

# Rank-76292 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1361351246

## General information

Number of lines	2
Number of points	73
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^2$
Type of lines on points	$2, 1^{16}, 0^{56}$

## Singular Points

The surface has 2 singular points:

$$0 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$1 : P_{139} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1)$$

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{10}$$

Rank of lines: ( 4744, 648 )

Rank of points on Klein quadric: ( 1, 10 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

### Single Points

The surface has 16 single points:

The single points on the surface are:

- |  |   |
|--|---|
| 0 : $P_3 = (0, 0, 0, 1)$ lies on line $\ell_0$     | 9 : $P_{330} = (1, 0, 4, 1)$ lies on line $\ell_1$  |
| 1 : $P_{75} = (1, 0, 0, 1)$ lies on line $\ell_1$  | 10 : $P_{393} = (0, 0, 5, 1)$ lies on line $\ell_0$ |
| 2 : $P_{138} = (0, 0, 1, 1)$ lies on line $\ell_0$ | 11 : $P_{394} = (1, 0, 5, 1)$ lies on line $\ell_1$ |
| 3 : $P_{139} = (1, 0, 1, 1)$ lies on line $\ell_1$ | 12 : $P_{457} = (0, 0, 6, 1)$ lies on line $\ell_0$ |
| 4 : $P_{201} = (0, 0, 2, 1)$ lies on line $\ell_0$ | 13 : $P_{458} = (1, 0, 6, 1)$ lies on line $\ell_1$ |
| 5 : $P_{202} = (1, 0, 2, 1)$ lies on line $\ell_1$ | 14 : $P_{521} = (0, 0, 7, 1)$ lies on line $\ell_0$ |
| 6 : $P_{265} = (0, 0, 3, 1)$ lies on line $\ell_0$ | 15 : $P_{522} = (1, 0, 7, 1)$ lies on line $\ell_1$ |
| 7 : $P_{266} = (1, 0, 3, 1)$ lies on line $\ell_1$ |   |
| 8 : $P_{329} = (0, 0, 4, 1)$ lies on line $\ell_0$ |   |

The single points on the surface are:

### Points on surface but on no line

The surface has 56 points not on any line:

The points on the surface but not on lines are:

- |                              |                               |
|------------------------------|-------------------------------|
| 0 : $P_1 = (0, 1, 0, 0)$     | 11 : $P_{108} = (2, 4, 0, 1)$ |
| 1 : $P_{22} = (3, 1, 1, 0)$  | 12 : $P_{134} = (4, 7, 0, 1)$ |
| 2 : $P_{24} = (5, 1, 1, 0)$  | 13 : $P_{146} = (0, 1, 1, 1)$ |
| 3 : $P_{25} = (6, 1, 1, 0)$  | 14 : $P_{158} = (5, 2, 1, 1)$ |
| 4 : $P_{37} = (2, 3, 1, 0)$  | 15 : $P_{167} = (6, 3, 1, 1)$ |
| 5 : $P_{55} = (4, 5, 1, 0)$  | 16 : $P_{175} = (6, 4, 1, 1)$ |
| 6 : $P_{66} = (7, 6, 1, 0)$  | 17 : $P_{180} = (3, 5, 1, 1)$ |
| 7 : $P_{85} = (3, 1, 0, 1)$  | 18 : $P_{190} = (5, 6, 1, 1)$ |
| 8 : $P_{87} = (5, 1, 0, 1)$  | 19 : $P_{196} = (3, 7, 1, 1)$ |
| 9 : $P_{88} = (6, 1, 0, 1)$  | 20 : $P_{233} = (0, 4, 2, 1)$ |
| 10 : $P_{97} = (7, 2, 0, 1)$ | 21 : $P_{235} = (2, 4, 2, 1)$ |

22 :  $P_{244} = (3, 5, 2, 1)$   
 23 :  $P_{250} = (1, 6, 2, 1)$   
 24 :  $P_{260} = (3, 7, 2, 1)$   
 25 :  $P_{276} = (3, 1, 3, 1)$   
 26 :  $P_{294} = (5, 3, 3, 1)$   
 27 :  $P_{302} = (5, 4, 3, 1)$   
 28 :  $P_{305} = (0, 5, 3, 1)$   
 29 :  $P_{311} = (6, 5, 3, 1)$   
 30 :  $P_{314} = (1, 6, 3, 1)$   
 31 :  $P_{327} = (6, 7, 3, 1)$   
 32 :  $P_{350} = (5, 2, 4, 1)$   
 33 :  $P_{354} = (1, 3, 4, 1)$   
 34 :  $P_{382} = (5, 6, 4, 1)$   
 35 :  $P_{385} = (0, 7, 4, 1)$   
 36 :  $P_{389} = (4, 7, 4, 1)$   
 37 :  $P_{406} = (5, 1, 5, 1)$   
 38 :  $P_{412} = (3, 2, 5, 1)$   
 39 :  $P_{418} = (1, 3, 5, 1)$

