Rank-76355 over GF(4)

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The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1503225177

General information

Number of lines	7
Number of points	29
Number of singular points	0
Number of Eckardt points	1
Number of double points	6
Number of single points	20
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	3, 2^6, 1^{20}, 0^2

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{21} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{21} = \mathbf{Pl}(1,0,0,0,0,1)_{102}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_{6}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{68} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{68} = \mathbf{Pl}(1,1,1,0,2,1)_{240}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{47} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{47} = \mathbf{Pl}(1,1,1,0,3,1)_{300}$$

Rank of lines: (0, 16, 21, 356, 100, 68, 47)
Rank of points on Klein quadric: (0, 2, 102, 1, 6, 240, 300)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_2$
$P_9 = (2,0,1,0) = \ell_1 \cap \ell_5$
$P_{10} = (3,0,1,0) = \ell_1 \cap \ell_6$
$P_{42} = (0,1,1,1) = \ell_5 \cap \ell_6$

Single Points

The surface has 20 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_3
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_{12} = (1,1,1,0)$ lies on line ℓ_2
$5 : P_{16} = (1,2,1,0)$ lies on line ℓ_2
$6 : P_{20} = (1,3,1,0)$ lies on line ℓ_2
$7 : P_{23} = (1,0,0,1)$ lies on line ℓ_4
$8 : P_{28} = (2,1,0,1)$ lies on line ℓ_5
$9 : P_{29} = (3,1,0,1)$ lies on line ℓ_6
$10 : P_{38} = (0,0,1,1)$ lies on line ℓ_3
$11 : P_{39} = (1,0,1,1)$ lies on line ℓ_4
$12 : P_{53} = (0,0,2,1)$ lies on line ℓ_3
$13 : P_{54} = (1,0,2,1)$ lies on line ℓ_4
$14 : P_{58} = (1,1,2,1)$ lies on line ℓ_5
$15 : P_{59} = (2,1,2,1)$ lies on line ℓ_6
$16 : P_{69} = (0,0,3,1)$ lies on line ℓ_3
$17 : P_{70} = (1,0,3,1)$ lies on line ℓ_4
$18 : P_{74} = (1,1,3,1)$ lies on line ℓ_6
$19 : P_{76} = (3,1,3,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

$0:\ P_{65} = (0, 3, 2, 1)$
$1:\ P_{77} = (0, 2, 3, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 0 0 0 0
1	1 0 1 1 1 1 1
2	1 1 0 0 0 0 0
3	0 1 0 0 1 0 0
4	0 1 0 1 0 0 0
5	0 1 0 0 0 0 1
6	0 1 0 0 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_8	P_2	P_2	P_9	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_8

Line 3 intersects

Line	ℓ_1	ℓ_4
in point	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_2	P_2

Line 5 intersects

Line	ℓ_1	ℓ_6
in point	P_9	P_{42}

Line 6 intersects

Line	ℓ_1	ℓ_5
in point	P_{10}	P_{42}

The surface has 29 points:
The points on the surface are:

$0:\ P_0 = (1, 0, 0, 0)$
$1:\ P_1 = (0, 1, 0, 0)$
$2:\ P_2 = (0, 0, 1, 0)$
$3:\ P_3 = (0, 0, 0, 1)$
$4:\ P_5 = (1, 1, 0, 0)$
$5:\ P_6 = (2, 1, 0, 0)$
$6:\ P_7 = (3, 1, 0, 0)$
$7:\ P_8 = (1, 0, 1, 0)$
$8:\ P_9 = (2, 0, 1, 0)$
$9:\ P_{10} = (3, 0, 1, 0)$
$10:\ P_{12} = (1, 1, 1, 0)$
$11:\ P_{16} = (1, 2, 1, 0)$
$12:\ P_{20} = (1, 3, 1, 0)$
$13:\ P_{23} = (1, 0, 0, 1)$
$14:\ P_{28} = (2, 1, 0, 1)$
$15:\ P_{29} = (3, 1, 0, 1)$
$16:\ P_{38} = (0, 0, 1, 1)$
$17:\ P_{39} = (1, 0, 1, 1)$
$18:\ P_{42} = (0, 1, 1, 1)$
$19:\ P_{53} = (0, 0, 2, 1)$
$20:\ P_{54} = (1, 0, 2, 1)$

$21: P_{58} = (1, 1, 2, 1)$
$22: P_{59} = (2, 1, 2, 1)$
$23: P_{65} = (0, 3, 2, 1)$
$24: P_{69} = (0, 0, 3, 1)$
$25: P_{70} = (1, 0, 3, 1)$
$26: P_{74} = (1, 1, 3, 1)$
$27: P_{76} = (3, 1, 3, 1)$
$28: P_{77} = (0, 2, 3, 1)$