Rank-35 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 35

General information

Number of lines	3
Number of points	7
Number of singular points	3
Number of Eckardt points	1
Number of double points	0
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	3, 1^6

Singular Points

The surface has 3 singular points:

$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$2 : P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0,0,0,0,1,0)_9$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0,1,0,1,1,0)_{17}$$

Rank of lines: (6, 30, 27)
Rank of points on Klein quadric: (9, 5, 17)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1).\ T = -1$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 6 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_1 = (0,1,0,0)$ lies on line ℓ_1
$2 :\ P_4 = (1,1,1,1)$ lies on line ℓ_2
$3 :\ P_8 = (1,1,1,0)$ lies on line ℓ_2
$4 :\ P_9 = (1,0,0,1)$ lies on line ℓ_0
$5 :\ P_{10} = (0,1,0,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 7 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_8 = (1,1,1,0)$
$5: P_9 = (1,0,0,1)$
$6: P_{10} = (0,1,0,1)$