Rank-65915 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is -2113400763

General information

Number of lines	3
Number of points	4161
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	3969
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^3, 1^{189}, 0^{3969}

Singular Points

The surface has 1 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & \epsilon^{21} & 0 & \epsilon^{42} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15154297} = \begin{bmatrix} 1 & 57 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15154297} = \mathbf{Pl}(0,57,56,0,0,1)_{277633}$$

$$\ell_1 = \begin{bmatrix} 1 & \epsilon^{42} & 0 & \epsilon^{21} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15416440} = \begin{bmatrix} 1 & 56 & 0 & 57 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15416440} = \mathbf{Pl}(0,56,57,0,0,1)_{277759}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \mathbf{Pl}(1,1,1,1,1,0)_{20354}$$

Rank of lines: (15154297, 15416440, 266369)
Rank of points on Klein quadric: (277633, 277759, 20354)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$　　　　　　　　　　$P_{241337} = (56,57,57,1) = \ell_1 \cap \ell_2$
$P_{237178} = (57,56,56,1) = \ell_0 \cap \ell_2$

Single Points

The surface has 189 single points:
The single points on the surface are:

$0 : P_{132} = (1,1,1,0)$ lies on line ℓ_2　　　　$26 : P_{40570} = (57,56,8,1)$ lies on line ℓ_0
$1 : P_{4163} = (1,0,0,1)$ lies on line ℓ_2　　　　$27 : P_{40633} = (56,57,8,1)$ lies on line ℓ_1
$2 : P_{7803} = (57,56,0,1)$ lies on line ℓ_0　　　$28 : P_{41609} = (8,9,9,1)$ lies on line ℓ_2
$3 : P_{7866} = (56,57,0,1)$ lies on line ℓ_1　　　$29 : P_{44666} = (57,56,9,1)$ lies on line ℓ_0
$4 : P_{8322} = (0,1,1,1)$ lies on line ℓ_2　　　　$30 : P_{44729} = (56,57,9,1)$ lies on line ℓ_1
$5 : P_{11898} = (57,56,1,1)$ lies on line ℓ_0　　$31 : P_{45772} = (11,10,10,1)$ lies on line ℓ_2
$6 : P_{11961} = (56,57,1,1)$ lies on line ℓ_1　　$32 : P_{48762} = (57,56,10,1)$ lies on line ℓ_0
$7 : P_{12484} = (3,2,2,1)$ lies on line ℓ_2　　　$33 : P_{48825} = (56,57,10,1)$ lies on line ℓ_1
$8 : P_{15994} = (57,56,2,1)$ lies on line ℓ_0　　$34 : P_{49931} = (10,11,11,1)$ lies on line ℓ_2
$9 : P_{16057} = (56,57,2,1)$ lies on line ℓ_1　　$35 : P_{52858} = (57,56,11,1)$ lies on line ℓ_0
$10 : P_{16643} = (2,3,3,1)$ lies on line ℓ_2　　$36 : P_{52921} = (56,57,11,1)$ lies on line ℓ_1
$11 : P_{20090} = (57,56,3,1)$ lies on line ℓ_0　$37 : P_{54094} = (13,12,12,1)$ lies on line ℓ_2
$12 : P_{20153} = (56,57,3,1)$ lies on line ℓ_1　$38 : P_{56954} = (57,56,12,1)$ lies on line ℓ_0
$13 : P_{20806} = (5,4,4,1)$ lies on line ℓ_2　　$39 : P_{57017} = (56,57,12,1)$ lies on line ℓ_1
$14 : P_{24186} = (57,56,4,1)$ lies on line ℓ_0　$40 : P_{58253} = (12,13,13,1)$ lies on line ℓ_2
$15 : P_{24249} = (56,57,4,1)$ lies on line ℓ_1　$41 : P_{61050} = (57,56,13,1)$ lies on line ℓ_0
$16 : P_{24965} = (4,5,5,1)$ lies on line ℓ_2　　$42 : P_{61113} = (56,57,13,1)$ lies on line ℓ_1
$17 : P_{28282} = (57,56,5,1)$ lies on line ℓ_0　$43 : P_{62416} = (15,14,14,1)$ lies on line ℓ_2
$18 : P_{28345} = (56,57,5,1)$ lies on line ℓ_1　$44 : P_{65146} = (57,56,14,1)$ lies on line ℓ_0
$19 : P_{29128} = (7,6,6,1)$ lies on line ℓ_2　　$45 : P_{65209} = (56,57,14,1)$ lies on line ℓ_1
$20 : P_{32378} = (57,56,6,1)$ lies on line ℓ_0　$46 : P_{66575} = (14,15,15,1)$ lies on line ℓ_2
$21 : P_{32441} = (56,57,6,1)$ lies on line ℓ_1　$47 : P_{69242} = (57,56,15,1)$ lies on line ℓ_0
$22 : P_{33287} = (6,7,7,1)$ lies on line ℓ_2　　$48 : P_{69305} = (56,57,15,1)$ lies on line ℓ_1
$23 : P_{36474} = (57,56,7,1)$ lies on line ℓ_0　$49 : P_{70738} = (17,16,16,1)$ lies on line ℓ_2
$24 : P_{36537} = (56,57,7,1)$ lies on line ℓ_1　$50 : P_{73338} = (57,56,16,1)$ lies on line ℓ_0
$25 : P_{37450} = (9,8,8,1)$ lies on line ℓ_2　　$51 : P_{73401} = (56,57,16,1)$ lies on line ℓ_1

