# Rank-74106 over GF(8)

January 15, 2021

# The equation

The equation of the surface is:

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244209814

# General information

Number of lines	27
Number of points	121
Number of singular points	0
Number of Eckardt points	13
Number of double points	96
Number of single points	12
Number of points off lines	0
Number of Hesse planes	0
Number of axes	16
Type of points on lines	$9^{27}$
Type of lines on points	$3^{13}, 2^{96}, 1^{12}$

#### Singular Points

The surface has 0 singular points:

#### The 27 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = a_1 = \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{368} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{368} = \mathbf{Pl}(7, 0, 4, 0, 0, 1)_{716}$$

$$\ell_1 = a_2 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^4 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{4626} = \begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{4626} = \mathbf{Pl}(4, 0, 1, 1, 2, 1)_{1830}$$

$$\begin{split} &\ell_2 = a_3 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^2 \\ 0 & 1 & \gamma^5 & \gamma \end{bmatrix}_{2647} = \begin{bmatrix} 1 & 0 & 4 & 4 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{2647} = \mathbf{PI}(1,1,1,1,6,1)_{3982} \\ &\ell_3 = a_4 = \begin{bmatrix} 0 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \mathbf{PI}(0,6,0,1,0,0)_{30} \\ &\ell_4 = a_5 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^5 & \gamma^2 \end{bmatrix}_{4652} = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4052} = \mathbf{PI}(5,7,3,4,3,1)_{2544} \\ &\ell_5 = a_6 = \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^5 & 1 \end{bmatrix}_{449} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{449} = \mathbf{PI}(1,1,3,0,2,1)_{1742} \\ &\ell_6 = b_1 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^2 \\ 0 & 1 & \gamma^6 & \gamma^6 \end{bmatrix}_{2682} = \begin{bmatrix} 1 & 0 & 4 & 4 \\ 0 & 1 & 6 & 6 \end{bmatrix}_{2682} = \mathbf{PI}(2,0,1,1,7,1)_{4348} \\ &\ell_7 = b_2 = \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^6 & 1 \end{bmatrix}_{379} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 6 & 1 \end{bmatrix}_{379} = \mathbf{PI}(1,1,6,0,7,1)_{4283} \\ &\ell_8 = b_3 = \begin{bmatrix} 1 & 0 & \gamma^7 & \gamma^5 \\ 0 & 1 & \gamma^6 & \gamma^4 \end{bmatrix}_{1960} = \begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4636} = \mathbf{PI}(1,1,1,1,3,1)_{2435} \\ &\ell_{10} = b_5 = \begin{bmatrix} 0 & 1 & \gamma^5 & 0 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{bmatrix}_{4636} = \begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4636} = \mathbf{PI}(1,1,1,1,3,1)_{2435} \\ &\ell_{11} = b_6 = \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{225} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{225} = \mathbf{PI}(4,0,2,0,0,1)_{683} \\ &\ell_{12} = c_{12} = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^3 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{326} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{1359} = \mathbf{PI}(7,0,1,1,4,1)_{2841} \\ &\ell_{13} = c_{13} = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^3 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{3266} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{3266} = \mathbf{PI}(5,7,6,2,7,1)_{4665} \\ &\ell_{15} = c_{15} = \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{326} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{443} = \mathbf{PI}(2,0,7,0,0,1)_{756} \\ &\ell_{17} = c_{23} = \begin{bmatrix} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{362} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{443} = \mathbf{PI}(5,7,7,2,0,1)_{969} \\ &\ell_{18} = c_{24} = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{4042} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{443} = \mathbf{PI}(6,2,3,4,2,1)_{2034} \\ &\ell_{19} = c_{25} = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{4042} = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{4042} = \mathbf{PI}(1,1,5,0,4,1)_{2764} \\ &\ell_{20} = c_{36} = \begin{bmatrix} 1 & 0$$

$$\ell_{23} = c_{36} = \begin{bmatrix} 1 & 0 & \gamma & \gamma^5 \\ 0 & 1 & \gamma^3 & \gamma^3 \end{bmatrix}_{1943} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{1943} = \mathbf{Pl}(3, 4, 5, 7, 4, 1)_{3144}$$

$$\ell_{24} = c_{45} = \begin{bmatrix} 0 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \mathbf{Pl}(0, 5, 0, 1, 0, 0)_{29}$$

$$\ell_{25} = c_{46} = \begin{bmatrix} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{809} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{809} = \mathbf{Pl}(6, 2, 2, 4, 0, 1)_{1033}$$

$$\ell_{26} = c_{56} = \begin{bmatrix} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^5 & 1 \end{bmatrix}_{1033} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{1033} = \mathbf{Pl}(5, 7, 6, 2, 2, 1)_{2173}$$

