Rank-65611 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_3 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303112469

General information

Number of lines	5
Number of points	289
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	75
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^5
Type of lines on points	3^2, 2^2, 1^{75}, 0^{210}

Singular Points

The surface has 3 singular points:

$0:\ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$2:\ P_{275} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \mathbf{Pl}(0,0,0,0,0,1)_{4625}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1,0,0,1,0,0)_{34}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0,1,1,0,0,0)_{18}$$

Rank of lines: (0, 256, 69888, 4368, 4624)
Rank of points on Klein quadric: (0, 2, 4625, 34, 18)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{275} = (1,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 75 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$3 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$4 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$5 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$6 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$7 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$8 : P_{13} = (9,1,0,0)$ lies on line ℓ_0
$9 : P_{14} = (10,1,0,0)$ lies on line ℓ_0
$10 : P_{15} = (11,1,0,0)$ lies on line ℓ_0
$11 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$12 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$13 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$14 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$15 : P_{20} = (1,0,1,0)$ lies on line ℓ_1
$16 : P_{21} = (2,0,1,0)$ lies on line ℓ_1
$17 : P_{22} = (3,0,1,0)$ lies on line ℓ_1
$18 : P_{23} = (4,0,1,0)$ lies on line ℓ_1
$19 : P_{24} = (5,0,1,0)$ lies on line ℓ_1
$20 : P_{25} = (6,0,1,0)$ lies on line ℓ_1
$21 : P_{26} = (7,0,1,0)$ lies on line ℓ_1
$22 : P_{27} = (8,0,1,0)$ lies on line ℓ_1
$23 : P_{28} = (9,0,1,0)$ lies on line ℓ_1
$24 : P_{29} = (10,0,1,0)$ lies on line ℓ_1
$25 : P_{30} = (11,0,1,0)$ lies on line ℓ_1
$26 : P_{31} = (12,0,1,0)$ lies on line ℓ_1
$27 : P_{32} = (13,0,1,0)$ lies on line ℓ_1
$28 : P_{33} = (14,0,1,0)$ lies on line ℓ_1
$29 : P_{34} = (15,0,1,0)$ lies on line ℓ_1
$30 : P_{35} = (0,1,1,0)$ lies on line ℓ_2
$31 : P_{51} = (0,2,1,0)$ lies on line ℓ_2
$32 : P_{67} = (0,3,1,0)$ lies on line ℓ_2
$33 : P_{83} = (0,4,1,0)$ lies on line ℓ_2
$34 : P_{99} = (0,5,1,0)$ lies on line ℓ_2
$35 : P_{115} = (0,6,1,0)$ lies on line ℓ_2

