Rank-67115 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is -1859840947

General information

Number of lines	6
Number of points	81
Number of singular points	3
Number of Eckardt points	0
Number of double points	9
Number of single points	36
Number of points off lines	36
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$2^9, 1^{36}, 0^{36}$

Singular Points

The surface has 3 singular points:

0 : $P_{82} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$ 2 : $P_{146} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$
1 : $P_{83} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{8} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{8} = \mathbf{Pl}(1,0,0,0,1,0)_{82}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \mathbf{Pl}(0,1,0,0,0,1)_{657}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \mathbf{Pl}(1,1,1,0,1,1)_{1224}$$

Rank of lines: (0, 74, 4672, 8, 4681, 82)
Rank of points on Klein quadric: (0, 665, 649, 82, 657, 1224)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_3$

$P_{19} = (0,1,1,0) = \ell_1 \cap \ell_2$

$P_{12} = (1,0,1,0) = \ell_1 \cap \ell_5$

$P_2 = (0,0,1,0) = \ell_2 \cap \ell_4$

$P_{82} = (0,1,0,1) = \ell_3 \cap \ell_4$

$P_{83} = (1,1,0,1) = \ell_3 \cap \ell_5$

$P_{146} = (0,1,1,1) = \ell_4 \cap \ell_5$

Single Points

The surface has 36 single points:
The single points on the surface are:

0 : $P_6 = (2,1,0,0)$ lies on line ℓ_0

1 : $P_7 = (3,1,0,0)$ lies on line ℓ_0

2 : $P_8 = (4,1,0,0)$ lies on line ℓ_0

3 : $P_9 = (5,1,0,0)$ lies on line ℓ_0

4 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0

5 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0

6 : $P_{27} = (0,2,1,0)$ lies on line ℓ_2

7 : $P_{30} = (3,2,1,0)$ lies on line ℓ_1

8 : $P_{35} = (0,3,1,0)$ lies on line ℓ_2

9 : $P_{37} = (2,3,1,0)$ lies on line ℓ_1

10 : $P_{43} = (0,4,1,0)$ lies on line ℓ_2

11 : $P_{48} = (5,4,1,0)$ lies on line ℓ_1

12 : $P_{51} = (0,5,1,0)$ lies on line ℓ_2

13 : $P_{55} = (4,5,1,0)$ lies on line ℓ_1

14 : $P_{59} = (0,6,1,0)$ lies on line ℓ_2

15 : $P_{66} = (7,6,1,0)$ lies on line ℓ_1

16 : $P_{67} = (0,7,1,0)$ lies on line ℓ_2

17 : $P_{73} = (6,7,1,0)$ lies on line ℓ_1

18 : $P_{84} = (2,1,0,1)$ lies on line ℓ_3

19 : $P_{85} = (3,1,0,1)$ lies on line ℓ_3

20 : $P_{86} = (4,1,0,1)$ lies on line ℓ_3

21 : $P_{87} = (5,1,0,1)$ lies on line ℓ_3

22 : $P_{88} = (6,1,0,1)$ lies on line ℓ_3

23 : $P_{89} = (7,1,0,1)$ lies on line ℓ_3

24 : $P_{209} = (0,1,2,1)$ lies on line ℓ_4

25 : $P_{212} = (3,1,2,1)$ lies on line ℓ_5

26 : $P_{273} = (0,1,3,1)$ lies on line ℓ_4

27 : $P_{275} = (2,1,3,1)$ lies on line ℓ_5

28 : $P_{337} = (0,1,4,1)$ lies on line ℓ_4

29 : $P_{342} = (5,1,4,1)$ lies on line ℓ_5

$30: P_{401} = (0,1,5,1)$ lies on line ℓ_4 $34: P_{529} = (0,1,7,1)$ lies on line ℓ_4
$31: P_{405} = (4,1,5,1)$ lies on line ℓ_5 $35: P_{535} = (6,1,7,1)$ lies on line ℓ_5
$32: P_{465} = (0,1,6,1)$ lies on line ℓ_4
$33: P_{472} = (7,1,6,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 36 points not on any line:
The points on the surface but not on lines are:

$0: P_{219} = (2,2,2,1)$ $19: P_{412} = (3,2,5,1)$
$1: P_{251} = (2,6,2,1)$ $20: P_{413} = (4,2,5,1)$
$2: P_{255} = (6,6,2,1)$ $21: P_{431} = (6,4,5,1)$
$3: P_{260} = (3,7,2,1)$ $22: P_{432} = (7,4,5,1)$
$4: P_{263} = (6,7,2,1)$ $23: P_{440} = (7,5,5,1)$
$5: P_{270} = (5,0,3,1)$ $24: P_{460} = (3,0,6,1)$
$6: P_{271} = (6,0,3,1)$ $25: P_{462} = (5,0,6,1)$
$7: P_{285} = (4,2,3,1)$ $26: P_{494} = (5,4,6,1)$
$8: P_{286} = (5,2,3,1)$ $27: P_{496} = (7,4,6,1)$
$9: P_{293} = (4,3,3,1)$ $28: P_{507} = (2,6,6,1)$
$10: P_{323} = (2,7,3,1)$ $29: P_{515} = (2,7,6,1)$
$11: P_{327} = (6,7,3,1)$ $30: P_{516} = (3,7,6,1)$
$12: P_{348} = (3,2,4,1)$ $31: P_{558} = (5,4,7,1)$
$13: P_{350} = (5,2,4,1)$ $32: P_{559} = (6,4,7,1)$
$14: P_{356} = (3,3,4,1)$ $33: P_{566} = (5,5,7,1)$
$15: P_{357} = (4,3,4,1)$ $34: P_{568} = (7,5,7,1)$
$16: P_{365} = (4,4,4,1)$ $35: P_{584} = (7,7,7,1)$
$17: P_{396} = (3,0,5,1)$
$18: P_{399} = (6,0,5,1)$

