Rank-67107 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_2 + X_1^2X_3 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 287379733

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	87
Number of points off lines	211
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^6
Type of lines on points	3, 2^6, 1^{87}, 0^{211}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(1,0,1,0,0,1)_{4657}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \mathbf{Pl}(0,0,0,0,0,1)_{4625}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \mathbf{Pl}(0,0,0,0,1,0)_{289}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4640} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4640} = \mathbf{Pl}(0,1,0,0,1,0)_{305}$$

Rank of lines: (0, 274, 69888, 272, 70160, 4640)
Rank of points on Klein quadric: (0, 4657, 4625, 289, 1, 305)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_0 = (1,0,0,0) = \ell_0 \cap \ell_3$
$P_{35} = (0,1,1,0) = \ell_1 \cap \ell_2$
$P_{20} = (1,0,1,0) = \ell_1 \cap \ell_5$
$P_2 = (0,0,1,0) = \ell_2 \cap \ell_4$

Single Points

The surface has 87 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$1 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$2 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$3 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$4 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$5 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$6 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$7 : P_{13} = (9,1,0,0)$ lies on line ℓ_0
$8 : P_{14} = (10,1,0,0)$ lies on line ℓ_0
$9 : P_{15} = (11,1,0,0)$ lies on line ℓ_0
$10 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$11 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$12 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$13 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$14 : P_{51} = (0,2,1,0)$ lies on line ℓ_2
$15 : P_{54} = (3,2,1,0)$ lies on line ℓ_1
$16 : P_{67} = (0,3,1,0)$ lies on line ℓ_2
$17 : P_{69} = (2,3,1,0)$ lies on line ℓ_1
$18 : P_{83} = (0,4,1,0)$ lies on line ℓ_2
$19 : P_{88} = (5,4,1,0)$ lies on line ℓ_1
$20 : P_{99} = (0,5,1,0)$ lies on line ℓ_2
$21 : P_{103} = (4,5,1,0)$ lies on line ℓ_1
$22 : P_{115} = (0,6,1,0)$ lies on line ℓ_2
$23 : P_{122} = (7,6,1,0)$ lies on line ℓ_1
$24 : P_{131} = (0,7,1,0)$ lies on line ℓ_2
$25 : P_{137} = (6,7,1,0)$ lies on line ℓ_1
$26 : P_{147} = (0,8,1,0)$ lies on line ℓ_2
$27 : P_{156} = (9,8,1,0)$ lies on line ℓ_1
$28 : P_{163} = (0,9,1,0)$ lies on line ℓ_2
$29 : P_{171} = (8,9,1,0)$ lies on line ℓ_1

30 : $P_{179} = (0, 10, 1, 0)$ lies on line ℓ_2
31 : $P_{190} = (11, 10, 1, 0)$ lies on line ℓ_1
32 : $P_{195} = (0, 11, 1, 0)$ lies on line ℓ_2
33 : $P_{205} = (10, 11, 1, 0)$ lies on line ℓ_1
34 : $P_{211} = (0, 12, 1, 0)$ lies on line ℓ_2
