Rank-65614 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303112486

General information

Number of lines	3
Number of points	289
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	48
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	3, 1^{48}, 0^{240}

Singular Points

The surface has 1 singular points:

$0 : \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69905} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69905} = \mathbf{Pl}(0,1,0,0,0,1)_{4641}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & \delta^5 \\ 0 & 0 & 1 & 0 \end{array}\right]_{70075} = \left[\begin{array}{cccc} 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{array}\right]_{70075} = \mathbf{Pl}(0,11,0,0,0,1)_{4651}$$
$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{array}\right]_{70058} = \left[\begin{array}{cccc} 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{array}\right]_{70058} = \mathbf{Pl}(0,10,0,0,0,1)_{4650}$$

Rank of lines: (69905, 70075, 70058)
Rank of points on Klein quadric: (4641, 4651, 4650)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 48 single points:
The single points on the surface are:

$0 : P_{290} = (0,1,0,1)$ lies on line ℓ_0
$1 : P_{434} = (0,10,0,1)$ lies on line ℓ_1
$2 : P_{450} = (0,11,0,1)$ lies on line ℓ_2
$3 : P_{546} = (0,1,1,1)$ lies on line ℓ_0
$4 : P_{689} = (0,10,1,1)$ lies on line ℓ_1
$5 : P_{705} = (0,11,1,1)$ lies on line ℓ_2
$6 : P_{801} = (0,1,2,1)$ lies on line ℓ_0
$7 : P_{945} = (0,10,2,1)$ lies on line ℓ_1
$8 : P_{961} = (0,11,2,1)$ lies on line ℓ_2
$9 : P_{1057} = (0,1,3,1)$ lies on line ℓ_0
$10 : P_{1201} = (0,10,3,1)$ lies on line ℓ_1
$11 : P_{1217} = (0,11,3,1)$ lies on line ℓ_2
$12 : P_{1313} = (0,1,4,1)$ lies on line ℓ_0
$13 : P_{1457} = (0,10,4,1)$ lies on line ℓ_1
$14 : P_{1473} = (0,11,4,1)$ lies on line ℓ_2
$15 : P_{1569} = (0,1,5,1)$ lies on line ℓ_0
$16 : P_{1713} = (0,10,5,1)$ lies on line ℓ_1
$17 : P_{1729} = (0,11,5,1)$ lies on line ℓ_2
$18 : P_{1825} = (0,1,6,1)$ lies on line ℓ_0
$19 : P_{1969} = (0,10,6,1)$ lies on line ℓ_1
$20 : P_{1985} = (0,11,6,1)$ lies on line ℓ_2
$21 : P_{2081} = (0,1,7,1)$ lies on line ℓ_0
$22 : P_{2225} = (0,10,7,1)$ lies on line ℓ_1
$23 : P_{2241} = (0,11,7,1)$ lies on line ℓ_2
$24 : P_{2337} = (0,1,8,1)$ lies on line ℓ_0
$25 : P_{2481} = (0,10,8,1)$ lies on line ℓ_1
$26 : P_{2497} = (0,11,8,1)$ lies on line ℓ_2
$27 : P_{2593} = (0,1,9,1)$ lies on line ℓ_0
$28 : P_{2737} = (0,10,9,1)$ lies on line ℓ_1
$29 : P_{2753} = (0,11,9,1)$ lies on line ℓ_2
$30 : P_{2849} = (0,1,10,1)$ lies on line ℓ_0
