

# Rank-76100 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(4) is 1503028570

## General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	21
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^9$
Type of lines on points	$3^4, 2^6, 1^{21}, 0^2$

## Singular Points

The surface has 0 singular points:

## The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned} \ell_0 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_9 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{252} = \mathbf{Pl}(2, 0, 0, 1, 0, 0)_{11} \end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{168} = \mathbf{Pl}(3, 0, 0, 1, 0, 0)_{12} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \mathbf{Pl}(0, 3, 1, 0, 0, 0)_8 \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \mathbf{Pl}(0, 2, 1, 0, 0, 0)_7 \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{13} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(2, 3, 1, 3, 0, 1)_{169} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 1 & 0 \end{bmatrix}_{190} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{190} = \mathbf{Pl}(3, 2, 1, 2, 0, 1)_{161}
\end{aligned}$$

Rank of lines: ( 340, 252, 168, 356, 268, 184, 345, 274, 190 )

Rank of points on Klein quadric: ( 9, 11, 12, 1, 8, 7, 13, 169, 161 )

### Eckardt Points

The surface has 4 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0),$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1),$$

$$3 : P_{11} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0).$$

### Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_{24} = (2, 0, 0, 1) = \ell_1 \cap \ell_4$$

$$P_{32} = (2, 2, 0, 1) = \ell_1 \cap \ell_7$$

$$P_{25} = (3, 0, 0, 1) = \ell_2 \cap \ell_5$$

$$P_{37} = (3, 3, 0, 1) = \ell_2 \cap \ell_8$$

$$P_{55} = (2, 0, 2, 1) = \ell_4 \cap \ell_7$$

$$P_{72} = (3, 0, 3, 1) = \ell_5 \cap \ell_8$$

### Single Points

The surface has 21 single points:

The single points on the surface are:

$$0 : P_{26} = (0, 1, 0, 1) \text{ lies on line } \ell_0$$

$$1 : P_{28} = (2, 1, 0, 1) \text{ lies on line } \ell_1$$

$$2 : P_{29} = (3, 1, 0, 1) \text{ lies on line } \ell_2$$

$$3 : P_{30} = (0, 2, 0, 1) \text{ lies on line } \ell_0$$

$$4 : P_{33} = (3, 2, 0, 1) \text{ lies on line } \ell_2$$

$$5 : P_{34} = (0, 3, 0, 1) \text{ lies on line } \ell_0$$

$$6 : P_{36} = (2, 3, 0, 1) \text{ lies on line } \ell_1$$

$$7 : P_{38} = (0, 0, 1, 1) \text{ lies on line } \ell_3$$

$$8 : P_{40} = (2, 0, 1, 1) \text{ lies on line } \ell_4$$

$$9 : P_{41} = (3, 0, 1, 1) \text{ lies on line } \ell_5$$

$$10 : P_{42} = (0, 1, 1, 1) \text{ lies on line } \ell_6$$

$$11 : P_{48} = (3, 2, 1, 1) \text{ lies on line } \ell_8$$

$$12 : P_{51} = (2, 3, 1, 1) \text{ lies on line } \ell_7$$

$$13 : P_{53} = (0, 0, 2, 1) \text{ lies on line } \ell_3$$

14 :  $P_{56} = (3, 0, 2, 1)$  lies on line  $\ell_5$   
15 :  $P_{60} = (3, 1, 2, 1)$  lies on line  $\ell_8$   
16 :  $P_{61} = (0, 2, 2, 1)$  lies on line  $\ell_6$   
17 :  $P_{69} = (0, 0, 3, 1)$  lies on line  $\ell_3$

18 :  $P_{71} = (2, 0, 3, 1)$  lies on line  $\ell_4$   
19 :  $P_{75} = (2, 1, 3, 1)$  lies on line  $\ell_7$   
20 :  $P_{81} = (0, 3, 3, 1)$  lies on line  $\ell_6$

The single points on the surface are:

### Points on surface but on no line

The surface has 2 points not on any line:  
The points on the surface but not on lines are:

0 :  $P_4 = (1, 1, 1, 1)$   
1 :  $P_{12} = (1, 1, 1, 0)$

### Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	1	0	0
1	1	0	1	0	1	0	0	1	0
2	1	1	0	0	0	1	0	0	1
3	1	0	0	0	1	1	1	0	0
4	0	1	0	1	0	1	0	1	0
5	0	0	1	1	1	0	0	0	1
6	1	0	0	1	0	0	0	1	1
7	0	1	0	0	1	0	1	0	1
8	0	0	1	0	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_6$
in point	$P_1$	$P_1$	$P_3$	$P_3$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_7$
in point	$P_1$	$P_1$	$P_{24}$	$P_{32}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_5$	$\ell_8$
in point	$P_1$	$P_1$	$P_{25}$	$P_{37}$

Line 3 intersects

Line	$\ell_0$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_3$	$P_2$	$P_2$	$P_3$

Line 4 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_7$
in point	$P_{24}$	$P_2$	$P_2$	$P_{55}$

Line 5 intersects

Line	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_8$
in point	$P_{25}$	$P_2$	$P_2$	$P_{72}$

Line 6 intersects

Line	$\ell_0$	$\ell_3$	$\ell_7$	$\ell_8$
in point	$P_3$	$P_3$	$P_{11}$	$P_{11}$

Line 7 intersects

Line	$\ell_1$	$\ell_4$	$\ell_6$	$\ell_8$
in point	$P_{32}$	$P_{55}$	$P_{11}$	$P_{11}$

Line 8 intersects

Line	$\ell_2$	$\ell_5$	$\ell_6$	$\ell_7$
in point	$P_{37}$	$P_{72}$	$P_{11}$	$P_{11}$

The surface has 33 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	12 : $P_{32} = (2, 2, 0, 1)$	24 : $P_{55} = (2, 0, 2, 1)$
1 : $P_2 = (0, 0, 1, 0)$	13 : $P_{33} = (3, 2, 0, 1)$	25 : $P_{56} = (3, 0, 2, 1)$
2 : $P_3 = (0, 0, 0, 1)$	14 : $P_{34} = (0, 3, 0, 1)$	26 : $P_{60} = (3, 1, 2, 1)$
3 : $P_4 = (1, 1, 1, 1)$	15 : $P_{36} = (2, 3, 0, 1)$	27 : $P_{61} = (0, 2, 2, 1)$
4 : $P_{11} = (0, 1, 1, 0)$	16 : $P_{37} = (3, 3, 0, 1)$	28 : $P_{69} = (0, 0, 3, 1)$
5 : $P_{12} = (1, 1, 1, 0)$	17 : $P_{38} = (0, 0, 1, 1)$	29 : $P_{71} = (2, 0, 3, 1)$
6 : $P_{24} = (2, 0, 0, 1)$	18 : $P_{40} = (2, 0, 1, 1)$	30 : $P_{72} = (3, 0, 3, 1)$
7 : $P_{25} = (3, 0, 0, 1)$	19 : $P_{41} = (3, 0, 1, 1)$	31 : $P_{75} = (2, 1, 3, 1)$
8 : $P_{26} = (0, 1, 0, 1)$	20 : $P_{42} = (0, 1, 1, 1)$	32 : $P_{81} = (0, 3, 3, 1)$
9 : $P_{28} = (2, 1, 0, 1)$	21 : $P_{48} = (3, 2, 1, 1)$	
10 : $P_{29} = (3, 1, 0, 1)$	22 : $P_{51} = (2, 3, 1, 1)$	
11 : $P_{30} = (0, 2, 0, 1)$	23 : $P_{53} = (0, 0, 2, 1)$	