

# Rank-65858 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1243948182

## General information

Number of lines	3
Number of points	73
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^3$
Type of lines on points	$2^3, 1^{21}, 0^{49}$

## Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_{41} &= \mathbf{P}(\gamma^6, \gamma^5, 1, 0) = \mathbf{P}(6, 3, 1, 0) \\ 1 : P_{54} &= \mathbf{P}(\gamma^5, \gamma^3, 1, 0) = \mathbf{P}(3, 5, 1, 0) \end{aligned}$$

$$2 : P_{64} = \mathbf{P}(\gamma^3, \gamma^6, 1, 0) = \mathbf{P}(5, 6, 1, 0)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^6 & 0 \end{array} \right]_{225} = \left[ \begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 6 & 0 \end{array} \right]_{225} = \mathbf{Pl}(4, 0, 2, 0, 0, 1)_{683}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{368} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{368} = \mathbf{Pl}(7, 0, 4, 0, 0, 1)_{716}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{443} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{443} = \mathbf{Pl}(2, 0, 7, 0, 0, 1)_{756}$$

Rank of lines: ( 225, 368, 443 )

Rank of points on Klein quadric: ( 683, 716, 756 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{64} = (5, 6, 1, 0) = \ell_0 \cap \ell_1$$

$$P_{54} = (3, 5, 1, 0) = \ell_0 \cap \ell_2$$

$$P_{41} = (6, 3, 1, 0) = \ell_1 \cap \ell_2$$

### Single Points

The surface has 21 single points:

The single points on the surface are:

0 :  $P_6 = (2, 1, 0, 0)$  lies on line  $\ell_0$

1 :  $P_8 = (4, 1, 0, 0)$  lies on line  $\ell_1$

2 :  $P_{11} = (7, 1, 0, 0)$  lies on line  $\ell_2$

3 :  $P_{13} = (2, 0, 1, 0)$  lies on line  $\ell_2$

4 :  $P_{15} = (4, 0, 1, 0)$  lies on line  $\ell_0$

5 :  $P_{18} = (7, 0, 1, 0)$  lies on line  $\ell_1$

6 :  $P_{22} = (3, 1, 1, 0)$  lies on line  $\ell_1$

7 :  $P_{24} = (5, 1, 1, 0)$  lies on line  $\ell_2$

8 :  $P_{25} = (6, 1, 1, 0)$  lies on line  $\ell_0$

9 :  $P_{27} = (0, 2, 1, 0)$  lies on line  $\ell_0$

10 :  $P_{28} = (1, 2, 1, 0)$  lies on line  $\ell_2$

11 :  $P_{29} = (2, 2, 1, 0)$  lies on line  $\ell_1$

12 :  $P_{37} = (2, 3, 1, 0)$  lies on line  $\ell_0$

13 :  $P_{43} = (0, 4, 1, 0)$  lies on line  $\ell_1$

14 :  $P_{44} = (1, 4, 1, 0)$  lies on line  $\ell_0$

15 :  $P_{47} = (4, 4, 1, 0)$  lies on line  $\ell_2$

16 :  $P_{55} = (4, 5, 1, 0)$  lies on line  $\ell_1$

17 :  $P_{66} = (7, 6, 1, 0)$  lies on line  $\ell_2$

18 :  $P_{67} = (0, 7, 1, 0)$  lies on line  $\ell_2$

19 :  $P_{68} = (1, 7, 1, 0)$  lies on line  $\ell_1$

20 :  $P_{74} = (7, 7, 1, 0)$  lies on line  $\ell_0$

The single points on the surface are:

### Points on surface but on no line

The surface has 49 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_{75} = (1, 0, 0, 1)$$

