

# Rank-355 over GF(16)

January 15, 2021

## The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_1^2 X_2 = 0$$

( 0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(16) is 303178013

## General information

|                            |                        |
|----------------------------|------------------------|
| Number of lines            | 3                      |
| Number of points           | 289                    |
| Number of singular points  | 1                      |
| Number of Eckardt points   | 0                      |
| Number of double points    | 2                      |
| Number of single points    | 47                     |
| Number of points off lines | 240                    |
| Number of Hesse planes     | 0                      |
| Number of axes             | 0                      |
| Type of points on lines    | $17^3$                 |
| Type of lines on points    | $2^2, 1^{47}, 0^{240}$ |

## Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{256} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{321}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{18}$$

Rank of lines: ( 256, 17, 4624 )

Rank of points on Klein quadric: ( 2, 321, 18 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_2$$

### Single Points

The surface has 47 single points:

The single points on the surface are:

- 0 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_1$
- 1 :  $P_{20} = (1, 0, 1, 0)$  lies on line  $\ell_0$
- 2 :  $P_{21} = (2, 0, 1, 0)$  lies on line  $\ell_0$
- 3 :  $P_{22} = (3, 0, 1, 0)$  lies on line  $\ell_0$
- 4 :  $P_{23} = (4, 0, 1, 0)$  lies on line  $\ell_0$
- 5 :  $P_{24} = (5, 0, 1, 0)$  lies on line  $\ell_0$
- 6 :  $P_{25} = (6, 0, 1, 0)$  lies on line  $\ell_0$
- 7 :  $P_{26} = (7, 0, 1, 0)$  lies on line  $\ell_0$
- 8 :  $P_{27} = (8, 0, 1, 0)$  lies on line  $\ell_0$
- 9 :  $P_{28} = (9, 0, 1, 0)$  lies on line  $\ell_0$
- 10 :  $P_{29} = (10, 0, 1, 0)$  lies on line  $\ell_0$
- 11 :  $P_{30} = (11, 0, 1, 0)$  lies on line  $\ell_0$
- 12 :  $P_{31} = (12, 0, 1, 0)$  lies on line  $\ell_0$
- 13 :  $P_{32} = (13, 0, 1, 0)$  lies on line  $\ell_0$
- 14 :  $P_{33} = (14, 0, 1, 0)$  lies on line  $\ell_0$
- 15 :  $P_{34} = (15, 0, 1, 0)$  lies on line  $\ell_0$
- 16 :  $P_{275} = (1, 0, 0, 1)$  lies on line  $\ell_2$
- 17 :  $P_{531} = (1, 0, 1, 1)$  lies on line  $\ell_2$
- 18 :  $P_{546} = (0, 1, 1, 1)$  lies on line  $\ell_1$
- 19 :  $P_{547} = (2, 1, 1, 1)$  lies on line  $\ell_1$
- 20 :  $P_{548} = (3, 1, 1, 1)$  lies on line  $\ell_1$
- 21 :  $P_{549} = (4, 1, 1, 1)$  lies on line  $\ell_1$
- 22 :  $P_{550} = (5, 1, 1, 1)$  lies on line  $\ell_1$
- 23 :  $P_{551} = (6, 1, 1, 1)$  lies on line  $\ell_1$

- 24 :  $P_{552} = (7, 1, 1, 1)$  lies on line  $\ell_1$
- 25 :  $P_{553} = (8, 1, 1, 1)$  lies on line  $\ell_1$
- 26 :  $P_{554} = (9, 1, 1, 1)$  lies on line  $\ell_1$
- 27 :  $P_{555} = (10, 1, 1, 1)$  lies on line  $\ell_1$
- 28 :  $P_{556} = (11, 1, 1, 1)$  lies on line  $\ell_1$
- 29 :  $P_{557} = (12, 1, 1, 1)$  lies on line  $\ell_1$
- 30 :  $P_{558} = (13, 1, 1, 1)$  lies on line  $\ell_1$
- 31 :  $P_{559} = (14, 1, 1, 1)$  lies on line  $\ell_1$
- 32 :  $P_{560} = (15, 1, 1, 1)$  lies on line  $\ell_1$
- 33 :  $P_{786} = (1, 0, 2, 1)$  lies on line  $\ell_2$
- 34 :  $P_{1042} = (1, 0, 3, 1)$  lies on line  $\ell_2$
- 35 :  $P_{1298} = (1, 0, 4, 1)$  lies on line  $\ell_2$
- 36 :  $P_{1554} = (1, 0, 5, 1)$  lies on line  $\ell_2$
- 37 :  $P_{1810} = (1, 0, 6, 1)$  lies on line  $\ell_2$
- 38 :  $P_{2066} = (1, 0, 7, 1)$  lies on line  $\ell_2$
- 39 :  $P_{2322} = (1, 0, 8, 1)$  lies on line  $\ell_2$
- 40 :  $P_{2578} = (1, 0, 9, 1)$  lies on line  $\ell_2$
- 41 :  $P_{2834} = (1, 0, 10, 1)$  lies on line  $\ell_2$
- 42 :  $P_{3090} = (1, 0, 11, 1)$  lies on line  $\ell_2$
- 43 :  $P_{3346} = (1, 0, 12, 1)$  lies on line  $\ell_2$
- 44 :  $P_{3602} = (1, 0, 13, 1)$  lies on line  $\ell_2$
- 45 :  $P_{3858} = (1, 0, 14, 1)$  lies on line  $\ell_2$
- 46 :  $P_{4114} = (1, 0, 15, 1)$  lies on line  $\ell_2$

