

Rank-74248 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1361351310

General information

Number of lines	3
Number of points	65
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	41
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$2^3, 1^{21}, 0^{41}$

Singular Points

The surface has 1 singular points:

$$0 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^4 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{bmatrix}_{4636} = \begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4636} = \mathbf{Pl}(1, 1, 1, 1, 3, 1)_{2435}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma & \gamma \\ 0 & 1 & \gamma^6 & \gamma^4 \end{bmatrix}_{1376} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 6 & 7 \end{bmatrix}_{1376} = \mathbf{Pl}(1, 1, 1, 1, 5, 1)_{3457}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^2 \\ 0 & 1 & \gamma^5 & \gamma \end{bmatrix}_{2647} = \begin{bmatrix} 1 & 0 & 4 & 4 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{2647} = \mathbf{Pl}(1, 1, 1, 1, 6, 1)_{3982}$$

Rank of lines: (4636, 1376, 2647)

Rank of points on Klein quadric: (2435, 3457, 3982)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{571} = (2, 6, 7, 1) = \ell_0 \cap \ell_1$$

$$P_{376} = (7, 5, 4, 1) = \ell_0 \cap \ell_2$$

$$P_{229} = (4, 3, 2, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 21 single points:

The single points on the surface are:

$$0 : P_{22} = (3, 1, 1, 0) \text{ lies on line } \ell_0$$

$$1 : P_{24} = (5, 1, 1, 0) \text{ lies on line } \ell_1$$

$$2 : P_{25} = (6, 1, 1, 0) \text{ lies on line } \ell_2$$

$$3 : P_{85} = (3, 1, 0, 1) \text{ lies on line } \ell_1$$

$$4 : P_{87} = (5, 1, 0, 1) \text{ lies on line } \ell_2$$

$$5 : P_{88} = (6, 1, 0, 1) \text{ lies on line } \ell_0$$

$$6 : P_{141} = (3, 0, 1, 1) \text{ lies on line } \ell_2$$

$$7 : P_{143} = (5, 0, 1, 1) \text{ lies on line } \ell_0$$

$$8 : P_{144} = (6, 0, 1, 1) \text{ lies on line } \ell_1$$

$$9 : P_{225} = (0, 3, 2, 1) \text{ lies on line } \ell_0$$

$$10 : P_{282} = (1, 2, 3, 1) \text{ lies on line } \ell_1$$

$$11 : P_{283} = (2, 2, 3, 1) \text{ lies on line } \ell_2$$

$$12 : P_{284} = (3, 2, 3, 1) \text{ lies on line } \ell_0$$

$$13 : P_{369} = (0, 5, 4, 1) \text{ lies on line } \ell_1$$

$$14 : P_{426} = (1, 4, 5, 1) \text{ lies on line } \ell_2$$

$$15 : P_{429} = (4, 4, 5, 1) \text{ lies on line } \ell_0$$

$$16 : P_{430} = (5, 4, 5, 1) \text{ lies on line } \ell_1$$

$$17 : P_{514} = (1, 7, 6, 1) \text{ lies on line } \ell_0$$

$$18 : P_{519} = (6, 7, 6, 1) \text{ lies on line } \ell_2$$

$$19 : P_{520} = (7, 7, 6, 1) \text{ lies on line } \ell_1$$

$$20 : P_{569} = (0, 6, 7, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

Points on surface but on no line

The surface has 41 points not on any line:

The points on the surface but not on lines are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_3 = (0, 0, 0, 1)$$

$$2 : P_{12} = (1, 0, 1, 0)$$

$$3 : P_{29} = (2, 2, 1, 0)$$

$$4 : P_{47} = (4, 4, 1, 0)$$

$$5 : P_{74} = (7, 7, 1, 0)$$

$$6 : P_{75} = (1, 0, 0, 1)$$

$$7 : P_{97} = (7, 2, 0, 1)$$

$$8 : P_{108} = (2, 4, 0, 1)$$

$$9 : P_{134} = (4, 7, 0, 1)$$

10 : $P_{146} = (0, 1, 1, 1)$	26 : $P_{407} = (6, 1, 5, 1)$
11 : $P_{164} = (3, 3, 1, 1)$	27 : $P_{408} = (7, 1, 5, 1)$
12 : $P_{182} = (5, 5, 1, 1)$	28 : $P_{409} = (0, 2, 5, 1)$
13 : $P_{191} = (6, 6, 1, 1)$	29 : $P_{414} = (5, 2, 5, 1)$
14 : $P_{253} = (4, 6, 2, 1)$	30 : $P_{439} = (6, 5, 5, 1)$
15 : $P_{269} = (4, 0, 3, 1)$	31 : $P_{453} = (4, 7, 5, 1)$
16 : $P_{274} = (1, 1, 3, 1)$	32 : $P_{459} = (2, 0, 6, 1)$
17 : $P_{277} = (4, 1, 3, 1)$	33 : $P_{466} = (1, 1, 6, 1)$
18 : $P_{278} = (5, 1, 3, 1)$	34 : $P_{467} = (2, 1, 6, 1)$
19 : $P_{294} = (5, 3, 3, 1)$	35 : $P_{468} = (3, 1, 6, 1)$
20 : $P_{299} = (2, 4, 3, 1)$	36 : $P_{480} = (7, 2, 6, 1)$
21 : $P_{321} = (0, 7, 3, 1)$	37 : $P_{489} = (0, 4, 6, 1)$
22 : $P_{324} = (3, 7, 3, 1)$	38 : $P_{495} = (6, 4, 6, 1)$
23 : $P_{360} = (7, 3, 4, 1)$	39 : $P_{508} = (3, 6, 6, 1)$
24 : $P_{400} = (7, 0, 5, 1)$	40 : $P_{563} = (2, 5, 7, 1)$
25 : $P_{402} = (1, 1, 5, 1)$	

