

# Rank-65859 over GF(64)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(64) is 1090785349

## General information

Number of lines	67
Number of points	4225
Number of singular points	65
Number of Eckardt points	32
Number of double points	66
Number of single points	4127
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^{67}$
Type of lines on points	$3^{32}, 2^{66}, 1^{4127}$

## Singular Points

The surface has 65 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{12353} = \mathbf{P}(0, 0, \epsilon, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{16449} = \mathbf{P}(0, 0, \epsilon^{58}, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{20545} = \mathbf{P}(0, 0, \epsilon^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{24641} = \mathbf{P}(0, 0, \epsilon^{53}, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{28737} = \mathbf{P}(0, 0, \epsilon^{59}, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{32833} = \mathbf{P}(0, 0, \epsilon^{39}, 1) = \mathbf{P}(0, 0, 7, 1)$$

$$9 : P_{36929} = \mathbf{P}(0, 0, \epsilon^3, 1) = \mathbf{P}(0, 0, 8, 1)$$

$$10 : P_{41025} = \mathbf{P}(0, 0, \epsilon^{34}, 1) = \mathbf{P}(0, 0, 9, 1)$$

$$11 : P_{45121} = \mathbf{P}(0, 0, \epsilon^{54}, 1) = \mathbf{P}(0, 0, 10, 1)$$

$$12 : P_{49217} = \mathbf{P}(0, 0, \epsilon^{18}, 1) = \mathbf{P}(0, 0, 11, 1)$$

$$13 : P_{53313} = \mathbf{P}(0, 0, \epsilon^{60}, 1) = \mathbf{P}(0, 0, 12, 1)$$

$$14 : P_{57409} = \mathbf{P}(0, 0, \epsilon^{31}, 1) = \mathbf{P}(0, 0, 13, 1)$$

$$15 : P_{61505} = \mathbf{P}(0, 0, \epsilon^{40}, 1) = \mathbf{P}(0, 0, 14, 1)$$

$$16 : P_{65601} = \mathbf{P}(0, 0, \epsilon^{48}, 1) = \mathbf{P}(0, 0, 15, 1)$$

$$17 : P_{69697} = \mathbf{P}(0, 0, \epsilon^4, 1) = \mathbf{P}(0, 0, 16, 1)$$

$$\begin{aligned}
18 : P_{73793} &= \mathbf{P}(0, 0, \epsilon^{43}, 1) = \mathbf{P}(0, 0, 17, 1) \\
19 : P_{77889} &= \mathbf{P}(0, 0, \epsilon^{35}, 1) = \mathbf{P}(0, 0, 18, 1) \\
20 : P_{81985} &= \mathbf{P}(0, 0, \epsilon^{22}, 1) = \mathbf{P}(0, 0, 19, 1) \\
21 : P_{86081} &= \mathbf{P}(0, 0, \epsilon^{55}, 1) = \mathbf{P}(0, 0, 20, 1) \\
22 : P_{90177} &= \mathbf{P}(0, 0, \epsilon^{15}, 1) = \mathbf{P}(0, 0, 21, 1) \\
23 : P_{94273} &= \mathbf{P}(0, 0, \epsilon^{19}, 1) = \mathbf{P}(0, 0, 22, 1) \\
24 : P_{98369} &= \mathbf{P}(0, 0, \epsilon^{26}, 1) = \mathbf{P}(0, 0, 23, 1) \\
25 : P_{102465} &= \mathbf{P}(0, 0, \epsilon^{61}, 1) = \mathbf{P}(0, 0, 24, 1) \\
26 : P_{106561} &= \mathbf{P}(0, 0, \epsilon^{51}, 1) = \mathbf{P}(0, 0, 25, 1) \\
27 : P_{110657} &= \mathbf{P}(0, 0, \epsilon^{32}, 1) = \mathbf{P}(0, 0, 26, 1) \\
28 : P_{114753} &= \mathbf{P}(0, 0, \epsilon^{29}, 1) = \mathbf{P}(0, 0, 27, 1) \\
29 : P_{118849} &= \mathbf{P}(0, 0, \epsilon^{41}, 1) = \mathbf{P}(0, 0, 28, 1) \\
30 : P_{122945} &= \mathbf{P}(0, 0, \epsilon^{13}, 1) = \mathbf{P}(0, 0, 29, 1) \\
31 : P_{127041} &= \mathbf{P}(0, 0, \epsilon^{49}, 1) = \mathbf{P}(0, 0, 30, 1) \\
32 : P_{131137} &= \mathbf{P}(0, 0, \epsilon^{11}, 1) = \mathbf{P}(0, 0, 31, 1) \\
33 : P_{135233} &= \mathbf{P}(0, 0, \epsilon^5, 1) = \mathbf{P}(0, 0, 32, 1) \\
34 : P_{139329} &= \mathbf{P}(0, 0, \epsilon^6, 1) = \mathbf{P}(0, 0, 33, 1) \\
35 : P_{143425} &= \mathbf{P}(0, 0, \epsilon^{44}, 1) = \mathbf{P}(0, 0, 34, 1) \\
36 : P_{147521} &= \mathbf{P}(0, 0, \epsilon^7, 1) = \mathbf{P}(0, 0, 35, 1) \\
37 : P_{151617} &= \mathbf{P}(0, 0, \epsilon^{36}, 1) = \mathbf{P}(0, 0, 36, 1) \\
38 : P_{155713} &= \mathbf{P}(0, 0, \epsilon^{45}, 1) = \mathbf{P}(0, 0, 37, 1) \\
39 : P_{159809} &= \mathbf{P}(0, 0, \epsilon^{23}, 1) = \mathbf{P}(0, 0, 38, 1) \\
40 : P_{163905} &= \mathbf{P}(0, 0, \epsilon^8, 1) = \mathbf{P}(0, 0, 39, 1) \\
41 : P_{168001} &= \mathbf{P}(0, 0, \epsilon^{56}, 1) = \mathbf{P}(0, 0, 40, 1) \\
42 : P_{172097} &= \mathbf{P}(0, 0, \epsilon^{37}, 1) = \mathbf{P}(0, 0, 41, 1) \\
43 : P_{176193} &= \mathbf{P}(0, 0, \epsilon^{16}, 1) = \mathbf{P}(0, 0, 42, 1) \\
44 : P_{180289} &= \mathbf{P}(0, 0, \epsilon^{46}, 1) = \mathbf{P}(0, 0, 43, 1) \\
45 : P_{184385} &= \mathbf{P}(0, 0, \epsilon^{20}, 1) = \mathbf{P}(0, 0, 44, 1) \\
46 : P_{188481} &= \mathbf{P}(0, 0, \epsilon^{24}, 1) = \mathbf{P}(0, 0, 45, 1) \\
47 : P_{192577} &= \mathbf{P}(0, 0, \epsilon^{27}, 1) = \mathbf{P}(0, 0, 46, 1) \\
48 : P_{196673} &= \mathbf{P}(0, 0, \epsilon^9, 1) = \mathbf{P}(0, 0, 47, 1) \\
49 : P_{200769} &= \mathbf{P}(0, 0, \epsilon^{62}, 1) = \mathbf{P}(0, 0, 48, 1) \\
50 : P_{204865} &= \mathbf{P}(0, 0, \epsilon^{57}, 1) = \mathbf{P}(0, 0, 49, 1) \\
51 : P_{208961} &= \mathbf{P}(0, 0, \epsilon^{52}, 1) = \mathbf{P}(0, 0, 50, 1) \\
52 : P_{213057} &= \mathbf{P}(0, 0, \epsilon^{38}, 1) = \mathbf{P}(0, 0, 51, 1) \\
53 : P_{217153} &= \mathbf{P}(0, 0, \epsilon^{33}, 1) = \mathbf{P}(0, 0, 52, 1) \\
54 : P_{221249} &= \mathbf{P}(0, 0, \epsilon^{17}, 1) = \mathbf{P}(0, 0, 53, 1) \\
55 : P_{225345} &= \mathbf{P}(0, 0, \epsilon^{30}, 1) = \mathbf{P}(0, 0, 54, 1) \\
56 : P_{229441} &= \mathbf{P}(0, 0, \epsilon^{47}, 1) = \mathbf{P}(0, 0, 55, 1) \\
57 : P_{233537} &= \mathbf{P}(0, 0, \epsilon^{42}, 1) = \mathbf{P}(0, 0, 56, 1) \\
58 : P_{237633} &= \mathbf{P}(0, 0, \epsilon^{21}, 1) = \mathbf{P}(0, 0, 57, 1) \\
59 : P_{241729} &= \mathbf{P}(0, 0, \epsilon^{14}, 1) = \mathbf{P}(0, 0, 58, 1) \\
60 : P_{245825} &= \mathbf{P}(0, 0, \epsilon^{25}, 1) = \mathbf{P}(0, 0, 59, 1) \\
61 : P_{249921} &= \mathbf{P}(0, 0, \epsilon^{50}, 1) = \mathbf{P}(0, 0, 60, 1) \\
62 : P_{254017} &= \mathbf{P}(0, 0, \epsilon^{28}, 1) = \mathbf{P}(0, 0, 61, 1) \\
63 : P_{258113} &= \mathbf{P}(0, 0, \epsilon^{12}, 1) = \mathbf{P}(0, 0, 62, 1) \\
64 : P_{262209} &= \mathbf{P}(0, 0, \epsilon^{10}, 1) = \mathbf{P}(0, 0, 63, 1)
\end{aligned}$$

