

# Rank-67117 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

( 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is -1859840939

## General information

Number of lines	21
Number of points	113
Number of singular points	1
Number of Eckardt points	3
Number of double points	66
Number of single points	42
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^{21}$
Type of lines on points	$6, 3^3, 2^{66}, 1^{42}, 0$

## Singular Points

The surface has 1 singular points:

$$0 : P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

## The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \gamma^6 \end{bmatrix}_{48} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}_{48} = \mathbf{Pl}(2, 0, 0, 0, 1, 0)_{83}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \gamma^5 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{24} = \mathbf{Pl}(4, 0, 0, 0, 1, 0)_{85} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \gamma^3 \end{bmatrix}_{40} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}_{40} = \mathbf{Pl}(7, 0, 0, 0, 1, 0)_{88} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4726} = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4726} = \mathbf{Pl}(0, 6, 0, 0, 0, 1)_{662} \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4699} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4699} = \mathbf{Pl}(0, 3, 0, 0, 0, 1)_{659} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4717} = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4717} = \mathbf{Pl}(0, 5, 0, 0, 0, 1)_{661} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & \gamma^6 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 6 \end{bmatrix}_{122} = \mathbf{Pl}(2, 6, 2, 0, 6, 1)_{3752} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & \gamma^5 \end{bmatrix}_{98} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{98} = \mathbf{Pl}(4, 3, 4, 0, 3, 1)_{2256} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & \gamma^3 \end{bmatrix}_{114} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 5 \end{bmatrix}_{114} = \mathbf{Pl}(7, 5, 7, 0, 5, 1)_{3288} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma \\ 0 & 1 & \gamma^6 & \gamma^5 \end{bmatrix}_{1709} = \begin{bmatrix} 1 & 0 & 7 & 2 \\ 0 & 1 & 6 & 3 \end{bmatrix}_{1709} = \mathbf{Pl}(2, 6, 5, 7, 2, 1)_{2107} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^2 \\ 0 & 1 & \gamma^4 & \gamma^6 \end{bmatrix}_{2537} = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 7 & 6 \end{bmatrix}_{2537} = \mathbf{Pl}(7, 5, 6, 2, 3, 1)_{2679} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^2 \\ 0 & 1 & \gamma & \gamma^3 \end{bmatrix}_{2597} = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 5 \end{bmatrix}_{2597} = \mathbf{Pl}(2, 6, 5, 7, 3, 1)_{2618} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^4 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{4480} = \begin{bmatrix} 1 & 0 & 5 & 7 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{4480} = \mathbf{Pl}(7, 5, 6, 2, 4, 1)_{3155} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^4 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{bmatrix}_{4505} = \begin{bmatrix} 1 & 0 & 5 & 7 \\ 0 & 1 & 4 & 6 \end{bmatrix}_{4505} = \mathbf{Pl}(4, 3, 6, 2, 5, 1)_{3691} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma \\ 0 & 1 & \gamma^3 & \gamma^3 \end{bmatrix}_{1651} = \begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{1651} = \mathbf{Pl}(2, 6, 3, 4, 7, 1)_{4550} \\
\ell_{15} &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^2 \\ 0 & 1 & \gamma^5 & \gamma^3 \end{bmatrix}_{2525} = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 5 \end{bmatrix}_{2525} = \mathbf{Pl}(4, 3, 6, 2, 4, 1)_{3173} \\
\ell_{16} &= \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^4 \\ 0 & 1 & \gamma & \gamma^5 \end{bmatrix}_{4406} = \begin{bmatrix} 1 & 0 & 4 & 7 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{4406} = \mathbf{Pl}(2, 6, 3, 4, 5, 1)_{3535} \\
\ell_{17} &= \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^4 \\ 0 & 1 & \gamma^3 & \gamma^6 \end{bmatrix}_{4433} = \begin{bmatrix} 1 & 0 & 4 & 7 \\ 0 & 1 & 5 & 6 \end{bmatrix}_{4433} = \mathbf{Pl}(7, 5, 3, 4, 7, 1)_{4569} \\
\ell_{18} &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma \\ 0 & 1 & \gamma^2 & \gamma^3 \end{bmatrix}_{1723} = \begin{bmatrix} 1 & 0 & 7 & 2 \\ 0 & 1 & 4 & 5 \end{bmatrix}_{1723} = \mathbf{Pl}(4, 3, 5, 7, 6, 1)_{4139} \\
\ell_{19} &= \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma \\ 0 & 1 & \gamma^4 & \gamma^5 \end{bmatrix}_{1637} = \begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & 7 & 3 \end{bmatrix}_{1637} = \mathbf{Pl}(7, 5, 3, 4, 6, 1)_{4037} \\
\ell_{20} &= \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^2 \\ 0 & 1 & \gamma^6 & \gamma^6 \end{bmatrix}_{2609} = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 6 & 6 \end{bmatrix}_{2609} = \mathbf{Pl}(4, 3, 5, 7, 2, 1)_{2137}
\end{aligned}$$

