Rank-331 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(64) is 1090785357

General information

Number of lines	67
Number of points	4225
Number of singular points	65
Number of Eckardt points	0
Number of double points	130
Number of single points	4095
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{67}
Type of lines on points	2^{130}, 1^{4095}

Singular Points

The surface has 65 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
1 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
2 : $P_{8258} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
3 : $P_{12353} = \mathbf{P}(0,0,\epsilon,1) = \mathbf{P}(0,0,2,1)$
4 : $P_{16449} = \mathbf{P}(0,0,\epsilon^{58},1) = \mathbf{P}(0,0,3,1)$
5 : $P_{20545} = \mathbf{P}(0,0,\epsilon^{2},1) = \mathbf{P}(0,0,4,1)$
6 : $P_{24641} = \mathbf{P}(0,0,\epsilon^{53},1) = \mathbf{P}(0,0,5,1)$
7 : $P_{28737} = \mathbf{P}(0,0,\epsilon^{59},1) = \mathbf{P}(0,0,6,1)$
8 : $P_{32833} = \mathbf{P}(0,0,\epsilon^{39},1) = \mathbf{P}(0,0,7,1)$
9 : $P_{36929} = \mathbf{P}(0,0,\epsilon^{3},1) = \mathbf{P}(0,0,8,1)$
10 : $P_{41025} = \mathbf{P}(0,0,\epsilon^{34},1) = \mathbf{P}(0,0,9,1)$
11 : $P_{45121} = \mathbf{P}(0,0,\epsilon^{54},1) = \mathbf{P}(0,0,10,1)$
12 : $P_{49217} = \mathbf{P}(0,0,\epsilon^{18},1) = \mathbf{P}(0,0,11,1)$
13 : $P_{53313} = \mathbf{P}(0,0,\epsilon^{60},1) = \mathbf{P}(0,0,12,1)$
14 : $P_{57409} = \mathbf{P}(0,0,\epsilon^{31},1) = \mathbf{P}(0,0,13,1)$
15 : $P_{61505} = \mathbf{P}(0,0,\epsilon^{40},1) = \mathbf{P}(0,0,14,1)$
16 : $P_{65601} = \mathbf{P}(0,0,\epsilon^{48},1) = \mathbf{P}(0,0,15,1)$
17 : $P_{69697} = \mathbf{P}(0,0,\epsilon^{4},1) = \mathbf{P}(0,0,16,1)$

$18:\ P_{73793} = \mathbf{P}(0,0,\epsilon^{43},1) = \mathbf{P}(0,0,17,1)$
$19:\ P_{77889} = \mathbf{P}(0,0,\epsilon^{35},1) = \mathbf{P}(0,0,18,1)$
$20:\ P_{81985} = \mathbf{P}(0,0,\epsilon^{22},1) = \mathbf{P}(0,0,19,1)$
$21:\ P_{86081} = \mathbf{P}(0,0,\epsilon^{55},1) = \mathbf{P}(0,0,20,1)$
$22:\ P_{90177} = \mathbf{P}(0,0,\epsilon^{15},1) = \mathbf{P}(0,0,21,1)$
$23:\ P_{94273} = \mathbf{P}(0,0,\epsilon^{19},1) = \mathbf{P}(0,0,22,1)$
$24:\ P_{98369} = \mathbf{P}(0,0,\epsilon^{26},1) = \mathbf{P}(0,0,23,1)$
$25:\ P_{102465} = \mathbf{P}(0,0,\epsilon^{61},1) = \mathbf{P}(0,0,24,1)$
$26:\ P_{106561} = \mathbf{P}(0,0,\epsilon^{51},1) = \mathbf{P}(0,0,25,1)$
$27:\ P_{110657} = \mathbf{P}(0,0,\epsilon^{32},1) = \mathbf{P}(0,0,26,1)$
$28:\ P_{114753} = \mathbf{P}(0,0,\epsilon^{29},1) = \mathbf{P}(0,0,27,1)$
$29:\ P_{118849} = \mathbf{P}(0,0,\epsilon^{41},1) = \mathbf{P}(0,0,28,1)$
$30:\ P_{122945} = \mathbf{P}(0,0,\epsilon^{13},1) = \mathbf{P}(0,0,29,1)$
$31:\ P_{127041} = \mathbf{P}(0,0,\epsilon^{49},1) = \mathbf{P}(0,0,30,1)$
$32:\ P_{131137} = \mathbf{P}(0,0,\epsilon^{11},1) = \mathbf{P}(0,0,31,1)$
$33:\ P_{135233} = \mathbf{P}(0,0,\epsilon^{5},1) = \mathbf{P}(0,0,32,1)$
$34:\ P_{139329} = \mathbf{P}(0,0,\epsilon^{6},1) = \mathbf{P}(0,0,33,1)$
$35:\ P_{143425} = \mathbf{P}(0,0,\epsilon^{44},1) = \mathbf{P}(0,0,34,1)$
$36:\ P_{147521} = \mathbf{P}(0,0,\epsilon^{7},1) = \mathbf{P}(0,0,35,1)$
$37:\ P_{151617} = \mathbf{P}(0,0,\epsilon^{36},1) = \mathbf{P}(0,0,36,1)$
$38:\ P_{155713} = \mathbf{P}(0,0,\epsilon^{45},1) = \mathbf{P}(0,0,37,1)$
$39:\ P_{159809} = \mathbf{P}(0,0,\epsilon^{23},1) = \mathbf{P}(0,0,38,1)$
$40:\ P_{163905} = \mathbf{P}(0,0,\epsilon^{8},1) = \mathbf{P}(0,0,39,1)$
$41:\ P_{168001} = \mathbf{P}(0,0,\epsilon^{56},1) = \mathbf{P}(0,0,40,1)$
$42:\ P_{172097} = \mathbf{P}(0,0,\epsilon^{37},1) = \mathbf{P}(0,0,41,1)$
$43:\ P_{176193} = \mathbf{P}(0,0,\epsilon^{16},1) = \mathbf{P}(0,0,42,1)$
$44:\ P_{180289} = \mathbf{P}(0,0,\epsilon^{46},1) = \mathbf{P}(0,0,43,1)$
$45:\ P_{184385} = \mathbf{P}(0,0,\epsilon^{20},1) = \mathbf{P}(0,0,44,1)$
$46:\ P_{188481} = \mathbf{P}(0,0,\epsilon^{24},1) = \mathbf{P}(0,0,45,1)$
$47:\ P_{192577} = \mathbf{P}(0,0,\epsilon^{27},1) = \mathbf{P}(0,0,46,1)$
$48:\ P_{196673} = \mathbf{P}(0,0,\epsilon^{9},1) = \mathbf{P}(0,0,47,1)$
$49:\ P_{200769} = \mathbf{P}(0,0,\epsilon^{62},1) = \mathbf{P}(0,0,48,1)$
$50:\ P_{204865} = \mathbf{P}(0,0,\epsilon^{57},1) = \mathbf{P}(0,0,49,1)$
$51:\ P_{208961} = \mathbf{P}(0,0,\epsilon^{52},1) = \mathbf{P}(0,0,50,1)$
$52:\ P_{213057} = \mathbf{P}(0,0,\epsilon^{38},1) = \mathbf{P}(0,0,51,1)$
$53:\ P_{217153} = \mathbf{P}(0,0,\epsilon^{33},1) = \mathbf{P}(0,0,52,1)$
$54:\ P_{221249} = \mathbf{P}(0,0,\epsilon^{17},1) = \mathbf{P}(0,0,53,1)$
$55:\ P_{225345} = \mathbf{P}(0,0,\epsilon^{30},1) = \mathbf{P}(0,0,54,1)$
$56:\ P_{229441} = \mathbf{P}(0,0,\epsilon^{47},1) = \mathbf{P}(0,0,55,1)$
$57:\ P_{233537} = \mathbf{P}(0,0,\epsilon^{42},1) = \mathbf{P}(0,0,56,1)$
$58:\ P_{237633} = \mathbf{P}(0,0,\epsilon^{21},1) = \mathbf{P}(0,0,57,1)$
$59:\ P_{241729} = \mathbf{P}(0,0,\epsilon^{14},1) = \mathbf{P}(0,0,58,1)$
$60:\ P_{245825} = \mathbf{P}(0,0,\epsilon^{25},1) = \mathbf{P}(0,0,59,1)$
$61:\ P_{249921} = \mathbf{P}(0,0,\epsilon^{50},1) = \mathbf{P}(0,0,60,1)$
$62:\ P_{254017} = \mathbf{P}(0,0,\epsilon^{28},1) = \mathbf{P}(0,0,61,1)$
$63:\ P_{258113} = \mathbf{P}(0,0,\epsilon^{12},1) = \mathbf{P}(0,0,62,1)$
$64:\ P_{262209} = \mathbf{P}(0,0,\epsilon^{10},1) = \mathbf{P}(0,0,63,1)$