36 : $P_{131} = (0, 7, 1, 0)$ lies on line ℓ_2
37 : $P_{147} = (0, 8, 1, 0)$ lies on line ℓ_2
38 : $P_{163} = (0, 9, 1, 0)$ lies on line ℓ_2
39 : $P_{179} = (0, 10, 1, 0)$ lies on line ℓ_2
40 : $P_{195} = (0, 11, 1, 0)$ lies on line ℓ_2
41 : $P_{211} = (0, 12, 1, 0)$ lies on line ℓ_2
42 : $P_{227} = (0, 13, 1, 0)$ lies on line ℓ_2
43 : $P_{243} = (0, 14, 1, 0)$ lies on line ℓ_2
44 : $P_{259} = (0, 15, 1, 0)$ lies on line ℓ_2
45 : $P_{291} = (1, 1, 0, 1)$ lies on line ℓ_3
46 : $P_{307} = (1, 2, 0, 1)$ lies on line ℓ_3
47 : $P_{323} = (1, 3, 0, 1)$ lies on line ℓ_3
48 : $P_{339} = (1, 4, 0, 1)$ lies on line ℓ_3
49 : $P_{355} = (1, 5, 0, 1)$ lies on line ℓ_3
50 : $P_{371} = (1, 6, 0, 1)$ lies on line ℓ_3
51 : $P_{387} = (1, 7, 0, 1)$ lies on line ℓ_3
52 : $P_{403} = (1, 8, 0, 1)$ lies on line ℓ_3
53 : $P_{419} = (1, 9, 0, 1)$ lies on line ℓ_3
54 : $P_{435} = (1, 10, 0, 1)$ lies on line ℓ_3
55 : $P_{451} = (1, 11, 0, 1)$ lies on line ℓ_3
56 : $P_{467} = (1, 12, 0, 1)$ lies on line ℓ_3
57 : $P_{483} = (1, 13, 0, 1)$ lies on line ℓ_3
58 : $P_{499} = (1, 14, 0, 1)$ lies on line ℓ_3
59 : $P_{515} = (1, 15, 0, 1)$ lies on line ℓ_3
60 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_4
61 : $P_{786} = (1, 0, 2, 1)$ lies on line ℓ_4
62 : $P_{1042} = (1, 0, 3, 1)$ lies on line ℓ_4
63 : $P_{1298} = (1, 0, 4, 1)$ lies on line ℓ_4
64 : $P_{1554} = (1, 0, 5, 1)$ lies on line ℓ_4
65 : $P_{1810} = (1, 0, 6, 1)$ lies on line ℓ_4
66 : $P_{2066} = (1, 0, 7, 1)$ lies on line ℓ_4
67 : $P_{2322} = (1, 0, 8, 1)$ lies on line ℓ_4
68 : $P_{2578} = (1, 0, 9, 1)$ lies on line ℓ_4
69 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_4
70 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_4
71 : $P_{3346} = (1, 0, 12, 1)$ lies on line ℓ_4
72 : $P_{3602} = (1, 0, 13, 1)$ lies on line ℓ_4
73 : $P_{3858} = (1, 0, 14, 1)$ lies on line ℓ_4
74 : $P_{4114} = (1, 0, 15, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_{555} = (10, 1, 1, 1)$
1 : $P_{556} = (11, 1, 1, 1)$
2 : $P_{565} = (4, 2, 1, 1)$
3 : $P_{567} = (6, 2, 1, 1)$
4 : $P_{602} = (9, 4, 1, 1)$
5 : $P_{606} = (13, 4, 1, 1)$
6 : $P_{680} = (7, 9, 1, 1)$
7 : $P_{687} = (14, 9, 1, 1)$
8 : $P_{694} = (5, 10, 1, 1)$
9 : $P_{704} = (15, 10, 1, 1)$
10 : $P_{708} = (3, 11, 1, 1)$
11 : $P_{713} = (8, 11, 1, 1)$
12 : $P_{755} = (2, 14, 1, 1)$
13 : $P_{765} = (12, 14, 1, 1)$
14 : $P_{805} = (4, 1, 2, 1)$
15 : $P_{807} = (6, 1, 2, 1)$
16 : $P_{826} = (9, 2, 2, 1)$
17 : $P_{830} = (13, 2, 2, 1)$
18 : $P_{870} = (5, 5, 2, 1)$
