Rank-65831 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_2 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431722345

General information

Number of lines	24
Number of points	33
Number of singular points	9
Number of Eckardt points	0
Number of double points	0
Number of single points	12
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{24}
Type of lines on points	8, 5^{20}, 1^{12}

Singular Points

The surface has 9 singular points:

0 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
1 : $P_6 = \mathbf{P}(\omega,1,0,0) = \mathbf{P}(2,1,0,0)$
2 : $P_7 = \mathbf{P}(\omega^2,1,0,0) = \mathbf{P}(3,1,0,0)$
3 : $P_{28} = \mathbf{P}(\omega,1,0,1) = \mathbf{P}(2,1,0,1)$
4 : $P_{29} = \mathbf{P}(\omega^2,1,0,1) = \mathbf{P}(3,1,0,1)$
5 : $P_{31} = \mathbf{P}(1,\omega,0,1) = \mathbf{P}(1,2,0,1)$
6 : $P_{33} = \mathbf{P}(\omega^2,\omega,0,1) = \mathbf{P}(3,2,0,1)$
7 : $P_{35} = \mathbf{P}(1,\omega^2,0,1) = \mathbf{P}(1,3,0,1)$
8 : $P_{36} = \mathbf{P}(\omega,\omega^2,0,1) = \mathbf{P}(2,3,0,1)$

The 24 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \mathbf{Pl}(0,0,0,0,1,0)_{25}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_4 = \mathbf{Pl}(1,0,0,0,1,0)_{26}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{12} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right]_{12} = \mathbf{Pl}(2,0,0,0,1,0)_{27}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \omega \end{array}\right]_8 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]_8 = \mathbf{Pl}(3,0,0,0,1,0)_{28}$$

$$\ell_5 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_6 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{84} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 0 \end{array}\right]_{252} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]_{252} = \mathbf{Pl}(2,0,0,1,0,0)_{11}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 0 \end{array}\right]_{168} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{array}\right]_{168} = \mathbf{Pl}(3,0,0,1,0,0)_{12}$$

$$\ell_9 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{41} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{88} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{88} = \mathbf{Pl}(1,0,0,1,1,0)_{54}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{264} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \end{array}\right]_{264} = \mathbf{Pl}(2,0,0,1,1,0)_{55}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & \omega \end{array}\right]_{176} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \end{array}\right]_{176} = \mathbf{Pl}(3,0,0,1,1,0)_{56}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{83} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{83} = \mathbf{Pl}(0,0,0,3,1,0)_{67}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \omega \end{array}\right]_{92} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{array}\right]_{92} = \mathbf{Pl}(3,0,0,3,1,0)_{70}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 1 \end{array}\right]_{256} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \end{array}\right]_{256} = \mathbf{Pl}(1,0,0,3,1,0)_{68}$$

$$\ell_{16} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{180} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \end{array}\right]_{180} = \mathbf{Pl}(2,0,0,3,1,0)_{69}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & \omega & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{62} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{62} = \mathbf{Pl}(0,0,0,2,1,0)_{60}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{96} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \end{array}\right]_{96} = \mathbf{Pl}(2,0,0,2,1,0)_{62}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & \omega \end{array}\right]_{260} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right]_{260} = \mathbf{Pl}(3,0,0,2,1,0)_{63}$$

$$\ell_{20} = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 1 \end{bmatrix}_{172} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{172} = \mathbf{Pl}(1,0,0,2,1,0)_{61}$$

$$\ell_{21} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_{22} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_{23} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

