Cheat Sheet GF(64)

January 16, 2021

polynomial: $X^6 + X^5 + 1 = 97$
$Z_i = \log_\alpha(1 + \alpha^i)$
Subfields:

Subfield	Polynomial	Numerical Rank
$\mathbb{F}_4$	$X_2 + X_1 + 1$	7
$\mathbb{F}_8$	$X_3 + X_1 + 1$	11

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$	N
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0	0	0	0	
1	$1 = 1$	1	1	63	2	58	1	0	1	0	1	1	
2	$\alpha = \epsilon$	2	48	1	4	53	4	1	1	37	47	56	
3	$\alpha + 1 = \epsilon^{58}$	3	32	58	8	34	5	1	1	37	11	57	
4	$\alpha^2 = \epsilon^2$	4	24	2	16	43	16	1	1	46	11	57	
5	$\alpha^2 + 1 = \epsilon^{53}$	5	63	53	32	6	17	1	1	46	36	56	
6	$\alpha^2 + \alpha = \epsilon^{59}$	6	16	59	33	5	20	0	1	11	46	1	
7	$\alpha^2 + \alpha + 1 = \epsilon^{39}$	7	45	39	35	44	21	0	1	11	36	0	
8	$\alpha^3 = \epsilon^3$	8	12	3	39	23	33	1	1	37	46	57	
9	$\alpha^3 + 1 = \epsilon^{34}$	9	27	34	47	27	32	1	1	37	10	56	
10	$\alpha^3 + \alpha = \epsilon^{54}$	10	47	54	63	12	37	0	1	0	37	1	
11	$\alpha^3 + \alpha + 1 = \epsilon^{18}$	11	37	18	31	49	36	0	1	0	36	0	
12	$\alpha^3 + \alpha^2 = \epsilon^{60}$	12	8	60	62	10	49	0	1	11	36	0	
13	$\alpha^3 + \alpha^2 + 1 = \epsilon^{31}$	13	26	31	29	41	48	0	1	11	46	1	
14	$\alpha^3 + \alpha^2 + \alpha = \epsilon^{40}$	14	38	40	58	25	53	1	1	46	37	56	
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^{48}$	15	21	48	21	55	52	1	1	46	10	57	
16	$\alpha^4 = \epsilon^4$	16	6	4	42	46	39	1	1	10	36	56	
17	$\alpha^4 + 1 = \epsilon^{43}$	17	44	43	53	33	38	1	1	10	47	57	
18	$\alpha^4 + \alpha = \epsilon^{35}$	18	61	35	11	54	35	0	1	47	1	0	
19	$\alpha^4 + \alpha + 1 = \epsilon^{22}$	19	28	22	22	26	34	0	1	47	47	1	
20	$\alpha^4 + \alpha^2 = \epsilon^{55}$	20	39	55	44	24	55	0	1	36	10	1	
21	$\alpha^4 + \alpha^2 + 1 = \epsilon^{15}$	21	15	15	57	42	54	0	1	36	47	0	
22	$\alpha^4 + \alpha^2 + \alpha = \epsilon^{19}$	22	34	19	19	35	51	1	1	1	37	57	
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \epsilon^{26}$	23	41	26	38	8	50	1	1	1	37	56	
24	$\alpha^4 + \alpha^3 = \epsilon^{61}$	24	4	61	45	20	6	0	1	47	37	1	
25	$\alpha^4 + \alpha^3 + 1 = \epsilon^{51}$	25	62	51	59	14	7	0	1	47	11	0	

