Rank-139 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 2236975

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	90
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^6
Type of lines on points	3^2, 2^3, 1^{90}, 0^{210}

Singular Points

The surface has 1 singular points:

$0: \ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \mathbf{Pl}(1,0,0,0,1,0)_{290}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \mathbf{Pl}(0,0,1,0,1,0)_{320}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \mathbf{Pl}(1,1,0,0,1,1)_{8961}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \mathbf{Pl}(1,1,1,1,0,0)_{64}$$

Rank of lines: (1, 16, 257, 530, 289, 4369)
Rank of points on Klein quadric: (3, 290, 320, 9426, 8961, 64)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{35} = (0,1,1,0) = \ell_0 \cap \ell_5$
$P_{290} = (0,1,0,1) = \ell_1 \cap \ell_4$
$P_{530} = (0,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 90 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_3
$1 : P_{20} = (1,0,1,0)$ lies on line ℓ_4
$2 : P_{36} = (1,1,1,0)$ lies on line ℓ_0
$3 : P_{37} = (2,1,1,0)$ lies on line ℓ_0
$4 : P_{38} = (3,1,1,0)$ lies on line ℓ_0
$5 : P_{39} = (4,1,1,0)$ lies on line ℓ_0
$6 : P_{40} = (5,1,1,0)$ lies on line ℓ_0
$7 : P_{41} = (6,1,1,0)$ lies on line ℓ_0
$8 : P_{42} = (7,1,1,0)$ lies on line ℓ_0
$9 : P_{43} = (8,1,1,0)$ lies on line ℓ_0
$10 : P_{44} = (9,1,1,0)$ lies on line ℓ_0
$11 : P_{45} = (10,1,1,0)$ lies on line ℓ_0
$12 : P_{46} = (11,1,1,0)$ lies on line ℓ_0
$13 : P_{47} = (12,1,1,0)$ lies on line ℓ_0
$14 : P_{48} = (13,1,1,0)$ lies on line ℓ_0
$15 : P_{49} = (14,1,1,0)$ lies on line ℓ_0
$16 : P_{50} = (15,1,1,0)$ lies on line ℓ_0
$17 : P_{275} = (1,0,0,1)$ lies on line ℓ_5
$18 : P_{291} = (1,1,0,1)$ lies on line ℓ_1
$19 : P_{292} = (2,1,0,1)$ lies on line ℓ_1
$20 : P_{293} = (3,1,0,1)$ lies on line ℓ_1
$21 : P_{294} = (4,1,0,1)$ lies on line ℓ_1
$22 : P_{295} = (5,1,0,1)$ lies on line ℓ_1
$23 : P_{296} = (6,1,0,1)$ lies on line ℓ_1
$24 : P_{297} = (7,1,0,1)$ lies on line ℓ_1
$25 : P_{298} = (8,1,0,1)$ lies on line ℓ_1
$26 : P_{299} = (9,1,0,1)$ lies on line ℓ_1
$27 : P_{300} = (10,1,0,1)$ lies on line ℓ_1
$28 : P_{301} = (11,1,0,1)$ lies on line ℓ_1
$29 : P_{302} = (12,1,0,1)$ lies on line ℓ_1
$30 : P_{303} = (13,1,0,1)$ lies on line ℓ_1
$31 : P_{304} = (14,1,0,1)$ lies on line ℓ_1

32 : $P_{305} = (15, 1, 0, 1)$ lies on line ℓ_1
33 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_2
34 : $P_{532} = (2, 0, 1, 1)$ lies on line ℓ_2
35 : $P_{533} = (3, 0, 1, 1)$ lies on line ℓ_2
36 : $P_{534} = (4, 0, 1, 1)$ lies on line ℓ_2
37 : $P_{535} = (5, 0, 1, 1)$ lies on line ℓ_2
38 : $P_{536} = (6, 0, 1, 1)$ lies on line ℓ_2
39 : $P_{537} = (7, 0, 1, 1)$ lies on line ℓ_2
