Rank-65547 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286335253

General information

Number of lines	3
Number of points	273
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	45
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	2^3, 1^{45}, 0^{225}

Singular Points

The surface has 3 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$2: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \mathbf{Pl}(0,0,0,0,0,1)_{4625}$$

Rank of lines: (0, 256, 69888)
Rank of points on Klein quadric: (0, 2, 4625)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 45 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
2 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
3 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
4 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
5 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
6 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
7 : $P_{12} = (8,1,0,0)$ lies on line ℓ_0
8 : $P_{13} = (9,1,0,0)$ lies on line ℓ_0
9 : $P_{14} = (10,1,0,0)$ lies on line ℓ_0
10 : $P_{15} = (11,1,0,0)$ lies on line ℓ_0
11 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0
12 : $P_{17} = (13,1,0,0)$ lies on line ℓ_0
13 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0
14 : $P_{19} = (15,1,0,0)$ lies on line ℓ_0
15 : $P_{20} = (1,0,1,0)$ lies on line ℓ_1
16 : $P_{21} = (2,0,1,0)$ lies on line ℓ_1
17 : $P_{22} = (3,0,1,0)$ lies on line ℓ_1
18 : $P_{23} = (4,0,1,0)$ lies on line ℓ_1
19 : $P_{24} = (5,0,1,0)$ lies on line ℓ_1
20 : $P_{25} = (6,0,1,0)$ lies on line ℓ_1
21 : $P_{26} = (7,0,1,0)$ lies on line ℓ_1
22 : $P_{27} = (8,0,1,0)$ lies on line ℓ_1
23 : $P_{28} = (9,0,1,0)$ lies on line ℓ_1
24 : $P_{29} = (10,0,1,0)$ lies on line ℓ_1
25 : $P_{30} = (11,0,1,0)$ lies on line ℓ_1
26 : $P_{31} = (12,0,1,0)$ lies on line ℓ_1
27 : $P_{32} = (13,0,1,0)$ lies on line ℓ_1
28 : $P_{33} = (14,0,1,0)$ lies on line ℓ_1
29 : $P_{34} = (15,0,1,0)$ lies on line ℓ_1
30 : $P_{35} = (0,1,1,0)$ lies on line ℓ_2
31 : $P_{51} = (0,2,1,0)$ lies on line ℓ_2
32 : $P_{67} = (0,3,1,0)$ lies on line ℓ_2
33 : $P_{83} = (0,4,1,0)$ lies on line ℓ_2
34 : $P_{99} = (0,5,1,0)$ lies on line ℓ_2
35 : $P_{115} = (0,6,1,0)$ lies on line ℓ_2
36 : $P_{131} = (0,7,1,0)$ lies on line ℓ_2
37 : $P_{147} = (0,8,1,0)$ lies on line ℓ_2
