Rank-74275 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(16) is 287379733

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	90
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{6}
Type of lines on points	$4, 2^4, 1^{90}, 0^{210}$

Singular Points

The surface has 1 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{4656}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{4625}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_{1}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{4672}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{5586}$$

Rank of lines: (0, 529, 69888, 70160, 4897, 4642)

Rank of points on Klein quadric: (0, 4656, 4625, 1, 4672, 5586)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points: The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{35} = (0, 1, 1, 0) = \ell_2 \cap \ell_5$$

 $P_{291} = (1, 1, 0, 1) = \ell_4 \cap \ell_5$

Single Points

The surface has 90 single points: The single points on the surface are:

 $0: P_0 = (1,0,0,0)$ lies on line ℓ_0 1: $P_3 = (0,0,0,1)$ lies on line ℓ_3 2: $P_4 = (1, 1, 1, 1)$ lies on line ℓ_4 $3: P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 4: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 6: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 7: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 8: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 9: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 10: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 11: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 12: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 13: $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 14: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 15: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 16: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 17: $P_{36} = (1, 1, 1, 0)$ lies on line ℓ_1 18: $P_{51} = (0, 2, 1, 0)$ lies on line ℓ_2 19: $P_{53} = (2, 2, 1, 0)$ lies on line ℓ_1 20: $P_{67} = (0, 3, 1, 0)$ lies on line ℓ_2 21: $P_{70} = (3, 3, 1, 0)$ lies on line ℓ_1 22: $P_{83} = (0, 4, 1, 0)$ lies on line ℓ_2 23 : $P_{87} = (4,4,1,0)$ lies on line ℓ_1 24: $P_{99} = (0, 5, 1, 0)$ lies on line ℓ_2 25: $P_{104} = (5, 5, 1, 0)$ lies on line ℓ_1 26: $P_{115} = (0, 6, 1, 0)$ lies on line ℓ_2 27: $P_{121} = (6, 6, 1, 0)$ lies on line ℓ_1 28: $P_{131} = (0, 7, 1, 0)$ lies on line ℓ_2 29: $P_{138} = (7,7,1,0)$ lies on line ℓ_1 $30: P_{147} = (0, 8, 1, 0)$ lies on line ℓ_2 31: $P_{155} = (8, 8, 1, 0)$ lies on line ℓ_1 $32: P_{163} = (0, 9, 1, 0)$ lies on line ℓ_2 33: $P_{172} = (9, 9, 1, 0)$ lies on line ℓ_1

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63: P_{2065} = (0, 0, 7, 1) lies on line \ell_3
34: P_{179} = (0, 10, 1, 0) lies on line \ell_2
                                                                      64: P_{2082} = (1, 1, 7, 1) lies on line \ell_4
35: P_{189} = (10, 10, 1, 0) lies on line \ell_1
36: P_{195} = (0, 11, 1, 0) lies on line \ell_2
                                                                      65: P_{2162} = (1, 6, 7, 1) lies on line \ell_5
37: P_{206} = (11, 11, 1, 0) lies on line \ell_1
                                                                      66: P_{2321} = (0, 0, 8, 1) lies on line \ell_3
                                                                      67 : P_{2338} = (1, 1, 8, 1) lies on line \ell_4
38: P_{211} = (0, 12, 1, 0) lies on line \ell_2
39: P_{223} = (12, 12, 1, 0) lies on line \ell_1
                                                                      68: P_{2466} = (1, 9, 8, 1) lies on line \ell_5
40: P_{227} = (0, 13, 1, 0) lies on line \ell_2
                                                                      69: P_{2577} = (0,0,9,1) lies on line \ell_3
41: P_{240} = (13, 13, 1, 0) lies on line \ell_1
                                                                      70: P_{2594} = (1, 1, 9, 1) lies on line \ell_4
42: P_{243} = (0, 14, 1, 0) lies on line \ell_2
                                                                      71: P_{2706} = (1, 8, 9, 1) lies on line \ell_5
43: P_{257} = (14, 14, 1, 0) lies on line \ell_1
                                                                      72: P_{2833} = (0, 0, 10, 1) lies on line \ell_3
44: P_{259} = (0, 15, 1, 0) lies on line \ell_2
                                                                      73: P_{2850} = (1, 1, 10, 1) lies on line \ell_4
45: P_{274} = (15, 15, 1, 0) lies on line \ell_1
                                                                      74: P_{3010} = (1, 11, 10, 1) lies on line \ell_5
46: P_{530} = (0,0,1,1) lies on line \ell_3
                                                                      75: P_{3089} = (0, 0, 11, 1) lies on line \ell_3
47: P_{531} = (1,0,1,1) lies on line \ell_5