40 : $P_{538} = (8, 0, 1, 1)$ lies on line ℓ_2
41 : $P_{539} = (9, 0, 1, 1)$ lies on line ℓ_2
42 : $P_{540} = (10, 0, 1, 1)$ lies on line ℓ_2
43 : $P_{541} = (11, 0, 1, 1)$ lies on line ℓ_2
44 : $P_{542} = (12, 0, 1, 1)$ lies on line ℓ_2
45 : $P_{543} = (13, 0, 1, 1)$ lies on line ℓ_2
46 : $P_{544} = (14, 0, 1, 1)$ lies on line ℓ_2
47 : $P_{545} = (15, 0, 1, 1)$ lies on line ℓ_2
48 : $P_{563} = (2, 2, 1, 1)$ lies on line ℓ_3
49 : $P_{580} = (3, 3, 1, 1)$ lies on line ℓ_3
50 : $P_{597} = (4, 4, 1, 1)$ lies on line ℓ_3
51 : $P_{614} = (5, 5, 1, 1)$ lies on line ℓ_3
52 : $P_{631} = (6, 6, 1, 1)$ lies on line ℓ_3
53 : $P_{648} = (7, 7, 1, 1)$ lies on line ℓ_3
54 : $P_{665} = (8, 8, 1, 1)$ lies on line ℓ_3
55 : $P_{682} = (9, 9, 1, 1)$ lies on line ℓ_3
56 : $P_{699} = (10, 10, 1, 1)$ lies on line ℓ_3
57 : $P_{716} = (11, 11, 1, 1)$ lies on line ℓ_3
58 : $P_{733} = (12, 12, 1, 1)$ lies on line ℓ_3
59 : $P_{750} = (13, 13, 1, 1)$ lies on line ℓ_3
60 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_3
61 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_3
62 : $P_{803} = (2, 1, 2, 1)$ lies on line ℓ_4
63 : $P_{818} = (1, 2, 2, 1)$ lies on line ℓ_5
64 : $P_{1060} = (3, 1, 3, 1)$ lies on line ℓ_4
65 : $P_{1090} = (1, 3, 3, 1)$ lies on line ℓ_5
66 : $P_{1317} = (4, 1, 4, 1)$ lies on line ℓ_4
67 : $P_{1362} = (1, 4, 4, 1)$ lies on line ℓ_5
68 : $P_{1574} = (5, 1, 5, 1)$ lies on line ℓ_4
69 : $P_{1634} = (1, 5, 5, 1)$ lies on line ℓ_5
70 : $P_{1831} = (6, 1, 6, 1)$ lies on line ℓ_4
71 : $P_{1906} = (1, 6, 6, 1)$ lies on line ℓ_5
72 : $P_{2088} = (7, 1, 7, 1)$ lies on line ℓ_4
73 : $P_{2178} = (1, 7, 7, 1)$ lies on line ℓ_5
74 : $P_{2345} = (8, 1, 8, 1)$ lies on line ℓ_4
75 : $P_{2450} = (1, 8, 8, 1)$ lies on line ℓ_5
76 : $P_{2602} = (9, 1, 9, 1)$ lies on line ℓ_4
77 : $P_{2722} = (1, 9, 9, 1)$ lies on line ℓ_5
78 : $P_{2859} = (10, 1, 10, 1)$ lies on line ℓ_4
79 : $P_{2994} = (1, 10, 10, 1)$ lies on line ℓ_5
80 : $P_{3116} = (11, 1, 11, 1)$ lies on line ℓ_4
81 : $P_{3266} = (1, 11, 11, 1)$ lies on line ℓ_5
82 : $P_{3373} = (12, 1, 12, 1)$ lies on line ℓ_4
83 : $P_{3538} = (1, 12, 12, 1)$ lies on line ℓ_5
84 : $P_{3630} = (13, 1, 13, 1)$ lies on line ℓ_4
85 : $P_{3810} = (1, 13, 13, 1)$ lies on line ℓ_5
86 : $P_{3887} = (14, 1, 14, 1)$ lies on line ℓ_4
87 : $P_{4082} = (1, 14, 14, 1)$ lies on line ℓ_5
88 : $P_{4144} = (15, 1, 15, 1)$ lies on line ℓ_4
89 : $P_{4354} = (1, 15, 15, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_{64} = (13, 2, 1, 0)$
1 : $P_{80} = (13, 3, 1, 0)$
2 : $P_{90} = (7, 4, 1, 0)$
3 : $P_{106} = (7, 5, 1, 0)$
4 : $P_{126} = (11, 6, 1, 0)$
5 : $P_{142} = (11, 7, 1, 0)$
6 : $P_{159} = (12, 8, 1, 0)$
7 : $P_{175} = (12, 9, 1, 0)$
8 : $P_{179} = (0, 10, 1, 0)$