The 67 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \mathbf{Pl}(0,0,0,1,0,0)_{129}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0,0,1,1,1,1)_{544578}$$

$$\ell_5 = \begin{bmatrix} 1 & \epsilon^{62} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{61} \end{bmatrix}_{203848} = \begin{bmatrix} 1 & 48 & 0 & 0 \\ 0 & 0 & 1 & 24 \end{bmatrix}_{203848} = \mathbf{Pl}(0,0,4,24,48,1)_{12862719}$$

$$\ell_6 = \begin{bmatrix} 1 & \epsilon^{5} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{10} \end{bmatrix}_{137311} = \begin{bmatrix} 1 & 32 & 0 & 0 \\ 0 & 0 & 1 & 63 \end{bmatrix}_{137311} = \mathbf{Pl}(0,0,5,63,32,1)_{8669566}$$

$$\ell_7 = \begin{bmatrix} 1 & \epsilon^{61} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{59} \end{bmatrix}_{103966} = \begin{bmatrix} 1 & 24 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{103966} = \mathbf{Pl}(0,0,16,6,24,1)_{6574323}$$

$$\ell_8 = \begin{bmatrix} 1 & \epsilon^{10} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{20} \end{bmatrix}_{266283} = \begin{bmatrix} 1 & 63 & 0 & 0 \\ 0 & 0 & 1 & 44 \end{bmatrix}_{266283} = \mathbf{Pl}(0,0,17,44,63,1)_{16795570}$$

$$\ell_9 = \begin{bmatrix} 1 & \epsilon^{4} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{8} \end{bmatrix}_{70711} = \begin{bmatrix} 1 & 16 & 0 & 0 \\ 0 & 0 & 1 & 39 \end{bmatrix}_{70711} = \mathbf{Pl}(0,0,20,39,16,1)_{4478191}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & \epsilon^{24} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{48} \end{array}\right]_{191356} = \left[\begin{array}{cccc} 1 & 45 & 0 & 0 \\ 0 & 0 & 1 & 15 \end{array}\right]_{191356} = \mathbf{Pl}(0,0,21,15,45,1)_{12078638}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & \epsilon^{60} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{57} \end{array}\right]_{54077} = \left[\begin{array}{cccc} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 49 \end{array}\right]_{54077} = \mathbf{Pl}(0,0,33,49,12,1)_{3431522}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & \epsilon^{29} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{58} \end{array}\right]_{116446} = \left[\begin{array}{cccc} 1 & 27 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{116446} = \mathbf{Pl}(0,0,32,3,27,1)_{7362595}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & \epsilon^{9} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{18} \end{array}\right]_{199674} = \left[\begin{array}{cccc} 1 & 47 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{array}\right]_{199674} = \mathbf{Pl}(0,0,37,11,47,1)_{12604830}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & \epsilon^{45} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{27} \end{array}\right]_{158099} = \left[\begin{array}{cccc} 1 & 37 & 0 & 0 \\ 0 & 0 & 1 & 46 \end{array}\right]_{158099} = \mathbf{Pl}(0,0,36,46,37,1)_{9983903}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & \epsilon^{3} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{6} \end{array}\right]_{37417} = \left[\begin{array}{cccc} 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 33 \end{array}\right]_{37417} = \mathbf{Pl}(0,0,49,33,8,1)_{2385234}$$

$$\ell_{16} = \left[\begin{array}{cccc} 1 & \epsilon^{32} & 0 & 0 \\ 0 & 0 & 1 & \epsilon \end{array}\right]_{112284} = \left[\begin{array}{cccc} 1 & 26 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{112284} = \mathbf{Pl}(0,0,48,2,26,1)_{7102547}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & \epsilon^{23} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{46} \end{array}\right]_{162257} = \left[\begin{array}{cccc} 1 & 38 & 0 & 0 \\ 0 & 0 & 1 & 43 \end{array}\right]_{162257} = \mathbf{Pl}(0,0,53,43,38,1)_{10248142}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & \epsilon^{15} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{30} \end{array}\right]_{91531} = \left[\begin{array}{cccc} 1 & 21 & 0 & 0 \\ 0 & 0 & 1 & 54 \end{array}\right]_{91531} = \mathbf{Pl}(0,0,52,54,21,1)_{5792655}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & \epsilon^{59} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{55} \end{array}\right]_{29082} = \left[\begin{array}{cccc} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{array}\right]_{29082} = \mathbf{Pl}(0,0,39,20,6,1)_{1859804}$$

