Rank-74099 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 304222485

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	141
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	5, 3^2, 2^9, 1^{141}, 0^{168}

Singular Points

The surface has 1 singular points:

$0 :\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69904} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^5 & 0 & \delta^7 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{33835} = \begin{bmatrix} 1 & 11 & 0 & 7 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{33835} = \mathbf{Pl}(0,4,10,0,0,1)_{4954}$$

$$\ell_3 = \begin{bmatrix} 1 & \delta^5 & 0 & \delta^{13} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{29467} = \begin{bmatrix} 1 & 11 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{29467} = \mathbf{Pl}(0,14,10,0,0,1)_{4964}$$

$$\ell_4 = \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^{11} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{59770} = \begin{bmatrix} 1 & 10 & 0 & 13 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{59770} = \mathbf{Pl}(0,2,11,0,0,1)_{4983}$$

$$\ell_5 = \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^{14} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{55402} = \begin{bmatrix} 1 & 10 & 0 & 12 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{55402} = \mathbf{Pl}(0,9,11,0,0,1)_{4990}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^5 & \delta^{13} \\ 0 & 1 & 0 & \delta^{10} \end{bmatrix}_{29371} = \begin{bmatrix} 1 & 0 & 11 & 6 \\ 0 & 1 & 0 & 10 \end{bmatrix}_{29371} = \mathbf{Pl}(11,10,0,14,11,1)_{50206}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^5 & \delta^7 \\ 0 & 1 & 0 & \delta^{10} \end{bmatrix}_{33739} = \begin{bmatrix} 1 & 0 & 11 & 7 \\ 0 & 1 & 0 & 10 \end{bmatrix}_{33739} = \mathbf{Pl}(11,10,0,4,11,1)_{50056}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{14} \\ 0 & 1 & 0 & \delta^5 \end{bmatrix}_{55322} = \begin{bmatrix} 1 & 0 & 10 & 12 \\ 0 & 1 & 0 & 11 \end{bmatrix}_{55322} = \mathbf{Pl}(10,11,0,9,10,1)_{46050}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{11} \\ 0 & 1 & 0 & \delta^5 \end{bmatrix}_{59690} = \begin{bmatrix} 1 & 0 & 10 & 13 \\ 0 & 1 & 0 & 11 \end{bmatrix}_{59690} = \mathbf{Pl}(10,11,0,2,10,1)_{45945}$$

Rank of lines: (69904, 70160, 33835, 29467, 59770, 55402, 29371, 33739, 55322, 59690)
Rank of points on Klein quadric: (33, 1, 4954, 4964, 4983, 4990, 50206, 50056, 46050, 45945)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{434} = \mathbf{P}(0, \delta^{10}, 0, 1) = \mathbf{P}(0, 10, 0, 1)$,
$1 : P_{450} = \mathbf{P}(0, \delta^5, 0, 1) = \mathbf{P}(0, 11, 0, 1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$
$P_{1919} = (14,6,6,1) = \ell_2 \cap \ell_6$
$P_{2431} = (14,6,8,1) = \ell_2 \cap \ell_9$
$P_{2181} = (4,7,7,1) = \ell_3 \cap \ell_7$
$P_{1157} = (4,7,3,1) = \ell_3 \cap \ell_8$
$P_{1754} = (9,12,5,1) = \ell_4 \cap \ell_6$
$P_{3546} = (9,12,12,1) = \ell_4 \cap \ell_8$
$P_{4323} = (2,13,15,1) = \ell_5 \cap \ell_7$
$P_{3811} = (2,13,13,1) = \ell_5 \cap \ell_9$

Single Points

The surface has 141 single points:
The single points on the surface are:

0 : $P_{1} = (0, 1, 0, 0)$ lies on line ℓ_{0}
1 : $P_{125} = (10, 6, 1, 0)$ lies on line ℓ_{6}
2 : $P_{141} = (10, 7, 1, 0)$ lies on line ℓ_{7}
3 : $P_{222} = (11, 12, 1, 0)$ lies on line ℓ_{8}
4 : $P_{238} = (11, 13, 1, 0)$ lies on line ℓ_{9}
5 : $P_{290} = (0, 1, 0, 1)$ lies on line ℓ_{0}
6 : $P_{306} = (0, 2, 0, 1)$ lies on line ℓ_{0}
7 : $P_{322} = (0, 3, 0, 1)$ lies on line ℓ_{0}
8 : $P_{338} = (0, 4, 0, 1)$ lies on line ℓ_{0}
9 : $P_{354} = (0, 5, 0, 1)$ lies on line ℓ_{0}
10 : $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_{0}
11 : $P_{384} = (14, 6, 0, 1)$ lies on line ℓ_{2}
12 : $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_{0}
13 : $P_{390} = (4, 7, 0, 1)$ lies on line ℓ_{3}
14 : $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_{0}
15 : $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_{0}
16 : $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_{0}
17 : $P_{475} = (9, 12, 0, 1)$ lies on line ℓ_{4}
18 : $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_{0}
19 : $P_{484} = (2, 13, 0, 1)$ lies on line ℓ_{5}
20 : $P_{498} = (0, 14, 0, 1)$ lies on line ℓ_{0}
21 : $P_{514} = (0, 15, 0, 1)$ lies on line ℓ_{0}
22 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_{1}
23 : $P_{636} = (11, 6, 1, 1)$ lies on line ℓ_{8}
24 : $P_{639} = (14, 6, 1, 1)$ lies on line ℓ_{2}
25 : $P_{645} = (4, 7, 1, 1)$ lies on line ℓ_{3}
26 : $P_{652} = (11, 7, 1, 1)$ lies on line ℓ_{9}
27 : $P_{730} = (9, 12, 1, 1)$ lies on line ℓ_{4}
28 : $P_{731} = (10, 12, 1, 1)$ lies on line ℓ_{7}
29 : $P_{739} = (2, 13, 1, 1)$ lies on line ℓ_{5}
30 : $P_{747} = (10, 13, 1, 1)$ lies on line ℓ_{6}
31 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_{1}
32 : $P_{878} = (13, 5, 2, 1)$ lies on line ℓ_{7}
33 : $P_{895} = (14, 6, 2, 1)$ lies on line ℓ_{2}
34 : $P_{901} = (4, 7, 2, 1)$ lies on line ℓ_{3}
35 : $P_{910} = (13, 7, 2, 1)$ lies on line ℓ_{6}
36 : $P_{944} = (15, 9, 2, 1)$ lies on line ℓ_{9}
37 : $P_{976} = (15, 11, 2, 1)$ lies on line ℓ_{8}
38 : $P_{986} = (9, 12, 2, 1)$ lies on line ℓ_{4}
39 : $P_{995} = (2, 13, 2, 1)$ lies on line ℓ_{5}
40 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_{1}
41 : $P_{1064} = (7, 1, 3, 1)$ lies on line ℓ_{6}
42 : $P_{1080} = (7, 2, 3, 1)$ lies on line ℓ_{7}
43 : $P_{1109} = (4, 4, 3, 1)$ lies on line ℓ_{9}
44 : $P_{1151} = (14, 6, 3, 1)$ lies on line ℓ_{2}
45 : $P_{1242} = (9, 12, 3, 1)$ lies on line ℓ_{4}
46 : $P_{1251} = (2, 13, 3, 1)$ lies on line ℓ_{5}
47 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_{1}
48 : $P_{1407} = (14, 6, 4, 1)$ lies on line ℓ_{2}
49 : $P_{1413} = (4, 7, 4, 1)$ lies on line ℓ_{3}
50 : $P_{1432} = (7, 8, 4, 1)$ lies on line ℓ_{8}
51 : $P_{1460} = (3, 10, 4, 1)$ lies on line ℓ_{6}
52 : $P_{1496} = (7, 12, 4, 1)$ lies on line ℓ_{9}
53 : $P_{1498} = (9, 12, 4, 1)$ lies on line ℓ_{4}
54 : $P_{1507} = (2, 13, 4, 1)$ lies on line ℓ_{5}
55 : $P_{1524} = (3, 14, 4, 1)$ lies on line ℓ_{7}
56 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_{1}
57 : $P_{1581} = (12, 1, 5, 1)$ lies on line ℓ_{9}
58 : $P_{1629} = (12, 4, 5, 1)$ lies on line ℓ_{8}
59 : $P_{1663} = (14, 6, 5, 1)$ lies on line ℓ_{2}
60 : $P_{1669} = (4, 7, 5, 1)$ lies on line ℓ_{3}
61 : $P_{1706} = (9, 9, 5, 1)$ lies on line ℓ_{7}
62 : $P_{1763} = (2, 13, 5, 1)$ lies on line ℓ_{5}
63 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_{1}
64 : $P_{1823} = (14, 0, 6, 1)$ lies on line ℓ_{7}
65 : $P_{1925} = (4, 7, 6, 1)$ lies on line ℓ_{3}
66 : $P_{1961} = (8, 9, 6, 1)$ lies on line ℓ_{8}
67 : $P_{2010} = (9, 12, 6, 1)$ lies on line ℓ_{4}
68 : $P_{2019} = (2, 13, 6, 1)$ lies on line ℓ_{5}
69 : $P_{2057} = (8, 15, 6, 1)$ lies on line ℓ_{9}
70 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_{1}
71 : $P_{2069} = (4, 0, 7, 1)$ lies on line ℓ_{6}
72 : $P_{2100} = (3, 2, 7, 1)$ lies on line ℓ_{9}
73 : $P_{2148} = (3, 5, 7, 1)$ lies on line ℓ_{8}
74 : $P_{2175} = (14, 6, 7, 1)$ lies on line ℓ_{2}
75 : $P_{2266} = (9, 12, 7, 1)$ lies on line ℓ_{4}
76 : $P_{2275} = (2, 13, 7, 1)$ lies on line ℓ_{5}
77 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_{1}
78 : $P_{2343} = (6, 1, 8, 1)$ lies on line ℓ_{7}
79 : $P_{2437} = (4, 7, 8, 1)$ lies on line ℓ_{3}
80 : $P_{2471} = (6, 9, 8, 1)$ lies on line ℓ_{6}
81 : $P_{2522} = (9, 12, 8, 1)$ lies on line ℓ_{4}
82 : $P_{2531} = (2, 13, 8, 1)$ lies on line ℓ_{5}
83 : $P_{2559} = (14, 14, 8, 1)$ lies on line ℓ_{8}
84 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_{1}
85 : $P_{2614} = (5, 2, 9, 1)$ lies on line ℓ_{8}
86 : $P_{2685} = (12, 6, 9, 1)$ lies on line ℓ_{7}
87 : $P_{2687} = (14, 6, 9, 1)$ lies on line ℓ_{2}
88 : $P_{2693} = (4, 7, 9, 1)$ lies on line ℓ_{3}
89 : $P_{2758} = (5, 11, 9, 1)$ lies on line ℓ_{9}
90 : $P_{2778} = (9, 12, 9, 1)$ lies on line ℓ_{4}
91 : $P_{2787} = (2, 13, 9, 1)$ lies on line ℓ_{5}
92 : $P_{2829} = (12, 15, 9, 1)$ lies on line ℓ_{6}
93 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_{1}
94 : $P_{2914} = (1, 5, 10, 1)$ lies on line ℓ_{9}
95 : $P_{2924} = (11, 5, 10, 1)$ lies on line ℓ_{6}
96 : $P_{2943} = (14, 6, 10, 1)$ lies on line ℓ_{2}
97 : $P_{2949} = (4, 7, 10, 1)$ lies on line ℓ_{3}
98 : $P_{3034} = (9, 12, 10, 1)$ lies on line ℓ_{4}
99 : $P_{3043} = (2, 13, 10, 1)$ lies on line ℓ_{5}
100 : $P_{3074} = (1, 15, 10, 1)$ lies on line ℓ_{8}
101 : $P_{3084} = (11, 15, 10, 1)$ lies on line ℓ_{7}
102 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_{1}
103 : $P_{3138} = (1, 3, 11, 1)$ lies on line ℓ_{6}
104 : $P_{3147} = (10, 3, 11, 1)$ lies on line ℓ_{8}
105 : $P_{3199} = (14, 6, 11, 1)$ lies on line ℓ_{2}
106 : $P_{3205} = (4, 7, 11, 1)$ lies on line ℓ_{3}
107 : $P_{3218} = (1, 8, 11, 1)$ lies on line ℓ_{7}