9 : $P_{195} = (0, 11, 1, 0)$
10 : $P_{221} = (10, 12, 1, 0)$
11 : $P_{237} = (10, 13, 1, 0)$
12 : $P_{249} = (6, 14, 1, 0)$
13 : $P_{265} = (6, 15, 1, 0)$
14 : $P_{319} = (13, 2, 0, 1)$
15 : $P_{335} = (13, 3, 0, 1)$
16 : $P_{345} = (7, 4, 0, 1)$
17 : $P_{361} = (7, 5, 0, 1)$
18 : $P_{381} = (11, 6, 0, 1)$
19 : $P_{397} = (11, 7, 0, 1)$
20 : $P_{414} = (12, 8, 0, 1)$
21 : $P_{430} = (12, 9, 0, 1)$
22 : $P_{434} = (0, 10, 0, 1)$
23 : $P_{450} = (0, 11, 0, 1)$
24 : $P_{476} = (10, 12, 0, 1)$
25 : $P_{492} = (10, 13, 0, 1)$
26 : $P_{504} = (6, 14, 0, 1)$
27 : $P_{520} = (6, 15, 0, 1)$

28 : $P_{798} = (13, 0, 2, 1)$
29 : $P_{857} = (8, 4, 2, 1)$
30 : $P_{880} = (15, 5, 2, 1)$
31 : $P_{891} = (10, 6, 2, 1)$
32 : $P_{903} = (6, 7, 2, 1)$
33 : $P_{916} = (3, 8, 2, 1)$
34 : $P_{930} = (1, 9, 2, 1)$
35 : $P_{952} = (7, 10, 2, 1)$
36 : $P_{962} = (1, 11, 2, 1)$
37 : $P_{981} = (4, 12, 2, 1)$
38 : $P_{1006} = (13, 13, 2, 1)$
39 : $P_{1014} = (5, 14, 2, 1)$
40 : $P_{1039} = (14, 15, 2, 1)$
41 : $P_{1054} = (13, 0, 3, 1)$
42 : $P_{1107} = (2, 4, 3, 1)$
43 : $P_{1125} = (4, 5, 3, 1)$
44 : $P_{1142} = (5, 6, 3, 1)$
45 : $P_{1161} = (8, 7, 3, 1)$
46 : $P_{1180} = (11, 8, 3, 1)$
47 : $P_{1193} = (8, 9, 3, 1)$
48 : $P_{1206} = (5, 10, 3, 1)$
49 : $P_{1219} = (2, 11, 3, 1)$
50 : $P_{1238} = (5, 12, 3, 1)$
51 : $P_{1262} = (13, 13, 3, 1)$
52 : $P_{1273} = (8, 14, 3, 1)$
53 : $P_{1283} = (2, 15, 3, 1)$
54 : $P_{1304} = (7, 0, 4, 1)$
55 : $P_{1337} = (8, 2, 4, 1)$
56 : $P_{1347} = (2, 3, 4, 1)$
57 : $P_{1402} = (9, 6, 4, 1)$
58 : $P_{1416} = (7, 7, 4, 1)$
59 : $P_{1428} = (3, 8, 4, 1)$
60 : $P_{1456} = (15, 9, 4, 1)$
61 : $P_{1458} = (1, 10, 4, 1)$
62 : $P_{1485} = (12, 11, 4, 1)$
63 : $P_{1502} = (13, 12, 4, 1)$
64 : $P_{1516} = (11, 13, 4, 1)$
65 : $P_{1522} = (1, 14, 4, 1)$
66 : $P_{1542} = (5, 15, 4, 1)$
67 : $P_{1560} = (7, 0, 5, 1)$
68 : $P_{1600} = (15, 2, 5, 1)$
69 : $P_{1605} = (4, 3, 5, 1)$
70 : $P_{1657} = (8, 6, 5, 1)$
71 : $P_{1672} = (7, 7, 5, 1)$
72 : $P_{1690} = (9, 8, 5, 1)$
73 : $P_{1701} = (4, 9, 5, 1)$
74 : $P_{1717} = (4, 10, 5, 1)$
75 : $P_{1737} = (8, 11, 5, 1)$
76 : $P_{1760} = (15, 12, 5, 1)$
77 : $P_{1769} = (8, 13, 5, 1)$
78 : $P_{1792} = (15, 14, 5, 1)$
79 : $P_{1803} = (10, 15, 5, 1)$
80 : $P_{1820} = (11, 0, 6, 1)$
81 : $P_{1851} = (10, 2, 6, 1)$
82 : $P_{1862} = (5, 3, 6, 1)$
83 : $P_{1882} = (9, 4, 6, 1)$
84 : $P_{1897} = (8, 5, 6, 1)$
85 : $P_{1940} = (3, 8, 6, 1)$
86 : $P_{1960} = (7, 9, 6, 1)$
87 : $P_{1981} = (12, 10, 6, 1)$
88 : $P_{1996} = (11, 11, 6, 1)$
89 : $P_{2003} = (2, 12, 6, 1)$
90 : $P_{2021} = (4, 13, 6, 1)$
91 : $P_{2039} = (6, 14, 6, 1)$
92 : $P_{2055} = (6, 15, 6, 1)$
93 : $P_{2076} = (11, 0, 7, 1)$
94 : $P_{2103} = (6, 2, 7, 1)$
95 : $P_{2121} = (8, 3, 7, 1)$
