

Rank-73798 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108390

General information

Number of lines	3
Number of points	289
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	48
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	$3, 1^{48}, 0^{240}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & \delta^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \mathbf{PI}(0, 11, 1, 0, 0, 0)_{28} \\ \ell_2 &= \begin{bmatrix} 1 & 0 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \mathbf{PI}(0, 10, 1, 0, 0, 0)_{27}\end{aligned}$$

Rank of lines: (70160, 48304, 43936)

Rank of points on Klein quadric: (1, 28, 27)

Eckardt Points

The surface has 1 Eckardt points:

$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 48 single points:

The single points on the surface are:

- | | |
|---|--|
| 0 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0 | 25 : $P_{2331} = (10, 0, 8, 1)$ lies on line ℓ_1 |
| 1 : $P_{284} = (10, 0, 0, 1)$ lies on line ℓ_1 | 26 : $P_{2332} = (11, 0, 8, 1)$ lies on line ℓ_2 |
| 2 : $P_{285} = (11, 0, 0, 1)$ lies on line ℓ_2 | 27 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_0 |
| 3 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_0 | 28 : $P_{2587} = (10, 0, 9, 1)$ lies on line ℓ_1 |
| 4 : $P_{540} = (10, 0, 1, 1)$ lies on line ℓ_1 | 29 : $P_{2588} = (11, 0, 9, 1)$ lies on line ℓ_2 |
| 5 : $P_{541} = (11, 0, 1, 1)$ lies on line ℓ_2 | 30 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_0 |
| 6 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_0 | 31 : $P_{2843} = (10, 0, 10, 1)$ lies on line ℓ_1 |
| 7 : $P_{795} = (10, 0, 2, 1)$ lies on line ℓ_1 | 32 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_2 |
| 8 : $P_{796} = (11, 0, 2, 1)$ lies on line ℓ_2 | 33 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_0 |
| 9 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_0 | 34 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_1 |
| 10 : $P_{1051} = (10, 0, 3, 1)$ lies on line ℓ_1 | 35 : $P_{3100} = (11, 0, 11, 1)$ lies on line ℓ_2 |
| 11 : $P_{1052} = (11, 0, 3, 1)$ lies on line ℓ_2 | 36 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_0 |
| 12 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_0 | 37 : $P_{3355} = (10, 0, 12, 1)$ lies on line ℓ_1 |
| 13 : $P_{1307} = (10, 0, 4, 1)$ lies on line ℓ_1 | 38 : $P_{3356} = (11, 0, 12, 1)$ lies on line ℓ_2 |
| 14 : $P_{1308} = (11, 0, 4, 1)$ lies on line ℓ_2 | 39 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_0 |
| 15 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_0 | 40 : $P_{3611} = (10, 0, 13, 1)$ lies on line ℓ_1 |
| 16 : $P_{1563} = (10, 0, 5, 1)$ lies on line ℓ_1 | 41 : $P_{3612} = (11, 0, 13, 1)$ lies on line ℓ_2 |
| 17 : $P_{1564} = (11, 0, 5, 1)$ lies on line ℓ_2 | 42 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_0 |
| 18 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_0 | 43 : $P_{3867} = (10, 0, 14, 1)$ lies on line ℓ_1 |
| 19 : $P_{1819} = (10, 0, 6, 1)$ lies on line ℓ_1 | 44 : $P_{3868} = (11, 0, 14, 1)$ lies on line ℓ_2 |
| 20 : $P_{1820} = (11, 0, 6, 1)$ lies on line ℓ_2 | 45 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_0 |
| 21 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_0 | 46 : $P_{4123} = (10, 0, 15, 1)$ lies on line ℓ_1 |
| 22 : $P_{2075} = (10, 0, 7, 1)$ lies on line ℓ_1 | 47 : $P_{4124} = (11, 0, 15, 1)$ lies on line ℓ_2 |
| 23 : $P_{2076} = (11, 0, 7, 1)$ lies on line ℓ_2 | |
| 24 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_0 | |

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line:

The points on the surface but not on lines are:

0 : $P_5 = (1, 1, 0, 0)$	48 : $P_{1117} = (12, 4, 3, 1)$
1 : $P_{14} = (10, 1, 0, 0)$	49 : $P_{1120} = (15, 4, 3, 1)$
2 : $P_{15} = (11, 1, 0, 0)$	50 : $P_{1181} = (12, 8, 3, 1)$
3 : $P_{60} = (9, 2, 1, 0)$	51 : $P_{1186} = (1, 9, 3, 1)$
4 : $P_{73} = (6, 3, 1, 0)$	52 : $P_{1200} = (15, 9, 3, 1)$
5 : $P_{97} = (14, 4, 1, 0)$	53 : $P_{1224} = (7, 11, 3, 1)$
6 : $P_{112} = (13, 5, 1, 0)$	54 : $P_{1245} = (12, 12, 3, 1)$
7 : $P_{118} = (3, 6, 1, 0)$	55 : $P_{1253} = (4, 13, 3, 1)$
8 : $P_{139} = (8, 7, 1, 0)$	56 : $P_{1259} = (10, 13, 3, 1)$
9 : $P_{154} = (7, 8, 1, 0)$	57 : $P_{1264} = (15, 13, 3, 1)$
10 : $P_{165} = (2, 9, 1, 0)$	58 : $P_{1269} = (4, 14, 3, 1)$
11 : $P_{226} = (15, 12, 1, 0)$	59 : $P_{1284} = (3, 15, 3, 1)$
12 : $P_{232} = (5, 13, 1, 0)$	60 : $P_{1320} = (7, 1, 4, 1)$
13 : $P_{247} = (4, 14, 1, 0)$	61 : $P_{1332} = (3, 2, 4, 1)$
14 : $P_{271} = (12, 15, 1, 0)$	62 : $P_{1347} = (2, 3, 4, 1)$
15 : $P_{291} = (1, 1, 0, 1)$	63 : $P_{1357} = (12, 3, 4, 1)$
16 : $P_{435} = (1, 10, 0, 1)$	64 : $P_{1360} = (15, 3, 4, 1)$
17 : $P_{451} = (1, 11, 0, 1)$	65 : $P_{1389} = (12, 5, 4, 1)$
18 : $P_{573} = (12, 2, 1, 1)$	66 : $P_{1405} = (12, 6, 4, 1)$
19 : $P_{599} = (6, 4, 1, 1)$	67 : $P_{1413} = (4, 7, 4, 1)$
20 : $P_{686} = (13, 9, 1, 1)$	68 : $P_{1429} = (4, 8, 4, 1)$
21 : $P_{693} = (4, 10, 1, 1)$	69 : $P_{1462} = (5, 10, 4, 1)$
22 : $P_{700} = (11, 10, 1, 1)$	70 : $P_{1490} = (1, 12, 4, 1)$
23 : $P_{703} = (14, 10, 1, 1)$	71 : $P_{1504} = (15, 12, 4, 1)$
24 : $P_{707} = (2, 11, 1, 1)$	72 : $P_{1516} = (11, 13, 4, 1)$
25 : $P_{714} = (9, 11, 1, 1)$	73 : $P_{1541} = (4, 15, 4, 1)$
26 : $P_{715} = (10, 11, 1, 1)$	74 : $P_{1547} = (10, 15, 4, 1)$
27 : $P_{760} = (7, 14, 1, 1)$	75 : $P_{1552} = (15, 15, 4, 1)$
28 : $P_{814} = (13, 1, 2, 1)$	76 : $P_{1577} = (8, 1, 5, 1)$
29 : $P_{840} = (7, 3, 2, 1)$	77 : $P_{1594} = (9, 2, 5, 1)$
30 : $P_{867} = (2, 5, 2, 1)$	78 : $P_{1606} = (5, 3, 5, 1)$
31 : $P_{891} = (10, 6, 2, 1)$	79 : $P_{1627} = (10, 4, 5, 1)$
32 : $P_{898} = (1, 7, 2, 1)$	80 : $P_{1642} = (9, 5, 5, 1)$
33 : $P_{905} = (8, 7, 2, 1)$	81 : $P_{1655} = (6, 6, 5, 1)$
34 : $P_{915} = (2, 8, 2, 1)$	82 : $P_{1668} = (3, 7, 5, 1)$
35 : $P_{921} = (8, 8, 2, 1)$	83 : $P_{1674} = (9, 7, 5, 1)$
36 : $P_{924} = (11, 8, 2, 1)$	84 : $P_{1676} = (11, 7, 5, 1)$
37 : $P_{964} = (3, 11, 2, 1)$	85 : $P_{1700} = (3, 9, 5, 1)$
38 : $P_{984} = (7, 12, 2, 1)$	86 : $P_{1701} = (4, 9, 5, 1)$
39 : $P_{995} = (2, 13, 2, 1)$	87 : $P_{1703} = (6, 9, 5, 1)$
40 : $P_{1024} = (15, 14, 2, 1)$	88 : $P_{1725} = (12, 10, 5, 1)$
41 : $P_{1032} = (7, 15, 2, 1)$	89 : $P_{1778} = (1, 14, 5, 1)$
42 : $P_{1033} = (8, 15, 2, 1)$	90 : $P_{1780} = (3, 14, 5, 1)$
43 : $P_{1039} = (14, 15, 2, 1)$	91 : $P_{1799} = (6, 15, 5, 1)$
44 : $P_{1062} = (5, 1, 3, 1)$	92 : $P_{1854} = (13, 2, 6, 1)$
45 : $P_{1084} = (11, 2, 3, 1)$	93 : $P_{1886} = (13, 4, 6, 1)$
46 : $P_{1093} = (4, 3, 3, 1)$	94 : $P_{1900} = (11, 5, 6, 1)$
47 : $P_{1107} = (2, 4, 3, 1)$	95 : $P_{1910} = (5, 6, 6, 1)$