## The 67 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{270528} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{129} \\
\ell_4 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_5 &= \begin{bmatrix} 1 & \epsilon^{62} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{56} \end{bmatrix}_{203864} = \begin{bmatrix} 1 & 48 & 0 & 0 \\ 0 & 0 & 1 & 40 \end{bmatrix}_{203864} = \mathbf{Pl}(0, 0, 35, 40, 49, 1)_{13128736} \\
\ell_6 &= \begin{bmatrix} 1 & \epsilon^5 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{11} \end{bmatrix}_{137279} = \begin{bmatrix} 1 & 32 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{137279} = \mathbf{Pl}(0, 0, 50, 31, 33, 1)_{8937361} \\
\ell_7 &= \begin{bmatrix} 1 & \epsilon^{61} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{49} \end{bmatrix}_{103990} = \begin{bmatrix} 1 & 24 & 0 & 0 \\ 0 & 0 & 1 & 30 \end{bmatrix}_{103990} = \mathbf{Pl}(0, 0, 58, 30, 25, 1)_{6841737} \\
\ell_8 &= \begin{bmatrix} 1 & \epsilon^{10} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{22} \end{bmatrix}_{266258} = \begin{bmatrix} 1 & 63 & 0 & 0 \\ 0 & 0 & 1 & 19 \end{bmatrix}_{266258} = \mathbf{Pl}(0, 0, 28, 19, 62, 1)_{16534887} \\
\ell_9 &= \begin{bmatrix} 1 & \epsilon^4 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{47} \end{bmatrix}_{70727} = \begin{bmatrix} 1 & 16 & 0 & 0 \\ 0 & 0 & 1 & 55 \end{bmatrix}_{70727} = \mathbf{Pl}(0, 0, 42, 55, 17, 1)_{4743065}
\end{aligned}$$