$31 : P_{2993} = (0,10,10,1)$ lies on line ℓ_1
$32 : P_{3009} = (0,11,10,1)$ lies on line ℓ_2
$33 : P_{3105} = (0,1,11,1)$ lies on line ℓ_0
$34 : P_{3249} = (0,10,11,1)$ lies on line ℓ_1
$35 : P_{3265} = (0,11,11,1)$ lies on line ℓ_2
$36 : P_{3361} = (0,1,12,1)$ lies on line ℓ_0
$37 : P_{3505} = (0,10,12,1)$ lies on line ℓ_1
$38 : P_{3521} = (0,11,12,1)$ lies on line ℓ_2
$39 : P_{3617} = (0,1,13,1)$ lies on line ℓ_0
$40 : P_{3761} = (0,10,13,1)$ lies on line ℓ_1
$41 : P_{3777} = (0,11,13,1)$ lies on line ℓ_2
$42 : P_{3873} = (0,1,14,1)$ lies on line ℓ_0
$43 : P_{4017} = (0,10,14,1)$ lies on line ℓ_1
$44 : P_{4033} = (0,11,14,1)$ lies on line ℓ_2
$45 : P_{4129} = (0,1,15,1)$ lies on line ℓ_0
$46 : P_{4273} = (0,10,15,1)$ lies on line ℓ_1
$47 : P_{4289} = (0,11,15,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line:
The points on the surface but not on lines are:

$0: P_{5} = (1,1,0,0)$
$1: P_{14} = (10,1,0,0)$
$2: P_{15} = (11,1,0,0)$
$3: P_{60} = (9,2,1,0)$
$4: P_{73} = (6,3,1,0)$
$5: P_{97} = (14,4,1,0)$
$6: P_{112} = (13,5,1,0)$
$7: P_{118} = (3,6,1,0)$
$8: P_{139} = (8,7,1,0)$
$9: P_{154} = (7,8,1,0)$
$10: P_{165} = (2,9,1,0)$
$11: P_{226} = (15,12,1,0)$
$12: P_{232} = (5,13,1,0)$
$13: P_{247} = (4,14,1,0)$
$14: P_{271} = (12,15,1,0)$
$15: P_{291} = (1,1,0,1)$
$16: P_{435} = (1,10,0,1)$
$17: P_{451} = (1,11,0,1)$
$18: P_{555} = (10,1,1,1)$
$19: P_{556} = (11,1,1,1)$
$20: P_{635} = (10,6,1,1)$
$21: P_{651} = (10,7,1,1)$
$22: P_{701} = (12,10,1,1)$
$23: P_{702} = (13,10,1,1)$
$24: P_{711} = (6,11,1,1)$
$25: P_{712} = (7,11,1,1)$
$26: P_{732} = (11,12,1,1)$
$27: P_{748} = (11,13,1,1)$
$28: P_{823} = (6,2,2,1)$
$29: P_{827} = (10,2,2,1)$
$30: P_{830} = (13,2,2,1)$
$31: P_{864} = (15,4,2,1)$
$32: P_{870} = (5,5,2,1)$
$33: P_{885} = (4,6,2,1)$
$34: P_{898} = (1,7,2,1)$
$35: P_{906} = (9,7,2,1)$
$36: P_{927} = (14,8,2,1)$
$37: P_{949} = (4,10,2,1)$
$38: P_{950} = (5,10,2,1)$
$39: P_{981} = (4,12,2,1)$
$40: P_{985} = (8,12,2,1)$
$41: P_{990} = (13,12,2,1)$
$42: P_{1022} = (13,14,2,1)$
$43: P_{1030} = (5,15,2,1)$
$44: P_{1082} = (9,2,3,1)$
$45: P_{1113} = (8,4,3,1)$
$46: P_{1124} = (3,5,3,1)$
$47: P_{1160} = (7,7,3,1)$
$48: P_{1164} = (11,7,3,1)$
$49: P_{1166} = (13,7,3,1)$
$50: P_{1176} = (7,8,3,1)$
$51: P_{1178} = (9,8,3,1)$
$52: P_{1184} = (15,8,3,1)$
$53: P_{1186} = (1,9,3,1)$
$54: P_{1199} = (14,9,3,1)$
$55: P_{1209} = (8,10,3,1)$
$56: P_{1210} = (9,10,3,1)$
$57: P_{1251} = (2,13,3,1)$
$58: P_{1273} = (8,14,3,1)$
$59: P_{1288} = (7,15,3,1)$
$60: P_{1336} = (7,2,4,1)$
$61: P_{1353} = (8,3,4,1)$
