Rank-65851 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_1 + X_0^2X_2 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1243948109

General information

Number of lines	3
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	2^3, 1^{21}, 0^{49}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^2 & \gamma^3 \end{array}\right]_{4059} = \left[\begin{array}{cccc} 1 & 0 & 7 & 6 \\ 0 & 1 & 4 & 5 \end{array}\right]_{4059} = \mathbf{Pl}(6,2,3,4,6,1)_{4036}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma & \gamma^5 \\ 0 & 1 & \gamma^4 & \gamma^6 \end{bmatrix}_{1953} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 7 & 6 \end{bmatrix}_{1953} = \mathbf{Pl}(3,4,5,7,3,1)_{2619}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^3 \\ 0 & 1 & \gamma & \gamma^5 \end{bmatrix}_{3238} = \begin{bmatrix} 1 & 0 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{3238} = \mathbf{Pl}(5,7,6,2,5,1)_{3692}$$

Rank of lines: (4059, 1953, 3238)
Rank of points on Klein quadric: (4036, 2619, 3692)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{564} = (3,5,7,1) = \ell_0 \cap \ell_1$
$P_{359} = (6,3,4,1) = \ell_0 \cap \ell_2$
$P_{254} = (5,6,2,1) = \ell_1 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_{30} = (3,2,1,0)$ lies on line ℓ_0
1 : $P_{48} = (5,4,1,0)$ lies on line ℓ_1
2 : $P_{73} = (6,7,1,0)$ lies on line ℓ_2
3 : $P_{100} = (2,3,0,1)$ lies on line ℓ_1
4 : $P_{118} = (4,5,0,1)$ lies on line ℓ_2
5 : $P_{129} = (7,6,0,1)$ lies on line ℓ_0
6 : $P_{155} = (2,2,1,1)$ lies on line ℓ_2
7 : $P_{173} = (4,4,1,1)$ lies on line ℓ_0
8 : $P_{200} = (7,7,1,1)$ lies on line ℓ_1
9 : $P_{218} = (1,2,2,1)$ lies on line ℓ_0
10 : $P_{267} = (2,0,3,1)$ lies on line ℓ_0
11 : $P_{276} = (3,1,3,1)$ lies on line ℓ_2
12 : $P_{281} = (0,2,3,1)$ lies on line ℓ_1
13 : $P_{362} = (1,4,4,1)$ lies on line ℓ_1
14 : $P_{397} = (4,0,5,1)$ lies on line ℓ_1
15 : $P_{406} = (5,1,5,1)$ lies on line ℓ_0
16 : $P_{425} = (0,4,5,1)$ lies on line ℓ_2
17 : $P_{464} = (7,0,6,1)$ lies on line ℓ_2
18 : $P_{471} = (6,1,6,1)$ lies on line ℓ_1
19 : $P_{513} = (0,7,6,1)$ lies on line ℓ_0
20 : $P_{578} = (1,7,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_{20} = (1,1,1,0)$
4 : $P_{33} = (6,2,1,0)$
5 : $P_{46} = (3,4,1,0)$
6 : $P_{72} = (5,7,1,0)$
7 : $P_{83} = (1,1,0,1)$
8 : $P_{95} = (5,2,0,1)$
9 : $P_{112} = (6,4,0,1)$

10 : $P_{133} = (3, 7, 0, 1)$
11 : $P_{139} = (1, 0, 1, 1)$
12 : $P_{146} = (0, 1, 1, 1)$
13 : $P_{160} = (7, 2, 1, 1)$
14 : $P_{171} = (2, 4, 1, 1)$
15 : $P_{197} = (4, 7, 1, 1)$
16 : $P_{206} = (5, 0, 2, 1)$
17 : $P_{241} = (0, 5, 2, 1)$
18 : $P_{242} = (1, 5, 2, 1)$
19 : $P_{255} = (6, 6, 2, 1)$
20 : $P_{259} = (2, 7, 2, 1)$
21 : $P_{263} = (6, 7, 2, 1)$
22 : $P_{277} = (4, 1, 3, 1)$
23 : $P_{287} = (6, 2, 3, 1)$
24 : $P_{293} = (4, 3, 3, 1)$
25 : $P_{307} = (2, 5, 3, 1)$
26 : $P_{311} = (6, 5, 3, 1)$
27 : $P_{335} = (6, 0, 4, 1)$
28 : $P_{348} = (3, 2, 4, 1)$
29 : $P_{349} = (4, 2, 4, 1)$
30 : $P_{356} = (3, 3, 4, 1)$
31 : $P_{377} = (0, 6, 4, 1)$
32 : $P_{378} = (1, 6, 4, 1)$
33 : $P_{408} = (7, 1, 5, 1)$
34 : $P_{428} = (3, 4, 5, 1)$
35 : $P_{440} = (7, 5, 5, 1)$
36 : $P_{444} = (3, 6, 5, 1)$
37 : $P_{445} = (4, 6, 5, 1)$
38 : $P_{467} = (2, 1, 6, 1)$
39 : $P_{486} = (5, 3, 6, 1)$
40 : $P_{488} = (7, 3, 6, 1)$
41 : $P_{507} = (2, 6, 6, 1)$
42 : $P_{518} = (5, 7, 6, 1)$
43 : $P_{524} = (3, 0, 7, 1)$
44 : $P_{545} = (0, 3, 7, 1)$
45 : $P_{546} = (1, 3, 7, 1)$
46 : $P_{558} = (5, 4, 7, 1)$
47 : $P_{560} = (7, 4, 7, 1)$
48 : $P_{566} = (5, 5, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{564}	P_{359}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{564}	P_{254}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{359}	P_{254}

The surface has 73 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_{20} = (1, 1, 1, 0)$
4 : $P_{30} = (3, 2, 1, 0)$
5 : $P_{33} = (6, 2, 1, 0)$
6 : $P_{46} = (3, 4, 1, 0)$
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