

# Rank-76308 over GF(2)

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## The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_1 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(2) is 76308

## General information

Number of lines	4
Number of points	11
Number of singular points	0
Number of Eckardt points	1
Number of double points	1
Number of single points	7
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$3^4$
Type of lines on points	$3, 2, 1^7, 0^2$

## Singular Points

The surface has 0 singular points:

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{32}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{34}$$

Rank of lines: ( 34, 18, 12, 26 )

Rank of points on Klein quadric: ( 1, 4, 32, 34 )

### Eckardt Points

The surface has 1 Eckardt points:

0 :  $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$ .  $T = 13$

### Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

### Single Points

The surface has 7 single points:

The single points on the surface are:

0 :  $P_3 = (0, 0, 0, 1)$  lies on line  $\ell_0$

1 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_2$

2 :  $P_5 = (1, 1, 0, 0)$  lies on line  $\ell_2$

3 :  $P_8 = (1, 1, 1, 0)$  lies on line  $\ell_3$

4 :  $P_9 = (1, 0, 0, 1)$  lies on line  $\ell_1$

5 :  $P_{11} = (1, 1, 0, 1)$  lies on line  $\ell_3$

6 :  $P_{13} = (1, 0, 1, 1)$  lies on line  $\ell_1$

The single points on the surface are:

### Points on surface but on no line

The surface has 2 points not on any line:

The points on the surface but not on lines are:

0 :  $P_1 = (0, 1, 0, 0)$

1 :  $P_{14} = (0, 1, 1, 1)$

### Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	0	0
2	1	0	0	1
3	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$
in point	$P_2$	$P_{12}$	$P_{12}$

Line 1 intersects

Line	$\ell_0$
in point	$P_2$

Line 2 intersects

Line	$\ell_0$	$\ell_3$
in point	$P_{12}$	$P_{12}$

Line 3 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{12}$	$P_{12}$

The surface has 11 points:

The points on the surface are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_5 = (1, 1, 0, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_9 = (1, 0, 0, 1)$$

$$7 : P_{11} = (1, 1, 0, 1)$$

$$8 : P_{12} = (0, 0, 1, 1)$$

$$9 : P_{13} = (1, 0, 1, 1)$$

$$10 : P_{14} = (0, 1, 1, 1)$$