Rank-74243 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(64) is 1090785349

General information

Number of lines	4
Number of points	4225
Number of singular points	2
Number of Eckardt points	1
Number of double points	2
Number of single points	253
Number of points off lines	3969
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{4}
Type of lines on points	$3, 2^2, 1^{253}, 0^{3969}$

Singular Points

The surface has 2 singular points:

$$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0) 1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{270401}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (0, 4096, 17043456, 17047616) Rank of points on Klein quadric: (0, 2, 270401, 1)

Eckardt Points

The surface has 1 Eckardt points: $0: P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0).$

Double Points

The surface has 2 Double points: The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

Single Points

The surface has 253 single points: The single points on the surface are:

 $0: P_3 = (0,0,0,1)$ lies on line ℓ_3 $22: P_{26} = (22, 1, 0, 0)$ lies on line ℓ_0 1: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 23: $P_{27} = (23, 1, 0, 0)$ lies on line ℓ_0 2: $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 24: $P_{28} = (24, 1, 0, 0)$ lies on line ℓ_0 $3: P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 25: $P_{29} = (25, 1, 0, 0)$ lies on line ℓ_0 4: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 26: $P_{30} = (26, 1, 0, 0)$ lies on line ℓ_0 5: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 27: $P_{31} = (27, 1, 0, 0)$ lies on line ℓ_0 6: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 28: $P_{32} = (28, 1, 0, 0)$ lies on line ℓ_0 7: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 29: $P_{33} = (29, 1, 0, 0)$ lies on line ℓ_0 8: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 $30: P_{34} = (30, 1, 0, 0)$ lies on line ℓ_0 9: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 $31: P_{35} = (31, 1, 0, 0)$ lies on line ℓ_0 10: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 $32: P_{36} = (32, 1, 0, 0)$ lies on line ℓ_0 11: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 33: $P_{37} = (33, 1, 0, 0)$ lies on line ℓ_0 12: $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 $34: P_{38} = (34, 1, 0, 0)$ lies on line ℓ_0 13: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 $35: P_{39} = (35, 1, 0, 0)$ lies on line ℓ_0 14: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 $36: P_{40} = (36, 1, 0, 0)$ lies on line ℓ_0 37 : $P_{41} = (37, 1, 0, 0)$ lies on line ℓ_0 15: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 16: $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0 $38: P_{42} = (38, 1, 0, 0)$ lies on line ℓ_0 17: $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0 39: $P_{43} = (39, 1, 0, 0)$ lies on line ℓ_0 18 : $P_{22} = (18, 1, 0, 0)$ lies on line ℓ_0 40: $P_{44} = (40, 1, 0, 0)$ lies on line ℓ_0 19 : $P_{23} = (19,1,0,0)$ lies on line ℓ_0 41: $P_{45} = (41, 1, 0, 0)$ lies on line ℓ_0 20: $P_{24} = (20, 1, 0, 0)$ lies on line ℓ_0 42: $P_{46} = (42, 1, 0, 0)$ lies on line ℓ_0 21: $P_{25} = (21, 1, 0, 0)$ lies on line ℓ_0 43: $P_{47} = (43, 1, 0, 0)$ lies on line ℓ_0

