Rank-65760 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(4)$ is 1431676586

General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	1
Number of double points	15
Number of single points	12
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	$3, 2^{15}, 1^{12}, 0^5$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_3 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{82} = \mathbf{Pl}(0,0,2,3,1,1)_{205}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{156} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{156} = \mathbf{Pl}(3,0,3,2,3,1)_{335}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{139} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{139} = \mathbf{Pl}(2,0,2,3,2,1)_{267}$$

$$\ell_6 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{60} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{60} = \mathbf{Pl}(0,0,3,2,1,1)_{212}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{36} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{36} = \mathbf{Pl}(2,3,1,0,3,1)_{301}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega & \omega \end{bmatrix}_{31} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{31} = \mathbf{Pl}(3,2,1,0,2,1)_{242}$$

Rank of lines: (337, 339, 338, 82, 156, 139, 60, 36, 31)
Rank of points on Klein quadric: (129, 143, 136, 205, 335, 267, 212, 301, 242)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{45} = (0,2,1,1) = \ell_0 \cap \ell_7$

$P_{49} = (0,3,1,1) = \ell_0 \cap \ell_8$

$P_{53} = (0,0,2,1) = \ell_1 \cap \ell_3$

$P_{61} = (0,2,2,1) = \ell_1 \cap \ell_5$

$P_{81} = (0,3,3,1) = \ell_2 \cap \ell_4$

$P_{69} = (0,0,3,1) = \ell_2 \cap \ell_6$

$P_6 = (2,1,0,0) = \ell_3 \cap \ell_4$

$P_{59} = (2,1,2,1) = \ell_3 \cap \ell_5$

$P_{64} = (3,2,2,1) = \ell_3 \cap \ell_7$

$P_{76} = (3,1,3,1) = \ell_4 \cap \ell_6$

$P_{79} = (2,2,3,1) = \ell_4 \cap \ell_7$

$P_7 = (3,1,0,0) = \ell_5 \cap \ell_6$

$P_{68} = (3,3,2,1) = \ell_5 \cap \ell_8$

$P_{83} = (2,3,3,1) = \ell_6 \cap \ell_8$

$P_8 = (1,0,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 12 single points:
The single points on the surface are:

$0 : P_{31} = (1,2,0,1)$ lies on line ℓ_7

$1 : P_{35} = (1,3,0,1)$ lies on line ℓ_8

$2 : P_{38} = (0,0,1,1)$ lies on line ℓ_0

$3 : P_{42} = (0,1,1,1)$ lies on line ℓ_0

$4 : P_{54} = (1,0,2,1)$ lies on line ℓ_5

$5 : P_{57} = (0,1,2,1)$ lies on line ℓ_1

$6 : P_{65} = (0,3,2,1)$ lies on line ℓ_1

$7 : P_{66} = (1,3,2,1)$ lies on line ℓ_3

$8 : P_{70} = (1,0,3,1)$ lies on line ℓ_4

$9 : P_{73} = (0,1,3,1)$ lies on line ℓ_2

$10 : P_{77} = (0,2,3,1)$ lies on line ℓ_2

$11 : P_{78} = (1,2,3,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 5 points not on any line:
The points on the surface but not on lines are:

$0: \ P_9 = (2,0,1,0)$ $3: \ P_{21} = (2,3,1,0)$

$1: \ P_{10} = (3,0,1,0)$ $4: \ P_{39} = (1,0,1,1)$

$2: \ P_{18} = (3,2,1,0)$

Line Intersection Graph

```
 |0 1 2 3 4 5 6 7 8
0|0 1 1 0 0 0 0 1 1
1|1 0 1 1 0 1 0 0 0
2|1 1 0 0 1 0 1 0 0
3|0 1 0 0 1 1 0 1 0
4|0 0 1 1 0 0 1 1 0
5|0 1 0 1 0 0 1 0 1
6|0 0 1 0 1 1 0 0 1
7|1 0 0 1 1 0 0 0 1
8|1 0 0 0 0 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8
in point	P_1	P_1	P_{45}	P_{49}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_1	P_1	P_{53}	P_{61}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_1	P_1	P_{81}	P_{69}

Line 3 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_{53}	P_6	P_{59}	P_{64}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{81}	P_6	P_{76}	P_{79}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_{61}	P_{59}	P_7	P_{68}

Line 6 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_{69}	P_{76}	P_7	P_{83}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_8
in point	P_{45}	P_{64}	P_{79}	P_8

Line 8 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_7
in point	P_{49}	P_{68}	P_{83}	P_8

The surface has **33** points:
The points on the surface are:

$0 :\ P_1 = (0,1,0,0)$

$1 :\ P_6 = (2,1,0,0)$

$2 :\ P_7 = (3,1,0,0)$

$3 :\ P_8 = (1,0,1,0)$

$4 :\ P_9 = (2,0,1,0)$

$5 :\ P_{10} = (3,0,1,0)$

$6 :\ P_{18} = (3,2,1,0)$

$7 :\ P_{21} = (2,3,1,0)$

$8 :\ P_{31} = (1,2,0,1)$

$9 :\ P_{35} = (1,3,0,1)$

$10 :\ P_{38} = (0,0,1,1)$

$11 :\ P_{39} = (1,0,1,1)$

$12 :\ P_{42} = (0,1,1,1)$

$13 :\ P_{45} = (0,2,1,1)$

$14 :\ P_{49} = (0,3,1,1)$

$15 :\ P_{53} = (0,0,2,1)$

$16 :\ P_{54} = (1,0,2,1)$

$17 :\ P_{57} = (0,1,2,1)$

$18 :\ P_{59} = (2,1,2,1)$

$19 :\ P_{61} = (0,2,2,1)$

$20 :\ P_{64} = (3,2,2,1)$

$21 :\ P_{65} = (0,3,2,1)$

$22 :\ P_{66} = (1,3,2,1)$

$23 :\ P_{68} = (3,3,2,1)$

$24 :\ P_{69} = (0,0,3,1)$

$25 :\ P_{70} = (1,0,3,1)$

$26 :\ P_{73} = (0,1,3,1)$

$27 :\ P_{76} = (3,1,3,1)$

$28 :\ P_{77} = (0,2,3,1)$

$29 :\ P_{78} = (1,2,3,1)$

$30 :\ P_{79} = (2,2,3,1)$

$31 :\ P_{81} = (0,3,3,1)$

$32 :\ P_{83} = (2,3,3,1)$