Rank-331 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 2658901

General information

Number of lines	11
Number of points	81
Number of singular points	9
Number of Eckardt points	0
Number of double points	18
Number of single points	63
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{11}
Type of lines on points	2^{18}, 1^{63}

Singular Points

The surface has 9 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
1 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
2 : $P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
3 : $P_{201} = \mathbf{P}(0,0,\gamma,1) = \mathbf{P}(0,0,2,1)$
4 : $P_{265} = \mathbf{P}(0,0,\gamma^5,1) = \mathbf{P}(0,0,3,1)$
5 : $P_{329} = \mathbf{P}(0,0,\gamma^2,1) = \mathbf{P}(0,0,4,1)$
6 : $P_{393} = \mathbf{P}(0,0,\gamma^3,1) = \mathbf{P}(0,0,5,1)$
7 : $P_{457} = \mathbf{P}(0,0,\gamma^6,1) = \mathbf{P}(0,0,6,1)$
8 : $P_{521} = \mathbf{P}(0,0,\gamma^4,1) = \mathbf{P}(0,0,7,1)$

The 11 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_5 = \begin{bmatrix} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 1 & \gamma^5 \end{bmatrix}_{505} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{505} = \mathbf{Pl}(0,0,4,3,6,1)_{3887}$$

$$\ell_6 = \begin{bmatrix} 1 & \gamma^2 & 0 & 0 \\ 0 & 0 & 1 & \gamma^4 \end{bmatrix}_{363} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{363} = \mathbf{Pl}(0,0,5,7,4,1)_{2894}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 1 & \gamma^3 \end{bmatrix}_{288} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{288} = \mathbf{Pl}(0,0,7,5,3,1)_{2420}$$

$$\ell_8 = \begin{bmatrix} 1 & \gamma^4 & 0 & 0 \\ 0 & 0 & 1 & \gamma \end{bmatrix}_{577} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{577} = \mathbf{Pl}(0,0,6,2,7,1)_{4421}$$

$$\ell_9 = \begin{bmatrix} 1 & \gamma & 0 & 0 \\ 0 & 0 & 1 & \gamma^2 \end{bmatrix}_{214} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{214} = \mathbf{Pl}(0,0,3,4,2,1)_{1856}$$

$$\ell_{10} = \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 1 & \gamma^6 \end{bmatrix}_{435} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{435} = \mathbf{Pl}(0,0,2,6,5,1)_{3353}$$

Rank of lines: (0, 64, 4680, 4744, 138, 505, 363, 288, 577, 214, 435)
Rank of points on Klein quadric: (0, 2, 17, 1, 1322, 3887, 2894, 2420, 4421, 1856, 3353)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 18 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_4$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_5$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_6$
$P_8 = (4,1,0,0) = \ell_0 \cap \ell_7$
$P_9 = (5,1,0,0) = \ell_0 \cap \ell_8$
$P_{10} = (6,1,0,0) = \ell_0 \cap \ell_9$
$P_{11} = (7,1,0,0) = \ell_0 \cap \ell_{10}$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_2 \cap \ell_3$
$P_{138} = (0,0,1,1) = \ell_3 \cap \ell_4$
$P_{329} = (0,0,4,1) = \ell_3 \cap \ell_5$
$P_{393} = (0,0,5,1) = \ell_3 \cap \ell_6$

$P_{521} = (0,0,7,1) = \ell_3 \cap \ell_7$
$P_{457} = (0,0,6,1) = \ell_3 \cap \ell_8$
$P_{265} = (0,0,3,1) = \ell_3 \cap \ell_9$
$P_{201} = (0,0,2,1) = \ell_3 \cap \ell_{10}$