108 : $P_{3227} = (10, 8, 11, 1)$ lies on line ℓ_9
109 : $P_{3290} = (9, 12, 11, 1)$ lies on line ℓ_4
110 : $P_{3299} = (2, 13, 11, 1)$ lies on line ℓ_5
111 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_1
112 : $P_{3354} = (9, 0, 12, 1)$ lies on line ℓ_9
113 : $P_{3414} = (5, 4, 12, 1)$ lies on line ℓ_7
114 : $P_{3455} = (14, 6, 12, 1)$ lies on line ℓ_2
115 : $P_{3461} = (4, 7, 12, 1)$ lies on line ℓ_3
116 : $P_{3478} = (5, 8, 12, 1)$ lies on line ℓ_6
117 : $P_{3555} = (2, 13, 12, 1)$ lies on line ℓ_5
118 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_1
119 : $P_{3603} = (2, 0, 13, 1)$ lies on line ℓ_8
120 : $P_{3664} = (15, 3, 13, 1)$ lies on line ℓ_7
121 : $P_{3711} = (14, 6, 13, 1)$ lies on line ℓ_2
122 : $P_{3717} = (4, 7, 13, 1)$ lies on line ℓ_3
123 : $P_{3802} = (9, 12, 13, 1)$ lies on line ℓ_4
124 : $P_{3840} = (15, 14, 13, 1)$ lies on line ℓ_6
125 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_1
126 : $P_{3911} = (6, 3, 14, 1)$ lies on line ℓ_9
127 : $P_{3929} = (8, 4, 14, 1)$ lies on line ℓ_6
128 : $P_{3967} = (14, 6, 14, 1)$ lies on line ℓ_2
129 : $P_{3973} = (4, 7, 14, 1)$ lies on line ℓ_3
130 : $P_{4025} = (8, 10, 14, 1)$ lies on line ℓ_7
131 : $P_{4058} = (9, 12, 14, 1)$ lies on line ℓ_4
132 : $P_{4067} = (2, 13, 14, 1)$ lies on line ℓ_5
133 : $P_{4071} = (6, 13, 14, 1)$ lies on line ℓ_8
134 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_1
135 : $P_{4142} = (13, 1, 15, 1)$ lies on line ℓ_8
136 : $P_{4147} = (2, 2, 15, 1)$ lies on line ℓ_6
137 : $P_{4223} = (14, 6, 15, 1)$ lies on line ℓ_2
138 : $P_{4229} = (4, 7, 15, 1)$ lies on line ℓ_3
139 : $P_{4314} = (9, 12, 15, 1)$ lies on line ℓ_4
140 : $P_{4350} = (13, 14, 15, 1)$ lies on line ℓ_9

