Rank-73737 over GF(16)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286331429

General information

Number of lines	9
Number of points	305
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	126
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^5, 2^6, 1^{126}, 0^{168}

Singular Points

The surface has 4 singular points:

$0:\ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1:\ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
$2:\ P_{3010} = \mathbf{P}(1,\delta^5,\delta^{10},1) = \mathbf{P}(1,11,10,1)$
$3:\ P_{3250} = \mathbf{P}(1,\delta^{10},\delta^5,1) = \mathbf{P}(1,10,11,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,0,1,0,1,0)_{321}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^5 & \delta^{10} \end{bmatrix}_{171} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{171} = \mathbf{Pl}(11,0,10,0,1,0)_{610}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{10} & \delta^5 \end{bmatrix}_{186} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 10 & 11 \end{bmatrix}_{186} = \mathbf{Pl}(10,0,11,0,1,0)_{640}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1,1,1,1,0,1)_{5586}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \delta^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \mathbf{Pl}(0,11,0,1,0,0)_{59}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^5 & 0 \end{bmatrix}_{7109} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{7109} = \mathbf{Pl}(11,10,10,11,0,1)_{7981}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \begin{bmatrix} 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \mathbf{Pl}(0,10,0,1,0,0)_{58}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^5 & 1 \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{7381} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{7381} = \mathbf{Pl}(10,11,11,10,0,1)_{7770}$$

Rank of lines: (17, 171, 186, 69921, 4642, 70091, 7109, 70074, 7381)
Rank of points on Klein quadric: (321, 610, 640, 49, 5586, 59, 7981, 58, 7770)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$2 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$,
$3 : P_{3010} = \mathbf{P}(1,\delta^5,\delta^{10},1) = \mathbf{P}(1,11,10,1)$,
$4 : P_{3250} = \mathbf{P}(1,\delta^{10},\delta^5,1) = \mathbf{P}(1,10,11,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{546} = (0,1,1,1) = \ell_0 \cap \ell_3$
$P_{3009} = (0,11,10,1) = \ell_1 \cap \ell_5$
$P_{3249} = (0,10,11,1) = \ell_2 \cap \ell_7$
$P_{35} = (0,1,1,0) = \ell_3 \cap \ell_4$
$P_{179} = (0,10,1,0) = \ell_5 \cap \ell_6$
$P_{195} = (0,11,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 126 single points:
The single points on the surface are:

$0 :\ P_{291} = (1,1,0,1)$ lies on line ℓ_4
$1 :\ P_{435} = (1,10,0,1)$ lies on line ℓ_8
$2 :\ P_{451} = (1,11,0,1)$ lies on line ℓ_6
$3 :\ P_{531} = (1,0,1,1)$ lies on line ℓ_4
$4 :\ P_{547} = (2,1,1,1)$ lies on line ℓ_0
$5 :\ P_{548} = (3,1,1,1)$ lies on line ℓ_0