$$\begin{aligned}
\ell_{10} &= \begin{bmatrix} 1 & \epsilon^{24} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{44} \end{bmatrix}_{191375} = \begin{bmatrix} 1 & 45 & 0 & 0 \\ 0 & 0 & 1 & 34 \end{bmatrix}_{191375} = \mathbf{Pl}(0, 0, 22, 34, 44, 1)_{11816685} \\
\ell_{11} &= \begin{bmatrix} 1 & \epsilon^{60} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{28} \end{bmatrix}_{54089} = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 61 \end{bmatrix}_{54089} = \mathbf{Pl}(0, 0, 18, 61, 13, 1)_{3691697} \\
\ell_{12} &= \begin{bmatrix} 1 & \epsilon^{29} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{61} \end{bmatrix}_{116467} = \begin{bmatrix} 1 & 27 & 0 & 0 \\ 0 & 0 & 1 & 24 \end{bmatrix}_{116467} = \mathbf{Pl}(0, 0, 4, 24, 26, 1)_{7096959} \\
\ell_{13} &= \begin{bmatrix} 1 & \epsilon^9 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{36} \end{bmatrix}_{199699} = \begin{bmatrix} 1 & 47 & 0 & 0 \\ 0 & 0 & 1 & 36 \end{bmatrix}_{199699} = \mathbf{Pl}(0, 0, 46, 36, 46, 1)_{12343893} \\
\ell_{14} &= \begin{bmatrix} 1 & \epsilon^{45} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{18} \end{bmatrix}_{158064} = \begin{bmatrix} 1 & 37 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{158064} = \mathbf{Pl}(0, 0, 37, 11, 36, 1)_{9721950} \\
\ell_{15} &= \begin{bmatrix} 1 & \epsilon^3 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{37} \end{bmatrix}_{37425} = \begin{bmatrix} 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 41 \end{bmatrix}_{37425} = \mathbf{Pl}(0, 0, 23, 41, 9, 1)_{2644012} \\
\ell_{16} &= \begin{bmatrix} 1 & \epsilon^{32} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{61} \end{bmatrix}_{112306} = \begin{bmatrix} 1 & 26 & 0 & 0 \\ 0 & 0 & 1 & 24 \end{bmatrix}_{112306} = \mathbf{Pl}(0, 0, 4, 24, 27, 1)_{7359039} \\
\ell_{17} &= \begin{bmatrix} 1 & \epsilon^{23} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{31} \end{bmatrix}_{162227} = \begin{bmatrix} 1 & 38 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{162227} = \mathbf{Pl}(0, 0, 26, 13, 39, 1)_{10506793} \\
\ell_{18} &= \begin{bmatrix} 1 & \epsilon^{15} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^7 \end{bmatrix}_{91512} = \begin{bmatrix} 1 & 21 & 0 & 0 \\ 0 & 0 & 1 & 35 \end{bmatrix}_{91512} = \mathbf{Pl}(0, 0, 40, 35, 20, 1)_{5529051} \\
\ell_{19} &= \begin{bmatrix} 1 & \epsilon^{59} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{35} \end{bmatrix}_{29080} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{29080} = \mathbf{Pl}(0, 0, 61, 18, 7, 1)_{2124678} \\
\ell_{20} &= \begin{bmatrix} 1 & \epsilon^{20} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{44} \end{bmatrix}_{187214} = \begin{bmatrix} 1 & 44 & 0 & 0 \\ 0 & 0 & 1 & 34 \end{bmatrix}_{187214} = \mathbf{Pl}(0, 0, 22, 34, 45, 1)_{12078765} \\
\ell_{21} &= \begin{bmatrix} 1 & \epsilon^{28} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{15} \end{bmatrix}_{257938} = \begin{bmatrix} 1 & 61 & 0 & 0 \\ 0 & 0 & 1 & 21 \end{bmatrix}_{257938} = \mathbf{Pl}(0, 0, 15, 21, 60, 1)_{16009076} \\
\ell_{22} &= \begin{bmatrix} 1 & \epsilon^{41} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{54} \end{bmatrix}_{120614} = \begin{bmatrix} 1 & 28 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{120614} = \mathbf{Pl}(0, 0, 47, 10, 29, 1)_{7888660} \\
\ell_{23} &= \begin{bmatrix} 1 & \epsilon^8 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{31} \end{bmatrix}_{166388} = \begin{bmatrix} 1 & 39 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{166388} = \mathbf{Pl}(0, 0, 26, 13, 38, 1)_{10244713} \\
\ell_{24} &= \begin{bmatrix} 1 & \epsilon^{48} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{25} \end{bmatrix}_{66570} = \begin{bmatrix} 1 & 15 & 0 & 0 \\ 0 & 0 & 1 & 59 \end{bmatrix}_{66570} = \mathbf{Pl}(0, 0, 51, 59, 14, 1)_{3957968} \\
\ell_{25} &= \begin{bmatrix} 1 & \epsilon^{44} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{51} \end{bmatrix}_{145595} = \begin{bmatrix} 1 & 34 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{145595} = \mathbf{Pl}(0, 0, 62, 25, 35, 1)_{9463045} \\
\ell_{26} &= \begin{bmatrix} 1 & \epsilon^{37} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{30} \end{bmatrix}_{174751} = \begin{bmatrix} 1 & 41 & 0 & 0 \\ 0 & 0 & 1 & 54 \end{bmatrix}_{174751} = \mathbf{Pl}(0, 0, 52, 54, 40, 1)_{10772175} \\
\ell_{27} &= \begin{bmatrix} 1 & \epsilon^2 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{55} \end{bmatrix}_{20760} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{20760} = \mathbf{Pl}(0, 0, 39, 20, 5, 1)_{1597724} \\
\ell_{28} &= \begin{bmatrix} 1 & \epsilon^{12} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{22} \end{bmatrix}_{262097} = \begin{bmatrix} 1 & 62 & 0 & 0 \\ 0 & 0 & 1 & 19 \end{bmatrix}_{262097} = \mathbf{Pl}(0, 0, 28, 19, 63, 1)_{16796967} \\
\ell_{29} &= \begin{bmatrix} 1 & \epsilon^{31} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{28} \end{bmatrix}_{58250} = \begin{bmatrix} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 61 \end{bmatrix}_{58250} = \mathbf{Pl}(0, 0, 18, 61, 12, 1)_{3429617} \\
\ell_{30} &= \begin{bmatrix} 1 & \epsilon^{34} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{37} \end{bmatrix}_{41586} = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 41 \end{bmatrix}_{41586} = \mathbf{Pl}(0, 0, 23, 41, 8, 1)_{2381932}
\end{aligned}$$

