

Rank-74275 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1499027801

General information

Number of lines	6
Number of points	29
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	18
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^6
Type of lines on points	$4, 2^4, 1^{18}, 0^6$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{108} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{101} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{112} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{150}
\end{aligned}$$

Rank of lines: (0, 37, 336, 356, 121, 106)

Rank of points on Klein quadric: (0, 108, 101, 1, 112, 150)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{11} = (0, 1, 1, 0) = \ell_2 \cap \ell_5$$

$$P_{27} = (1, 1, 0, 1) = \ell_4 \cap \ell_5$$

Single Points

The surface has 18 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_3 = (0, 0, 0, 1) \text{ lies on line } \ell_3$$

$$2 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_4$$

$$3 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{12} = (1, 1, 1, 0) \text{ lies on line } \ell_1$$

$$6 : P_{15} = (0, 2, 1, 0) \text{ lies on line } \ell_2$$

$$7 : P_{17} = (2, 2, 1, 0) \text{ lies on line } \ell_1$$

$$8 : P_{19} = (0, 3, 1, 0) \text{ lies on line } \ell_2$$

$$9 : P_{22} = (3, 3, 1, 0) \text{ lies on line } \ell_1$$

$$10 : P_{38} = (0, 0, 1, 1) \text{ lies on line } \ell_3$$

$$11 : P_{39} = (1, 0, 1, 1) \text{ lies on line } \ell_5$$

$$12 : P_{53} = (0, 0, 2, 1) \text{ lies on line } \ell_3$$

$$13 : P_{58} = (1, 1, 2, 1) \text{ lies on line } \ell_4$$

$$14 : P_{66} = (1, 3, 2, 1) \text{ lies on line } \ell_5$$

$$15 : P_{69} = (0, 0, 3, 1) \text{ lies on line } \ell_3$$

$$16 : P_{74} = (1, 1, 3, 1) \text{ lies on line } \ell_4$$

$$17 : P_{78} = (1, 2, 3, 1) \text{ lies on line } \ell_5$$

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:

The points on the surface but not on lines are:

0 : $P_{33} = (3, 2, 0, 1)$
 1 : $P_{36} = (2, 3, 0, 1)$
 2 : $P_{56} = (3, 0, 2, 1)$
 3 : $P_{60} = (3, 1, 2, 1)$

4 : $P_{71} = (2, 0, 3, 1)$
 5 : $P_{75} = (2, 1, 3, 1)$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	0	0	0
1	1	0	1	1	1	0
2	1	1	0	1	1	1
3	0	1	1	0	1	0
4	0	1	1	1	0	1
5	0	0	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_5	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5
in point	P_1	P_2	P_2	P_2	P_{11}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_2	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{27}

Line 5 intersects

Line	ℓ_2	ℓ_4
in point	P_{11}	P_{27}

The surface has 29 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_5 = (1, 1, 0, 0)$
 6 : $P_6 = (2, 1, 0, 0)$
 7 : $P_7 = (3, 1, 0, 0)$
 8 : $P_{11} = (0, 1, 1, 0)$
 9 : $P_{12} = (1, 1, 1, 0)$

10 : $P_{15} = (0, 2, 1, 0)$
 11 : $P_{17} = (2, 2, 1, 0)$
 12 : $P_{19} = (0, 3, 1, 0)$
 13 : $P_{22} = (3, 3, 1, 0)$
 14 : $P_{27} = (1, 1, 0, 1)$
 15 : $P_{33} = (3, 2, 0, 1)$
 16 : $P_{36} = (2, 3, 0, 1)$
 17 : $P_{38} = (0, 0, 1, 1)$
 18 : $P_{39} = (1, 0, 1, 1)$
 19 : $P_{53} = (0, 0, 2, 1)$

20 : $P_{56} = (3, 0, 2, 1)$
 21 : $P_{58} = (1, 1, 2, 1)$
 22 : $P_{60} = (3, 1, 2, 1)$
 23 : $P_{66} = (1, 3, 2, 1)$
 24 : $P_{69} = (0, 0, 3, 1)$
 25 : $P_{71} = (2, 0, 3, 1)$
 26 : $P_{74} = (1, 1, 3, 1)$
 27 : $P_{75} = (2, 1, 3, 1)$
 28 : $P_{78} = (1, 2, 3, 1)$