Rank of lines: ( 48, 24, 40, 4726, 4699, 4717, 122, 98, 114, 1709, 2537, 2597, 4480, 4505, 1651, 2525, 4406, 4433, 1723, 1637, 2609 )

Rank of points on Klein quadric: ( 83, 85, 88, 662, 659, 661, 3752, 2256, 3288, 2107, 2679, 2618, 3155, 3691, 4550, 3173, 3535, 4569, 4139, 4037, 2137 )

### Eckardt Points

The surface has 3 Eckardt points:

$$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0),$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0),$$

$$2 : P_{12} = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0).$$

### Double Points

The surface has 66 Double points:

The double points on the surface are:

$$\begin{aligned} P_{90} &= (0, 2, 0, 1) = \ell_0 \cap \ell_3 \\ P_{92} &= (2, 2, 0, 1) = \ell_0 \cap \ell_6 \\ P_{95} &= (5, 2, 0, 1) = \ell_0 \cap \ell_9 \\ P_{97} &= (7, 2, 0, 1) = \ell_0 \cap \ell_{11} \\ P_{93} &= (3, 2, 0, 1) = \ell_0 \cap \ell_{14} \\ P_{91} &= (1, 2, 0, 1) = \ell_0 \cap \ell_{16} \\ P_{106} &= (0, 4, 0, 1) = \ell_1 \cap \ell_4 \\ P_{110} &= (4, 4, 0, 1) = \ell_1 \cap \ell_7 \\ P_{108} &= (2, 4, 0, 1) = \ell_1 \cap \ell_{13} \\ P_{112} &= (6, 4, 0, 1) = \ell_1 \cap \ell_{15} \\ P_{107} &= (1, 4, 0, 1) = \ell_1 \cap \ell_{18} \\ P_{111} &= (5, 4, 0, 1) = \ell_1 \cap \ell_{20} \\ P_{130} &= (0, 7, 0, 1) = \ell_2 \cap \ell_5 \\ P_{137} &= (7, 7, 0, 1) = \ell_2 \cap \ell_8 \\ P_{131} &= (1, 7, 0, 1) = \ell_2 \cap \ell_{10} \\ P_{136} &= (6, 7, 0, 1) = \ell_2 \cap \ell_{12} \\ P_{133} &= (3, 7, 0, 1) = \ell_2 \cap \ell_{17} \\ P_{134} &= (4, 7, 0, 1) = \ell_2 \cap \ell_{19} \\ P_{217} &= (0, 2, 2, 1) = \ell_3 \cap \ell_6 \\ P_{281} &= (0, 2, 3, 1) = \ell_3 \cap \ell_{10} \\ P_{409} &= (0, 2, 5, 1) = \ell_3 \cap \ell_{13} \\ P_{537} &= (0, 2, 7, 1) = \ell_3 \cap \ell_{17} \\ P_{153} &= (0, 2, 1, 1) = \ell_3 \cap \ell_{20} \\ P_{361} &= (0, 4, 4, 1) = \ell_4 \cap \ell_7 \\ P_{233} &= (0, 4, 2, 1) = \ell_4 \cap \ell_9 \\ P_{169} &= (0, 4, 1, 1) = \ell_4 \cap \ell_{12} \\ P_{425} &= (0, 4, 5, 1) = \ell_4 \cap \ell_{16} \\ P_{489} &= (0, 4, 6, 1) = \ell_4 \cap \ell_{19} \\ P_{577} &= (0, 7, 7, 1) = \ell_5 \cap \ell_8 \\ P_{321} &= (0, 7, 3, 1) = \ell_5 \cap \ell_{11} \\ P_{193} &= (0, 7, 1, 1) = \ell_5 \cap \ell_{14} \\ P_{385} &= (0, 7, 4, 1) = \ell_5 \cap \ell_{15} \\ P_{513} &= (0, 7, 6, 1) = \ell_5 \cap \ell_{18} \\ P_{282} &= (1, 2, 3, 1) = \ell_6 \cap \ell_{12} \end{aligned}$$

