Rank-66755 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_0X_1^2 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1432724825

General information

Number of lines	29
Number of points	37
Number of singular points	9
Number of Eckardt points	0
Number of double points	16
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{29}
Type of lines on points	6^8, 5^{13}, 2^{16}

Singular Points

The surface has 9 singular points:

0 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
1 : $P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0)$
2 : $P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0)$
3 : $P_{45} = \mathbf{P}(0,\omega,1,1) = \mathbf{P}(0,2,1,1)$
4 : $P_{49} = \mathbf{P}(0,\omega^2,1,1) = \mathbf{P}(0,3,1,1)$
5 : $P_{57} = \mathbf{P}(0,1,\omega,1) = \mathbf{P}(0,1,2,1)$
6 : $P_{65} = \mathbf{P}(0,\omega^2,\omega,1) = \mathbf{P}(0,3,2,1)$
7 : $P_{73} = \mathbf{P}(0,1,\omega^2,1) = \mathbf{P}(0,1,3,1)$
8 : $P_{77} = \mathbf{P}(0,\omega,\omega^2,1) = \mathbf{P}(0,2,3,1)$

The 29 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega & 0 \end{array}\right]_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]_2 = \mathbf{Pl}(3,0,1,0,0,0)_5$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega^2 & 0 \end{array}\right]_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right]_3 = \mathbf{Pl}(2,0,1,0,0,0)_4$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_4 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_5 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{339} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_6 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{338} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_7 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_8 = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{341} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_9 = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{351} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]_{351} = \mathbf{Pl}(0,3,0,0,0,1)_{107}$$

$$\ell_{10} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{array}\right]_{346} = \left[\begin{array}{cccc} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{346} = \mathbf{Pl}(0,2,0,0,0,1)_{106}$$

$$\ell_{11} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_{12} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{342} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{342} = \mathbf{Pl}(0,1,0,1,0,1)_{133}$$

$$\ell_{13} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{354} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]_{354} = \mathbf{Pl}(0,3,0,3,0,1)_{149}$$

$$\ell_{14} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{array}\right]_{348} = \left[\begin{array}{cccc} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]_{348} = \mathbf{Pl}(0,2,0,2,0,1)_{141}$$

$$\ell_{15} = \left[\begin{array}{cccc} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{355} = \left[\begin{array}{cccc} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_{16} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{array}\right]_{343} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]_{343} = \mathbf{Pl}(0,1,0,2,0,1)_{140}$$

$$\ell_{17} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{array}\right]_{352} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]_{352} = \mathbf{Pl}(0,3,0,1,0,1)_{135}$$

$$\ell_{18} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{349} = \left[\begin{array}{cccc} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]_{349} = \mathbf{Pl}(0,2,0,3,0,1)_{148}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega & \omega \end{array}\right]_{304} = \left[\begin{array}{cccc} 1 & 0 & 2 & 3 \\ 0 & 1 & 2 & 2 \end{array}\right]_{304} = \mathbf{Pl}(1,1,3,2,1,1)_{231}$$

$$\ell_{20} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \omega & 1 \end{bmatrix}_{111} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{111} = \mathbf{Pl}(2,3,1,1,1,1)_{220}$$

$$\ell_{21} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{245} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{245} = \mathbf{Pl}(3,2,2,3,1,1)_{227}$$

$$\ell_{22} = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_{23} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{344} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{344} = \mathbf{Pl}(0,1,0,3,0,1)_{147}$$

$$\ell_{24} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{353} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{353} = \mathbf{Pl}(0,3,0,2,0,1)_{142}$$

$$\ell_{25} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \mathbf{Pl}(0,2,0,1,0,1)_{134}$$

$$\ell_{26} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{246} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{246} = \mathbf{Pl}(1,1,2,3,1,1)_{228}$$

$$\ell_{27} = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{305} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{305} = \mathbf{Pl}(2,3,3,2,1,1)_{235}$$

$$\ell_{28} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{112} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{112} = \mathbf{Pl}(3,2,1,1,1,1)_{224}$$

