# Rank-65605 over GF(32)

January 15, 2021

# The equation

The equation of the surface is:

$$X_1^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)The point rank of the equation over GF(32) is -2112846779

## General information

Number of lines	33
Number of points	1057
Number of singular points	33
Number of Eckardt points	0
Number of double points	32
Number of single points	1025
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$33^{33}$
Type of lines on points	$2^{32}, 1^{1025}$

### Singular Points

The surface has 33 singular points:

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\begin{array}{lll} 0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0) & 9: \ P_{9249} = \mathbf{P}(0,0,\eta^3,1) = \mathbf{P}(0,0,8,1) \\ 1: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1) & 10: \ P_{10273} = \mathbf{P}(0,0,\eta^{29},1) = \mathbf{P}(0,0,9,1) \\ 2: \ P_{2082} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1) & 11: \ P_{11297} = \mathbf{P}(0,0,\eta^6,1) = \mathbf{P}(0,0,10,1) \\ 3: \ P_{3105} = \mathbf{P}(0,0,\eta,1) = \mathbf{P}(0,0,2,1) & 12: \ P_{12321} = \mathbf{P}(0,0,\eta^{27},1) = \mathbf{P}(0,0,11,1) \\ 4: \ P_{4129} = \mathbf{P}(0,0,\eta^{18},1) = \mathbf{P}(0,0,3,1) & 13: \ P_{13345} = \mathbf{P}(0,0,\eta^{20},1) = \mathbf{P}(0,0,12,1) \\ 5: \ P_{5153} = \mathbf{P}(0,0,\eta^2,1) = \mathbf{P}(0,0,4,1) & 14: \ P_{14369} = \mathbf{P}(0,0,\eta^8,1) = \mathbf{P}(0,0,13,1) \\ 6: \ P_{6177} = \mathbf{P}(0,0,\eta^5,1) = \mathbf{P}(0,0,5,1) & 15: \ P_{15393} = \mathbf{P}(0,0,\eta^{12},1) = \mathbf{P}(0,0,14,1) \\ 7: \ P_{7201} = \mathbf{P}(0,0,\eta^{19},1) = \mathbf{P}(0,0,6,1) & 16: \ P_{16417} = \mathbf{P}(0,0,\eta^{23},1) = \mathbf{P}(0,0,15,1) \\ 8: \ P_{8225} = \mathbf{P}(0,0,\eta^{11},1) = \mathbf{P}(0,0,7,1) & 17: \ P_{17441} = \mathbf{P}(0,0,\eta^4,1) = \mathbf{P}(0,0,16,1) \end{array}
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\begin{array}{lll} 18: \ P_{18465} = \mathbf{P}(0,0,\eta^{10},1) = \mathbf{P}(0,0,17,1) \\ 19: \ P_{19489} = \mathbf{P}(0,0,\eta^{30},1) = \mathbf{P}(0,0,18,1) \\ 20: \ P_{20513} = \mathbf{P}(0,0,\eta^{17},1) = \mathbf{P}(0,0,19,1) \\ 21: \ P_{21537} = \mathbf{P}(0,0,\eta^{7},1) = \mathbf{P}(0,0,20,1) \\ 22: \ P_{22561} = \mathbf{P}(0,0,\eta^{22},1) = \mathbf{P}(0,0,21,1) \\ 23: \ P_{23585} = \mathbf{P}(0,0,\eta^{28},1) = \mathbf{P}(0,0,22,1) \\ 24: \ P_{24609} = \mathbf{P}(0,0,\eta^{26},1) = \mathbf{P}(0,0,24,1) \\ 25: \ P_{25633} = \mathbf{P}(0,0,\eta^{21},1) = \mathbf{P}(0,0,24,1) \\ 26: \ P_{26657} = \mathbf{P}(0,0,\eta^{25},1) = \mathbf{P}(0,0,26,1) \\ 27: \ P_{27681} = \mathbf{P}(0,0,\eta^{16},1) = \mathbf{P}(0,0,27,1) \\ 28: \ P_{28705} = \mathbf{P}(0,0,\eta^{16},1) = \mathbf{P}(0,0,27,1) \\ 29: \ P_{29729} = \mathbf{P}(0,0,\eta^{13},1) = \mathbf{P}(0,0,28,1) \\ 30: \ P_{30753} = \mathbf{P}(0,0,\eta^{14},1) = \mathbf{P}(0,0,29,1) \\ 31: \ P_{31777} = \mathbf{P}(0,0,\eta^{24},1) = \mathbf{P}(0,0,30,1) \\ 32: \ P_{32801} = \mathbf{P}(0,0,\eta^{15},1) = \mathbf{P}(0,0,31,1) \end{array}
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### The 33 Lines

