Rank-74532 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_2 + X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 287379734

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	0
Number of double points	15
Number of single points	135
Number of points off lines	170
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	5, 2^{15}, 1^{135}, 0^{170}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{69889} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{69889} = \mathbf{Pl}(0,0,0,1,0,1)_{5121}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^{12} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16090} = \begin{bmatrix} 1 & 10 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16090} = \mathbf{Pl}(0,4,11,0,0,1)_{4985}$$

$$\ell_3 = \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^{3} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37930} = \begin{bmatrix} 1 & 10 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37930} = \mathbf{Pl}(0,14,11,0,0,1)_{4995}$$

$$\ell_4 = \begin{bmatrix} 1 & \delta^{5} & 0 & \delta^{6} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{68779} = \begin{bmatrix} 1 & 11 & 0 & 15 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{68779} = \mathbf{Pl}(0,2,10,0,0,1)_{4952}$$

$$\ell_5 = \begin{bmatrix} 1 & \delta^{5} & 0 & \delta^{9} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25099} = \begin{bmatrix} 1 & 11 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25099} = \mathbf{Pl}(0,9,10,0,0,1)_{4959}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^{11} & \delta^{7} \\ 0 & 1 & \delta^{11} & \delta^{11} \end{bmatrix}_{34346} = \begin{bmatrix} 1 & 0 & 13 & 7 \\ 0 & 1 & 13 & 13 \end{bmatrix}_{34346} = \mathbf{Pl}(11,10,9,13,1,1)_{11746}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^{7} & \delta^{14} \\ 0 & 1 & \delta^{7} & \delta^{7} \end{bmatrix}_{54446} = \begin{bmatrix} 1 & 0 & 7 & 12 \\ 0 & 1 & 7 & 7 \end{bmatrix}_{54446} = \mathbf{Pl}(10,11,14,7,1,1)_{12705}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^{14} & \delta^{13} \\ 0 & 1 & \delta^{14} & \delta^{14} \end{bmatrix}_{29688} = \begin{bmatrix} 1 & 0 & 12 & 6 \\ 0 & 1 & 12 & 12 \end{bmatrix}_{29688} = \mathbf{Pl}(11,10,2,12,1,1)_{10261}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \delta^{13} & \delta^{11} \\ 0 & 1 & \delta^{13} & \delta^{13} \end{bmatrix}_{58524} = \begin{bmatrix} 1 & 0 & 6 & 13 \\ 0 & 1 & 6 & 6 \end{bmatrix}_{58524} = \mathbf{Pl}(10,11,4,6,1,1)_{10590}$$

Rank of lines: (69889, 70160, 16090, 37930, 68779, 25099, 34346, 54446, 29688, 58524)
Rank of points on Klein quadric: (5121, 1, 4985, 4995, 4952, 4959, 11746, 12705, 10261, 10590)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{530} = (0,0,1,1) = \ell_0 \cap \ell_1$
$P_{673} = (0,9,1,1) = \ell_0 \cap \ell_6$
$P_{753} = (0,14,1,1) = \ell_0 \cap \ell_7$
$P_{561} = (0,2,1,1) = \ell_0 \cap \ell_8$
$P_{593} = (0,4,1,1) = \ell_0 \cap \ell_9$
$P_{2169} = (8,6,7,1) = \ell_2 \cap \ell_8$
$P_{4217} = (8,6,15,1) = \ell_2 \cap \ell_9$
$P_{1924} = (3,7,6,1) = \ell_3 \cap \ell_6$
$P_{1668} = (3,7,5,1) = \ell_3 \cap \ell_7$
$P_{3798} = (5,12,13,1) = \ell_4 \cap \ell_7$
$P_{2518} = (5,12,8,1) = \ell_4 \cap \ell_8$
$P_{1264} = (15,13,3,1) = \ell_5 \cap \ell_6$
$P_{3568} = (15,13,12,1) = \ell_5 \cap \ell_9$
$P_{461} = (11,11,0,1) = \ell_6 \cap \ell_8$
$P_{444} = (10,10,0,1) = \ell_7 \cap \ell_9$

Single Points

The surface has 135 single points:
The single points on the surface are:

0 : $P_{1} = (0, 1, 0, 0)$ lies on line ℓ_{0}
1 : $P_{3} = (0, 0, 0, 1)$ lies on line ℓ_{1}
2 : $P_{62} = (11, 2, 1, 0)$ lies on line ℓ_{6}
3 : $P_{93} = (10, 4, 1, 0)$ lies on line ℓ_{7}
4 : $P_{174} = (11, 9, 1, 0)$ lies on line ℓ_{8}
5 : $P_{253} = (10, 14, 1, 0)$ lies on line ℓ_{9}
6 : $P_{378} = (8, 6, 0, 1)$ lies on line ℓ_{2}
7 : $P_{389} = (3, 7, 0, 1)$ lies on line ℓ_{3}
8 : $P_{471} = (5, 12, 0, 1)$ lies on line ℓ_{4}
9 : $P_{497} = (15, 13, 0, 1)$ lies on line ℓ_{5}
10 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_{0}
11 : $P_{577} = (0, 3, 1, 1)$ lies on line ℓ_{0}
12 : $P_{609} = (0, 5, 1, 1)$ lies on line ℓ_{0}
13 : $P_{625} = (0, 6, 1, 1)$ lies on line ℓ_{0}
14 : $P_{633} = (8, 6, 1, 1)$ lies on line ℓ_{2}
15 : $P_{641} = (0, 7, 1, 1)$ lies on line ℓ_{0}
16 : $P_{644} = (3, 7, 1, 1)$ lies on line ℓ_{3}
17 : $P_{657} = (0, 8, 1, 1)$ lies on line ℓ_{0}
18 : $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_{0}
19 : $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_{0}
20 : $P_{721} = (0, 12, 1, 1)$ lies on line ℓ_{0}
21 : $P_{726} = (5, 12, 1, 1)$ lies on line ℓ_{4}
22 : $P_{737} = (0, 13, 1, 1)$ lies on line ℓ_{0}
23 : $P_{752} = (15, 13, 1, 1)$ lies on line ℓ_{5}
24 : $P_{769} = (0, 15, 1, 1)$ lies on line ℓ_{0}
25 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_{1}
26 : $P_{789} = (4, 0, 2, 1)$ lies on line ℓ_{8}
27 : $P_{824} = (7, 2, 2, 1)$ lies on line ℓ_{7}
28 : $P_{889} = (8, 6, 2, 1)$ lies on line ℓ_{2}
29 : $P_{900} = (3, 7, 2, 1)$ lies on line ℓ_{3}
30 : $P_{982} = (5, 12, 2, 1)$ lies on line ℓ_{4}
31 : $P_{1008} = (15, 13, 2, 1)$ lies on line ℓ_{5}
32 : $P_{1029} = (4, 15, 2, 1)$ lies on line ℓ_{6}
33 : $P_{1032} = (7, 15, 2, 1)$ lies on line ℓ_{9}
34 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_{1}
35 : $P_{1070} = (13, 1, 3, 1)$ lies on line ℓ_{9}
36 : $P_{1145} = (8, 6, 3, 1)$ lies on line ℓ_{2}
37 : $P_{1150} = (13, 6, 3, 1)$ lies on line ℓ_{7}
38 : $P_{1156} = (3, 7, 3, 1)$ lies on line ℓ_{3}
39 : $P_{1200} = (15, 9, 3, 1)$ lies on line ℓ_{8}
40 : $P_{1238} = (5, 12, 3, 1)$ lies on line ℓ_{4}
41 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_{1}
42 : $P_{1306} = (9, 0, 4, 1)$ lies on line ℓ_{9}
43 : $P_{1354} = (9, 3, 4, 1)$ lies on line ℓ_{7}
44 : $P_{1357} = (12, 3, 4, 1)$ lies on line ℓ_{6}
45 : $P_{1373} = (12, 4, 4, 1)$ lies on line ℓ_{8}
46 : $P_{1401} = (8, 6, 4, 1)$ lies on line ℓ_{2}
47 : $P_{1412} = (3, 7, 4, 1)$ lies on line ℓ_{3}
48 : $P_{1494} = (5, 12, 4, 1)$ lies on line ℓ_{4}
49 : $P_{1520} = (15, 13, 4, 1)$ lies on line ℓ_{5}
50 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_{1}
51 : $P_{1576} = (7, 1, 5, 1)$ lies on line ℓ_{6}
52 : $P_{1657} = (8, 6, 5, 1)$ lies on line ℓ_{2}
53 : $P_{1750} = (5, 12, 5, 1)$ lies on line ℓ_{4}
54 : $P_{1768} = (7, 13, 5, 1)$ lies on line ℓ_{8}
55 : $P_{1776} = (15, 13, 5, 1)$ lies on line ℓ_{5}
56 : $P_{1780} = (3, 14, 5, 1)$ lies on line ℓ_{9}
57 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_{1}
58 : $P_{1893} = (4, 5, 6, 1)$ lies on line ℓ_{9}
59 : $P_{1913} = (8, 6, 6, 1)$ lies on line ℓ_{2}
60 : $P_{1989} = (4, 11, 6, 1)$ lies on line ℓ_{7}
61 : $P_{2006} = (5, 12, 6, 1)$ lies on line ℓ_{4}
62 : $P_{2032} = (15, 13, 6, 1)$ lies on line ℓ_{5}
63 : $P_{2052} = (3, 15, 6, 1)$ lies on line ℓ_{8}
64 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_{1}
65 : $P_{2153} = (8, 5, 7, 1)$ lies on line ℓ_{6}
66 : $P_{2180} = (3, 7, 7, 1)$ lies on line ℓ_{3}
67 : $P_{2255} = (14, 11, 7, 1)$ lies on line ℓ_{9}
68 : $P_{2262} = (5, 12, 7, 1)$ lies on line ℓ_{4}
69 : $P_{2288} = (15, 13, 7, 1)$ lies on line ℓ_{5}
70 : $P_{2319} = (14, 15, 7, 1)$ lies on line ℓ_{7}
71 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_{1}
72 : $P_{2349} = (12, 1, 8, 1)$ lies on line ℓ_{7}
73 : $P_{2358} = (5, 2, 8, 1)$ lies on line ℓ_{6}
74 : $P_{2425} = (8, 6, 8, 1)$ lies on line ℓ_{2}
75 : $P_{2436} = (3, 7, 8, 1)$ lies on line ℓ_{3}
76 : $P_{2445} = (12, 7, 8, 1)$ lies on line ℓ_{9}
77 : $P_{2544} = (15, 13, 8, 1)$ lies on line ℓ_{5}
78 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_{1}
79 : $P_{2591} = (14, 0, 9, 1)$ lies on line ℓ_{6}
80 : $P_{2663} = (6, 5, 9, 1)$ lies on line ℓ_{7}
81 : $P_{2671} = (14, 5, 9, 1)$ lies on line ℓ_{8}
82 : $P_{2681} = (8, 6, 9, 1)$ lies on line ℓ_{2}
83 : $P_{2692} = (3, 7, 9, 1)$ lies on line ℓ_{3}
84 : $P_{2727} = (6, 9, 9, 1)$ lies on line ℓ_{9}
85 : $P_{2774} = (5, 12, 9, 1)$ lies on line ℓ_{4}
86 : $P_{2800} = (15, 13, 9, 1)$ lies on line ℓ_{5}
87 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_{1}
88 : $P_{2866} = (1, 2, 10, 1)$ lies on line ℓ_{9}
89 : $P_{2937} = (8, 6, 10, 1)$ lies on line ℓ_{2}
90 : $P_{2939} = (10, 6, 10, 1)$ lies on line ℓ_{6}
91 : $P_{2948} = (3, 7, 10, 1)$ lies on line ℓ_{3}
92 : $P_{2955} = (10, 7, 10, 1)$ lies on line ℓ_{8}
93 : $P_{2978} = (1, 9, 10, 1)$ lies on line ℓ_{7}
94 : $P_{3030} = (5, 12, 10, 1)$ lies on line ℓ_{4}
95 : $P_{3056} = (15, 13, 10, 1)$ lies on line ℓ_{5}
96 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_{1}
97 : $P_{3154} = (1, 4, 11, 1)$ lies on line ℓ_{6}
98 : $P_{3193} = (8, 6, 11, 1)$ lies on line ℓ_{2}
99 : $P_{3204} = (3, 7, 11, 1)$ lies on line ℓ_{3}
100 : $P_{3286} = (5, 12, 11, 1)$ lies on line ℓ_{4}
101 : $P_{3292} = (11, 12, 11, 1)$ lies on line ℓ_{9}
102 : $P_{3308} = (11, 13, 11, 1)$ lies on line ℓ_{7}
103 : $P_{3312} = (15, 13, 11, 1)$ lies on line ℓ_{5}
104 : $P_{3314} = (1, 14, 11, 1)$ lies on line ℓ_{8}
105 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_{1}
106 : $P_{3395} = (2, 3, 12, 1)$ lies on line ℓ_{8}
107 : $P_{3449} = (8, 6, 12, 1)$ lies on line ℓ_{2}

