

Rank-76099 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 76099

General information

Number of lines	9
Number of points	13
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	3
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^9
Type of lines on points	$3^4, 2^6, 1^3$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \mathbf{Pl}(1, 0, 1, 0, 0, 1)_{23} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_6 \\
\ell_5 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4 \\
\ell_7 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{28}
\end{aligned}$$

Rank of lines: (0, 4, 8, 30, 14, 34, 18, 33, 22)

Rank of points on Klein quadric: (0, 2, 23, 5, 6, 1, 4, 7, 28)

Eckardt Points

The surface has 4 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0), \quad T = 2$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0), \quad T = 6$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1), \quad T = 14$$

$$3 : P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0), \quad T = 12$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_6 = (1, 0, 1, 0) = \ell_1 \cap \ell_2$$

$$P_9 = (1, 0, 0, 1) = \ell_4 \cap \ell_6$$

$$P_{11} = (1, 1, 0, 1) = \ell_4 \cap \ell_8$$

$$P_{13} = (1, 0, 1, 1) = \ell_6 \cap \ell_8$$

Single Points

The surface has 3 single points:

The single points on the surface are:

$$0 : P_{10} = (0, 1, 0, 1) \text{ lies on line } \ell_3$$

$$1 : P_{12} = (0, 0, 1, 1) \text{ lies on line } \ell_5$$

$$2 : P_{14} = (0, 1, 1, 1) \text{ lies on line } \ell_7$$

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	0	0	0	0
1	1	0	1	0	0	1	1	0	0
2	1	1	0	0	0	0	0	1	1
3	1	0	0	0	1	1	0	1	0
4	1	0	0	1	0	0	1	0	1
5	0	1	0	1	0	0	1	1	0
6	0	1	0	0	1	1	0	0	1
7	0	0	1	1	0	1	0	0	1
8	0	0	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_0	P_6	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_5	P_6	P_7	P_7

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_1	P_1	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_8
in point	P_1	P_1	P_9	P_{11}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_3	P_2	P_3

Line 6 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_8
in point	P_2	P_9	P_2	P_{13}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_7	P_3	P_3	P_7

Line 8 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_7
in point	P_7	P_{11}	P_{13}	P_7

The surface has 13 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_5 = (1, 1, 0, 0)$

5 : $P_6 = (1, 0, 1, 0)$
6 : $P_7 = (0, 1, 1, 0)$
7 : $P_9 = (1, 0, 0, 1)$
8 : $P_{10} = (0, 1, 0, 1)$
9 : $P_{11} = (1, 1, 0, 1)$

10 : $P_{12} = (0, 0, 1, 1)$
11 : $P_{13} = (1, 0, 1, 1)$
12 : $P_{14} = (0, 1, 1, 1)$