The lines and their Pluecker coordinates are:

$$\begin{split} \ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \mathbf{Pl}(0,0,1,0,0,0)_2 \\ \ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{Pl}(0,1,0,0,0)_1 \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{35906} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{35906} = \mathbf{Pl}(0,1,1,1,1,1)_{70594} \\ \ell_3 &= \begin{bmatrix} 1 & \eta^{30} & 0 & \eta^{28} \\ 0 & 0 & 1 & \eta^{30} \end{bmatrix}_{764196} = \begin{bmatrix} 1 & 18 & 0 & 22 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{764196} = \mathbf{Pl}(0,9,2,18,1,1)_{70665} \\ \ell_4 &= \begin{bmatrix} 1 & \eta^{13} & 0 & \eta^{8} \\ 0 & 0 & 1 & \eta^{13} \end{bmatrix}_{470360} = \begin{bmatrix} 1 & 28 & 0 & 13 \\ 0 & 0 & 1 & 28 \end{bmatrix}_{470360} = \mathbf{Pl}(0,23,3,28,1,1)_{70742} \\ \ell_5 &= \begin{bmatrix} 1 & \eta^{29} & 0 & \eta^{25} \\ 0 & 0 & 1 & \eta^{29} \end{bmatrix}_{856146} = \begin{bmatrix} 1 & 9 & 0 & 25 \\ 0 & 0 & 1 & 9 \end{bmatrix}_{856146} = \mathbf{Pl}(0,11,4,9,1,1)_{70793} \\ \ell_6 &= \begin{bmatrix} 1 & \eta^{26} & 0 & \eta^{16} \\ 0 & 0 & 1 & \eta^{26} \end{bmatrix}_{938606} = \begin{bmatrix} 1 & 23 & 0 & 27 \\ 0 & 0 & 1 & 23 \end{bmatrix}_{938606} = \mathbf{Pl}(0,24,5,23,1,1)_{70869} \\ \ell_7 &= \begin{bmatrix} 1 & \eta^{12} & 0 & \eta^5 \\ 0 & 0 & 1 & \eta^{12} \end{bmatrix}_{184956} = \begin{bmatrix} 1 & 14 & 0 & 5 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{184956} = \mathbf{Pl}(0,30,6,14,1,1)_{70997} \\ \ell_8 &= \begin{bmatrix} 1 & \eta^{20} & 0 & \eta^{29} \\ 0 & 0 & 1 & \eta^{20} \end{bmatrix}_{318136} = \begin{bmatrix} 1 & 12 & 0 & 9 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{318136} = \mathbf{Pl}(0,26,7,12,1,1)_{70997} \\ \ell_{10} &= \begin{bmatrix} 1 & \eta^{28} & 0 & \eta^{22} \\ 0 & 0 & 1 & \eta^{28} \end{bmatrix}_{734604} = \begin{bmatrix} 1 & 22 & 0 & 21 \\ 0 & 0 & 1 & 22 \end{bmatrix}_{734604} = \mathbf{Pl}(0,16,9,4,1,1)_{71113} \\ \ell_{11} &= \begin{bmatrix} 1 & \eta^{25} & 0 & \eta^{13} \\ 0 & 0 & 1 & \eta^{2} \end{bmatrix}_{343496} = \begin{bmatrix} 1 & 16 & 0 & 14 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{974546} = \mathbf{Pl}(0,6,10,25,1,1)_{71166} \\ \ell_{12} &= \begin{bmatrix} 1 & \eta^4 & 0 & \eta^2 \\ 0 & 0 & 1 & \eta^{11} \end{bmatrix}_{491488} = \begin{bmatrix} 1 & 16 & 0 & 14 \\ 0 & 0 & 1 & 16 \end{bmatrix}_{491488} = \mathbf{Pl}(0,21,12,7,1,1)_{71307} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^{11} & 0 & \eta^2 \\ 0 & 0 & 1 & \eta^{13} \end{bmatrix}_{693374} = \begin{bmatrix} 1 & 15 & 0 & 20 \\ 0 & 0 & 1 & 15 \end{bmatrix}_{693374} = \mathbf{Pl}(0,31,13,15,1,1)_{71330} \\ \ell_{15} &= \begin{bmatrix} 1 & \eta^{19} & 0 & \eta^{26} \\ 0 & 0 & 1 & \eta^{13} \end{bmatrix}_{785324} = \begin{bmatrix} 1 & 6 & 0 & 23 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{785324} = \mathbf{Pl}(0,20,14,6,1,1)_{71432} \end{aligned}$$

