# Rank-65915 over GF(16)

January 15, 2021

# The equation

The equation of the surface is:

$$X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)The point rank of the equation over GF(16) is 304226581

### General information

Number of lines	15
Number of points	337
Number of singular points	1
Number of Eckardt points	0
Number of double points	36
Number of single points	177
Number of points off lines	123
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$17^{15}$
Type of lines on points	$6, 2^{36}, 1^{177}, 0^{123}$

# Singular Points

The surface has 1 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

## The 15 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & \delta^5 & 0 & \delta^8 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64411} = \begin{bmatrix} 1 & 11 & 0 & 14 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64411} = \mathbf{Pl}(0, 8, 10, 0, 0, 1)_{4958}$$

$$\begin{split} \ell_1 &= \begin{bmatrix} 1 & \delta^{10} & 0 & \delta \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11722} = \begin{bmatrix} 1 & 10 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11722} = \mathbf{PI}(0,15,11,0,0,1)_{4996} \\ \ell_2 &= \begin{bmatrix} 1 & \delta^5 & 0 & \delta^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{20731} = \begin{bmatrix} 1 & 11 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{20731} = \mathbf{PI}(0,3,10,0,0,1)_{4953} \\ \ell_3 &= \begin{bmatrix} 1 & \delta^5 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{46939} = \begin{bmatrix} 1 & 11 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{46939} = \mathbf{PI}(0,11,10,0,0,1)_{4961} \\ \ell_4 &= \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{51034} = \begin{bmatrix} 1 & 10 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{51034} = \mathbf{PI}(0,10,11,0,0,1)_{4991} \\ \ell_5 &= \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^4 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{42298} = \begin{bmatrix} 1 & 10 & 0 & 9 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{42298} = \mathbf{PI}(0,5,11,0,0,1)_{4986} \\ \ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \mathbf{PI}(1,1,1,1,1,0)_{1250} \\ \ell_7 &= \begin{bmatrix} 1 & 0 & \delta^9 & \delta^{11} \\ 0 & 1 & \delta^6 & \delta^7 \end{bmatrix}_{58276} = \begin{bmatrix} 1 & 0 & 5 & 13 \\ 0 & 1 & 15 & 7 \end{bmatrix}_{58276} = \mathbf{PI}(13,9,6,4,6,1)_{31353} \\ \ell_8 &= \begin{bmatrix} 1 & 0 & \delta^6 & \delta^{14} \\ 0 & 1 & \delta^9 & \delta^{13} \end{bmatrix}_{56612} = \begin{bmatrix} 1 & 0 & 15 & 12 \\ 0 & 1 & 5 & 6 \end{bmatrix}_{56212} = \mathbf{PI}(12,2,7,14,7,1)_{35747} \\ \ell_9 &= \begin{bmatrix} 1 & 0 & \delta^{10} & \delta \\ 0 & 1 & \delta^3 & \delta^6 \end{bmatrix}_{42290} = \begin{bmatrix} 1 & 0 & 10 & 9 \\ 0 & 1 & 8 & 15 \end{bmatrix}_{42290} = \mathbf{PI}(2,12,15,5,13,1)_{61987} \\ \ell_{10} &= \begin{bmatrix} 1 & 0 & \delta^{10} & \delta \\ 0 & 1 & \delta^{12} & \delta^9 \end{bmatrix}_{11549} = \begin{bmatrix} 1 & 0 & 10 & 2 \\ 0 & 1 & 3 & 5 \end{bmatrix}_{11549} = \mathbf{PI}(9,13,5,15,12,1)_{55664} \\ \ell_{11} &= \begin{bmatrix} 1 & 0 & \delta^5 & \delta^2 \\ 0 & 1 & \delta^9 & \delta^3 \end{bmatrix}_{20608} = \begin{bmatrix} 1 & 0 & 11 & 4 \\ 0 & 1 & 15 & 3 \end{bmatrix}_{64218} = \mathbf{PI}(4,6,3,8,7,1)_{34809} \\ \ell_{12} &= \begin{bmatrix} 1 & 0 & \delta^{12} & \delta^{13} \\ 0 & 1 & \delta^3 & \delta^{11} \end{bmatrix}_{27243} = \begin{bmatrix} 1 & 0 & 8 & 7 \\ 0 & 1 & 8 & 13 \end{bmatrix}_{27243} = \mathbf{PI}(6,4,12,2,12,1)_{57266} \\ \ell_{14} &= \begin{bmatrix} 1 & 0 & \delta^3 & \delta^7 \\ 0 & 1 & \delta^{12} & \delta^{14} \end{bmatrix}_{32955} = \begin{bmatrix} 1 & 0 & 8 & 7 \\ 0 & 1 & 3 & 12 \end{bmatrix}_{32955} = \mathbf{PI}(7,14,13,9,13,1)_{61407} \end{aligned}$$

Rank of lines: (64411, 11722, 20731, 46939, 51034, 42298, 4385, 58276, 56612, 42290, 11549, 64218, 20608, 27243, 32955)

Rank of points on Klein quadric: (4958, 4996, 4953, 4961, 4991, 4986, 1250, 31353, 35747, 61987, 55664, 34809, 31864, 57266, 61407)

#### **Eckardt Points**

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 36 Double points:

The double points on the surface are:

$P_{2888} = (7, 3, 10, 1) = \ell_0 \cap \ell_7$
$P_{584} = (7, 3, 1, 1) = \ell_0 \cap \ell_{10}$
$P_{4168} = (7, 3, 15, 1) = \ell_0 \cap \ell_{14}$
$P_{1133} = (12, 5, 3, 1) = \ell_1 \cap \ell_8$
$P_{621} = (12, 5, 1, 1) = \ell_1 \cap \ell_{12}$
$P_{3181} = (12, 5, 11, 1) = \ell_1 \cap \ell_{14}$
$P_{2967} = (6, 8, 10, 1) = \ell_2 \cap \ell_8$
$P_{663} = (6, 8, 1, 1) = \ell_2 \cap \ell_9$
$P_{1687} = (6, 8, 5, 1) = \ell_2 \cap \ell_{13}$
$P_{3004} = (11, 10, 10, 1) = \ell_3 \cap \ell_6$
$P_{2492} = (11, 10, 8, 1) = \ell_3 \cap \ell_{11}$
$P_{1212} = (11, 10, 3, 1) = \ell_3 \cap \ell_{12}$
$P_{3275} = (10, 11, 11, 1) = \ell_4 \cap \ell_6$
$P_{1739} = (10, 11, 5, 1) = \ell_4 \cap \ell_9$
$P_{4299} = (10, 11, 15, 1) = \ell_4 \cap \ell_{10}$
$P_{2574} = (13, 15, 8, 1) = \ell_5 \cap \ell_7$
$P_{782} = (13, 15, 1, 1) = \ell_5 \cap \ell_{11}$
$P_{3342} = (13, 15, 11, 1) = \ell_5 \cap \ell_{13}$
$P_{2729} = (8, 9, 9, 1) = \ell_6 \cap \ell_7$

