Rank-355 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 2663509

General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	23
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$2^2, 1^{23}, 0^{56}$

Singular Points

The surface has 1 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

Rank of lines: (64, 9, 648)
Rank of points on Klein quadric: (2, 97, 10)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 23 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_1
1 : $P_{12} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{13} = (2,0,1,0)$ lies on line ℓ_0
3 : $P_{14} = (3,0,1,0)$ lies on line ℓ_0
4 : $P_{15} = (4,0,1,0)$ lies on line ℓ_0
5 : $P_{16} = (5,0,1,0)$ lies on line ℓ_0
6 : $P_{17} = (6,0,1,0)$ lies on line ℓ_0
7 : $P_{18} = (7,0,1,0)$ lies on line ℓ_0
8 : $P_{75} = (1,0,0,1)$ lies on line ℓ_2
9 : $P_{139} = (1,0,1,1)$ lies on line ℓ_2
10 : $P_{146} = (0,1,1,1)$ lies on line ℓ_1
11 : $P_{147} = (2,1,1,1)$ lies on line ℓ_1

12 : $P_{148} = (3,1,1,1)$ lies on line ℓ_1
13 : $P_{149} = (4,1,1,1)$ lies on line ℓ_1
14 : $P_{150} = (5,1,1,1)$ lies on line ℓ_1
15 : $P_{151} = (6,1,1,1)$ lies on line ℓ_1
16 : $P_{152} = (7,1,1,1)$ lies on line ℓ_1
17 : $P_{202} = (1,0,2,1)$ lies on line ℓ_2
18 : $P_{266} = (1,0,3,1)$ lies on line ℓ_2
19 : $P_{330} = (1,0,4,1)$ lies on line ℓ_2
20 : $P_{394} = (1,0,5,1)$ lies on line ℓ_2
21 : $P_{458} = (1,0,6,1)$ lies on line ℓ_2
22 : $P_{522} = (1,0,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_1 = (0,1,0,0)$
1 : $P_{20} = (1,1,1,0)$
2 : $P_{34} = (7,2,1,0)$
3 : $P_{41} = (6,3,1,0)$

4 : $P_{45} = (2,4,1,0)$
5 : $P_{54} = (3,5,1,0)$
6 : $P_{64} = (5,6,1,0)$
7 : $P_{71} = (4,7,1,0)$

2

$8 : P_{92} = (2, 2, 0, 1)$
$9 : P_{103} = (5, 3, 0, 1)$
$10 : P_{110} = (4, 4, 0, 1)$
$11 : P_{120} = (6, 5, 0, 1)$
$12 : P_{125} = (3, 6, 0, 1)$
$13 : P_{137} = (7, 7, 0, 1)$
$14 : P_{159} = (6, 2, 1, 1)$
$15 : P_{168} = (7, 3, 1, 1)$
$16 : P_{172} = (3, 4, 1, 1)$
$17 : P_{179} = (2, 5, 1, 1)$
$18 : P_{189} = (4, 6, 1, 1)$
$19 : P_{198} = (5, 7, 1, 1)$
$20 : P_{221} = (4, 2, 2, 1)$
$21 : P_{231} = (6, 3, 2, 1)$
$22 : P_{239} = (6, 4, 2, 1)$
$23 : P_{241} = (0, 5, 2, 1)$
$24 : P_{250} = (1, 6, 2, 1)$
$25 : P_{261} = (4, 7, 2, 1)$
$26 : P_{281} = (0, 2, 3, 1)$
$27 : P_{293} = (4, 3, 3, 1)$
$28 : P_{298} = (1, 4, 3, 1)$
$29 : P_{309} = (4, 5, 3, 1)$
$30 : P_{319} = (6, 6, 3, 1)$
$31 : P_{327} = (6, 7, 3, 1)$
$32 : P_{352} = (7, 2, 4, 1)$

$33 : P_{354} = (1, 3, 4, 1)$
$34 : P_{368} = (7, 4, 4, 1)$
$35 : P_{372} = (3, 5, 4, 1)$
$36 : P_{377} = (0, 6, 4, 1)$
$37 : P_{388} = (3, 7, 4, 1)$
$38 : P_{412} = (3, 2, 5, 1)$
$39 : P_{420} = (3, 3, 5, 1)$
$40 : P_{425} = (0, 4, 5, 1)$
$41 : P_{440} = (7, 5, 5, 1)$
$42 : P_{448} = (7, 6, 5, 1)$
$43 : P_{450} = (1, 7, 5, 1)$
$44 : P_{474} = (1, 2, 6, 1)$
$45 : P_{483} = (2, 3, 6, 1)$
$46 : P_{494} = (5, 4, 6, 1)$
$47 : P_{502} = (5, 5, 6, 1)$
$48 : P_{507} = (2, 6, 6, 1)$
$49 : P_{513} = (0, 7, 6, 1)$
$50 : P_{542} = (5, 2, 7, 1)$
$51 : P_{545} = (0, 3, 7, 1)$
$52 : P_{555} = (2, 4, 7, 1)$
$53 : P_{562} = (1, 5, 7, 1)$
$54 : P_{574} = (5, 6, 7, 1)$
$55 : P_{579} = (2, 7, 7, 1)$