38 : $P_{163} = (0,9,1,0)$ lies on line ℓ_2
39 : $P_{179} = (0,10,1,0)$ lies on line ℓ_2
40 : $P_{195} = (0,11,1,0)$ lies on line ℓ_2
41 : $P_{211} = (0,12,1,0)$ lies on line ℓ_2
42 : $P_{227} = (0,13,1,0)$ lies on line ℓ_2
43 : $P_{243} = (0,14,1,0)$ lies on line ℓ_2
44 : $P_{259} = (0,15,1,0)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

$0: P_4 = (1,1,1,1)$
$1: P_{573} = (12,2,1,1)$
$2: P_{585} = (8,3,1,1)$
$3: P_{599} = (6,4,1,1)$
$4: P_{624} = (15,5,1,1)$
$5: P_{629} = (4,6,1,1)$
$6: P_{655} = (14,7,1,1)$
$7: P_{660} = (3,8,1,1)$
$8: P_{686} = (13,9,1,1)$
$9: P_{700} = (11,10,1,1)$
$10: P_{715} = (10,11,1,1)$
$11: P_{723} = (2,12,1,1)$
$12: P_{746} = (9,13,1,1)$
$13: P_{760} = (7,14,1,1)$
$14: P_{774} = (5,15,1,1)$
$15: P_{813} = (12,1,2,1)$
$16: P_{823} = (6,2,2,1)$
$17: P_{837} = (4,3,2,1)$
$18: P_{852} = (3,4,2,1)$
$19: P_{876} = (11,5,2,1)$
$20: P_{883} = (2,6,2,1)$
$21: P_{904} = (7,7,2,1)$
$22: P_{926} = (13,8,2,1)$
$23: P_{939} = (10,9,2,1)$
$24: P_{954} = (9,10,2,1)$
$25: P_{966} = (5,11,2,1)$
$26: P_{978} = (1,12,2,1)$
$27: P_{1001} = (8,13,2,1)$
$28: P_{1024} = (15,14,2,1)$
$29: P_{1039} = (14,15,2,1)$
$30: P_{1065} = (8,1,3,1)$
$31: P_{1077} = (4,2,3,1)$
$32: P_{1104} = (15,3,3,1)$
$33: P_{1107} = (2,4,3,1)$
$34: P_{1126} = (5,5,3,1)$
$35: P_{1148} = (11,6,3,1)$
$36: P_{1166} = (13,7,3,1)$
$37: P_{1170} = (1,8,3,1)$
$38: P_{1197} = (12,9,3,1)$
$39: P_{1215} = (14,10,3,1)$
$40: P_{1223} = (6,11,3,1)$
$41: P_{1242} = (9,12,3,1)$
$42: P_{1256} = (7,13,3,1)$
$43: P_{1275} = (10,14,3,1)$
$44: P_{1284} = (3,15,3,1)$
$45: P_{1319} = (6,1,4,1)$
$46: P_{1332} = (3,2,4,1)$
$47: P_{1347} = (2,3,4,1)$
$48: P_{1374} = (13,4,4,1)$
$49: P_{1386} = (9,5,4,1)$
$50: P_{1394} = (1,6,4,1)$
$51: P_{1424} = (15,7,4,1)$
$52: P_{1435} = (10,8,4,1)$
$53: P_{1446} = (5,9,4,1)$
$54: P_{1465} = (8,10,4,1)$
$55: P_{1487} = (14,11,4,1)$
$56: P_{1501} = (12,12,4,1)$
$57: P_{1509} = (4,13,4,1)$
$58: P_{1532} = (11,14,4,1)$
$59: P_{1544} = (7,15,4,1)$
$60: P_{1584} = (15,1,5,1)$
$61: P_{1596} = (11,2,5,1)$
$62: P_{1606} = (5,3,5,1)$
$63: P_{1626} = (9,4,5,1)$
$64: P_{1636} = (3,5,5,1)$
$65: P_{1663} = (14,6,5,1)$
$66: P_{1677} = (12,7,5,1)$
$67: P_{1689} = (8,8,5,1)$
$68: P_{1701} = (4,9,5,1)$
$69: P_{1726} = (13,10,5,1)$
$70: P_{1731} = (2,11,5,1)$
$71: P_{1752} = (7,12,5,1)$