35 : $P_{224} = (13, 12, 1, 0)$ lies on line ℓ_1
36 : $P_{227} = (0, 13, 1, 0)$ lies on line ℓ_2
37 : $P_{239} = (12, 13, 1, 0)$ lies on line ℓ_1
38 : $P_{243} = (0, 14, 1, 0)$ lies on line ℓ_2
39 : $P_{258} = (15, 14, 1, 0)$ lies on line ℓ_1
40 : $P_{259} = (0, 15, 1, 0)$ lies on line ℓ_2
41 : $P_{273} = (14, 15, 1, 0)$ lies on line ℓ_1
42 : $P_{275} = (1, 0, 0, 1)$ lies on line ℓ_3
43 : $P_{276} = (2, 0, 0, 1)$ lies on line ℓ_3
44 : $P_{277} = (3, 0, 0, 1)$ lies on line ℓ_3
45 : $P_{278} = (4, 0, 0, 1)$ lies on line ℓ_3
46 : $P_{279} = (5, 0, 0, 1)$ lies on line ℓ_3
47 : $P_{280} = (6, 0, 0, 1)$ lies on line ℓ_3
48 : $P_{281} = (7, 0, 0, 1)$ lies on line ℓ_3
49 : $P_{282} = (8, 0, 0, 1)$ lies on line ℓ_3
50 : $P_{283} = (9, 0, 0, 1)$ lies on line ℓ_3
51 : $P_{284} = (10, 0, 0, 1)$ lies on line ℓ_3
52 : $P_{285} = (11, 0, 0, 1)$ lies on line ℓ_3
53 : $P_{286} = (12, 0, 0, 1)$ lies on line ℓ_3
54 : $P_{287} = (13, 0, 0, 1)$ lies on line ℓ_3
55 : $P_{288} = (14, 0, 0, 1)$ lies on line ℓ_3
56 : $P_{289} = (15, 0, 0, 1)$ lies on line ℓ_3
57 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_4
58 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_5
59 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_4
60 : $P_{787} = (2, 0, 2, 1)$ lies on line ℓ_5
61 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_4
62 : $P_{1044} = (3, 0, 3, 1)$ lies on line ℓ_5
63 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_4
64 : $P_{1301} = (4, 0, 4, 1)$ lies on line ℓ_5
65 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_4
66 : $P_{1558} = (5, 0, 5, 1)$ lies on line ℓ_5
67 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_4
68 : $P_{1815} = (6, 0, 6, 1)$ lies on line ℓ_5
69 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_4
70 : $P_{2072} = (7, 0, 7, 1)$ lies on line ℓ_5
71 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_4
72 : $P_{2329} = (8, 0, 8, 1)$ lies on line ℓ_5
73 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_4
74 : $P_{2586} = (9, 0, 9, 1)$ lies on line ℓ_5
75 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_4
76 : $P_{2843} = (10, 0, 10, 1)$ lies on line ℓ_5
77 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_4
78 : $P_{3100} = (11, 0, 11, 1)$ lies on line ℓ_5
79 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_4
80 : $P_{3357} = (12, 0, 12, 1)$ lies on line ℓ_5
81 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_4
82 : $P_{3614} = (13, 0, 13, 1)$ lies on line ℓ_5
83 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_4
84 : $P_{3871} = (14, 0, 14, 1)$ lies on line ℓ_5
85 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_4
86 : $P_{4128} = (15, 0, 15, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 211 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_{586} = (9, 3, 1, 1)$
