Rank-67115 over GF(2)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_1^2X_3 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 67115

General information

Number of lines	6
Number of points	9
Number of singular points	3
Number of Eckardt points	0
Number of double points	9
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^6
Type of lines on points	2^9

Singular Points

The surface has 3 singular points:

$0:\ P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$
$1:\ P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
$2:\ P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \mathbf{Pl}(1,0,1,0,0,1)_{23}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \mathbf{Pl}(1,0,0,0,1,0)_{10}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0,1,0,0,0,1)_{21}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1,1,1,0,1,1)_{30}$$

Rank of lines: (0, 8, 28, 2, 31, 10)
Rank of points on Klein quadric: (0, 23, 19, 10, 21, 30)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_0 = (1,0,0,0) = \ell_0 \cap \ell_3$
$P_7 = (0,1,1,0) = \ell_1 \cap \ell_2$
$P_6 = (1,0,1,0) = \ell_1 \cap \ell_5$
$P_2 = (0,0,1,0) = \ell_2 \cap \ell_4$
$P_{10} = (0,1,0,1) = \ell_3 \cap \ell_4$
$P_{11} = (1,1,0,1) = \ell_3 \cap \ell_5$
$P_{14} = (0,1,1,1) = \ell_4 \cap \ell_5$

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5
0	0 1 1 1 0 0
1	1 0 1 0 0 1
2	1 1 0 0 1 0
3	1 0 0 0 1 1
4	0 0 1 1 0 1
5	0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_1	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_5	P_7	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_1	P_7	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_0	P_{10}	P_{11}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_{10}	P_{14}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_6	P_{11}	P_{14}

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (1,0,1,0)$
$5: P_7 = (0,1,1,0)$
$6: P_{10} = (0,1,0,1)$
$7: P_{11} = (1,1,0,1)$
$8: P_{14} = (0,1,1,1)$