$$\begin{aligned}
\ell_{31} &= \begin{bmatrix} 1 & \epsilon^{22} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{57} \end{bmatrix}_{83204} = \begin{bmatrix} 1 & 19 & 0 & 0 \\ 0 & 0 & 1 & 49 \end{bmatrix}_{83204} = \mathbf{Pl}(0, 0, 33, 49, 18, 1)_{5004002} \\
\ell_{32} &= \begin{bmatrix} 1 & \epsilon^{50} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{15} \end{bmatrix}_{253777} = \begin{bmatrix} 1 & 60 & 0 & 0 \\ 0 & 0 & 1 & 21 \end{bmatrix}_{253777} = \mathbf{Pl}(0, 0, 15, 21, 61, 1)_{16271156} \\
\ell_{33} &= \begin{bmatrix} 1 & \epsilon^{14} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{39} \end{bmatrix}_{245441} = \begin{bmatrix} 1 & 58 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{245441} = \mathbf{Pl}(0, 0, 45, 7, 59, 1)_{15750806} \\
\ell_{34} &= \begin{bmatrix} 1 & \epsilon^{52} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{27} \end{bmatrix}_{212192} = \begin{bmatrix} 1 & 50 & 0 & 0 \\ 0 & 0 & 1 & 46 \end{bmatrix}_{212192} = \mathbf{Pl}(0, 0, 36, 46, 51, 1)_{13653023} \\
\ell_{35} &= \begin{bmatrix} 1 & \epsilon^{58} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{59} \end{bmatrix}_{16585} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{16585} = \mathbf{Pl}(0, 0, 16, 6, 2, 1)_{808563} \\
\ell_{36} &= \begin{bmatrix} 1 & \epsilon^{57} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{56} \end{bmatrix}_{208025} = \begin{bmatrix} 1 & 49 & 0 & 0 \\ 0 & 0 & 1 & 40 \end{bmatrix}_{208025} = \mathbf{Pl}(0, 0, 35, 40, 48, 1)_{12866656} \\
\ell_{37} &= \begin{bmatrix} 1 & \epsilon^{19} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{45} \end{bmatrix}_{95675} = \begin{bmatrix} 1 & 22 & 0 & 0 \\ 0 & 0 & 1 & 37 \end{bmatrix}_{95675} = \mathbf{Pl}(0, 0, 11, 37, 23, 1)_{6311608} \\
\ell_{38} &= \begin{bmatrix} 1 & \epsilon^{56} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{30} \end{bmatrix}_{170590} = \begin{bmatrix} 1 & 40 & 0 & 0 \\ 0 & 0 & 1 & 54 \end{bmatrix}_{170590} = \mathbf{Pl}(0, 0, 52, 54, 41, 1)_{11034255} \\
\ell_{39} &= \begin{bmatrix} 1 & \epsilon^{27} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{36} \end{bmatrix}_{195538} = \begin{bmatrix} 1 & 46 & 0 & 0 \\ 0 & 0 & 1 & 36 \end{bmatrix}_{195538} = \mathbf{Pl}(0, 0, 46, 36, 47, 1)_{12605973} \\
\ell_{40} &= \begin{bmatrix} 1 & \epsilon^{18} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^9 \end{bmatrix}_{49914} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 47 \end{bmatrix}_{49914} = \mathbf{Pl}(0, 0, 10, 47, 10, 1)_{2904441} \\
\ell_{41} &= \begin{bmatrix} 1 & \epsilon^{40} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{25} \end{bmatrix}_{62409} = \begin{bmatrix} 1 & 14 & 0 & 0 \\ 0 & 0 & 1 & 59 \end{bmatrix}_{62409} = \mathbf{Pl}(0, 0, 51, 59, 15, 1)_{4220048} \\
\ell_{42} &= \begin{bmatrix} 1 & \epsilon^{55} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^7 \end{bmatrix}_{87351} = \begin{bmatrix} 1 & 20 & 0 & 0 \\ 0 & 0 & 1 & 35 \end{bmatrix}_{87351} = \mathbf{Pl}(0, 0, 40, 35, 21, 1)_{5791131} \\
\ell_{43} &= \begin{bmatrix} 1 & \epsilon^7 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{51} \end{bmatrix}_{149756} = \begin{bmatrix} 1 & 35 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{149756} = \mathbf{Pl}(0, 0, 62, 25, 34, 1)_{9200965} \\
\ell_{44} &= \begin{bmatrix} 1 & \epsilon^{26} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{45} \end{bmatrix}_{99836} = \begin{bmatrix} 1 & 23 & 0 & 0 \\ 0 & 0 & 1 & 37 \end{bmatrix}_{99836} = \mathbf{Pl}(0, 0, 11, 37, 22, 1)_{6049528} \\
\ell_{45} &= \begin{bmatrix} 1 & \epsilon^{47} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{14} \end{bmatrix}_{233009} = \begin{bmatrix} 1 & 55 & 0 & 0 \\ 0 & 0 & 1 & 58 \end{bmatrix}_{233009} = \mathbf{Pl}(0, 0, 30, 58, 54, 1)_{14438501} \\
\ell_{46} &= \begin{bmatrix} 1 & \epsilon^{17} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{50} \end{bmatrix}_{224689} = \begin{bmatrix} 1 & 53 & 0 & 0 \\ 0 & 0 & 1 & 60 \end{bmatrix}_{224689} = \mathbf{Pl}(0, 0, 29, 60, 52, 1)_{13914214} \\
\ell_{47} &= \begin{bmatrix} 1 & \epsilon^{43} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{47} \end{bmatrix}_{74888} = \begin{bmatrix} 1 & 17 & 0 & 0 \\ 0 & 0 & 1 & 55 \end{bmatrix}_{74888} = \mathbf{Pl}(0, 0, 42, 55, 16, 1)_{4480985} \\
\ell_{48} &= \begin{bmatrix} 1 & \epsilon^{39} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{35} \end{bmatrix}_{33241} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{33241} = \mathbf{Pl}(0, 0, 61, 18, 6, 1)_{1862598} \\
\ell_{49} &= \begin{bmatrix} 1 & \epsilon^{36} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{18} \end{bmatrix}_{153903} = \begin{bmatrix} 1 & 36 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{153903} = \mathbf{Pl}(0, 0, 37, 11, 37, 1)_{9984030} \\
\ell_{50} &= \begin{bmatrix} 1 & \epsilon^{54} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^9 \end{bmatrix}_{45753} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 47 \end{bmatrix}_{45753} = \mathbf{Pl}(0, 0, 10, 47, 11, 1)_{3166521} \\
\ell_{51} &= \begin{bmatrix} 1 & \epsilon & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{59} \end{bmatrix}_{12424} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{12424} = \mathbf{Pl}(0, 0, 16, 6, 3, 1)_{1070643}
\end{aligned}$$

