

Rank-65874 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is 1091051654

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	4097
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$2^3, 1^{189}, 0^{4097}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right]_{270530} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right]_{270530} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{544579} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{241274} = \left[\begin{array}{cccc} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{241274} = \mathbf{Pl}(0, 0, 1, 1, 56, 1)_{14958978}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \mathbf{Pl}(0, 0, 1, 1, 57, 1)_{15221058}$$

Rank of lines: (270530, 241274, 237113)

Rank of points on Klein quadric: (544579, 14958978, 15221058)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{11898} = (57, 56, 1, 1) = \ell_0 \cap \ell_1$$

$$P_{11961} = (56, 57, 1, 1) = \ell_0 \cap \ell_2$$

$$P_{8258} = (0, 0, 1, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 189 single points:

The single points on the surface are:

- 0 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0
- 1 : $P_{60} = (56, 1, 0, 0)$ lies on line ℓ_1
- 2 : $P_{61} = (57, 1, 0, 0)$ lies on line ℓ_2
- 3 : $P_{8259} = (1, 0, 1, 1)$ lies on line ℓ_0
- 4 : $P_{8322} = (0, 1, 1, 1)$ lies on line ℓ_0
- 5 : $P_{8377} = (56, 1, 1, 1)$ lies on line ℓ_1
- 6 : $P_{8378} = (57, 1, 1, 1)$ lies on line ℓ_2
- 7 : $P_{8388} = (3, 2, 1, 1)$ lies on line ℓ_0
- 8 : $P_{8402} = (17, 2, 1, 1)$ lies on line ℓ_1
- 9 : $P_{8404} = (19, 2, 1, 1)$ lies on line ℓ_2
- 10 : $P_{8451} = (2, 3, 1, 1)$ lies on line ℓ_0
- 11 : $P_{8490} = (41, 3, 1, 1)$ lies on line ℓ_1
- 12 : $P_{8491} = (42, 3, 1, 1)$ lies on line ℓ_2
- 13 : $P_{8518} = (5, 4, 1, 1)$ lies on line ℓ_0
- 14 : $P_{8547} = (34, 4, 1, 1)$ lies on line ℓ_1
- 15 : $P_{8551} = (38, 4, 1, 1)$ lies on line ℓ_2
- 16 : $P_{8581} = (4, 5, 1, 1)$ lies on line ℓ_0
- 17 : $P_{8603} = (26, 5, 1, 1)$ lies on line ℓ_1
- 18 : $P_{8608} = (31, 5, 1, 1)$ lies on line ℓ_2
- 19 : $P_{8648} = (7, 6, 1, 1)$ lies on line ℓ_0
- 20 : $P_{8692} = (51, 6, 1, 1)$ lies on line ℓ_1
- 21 : $P_{8694} = (53, 6, 1, 1)$ lies on line ℓ_2
- 22 : $P_{8711} = (6, 7, 1, 1)$ lies on line ℓ_0
- 23 : $P_{8716} = (11, 7, 1, 1)$ lies on line ℓ_1
- 24 : $P_{8717} = (12, 7, 1, 1)$ lies on line ℓ_2
- 25 : $P_{8778} = (9, 8, 1, 1)$ lies on line ℓ_0
- 26 : $P_{8806} = (37, 8, 1, 1)$ lies on line ℓ_1
- 27 : $P_{8814} = (45, 8, 1, 1)$ lies on line ℓ_2

