Rank-65735 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)The point rank of the equation over GF(8) is 1229492877

General information

Number of lines	1
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

Singular Points

The surface has 1 singular points:

$$0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17}$$

Rank of lines: (4680)

Rank of points on Klein quadric: (17)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

 $\begin{array}{lll} 0: \ P_1 = (0,1,0,0) \ \mbox{lies on line} \ \ell_0 \\ 1: \ P_3 = (0,0,0,1) \ \mbox{lies on line} \ \ell_0 \\ 2: \ P_{82} = (0,1,0,1) \ \mbox{lies on line} \ \ell_0 \\ 3: \ P_{90} = (0,2,0,1) \ \mbox{lies on line} \ \ell_0 \\ 4: \ P_{98} = (0,3,0,1) \ \mbox{lies on line} \ \ell_0 \\ \end{array}$

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line: The points on the surface but not on lines are:

 $0: P_0 = (1,0,0,0)$ 18: $P_{247} = (6, 5, 2, 1)$ $1: P_4 = (1, 1, 1, 1)$ 19: $P_{248} = (7, 5, 2, 1)$ $2: P_{29} = (2, 2, 1, 0)$ $20: P_{263} = (6, 7, 2, 1)$ $3: P_{37} = (2,3,1,0)$ $21: P_{264} = (7,7,2,1)$ $4: P_{47} = (4, 4, 1, 0)$ $22: P_{272} = (7,0,3,1)$ $5: P_{55} = (4, 5, 1, 0)$ 23: $P_{277} = (4, 1, 3, 1)$ $6: P_{66} = (7, 6, 1, 0)$ $24: P_{279} = (6, 1, 3, 1)$ 7: $P_{74} = (7, 7, 1, 0)$ $25: P_{285} = (4, 2, 3, 1)$ $8: P_{83} = (1, 1, 0, 1)$ $26: P_{287} = (6, 2, 3, 1)$ 9: $P_{94} = (4, 2, 0, 1)$ $27: P_{296} = (7, 3, 3, 1)$ 10: $P_{103} = (5, 3, 0, 1)$ $28: P_{300} = (3,4,3,1)$ 11: $P_{113} = (7, 4, 0, 1)$ 29: $P_{302} = (5, 4, 3, 1)$ $12: P_{120} = (6, 5, 0, 1)$ $30: P_{324} = (3,7,3,1)$ 13: $P_{125} = (3, 6, 0, 1)$ $31: P_{326} = (5,7,3,1)$ 14: $P_{132} = (2, 7, 0, 1)$ $32: P_{334} = (5, 0, 4, 1)$ 15: $P_{139} = (1, 0, 1, 1)$ $33: P_{347} = (2, 2, 4, 1)$ 16: $P_{204} = (3, 0, 2, 1)$ $34: P_{348} = (3, 2, 4, 1)$ 17: $P_{220} = (3, 2, 2, 1)$ $35: P_{366} = (5, 4, 4, 1)$

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36: P_{379} = (2, 6, 4, 1)
                                                                   51: P_{476} = (3, 2, 6, 1)
                                                                   52: P_{479} = (6, 2, 6, 1)
37: P_{380} = (3, 6, 4, 1)
38: P_{395} = (2,0,5,1)
                                                                   53: P_{492} = (3, 4, 6, 1)
                                                                   54: P_{495} = (6, 4, 6, 1)
39: P_{404} = (3, 1, 5, 1)
40: P_{408} = (7, 1, 5, 1)
                                                                   55: P_{509} = (4, 6, 6, 1)
41: P_{414} = (5, 2, 5, 1)
                                                                   56: P_{515} = (2,7,6,1)
42: P_{415} = (6, 2, 5, 1)
                                                                   57: P_{518} = (5, 7, 6, 1)
                                                                   58: P_{527} = (6, 0, 7, 1)
43: P_{428} = (3, 4, 5, 1)
44: P_{432} = (7, 4, 5, 1)
                                                                   59: P_{549} = (4, 3, 7, 1)
                                                                   60: P_{550} = (5, 3, 7, 1)
45: P_{435} = (2, 5, 5, 1)
46: P_{454} = (5, 7, 5, 1)
                                                                   61: P_{557} = (4, 4, 7, 1)
                                                                   62: P_{558} = (5, 4, 7, 1)
47: P_{455} = (6, 7, 5, 1)
                                                                   63: P_{583} = (6, 7, 7, 1)
48: P_{461} = (4, 0, 6, 1)
49: P_{467} = (2, 1, 6, 1)
50: P_{470} = (5, 1, 6, 1)
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Line Intersection Graph