$$\begin{split} &\ell_{16} = \begin{bmatrix} 1 & \eta^8 & 0 & \eta^{24} \\ 0 & 0 & 1 & \eta^8 \end{bmatrix}_{1029498} = \begin{bmatrix} 1 & 13 & 0 & 30 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{1029498} = \mathbf{PI}(0, 27, 15, 13, 1, 1)_{71502} \\ &\ell_{17} = \begin{bmatrix} 1 & \eta^{27} & 0 & \eta^{19} \\ 0 & 0 & 1 & \eta^{27} \end{bmatrix}_{215606} = \begin{bmatrix} 1 & 11 & 0 & 6 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{215606} = \mathbf{PI}(0, 15, 16, 11, 1, 1)_{71553} \\ &\ell_{18} = \begin{bmatrix} 1 & \eta^{21} & 0 & \eta \\ 0 & 0 & 1 & \eta^{21} \end{bmatrix}_{94064} = \begin{bmatrix} 1 & 24 & 0 & 2 \\ 0 & 0 & 1 & 24 \end{bmatrix}_{94064} = \mathbf{PI}(0, 7, 17, 24, 1, 1)_{71608} \\ &\ell_{19} = \begin{bmatrix} 1 & \eta & 0 & \eta^3 \\ 0 & 0 & 1 & \eta \end{bmatrix}_{273732} = \begin{bmatrix} 1 & 2 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{273732} = \mathbf{PI}(0, 4, 18, 2, 1, 1)_{71608} \\ &\ell_{20} = \begin{bmatrix} 1 & \eta^{14} & 0 & \eta^{11} \\ 0 & 0 & 1 & \eta^{14} \end{bmatrix}_{268474} = \begin{bmatrix} 1 & 29 & 0 & 7 \\ 0 & 0 & 1 & 29 \end{bmatrix}_{268474} = \mathbf{PI}(0, 22, 19, 29, 1, 1)_{71749} \\ &\ell_{21} = \begin{bmatrix} 1 & \eta^{24} & 0 & \eta^{10} \\ 0 & 0 & 1 & \eta^{24} \end{bmatrix}_{607772} = \begin{bmatrix} 1 & 30 & 0 & 17 \\ 0 & 0 & 1 & 30 \end{bmatrix}_{607772} = \mathbf{PI}(0, 19, 20, 30, 1, 1)_{71809} \\ &\ell_{22} = \begin{bmatrix} 1 & \eta^3 & 0 & \eta^9 \\ 0 & 0 & 1 & \eta^9 \end{bmatrix}_{300596} = \begin{bmatrix} 1 & 26 & 0 & 11 \\ 0 & 0 & 1 & 26 \end{bmatrix}_{400596} = \mathbf{PI}(0, 3, 21, 26, 1, 1)_{71856} \\ &\ell_{23} = \begin{bmatrix} 1 & \eta^3 & 0 & \eta^9 \\ 0 & 0 & 1 & \eta^5 \end{bmatrix}_{1054858} = \begin{bmatrix} 1 & 5 & 0 & 31 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{1054858} = \mathbf{PI}(0, 17, 23, 5, 1, 1)_{71996} \\ &\ell_{24} = \begin{bmatrix} 1 & \eta^{10} & 0 & \eta^{18} \\ 0 & 0 & 1 & \eta^{10} \end{bmatrix}_{627842} = \begin{bmatrix} 1 & 17 & 0 & 18 \\ 0 & 0 & 1 & 17 \end{bmatrix}_{627842} = \mathbf{PI}(0, 14, 25, 10, 1, 1)_{72119} \\ &\ell_{26} = \begin{bmatrix} 1 & \eta^{6} & 0 & \eta^{18} \\ 0 & 0 & 1 & \eta^{20} \end{bmatrix}_{130478} = \begin{bmatrix} 1 & 10 & 0 & 3 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{113076} = \mathbf{PI}(0, 18, 27, 31, 1, 1)_{72249} \\ &\ell_{28} = \begin{bmatrix} 1 & \eta^{15} & 0 & \eta^{14} \\ 0 & 0 & 1 & \eta^{15} \end{bmatrix}_{1014718} = \begin{bmatrix} 1 & 31 & 0 & 29 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{1014718} = \mathbf{PI}(0, 18, 27, 31, 1, 1)_{72249} \\ &\ell_{29} = \begin{bmatrix} 1 & \eta^{18} & 0 & \eta^{23} \\ 0 & 0 & 1 & \eta^{18} \end{bmatrix}_{131558} = \begin{bmatrix} 1 & 3 & 0 & 15 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{1014718} = \mathbf{PI}(0, 8, 29, 19, 1, 1)_{72249} \\ &\ell_{29} = \begin{bmatrix} 1 & \eta^{17} & 0 & \eta^{20} \\ 0 & 0 & 1 & \eta^{17} \end{bmatrix}_{427014} = \begin{bmatrix} 1 & 19 & 0 & 12 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{833960} = \mathbf{PI}(0, 2, 30, 20, 1, 1)_{72249} \\ &\ell_{31} = \begin{bmatrix} 1 & \eta^{7} & 0 & \eta^{21} \\ 0 & 0 & 1 & \eta^{15} \end{bmatrix}_{672246} = \begin{bmatrix} 1 & 27 & 0 &$$

