Rank-74243 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286331157

General information

Number of lines	4
Number of points	289
Number of singular points	2
Number of Eckardt points	1
Number of double points	2
Number of single points	61
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^4
Type of lines on points	3, 2^2, 1^{61}, 0^{225}

Singular Points

The surface has 2 singular points:

$0:\ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \mathbf{Pl}(0,0,0,0,0,1)_{4625}$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 256, 69888, 70160)
Rank of points on Klein quadric: (0, 2, 4625, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 61 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_3
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$5 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$6 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$7 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$8 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$9 : P_{13} = (9,1,0,0)$ lies on line ℓ_0
$10 : P_{14} = (10,1,0,0)$ lies on line ℓ_0
$11 : P_{15} = (11,1,0,0)$ lies on line ℓ_0
$12 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$13 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$14 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$15 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$16 : P_{20} = (1,0,1,0)$ lies on line ℓ_1
$17 : P_{21} = (2,0,1,0)$ lies on line ℓ_1
$18 : P_{22} = (3,0,1,0)$ lies on line ℓ_1
$19 : P_{23} = (4,0,1,0)$ lies on line ℓ_1
$20 : P_{24} = (5,0,1,0)$ lies on line ℓ_1
$21 : P_{25} = (6,0,1,0)$ lies on line ℓ_1
$22 : P_{26} = (7,0,1,0)$ lies on line ℓ_1
$23 : P_{27} = (8,0,1,0)$ lies on line ℓ_1
$24 : P_{28} = (9,0,1,0)$ lies on line ℓ_1
$25 : P_{29} = (10,0,1,0)$ lies on line ℓ_1
$26 : P_{30} = (11,0,1,0)$ lies on line ℓ_1
$27 : P_{31} = (12,0,1,0)$ lies on line ℓ_1
$28 : P_{32} = (13,0,1,0)$ lies on line ℓ_1
$29 : P_{33} = (14,0,1,0)$ lies on line ℓ_1
$30 : P_{34} = (15,0,1,0)$ lies on line ℓ_1
$31 : P_{35} = (0,1,1,0)$ lies on line ℓ_2
$32 : P_{51} = (0,2,1,0)$ lies on line ℓ_2
$33 : P_{67} = (0,3,1,0)$ lies on line ℓ_2
$34 : P_{83} = (0,4,1,0)$ lies on line ℓ_2
$35 : P_{99} = (0,5,1,0)$ lies on line ℓ_2
$36 : P_{115} = (0,6,1,0)$ lies on line ℓ_2
$37 : P_{131} = (0,7,1,0)$ lies on line ℓ_2
$38 : P_{147} = (0,8,1,0)$ lies on line ℓ_2
$39 : P_{163} = (0,9,1,0)$ lies on line ℓ_2
$40 : P_{179} = (0,10,1,0)$ lies on line ℓ_2