108 : $P_{3460} = (3, 7, 12, 1)$ lies on line ℓ_3
109 : $P_{3488} = (15, 8, 12, 1)$ lies on line ℓ_7
110 : $P_{3507} = (2, 10, 12, 1)$ lies on line ℓ_6
111 : $P_{3542} = (5, 12, 12, 1)$ lies on line ℓ_4
112 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_1
113 : $P_{3654} = (5, 3, 13, 1)$ lies on line ℓ_9
114 : $P_{3705} = (8, 6, 13, 1)$ lies on line ℓ_2
115 : $P_{3716} = (3, 7, 13, 1)$ lies on line ℓ_3
116 : $P_{3738} = (9, 8, 13, 1)$ lies on line ℓ_6
117 : $P_{3770} = (9, 10, 13, 1)$ lies on line ℓ_8
118 : $P_{3824} = (15, 13, 13, 1)$ lies on line ℓ_5
119 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_1
120 : $P_{3859} = (2, 0, 14, 1)$ lies on line ℓ_7
121 : $P_{3961} = (8, 6, 14, 1)$ lies on line ℓ_2
122 : $P_{3972} = (3, 7, 14, 1)$ lies on line ℓ_3
123 : $P_{3987} = (2, 8, 14, 1)$ lies on line ℓ_9
124 : $P_{3998} = (13, 8, 14, 1)$ lies on line ℓ_8
125 : $P_{4054} = (5, 12, 14, 1)$ lies on line ℓ_4
126 : $P_{4080} = (15, 13, 14, 1)$ lies on line ℓ_5
127 : $P_{4094} = (13, 14, 14, 1)$ lies on line ℓ_6
128 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_1
129 : $P_{4135} = (6, 1, 15, 1)$ lies on line ℓ_8
130 : $P_{4185} = (8, 4, 15, 1)$ lies on line ℓ_7
131 : $P_{4228} = (3, 7, 15, 1)$ lies on line ℓ_3
132 : $P_{4310} = (5, 12, 15, 1)$ lies on line ℓ_4
133 : $P_{4311} = (6, 12, 15, 1)$ lies on line ℓ_6
134 : $P_{4336} = (15, 13, 15, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 170 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_{20} = (1, 0, 1, 0)$
2 : $P_{36} = (1, 1, 1, 0)$
3 : $P_{57} = (6, 2, 1, 0)$
4 : $P_{63} = (12, 2, 1, 0)$
5 : $P_{89} = (6, 4, 1, 0)$
6 : $P_{96} = (13, 4, 1, 0)$
7 : $P_{170} = (7, 9, 1, 0)$
8 : $P_{176} = (13, 9, 1, 0)$
9 : $P_{250} = (7, 14, 1, 0)$
10 : $P_{255} = (12, 14, 1, 0)$
11 : $P_{275} = (1, 0, 0, 1)$
12 : $P_{335} = (13, 3, 0, 1)$
13 : $P_{361} = (7, 5, 0, 1)$
14 : $P_{414} = (12, 8, 0, 1)$
15 : $P_{438} = (4, 10, 0, 1)$
16 : $P_{448} = (14, 10, 0, 1)$
17 : $P_{452} = (2, 11, 0, 1)$
18 : $P_{459} = (9, 11, 0, 1)$
19 : $P_{520} = (6, 15, 0, 1)$
20 : $P_{540} = (10, 0, 1, 1)$
21 : $P_{541} = (11, 0, 1, 1)$
22 : $P_{634} = (9, 6, 1, 1)$
23 : $P_{643} = (2, 7, 1, 1)$
24 : $P_{695} = (6, 10, 1, 1)$
25 : $P_{696} = (7, 10, 1, 1)$
26 : $P_{717} = (12, 11, 1, 1)$
27 : $P_{718} = (13, 11, 1, 1)$
28 : $P_{725} = (4, 12, 1, 1)$
29 : $P_{751} = (14, 13, 1, 1)$
30 : $P_{791} = (6, 0, 2, 1)$
31 : $P_{809} = (8, 1, 2, 1)$
32 : $P_{844} = (11, 3, 2, 1)$
33 : $P_{855} = (6, 4, 2, 1)$
34 : $P_{859} = (10, 4, 2, 1)$
35 : $P_{863} = (14, 4, 2, 1)$
36 : $P_{868} = (3, 5, 2, 1)$
37 : $P_{882} = (1, 6, 2, 1)$
38 : $P_{892} = (11, 6, 2, 1)$
39 : $P_{911} = (14, 7, 2, 1)$
40 : $P_{912} = (15, 7, 2, 1)$
41 : $P_{923} = (10, 8, 2, 1)$
42 : $P_{950} = (5, 10, 2, 1)$
43 : $P_{1026} = (1, 15, 2, 1)$
44 : $P_{1077} = (4, 2, 3, 1)$
45 : $P_{1095} = (6, 3, 3, 1)$
46 : $P_{1109} = (4, 4, 3, 1)$
47 : $P_{1143} = (6, 6, 3, 1)$
48 : $P_{1158} = (5, 7, 3, 1)$
49 : $P_{1188} = (3, 9, 3, 1)$
50 : $P_{1209} = (8, 10, 3, 1)$
51 : $P_{1274} = (9, 14, 3, 1)$
52 : $P_{1290} = (9, 15, 3, 1)$
53 : $P_{1310} = (13, 0, 4, 1)$
54 : $P_{1328} = (15, 1, 4, 1)$
55 : $P_{1346} = (1, 3, 4, 1)$
56 : $P_{1387} = (10, 5, 4, 1)$
57 : $P_{1430} = (5, 8, 4, 1)$
58 : $P_{1443} = (2, 9, 4, 1)$
59 : $P_{1452} = (11, 9, 4, 1)$