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$
26	$\alpha^4+\alpha^3+\alpha=\epsilon^{32}$	26	13	32	23	19	2	1	1	10	36	57
27	$\alpha^4+\alpha^3+\alpha+1=\epsilon^{29}$	27	9	29	46	9	3	1	1	10	47	56
28	$\alpha^4+\alpha^3+\alpha^2=\epsilon^{41}$	28	19	41	61	50	22	1	1	1	10	56
29	$\alpha^4+\alpha^3+\alpha^2+1=\epsilon^{13}$	29	60	13	27	32	23	1	1	1	10	57
30	$\alpha^4+\alpha^3+\alpha^2+\alpha=\epsilon^{49}$	30	58	49	54	47	18	0	1	36	1	0
31	$\alpha^4+\alpha^3+\alpha^2+\alpha+1=\epsilon^{11}$	31	50	11	13	60	19	0	1	36	36	1
32	$\alpha^5=\epsilon^5$	32	3	5	26	29	63	1	1	46	37	57
33	$\alpha^5+1=\epsilon^6$	33	49	6	52	17	62	1	1	46	10	56
34	$\alpha^5+\alpha=\epsilon^{44}$	34	22	44	9	3	59	0	1	11	11	1
35	$\alpha^5+\alpha+1=\epsilon^7$	35	40	7	18	22	58	0	1	11	1	0
36	$\alpha^5+\alpha^2=\epsilon^{36}$	36	46	36	36	45	47	0	1	0	47	0
37	$\alpha^5+\alpha^2+1=\epsilon^{45}$	37	11	45	41	56	46	0	1	0	46	1
38	$\alpha^5+\alpha^2+\alpha=\epsilon^{23}$	38	14	23	51	52	43	1	1	37	11	56
39	$\alpha^5+\alpha^2+\alpha+1=\epsilon^8$	39	20	8	7	59	42	1	1	37	47	57
40	$\alpha^5+\alpha^3=\epsilon^{56}$	40	35	56	14	48	30	0	1	11	1	0
41	$\alpha^5+\alpha^3+1=\epsilon^{37}$	41	23	37	28	13	31	0	1	11	11	1
42	$\alpha^5+\alpha^3+\alpha=\epsilon^{16}$	42	55	16	56	21	26	1	1	46	11	56
43	$\alpha^5+\alpha^3+\alpha+1=\epsilon^{46}$	43	53	46	17	4	27	1	1	46	36	57
44	$\alpha^5+\alpha^3+\alpha^2=\epsilon^{20}$	44	17	20	34	7	14	1	1	37	10	57
45	$\alpha^5+\alpha^3+\alpha^2+1=\epsilon^{24}$	45	7	24	37	36	15	1	1	37	46	56
46	$\alpha^5+\alpha^3+\alpha^2+\alpha=\epsilon^{27}$	46	36	27	43	16	10	0	1	0	10	1
47	$\alpha^5+\alpha^3+\alpha^2+\alpha+1=\epsilon^9$	47	10	9	55	30	11	0	1	0	11	0
48	$\alpha^5+\alpha^4=\epsilon^{62}$	48	2	62	15	40	24	0	1	36	10	1
49	$\alpha^5+\alpha^4+1=\epsilon^{57}$	49	33	57	30	11	25	0	1	36	47	0
50	$\alpha^5+\alpha^4+\alpha=\epsilon^{52}$	50	31	52	60	28	28	1	1	1	46	57

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	T
51	$\alpha^5+\alpha^4+\alpha+1=\epsilon^{38}$	51	59	38	25	61	29	1	1	1	46	
52	$\alpha^5+\alpha^4+\alpha^2=\epsilon^{33}$	52	54	33	50	38	8	1	1	10	37	
53	$\alpha^5+\alpha^4+\alpha^2+1=\epsilon^{17}$	53	43	17	5	2	9	1	1	10	46	
54	$\alpha^5+\alpha^4+\alpha^2+\alpha=\epsilon^{30}$	54	52	30	10	18	12	0	1	47	11	
55	$\alpha^5+\alpha^4+\alpha^2+\alpha+1=\epsilon^{47}$	55	42	47	20	15	13	0	1	47	37	
56	$\alpha^5+\alpha^4+\alpha^3=\epsilon^{42}$	56	57	42	40	37	57	1	1	1	1	
57	$\alpha^5+\alpha^4+\alpha^3+1=\epsilon^{21}$	57	56	21	49	62	56	1	1	1	1	
58	$\alpha^5+\alpha^4+\alpha^3+\alpha=\epsilon^{14}$	58	30	14	3	1	61	0	1	36	1	
59	$\alpha^5+\alpha^4+\alpha^3+\alpha+1=\epsilon^{25}$	59	51	25	6	39	60	0	1	36	36	
60	$\alpha^5+\alpha^4+\alpha^3+\alpha^2=\epsilon^{50}$	60	29	50	12	31	41	0	1	47	47	
61	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+1=\epsilon^{28}$	61	18	28	24	51	40	0	1	47	1	
62	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+\alpha=\epsilon^{12}$	62	25	12	48	57	45	1	1	10	37	
63	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+\alpha+1=\epsilon^{10}$	63	5	10	1	DNE	44	1	1	10	46	