$$\begin{aligned} P_{416} &= (7, 2, 5, 1) = \ell_6 \cap \ell_{15} \\ P_{156} &= (3, 2, 1, 1) = \ell_6 \cap \ell_{18} \\ P_{542} &= (5, 2, 7, 1) = \ell_6 \cap \ell_{19} \\ P_{174} &= (5, 4, 1, 1) = \ell_7 \cap \ell_{10} \\ P_{239} &= (6, 4, 2, 1) = \ell_7 \cap \ell_{11} \\ P_{426} &= (1, 4, 5, 1) = \ell_7 \cap \ell_{14} \\ P_{491} &= (2, 4, 6, 1) = \ell_7 \cap \ell_{17} \\ P_{325} &= (4, 7, 3, 1) = \ell_8 \cap \ell_9 \\ P_{388} &= (3, 7, 4, 1) = \ell_8 \cap \ell_{13} \\ P_{199} &= (6, 7, 1, 1) = \ell_8 \cap \ell_{16} \\ P_{514} &= (1, 7, 6, 1) = \ell_8 \cap \ell_{20} \\ P_{39} &= (4, 3, 1, 0) = \ell_9 \cap \ell_{10} \\ P_{399} &= (6, 0, 5, 1) = \ell_9 \cap \ell_{18} \\ P_{435} &= (2, 5, 5, 1) = \ell_{10} \cap \ell_{12} \\ P_{552} &= (7, 3, 7, 1) = \ell_{10} \cap \ell_{14} \\ P_{460} &= (3, 0, 6, 1) = \ell_{10} \cap \ell_{15} \\ P_{383} &= (6, 6, 4, 1) = \ell_{10} \cap \ell_{20} \\ P_{41} &= (6, 3, 1, 0) = \ell_{11} \cap \ell_{12} \\ P_{396} &= (3, 0, 5, 1) = \ell_{11} \cap \ell_{20} \\ P_{462} &= (5, 0, 6, 1) = \ell_{12} \cap \ell_{13} \\ P_{548} &= (3, 3, 7, 1) = \ell_{12} \cap \ell_{16} \\ P_{381} &= (4, 6, 4, 1) = \ell_{12} \cap \ell_{18} \\ P_{54} &= (3, 5, 1, 0) = \ell_{13} \cap \ell_{14} \\ P_{509} &= (4, 6, 6, 1) = \ell_{14} \cap \ell_{16} \\ P_{246} &= (5, 5, 2, 1) = \ell_{14} \cap \ell_{18} \\ P_{271} &= (6, 0, 3, 1) = \ell_{14} \cap \ell_{19} \\ P_{58} &= (7, 5, 1, 0) = \ell_{15} \cap \ell_{16} \\ P_{270} &= (5, 0, 3, 1) = \ell_{16} \cap \ell_{17} \\ P_{243} &= (2, 5, 2, 1) = \ell_{16} \cap \ell_{20} \\ P_{61} &= (2, 6, 1, 0) = \ell_{17} \cap \ell_{18} \\ P_{296} &= (7, 3, 3, 1) = \ell_{18} \cap \ell_{20} \\ P_{64} &= (5, 6, 1, 0) = \ell_{19} \cap \ell_{20} \end{aligned}$$

