Rank-65921 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(16) is 304226853

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	87
Number of points off lines	211
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 ⁶
Type of lines on points	$3, 2^6, 1^{87}, 0^{211}$

Singular Points

The surface has 1 singular points:

$$0: P_{531} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \mathbf{Pl}(1, 0, 0, 0, 1, 0)_{290}$$

$$\ell_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{320}$$

$$\ell_{2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{9426}$$

$$\ell_{3} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{9427}$$

$$\ell_{4} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \mathbf{Pl}(1, 1, 1, 1, 1, 1, 1)_{1250}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \mathbf{Pl}(1, 1, 1, 1, 1, 1, 1, 1)_{9201}$$

Rank of lines: (16, 257, 530, 4658, 4385, 4657)

Rank of points on Klein quadric: (290, 320, 9426, 9427, 1250, 9201)

Eckardt Points

The surface has 1 Eckardt points: $0: P_{531} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1).$

Double Points

The surface has 6 Double points: The double points on the surface are:

$$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$$

$$P_{290} = (0,1,0,1) = \ell_0 \cap \ell_5$$

$$P_{530} = (0,0,1,1) = \ell_1 \cap \ell_2$$

$$P_5 = (1,1,0,0) = \ell_2 \cap \ell_3$$

$P_{546} = (0, 1, 1, 1) = \ell_3 \cap \ell_4$ $P_{36} = (1, 1, 1, 0) = \ell_4 \cap \ell_5$

Single Points

The surface has 87 single points: The single points on the surface are:

 $0: P_4 = (1, 1, 1, 1)$ lies on line ℓ_2 15: $P_{304} = (14, 1, 0, 1)$ lies on line ℓ_0 1: $P_{275} = (1,0,0,1)$ lies on line ℓ_4 16: $P_{305} = (15, 1, 0, 1)$ lies on line ℓ_0 2: $P_{291} = (1, 1, 0, 1)$ lies on line ℓ_0 17: $P_{532} = (2,0,1,1)$ lies on line ℓ_1 18: $P_{533} = (3,0,1,1)$ lies on line ℓ_1 $3: P_{292} = (2,1,0,1)$ lies on line ℓ_0 4: $P_{293} = (3, 1, 0, 1)$ lies on line ℓ_0 19: $P_{534} = (4,0,1,1)$ lies on line ℓ_1 5: $P_{294} = (4, 1, 0, 1)$ lies on line ℓ_0 20: $P_{535} = (5, 0, 1, 1)$ lies on line ℓ_1 6: $P_{295} = (5, 1, 0, 1)$ lies on line ℓ_0 21: $P_{536} = (6,0,1,1)$ lies on line ℓ_1 7: $P_{296} = (6, 1, 0, 1)$ lies on line ℓ_0 22: $P_{537} = (7,0,1,1)$ lies on line ℓ_1 8: $P_{297} = (7, 1, 0, 1)$ lies on line ℓ_0 23: $P_{538} = (8, 0, 1, 1)$ lies on line ℓ_1 9 : $P_{298} = (8, 1, 0, 1)$ lies on line ℓ_0 24: $P_{539} = (9,0,1,1)$ lies on line ℓ_1 10: $P_{299} = (9, 1, 0, 1)$ lies on line ℓ_0 25: $P_{540} = (10, 0, 1, 1)$ lies on line ℓ_1 11: $P_{300} = (10, 1, 0, 1)$ lies on line ℓ_0 26: $P_{541} = (11, 0, 1, 1)$ lies on line ℓ_1 12: $P_{301} = (11, 1, 0, 1)$ lies on line ℓ_0 27: $P_{542} = (12, 0, 1, 1)$ lies on line ℓ_1 13: $P_{302} = (12, 1, 0, 1)$ lies on line ℓ_0 28: $P_{543} = (13, 0, 1, 1)$ lies on line ℓ_1 29 : $P_{544} = (14, 0, 1, 1)$ lies on line ℓ_1 14: $P_{303} = (13, 1, 0, 1)$ lies on line ℓ_0

