

Rank-346 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(4) is 26038

General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	6
Number of double points	0
Number of single points	27
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	$3^6, 1^{27}$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{198} \\ \ell_1 &= \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{81} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{81} = \mathbf{Pl}(0, 0, 3, 2, 3, 1)_{332}\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{61} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{61} = \mathbf{Pl}(0, 0, 2, 3, 2, 1)_{265} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{180} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{72} = \mathbf{Pl}(3, 2, 3, 0, 3, 1)_{308} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{55} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{55} = \mathbf{Pl}(2, 3, 2, 0, 2, 1)_{244} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1, 1, 0, 1, 1, 1)_{189} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{155} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{155} = \mathbf{Pl}(3, 2, 0, 2, 3, 1)_{314} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{138} = \mathbf{Pl}(2, 3, 0, 3, 2, 1)_{256}
\end{aligned}$$

Rank of lines: (38, 81, 61, 26, 72, 55, 109, 155, 138)

Rank of points on Klein quadric: (198, 332, 265, 180, 308, 244, 189, 314, 256)

Eckardt Points

The surface has 6 Eckardt points:

- 0 : $P_{47} = \mathbf{P}(\omega, \omega, 1, 1) = \mathbf{P}(2, 2, 1, 1)$,
- 1 : $P_{52} = \mathbf{P}(\omega^2, \omega^2, 1, 1) = \mathbf{P}(3, 3, 1, 1)$,
- 2 : $P_{60} = \mathbf{P}(\omega^2, 1, \omega, 1) = \mathbf{P}(3, 1, 2, 1)$,
- 3 : $P_{67} = \mathbf{P}(\omega, \omega^2, \omega, 1) = \mathbf{P}(2, 3, 2, 1)$,
- 4 : $P_{75} = \mathbf{P}(\omega, 1, \omega^2, 1) = \mathbf{P}(2, 1, 3, 1)$,
- 5 : $P_{80} = \mathbf{P}(\omega^2, \omega, \omega^2, 1) = \mathbf{P}(3, 2, 3, 1)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 27 single points:

The single points on the surface are:

- | | |
|---|--|
| 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_0 | 8 : $P_{13} = (2, 1, 1, 0)$ lies on line ℓ_7 |
| 1 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 | 9 : $P_{14} = (3, 1, 1, 0)$ lies on line ℓ_8 |
| 2 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_1 | 10 : $P_{26} = (0, 1, 0, 1)$ lies on line ℓ_6 |
| 3 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_2 | 11 : $P_{27} = (1, 1, 0, 1)$ lies on line ℓ_3 |
| 4 : $P_8 = (1, 0, 1, 0)$ lies on line ℓ_3 | 12 : $P_{30} = (0, 2, 0, 1)$ lies on line ℓ_8 |
| 5 : $P_9 = (2, 0, 1, 0)$ lies on line ℓ_4 | 13 : $P_{31} = (1, 2, 0, 1)$ lies on line ℓ_5 |
| 6 : $P_{10} = (3, 0, 1, 0)$ lies on line ℓ_5 | 14 : $P_{34} = (0, 3, 0, 1)$ lies on line ℓ_7 |
| 7 : $P_{12} = (1, 1, 1, 0)$ lies on line ℓ_6 | 15 : $P_{35} = (1, 3, 0, 1)$ lies on line ℓ_4 |

16 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_0
17 : $P_{39} = (1, 0, 1, 1)$ lies on line ℓ_6
18 : $P_{42} = (0, 1, 1, 1)$ lies on line ℓ_3
19 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_2
20 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_8
21 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_5

22 : $P_{62} = (1, 2, 2, 1)$ lies on line ℓ_2
23 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_1
24 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_7
25 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_4
26 : $P_{82} = (1, 3, 3, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	1	1	0	1	1
1	0	0	0	1	0	1	1	0	1
2	0	0	0	1	1	0	1	1	0
3	0	1	1	0	0	0	0	1	1
4	1	0	1	0	0	0	1	0	1
5	1	1	0	0	0	0	1	1	0
6	0	1	1	0	1	1	0	0	0
7	1	0	1	1	0	1	0	0	0
8	1	1	0	1	1	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{52}	P_{47}	P_{47}	P_{52}

Line 1 intersects

Line	ℓ_3	ℓ_5	ℓ_6	ℓ_8
in point	P_{75}	P_{80}	P_{80}	P_{75}

Line 2 intersects

Line	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_{60}	P_{67}	P_{67}	P_{60}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8
in point	P_{75}	P_{60}	P_{60}	P_{75}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8
in point	P_{52}	P_{67}	P_{67}	P_{52}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7
in point	P_{47}	P_{80}	P_{80}	P_{47}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{80}	P_{67}	P_{67}	P_{80}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_{47}	P_{60}	P_{60}	P_{47}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{52}	P_{75}	P_{75}	P_{52}

The surface has 33 points:

The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$	12 : $P_{30} = (0, 2, 0, 1)$	24 : $P_{61} = (0, 2, 2, 1)$
1 : $P_5 = (1, 1, 0, 0)$	13 : $P_{31} = (1, 2, 0, 1)$	25 : $P_{62} = (1, 2, 2, 1)$
2 : $P_6 = (2, 1, 0, 0)$	14 : $P_{34} = (0, 3, 0, 1)$	26 : $P_{67} = (2, 3, 2, 1)$
3 : $P_7 = (3, 1, 0, 0)$	15 : $P_{35} = (1, 3, 0, 1)$	27 : $P_{69} = (0, 0, 3, 1)$
4 : $P_8 = (1, 0, 1, 0)$	16 : $P_{38} = (0, 0, 1, 1)$	28 : $P_{70} = (1, 0, 3, 1)$
5 : $P_9 = (2, 0, 1, 0)$	17 : $P_{39} = (1, 0, 1, 1)$	29 : $P_{75} = (2, 1, 3, 1)$
6 : $P_{10} = (3, 0, 1, 0)$	18 : $P_{42} = (0, 1, 1, 1)$	30 : $P_{80} = (3, 2, 3, 1)$
7 : $P_{12} = (1, 1, 1, 0)$	19 : $P_{47} = (2, 2, 1, 1)$	31 : $P_{81} = (0, 3, 3, 1)$
8 : $P_{13} = (2, 1, 1, 0)$	20 : $P_{52} = (3, 3, 1, 1)$	32 : $P_{82} = (1, 3, 3, 1)$
9 : $P_{14} = (3, 1, 1, 0)$	21 : $P_{53} = (0, 0, 2, 1)$	
10 : $P_{26} = (0, 1, 0, 1)$	22 : $P_{54} = (1, 0, 2, 1)$	
11 : $P_{27} = (1, 1, 0, 1)$	23 : $P_{60} = (3, 1, 2, 1)$	