                                                                      76: P_{3106} = (1, 1, 11, 1) lies on line \ell_4
48: P_{785} = (0, 0, 2, 1) lies on line \ell_3
                                                                      77: P_{3250} = (1, 10, 11, 1) lies on line \ell_5
49: P_{802} = (1, 1, 2, 1) lies on line \ell_4
                                                                      78: P_{3345} = (0, 0, 12, 1) lies on line \ell_3
50: P_{834} = (1, 3, 2, 1) lies on line \ell_5
                                                                      79: P_{3362} = (1, 1, 12, 1) lies on line \ell_4
51: P_{1041} = (0,0,3,1) lies on line \ell_3
                                                                      80: P_{3554} = (1, 13, 12, 1) lies on line \ell_5
52: P_{1058} = (1, 1, 3, 1) lies on line \ell_4
                                                                      81: P_{3601} = (0, 0, 13, 1) lies on line \ell_3
                                                                      82 : P_{3618} = (1, 1, 13, 1) lies on line \ell_4
53: P_{1074} = (1, 2, 3, 1) lies on line \ell_5
54: P_{1297} = (0,0,4,1) lies on line \ell_3
                                                                      83: P_{3794} = (1, 12, 13, 1) lies on line \ell_5
55: P_{1314} = (1, 1, 4, 1) lies on line \ell_4
                                                                      84: P_{3857} = (0, 0, 14, 1) lies on line \ell_3
56: P_{1378} = (1, 5, 4, 1) lies on line \ell_5
                                                                      85: P_{3874} = (1, 1, 14, 1) lies on line \ell_4
                                                                      86 : P_{4098} = (1, 15, 14, 1) lies on line \ell_5
57: P_{1553} = (0,0,5,1) lies on line \ell_3
58: P_{1570} = (1, 1, 5, 1) lies on line \ell_4
                                                                      87: P_{4113} = (0, 0, 15, 1) lies on line \ell_3
59: P_{1618} = (1, 4, 5, 1) lies on line \ell_5
                                                                      88: P_{4130} = (1, 1, 15, 1) lies on line \ell_4
60: P_{1809} = (0,0,6,1) lies on line \ell_3
                                                                      89: P_{4338} = (1, 14, 15, 1) lies on line \ell_5
61: P_{1826} = (1, 1, 6, 1) lies on line \ell_4
62: P_{1922} = (1,7,6,1) lies on line \ell_5
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The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line: The points on the surface but not on lines are:

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15: P_{588} = (11, 3, 1, 1)
0: P_{310} = (4, 2, 0, 1)
1: P_{327} = (5, 3, 0, 1)
                                                                   16: P_{619} = (10, 5, 1, 1)
2: P_{347} = (9, 4, 0, 1)
                                                                   17: P_{623} = (14, 5, 1, 1)
3: P_{362} = (8, 5, 0, 1)
                                                                   18: P_{659} = (2, 8, 1, 1)
                                                                   19: P_{668} = (11, 8, 1, 1)
4: P_{383} = (13, 6, 0, 1)
5: P_{398} = (12, 7, 0, 1)
                                                                   20: P_{691} = (2, 10, 1, 1)
6: P_{417} = (15, 8, 0, 1)
                                                                   21: P_{698} = (9, 10, 1, 1)
7: P_{432} = (14, 9, 0, 1)
                                                                   22: P_{709} = (4, 11, 1, 1)
8: P_{445} = (11, 10, 0, 1)
                                                                   23: P_{719} = (14, 11, 1, 1)
9: P_{460} = (10, 11, 0, 1)
                                                                   24: P_{773} = (4, 15, 1, 1)
                                                                   25: P_{779} = (10, 15, 1, 1)
10: P_{472} = (6, 12, 0, 1)
11: P_{489} = (7, 13, 0, 1)
                                                                   26: P_{797} = (12, 0, 2, 1)
12: P_{500} = (2, 14, 0, 1)
                                                                   27: P_{813} = (12, 1, 2, 1)
13: P_{517} = (3, 15, 0, 1)
                                                                   28: P_{823} = (6, 2, 2, 1)
14: P_{586} = (9, 3, 1, 1)
                                                                   29: P_{825} = (8, 2, 2, 1)
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84: P_{1915} = (10, 6, 6, 1)
30: P_{847} = (14, 3, 2, 1)
31: P_{885} = (4, 6, 2, 1)
                                                                 85: P_{1923} = (2,7,6,1)
32: P_{895} = (14, 6, 2, 1)
                                                                 86: P_{1943} = (6, 8, 6, 1)
                                                                 87: P_{1947} = (10, 8, 6, 1)
