# Rank-74499 over GF(16)

January 15, 2021

# The equation

The equation of the surface is:

$$X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 ) The point rank of the equation over  ${\rm GF}(16)$  is 286331157

## General information

Number of lines	7
Number of points	305
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	102
Number of points off lines	196
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 <sup>7</sup>
Type of lines on points	$4, 3, 2^5, 1^{102}, 0^{196}$

# Singular Points

The surface has 2 singular points:

$$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0) 1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

## The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_{2}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{4656}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{321}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{5121}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_{1}$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{4672}$$

Rank of lines: (0, 256, 529, 17, 69889, 70160, 4897)

Rank of points on Klein quadric: (0, 2, 4656, 321, 5121, 1, 4672)

## **Eckardt Points**

The surface has 1 Eckardt points:  $0: P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0).$ 

#### **Double Points**

The surface has 5 Double points: The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$
  

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_4$$
  

$$P_{546} = (0, 1, 1, 1) = \ell_3 \cap \ell_4$$

 $P_4 = (1, 1, 1, 1) = \ell_3 \cap \ell_6$  $P_{530} = (0, 0, 1, 1) = \ell_4 \cap \ell_5$ 

#### Single Points

The surface has 102 single points: The single points on the surface are:

 $0: P_3 = (0,0,0,1)$  lies on line  $\ell_5$ 13:  $P_{18} = (14, 1, 0, 0)$  lies on line  $\ell_0$ 14:  $P_{19} = (15, 1, 0, 0)$  lies on line  $\ell_0$ 1:  $P_6 = (2, 1, 0, 0)$  lies on line  $\ell_0$ 2:  $P_7 = (3, 1, 0, 0)$  lies on line  $\ell_0$ 15:  $P_{20} = (1, 0, 1, 0)$  lies on line  $\ell_1$ 3:  $P_8 = (4, 1, 0, 0)$  lies on line  $\ell_0$ 16:  $P_{21} = (2, 0, 1, 0)$  lies on line  $\ell_1$ 4:  $P_9 = (5, 1, 0, 0)$  lies on line  $\ell_0$ 17:  $P_{22} = (3, 0, 1, 0)$  lies on line  $\ell_1$ 18:  $P_{23} = (4, 0, 1, 0)$  lies on line  $\ell_1$ 5:  $P_{10} = (6, 1, 0, 0)$  lies on line  $\ell_0$ 19:  $P_{24} = (5, 0, 1, 0)$  lies on line  $\ell_1$ 6:  $P_{11} = (7, 1, 0, 0)$  lies on line  $\ell_0$ 7:  $P_{12} = (8, 1, 0, 0)$  lies on line  $\ell_0$ 20:  $P_{25} = (6,0,1,0)$  lies on line  $\ell_1$ 8:  $P_{13} = (9, 1, 0, 0)$  lies on line  $\ell_0$ 21 :  $P_{26} = (7, 0, 1, 0)$  lies on line  $\ell_1$ 9:  $P_{14} = (10, 1, 0, 0)$  lies on line  $\ell_0$ 10:  $P_{15} = (11, 1, 0, 0)$  lies on line  $\ell_0$ 11:  $P_{16} = (12, 1, 0, 0)$  lies on line  $\ell_0$ 12 :  $P_{17} = (13, 1, 0, 0)$  lies on line  $\ell_0$ 

