

Rank-4 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_1 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(2) is 4

General information

Number of lines	13
Number of points	11
Number of singular points	7
Number of Eckardt points	8
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{13}
Type of lines on points	$5^3, 3^8$

Singular Points

The surface has 7 singular points:

0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$
1 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$
2 : $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$
3 : $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$

4 : $P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$
5 : $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$
6 : $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$

The 13 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_9 \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{12} \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{25} \\
\ell_6 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4 \\
\ell_8 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{11} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{19} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{19} = \mathbf{Pl}(0, 1, 1, 0, 1, 0)_{14} \\
\ell_{11} &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_{12} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{27}
\end{aligned}$$

Rank of lines: (4, 28, 6, 5, 30, 29, 34, 18, 31, 20, 19, 33, 32)

Rank of points on Klein quadric: (2, 19, 9, 12, 5, 25, 1, 4, 21, 11, 14, 7, 27)

Eckardt Points

The surface has 8 Eckardt points:

$$\begin{aligned}
0 : P_0 &= \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0), \quad T = 6 \\
1 : P_1 &= \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0), \quad T = -1 \\
2 : P_6 &= \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0), \quad T = 6 \\
3 : P_7 &= \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0), \quad T = -1 \\
4 : P_9 &= \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1), \quad T = 6 \\
5 : P_{10} &= \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1), \quad T = -1 \\
6 : P_{13} &= \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1), \quad T = 6 \\
7 : P_{14} &= \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1), \quad T = -1
\end{aligned}$$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	1	1	0	0	1	1	1	1	1	0	0
1	1	0	0	0	1	1	1	1	1	0	0	1	1
2	1	0	0	1	1	0	1	1	0	1	1	1	0
3	1	0	1	0	0	1	1	1	0	1	1	0	1
4	0	1	1	0	0	1	1	0	1	1	0	1	1
5	0	1	0	1	1	0	1	0	1	0	1	1	1
6	1	1	1	1	1	1	0	1	1	1	1	1	1
7	1	1	1	1	0	0	1	0	1	1	1	0	0
8	1	1	0	0	1	1	1	1	0	0	0	1	1
9	1	0	1	1	1	0	1	1	0	0	1	1	0
10	1	0	1	1	0	1	1	1	0	1	0	0	1
11	0	1	1	0	1	1	1	0	1	1	0	0	1
12	0	1	0	1	1	1	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_0	P_0	P_2	P_2	P_2	P_6	P_6

Line 1 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}
in point	P_2	P_1	P_1	P_2	P_2	P_2	P_7	P_7

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}
in point	P_0	P_0	P_3	P_3	P_9	P_3	P_9	P_3

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}
in point	P_0	P_0	P_{12}	P_{12}	P_{13}	P_{13}	P_{12}	P_{12}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}
in point	P_1	P_3	P_1	P_3	P_{10}	P_3	P_3	P_{10}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_1	P_{12}	P_1	P_{12}	P_{14}	P_{12}	P_{14}	P_{12}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_2	P_2	P_3	P_{12}	P_3	P_{12}	P_2	P_2	P_3	P_{12}	P_3	P_{12}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_2	P_9	P_{13}	P_2	P_2	P_{13}	P_9

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{12}
in point	P_2	P_2	P_{10}	P_{14}	P_2	P_2	P_{14}	P_{10}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}
in point	P_6	P_3	P_{13}	P_3	P_3	P_{13}	P_6	P_3

Line 10 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{12}
in point	P_6	P_9	P_{12}	P_{12}	P_{12}	P_9	P_6	P_{12}

Line 11 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{12}
in point	P_7	P_3	P_3	P_{14}	P_3	P_{14}	P_3	P_7

Line 12 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}
in point	P_7	P_{12}	P_{10}	P_{12}	P_{12}	P_{10}	P_{12}	P_7

The surface has 11 points:

The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
 $1 : P_1 = (0, 1, 0, 0)$
 $2 : P_2 = (0, 0, 1, 0)$
 $3 : P_3 = (0, 0, 0, 1)$

$4 : P_6 = (1, 0, 1, 0)$
 $5 : P_7 = (0, 1, 1, 0)$
 $6 : P_9 = (1, 0, 0, 1)$
 $7 : P_{10} = (0, 1, 0, 1)$

$8 : P_{12} = (0, 0, 1, 1)$
 $9 : P_{13} = (1, 0, 1, 1)$
 $10 : P_{14} = (0, 1, 1, 1)$