Rank-74051 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74051

General information

Number of lines	10
Number of points	13
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	1
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{10}
Type of lines on points	5, 3^2, 2^9, 1

Singular Points

The surface has 1 singular points:

$0 :\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0,0,1,0,0,1)_{22}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \mathbf{Pl}(0,1,1,0,0,1)_{24}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,1,0,0,1,1)_{29}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

Rank of lines: (0, 4, 11, 30, 14, 34, 25, 18, 9, 23)
Rank of points on Klein quadric: (0, 2, 22, 5, 6, 1, 24, 4, 29, 31)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$, $T = 2$
$1 : P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$. $T = 9$

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_{11} = (1,1,0,1) = \ell_4 \cap \ell_6$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$ $P_9 = (1,0,0,1) = \ell_4 \cap \ell_7$
$P_6 = (1,0,1,0) = \ell_1 \cap \ell_8$ $P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$
$P_8 = (1,1,1,0) = \ell_2 \cap \ell_9$ $P_{13} = (1,0,1,1) = \ell_7 \cap \ell_9$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_5$

Single Points

The surface has 1 single points:
The single points on the surface are:

$0: P_{12} = (0, 0, 1, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7 8 9
--+-------------------
0 |0 1 1 1 1 0 0 0 0 0
1 |1 0 1 0 0 1 1 1 1 0
2 |1 1 0 0 0 1 1 1 0 1
3 |1 0 0 0 1 1 0 0 1 1
4 |1 0 0 1 0 0 1 1 0 0
5 |0 1 1 1 0 0 1 1 0 0
6 |0 1 1 0 1 1 0 1 1 0
7 |0 1 1 0 1 1 1 0 0 1
8 |0 1 0 1 0 0 1 0 0 1
9 |0 0 1 1 0 0 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_2	P_2	P_2	P_2	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_5	P_2	P_2	P_2	P_2	P_8

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{10}	P_{10}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7
in point	P_1	P_1	P_{11}	P_9

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_2	P_3	P_2	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_{11}	P_2	P_2	P_4

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_2	P_2	P_9	P_2	P_2	P_{13}

3

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_9
in point	P_6	P_{10}	P_4	P_{10}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_8	P_{10}	P_{13}	P_{10}

The surface has 13 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$

$1 : P_1 = (0, 1, 0, 0)$

$2 : P_2 = (0, 0, 1, 0)$

$3 : P_3 = (0, 0, 0, 1)$

$4 : P_4 = (1, 1, 1, 1)$

$5 : P_5 = (1, 1, 0, 0)$

$6 : P_6 = (1, 0, 1, 0)$

$7 : P_8 = (1, 1, 1, 0)$

$8 : P_9 = (1, 0, 0, 1)$

$9 : P_{10} = (0, 1, 0, 1)$

$10 : P_{11} = (1, 1, 0, 1)$

$11 : P_{12} = (0, 0, 1, 1)$

$12 : P_{13} = (1, 0, 1, 1)$