Rank-65839 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(16)$ is 287384085

General information

Number of lines	7
Number of points	305
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	105
Number of points off lines	195
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 ⁷
Type of lines on points	$4^2, 2^3, 1^{105}, 0^{195}$

Singular Points

The surface has 2 singular points:

$$0: P_{14} = \mathbf{P}(\delta^{10}, 1, 0, 0) = \mathbf{P}(10, 1, 0, 0) 1: P_{15} = \mathbf{P}(\delta^{5}, 1, 0, 0) = \mathbf{P}(11, 1, 0, 0)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \mathbf{Pl}(0, 0, 1, 1, 10, 1)_{46146}$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{3270} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{3270} = \mathbf{Pl}(0, 0, 10, 11, 1, 1)_{9705}$$

$$\ell_3 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3269} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3269} = \mathbf{Pl}(0, 0, 11, 10, 11, 1)_{50536}$$

$$\ell_4 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \mathbf{Pl}(0, 0, 1, 1, 11, 1)_{50226}$$

$$\ell_5 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{2997} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{2997} = \mathbf{Pl}(0, 0, 10, 11, 10, 1)_{46425}$$

$$\ell_6 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{2996} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{2996} = \mathbf{Pl}(0, 0, 11, 10, 1, 1)_{9736}$$

Rank of lines: (0, 3260, 3270, 3269, 2987, 2997, 2996)

Rank of points on Klein quadric: (0, 46146, 9705, 50536, 50226, 46425, 9736)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points: The double points on the surface are:

$$P_{530} = (0, 0, 1, 1) = \ell_1 \cap \ell_4$$

$$P_{2833} = (0, 0, 10, 1) = \ell_2 \cap \ell_5$$

 $P_{3089} = (0, 0, 11, 1) = \ell_3 \cap \ell_6$

Single Points

The surface has 105 single points: The single points on the surface are:

 $0: P_0 = (1,0,0,0)$ lies on line ℓ_0 15: $P_{555} = (10, 1, 1, 1)$ lies on line ℓ_1 1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0 16: $P_{556} = (11, 1, 1, 1)$ lies on line ℓ_4 2: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 17: $P_{574} = (13, 2, 1, 1)$ lies on line ℓ_1 3 : $P_6=(2,1,0,0)$ lies on line ℓ_0 18: $P_{576} = (15, 2, 1, 1)$ lies on line ℓ_4 4: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 19: $P_{581} = (4, 3, 1, 1)$ lies on line ℓ_4 5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 20: $P_{584} = (7, 3, 1, 1)$ lies on line ℓ_1 6: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 21: $P_{596} = (3, 4, 1, 1)$ lies on line ℓ_1 7: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 22: $P_{600} = (7, 4, 1, 1)$ lies on line ℓ_4 8: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 23: $P_{618} = (9, 5, 1, 1)$ lies on line ℓ_1 9: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 24: $P_{621} = (12, 5, 1, 1)$ lies on line ℓ_4 10: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 25: $P_{633} = (8, 6, 1, 1)$ lies on line ℓ_4 11: $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 26: $P_{639} = (14, 6, 1, 1)$ lies on line ℓ_1 12: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 27: $P_{644} = (3,7,1,1)$ lies on line ℓ_4 13 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0 28 : $P_{645} = (4,7,1,1)$ lies on line ℓ_1 14: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 29: $P_{663} = (6, 8, 1, 1)$ lies on line ℓ_1

