

# Rank-65874 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(4) is 1431725486

## General information

Number of lines	3
Number of points	29
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	9
Number of points off lines	17
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^3$
Type of lines on points	$2^3, 1^9, 0^{17}$

## Singular Points

The surface has 0 singular points:

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right]_{110} = \left[ \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array} \right]_{110} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{199} \\ \ell_1 &= \left[ \begin{array}{cccc} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{80} = \left[ \begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{80} = \mathbf{Pl}(0, 0, 1, 1, 2, 1)_{258}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \mathbf{PI}(0, 0, 1, 1, 3, 1)_{318}$$

Rank of lines: ( 110, 80, 59 )

Rank of points on Klein quadric: ( 199, 258, 318 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{48} = (3, 2, 1, 1) = \ell_0 \cap \ell_1$$

$$P_{51} = (2, 3, 1, 1) = \ell_0 \cap \ell_2$$

$$P_{38} = (0, 0, 1, 1) = \ell_1 \cap \ell_2$$

### Single Points

The surface has 9 single points:

The single points on the surface are:

$$0 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_1$$

$$2 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_2$$

$$3 : P_{39} = (1, 0, 1, 1) \text{ lies on line } \ell_0$$

$$4 : P_{42} = (0, 1, 1, 1) \text{ lies on line } \ell_0$$

$$5 : P_{43} = (2, 1, 1, 1) \text{ lies on line } \ell_1$$

$$6 : P_{44} = (3, 1, 1, 1) \text{ lies on line } \ell_2$$

$$7 : P_{46} = (1, 2, 1, 1) \text{ lies on line } \ell_2$$

$$8 : P_{50} = (1, 3, 1, 1) \text{ lies on line } \ell_1$$

The single points on the surface are:

### Points on surface but on no line

The surface has 17 points not on any line:

The points on the surface but not on lines are:

$$0 : P_8 = (1, 0, 1, 0)$$

$$1 : P_9 = (2, 0, 1, 0)$$

$$2 : P_{10} = (3, 0, 1, 0)$$

$$3 : P_{16} = (1, 2, 1, 0)$$

$$4 : P_{20} = (1, 3, 1, 0)$$

$$5 : P_{26} = (0, 1, 0, 1)$$

$$6 : P_{27} = (1, 1, 0, 1)$$

$$7 : P_{30} = (0, 2, 0, 1)$$

$$8 : P_{31} = (1, 2, 0, 1)$$

$$9 : P_{34} = (0, 3, 0, 1)$$

$$10 : P_{35} = (1, 3, 0, 1)$$

$$11 : P_{53} = (0, 0, 2, 1)$$

$$12 : P_{54} = (1, 0, 2, 1)$$

$$13 : P_{61} = (0, 2, 2, 1)$$

$$14 : P_{69} = (0, 0, 3, 1)$$

$$15 : P_{70} = (1, 0, 3, 1)$$

$$16 : P_{81} = (0, 3, 3, 1)$$

## Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_{48}$	$P_{51}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{48}$	$P_{38}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_{51}$	$P_{38}$

The surface has 29 points:

The points on the surface are:

0 :  $P_5 = (1, 1, 0, 0)$   
 1 :  $P_6 = (2, 1, 0, 0)$   
 2 :  $P_7 = (3, 1, 0, 0)$   
 3 :  $P_8 = (1, 0, 1, 0)$   
 4 :  $P_9 = (2, 0, 1, 0)$   
 5 :  $P_{10} = (3, 0, 1, 0)$   
 6 :  $P_{16} = (1, 2, 1, 0)$   
 7 :  $P_{20} = (1, 3, 1, 0)$   
 8 :  $P_{26} = (0, 1, 0, 1)$   
 9 :  $P_{27} = (1, 1, 0, 1)$

10 :  $P_{30} = (0, 2, 0, 1)$   
 11 :  $P_{31} = (1, 2, 0, 1)$   
 12 :  $P_{34} = (0, 3, 0, 1)$   
 13 :  $P_{35} = (1, 3, 0, 1)$   
 14 :  $P_{38} = (0, 0, 1, 1)$   
 15 :  $P_{39} = (1, 0, 1, 1)$   
 16 :  $P_{42} = (0, 1, 1, 1)$   
 17 :  $P_{43} = (2, 1, 1, 1)$   
 18 :  $P_{44} = (3, 1, 1, 1)$   
 19 :  $P_{46} = (1, 2, 1, 1)$

20 :  $P_{48} = (3, 2, 1, 1)$   
 21 :  $P_{50} = (1, 3, 1, 1)$   
 22 :  $P_{51} = (2, 3, 1, 1)$   
 23 :  $P_{53} = (0, 0, 2, 1)$   
 24 :  $P_{54} = (1, 0, 2, 1)$   
 25 :  $P_{61} = (0, 2, 2, 1)$   
 26 :  $P_{69} = (0, 0, 3, 1)$   
 27 :  $P_{70} = (1, 0, 3, 1)$   
 28 :  $P_{81} = (0, 3, 3, 1)$