19 : $P_{880} = (15, 5, 2, 1)$
20 : $P_{899} = (2, 7, 2, 1)$
21 : $P_{909} = (12, 7, 2, 1)$
22 : $P_{920} = (7, 8, 2, 1)$
23 : $P_{927} = (14, 8, 2, 1)$
24 : $P_{932} = (3, 9, 2, 1)$
25 : $P_{937} = (8, 9, 2, 1)$
26 : $P_{987} = (10, 12, 2, 1)$
27 : $P_{988} = (11, 12, 2, 1)$
28 : $P_{1142} = (5, 6, 3, 1)$
29 : $P_{1152} = (15, 6, 3, 1)$
30 : $P_{1160} = (7, 7, 3, 1)$
31 : $P_{1167} = (14, 7, 3, 1)$
32 : $P_{1179} = (10, 8, 3, 1)$
33 : $P_{1180} = (11, 8, 3, 1)$
34 : $P_{1189} = (4, 9, 3, 1)$
35 : $P_{1191} = (6, 9, 3, 1)$
36 : $P_{1226} = (9, 11, 3, 1)$
37 : $P_{1230} = (13, 11, 3, 1)$
38 : $P_{1251} = (2, 13, 3, 1)$
39 : $P_{1261} = (12, 13, 3, 1)$
40 : $P_{1268} = (3, 14, 3, 1)$
41 : $P_{1273} = (8, 14, 3, 1)$
42 : $P_{1322} = (9, 1, 4, 1)$
43 : $P_{1326} = (13, 1, 4, 1)$
44 : $P_{1368} = (7, 4, 4, 1)$
45 : $P_{1375} = (14, 4, 4, 1)$
46 : $P_{1403} = (10, 6, 4, 1)$
47 : $P_{1404} = (11, 6, 4, 1)$

$48: P_{1428} = (3, 8, 4, 1)$
$49: P_{1433} = (8, 8, 4, 1)$
$50: P_{1493} = (4, 12, 4, 1)$
$51: P_{1495} = (6, 12, 4, 1)$
$52: P_{1526} = (5, 14, 4, 1)$
$53: P_{1536} = (15, 14, 4, 1)$
$54: P_{1539} = (2, 15, 4, 1)$
$55: P_{1549} = (12, 15, 4, 1)$
$56: P_{1590} = (5, 2, 5, 1)$
$57: P_{1600} = (15, 2, 5, 1)$
$58: P_{1669} = (4, 7, 5, 1)$
$59: P_{1671} = (6, 7, 5, 1)$
$60: P_{1720} = (7, 10, 5, 1)$
$61: P_{1727} = (14, 10, 5, 1)$
$62: P_{1747} = (2, 12, 5, 1)$
$63: P_{1757} = (12, 12, 5, 1)$
$64: P_{1764} = (3, 13, 5, 1)$
$65: P_{1769} = (8, 13, 5, 1)$
$66: P_{1786} = (9, 14, 5, 1)$
$67: P_{1790} = (13, 14, 5, 1)$
$68: P_{1803} = (10, 15, 5, 1)$
$69: P_{1804} = (11, 15, 5, 1)$
$70: P_{1862} = (5, 3, 6, 1)$
$71: P_{1872} = (15, 3, 6, 1)$
$72: P_{1883} = (10, 4, 6, 1)$
$73: P_{1884} = (11, 4, 6, 1)$
$74: P_{1924} = (3, 7, 6, 1)$
$75: P_{1929} = (8, 7, 6, 1)$
$76: P_{1941} = (4, 8, 6, 1)$
$77: P_{1943} = (6, 8, 6, 1)$
$78: P_{1962} = (9, 9, 6, 1)$
$79: P_{1966} = (13, 9, 6, 1)$
$80: P_{1971} = (2, 10, 6, 1)$
$81: P_{1981} = (12, 10, 6, 1)$
$82: P_{2056} = (7, 15, 6, 1)$
$83: P_{2063} = (14, 15, 6, 1)$
$84: P_{2099} = (2, 2, 7, 1)$
$85: P_{2109} = (12, 2, 7, 1)$
$86: P_{2120} = (7, 3, 7, 1)$
$87: P_{2127} = (14, 3, 7, 1)$
$88: P_{2149} = (4, 5, 7, 1)$
$89: P_{2151} = (6, 5, 7, 1)$
$90: P_{2164} = (3, 6, 7, 1)$
$91: P_{2169} = (8, 6, 7, 1)$
$92: P_{2198} = (5, 8, 7, 1)$
$93: P_{2208} = (15, 8, 7, 1)$
$94: P_{2234} = (9, 10, 7, 1)$
$95: P_{2238} = (13, 10, 7, 1)$
$96: P_{2299} = (10, 14, 7, 1)$
$97: P_{2300} = (11, 14, 7, 1)$
$98: P_{2360} = (7, 2, 8, 1)$
$99: P_{2367} = (14, 2, 8, 1)$