Rank of lines: (2, 3, 336, 340, 337, 339, 338, 356, 341, 351, 346, 345, 342, 354, 348, 355, 343, 352, 349, 304, 111, 245, 350, 344, 353, 347, 246, 305, 112)
Rank of points on Klein quadric: (5, 4, 101, 9, 129, 143, 136, 1, 105, 107, 106, 13, 133, 149, 141, 15, 140, 135, 148, 231, 220, 227, 14, 147, 142, 134, 228, 235, 224)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 16 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{20} = (1,3,1,0) = \ell_0 \cap \ell_{26}$
$P_{21} = (2,3,1,0) = \ell_0 \cap \ell_{27}$
$P_{22} = (3,3,1,0) = \ell_0 \cap \ell_{28}$
$P_{16} = (1,2,1,0) = \ell_1 \cap \ell_{19}$
$P_{17} = (2,2,1,0) = \ell_1 \cap \ell_{20}$
$P_{18} = (3,2,1,0) = \ell_1 \cap \ell_{21}$
$P_{27} = (1,1,0,1) = \ell_{19} \cap \ell_{26}$
$P_{71} = (2,0,3,1) = \ell_{19} \cap \ell_{27}$
$P_{64} = (3,2,2,1) = \ell_{19} \cap \ell_{28}$
$P_{83} = (2,3,3,1) = \ell_{20} \cap \ell_{26}$
$P_{33} = (3,2,0,1) = \ell_{20} \cap \ell_{27}$
$P_{39} = (1,0,1,1) = \ell_{20} \cap \ell_{28}$
$P_{56} = (3,0,2,1) = \ell_{21} \cap \ell_{26}$
$P_4 = (1,1,1,1) = \ell_{21} \cap \ell_{27}$
$P_{36} = (2,3,0,1) = \ell_{21} \cap \ell_{28}$

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
2	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	0
3	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	0
4	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0
5	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1	0
6	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	1
7	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	0
8	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	1
9	0	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0
10	0	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	1	0
11	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	0
12	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	1	1	1	1	0	1	0
13	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0	1
14	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	0	1	1	1	1	1	0	0
15	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	1	1	1	1
16	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	0	0
17	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1	0	0	0
18	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	0	0	0
19	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	1	1	1
20	0	1	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	1	1	1
21	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1
22	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0
23	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	1	1	1	0	0
24	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	0	1	0
25	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	0	0	1
26	1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	1	1	0	1	0	0	0	0	0
27	1	0	0	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	0	0	0
28	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_0	P_{19}	P_{19}	P_{19}	P_{19}	P_{19}	P_{20}	P_{21}	P_{22}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_0	P_{15}	P_{15}	P_{15}	P_{15}	P_{15}	P_{16}	P_{17}	P_{18}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}	ℓ_{24}
in point	P_{19}	P_{15}	P_1	P_1	P_1	P_1	P_2	P_2	P_2	P_2	P_{11}	P_{11}	P_{11}	P_{11}	P_{15}	P_{15}	P_{15}	P_{15}	P_{19}	P_{19}	P_{19}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_1	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_1	P_1	P_1	P_1	P_{38}	P_{42}	P_{45}	P_{49}	P_{42}	P_{38}	P_{49}	P_{45}	P_{45}	P_{49}	P_{38}	P_{42}	P_{49}	P_{49}	P_{45}	P_{42}	P

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_1	P_1	P_1	P_1	P_{53}	P_{57}	P_{61}	P_{65}	P_{61}	P_{65}	P_{53}	P_{57}	P_{65}	P_{61}	P_{57}	P_{53}	P_{57}	P_{57}	P_{53}	P_{65}	P

Line 6 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_1	P_1	P_1	P_1	P_{69}	P_{73}	P_{77}	P_{81}	P_{81}	P_{77}	P_{73}	P_{69}	P_{73}	P_{69}	P_{81}	P_{77}	P_{77}	P_{77}	P_{81}	P_{69}	P

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_2	P_3	P_{38}	P_{53}	P_{69}	P_2	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_3	P_{69}	P_{38}	P_{53}	P_3	P_{53}	P_{69}	P_{38}