$52: P_{74897} = (16, 17, 17, 1)$ lies on line ℓ_2
$53: P_{77434} = (57, 56, 17, 1)$ lies on line ℓ_0
$54: P_{77497} = (56, 57, 17, 1)$ lies on line ℓ_1
$55: P_{79060} = (19, 18, 18, 1)$ lies on line ℓ_2
$56: P_{81530} = (57, 56, 18, 1)$ lies on line ℓ_0
$57: P_{81593} = (56, 57, 18, 1)$ lies on line ℓ_1
$58: P_{83219} = (18, 19, 19, 1)$ lies on line ℓ_2
$59: P_{85626} = (57, 56, 19, 1)$ lies on line ℓ_0
$60: P_{85689} = (56, 57, 19, 1)$ lies on line ℓ_1
$61: P_{87382} = (21, 20, 20, 1)$ lies on line ℓ_2
$62: P_{89722} = (57, 56, 20, 1)$ lies on line ℓ_0
$63: P_{89785} = (56, 57, 20, 1)$ lies on line ℓ_1
$64: P_{91541} = (20, 21, 21, 1)$ lies on line ℓ_2
$65: P_{93818} = (57, 56, 21, 1)$ lies on line ℓ_0
$66: P_{93881} = (56, 57, 21, 1)$ lies on line ℓ_1
$67: P_{95704} = (23, 22, 22, 1)$ lies on line ℓ_2
$68: P_{97914} = (57, 56, 22, 1)$ lies on line ℓ_0
$69: P_{97977} = (56, 57, 22, 1)$ lies on line ℓ_1
$70: P_{99863} = (22, 23, 23, 1)$ lies on line ℓ_2
$71: P_{102010} = (57, 56, 23, 1)$ lies on line ℓ_0
$72: P_{102073} = (56, 57, 23, 1)$ lies on line ℓ_1
$73: P_{104026} = (25, 24, 24, 1)$ lies on line ℓ_2
$74: P_{106106} = (57, 56, 24, 1)$ lies on line ℓ_0
$75: P_{106169} = (56, 57, 24, 1)$ lies on line ℓ_1
$76: P_{108185} = (24, 25, 25, 1)$ lies on line ℓ_2
$77: P_{110202} = (57, 56, 25, 1)$ lies on line ℓ_0
$78: P_{110265} = (56, 57, 25, 1)$ lies on line ℓ_1
$79: P_{112348} = (27, 26, 26, 1)$ lies on line ℓ_2
$80: P_{114298} = (57, 56, 26, 1)$ lies on line ℓ_0
$81: P_{114361} = (56, 57, 26, 1)$ lies on line ℓ_1
$82: P_{116507} = (26, 27, 27, 1)$ lies on line ℓ_2
$83: P_{118394} = (57, 56, 27, 1)$ lies on line ℓ_0
$84: P_{118457} = (56, 57, 27, 1)$ lies on line ℓ_1
$85: P_{120670} = (29, 28, 28, 1)$ lies on line ℓ_2
$86: P_{122490} = (57, 56, 28, 1)$ lies on line ℓ_0
$87: P_{122553} = (56, 57, 28, 1)$ lies on line ℓ_1
$88: P_{124829} = (28, 29, 29, 1)$ lies on line ℓ_2
$89: P_{126586} = (57, 56, 29, 1)$ lies on line ℓ_0
$90: P_{126649} = (56, 57, 29, 1)$ lies on line ℓ_1
$91: P_{128992} = (31, 30, 30, 1)$ lies on line ℓ_2
$92: P_{130682} = (57, 56, 30, 1)$ lies on line ℓ_0
$93: P_{130745} = (56, 57, 30, 1)$ lies on line ℓ_1
$94: P_{133151} = (30, 31, 31, 1)$ lies on line ℓ_2
$95: P_{134778} = (57, 56, 31, 1)$ lies on line ℓ_0
$96: P_{134841} = (56, 57, 31, 1)$ lies on line ℓ_1
$97: P_{137314} = (33, 32, 32, 1)$ lies on line ℓ_2
$98: P_{138874} = (57, 56, 32, 1)$ lies on line ℓ_0
$99: P_{138937} = (56, 57, 32, 1)$ lies on line ℓ_1
$100: P_{141473} = (32, 33, 33, 1)$ lies on line ℓ_2
$101: P_{142970} = (57, 56, 33, 1)$ lies on line ℓ_0
$102: P_{143033} = (56, 57, 33, 1)$ lies on line ℓ_1
$103: P_{145636} = (35, 34, 34, 1)$ lies on line ℓ_2
$104: P_{147066} = (57, 56, 34, 1)$ lies on line ℓ_0
$105: P_{147129} = (56, 57, 34, 1)$ lies on line ℓ_1