$62: P_{1368} = (7,4,4,1)$
$63: P_{1372} = (11,4,4,1)$
$64: P_{1374} = (13,4,4,1)$
$65: P_{1400} = (7,6,4,1)$
$66: P_{1402} = (9,6,4,1)$
$67: P_{1408} = (15,6,4,1)$
$68: P_{1433} = (8,8,4,1)$
$69: P_{1444} = (3,9,4,1)$
$70: P_{1481} = (8,11,4,1)$
$71: P_{1482} = (9,11,4,1)$
$72: P_{1490} = (1,12,4,1)$
$73: P_{1503} = (14,12,4,1)$
$74: P_{1514} = (9,13,4,1)$
$75: P_{1539} = (2,15,4,1)$
$76: P_{1600} = (15,2,5,1)$
$77: P_{1613} = (12,3,5,1)$
$78: P_{1631} = (14,4,5,1)$
$79: P_{1669} = (4,7,5,1)$
$80: P_{1686} = (5,8,5,1)$
$81: P_{1712} = (15,9,5,1)$
$82: P_{1743} = (14,11,5,1)$
$83: P_{1744} = (15,11,5,1)$
$84: P_{1752} = (7,12,5,1)$
$85: P_{1755} = (10,12,5,1)$
$86: P_{1757} = (12,12,5,1)$
$87: P_{1778} = (1,14,5,1)$
$88: P_{1779} = (2,14,5,1)$
$89: P_{1796} = (3,15,5,1)$
$90: P_{1805} = (12,15,5,1)$
$91: P_{1807} = (14,15,5,1)$
$92: P_{1827} = (2,1,6,1)$
$93: P_{1828} = (3,1,6,1)$
$94: P_{1875} = (2,4,6,1)$
$95: P_{1878} = (5,4,6,1)$

96 : $P_{1879} = (6, 4, 6, 1)$
97 : $P_{1891} = (2, 5, 6, 1)$
98 : $P_{1908} = (3, 6, 6, 1)$
99 : $P_{1924} = (3, 7, 6, 1)$
100 : $P_{1943} = (6, 8, 6, 1)$
101 : $P_{1948} = (11, 8, 6, 1)$
102 : $P_{1949} = (12, 8, 6, 1)$
103 : $P_{1962} = (9, 9, 6, 1)$
104 : $P_{2007} = (6, 12, 6, 1)$
105 : $P_{2025} = (8, 13, 6, 1)$
106 : $P_{2050} = (1, 15, 6, 1)$
107 : $P_{2053} = (4, 15, 6, 1)$
108 : $P_{2089} = (8, 1, 7, 1)$
109 : $P_{2090} = (9, 1, 7, 1)$
110 : $P_{2099} = (2, 2, 7, 1)$
111 : $P_{2120} = (7, 3, 7, 1)$
112 : $P_{2124} = (11, 3, 7, 1)$
113 : $P_{2126} = (13, 3, 7, 1)$
114 : $P_{2146} = (1, 5, 7, 1)$
115 : $P_{2159} = (14, 5, 7, 1)$
116 : $P_{2169} = (8, 6, 7, 1)$
117 : $P_{2185} = (8, 7, 7, 1)$
118 : $P_{2260} = (3, 12, 7, 1)$
119 : $P_{2280} = (7, 13, 7, 1)$
120 : $P_{2296} = (7, 14, 7, 1)$
121 : $P_{2298} = (9, 14, 7, 1)$
122 : $P_{2304} = (15, 14, 7, 1)$
123 : $P_{2314} = (9, 15, 7, 1)$
124 : $P_{2354} = (1, 2, 8, 1)$
125 : $P_{2357} = (4, 2, 8, 1)$
126 : $P_{2371} = (2, 3, 8, 1)$
127 : $P_{2374} = (5, 3, 8, 1)$
128 : $P_{2375} = (6, 3, 8, 1)$
129 : $P_{2388} = (3, 4, 8, 1)$
130 : $P_{2407} = (6, 5, 8, 1)$
131 : $P_{2423} = (6, 6, 8, 1)$
132 : $P_{2428} = (11, 6, 8, 1)$
133 : $P_{2429} = (12, 6, 8, 1)$
134 : $P_{2467} = (2, 9, 8, 1)$
135 : $P_{2483} = (2, 10, 8, 1)$
136 : $P_{2484} = (3, 10, 8, 1)$
137 : $P_{2522} = (9, 12, 8, 1)$
138 : $P_{2548} = (3, 14, 8, 1)$
139 : $P_{2569} = (8, 15, 8, 1)$
140 : $P_{2629} = (4, 3, 9, 1)$
141 : $P_{2653} = (12, 4, 9, 1)$
142 : $P_{2672} = (15, 5, 9, 1)$
143 : $P_{2674} = (1, 6, 9, 1)$
144 : $P_{2675} = (2, 6, 9, 1)$