33: P_{969} = (8, 11, 2, 1)
34: P_{976} = (15, 11, 2, 1)
                                                                 88: P_{1987} = (2, 11, 6, 1)
35: P_{992} = (15, 12, 2, 1)
                                                                 89: P_{1998} = (13, 11, 6, 1)
36: P_{1013} = (4, 14, 2, 1)
                                                                 90: P_{2040} = (7, 14, 6, 1)
37: P_{1015} = (6, 14, 2, 1)
                                                                 91: P_{2046} = (13, 14, 6, 1)
38: P_{1049} = (8,0,3,1)
                                                                 92: P_{2079} = (14, 0, 7, 1)
39: P_{1065} = (8, 1, 3, 1)
                                                                 93: P_{2095} = (14, 1, 7, 1)
40: P_{1084} = (11, 2, 3, 1)
                                                                 94: P_{2120} = (7, 3, 7, 1)
41: P_{1125} = (4,5,3,1)
                                                                 95: P_{2123} = (10, 3, 7, 1)
42: P_{1130} = (9, 5, 3, 1)
                                                                 96: P_{2135} = (6, 4, 7, 1)
43: P_{1142} = (5, 6, 3, 1)
                                                                 97: P_{2141} = (12, 4, 7, 1)
44: P_{1148} = (11, 6, 3, 1)
                                                                 98: P_{2170} = (9, 6, 7, 1)
45: P_{1159} = (6,7,3,1)
                                                                 99: P_{2180} = (3,7,7,1)
46: P_{1162} = (9,7,3,1)
                                                                 100: P_{2187} = (10, 7, 7, 1)
47: P_{1172} = (3, 8, 3, 1)
                                                                 101: P_{2250} = (9, 11, 7, 1)
                                                                 102: P_{2253} = (12, 11, 7, 1)
48: P_{1189} = (4, 9, 3, 1)
49: P_{1190} = (5, 9, 3, 1)
                                                                 103: P_{2292} = (3, 14, 7, 1)
50: P_{1252} = (3, 13, 3, 1)
                                                                 104: P_{2311} = (6, 15, 7, 1)
                                                                 105: P_{2312} = (7, 15, 7, 1)
51: P_{1255} = (6, 13, 3, 1)
                                                                 106: P_{2324} = (3, 0, 8, 1)
52: P_{1303} = (6,0,4,1)
53: P_{1319} = (6, 1, 4, 1)
                                                                 107: P_{2340} = (3, 1, 8, 1)
54: P_{1338} = (9, 2, 4, 1)
                                                                 108: P_{2367} = (14, 2, 8, 1)
55: P_{1342} = (13, 2, 4, 1)
                                                                 109: P_{2368} = (15, 2, 8, 1)
                                                                 110: P_{2377} = (8, 3, 8, 1)
56: P_{1374} = (13, 4, 4, 1)
57: P_{1376} = (15, 4, 4, 1)
                                                                 111: P_{2419} = (2, 6, 8, 1)
58: P_{1379} = (2, 5, 4, 1)
                                                                 112: P_{2424} = (7, 6, 8, 1)
59: P_{1396} = (3, 6, 4, 1)
                                                                 113: P_{2444} = (11, 7, 8, 1)
60: P_{1460} = (3, 10, 4, 1)
                                                                 114: P_{2448} = (15, 7, 8, 1)
61: P_{1472} = (15, 10, 4, 1)
                                                                 115: P_{2476} = (11, 9, 8, 1)
62: P_{1507} = (2, 13, 4, 1)
                                                                 116: P_{2520} = (7, 12, 8, 1)
63: P_{1514} = (9, 13, 4, 1)
                                                                 117: P_{2521} = (8, 12, 8, 1)
64: P_{1568} = (15, 0, 5, 1)
                                                                 118: P_{2563} = (2, 15, 8, 1)
                                                                 119: P_{2575} = (14, 15, 8, 1)
65: P_{1584} = (15, 1, 5, 1)
66: P_{1627} = (10, 4, 5, 1)
                                                                 120: P_{2590} = (13, 0, 9, 1)
                                                                 121: P_{2606} = (13, 1, 9, 1)
67: P_{1670} = (5, 7, 5, 1)
68: P_{1678} = (13, 7, 5, 1)
                                                                 122: P_{2648} = (7, 4, 9, 1)
69: P_{1690} = (9, 8, 5, 1)
                                                                 123: P_{2655} = (14, 4, 9, 1)
70: P_{1695} = (14, 8, 5, 1)
                                                                 124: P_{2693} = (4,7,9,1)
                                                                 125: P_{2703} = (14, 7, 9, 1)
71: P_{1758} = (13, 12, 5, 1)
72: P_{1759} = (14, 12, 5, 1)
                                                                 126: P_{2709} = (4, 8, 9, 1)