$$\begin{aligned}
\ell_{52} &= \begin{bmatrix} 1 & \epsilon^6 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{11} \end{bmatrix}_{141440} = \begin{bmatrix} 1 & 33 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{141440} = \mathbf{Pl}(0, 0, 50, 31, 32, 1)_{8675281} \\
\ell_{53} &= \begin{bmatrix} 1 & \epsilon^{11} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{60} \end{bmatrix}_{133099} = \begin{bmatrix} 1 & 31 & 0 & 0 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{133099} = \mathbf{Pl}(0, 0, 8, 12, 30, 1)_{8145787} \\
\ell_{54} &= \begin{bmatrix} 1 & \epsilon^{25} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{39} \end{bmatrix}_{249602} = \begin{bmatrix} 1 & 59 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{249602} = \mathbf{Pl}(0, 0, 45, 7, 58, 1)_{15488726} \\
\ell_{55} &= \begin{bmatrix} 1 & \epsilon^{30} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{14} \end{bmatrix}_{228848} = \begin{bmatrix} 1 & 54 & 0 & 0 \\ 0 & 0 & 1 & 58 \end{bmatrix}_{228848} = \mathbf{Pl}(0, 0, 30, 58, 55, 1)_{14700581} \\
\ell_{56} &= \begin{bmatrix} 1 & \epsilon^{46} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{62} \end{bmatrix}_{183067} = \begin{bmatrix} 1 & 43 & 0 & 0 \\ 0 & 0 & 1 & 48 \end{bmatrix}_{183067} = \mathbf{Pl}(0, 0, 2, 48, 42, 1)_{11289985} \\
\ell_{57} &= \begin{bmatrix} 1 & \epsilon^{33} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{50} \end{bmatrix}_{220528} = \begin{bmatrix} 1 & 52 & 0 & 0 \\ 0 & 0 & 1 & 60 \end{bmatrix}_{220528} = \mathbf{Pl}(0, 0, 29, 60, 53, 1)_{14176294} \\
\ell_{58} &= \begin{bmatrix} 1 & \epsilon^{16} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{62} \end{bmatrix}_{178906} = \begin{bmatrix} 1 & 42 & 0 & 0 \\ 0 & 0 & 1 & 48 \end{bmatrix}_{178906} = \mathbf{Pl}(0, 0, 2, 48, 43, 1)_{11552065} \\
\ell_{59} &= \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \mathbf{Pl}(0, 0, 1, 1, 56, 1)_{14958978} \\
\ell_{60} &= \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \mathbf{Pl}(0, 0, 1, 1, 57, 1)_{15221058} \\
\ell_{61} &= \begin{bmatrix} 1 & \epsilon^{49} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{60} \end{bmatrix}_{128938} = \begin{bmatrix} 1 & 30 & 0 & 0 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{128938} = \mathbf{Pl}(0, 0, 8, 12, 31, 1)_{8407867} \\
\ell_{62} &= \begin{bmatrix} 1 & \epsilon^{38} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{27} \end{bmatrix}_{216353} = \begin{bmatrix} 1 & 51 & 0 & 0 \\ 0 & 0 & 1 & 46 \end{bmatrix}_{216353} = \mathbf{Pl}(0, 0, 36, 46, 50, 1)_{13390943} \\
\ell_{63} &= \begin{bmatrix} 1 & \epsilon^{13} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{54} \end{bmatrix}_{124775} = \begin{bmatrix} 1 & 29 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{124775} = \mathbf{Pl}(0, 0, 47, 10, 28, 1)_{7626580} \\
\ell_{64} &= \begin{bmatrix} 1 & \epsilon^{35} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{57} \end{bmatrix}_{79043} = \begin{bmatrix} 1 & 18 & 0 & 0 \\ 0 & 0 & 1 & 49 \end{bmatrix}_{79043} = \mathbf{Pl}(0, 0, 33, 49, 19, 1)_{5266082} \\
\ell_{65} &= \begin{bmatrix} 1 & \epsilon^{51} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{49} \end{bmatrix}_{108151} = \begin{bmatrix} 1 & 25 & 0 & 0 \\ 0 & 0 & 1 & 30 \end{bmatrix}_{108151} = \mathbf{Pl}(0, 0, 58, 30, 24, 1)_{6579657} \\
\ell_{66} &= \begin{bmatrix} 1 & \epsilon^{53} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{55} \end{bmatrix}_{24921} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{24921} = \mathbf{Pl}(0, 0, 39, 20, 4, 1)_{1335644}
\end{aligned}$$

Rank of lines: ( 0, 4096, 8257, 17043520, 17047616, 203864, 137279, 103990, 266258, 70727, 191375, 54089, 116467, 199699, 158064, 37425, 112306, 162227, 91512, 29080, 187214, 257938, 120614, 166388, 66570, 145595, 174751, 20760, 262097, 58250, 41586, 83204, 253777, 245441, 212192, 16585, 208025, 95675, 170590, 195538, 49914, 62409, 87351, 149756, 99836, 233009, 224689, 74888, 33241, 153903, ...79043, 108151, 24921 )

Rank of points on Klein quadric: ( 0, 2, 270528, 129, 1, 13128736, 8937361, 6841737, 16534887, 4743065, 11816685, 3691697, 7096959, 12343893, 9721950, 2644012, 7359039, 10506793, 5529051, 2124678, 12078765, 16009076, 7888660, 10244713, 3957968, 9463045, 10772175, 1597724, 16796967, 3429617, 2381932, 5004002, 16271156, 15750806, 13653023, 808563, 12866656, 6311608, 11034255, 12605973, 2904441, 4220048, 5791131, 9200965, 6049528, 14438501, 13914214, 4480985, 1862598, 9984030, ...5266082, 6579657, 1335644 )

