Rank-76323 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^2 X_2 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(8) is 1361384013

General information

Number of lines	2
Number of points	81
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	18
Number of points off lines	63
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{2}
Type of lines on points	$1^{18}, 0^{63}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (0, 4744)

Rank of points on Klein quadric: (0, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 18 single points: The single points on the surface are:

$0: P_0 = (1, 0, 0, 0)$ lies on line ℓ_0	$10: P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0
1: $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0	11: $P_{138} = (0,0,1,1)$ lies on line ℓ_1
2: $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1	12: $P_{201} = (0,0,2,1)$ lies on line ℓ_1
$3: P_3 = (0,0,0,1)$ lies on line ℓ_1	13: $P_{265} = (0,0,3,1)$ lies on line ℓ_1
4: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0	14: $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_1
5: $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0	15: $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_1
6: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0	16: $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_1
7: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0	17: $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_1
8: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0	
9: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0	

The single points on the surface are:

Points on surface but on no line

The surface has 63 points not on any line: The points on the surface but not on lines are:

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0: P_{31} = (4, 2, 1, 0)
                                                                  13: P_{139} = (1, 0, 1, 1)
1: P_{33} = (6, 2, 1, 0)
                                                                  14: P_{146} = (0, 1, 1, 1)
2: P_{46} = (3, 4, 1, 0)
                                                                  15: P_{157} = (4, 2, 1, 1)
3: P_{50} = (7, 4, 1, 0)
                                                                  16: P_{160} = (7, 2, 1, 1)
4: P_{69} = (2,7,1,0)
                                                                  17: P_{171} = (2, 4, 1, 1)
5: P_{72} = (5, 7, 1, 0)
                                                                  18: P_{176} = (7, 4, 1, 1)
6: P_{83} = (1, 1, 0, 1)
                                                                  19: P_{195} = (2, 7, 1, 1)
7: P_{94} = (4, 2, 0, 1)
                                                                  20: P_{197} = (4,7,1,1)
8: P_{103} = (5, 3, 0, 1)
                                                                  21: P_{207} = (6,0,2,1)
9: P_{113} = (7, 4, 0, 1)
                                                                  22: P_{233} = (0,4,2,1)
10: P_{120} = (6, 5, 0, 1)
                                                                  23: P_{235} = (2,4,2,1)
11: P_{125} = (3, 6, 0, 1)
                                                                  24: P_{246} = (5, 5, 2, 1)
                                                                  25: P_{247} = (6, 5, 2, 1)
12: P_{132} = (2,7,0,1)
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45: P_{436} = (3, 5, 5, 1)
26: P_{254} = (5, 6, 2, 1)
27: P_{269} = (4,0,3,1)
                                                                  46: P_{441} = (0, 6, 5, 1)
28: P_{276} = (3, 1, 3, 1)
                                                                  47: P_{442} = (1, 6, 5, 1)
29: P_{279} = (6, 1, 3, 1)
                                                                  48: P_{456} = (7, 7, 5, 1)
30: P_{290} = (1, 3, 3, 1)
                                                                  49: P_{459} = (2, 0, 6, 1)
31: P_{295} = (6, 3, 3, 1)
                                                                  50: P_{470} = (5, 1, 6, 1)
32: P_{301} = (4,4,3,1)
                                                                  51: P_{471} = (6, 1, 6, 1)
33: P_{305} = (0, 5, 3, 1)
                                                                  52: P_{475} = (2, 2, 6, 1)
34: P_{306} = (1, 5, 3, 1)
                                                                  53: P_{481} = (0, 3, 6, 1)
35: P_{332} = (3,0,4,1)
                                                                  54: P_{482} = (1, 3, 6, 1)
36: P_{359} = (6, 3, 4, 1)
                                                                  55: P_{506} = (1, 6, 6, 1)
                                                                  56: P_{510} = (5, 6, 6, 1)
37: P_{380} = (3, 6, 4, 1)
38: P_{383} = (6, 6, 4, 1)
                                                                  57: P_{526} = (5, 0, 7, 1)
                                                                  58: P_{537} = (0, 2, 7, 1)
39: P_{385} = (0, 7, 4, 1)
40: P_{389} = (4,7,4,1)
                                                                  59: P_{544} = (7, 2, 7, 1)
41: P_{400} = (7, 0, 5, 1)
                                                                  60: P_{548} = (3, 3, 7, 1)
42: P_{404} = (3, 1, 5, 1)
                                                                  61: P_{550} = (5, 3, 7, 1)
43: P_{406} = (5, 1, 5, 1)
                                                                  62: P_{564} = (3, 5, 7, 1)
44: P_{434} = (1, 5, 5, 1)
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Line Intersection Graph