$106: P_{149795} = (34, 35, 35, 1)$ lies on line ℓ_2
$107: P_{151162} = (57, 56, 35, 1)$ lies on line ℓ_0
$108: P_{151225} = (56, 57, 35, 1)$ lies on line ℓ_1
$109: P_{153958} = (37, 36, 36, 1)$ lies on line ℓ_2
$110: P_{155258} = (57, 56, 36, 1)$ lies on line ℓ_0
$111: P_{155321} = (56, 57, 36, 1)$ lies on line ℓ_1
$112: P_{158117} = (36, 37, 37, 1)$ lies on line ℓ_2
$113: P_{159354} = (57, 56, 37, 1)$ lies on line ℓ_0
$114: P_{159417} = (56, 57, 37, 1)$ lies on line ℓ_1
$115: P_{162280} = (39, 38, 38, 1)$ lies on line ℓ_2
$116: P_{163450} = (57, 56, 38, 1)$ lies on line ℓ_0
$117: P_{163513} = (56, 57, 38, 1)$ lies on line ℓ_1
$118: P_{166439} = (38, 39, 39, 1)$ lies on line ℓ_2
$119: P_{167546} = (57, 56, 39, 1)$ lies on line ℓ_0
$120: P_{167609} = (56, 57, 39, 1)$ lies on line ℓ_1
$121: P_{170602} = (41, 40, 40, 1)$ lies on line ℓ_2
$122: P_{171642} = (57, 56, 40, 1)$ lies on line ℓ_0
$123: P_{171705} = (56, 57, 40, 1)$ lies on line ℓ_1
$124: P_{174761} = (40, 41, 41, 1)$ lies on line ℓ_2
$125: P_{175738} = (57, 56, 41, 1)$ lies on line ℓ_0
$126: P_{175801} = (56, 57, 41, 1)$ lies on line ℓ_1
$127: P_{178924} = (43, 42, 42, 1)$ lies on line ℓ_2
$128: P_{179834} = (57, 56, 42, 1)$ lies on line ℓ_0
$129: P_{179897} = (56, 57, 42, 1)$ lies on line ℓ_1
$130: P_{183083} = (42, 43, 43, 1)$ lies on line ℓ_2
$131: P_{183930} = (57, 56, 43, 1)$ lies on line ℓ_0
$132: P_{183993} = (56, 57, 43, 1)$ lies on line ℓ_1
$133: P_{187246} = (45, 44, 44, 1)$ lies on line ℓ_2
$134: P_{188026} = (57, 56, 44, 1)$ lies on line ℓ_0
$135: P_{188089} = (56, 57, 44, 1)$ lies on line ℓ_1
$136: P_{191405} = (44, 45, 45, 1)$ lies on line ℓ_2
$137: P_{192122} = (57, 56, 45, 1)$ lies on line ℓ_0
$138: P_{192185} = (56, 57, 45, 1)$ lies on line ℓ_1
$139: P_{195568} = (47, 46, 46, 1)$ lies on line ℓ_2
$140: P_{196218} = (57, 56, 46, 1)$ lies on line ℓ_0
$141: P_{196281} = (56, 57, 46, 1)$ lies on line ℓ_1
$142: P_{199727} = (46, 47, 47, 1)$ lies on line ℓ_2
$143: P_{200314} = (57, 56, 47, 1)$ lies on line ℓ_0
$144: P_{200377} = (56, 57, 47, 1)$ lies on line ℓ_1
$145: P_{203890} = (49, 48, 48, 1)$ lies on line ℓ_2
$146: P_{204410} = (57, 56, 48, 1)$ lies on line ℓ_0
$147: P_{204473} = (56, 57, 48, 1)$ lies on line ℓ_1
$148: P_{208049} = (48, 49, 49, 1)$ lies on line ℓ_2
$149: P_{208506} = (57, 56, 49, 1)$ lies on line ℓ_0
$150: P_{208569} = (56, 57, 49, 1)$ lies on line ℓ_1
$151: P_{212212} = (51, 50, 50, 1)$ lies on line ℓ_2
$152: P_{212602} = (57, 56, 50, 1)$ lies on line ℓ_0
$153: P_{212665} = (56, 57, 50, 1)$ lies on line ℓ_1
$154: P_{216371} = (50, 51, 51, 1)$ lies on line ℓ_2
$155: P_{216698} = (57, 56, 51, 1)$ lies on line ℓ_0
$156: P_{216761} = (56, 57, 51, 1)$ lies on line ℓ_1
$157: P_{220534} = (53, 52, 52, 1)$ lies on line ℓ_2
$158: P_{220794} = (57, 56, 52, 1)$ lies on line ℓ_0
$159: P_{220857} = (56, 57, 52, 1)$ lies on line ℓ_1