96 : $P_{2136} = (7, 4, 7, 1)$
97 : $P_{2152} = (7, 5, 7, 1)$
98 : $P_{2208} = (15, 8, 7, 1)$
99 : $P_{2219} = (10, 9, 7, 1)$
100 : $P_{2238} = (13, 10, 7, 1)$
101 : $P_{2252} = (11, 11, 7, 1)$
102 : $P_{2271} = (14, 12, 7, 1)$
103 : $P_{2282} = (9, 13, 7, 1)$
104 : $P_{2291} = (2, 14, 7, 1)$
105 : $P_{2308} = (3, 15, 7, 1)$
106 : $P_{2333} = (12, 0, 8, 1)$
107 : $P_{2356} = (3, 2, 8, 1)$
108 : $P_{2380} = (11, 3, 8, 1)$
109 : $P_{2388} = (3, 4, 8, 1)$
110 : $P_{2410} = (9, 5, 8, 1)$
111 : $P_{2420} = (3, 6, 8, 1)$
112 : $P_{2448} = (15, 7, 8, 1)$
113 : $P_{2496} = (15, 10, 8, 1)$
114 : $P_{2506} = (9, 11, 8, 1)$
115 : $P_{2525} = (12, 12, 8, 1)$
116 : $P_{2544} = (15, 13, 8, 1)$
117 : $P_{2554} = (9, 14, 8, 1)$
118 : $P_{2575} = (14, 15, 8, 1)$
119 : $P_{2589} = (12, 0, 9, 1)$
120 : $P_{2610} = (1, 2, 9, 1)$
121 : $P_{2633} = (8, 3, 9, 1)$
122 : $P_{2656} = (15, 4, 9, 1)$
123 : $P_{2661} = (4, 5, 9, 1)$
124 : $P_{2680} = (7, 6, 9, 1)$
125 : $P_{2699} = (10, 7, 9, 1)$
126 : $P_{2743} = (6, 10, 9, 1)$
127 : $P_{2754} = (1, 11, 9, 1)$
128 : $P_{2781} = (12, 12, 9, 1)$
129 : $P_{2799} = (14, 13, 9, 1)$
130 : $P_{2804} = (3, 14, 9, 1)$
131 : $P_{2822} = (5, 15, 9, 1)$
132 : $P_{2833} = (0, 0, 10, 1)$
133 : $P_{2872} = (7, 2, 10, 1)$
134 : $P_{2886} = (5, 3, 10, 1)$
135 : $P_{2898} = (1, 4, 10, 1)$

136 : $P_{2917} = (4, 5, 10, 1)$
137 : $P_{2941} = (12, 6, 10, 1)$
138 : $P_{2958} = (13, 7, 10, 1)$
139 : $P_{2976} = (15, 8, 10, 1)$
140 : $P_{2983} = (6, 9, 10, 1)$
141 : $P_{3035} = (10, 12, 10, 1)$
142 : $P_{3051} = (10, 13, 10, 1)$
143 : $P_{3058} = (1, 14, 10, 1)$
144 : $P_{3087} = (14, 15, 10, 1)$
145 : $P_{3089} = (0, 0, 11, 1)$
146 : $P_{3122} = (1, 2, 11, 1)$
147 : $P_{3139} = (2, 3, 11, 1)$
148 : $P_{3165} = (12, 4, 11, 1)$
149 : $P_{3177} = (8, 5, 11, 1)$
150 : $P_{3196} = (11, 6, 11, 1)$
151 : $P_{3212} = (11, 7, 11, 1)$
152 : $P_{3226} = (9, 8, 11, 1)$
153 : $P_{3234} = (1, 9, 11, 1)$
154 : $P_{3288} = (7, 12, 11, 1)$
155 : $P_{3303} = (6, 13, 11, 1)$
156 : $P_{3326} = (13, 14, 11, 1)$
157 : $P_{3332} = (3, 15, 11, 1)$
158 : $P_{3355} = (10, 0, 12, 1)$
159 : $P_{3381} = (4, 2, 12, 1)$
160 : $P_{3398} = (5, 3, 12, 1)$
161 : $P_{3422} = (13, 4, 12, 1)$
162 : $P_{3440} = (15, 5, 12, 1)$
163 : $P_{3443} = (2, 6, 12, 1)$
164 : $P_{3471} = (14, 7, 12, 1)$
165 : $P_{3485} = (12, 8, 12, 1)$
166 : $P_{3501} = (12, 9, 12, 1)$
167 : $P_{3515} = (10, 10, 12, 1)$
168 : $P_{3528} = (7, 11, 12, 1)$
169 : $P_{3580} = (11, 14, 12, 1)$
170 : $P_{3588} = (3, 15, 12, 1)$
171 : $P_{3611} = (10, 0, 13, 1)$
172 : $P_{3646} = (13, 2, 13, 1)$
173 : $P_{3662} = (13, 3, 13, 1)$
174 : $P_{3676} = (11, 4, 13, 1)$
175 : $P_{3689} = (8, 5, 13, 1)$
176 : $P_{3701} = (4, 6, 13, 1)$
177 : $P_{3722} = (9, 7, 13, 1)$