### Single Points

The surface has 42 single points:

The single points on the surface are:

- |  |  |
|--|--|
| 0 : $P_{94} = (4, 2, 0, 1)$ lies on line $\ell_0$      | 22 : $P_{355} = (2, 3, 4, 1)$ lies on line $\ell_9$    |
| 1 : $P_{96} = (6, 2, 0, 1)$ lies on line $\ell_0$      | 23 : $P_{358} = (5, 3, 4, 1)$ lies on line $\ell_{11}$ |
| 2 : $P_{109} = (3, 4, 0, 1)$ lies on line $\ell_1$     | 24 : $P_{375} = (6, 5, 4, 1)$ lies on line $\ell_{17}$ |
| 3 : $P_{113} = (7, 4, 0, 1)$ lies on line $\ell_1$     | 25 : $P_{376} = (7, 5, 4, 1)$ lies on line $\ell_{19}$ |
| 4 : $P_{132} = (2, 7, 0, 1)$ lies on line $\ell_2$     | 26 : $P_{419} = (2, 3, 5, 1)$ lies on line $\ell_{19}$ |
| 5 : $P_{135} = (5, 7, 0, 1)$ lies on line $\ell_2$     | 27 : $P_{421} = (4, 3, 5, 1)$ lies on line $\ell_{17}$ |
| 6 : $P_{213} = (4, 1, 2, 1)$ lies on line $\ell_{10}$  | 28 : $P_{449} = (0, 7, 5, 1)$ lies on line $\ell_5$    |
| 7 : $P_{216} = (7, 1, 2, 1)$ lies on line $\ell_{12}$  | 29 : $P_{451} = (2, 7, 5, 1)$ lies on line $\ell_8$    |
| 8 : $P_{229} = (4, 3, 2, 1)$ lies on line $\ell_{13}$  | 30 : $P_{473} = (0, 2, 6, 1)$ lies on line $\ell_3$    |
| 9 : $P_{230} = (5, 3, 2, 1)$ lies on line $\ell_{15}$  | 31 : $P_{477} = (4, 2, 6, 1)$ lies on line $\ell_6$    |
| 10 : $P_{252} = (3, 6, 2, 1)$ lies on line $\ell_{19}$ | 32 : $P_{501} = (4, 5, 6, 1)$ lies on line $\ell_{11}$ |
| 11 : $P_{256} = (7, 6, 2, 1)$ lies on line $\ell_{17}$ | 33 : $P_{504} = (7, 5, 6, 1)$ lies on line $\ell_9$    |
| 12 : $P_{257} = (0, 7, 2, 1)$ lies on line $\ell_5$    | 34 : $P_{531} = (2, 1, 7, 1)$ lies on line $\ell_{18}$ |
| 13 : $P_{262} = (5, 7, 2, 1)$ lies on line $\ell_8$    | 35 : $P_{533} = (4, 1, 7, 1)$ lies on line $\ell_{20}$ |
| 14 : $P_{297} = (0, 4, 3, 1)$ lies on line $\ell_4$    | 36 : $P_{553} = (0, 4, 7, 1)$ lies on line $\ell_4$    |
| 15 : $P_{304} = (7, 4, 3, 1)$ lies on line $\ell_7$    | 37 : $P_{556} = (3, 4, 7, 1)$ lies on line $\ell_7$    |
| 16 : $P_{315} = (2, 6, 3, 1)$ lies on line $\ell_{15}$ | 38 : $P_{565} = (4, 5, 7, 1)$ lies on line $\ell_{15}$ |
| 17 : $P_{320} = (7, 6, 3, 1)$ lies on line $\ell_{13}$ | 39 : $P_{567} = (6, 5, 7, 1)$ lies on line $\ell_{13}$ |
| 18 : $P_{339} = (2, 1, 4, 1)$ lies on line $\ell_{14}$ | 40 : $P_{571} = (2, 6, 7, 1)$ lies on line $\ell_{11}$ |
| 19 : $P_{344} = (7, 1, 4, 1)$ lies on line $\ell_{16}$ | 41 : $P_{572} = (3, 6, 7, 1)$ lies on line $\ell_9$    |
| 20 : $P_{345} = (0, 2, 4, 1)$ lies on line $\ell_3$    |  |
| 21 : $P_{351} = (6, 2, 4, 1)$ lies on line $\ell_6$    |  |

