Rank-74295 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_0^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361388173

General information

Number of lines	11
Number of points	97
Number of singular points	1
Number of Eckardt points	0
Number of double points	21
Number of single points	53
Number of points off lines	22
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{11}
Type of lines on points	4, 2^{21}, 1^{53}, 0^{22}

Singular Points

The surface has 1 singular points:

$0: \ P_{12} = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$

The 11 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \mathbf{Pl}(1,0,0,0,0,1)_{650}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_9 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_9 = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{array}\right]_{125} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 4 & 6 \end{array}\right]_{125} = \mathbf{Pl}(2,6,5,0,6,1)_{3773}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & \gamma^4 & \gamma^5 \end{array}\right]_{104} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 7 & 3 \end{array}\right]_{104} = \mathbf{Pl}(4,3,6,0,3,1)_{2270}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & \gamma & \gamma^3 \end{array}\right]_{115} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 5 \end{array}\right]_{115} = \mathbf{Pl}(7,5,3,0,5,1)_{3260}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & \gamma^4 & \gamma \\ 0 & 1 & \gamma & \gamma^3 \end{array}\right]_{1721} = \left[\begin{array}{cccc} 1 & 0 & 7 & 2 \\ 0 & 1 & 2 & 5 \end{array}\right]_{1721} = \mathbf{Pl}(2,6,5,7,6,1)_{4130}$$

$$\ell_6 = \left[\begin{array}{cccc} 1 & 0 & \gamma^3 & 1 \\ 0 & 1 & \gamma^5 & \gamma \end{array}\right]_{968} = \left[\begin{array}{cccc} 1 & 0 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{array}\right]_{968} = \mathbf{Pl}(2,6,5,7,3,1)_{2604}$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 0 & \gamma & \gamma^2 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{array}\right]_{2534} = \left[\begin{array}{cccc} 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & 6 \end{array}\right]_{2534} = \mathbf{Pl}(4,3,6,2,3,1)_{2683}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{array}\right]_{1059} = \left[\begin{array}{cccc} 1 & 0 & 6 & 1 \\ 0 & 1 & 5 & 4 \end{array}\right]_{1059} = \mathbf{Pl}(4,3,6,2,5,1)_{3670}$$

$$\ell_9 = \left[\begin{array}{cccc} 1 & 0 & \gamma^2 & \gamma^4 \\ 0 & 1 & \gamma^4 & \gamma^5 \end{array}\right]_{4411} = \left[\begin{array}{cccc} 1 & 0 & 4 & 7 \\ 0 & 1 & 7 & 3 \end{array}\right]_{4411} = \mathbf{Pl}(7,5,3,4,5,1)_{3533}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & \gamma^6 & \gamma^4 \end{array}\right]_{865} = \left[\begin{array}{cccc} 1 & 0 & 3 & 1 \\ 0 & 1 & 6 & 7 \end{array}\right]_{865} = \mathbf{Pl}(7,5,3,4,6,1)_{4072}$$

Rank of lines: (73, 9, 125, 104, 115, 1721, 968, 2534, 1059, 4411, 865)
Rank of points on Klein quadric: (650, 97, 3773, 2270, 3260, 4130, 2604, 2683, 3670, 3533, 4072)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 21 Double points:
The double points on the surface are:

$P_{36} = (1,3,1,0) = \ell_0 \cap \ell_5$
$P_{52} = (1,5,1,0) = \ell_0 \cap \ell_7$
$P_{60} = (1,6,1,0) = \ell_0 \cap \ell_9$
$P_{147} = (2,1,1,1) = \ell_1 \cap \ell_5$
$P_{148} = (3,1,1,1) = \ell_1 \cap \ell_6$
$P_{149} = (4,1,1,1) = \ell_1 \cap \ell_7$
$P_{150} = (5,1,1,1) = \ell_1 \cap \ell_8$
$P_{152} = (7,1,1,1) = \ell_1 \cap \ell_9$
$P_{151} = (6,1,1,1) = \ell_1 \cap \ell_{10}$
$P_{95} = (5,2,0,1) = \ell_2 \cap \ell_6$
$P_{409} = (0,2,5,1) = \ell_2 \cap \ell_7$
$P_{539} = (2,2,7,1) = \ell_2 \cap \ell_{10}$
$P_{237} = (4,4,2,1) = \ell_3 \cap \ell_6$
$P_{112} = (6,4,0,1) = \ell_3 \cap \ell_8$
$P_{489} = (0,4,6,1) = \ell_3 \cap \ell_9$
$P_{321} = (0,7,3,1) = \ell_4 \cap \ell_5$
$P_{392} = (7,7,4,1) = \ell_4 \cap \ell_8$
$P_{133} = (3,7,0,1) = \ell_4 \cap \ell_{10}$
$P_{360} = (7,3,4,1) = \ell_5 \cap \ell_6$
$P_{563} = (2,5,7,1) = \ell_7 \cap \ell_8$
$P_{253} = (4,6,2,1) = \ell_9 \cap \ell_{10}$