Rank of lines: (0, 20, 4, 12, 8, 340, 84, 252, 168, 41, 88, 264, 176, 83, 92, 256, 180, 62, 96, 260, 172, 104, 345, 125)
Rank of points on Klein quadric: (0, 25, 26, 27, 28, 9, 10, 11, 12, 53, 54, 55, 56, 67, 70, 68, 69, 60, 62, 63, 61, 29, 13, 57)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 12 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_{23}
1 : $P_8 = (1,0,1,0)$ lies on line ℓ_{21}
2 : $P_{11} = (0,1,1,0)$ lies on line ℓ_{22}
3 : $P_{12} = (1,1,1,0)$ lies on line ℓ_{23}
4 : $P_{39} = (1,0,1,1)$ lies on line ℓ_{21}
5 : $P_{42} = (0,1,1,1)$ lies on line ℓ_{22}
6 : $P_{55} = (2,0,2,1)$ lies on line ℓ_{21}
7 : $P_{61} = (0,2,2,1)$ lies on line ℓ_{22}
8 : $P_{63} = (2,2,2,1)$ lies on line ℓ_{23}
9 : $P_{72} = (3,0,3,1)$ lies on line ℓ_{21}
10 : $P_{81} = (0,3,3,1)$ lies on line ℓ_{22}
11 : $P_{84} = (3,3,3,1)$ lies on line ℓ_{23}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
3	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
7	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
8	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
21	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	1	1
22	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	1
23	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_1	P_1	P_1	P_1	P_5	P_5	P_5	P_5	P_6	P_6	P_6	P_6	P_7	P_7	P_7	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_0	P_0	P_0	P_0	P_3	P_{23}	P_{24}	P_{25}	P_3	P_{23}	P_{24}	P_{25}	P_3	P_{23}	P_{24}	P_{25}	P_3	P_{23}	P_{24}	P_{25}	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_{26}	P_{27}	P_{28}	P_{29}	P_{27}	P_{26}	P_{29}	P_{28}	P_{28}	P_{29}	P_{26}	P_{27}	P_{29}	P_{28}	P_{27}	P_{26}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_{30}	P_{31}	P_{32}	P_{33}	P_{32}	P_{33}	P_{30}	P_{31}	P_{33}	P_{32}	P_{31}	P_{30}	P_{31}	P_{30}	P_{33}	P_{32}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_{34}	P_{35}	P_{36}	P_{37}	P_{37}	P_{36}	P_{35}	P_{34}	P_{35}	P_{34}	P_{37}	P_{36}	P_{36}	P_{37}	P_{34}	P_{35}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_1	P_3	P_{26}	P_{30}	P_{34}	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{34}	P_{26}	P_{30}	P_3	P_{30}	P_{34}	P_{26}	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_{23}	P_{27}	P_{31}	P_{35}	P_1	P_1	P_1	P_{27}	P_{23}	P_{35}	P_{31}	P_{35}	P_{23}	P_{31}	P_{27}	P_{31}	P_{23}	P_{27}	P_{35}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_{24}	P_{28}	P_{32}	P_{36}	P_1	P_1	P_1	P_{32}	P_{36}	P_{24}	P_{28}	P_{28}	P_{32}	P_{24}	P_{36}	P_{36}	P_{28}	P_{24}	P_{32}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_{25}	P_{29}	P_{33}	P_{37}	P_1	P_1	P_1	P_{37}	P_{33}	P_{29}	P_{25}	P_{33}	P_{29}	P_{37}	P_{25}	P_{29}	P_{37}	P_{33}	P_{25}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_2
in point	P_5	P_3	P_{27}	P_{32}	P_{37}	P_3	P_{27}	P_{32}	P_{37}	P_5	P_5	P_5	P_3	P_{32}	P_{37}	P_{27}	P_3	P_{37}	P_{27}	P_{32}	P_3

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_5	P_{23}	P_{26}	P_{33}	P_{36}	P_{26}	P_{23}	P_{36}	P_{33}	P_5	P_5	P_5	P_{33}	P_{23}	P_{26}	P_{36}	P_{36}	P_{23}	P_{33}	P_{26}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_5	P_{24}	P_{29}	P_{30}	P_{35}	P_{30}	P_{35}	P_{24}	P_{29}	P_5	P_5	P_5	P_{35}	P_{29}	P_{24}	P_{30}	P_{29}	P_{30}	P_{24}	P_{35}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_5	P_{25}	P_{28}	P_{31}	P_{34}	P_{34}	P_{31}	P_{28}	P_{25}	P_5	P_5	P_5	P_{28}	P_{34}	P_{31}	P_{25}	P_{31}	P_{28}	P_{34}	P_{25}

