

Rank-74007 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1243914893

General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	23
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$2^2, 1^{23}, 0^{56}$

Singular Points

The surface has 1 singular points:

$$0 : P_{146} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \mathbf{Pl}(1, 1, 0, 1, 1, 1)_{1273}$$

Rank of lines: (4680, 4689, 665)

Rank of points on Klein quadric: (17, 25, 1273)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_3 = (0, 0, 0, 1) = \ell_0 \cap \ell_1$$

$$P_{82} = (0, 1, 0, 1) = \ell_0 \cap \ell_2$$

Single Points

The surface has 23 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

1 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_1

2 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_2

3 : $P_{90} = (0, 2, 0, 1)$ lies on line ℓ_0

4 : $P_{98} = (0, 3, 0, 1)$ lies on line ℓ_0

5 : $P_{106} = (0, 4, 0, 1)$ lies on line ℓ_0

6 : $P_{114} = (0, 5, 0, 1)$ lies on line ℓ_0

7 : $P_{122} = (0, 6, 0, 1)$ lies on line ℓ_0

8 : $P_{130} = (0, 7, 0, 1)$ lies on line ℓ_0

9 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_2

10 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_1

11 : $P_{217} = (0, 2, 2, 1)$ lies on line ℓ_1

12 : $P_{227} = (2, 3, 2, 1)$ lies on line ℓ_2

13 : $P_{284} = (3, 2, 3, 1)$ lies on line ℓ_2

14 : $P_{289} = (0, 3, 3, 1)$ lies on line ℓ_1

15 : $P_{361} = (0, 4, 4, 1)$ lies on line ℓ_1

16 : $P_{373} = (4, 5, 4, 1)$ lies on line ℓ_2

17 : $P_{430} = (5, 4, 5, 1)$ lies on line ℓ_2

18 : $P_{433} = (0, 5, 5, 1)$ lies on line ℓ_1

19 : $P_{505} = (0, 6, 6, 1)$ lies on line ℓ_1

20 : $P_{519} = (6, 7, 6, 1)$ lies on line ℓ_2

21 : $P_{576} = (7, 6, 7, 1)$ lies on line ℓ_2

22 : $P_{577} = (0, 7, 7, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:

The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$

1 : $P_{83} = (1, 1, 0, 1)$

2 : $P_{96} = (6, 2, 0, 1)$

3 : $P_{102} = (4, 3, 0, 1)$

4 : $P_{109} = (3, 4, 0, 1)$

5 : $P_{121} = (7, 5, 0, 1)$

6 : $P_{124} = (2, 6, 0, 1)$

7 : $P_{135} = (5, 7, 0, 1)$

8 : $P_{164} = (3, 3, 1, 1)$	33 : $P_{403} = (2, 1, 5, 1)$
9 : $P_{167} = (6, 3, 1, 1)$	34 : $P_{407} = (6, 1, 5, 1)$
10 : $P_{180} = (3, 5, 1, 1)$	35 : $P_{423} = (6, 3, 5, 1)$
11 : $P_{182} = (5, 5, 1, 1)$	36 : $P_{424} = (7, 3, 5, 1)$
12 : $P_{190} = (5, 6, 1, 1)$	37 : $P_{428} = (3, 4, 5, 1)$
13 : $P_{191} = (6, 6, 1, 1)$	38 : $P_{435} = (2, 5, 5, 1)$
14 : $P_{206} = (5, 0, 2, 1)$	39 : $P_{444} = (3, 6, 5, 1)$
15 : $P_{221} = (4, 2, 2, 1)$	40 : $P_{445} = (4, 6, 5, 1)$
16 : $P_{229} = (4, 3, 2, 1)$	41 : $P_{456} = (7, 7, 5, 1)$
17 : $P_{254} = (5, 6, 2, 1)$	42 : $P_{464} = (7, 0, 6, 1)$
18 : $P_{267} = (2, 0, 3, 1)$	43 : $P_{468} = (3, 1, 6, 1)$
19 : $P_{278} = (5, 1, 3, 1)$	44 : $P_{469} = (4, 1, 6, 1)$
20 : $P_{280} = (7, 1, 3, 1)$	45 : $P_{475} = (2, 2, 6, 1)$
21 : $P_{287} = (6, 2, 3, 1)$	46 : $P_{486} = (5, 3, 6, 1)$
22 : $P_{296} = (7, 3, 3, 1)$	47 : $P_{488} = (7, 3, 6, 1)$
23 : $P_{301} = (4, 4, 3, 1)$	48 : $P_{499} = (2, 5, 6, 1)$
24 : $P_{307} = (2, 5, 3, 1)$	49 : $P_{500} = (3, 5, 6, 1)$
25 : $P_{311} = (6, 5, 3, 1)$	50 : $P_{509} = (4, 6, 6, 1)$
26 : $P_{317} = (4, 6, 3, 1)$	51 : $P_{518} = (5, 7, 6, 1)$
27 : $P_{318} = (5, 6, 3, 1)$	52 : $P_{524} = (3, 0, 7, 1)$
28 : $P_{335} = (6, 0, 4, 1)$	53 : $P_{564} = (3, 5, 7, 1)$
29 : $P_{359} = (6, 3, 4, 1)$	54 : $P_{571} = (2, 6, 7, 1)$
30 : $P_{368} = (7, 4, 4, 1)$	55 : $P_{579} = (2, 7, 7, 1)$
31 : $P_{376} = (7, 5, 4, 1)$	
32 : $P_{397} = (4, 0, 5, 1)$	

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 0
2	1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_{82}

Line 1 intersects

Line	ℓ_0
in point	P_3

Line 2 intersects

Line	ℓ_0
in point	P_{82}

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	7 : $P_{90} = (0, 2, 0, 1)$	14 : $P_{121} = (7, 5, 0, 1)$
1 : $P_1 = (0, 1, 0, 0)$	8 : $P_{96} = (6, 2, 0, 1)$	15 : $P_{122} = (0, 6, 0, 1)$
2 : $P_3 = (0, 0, 0, 1)$	9 : $P_{98} = (0, 3, 0, 1)$	16 : $P_{124} = (2, 6, 0, 1)$
3 : $P_{19} = (0, 1, 1, 0)$	10 : $P_{102} = (4, 3, 0, 1)$	17 : $P_{130} = (0, 7, 0, 1)$
4 : $P_{20} = (1, 1, 1, 0)$	11 : $P_{106} = (0, 4, 0, 1)$	18 : $P_{135} = (5, 7, 0, 1)$
5 : $P_{82} = (0, 1, 0, 1)$	12 : $P_{109} = (3, 4, 0, 1)$	19 : $P_{139} = (1, 0, 1, 1)$
6 : $P_{83} = (1, 1, 0, 1)$	13 : $P_{114} = (0, 5, 0, 1)$	20 : $P_{146} = (0, 1, 1, 1)$

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