Rank-74276 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361384014

General information

Number of lines	2
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	2, 1^{16}, 0^{56}

Singular Points

The surface has 1 singular points:

$0 : \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (4672, 4744)
Rank of points on Klein quadric: (649, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
2 : $P_{19} = (0,1,1,0)$ lies on line ℓ_0
3 : $P_{27} = (0,2,1,0)$ lies on line ℓ_0
4 : $P_{35} = (0,3,1,0)$ lies on line ℓ_0
5 : $P_{43} = (0,4,1,0)$ lies on line ℓ_0
6 : $P_{51} = (0,5,1,0)$ lies on line ℓ_0
7 : $P_{59} = (0,6,1,0)$ lies on line ℓ_0
8 : $P_{67} = (0,7,1,0)$ lies on line ℓ_0
9 : $P_{138} = (0,0,1,1)$ lies on line ℓ_1
10 : $P_{201} = (0,0,2,1)$ lies on line ℓ_1
11 : $P_{265} = (0,0,3,1)$ lies on line ℓ_1
12 : $P_{329} = (0,0,4,1)$ lies on line ℓ_1
13 : $P_{393} = (0,0,5,1)$ lies on line ℓ_1
14 : $P_{457} = (0,0,6,1)$ lies on line ℓ_1
15 : $P_{521} = (0,0,7,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1,0,1,0)$
1 : $P_{39} = (4,3,1,0)$
2 : $P_{40} = (5,3,1,0)$
3 : $P_{57} = (6,5,1,0)$
4 : $P_{58} = (7,5,1,0)$
5 : $P_{61} = (2,6,1,0)$
6 : $P_{62} = (3,6,1,0)$
7 : $P_{75} = (1,0,0,1)$
8 : $P_{85} = (3,1,0,1)$
9 : $P_{87} = (5,1,0,1)$
10 : $P_{88} = (6,1,0,1)$
11 : $P_{97} = (7,2,0,1)$
12 : $P_{108} = (2,4,0,1)$
13 : $P_{134} = (4,7,0,1)$
14 : $P_{147} = (2,1,1,1)$
15 : $P_{149} = (4,1,1,1)$
16 : $P_{152} = (7,1,1,1)$
17 : $P_{163} = (2,3,1,1)$
18 : $P_{166} = (5,3,1,1)$
19 : $P_{167} = (6,3,1,1)$
20 : $P_{180} = (3,5,1,1)$
21 : $P_{181} = (4,5,1,1)$

22 : $P_{183} = (6, 5, 1, 1)$
23 : $P_{188} = (3, 6, 1, 1)$
24 : $P_{190} = (5, 6, 1, 1)$
25 : $P_{192} = (7, 6, 1, 1)$
26 : $P_{206} = (5, 0, 2, 1)$
27 : $P_{208} = (7, 0, 2, 1)$
28 : $P_{232} = (7, 3, 2, 1)$
29 : $P_{234} = (1, 4, 2, 1)$
30 : $P_{250} = (1, 6, 2, 1)$
31 : $P_{262} = (5, 7, 2, 1)$
32 : $P_{284} = (3, 2, 3, 1)$
33 : $P_{285} = (4, 2, 3, 1)$
34 : $P_{293} = (4, 3, 3, 1)$
35 : $P_{324} = (3, 7, 3, 1)$
36 : $P_{331} = (2, 0, 4, 1)$
37 : $P_{335} = (6, 0, 4, 1)$
38 : $P_{351} = (6, 2, 4, 1)$
39 : $P_{354} = (1, 3, 4, 1)$
40 : $P_{371} = (2, 5, 4, 1)$
41 : $P_{386} = (1, 7, 4, 1)$
42 : $P_{414} = (5, 2, 5, 1)$
43 : $P_{430} = (5, 4, 5, 1)$
44 : $P_{432} = (7, 4, 5, 1)$
45 : $P_{440} = (7, 5, 5, 1)$
46 : $P_{495} = (6, 4, 6, 1)$
47 : $P_{507} = (2, 6, 6, 1)$
48 : $P_{515} = (2, 7, 6, 1)$
49 : $P_{519} = (6, 7, 6, 1)$
50 : $P_{524} = (3, 0, 7, 1)$
51 : $P_{525} = (4, 0, 7, 1)$
52 : $P_{538} = (1, 2, 7, 1)$
53 : $P_{556} = (3, 4, 7, 1)$
54 : $P_{562} = (1, 5, 7, 1)$
55 : $P_{573} = (4, 6, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 73 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_{12} = (1, 0, 1, 0)$
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