

Rank-355 over GF(2)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(2) is 355

General information

Number of lines	3
Number of points	9
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	5
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	$2^2, 1^5, 0^2$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{13}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4$$

Rank of lines: (4, 3, 18)

Rank of points on Klein quadric: (2, 13, 4)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_2$$

Single Points

The surface has 5 single points:

The single points on the surface are:

$$0 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_1$$

$$1 : P_6 = (1, 0, 1, 0) \text{ lies on line } \ell_0$$

$$2 : P_9 = (1, 0, 0, 1) \text{ lies on line } \ell_2$$

$$3 : P_{13} = (1, 0, 1, 1) \text{ lies on line } \ell_2$$

$$4 : P_{14} = (0, 1, 1, 1) \text{ lies on line } \ell_1$$

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:

The points on the surface but not on lines are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_8 = (1, 1, 1, 0)$$

Line Intersection Graph

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_2

Line 1 intersects

Line	ℓ_0
in point	P_0

Line 2 intersects

Line	ℓ_0
in point	P_2

The surface has 9 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_6 = (1, 0, 1, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_9 = (1, 0, 0, 1)$$

$$7 : P_{13} = (1, 0, 1, 1)$$

$$8 : P_{14} = (0, 1, 1, 1)$$