2 : $P_{588} = (11, 3, 1, 1)$
3 : $P_{619} = (10, 5, 1, 1)$
4 : $P_{623} = (14, 5, 1, 1)$
5 : $P_{659} = (2, 8, 1, 1)$
6 : $P_{668} = (11, 8, 1, 1)$
7 : $P_{691} = (2, 10, 1, 1)$
8 : $P_{698} = (9, 10, 1, 1)$
9 : $P_{709} = (4, 11, 1, 1)$
10 : $P_{719} = (14, 11, 1, 1)$
11 : $P_{773} = (4, 15, 1, 1)$
12 : $P_{779} = (10, 15, 1, 1)$
13 : $P_{810} = (9, 1, 2, 1)$
14 : $P_{811} = (10, 1, 2, 1)$
15 : $P_{831} = (14, 2, 2, 1)$
16 : $P_{851} = (2, 4, 2, 1)$
17 : $P_{853} = (4, 4, 2, 1)$
18 : $P_{908} = (11, 7, 2, 1)$
19 : $P_{911} = (14, 7, 2, 1)$
20 : $P_{914} = (1, 8, 2, 1)$
21 : $P_{924} = (11, 8, 2, 1)$
22 : $P_{946} = (1, 10, 2, 1)$
23 : $P_{954} = (9, 10, 2, 1)$
24 : $P_{981} = (4, 12, 2, 1)$
25 : $P_{987} = (10, 12, 2, 1)$
26 : $P_{1079} = (6, 2, 3, 1)$
27 : $P_{1080} = (7, 2, 3, 1)$
28 : $P_{1104} = (15, 3, 3, 1)$
29 : $P_{1124} = (3, 5, 3, 1)$

30 : $P_{1126} = (5, 5, 3, 1)$
31 : $P_{1145} = (8, 6, 3, 1)$
32 : $P_{1150} = (13, 6, 3, 1)$
33 : $P_{1161} = (8, 7, 3, 1)$
34 : $P_{1165} = (12, 7, 3, 1)$
35 : $P_{1175} = (6, 8, 3, 1)$
36 : $P_{1182} = (13, 8, 3, 1)$
37 : $P_{1206} = (5, 10, 3, 1)$
38 : $P_{1213} = (12, 10, 3, 1)$
39 : $P_{1224} = (7, 11, 3, 1)$
40 : $P_{1232} = (15, 11, 3, 1)$
41 : $P_{1324} = (11, 1, 4, 1)$
42 : $P_{1327} = (14, 1, 4, 1)$
43 : $P_{1363} = (2, 4, 4, 1)$
44 : $P_{1402} = (9, 6, 4, 1)$
45 : $P_{1404} = (11, 6, 4, 1)$
46 : $P_{1445} = (4, 9, 4, 1)$
47 : $P_{1450} = (9, 9, 4, 1)$
48 : $P_{1474} = (1, 11, 4, 1)$
49 : $P_{1487} = (14, 11, 4, 1)$
50 : $P_{1491} = (2, 12, 4, 1)$
51 : $P_{1499} = (10, 12, 4, 1)$
52 : $P_{1538} = (1, 15, 4, 1)$
53 : $P_{1547} = (10, 15, 4, 1)$
54 : $P_{1629} = (12, 4, 5, 1)$
55 : $P_{1630} = (13, 4, 5, 1)$
56 : $P_{1636} = (3, 5, 5, 1)$
57 : $P_{1686} = (5, 8, 5, 1)$
58 : $P_{1689} = (8, 8, 5, 1)$
59 : $P_{1716} = (3, 10, 5, 1)$
60 : $P_{1725} = (12, 10, 5, 1)$
61 : $P_{1735} = (6, 11, 5, 1)$
62 : $P_{1737} = (8, 11, 5, 1)$
63 : $P_{1751} = (6, 12, 5, 1)$
64 : $P_{1760} = (15, 12, 5, 1)$
65 : $P_{1768} = (7, 13, 5, 1)$
66 : $P_{1776} = (15, 13, 5, 1)$
67 : $P_{1800} = (7, 15, 5, 1)$
68 : $P_{1806} = (13, 15, 5, 1)$
69 : $P_{1844} = (3, 2, 6, 1)$
70 : $P_{1848} = (7, 2, 6, 1)$
71 : $P_{1901} = (12, 5, 6, 1)$
72 : $P_{1904} = (15, 5, 6, 1)$
73 : $P_{1917} = (12, 6, 6, 1)$
74 : $P_{1940} = (3, 8, 6, 1)$
75 : $P_{1950} = (13, 8, 6, 1)$
76 : $P_{1960} = (7, 9, 6, 1)$
77 : $P_{1961} = (8, 9, 6, 1)$
78 : $P_{1990} = (5, 11, 6, 1)$
79 : $P_{1993} = (8, 11, 6, 1)$
80 : $P_{2006} = (5, 12, 6, 1)$
81 : $P_{2016} = (15, 12, 6, 1)$
82 : $P_{2023} = (6, 13, 6, 1)$
83 : $P_{2030} = (13, 13, 6, 1)$
84 : $P_{2100} = (3, 2, 7, 1)$
85 : $P_{2103} = (6, 2, 7, 1)$
86 : $P_{2121} = (8, 3, 7, 1)$
87 : $P_{2125} = (12, 3, 7, 1)$
88 : $P_{2190} = (13, 7, 7, 1)$
89 : $P_{2215} = (6, 9, 7, 1)$
90 : $P_{2217} = (8, 9, 7, 1)$