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30: P_{545} = (15, 0, 1, 1) lies on line \ell_1
                                                                      59: P_{820} = (3, 2, 2, 1) lies on line \ell_4
31: P_{563} = (2, 2, 1, 1) lies on line \ell_2
                                                                      60: P_{835} = (2, 3, 2, 1) lies on line \ell_5
32: P_{564} = (3, 2, 1, 1) lies on line \ell_3
                                                                      61: P_{1076} = (3, 2, 3, 1) lies on line \ell_5
33: P_{579} = (2, 3, 1, 1) lies on line \ell_3
                                                                      62: P_{1091} = (2, 3, 3, 1) lies on line \ell_4
34: P_{580} = (3, 3, 1, 1) lies on line \ell_2
                                                                      63: P_{1366} = (5, 4, 4, 1) lies on line \ell_4
35: P_{597} = (4, 4, 1, 1) lies on line \ell_2
                                                                      64: P_{1381} = (4, 5, 4, 1) lies on line \ell_5
36: P_{598} = (5, 4, 1, 1) lies on line \ell_3
                                                                      65: P_{1622} = (5, 4, 5, 1) lies on line \ell_5
                                                                      66: P_{1637} = (4, 5, 5, 1) lies on line \ell_4
37: P_{613} = (4,5,1,1) lies on line \ell_3
38: P_{614} = (5, 5, 1, 1) lies on line \ell_2
                                                                      67: P_{1912} = (7, 6, 6, 1) lies on line \ell_4
39: P_{631} = (6,6,1,1) lies on line \ell_2
                                                                      68: P_{1927} = (6,7,6,1) lies on line \ell_5
40: P_{632} = (7, 6, 1, 1) lies on line \ell_3
                                                                      69: P_{2168} = (7, 6, 7, 1) lies on line \ell_5
                                                                      70: P_{2183} = (6, 7, 7, 1) lies on line \ell_4
41: P_{647} = (6,7,1,1) lies on line \ell_3
42: P_{648} = (7,7,1,1) lies on line \ell_2
                                                                      71: P_{2458} = (9, 8, 8, 1) lies on line \ell_4
43: P_{665} = (8, 8, 1, 1) lies on line \ell_2
                                                                      72: P_{2473} = (8, 9, 8, 1) lies on line \ell_5
44 : P_{666} = (9, 8, 1, 1) lies on line \ell_3
                                                                      73 : P_{2714} = (9, 8, 9, 1) lies on line \ell_5
45: P_{681} = (8, 9, 1, 1) lies on line \ell_3
                                                                      74: P_{2729} = (8, 9, 9, 1) lies on line \ell_4
46: P_{682} = (9, 9, 1, 1) lies on line \ell_2
                                                                      75: P_{3004} = (11, 10, 10, 1) lies on line \ell_4
                                                                      76: P_{3019} = (10, 11, 10, 1) lies on line \ell_5
47: P_{699} = (10, 10, 1, 1) lies on line \ell_2
48: P_{700} = (11, 10, 1, 1) lies on line \ell_3
                                                                      77: P_{3260} = (11, 10, 11, 1) lies on line \ell_5
                                                                      78: P_{3275} = (10, 11, 11, 1) lies on line \ell_4
49: P_{715} = (10, 11, 1, 1) lies on line \ell_3
50: P_{716} = (11, 11, 1, 1) lies on line \ell_2
                                                                      79: P_{3550} = (13, 12, 12, 1) lies on line \ell_4
51: P_{733} = (12, 12, 1, 1) lies on line \ell_2
                                                                      80 : P_{3565} = (12, 13, 12, 1) lies on line \ell_5
52: P_{734} = (13, 12, 1, 1) lies on line \ell_3
                                                                      81: P_{3806} = (13, 12, 13, 1) lies on line \ell_5
                                                                      82 : P_{3821} = (12, 13, 13, 1) lies on line \ell_4
53: P_{749} = (12, 13, 1, 1) lies on line \ell_3
54: P_{750} = (13, 13, 1, 1) lies on line \ell_2
                                                                      83: P_{4096} = (15, 14, 14, 1) lies on line \ell_4
55: P_{767} = (14, 14, 1, 1) lies on line \ell_2
                                                                      84: P_{4111} = (14, 15, 14, 1) lies on line \ell_5
56: P_{768} = (15, 14, 1, 1) lies on line \ell_3
                                                                      85: P_{4352} = (15, 14, 15, 1) lies on line \ell_5
                                                                      86: P_{4367} = (14, 15, 15, 1) lies on line \ell_4
57: P_{783} = (14, 15, 1, 1) lies on line \ell_3
58: P_{784} = (15, 15, 1, 1) lies on line \ell_2
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The single points on the surface are:

Points on surface but on no line

The surface has 211 points not on any line: The points on the surface but not on lines are:

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15: P_{345} = (7, 4, 0, 1)
0: P_{20} = (1,0,1,0)
1: P_{117} = (2, 6, 1, 0)
                                                                  16: P_{361} = (7, 5, 0, 1)
2: P_{128} = (13, 6, 1, 0)
                                                                  17: P_{381} = (11, 6, 0, 1)
3: P_{140} = (9,7,1,0)
                                                                  18: P_{397} = (11, 7, 0, 1)
4: P_{143} = (12, 7, 1, 0)
                                                                  19: P_{414} = (12, 8, 0, 1)
                                                                  20: P_{430} = (12, 9, 0, 1)
5: P_{182} = (3, 10, 1, 0)
6: P_{187} = (8, 10, 1, 0)
                                                                  21: P_{434} = (0, 10, 0, 1)
7: P_{200} = (5, 11, 1, 0)
                                                                  22: P_{450} = (0, 11, 0, 1)
8: P_{210} = (15, 11, 1, 0)
                                                                  23: P_{476} = (10, 12, 0, 1)
9: P_{217} = (6, 12, 1, 0)
                                                                  24: P_{492} = (10, 13, 0, 1)
                                                                  25: P_{504} = (6, 14, 0, 1)
10: P_{225} = (14, 12, 1, 0)
11: P_{231} = (4, 13, 1, 0)
                                                                  26: P_{520} = (6, 15, 0, 1)
12: P_{234} = (7, 13, 1, 0)
                                                                  27: P_{798} = (13, 0, 2, 1)
13: P_{319} = (13, 2, 0, 1)
                                                                  28: P_{824} = (7, 2, 2, 1)
14: P_{335} = (13, 3, 0, 1)
                                                                  29: P_{874} = (9, 5, 2, 1)
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84: P_{1959} = (6, 9, 6, 1)
30: P_{875} = (10, 5, 2, 1)
                                                                  85: P_{1987} = (2, 11, 6, 1)
31: P_{967} = (6, 11, 2, 1)
32: P_{975} = (14, 11, 2, 1)
                                                                  86: P_{1996} = (11, 11, 6, 1)
33: P_{997} = (4, 13, 2, 1)
                                                                  87: P_{2024} = (7, 13, 6, 1)
34: P_{1006} = (13, 13, 2, 1)
                                                                  88: P_{2028} = (11, 13, 6, 1)
35: P_{1010} = (1, 14, 2, 1)
                                                                  89: P_{2076} = (11, 0, 7, 1)
36: P_{1016} = (7, 14, 2, 1)
                                                                  90: P_{2085} = (4, 1, 7, 1)
                                                                  91: P_{2086} = (5, 1, 7, 1)
37: P_{1028} = (3, 15, 2, 1)
38: P_{1038} = (13, 15, 2, 1)
                                                                  92: P_{2104} = (7, 2, 7, 1)
39: P_{1054} = (13, 0, 3, 1)
                                                                  93: P_{2105} = (8, 2, 7, 1)
40: P_{1096} = (7, 3, 3, 1)
                                                                  94: P_{2120} = (7, 3, 7, 1)
41: P_{1154} = (1,7,3,1)
                                                                  95: P_{2126} = (13, 3, 7, 1)
42: P_{1164} = (11, 7, 3, 1)
                                                                  96: P_{2187} = (10, 7, 7, 1)
43: P_{1174} = (5, 8, 3, 1)
                                                                  97: P_{2203} = (10, 8, 7, 1)
44: P_{1183} = (14, 8, 3, 1)
                                                                  98: P_{2207} = (14, 8, 7, 1)
45: P_{1258} = (9, 13, 3, 1)
                                                                  99: P_{2213} = (4, 9, 7, 1)
                                                                  100: P_{2224} = (15, 9, 7, 1)
46: P_{1262} = (13, 13, 3, 1)
                                                                  101: P_{2250} = (9, 11, 7, 1)
47: P_{1267} = (2, 14, 3, 1)
                                                                  102: P_{2252} = (11, 11, 7, 1)
48: P_{1278} = (13, 14, 3, 1)
49: P_{1304} = (7, 0, 4, 1)
                                                                  103: P_{2263} = (6, 12, 7, 1)
50: P_{1330} = (1, 2, 4, 1)
                                                                  104: P_{2268} = (11, 12, 7, 1)
