

Rank-74275 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1361384013

General information

Number of lines	6
Number of points	89
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	42
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$4, 2^4, 1^{42}, 0^{42}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{664} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{649} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{672} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{874}
\end{aligned}$$

Rank of lines: (0, 137, 4672, 4744, 721, 658)

Rank of points on Klein quadric: (0, 664, 649, 1, 672, 874)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{19} = (0, 1, 1, 0) = \ell_2 \cap \ell_5$$

$$P_{83} = (1, 1, 0, 1) = \ell_4 \cap \ell_5$$

Single Points

The surface has 42 single points:

The single points on the surface are:

- 0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
- 1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_3
- 2 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_4
- 3 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0
- 4 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0
- 5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0
- 6 : $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0
- 7 : $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0
- 8 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0
- 9 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_1
- 10 : $P_{27} = (0, 2, 1, 0)$ lies on line ℓ_2
- 11 : $P_{29} = (2, 2, 1, 0)$ lies on line ℓ_1
- 12 : $P_{35} = (0, 3, 1, 0)$ lies on line ℓ_2
- 13 : $P_{38} = (3, 3, 1, 0)$ lies on line ℓ_1
- 14 : $P_{43} = (0, 4, 1, 0)$ lies on line ℓ_2
- 15 : $P_{47} = (4, 4, 1, 0)$ lies on line ℓ_1
- 16 : $P_{51} = (0, 5, 1, 0)$ lies on line ℓ_2

- 17 : $P_{56} = (5, 5, 1, 0)$ lies on line ℓ_1
- 18 : $P_{59} = (0, 6, 1, 0)$ lies on line ℓ_2
- 19 : $P_{65} = (6, 6, 1, 0)$ lies on line ℓ_1
- 20 : $P_{67} = (0, 7, 1, 0)$ lies on line ℓ_2
- 21 : $P_{74} = (7, 7, 1, 0)$ lies on line ℓ_1
- 22 : $P_{138} = (0, 0, 1, 1)$ lies on line ℓ_3
- 23 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_5
- 24 : $P_{201} = (0, 0, 2, 1)$ lies on line ℓ_3
- 25 : $P_{210} = (1, 1, 2, 1)$ lies on line ℓ_4
- 26 : $P_{226} = (1, 3, 2, 1)$ lies on line ℓ_5
- 27 : $P_{265} = (0, 0, 3, 1)$ lies on line ℓ_3
- 28 : $P_{274} = (1, 1, 3, 1)$ lies on line ℓ_4
- 29 : $P_{282} = (1, 2, 3, 1)$ lies on line ℓ_5
- 30 : $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_3
- 31 : $P_{338} = (1, 1, 4, 1)$ lies on line ℓ_4
- 32 : $P_{370} = (1, 5, 4, 1)$ lies on line ℓ_5
- 33 : $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_3

34 : $P_{402} = (1, 1, 5, 1)$ lies on line ℓ_4
35 : $P_{426} = (1, 4, 5, 1)$ lies on line ℓ_5
36 : $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_3
37 : $P_{466} = (1, 1, 6, 1)$ lies on line ℓ_4
38 : $P_{514} = (1, 7, 6, 1)$ lies on line ℓ_5

39 : $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_3
40 : $P_{530} = (1, 1, 7, 1)$ lies on line ℓ_4
41 : $P_{570} = (1, 6, 7, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:

The points on the surface but not on lines are:

0 : $P_{94} = (4, 2, 0, 1)$	22 : $P_{332} = (3, 0, 4, 1)$
1 : $P_{103} = (5, 3, 0, 1)$	23 : $P_{340} = (3, 1, 4, 1)$
2 : $P_{113} = (7, 4, 0, 1)$	24 : $P_{360} = (7, 3, 4, 1)$
3 : $P_{120} = (6, 5, 0, 1)$	25 : $P_{376} = (7, 5, 4, 1)$
4 : $P_{125} = (3, 6, 0, 1)$	26 : $P_{400} = (7, 0, 5, 1)$
5 : $P_{132} = (2, 7, 0, 1)$	27 : $P_{408} = (7, 1, 5, 1)$
6 : $P_{158} = (5, 2, 1, 1)$	28 : $P_{419} = (2, 3, 5, 1)$
7 : $P_{159} = (6, 2, 1, 1)$	29 : $P_{423} = (6, 3, 5, 1)$
8 : $P_{172} = (3, 4, 1, 1)$	30 : $P_{427} = (2, 4, 5, 1)$
9 : $P_{175} = (6, 4, 1, 1)$	31 : $P_{455} = (6, 7, 5, 1)$
10 : $P_{196} = (3, 7, 1, 1)$	32 : $P_{459} = (2, 0, 6, 1)$
11 : $P_{198} = (5, 7, 1, 1)$	33 : $P_{467} = (2, 1, 6, 1)$
12 : $P_{207} = (6, 0, 2, 1)$	34 : $P_{476} = (3, 2, 6, 1)$
13 : $P_{215} = (6, 1, 2, 1)$	35 : $P_{500} = (3, 5, 6, 1)$
14 : $P_{229} = (4, 3, 2, 1)$	36 : $P_{501} = (4, 5, 6, 1)$
15 : $P_{253} = (4, 6, 2, 1)$	37 : $P_{517} = (4, 7, 6, 1)$
16 : $P_{269} = (4, 0, 3, 1)$	38 : $P_{526} = (5, 0, 7, 1)$
17 : $P_{277} = (4, 1, 3, 1)$	39 : $P_{534} = (5, 1, 7, 1)$
18 : $P_{288} = (7, 2, 3, 1)$	40 : $P_{563} = (2, 5, 7, 1)$
19 : $P_{302} = (5, 4, 3, 1)$	41 : $P_{571} = (2, 6, 7, 1)$
20 : $P_{318} = (5, 6, 3, 1)$	
21 : $P_{320} = (7, 6, 3, 1)$	

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	0	0	0
1	1	0	1	1	1	0
2	1	1	0	1	1	1
3	0	1	1	0	1	0
4	0	1	1	1	0	1
5	0	0	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_5	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5
in point	P_1	P_2	P_2	P_2	P_{19}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_2	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{83}

Line 5 intersects

Line	ℓ_2	ℓ_4
in point	P_{19}	P_{83}

The surface has 89 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_5 = (1, 1, 0, 0)$
 6 : $P_6 = (2, 1, 0, 0)$
 7 : $P_7 = (3, 1, 0, 0)$
 8 : $P_8 = (4, 1, 0, 0)$
 9 : $P_9 = (5, 1, 0, 0)$
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