Rank-139 over GF(8)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 74911

General information

Number of lines	6
Number of points	89
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	42
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$3^2, 2^3, 1^{42}, 0^{42}$

Singular Points

The surface has 1 singular points:

$0:\ P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1, 0, 1, 0, 0, 0)_3$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \mathbf{Pl}(1,0,0,0,1,0)_{82}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0,0,1,0,1,0)_{96}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \mathbf{Pl}(1,1,0,0,1,1)_{1217}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \mathbf{Pl}(1,1,1,1,0,0)_{32}$$

Rank of lines: (1, 8, 65, 138, 81, 585)
Rank of points on Klein quadric: (3, 82, 96, 1322, 1217, 32)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0),$
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1).$

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{19} = (0,1,1,0) = \ell_0 \cap \ell_5$ $P_{138} = (0,0,1,1) = \ell_2 \cap \ell_3$
$P_{82} = (0,1,0,1) = \ell_1 \cap \ell_4$

Single Points

The surface has 42 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_3 $16 : P_{89} = (7,1,0,1)$ lies on line ℓ_1
$1 : P_{12} = (1,0,1,0)$ lies on line ℓ_4 $17 : P_{139} = (1,0,1,1)$ lies on line ℓ_2
$2 : P_{20} = (1,1,1,0)$ lies on line ℓ_0 $18 : P_{140} = (2,0,1,1)$ lies on line ℓ_2
$3 : P_{21} = (2,1,1,0)$ lies on line ℓ_0 $19 : P_{141} = (3,0,1,1)$ lies on line ℓ_2
$4 : P_{22} = (3,1,1,0)$ lies on line ℓ_0 $20 : P_{142} = (4,0,1,1)$ lies on line ℓ_2
$5 : P_{23} = (4,1,1,0)$ lies on line ℓ_0 $21 : P_{143} = (5,0,1,1)$ lies on line ℓ_2
$6 : P_{24} = (5,1,1,0)$ lies on line ℓ_0 $22 : P_{144} = (6,0,1,1)$ lies on line ℓ_2
$7 : P_{25} = (6,1,1,0)$ lies on line ℓ_0 $23 : P_{145} = (7,0,1,1)$ lies on line ℓ_2
$8 : P_{26} = (7,1,1,0)$ lies on line ℓ_0 $24 : P_{155} = (2,2,1,1)$ lies on line ℓ_3
$9 : P_{75} = (1,0,0,1)$ lies on line ℓ_5 $25 : P_{164} = (3,3,1,1)$ lies on line ℓ_3
$10 : P_{83} = (1,1,0,1)$ lies on line ℓ_1 $26 : P_{173} = (4,4,1,1)$ lies on line ℓ_3
$11 : P_{84} = (2,1,0,1)$ lies on line ℓ_1 $27 : P_{182} = (5,5,1,1)$ lies on line ℓ_3
$12 : P_{85} = (3,1,0,1)$ lies on line ℓ_1 $28 : P_{191} = (6,6,1,1)$ lies on line ℓ_3
$13 : P_{86} = (4,1,0,1)$ lies on line ℓ_1 $29 : P_{200} = (7,7,1,1)$ lies on line ℓ_3
$14 : P_{87} = (5,1,0,1)$ lies on line ℓ_1 $30 : P_{211} = (2,1,2,1)$ lies on line ℓ_4
$15 : P_{88} = (6,1,0,1)$ lies on line ℓ_1 $31 : P_{218} = (1,2,2,1)$ lies on line ℓ_5

$32 : P_{276} = (3,1,3,1)$ lies on line ℓ_4
$33 : P_{290} = (1,3,3,1)$ lies on line ℓ_5
$34 : P_{341} = (4,1,4,1)$ lies on line ℓ_4
$35 : P_{362} = (1,4,4,1)$ lies on line ℓ_5
$36 : P_{406} = (5,1,5,1)$ lies on line ℓ_4
$37 : P_{434} = (1,5,5,1)$ lies on line ℓ_5

$38 : P_{471} = (6,1,6,1)$ lies on line ℓ_4
$39 : P_{506} = (1,6,6,1)$ lies on line ℓ_5
$40 : P_{536} = (7,1,7,1)$ lies on line ℓ_4
$41 : P_{578} = (1,7,7,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:
The points on the surface but not on lines are:

$0 : P_{31} = (4,2,1,0)$
$1 : P_{39} = (4,3,1,0)$
$2 : P_{50} = (7,4,1,0)$
$3 : P_{58} = (7,5,1,0)$
$4 : P_{61} = (2,6,1,0)$
$5 : P_{69} = (2,7,1,0)$
$6 : P_{94} = (4,2,0,1)$
$7 : P_{102} = (4,3,0,1)$
$8 : P_{113} = (7,4,0,1)$
$9 : P_{121} = (7,5,0,1)$
$10 : P_{124} = (2,6,0,1)$
$11 : P_{132} = (2,7,0,1)$
$12 : P_{205} = (4,0,2,1)$
$13 : P_{237} = (4,4,2,1)$
$14 : P_{241} = (0,5,2,1)$
$15 : P_{251} = (2,6,2,1)$
$16 : P_{259} = (2,7,2,1)$
$17 : P_{269} = (4,0,3,1)$
$18 : P_{301} = (4,4,3,1)$
$19 : P_{306} = (1,5,3,1)$
$20 : P_{314} = (1,6,3,1)$
$21 : P_{321} = (0,7,3,1)$