91 : $P_{2244} = (3, 11, 7, 1)$
92 : $P_{2256} = (15, 11, 7, 1)$
93 : $P_{2264} = (7, 12, 7, 1)$
94 : $P_{2269} = (12, 12, 7, 1)$
95 : $P_{2278} = (5, 13, 7, 1)$
96 : $P_{2288} = (15, 13, 7, 1)$
97 : $P_{2310} = (5, 15, 7, 1)$
98 : $P_{2318} = (13, 15, 7, 1)$
99 : $P_{2376} = (7, 3, 8, 1)$
100 : $P_{2381} = (12, 3, 8, 1)$
101 : $P_{2420} = (3, 6, 8, 1)$
102 : $P_{2430} = (13, 6, 8, 1)$
103 : $P_{2436} = (3, 7, 8, 1)$
104 : $P_{2445} = (12, 7, 8, 1)$
105 : $P_{2454} = (5, 8, 8, 1)$
106 : $P_{2471} = (6, 9, 8, 1)$
107 : $P_{2472} = (7, 9, 8, 1)$
108 : $P_{2494} = (13, 10, 8, 1)$
109 : $P_{2496} = (15, 10, 8, 1)$
110 : $P_{2502} = (5, 11, 8, 1)$
111 : $P_{2503} = (6, 11, 8, 1)$
112 : $P_{2569} = (8, 15, 8, 1)$
113 : $P_{2576} = (15, 15, 8, 1)$
114 : $P_{2595} = (2, 1, 9, 1)$
115 : $P_{2603} = (10, 1, 9, 1)$
116 : $P_{2626} = (1, 3, 9, 1)$
117 : $P_{2636} = (11, 3, 9, 1)$
118 : $P_{2677} = (4, 6, 9, 1)$
119 : $P_{2684} = (11, 6, 9, 1)$
120 : $P_{2725} = (4, 9, 9, 1)$
121 : $P_{2738} = (1, 10, 9, 1)$
122 : $P_{2739} = (2, 10, 9, 1)$
123 : $P_{2795} = (10, 13, 9, 1)$
124 : $P_{2799} = (14, 13, 9, 1)$
125 : $P_{2810} = (9, 14, 9, 1)$
126 : $P_{2815} = (14, 14, 9, 1)$
127 : $P_{2851} = (2, 1, 10, 1)$
128 : $P_{2858} = (9, 1, 10, 1)$
129 : $P_{2914} = (1, 5, 10, 1)$
130 : $P_{2927} = (14, 5, 10, 1)$
131 : $P_{3004} = (11, 10, 10, 1)$
132 : $P_{3019} = (10, 11, 10, 1)$
133 : $P_{3020} = (11, 11, 10, 1)$
134 : $P_{3027} = (2, 12, 10, 1)$
135 : $P_{3029} = (4, 12, 10, 1)$
136 : $P_{3050} = (9, 13, 10, 1)$
137 : $P_{3055} = (14, 13, 10, 1)$

138 : $P_{3074} = (1, 15, 10, 1)$
139 : $P_{3077} = (4, 15, 10, 1)$
140 : $P_{3109} = (4, 1, 11, 1)$
141 : $P_{3119} = (14, 1, 11, 1)$
142 : $P_{3138} = (1, 3, 11, 1)$
143 : $P_{3146} = (9, 3, 11, 1)$
144 : $P_{3189} = (4, 6, 11, 1)$
145 : $P_{3194} = (9, 6, 11, 1)$
146 : $P_{3203} = (2, 7, 11, 1)$
147 : $P_{3215} = (14, 7, 11, 1)$
148 : $P_{3218} = (1, 8, 11, 1)$
149 : $P_{3219} = (2, 8, 11, 1)$
150 : $P_{3259} = (10, 10, 11, 1)$
151 : $P_{3260} = (11, 10, 11, 1)$
152 : $P_{3275} = (10, 11, 11, 1)$
153 : $P_{3400} = (7, 3, 12, 1)$
154 : $P_{3401} = (8, 3, 12, 1)$
155 : $P_{3414} = (5, 4, 12, 1)$
156 : $P_{3422} = (13, 4, 12, 1)$
157 : $P_{3431} = (6, 5, 12, 1)$
158 : $P_{3440} = (15, 5, 12, 1)$
159 : $P_{3447} = (6, 6, 12, 1)$
160 : $P_{3453} = (12, 6, 12, 1)$
161 : $P_{3460} = (3, 7, 12, 1)$
162 : $P_{3465} = (8, 7, 12, 1)$
163 : $P_{3508} = (3, 10, 12, 1)$
164 : $P_{3510} = (5, 10, 12, 1)$
165 : $P_{3544} = (7, 12, 12, 1)$
166 : $P_{3582} = (13, 14, 12, 1)$
167 : $P_{3584} = (15, 14, 12, 1)$
168 : $P_{3670} = (5, 4, 13, 1)$
169 : $P_{3677} = (12, 4, 13, 1)$
170 : $P_{3700} = (3, 6, 13, 1)$
171 : $P_{3705} = (8, 6, 13, 1)$
172 : $P_{3720} = (7, 7, 13, 1)$
173 : $P_{3726} = (13, 7, 13, 1)$
