# Rank-65903 over GF(64)

January 15, 2021

# The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is -2130173883

### General information

Number of lines	4
Number of points	4161
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	252
Number of points off lines	3905
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^{4}$
Type of lines on points	$2^4, 1^{252}, 0^{3905}$

### Singular Points

The surface has 2 singular points:

$$\begin{array}{l} 0: \ P_{7747} = \mathbf{P}(1, \epsilon^{42}, 0, 1) = \mathbf{P}(1, 56, 0, 1) \\ 1: \ P_{7811} = \mathbf{P}(1, \epsilon^{21}, 0, 1) = \mathbf{P}(1, 57, 0, 1) \end{array}$$

### The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{4352}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{130}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578}$$

Rank of lines: (0, 4097, 266304, 8258)

Rank of points on Klein quadric: (0, 4352, 130, 544578)

#### **Eckardt Points**

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 4 Double points: The double points on the surface are:

$$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$$
  

$$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$$
  

$$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$$

## $P_{8258} = (0,0,1,1) = \ell_1 \cap \ell_3$

#### Single Points

The surface has 252 single points: The single points on the surface are:

 $0: P_4 = (1, 1, 1, 1)$  lies on line  $\ell_3$ 22:  $P_{27} = (23, 1, 0, 0)$  lies on line  $\ell_0$ 1:  $P_6 = (2, 1, 0, 0)$  lies on line  $\ell_0$ 23:  $P_{28} = (24, 1, 0, 0)$  lies on line  $\ell_0$ 2:  $P_7 = (3, 1, 0, 0)$  lies on line  $\ell_0$ 24:  $P_{29} = (25, 1, 0, 0)$  lies on line  $\ell_0$ 3:  $P_8 = (4, 1, 0, 0)$  lies on line  $\ell_0$ 25:  $P_{30} = (26, 1, 0, 0)$  lies on line  $\ell_0$ 4:  $P_9 = (5, 1, 0, 0)$  lies on line  $\ell_0$ 26:  $P_{31} = (27, 1, 0, 0)$  lies on line  $\ell_0$ 5:  $P_{10} = (6, 1, 0, 0)$  lies on line  $\ell_0$ 27:  $P_{32} = (28, 1, 0, 0)$  lies on line  $\ell_0$ 6:  $P_{11} = (7, 1, 0, 0)$  lies on line  $\ell_0$ 28:  $P_{33} = (29, 1, 0, 0)$  lies on line  $\ell_0$ 29:  $P_{34} = (30, 1, 0, 0)$  lies on line  $\ell_0$ 7:  $P_{12} = (8, 1, 0, 0)$  lies on line  $\ell_0$ 8:  $P_{13} = (9, 1, 0, 0)$  lies on line  $\ell_0$  $30: P_{35} = (31, 1, 0, 0)$  lies on line  $\ell_0$ 9:  $P_{14} = (10, 1, 0, 0)$  lies on line  $\ell_0$  $31: P_{36} = (32, 1, 0, 0)$  lies on line  $\ell_0$ 10:  $P_{15} = (11, 1, 0, 0)$  lies on line  $\ell_0$  $32: P_{37} = (33, 1, 0, 0)$  lies on line  $\ell_0$ 11:  $P_{16} = (12, 1, 0, 0)$  lies on line  $\ell_0$ 33:  $P_{38} = (34, 1, 0, 0)$  lies on line  $\ell_0$ 12 :  $P_{17} = (13, 1, 0, 0)$  lies on line  $\ell_0$  $34: P_{39} = (35, 1, 0, 0)$  lies on line  $\ell_0$ 13:  $P_{18} = (14, 1, 0, 0)$  lies on line  $\ell_0$  $35: P_{40} = (36, 1, 0, 0)$  lies on line  $\ell_0$ 14 :  $P_{19} = (15, 1, 0, 0)$  lies on line  $\ell_0$  $36: P_{41} = (37, 1, 0, 0)$  lies on line  $\ell_0$ 15:  $P_{20} = (16, 1, 0, 0)$  lies on line  $\ell_0$  $37: P_{42} = (38, 1, 0, 0)$  lies on line  $\ell_0$ 16:  $P_{21} = (17, 1, 0, 0)$  lies on line  $\ell_0$  $38: P_{43} = (39, 1, 0, 0)$  lies on line  $\ell_0$ 17:  $P_{22} = (18, 1, 0, 0)$  lies on line  $\ell_0$ 39:  $P_{44} = (40, 1, 0, 0)$  lies on line  $\ell_0$ 18 :  $P_{23} = (19, 1, 0, 0)$  lies on line  $\ell_0$  $40: P_{45} = (41, 1, 0, 0)$  lies on line  $\ell_0$ 19 :  $P_{24} = (20,1,0,0)$  lies on line  $\ell_0$ 41:  $P_{46} = (42, 1, 0, 0)$  lies on line  $\ell_0$ 20:  $P_{25} = (21, 1, 0, 0)$  lies on line  $\ell_0$  $42: P_{47} = (43, 1, 0, 0)$  lies on line  $\ell_0$ 21 :  $P_{26} = (22,1,0,0)$  lies on line  $\ell_0$ 43:  $P_{48} = (44, 1, 0, 0)$  lies on line  $\ell_0$ 