51: P_{1341} = (12, 2, 4, 1)
                                                                  105: P_{2306} = (1, 15, 7, 1)
                                                                  106: P_{2309} = (4, 15, 7, 1)
52: P_{1350} = (5, 3, 4, 1)
53: P_{1352} = (7, 3, 4, 1)
                                                                  107: P_{2333} = (12, 0, 8, 1)
54: P_{1373} = (12, 4, 4, 1)
                                                                  108: P_{2373} = (4, 3, 8, 1)
55: P_{1416} = (7,7,4,1)
                                                                  109: P_{2384} = (15, 3, 8, 1)
                                                                  110: P_{2394} = (9, 4, 8, 1)
56: P_{1418} = (9,7,4,1)
57: P_{1436} = (11, 8, 4, 1)
                                                                  111: P_{2397} = (12, 4, 8, 1)
58: P_{1439} = (14, 8, 4, 1)
                                                                  112: P_{2418} = (1, 6, 8, 1)
59: P_{1459} = (2, 10, 4, 1)
                                                                  113: P_{2428} = (11, 6, 8, 1)
60: P_{1470} = (13, 10, 4, 1)
                                                                  114: P_{2455} = (6, 8, 8, 1)
61: P_{1560} = (7, 0, 5, 1)
                                                                  115: P_{2515} = (2, 12, 8, 1)
62: P_{1589} = (4, 2, 5, 1)
                                                                  116: P_{2525} = (12, 12, 8, 1)
63: P_{1592} = (7, 2, 5, 1)
                                                                  117: P_{2589} = (12, 0, 9, 1)
64: P_{1645} = (12, 5, 5, 1)
                                                                  118: P_{2642} = (1, 4, 9, 1)
                                                                  119: P_{2647} = (6, 4, 9, 1)
65: P_{1672} = (7, 7, 5, 1)
66: P_{1679} = (14, 7, 5, 1)
                                                                  120: P_{2665} = (8, 5, 9, 1)
                                                                  121: P_{2669} = (12, 5, 9, 1)
67: P_{1746} = (1, 12, 5, 1)
68: P_{1755} = (10, 12, 5, 1)
                                                                  122: P_{2727} = (6, 9, 9, 1)
69: P_{1795} = (2, 15, 5, 1)
                                                                  123: P_{2757} = (4, 11, 9, 1)
70: P_{1801} = (8, 15, 5, 1)
                                                                  124: P_{2760} = (7, 11, 9, 1)
                                                                  125: P_{2781} = (12, 12, 9, 1)
71: P_{1820} = (11, 0, 6, 1)
72: P_{1839} = (14, 1, 6, 1)
                                                                  126: P_{2783} = (14, 12, 9, 1)
                                                                  127: P_{2819} = (2, 15, 9, 1)
73: P_{1840} = (15, 1, 6, 1)
74: P_{1846} = (5, 2, 6, 1)
                                                                  128: P_{2827} = (10, 15, 9, 1)
                                                                  129 : P_{2833} = (0, 0, 10, 1)
75: P_{1855} = (14, 2, 6, 1)
76: P_{1861} = (4, 3, 6, 1)
                                                                  130: P_{2861} = (12, 1, 10, 1)
77: P_{1867} = (10, 3, 6, 1)
                                                                  131: P_{2862} = (13, 1, 10, 1)
78: P_{1890} = (1, 5, 6, 1)
                                                                  132: P_{2938} = (9, 6, 10, 1)
79: P_{1903} = (14, 5, 6, 1)
                                                                  133: P_{2939} = (10, 6, 10, 1)
80: P_{1915} = (10, 6, 6, 1)
                                                                  134: P_{2947} = (2, 7, 10, 1)
81: P_{1943} = (6, 8, 6, 1)
                                                                  135: P_{2955} = (10, 7, 10, 1)
82: P_{1949} = (12, 8, 6, 1)
                                                                  136: P_{2993} = (0, 10, 10, 1)
83: P_{1956} = (3, 9, 6, 1)
                                                                  137: P_{3028} = (3, 12, 10, 1)
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138: P_{3032} = (7, 12, 10, 1)
                                                                175: P_{3673} = (8, 4, 13, 1)
139: P_{3047} = (6, 13, 10, 1)
                                                                176: P_{3690} = (9, 5, 13, 1)