$60: P_{1454} = (13, 9, 4, 1)$
$61: P_{1481} = (8, 11, 4, 1)$
$62: P_{1491} = (2, 12, 4, 1)$
$63: P_{1492} = (3, 12, 4, 1)$
$64: P_{1506} = (1, 13, 4, 1)$
$65: P_{1515} = (10, 13, 4, 1)$
$66: P_{1548} = (11, 15, 4, 1)$
$67: P_{1599} = (14, 2, 5, 1)$
$68: P_{1615} = (14, 3, 5, 1)$
$69: P_{1626} = (9, 4, 5, 1)$
$70: P_{1646} = (13, 5, 5, 1)$
$71: P_{1706} = (9, 9, 5, 1)$
$72: P_{1744} = (15, 11, 5, 1)$
$73: P_{1753} = (8, 12, 5, 1)$
$74: P_{1774} = (13, 13, 5, 1)$
$75: P_{1782} = (5, 14, 5, 1)$
$76: P_{1827} = (2, 1, 6, 1)$
$77: P_{1847} = (6, 2, 6, 1)$
$78: P_{1865} = (8, 3, 6, 1)$
$79: P_{1907} = (2, 6, 6, 1)$
$80: P_{1917} = (12, 6, 6, 1)$
$81: P_{1991} = (6, 11, 6, 1)$
$82: P_{2022} = (5, 13, 6, 1)$
$83: P_{2029} = (12, 13, 6, 1)$
$84: P_{2048} = (15, 14, 6, 1)$
$85: P_{2090} = (9, 1, 7, 1)$
$86: P_{2134} = (5, 4, 7, 1)$
$87: P_{2186} = (9, 7, 7, 1)$
$88: P_{2190} = (13, 7, 7, 1)$
$89: P_{2196} = (3, 8, 7, 1)$
$90: P_{2216} = (7, 9, 7, 1)$
$91: P_{2248} = (7, 11, 7, 1)$
$92: P_{2270} = (13, 12, 7, 1)$
$93: P_{2272} = (15, 12, 7, 1)$
$94: P_{2361} = (8, 2, 8, 1)$
$95: P_{2387} = (2, 4, 8, 1)$
$96: P_{2403} = (2, 5, 8, 1)$
$97: P_{2432} = (15, 6, 8, 1)$
$98: P_{2440} = (7, 7, 8, 1)$
$99: P_{2456} = (7, 8, 8, 1)$
$100: P_{2479} = (14, 9, 8, 1)$
$101: P_{2484} = (3, 10, 8, 1)$
$102: P_{2559} = (14, 14, 8, 1)$
$103: P_{2584} = (7, 0, 9, 1)$
$104: P_{2596} = (3, 1, 9, 1)$
$105: P_{2635} = (10, 3, 9, 1)$
$106: P_{2658} = (1, 5, 9, 1)$
$107: P_{2677} = (4, 6, 9, 1)$
$108: P_{2678} = (5, 6, 9, 1)$
$109: P_{2690} = (1, 7, 9, 1)$
$110: P_{2700} = (11, 7, 9, 1)$
$111: P_{2716} = (11, 8, 9, 1)$
$112: P_{2752} = (15, 10, 9, 1)$
$113: P_{2805} = (4, 14, 9, 1)$
$114: P_{2808} = (7, 14, 9, 1)$
$115: P_{2811} = (10, 14, 9, 1)$
$116: P_{2825} = (8, 15, 9, 1)$
$117: P_{2838} = (5, 0, 10, 1)$
$118: P_{2848} = (15, 0, 10, 1)$
$119: P_{2884} = (3, 3, 10, 1)$
$120: P_{2908} = (11, 4, 10, 1)$
$121: P_{2969} = (8, 8, 10, 1)$
$122: P_{3068} = (11, 14, 10, 1)$
$123: P_{3092} = (3, 0, 11, 1)$
$124: P_{3097} = (8, 0, 11, 1)$
$125: P_{3131} = (10, 2, 11, 1)$
$126: P_{3174} = (5, 5, 11, 1)$
$127: P_{3243} = (10, 9, 11, 1)$
$128: P_{3344} = (15, 15, 11, 1)$
$129: P_{3375} = (14, 1, 12, 1)$
$130: P_{3444} = (3, 6, 12, 1)$
$131: P_{3448} = (7, 6, 12, 1)$
$132: P_{3497} = (8, 9, 12, 1)$
$133: P_{3517} = (12, 10, 12, 1)$
$134: P_{3544} = (7, 12, 12, 1)$
$135: P_{3551} = (14, 12, 12, 1)$
$136: P_{3581} = (12, 14, 12, 1)$
$137: P_{3590} = (5, 15, 12, 1)$
$138: P_{3621} = (4, 1, 13, 1)$
$139: P_{3636} = (3, 2, 13, 1)$
$140: P_{3678} = (13, 4, 13, 1)$
$141: P_{3696} = (15, 5, 13, 1)$
$142: P_{3719} = (6, 7, 13, 1)$
$143: P_{3721} = (8, 7, 13, 1)$
$144: P_{3774} = (13, 10, 13, 1)$
$145: P_{3813} = (4, 13, 13, 1)$
$146: P_{3815} = (6, 13, 13, 1)$
$147: P_{3869} = (12, 0, 14, 1)$
$148: P_{3878} = (5, 1, 14, 1)$
$149: P_{3898} = (9, 2, 14, 1)$
$150: P_{3900} = (11, 2, 14, 1)$
$151: P_{3901} = (12, 2, 14, 1)$
$152: P_{3920} = (15, 3, 14, 1)$
$153: P_{3948} = (11, 5, 14, 1)$
$154: P_{3986} = (1, 8, 14, 1)$
$155: P_{4036} = (3, 11, 14, 1)$
$156: P_{4050} = (1, 12, 14, 1)$
$157: P_{4059} = (10, 12, 14, 1)$
$158: P_{4073} = (8, 13, 14, 1)$
$159: P_{4074} = (9, 13, 14, 1)$
$160: P_{4107} = (10, 15, 14, 1)$
$161: P_{4147} = (2, 2, 15, 1)$
$162: P_{4192} = (15, 4, 15, 1)$
$163: P_{4245} = (4, 8, 15, 1)$
$164: P_{4261} = (4, 9, 15, 1)$
$165: P_{4294} = (5, 11, 15, 1)$
$166: P_{4317} = (12, 12, 15, 1)$
$167: P_{4324} = (3, 13, 15, 1)$