$72: P_{1771} = (10,13,5,1)$
$73: P_{1783} = (6,14,5,1)$
$74: P_{1794} = (1,15,5,1)$
$75: P_{1829} = (4,1,6,1)$
$76: P_{1843} = (2,2,6,1)$
$77: P_{1868} = (11,3,6,1)$
$78: P_{1874} = (1,4,6,1)$
$79: P_{1903} = (14,5,6,1)$
$80: P_{1914} = (9,6,6,1)$
$81: P_{1931} = (10,7,6,1)$
$82: P_{1949} = (12,8,6,1)$
$83: P_{1959} = (6,9,6,1)$
$84: P_{1976} = (7,10,6,1)$
$85: P_{1988} = (3,11,6,1)$
$86: P_{2009} = (8,12,6,1)$
$87: P_{2032} = (15,13,6,1)$
$88: P_{2038} = (5,14,6,1)$
$89: P_{2062} = (13,15,6,1)$
$90: P_{2095} = (14,1,7,1)$
$91: P_{2104} = (7,2,7,1)$
$92: P_{2126} = (13,3,7,1)$
$93: P_{2144} = (15,4,7,1)$
$94: P_{2157} = (12,5,7,1)$
$95: P_{2171} = (10,6,7,1)$

96 : $P_{2179} = (2, 7, 7, 1)$
97 : $P_{2204} = (11, 8, 7, 1)$
98 : $P_{2218} = (9, 9, 7, 1)$
99 : $P_{2231} = (6, 10, 7, 1)$
100 : $P_{2249} = (8, 11, 7, 1)$
101 : $P_{2262} = (5, 12, 7, 1)$
102 : $P_{2276} = (3, 13, 7, 1)$
103 : $P_{2290} = (1, 14, 7, 1)$
104 : $P_{2309} = (4, 15, 7, 1)$
105 : $P_{2340} = (3, 1, 8, 1)$
106 : $P_{2366} = (13, 2, 8, 1)$
107 : $P_{2370} = (1, 3, 8, 1)$
108 : $P_{2395} = (10, 4, 8, 1)$
109 : $P_{2409} = (8, 5, 8, 1)$
110 : $P_{2429} = (12, 6, 8, 1)$
111 : $P_{2444} = (11, 7, 8, 1)$
112 : $P_{2454} = (5, 8, 8, 1)$
113 : $P_{2479} = (14, 9, 8, 1)$
114 : $P_{2485} = (4, 10, 8, 1)$
115 : $P_{2504} = (7, 11, 8, 1)$
116 : $P_{2519} = (6, 12, 8, 1)$
117 : $P_{2531} = (2, 13, 8, 1)$
118 : $P_{2554} = (9, 14, 8, 1)$
119 : $P_{2576} = (15, 15, 8, 1)$
120 : $P_{2606} = (13, 1, 9, 1)$
121 : $P_{2619} = (10, 2, 9, 1)$
122 : $P_{2637} = (12, 3, 9, 1)$
123 : $P_{2646} = (5, 4, 9, 1)$
124 : $P_{2661} = (4, 5, 9, 1)$
125 : $P_{2679} = (6, 6, 9, 1)$
126 : $P_{2698} = (9, 7, 9, 1)$
127 : $P_{2719} = (14, 8, 9, 1)$
128 : $P_{2728} = (7, 9, 9, 1)$
129 : $P_{2739} = (2, 10, 9, 1)$
130 : $P_{2768} = (15, 11, 9, 1)$
131 : $P_{2772} = (3, 12, 9, 1)$
132 : $P_{2786} = (1, 13, 9, 1)$
133 : $P_{2809} = (8, 14, 9, 1)$
134 : $P_{2828} = (11, 15, 9, 1)$
135 : $P_{2860} = (11, 1, 10, 1)$
136 : $P_{2874} = (9, 2, 10, 1)$
137 : $P_{2895} = (14, 3, 10, 1)$
138 : $P_{2905} = (8, 4, 10, 1)$
139 : $P_{2926} = (13, 5, 10, 1)$
140 : $P_{2936} = (7, 6, 10, 1)$
141 : $P_{2951} = (6, 7, 10, 1)$
142 : $P_{2965} = (4, 8, 10, 1)$
143 : $P_{2979} = (2, 9, 10, 1)$
144 : $P_{3003} = (10, 10, 10, 1)$