                                                                177: P_{3692} = (11, 5, 13, 1)
140: P_{3049} = (8, 13, 10, 1)
                                                                178: P_{3723} = (10, 7, 13, 1)
141: P_{3089} = (0, 0, 11, 1)
142: P_{3111} = (6, 1, 11, 1)
                                                                179: P_{3725} = (12, 7, 13, 1)
143: P_{3112} = (7, 1, 11, 1)
                                                                180: P_{3730} = (1, 8, 13, 1)
144: P_{3190} = (5, 6, 11, 1)
                                                                181: P_{3731} = (2, 8, 13, 1)
145: P_{3197} = (12, 6, 11, 1)
                                                                182: P_{3765} = (4, 10, 13, 1)
146: P_{3214} = (13, 7, 11, 1)
                                                                183: P_{3771} = (10, 10, 13, 1)
147: P_{3216} = (15, 7, 11, 1)
                                                                184: P_{3820} = (11, 13, 13, 1)
148: P_{3265} = (0, 11, 11, 1)
                                                                185: P_{3830} = (5, 14, 13, 1)
149: P_{3285} = (4, 12, 11, 1)
                                                                186: P_{3838} = (13, 14, 13, 1)
150: P_{3292} = (11, 12, 11, 1)
                                                                187: P_{3847} = (6, 15, 13, 1)
151: P_{3308} = (11, 13, 11, 1)
                                                                188: P_{3854} = (13, 15, 13, 1)
                                                                189: P_{3863} = (6, 0, 14, 1)
152: P_{3311} = (14, 13, 11, 1)
153: P_{3355} = (10, 0, 12, 1)
                                                                190: P_{3909} = (4, 3, 14, 1)
                                                                191: P_{3916} = (11, 3, 14, 1)
154: P_{3369} = (8, 1, 12, 1)
                                                                192: P_{3955} = (2, 6, 14, 1)
155: P_{3370} = (9, 1, 12, 1)
156: P_{3394} = (1, 3, 12, 1)
                                                                193: P_{3959} = (6, 6, 14, 1)
                                                                194: P_{3991} = (6, 8, 14, 1)
157: P_{3402} = (9, 3, 12, 1)
158: P_{3421} = (12, 4, 12, 1)
                                                                195: P_{4000} = (15, 8, 14, 1)
159: P_{3424} = (15, 4, 12, 1)
                                                                196: P_{4002} = (1, 9, 14, 1)
160: P_{3432} = (7, 5, 12, 1)
                                                                197: P_{4014} = (13, 9, 14, 1)
161: P_{3437} = (12, 5, 12, 1)
                                                                198: P_{4026} = (9, 10, 14, 1)
162: P_{3451} = (10, 6, 12, 1)
                                                                199: P_{4029} = (12, 10, 14, 1)
163: P_{3454} = (13, 6, 12, 1)
                                                                200: P_{4094} = (13, 14, 14, 1)
164: P_{3515} = (10, 10, 12, 1)
                                                                201: P_{4119} = (6, 0, 15, 1)
165: P_{3519} = (14, 10, 12, 1)
                                                                202: P_{4196} = (3, 5, 15, 1)
                                                                203: P_{4202} = (9, 5, 15, 1)
166: P_{3548} = (11, 12, 12, 1)
167: P_{3572} = (3, 14, 12, 1)
                                                                204: P_{4213} = (4, 6, 15, 1)
168: P_{3578} = (9, 14, 12, 1)
                                                                205: P_{4215} = (6, 6, 15, 1)
169: P_{3587} = (2, 15, 12, 1)
                                                                206: P_{4263} = (6, 9, 15, 1)
170: P_{3596} = (11, 15, 12, 1)
                                                                207: P_{4271} = (14, 9, 15, 1)
171: P_{3611} = (10, 0, 13, 1)
                                                                208: P_{4322} = (1, 13, 15, 1)
172: P_{3619} = (2, 1, 13, 1)
                                                                209: P_{4331} = (10, 13, 15, 1)
173: P_{3620} = (3, 1, 13, 1)
                                                                210: P_{4366} = (13, 15, 15, 1)
174: P_{3667} = (2, 4, 13, 1)
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Line Intersection Graph