$100: P_{2379} = (10, 3, 8, 1)$
$101: P_{2380} = (11, 3, 8, 1)$
$102: P_{2388} = (3, 4, 8, 1)$
$103: P_{2393} = (8, 4, 8, 1)$
$104: P_{2421} = (4, 6, 8, 1)$
$105: P_{2423} = (6, 6, 8, 1)$
$106: P_{2438} = (5, 7, 8, 1)$
$107: P_{2448} = (15, 7, 8, 1)$
$108: P_{2499} = (2, 11, 8, 1)$
$109: P_{2509} = (12, 11, 8, 1)$
$110: P_{2522} = (9, 12, 8, 1)$
$111: P_{2526} = (13, 12, 8, 1)$
$112: P_{2600} = (7, 1, 9, 1)$
$113: P_{2607} = (14, 1, 9, 1)$
$114: P_{2612} = (3, 2, 9, 1)$
$115: P_{2617} = (8, 2, 9, 1)$
$116: P_{2629} = (4, 3, 9, 1)$
$117: P_{2631} = (6, 3, 9, 1)$
$118: P_{2682} = (9, 6, 9, 1)$
$119: P_{2686} = (13, 6, 9, 1)$
$120: P_{2723} = (2, 9, 9, 1)$
$121: P_{2733} = (12, 9, 9, 1)$
$122: P_{2795} = (10, 13, 9, 1)$
$123: P_{2796} = (11, 13, 9, 1)$
$124: P_{2822} = (5, 15, 9, 1)$
$125: P_{2832} = (15, 15, 9, 1)$
$126: P_{2854} = (5, 1, 10, 1)$
$127: P_{2864} = (15, 1, 10, 1)$
$128: P_{2920} = (7, 5, 10, 1)$
$129: P_{2927} = (14, 5, 10, 1)$
$130: P_{2931} = (2, 6, 10, 1)$
$131: P_{2941} = (12, 6, 10, 1)$
$132: P_{2954} = (9, 7, 10, 1)$
$133: P_{2958} = (13, 7, 10, 1)$
$134: P_{2996} = (3, 10, 10, 1)$
$135: P_{3001} = (8, 10, 10, 1)$
$136: P_{3019} = (10, 11, 10, 1)$
$137: P_{3020} = (11, 11, 10, 1)$
$138: P_{3077} = (4, 15, 10, 1)$
$139: P_{3079} = (6, 15, 10, 1)$
$140: P_{3108} = (3, 1, 11, 1)$
$141: P_{3113} = (8, 1, 11, 1)$
$142: P_{3146} = (9, 3, 11, 1)$
$143: P_{3150} = (13, 3, 11, 1)$
$144: P_{3219} = (2, 8, 11, 1)$
$145: P_{3229} = (12, 8, 11, 1)$
$146: P_{3259} = (10, 10, 11, 1)$
$147: P_{3260} = (11, 10, 11, 1)$
$148: P_{3270} = (5, 11, 11, 1)$
$149: P_{3280} = (15, 11, 11, 1)$
$150: P_{3288} = (7, 12, 11, 1)$
$151: P_{3295} = (14, 12, 11, 1)$
$152: P_{3301} = (4, 13, 11, 1)$
$153: P_{3303} = (6, 13, 11, 1)$
$154: P_{3387} = (10, 2, 12, 1)$
$155: P_{3388} = (11, 2, 12, 1)$

156 : $P_{3413} = (4, 4, 12, 1)$
157 : $P_{3415} = (6, 4, 12, 1)$
158 : $P_{3427} = (2, 5, 12, 1)$
159 : $P_{3437} = (12, 5, 12, 1)$
160 : $P_{3482} = (9, 8, 12, 1)$
161 : $P_{3486} = (13, 8, 12, 1)$
162 : $P_{3528} = (7, 11, 12, 1)$
163 : $P_{3535} = (14, 11, 12, 1)$
164 : $P_{3558} = (5, 13, 12, 1)$
165 : $P_{3568} = (15, 13, 12, 1)$
166 : $P_{3588} = (3, 15, 12, 1)$
167 : $P_{3593} = (8, 15, 12, 1)$
168 : $P_{3651} = (2, 3, 13, 1)$
169 : $P_{3661} = (12, 3, 13, 1)$
170 : $P_{3684} = (3, 5, 13, 1)$
171 : $P_{3689} = (8, 5, 13, 1)$
172 : $P_{3755} = (10, 9, 13, 1)$
173 : $P_{3756} = (11, 9, 13, 1)$
174 : $P_{3781} = (4, 11, 13, 1)$
175 : $P_{3783} = (6, 11, 13, 1)$
176 : $P_{3798} = (5, 12, 13, 1)$
