Rank-139 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 139

General information

Number of lines	6
Number of points	11
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^6
Type of lines on points	3^2, 2^3, 1^6

Singular Points

The surface has 1 singular points:

$$0: \ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{2} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{2} = \mathbf{Pl}(1,0,0,0,1,0)_{10}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{12} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{9} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{9} = \mathbf{Pl}(1,1,0,0,1,1)_{29}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{15} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{15} = \mathbf{Pl}(1,1,1,1,0,0)_{8}$$

Rank of lines: (1, 2, 5, 12, 9, 15)
Rank of points on Klein quadric: (3, 10, 12, 32, 29, 8)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0),\ T = 4$
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1).\ T = -1$

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_7 = (0,1,1,0) = \ell_0 \cap \ell_5$
$P_{10} = (0,1,0,1) = \ell_1 \cap \ell_4$
$P_{12} = (0,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 6 single points:
The single points on the surface are:

$0 :\ P_5 = (1,1,0,0)$ lies on line ℓ_3
$1 :\ P_6 = (1,0,1,0)$ lies on line ℓ_4
$2 :\ P_8 = (1,1,1,0)$ lies on line ℓ_0
$3 :\ P_9 = (1,0,0,1)$ lies on line ℓ_5
$4 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_1
$5 :\ P_{13} = (1,0,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	0	0	1
1	1	0	1	0	1	0
2	1	1	0	1	0	0
3	0	0	1	0	1	1
4	0	1	0	1	0	1
5	1	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_0	P_0	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_0	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_0	P_0	P_{12}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5
in point	P_{12}	P_4	P_4

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{10}	P_4	P_4

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_7	P_4	P_4

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_4 = (1,1,1,1)$
$2: P_5 = (1,1,0,0)$
$3: P_6 = (1,0,1,0)$
$4: P_7 = (0,1,1,0)$
$5: P_8 = (1,1,1,0)$
$6: P_9 = (1,0,0,1)$
$7: P_{10} = (0,1,0,1)$
$8: P_{11} = (1,1,0,1)$
$9: P_{12} = (0,0,1,1)$
$10: P_{13} = (1,0,1,1)$