

Rank-38 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(8) is 674

General information

Number of lines	9
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	72
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^9
Type of lines on points	$9, 1^{72}$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned} \ell_0 &= \left[\begin{array}{cccc} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{510} = \left[\begin{array}{cccc} 1 & 6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{510} = \mathbf{Pl}(0, 0, 0, 6, 1, 0)_{276} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{291} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{291} = \mathbf{Pl}(0, 0, 0, 3, 1, 0)_{231} \end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{437} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{437} = \mathbf{Pl}(0, 0, 0, 5, 1, 0)_{261} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{89} \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25} \\
\ell_5 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{209} \\
\ell_6 &= \begin{bmatrix} 1 & \gamma^2 & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4452} = \begin{bmatrix} 1 & 4 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4452} = \mathbf{Pl}(0, 7, 0, 4, 1, 0)_{260} \\
\ell_7 &= \begin{bmatrix} 1 & \gamma^4 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1751} = \begin{bmatrix} 1 & 7 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1751} = \mathbf{Pl}(0, 2, 0, 7, 1, 0)_{300} \\
\ell_8 &= \begin{bmatrix} 1 & \gamma & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2554} = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2554} = \mathbf{Pl}(0, 4, 0, 2, 1, 0)_{227}
\end{aligned}$$

Rank of lines: (510, 291, 437, 656, 4689, 729, 4452, 1751, 2554)

Rank of points on Klein quadric: (276, 231, 261, 89, 25, 209, 260, 300, 227)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 72 single points:

The single points on the surface are:

- 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_5
- 1 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0
- 2 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_1
- 3 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_2
- 4 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_3
- 5 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_4
- 6 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_5
- 7 : $P_{40} = (5, 3, 1, 0)$ lies on line ℓ_6
- 8 : $P_{57} = (6, 5, 1, 0)$ lies on line ℓ_7
- 9 : $P_{62} = (3, 6, 1, 0)$ lies on line ℓ_8
- 10 : $P_{84} = (2, 1, 0, 1)$ lies on line ℓ_0
- 11 : $P_{86} = (4, 1, 0, 1)$ lies on line ℓ_1
- 12 : $P_{89} = (7, 1, 0, 1)$ lies on line ℓ_2
- 13 : $P_{93} = (3, 2, 0, 1)$ lies on line ℓ_2

- 14 : $P_{94} = (4, 2, 0, 1)$ lies on line ℓ_0
- 15 : $P_{95} = (5, 2, 0, 1)$ lies on line ℓ_1
- 16 : $P_{99} = (1, 3, 0, 1)$ lies on line ℓ_1
- 17 : $P_{102} = (4, 3, 0, 1)$ lies on line ℓ_2
- 18 : $P_{104} = (6, 3, 0, 1)$ lies on line ℓ_0
- 19 : $P_{111} = (5, 4, 0, 1)$ lies on line ℓ_0
- 20 : $P_{112} = (6, 4, 0, 1)$ lies on line ℓ_2
- 21 : $P_{113} = (7, 4, 0, 1)$ lies on line ℓ_1
- 22 : $P_{115} = (1, 5, 0, 1)$ lies on line ℓ_2
- 23 : $P_{117} = (3, 5, 0, 1)$ lies on line ℓ_1
- 24 : $P_{121} = (7, 5, 0, 1)$ lies on line ℓ_0
- 25 : $P_{123} = (1, 6, 0, 1)$ lies on line ℓ_0
- 26 : $P_{124} = (2, 6, 0, 1)$ lies on line ℓ_1
- 27 : $P_{127} = (5, 6, 0, 1)$ lies on line ℓ_2