$41 : P_{195} = (0,11,1,0)$ lies on line ℓ_2
$42 : P_{211} = (0,12,1,0)$ lies on line ℓ_2
$43 : P_{227} = (0,13,1,0)$ lies on line ℓ_2

44 : $P_{243} = (0, 14, 1, 0)$ lies on line ℓ_2
45 : $P_{259} = (0, 15, 1, 0)$ lies on line ℓ_2
46 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_3
47 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_3
48 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_3
49 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_3
50 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_3
51 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_3
52 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_3
53 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_3
54 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_3
55 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_3
56 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_3
57 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_3
58 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_3
59 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_3
60 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

0 : $P_{291} = (1, 1, 0, 1)$
1 : $P_{310} = (4, 2, 0, 1)$
2 : $P_{327} = (5, 3, 0, 1)$
3 : $P_{347} = (9, 4, 0, 1)$
4 : $P_{362} = (8, 5, 0, 1)$
5 : $P_{383} = (13, 6, 0, 1)$
6 : $P_{398} = (12, 7, 0, 1)$
7 : $P_{417} = (15, 8, 0, 1)$
8 : $P_{432} = (14, 9, 0, 1)$
9 : $P_{445} = (11, 10, 0, 1)$
10 : $P_{460} = (10, 11, 0, 1)$
11 : $P_{472} = (6, 12, 0, 1)$
12 : $P_{489} = (7, 13, 0, 1)$
13 : $P_{500} = (2, 14, 0, 1)$
14 : $P_{517} = (3, 15, 0, 1)$
15 : $P_{572} = (11, 2, 1, 1)$
16 : $P_{591} = (14, 3, 1, 1)$
17 : $P_{603} = (10, 4, 1, 1)$
18 : $P_{611} = (2, 5, 1, 1)$
19 : $P_{634} = (9, 6, 1, 1)$
20 : $P_{643} = (2, 7, 1, 1)$
21 : $P_{661} = (4, 8, 1, 1)$
22 : $P_{684} = (11, 9, 1, 1)$
23 : $P_{690} = (1, 10, 1, 1)$
24 : $P_{706} = (1, 11, 1, 1)$
25 : $P_{725} = (4, 12, 1, 1)$
26 : $P_{751} = (14, 13, 1, 1)$
27 : $P_{763} = (10, 14, 1, 1)$
28 : $P_{778} = (9, 15, 1, 1)$
29 : $P_{809} = (8, 1, 2, 1)$
30 : $P_{831} = (14, 2, 2, 1)$
31 : $P_{837} = (4, 3, 2, 1)$
32 : $P_{850} = (1, 4, 2, 1)$
33 : $P_{871} = (6, 5, 2, 1)$
34 : $P_{882} = (1, 6, 2, 1)$
35 : $P_{911} = (14, 7, 2, 1)$
36 : $P_{921} = (8, 8, 2, 1)$
37 : $P_{935} = (6, 9, 2, 1)$
38 : $P_{960} = (15, 10, 2, 1)$
39 : $P_{965} = (4, 11, 2, 1)$
40 : $P_{1008} = (15, 13, 2, 1)$
41 : $P_{1021} = (12, 14, 2, 1)$
42 : $P_{1037} = (12, 15, 2, 1)$