168 : $P_{4339} = (2, 14, 15, 1)$
169 : $P_{4365} = (12, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 0 0 0 0 1 1 1 1
1	1 0 1 1 1 1 0 0 0 0
2	0 1 0 1 1 1 0 0 1 1
3	0 1 1 0 1 1 1 1 0 0
4	0 1 1 1 0 1 0 1 1 0
5	0 1 1 1 1 0 1 0 0 1
6	1 0 0 1 0 1 0 0 1 0
7	1 0 0 1 1 0 0 0 0 1
8	1 0 1 0 1 0 1 0 0 0
9	1 0 1 0 0 1 0 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_{530}	P_{673}	P_{753}	P_{561}	P_{593}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_{530}	P_2	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{2169}	P_{4217}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_2	P_2	P_2	P_2	P_{1924}	P_{1668}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{3798}	P_{2518}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{1264}	P_{3568}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_8
in point	P_{673}	P_{1924}	P_{1264}	P_{461}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_9
in point	P_{753}	P_{1668}	P_{3798}	P_{444}

Line 8 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_{561}	P_{2169}	P_{2518}	P_{461}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_7
in point	P_{593}	P_{4217}	P_{3568}	P_{444}

The surface has 321 points:
The points on the surface are:

$0:\ P_{1} = (0,1,0,0)$
$1:\ P_{2} = (0,0,1,0)$
$2:\ P_{3} = (0,0,0,1)$
$3:\ P_{4} = (1,1,1,1)$
$4:\ P_{20} = (1,0,1,0)$
$5:\ P_{36} = (1,1,1,0)$
$6:\ P_{57} = (6,2,1,0)$
$7:\ P_{62} = (11,2,1,0)$
$8:\ P_{63} = (12,2,1,0)$
$9:\ P_{89} = (6,4,1,0)$
$10:\ P_{93} = (10,4,1,0)$
$11:\ P_{96} = (13,4,1,0)$
$12:\ P_{170} = (7,9,1,0)$
$13:\ P_{174} = (11,9,1,0)$
$14:\ P_{176} = (13,9,1,0)$
$15:\ P_{250} = (7,14,1,0)$
$16:\ P_{253} = (10,14,1,0)$
$17:\ P_{255} = (12,14,1,0)$
$18:\ P_{275} = (1,0,0,1)$
$19:\ P_{335} = (13,3,0,1)$
$20:\ P_{361} = (7,5,0,1)$
$21:\ P_{378} = (8,6,0,1)$
$22:\ P_{389} = (3,7,0,1)$
$23:\ P_{414} = (12,8,0,1)$
$24:\ P_{438} = (4,10,0,1)$
$25:\ P_{444} = (10,10,0,1)$
$26:\ P_{448} = (14,10,0,1)$
$27:\ P_{452} = (2,11,0,1)$
$28:\ P_{459} = (9,11,0,1)$
$29:\ P_{461} = (11,11,0,1)$
$30:\ P_{471} = (5,12,0,1)$
$31:\ P_{497} = (15,13,0,1)$
$32:\ P_{520} = (6,15,0,1)$
$33:\ P_{530} = (0,0,1,1)$
$34:\ P_{540} = (10,0,1,1)$
$35:\ P_{541} = (11,0,1,1)$
$36:\ P_{546} = (0,1,1,1)$
$37:\ P_{561} = (0,2,1,1)$
$38:\ P_{577} = (0,3,1,1)$
$39:\ P_{593} = (0,4,1,1)$
$40:\ P_{609} = (0,5,1,1)$
$41:\ P_{625} = (0,6,1,1)$
$42:\ P_{633} = (8,6,1,1)$
$43:\ P_{634} = (9,6,1,1)$
$44:\ P_{641} = (0,7,1,1)$
$45:\ P_{643} = (2,7,1,1)$
$46:\ P_{644} = (3,7,1,1)$
$47:\ P_{657} = (0,8,1,1)$
$48:\ P_{673} = (0,9,1,1)$
$49:\ P_{689} = (0,10,1,1)$
$50:\ P_{695} = (6,10,1,1)$
$51:\ P_{696} = (7,10,1,1)$
$52:\ P_{705} = (0,11,1,1)$
$53:\ P_{717} = (12,11,1,1)$
$54:\ P_{718} = (13,11,1,1)$
$55:\ P_{721} = (0,12,1,1)$
$56:\ P_{725} = (4,12,1,1)$
$57:\ P_{726} = (5,12,1,1)$
$58:\ P_{737} = (0,13,1,1)$
$59:\ P_{751} = (14,13,1,1)$
$60:\ P_{752} = (15,13,1,1)$
$61:\ P_{753} = (0,14,1,1)$