$$\ell_{20} = \left[\begin{array}{cccc} 1 & \epsilon^{20} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{40} \end{array}\right]_{187194} = \left[\begin{array}{cccc} 1 & 44 & 0 & 0 \\ 0 & 0 & 1 & 14 \end{array}\right]_{187194} = \mathbf{Pl}(0,0,38,14,44,1)_{11818717}$$

$$\ell_{21} = \left[\begin{array}{cccc} 1 & \epsilon^{28} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{56} \end{array}\right]_{257957} = \left[\begin{array}{cccc} 1 & 61 & 0 & 0 \\ 0 & 0 & 1 & 40 \end{array}\right]_{257957} = \mathbf{Pl}(0,0,35,40,61,1)_{16273696}$$

$$\ell_{22} = \left[\begin{array}{cccc} 1 & \epsilon^{41} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{19} \end{array}\right]_{120626} = \left[\begin{array}{cccc} 1 & 28 & 0 & 0 \\ 0 & 0 & 1 & 22 \end{array}\right]_{120626} = \mathbf{Pl}(0,0,34,22,28,1)_{7624929}$$

$$\ell_{23} = \left[\begin{array}{cccc} 1 & \epsilon^{8} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{16} \end{array}\right]_{166417} = \left[\begin{array}{cccc} 1 & 39 & 0 & 0 \\ 0 & 0 & 1 & 42 \end{array}\right]_{166417} = \mathbf{Pl}(0,0,55,42,39,1)_{10510476}$$

$$\ell_{24} = \left[\begin{array}{cccc} 1 & \epsilon^{48} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{33} \end{array}\right]_{66563} = \left[\begin{array}{cccc} 1 & 15 & 0 & 0 \\ 0 & 0 & 1 & 52 \end{array}\right]_{66563} = \mathbf{Pl}(0,0,54,52,15,1)_{4220429}$$

$$\ell_{25} = \left[\begin{array}{cccc} 1 & \epsilon^{44} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{25} \end{array}\right]_{145629} = \left[\begin{array}{cccc} 1 & 34 & 0 & 0 \\ 0 & 0 & 1 & 59 \end{array}\right]_{145629} = \mathbf{Pl}(0,0,51,59,34,1)_{9199568}$$

$$\ell_{26} = \left[\begin{array}{cccc} 1 & \epsilon^{37} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{11} \end{array}\right]_{174728} = \left[\begin{array}{cccc} 1 & 41 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{array}\right]_{174728} = \mathbf{Pl}(0,0,50,31,41,1)_{11034001}$$

$$\ell_{27} = \left[\begin{array}{cccc} 1 & \epsilon^{2} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{4} \end{array}\right]_{20756} = \left[\begin{array}{cccc} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 16 \end{array}\right]_{20756} = \mathbf{Pl}(0,0,6,16,4,1)_{1331453}$$

$$\ell_{28} = \left[\begin{array}{cccc} 1 & \epsilon^{12} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{24} \end{array}\right]_{262123} = \left[\begin{array}{cccc} 1 & 62 & 0 & 0 \\ 0 & 0 & 1 & 45 \end{array}\right]_{262123} = \mathbf{Pl}(0,0,7,45,62,1)_{16532220}$$

$$\ell_{29} = \left[\begin{array}{cccc} 1 & \epsilon^{31} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{62} \end{array}\right]_{58237} = \left[\begin{array}{cccc} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 48 \end{array}\right]_{58237} = \mathbf{Pl}(0,0,2,48,13,1)_{3689665}$$

$$\ell_{30} = \left[\begin{array}{cccc} 1 & \epsilon^{34} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{5} \end{array}\right]_{41577} = \left[\begin{array}{cccc} 1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 32 \end{array}\right]_{41577} = \mathbf{Pl}(0,0,3,32,9,1)_{2641472}$$

$$\ell_{31} = \left[\begin{array}{cccc} 1 & \epsilon^{22} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{44} \end{array}\right]_{83189} = \left[\begin{array}{cccc} 1 & 19 & 0 & 0 \\ 0 & 0 & 1 & 34 \end{array}\right]_{83189} = \mathbf{Pl}(0,0,22,34,19,1)_{5264685}$$

$$\ell_{32} = \left[\begin{array}{cccc} 1 & \epsilon^{50} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{37} \end{array}\right]_{253797} = \left[\begin{array}{cccc} 1 & 60 & 0 & 0 \\ 0 & 0 & 1 & 41 \end{array}\right]_{253797} = \mathbf{Pl}(0,0,23,41,60,1)_{16010092}$$

$$\ell_{33} = \left[\begin{array}{cccc} 1 & \epsilon^{14} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{28} \end{array}\right]_{245495} = \left[\begin{array}{cccc} 1 & 58 & 0 & 0 \\ 0 & 0 & 1 & 61 \end{array}\right]_{245495} = \mathbf{Pl}(0,0,18,61,58,1)_{15485297}$$

$$\ell_{34} = \left[\begin{array}{cccc} 1 & \epsilon^{52} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{41} \end{array}\right]_{212174} = \left[\begin{array}{cccc} 1 & 50 & 0 & 0 \\ 0 & 0 & 1 & 28 \end{array}\right]_{212174} = \mathbf{Pl}(0,0,19,28,50,1)_{13388784}$$

$$\ell_{35} = \left[\begin{array}{cccc} 1 & \epsilon^{58} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{53} \end{array}\right]_{16584} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{array}\right]_{16584} = \mathbf{Pl}(0,0,63,5,3,1)_{1076612}$$

$$\ell_{36} = \left[\begin{array}{cccc} 1 & \epsilon^{57} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{51} \end{array}\right]_{208010} = \left[\begin{array}{cccc} 1 & 49 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{array}\right]_{208010} = \mathbf{Pl}(0,0,62,25,49,1)_{13132165}$$