145 : $P_{3010} = (1, 11, 10, 1)$
146 : $P_{3040} = (15, 12, 10, 1)$
147 : $P_{3046} = (5, 13, 10, 1)$
148 : $P_{3060} = (3, 14, 10, 1)$
149 : $P_{3085} = (12, 15, 10, 1)$
150 : $P_{3115} = (10, 1, 11, 1)$
151 : $P_{3126} = (5, 2, 11, 1)$
152 : $P_{3143} = (6, 3, 11, 1)$
153 : $P_{3167} = (14, 4, 11, 1)$
154 : $P_{3171} = (2, 5, 11, 1)$
155 : $P_{3188} = (3, 6, 11, 1)$
156 : $P_{3209} = (8, 7, 11, 1)$
157 : $P_{3224} = (7, 8, 11, 1)$
158 : $P_{3248} = (15, 9, 11, 1)$
159 : $P_{3250} = (1, 10, 11, 1)$
160 : $P_{3276} = (11, 11, 11, 1)$
161 : $P_{3294} = (13, 12, 11, 1)$
162 : $P_{3309} = (12, 13, 11, 1)$
163 : $P_{3317} = (4, 14, 11, 1)$
164 : $P_{3338} = (9, 15, 11, 1)$
165 : $P_{3363} = (2, 1, 12, 1)$
166 : $P_{3378} = (1, 2, 12, 1)$
167 : $P_{3402} = (9, 3, 12, 1)$
168 : $P_{3421} = (12, 4, 12, 1)$
169 : $P_{3432} = (7, 5, 12, 1)$
170 : $P_{3449} = (8, 6, 12, 1)$
171 : $P_{3462} = (5, 7, 12, 1)$
172 : $P_{3479} = (6, 8, 12, 1)$
173 : $P_{3492} = (3, 9, 12, 1)$
174 : $P_{3520} = (15, 10, 12, 1)$
175 : $P_{3534} = (13, 11, 12, 1)$
176 : $P_{3541} = (4, 12, 12, 1)$
177 : $P_{3564} = (11, 13, 12, 1)$
178 : $P_{3583} = (14, 14, 12, 1)$
179 : $P_{3595} = (10, 15, 12, 1)$
180 : $P_{3626} = (9, 1, 13, 1)$
181 : $P_{3641} = (8, 2, 13, 1)$
182 : $P_{3656} = (7, 3, 13, 1)$
183 : $P_{3669} = (4, 4, 13, 1)$
184 : $P_{3691} = (10, 5, 13, 1)$
185 : $P_{3712} = (15, 6, 13, 1)$
186 : $P_{3716} = (3, 7, 13, 1)$
187 : $P_{3731} = (2, 8, 13, 1)$
188 : $P_{3746} = (1, 9, 13, 1)$
189 : $P_{3766} = (5, 10, 13, 1)$
190 : $P_{3789} = (12, 11, 13, 1)$
191 : $P_{3804} = (11, 12, 13, 1)$
192 : $P_{3823} = (14, 13, 13, 1)$
193 : $P_{3838} = (13, 14, 13, 1)$
194 : $P_{3847} = (6, 15, 13, 1)$
195 : $P_{3880} = (7, 1, 14, 1)$
196 : $P_{3904} = (15, 2, 14, 1)$
197 : $P_{3915} = (10, 3, 14, 1)$
198 : $P_{3932} = (11, 4, 14, 1)$
199 : $P_{3943} = (6, 5, 14, 1)$
200 : $P_{3958} = (5, 6, 14, 1)$
201 : $P_{3970} = (1, 7, 14, 1)$
202 : $P_{3994} = (9, 8, 14, 1)$
203 : $P_{4009} = (8, 9, 14, 1)$

204 : $P_{4020} = (3, 10, 14, 1)$
205 : $P_{4037} = (4, 11, 14, 1)$
206 : $P_{4063} = (14, 12, 14, 1)$
207 : $P_{4078} = (13, 13, 14, 1)$
208 : $P_{4093} = (12, 14, 14, 1)$
209 : $P_{4099} = (2, 15, 14, 1)$
210 : $P_{4134} = (5, 1, 15, 1)$
211 : $P_{4159} = (14, 2, 15, 1)$
212 : $P_{4164} = (3, 3, 15, 1)$