The single points on the surface are:

#### Points on surface but on no line

The surface has 1 points not on any line:

The points on the surface but not on lines are:

- 0 :  $P_{20} = (1, 1, 1, 0)$

## Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	1	1	0	0	1	0	0	1	0	1	0	0	1	0	1	0	0	0	0
1	1	0	1	0	1	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	1
2	1	1	0	0	0	1	0	0	1	0	1	0	0	0	0	0	1	0	1	0	0
3	1	0	0	0	1	1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1
4	0	1	0	1	0	1	0	1	0	0	1	0	0	0	1	0	0	1	0	1	0
5	0	0	1	1	1	0	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0
6	1	0	0	1	0	0	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0
7	0	1	0	0	1	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0
8	0	0	1	0	0	1	1	1	0	1	0	0	0	1	0	0	1	0	0	0	1
9	1	0	0	0	1	0	0	0	1	1	0	1	0	1	0	1	0	1	1	1	0
10	0	0	1	1	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	1	1
11	1	0	0	0	0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	1	1
12	0	0	1	0	1	0	1	0	0	1	1	0	1	0	0	1	0	1	0	0	0
13	0	1	0	1	0	0	0	0	1	1	0	1	1	0	1	1	0	1	0	1	0
14	1	0	0	0	0	1	0	1	0	0	1	0	0	1	0	0	1	1	1	0	0
15	0	1	0	0	0	1	1	0	0	1	1	0	1	0	0	1	1	0	1	0	0
16	1	0	0	0	1	0	0	0	1	0	1	1	0	1	1	0	1	0	0	1	1
17	0	0	1	1	0	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0
18	0	1	0	0	0	1	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1
19	0	0	1	0	1	0	1	0	0	1	0	1	1	1	1	0	1	0	0	0	1
20	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	1	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_6$	$\ell_9$	$\ell_{11}$	$\ell_{14}$	$\ell_{16}$
in point	$P_0$	$P_0$	$P_{90}$	$P_{92}$	$P_{95}$	$P_{97}$	$P_{93}$	$P_{91}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_7$	$\ell_{13}$	$\ell_{15}$	$\ell_{18}$	$\ell_{20}$
in point	$P_0$	$P_0$	$P_{106}$	$P_{110}$	$P_{108}$	$P_{112}$	$P_{107}$	$P_{111}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_5$	$\ell_8$	$\ell_{10}$	$\ell_{12}$	$\ell_{17}$	$\ell_{19}$
in point	$P_0$	$P_0$	$P_{130}$	$P_{137}$	$P_{131}$	$P_{136}$	$P_{133}$	$P_{134}$

Line 3 intersects

Line	$\ell_0$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{13}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{90}$	$P_2$	$P_2$	$P_{217}$	$P_{281}$	$P_{409}$	$P_{537}$	$P_{153}$

Line 4 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{12}$	$\ell_{16}$	$\ell_{19}$
in point	$P_{106}$	$P_2$	$P_2$	$P_{361}$	$P_{233}$	$P_{169}$	$P_{425}$	$P_{489}$

