

# Rank-74531 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(4) is 1499093337

## General information

Number of lines	10
Number of points	33
Number of singular points	1
Number of Eckardt points	0
Number of double points	15
Number of single points	15
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^{10}$
Type of lines on points	$5, 2^{15}, 1^{15}, 0^2$

## Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

## The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{58} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{58} = \mathbf{Pl}(0, 0, 3, 0, 0, 1)_{122} \\
\ell_2 &= \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{79} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{79} = \mathbf{Pl}(0, 0, 2, 0, 0, 1)_{115} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{129} \\
\ell_4 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_5 &= \begin{bmatrix} 1 & \omega^2 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{247} = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{247} = \mathbf{Pl}(0, 3, 2, 0, 0, 1)_{121} \\
\ell_6 &= \begin{bmatrix} 1 & \omega & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{310} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{310} = \mathbf{Pl}(0, 2, 3, 0, 0, 1)_{127} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{199} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega & \omega \end{bmatrix}_{304} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{304} = \mathbf{Pl}(1, 1, 3, 2, 1, 1)_{231} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{246} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{246} = \mathbf{Pl}(1, 1, 2, 3, 1, 1)_{228}
\end{aligned}$$

Rank of lines: ( 0, 58, 79, 337, 356, 247, 310, 110, 304, 246 )

Rank of points on Klein quadric: ( 0, 122, 115, 129, 1, 121, 127, 199, 231, 228 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 15 Double points:

The double points on the surface are:

$$\begin{aligned}
P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_1 & P_{49} &= (0, 3, 1, 1) = \ell_3 \cap \ell_8 \\
P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_2 & P_{45} &= (0, 2, 1, 1) = \ell_3 \cap \ell_9 \\
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_3 & P_{48} &= (3, 2, 1, 1) = \ell_5 \cap \ell_7 \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_7 & P_{64} &= (3, 2, 2, 1) = \ell_5 \cap \ell_8 \\
P_{16} &= (1, 2, 1, 0) = \ell_1 \cap \ell_8 & P_{51} &= (2, 3, 1, 1) = \ell_6 \cap \ell_7 \\
P_{20} &= (1, 3, 1, 0) = \ell_2 \cap \ell_9 & P_{83} &= (2, 3, 3, 1) = \ell_6 \cap \ell_9 \\
P_{38} &= (0, 0, 1, 1) = \ell_3 \cap \ell_4 & P_{27} &= (1, 1, 0, 1) = \ell_8 \cap \ell_9 \\
P_{42} &= (0, 1, 1, 1) = \ell_3 \cap \ell_7
\end{aligned}$$

### Single Points

The surface has 15 single points:

The single points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$  lies on line  $\ell_0$   
 1 :  $P_3 = (0, 0, 0, 1)$  lies on line  $\ell_4$   
 2 :  $P_{13} = (2, 1, 1, 0)$  lies on line  $\ell_2$   
 3 :  $P_{14} = (3, 1, 1, 0)$  lies on line  $\ell_1$   
 4 :  $P_{18} = (3, 2, 1, 0)$  lies on line  $\ell_2$   
 5 :  $P_{21} = (2, 3, 1, 0)$  lies on line  $\ell_1$   
 6 :  $P_{33} = (3, 2, 0, 1)$  lies on line  $\ell_5$   
 7 :  $P_{36} = (2, 3, 0, 1)$  lies on line  $\ell_6$

8 :  $P_{39} = (1, 0, 1, 1)$  lies on line  $\ell_7$   
 9 :  $P_{53} = (0, 0, 2, 1)$  lies on line  $\ell_4$   
 10 :  $P_{56} = (3, 0, 2, 1)$  lies on line  $\ell_9$   
 11 :  $P_{67} = (2, 3, 2, 1)$  lies on line  $\ell_6$   
 12 :  $P_{69} = (0, 0, 3, 1)$  lies on line  $\ell_4$   
 13 :  $P_{71} = (2, 0, 3, 1)$  lies on line  $\ell_8$   
 14 :  $P_{80} = (3, 2, 3, 1)$  lies on line  $\ell_5$

The single points on the surface are:

### Points on surface but on no line

The surface has 2 points not on any line:

The points on the surface but not on lines are:

0 :  $P_{63} = (2, 2, 2, 1)$   
 1 :  $P_{84} = (3, 3, 3, 1)$

### Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	0	0	0	1	0	0
1	1	0	1	0	1	1	1	0	1	0
2	1	1	0	0	1	1	1	0	0	1
3	1	0	0	0	1	0	0	1	1	1
4	0	1	1	1	0	1	1	0	0	0
5	0	1	1	0	1	0	1	1	1	0
6	0	1	1	0	1	1	0	1	0	1
7	1	0	0	1	0	1	1	0	0	0
8	0	1	0	1	0	1	0	0	0	1
9	0	0	1	1	0	0	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_7$
in point	$P_7$	$P_6$	$P_1$	$P_5$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_8$
in point	$P_7$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{16}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_9$
in point	$P_6$	$P_2$	$P_2$	$P_2$	$P_2$	$P_{20}$

Line 3 intersects

Line	$\ell_0$	$\ell_4$	$\ell_7$	$\ell_8$	$\ell_9$
in point	$P_1$	$P_{38}$	$P_{42}$	$P_{49}$	$P_{45}$

Line 4 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$
in point	$P_2$	$P_2$	$P_{38}$	$P_2$	$P_2$

Line 5 intersects

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_6$	$\ell_7$	$\ell_8$
in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_{48}$	$P_{64}$

Line 6 intersects

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_7$	$\ell_9$
in point	$P_2$	$P_2$	$P_2$	$P_2$	$P_{51}$	$P_{83}$

Line 7 intersects

Line	$\ell_0$	$\ell_3$	$\ell_5$	$\ell_6$
in point	$P_5$	$P_{42}$	$P_{48}$	$P_{51}$

Line 8 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_9$
in point	$P_{16}$	$P_{49}$	$P_{64}$	$P_{27}$

Line 9 intersects

Line	$\ell_2$	$\ell_3$	$\ell_6$	$\ell_8$
in point	$P_{20}$	$P_{45}$	$P_{83}$	$P_{27}$

The surface has 33 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_1 = (0, 1, 0, 0)$   
 2 :  $P_2 = (0, 0, 1, 0)$   
 3 :  $P_3 = (0, 0, 0, 1)$   
 4 :  $P_5 = (1, 1, 0, 0)$   
 5 :  $P_6 = (2, 1, 0, 0)$   
 6 :  $P_7 = (3, 1, 0, 0)$   
 7 :  $P_{13} = (2, 1, 1, 0)$   
 8 :  $P_{14} = (3, 1, 1, 0)$   
 9 :  $P_{16} = (1, 2, 1, 0)$   
 10 :  $P_{18} = (3, 2, 1, 0)$   
 11 :  $P_{20} = (1, 3, 1, 0)$

12 :  $P_{21} = (2, 3, 1, 0)$   
 13 :  $P_{27} = (1, 1, 0, 1)$   
 14 :  $P_{33} = (3, 2, 0, 1)$   
 15 :  $P_{36} = (2, 3, 0, 1)$   
 16 :  $P_{38} = (0, 0, 1, 1)$   
 17 :  $P_{39} = (1, 0, 1, 1)$   
 18 :  $P_{42} = (0, 1, 1, 1)$   
 19 :  $P_{45} = (0, 2, 1, 1)$   
 20 :  $P_{48} = (3, 2, 1, 1)$   
 21 :  $P_{49} = (0, 3, 1, 1)$   
 22 :  $P_{51} = (2, 3, 1, 1)$   
 23 :  $P_{53} = (0, 0, 2, 1)$

24 :  $P_{56} = (3, 0, 2, 1)$   
 25 :  $P_{63} = (2, 2, 2, 1)$   
 26 :  $P_{64} = (3, 2, 2, 1)$   
 27 :  $P_{67} = (2, 3, 2, 1)$   
 28 :  $P_{69} = (0, 0, 3, 1)$   
 29 :  $P_{71} = (2, 0, 3, 1)$   
 30 :  $P_{80} = (3, 2, 3, 1)$   
 31 :  $P_{83} = (2, 3, 3, 1)$   
 32 :  $P_{84} = (3, 3, 3, 1)$