40 :  $P_{439} = (6, 5, 5, 1)$   
 41 :  $P_{441} = (0, 6, 5, 1)$   
 42 :  $P_{444} = (3, 6, 5, 1)$   
 43 :  $P_{455} = (6, 7, 5, 1)$   
 44 :  $P_{471} = (6, 1, 6, 1)$   
 45 :  $P_{476} = (3, 2, 6, 1)$   
 46 :  $P_{481} = (0, 3, 6, 1)$   
 47 :  $P_{486} = (5, 3, 6, 1)$   
 48 :  $P_{494} = (5, 4, 6, 1)$   
 49 :  $P_{498} = (1, 5, 6, 1)$   
 50 :  $P_{508} = (3, 6, 6, 1)$   
 51 :  $P_{537} = (0, 2, 7, 1)$   
 52 :  $P_{544} = (7, 2, 7, 1)$   
 53 :  $P_{551} = (6, 3, 7, 1)$   
 54 :  $P_{559} = (6, 4, 7, 1)$   
 55 :  $P_{562} = (1, 5, 7, 1)$

## Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$
in point	$P_2$

Line 1 intersects

Line	$\ell_0$
in point	$P_2$

The surface has 73 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	18 : $P_{146} = (0, 1, 1, 1)$	36 : $P_{302} = (5, 4, 3, 1)$
1 : $P_2 = (0, 0, 1, 0)$	19 : $P_{158} = (5, 2, 1, 1)$	37 : $P_{305} = (0, 5, 3, 1)$
2 : $P_3 = (0, 0, 0, 1)$	20 : $P_{167} = (6, 3, 1, 1)$	38 : $P_{311} = (6, 5, 3, 1)$
3 : $P_{22} = (3, 1, 1, 0)$	21 : $P_{175} = (6, 4, 1, 1)$	39 : $P_{314} = (1, 6, 3, 1)$
4 : $P_{24} = (5, 1, 1, 0)$	22 : $P_{180} = (3, 5, 1, 1)$	40 : $P_{327} = (6, 7, 3, 1)$
5 : $P_{25} = (6, 1, 1, 0)$	23 : $P_{190} = (5, 6, 1, 1)$	41 : $P_{329} = (0, 0, 4, 1)$
6 : $P_{37} = (2, 3, 1, 0)$	24 : $P_{196} = (3, 7, 1, 1)$	42 : $P_{330} = (1, 0, 4, 1)$
7 : $P_{55} = (4, 5, 1, 0)$	25 : $P_{201} = (0, 0, 2, 1)$	43 : $P_{350} = (5, 2, 4, 1)$
8 : $P_{66} = (7, 6, 1, 0)$	26 : $P_{202} = (1, 0, 2, 1)$	44 : $P_{354} = (1, 3, 4, 1)$
9 : $P_{75} = (1, 0, 0, 1)$	27 : $P_{233} = (0, 4, 2, 1)$	45 : $P_{382} = (5, 6, 4, 1)$
10 : $P_{85} = (3, 1, 0, 1)$	28 : $P_{235} = (2, 4, 2, 1)$	46 : $P_{385} = (0, 7, 4, 1)$
11 : $P_{87} = (5, 1, 0, 1)$	29 : $P_{244} = (3, 5, 2, 1)$	47 : $P_{389} = (4, 7, 4, 1)$
12 : $P_{88} = (6, 1, 0, 1)$	30 : $P_{250} = (1, 6, 2, 1)$	48 : $P_{393} = (0, 0, 5, 1)$
13 : $P_{97} = (7, 2, 0, 1)$	31 : $P_{260} = (3, 7, 2, 1)$	49 : $P_{394} = (1, 0, 5, 1)$
14 : $P_{108} = (2, 4, 0, 1)$	32 : $P_{265} = (0, 0, 3, 1)$	50 : $P_{406} = (5, 1, 5, 1)$
15 : $P_{134} = (4, 7, 0, 1)$	33 : $P_{266} = (1, 0, 3, 1)$	51 : $P_{412} = (3, 2, 5, 1)$
16 : $P_{138} = (0, 0, 1, 1)$	34 : $P_{276} = (3, 1, 3, 1)$	52 : $P_{418} = (1, 3, 5, 1)$
17 : $P_{139} = (1, 0, 1, 1)$	35 : $P_{294} = (5, 3, 3, 1)$	53 : $P_{439} = (6, 5, 5, 1)$

54 :  $P_{441} = (0, 6, 5, 1)$   
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 56 :  $P_{455} = (6, 7, 5, 1)$   
 57 :  $P_{457} = (0, 0, 6, 1)$   
 58 :  $P_{458} = (1, 0, 6, 1)$   
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 63 :  $P_{494} = (5, 4, 6, 1)$   
 64 :  $P_{498} = (1, 5, 6, 1)$   
 65 :  $P_{508} = (3, 6, 6, 1)$   
 66 :  $P_{521} = (0, 0, 7, 1)$   
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 72 :  $P_{562} = (1, 5, 7, 1)$