

Rank-76051 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1243914829

General information

Number of lines	5
Number of points	89
Number of singular points	0
Number of Eckardt points	1
Number of double points	2
Number of single points	38
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	$3, 2^2, 1^{38}, 0^{48}$

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{64} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\ \ell_1 &= \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4680} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17}\end{aligned}$$

$$\begin{aligned}\ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_3 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25} \\ \ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{1330}\end{aligned}$$

Rank of lines: (64, 4680, 4744, 4689, 722)

Rank of points on Klein quadric: (2, 17, 1, 25, 1330)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1).$$

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_2$$

$$P_{138} = (0, 0, 1, 1) = \ell_2 \cap \ell_4$$

Single Points

The surface has 38 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_1 = (0, 1, 0, 0) \text{ lies on line } \ell_1$$

$$2 : P_{12} = (1, 0, 1, 0) \text{ lies on line } \ell_0$$

$$3 : P_{13} = (2, 0, 1, 0) \text{ lies on line } \ell_0$$

$$4 : P_{14} = (3, 0, 1, 0) \text{ lies on line } \ell_0$$

$$5 : P_{15} = (4, 0, 1, 0) \text{ lies on line } \ell_0$$

$$6 : P_{16} = (5, 0, 1, 0) \text{ lies on line } \ell_0$$

$$7 : P_{17} = (6, 0, 1, 0) \text{ lies on line } \ell_0$$

$$8 : P_{18} = (7, 0, 1, 0) \text{ lies on line } \ell_0$$

$$9 : P_{19} = (0, 1, 1, 0) \text{ lies on line } \ell_3$$

$$10 : P_{20} = (1, 1, 1, 0) \text{ lies on line } \ell_4$$

$$11 : P_{82} = (0, 1, 0, 1) \text{ lies on line } \ell_1$$

$$12 : P_{83} = (1, 1, 0, 1) \text{ lies on line } \ell_4$$

$$13 : P_{90} = (0, 2, 0, 1) \text{ lies on line } \ell_1$$

$$14 : P_{98} = (0, 3, 0, 1) \text{ lies on line } \ell_1$$

$$15 : P_{106} = (0, 4, 0, 1) \text{ lies on line } \ell_1$$

$$16 : P_{114} = (0, 5, 0, 1) \text{ lies on line } \ell_1$$

$$17 : P_{122} = (0, 6, 0, 1) \text{ lies on line } \ell_1$$

$$18 : P_{130} = (0, 7, 0, 1) \text{ lies on line } \ell_1$$

$$19 : P_{146} = (0, 1, 1, 1) \text{ lies on line } \ell_3$$

$$20 : P_{201} = (0, 0, 2, 1) \text{ lies on line } \ell_2$$

$$21 : P_{217} = (0, 2, 2, 1) \text{ lies on line } \ell_3$$

$$22 : P_{228} = (3, 3, 2, 1) \text{ lies on line } \ell_4$$

$$23 : P_{265} = (0, 0, 3, 1) \text{ lies on line } \ell_2$$