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30: P_{671} = (14, 8, 1, 1) lies on line \ell_4
                                                                     68: P_{3034} = (9, 12, 10, 1) lies on line \ell_5
31: P_{678} = (5, 9, 1, 1) lies on line \ell_4
                                                                     69: P_{3043} = (2, 13, 10, 1) lies on line \ell_5
                                                                     70: P_{3056} = (15, 13, 10, 1) lies on line \ell_2
32: P_{685} = (12, 9, 1, 1) lies on line \ell_1
33: P_{690} = (1, 10, 1, 1) lies on line \ell_4
                                                                     71: P_{3063} = (6, 14, 10, 1) lies on line \ell_5
34: P_{700} = (11, 10, 1, 1) lies on line \ell_1
                                                                     72: P_{3065} = (8, 14, 10, 1) lies on line \ell_2
35: P_{706} = (1, 11, 1, 1) lies on line \ell_1
                                                                     73: P_{3075} = (2, 15, 10, 1) lies on line \ell_2
36: P_{715} = (10, 11, 1, 1) lies on line \ell_4
                                                                     74: P_{3086} = (13, 15, 10, 1) lies on line \ell_5
37: P_{726} = (5, 12, 1, 1) lies on line \ell_1
                                                                     75: P_{3115} = (10, 1, 11, 1) lies on line \ell_3
38: P_{730} = (9, 12, 1, 1) lies on line \ell_4
                                                                     76: P_{3116} = (11, 1, 11, 1) lies on line \ell_6
39: P_{739} = (2, 13, 1, 1) lies on line \ell_4
                                                                     77: P_{3134} = (13, 2, 11, 1) lies on line \ell_3
40: P_{752} = (15, 13, 1, 1) lies on line \ell_1
                                                                     78: P_{3136} = (15, 2, 11, 1) lies on line \ell_6
                                                                     79: P_{3141} = (4, 3, 11, 1) lies on line \ell_6
41: P_{759} = (6, 14, 1, 1) lies on line \ell_4
42: P_{761} = (8, 14, 1, 1) lies on line \ell_1
                                                                     80: P_{3144} = (7, 3, 11, 1) lies on line \ell_3
                                                                     81: P_{3156} = (3, 4, 11, 1) lies on line \ell_3
43: P_{771} = (2, 15, 1, 1) lies on line \ell_1
44: P_{782} = (13, 15, 1, 1) lies on line \ell_4
                                                                     82 : P_{3160} = (7, 4, 11, 1) lies on line \ell_6
45: P_{2859} = (10, 1, 10, 1) lies on line \ell_2
                                                                     83: P_{3178} = (9, 5, 11, 1) lies on line \ell_3
46: P_{2860} = (11, 1, 10, 1) lies on line \ell_5
                                                                     84: P_{3181} = (12, 5, 11, 1) lies on line \ell_6
47: P_{2878} = (13, 2, 10, 1) lies on line \ell_2
                                                                     85: P_{3193} = (8, 6, 11, 1) lies on line \ell_6
48: P_{2880} = (15, 2, 10, 1) lies on line \ell_5
                                                                     86: P_{3199} = (14, 6, 11, 1) lies on line \ell_3
49: P_{2885} = (4, 3, 10, 1) lies on line \ell_5
                                                                     87: P_{3204} = (3, 7, 11, 1) lies on line \ell_6
50: P_{2888} = (7, 3, 10, 1) lies on line \ell_2
                                                                     88: P_{3205} = (4, 7, 11, 1) lies on line \ell_3
                                                                     89: P_{3223} = (6, 8, 11, 1) lies on line \ell_3
51: P_{2900} = (3, 4, 10, 1) lies on line \ell_2
                                                                     90: P_{3231} = (14, 8, 11, 1) lies on line \ell_6
52: P_{2904} = (7, 4, 10, 1) lies on line \ell_5
                                                                     91 : P_{3238} = (5, 9, 11, 1) lies on line \ell_6
53: P_{2922} = (9, 5, 10, 1) lies on line \ell_2
54: P_{2925} = (12, 5, 10, 1) lies on line \ell_5
                                                                     92: P_{3245} = (12, 9, 11, 1) lies on line \ell_3
55: P_{2937} = (8, 6, 10, 1) lies on line \ell_5
                                                                     93: P_{3250} = (1, 10, 11, 1) lies on line \ell_6
56: P_{2943} = (14, 6, 10, 1) lies on line \ell_2
                                                                     94: P_{3260} = (11, 10, 11, 1) lies on line \ell_3
57: P_{2948} = (3,7,10,1) lies on line \ell_5
                                                                     95: P_{3266} = (1, 11, 11, 1) lies on line \ell_3
58: P_{2949} = (4,7,10,1) lies on line \ell_2
                                                                     96: P_{3275} = (10, 11, 11, 1) lies on line \ell_6
59: P_{2967} = (6, 8, 10, 1) lies on line \ell_2
                                                                     97: P_{3286} = (5, 12, 11, 1) lies on line \ell_3
60: P_{2975} = (14, 8, 10, 1) lies on line \ell_5
                                                                     98: P_{3290} = (9, 12, 11, 1) lies on line \ell_6
61: P_{2982} = (5, 9, 10, 1) lies on line \ell_5
                                                                     99: P_{3299} = (2, 13, 11, 1) lies on line \ell_6
62: P_{2989} = (12, 9, 10, 1) lies on line \ell_2
                                                                     100: P_{3312} = (15, 13, 11, 1) lies on line \ell_3
63: P_{2994} = (1, 10, 10, 1) lies on line \ell_5
                                                                     101: P_{3319} = (6, 14, 11, 1) lies on line \ell_6
64: P_{3004} = (11, 10, 10, 1) lies on line \ell_2
                                                                     102: P_{3321} = (8, 14, 11, 1) lies on line \ell_3
65: P_{3010} = (1, 11, 10, 1) lies on line \ell_2
                                                                     103: P_{3331} = (2, 15, 11, 1) lies on line \ell_3
66: P_{3019} = (10, 11, 10, 1) lies on line \ell_5
                                                                     104: P_{3342} = (13, 15, 11, 1) lies on line \ell_6
67: P_{3030} = (5, 12, 10, 1) lies on line \ell_2
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The single points on the surface are:

Points on surface but on no line

The surface has 195 points not on any line: The points on the surface but not on lines are:

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\begin{array}{lll} 0: \ P_{20} = (1,0,1,0) & 6: \ P_{97} = (14,4,1,0) \\ 1: \ P_{35} = (0,1,1,0) & 7: \ P_{165} = (2,9,1,0) \\ 2: \ P_{36} = (1,1,1,0) & 8: \ P_{174} = (11,9,1,0) \\ 3: \ P_{60} = (9,2,1,0) & 9: \ P_{183} = (4,10,1,0) \\ 4: \ P_{62} = (11,2,1,0) & 10: \ P_{193} = (14,10,1,0) \\ 5: \ P_{93} = (10,4,1,0) & 11: \ P_{197} = (2,11,1,0) \end{array}
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12: P_{204} = (9, 11, 1, 0)
                                                                  66: P_{1603} = (2, 3, 5, 1)
13: P_{247} = (4, 14, 1, 0)
                                                                  67: P_{1658} = (9, 6, 5, 1)
14: P_{253} = (10, 14, 1, 0)
                                                                  68: P_{1664} = (15, 6, 5, 1)
                                                                  69: P_{1703} = (6, 9, 5, 1)
15: P_{790} = (5, 0, 2, 1)
                                                                  70: P_{1712} = (15, 9, 5, 1)
16: P_{858} = (9, 4, 2, 1)
                                                                  71: P_{1761} = (0, 13, 5, 1)
17: P_{862} = (13, 4, 2, 1)
18: P_{865} = (0, 5, 2, 1)
                                                                  72: P_{1774} = (13, 13, 5, 1)
                                                                  73: P_{1799} = (6, 15, 5, 1)
19: P_{870} = (5, 5, 2, 1)
20: P_{891} = (10, 6, 2, 1)
                                                                  74: P_{1802} = (9, 15, 5, 1)
21: P_{893} = (12, 6, 2, 1)
                                                                  75: P_{1813} = (4, 0, 6, 1)
22: P_{933} = (4, 9, 2, 1)
                                                                  76: P_{1866} = (9, 3, 6, 1)
23: P_{942} = (13, 9, 2, 1)
                                                                  77: P_{1867} = (10, 3, 6, 1)
24: P_{951} = (6, 10, 2, 1)
                                                                  78: P_{1873} = (0, 4, 6, 1)
25: P_{957} = (12, 10, 2, 1)
                                                                  79: P_{1877} = (4, 4, 6, 1)
26: P_{983} = (6, 12, 2, 1)
                                                                  80: P_{1929} = (8, 7, 6, 1)
27: P_{987} = (10, 12, 2, 1)
                                                                  81: P_{1936} = (15, 7, 6, 1)
28: P_{997} = (4, 13, 2, 1)
                                                                  82: P_{1944} = (7, 8, 6, 1)
29: P_{1002} = (9, 13, 2, 1)
                                                                  83: P_{1952} = (15, 8, 6, 1)
30: P_{1047} = (6,0,3,1)
                                                                  84: P_{1956} = (3, 9, 6, 1)
31: P_{1071} = (14, 1, 3, 1)
                                                                  85: P_{1963} = (10, 9, 6, 1)
32: P_{1072} = (15, 1, 3, 1)
                                                                  86: P_{1972} = (3, 10, 6, 1)
33: P_{1113} = (8,4,3,1)
                                                                  87: P_{1978} = (9, 10, 6, 1)
34: P_{1117} = (12, 4, 3, 1)
                                                                  88: P_{2056} = (7, 15, 6, 1)
35: P_{1137} = (0,6,3,1)
                                                                  89: P_{2057} = (8, 15, 6, 1)
36: P_{1143} = (6,6,3,1)
                                                                  90: P_{2079} = (14, 0, 7, 1)
37: P_{1173} = (4, 8, 3, 1)
                                                                  91: P_{2105} = (8, 2, 7, 1)
38: P_{1181} = (12, 8, 3, 1)
                                                                  92: P_{2107} = (10, 2, 7, 1)
39: P_{1237} = (4, 12, 3, 1)
                                                                  93: P_{2118} = (5, 3, 7, 1)
                                                                  94: P_{2119} = (6, 3, 7, 1)
40: P_{1241} = (8, 12, 3, 1)
41: P_{1266} = (1, 14, 3, 1)
                                                                  95: P_{2148} = (3, 5, 7, 1)
42: P_{1280} = (15, 14, 3, 1)
                                                                  96: P_{2151} = (6, 5, 7, 1)
43: P_{1282} = (1, 15, 3, 1)
                                                                  97: P_{2164} = (3, 6, 7, 1)
44: P_{1295} = (14, 15, 3, 1)
                                                                  98: P_{2166} = (5, 6, 7, 1)
                                                                  99: P_{2195} = (2, 8, 7, 1)
45: P_{1305} = (8,0,4,1)
                                                                  100: P_{2203} = (10, 8, 7, 1)
46: P_{1404} = (11, 6, 4, 1)
47: P_{1406} = (13, 6, 4, 1)
                                                                  101: P_{2227} = (2, 10, 7, 1)
48: P_{1418} = (9,7,4,1)
                                                                  102: P_{2233} = (8, 10, 7, 1)
                                                                  103: P_{2289} = (0, 14, 7, 1)
49: P_{1423} = (14, 7, 4, 1)
50: P_{1425} = (0, 8, 4, 1)
                                                                  104: P_{2303} = (14, 14, 7, 1)
51: P_{1433} = (8, 8, 4, 1)
                                                                  105: P_{2328} = (7,0,8,1)
52: P_{1448} = (7, 9, 4, 1)
                                                                  106: P_{2341} = (4, 1, 8, 1)
                                                                  107: P_{2342} = (5, 1, 8, 1)
53: P_{1455} = (14, 9, 4, 1)
54: P_{1479} = (6, 11, 4, 1)
                                                                  108: P_{2382} = (13, 3, 8, 1)
                                                                  109: P_{2383} = (14, 3, 8, 1)
55: P_{1486} = (13, 11, 4, 1)
56: P_{1511} = (6, 13, 4, 1)
                                                                  110: P_{2386} = (1, 4, 8, 1)