                                                                 127: P_{2724} = (3, 9, 9, 1)
73: P_{1769} = (8, 13, 5, 1)
74: P_{1771} = (10, 13, 5, 1)
                                                                 128: P_{2728} = (7, 9, 9, 1)
                                                                 129: P_{2756} = (3, 11, 9, 1)
75: P_{1785} = (8, 14, 5, 1)
76: P_{1786} = (9, 14, 5, 1)
                                                                 130: P_{2758} = (5, 11, 9, 1)
77: P_{1798} = (5, 15, 5, 1)
                                                                 131: P_{2790} = (5, 13, 9, 1)
                                                                 132: P_{2844} = (11, 0, 10, 1)
78: P_{1813} = (4, 0, 6, 1)
79: P_{1829} = (4, 1, 6, 1)
                                                                 133: P_{2860} = (11, 1, 10, 1)
80: P_{1881} = (8, 4, 6, 1)
                                                                 134: P_{2868} = (3, 2, 10, 1)
81: P_{1895} = (6,5,6,1)
                                                                 135: P_{2875} = (10, 2, 10, 1)
82: P_{1896} = (7, 5, 6, 1)
                                                                 136: P_{2916} = (3, 5, 10, 1)
83: P_{1913} = (8, 6, 6, 1)
                                                                 137: P_{2926} = (13, 5, 10, 1)
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138: P_{2985} = (8, 9, 10, 1)
                                                                175: P_{3742} = (13, 8, 13, 1)
139: P_{2987} = (10, 9, 10, 1)
                                                                176: P_{3760} = (15, 9, 13, 1)
140: P_{3005} = (12, 10, 10, 1)
                                                                177: P_{3765} = (4, 10, 13, 1)
141: P_{3006} = (13, 10, 10, 1)
                                                                178: P_{3768} = (7, 10, 13, 1)
142: P_{3081} = (8, 15, 10, 1)
                                                                179: P_{3797} = (4, 12, 13, 1)
143: P_{3085} = (12, 15, 10, 1)
                                                                180: P_{3820} = (11, 13, 13, 1)
144: P_{3099} = (10, 0, 11, 1)
                                                                181: P_{3824} = (15, 13, 13, 1)
145: P_{3115} = (10, 1, 11, 1)
                                                                182: P_{3852} = (11, 15, 13, 1)
146: P_{3143} = (6, 3, 11, 1)
                                                                183: P_{3854} = (13, 15, 13, 1)
147: P_{3152} = (15, 3, 11, 1)
                                                                184: P_{3864} = (7, 0, 14, 1)
148: P_{3158} = (5, 4, 11, 1)
                                                                185 : P_{3880} = (7, 1, 14, 1)
                                                                186: P_{3977} = (8, 7, 14, 1)
149: P_{3164} = (11, 4, 11, 1)
150: P_{3222} = (5, 8, 11, 1)
                                                                187: P_{4003} = (2, 9, 14, 1)
151: P_{3224} = (7, 8, 11, 1)
                                                                188: P_{4013} = (12, 9, 14, 1)
                                                                189: P_{4022} = (5, 10, 14, 1)
152: P_{3271} = (6, 11, 11, 1)
153: P_{3272} = (7, 11, 11, 1)
                                                                190: P_{4025} = (8, 10, 14, 1)
154: P_{3324} = (11, 14, 11, 1)
                                                                191: P_{4051} = (2, 12, 14, 1)
155: P_{3328} = (15, 14, 11, 1)
                                                                192: P_{4058} = (9, 12, 14, 1)
156: P_{3347} = (2, 0, 12, 1)
                                                                193: P_{4086} = (5, 14, 14, 1)
157: P_{3363} = (2, 1, 12, 1)
                                                                194: P_{4093} = (12, 14, 14, 1)
158: P_{3382} = (5, 2, 12, 1)
                                                                195: P_{4106} = (9, 15, 14, 1)
                                                                196: P_{4118} = (5, 0, 15, 1)
159: P_{3405} = (12, 3, 12, 1)
                                                                197: P_{4134} = (5, 1, 15, 1)
160: P_{3406} = (13, 3, 12, 1)
161: P_{3436} = (11, 5, 12, 1)
                                                                198: P_{4163} = (2, 3, 15, 1)
162: P_{3437} = (12, 5, 12, 1)
                                                                199: P_{4165} = (4, 3, 15, 1)
163: P_{3495} = (6, 9, 12, 1)
                                                                200: P_{4179} = (2, 4, 15, 1)
164: P_{3502} = (13, 9, 12, 1)
                                                                201: P_{4180} = (3, 4, 15, 1)