$$2 : P_{82} = (0, 1, 0, 1)$$

$$3 : P_{83} = (1, 1, 0, 1)$$

$$4 : P_{103} = (5, 3, 0, 1)$$

$$5 : P_{120} = (6, 5, 0, 1)$$

$$6 : P_{125} = (3, 6, 0, 1)$$

$$7 : P_{138} = (0, 0, 1, 1)$$

$$8 : P_{139} = (1, 0, 1, 1)$$

$$9 : P_{146} = (0, 1, 1, 1)$$

10 : $P_{166} = (5, 3, 1, 1)$	30 : $P_{424} = (7, 3, 5, 1)$
11 : $P_{183} = (6, 5, 1, 1)$	31 : $P_{428} = (3, 4, 5, 1)$
12 : $P_{188} = (3, 6, 1, 1)$	32 : $P_{436} = (3, 5, 5, 1)$
13 : $P_{213} = (4, 1, 2, 1)$	33 : $P_{441} = (0, 6, 5, 1)$
14 : $P_{238} = (5, 4, 2, 1)$	34 : $P_{446} = (5, 6, 5, 1)$
15 : $P_{242} = (1, 5, 2, 1)$	35 : $P_{447} = (6, 6, 5, 1)$
16 : $P_{270} = (5, 0, 3, 1)$	36 : $P_{453} = (4, 7, 5, 1)$
17 : $P_{287} = (6, 2, 3, 1)$	37 : $P_{460} = (3, 0, 6, 1)$
18 : $P_{295} = (6, 3, 3, 1)$	38 : $P_{480} = (7, 2, 6, 1)$
19 : $P_{299} = (2, 4, 3, 1)$	39 : $P_{481} = (0, 3, 6, 1)$
20 : $P_{305} = (0, 5, 3, 1)$	40 : $P_{484} = (3, 3, 6, 1)$
21 : $P_{308} = (3, 5, 3, 1)$	41 : $P_{487} = (6, 3, 6, 1)$
22 : $P_{310} = (5, 5, 3, 1)$	42 : $P_{499} = (2, 5, 6, 1)$
23 : $P_{317} = (4, 6, 3, 1)$	43 : $P_{500} = (3, 5, 6, 1)$
24 : $P_{318} = (5, 6, 3, 1)$	44 : $P_{510} = (5, 6, 6, 1)$
25 : $P_{344} = (7, 1, 4, 1)$	45 : $P_{518} = (5, 7, 6, 1)$
26 : $P_{378} = (1, 6, 4, 1)$	46 : $P_{531} = (2, 1, 7, 1)$
27 : $P_{391} = (6, 7, 4, 1)$	47 : $P_{540} = (3, 2, 7, 1)$
28 : $P_{399} = (6, 0, 5, 1)$	48 : $P_{546} = (1, 3, 7, 1)$
29 : $P_{423} = (6, 3, 5, 1)$	

## Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_{64}$	$P_{54}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{64}$	$P_{41}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_{54}$	$P_{41}$

The surface has 73 points:

The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$	12 : $P_{29} = (2, 2, 1, 0)$	24 : $P_{74} = (7, 7, 1, 0)$
1 : $P_6 = (2, 1, 0, 0)$	13 : $P_{37} = (2, 3, 1, 0)$	25 : $P_{75} = (1, 0, 0, 1)$
2 : $P_8 = (4, 1, 0, 0)$	14 : $P_{41} = (6, 3, 1, 0)$	26 : $P_{82} = (0, 1, 0, 1)$
3 : $P_{11} = (7, 1, 0, 0)$	15 : $P_{43} = (0, 4, 1, 0)$	27 : $P_{83} = (1, 1, 0, 1)$
4 : $P_{13} = (2, 0, 1, 0)$	16 : $P_{44} = (1, 4, 1, 0)$	28 : $P_{103} = (5, 3, 0, 1)$
5 : $P_{15} = (4, 0, 1, 0)$	17 : $P_{47} = (4, 4, 1, 0)$	29 : $P_{120} = (6, 5, 0, 1)$
6 : $P_{18} = (7, 0, 1, 0)$	18 : $P_{54} = (3, 5, 1, 0)$	30 : $P_{125} = (3, 6, 0, 1)$
7 : $P_{22} = (3, 1, 1, 0)$	19 : $P_{55} = (4, 5, 1, 0)$	31 : $P_{138} = (0, 0, 1, 1)$
8 : $P_{24} = (5, 1, 1, 0)$	20 : $P_{64} = (5, 6, 1, 0)$	32 : $P_{139} = (1, 0, 1, 1)$
9 : $P_{25} = (6, 1, 1, 0)$	21 : $P_{66} = (7, 6, 1, 0)$	33 : $P_{146} = (0, 1, 1, 1)$
10 : $P_{27} = (0, 2, 1, 0)$	22 : $P_{67} = (0, 7, 1, 0)$	34 : $P_{166} = (5, 3, 1, 1)$
11 : $P_{28} = (1, 2, 1, 0)$	23 : $P_{68} = (1, 7, 1, 0)$	35 : $P_{183} = (6, 5, 1, 1)$

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