

Rank-34 over GF(16)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_1 = 0$$

(1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(16) is 4402

General information

Number of lines	1
Number of points	17
Number of singular points	17
Number of Eckardt points	0
Number of double points	0
Number of single points	17
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17
Type of lines on points	1^{17}

Singular Points

The surface has 17 singular points:

0 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$
1 : $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$
2 : $P_{530} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$
3 : $P_{785} = \mathbf{P}(0, 0, \delta, 1) = \mathbf{P}(0, 0, 2, 1)$
4 : $P_{1041} = \mathbf{P}(0, 0, \delta^{12}, 1) = \mathbf{P}(0, 0, 3, 1)$
5 : $P_{1297} = \mathbf{P}(0, 0, \delta^2, 1) = \mathbf{P}(0, 0, 4, 1)$
6 : $P_{1553} = \mathbf{P}(0, 0, \delta^9, 1) = \mathbf{P}(0, 0, 5, 1)$
7 : $P_{1809} = \mathbf{P}(0, 0, \delta^{13}, 1) = \mathbf{P}(0, 0, 6, 1)$
8 : $P_{2065} = \mathbf{P}(0, 0, \delta^7, 1) = \mathbf{P}(0, 0, 7, 1)$

9 : $P_{2321} = \mathbf{P}(0, 0, \delta^3, 1) = \mathbf{P}(0, 0, 8, 1)$
10 : $P_{2577} = \mathbf{P}(0, 0, \delta^4, 1) = \mathbf{P}(0, 0, 9, 1)$
11 : $P_{2833} = \mathbf{P}(0, 0, \delta^{10}, 1) = \mathbf{P}(0, 0, 10, 1)$
12 : $P_{3089} = \mathbf{P}(0, 0, \delta^5, 1) = \mathbf{P}(0, 0, 11, 1)$
13 : $P_{3345} = \mathbf{P}(0, 0, \delta^{14}, 1) = \mathbf{P}(0, 0, 12, 1)$
14 : $P_{3601} = \mathbf{P}(0, 0, \delta^{11}, 1) = \mathbf{P}(0, 0, 13, 1)$
15 : $P_{3857} = \mathbf{P}(0, 0, \delta^8, 1) = \mathbf{P}(0, 0, 14, 1)$
16 : $P_{4113} = \mathbf{P}(0, 0, \delta^6, 1) = \mathbf{P}(0, 0, 15, 1)$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{70160} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (70160)

Rank of points on Klein quadric: (1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 17 single points:

The single points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_0

1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0

2 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_0

3 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_0

4 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_0

5 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_0

6 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_0

7 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_0

8 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_0

9 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_0

10 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_0

11 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_0

12 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_0

13 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_0

14 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_0

15 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_0

16 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 17 points:
The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$	6 : $P_{1553} = (0, 0, 5, 1)$	12 : $P_{3089} = (0, 0, 11, 1)$
1 : $P_3 = (0, 0, 0, 1)$	7 : $P_{1809} = (0, 0, 6, 1)$	13 : $P_{3345} = (0, 0, 12, 1)$
2 : $P_{530} = (0, 0, 1, 1)$	8 : $P_{2065} = (0, 0, 7, 1)$	14 : $P_{3601} = (0, 0, 13, 1)$
3 : $P_{785} = (0, 0, 2, 1)$	9 : $P_{2321} = (0, 0, 8, 1)$	15 : $P_{3857} = (0, 0, 14, 1)$
4 : $P_{1041} = (0, 0, 3, 1)$	10 : $P_{2577} = (0, 0, 9, 1)$	16 : $P_{4113} = (0, 0, 15, 1)$
5 : $P_{1297} = (0, 0, 4, 1)$	11 : $P_{2833} = (0, 0, 10, 1)$	