Rank-74053 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172885

General information

Number of lines	4
Number of points	81
Number of singular points	1
Number of Eckardt points	1
Number of double points	1
Number of single points	31
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	3, 2, 1^{31}, 0^{48}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$
$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$
$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

Rank of lines: (64, 4744, 648, 4689)
Rank of points on Klein quadric: (2, 1, 10, 25)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).$

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 31 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_{12} = (1,0,1,0)$ lies on line ℓ_0
$2 : P_{13} = (2,0,1,0)$ lies on line ℓ_0
$3 : P_{14} = (3,0,1,0)$ lies on line ℓ_0
$4 : P_{15} = (4,0,1,0)$ lies on line ℓ_0
$5 : P_{16} = (5,0,1,0)$ lies on line ℓ_0
$6 : P_{17} = (6,0,1,0)$ lies on line ℓ_0
$7 : P_{18} = (7,0,1,0)$ lies on line ℓ_0
$8 : P_{19} = (0,1,1,0)$ lies on line ℓ_3
$9 : P_{75} = (1,0,0,1)$ lies on line ℓ_2
$10 : P_{138} = (0,0,1,1)$ lies on line ℓ_1
$11 : P_{139} = (1,0,1,1)$ lies on line ℓ_2
$12 : P_{146} = (0,1,1,1)$ lies on line ℓ_3
$13 : P_{201} = (0,0,2,1)$ lies on line ℓ_1
$14 : P_{202} = (1,0,2,1)$ lies on line ℓ_2
$15 : P_{217} = (0,2,2,1)$ lies on line ℓ_3
$16 : P_{265} = (0,0,3,1)$ lies on line ℓ_1
$17 : P_{266} = (1,0,3,1)$ lies on line ℓ_2
$18 : P_{289} = (0,3,3,1)$ lies on line ℓ_3
$19 : P_{329} = (0,0,4,1)$ lies on line ℓ_1
$20 : P_{330} = (1,0,4,1)$ lies on line ℓ_2
$21 : P_{361} = (0,4,4,1)$ lies on line ℓ_3
$22 : P_{393} = (0,0,5,1)$ lies on line ℓ_1
$23 : P_{394} = (1,0,5,1)$ lies on line ℓ_2
$24 : P_{433} = (0,5,5,1)$ lies on line ℓ_3
$25 : P_{457} = (0,0,6,1)$ lies on line ℓ_1
$26 : P_{458} = (1,0,6,1)$ lies on line ℓ_2
$27 : P_{505} = (0,6,6,1)$ lies on line ℓ_3
$28 : P_{521} = (0,0,7,1)$ lies on line ℓ_1
$29 : P_{522} = (1,0,7,1)$ lies on line ℓ_2
$30 : P_{577} = (0,7,7,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:
The points on the surface but not on lines are:

0 : $P_{33} = (6, 2, 1, 0)$
1 : $P_{41} = (6, 3, 1, 0)$
2 : $P_{46} = (3, 4, 1, 0)$
3 : $P_{54} = (3, 5, 1, 0)$
4 : $P_{64} = (5, 6, 1, 0)$
5 : $P_{72} = (5, 7, 1, 0)$
6 : $P_{96} = (6, 2, 0, 1)$
7 : $P_{97} = (7, 2, 0, 1)$
8 : $P_{108} = (2, 4, 0, 1)$
9 : $P_{109} = (3, 4, 0, 1)$
10 : $P_{134} = (4, 7, 0, 1)$
11 : $P_{135} = (5, 7, 0, 1)$
12 : $P_{222} = (5, 2, 2, 1)$
13 : $P_{234} = (1, 4, 2, 1)$
14 : $P_{238} = (5, 4, 2, 1)$
15 : $P_{243} = (2, 5, 2, 1)$
16 : $P_{245} = (4, 5, 2, 1)$
17 : $P_{250} = (1, 6, 2, 1)$
18 : $P_{277} = (4, 1, 3, 1)$
19 : $P_{279} = (6, 1, 3, 1)$
20 : $P_{293} = (4, 3, 3, 1)$
21 : $P_{304} = (7, 4, 3, 1)$
22 : $P_{316} = (3, 6, 3, 1)$
23 : $P_{318} = (5, 6, 3, 1)$
24 : $P_{354} = (1, 3, 4, 1)$
25 : $P_{367} = (6, 4, 4, 1)$
26 : $P_{381} = (4, 6, 4, 1)$
27 : $P_{384} = (7, 6, 4, 1)$
28 : $P_{386} = (1, 7, 4, 1)$
29 : $P_{391} = (6, 7, 4, 1)$
30 : $P_{404} = (3, 1, 5, 1)$
31 : $P_{408} = (7, 1, 5, 1)$
32 : $P_{422} = (5, 3, 5, 1)$
33 : $P_{423} = (6, 3, 5, 1)$
34 : $P_{440} = (7, 5, 5, 1)$
35 : $P_{451} = (2, 7, 5, 1)$
36 : $P_{467} = (2, 1, 6, 1)$
37 : $P_{470} = (5, 1, 6, 1)$
38 : $P_{477} = (4, 2, 6, 1)$
39 : $P_{500} = (3, 5, 6, 1)$
40 : $P_{503} = (6, 5, 6, 1)$
41 : $P_{507} = (2, 6, 6, 1)$
42 : $P_{538} = (1, 2, 7, 1)$
43 : $P_{540} = (3, 2, 7, 1)$
44 : $P_{547} = (2, 3, 7, 1)$
45 : $P_{552} = (7, 3, 7, 1)$
46 : $P_{562} = (1, 5, 7, 1)$
47 : $P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

	0	1	2	3
0	0	1	1	0
1	1	0	1	1
2	1	1	0	0
3	0	1	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

Line 3 intersects

Line	ℓ_1
in point	P_3

The surface has 81 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_{12} = (1,0,1,0)$
$4: P_{13} = (2,0,1,0)$
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$12: P_{41} = (6,3,1,0)$
$13: P_{46} = (3,4,1,0)$
$14: P_{54} = (3,5,1,0)$
$15: P_{64} = (5,6,1,0)$
$16: P_{72} = (5,7,1,0)$
$17: P_{75} = (1,0,0,1)$
$18: P_{96} = (6,2,0,1)$
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