The single points on the surface are:

### Points on surface but on no line

The surface has 240 points not on any line:

The points on the surface but not on lines are:

- |                                 |                                  |
|---------------------------------|----------------------------------|
| 0 : $P_1 = (0, 1, 0, 0)$        | 48 : $P_{884} = (3, 6, 2, 1)$    |
| 1 : $P_{36} = (1, 1, 1, 0)$     | 49 : $P_{897} = (0, 7, 2, 1)$    |
| 2 : $P_{65} = (14, 2, 1, 0)$    | 50 : $P_{916} = (3, 8, 2, 1)$    |
| 3 : $P_{82} = (15, 3, 1, 0)$    | 51 : $P_{936} = (7, 9, 2, 1)$    |
| 4 : $P_{85} = (2, 4, 1, 0)$     | 52 : $P_{950} = (5, 10, 2, 1)$   |
| 5 : $P_{102} = (3, 5, 1, 0)$    | 53 : $P_{965} = (4, 11, 2, 1)$   |
| 6 : $P_{127} = (12, 6, 1, 0)$   | 54 : $P_{978} = (1, 12, 2, 1)$   |
| 7 : $P_{144} = (13, 7, 1, 0)$   | 55 : $P_{1006} = (13, 13, 2, 1)$ |
| 8 : $P_{152} = (5, 8, 1, 0)$    | 56 : $P_{1014} = (5, 14, 2, 1)$  |
| 9 : $P_{167} = (4, 9, 1, 0)$    | 57 : $P_{1032} = (7, 15, 2, 1)$  |
| 10 : $P_{190} = (11, 10, 1, 0)$ | 58 : $P_{1080} = (7, 2, 3, 1)$   |
| 11 : $P_{205} = (10, 11, 1, 0)$ | 59 : $P_{1102} = (13, 3, 3, 1)$  |
| 12 : $P_{218} = (7, 12, 1, 0)$  | 60 : $P_{1110} = (5, 4, 3, 1)$   |
| 13 : $P_{233} = (6, 13, 1, 0)$  | 61 : $P_{1121} = (0, 5, 3, 1)$   |
| 14 : $P_{252} = (9, 14, 1, 0)$  | 62 : $P_{1144} = (7, 6, 3, 1)$   |
| 15 : $P_{267} = (8, 15, 1, 0)$  | 63 : $P_{1167} = (14, 7, 3, 1)$  |
| 16 : $P_{311} = (5, 2, 0, 1)$   | 64 : $P_{1170} = (1, 8, 3, 1)$   |
| 17 : $P_{329} = (7, 3, 0, 1)$   | 65 : $P_{1198} = (13, 9, 3, 1)$  |
| 18 : $P_{346} = (8, 4, 0, 1)$   | 66 : $P_{1205} = (4, 10, 3, 1)$  |
| 19 : $P_{366} = (12, 5, 0, 1)$  | 67 : $P_{1222} = (5, 11, 3, 1)$  |
| 20 : $P_{379} = (9, 6, 0, 1)$   | 68 : $P_{1236} = (3, 12, 3, 1)$  |
| 21 : $P_{388} = (2, 7, 0, 1)$   | 69 : $P_{1253} = (4, 13, 3, 1)$  |
| 22 : $P_{408} = (6, 8, 0, 1)$   | 70 : $P_{1279} = (14, 14, 3, 1)$ |
| 23 : $P_{433} = (15, 9, 0, 1)$  | 71 : $P_{1284} = (3, 15, 3, 1)$  |
| 24 : $P_{445} = (11, 10, 0, 1)$ | 72 : $P_{1337} = (8, 2, 4, 1)$   |
| 25 : $P_{460} = (10, 11, 0, 1)$ | 73 : $P_{1357} = (12, 3, 4, 1)$  |
| 26 : $P_{470} = (4, 12, 0, 1)$  | 74 : $P_{1368} = (7, 4, 4, 1)$   |
| 27 : $P_{496} = (14, 13, 0, 1)$ | 75 : $P_{1386} = (9, 5, 4, 1)$   |
| 28 : $P_{501} = (3, 14, 0, 1)$  | 76 : $P_{1394} = (1, 6, 4, 1)$   |
| 29 : $P_{527} = (13, 15, 0, 1)$ | 77 : $P_{1416} = (7, 7, 4, 1)$   |
| 30 : $P_{576} = (15, 2, 1, 1)$  | 78 : $P_{1427} = (2, 8, 4, 1)$   |
| 31 : $P_{591} = (14, 3, 1, 1)$  | 79 : $P_{1443} = (2, 9, 4, 1)$   |
| 32 : $P_{596} = (3, 4, 1, 1)$   | 80 : $P_{1466} = (9, 10, 4, 1)$  |
| 33 : $P_{611} = (2, 5, 1, 1)$   | 81 : $P_{1481} = (8, 11, 4, 1)$  |
| 34 : $P_{638} = (13, 6, 1, 1)$  | 82 : $P_{1489} = (0, 12, 4, 1)$  |
| 35 : $P_{653} = (12, 7, 1, 1)$  | 83 : $P_{1510} = (5, 13, 4, 1)$  |
| 36 : $P_{661} = (4, 8, 1, 1)$   | 84 : $P_{1533} = (12, 14, 4, 1)$ |
| 37 : $P_{678} = (5, 9, 1, 1)$   | 85 : $P_{1542} = (5, 15, 4, 1)$  |
| 38 : $P_{699} = (10, 10, 1, 1)$ | 86 : $P_{1587} = (2, 2, 5, 1)$   |
| 39 : $P_{716} = (11, 11, 1, 1)$ | 87 : $P_{1606} = (5, 3, 5, 1)$   |
| 40 : $P_{727} = (6, 12, 1, 1)$  | 88 : $P_{1629} = (12, 4, 5, 1)$  |
| 41 : $P_{744} = (7, 13, 1, 1)$  | 89 : $P_{1640} = (7, 5, 5, 1)$   |
| 42 : $P_{761} = (8, 14, 1, 1)$  | 90 : $P_{1654} = (5, 6, 5, 1)$   |
| 43 : $P_{778} = (9, 15, 1, 1)$  | 91 : $P_{1674} = (9, 7, 5, 1)$   |
| 44 : $P_{830} = (13, 2, 2, 1)$  | 92 : $P_{1681} = (0, 8, 5, 1)$   |
| 45 : $P_{837} = (4, 3, 2, 1)$   | 93 : $P_{1705} = (8, 9, 5, 1)$   |
| 46 : $P_{863} = (14, 4, 2, 1)$  | 94 : $P_{1721} = (8, 10, 5, 1)$  |
| 47 : $P_{879} = (14, 5, 2, 1)$  | 95 : $P_{1738} = (9, 11, 5, 1)$  |