	012345
0	010001
1	101101
2	010100
3	011011
4	000101
5	$\begin{array}{c} 0 \ 1 \ 2 \ 3 \ 4 \ 5 \\ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \\ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \\ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \\ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \\ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_5
in point	P_0	P_{290}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_0	P_{530}	P_{531}	P_{531}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{530}	P_5

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{531}	P_5	P_{546}	P_{531}

Line 4 intersects

Line	ℓ_3	ℓ_5
in point	P_{546}	P_{36}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{290}	P_{531}	P_{531}	P_{36}

The surface has 305 points: The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	$37: P_{361} = (7, 5, 0, 1)$	$74: P_{632} = (7, 6, 1, 1)$
$1: P_4 = (1, 1, 1, 1)$	$38: P_{381} = (1, 6, 0, 1)$	$75: P_{647} = (6, 7, 1, 1)$
$\begin{array}{ccc} 1 & 1 & 4 & -(1, 1, 1, 1) \\ 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \end{array}$	$39: P_{397} = (11, 7, 0, 1)$	$76: P_{648} = (7,7,1,1)$
$3: P_{20} = (1, 0, 1, 0)$	$40: P_{414} = (12, 8, 0, 1)$	$77: P_{665} = (8, 8, 1, 1)$
$A: P_{36} = (1, 0, 1, 0)$	$41: P_{430} = (12, 9, 0, 1)$	$78: P_{666} = (9, 8, 1, 1)$
$5: P_{117} = (2, 6, 1, 0)$	41: 1430 = (12, 3, 0, 1) $42: P_{434} = (0, 10, 0, 1)$	$79: P_{681} = (8, 9, 1, 1)$
$6: P_{117} = (2, 6, 1, 6)$ $6: P_{128} = (13, 6, 1, 0)$	$43: P_{450} = (0, 11, 0, 1)$	$80: P_{682} = (9, 9, 1, 1)$
$7: P_{140} = (9, 7, 1, 0)$	$44: P_{476} = (10, 12, 0, 1)$	$81: P_{699} = (10, 10, 1, 1)$
$8: P_{143} = (12, 7, 1, 0)$	$45: P_{492} = (10, 13, 0, 1)$	$82: P_{700} = (11, 10, 1, 1)$
9: $P_{182} = (3, 10, 1, 0)$	$46: P_{504} = (6, 14, 0, 1)$	83: $P_{715} = (10, 11, 1, 1)$
$10: P_{187} = (8, 10, 1, 0)$	$47: P_{520} = (6, 15, 0, 1)$	$84: P_{716} = (11, 11, 1, 1)$
$11: P_{200} = (5, 11, 1, 0)$	$48: P_{530} = (0, 0, 1, 1)$	$85: P_{733} = (12, 12, 1, 1)$
$12: P_{210} = (15, 11, 1, 0)$	$49: P_{531} = (1, 0, 1, 1)$	$86: P_{734} = (13, 12, 1, 1)$
$13: P_{217} = (6, 12, 1, 0)$	$50: P_{532} = (2,0,1,1)$	$87: P_{749} = (12, 13, 1, 1)$
$14: P_{225} = (14, 12, 1, 0)$	$51: P_{533} = (3, 0, 1, 1)$	$88: P_{750} = (13, 13, 1, 1)$
$15: P_{231} = (4, 13, 1, 0)$	$52: P_{534} = (4, 0, 1, 1)$	$89: P_{767} = (14, 14, 1, 1)$
$16: P_{234} = (7, 13, 1, 0)$	$53: P_{535} = (5, 0, 1, 1)$	$90: P_{768} = (15, 14, 1, 1)$
$17: P_{275} = (1,0,0,1)$	$54: P_{536} = (6,0,1,1)$	$91: P_{783} = (14, 15, 1, 1)$
18: $P_{290} = (0, 1, 0, 1)$	$55: P_{537} = (7,0,1,1)$	$92: P_{784} = (15, 15, 1, 1)$
19: $P_{291} = (1, 1, 0, 1)$	$56: P_{538} = (8, 0, 1, 1)$	$93: P_{798} = (13, 0, 2, 1)$
$20: P_{292} = (2, 1, 0, 1)$	$57: P_{539} = (9, 0, 1, 1)$	$94: P_{820} = (3, 2, 2, 1)$
$21: P_{293} = (3, 1, 0, 1)$	$58: P_{540} = (10, 0, 1, 1)$	$95: P_{824} = (7, 2, 2, 1)$
$22: P_{294} = (4, 1, 0, 1)$	$59: P_{541} = (11, 0, 1, 1)$	$96: P_{835} = (2, 3, 2, 1)$
$23: P_{295} = (5, 1, 0, 1)$	$60: P_{542} = (12, 0, 1, 1)$	$97: P_{874} = (9, 5, 2, 1)$
$24: P_{296} = (6, 1, 0, 1)$	$61: P_{543} = (13, 0, 1, 1)$	$98: P_{875} = (10, 5, 2, 1)$
$25: P_{297} = (7, 1, 0, 1)$	$62: P_{544} = (14, 0, 1, 1)$	$99: P_{967} = (6, 11, 2, 1)$
$26: P_{298} = (8, 1, 0, 1)$	$63: P_{545} = (15, 0, 1, 1)$	$100: P_{975} = (14, 11, 2, 1)$
$27: P_{299} = (9, 1, 0, 1)$	$64: P_{546} = (0, 1, 1, 1)$	$101: P_{997} = (4, 13, 2, 1)$
$28: P_{300} = (10, 1, 0, 1)$	$65: P_{563} = (2, 2, 1, 1)$	$102: P_{1006} = (13, 13, 2, 1)$
$29: P_{301} = (11, 1, 0, 1)$	$66: P_{564} = (3, 2, 1, 1)$	$103: P_{1010} = (1, 14, 2, 1)$
$30: P_{302} = (12, 1, 0, 1)$	$67: P_{579} = (2, 3, 1, 1)$	$104: P_{1016} = (7, 14, 2, 1)$
$31: P_{303} = (13, 1, 0, 1)$	$68: P_{580} = (3, 3, 1, 1)$	$105: P_{1028} = (3, 15, 2, 1)$
$32: P_{304} = (14, 1, 0, 1)$	$69: P_{597} = (4, 4, 1, 1)$	$106: P_{1038} = (13, 15, 2, 1)$
$33: P_{305} = (15, 1, 0, 1)$	$70: P_{598} = (5, 4, 1, 1)$	$107: P_{1054} = (13, 0, 3, 1)$
$34: P_{319} = (13, 2, 0, 1)$	$71: P_{613} = (4, 5, 1, 1)$	$108: P_{1076} = (3, 2, 3, 1)$
$35: P_{335} = (13, 3, 0, 1)$	$72: P_{614} = (5, 5, 1, 1)$	$109: P_{1091} = (2, 3, 3, 1)$
$36: P_{345} = (7, 4, 0, 1)$	$73: P_{631} = (6, 6, 1, 1)$	$110: P_{1096} = (7, 3, 3, 1)$

