

# Rank-74500 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1378128462

## General information

Number of lines	3
Number of points	73
Number of singular points	2
Number of Eckardt points	0
Number of double points	2
Number of single points	23
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^3$
Type of lines on points	$2^2, 1^{23}, 0^{48}$

## Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{4673} = \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{4673} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{769}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0)_{10}$$

Rank of lines: ( 4673, 4744, 648 )

Rank of points on Klein quadric: ( 769, 1, 10 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_{138} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

$$P_2 = (0, 0, 1, 0) = \ell_1 \cap \ell_2$$

### Single Points

The surface has 23 single points:

The single points on the surface are:

$$0 : P_1 = (0, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_3 = (0, 0, 0, 1) \text{ lies on line } \ell_1$$

$$2 : P_{75} = (1, 0, 0, 1) \text{ lies on line } \ell_2$$

$$3 : P_{139} = (1, 0, 1, 1) \text{ lies on line } \ell_2$$

$$4 : P_{146} = (0, 1, 1, 1) \text{ lies on line } \ell_0$$

$$5 : P_{153} = (0, 2, 1, 1) \text{ lies on line } \ell_0$$

$$6 : P_{161} = (0, 3, 1, 1) \text{ lies on line } \ell_0$$

$$7 : P_{169} = (0, 4, 1, 1) \text{ lies on line } \ell_0$$

$$8 : P_{177} = (0, 5, 1, 1) \text{ lies on line } \ell_0$$

$$9 : P_{185} = (0, 6, 1, 1) \text{ lies on line } \ell_0$$

$$10 : P_{193} = (0, 7, 1, 1) \text{ lies on line } \ell_0$$

$$11 : P_{201} = (0, 0, 2, 1) \text{ lies on line } \ell_1$$

$$12 : P_{202} = (1, 0, 2, 1) \text{ lies on line } \ell_2$$

$$13 : P_{265} = (0, 0, 3, 1) \text{ lies on line } \ell_1$$

$$14 : P_{266} = (1, 0, 3, 1) \text{ lies on line } \ell_2$$

$$15 : P_{329} = (0, 0, 4, 1) \text{ lies on line } \ell_1$$

$$16 : P_{330} = (1, 0, 4, 1) \text{ lies on line } \ell_2$$

$$17 : P_{393} = (0, 0, 5, 1) \text{ lies on line } \ell_1$$

$$18 : P_{394} = (1, 0, 5, 1) \text{ lies on line } \ell_2$$

$$19 : P_{457} = (0, 0, 6, 1) \text{ lies on line } \ell_1$$

$$20 : P_{458} = (1, 0, 6, 1) \text{ lies on line } \ell_2$$

$$21 : P_{521} = (0, 0, 7, 1) \text{ lies on line } \ell_1$$

$$22 : P_{522} = (1, 0, 7, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

### Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

$$0 : P_{22} = (3, 1, 1, 0)$$

$$1 : P_{24} = (5, 1, 1, 0)$$

$$2 : P_{25} = (6, 1, 1, 0)$$

$$3 : P_{30} = (3, 2, 1, 0)$$

$$4 : P_{48} = (5, 4, 1, 0)$$

$$5 : P_{73} = (6, 7, 1, 0)$$