|                                   |                                    |
|-----------------------------------|------------------------------------|
| 96 : $P_{1747} = (2, 12, 5, 1)$   | 150 : $P_{2752} = (15, 10, 9, 1)$  |
| 97 : $P_{1773} = (12, 13, 5, 1)$  | 151 : $P_{2767} = (14, 11, 9, 1)$  |
| 98 : $P_{1784} = (7, 14, 5, 1)$   | 152 : $P_{2781} = (12, 12, 9, 1)$  |
| 99 : $P_{1794} = (1, 15, 5, 1)$   | 153 : $P_{2786} = (1, 13, 9, 1)$   |
| 100 : $P_{1841} = (0, 2, 6, 1)$   | 154 : $P_{2805} = (4, 14, 9, 1)$   |
| 101 : $P_{1872} = (15, 3, 6, 1)$  | 155 : $P_{2821} = (4, 15, 9, 1)$   |
| 102 : $P_{1874} = (1, 4, 6, 1)$   | 156 : $P_{2869} = (4, 2, 10, 1)$   |
| 103 : $P_{1900} = (11, 5, 6, 1)$  | 157 : $P_{2883} = (2, 3, 10, 1)$   |
| 104 : $P_{1916} = (11, 6, 6, 1)$  | 158 : $P_{2899} = (2, 4, 10, 1)$   |
| 105 : $P_{1926} = (5, 7, 6, 1)$   | 159 : $P_{2923} = (10, 5, 10, 1)$  |
| 106 : $P_{1944} = (7, 8, 6, 1)$   | 160 : $P_{2943} = (14, 6, 10, 1)$  |
| 107 : $P_{1963} = (10, 9, 6, 1)$  | 161 : $P_{2949} = (4, 7, 10, 1)$   |
| 108 : $P_{1976} = (7, 10, 6, 1)$  | 162 : $P_{2970} = (9, 8, 10, 1)$   |
| 109 : $P_{1991} = (6, 11, 6, 1)$  | 163 : $P_{2991} = (14, 9, 10, 1)$  |
| 110 : $P_{2006} = (5, 12, 6, 1)$  | 164 : $P_{2993} = (0, 10, 10, 1)$  |
| 111 : $P_{2023} = (6, 13, 6, 1)$  | 165 : $P_{3010} = (1, 11, 10, 1)$  |
| 112 : $P_{2043} = (10, 14, 6, 1)$ | 166 : $P_{3036} = (11, 12, 10, 1)$ |
| 113 : $P_{2064} = (15, 15, 6, 1)$ | 167 : $P_{3052} = (11, 13, 10, 1)$ |
| 114 : $P_{2107} = (10, 2, 7, 1)$  | 168 : $P_{3066} = (9, 14, 10, 1)$  |
| 115 : $P_{2119} = (6, 3, 7, 1)$   | 169 : $P_{3083} = (10, 15, 10, 1)$ |
| 116 : $P_{2139} = (10, 4, 7, 1)$  | 170 : $P_{3135} = (14, 2, 11, 1)$  |
| 117 : $P_{2150} = (5, 5, 7, 1)$   | 171 : $P_{3148} = (11, 3, 11, 1)$  |
| 118 : $P_{2176} = (15, 6, 7, 1)$  | 172 : $P_{3162} = (9, 4, 11, 1)$   |
| 119 : $P_{2188} = (11, 7, 7, 1)$  | 173 : $P_{3173} = (4, 5, 11, 1)$   |
| 120 : $P_{2198} = (5, 8, 7, 1)$   | 174 : $P_{3195} = (10, 6, 11, 1)$  |
| 121 : $P_{2209} = (0, 9, 7, 1)$   | 175 : $P_{3211} = (10, 7, 11, 1)$  |
| 122 : $P_{2231} = (6, 10, 7, 1)$  | 176 : $P_{3228} = (11, 8, 11, 1)$  |
| 123 : $P_{2248} = (7, 11, 7, 1)$  | 177 : $P_{3237} = (4, 9, 11, 1)$   |
| 124 : $P_{2264} = (7, 12, 7, 1)$  | 178 : $P_{3250} = (1, 10, 11, 1)$  |
| 125 : $P_{2288} = (15, 13, 7, 1)$ | 179 : $P_{3265} = (0, 11, 11, 1)$  |
| 126 : $P_{2290} = (1, 14, 7, 1)$  | 180 : $P_{3290} = (9, 12, 11, 1)$  |
| 127 : $P_{2316} = (11, 15, 7, 1)$ | 181 : $P_{3299} = (2, 13, 11, 1)$  |
| 128 : $P_{2365} = (12, 2, 8, 1)$  | 182 : $P_{3315} = (2, 14, 11, 1)$  |
| 129 : $P_{2370} = (1, 3, 8, 1)$   | 183 : $P_{3343} = (14, 15, 11, 1)$ |
| 130 : $P_{2389} = (4, 4, 8, 1)$   | 184 : $P_{3378} = (1, 2, 12, 1)$   |
| 131 : $P_{2409} = (8, 5, 8, 1)$   | 185 : $P_{3403} = (10, 3, 12, 1)$  |
| 132 : $P_{2421} = (4, 6, 8, 1)$   | 186 : $P_{3420} = (11, 4, 12, 1)$  |
| 133 : $P_{2439} = (6, 7, 8, 1)$   | 187 : $P_{3438} = (13, 5, 12, 1)$  |
| 134 : $P_{2461} = (12, 8, 8, 1)$  | 188 : $P_{3453} = (12, 6, 12, 1)$  |
| 135 : $P_{2471} = (6, 9, 8, 1)$   | 189 : $P_{3460} = (3, 7, 12, 1)$   |
| 136 : $P_{2495} = (14, 10, 8, 1)$ | 190 : $P_{3481} = (8, 8, 12, 1)$   |
| 137 : $P_{2512} = (15, 11, 8, 1)$ | 191 : $P_{3500} = (11, 9, 12, 1)$  |
| 138 : $P_{2527} = (14, 12, 8, 1)$ | 192 : $P_{3517} = (12, 10, 12, 1)$ |
| 139 : $P_{2537} = (8, 13, 8, 1)$  | 193 : $P_{3534} = (13, 11, 12, 1)$ |
| 140 : $P_{2560} = (15, 14, 8, 1)$ | 194 : $P_{3547} = (10, 12, 12, 1)$ |
| 141 : $P_{2561} = (0, 15, 8, 1)$  | 195 : $P_{3556} = (3, 13, 12, 1)$  |
| 142 : $P_{2615} = (6, 2, 9, 1)$   | 196 : $P_{3569} = (0, 14, 12, 1)$  |
| 143 : $P_{2633} = (8, 3, 9, 1)$   | 197 : $P_{3593} = (8, 15, 12, 1)$  |
| 144 : $P_{2656} = (15, 4, 9, 1)$  | 198 : $P_{3644} = (11, 2, 13, 1)$  |
| 145 : $P_{2663} = (6, 5, 9, 1)$   | 199 : $P_{3652} = (3, 3, 13, 1)$   |
| 146 : $P_{2673} = (0, 6, 9, 1)$   | 200 : $P_{3665} = (0, 4, 13, 1)$   |
| 147 : $P_{2697} = (8, 7, 9, 1)$   | 201 : $P_{3684} = (3, 5, 13, 1)$   |
| 148 : $P_{2719} = (14, 8, 9, 1)$  | 202 : $P_{3705} = (8, 6, 13, 1)$   |
| 149 : $P_{2733} = (12, 9, 9, 1)$  | 203 : $P_{3726} = (13, 7, 13, 1)$  |

