# Rank-43 over GF(4)

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## The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_1 = 0$$

( 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )
The point rank of the equation over GF(4) is 181

## General information

| | |
|---|---|
| Number of lines | 6 |
| Number of points | 29 |
| Number of singular points | 1 |
| Number of Eckardt points | 2 |
| Number of double points | 3 |
| Number of single points | 18 |
| Number of points off lines | 6 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^6$ |
| Type of lines on points | $3^2$, $2^3$, $1^{18}$, $0^6$ |

### Singular Points

The surface has 1 singular points:

$0: \ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

## The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{19} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{19} = \mathbf{Pl}(0,0,2,0,1,0)_{39}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{18} = \mathbf{Pl}(0,0,3,0,1,0)_{46}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

Rank of lines: ( 17, 19, 18, 337, 339, 338 )
Rank of points on Klein quadric: ( 32, 39, 46, 129, 143, 136 )

### Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

### Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{38} = (0,0,1,1) = \ell_0 \cap \ell_3$
$P_{53} = (0,0,2,1) = \ell_1 \cap \ell_4$
$P_{69} = (0,0,3,1) = \ell_2 \cap \ell_5$

### Single Points

The surface has 18 single points:
The single points on the surface are:

$0 : P_{39} = (1,0,1,1)$ lies on line $\ell_0$
$1 : P_{40} = (2,0,1,1)$ lies on line $\ell_0$
$2 : P_{41} = (3,0,1,1)$ lies on line $\ell_0$
$3 : P_{42} = (0,1,1,1)$ lies on line $\ell_3$
$4 : P_{45} = (0,2,1,1)$ lies on line $\ell_3$
$5 : P_{49} = (0,3,1,1)$ lies on line $\ell_3$
$6 : P_{54} = (1,0,2,1)$ lies on line $\ell_1$
$7 : P_{55} = (2,0,2,1)$ lies on line $\ell_1$
$8 : P_{56} = (3,0,2,1)$ lies on line $\ell_1$
$9 : P_{57} = (0,1,2,1)$ lies on line $\ell_4$
$10 : P_{61} = (0,2,2,1)$ lies on line $\ell_4$
$11 : P_{65} = (0,3,2,1)$ lies on line $\ell_4$
$12 : P_{70} = (1,0,3,1)$ lies on line $\ell_2$
$13 : P_{71} = (2,0,3,1)$ lies on line $\ell_2$
$14 : P_{72} = (3,0,3,1)$ lies on line $\ell_2$
$15 : P_{73} = (0,1,3,1)$ lies on line $\ell_5$
$16 : P_{77} = (0,2,3,1)$ lies on line $\ell_5$
$17 : P_{81} = (0,3,3,1)$ lies on line $\ell_5$

The single points on the surface are:

### Points on surface but on no line

The surface has 6 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1, 1, 1, 0)$
1 : $P_{17} = (2, 2, 1, 0)$
2 : $P_{22} = (3, 3, 1, 0)$
3 : $P_{27} = (1, 1, 0, 1)$
4 : $P_{32} = (2, 2, 0, 1)$
5 : $P_{37} = (3, 3, 0, 1)$

## Line Intersection Graph

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 1 | 0 | 1 |
| 5 | 0 | 0 | 1 | 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ |
|---|---|---|---|
| in point | $P_0$ | $P_0$ | $P_{38}$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_4$ |
|---|---|---|---|
| in point | $P_0$ | $P_0$ | $P_{53}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_5$ |
|---|---|---|---|
| in point | $P_0$ | $P_0$ | $P_{69}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_4$ | $\ell_5$ |
|---|---|---|---|
| in point | $P_{38}$ | $P_1$ | $P_1$ |

Line 4 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_5$ |
|---|---|---|---|
| in point | $P_{53}$ | $P_1$ | $P_1$ |

Line 5 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|
| in point | $P_{69}$ | $P_1$ | $P_1$ |

The surface has 29 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_{12} = (1, 1, 1, 0)$
3 : $P_{17} = (2, 2, 1, 0)$
4 : $P_{22} = (3, 3, 1, 0)$
5 : $P_{27} = (1, 1, 0, 1)$
6 : $P_{32} = (2, 2, 0, 1)$
7 : $P_{37} = (3, 3, 0, 1)$
8 : $P_{38} = (0, 0, 1, 1)$
9 : $P_{39} = (1, 0, 1, 1)$
10 : $P_{40} = (2, 0, 1, 1)$
11 : $P_{41} = (3, 0, 1, 1)$
12 : $P_{42} = (0, 1, 1, 1)$
13 : $P_{45} = (0, 2, 1, 1)$
14 : $P_{49} = (0, 3, 1, 1)$
15 : $P_{53} = (0, 0, 2, 1)$
16 : $P_{54} = (1, 0, 2, 1)$
17 : $P_{55} = (2, 0, 2, 1)$
18 : $P_{56} = (3, 0, 2, 1)$
19 : $P_{57} = (0, 1, 2, 1)$
20 : $P_{61} = (0, 2, 2, 1)$
21 : $P_{65} = (0, 3, 2, 1)$
22 : $P_{69} = (0, 0, 3, 1)$
23 : $P_{70} = (1, 0, 3, 1)$
24 : $P_{71} = (2, 0, 3, 1)$
25 : $P_{72} = (3, 0, 3, 1)$
26 : $P_{73} = (0, 1, 3, 1)$
27 : $P_{77} = (0, 2, 3, 1)$
28 : $P_{81} = (0, 3, 3, 1)$