Rank-65618 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431659950

General information

Number of lines	12
Number of points	33
Number of singular points	3
Number of Eckardt points	0
Number of double points	18
Number of single points	12
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{12}
Type of lines on points	4^3, 2^{18}, 1^{12}

Singular Points

The surface has 3 singular points:

0 : $P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

1 : $P_{66} = \mathbf{P}(1,\omega^2,\omega,1) = \mathbf{P}(1,3,2,1)$

2 : $P_{78} = \mathbf{P}(1,\omega,\omega^2,1) = \mathbf{P}(1,2,3,1)$

The 12 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{45} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{45} = \mathbf{Pl}(3,0,2,0,0,1)_{118}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{65} = \mathbf{Pl}(2,0,3,0,0,1)_{124}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_4 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{82} = \mathbf{Pl}(0,0,2,3,1,1)_{205}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{60} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{60} = \mathbf{Pl}(0,0,3,2,1,1)_{212}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{75} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{75} = \mathbf{Pl}(2,3,0,0,1,1)_{178}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{50} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{50} = \mathbf{Pl}(3,2,0,0,1,1)_{179}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

$$\ell_{10} = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{129} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{129} = \mathbf{Pl}(3,2,2,3,0,1)_{173}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{149} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{149} = \mathbf{Pl}(2,3,3,2,0,1)_{166}$$

Rank of lines: (22, 45, 65, 38, 82, 60, 25, 75, 50, 106, 129, 149)
Rank of points on Klein quadric: (109, 118, 124, 198, 205, 212, 177, 178, 179, 150, 173, 166)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 18 Double points:
The double points on the surface are:

$P_{21} = (2,3,1,0) = \ell_0 \cap \ell_1$ $\qquad$ $P_9 = (2,0,1,0) = \ell_2 \cap \ell_7$
$P_{18} = (3,2,1,0) = \ell_0 \cap \ell_2$ $\qquad$ $P_{19} = (0,3,1,0) = \ell_2 \cap \ell_{11}$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$ $\qquad$ $P_{47} = (2,2,1,1) = \ell_3 \cap \ell_7$
$P_8 = (1,0,1,0) = \ell_0 \cap \ell_6$ $\qquad$ $P_{52} = (3,3,1,1) = \ell_3 \cap \ell_8$
$P_{11} = (0,1,1,0) = \ell_0 \cap \ell_9$ $\qquad$ $P_{59} = (2,1,2,1) = \ell_4 \cap \ell_6$
$P_{12} = (1,1,1,0) = \ell_1 \cap \ell_2$ $\qquad$ $P_{64} = (3,2,2,1) = \ell_4 \cap \ell_7$
$P_6 = (2,1,0,0) = \ell_1 \cap \ell_4$ $\qquad$ $P_{76} = (3,1,3,1) = \ell_5 \cap \ell_6$
$P_{10} = (3,0,1,0) = \ell_1 \cap \ell_8$ $\qquad$ $P_{83} = (2,3,3,1) = \ell_5 \cap \ell_8$
$P_{15} = (0,2,1,0) = \ell_1 \cap \ell_{10}$
$P_7 = (3,1,0,0) = \ell_2 \cap \ell_5$

Single Points

The surface has 12 single points:
The single points on the surface are:

$0 : P_{26} = (0, 1, 0, 1)$ lies on line ℓ_6

$1 : P_{27} = (1, 1, 0, 1)$ lies on line ℓ_9

$2 : P_{30} = (0, 2, 0, 1)$ lies on line ℓ_7

$3 : P_{31} = (1, 2, 0, 1)$ lies on line ℓ_{11}

$4 : P_{34} = (0, 3, 0, 1)$ lies on line ℓ_8

$5 : P_{35} = (1, 3, 0, 1)$ lies on line ℓ_{10}

$6 : P_{38} = (0, 0, 1, 1)$ lies on line ℓ_3

$7 : P_{39} = (1, 0, 1, 1)$ lies on line ℓ_9

$8 : P_{53} = (0, 0, 2, 1)$ lies on line ℓ_4

$9 : P_{54} = (1, 0, 2, 1)$ lies on line ℓ_{10}

$10 : P_{69} = (0, 0, 3, 1)$ lies on line ℓ_5

$11 : P_{70} = (1, 0, 3, 1)$ lies on line ℓ_{11}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	1	1	0	0	1	0	0	1	0	0
1	1	0	1	0	1	0	0	0	1	0	1	0
2	1	1	0	0	0	1	0	1	0	0	0	1
3	1	0	0	0	0	0	1	1	1	0	1	1
4	0	1	0	0	0	0	1	1	1	1	0	1
5	0	0	1	0	0	0	1	1	1	1	1	0
6	1	0	0	1	1	1	0	0	0	0	1	1
7	0	0	1	1	1	1	0	0	0	1	1	0
8	0	1	0	1	1	1	0	0	0	1	0	1
9	1	0	0	0	1	1	0	1	1	0	1	1
10	0	1	0	1	0	1	1	1	0	1	0	1
11	0	0	1	1	1	0	1	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_9
in point	P_{21}	P_{18}	P_5	P_8	P_{11}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}
in point	P_{21}	P_{12}	P_6	P_{10}	P_{15}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}
in point	P_{18}	P_{12}	P_7	P_9	P_{19}

Line 3 intersects

Line	ℓ_0	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}
in point	P_5	P_4	P_{47}	P_{52}	P_4	P_4

Line 4 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}
in point	P_6	P_{59}	P_{64}	P_{66}	P_{66}	P_{66}

Line 5 intersects

Line	ℓ_2	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_7	P_{76}	P_{78}	P_{83}	P_{78}	P_{78}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}
in point	P_8	P_4	P_{59}	P_{76}	P_4	P_4

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_9	ℓ_{10}
in point	P_9	P_{47}	P_{64}	P_{78}	P_{78}	P_{78}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_9	ℓ_{11}
in point	P_{10}	P_{52}	P_{66}	P_{83}	P_{66}	P_{66}

Line 9 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}
in point	P_{11}	P_{66}	P_{78}	P_{78}	P_{66}	P_{78}	P_{66}

Line 10 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{11}
in point	P_{15}	P_4	P_{78}	P_4	P_{78}	P_{78}	P_4

Line 11 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}
in point	P_{19}	P_4	P_{66}	P_4	P_{66}	P_{66}	P_4

The surface has **33** points:
The points on the surface are:

$0: P_4 = (1,1,1,1)$

$1: P_5 = (1,1,0,0)$

$2: P_6 = (2,1,0,0)$

$3: P_7 = (3,1,0,0)$

$4: P_8 = (1,0,1,0)$

$5: P_9 = (2,0,1,0)$

$6: P_{10} = (3,0,1,0)$

$7: P_{11} = (0,1,1,0)$

$8: P_{12} = (1,1,1,0)$

$9: P_{15} = (0,2,1,0)$

$10: P_{18} = (3,2,1,0)$

$11: P_{19} = (0,3,1,0)$

$12: P_{21} = (2,3,1,0)$

$13: P_{26} = (0,1,0,1)$

$14: P_{27} = (1,1,0,1)$

$15: P_{30} = (0,2,0,1)$

$16: P_{31} = (1,2,0,1)$

$17: P_{34} = (0,3,0,1)$

$18: P_{35} = (1,3,0,1)$

$19: P_{38} = (0,0,1,1)$

$20: P_{39} = (1,0,1,1)$

$21: P_{47} = (2,2,1,1)$

$22: P_{52} = (3,3,1,1)$

$23: P_{53} = (0,0,2,1)$

$24: P_{54} = (1,0,2,1)$

$25: P_{59} = (2,1,2,1)$

$26: P_{64} = (3,2,2,1)$

$27: P_{66} = (1,3,2,1)$

$28: P_{69} = (0,0,3,1)$

$29: P_{70} = (1,0,3,1)$

$30: P_{76} = (3,1,3,1)$

$31: P_{78} = (1,2,3,1)$

$32: P_{83} = (2,3,3,1)$