Rank-65859 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431725401

General information

Number of lines	7
Number of points	25
Number of singular points	5
Number of Eckardt points	2
Number of double points	6
Number of single points	17
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	$3^2, 2^6, 1^{17}$

Singular Points

The surface has 5 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$ 3 : $P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$
1 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$ 4 : $P_{69} = \mathbf{P}(0,0,\omega^2,1) = \mathbf{P}(0,0,3,1)$
2 : $P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \mathbf{Pl}(0,0,1,0,0,1)_{108}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_5 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \mathbf{Pl}(0,0,1,1,2,1)_{258}$$

$$\ell_6 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \mathbf{Pl}(0,0,1,1,3,1)_{318}$$

Rank of lines: (0, 16, 37, 340, 356, 80, 59)
Rank of points on Klein quadric: (0, 2, 108, 9, 1, 258, 318)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$,
$1 : P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $\qquad\qquad$ $P_7 = (3,1,0,0) = \ell_0 \cap \ell_6$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$ $\qquad\qquad$ $P_3 = (0,0,0,1) = \ell_3 \cap \ell_4$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_5$

Single Points

The surface has 17 single points:
The single points on the surface are:

$0 : P_8 = (1,0,1,0)$ lies on line ℓ_1 $\qquad$ $9 : P_{43} = (2,1,1,1)$ lies on line ℓ_5
$1 : P_9 = (2,0,1,0)$ lies on line ℓ_1 $\qquad$ $10 : P_{44} = (3,1,1,1)$ lies on line ℓ_6
$2 : P_{10} = (3,0,1,0)$ lies on line ℓ_1 $\qquad$ $11 : P_{46} = (1,2,1,1)$ lies on line ℓ_6
$3 : P_{12} = (1,1,1,0)$ lies on line ℓ_2 $\qquad$ $12 : P_{48} = (3,2,1,1)$ lies on line ℓ_5
$4 : P_{17} = (2,2,1,0)$ lies on line ℓ_2 $\qquad$ $13 : P_{50} = (1,3,1,1)$ lies on line ℓ_5
$5 : P_{22} = (3,3,1,0)$ lies on line ℓ_2 $\qquad$ $14 : P_{51} = (2,3,1,1)$ lies on line ℓ_6
$6 : P_{26} = (0,1,0,1)$ lies on line ℓ_3 $\qquad$ $15 : P_{53} = (0,0,2,1)$ lies on line ℓ_4
$7 : P_{30} = (0,2,0,1)$ lies on line ℓ_3 $\qquad$ $16 : P_{69} = (0,0,3,1)$ lies on line ℓ_4
$8 : P_{34} = (0,3,0,1)$ lies on line ℓ_3

The single points on the surface are:

2

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

$$
\begin{array}{c|ccccccc}
 & 0\,1\,2\,3\,4\,5\,6 \\
\hline
0 & 0\,1\,1\,1\,0\,1\,1 \\
1 & 1\,0\,1\,0\,1\,0\,0 \\
2 & 1\,1\,0\,0\,1\,0\,0 \\
3 & 1\,0\,0\,0\,1\,0\,0 \\
4 & 0\,1\,1\,1\,0\,1\,1 \\
5 & 1\,0\,0\,0\,1\,0\,1 \\
6 & 1\,0\,0\,0\,1\,1\,0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_0	P_5	P_1	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_3	P_{38}	P_{38}

Line 5 intersects

Line	ℓ_0	ℓ_4	ℓ_6
in point	P_6	P_{38}	P_{38}

Line 6 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_7	P_{38}	P_{38}

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_5 = (1,1,0,0)$
5 : $P_6 = (2,1,0,0)$
6 : $P_7 = (3,1,0,0)$
7 : $P_8 = (1,0,1,0)$
8 : $P_9 = (2,0,1,0)$

9 : $P_{10} = (3,0,1,0)$
10 : $P_{12} = (1,1,1,0)$
11 : $P_{17} = (2,2,1,0)$
12 : $P_{22} = (3,3,1,0)$
13 : $P_{26} = (0,1,0,1)$
14 : $P_{30} = (0,2,0,1)$
15 : $P_{34} = (0,3,0,1)$
16 : $P_{38} = (0,0,1,1)$
17 : $P_{43} = (2,1,1,1)$

18 : $P_{44} = (3,1,1,1)$
19 : $P_{46} = (1,2,1,1)$
20 : $P_{48} = (3,2,1,1)$
21 : $P_{50} = (1,3,1,1)$
22 : $P_{51} = (2,3,1,1)$
23 : $P_{53} = (0,0,2,1)$
24 : $P_{69} = (0,0,3,1)$