Line Intersection Graph

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 |0 1 2 3 4 5
─┼───────────
0|0 1 1 1 0 0
1|1 0 1 0 0 1
2|1 1 0 0 1 0
3|1 0 0 0 1 1
4|0 0 1 1 0 1
5|0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_1	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_5	P_{19}	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_1	P_{19}	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_0	P_{82}	P_{83}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_{82}	P_{146}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{12}	P_{83}	P_{146}

The surface has 81 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_1 = (0,1,0,0)$

$2 : P_2 = (0,0,1,0)$

$3 : P_5 = (1,1,0,0)$

$4 : P_6 = (2,1,0,0)$

$5 : P_7 = (3,1,0,0)$

$6 : P_8 = (4,1,0,0)$

$7 : P_9 = (5,1,0,0)$

$8 : P_{10} = (6,1,0,0)$

$9 : P_{11} = (7,1,0,0)$

$10 : P_{12} = (1,0,1,0)$

$11 : P_{19} = (0,1,1,0)$

$12 : P_{27} = (0,2,1,0)$

$13 : P_{30} = (3,2,1,0)$

$14 : P_{35} = (0,3,1,0)$

$15 : P_{37} = (2,3,1,0)$

$16 : P_{43} = (0,4,1,0)$

$17 : P_{48} = (5,4,1,0)$

$18 : P_{51} = (0,5,1,0)$

$19 : P_{55} = (4,5,1,0)$

$20 : P_{59} = (0,6,1,0)$

$21 : P_{66} = (7,6,1,0)$

$22 : P_{67} = (0,7,1,0)$

$23 : P_{73} = (6,7,1,0)$

$24 : P_{82} = (0,1,0,1)$

$25 : P_{83} = (1,1,0,1)$

$26 : P_{84} = (2,1,0,1)$

$27 : P_{85} = (3,1,0,1)$

$28 : P_{86} = (4,1,0,1)$

$29 : P_{87} = (5,1,0,1)$

$30 : P_{88} = (6,1,0,1)$

$31 : P_{89} = (7,1,0,1)$

$32 : P_{146} = (0,1,1,1)$

$33 : P_{209} = (0,1,2,1)$

$34 : P_{212} = (3,1,2,1)$

$35 : P_{219} = (2,2,2,1)$

$36 : P_{251} = (2,6,2,1)$

$37 : P_{255} = (6,6,2,1)$

$38 : P_{260} = (3,7,2,1)$

$39 : P_{263} = (6,7,2,1)$

$40 : P_{270} = (5,0,3,1)$

$41 : P_{271} = (6,0,3,1)$

$42 : P_{273} = (0,1,3,1)$

$43 : P_{275} = (2,1,3,1)$

$44 : P_{285} = (4,2,3,1)$

$45 : P_{286} = (5,2,3,1)$

$46 : P_{293} = (4,3,3,1)$

$47 : P_{323} = (2,7,3,1)$

$48 : P_{327} = (6,7,3,1)$

$49 : P_{337} = (0,1,4,1)$

$50 : P_{342} = (5,1,4,1)$

$51 : P_{348} = (3,2,4,1)$

$52 : P_{350} = (5,2,4,1)$

$53 : P_{356} = (3,3,4,1)$

$54 : P_{357} = (4,3,4,1)$

$55 : P_{365} = (4,4,4,1)$

$56 : P_{396} = (3,0,5,1)$

$57 : P_{399} = (6,0,5,1)$

$58 : P_{401} = (0,1,5,1)$

$59 : P_{405} = (4,1,5,1)$

$60 : P_{412} = (3,2,5,1)$

$61 : P_{413} = (4,2,5,1)$

$62 : P_{431} = (6,4,5,1)$

$63 : P_{432} = (7,4,5,1)$

$64 : P_{440} = (7,5,5,1)$

$65 : P_{460} = (3,0,6,1)$

$66 : P_{462} = (5,0,6,1)$

$67 : P_{465} = (0,1,6,1)$

$68 : P_{472} = (7,1,6,1)$

$69 : P_{494} = (5,4,6,1)$

$70 : P_{496} = (7,4,6,1)$

$71 : P_{507} = (2,6,6,1)$

$72 : P_{515} = (2,7,6,1)$

$73 : P_{516} = (3,7,6,1)$

$74 : P_{529} = (0,1,7,1)$

$75 : P_{535} = (6,1,7,1)$

$76 : P_{558} = (5,4,7,1)$

$77 : P_{559} = (6,4,7,1)$

$78 : P_{566} = (5,5,7,1)$

$79 : P_{568} = (7,5,7,1)$

$80 : P_{584} = (7,7,7,1)$