177 : $P_{3808} = (15, 12, 13, 1)$
178 : $P_{3832} = (7, 14, 13, 1)$
179 : $P_{3839} = (14, 14, 13, 1)$
180 : $P_{3850} = (9, 15, 13, 1)$
181 : $P_{3854} = (13, 15, 13, 1)$
182 : $P_{3875} = (2, 1, 14, 1)$
183 : $P_{3885} = (12, 1, 14, 1)$
184 : $P_{3908} = (3, 3, 14, 1)$
185 : $P_{3913} = (8, 3, 14, 1)$
186 : $P_{3926} = (5, 4, 14, 1)$
187 : $P_{3936} = (15, 4, 14, 1)$
188 : $P_{3946} = (9, 5, 14, 1)$
189 : $P_{3950} = (13, 5, 14, 1)$
190 : $P_{3979} = (10, 7, 14, 1)$
191 : $P_{3980} = (11, 7, 14, 1)$
192 : $P_{4072} = (7, 13, 14, 1)$
193 : $P_{4079} = (14, 13, 14, 1)$
194 : $P_{4085} = (4, 14, 14, 1)$
195 : $P_{4087} = (6, 14, 14, 1)$
196 : $P_{4179} = (2, 4, 15, 1)$
197 : $P_{4189} = (12, 4, 15, 1)$
198 : $P_{4203} = (10, 5, 15, 1)$
199 : $P_{4204} = (11, 5, 15, 1)$
200 : $P_{4216} = (7, 6, 15, 1)$
201 : $P_{4223} = (14, 6, 15, 1)$
202 : $P_{4262} = (5, 9, 15, 1)$
203 : $P_{4272} = (15, 9, 15, 1)$
204 : $P_{4277} = (4, 10, 15, 1)$
205 : $P_{4279} = (6, 10, 15, 1)$
206 : $P_{4308} = (3, 12, 15, 1)$
207 : $P_{4313} = (8, 12, 15, 1)$
208 : $P_{4330} = (9, 13, 15, 1)$
209 : $P_{4334} = (13, 13, 15, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	1	1
3	1	0	1	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_1	P_2	P_1	P_2

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_1	P_1	P_{275}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_{275}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_5 = (1,1,0,0)$
4 : $P_6 = (2,1,0,0)$
5 : $P_7 = (3,1,0,0)$
6 : $P_8 = (4,1,0,0)$
7 : $P_9 = (5,1,0,0)$
8 : $P_{10} = (6,1,0,0)$
9 : $P_{11} = (7,1,0,0)$
10 : $P_{12} = (8,1,0,0)$
11 : $P_{13} = (9,1,0,0)$
12 : $P_{14} = (10,1,0,0)$
13 : $P_{15} = (11,1,0,0)$
14 : $P_{16} = (12,1,0,0)$
15 : $P_{17} = (13,1,0,0)$
16 : $P_{18} = (14,1,0,0)$
17 : $P_{19} = (15,1,0,0)$
18 : $P_{20} = (1,0,1,0)$
19 : $P_{21} = (2,0,1,0)$
20 : $P_{22} = (3,0,1,0)$
21 : $P_{23} = (4,0,1,0)$
22 : $P_{24} = (5,0,1,0)$
23 : $P_{25} = (6,0,1,0)$
24 : $P_{26} = (7,0,1,0)$
25 : $P_{27} = (8,0,1,0)$
26 : $P_{28} = (9,0,1,0)$
27 : $P_{29} = (10,0,1,0)$
28 : $P_{30} = (11,0,1,0)$
29 : $P_{31} = (12,0,1,0)$
30 : $P_{32} = (13,0,1,0)$
31 : $P_{33} = (14,0,1,0)$
32 : $P_{34} = (15,0,1,0)$
33 : $P_{35} = (0,1,1,0)$
34 : $P_{51} = (0,2,1,0)$
35 : $P_{67} = (0,3,1,0)$
36 : $P_{83} = (0,4,1,0)$
37 : $P_{99} = (0,5,1,0)$
38 : $P_{115} = (0,6,1,0)$
39 : $P_{131} = (0,7,1,0)$
40 : $P_{147} = (0,8,1,0)$
41 : $P_{163} = (0,9,1,0)$
42 : $P_{179} = (0,10,1,0)$
43 : $P_{195} = (0,11,1,0)$
44 : $P_{211} = (0,12,1,0)$