204 :  $P_{3739} = (10, 8, 13, 1)$   
 205 :  $P_{3746} = (1, 9, 13, 1)$   
 206 :  $P_{3774} = (13, 10, 13, 1)$   
 207 :  $P_{3789} = (12, 11, 13, 1)$   
 208 :  $P_{3801} = (8, 12, 13, 1)$   
 209 :  $P_{3819} = (10, 13, 13, 1)$   
 210 :  $P_{3836} = (11, 14, 13, 1)$   
 211 :  $P_{3853} = (12, 15, 13, 1)$   
 212 :  $P_{3898} = (9, 2, 14, 1)$   
 213 :  $P_{3914} = (9, 3, 14, 1)$   
 214 :  $P_{3934} = (13, 4, 14, 1)$   
 215 :  $P_{3952} = (15, 5, 14, 1)$   
 216 :  $P_{3959} = (6, 6, 14, 1)$   
 217 :  $P_{3970} = (1, 7, 14, 1)$   
 218 :  $P_{3998} = (13, 8, 14, 1)$   
 219 :  $P_{4004} = (3, 9, 14, 1)$   
 220 :  $P_{4019} = (2, 10, 14, 1)$   
 221 :  $P_{4036} = (3, 11, 14, 1)$   
 222 :  $P_{4064} = (15, 12, 14, 1)$

