Rank-74057 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1090789509

General information

Number of lines	3
Number of points	4033
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	3840
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$3, 1^{192}, 0^{3840}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & \epsilon^9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17046575} = \begin{bmatrix} 0 & 1 & 47 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17046575} = \mathbf{Pl}(0, 47, 0, 1, 0, 0)_{239}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & \epsilon^{18} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17044235} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17044235} = \mathbf{Pl}(0, 11, 0, 1, 0, 0)_{203}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & \epsilon^{36} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17045860} = \begin{bmatrix} 0 & 1 & 36 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17045860} = \mathbf{Pl}(0, 36, 0, 1, 0, 0)_{228}$$

Rank of lines: (17046575, 17044235, 17045860)
Rank of points on Klein quadric: (239, 203, 228)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1).$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_{707} = (0, 10, 1, 0)$ lies on line ℓ_0
$1 : P_{2435} = (0, 37, 1, 0)$ lies on line ℓ_1
$2 : P_{3011} = (0, 46, 1, 0)$ lies on line ℓ_2
$3 : P_{8897} = (0, 10, 1, 1)$ lies on line ℓ_0
$4 : P_{10625} = (0, 37, 1, 1)$ lies on line ℓ_1
$5 : P_{11201} = (0, 46, 1, 1)$ lies on line ℓ_2
$6 : P_{13633} = (0, 20, 2, 1)$ lies on line ℓ_0
$7 : P_{15105} = (0, 43, 2, 1)$ lies on line ℓ_1
$8 : P_{16257} = (0, 61, 2, 1)$ lies on line ℓ_2
$9 : P_{17345} = (0, 14, 3, 1)$ lies on line ℓ_1
$10 : P_{17665} = (0, 19, 3, 1)$ lies on line ℓ_2
$11 : P_{18369} = (0, 30, 3, 1)$ lies on line ℓ_0
$12 : P_{22273} = (0, 27, 4, 1)$ lies on line ℓ_2
$13 : P_{23105} = (0, 40, 4, 1)$ lies on line ℓ_0
$14 : P_{24065} = (0, 55, 4, 1)$ lies on line ℓ_1
$15 : P_{25793} = (0, 18, 5, 1)$ lies on line ℓ_1
$16 : P_{26817} = (0, 34, 5, 1)$ lies on line ℓ_0
$17 : P_{28033} = (0, 53, 5, 1)$ lies on line ℓ_2
$18 : P_{30529} = (0, 28, 6, 1)$ lies on line ℓ_1
$19 : P_{31169} = (0, 38, 6, 1)$ lies on line ℓ_2
$20 : P_{32577} = (0, 60, 6, 1)$ lies on line ℓ_0
$21 : P_{33345} = (0, 8, 7, 1)$ lies on line ℓ_2
$22 : P_{36289} = (0, 54, 7, 1)$ lies on line ℓ_0
$23 : P_{36481} = (0, 57, 7, 1)$ lies on line ℓ_1
$24 : P_{37889} = (0, 15, 8, 1)$ lies on line ℓ_1
$25 : P_{40065} = (0, 49, 8, 1)$ lies on line ℓ_0
$26 : P_{40385} = (0, 54, 8, 1)$ lies on line ℓ_2
$27 : P_{42561} = (0, 24, 9, 1)$ lies on line ℓ_2
$28 : P_{43713} = (0, 42, 9, 1)$ lies on line ℓ_1