$62:\ P_{769} = (0,15,1,1)$
$63:\ P_{785} = (0,0,2,1)$
$64:\ P_{789} = (4,0,2,1)$
$65:\ P_{791} = (6,0,2,1)$
$66:\ P_{809} = (8,1,2,1)$
$67:\ P_{824} = (7,2,2,1)$
$68:\ P_{844} = (11,3,2,1)$
$69:\ P_{855} = (6,4,2,1)$
$70:\ P_{859} = (10,4,2,1)$
$71:\ P_{863} = (14,4,2,1)$
$72:\ P_{868} = (3,5,2,1)$
$73:\ P_{882} = (1,6,2,1)$
$74:\ P_{889} = (8,6,2,1)$
$75:\ P_{892} = (11,6,2,1)$
$76:\ P_{900} = (3,7,2,1)$
$77:\ P_{911} = (14,7,2,1)$
$78:\ P_{912} = (15,7,2,1)$
$79:\ P_{923} = (10,8,2,1)$
$80:\ P_{950} = (5,10,2,1)$
$81:\ P_{982} = (5,12,2,1)$
$82:\ P_{1008} = (15,13,2,1)$
$83:\ P_{1026} = (1,15,2,1)$
$84:\ P_{1029} = (4,15,2,1)$
$85:\ P_{1032} = (7,15,2,1)$
$86:\ P_{1041} = (0,0,3,1)$
$87:\ P_{1070} = (13,1,3,1)$
$88:\ P_{1077} = (4,2,3,1)$
$89:\ P_{1095} = (6,3,3,1)$
$90:\ P_{1109} = (4,4,3,1)$
$91:\ P_{1143} = (6,6,3,1)$
$92:\ P_{1145} = (8,6,3,1)$
$93:\ P_{1150} = (13,6,3,1)$
$94:\ P_{1156} = (3,7,3,1)$
$95:\ P_{1158} = (5,7,3,1)$
$96:\ P_{1188} = (3,9,3,1)$
$97:\ P_{1200} = (15,9,3,1)$
$98:\ P_{1209} = (8,10,3,1)$
$99:\ P_{1238} = (5,12,3,1)$
$100:\ P_{1264} = (15,13,3,1)$
$101:\ P_{1274} = (9,14,3,1)$
$102:\ P_{1290} = (9,15,3,1)$
$103:\ P_{1297} = (0,0,4,1)$
$104:\ P_{1306} = (9,0,4,1)$
$105:\ P_{1310} = (13,0,4,1)$
$106:\ P_{1328} = (15,1,4,1)$
$107:\ P_{1346} = (1,3,4,1)$
$108:\ P_{1354} = (9,3,4,1)$
$109:\ P_{1357} = (12,3,4,1)$
$110:\ P_{1373} = (12,4,4,1)$
$111:\ P_{1387} = (10,5,4,1)$
$112:\ P_{1401} = (8,6,4,1)$
$113:\ P_{1412} = (3,7,4,1)$
$114:\ P_{1430} = (5,8,4,1)$
$115:\ P_{1443} = (2,9,4,1)$
$116:\ P_{1452} = (11,9,4,1)$
$117:\ P_{1454} = (13,9,4,1)$
$118:\ P_{1481} = (8,11,4,1)$
$119:\ P_{1491} = (2,12,4,1)$
$120:\ P_{1492} = (3,12,4,1)$
$121:\ P_{1494} = (5,12,4,1)$
$122:\ P_{1506} = (1,13,4,1)$
$123:\ P_{1515} = (10,13,4,1)$
$124:\ P_{1520} = (15,13,4,1)$
$125:\ P_{1548} = (11,15,4,1)$
$126:\ P_{1553} = (0,0,5,1)$
$127:\ P_{1576} = (7,1,5,1)$
$128:\ P_{1599} = (14,2,5,1)$
$129:\ P_{1615} = (14,3,5,1)$
$130:\ P_{1626} = (9,4,5,1)$
$131:\ P_{1646} = (13,5,5,1)$
$132:\ P_{1657} = (8,6,5,1)$
$133:\ P_{1668} = (3,7,5,1)$
$134:\ P_{1706} = (9,9,5,1)$
$135:\ P_{1744} = (15,11,5,1)$
$136:\ P_{1750} = (5,12,5,1)$
$137:\ P_{1753} = (8,12,5,1)$
$138:\ P_{1768} = (7,13,5,1)$
$139:\ P_{1774} = (13,13,5,1)$
$140:\ P_{1776} = (15,13,5,1)$
$141:\ P_{1780} = (3,14,5,1)$
$142:\ P_{1782} = (5,14,5,1)$
$143:\ P_{1809} = (0,0,6,1)$
$144:\ P_{1827} = (2,1,6,1)$
$145:\ P_{1847} = (6,2,6,1)$
$146:\ P_{1865} = (8,3,6,1)$
$147:\ P_{1893} = (4,5,6,1)$
$148:\ P_{1907} = (2,6,6,1)$
$149:\ P_{1913} = (8,6,6,1)$
$150:\ P_{1917} = (12,6,6,1)$
$151:\ P_{1924} = (3,7,6,1)$
$152:\ P_{1989} = (4,11,6,1)$
$153:\ P_{1991} = (6,11,6,1)$
$154:\ P_{2006} = (5,12,6,1)$
$155:\ P_{2022} = (5,13,6,1)$
$156:\ P_{2029} = (12,13,6,1)$
$157:\ P_{2032} = (15,13,6,1)$
$158:\ P_{2048} = (15,14,6,1)$
$159:\ P_{2052} = (3,15,6,1)$
$160:\ P_{2065} = (0,0,7,1)$
$161:\ P_{2090} = (9,1,7,1)$