Rank of lines: ( 368, 4626, 2647, 4734, 4052, 449, 2682, 379, 1960, 4636, 4707, 225, 1359, 816, 3266, 952, 443, 1027, 963, 4042, 232, 1376, 3231, 1943, 4725, 809, 1033 )

Rank of points on Klein quadric: ( 716, 1830, 3982, 30, 2544, 1742, 4348, 4283, 3644, 2435, 27, 683, 2841, 3035, 4665, 1191, 756, 969, 4642, 2034, 2764, 3457, 4166, 3144, 29, 1033, 2173)

#### **Eckardt Points**

The surface has 13 Eckardt points:

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0: E_{45} = a_4 \cap b_5 \cap c_{45} = P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1), \\ 1: E_{46} = a_4 \cap b_6 \cap c_{46} = P_{27} = \mathbf{P}(0,\gamma,1,0) = \mathbf{P}(0,2,1,0), \\ 2: E_{15} = a_1 \cap b_5 \cap c_{15} = P_{43} = \mathbf{P}(0,\gamma^2,1,0) = \mathbf{P}(0,4,1,0), \\ 3: E_{16,23,45} = c_{16} \cap c_{23} \cap c_{45} = P_{67} = \mathbf{P}(0,\gamma^4,1,0) = \mathbf{P}(0,7,1,0), \\ 4: E_{41} = a_4 \cap b_1 \cap c_{14} = P_{153} = \mathbf{P}(0,\gamma,1,1) = \mathbf{P}(0,2,1,1), \\ 5: E_{25} = a_2 \cap b_5 \cap c_{25} = P_{169} = \mathbf{P}(0,\gamma^2,1,1) = \mathbf{P}(0,4,1,1), \\ 6: E_{12,36,45} = c_{12} \cap c_{36} \cap c_{45} = P_{193} = \mathbf{P}(0,\gamma^4,1,1) = \mathbf{P}(0,7,1,1), \\ 7: E_{54} = a_5 \cap b_4 \cap c_{45} = P_{225} = \mathbf{P}(0,\gamma^5,\gamma,1) = \mathbf{P}(0,3,2,1), \\ 8: E_{65} = a_6 \cap b_5 \cap c_{56} = P_{273} = \mathbf{P}(0,1,\gamma^5,1) = \mathbf{P}(0,1,3,1), \\ 9: E_{43} = a_4 \cap b_3 \cap c_{34} = P_{369} = \mathbf{P}(0,\gamma^3,\gamma^2,1) = \mathbf{P}(0,5,4,1), \\ 10: E_{13,26,45} = c_{13} \cap c_{26} \cap c_{45} = P_{401} = \mathbf{P}(0,1,\gamma^3,1) = \mathbf{P}(0,1,5,1), \\ 11: E_{42} = a_4 \cap b_2 \cap c_{24} = P_{465} = \mathbf{P}(0,1,\gamma^6,1) = \mathbf{P}(0,1,6,1), \\ 12: E_{35} = a_3 \cap b_5 \cap c_{35} = P_{569} = \mathbf{P}(0,\gamma^6,\gamma^4,1) = \mathbf{P}(0,6,7,1).
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#### **Double Points**

The surface has 96 Double points: The double points on the surface are:

$$\begin{split} P_{18} &= (7,0,1,0) = \ell_0 \cap \ell_7 = a_1 \cap b_2 \\ P_{55} &= (4,5,1,0) = \ell_0 \cap \ell_8 = a_1 \cap b_3 \\ P_{22} &= (3,1,1,0) = \ell_0 \cap \ell_9 = a_1 \cap b_4 \\ P_{64} &= (5,6,1,0) = \ell_0 \cap \ell_{11} = a_1 \cap b_6 \\ P_8 &= (4,1,0,0) = \ell_0 \cap \ell_{12} = a_1 \cap c_{12} \\ P_{29} &= (2,2,1,0) = \ell_0 \cap \ell_{13} = a_1 \cap c_{13} \\ P_{68} &= (1,7,1,0) = \ell_0 \cap \ell_{14} = a_1 \cap c_{14} \\ P_{41} &= (6,3,1,0) = \ell_0 \cap \ell_{16} = a_1 \cap c_{16} \\ P_{179} &= (2,5,1,1) = \ell_1 \cap \ell_6 = a_2 \cap b_1 \\ P_{164} &= (3,3,1,1) = \ell_1 \cap \ell_8 = a_2 \cap b_3 \\ P_{143} &= (5,0,1,1) = \ell_1 \cap \ell_9 = a_2 \cap b_4 \\ P_6 &= (2,1,0,0) = \ell_1 \cap \ell_{11} = a_2 \cap b_6 \\ P_{189} &= (4,6,1,1) = \ell_1 \cap \ell_{12} = a_2 \cap c_{12} \\ P_{154} &= (1,2,1,1) = \ell_1 \cap \ell_{17} = a_2 \cap c_{23} \end{split}$$