28 : $P_{132} = (2, 7, 0, 1)$ lies on line ℓ_2
 29 : $P_{133} = (3, 7, 0, 1)$ lies on line ℓ_0
 30 : $P_{136} = (6, 7, 0, 1)$ lies on line ℓ_1
 31 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_3
 32 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_4
 33 : $P_{166} = (5, 3, 1, 1)$ lies on line ℓ_6
 34 : $P_{183} = (6, 5, 1, 1)$ lies on line ℓ_7
 35 : $P_{188} = (3, 6, 1, 1)$ lies on line ℓ_8
 36 : $P_{203} = (2, 0, 2, 1)$ lies on line ℓ_3
 37 : $P_{215} = (6, 1, 2, 1)$ lies on line ℓ_8
 38 : $P_{217} = (0, 2, 2, 1)$ lies on line ℓ_4
 39 : $P_{219} = (2, 2, 2, 1)$ lies on line ℓ_5
 40 : $P_{256} = (7, 6, 2, 1)$ lies on line ℓ_6
 41 : $P_{258} = (1, 7, 2, 1)$ lies on line ℓ_7
 42 : $P_{268} = (3, 0, 3, 1)$ lies on line ℓ_3
 43 : $P_{288} = (7, 2, 3, 1)$ lies on line ℓ_7
 44 : $P_{289} = (0, 3, 3, 1)$ lies on line ℓ_4
 45 : $P_{292} = (3, 3, 3, 1)$ lies on line ℓ_5
 46 : $P_{307} = (2, 5, 3, 1)$ lies on line ℓ_6
 47 : $P_{326} = (5, 7, 3, 1)$ lies on line ℓ_8
 48 : $P_{333} = (4, 0, 4, 1)$ lies on line ℓ_3
 49 : $P_{340} = (3, 1, 4, 1)$ lies on line ℓ_6
 50 : $P_{346} = (1, 2, 4, 1)$ lies on line ℓ_8

51 : $P_{355} = (2, 3, 4, 1)$ lies on line ℓ_7
 52 : $P_{361} = (0, 4, 4, 1)$ lies on line ℓ_4
 53 : $P_{365} = (4, 4, 4, 1)$ lies on line ℓ_5
 54 : $P_{398} = (5, 0, 5, 1)$ lies on line ℓ_3
 55 : $P_{415} = (6, 2, 5, 1)$ lies on line ℓ_6
 56 : $P_{427} = (2, 4, 5, 1)$ lies on line ℓ_8
 57 : $P_{433} = (0, 5, 5, 1)$ lies on line ℓ_4
 58 : $P_{438} = (5, 5, 5, 1)$ lies on line ℓ_5
 59 : $P_{445} = (4, 6, 5, 1)$ lies on line ℓ_7
 60 : $P_{463} = (6, 0, 6, 1)$ lies on line ℓ_3
 61 : $P_{488} = (7, 3, 6, 1)$ lies on line ℓ_8
 62 : $P_{492} = (3, 4, 6, 1)$ lies on line ℓ_7
 63 : $P_{505} = (0, 6, 6, 1)$ lies on line ℓ_4
 64 : $P_{511} = (6, 6, 6, 1)$ lies on line ℓ_5
 65 : $P_{517} = (4, 7, 6, 1)$ lies on line ℓ_6
 66 : $P_{528} = (7, 0, 7, 1)$ lies on line ℓ_3
 67 : $P_{534} = (5, 1, 7, 1)$ lies on line ℓ_7
 68 : $P_{554} = (1, 4, 7, 1)$ lies on line ℓ_6
 69 : $P_{565} = (4, 5, 7, 1)$ lies on line ℓ_8
 70 : $P_{577} = (0, 7, 7, 1)$ lies on line ℓ_4
 71 : $P_{584} = (7, 7, 7, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1
5	1	1	1	1	1	0	1	1	1
6	1	1	1	1	1	1	0	1	1
7	1	1	1	1	1	1	1	0	1
8	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 73 points:

The points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$	25 : $P_{121} = (7, 5, 0, 1)$	50 : $P_{340} = (3, 1, 4, 1)$
1 : $P_4 = (1, 1, 1, 1)$	26 : $P_{123} = (1, 6, 0, 1)$	51 : $P_{346} = (1, 2, 4, 1)$
2 : $P_6 = (2, 1, 0, 0)$	27 : $P_{124} = (2, 6, 0, 1)$	52 : $P_{355} = (2, 3, 4, 1)$
3 : $P_8 = (4, 1, 0, 0)$	28 : $P_{127} = (5, 6, 0, 1)$	53 : $P_{361} = (0, 4, 4, 1)$
4 : $P_{11} = (7, 1, 0, 0)$	29 : $P_{132} = (2, 7, 0, 1)$	54 : $P_{365} = (4, 4, 4, 1)$
5 : $P_{12} = (1, 0, 1, 0)$	30 : $P_{133} = (3, 7, 0, 1)$	55 : $P_{398} = (5, 0, 5, 1)$
6 : $P_{19} = (0, 1, 1, 0)$	31 : $P_{136} = (6, 7, 0, 1)$	56 : $P_{415} = (6, 2, 5, 1)$
7 : $P_{20} = (1, 1, 1, 0)$	32 : $P_{139} = (1, 0, 1, 1)$	57 : $P_{427} = (2, 4, 5, 1)$
8 : $P_{40} = (5, 3, 1, 0)$	33 : $P_{146} = (0, 1, 1, 1)$	58 : $P_{433} = (0, 5, 5, 1)$
9 : $P_{57} = (6, 5, 1, 0)$	34 : $P_{166} = (5, 3, 1, 1)$	59 : $P_{438} = (5, 5, 5, 1)$
10 : $P_{62} = (3, 6, 1, 0)$	35 : $P_{183} = (6, 5, 1, 1)$	60 : $P_{445} = (4, 6, 5, 1)$
11 : $P_{84} = (2, 1, 0, 1)$	36 : $P_{188} = (3, 6, 1, 1)$	61 : $P_{463} = (6, 0, 6, 1)$
12 : $P_{86} = (4, 1, 0, 1)$	37 : $P_{203} = (2, 0, 2, 1)$	62 : $P_{488} = (7, 3, 6, 1)$
13 : $P_{89} = (7, 1, 0, 1)$	38 : $P_{215} = (6, 1, 2, 1)$	63 : $P_{492} = (3, 4, 6, 1)$
14 : $P_{93} = (3, 2, 0, 1)$	39 : $P_{217} = (0, 2, 2, 1)$	64 : $P_{505} = (0, 6, 6, 1)$
15 : $P_{94} = (4, 2, 0, 1)$	40 : $P_{219} = (2, 2, 2, 1)$	65 : $P_{511} = (6, 6, 6, 1)$
16 : $P_{95} = (5, 2, 0, 1)$	41 : $P_{256} = (7, 6, 2, 1)$	66 : $P_{517} = (4, 7, 6, 1)$
17 : $P_{99} = (1, 3, 0, 1)$	42 : $P_{258} = (1, 7, 2, 1)$	67 : $P_{528} = (7, 0, 7, 1)$
18 : $P_{102} = (4, 3, 0, 1)$	43 : $P_{268} = (3, 0, 3, 1)$	68 : $P_{534} = (5, 1, 7, 1)$
19 : $P_{104} = (6, 3, 0, 1)$	44 : $P_{288} = (7, 2, 3, 1)$	69 : $P_{554} = (1, 4, 7, 1)$
20 : $P_{111} = (5, 4, 0, 1)$	45 : $P_{289} = (0, 3, 3, 1)$	70 : $P_{565} = (4, 5, 7, 1)$
21 : $P_{112} = (6, 4, 0, 1)$	46 : $P_{292} = (3, 3, 3, 1)$	71 : $P_{577} = (0, 7, 7, 1)$
22 : $P_{113} = (7, 4, 0, 1)$	47 : $P_{307} = (2, 5, 3, 1)$	72 : $P_{584} = (7, 7, 7, 1)$
23 : $P_{115} = (1, 5, 0, 1)$	48 : $P_{326} = (5, 7, 3, 1)$	
24 : $P_{117} = (3, 5, 0, 1)$	49 : $P_{333} = (4, 0, 4, 1)$	