Rank-65921 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65921

General information

Number of lines	6
Number of points	11
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	3
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^6
Type of lines on points	$3, 2^6, 1^3, 0$

Singular Points

The surface has 1 singular points:

$$0 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \mathbf{Pl}(1,0,0,0,1,0)_{10}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1,0,1,1,1,1)_{33}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,1,1,1,1,0)_{18}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

Rank of lines: (2, 5, 12, 24, 17, 23)
Rank of points on Klein quadric: (10, 12, 32, 33, 18, 31)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1). \ T = -1$

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_{14} = (0,1,1,1) = \ell_3 \cap \ell_4$
$P_{10} = (0,1,0,1) = \ell_0 \cap \ell_5$ $P_8 = (1,1,1,0) = \ell_4 \cap \ell_5$
$P_{12} = (0,0,1,1) = \ell_1 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_2 \cap \ell_3$

Single Points

The surface has 3 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_2 $2 : P_{11} = (1,1,0,1)$ lies on line ℓ_0
$1 : P_9 = (1,0,0,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0 : P_6 = (1,0,1,0)$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	0	0	0	1
1	1	0	1	1	0	1
2	0	1	0	1	0	0
3	0	1	1	0	1	1
4	0	0	0	1	0	1
5	1	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_5
in point	P_0	P_{10}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_0	P_{12}	P_{13}	P_{13}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{12}	P_5

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{13}	P_5	P_{14}	P_{13}

Line 4 intersects

Line	ℓ_3	ℓ_5
in point	P_{14}	P_8

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{10}	P_{13}	P_{13}	P_8

The surface has 11 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_4 = (1,1,1,1)$

$2 : P_5 = (1,1,0,0)$

$3 : P_6 = (1,0,1,0)$

$4 : P_8 = (1,1,1,0)$

$5 : P_9 = (1,0,0,1)$

$6 : P_{10} = (0,1,0,1)$

$7 : P_{11} = (1,1,0,1)$

$8 : P_{12} = (0,0,1,1)$

$9 : P_{13} = (1,0,1,1)$

$10 : P_{14} = (0,1,1,1)$