145 : $P_{2703} = (14, 7, 9, 1)$
146 : $P_{2728} = (7, 9, 9, 1)$
147 : $P_{2731} = (10, 9, 9, 1)$
148 : $P_{2733} = (12, 9, 9, 1)$
149 : $P_{2751} = (14, 10, 9, 1)$
150 : $P_{2752} = (15, 10, 9, 1)$
151 : $P_{2788} = (3, 13, 9, 1)$
152 : $P_{2797} = (12, 13, 9, 1)$
153 : $P_{2799} = (14, 13, 9, 1)$
154 : $P_{2806} = (5, 14, 9, 1)$
155 : $P_{2832} = (15, 15, 9, 1)$
156 : $P_{2861} = (12, 1, 10, 1)$
157 : $P_{2862} = (13, 1, 10, 1)$
158 : $P_{2876} = (11, 2, 10, 1)$
159 : $P_{2891} = (10, 3, 10, 1)$
160 : $P_{2971} = (10, 8, 10, 1)$
161 : $P_{2988} = (11, 9, 10, 1)$
162 : $P_{2999} = (6, 10, 10, 1)$
163 : $P_{3000} = (7, 10, 10, 1)$
164 : $P_{3019} = (10, 11, 10, 1)$
165 : $P_{3020} = (11, 11, 10, 1)$
166 : $P_{3111} = (6, 1, 11, 1)$
167 : $P_{3112} = (7, 1, 11, 1)$
168 : $P_{3163} = (10, 4, 11, 1)$
169 : $P_{3180} = (11, 5, 11, 1)$
170 : $P_{3259} = (10, 10, 11, 1)$
171 : $P_{3260} = (11, 10, 11, 1)$
172 : $P_{3277} = (12, 11, 11, 1)$
173 : $P_{3278} = (13, 11, 11, 1)$
174 : $P_{3323} = (10, 14, 11, 1)$
175 : $P_{3340} = (11, 15, 11, 1)$
176 : $P_{3375} = (14, 1, 12, 1)$
177 : $P_{3376} = (15, 1, 12, 1)$
178 : $P_{3380} = (3, 2, 12, 1)$
179 : $P_{3389} = (12, 2, 12, 1)$
180 : $P_{3391} = (14, 2, 12, 1)$
181 : $P_{3407} = (14, 3, 12, 1)$
182 : $P_{3413} = (4, 4, 12, 1)$
183 : $P_{3432} = (7, 5, 12, 1)$
184 : $P_{3435} = (10, 5, 12, 1)$
185 : $P_{3437} = (12, 5, 12, 1)$
186 : $P_{3446} = (5, 6, 12, 1)$
187 : $P_{3469} = (12, 7, 12, 1)$
188 : $P_{3474} = (1, 8, 12, 1)$
189 : $P_{3475} = (2, 8, 12, 1)$
190 : $P_{3552} = (15, 12, 12, 1)$
191 : $P_{3568} = (15, 13, 12, 1)$
192 : $P_{3621} = (4, 1, 13, 1)$
193 : $P_{3622} = (5, 1, 13, 1)$
194 : $P_{3650} = (1, 3, 13, 1)$
195 : $P_{3658} = (9, 3, 13, 1)$
196 : $P_{3710} = (13, 6, 13, 1)$
197 : $P_{3728} = (15, 7, 13, 1)$
198 : $P_{3733} = (4, 8, 13, 1)$
199 : $P_{3749} = (4, 9, 13, 1)$
200 : $P_{3753} = (8, 9, 13, 1)$
201 : $P_{3758} = (13, 9, 13, 1)$
202 : $P_{3798} = (5, 12, 13, 1)$
203 : $P_{3814} = (5, 13, 13, 1)$

204 : $P_{3839} = (14, 14, 13, 1)$
205 : $P_{3847} = (6, 15, 13, 1)$
206 : $P_{3851} = (10, 15, 13, 1)$
207 : $P_{3854} = (13, 15, 13, 1)$
208 : $P_{3897} = (8, 2, 14, 1)$
209 : $P_{3908} = (3, 3, 14, 1)$
210 : $P_{3946} = (9, 5, 14, 1)$
211 : $P_{3971} = (2, 7, 14, 1)$
212 : $P_{3974} = (5, 7, 14, 1)$
213 : $P_{3975} = (6, 7, 14, 1)$
214 : $P_{3988} = (3, 8, 14, 1)$
215 : $P_{4007} = (6, 9, 14, 1)$
216 : $P_{4035} = (2, 11, 14, 1)$
217 : $P_{4036} = (3, 11, 14, 1)$
218 : $P_{4051} = (2, 12, 14, 1)$
219 : $P_{4066} = (1, 13, 14, 1)$
