Rank-264 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1^2 = 0$$

(0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 10930

General information

Number of lines	7
Number of points	29
Number of singular points	0
Number of Eckardt points	3
Number of double points	0
Number of single points	26
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	3^3, 1^{26}

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$

Rank of lines: (17, 337, 339, 338, 38, 26, 89)
Rank of points on Klein quadric: (32, 129, 143, 136, 198, 180, 74)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$,
$2 : P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 26 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_4 = (1,1,1,1)$ lies on line ℓ_4
$2 : P_5 = (1,1,0,0)$ lies on line ℓ_4
$3 : P_8 = (1,0,1,0)$ lies on line ℓ_5
$4 : P_{12} = (1,1,1,0)$ lies on line ℓ_6
$5 : P_{23} = (1,0,0,1)$ lies on line ℓ_6
$6 : P_{27} = (1,1,0,1)$ lies on line ℓ_5
$7 : P_{39} = (1,0,1,1)$ lies on line ℓ_0
$8 : P_{40} = (2,0,1,1)$ lies on line ℓ_0
$9 : P_{41} = (3,0,1,1)$ lies on line ℓ_0
$10 : P_{45} = (0,2,1,1)$ lies on line ℓ_1
$11 : P_{47} = (2,2,1,1)$ lies on line ℓ_4
$12 : P_{49} = (0,3,1,1)$ lies on line ℓ_1
$13 : P_{52} = (3,3,1,1)$ lies on line ℓ_4
$14 : P_{53} = (0,0,2,1)$ lies on line ℓ_2
$15 : P_{57} = (0,1,2,1)$ lies on line ℓ_2
$16 : P_{60} = (3,1,2,1)$ lies on line ℓ_5
$17 : P_{61} = (0,2,2,1)$ lies on line ℓ_2
$18 : P_{64} = (3,2,2,1)$ lies on line ℓ_6
$19 : P_{65} = (0,3,2,1)$ lies on line ℓ_2
$20 : P_{69} = (0,0,3,1)$ lies on line ℓ_3
$21 : P_{73} = (0,1,3,1)$ lies on line ℓ_3
$22 : P_{75} = (2,1,3,1)$ lies on line ℓ_5
$23 : P_{77} = (0,2,3,1)$ lies on line ℓ_3
$24 : P_{81} = (0,3,3,1)$ lies on line ℓ_3
$25 : P_{83} = (2,3,3,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 0 0 1 0 0
1	1 0 1 1 1 1 1
2	0 1 0 1 0 0 0
3	0 1 1 0 0 0 0
4	1 1 0 0 0 0 0
5	0 1 0 0 0 0 1
6	0 1 0 0 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_4
in point	P_{38}	P_{38}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{38}	P_1	P_1	P_{38}	P_{42}	P_{42}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_1	P_1

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 4 intersects

Line	ℓ_0	ℓ_1
in point	P_{38}	P_{38}

Line 5 intersects

Line	ℓ_1	ℓ_6
in point	P_{42}	P_{42}

Line 6 intersects

Line	ℓ_1	ℓ_5
in point	P_{42}	P_{42}

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_8 = (1,0,1,0)$
$5: P_{12} = (1,1,1,0)$
$6: P_{23} = (1,0,0,1)$
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$25:\ P_{75} = (2,1,3,1)$
$26:\ P_{77} = (0,2,3,1)$
$27:\ P_{81} = (0,3,3,1)$
$28:\ P_{83} = (2,3,3,1)$