Rank-16 over GF(2)

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The equation

The equation of the surface is :

$$X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 16

General information

Number of lines	18
Number of points	13
Number of singular points	7
Number of Eckardt points	6
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{18}
Type of lines on points	6, 5^6, 3^6

Singular Points

The surface has 7 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$2: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$3: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$4: P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$
$5: P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$
$6: P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$

The 18 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{28} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_6 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_6 = \mathbf{Pl}(0,0,0,0,1,0)_9$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_2 = \mathbf{Pl}(1,0,0,0,1,0)_{10}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_5 = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_6 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$

$$\ell_8 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_9 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{34} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{18} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

$$\ell_{11} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{31} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{31} = \mathbf{Pl}(0,1,0,0,0,1)_{21}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{16} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{16} = \mathbf{Pl}(1,0,0,1,1,0)_{16}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \mathbf{Pl}(0,1,0,0,1,0)_{11}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{19} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{19} = \mathbf{Pl}(0,1,1,0,1,0)_{14}$$

$$\ell_{16} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

$$\ell_{17} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{32} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{32} = \mathbf{Pl}(0,1,0,1,0,1)_{27}$$

Rank of lines: (0, 4, 28, 6, 2, 5, 30, 14, 29, 34, 18, 31, 13, 16, 20, 19, 33, 32)
Rank of points on Klein quadric: (0, 2, 19, 9, 10, 12, 5, 6, 25, 1, 4, 21, 15, 16, 11, 14, 7, 27)

Eckardt Points

The surface has 6 Eckardt points:
$0 : P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0),\ T = 2$
$1 : P_6 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0),\ T = 6$
$2 : P_7 = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0),\ T = 14$
$3 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1),\ T = 2$
$4 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1),\ T = 6$
$5 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 14$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0
1	1	0	1	1	1	1	0	0	0	1	1	1	0	0	1	1	0	0
2	1	1	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1	1
3	1	1	0	0	1	1	1	1	0	1	1	0	1	1	1	1	1	0
4	1	1	0	1	0	1	1	1	0	0	0	1	1	1	0	0	0	1
5	1	1	0	1	1	0	0	0	1	1	1	0	0	0	1	1	0	1
6	1	0	1	1	1	0	0	1	1	1	0	1	1	1	1	0	1	1
7	1	0	1	1	1	0	1	0	1	0	1	0	1	1	0	1	0	0
8	1	0	1	0	0	1	1	1	0	1	0	1	0	0	0	1	1	1
9	0	1	1	1	0	1	1	0	1	0	1	1	1	0	1	1	1	1
10	0	1	1	1	0	1	0	1	0	1	0	1	0	1	1	1	0	0
11	0	1	1	0	1	0	1	0	1	1	1	0	0	1	0	0	1	1
12	1	0	0	1	1	0	1	1	0	1	0	0	0	1	1	0	1	0
13	1	0	0	1	1	0	1	1	0	0	1	1	1	0	0	1	0	1
14	0	1	0	1	0	1	1	0	0	1	1	0	1	0	0	1	1	0
15	0	1	0	1	0	1	0	1	1	1	1	0	0	1	1	0	0	1
16	0	0	1	1	0	0	1	0	1	1	0	1	1	0	1	0	0	1
17	0	0	1	0	1	1	1	0	1	1	0	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{12}	ℓ_{13}
in point	P_0	P_1	P_0	P_0	P_0	P_1	P_1	P_1	P_5	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{15}
in point	P_0	P_2	P_0	P_0	P_0	P_2	P_2	P_2	P_6	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{16}	ℓ_{17}
in point	P_1	P_2	P_1	P_1	P_1	P_2	P_2	P_2	P_7	P_7

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_0	P_0	P_0	P_0	P_3	P_9	P_3	P_9	P_3	P_9	P_3	P_9	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{17}
in point	P_0	P_0	P_0	P_0	P_{10}	P_{11}	P_{10}	P_{11}	P_{10}	P_{10}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_0	P_0	P_0	P_0	P_{12}	P_{12}	P_{13}	P_{13}	P_{12}	P_{12}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}
in point	P_1	P_1	P_3	P_{10}	P_1	P_1	P_3	P_{10}	P_3	P_{10}	P_3	P_3	P_{10}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{15}
in point	P_1	P_1	P_9	P_{11}	P_1	P_1	P_9	P_{11}	P_9	P_9

Line 8 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{16}	ℓ_{17}
in point	P_1	P_1	P_{12}	P_1	P_1	P_{12}	P_{14}	P_{12}	P_{14}	P_{12}

Line 9 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}
in point	P_2	P_2	P_3	P_{12}	P_3	P_{12}	P_2	P_2	P_3	P_3	P_{12}	P_3	P_{12}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_2	P_2	P_9	P_{13}	P_9	P_2	P_2	P_9	P_{13}	P_9

Line 11 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{16}	ℓ_{17}
in point	P_2	P_2	P_{10}	P_{10}	P_{14}	P_2	P_2	P_{10}	P_{14}	P_{10}

Line 12 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{16}
in point	P_5	P_3	P_{11}	P_3	P_{11}	P_3	P_5	P_3	P_3

Line 13 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{15}	ℓ_{17}
in point	P_5	P_9	P_{10}	P_{10}	P_9	P_9	P_{10}	P_5	P_9	P_{10}

Line 14 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{16}
in point	P_6	P_3	P_{13}	P_3	P_3	P_{13}	P_3	P_6	P_3

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{17}
in point	P_6	P_9	P_{12}	P_9	P_{12}	P_{12}	P_9	P_9	P_6	P_{12}

Line 16 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{17}
in point	P_7	P_3	P_3	P_{14}	P_3	P_{14}	P_3	P_3	P_7

Line 17 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{16}
in point	P_7	P_{10}	P_{12}	P_{10}	P_{12}	P_{12}	P_{10}	P_{10}	P_{12}	P_7

The surface has 13 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (1,0,1,0)$
$6: P_7 = (0,1,1,0)$
$7: P_9 = (1,0,0,1)$
$8: P_{10} = (0,1,0,1)$
$9: P_{11} = (1,1,0,1)$
$10: P_{12} = (0,0,1,1)$
$11: P_{13} = (1,0,1,1)$
$12: P_{14} = (0,1,1,1)$