Rank-337 over GF(8)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 2658973

General information

Number of lines	6
Number of points	89
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	42
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$3^2, 2^3, 1^{42}, 0^{42}$

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_3 = \mathbf{Pl}(4, 0, 1, 0, 0, 0)_6$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix}_5 = \mathbf{Pl}(7, 0, 1, 0, 0, 0)_9$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & 0 \end{bmatrix}_6 = \mathbf{Pl}(2,0,1,0,0,0)_4$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \mathbf{Pl}(0,6,0,1,0,0)_{30}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \mathbf{Pl}(0,3,0,1,0,0)_{27}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \mathbf{Pl}(0,5,0,1,0,0)_{29}$$

Rank of lines: (3, 5, 6, 4734, 4707, 4725)
Rank of points on Klein quadric: (6, 9, 4, 30, 27, 29)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{43} = (0,4,1,0) = \ell_0 \cap \ell_4$ $\qquad\qquad$ $P_{27} = (0,2,1,0) = \ell_2 \cap \ell_3$
$P_{67} = (0,7,1,0) = \ell_1 \cap \ell_5$

Single Points

The surface has 42 single points:
The single points on the surface are:

$0 :\ P_{28} = (1,2,1,0)$ lies on line ℓ_2 $\qquad$ $18 :\ P_{72} = (5,7,1,0)$ lies on line ℓ_1
$1 :\ P_{29} = (2,2,1,0)$ lies on line ℓ_2 $\qquad$ $19 :\ P_{73} = (6,7,1,0)$ lies on line ℓ_1
$2 :\ P_{30} = (3,2,1,0)$ lies on line ℓ_2 $\qquad$ $20 :\ P_{74} = (7,7,1,0)$ lies on line ℓ_1
$3 :\ P_{31} = (4,2,1,0)$ lies on line ℓ_2 $\qquad$ $21 :\ P_{153} = (0,2,1,1)$ lies on line ℓ_3
$4 :\ P_{32} = (5,2,1,0)$ lies on line ℓ_2 $\qquad$ $22 :\ P_{169} = (0,4,1,1)$ lies on line ℓ_4
$5 :\ P_{33} = (6,2,1,0)$ lies on line ℓ_2 $\qquad$ $23 :\ P_{193} = (0,7,1,1)$ lies on line ℓ_5
$6 :\ P_{34} = (7,2,1,0)$ lies on line ℓ_2 $\qquad$ $24 :\ P_{225} = (0,3,2,1)$ lies on line ℓ_5
$7 :\ P_{44} = (1,4,1,0)$ lies on line ℓ_0 $\qquad$ $25 :\ P_{233} = (0,4,2,1)$ lies on line ℓ_3
$8 :\ P_{45} = (2,4,1,0)$ lies on line ℓ_0 $\qquad$ $26 :\ P_{241} = (0,5,2,1)$ lies on line ℓ_4
$9 :\ P_{46} = (3,4,1,0)$ lies on line ℓ_0 $\qquad$ $27 :\ P_{273} = (0,1,3,1)$ lies on line ℓ_4
$10 :\ P_{47} = (4,4,1,0)$ lies on line ℓ_0 $\qquad$ $28 :\ P_{297} = (0,4,3,1)$ lies on line ℓ_5
$11 :\ P_{48} = (5,4,1,0)$ lies on line ℓ_0 $\qquad$ $29 :\ P_{313} = (0,6,3,1)$ lies on line ℓ_3
$12 :\ P_{49} = (6,4,1,0)$ lies on line ℓ_0 $\qquad$ $30 :\ P_{369} = (0,5,4,1)$ lies on line ℓ_3
$13 :\ P_{50} = (7,4,1,0)$ lies on line ℓ_0 $\qquad$ $31 :\ P_{377} = (0,6,4,1)$ lies on line ℓ_5
$14 :\ P_{68} = (1,7,1,0)$ lies on line ℓ_1 $\qquad$ $32 :\ P_{385} = (0,7,4,1)$ lies on line ℓ_4
$15 :\ P_{69} = (2,7,1,0)$ lies on line ℓ_1 $\qquad$ $33 :\ P_{401} = (0,1,5,1)$ lies on line ℓ_5
$16 :\ P_{70} = (3,7,1,0)$ lies on line ℓ_1 $\qquad$ $34 :\ P_{417} = (0,3,5,1)$ lies on line ℓ_4
$17 :\ P_{71} = (4,7,1,0)$ lies on line ℓ_1 $\qquad$ $35 :\ P_{449} = (0,7,5,1)$ lies on line ℓ_3

$36 : P_{465} = (0, 1, 6, 1)$ lies on line ℓ_3

$37 : P_{473} = (0, 2, 6, 1)$ lies on line ℓ_4

$38 : P_{497} = (0, 5, 6, 1)$ lies on line ℓ_5

$39 : P_{537} = (0, 2, 7, 1)$ lies on line ℓ_5

$40 : P_{545} = (0, 3, 7, 1)$ lies on line ℓ_3

$41 : P_{569} = (0, 6, 7, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1, 1, 1, 1)$