Single Points

The surface has 53 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_1
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
2 : $P_4 = (1,1,1,1)$ lies on line ℓ_1
3 : $P_{20} = (1,1,1,0)$ lies on line ℓ_0
4 : $P_{28} = (1,2,1,0)$ lies on line ℓ_0
5 : $P_{41} = (6,3,1,0)$ lies on line ℓ_6
6 : $P_{44} = (1,4,1,0)$ lies on line ℓ_0
7 : $P_{54} = (3,5,1,0)$ lies on line ℓ_8
8 : $P_{64} = (5,6,1,0)$ lies on line ℓ_{10}
9 : $P_{68} = (1,7,1,0)$ lies on line ℓ_0
10 : $P_{93} = (3,2,0,1)$ lies on line ℓ_5
11 : $P_{111} = (5,4,0,1)$ lies on line ℓ_7
12 : $P_{136} = (6,7,0,1)$ lies on line ℓ_9
13 : $P_{146} = (0,1,1,1)$ lies on line ℓ_1
14 : $P_{157} = (4,2,1,1)$ lies on line ℓ_2
15 : $P_{176} = (7,4,1,1)$ lies on line ℓ_3
16 : $P_{195} = (2,7,1,1)$ lies on line ℓ_4
17 : $P_{224} = (7,2,2,1)$ lies on line ℓ_2
18 : $P_{225} = (0,3,2,1)$ lies on line ℓ_8
19 : $P_{232} = (7,3,2,1)$ lies on line ℓ_7
20 : $P_{234} = (1,4,2,1)$ lies on line ℓ_5
21 : $P_{258} = (1,7,2,1)$ lies on line ℓ_4
22 : $P_{266} = (1,0,3,1)$ lies on line ℓ_{10}
23 : $P_{270} = (5,0,3,1)$ lies on line ℓ_9
24 : $P_{287} = (6,2,3,1)$ lies on line ℓ_2
25 : $P_{302} = (5,4,3,1)$ lies on line ℓ_3
26 : $P_{316} = (3,6,3,1)$ lies on line ℓ_8
27 : $P_{319} = (6,6,3,1)$ lies on line ℓ_7
28 : $P_{323} = (2,7,3,1)$ lies on line ℓ_6
29 : $P_{346} = (1,2,4,1)$ lies on line ℓ_2
30 : $P_{363} = (2,4,4,1)$ lies on line ℓ_3
31 : $P_{369} = (0,5,4,1)$ lies on line ℓ_{10}
32 : $P_{371} = (2,5,4,1)$ lies on line ℓ_9
33 : $P_{386} = (1,7,4,1)$ lies on line ℓ_7
34 : $P_{394} = (1,0,5,1)$ lies on line ℓ_6
35 : $P_{399} = (6,0,5,1)$ lies on line ℓ_5
36 : $P_{413} = (4,2,5,1)$ lies on line ℓ_8
37 : $P_{420} = (3,3,5,1)$ lies on line ℓ_9
38 : $P_{422} = (5,3,5,1)$ lies on line ℓ_{10}
39 : $P_{428} = (3,4,5,1)$ lies on line ℓ_3
40 : $P_{455} = (6,7,5,1)$ lies on line ℓ_4
41 : $P_{458} = (1,0,6,1)$ lies on line ℓ_8
42 : $P_{460} = (3,0,6,1)$ lies on line ℓ_7
43 : $P_{476} = (3,2,6,1)$ lies on line ℓ_2
44 : $P_{496} = (7,4,6,1)$ lies on line ℓ_{10}
45 : $P_{502} = (5,5,6,1)$ lies on line ℓ_5
46 : $P_{503} = (6,5,6,1)$ lies on line ℓ_6
47 : $P_{518} = (5,7,6,1)$ lies on line ℓ_4
48 : $P_{538} = (1,2,7,1)$ lies on line ℓ_9
49 : $P_{554} = (1,4,7,1)$ lies on line ℓ_3
50 : $P_{569} = (0,6,7,1)$ lies on line ℓ_6
51 : $P_{573} = (4,6,7,1)$ lies on line ℓ_5
52 : $P_{581} = (4,7,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 22 points not on any line:
The points on the surface but not on lines are:

0 : $P_3 = (0,0,0,1)$
1 : $P_{31} = (4,2,1,0)$
2 : $P_{50} = (7,4,1,0)$
3 : $P_{69} = (2,7,1,0)$
4 : $P_{158} = (5,2,1,1)$
5 : $P_{175} = (6,4,1,1)$
6 : $P_{196} = (3,7,1,1)$
7 : $P_{259} = (2,7,2,1)$
8 : $P_{282} = (1,2,3,1)$
9 : $P_{293} = (4,3,3,1)$
10 : $P_{307} = (2,5,3,1)$
11 : $P_{309} = (4,5,3,1)$
12 : $P_{349} = (4,2,4,1)$
13 : $P_{426} = (1,4,5,1)$
14 : $P_{440} = (7,5,5,1)$
15 : $P_{445} = (4,6,5,1)$
16 : $P_{448} = (7,6,5,1)$
17 : $P_{483} = (2,3,6,1)$
18 : $P_{488} = (7,3,6,1)$
19 : $P_{507} = (2,6,6,1)$
20 : $P_{514} = (1,7,6,1)$
21 : $P_{560} = (7,4,7,1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10
0	0	0	1	1	1	1	0	1	0	1	0
1	0	0	0	0	0	1	1	1	1	1	1
2	1	0	0	1	1	0	1	1	0	0	1
3	1	0	1	0	1	0	1	0	1	1	0
4	1	0	1	1	0	1	0	0	1	0	1
5	1	1	0	0	1	0	1	0	0	0	0
6	0	1	1	1	0	1	0	0	0	0	0
7	1	1	1	0	0	0	0	0	1	0	0
8	0	1	0	1	1	0	0	1	0	0	0
9	1	1	0	1	0	0	0	0	0	0	1
10	0	1	1	0	1	0	0	0	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_9
in point	P_{12}	P_{12}	P_{12}	P_{36}	P_{52}	P_{60}

Line 1 intersects

Line	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_{147}	P_{148}	P_{149}	P_{150}	P_{152}	P_{151}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_{10}
in point	P_{12}	P_{12}	P_{12}	P_{95}	P_{409}	P_{539}

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6	ℓ_8	ℓ_9
in point	P_{12}	P_{12}	P_{12}	P_{237}	P_{112}	P_{489}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_8	ℓ_{10}
in point	P_{12}	P_{12}	P_{12}	P_{321}	P_{392}	P_{133}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_{36}	P_{147}	P_{321}	P_{360}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_{148}	P_{95}	P_{237}	P_{360}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_8
in point	P_{52}	P_{149}	P_{409}	P_{563}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_{150}	P_{112}	P_{392}	P_{563}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_{10}
in point	P_{60}	P_{152}	P_{489}	P_{253}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_9
in point	P_{151}	P_{539}	P_{133}	P_{253}

The surface has 97 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_{12} = (1,0,1,0)$
$5: P_{20} = (1,1,1,0)$
$6: P_{28} = (1,2,1,0)$
$7: P_{31} = (4,2,1,0)$
$8: P_{36} = (1,3,1,0)$
$9: P_{41} = (6,3,1,0)$
$10: P_{44} = (1,4,1,0)$
$11: P_{50} = (7,4,1,0)$
$12: P_{52} = (1,5,1,0)$
$13: P_{54} = (3,5,1,0)$
$14: P_{60} = (1,6,1,0)$
$15: P_{64} = (5,6,1,0)$
$16: P_{68} = (1,7,1,0)$
$17: P_{69} = (2,7,1,0)$
$18: P_{93} = (3,2,0,1)$
$19: P_{95} = (5,2,0,1)$
$20: P_{111} = (5,4,0,1)$
$21: P_{112} = (6,4,0,1)$
$22: P_{133} = (3,7,0,1)$
$23: P_{136} = (6,7,0,1)$
$24: P_{146} = (0,1,1,1)$
$25: P_{147} = (2,1,1,1)$
$26: P_{148} = (3,1,1,1)$
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$44: P_{259} = (2,7,2,1)$
$45: P_{266} = (1,0,3,1)$
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$47: P_{282} = (1,2,3,1)$
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$59: P_{360} = (7,3,4,1)$
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$62: P_{371} = (2,5,4,1)$
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