Rank of points on Klein quadric: (2, 1, 70594, 70665, 70742, 70793, 70869, 70938, 70997, 71059, 71113, 71166, 71236, 71307, 71380, 71432, 71502, 71553, 71608, 71668, 71749, 71809, 71856, 71926, 71996, 72054, 72119, 72196, 72249, 72299, 72365, 72449, 72485)

### **Eckardt Points**

The surface has 0 Eckardt points:

### **Double Points**

The surface has 32 Double points: The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$
$P_{2082} = (0, 0, 1, 1) = \ell_1 \cap \ell_2$
$P_{3105} = (0, 0, 2, 1) = \ell_1 \cap \ell_3$
$P_{4129} = (0,0,3,1) = \ell_1 \cap \ell_4$
$P_{5153} = (0, 0, 4, 1) = \ell_1 \cap \ell_5$
$P_{6177} = (0, 0, 5, 1) = \ell_1 \cap \ell_6$
$P_{7201} = (0, 0, 6, 1) = \ell_1 \cap \ell_7$
$P_{8225} = (0, 0, 7, 1) = \ell_1 \cap \ell_8$
$P_{9249} = (0, 0, 8, 1) = \ell_1 \cap \ell_9$
$P_{10273} = (0, 0, 9, 1) = \ell_1 \cap \ell_{10}$
$P_{11297} = (0, 0, 10, 1) = \ell_1 \cap \ell_{11}$
$P_{12321} = (0, 0, 11, 1) = \ell_1 \cap \ell_{12}$
$P_{13345} = (0, 0, 12, 1) = \ell_1 \cap \ell_{13}$
$P_{14369} = (0, 0, 13, 1) = \ell_1 \cap \ell_{14}$
$P_{15393} = (0, 0, 14, 1) = \ell_1 \cap \ell_{15}$
$P_{16417} = (0, 0, 15, 1) = \ell_1 \cap \ell_{16}$
$P_{17441} = (0, 0, 16, 1) = \ell_1 \cap \ell_{17}$