Line 5 intersects

Line	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_8$	$\ell_{11}$	$\ell_{14}$	$\ell_{15}$	$\ell_{18}$
in point	$P_{130}$	$P_2$	$P_2$	$P_{577}$	$P_{321}$	$P_{193}$	$P_{385}$	$P_{513}$

Line 6 intersects

Line	$\ell_0$	$\ell_3$	$\ell_7$	$\ell_8$	$\ell_{12}$	$\ell_{15}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{92}$	$P_{217}$	$P_{12}$	$P_{12}$	$P_{282}$	$P_{416}$	$P_{156}$	$P_{542}$

Line 7 intersects

Line	$\ell_1$	$\ell_4$	$\ell_6$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$	$\ell_{17}$
in point	$P_{110}$	$P_{361}$	$P_{12}$	$P_{12}$	$P_{174}$	$P_{239}$	$P_{426}$	$P_{491}$

Line 8 intersects

Line	$\ell_2$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_9$	$\ell_{13}$	$\ell_{16}$	$\ell_{20}$
in point	$P_{137}$	$P_{577}$	$P_{12}$	$P_{12}$	$P_{325}$	$P_{388}$	$P_{199}$	$P_{514}$

Line 9 intersects

Line	$\ell_0$	$\ell_4$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{15}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{95}$	$P_{233}$	$P_{325}$	$P_{39}$	$P_4$	$P_4$	$P_4$	$P_4$	$P_{399}$	$P_4$

Line 10 intersects

Line	$\ell_2$	$\ell_3$	$\ell_7$	$\ell_9$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{20}$
in point	$P_{131}$	$P_{281}$	$P_{174}$	$P_{39}$	$P_{435}$	$P_{552}$	$P_{460}$	$P_{383}$

Line 11 intersects

Line	$\ell_0$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{97}$	$P_{321}$	$P_{239}$	$P_4$	$P_{41}$	$P_4$	$P_4$	$P_4$	$P_4$	$P_{396}$

Line 12 intersects

Line	$\ell_2$	$\ell_4$	$\ell_6$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{16}$	$\ell_{18}$
in point	$P_{136}$	$P_{169}$	$P_{282}$	$P_{435}$	$P_{41}$	$P_{462}$	$P_{548}$	$P_{381}$

Line 13 intersects

Line	$\ell_1$	$\ell_3$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{17}$	$\ell_{19}$
in point	$P_{108}$	$P_{409}$	$P_{388}$	$P_4$	$P_4$	$P_{462}$	$P_{54}$	$P_4$	$P_4$	$P_4$

Line 14 intersects

Line	$\ell_0$	$\ell_5$	$\ell_7$	$\ell_{10}$	$\ell_{13}$	$\ell_{16}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{93}$	$P_{193}$	$P_{426}$	$P_{552}$	$P_{54}$	$P_{509}$	$P_{246}$	$P_{271}$

Line 15 intersects

Line	$\ell_1$	$\ell_5$	$\ell_6$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{16}$	$\ell_{17}$	$\ell_{19}$
in point	$P_{112}$	$P_{385}$	$P_{416}$	$P_4$	$P_{460}$	$P_4$	$P_4$	$P_{58}$	$P_4$	$P_4$

Line 16 intersects

Line	$\ell_0$	$\ell_4$	$\ell_8$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{91}$	$P_{425}$	$P_{199}$	$P_{548}$	$P_{509}$	$P_{58}$	$P_{270}$	$P_{243}$

Line 17 intersects

Line	$\ell_2$	$\ell_3$	$\ell_7$	$\ell_9$	$\ell_{11}$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{133}$	$P_{537}$	$P_{491}$	$P_4$	$P_4$	$P_4$	$P_4$	$P_{270}$	$P_{61}$	$P_4$