                                                                  111: P_{2390} = (5, 4, 8, 1)
57: P_{1516} = (11, 13, 4, 1)
58: P_{1528} = (7, 14, 4, 1)
                                                                  112: P_{2402} = (1, 5, 8, 1)
59: P_{1530} = (9, 14, 4, 1)
                                                                  113: P_{2405} = (4, 5, 8, 1)
                                                                  114: P_{2433} = (0, 7, 8, 1)
60: P_{1566} = (13, 0, 5, 1)
61: P_{1571} = (2, 1, 5, 1)
                                                                  115: P_{2440} = (7,7,8,1)
62: P_{1572} = (3, 1, 5, 1)
                                                                  116: P_{2532} = (3, 13, 8, 1)
63: P_{1586} = (1, 2, 5, 1)
                                                                  117: P_{2543} = (14, 13, 8, 1)
64: P_{1588} = (3, 2, 5, 1)
                                                                  118: P_{2548} = (3, 14, 8, 1)
65: P_{1602} = (1, 3, 5, 1)
                                                                  119: P_{2558} = (13, 14, 8, 1)
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120: P_{2592} = (15, 0, 9, 1)
                                                                 158: P_{3791} = (14, 11, 13, 1)
121: P_{2621} = (12, 2, 9, 1)
                                                                 159: P_{3796} = (3, 12, 13, 1)
                                                                 160: P_{3808} = (15, 12, 13, 1)
122: P_{2623} = (14, 2, 9, 1)
123: P_{2699} = (10, 7, 9, 1)
                                                                 161: P_{3830} = (5, 14, 13, 1)
124: P_{2702} = (13, 7, 9, 1)
                                                                 162: P_{3836} = (11, 14, 13, 1)
125: P_{2744} = (7, 10, 9, 1)
                                                                 163: P_{3844} = (3, 15, 13, 1)
126: P_{2750} = (13, 10, 9, 1)
                                                                 164: P_{3853} = (12, 15, 13, 1)
127: P_{2771} = (2, 12, 9, 1)
                                                                 165: P_{3860} = (3, 0, 14, 1)
128: P_{2783} = (14, 12, 9, 1)
                                                                 166: P_{3893} = (4, 2, 14, 1)
129: P_{2792} = (7, 13, 9, 1)
                                                                 167: P_{3895} = (6, 2, 14, 1)
130: P_{2795} = (10, 13, 9, 1)
                                                                 168: P_{3905} = (0, 3, 14, 1)
131: P_{2803} = (2, 14, 9, 1)
                                                                 169: P_{3908} = (3, 3, 14, 1)
132: P_{2813} = (12, 14, 9, 1)
                                                                 170: P_{3923} = (2, 4, 14, 1)
133: P_{2817} = (0, 15, 9, 1)
                                                                 171: P_{3927} = (6, 4, 14, 1)
                                                                 172: P_{3955} = (2, 6, 14, 1)
134: P_{2832} = (15, 15, 9, 1)
135: P_{3347} = (2, 0, 12, 1)
                                                                 173: P_{3957} = (4, 6, 14, 1)
136: P_{3377} = (0, 2, 12, 1)
                                                                 174: P_{3980} = (11, 7, 14, 1)
137: P_{3379} = (2, 2, 12, 1)
                                                                 175: P_{3981} = (12, 7, 14, 1)
138: P_{3420} = (11, 4, 12, 1)
                                                                 176: P_{4040} = (7, 11, 14, 1)
139: P_{3424} = (15, 4, 12, 1)
                                                                 177: P_{4045} = (12, 11, 14, 1)
140: P_{3433} = (8, 5, 12, 1)
                                                                 178: P_{4056} = (7, 12, 14, 1)
141: P_{3438} = (13, 5, 12, 1)
                                                                 179: P_{4060} = (11, 12, 14, 1)
142: P_{3478} = (5, 8, 12, 1)
                                                                 180: P_{4125} = (12, 0, 15, 1)
143: P_{3486} = (13, 8, 12, 1)
                                                                 181: P_{4137} = (8, 1, 15, 1)
144: P_{3525} = (4, 11, 12, 1)
                                                                 182: P_{4138} = (9, 1, 15, 1)
145: P_{3536} = (15, 11, 12, 1)
                                                                 183: P_{4150} = (5, 2, 15, 1)
146: P_{3558} = (5, 13, 12, 1)
                                                                 184: P_{4152} = (7, 2, 15, 1)
147: P_{3561} = (8, 13, 12, 1)
                                                                 185 : P_{4195} = (2, 5, 15, 1)
                                                                 186: P_{4200} = (7, 5, 15, 1)
148: P_{3589} = (4, 15, 12, 1)
149: P_{3596} = (11, 15, 12, 1)
                                                                 187: P_{4227} = (2, 7, 15, 1)
150: P_{3610} = (9, 0, 13, 1)
                                                                 188: P_{4230} = (5, 7, 15, 1)
151: P_{3661} = (12, 3, 13, 1)
                                                                 189: P_{4242} = (1, 8, 15, 1)
152: P_{3664} = (15, 3, 13, 1)
                                                                 190: P_{4250} = (9, 8, 15, 1)
                                                                 191: P_{4258} = (1, 9, 15, 1)
153: P_{3692} = (11, 5, 13, 1)
154: P_{3695} = (14, 5, 13, 1)
                                                                 192: P_{4265} = (8, 9, 15, 1)
155: P_{3745} = (0, 9, 13, 1)
                                                                 193: P_{4305} = (0, 12, 15, 1)
156: P_{3754} = (9, 9, 13, 1)
                                                                 194: P_{4317} = (12, 12, 15, 1)
157: P_{3782} = (5, 11, 13, 1)
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Line Intersection Graph