$22 : P_{336} = (7,0,4,1)$
$23 : P_{349} = (4,2,4,1)$
$24 : P_{357} = (4,3,4,1)$
$25 : P_{377} = (0,6,4,1)$
$26 : P_{392} = (7,7,4,1)$
$27 : P_{400} = (7,0,5,1)$
$28 : P_{409} = (0,2,5,1)$
$29 : P_{418} = (1,3,5,1)$
$30 : P_{442} = (1,6,5,1)$
$31 : P_{456} = (7,7,5,1)$
$32 : P_{459} = (2,0,6,1)$
$33 : P_{475} = (2,2,6,1)$
$34 : P_{482} = (1,3,6,1)$
$35 : P_{489} = (0,4,6,1)$
$36 : P_{498} = (1,5,6,1)$
$37 : P_{523} = (2,0,7,1)$
$38 : P_{539} = (2,2,7,1)$
$39 : P_{545} = (0,3,7,1)$
$40 : P_{560} = (7,4,7,1)$
$41 : P_{568} = (7,5,7,1)$

Line Intersection Graph

```
 |0 1 2 3 4 5
--+-----------
0|0 1 1 0 0 1
1|1 0 1 0 1 0
2|1 1 0 1 0 0
3|0 0 1 0 1 1
4|0 1 0 1 0 1
5|1 0 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_0	P_0	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_0	P_{82}

3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_0	P_0	P_{138}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5
in point	P_{138}	P_4	P_4

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{82}	P_4	P_4

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_{19}	P_4	P_4

The surface has 89 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
$1 : P_4 = (1, 1, 1, 1)$
$2 : P_5 = (1, 1, 0, 0)$
$3 : P_{12} = (1, 0, 1, 0)$
$4 : P_{19} = (0, 1, 1, 0)$
$5 : P_{20} = (1, 1, 1, 0)$
$6 : P_{21} = (2, 1, 1, 0)$
$7 : P_{22} = (3, 1, 1, 0)$
$8 : P_{23} = (4, 1, 1, 0)$
$9 : P_{24} = (5, 1, 1, 0)$
$10 : P_{25} = (6, 1, 1, 0)$
$11 : P_{26} = (7, 1, 1, 0)$
$12 : P_{31} = (4, 2, 1, 0)$
$13 : P_{39} = (4, 3, 1, 0)$
$14 : P_{50} = (7, 4, 1, 0)$
$15 : P_{58} = (7, 5, 1, 0)$
$16 : P_{61} = (2, 6, 1, 0)$
$17 : P_{69} = (2, 7, 1, 0)$
$18 : P_{75} = (1, 0, 0, 1)$
$19 : P_{82} = (0, 1, 0, 1)$
$20 : P_{83} = (1, 1, 0, 1)$
$21 : P_{84} = (2, 1, 0, 1)$
$22 : P_{85} = (3, 1, 0, 1)$
$23 : P_{86} = (4, 1, 0, 1)$
$24 : P_{87} = (5, 1, 0, 1)$
$25 : P_{88} = (6, 1, 0, 1)$
$26 : P_{89} = (7, 1, 0, 1)$
$27 : P_{94} = (4, 2, 0, 1)$
$28 : P_{102} = (4, 3, 0, 1)$
$29 : P_{113} = (7, 4, 0, 1)$

$30 : P_{121} = (7, 5, 0, 1)$
$31 : P_{124} = (2, 6, 0, 1)$
$32 : P_{132} = (2, 7, 0, 1)$
$33 : P_{138} = (0, 0, 1, 1)$
$34 : P_{139} = (1, 0, 1, 1)$
$35 : P_{140} = (2, 0, 1, 1)$
$36 : P_{141} = (3, 0, 1, 1)$
$37 : P_{142} = (4, 0, 1, 1)$
$38 : P_{143} = (5, 0, 1, 1)$
$39 : P_{144} = (6, 0, 1, 1)$
$40 : P_{145} = (7, 0, 1, 1)$
$41 : P_{155} = (2, 2, 1, 1)$
$42 : P_{164} = (3, 3, 1, 1)$
$43 : P_{173} = (4, 4, 1, 1)$
$44 : P_{182} = (5, 5, 1, 1)$
$45 : P_{191} = (6, 6, 1, 1)$
$46 : P_{200} = (7, 7, 1, 1)$
$47 : P_{205} = (4, 0, 2, 1)$
$48 : P_{211} = (2, 1, 2, 1)$
$49 : P_{218} = (1, 2, 2, 1)$
$50 : P_{237} = (4, 4, 2, 1)$
$51 : P_{241} = (0, 5, 2, 1)$
$52 : P_{251} = (2, 6, 2, 1)$
$53 : P_{259} = (2, 7, 2, 1)$
$54 : P_{269} = (4, 0, 3, 1)$
$55 : P_{276} = (3, 1, 3, 1)$
$56 : P_{290} = (1, 3, 3, 1)$
$57 : P_{301} = (4, 4, 3, 1)$
$58 : P_{306} = (1, 5, 3, 1)$
$59 : P_{314} = (1, 6, 3, 1)$

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$62 : P_{341} = (4, 1, 4, 1)$
$63 : P_{349} = (4, 2, 4, 1)$
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$77 : P_{475} = (2, 2, 6, 1)$
$78 : P_{482} = (1, 3, 6, 1)$
$79 : P_{489} = (0, 4, 6, 1)$
$80 : P_{498} = (1, 5, 6, 1)$
$81 : P_{506} = (1, 6, 6, 1)$
$82 : P_{523} = (2, 0, 7, 1)$
$83 : P_{536} = (7, 1, 7, 1)$
$84 : P_{539} = (2, 2, 7, 1)$
$85 : P_{545} = (0, 3, 7, 1)$
$86 : P_{560} = (7, 4, 7, 1)$
$87 : P_{568} = (7, 5, 7, 1)$
$88 : P_{578} = (1, 7, 7, 1)$