220 : $P_{4069} = (4, 13, 14, 1)$
221 : $P_{4087} = (6, 14, 14, 1)$
222 : $P_{4092} = (11, 14, 14, 1)$
223 : $P_{4093} = (12, 14, 14, 1)$
224 : $P_{4150} = (5, 2, 15, 1)$
225 : $P_{4176} = (15, 3, 15, 1)$
226 : $P_{4178} = (1, 4, 15, 1)$
227 : $P_{4186} = (9, 4, 15, 1)$
228 : $P_{4197} = (4, 5, 15, 1)$
229 : $P_{4201} = (8, 5, 15, 1)$
230 : $P_{4206} = (13, 5, 15, 1)$
231 : $P_{4223} = (14, 6, 15, 1)$
232 : $P_{4254} = (13, 8, 15, 1)$
233 : $P_{4262} = (5, 9, 15, 1)$
234 : $P_{4293} = (4, 11, 15, 1)$
235 : $P_{4294} = (5, 11, 15, 1)$
236 : $P_{4327} = (6, 13, 15, 1)$
237 : $P_{4331} = (10, 13, 15, 1)$
238 : $P_{4334} = (13, 13, 15, 1)$
239 : $P_{4341} = (4, 14, 15, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 289 points:
The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{14} = (10, 1, 0, 0)$
3 : $P_{15} = (11, 1, 0, 0)$
4 : $P_{60} = (9, 2, 1, 0)$
5 : $P_{73} = (6, 3, 1, 0)$
6 : $P_{97} = (14, 4, 1, 0)$
7 : $P_{112} = (13, 5, 1, 0)$
8 : $P_{118} = (3, 6, 1, 0)$
9 : $P_{139} = (8, 7, 1, 0)$
10 : $P_{154} = (7, 8, 1, 0)$
11 : $P_{165} = (2, 9, 1, 0)$
12 : $P_{226} = (15, 12, 1, 0)$
13 : $P_{232} = (5, 13, 1, 0)$
14 : $P_{247} = (4, 14, 1, 0)$
15 : $P_{271} = (12, 15, 1, 0)$
16 : $P_{290} = (0, 1, 0, 1)$
17 : $P_{291} = (1, 1, 0, 1)$
18 : $P_{434} = (0, 10, 0, 1)$
19 : $P_{435} = (1, 10, 0, 1)$
20 : $P_{450} = (0, 11, 0, 1)$
21 : $P_{451} = (1, 11, 0, 1)$
22 : $P_{546} = (0, 1, 1, 1)$
23 : $P_{555} = (10, 1, 1, 1)$
24 : $P_{556} = (11, 1, 1, 1)$
25 : $P_{635} = (10, 6, 1, 1)$
26 : $P_{651} = (10, 7, 1, 1)$
27 : $P_{689} = (0, 10, 1, 1)$
28 : $P_{701} = (12, 10, 1, 1)$
29 : $P_{702} = (13, 10, 1, 1)$
30 : $P_{705} = (0, 11, 1, 1)$
31 : $P_{711} = (6, 11, 1, 1)$
32 : $P_{712} = (7, 11, 1, 1)$
33 : $P_{732} = (11, 12, 1, 1)$
34 : $P_{748} = (11, 13, 1, 1)$
35 : $P_{801} = (0, 1, 2, 1)$
36 : $P_{823} = (6, 2, 2, 1)$
37 : $P_{827} = (10, 2, 2, 1)$
38 : $P_{830} = (13, 2, 2, 1)$

39 : $P_{864} = (15, 4, 2, 1)$
40 : $P_{870} = (5, 5, 2, 1)$
41 : $P_{885} = (4, 6, 2, 1)$
42 : $P_{898} = (1, 7, 2, 1)$
43 : $P_{906} = (9, 7, 2, 1)$
44 : $P_{927} = (14, 8, 2, 1)$
45 : $P_{945} = (0, 10, 2, 1)$
46 : $P_{949} = (4, 10, 2, 1)$
47 : $P_{950} = (5, 10, 2, 1)$
48 : $P_{961} = (0, 11, 2, 1)$
49 : $P_{981} = (4, 12, 2, 1)$
50 : $P_{985} = (8, 12, 2, 1)$
51 : $P_{990} = (13, 12, 2, 1)$
52 : $P_{1022} = (13, 14, 2, 1)$
53 : $P_{1030} = (5, 15, 2, 1)$
54 : $P_{1057} = (0, 1, 3, 1)$
55 : $P_{1082} = (9, 2, 3, 1)$
56 : $P_{1113} = (8, 4, 3, 1)$
57 : $P_{1124} = (3, 5, 3, 1)$
58 : $P_{1160} = (7, 7, 3, 1)$