$$24 : P_{283} = (2, 2, 3, 1) \text{ lies on line } \ell_4$$

$$25 : P_{289} = (0, 3, 3, 1) \text{ lies on line } \ell_3$$

$$26 : P_{329} = (0, 0, 4, 1) \text{ lies on line } \ell_2$$

$$27 : P_{361} = (0, 4, 4, 1) \text{ lies on line } \ell_3$$

$$28 : P_{374} = (5, 5, 4, 1) \text{ lies on line } \ell_4$$

$$29 : P_{393} = (0, 0, 5, 1) \text{ lies on line } \ell_2$$

$$30 : P_{429} = (4, 4, 5, 1) \text{ lies on line } \ell_4$$

$$31 : P_{433} = (0, 5, 5, 1) \text{ lies on line } \ell_3$$

$$32 : P_{457} = (0, 0, 6, 1) \text{ lies on line } \ell_2$$

$$33 : P_{505} = (0, 6, 6, 1) \text{ lies on line } \ell_3$$

$$34 : P_{520} = (7, 7, 6, 1) \text{ lies on line } \ell_4$$

$$35 : P_{521} = (0, 0, 7, 1) \text{ lies on line } \ell_2$$

$$36 : P_{575} = (6, 6, 7, 1) \text{ lies on line } \ell_4$$

$$37 : P_{577} = (0, 7, 7, 1) \text{ lies on line } \ell_3$$

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

0 : $P_{31} = (4, 2, 1, 0)$	25 : $P_{344} = (7, 1, 4, 1)$
1 : $P_{32} = (5, 2, 1, 0)$	26 : $P_{349} = (4, 2, 4, 1)$
2 : $P_{49} = (6, 4, 1, 0)$	27 : $P_{351} = (6, 2, 4, 1)$
3 : $P_{50} = (7, 4, 1, 0)$	28 : $P_{358} = (5, 3, 4, 1)$
4 : $P_{69} = (2, 7, 1, 0)$	29 : $P_{368} = (7, 4, 4, 1)$
5 : $P_{70} = (3, 7, 1, 0)$	30 : $P_{375} = (6, 5, 4, 1)$
6 : $P_{96} = (6, 2, 0, 1)$	31 : $P_{379} = (2, 6, 4, 1)$
7 : $P_{102} = (4, 3, 0, 1)$	32 : $P_{381} = (4, 6, 4, 1)$
8 : $P_{109} = (3, 4, 0, 1)$	33 : $P_{427} = (2, 4, 5, 1)$
9 : $P_{121} = (7, 5, 0, 1)$	34 : $P_{435} = (2, 5, 5, 1)$
10 : $P_{124} = (2, 6, 0, 1)$	35 : $P_{453} = (4, 7, 5, 1)$
11 : $P_{135} = (5, 7, 0, 1)$	36 : $P_{480} = (7, 2, 6, 1)$
12 : $P_{213} = (4, 1, 2, 1)$	37 : $P_{509} = (4, 6, 6, 1)$
13 : $P_{216} = (7, 1, 2, 1)$	38 : $P_{517} = (4, 7, 6, 1)$
14 : $P_{221} = (4, 2, 2, 1)$	39 : $P_{531} = (2, 1, 7, 1)$
15 : $P_{230} = (5, 3, 2, 1)$	40 : $P_{533} = (4, 1, 7, 1)$
16 : $P_{243} = (2, 5, 2, 1)$	41 : $P_{549} = (4, 3, 7, 1)$
17 : $P_{248} = (7, 5, 2, 1)$	42 : $P_{552} = (7, 3, 7, 1)$
18 : $P_{252} = (3, 6, 2, 1)$	43 : $P_{556} = (3, 4, 7, 1)$
19 : $P_{259} = (2, 7, 2, 1)$	44 : $P_{560} = (7, 4, 7, 1)$
20 : $P_{262} = (5, 7, 2, 1)$	45 : $P_{567} = (6, 5, 7, 1)$
21 : $P_{288} = (7, 2, 3, 1)$	46 : $P_{572} = (3, 6, 7, 1)$
22 : $P_{296} = (7, 3, 3, 1)$	47 : $P_{579} = (2, 7, 7, 1)$
23 : $P_{299} = (2, 4, 3, 1)$	
24 : $P_{339} = (2, 1, 4, 1)$	

Line Intersection Graph

	0 1 2 3 4
0	0 0 1 0 0
1	0 0 1 1 0
2	1 1 0 1 1
3	0 1 1 0 0
4	0 0 1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_2
in point	P_2