223 :  $P_{4065} = (0, 13, 14, 1)$   
 224 :  $P_{4087} = (6, 14, 14, 1)$   
 225 :  $P_{4099} = (2, 15, 14, 1)$   
 226 :  $P_{4148} = (3, 2, 15, 1)$   
 227 :  $P_{4161} = (0, 3, 15, 1)$   
 228 :  $P_{4183} = (6, 4, 15, 1)$   
 229 :  $P_{4194} = (1, 5, 15, 1)$   
 230 :  $P_{4211} = (2, 6, 15, 1)$   
 231 :  $P_{4240} = (15, 7, 15, 1)$   
 232 :  $P_{4256} = (15, 8, 15, 1)$   
 233 :  $P_{4266} = (9, 9, 15, 1)$   
 234 :  $P_{4276} = (3, 10, 15, 1)$   
 235 :  $P_{4291} = (2, 11, 15, 1)$   
 236 :  $P_{4318} = (13, 12, 15, 1)$   
 237 :  $P_{4330} = (9, 13, 15, 1)$   
 238 :  $P_{4350} = (13, 14, 15, 1)$   
 239 :  $P_{4359} = (6, 15, 15, 1)$

## Line Intersection Graph

|   |   |   |   |
|---|---|---|---|
|   | 0 | 1 | 2 |
| 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 2 | 1 | 0 | 0 |

Neighbor sets in the line intersection graph:

Line 0 intersects

| Line     | $\ell_1$ | $\ell_2$ |
|----------|----------|----------|
| in point | $P_0$    | $P_2$    |

Line 1 intersects

| Line     | $\ell_0$ |
|----------|----------|
| in point | $P_0$    |

Line 2 intersects

| Line     | $\ell_0$ |
|----------|----------|
| in point | $P_2$    |

The surface has 289 points:

The points on the surface are:

|                              |                                |                                 |
|------------------------------|--------------------------------|---------------------------------|
| 0 : $P_0 = (1, 0, 0, 0)$     | 13 : $P_{29} = (10, 0, 1, 0)$  | 26 : $P_{152} = (5, 8, 1, 0)$   |
| 1 : $P_1 = (0, 1, 0, 0)$     | 14 : $P_{30} = (11, 0, 1, 0)$  | 27 : $P_{167} = (4, 9, 1, 0)$   |
| 2 : $P_2 = (0, 0, 1, 0)$     | 15 : $P_{31} = (12, 0, 1, 0)$  | 28 : $P_{190} = (11, 10, 1, 0)$ |
| 3 : $P_4 = (1, 1, 1, 1)$     | 16 : $P_{32} = (13, 0, 1, 0)$  | 29 : $P_{205} = (10, 11, 1, 0)$ |
| 4 : $P_{20} = (1, 0, 1, 0)$  | 17 : $P_{33} = (14, 0, 1, 0)$  | 30 : $P_{218} = (7, 12, 1, 0)$  |
| 5 : $P_{21} = (2, 0, 1, 0)$  | 18 : $P_{34} = (15, 0, 1, 0)$  | 31 : $P_{233} = (6, 13, 1, 0)$  |
| 6 : $P_{22} = (3, 0, 1, 0)$  | 19 : $P_{36} = (1, 1, 1, 0)$   | 32 : $P_{252} = (9, 14, 1, 0)$  |
| 7 : $P_{23} = (4, 0, 1, 0)$  | 20 : $P_{65} = (14, 2, 1, 0)$  | 33 : $P_{267} = (8, 15, 1, 0)$  |
| 8 : $P_{24} = (5, 0, 1, 0)$  | 21 : $P_{82} = (15, 3, 1, 0)$  | 34 : $P_{275} = (1, 0, 0, 1)$   |
| 9 : $P_{25} = (6, 0, 1, 0)$  | 22 : $P_{85} = (2, 4, 1, 0)$   | 35 : $P_{311} = (5, 2, 0, 1)$   |
| 10 : $P_{26} = (7, 0, 1, 0)$ | 23 : $P_{102} = (3, 5, 1, 0)$  | 36 : $P_{329} = (7, 3, 0, 1)$   |
| 11 : $P_{27} = (8, 0, 1, 0)$ | 24 : $P_{127} = (12, 6, 1, 0)$ | 37 : $P_{346} = (8, 4, 0, 1)$   |
| 12 : $P_{28} = (9, 0, 1, 0)$ | 25 : $P_{144} = (13, 7, 1, 0)$ | 38 : $P_{366} = (12, 5, 0, 1)$  |

