

Rank-76 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(64) is 34087056

General information

Number of lines	1
Number of points	4161
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	65
Number of points off lines	4096
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65
Type of lines on points	$1^{65}, 0^{4096}$

Singular Points

The surface has 1 singular points:

$$0 : P_{132} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{PI}(1, 0, 1, 0, 0, 0)_3$$

Rank of lines: (1)
Rank of points on Klein quadric: (3)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 65 single points:
The single points on the surface are:

- | | |
|--|--|
| 0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0 | 33 : $P_{163} = (32, 1, 1, 0)$ lies on line ℓ_0 |
| 1 : $P_{131} = (0, 1, 1, 0)$ lies on line ℓ_0 | 34 : $P_{164} = (33, 1, 1, 0)$ lies on line ℓ_0 |
| 2 : $P_{132} = (1, 1, 1, 0)$ lies on line ℓ_0 | 35 : $P_{165} = (34, 1, 1, 0)$ lies on line ℓ_0 |
| 3 : $P_{133} = (2, 1, 1, 0)$ lies on line ℓ_0 | 36 : $P_{166} = (35, 1, 1, 0)$ lies on line ℓ_0 |
| 4 : $P_{134} = (3, 1, 1, 0)$ lies on line ℓ_0 | 37 : $P_{167} = (36, 1, 1, 0)$ lies on line ℓ_0 |
| 5 : $P_{135} = (4, 1, 1, 0)$ lies on line ℓ_0 | 38 : $P_{168} = (37, 1, 1, 0)$ lies on line ℓ_0 |
| 6 : $P_{136} = (5, 1, 1, 0)$ lies on line ℓ_0 | 39 : $P_{169} = (38, 1, 1, 0)$ lies on line ℓ_0 |
| 7 : $P_{137} = (6, 1, 1, 0)$ lies on line ℓ_0 | 40 : $P_{170} = (39, 1, 1, 0)$ lies on line ℓ_0 |
| 8 : $P_{138} = (7, 1, 1, 0)$ lies on line ℓ_0 | 41 : $P_{171} = (40, 1, 1, 0)$ lies on line ℓ_0 |
| 9 : $P_{139} = (8, 1, 1, 0)$ lies on line ℓ_0 | 42 : $P_{172} = (41, 1, 1, 0)$ lies on line ℓ_0 |
| 10 : $P_{140} = (9, 1, 1, 0)$ lies on line ℓ_0 | 43 : $P_{173} = (42, 1, 1, 0)$ lies on line ℓ_0 |
| 11 : $P_{141} = (10, 1, 1, 0)$ lies on line ℓ_0 | 44 : $P_{174} = (43, 1, 1, 0)$ lies on line ℓ_0 |
| 12 : $P_{142} = (11, 1, 1, 0)$ lies on line ℓ_0 | 45 : $P_{175} = (44, 1, 1, 0)$ lies on line ℓ_0 |
| 13 : $P_{143} = (12, 1, 1, 0)$ lies on line ℓ_0 | 46 : $P_{176} = (45, 1, 1, 0)$ lies on line ℓ_0 |
| 14 : $P_{144} = (13, 1, 1, 0)$ lies on line ℓ_0 | 47 : $P_{177} = (46, 1, 1, 0)$ lies on line ℓ_0 |
| 15 : $P_{145} = (14, 1, 1, 0)$ lies on line ℓ_0 | 48 : $P_{178} = (47, 1, 1, 0)$ lies on line ℓ_0 |
| 16 : $P_{146} = (15, 1, 1, 0)$ lies on line ℓ_0 | 49 : $P_{179} = (48, 1, 1, 0)$ lies on line ℓ_0 |
| 17 : $P_{147} = (16, 1, 1, 0)$ lies on line ℓ_0 | 50 : $P_{180} = (49, 1, 1, 0)$ lies on line ℓ_0 |
| 18 : $P_{148} = (17, 1, 1, 0)$ lies on line ℓ_0 | 51 : $P_{181} = (50, 1, 1, 0)$ lies on line ℓ_0 |
| 19 : $P_{149} = (18, 1, 1, 0)$ lies on line ℓ_0 | 52 : $P_{182} = (51, 1, 1, 0)$ lies on line ℓ_0 |
| 20 : $P_{150} = (19, 1, 1, 0)$ lies on line ℓ_0 | 53 : $P_{183} = (52, 1, 1, 0)$ lies on line ℓ_0 |
| 21 : $P_{151} = (20, 1, 1, 0)$ lies on line ℓ_0 | 54 : $P_{184} = (53, 1, 1, 0)$ lies on line ℓ_0 |
| 22 : $P_{152} = (21, 1, 1, 0)$ lies on line ℓ_0 | 55 : $P_{185} = (54, 1, 1, 0)$ lies on line ℓ_0 |
| 23 : $P_{153} = (22, 1, 1, 0)$ lies on line ℓ_0 | 56 : $P_{186} = (55, 1, 1, 0)$ lies on line ℓ_0 |
| 24 : $P_{154} = (23, 1, 1, 0)$ lies on line ℓ_0 | 57 : $P_{187} = (56, 1, 1, 0)$ lies on line ℓ_0 |
| 25 : $P_{155} = (24, 1, 1, 0)$ lies on line ℓ_0 | 58 : $P_{188} = (57, 1, 1, 0)$ lies on line ℓ_0 |
| 26 : $P_{156} = (25, 1, 1, 0)$ lies on line ℓ_0 | 59 : $P_{189} = (58, 1, 1, 0)$ lies on line ℓ_0 |
| 27 : $P_{157} = (26, 1, 1, 0)$ lies on line ℓ_0 | 60 : $P_{190} = (59, 1, 1, 0)$ lies on line ℓ_0 |
| 28 : $P_{158} = (27, 1, 1, 0)$ lies on line ℓ_0 | 61 : $P_{191} = (60, 1, 1, 0)$ lies on line ℓ_0 |
| 29 : $P_{159} = (28, 1, 1, 0)$ lies on line ℓ_0 | 62 : $P_{192} = (61, 1, 1, 0)$ lies on line ℓ_0 |
| 30 : $P_{160} = (29, 1, 1, 0)$ lies on line ℓ_0 | 63 : $P_{193} = (62, 1, 1, 0)$ lies on line ℓ_0 |
| 31 : $P_{161} = (30, 1, 1, 0)$ lies on line ℓ_0 | 64 : $P_{194} = (63, 1, 1, 0)$ lies on line ℓ_0 |
| 32 : $P_{162} = (31, 1, 1, 0)$ lies on line ℓ_0 | |

The single points on the surface are:

Points on surface but on no line

The surface has 4096 points not on any line:
Too many to print.

Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & \end{array}$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 4161 points:
Too many to print.