6 : $P_{549} = (4, 1, 1, 1)$ lies on line ℓ_0
7 : $P_{550} = (5, 1, 1, 1)$ lies on line ℓ_0
8 : $P_{551} = (6, 1, 1, 1)$ lies on line ℓ_0
9 : $P_{552} = (7, 1, 1, 1)$ lies on line ℓ_0
10 : $P_{553} = (8, 1, 1, 1)$ lies on line ℓ_0
11 : $P_{554} = (9, 1, 1, 1)$ lies on line ℓ_0
12 : $P_{555} = (10, 1, 1, 1)$ lies on line ℓ_0
13 : $P_{556} = (11, 1, 1, 1)$ lies on line ℓ_0
14 : $P_{557} = (12, 1, 1, 1)$ lies on line ℓ_0
15 : $P_{558} = (13, 1, 1, 1)$ lies on line ℓ_0
16 : $P_{559} = (14, 1, 1, 1)$ lies on line ℓ_0
17 : $P_{560} = (15, 1, 1, 1)$ lies on line ℓ_0
18 : $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_5
19 : $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_7
20 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_3
21 : $P_{834} = (1, 3, 2, 1)$ lies on line ℓ_4
22 : $P_{866} = (1, 5, 2, 1)$ lies on line ℓ_8
23 : $P_{882} = (1, 6, 2, 1)$ lies on line ℓ_6
24 : $P_{993} = (0, 13, 2, 1)$ lies on line ℓ_5
25 : $P_{1025} = (0, 15, 2, 1)$ lies on line ℓ_7
26 : $P_{1074} = (1, 2, 3, 1)$ lies on line ℓ_4
27 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_3
28 : $P_{1105} = (0, 4, 3, 1)$ lies on line ℓ_7
29 : $P_{1153} = (0, 7, 3, 1)$ lies on line ℓ_5
30 : $P_{1234} = (1, 12, 3, 1)$ lies on line ℓ_6
31 : $P_{1266} = (1, 14, 3, 1)$ lies on line ℓ_8
32 : $P_{1345} = (0, 3, 4, 1)$ lies on line ℓ_5
33 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_3
34 : $P_{1378} = (1, 5, 4, 1)$ lies on line ℓ_4
35 : $P_{1409} = (0, 7, 4, 1)$ lies on line ℓ_7
36 : $P_{1426} = (1, 8, 4, 1)$ lies on line ℓ_6
37 : $P_{1506} = (1, 13, 4, 1)$ lies on line ℓ_8
38 : $P_{1586} = (1, 2, 5, 1)$ lies on line ℓ_6
39 : $P_{1618} = (1, 4, 5, 1)$ lies on line ℓ_4
40 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_3
41 : $P_{1650} = (1, 6, 5, 1)$ lies on line ℓ_8
42 : $P_{1697} = (0, 9, 5, 1)$ lies on line ℓ_5
43 : $P_{1745} = (0, 12, 5, 1)$ lies on line ℓ_7
44 : $P_{1842} = (1, 2, 6, 1)$ lies on line ℓ_8
45 : $P_{1890} = (1, 5, 6, 1)$ lies on line ℓ_6
46 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_3
47 : $P_{1922} = (1, 7, 6, 1)$ lies on line ℓ_4
48 : $P_{1937} = (0, 8, 6, 1)$ lies on line ℓ_7
49 : $P_{2033} = (0, 14, 6, 1)$ lies on line ℓ_5
50 : $P_{2113} = (0, 3, 7, 1)$ lies on line ℓ_7
51 : $P_{2129} = (0, 4, 7, 1)$ lies on line ℓ_5
52 : $P_{2162} = (1, 6, 7, 1)$ lies on line ℓ_4
53 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_3
54 : $P_{2210} = (1, 9, 7, 1)$ lies on line ℓ_8
55 : $P_{2306} = (1, 15, 7, 1)$ lies on line ℓ_6
56 : $P_{2386} = (1, 4, 8, 1)$ lies on line ℓ_8
57 : $P_{2417} = (0, 6, 8, 1)$ lies on line ℓ_5
58 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_3
59 : $P_{2466} = (1, 9, 8, 1)$ lies on line ℓ_4
60 : $P_{2530} = (1, 13, 8, 1)$ lies on line ℓ_6
61 : $P_{2545} = (0, 14, 8, 1)$ lies on line ℓ_7
62 : $P_{2657} = (0, 5, 9, 1)$ lies on line ℓ_7
63 : $P_{2690} = (1, 7, 9, 1)$ lies on line ℓ_6
64 : $P_{2706} = (1, 8, 9, 1)$ lies on line ℓ_4
65 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_3
66 : $P_{2769} = (0, 12, 9, 1)$ lies on line ℓ_5
67 : $P_{2818} = (1, 15, 9, 1)$ lies on line ℓ_8
68 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_6
69 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_7
70 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_3
71 : $P_{3011} = (2, 11, 10, 1)$ lies on line ℓ_1
72 : $P_{3012} = (3, 11, 10, 1)$ lies on line ℓ_1
73 : $P_{3013} = (4, 11, 10, 1)$ lies on line ℓ_1
74 : $P_{3014} = (5, 11, 10, 1)$ lies on line ℓ_1
75 : $P_{3015} = (6, 11, 10, 1)$ lies on line ℓ_1
76 : $P_{3016} = (7, 11, 10, 1)$ lies on line ℓ_1
77 : $P_{3017} = (8, 11, 10, 1)$ lies on line ℓ_1
78 : $P_{3018} = (9, 11, 10, 1)$ lies on line ℓ_1
79 : $P_{3019} = (10, 11, 10, 1)$ lies on line ℓ_1
80 : $P_{3020} = (11, 11, 10, 1)$ lies on line ℓ_1
81 : $P_{3021} = (12, 11, 10, 1)$ lies on line ℓ_1
82 : $P_{3022} = (13, 11, 10, 1)$ lies on line ℓ_1
83 : $P_{3023} = (14, 11, 10, 1)$ lies on line ℓ_1
84 : $P_{3024} = (15, 11, 10, 1)$ lies on line ℓ_1
85 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_8
86 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_5
87 : $P_{3251} = (2, 10, 11, 1)$ lies on line ℓ_2
88 : $P_{3252} = (3, 10, 11, 1)$ lies on line ℓ_2
89 : $P_{3253} = (4, 10, 11, 1)$ lies on line ℓ_2
90 : $P_{3254} = (5, 10, 11, 1)$ lies on line ℓ_2
91 : $P_{3255} = (6, 10, 11, 1)$ lies on line ℓ_2
92 : $P_{3256} = (7, 10, 11, 1)$ lies on line ℓ_2
93 : $P_{3257} = (8, 10, 11, 1)$ lies on line ℓ_2
94 : $P_{3258} = (9, 10, 11, 1)$ lies on line ℓ_2
95 : $P_{3259} = (10, 10, 11, 1)$ lies on line ℓ_2
96 : $P_{3260} = (11, 10, 11, 1)$ lies on line ℓ_2
97 : $P_{3261} = (12, 10, 11, 1)$ lies on line ℓ_2
98 : $P_{3262} = (13, 10, 11, 1)$ lies on line ℓ_2
99 : $P_{3263} = (14, 10, 11, 1)$ lies on line ℓ_2
100 : $P_{3264} = (15, 10, 11, 1)$ lies on line ℓ_2
101 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_3
102 : $P_{3394} = (1, 3, 12, 1)$ lies on line ℓ_8
103 : $P_{3425} = (0, 5, 12, 1)$ lies on line ℓ_5
104 : $P_{3489} = (0, 9, 12, 1)$ lies on line ℓ_7
105 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_3
106 : $P_{3554} = (1, 13, 12, 1)$ lies on line ℓ_4
107 : $P_{3570} = (1, 14, 12, 1)$ lies on line ℓ_6
108 : $P_{3633} = (0, 2, 13, 1)$ lies on line ℓ_7
109 : $P_{3666} = (1, 4, 13, 1)$ lies on line ℓ_6
110 : $P_{3730} = (1, 8, 13, 1)$ lies on line ℓ_8
111 : $P_{3794} = (1, 12, 13, 1)$ lies on line ℓ_4
112 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_3
113 : $P_{3841} = (0, 15, 13, 1)$ lies on line ℓ_5

