Rank-65554 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431655854

General information

Number of lines	3
Number of points	21
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	9
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	2^3, 1^9, 0^9

Singular Points

The surface has 3 singular points:

$0:\ P_{12} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
$1:\ P_{18} = \mathbf{P}(\omega^2,\omega,1,0) = \mathbf{P}(3,2,1,0)$
$2:\ P_{21} = \mathbf{P}(\omega,\omega^2,1,0) = \mathbf{P}(2,3,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{45} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{45} = \mathbf{Pl}(3,0,2,0,0,1)_{118}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{65} = \mathbf{Pl}(2,0,3,0,0,1)_{124}$$

Rank of lines: (22, 45, 65)
Rank of points on Klein quadric: (109, 118, 124)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{21} = (2,3,1,0) = \ell_0 \cap \ell_1$
$P_{12} = (1,1,1,0) = \ell_1 \cap \ell_2$
$P_{18} = (3,2,1,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 9 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_6 = (2,1,0,0)$ lies on line ℓ_1
2 : $P_7 = (3,1,0,0)$ lies on line ℓ_2
3 : $P_8 = (1,0,1,0)$ lies on line ℓ_0
4 : $P_9 = (2,0,1,0)$ lies on line ℓ_2
5 : $P_{10} = (3,0,1,0)$ lies on line ℓ_1
6 : $P_{11} = (0,1,1,0)$ lies on line ℓ_0
7 : $P_{15} = (0,2,1,0)$ lies on line ℓ_1
8 : $P_{19} = (0,3,1,0)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

0 : $P_{23} = (1,0,0,1)$
1 : $P_{24} = (2,0,0,1)$
2 : $P_{25} = (3,0,0,1)$
3 : $P_{26} = (0,1,0,1)$
4 : $P_{30} = (0,2,0,1)$
5 : $P_{34} = (0,3,0,1)$
6 : $P_{38} = (0,0,1,1)$
7 : $P_{53} = (0,0,2,1)$
8 : $P_{69} = (0,0,3,1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{21}	P_{18}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{21}	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{18}	P_{12}

The surface has 21 points:
The points on the surface are:

$0: P_5 = (1,1,0,0)$
$1: P_6 = (2,1,0,0)$
$2: P_7 = (3,1,0,0)$
$3: P_8 = (1,0,1,0)$
$4: P_9 = (2,0,1,0)$
$5: P_{10} = (3,0,1,0)$
$6: P_{11} = (0,1,1,0)$
$7: P_{12} = (1,1,1,0)$
$8: P_{15} = (0,2,1,0)$
$9: P_{18} = (3,2,1,0)$
$10: P_{19} = (0,3,1,0)$
$11: P_{21} = (2,3,1,0)$
$12: P_{23} = (1,0,0,1)$
$13: P_{24} = (2,0,0,1)$
$14: P_{25} = (3,0,0,1)$
$15: P_{26} = (0,1,0,1)$
$16: P_{30} = (0,2,0,1)$
$17: P_{34} = (0,3,0,1)$
$18: P_{38} = (0,0,1,1)$
$19: P_{53} = (0,0,2,1)$
$20: P_{69} = (0,0,3,1)$