$$\begin{split} P_{199} &= (6,7,1,1) = \ell_1 \cap \ell_{18} = a_2 \cap c_{24} \\ P_{152} &= (7,1,1,1) = \ell_1 \cap \ell_{20} = a_2 \cap c_{26} \\ P_{141} &= (3,0,1,1) = \ell_2 \cap \ell_6 = a_3 \cap b_1 \\ P_{87} &= (5,1,0,1) = \ell_2 \cap \ell_7 = a_3 \cap b_2 \\ P_{376} &= (7,5,4,1) = \ell_2 \cap \ell_9 = a_3 \cap b_4 \\ P_{25} &= (6,1,1,0) = \ell_2 \cap \ell_{11} = a_3 \cap b_6 \\ P_{519} &= (6,7,6,1) = \ell_2 \cap \ell_{13} = a_3 \cap c_{13} \\ P_{426} &= (1,4,5,1) = \ell_2 \cap \ell_{17} = a_3 \cap c_{23} \\ P_{229} &= (4,3,2,1) = \ell_2 \cap \ell_{21} = a_3 \cap c_{34} \\ P_{283} &= (2,2,3,1) = \ell_2 \cap \ell_{23} = a_3 \cap c_{36} \\ P_{191} &= (6,6,1,1) = \ell_4 \cap \ell_6 = a_5 \cap b_1 \\ P_{536} &= (7,1,7,1) = \ell_4 \cap \ell_7 = a_5 \cap b_2 \\ P_{478} &= (5,2,6,1) = \ell_4 \cap \ell_8 = a_5 \cap b_3 \\ P_{37} &= (2,3,1,0) = \ell_4 \cap \ell_{11} = a_5 \cap b_6 \end{split}$$

