Rank-156 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_0 X_1^2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(2)$ is 156

General information

Number of lines	19
Number of points	15
Number of singular points	3
Number of Eckardt points	12
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{19}
Type of lines on points	$7^3, 3^{12}$

Singular Points

The surface has 3 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$ $2: \ P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$

$1: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 19 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0,0,1,0,0,1)_{22}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6} = \mathbf{Pl}(0,0,0,0,1,0)_{9}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{5} = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_{5}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_7 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \mathbf{Pl}(0,1,1,0,0,1)_{24}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_{4}$$

$$\ell_{10} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0,1,0,0,0,1)_{21}$$

$$\ell_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

$$\ell_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0,1,0,0,1,0)_{11}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{19} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{19} = \mathbf{Pl}(0,1,1,0,1,0)_{14}$$

$$\ell_{15} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_{7}$$

$$\ell_{16} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0,1,0,1,0,1)_{27}$$

$$\ell_{17} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0,1,0,1,1,0)_{17}$$

$$\ell_{18} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(0,1,1,1,1,1)_{34}$$

Rank of lines: (4, 11, 28, 6, 5, 30, 29, 34, 25, 18, 31, 13, 12, 20, 19, 33, 32, 27, 26)
Rank of points on Klein quadric: (2, 22, 19, 9, 12, 5, 25, 1, 24, 4, 21, 15, 32, 11, 14, 7, 27, 17, 34)

Eckardt Points

The surface has 12 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0),\ T = 6$
$1 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),\ T = 14$
$2 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1),\ T = 13$
$3 : P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0),\ T = 13$
$4 : P_6 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0),\ T = 6$
$5 : P_7 = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0),\ T = 14$
$6 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0),\ T = 13$
$7 : P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1),\ T = 6$
$8 : P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1),\ T = 14$
$9 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1),\ T = 13$
$10 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1),\ T = 6$
$11 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 14$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	1	1	1	1	0	0	1	1	1	1	0	0	1	1	0	0	0	0
1	1	0	1	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1	1
2	1	1	0	0	0	1	1	1	1	1	1	0	0	0	0	1	1	0	0
3	1	0	0	0	1	1	0	1	0	1	0	1	0	1	1	1	0	1	0
4	1	0	0	1	0	0	1	1	0	1	0	0	1	1	1	0	1	0	1
5	0	0	1	1	0	0	1	1	0	0	1	1	0	1	0	1	1	1	0
6	0	0	1	0	1	1	0	1	0	0	1	0	1	0	1	1	1	0	1
7	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	0	0	0	0	1	0	1	1	1	1	0	0	0	0	1	1
9	1	1	1	1	1	0	0	1	1	0	1	0	0	1	1	0	0	0	0
10	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	0
11	0	1	0	1	0	1	0	1	1	0	0	0	1	1	0	1	0	1	1
12	0	1	0	0	1	0	1	1	1	0	0	1	0	0	1	0	1	1	1
13	1	0	0	1	1	1	0	1	0	1	0	1	0	0	1	1	0	1	0
14	1	0	0	1	1	0	1	1	0	1	0	0	1	1	0	0	1	0	1
15	0	0	1	1	0	1	1	1	0	0	1	1	0	1	0	0	1	1	0
16	0	0	1	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	1
17	0	1	0	1	0	1	0	1	1	0	0	1	1	1	0	1	0	0	1
18	0	1	0	0	1	0	1	1	1	0	0	1	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}
in point	P_2	P_2	P_0	P_0	P_2	P_2	P_2	P_2	P_6	P_6

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}
in point	P_2	P_2	P_2	P_2	P_2	P_2	P_5	P_5	P_8	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{15}	ℓ_{16}
in point	P_2	P_2	P_1	P_1	P_2	P_2	P_2	P_2	P_7	P_7

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_0	P_0	P_3	P_3	P_9	P_3	P_3	P_9	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{18}
in point	P_0	P_0	P_{12}	P_{12}	P_{13}	P_{12}	P_{13}	P_{12}	P_{12}	P_{12}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}
in point	P_1	P_3	P_1	P_3	P_{10}	P_3	P_3	P_3	P_{10}	P_3

Line 6 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}
in point	P_1	P_{12}	P_1	P_{12}	P_{14}	P_{12}	P_{12}	P_{14}	P_{12}	P_{12}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}
in point	P_2	P_2	P_2	P_3	P_{12}	P_3	P_{12}	P_2	P_2	P_2	P_3	P_{12}	P_3	P_{12}	P_3	P_{12}	P_3	P_{12}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}
in point	P_2	P_2	P_2	P_2	P_2	P_2	P_{11}	P_4	P_4	P_{11}

4

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{13}	ℓ_{14}
in point	P_2	P_2	P_2	P_9	P_{13}	P_2	P_2	P_2	P_{13}	P_9

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{15}	ℓ_{16}
in point	P_2	P_2	P_2	P_{10}	P_{14}	P_2	P_2	P_2	P_{14}	P_{10}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{18}
in point	P_5	P_3	P_3	P_3	P_{11}	P_5	P_3	P_3	P_3	P_{11}

Line 12 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{18}
in point	P_5	P_{12}	P_{12}	P_{12}	P_4	P_5	P_{12}	P_{12}	P_4	P_{12}

Line 13 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_6	P_3	P_{13}	P_3	P_3	P_{13}	P_3	P_6	P_3	P_3

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{18}
in point	P_6	P_9	P_{12}	P_{12}	P_{12}	P_9	P_{12}	P_6	P_{12}	P_{12}

Line 15 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{16}	ℓ_{17}
in point	P_7	P_3	P_3	P_{14}	P_3	P_{14}	P_3	P_3	P_7	P_3

Line 16 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}
in point	P_7	P_{12}	P_{10}	P_{12}	P_{12}	P_{10}	P_{12}	P_{12}	P_7	P_{12}

Line 17 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{18}
in point	P_8	P_3	P_3	P_3	P_4	P_3	P_4	P_3	P_3	P_8

Line 18 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{17}
in point	P_8	P_{12}	P_{12}	P_{12}	P_{11}	P_{11}	P_{12}	P_{12}	P_{12}	P_8

The surface has 15 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$

1 : $P_1 = (0, 1, 0, 0)$

2 : $P_2 = (0, 0, 1, 0)$

3 : $P_3 = (0, 0, 0, 1)$

4 : $P_4 = (1, 1, 1, 1)$

5 : $P_5 = (1, 1, 0, 0)$

6 : $P_6 = (1, 0, 1, 0)$

7 : $P_7 = (0, 1, 1, 0)$

8 : $P_8 = (1, 1, 1, 0)$

9 : $P_9 = (1, 0, 0, 1)$

10 : $P_{10} = (0, 1, 0, 1)$

11 : $P_{11} = (1, 1, 0, 1)$

12 : $P_{12} = (0, 0, 1, 1)$

13 : $P_{13} = (1, 0, 1, 1)$

14 : $P_{14} = (0, 1, 1, 1)$