174 : $P_{3732} = (3, 8, 13, 1)$
175 : $P_{3735} = (6, 8, 13, 1)$
176 : $P_{3769} = (8, 10, 13, 1)$
177 : $P_{3776} = (15, 10, 13, 1)$
178 : $P_{3815} = (6, 13, 13, 1)$
179 : $P_{3837} = (12, 14, 13, 1)$
180 : $P_{3840} = (15, 14, 13, 1)$
181 : $P_{3846} = (5, 15, 13, 1)$
182 : $P_{3848} = (7, 15, 13, 1)$
183 : $P_{3877} = (4, 1, 14, 1)$
184 : $P_{3884} = (11, 1, 14, 1)$
185 : $P_{3891} = (2, 2, 14, 1)$
186 : $P_{3903} = (14, 2, 14, 1)$
187 : $P_{3938} = (1, 5, 14, 1)$
188 : $P_{3947} = (10, 5, 14, 1)$
189 : $P_{3971} = (2, 7, 14, 1)$
190 : $P_{3980} = (11, 7, 14, 1)$
191 : $P_{4034} = (1, 11, 14, 1)$
192 : $P_{4037} = (4, 11, 14, 1)$
193 : $P_{4074} = (9, 13, 14, 1)$
194 : $P_{4075} = (10, 13, 14, 1)$
195 : $P_{4090} = (9, 14, 14, 1)$
196 : $P_{4164} = (3, 3, 15, 1)$
197 : $P_{4176} = (15, 3, 15, 1)$
198 : $P_{4199} = (6, 5, 15, 1)$
199 : $P_{4205} = (12, 5, 15, 1)$
200 : $P_{4281} = (8, 10, 15, 1)$
201 : $P_{4286} = (13, 10, 15, 1)$
202 : $P_{4292} = (3, 11, 15, 1)$
203 : $P_{4296} = (7, 11, 15, 1)$
204 : $P_{4310} = (5, 12, 15, 1)$
205 : $P_{4311} = (6, 12, 15, 1)$
206 : $P_{4326} = (5, 13, 15, 1)$
207 : $P_{4328} = (7, 13, 15, 1)$
208 : $P_{4349} = (12, 14, 15, 1)$
209 : $P_{4350} = (13, 14, 15, 1)$
210 : $P_{4361} = (8, 15, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	0	1
2	1	1	0	0	1	0
3	1	0	0	0	1	1
4	0	0	1	1	0	1
5	0	1	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_1	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_5	P_{35}	P_{20}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_1	P_{35}	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_0	P_3	P_3

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_3	P_3

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{20}	P_3	P_3

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_4 = (1,1,1,1)$
5 : $P_5 = (1,1,0,0)$
6 : $P_6 = (2,1,0,0)$
7 : $P_7 = (3,1,0,0)$
8 : $P_8 = (4,1,0,0)$
9 : $P_9 = (5,1,0,0)$
10 : $P_{10} = (6,1,0,0)$
11 : $P_{11} = (7,1,0,0)$
12 : $P_{12} = (8,1,0,0)$
13 : $P_{13} = (9,1,0,0)$
14 : $P_{14} = (10,1,0,0)$
15 : $P_{15} = (11,1,0,0)$
16 : $P_{16} = (12,1,0,0)$
17 : $P_{17} = (13,1,0,0)$
18 : $P_{18} = (14,1,0,0)$
19 : $P_{19} = (15,1,0,0)$
20 : $P_{20} = (1,0,1,0)$
21 : $P_{35} = (0,1,1,0)$
22 : $P_{51} = (0,2,1,0)$
23 : $P_{54} = (3,2,1,0)$
24 : $P_{67} = (0,3,1,0)$
25 : $P_{69} = (2,3,1,0)$
26 : $P_{83} = (0,4,1,0)$
27 : $P_{88} = (5,4,1,0)$
28 : $P_{99} = (0,5,1,0)$
29 : $P_{103} = (4,5,1,0)$
30 : $P_{115} = (0,6,1,0)$
31 : $P_{122} = (7,6,1,0)$
32 : $P_{131} = (0,7,1,0)$
33 : $P_{137} = (6,7,1,0)$
34 : $P_{147} = (0,8,1,0)$
35 : $P_{156} = (9,8,1,0)$
36 : $P_{163} = (0,9,1,0)$
37 : $P_{171} = (8,9,1,0)$
38 : $P_{179} = (0,10,1,0)$
39 : $P_{190} = (11,10,1,0)$
40 : $P_{195} = (0,11,1,0)$
41 : $P_{205} = (10,11,1,0)$
42 : $P_{211} = (0,12,1,0)$