$29 : P_{44801} = (0, 59, 9, 1)$ lies on line ℓ_0
$30 : P_{45825} = (0, 11, 10, 1)$ lies on line ℓ_2
$31 : P_{47425} = (0, 36, 10, 1)$ lies on line ℓ_1
$32 : P_{47489} = (0, 37, 10, 1)$ lies on line ℓ_0
$33 : P_{49281} = (0, 1, 11, 1)$ lies on line ℓ_1
$34 : P_{51585} = (0, 37, 11, 1)$ lies on line ℓ_2
$35 : P_{52225} = (0, 47, 11, 1)$ lies on line ℓ_0
$36 : P_{54913} = (0, 25, 12, 1)$ lies on line ℓ_0
$37 : P_{56193} = (0, 45, 12, 1)$ lies on line ℓ_2
$38 : P_{56897} = (0, 56, 12, 1)$ lies on line ℓ_1
$39 : P_{57601} = (0, 3, 13, 1)$ lies on line ℓ_2
$40 : P_{58625} = (0, 19, 13, 1)$ lies on line ℓ_0
$41 : P_{59265} = (0, 29, 13, 1)$ lies on line ℓ_1
$42 : P_{62337} = (0, 13, 14, 1)$ lies on line ℓ_0
$43 : P_{62529} = (0, 16, 14, 1)$ lies on line ℓ_2
$44 : P_{62721} = (0, 19, 14, 1)$ lies on line ℓ_1
$45 : P_{66049} = (0, 7, 15, 1)$ lies on line ℓ_0
$46 : P_{69057} = (0, 54, 15, 1)$ lies on line ℓ_1
$47 : P_{69569} = (0, 62, 15, 1)$ lies on line ℓ_2
$48 : P_{69889} = (0, 3, 16, 1)$ lies on line ℓ_0
$49 : P_{70529} = (0, 13, 16, 1)$ lies on line ℓ_2
$50 : P_{71617} = (0, 30, 16, 1)$ lies on line ℓ_1
$51 : P_{74369} = (0, 9, 17, 1)$ lies on line ℓ_0
$52 : P_{76033} = (0, 35, 17, 1)$ lies on line ℓ_2
$53 : P_{77569} = (0, 59, 17, 1)$ lies on line ℓ_1
$54 : P_{79361} = (0, 23, 18, 1)$ lies on line ℓ_0
$55 : P_{80961} = (0, 48, 18, 1)$ lies on line ℓ_2
$56 : P_{81281} = (0, 53, 18, 1)$ lies on line ℓ_1
$57 : P_{83009} = (0, 16, 19, 1)$ lies on line ℓ_1

