

Rank-65922 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244210326

General information

Number of lines	6
Number of points	81
Number of singular points	3
Number of Eckardt points	0
Number of double points	9
Number of single points	36
Number of points off lines	36
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$2^9, 1^{36}, 0^{36}$

Singular Points

The surface has 3 singular points:

$$0 : P_{250} = \mathbf{P}(1, \gamma^6, \gamma, 1) = \mathbf{P}(1, 6, 2, 1)$$

$$2 : P_{562} = \mathbf{P}(1, \gamma^3, \gamma^4, 1) = \mathbf{P}(1, 5, 7, 1)$$

$$1 : P_{354} = \mathbf{P}(1, \gamma^5, \gamma^2, 1) = \mathbf{P}(1, 3, 4, 1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^6 & 0 \end{array} \right]_{225} = \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 6 & 0 \end{array} \right]_{225} = \mathbf{Pl}(4, 0, 2, 0, 0, 1)_{683}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{368} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{368} = \mathbf{Pl}(7, 0, 4, 0, 0, 1)_{716} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{443} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{443} = \mathbf{Pl}(2, 0, 7, 0, 0, 1)_{756} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{809} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{809} = \mathbf{Pl}(6, 2, 2, 4, 0, 1)_{1033} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \gamma^3 & 1 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{952} = \begin{bmatrix} 1 & 0 & 5 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{952} = \mathbf{Pl}(3, 4, 4, 7, 0, 1)_{1191} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{1027} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{1027} = \mathbf{Pl}(5, 7, 7, 2, 0, 1)_{969}
\end{aligned}$$

Rank of lines: (225, 368, 443, 809, 952, 1027)

Rank of points on Klein quadric: (683, 716, 756, 1033, 1191, 969)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 9 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{64} &= (5, 6, 1, 0) = \ell_0 \cap \ell_1 & P_{67} &= (0, 7, 1, 0) = \ell_2 \cap \ell_5 \\
P_{54} &= (3, 5, 1, 0) = \ell_0 \cap \ell_2 & P_{562} &= (1, 5, 7, 1) = \ell_3 \cap \ell_4 \\
P_{27} &= (0, 2, 1, 0) = \ell_0 \cap \ell_3 & P_{354} &= (1, 3, 4, 1) = \ell_3 \cap \ell_5 \\
P_{41} &= (6, 3, 1, 0) = \ell_1 \cap \ell_2 & P_{250} &= (1, 6, 2, 1) = \ell_4 \cap \ell_5 \\
P_{43} &= (0, 4, 1, 0) = \ell_1 \cap \ell_4
\end{aligned}$$

Single Points

The surface has 36 single points:

The single points on the surface are:

$$\begin{aligned}
0 : P_6 &= (2, 1, 0, 0) \text{ lies on line } \ell_0 & 15 : P_{66} &= (7, 6, 1, 0) \text{ lies on line } \ell_2 \\
1 : P_8 &= (4, 1, 0, 0) \text{ lies on line } \ell_1 & 16 : P_{68} &= (1, 7, 1, 0) \text{ lies on line } \ell_1 \\
2 : P_{11} &= (7, 1, 0, 0) \text{ lies on line } \ell_2 & 17 : P_{74} &= (7, 7, 1, 0) \text{ lies on line } \ell_0 \\
3 : P_{13} &= (2, 0, 1, 0) \text{ lies on line } \ell_2 & 18 : P_{99} &= (1, 3, 0, 1) \text{ lies on line } \ell_4 \\
4 : P_{15} &= (4, 0, 1, 0) \text{ lies on line } \ell_0 & 19 : P_{115} &= (1, 5, 0, 1) \text{ lies on line } \ell_5 \\
5 : P_{18} &= (7, 0, 1, 0) \text{ lies on line } \ell_1 & 20 : P_{123} &= (1, 6, 0, 1) \text{ lies on line } \ell_3 \\
6 : P_{22} &= (3, 1, 1, 0) \text{ lies on line } \ell_1 & 21 : P_{154} &= (1, 2, 1, 1) \text{ lies on line } \ell_5 \\
7 : P_{24} &= (5, 1, 1, 0) \text{ lies on line } \ell_2 & 22 : P_{170} &= (1, 4, 1, 1) \text{ lies on line } \ell_3 \\
8 : P_{25} &= (6, 1, 1, 0) \text{ lies on line } \ell_0 & 23 : P_{194} &= (1, 7, 1, 1) \text{ lies on line } \ell_4 \\
9 : P_{28} &= (1, 2, 1, 0) \text{ lies on line } \ell_2 & 24 : P_{218} &= (1, 2, 2, 1) \text{ lies on line } \ell_3 \\
10 : P_{29} &= (2, 2, 1, 0) \text{ lies on line } \ell_1 & 25 : P_{266} &= (1, 0, 3, 1) \text{ lies on line } \ell_3 \\
11 : P_{37} &= (2, 3, 1, 0) \text{ lies on line } \ell_0 & 26 : P_{274} &= (1, 1, 3, 1) \text{ lies on line } \ell_5 \\
12 : P_{44} &= (1, 4, 1, 0) \text{ lies on line } \ell_0 & 27 : P_{282} &= (1, 2, 3, 1) \text{ lies on line } \ell_4 \\
13 : P_{47} &= (4, 4, 1, 0) \text{ lies on line } \ell_2 & 28 : P_{362} &= (1, 4, 4, 1) \text{ lies on line } \ell_4 \\
14 : P_{55} &= (4, 5, 1, 0) \text{ lies on line } \ell_1 & 29 : P_{394} &= (1, 0, 5, 1) \text{ lies on line } \ell_4
\end{aligned}$$