## Eckardt Points

The surface has 32 Eckardt points:

$$\begin{aligned}
0 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0), \\
1 : P_{8258} &= \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1), \\
2 : P_{12353} &= \mathbf{P}(0, 0, \epsilon, 1) = \mathbf{P}(0, 0, 2, 1), \\
3 : P_{20545} &= \mathbf{P}(0, 0, \epsilon^2, 1) = \mathbf{P}(0, 0, 4, 1), \\
4 : P_{36929} &= \mathbf{P}(0, 0, \epsilon^3, 1) = \mathbf{P}(0, 0, 8, 1), \\
5 : P_{45121} &= \mathbf{P}(0, 0, \epsilon^{54}, 1) = \mathbf{P}(0, 0, 10, 1), \\
6 : P_{49217} &= \mathbf{P}(0, 0, \epsilon^{18}, 1) = \mathbf{P}(0, 0, 11, 1), \\
7 : P_{65601} &= \mathbf{P}(0, 0, \epsilon^{48}, 1) = \mathbf{P}(0, 0, 15, 1), \\
8 : P_{69697} &= \mathbf{P}(0, 0, \epsilon^4, 1) = \mathbf{P}(0, 0, 16, 1), \\
9 : P_{77889} &= \mathbf{P}(0, 0, \epsilon^{35}, 1) = \mathbf{P}(0, 0, 18, 1), \\
10 : P_{94273} &= \mathbf{P}(0, 0, \epsilon^{19}, 1) = \mathbf{P}(0, 0, 22, 1), \\
11 : P_{98369} &= \mathbf{P}(0, 0, \epsilon^{26}, 1) = \mathbf{P}(0, 0, 23, 1), \\
12 : P_{110657} &= \mathbf{P}(0, 0, \epsilon^{32}, 1) = \mathbf{P}(0, 0, 26, 1), \\
13 : P_{118849} &= \mathbf{P}(0, 0, \epsilon^{41}, 1) = \mathbf{P}(0, 0, 28, 1), \\
14 : P_{122945} &= \mathbf{P}(0, 0, \epsilon^{13}, 1) = \mathbf{P}(0, 0, 29, 1), \\
15 : P_{127041} &= \mathbf{P}(0, 0, \epsilon^{49}, 1) = \mathbf{P}(0, 0, 30, 1), \\
16 : P_{139329} &= \mathbf{P}(0, 0, \epsilon^6, 1) = \mathbf{P}(0, 0, 33, 1), \\
17 : P_{147521} &= \mathbf{P}(0, 0, \epsilon^7, 1) = \mathbf{P}(0, 0, 35, 1), \\
18 : P_{151617} &= \mathbf{P}(0, 0, \epsilon^{36}, 1) = \mathbf{P}(0, 0, 36, 1), \\
19 : P_{155713} &= \mathbf{P}(0, 0, \epsilon^{45}, 1) = \mathbf{P}(0, 0, 37, 1), \\
20 : P_{163905} &= \mathbf{P}(0, 0, \epsilon^8, 1) = \mathbf{P}(0, 0, 39, 1), \\
21 : P_{168001} &= \mathbf{P}(0, 0, \epsilon^{56}, 1) = \mathbf{P}(0, 0, 40, 1), \\
22 : P_{176193} &= \mathbf{P}(0, 0, \epsilon^{16}, 1) = \mathbf{P}(0, 0, 42, 1), \\
23 : P_{188481} &= \mathbf{P}(0, 0, \epsilon^{24}, 1) = \mathbf{P}(0, 0, 45, 1), \\
24 : P_{192577} &= \mathbf{P}(0, 0, \epsilon^{27}, 1) = \mathbf{P}(0, 0, 46, 1), \\
25 : P_{196673} &= \mathbf{P}(0, 0, \epsilon^9, 1) = \mathbf{P}(0, 0, 47, 1), \\
26 : P_{208961} &= \mathbf{P}(0, 0, \epsilon^{52}, 1) = \mathbf{P}(0, 0, 50, 1), \\
27 : P_{213057} &= \mathbf{P}(0, 0, \epsilon^{38}, 1) = \mathbf{P}(0, 0, 51, 1), \\
28 : P_{217153} &= \mathbf{P}(0, 0, \epsilon^{33}, 1) = \mathbf{P}(0, 0, 52, 1), \\
29 : P_{241729} &= \mathbf{P}(0, 0, \epsilon^{14}, 1) = \mathbf{P}(0, 0, 58, 1), \\
30 : P_{254017} &= \mathbf{P}(0, 0, \epsilon^{28}, 1) = \mathbf{P}(0, 0, 61, 1), \\
31 : P_{258113} &= \mathbf{P}(0, 0, \epsilon^{12}, 1) = \mathbf{P}(0, 0, 62, 1).
\end{aligned}$$

## Double Points

The surface has 66 Double points:

The double points on the surface are:

$$\begin{aligned}
P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 & P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_{14} \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_2 & P_{16} &= (12, 1, 0, 0) = \ell_0 \cap \ell_{15} \\
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_3 & P_{17} &= (13, 1, 0, 0) = \ell_0 \cap \ell_{16} \\
P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_5 & P_{18} &= (14, 1, 0, 0) = \ell_0 \cap \ell_{17} \\
P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_6 & P_{19} &= (15, 1, 0, 0) = \ell_0 \cap \ell_{18} \\
P_8 &= (4, 1, 0, 0) = \ell_0 \cap \ell_7 & P_{20} &= (16, 1, 0, 0) = \ell_0 \cap \ell_{19} \\
P_9 &= (5, 1, 0, 0) = \ell_0 \cap \ell_8 & P_{21} &= (17, 1, 0, 0) = \ell_0 \cap \ell_{20} \\
P_{10} &= (6, 1, 0, 0) = \ell_0 \cap \ell_9 & P_{22} &= (18, 1, 0, 0) = \ell_0 \cap \ell_{21} \\
P_{11} &= (7, 1, 0, 0) = \ell_0 \cap \ell_{10} & P_{23} &= (19, 1, 0, 0) = \ell_0 \cap \ell_{22} \\
P_{12} &= (8, 1, 0, 0) = \ell_0 \cap \ell_{11} & P_{24} &= (20, 1, 0, 0) = \ell_0 \cap \ell_{23} \\
P_{13} &= (9, 1, 0, 0) = \ell_0 \cap \ell_{12} & P_{25} &= (21, 1, 0, 0) = \ell_0 \cap \ell_{24} \\
P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_{13} & P_{26} &= (22, 1, 0, 0) = \ell_0 \cap \ell_{25}
\end{aligned}$$