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26: P_{31} = (12, 0, 1, 0) lies on line \ell_1
                                                                      65: P_{641} = (0, 7, 1, 1) lies on line \ell_4
27: P_{32} = (13, 0, 1, 0) lies on line \ell_1
                                                                      66: P_{657} = (0, 8, 1, 1) lies on line \ell_4
28: P_{33} = (14, 0, 1, 0) lies on line \ell_1
                                                                      67: P_{673} = (0, 9, 1, 1) lies on line \ell_4
29: P_{34} = (15, 0, 1, 0) lies on line \ell_1
                                                                      68: P_{689} = (0, 10, 1, 1) lies on line \ell_4
30: P_{36} = (1, 1, 1, 0) lies on line \ell_2
                                                                      69: P_{705} = (0, 11, 1, 1) lies on line \ell_4
31: P_{53} = (2, 2, 1, 0) lies on line \ell_2
                                                                      70: P_{721} = (0, 12, 1, 1) lies on line \ell_4
                                                                      71 : P_{737} = (0, 13, 1, 1) lies on line \ell_4
32: P_{70} = (3, 3, 1, 0) lies on line \ell_2
33: P_{87} = (4, 4, 1, 0) lies on line \ell_2
                                                                      72: P_{753} = (0, 14, 1, 1) lies on line \ell_4
34: P_{104} = (5, 5, 1, 0) lies on line \ell_2
                                                                      73: P_{769} = (0, 15, 1, 1) lies on line \ell_4
35: P_{121} = (6, 6, 1, 0) lies on line \ell_2
                                                                      74: P_{785} = (0,0,2,1) lies on line \ell_5
36: P_{138} = (7,7,1,0) lies on line \ell_2
                                                                      75: P_{802} = (1, 1, 2, 1) lies on line \ell_6
37: P_{155} = (8, 8, 1, 0) lies on line \ell_2
                                                                      76: P_{1041} = (0,0,3,1) lies on line \ell_5
38: P_{172} = (9, 9, 1, 0) lies on line \ell_2
                                                                      77: P_{1058} = (1, 1, 3, 1) lies on line \ell_6
39: P_{189} = (10, 10, 1, 0) lies on line \ell_2
                                                                      78: P_{1297} = (0, 0, 4, 1) lies on line \ell_5
                                                                      79: P_{1314} = (1, 1, 4, 1) lies on line \ell_6
40: P_{206} = (11, 11, 1, 0) lies on line \ell_2
41: P_{223} = (12, 12, 1, 0) lies on line \ell_2
                                                                      80: P_{1553} = (0, 0, 5, 1) lies on line \ell_5
42: P_{240} = (13, 13, 1, 0) lies on line \ell_2
                                                                      81: P_{1570} = (1, 1, 5, 1) lies on line \ell_6
43: P_{257} = (14, 14, 1, 0) lies on line \ell_2
                                                                      82: P_{1809} = (0,0,6,1) lies on line \ell_5
                                                                      83 : P_{1826} = (1, 1, 6, 1) lies on line \ell_6
44: P_{274} = (15, 15, 1, 0) lies on line \ell_2
45: P_{291} = (1, 1, 0, 1) lies on line \ell_6
                                                                      84: P_{2065} = (0, 0, 7, 1) lies on line \ell_5
46: P_{547} = (2, 1, 1, 1) lies on line \ell_3
                                                                      85: P_{2082} = (1, 1, 7, 1) lies on line \ell_6
47: P_{548} = (3, 1, 1, 1) lies on line \ell_3
                                                                      86: P_{2321} = (0, 0, 8, 1) lies on line \ell_5
48: P_{549} = (4, 1, 1, 1) lies on line \ell_3
                                                                      87: P_{2338} = (1, 1, 8, 1) lies on line \ell_6
                                                                      88 : P_{2577} = (0, 0, 9, 1) lies on line \ell_5
49: P_{550} = (5, 1, 1, 1) lies on line \ell_3
50: P_{551} = (6, 1, 1, 1) lies on line \ell_3
                                                                      89: P_{2594} = (1, 1, 9, 1) lies on line \ell_6
51: P_{552} = (7, 1, 1, 1) lies on line \ell_3
                                                                      90: P_{2833} = (0, 0, 10, 1) lies on line \ell_5
52: P_{553} = (8, 1, 1, 1) lies on line \ell_3
                                                                      91: P_{2850} = (1, 1, 10, 1) lies on line \ell_6
53: P_{554} = (9, 1, 1, 1) lies on line \ell_3
                                                                      92: P_{3089} = (0, 0, 11, 1) lies on line \ell_5
54: P_{555} = (10, 1, 1, 1) lies on line \ell_3
                                                                      93: P_{3106} = (1, 1, 11, 1) lies on line \ell_6
55: P_{556} = (11, 1, 1, 1) lies on line \ell_3
                                                                      94: P_{3345} = (0, 0, 12, 1) lies on line \ell_5
                                                                      95: P_{3362} = (1, 1, 12, 1) lies on line \ell_6
56: P_{557} = (12, 1, 1, 1) lies on line \ell_3
57: P_{558} = (13, 1, 1, 1) lies on line \ell_3
                                                                      96: P_{3601} = (0, 0, 13, 1) lies on line \ell_5
58: P_{559} = (14, 1, 1, 1) lies on line \ell_3
                                                                      97: P_{3618} = (1, 1, 13, 1) lies on line \ell_6
                                                                      98 : P_{3857} = (0, 0, 14, 1) lies on line \ell_5
59: P_{560} = (15, 1, 1, 1) lies on line \ell_3
60: P_{561} = (0, 2, 1, 1) lies on line \ell_4
                                                                      99: P_{3874} = (1, 1, 14, 1) lies on line \ell_6
                                                                      100 : P_{4113} = (0, 0, 15, 1) lies on line \ell_5
61: P_{577} = (0, 3, 1, 1) lies on line \ell_4
62: P_{593} = (0, 4, 1, 1) lies on line \ell_4
                                                                      101: P_{4130} = (1, 1, 15, 1) lies on line \ell_6
63: P_{609} = (0, 5, 1, 1) lies on line \ell_4
64: P_{625} = (0, 6, 1, 1) lies on line \ell_4
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The single points on the surface are:

#### Points on surface but on no line

The surface has 196 points not on any line: The points on the surface but not on lines are:

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\begin{array}{lll} 0: \, P_{310} = (4,2,0,1) & 5: \, P_{398} = (12,7,0,1) \\ 1: \, P_{327} = (5,3,0,1) & 6: \, P_{417} = (15,8,0,1) \\ 2: \, P_{347} = (9,4,0,1) & 7: \, P_{432} = (14,9,0,1) \\ 3: \, P_{362} = (8,5,0,1) & 8: \, P_{445} = (11,10,0,1) \\ 4: \, P_{383} = (13,6,0,1) & 9: \, P_{460} = (10,11,0,1) \end{array}
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64: P_{1773} = (12, 13, 5, 1)
10: P_{472} = (6, 12, 0, 1)
11: P_{489} = (7, 13, 0, 1)
                                                                  65: P_{1782} = (5, 14, 5, 1)
12: P_{500} = (2, 14, 0, 1)
                                                                  66: P_{1847} = (6, 2, 6, 1)
13: P_{517} = (3, 15, 0, 1)
                                                                  67: P_{1870} = (13, 3, 6, 1)
14: P_{828} = (11, 2, 2, 1)
                                                                  68: P_{1892} = (3, 5, 6, 1)
                                                                  69: P_{1914} = (9, 6, 6, 1)
15: P_{845} = (12, 3, 2, 1)
16: P_{852} = (3, 4, 2, 1)
                                                                  70: P_{1934} = (13, 7, 6, 1)
17: P_{875} = (10, 5, 2, 1)
                                                                  71: P_{1939} = (2, 8, 6, 1)
18: P_{884} = (3, 6, 2, 1)
                                                                  72: P_{1968} = (15, 9, 6, 1)
19: P_{908} = (11, 7, 2, 1)
                                                                  73: P_{1984} = (15, 10, 6, 1)
20: P_{914} = (1, 8, 2, 1)
                                                                  74: P_{1991} = (6, 11, 6, 1)
21: P_{939} = (10, 9, 2, 1)
                                                                  75: P_{2010} = (9, 12, 6, 1)
22: P_{953} = (8, 10, 2, 1)
                                                                  76: P_{2020} = (3, 13, 6, 1)
23: P_{973} = (12, 11, 2, 1)
                                                                  77: P_{2034} = (1, 14, 6, 1)
24: P_{1001} = (8, 13, 2, 1)
                                                                  78: P_{2051} = (2, 15, 6, 1)
25: P_{1022} = (13, 14, 2, 1)
                                                                  79: P_{2102} = (5, 2, 7, 1)
26: P_{1038} = (13, 15, 2, 1)
                                                                  80: P_{2122} = (9, 3, 7, 1)
                                                                  81: P_{2130} = (1, 4, 7, 1)
27: P_{1086} = (13, 2, 3, 1)
                                                                  82: P_{2154} = (9, 5, 7, 1)
28: P_{1103} = (14, 3, 3, 1)
29: P_{1110} = (5, 4, 3, 1)
                                                                  83: P_{2173} = (12, 6, 7, 1)
30: P_{1134} = (13, 5, 3, 1)
                                                                  84: P_{2179} = (2,7,7,1)
31: P_{1144} = (7,6,3,1)
                                                                  85: P_{2205} = (12, 8, 7, 1)
32: P_{1156} = (3,7,3,1)
                                                                  86: P_{2216} = (7, 9, 7, 1)
33: P_{1188} = (3, 9, 3, 1)
                                                                  87: P_{2230} = (5, 10, 7, 1)
34: P_{1215} = (14, 10, 3, 1)
                                                                  88: P_{2248} = (7, 11, 7, 1)
35: P_{1221} = (4, 11, 3, 1)
                                                                  89: P_{2265} = (8, 12, 7, 1)
36: P_{1234} = (1, 12, 3, 1)
                                                                  90: P_{2275} = (2, 13, 7, 1)
37: P_{1253} = (4, 13, 3, 1)
                                                                  91: P_{2313} = (8, 15, 7, 1)
38: P_{1272} = (7, 14, 3, 1)
                                                                  92: P_{2361} = (8, 2, 8, 1)
39: P_{1286} = (5, 15, 3, 1)
                                                                  93: P_{2391} = (6, 4, 8, 1)
40: P_{1336} = (7, 2, 4, 1)
                                                                  94: P_{2416} = (15, 5, 8, 1)
41: P_{1352} = (7, 3, 4, 1)
                                                                  95: P_{2425} = (8, 6, 8, 1)
42: P_{1371} = (10, 4, 4, 1)
                                                                  96: P_{2439} = (6,7,8,1)
                                                                  97: P_{2453} = (4, 8, 8, 1)
43: P_{1383} = (6, 5, 4, 1)
44: P_{1424} = (15, 7, 4, 1)
                                                                  98: P_{2477} = (12, 9, 8, 1)
45: P_{1436} = (11, 8, 4, 1)
                                                                  99: P_{2485} = (4, 10, 8, 1)
46: P_{1446} = (5, 9, 4, 1)
                                                                  100: P_{2511} = (14, 11, 8, 1)
                                                                  101: P_{2527} = (14, 12, 8, 1)
47: P_{1463} = (6, 10, 4, 1)
48: P_{1488} = (15, 11, 4, 1)
                                                                  102: P_{2530} = (1, 13, 8, 1)
49: P_{1499} = (10, 12, 4, 1)
                                                                  103: P_{2560} = (15, 14, 8, 1)
50: P_{1510} = (5, 13, 4, 1)
                                                                  104: P_{2573} = (12, 15, 8, 1)
                                                                  105: P_{2619} = (10, 2, 9, 1)
51: P_{1532} = (11, 14, 4, 1)
52: P_{1538} = (1, 15, 4, 1)
                                                                  106: P_{2626} = (1, 3, 9, 1)
                                                                  107: P_{2653} = (12, 4, 9, 1)
53: P_{1597} = (12, 2, 5, 1)
54: P_{1609} = (8, 3, 5, 1)
                                                                  108: P_{2669} = (12, 5, 9, 1)
55: P_{1624} = (7, 4, 5, 1)
                                                                  109: P_{2684} = (11, 6, 9, 1)