114 : $P_{3906} = (1, 3, 14, 1)$ lies on line ℓ_6
115 : $P_{3953} = (0, 6, 14, 1)$ lies on line ℓ_7
116 : $P_{3985} = (0, 8, 14, 1)$ lies on line ℓ_5
117 : $P_{4050} = (1, 12, 14, 1)$ lies on line ℓ_8
118 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_3
119 : $P_{4098} = (1, 15, 14, 1)$ lies on line ℓ_4
120 : $P_{4145} = (0, 2, 15, 1)$ lies on line ℓ_5
121 : $P_{4226} = (1, 7, 15, 1)$ lies on line ℓ_8
122 : $P_{4258} = (1, 9, 15, 1)$ lies on line ℓ_6
123 : $P_{4321} = (0, 13, 15, 1)$ lies on line ℓ_7
124 : $P_{4338} = (1, 14, 15, 1)$ lies on line ℓ_4
125 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_{59} = (8, 2, 1, 0)$
1 : $P_{80} = (13, 3, 1, 0)$
2 : $P_{98} = (15, 4, 1, 0)$
3 : $P_{106} = (7, 5, 1, 0)$
4 : $P_{124} = (9, 6, 1, 0)$
5 : $P_{133} = (2, 7, 1, 0)$
6 : $P_{159} = (12, 8, 1, 0)$
7 : $P_{166} = (3, 9, 1, 0)$
8 : $P_{215} = (4, 12, 1, 0)$
9 : $P_{241} = (14, 13, 1, 0)$
10 : $P_{248} = (5, 14, 1, 0)$
11 : $P_{265} = (6, 15, 1, 0)$
12 : $P_{314} = (8, 2, 0, 1)$
13 : $P_{337} = (15, 3, 0, 1)$
14 : $P_{353} = (15, 4, 0, 1)$
15 : $P_{357} = (3, 5, 0, 1)$
16 : $P_{375} = (5, 6, 0, 1)$
17 : $P_{401} = (15, 7, 0, 1)$
18 : $P_{407} = (5, 8, 0, 1)$
19 : $P_{421} = (3, 9, 0, 1)$
20 : $P_{469} = (3, 12, 0, 1)$
21 : $P_{490} = (8, 13, 0, 1)$
22 : $P_{503} = (5, 14, 0, 1)$
23 : $P_{522} = (8, 15, 0, 1)$
24 : $P_{568} = (7, 2, 1, 1)$
25 : $P_{584} = (7, 3, 1, 1)$
26 : $P_{605} = (12, 4, 1, 1)$
27 : $P_{621} = (12, 5, 1, 1)$
28 : $P_{635} = (10, 6, 1, 1)$
29 : $P_{651} = (10, 7, 1, 1)$
30 : $P_{663} = (6, 8, 1, 1)$
31 : $P_{679} = (6, 9, 1, 1)$
32 : $P_{732} = (11, 12, 1, 1)$
33 : $P_{748} = (11, 13, 1, 1)$
34 : $P_{766} = (13, 14, 1, 1)$
35 : $P_{782} = (13, 15, 1, 1)$
36 : $P_{793} = (8, 0, 2, 1)$
37 : $P_{808} = (7, 1, 2, 1)$
38 : $P_{857} = (8, 4, 2, 1)$
39 : $P_{899} = (2, 7, 2, 1)$
40 : $P_{927} = (14, 8, 2, 1)$
41 : $P_{939} = (10, 9, 2, 1)$
42 : $P_{956} = (11, 10, 2, 1)$
43 : $P_{974} = (13, 11, 2, 1)$
44 : $P_{1014} = (5, 14, 2, 1)$
45 : $P_{1056} = (15, 0, 3, 1)$
46 : $P_{1064} = (7, 1, 3, 1)$
47 : $P_{1136} = (15, 5, 3, 1)$
48 : $P_{1148} = (11, 6, 3, 1)$
49 : $P_{1189} = (4, 9, 3, 1)$
50 : $P_{1211} = (10, 10, 3, 1)$
51 : $P_{1229} = (12, 11, 3, 1)$
52 : $P_{1251} = (2, 13, 3, 1)$
53 : $P_{1289} = (8, 15, 3, 1)$
54 : $P_{1312} = (15, 0, 4, 1)$
55 : $P_{1325} = (12, 1, 4, 1)$
56 : $P_{1337} = (8, 2, 4, 1)$
57 : $P_{1456} = (15, 9, 4, 1)$
58 : $P_{1464} = (7, 10, 4, 1)$
59 : $P_{1483} = (10, 11, 4, 1)$
60 : $P_{1493} = (4, 12, 4, 1)$
61 : $P_{1532} = (11, 14, 4, 1)$
62 : $P_{1539} = (2, 15, 4, 1)$
63 : $P_{1556} = (3, 0, 5, 1)$
64 : $P_{1581} = (12, 1, 5, 1)$
65 : $P_{1616} = (15, 3, 5, 1)$
66 : $P_{1669} = (4, 7, 5, 1)$
67 : $P_{1684} = (3, 8, 5, 1)$
68 : $P_{1719} = (6, 10, 5, 1)$
69 : $P_{1740} = (11, 11, 5, 1)$
70 : $P_{1771} = (10, 13, 5, 1)$
71 : $P_{1786} = (9, 14, 5, 1)$
72 : $P_{1814} = (5, 0, 6, 1)$
73 : $P_{1835} = (10, 1, 6, 1)$