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44: P_{48} = (44, 1, 0, 0) lies on line \ell_0
                                                                      98: P_{102} = (35, 0, 1, 0) lies on line \ell_1
45: P_{49} = (45, 1, 0, 0) lies on line \ell_0
                                                                      99: P_{103} = (36, 0, 1, 0) lies on line \ell_1
46: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                      100: P_{104} = (37, 0, 1, 0) lies on line \ell_1
47: P_{51} = (47, 1, 0, 0) lies on line \ell_0
                                                                      101: P_{105} = (38, 0, 1, 0) lies on line \ell_1
48: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                      102: P_{106} = (39, 0, 1, 0) lies on line \ell_1
49: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                      103: P_{107} = (40, 0, 1, 0) lies on line \ell_1
50 : P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                      104: P_{108} = (41, 0, 1, 0) lies on line \ell_1
51: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                      105: P_{109} = (42, 0, 1, 0) lies on line \ell_1
52: P_{56} = (52, 1, 0, 0) lies on line \ell_0
                                                                      106: P_{110} = (43, 0, 1, 0) lies on line \ell_1
53: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                      107: P_{111} = (44, 0, 1, 0) lies on line \ell_1
54: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                      108: P_{112} = (45, 0, 1, 0) lies on line \ell_1
55: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                      109: P_{113} = (46, 0, 1, 0) lies on line \ell_1
56: P_{60} = (56, 1, 0, 0) lies on line \ell_0
                                                                      110: P_{114} = (47, 0, 1, 0) lies on line \ell_1
57: P_{61} = (57, 1, 0, 0) lies on line \ell_0
                                                                      111: P_{115} = (48, 0, 1, 0) lies on line \ell_1
58: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                      112: P_{116} = (49, 0, 1, 0) lies on line \ell_1
59: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                      113: P_{117} = (50, 0, 1, 0) lies on line \ell_1
60: P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                      114: P_{118} = (51, 0, 1, 0) lies on line \ell_1
61: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                      115: P_{119} = (52, 0, 1, 0) lies on line \ell_1
62: P_{66} = (62, 1, 0, 0) lies on line \ell_0
                                                                      116: P_{120} = (53, 0, 1, 0) lies on line \ell_1
63: P_{67} = (63, 1, 0, 0) lies on line \ell_0
                                                                      117: P_{121} = (54, 0, 1, 0) lies on line \ell_1
64: P_{68} = (1, 0, 1, 0) lies on line \ell_1
                                                                      118: P_{122} = (55, 0, 1, 0) lies on line \ell_1
65: P_{69} = (2, 0, 1, 0) lies on line \ell_1
                                                                      119: P_{123} = (56, 0, 1, 0) lies on line \ell_1
66: P_{70} = (3, 0, 1, 0) lies on line \ell_1
                                                                      120: P_{124} = (57, 0, 1, 0) lies on line \ell_1
                                                                      121 : P_{125} = (58, 0, 1, 0) lies on line \ell_1
67: P_{71} = (4, 0, 1, 0) lies on line \ell_1
68: P_{72} = (5, 0, 1, 0) lies on line \ell_1
                                                                      122: P_{126} = (59, 0, 1, 0) lies on line \ell_1
69: P_{73} = (6,0,1,0) lies on line \ell_1
                                                                      123: P_{127} = (60, 0, 1, 0) lies on line \ell_1
70: P_{74} = (7, 0, 1, 0) lies on line \ell_1
                                                                      124: P_{128} = (61, 0, 1, 0) lies on line \ell_1
71: P_{75} = (8, 0, 1, 0) lies on line \ell_1
                                                                      125: P_{129} = (62, 0, 1, 0) lies on line \ell_1
72: P_{76} = (9, 0, 1, 0) lies on line \ell_1
                                                                      126: P_{130} = (63, 0, 1, 0) lies on line \ell_1
73: P_{77} = (10, 0, 1, 0) lies on line \ell_1