165: P_{3511} = (6, 10, 12, 1)
                                                                202: P_{4208} = (15, 5, 15, 1)
166: P_{3519} = (14, 10, 12, 1)
                                                                203: P_{4221} = (12, 6, 15, 1)
167: P_{3542} = (5, 12, 12, 1)
                                                                204: P_{4224} = (15, 6, 15, 1)
168: P_{3548} = (11, 12, 12, 1)
                                                                205: P_{4308} = (3, 12, 15, 1)
169: P_{3567} = (14, 13, 12, 1)
                                                                206: P_{4315} = (10, 12, 15, 1)
170: P_{3610} = (9, 0, 13, 1)
                                                                207: P_{4325} = (4, 13, 15, 1)
171: P_{3626} = (9, 1, 13, 1)
                                                                208: P_{4333} = (12, 13, 15, 1)
172: P_{3640} = (7, 2, 13, 1)
                                                                209: P_{4347} = (10, 14, 15, 1)
173: P_{3645} = (12, 2, 13, 1)
174: P_{3741} = (12, 8, 13, 1)
```

Line Intersection Graph

	012345
0	011000
1	101110
2	110111
3	011010
4	011101
5	$\begin{matrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{matrix}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_5	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5
in point	P_1	P_2	P_2	P_2	P_{35}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_2	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{291}

Line 5 intersects

Line	ℓ_2	ℓ_4
in point	P_{35}	P_{291}

The surface has 305 points: The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	$37: P_{172} = (9, 9, 1, 0)$	$74: P_{698} = (9, 10, 1, 1)$
$1: P_1 = (0, 1, 0, 0)$	$38: P_{179} = (0, 10, 1, 0)$	$75: P_{709} = (4, 11, 1, 1)$
$\begin{array}{l} 1: T_1 = (0, 1, 0, 0) \\ 2: P_2 = (0, 0, 1, 0) \end{array}$	$39: P_{189} = (0, 10, 1, 0)$	$76: P_{719} = (4, 11, 1, 1)$ $76: P_{719} = (14, 11, 1, 1)$
$3: P_3 = (0,0,1,0)$		
- (, , , ,	$40: P_{195} = (0, 11, 1, 0)$	$77: P_{773} = (4, 15, 1, 1)$
$4: P_4 = (1, 1, 1, 1)$	$41: P_{206} = (11, 11, 1, 0)$	$78: P_{779} = (10, 15, 1, 1)$
$5: P_5 = (1, 1, 0, 0)$	$42: P_{211} = (0, 12, 1, 0)$	$79: P_{785} = (0,0,2,1)$
$6: P_6 = (2, 1, 0, 0)$	$43: P_{223} = (12, 12, 1, 0)$	$80: P_{797} = (12, 0, 2, 1)$
$7: P_7 = (3, 1, 0, 0)$	$44: P_{227} = (0, 13, 1, 0)$	$81: P_{802} = (1, 1, 2, 1)$
$8: P_8 = (4, 1, 0, 0)$	$45: P_{240} = (13, 13, 1, 0)$	82: $P_{813} = (12, 1, 2, 1)$
9: $P_9 = (5, 1, 0, 0)$	$46: P_{243} = (0, 14, 1, 0)$	83: $P_{823} = (6, 2, 2, 1)$
$10: P_{10} = (6, 1, 0, 0)$	$47: P_{257} = (14, 14, 1, 0)$	$84: P_{825} = (8, 2, 2, 1)$
$11: P_{11} = (7, 1, 0, 0)$	$48: P_{259} = (0, 15, 1, 0)$	$85: P_{834} = (1,3,2,1)$
$12: P_{12} = (8, 1, 0, 0)$	$49: P_{274} = (15, 15, 1, 0)$	$86: P_{847} = (14, 3, 2, 1)$
13: $P_{13} = (9, 1, 0, 0)$	$50: P_{291} = (1, 1, 0, 1)$	$87: P_{885} = (4,6,2,1)$
$14: P_{14} = (10, 1, 0, 0)$	$51: P_{310} = (4, 2, 0, 1)$	$88: P_{895} = (14, 6, 2, 1)$
$15: P_{15} = (11, 1, 0, 0)$	$52: P_{327} = (5, 3, 0, 1)$	$89: P_{969} = (8, 11, 2, 1)$
$16: P_{16} = (12, 1, 0, 0)$	$53: P_{347} = (9, 4, 0, 1)$	$90: P_{976} = (15, 11, 2, 1)$
$17: P_{17} = (13, 1, 0, 0)$	$54: P_{362} = (8, 5, 0, 1)$	$91: P_{992} = (15, 12, 2, 1)$
$18: P_{18} = (14, 1, 0, 0)$	$55: P_{383} = (13, 6, 0, 1)$	$92: P_{1013} = (4, 14, 2, 1)$
$19: P_{19} = (15, 1, 0, 0)$	$56: P_{398} = (12, 7, 0, 1)$	$93: P_{1015} = (6, 14, 2, 1)$
$20: P_{35} = (0, 1, 1, 0)$	$57: P_{417} = (15, 8, 0, 1)$	$94: P_{1041} = (0,0,3,1)$
$21: P_{36} = (1, 1, 1, 0)$	$58: P_{432} = (14, 9, 0, 1)$	$95: P_{1049} = (8, 0, 3, 1)$
$22: P_{51} = (0, 2, 1, 0)$	$59: P_{445} = (11, 10, 0, 1)$	$96: P_{1058} = (1, 1, 3, 1)$
$23: P_{53} = (2, 2, 1, 0)$	$60: P_{460} = (10, 11, 0, 1)$	$97: P_{1065} = (8, 1, 3, 1)$
$24: P_{67} = (0, 3, 1, 0)$	$61: P_{472} = (6, 12, 0, 1)$	$98: P_{1074} = (1, 2, 3, 1)$
$25: P_{70} = (3, 3, 1, 0)$	$62: P_{489} = (7, 13, 0, 1)$	$99: P_{1084} = (11, 2, 3, 1)$
$26: P_{83} = (0, 4, 1, 0)$	$63: P_{500} = (2, 14, 0, 1)$	$100: P_{1125} = (4, 5, 3, 1)$
$27: P_{87} = (4, 4, 1, 0)$	$64: P_{517} = (3, 15, 0, 1)$	$101: P_{1130} = (9, 5, 3, 1)$
$28: P_{99} = (0, 5, 1, 0)$	$65: P_{530} = (0, 0, 1, 1)$	$102: P_{1142} = (5, 6, 3, 1)$
$29: P_{104} = (5, 5, 1, 0)$	$66: P_{531} = (1, 0, 1, 1)$	$103: P_{1148} = (11, 6, 3, 1)$
$30: P_{115} = (0, 6, 1, 0)$	$67: P_{586} = (9, 3, 1, 1)$	$104: P_{1159} = (6,7,3,1)$
$31: P_{121} = (6, 6, 1, 0)$	$68: P_{588} = (11, 3, 1, 1)$	$105: P_{1162} = (9,7,3,1)$
$32: P_{131} = (0,7,1,0)$	$69: P_{619} = (10, 5, 1, 1)$	$106: P_{1172} = (3, 8, 3, 1)$
$33: P_{138} = (7,7,1,0)$	$70: P_{623} = (14, 5, 1, 1)$	$107: P_{1189} = (4, 9, 3, 1)$
$34: P_{147} = (0, 8, 1, 0)$	71: $P_{659} = (2, 8, 1, 1)$	$108: P_{1190} = (5, 9, 3, 1)$
$35: P_{155} = (8, 8, 1, 0)$	$72: P_{668} = (11, 8, 1, 1)$	$109: P_{1252} = (3, 13, 3, 1)$
