 : $P_{3232} = (29, 49, 1, 0)$ lies on line ℓ_1
149 : $P_{3248} = (45, 49, 1, 0)$ lies on line ℓ_2
150 : $P_{3251} = (48, 49, 1, 0)$ lies on line ℓ_0
151 : $P_{3274} = (7, 50, 1, 0)$ lies on line ℓ_2
152 : $P_{3318} = (51, 50, 1, 0)$ lies on line ℓ_0
153 : $P_{3319} = (52, 50, 1, 0)$ lies on line ℓ_1
154 : $P_{3343} = (12, 51, 1, 0)$ lies on line ℓ_1
155 : $P_{3381} = (50, 51, 1, 0)$ lies on line ℓ_0
156 : $P_{3393} = (62, 51, 1, 0)$ lies on line ℓ_2
157 : $P_{3402} = (7, 52, 1, 0)$ lies on line ℓ_1
158 : $P_{3445} = (50, 52, 1, 0)$ lies on line ℓ_2
159 : $P_{3448} = (53, 52, 1, 0)$ lies on line ℓ_0

160 : $P_{3470} = (11, 53, 1, 0)$ lies on line ℓ_2
161 : $P_{3511} = (52, 53, 1, 0)$ lies on line ℓ_0
162 : $P_{3522} = (63, 53, 1, 0)$ lies on line ℓ_1
163 : $P_{3545} = (22, 54, 1, 0)$ lies on line ℓ_1
164 : $P_{3556} = (33, 54, 1, 0)$ lies on line ℓ_2
165 : $P_{3578} = (55, 54, 1, 0)$ lies on line ℓ_0
166 : $P_{3611} = (24, 55, 1, 0)$ lies on line ℓ_2
167 : $P_{3633} = (46, 55, 1, 0)$ lies on line ℓ_1
168 : $P_{3641} = (54, 55, 1, 0)$ lies on line ℓ_0
169 : $P_{3651} = (0, 56, 1, 0)$ lies on line ℓ_1
170 : $P_{3715} = (0, 57, 1, 0)$ lies on line ℓ_2
171 : $P_{3796} = (17, 58, 1, 0)$ lies on line ℓ_1
172 : $P_{3821} = (42, 58, 1, 0)$ lies on line ℓ_2
173 : $P_{3838} = (59, 58, 1, 0)$ lies on line ℓ_0
174 : $P_{3862} = (19, 59, 1, 0)$ lies on line ℓ_2
175 : $P_{3884} = (41, 59, 1, 0)$ lies on line ℓ_1
176 : $P_{3901} = (58, 59, 1, 0)$ lies on line ℓ_0
177 : $P_{3938} = (31, 60, 1, 0)$ lies on line ℓ_2
178 : $P_{3941} = (34, 60, 1, 0)$ lies on line ℓ_1
179 : $P_{3968} = (61, 60, 1, 0)$ lies on line ℓ_0
180 : $P_{3997} = (26, 61, 1, 0)$ lies on line ℓ_1
181 : $P_{4009} = (38, 61, 1, 0)$ lies on line ℓ_2
182 : $P_{4031} = (60, 61, 1, 0)$ lies on line ℓ_0
183 : $P_{4047} = (12, 62, 1, 0)$ lies on line ℓ_2
184 : $P_{4086} = (51, 62, 1, 0)$ lies on line ℓ_1
185 : $P_{4098} = (63, 62, 1, 0)$ lies on line ℓ_0
186 : $P_{4110} = (11, 63, 1, 0)$ lies on line ℓ_1
187 : $P_{4152} = (53, 63, 1, 0)$ lies on line ℓ_2
188 : $P_{4161} = (62, 63, 1, 0)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 3969 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{3771}	P_{3708}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{3771}	P_{132}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{3708}	P_{132}

The surface has 4161 points:
Too many to print.