$1 : P_{83} = (1, 1, 0, 1)$

$2 : P_{93} = (3, 2, 0, 1)$

$3 : P_{105} = (7, 3, 0, 1)$

$4 : P_{111} = (5, 4, 0, 1)$

$5 : P_{116} = (2, 5, 0, 1)$

$6 : P_{126} = (4, 6, 0, 1)$

$7 : P_{136} = (6, 7, 0, 1)$

$8 : P_{139} = (1, 0, 1, 1)$

$9 : P_{166} = (5, 3, 1, 1)$

$10 : P_{183} = (6, 5, 1, 1)$

$11 : P_{188} = (3, 6, 1, 1)$

$12 : P_{204} = (3, 0, 2, 1)$

$13 : P_{214} = (5, 1, 2, 1)$

$14 : P_{220} = (3, 2, 2, 1)$

$15 : P_{251} = (2, 6, 2, 1)$

$16 : P_{264} = (7, 7, 2, 1)$

$17 : P_{272} = (7, 0, 3, 1)$

$18 : P_{286} = (5, 2, 3, 1)$

$19 : P_{296} = (7, 3, 3, 1)$

$20 : P_{306} = (1, 5, 3, 1)$

$21 : P_{325} = (4, 7, 3, 1)$

$22 : P_{334} = (5, 0, 4, 1)$

$23 : P_{343} = (6, 1, 4, 1)$

$24 : P_{347} = (2, 2, 4, 1)$

$25 : P_{357} = (4, 3, 4, 1)$

$26 : P_{366} = (5, 4, 4, 1)$

$27 : P_{395} = (2, 0, 5, 1)$

$28 : P_{416} = (7, 2, 5, 1)$

$29 : P_{431} = (6, 4, 5, 1)$

$30 : P_{435} = (2, 5, 5, 1)$

$31 : P_{442} = (1, 6, 5, 1)$

$32 : P_{461} = (4, 0, 6, 1)$

$33 : P_{482} = (1, 3, 6, 1)$

$34 : P_{491} = (2, 4, 6, 1)$

$35 : P_{509} = (4, 6, 6, 1)$

$36 : P_{516} = (3, 7, 6, 1)$

$37 : P_{527} = (6, 0, 7, 1)$

$38 : P_{532} = (3, 1, 7, 1)$

$39 : P_{557} = (4, 4, 7, 1)$

$40 : P_{568} = (7, 5, 7, 1)$

$41 : P_{583} = (6, 7, 7, 1)$

Line Intersection Graph

```
 |0 1 2 3 4 5
0|0 1 1 0 1 0
1|1 0 1 0 0 1
2|1 1 0 1 0 0
3|0 0 1 0 1 1
4|1 0 0 1 0 1
5|0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_0	P_0	P_{43}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_0	P_0	P_{67}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_0	P_0	P_{27}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5
in point	P_{27}	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_{43}	P_3	P_3

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{67}	P_3	P_3

The surface has 89 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_3 = (0,0,0,1)$

$2 : P_4 = (1,1,1,1)$

$3 : P_{27} = (0,2,1,0)$

$4 : P_{28} = (1,2,1,0)$

$5 : P_{29} = (2,2,1,0)$

$6 : P_{30} = (3,2,1,0)$

$7 : P_{31} = (4,2,1,0)$

$8 : P_{32} = (5,2,1,0)$

$9 : P_{33} = (6,2,1,0)$

$10 : P_{34} = (7,2,1,0)$

$11 : P_{43} = (0,4,1,0)$

$12 : P_{44} = (1,4,1,0)$

$13 : P_{45} = (2,4,1,0)$

$14 : P_{46} = (3,4,1,0)$

$15 : P_{47} = (4,4,1,0)$

$16 : P_{48} = (5,4,1,0)$

$17 : P_{49} = (6,4,1,0)$

$18 : P_{50} = (7,4,1,0)$

$19 : P_{67} = (0,7,1,0)$

$20 : P_{68} = (1,7,1,0)$

$21 : P_{69} = (2,7,1,0)$

$22 : P_{70} = (3,7,1,0)$

$23 : P_{71} = (4,7,1,0)$

$24 : P_{72} = (5,7,1,0)$

$25 : P_{73} = (6,7,1,0)$

$26 : P_{74} = (7,7,1,0)$

$27 : P_{83} = (1,1,0,1)$

$28 : P_{93} = (3,2,0,1)$

$29 : P_{105} = (7,3,0,1)$

$30 : P_{111} = (5,4,0,1)$

$31 : P_{116} = (2,5,0,1)$

$32 : P_{126} = (4,6,0,1)$

$33 : P_{136} = (6,7,0,1)$

$34 : P_{139} = (1,0,1,1)$

$35 : P_{153} = (0,2,1,1)$

$36 : P_{166} = (5,3,1,1)$

$37 : P_{169} = (0,4,1,1)$

$38 : P_{183} = (6,5,1,1)$

$39 : P_{188} = (3,6,1,1)$

$40 : P_{193} = (0,7,1,1)$

$41 : P_{204} = (3,0,2,1)$

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$43 : P_{220} = (3,2,2,1)$

$44 : P_{225} = (0,3,2,1)$

$45 : P_{233} = (0,4,2,1)$

$46 : P_{241} = (0,5,2,1)$

$47 : P_{251} = (2,6,2,1)$

$48 : P_{264} = (7,7,2,1)$

$49 : P_{272} = (7,0,3,1)$

$50 : P_{273} = (0,1,3,1)$

$51 : P_{286} = (5,2,3,1)$

$52 : P_{296} = (7,3,3,1)$

$53 : P_{297} = (0,4,3,1)$

$54 : P_{306} = (1,5,3,1)$

$55 : P_{313} = (0,6,3,1)$

$56 : P_{325} = (4,7,3,1)$

$57 : P_{334} = (5,0,4,1)$

$58 : P_{343} = (6,1,4,1)$

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$60 : P_{357} = (4,3,4,1)$

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$63 : P_{377} = (0,6,4,1)$

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$78 : P_{497} = (0,5,6,1)$

$79 : P_{509} = (4,6,6,1)$

$80 : P_{516} = (3,7,6,1)$

$81 : P_{527} = (6,0,7,1)$

$82 : P_{532} = (3,1,7,1)$

$83 : P_{537} = (0,2,7,1)$

$84 : P_{545} = (0,3,7,1)$

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$87 : P_{569} = (0,6,7,1)$

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