43 : $P_{1069} = (12, 1, 3, 1)$
44 : $P_{1083} = (10, 2, 3, 1)$
45 : $P_{1096} = (7, 3, 3, 1)$
46 : $P_{1119} = (14, 4, 3, 1)$
47 : $P_{1131} = (10, 5, 3, 1)$
48 : $P_{1152} = (15, 6, 3, 1)$
49 : $P_{1166} = (13, 7, 3, 1)$
50 : $P_{1198} = (13, 9, 3, 1)$
51 : $P_{1208} = (7, 10, 3, 1)$
52 : $P_{1219} = (2, 11, 3, 1)$
53 : $P_{1245} = (12, 12, 3, 1)$
54 : $P_{1251} = (2, 13, 3, 1)$
55 : $P_{1280} = (15, 14, 3, 1)$
56 : $P_{1295} = (14, 15, 3, 1)$
57 : $P_{1328} = (15, 1, 4, 1)$
58 : $P_{1335} = (6, 2, 4, 1)$
59 : $P_{1351} = (6, 3, 4, 1)$
60 : $P_{1363} = (2, 4, 4, 1)$
61 : $P_{1386} = (9, 5, 4, 1)$
62 : $P_{1412} = (3, 7, 4, 1)$
63 : $P_{1438} = (13, 8, 4, 1)$
64 : $P_{1442} = (1, 9, 4, 1)$
65 : $P_{1466} = (9, 10, 4, 1)$
66 : $P_{1476} = (3, 11, 4, 1)$
67 : $P_{1491} = (2, 12, 4, 1)$
68 : $P_{1506} = (1, 13, 4, 1)$
69 : $P_{1534} = (13, 14, 4, 1)$

$70: P_{1552} = (15, 15, 4, 1)$
$71: P_{1575} = (6, 1, 5, 1)$
$72: P_{1588} = (3, 2, 5, 1)$
$73: P_{1603} = (2, 3, 5, 1)$
$74: P_{1628} = (11, 4, 5, 1)$
$75: P_{1645} = (12, 5, 5, 1)$
$76: P_{1655} = (6, 6, 5, 1)$
$77: P_{1669} = (4, 7, 5, 1)$
$78: P_{1692} = (11, 8, 5, 1)$
$79: P_{1699} = (2, 9, 5, 1)$
$80: P_{1717} = (4, 10, 5, 1)$
$81: P_{1741} = (12, 11, 5, 1)$
$82: P_{1752} = (7, 12, 5, 1)$
$83: P_{1764} = (3, 13, 5, 1)$
$84: P_{1784} = (7, 14, 5, 1)$
$85: P_{1839} = (14, 1, 6, 1)$
$86: P_{1856} = (15, 2, 6, 1)$
$87: P_{1866} = (9, 3, 6, 1)$
$88: P_{1900} = (11, 5, 6, 1)$
$89: P_{1908} = (3, 6, 6, 1)$
$90: P_{1930} = (9, 7, 6, 1)$
$91: P_{1942} = (5, 8, 6, 1)$
$92: P_{1965} = (12, 9, 6, 1)$
$93: P_{1981} = (12, 10, 6, 1)$
$94: P_{2000} = (15, 11, 6, 1)$
$95: P_{2004} = (3, 12, 6, 1)$
$96: P_{2028} = (11, 13, 6, 1)$
$97: P_{2047} = (14, 14, 6, 1)$
$98: P_{2054} = (5, 15, 6, 1)$
$99: P_{2085} = (4, 1, 7, 1)$
$100: P_{2110} = (13, 2, 7, 1)$
$101: P_{2128} = (15, 3, 7, 1)$
$102: P_{2133} = (4, 4, 7, 1)$
$103: P_{2160} = (15, 5, 7, 1)$
$104: P_{2163} = (2, 6, 7, 1)$
$105: P_{2185} = (8, 7, 7, 1)$
$106: P_{2195} = (2, 8, 7, 1)$
$107: P_{2214} = (5, 9, 7, 1)$
$108: P_{2238} = (13, 10, 7, 1)$
$109: P_{2246} = (5, 11, 7, 1)$
$110: P_{2268} = (11, 12, 7, 1)$
$111: P_{2281} = (8, 13, 7, 1)$
$112: P_{2316} = (11, 15, 7, 1)$
$113: P_{2350} = (13, 1, 8, 1)$
$114: P_{2365} = (12, 2, 8, 1)$
$115: P_{2390} = (5, 4, 8, 1)$
$116: P_{2405} = (4, 5, 8, 1)$
$117: P_{2429} = (12, 6, 8, 1)$
$118: P_{2438} = (5, 7, 8, 1)$
$119: P_{2455} = (6, 8, 8, 1)$