|                                  |                                   |                                   |
|----------------------------------|-----------------------------------|-----------------------------------|
| 39 : $P_{379} = (9, 6, 0, 1)$    | 93 : $P_{1032} = (7, 15, 2, 1)$   | 147 : $P_{1963} = (10, 9, 6, 1)$  |
| 40 : $P_{388} = (2, 7, 0, 1)$    | 94 : $P_{1042} = (1, 0, 3, 1)$    | 148 : $P_{1976} = (7, 10, 6, 1)$  |
| 41 : $P_{408} = (6, 8, 0, 1)$    | 95 : $P_{1080} = (7, 2, 3, 1)$    | 149 : $P_{1991} = (6, 11, 6, 1)$  |
| 42 : $P_{433} = (15, 9, 0, 1)$   | 96 : $P_{1102} = (13, 3, 3, 1)$   | 150 : $P_{2006} = (5, 12, 6, 1)$  |
| 43 : $P_{445} = (11, 10, 0, 1)$  | 97 : $P_{1110} = (5, 4, 3, 1)$    | 151 : $P_{2023} = (6, 13, 6, 1)$  |
| 44 : $P_{460} = (10, 11, 0, 1)$  | 98 : $P_{1121} = (0, 5, 3, 1)$    | 152 : $P_{2043} = (10, 14, 6, 1)$ |
| 45 : $P_{470} = (4, 12, 0, 1)$   | 99 : $P_{1144} = (7, 6, 3, 1)$    | 153 : $P_{2064} = (15, 15, 6, 1)$ |
| 46 : $P_{496} = (14, 13, 0, 1)$  | 100 : $P_{1167} = (14, 7, 3, 1)$  | 154 : $P_{2066} = (1, 0, 7, 1)$   |
| 47 : $P_{501} = (3, 14, 0, 1)$   | 101 : $P_{1170} = (1, 8, 3, 1)$   | 155 : $P_{2107} = (10, 2, 7, 1)$  |
| 48 : $P_{527} = (13, 15, 0, 1)$  | 102 : $P_{1198} = (13, 9, 3, 1)$  | 156 : $P_{2119} = (6, 3, 7, 1)$   |
| 49 : $P_{531} = (1, 0, 1, 1)$    | 103 : $P_{1205} = (4, 10, 3, 1)$  | 157 : $P_{2139} = (10, 4, 7, 1)$  |
| 50 : $P_{546} = (0, 1, 1, 1)$    | 104 : $P_{1222} = (5, 11, 3, 1)$  | 158 : $P_{2150} = (5, 5, 7, 1)$   |
| 51 : $P_{547} = (2, 1, 1, 1)$    | 105 : $P_{1236} = (3, 12, 3, 1)$  | 159 : $P_{2176} = (15, 6, 7, 1)$  |
| 52 : $P_{548} = (3, 1, 1, 1)$    | 106 : $P_{1253} = (4, 13, 3, 1)$  | 160 : $P_{2188} = (11, 7, 7, 1)$  |
| 53 : $P_{549} = (4, 1, 1, 1)$    | 107 : $P_{1279} = (14, 14, 3, 1)$ | 161 : $P_{2198} = (5, 8, 7, 1)$   |
| 54 : $P_{550} = (5, 1, 1, 1)$    | 108 : $P_{1284} = (3, 15, 3, 1)$  | 162 : $P_{2209} = (0, 9, 7, 1)$   |
| 55 : $P_{551} = (6, 1, 1, 1)$    | 109 : $P_{1298} = (1, 0, 4, 1)$   | 163 : $P_{2231} = (6, 10, 7, 1)$  |
| 56 : $P_{552} = (7, 1, 1, 1)$    | 110 : $P_{1337} = (8, 2, 4, 1)$   | 164 : $P_{2248} = (7, 11, 7, 1)$  |
| 57 : $P_{553} = (8, 1, 1, 1)$    | 111 : $P_{1357} = (12, 3, 4, 1)$  | 165 : $P_{2264} = (7, 12, 7, 1)$  |
| 58 : $P_{554} = (9, 1, 1, 1)$    | 112 : $P_{1368} = (7, 4, 4, 1)$   | 166 : $P_{2288} = (15, 13, 7, 1)$ |
| 59 : $P_{555} = (10, 1, 1, 1)$   | 113 : $P_{1386} = (9, 5, 4, 1)$   | 167 : $P_{2290} = (1, 14, 7, 1)$  |
| 60 : $P_{556} = (11, 1, 1, 1)$   | 114 : $P_{1394} = (1, 6, 4, 1)$   | 168 : $P_{2316} = (11, 15, 7, 1)$ |
| 61 : $P_{557} = (12, 1, 1, 1)$   | 115 : $P_{1416} = (7, 7, 4, 1)$   | 169 : $P_{2322} = (1, 0, 8, 1)$   |
| 62 : $P_{558} = (13, 1, 1, 1)$   | 116 : $P_{1427} = (2, 8, 4, 1)$   | 170 : $P_{2365} = (12, 2, 8, 1)$  |
| 63 : $P_{559} = (14, 1, 1, 1)$   | 117 : $P_{1443} = (2, 9, 4, 1)$   | 171 : $P_{2370} = (1, 3, 8, 1)$   |
| 64 : $P_{560} = (15, 1, 1, 1)$   | 118 : $P_{1466} = (9, 10, 4, 1)$  | 172 : $P_{2389} = (4, 4, 8, 1)$   |