59 : $P_{1164} = (11, 7, 3, 1)$
60 : $P_{1166} = (13, 7, 3, 1)$
61 : $P_{1176} = (7, 8, 3, 1)$
62 : $P_{1178} = (9, 8, 3, 1)$
63 : $P_{1184} = (15, 8, 3, 1)$
64 : $P_{1186} = (1, 9, 3, 1)$
65 : $P_{1199} = (14, 9, 3, 1)$
66 : $P_{1201} = (0, 10, 3, 1)$
67 : $P_{1209} = (8, 10, 3, 1)$
68 : $P_{1210} = (9, 10, 3, 1)$
69 : $P_{1217} = (0, 11, 3, 1)$
70 : $P_{1251} = (2, 13, 3, 1)$
71 : $P_{1273} = (8, 14, 3, 1)$
72 : $P_{1288} = (7, 15, 3, 1)$
73 : $P_{1313} = (0, 1, 4, 1)$
74 : $P_{1336} = (7, 2, 4, 1)$
75 : $P_{1353} = (8, 3, 4, 1)$
76 : $P_{1368} = (7, 4, 4, 1)$
77 : $P_{1372} = (11, 4, 4, 1)$
78 : $P_{1374} = (13, 4, 4, 1)$
79 : $P_{1400} = (7, 6, 4, 1)$
80 : $P_{1402} = (9, 6, 4, 1)$
81 : $P_{1408} = (15, 6, 4, 1)$
82 : $P_{1433} = (8, 8, 4, 1)$
83 : $P_{1444} = (3, 9, 4, 1)$
84 : $P_{1457} = (0, 10, 4, 1)$
85 : $P_{1473} = (0, 11, 4, 1)$
86 : $P_{1481} = (8, 11, 4, 1)$
87 : $P_{1482} = (9, 11, 4, 1)$
88 : $P_{1490} = (1, 12, 4, 1)$
89 : $P_{1503} = (14, 12, 4, 1)$
90 : $P_{1514} = (9, 13, 4, 1)$
91 : $P_{1539} = (2, 15, 4, 1)$
92 : $P_{1569} = (0, 1, 5, 1)$
93 : $P_{1600} = (15, 2, 5, 1)$
94 : $P_{1613} = (12, 3, 5, 1)$
95 : $P_{1631} = (14, 4, 5, 1)$
96 : $P_{1669} = (4, 7, 5, 1)$
97 : $P_{1686} = (5, 8, 5, 1)$
98 : $P_{1712} = (15, 9, 5, 1)$
99 : $P_{1713} = (0, 10, 5, 1)$
100 : $P_{1729} = (0, 11, 5, 1)$
101 : $P_{1743} = (14, 11, 5, 1)$
102 : $P_{1744} = (15, 11, 5, 1)$
103 : $P_{1752} = (7, 12, 5, 1)$
104 : $P_{1755} = (10, 12, 5, 1)$
105 : $P_{1757} = (12, 12, 5, 1)$
106 : $P_{1778} = (1, 14, 5, 1)$
107 : $P_{1779} = (2, 14, 5, 1)$
108 : $P_{1796} = (3, 15, 5, 1)$
109 : $P_{1805} = (12, 15, 5, 1)$
110 : $P_{1807} = (14, 15, 5, 1)$
111 : $P_{1825} = (0, 1, 6, 1)$
112 : $P_{1827} = (2, 1, 6, 1)$
113 : $P_{1828} = (3, 1, 6, 1)$
114 : $P_{1875} = (2, 4, 6, 1)$
115 : $P_{1878} = (5, 4, 6, 1)$
116 : $P_{1879} = (6, 4, 6, 1)$
117 : $P_{1891} = (2, 5, 6, 1)$
118 : $P_{1908} = (3, 6, 6, 1)$
119 : $P_{1924} = (3, 7, 6, 1)$
120 : $P_{1943} = (6, 8, 6, 1)$
121 : $P_{1948} = (11, 8, 6, 1)$
122 : $P_{1949} = (12, 8, 6, 1)$
123 : $P_{1962} = (9, 9, 6, 1)$
124 : $P_{1969} = (0, 10, 6, 1)$
125 : $P_{1985} = (0, 11, 6, 1)$
126 : $P_{2007} = (6, 12, 6, 1)$
127 : $P_{2025} = (8, 13, 6, 1)$
128 : $P_{2050} = (1, 15, 6, 1)$
129 : $P_{2053} = (4, 15, 6, 1)$
130 : $P_{2081} = (0, 1, 7, 1)$
131 : $P_{2089} = (8, 1, 7, 1)$
132 : $P_{2090} = (9, 1, 7, 1)$
133 : $P_{2099} = (2, 2, 7, 1)$
134 : $P_{2120} = (7, 3, 7, 1)$
135 : $P_{2124} = (11, 3, 7, 1)$
136 : $P_{2126} = (13, 3, 7, 1)$