43 : $P_{224} = (13,12,1,0)$
44 : $P_{227} = (0,13,1,0)$
45 : $P_{239} = (12,13,1,0)$
46 : $P_{243} = (0,14,1,0)$
47 : $P_{258} = (15,14,1,0)$
48 : $P_{259} = (0,15,1,0)$
49 : $P_{273} = (14,15,1,0)$
50 : $P_{275} = (1,0,0,1)$
51 : $P_{276} = (2,0,0,1)$
52 : $P_{277} = (3,0,0,1)$
53 : $P_{278} = (4,0,0,1)$
54 : $P_{279} = (5,0,0,1)$
55 : $P_{280} = (6,0,0,1)$
56 : $P_{281} = (7,0,0,1)$
57 : $P_{282} = (8,0,0,1)$
58 : $P_{283} = (9,0,0,1)$
59 : $P_{284} = (10,0,0,1)$
60 : $P_{285} = (11,0,0,1)$
61 : $P_{286} = (12,0,0,1)$
62 : $P_{287} = (13,0,0,1)$
63 : $P_{288} = (14,0,0,1)$
64 : $P_{289} = (15,0,0,1)$
65 : $P_{530} = (0,0,1,1)$
66 : $P_{531} = (1,0,1,1)$
67 : $P_{586} = (9,3,1,1)$
68 : $P_{588} = (11,3,1,1)$
69 : $P_{619} = (10,5,1,1)$
70 : $P_{623} = (14,5,1,1)$
71 : $P_{659} = (2,8,1,1)$
72 : $P_{668} = (11,8,1,1)$
73 : $P_{691} = (2,10,1,1)$
74 : $P_{698} = (9,10,1,1)$
75 : $P_{709} = (4,11,1,1)$
76 : $P_{719} = (14,11,1,1)$
77 : $P_{773} = (4,15,1,1)$
78 : $P_{779} = (10,15,1,1)$
79 : $P_{785} = (0,0,2,1)$
80 : $P_{787} = (2,0,2,1)$
81 : $P_{810} = (9,1,2,1)$
82 : $P_{811} = (10,1,2,1)$
83 : $P_{831} = (14,2,2,1)$
84 : $P_{851} = (2,4,2,1)$
85 : $P_{853} = (4,4,2,1)$
86 : $P_{908} = (11,7,2,1)$
87 : $P_{911} = (14,7,2,1)$
88 : $P_{914} = (1,8,2,1)$
89 : $P_{924} = (11,8,2,1)$
90 : $P_{946} = (1,10,2,1)$
91 : $P_{954} = (9,10,2,1)$
92 : $P_{981} = (4,12,2,1)$
93 : $P_{987} = (10,12,2,1)$
94 : $P_{1041} = (0,0,3,1)$
95 : $P_{1044} = (3,0,3,1)$
96 : $P_{1079} = (6,2,3,1)$
97 : $P_{1080} = (7,2,3,1)$
98 : $P_{1104} = (15,3,3,1)$
99 : $P_{1124} = (3,5,3,1)$
100 : $P_{1126} = (5,5,3,1)$
101 : $P_{1145} = (8,6,3,1)$
102 : $P_{1150} = (13,6,3,1)$
103 : $P_{1161} = (8,7,3,1)$
104 : $P_{1165} = (12,7,3,1)$
105 : $P_{1175} = (6,8,3,1)$
106 : $P_{1182} = (13,8,3,1)$
107 : $P_{1206} = (5,10,3,1)$
108 : $P_{1213} = (12,10,3,1)$
109 : $P_{1224} = (7,11,3,1)$
110 : $P_{1232} = (15,11,3,1)$

111 : $P_{1297} = (0, 0, 4, 1)$
112 : $P_{1301} = (4, 0, 4, 1)$
113 : $P_{1324} = (11, 1, 4, 1)$
114 : $P_{1327} = (14, 1, 4, 1)$
115 : $P_{1363} = (2, 4, 4, 1)$
116 : $P_{1402} = (9, 6, 4, 1)$
117 : $P_{1404} = (11, 6, 4, 1)$
118 : $P_{1445} = (4, 9, 4, 1)$
119 : $P_{1450} = (9, 9, 4, 1)$
120 : $P_{1474} = (1, 11, 4, 1)$
121 : $P_{1487} = (14, 11, 4, 1)$
122 : $P_{1491} = (2, 12, 4, 1)$
123 : $P_{1499} = (10, 12, 4, 1)$
124 : $P_{1538} = (1, 15, 4, 1)$
125 : $P_{1547} = (10, 15, 4, 1)$
126 : $P_{1553} = (0, 0, 5, 1)$
127 : $P_{1558} = (5, 0, 5, 1)$
128 : $P_{1629} = (12, 4, 5, 1)$
129 : $P_{1630} = (13, 4, 5, 1)$
130 : $P_{1636} = (3, 5, 5, 1)$
131 : $P_{1686} = (5, 8, 5, 1)$
132 : $P_{1689} = (8, 8, 5, 1)$
133 : $P_{1716} = (3, 10, 5, 1)$
134 : $P_{1725} = (12, 10, 5, 1)$
135 : $P_{1735} = (6, 11, 5, 1)$
136 : $P_{1737} = (8, 11, 5, 1)$