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_{36} = (1, 1, 1, 0)$
3 : $P_{120} = (5, 6, 1, 0)$
4 : $P_{146} = (15, 7, 1, 0)$
5 : $P_{182} = (3, 10, 1, 0)$
6 : $P_{187} = (8, 10, 1, 0)$
7 : $P_{200} = (5, 11, 1, 0)$
8 : $P_{210} = (15, 11, 1, 0)$
9 : $P_{214} = (3, 12, 1, 0)$
10 : $P_{235} = (8, 13, 1, 0)$
11 : $P_{275} = (1, 0, 0, 1)$
12 : $P_{314} = (8, 2, 0, 1)$
13 : $P_{334} = (12, 3, 0, 1)$
14 : $P_{353} = (15, 4, 0, 1)$
15 : $P_{360} = (6, 5, 0, 1)$
16 : $P_{415} = (13, 8, 0, 1)$
17 : $P_{421} = (3, 9, 0, 1)$
18 : $P_{444} = (10, 10, 0, 1)$
19 : $P_{461} = (11, 11, 0, 1)$
20 : $P_{503} = (5, 14, 0, 1)$
21 : $P_{521} = (7, 15, 0, 1)$
22 : $P_{693} = (4, 10, 1, 1)$
23 : $P_{703} = (14, 10, 1, 1)$
24 : $P_{707} = (2, 11, 1, 1)$
25 : $P_{714} = (9, 11, 1, 1)$
26 : $P_{793} = (8, 0, 2, 1)$
27 : $P_{826} = (9, 2, 2, 1)$
28 : $P_{829} = (12, 2, 2, 1)$
29 : $P_{841} = (8, 3, 2, 1)$
30 : $P_{875} = (10, 5, 2, 1)$
31 : $P_{891} = (10, 6, 2, 1)$
32 : $P_{943} = (14, 9, 2, 1)$
33 : $P_{965} = (4, 11, 2, 1)$
34 : $P_{1005} = (12, 13, 2, 1)$
35 : $P_{1053} = (12, 0, 3, 1)$
36 : $P_{1071} = (14, 1, 3, 1)$
37 : $P_{1107} = (2, 4, 3, 1)$
38 : $P_{1143} = (6, 6, 3, 1)$
39 : $P_{1154} = (1, 7, 3, 1)$
40 : $P_{1178} = (9, 8, 3, 1)$
41 : $P_{1239} = (6, 12, 3, 1)$
42 : $P_{1252} = (3, 13, 3, 1)$
43 : $P_{1266} = (1, 14, 3, 1)$
44 : $P_{1277} = (12, 14, 3, 1)$
45 : $P_{1312} = (15, 0, 4, 1)$
46 : $P_{1367} = (6, 4, 4, 1)$
47 : $P_{1375} = (14, 4, 4, 1)$
48 : $P_{1392} = (15, 5, 4, 1)$
49 : $P_{1415} = (6, 7, 4, 1)$
50 : $P_{1436} = (11, 8, 4, 1)$
51 : $P_{1466} = (9, 10, 4, 1)$
52 : $P_{1516} = (11, 13, 4, 1)$
53 : $P_{1523} = (2, 14, 4, 1)$

$54: P_{1559} = (6, 0, 5, 1)$
$55: P_{1571} = (2, 1, 5, 1)$
$56: P_{1586} = (1, 2, 5, 1)$
$57: P_{1591} = (6, 2, 5, 1)$
$58: P_{1662} = (13, 6, 5, 1)$
$59: P_{1670} = (5, 7, 5, 1)$
$60: P_{1701} = (4, 9, 5, 1)$
$61: P_{1746} = (1, 12, 5, 1)$
$62: P_{1774} = (13, 13, 5, 1)$
$63: P_{1807} = (14, 15, 5, 1)$
$64: P_{1862} = (5, 3, 6, 1)$
$65: P_{1870} = (13, 3, 6, 1)$
$66: P_{1883} = (10, 4, 6, 1)$
$67: P_{1907} = (2, 6, 6, 1)$
$68: P_{1963} = (10, 9, 6, 1)$
$69: P_{1991} = (6, 11, 6, 1)$
$70: P_{1998} = (13, 11, 6, 1)$
$71: P_{2005} = (4, 12, 6, 1)$
$72: P_{2022} = (5, 13, 6, 1)$
$73: P_{2058} = (9, 15, 6, 1)$
$74: P_{2107} = (10, 2, 7, 1)$
$75: P_{2147} = (2, 5, 7, 1)$
$76: P_{2186} = (9, 7, 7, 1)$
$77: P_{2205} = (12, 8, 7, 1)$
$78: P_{2208} = (15, 8, 7, 1)$
$79: P_{2248} = (7, 11, 7, 1)$
$80: P_{2253} = (12, 11, 7, 1)$
$81: P_{2272} = (15, 12, 7, 1)$
$82: P_{2287} = (14, 13, 7, 1)$