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{571}	P_{376}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{571}	P_{229}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{376}	P_{229}

The surface has 65 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	16 : $P_{141} = (3, 0, 1, 1)$	32 : $P_{284} = (3, 2, 3, 1)$
1 : $P_3 = (0, 0, 0, 1)$	17 : $P_{143} = (5, 0, 1, 1)$	33 : $P_{294} = (5, 3, 3, 1)$
2 : $P_{12} = (1, 0, 1, 0)$	18 : $P_{144} = (6, 0, 1, 1)$	34 : $P_{299} = (2, 4, 3, 1)$
3 : $P_{22} = (3, 1, 1, 0)$	19 : $P_{146} = (0, 1, 1, 1)$	35 : $P_{321} = (0, 7, 3, 1)$
4 : $P_{24} = (5, 1, 1, 0)$	20 : $P_{164} = (3, 3, 1, 1)$	36 : $P_{324} = (3, 7, 3, 1)$
5 : $P_{25} = (6, 1, 1, 0)$	21 : $P_{182} = (5, 5, 1, 1)$	37 : $P_{360} = (7, 3, 4, 1)$
6 : $P_{29} = (2, 2, 1, 0)$	22 : $P_{191} = (6, 6, 1, 1)$	38 : $P_{369} = (0, 5, 4, 1)$
7 : $P_{47} = (4, 4, 1, 0)$	23 : $P_{225} = (0, 3, 2, 1)$	39 : $P_{376} = (7, 5, 4, 1)$
8 : $P_{74} = (7, 7, 1, 0)$	24 : $P_{229} = (4, 3, 2, 1)$	40 : $P_{400} = (7, 0, 5, 1)$
9 : $P_{75} = (1, 0, 0, 1)$	25 : $P_{253} = (4, 6, 2, 1)$	41 : $P_{402} = (1, 1, 5, 1)$
10 : $P_{85} = (3, 1, 0, 1)$	26 : $P_{269} = (4, 0, 3, 1)$	42 : $P_{407} = (6, 1, 5, 1)$
11 : $P_{87} = (5, 1, 0, 1)$	27 : $P_{274} = (1, 1, 3, 1)$	43 : $P_{408} = (7, 1, 5, 1)$
12 : $P_{88} = (6, 1, 0, 1)$	28 : $P_{277} = (4, 1, 3, 1)$	44 : $P_{409} = (0, 2, 5, 1)$
13 : $P_{97} = (7, 2, 0, 1)$	29 : $P_{278} = (5, 1, 3, 1)$	45 : $P_{414} = (5, 2, 5, 1)$
14 : $P_{108} = (2, 4, 0, 1)$	30 : $P_{282} = (1, 2, 3, 1)$	46 : $P_{426} = (1, 4, 5, 1)$
15 : $P_{134} = (4, 7, 0, 1)$	31 : $P_{283} = (2, 2, 3, 1)$	47 : $P_{429} = (4, 4, 5, 1)$

$$\begin{aligned}
48 : P_{430} &= (5, 4, 5, 1) \\
49 : P_{439} &= (6, 5, 5, 1) \\
50 : P_{453} &= (4, 7, 5, 1) \\
51 : P_{459} &= (2, 0, 6, 1) \\
52 : P_{466} &= (1, 1, 6, 1) \\
53 : P_{467} &= (2, 1, 6, 1)
\end{aligned}$$

$$\begin{aligned}
54 : P_{468} &= (3, 1, 6, 1) \\
55 : P_{480} &= (7, 2, 6, 1) \\
56 : P_{489} &= (0, 4, 6, 1) \\
57 : P_{495} &= (6, 4, 6, 1) \\
58 : P_{508} &= (3, 6, 6, 1) \\
59 : P_{514} &= (1, 7, 6, 1)
\end{aligned}$$

$$\begin{aligned}
60 : P_{519} &= (6, 7, 6, 1) \\
61 : P_{520} &= (7, 7, 6, 1) \\
62 : P_{563} &= (2, 5, 7, 1) \\
63 : P_{569} &= (0, 6, 7, 1) \\
64 : P_{571} &= (2, 6, 7, 1)
\end{aligned}$$