$$\begin{aligned}
P_{27} &= (23, 1, 0, 0) = \ell_0 \cap \ell_{26} \\
P_{28} &= (24, 1, 0, 0) = \ell_0 \cap \ell_{27} \\
P_{29} &= (25, 1, 0, 0) = \ell_0 \cap \ell_{28} \\
P_{30} &= (26, 1, 0, 0) = \ell_0 \cap \ell_{29} \\
P_{31} &= (27, 1, 0, 0) = \ell_0 \cap \ell_{30} \\
P_{32} &= (28, 1, 0, 0) = \ell_0 \cap \ell_{31} \\
P_{33} &= (29, 1, 0, 0) = \ell_0 \cap \ell_{32} \\
P_{34} &= (30, 1, 0, 0) = \ell_0 \cap \ell_{33} \\
P_{35} &= (31, 1, 0, 0) = \ell_0 \cap \ell_{34} \\
P_{36} &= (32, 1, 0, 0) = \ell_0 \cap \ell_{35} \\
P_{37} &= (33, 1, 0, 0) = \ell_0 \cap \ell_{36} \\
P_{38} &= (34, 1, 0, 0) = \ell_0 \cap \ell_{37} \\
P_{39} &= (35, 1, 0, 0) = \ell_0 \cap \ell_{38} \\
P_{40} &= (36, 1, 0, 0) = \ell_0 \cap \ell_{39} \\
P_{41} &= (37, 1, 0, 0) = \ell_0 \cap \ell_{40} \\
P_{42} &= (38, 1, 0, 0) = \ell_0 \cap \ell_{41} \\
P_{43} &= (39, 1, 0, 0) = \ell_0 \cap \ell_{42} \\
P_{44} &= (40, 1, 0, 0) = \ell_0 \cap \ell_{43} \\
P_{45} &= (41, 1, 0, 0) = \ell_0 \cap \ell_{44} \\
P_{46} &= (42, 1, 0, 0) = \ell_0 \cap \ell_{45} \\
P_{47} &= (43, 1, 0, 0) = \ell_0 \cap \ell_{46} \\
P_{48} &= (44, 1, 0, 0) = \ell_0 \cap \ell_{47}
\end{aligned}$$

$$\begin{aligned}
P_{49} &= (45, 1, 0, 0) = \ell_0 \cap \ell_{48} \\
P_{50} &= (46, 1, 0, 0) = \ell_0 \cap \ell_{49} \\
P_{51} &= (47, 1, 0, 0) = \ell_0 \cap \ell_{50} \\
P_{52} &= (48, 1, 0, 0) = \ell_0 \cap \ell_{51} \\
P_{53} &= (49, 1, 0, 0) = \ell_0 \cap \ell_{52} \\
P_{54} &= (50, 1, 0, 0) = \ell_0 \cap \ell_{53} \\
P_{55} &= (51, 1, 0, 0) = \ell_0 \cap \ell_{54} \\
P_{56} &= (52, 1, 0, 0) = \ell_0 \cap \ell_{55} \\
P_{57} &= (53, 1, 0, 0) = \ell_0 \cap \ell_{56} \\
P_{58} &= (54, 1, 0, 0) = \ell_0 \cap \ell_{57} \\
P_{59} &= (55, 1, 0, 0) = \ell_0 \cap \ell_{58} \\
P_{60} &= (56, 1, 0, 0) = \ell_0 \cap \ell_{59} \\
P_{61} &= (57, 1, 0, 0) = \ell_0 \cap \ell_{60} \\
P_{62} &= (58, 1, 0, 0) = \ell_0 \cap \ell_{61} \\
P_{63} &= (59, 1, 0, 0) = \ell_0 \cap \ell_{62} \\
P_{64} &= (60, 1, 0, 0) = \ell_0 \cap \ell_{63} \\
P_{65} &= (61, 1, 0, 0) = \ell_0 \cap \ell_{64} \\
P_{66} &= (62, 1, 0, 0) = \ell_0 \cap \ell_{65} \\
P_{67} &= (63, 1, 0, 0) = \ell_0 \cap \ell_{66} \\
P_3 &= (0, 0, 0, 1) = \ell_3 \cap \ell_4
\end{aligned}$$

### Single Points

The surface has 4127 single points:  
Too many to print.

### Points on surface but on no line

The surface has 0 points not on any line:  
The points on the surface but not on lines are:

## Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	
0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0</																							



Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
in point	$P_0$	$P_5$	$P_1$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{16}$	$P_{17}$	$P_{18}$	$P_{19}$	$P_{20}$	$P_{21}$	$P_{22}$	$P_{23}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$
in point	$P_0$	$P_2$	$P_2$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_4$
in point	$P_5$	$P_2$	$P_2$

Line 3 intersects

Line	$\ell_0$	$\ell_4$
in point	$P_1$	$P_3$

Line 4 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_2$	$P_2$	$P_3$	$P_{147521}$	$P_{208961}$	$P_{241729}$	$P_{118849}$	$P_{176193}$	$P_{94273}$	$P_{77889}$	$P_{20545}$	$P_{192577}$	$P_{155713}$	$P_{98369}$

Line 5 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{36}$
in point	$P_6$	$P_{147521}$	$P_{147521}$

Line 6 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{52}$
in point	$P_7$	$P_{208961}$	$P_{208961}$

Line 7 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{65}$
in point	$P_8$	$P_{241729}$	$P_{241729}$

Line 8 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{28}$
in point	$P_9$	$P_{118849}$	$P_{118849}$

Line 9 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{47}$
in point	$P_{10}$	$P_{176193}$	$P_{176193}$