$74: P_{1868} = (11, 3, 6, 1)$
$75: P_{1962} = (9, 9, 6, 1)$
$76: P_{1982} = (13, 10, 6, 1)$
$77: P_{1991} = (6, 11, 6, 1)$
$78: P_{2004} = (3, 12, 6, 1)$
$79: P_{2022} = (5, 13, 6, 1)$
$80: P_{2063} = (14, 15, 6, 1)$
$81: P_{2080} = (15, 0, 7, 1)$
$82: P_{2091} = (10, 1, 7, 1)$
$83: P_{2099} = (2, 2, 7, 1)$
$84: P_{2149} = (4, 5, 7, 1)$
$85: P_{2204} = (11, 8, 7, 1)$
$86: P_{2237} = (12, 10, 7, 1)$
$87: P_{2248} = (7, 11, 7, 1)$
$88: P_{2272} = (15, 12, 7, 1)$
$89: P_{2281} = (8, 13, 7, 1)$
$90: P_{2326} = (5, 0, 8, 1)$
$91: P_{2343} = (6, 1, 8, 1)$
$92: P_{2367} = (14, 2, 8, 1)$
$93: P_{2404} = (3, 5, 8, 1)$
$94: P_{2444} = (11, 7, 8, 1)$
$95: P_{2491} = (10, 10, 8, 1)$
$96: P_{2510} = (13, 11, 8, 1)$
$97: P_{2522} = (9, 12, 8, 1)$
$98: P_{2566} = (5, 15, 8, 1)$
$99: P_{2580} = (3, 0, 9, 1)$
$100: P_{2599} = (6, 1, 9, 1)$
$101: P_{2619} = (10, 2, 9, 1)$
$102: P_{2629} = (4, 3, 9, 1)$
$103: P_{2656} = (15, 4, 9, 1)$
$104: P_{2682} = (9, 6, 9, 1)$
$105: P_{2748} = (11, 10, 9, 1)$
$106: P_{2765} = (12, 11, 9, 1)$
$107: P_{2804} = (3, 14, 9, 1)$
$108: P_{2876} = (11, 2, 10, 1)$
$109: P_{2891} = (10, 3, 10, 1)$
$110: P_{2904} = (7, 4, 10, 1)$
$111: P_{2919} = (6, 5, 10, 1)$
$112: P_{2942} = (13, 6, 10, 1)$
$113: P_{2957} = (12, 7, 10, 1)$
$114: P_{2971} = (10, 8, 10, 1)$
$115: P_{2988} = (11, 9, 10, 1)$
$116: P_{3037} = (12, 12, 10, 1)$
$117: P_{3054} = (13, 13, 10, 1)$
$118: P_{3063} = (6, 14, 10, 1)$
$119: P_{3080} = (7, 15, 10, 1)$
$120: P_{3134} = (13, 2, 11, 1)$
$121: P_{3149} = (12, 3, 11, 1)$
$122: P_{3163} = (10, 4, 11, 1)$
$123: P_{3180} = (11, 5, 11, 1)$
$124: P_{3191} = (6, 6, 11, 1)$
$125: P_{3208} = (7, 7, 11, 1)$
$126: P_{3230} = (13, 8, 11, 1)$
$127: P_{3245} = (12, 9, 11, 1)$
$128: P_{3287} = (6, 12, 11, 1)$
$129: P_{3304} = (7, 13, 11, 1)$
$130: P_{3323} = (10, 14, 11, 1)$
$131: P_{3340} = (11, 15, 11, 1)$
$132: P_{3348} = (3, 0, 12, 1)$
$133: P_{3372} = (11, 1, 12, 1)$
$134: P_{3413} = (4, 4, 12, 1)$
$135: P_{3444} = (3, 6, 12, 1)$
$136: P_{3472} = (15, 7, 12, 1)$
$137: P_{3482} = (9, 8, 12, 1)$
$138: P_{3517} = (12, 10, 12, 1)$
$139: P_{3527} = (6, 11, 12, 1)$
$140: P_{3595} = (10, 15, 12, 1)$
$141: P_{3609} = (8, 0, 13, 1)$
$142: P_{3628} = (11, 1, 13, 1)$
$143: P_{3651} = (2, 3, 13, 1)$
$144: P_{3691} = (10, 5, 13, 1)$
$145: P_{3702} = (5, 6, 13, 1)$
$146: P_{3721} = (8, 7, 13, 1)$
$147: P_{3774} = (13, 10, 13, 1)$
$148: P_{3784} = (7, 11, 13, 1)$
$149: P_{3839} = (14, 14, 13, 1)$
$150: P_{3862} = (5, 0, 14, 1)$
$151: P_{3886} = (13, 1, 14, 1)$
$152: P_{3894} = (5, 2, 14, 1)$
$153: P_{3932} = (11, 4, 14, 1)$
$154: P_{3946} = (9, 5, 14, 1)$
$155: P_{4004} = (3, 9, 14, 1)$
$156: P_{4023} = (6, 10, 14, 1)$
$157: P_{4043} = (10, 11, 14, 1)$
$158: P_{4079} = (14, 13, 14, 1)$
$159: P_{4121} = (8, 0, 15, 1)$
$160: P_{4142} = (13, 1, 15, 1)$
$161: P_{4169} = (8, 3, 15, 1)$
$162: P_{4179} = (2, 4, 15, 1)$
$163: P_{4223} = (14, 6, 15, 1)$
$164: P_{4246} = (5, 8, 15, 1)$
$165: P_{4280} = (7, 10, 15, 1)$
$166: P_{4300} = (11, 11, 15, 1)$
$167: P_{4315} = (10, 12, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 0 0 1 0 1
1	1 0 1 0 1 1 0 0 1
2	1 1 0 0 1 0 1 1 0
3	1 0 0 0 1 1 0 1 0
4	0 1 1 1 0 0 1 0 1
5	0 1 0 1 0 0 1 1 0
6	1 0 1 0 1 1 0 0 1
7	0 0 1 1 0 1 0 0 1
8	1 1 0 0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_0	P_0	P_{546}	P_4	P_4