45 : $P_{227} = (0,13,1,0)$
46 : $P_{243} = (0,14,1,0)$
47 : $P_{259} = (0,15,1,0)$
48 : $P_{275} = (1,0,0,1)$
49 : $P_{291} = (1,1,0,1)$
50 : $P_{307} = (1,2,0,1)$
51 : $P_{323} = (1,3,0,1)$
52 : $P_{339} = (1,4,0,1)$
53 : $P_{355} = (1,5,0,1)$
54 : $P_{371} = (1,6,0,1)$
55 : $P_{387} = (1,7,0,1)$
56 : $P_{403} = (1,8,0,1)$
57 : $P_{419} = (1,9,0,1)$
58 : $P_{435} = (1,10,0,1)$
59 : $P_{451} = (1,11,0,1)$
60 : $P_{467} = (1,12,0,1)$
61 : $P_{483} = (1,13,0,1)$
62 : $P_{499} = (1,14,0,1)$
63 : $P_{515} = (1,15,0,1)$
64 : $P_{531} = (1,0,1,1)$
65 : $P_{555} = (10,1,1,1)$
66 : $P_{556} = (11,1,1,1)$
67 : $P_{565} = (4,2,1,1)$
68 : $P_{567} = (6,2,1,1)$
69 : $P_{602} = (9,4,1,1)$
70 : $P_{606} = (13,4,1,1)$
71 : $P_{680} = (7,9,1,1)$
72 : $P_{687} = (14,9,1,1)$
73 : $P_{694} = (5,10,1,1)$
74 : $P_{704} = (15,10,1,1)$
75 : $P_{708} = (3,11,1,1)$
76 : $P_{713} = (8,11,1,1)$
77 : $P_{755} = (2,14,1,1)$
78 : $P_{765} = (12,14,1,1)$
79 : $P_{786} = (1,0,2,1)$
80 : $P_{805} = (4,1,2,1)$
81 : $P_{807} = (6,1,2,1)$
82 : $P_{826} = (9,2,2,1)$
83 : $P_{830} = (13,2,2,1)$
84 : $P_{870} = (5,5,2,1)$
85 : $P_{880} = (15,5,2,1)$
86 : $P_{899} = (2,7,2,1)$
87 : $P_{909} = (12,7,2,1)$
88 : $P_{920} = (7,8,2,1)$
89 : $P_{927} = (14,8,2,1)$
90 : $P_{932} = (3,9,2,1)$
91 : $P_{937} = (8,9,2,1)$
92 : $P_{987} = (10,12,2,1)$
93 : $P_{988} = (11,12,2,1)$
94 : $P_{1042} = (1,0,3,1)$
95 : $P_{1142} = (5,6,3,1)$
96 : $P_{1152} = (15,6,3,1)$
97 : $P_{1160} = (7,7,3,1)$
98 : $P_{1167} = (14,7,3,1)$
99 : $P_{1179} = (10,8,3,1)$
100 : $P_{1180} = (11,8,3,1)$
101 : $P_{1189} = (4,9,3,1)$
102 : $P_{1191} = (6,9,3,1)$
103 : $P_{1226} = (9,11,3,1)$
104 : $P_{1230} = (13,11,3,1)$
105 : $P_{1251} = (2,13,3,1)$
106 : $P_{1261} = (12,13,3,1)$
107 : $P_{1268} = (3,14,3,1)$
108 : $P_{1273} = (8,14,3,1)$
109 : $P_{1298} = (1,0,4,1)$
110 : $P_{1322} = (9,1,4,1)$
111 : $P_{1326} = (13,1,4,1)$
112 : $P_{1368} = (7,4,4,1)$
113 : $P_{1375} = (14,4,4,1)$
114 : $P_{1403} = (10,6,4,1)$
115 : $P_{1404} = (11,6,4,1)$
116 : $P_{1428} = (3,8,4,1)$
117 : $P_{1433} = (8,8,4,1)$
118 : $P_{1493} = (4,12,4,1)$
119 : $P_{1495} = (6,12,4,1)$
120 : $P_{1526} = (5,14,4,1)$
121 : $P_{1536} = (15,14,4,1)$
122 : $P_{1539} = (2,15,4,1)$
123 : $P_{1549} = (12,15,4,1)$
124 : $P_{1554} = (1,0,5,1)$
125 : $P_{1590} = (5,2,5,1)$
126 : $P_{1600} = (15,2,5,1)$
127 : $P_{1669} = (4,7,5,1)$
128 : $P_{1671} = (6,7,5,1)$
129 : $P_{1720} = (7,10,5,1)$
130 : $P_{1727} = (14,10,5,1)$
131 : $P_{1747} = (2,12,5,1)$
132 : $P_{1757} = (12,12,5,1)$
