

Rank-67107 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 67107

General information

Number of lines	6
Number of points	11
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	3
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^6
Type of lines on points	$3, 2^6, 1^3, 0$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_8 = \mathbf{Pl}(1, 0, 1, 0, 0, 1)_{23} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_9 \\
\ell_4 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{11}
\end{aligned}$$

Rank of lines: (0, 8, 28, 6, 34, 20)

Rank of points on Klein quadric: (0, 23, 19, 9, 1, 11)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1). T = -1$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$\begin{aligned}
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_1 \\
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_2 \\
P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_3 \\
P_7 &= (0, 1, 1, 0) = \ell_1 \cap \ell_2
\end{aligned}$$

$$\begin{aligned}
P_6 &= (1, 0, 1, 0) = \ell_1 \cap \ell_5 \\
P_2 &= (0, 0, 1, 0) = \ell_2 \cap \ell_4
\end{aligned}$$

Single Points

The surface has 3 single points:

The single points on the surface are:

$$\begin{aligned}
0 : P_9 &= (1, 0, 0, 1) \text{ lies on line } \ell_3 \\
1 : P_{12} &= (0, 0, 1, 1) \text{ lies on line } \ell_4
\end{aligned}$$

$$2 : P_{13} = (1, 0, 1, 1) \text{ lies on line } \ell_5$$

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	0	1
2	1	1	0	0	1	0
3	1	0	0	0	1	1
4	0	0	1	1	0	1
5	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_1	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_5	P_7	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_1	P_7	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_0	P_3	P_3

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_3	P_3

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_6	P_3	P_3

The surface has 11 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_3 = (0, 0, 0, 1)$$

$$4 : P_4 = (1, 1, 1, 1)$$

$$5 : P_5 = (1, 1, 0, 0)$$

$$6 : P_6 = (1, 0, 1, 0)$$

$$7 : P_7 = (0, 1, 1, 0)$$

$$8 : P_8 = (1, 0, 0, 1)$$

$$9 : P_{12} = (0, 0, 1, 1)$$

$$10 : P_{13} = (1, 0, 1, 1)$$