Rank-74532 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_2 + X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74532

General information

Number of lines	2
Number of points	9
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	4
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^2
Type of lines on points	2, 1^4, 0^4

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (29, 34)
Rank of points on Klein quadric: (25, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_{12} = (0,0,1,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 4 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_2 = (0,0,1,0)$ lies on line ℓ_1
2 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
3 : $P_{14} = (0,1,1,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$
1 : $P_6 = (1,0,1,0)$
2 : $P_8 = (1,1,1,0)$
3 : $P_9 = (1,0,0,1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{12}

Line 1 intersects

Line	ℓ_0
in point	P_{12}

The surface has 9 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_6 = (1,0,1,0)$
$5: P_8 = (1,1,1,0)$
$6: P_9 = (1,0,0,1)$
$7: P_{12} = (0,0,1,1)$
$8: P_{14} = (0,1,1,1)$