56: P_{1635} = (2, 5, 5, 1)
                                                                  110: P_{2697} = (8, 7, 9, 1)
57: P_{1650} = (1, 6, 5, 1)
                                                                  111: P_{2718} = (13, 8, 9, 1)
58: P_{1674} = (9, 7, 5, 1)
                                                                  112: P_{2732} = (11, 9, 9, 1)
59: P_{1688} = (7, 8, 5, 1)
                                                                  113: P_{2740} = (3, 10, 9, 1)
60: P_{1705} = (8, 9, 5, 1)
                                                                  114: P_{2766} = (13, 11, 9, 1)
61: P_{1722} = (9, 10, 5, 1)
                                                                  115: P_{2772} = (3, 12, 9, 1)
62: P_{1731} = (2, 11, 5, 1)
                                                                  116: P_{2809} = (8, 14, 9, 1)
63: P_{1750} = (5, 12, 5, 1)
                                                                  117: P_{2827} = (10, 15, 9, 1)
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118: P_{2879} = (14, 2, 10, 1)
                                                                158: P_{3653} = (4, 3, 13, 1)
119: P_{2883} = (2, 3, 10, 1)
                                                                159: P_{3678} = (13, 4, 13, 1)
                                                                160: P_{3688} = (7, 5, 13, 1)
120: P_{2911} = (14, 4, 10, 1)
121: P_{2924} = (11, 5, 10, 1)
                                                                161: P_{3711} = (14, 6, 13, 1)
122: P_{2939} = (10, 6, 10, 1)
                                                                162: P_{3718} = (5, 7, 13, 1)
123: P_{2955} = (10, 7, 10, 1)
                                                                163: P_{3734} = (5, 8, 13, 1)
124: P_{2970} = (9, 8, 10, 1)
                                                                164: P_{3774} = (13, 10, 13, 1)
125: P_{2981} = (4, 9, 10, 1)
                                                                165: P_{3780} = (3, 11, 13, 1)
126: P_{2994} = (1, 10, 10, 1)
                                                                166: P_{3800} = (7, 12, 13, 1)
127: P_{3027} = (2, 12, 10, 1)
                                                                167: P_{3823} = (14, 13, 13, 1)
128: P_{3050} = (9, 13, 10, 1)
                                                                168: P_{3828} = (3, 14, 13, 1)
129: P_{3061} = (4, 14, 10, 1)
                                                                169: P_{3845} = (4, 15, 13, 1)
130: P_{3084} = (11, 15, 10, 1)
                                                                170: P_{3904} = (15, 2, 14, 1)
131: P_{3130} = (9, 2, 11, 1)
                                                                171: P_{3916} = (11, 3, 14, 1)
132: P_{3147} = (10, 3, 11, 1)
                                                                172: P_{3932} = (11, 4, 14, 1)
133: P_{3155} = (2, 4, 11, 1)
                                                                173: P_{3938} = (1, 5, 14, 1)
134: P_{3173} = (4, 5, 11, 1)
                                                                174: P_{3958} = (5, 6, 14, 1)
                                                                175: P_{3991} = (6, 8, 14, 1)
135: P_{3189} = (4, 6, 11, 1)
136: P_{3215} = (14, 7, 11, 1)
                                                                176: P_{4007} = (6, 9, 14, 1)
137: P_{3227} = (10, 8, 11, 1)
                                                                177: P_{4024} = (7, 10, 14, 1)
138: P_{3235} = (2, 9, 11, 1)
                                                                178: P_{4038} = (5, 11, 14, 1)
139: P_{3266} = (1, 11, 11, 1)
                                                                179: P_{4064} = (15, 12, 14, 1)
140: P_{3292} = (11, 12, 11, 1)
                                                                180: P_{4075} = (10, 13, 14, 1)
141: P_{3308} = (11, 13, 11, 1)
                                                                181: P_{4091} = (10, 14, 14, 1)
142: P_{3322} = (9, 14, 11, 1)
                                                                182: P_{4104} = (7, 15, 14, 1)
143: P_{3343} = (14, 15, 11, 1)
                                                                183: P_{4148} = (3, 2, 15, 1)
                                                                184: P_{4167} = (6, 3, 15, 1)
144: P_{3408} = (15, 3, 12, 1)
145: P_{3417} = (8, 4, 12, 1)
                                                                185: P_{4192} = (15, 4, 15, 1)
146: P_{3439} = (14, 5, 12, 1)
                                                                186: P_{4211} = (2, 6, 15, 1)
147: P_{3456} = (15, 6, 12, 1)
                                                                187: P_{4226} = (1, 7, 15, 1)
148: P_{3461} = (4, 7, 12, 1)
                                                                188: P_{4244} = (3, 8, 15, 1)
149: P_{3487} = (14, 8, 12, 1)
                                                                189: P_{4270} = (13, 9, 15, 1)
150: P_{3490} = (1, 9, 12, 1)
                                                                190: P_{4275} = (2, 10, 15, 1)
151: P_{3517} = (12, 10, 12, 1)
                                                                191: P_{4298} = (9, 11, 15, 1)
152: P_{3529} = (8, 11, 12, 1)
                                                                192: P_{4318} = (13, 12, 15, 1)
153: P_{3541} = (4, 12, 12, 1)
                                                                193: P_{4336} = (15, 13, 15, 1)
154: P_{3559} = (6, 13, 12, 1)
                                                                194: P_{4343} = (6, 14, 15, 1)
                                                                195: P_{4362} = (9, 15, 15, 1)
155: P_{3581} = (12, 14, 12, 1)
156: P_{3591} = (6, 15, 12, 1)
157: P_{3634} = (1, 2, 13, 1)
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#### Line Intersection Graph