$$\ell_{37} = \left[\begin{array}{cccc} 1 & \epsilon^{19} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{38} \end{array}\right]_{95689} = \left[\begin{array}{cccc} 1 & 22 & 0 & 0 \\ 0 & 0 & 1 & 51 \end{array}\right]_{95689} = \mathbf{Pl}(0,0,59,51,22,1)_{6055624}$$

$$\ell_{38} = \left[\begin{array}{cccc} 1 & \epsilon^{56} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{49} \end{array}\right]_{170566} = \left[\begin{array}{cccc} 1 & 40 & 0 & 0 \\ 0 & 0 & 1 & 30 \end{array}\right]_{170566} = \mathbf{Pl}(0,0,58,30,40,1)_{10772937}$$

$$\ell_{39} = \left[\begin{array}{cccc} 1 & \epsilon^{27} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{54} \end{array}\right]_{195512} = \left[\begin{array}{cccc} 1 & 46 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{array}\right]_{195512} = \mathbf{Pl}(0,0,47,10,46,1)_{12344020}$$

$$\ell_{40} = \left[\begin{array}{cccc} 1 & \epsilon^{18} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{36} \end{array}\right]_{49903} = \left[\begin{array}{cccc} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 36 \end{array}\right]_{49903} = \mathbf{Pl}(0,0,46,36,11,1)_{3171093}$$

$$\ell_{41} = \left[\begin{array}{cccc} 1 & \epsilon^{40} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{17} \end{array}\right]_{62403} = \left[\begin{array}{cccc} 1 & 14 & 0 & 0 \\ 0 & 0 & 1 & 53 \end{array}\right]_{62403} = \mathbf{Pl}(0,0,43,53,14,1)_{3956952}$$

$$\ell_{42} = \left[\begin{array}{cccc} 1 & \epsilon^{55} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{47} \end{array}\right]_{87371} = \left[\begin{array}{cccc} 1 & 20 & 0 & 0 \\ 0 & 0 & 1 & 55 \end{array}\right]_{87371} = \mathbf{Pl}(0,0,42,55,20,1)_{5529305}$$

$$\ell_{43} = \left[\begin{array}{cccc} 1 & \epsilon^{7} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{14} \end{array}\right]_{149789} = \left[\begin{array}{cccc} 1 & 35 & 0 & 0 \\ 0 & 0 & 1 & 58 \end{array}\right]_{149789} = \mathbf{Pl}(0,0,30,58,35,1)_{9458981}$$

$$\ell_{44} = \left[\begin{array}{cccc} 1 & \epsilon^{26} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{52} \end{array}\right]_{99849} = \left[\begin{array}{cccc} 1 & 23 & 0 & 0 \\ 0 & 0 & 1 & 50 \end{array}\right]_{99849} = \mathbf{Pl}(0,0,31,50,23,1)_{6314148}$$

$$\ell_{45} = \left[\begin{array}{cccc} 1 & \epsilon^{47} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{31} \end{array}\right]_{232964} = \left[\begin{array}{cccc} 1 & 55 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{array}\right]_{232964} = \mathbf{Pl}(0,0,26,13,55,1)_{14700073}$$

$$\ell_{46} = \left[\begin{array}{cccc} 1 & \epsilon^{17} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{34} \end{array}\right]_{224638} = \left[\begin{array}{cccc} 1 & 53 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{array}\right]_{224638} = \mathbf{Pl}(0,0,27,9,53,1)_{14176040}$$

$$\ell_{47} = \left[\begin{array}{cccc} 1 & \epsilon^{43} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{23} \end{array}\right]_{74871} = \left[\begin{array}{cccc} 1 & 17 & 0 & 0 \\ 0 & 0 & 1 & 38 \end{array}\right]_{74871} = \mathbf{Pl}(0,0,14,38,17,1)_{4739509}$$

$$\ell_{48} = \left[\begin{array}{cccc} 1 & \epsilon^{39} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{15} \end{array}\right]_{33244} = \left[\begin{array}{cccc} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 21 \end{array}\right]_{33244} = \mathbf{Pl}(0,0,15,21,7,1)_{2118836}$$

$$\ell_{49} = \left[\begin{array}{cccc} 1 & \epsilon^{36} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{9} \end{array}\right]_{153939} = \left[\begin{array}{cccc} 1 & 36 & 0 & 0 \\ 0 & 0 & 1 & 47 \end{array}\right]_{153939} = \mathbf{Pl}(0,0,10,47,36,1)_{9718521}$$

$$\ell_{50} = \left[\begin{array}{cccc} 1 & \epsilon^{54} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{45} \end{array}\right]_{45743} = \left[\begin{array}{cccc} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 37 \end{array}\right]_{45743} = \mathbf{Pl}(0,0,11,37,10,1)_{2904568}$$

$$\ell_{51} = \left[\begin{array}{cccc} 1 & \epsilon & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{2} \end{array}\right]_{12422} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]_{12422} = \mathbf{Pl}(0,0,24,4,2,1)_{809579}$$

$$\ell_{52} = \begin{bmatrix} 1 & \epsilon^{6} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{12} \end{bmatrix}_{141471} = \begin{bmatrix} 1 & 33 & 0 & 0 \\ 0 & 0 & 1 & 62 \end{bmatrix}_{141471} = \mathbf{Pl}(0,0,25,62,33,1)_{8934186}$$

$$\ell_{53} = \begin{bmatrix} 1 & \epsilon^{11} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{22} \end{bmatrix}_{133106} = \begin{bmatrix} 1 & 31 & 0 & 0 \\ 0 & 0 & 1 & 19 \end{bmatrix}_{133106} = \mathbf{Pl}(0,0,28,19,31,1)_{8410407}$$

$$\ell_{54} = \begin{bmatrix} 1 & \epsilon^{25} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{50} \end{bmatrix}_{249655} = \begin{bmatrix} 1 & 59 & 0 & 0 \\ 0 & 0 & 1 & 60 \end{bmatrix}_{249655} = \mathbf{Pl}(0,0,29,60,59,1)_{15748774}$$

$$\ell_{55} = \begin{bmatrix} 1 & \epsilon^{30} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{60} \end{bmatrix}_{228802} = \begin{bmatrix} 1 & 54 & 0 & 0 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{228802} = \mathbf{Pl}(0,0,8,12,54,1)_{14435707}$$

