

# Rank-192 over GF(8)

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## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_0 X_1^2 = 0$$

( 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )  
The point rank of the equation over GF(8) is 336534

## General information

Number of lines	5
Number of points	41
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	40
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^5$
Type of lines on points	$5, 1^{40}$

## Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

## The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{72} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{72} = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_{81}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17} \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{201} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{89} \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{209}
\end{aligned}$$

Rank of lines: ( 72, 4680, 145, 656, 729 )

Rank of points on Klein quadric: ( 81, 17, 201, 89, 209 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 40 single points:

The single points on the surface are:

- |   |   |
|---|---|
| 0 : $P_0 = (1, 0, 0, 0)$ lies on line $\ell_0$      | 21 : $P_{114} = (0, 5, 0, 1)$ lies on line $\ell_1$ |
| 1 : $P_1 = (0, 1, 0, 0)$ lies on line $\ell_1$      | 22 : $P_{119} = (5, 5, 0, 1)$ lies on line $\ell_2$ |
| 2 : $P_4 = (1, 1, 1, 1)$ lies on line $\ell_4$      | 23 : $P_{122} = (0, 6, 0, 1)$ lies on line $\ell_1$ |
| 3 : $P_5 = (1, 1, 0, 0)$ lies on line $\ell_2$      | 24 : $P_{128} = (6, 6, 0, 1)$ lies on line $\ell_2$ |
| 4 : $P_{12} = (1, 0, 1, 0)$ lies on line $\ell_3$   | 25 : $P_{130} = (0, 7, 0, 1)$ lies on line $\ell_1$ |
| 5 : $P_{20} = (1, 1, 1, 0)$ lies on line $\ell_4$   | 26 : $P_{137} = (7, 7, 0, 1)$ lies on line $\ell_2$ |
| 6 : $P_{75} = (1, 0, 0, 1)$ lies on line $\ell_0$   | 27 : $P_{139} = (1, 0, 1, 1)$ lies on line $\ell_3$ |
| 7 : $P_{76} = (2, 0, 0, 1)$ lies on line $\ell_0$   | 28 : $P_{203} = (2, 0, 2, 1)$ lies on line $\ell_3$ |
| 8 : $P_{77} = (3, 0, 0, 1)$ lies on line $\ell_0$   | 29 : $P_{219} = (2, 2, 2, 1)$ lies on line $\ell_4$ |
| 9 : $P_{78} = (4, 0, 0, 1)$ lies on line $\ell_0$   | 30 : $P_{268} = (3, 0, 3, 1)$ lies on line $\ell_3$ |
| 10 : $P_{79} = (5, 0, 0, 1)$ lies on line $\ell_0$  | 31 : $P_{292} = (3, 3, 3, 1)$ lies on line $\ell_4$ |
| 11 : $P_{80} = (6, 0, 0, 1)$ lies on line $\ell_0$  | 32 : $P_{333} = (4, 0, 4, 1)$ lies on line $\ell_3$ |
| 12 : $P_{81} = (7, 0, 0, 1)$ lies on line $\ell_0$  | 33 : $P_{365} = (4, 4, 4, 1)$ lies on line $\ell_4$ |
| 13 : $P_{82} = (0, 1, 0, 1)$ lies on line $\ell_1$  | 34 : $P_{398} = (5, 0, 5, 1)$ lies on line $\ell_3$ |
| 14 : $P_{83} = (1, 1, 0, 1)$ lies on line $\ell_2$  | 35 : $P_{438} = (5, 5, 5, 1)$ lies on line $\ell_4$ |
| 15 : $P_{90} = (0, 2, 0, 1)$ lies on line $\ell_1$  | 36 : $P_{463} = (6, 0, 6, 1)$ lies on line $\ell_3$ |
| 16 : $P_{92} = (2, 2, 0, 1)$ lies on line $\ell_2$  | 37 : $P_{511} = (6, 6, 6, 1)$ lies on line $\ell_4$ |
| 17 : $P_{98} = (0, 3, 0, 1)$ lies on line $\ell_1$  | 38 : $P_{528} = (7, 0, 7, 1)$ lies on line $\ell_3$ |
| 18 : $P_{101} = (3, 3, 0, 1)$ lies on line $\ell_2$ | 39 : $P_{584} = (7, 7, 7, 1)$ lies on line $\ell_4$ |
| 19 : $P_{106} = (0, 4, 0, 1)$ lies on line $\ell_1$ |   |
| 20 : $P_{110} = (4, 4, 0, 1)$ lies on line $\ell_2$ |   |

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

### Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	1	1	1
2	1	1	0	1	1
3	1	1	1	0	1
4	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_3$	$P_3$	$P_3$	$P_3$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_3$	$P_3$	$P_3$	$P_3$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$
in point	$P_3$	$P_3$	$P_3$	$P_3$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$
in point	$P_3$	$P_3$	$P_3$	$P_3$

Line 4 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$	$P_3$

The surface has 41 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	14 : $P_{82} = (0, 1, 0, 1)$	28 : $P_{139} = (1, 0, 1, 1)$
1 : $P_1 = (0, 1, 0, 0)$	15 : $P_{83} = (1, 1, 0, 1)$	29 : $P_{203} = (2, 0, 2, 1)$
2 : $P_3 = (0, 0, 0, 1)$	16 : $P_{90} = (0, 2, 0, 1)$	30 : $P_{219} = (2, 2, 2, 1)$
3 : $P_4 = (1, 1, 1, 1)$	17 : $P_{92} = (2, 2, 0, 1)$	31 : $P_{268} = (3, 0, 3, 1)$
4 : $P_5 = (1, 1, 0, 0)$	18 : $P_{98} = (0, 3, 0, 1)$	32 : $P_{292} = (3, 3, 3, 1)$
5 : $P_{12} = (1, 0, 1, 0)$	19 : $P_{101} = (3, 3, 0, 1)$	33 : $P_{333} = (4, 0, 4, 1)$
6 : $P_{20} = (1, 1, 1, 0)$	20 : $P_{106} = (0, 4, 0, 1)$	34 : $P_{365} = (4, 4, 4, 1)$
7 : $P_{75} = (1, 0, 0, 1)$	21 : $P_{110} = (4, 4, 0, 1)$	35 : $P_{398} = (5, 0, 5, 1)$
8 : $P_{76} = (2, 0, 0, 1)$	22 : $P_{114} = (0, 5, 0, 1)$	36 : $P_{438} = (5, 5, 5, 1)$
9 : $P_{77} = (3, 0, 0, 1)$	23 : $P_{119} = (5, 5, 0, 1)$	37 : $P_{463} = (6, 0, 6, 1)$
10 : $P_{78} = (4, 0, 0, 1)$	24 : $P_{122} = (0, 6, 0, 1)$	38 : $P_{511} = (6, 6, 6, 1)$
11 : $P_{79} = (5, 0, 0, 1)$	25 : $P_{128} = (6, 6, 0, 1)$	39 : $P_{528} = (7, 0, 7, 1)$
12 : $P_{80} = (6, 0, 0, 1)$	26 : $P_{130} = (0, 7, 0, 1)$	40 : $P_{584} = (7, 7, 7, 1)$
13 : $P_{81} = (7, 0, 0, 1)$	27 : $P_{137} = (7, 7, 0, 1)$	