133 : $P_{1764} = (3,13,5,1)$
134 : $P_{1769} = (8,13,5,1)$
135 : $P_{1786} = (9,14,5,1)$
136 : $P_{1790} = (13,14,5,1)$
137 : $P_{1803} = (10,15,5,1)$
138 : $P_{1804} = (11,15,5,1)$
139 : $P_{1810} = (1,0,6,1)$
140 : $P_{1862} = (5,3,6,1)$
141 : $P_{1872} = (15,3,6,1)$
142 : $P_{1883} = (10,4,6,1)$
143 : $P_{1884} = (11,4,6,1)$
144 : $P_{1924} = (3,7,6,1)$
145 : $P_{1929} = (8,7,6,1)$
146 : $P_{1941} = (4,8,6,1)$
147 : $P_{1943} = (6,8,6,1)$
148 : $P_{1962} = (9,9,6,1)$
149 : $P_{1966} = (13,9,6,1)$

$150: P_{1971} = (2, 10, 6, 1)$
$151: P_{1981} = (12, 10, 6, 1)$
$152: P_{2056} = (7, 15, 6, 1)$
$153: P_{2063} = (14, 15, 6, 1)$
$154: P_{2066} = (1, 0, 7, 1)$
$155: P_{2099} = (2, 2, 7, 1)$
$156: P_{2109} = (12, 2, 7, 1)$
$157: P_{2120} = (7, 3, 7, 1)$
$158: P_{2127} = (14, 3, 7, 1)$
$159: P_{2149} = (4, 5, 7, 1)$
$160: P_{2151} = (6, 5, 7, 1)$
$161: P_{2164} = (3, 6, 7, 1)$
$162: P_{2169} = (8, 6, 7, 1)$
$163: P_{2198} = (5, 8, 7, 1)$
$164: P_{2208} = (15, 8, 7, 1)$
$165: P_{2234} = (9, 10, 7, 1)$
$166: P_{2238} = (13, 10, 7, 1)$
$167: P_{2299} = (10, 14, 7, 1)$
$168: P_{2300} = (11, 14, 7, 1)$
$169: P_{2322} = (1, 0, 8, 1)$
$170: P_{2360} = (7, 2, 8, 1)$
$171: P_{2367} = (14, 2, 8, 1)$
$172: P_{2379} = (10, 3, 8, 1)$
$173: P_{2380} = (11, 3, 8, 1)$
$174: P_{2388} = (3, 4, 8, 1)$
$175: P_{2393} = (8, 4, 8, 1)$
$176: P_{2421} = (4, 6, 8, 1)$
$177: P_{2423} = (6, 6, 8, 1)$
$178: P_{2438} = (5, 7, 8, 1)$
$179: P_{2448} = (15, 7, 8, 1)$
$180: P_{2499} = (2, 11, 8, 1)$
$181: P_{2509} = (12, 11, 8, 1)$
$182: P_{2522} = (9, 12, 8, 1)$
$183: P_{2526} = (13, 12, 8, 1)$
$184: P_{2578} = (1, 0, 9, 1)$
$185: P_{2600} = (7, 1, 9, 1)$
$186: P_{2607} = (14, 1, 9, 1)$
$187: P_{2612} = (3, 2, 9, 1)$
$188: P_{2617} = (8, 2, 9, 1)$
$189: P_{2629} = (4, 3, 9, 1)$
$190: P_{2631} = (6, 3, 9, 1)$
$191: P_{2682} = (9, 6, 9, 1)$
$192: P_{2686} = (13, 6, 9, 1)$
$193: P_{2723} = (2, 9, 9, 1)$
$194: P_{2733} = (12, 9, 9, 1)$
$195: P_{2795} = (10, 13, 9, 1)$
$196: P_{2796} = (11, 13, 9, 1)$
$197: P_{2822} = (5, 15, 9, 1)$
$198: P_{2832} = (15, 15, 9, 1)$
$199: P_{2834} = (1, 0, 10, 1)$
$200: P_{2854} = (5, 1, 10, 1)$
$201: P_{2864} = (15, 1, 10, 1)$
$202: P_{2920} = (7, 5, 10, 1)$
$203: P_{2927} = (14, 5, 10, 1)$
$204: P_{2931} = (2, 6, 10, 1)$
$205: P_{2941} = (12, 6, 10, 1)$
$206: P_{2954} = (9, 7, 10, 1)$
$207: P_{2958} = (13, 7, 10, 1)$
$208: P_{2996} = (3, 10, 10, 1)$
$209: P_{3001} = (8, 10, 10, 1)$
$210: P_{3019} = (10, 11, 10, 1)$
$211: P_{3020} = (11, 11, 10, 1)$
