Rank-24 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 = 0$$

(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 24

General information

Number of lines	3
Number of points	7
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	3, 1^6

Singular Points

The surface has 1 singular points:

$0 : \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0,1,0,0,1,0)_{11}$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

Rank of lines: (13, 20, 33)
Rank of points on Klein quadric: (15, 11, 7)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1).\ T = -1$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 6 single points:
The single points on the surface are:

$0 :\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$1 :\ P_6 = (1,0,1,0)$ lies on line ℓ_1
$2 :\ P_7 = (0,1,1,0)$ lies on line ℓ_2
$3 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_0
$4 :\ P_{13} = (1,0,1,1)$ lies on line ℓ_1
$5 :\ P_{14} = (0,1,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 7 points:
The points on the surface are:

$0:\ P_3 = (0,0,0,1)$
$1:\ P_5 = (1,1,0,0)$
$2:\ P_6 = (1,0,1,0)$
$3:\ P_7 = (0,1,1,0)$
$4:\ P_{11} = (1,1,0,1)$
$5:\ P_{13} = (1,0,1,1)$
$6:\ P_{14} = (0,1,1,1)$