$162: P_{2134} = (5, 4, 7, 1)$
$163: P_{2153} = (8, 5, 7, 1)$
$164: P_{2169} = (8, 6, 7, 1)$
$165: P_{2180} = (3, 7, 7, 1)$
$166: P_{2186} = (9, 7, 7, 1)$
$167: P_{2190} = (13, 7, 7, 1)$
$168: P_{2196} = (3, 8, 7, 1)$
$169: P_{2216} = (7, 9, 7, 1)$
$170: P_{2248} = (7, 11, 7, 1)$
$171: P_{2255} = (14, 11, 7, 1)$
$172: P_{2262} = (5, 12, 7, 1)$
$173: P_{2270} = (13, 12, 7, 1)$
$174: P_{2272} = (15, 12, 7, 1)$
$175: P_{2288} = (15, 13, 7, 1)$
$176: P_{2319} = (14, 15, 7, 1)$
$177: P_{2321} = (0, 0, 8, 1)$
$178: P_{2349} = (12, 1, 8, 1)$
$179: P_{2358} = (5, 2, 8, 1)$
$180: P_{2361} = (8, 2, 8, 1)$
$181: P_{2387} = (2, 4, 8, 1)$
$182: P_{2403} = (2, 5, 8, 1)$
$183: P_{2425} = (8, 6, 8, 1)$
$184: P_{2432} = (15, 6, 8, 1)$
$185: P_{2436} = (3, 7, 8, 1)$
$186: P_{2440} = (7, 7, 8, 1)$
$187: P_{2445} = (12, 7, 8, 1)$
$188: P_{2456} = (7, 8, 8, 1)$
$189: P_{2479} = (14, 9, 8, 1)$
$190: P_{2484} = (3, 10, 8, 1)$
$191: P_{2518} = (5, 12, 8, 1)$
$192: P_{2544} = (15, 13, 8, 1)$
$193: P_{2559} = (14, 14, 8, 1)$
$194: P_{2577} = (0, 0, 9, 1)$
$195: P_{2584} = (7, 0, 9, 1)$
$196: P_{2591} = (14, 0, 9, 1)$
$197: P_{2596} = (3, 1, 9, 1)$
$198: P_{2635} = (10, 3, 9, 1)$
$199: P_{2658} = (1, 5, 9, 1)$
$200: P_{2663} = (6, 5, 9, 1)$
$201: P_{2671} = (14, 5, 9, 1)$
$202: P_{2677} = (4, 6, 9, 1)$
$203: P_{2678} = (5, 6, 9, 1)$
$204: P_{2681} = (8, 6, 9, 1)$
$205: P_{2690} = (1, 7, 9, 1)$
$206: P_{2692} = (3, 7, 9, 1)$
$207: P_{2700} = (11, 7, 9, 1)$
$208: P_{2716} = (11, 8, 9, 1)$
$209: P_{2727} = (6, 9, 9, 1)$
$210: P_{2752} = (15, 10, 9, 1)$
$211: P_{2774} = (5, 12, 9, 1)$
$212: P_{2800} = (15, 13, 9, 1)$
$213: P_{2805} = (4, 14, 9, 1)$
$214: P_{2808} = (7, 14, 9, 1)$
$215: P_{2811} = (10, 14, 9, 1)$
$216: P_{2825} = (8, 15, 9, 1)$
$217: P_{2833} = (0, 0, 10, 1)$
$218: P_{2838} = (5, 0, 10, 1)$
$219: P_{2848} = (15, 0, 10, 1)$
$220: P_{2866} = (1, 2, 10, 1)$
$221: P_{2884} = (3, 3, 10, 1)$
$222: P_{2908} = (11, 4, 10, 1)$
$223: P_{2937} = (8, 6, 10, 1)$
$224: P_{2939} = (10, 6, 10, 1)$
$225: P_{2948} = (3, 7, 10, 1)$
$226: P_{2955} = (10, 7, 10, 1)$
$227: P_{2969} = (8, 8, 10, 1)$
$228: P_{2978} = (1, 9, 10, 1)$
$229: P_{3030} = (5, 12, 10, 1)$
$230: P_{3056} = (15, 13, 10, 1)$
$231: P_{3068} = (11, 14, 10, 1)$
$232: P_{3089} = (0, 0, 11, 1)$
$233: P_{3092} = (3, 0, 11, 1)$
$234: P_{3097} = (8, 0, 11, 1)$
$235: P_{3131} = (10, 2, 11, 1)$
$236: P_{3154} = (1, 4, 11, 1)$
$237: P_{3174} = (5, 5, 11, 1)$
$238: P_{3193} = (8, 6, 11, 1)$
$239: P_{3204} = (3, 7, 11, 1)$
$240: P_{3243} = (10, 9, 11, 1)$
$241: P_{3286} = (5, 12, 11, 1)$
$242: P_{3292} = (11, 12, 11, 1)$