Line 18 intersects

Line	$\ell_1$	$\ell_5$	$\ell_6$	$\ell_9$	$\ell_{12}$	$\ell_{14}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{107}$	$P_{513}$	$P_{156}$	$P_{399}$	$P_{381}$	$P_{246}$	$P_{61}$	$P_{296}$

Line 19 intersects

Line	$\ell_2$	$\ell_4$	$\ell_6$	$\ell_9$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{134}$	$P_{489}$	$P_{542}$	$P_4$	$P_4$	$P_4$	$P_{271}$	$P_4$	$P_4$	$P_{64}$

Line 20 intersects

Line	$\ell_1$	$\ell_3$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{16}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{111}$	$P_{153}$	$P_{514}$	$P_{383}$	$P_{396}$	$P_{243}$	$P_{296}$	$P_{64}$

The surface has 113 points:

The points on the surface are:

$$\begin{aligned} 0 : P_0 &= (1, 0, 0, 0) \\ 1 : P_2 &= (0, 0, 1, 0) \\ 2 : P_4 &= (1, 1, 1, 1) \end{aligned}$$

$$\begin{aligned} 3 : P_{12} &= (1, 0, 1, 0) \\ 4 : P_{20} &= (1, 1, 1, 0) \\ 5 : P_{39} &= (4, 3, 1, 0) \end{aligned}$$

$$\begin{aligned} 6 : P_{41} &= (6, 3, 1, 0) \\ 7 : P_{54} &= (3, 5, 1, 0) \\ 8 : P_{58} &= (7, 5, 1, 0) \end{aligned}$$

