Rank-34 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_1 = 0$$

(1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(2)$ is 34

General information

Number of lines	1
Number of points	3
Number of singular points	3
Number of Eckardt points	0
Number of double points	0
Number of single points	3
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3
Type of lines on points	1^3

Singular Points

The surface has 3 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$2 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (34)
Rank of points on Klein quadric: (1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 3 single points:
The single points on the surface are:

$0 : P_2 = (0, 0, 1, 0)$ lies on line ℓ_0 $2 : P_{12} = (0, 0, 1, 1)$ lies on line ℓ_0
$1 : P_3 = (0, 0, 0, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 3 points:
The points on the surface are:

$0 : P_2 = (0, 0, 1, 0)$ $2 : P_{12} = (0, 0, 1, 1)$
$1 : P_3 = (0, 0, 0, 1)$