# $\begin{array}{l} P_{18465} = (0,0,17,1) = \ell_1 \cap \ell_{18} \\ P_{19489} = (0,0,18,1) = \ell_1 \cap \ell_{19} \\ P_{20513} = (0,0,19,1) = \ell_1 \cap \ell_{20} \\ P_{21537} = (0,0,20,1) = \ell_1 \cap \ell_{21} \\ P_{22561} = (0,0,21,1) = \ell_1 \cap \ell_{22} \\ P_{23585} = (0,0,22,1) = \ell_1 \cap \ell_{23} \\ P_{24609} = (0,0,23,1) = \ell_1 \cap \ell_{24} \\ P_{25633} = (0,0,24,1) = \ell_1 \cap \ell_{25} \\ P_{26657} = (0,0,25,1) = \ell_1 \cap \ell_{26} \\ P_{27681} = (0,0,26,1) = \ell_1 \cap \ell_{27} \\ P_{28705} = (0,0,27,1) = \ell_1 \cap \ell_{28} \\ P_{29729} = (0,0,28,1) = \ell_1 \cap \ell_{29} \\ P_{30753} = (0,0,29,1) = \ell_1 \cap \ell_{30} \\ P_{31777} = (0,0,30,1) = \ell_1 \cap \ell_{31} \\ P_{32801} = (0,0,31,1) = \ell_1 \cap \ell_{32} \end{array}$

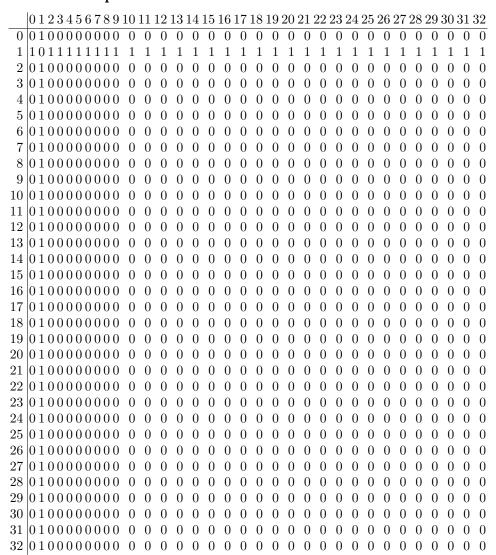
# Single Points

The surface has 1025 single points: Too many to print.

### Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

# Line Intersection Graph



Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$
in point	$P_2$

Line 1 intersects

	Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_1$
Ì	in point	$P_2$	$P_{2082}$	$P_{3105}$	$P_{4129}$	$P_{5153}$	$P_{6177}$	$P_{7201}$	$P_{8225}$	$P_{9249}$	$P_{10273}$	$P_{11297}$	$P_{12321}$	$P_{13345}$	$P_{14369}$	$P_{153}$

Line 2 intersects

 $\begin{array}{c|c} \text{Line} & \ell_1 \\ \text{in point} & P_{2082} \end{array}$ 

Line 3 intersects

 $\begin{array}{|c|c|c|} \hline \text{Line} & \ell_1 \\ \hline \text{in point} & P_{3105} \\ \hline \end{array}$ 

Line 4 intersects

 $\begin{array}{c|c} \text{Line} & \ell_1 \\ \text{in point} & P_{4129} \end{array}$ 