178 : $P_{3744} = (15, 8, 13, 1)$
179 : $P_{3759} = (14, 9, 13, 1)$
180 : $P_{3771} = (10, 10, 13, 1)$
181 : $P_{3783} = (6, 11, 13, 1)$
182 : $P_{3837} = (12, 14, 13, 1)$
183 : $P_{3846} = (5, 15, 13, 1)$
184 : $P_{3863} = (6, 0, 14, 1)$
185 : $P_{3894} = (5, 2, 14, 1)$
186 : $P_{3913} = (8, 3, 14, 1)$
187 : $P_{3922} = (1, 4, 14, 1)$
188 : $P_{3952} = (15, 5, 14, 1)$
189 : $P_{3959} = (6, 6, 14, 1)$
190 : $P_{3971} = (2, 7, 14, 1)$
191 : $P_{3994} = (9, 8, 14, 1)$
192 : $P_{4004} = (3, 9, 14, 1)$
193 : $P_{4018} = (1, 10, 14, 1)$
194 : $P_{4046} = (13, 11, 14, 1)$
195 : $P_{4060} = (11, 12, 14, 1)$
196 : $P_{4077} = (12, 13, 14, 1)$
197 : $P_{4119} = (6, 0, 15, 1)$
198 : $P_{4159} = (14, 2, 15, 1)$
199 : $P_{4163} = (2, 3, 15, 1)$
200 : $P_{4182} = (5, 4, 15, 1)$
201 : $P_{4203} = (10, 5, 15, 1)$
202 : $P_{4215} = (6, 6, 15, 1)$
203 : $P_{4228} = (3, 7, 15, 1)$
204 : $P_{4255} = (14, 8, 15, 1)$
205 : $P_{4262} = (5, 9, 15, 1)$
206 : $P_{4287} = (14, 10, 15, 1)$
207 : $P_{4292} = (3, 11, 15, 1)$
208 : $P_{4308} = (3, 12, 15, 1)$
209 : $P_{4326} = (5, 13, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	0	0	1
1	1	0	1	0	1	0
2	1	1	0	1	0	0
3	0	0	1	0	1	1
4	0	1	0	1	0	1
5	1	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_0	P_0	P_{35}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_0	P_{290}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_0	P_0	P_{530}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5
in point	P_{530}	P_4	P_4

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{290}	P_4	P_4

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_{35}	P_4	P_4

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_5 = (1,1,0,0)$
3 : $P_{20} = (1,0,1,0)$
4 : $P_{35} = (0,1,1,0)$
5 : $P_{36} = (1,1,1,0)$
6 : $P_{37} = (2,1,1,0)$
7 : $P_{38} = (3,1,1,0)$
8 : $P_{39} = (4,1,1,0)$
9 : $P_{40} = (5,1,1,0)$
10 : $P_{41} = (6,1,1,0)$
11 : $P_{42} = (7,1,1,0)$
12 : $P_{43} = (8,1,1,0)$
13 : $P_{44} = (9,1,1,0)$
14 : $P_{45} = (10,1,1,0)$
15 : $P_{46} = (11,1,1,0)$
16 : $P_{47} = (12,1,1,0)$
17 : $P_{48} = (13,1,1,0)$
18 : $P_{49} = (14,1,1,0)$
19 : $P_{50} = (15,1,1,0)$
20 : $P_{64} = (13,2,1,0)$
21 : $P_{80} = (13,3,1,0)$
22 : $P_{90} = (7,4,1,0)$
23 : $P_{106} = (7,5,1,0)$
24 : $P_{126} = (11,6,1,0)$
25 : $P_{142} = (11,7,1,0)$
26 : $P_{159} = (12,8,1,0)$
27 : $P_{175} = (12,9,1,0)$
28 : $P_{179} = (0,10,1,0)$
29 : $P_{195} = (0,11,1,0)$
30 : $P_{221} = (10,12,1,0)$
31 : $P_{237} = (10,13,1,0)$
32 : $P_{249} = (6,14,1,0)$
33 : $P_{265} = (6,15,1,0)$
34 : $P_{275} = (1,0,0,1)$
35 : $P_{290} = (0,1,0,1)$
36 : $P_{291} = (1,1,0,1)$
37 : $P_{292} = (2,1,0,1)$
38 : $P_{293} = (3,1,0,1)$
39 : $P_{294} = (4,1,0,1)$
40 : $P_{295} = (5,1,0,1)$
41 : $P_{296} = (6,1,0,1)$
42 : $P_{297} = (7,1,0,1)$