$212: P_{3077} = (4, 15, 10, 1)$
$213: P_{3079} = (6, 15, 10, 1)$
$214: P_{3090} = (1, 0, 11, 1)$
$215: P_{3108} = (3, 1, 11, 1)$
$216: P_{3113} = (8, 1, 11, 1)$
$217: P_{3146} = (9, 3, 11, 1)$
$218: P_{3150} = (13, 3, 11, 1)$
$219: P_{3219} = (2, 8, 11, 1)$
$220: P_{3229} = (12, 8, 11, 1)$
$221: P_{3259} = (10, 10, 11, 1)$
$222: P_{3260} = (11, 10, 11, 1)$
$223: P_{3270} = (5, 11, 11, 1)$
$224: P_{3280} = (15, 11, 11, 1)$
$225: P_{3288} = (7, 12, 11, 1)$
$226: P_{3295} = (14, 12, 11, 1)$
$227: P_{3301} = (4, 13, 11, 1)$
$228: P_{3303} = (6, 13, 11, 1)$
$229: P_{3346} = (1, 0, 12, 1)$
$230: P_{3387} = (10, 2, 12, 1)$
$231: P_{3388} = (11, 2, 12, 1)$
$232: P_{3413} = (4, 4, 12, 1)$
$233: P_{3415} = (6, 4, 12, 1)$
$234: P_{3427} = (2, 5, 12, 1)$
$235: P_{3437} = (12, 5, 12, 1)$
$236: P_{3482} = (9, 8, 12, 1)$
$237: P_{3486} = (13, 8, 12, 1)$
$238: P_{3528} = (7, 11, 12, 1)$
$239: P_{3535} = (14, 11, 12, 1)$
$240: P_{3558} = (5, 13, 12, 1)$
$241: P_{3568} = (15, 13, 12, 1)$
$242: P_{3588} = (3, 15, 12, 1)$
$243: P_{3593} = (8, 15, 12, 1)$
$244: P_{3602} = (1, 0, 13, 1)$
$245: P_{3651} = (2, 3, 13, 1)$
$246: P_{3661} = (12, 3, 13, 1)$
$247: P_{3684} = (3, 5, 13, 1)$
$248: P_{3689} = (8, 5, 13, 1)$
$249: P_{3755} = (10, 9, 13, 1)$
$250: P_{3756} = (11, 9, 13, 1)$
$251: P_{3781} = (4, 11, 13, 1)$
$252: P_{3783} = (6, 11, 13, 1)$
$253: P_{3798} = (5, 12, 13, 1)$
$254: P_{3808} = (15, 12, 13, 1)$
$255: P_{3832} = (7, 14, 13, 1)$
$256: P_{3839} = (14, 14, 13, 1)$
$257: P_{3850} = (9, 15, 13, 1)$
$258: P_{3854} = (13, 15, 13, 1)$
$259: P_{3858} = (1, 0, 14, 1)$
$260: P_{3875} = (2, 1, 14, 1)$
$261: P_{3885} = (12, 1, 14, 1)$
$262: P_{3908} = (3, 3, 14, 1)$
$263: P_{3913} = (8, 3, 14, 1)$
$264: P_{3926} = (5, 4, 14, 1)$
$265: P_{3936} = (15, 4, 14, 1)$
$266: P_{3946} = (9, 5, 14, 1)$
$267: P_{3950} = (13, 5, 14, 1)$
$268: P_{3979} = (10, 7, 14, 1)$
$269: P_{3980} = (11, 7, 14, 1)$
$270: P_{4072} = (7, 13, 14, 1)$
$271: P_{4079} = (14, 13, 14, 1)$
$272: P_{4085} = (4, 14, 14, 1)$
$273: P_{4087} = (6, 14, 14, 1)$
$274: P_{4114} = (1, 0, 15, 1)$
$275: P_{4179} = (2, 4, 15, 1)$
$276: P_{4189} = (12, 4, 15, 1)$
$277: P_{4203} = (10, 5, 15, 1)$
$278: P_{4204} = (11, 5, 15, 1)$
$279: P_{4216} = (7, 6, 15, 1)$
$280: P_{4223} = (14, 6, 15, 1)$
$281: P_{4262} = (5, 9, 15, 1)$
$282: P_{4272} = (15, 9, 15, 1)$
$283: P_{4277} = (4, 10, 15, 1)$
$284: P_{4279} = (6, 10, 15, 1)$
$285: P_{4308} = (3, 12, 15, 1)$
$286: P_{4313} = (8, 12, 15, 1)$
$287: P_{4330} = (9, 13, 15, 1)$
$288: P_{4334} = (13, 13, 15, 1)$