Single Points

The surface has 63 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_4
1 : $P_{12} = (1,0,1,0)$ lies on line ℓ_1
2 : $P_{13} = (2,0,1,0)$ lies on line ℓ_1
3 : $P_{14} = (3,0,1,0)$ lies on line ℓ_1
4 : $P_{15} = (4,0,1,0)$ lies on line ℓ_1
5 : $P_{16} = (5,0,1,0)$ lies on line ℓ_1
6 : $P_{17} = (6,0,1,0)$ lies on line ℓ_1
7 : $P_{18} = (7,0,1,0)$ lies on line ℓ_1
8 : $P_{82} = (0,1,0,1)$ lies on line ℓ_2
9 : $P_{90} = (0,2,0,1)$ lies on line ℓ_2
10 : $P_{98} = (0,3,0,1)$ lies on line ℓ_2
11 : $P_{106} = (0,4,0,1)$ lies on line ℓ_2
12 : $P_{114} = (0,5,0,1)$ lies on line ℓ_2
13 : $P_{122} = (0,6,0,1)$ lies on line ℓ_2
14 : $P_{130} = (0,7,0,1)$ lies on line ℓ_2
15 : $P_{155} = (2,2,1,1)$ lies on line ℓ_4
16 : $P_{164} = (3,3,1,1)$ lies on line ℓ_4
17 : $P_{173} = (4,4,1,1)$ lies on line ℓ_4
18 : $P_{182} = (5,5,1,1)$ lies on line ℓ_4
19 : $P_{191} = (6,6,1,1)$ lies on line ℓ_4
20 : $P_{200} = (7,7,1,1)$ lies on line ℓ_4
21 : $P_{216} = (7,1,2,1)$ lies on line ℓ_{10}
22 : $P_{220} = (3,2,2,1)$ lies on line ℓ_{10}
23 : $P_{229} = (4,3,2,1)$ lies on line ℓ_{10}
24 : $P_{239} = (6,4,2,1)$ lies on line ℓ_{10}
25 : $P_{242} = (1,5,2,1)$ lies on line ℓ_{10}
26 : $P_{254} = (5,6,2,1)$ lies on line ℓ_{10}
27 : $P_{259} = (2,7,2,1)$ lies on line ℓ_{10}
28 : $P_{279} = (6,1,3,1)$ lies on line ℓ_9
29 : $P_{282} = (1,2,3,1)$ lies on line ℓ_9
30 : $P_{296} = (7,3,3,1)$ lies on line ℓ_9
31 : $P_{299} = (2,4,3,1)$ lies on line ℓ_9
32 : $P_{309} = (4,5,3,1)$ lies on line ℓ_9
33 : $P_{316} = (3,6,3,1)$ lies on line ℓ_9
34 : $P_{326} = (5,7,3,1)$ lies on line ℓ_9
35 : $P_{339} = (2,1,4,1)$ lies on line ℓ_5
36 : $P_{349} = (4,2,4,1)$ lies on line ℓ_5
37 : $P_{359} = (6,3,4,1)$ lies on line ℓ_5
38 : $P_{366} = (5,4,4,1)$ lies on line ℓ_5
39 : $P_{376} = (7,5,4,1)$ lies on line ℓ_5
40 : $P_{378} = (1,6,4,1)$ lies on line ℓ_5
41 : $P_{388} = (3,7,4,1)$ lies on line ℓ_5
42 : $P_{404} = (3,1,5,1)$ lies on line ℓ_6
43 : $P_{415} = (6,2,5,1)$ lies on line ℓ_6
44 : $P_{422} = (5,3,5,1)$ lies on line ℓ_6
45 : $P_{426} = (1,4,5,1)$ lies on line ℓ_6
46 : $P_{435} = (2,5,5,1)$ lies on line ℓ_6
47 : $P_{448} = (7,6,5,1)$ lies on line ℓ_6
48 : $P_{453} = (4,7,5,1)$ lies on line ℓ_6
49 : $P_{470} = (5,1,6,1)$ lies on line ℓ_8
50 : $P_{480} = (7,2,6,1)$ lies on line ℓ_8
51 : $P_{483} = (2,3,6,1)$ lies on line ℓ_8
52 : $P_{492} = (3,4,6,1)$ lies on line ℓ_8
53 : $P_{503} = (6,5,6,1)$ lies on line ℓ_8
54 : $P_{509} = (4,6,6,1)$ lies on line ℓ_8
55 : $P_{514} = (1,7,6,1)$ lies on line ℓ_8
56 : $P_{533} = (4,1,7,1)$ lies on line ℓ_7
57 : $P_{542} = (5,2,7,1)$ lies on line ℓ_7
58 : $P_{546} = (1,3,7,1)$ lies on line ℓ_7
59 : $P_{560} = (7,4,7,1)$ lies on line ℓ_7
60 : $P_{564} = (3,5,7,1)$ lies on line ℓ_7
61 : $P_{571} = (2,6,7,1)$ lies on line ℓ_7
62 : $P_{583} = (6,7,7,1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10
0	0	1	1	0	1	1	1	1	1	1	1
1	1	0	0	1	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0	0
3	0	1	1	0	1	1	1	1	1	1	1
4	1	0	0	1	0	0	0	0	0	0	0
5	1	0	0	1	0	0	0	0	0	0	0
6	1	0	0	1	0	0	0	0	0	0	0
7	1	0	0	1	0	0	0	0	0	0	0
8	1	0	0	1	0	0	0	0	0	0	0
9	1	0	0	1	0	0	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_0	P_1	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_3	P_{138}	P_{329}	P_{393}	P_{521}	P_{457}	P_{265}	P_{201}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{138}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{329}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_7	P_{393}

Line 7 intersects

Line	ℓ_0	ℓ_3
in point	P_8	P_{521}

Line 8 intersects

Line	ℓ_0	ℓ_3
in point	P_9	P_{457}

Line 9 intersects

Line	ℓ_0	ℓ_3
in point	P_{10}	P_{265}

Line 10 intersects

Line	ℓ_0	ℓ_3
in point	P_{11}	P_{201}

The surface has 81 points:

The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (4,1,0,0)$
$9: P_9 = (5,1,0,0)$
$10: P_{10} = (6,1,0,0)$
$11: P_{11} = (7,1,0,0)$
$12: P_{12} = (1,0,1,0)$
$13: P_{13} = (2,0,1,0)$
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