

Rank-65611 over GF(8)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227396173

General information

Number of lines	5
Number of points	81
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	35
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	$3^2, 2^2, 1^{35}, 0^{42}$

Singular Points

The surface has 3 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{649} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{18} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{10}
\end{aligned}$$

Rank of lines: (0, 64, 4672, 584, 648)

Rank of points on Klein quadric: (0, 2, 649, 18, 10)

Eckardt Points

The surface has 2 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0).$$

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{75} = (1, 0, 0, 1) = \ell_3 \cap \ell_4$$

Single Points

The surface has 35 single points:

The single points on the surface are:

$$0 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_8 = (4, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_9 = (5, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{10} = (6, 1, 0, 0) \text{ lies on line } \ell_0$$

$$6 : P_{11} = (7, 1, 0, 0) \text{ lies on line } \ell_0$$

$$7 : P_{12} = (1, 0, 1, 0) \text{ lies on line } \ell_1$$

$$8 : P_{13} = (2, 0, 1, 0) \text{ lies on line } \ell_1$$

$$9 : P_{14} = (3, 0, 1, 0) \text{ lies on line } \ell_1$$

$$10 : P_{15} = (4, 0, 1, 0) \text{ lies on line } \ell_1$$

$$11 : P_{16} = (5, 0, 1, 0) \text{ lies on line } \ell_1$$

$$12 : P_{17} = (6, 0, 1, 0) \text{ lies on line } \ell_1$$

$$13 : P_{18} = (7, 0, 1, 0) \text{ lies on line } \ell_1$$

$$14 : P_{19} = (0, 1, 1, 0) \text{ lies on line } \ell_2$$

$$15 : P_{27} = (0, 2, 1, 0) \text{ lies on line } \ell_2$$

$$16 : P_{35} = (0, 3, 1, 0) \text{ lies on line } \ell_2$$

$$17 : P_{43} = (0, 4, 1, 0) \text{ lies on line } \ell_2$$

$$18 : P_{51} = (0, 5, 1, 0) \text{ lies on line } \ell_2$$

$$19 : P_{59} = (0, 6, 1, 0) \text{ lies on line } \ell_2$$

$$20 : P_{67} = (0, 7, 1, 0) \text{ lies on line } \ell_2$$

$$21 : P_{83} = (1, 1, 0, 1) \text{ lies on line } \ell_3$$

$$22 : P_{91} = (1, 2, 0, 1) \text{ lies on line } \ell_3$$

$$23 : P_{99} = (1, 3, 0, 1) \text{ lies on line } \ell_3$$

$$24 : P_{107} = (1, 4, 0, 1) \text{ lies on line } \ell_3$$

$$25 : P_{115} = (1, 5, 0, 1) \text{ lies on line } \ell_3$$

$$26 : P_{123} = (1, 6, 0, 1) \text{ lies on line } \ell_3$$

$$27 : P_{131} = (1, 7, 0, 1) \text{ lies on line } \ell_3$$

$$28 : P_{139} = (1, 0, 1, 1) \text{ lies on line } \ell_4$$

$$29 : P_{202} = (1, 0, 2, 1) \text{ lies on line } \ell_4$$

$$30 : P_{266} = (1, 0, 3, 1) \text{ lies on line } \ell_4$$

$$31 : P_{330} = (1, 0, 4, 1) \text{ lies on line } \ell_4$$

$$32 : P_{394} = (1, 0, 5, 1) \text{ lies on line } \ell_4$$

$$33 : P_{458} = (1, 0, 6, 1) \text{ lies on line } \ell_4$$

$$34 : P_{522} = (1, 0, 7, 1) \text{ lies on line } \ell_4$$

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:

The points on the surface but not on lines are:

0 : $P_{158} = (5, 2, 1, 1)$	22 : $P_{382} = (5, 6, 4, 1)$
1 : $P_{160} = (7, 2, 1, 1)$	23 : $P_{384} = (7, 6, 4, 1)$
2 : $P_{171} = (2, 4, 1, 1)$	24 : $P_{412} = (3, 2, 5, 1)$
3 : $P_{175} = (6, 4, 1, 1)$	25 : $P_{413} = (4, 2, 5, 1)$
4 : $P_{196} = (3, 7, 1, 1)$	26 : $P_{422} = (5, 3, 5, 1)$
5 : $P_{197} = (4, 7, 1, 1)$	27 : $P_{424} = (7, 3, 5, 1)$
6 : $P_{214} = (5, 1, 2, 1)$	28 : $P_{443} = (2, 6, 5, 1)$
7 : $P_{216} = (7, 1, 2, 1)$	29 : $P_{447} = (6, 6, 5, 1)$
8 : $P_{219} = (2, 2, 2, 1)$	30 : $P_{484} = (3, 3, 6, 1)$
9 : $P_{223} = (6, 2, 2, 1)$	31 : $P_{485} = (4, 3, 6, 1)$
10 : $P_{244} = (3, 5, 2, 1)$	32 : $P_{494} = (5, 4, 6, 1)$
11 : $P_{245} = (4, 5, 2, 1)$	33 : $P_{496} = (7, 4, 6, 1)$
12 : $P_{310} = (5, 5, 3, 1)$	34 : $P_{499} = (2, 5, 6, 1)$
13 : $P_{312} = (7, 5, 3, 1)$	35 : $P_{503} = (6, 5, 6, 1)$
14 : $P_{316} = (3, 6, 3, 1)$	36 : $P_{532} = (3, 1, 7, 1)$
15 : $P_{317} = (4, 6, 3, 1)$	37 : $P_{533} = (4, 1, 7, 1)$
16 : $P_{323} = (2, 7, 3, 1)$	38 : $P_{547} = (2, 3, 7, 1)$
17 : $P_{327} = (6, 7, 3, 1)$	39 : $P_{551} = (6, 3, 7, 1)$
18 : $P_{339} = (2, 1, 4, 1)$	40 : $P_{582} = (5, 7, 7, 1)$
19 : $P_{343} = (6, 1, 4, 1)$	41 : $P_{584} = (7, 7, 7, 1)$
20 : $P_{364} = (3, 4, 4, 1)$	
21 : $P_{365} = (4, 4, 4, 1)$	

