Rank-65865 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172949

General information

Number of lines	6
Number of points	89
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	39
Number of points off lines	43
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$3, 2^6, 1^{39}, 0^{43}$

Singular Points

The surface has 1 singular points:

$0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \mathbf{Pl}(0,6,0,1,0,0)_{30}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma \\ 0 & 1 & \gamma^6 & \gamma^4 \end{bmatrix}_{1449} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 6 & 7 \end{bmatrix}_{1449} = \mathbf{Pl}(4,3,7,5,6,1)_{4209}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \mathbf{Pl}(0,3,0,1,0,0)_{27}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^2 \\ 0 & 1 & \gamma^5 & \gamma \end{bmatrix}_{2720} = \begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{2720} = \mathbf{Pl}(7,5,2,6,3,1)_{2504}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^4 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{bmatrix}_{4563} = \begin{bmatrix} 1 & 0 & 6 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4563} = \mathbf{Pl}(2,6,4,3,5,1)_{3598}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \mathbf{Pl}(0,5,0,1,0,0)_{29}$$

Rank of lines: (4734, 1449, 4707, 2720, 4563, 4725)
Rank of points on Klein quadric: (30, 4209, 27, 2504, 3598, 29)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{369} = (0,5,4,1) = \ell_0 \cap \ell_1$ $P_{312} = (7,5,3,1) = \ell_3 \cap \ell_4$
$P_{485} = (4,3,6,1) = \ell_1 \cap \ell_3$ $P_{225} = (0,3,2,1) = \ell_4 \cap \ell_5$
$P_{443} = (2,6,5,1) = \ell_1 \cap \ell_4$
$P_{569} = (0,6,7,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 39 single points:
The single points on the surface are:

$0 : P_{27} = (0,2,1,0)$ lies on line ℓ_0 $15 : P_{204} = (3,0,2,1)$ lies on line ℓ_3
$1 : P_{37} = (2,3,1,0)$ lies on line ℓ_1 $16 : P_{218} = (1,2,2,1)$ lies on line ℓ_1
$2 : P_{43} = (0,4,1,0)$ lies on line ℓ_2 $17 : P_{233} = (0,4,2,1)$ lies on line ℓ_0
$3 : P_{55} = (4,5,1,0)$ lies on line ℓ_3 $18 : P_{241} = (0,5,2,1)$ lies on line ℓ_2
$4 : P_{66} = (7,6,1,0)$ lies on line ℓ_4 $19 : P_{273} = (0,1,3,1)$ lies on line ℓ_2
$5 : P_{67} = (0,7,1,0)$ lies on line ℓ_5 $20 : P_{276} = (3,1,3,1)$ lies on line ℓ_1
$6 : P_{93} = (3,2,0,1)$ lies on line ℓ_4 $21 : P_{297} = (0,4,3,1)$ lies on line ℓ_5
$7 : P_{111} = (5,4,0,1)$ lies on line ℓ_1 $22 : P_{313} = (0,6,3,1)$ lies on line ℓ_0
$8 : P_{136} = (6,7,0,1)$ lies on line ℓ_3 $23 : P_{334} = (5,0,4,1)$ lies on line ℓ_4
$9 : P_{153} = (0,2,1,1)$ lies on line ℓ_0 $24 : P_{362} = (1,4,4,1)$ lies on line ℓ_3
$10 : P_{155} = (2,2,1,1)$ lies on line ℓ_3 $25 : P_{377} = (0,6,4,1)$ lies on line ℓ_5
$11 : P_{169} = (0,4,1,1)$ lies on line ℓ_2 $26 : P_{385} = (0,7,4,1)$ lies on line ℓ_2
$12 : P_{173} = (4,4,1,1)$ lies on line ℓ_4 $27 : P_{401} = (0,1,5,1)$ lies on line ℓ_5
$13 : P_{193} = (0,7,1,1)$ lies on line ℓ_5 $28 : P_{406} = (5,1,5,1)$ lies on line ℓ_3
$14 : P_{200} = (7,7,1,1)$ lies on line ℓ_1 $29 : P_{417} = (0,3,5,1)$ lies on line ℓ_2

30 : $P_{449} = (0,7,5,1)$ lies on line ℓ_0
31 : $P_{465} = (0,1,6,1)$ lies on line ℓ_0
32 : $P_{471} = (6,1,6,1)$ lies on line ℓ_4
33 : $P_{473} = (0,2,6,1)$ lies on line ℓ_2
34 : $P_{497} = (0,5,6,1)$ lies on line ℓ_5