96 : $P_{1914} = (9, 6, 6, 1)$	150 : $P_{2699} = (10, 7, 9, 1)$
97 : $P_{1918} = (13, 6, 6, 1)$	151 : $P_{2711} = (6, 8, 9, 1)$
98 : $P_{1935} = (14, 7, 6, 1)$	152 : $P_{2761} = (8, 11, 9, 1)$
99 : $P_{1958} = (5, 9, 6, 1)$	153 : $P_{2778} = (9, 12, 9, 1)$
100 : $P_{1963} = (10, 9, 6, 1)$	154 : $P_{2791} = (6, 13, 9, 1)$
101 : $P_{1967} = (14, 9, 6, 1)$	155 : $P_{2826} = (9, 15, 9, 1)$
102 : $P_{1975} = (6, 10, 6, 1)$	156 : $P_{2853} = (4, 1, 10, 1)$
103 : $P_{2000} = (15, 11, 6, 1)$	157 : $P_{2860} = (11, 1, 10, 1)$
104 : $P_{2009} = (8, 12, 6, 1)$	158 : $P_{2863} = (14, 1, 10, 1)$
105 : $P_{2047} = (14, 14, 6, 1)$	159 : $P_{2926} = (13, 5, 10, 1)$
106 : $P_{2050} = (1, 15, 6, 1)$	160 : $P_{2936} = (7, 6, 10, 1)$
107 : $P_{2054} = (5, 15, 6, 1)$	161 : $P_{2951} = (6, 7, 10, 1)$
108 : $P_{2101} = (4, 2, 7, 1)$	162 : $P_{2995} = (2, 10, 10, 1)$
109 : $P_{2107} = (10, 2, 7, 1)$	163 : $P_{3002} = (9, 10, 10, 1)$
110 : $P_{2112} = (15, 2, 7, 1)$	164 : $P_{3003} = (10, 10, 10, 1)$
111 : $P_{2133} = (4, 4, 7, 1)$	165 : $P_{3085} = (12, 15, 10, 1)$
112 : $P_{2146} = (1, 5, 7, 1)$	166 : $P_{3107} = (2, 1, 11, 1)$
113 : $P_{2160} = (15, 5, 7, 1)$	167 : $P_{3114} = (9, 1, 11, 1)$
114 : $P_{2165} = (4, 6, 7, 1)$	168 : $P_{3115} = (10, 1, 11, 1)$
115 : $P_{2179} = (2, 7, 7, 1)$	169 : $P_{3143} = (6, 3, 11, 1)$
116 : $P_{2189} = (12, 7, 7, 1)$	170 : $P_{3224} = (7, 8, 11, 1)$
117 : $P_{2192} = (15, 7, 7, 1)$	171 : $P_{3269} = (4, 11, 11, 1)$
118 : $P_{2221} = (12, 9, 7, 1)$	172 : $P_{3276} = (11, 11, 11, 1)$
119 : $P_{2232} = (7, 10, 7, 1)$	173 : $P_{3279} = (14, 11, 11, 1)$
120 : $P_{2246} = (5, 11, 7, 1)$	174 : $P_{3294} = (13, 12, 11, 1)$
121 : $P_{2276} = (3, 13, 7, 1)$	175 : $P_{3309} = (12, 13, 11, 1)$
122 : $P_{2301} = (12, 14, 7, 1)$	176 : $P_{3383} = (6, 2, 12, 1)$