$$\ell_{56} = \begin{bmatrix} 1 & \epsilon^{46} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{29} \end{bmatrix}_{183046} = \begin{bmatrix} 1 & 43 & 0 & 0 \\ 0 & 0 & 1 & 27 \end{bmatrix}_{183046} = \mathbf{Pl}(0,0,9,27,43,1)_{11552954}$$

$$\ell_{57} = \begin{bmatrix} 1 & \epsilon^{33} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{3} \end{bmatrix}_{220476} = \begin{bmatrix} 1 & 52 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{bmatrix}_{220476} = \mathbf{Pl}(0,0,12,8,52,1)_{13912055}$$

$$\ell_{58} = \begin{bmatrix} 1 & \epsilon^{16} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{32} \end{bmatrix}_{178884} = \begin{bmatrix} 1 & 42 & 0 & 0 \\ 0 & 0 & 1 & 26 \end{bmatrix}_{178884} = \mathbf{Pl}(0,0,13,26,42,1)_{11291382}$$

$$\ell_{59} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{241329} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{241329} = \mathbf{Pl}(0,0,57,56,57,1)_{15228170}$$

$$\ell_{60} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{237169} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{237169} = \mathbf{Pl}(0,0,56,57,56,1)_{14965963}$$

$$\ell_{61} = \begin{bmatrix} 1 & \epsilon^{49} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{35} \end{bmatrix}_{128944} = \begin{bmatrix} 1 & 30 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{128944} = \mathbf{Pl}(0,0,61,18,30,1)_{8152518}$$

$$\ell_{62} = \begin{bmatrix} 1 & \epsilon^{38} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{13} \end{bmatrix}_{216336} = \begin{bmatrix} 1 & 51 & 0 & 0 \\ 0 & 0 & 1 & 29 \end{bmatrix}_{216336} = \mathbf{Pl}(0,0,60,29,51,1)_{13656071}$$

$$\ell_{63} = \begin{bmatrix} 1 & \epsilon^{13} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{26} \end{bmatrix}_{124788} = \begin{bmatrix} 1 & 29 & 0 & 0 \\ 0 & 0 & 1 & 23 \end{bmatrix}_{124788} = \mathbf{Pl}(0,0,41,23,29,1)_{7887898}$$

$$\ell_{64} = \begin{bmatrix} 1 & \epsilon^{35} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{7} \end{bmatrix}_{79029} = \begin{bmatrix} 1 & 18 & 0 & 0 \\ 0 & 0 & 1 & 35 \end{bmatrix}_{79029} = \mathbf{Pl}(0,0,40,35,18,1)_{5004891}$$

$$\ell_{65} = \begin{bmatrix} 1 & \epsilon^{51} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{39} \end{bmatrix}_{108128} = \begin{bmatrix} 1 & 25 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{108128} = \mathbf{Pl}(0,0,45,7,25,1)_{6840086}$$

$$\ell_{66} = \begin{bmatrix} 1 & \epsilon^{53} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{43} \end{bmatrix}_{24918} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 17 \end{bmatrix}_{24918} = \mathbf{Pl}(0,0,44,17,5,1)_{1598359}$$

Rank of lines: (0, 4096, 17043520, 17047616, 8258, 203848, 137311, 103966, 266283, 70711, 191356, 54077, 116446, 199674, 158099, 37417, 112284, 162257, 91531, 29082, 187194, 257957, 120626, 166417, 66563, 145629, 174728, 20756, 262123, 58237, 41577, 83189, 253797, 245495, 212174, 16584, 208010, 95689, 170566, 195512, 49903, 62403, 87371, 149789, 99849, 232964, 224638, 74871, 33244, 153939, ...79029, 108128, 24918)