# $P_{820} = (3, 2, 2, 1) = \ell_6 \cap \ell_8$ $P_{1366} = (5, 4, 4, 1) = \ell_6 \cap \ell_{13}$ $P_{4096} = (15, 14, 14, 1) = \ell_6 \cap \ell_{14}$ $P_{716} = (11, 11, 1, 1) = \ell_7 \cap \ell_8$ $P_{2173} = (12, 6, 7, 1) = \ell_7 \cap \ell_9$ $P_{805} = (4, 1, 2, 1) = \ell_7 \cap \ell_{12}$ $P_{1934} = (13, 7, 6, 1) = \ell_8 \cap \ell_{10}$ $P_{2607} = (14, 1, 9, 1) = \ell_8 \cap \ell_{11}$ $P_{2016} = (15, 12, 6, 1) = \ell_9 \cap \ell_{11}$ $P_{3465} = (8, 7, 12, 1) = \ell_9 \cap \ell_{12}$ $P_{1322} = (9, 1, 4, 1) = \ell_9 \cap \ell_{14}$ $P_{3700} = (3, 6, 13, 1) = \ell_{10} \cap \ell_{11}$ $P_{2278} = (5, 13, 7, 1) = \ell_{10} \cap \ell_{12}$ $P_{3875} = (2, 1, 14, 1) = \ell_{10} \cap \ell_{13}$ $P_{3559} = (6, 13, 12, 1) = \ell_{11} \cap \ell_{14}$ $P_{3800} = (7, 12, 13, 1) = \ell_{12} \cap \ell_{13}$ $P_{699} = (10, 10, 1, 1) = \ell_{13} \cap \ell_{14}$

## Single Points

The surface has 177 single points: The single points on the surface are:

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0: P_{36} = (1, 1, 1, 0) lies on line \ell_6
1: P_{120} = (5, 6, 1, 0) lies on line \ell_7
2: P_{146} = (15, 7, 1, 0) lies on line \ell_8
3: P_{182} = (3, 10, 1, 0) lies on line \ell_9
4: P_{187} = (8, 10, 1, 0) lies on line \ell_{10}
5: P_{200} = (5, 11, 1, 0) lies on line \ell_{11}
6: P_{210} = (15, 11, 1, 0) lies on line \ell_{12}
7: P_{214} = (3, 12, 1, 0) lies on line \ell_{13}
8: P_{235} = (8, 13, 1, 0) lies on line \ell_{14}
9: P_{275} = (1,0,0,1) lies on line \ell_6
10: P_{311} = (5, 2, 0, 1) lies on line \ell_9
11: P_{329} = (7, 3, 0, 1) lies on line \ell_0
12: P_{346} = (8, 4, 0, 1) lies on line \ell_{11}
13: P_{366} = (12, 5, 0, 1) lies on line \ell_1
14: P_{379} = (9, 6, 0, 1) lies on line \ell_{13}
15: P_{388} = (2,7,0,1) lies on line \ell_{14}
16: P_{408} = (6, 8, 0, 1) lies on line \ell_2
17: P_{433} = (15, 9, 0, 1) lies on line \ell_{10}
18: P_{445} = (11, 10, 0, 1) lies on line \ell_3
19: P_{460} = (10, 11, 0, 1) lies on line \ell_4
20: P_{470} = (4, 12, 0, 1) lies on line \ell_8
21: P_{496} = (14, 13, 0, 1) lies on line \ell_7
22: P_{501} = (3, 14, 0, 1) lies on line \ell_{12}
23: P_{527} = (13, 15, 0, 1) lies on line \ell_5
24: P_{546} = (0, 1, 1, 1) lies on line \ell_6
25: P_{700} = (11, 10, 1, 1) lies on line \ell_3
26: P_{715} = (10, 11, 1, 1) lies on line \ell_4
27: P_{840} = (7, 3, 2, 1) lies on line \ell_0
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28: P_{855} = (6, 4, 2, 1) lies on line \ell_{10}
29: P_{860} = (11, 4, 2, 1) lies on line \ell_{14}
30: P_{877} = (12, 5, 2, 1) lies on line \ell_1
31: P_{912} = (15, 7, 2, 1) lies on line \ell_{13}
32: P_{919} = (6, 8, 2, 1) lies on line \ell_2
33: P_{956} = (11, 10, 2, 1) lies on line \ell_3
34: P_{963} = (2, 11, 2, 1) lies on line \ell_{11}
35: P_{971} = (10, 11, 2, 1) lies on line \ell_4
36: P_{1028} = (3, 15, 2, 1) lies on line \ell_9
37: P_{1038} = (13, 15, 2, 1) lies on line \ell_5
38: P_{1048} = (7,0,3,1) lies on line \ell_{11}
39: P_{1091} = (2,3,3,1) lies on line \ell_6
40: P_{1096} = (7, 3, 3, 1) lies on line \ell_0
41: P_{1121} = (0, 5, 3, 1) lies on line \ell_9
42: P_{1154} = (1,7,3,1) lies on line \ell_7
43: P_{1175} = (6, 8, 3, 1) lies on line \ell_2
44: P_{1188} = (3, 9, 3, 1) lies on line \ell_{14}
45: P_{1227} = (10, 11, 3, 1) lies on line \ell_4
46: P_{1229} = (12, 11, 3, 1) lies on line \ell_{13}
47: P_{1279} = (14, 14, 3, 1) lies on line \ell_{10}
48: P_{1294} = (13, 15, 3, 1) lies on line \ell_5
49: P_{1350} = (5, 3, 4, 1) lies on line \ell_{11}
50: P_{1352} = (7, 3, 4, 1) lies on line \ell_0
51: P_{1389} = (12, 5, 4, 1) lies on line \ell_1
52: P_{1431} = (6, 8, 4, 1) lies on line \ell_2
53: P_{1451} = (10, 9, 4, 1) lies on line \ell_8
54: P_{1454} = (13, 9, 4, 1) lies on line \ell_{12}
55: P_{1461} = (4, 10, 4, 1) lies on line \ell_{10}
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56: P_{1468} = (11, 10, 4, 1) lies on line \ell_3
                                                                      110: P_{2762} = (9, 11, 9, 1) lies on line \ell_{12}
                                                                      111: P_{2763} = (10, 11, 9, 1) lies on line \ell_4
57: P_{1483} = (10, 11, 4, 1) lies on line \ell_4
58: P_{1492} = (3, 12, 4, 1) lies on line \ell_7
                                                                      112: P_{2808} = (7, 14, 9, 1) lies on line \ell_9
59: P_{1550} = (13, 15, 4, 1) lies on line \ell_5
                                                                      113: P_{2812} = (11, 14, 9, 1) lies on line \ell_{13}
60: P_{1565} = (12, 0, 5, 1) lies on line \ell_{10}
                                                                      114: P_{2830} = (13, 15, 9, 1) lies on line \ell_5
61: P_{1587} = (2, 2, 5, 1) lies on line \ell_{12}
                                                                      115: P_{2874} = (9, 2, 10, 1) lies on line \ell_{10}
62: P_{1608} = (7, 3, 5, 1) lies on line \ell_0
                                                                      116: P_{2895} = (14, 3, 10, 1) lies on line \ell_{13}
63: P_{1637} = (4, 5, 5, 1) lies on line \ell_6
                                                                      117: P_{2914} = (1, 5, 10, 1) lies on line \ell_{11}
64: P_{1645} = (12, 5, 5, 1) lies on line \ell_1
                                                                      118: P_{2925} = (12, 5, 10, 1) lies on line \ell_1
65: P_{1681} = (0, 8, 5, 1) lies on line \ell_{11}
                                                                      119: P_{2965} = (4, 8, 10, 1) lies on line \ell_{14}
66: P_{1719} = (6, 10, 5, 1) lies on line \ell_7
                                                                      120: P_{2979} = (2, 9, 10, 1) lies on line \ell_9
67: P_{1724} = (11, 10, 5, 1) lies on line \ell_3
                                                                      121: P_{3019} = (10, 11, 10, 1) lies on line \ell_4
68: P_{1746} = (1, 12, 5, 1) lies on line \ell_{14}
                                                                      122: P_{3074} = (1, 15, 10, 1) lies on line \ell_{12}
69: P_{1782} = (5, 14, 5, 1) lies on line \ell_8
                                                                      123: P_{3086} = (13, 15, 10, 1) lies on line \ell_5
70: P_{1806} = (13, 15, 5, 1) lies on line \ell_5
                                                                      124: P_{3138} = (1, 3, 11, 1) lies on line \ell_9
71: P_{1818} = (9,0,6,1) lies on line \ell_7
                                                                      125: P_{3144} = (7, 3, 11, 1) lies on line \ell_0
72: P_{1841} = (0, 2, 6, 1) lies on line \ell_{14}
                                                                      126: P_{3167} = (14, 4, 11, 1) lies on line \ell_{12}
73: P_{1864} = (7, 3, 6, 1) lies on line \ell_0
                                                                      127: P_{3171} = (2, 5, 11, 1) lies on line \ell_7
                                                                      128: P_{3218} = (1, 8, 11, 1) lies on line \ell_{10}
74: P_{1892} = (3, 5, 6, 1) lies on line \ell_{13}
75: P_{1901} = (12, 5, 6, 1) lies on line \ell_1
                                                                      129: P_{3223} = (6, 8, 11, 1) lies on line \ell_2
76: P_{1912} = (7, 6, 6, 1) lies on line \ell_6
                                                                      130 : P_{3260} = (11, 10, 11, 1) lies on line \ell_3
                                                                      131: P_{3317} = (4, 14, 11, 1) lies on line \ell_{11}
77: P_{1915} = (10, 6, 6, 1) lies on line \ell_{12}
78: P_{1943} = (6, 8, 6, 1) lies on line \ell_2
                                                                      132: P_{3338} = (9, 15, 11, 1) lies on line \ell_8
79: P_{1980} = (11, 10, 6, 1) lies on line \ell_3
                                                                      133: P_{3349} = (4, 0, 12, 1) lies on line \ell_{13}
80: P_{1995} = (10, 11, 6, 1) lies on line \ell_4
                                                                      134: P_{3400} = (7, 3, 12, 1) lies on line \ell_0
81: P_{2062} = (13, 15, 6, 1) lies on line \ell_5
                                                                      135 : P_{3408} = (15, 3, 12, 1) lies on line \ell_8
82: P_{2067} = (2,0,7,1) lies on line \ell_8
                                                                      136: P_{3437} = (12, 5, 12, 1) lies on line \ell_1
83: P_{2120} = (7, 3, 7, 1) lies on line \ell_0
                                                                      137: P_{3479} = (6, 8, 12, 1) lies on line \ell_2
                                                                      138: P_{3516} = (11, 10, 12, 1) lies on line \ell_3
84: P_{2157} = (12, 5, 7, 1) lies on line \ell_1
85: P_{2183} = (6,7,7,1) lies on line \ell_6
                                                                      139: P_{3531} = (10, 11, 12, 1) lies on line \ell_4
86: P_{2187} = (10, 7, 7, 1) lies on line \ell_{11}
                                                                      140: P_{3548} = (11, 12, 12, 1) lies on line \ell_{10}
87: P_{2199} = (6, 8, 7, 1) lies on line \ell_2
                                                                      141: P_{3550} = (13, 12, 12, 1) lies on line \ell_6
88: P_{2209} = (0, 9, 7, 1) lies on line \ell_{13}
                                                                      142: P_{3569} = (0, 14, 12, 1) lies on line \ell_7
                                                                      143: P_{3598} = (13, 15, 12, 1) lies on line \ell_5
89: P_{2236} = (11, 10, 7, 1) lies on line \ell_3
90: P_{2251} = (10, 11, 7, 1) lies on line \ell_4
                                                                      144: P_{3615} = (14, 0, 13, 1) lies on line \ell_{14}
91: P_{2313} = (8, 15, 7, 1) lies on line \ell_{14}
                                                                      145: P_{3656} = (7, 3, 13, 1) lies on line \ell_0
92: P_{2318} = (13, 15, 7, 1) lies on line \ell_5
                                                                      146: P_{3665} = (0, 4, 13, 1) lies on line \ell_8
93: P_{2327} = (6,0,8,1) lies on line \ell_{12}
                                                                      147: P_{3693} = (12, 5, 13, 1) lies on line \ell_1
94: P_{2361} = (8, 2, 8, 1) lies on line \ell_{13}
                                                                      148: P_{3734} = (5, 8, 13, 1) lies on line \ell_7
95: P_{2376} = (7, 3, 8, 1) lies on line \ell_0
                                                                      149: P_{3735} = (6, 8, 13, 1) lies on line \ell_2
96 : P_{2389} = (4, 4, 8, 1) lies on line \ell_9
                                                                      150: P_{3772} = (11, 10, 13, 1) lies on line \ell_3
97: P_{2413} = (12, 5, 8, 1) lies on line \ell_1
                                                                      151: P_{3787} = (10, 11, 13, 1) lies on line \ell_4
98: P_{2418} = (1, 6, 8, 1) lies on line \ell_8
                                                                      152: P_{3820} = (11, 13, 13, 1) lies on line \ell_9
99: P_{2455} = (6, 8, 8, 1) lies on line \ell_2
                                                                      153: P_{3821} = (12, 13, 13, 1) lies on line \ell_6
100: P_{2458} = (9, 8, 8, 1) lies on line \ell_6
                                                                      154: P_{3854} = (13, 15, 13, 1) lies on line \ell_5
101: P_{2507} = (10, 11, 8, 1) lies on line \ell_4
                                                                      155: P_{3899} = (10, 2, 14, 1) lies on line \ell_7
102: P_{2510} = (13, 11, 8, 1) lies on line \ell_{14}
                                                                      156: P_{3901} = (12, 2, 14, 1) lies on line \ell_{11}
103: P_{2561} = (0, 15, 8, 1) lies on line \ell_{10}
                                                                      157: P_{3912} = (7, 3, 14, 1) lies on line \ell_0
104 : P_{2632} = (7, 3, 9, 1) lies on line \ell_0
                                                                      158: P_{3949} = (12, 5, 14, 1) lies on line \ell_1
105: P_{2665} = (8, 5, 9, 1) lies on line \ell_{10}
                                                                      159: P_{3991} = (6, 8, 14, 1) lies on line \ell_2
106: P_{2669} = (12, 5, 9, 1) lies on line \ell_1
                                                                      160: P_{4000} = (15, 8, 14, 1) lies on line \ell_{12}
107: P_{2678} = (5, 6, 9, 1) lies on line \ell_{14}
                                                                      161: P_{4028} = (11, 10, 14, 1) lies on line \ell_3
                                                                      162 : P_{4031} = (14, 10, 14, 1) lies on line \ell_9
108: P_{2711} = (6, 8, 9, 1) lies on line \ell_2
109: P_{2748} = (11, 10, 9, 1) lies on line \ell_3
                                                                      163: P_{4043} = (10, 11, 14, 1) lies on line \ell_4
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\begin{array}{llll} 164: \ P_{4073} = (8,13,14,1) \ \text{lies on line} \ \ell_8 \\ 165: \ P_{4110} = (13,15,14,1) \ \text{lies on line} \ \ell_5 \\ 166: \ P_{4126} = (13,0,15,1) \ \text{lies on line} \ \ell_9 \\ 167: \ P_{4161} = (0,3,15,1) \ \text{lies on line} \ \ell_1 \\ 168: \ P_{4192} = (15,4,15,1) \ \text{lies on line} \ \ell_1 \\ 169: \ P_{4205} = (12,5,15,1) \ \text{lies on line} \ \ell_1 \\ 170: \ P_{4247} = (6,8,15,1) \ \text{lies on line} \ \ell_2 \\ \end{array}
\begin{array}{ll} 171: \ P_{4266} = (9,9,15,1) \ \text{lies on line} \ \ell_8 \\ 172: \ P_{4280} = (7,10,15,1) \ \text{lies on line} \ \ell_3 \\ 173: \ P_{4284} = (11,10,15,1) \ \text{lies on line} \ \ell_3 \\ 174: \ P_{4322} = (1,13,15,1) \ \text{lies on line} \ \ell_1 \\ 175: \ P_{4366} = (13,15,15,1) \ \text{lies on line} \ \ell_5 \\ 176: \ P_{4367} = (14,15,15,1) \ \text{lies on line} \ \ell_6 \\ \end{array}
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The single points on the surface are:

#### Points on surface but on no line

The surface has 123 points not on any line: The points on the surface but not on lines are:

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0: P_0 = (1,0,0,0)
                                                                  37: P_{1612} = (11, 3, 5, 1)
1: P_1 = (0, 1, 0, 0)
                                                                  38: P_{1630} = (13, 4, 5, 1)
2: P_4 = (1, 1, 1, 1)
                                                                  39: P_{1733} = (4, 11, 5, 1)
3: P_{125} = (10, 6, 1, 0)
                                                                  40: P_{1755} = (10, 12, 5, 1)
                                                                  41: P_{1779} = (2, 14, 5, 1)
4: P_{141} = (10, 7, 1, 0)
5: P_{222} = (11, 12, 1, 0)
                                                                  42: P_{1800} = (7, 15, 5, 1)
                                                                  43: P_{1852} = (11, 2, 6, 1)
6: P_{238} = (11, 13, 1, 0)
7: P_{583} = (6, 3, 1, 1)
                                                                  44: P_{1866} = (9, 3, 6, 1)
8: P_{622} = (13, 5, 1, 1)
                                                                  45: P_{1949} = (12, 8, 6, 1)
                                                                  46: P_{1977} = (8, 10, 6, 1)
9: P_{664} = (7, 8, 1, 1)
10: P_{781} = (12, 15, 1, 1)
                                                                  47: P_{1988} = (3, 11, 6, 1)
11: P_{790} = (5, 0, 2, 1)
                                                                  48: P_{2009} = (8, 12, 6, 1)
12: P_{806} = (5, 1, 2, 1)
                                                                  49: P_{2064} = (15, 15, 6, 1)
13: P_{824} = (7, 2, 2, 1)
                                                                  50: P_{2126} = (13, 3, 7, 1)
14: P_{880} = (15, 5, 2, 1)
                                                                  51: P_{2150} = (5, 5, 7, 1)
15: P_{897} = (0, 7, 2, 1)
                                                                  52: P_{2195} = (2, 8, 7, 1)
16: P_{923} = (10, 8, 2, 1)
                                                                  53: P_{2220} = (11, 9, 7, 1)
17: P_{957} = (12, 10, 2, 1)
                                                                  54: P_{2228} = (3, 10, 7, 1)
18: P_{997} = (4, 13, 2, 1)
                                                                  55: P_{2249} = (8, 11, 7, 1)
19: P_{1006} = (13, 13, 2, 1)
                                                                  56: P_{2276} = (3, 13, 7, 1)
20: P_{1079} = (6, 2, 3, 1)
                                                                  57: P_{2357} = (4, 2, 8, 1)
                                                                  58: P_{2381} = (12, 3, 8, 1)
21: P_{1164} = (11, 7, 3, 1)
22: P_{1182} = (13, 8, 3, 1)
                                                                  59: P_{2386} = (1, 4, 8, 1)
                                                                  60: P_{2411} = (10, 5, 8, 1)
23: P_{1199} = (14, 9, 3, 1)
24: P_{1203} = (2, 10, 3, 1)
                                                                  61: P_{2428} = (11, 6, 8, 1)
25: P_{1266} = (1, 14, 3, 1)
                                                                  62: P_{2472} = (7, 9, 8, 1)
26: P_{1291} = (10, 15, 3, 1)
                                                                  63: P_{2490} = (9, 10, 8, 1)
27: P_{1305} = (8, 0, 4, 1)
                                                                  64: P_{2592} = (15, 0, 9, 1)
28: P_{1321} = (8, 1, 4, 1)
                                                                  65: P_{2608} = (15, 1, 9, 1)
29: P_{1373} = (12, 4, 4, 1)
                                                                  66: P_{2635} = (10, 3, 9, 1)
30: P_{1416} = (7,7,4,1)
                                                                  67: P_{2673} = (0, 6, 9, 1)
31: P_{1418} = (9,7,4,1)
                                                                  68: P_{2727} = (6, 9, 9, 1)
32: P_{1428} = (3, 8, 4, 1)
                                                                  69: P_{2750} = (13, 10, 9, 1)
33: P_{1479} = (6, 11, 4, 1)
                                                                  70: P_{2781} = (12, 12, 9, 1)
34: P_{1489} = (0, 12, 4, 1)
                                                                  71: P_{2783} = (14, 12, 9, 1)
35: P_{1548} = (11, 15, 4, 1)
                                                                  72: P_{2822} = (5, 15, 9, 1)
                                                                  73: P_{2844} = (11, 0, 10, 1)
36: P_{1586} = (1, 2, 5, 1)
```

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74: P_{2861} = (12, 1, 10, 1)
                                                                 99: P_{3589} = (4, 15, 12, 1)
75: P_{2862} = (13, 1, 10, 1)
                                                                 100: P_{3652} = (3, 3, 13, 1)
76: P_{2879} = (14, 2, 10, 1)
                                                                 101: P_{3675} = (10, 4, 13, 1)
77: P_{2981} = (4, 9, 10, 1)
                                                                 102: P_{3695} = (14, 5, 13, 1)
78: P_{2993} = (0, 10, 10, 1)
                                                                 103: P_{3712} = (15, 6, 13, 1)
79: P_{3027} = (2, 12, 10, 1)
                                                                 104: P_{3766} = (5, 10, 13, 1)
80: P_{3031} = (6, 12, 10, 1)
                                                                 105: P_{3792} = (15, 11, 13, 1)
                                                                 106: P_{3847} = (6, 15, 13, 1)
81: P_{3048} = (7, 13, 10, 1)
82: P_{3050} = (9, 13, 10, 1)
                                                                 107: P_{3860} = (3, 0, 14, 1)
83: P_{3099} = (10, 0, 11, 1)
                                                                 108: P_{3876} = (3, 1, 14, 1)
84: P_{3111} = (6, 1, 11, 1)
                                                                 109: P_{3913} = (8, 3, 14, 1)
85: P_{3112} = (7, 1, 11, 1)
                                                                 110: P_{3948} = (11, 5, 14, 1)
86: P_{3155} = (2, 4, 11, 1)
                                                                 111: P_{3955} = (2, 6, 14, 1)
87: P_{3189} = (4, 6, 11, 1)
                                                                 112: P_{3959} = (6, 6, 14, 1)
88: P_{3198} = (13, 6, 11, 1)
                                                                 113: P_{4040} = (7, 11, 14, 1)
89: P_{3213} = (12, 7, 11, 1)
                                                                 114: P_{4065} = (0, 13, 14, 1)
90: P_{3215} = (14, 7, 11, 1)
                                                                 115: P_{4094} = (13, 14, 14, 1)
91: P_{3265} = (0, 11, 11, 1)
                                                                 116: P_{4186} = (9, 4, 15, 1)
92: P_{3322} = (9, 14, 11, 1)
                                                                 117: P_{4199} = (6, 5, 15, 1)
93: P_{3432} = (7, 5, 12, 1)
                                                                 118: P_{4252} = (11, 8, 15, 1)
94: P_{3462} = (5, 7, 12, 1)
                                                                 119: P_{4258} = (1, 9, 15, 1)
95: P_{3481} = (8, 8, 12, 1)
                                                                 120: P_{4303} = (14, 11, 15, 1)
96: P_{3520} = (15, 10, 12, 1)
                                                                 121: P_{4331} = (10, 13, 15, 1)
97: P_{3526} = (5, 11, 12, 1)
                                                                 122: P_{4349} = (12, 14, 15, 1)
98: P_{3579} = (10, 14, 12, 1)
```