$83: P_{2299} = (10, 14, 7, 1)$
$84: P_{2334} = (13, 0, 8, 1)$
$85: P_{2341} = (4, 1, 8, 1)$
$86: P_{2371} = (2, 3, 8, 1)$
$87: P_{2386} = (1, 4, 8, 1)$
$88: P_{2398} = (13, 4, 8, 1)$
$89: P_{2418} = (1, 6, 8, 1)$
$90: P_{2440} = (7, 7, 8, 1)$
$91: P_{2521} = (8, 12, 8, 1)$
$92: P_{2536} = (7, 13, 8, 1)$
$93: P_{2554} = (9, 14, 8, 1)$
$94: P_{2580} = (3, 0, 9, 1)$
$95: P_{2613} = (4, 2, 9, 1)$
$96: P_{2699} = (10, 7, 9, 1)$
$97: P_{2708} = (3, 8, 9, 1)$
$98: P_{2723} = (2, 9, 9, 1)$
$99: P_{2734} = (13, 9, 9, 1)$
$100: P_{2767} = (14, 11, 9, 1)$
$101: P_{2782} = (13, 12, 9, 1)$
$102: P_{2827} = (10, 15, 9, 1)$
$103: P_{2843} = (10, 0, 10, 1)$
$104: P_{2854} = (5, 1, 10, 1)$
$105: P_{2864} = (15, 1, 10, 1)$
$106: P_{2884} = (3, 3, 10, 1)$
$107: P_{2890} = (9, 3, 10, 1)$
$108: P_{2933} = (4, 6, 10, 1)$
$109: P_{2959} = (14, 7, 10, 1)$
$110: P_{2963} = (2, 8, 10, 1)$
$111: P_{2969} = (8, 8, 10, 1)$
$112: P_{2998} = (5, 10, 10, 1)$
$113: P_{3008} = (15, 10, 10, 1)$
$114: P_{3028} = (3, 12, 10, 1)$
$115: P_{3049} = (8, 13, 10, 1)$
$116: P_{3100} = (11, 0, 11, 1)$
$117: P_{3108} = (3, 1, 11, 1)$
$118: P_{3113} = (8, 1, 11, 1)$
$119: P_{3174} = (5, 5, 11, 1)$
$120: P_{3183} = (14, 5, 11, 1)$
$121: P_{3190} = (5, 6, 11, 1)$
$122: P_{3216} = (15, 7, 11, 1)$
$123: P_{3268} = (3, 11, 11, 1)$
$124: P_{3273} = (8, 11, 11, 1)$
$125: P_{3283} = (2, 12, 11, 1)$
$126: P_{3306} = (9, 13, 11, 1)$
$127: P_{3333} = (4, 15, 11, 1)$
$128: P_{3344} = (15, 15, 11, 1)$
$129: P_{3388} = (11, 2, 12, 1)$
$130: P_{3420} = (11, 4, 12, 1)$
$131: P_{3444} = (3, 6, 12, 1)$
$132: P_{3459} = (2, 7, 12, 1)$
$133: P_{3477} = (4, 8, 12, 1)$
$134: P_{3511} = (6, 10, 12, 1)$
$135: P_{3517} = (12, 10, 12, 1)$
$136: P_{3551} = (14, 12, 12, 1)$
$137: P_{3588} = (3, 15, 12, 1)$
$138: P_{3591} = (6, 15, 12, 1)$
$139: P_{3663} = (14, 3, 13, 1)$
$140: P_{3688} = (7, 5, 13, 1)$
$141: P_{3689} = (8, 5, 13, 1)$
$142: P_{3706} = (9, 6, 13, 1)$
$143: P_{3721} = (8, 7, 13, 1)$
$144: P_{3756} = (11, 9, 13, 1)$
$145: P_{3768} = (7, 10, 13, 1)$
$146: P_{3774} = (13, 10, 13, 1)$
$147: P_{3813} = (4, 13, 13, 1)$
$148: P_{3836} = (11, 14, 13, 1)$
$149: P_{3862} = (5, 0, 14, 1)$
$150: P_{3916} = (11, 3, 14, 1)$
$151: P_{3930} = (9, 4, 14, 1)$
$152: P_{3960} = (7, 6, 14, 1)$
$153: P_{4019} = (2, 10, 14, 1)$
$154: P_{4060} = (11, 12, 14, 1)$
$155: P_{4085} = (4, 14, 14, 1)$
$156: P_{4088} = (7, 14, 14, 1)$
$157: P_{4102} = (5, 15, 14, 1)$
$158: P_{4120} = (7, 0, 15, 1)$
$159: P_{4138} = (9, 1, 15, 1)$
$160: P_{4159} = (14, 2, 15, 1)$
$161: P_{4197} = (4, 5, 15, 1)$

