

# Rank-65695 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_1 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1229235341

## General information

Number of lines	3
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^3$
Type of lines on points	$2^3, 1^{21}, 0^{49}$

## Singular Points

The surface has 1 singular points:

$$0 : P_{20} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{65} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{65} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{96}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{769}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{1323}$$

Rank of lines: ( 65, 4673, 666 )

Rank of points on Klein quadric: ( 96, 769, 1323 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{138} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

$$P_{139} = (1, 0, 1, 1) = \ell_0 \cap \ell_2$$

$$P_{146} = (0, 1, 1, 1) = \ell_1 \cap \ell_2$$

### Single Points

The surface has 21 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_1 = (0, 1, 0, 0) \text{ lies on line } \ell_1$$

$$2 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_2$$

$$3 : P_{140} = (2, 0, 1, 1) \text{ lies on line } \ell_0$$

$$4 : P_{141} = (3, 0, 1, 1) \text{ lies on line } \ell_0$$

$$5 : P_{142} = (4, 0, 1, 1) \text{ lies on line } \ell_0$$

$$6 : P_{143} = (5, 0, 1, 1) \text{ lies on line } \ell_0$$

$$7 : P_{144} = (6, 0, 1, 1) \text{ lies on line } \ell_0$$

$$8 : P_{145} = (7, 0, 1, 1) \text{ lies on line } \ell_0$$

$$9 : P_{153} = (0, 2, 1, 1) \text{ lies on line } \ell_1$$

$$10 : P_{156} = (3, 2, 1, 1) \text{ lies on line } \ell_2$$

$$11 : P_{161} = (0, 3, 1, 1) \text{ lies on line } \ell_1$$

$$12 : P_{163} = (2, 3, 1, 1) \text{ lies on line } \ell_2$$

$$13 : P_{169} = (0, 4, 1, 1) \text{ lies on line } \ell_1$$

$$14 : P_{174} = (5, 4, 1, 1) \text{ lies on line } \ell_2$$

$$15 : P_{177} = (0, 5, 1, 1) \text{ lies on line } \ell_1$$

$$16 : P_{181} = (4, 5, 1, 1) \text{ lies on line } \ell_2$$

$$17 : P_{185} = (0, 6, 1, 1) \text{ lies on line } \ell_1$$

$$18 : P_{192} = (7, 6, 1, 1) \text{ lies on line } \ell_2$$

$$19 : P_{193} = (0, 7, 1, 1) \text{ lies on line } \ell_1$$

$$20 : P_{199} = (6, 7, 1, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

### Points on surface but on no line

The surface has 49 points not on any line:

The points on the surface but not on lines are:

$$0 : P_{20} = (1, 1, 1, 0)$$

$$1 : P_{31} = (4, 2, 1, 0)$$

$$2 : P_{34} = (7, 2, 1, 0)$$

$$3 : P_{45} = (2, 4, 1, 0)$$

$$4 : P_{50} = (7, 4, 1, 0)$$

$$5 : P_{69} = (2, 7, 1, 0)$$

$$6 : P_{71} = (4, 7, 1, 0)$$

$$7 : P_{103} = (5, 3, 0, 1)$$

$$8 : P_{104} = (6, 3, 0, 1)$$

$$9 : P_{117} = (3, 5, 0, 1)$$

10 : $P_{120} = (6, 5, 0, 1)$	30 : $P_{359} = (6, 3, 4, 1)$
11 : $P_{125} = (3, 6, 0, 1)$	31 : $P_{363} = (2, 4, 4, 1)$
12 : $P_{127} = (5, 6, 0, 1)$	32 : $P_{378} = (1, 6, 4, 1)$
13 : $P_{214} = (5, 1, 2, 1)$	33 : $P_{380} = (3, 6, 4, 1)$
14 : $P_{215} = (6, 1, 2, 1)$	34 : $P_{402} = (1, 1, 5, 1)$
15 : $P_{224} = (7, 2, 2, 1)$	35 : $P_{406} = (5, 1, 5, 1)$
16 : $P_{242} = (1, 5, 2, 1)$	36 : $P_{434} = (1, 5, 5, 1)$
17 : $P_{247} = (6, 5, 2, 1)$	37 : $P_{466} = (1, 1, 6, 1)$
18 : $P_{250} = (1, 6, 2, 1)$	38 : $P_{471} = (6, 1, 6, 1)$
19 : $P_{254} = (5, 6, 2, 1)$	39 : $P_{506} = (1, 6, 6, 1)$
20 : $P_{259} = (2, 7, 2, 1)$	40 : $P_{532} = (3, 1, 7, 1)$
21 : $P_{264} = (7, 7, 2, 1)$	41 : $P_{534} = (5, 1, 7, 1)$
22 : $P_{274} = (1, 1, 3, 1)$	42 : $P_{546} = (1, 3, 7, 1)$
23 : $P_{276} = (3, 1, 3, 1)$	43 : $P_{550} = (5, 3, 7, 1)$
24 : $P_{290} = (1, 3, 3, 1)$	44 : $P_{557} = (4, 4, 7, 1)$
25 : $P_{340} = (3, 1, 4, 1)$	45 : $P_{560} = (7, 4, 7, 1)$
26 : $P_{343} = (6, 1, 4, 1)$	46 : $P_{562} = (1, 5, 7, 1)$
27 : $P_{347} = (2, 2, 4, 1)$	47 : $P_{564} = (3, 5, 7, 1)$
28 : $P_{349} = (4, 2, 4, 1)$	48 : $P_{581} = (4, 7, 7, 1)$
29 : $P_{354} = (1, 3, 4, 1)$	

## Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_{138}$	$P_{139}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{138}$	$P_{146}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_{139}$	$P_{146}$

The surface has 73 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	12 : $P_{117} = (3, 5, 0, 1)$	24 : $P_{146} = (0, 1, 1, 1)$
1 : $P_1 = (0, 1, 0, 0)$	13 : $P_{120} = (6, 5, 0, 1)$	25 : $P_{153} = (0, 2, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$	14 : $P_{125} = (3, 6, 0, 1)$	26 : $P_{156} = (3, 2, 1, 1)$
3 : $P_{20} = (1, 1, 1, 0)$	15 : $P_{127} = (5, 6, 0, 1)$	27 : $P_{161} = (0, 3, 1, 1)$
4 : $P_{31} = (4, 2, 1, 0)$	16 : $P_{138} = (0, 0, 1, 1)$	28 : $P_{163} = (2, 3, 1, 1)$
5 : $P_{34} = (7, 2, 1, 0)$	17 : $P_{139} = (1, 0, 1, 1)$	29 : $P_{169} = (0, 4, 1, 1)$
6 : $P_{45} = (2, 4, 1, 0)$	18 : $P_{140} = (2, 0, 1, 1)$	30 : $P_{174} = (5, 4, 1, 1)$
7 : $P_{50} = (7, 4, 1, 0)$	19 : $P_{141} = (3, 0, 1, 1)$	31 : $P_{177} = (0, 5, 1, 1)$
8 : $P_{69} = (2, 7, 1, 0)$	20 : $P_{142} = (4, 0, 1, 1)$	32 : $P_{181} = (4, 5, 1, 1)$
9 : $P_{71} = (4, 7, 1, 0)$	21 : $P_{143} = (5, 0, 1, 1)$	33 : $P_{185} = (0, 6, 1, 1)$
10 : $P_{103} = (5, 3, 0, 1)$	22 : $P_{144} = (6, 0, 1, 1)$	34 : $P_{192} = (7, 6, 1, 1)$
11 : $P_{104} = (6, 3, 0, 1)$	23 : $P_{145} = (7, 0, 1, 1)$	35 : $P_{193} = (0, 7, 1, 1)$

36 :  $P_{199} = (6, 7, 1, 1)$   
 37 :  $P_{214} = (5, 1, 2, 1)$   
 38 :  $P_{215} = (6, 1, 2, 1)$   
 39 :  $P_{224} = (7, 2, 2, 1)$   
 40 :  $P_{242} = (1, 5, 2, 1)$   
 41 :  $P_{247} = (6, 5, 2, 1)$   
 42 :  $P_{250} = (1, 6, 2, 1)$   
 43 :  $P_{254} = (5, 6, 2, 1)$   
 44 :  $P_{259} = (2, 7, 2, 1)$   
 45 :  $P_{264} = (7, 7, 2, 1)$   
 46 :  $P_{274} = (1, 1, 3, 1)$   
 47 :  $P_{276} = (3, 1, 3, 1)$   
 48 :  $P_{290} = (1, 3, 3, 1)$

49 :  $P_{340} = (3, 1, 4, 1)$   
 50 :  $P_{343} = (6, 1, 4, 1)$   
 51 :  $P_{347} = (2, 2, 4, 1)$   
 52 :  $P_{349} = (4, 2, 4, 1)$   
 53 :  $P_{354} = (1, 3, 4, 1)$   
 54 :  $P_{359} = (6, 3, 4, 1)$   
 55 :  $P_{363} = (2, 4, 4, 1)$   
 56 :  $P_{378} = (1, 6, 4, 1)$   
 57 :  $P_{380} = (3, 6, 4, 1)$   
 58 :  $P_{402} = (1, 1, 5, 1)$   
 59 :  $P_{406} = (5, 1, 5, 1)$   
 60 :  $P_{434} = (1, 5, 5, 1)$   
 61 :  $P_{466} = (1, 1, 6, 1)$

62 :  $P_{471} = (6, 1, 6, 1)$   
 63 :  $P_{506} = (1, 6, 6, 1)$   
 64 :  $P_{532} = (3, 1, 7, 1)$   
 65 :  $P_{534} = (5, 1, 7, 1)$   
 66 :  $P_{546} = (1, 3, 7, 1)$   
 67 :  $P_{550} = (5, 3, 7, 1)$   
 68 :  $P_{557} = (4, 4, 7, 1)$   
 69 :  $P_{560} = (7, 4, 7, 1)$   
 70 :  $P_{562} = (1, 5, 7, 1)$   
 71 :  $P_{564} = (3, 5, 7, 1)$   
 72 :  $P_{581} = (4, 7, 7, 1)$