

Rank-65760 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1229497486

General information

Number of lines	7
Number of points	89
Number of singular points	0
Number of Eckardt points	0
Number of double points	9
Number of single points	45
Number of points off lines	35
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^7
Type of lines on points	$2^9, 1^{45}, 0^{35}$

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{4673} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{4673} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{769} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & 0 & \gamma^2 & \gamma \\ 0 & 1 & \gamma^2 & \gamma^2 \end{array} \right]_{1496} = \left[\begin{array}{cccc} 1 & 0 & 4 & 2 \\ 0 & 1 & 4 & 4 \end{array} \right]_{1496} = \mathbf{Pl}(2, 6, 2, 6, 1, 1)_{1498}\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^2 \\ 0 & 1 & \gamma^4 & \gamma^4 \end{bmatrix}_{2910} = \begin{bmatrix} 1 & 0 & 7 & 4 \\ 0 & 1 & 7 & 7 \end{bmatrix}_{2910} = \mathbf{Pl}(4, 3, 4, 3, 1, 1)_{1563} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^4 \\ 0 & 1 & \gamma & \gamma \end{bmatrix}_{4252} = \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{4252} = \mathbf{Pl}(7, 5, 7, 5, 1, 1)_{1706} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & \gamma^6 & \gamma^6 \end{bmatrix}_{3558} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 6 & 6 \end{bmatrix}_{3558} = \mathbf{Pl}(2, 6, 1, 1, 1, 0)_{307} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{1779} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{1779} = \mathbf{Pl}(4, 3, 1, 1, 1, 0)_{309} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & \gamma^3 & \gamma^3 \end{bmatrix}_{2965} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{2965} = \mathbf{Pl}(7, 5, 1, 1, 1, 0)_{312}
\end{aligned}$$

Rank of lines: (4673, 1496, 2910, 4252, 3558, 1779, 2965)

Rank of points on Klein quadric: (769, 1498, 1563, 1706, 307, 309, 312)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 9 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{161} &= (0, 3, 1, 1) = \ell_0 \cap \ell_1 & P_{193} &= (0, 7, 1, 1) = \ell_0 \cap \ell_6 \\
P_{177} &= (0, 5, 1, 1) = \ell_0 \cap \ell_2 & P_{454} &= (5, 7, 5, 1) = \ell_1 \cap \ell_4 \\
P_{185} &= (0, 6, 1, 1) = \ell_0 \cap \ell_3 & P_{479} &= (6, 2, 6, 1) = \ell_2 \cap \ell_5 \\
P_{153} &= (0, 2, 1, 1) = \ell_0 \cap \ell_4 & P_{300} &= (3, 4, 3, 1) = \ell_3 \cap \ell_6 \\
P_{169} &= (0, 4, 1, 1) = \ell_0 \cap \ell_5
\end{aligned}$$

Single Points

The surface has 45 single points:

The single points on the surface are:

$$\begin{aligned}
0 : P_1 &= (0, 1, 0, 0) \text{ lies on line } \ell_0 & 15 : P_{207} &= (6, 0, 2, 1) \text{ lies on line } \ell_1 \\
1 : P_{21} &= (2, 1, 1, 0) \text{ lies on line } \ell_1 & 16 : P_{229} &= (4, 3, 2, 1) \text{ lies on line } \ell_6 \\
2 : P_{23} &= (4, 1, 1, 0) \text{ lies on line } \ell_2 & 17 : P_{239} &= (6, 4, 2, 1) \text{ lies on line } \ell_4 \\
3 : P_{26} &= (7, 1, 1, 0) \text{ lies on line } \ell_3 & 18 : P_{242} &= (1, 5, 2, 1) \text{ lies on line } \ell_5 \\
4 : P_{29} &= (2, 2, 1, 0) \text{ lies on line } \ell_4 & 19 : P_{245} &= (4, 5, 2, 1) \text{ lies on line } \ell_3 \\
5 : P_{47} &= (4, 4, 1, 0) \text{ lies on line } \ell_5 & 20 : P_{250} &= (1, 6, 2, 1) \text{ lies on line } \ell_2 \\
6 : P_{74} &= (7, 7, 1, 0) \text{ lies on line } \ell_6 & 21 : P_{277} &= (4, 1, 3, 1) \text{ lies on line } \ell_1 \\
7 : P_{76} &= (2, 0, 0, 1) \text{ lies on line } \ell_4 & 22 : P_{278} &= (5, 1, 3, 1) \text{ lies on line } \ell_5 \\
8 : P_{78} &= (4, 0, 0, 1) \text{ lies on line } \ell_5 & 23 : P_{317} &= (4, 6, 3, 1) \text{ lies on line } \ell_4 \\
9 : P_{81} &= (7, 0, 0, 1) \text{ lies on line } \ell_6 & 24 : P_{326} &= (5, 7, 3, 1) \text{ lies on line } \ell_2 \\
10 : P_{92} &= (2, 2, 0, 1) \text{ lies on line } \ell_1 & 25 : P_{332} &= (3, 0, 4, 1) \text{ lies on line } \ell_2 \\
11 : P_{110} &= (4, 4, 0, 1) \text{ lies on line } \ell_2 & 26 : P_{354} &= (1, 3, 4, 1) \text{ lies on line } \ell_3 \\
12 : P_{137} &= (7, 7, 0, 1) \text{ lies on line } \ell_3 & 27 : P_{376} &= (7, 5, 4, 1) \text{ lies on line } \ell_4 \\
13 : P_{138} &= (0, 0, 1, 1) \text{ lies on line } \ell_0 & 28 : P_{378} &= (1, 6, 4, 1) \text{ lies on line } \ell_6 \\
14 : P_{146} &= (0, 1, 1, 1) \text{ lies on line } \ell_0 & 29 : P_{384} &= (7, 6, 4, 1) \text{ lies on line } \ell_1
\end{aligned}$$

30 : $P_{388} = (3, 7, 4, 1)$ lies on line ℓ_5
 31 : $P_{407} = (6, 1, 5, 1)$ lies on line ℓ_6
 32 : $P_{408} = (7, 1, 5, 1)$ lies on line ℓ_2
 33 : $P_{415} = (6, 2, 5, 1)$ lies on line ℓ_3
 34 : $P_{424} = (7, 3, 5, 1)$ lies on line ℓ_5
 35 : $P_{467} = (2, 1, 6, 1)$ lies on line ℓ_3
 36 : $P_{468} = (3, 1, 6, 1)$ lies on line ℓ_4
 37 : $P_{492} = (3, 4, 6, 1)$ lies on line ℓ_1

38 : $P_{499} = (2, 5, 6, 1)$ lies on line ℓ_6
 39 : $P_{526} = (5, 0, 7, 1)$ lies on line ℓ_3
 40 : $P_{542} = (5, 2, 7, 1)$ lies on line ℓ_6
 41 : $P_{546} = (1, 3, 7, 1)$ lies on line ℓ_4
 42 : $P_{547} = (2, 3, 7, 1)$ lies on line ℓ_2
 43 : $P_{562} = (1, 5, 7, 1)$ lies on line ℓ_1
 44 : $P_{571} = (2, 6, 7, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 35 points not on any line:

The points on the surface but not on lines are:

0 : $P_{12} = (1, 0, 1, 0)$
 1 : $P_{32} = (5, 2, 1, 0)$
 2 : $P_{41} = (6, 3, 1, 0)$
 3 : $P_{49} = (6, 4, 1, 0)$
 4 : $P_{54} = (3, 5, 1, 0)$
 5 : $P_{64} = (5, 6, 1, 0)$
 6 : $P_{70} = (3, 7, 1, 0)$
 7 : $P_{85} = (3, 1, 0, 1)$
 8 : $P_{87} = (5, 1, 0, 1)$
 9 : $P_{88} = (6, 1, 0, 1)$
 10 : $P_{93} = (3, 2, 0, 1)$
 11 : $P_{111} = (5, 4, 0, 1)$
 12 : $P_{136} = (6, 7, 0, 1)$
 13 : $P_{139} = (1, 0, 1, 1)$
 14 : $P_{157} = (4, 2, 1, 1)$
 15 : $P_{160} = (7, 2, 1, 1)$
 16 : $P_{171} = (2, 4, 1, 1)$
 17 : $P_{176} = (7, 4, 1, 1)$

18 : $P_{195} = (2, 7, 1, 1)$
 19 : $P_{197} = (4, 7, 1, 1)$
 20 : $P_{219} = (2, 2, 2, 1)$
 21 : $P_{274} = (1, 1, 3, 1)$
 22 : $P_{287} = (6, 2, 3, 1)$
 23 : $P_{290} = (1, 3, 3, 1)$
 24 : $P_{327} = (6, 7, 3, 1)$
 25 : $P_{365} = (4, 4, 4, 1)$
 26 : $P_{402} = (1, 1, 5, 1)$
 27 : $P_{412} = (3, 2, 5, 1)$
 28 : $P_{428} = (3, 4, 5, 1)$
 29 : $P_{434} = (1, 5, 5, 1)$
 30 : $P_{466} = (1, 1, 6, 1)$
 31 : $P_{494} = (5, 4, 6, 1)$
 32 : $P_{506} = (1, 6, 6, 1)$
 33 : $P_{518} = (5, 7, 6, 1)$
 34 : $P_{584} = (7, 7, 7, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	0	0	1	0	0
2	1	0	0	0	0	1	0
3	1	0	0	0	0	0	1
4	1	1	0	0	0	0	0
5	1	0	1	0	0	0	0
6	1	0	0	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{161}	P_{177}	P_{185}	P_{153}	P_{169}	P_{193}

Line 1 intersects

Line	ℓ_0	ℓ_4
in point	P_{161}	P_{454}

Line 2 intersects

Line	ℓ_0	ℓ_5
in point	P_{177}	P_{479}

Line 3 intersects

Line	ℓ_0	ℓ_6
in point	P_{185}	P_{300}

Line 4 intersects

Line	ℓ_0	ℓ_1
in point	P_{153}	P_{454}

Line 5 intersects

Line	ℓ_0	ℓ_2
in point	P_{169}	P_{479}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_{193}	P_{300}