$120: P_{2475} = (10, 9, 8, 1)$
$121: P_{2487} = (6, 10, 8, 1)$
$122: P_{2506} = (9, 11, 8, 1)$
$123: P_{2522} = (9, 12, 8, 1)$
$124: P_{2542} = (13, 13, 8, 1)$
$125: P_{2549} = (4, 14, 8, 1)$
$126: P_{2571} = (10, 15, 8, 1)$
$127: P_{2596} = (3, 1, 9, 1)$
$128: P_{2616} = (7, 2, 9, 1)$
$129: P_{2628} = (3, 3, 9, 1)$
$130: P_{2654} = (13, 4, 9, 1)$
$131: P_{2670} = (13, 5, 9, 1)$
$132: P_{2677} = (4, 6, 9, 1)$
$133: P_{2690} = (1, 7, 9, 1)$
$134: P_{2719} = (14, 8, 9, 1)$
$135: P_{2725} = (4, 9, 9, 1)$
$136: P_{2742} = (5, 10, 9, 1)$
$137: P_{2767} = (14, 11, 9, 1)$
$138: P_{2774} = (5, 12, 9, 1)$
$139: P_{2802} = (1, 14, 9, 1)$
$140: P_{2824} = (7, 15, 9, 1)$
$141: P_{2859} = (10, 1, 10, 1)$
$142: P_{2873} = (8, 2, 10, 1)$
$143: P_{2894} = (13, 3, 10, 1)$
$144: P_{2905} = (8, 4, 10, 1)$
$145: P_{2914} = (1, 5, 10, 1)$
$146: P_{2940} = (11, 6, 10, 1)$
$147: P_{2956} = (11, 7, 10, 1)$
$148: P_{2973} = (12, 8, 10, 1)$
$149: P_{2980} = (3, 9, 10, 1)$
$150: P_{3003} = (10, 10, 10, 1)$
$151: P_{3038} = (13, 12, 10, 1)$
$152: P_{3053} = (12, 13, 10, 1)$
$153: P_{3060} = (3, 14, 10, 1)$
$154: P_{3074} = (1, 15, 10, 1)$
$155: P_{3116} = (11, 1, 11, 1)$
$156: P_{3126} = (5, 2, 11, 1)$
$157: P_{3138} = (1, 3, 11, 1)$
$158: P_{3168} = (15, 4, 11, 1)$
$159: P_{3176} = (7, 5, 11, 1)$
$160: P_{3192} = (7, 6, 11, 1)$
$161: P_{3207} = (6, 7, 11, 1)$
$162: P_{3218} = (1, 8, 11, 1)$
$163: P_{3248} = (15, 9, 11, 1)$
$164: P_{3276} = (11, 11, 11, 1)$
$165: P_{3291} = (10, 12, 11, 1)$
$166: P_{3307} = (10, 13, 11, 1)$
$167: P_{3318} = (5, 14, 11, 1)$
$168: P_{3335} = (6, 15, 11, 1)$
$169: P_{3370} = (9, 1, 12, 1)$
$170: P_{3403} = (10, 3, 12, 1)$
$171: P_{3416} = (7, 4, 12, 1)$
$172: P_{3428} = (3, 5, 12, 1)$
$173: P_{3451} = (10, 6, 12, 1)$
$174: P_{3472} = (15, 7, 12, 1)$
$175: P_{3476} = (3, 8, 12, 1)$
$176: P_{3498} = (9, 9, 12, 1)$
$177: P_{3513} = (8, 10, 12, 1)$

178 : $P_{3528} = (7, 11, 12, 1)$
179 : $P_{3552} = (15, 12, 12, 1)$
180 : $P_{3557} = (4, 13, 12, 1)$
181 : $P_{3577} = (8, 14, 12, 1)$
182 : $P_{3589} = (4, 15, 12, 1)$
183 : $P_{3619} = (2, 1, 13, 1)$
184 : $P_{3635} = (2, 2, 13, 1)$
185 : $P_{3657} = (8, 3, 13, 1)$
186 : $P_{3668} = (3, 4, 13, 1)$
187 : $P_{3695} = (14, 5, 13, 1)$
188 : $P_{3702} = (5, 6, 13, 1)$
189 : $P_{3723} = (10, 7, 13, 1)$
190 : $P_{3739} = (10, 8, 13, 1)$
191 : $P_{3764} = (3, 10, 13, 1)$
192 : $P_{3783} = (6, 11, 13, 1)$
193 : $P_{3807} = (14, 12, 13, 1)$