 $\begin{array}{c|c} 0 \ 1 \\ \hline 0 \ 0 \ 0 \\ 1 \ 0 \ 0 \end{array}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

Line 1 intersects

Line in point

The surface has 81 points:

The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	17: $P_{83} = (1, 1, 0, 1)$	$34: P_{207} = (6, 0, 2, 1)$
$1: P_1 = (0, 1, 0, 0)$	18: $P_{94} = (4, 2, 0, 1)$	$35: P_{233} = (0,4,2,1)$
$2: P_2 = (0, 0, 1, 0)$	19: $P_{103} = (5, 3, 0, 1)$	$36: P_{235} = (2,4,2,1)$
$3: P_3 = (0,0,0,1)$	$20: P_{113} = (7, 4, 0, 1)$	$37: P_{246} = (5, 5, 2, 1)$
$4: P_5 = (1, 1, 0, 0)$	$21: P_{120} = (6, 5, 0, 1)$	$38: P_{247} = (6, 5, 2, 1)$
$5: P_6 = (2, 1, 0, 0)$	$22: P_{125} = (3, 6, 0, 1)$	$39: P_{254} = (5, 6, 2, 1)$
$6: P_7 = (3, 1, 0, 0)$	$23: P_{132} = (2,7,0,1)$	$40: P_{265} = (0, 0, 3, 1)$
$7: P_8 = (4, 1, 0, 0)$	$24: P_{138} = (0,0,1,1)$	$41: P_{269} = (4, 0, 3, 1)$
$8: P_9 = (5, 1, 0, 0)$	$25: P_{139} = (1, 0, 1, 1)$	$42: P_{276} = (3, 1, 3, 1)$
$9: P_{10} = (6, 1, 0, 0)$	$26: P_{146} = (0, 1, 1, 1)$	$43: P_{279} = (6, 1, 3, 1)$
$10: P_{11} = (7, 1, 0, 0)$	$27: P_{157} = (4, 2, 1, 1)$	$44: P_{290} = (1, 3, 3, 1)$
11: $P_{31} = (4, 2, 1, 0)$	28: $P_{160} = (7, 2, 1, 1)$	$45: P_{295} = (6, 3, 3, 1)$
$12: P_{33} = (6, 2, 1, 0)$	$29: P_{171} = (2, 4, 1, 1)$	$46: P_{301} = (4, 4, 3, 1)$
13: $P_{46} = (3, 4, 1, 0)$	$30: P_{176} = (7, 4, 1, 1)$	$47: P_{305} = (0, 5, 3, 1)$
$14: P_{50} = (7, 4, 1, 0)$	$31: P_{195} = (2,7,1,1)$	$48: P_{306} = (1, 5, 3, 1)$
$15: P_{69} = (2, 7, 1, 0)$	$32: P_{197} = (4,7,1,1)$	$49: P_{329} = (0, 0, 4, 1)$
$16: P_{72} = (5, 7, 1, 0)$	$33: P_{201} = (0,0,2,1)$	$50: P_{332} = (3, 0, 4, 1)$

$51: P_{359} = (6, 3, 4, 1)$	$62: P_{441} = (0, 6, 5, 1)$	73: $P_{510} = (5, 6, 6, 1)$
$52: P_{380} = (3, 6, 4, 1)$	$63: P_{442} = (1, 6, 5, 1)$	$74: P_{521} = (0, 0, 7, 1)$
$53: P_{383} = (6, 6, 4, 1)$	$64: P_{456} = (7, 7, 5, 1)$	$75: P_{526} = (5, 0, 7, 1)$
$54: P_{385} = (0,7,4,1)$	$65: P_{457} = (0, 0, 6, 1)$	$76: P_{537} = (0, 2, 7, 1)$
$55: P_{389} = (4,7,4,1)$	$66: P_{459} = (2, 0, 6, 1)$	77: $P_{544} = (7, 2, 7, 1)$
$56: P_{393} = (0,0,5,1)$	$67: P_{470} = (5, 1, 6, 1)$	$78: P_{548} = (3, 3, 7, 1)$
$57: P_{400} = (7, 0, 5, 1)$	$68: P_{471} = (6, 1, 6, 1)$	$79: P_{550} = (5, 3, 7, 1)$
$58: P_{404} = (3, 1, 5, 1)$	$69: P_{475} = (2, 2, 6, 1)$	$80: P_{564} = (3, 5, 7, 1)$
$59: P_{406} = (5, 1, 5, 1)$	$70: P_{481} = (0, 3, 6, 1)$	
$60: P_{434} = (1, 5, 5, 1)$	$71: P_{482} = (1, 3, 6, 1)$	
$61: P_{436} = (3, 5, 5, 1)$	$72: P_{506} = (1, 6, 6, 1)$	