                                                                      127: P_{131} = (0, 1, 1, 0) lies on line \ell_2
74: P_{78} = (11, 0, 1, 0) lies on line \ell_1
                                                                      128: P_{195} = (0, 2, 1, 0) lies on line \ell_2
75: P_{79} = (12, 0, 1, 0) lies on line \ell_1
                                                                      129: P_{259} = (0, 3, 1, 0) lies on line \ell_2
76: P_{80} = (13, 0, 1, 0) lies on line \ell_1
                                                                      130: P_{323} = (0, 4, 1, 0) lies on line \ell_2
                                                                      131: P_{387} = (0, 5, 1, 0) lies on line \ell_2
77: P_{81} = (14, 0, 1, 0) lies on line \ell_1
78: P_{82} = (15, 0, 1, 0) lies on line \ell_1
                                                                      132: P_{451} = (0, 6, 1, 0) lies on line \ell_2
79: P_{83} = (16, 0, 1, 0) lies on line \ell_1
                                                                      133: P_{515} = (0, 7, 1, 0) lies on line \ell_2
80: P_{84} = (17, 0, 1, 0) lies on line \ell_1
                                                                      134: P_{579} = (0, 8, 1, 0) lies on line \ell_2
81: P_{85} = (18, 0, 1, 0) lies on line \ell_1
                                                                      135: P_{643} = (0, 9, 1, 0) lies on line \ell_2
82: P_{86} = (19, 0, 1, 0) lies on line \ell_1
                                                                      136: P_{707} = (0, 10, 1, 0) lies on line \ell_2
83: P_{87} = (20, 0, 1, 0) lies on line \ell_1
                                                                      137: P_{771} = (0, 11, 1, 0) lies on line \ell_2
84: P_{88} = (21, 0, 1, 0) lies on line \ell_1
                                                                      138: P_{835} = (0, 12, 1, 0) lies on line \ell_2
85: P_{89} = (22, 0, 1, 0) lies on line \ell_1
                                                                      139: P_{899} = (0, 13, 1, 0) lies on line \ell_2
86: P_{90} = (23, 0, 1, 0) lies on line \ell_1
                                                                      140: P_{963} = (0, 14, 1, 0) lies on line \ell_2
87: P_{91} = (24, 0, 1, 0) lies on line \ell_1
                                                                      141: P_{1027} = (0, 15, 1, 0) lies on line \ell_2
88: P_{92} = (25, 0, 1, 0) lies on line \ell_1
                                                                      142: P_{1091} = (0, 16, 1, 0) lies on line \ell_2
89: P_{93} = (26, 0, 1, 0) lies on line \ell_1
                                                                      143: P_{1155} = (0, 17, 1, 0) lies on line \ell_2
90: P_{94} = (27, 0, 1, 0) lies on line \ell_1
                                                                      144: P_{1219} = (0, 18, 1, 0) lies on line \ell_2
91: P_{95} = (28, 0, 1, 0) lies on line \ell_1
                                                                      145: P_{1283} = (0, 19, 1, 0) lies on line \ell_2
92: P_{96} = (29, 0, 1, 0) lies on line \ell_1
                                                                      146: P_{1347} = (0, 20, 1, 0) lies on line \ell_2
93: P_{97} = (30, 0, 1, 0) lies on line \ell_1
                                                                      147: P_{1411} = (0, 21, 1, 0) lies on line \ell_2
94: P_{98} = (31, 0, 1, 0) lies on line \ell_1
                                                                      148: P_{1475} = (0, 22, 1, 0) lies on line \ell_2
95: P_{99} = (32, 0, 1, 0) lies on line \ell_1
                                                                      149: P_{1539} = (0, 23, 1, 0) lies on line \ell_2
96: P_{100} = (33, 0, 1, 0) lies on line \ell_1
                                                                      150: P_{1603} = (0, 24, 1, 0) lies on line \ell_2
97 : P_{101} = (34, 0, 1, 0) lies on line \ell_1
                                                                      151: P_{1667} = (0, 25, 1, 0) lies on line \ell_2
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152 : P_{1731} = (0, 26, 1, 0) lies on line \ell_2
                                                                     203 : P_{61505} = (0, 0, 14, 1) lies on line \ell_3
                                                                     204: P_{65601} = (0, 0, 15, 1) lies on line \ell_3