	0123456
$\overline{0}$	0111111
1	$\begin{array}{c} 0111111\\ 101110\\ 110101\\ 1110001\\ 1100011 \end{array}$
2	1101010
3	1110001
4	1100011
5	1010101
6	$\begin{vmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{vmatrix}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{14}	P_{14}	P_{14}	P_{15}	P_{15}	P_{15}

Line	1	intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{14}	P_{14}	P_{14}	P_{530}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_{14}	P_{14}	P_{14}	P_{2833}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_{14}	P_{14}	P_{14}	P_{3089}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_{15}	P_{530}	P_{15}	P_{15}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_{15}	P_{2833}	P_{15}	P_{15}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_{15}	P_{3089}	P_{15}	P_{15}

The surface has 305 points: The points on the surface are:

$0: P_0 = (1,0,0,0)$	$30: P_{247} = (4, 14, 1, 0)$	$60: P_{761} = (8, 14, 1, 1)$
$1: P_1 = (0, 1, 0, 0)$	$31: P_{253} = (10, 14, 1, 0)$	$61: P_{771} = (2, 15, 1, 1)$
$2: P_5 = (1, 1, 0, 0)$	$32: P_{530} = (0, 0, 1, 1)$	$62: P_{782} = (13, 15, 1, 1)$
$3: P_6 = (2, 1, 0, 0)$	$33: P_{555} = (10, 1, 1, 1)$	$63: P_{790} = (5,0,2,1)$
$4: P_7 = (3, 1, 0, 0)$	$34: P_{556} = (11, 1, 1, 1)$	$64: P_{858} = (9, 4, 2, 1)$
$5: P_8 = (4, 1, 0, 0)$	$35: P_{574} = (13, 2, 1, 1)$	$65: P_{862} = (13, 4, 2, 1)$
$6: P_9 = (5, 1, 0, 0)$	$36: P_{576} = (15, 2, 1, 1)$	$66: P_{865} = (0, 5, 2, 1)$
7: $P_{10} = (6, 1, 0, 0)$	$37: P_{581} = (4,3,1,1)$	$67: P_{870} = (5, 5, 2, 1)$
$8: P_{11} = (7, 1, 0, 0)$	$38: P_{584} = (7, 3, 1, 1)$	$68: P_{891} = (10, 6, 2, 1)$
$9: P_{12} = (8, 1, 0, 0)$	$39: P_{596} = (3, 4, 1, 1)$	$69: P_{893} = (12, 6, 2, 1)$
$10: P_{13} = (9, 1, 0, 0)$	$40: P_{600} = (7, 4, 1, 1)$	$70: P_{933} = (4, 9, 2, 1)$
$11: P_{14} = (10, 1, 0, 0)$	$41: P_{618} = (9, 5, 1, 1)$	$71: P_{942} = (13, 9, 2, 1)$
$12: P_{15} = (11, 1, 0, 0)$	$42: P_{621} = (12, 5, 1, 1)$	$72: P_{951} = (6, 10, 2, 1)$
13: $P_{16} = (12, 1, 0, 0)$	$43: P_{633} = (8, 6, 1, 1)$	73: $P_{957} = (12, 10, 2, 1)$
$14: P_{17} = (13, 1, 0, 0)$	$44: P_{639} = (14, 6, 1, 1)$	$74: P_{983} = (6, 12, 2, 1)$
15: $P_{18} = (14, 1, 0, 0)$	$45: P_{644} = (3, 7, 1, 1)$	$75: P_{987} = (10, 12, 2, 1)$
$16: P_{19} = (15, 1, 0, 0)$	$46: P_{645} = (4,7,1,1)$	$76: P_{997} = (4, 13, 2, 1)$
17: $P_{20} = (1, 0, 1, 0)$	$47: P_{663} = (6, 8, 1, 1)$	$77: P_{1002} = (9, 13, 2, 1)$
$18: P_{35} = (0, 1, 1, 0)$	$48: P_{671} = (14, 8, 1, 1)$	$78: P_{1047} = (6, 0, 3, 1)$
$19: P_{36} = (1, 1, 1, 0)$	$49: P_{678} = (5, 9, 1, 1)$	$79: P_{1071} = (14, 1, 3, 1)$
$20: P_{60} = (9, 2, 1, 0)$	$50: P_{685} = (12, 9, 1, 1)$	$80: P_{1072} = (15, 1, 3, 1)$
$21: P_{62} = (11, 2, 1, 0)$	$51: P_{690} = (1, 10, 1, 1)$	$81: P_{1113} = (8,4,3,1)$
$22: P_{93} = (10, 4, 1, 0)$	$52: P_{700} = (11, 10, 1, 1)$	$82: P_{1117} = (12, 4, 3, 1)$
$23: P_{97} = (14, 4, 1, 0)$	$53: P_{706} = (1, 11, 1, 1)$	$83: P_{1137} = (0, 6, 3, 1)$
$24: P_{165} = (2, 9, 1, 0)$	$54: P_{715} = (10, 11, 1, 1)$	$84: P_{1143} = (6, 6, 3, 1)$
$25: P_{174} = (11, 9, 1, 0)$	$55: P_{726} = (5, 12, 1, 1)$	$85: P_{1173} = (4, 8, 3, 1)$
$26: P_{183} = (4, 10, 1, 0)$	$56: P_{730} = (9, 12, 1, 1)$	$86: P_{1181} = (12, 8, 3, 1)$
$27: P_{193} = (14, 10, 1, 0)$	$57: P_{739} = (2, 13, 1, 1)$	$87: P_{1237} = (4, 12, 3, 1)$
$28: P_{197} = (2, 11, 1, 0)$	$58: P_{752} = (15, 13, 1, 1)$	$88: P_{1241} = (8, 12, 3, 1)$
$29: P_{204} = (9, 11, 1, 0)$	$59: P_{759} = (6, 14, 1, 1)$	$89: P_{1266} = (1, 14, 3, 1)$

