Rank-65759 over GF(4)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431676585

General information

Number of lines	5
Number of points	25
Number of singular points	1
Number of Eckardt points	1
Number of double points	3
Number of single points	16
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	3, 2^3, 1^{16}, 0^5

Singular Points

The surface has 1 singular points:

$0: \ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

Rank of lines: (5, 337, 339, 338, 38)
Rank of points on Klein quadric: (33, 129, 143, 136, 198)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{42} = (0,1,1,1) = \ell_0 \cap \ell_1$
$P_4 = (1,1,1,1) = \ell_0 \cap \ell_4$
$P_{38} = (0,0,1,1) = \ell_1 \cap \ell_4$

Single Points

The surface has 16 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_4
$2 : P_{43} = (2,1,1,1)$ lies on line ℓ_0
$3 : P_{44} = (3,1,1,1)$ lies on line ℓ_0
$4 : P_{45} = (0,2,1,1)$ lies on line ℓ_1
$5 : P_{47} = (2,2,1,1)$ lies on line ℓ_4
$6 : P_{49} = (0,3,1,1)$ lies on line ℓ_1
$7 : P_{52} = (3,3,1,1)$ lies on line ℓ_4
$8 : P_{53} = (0,0,2,1)$ lies on line ℓ_2
$9 : P_{57} = (0,1,2,1)$ lies on line ℓ_2
$10 : P_{61} = (0,2,2,1)$ lies on line ℓ_2
$11 : P_{65} = (0,3,2,1)$ lies on line ℓ_2
$12 : P_{69} = (0,0,3,1)$ lies on line ℓ_3
$13 : P_{73} = (0,1,3,1)$ lies on line ℓ_3
$14 : P_{77} = (0,2,3,1)$ lies on line ℓ_3
$15 : P_{81} = (0,3,3,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 5 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1, 1, 1, 0)$
1 : $P_{23} = (1, 0, 0, 1)$
2 : $P_{27} = (1, 1, 0, 1)$
3 : $P_{67} = (2, 3, 2, 1)$
4 : $P_{80} = (3, 2, 3, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	0	0	1
1	1	0	1	1	1
2	0	1	0	1	0
3	0	1	1	0	0
4	1	1	0	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_4
in point	P_{42}	P_4

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{42}	P_1	P_1	P_{38}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_1	P_1

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 4 intersects

Line	ℓ_0	ℓ_1
in point	P_4	P_{38}

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_{12} = (1, 1, 1, 0)$
5 : $P_{23} = (1, 0, 0, 1)$
6 : $P_{27} = (1, 1, 0, 1)$
7 : $P_{38} = (0, 0, 1, 1)$
8 : $P_{42} = (0, 1, 1, 1)$
9 : $P_{43} = (2, 1, 1, 1)$
10 : $P_{44} = (3, 1, 1, 1)$
11 : $P_{45} = (0, 2, 1, 1)$
12 : $P_{47} = (2, 2, 1, 1)$
13 : $P_{49} = (0, 3, 1, 1)$
14 : $P_{52} = (3, 3, 1, 1)$
15 : $P_{53} = (0, 0, 2, 1)$
16 : $P_{57} = (0, 1, 2, 1)$
17 : $P_{61} = (0, 2, 2, 1)$
18 : $P_{65} = (0, 3, 2, 1)$
19 : $P_{67} = (2, 3, 2, 1)$
20 : $P_{69} = (0, 0, 3, 1)$
21 : $P_{73} = (0, 1, 3, 1)$
22 : $P_{77} = (0, 2, 3, 1)$
23 : $P_{80} = (3, 2, 3, 1)$
24 : $P_{81} = (0, 3, 3, 1)$