137 : $P_{2146} = (1, 5, 7, 1)$
138 : $P_{2159} = (14, 5, 7, 1)$
139 : $P_{2169} = (8, 6, 7, 1)$
140 : $P_{2185} = (8, 7, 7, 1)$
141 : $P_{2225} = (0, 10, 7, 1)$
142 : $P_{2241} = (0, 11, 7, 1)$
143 : $P_{2260} = (3, 12, 7, 1)$
144 : $P_{2280} = (7, 13, 7, 1)$
145 : $P_{2296} = (7, 14, 7, 1)$
146 : $P_{2298} = (9, 14, 7, 1)$
147 : $P_{2304} = (15, 14, 7, 1)$
148 : $P_{2314} = (9, 15, 7, 1)$
149 : $P_{2337} = (0, 1, 8, 1)$
150 : $P_{2354} = (1, 2, 8, 1)$
151 : $P_{2357} = (4, 2, 8, 1)$
152 : $P_{2371} = (2, 3, 8, 1)$
153 : $P_{2374} = (5, 3, 8, 1)$
154 : $P_{2375} = (6, 3, 8, 1)$
155 : $P_{2388} = (3, 4, 8, 1)$
156 : $P_{2407} = (6, 5, 8, 1)$
157 : $P_{2423} = (6, 6, 8, 1)$
158 : $P_{2428} = (11, 6, 8, 1)$
159 : $P_{2429} = (12, 6, 8, 1)$
160 : $P_{2467} = (2, 9, 8, 1)$
161 : $P_{2481} = (0, 10, 8, 1)$
162 : $P_{2483} = (2, 10, 8, 1)$
163 : $P_{2484} = (3, 10, 8, 1)$
164 : $P_{2497} = (0, 11, 8, 1)$
165 : $P_{2522} = (9, 12, 8, 1)$
166 : $P_{2548} = (3, 14, 8, 1)$
167 : $P_{2569} = (8, 15, 8, 1)$
168 : $P_{2593} = (0, 1, 9, 1)$
169 : $P_{2629} = (4, 3, 9, 1)$
170 : $P_{2653} = (12, 4, 9, 1)$
171 : $P_{2672} = (15, 5, 9, 1)$
172 : $P_{2674} = (1, 6, 9, 1)$
173 : $P_{2675} = (2, 6, 9, 1)$
174 : $P_{2703} = (14, 7, 9, 1)$
175 : $P_{2728} = (7, 9, 9, 1)$
176 : $P_{2731} = (10, 9, 9, 1)$
177 : $P_{2733} = (12, 9, 9, 1)$
178 : $P_{2737} = (0, 10, 9, 1)$
179 : $P_{2751} = (14, 10, 9, 1)$
180 : $P_{2752} = (15, 10, 9, 1)$
181 : $P_{2753} = (0, 11, 9, 1)$
182 : $P_{2788} = (3, 13, 9, 1)$
183 : $P_{2797} = (12, 13, 9, 1)$
184 : $P_{2799} = (14, 13, 9, 1)$
185 : $P_{2806} = (5, 14, 9, 1)$
186 : $P_{2832} = (15, 15, 9, 1)$
187 : $P_{2849} = (0, 1, 10, 1)$
188 : $P_{2861} = (12, 1, 10, 1)$
189 : $P_{2862} = (13, 1, 10, 1)$
190 : $P_{2876} = (11, 2, 10, 1)$
191 : $P_{2891} = (10, 3, 10, 1)$
192 : $P_{2971} = (10, 8, 10, 1)$
193 : $P_{2988} = (11, 9, 10, 1)$
194 : $P_{2993} = (0, 10, 10, 1)$
195 : $P_{2999} = (6, 10, 10, 1)$
196 : $P_{3000} = (7, 10, 10, 1)$
197 : $P_{3009} = (0, 11, 10, 1)$
198 : $P_{3019} = (10, 11, 10, 1)$
199 : $P_{3020} = (11, 11, 10, 1)$
200 : $P_{3105} = (0, 1, 11, 1)$

201 : $P_{3111} = (6, 1, 11, 1)$
202 : $P_{3112} = (7, 1, 11, 1)$
203 : $P_{3163} = (10, 4, 11, 1)$
204 : $P_{3180} = (11, 5, 11, 1)$
205 : $P_{3249} = (0, 10, 11, 1)$
206 : $P_{3259} = (10, 10, 11, 1)$
207 : $P_{3260} = (11, 10, 11, 1)$
208 : $P_{3265} = (0, 11, 11, 1)$
209 : $P_{3277} = (12, 11, 11, 1)$
210 : $P_{3278} = (13, 11, 11, 1)$
211 : $P_{3323} = (10, 14, 11, 1)$
212 : $P_{3340} = (11, 15, 11, 1)$
213 : $P_{3361} = (0, 1, 12, 1)$
