

Rank-38 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(2) is 38

General information

Number of lines	3
Number of points	7
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	$3, 1^6$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{11} \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}$$

Rank of lines: (20, 33, 27)

Rank of points on Klein quadric: (11, 7, 17)

Eckardt Points

The surface has 1 Eckardt points:

$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$. $T = -1$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 6 single points:

The single points on the surface are:

$0 : P_4 = (1, 1, 1, 1)$ lies on line ℓ_2

$1 : P_6 = (1, 0, 1, 0)$ lies on line ℓ_0

$2 : P_7 = (0, 1, 1, 0)$ lies on line ℓ_1

$3 : P_8 = (1, 1, 1, 0)$ lies on line ℓ_2

$4 : P_{13} = (1, 0, 1, 1)$ lies on line ℓ_0

$5 : P_{14} = (0, 1, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 7 points:

The points on the surface are:

$$0 : P_3 = (0, 0, 0, 1)$$

$$1 : P_4 = (1, 1, 1, 1)$$

$$2 : P_6 = (1, 0, 1, 0)$$

$$3 : P_7 = (0, 1, 1, 0)$$

$$4 : P_8 = (1, 1, 1, 0)$$

$$5 : P_{13} = (1, 0, 1, 1)$$

$$6 : P_{14} = (0, 1, 1, 1)$$