Line 13 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_2
in point	P_6	P_3	P_{28}	P_{33}	P_{35}	P_3	P_{35}	P_{28}	P_{33}	P_3	P_{33}	P_{35}	P_{28}	P_6	P_6	P_6	P_3	P_{28}	P_{33}	P_{35}	P_3

Line 14 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_6	P_{23}	P_{29}	P_{32}	P_{34}	P_{34}	P_{23}	P_{32}	P_{29}	P_{32}	P_{23}	P_{29}	P_{34}	P_6	P_6	P_6	P_{29}	P_{23}	P_{34}	P_{32}

Line 15 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_6	P_{24}	P_{26}	P_{31}	P_{37}	P_{26}	P_{31}	P_{24}	P_{37}	P_{37}	P_{26}	P_{24}	P_{31}	P_6	P_6	P_6	P_{31}	P_{37}	P_{24}	P_{26}

Line 16 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_6	P_{25}	P_{27}	P_{30}	P_{36}	P_{30}	P_{27}	P_{36}	P_{25}	P_{27}	P_{36}	P_{30}	P_{25}	P_6	P_6	P_6	P_{36}	P_{30}	P_{27}	P_{25}

Line 17 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_2
in point	P_7	P_3	P_{29}	P_{31}	P_{36}	P_3	P_{31}	P_{36}	P_{29}	P_3	P_{36}	P_{29}	P_{31}	P_3	P_{29}	P_{31}	P_{36}	P_7	P_7	P_7	P_3

Line 18 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}
in point	P_7	P_{23}	P_{28}	P_{30}	P_{37}	P_{30}	P_{23}	P_{28}	P_{37}	P_{37}	P_{23}	P_{30}	P_{28}	P_{28}	P_{23}	P_{37}	P_{30}	P_7	P_7	P_7

Line 19 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}
in point	P_7	P_{24}	P_{27}	P_{33}	P_{34}	P_{34}	P_{27}	P_{24}	P_{33}	P_{27}	P_{33}	P_{24}	P_{34}	P_{33}	P_{34}	P_{24}	P_{27}	P_7	P_7	P_7

Line 20 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}
in point	P_7	P_{25}	P_{26}	P_{32}	P_{35}	P_{26}	P_{35}	P_{32}	P_{25}	P_{32}	P_{26}	P_{35}	P_{25}	P_{35}	P_{32}	P_{26}	P_{25}	P_7	P_7	P_7

Line 21 intersects

Line	ℓ_1	ℓ_5	ℓ_9	ℓ_{13}	ℓ_{17}	ℓ_{22}	ℓ_{23}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 22 intersects

Line	ℓ_1	ℓ_5	ℓ_9	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{23}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 23 intersects

Line	ℓ_1	ℓ_5	ℓ_9	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 33 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (1,0,1,0)$
$8: P_{11} = (0,1,1,0)$
$9: P_{12} = (1,1,1,0)$
$10: P_{23} = (1,0,0,1)$
$11: P_{24} = (2,0,0,1)$
$12: P_{25} = (3,0,0,1)$
$13: P_{26} = (0,1,0,1)$
$14: P_{27} = (1,1,0,1)$
$15: P_{28} = (2,1,0,1)$
$16: P_{29} = (3,1,0,1)$
$17: P_{30} = (0,2,0,1)$
$18: P_{31} = (1,2,0,1)$
$19: P_{32} = (2,2,0,1)$
$20: P_{33} = (3,2,0,1)$
$21: P_{34} = (0,3,0,1)$
$22: P_{35} = (1,3,0,1)$
$23: P_{36} = (2,3,0,1)$
$24: P_{37} = (3,3,0,1)$
$25: P_{39} = (1,0,1,1)$
$26: P_{42} = (0,1,1,1)$
$27: P_{55} = (2,0,2,1)$
$28: P_{61} = (0,2,2,1)$
$29: P_{63} = (2,2,2,1)$
$30: P_{72} = (3,0,3,1)$
$31: P_{81} = (0,3,3,1)$
$32: P_{84} = (3,3,3,1)$