$58 : P_{83841} = (0, 29, 19, 1)$ lies on line ℓ_0

$59 : P_{83905} = (0, 30, 19, 1)$ lies on line ℓ_2

$60 : P_{87489} = (0, 22, 20, 1)$ lies on line ℓ_2

$61 : P_{88705} = (0, 41, 20, 1)$ lies on line ℓ_1

$62 : P_{88833} = (0, 43, 20, 1)$ lies on line ℓ_0

$63 : P_{90945} = (0, 12, 21, 1)$ lies on line ℓ_1

$64 : P_{92289} = (0, 33, 21, 1)$ lies on line ℓ_0

$65 : P_{93761} = (0, 56, 21, 1)$ lies on line ℓ_2

$66 : P_{94401} = (0, 2, 22, 1)$ lies on line ℓ_1

$67 : P_{97025} = (0, 43, 22, 1)$ lies on line ℓ_2

$68 : P_{98305} = (0, 63, 22, 1)$ lies on line ℓ_0

$69 : P_{98689} = (0, 5, 23, 1)$ lies on line ℓ_2

$70 : P_{100865} = (0, 39, 23, 1)$ lies on line ℓ_1

$71 : P_{101761} = (0, 53, 23, 1)$ lies on line ℓ_0

$72 : P_{103553} = (0, 17, 24, 1)$ lies on line ℓ_1

$73 : P_{105665} = (0, 50, 24, 1)$ lies on line ℓ_0

$74 : P_{106241} = (0, 59, 24, 1)$ lies on line ℓ_2

$75 : P_{107905} = (0, 21, 25, 1)$ lies on line ℓ_2

$76 : P_{109889} = (0, 52, 25, 1)$ lies on line ℓ_1

$77 : P_{110145} = (0, 56, 25, 1)$ lies on line ℓ_0

$78 : P_{111041} = (0, 6, 26, 1)$ lies on line ℓ_2

$79 : P_{113089} = (0, 38, 26, 1)$ lies on line ℓ_0

$80 : P_{114369} = (0, 58, 26, 1)$ lies on line ℓ_1

$81 : P_{116737} = (0, 31, 27, 1)$ lies on line ℓ_1

$82 : P_{117313} = (0, 40, 27, 1)$ lies on line ℓ_2

$83 : P_{117569} = (0, 44, 27, 1)$ lies on line ℓ_0

$84 : P_{120513} = (0, 26, 28, 1)$ lies on line ℓ_0

$85 : P_{120897} = (0, 32, 28, 1)$ lies on line ℓ_2

$86 : P_{121281} = (0, 38, 28, 1)$ lies on line ℓ_1

$87 : P_{123137} = (0, 3, 29, 1)$ lies on line ℓ_1

$88 : P_{123841} = (0, 14, 29, 1)$ lies on line ℓ_2

$89 : P_{123969} = (0, 16, 29, 1)$ lies on line ℓ_0

$90 : P_{127873} = (0, 13, 30, 1)$ lies on line ℓ_1

$91 : P_{127937} = (0, 14, 30, 1)$ lies on line ℓ_0

$92 : P_{128897} = (0, 29, 30, 1)$ lies on line ℓ_2

$93 : P_{131393} = (0, 4, 31, 1)$ lies on line ℓ_0

$94 : P_{133697} = (0, 40, 31, 1)$ lies on line ℓ_1

$95 : P_{134401} = (0, 51, 31, 1)$ lies on line ℓ_2

$96 : P_{135617} = (0, 6, 32, 1)$ lies on line ℓ_0

$97 : P_{136897} = (0, 26, 32, 1)$ lies on line ℓ_2

$98 : P_{139073} = (0, 60, 32, 1)$ lies on line ℓ_1

$99 : P_{140097} = (0, 12, 33, 1)$ lies on line ℓ_0

$100 : P_{140929} = (0, 25, 33, 1)$ lies on line ℓ_1

$101 : P_{142657} = (0, 52, 33, 1)$ lies on line ℓ_2

$102 : P_{144577} = (0, 18, 34, 1)$ lies on line ℓ_0

$103 : P_{144897} = (0, 23, 34, 1)$ lies on line ℓ_1

$104 : P_{145921} = (0, 39, 34, 1)$ lies on line ℓ_2

$105 : P_{148097} = (0, 9, 35, 1)$ lies on line ℓ_2

$106 : P_{149057} = (0, 24, 35, 1)$ lies on line ℓ_0

$107 : P_{150721} = (0, 50, 35, 1)$ lies on line ℓ_1

$108 : P_{151681} = (0, 1, 36, 1)$ lies on line ℓ_2

$109 : P_{152321} = (0, 11, 36, 1)$ lies on line ℓ_1

$110 : P_{154561} = (0, 46, 36, 1)$ lies on line ℓ_0

$111 : P_{158017} = (0, 36, 37, 1)$ lies on line ℓ_0

$112 : P_{158657} = (0, 46, 37, 1)$ lies on line ℓ_1

$113 : P_{158721} = (0, 47, 37, 1)$ lies on line ℓ_2

$114 : P_{161857} = (0, 32, 38, 1)$ lies on line ℓ_1

$115 : P_{163521} = (0, 58, 38, 1)$ lies on line ℓ_0

$116 : P_{163649} = (0, 60, 38, 1)$ lies on line ℓ_2

$117 : P_{164225} = (0, 5, 39, 1)$ lies on line ℓ_1

$118 : P_{165057} = (0, 18, 39, 1)$ lies on line ℓ_2

$119 : P_{166977} = (0, 48, 39, 1)$ lies on line ℓ_0

$120 : P_{170817} = (0, 44, 40, 1)$ lies on line ℓ_2

$121 : P_{171265} = (0, 51, 40, 1)$ lies on line ℓ_1

$122 : P_{171521} = (0, 55, 40, 1)$ lies on line ℓ_0

$123 : P_{172225} = (0, 2, 41, 1)$ lies on line ℓ_2

$124 : P_{173505} = (0, 22, 41, 1)$ lies on line ℓ_1

$125 : P_{176001} = (0, 61, 41, 1)$ lies on line ℓ_0

$126 : P_{177281} = (0, 17, 42, 1)$ lies on line ℓ_2