153: P_{1795} = (0, 27, 1, 0) lies on line \ell_2
154: P_{1859} = (0, 28, 1, 0) lies on line \ell_2
                                                                     205: P_{69697} = (0, 0, 16, 1) lies on line \ell_3
155: P_{1923} = (0, 29, 1, 0) lies on line \ell_2
                                                                     206: P_{73793} = (0, 0, 17, 1) lies on line \ell_3
                                                                     207 : P_{77889} = (0, 0, 18, 1) lies on line \ell_3
156: P_{1987} = (0, 30, 1, 0) lies on line \ell_2
157: P_{2051} = (0, 31, 1, 0) lies on line \ell_2
                                                                     208: P_{81985} = (0, 0, 19, 1) lies on line \ell_3
158: P_{2115} = (0, 32, 1, 0) lies on line \ell_2
                                                                     209: P_{86081} = (0, 0, 20, 1) lies on line \ell_3
159: P_{2179} = (0, 33, 1, 0) lies on line \ell_2
                                                                     210: P_{90177} = (0, 0, 21, 1) lies on line \ell_3
160: P_{2243} = (0, 34, 1, 0) lies on line \ell_2
                                                                     211: P_{94273} = (0, 0, 22, 1) lies on line \ell_3
161: P_{2307} = (0, 35, 1, 0) lies on line \ell_2
                                                                     212: P_{98369} = (0, 0, 23, 1) lies on line \ell_3
162: P_{2371} = (0, 36, 1, 0) lies on line \ell_2
                                                                     213: P_{102465} = (0, 0, 24, 1) lies on line \ell_3
163: P_{2435} = (0, 37, 1, 0) lies on line \ell_2
                                                                     214: P_{106561} = (0, 0, 25, 1) lies on line \ell_3
164: P_{2499} = (0, 38, 1, 0) lies on line \ell_2
                                                                     215: P_{110657} = (0, 0, 26, 1) lies on line \ell_3
165: P_{2563} = (0, 39, 1, 0) lies on line \ell_2
                                                                     216: P_{114753} = (0, 0, 27, 1) lies on line \ell_3
166: P_{2627} = (0, 40, 1, 0) lies on line \ell_2
                                                                     217: P_{118849} = (0, 0, 28, 1) lies on line \ell_3
167: P_{2691} = (0, 41, 1, 0) lies on line \ell_2
                                                                     218: P_{122945} = (0, 0, 29, 1) lies on line \ell_3
                                                                     219 : P_{127041} = (0, 0, 30, 1) lies on line \ell_3
168: P_{2755} = (0, 42, 1, 0) lies on line \ell_2
169: P_{2819} = (0, 43, 1, 0) lies on line \ell_2
                                                                     220: P_{131137} = (0, 0, 31, 1) lies on line \ell_3
                                                                     221 : P_{135233} = (0, 0, 32, 1) lies on line \ell_3
170: P_{2883} = (0, 44, 1, 0) lies on line \ell_2
                                                                     222: P_{139329} = (0, 0, 33, 1) lies on line \ell_3
171: P_{2947} = (0, 45, 1, 0) lies on line \ell_2
172: P_{3011} = (0, 46, 1, 0) lies on line \ell_2
                                                                     223: P_{143425} = (0, 0, 34, 1) lies on line \ell_3
173: P_{3075} = (0, 47, 1, 0) lies on line \ell_2
                                                                     224: P_{147521} = (0, 0, 35, 1) lies on line \ell_3
174: P_{3139} = (0, 48, 1, 0) lies on line \ell_2
                                                                     225: P_{151617} = (0, 0, 36, 1) lies on line \ell_3
                                                                     226 : P_{155713} = (0, 0, 37, 1) lies on line \ell_3
175: P_{3203} = (0, 49, 1, 0) lies on line \ell_2
176: P_{3267} = (0, 50, 1, 0) lies on line \ell_2
                                                                     227: P_{159809} = (0, 0, 38, 1) lies on line \ell_3
177: P_{3331} = (0, 51, 1, 0) lies on line \ell_2
                                                                     228: P_{163905} = (0, 0, 39, 1) lies on line \ell_3
178: P_{3395} = (0, 52, 1, 0) lies on line \ell_2
                                                                     229: P_{168001} = (0, 0, 40, 1) lies on line \ell_3
179: P_{3459} = (0, 53, 1, 0) lies on line \ell_2
                                                                     230: P_{172097} = (0, 0, 41, 1) lies on line \ell_3