$36: P_{163} = (0, 9, 1, 0)$	$73: P_{691} = (2, 10, 1, 1)$	$110: P_{1255} = (6, 13, 3, 1)$

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219: P_{3005} = (12, 10, 10, 1)
111: P_{1297} = (0, 0, 4, 1)
                                           165: P_{2123} = (10, 3, 7, 1)
112: P_{1303} = (6, 0, 4, 1)
                                           166: P_{2135} = (6, 4, 7, 1)
                                                                                       220: P_{3006} = (13, 10, 10, 1)
                                           167: P_{2141} = (12, 4, 7, 1)
                                                                                       221: P_{3010} = (1, 11, 10, 1)
113: P_{1314} = (1, 1, 4, 1)
114: P_{1319} = (6, 1, 4, 1)
                                           168: P_{2162} = (1, 6, 7, 1)
                                                                                       222: P_{3081} = (8, 15, 10, 1)
115: P_{1338} = (9, 2, 4, 1)
                                           169: P_{2170} = (9, 6, 7, 1)
                                                                                       223: P_{3085} = (12, 15, 10, 1)
116: P_{1342} = (13, 2, 4, 1)
                                           170: P_{2180} = (3,7,7,1)
                                                                                       224: P_{3089} = (0, 0, 11, 1)
117: P_{1374} = (13, 4, 4, 1)
                                           171: P_{2187} = (10, 7, 7, 1)
                                                                                       225: P_{3099} = (10, 0, 11, 1)
118: P_{1376} = (15, 4, 4, 1)
                                           172: P_{2250} = (9, 11, 7, 1)
                                                                                       226: P_{3106} = (1, 1, 11, 1)
119: P_{1378} = (1, 5, 4, 1)
                                           173: P_{2253} = (12, 11, 7, 1)
                                                                                       227: P_{3115} = (10, 1, 11, 1)
120: P_{1379} = (2, 5, 4, 1)
                                           174: P_{2292} = (3, 14, 7, 1)
                                                                                       228: P_{3143} = (6, 3, 11, 1)
121: P_{1396} = (3, 6, 4, 1)
                                           175: P_{2311} = (6, 15, 7, 1)
                                                                                       229: P_{3152} = (15, 3, 11, 1)
122: P_{1460} = (3, 10, 4, 1)
                                           176: P_{2312} = (7, 15, 7, 1)
                                                                                       230: P_{3158} = (5, 4, 11, 1)
123: P_{1472} = (15, 10, 4, 1)
                                           177: P_{2321} = (0,0,8,1)
                                                                                       231: P_{3164} = (11, 4, 11, 1)
124: P_{1507} = (2, 13, 4, 1)
                                           178: P_{2324} = (3, 0, 8, 1)
                                                                                       232: P_{3222} = (5, 8, 11, 1)
                                                                                       233: P_{3224} = (7, 8, 11, 1)
                                           179: P_{2338} = (1, 1, 8, 1)
125: P_{1514} = (9, 13, 4, 1)
126: P_{1553} = (0, 0, 5, 1)
                                           180: P_{2340} = (3, 1, 8, 1)
                                                                                       234: P_{3250} = (1, 10, 11, 1)
                                           181: P_{2367} = (14, 2, 8, 1)
                                                                                       235: P_{3271} = (6, 11, 11, 1)
127: P_{1568} = (15, 0, 5, 1)
                                           182: P_{2368} = (15, 2, 8, 1)
128: P_{1570} = (1, 1, 5, 1)
                                                                                       236: P_{3272} = (7, 11, 11, 1)
129: P_{1584} = (15, 1, 5, 1)
                                           183: P_{2377} = (8, 3, 8, 1)
                                                                                       237: P_{3324} = (11, 14, 11, 1)
130: P_{1618} = (1, 4, 5, 1)
                                           184: P_{2419} = (2, 6, 8, 1)
                                                                                       238: P_{3328} = (15, 14, 11, 1)
131: P_{1627} = (10, 4, 5, 1)
                                           185: P_{2424} = (7, 6, 8, 1)
                                                                                       239: P_{3345} = (0, 0, 12, 1)
                                                                                       240: P_{3347} = (2, 0, 12, 1)