123 : $P_{2316} = (11, 15, 7, 1)$	177 : $P_{3403} = (10, 3, 12, 1)$
124 : $P_{2352} = (15, 1, 8, 1)$	178 : $P_{3412} = (3, 4, 12, 1)$
125 : $P_{2354} = (1, 2, 8, 1)$	179 : $P_{3418} = (9, 4, 12, 1)$
126 : $P_{2358} = (5, 2, 8, 1)$	180 : $P_{3420} = (11, 4, 12, 1)$
127 : $P_{2382} = (13, 3, 8, 1)$	181 : $P_{3462} = (5, 7, 12, 1)$
128 : $P_{2399} = (14, 4, 8, 1)$	182 : $P_{3474} = (1, 8, 12, 1)$
129 : $P_{2409} = (8, 5, 8, 1)$	183 : $P_{3476} = (3, 8, 12, 1)$
130 : $P_{2463} = (14, 8, 8, 1)$	184 : $P_{3498} = (9, 9, 12, 1)$
131 : $P_{2476} = (11, 9, 8, 1)$	185 : $P_{3513} = (8, 10, 12, 1)$
132 : $P_{2503} = (6, 11, 8, 1)$	186 : $P_{3533} = (12, 11, 12, 1)$
133 : $P_{2518} = (5, 12, 8, 1)$	187 : $P_{3540} = (3, 12, 12, 1)$
134 : $P_{2523} = (10, 12, 8, 1)$	188 : $P_{3541} = (4, 12, 12, 1)$
135 : $P_{2527} = (14, 12, 8, 1)$	189 : $P_{3543} = (6, 12, 12, 1)$
136 : $P_{2542} = (13, 13, 8, 1)$	190 : $P_{3562} = (9, 13, 12, 1)$
137 : $P_{2550} = (5, 14, 8, 1)$	191 : $P_{3575} = (6, 14, 12, 1)$
138 : $P_{2554} = (9, 14, 8, 1)$	192 : $P_{3635} = (2, 2, 13, 1)$
139 : $P_{2558} = (13, 14, 8, 1)$	193 : $P_{3650} = (1, 3, 13, 1)$
140 : $P_{2605} = (12, 1, 9, 1)$	194 : $P_{3657} = (8, 3, 13, 1)$
141 : $P_{2628} = (3, 3, 9, 1)$	195 : $P_{3672} = (7, 4, 13, 1)$
142 : $P_{2634} = (9, 3, 9, 1)$	196 : $P_{3712} = (15, 6, 13, 1)$
143 : $P_{2636} = (11, 3, 9, 1)$	197 : $P_{3739} = (10, 8, 13, 1)$
144 : $P_{2646} = (5, 4, 9, 1)$	198 : $P_{3752} = (7, 9, 13, 1)$
145 : $P_{2660} = (3, 5, 9, 1)$	199 : $P_{3764} = (3, 10, 13, 1)$
146 : $P_{2661} = (4, 5, 9, 1)$	200 : $P_{3790} = (13, 11, 13, 1)$
147 : $P_{2663} = (6, 5, 9, 1)$	201 : $P_{3795} = (2, 12, 13, 1)$
148 : $P_{2674} = (1, 6, 9, 1)$	202 : $P_{3816} = (7, 13, 13, 1)$
149 : $P_{2676} = (3, 6, 9, 1)$	203 : $P_{3817} = (8, 13, 13, 1)$

