

Rank-74500 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1499092314

General information

Number of lines	7
Number of points	29
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	18
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	$4, 3, 2^5, 1^{18}, 0^4$

Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{129}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0)_1 \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0)_6 \\
\ell_3 &= \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \mathbf{Pl}(0, 3, 1, 0, 0)_114 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \mathbf{Pl}(0, 2, 1, 0, 0)_113 \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{99} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{99} = \mathbf{Pl}(1, 1, 1, 2, 1)_83 \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega & \omega \end{bmatrix}_{94} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{94} = \mathbf{Pl}(1, 1, 1, 3, 1)_92
\end{aligned}$$

Rank of lines: (337, 356, 100, 289, 205, 99, 94)

Rank of points on Klein quadric: (129, 1, 6, 114, 113, 83, 92)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1).$$

Double Points

The surface has 5 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{38} &= (0, 0, 1, 1) = \ell_0 \cap \ell_1 & P_{79} &= (2, 2, 3, 1) = \ell_3 \cap \ell_6 \\
P_{45} &= (0, 2, 1, 1) = \ell_0 \cap \ell_5 & P_{68} &= (3, 3, 2, 1) = \ell_4 \cap \ell_5 \\
P_{49} &= (0, 3, 1, 1) = \ell_0 \cap \ell_6
\end{aligned}$$

Single Points

The surface has 18 single points:

The single points on the surface are:

$$\begin{aligned}
0 : P_1 &= (0, 1, 0, 0) \text{ lies on line } \ell_0 & 10 : P_{53} &= (0, 0, 2, 1) \text{ lies on line } \ell_1 \\
1 : P_3 &= (0, 0, 0, 1) \text{ lies on line } \ell_1 & 11 : P_{54} &= (1, 0, 2, 1) \text{ lies on line } \ell_2 \\
2 : P_{16} &= (1, 2, 1, 0) \text{ lies on line } \ell_5 & 12 : P_{60} &= (3, 1, 2, 1) \text{ lies on line } \ell_6 \\
3 : P_{20} &= (1, 3, 1, 0) \text{ lies on line } \ell_6 & 13 : P_{63} &= (2, 2, 2, 1) \text{ lies on line } \ell_3 \\
4 : P_{32} &= (2, 2, 0, 1) \text{ lies on line } \ell_3 & 14 : P_{69} &= (0, 0, 3, 1) \text{ lies on line } \ell_1 \\
5 : P_{37} &= (3, 3, 0, 1) \text{ lies on line } \ell_4 & 15 : P_{70} &= (1, 0, 3, 1) \text{ lies on line } \ell_2 \\
6 : P_{39} &= (1, 0, 1, 1) \text{ lies on line } \ell_2 & 16 : P_{75} &= (2, 1, 3, 1) \text{ lies on line } \ell_5 \\
7 : P_{42} &= (0, 1, 1, 1) \text{ lies on line } \ell_0 & 17 : P_{84} &= (3, 3, 3, 1) \text{ lies on line } \ell_4 \\
8 : P_{47} &= (2, 2, 1, 1) \text{ lies on line } \ell_3 \\
9 : P_{52} &= (3, 3, 1, 1) \text{ lies on line } \ell_4
\end{aligned}$$

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:

The points on the surface but not on lines are:

$$0 : P_{66} = (1, 3, 2, 1)$$

$$1 : P_{67} = (2, 3, 2, 1)$$

$$2 : P_{78} = (1, 2, 3, 1)$$

$$3 : P_{80} = (3, 2, 3, 1)$$

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	0	0	0	1	1
1	1	0	1	1	1	0	0
2	0	1	0	1	1	1	1
3	0	1	1	0	1	0	1
4	0	1	1	1	0	1	0
5	1	0	1	0	1	0	1
6	1	0	1	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_5	ℓ_6
in point	P_{38}	P_{45}	P_{49}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{38}	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_2	P_{23}	P_{23}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_2	P_2	P_{79}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{68}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_{45}	P_{23}	P_{68}	P_{23}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_{49}	P_{23}	P_{79}	P_{23}

The surface has 29 points:

The points on the surface are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_{16} = (1, 2, 1, 0)$$

$$4 : P_{20} = (1, 3, 1, 0)$$

$$5 : P_{23} = (1, 0, 0, 1)$$

$$6 : P_{32} = (2, 2, 0, 1)$$

$$7 : P_{37} = (3, 3, 0, 1)$$

$$8 : P_{38} = (0, 0, 1, 1)$$

$$9 : P_{39} = (1, 0, 1, 1)$$

$$10 : P_{42} = (0, 1, 1, 1)$$

$$11 : P_{45} = (0, 2, 1, 1)$$

$$12 : P_{47} = (2, 2, 1, 1)$$

$$13 : P_{49} = (0, 3, 1, 1)$$

$$14 : P_{52} = (3, 3, 1, 1)$$

$$15 : P_{53} = (0, 0, 2, 1)$$

$$16 : P_{54} = (1, 0, 2, 1)$$

$$17 : P_{60} = (3, 1, 2, 1)$$

$$\begin{aligned}
18 : P_{63} &= (2, 2, 2, 1) \\
19 : P_{66} &= (1, 3, 2, 1) \\
20 : P_{67} &= (2, 3, 2, 1) \\
21 : P_{68} &= (3, 3, 2, 1)
\end{aligned}$$

$$\begin{aligned}
22 : P_{69} &= (0, 0, 3, 1) \\
23 : P_{70} &= (1, 0, 3, 1) \\
24 : P_{75} &= (2, 1, 3, 1) \\
25 : P_{78} &= (1, 2, 3, 1)
\end{aligned}$$

$$\begin{aligned}
26 : P_{79} &= (2, 2, 3, 1) \\
27 : P_{80} &= (3, 2, 3, 1) \\
28 : P_{84} &= (3, 3, 3, 1)
\end{aligned}$$