- 28 : $P_{8841} = (8, 9, 1, 1)$ lies on line ℓ_0
- 29 : $P_{8853} = (20, 9, 1, 1)$ lies on line ℓ_2
- 30 : $P_{8862} = (29, 9, 1, 1)$ lies on line ℓ_1
- 31 : $P_{8908} = (11, 10, 1, 1)$ lies on line ℓ_0
- 32 : $P_{8949} = (52, 10, 1, 1)$ lies on line ℓ_1
- 33 : $P_{8959} = (62, 10, 1, 1)$ lies on line ℓ_2
- 34 : $P_{8968} = (7, 11, 1, 1)$ lies on line ℓ_2
- 35 : $P_{8971} = (10, 11, 1, 1)$ lies on line ℓ_0
- 36 : $P_{8973} = (12, 11, 1, 1)$ lies on line ℓ_1
- 37 : $P_{9032} = (7, 12, 1, 1)$ lies on line ℓ_1
- 38 : $P_{9036} = (11, 12, 1, 1)$ lies on line ℓ_2
- 39 : $P_{9038} = (13, 12, 1, 1)$ lies on line ℓ_0
- 40 : $P_{9101} = (12, 13, 1, 1)$ lies on line ℓ_0
- 41 : $P_{9139} = (50, 13, 1, 1)$ lies on line ℓ_2
- 42 : $P_{9152} = (63, 13, 1, 1)$ lies on line ℓ_1
- 43 : $P_{9168} = (15, 14, 1, 1)$ lies on line ℓ_0
- 44 : $P_{9175} = (22, 14, 1, 1)$ lies on line ℓ_1
- 45 : $P_{9177} = (24, 14, 1, 1)$ lies on line ℓ_2
- 46 : $P_{9231} = (14, 15, 1, 1)$ lies on line ℓ_0
- 47 : $P_{9250} = (33, 15, 1, 1)$ lies on line ℓ_2
- 48 : $P_{9263} = (46, 15, 1, 1)$ lies on line ℓ_1
- 49 : $P_{9298} = (17, 16, 1, 1)$ lies on line ℓ_0
- 50 : $P_{9324} = (43, 16, 1, 1)$ lies on line ℓ_1
- 51 : $P_{9340} = (59, 16, 1, 1)$ lies on line ℓ_2
- 52 : $P_{9347} = (2, 17, 1, 1)$ lies on line ℓ_2
- 53 : $P_{9361} = (16, 17, 1, 1)$ lies on line ℓ_0
- 54 : $P_{9364} = (19, 17, 1, 1)$ lies on line ℓ_1
- 55 : $P_{9428} = (19, 18, 1, 1)$ lies on line ℓ_0

56 : $P_{9449} = (40, 18, 1, 1)$ lies on line ℓ_2
 57 : $P_{9467} = (58, 18, 1, 1)$ lies on line ℓ_1
 58 : $P_{9475} = (2, 19, 1, 1)$ lies on line ℓ_1
 59 : $P_{9490} = (17, 19, 1, 1)$ lies on line ℓ_2
 60 : $P_{9491} = (18, 19, 1, 1)$ lies on line ℓ_0
 61 : $P_{9546} = (9, 20, 1, 1)$ lies on line ℓ_1
 62 : $P_{9558} = (21, 20, 1, 1)$ lies on line ℓ_0
 63 : $P_{9566} = (29, 20, 1, 1)$ lies on line ℓ_2
 64 : $P_{9621} = (20, 21, 1, 1)$ lies on line ℓ_0
 65 : $P_{9637} = (36, 21, 1, 1)$ lies on line ℓ_2
 66 : $P_{9650} = (49, 21, 1, 1)$ lies on line ℓ_1
 67 : $P_{9679} = (14, 22, 1, 1)$ lies on line ℓ_2
 68 : $P_{9688} = (23, 22, 1, 1)$ lies on line ℓ_0
 69 : $P_{9689} = (24, 22, 1, 1)$ lies on line ℓ_1
 70 : $P_{9751} = (22, 23, 1, 1)$ lies on line ℓ_0
 71 : $P_{9761} = (32, 23, 1, 1)$ lies on line ℓ_1
 72 : $P_{9784} = (55, 23, 1, 1)$ lies on line ℓ_2
 73 : $P_{9807} = (14, 24, 1, 1)$ lies on line ℓ_1
 74 : $P_{9815} = (22, 24, 1, 1)$ lies on line ℓ_2
 75 : $P_{9818} = (25, 24, 1, 1)$ lies on line ℓ_0
 76 : $P_{9881} = (24, 25, 1, 1)$ lies on line ℓ_0
 77 : $P_{9904} = (47, 25, 1, 1)$ lies on line ℓ_2
 78 : $P_{9911} = (54, 25, 1, 1)$ lies on line ℓ_1
 79 : $P_{9926} = (5, 26, 1, 1)$ lies on line ℓ_2
 80 : $P_{9948} = (27, 26, 1, 1)$ lies on line ℓ_0
 81 : $P_{9952} = (31, 26, 1, 1)$ lies on line ℓ_1
 82 : $P_{10011} = (26, 27, 1, 1)$ lies on line ℓ_0
 83 : $P_{10024} = (39, 27, 1, 1)$ lies on line ℓ_1
 84 : $P_{10045} = (60, 27, 1, 1)$ lies on line ℓ_2
 85 : $P_{10078} = (29, 28, 1, 1)$ lies on line ℓ_0
 86 : $P_{10093} = (44, 28, 1, 1)$ lies on line ℓ_1
 87 : $P_{10097} = (48, 28, 1, 1)$ lies on line ℓ_2
 88 : $P_{10122} = (9, 29, 1, 1)$ lies on line ℓ_2
 89 : $P_{10133} = (20, 29, 1, 1)$ lies on line ℓ_1
 90 : $P_{10141} = (28, 29, 1, 1)$ lies on line ℓ_0
 91 : $P_{10208} = (31, 30, 1, 1)$ lies on line ℓ_0
 92 : $P_{10212} = (35, 30, 1, 1)$ lies on line ℓ_2
 93 : $P_{10238} = (61, 30, 1, 1)$ lies on line ℓ_1
 94 : $P_{10246} = (5, 31, 1, 1)$ lies on line ℓ_1
 95 : $P_{10267} = (26, 31, 1, 1)$ lies on line ℓ_2
 96 : $P_{10271} = (30, 31, 1, 1)$ lies on line ℓ_0
 97 : $P_{10328} = (23, 32, 1, 1)$ lies on line ℓ_2
 98 : $P_{10338} = (33, 32, 1, 1)$ lies on line ℓ_0
 99 : $P_{10360} = (55, 32, 1, 1)$ lies on line ℓ_1
 100 : $P_{10384} = (15, 33, 1, 1)$ lies on line ℓ_1
 101 : $P_{10401} = (32, 33, 1, 1)$ lies on line ℓ_0
 102 : $P_{10415} = (46, 33, 1, 1)$ lies on line ℓ_2
 103 : $P_{10437} = (4, 34, 1, 1)$ lies on line ℓ_2
 104 : $P_{10468} = (35, 34, 1, 1)$ lies on line ℓ_0
 105 : $P_{10471} = (38, 34, 1, 1)$ lies on line ℓ_1
 106 : $P_{10527} = (30, 35, 1, 1)$ lies on line ℓ_1
 107 : $P_{10531} = (34, 35, 1, 1)$ lies on line ℓ_0
 108 : $P_{10558} = (61, 35, 1, 1)$ lies on line ℓ_2
 109 : $P_{10582} = (21, 36, 1, 1)$ lies on line ℓ_1