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_0	P_0	P_{3010}	P_{3009}	P_{3010}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_0	P_0	P_{3250}	P_{3250}	P_{3249}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_{546}	P_{35}	P_3	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_{3010}	P_{3250}	P_{35}	P_{3250}	P_{3010}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_{3009}	P_3	P_{179}	P_3

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_4	P_{3250}	P_{3250}	P_{179}	P_4

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_{3249}	P_3	P_3	P_{195}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_4	P_{3010}	P_{3010}	P_4	P_{195}

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_{35} = (0, 1, 1, 0)$
4 : $P_{59} = (8, 2, 1, 0)$
5 : $P_{80} = (13, 3, 1, 0)$
6 : $P_{98} = (15, 4, 1, 0)$
7 : $P_{106} = (7, 5, 1, 0)$
8 : $P_{124} = (9, 6, 1, 0)$
9 : $P_{133} = (2, 7, 1, 0)$
10 : $P_{159} = (12, 8, 1, 0)$
11 : $P_{166} = (3, 9, 1, 0)$
12 : $P_{179} = (0, 10, 1, 0)$
13 : $P_{195} = (0, 11, 1, 0)$
14 : $P_{215} = (4, 12, 1, 0)$
15 : $P_{241} = (14, 13, 1, 0)$
16 : $P_{248} = (5, 14, 1, 0)$
17 : $P_{265} = (6, 15, 1, 0)$
18 : $P_{291} = (1, 1, 0, 1)$
19 : $P_{314} = (8, 2, 0, 1)$
20 : $P_{337} = (15, 3, 0, 1)$