Line 5 intersects	
Ellie 3 Milesbessis	$\boxed{ \text{Line} \mid  \ell_1  }$
	in point $P_{5153}$
Line 6 intersects	
	Line $\ell_1$
	in point $P_{6177}$
Line 7 intersects	
	Line $\ell_1$
	in point $P_{7201}$
Line 8 intersects	
	Line $\ell_1$
	in point $P_{8225}$
Line 9 intersects	
	Line $\ell_1$
	in point $P_{9249}$
Line 10 intersects	T' (
	$\begin{array}{ c c c c }\hline \text{Line} & \ell_1\\ \hline \text{in point} & P_{10273}\\ \hline \end{array}$
	in point $P_{10273}$
Line 11 intersects	$oxed{ ext{Line} \ \ell_1}$
	in point $P_{11297}$
T1 401	11 point 1 11297
Line 12 intersects	Line $\ell_1$
	in point $P_{12321}$
I : 19 : t	1 12021
Line 13 intersects	$oxed{ ext{Line} \ \ell_1}$
	in point $P_{13345}$
Line 14 intersects	
Line 14 intersects	Line $\ell_1$
	in point $P_{14369}$
Line 15 intersects	
	Line $\ell_1$
	in point $P_{15393}$
Line 16 intersects	
	Line $\ell_1$
	$\begin{array}{c c} \text{Line} & \ell_1 \\ \text{in point} & P_{16417} \end{array}$
Line 17 intersects	in point $P_{16417}$
Line 17 intersects	in point $P_{16417}$ Line $\ell_1$
Line 17 intersects	in point $P_{16417}$
Line 17 intersects  Line 18 intersects	in point $P_{16417}$ Line $\ell_1$ in point $P_{17441}$
	$ \begin{array}{ c c c c c } \hline \text{in point} & P_{16417} \\ \hline \\ \hline & \text{Line} & \ell_1 \\ \hline & \text{in point} & P_{17441} \\ \hline \\ \hline \\ \hline \\ \hline & \text{Line} & \ell_1 \\ \hline \end{array} $
	in point $P_{16417}$ Line $\ell_1$ in point $P_{17441}$
	$\begin{array}{ c c c c }\hline \text{in point} & P_{16417}\\ \hline & \text{Line} & \ell_1\\ \hline \text{in point} & P_{17441}\\ \hline \\ & \text{Line} & \ell_1\\ \hline \text{in point} & P_{18465}\\ \hline \end{array}$
Line 18 intersects	$\begin{array}{ c c c c }\hline \text{in point} & P_{16417}\\\hline\\ & \text{Line} & \ell_1\\\hline\\ & \text{in point} & P_{17441}\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\ & \text{in point} & P_{18465}\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\$
Line 18 intersects  Line 19 intersects	$\begin{array}{ c c c c }\hline \text{in point} & P_{16417}\\ \hline & \text{Line} & \ell_1\\ \hline \text{in point} & P_{17441}\\ \hline \\ & \text{Line} & \ell_1\\ \hline \text{in point} & P_{18465}\\ \hline \end{array}$
Line 18 intersects	$\begin{array}{c c} \text{in point} & P_{16417} \\ \hline \\ \text{Line} & \ell_1 \\ \text{in point} & P_{17441} \\ \hline \\ \\ \text{Line} & \ell_1 \\ \text{in point} & P_{18465} \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $
Line 18 intersects  Line 19 intersects	$\begin{array}{ c c c c }\hline \text{in point} & P_{16417}\\\hline\\ & \text{Line} & \ell_1\\\hline\\ & \text{in point} & P_{17441}\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\ & \text{in point} & P_{18465}\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\\hline\\ & \text{Line} & \ell_1\\\hline\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\\\\\\\ & \text{Line} & \ell_1\\\hline\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\$

Line 21 intersects		
	Line	$\ell_1$
	in point	$P_{21537}$
Line 22 intersects		
	Line	$\ell_1$
	in point	$P_{22561}$
Line 23 intersects		
	Line	$\ell_1$
	in point	$P_{23585}$

Line 24 intersects		
	Line	$\ell_1$
	in point	$P_{24609}$

Line 25 intersects		
	Line	$\ell_1$
	in point	$P_{25633}$

Line 26 intersects		
	Line	$\ell_1$
	in point	$P_{26657}$

Line 27 intersects		
	Line	$\ell_1$
	in point	$P_{27681}$

Line 28 intersects		
	Line	$\ell_1$
	in point	$P_{28705}$

Line 29 intersects		
	Line	$\ell_1$
	in point	$P_{29729}$

Line 30 intersects		
	Line	$\ell_1$
	in point	$P_{30753}$

Line 31 intersects		
	Line	$\ell_1$
	in point	$P_{31777}$

Line 32 intersects		
	Line	$\ell_1$
	in point	$P_{32801}$

The surface has 1057 points: Too many to print.