Rank-65548 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227134030

General information

Number of lines	3
Number of points	73
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$2^3, 1^{21}, 0^{49}$

Singular Points

The surface has 2 singular points:

$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1,0,0,1,0,0)_{18}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

Rank of lines: (4672, 584, 648)
Rank of points on Klein quadric: (649, 18, 10)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_1$ $\qquad\qquad\qquad$ $P_{75} = (1,0,0,1) = \ell_1 \cap \ell_2$
$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_{19} = (0,1,1,0)$ lies on line ℓ_0 $\qquad$ 11 : $P_{115} = (1,5,0,1)$ lies on line ℓ_1
1 : $P_{27} = (0,2,1,0)$ lies on line ℓ_0 $\qquad$ 12 : $P_{123} = (1,6,0,1)$ lies on line ℓ_1
2 : $P_{35} = (0,3,1,0)$ lies on line ℓ_0 $\qquad$ 13 : $P_{131} = (1,7,0,1)$ lies on line ℓ_1
3 : $P_{43} = (0,4,1,0)$ lies on line ℓ_0 $\qquad$ 14 : $P_{139} = (1,0,1,1)$ lies on line ℓ_2
4 : $P_{51} = (0,5,1,0)$ lies on line ℓ_0 $\qquad$ 15 : $P_{202} = (1,0,2,1)$ lies on line ℓ_2
5 : $P_{59} = (0,6,1,0)$ lies on line ℓ_0 $\qquad$ 16 : $P_{266} = (1,0,3,1)$ lies on line ℓ_2
6 : $P_{67} = (0,7,1,0)$ lies on line ℓ_0 $\qquad$ 17 : $P_{330} = (1,0,4,1)$ lies on line ℓ_2
7 : $P_{83} = (1,1,0,1)$ lies on line ℓ_1 $\qquad$ 18 : $P_{394} = (1,0,5,1)$ lies on line ℓ_2
8 : $P_{91} = (1,2,0,1)$ lies on line ℓ_1 $\qquad$ 19 : $P_{458} = (1,0,6,1)$ lies on line ℓ_2
9 : $P_{99} = (1,3,0,1)$ lies on line ℓ_1 $\qquad$ 20 : $P_{522} = (1,0,7,1)$ lies on line ℓ_2
10 : $P_{107} = (1,4,0,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:
The points on the surface but not on lines are:

0 : $P_{20} = (1,1,1,0)$ $\qquad\qquad\qquad$ 5 : $P_{64} = (5,6,1,0)$
1 : $P_{34} = (7,2,1,0)$ $\qquad\qquad\qquad$ 6 : $P_{71} = (4,7,1,0)$
2 : $P_{41} = (6,3,1,0)$ $\qquad\qquad\qquad$ 7 : $P_{148} = (3,1,1,1)$
3 : $P_{45} = (2,4,1,0)$ $\qquad\qquad\qquad$ 8 : $P_{150} = (5,1,1,1)$
4 : $P_{54} = (3,5,1,0)$ $\qquad\qquad\qquad$ 9 : $P_{151} = (6,1,1,1)$

$10: P_{155} = (2, 2, 1, 1)$
$11: P_{173} = (4, 4, 1, 1)$
$12: P_{200} = (7, 7, 1, 1)$
$13: P_{211} = (2, 1, 2, 1)$
$14: P_{221} = (4, 2, 2, 1)$
$15: P_{248} = (7, 5, 2, 1)$
$16: P_{252} = (3, 6, 2, 1)$
$17: P_{254} = (5, 6, 2, 1)$
$18: P_{255} = (6, 6, 2, 1)$
$19: P_{300} = (3, 4, 3, 1)$
$20: P_{302} = (5, 4, 3, 1)$
$21: P_{303} = (6, 4, 3, 1)$
$22: P_{307} = (2, 5, 3, 1)$
$23: P_{320} = (7, 6, 3, 1)$
$24: P_{325} = (4, 7, 3, 1)$
$25: P_{341} = (4, 1, 4, 1)$
$26: P_{356} = (3, 3, 4, 1)$
$27: P_{358} = (5, 3, 4, 1)$
$28: P_{359} = (6, 3, 4, 1)$
$29: P_{368} = (7, 4, 4, 1)$