Rank of points on Klein quadric: (0, 2, 129, 1, 544578, 12862719, 8669566, 6574323, 16795570, 4478191, 12078638, 3431522, 7362595, 12604830, 9983903, 2385234, 7102547, 10248142, 5792655, 1859804, 11818717, 16273696, 7624929, 10510476, 4220429, 9199568, 11034001, 1331453, 16532220, 3689665, 2641472, 5264685, 16010092, 15485297, 13388784, 1076612, 13132165, 6055624, 10772937, 12344020, 3171093, 3956952, 5529305, 9458981, 6314148, 14700073, 14176040, 4739509, 2118836, 9718521, ...5004891, 6840086, 1598359)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 130 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_4$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_5$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_6$
$P_8 = (4,1,0,0) = \ell_0 \cap \ell_7$
$P_9 = (5,1,0,0) = \ell_0 \cap \ell_8$
$P_{10} = (6,1,0,0) = \ell_0 \cap \ell_9$
$P_{11} = (7,1,0,0) = \ell_0 \cap \ell_{10}$
$P_{12} = (8,1,0,0) = \ell_0 \cap \ell_{11}$
$P_{13} = (9,1,0,0) = \ell_0 \cap \ell_{12}$
$P_{14} = (10,1,0,0) = \ell_0 \cap \ell_{13}$
$P_{15} = (11,1,0,0) = \ell_0 \cap \ell_{14}$
$P_{16} = (12,1,0,0) = \ell_0 \cap \ell_{15}$
$P_{17} = (13,1,0,0) = \ell_0 \cap \ell_{16}$
$P_{18} = (14,1,0,0) = \ell_0 \cap \ell_{17}$
$P_{19} = (15,1,0,0) = \ell_0 \cap \ell_{18}$
$P_{20} = (16,1,0,0) = \ell_0 \cap \ell_{19}$
$P_{21} = (17,1,0,0) = \ell_0 \cap \ell_{20}$
$P_{22} = (18,1,0,0) = \ell_0 \cap \ell_{21}$
$P_{23} = (19,1,0,0) = \ell_0 \cap \ell_{22}$
$P_{24} = (20,1,0,0) = \ell_0 \cap \ell_{23}$
$P_{25} = (21,1,0,0) = \ell_0 \cap \ell_{24}$
$P_{26} = (22,1,0,0) = \ell_0 \cap \ell_{25}$
$P_{27} = (23,1,0,0) = \ell_0 \cap \ell_{26}$
$P_{28} = (24,1,0,0) = \ell_0 \cap \ell_{27}$
$P_{29} = (25,1,0,0) = \ell_0 \cap \ell_{28}$
$P_{30} = (26,1,0,0) = \ell_0 \cap \ell_{29}$
$P_{31} = (27,1,0,0) = \ell_0 \cap \ell_{30}$
$P_{32} = (28,1,0,0) = \ell_0 \cap \ell_{31}$
$P_{33} = (29,1,0,0) = \ell_0 \cap \ell_{32}$
$P_{34} = (30,1,0,0) = \ell_0 \cap \ell_{33}$
$P_{35} = (31,1,0,0) = \ell_0 \cap \ell_{34}$
$P_{36} = (32,1,0,0) = \ell_0 \cap \ell_{35}$
$P_{37} = (33,1,0,0) = \ell_0 \cap \ell_{36}$
$P_{38} = (34,1,0,0) = \ell_0 \cap \ell_{37}$
$P_{39} = (35,1,0,0) = \ell_0 \cap \ell_{38}$
$P_{40} = (36,1,0,0) = \ell_0 \cap \ell_{39}$
$P_{41} = (37,1,0,0) = \ell_0 \cap \ell_{40}$
$P_{42} = (38,1,0,0) = \ell_0 \cap \ell_{41}$
$P_{43} = (39,1,0,0) = \ell_0 \cap \ell_{42}$
$P_{44} = (40,1,0,0) = \ell_0 \cap \ell_{43}$
$P_{45} = (41,1,0,0) = \ell_0 \cap \ell_{44}$
$P_{46} = (42,1,0,0) = \ell_0 \cap \ell_{45}$
$P_{47} = (43,1,0,0) = \ell_0 \cap \ell_{46}$
$P_{48} = (44,1,0,0) = \ell_0 \cap \ell_{47}$
$P_{49} = (45,1,0,0) = \ell_0 \cap \ell_{48}$
$P_{50} = (46,1,0,0) = \ell_0 \cap \ell_{49}$
$P_{51} = (47,1,0,0) = \ell_0 \cap \ell_{50}$
$P_{52} = (48,1,0,0) = \ell_0 \cap \ell_{51}$
$P_{53} = (49,1,0,0) = \ell_0 \cap \ell_{52}$
$P_{54} = (50,1,0,0) = \ell_0 \cap \ell_{53}$
$P_{55} = (51,1,0,0) = \ell_0 \cap \ell_{54}$
$P_{56} = (52,1,0,0) = \ell_0 \cap \ell_{55}$
$P_{57} = (53,1,0,0) = \ell_0 \cap \ell_{56}$
$P_{58} = (54,1,0,0) = \ell_0 \cap \ell_{57}$
$P_{59} = (55,1,0,0) = \ell_0 \cap \ell_{58}$
$P_{60} = (56,1,0,0) = \ell_0 \cap \ell_{59}$
$P_{61} = (57,1,0,0) = \ell_0 \cap \ell_{60}$
$P_{62} = (58,1,0,0) = \ell_0 \cap \ell_{61}$
$P_{63} = (59,1,0,0) = \ell_0 \cap \ell_{62}$
$P_{64} = (60,1,0,0) = \ell_0 \cap \ell_{63}$
$P_{65} = (61,1,0,0) = \ell_0 \cap \ell_{64}$
$P_{66} = (62,1,0,0) = \ell_0 \cap \ell_{65}$
$P_{67} = (63,1,0,0) = \ell_0 \cap \ell_{66}$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_2 \cap \ell_3$
$P_{8258} = (0,0,1,1) = \ell_3 \cap \ell_4$
$P_{20545} = (0,0,4,1) = \ell_3 \cap \ell_5$
$P_{24641} = (0,0,5,1) = \ell_3 \cap \ell_6$
$P_{69697} = (0,0,16,1) = \ell_3 \cap \ell_7$
$P_{73793} = (0,0,17,1) = \ell_3 \cap \ell_8$
$P_{86081} = (0,0,20,1) = \ell_3 \cap \ell_9$
$P_{90177} = (0,0,21,1) = \ell_3 \cap \ell_{10}$
$P_{139329} = (0,0,33,1) = \ell_3 \cap \ell_{11}$
$P_{135233} = (0,0,32,1) = \ell_3 \cap \ell_{12}$
$P_{155713} = (0,0,37,1) = \ell_3 \cap \ell_{13}$
$P_{151617} = (0,0,36,1) = \ell_3 \cap \ell_{14}$
$P_{204865} = (0,0,49,1) = \ell_3 \cap \ell_{15}$
$P_{200769} = (0,0,48,1) = \ell_3 \cap \ell_{16}$
$P_{221249} = (0,0,53,1) = \ell_3 \cap \ell_{17}$
$P_{217153} = (0,0,52,1) = \ell_3 \cap \ell_{18}$
$P_{163905} = (0,0,39,1) = \ell_3 \cap \ell_{19}$
$P_{159809} = (0,0,38,1) = \ell_3 \cap \ell_{20}$
$P_{147521} = (0,0,35,1) = \ell_3 \cap \ell_{21}$
$P_{143425} = (0,0,34,1) = \ell_3 \cap \ell_{22}$
$P_{229441} = (0,0,55,1) = \ell_3 \cap \ell_{23}$
$P_{225345} = (0,0,54,1) = \ell_3 \cap \ell_{24}$
$P_{213057} = (0,0,51,1) = \ell_3 \cap \ell_{25}$
$P_{208961} = (0,0,50,1) = \ell_3 \cap \ell_{26}$
$P_{28737} = (0,0,6,1) = \ell_3 \cap \ell_{27}$
$P_{32833} = (0,0,7,1) = \ell_3 \cap \ell_{28}$
$P_{12353} = (0,0,2,1) = \ell_3 \cap \ell_{29}$
$P_{16449} = (0,0,3,1) = \ell_3 \cap \ell_{30}$
$P_{94273} = (0,0,22,1) = \ell_3 \cap \ell_{31}$
$P_{98369} = (0,0,23,1) = \ell_3 \cap \ell_{32}$