Line 10 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{20}$
in point	$P_{11}$	$P_{94273}$	$P_{94273}$

Line 11 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{29}$
in point	$P_{12}$	$P_{77889}$	$P_{77889}$

Line 12 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{16}$
in point	$P_{13}$	$P_{20545}$	$P_{20545}$

Line 13 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{39}$
in point	$P_{14}$	$P_{192577}$	$P_{192577}$

Line 14 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{49}$
in point	$P_{15}$	$P_{155713}$	$P_{155713}$

Line 15 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{30}$
in point	$P_{16}$	$P_{98369}$	$P_{98369}$

Line 16 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{12}$
in point	$P_{17}$	$P_{20545}$	$P_{20545}$

Line 17 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{23}$
in point	$P_{18}$	$P_{110657}$	$P_{110657}$

Line 18 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{42}$
in point	$P_{19}$	$P_{168001}$	$P_{168001}$

Line 19 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{48}$
in point	$P_{20}$	$P_{254017}$	$P_{254017}$

Line 20 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{10}$
in point	$P_{21}$	$P_{94273}$	$P_{94273}$

Line 21 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{32}$
in point	$P_{22}$	$P_{65601}$	$P_{65601}$

Line 22 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{63}$
in point	$P_{23}$	$P_{196673}$	$P_{196673}$

Line 23 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{17}$
in point	$P_{24}$	$P_{110657}$	$P_{110657}$

Line 24 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{41}$
in point	$P_{25}$	$P_{213057}$	$P_{213057}$

Line 25 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{43}$
in point	$P_{26}$	$P_{258113}$	$P_{258113}$

Line 26 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{38}$
in point	$P_{27}$	$P_{217153}$	$P_{217153}$

Line 27 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{66}$
in point	$P_{28}$	$P_{163905}$	$P_{163905}$

Line 28 intersects

Line	$\ell_0$	$\ell_4$	$\ell_8$
in point	$P_{29}$	$P_{118849}$	$P_{118849}$

Line 29 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{11}$
in point	$P_{30}$	$P_{77889}$	$P_{77889}$

Line 30 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{15}$
in point	$P_{31}$	$P_{98369}$	$P_{98369}$

Line 31 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{64}$
in point	$P_{32}$	$P_{139329}$	$P_{139329}$

Line 32 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{21}$
in point	$P_{33}$	$P_{65601}$	$P_{65601}$

Line 33 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{54}$
in point	$P_{34}$	$P_{188481}$	$P_{188481}$

Line 34 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{62}$
in point	$P_{35}$	$P_{151617}$	$P_{151617}$

Line 35 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{51}$
in point	$P_{36}$	$P_{69697}$	$P_{69697}$

Line 36 intersects

Line	$\ell_0$	$\ell_4$	$\ell_5$
in point	$P_{37}$	$P_{147521}$	$P_{147521}$

Line 37 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{44}$
in point	$P_{38}$	$P_{49217}$	$P_{49217}$

Line 38 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{26}$
in point	$P_{39}$	$P_{217153}$	$P_{217153}$

Line 39 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{13}$
in point	$P_{40}$	$P_{192577}$	$P_{192577}$

Line 40 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{50}$
in point	$P_{41}$	$P_{45121}$	$P_{45121}$

Line 41 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{24}$
in point	$P_{42}$	$P_{213057}$	$P_{213057}$

Line 42 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{18}$
in point	$P_{43}$	$P_{168001}$	$P_{168001}$

Line 43 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{25}$
in point	$P_{44}$	$P_{258113}$	$P_{258113}$

Line 44 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{37}$
in point	$P_{45}$	$P_{49217}$	$P_{49217}$

Line 45 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{55}$
in point	$P_{46}$	$P_{127041}$	$P_{127041}$

Line 46 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{57}$
in point	$P_{47}$	$P_{122945}$	$P_{122945}$

Line 47 intersects

Line	$\ell_0$	$\ell_4$	$\ell_9$
in point	$P_{48}$	$P_{176193}$	$P_{176193}$

Line 48 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{19}$
in point	$P_{49}$	$P_{254017}$	$P_{254017}$

Line 49 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{14}$
in point	$P_{50}$	$P_{155713}$	$P_{155713}$

Line 50 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{40}$
in point	$P_{51}$	$P_{45121}$	$P_{45121}$

Line 51 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{35}$
in point	$P_{52}$	$P_{69697}$	$P_{69697}$

Line 52 intersects

Line	$\ell_0$	$\ell_4$	$\ell_6$
in point	$P_{53}$	$P_{208961}$	$P_{208961}$

Line 53 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{61}$
in point	$P_{54}$	$P_{36929}$	$P_{36929}$

Line 54 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{33}$
in point	$P_{55}$	$P_{188481}$	$P_{188481}$

Line 55 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{45}$
in point	$P_{56}$	$P_{127041}$	$P_{127041}$

Line 56 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{58}$
in point	$P_{57}$	$P_{12353}$	$P_{12353}$

Line 57 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{46}$
in point	$P_{58}$	$P_{122945}$	$P_{122945}$

Line 58 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{56}$
in point	$P_{59}$	$P_{12353}$	$P_{12353}$

Line 59 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{60}$
in point	$P_{60}$	$P_{8258}$	$P_{8258}$

Line 60 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{59}$
in point	$P_{61}$	$P_{8258}$	$P_{8258}$

Line 61 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{53}$
in point	$P_{62}$	$P_{36929}$	$P_{36929}$

Line 62 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{34}$
in point	$P_{63}$	$P_{151617}$	$P_{151617}$

Line 63 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{22}$
in point	$P_{64}$	$P_{196673}$	$P_{196673}$

Line 64 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{31}$
in point	$P_{65}$	$P_{139329}$	$P_{139329}$

Line 65 intersects

Line	$\ell_0$	$\ell_4$	$\ell_7$
in point	$P_{66}$	$P_{241729}$	$P_{241729}$

Line 66 intersects

Line	$\ell_0$	$\ell_4$	$\ell_{27}$
in point	$P_{67}$	$P_{163905}$	$P_{163905}$

The surface has 4225 points:  
Too many to print.