137 : $P_{1751} = (6, 12, 5, 1)$
138 : $P_{1760} = (15, 12, 5, 1)$
139 : $P_{1768} = (7, 13, 5, 1)$
140 : $P_{1776} = (15, 13, 5, 1)$
141 : $P_{1800} = (7, 15, 5, 1)$
142 : $P_{1806} = (13, 15, 5, 1)$
143 : $P_{1809} = (0, 0, 6, 1)$
144 : $P_{1815} = (6, 0, 6, 1)$
145 : $P_{1844} = (3, 2, 6, 1)$
146 : $P_{1848} = (7, 2, 6, 1)$
147 : $P_{1901} = (12, 5, 6, 1)$
148 : $P_{1904} = (15, 5, 6, 1)$
149 : $P_{1917} = (12, 6, 6, 1)$
150 : $P_{1940} = (3, 8, 6, 1)$
151 : $P_{1950} = (13, 8, 6, 1)$
152 : $P_{1960} = (7, 9, 6, 1)$
153 : $P_{1961} = (8, 9, 6, 1)$
154 : $P_{1990} = (5, 11, 6, 1)$
155 : $P_{1993} = (8, 11, 6, 1)$
156 : $P_{2006} = (5, 12, 6, 1)$
157 : $P_{2016} = (15, 12, 6, 1)$
158 : $P_{2023} = (6, 13, 6, 1)$
159 : $P_{2030} = (13, 13, 6, 1)$
160 : $P_{2065} = (0, 0, 7, 1)$
161 : $P_{2072} = (7, 0, 7, 1)$
162 : $P_{2100} = (3, 2, 7, 1)$
163 : $P_{2103} = (6, 2, 7, 1)$
164 : $P_{2121} = (8, 3, 7, 1)$
165 : $P_{2125} = (12, 3, 7, 1)$
166 : $P_{2190} = (13, 7, 7, 1)$
167 : $P_{2215} = (6, 9, 7, 1)$
168 : $P_{2217} = (8, 9, 7, 1)$
169 : $P_{2244} = (3, 11, 7, 1)$
170 : $P_{2256} = (15, 11, 7, 1)$
171 : $P_{2264} = (7, 12, 7, 1)$
172 : $P_{2269} = (12, 12, 7, 1)$
173 : $P_{2278} = (5, 13, 7, 1)$
174 : $P_{2288} = (15, 13, 7, 1)$
175 : $P_{2310} = (5, 15, 7, 1)$
176 : $P_{2318} = (13, 15, 7, 1)$
177 : $P_{2321} = (0, 0, 8, 1)$
178 : $P_{2329} = (8, 0, 8, 1)$
179 : $P_{2376} = (7, 3, 8, 1)$
180 : $P_{2381} = (12, 3, 8, 1)$
181 : $P_{2420} = (3, 6, 8, 1)$
182 : $P_{2430} = (13, 6, 8, 1)$
183 : $P_{2436} = (3, 7, 8, 1)$
184 : $P_{2445} = (12, 7, 8, 1)$
185 : $P_{2454} = (5, 8, 8, 1)$
186 : $P_{2471} = (6, 9, 8, 1)$
187 : $P_{2472} = (7, 9, 8, 1)$
188 : $P_{2494} = (13, 10, 8, 1)$
189 : $P_{2496} = (15, 10, 8, 1)$
190 : $P_{2502} = (5, 11, 8, 1)$
191 : $P_{2503} = (6, 11, 8, 1)$
192 : $P_{2569} = (8, 15, 8, 1)$
193 : $P_{2576} = (15, 15, 8, 1)$
194 : $P_{2577} = (0, 0, 9, 1)$
195 : $P_{2586} = (9, 0, 9, 1)$
196 : $P_{2595} = (2, 1, 9, 1)$
197 : $P_{2603} = (10, 1, 9, 1)$
198 : $P_{2626} = (1, 3, 9, 1)$
199 : $P_{2636} = (11, 3, 9, 1)$
200 : $P_{2677} = (4, 6, 9, 1)$
201 : $P_{2684} = (11, 6, 9, 1)$
202 : $P_{2725} = (4, 9, 9, 1)$
203 : $P_{2738} = (1, 10, 9, 1)$
204 : $P_{2739} = (2, 10, 9, 1)$
205 : $P_{2795} = (10, 13, 9, 1)$
206 : $P_{2799} = (14, 13, 9, 1)$
207 : $P_{2810} = (9, 14, 9, 1)$
208 : $P_{2815} = (14, 14, 9, 1)$
209 : $P_{2833} = (0, 0, 10, 1)$
210 : $P_{2843} = (10, 0, 10, 1)$
211 : $P_{2851} = (2, 1, 10, 1)$
212 : $P_{2858} = (9, 1, 10, 1)$
213 : $P_{2914} = (1, 5, 10, 1)$
214 : $P_{2927} = (14, 5, 10, 1)$
215 : $P_{3004} = (11, 10, 10, 1)$
216 : $P_{3019} = (10, 11, 10, 1)$
217 : $P_{3020} = (11, 11, 10, 1)$
218 : $P_{3027} = (2, 12, 10, 1)$
219 : $P_{3029} = (4, 12, 10, 1)$
220 : $P_{3050} = (9, 13, 10, 1)$
221 : $P_{3055} = (14, 13, 10, 1)$