110 : $P_{10598} = (37, 36, 1, 1)$ lies on line ℓ_0
 111 : $P_{10610} = (49, 36, 1, 1)$ lies on line ℓ_2
 112 : $P_{10633} = (8, 37, 1, 1)$ lies on line ℓ_2
 113 : $P_{10661} = (36, 37, 1, 1)$ lies on line ℓ_0
 114 : $P_{10670} = (45, 37, 1, 1)$ lies on line ℓ_1
 115 : $P_{10693} = (4, 38, 1, 1)$ lies on line ℓ_1
 116 : $P_{10723} = (34, 38, 1, 1)$ lies on line ℓ_2
 117 : $P_{10728} = (39, 38, 1, 1)$ lies on line ℓ_0
 118 : $P_{10780} = (27, 39, 1, 1)$ lies on line ℓ_2
 119 : $P_{10791} = (38, 39, 1, 1)$ lies on line ℓ_0
 120 : $P_{10813} = (60, 39, 1, 1)$ lies on line ℓ_1
 121 : $P_{10835} = (18, 40, 1, 1)$ lies on line ℓ_1
 122 : $P_{10858} = (41, 40, 1, 1)$ lies on line ℓ_0
 123 : $P_{10875} = (58, 40, 1, 1)$ lies on line ℓ_2
 124 : $P_{10884} = (3, 41, 1, 1)$ lies on line ℓ_2
 125 : $P_{10921} = (40, 41, 1, 1)$ lies on line ℓ_0
 126 : $P_{10923} = (42, 41, 1, 1)$ lies on line ℓ_1
 127 : $P_{10948} = (3, 42, 1, 1)$ lies on line ℓ_1
 128 : $P_{10986} = (41, 42, 1, 1)$ lies on line ℓ_2
 129 : $P_{10988} = (43, 42, 1, 1)$ lies on line ℓ_0
 130 : $P_{11025} = (16, 43, 1, 1)$ lies on line ℓ_2
 131 : $P_{11051} = (42, 43, 1, 1)$ lies on line ℓ_0
 132 : $P_{11068} = (59, 43, 1, 1)$ lies on line ℓ_1
 133 : $P_{11101} = (28, 44, 1, 1)$ lies on line ℓ_2
 134 : $P_{11118} = (45, 44, 1, 1)$ lies on line ℓ_0
 135 : $P_{11121} = (48, 44, 1, 1)$ lies on line ℓ_1
 136 : $P_{11145} = (8, 45, 1, 1)$ lies on line ℓ_1
 137 : $P_{11174} = (37, 45, 1, 1)$ lies on line ℓ_2
 138 : $P_{11181} = (44, 45, 1, 1)$ lies on line ℓ_0
 139 : $P_{11216} = (15, 46, 1, 1)$ lies on line ℓ_2
 140 : $P_{11234} = (33, 46, 1, 1)$ lies on line ℓ_1
 141 : $P_{11248} = (47, 46, 1, 1)$ lies on line ℓ_0
 142 : $P_{11290} = (25, 47, 1, 1)$ lies on line ℓ_1
 143 : $P_{11311} = (46, 47, 1, 1)$ lies on line ℓ_0
 144 : $P_{11319} = (54, 47, 1, 1)$ lies on line ℓ_2
 145 : $P_{11357} = (28, 48, 1, 1)$ lies on line ℓ_1
 146 : $P_{11373} = (44, 48, 1, 1)$ lies on line ℓ_2
 147 : $P_{11378} = (49, 48, 1, 1)$ lies on line ℓ_0
 148 : $P_{11414} = (21, 49, 1, 1)$ lies on line ℓ_2
 149 : $P_{11429} = (36, 49, 1, 1)$ lies on line ℓ_1
 150 : $P_{11441} = (48, 49, 1, 1)$ lies on line ℓ_0
 151 : $P_{11470} = (13, 50, 1, 1)$ lies on line ℓ_1
 152 : $P_{11508} = (51, 50, 1, 1)$ lies on line ℓ_0
 153 : $P_{11520} = (63, 50, 1, 1)$ lies on line ℓ_2
 154 : $P_{11527} = (6, 51, 1, 1)$ lies on line ℓ_2
 155 : $P_{11571} = (50, 51, 1, 1)$ lies on line ℓ_0
 156 : $P_{11574} = (53, 51, 1, 1)$ lies on line ℓ_1
 157 : $P_{11595} = (10, 52, 1, 1)$ lies on line ℓ_2
 158 : $P_{11638} = (53, 52, 1, 1)$ lies on line ℓ_0
 159 : $P_{11647} = (62, 52, 1, 1)$ lies on line ℓ_1
 160 : $P_{11655} = (6, 53, 1, 1)$ lies on line ℓ_1
 161 : $P_{11700} = (51, 53, 1, 1)$ lies on line ℓ_2
 162 : $P_{11701} = (52, 53, 1, 1)$ lies on line ℓ_0
 163 : $P_{11738} = (25, 54, 1, 1)$ lies on line ℓ_2