$P_{77889} = (0,0,18,1) = \ell_3 \cap \ell_{33}$
$P_{81985} = (0,0,19,1) = \ell_3 \cap \ell_{34}$
$P_{262209} = (0,0,63,1) = \ell_3 \cap \ell_{35}$
$P_{258113} = (0,0,62,1) = \ell_3 \cap \ell_{36}$
$P_{245825} = (0,0,59,1) = \ell_3 \cap \ell_{37}$
$P_{241729} = (0,0,58,1) = \ell_3 \cap \ell_{38}$
$P_{196673} = (0,0,47,1) = \ell_3 \cap \ell_{39}$
$P_{192577} = (0,0,46,1) = \ell_3 \cap \ell_{40}$
$P_{180289} = (0,0,43,1) = \ell_3 \cap \ell_{41}$
$P_{176193} = (0,0,42,1) = \ell_3 \cap \ell_{42}$
$P_{127041} = (0,0,30,1) = \ell_3 \cap \ell_{43}$
$P_{131137} = (0,0,31,1) = \ell_3 \cap \ell_{44}$
$P_{110657} = (0,0,26,1) = \ell_3 \cap \ell_{45}$
$P_{114753} = (0,0,27,1) = \ell_3 \cap \ell_{46}$
$P_{61505} = (0,0,14,1) = \ell_3 \cap \ell_{47}$
$P_{65601} = (0,0,15,1) = \ell_3 \cap \ell_{48}$
$P_{45121} = (0,0,10,1) = \ell_3 \cap \ell_{49}$
$P_{49217} = (0,0,11,1) = \ell_3 \cap \ell_{50}$
$P_{102465} = (0,0,24,1) = \ell_3 \cap \ell_{51}$
$P_{106561} = (0,0,25,1) = \ell_3 \cap \ell_{52}$
$P_{118849} = (0,0,28,1) = \ell_3 \cap \ell_{53}$
$P_{122945} = (0,0,29,1) = \ell_3 \cap \ell_{54}$
$P_{36929} = (0,0,8,1) = \ell_3 \cap \ell_{55}$
$P_{41025} = (0,0,9,1) = \ell_3 \cap \ell_{56}$
$P_{53313} = (0,0,12,1) = \ell_3 \cap \ell_{57}$
$P_{57409} = (0,0,13,1) = \ell_3 \cap \ell_{58}$
$P_{237633} = (0,0,57,1) = \ell_3 \cap \ell_{59}$
$P_{233537} = (0,0,56,1) = \ell_3 \cap \ell_{60}$
$P_{254017} = (0,0,61,1) = \ell_3 \cap \ell_{61}$
$P_{249921} = (0,0,60,1) = \ell_3 \cap \ell_{62}$
$P_{172097} = (0,0,41,1) = \ell_3 \cap \ell_{63}$
$P_{168001} = (0,0,40,1) = \ell_3 \cap \ell_{64}$
$P_{188481} = (0,0,45,1) = \ell_3 \cap \ell_{65}$
$P_{184385} = (0,0,44,1) = \ell_3 \cap \ell_{66}$

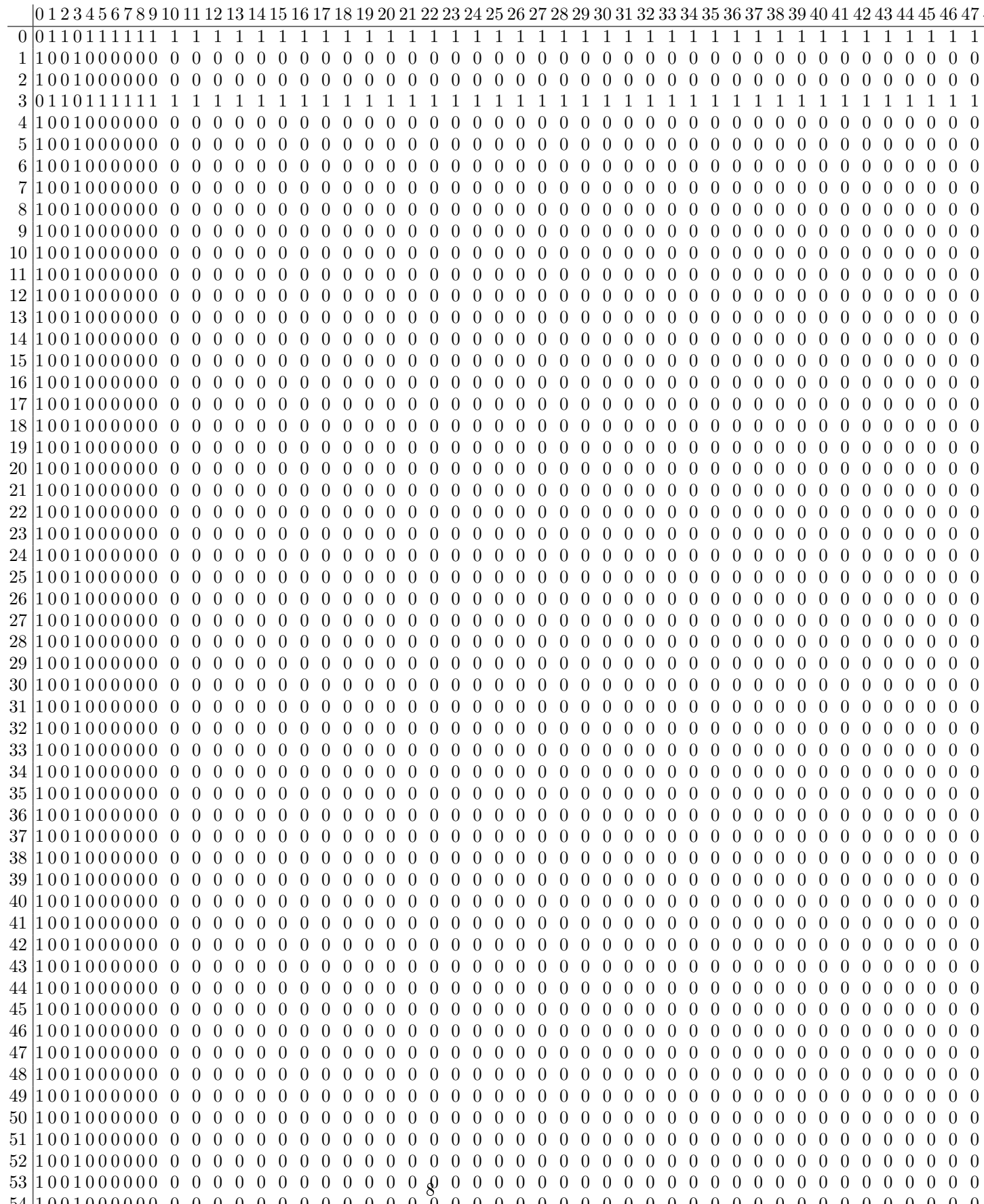
Single Points

The surface has 4095 single points:
Too many to print.