	0123456
0	0111100
1	1011011
2	$0111100 \\ 1011011 \\ 1100011$
3	1100101
4	1001010
5	0110101
6	$ \begin{array}{c} 1100101\\ 1001010\\ 0110101\\ 0111010 \end{array} $

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_0$	$P_5$	$P_0$	$P_1$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$
in point	$P_0$	$P_2$	$P_0$	$P_2$	$P_2$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_5$	$\ell_6$
in point	$P_5$	$P_2$	$P_2$	$P_2$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_4$	$\ell_6$
in point	$P_0$	$P_0$	$P_{546}$	$P_4$

Line 4 intersects

Line	$\ell_0$	$\ell_3$	$\ell_5$
in point	$P_1$	$P_{546}$	$P_{530}$

 ${\bf Line~5~intersects}$ 

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_6$
in point	$P_2$	$P_2$	$P_{530}$	$P_2$

Line 6 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$
in point	$P_2$	$P_2$	$P_4$	$P_2$

The surface has 305 points:

The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	$25: P_{25} = (6, 0, 1, 0)$	$50: P_{291} = (1, 1, 0, 1)$
$1: P_1 = (0, 1, 0, 0)$	$26: P_{26} = (7,0,1,0)$	$51: P_{310} = (4, 2, 0, 1)$
$2: P_2 = (0, 0, 1, 0)$	$27: P_{27} = (8,0,1,0)$	$52: P_{327} = (5, 3, 0, 1)$
$3: P_3 = (0,0,0,1)$	$28: P_{28} = (9, 0, 1, 0)$	$53: P_{347} = (9,4,0,1)$
$4: P_4 = (1, 1, 1, 1)$	$29: P_{29} = (10, 0, 1, 0)$	$54: P_{362} = (8, 5, 0, 1)$
$5: P_5 = (1, 1, 0, 0)$	$30: P_{30} = (11, 0, 1, 0)$	$55: P_{383} = (13, 6, 0, 1)$
$6: P_6 = (2, 1, 0, 0)$	$31: P_{31} = (12, 0, 1, 0)$	$56: P_{398} = (12, 7, 0, 1)$
$7: P_7 = (3, 1, 0, 0)$	$32: P_{32} = (13, 0, 1, 0)$	$57: P_{417} = (15, 8, 0, 1)$
$8: P_8 = (4, 1, 0, 0)$	$33: P_{33} = (14, 0, 1, 0)$	$58: P_{432} = (14, 9, 0, 1)$
$9: P_9 = (5, 1, 0, 0)$	$34: P_{34} = (15, 0, 1, 0)$	$59: P_{445} = (11, 10, 0, 1)$
$10: P_{10} = (6, 1, 0, 0)$	$35: P_{36} = (1, 1, 1, 0)$	$60: P_{460} = (10, 11, 0, 1)$
$11: P_{11} = (7, 1, 0, 0)$	$36: P_{53} = (2, 2, 1, 0)$	$61: P_{472} = (6, 12, 0, 1)$
$12: P_{12} = (8, 1, 0, 0)$	$37: P_{70} = (3, 3, 1, 0)$	$62: P_{489} = (7, 13, 0, 1)$
13: $P_{13} = (9, 1, 0, 0)$	$38: P_{87} = (4, 4, 1, 0)$	$63: P_{500} = (2, 14, 0, 1)$
$14: P_{14} = (10, 1, 0, 0)$	$39: P_{104} = (5, 5, 1, 0)$	$64: P_{517} = (3, 15, 0, 1)$
$15: P_{15} = (11, 1, 0, 0)$	$40: P_{121} = (6, 6, 1, 0)$	$65: P_{530} = (0,0,1,1)$
$16: P_{16} = (12, 1, 0, 0)$	$41: P_{138} = (7, 7, 1, 0)$	$66: P_{546} = (0, 1, 1, 1)$
17: $P_{17} = (13, 1, 0, 0)$	$42: P_{155} = (8, 8, 1, 0)$	$67: P_{547} = (2, 1, 1, 1)$
$18: P_{18} = (14, 1, 0, 0)$	$43: P_{172} = (9, 9, 1, 0)$	$68: P_{548} = (3, 1, 1, 1)$
$19: P_{19} = (15, 1, 0, 0)$	$44: P_{189} = (10, 10, 1, 0)$	$69: P_{549} = (4, 1, 1, 1)$
$20: P_{20} = (1,0,1,0)$	$45: P_{206} = (11, 11, 1, 0)$	$70: P_{550} = (5, 1, 1, 1)$
$21: P_{21} = (2,0,1,0)$	$46: P_{223} = (12, 12, 1, 0)$	$71: P_{551} = (6, 1, 1, 1)$
$22: P_{22} = (3, 0, 1, 0)$	$47: P_{240} = (13, 13, 1, 0)$	$72: P_{552} = (7, 1, 1, 1)$
$23: P_{23} = (4,0,1,0)$	$48: P_{257} = (14, 14, 1, 0)$	$73: P_{553} = (8, 1, 1, 1)$
$24: P_{24} = (5, 0, 1, 0)$	$49: P_{274} = (15, 15, 1, 0)$	$74: P_{554} = (9, 1, 1, 1)$
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183: P_{2275} = (2, 13, 7, 1)
75: P_{555} = (10, 1, 1, 1)
                                           129: P_{1371} = (10, 4, 4, 1)
                                                                                        184: P_{2313} = (8, 15, 7, 1)
76: P_{556} = (11, 1, 1, 1)
                                           130: P_{1383} = (6, 5, 4, 1)
77: P_{557} = (12, 1, 1, 1)
                                                                                        185: P_{2321} = (0, 0, 8, 1)
                                            131: P_{1424} = (15, 7, 4, 1)
78: P_{558} = (13, 1, 1, 1)
                                                                                       186: P_{2338} = (1, 1, 8, 1)
                                           132: P_{1436} = (11, 8, 4, 1)
79: P_{559} = (14, 1, 1, 1)
                                           133: P_{1446} = (5, 9, 4, 1)
                                                                                       187: P_{2361} = (8, 2, 8, 1)
80: P_{560} = (15, 1, 1, 1)
                                           134: P_{1463} = (6, 10, 4, 1)
                                                                                       188: P_{2391} = (6, 4, 8, 1)
81: P_{561} = (0, 2, 1, 1)
                                           135: P_{1488} = (15, 11, 4, 1)
                                                                                       189: P_{2416} = (15, 5, 8, 1)
                                            136: P_{1499} = (10, 12, 4, 1)
82: P_{577} = (0, 3, 1, 1)
                                                                                       190: P_{2425} = (8, 6, 8, 1)
83: P_{593} = (0, 4, 1, 1)
                                           137: P_{1510} = (5, 13, 4, 1)
                                                                                       191: P_{2439} = (6,7,8,1)
84: P_{609} = (0, 5, 1, 1)