43 : $P_{298} = (8,1,0,1)$
44 : $P_{299} = (9,1,0,1)$
45 : $P_{300} = (10,1,0,1)$
46 : $P_{301} = (11,1,0,1)$
47 : $P_{302} = (12,1,0,1)$
48 : $P_{303} = (13,1,0,1)$
49 : $P_{304} = (14,1,0,1)$
50 : $P_{305} = (15,1,0,1)$
51 : $P_{319} = (13,2,0,1)$
52 : $P_{335} = (13,3,0,1)$
53 : $P_{345} = (7,4,0,1)$
54 : $P_{361} = (7,5,0,1)$
55 : $P_{381} = (11,6,0,1)$
56 : $P_{397} = (11,7,0,1)$
57 : $P_{414} = (12,8,0,1)$
58 : $P_{430} = (12,9,0,1)$
59 : $P_{434} = (0,10,0,1)$
60 : $P_{450} = (0,11,0,1)$
61 : $P_{476} = (10,12,0,1)$
62 : $P_{492} = (10,13,0,1)$
63 : $P_{504} = (6,14,0,1)$
64 : $P_{520} = (6,15,0,1)$
65 : $P_{530} = (0,0,1,1)$
66 : $P_{531} = (1,0,1,1)$
67 : $P_{532} = (2,0,1,1)$
68 : $P_{533} = (3,0,1,1)$
69 : $P_{534} = (4,0,1,1)$
70 : $P_{535} = (5,0,1,1)$
71 : $P_{536} = (6,0,1,1)$
72 : $P_{537} = (7,0,1,1)$
73 : $P_{538} = (8,0,1,1)$
74 : $P_{539} = (9,0,1,1)$
75 : $P_{540} = (10,0,1,1)$
76 : $P_{541} = (11,0,1,1)$
77 : $P_{542} = (12,0,1,1)$
78 : $P_{543} = (13,0,1,1)$
79 : $P_{544} = (14,0,1,1)$
80 : $P_{545} = (15,0,1,1)$
81 : $P_{563} = (2,2,1,1)$
82 : $P_{580} = (3,3,1,1)$
83 : $P_{597} = (4,4,1,1)$
84 : $P_{614} = (5,5,1,1)$
85 : $P_{631} = (6,6,1,1)$
86 : $P_{648} = (7,7,1,1)$
87 : $P_{665} = (8,8,1,1)$
88 : $P_{682} = (9,9,1,1)$
89 : $P_{699} = (10,10,1,1)$
90 : $P_{716} = (11,11,1,1)$
91 : $P_{733} = (12,12,1,1)$
92 : $P_{750} = (13,13,1,1)$
93 : $P_{767} = (14,14,1,1)$
94 : $P_{784} = (15,15,1,1)$
95 : $P_{798} = (13,0,2,1)$
96 : $P_{803} = (2,1,2,1)$
97 : $P_{818} = (1,2,2,1)$
98 : $P_{857} = (8,4,2,1)$

$99: P_{880} = (15, 5, 2, 1)$
$100: P_{891} = (10, 6, 2, 1)$
$101: P_{903} = (6, 7, 2, 1)$
$102: P_{916} = (3, 8, 2, 1)$
$103: P_{930} = (1, 9, 2, 1)$
$104: P_{952} = (7, 10, 2, 1)$
$105: P_{962} = (1, 11, 2, 1)$
$106: P_{981} = (4, 12, 2, 1)$
$107: P_{1006} = (13, 13, 2, 1)$
$108: P_{1014} = (5, 14, 2, 1)$
$109: P_{1039} = (14, 15, 2, 1)$
$110: P_{1054} = (13, 0, 3, 1)$
$111: P_{1060} = (3, 1, 3, 1)$
$112: P_{1090} = (1, 3, 3, 1)$
$113: P_{1107} = (2, 4, 3, 1)$
$114: P_{1125} = (4, 5, 3, 1)$
$115: P_{1142} = (5, 6, 3, 1)$
$116: P_{1161} = (8, 7, 3, 1)$
$117: P_{1180} = (11, 8, 3, 1)$
$118: P_{1193} = (8, 9, 3, 1)$
$119: P_{1206} = (5, 10, 3, 1)$
$120: P_{1219} = (2, 11, 3, 1)$
$121: P_{1238} = (5, 12, 3, 1)$
$122: P_{1262} = (13, 13, 3, 1)$
$123: P_{1273} = (8, 14, 3, 1)$
$124: P_{1283} = (2, 15, 3, 1)$
$125: P_{1304} = (7, 0, 4, 1)$
$126: P_{1317} = (4, 1, 4, 1)$
$127: P_{1337} = (8, 2, 4, 1)$
$128: P_{1347} = (2, 3, 4, 1)$
$129: P_{1362} = (1, 4, 4, 1)$
$130: P_{1402} = (9, 6, 4, 1)$
$131: P_{1416} = (7, 7, 4, 1)$
$132: P_{1428} = (3, 8, 4, 1)$
$133: P_{1456} = (15, 9, 4, 1)$
$134: P_{1458} = (1, 10, 4, 1)$
$135: P_{1485} = (12, 11, 4, 1)$
$136: P_{1502} = (13, 12, 4, 1)$
