Rank-65871 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244173453

General information

Number of lines	2
Number of points	73
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	2, 1^{16}, 0^{56}

Singular Points

The surface has 2 singular points:

0 : $P_{75} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$
1 : $P_{83} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1,0,0,1,0,0)_{18}$$

Rank of lines: (0, 584)
Rank of points on Klein quadric: (0, 18)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
4 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
5 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
6 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
7 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
8 : $P_{75} = (1,0,0,1)$ lies on line ℓ_1
9 : $P_{83} = (1,1,0,1)$ lies on line ℓ_1
10 : $P_{91} = (1,2,0,1)$ lies on line ℓ_1
11 : $P_{99} = (1,3,0,1)$ lies on line ℓ_1
12 : $P_{107} = (1,4,0,1)$ lies on line ℓ_1
13 : $P_{115} = (1,5,0,1)$ lies on line ℓ_1
14 : $P_{123} = (1,6,0,1)$ lies on line ℓ_1
15 : $P_{131} = (1,7,0,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_{19} = (0,1,1,0)$
1 : $P_{31} = (4,2,1,0)$
2 : $P_{42} = (7,3,1,0)$
3 : $P_{50} = (7,4,1,0)$
4 : $P_{53} = (2,5,1,0)$
5 : $P_{63} = (4,6,1,0)$
6 : $P_{69} = (2,7,1,0)$
7 : $P_{138} = (0,0,1,1)$
8 : $P_{203} = (2,0,2,1)$
9 : $P_{219} = (2,2,2,1)$
10 : $P_{223} = (6,2,2,1)$
11 : $P_{236} = (3,4,2,1)$
12 : $P_{239} = (6,4,2,1)$
13 : $P_{257} = (0,7,2,1)$
14 : $P_{260} = (3,7,2,1)$
15 : $P_{271} = (6,0,3,1)$
16 : $P_{273} = (0,1,3,1)$
17 : $P_{276} = (3,1,3,1)$
18 : $P_{284} = (3,2,3,1)$
19 : $P_{286} = (5,2,3,1)$
20 : $P_{314} = (1,6,3,1)$
21 : $P_{319} = (6,6,3,1)$

22 : $P_{322} = (1, 7, 3, 1)$
23 : $P_{326} = (5, 7, 3, 1)$
24 : $P_{333} = (4, 0, 4, 1)$
25 : $P_{345} = (0, 2, 4, 1)$
26 : $P_{350} = (5, 2, 4, 1)$
27 : $P_{364} = (3, 4, 4, 1)$
28 : $P_{365} = (4, 4, 4, 1)$
29 : $P_{388} = (3, 7, 4, 1)$
30 : $P_{390} = (5, 7, 4, 1)$
31 : $P_{396} = (3, 0, 5, 1)$
32 : $P_{401} = (0, 1, 5, 1)$
33 : $P_{406} = (5, 1, 5, 1)$
34 : $P_{410} = (1, 2, 5, 1)$
35 : $P_{415} = (6, 2, 5, 1)$
36 : $P_{418} = (1, 3, 5, 1)$
37 : $P_{420} = (3, 3, 5, 1)$
38 : $P_{430} = (5, 4, 5, 1)$
39 : $P_{431} = (6, 4, 5, 1)$
40 : $P_{462} = (5, 0, 6, 1)$
41 : $P_{465} = (0, 1, 6, 1)$
42 : $P_{471} = (6, 1, 6, 1)$
43 : $P_{490} = (1, 4, 6, 1)$
44 : $P_{492} = (3, 4, 6, 1)$
45 : $P_{498} = (1, 5, 6, 1)$
46 : $P_{502} = (5, 5, 6, 1)$
47 : $P_{516} = (3, 7, 6, 1)$
48 : $P_{519} = (6, 7, 6, 1)$
49 : $P_{528} = (7, 0, 7, 1)$
50 : $P_{542} = (5, 2, 7, 1)$
51 : $P_{543} = (6, 2, 7, 1)$
52 : $P_{553} = (0, 4, 7, 1)$
53 : $P_{559} = (6, 4, 7, 1)$
54 : $P_{582} = (5, 7, 7, 1)$
55 : $P_{584} = (7, 7, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_1

Line 1 intersects

Line	ℓ_0
in point	P_1

The surface has 73 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_6 = (2, 1, 0, 0)$
4 : $P_7 = (3, 1, 0, 0)$
5 : $P_8 = (4, 1, 0, 0)$
6 : $P_9 = (5, 1, 0, 0)$
7 : $P_{10} = (6, 1, 0, 0)$
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$70: P_{559} = (6, 4, 7, 1)$
$71: P_{582} = (5, 7, 7, 1)$
$72: P_{584} = (7, 7, 7, 1)$