

Rank-65603 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65603

General information

Number of lines	11
Number of points	11
Number of singular points	5
Number of Eckardt points	3
Number of double points	4
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{11}
Type of lines on points	$4^4, 3^3, 2^4$

Singular Points

The surface has 5 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$3 : P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$$

$$4 : P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

The 11 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{25} \\
\ell_5 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21} \\
\ell_7 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{32} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{29} \\
\ell_9 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_{10} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{27}
\end{aligned}$$

Rank of lines: (0, 4, 28, 30, 29, 34, 31, 12, 9, 33, 32)

Rank of points on Klein quadric: (0, 2, 19, 5, 25, 1, 21, 32, 29, 7, 27)

Eckardt Points

The surface has 3 Eckardt points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1), \quad T = -1$$

$$1 : P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0), \quad T = 14$$

$$2 : P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1), \quad T = 14$$

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_7$$

$$P_6 = (1, 0, 1, 0) = \ell_1 \cap \ell_8$$

$$P_4 = (1, 1, 1, 1) = \ell_7 \cap \ell_8$$

Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10
0	0	1	1	1	1	0	0	1	0	0	0
1	1	0	1	0	0	1	1	0	1	0	0
2	1	1	0	1	1	1	1	0	0	1	1
3	1	0	1	0	1	1	1	0	1	1	1
4	1	0	1	1	0	1	1	1	0	1	1
5	0	1	1	1	1	0	1	1	0	1	1
6	0	1	1	1	1	1	0	0	1	1	1
7	1	0	0	0	1	1	0	0	1	0	1
8	0	1	0	1	0	0	1	1	0	0	1
9	0	0	1	1	1	1	1	0	0	0	1
10	0	0	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_0	P_1	P_1	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_0	P_2	P_2	P_2	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}
in point	P_1	P_2	P_1	P_1	P_2	P_2	P_7	P_7

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}
in point	P_1	P_1	P_1	P_3	P_{10}	P_{10}	P_3	P_{10}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}
in point	P_1	P_1	P_1	P_{12}	P_{14}	P_{12}	P_{14}	P_{12}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}
in point	P_2	P_2	P_3	P_{12}	P_2	P_{12}	P_3	P_{12}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_2	P_{10}	P_{14}	P_2	P_{10}	P_{14}	P_{10}

Line 7 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_{10}
in point	P_5	P_{12}	P_{12}	P_4	P_{12}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7	ℓ_{10}
in point	P_6	P_{10}	P_{10}	P_4	P_{10}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_{10}
in point	P_7	P_3	P_{14}	P_3	P_{14}	P_7

Line 10 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_7	P_{10}	P_{12}	P_{12}	P_{10}	P_{12}	P_{10}	P_7

The surface has 11 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_3 = (0, 0, 0, 1)$$

$$4 : P_4 = (1, 1, 1, 1)$$

$$5 : P_5 = (1, 1, 0, 0)$$

$$6 : P_6 = (1, 0, 1, 0)$$

$$7 : P_7 = (0, 1, 1, 0)$$

$$8 : P_{10} = (0, 1, 0, 1)$$

$$9 : P_{12} = (0, 0, 1, 1)$$

$$10 : P_{14} = (0, 1, 1, 1)$$