Rank-65666 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227433110

General information

Number of lines	3
Number of points	89
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	3, 1^{24}, 0^{64}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & \gamma^6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3578} = \begin{bmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3578} = \mathbf{Pl}(2,6,1,6,0,1)_{1120}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & \gamma^5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1826} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1826} = \mathbf{Pl}(4,3,1,3,0,1)_{975}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & \gamma^3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2994} = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2994} = \mathbf{Pl}(7,5,1,5,0,1)_{1076}$$

Rank of lines: (3578, 1826, 2994)
Rank of points on Klein quadric: (1120, 975, 1076)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{19} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_{92} = (2,2,0,1)$ lies on line ℓ_0
$1 : P_{110} = (4,4,0,1)$ lies on line ℓ_1
$2 : P_{137} = (7,7,0,1)$ lies on line ℓ_2
$3 : P_{163} = (2,3,1,1)$ lies on line ℓ_0
$4 : P_{181} = (4,5,1,1)$ lies on line ℓ_1
$5 : P_{192} = (7,6,1,1)$ lies on line ℓ_2
$6 : P_{203} = (2,0,2,1)$ lies on line ℓ_0
$7 : P_{248} = (7,5,2,1)$ lies on line ℓ_2
$8 : P_{253} = (4,6,2,1)$ lies on line ℓ_1
$9 : P_{275} = (2,1,3,1)$ lies on line ℓ_0
$10 : P_{304} = (7,4,3,1)$ lies on line ℓ_2
$11 : P_{325} = (4,7,3,1)$ lies on line ℓ_1
$12 : P_{333} = (4,0,4,1)$ lies on line ℓ_1
$13 : P_{360} = (7,3,4,1)$ lies on line ℓ_2
$14 : P_{379} = (2,6,4,1)$ lies on line ℓ_0
$15 : P_{405} = (4,1,5,1)$ lies on line ℓ_1
$16 : P_{416} = (7,2,5,1)$ lies on line ℓ_2
$17 : P_{451} = (2,7,5,1)$ lies on line ℓ_0
$18 : P_{472} = (7,1,6,1)$ lies on line ℓ_2
$19 : P_{477} = (4,2,6,1)$ lies on line ℓ_1
$20 : P_{491} = (2,4,6,1)$ lies on line ℓ_0
$21 : P_{528} = (7,0,7,1)$ lies on line ℓ_2
$22 : P_{549} = (4,3,7,1)$ lies on line ℓ_1
$23 : P_{563} = (2,5,7,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1,1,1,1)$
$1 : P_6 = (2,1,0,0)$
$2 : P_8 = (4,1,0,0)$
$3 : P_{11} = (7,1,0,0)$
$4 : P_{13} = (2,0,1,0)$
$5 : P_{15} = (4,0,1,0)$
$6 : P_{18} = (7,0,1,0)$
$7 : P_{20} = (1,1,1,0)$
$8 : P_{32} = (5,2,1,0)$
$9 : P_{42} = (7,3,1,0)$
$10 : P_{49} = (6,4,1,0)$
$11 : P_{53} = (2,5,1,0)$

12 : $P_{63} = (4, 6, 1, 0)$
13 : $P_{70} = (3, 7, 1, 0)$
14 : $P_{76} = (2, 0, 0, 1)$
15 : $P_{78} = (4, 0, 0, 1)$
16 : $P_{81} = (7, 0, 0, 1)$
17 : $P_{82} = (0, 1, 0, 1)$
18 : $P_{99} = (1, 3, 0, 1)$
19 : $P_{115} = (1, 5, 0, 1)$
20 : $P_{123} = (1, 6, 0, 1)$
21 : $P_{138} = (0, 0, 1, 1)$
22 : $P_{222} = (5, 2, 2, 1)$
23 : $P_{232} = (7, 3, 2, 1)$
24 : $P_{241} = (0, 5, 2, 1)$
25 : $P_{242} = (1, 5, 2, 1)$
26 : $P_{255} = (6, 6, 2, 1)$
27 : $P_{256} = (7, 6, 2, 1)$
28 : $P_{266} = (1, 0, 3, 1)$
29 : $P_{288} = (7, 2, 3, 1)$
30 : $P_{299} = (2, 4, 3, 1)$
31 : $P_{300} = (3, 4, 3, 1)$
32 : $P_{307} = (2, 5, 3, 1)$
33 : $P_{320} = (7, 6, 3, 1)$
34 : $P_{321} = (0, 7, 3, 1)$
35 : $P_{322} = (1, 7, 3, 1)$
36 : $P_{355} = (2, 3, 4, 1)$
37 : $P_{356} = (3, 3, 4, 1)$
38 : $P_{367} = (6, 4, 4, 1)$
39 : $P_{371} = (2, 5, 4, 1)$
40 : $P_{377} = (0, 6, 4, 1)$
41 : $P_{378} = (1, 6, 4, 1)$
42 : $P_{394} = (1, 0, 5, 1)$
43 : $P_{409} = (0, 2, 5, 1)$
44 : $P_{410} = (1, 2, 5, 1)$
45 : $P_{419} = (2, 3, 5, 1)$
46 : $P_{427} = (2, 4, 5, 1)$
47 : $P_{445} = (4, 6, 5, 1)$
48 : $P_{453} = (4, 7, 5, 1)$
49 : $P_{454} = (5, 7, 5, 1)$
50 : $P_{458} = (1, 0, 6, 1)$
51 : $P_{479} = (6, 2, 6, 1)$
52 : $P_{480} = (7, 2, 6, 1)$
53 : $P_{488} = (7, 3, 6, 1)$
54 : $P_{489} = (0, 4, 6, 1)$
55 : $P_{490} = (1, 4, 6, 1)$
56 : $P_{501} = (4, 5, 6, 1)$
57 : $P_{517} = (4, 7, 6, 1)$
58 : $P_{545} = (0, 3, 7, 1)$
59 : $P_{546} = (1, 3, 7, 1)$
60 : $P_{565} = (4, 5, 7, 1)$
61 : $P_{566} = (5, 5, 7, 1)$
62 : $P_{573} = (4, 6, 7, 1)$
63 : $P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{19}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{19}	P_{19}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{19}	P_{19}