222 : $P_{3074} = (1, 15, 10, 1)$
223 : $P_{3077} = (4, 15, 10, 1)$
224 : $P_{3089} = (0, 0, 11, 1)$
225 : $P_{3100} = (11, 0, 11, 1)$
226 : $P_{3109} = (4, 1, 11, 1)$
227 : $P_{3119} = (14, 1, 11, 1)$
228 : $P_{3138} = (1, 3, 11, 1)$
229 : $P_{3146} = (9, 3, 11, 1)$
230 : $P_{3189} = (4, 6, 11, 1)$
231 : $P_{3194} = (9, 6, 11, 1)$
232 : $P_{3203} = (2, 7, 11, 1)$
233 : $P_{3215} = (14, 7, 11, 1)$
234 : $P_{3218} = (1, 8, 11, 1)$
235 : $P_{3219} = (2, 8, 11, 1)$
236 : $P_{3259} = (10, 10, 11, 1)$
237 : $P_{3260} = (11, 10, 11, 1)$
238 : $P_{3275} = (10, 11, 11, 1)$
239 : $P_{3345} = (0, 0, 12, 1)$
240 : $P_{3357} = (12, 0, 12, 1)$
241 : $P_{3400} = (7, 3, 12, 1)$
242 : $P_{3401} = (8, 3, 12, 1)$
243 : $P_{3414} = (5, 4, 12, 1)$
244 : $P_{3422} = (13, 4, 12, 1)$
245 : $P_{3431} = (6, 5, 12, 1)$
246 : $P_{3440} = (15, 5, 12, 1)$
247 : $P_{3447} = (6, 6, 12, 1)$
248 : $P_{3453} = (12, 6, 12, 1)$
249 : $P_{3460} = (3, 7, 12, 1)$
250 : $P_{3465} = (8, 7, 12, 1)$
251 : $P_{3508} = (3, 10, 12, 1)$
252 : $P_{3510} = (5, 10, 12, 1)$
253 : $P_{3544} = (7, 12, 12, 1)$
254 : $P_{3582} = (13, 14, 12, 1)$
255 : $P_{3584} = (15, 14, 12, 1)$
256 : $P_{3601} = (0, 0, 13, 1)$
257 : $P_{3614} = (13, 0, 13, 1)$
258 : $P_{3670} = (5, 4, 13, 1)$
259 : $P_{3677} = (12, 4, 13, 1)$
260 : $P_{3700} = (3, 6, 13, 1)$
261 : $P_{3705} = (8, 6, 13, 1)$
262 : $P_{3720} = (7, 7, 13, 1)$
263 : $P_{3726} = (13, 7, 13, 1)$
264 : $P_{3732} = (3, 8, 13, 1)$
265 : $P_{3735} = (6, 8, 13, 1)$
266 : $P_{3769} = (8, 10, 13, 1)$
267 : $P_{3776} = (15, 10, 13, 1)$
268 : $P_{3815} = (6, 13, 13, 1)$
269 : $P_{3837} = (12, 14, 13, 1)$
270 : $P_{3840} = (15, 14, 13, 1)$
271 : $P_{3846} = (5, 15, 13, 1)$
272 : $P_{3848} = (7, 15, 13, 1)$

$273: P_{3857} = (0, 0, 14, 1)$
$274: P_{3871} = (14, 0, 14, 1)$
$275: P_{3877} = (4, 1, 14, 1)$
$276: P_{3884} = (11, 1, 14, 1)$
$277: P_{3891} = (2, 2, 14, 1)$
$278: P_{3903} = (14, 2, 14, 1)$
$279: P_{3938} = (1, 5, 14, 1)$
$280: P_{3947} = (10, 5, 14, 1)$
$281: P_{3971} = (2, 7, 14, 1)$
$282: P_{3980} = (11, 7, 14, 1)$
$283: P_{4034} = (1, 11, 14, 1)$
$284: P_{4037} = (4, 11, 14, 1)$
$285: P_{4074} = (9, 13, 14, 1)$
$286: P_{4075} = (10, 13, 14, 1)$
$287: P_{4090} = (9, 14, 14, 1)$
$288: P_{4113} = (0, 0, 15, 1)$
$289: P_{4128} = (15, 0, 15, 1)$
$290: P_{4164} = (3, 3, 15, 1)$
$291: P_{4176} = (15, 3, 15, 1)$
$292: P_{4199} = (6, 5, 15, 1)$
$293: P_{4205} = (12, 5, 15, 1)$
$294: P_{4281} = (8, 10, 15, 1)$
$295: P_{4286} = (13, 10, 15, 1)$
$296: P_{4292} = (3, 11, 15, 1)$
$297: P_{4296} = (7, 11, 15, 1)$
$298: P_{4310} = (5, 12, 15, 1)$
$299: P_{4311} = (6, 12, 15, 1)$
$300: P_{4326} = (5, 13, 15, 1)$
$301: P_{4328} = (7, 13, 15, 1)$
$302: P_{4349} = (12, 14, 15, 1)$
$303: P_{4350} = (13, 14, 15, 1)$
$304: P_{4361} = (8, 15, 15, 1)$