Line 8 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_2	P_{26}	P_{42}	P_{57}	P_{73}	P_2	P_2	P_2	P_{42}	P_{26}	P_{73}	P_{57}	P_{73}	P_{26}	P_{57}	P_{42}	P_{57}	P_{57}	P_{26}	P_{42}	P

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_2	P_{30}	P_{45}	P_{61}	P_{77}	P_2	P_2	P_2	P_{61}	P_{77}	P_{30}	P_{45}	P_{45}	P_{61}	P_{30}	P_{77}	P_{77}	P_{77}	P_{45}	P_{30}	P

Line 10 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ
in point	P_2	P_{34}	P_{49}	P_{65}	P_{81}	P_2	P_2	P_2	P_{81}	P_{65}	P_{49}	P_{34}	P_{65}	P_{49}	P_{81}	P_{34}	P_{49}	P_{49}	P_{81}	P_{65}	P

Line 11 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{42}	P_{61}	P_{81}	P_{11}	P_{11}	P_{11}	P_3	P_{61}	P_{81}	P_{42}	P_3	P_{81}	P_{42}	P_{61}

Line 12 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{2}
in point	P_{11}	P_{26}	P_{38}	P_{65}	P_{77}	P_{38}	P_{26}	P_{77}	P_{65}	P_{11}	P_{11}	P_{11}	P_{65}	P_{26}	P_{38}	P_{77}	P_{77}	P_{77}	P_{26}	P_{6}

Line 13 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{22}	ℓ_{23}	ℓ_{2}
in point	P_{11}	P_{30}	P_{49}	P_{53}	P_{73}	P_{53}	P_{73}	P_{30}	P_{49}	P_{11}	P_{11}	P_{11}	P_{73}	P_{49}	P_{30}	P_{53}	P_{49}	P_{49}	P_{53}	P_{3}

Line 14 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{2}
in point	P_{11}	P_{34}	P_{45}	P_{57}	P_{69}	P_{69}	P_{57}	P_{45}	P_{34}	P_{11}	P_{11}	P_{11}	P_{45}	P_{69}	P_{57}	P_{34}	P_{57}	P_{57}	P_{45}	P_{6}

Line 15 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}	ℓ_{24}
in point	P_{15}	P_{15}	P_3	P_{45}	P_{65}	P_{73}	P_3	P_{73}	P_{45}	P_{65}	P_3	P_{65}	P_{73}	P_{45}	P_{15}	P_{15}	P_{15}	P_3	P_{45}	P_{65}

Line 16 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{22}	ℓ_{23}
in point	P_{15}	P_{15}	P_{26}	P_{49}	P_{61}	P_{69}	P_{69}	P_{26}	P_{61}	P_{49}	P_{61}	P_{26}	P_{49}	P_{69}	P_{15}	P_{15}	P_{15}	P_{49}	P_{49}	P_{2}

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{20}	ℓ_{22}	ℓ_{23}
in point	P_{15}	P_{15}	P_{30}	P_{38}	P_{57}	P_{81}	P_{38}	P_{57}	P_{30}	P_{81}	P_{81}	P_{38}	P_{30}	P_{57}	P_{15}	P_{15}	P_{15}	P_{57}	P_{57}	P_{8}

Line 18 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{15}	P_{15}	P_{34}	P_{42}	P_{53}	P_{77}	P_{53}	P_{42}	P_{77}	P_{34}	P_{42}	P_{77}	P_{53}	P_{34}	P_{15}	P_{15}	P_{15}	P_{77}	P_{77}	P_{5}

Line 19 intersects

Line	ℓ_1	ℓ_4	ℓ_{10}	ℓ_{13}	ℓ_{16}	ℓ_{22}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{16}	P_{49}	P_{49}	P_{49}	P_{49}	P_{49}	P_{27}	P_{71}	P_{64}

Line 20 intersects

Line	ℓ_1	ℓ_5	ℓ_8	ℓ_{14}	ℓ_{17}	ℓ_{22}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{17}	P_{57}	P_{57}	P_{57}	P_{57}	P_{57}	P_{83}	P_{33}	P_{39}