162 : $P_{4224} = (15, 6, 15, 1)$
163 : $P_{4237} = (12, 7, 15, 1)$
164 : $P_{4258} = (1, 9, 15, 1)$
165 : $P_{4264} = (7, 9, 15, 1)$
166 : $P_{4317} = (12, 12, 15, 1)$
167 : $P_{4322} = (1, 13, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 0 0 0 0 1 1 1 1
1	1 0 1 1 1 1 0 0 0 0
2	0 1 0 1 1 1 1 0 0 1
3	0 1 1 0 1 1 0 1 1 0
4	0 1 1 1 0 1 1 0 1 0
5	0 1 1 1 1 0 0 1 0 1
6	1 0 1 0 1 0 0 1 0 0
7	1 0 0 1 0 1 1 0 0 0
8	1 0 0 1 1 0 0 0 0 1
9	1 0 1 0 0 1 0 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_{450}	P_{450}	P_{434}	P_{434}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_3	P_2	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{1919}	P_{2431}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{2181}	P_{1157}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{1754}	P_{3546}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{4323}	P_{3811}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7
in point	P_{450}	P_{1919}	P_{1754}	P_{450}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6
in point	P_{450}	P_{2181}	P_{4323}	P_{450}

Line 8 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_9
in point	P_{434}	P_{1157}	P_{3546}	P_{434}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_8
in point	P_{434}	P_{2431}	P_{3811}	P_{434}