Line Intersection Graph

```
  |0 1 2
 ─────────
0 |0 1 1
1 |1 0 0
2 |1 0 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_2

Line 1 intersects

Line	ℓ_0
in point	P_0

Line 2 intersects

Line	ℓ_0
in point	P_2

The surface has 81 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
$1 : P_1 = (0, 1, 0, 0)$
$2 : P_2 = (0, 0, 1, 0)$
$3 : P_4 = (1, 1, 1, 1)$
$4 : P_{12} = (1, 0, 1, 0)$
$5 : P_{13} = (2, 0, 1, 0)$
$6 : P_{14} = (3, 0, 1, 0)$

$7 : P_{15} = (4, 0, 1, 0)$
$8 : P_{16} = (5, 0, 1, 0)$
$9 : P_{17} = (6, 0, 1, 0)$
$10 : P_{18} = (7, 0, 1, 0)$
$11 : P_{20} = (1, 1, 1, 0)$
$12 : P_{34} = (7, 2, 1, 0)$
$13 : P_{41} = (6, 3, 1, 0)$

$14 : P_{45} = (2, 4, 1, 0)$
$15 : P_{54} = (3, 5, 1, 0)$
$16 : P_{64} = (5, 6, 1, 0)$
$17 : P_{71} = (4, 7, 1, 0)$
$18 : P_{75} = (1, 0, 0, 1)$
$19 : P_{92} = (2, 2, 0, 1)$
$20 : P_{103} = (5, 3, 0, 1)$

3

$21: P_{110} = (4, 4, 0, 1)$

$22: P_{120} = (6, 5, 0, 1)$

$23: P_{125} = (3, 6, 0, 1)$

$24: P_{137} = (7, 7, 0, 1)$

$25: P_{139} = (1, 0, 1, 1)$

$26: P_{146} = (0, 1, 1, 1)$

$27: P_{147} = (2, 1, 1, 1)$

$28: P_{148} = (3, 1, 1, 1)$

$29: P_{149} = (4, 1, 1, 1)$

$30: P_{150} = (5, 1, 1, 1)$

$31: P_{151} = (6, 1, 1, 1)$

$32: P_{152} = (7, 1, 1, 1)$

$33: P_{159} = (6, 2, 1, 1)$

$34: P_{168} = (7, 3, 1, 1)$

$35: P_{172} = (3, 4, 1, 1)$

$36: P_{179} = (2, 5, 1, 1)$

$37: P_{189} = (4, 6, 1, 1)$

$38: P_{198} = (5, 7, 1, 1)$

$39: P_{202} = (1, 0, 2, 1)$

$40: P_{221} = (4, 2, 2, 1)$

$41: P_{231} = (6, 3, 2, 1)$

$42: P_{239} = (6, 4, 2, 1)$

$43: P_{241} = (0, 5, 2, 1)$

$44: P_{250} = (1, 6, 2, 1)$

$45: P_{261} = (4, 7, 2, 1)$

$46: P_{266} = (1, 0, 3, 1)$

$47: P_{281} = (0, 2, 3, 1)$

$48: P_{293} = (4, 3, 3, 1)$

$49: P_{298} = (1, 4, 3, 1)$

$50: P_{309} = (4, 5, 3, 1)$

$51: P_{319} = (6, 6, 3, 1)$

$52: P_{327} = (6, 7, 3, 1)$

$53: P_{330} = (1, 0, 4, 1)$

$54: P_{352} = (7, 2, 4, 1)$

$55: P_{354} = (1, 3, 4, 1)$

$56: P_{368} = (7, 4, 4, 1)$

$57: P_{372} = (3, 5, 4, 1)$

$58: P_{377} = (0, 6, 4, 1)$

$59: P_{388} = (3, 7, 4, 1)$

$60: P_{394} = (1, 0, 5, 1)$

$61: P_{412} = (3, 2, 5, 1)$

$62: P_{420} = (3, 3, 5, 1)$

$63: P_{425} = (0, 4, 5, 1)$

$64: P_{440} = (7, 5, 5, 1)$

$65: P_{448} = (7, 6, 5, 1)$

$66: P_{450} = (1, 7, 5, 1)$

$67: P_{458} = (1, 0, 6, 1)$

$68: P_{474} = (1, 2, 6, 1)$

$69: P_{483} = (2, 3, 6, 1)$

$70: P_{494} = (5, 4, 6, 1)$

$71: P_{502} = (5, 5, 6, 1)$

$72: P_{507} = (2, 6, 6, 1)$

$73: P_{513} = (0, 7, 6, 1)$

$74: P_{522} = (1, 0, 7, 1)$

$75: P_{542} = (5, 2, 7, 1)$

$76: P_{545} = (0, 3, 7, 1)$

$77: P_{555} = (2, 4, 7, 1)$

$78: P_{562} = (1, 5, 7, 1)$

$79: P_{574} = (5, 6, 7, 1)$

$80: P_{579} = (2, 7, 7, 1)$