194 : $P_{3814} = (5, 13, 13, 1)$
195 : $P_{3831} = (6, 14, 13, 1)$
196 : $P_{3849} = (8, 15, 13, 1)$
197 : $P_{3878} = (5, 1, 14, 1)$
198 : $P_{3890} = (1, 2, 14, 1)$
199 : $P_{3917} = (12, 3, 14, 1)$
200 : $P_{3933} = (12, 4, 14, 1)$
201 : $P_{3942} = (5, 5, 14, 1)$
202 : $P_{3961} = (8, 6, 14, 1)$
203 : $P_{3992} = (7, 8, 14, 1)$
204 : $P_{4008} = (7, 9, 14, 1)$
205 : $P_{4019} = (2, 10, 14, 1)$
206 : $P_{4041} = (8, 11, 14, 1)$
207 : $P_{4050} = (1, 12, 14, 1)$
208 : $P_{4074} = (9, 13, 14, 1)$
209 : $P_{4090} = (9, 14, 14, 1)$
210 : $P_{4099} = (2, 15, 14, 1)$
211 : $P_{4136} = (7, 1, 15, 1)$
212 : $P_{4154} = (9, 2, 15, 1)$
213 : $P_{4172} = (11, 3, 15, 1)$
214 : $P_{4183} = (6, 4, 15, 1)$
215 : $P_{4223} = (14, 6, 15, 1)$
216 : $P_{4232} = (7, 7, 15, 1)$
217 : $P_{4250} = (9, 8, 15, 1)$
218 : $P_{4265} = (8, 9, 15, 1)$
219 : $P_{4287} = (14, 10, 15, 1)$
220 : $P_{4302} = (13, 11, 15, 1)$
221 : $P_{4313} = (8, 12, 15, 1)$
222 : $P_{4327} = (6, 13, 15, 1)$
223 : $P_{4348} = (11, 14, 15, 1)$
224 : $P_{4366} = (13, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3
0	0 1 1 0
1	1 0 1 1
2	1 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_1	P_2	P_2

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_5 = (1, 1, 0, 0)$
5 : $P_6 = (2, 1, 0, 0)$
6 : $P_7 = (3, 1, 0, 0)$
7 : $P_8 = (4, 1, 0, 0)$
8 : $P_9 = (5, 1, 0, 0)$

$9: P_{10} = (6,1,0,0)$
$10: P_{11} = (7,1,0,0)$
$11: P_{12} = (8,1,0,0)$
$12: P_{13} = (9,1,0,0)$
$13: P_{14} = (10,1,0,0)$
$14: P_{15} = (11,1,0,0)$
$15: P_{16} = (12,1,0,0)$
$16: P_{17} = (13,1,0,0)$
$17: P_{18} = (14,1,0,0)$
$18: P_{19} = (15,1,0,0)$
$19: P_{20} = (1,0,1,0)$
$20: P_{21} = (2,0,1,0)$
$21: P_{22} = (3,0,1,0)$
$22: P_{23} = (4,0,1,0)$
$23: P_{24} = (5,0,1,0)$
$24: P_{25} = (6,0,1,0)$
$25: P_{26} = (7,0,1,0)$
$26: P_{27} = (8,0,1,0)$
$27: P_{28} = (9,0,1,0)$
$28: P_{29} = (10,0,1,0)$
$29: P_{30} = (11,0,1,0)$
$30: P_{31} = (12,0,1,0)$
$31: P_{32} = (13,0,1,0)$
$32: P_{33} = (14,0,1,0)$
$33: P_{34} = (15,0,1,0)$
$34: P_{35} = (0,1,1,0)$
$35: P_{51} = (0,2,1,0)$
$36: P_{67} = (0,3,1,0)$
$37: P_{83} = (0,4,1,0)$
$38: P_{99} = (0,5,1,0)$
$39: P_{115} = (0,6,1,0)$
$40: P_{131} = (0,7,1,0)$
$41: P_{147} = (0,8,1,0)$
$42: P_{163} = (0,9,1,0)$
$43: P_{179} = (0,10,1,0)$
$44: P_{195} = (0,11,1,0)$
$45: P_{211} = (0,12,1,0)$
$46: P_{227} = (0,13,1,0)$
$47: P_{243} = (0,14,1,0)$
$48: P_{259} = (0,15,1,0)$
$49: P_{291} = (1,1,0,1)$
$50: P_{310} = (4,2,0,1)$
$51: P_{327} = (5,3,0,1)$