30 : $P_{402} = (1, 1, 5, 1)$ lies on line ℓ_3
31 : $P_{426} = (1, 4, 5, 1)$ lies on line ℓ_5
32 : $P_{458} = (1, 0, 6, 1)$ lies on line ℓ_5
33 : $P_{466} = (1, 1, 6, 1)$ lies on line ℓ_4

34 : $P_{514} = (1, 7, 6, 1)$ lies on line ℓ_3
35 : $P_{578} = (1, 7, 7, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 36 points not on any line:

The points on the surface but not on lines are:

0 : $P_{76} = (2, 0, 0, 1)$	19 : $P_{359} = (6, 3, 4, 1)$
1 : $P_{78} = (4, 0, 0, 1)$	20 : $P_{365} = (4, 4, 4, 1)$
2 : $P_{81} = (7, 0, 0, 1)$	21 : $P_{387} = (2, 7, 4, 1)$
3 : $P_{82} = (0, 1, 0, 1)$	22 : $P_{390} = (5, 7, 4, 1)$
4 : $P_{92} = (2, 2, 0, 1)$	23 : $P_{403} = (2, 1, 5, 1)$
5 : $P_{110} = (4, 4, 0, 1)$	24 : $P_{407} = (6, 1, 5, 1)$
6 : $P_{137} = (7, 7, 0, 1)$	25 : $P_{421} = (4, 3, 5, 1)$
7 : $P_{138} = (0, 0, 1, 1)$	26 : $P_{441} = (0, 6, 5, 1)$
8 : $P_{146} = (0, 1, 1, 1)$	27 : $P_{468} = (3, 1, 6, 1)$
9 : $P_{203} = (2, 0, 2, 1)$	28 : $P_{469} = (4, 1, 6, 1)$
10 : $P_{219} = (2, 2, 2, 1)$	29 : $P_{481} = (0, 3, 6, 1)$
11 : $P_{236} = (3, 4, 2, 1)$	30 : $P_{504} = (7, 5, 6, 1)$
12 : $P_{240} = (7, 4, 2, 1)$	31 : $P_{528} = (7, 0, 7, 1)$
13 : $P_{254} = (5, 6, 2, 1)$	32 : $P_{541} = (4, 2, 7, 1)$
14 : $P_{278} = (5, 1, 3, 1)$	33 : $P_{543} = (6, 2, 7, 1)$
15 : $P_{280} = (7, 1, 3, 1)$	34 : $P_{564} = (3, 5, 7, 1)$
16 : $P_{305} = (0, 5, 3, 1)$	35 : $P_{584} = (7, 7, 7, 1)$
17 : $P_{315} = (2, 6, 3, 1)$	
18 : $P_{333} = (4, 0, 4, 1)$	

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	1	0
2	1	1	0	0	0	1
3	1	0	0	0	1	1
4	0	1	0	1	0	1
5	0	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_{64}	P_{54}	P_{27}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_{64}	P_{41}	P_{43}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5
in point	P_{54}	P_{41}	P_{67}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_{27}	P_{562}	P_{354}

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{43}	P_{562}	P_{250}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4
in point	P_{67}	P_{354}	P_{250}

The surface has 81 points:

The points on the surface are:

0 : $P_6 = (2, 1, 0, 0)$	28 : $P_{92} = (2, 2, 0, 1)$	56 : $P_{362} = (1, 4, 4, 1)$
1 : $P_8 = (4, 1, 0, 0)$	29 : $P_{99} = (1, 3, 0, 1)$	57 : $P_{365} = (4, 4, 4, 1)$
2 : $P_{11} = (7, 1, 0, 0)$	30 : $P_{110} = (4, 4, 0, 1)$	58 : $P_{387} = (2, 7, 4, 1)$
3 : $P_{13} = (2, 0, 1, 0)$	31 : $P_{115} = (1, 5, 0, 1)$	59 : $P_{390} = (5, 7, 4, 1)$
4 : $P_{15} = (4, 0, 1, 0)$	32 : $P_{123} = (1, 6, 0, 1)$	60 : $P_{394} = (1, 0, 5, 1)$
5 : $P_{18} = (7, 0, 1, 0)$	33 : $P_{137} = (7, 7, 0, 1)$	61 : $P_{402} = (1, 1, 5, 1)$
6 : $P_{22} = (3, 1, 1, 0)$	34 : $P_{138} = (0, 0, 1, 1)$	62 : $P_{403} = (2, 1, 5, 1)$
7 : $P_{24} = (5, 1, 1, 0)$	35 : $P_{146} = (0, 1, 1, 1)$	63 : $P_{407} = (6, 1, 5, 1)$
8 : $P_{25} = (6, 1, 1, 0)$	36 : $P_{154} = (1, 2, 1, 1)$	64 : $P_{421} = (4, 3, 5, 1)$
9 : $P_{27} = (0, 2, 1, 0)$	37 : $P_{170} = (1, 4, 1, 1)$	65 : $P_{426} = (1, 4, 5, 1)$
10 : $P_{28} = (1, 2, 1, 0)$	38 : $P_{194} = (1, 7, 1, 1)$	66 : $P_{441} = (0, 6, 5, 1)$
11 : $P_{29} = (2, 2, 1, 0)$	39 : $P_{203} = (2, 0, 2, 1)$	67 : $P_{458} = (1, 0, 6, 1)$
12 : $P_{37} = (2, 3, 1, 0)$	40 : $P_{218} = (1, 2, 2, 1)$	68 : $P_{466} = (1, 1, 6, 1)$
13 : $P_{41} = (6, 3, 1, 0)$	41 : $P_{219} = (2, 2, 2, 1)$	69 : $P_{468} = (3, 1, 6, 1)$
14 : $P_{43} = (0, 4, 1, 0)$	42 : $P_{236} = (3, 4, 2, 1)$	70 : $P_{469} = (4, 1, 6, 1)$
15 : $P_{44} = (1, 4, 1, 0)$	43 : $P_{240} = (7, 4, 2, 1)$	71 : $P_{481} = (0, 3, 6, 1)$
16 : $P_{47} = (4, 4, 1, 0)$	44 : $P_{250} = (1, 6, 2, 1)$	72 : $P_{504} = (7, 5, 6, 1)$
17 : $P_{54} = (3, 5, 1, 0)$	45 : $P_{254} = (5, 6, 2, 1)$	73 : $P_{514} = (1, 7, 6, 1)$
18 : $P_{55} = (4, 5, 1, 0)$	46 : $P_{266} = (1, 0, 3, 1)$	74 : $P_{528} = (7, 0, 7, 1)$
19 : $P_{64} = (5, 6, 1, 0)$	47 : $P_{274} = (1, 1, 3, 1)$	75 : $P_{541} = (4, 2, 7, 1)$
20 : $P_{66} = (7, 6, 1, 0)$	48 : $P_{278} = (5, 1, 3, 1)$	76 : $P_{543} = (6, 2, 7, 1)$
21 : $P_{67} = (0, 7, 1, 0)$	49 : $P_{280} = (7, 1, 3, 1)$	77 : $P_{562} = (1, 5, 7, 1)$
22 : $P_{68} = (1, 7, 1, 0)$	50 : $P_{282} = (1, 2, 3, 1)$	78 : $P_{564} = (3, 5, 7, 1)$
23 : $P_{74} = (7, 7, 1, 0)$	51 : $P_{305} = (0, 5, 3, 1)$	79 : $P_{578} = (1, 7, 7, 1)$
24 : $P_{76} = (2, 0, 0, 1)$	52 : $P_{315} = (2, 6, 3, 1)$	80 : $P_{584} = (7, 7, 7, 1)$
25 : $P_{78} = (4, 0, 0, 1)$	53 : $P_{333} = (4, 0, 4, 1)$	
26 : $P_{81} = (7, 0, 0, 1)$	54 : $P_{354} = (1, 3, 4, 1)$	
27 : $P_{82} = (0, 1, 0, 1)$	55 : $P_{359} = (6, 3, 4, 1)$	