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198: P_{2967} = (6, 8, 10, 1)
90: P_{1280} = (15, 14, 3, 1)
                                           144: P_{2151} = (6, 5, 7, 1)
91: P_{1282} = (1, 15, 3, 1)
                                           145: P_{2164} = (3, 6, 7, 1)
                                                                                       199: P_{2975} = (14, 8, 10, 1)
                                                                                       200: P_{2982} = (5, 9, 10, 1)
92: P_{1295} = (14, 15, 3, 1)
                                           146: P_{2166} = (5, 6, 7, 1)
                                           147: P_{2195} = (2, 8, 7, 1)
                                                                                       201: P_{2989} = (12, 9, 10, 1)
93: P_{1305} = (8, 0, 4, 1)
94: P_{1404} = (11, 6, 4, 1)
                                           148: P_{2203} = (10, 8, 7, 1)
                                                                                      202: P_{2994} = (1, 10, 10, 1)
95: P_{1406} = (13, 6, 4, 1)
                                           149: P_{2227} = (2, 10, 7, 1)
                                                                                      203: P_{3004} = (11, 10, 10, 1)
96: P_{1418} = (9,7,4,1)
                                           150: P_{2233} = (8, 10, 7, 1)
                                                                                      204: P_{3010} = (1, 11, 10, 1)
97: P_{1423} = (14, 7, 4, 1)
                                           151: P_{2289} = (0, 14, 7, 1)
                                                                                       205: P_{3019} = (10, 11, 10, 1)
98: P_{1425} = (0, 8, 4, 1)
                                           152: P_{2303} = (14, 14, 7, 1)
                                                                                      206: P_{3030} = (5, 12, 10, 1)
99: P_{1433} = (8, 8, 4, 1)
                                           153: P_{2328} = (7, 0, 8, 1)
                                                                                       207: P_{3034} = (9, 12, 10, 1)
100: P_{1448} = (7, 9, 4, 1)
                                           154: P_{2341} = (4, 1, 8, 1)
                                                                                       208: P_{3043} = (2, 13, 10, 1)
101: P_{1455} = (14, 9, 4, 1)
                                           155: P_{2342} = (5, 1, 8, 1)
                                                                                       209: P_{3056} = (15, 13, 10, 1)
102: P_{1479} = (6, 11, 4, 1)
                                           156: P_{2382} = (13, 3, 8, 1)
                                                                                      210: P_{3063} = (6, 14, 10, 1)
103: P_{1486} = (13, 11, 4, 1)
                                           157: P_{2383} = (14, 3, 8, 1)
                                                                                       211: P_{3065} = (8, 14, 10, 1)
104: P_{1511} = (6, 13, 4, 1)
                                           158: P_{2386} = (1, 4, 8, 1)
                                                                                       212: P_{3075} = (2, 15, 10, 1)
105: P_{1516} = (11, 13, 4, 1)
                                           159: P_{2390} = (5, 4, 8, 1)
                                                                                       213: P_{3086} = (13, 15, 10, 1)
106: P_{1528} = (7, 14, 4, 1)
                                           160: P_{2402} = (1, 5, 8, 1)
                                                                                       214: P_{3089} = (0, 0, 11, 1)
                                           161: P_{2405} = (4, 5, 8, 1)
107: P_{1530} = (9, 14, 4, 1)
                                                                                       215: P_{3115} = (10, 1, 11, 1)
                                           162: P_{2433} = (0,7,8,1)
                                                                                       216: P_{3116} = (11, 1, 11, 1)
108: P_{1566} = (13, 0, 5, 1)
109: P_{1571} = (2, 1, 5, 1)
                                           163: P_{2440} = (7,7,8,1)
                                                                                       217: P_{3134} = (13, 2, 11, 1)
110: P_{1572} = (3, 1, 5, 1)
                                           164: P_{2532} = (3, 13, 8, 1)
                                                                                      218: P_{3136} = (15, 2, 11, 1)
                                                                                       219: P_{3141} = (4, 3, 11, 1)
111: P_{1586} = (1, 2, 5, 1)
                                           165: P_{2543} = (14, 13, 8, 1)
                                                                                       220: P_{3144} = (7, 3, 11, 1)
112: P_{1588} = (3, 2, 5, 1)
                                           166: P_{2548} = (3, 14, 8, 1)
113: P_{1602} = (1, 3, 5, 1)
                                           167: P_{2558} = (13, 14, 8, 1)
                                                                                      221: P_{3156} = (3, 4, 11, 1)
114: P_{1603} = (2, 3, 5, 1)
