

Rank-66764 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is -1991732146

General information

Number of lines	1
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{48}$

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{649}$$

Rank of lines: (4672)

Rank of points on Klein quadric: (649)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 9 single points:

The single points on the surface are:

- 0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0
- 1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_0
- 2 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_0
- 3 : $P_{27} = (0, 2, 1, 0)$ lies on line ℓ_0
- 4 : $P_{35} = (0, 3, 1, 0)$ lies on line ℓ_0

- 5 : $P_{43} = (0, 4, 1, 0)$ lies on line ℓ_0
- 6 : $P_{51} = (0, 5, 1, 0)$ lies on line ℓ_0
- 7 : $P_{59} = (0, 6, 1, 0)$ lies on line ℓ_0
- 8 : $P_{67} = (0, 7, 1, 0)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

- | | |
|-------------------------------|-------------------------------|
| 0 : $P_4 = (1, 1, 1, 1)$ | 22 : $P_{190} = (5, 6, 1, 1)$ |
| 1 : $P_5 = (1, 1, 0, 0)$ | 23 : $P_{198} = (5, 7, 1, 1)$ |
| 2 : $P_{12} = (1, 0, 1, 0)$ | 24 : $P_{206} = (5, 0, 2, 1)$ |
| 3 : $P_{20} = (1, 1, 1, 0)$ | 25 : $P_{215} = (6, 1, 2, 1)$ |
| 4 : $P_{31} = (4, 2, 1, 0)$ | 26 : $P_{222} = (5, 2, 2, 1)$ |
| 5 : $P_{39} = (4, 3, 1, 0)$ | 27 : $P_{231} = (6, 3, 2, 1)$ |
| 6 : $P_{50} = (7, 4, 1, 0)$ | 28 : $P_{279} = (6, 1, 3, 1)$ |
| 7 : $P_{58} = (7, 5, 1, 0)$ | 29 : $P_{287} = (6, 2, 3, 1)$ |
| 8 : $P_{61} = (2, 6, 1, 0)$ | 30 : $P_{306} = (1, 5, 3, 1)$ |
| 9 : $P_{69} = (2, 7, 1, 0)$ | 31 : $P_{314} = (1, 6, 3, 1)$ |
| 10 : $P_{76} = (2, 0, 0, 1)$ | 32 : $P_{335} = (6, 0, 4, 1)$ |
| 11 : $P_{78} = (4, 0, 0, 1)$ | 33 : $P_{340} = (3, 1, 4, 1)$ |
| 12 : $P_{81} = (7, 0, 0, 1)$ | 34 : $P_{367} = (6, 4, 4, 1)$ |
| 13 : $P_{83} = (1, 1, 0, 1)$ | 35 : $P_{372} = (3, 5, 4, 1)$ |
| 14 : $P_{95} = (5, 2, 0, 1)$ | 36 : $P_{404} = (3, 1, 5, 1)$ |
| 15 : $P_{112} = (6, 4, 0, 1)$ | 37 : $P_{418} = (1, 3, 5, 1)$ |
| 16 : $P_{133} = (3, 7, 0, 1)$ | 38 : $P_{428} = (3, 4, 5, 1)$ |
| 17 : $P_{139} = (1, 0, 1, 1)$ | 39 : $P_{442} = (1, 6, 5, 1)$ |
| 18 : $P_{159} = (6, 2, 1, 1)$ | 40 : $P_{470} = (5, 1, 6, 1)$ |
| 19 : $P_{167} = (6, 3, 1, 1)$ | 41 : $P_{482} = (1, 3, 6, 1)$ |
| 20 : $P_{172} = (3, 4, 1, 1)$ | 42 : $P_{498} = (1, 5, 6, 1)$ |
| 21 : $P_{180} = (3, 5, 1, 1)$ | 43 : $P_{518} = (5, 7, 6, 1)$ |

44 : $P_{524} = (3, 0, 7, 1)$
 45 : $P_{534} = (5, 1, 7, 1)$
 46 : $P_{574} = (5, 6, 7, 1)$

47 : $P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

$$\frac{0}{0} \bigg| \frac{0}{0}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 57 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	20 : $P_{78} = (4, 0, 0, 1)$	40 : $P_{314} = (1, 6, 3, 1)$
1 : $P_2 = (0, 0, 1, 0)$	21 : $P_{81} = (7, 0, 0, 1)$	41 : $P_{335} = (6, 0, 4, 1)$
2 : $P_4 = (1, 1, 1, 1)$	22 : $P_{83} = (1, 1, 0, 1)$	42 : $P_{340} = (3, 1, 4, 1)$
3 : $P_5 = (1, 1, 0, 0)$	23 : $P_{95} = (5, 2, 0, 1)$	43 : $P_{367} = (6, 4, 4, 1)$
4 : $P_{12} = (1, 0, 1, 0)$	24 : $P_{112} = (6, 4, 0, 1)$	44 : $P_{372} = (3, 5, 4, 1)$
5 : $P_{19} = (0, 1, 1, 0)$	25 : $P_{133} = (3, 7, 0, 1)$	45 : $P_{404} = (3, 1, 5, 1)$
6 : $P_{20} = (1, 1, 1, 0)$	26 : $P_{139} = (1, 0, 1, 1)$	46 : $P_{418} = (1, 3, 5, 1)$
7 : $P_{27} = (0, 2, 1, 0)$	27 : $P_{159} = (6, 2, 1, 1)$	47 : $P_{428} = (3, 4, 5, 1)$
8 : $P_{31} = (4, 2, 1, 0)$	28 : $P_{167} = (6, 3, 1, 1)$	48 : $P_{442} = (1, 6, 5, 1)$
9 : $P_{35} = (0, 3, 1, 0)$	29 : $P_{172} = (3, 4, 1, 1)$	49 : $P_{470} = (5, 1, 6, 1)$
10 : $P_{39} = (4, 3, 1, 0)$	30 : $P_{180} = (3, 5, 1, 1)$	50 : $P_{482} = (1, 3, 6, 1)$
11 : $P_{43} = (0, 4, 1, 0)$	31 : $P_{190} = (5, 6, 1, 1)$	51 : $P_{498} = (1, 5, 6, 1)$
12 : $P_{50} = (7, 4, 1, 0)$	32 : $P_{198} = (5, 7, 1, 1)$	52 : $P_{518} = (5, 7, 6, 1)$
13 : $P_{51} = (0, 5, 1, 0)$	33 : $P_{206} = (5, 0, 2, 1)$	53 : $P_{524} = (3, 0, 7, 1)$
14 : $P_{58} = (7, 5, 1, 0)$	34 : $P_{215} = (6, 1, 2, 1)$	54 : $P_{534} = (5, 1, 7, 1)$
15 : $P_{59} = (0, 6, 1, 0)$	35 : $P_{222} = (5, 2, 2, 1)$	55 : $P_{574} = (5, 6, 7, 1)$
16 : $P_{61} = (2, 6, 1, 0)$	36 : $P_{231} = (6, 3, 2, 1)$	56 : $P_{580} = (3, 7, 7, 1)$
17 : $P_{67} = (0, 7, 1, 0)$	37 : $P_{279} = (6, 1, 3, 1)$	
18 : $P_{69} = (2, 7, 1, 0)$	38 : $P_{287} = (6, 2, 3, 1)$	
19 : $P_{76} = (2, 0, 0, 1)$	39 : $P_{306} = (1, 5, 3, 1)$	