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44: P_{49} = (45, 1, 0, 0) lies on line \ell_0
                                                                      98: P_{6403} = (1, 35, 0, 1) lies on line \ell_2
45: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                      99: P_{6467} = (1, 36, 0, 1) lies on line \ell_2
                                                                      100: P_{6531} = (1, 37, 0, 1) lies on line \ell_2
46: P_{51} = (47, 1, 0, 0) lies on line \ell_0
47: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                      101: P_{6595} = (1, 38, 0, 1) lies on line \ell_2
48: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                      102: P_{6659} = (1, 39, 0, 1) lies on line \ell_2
49: P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                      103: P_{6723} = (1, 40, 0, 1) lies on line \ell_2
50: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                      104: P_{6787} = (1, 41, 0, 1) lies on line \ell_2
51: P_{56} = (52, 1, 0, 0) lies on line \ell_0
                                                                      105: P_{6851} = (1, 42, 0, 1) lies on line \ell_2
52: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                      106: P_{6915} = (1, 43, 0, 1) lies on line \ell_2
53: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                      107: P_{6979} = (1, 44, 0, 1) lies on line \ell_2
54: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                      108: P_{7043} = (1, 45, 0, 1) lies on line \ell_2
55: P_{60} = (56, 1, 0, 0) lies on line \ell_0
                                                                      109: P_{7107} = (1, 46, 0, 1) lies on line \ell_2
56: P_{61} = (57, 1, 0, 0) lies on line \ell_0
                                                                      110: P_{7171} = (1, 47, 0, 1) lies on line \ell_2
57: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                      111: P_{7235} = (1, 48, 0, 1) lies on line \ell_2
58: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                      112: P_{7299} = (1, 49, 0, 1) lies on line \ell_2
59: P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                      113: P_{7363} = (1, 50, 0, 1) lies on line \ell_2
60: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                      114: P_{7427} = (1, 51, 0, 1) lies on line \ell_2
61: P_{66} = (62, 1, 0, 0) lies on line \ell_0
                                                                      115: P_{7491} = (1, 52, 0, 1) lies on line \ell_2
                                                                      116 : P_{7555} = (1, 53, 0, 1) lies on line \ell_2
62: P_{67} = (63, 1, 0, 0) lies on line \ell_0
63: P_{4163} = (1,0,0,1) lies on line \ell_2
                                                                      117: P_{7619} = (1, 54, 0, 1) lies on line \ell_2
64: P_{4227} = (1, 1, 0, 1) lies on line \ell_2
                                                                      118: P_{7683} = (1, 55, 0, 1) lies on line \ell_2
65: P_{4291} = (1, 2, 0, 1) lies on line \ell_2
                                                                      119: P_{7747} = (1, 56, 0, 1) lies on line \ell_2
66: P_{4355} = (1, 3, 0, 1) lies on line \ell_2
                                                                      120: P_{7811} = (1, 57, 0, 1) lies on line \ell_2
67: P_{4419} = (1, 4, 0, 1) lies on line \ell_2
                                                                      121: P_{7875} = (1, 58, 0, 1) lies on line \ell_2
68: P_{4483} = (1, 5, 0, 1) lies on line \ell_2
                                                                      122: P_{7939} = (1, 59, 0, 1) lies on line \ell_2
69: P_{4547} = (1, 6, 0, 1) lies on line \ell_2
                                                                      123: P_{8003} = (1, 60, 0, 1) lies on line \ell_2
70: P_{4611} = (1,7,0,1) lies on line \ell_2
                                                                      124: P_{8067} = (1, 61, 0, 1) lies on line \ell_2
71: P_{4675} = (1, 8, 0, 1) lies on line \ell_2
                                                                      125: P_{8131} = (1, 62, 0, 1) lies on line \ell_2
72: P_{4739} = (1, 9, 0, 1) lies on line \ell_2
                                                                      126: P_{8195} = (1, 63, 0, 1) lies on line \ell_2