The surface has 89 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	30 : $P_{157} = (4, 2, 1, 1)$	60 : $P_{376} = (7, 5, 4, 1)$
1 : $P_{12} = (1, 0, 1, 0)$	31 : $P_{160} = (7, 2, 1, 1)$	61 : $P_{378} = (1, 6, 4, 1)$
2 : $P_{21} = (2, 1, 1, 0)$	32 : $P_{161} = (0, 3, 1, 1)$	62 : $P_{384} = (7, 6, 4, 1)$
3 : $P_{23} = (4, 1, 1, 0)$	33 : $P_{169} = (0, 4, 1, 1)$	63 : $P_{388} = (3, 7, 4, 1)$
4 : $P_{26} = (7, 1, 1, 0)$	34 : $P_{171} = (2, 4, 1, 1)$	64 : $P_{402} = (1, 1, 5, 1)$
5 : $P_{29} = (2, 2, 1, 0)$	35 : $P_{176} = (7, 4, 1, 1)$	65 : $P_{407} = (6, 1, 5, 1)$
6 : $P_{32} = (5, 2, 1, 0)$	36 : $P_{177} = (0, 5, 1, 1)$	66 : $P_{408} = (7, 1, 5, 1)$
7 : $P_{41} = (6, 3, 1, 0)$	37 : $P_{185} = (0, 6, 1, 1)$	67 : $P_{412} = (3, 2, 5, 1)$
8 : $P_{47} = (4, 4, 1, 0)$	38 : $P_{193} = (0, 7, 1, 1)$	68 : $P_{415} = (6, 2, 5, 1)$
9 : $P_{49} = (6, 4, 1, 0)$	39 : $P_{195} = (2, 7, 1, 1)$	69 : $P_{424} = (7, 3, 5, 1)$
10 : $P_{54} = (3, 5, 1, 0)$	40 : $P_{197} = (4, 7, 1, 1)$	70 : $P_{428} = (3, 4, 5, 1)$
11 : $P_{64} = (5, 6, 1, 0)$	41 : $P_{207} = (6, 0, 2, 1)$	71 : $P_{434} = (1, 5, 5, 1)$
12 : $P_{70} = (3, 7, 1, 0)$	42 : $P_{219} = (2, 2, 2, 1)$	72 : $P_{454} = (5, 7, 5, 1)$
13 : $P_{74} = (7, 7, 1, 0)$	43 : $P_{229} = (4, 3, 2, 1)$	73 : $P_{466} = (1, 1, 6, 1)$
14 : $P_{76} = (2, 0, 0, 1)$	44 : $P_{239} = (6, 4, 2, 1)$	74 : $P_{467} = (2, 1, 6, 1)$
15 : $P_{78} = (4, 0, 0, 1)$	45 : $P_{242} = (1, 5, 2, 1)$	75 : $P_{468} = (3, 1, 6, 1)$
16 : $P_{81} = (7, 0, 0, 1)$	46 : $P_{245} = (4, 5, 2, 1)$	76 : $P_{479} = (6, 2, 6, 1)$
17 : $P_{85} = (3, 1, 0, 1)$	47 : $P_{250} = (1, 6, 2, 1)$	77 : $P_{492} = (3, 4, 6, 1)$
18 : $P_{87} = (5, 1, 0, 1)$	48 : $P_{274} = (1, 1, 3, 1)$	78 : $P_{494} = (5, 4, 6, 1)$
19 : $P_{88} = (6, 1, 0, 1)$	49 : $P_{277} = (4, 1, 3, 1)$	79 : $P_{499} = (2, 5, 6, 1)$
20 : $P_{92} = (2, 2, 0, 1)$	50 : $P_{278} = (5, 1, 3, 1)$	80 : $P_{506} = (1, 6, 6, 1)$
21 : $P_{93} = (3, 2, 0, 1)$	51 : $P_{287} = (6, 2, 3, 1)$	81 : $P_{518} = (5, 7, 6, 1)$
22 : $P_{110} = (4, 4, 0, 1)$	52 : $P_{290} = (1, 3, 3, 1)$	82 : $P_{526} = (5, 0, 7, 1)$
23 : $P_{111} = (5, 4, 0, 1)$	53 : $P_{300} = (3, 4, 3, 1)$	83 : $P_{542} = (5, 2, 7, 1)$
24 : $P_{136} = (6, 7, 0, 1)$	54 : $P_{317} = (4, 6, 3, 1)$	84 : $P_{546} = (1, 3, 7, 1)$
25 : $P_{137} = (7, 7, 0, 1)$	55 : $P_{326} = (5, 7, 3, 1)$	85 : $P_{547} = (2, 3, 7, 1)$
26 : $P_{138} = (0, 0, 1, 1)$	56 : $P_{327} = (6, 7, 3, 1)$	86 : $P_{562} = (1, 5, 7, 1)$
27 : $P_{139} = (1, 0, 1, 1)$	57 : $P_{332} = (3, 0, 4, 1)$	87 : $P_{571} = (2, 6, 7, 1)$
28 : $P_{146} = (0, 1, 1, 1)$	58 : $P_{354} = (1, 3, 4, 1)$	88 : $P_{584} = (7, 7, 7, 1)$
29 : $P_{153} = (0, 2, 1, 1)$	59 : $P_{365} = (4, 4, 4, 1)$	