| 65 : $P_{576} = (15, 2, 1, 1)$   | 119 : $P_{1481} = (8, 11, 4, 1)$  | 173 : $P_{2409} = (8, 5, 8, 1)$   |
| 66 : $P_{591} = (14, 3, 1, 1)$   | 120 : $P_{1489} = (0, 12, 4, 1)$  | 174 : $P_{2421} = (4, 6, 8, 1)$   |
| 67 : $P_{596} = (3, 4, 1, 1)$    | 121 : $P_{1510} = (5, 13, 4, 1)$  | 175 : $P_{2439} = (6, 7, 8, 1)$   |
| 68 : $P_{611} = (2, 5, 1, 1)$    | 122 : $P_{1533} = (12, 14, 4, 1)$ | 176 : $P_{2461} = (12, 8, 8, 1)$  |
| 69 : $P_{638} = (13, 6, 1, 1)$   | 123 : $P_{1542} = (5, 15, 4, 1)$  | 177 : $P_{2471} = (6, 9, 8, 1)$   |
| 70 : $P_{653} = (12, 7, 1, 1)$   | 124 : $P_{1554} = (1, 0, 5, 1)$   | 178 : $P_{2495} = (14, 10, 8, 1)$ |
| 71 : $P_{661} = (4, 8, 1, 1)$    | 125 : $P_{1587} = (2, 2, 5, 1)$   | 179 : $P_{2512} = (15, 11, 8, 1)$ |
| 72 : $P_{678} = (5, 9, 1, 1)$    | 126 : $P_{1606} = (5, 3, 5, 1)$   | 180 : $P_{2527} = (14, 12, 8, 1)$ |
| 73 : $P_{699} = (10, 10, 1, 1)$  | 127 : $P_{1629} = (12, 4, 5, 1)$  | 181 : $P_{2537} = (8, 13, 8, 1)$  |
| 74 : $P_{716} = (11, 11, 1, 1)$  | 128 : $P_{1640} = (7, 5, 5, 1)$   | 182 : $P_{2560} = (15, 14, 8, 1)$ |
| 75 : $P_{727} = (6, 12, 1, 1)$   | 129 : $P_{1654} = (5, 6, 5, 1)$   | 183 : $P_{2561} = (0, 15, 8, 1)$  |
| 76 : $P_{744} = (7, 13, 1, 1)$   | 130 : $P_{1674} = (9, 7, 5, 1)$   | 184 : $P_{2578} = (1, 0, 9, 1)$   |
| 77 : $P_{761} = (8, 14, 1, 1)$   | 131 : $P_{1681} = (0, 8, 5, 1)$   | 185 : $P_{2615} = (6, 2, 9, 1)$   |
| 78 : $P_{778} = (9, 15, 1, 1)$   | 132 : $P_{1705} = (8, 9, 5, 1)$   | 186 : $P_{2633} = (8, 3, 9, 1)$   |
| 79 : $P_{786} = (1, 0, 2, 1)$    | 133 : $P_{1721} = (8, 10, 5, 1)$  | 187 : $P_{2656} = (15, 4, 9, 1)$  |
| 80 : $P_{830} = (13, 2, 2, 1)$   | 134 : $P_{1738} = (9, 11, 5, 1)$  | 188 : $P_{2663} = (6, 5, 9, 1)$   |
| 81 : $P_{837} = (4, 3, 2, 1)$    | 135 : $P_{1747} = (2, 12, 5, 1)$  | 189 : $P_{2673} = (0, 6, 9, 1)$   |
| 82 : $P_{863} = (14, 4, 2, 1)$   | 136 : $P_{1773} = (12, 13, 5, 1)$ | 190 : $P_{2697} = (8, 7, 9, 1)$   |
| 83 : $P_{879} = (14, 5, 2, 1)$   | 137 : $P_{1784} = (7, 14, 5, 1)$  | 191 : $P_{2719} = (14, 8, 9, 1)$  |
| 84 : $P_{884} = (3, 6, 2, 1)$    | 138 : $P_{1794} = (1, 15, 5, 1)$  | 192 : $P_{2733} = (12, 9, 9, 1)$  |
| 85 : $P_{897} = (0, 7, 2, 1)$    | 139 : $P_{1810} = (1, 0, 6, 1)$   | 193 : $P_{2752} = (15, 10, 9, 1)$ |
| 86 : $P_{916} = (3, 8, 2, 1)$    | 140 : $P_{1841} = (0, 2, 6, 1)$   | 194 : $P_{2767} = (14, 11, 9, 1)$ |
| 87 : $P_{936} = (7, 9, 2, 1)$    | 141 : $P_{1872} = (15, 3, 6, 1)$  | 195 : $P_{2781} = (12, 12, 9, 1)$ |
| 88 : $P_{950} = (5, 10, 2, 1)$   | 142 : $P_{1874} = (1, 4, 6, 1)$   | 196 : $P_{2786} = (1, 13, 9, 1)$  |
| 89 : $P_{965} = (4, 11, 2, 1)$   | 143 : $P_{1900} = (11, 5, 6, 1)$  | 197 : $P_{2805} = (4, 14, 9, 1)$  |
| 90 : $P_{978} = (1, 12, 2, 1)$   | 144 : $P_{1916} = (11, 6, 6, 1)$  | 198 : $P_{2821} = (4, 15, 9, 1)$  |
| 91 : $P_{1006} = (13, 13, 2, 1)$ | 145 : $P_{1926} = (5, 7, 6, 1)$   | 199 : $P_{2834} = (1, 0, 10, 1)$  |
| 92 : $P_{1014} = (5, 14, 2, 1)$  | 146 : $P_{1944} = (7, 8, 6, 1)$   | 200 : $P_{2869} = (4, 2, 10, 1)$  |