$52: P_{347} = (9,4,0,1)$
$53: P_{362} = (8,5,0,1)$
$54: P_{383} = (13,6,0,1)$
$55: P_{398} = (12,7,0,1)$
$56: P_{417} = (15,8,0,1)$
$57: P_{432} = (14,9,0,1)$
$58: P_{445} = (11,10,0,1)$
$59: P_{460} = (10,11,0,1)$
$60: P_{472} = (6,12,0,1)$
$61: P_{489} = (7,13,0,1)$
$62: P_{500} = (2,14,0,1)$
$63: P_{517} = (3,15,0,1)$
$64: P_{530} = (0,0,1,1)$
$65: P_{572} = (11,2,1,1)$
$66: P_{591} = (14,3,1,1)$
$67: P_{603} = (10,4,1,1)$
$68: P_{611} = (2,5,1,1)$
$69: P_{634} = (9,6,1,1)$
$70: P_{643} = (2,7,1,1)$
$71: P_{661} = (4,8,1,1)$
$72: P_{684} = (11,9,1,1)$
$73: P_{690} = (1,10,1,1)$
$74: P_{706} = (1,11,1,1)$
$75: P_{725} = (4,12,1,1)$
$76: P_{751} = (14,13,1,1)$
$77: P_{763} = (10,14,1,1)$
$78: P_{778} = (9,15,1,1)$
$79: P_{785} = (0,0,2,1)$
$80: P_{809} = (8,1,2,1)$
$81: P_{831} = (14,2,2,1)$
$82: P_{837} = (4,3,2,1)$
$83: P_{850} = (1,4,2,1)$
$84: P_{871} = (6,5,2,1)$
$85: P_{882} = (1,6,2,1)$
$86: P_{911} = (14,7,2,1)$
$87: P_{921} = (8,8,2,1)$
$88: P_{935} = (6,9,2,1)$
$89: P_{960} = (15,10,2,1)$
$90: P_{965} = (4,11,2,1)$
$91: P_{1008} = (15,13,2,1)$
$92: P_{1021} = (12,14,2,1)$
$93: P_{1037} = (12,15,2,1)$
$94: P_{1041} = (0,0,3,1)$
$95: P_{1069} = (12,1,3,1)$
$96: P_{1083} = (10,2,3,1)$
$97: P_{1096} = (7,3,3,1)$
$98: P_{1119} = (14,4,3,1)$
$99: P_{1131} = (10,5,3,1)$
$100: P_{1152} = (15,6,3,1)$
$101: P_{1166} = (13,7,3,1)$
$102: P_{1198} = (13,9,3,1)$
$103: P_{1208} = (7,10,3,1)$
$104: P_{1219} = (2,11,3,1)$
$105: P_{1245} = (12,12,3,1)$
$106: P_{1251} = (2,13,3,1)$
$107: P_{1280} = (15,14,3,1)$
$108: P_{1295} = (14,15,3,1)$
$109: P_{1297} = (0,0,4,1)$
$110: P_{1328} = (15,1,4,1)$
$111: P_{1335} = (6,2,4,1)$
$112: P_{1351} = (6,3,4,1)$
$113: P_{1363} = (2,4,4,1)$
$114: P_{1386} = (9,5,4,1)$
$115: P_{1412} = (3,7,4,1)$
$116: P_{1438} = (13,8,4,1)$
$117: P_{1442} = (1,9,4,1)$
$118: P_{1466} = (9,10,4,1)$
$119: P_{1476} = (3,11,4,1)$
$120: P_{1491} = (2,12,4,1)$
$121: P_{1506} = (1,13,4,1)$
$122: P_{1534} = (13,14,4,1)$
$123: P_{1552} = (15,15,4,1)$
$124: P_{1553} = (0,0,5,1)$
$125: P_{1575} = (6,1,5,1)$
$126: P_{1588} = (3,2,5,1)$
$127: P_{1603} = (2,3,5,1)$
$128: P_{1628} = (11,4,5,1)$
$129: P_{1645} = (12,5,5,1)$
$130: P_{1655} = (6,6,5,1)$
$131: P_{1669} = (4,7,5,1)$
$132: P_{1692} = (11,8,5,1)$
$133: P_{1699} = (2,9,5,1)$
$134: P_{1717} = (4,10,5,1)$
$135: P_{1741} = (12,11,5,1)$
$136: P_{1752} = (7,12,5,1)$
$137: P_{1764} = (3,13,5,1)$
$138: P_{1784} = (7,14,5,1)$
