

Rank-65546 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0X_1X_2 = 0$$

(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65546

General information

Number of lines	8
Number of points	9
Number of singular points	3
Number of Eckardt points	6
Number of double points	0
Number of single points	2
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^8
Type of lines on points	$4, 3^6, 1^2$

Singular Points

The surface has 3 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_8 = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

$$1 : P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \mathbf{Pl}(1, 0, 1, 0, 0, 1)_{23}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{15} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{33} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{11} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{30} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{28} \\
\ell_7 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}
\end{aligned}$$

Rank of lines: (8, 13, 24, 20, 10, 33, 22, 27)

Rank of points on Klein quadric: (23, 15, 33, 11, 30, 7, 28, 17)

Eckardt Points

The surface has 6 Eckardt points:

- 0 : $P_5 = \mathbf{P}(1, 1, 0, 0) = \mathbf{P}(1, 1, 0, 0)$, $T = 12$
- 1 : $P_6 = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0)$, $T = 12$
- 2 : $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$, $T = 12$
- 3 : $P_{11} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$, $T = 12$
- 4 : $P_{13} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1)$, $T = 12$
- 5 : $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$, $T = 12$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 2 single points:

The single points on the surface are:

- 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_7
- 1 : $P_8 = (1, 1, 1, 0)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	1	1	1	0
1	1	0	1	1	1	1	1	1
2	1	1	0	1	1	1	1	0
3	1	1	1	0	1	1	1	1
4	1	1	1	1	0	1	1	0
5	1	1	1	1	1	0	1	1
6	1	1	1	1	1	1	0	0
7	0	1	0	1	0	1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_5	P_5	P_6	P_6	P_7	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_5	P_5	P_3	P_{11}	P_3	P_{11}	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_5	P_5	P_{13}	P_{14}	P_{14}	P_{13}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_6	P_3	P_{13}	P_6	P_3	P_{13}	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_6	P_{11}	P_{14}	P_6	P_{14}	P_{11}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_7	P_3	P_{14}	P_3	P_{14}	P_7	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_7	P_{11}	P_{13}	P_{13}	P_{11}	P_7

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_3	P_3	P_3

The surface has 9 points:

The points on the surface are:

$$0 : P_3 = (0, 0, 0, 1)$$

$$1 : P_4 = (1, 1, 1, 1)$$

$$2 : P_5 = (1, 1, 0, 0)$$

$$3 : P_6 = (1, 0, 1, 0)$$

$$4 : P_7 = (0, 1, 1, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_{11} = (1, 1, 0, 1)$$

$$7 : P_{13} = (1, 0, 1, 1)$$

$$8 : P_{14} = (0, 1, 1, 1)$$