$$6 : P_{85} = (3, 1, 0, 1)$$

$$7 : P_{87} = (5, 1, 0, 1)$$

8 : $P_{88} = (6, 1, 0, 1)$	29 : $P_{348} = (3, 2, 4, 1)$
9 : $P_{97} = (7, 2, 0, 1)$	30 : $P_{357} = (4, 3, 4, 1)$
10 : $P_{108} = (2, 4, 0, 1)$	31 : $P_{372} = (3, 5, 4, 1)$
11 : $P_{134} = (4, 7, 0, 1)$	32 : $P_{378} = (1, 6, 4, 1)$
12 : $P_{159} = (6, 2, 1, 1)$	33 : $P_{410} = (1, 2, 5, 1)$
13 : $P_{168} = (7, 3, 1, 1)$	34 : $P_{419} = (2, 3, 5, 1)$
14 : $P_{172} = (3, 4, 1, 1)$	35 : $P_{431} = (6, 4, 5, 1)$
15 : $P_{179} = (2, 5, 1, 1)$	36 : $P_{439} = (6, 5, 5, 1)$
16 : $P_{189} = (4, 6, 1, 1)$	37 : $P_{451} = (2, 7, 5, 1)$
17 : $P_{198} = (5, 7, 1, 1)$	38 : $P_{477} = (4, 2, 6, 1)$
18 : $P_{211} = (2, 1, 2, 1)$	39 : $P_{490} = (1, 4, 6, 1)$
19 : $P_{231} = (6, 3, 2, 1)$	40 : $P_{501} = (4, 5, 6, 1)$
20 : $P_{242} = (1, 5, 2, 1)$	41 : $P_{508} = (3, 6, 6, 1)$
21 : $P_{251} = (2, 6, 2, 1)$	42 : $P_{516} = (3, 7, 6, 1)$
22 : $P_{263} = (6, 7, 2, 1)$	43 : $P_{536} = (7, 1, 7, 1)$
23 : $P_{286} = (5, 2, 3, 1)$	44 : $P_{546} = (1, 3, 7, 1)$
24 : $P_{294} = (5, 3, 3, 1)$	45 : $P_{558} = (5, 4, 7, 1)$
25 : $P_{304} = (7, 4, 3, 1)$	46 : $P_{568} = (7, 5, 7, 1)$
26 : $P_{320} = (7, 6, 3, 1)$	47 : $P_{574} = (5, 6, 7, 1)$
27 : $P_{322} = (1, 7, 3, 1)$	
28 : $P_{341} = (4, 1, 4, 1)$	

## Line Intersection Graph

	0 1 2
0	0 1 0
1	1 0 1
2	0 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$
in point	$P_{138}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{138}$	$P_2$

Line 2 intersects

Line	$\ell_1$
in point	$P_2$

The surface has 73 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	11 : $P_{87} = (5, 1, 0, 1)$	22 : $P_{168} = (7, 3, 1, 1)$
1 : $P_2 = (0, 0, 1, 0)$	12 : $P_{88} = (6, 1, 0, 1)$	23 : $P_{169} = (0, 4, 1, 1)$
2 : $P_3 = (0, 0, 0, 1)$	13 : $P_{97} = (7, 2, 0, 1)$	24 : $P_{172} = (3, 4, 1, 1)$
3 : $P_{22} = (3, 1, 1, 0)$	14 : $P_{108} = (2, 4, 0, 1)$	25 : $P_{177} = (0, 5, 1, 1)$
4 : $P_{24} = (5, 1, 1, 0)$	15 : $P_{134} = (4, 7, 0, 1)$	26 : $P_{179} = (2, 5, 1, 1)$
5 : $P_{25} = (6, 1, 1, 0)$	16 : $P_{138} = (0, 0, 1, 1)$	27 : $P_{185} = (0, 6, 1, 1)$
6 : $P_{30} = (3, 2, 1, 0)$	17 : $P_{139} = (1, 0, 1, 1)$	28 : $P_{189} = (4, 6, 1, 1)$
7 : $P_{48} = (5, 4, 1, 0)$	18 : $P_{146} = (0, 1, 1, 1)$	29 : $P_{193} = (0, 7, 1, 1)$
8 : $P_{73} = (6, 7, 1, 0)$	19 : $P_{153} = (0, 2, 1, 1)$	30 : $P_{198} = (5, 7, 1, 1)$
9 : $P_{75} = (1, 0, 0, 1)$	20 : $P_{159} = (6, 2, 1, 1)$	31 : $P_{201} = (0, 0, 2, 1)$
10 : $P_{85} = (3, 1, 0, 1)$	21 : $P_{161} = (0, 3, 1, 1)$	32 : $P_{202} = (1, 0, 2, 1)$

33 :  $P_{211} = (2, 1, 2, 1)$   
 34 :  $P_{231} = (6, 3, 2, 1)$   
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