The surface has 321 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_{36} = (1,1,1,0)$
$6: P_{120} = (5,6,1,0)$
$7: P_{125} = (10,6,1,0)$
$8: P_{141} = (10,7,1,0)$
$9: P_{146} = (15,7,1,0)$
$10: P_{182} = (3,10,1,0)$
$11: P_{187} = (8,10,1,0)$
$12: P_{200} = (5,11,1,0)$
$13: P_{210} = (15,11,1,0)$
$14: P_{214} = (3,12,1,0)$
$15: P_{222} = (11,12,1,0)$
$16: P_{235} = (8,13,1,0)$
$17: P_{238} = (11,13,1,0)$
$18: P_{275} = (1,0,0,1)$
$19: P_{290} = (0,1,0,1)$
$20: P_{306} = (0,2,0,1)$
$21: P_{314} = (8,2,0,1)$
$22: P_{322} = (0,3,0,1)$
$23: P_{334} = (12,3,0,1)$
$24: P_{338} = (0,4,0,1)$
$25: P_{353} = (15,4,0,1)$
$26: P_{354} = (0,5,0,1)$
$27: P_{360} = (6,5,0,1)$
$28: P_{370} = (0,6,0,1)$
$29: P_{384} = (14,6,0,1)$
$30: P_{386} = (0,7,0,1)$
$31: P_{390} = (4,7,0,1)$
$32: P_{402} = (0,8,0,1)$
$33: P_{415} = (13,8,0,1)$
$34: P_{418} = (0,9,0,1)$
$35: P_{421} = (3,9,0,1)$
$36: P_{434} = (0,10,0,1)$
$37: P_{444} = (10,10,0,1)$
$38: P_{450} = (0,11,0,1)$
$39: P_{461} = (11,11,0,1)$
$40: P_{466} = (0,12,0,1)$
$41: P_{475} = (9,12,0,1)$
$42: P_{482} = (0,13,0,1)$
$43: P_{484} = (2,13,0,1)$
$44: P_{498} = (0,14,0,1)$
$45: P_{503} = (5,14,0,1)$
$46: P_{514} = (0,15,0,1)$
$47: P_{521} = (7,15,0,1)$
$48: P_{530} = (0,0,1,1)$
$49: P_{636} = (11,6,1,1)$
$50: P_{639} = (14,6,1,1)$
$51: P_{645} = (4,7,1,1)$
$52: P_{652} = (11,7,1,1)$
$53: P_{693} = (4,10,1,1)$
$54: P_{703} = (14,10,1,1)$
$55: P_{707} = (2,11,1,1)$
$56: P_{714} = (9,11,1,1)$
$57: P_{730} = (9,12,1,1)$
$58: P_{731} = (10,12,1,1)$
$59: P_{739} = (2,13,1,1)$
$60: P_{747} = (10,13,1,1)$
$61: P_{785} = (0,0,2,1)$
$62: P_{793} = (8,0,2,1)$
$63: P_{826} = (9,2,2,1)$
$64: P_{829} = (12,2,2,1)$
$65: P_{841} = (8,3,2,1)$
$66: P_{875} = (10,5,2,1)$
$67: P_{878} = (13,5,2,1)$
$68: P_{891} = (10,6,2,1)$
$69: P_{895} = (14,6,2,1)$
$70: P_{901} = (4,7,2,1)$
$71: P_{910} = (13,7,2,1)$
$72: P_{943} = (14,9,2,1)$
$73: P_{944} = (15,9,2,1)$
$74: P_{965} = (4,11,2,1)$
$75: P_{976} = (15,11,2,1)$
$76: P_{986} = (9,12,2,1)$
$77: P_{995} = (2,13,2,1)$
$78: P_{1005} = (12,13,2,1)$
$79: P_{1041} = (0,0,3,1)$
$80: P_{1053} = (12,0,3,1)$
$81: P_{1064} = (7,1,3,1)$
$82: P_{1071} = (14,1,3,1)$
$83: P_{1080} = (7,2,3,1)$
$84: P_{1107} = (2,4,3,1)$
$85: P_{1109} = (4,4,3,1)$
$86: P_{1143} = (6,6,3,1)$
$87: P_{1151} = (14,6,3,1)$
$88: P_{1154} = (1,7,3,1)$
$89: P_{1157} = (4,7,3,1)$
$90: P_{1178} = (9,8,3,1)$
$91: P_{1239} = (6,12,3,1)$
$92: P_{1242} = (9,12,3,1)$
$93: P_{1251} = (2,13,3,1)$
$94: P_{1252} = (3,13,3,1)$
$95: P_{1266} = (1,14,3,1)$
$96: P_{1277} = (12,14,3,1)$
$97: P_{1297} = (0,0,4,1)$
$98: P_{1312} = (15,0,4,1)$
$99: P_{1367} = (6,4,4,1)$
$100: P_{1375} = (14,4,4,1)$
$101: P_{1392} = (15,5,4,1)$
$102: P_{1407} = (14,6,4,1)$
$103: P_{1413} = (4,7,4,1)$
$104: P_{1415} = (6,7,4,1)$
$105: P_{1432} = (7,8,4,1)$
$106: P_{1436} = (11,8,4,1)$
$107: P_{1460} = (3,10,4,1)$
$108: P_{1466} = (9,10,4,1)$
$109: P_{1496} = (7,12,4,1)$
$110: P_{1498} = (9,12,4,1)$
$111: P_{1507} = (2,13,4,1)$
$112: P_{1516} = (11,13,4,1)$
$113: P_{1523} = (2,14,4,1)$
$114: P_{1524} = (3,14,4,1)$
$115: P_{1553} = (0,0,5,1)$
$116: P_{1559} = (6,0,5,1)$
$117: P_{1571} = (2,1,5,1)$
$118: P_{1581} = (12,1,5,1)$
$119: P_{1586} = (1,2,5,1)$
$120: P_{1591} = (6,2,5,1)$
$121: P_{1629} = (12,4,5,1)$
$122: P_{1662} = (13,6,5,1)$
$123: P_{1663} = (14,6,5,1)$
$124: P_{1669} = (4,7,5,1)$
$125: P_{1670} = (5,7,5,1)$
$126: P_{1701} = (4,9,5,1)$
$127: P_{1706} = (9,9,5,1)$
$128: P_{1746} = (1,12,5,1)$
$129: P_{1754} = (9,12,5,1)$
$130: P_{1763} = (2,13,5,1)$
$131: P_{1774} = (13,13,5,1)$
$132: P_{1807} = (14,15,5,1)$
$133: P_{1809} = (0,0,6,1)$
$134: P_{1823} = (14,0,6,1)$
$135: P_{1862} = (5,3,6,1)$
$136: P_{1870} = (13,3,6,1)$
$137: P_{1883} = (10,4,6,1)$
$138: P_{1907} = (2,6,6,1)$
$139: P_{1919} = (14,6,6,1)$
$140: P_{1925} = (4,7,6,1)$
$141: P_{1961} = (8,9,6,1)$
$142: P_{1963} = (10,9,6,1)$
$143: P_{1991} = (6,11,6,1)$
$144: P_{1998} = (13,11,6,1)$
$145: P_{2005} = (4,12,6,1)$
$146: P_{2010} = (9,12,6,1)$
$147: P_{2019} = (2,13,6,1)$
$148: P_{2022} = (5,13,6,1)$
$149: P_{2057} = (8,15,6,1)$
$150: P_{2058} = (9,15,6,1)$
$151: P_{2065} = (0,0,7,1)$
$152: P_{2069} = (4,0,7,1)$
$153: P_{2100} = (3,2,7,1)$
$154: P_{2107} = (10,2,7,1)$
$155: P_{2147} = (2,5,7,1)$
$156: P_{2148} = (3,5,7,1)$
$157: P_{2175} = (14,6,7,1)$
$158: P_{2181} = (4,7,7,1)$
$159: P_{2186} = (9,7,7,1)$
$160: P_{2205} = (12,8,7,1)$
$161: P_{2208} = (15,8,7,1)$

