Rank-73801 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498768749

General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	21
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	$3^4,\, 2^6,\, 1^{21},\, 0^2$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(2,3,1,3,0,1)_{169}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 1 & 0 \end{bmatrix}_{190} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{190} = \mathbf{Pl}(3,2,1,2,0,1)_{161}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{297} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{297} = \mathbf{Pl}(1,1,2,2,0,1)_{162}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{213} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{213} = \mathbf{Pl}(2,3,2,1,0,1)_{154}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega & 0 \end{bmatrix}_{233} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{233} = \mathbf{Pl}(1,1,3,3,0,1)_{174}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{317} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{317} = \mathbf{Pl}(3,2,3,1,0,1)_{158}$$

Rank of lines: (345, 274, 190, 355, 297, 213, 350, 233, 317)
Rank of points on Klein quadric: (13, 169, 161, 15, 162, 154, 14, 174, 158)

Eckardt Points

The surface has 4 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_{11} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$2 : P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0)$,
$3 : P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{51} = (2,3,1,1) = \ell_1 \cap \ell_4$ $P_{63} = (2,2,2,1) = \ell_4 \cap \ell_8$
$P_{75} = (2,1,3,1) = \ell_1 \cap \ell_8$ $P_{84} = (3,3,3,1) = \ell_5 \cap \ell_7$
$P_{60} = (3,1,2,1) = \ell_2 \cap \ell_5$
$P_{48} = (3,2,1,1) = \ell_2 \cap \ell_7$

Single Points

The surface has 21 single points:
The single points on the surface are:

$0 : P_{28} = (2,1,0,1)$ lies on line ℓ_4 $7 : P_{41} = (3,0,1,1)$ lies on line ℓ_5
$1 : P_{29} = (3,1,0,1)$ lies on line ℓ_7 $8 : P_{42} = (0,1,1,1)$ lies on line ℓ_0
$2 : P_{32} = (2,2,0,1)$ lies on line ℓ_1 $9 : P_{45} = (0,2,1,1)$ lies on line ℓ_3
$3 : P_{33} = (3,2,0,1)$ lies on line ℓ_5 $10 : P_{49} = (0,3,1,1)$ lies on line ℓ_6
$4 : P_{36} = (2,3,0,1)$ lies on line ℓ_8 $11 : P_{55} = (2,0,2,1)$ lies on line ℓ_1
$5 : P_{37} = (3,3,0,1)$ lies on line ℓ_2 $12 : P_{56} = (3,0,2,1)$ lies on line ℓ_7
$6 : P_{40} = (2,0,1,1)$ lies on line ℓ_8 $13 : P_{57} = (0,1,2,1)$ lies on line ℓ_6

14 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_0
15 : $P_{65} = (0, 3, 2, 1)$ lies on line ℓ_3
16 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_4
17 : $P_{72} = (3, 0, 3, 1)$ lies on line ℓ_2

18 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_3
19 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_6
20 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{23} = (1, 0, 0, 1)$

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7 8
 0|0 1 1 1 0 0 1 0 0
 1|1 0 1 0 1 0 0 0 1
 2|1 1 0 0 0 1 0 1 0
 3|1 0 0 0 1 1 1 0 0
 4|0 1 0 1 0 1 0 0 1
 5|0 0 1 1 1 0 0 1 0
 6|1 0 0 1 0 0 0 1 1
 7|0 0 1 0 0 1 1 0 1
 8|0 1 0 0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6
in point	P_{11}	P_{11}	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_8
in point	P_{11}	P_{11}	P_{51}	P_{75}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_{11}	P_{11}	P_{60}	P_{48}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6
in point	P_3	P_{15}	P_{15}	P_3

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8
in point	P_{51}	P_{15}	P_{15}	P_{63}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_{60}	P_{15}	P_{15}	P_{84}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8
in point	P_3	P_3	P_{19}	P_{19}

Line 7 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{48}	P_{84}	P_{19}	P_{19}

Line 8 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_{75}	P_{63}	P_{19}	P_{19}

The surface has **33** points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_3 = (0,0,0,1)$

$2 : P_{11} = (0,1,1,0)$

$3 : P_{15} = (0,2,1,0)$

$4 : P_{19} = (0,3,1,0)$

$5 : P_{23} = (1,0,0,1)$

$6 : P_{28} = (2,1,0,1)$

$7 : P_{29} = (3,1,0,1)$

$8 : P_{32} = (2,2,0,1)$

$9 : P_{33} = (3,2,0,1)$

$10 : P_{36} = (2,3,0,1)$

$11 : P_{37} = (3,3,0,1)$

$12 : P_{40} = (2,0,1,1)$

$13 : P_{41} = (3,0,1,1)$

$14 : P_{42} = (0,1,1,1)$

$15 : P_{45} = (0,2,1,1)$

$16 : P_{48} = (3,2,1,1)$

$17 : P_{49} = (0,3,1,1)$

$18 : P_{51} = (2,3,1,1)$

$19 : P_{55} = (2,0,2,1)$

$20 : P_{56} = (3,0,2,1)$

$21 : P_{57} = (0,1,2,1)$

$22 : P_{60} = (3,1,2,1)$

$23 : P_{61} = (0,2,2,1)$

$24 : P_{63} = (2,2,2,1)$

$25 : P_{65} = (0,3,2,1)$

$26 : P_{71} = (2,0,3,1)$

$27 : P_{72} = (3,0,3,1)$

$28 : P_{73} = (0,1,3,1)$

$29 : P_{75} = (2,1,3,1)$

$30 : P_{77} = (0,2,3,1)$

$31 : P_{81} = (0,3,3,1)$

$32 : P_{84} = (3,3,3,1)$