204 : $P_{3823} = (14, 13, 13, 1)$
 205 : $P_{3827} = (2, 14, 13, 1)$
 206 : $P_{3833} = (8, 14, 13, 1)$
 207 : $P_{3836} = (11, 14, 13, 1)$
 208 : $P_{3879} = (6, 1, 14, 1)$
 209 : $P_{3919} = (14, 3, 14, 1)$
 210 : $P_{3942} = (5, 5, 14, 1)$
 211 : $P_{3947} = (10, 5, 14, 1)$
 212 : $P_{3951} = (14, 5, 14, 1)$
 213 : $P_{3967} = (14, 6, 14, 1)$
 214 : $P_{3982} = (13, 7, 14, 1)$
 215 : $P_{3990} = (5, 8, 14, 1)$
 216 : $P_{3994} = (9, 8, 14, 1)$
 217 : $P_{3998} = (13, 8, 14, 1)$
 218 : $P_{4009} = (8, 9, 14, 1)$
 219 : $P_{4032} = (15, 10, 14, 1)$
 220 : $P_{4060} = (11, 12, 14, 1)$
 221 : $P_{4066} = (1, 13, 14, 1)$
 222 : $P_{4070} = (5, 13, 14, 1)$

223 : $P_{4110} = (13, 15, 14, 1)$
 224 : $P_{4132} = (3, 1, 15, 1)$
 225 : $P_{4152} = (7, 2, 15, 1)$
 226 : $P_{4153} = (8, 2, 15, 1)$
 227 : $P_{4159} = (14, 2, 15, 1)$
 228 : $P_{4178} = (1, 4, 15, 1)$
 229 : $P_{4185} = (8, 4, 15, 1)$
 230 : $P_{4200} = (7, 5, 15, 1)$
 231 : $P_{4211} = (2, 6, 15, 1)$
 232 : $P_{4217} = (8, 6, 15, 1)$
 233 : $P_{4220} = (11, 6, 15, 1)$
 234 : $P_{4232} = (7, 7, 15, 1)$
 235 : $P_{4256} = (15, 8, 15, 1)$
 236 : $P_{4259} = (2, 9, 15, 1)$
 237 : $P_{4286} = (13, 10, 15, 1)$
 238 : $P_{4347} = (10, 14, 15, 1)$
 239 : $P_{4355} = (2, 15, 15, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 289 points:

The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$	13 : $P_{226} = (15, 12, 1, 0)$	26 : $P_{599} = (6, 4, 1, 1)$
1 : $P_3 = (0, 0, 0, 1)$	14 : $P_{232} = (5, 13, 1, 0)$	27 : $P_{686} = (13, 9, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$	15 : $P_{247} = (4, 14, 1, 0)$	28 : $P_{693} = (4, 10, 1, 1)$
3 : $P_{14} = (10, 1, 0, 0)$	16 : $P_{271} = (12, 15, 1, 0)$	29 : $P_{700} = (11, 10, 1, 1)$
4 : $P_{15} = (11, 1, 0, 0)$	17 : $P_{284} = (10, 0, 0, 1)$	30 : $P_{703} = (14, 10, 1, 1)$
5 : $P_{60} = (9, 2, 1, 0)$	18 : $P_{285} = (11, 0, 0, 1)$	31 : $P_{707} = (2, 11, 1, 1)$
6 : $P_{73} = (6, 3, 1, 0)$	19 : $P_{291} = (1, 1, 0, 1)$	32 : $P_{714} = (9, 11, 1, 1)$
7 : $P_{97} = (14, 4, 1, 0)$	20 : $P_{435} = (1, 10, 0, 1)$	33 : $P_{715} = (10, 11, 1, 1)$
8 : $P_{112} = (13, 5, 1, 0)$	21 : $P_{451} = (1, 11, 0, 1)$	34 : $P_{760} = (7, 14, 1, 1)$
9 : $P_{118} = (3, 6, 1, 0)$	22 : $P_{530} = (0, 0, 1, 1)$	35 : $P_{785} = (0, 0, 2, 1)$
10 : $P_{139} = (8, 7, 1, 0)$	23 : $P_{540} = (10, 0, 1, 1)$	36 : $P_{795} = (10, 0, 2, 1)$
11 : $P_{154} = (7, 8, 1, 0)$	24 : $P_{541} = (11, 0, 1, 1)$	37 : $P_{796} = (11, 0, 2, 1)$
12 : $P_{165} = (2, 9, 1, 0)$	25 : $P_{573} = (12, 2, 1, 1)$	38 : $P_{814} = (13, 1, 2, 1)$