9 : $P_{61} = (2, 6, 1, 0)$	44 : $P_{229} = (4, 3, 2, 1)$	79 : $P_{399} = (6, 0, 5, 1)$
10 : $P_{64} = (5, 6, 1, 0)$	45 : $P_{230} = (5, 3, 2, 1)$	80 : $P_{409} = (0, 2, 5, 1)$
11 : $P_{90} = (0, 2, 0, 1)$	46 : $P_{233} = (0, 4, 2, 1)$	81 : $P_{416} = (7, 2, 5, 1)$
12 : $P_{91} = (1, 2, 0, 1)$	47 : $P_{239} = (6, 4, 2, 1)$	82 : $P_{419} = (2, 3, 5, 1)$
13 : $P_{92} = (2, 2, 0, 1)$	48 : $P_{243} = (2, 5, 2, 1)$	83 : $P_{421} = (4, 3, 5, 1)$
14 : $P_{93} = (3, 2, 0, 1)$	49 : $P_{246} = (5, 5, 2, 1)$	84 : $P_{425} = (0, 4, 5, 1)$
15 : $P_{94} = (4, 2, 0, 1)$	50 : $P_{252} = (3, 6, 2, 1)$	85 : $P_{426} = (1, 4, 5, 1)$
16 : $P_{95} = (5, 2, 0, 1)$	51 : $P_{256} = (7, 6, 2, 1)$	86 : $P_{435} = (2, 5, 5, 1)$
17 : $P_{96} = (6, 2, 0, 1)$	52 : $P_{257} = (0, 7, 2, 1)$	87 : $P_{449} = (0, 7, 5, 1)$
18 : $P_{97} = (7, 2, 0, 1)$	53 : $P_{262} = (5, 7, 2, 1)$	88 : $P_{451} = (2, 7, 5, 1)$
19 : $P_{106} = (0, 4, 0, 1)$	54 : $P_{270} = (5, 0, 3, 1)$	89 : $P_{460} = (3, 0, 6, 1)$
20 : $P_{107} = (1, 4, 0, 1)$	55 : $P_{271} = (6, 0, 3, 1)$	90 : $P_{462} = (5, 0, 6, 1)$
21 : $P_{108} = (2, 4, 0, 1)$	56 : $P_{281} = (0, 2, 3, 1)$	91 : $P_{473} = (0, 2, 6, 1)$
22 : $P_{109} = (3, 4, 0, 1)$	57 : $P_{282} = (1, 2, 3, 1)$	92 : $P_{477} = (4, 2, 6, 1)$
23 : $P_{110} = (4, 4, 0, 1)$	58 : $P_{296} = (7, 3, 3, 1)$	93 : $P_{489} = (0, 4, 6, 1)$
24 : $P_{111} = (5, 4, 0, 1)$	59 : $P_{297} = (0, 4, 3, 1)$	94 : $P_{491} = (2, 4, 6, 1)$
25 : $P_{112} = (6, 4, 0, 1)$	60 : $P_{304} = (7, 4, 3, 1)$	95 : $P_{501} = (4, 5, 6, 1)$
26 : $P_{113} = (7, 4, 0, 1)$	61 : $P_{315} = (2, 6, 3, 1)$	96 : $P_{504} = (7, 5, 6, 1)$
27 : $P_{130} = (0, 7, 0, 1)$	62 : $P_{320} = (7, 6, 3, 1)$	97 : $P_{509} = (4, 6, 6, 1)$
28 : $P_{131} = (1, 7, 0, 1)$	63 : $P_{321} = (0, 7, 3, 1)$	98 : $P_{513} = (0, 7, 6, 1)$
29 : $P_{132} = (2, 7, 0, 1)$	64 : $P_{325} = (4, 7, 3, 1)$	99 : $P_{514} = (1, 7, 6, 1)$
30 : $P_{133} = (3, 7, 0, 1)$	65 : $P_{339} = (2, 1, 4, 1)$	100 : $P_{531} = (2, 1, 7, 1)$
31 : $P_{134} = (4, 7, 0, 1)$	66 : $P_{344} = (7, 1, 4, 1)$	101 : $P_{533} = (4, 1, 7, 1)$
32 : $P_{135} = (5, 7, 0, 1)$	67 : $P_{345} = (0, 2, 4, 1)$	102 : $P_{537} = (0, 2, 7, 1)$
33 : $P_{136} = (6, 7, 0, 1)$	68 : $P_{351} = (6, 2, 4, 1)$	103 : $P_{542} = (5, 2, 7, 1)$
34 : $P_{137} = (7, 7, 0, 1)$	69 : $P_{355} = (2, 3, 4, 1)$	104 : $P_{548} = (3, 3, 7, 1)$
35 : $P_{153} = (0, 2, 1, 1)$	70 : $P_{358} = (5, 3, 4, 1)$	105 : $P_{552} = (7, 3, 7, 1)$
36 : $P_{156} = (3, 2, 1, 1)$	71 : $P_{361} = (0, 4, 4, 1)$	106 : $P_{553} = (0, 4, 7, 1)$
37 : $P_{169} = (0, 4, 1, 1)$	72 : $P_{375} = (6, 5, 4, 1)$	107 : $P_{556} = (3, 4, 7, 1)$
38 : $P_{174} = (5, 4, 1, 1)$	73 : $P_{376} = (7, 5, 4, 1)$	108 : $P_{565} = (4, 5, 7, 1)$
39 : $P_{193} = (0, 7, 1, 1)$	74 : $P_{381} = (4, 6, 4, 1)$	109 : $P_{567} = (6, 5, 7, 1)$
40 : $P_{199} = (6, 7, 1, 1)$	75 : $P_{383} = (6, 6, 4, 1)$	110 : $P_{571} = (2, 6, 7, 1)$
41 : $P_{213} = (4, 1, 2, 1)$	76 : $P_{385} = (0, 7, 4, 1)$	111 : $P_{572} = (3, 6, 7, 1)$
42 : $P_{216} = (7, 1, 2, 1)$	77 : $P_{388} = (3, 7, 4, 1)$	112 : $P_{577} = (0, 7, 7, 1)$
43 : $P_{217} = (0, 2, 2, 1)$	78 : $P_{396} = (3, 0, 5, 1)$	