

Rank-65859 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65859

General information

Number of lines	5
Number of points	9
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	4
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	$3, 2^4, 1^4$

Singular Points

The surface has 3 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{22} \\ \ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\ \ell_4 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1\end{aligned}$$

Rank of lines: (0, 4, 11, 30, 34)

Rank of points on Klein quadric: (0, 2, 22, 5, 1)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0). T = -1$$

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$\begin{aligned}P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 \\ P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_2 \\ P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_3\end{aligned}$$

$$P_3 = (0, 0, 0, 1) = \ell_3 \cap \ell_4$$

Single Points

The surface has 4 single points:

The single points on the surface are:

$$\begin{aligned}0 : P_6 &= (1, 0, 1, 0) \text{ lies on line } \ell_1 \\ 1 : P_8 &= (1, 1, 1, 0) \text{ lies on line } \ell_2 \\ 2 : P_{10} &= (0, 1, 0, 1) \text{ lies on line } \ell_3\end{aligned}$$

$$3 : P_{12} = (0, 0, 1, 1) \text{ lies on line } \ell_4$$

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	0	1
3	1	0	0	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

The surface has 9 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_3 = (0, 0, 0, 1)$$

$$4 : P_5 = (1, 1, 0, 0)$$

$$5 : P_6 = (1, 0, 1, 0)$$

$$6 : P_8 = (1, 1, 1, 0)$$

$$7 : P_{10} = (0, 1, 0, 1)$$

$$8 : P_{12} = (0, 0, 1, 1)$$