# Rank-74055 over GF(4)

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## The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

( 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(4) is 1498834281

## General information

| | |
|---|---|
| Number of lines | 10 |
| Number of points | 33 |
| Number of singular points | 1 |
| Number of Eckardt points | 2 |
| Number of double points | 9 |
| Number of single points | 21 |
| Number of points off lines | 0 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^{10}$ |
| Type of lines on points | 5, $3^2$, $2^9$, $1^{21}$ |

### Singular Points

The surface has 1 singular points:

$0: \ P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

## The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \mathbf{Pl}(1,0,1,1,2,1)_{259}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \mathbf{Pl}(1,0,1,1,3,1)_{319}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 0 & 1 \end{bmatrix}_{193} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{193} = \mathbf{Pl}(1,1,0,2,1,1)_{192}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \mathbf{Pl}(3,2,2,3,2,1)_{290}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{277} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{277} = \mathbf{Pl}(1,1,0,3,1,1)_{195}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \mathbf{Pl}(2,3,3,2,3,1)_{352}$$

Rank of lines: ( 0, 340, 84, 320, 215, 345, 193, 236, 277, 299 )
Rank of points on Klein quadric: ( 0, 9, 10, 259, 319, 13, 192, 290, 195, 352 )

### Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

### Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_6 = (2,1,0,0) = \ell_0 \cap \ell_3$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_4$
$P_3 = (0,0,0,1) = \ell_1 \cap \ell_5$
$P_{35} = (1,3,0,1) = \ell_2 \cap \ell_7$
$P_{31} = (1,2,0,1) = \ell_2 \cap \ell_9$
$P_{46} = (1,2,1,1) = \ell_3 \cap \ell_8$
$P_{50} = (1,3,1,1) = \ell_4 \cap \ell_6$
$P_{16} = (1,2,1,0) = \ell_6 \cap \ell_7$
$P_{20} = (1,3,1,0) = \ell_8 \cap \ell_9$

### Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line $\ell_0$
1 : $P_5 = (1,1,0,0)$ lies on line $\ell_0$
2 : $P_{11} = (0,1,1,0)$ lies on line $\ell_5$
3 : $P_{23} = (1,0,0,1)$ lies on line $\ell_2$
4 : $P_{27} = (1,1,0,1)$ lies on line $\ell_2$
5 : $P_{30} = (0,2,0,1)$ lies on line $\ell_1$
6 : $P_{34} = (0,3,0,1)$ lies on line $\ell_1$
7 : $P_{40} = (2,0,1,1)$ lies on line $\ell_3$
8 : $P_{41} = (3,0,1,1)$ lies on line $\ell_4$
9 : $P_{47} = (2,2,1,1)$ lies on line $\ell_4$
10 : $P_{52} = (3,3,1,1)$ lies on line $\ell_3$
11 : $P_{55} = (2,0,2,1)$ lies on line $\ell_8$
12 : $P_{56} = (3,0,2,1)$ lies on line $\ell_7$
13 : $P_{61} = (0,2,2,1)$ lies on line $\ell_5$
14 : $P_{63} = (2,2,2,1)$ lies on line $\ell_6$
15 : $P_{68} = (3,3,2,1)$ lies on line $\ell_9$
16 : $P_{71} = (2,0,3,1)$ lies on line $\ell_9$
17 : $P_{72} = (3,0,3,1)$ lies on line $\ell_6$
18 : $P_{79} = (2,2,3,1)$ lies on line $\ell_7$
19 : $P_{81} = (0,3,3,1)$ lies on line $\ell_5$
20 : $P_{84} = (3,3,3,1)$ lies on line $\ell_8$

The single points on the surface are:

**Points on surface but on no line**

The surface has 0 points not on any line:
The points on the surface but not on lines are:

## Line Intersection Graph

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 3 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 7 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 8 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 9 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|---|
| in point | $P_1$ | $P_1$ | $P_6$ | $P_7$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_5$ | $\ell_6$ | $\ell_8$ |
|---|---|---|---|---|---|
| in point | $P_1$ | $P_1$ | $P_3$ | $P_{26}$ | $P_{26}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_7$ | $\ell_9$ |
|---|---|---|---|---|
| in point | $P_1$ | $P_1$ | $P_{35}$ | $P_{31}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_4$ | $\ell_5$ | $\ell_7$ | $\ell_8$ | $\ell_9$ |
|---|---|---|---|---|---|---|
| in point | $P_6$ | $P_{42}$ | $P_{42}$ | $P_{42}$ | $P_{46}$ | $P_{42}$ |

Line 4 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_5$ | $\ell_6$ | $\ell_7$ | $\ell_9$ |
|---|---|---|---|---|---|---|
| in point | $P_7$ | $P_{42}$ | $P_{42}$ | $P_{50}$ | $P_{42}$ | $P_{42}$ |

Line 5 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_4$ | $\ell_7$ | $\ell_9$ |
|---|---|---|---|---|---|
| in point | $P_3$ | $P_{42}$ | $P_{42}$ | $P_{42}$ | $P_{42}$ |

Line 6 intersects

| Line | $\ell_1$ | $\ell_4$ | $\ell_7$ | $\ell_8$ |
|---|---|---|---|---|
| in point | $P_{26}$ | $P_{50}$ | $P_{16}$ | $P_{26}$ |

Line 7 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_4$ | $\ell_5$ | $\ell_6$ | $\ell_9$ |
|---|---|---|---|---|---|---|
| in point | $P_{35}$ | $P_{42}$ | $P_{42}$ | $P_{42}$ | $P_{16}$ | $P_{42}$ |

Line 8 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_6$ | $\ell_9$ |
|---|---|---|---|---|
| in point | $P_{26}$ | $P_{46}$ | $P_{26}$ | $P_{20}$ |

Line 9 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_4$ | $\ell_5$ | $\ell_7$ | $\ell_8$ |
|---|---|---|---|---|---|---|
| in point | $P_{31}$ | $P_{42}$ | $P_{42}$ | $P_{42}$ | $P_{42}$ | $P_{20}$ |

The surface has 33 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (2,1,0,0)$
$5: P_7 = (3,1,0,0)$
$6: P_{11} = (0,1,1,0)$
$7: P_{16} = (1,2,1,0)$
$8: P_{20} = (1,3,1,0)$
$9: P_{23} = (1,0,0,1)$
$10: P_{26} = (0,1,0,1)$
$11: P_{27} = (1,1,0,1)$
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$18: P_{42} = (0,1,1,1)$
$19: P_{46} = (1,2,1,1)$
$20: P_{47} = (2,2,1,1)$
$21: P_{50} = (1,3,1,1)$
$22: P_{52} = (3,3,1,1)$
$23: P_{55} = (2,0,2,1)$
$24: P_{56} = (3,0,2,1)$
$25: P_{61} = (0,2,2,1)$
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$27: P_{68} = (3,3,2,1)$
$28: P_{71} = (2,0,3,1)$
$29: P_{72} = (3,0,3,1)$
$30: P_{79} = (2,2,3,1)$
$31: P_{81} = (0,3,3,1)$
$32: P_{84} = (3,3,3,1)$