Rank-65903 over GF(4)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431726505

General information

Number of lines	4
Number of points	21
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	12
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	2^4, 1^{12}, 0^5

Singular Points

The surface has 2 singular points:

$0: \ P_{31} = \mathbf{P}(1, \omega, 0, 1) = \mathbf{P}(1, 2, 0, 1)$
$1: \ P_{35} = \mathbf{P}(1, \omega^2, 0, 1) = \mathbf{P}(1, 3, 0, 1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$
$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$
$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

Rank of lines: (0, 17, 84, 38)
Rank of points on Klein quadric: (0, 32, 10, 198)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$
$P_{38} = (0,0,1,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 12 single points:
The single points on the surface are:

$0: P_4 = (1,1,1,1)$ lies on line ℓ_3
$1: P_6 = (2,1,0,0)$ lies on line ℓ_0
$2: P_7 = (3,1,0,0)$ lies on line ℓ_0
$3: P_{23} = (1,0,0,1)$ lies on line ℓ_2
$4: P_{27} = (1,1,0,1)$ lies on line ℓ_2
$5: P_{31} = (1,2,0,1)$ lies on line ℓ_2
$6: P_{35} = (1,3,0,1)$ lies on line ℓ_2
$7: P_{39} = (1,0,1,1)$ lies on line ℓ_1
$8: P_{40} = (2,0,1,1)$ lies on line ℓ_1
$9: P_{41} = (3,0,1,1)$ lies on line ℓ_1
$10: P_{47} = (2,2,1,1)$ lies on line ℓ_3
$11: P_{52} = (3,3,1,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 5 points not on any line:
The points on the surface but not on lines are:

$0: P_8 = (1,0,1,0)$
$1: P_{11} = (0,1,1,0)$
$2: P_{12} = (1,1,1,0)$
$3: P_{53} = (0,0,2,1)$
$4: P_{69} = (0,0,3,1)$

Line Intersection Graph

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_{38}

Line 2 intersects

Line	ℓ_0
in point	P_1

Line 3 intersects

Line	ℓ_0	ℓ_1
in point	P_5	P_{38}

The surface has 21 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (2,1,0,0)$
$5: P_7 = (3,1,0,0)$
$6: P_8 = (1,0,1,0)$
$7: P_{11} = (0,1,1,0)$
$8: P_{12} = (1,1,1,0)$
$9: P_{23} = (1,0,0,1)$
$10: P_{27} = (1,1,0,1)$
$11: P_{31} = (1,2,0,1)$
$12: P_{35} = (1,3,0,1)$
$13: P_{38} = (0,0,1,1)$
$14: P_{39} = (1,0,1,1)$
$15: P_{40} = (2,0,1,1)$
$16: P_{41} = (3,0,1,1)$
$17: P_{47} = (2,2,1,1)$
$18: P_{52} = (3,3,1,1)$
$19: P_{53} = (0,0,2,1)$
$20: P_{69} = (0,0,3,1)$