

Rank-74099 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244209741

General information

Number of lines	2
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	$2, 1^{16}, 0^{64}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4680} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{PI}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (4680, 4744)

Rank of points on Klein quadric: (17, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_3 = (0, 0, 0, 1) = \ell_0 \cap \ell_1$$

Single Points

The surface has 16 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1

2 : $P_{82} = (0, 1, 0, 1)$ lies on line ℓ_0

3 : $P_{90} = (0, 2, 0, 1)$ lies on line ℓ_0

4 : $P_{98} = (0, 3, 0, 1)$ lies on line ℓ_0

5 : $P_{106} = (0, 4, 0, 1)$ lies on line ℓ_0

6 : $P_{114} = (0, 5, 0, 1)$ lies on line ℓ_0

7 : $P_{122} = (0, 6, 0, 1)$ lies on line ℓ_0

8 : $P_{130} = (0, 7, 0, 1)$ lies on line ℓ_0

9 : $P_{138} = (0, 0, 1, 1)$ lies on line ℓ_1

10 : $P_{201} = (0, 0, 2, 1)$ lies on line ℓ_1

11 : $P_{265} = (0, 0, 3, 1)$ lies on line ℓ_1

12 : $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_1

13 : $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_1

14 : $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_1

15 : $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:

The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$

1 : $P_4 = (1, 1, 1, 1)$

2 : $P_{20} = (1, 1, 1, 0)$

3 : $P_{30} = (3, 2, 1, 0)$

4 : $P_{33} = (6, 2, 1, 0)$

5 : $P_{46} = (3, 4, 1, 0)$

6 : $P_{48} = (5, 4, 1, 0)$

7 : $P_{72} = (5, 7, 1, 0)$

8 : $P_{73} = (6, 7, 1, 0)$

9 : $P_{75} = (1, 0, 0, 1)$

10 : $P_{94} = (4, 2, 0, 1)$

11 : $P_{104} = (6, 3, 0, 1)$

12 : $P_{113} = (7, 4, 0, 1)$

13 : $P_{117} = (3, 5, 0, 1)$

14 : $P_{127} = (5, 6, 0, 1)$

15 : $P_{132} = (2, 7, 0, 1)$

16 : $P_{163} = (2, 3, 1, 1)$

17 : $P_{168} = (7, 3, 1, 1)$

18 : $P_{179} = (2, 5, 1, 1)$

19 : $P_{181} = (4, 5, 1, 1)$

20 : $P_{189} = (4, 6, 1, 1)$

21 : $P_{192} = (7, 6, 1, 1)$

22 : $P_{205} = (4, 0, 2, 1)$
 23 : $P_{212} = (3, 1, 2, 1)$
 24 : $P_{213} = (4, 1, 2, 1)$
 25 : $P_{226} = (1, 3, 2, 1)$
 26 : $P_{234} = (1, 4, 2, 1)$
 27 : $P_{235} = (2, 4, 2, 1)$
 28 : $P_{252} = (3, 6, 2, 1)$
 29 : $P_{271} = (6, 0, 3, 1)$
 30 : $P_{286} = (5, 2, 3, 1)$
 31 : $P_{300} = (3, 4, 3, 1)$
 32 : $P_{310} = (5, 5, 3, 1)$
 33 : $P_{312} = (7, 5, 3, 1)$
 34 : $P_{327} = (6, 7, 3, 1)$
 35 : $P_{328} = (7, 7, 3, 1)$
 36 : $P_{336} = (7, 0, 4, 1)$
 37 : $P_{342} = (5, 1, 4, 1)$
 38 : $P_{344} = (7, 1, 4, 1)$
 39 : $P_{358} = (5, 3, 4, 1)$
 40 : $P_{370} = (1, 5, 4, 1)$
 41 : $P_{386} = (1, 7, 4, 1)$
 42 : $P_{389} = (4, 7, 4, 1)$
 43 : $P_{396} = (3, 0, 5, 1)$

44 : $P_{411} = (2, 2, 5, 1)$
 45 : $P_{412} = (3, 2, 5, 1)$
 46 : $P_{431} = (6, 4, 5, 1)$
 47 : $P_{443} = (2, 6, 5, 1)$
 48 : $P_{447} = (6, 6, 5, 1)$
 49 : $P_{454} = (5, 7, 5, 1)$
 50 : $P_{462} = (5, 0, 6, 1)$
 51 : $P_{479} = (6, 2, 6, 1)$
 52 : $P_{484} = (3, 3, 6, 1)$
 53 : $P_{485} = (4, 3, 6, 1)$
 54 : $P_{493} = (4, 4, 6, 1)$
 55 : $P_{494} = (5, 4, 6, 1)$
 56 : $P_{516} = (3, 7, 6, 1)$
 57 : $P_{523} = (2, 0, 7, 1)$
 58 : $P_{531} = (2, 1, 7, 1)$
 59 : $P_{535} = (6, 1, 7, 1)$
 60 : $P_{538} = (1, 2, 7, 1)$
 61 : $P_{544} = (7, 2, 7, 1)$
 62 : $P_{567} = (6, 5, 7, 1)$
 63 : $P_{570} = (1, 6, 7, 1)$

Line Intersection Graph

$$\begin{array}{c|c} & 0 \ 1 \\ \hline 0 & 0 \ 1 \\ 1 & 1 \ 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_3

Line 1 intersects

Line	ℓ_0
in point	P_3

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_{20} = (1, 1, 1, 0)$
 6 : $P_{30} = (3, 2, 1, 0)$
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 10 : $P_{72} = (5, 7, 1, 0)$
 11 : $P_{73} = (6, 7, 1, 0)$
 12 : $P_{75} = (1, 0, 0, 1)$
 13 : $P_{82} = (0, 1, 0, 1)$

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