39 : $P_{840} = (7, 3, 2, 1)$	93 : $P_{1563} = (10, 0, 5, 1)$	147 : $P_{2301} = (12, 14, 7, 1)$
40 : $P_{867} = (2, 5, 2, 1)$	94 : $P_{1564} = (11, 0, 5, 1)$	148 : $P_{2316} = (11, 15, 7, 1)$
41 : $P_{891} = (10, 6, 2, 1)$	95 : $P_{1577} = (8, 1, 5, 1)$	149 : $P_{2321} = (0, 0, 8, 1)$
42 : $P_{898} = (1, 7, 2, 1)$	96 : $P_{1594} = (9, 2, 5, 1)$	150 : $P_{2331} = (10, 0, 8, 1)$
43 : $P_{905} = (8, 7, 2, 1)$	97 : $P_{1606} = (5, 3, 5, 1)$	151 : $P_{2332} = (11, 0, 8, 1)$
44 : $P_{915} = (2, 8, 2, 1)$	98 : $P_{1627} = (10, 4, 5, 1)$	152 : $P_{2352} = (15, 1, 8, 1)$
45 : $P_{921} = (8, 8, 2, 1)$	99 : $P_{1642} = (9, 5, 5, 1)$	153 : $P_{2354} = (1, 2, 8, 1)$
46 : $P_{924} = (11, 8, 2, 1)$	100 : $P_{1655} = (6, 6, 5, 1)$	154 : $P_{2358} = (5, 2, 8, 1)$
47 : $P_{964} = (3, 11, 2, 1)$	101 : $P_{1668} = (3, 7, 5, 1)$	155 : $P_{2382} = (13, 3, 8, 1)$
48 : $P_{984} = (7, 12, 2, 1)$	102 : $P_{1674} = (9, 7, 5, 1)$	156 : $P_{2399} = (14, 4, 8, 1)$
49 : $P_{995} = (2, 13, 2, 1)$	103 : $P_{1676} = (11, 7, 5, 1)$	157 : $P_{2409} = (8, 5, 8, 1)$
50 : $P_{1024} = (15, 14, 2, 1)$	104 : $P_{1700} = (3, 9, 5, 1)$	158 : $P_{2463} = (14, 8, 8, 1)$
51 : $P_{1032} = (7, 15, 2, 1)$	105 : $P_{1701} = (4, 9, 5, 1)$	159 : $P_{2476} = (11, 9, 8, 1)$
52 : $P_{1033} = (8, 15, 2, 1)$	106 : $P_{1703} = (6, 9, 5, 1)$	160 : $P_{2503} = (6, 11, 8, 1)$
53 : $P_{1039} = (14, 15, 2, 1)$	107 : $P_{1725} = (12, 10, 5, 1)$	161 : $P_{2518} = (5, 12, 8, 1)$
54 : $P_{1041} = (0, 0, 3, 1)$	108 : $P_{1778} = (1, 14, 5, 1)$	162 : $P_{2523} = (10, 12, 8, 1)$
55 : $P_{1051} = (10, 0, 3, 1)$	109 : $P_{1780} = (3, 14, 5, 1)$	163 : $P_{2527} = (14, 12, 8, 1)$
56 : $P_{1052} = (11, 0, 3, 1)$	110 : $P_{1799} = (6, 15, 5, 1)$	164 : $P_{2542} = (13, 13, 8, 1)$
57 : $P_{1062} = (5, 1, 3, 1)$	111 : $P_{1809} = (0, 0, 6, 1)$	165 : $P_{2550} = (5, 14, 8, 1)$
58 : $P_{1084} = (11, 2, 3, 1)$	112 : $P_{1819} = (10, 0, 6, 1)$	166 : $P_{2554} = (9, 14, 8, 1)$
59 : $P_{1093} = (4, 3, 3, 1)$	113 : $P_{1820} = (11, 0, 6, 1)$	167 : $P_{2558} = (13, 14, 8, 1)$
60 : $P_{1107} = (2, 4, 3, 1)$	114 : $P_{1854} = (13, 2, 6, 1)$	168 : $P_{2577} = (0, 0, 9, 1)$
61 : $P_{1117} = (12, 4, 3, 1)$	115 : $P_{1886} = (13, 4, 6, 1)$	169 : $P_{2587} = (10, 0, 9, 1)$
62 : $P_{1120} = (15, 4, 3, 1)$	116 : $P_{1900} = (11, 5, 6, 1)$	170 : $P_{2588} = (11, 0, 9, 1)$
63 : $P_{1181} = (12, 8, 3, 1)$	117 : $P_{1910} = (5, 6, 6, 1)$	171 : $P_{2605} = (12, 1, 9, 1)$
64 : $P_{1186} = (1, 9, 3, 1)$	118 : $P_{1914} = (9, 6, 6, 1)$	172 : $P_{2628} = (3, 3, 9, 1)$
65 : $P_{1200} = (15, 9, 3, 1)$	119 : $P_{1918} = (13, 6, 6, 1)$	173 : $P_{2634} = (9, 3, 9, 1)$