# Line Intersection Graph

	0123456789	10	11	12	13	14
0	0111110100	1	0	0	0	1
1	1011110010	0	0	1	0	1
2	1101110011	0	0	0	1	0
3	1110111000	0	1	1	0	0
4	11111011001	1	0	0	0	0
5	11111100100	0	1	0	1	0
6	0001100110	0	0	0	1	1
7	1000011011	0	0	1	0	0
8	0110001100	1	1	0	0	0
9	0010100100	0	1	1	0	1
10	1000100010	0	1	1	1	0
11	0001010011	1	0	0	0	1
12	0101000101	1	0	0	1	0
13	0010011000	1	0	1	0	1
14	1100001001	0	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_7$	$\ell_{10}$	$\ell_{14}$
in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{2888}$	$P_{584}$	$P_{4168}$

Line 1 intersects

	Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_8$	$\ell_{12}$	$\ell_{14}$
i	n point	$P_2$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{1133}$	$P_{621}$	$P_{3181}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_8$	$\ell_9$	$\ell_{13}$
in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{2967}$	$P_{663}$	$P_{1687}$

Line 3 intersects									
Line o intersects	Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_{11}$	$\ell_{12}$
	in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{3004}$	$P_{2492}$	$P_{1212}$
Line 4 intersects				•					•
Line 4 intersects	Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_9$	$\ell_{10}$
	in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{3275}$	$P_{1739}$	$P_{4299}$
Line 5 intersects				'					•
Line o intersects	Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_7$	$\ell_{11}$	$\ell_{13}$
	in point	$P_2$	$P_2$		$P_2$	$P_2$	$P_{2574}$		$P_{3342}$
Line 6 intersects							'		
Line o intersects	Line	1	3	$\ell_4$		$\ell_7$	$\ell_8$	$\ell_{13}$	$\ell_{14}$
	in point		004	$P_{3275}$		2729	$P_{820}$	$P_{1366}$	$P_{4096}$
Line 7 intersects									
Line / intersects	Line	,	$\ell_0$	$\ell_5$		$\ell_6$	$\ell_8$	$\ell_9$	$\ell_{12}$
	in point		2888	$P_{2574}$	1 F	2729	$P_{716}$	$P_{2173}$	$P_{805}$
Line 8 intersects							1 ,		
Line o intersects	Line	, ,	$\overline{\ell_1}$	$\ell_2$		$\ell_6$	$\ell_7$	$\ell_{10}$	$\ell_{11}$
	in point		1133	$P_{2967}$		820	$P_{716}$	$P_{1934}$	$P_{2607}$
Line 9 intersects									
Line 9 intersects	Line	$\ell$	2	$\ell_4$	$\ell$	7	$\ell_{11}$	$\ell_{12}$	$\ell_{14}$
	in point	$P_6$		$P_{1739}$		173	$P_{2016}$	$P_{3465}$	$P_{1322}$
Line 10 intersects									
Line 10 intersects	Line	$\ell$	0	$\ell_4$	$\ell$	8	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$
	in point	$P_5$	-	$P_{4299}$	$P_1$	_	$P_{3700}$	$P_{2278}$	$P_{3875}$
Line 11 intersects									
Line 11 intersects	Line	1 1	3	$\ell_5$	$\ell$	8	$\ell_9$	$\ell_{10}$	$\ell_{14}$
	in point		492	$P_{782}$	_	607	$P_{2016}$	$P_{3700}$	$P_{3559}$
Line 12 intersects								0.00	
Line 12 intersects	Line		21	$\ell_3$	$\ell$	7	$\ell_9$	$\ell_{10}$	$\ell_{13}$
	in point		521	$P_{1212}$	$P_8$		$P_{3465}$	$P_{2278}$	$P_{3800}$
Line 13 intersects						-	0.000		
Line 15 intersects	Line	1	2	$\ell_5$		$\ell_6$	$\ell_{10}$	$\ell_{12}$	$\ell_{14}$
	in point		687	$P_{3342}$		1366	$P_{3875}$	$P_{3800}$	$P_{699}$
Line 14 intercepts			JO1	0012		1000	0010	1 3030	000
Line 14 intersects	Line	·	20	$\ell_1$		$\ell_6$	$\ell_9$	$\ell_{11}$	$\ell_{13}$
		1 "	~	1 1	- 1	•		1 11	10