$30: P_{379} = (2, 6, 4, 1)$
$31: P_{416} = (7, 2, 5, 1)$
$32: P_{419} = (2, 3, 5, 1)$
$33: P_{445} = (4, 6, 5, 1)$
$34: P_{452} = (3, 7, 5, 1)$
$35: P_{454} = (5, 7, 5, 1)$
$36: P_{455} = (6, 7, 5, 1)$
$37: P_{476} = (3, 2, 6, 1)$
$38: P_{478} = (5, 2, 6, 1)$
$39: P_{479} = (6, 2, 6, 1)$
$40: P_{488} = (7, 3, 6, 1)$
$41: P_{491} = (2, 4, 6, 1)$
$42: P_{501} = (4, 5, 6, 1)$
$43: P_{536} = (7, 1, 7, 1)$
$44: P_{549} = (4, 3, 7, 1)$
$45: P_{564} = (3, 5, 7, 1)$
$46: P_{566} = (5, 5, 7, 1)$
$47: P_{567} = (6, 5, 7, 1)$
$48: P_{579} = (2, 7, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_{75}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_{75}

The surface has 73 points:
The points on the surface are:

$0: P_1 = (0, 1, 0, 0)$
$1: P_2 = (0, 0, 1, 0)$
$2: P_{19} = (0, 1, 1, 0)$
$3: P_{20} = (1, 1, 1, 0)$
$4: P_{27} = (0, 2, 1, 0)$
$5: P_{34} = (7, 2, 1, 0)$
$6: P_{35} = (0, 3, 1, 0)$
$7: P_{41} = (6, 3, 1, 0)$
$8: P_{43} = (0, 4, 1, 0)$
$9: P_{45} = (2, 4, 1, 0)$
$10: P_{51} = (0, 5, 1, 0)$
$11: P_{54} = (3, 5, 1, 0)$

$12: P_{59} = (0, 6, 1, 0)$
$13: P_{64} = (5, 6, 1, 0)$
$14: P_{67} = (0, 7, 1, 0)$
$15: P_{71} = (4, 7, 1, 0)$
$16: P_{75} = (1, 0, 0, 1)$
$17: P_{83} = (1, 1, 0, 1)$
$18: P_{91} = (1, 2, 0, 1)$
$19: P_{99} = (1, 3, 0, 1)$
$20: P_{107} = (1, 4, 0, 1)$
$21: P_{115} = (1, 5, 0, 1)$
$22: P_{123} = (1, 6, 0, 1)$
$23: P_{131} = (1, 7, 0, 1)$

$24: P_{139} = (1, 0, 1, 1)$
$25: P_{148} = (3, 1, 1, 1)$
$26: P_{150} = (5, 1, 1, 1)$
$27: P_{151} = (6, 1, 1, 1)$
$28: P_{155} = (2, 2, 1, 1)$
$29: P_{173} = (4, 4, 1, 1)$
$30: P_{200} = (7, 7, 1, 1)$
$31: P_{202} = (1, 0, 2, 1)$
$32: P_{211} = (2, 1, 2, 1)$
$33: P_{221} = (4, 2, 2, 1)$
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$35: P_{252} = (3, 6, 2, 1)$

$36: P_{254} = (5,6,2,1)$

$37: P_{255} = (6,6,2,1)$

$38: P_{266} = (1,0,3,1)$

$39: P_{300} = (3,4,3,1)$

$40: P_{302} = (5,4,3,1)$

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$43: P_{320} = (7,6,3,1)$

$44: P_{325} = (4,7,3,1)$

$45: P_{330} = (1,0,4,1)$

$46: P_{341} = (4,1,4,1)$

$47: P_{356} = (3,3,4,1)$

$48: P_{358} = (5,3,4,1)$

$49: P_{359} = (6,3,4,1)$

$50: P_{368} = (7,4,4,1)$

$51: P_{379} = (2,6,4,1)$

$52: P_{394} = (1,0,5,1)$

$53: P_{416} = (7,2,5,1)$

$54: P_{419} = (2,3,5,1)$

$55: P_{445} = (4,6,5,1)$

$56: P_{452} = (3,7,5,1)$

$57: P_{454} = (5,7,5,1)$

$58: P_{455} = (6,7,5,1)$

$59: P_{458} = (1,0,6,1)$

$60: P_{476} = (3,2,6,1)$

$61: P_{478} = (5,2,6,1)$

$62: P_{479} = (6,2,6,1)$

$63: P_{488} = (7,3,6,1)$

$64: P_{491} = (2,4,6,1)$

$65: P_{501} = (4,5,6,1)$

$66: P_{522} = (1,0,7,1)$

$67: P_{536} = (7,1,7,1)$

$68: P_{549} = (4,3,7,1)$

$69: P_{564} = (3,5,7,1)$

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