The surface has 89 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_6 = (2, 1, 0, 0)$
2 : $P_8 = (4, 1, 0, 0)$
3 : $P_{11} = (7, 1, 0, 0)$
4 : $P_{13} = (2, 0, 1, 0)$
5 : $P_{15} = (4, 0, 1, 0)$
6 : $P_{18} = (7, 0, 1, 0)$
7 : $P_{19} = (0, 1, 1, 0)$
8 : $P_{20} = (1, 1, 1, 0)$
9 : $P_{32} = (5, 2, 1, 0)$
10 : $P_{42} = (7, 3, 1, 0)$
11 : $P_{49} = (6, 4, 1, 0)$
12 : $P_{53} = (2, 5, 1, 0)$
13 : $P_{63} = (4, 6, 1, 0)$
14 : $P_{70} = (3, 7, 1, 0)$

15 : $P_{76} = (2, 0, 0, 1)$
16 : $P_{78} = (4, 0, 0, 1)$
17 : $P_{81} = (7, 0, 0, 1)$
18 : $P_{82} = (0, 1, 0, 1)$
19 : $P_{92} = (2, 2, 0, 1)$
20 : $P_{99} = (1, 3, 0, 1)$
21 : $P_{110} = (4, 4, 0, 1)$
22 : $P_{115} = (1, 5, 0, 1)$
23 : $P_{123} = (1, 6, 0, 1)$
24 : $P_{137} = (7, 7, 0, 1)$
25 : $P_{138} = (0, 0, 1, 1)$
26 : $P_{163} = (2, 3, 1, 1)$
27 : $P_{181} = (4, 5, 1, 1)$
28 : $P_{192} = (7, 6, 1, 1)$
29 : $P_{203} = (2, 0, 2, 1)$
30 : $P_{222} = (5, 2, 2, 1)$
31 : $P_{232} = (7, 3, 2, 1)$
32 : $P_{241} = (0, 5, 2, 1)$
33 : $P_{242} = (1, 5, 2, 1)$
34 : $P_{248} = (7, 5, 2, 1)$
35 : $P_{253} = (4, 6, 2, 1)$
36 : $P_{255} = (6, 6, 2, 1)$
37 : $P_{256} = (7, 6, 2, 1)$
38 : $P_{266} = (1, 0, 3, 1)$
39 : $P_{275} = (2, 1, 3, 1)$
40 : $P_{288} = (7, 2, 3, 1)$
41 : $P_{299} = (2, 4, 3, 1)$
42 : $P_{300} = (3, 4, 3, 1)$
43 : $P_{304} = (7, 4, 3, 1)$
44 : $P_{307} = (2, 5, 3, 1)$
45 : $P_{320} = (7, 6, 3, 1)$
46 : $P_{321} = (0, 7, 3, 1)$
47 : $P_{322} = (1, 7, 3, 1)$
48 : $P_{325} = (4, 7, 3, 1)$
49 : $P_{333} = (4, 0, 4, 1)$
50 : $P_{355} = (2, 3, 4, 1)$
51 : $P_{356} = (3, 3, 4, 1)$
52 : $P_{360} = (7, 3, 4, 1)$
53 : $P_{367} = (6, 4, 4, 1)$
54 : $P_{371} = (2, 5, 4, 1)$
55 : $P_{377} = (0, 6, 4, 1)$
56 : $P_{378} = (1, 6, 4, 1)$
57 : $P_{379} = (2, 6, 4, 1)$
58 : $P_{394} = (1, 0, 5, 1)$
59 : $P_{405} = (4, 1, 5, 1)$
60 : $P_{409} = (0, 2, 5, 1)$
61 : $P_{410} = (1, 2, 5, 1)$
62 : $P_{416} = (7, 2, 5, 1)$
63 : $P_{419} = (2, 3, 5, 1)$
64 : $P_{427} = (2, 4, 5, 1)$
65 : $P_{445} = (4, 6, 5, 1)$
66 : $P_{451} = (2, 7, 5, 1)$
67 : $P_{453} = (4, 7, 5, 1)$
68 : $P_{454} = (5, 7, 5, 1)$
69 : $P_{458} = (1, 0, 6, 1)$
70 : $P_{472} = (7, 1, 6, 1)$
71 : $P_{477} = (4, 2, 6, 1)$
72 : $P_{479} = (6, 2, 6, 1)$
73 : $P_{480} = (7, 2, 6, 1)$
74 : $P_{488} = (7, 3, 6, 1)$
75 : $P_{489} = (0, 4, 6, 1)$
76 : $P_{490} = (1, 4, 6, 1)$
77 : $P_{491} = (2, 4, 6, 1)$
78 : $P_{501} = (4, 5, 6, 1)$
79 : $P_{517} = (4, 7, 6, 1)$
80 : $P_{528} = (7, 0, 7, 1)$
81 : $P_{545} = (0, 3, 7, 1)$
82 : $P_{546} = (1, 3, 7, 1)$
83 : $P_{549} = (4, 3, 7, 1)$
84 : $P_{563} = (2, 5, 7, 1)$
85 : $P_{565} = (4, 5, 7, 1)$
86 : $P_{566} = (5, 5, 7, 1)$
87 : $P_{573} = (4, 6, 7, 1)$
88 : $P_{580} = (3, 7, 7, 1)$