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
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$2^3 = 8$
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$2^{61} = 24$
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i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$	N
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0	0	0	0	
1	$1 = 1$	1	1	63	2	58	1	0	1	0	1	1	
2	$\alpha = \epsilon$	2	48	1	4	53	4	1	1	37	47	56	
3	$\alpha + 1 = \epsilon^{58}$	3	32	58	8	34	5	1	1	37	11	57	
4	$\alpha^2 = \epsilon^2$	4	24	2	16	43	16	1	1	46	11	57	
5	$\alpha^2 + 1 = \epsilon^{53}$	5	63	53	32	6	17	1	1	46	36	56	
6	$\alpha^2 + \alpha = \epsilon^{59}$	6	16	59	33	5	20	0	1	11	46	1	
7	$\alpha^2 + \alpha + 1 = \epsilon^{39}$	7	45	39	35	44	21	0	1	11	36	0	
8	$\alpha^3 = \epsilon^3$	8	12	3	39	23	33	1	1	37	46	57	
9	$\alpha^3 + 1 = \epsilon^{34}$	9	27	34	47	27	32	1	1	37	10	56	
10	$\alpha^3 + \alpha = \epsilon^{54}$	10	47	54	63	12	37	0	1	0	37	1	
11	$\alpha^3 + \alpha + 1 = \epsilon^{18}$	11	37	18	31	49	36	0	1	0	36	0	
12	$\alpha^3 + \alpha^2 = \epsilon^{60}$	12	8	60	62	10	49	0	1	11	36	0	
13	$\alpha^3 + \alpha^2 + 1 = \epsilon^{31}$	13	26	31	29	41	48	0	1	11	46	1	
14	$\alpha^3 + \alpha^2 + \alpha = \epsilon^{40}$	14	38	40	58	25	53	1	1	46	37	56	
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^{48}$	15	21	48	21	55	52	1	1	46	10	57	
16	$\alpha^4 = \epsilon^4$	16	6	4	42	46	39	1	1	10	36	56	
17	$\alpha^4 + 1 = \epsilon^{43}$	17	44	43	53	33	38	1	1	10	47	57	
18	$\alpha^4 + \alpha = \epsilon^{35}$	18	61	35	11	54	35	0	1	47	1	0	
19	$\alpha^4 + \alpha + 1 = \epsilon^{22}$	19	28	22	22	26	34	0	1	47	47	1	
20	$\alpha^4 + \alpha^2 = \epsilon^{55}$	20	39	55	44	24	55	0	1	36	10	1	
21	$\alpha^4 + \alpha^2 + 1 = \epsilon^{15}$	21	15	15	57	42	54	0	1	36	47	0	
22	$\alpha^4 + \alpha^2 + \alpha = \epsilon^{19}$	22	34	19	19	35	51	1	1	1	37	57	
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \epsilon^{26}$	23	41	26	38	8	50	1	1	1	37	56	
24	$\alpha^4 + \alpha^3 = \epsilon^{61}$	24	4	61	45	20	6	0	1	47	37	1	
25	$\alpha^4 + \alpha^3 + 1 = \epsilon^{51}$	25	62	51	59	14	7	0	1	47	11	0	