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P_{362} = (1, 4, 4, 1) = \ell_4 \cap \ell_{15} = a_5 \cap c_{15}
P_{267} = (2,0,3,1) = \ell_4 \cap \ell_{19} = a_5 \cap c_{25}
P_{452} = (3,7,5,1) = \ell_4 \cap \ell_{22} = a_5 \cap c_{35}
P_{118} = (4, 5, 0, 1) = \ell_4 \cap \ell_{26} = a_5 \cap c_{56}
P_{149} = (4, 1, 1, 1) = \ell_5 \cap \ell_6 = a_6 \cap b_1
P_{340} = (3, 1, 4, 1) = \ell_5 \cap \ell_7 = a_6 \cap b_2
P_{211} = (2, 1, 2, 1) = \ell_5 \cap \ell_8 = a_6 \cap b_3
P_{88} = (6,1,0,1) = \ell_5 \cap \ell_9 = a_6 \cap b_4
P_{13} = (2, 0, 1, 0) = \ell_5 \cap \ell_{16} = a_6 \cap c_{16}
P_{534} = (5, 1, 7, 1) = \ell_5 \cap \ell_{20} = a_6 \cap c_{26}
P_{472} = (7, 1, 6, 1) = \ell_5 \cap \ell_{23} = a_6 \cap c_{36}
P_{402} = (1, 1, 5, 1) = \ell_5 \cap \ell_{25} = a_6 \cap c_{46}
P_{168} = (7, 3, 1, 1) = \ell_6 \cap \ell_{12} = b_1 \cap c_{12}
P_{174} = (5, 4, 1, 1) = \ell_6 \cap \ell_{13} = b_1 \cap c_{13}
P_{194} = (1, 7, 1, 1) = \ell_6 \cap \ell_{15} = b_1 \cap c_{15}
P_{11} = (7, 1, 0, 0) = \ell_6 \cap \ell_{16} = b_1 \cap c_{16}
P_{147} = (2, 1, 1, 1) = \ell_7 \cap \ell_{12} = b_2 \cap c_{12}
P_{274} = (1, 1, 3, 1) = \ell_7 \cap \ell_{17} = b_2 \cap c_{23}
P_{405} = (4, 1, 5, 1) = \ell_7 \cap \ell_{19} = b_2 \cap c_{25}
P_{215} = (6, 1, 2, 1) = \ell_7 \cap \ell_{20} = b_2 \cap c_{26}
P_{129} = (7, 6, 0, 1) = \ell_8 \cap \ell_{13} = b_3 \cap c_{13}
P_{578} = (1, 7, 7, 1) = \ell_8 \cap \ell_{17} = b_3 \cap c_{23}
P_{303} = (6, 4, 3, 1) = \ell_8 \cap \ell_{22} = b_3 \cap c_{35}
P_{397} = (4,0,5,1) = \ell_8 \cap \ell_{23} = b_3 \cap c_{36}
P_{429} = (4, 4, 5, 1) = \ell_9 \cap \ell_{14} = b_4 \cap c_{14}
P_{284} = (3, 2, 3, 1) = \ell_9 \cap \ell_{18} = b_4 \cap c_{24}
P_{571} = (2, 6, 7, 1) = \ell_9 \cap \ell_{21} = b_4 \cap c_{34}
P_{514} = (1,7,6,1) = \ell_9 \cap \ell_{25} = b_4 \cap c_{46}
P_{54} = (3, 5, 1, 0) = \ell_{11} \cap \ell_{16} = b_6 \cap c_{16}
P_{15} = (4,0,1,0) = \ell_{11} \cap \ell_{20} = b_6 \cap c_{26}
P_{44} = (1, 4, 1, 0) = \ell_{11} \cap \ell_{23} = b_6 \cap c_{36}
P_{74} = (7, 7, 1, 0) = \ell_{11} \cap \ell_{26} = b_6 \cap c_{56}
P_{144} = (6,0,1,1) = \ell_{12} \cap \ell_{21} = c_{12} \cap c_{34}
P_{182} = (5, 5, 1, 1) = \ell_{12} \cap \ell_{22} = c_{12} \cap c_{35}
P_{170} = (1, 4, 1, 1) = \ell_{12} \cap \ell_{25} = c_{12} \cap c_{46}
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# $P_{156} = (3, 2, 1, 1) = \ell_{12} \cap \ell_{26} = c_{12} \cap c_{56}$ $P_{565} = (4, 5, 7, 1) = \ell_{13} \cap \ell_{18} = c_{13} \cap c_{24}$ $P_{220} = (3, 2, 2, 1) = \ell_{13} \cap \ell_{19} = c_{13} \cap c_{25}$ $P_{266} = (1, 0, 3, 1) = \ell_{13} \cap \ell_{25} = c_{13} \cap c_{46}$ $P_{355} = (2, 3, 4, 1) = \ell_{13} \cap \ell_{26} = c_{13} \cap c_{56}$ $P_{115} = (1, 5, 0, 1) = \ell_{14} \cap \ell_{17} = c_{14} \cap c_{23}$ $P_{358} = (5, 3, 4, 1) = \ell_{14} \cap \ell_{19} = c_{14} \cap c_{25}$ $P_{275} = (2, 1, 3, 1) = \ell_{14} \cap \ell_{20} = c_{14} \cap c_{26}$ $P_{464} = (7, 0, 6, 1) = \ell_{14} \cap \ell_{22} = c_{14} \cap c_{35}$ $P_{252} = (3, 6, 2, 1) = \ell_{14} \cap \ell_{23} = c_{14} \cap c_{36}$ $P_{583} = (6,7,7,1) = \ell_{14} \cap \ell_{26} = c_{14} \cap c_{56}$ $P_{250} = (1, 6, 2, 1) = \ell_{15} \cap \ell_{17} = c_{15} \cap c_{23}$ $P_{394} = (1,0,5,1) = \ell_{15} \cap \ell_{18} = c_{15} \cap c_{24}$ $P_{466} = (1, 1, 6, 1) = \ell_{15} \cap \ell_{20} = c_{15} \cap c_{26}$ $P_{282} = (1, 2, 3, 1) = \ell_{15} \cap \ell_{21} = c_{15} \cap c_{34}$ $P_{99} = (1,3,0,1) = \ell_{15} \cap \ell_{23} = c_{15} \cap c_{36}$ $P_{562} = (1, 5, 7, 1) = \ell_{15} \cap \ell_{25} = c_{15} \cap c_{46}$ $P_{47} = (4, 4, 1, 0) = \ell_{16} \cap \ell_{18} = c_{16} \cap c_{24}$ $P_{28} = (1, 2, 1, 0) = \ell_{16} \cap \ell_{19} = c_{16} \cap c_{25}$ $P_{24} = (5, 1, 1, 0) = \ell_{16} \cap \ell_{21} = c_{16} \cap c_{34}$ $P_{66} = (7, 6, 1, 0) = \ell_{16} \cap \ell_{22} = c_{16} \cap c_{35}$ $P_{354} = (1, 3, 4, 1) = \ell_{17} \cap \ell_{25} = c_{23} \cap c_{46}$ $P_{458} = (1,0,6,1) = \ell_{17} \cap \ell_{26} = c_{23} \cap c_{56}$ $P_{100} = (2,3,0,1) = \ell_{18} \cap \ell_{22} = c_{24} \cap c_{35}$ $P_{366} = (5, 4, 4, 1) = \ell_{18} \cap \ell_{23} = c_{24} \cap c_{36}$ $P_{256} = (7, 6, 2, 1) = \ell_{18} \cap \ell_{26} = c_{24} \cap c_{56}$ $P_{520} = (7,7,6,1) = \ell_{19} \cap \ell_{21} = c_{25} \cap c_{34}$ $P_{567} = (6, 5, 7, 1) = \ell_{19} \cap \ell_{23} = c_{25} \cap c_{36}$ $P_{123} = (1, 6, 0, 1) = \ell_{19} \cap \ell_{25} = c_{25} \cap c_{46}$ $P_{85} = (3, 1, 0, 1) = \ell_{20} \cap \ell_{21} = c_{26} \cap c_{34}$ $P_{341} = (4, 1, 4, 1) = \ell_{20} \cap \ell_{22} = c_{26} \cap c_{35}$ $P_{430} = (5, 4, 5, 1) = \ell_{21} \cap \ell_{26} = c_{34} \cap c_{56}$ $P_{218} = (1, 2, 2, 1) = \ell_{22} \cap \ell_{25} = c_{35} \cap c_{46}$