214 : $P_{3375} = (14, 1, 12, 1)$
215 : $P_{3376} = (15, 1, 12, 1)$
216 : $P_{3380} = (3, 2, 12, 1)$
217 : $P_{3389} = (12, 2, 12, 1)$
218 : $P_{3391} = (14, 2, 12, 1)$
219 : $P_{3407} = (14, 3, 12, 1)$
220 : $P_{3413} = (4, 4, 12, 1)$
221 : $P_{3432} = (7, 5, 12, 1)$
222 : $P_{3435} = (10, 5, 12, 1)$
223 : $P_{3437} = (12, 5, 12, 1)$
224 : $P_{3446} = (5, 6, 12, 1)$
225 : $P_{3469} = (12, 7, 12, 1)$
226 : $P_{3474} = (1, 8, 12, 1)$
227 : $P_{3475} = (2, 8, 12, 1)$
228 : $P_{3505} = (0, 10, 12, 1)$
229 : $P_{3521} = (0, 11, 12, 1)$
230 : $P_{3552} = (15, 12, 12, 1)$
231 : $P_{3568} = (15, 13, 12, 1)$
232 : $P_{3617} = (0, 1, 13, 1)$
233 : $P_{3621} = (4, 1, 13, 1)$
234 : $P_{3622} = (5, 1, 13, 1)$
235 : $P_{3650} = (1, 3, 13, 1)$
236 : $P_{3658} = (9, 3, 13, 1)$
237 : $P_{3710} = (13, 6, 13, 1)$
238 : $P_{3728} = (15, 7, 13, 1)$
239 : $P_{3733} = (4, 8, 13, 1)$
240 : $P_{3749} = (4, 9, 13, 1)$
241 : $P_{3753} = (8, 9, 13, 1)$
242 : $P_{3758} = (13, 9, 13, 1)$
243 : $P_{3761} = (0, 10, 13, 1)$
244 : $P_{3777} = (0, 11, 13, 1)$
245 : $P_{3798} = (5, 12, 13, 1)$
246 : $P_{3814} = (5, 13, 13, 1)$
247 : $P_{3839} = (14, 14, 13, 1)$
248 : $P_{3847} = (6, 15, 13, 1)$
249 : $P_{3851} = (10, 15, 13, 1)$
250 : $P_{3854} = (13, 15, 13, 1)$
251 : $P_{3873} = (0, 1, 14, 1)$
252 : $P_{3897} = (8, 2, 14, 1)$
253 : $P_{3908} = (3, 3, 14, 1)$
254 : $P_{3946} = (9, 5, 14, 1)$
255 : $P_{3971} = (2, 7, 14, 1)$
256 : $P_{3974} = (5, 7, 14, 1)$
257 : $P_{3975} = (6, 7, 14, 1)$
258 : $P_{3988} = (3, 8, 14, 1)$
259 : $P_{4007} = (6, 9, 14, 1)$
260 : $P_{4017} = (0, 10, 14, 1)$
261 : $P_{4033} = (0, 11, 14, 1)$
262 : $P_{4035} = (2, 11, 14, 1)$
263 : $P_{4036} = (3, 11, 14, 1)$
264 : $P_{4051} = (2, 12, 14, 1)$
265 : $P_{4066} = (1, 13, 14, 1)$
266 : $P_{4069} = (4, 13, 14, 1)$
267 : $P_{4087} = (6, 14, 14, 1)$
268 : $P_{4092} = (11, 14, 14, 1)$
269 : $P_{4093} = (12, 14, 14, 1)$
270 : $P_{4129} = (0, 1, 15, 1)$
271 : $P_{4150} = (5, 2, 15, 1)$
272 : $P_{4176} = (15, 3, 15, 1)$
273 : $P_{4178} = (1, 4, 15, 1)$
274 : $P_{4186} = (9, 4, 15, 1)$
275 : $P_{4197} = (4, 5, 15, 1)$
276 : $P_{4201} = (8, 5, 15, 1)$
277 : $P_{4206} = (13, 5, 15, 1)$
278 : $P_{4223} = (14, 6, 15, 1)$
279 : $P_{4254} = (13, 8, 15, 1)$
280 : $P_{4262} = (5, 9, 15, 1)$
281 : $P_{4273} = (0, 10, 15, 1)$
282 : $P_{4289} = (0, 11, 15, 1)$
283 : $P_{4293} = (4, 11, 15, 1)$
284 : $P_{4294} = (5, 11, 15, 1)$
285 : $P_{4327} = (6, 13, 15, 1)$
286 : $P_{4331} = (10, 13, 15, 1)$
287 : $P_{4334} = (13, 13, 15, 1)$
288 : $P_{4341} = (4, 14, 15, 1)$