35 : $P_{527} = (6,0,7,1)$ lies on line ℓ_1
36 : $P_{537} = (0,2,7,1)$ lies on line ℓ_5
37 : $P_{545} = (0,3,7,1)$ lies on line ℓ_0
38 : $P_{578} = (1,7,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 43 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_{20} = (1,1,1,0)$
2 : $P_{83} = (1,1,0,1)$
3 : $P_{105} = (7,3,0,1)$
4 : $P_{116} = (2,5,0,1)$
5 : $P_{126} = (4,6,0,1)$
6 : $P_{139} = (1,0,1,1)$
7 : $P_{165} = (4,3,1,1)$
8 : $P_{168} = (7,3,1,1)$
9 : $P_{179} = (2,5,1,1)$
10 : $P_{184} = (7,5,1,1)$
11 : $P_{187} = (2,6,1,1)$
12 : $P_{189} = (4,6,1,1)$
13 : $P_{222} = (5,2,2,1)$
14 : $P_{231} = (6,3,2,1)$
15 : $P_{238} = (5,4,2,1)$
16 : $P_{248} = (7,5,2,1)$
17 : $P_{258} = (1,7,2,1)$
18 : $P_{259} = (2,7,2,1)$
19 : $P_{272} = (7,0,3,1)$
20 : $P_{298} = (1,4,3,1)$
21 : $P_{310} = (5,5,3,1)$

22 : $P_{320} = (7,6,3,1)$
23 : $P_{346} = (1,2,4,1)$
24 : $P_{349} = (4,2,4,1)$
25 : $P_{367} = (6,4,4,1)$
26 : $P_{372} = (3,5,4,1)$
27 : $P_{379} = (2,6,4,1)$
28 : $P_{391} = (6,7,4,1)$
29 : $P_{395} = (2,0,5,1)$
30 : $P_{419} = (2,3,5,1)$
31 : $P_{447} = (6,6,5,1)$
32 : $P_{450} = (1,7,5,1)$
33 : $P_{461} = (4,0,6,1)$
34 : $P_{474} = (1,2,6,1)$
35 : $P_{484} = (3,3,6,1)$
36 : $P_{501} = (4,5,6,1)$
37 : $P_{540} = (3,2,7,1)$
38 : $P_{549} = (4,3,7,1)$
39 : $P_{554} = (1,4,7,1)$
40 : $P_{560} = (7,4,7,1)$
41 : $P_{574} = (5,6,7,1)$
42 : $P_{580} = (3,7,7,1)$

Line Intersection Graph

```
 |0 1 2 3 4 5
0|0 1 1 0 0 1
1|1 0 0 1 1 0
2|1 0 0 1 0 1
3|0 1 1 0 1 0
4|0 1 0 1 0 1
5|1 0 1 0 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_{369}	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_{369}	P_{485}	P_{443}

3

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_3	P_{569}	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{485}	P_{569}	P_{312}

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{443}	P_{312}	P_{225}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_3	P_3	P_{225}

The surface has 89 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_3 = (0,0,0,1)$
$2: P_{20} = (1,1,1,0)$
$3: P_{27} = (0,2,1,0)$
$4: P_{37} = (2,3,1,0)$
$5: P_{43} = (0,4,1,0)$
$6: P_{55} = (4,5,1,0)$
$7: P_{66} = (7,6,1,0)$
$8: P_{67} = (0,7,1,0)$
$9: P_{83} = (1,1,0,1)$
$10: P_{93} = (3,2,0,1)$
$11: P_{105} = (7,3,0,1)$
$12: P_{111} = (5,4,0,1)$
$13: P_{116} = (2,5,0,1)$
$14: P_{126} = (4,6,0,1)$
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$76: P_{497} = (0,5,6,1)$
$77: P_{501} = (4,5,6,1)$
$78: P_{527} = (6,0,7,1)$
$79: P_{537} = (0,2,7,1)$
$80: P_{540} = (3,2,7,1)$
$81: P_{545} = (0,3,7,1)$
$82: P_{549} = (4,3,7,1)$
$83: P_{554} = (1,4,7,1)$
$84: P_{560} = (7,4,7,1)$
$85: P_{569} = (0,6,7,1)$
$86: P_{574} = (5,6,7,1)$
$87: P_{578} = (1,7,7,1)$
$88: P_{580} = (3,7,7,1)$