 $\begin{array}{c|c} 0 \\ \hline 0 & 0 \end{array}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 73 points: The points on the surface are:

$0: P_0 = (1,0,0,0)$	$25: P_{204} = (3, 0, 2, 1)$	$50: P_{414} = (5, 2, 5, 1)$
$1: P_1 = (0, 1, 0, 0)$	$26: P_{220} = (3, 2, 2, 1)$	$51: P_{415} = (6, 2, 5, 1)$
$2: P_3 = (0,0,0,1)$	$27: P_{247} = (6, 5, 2, 1)$	$52: P_{428} = (3, 4, 5, 1)$
$3: P_4 = (1, 1, 1, 1)$	$28: P_{248} = (7, 5, 2, 1)$	$53: P_{432} = (7, 4, 5, 1)$
$4: P_{29} = (2, 2, 1, 0)$	$29: P_{263} = (6,7,2,1)$	$54: P_{435} = (2, 5, 5, 1)$
$5: P_{37} = (2,3,1,0)$	$30: P_{264} = (7,7,2,1)$	$55: P_{454} = (5, 7, 5, 1)$
$6: P_{47} = (4, 4, 1, 0)$	$31: P_{272} = (7, 0, 3, 1)$	$56: P_{455} = (6, 7, 5, 1)$
$7: P_{55} = (4, 5, 1, 0)$	$32: P_{277} = (4, 1, 3, 1)$	$57: P_{461} = (4, 0, 6, 1)$
$8: P_{66} = (7, 6, 1, 0)$	$33: P_{279} = (6, 1, 3, 1)$	$58: P_{467} = (2, 1, 6, 1)$
$9: P_{74} = (7,7,1,0)$	$34: P_{285} = (4, 2, 3, 1)$	$59: P_{470} = (5, 1, 6, 1)$
$10: P_{82} = (0, 1, 0, 1)$	$35: P_{287} = (6, 2, 3, 1)$	$60: P_{476} = (3, 2, 6, 1)$
$11: P_{83} = (1, 1, 0, 1)$	$36: P_{296} = (7, 3, 3, 1)$	$61: P_{479} = (6, 2, 6, 1)$
$12: P_{90} = (0, 2, 0, 1)$	$37: P_{300} = (3, 4, 3, 1)$	$62: P_{492} = (3, 4, 6, 1)$
13: $P_{94} = (4, 2, 0, 1)$	$38: P_{302} = (5, 4, 3, 1)$	$63: P_{495} = (6,4,6,1)$
$14: P_{98} = (0, 3, 0, 1)$	$39: P_{324} = (3,7,3,1)$	$64: P_{509} = (4, 6, 6, 1)$
15: $P_{103} = (5, 3, 0, 1)$	$40: P_{326} = (5, 7, 3, 1)$	$65: P_{515} = (2, 7, 6, 1)$
16: $P_{106} = (0, 4, 0, 1)$	$41: P_{334} = (5, 0, 4, 1)$	$66: P_{518} = (5, 7, 6, 1)$
17: $P_{113} = (7, 4, 0, 1)$	$42: P_{347} = (2, 2, 4, 1)$	$67: P_{527} = (6, 0, 7, 1)$
18: $P_{114} = (0, 5, 0, 1)$	$43: P_{348} = (3, 2, 4, 1)$	$68: P_{549} = (4, 3, 7, 1)$
$19: P_{120} = (6, 5, 0, 1)$	$44: P_{366} = (5, 4, 4, 1)$	$69: P_{550} = (5, 3, 7, 1)$
$20: P_{122} = (0, 6, 0, 1)$	$45: P_{379} = (2, 6, 4, 1)$	$70: P_{557} = (4, 4, 7, 1)$
$21: P_{125} = (3, 6, 0, 1)$	$46: P_{380} = (3, 6, 4, 1)$	71: $P_{558} = (5, 4, 7, 1)$
$22: P_{130} = (0,7,0,1)$	$47: P_{395} = (2, 0, 5, 1)$	$72: P_{583} = (6, 7, 7, 1)$
$23: P_{132} = (2,7,0,1)$	$48: P_{404} = (3, 1, 5, 1)$	
$24: P_{139} = (1, 0, 1, 1)$	$49: P_{408} = (7, 1, 5, 1)$	