                                           168: P_{2592} = (15, 0, 9, 1)
                                                                                       222: P_{3160} = (7, 4, 11, 1)
115: P_{1658} = (9, 6, 5, 1)
                                           169: P_{2621} = (12, 2, 9, 1)
                                                                                       223: P_{3178} = (9, 5, 11, 1)
116: P_{1664} = (15, 6, 5, 1)
                                           170: P_{2623} = (14, 2, 9, 1)
                                                                                       224: P_{3181} = (12, 5, 11, 1)
117: P_{1703} = (6, 9, 5, 1)
                                           171: P_{2699} = (10, 7, 9, 1)
                                                                                       225: P_{3193} = (8, 6, 11, 1)
                                                                                       226: P_{3199} = (14, 6, 11, 1)
118: P_{1712} = (15, 9, 5, 1)
                                           172: P_{2702} = (13, 7, 9, 1)
119: P_{1761} = (0, 13, 5, 1)
                                           173: P_{2744} = (7, 10, 9, 1)
                                                                                       227: P_{3204} = (3,7,11,1)
120: P_{1774} = (13, 13, 5, 1)
                                           174: P_{2750} = (13, 10, 9, 1)
                                                                                       228: P_{3205} = (4,7,11,1)
121: P_{1799} = (6, 15, 5, 1)
                                           175: P_{2771} = (2, 12, 9, 1)
                                                                                       229: P_{3223} = (6, 8, 11, 1)
122: P_{1802} = (9, 15, 5, 1)
                                           176: P_{2783} = (14, 12, 9, 1)
                                                                                       230: P_{3231} = (14, 8, 11, 1)
123: P_{1813} = (4, 0, 6, 1)
                                           177: P_{2792} = (7, 13, 9, 1)
                                                                                       231: P_{3238} = (5, 9, 11, 1)
                                           178: P_{2795} = (10, 13, 9, 1)
124: P_{1866} = (9, 3, 6, 1)
                                                                                       232: P_{3245} = (12, 9, 11, 1)
125: P_{1867} = (10, 3, 6, 1)
                                           179: P_{2803} = (2, 14, 9, 1)
                                                                                      233: P_{3250} = (1, 10, 11, 1)
126: P_{1873} = (0, 4, 6, 1)
                                           180: P_{2813} = (12, 14, 9, 1)
                                                                                       234: P_{3260} = (11, 10, 11, 1)
127: P_{1877} = (4, 4, 6, 1)
                                                                                       235: P_{3266} = (1, 11, 11, 1)
                                           181: P_{2817} = (0, 15, 9, 1)
128: P_{1929} = (8,7,6,1)
                                           182: P_{2832} = (15, 15, 9, 1)
                                                                                      236: P_{3275} = (10, 11, 11, 1)
129: P_{1936} = (15, 7, 6, 1)
                                           183: P_{2833} = (0, 0, 10, 1)
                                                                                       237: P_{3286} = (5, 12, 11, 1)
130: P_{1944} = (7, 8, 6, 1)
                                           184: P_{2859} = (10, 1, 10, 1)
                                                                                       238: P_{3290} = (9, 12, 11, 1)
                                           185: P_{2860} = (11, 1, 10, 1)
                                                                                       239: P_{3299} = (2, 13, 11, 1)
131: P_{1952} = (15, 8, 6, 1)
132: P_{1956} = (3, 9, 6, 1)
                                           186: P_{2878} = (13, 2, 10, 1)
                                                                                      240: P_{3312} = (15, 13, 11, 1)
                                           187: P_{2880} = (15, 2, 10, 1)
                                                                                       241: P_{3319} = (6, 14, 11, 1)
133: P_{1963} = (10, 9, 6, 1)
134: P_{1972} = (3, 10, 6, 1)
                                           188: P_{2885} = (4, 3, 10, 1)
                                                                                       242: P_{3321} = (8, 14, 11, 1)
                                           189: P_{2888} = (7, 3, 10, 1)
                                                                                       243: P_{3331} = (2, 15, 11, 1)
135: P_{1978} = (9, 10, 6, 1)
136: P_{2056} = (7, 15, 6, 1)
                                           190: P_{2900} = (3, 4, 10, 1)
                                                                                      244: P_{3342} = (13, 15, 11, 1)
137: P_{2057} = (8, 15, 6, 1)
                                           191: P_{2904} = (7, 4, 10, 1)
                                                                                       245: P_{3347} = (2,0,12,1)
138: P_{2079} = (14, 0, 7, 1)