180: P_{3523} = (0, 54, 1, 0) lies on line \ell_2
                                                                     231: P_{176193} = (0, 0, 42, 1) lies on line \ell_3
181: P_{3587} = (0, 55, 1, 0) lies on line \ell_2
                                                                     232: P_{180289} = (0, 0, 43, 1) lies on line \ell_3
182: P_{3651} = (0, 56, 1, 0) lies on line \ell_2
                                                                     233: P_{184385} = (0, 0, 44, 1) lies on line \ell_3
183 : P_{3715} = (0, 57, 1, 0) lies on line \ell_2
                                                                     234 : P_{188481} = (0, 0, 45, 1) lies on line \ell_3
184: P_{3779} = (0, 58, 1, 0) lies on line \ell_2
                                                                     235: P_{192577} = (0, 0, 46, 1) lies on line \ell_3
                                                                     236: P_{196673} = (0, 0, 47, 1) lies on line \ell_3
185: P_{3843} = (0, 59, 1, 0) lies on line \ell_2
186: P_{3907} = (0, 60, 1, 0) lies on line \ell_2
                                                                     237: P_{200769} = (0, 0, 48, 1) lies on line \ell_3
                                                                     238 : P_{204865} = (0, 0, 49, 1) lies on line \ell_3
187: P_{3971} = (0, 61, 1, 0) lies on line \ell_2
188: P_{4035} = (0, 62, 1, 0) lies on line \ell_2
                                                                     239: P_{208961} = (0, 0, 50, 1) lies on line \ell_3
189 : P_{4099} = (0, 63, 1, 0) lies on line \ell_2
                                                                     240: P_{213057} = (0, 0, 51, 1) lies on line \ell_3
190 : P_{8258} = (0, 0, 1, 1) lies on line \ell_3
                                                                     241: P_{217153} = (0, 0, 52, 1) lies on line \ell_3
191: P_{12353} = (0, 0, 2, 1) lies on line \ell_3
                                                                     242: P_{221249} = (0, 0, 53, 1) lies on line \ell_3
192: P_{16449} = (0,0,3,1) lies on line \ell_3
                                                                     243: P_{225345} = (0, 0, 54, 1) lies on line \ell_3
193: P_{20545} = (0, 0, 4, 1) lies on line \ell_3
                                                                     244: P_{229441} = (0, 0, 55, 1) lies on line \ell_3
194 : P_{24641} = (0, 0, 5, 1) lies on line \ell_3
                                                                     245 : P_{233537} = (0, 0, 56, 1) lies on line \ell_3
195: P_{28737} = (0, 0, 6, 1) lies on line \ell_3
                                                                     246: P_{237633} = (0, 0, 57, 1) lies on line \ell_3
196: P_{32833} = (0, 0, 7, 1) lies on line \ell_3
                                                                     247: P_{241729} = (0, 0, 58, 1) lies on line \ell_3
                                                                     248 : P_{245825} = (0, 0, 59, 1) lies on line \ell_3
197: P_{36929} = (0, 0, 8, 1) lies on line \ell_3
198: P_{41025} = (0, 0, 9, 1) lies on line \ell_3
                                                                     249: P_{249921} = (0, 0, 60, 1) lies on line \ell_3
199: P_{45121} = (0, 0, 10, 1) lies on line \ell_3
                                                                     250: P_{254017} = (0, 0, 61, 1) lies on line \ell_3
200: P_{49217} = (0, 0, 11, 1) lies on line \ell_3
                                                                     251: P_{258113} = (0, 0, 62, 1) lies on line \ell_3
201: P_{53313} = (0, 0, 12, 1) lies on line \ell_3
                                                                     252: P_{262209} = (0, 0, 63, 1) lies on line \ell_3
202: P_{57409} = (0, 0, 13, 1) lies on line \ell_3
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The single points on the surface are:

Points on surface but on no line

The surface has 3969 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c|c} 0123 \\ \hline 0 & 0110 \\ 1 & 1011 \\ 2 & 1101 \\ 3 & 0110 \\ \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

 $\begin{array}{|c|c|c|c|c|}\hline \text{Line} & \ell_1 & \ell_2\\\hline \text{in point} & P_0 & P_1\\\hline \end{array}$

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_1	P_2	P_2

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

The surface has 4225 points:

Too many to print.