

Rank-65547 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227134029

General information

Number of lines	3
Number of points	73
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$2^3, 1^{21}, 0^{49}$

Singular Points

The surface has 3 singular points:

$$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0)$$

$$1 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$2 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{649}$$

Rank of lines: (0, 64, 4672)

Rank of points on Klein quadric: (0, 2, 649)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_2 = (0, 0, 1, 0) = \ell_1 \cap \ell_2$$

Single Points

The surface has 21 single points:

The single points on the surface are:

0 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0

1 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0

2 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0

3 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0

4 : $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0

5 : $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0

6 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0

7 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_1

8 : $P_{13} = (2, 0, 1, 0)$ lies on line ℓ_1

9 : $P_{14} = (3, 0, 1, 0)$ lies on line ℓ_1

10 : $P_{15} = (4, 0, 1, 0)$ lies on line ℓ_1

11 : $P_{16} = (5, 0, 1, 0)$ lies on line ℓ_1

12 : $P_{17} = (6, 0, 1, 0)$ lies on line ℓ_1

13 : $P_{18} = (7, 0, 1, 0)$ lies on line ℓ_1

14 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_2

15 : $P_{27} = (0, 2, 1, 0)$ lies on line ℓ_2

16 : $P_{35} = (0, 3, 1, 0)$ lies on line ℓ_2

17 : $P_{43} = (0, 4, 1, 0)$ lies on line ℓ_2

18 : $P_{51} = (0, 5, 1, 0)$ lies on line ℓ_2

19 : $P_{59} = (0, 6, 1, 0)$ lies on line ℓ_2

20 : $P_{67} = (0, 7, 1, 0)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_{159} = (6, 2, 1, 1)$$

$$2 : P_{165} = (4, 3, 1, 1)$$

$$3 : P_{172} = (3, 4, 1, 1)$$

$$4 : P_{184} = (7, 5, 1, 1)$$

$$5 : P_{187} = (2, 6, 1, 1)$$

$$6 : P_{198} = (5, 7, 1, 1)$$

$$7 : P_{215} = (6, 1, 2, 1)$$

$$8 : P_{220} = (3, 2, 2, 1)$$

$$9 : P_{227} = (2, 3, 2, 1)$$

10 : $P_{240} = (7, 4, 2, 1)$	30 : $P_{423} = (6, 3, 5, 1)$
11 : $P_{246} = (5, 5, 2, 1)$	31 : $P_{429} = (4, 4, 5, 1)$
12 : $P_{250} = (1, 6, 2, 1)$	32 : $P_{435} = (2, 5, 5, 1)$
13 : $P_{261} = (4, 7, 2, 1)$	33 : $P_{444} = (3, 6, 5, 1)$
14 : $P_{277} = (4, 1, 3, 1)$	34 : $P_{450} = (1, 7, 5, 1)$
15 : $P_{283} = (2, 2, 3, 1)$	35 : $P_{467} = (2, 1, 6, 1)$
16 : $P_{296} = (7, 3, 3, 1)$	36 : $P_{474} = (1, 2, 6, 1)$
17 : $P_{298} = (1, 4, 3, 1)$	37 : $P_{486} = (5, 3, 6, 1)$
18 : $P_{311} = (6, 5, 3, 1)$	38 : $P_{495} = (6, 4, 6, 1)$
19 : $P_{318} = (5, 6, 3, 1)$	39 : $P_{500} = (3, 5, 6, 1)$
20 : $P_{324} = (3, 7, 3, 1)$	40 : $P_{509} = (4, 6, 6, 1)$
21 : $P_{340} = (3, 1, 4, 1)$	41 : $P_{520} = (7, 7, 6, 1)$
22 : $P_{352} = (7, 2, 4, 1)$	42 : $P_{534} = (5, 1, 7, 1)$
23 : $P_{354} = (1, 3, 4, 1)$	43 : $P_{541} = (4, 2, 7, 1)$
24 : $P_{366} = (5, 4, 4, 1)$	44 : $P_{548} = (3, 3, 7, 1)$
25 : $P_{373} = (4, 5, 4, 1)$	45 : $P_{555} = (2, 4, 7, 1)$
26 : $P_{383} = (6, 6, 4, 1)$	46 : $P_{562} = (1, 5, 7, 1)$
27 : $P_{387} = (2, 7, 4, 1)$	47 : $P_{576} = (7, 6, 7, 1)$
28 : $P_{408} = (7, 1, 5, 1)$	48 : $P_{583} = (6, 7, 7, 1)$
29 : $P_{414} = (5, 2, 5, 1)$	

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_2

The surface has 73 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	12 : $P_{13} = (2, 0, 1, 0)$	24 : $P_{67} = (0, 7, 1, 0)$
1 : $P_1 = (0, 1, 0, 0)$	13 : $P_{14} = (3, 0, 1, 0)$	25 : $P_{159} = (6, 2, 1, 1)$
2 : $P_2 = (0, 0, 1, 0)$	14 : $P_{15} = (4, 0, 1, 0)$	26 : $P_{165} = (4, 3, 1, 1)$
3 : $P_4 = (1, 1, 1, 1)$	15 : $P_{16} = (5, 0, 1, 0)$	27 : $P_{172} = (3, 4, 1, 1)$
4 : $P_5 = (1, 1, 0, 0)$	16 : $P_{17} = (6, 0, 1, 0)$	28 : $P_{184} = (7, 5, 1, 1)$
5 : $P_6 = (2, 1, 0, 0)$	17 : $P_{18} = (7, 0, 1, 0)$	29 : $P_{187} = (2, 6, 1, 1)$
6 : $P_7 = (3, 1, 0, 0)$	18 : $P_{19} = (0, 1, 1, 0)$	30 : $P_{198} = (5, 7, 1, 1)$
7 : $P_8 = (4, 1, 0, 0)$	19 : $P_{27} = (0, 2, 1, 0)$	31 : $P_{215} = (6, 1, 2, 1)$
8 : $P_9 = (5, 1, 0, 0)$	20 : $P_{35} = (0, 3, 1, 0)$	32 : $P_{220} = (3, 2, 2, 1)$
9 : $P_{10} = (6, 1, 0, 0)$	21 : $P_{43} = (0, 4, 1, 0)$	33 : $P_{227} = (2, 3, 2, 1)$
10 : $P_{11} = (7, 1, 0, 0)$	22 : $P_{51} = (0, 5, 1, 0)$	34 : $P_{240} = (7, 4, 2, 1)$
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