

Rank-65618 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65618

General information

Number of lines	4
Number of points	9
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	4
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^4
Type of lines on points	$2^4, 1^4, 0$

Singular Points

The surface has 1 singular points:

$$0 : P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \mathbf{Pl}(1, 0, 1, 0, 0, 1)_{23}$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{32} \\ \ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{29} \\ \ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{28}\end{aligned}$$

Rank of lines: (8, 12, 9, 22)

Rank of points on Klein quadric: (23, 32, 29, 28)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_6 = (1, 0, 1, 0) = \ell_0 \cap \ell_2$$

$$P_7 = (0, 1, 1, 0) = \ell_0 \cap \ell_3$$

$$P_4 = (1, 1, 1, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 4 single points:

The single points on the surface are:

$$0 : P_{10} = (0, 1, 0, 1) \text{ lies on line } \ell_2$$

$$1 : P_{11} = (1, 1, 0, 1) \text{ lies on line } \ell_3$$

$$2 : P_{12} = (0, 0, 1, 1) \text{ lies on line } \ell_1$$

$$3 : P_{13} = (1, 0, 1, 1) \text{ lies on line } \ell_3$$

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:

The points on the surface but not on lines are:

$$0 : P_8 = (1, 1, 1, 0)$$

Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	1	0
2	1	1	0	0
3	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_5	P_4

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_6	P_4

Line 3 intersects

Line	ℓ_0
in point	P_7

The surface has 9 points:

The points on the surface are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_5 = (1, 1, 0, 0)$$

$$2 : P_6 = (1, 0, 1, 0)$$

$$3 : P_7 = (0, 1, 1, 0)$$

$$4 : P_8 = (1, 1, 1, 0)$$

$$5 : P_{10} = (0, 1, 0, 1)$$

$$6 : P_{11} = (1, 1, 0, 1)$$

$$7 : P_{12} = (0, 0, 1, 1)$$

$$8 : P_{13} = (1, 0, 1, 1)$$