#### Single Points

The surface has 12 single points: The single points on the surface are:

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0: P_{233} = (0, 4, 2, 1) lies on line a_4

1: P_{241} = (0, 5, 2, 1) lies on line b_5

2: P_{297} = (0, 4, 3, 1) lies on line c_{45}

3: P_{313} = (0, 6, 3, 1) lies on line a_4

4: P_{377} = (0, 6, 4, 1) lies on line c_{45}

5: P_{385} = (0, 7, 4, 1) lies on line b_5

6: P_{417} = (0, 3, 5, 1) lies on line b_5
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The single points on the surface are:

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7: P_{449} = (0,7,5,1) lies on line a_4

8: P_{473} = (0,2,6,1) lies on line b_5

9: P_{497} = (0,5,6,1) lies on line c_{45}

10: P_{537} = (0,2,7,1) lies on line c_{45}

11: P_{545} = (0,3,7,1) lies on line a_4
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# Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

# Line Intersection Graph

	0																								24		
	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$c_{12}$	$c_{13}$	$c_{14}$	C <sub>15</sub> (	C16	$c_{23}$	$c_{24}$	$c_{25}$	$c_{26}$	$c_{34}$	$c_{35}$	$c_{36}$	$c_{45}$ (	$c_{46}$ (	<sup>2</sup> 56
$0 a_1$	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
$1 a_2$	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
$2 a_3$	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
$3 a_4$	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
$4 a_5$	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
$5 \ a_6$	1	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6 $b_1$	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7 $b_2$		0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8 $b_3$	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9 $b_4$	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10 $b_5$		1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11 $b_6$	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
$12 c_{12}$		1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
$13 c_{13}$	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
$14 c_{14}$		0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
$15 c_{15}$		0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
$16 c_{16}$		0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
$17 c_{23}$		1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
$18 c_{24}$	1	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
$19 c_{25}$	1	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
$20 c_{26}$	1	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
$21 c_{34}$		0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
$22 c_{35}$		0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
$23 c_{36}$		0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
$24 c_{45}$		0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
$25 c_{46}$		0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
$26 c_{56}$	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$
in point	$P_{18}$	$P_{55}$	$P_{22}$	$P_{43}$	$P_{64}$	$P_8$	$P_{29}$	$P_{68}$	$P_{43}$	$P_{41}$

Line 1 intersects

Line	$\ell_6$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{179}$	$P_{164}$	$P_{143}$	$P_{169}$	$P_6$	$P_{189}$	$P_{154}$	$P_{199}$	$P_{169}$	$P_{152}$

Line 2 intersects

Line	$\ell_6$	$\ell_7$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{17}$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$
$\operatorname{point}$	$P_{141}$	$P_{87}$	$P_{376}$	$P_{569}$	$P_{25}$	$P_{519}$	$P_{426}$	$P_{229}$	$P_{569}$	$P_{283}$

Line 3 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$	$\ell_{18}$	$\ell_{21}$	$\ell_{24}$	$\ell_{25}$
in point	$P_{153}$	$P_{465}$	$P_{369}$	$P_3$	$P_{27}$	$P_{153}$	$P_{465}$	$P_{369}$	$P_3$	$P_{27}$

#### Line 4 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{15}$	$\ell_{19}$	$\ell_{22}$	$\ell_{24}$	$\ell_{26}$
in point	$P_{191}$	$P_{536}$	$P_{478}$	$P_{225}$	$P_{37}$	$P_{362}$	$P_{267}$	$P_{452}$	$P_{225}$	$P_{118}$

#### Line 5 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{16}$	$\ell_{20}$	$\ell_{23}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{149}$	$P_{340}$	$P_{211}$	$P_{88}$	$P_{273}$	$P_{13}$	$P_{534}$	$P_{472}$	$P_{402}$	$P_{273}$

# ${\bf Line~6~intersects}$

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$
in point	$P_{179}$	$P_{141}$	$P_{153}$	$P_{191}$	$P_{149}$	$P_{168}$	$P_{174}$	$P_{153}$	$P_{194}$	$P_{11}$

# ${\bf Line}~7~{\bf intersects}$

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{12}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{18}$	$P_{87}$	$P_{465}$	$P_{536}$	$P_{340}$	$P_{147}$	$P_{274}$	$P_{465}$	$P_{405}$	$P_{215}$