$137: P_{1516} = (11, 13, 4, 1)$
$138: P_{1522} = (1, 14, 4, 1)$
$139: P_{1542} = (5, 15, 4, 1)$
$140: P_{1560} = (7, 0, 5, 1)$
$141: P_{1574} = (5, 1, 5, 1)$
$142: P_{1600} = (15, 2, 5, 1)$
$143: P_{1605} = (4, 3, 5, 1)$
$144: P_{1634} = (1, 5, 5, 1)$
$145: P_{1657} = (8, 6, 5, 1)$
$146: P_{1672} = (7, 7, 5, 1)$
$147: P_{1690} = (9, 8, 5, 1)$
$148: P_{1701} = (4, 9, 5, 1)$
$149: P_{1717} = (4, 10, 5, 1)$
$150: P_{1737} = (8, 11, 5, 1)$
$151: P_{1760} = (15, 12, 5, 1)$
$152: P_{1769} = (8, 13, 5, 1)$
$153: P_{1792} = (15, 14, 5, 1)$
$154: P_{1803} = (10, 15, 5, 1)$
$155: P_{1820} = (11, 0, 6, 1)$
$156: P_{1831} = (6, 1, 6, 1)$
$157: P_{1851} = (10, 2, 6, 1)$
$158: P_{1862} = (5, 3, 6, 1)$
$159: P_{1882} = (9, 4, 6, 1)$
$160: P_{1897} = (8, 5, 6, 1)$
$161: P_{1906} = (1, 6, 6, 1)$
$162: P_{1940} = (3, 8, 6, 1)$
$163: P_{1960} = (7, 9, 6, 1)$
$164: P_{1981} = (12, 10, 6, 1)$
$165: P_{1996} = (11, 11, 6, 1)$
$166: P_{2003} = (2, 12, 6, 1)$
$167: P_{2021} = (4, 13, 6, 1)$
$168: P_{2039} = (6, 14, 6, 1)$
$169: P_{2055} = (6, 15, 6, 1)$
$170: P_{2076} = (11, 0, 7, 1)$
$171: P_{2088} = (7, 1, 7, 1)$
$172: P_{2103} = (6, 2, 7, 1)$
$173: P_{2121} = (8, 3, 7, 1)$
$174: P_{2136} = (7, 4, 7, 1)$
$175: P_{2152} = (7, 5, 7, 1)$
$176: P_{2178} = (1, 7, 7, 1)$
$177: P_{2208} = (15, 8, 7, 1)$
$178: P_{2219} = (10, 9, 7, 1)$
$179: P_{2238} = (13, 10, 7, 1)$
$180: P_{2252} = (11, 11, 7, 1)$
$181: P_{2271} = (14, 12, 7, 1)$
$182: P_{2282} = (9, 13, 7, 1)$
$183: P_{2291} = (2, 14, 7, 1)$
$184: P_{2308} = (3, 15, 7, 1)$
$185: P_{2333} = (12, 0, 8, 1)$
$186: P_{2345} = (8, 1, 8, 1)$
$187: P_{2356} = (3, 2, 8, 1)$
$188: P_{2380} = (11, 3, 8, 1)$
$189: P_{2388} = (3, 4, 8, 1)$
$190: P_{2410} = (9, 5, 8, 1)$
$191: P_{2420} = (3, 6, 8, 1)$
$192: P_{2448} = (15, 7, 8, 1)$
$193: P_{2450} = (1, 8, 8, 1)$
$194: P_{2496} = (15, 10, 8, 1)$
$195: P_{2506} = (9, 11, 8, 1)$
$196: P_{2525} = (12, 12, 8, 1)$
$197: P_{2544} = (15, 13, 8, 1)$
$198: P_{2554} = (9, 14, 8, 1)$
$199: P_{2575} = (14, 15, 8, 1)$
$200: P_{2589} = (12, 0, 9, 1)$
$201: P_{2602} = (9, 1, 9, 1)$
$202: P_{2610} = (1, 2, 9, 1)$
$203: P_{2633} = (8, 3, 9, 1)$
$204: P_{2656} = (15, 4, 9, 1)$
$205: P_{2661} = (4, 5, 9, 1)$
$206: P_{2680} = (7, 6, 9, 1)$
$207: P_{2699} = (10, 7, 9, 1)$
$208: P_{2722} = (1, 9, 9, 1)$
$209: P_{2743} = (6, 10, 9, 1)$
$210: P_{2754} = (1, 11, 9, 1)$
$211: P_{2781} = (12, 12, 9, 1)$
$212: P_{2799} = (14, 13, 9, 1)$
$213: P_{2804} = (3, 14, 9, 1)$
$214: P_{2822} = (5, 15, 9, 1)$
$215: P_{2833} = (0, 0, 10, 1)$
$216: P_{2859} = (10, 1, 10, 1)$
$217: P_{2872} = (7, 2, 10, 1)$
$218: P_{2886} = (5, 3, 10, 1)$
$219: P_{2898} = (1, 4, 10, 1)$
$220: P_{2917} = (4, 5, 10, 1)$
$221: P_{2941} = (12, 6, 10, 1)$
$222: P_{2958} = (13, 7, 10, 1)$