                                           192: P_{2922} = (9, 5, 10, 1)
                                                                                       246: P_{3377} = (0, 2, 12, 1)
                                           193: P_{2925} = (12, 5, 10, 1)
139: P_{2105} = (8, 2, 7, 1)
                                                                                       247: P_{3379} = (2, 2, 12, 1)
140: P_{2107} = (10, 2, 7, 1)
                                           194: P_{2937} = (8, 6, 10, 1)
                                                                                       248: P_{3420} = (11, 4, 12, 1)
141: P_{2118} = (5, 3, 7, 1)
                                           195: P_{2943} = (14, 6, 10, 1)
                                                                                       249: P_{3424} = (15, 4, 12, 1)
142: P_{2119} = (6, 3, 7, 1)
                                           196: P_{2948} = (3, 7, 10, 1)
                                                                                       250: P_{3433} = (8, 5, 12, 1)
143: P_{2148} = (3, 5, 7, 1)
                                           197: P_{2949} = (4,7,10,1)
                                                                                      251: P_{3438} = (13, 5, 12, 1)
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252: P_{3478} = (5, 8, 12, 1)
                                           270: P_{3808} = (15, 12, 13, 1)
                                                                                      288: P_{4056} = (7, 12, 14, 1)
253: P_{3486} = (13, 8, 12, 1)
                                           271: P_{3830} = (5, 14, 13, 1)
                                                                                      289: P_{4060} = (11, 12, 14, 1)
254: P_{3525} = (4, 11, 12, 1)
                                           272: P_{3836} = (11, 14, 13, 1)
                                                                                      290: P_{4125} = (12, 0, 15, 1)
255: P_{3536} = (15, 11, 12, 1)
                                           273: P_{3844} = (3, 15, 13, 1)
                                                                                      291: P_{4137} = (8, 1, 15, 1)
256: P_{3558} = (5, 13, 12, 1)
                                           274: P_{3853} = (12, 15, 13, 1)
                                                                                      292: P_{4138} = (9, 1, 15, 1)
                                                                                      293: P_{4150} = (5, 2, 15, 1)
257: P_{3561} = (8, 13, 12, 1)
                                           275: P_{3860} = (3, 0, 14, 1)
                                                                                      294: P_{4152} = (7, 2, 15, 1)
258: P_{3589} = (4, 15, 12, 1)
                                           276: P_{3893} = (4, 2, 14, 1)
                                           277: P_{3895} = (6, 2, 14, 1)
                                                                                      295: P_{4195} = (2, 5, 15, 1)
259: P_{3596} = (11, 15, 12, 1)
260: P_{3610} = (9, 0, 13, 1)
                                           278: P_{3905} = (0, 3, 14, 1)
                                                                                      296: P_{4200} = (7, 5, 15, 1)
261: P_{3661} = (12, 3, 13, 1)
                                           279: P_{3908} = (3, 3, 14, 1)
                                                                                      297: P_{4227} = (2, 7, 15, 1)
262: P_{3664} = (15, 3, 13, 1)
                                           280: P_{3923} = (2, 4, 14, 1)
                                                                                      298: P_{4230} = (5, 7, 15, 1)
263: P_{3692} = (11, 5, 13, 1)
                                           281: P_{3927} = (6, 4, 14, 1)
                                                                                      299: P_{4242} = (1, 8, 15, 1)
264: P_{3695} = (14, 5, 13, 1)
                                           282: P_{3955} = (2, 6, 14, 1)
                                                                                      300: P_{4250} = (9, 8, 15, 1)
                                           283: P_{3957} = (4, 6, 14, 1)
265: P_{3745} = (0, 9, 13, 1)
                                                                                      301: P_{4258} = (1, 9, 15, 1)
266: P_{3754} = (9, 9, 13, 1)
                                           284: P_{3980} = (11, 7, 14, 1)
                                                                                      302: P_{4265} = (8, 9, 15, 1)
267: P_{3782} = (5, 11, 13, 1)
                                           285: P_{3981} = (12, 7, 14, 1)
                                                                                      303: P_{4305} = (0, 12, 15, 1)
                                                                                      304: P_{4317} = (12, 12, 15, 1)
268: P_{3791} = (14, 11, 13, 1)
                                           286: P_{4040} = (7, 11, 14, 1)
269: P_{3796} = (3, 12, 13, 1)
                                           287: P_{4045} = (12, 11, 14, 1)
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