160 : $P_{224693} = (52, 53, 53, 1)$ lies on line ℓ_2
161 : $P_{224890} = (57, 56, 53, 1)$ lies on line ℓ_0
162 : $P_{224953} = (56, 57, 53, 1)$ lies on line ℓ_1
163 : $P_{228856} = (55, 54, 54, 1)$ lies on line ℓ_2
164 : $P_{228986} = (57, 56, 54, 1)$ lies on line ℓ_0
165 : $P_{229049} = (56, 57, 54, 1)$ lies on line ℓ_1
166 : $P_{233015} = (54, 55, 55, 1)$ lies on line ℓ_2
167 : $P_{233082} = (57, 56, 55, 1)$ lies on line ℓ_0
168 : $P_{233145} = (56, 57, 55, 1)$ lies on line ℓ_1
169 : $P_{237241} = (56, 57, 56, 1)$ lies on line ℓ_1
170 : $P_{241274} = (57, 56, 57, 1)$ lies on line ℓ_0
171 : $P_{245370} = (57, 56, 58, 1)$ lies on line ℓ_0
172 : $P_{245433} = (56, 57, 58, 1)$ lies on line ℓ_1
173 : $P_{245500} = (59, 58, 58, 1)$ lies on line ℓ_2
174 : $P_{249466} = (57, 56, 59, 1)$ lies on line ℓ_0

175 : $P_{249529} = (56, 57, 59, 1)$ lies on line ℓ_1
176 : $P_{249659} = (58, 59, 59, 1)$ lies on line ℓ_2
177 : $P_{253562} = (57, 56, 60, 1)$ lies on line ℓ_0
178 : $P_{253625} = (56, 57, 60, 1)$ lies on line ℓ_1
179 : $P_{253822} = (61, 60, 60, 1)$ lies on line ℓ_2
180 : $P_{257658} = (57, 56, 61, 1)$ lies on line ℓ_0
181 : $P_{257721} = (56, 57, 61, 1)$ lies on line ℓ_1
182 : $P_{257981} = (60, 61, 61, 1)$ lies on line ℓ_2
183 : $P_{261754} = (57, 56, 62, 1)$ lies on line ℓ_0
184 : $P_{261817} = (56, 57, 62, 1)$ lies on line ℓ_1
185 : $P_{262144} = (63, 62, 62, 1)$ lies on line ℓ_2
186 : $P_{265850} = (57, 56, 63, 1)$ lies on line ℓ_0
187 : $P_{265913} = (56, 57, 63, 1)$ lies on line ℓ_1
188 : $P_{266303} = (62, 63, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has **3969** points not on any line:
Too many to print.

Line Intersection Graph

$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
2 & 1 & 1 & 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_{237178}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_{241337}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{237178}	P_{241337}

The surface has **4161** points:
Too many to print.