$223: P_{2976} = (15, 8, 10, 1)$
$224: P_{2983} = (6, 9, 10, 1)$
$225: P_{2994} = (1, 10, 10, 1)$
$226: P_{3035} = (10, 12, 10, 1)$
$227: P_{3051} = (10, 13, 10, 1)$
$228: P_{3058} = (1, 14, 10, 1)$
$229: P_{3087} = (14, 15, 10, 1)$
$230: P_{3089} = (0, 0, 11, 1)$
$231: P_{3116} = (11, 1, 11, 1)$
$232: P_{3122} = (1, 2, 11, 1)$
$233: P_{3139} = (2, 3, 11, 1)$
$234: P_{3165} = (12, 4, 11, 1)$
$235: P_{3177} = (8, 5, 11, 1)$
$236: P_{3196} = (11, 6, 11, 1)$
$237: P_{3212} = (11, 7, 11, 1)$
$238: P_{3226} = (9, 8, 11, 1)$
$239: P_{3234} = (1, 9, 11, 1)$
$240: P_{3266} = (1, 11, 11, 1)$
$241: P_{3288} = (7, 12, 11, 1)$
$242: P_{3303} = (6, 13, 11, 1)$
$243: P_{3326} = (13, 14, 11, 1)$
$244: P_{3332} = (3, 15, 11, 1)$
$245: P_{3355} = (10, 0, 12, 1)$
$246: P_{3373} = (12, 1, 12, 1)$
$247: P_{3381} = (4, 2, 12, 1)$
$248: P_{3398} = (5, 3, 12, 1)$
$249: P_{3422} = (13, 4, 12, 1)$
$250: P_{3440} = (15, 5, 12, 1)$
$251: P_{3443} = (2, 6, 12, 1)$
$252: P_{3471} = (14, 7, 12, 1)$
$253: P_{3485} = (12, 8, 12, 1)$
$254: P_{3501} = (12, 9, 12, 1)$
$255: P_{3515} = (10, 10, 12, 1)$
$256: P_{3528} = (7, 11, 12, 1)$
$257: P_{3538} = (1, 12, 12, 1)$
$258: P_{3580} = (11, 14, 12, 1)$
$259: P_{3588} = (3, 15, 12, 1)$
$260: P_{3611} = (10, 0, 13, 1)$

261 : $P_{3630} = (13, 1, 13, 1)$
262 : $P_{3646} = (13, 2, 13, 1)$
263 : $P_{3662} = (13, 3, 13, 1)$
264 : $P_{3676} = (11, 4, 13, 1)$
265 : $P_{3689} = (8, 5, 13, 1)$
266 : $P_{3701} = (4, 6, 13, 1)$
267 : $P_{3722} = (9, 7, 13, 1)$
268 : $P_{3744} = (15, 8, 13, 1)$
269 : $P_{3759} = (14, 9, 13, 1)$
270 : $P_{3771} = (10, 10, 13, 1)$
271 : $P_{3783} = (6, 11, 13, 1)$
272 : $P_{3810} = (1, 13, 13, 1)$
273 : $P_{3837} = (12, 14, 13, 1)$
274 : $P_{3846} = (5, 15, 13, 1)$
275 : $P_{3863} = (6, 0, 14, 1)$
276 : $P_{3887} = (14, 1, 14, 1)$
277 : $P_{3894} = (5, 2, 14, 1)$
278 : $P_{3913} = (8, 3, 14, 1)$
279 : $P_{3922} = (1, 4, 14, 1)$
280 : $P_{3952} = (15, 5, 14, 1)$
281 : $P_{3959} = (6, 6, 14, 1)$
282 : $P_{3971} = (2, 7, 14, 1)$
283 : $P_{3994} = (9, 8, 14, 1)$
284 : $P_{4004} = (3, 9, 14, 1)$
285 : $P_{4018} = (1, 10, 14, 1)$
286 : $P_{4046} = (13, 11, 14, 1)$
287 : $P_{4060} = (11, 12, 14, 1)$
288 : $P_{4077} = (12, 13, 14, 1)$
289 : $P_{4082} = (1, 14, 14, 1)$
290 : $P_{4119} = (6, 0, 15, 1)$
291 : $P_{4144} = (15, 1, 15, 1)$
292 : $P_{4159} = (14, 2, 15, 1)$
293 : $P_{4163} = (2, 3, 15, 1)$
294 : $P_{4182} = (5, 4, 15, 1)$
295 : $P_{4203} = (10, 5, 15, 1)$
296 : $P_{4215} = (6, 6, 15, 1)$
297 : $P_{4228} = (3, 7, 15, 1)$
298 : $P_{4255} = (14, 8, 15, 1)$
299 : $P_{4262} = (5, 9, 15, 1)$
300 : $P_{4287} = (14, 10, 15, 1)$
301 : $P_{4292} = (3, 11, 15, 1)$
302 : $P_{4308} = (3, 12, 15, 1)$
303 : $P_{4326} = (5, 13, 15, 1)$
304 : $P_{4354} = (1, 15, 15, 1)$