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$
26	$\alpha^4+\alpha^3+\alpha=\epsilon^{32}$	26	13	32	23	19	2	1	1	10	36	57
27	$\alpha^4+\alpha^3+\alpha+1=\epsilon^{29}$	27	9	29	46	9	3	1	1	10	47	56
28	$\alpha^4+\alpha^3+\alpha^2=\epsilon^{41}$	28	19	41	61	50	22	1	1	1	10	56
29	$\alpha^4+\alpha^3+\alpha^2+1=\epsilon^{13}$	29	60	13	27	32	23	1	1	1	10	57
30	$\alpha^4+\alpha^3+\alpha^2+\alpha=\epsilon^{49}$	30	58	49	54	47	18	0	1	36	1	0
31	$\alpha^4+\alpha^3+\alpha^2+\alpha+1=\epsilon^{11}$	31	50	11	13	60	19	0	1	36	36	1
32	$\alpha^5=\epsilon^5$	32	3	5	26	29	63	1	1	46	37	57
33	$\alpha^5+1=\epsilon^6$	33	49	6	52	17	62	1	1	46	10	56
34	$\alpha^5+\alpha=\epsilon^{44}$	34	22	44	9	3	59	0	1	11	11	1
35	$\alpha^5+\alpha+1=\epsilon^7$	35	40	7	18	22	58	0	1	11	1	0
36	$\alpha^5+\alpha^2=\epsilon^{36}$	36	46	36	36	45	47	0	1	0	47	0
37	$\alpha^5+\alpha^2+1=\epsilon^{45}$	37	11	45	41	56	46	0	1	0	46	1
38	$\alpha^5+\alpha^2+\alpha=\epsilon^{23}$	38	14	23	51	52	43	1	1	37	11	56
39	$\alpha^5+\alpha^2+\alpha+1=\epsilon^8$	39	20	8	7	59	42	1	1	37	47	57
40	$\alpha^5+\alpha^3=\epsilon^{56}$	40	35	56	14	48	30	0	1	11	1	0
41	$\alpha^5+\alpha^3+1=\epsilon^{37}$	41	23	37	28	13	31	0	1	11	11	1
42	$\alpha^5+\alpha^3+\alpha=\epsilon^{16}$	42	55	16	56	21	26	1	1	46	11	56
43	$\alpha^5+\alpha^3+\alpha+1=\epsilon^{46}$	43	53	46	17	4	27	1	1	46	36	57
44	$\alpha^5+\alpha^3+\alpha^2=\epsilon^{20}$	44	17	20	34	7	14	1	1	37	10	57
45	$\alpha^5+\alpha^3+\alpha^2+1=\epsilon^{24}$	45	7	24	37	36	15	1	1	37	46	56
46	$\alpha^5+\alpha^3+\alpha^2+\alpha=\epsilon^{27}$	46	36	27	43	16	10	0	1	0	10	1
47	$\alpha^5+\alpha^3+\alpha^2+\alpha+1=\epsilon^9$	47	10	9	55	30	11	0	1	0	11	0
48	$\alpha^5+\alpha^4=\epsilon^{62}$	48	2	62	15	40	24	0	1	36	10	1
49	$\alpha^5+\alpha^4+1=\epsilon^{57}$	49	33	57	30	11	25	0	1	36	47	0
50	$\alpha^5+\alpha^4+\alpha=\epsilon^{52}$	50	31	52	60	28	28	1	1	1	46	57

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	T
51	$\alpha^5+\alpha^4+\alpha+1=\epsilon^{38}$	51	59	38	25	61	29	1	1	1	46	
52	$\alpha^5+\alpha^4+\alpha^2=\epsilon^{33}$	52	54	33	50	38	8	1	1	10	37	
53	$\alpha^5+\alpha^4+\alpha^2+1=\epsilon^{17}$	53	43	17	5	2	9	1	1	10	46	
54	$\alpha^5+\alpha^4+\alpha^2+\alpha=\epsilon^{30}$	54	52	30	10	18	12	0	1	47	11	
55	$\alpha^5+\alpha^4+\alpha^2+\alpha+1=\epsilon^{47}$	55	42	47	20	15	13	0	1	47	37	
56	$\alpha^5+\alpha^4+\alpha^3=\epsilon^{42}$	56	57	42	40	37	57	1	1	1	1	
57	$\alpha^5+\alpha^4+\alpha^3+1=\epsilon^{21}$	57	56	21	49	62	56	1	1	1	1	
58	$\alpha^5+\alpha^4+\alpha^3+\alpha=\epsilon^{14}$	58	30	14	3	1	61	0	1	36	1	
59	$\alpha^5+\alpha^4+\alpha^3+\alpha+1=\epsilon^{25}$	59	51	25	6	39	60	0	1	36	36	
60	$\alpha^5+\alpha^4+\alpha^3+\alpha^2=\epsilon^{50}$	60	29	50	12	31	41	0	1	47	47	
61	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+1=\epsilon^{28}$	61	18	28	24	51	40	0	1	47	1	
62	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+\alpha=\epsilon^{12}$	62	25	12	48	57	45	1	1	10	37	
63	$\alpha^5+\alpha^4+\alpha^3+\alpha^2+\alpha+1=\epsilon^{10}$	63	5	10	1	DNE	44	1	1	10	46	

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