73: P_{4803} = (1, 10, 0, 1) lies on line \ell_2
                                                                      127: P_{8259} = (1, 0, 1, 1) lies on line \ell_1
74: P_{4867} = (1, 11, 0, 1) lies on line \ell_2
                                                                      128: P_{8260} = (2, 0, 1, 1) lies on line \ell_1
75: P_{4931} = (1, 12, 0, 1) lies on line \ell_2
                                                                      129: P_{8261} = (3,0,1,1) lies on line \ell_1
76: P_{4995} = (1, 13, 0, 1) lies on line \ell_2
                                                                      130: P_{8262} = (4, 0, 1, 1) lies on line \ell_1
77: P_{5059} = (1, 14, 0, 1) lies on line \ell_2
                                                                      131: P_{8263} = (5, 0, 1, 1) lies on line \ell_1
78: P_{5123} = (1, 15, 0, 1) lies on line \ell_2
                                                                      132: P_{8264} = (6,0,1,1) lies on line \ell_1
79: P_{5187} = (1, 16, 0, 1) lies on line \ell_2
                                                                      133: P_{8265} = (7,0,1,1) lies on line \ell_1
80: P_{5251} = (1, 17, 0, 1) lies on line \ell_2
                                                                      134: P_{8266} = (8, 0, 1, 1) lies on line \ell_1
81: P_{5315} = (1, 18, 0, 1) lies on line \ell_2
                                                                      135 : P_{8267} = (9, 0, 1, 1) lies on line \ell_1
82: P_{5379} = (1, 19, 0, 1) lies on line \ell_2
                                                                      136: P_{8268} = (10, 0, 1, 1) lies on line \ell_1
83: P_{5443} = (1, 20, 0, 1) lies on line \ell_2
                                                                      137: P_{8269} = (11, 0, 1, 1) lies on line \ell_1
                                                                      138 : P_{8270} = (12, 0, 1, 1) lies on line \ell_1
84: P_{5507} = (1, 21, 0, 1) lies on line \ell_2
85: P_{5571} = (1, 22, 0, 1) lies on line \ell_2
                                                                      139: P_{8271} = (13, 0, 1, 1) lies on line \ell_1
86 : P_{5635} = (1, 23, 0, 1) lies on line \ell_2
                                                                      140: P_{8272} = (14, 0, 1, 1) lies on line \ell_1
87: P_{5699} = (1, 24, 0, 1) lies on line \ell_2
                                                                      141: P_{8273} = (15, 0, 1, 1) lies on line \ell_1
88: P_{5763} = (1, 25, 0, 1) lies on line \ell_2
                                                                      142: P_{8274} = (16, 0, 1, 1) lies on line \ell_1
89: P_{5827} = (1, 26, 0, 1) lies on line \ell_2
                                                                      143: P_{8275} = (17, 0, 1, 1) lies on line \ell_1
90: P_{5891} = (1, 27, 0, 1) lies on line \ell_2
                                                                      144: P_{8276} = (18, 0, 1, 1) lies on line \ell_1
91: P_{5955} = (1, 28, 0, 1) lies on line \ell_2
                                                                      145: P_{8277} = (19, 0, 1, 1) lies on line \ell_1
92: P_{6019} = (1, 29, 0, 1) lies on line \ell_2
                                                                      146: P_{8278} = (20, 0, 1, 1) lies on line \ell_1
93: P_{6083} = (1, 30, 0, 1) lies on line \ell_2
                                                                      147: P_{8279} = (21, 0, 1, 1) lies on line \ell_1
94: P_{6147} = (1, 31, 0, 1) lies on line \ell_2
                                                                      148: P_{8280} = (22, 0, 1, 1) lies on line \ell_1
95: P_{6211} = (1, 32, 0, 1) lies on line \ell_2
                                                                      149: P_{8281} = (23, 0, 1, 1) lies on line \ell_1
                                                                      150 : P_{8282} = (24, 0, 1, 1) lies on line \ell_1
96: P_{6275} = (1, 33, 0, 1) lies on line \ell_2
97: P_{6339} = (1, 34, 0, 1) lies on line \ell_2
                                                                      151: P_{8283} = (25, 0, 1, 1) lies on line \ell_1
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152: P_{8284} = (26, 0, 1, 1) lies on line \ell_1
                                                                     203: P_{9232} = (15, 15, 1, 1) lies on line \ell_3
153: P_{8285} = (27, 0, 1, 1) lies on line \ell_1