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph



Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_1	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_2	P_3	P_{8258}	P_{20545}	P_{24641}	P_{69697}	P_{73793}	P_{86081}	P_{90177}	P_{139329}	P_{135233}	P_{155713}	P_{151617}	P_{20486}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{8258}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{20545}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_7	P_{24641}

Line 7 intersects

Line	ℓ_0	ℓ_3
in point	P_8	P_{69697}

Line 8 intersects

Line	ℓ_0	ℓ_3
in point	P_9	P_{73793}

Line 9 intersects

Line	ℓ_0	ℓ_3
in point	P_{10}	P_{86081}

Line 10 intersects

Line	ℓ_0	ℓ_3
in point	P_{11}	P_{90177}

Line 11 intersects

Line	ℓ_0	ℓ_3
in point	P_{12}	P_{139329}

Line 12 intersects

Line	ℓ_0	ℓ_3
in point	P_{13}	P_{135233}

Line 13 intersects

Line	ℓ_0	ℓ_3
in point	P_{14}	P_{155713}

Line 14 intersects

Line	ℓ_0	ℓ_3
in point	P_{15}	P_{151617}

Line 15 intersects

Line	ℓ_0	ℓ_3
in point	P_{16}	P_{204865}

Line 16 intersects

Line	ℓ_0	ℓ_3
in point	P_{17}	P_{200769}

Line 17 intersects

Line	ℓ_0	ℓ_3
in point	P_{18}	P_{221249}

Line 18 intersects

Line	ℓ_0	ℓ_3
in point	P_{19}	P_{217153}

Line 19 intersects

Line	ℓ_0	ℓ_3
in point	P_{20}	P_{163905}

Line 20 intersects

Line	ℓ_0	ℓ_3
in point	P_{21}	P_{159809}

Line 21 intersects

Line	ℓ_0	ℓ_3
in point	P_{22}	P_{147521}

Line 22 intersects

Line	ℓ_0	ℓ_3
in point	P_{23}	P_{143425}

Line 23 intersects

Line	ℓ_0	ℓ_3
in point	P_{24}	P_{229441}

Line 24 intersects

Line	ℓ_0	ℓ_3
in point	P_{25}	P_{225345}

Line 25 intersects

Line	ℓ_0	ℓ_3
in point	P_{26}	P_{213057}

Line 26 intersects

Line	ℓ_0	ℓ_3
in point	P_{27}	P_{208961}

Line 27 intersects

Line	ℓ_0	ℓ_3
in point	P_{28}	P_{28737}

Line 28 intersects

Line	ℓ_0	ℓ_3
in point	P_{29}	P_{32833}

Line 29 intersects

Line	ℓ_0	ℓ_3
in point	P_{30}	P_{12353}

Line 30 intersects

Line	ℓ_0	ℓ_3
in point	P_{31}	P_{16449}

Line 31 intersects

Line	ℓ_0	ℓ_3
in point	P_{32}	P_{94273}

Line 32 intersects

Line	ℓ_0	ℓ_3
in point	P_{33}	P_{98369}

Line 33 intersects

Line	ℓ_0	ℓ_3
in point	P_{34}	P_{77889}

Line 34 intersects

Line	ℓ_0	ℓ_3
in point	P_{35}	P_{81985}

Line 35 intersects

Line	ℓ_0	ℓ_3
in point	P_{36}	P_{262209}

Line 36 intersects

Line	ℓ_0	ℓ_3
in point	P_{37}	P_{258113}

Line 37 intersects

Line	ℓ_0	ℓ_3
in point	P_{38}	P_{245825}

Line 38 intersects

Line	ℓ_0	ℓ_3
in point	P_{39}	P_{241729}

Line 39 intersects

Line	ℓ_0	ℓ_3
in point	P_{40}	P_{196673}

Line 40 intersects

Line	ℓ_0	ℓ_3
in point	P_{41}	P_{192577}

Line 41 intersects

Line	ℓ_0	ℓ_3
in point	P_{42}	P_{180289}

Line 42 intersects

Line	ℓ_0	ℓ_3
in point	P_{43}	P_{176193}

Line 43 intersects

Line	ℓ_0	ℓ_3
in point	P_{44}	P_{127041}

Line 44 intersects

Line	ℓ_0	ℓ_3
in point	P_{45}	P_{131137}

Line 45 intersects

Line	ℓ_0	ℓ_3
in point	P_{46}	P_{110657}

Line 46 intersects

Line	ℓ_0	ℓ_3
in point	P_{47}	P_{114753}

Line 47 intersects

Line	ℓ_0	ℓ_3
in point	P_{48}	P_{61505}

Line 48 intersects

Line	ℓ_0	ℓ_3
in point	P_{49}	P_{65601}

Line 49 intersects

Line	ℓ_0	ℓ_3
in point	P_{50}	P_{45121}

Line 50 intersects

Line	ℓ_0	ℓ_3
in point	P_{51}	P_{49217}

Line 51 intersects

Line	ℓ_0	ℓ_3
in point	P_{52}	P_{102465}

Line 52 intersects

Line	ℓ_0	ℓ_3
in point	P_{53}	P_{106561}

Line 53 intersects

Line	ℓ_0	ℓ_3
in point	P_{54}	P_{118849}

Line 54 intersects

Line	ℓ_0	ℓ_3
in point	P_{55}	P_{122945}

Line 55 intersects

Line	ℓ_0	ℓ_3
in point	P_{56}	P_{36929}

Line 56 intersects

Line	ℓ_0	ℓ_3
in point	P_{57}	P_{41025}

Line 57 intersects

Line	ℓ_0	ℓ_3
in point	P_{58}	P_{53313}

Line 58 intersects

Line	ℓ_0	ℓ_3
in point	P_{59}	P_{57409}

Line 59 intersects

Line	ℓ_0	ℓ_3
in point	P_{60}	P_{237633}

Line 60 intersects

Line	ℓ_0	ℓ_3
in point	P_{61}	P_{233537}

Line 61 intersects

Line	ℓ_0	ℓ_3
in point	P_{62}	P_{254017}

Line 62 intersects

Line	ℓ_0	ℓ_3
in point	P_{63}	P_{249921}

Line 63 intersects

Line	ℓ_0	ℓ_3
in point	P_{64}	P_{172097}

Line 64 intersects

Line	ℓ_0	ℓ_3
in point	P_{65}	P_{168001}

Line 65 intersects

Line	ℓ_0	ℓ_3
in point	P_{66}	P_{188481}

Line 66 intersects

Line	ℓ_0	ℓ_3
in point	P_{67}	P_{184385}

The surface has 4225 points:
Too many to print.