Line 1 intersects

Line	ℓ_2	ℓ_3
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_3	P_3	P_{138}

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 4 intersects

Line	ℓ_2
in point	P_{138}

The surface has 89 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	30 : $P_{124} = (2, 6, 0, 1)$	60 : $P_{368} = (7, 4, 4, 1)$
1 : $P_1 = (0, 1, 0, 0)$	31 : $P_{130} = (0, 7, 0, 1)$	61 : $P_{374} = (5, 5, 4, 1)$
2 : $P_2 = (0, 0, 1, 0)$	32 : $P_{135} = (5, 7, 0, 1)$	62 : $P_{375} = (6, 5, 4, 1)$
3 : $P_3 = (0, 0, 0, 1)$	33 : $P_{138} = (0, 0, 1, 1)$	63 : $P_{379} = (2, 6, 4, 1)$
4 : $P_{12} = (1, 0, 1, 0)$	34 : $P_{146} = (0, 1, 1, 1)$	64 : $P_{381} = (4, 6, 4, 1)$
5 : $P_{13} = (2, 0, 1, 0)$	35 : $P_{201} = (0, 0, 2, 1)$	65 : $P_{393} = (0, 0, 5, 1)$
6 : $P_{14} = (3, 0, 1, 0)$	36 : $P_{213} = (4, 1, 2, 1)$	66 : $P_{427} = (2, 4, 5, 1)$
7 : $P_{15} = (4, 0, 1, 0)$	37 : $P_{216} = (7, 1, 2, 1)$	67 : $P_{429} = (4, 4, 5, 1)$
8 : $P_{16} = (5, 0, 1, 0)$	38 : $P_{217} = (0, 2, 2, 1)$	68 : $P_{433} = (0, 5, 5, 1)$
9 : $P_{17} = (6, 0, 1, 0)$	39 : $P_{221} = (4, 2, 2, 1)$	69 : $P_{435} = (2, 5, 5, 1)$
10 : $P_{18} = (7, 0, 1, 0)$	40 : $P_{228} = (3, 3, 2, 1)$	70 : $P_{453} = (4, 7, 5, 1)$
11 : $P_{19} = (0, 1, 1, 0)$	41 : $P_{230} = (5, 3, 2, 1)$	71 : $P_{457} = (0, 0, 6, 1)$
12 : $P_{20} = (1, 1, 1, 0)$	42 : $P_{243} = (2, 5, 2, 1)$	72 : $P_{480} = (7, 2, 6, 1)$
13 : $P_{31} = (4, 2, 1, 0)$	43 : $P_{248} = (7, 5, 2, 1)$	73 : $P_{505} = (0, 6, 6, 1)$
14 : $P_{32} = (5, 2, 1, 0)$	44 : $P_{252} = (3, 6, 2, 1)$	74 : $P_{509} = (4, 6, 6, 1)$
15 : $P_{49} = (6, 4, 1, 0)$	45 : $P_{259} = (2, 7, 2, 1)$	75 : $P_{517} = (4, 7, 6, 1)$
16 : $P_{50} = (7, 4, 1, 0)$	46 : $P_{262} = (5, 7, 2, 1)$	76 : $P_{520} = (7, 7, 6, 1)$
17 : $P_{69} = (2, 7, 1, 0)$	47 : $P_{265} = (0, 0, 3, 1)$	77 : $P_{521} = (0, 0, 7, 1)$
18 : $P_{70} = (3, 7, 1, 0)$	48 : $P_{283} = (2, 2, 3, 1)$	78 : $P_{531} = (2, 1, 7, 1)$
19 : $P_{82} = (0, 1, 0, 1)$	49 : $P_{288} = (7, 2, 3, 1)$	79 : $P_{533} = (4, 1, 7, 1)$
20 : $P_{83} = (1, 1, 0, 1)$	50 : $P_{289} = (0, 3, 3, 1)$	80 : $P_{549} = (4, 3, 7, 1)$
21 : $P_{90} = (0, 2, 0, 1)$	51 : $P_{296} = (7, 3, 3, 1)$	81 : $P_{552} = (7, 3, 7, 1)$
22 : $P_{96} = (6, 2, 0, 1)$	52 : $P_{299} = (2, 4, 3, 1)$	82 : $P_{556} = (3, 4, 7, 1)$
23 : $P_{98} = (0, 3, 0, 1)$	53 : $P_{329} = (0, 0, 4, 1)$	83 : $P_{560} = (7, 4, 7, 1)$
24 : $P_{102} = (4, 3, 0, 1)$	54 : $P_{339} = (2, 1, 4, 1)$	84 : $P_{567} = (6, 5, 7, 1)$
25 : $P_{106} = (0, 4, 0, 1)$	55 : $P_{344} = (7, 1, 4, 1)$	85 : $P_{572} = (3, 6, 7, 1)$
26 : $P_{109} = (3, 4, 0, 1)$	56 : $P_{349} = (4, 2, 4, 1)$	86 : $P_{575} = (6, 6, 7, 1)$
27 : $P_{114} = (0, 5, 0, 1)$	57 : $P_{351} = (6, 2, 4, 1)$	87 : $P_{577} = (0, 7, 7, 1)$
28 : $P_{121} = (7, 5, 0, 1)$	58 : $P_{358} = (5, 3, 4, 1)$	88 : $P_{579} = (2, 7, 7, 1)$
29 : $P_{122} = (0, 6, 0, 1)$	59 : $P_{361} = (0, 4, 4, 1)$	