$162: P_{2248} = (7, 11, 7, 1)$
$163: P_{2253} = (12, 11, 7, 1)$
$164: P_{2266} = (9, 12, 7, 1)$
$165: P_{2272} = (15, 12, 7, 1)$
$166: P_{2275} = (2, 13, 7, 1)$
$167: P_{2287} = (14, 13, 7, 1)$
$168: P_{2299} = (10, 14, 7, 1)$
$169: P_{2321} = (0, 0, 8, 1)$
$170: P_{2334} = (13, 0, 8, 1)$
$171: P_{2341} = (4, 1, 8, 1)$
$172: P_{2343} = (6, 1, 8, 1)$
$173: P_{2371} = (2, 3, 8, 1)$
$174: P_{2386} = (1, 4, 8, 1)$
$175: P_{2398} = (13, 4, 8, 1)$
$176: P_{2418} = (1, 6, 8, 1)$
$177: P_{2431} = (14, 6, 8, 1)$
$178: P_{2437} = (4, 7, 8, 1)$
$179: P_{2440} = (7, 7, 8, 1)$
$180: P_{2471} = (6, 9, 8, 1)$
$181: P_{2521} = (8, 12, 8, 1)$
$182: P_{2522} = (9, 12, 8, 1)$
$183: P_{2531} = (2, 13, 8, 1)$
$184: P_{2536} = (7, 13, 8, 1)$
$185: P_{2554} = (9, 14, 8, 1)$
$186: P_{2559} = (14, 14, 8, 1)$
$187: P_{2577} = (0, 0, 9, 1)$
$188: P_{2580} = (3, 0, 9, 1)$
$189: P_{2613} = (4, 2, 9, 1)$
$190: P_{2614} = (5, 2, 9, 1)$
$191: P_{2685} = (12, 6, 9, 1)$
$192: P_{2687} = (14, 6, 9, 1)$
$193: P_{2693} = (4, 7, 9, 1)$
$194: P_{2699} = (10, 7, 9, 1)$
$195: P_{2708} = (3, 8, 9, 1)$
$196: P_{2723} = (2, 9, 9, 1)$
$197: P_{2734} = (13, 9, 9, 1)$
$198: P_{2758} = (5, 11, 9, 1)$
$199: P_{2767} = (14, 11, 9, 1)$
$200: P_{2778} = (9, 12, 9, 1)$
$201: P_{2782} = (13, 12, 9, 1)$
$202: P_{2787} = (2, 13, 9, 1)$
$203: P_{2827} = (10, 15, 9, 1)$
$204: P_{2829} = (12, 15, 9, 1)$
$205: P_{2833} = (0, 0, 10, 1)$
$206: P_{2843} = (10, 0, 10, 1)$
$207: P_{2854} = (5, 1, 10, 1)$
$208: P_{2864} = (15, 1, 10, 1)$
$209: P_{2884} = (3, 3, 10, 1)$
$210: P_{2890} = (9, 3, 10, 1)$
$211: P_{2914} = (1, 5, 10, 1)$
$212: P_{2924} = (11, 5, 10, 1)$
$213: P_{2933} = (4, 6, 10, 1)$
$214: P_{2943} = (14, 6, 10, 1)$
$215: P_{2949} = (4, 7, 10, 1)$
$216: P_{2959} = (14, 7, 10, 1)$
$217: P_{2963} = (2, 8, 10, 1)$
$218: P_{2969} = (8, 8, 10, 1)$
$219: P_{2998} = (5, 10, 10, 1)$
$220: P_{3008} = (15, 10, 10, 1)$
$221: P_{3028} = (3, 12, 10, 1)$
$222: P_{3034} = (9, 12, 10, 1)$
$223: P_{3043} = (2, 13, 10, 1)$
$224: P_{3049} = (8, 13, 10, 1)$
$225: P_{3074} = (1, 15, 10, 1)$
$226: P_{3084} = (11, 15, 10, 1)$
$227: P_{3089} = (0, 0, 11, 1)$
$228: P_{3100} = (11, 0, 11, 1)$
$229: P_{3108} = (3, 1, 11, 1)$
$230: P_{3113} = (8, 1, 11, 1)$
$231: P_{3138} = (1, 3, 11, 1)$
$232: P_{3147} = (10, 3, 11, 1)$
$233: P_{3174} = (5, 5, 11, 1)$
$234: P_{3183} = (14, 5, 11, 1)$
$235: P_{3190} = (5, 6, 11, 1)$
$236: P_{3199} = (14, 6, 11, 1)$
$237: P_{3205} = (4, 7, 11, 1)$
$238: P_{3216} = (15, 7, 11, 1)$
$239: P_{3218} = (1, 8, 11, 1)$
$240: P_{3227} = (10, 8, 11, 1)$
$241: P_{3268} = (3, 11, 11, 1)$
$242: P_{3273} = (8, 11, 11, 1)$
$243: P_{3283} = (2, 12, 11, 1)$
$244: P_{3290} = (9, 12, 11, 1)$
$245: P_{3299} = (2, 13, 11, 1)$
$246: P_{3306} = (9, 13, 11, 1)$
$247: P_{3333} = (4, 15, 11, 1)$
$248: P_{3344} = (15, 15, 11, 1)$
$249: P_{3345} = (0, 0, 12, 1)$
$250: P_{3354} = (9, 0, 12, 1)$
$251: P_{3388} = (11, 2, 12, 1)$
$252: P_{3414} = (5, 4, 12, 1)$
$253: P_{3420} = (11, 4, 12, 1)$
$254: P_{3444} = (3, 6, 12, 1)$
$255: P_{3455} = (14, 6, 12, 1)$
$256: P_{3459} = (2, 7, 12, 1)$
$257: P_{3461} = (4, 7, 12, 1)$
$258: P_{3477} = (4, 8, 12, 1)$
$259: P_{3478} = (5, 8, 12, 1)$
$260: P_{3511} = (6, 10, 12, 1)$
$261: P_{3517} = (12, 10, 12, 1)$
$262: P_{3546} = (9, 12, 12, 1)$
$263: P_{3551} = (14, 12, 12, 1)$
$264: P_{3555} = (2, 13, 12, 1)$
$265: P_{3588} = (3, 15, 12, 1)$
$266: P_{3591} = (6, 15, 12, 1)$
$267: P_{3601} = (0, 0, 13, 1)$
$268: P_{3603} = (2, 0, 13, 1)$
$269: P_{3663} = (14, 3, 13, 1)$
$270: P_{3664} = (15, 3, 13, 1)$
$271: P_{3688} = (7, 5, 13, 1)$
$272: P_{3689} = (8, 5, 13, 1)$
$273: P_{3706} = (9, 6, 13, 1)$
$274: P_{3711} = (14, 6, 13, 1)$
$275: P_{3717} = (4, 7, 13, 1)$
$276: P_{3721} = (8, 7, 13, 1)$
$277: P_{3756} = (11, 9, 13, 1)$
$278: P_{3768} = (7, 10, 13, 1)$
$279: P_{3774} = (13, 10, 13, 1)$
$280: P_{3802} = (9, 12, 13, 1)$
$281: P_{3811} = (2, 13, 13, 1)$
$282: P_{3813} = (4, 13, 13, 1)$
$283: P_{3836} = (11, 14, 13, 1)$
$284: P_{3840} = (15, 14, 13, 1)$
$285: P_{3857} = (0, 0, 14, 1)$
$286: P_{3862} = (5, 0, 14, 1)$
$287: P_{3911} = (6, 3, 14, 1)$
$288: P_{3916} = (11, 3, 14, 1)$
$289: P_{3929} = (8, 4, 14, 1)$
$290: P_{3930} = (9, 4, 14, 1)$
$291: P_{3960} = (7, 6, 14, 1)$
$292: P_{3967} = (14, 6, 14, 1)$
$293: P_{3973} = (4, 7, 14, 1)$
$294: P_{4019} = (2, 10, 14, 1)$
$295: P_{4025} = (8, 10, 14, 1)$
$296: P_{4058} = (9, 12, 14, 1)$
$297: P_{4060} = (11, 12, 14, 1)$
$298: P_{4067} = (2, 13, 14, 1)$
$299: P_{4071} = (6, 13, 14, 1)$
$300: P_{4085} = (4, 14, 14, 1)$
$301: P_{4088} = (7, 14, 14, 1)$
$302: P_{4102} = (5, 15, 14, 1)$
$303: P_{4113} = (0, 0, 15, 1)$
$304: P_{4120} = (7, 0, 15, 1)$
$305: P_{4138} = (9, 1, 15, 1)$
$306: P_{4142} = (13, 1, 15, 1)$
$307: P_{4147} = (2, 2, 15, 1)$
$308: P_{4159} = (14, 2, 15, 1)$
$309: P_{4197} = (4, 5, 15, 1)$
$310: P_{4223} = (14, 6, 15, 1)$
$311: P_{4224} = (15, 6, 15, 1)$
$312: P_{4229} = (4, 7, 15, 1)$
$313: P_{4237} = (12, 7, 15, 1)$
$314: P_{4258} = (1, 9, 15, 1)$
$315: P_{4264} = (7, 9, 15, 1)$
$316: P_{4314} = (9, 12, 15, 1)$
$317: P_{4317} = (12, 12, 15, 1)$
$318: P_{4322} = (1, 13, 15, 1)$
$319: P_{4323} = (2, 13, 15, 1)$
$320: P_{4350} = (13, 14, 15, 1)$