66 : $P_{1224} = (7, 11, 3, 1)$	120 : $P_{1935} = (14, 7, 6, 1)$	174 : $P_{2636} = (11, 3, 9, 1)$
67 : $P_{1245} = (12, 12, 3, 1)$	121 : $P_{1958} = (5, 9, 6, 1)$	175 : $P_{2646} = (5, 4, 9, 1)$
68 : $P_{1253} = (4, 13, 3, 1)$	122 : $P_{1963} = (10, 9, 6, 1)$	176 : $P_{2660} = (3, 5, 9, 1)$
69 : $P_{1259} = (10, 13, 3, 1)$	123 : $P_{1967} = (14, 9, 6, 1)$	177 : $P_{2661} = (4, 5, 9, 1)$
70 : $P_{1264} = (15, 13, 3, 1)$	124 : $P_{1975} = (6, 10, 6, 1)$	178 : $P_{2663} = (6, 5, 9, 1)$
71 : $P_{1269} = (4, 14, 3, 1)$	125 : $P_{2000} = (15, 11, 6, 1)$	179 : $P_{2674} = (1, 6, 9, 1)$
72 : $P_{1284} = (3, 15, 3, 1)$	126 : $P_{2009} = (8, 12, 6, 1)$	180 : $P_{2676} = (3, 6, 9, 1)$
73 : $P_{1297} = (0, 0, 4, 1)$	127 : $P_{2047} = (14, 14, 6, 1)$	181 : $P_{2699} = (10, 7, 9, 1)$
74 : $P_{1307} = (10, 0, 4, 1)$	128 : $P_{2050} = (1, 15, 6, 1)$	182 : $P_{2711} = (6, 8, 9, 1)$
75 : $P_{1308} = (11, 0, 4, 1)$	129 : $P_{2054} = (5, 15, 6, 1)$	183 : $P_{2761} = (8, 11, 9, 1)$
76 : $P_{1320} = (7, 1, 4, 1)$	130 : $P_{2065} = (0, 0, 7, 1)$	184 : $P_{2778} = (9, 12, 9, 1)$
77 : $P_{1332} = (3, 2, 4, 1)$	131 : $P_{2075} = (10, 0, 7, 1)$	185 : $P_{2791} = (6, 13, 9, 1)$
78 : $P_{1347} = (2, 3, 4, 1)$	132 : $P_{2076} = (11, 0, 7, 1)$	186 : $P_{2826} = (9, 15, 9, 1)$
79 : $P_{1357} = (12, 3, 4, 1)$	133 : $P_{2101} = (4, 2, 7, 1)$	187 : $P_{2833} = (0, 0, 10, 1)$
80 : $P_{1360} = (15, 3, 4, 1)$	134 : $P_{2107} = (10, 2, 7, 1)$	188 : $P_{2843} = (10, 0, 10, 1)$
81 : $P_{1389} = (12, 5, 4, 1)$	135 : $P_{2112} = (15, 2, 7, 1)$	189 : $P_{2844} = (11, 0, 10, 1)$
82 : $P_{1405} = (12, 6, 4, 1)$	136 : $P_{2133} = (4, 4, 7, 1)$	190 : $P_{2853} = (4, 1, 10, 1)$
83 : $P_{1413} = (4, 7, 4, 1)$	137 : $P_{2146} = (1, 5, 7, 1)$	191 : $P_{2860} = (11, 1, 10, 1)$
84 : $P_{1429} = (4, 8, 4, 1)$	138 : $P_{2160} = (15, 5, 7, 1)$	192 : $P_{2863} = (14, 1, 10, 1)$
85 : $P_{1462} = (5, 10, 4, 1)$	139 : $P_{2165} = (4, 6, 7, 1)$	193 : $P_{2926} = (13, 5, 10, 1)$
86 : $P_{1490} = (1, 12, 4, 1)$	140 : $P_{2179} = (2, 7, 7, 1)$	194 : $P_{2936} = (7, 6, 10, 1)$
87 : $P_{1504} = (15, 12, 4, 1)$	141 : $P_{2189} = (12, 7, 7, 1)$	195 : $P_{2951} = (6, 7, 10, 1)$
88 : $P_{1516} = (11, 13, 4, 1)$	142 : $P_{2192} = (15, 7, 7, 1)$	196 : $P_{2995} = (2, 10, 10, 1)$
89 : $P_{1541} = (4, 15, 4, 1)$	143 : $P_{2221} = (12, 9, 7, 1)$	197 : $P_{3002} = (9, 10, 10, 1)$
90 : $P_{1547} = (10, 15, 4, 1)$	144 : $P_{2232} = (7, 10, 7, 1)$	198 : $P_{3003} = (10, 10, 10, 1)$
91 : $P_{1552} = (15, 15, 4, 1)$	145 : $P_{2246} = (5, 11, 7, 1)$	199 : $P_{3085} = (12, 15, 10, 1)$
92 : $P_{1553} = (0, 0, 5, 1)$	146 : $P_{2276} = (3, 13, 7, 1)$	200 : $P_{3089} = (0, 0, 11, 1)$