$243: P_{3308} = (11, 13, 11, 1)$
$244: P_{3312} = (15, 13, 11, 1)$
$245: P_{3314} = (1, 14, 11, 1)$
$246: P_{3344} = (15, 15, 11, 1)$
$247: P_{3345} = (0, 0, 12, 1)$
$248: P_{3375} = (14, 1, 12, 1)$
$249: P_{3395} = (2, 3, 12, 1)$
$250: P_{3444} = (3, 6, 12, 1)$
$251: P_{3448} = (7, 6, 12, 1)$
$252: P_{3449} = (8, 6, 12, 1)$
$253: P_{3460} = (3, 7, 12, 1)$
$254: P_{3488} = (15, 8, 12, 1)$
$255: P_{3497} = (8, 9, 12, 1)$
$256: P_{3507} = (2, 10, 12, 1)$
$257: P_{3517} = (12, 10, 12, 1)$
$258: P_{3542} = (5, 12, 12, 1)$
$259: P_{3544} = (7, 12, 12, 1)$
$260: P_{3551} = (14, 12, 12, 1)$
$261: P_{3568} = (15, 13, 12, 1)$
$262: P_{3581} = (12, 14, 12, 1)$
$263: P_{3590} = (5, 15, 12, 1)$
$264: P_{3601} = (0, 0, 13, 1)$
$265: P_{3621} = (4, 1, 13, 1)$
$266: P_{3636} = (3, 2, 13, 1)$
$267: P_{3654} = (5, 3, 13, 1)$
$268: P_{3678} = (13, 4, 13, 1)$
$269: P_{3696} = (15, 5, 13, 1)$
$270: P_{3705} = (8, 6, 13, 1)$
$271: P_{3716} = (3, 7, 13, 1)$
$272: P_{3719} = (6, 7, 13, 1)$
$273: P_{3721} = (8, 7, 13, 1)$
$274: P_{3738} = (9, 8, 13, 1)$
$275: P_{3770} = (9, 10, 13, 1)$
$276: P_{3774} = (13, 10, 13, 1)$
$277: P_{3798} = (5, 12, 13, 1)$
$278: P_{3813} = (4, 13, 13, 1)$
$279: P_{3815} = (6, 13, 13, 1)$
$280: P_{3824} = (15, 13, 13, 1)$
$281: P_{3857} = (0, 0, 14, 1)$
$282: P_{3859} = (2, 0, 14, 1)$
$283: P_{3869} = (12, 0, 14, 1)$
$284: P_{3878} = (5, 1, 14, 1)$
$285: P_{3898} = (9, 2, 14, 1)$
$286: P_{3900} = (11, 2, 14, 1)$
$287: P_{3901} = (12, 2, 14, 1)$
$288: P_{3920} = (15, 3, 14, 1)$
$289: P_{3948} = (11, 5, 14, 1)$
$290: P_{3961} = (8, 6, 14, 1)$
$291: P_{3972} = (3, 7, 14, 1)$
$292: P_{3986} = (1, 8, 14, 1)$
$293: P_{3987} = (2, 8, 14, 1)$
$294: P_{3998} = (13, 8, 14, 1)$
$295: P_{4036} = (3, 11, 14, 1)$
$296: P_{4050} = (1, 12, 14, 1)$
$297: P_{4054} = (5, 12, 14, 1)$
$298: P_{4059} = (10, 12, 14, 1)$
$299: P_{4073} = (8, 13, 14, 1)$
$300: P_{4074} = (9, 13, 14, 1)$
$301: P_{4080} = (15, 13, 14, 1)$
$302: P_{4094} = (13, 14, 14, 1)$
$303: P_{4107} = (10, 15, 14, 1)$
$304: P_{4113} = (0, 0, 15, 1)$
$305: P_{4135} = (6, 1, 15, 1)$
$306: P_{4147} = (2, 2, 15, 1)$
$307: P_{4185} = (8, 4, 15, 1)$
$308: P_{4192} = (15, 4, 15, 1)$
$309: P_{4217} = (8, 6, 15, 1)$
$310: P_{4228} = (3, 7, 15, 1)$
$311: P_{4245} = (4, 8, 15, 1)$
$312: P_{4261} = (4, 9, 15, 1)$
$313: P_{4294} = (5, 11, 15, 1)$
$314: P_{4310} = (5, 12, 15, 1)$
$315: P_{4311} = (6, 12, 15, 1)$
$316: P_{4317} = (12, 12, 15, 1)$
$317: P_{4324} = (3, 13, 15, 1)$
$318: P_{4336} = (15, 13, 15, 1)$
$319: P_{4339} = (2, 14, 15, 1)$
$320: P_{4365} = (12, 15, 15, 1)$