Rank-74500 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74500

General information

Number of lines	3
Number of points	7
Number of singular points	2
Number of Eckardt points	0
Number of double points	2
Number of single points	5
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	2^2, 1^5

Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$
$$1 : P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

Rank of lines: (29, 34, 18)
Rank of points on Klein quadric: (25, 1, 4)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_{12} = (0,0,1,1) = \ell_0 \cap \ell_1$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 5 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0 3 : $P_{13} = (1,0,1,1)$ lies on line ℓ_2
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1 4 : $P_{14} = (0,1,1,1)$ lies on line ℓ_0
2 : $P_9 = (1,0,0,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
 |0 1 2
0|0 1 0
1|1 0 1
2|0 1 0
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Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{12}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{12}	P_2

Line 2 intersects

Line	ℓ_1
in point	P_2

The surface has 7 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$ $3: P_9 = (1,0,0,1)$ $6: P_{14} = (0,1,1,1)$

$1: P_2 = (0,0,1,0)$ $4: P_{12} = (0,0,1,1)$

$2: P_3 = (0,0,0,1)$ $5: P_{13} = (1,0,1,1)$