201 : $P_{3099} = (10, 0, 11, 1)$	231 : $P_{3575} = (6, 14, 12, 1)$	261 : $P_{3990} = (5, 8, 14, 1)$
202 : $P_{3100} = (11, 0, 11, 1)$	232 : $P_{3601} = (0, 0, 13, 1)$	262 : $P_{3994} = (9, 8, 14, 1)$
203 : $P_{3107} = (2, 1, 11, 1)$	233 : $P_{3611} = (10, 0, 13, 1)$	263 : $P_{3998} = (13, 8, 14, 1)$
204 : $P_{3114} = (9, 1, 11, 1)$	234 : $P_{3612} = (11, 0, 13, 1)$	264 : $P_{4009} = (8, 9, 14, 1)$
205 : $P_{3115} = (10, 1, 11, 1)$	235 : $P_{3635} = (2, 2, 13, 1)$	265 : $P_{4032} = (15, 10, 14, 1)$
206 : $P_{3143} = (6, 3, 11, 1)$	236 : $P_{3650} = (1, 3, 13, 1)$	266 : $P_{4060} = (11, 12, 14, 1)$
207 : $P_{3224} = (7, 8, 11, 1)$	237 : $P_{3657} = (8, 3, 13, 1)$	267 : $P_{4066} = (1, 13, 14, 1)$
208 : $P_{3269} = (4, 11, 11, 1)$	238 : $P_{3672} = (7, 4, 13, 1)$	268 : $P_{4070} = (5, 13, 14, 1)$
209 : $P_{3276} = (11, 11, 11, 1)$	239 : $P_{3712} = (15, 6, 13, 1)$	269 : $P_{4110} = (13, 15, 14, 1)$
210 : $P_{3279} = (14, 11, 11, 1)$	240 : $P_{3739} = (10, 8, 13, 1)$	270 : $P_{4113} = (0, 0, 15, 1)$
211 : $P_{3294} = (13, 12, 11, 1)$	241 : $P_{3752} = (7, 9, 13, 1)$	271 : $P_{4123} = (10, 0, 15, 1)$
212 : $P_{3309} = (12, 13, 11, 1)$	242 : $P_{3764} = (3, 10, 13, 1)$	272 : $P_{4124} = (11, 0, 15, 1)$
213 : $P_{3345} = (0, 0, 12, 1)$	243 : $P_{3790} = (13, 11, 13, 1)$	273 : $P_{4132} = (3, 1, 15, 1)$
214 : $P_{3355} = (10, 0, 12, 1)$	244 : $P_{3795} = (2, 12, 13, 1)$	274 : $P_{4152} = (7, 2, 15, 1)$
215 : $P_{3356} = (11, 0, 12, 1)$	245 : $P_{3816} = (7, 13, 13, 1)$	275 : $P_{4153} = (8, 2, 15, 1)$
216 : $P_{3383} = (6, 2, 12, 1)$	246 : $P_{3817} = (8, 13, 13, 1)$	276 : $P_{4159} = (14, 2, 15, 1)$
217 : $P_{3403} = (10, 3, 12, 1)$	247 : $P_{3823} = (14, 13, 13, 1)$	277 : $P_{4178} = (1, 4, 15, 1)$
218 : $P_{3412} = (3, 4, 12, 1)$	248 : $P_{3827} = (2, 14, 13, 1)$	278 : $P_{4185} = (8, 4, 15, 1)$
219 : $P_{3418} = (9, 4, 12, 1)$	249 : $P_{3833} = (8, 14, 13, 1)$	279 : $P_{4200} = (7, 5, 15, 1)$
220 : $P_{3420} = (11, 4, 12, 1)$	250 : $P_{3836} = (11, 14, 13, 1)$	280 : $P_{4211} = (2, 6, 15, 1)$
221 : $P_{3462} = (5, 7, 12, 1)$	251 : $P_{3857} = (0, 0, 14, 1)$	281 : $P_{4217} = (8, 6, 15, 1)$
222 : $P_{3474} = (1, 8, 12, 1)$	252 : $P_{3867} = (10, 0, 14, 1)$	282 : $P_{4220} = (11, 6, 15, 1)$
223 : $P_{3476} = (3, 8, 12, 1)$	253 : $P_{3868} = (11, 0, 14, 1)$	283 : $P_{4232} = (7, 7, 15, 1)$
224 : $P_{3498} = (9, 9, 12, 1)$	254 : $P_{3879} = (6, 1, 14, 1)$	284 : $P_{4256} = (15, 8, 15, 1)$
225 : $P_{3513} = (8, 10, 12, 1)$	255 : $P_{3919} = (14, 3, 14, 1)$	285 : $P_{4259} = (2, 9, 15, 1)$
226 : $P_{3533} = (12, 11, 12, 1)$	256 : $P_{3942} = (5, 5, 14, 1)$	286 : $P_{4286} = (13, 10, 15, 1)$
227 : $P_{3540} = (3, 12, 12, 1)$	257 : $P_{3947} = (10, 5, 14, 1)$	287 : $P_{4347} = (10, 14, 15, 1)$
228 : $P_{3541} = (4, 12, 12, 1)$	258 : $P_{3951} = (14, 5, 14, 1)$	288 : $P_{4355} = (2, 15, 15, 1)$
229 : $P_{3543} = (6, 12, 12, 1)$	259 : $P_{3967} = (14, 6, 14, 1)$	
230 : $P_{3562} = (9, 13, 12, 1)$	260 : $P_{3982} = (13, 7, 14, 1)$	