                                                                     204 : P_{9297} = (16, 16, 1, 1) lies on line \ell_3
154: P_{8286} = (28, 0, 1, 1) lies on line \ell_1
                                                                     205: P_{9362} = (17, 17, 1, 1) lies on line \ell_3
155: P_{8287} = (29, 0, 1, 1) lies on line \ell_1
                                                                     206: P_{9427} = (18, 18, 1, 1) lies on line \ell_3
156: P_{8288} = (30, 0, 1, 1) lies on line \ell_1
                                                                     207: P_{9492} = (19, 19, 1, 1) lies on line \ell_3
                                                                     208: P_{9557} = (20, 20, 1, 1) lies on line \ell_3
157: P_{8289} = (31, 0, 1, 1) lies on line \ell_1
158: P_{8290} = (32, 0, 1, 1) lies on line \ell_1
                                                                     209: P_{9622} = (21, 21, 1, 1) lies on line \ell_3
159: P_{8291} = (33, 0, 1, 1) lies on line \ell_1
                                                                     210: P_{9687} = (22, 22, 1, 1) lies on line \ell_3
160: P_{8292} = (34, 0, 1, 1) lies on line \ell_1
                                                                     211: P_{9752} = (23, 23, 1, 1) lies on line \ell_3
161: P_{8293} = (35, 0, 1, 1) lies on line \ell_1
                                                                     212: P_{9817} = (24, 24, 1, 1) lies on line \ell_3
162: P_{8294} = (36, 0, 1, 1) lies on line \ell_1
                                                                     213: P_{9882} = (25, 25, 1, 1) lies on line \ell_3
163: P_{8295} = (37, 0, 1, 1) lies on line \ell_1
                                                                     214: P_{9947} = (26, 26, 1, 1) lies on line \ell_3
164: P_{8296} = (38, 0, 1, 1) lies on line \ell_1
                                                                     215: P_{10012} = (27, 27, 1, 1) lies on line \ell_3
165: P_{8297} = (39, 0, 1, 1) lies on line \ell_1
                                                                     216: P_{10077} = (28, 28, 1, 1) lies on line \ell_3
                                                                     217: P_{10142} = (29, 29, 1, 1) lies on line \ell_3
166: P_{8298} = (40, 0, 1, 1) lies on line \ell_1
167: P_{8299} = (41, 0, 1, 1) lies on line \ell_1
                                                                     218: P_{10207} = (30, 30, 1, 1) lies on line \ell_3
168: P_{8300} = (42, 0, 1, 1) lies on line \ell_1
                                                                     219: P_{10272} = (31, 31, 1, 1) lies on line \ell_3
169: P_{8301} = (43, 0, 1, 1) lies on line \ell_1
                                                                     220: P_{10337} = (32, 32, 1, 1) lies on line \ell_3
170: P_{8302} = (44, 0, 1, 1) lies on line \ell_1
                                                                     221: P_{10402} = (33, 33, 1, 1) lies on line \ell_3
                                                                     222: P_{10467} = (34, 34, 1, 1) lies on line \ell_3
171: P_{8303} = (45, 0, 1, 1) lies on line \ell_1
172: P_{8304} = (46, 0, 1, 1) lies on line \ell_1
                                                                     223: P_{10532} = (35, 35, 1, 1) lies on line \ell_3
173: P_{8305} = (47, 0, 1, 1) lies on line \ell_1
                                                                     224: P_{10597} = (36, 36, 1, 1) lies on line \ell_3
                                                                     225: P_{10662} = (37, 37, 1, 1) lies on line \ell_3
174: P_{8306} = (48, 0, 1, 1) lies on line \ell_1
175: P_{8307} = (49, 0, 1, 1) lies on line \ell_1
                                                                     226: P_{10727} = (38, 38, 1, 1) lies on line \ell_3
176: P_{8308} = (50, 0, 1, 1) lies on line \ell_1
                                                                     227: P_{10792} = (39, 39, 1, 1) lies on line \ell_3
177: P_{8309} = (51, 0, 1, 1) lies on line \ell_1
                                                                     228: P_{10857} = (40, 40, 1, 1) lies on line \ell_3
178: P_{8310} = (52, 0, 1, 1) lies on line \ell_1
                                                                     229: P_{10922} = (41, 41, 1, 1) lies on line \ell_3
179: P_{8311} = (53, 0, 1, 1) lies on line \ell_1