213 : $P_{4184} = (7, 4, 15, 1)$
214 : $P_{4194} = (1, 5, 15, 1)$
215 : $P_{4222} = (13, 6, 15, 1)$
216 : $P_{4229} = (4, 7, 15, 1)$
217 : $P_{4256} = (15, 8, 15, 1)$
218 : $P_{4268} = (11, 9, 15, 1)$
219 : $P_{4285} = (12, 10, 15, 1)$
220 : $P_{4298} = (9, 11, 15, 1)$
221 : $P_{4315} = (10, 12, 15, 1)$
222 : $P_{4327} = (6, 13, 15, 1)$
223 : $P_{4339} = (2, 14, 15, 1)$
224 : $P_{4361} = (8, 15, 15, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_2

The surface has 273 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_5 = (1, 1, 0, 0)$
5 : $P_6 = (2, 1, 0, 0)$
6 : $P_7 = (3, 1, 0, 0)$
7 : $P_8 = (4, 1, 0, 0)$
8 : $P_9 = (5, 1, 0, 0)$
9 : $P_{10} = (6, 1, 0, 0)$
10 : $P_{11} = (7, 1, 0, 0)$
11 : $P_{12} = (8, 1, 0, 0)$
12 : $P_{13} = (9, 1, 0, 0)$
13 : $P_{14} = (10, 1, 0, 0)$
14 : $P_{15} = (11, 1, 0, 0)$
15 : $P_{16} = (12, 1, 0, 0)$
16 : $P_{17} = (13, 1, 0, 0)$
17 : $P_{18} = (14, 1, 0, 0)$
18 : $P_{19} = (15, 1, 0, 0)$
19 : $P_{20} = (1, 0, 1, 0)$
20 : $P_{21} = (2, 0, 1, 0)$
21 : $P_{22} = (3, 0, 1, 0)$
22 : $P_{23} = (4, 0, 1, 0)$
23 : $P_{24} = (5, 0, 1, 0)$
24 : $P_{25} = (6, 0, 1, 0)$
25 : $P_{26} = (7, 0, 1, 0)$
26 : $P_{27} = (8, 0, 1, 0)$
27 : $P_{28} = (9, 0, 1, 0)$
28 : $P_{29} = (10, 0, 1, 0)$
29 : $P_{30} = (11, 0, 1, 0)$
30 : $P_{31} = (12, 0, 1, 0)$
31 : $P_{32} = (13, 0, 1, 0)$
32 : $P_{33} = (14, 0, 1, 0)$
33 : $P_{34} = (15, 0, 1, 0)$
34 : $P_{35} = (0, 1, 1, 0)$
35 : $P_{51} = (0, 2, 1, 0)$
36 : $P_{67} = (0, 3, 1, 0)$
37 : $P_{83} = (0, 4, 1, 0)$
38 : $P_{99} = (0, 5, 1, 0)$
39 : $P_{115} = (0, 6, 1, 0)$
40 : $P_{131} = (0, 7, 1, 0)$
41 : $P_{147} = (0, 8, 1, 0)$
42 : $P_{163} = (0, 9, 1, 0)$
43 : $P_{179} = (0, 10, 1, 0)$
44 : $P_{195} = (0, 11, 1, 0)$
45 : $P_{211} = (0, 12, 1, 0)$
46 : $P_{227} = (0, 13, 1, 0)$
47 : $P_{243} = (0, 14, 1, 0)$
48 : $P_{259} = (0, 15, 1, 0)$
49 : $P_{573} = (12, 2, 1, 1)$
50 : $P_{585} = (8, 3, 1, 1)$
51 : $P_{599} = (6, 4, 1, 1)$
52 : $P_{624} = (15, 5, 1, 1)$
53 : $P_{629} = (4, 6, 1, 1)$
54 : $P_{655} = (14, 7, 1, 1)$
55 : $P_{660} = (3, 8, 1, 1)$
56 : $P_{686} = (13, 9, 1, 1)$
57 : $P_{700} = (11, 10, 1, 1)$
58 : $P_{715} = (10, 11, 1, 1)$
59 : $P_{723} = (2, 12, 1, 1)$
60 : $P_{746} = (9, 13, 1, 1)$
61 : $P_{760} = (7, 14, 1, 1)$
62 : $P_{774} = (5, 15, 1, 1)$

63 : $P_{813} = (12, 1, 2, 1)$
