# Rank-65550 over GF(8)

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0 X_1 X_2 = 0$$

( 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over $GF(8)$ is 1227134038

## General information

| | |
|---|---:|
| Number of lines | 3 |
| Number of points | 73 |
| Number of singular points | 1 |
| Number of Eckardt points | 0 |
| Number of double points | 3 |
| Number of single points | 21 |
| Number of points off lines | 49 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^3$ |
| Type of lines on points | $2^3, 1^{21}, 0^{49}$ |

### Singular Points

The surface has 1 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \mathbf{Pl}(0,1,0,0,0,1)_{657}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1,0,1,1,1,1)_{1323}$$

Rank of lines: ( 648, 4681, 666 )
Rank of points on Klein quadric: ( 10, 657, 1323 )

**Eckardt Points**

The surface has 0 Eckardt points:

**Double Points**

The surface has 3 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$ $\qquad\qquad$ $P_{146} = (0,1,1,1) = \ell_1 \cap \ell_2$
$P_{139} = (1,0,1,1) = \ell_0 \cap \ell_2$

**Single Points**

The surface has 21 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line $\ell_2$ $\qquad$ 11 : $P_{266} = (1,0,3,1)$ lies on line $\ell_0$
1 : $P_{75} = (1,0,0,1)$ lies on line $\ell_0$ $\qquad$ 12 : $P_{273} = (0,1,3,1)$ lies on line $\ell_1$
2 : $P_{82} = (0,1,0,1)$ lies on line $\ell_1$ $\qquad$ 13 : $P_{330} = (1,0,4,1)$ lies on line $\ell_0$
3 : $P_{156} = (3,2,1,1)$ lies on line $\ell_2$ $\qquad$ 14 : $P_{337} = (0,1,4,1)$ lies on line $\ell_1$
4 : $P_{163} = (2,3,1,1)$ lies on line $\ell_2$ $\qquad$ 15 : $P_{394} = (1,0,5,1)$ lies on line $\ell_0$
5 : $P_{174} = (5,4,1,1)$ lies on line $\ell_2$ $\qquad$ 16 : $P_{401} = (0,1,5,1)$ lies on line $\ell_1$
6 : $P_{181} = (4,5,1,1)$ lies on line $\ell_2$ $\qquad$ 17 : $P_{458} = (1,0,6,1)$ lies on line $\ell_0$
7 : $P_{192} = (7,6,1,1)$ lies on line $\ell_2$ $\qquad$ 18 : $P_{465} = (0,1,6,1)$ lies on line $\ell_1$
8 : $P_{199} = (6,7,1,1)$ lies on line $\ell_2$ $\qquad$ 19 : $P_{522} = (1,0,7,1)$ lies on line $\ell_0$
9 : $P_{202} = (1,0,2,1)$ lies on line $\ell_0$ $\qquad$ 20 : $P_{529} = (0,1,7,1)$ lies on line $\ell_1$
10 : $P_{209} = (0,1,2,1)$ lies on line $\ell_1$

The single points on the surface are:

**Points on surface but on no line**

The surface has 49 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$ $\qquad\qquad$ 8 : $P_{105} = (7,3,0,1)$
1 : $P_{22} = (3,1,1,0)$ $\qquad\qquad$ 9 : $P_{112} = (6,4,0,1)$
2 : $P_{24} = (5,1,1,0)$ $\qquad\qquad$ 10 : $P_{116} = (2,5,0,1)$
3 : $P_{25} = (6,1,1,0)$ $\qquad\qquad$ 11 : $P_{126} = (4,6,0,1)$
4 : $P_{36} = (1,3,1,0)$ $\qquad\qquad$ 12 : $P_{133} = (3,7,0,1)$
5 : $P_{52} = (1,5,1,0)$ $\qquad\qquad$ 13 : $P_{216} = (7,1,2,1)$
6 : $P_{60} = (1,6,1,0)$ $\qquad\qquad$ 14 : $P_{246} = (5,5,2,1)$
7 : $P_{95} = (5,2,0,1)$ $\qquad\qquad$ 15 : $P_{258} = (1,7,2,1)$

16 : $P_{279} = (6,1,3,1)$
17 : $P_{283} = (2,2,3,1)$
18 : $P_{285} = (4,2,3,1)$
19 : $P_{287} = (6,2,3,1)$
20 : $P_{295} = (6,3,3,1)$
21 : $P_{299} = (2,4,3,1)$
22 : $P_{314} = (1,6,3,1)$
23 : $P_{315} = (2,6,3,1)$
24 : $P_{316} = (3,6,3,1)$
25 : $P_{339} = (2,1,4,1)$
26 : $P_{346} = (1,2,4,1)$
27 : $P_{383} = (6,6,4,1)$
28 : $P_{404} = (3,1,5,1)$
29 : $P_{418} = (1,3,5,1)$
30 : $P_{421} = (4,3,5,1)$
31 : $P_{422} = (5,3,5,1)$
32 : $P_{428} = (3,4,5,1)$