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219: P_{3004} = (11, 10, 10, 1)
111: P_{1154} = (1, 7, 3, 1)
                                           165: P_{2076} = (11, 0, 7, 1)
112: P_{1164} = (11, 7, 3, 1)
                                           166: P_{2085} = (4, 1, 7, 1)
                                                                                       220: P_{3019} = (10, 11, 10, 1)
                                                                                       221: P_{3028} = (3, 12, 10, 1)
113: P_{1174} = (5, 8, 3, 1)
                                           167: P_{2086} = (5, 1, 7, 1)
                                           168: P_{2104} = (7, 2, 7, 1)
114: P_{1183} = (14, 8, 3, 1)
                                                                                       222: P_{3032} = (7, 12, 10, 1)
115: P_{1258} = (9, 13, 3, 1)
                                           169: P_{2105} = (8, 2, 7, 1)
                                                                                       223: P_{3047} = (6, 13, 10, 1)
116: P_{1262} = (13, 13, 3, 1)
                                           170: P_{2120} = (7, 3, 7, 1)
                                                                                       224: P_{3049} = (8, 13, 10, 1)
117: P_{1267} = (2, 14, 3, 1)
                                           171: P_{2126} = (13, 3, 7, 1)
                                                                                       225: P_{3089} = (0, 0, 11, 1)
                                                                                       226: P_{3111} = (6, 1, 11, 1)
118: P_{1278} = (13, 14, 3, 1)
                                           172: P_{2168} = (7, 6, 7, 1)
119: P_{1304} = (7, 0, 4, 1)
                                           173: P_{2183} = (6,7,7,1)
                                                                                       227: P_{3112} = (7, 1, 11, 1)
120: P_{1330} = (1, 2, 4, 1)
                                           174: P_{2187} = (10, 7, 7, 1)
                                                                                       228: P_{3190} = (5, 6, 11, 1)
121: P_{1341} = (12, 2, 4, 1)
                                           175: P_{2203} = (10, 8, 7, 1)
                                                                                       229: P_{3197} = (12, 6, 11, 1)
122: P_{1350} = (5, 3, 4, 1)
                                           176: P_{2207} = (14, 8, 7, 1)
                                                                                       230: P_{3214} = (13, 7, 11, 1)
123: P_{1352} = (7, 3, 4, 1)
                                           177: P_{2213} = (4, 9, 7, 1)
                                                                                       231: P_{3216} = (15, 7, 11, 1)
                                           178: P_{2224} = (15, 9, 7, 1)
124: P_{1366} = (5, 4, 4, 1)
                                                                                       232: P_{3260} = (11, 10, 11, 1)
                                                                                       233:\ P_{3265}=(0,11,11,1)
125: P_{1373} = (12, 4, 4, 1)
                                           179: P_{2250} = (9, 11, 7, 1)
126: P_{1381} = (4, 5, 4, 1)
                                           180: P_{2252} = (11, 11, 7, 1)
                                                                                       234: P_{3275} = (10, 11, 11, 1)
127: P_{1416} = (7,7,4,1)
                                           181: P_{2263} = (6, 12, 7, 1)
                                                                                       235: P_{3285} = (4, 12, 11, 1)
                                           182: P_{2268} = (11, 12, 7, 1)
128: P_{1418} = (9,7,4,1)
                                                                                       236: P_{3292} = (11, 12, 11, 1)
129: P_{1436} = (11, 8, 4, 1)
                                                                                       237: P_{3308} = (11, 13, 11, 1)
                                           183: P_{2306} = (1, 15, 7, 1)
130: P_{1439} = (14, 8, 4, 1)
                                           184: P_{2309} = (4, 15, 7, 1)
                                                                                       238: P_{3311} = (14, 13, 11, 1)
131: P_{1459} = (2, 10, 4, 1)
                                           185: P_{2333} = (12, 0, 8, 1)
                                                                                       239: P_{3355} = (10, 0, 12, 1)
132: P_{1470} = (13, 10, 4, 1)
                                           186: P_{2373} = (4, 3, 8, 1)
                                                                                       240: P_{3369} = (8, 1, 12, 1)
                                           187: P_{2384} = (15, 3, 8, 1)
                                                                                       241: P_{3370} = (9, 1, 12, 1)
133: P_{1560} = (7, 0, 5, 1)
134: P_{1589} = (4, 2, 5, 1)
                                           188: P_{2394} = (9, 4, 8, 1)
                                                                                       242: P_{3394} = (1, 3, 12, 1)
135: P_{1592} = (7, 2, 5, 1)
                                           189: P_{2397} = (12, 4, 8, 1)
                                                                                       243: P_{3402} = (9, 3, 12, 1)
136: P_{1622} = (5, 4, 5, 1)
                                           190 : P_{2418} = (1, 6, 8, 1)
                                                                                       244: P_{3421} = (12, 4, 12, 1)
                                                                                       245: P_{3424} = (15, 4, 12, 1)
137: P_{1637} = (4, 5, 5, 1)
                                           191: P_{2428} = (11, 6, 8, 1)
138: P_{1645} = (12, 5, 5, 1)
                                           192: P_{2455} = (6, 8, 8, 1)
                                                                                       246: P_{3432} = (7, 5, 12, 1)
                                                                                       247: P_{3437} = (12, 5, 12, 1)
139: P_{1672} = (7, 7, 5, 1)
                                           193: P_{2458} = (9, 8, 8, 1)
140: P_{1679} = (14, 7, 5, 1)
                                           194 : P_{2473} = (8, 9, 8, 1)
                                                                                       248: P_{3451} = (10, 6, 12, 1)
141: P_{1746} = (1, 12, 5, 1)
                                           195: P_{2515} = (2, 12, 8, 1)
                                                                                       249: P_{3454} = (13, 6, 12, 1)
142: P_{1755} = (10, 12, 5, 1)
                                           196: P_{2525} = (12, 12, 8, 1)
                                                                                       250: P_{3515} = (10, 10, 12, 1)
                                           197: P_{2589} = (12, 0, 9, 1)
