Rank-74052 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498834266

General information

Number of lines	10
Number of points	33
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	21
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{10}
Type of lines on points	5, 3^2, 2^9, 1^{21}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \mathbf{Pl}(2,0,0,1,0,0)_{11}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \mathbf{Pl}(3,0,0,1,0,0)_{12}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \mathbf{Pl}(0,3,1,0,0,0)_{8}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \mathbf{Pl}(0,2,1,0,0,0)_{7}$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \mathbf{Pl}(0,3,1,0,0,1)_{114}$$

$$\ell_7 = \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \mathbf{Pl}(0,2,1,0,0,1)_{113}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 0 & 1 \end{bmatrix}_{193} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{193} = \mathbf{Pl}(1,1,0,2,1,1)_{192}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{277} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{277} = \mathbf{Pl}(1,1,0,3,1,1)_{195}$$

Rank of lines: (340, 252, 168, 356, 268, 184, 289, 205, 193, 277)
Rank of points on Klein quadric: (9, 11, 12, 1, 8, 7, 114, 113, 192, 195)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_3$
$P_{24} = (2,0,0,1) = \ell_1 \cap \ell_4$
$P_{32} = (2,2,0,1) = \ell_1 \cap \ell_6$
$P_{25} = (3,0,0,1) = \ell_2 \cap \ell_5$
$P_{37} = (3,3,0,1) = \ell_2 \cap \ell_7$
$P_{55} = (2,0,2,1) = \ell_4 \cap \ell_9$
$P_{72} = (3,0,3,1) = \ell_5 \cap \ell_8$
$P_{63} = (2,2,2,1) = \ell_6 \cap \ell_8$
$P_{84} = (3,3,3,1) = \ell_7 \cap \ell_9$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_{16} = (1, 2, 1, 0)$ lies on line ℓ_8
1 : $P_{20} = (1, 3, 1, 0)$ lies on line ℓ_9
2 : $P_{28} = (2, 1, 0, 1)$ lies on line ℓ_1
3 : $P_{29} = (3, 1, 0, 1)$ lies on line ℓ_2
4 : $P_{30} = (0, 2, 0, 1)$ lies on line ℓ_0
5 : $P_{33} = (3, 2, 0, 1)$ lies on line ℓ_2
6 : $P_{34} = (0, 3, 0, 1)$ lies on line ℓ_0
7 : $P_{36} = (2, 3, 0, 1)$ lies on line ℓ_1
8 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_3
9 : $P_{40} = (2, 0, 1, 1)$ lies on line ℓ_4
10 : $P_{41} = (3, 0, 1, 1)$ lies on line ℓ_5
11 : $P_{46} = (1, 2, 1, 1)$ lies on line ℓ_9
12 : $P_{47} = (2, 2, 1, 1)$ lies on line ℓ_6
13 : $P_{50} = (1, 3, 1, 1)$ lies on line ℓ_8
14 : $P_{52} = (3, 3, 1, 1)$ lies on line ℓ_7
15 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_3
16 : $P_{56} = (3, 0, 2, 1)$ lies on line ℓ_5
17 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_7
18 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_3
19 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_4
20 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	0	0	0	0	1	1
1	1	0	1	0	1	0	1	0	0	0
2	1	1	0	0	0	1	0	1	0	0
3	1	0	0	0	1	1	1	1	0	0
4	0	1	0	1	0	1	1	1	0	1
5	0	0	1	1	1	0	1	1	1	0
6	0	1	0	1	1	1	0	1	1	0
7	0	0	1	1	1	1	1	0	0	1
8	1	0	0	0	0	1	1	0	0	1
9	1	0	0	0	1	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{26}	P_{26}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_1	P_1	P_{24}	P_{32}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_1	P_1	P_{25}	P_{37}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_2	P_2	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_{24}	P_2	P_2	P_2	P_2	P_{55}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_{25}	P_2	P_2	P_2	P_2	P_{72}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{32}	P_2	P_2	P_2	P_2	P_{63}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_{37}	P_2	P_2	P_2	P_2	P_{84}

Line 8 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_9
in point	P_{26}	P_{72}	P_{63}	P_{26}

Line 9 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_8
in point	P_{26}	P_{55}	P_{84}	P_{26}

The surface has 33 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_{16} = (1,2,1,0)$
$4: P_{20} = (1,3,1,0)$
$5: P_{24} = (2,0,0,1)$
$6: P_{25} = (3,0,0,1)$
$7: P_{26} = (0,1,0,1)$
$8: P_{28} = (2,1,0,1)$
$9: P_{29} = (3,1,0,1)$
$10: P_{30} = (0,2,0,1)$
$11: P_{32} = (2,2,0,1)$
$12: P_{33} = (3,2,0,1)$
$13: P_{34} = (0,3,0,1)$
$14: P_{36} = (2,3,0,1)$
$15: P_{37} = (3,3,0,1)$
$16: P_{38} = (0,0,1,1)$
$17: P_{40} = (2,0,1,1)$
$18: P_{41} = (3,0,1,1)$
$19: P_{46} = (1,2,1,1)$
$20: P_{47} = (2,2,1,1)$
$21: P_{50} = (1,3,1,1)$
$22: P_{52} = (3,3,1,1)$
$23: P_{53} = (0,0,2,1)$
$24: P_{55} = (2,0,2,1)$
$25: P_{56} = (3,0,2,1)$
$26: P_{63} = (2,2,2,1)$
$27: P_{68} = (3,3,2,1)$
$28: P_{69} = (0,0,3,1)$
$29: P_{71} = (2,0,3,1)$
$30: P_{72} = (3,0,3,1)$
$31: P_{79} = (2,2,3,1)$
$32: P_{84} = (3,3,3,1)$