                                           138: P_{1532} = (11, 14, 4, 1)
                                                                                        192: P_{2453} = (4, 8, 8, 1)
85: P_{625} = (0, 6, 1, 1)
                                           139: P_{1538} = (1, 15, 4, 1)
                                                                                       193: P_{2477} = (12, 9, 8, 1)
86: P_{641} = (0, 7, 1, 1)
                                           140: P_{1553} = (0, 0, 5, 1)
                                                                                       194: P_{2485} = (4, 10, 8, 1)
87: P_{657} = (0, 8, 1, 1)
                                           141: P_{1570} = (1, 1, 5, 1)
                                                                                       195: P_{2511} = (14, 11, 8, 1)
                                                                                       196: P_{2527} = (14, 12, 8, 1)
88: P_{673} = (0, 9, 1, 1)
                                           142: P_{1597} = (12, 2, 5, 1)
                                                                                       197: P_{2530} = (1, 13, 8, 1)
89: P_{689} = (0, 10, 1, 1)
                                           143: P_{1609} = (8, 3, 5, 1)
90: P_{705} = (0, 11, 1, 1)
                                           144: P_{1624} = (7, 4, 5, 1)
                                                                                       198: P_{2560} = (15, 14, 8, 1)
91: P_{721} = (0, 12, 1, 1)
                                           145: P_{1635} = (2, 5, 5, 1)
                                                                                       199: P_{2573} = (12, 15, 8, 1)
92: P_{737} = (0, 13, 1, 1)
                                           146: P_{1650} = (1, 6, 5, 1)
                                                                                       200: P_{2577} = (0,0,9,1)
93: P_{753} = (0, 14, 1, 1)
                                                                                       201: P_{2594} = (1, 1, 9, 1)
                                           147: P_{1674} = (9, 7, 5, 1)
94: P_{769} = (0, 15, 1, 1)
                                           148: P_{1688} = (7, 8, 5, 1)
                                                                                       202: P_{2619} = (10, 2, 9, 1)
95: P_{785} = (0, 0, 2, 1)
                                           149: P_{1705} = (8, 9, 5, 1)
                                                                                       203: P_{2626} = (1, 3, 9, 1)
                                                                                       204: P_{2653} = (12, 4, 9, 1)
96: P_{802} = (1, 1, 2, 1)
                                           150: P_{1722} = (9, 10, 5, 1)
                                                                                       205: P_{2669} = (12, 5, 9, 1)
97: P_{828} = (11, 2, 2, 1)
                                            151: P_{1731} = (2, 11, 5, 1)
98: P_{845} = (12, 3, 2, 1)
                                           152: P_{1750} = (5, 12, 5, 1)
                                                                                       206: P_{2684} = (11, 6, 9, 1)
99: P_{852} = (3, 4, 2, 1)
                                           153: P_{1773} = (12, 13, 5, 1)
                                                                                       207: P_{2697} = (8,7,9,1)
100: P_{875} = (10, 5, 2, 1)
                                           154: P_{1782} = (5, 14, 5, 1)
                                                                                       208: P_{2718} = (13, 8, 9, 1)
                                                                                       209: P_{2732} = (11, 9, 9, 1)
101: P_{884} = (3, 6, 2, 1)
                                           155: P_{1809} = (0, 0, 6, 1)
102: P_{908} = (11, 7, 2, 1)
                                           156: P_{1826} = (1, 1, 6, 1)
                                                                                       210: P_{2740} = (3, 10, 9, 1)
                                           157: P_{1847} = (6, 2, 6, 1)
                                                                                       211: P_{2766} = (13, 11, 9, 1)
103: P_{914} = (1, 8, 2, 1)
104: P_{939} = (10, 9, 2, 1)
                                           158: P_{1870} = (13, 3, 6, 1)
                                                                                       212: P_{2772} = (3, 12, 9, 1)
105: P_{953} = (8, 10, 2, 1)
                                            159: P_{1892} = (3, 5, 6, 1)
                                                                                       213: P_{2809} = (8, 14, 9, 1)
106: P_{973} = (12, 11, 2, 1)
                                           160: P_{1914} = (9, 6, 6, 1)
                                                                                       214: P_{2827} = (10, 15, 9, 1)