                                                                     230: P_{10987} = (42, 42, 1, 1) lies on line \ell_3
180: P_{8312} = (54, 0, 1, 1) lies on line \ell_1
                                                                     231: P_{11052} = (43, 43, 1, 1) lies on line \ell_3
181: P_{8313} = (55, 0, 1, 1) lies on line \ell_1
                                                                     232 : P_{11117} = (44, 44, 1, 1) lies on line \ell_3
182: P_{8314} = (56, 0, 1, 1) lies on line \ell_1
                                                                     233: P_{11182} = (45, 45, 1, 1) lies on line \ell_3
183: P_{8315} = (57, 0, 1, 1) lies on line \ell_1
                                                                     234 : P_{11247} = (46, 46, 1, 1) lies on line \ell_3
184: P_{8316} = (58, 0, 1, 1) lies on line \ell_1
                                                                     235 : P_{11312} = (47, 47, 1, 1) lies on line \ell_3
185: P_{8317} = (59, 0, 1, 1) lies on line \ell_1
                                                                     236: P_{11377} = (48, 48, 1, 1) lies on line \ell_3
186: P_{8318} = (60, 0, 1, 1) lies on line \ell_1
                                                                     237: P_{11442} = (49, 49, 1, 1) lies on line \ell_3
187: P_{8319} = (61, 0, 1, 1) lies on line \ell_1
                                                                     238: P_{11507} = (50, 50, 1, 1) lies on line \ell_3
188: P_{8320} = (62, 0, 1, 1) lies on line \ell_1
                                                                     239: P_{11572} = (51, 51, 1, 1) lies on line \ell_3
189 : P_{8321} = (63, 0, 1, 1) lies on line \ell_1
                                                                     240 : P_{11637} = (52, 52, 1, 1) lies on line \ell_3
190 : P_{8387} = (2, 2, 1, 1) lies on line \ell_3
                                                                     241 : P_{11702} = (53, 53, 1, 1) lies on line \ell_3
191: P_{8452} = (3, 3, 1, 1) lies on line \ell_3
                                                                     242: P_{11767} = (54, 54, 1, 1) lies on line \ell_3
192: P_{8517} = (4, 4, 1, 1) lies on line \ell_3
                                                                     243: P_{11832} = (55, 55, 1, 1) lies on line \ell_3
193: P_{8582} = (5, 5, 1, 1) lies on line \ell_3
                                                                     244: P_{11897} = (56, 56, 1, 1) lies on line \ell_3
194: P_{8647} = (6, 6, 1, 1) lies on line \ell_3
                                                                     245: P_{11962} = (57, 57, 1, 1) lies on line \ell_3
195: P_{8712} = (7,7,1,1) lies on line \ell_3
                                                                     246: P_{12027} = (58, 58, 1, 1) lies on line \ell_3
196: P_{8777} = (8, 8, 1, 1) lies on line \ell_3
                                                                     247: P_{12092} = (59, 59, 1, 1) lies on line \ell_3
197: P_{8842} = (9, 9, 1, 1) lies on line \ell_3
                                                                     248 : P_{12157} = (60, 60, 1, 1) lies on line \ell_3
198: P_{8907} = (10, 10, 1, 1) lies on line \ell_3
                                                                     249: P_{12222} = (61, 61, 1, 1) lies on line \ell_3
199: P_{8972} = (11, 11, 1, 1) lies on line \ell_3
                                                                     250: P_{12287} = (62, 62, 1, 1) lies on line \ell_3
200: P_{9037} = (12, 12, 1, 1) lies on line \ell_3
                                                                     251: P_{12352} = (63, 63, 1, 1) lies on line \ell_3
201: P_{9102} = (13, 13, 1, 1) lies on line \ell_3
202: P_{9167} = (14, 14, 1, 1) lies on line \ell_3
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The single points on the surface are:

### Points on surface but on no line

The surface has 3905 points not on any line: Too many to print.

# Line Intersection Graph

 $\begin{array}{c|c} 0123\\ \hline 00111\\ 11001\\ 21000\\ 31100\\ \end{array}$ 

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$
in point	$P_0$	$P_1$	$P_5$

 ${\bf Line~1~intersects}$ 

Line	$\ell_0$	$\ell_3$
in point	$P_0$	$P_{8258}$

Line 2 intersects

Line	$\ell_0$
in point	$P_1$

Line 3 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_5$	$P_{8258}$

The surface has 4161 points:

Too many to print.