# Line 8 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{13}$	$\ell_{17}$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$
in point	$P_{55}$	$P_{164}$	$P_{369}$	$P_{478}$	$P_{211}$	$P_{129}$	$P_{578}$	$P_{369}$	$P_{303}$	$P_{397}$

#### Line 9 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_{14}$	$\ell_{18}$	$\ell_{21}$	$\ell_{24}$	$\ell_{25}$
in point	$P_{22}$	$P_{143}$	$P_{376}$	$P_{225}$	$P_{88}$	$P_{429}$	$P_{284}$	$P_{571}$	$P_{225}$	$P_{514}$

#### Line 10 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_{15}$	$\ell_{19}$	$\ell_{22}$	$\ell_{24}$	$\ell_{26}$
in point	$P_{43}$	$P_{169}$	$P_{569}$	$P_3$	$P_{273}$	$P_{43}$	$P_{169}$	$P_{569}$	$P_3$	$P_{273}$

#### ${\rm Line}\ 11\ {\rm intersects}$

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_{16}$	$\ell_{20}$	$\ell_{23}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{64}$	$P_6$	$P_{25}$	$P_{27}$	$P_{37}$	$P_{54}$	$P_{15}$	$P_{44}$	$P_{27}$	$P_{74}$

# Line 12 intersects

Line	$\ell_0$	$\ell_1$	$\ell_6$	$\ell_7$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
in point	$P_8$	$P_{189}$	$P_{168}$	$P_{147}$	$P_{144}$	$P_{182}$	$P_{193}$	$P_{193}$	$P_{170}$	$P_{156}$

### Line 13 intersects

	Line	$\ell_0$	$\ell_2$	$\ell_6$	$\ell_8$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
i	n point	$P_{29}$	$P_{519}$	$P_{174}$	$P_{129}$	$P_{565}$	$P_{220}$	$P_{401}$	$P_{401}$	$P_{266}$	$P_{355}$

#### Line 14 intersects

Line	$\ell_0$	$\ell_3$	$\ell_6$	$\ell_9$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$	$\ell_{22}$	$\ell_{23}$	$\ell_{26}$
in point	$P_{68}$	$P_{153}$	$P_{153}$	$P_{429}$	$P_{115}$	$P_{358}$	$P_{275}$	$P_{464}$	$P_{252}$	$P_{583}$

### Line 15 intersects

Line	$\ell_0$	$\ell_4$	$\ell_6$	$\ell_{10}$	$\ell_{17}$	$\ell_{18}$	$\ell_{20}$	$\ell_{21}$	$\ell_{23}$	$\ell_{25}$
in point	$P_{43}$	$P_{362}$	$P_{194}$	$P_{43}$	$P_{250}$	$P_{394}$	$P_{466}$	$P_{282}$	$P_{99}$	$P_{562}$

# Line 16 intersects

Line	$\ell_0$	$\ell_5$	$\ell_6$	$\ell_{11}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{21}$	$\ell_{22}$	$\ell_{24}$
in point	$P_{41}$	$P_{13}$	$P_{11}$	$P_{54}$	$P_{67}$	$P_{47}$	$P_{28}$	$P_{24}$	$P_{66}$	$P_{67}$

# Line 17 intersects

Line	$\ell_1$	$\ell_2$	$\ell_7$	$\ell_8$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{154}$	$P_{426}$	$P_{274}$	$P_{578}$	$P_{115}$	$P_{250}$	$P_{67}$	$P_{67}$	$P_{354}$	$P_{458}$

# Line 18 intersects

	Line	$\ell_1$	$\ell_3$	$\ell_7$	$\ell_9$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$	$\ell_{22}$	$\ell_{23}$	$\ell_{26}$
i	n point	$P_{199}$	$P_{465}$	$P_{465}$	$P_{284}$	$P_{565}$	$P_{394}$	$P_{47}$	$P_{100}$	$P_{366}$	$P_{256}$

# Line 19 intersects

Line	$\ell_1$	$\ell_4$	$\ell_7$	$\ell_{10}$	$\ell_{13}$	$\ell_{14}$	$\ell_{16}$	$\ell_{21}$	$\ell_{23}$	$\ell_{25}$
in point	$P_{169}$	$P_{267}$	$P_{405}$	$P_{169}$	$P_{220}$	$P_{358}$	$P_{28}$	$P_{520}$	$P_{567}$	$P_{123}$

#### Line 20 intersects

Line	$\ell_1$	$\ell_5$	$\ell_7$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{21}$	$\ell_{22}$	$\ell_{24}$
in point	$P_{152}$	$P_{534}$	$P_{215}$	$P_{15}$	$P_{401}$	$P_{275}$	$P_{466}$	$P_{85}$	$P_{341}$	$P_{401}$