21 : $P_{353} = (15, 4, 0, 1)$
22 : $P_{357} = (3, 5, 0, 1)$
23 : $P_{375} = (5, 6, 0, 1)$
24 : $P_{401} = (15, 7, 0, 1)$
25 : $P_{407} = (5, 8, 0, 1)$
26 : $P_{421} = (3, 9, 0, 1)$
27 : $P_{435} = (1, 10, 0, 1)$
28 : $P_{451} = (1, 11, 0, 1)$
29 : $P_{469} = (3, 12, 0, 1)$
30 : $P_{490} = (8, 13, 0, 1)$
31 : $P_{503} = (5, 14, 0, 1)$
32 : $P_{522} = (8, 15, 0, 1)$
33 : $P_{531} = (1, 0, 1, 1)$
34 : $P_{546} = (0, 1, 1, 1)$
35 : $P_{547} = (2, 1, 1, 1)$
36 : $P_{548} = (3, 1, 1, 1)$
37 : $P_{549} = (4, 1, 1, 1)$
38 : $P_{550} = (5, 1, 1, 1)$
39 : $P_{551} = (6, 1, 1, 1)$
40 : $P_{552} = (7, 1, 1, 1)$
41 : $P_{553} = (8, 1, 1, 1)$
42 : $P_{554} = (9, 1, 1, 1)$
43 : $P_{555} = (10, 1, 1, 1)$
44 : $P_{556} = (11, 1, 1, 1)$
45 : $P_{557} = (12, 1, 1, 1)$
46 : $P_{558} = (13, 1, 1, 1)$
47 : $P_{559} = (14, 1, 1, 1)$
48 : $P_{560} = (15, 1, 1, 1)$
49 : $P_{568} = (7, 2, 1, 1)$
50 : $P_{584} = (7, 3, 1, 1)$
51 : $P_{605} = (12, 4, 1, 1)$
52 : $P_{621} = (12, 5, 1, 1)$
53 : $P_{635} = (10, 6, 1, 1)$
54 : $P_{651} = (10, 7, 1, 1)$
55 : $P_{663} = (6, 8, 1, 1)$
56 : $P_{679} = (6, 9, 1, 1)$
57 : $P_{689} = (0, 10, 1, 1)$
58 : $P_{705} = (0, 11, 1, 1)$
59 : $P_{732} = (11, 12, 1, 1)$
60 : $P_{748} = (11, 13, 1, 1)$
61 : $P_{766} = (13, 14, 1, 1)$
62 : $P_{782} = (13, 15, 1, 1)$
63 : $P_{793} = (8, 0, 2, 1)$
64 : $P_{808} = (7, 1, 2, 1)$
65 : $P_{817} = (0, 2, 2, 1)$
66 : $P_{834} = (1, 3, 2, 1)$
67 : $P_{857} = (8, 4, 2, 1)$
68 : $P_{866} = (1, 5, 2, 1)$
69 : $P_{882} = (1, 6, 2, 1)$
70 : $P_{899} = (2, 7, 2, 1)$
71 : $P_{927} = (14, 8, 2, 1)$
72 : $P_{939} = (10, 9, 2, 1)$
73 : $P_{956} = (11, 10, 2, 1)$
74 : $P_{974} = (13, 11, 2, 1)$
75 : $P_{993} = (0, 13, 2, 1)$
76 : $P_{1014} = (5, 14, 2, 1)$
77 : $P_{1025} = (0, 15, 2, 1)$
78 : $P_{1056} = (15, 0, 3, 1)$
79 : $P_{1064} = (7, 1, 3, 1)$
80 : $P_{1074} = (1, 2, 3, 1)$
81 : $P_{1089} = (0, 3, 3, 1)$
82 : $P_{1105} = (0, 4, 3, 1)$
83 : $P_{1136} = (15, 5, 3, 1)$
84 : $P_{1148} = (11, 6, 3, 1)$
85 : $P_{1153} = (0, 7, 3, 1)$
86 : $P_{1189} = (4, 9, 3, 1)$
87 : $P_{1211} = (10, 10, 3, 1)$
88 : $P_{1229} = (12, 11, 3, 1)$
89 : $P_{1234} = (1, 12, 3, 1)$
90 : $P_{1251} = (2, 13, 3, 1)$
91 : $P_{1266} = (1, 14, 3, 1)$
92 : $P_{1289} = (8, 15, 3, 1)$
93 : $P_{1312} = (15, 0, 4, 1)$
94 : $P_{1325} = (12, 1, 4, 1)$
95 : $P_{1337} = (8, 2, 4, 1)$
96 : $P_{1345} = (0, 3, 4, 1)$
97 : $P_{1361} = (0, 4, 4, 1)$
98 : $P_{1378} = (1, 5, 4, 1)$
99 : $P_{1409} = (0, 7, 4, 1)$
100 : $P_{1426} = (1, 8, 4, 1)$
101 : $P_{1456} = (15, 9, 4, 1)$
102 : $P_{1464} = (7, 10, 4, 1)$
103 : $P_{1483} = (10, 11, 4, 1)$