132: P_{1670} = (5, 7, 5, 1)
                                           186: P_{2444} = (11, 7, 8, 1)
                                                                                       241: P_{3362} = (1, 1, 12, 1)
133: P_{1678} = (13, 7, 5, 1)
                                           187: P_{2448} = (15, 7, 8, 1)
134: P_{1690} = (9, 8, 5, 1)
                                           188: P_{2466} = (1, 9, 8, 1)
                                                                                       242: P_{3363} = (2, 1, 12, 1)
135: P_{1695} = (14, 8, 5, 1)
                                           189: P_{2476} = (11, 9, 8, 1)
                                                                                       243: P_{3382} = (5, 2, 12, 1)
136: P_{1758} = (13, 12, 5, 1)
                                           190: P_{2520} = (7, 12, 8, 1)
                                                                                       244: P_{3405} = (12, 3, 12, 1)
137: P_{1759} = (14, 12, 5, 1)
                                           191: P_{2521} = (8, 12, 8, 1)
                                                                                       245: P_{3406} = (13, 3, 12, 1)
138: P_{1769} = (8, 13, 5, 1)
                                           192: P_{2563} = (2, 15, 8, 1)
                                                                                       246: P_{3436} = (11, 5, 12, 1)
139: P_{1771} = (10, 13, 5, 1)
                                           193: P_{2575} = (14, 15, 8, 1)
                                                                                       247: P_{3437} = (12, 5, 12, 1)
140: P_{1785} = (8, 14, 5, 1)
                                           194: P_{2577} = (0, 0, 9, 1)
                                                                                       248: P_{3495} = (6, 9, 12, 1)
141: P_{1786} = (9, 14, 5, 1)
                                           195: P_{2590} = (13, 0, 9, 1)
                                                                                       249: P_{3502} = (13, 9, 12, 1)
142: P_{1798} = (5, 15, 5, 1)
                                           196: P_{2594} = (1, 1, 9, 1)
                                                                                       250: P_{3511} = (6, 10, 12, 1)
143: P_{1809} = (0, 0, 6, 1)
                                           197: P_{2606} = (13, 1, 9, 1)
                                                                                       251: P_{3519} = (14, 10, 12, 1)
144: P_{1813} = (4, 0, 6, 1)
                                           198: P_{2648} = (7, 4, 9, 1)
                                                                                       252: P_{3542} = (5, 12, 12, 1)
                                           199: P_{2655} = (14, 4, 9, 1)
145: P_{1826} = (1, 1, 6, 1)
                                                                                       253: P_{3548} = (11, 12, 12, 1)
146: P_{1829} = (4, 1, 6, 1)
                                           200: P_{2693} = (4,7,9,1)
                                                                                       254: P_{3554} = (1, 13, 12, 1)
147: P_{1881} = (8, 4, 6, 1)
                                           201: P_{2703} = (14, 7, 9, 1)
                                                                                       255: P_{3567} = (14, 13, 12, 1)
                                           202: P_{2706} = (1, 8, 9, 1)
                                                                                       256: P_{3601} = (0, 0, 13, 1)
148: P_{1895} = (6, 5, 6, 1)
149: P_{1896} = (7, 5, 6, 1)
                                           203: P_{2709} = (4, 8, 9, 1)
                                                                                       257: P_{3610} = (9, 0, 13, 1)
150: P_{1913} = (8, 6, 6, 1)
                                           204: P_{2724} = (3, 9, 9, 1)
                                                                                       258: P_{3618} = (1, 1, 13, 1)
151: P_{1915} = (10, 6, 6, 1)
                                           205: P_{2728} = (7, 9, 9, 1)
                                                                                       259: P_{3626} = (9, 1, 13, 1)
152: P_{1922} = (1, 7, 6, 1)
                                           206: P_{2756} = (3, 11, 9, 1)
                                                                                       260: P_{3640} = (7, 2, 13, 1)
153: P_{1923} = (2,7,6,1)
                                           207: P_{2758} = (5, 11, 9, 1)
                                                                                       261: P_{3645} = (12, 2, 13, 1)
                                           208: P_{2790} = (5, 13, 9, 1)