$139: P_{1809} = (0,0,6,1)$
$140: P_{1839} = (14,1,6,1)$
$141: P_{1856} = (15,2,6,1)$
$142: P_{1866} = (9,3,6,1)$
$143: P_{1900} = (11,5,6,1)$
$144: P_{1908} = (3,6,6,1)$
$145: P_{1930} = (9,7,6,1)$
$146: P_{1942} = (5,8,6,1)$
$147: P_{1965} = (12,9,6,1)$
$148: P_{1981} = (12,10,6,1)$
$149: P_{2000} = (15,11,6,1)$
$150: P_{2004} = (3,12,6,1)$
$151: P_{2028} = (11,13,6,1)$
$152: P_{2047} = (14,14,6,1)$
$153: P_{2054} = (5,15,6,1)$
$154: P_{2065} = (0,0,7,1)$
$155: P_{2085} = (4,1,7,1)$
$156: P_{2110} = (13,2,7,1)$
$157: P_{2128} = (15,3,7,1)$
$158: P_{2133} = (4,4,7,1)$
$159: P_{2160} = (15,5,7,1)$
$160: P_{2163} = (2,6,7,1)$
$161: P_{2185} = (8,7,7,1)$
$162: P_{2195} = (2,8,7,1)$
$163: P_{2214} = (5,9,7,1)$
$164: P_{2238} = (13,10,7,1)$
$165: P_{2246} = (5,11,7,1)$
$166: P_{2268} = (11,12,7,1)$
$167: P_{2281} = (8,13,7,1)$
$168: P_{2316} = (11,15,7,1)$
$169: P_{2321} = (0,0,8,1)$
$170: P_{2350} = (13,1,8,1)$

171 : $P_{2365} = (12, 2, 8, 1)$
172 : $P_{2390} = (5, 4, 8, 1)$
173 : $P_{2405} = (4, 5, 8, 1)$
174 : $P_{2429} = (12, 6, 8, 1)$
175 : $P_{2438} = (5, 7, 8, 1)$
176 : $P_{2455} = (6, 8, 8, 1)$
177 : $P_{2475} = (10, 9, 8, 1)$
178 : $P_{2487} = (6, 10, 8, 1)$
179 : $P_{2506} = (9, 11, 8, 1)$
180 : $P_{2522} = (9, 12, 8, 1)$
181 : $P_{2542} = (13, 13, 8, 1)$
182 : $P_{2549} = (4, 14, 8, 1)$
183 : $P_{2571} = (10, 15, 8, 1)$
184 : $P_{2577} = (0, 0, 9, 1)$
185 : $P_{2596} = (3, 1, 9, 1)$
186 : $P_{2616} = (7, 2, 9, 1)$
187 : $P_{2628} = (3, 3, 9, 1)$
188 : $P_{2654} = (13, 4, 9, 1)$
189 : $P_{2670} = (13, 5, 9, 1)$
190 : $P_{2677} = (4, 6, 9, 1)$
191 : $P_{2690} = (1, 7, 9, 1)$
192 : $P_{2719} = (14, 8, 9, 1)$
193 : $P_{2725} = (4, 9, 9, 1)$
194 : $P_{2742} = (5, 10, 9, 1)$
195 : $P_{2767} = (14, 11, 9, 1)$
196 : $P_{2774} = (5, 12, 9, 1)$
197 : $P_{2802} = (1, 14, 9, 1)$
198 : $P_{2824} = (7, 15, 9, 1)$
199 : $P_{2833} = (0, 0, 10, 1)$
200 : $P_{2859} = (10, 1, 10, 1)$
201 : $P_{2873} = (8, 2, 10, 1)$
202 : $P_{2894} = (13, 3, 10, 1)$
203 : $P_{2905} = (8, 4, 10, 1)$
204 : $P_{2914} = (1, 5, 10, 1)$
205 : $P_{2940} = (11, 6, 10, 1)$
206 : $P_{2956} = (11, 7, 10, 1)$
207 : $P_{2973} = (12, 8, 10, 1)$
208 : $P_{2980} = (3, 9, 10, 1)$
209 : $P_{3003} = (10, 10, 10, 1)$
210 : $P_{3038} = (13, 12, 10, 1)$
211 : $P_{3053} = (12, 13, 10, 1)$
212 : $P_{3060} = (3, 14, 10, 1)$
213 : $P_{3074} = (1, 15, 10, 1)$
214 : $P_{3089} = (0, 0, 11, 1)$
215 : $P_{3116} = (11, 1, 11, 1)$
216 : $P_{3126} = (5, 2, 11, 1)$