164 : $P_{11760} = (47, 54, 1, 1)$ lies on line ℓ_1
 165 : $P_{11768} = (55, 54, 1, 1)$ lies on line ℓ_0
 166 : $P_{11800} = (23, 55, 1, 1)$ lies on line ℓ_1
 167 : $P_{11809} = (32, 55, 1, 1)$ lies on line ℓ_2
 168 : $P_{11831} = (54, 55, 1, 1)$ lies on line ℓ_0
 169 : $P_{11842} = (1, 56, 1, 1)$ lies on line ℓ_2
 170 : $P_{11906} = (1, 57, 1, 1)$ lies on line ℓ_1
 171 : $P_{11987} = (18, 58, 1, 1)$ lies on line ℓ_2
 172 : $P_{12009} = (40, 58, 1, 1)$ lies on line ℓ_1
 173 : $P_{12028} = (59, 58, 1, 1)$ lies on line ℓ_0
 174 : $P_{12049} = (16, 59, 1, 1)$ lies on line ℓ_1
 175 : $P_{12076} = (43, 59, 1, 1)$ lies on line ℓ_2
 176 : $P_{12091} = (58, 59, 1, 1)$ lies on line ℓ_0

177 : $P_{12124} = (27, 60, 1, 1)$ lies on line ℓ_1
 178 : $P_{12136} = (39, 60, 1, 1)$ lies on line ℓ_2
 179 : $P_{12158} = (61, 60, 1, 1)$ lies on line ℓ_0
 180 : $P_{12191} = (30, 61, 1, 1)$ lies on line ℓ_2
 181 : $P_{12196} = (35, 61, 1, 1)$ lies on line ℓ_1
 182 : $P_{12221} = (60, 61, 1, 1)$ lies on line ℓ_0
 183 : $P_{12235} = (10, 62, 1, 1)$ lies on line ℓ_1
 184 : $P_{12277} = (52, 62, 1, 1)$ lies on line ℓ_2
 185 : $P_{12288} = (63, 62, 1, 1)$ lies on line ℓ_0
 186 : $P_{12302} = (13, 63, 1, 1)$ lies on line ℓ_2
 187 : $P_{12339} = (50, 63, 1, 1)$ lies on line ℓ_1
 188 : $P_{12351} = (62, 63, 1, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 4097 points not on any line:
 Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{11898}	P_{11961}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{11898}	P_{8258}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{11961}	P_{8258}

The surface has 4289 points:
 Too many to print.