143: P_{1795} = (2, 15, 5, 1)
                                                                                       251: P_{3519} = (14, 10, 12, 1)
144: P_{1801} = (8, 15, 5, 1)
                                           198: P_{2642} = (1, 4, 9, 1)
                                                                                       252: P_{3548} = (11, 12, 12, 1)
145: P_{1820} = (11, 0, 6, 1)
                                                                                       253: P_{3550} = (13, 12, 12, 1)
                                           199: P_{2647} = (6, 4, 9, 1)
                                           200: P_{2665} = (8, 5, 9, 1)
146: P_{1839} = (14, 1, 6, 1)
                                                                                       254: P_{3565} = (12, 13, 12, 1)
147: P_{1840} = (15, 1, 6, 1)
                                           201: P_{2669} = (12, 5, 9, 1)
                                                                                       255: P_{3572} = (3, 14, 12, 1)
                                           202: P_{2714} = (9, 8, 9, 1)
                                                                                       256: P_{3578} = (9, 14, 12, 1)
148: P_{1846} = (5, 2, 6, 1)
                                                                                       257: P_{3587} = (2, 15, 12, 1)
149: P_{1855} = (14, 2, 6, 1)
                                           203: P_{2727} = (6, 9, 9, 1)
150: P_{1861} = (4, 3, 6, 1)
                                           204: P_{2729} = (8, 9, 9, 1)
                                                                                       258: P_{3596} = (11, 15, 12, 1)
151: P_{1867} = (10, 3, 6, 1)
                                           205: P_{2757} = (4, 11, 9, 1)
                                                                                       259: P_{3611} = (10, 0, 13, 1)
                                           206: P_{2760} = (7, 11, 9, 1)
                                                                                       260: P_{3619} = (2, 1, 13, 1)
152: P_{1890} = (1, 5, 6, 1)
153: P_{1903} = (14, 5, 6, 1)
                                           207: P_{2781} = (12, 12, 9, 1)
                                                                                       261: P_{3620} = (3, 1, 13, 1)
                                           208: P_{2783} = (14, 12, 9, 1)
                                                                                       262: P_{3667} = (2, 4, 13, 1)
154: P_{1912} = (7, 6, 6, 1)
155: P_{1915} = (10, 6, 6, 1)
                                           209: P_{2819} = (2, 15, 9, 1)
                                                                                       263: P_{3673} = (8, 4, 13, 1)
                                                                                       264: P_{3690} = (9, 5, 13, 1)
156: P_{1927} = (6, 7, 6, 1)
                                           210: P_{2827} = (10, 15, 9, 1)
157: P_{1943} = (6, 8, 6, 1)
                                           211: P_{2833} = (0, 0, 10, 1)
                                                                                       265: P_{3692} = (11, 5, 13, 1)
158: P_{1949} = (12, 8, 6, 1)
                                           212: P_{2861} = (12, 1, 10, 1)
                                                                                       266: P_{3723} = (10, 7, 13, 1)
                                                                                       267: P_{3725} = (12, 7, 13, 1)
159: P_{1956} = (3, 9, 6, 1)
                                           213: P_{2862} = (13, 1, 10, 1)
160: P_{1959} = (6, 9, 6, 1)
                                           214: P_{2938} = (9, 6, 10, 1)
                                                                                       268: P_{3730} = (1, 8, 13, 1)
161: P_{1987} = (2, 11, 6, 1)
                                           215: P_{2939} = (10, 6, 10, 1)
                                                                                       269: P_{3731} = (2, 8, 13, 1)
162: P_{1996} = (11, 11, 6, 1)
                                           216: P_{2947} = (2,7,10,1)
                                                                                       270: P_{3765} = (4, 10, 13, 1)
163: P_{2024} = (7, 13, 6, 1)
                                           217: P_{2955} = (10, 7, 10, 1)
                                                                                       271: P_{3771} = (10, 10, 13, 1)
164: P_{2028} = (11, 13, 6, 1)
                                           218: P_{2993} = (0, 10, 10, 1)
                                                                                       272: P_{3806} = (13, 12, 13, 1)
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295: P_{4202} = (9, 5, 15, 1)
273: P_{3820} = (11, 13, 13, 1)
                                          284: P_{3991} = (6, 8, 14, 1)
                                          285: P_{4000} = (15, 8, 14, 1)
                                                                                     296: P_{4213} = (4, 6, 15, 1)
274: P_{3821} = (12, 13, 13, 1)
275: P_{3830} = (5, 14, 13, 1)
                                          286: P_{4002} = (1, 9, 14, 1)
                                                                                     297: P_{4215} = (6, 6, 15, 1)
276: P_{3838} = (13, 14, 13, 1)
                                          287: P_{4014} = (13, 9, 14, 1)
                                                                                     298: P_{4263} = (6, 9, 15, 1)
277: P_{3847} = (6, 15, 13, 1)
                                          288: P_{4026} = (9, 10, 14, 1)
                                                                                     299: P_{4271} = (14, 9, 15, 1)
278: P_{3854} = (13, 15, 13, 1)
                                          289: P_{4029} = (12, 10, 14, 1)
                                                                                     300: P_{4322} = (1, 13, 15, 1)
279: P_{3863} = (6, 0, 14, 1)
                                          290: P_{4094} = (13, 14, 14, 1)
                                                                                     301: P_{4331} = (10, 13, 15, 1)
280: P_{3909} = (4, 3, 14, 1)
                                          291: P_{4096} = (15, 14, 14, 1)
                                                                                     302: P_{4352} = (15, 14, 15, 1)
                                          292:\,P_{4111}=(14,15,14,1)
281: P_{3916} = (11, 3, 14, 1)
                                                                                     303: P_{4366} = (13, 15, 15, 1)
                                          293: P_{4119} = (6, 0, 15, 1)
                                                                                     304: P_{4367} = (14, 15, 15, 1)
282: P_{3955} = (2, 6, 14, 1)
283: P_{3959} = (6, 6, 14, 1)
                                          294: P_{4196} = (3, 5, 15, 1)
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