217 : $P_{3138} = (1, 3, 11, 1)$
218 : $P_{3168} = (15, 4, 11, 1)$
219 : $P_{3176} = (7, 5, 11, 1)$
220 : $P_{3192} = (7, 6, 11, 1)$
221 : $P_{3207} = (6, 7, 11, 1)$
222 : $P_{3218} = (1, 8, 11, 1)$
223 : $P_{3248} = (15, 9, 11, 1)$
224 : $P_{3276} = (11, 11, 11, 1)$
225 : $P_{3291} = (10, 12, 11, 1)$
226 : $P_{3307} = (10, 13, 11, 1)$
227 : $P_{3318} = (5, 14, 11, 1)$
228 : $P_{3335} = (6, 15, 11, 1)$
229 : $P_{3345} = (0, 0, 12, 1)$
230 : $P_{3370} = (9, 1, 12, 1)$
231 : $P_{3403} = (10, 3, 12, 1)$
232 : $P_{3416} = (7, 4, 12, 1)$
233 : $P_{3428} = (3, 5, 12, 1)$
234 : $P_{3451} = (10, 6, 12, 1)$
235 : $P_{3472} = (15, 7, 12, 1)$
236 : $P_{3476} = (3, 8, 12, 1)$
237 : $P_{3498} = (9, 9, 12, 1)$
238 : $P_{3513} = (8, 10, 12, 1)$
239 : $P_{3528} = (7, 11, 12, 1)$
240 : $P_{3552} = (15, 12, 12, 1)$
241 : $P_{3557} = (4, 13, 12, 1)$
242 : $P_{3577} = (8, 14, 12, 1)$
243 : $P_{3589} = (4, 15, 12, 1)$
244 : $P_{3601} = (0, 0, 13, 1)$
245 : $P_{3619} = (2, 1, 13, 1)$
246 : $P_{3635} = (2, 2, 13, 1)$
247 : $P_{3657} = (8, 3, 13, 1)$
248 : $P_{3668} = (3, 4, 13, 1)$
249 : $P_{3695} = (14, 5, 13, 1)$
250 : $P_{3702} = (5, 6, 13, 1)$
251 : $P_{3723} = (10, 7, 13, 1)$
252 : $P_{3739} = (10, 8, 13, 1)$
253 : $P_{3764} = (3, 10, 13, 1)$
254 : $P_{3783} = (6, 11, 13, 1)$
255 : $P_{3807} = (14, 12, 13, 1)$
256 : $P_{3814} = (5, 13, 13, 1)$
257 : $P_{3831} = (6, 14, 13, 1)$
258 : $P_{3849} = (8, 15, 13, 1)$
259 : $P_{3857} = (0, 0, 14, 1)$
260 : $P_{3878} = (5, 1, 14, 1)$
261 : $P_{3890} = (1, 2, 14, 1)$
262 : $P_{3917} = (12, 3, 14, 1)$
263 : $P_{3933} = (12, 4, 14, 1)$
264 : $P_{3942} = (5, 5, 14, 1)$
265 : $P_{3961} = (8, 6, 14, 1)$
266 : $P_{3992} = (7, 8, 14, 1)$
267 : $P_{4008} = (7, 9, 14, 1)$
268 : $P_{4019} = (2, 10, 14, 1)$
269 : $P_{4041} = (8, 11, 14, 1)$
270 : $P_{4050} = (1, 12, 14, 1)$
271 : $P_{4074} = (9, 13, 14, 1)$
272 : $P_{4090} = (9, 14, 14, 1)$
273 : $P_{4099} = (2, 15, 14, 1)$
274 : $P_{4113} = (0, 0, 15, 1)$
275 : $P_{4136} = (7, 1, 15, 1)$
276 : $P_{4154} = (9, 2, 15, 1)$
277 : $P_{4172} = (11, 3, 15, 1)$
278 : $P_{4183} = (6, 4, 15, 1)$
279 : $P_{4223} = (14, 6, 15, 1)$
280 : $P_{4232} = (7, 7, 15, 1)$
281 : $P_{4250} = (9, 8, 15, 1)$
282 : $P_{4265} = (8, 9, 15, 1)$
283 : $P_{4287} = (14, 10, 15, 1)$
284 : $P_{4302} = (13, 11, 15, 1)$
285 : $P_{4313} = (8, 12, 15, 1)$
286 : $P_{4327} = (6, 13, 15, 1)$
287 : $P_{4348} = (11, 14, 15, 1)$
288 : $P_{4366} = (13, 15, 15, 1)$