Line Intersection Graph

	0 1 2 3 4
0	0 1 1 1 0
1	1 0 1 0 1
2	1 1 0 1 1
3	1 0 1 0 1
4	0 1 1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_1	P_2	P_1	P_2

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_1	P_1	P_{75}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_{75}

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	28 : $P_{107} = (1, 4, 0, 1)$	56 : $P_{364} = (3, 4, 4, 1)$
1 : $P_1 = (0, 1, 0, 0)$	29 : $P_{115} = (1, 5, 0, 1)$	57 : $P_{365} = (4, 4, 4, 1)$
2 : $P_2 = (0, 0, 1, 0)$	30 : $P_{123} = (1, 6, 0, 1)$	58 : $P_{382} = (5, 6, 4, 1)$
3 : $P_3 = (1, 1, 0, 0)$	31 : $P_{131} = (1, 7, 0, 1)$	59 : $P_{384} = (7, 6, 4, 1)$
4 : $P_6 = (2, 1, 0, 0)$	32 : $P_{139} = (1, 0, 1, 1)$	60 : $P_{394} = (1, 0, 5, 1)$
5 : $P_7 = (3, 1, 0, 0)$	33 : $P_{158} = (5, 2, 1, 1)$	61 : $P_{412} = (3, 2, 5, 1)$
6 : $P_8 = (4, 1, 0, 0)$	34 : $P_{160} = (7, 2, 1, 1)$	62 : $P_{413} = (4, 2, 5, 1)$
7 : $P_9 = (5, 1, 0, 0)$	35 : $P_{171} = (2, 4, 1, 1)$	63 : $P_{422} = (5, 3, 5, 1)$
8 : $P_{10} = (6, 1, 0, 0)$	36 : $P_{175} = (6, 4, 1, 1)$	64 : $P_{424} = (7, 3, 5, 1)$
9 : $P_{11} = (7, 1, 0, 0)$	37 : $P_{196} = (3, 7, 1, 1)$	65 : $P_{443} = (2, 6, 5, 1)$
10 : $P_{12} = (1, 0, 1, 0)$	38 : $P_{197} = (4, 7, 1, 1)$	66 : $P_{447} = (6, 6, 5, 1)$
11 : $P_{13} = (2, 0, 1, 0)$	39 : $P_{202} = (1, 0, 2, 1)$	67 : $P_{458} = (1, 0, 6, 1)$
12 : $P_{14} = (3, 0, 1, 0)$	40 : $P_{214} = (5, 1, 2, 1)$	68 : $P_{484} = (3, 3, 6, 1)$
13 : $P_{15} = (4, 0, 1, 0)$	41 : $P_{216} = (7, 1, 2, 1)$	69 : $P_{485} = (4, 3, 6, 1)$
14 : $P_{16} = (5, 0, 1, 0)$	42 : $P_{219} = (2, 2, 2, 1)$	70 : $P_{494} = (5, 4, 6, 1)$
15 : $P_{17} = (6, 0, 1, 0)$	43 : $P_{223} = (6, 2, 2, 1)$	71 : $P_{496} = (7, 4, 6, 1)$
16 : $P_{18} = (7, 0, 1, 0)$	44 : $P_{244} = (3, 5, 2, 1)$	72 : $P_{499} = (2, 5, 6, 1)$
17 : $P_{19} = (0, 1, 1, 0)$	45 : $P_{245} = (4, 5, 2, 1)$	73 : $P_{503} = (6, 5, 6, 1)$
18 : $P_{27} = (0, 2, 1, 0)$	46 : $P_{266} = (1, 0, 3, 1)$	74 : $P_{522} = (1, 0, 7, 1)$
19 : $P_{35} = (0, 3, 1, 0)$	47 : $P_{310} = (5, 5, 3, 1)$	75 : $P_{532} = (3, 1, 7, 1)$
20 : $P_{43} = (0, 4, 1, 0)$	48 : $P_{312} = (7, 5, 3, 1)$	76 : $P_{533} = (4, 1, 7, 1)$
21 : $P_{51} = (0, 5, 1, 0)$	49 : $P_{316} = (3, 6, 3, 1)$	77 : $P_{547} = (2, 3, 7, 1)$
22 : $P_{59} = (0, 6, 1, 0)$	50 : $P_{317} = (4, 6, 3, 1)$	78 : $P_{551} = (6, 3, 7, 1)$
23 : $P_{67} = (0, 7, 1, 0)$	51 : $P_{323} = (2, 7, 3, 1)$	79 : $P_{582} = (5, 7, 7, 1)$
24 : $P_{75} = (1, 0, 0, 1)$	52 : $P_{327} = (6, 7, 3, 1)$	80 : $P_{584} = (7, 7, 7, 1)$
25 : $P_{83} = (1, 1, 0, 1)$	53 : $P_{330} = (1, 0, 4, 1)$	
26 : $P_{91} = (1, 2, 0, 1)$	54 : $P_{339} = (2, 1, 4, 1)$	
27 : $P_{99} = (1, 3, 0, 1)$	55 : $P_{343} = (6, 1, 4, 1)$	