Cheat Sheet GF(8)

January 16, 2021

polynomial: $X^3 + X^2 + 1 = 13$
$Z_i = \log_\alpha(1 + \alpha^i)$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0
1	$1 = 1$	1	1	7	2	5	1	1	1
2	$\alpha = \gamma$	2	6	1	4	3	4	1	1
3	$\alpha + 1 = \gamma^5$	3	4	5	5	2	5	0	1
4	$\alpha^2 = \gamma^2$	4	3	2	7	6	7	1	1
5	$\alpha^2 + 1 = \gamma^3$	5	7	3	3	1	6	0	1
6	$\alpha^2 + \alpha = \gamma^6$	6	2	6	6	4	3	0	1
7	$\alpha^2 + \alpha + 1 = \gamma^4$	7	5	4	1	DNE	2	1	1

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	3	2	5	4	7	6
2	2	3	0	1	6	7	4	5
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	5	4	7	6	1	0	3	2
6	6	7	4	5	2	3	0	1
7	7	6	5	4	3	2	1	0

·	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	5	7	1	3
3	3	6	5	1	2	7	4
4	4	5	1	7	3	2	6
5	5	7	2	3	6	4	1
6	6	1	7	2	4	3	5
7	7	3	4	6	1	5	2

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 5$
$2^4 = 7$
$2^5 = 3$
$2^6 = 6$
$2^7 = 1$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0
1	$1 = 1$	1	1	7	2	5	1	1	1
2	$\alpha = \gamma$	2	6	1	4	3	4	1	1
3	$\alpha + 1 = \gamma^5$	3	4	5	5	2	5	0	1
4	$\alpha^2 = \gamma^2$	4	3	2	7	6	7	1	1
5	$\alpha^2 + 1 = \gamma^3$	5	7	3	3	1	6	0	1
6	$\alpha^2 + \alpha = \gamma^6$	6	2	6	6	4	3	0	1
7	$\alpha^2 + \alpha + 1 = \gamma^4$	7	5	4	1	DNE	2	1	1

$+$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	3	2	5	4	7	6
2	2	3	0	1	6	7	4	5
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	5	4	7	6	1	0	3	2
6	6	7	4	5	2	3	0	1
7	7	6	5	4	3	2	1	0

$\cdot$	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	5	7	1	3
3	3	6	5	1	2	7	4
4	4	5	1	7	3	2	6
5	5	7	2	3	6	4	1
6	6	1	7	2	4	3	5
7	7	3	4	6	1	5	2

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 5$
$2^4 = 7$
$2^5 = 3$
$2^6 = 6$
$2^7 = 1$