104 : $P_{1493} = (4, 12, 4, 1)$
105 : $P_{1506} = (1, 13, 4, 1)$
106 : $P_{1532} = (11, 14, 4, 1)$
107 : $P_{1539} = (2, 15, 4, 1)$
108 : $P_{1556} = (3, 0, 5, 1)$
109 : $P_{1581} = (12, 1, 5, 1)$
110 : $P_{1586} = (1, 2, 5, 1)$
111 : $P_{1616} = (15, 3, 5, 1)$
112 : $P_{1618} = (1, 4, 5, 1)$
113 : $P_{1633} = (0, 5, 5, 1)$
114 : $P_{1650} = (1, 6, 5, 1)$
115 : $P_{1669} = (4, 7, 5, 1)$
116 : $P_{1684} = (3, 8, 5, 1)$
117 : $P_{1697} = (0, 9, 5, 1)$
118 : $P_{1719} = (6, 10, 5, 1)$
119 : $P_{1740} = (11, 11, 5, 1)$
120 : $P_{1745} = (0, 12, 5, 1)$
121 : $P_{1771} = (10, 13, 5, 1)$
122 : $P_{1786} = (9, 14, 5, 1)$
123 : $P_{1814} = (5, 0, 6, 1)$
124 : $P_{1835} = (10, 1, 6, 1)$
125 : $P_{1842} = (1, 2, 6, 1)$
126 : $P_{1868} = (11, 3, 6, 1)$
127 : $P_{1890} = (1, 5, 6, 1)$
128 : $P_{1905} = (0, 6, 6, 1)$
129 : $P_{1922} = (1, 7, 6, 1)$
130 : $P_{1937} = (0, 8, 6, 1)$
131 : $P_{1962} = (9, 9, 6, 1)$
132 : $P_{1982} = (13, 10, 6, 1)$
133 : $P_{1991} = (6, 11, 6, 1)$
134 : $P_{2004} = (3, 12, 6, 1)$
135 : $P_{2022} = (5, 13, 6, 1)$
136 : $P_{2033} = (0, 14, 6, 1)$
137 : $P_{2063} = (14, 15, 6, 1)$
138 : $P_{2080} = (15, 0, 7, 1)$
139 : $P_{2091} = (10, 1, 7, 1)$
140 : $P_{2099} = (2, 2, 7, 1)$
141 : $P_{2113} = (0, 3, 7, 1)$
142 : $P_{2129} = (0, 4, 7, 1)$
143 : $P_{2149} = (4, 5, 7, 1)$
144 : $P_{2162} = (1, 6, 7, 1)$
145 : $P_{2177} = (0, 7, 7, 1)$
146 : $P_{2204} = (11, 8, 7, 1)$
147 : $P_{2210} = (1, 9, 7, 1)$
148 : $P_{2237} = (12, 10, 7, 1)$
149 : $P_{2248} = (7, 11, 7, 1)$
150 : $P_{2272} = (15, 12, 7, 1)$
151 : $P_{2281} = (8, 13, 7, 1)$
152 : $P_{2306} = (1, 15, 7, 1)$
153 : $P_{2326} = (5, 0, 8, 1)$
154 : $P_{2343} = (6, 1, 8, 1)$
155 : $P_{2367} = (14, 2, 8, 1)$
156 : $P_{2386} = (1, 4, 8, 1)$
157 : $P_{2404} = (3, 5, 8, 1)$
158 : $P_{2417} = (0, 6, 8, 1)$
159 : $P_{2444} = (11, 7, 8, 1)$
160 : $P_{2449} = (0, 8, 8, 1)$
161 : $P_{2466} = (1, 9, 8, 1)$
162 : $P_{2491} = (10, 10, 8, 1)$
163 : $P_{2510} = (13, 11, 8, 1)$
164 : $P_{2522} = (9, 12, 8, 1)$
165 : $P_{2530} = (1, 13, 8, 1)$
166 : $P_{2545} = (0, 14, 8, 1)$
167 : $P_{2566} = (5, 15, 8, 1)$
168 : $P_{2580} = (3, 0, 9, 1)$
169 : $P_{2599} = (6, 1, 9, 1)$
170 : $P_{2619} = (10, 2, 9, 1)$
171 : $P_{2629} = (4, 3, 9, 1)$
172 : $P_{2656} = (15, 4, 9, 1)$
173 : $P_{2657} = (0, 5, 9, 1)$
174 : $P_{2682} = (9, 6, 9, 1)$
175 : $P_{2690} = (1, 7, 9, 1)$
176 : $P_{2706} = (1, 8, 9, 1)$
177 : $P_{2721} = (0, 9, 9, 1)$
178 : $P_{2748} = (11, 10, 9, 1)$
179 : $P_{2765} = (12, 11, 9, 1)$
180 : $P_{2769} = (0, 12, 9, 1)$
181 : $P_{2804} = (3, 14, 9, 1)$
182 : $P_{2818} = (1, 15, 9, 1)$

