# Rank-74007 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(4) is 1498830441

## General information

| | |
|---|---|
| Number of lines | 5 |
| Number of points | 25 |
| Number of singular points | 3 |
| Number of Eckardt points | 1 |
| Number of double points | 4 |
| Number of single points | 14 |
| Number of points off lines | 6 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^5$ |
| Type of lines on points | $3, 2^4, 1^{14}, 0^6$ |

### Singular Points

The surface has 3 singular points:

0 : $P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$          2 : $P_{80} = \mathbf{P}(\omega^2, \omega, \omega^2, 1) = \mathbf{P}(3,2,3,1)$
1 : $P_{67} = \mathbf{P}(\omega, \omega^2, \omega, 1) = \mathbf{P}(2,3,2,1)$

## The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1,1,0,1,1,1)_{189}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{131} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{131} = \mathbf{Pl}(3,2,2,3,3,1)_{350}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{152} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{152} = \mathbf{Pl}(2,3,3,2,2,1)_{292}$$

Rank of lines: ( 340, 345, 109, 131, 152 )
Rank of points on Klein quadric: ( 9, 13, 189, 350, 292 )

**Eckardt Points**

The surface has 1 Eckardt points:
$0 : P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

**Double Points**

The surface has 4 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$          $P_{67} = (2,3,2,1) = \ell_2 \cap \ell_4$
$P_{26} = (0,1,0,1) = \ell_0 \cap \ell_2$
$P_{80} = (3,2,3,1) = \ell_2 \cap \ell_3$

**Single Points**

The surface has 14 single points:
The single points on the surface are:

$0 : P_1 = (0,1,0,0)$ lies on line $\ell_0$          $8 : P_{36} = (2,3,0,1)$ lies on line $\ell_3$
$1 : P_{11} = (0,1,1,0)$ lies on line $\ell_1$          $9 : P_{39} = (1,0,1,1)$ lies on line $\ell_2$
$2 : P_{12} = (1,1,1,0)$ lies on line $\ell_2$          $10 : P_{54} = (1,0,2,1)$ lies on line $\ell_3$
$3 : P_{17} = (2,2,1,0)$ lies on line $\ell_3$          $11 : P_{61} = (0,2,2,1)$ lies on line $\ell_1$
$4 : P_{22} = (3,3,1,0)$ lies on line $\ell_4$          $12 : P_{70} = (1,0,3,1)$ lies on line $\ell_4$
$5 : P_{30} = (0,2,0,1)$ lies on line $\ell_0$          $13 : P_{81} = (0,3,3,1)$ lies on line $\ell_1$
$6 : P_{33} = (3,2,0,1)$ lies on line $\ell_4$
$7 : P_{34} = (0,3,0,1)$ lies on line $\ell_0$

The single points on the surface are:

**Points on surface but on no line**

The surface has 6 points not on any line:
The points on the surface but not on lines are:

$0 : P_0 = (1,0,0,0)$
$1 : P_{18} = (3,2,1,0)$
$2 : P_{21} = (2,3,1,0)$
$3 : P_{27} = (1,1,0,1)$

$4 : P_{62} = (1,2,2,1)$
$5 : P_{82} = (1,3,3,1)$

## Line Intersection Graph

```
  |0 1 2 3 4
--+---------
0 |0 1 1 0 0
1 |1 0 0 1 1
2 |1 0 0 1 1
3 |0 1 1 0 1
4 |0 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ |
|---|---|---|
| in point | $P_3$ | $P_{26}$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|
| in point | $P_3$ | $P_{42}$ | $P_{42}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|
| in point | $P_{26}$ | $P_{80}$ | $P_{67}$ |

Line 3 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_4$ |
|---|---|---|---|
| in point | $P_{42}$ | $P_{80}$ | $P_{42}$ |

Line 4 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ |
|---|---|---|---|
| in point | $P_{42}$ | $P_{67}$ | $P_{42}$ |

The surface has 25 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$
$1 : P_1 = (0,1,0,0)$
$2 : P_3 = (0,0,0,1)$
$3 : P_{11} = (0,1,1,0)$
$4 : P_{12} = (1,1,1,0)$
$5 : P_{17} = (2,2,1,0)$
$6 : P_{18} = (3,2,1,0)$
$7 : P_{21} = (2,3,1,0)$
$8 : P_{22} = (3,3,1,0)$

$9 : P_{26} = (0,1,0,1)$
$10 : P_{27} = (1,1,0,1)$
$11 : P_{30} = (0,2,0,1)$
$12 : P_{33} = (3,2,0,1)$
$13 : P_{34} = (0,3,0,1)$
$14 : P_{36} = (2,3,0,1)$
$15 : P_{39} = (1,0,1,1)$
$16 : P_{42} = (0,1,1,1)$
$17 : P_{54} = (1,0,2,1)$

$18 : P_{61} = (0,2,2,1)$
$19 : P_{62} = (1,2,2,1)$
$20 : P_{67} = (2,3,2,1)$
$21 : P_{70} = (1,0,3,1)$
$22 : P_{80} = (3,2,3,1)$
$23 : P_{81} = (0,3,3,1)$
$24 : P_{82} = (1,3,3,1)$