                                                                                       262: P_{3741} = (12, 8, 13, 1)
154: P_{1943} = (6, 8, 6, 1)
155: P_{1947} = (10, 8, 6, 1)
                                           209: P_{2833} = (0, 0, 10, 1)
                                                                                       263: P_{3742} = (13, 8, 13, 1)
                                           210: P_{2844} = (11, 0, 10, 1)
                                                                                       264: P_{3760} = (15, 9, 13, 1)
156: P_{1987} = (2, 11, 6, 1)
157: P_{1998} = (13, 11, 6, 1)
                                           211: P_{2850} = (1, 1, 10, 1)
                                                                                       265: P_{3765} = (4, 10, 13, 1)
158: P_{2040} = (7, 14, 6, 1)
                                           212: P_{2860} = (11, 1, 10, 1)
                                                                                       266: P_{3768} = (7, 10, 13, 1)
                                                                                       267: P_{3794} = (1, 12, 13, 1)
159: P_{2046} = (13, 14, 6, 1)
                                           213: P_{2868} = (3, 2, 10, 1)
160: P_{2065} = (0, 0, 7, 1)
                                           214: P_{2875} = (10, 2, 10, 1)
                                                                                       268: P_{3797} = (4, 12, 13, 1)
161: P_{2079} = (14, 0, 7, 1)
                                           215: P_{2916} = (3, 5, 10, 1)
                                                                                       269: P_{3820} = (11, 13, 13, 1)
                                           216: P_{2926} = (13, 5, 10, 1)
162: P_{2082} = (1, 1, 7, 1)
                                                                                       270: P_{3824} = (15, 13, 13, 1)
163: P_{2095} = (14, 1, 7, 1)
                                           217: P_{2985} = (8, 9, 10, 1)
                                                                                       271: P_{3852} = (11, 15, 13, 1)
164: P_{2120} = (7, 3, 7, 1)
                                           218: P_{2987} = (10, 9, 10, 1)
                                                                                       272: P_{3854} = (13, 15, 13, 1)
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273: P_{3857} = (0, 0, 14, 1)
                                           284: P_{4086} = (5, 14, 14, 1)
                                                                                      295: P_{4180} = (3, 4, 15, 1)
274: P_{3864} = (7, 0, 14, 1)
                                                                                      296: P_{4208} = (15, 5, 15, 1)
                                           285: P_{4093} = (12, 14, 14, 1)
275: P_{3874} = (1, 1, 14, 1)
                                           286: P_{4098} = (1, 15, 14, 1)
                                                                                      297: P_{4221} = (12, 6, 15, 1)
276: P_{3880} = (7, 1, 14, 1)
                                           287:\,P_{4106}=(9,15,14,1)
                                                                                      298 : P_{4224} = (15, 6, 15, 1)
277: P_{3977} = (8, 7, 14, 1)
                                           288: P_{4113} = (0, 0, 15, 1)
                                                                                      299: P_{4308} = (3, 12, 15, 1)
278: P_{4003} = (2, 9, 14, 1)
                                           289: P_{4118} = (5, 0, 15, 1)
                                                                                      300: P_{4315} = (10, 12, 15, 1)
279: P_{4013} = (12, 9, 14, 1)
                                           290: P_{4130} = (1, 1, 15, 1)
                                                                                      301: P_{4325} = (4, 13, 15, 1)
280: P_{4022} = (5, 10, 14, 1)
                                           291: P_{4134} = (5, 1, 15, 1)
                                                                                      302: P_{4333} = (12, 13, 15, 1)
281: P_{4025} = (8, 10, 14, 1)
                                           292: P_{4163} = (2, 3, 15, 1)
                                                                                      303: P_{4338} = (1, 14, 15, 1)
                                                                                      304: P_{4347} = (10, 14, 15, 1)
282: P_{4051} = (2, 12, 14, 1)
                                           293: P_{4165} = (4, 3, 15, 1)
283: P_{4058} = (9, 12, 14, 1)
                                           294: P_{4179} = (2, 4, 15, 1)
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