|                                    |                                    |                                    |
|------------------------------------|------------------------------------|------------------------------------|
| 201 : $P_{2883} = (2, 3, 10, 1)$   | 231 : $P_{3403} = (10, 3, 12, 1)$  | 261 : $P_{3914} = (9, 3, 14, 1)$   |
| 202 : $P_{2899} = (2, 4, 10, 1)$   | 232 : $P_{3420} = (11, 4, 12, 1)$  | 262 : $P_{3934} = (13, 4, 14, 1)$  |
| 203 : $P_{2923} = (10, 5, 10, 1)$  | 233 : $P_{3438} = (13, 5, 12, 1)$  | 263 : $P_{3952} = (15, 5, 14, 1)$  |
| 204 : $P_{2943} = (14, 6, 10, 1)$  | 234 : $P_{3453} = (12, 6, 12, 1)$  | 264 : $P_{3959} = (6, 6, 14, 1)$   |
| 205 : $P_{2949} = (4, 7, 10, 1)$   | 235 : $P_{3460} = (3, 7, 12, 1)$   | 265 : $P_{3970} = (1, 7, 14, 1)$   |
| 206 : $P_{2970} = (9, 8, 10, 1)$   | 236 : $P_{3481} = (8, 8, 12, 1)$   | 266 : $P_{3998} = (13, 8, 14, 1)$  |
| 207 : $P_{2991} = (14, 9, 10, 1)$  | 237 : $P_{3500} = (11, 9, 12, 1)$  | 267 : $P_{4004} = (3, 9, 14, 1)$   |
| 208 : $P_{2993} = (0, 10, 10, 1)$  | 238 : $P_{3517} = (12, 10, 12, 1)$ | 268 : $P_{4019} = (2, 10, 14, 1)$  |
| 209 : $P_{3010} = (1, 11, 10, 1)$  | 239 : $P_{3534} = (13, 11, 12, 1)$ | 269 : $P_{4036} = (3, 11, 14, 1)$  |
| 210 : $P_{3036} = (11, 12, 10, 1)$ | 240 : $P_{3547} = (10, 12, 12, 1)$ | 270 : $P_{4064} = (15, 12, 14, 1)$ |
| 211 : $P_{3052} = (11, 13, 10, 1)$ | 241 : $P_{3556} = (3, 13, 12, 1)$  | 271 : $P_{4065} = (0, 13, 14, 1)$  |
| 212 : $P_{3066} = (9, 14, 10, 1)$  | 242 : $P_{3569} = (0, 14, 12, 1)$  | 272 : $P_{4087} = (6, 14, 14, 1)$  |
| 213 : $P_{3083} = (10, 15, 10, 1)$ | 243 : $P_{3593} = (8, 15, 12, 1)$  | 273 : $P_{4099} = (2, 15, 14, 1)$  |
| 214 : $P_{3090} = (1, 0, 11, 1)$   | 244 : $P_{3602} = (1, 0, 13, 1)$   | 274 : $P_{4114} = (1, 0, 15, 1)$   |
| 215 : $P_{3135} = (14, 2, 11, 1)$  | 245 : $P_{3644} = (11, 2, 13, 1)$  | 275 : $P_{4148} = (3, 2, 15, 1)$   |
| 216 : $P_{3148} = (11, 3, 11, 1)$  | 246 : $P_{3652} = (3, 3, 13, 1)$   | 276 : $P_{4161} = (0, 3, 15, 1)$   |
| 217 : $P_{3162} = (9, 4, 11, 1)$   | 247 : $P_{3665} = (0, 4, 13, 1)$   | 277 : $P_{4183} = (6, 4, 15, 1)$   |
| 218 : $P_{3173} = (4, 5, 11, 1)$   | 248 : $P_{3684} = (3, 5, 13, 1)$   | 278 : $P_{4194} = (1, 5, 15, 1)$   |
| 219 : $P_{3195} = (10, 6, 11, 1)$  | 249 : $P_{3705} = (8, 6, 13, 1)$   | 279 : $P_{4211} = (2, 6, 15, 1)$   |
| 220 : $P_{3211} = (10, 7, 11, 1)$  | 250 : $P_{3726} = (13, 7, 13, 1)$  | 280 : $P_{4240} = (15, 7, 15, 1)$  |
| 221 : $P_{3228} = (11, 8, 11, 1)$  | 251 : $P_{3739} = (10, 8, 13, 1)$  | 281 : $P_{4256} = (15, 8, 15, 1)$  |
| 222 : $P_{3237} = (4, 9, 11, 1)$   | 252 : $P_{3746} = (1, 9, 13, 1)$   | 282 : $P_{4266} = (9, 9, 15, 1)$   |
| 223 : $P_{3250} = (1, 10, 11, 1)$  | 253 : $P_{3774} = (13, 10, 13, 1)$ | 283 : $P_{4276} = (3, 10, 15, 1)$  |
| 224 : $P_{3265} = (0, 11, 11, 1)$  | 254 : $P_{3789} = (12, 11, 13, 1)$ | 284 : $P_{4291} = (2, 11, 15, 1)$  |
| 225 : $P_{3290} = (9, 12, 11, 1)$  | 255 : $P_{3801} = (8, 12, 13, 1)$  | 285 : $P_{4318} = (13, 12, 15, 1)$ |
| 226 : $P_{3299} = (2, 13, 11, 1)$  | 256 : $P_{3819} = (10, 13, 13, 1)$ | 286 : $P_{4330} = (9, 13, 15, 1)$  |
| 227 : $P_{3315} = (2, 14, 11, 1)$  | 257 : $P_{3836} = (11, 14, 13, 1)$ | 287 : $P_{4350} = (13, 14, 15, 1)$ |
| 228 : $P_{3343} = (14, 15, 11, 1)$ | 258 : $P_{3853} = (12, 15, 13, 1)$ | 288 : $P_{4359} = (6, 15, 15, 1)$  |
| 229 : $P_{3346} = (1, 0, 12, 1)$   | 259 : $P_{3858} = (1, 0, 14, 1)$   |                                    |
| 230 : $P_{3378} = (1, 2, 12, 1)$   | 260 : $P_{3898} = (9, 2, 14, 1)$   |                                    |