183 : $P_{2834} = (1, 0, 10, 1)$
184 : $P_{2849} = (0, 1, 10, 1)$
185 : $P_{2876} = (11, 2, 10, 1)$
186 : $P_{2891} = (10, 3, 10, 1)$
187 : $P_{2904} = (7, 4, 10, 1)$
188 : $P_{2919} = (6, 5, 10, 1)$
189 : $P_{2942} = (13, 6, 10, 1)$
190 : $P_{2957} = (12, 7, 10, 1)$
191 : $P_{2971} = (10, 8, 10, 1)$
192 : $P_{2988} = (11, 9, 10, 1)$
193 : $P_{2993} = (0, 10, 10, 1)$
194 : $P_{3009} = (0, 11, 10, 1)$
195 : $P_{3010} = (1, 11, 10, 1)$
196 : $P_{3011} = (2, 11, 10, 1)$
197 : $P_{3012} = (3, 11, 10, 1)$
198 : $P_{3013} = (4, 11, 10, 1)$
199 : $P_{3014} = (5, 11, 10, 1)$
200 : $P_{3015} = (6, 11, 10, 1)$
201 : $P_{3016} = (7, 11, 10, 1)$
202 : $P_{3017} = (8, 11, 10, 1)$
203 : $P_{3018} = (9, 11, 10, 1)$
204 : $P_{3019} = (10, 11, 10, 1)$
205 : $P_{3020} = (11, 11, 10, 1)$
206 : $P_{3021} = (12, 11, 10, 1)$
207 : $P_{3022} = (13, 11, 10, 1)$
208 : $P_{3023} = (14, 11, 10, 1)$
209 : $P_{3024} = (15, 11, 10, 1)$
210 : $P_{3037} = (12, 12, 10, 1)$
211 : $P_{3054} = (13, 13, 10, 1)$
212 : $P_{3063} = (6, 14, 10, 1)$
213 : $P_{3080} = (7, 15, 10, 1)$
214 : $P_{3090} = (1, 0, 11, 1)$
215 : $P_{3105} = (0, 1, 11, 1)$
216 : $P_{3134} = (13, 2, 11, 1)$
217 : $P_{3149} = (12, 3, 11, 1)$
218 : $P_{3163} = (10, 4, 11, 1)$
219 : $P_{3180} = (11, 5, 11, 1)$
220 : $P_{3191} = (6, 6, 11, 1)$
221 : $P_{3208} = (7, 7, 11, 1)$
222 : $P_{3230} = (13, 8, 11, 1)$
223 : $P_{3245} = (12, 9, 11, 1)$
224 : $P_{3249} = (0, 10, 11, 1)$
225 : $P_{3250} = (1, 10, 11, 1)$
226 : $P_{3251} = (2, 10, 11, 1)$
227 : $P_{3252} = (3, 10, 11, 1)$
228 : $P_{3253} = (4, 10, 11, 1)$
229 : $P_{3254} = (5, 10, 11, 1)$
230 : $P_{3255} = (6, 10, 11, 1)$
231 : $P_{3256} = (7, 10, 11, 1)$
232 : $P_{3257} = (8, 10, 11, 1)$
233 : $P_{3258} = (9, 10, 11, 1)$
234 : $P_{3259} = (10, 10, 11, 1)$
235 : $P_{3260} = (11, 10, 11, 1)$
236 : $P_{3261} = (12, 10, 11, 1)$
237 : $P_{3262} = (13, 10, 11, 1)$
238 : $P_{3263} = (14, 10, 11, 1)$
239 : $P_{3264} = (15, 10, 11, 1)$
240 : $P_{3265} = (0, 11, 11, 1)$
241 : $P_{3287} = (6, 12, 11, 1)$
242 : $P_{3304} = (7, 13, 11, 1)$
243 : $P_{3323} = (10, 14, 11, 1)$
244 : $P_{3340} = (11, 15, 11, 1)$
245 : $P_{3348} = (3, 0, 12, 1)$
246 : $P_{3372} = (11, 1, 12, 1)$
247 : $P_{3394} = (1, 3, 12, 1)$
248 : $P_{3413} = (4, 4, 12, 1)$
249 : $P_{3425} = (0, 5, 12, 1)$
250 : $P_{3444} = (3, 6, 12, 1)$
251 : $P_{3472} = (15, 7, 12, 1)$
252 : $P_{3482} = (9, 8, 12, 1)$
253 : $P_{3489} = (0, 9, 12, 1)$
254 : $P_{3517} = (12, 10, 12, 1)$
255 : $P_{3527} = (6, 11, 12, 1)$
256 : $P_{3537} = (0, 12, 12, 1)$
257 : $P_{3554} = (1, 13, 12, 1)$
258 : $P_{3570} = (1, 14, 12, 1)$
259 : $P_{3595} = (10, 15, 12, 1)$
260 : $P_{3609} = (8, 0, 13, 1)$
261 : $P_{3628} = (11, 1, 13, 1)$
262 : $P_{3633} = (0, 2, 13, 1)$
263 : $P_{3651} = (2, 3, 13, 1)$
264 : $P_{3666} = (1, 4, 13, 1)$
265 : $P_{3691} = (10, 5, 13, 1)$
266 : $P_{3702} = (5, 6, 13, 1)$
267 : $P_{3721} = (8, 7, 13, 1)$
268 : $P_{3730} = (1, 8, 13, 1)$
269 : $P_{3774} = (13, 10, 13, 1)$
270 : $P_{3784} = (7, 11, 13, 1)$
271 : $P_{3794} = (1, 12, 13, 1)$
272 : $P_{3809} = (0, 13, 13, 1)$
273 : $P_{3839} = (14, 14, 13, 1)$
274 : $P_{3841} = (0, 15, 13, 1)$
275 : $P_{3862} = (5, 0, 14, 1)$
276 : $P_{3886} = (13, 1, 14, 1)$
277 : $P_{3894} = (5, 2, 14, 1)$
278 : $P_{3906} = (1, 3, 14, 1)$
279 : $P_{3932} = (11, 4, 14, 1)$
280 : $P_{3946} = (9, 5, 14, 1)$
281 : $P_{3953} = (0, 6, 14, 1)$
282 : $P_{3985} = (0, 8, 14, 1)$
283 : $P_{4004} = (3, 9, 14, 1)$
284 : $P_{4023} = (6, 10, 14, 1)$
285 : $P_{4043} = (10, 11, 14, 1)$
286 : $P_{4050} = (1, 12, 14, 1)$
287 : $P_{4079} = (14, 13, 14, 1)$
288 : $P_{4081} = (0, 14, 14, 1)$
289 : $P_{4098} = (1, 15, 14, 1)$
290 : $P_{4121} = (8, 0, 15, 1)$
291 : $P_{4142} = (13, 1, 15, 1)$
292 : $P_{4145} = (0, 2, 15, 1)$
293 : $P_{4169} = (8, 3, 15, 1)$
294 : $P_{4179} = (2, 4, 15, 1)$
295 : $P_{4223} = (14, 6, 15, 1)$
296 : $P_{4226} = (1, 7, 15, 1)$
297 : $P_{4246} = (5, 8, 15, 1)$
298 : $P_{4258} = (1, 9, 15, 1)$
299 : $P_{4280} = (7, 10, 15, 1)$
300 : $P_{4300} = (11, 11, 15, 1)$
301 : $P_{4315} = (10, 12, 15, 1)$
302 : $P_{4321} = (0, 13, 15, 1)$
303 : $P_{4338} = (1, 14, 15, 1)$
304 : $P_{4353} = (0, 15, 15, 1)$