64 : $P_{823} = (6, 2, 2, 1)$
65 : $P_{837} = (4, 3, 2, 1)$
66 : $P_{852} = (3, 4, 2, 1)$
67 : $P_{876} = (11, 5, 2, 1)$
68 : $P_{883} = (2, 6, 2, 1)$
69 : $P_{904} = (7, 7, 2, 1)$
70 : $P_{926} = (13, 8, 2, 1)$
71 : $P_{939} = (10, 9, 2, 1)$
72 : $P_{954} = (9, 10, 2, 1)$
73 : $P_{966} = (5, 11, 2, 1)$
74 : $P_{978} = (1, 12, 2, 1)$
75 : $P_{1001} = (8, 13, 2, 1)$
76 : $P_{1024} = (15, 14, 2, 1)$
77 : $P_{1039} = (14, 15, 2, 1)$
78 : $P_{1065} = (8, 1, 3, 1)$
79 : $P_{1077} = (4, 2, 3, 1)$
80 : $P_{1104} = (15, 3, 3, 1)$
81 : $P_{1107} = (2, 4, 3, 1)$
82 : $P_{1126} = (5, 5, 3, 1)$
83 : $P_{1148} = (11, 6, 3, 1)$
84 : $P_{1166} = (13, 7, 3, 1)$
85 : $P_{1170} = (1, 8, 3, 1)$
86 : $P_{1197} = (12, 9, 3, 1)$
87 : $P_{1215} = (14, 10, 3, 1)$
88 : $P_{1223} = (6, 11, 3, 1)$
89 : $P_{1242} = (9, 12, 3, 1)$
90 : $P_{1256} = (7, 13, 3, 1)$
91 : $P_{1275} = (10, 14, 3, 1)$
92 : $P_{1284} = (3, 15, 3, 1)$
93 : $P_{1319} = (6, 1, 4, 1)$
94 : $P_{1332} = (3, 2, 4, 1)$
95 : $P_{1347} = (2, 3, 4, 1)$
96 : $P_{1374} = (13, 4, 4, 1)$
97 : $P_{1386} = (9, 5, 4, 1)$
98 : $P_{1394} = (1, 6, 4, 1)$
99 : $P_{1424} = (15, 7, 4, 1)$
100 : $P_{1435} = (10, 8, 4, 1)$
101 : $P_{1446} = (5, 9, 4, 1)$
102 : $P_{1465} = (8, 10, 4, 1)$
103 : $P_{1487} = (14, 11, 4, 1)$
104 : $P_{1501} = (12, 12, 4, 1)$
105 : $P_{1509} = (4, 13, 4, 1)$
106 : $P_{1532} = (11, 14, 4, 1)$
107 : $P_{1544} = (7, 15, 4, 1)$
108 : $P_{1584} = (15, 1, 5, 1)$
109 : $P_{1596} = (11, 2, 5, 1)$
110 : $P_{1606} = (5, 3, 5, 1)$
111 : $P_{1626} = (9, 4, 5, 1)$
112 : $P_{1636} = (3, 5, 5, 1)$
113 : $P_{1663} = (14, 6, 5, 1)$
114 : $P_{1677} = (12, 7, 5, 1)$
115 : $P_{1689} = (8, 8, 5, 1)$
116 : $P_{1701} = (4, 9, 5, 1)$
117 : $P_{1726} = (13, 10, 5, 1)$
118 : $P_{1731} = (2, 11, 5, 1)$
119 : $P_{1752} = (7, 12, 5, 1)$
120 : $P_{1771} = (10, 13, 5, 1)$
121 : $P_{1783} = (6, 14, 5, 1)$
122 : $P_{1794} = (1, 15, 5, 1)$
123 : $P_{1829} = (4, 1, 6, 1)$
124 : $P_{1843} = (2, 2, 6, 1)$
125 : $P_{1868} = (11, 3, 6, 1)$
126 : $P_{1874} = (1, 4, 6, 1)$
127 : $P_{1903} = (14, 5, 6, 1)$
128 : $P_{1914} = (9, 6, 6, 1)$
129 : $P_{1931} = (10, 7, 6, 1)$
130 : $P_{1949} = (12, 8, 6, 1)$
131 : $P_{1959} = (6, 9, 6, 1)$
132 : $P_{1976} = (7, 10, 6, 1)$
133 : $P_{1988} = (3, 11, 6, 1)$
134 : $P_{2009} = (8, 12, 6, 1)$
135 : $P_{2032} = (15, 13, 6, 1)$
