

Rank-76323 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 76323

General information

Number of lines	2
Number of points	9
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	6
Number of points off lines	3
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^2
Type of lines on points	$1^6, 0^3$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1\end{aligned}$$

Rank of lines: (0, 34)

Rank of points on Klein quadric: (0, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 6 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

2 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1

3 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1

4 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0

5 : $P_{12} = (0, 0, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 3 points not on any line:

The points on the surface but not on lines are:

0 : $P_{11} = (1, 1, 0, 1)$

1 : $P_{13} = (1, 0, 1, 1)$

2 : $P_{14} = (0, 1, 1, 1)$

Line Intersection Graph

	0	1
0	0	0
1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 9 points:

The points on the surface are:

$$\begin{aligned}
0 : P_0 &= (1, 0, 0, 0) \\
1 : P_1 &= (0, 1, 0, 0) \\
2 : P_2 &= (0, 0, 1, 0) \\
3 : P_3 &= (0, 0, 0, 1)
\end{aligned}$$

$$\begin{aligned}
4 : P_5 &= (1, 1, 0, 0) \\
5 : P_{11} &= (1, 1, 0, 1) \\
6 : P_{12} &= (0, 0, 1, 1) \\
7 : P_{13} &= (1, 0, 1, 1)
\end{aligned}$$

$$8 : P_{14} = (0, 1, 1, 1)$$