The surface has 337 points: The points on the surface are:

in point

$0: P_0 = (1, 0, 0, 0)$ $1: P_1 = (0, 1, 0, 0)$ $2: P_2 = (0, 0, 1, 0)$ $3: P_4 = (1, 1, 1, 1)$	$ 10: P_{187} = (8, 10, 1, 0)  11: P_{200} = (5, 11, 1, 0)  12: P_{210} = (15, 11, 1, 0)  13: P_{214} = (3, 12, 1, 0) $	$ 20: P_{346} = (8, 4, 0, 1) $ $ 21: P_{366} = (12, 5, 0, 1) $ $ 22: P_{379} = (9, 6, 0, 1) $ $ 23: P_{388} = (2, 7, 0, 1) $
$4: P_{36} = (1, 1, 1, 0)$ $5: P_{120} = (5, 6, 1, 0)$ $6: P_{125} = (10, 6, 1, 0)$ $7: P_{141} = (10, 7, 1, 0)$ $8: P_{146} = (15, 7, 1, 0)$ $9: P_{182} = (3, 10, 1, 0)$	$14: P_{222} = (11, 12, 1, 0)$ $15: P_{235} = (8, 13, 1, 0)$ $16: P_{238} = (11, 13, 1, 0)$ $17: P_{275} = (1, 0, 0, 1)$ $18: P_{311} = (5, 2, 0, 1)$ $19: P_{329} = (7, 3, 0, 1)$	$24: P_{408} = (6, 8, 0, 1)$ $25: P_{433} = (15, 9, 0, 1)$ $26: P_{445} = (11, 10, 0, 1)$ $27: P_{460} = (10, 11, 0, 1)$ $28: P_{470} = (4, 12, 0, 1)$ $29: P_{496} = (14, 13, 0, 1)$