136 : $P_{2038} = (5, 14, 6, 1)$
137 : $P_{2062} = (13, 15, 6, 1)$
138 : $P_{2095} = (14, 1, 7, 1)$
139 : $P_{2104} = (7, 2, 7, 1)$
140 : $P_{2126} = (13, 3, 7, 1)$
141 : $P_{2144} = (15, 4, 7, 1)$
142 : $P_{2157} = (12, 5, 7, 1)$
143 : $P_{2171} = (10, 6, 7, 1)$
144 : $P_{2179} = (2, 7, 7, 1)$
145 : $P_{2204} = (11, 8, 7, 1)$
146 : $P_{2218} = (9, 9, 7, 1)$
147 : $P_{2231} = (6, 10, 7, 1)$
148 : $P_{2249} = (8, 11, 7, 1)$
149 : $P_{2262} = (5, 12, 7, 1)$
150 : $P_{2276} = (3, 13, 7, 1)$
151 : $P_{2290} = (1, 14, 7, 1)$
152 : $P_{2309} = (4, 15, 7, 1)$
153 : $P_{2340} = (3, 1, 8, 1)$
154 : $P_{2366} = (13, 2, 8, 1)$
155 : $P_{2370} = (1, 3, 8, 1)$
156 : $P_{2395} = (10, 4, 8, 1)$
157 : $P_{2409} = (8, 5, 8, 1)$
158 : $P_{2429} = (12, 6, 8, 1)$
159 : $P_{2444} = (11, 7, 8, 1)$
160 : $P_{2454} = (5, 8, 8, 1)$
161 : $P_{2479} = (14, 9, 8, 1)$
162 : $P_{2485} = (4, 10, 8, 1)$
163 : $P_{2504} = (7, 11, 8, 1)$
164 : $P_{2519} = (6, 12, 8, 1)$
165 : $P_{2531} = (2, 13, 8, 1)$
166 : $P_{2554} = (9, 14, 8, 1)$
167 : $P_{2576} = (15, 15, 8, 1)$
168 : $P_{2606} = (13, 1, 9, 1)$
169 : $P_{2619} = (10, 2, 9, 1)$
170 : $P_{2637} = (12, 3, 9, 1)$
171 : $P_{2646} = (5, 4, 9, 1)$
172 : $P_{2661} = (4, 5, 9, 1)$
173 : $P_{2679} = (6, 6, 9, 1)$
174 : $P_{2698} = (9, 7, 9, 1)$
175 : $P_{2719} = (14, 8, 9, 1)$
176 : $P_{2728} = (7, 9, 9, 1)$
177 : $P_{2739} = (2, 10, 9, 1)$
178 : $P_{2768} = (15, 11, 9, 1)$
179 : $P_{2772} = (3, 12, 9, 1)$
180 : $P_{2786} = (1, 13, 9, 1)$
181 : $P_{2809} = (8, 14, 9, 1)$
182 : $P_{2828} = (11, 15, 9, 1)$
183 : $P_{2860} = (11, 1, 10, 1)$
184 : $P_{2874} = (9, 2, 10, 1)$
185 : $P_{2895} = (14, 3, 10, 1)$
186 : $P_{2905} = (8, 4, 10, 1)$
187 : $P_{2926} = (13, 5, 10, 1)$
188 : $P_{2936} = (7, 6, 10, 1)$
189 : $P_{2951} = (6, 7, 10, 1)$
190 : $P_{2965} = (4, 8, 10, 1)$
191 : $P_{2979} = (2, 9, 10, 1)$
192 : $P_{3003} = (10, 10, 10, 1)$
193 : $P_{3010} = (1, 11, 10, 1)$
194 : $P_{3040} = (15, 12, 10, 1)$
195 : $P_{3046} = (5, 13, 10, 1)$
196 : $P_{3060} = (3, 14, 10, 1)$
197 : $P_{3085} = (12, 15, 10, 1)$
198 : $P_{3115} = (10, 1, 11, 1)$
199 : $P_{3126} = (5, 2, 11, 1)$
200 : $P_{3143} = (6, 3, 11, 1)$
201 : $P_{3167} = (14, 4, 11, 1)$
202 : $P_{3171} = (2, 5, 11, 1)$
203 : $P_{3188} = (3, 6, 11, 1)$
204 : $P_{3209} = (8, 7, 11, 1)$