# ${\bf Line~21~intersects}$

Line	$\ell_2$	$\ell_3$	$\ell_8$	$\ell_9$	$\ell_{12}$	$\ell_{15}$	$\ell_{16}$	$\ell_{19}$	$\ell_{20}$	$\ell_{26}$
in point	$P_{229}$	$P_{369}$	$P_{369}$	$P_{571}$	$P_{144}$	$P_{282}$	$P_{24}$	$P_{520}$	$P_{85}$	$P_{430}$

# Line 22 intersects

Line	$\ell_2$	$\ell_4$	$\ell_8$	$\ell_{10}$	$\ell_{12}$	$\ell_{14}$	$\ell_{16}$	$\ell_{18}$	$\ell_{20}$	$\ell_{25}$
in point	$P_{569}$	$P_{452}$	$P_{303}$	$P_{569}$	$P_{182}$	$P_{464}$	$P_{66}$	$P_{100}$	$P_{341}$	$P_{218}$

# Line 23 intersects

	Line	$\ell_2$	$\ell_5$	$\ell_8$	$\ell_{11}$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{18}$	$\ell_{19}$	$\ell_{24}$
in	n point	$P_{283}$	$P_{472}$	$P_{397}$	$P_{44}$	$P_{193}$	$P_{252}$	$P_{99}$	$P_{366}$	$P_{567}$	$P_{193}$

# Line 24 intersects

Line	$\ell_3$	$\ell_4$	$\ell_9$	$\ell_{10}$	$\ell_{12}$	$\ell_{13}$	$\ell_{16}$	$\ell_{17}$	$\ell_{20}$	$\ell_{23}$
in point	$P_3$	$P_{225}$	$P_{225}$	$P_3$	$P_{193}$	$P_{401}$	$P_{67}$	$P_{67}$	$P_{401}$	$P_{193}$

#### Line 25 intersects

Line	$\ell_3$	$\ell_5$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$	$\ell_{17}$	$\ell_{19}$	$\ell_{22}$
in point	$P_{27}$	$P_{402}$	$P_{514}$	$P_{27}$	$P_{170}$	$P_{266}$	$P_{562}$	$P_{354}$	$P_{123}$	$P_{218}$

# Line 26 intersects

Line	$\ell_4$	$\ell_5$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{17}$	$\ell_{18}$	$\ell_{21}$
in point	$P_{118}$	$P_{273}$	$P_{273}$	$P_{74}$	$P_{156}$	$P_{355}$	$P_{583}$	$P_{458}$	$P_{256}$	$P_{430}$

The surface has 121 points:

The points on the surface are:

$0: P_3 = (0,0,0,1)$	$5: P_{15} = (4,0,1,0)$	$10: P_{27} = (0, 2, 1, 0)$
$1: P_6 = (2, 1, 0, 0)$	$6: P_{18} = (7,0,1,0)$	$11: P_{28} = (1, 2, 1, 0)$
$2: P_8 = (4, 1, 0, 0)$	$7: P_{22} = (3, 1, 1, 0)$	$12: P_{29} = (2, 2, 1, 0)$
$3: P_{11} = (7, 1, 0, 0)$	$8: P_{24} = (5, 1, 1, 0)$	13: $P_{37} = (2, 3, 1, 0)$
$4: P_{13} = (2, 0, 1, 0)$	9: $P_{25} = (6, 1, 1, 0)$	$14: P_{41} = (6, 3, 1, 0)$

```
15: P_{43} = (0, 4, 1, 0)
                                            51: P_{191} = (6, 6, 1, 1)
                                                                                        87: P_{385} = (0,7,4,1)
16: P_{44} = (1, 4, 1, 0)
                                            52: P_{193} = (0,7,1,1)
                                                                                         88: P_{394} = (1, 0, 5, 1)
17: P_{47} = (4, 4, 1, 0)
                                            53: P_{194} = (1,7,1,1)
                                                                                        89: P_{397} = (4, 0, 5, 1)
18: P_{54} = (3, 5, 1, 0)
                                            54: P_{199} = (6,7,1,1)
                                                                                        90: P_{401} = (0, 1, 5, 1)
19: P_{55} = (4, 5, 1, 0)
                                            55: P_{211} = (2, 1, 2, 1)
                                                                                        91: P_{402} = (1, 1, 5, 1)
20: P_{64} = (5, 6, 1, 0)
                                            56: P_{215} = (6, 1, 2, 1)
                                                                                        92: P_{405} = (4, 1, 5, 1)
21: P_{66} = (7, 6, 1, 0)
                                            57: P_{218} = (1, 2, 2, 1)
                                                                                        93: P_{417} = (0, 3, 5, 1)
22: P_{67} = (0,7,1,0)
                                            58: P_{220} = (3, 2, 2, 1)
                                                                                        94: P_{426} = (1,4,5,1)
23: P_{68} = (1, 7, 1, 0)
                                            59: P_{225} = (0, 3, 2, 1)
                                                                                        95: P_{429} = (4, 4, 5, 1)
24: P_{74} = (7,7,1,0)
                                            60: P_{229} = (4, 3, 2, 1)
                                                                                        96: P_{430} = (5, 4, 5, 1)
25: P_{85} = (3, 1, 0, 1)
                                            61: P_{233} = (0, 4, 2, 1)
                                                                                        97: P_{449} = (0, 7, 5, 1)
26: P_{87} = (5, 1, 0, 1)
                                            62: P_{241} = (0, 5, 2, 1)
                                                                                        98: P_{452} = (3, 7, 5, 1)
27: P_{88} = (6, 1, 0, 1)
                                            63: P_{250} = (1, 6, 2, 1)
                                                                                        99: P_{458} = (1, 0, 6, 1)
28: P_{99} = (1, 3, 0, 1)
                                                                                        100: P_{464} = (7, 0, 6, 1)
                                            64: P_{252} = (3, 6, 2, 1)
29: P_{100} = (2, 3, 0, 1)
                                            65: P_{256} = (7, 6, 2, 1)
                                                                                        101: P_{465} = (0, 1, 6, 1)
30: P_{115} = (1, 5, 0, 1)
                                            66: P_{266} = (1, 0, 3, 1)
                                                                                        102: P_{466} = (1, 1, 6, 1)
                                                                                        103: P_{472} = (7, 1, 6, 1)
31: P_{118} = (4,5,0,1)
                                            67: P_{267} = (2,0,3,1)
32: P_{123} = (1, 6, 0, 1)
                                            68: P_{273} = (0, 1, 3, 1)
                                                                                        104: P_{473} = (0, 2, 6, 1)
                                                                                        105: P_{478} = (5, 2, 6, 1)
33: P_{129} = (7, 6, 0, 1)
                                            69: P_{274} = (1, 1, 3, 1)
34: P_{141} = (3,0,1,1)
                                            70: P_{275} = (2, 1, 3, 1)
                                                                                        106: P_{497} = (0, 5, 6, 1)
35: P_{143} = (5,0,1,1)
                                            71: P_{282} = (1, 2, 3, 1)
                                                                                        107: P_{514} = (1,7,6,1)
36: P_{144} = (6,0,1,1)
                                            72: P_{283} = (2, 2, 3, 1)
                                                                                        108: P_{519} = (6,7,6,1)
37: P_{147} = (2, 1, 1, 1)
                                                                                        109: P_{520} = (7,7,6,1)
                                            73: P_{284} = (3, 2, 3, 1)
38: P_{149} = (4, 1, 1, 1)
                                            74: P_{297} = (0, 4, 3, 1)
                                                                                        110: P_{534} = (5, 1, 7, 1)
                                                                                        111: P_{536} = (7, 1, 7, 1)
39: P_{152} = (7,1,1,1)
                                            75: P_{303} = (6, 4, 3, 1)
40: P_{153} = (0, 2, 1, 1)
                                            76: P_{313} = (0, 6, 3, 1)
                                                                                        112: P_{537} = (0, 2, 7, 1)
41: P_{154} = (1, 2, 1, 1)
                                            77: P_{340} = (3, 1, 4, 1)
                                                                                        113: P_{545} = (0, 3, 7, 1)
42: P_{156} = (3, 2, 1, 1)
                                            78: P_{341} = (4, 1, 4, 1)
                                                                                        114: P_{562} = (1, 5, 7, 1)
                                            79: P_{354} = (1, 3, 4, 1)
43: P_{164} = (3, 3, 1, 1)
                                                                                        115: P_{565} = (4, 5, 7, 1)
44: P_{168} = (7, 3, 1, 1)
                                            80: P_{355} = (2, 3, 4, 1)
                                                                                        116: P_{567} = (6, 5, 7, 1)
45: P_{169} = (0, 4, 1, 1)
                                                                                        117: P_{569} = (0, 6, 7, 1)
                                            81: P_{358} = (5, 3, 4, 1)
                                                                                        118: P_{571} = (2, 6, 7, 1)
46: P_{170} = (1, 4, 1, 1)
                                            82: P_{362} = (1, 4, 4, 1)
47: P_{174} = (5, 4, 1, 1)
                                            83: P_{366} = (5, 4, 4, 1)
                                                                                        119: P_{578} = (1,7,7,1)
                                                                                        120: P_{583} = (6,7,7,1)
48: P_{179} = (2, 5, 1, 1)
                                            84: P_{369} = (0, 5, 4, 1)
49: P_{182} = (5, 5, 1, 1)
                                            85: P_{376} = (7, 5, 4, 1)
50: P_{189} = (4, 6, 1, 1)
                                            86: P_{377} = (0, 6, 4, 1)
```