 $P_{4168}$   $P_{3181}$ 

 $P_{4096}$ 

 $P_{1322}$ 

 $P_{3559}$ 

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30: P_{501} = (3, 14, 0, 1)
                                                                                        138: P_{1934} = (13, 7, 6, 1)
                                            84: P_{1279} = (14, 14, 3, 1)
31: P_{527} = (13, 15, 0, 1)
                                                                                        139: P_{1943} = (6, 8, 6, 1)
                                            85: P_{1291} = (10, 15, 3, 1)
32: P_{546} = (0, 1, 1, 1)
                                                                                        140: P_{1949} = (12, 8, 6, 1)
                                            86: P_{1294} = (13, 15, 3, 1)
33: P_{583} = (6, 3, 1, 1)
                                            87: P_{1305} = (8, 0, 4, 1)
                                                                                        141: P_{1977} = (8, 10, 6, 1)
34: P_{584} = (7,3,1,1)
                                           88: P_{1321} = (8, 1, 4, 1)
                                                                                        142: P_{1980} = (11, 10, 6, 1)
                                            89: P_{1322} = (9, 1, 4, 1)
35: P_{621} = (12, 5, 1, 1)
                                                                                        143: P_{1988} = (3, 11, 6, 1)
36: P_{622} = (13, 5, 1, 1)
                                            90: P_{1350} = (5, 3, 4, 1)
                                                                                        144: P_{1995} = (10, 11, 6, 1)
37: P_{663} = (6, 8, 1, 1)
                                            91: P_{1352} = (7, 3, 4, 1)
                                                                                        145: P_{2009} = (8, 12, 6, 1)
38: P_{664} = (7, 8, 1, 1)
                                           92: P_{1366} = (5, 4, 4, 1)
                                                                                        146: P_{2016} = (15, 12, 6, 1)
39: P_{699} = (10, 10, 1, 1)
                                            93: P_{1373} = (12, 4, 4, 1)
                                                                                        147: P_{2062} = (13, 15, 6, 1)
40: P_{700} = (11, 10, 1, 1)
                                            94: P_{1389} = (12, 5, 4, 1)
                                                                                        148: P_{2064} = (15, 15, 6, 1)
41: P_{715} = (10, 11, 1, 1)
                                            95: P_{1416} = (7, 7, 4, 1)
                                                                                        149: P_{2067} = (2, 0, 7, 1)
42: P_{716} = (11, 11, 1, 1)
                                           96: P_{1418} = (9, 7, 4, 1)
                                                                                        150: P_{2120} = (7, 3, 7, 1)
43: P_{781} = (12, 15, 1, 1)
                                           97: P_{1428} = (3, 8, 4, 1)
                                                                                        151: P_{2126} = (13, 3, 7, 1)
                                                                                        152: P_{2150} = (5, 5, 7, 1)
44: P_{782} = (13, 15, 1, 1)
                                            98: P_{1431} = (6, 8, 4, 1)
45: P_{790} = (5,0,2,1)
                                           99: P_{1451} = (10, 9, 4, 1)
                                                                                        153: P_{2157} = (12, 5, 7, 1)
46: P_{805} = (4, 1, 2, 1)
                                           100: P_{1454} = (13, 9, 4, 1)
                                                                                        154: P_{2173} = (12, 6, 7, 1)
47: P_{806} = (5, 1, 2, 1)
                                                                                        155: P_{2183} = (6,7,7,1)
                                            101: P_{1461} = (4, 10, 4, 1)
48: P_{820} = (3, 2, 2, 1)
                                                                                        156: P_{2187} = (10, 7, 7, 1)
                                           102: P_{1468} = (11, 10, 4, 1)
49: P_{824} = (7, 2, 2, 1)
                                           103: P_{1479} = (6, 11, 4, 1)
                                                                                        157: P_{2195} = (2, 8, 7, 1)
50: P_{840} = (7, 3, 2, 1)
                                           104: P_{1483} = (10, 11, 4, 1)
                                                                                        158: P_{2199} = (6, 8, 7, 1)
51: P_{855} = (6, 4, 2, 1)
                                                                                        159: P_{2209} = (0, 9, 7, 1)
                                           105: P_{1489} = (0, 12, 4, 1)
                                                                                        160: P_{2220} = (11, 9, 7, 1)
52: P_{860} = (11, 4, 2, 1)
                                            106: P_{1492} = (3, 12, 4, 1)
53: P_{877} = (12, 5, 2, 1)
                                           107: P_{1548} = (11, 15, 4, 1)
                                                                                        161: P_{2228} = (3, 10, 7, 1)
54: P_{880} = (15, 5, 2, 1)
                                           108: P_{1550} = (13, 15, 4, 1)
                                                                                        162: P_{2236} = (11, 10, 7, 1)
55: P_{897} = (0,7,2,1)
                                           109: P_{1565} = (12, 0, 5, 1)
                                                                                        163: P_{2249} = (8, 11, 7, 1)
56: P_{912} = (15, 7, 2, 1)
                                           110: P_{1586} = (1, 2, 5, 1)
                                                                                        164: P_{2251} = (10, 11, 7, 1)
57: P_{919} = (6, 8, 2, 1)
                                           111: P_{1587} = (2, 2, 5, 1)
                                                                                        165: P_{2276} = (3, 13, 7, 1)
                                                                                        166: P_{2278} = (5, 13, 7, 1)
                                           112: P_{1608} = (7, 3, 5, 1)
58: P_{923} = (10, 8, 2, 1)
59: P_{956} = (11, 10, 2, 1)
                                           113: P_{1612} = (11, 3, 5, 1)
                                                                                        167: P_{2313} = (8, 15, 7, 1)
60: P_{957} = (12, 10, 2, 1)
                                           114: P_{1630} = (13, 4, 5, 1)
                                                                                        168: P_{2318} = (13, 15, 7, 1)
61: P_{963} = (2, 11, 2, 1)
                                           115: P_{1637} = (4, 5, 5, 1)
                                                                                        169: P_{2327} = (6,0,8,1)
62: P_{971} = (10, 11, 2, 1)
                                           116: P_{1645} = (12, 5, 5, 1)
                                                                                        170: P_{2357} = (4, 2, 8, 1)
                                                                                        171: P_{2361} = (8, 2, 8, 1)
63: P_{997} = (4, 13, 2, 1)
                                           117: P_{1681} = (0, 8, 5, 1)
64: P_{1006} = (13, 13, 2, 1)
                                                                                        172: P_{2376} = (7, 3, 8, 1)
                                           118: P_{1687} = (6, 8, 5, 1)
                                           119: P_{1719} = (6, 10, 5, 1)
65: P_{1028} = (3, 15, 2, 1)
                                                                                        173: P_{2381} = (12, 3, 8, 1)
66: P_{1038} = (13, 15, 2, 1)
                                           120: P_{1724} = (11, 10, 5, 1)
                                                                                        174: P_{2386} = (1, 4, 8, 1)
                                                                                        175: P_{2389} = (4, 4, 8, 1)
67: P_{1048} = (7,0,3,1)
                                           121: P_{1733} = (4, 11, 5, 1)
68: P_{1079} = (6, 2, 3, 1)
                                           122: P_{1739} = (10, 11, 5, 1)
                                                                                        176: P_{2411} = (10, 5, 8, 1)
69: P_{1091} = (2, 3, 3, 1)
                                           123: P_{1746} = (1, 12, 5, 1)
                                                                                        177: P_{2413} = (12, 5, 8, 1)
70: P_{1096} = (7, 3, 3, 1)
                                           124: P_{1755} = (10, 12, 5, 1)
                                                                                        178: P_{2418} = (1, 6, 8, 1)
                                                                                        179: P_{2428} = (11, 6, 8, 1)
71: P_{1121} = (0,5,3,1)
                                           125: P_{1779} = (2, 14, 5, 1)
72: P_{1133} = (12, 5, 3, 1)
                                           126: P_{1782} = (5, 14, 5, 1)
                                                                                        180: P_{2455} = (6, 8, 8, 1)
73: P_{1154} = (1, 7, 3, 1)
                                           127: P_{1800} = (7, 15, 5, 1)
                                                                                        181: P_{2458} = (9, 8, 8, 1)
74: P_{1164} = (11, 7, 3, 1)
                                           128: P_{1806} = (13, 15, 5, 1)
                                                                                        182: P_{2472} = (7, 9, 8, 1)
                                                                                        183: P_{2490} = (9, 10, 8, 1)
75: P_{1175} = (6, 8, 3, 1)
                                           129: P_{1818} = (9, 0, 6, 1)
                                           130 : P_{1841} = (0, 2, 6, 1)
76: P_{1182} = (13, 8, 3, 1)
                                                                                        184: P_{2492} = (11, 10, 8, 1)
77: P_{1188} = (3, 9, 3, 1)
                                           131: P_{1852} = (11, 2, 6, 1)
                                                                                        185: P_{2507} = (10, 11, 8, 1)
78: P_{1199} = (14, 9, 3, 1)
                                           132: P_{1864} = (7, 3, 6, 1)
                                                                                        186: P_{2510} = (13, 11, 8, 1)
79: P_{1203} = (2, 10, 3, 1)
                                           133: P_{1866} = (9, 3, 6, 1)
                                                                                        187: P_{2561} = (0, 15, 8, 1)
                                           134: P_{1892} = (3, 5, 6, 1)
                                                                                        188: P_{2574} = (13, 15, 8, 1)
80: P_{1212} = (11, 10, 3, 1)
81: P_{1227} = (10, 11, 3, 1)