Line 21 intersects

Line	ℓ_1	ℓ_6	ℓ_9	ℓ_{12}	ℓ_{18}	ℓ_{22}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{18}	P_{77}	P_{77}	P_{77}	P_{77}	P_{77}	P_{56}	P_{4}	P_{36}

Line 22 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{19}	P_{19}	P_{3}	P_{49}	P_{57}	P_{77}	P_{3}	P_{57}	P_{77}	P_{49}	P_{3}	P_{77}	P_{49}	P_{57}	P_{3}	P_{49}	P_{57}	P_{77}	P_{49}	P_{57}

Line 23 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{24}
in point	P_{19}	P_{19}	P_{26}	P_{45}	P_{53}	P_{81}	P_{53}	P_{26}	P_{45}	P_{81}	P_{81}	P_{26}	P_{53}	P_{45}	P_{45}	P_{26}	P_{81}	P_{53}	P_{19}	P_{1}

Line 24 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}
in point	P_{19}	P_{19}	P_{30}	P_{42}	P_{65}	P_{69}	P_{69}	P_{42}	P_{30}	P_{65}	P_{42}	P_{65}	P_{30}	P_{69}	P_{65}	P_{69}	P_{30}	P_{42}	P_{19}	P_{1}

Line 25 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{22}	ℓ_{23}
in point	P_{19}	P_{19}	P_{34}	P_{38}	P_{61}	P_{73}	P_{38}	P_{73}	P_{61}	P_{34}	P_{61}	P_{38}	P_{73}	P_{34}	P_{73}	P_{61}	P_{38}	P_{34}	P_{19}	P_{1}

Line 26 intersects

Line	ℓ_0	ℓ_4	ℓ_9	ℓ_{14}	ℓ_{15}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{23}
in point	P_{20}	P_{45}	P_{45}	P_{45}	P_{45}	P_{27}	P_{83}	P_{56}	P_{45}

Line 27 intersects

Line	ℓ_0	ℓ_5	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{24}
in point	P_{21}	P_{65}	P_{65}	P_{65}	P_{65}	P_{71}	P_{33}	P_{4}	P_{65}

Line 28 intersects

Line	ℓ_0	ℓ_6	ℓ_8	ℓ_{13}	ℓ_{15}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{25}
in point	P_{22}	P_{73}	P_{73}	P_{73}	P_{73}	P_{64}	P_{39}	P_{36}	P_{73}

The surface has 37 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_{11} = (0,1,1,0)$
$6: P_{15} = (0,2,1,0)$
$7: P_{16} = (1,2,1,0)$
$8: P_{17} = (2,2,1,0)$
$9: P_{18} = (3,2,1,0)$
$10: P_{19} = (0,3,1,0)$
$11: P_{20} = (1,3,1,0)$
$12: P_{21} = (2,3,1,0)$
$13: P_{22} = (3,3,1,0)$
$14: P_{26} = (0,1,0,1)$
$15: P_{27} = (1,1,0,1)$
$16: P_{30} = (0,2,0,1)$
$17: P_{33} = (3,2,0,1)$
$18: P_{34} = (0,3,0,1)$
$19: P_{36} = (2,3,0,1)$
$20: P_{38} = (0,0,1,1)$
$21: P_{39} = (1,0,1,1)$
$22: P_{42} = (0,1,1,1)$
$23: P_{45} = (0,2,1,1)$
$24: P_{49} = (0,3,1,1)$
$25: P_{53} = (0,0,2,1)$
$26: P_{56} = (3,0,2,1)$
$27: P_{57} = (0,1,2,1)$
$28: P_{61} = (0,2,2,1)$
$29: P_{64} = (3,2,2,1)$
$30: P_{65} = (0,3,2,1)$
$31: P_{69} = (0,0,3,1)$
$32: P_{71} = (2,0,3,1)$
$33: P_{73} = (0,1,3,1)$
$34: P_{77} = (0,2,3,1)$
$35: P_{81} = (0,3,3,1)$
$36: P_{83} = (2,3,3,1)$