107: P_{1001} = (8, 13, 2, 1)
                                           161: P_{1934} = (13, 7, 6, 1)
                                                                                       215: P_{2833} = (0, 0, 10, 1)
108: P_{1022} = (13, 14, 2, 1)
                                                                                       216: P_{2850} = (1, 1, 10, 1)
                                           162: P_{1939} = (2, 8, 6, 1)
                                           163: P_{1968} = (15, 9, 6, 1)
109: P_{1038} = (13, 15, 2, 1)
                                                                                       217: P_{2879} = (14, 2, 10, 1)
                                           164: P_{1984} = (15, 10, 6, 1)
110: P_{1041} = (0, 0, 3, 1)
                                                                                       218: P_{2883} = (2, 3, 10, 1)
111: P_{1058} = (1, 1, 3, 1)
                                           165: P_{1991} = (6, 11, 6, 1)
                                                                                       219: P_{2911} = (14, 4, 10, 1)
                                                                                       220: P_{2924} = (11, 5, 10, 1)
112: P_{1086} = (13, 2, 3, 1)
                                            166: P_{2010} = (9, 12, 6, 1)
113: P_{1103} = (14, 3, 3, 1)
                                            167: P_{2020} = (3, 13, 6, 1)
                                                                                       221: P_{2939} = (10, 6, 10, 1)
114: P_{1110} = (5, 4, 3, 1)
                                           168: P_{2034} = (1, 14, 6, 1)
                                                                                       222: P_{2955} = (10, 7, 10, 1)
115: P_{1134} = (13, 5, 3, 1)
                                           169: P_{2051} = (2, 15, 6, 1)
                                                                                       223: P_{2970} = (9, 8, 10, 1)
                                           170: P_{2065} = (0, 0, 7, 1)
                                                                                       224: P_{2981} = (4, 9, 10, 1)
116: P_{1144} = (7, 6, 3, 1)
117: P_{1156} = (3,7,3,1)
                                           171: P_{2082} = (1, 1, 7, 1)
                                                                                       225: P_{2994} = (1, 10, 10, 1)
                                           172: P_{2102} = (5, 2, 7, 1)
                                                                                       226: P_{3027} = (2, 12, 10, 1)
118: P_{1188} = (3, 9, 3, 1)
119: P_{1215} = (14, 10, 3, 1)
                                           173: P_{2122} = (9, 3, 7, 1)
                                                                                       227: P_{3050} = (9, 13, 10, 1)
                                                                                       228: P_{3061} = (4, 14, 10, 1)
120: P_{1221} = (4, 11, 3, 1)
                                           174: P_{2130} = (1, 4, 7, 1)
121: P_{1234} = (1, 12, 3, 1)
                                           175: P_{2154} = (9, 5, 7, 1)
                                                                                       229: P_{3084} = (11, 15, 10, 1)
122: P_{1253} = (4, 13, 3, 1)
                                           176: P_{2173} = (12, 6, 7, 1)
                                                                                       230: P_{3089} = (0, 0, 11, 1)
123: P_{1272} = (7, 14, 3, 1)
                                           177: P_{2179} = (2,7,7,1)
                                                                                       231: P_{3106} = (1, 1, 11, 1)
                                           178: P_{2205} = (12, 8, 7, 1)
124: P_{1286} = (5, 15, 3, 1)
                                                                                       232: P_{3130} = (9, 2, 11, 1)
125: P_{1297} = (0, 0, 4, 1)
                                           179: P_{2216} = (7, 9, 7, 1)
                                                                                       233: P_{3147} = (10, 3, 11, 1)
126: P_{1314} = (1, 1, 4, 1)
                                           180: P_{2230} = (5, 10, 7, 1)
                                                                                       234: P_{3155} = (2,4,11,1)
127: P_{1336} = (7, 2, 4, 1)
                                            181: P_{2248} = (7, 11, 7, 1)
                                                                                       235: P_{3173} = (4, 5, 11, 1)
128: P_{1352} = (7, 3, 4, 1)
                                           182: P_{2265} = (8, 12, 7, 1)
                                                                                       236: P_{3189} = (4, 6, 11, 1)
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237: P_{3215} = (14, 7, 11, 1)