                                           135: P_{1901} = (12, 5, 6, 1)
                                                                                        189: P_{2592} = (15, 0, 9, 1)
82: P_{1229} = (12, 11, 3, 1)
                                           136: P_{1912} = (7, 6, 6, 1)
                                                                                        190: P_{2607} = (14, 1, 9, 1)
83: P_{1266} = (1, 14, 3, 1)
                                           137: P_{1915} = (10, 6, 6, 1)
                                                                                        191: P_{2608} = (15, 1, 9, 1)
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192: P_{2632} = (7, 3, 9, 1)
                                                                                      290: P_{3800} = (7, 12, 13, 1)
                                           241: P_{3181} = (12, 5, 11, 1)
193: P_{2635} = (10, 3, 9, 1)
                                           242: P_{3189} = (4, 6, 11, 1)
                                                                                      291: P_{3820} = (11, 13, 13, 1)
194: P_{2665} = (8, 5, 9, 1)
                                           243: P_{3198} = (13, 6, 11, 1)
                                                                                      292: P_{3821} = (12, 13, 13, 1)
195: P_{2669} = (12, 5, 9, 1)
                                           244: P_{3213} = (12, 7, 11, 1)
                                                                                      293: P_{3847} = (6, 15, 13, 1)
196: P_{2673} = (0, 6, 9, 1)
                                           245: P_{3215} = (14, 7, 11, 1)
                                                                                      294: P_{3854} = (13, 15, 13, 1)
197: P_{2678} = (5, 6, 9, 1)
                                           246: P_{3218} = (1, 8, 11, 1)
                                                                                      295: P_{3860} = (3, 0, 14, 1)
198: P_{2711} = (6, 8, 9, 1)
                                           247: P_{3223} = (6, 8, 11, 1)
                                                                                      296: P_{3875} = (2, 1, 14, 1)
199: P_{2727} = (6, 9, 9, 1)
                                           248: P_{3260} = (11, 10, 11, 1)
                                                                                      297: P_{3876} = (3, 1, 14, 1)
200: P_{2729} = (8, 9, 9, 1)
                                           249: P_{3265} = (0, 11, 11, 1)
                                                                                      298: P_{3899} = (10, 2, 14, 1)
201: P_{2748} = (11, 10, 9, 1)
                                           250: P_{3275} = (10, 11, 11, 1)
                                                                                      299: P_{3901} = (12, 2, 14, 1)
202: P_{2750} = (13, 10, 9, 1)
                                           251: P_{3317} = (4, 14, 11, 1)
                                                                                      300: P_{3912} = (7, 3, 14, 1)
203: P_{2762} = (9, 11, 9, 1)
                                           252: P_{3322} = (9, 14, 11, 1)
                                                                                      301: P_{3913} = (8, 3, 14, 1)
204: P_{2763} = (10, 11, 9, 1)
                                           253: P_{3338} = (9, 15, 11, 1)
                                                                                      302: P_{3948} = (11, 5, 14, 1)
205: P_{2781} = (12, 12, 9, 1)
                                                                                      303: P_{3949} = (12, 5, 14, 1)
                                           254: P_{3342} = (13, 15, 11, 1)
206: P_{2783} = (14, 12, 9, 1)
                                           255: P_{3349} = (4, 0, 12, 1)
                                                                                      304: P_{3955} = (2, 6, 14, 1)
207: P_{2808} = (7, 14, 9, 1)
                                           256: P_{3400} = (7, 3, 12, 1)
                                                                                      305: P_{3959} = (6, 6, 14, 1)
                                           257: P_{3408} = (15, 3, 12, 1)
                                                                                      306: P_{3991} = (6, 8, 14, 1)
208: P_{2812} = (11, 14, 9, 1)
209: P_{2822} = (5, 15, 9, 1)
                                                                                      307: P_{4000} = (15, 8, 14, 1)
                                           258: P_{3432} = (7, 5, 12, 1)
210: P_{2830} = (13, 15, 9, 1)
                                           259: P_{3437} = (12, 5, 12, 1)
                                                                                      308: P_{4028} = (11, 10, 14, 1)
211: P_{2844} = (11, 0, 10, 1)
                                           260: P_{3462} = (5, 7, 12, 1)
                                                                                      309: P_{4031} = (14, 10, 14, 1)
212: P_{2861} = (12, 1, 10, 1)
                                           261: P_{3465} = (8, 7, 12, 1)
                                                                                      310: P_{4040} = (7, 11, 14, 1)
                                                                                      311: P_{4043} = (10, 11, 14, 1)
213: P_{2862} = (13, 1, 10, 1)
                                           262: P_{3479} = (6, 8, 12, 1)
214: P_{2874} = (9, 2, 10, 1)
                                           263: P_{3481} = (8, 8, 12, 1)
                                                                                      312: P_{4065} = (0, 13, 14, 1)
215: P_{2879} = (14, 2, 10, 1)
                                           264: P_{3516} = (11, 10, 12, 1)
                                                                                      313: P_{4073} = (8, 13, 14, 1)
216: P_{2888} = (7, 3, 10, 1)
                                           265: P_{3520} = (15, 10, 12, 1)
                                                                                      314: P_{4094} = (13, 14, 14, 1)
217: P_{2895} = (14, 3, 10, 1)
                                           266: P_{3526} = (5, 11, 12, 1)
                                                                                      315: P_{4096} = (15, 14, 14, 1)
218: P_{2914} = (1, 5, 10, 1)
                                           267: P_{3531} = (10, 11, 12, 1)
                                                                                      316: P_{4110} = (13, 15, 14, 1)
219: P_{2925} = (12, 5, 10, 1)
                                           268: P_{3548} = (11, 12, 12, 1)
                                                                                      317: P_{4126} = (13, 0, 15, 1)
                                                                                      318: P_{4161} = (0, 3, 15, 1)
220: P_{2965} = (4, 8, 10, 1)
                                           269: P_{3550} = (13, 12, 12, 1)
221: P_{2967} = (6, 8, 10, 1)
                                           270: P_{3559} = (6, 13, 12, 1)
                                                                                      319: P_{4168} = (7, 3, 15, 1)
222: P_{2979} = (2, 9, 10, 1)
                                           271: P_{3569} = (0, 14, 12, 1)
                                                                                      320: P_{4186} = (9, 4, 15, 1)
223: P_{2981} = (4, 9, 10, 1)
                                           272: P_{3579} = (10, 14, 12, 1)
                                                                                      321: P_{4192} = (15, 4, 15, 1)
224: P_{2993} = (0, 10, 10, 1)
                                           273: P_{3589} = (4, 15, 12, 1)
                                                                                      322: P_{4199} = (6, 5, 15, 1)
225: P_{3004} = (11, 10, 10, 1)
                                           274: P_{3598} = (13, 15, 12, 1)
                                                                                      323: P_{4205} = (12, 5, 15, 1)
226: P_{3019} = (10, 11, 10, 1)
                                           275: P_{3615} = (14, 0, 13, 1)
                                                                                      324: P_{4247} = (6, 8, 15, 1)
                                           276: P_{3652} = (3, 3, 13, 1)
227: P_{3027} = (2, 12, 10, 1)
                                                                                      325: P_{4252} = (11, 8, 15, 1)
228: P_{3031} = (6, 12, 10, 1)
                                           277: P_{3656} = (7, 3, 13, 1)
                                                                                      326: P_{4258} = (1, 9, 15, 1)
229: P_{3048} = (7, 13, 10, 1)
                                           278: P_{3665} = (0, 4, 13, 1)
                                                                                      327: P_{4266} = (9, 9, 15, 1)
230: P_{3050} = (9, 13, 10, 1)
                                                                                      328: P_{4280} = (7, 10, 15, 1)
                                           279: P_{3675} = (10, 4, 13, 1)
231: P_{3074} = (1, 15, 10, 1)
                                           280: P_{3693} = (12, 5, 13, 1)
                                                                                      329: P_{4284} = (11, 10, 15, 1)
232: P_{3086} = (13, 15, 10, 1)
                                           281: P_{3695} = (14, 5, 13, 1)
                                                                                      330: P_{4299} = (10, 11, 15, 1)
233: P_{3099} = (10, 0, 11, 1)
                                           282: P_{3700} = (3, 6, 13, 1)
                                                                                      331: P_{4303} = (14, 11, 15, 1)
234: P_{3111} = (6, 1, 11, 1)
                                           283: P_{3712} = (15, 6, 13, 1)
                                                                                      332: P_{4322} = (1, 13, 15, 1)
235: P_{3112} = (7, 1, 11, 1)
                                           284: P_{3734} = (5, 8, 13, 1)
                                                                                      333: P_{4331} = (10, 13, 15, 1)
236: P_{3138} = (1, 3, 11, 1)
                                           285: P_{3735} = (6, 8, 13, 1)
                                                                                      334: P_{4349} = (12, 14, 15, 1)
237: P_{3144} = (7, 3, 11, 1)
                                           286: P_{3766} = (5, 10, 13, 1)
                                                                                      335: P_{4366} = (13, 15, 15, 1)
238: P_{3155} = (2, 4, 11, 1)
                                           287: P_{3772} = (11, 10, 13, 1)
                                                                                      336: P_{4367} = (14, 15, 15, 1)
239: P_{3167} = (14, 4, 11, 1)
                                           288: P_{3787} = (10, 11, 13, 1)
240: P_{3171} = (2, 5, 11, 1)
                                           289: P_{3792} = (15, 11, 13, 1)
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