$127 : P_{177729} = (0, 24, 42, 1)$ lies on line ℓ_1

$128 : P_{178433} = (0, 35, 42, 1)$ lies on line ℓ_0

$129 : P_{182913} = (0, 41, 43, 1)$ lies on line ℓ_0

$130 : P_{184193} = (0, 61, 43, 1)$ lies on line ℓ_1

$131 : P_{184321} = (0, 63, 43, 1)$ lies on line ℓ_2

$132 : P_{184641} = (0, 4, 44, 1)$ lies on line ℓ_1

$133 : P_{186369} = (0, 31, 44, 1)$ lies on line ℓ_0

$134 : P_{187905} = (0, 55, 44, 1)$ lies on line ℓ_2

$135 : P_{189825} = (0, 21, 45, 1)$ lies on line ℓ_0

$136 : P_{190081} = (0, 25, 45, 1)$ lies on line ℓ_2

$137 : P_{190593} = (0, 33, 45, 1)$ lies on line ℓ_1

$138 : P_{193217} = (0, 10, 46, 1)$ lies on line ℓ_2

$139 : P_{193281} = (0, 11, 46, 1)$ lies on line ℓ_0

$140 : P_{195585} = (0, 47, 46, 1)$ lies on line ℓ_1

$141 : P_{196737} = (0, 1, 47, 1)$ lies on line ℓ_0

$142 : P_{197313} = (0, 10, 47, 1)$ lies on line ℓ_1

$143 : P_{198977} = (0, 36, 47, 1)$ lies on line ℓ_2

$144 : P_{201089} = (0, 5, 48, 1)$ lies on line ℓ_0

$145 : P_{202241} = (0, 23, 48, 1)$ lies on line ℓ_2

$146 : P_{202945} = (0, 34, 48, 1)$ lies on line ℓ_1

$147 : P_{205313} = (0, 7, 49, 1)$ lies on line ℓ_1

$148 : P_{205825} = (0, 15, 49, 1)$ lies on line ℓ_0

$149 : P_{208513} = (0, 57, 49, 1)$ lies on line ℓ_2

$150 : P_{209537} = (0, 9, 50, 1)$ lies on line ℓ_1

$151 : P_{210049} = (0, 17, 50, 1)$ lies on line ℓ_0

$152 : P_{211649} = (0, 42, 50, 1)$ lies on line ℓ_2

$153 : P_{213313} = (0, 4, 51, 1)$ lies on line ℓ_2

$154 : P_{214785} = (0, 27, 51, 1)$ lies on line ℓ_0

$155 : P_{215873} = (0, 44, 51, 1)$ lies on line ℓ_1

$156 : P_{217921} = (0, 12, 52, 1)$ lies on line ℓ_2

$157 : P_{218497} = (0, 21, 52, 1)$ lies on line ℓ_1

$158 : P_{220033} = (0, 45, 52, 1)$ lies on line ℓ_0

$159 : P_{223425} = (0, 34, 53, 1)$ lies on line ℓ_2

$160 : P_{223745} = (0, 39, 53, 1)$ lies on line ℓ_0

$161 : P_{224321} = (0, 48, 53, 1)$ lies on line ℓ_1

$162 : P_{228481} = (0, 49, 54, 1)$ lies on line ℓ_2

$163 : P_{228993} = (0, 57, 54, 1)$ lies on line ℓ_0

$164 : P_{229313} = (0, 62, 54, 1)$ lies on line ℓ_1

$165 : P_{231169} = (0, 27, 55, 1)$ lies on line ℓ_1

166 : $P_{231425} = (0, 31, 55, 1)$ lies on line ℓ_2
167 : $P_{232705} = (0, 51, 55, 1)$ lies on line ℓ_0
168 : $P_{235649} = (0, 33, 56, 1)$ lies on line ℓ_2
169 : $P_{236417} = (0, 45, 56, 1)$ lies on line ℓ_1
170 : $P_{236865} = (0, 52, 56, 1)$ lies on line ℓ_0
171 : $P_{238145} = (0, 8, 57, 1)$ lies on line ℓ_1
172 : $P_{238593} = (0, 15, 57, 1)$ lies on line ℓ_2
173 : $P_{241601} = (0, 62, 57, 1)$ lies on line ℓ_0
174 : $P_{242113} = (0, 6, 58, 1)$ lies on line ℓ_1
175 : $P_{243521} = (0, 28, 58, 1)$ lies on line ℓ_2
176 : $P_{243777} = (0, 32, 58, 1)$ lies on line ℓ_0
177 : $P_{248065} = (0, 35, 59, 1)$ lies on line ℓ_1
178 : $P_{248513} = (0, 42, 59, 1)$ lies on line ℓ_0
179 : $P_{249025} = (0, 50, 59, 1)$ lies on line ℓ_2

180 : $P_{251585} = (0, 26, 60, 1)$ lies on line ℓ_1
181 : $P_{251713} = (0, 28, 60, 1)$ lies on line ℓ_0
182 : $P_{253633} = (0, 58, 60, 1)$ lies on line ℓ_2
183 : $P_{255297} = (0, 20, 61, 1)$ lies on line ℓ_2
184 : $P_{255425} = (0, 22, 61, 1)$ lies on line ℓ_0
185 : $P_{258049} = (0, 63, 61, 1)$ lies on line ℓ_1
186 : $P_{258561} = (0, 7, 62, 1)$ lies on line ℓ_2
187 : $P_{258625} = (0, 8, 62, 1)$ lies on line ℓ_0
188 : $P_{261249} = (0, 49, 62, 1)$ lies on line ℓ_1
189 : $P_{262337} = (0, 2, 63, 1)$ lies on line ℓ_0
190 : $P_{263489} = (0, 20, 63, 1)$ lies on line ℓ_1
191 : $P_{264833} = (0, 41, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 3840 points not on any line:
Too many to print.

Line Intersection Graph

```
  |0 1 2
--+------
0 |0 1 1
1 |1 0 1
2 |1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 4033 points:
Too many to print.