                                           260: P_{3601} = (0, 0, 13, 1)
                                                                                      283: P_{4007} = (6, 9, 14, 1)
                                                                                      284: P_{4024} = (7, 10, 14, 1)
238: P_{3227} = (10, 8, 11, 1)
                                           261: P_{3618} = (1, 1, 13, 1)
239: P_{3235} = (2, 9, 11, 1)
                                           262: P_{3634} = (1, 2, 13, 1)
                                                                                      285: P_{4038} = (5, 11, 14, 1)
                                                                                      286: P_{4064} = (15, 12, 14, 1)
240: P_{3266} = (1, 11, 11, 1)
                                           263: P_{3653} = (4, 3, 13, 1)
241: P_{3292} = (11, 12, 11, 1)
                                           264: P_{3678} = (13, 4, 13, 1)
                                                                                      287: P_{4075} = (10, 13, 14, 1)
242: P_{3308} = (11, 13, 11, 1)
                                           265: P_{3688} = (7, 5, 13, 1)
                                                                                      288: P_{4091} = (10, 14, 14, 1)
243: P_{3322} = (9, 14, 11, 1)
                                           266: P_{3711} = (14, 6, 13, 1)
                                                                                      289: P_{4104} = (7, 15, 14, 1)
                                           267: P_{3718} = (5, 7, 13, 1)
244: P_{3343} = (14, 15, 11, 1)
                                                                                      290: P_{4113} = (0, 0, 15, 1)
245: P_{3345} = (0, 0, 12, 1)
                                           268: P_{3734} = (5, 8, 13, 1)
                                                                                      291: P_{4130} = (1, 1, 15, 1)
246: P_{3362} = (1, 1, 12, 1)
                                           269: P_{3774} = (13, 10, 13, 1)
                                                                                      292: P_{4148} = (3, 2, 15, 1)
247: P_{3408} = (15, 3, 12, 1)
                                           270: P_{3780} = (3, 11, 13, 1)
                                                                                      293: P_{4167} = (6, 3, 15, 1)
248: P_{3417} = (8, 4, 12, 1)
                                           271: P_{3800} = (7, 12, 13, 1)
                                                                                      294: P_{4192} = (15, 4, 15, 1)
249: P_{3439} = (14, 5, 12, 1)
                                           272: P_{3823} = (14, 13, 13, 1)
                                                                                      295: P_{4211} = (2, 6, 15, 1)
250: P_{3456} = (15, 6, 12, 1)
                                           273: P_{3828} = (3, 14, 13, 1)
                                                                                      296: P_{4226} = (1,7,15,1)
                                                                                      297: P_{4244} = (3, 8, 15, 1)
251: P_{3461} = (4, 7, 12, 1)
                                           274: P_{3845} = (4, 15, 13, 1)
252: P_{3487} = (14, 8, 12, 1)
                                           275: P_{3857} = (0, 0, 14, 1)
                                                                                      298: P_{4270} = (13, 9, 15, 1)
                                           276: P_{3874} = (1, 1, 14, 1)
                                                                                      299: P_{4275} = (2, 10, 15, 1)
253: P_{3490} = (1, 9, 12, 1)
254: P_{3517} = (12, 10, 12, 1)
                                                                                      300: P_{4298} = (9, 11, 15, 1)
                                           277: P_{3904} = (15, 2, 14, 1)
                                           278: P_{3916} = (11, 3, 14, 1)
                                                                                      301: P_{4318} = (13, 12, 15, 1)
255: P_{3529} = (8, 11, 12, 1)
256: P_{3541} = (4, 12, 12, 1)
                                           279: P_{3932} = (11, 4, 14, 1)
                                                                                      302: P_{4336} = (15, 13, 15, 1)
257: P_{3559} = (6, 13, 12, 1)
                                           280: P_{3938} = (1, 5, 14, 1)
                                                                                      303: P_{4343} = (6, 14, 15, 1)
                                                                                      304: P_{4362} = (9, 15, 15, 1)
258: P_{3581} = (12, 14, 12, 1)
                                           281: P_{3958} = (5, 6, 14, 1)
259: P_{3591} = (6, 15, 12, 1)
                                           282: P_{3991} = (6, 8, 14, 1)
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