33 : $P_{429} = (4,4,5,1)$
34 : $P_{432} = (7,4,5,1)$
35 : $P_{436} = (3,5,5,1)$
36 : $P_{453} = (4,7,5,1)$
37 : $P_{470} = (5,1,6,1)$
38 : $P_{480} = (7,2,6,1)$
39 : $P_{498} = (1,5,6,1)$
40 : $P_{503} = (6,5,6,1)$
41 : $P_{504} = (7,5,6,1)$
42 : $P_{510} = (5,6,6,1)$
43 : $P_{515} = (2,7,6,1)$
44 : $P_{518} = (5,7,6,1)$
45 : $P_{520} = (7,7,6,1)$
46 : $P_{533} = (4,1,7,1)$
47 : $P_{548} = (3,3,7,1)$
48 : $P_{554} = (1,4,7,1)$

## Line Intersection Graph

|   | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 2 | 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ |
|---|---|---|
| in point | $P_2$ | $P_{139}$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ |
|---|---|---|
| in point | $P_2$ | $P_{146}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ |
|---|---|---|
| in point | $P_{139}$ | $P_{146}$ |

The surface has 73 points:
The points on the surface are:

0 : $P_2 = (0,0,1,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_5 = (1,1,0,0)$
3 : $P_{22} = (3,1,1,0)$
4 : $P_{24} = (5,1,1,0)$
5 : $P_{25} = (6,1,1,0)$
6 : $P_{36} = (1,3,1,0)$
7 : $P_{52} = (1,5,1,0)$
8 : $P_{60} = (1,6,1,0)$
9 : $P_{75} = (1,0,0,1)$
10 : $P_{82} = (0,1,0,1)$
11 : $P_{95} = (5,2,0,1)$
12 : $P_{105} = (7,3,0,1)$
13 : $P_{112} = (6,4,0,1)$
14 : $P_{116} = (2,5,0,1)$

15 : $P_{126} = (4,6,0,1)$
16 : $P_{133} = (3,7,0,1)$
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26 : $P_{209} = (0,1,2,1)$
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30 : $P_{266} = (1,0,3,1)$
31 : $P_{273} = (0,1,3,1)$
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36 : $P_{295} = (6,3,3,1)$
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38 : $P_{314} = (1,6,3,1)$
39 : $P_{315} = (2,6,3,1)$
40 : $P_{316} = (3,6,3,1)$
41 : $P_{330} = (1,0,4,1)$
42 : $P_{337} = (0,1,4,1)$
43 : $P_{339} = (2,1,4,1)$
44 : $P_{346} = (1,2,4,1)$

$45: P_{383} = (6, 6, 4, 1)$

$46: P_{394} = (1, 0, 5, 1)$

$47: P_{401} = (0, 1, 5, 1)$

$48: P_{404} = (3, 1, 5, 1)$

$49: P_{418} = (1, 3, 5, 1)$

$50: P_{421} = (4, 3, 5, 1)$

$51: P_{422} = (5, 3, 5, 1)$

$52: P_{428} = (3, 4, 5, 1)$

$53: P_{429} = (4, 4, 5, 1)$

$54: P_{432} = (7, 4, 5, 1)$

$55: P_{436} = (3, 5, 5, 1)$

$56: P_{453} = (4, 7, 5, 1)$

$57: P_{458} = (1, 0, 6, 1)$

$58: P_{465} = (0, 1, 6, 1)$

$59: P_{470} = (5, 1, 6, 1)$

$60: P_{480} = (7, 2, 6, 1)$

$61: P_{498} = (1, 5, 6, 1)$

$62: P_{503} = (6, 5, 6, 1)$

$63: P_{504} = (7, 5, 6, 1)$

$64: P_{510} = (5, 6, 6, 1)$

$65: P_{515} = (2, 7, 6, 1)$

$66: P_{518} = (5, 7, 6, 1)$

$67: P_{520} = (7, 7, 6, 1)$

$68: P_{522} = (1, 0, 7, 1)$

$69: P_{529} = (0, 1, 7, 1)$

$70: P_{533} = (4, 1, 7, 1)$

$71: P_{548} = (3, 3, 7, 1)$

$72: P_{554} = (1, 4, 7, 1)$