205 : $P_{3224} = (7, 8, 11, 1)$
206 : $P_{3248} = (15, 9, 11, 1)$
207 : $P_{3250} = (1, 10, 11, 1)$
208 : $P_{3276} = (11, 11, 11, 1)$
209 : $P_{3294} = (13, 12, 11, 1)$
210 : $P_{3309} = (12, 13, 11, 1)$
211 : $P_{3317} = (4, 14, 11, 1)$
212 : $P_{3338} = (9, 15, 11, 1)$
213 : $P_{3363} = (2, 1, 12, 1)$
214 : $P_{3378} = (1, 2, 12, 1)$
215 : $P_{3402} = (9, 3, 12, 1)$
216 : $P_{3421} = (12, 4, 12, 1)$
217 : $P_{3432} = (7, 5, 12, 1)$
218 : $P_{3449} = (8, 6, 12, 1)$
219 : $P_{3462} = (5, 7, 12, 1)$
220 : $P_{3479} = (6, 8, 12, 1)$
221 : $P_{3492} = (3, 9, 12, 1)$
222 : $P_{3520} = (15, 10, 12, 1)$
223 : $P_{3534} = (13, 11, 12, 1)$
224 : $P_{3541} = (4, 12, 12, 1)$

225 : $P_{3564} = (11, 13, 12, 1)$
226 : $P_{3583} = (14, 14, 12, 1)$
227 : $P_{3595} = (10, 15, 12, 1)$
228 : $P_{3626} = (9, 1, 13, 1)$
229 : $P_{3641} = (8, 2, 13, 1)$
230 : $P_{3656} = (7, 3, 13, 1)$
231 : $P_{3669} = (4, 4, 13, 1)$
232 : $P_{3691} = (10, 5, 13, 1)$
233 : $P_{3712} = (15, 6, 13, 1)$
234 : $P_{3716} = (3, 7, 13, 1)$
235 : $P_{3731} = (2, 8, 13, 1)$
236 : $P_{3746} = (1, 9, 13, 1)$
237 : $P_{3766} = (5, 10, 13, 1)$
238 : $P_{3789} = (12, 11, 13, 1)$
239 : $P_{3804} = (11, 12, 13, 1)$
240 : $P_{3823} = (14, 13, 13, 1)$
241 : $P_{3838} = (13, 14, 13, 1)$
242 : $P_{3847} = (6, 15, 13, 1)$
243 : $P_{3880} = (7, 1, 14, 1)$
244 : $P_{3904} = (15, 2, 14, 1)$
245 : $P_{3915} = (10, 3, 14, 1)$
246 : $P_{3932} = (11, 4, 14, 1)$
247 : $P_{3943} = (6, 5, 14, 1)$
248 : $P_{3958} = (5, 6, 14, 1)$
249 : $P_{3970} = (1, 7, 14, 1)$
250 : $P_{3994} = (9, 8, 14, 1)$
251 : $P_{4009} = (8, 9, 14, 1)$
252 : $P_{4020} = (3, 10, 14, 1)$
253 : $P_{4037} = (4, 11, 14, 1)$
254 : $P_{4063} = (14, 12, 14, 1)$
255 : $P_{4078} = (13, 13, 14, 1)$
256 : $P_{4093} = (12, 14, 14, 1)$
257 : $P_{4099} = (2, 15, 14, 1)$
258 : $P_{4134} = (5, 1, 15, 1)$
259 : $P_{4159} = (14, 2, 15, 1)$
260 : $P_{4164} = (3, 3, 15, 1)$
261 : $P_{4184} = (7, 4, 15, 1)$
262 : $P_{4194} = (1, 5, 15, 1)$
263 : $P_{4222} = (13, 6, 15, 1)$
264 : $P_{4229} = (4, 7, 15, 1)$
265 : $P_{4256} = (15, 8, 15, 1)$
266 : $P_{4268} = (11, 9, 15, 1)$
267 : $P_{4285} = (12, 10, 15, 1)$
268 : $P_{4298} = (9, 11, 15, 1)$
269 : $P_{4315} = (10, 12, 15, 1)$
270 : $P_{4327} = (6, 13, 15, 1)$
271 : $P_{4339} = (2, 14, 15, 1)$
272 : $P_{4361} = (8, 15, 15, 1)$