

# Rank-65871 over GF(16)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(16) is 303112725

## General information

Number of lines	16
Number of points	337
Number of singular points	2
Number of Eckardt points	0
Number of double points	38
Number of single points	186
Number of points off lines	111
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$17^{16}$
Type of lines on points	$5^2, 2^{38}, 1^{186}, 0^{111}$

## Singular Points

The surface has 2 singular points:

$$0 : P_{275} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$1 : P_{291} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$$

## The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{34} \\
\ell_2 &= \begin{bmatrix} 1 & \delta^{14} & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3542} = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3542} = \mathbf{Pl}(0, 0, 11, 10, 13, 1)_{58696} \\
\ell_3 &= \begin{bmatrix} 1 & \delta^{13} & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{1905} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{1905} = \mathbf{Pl}(0, 0, 10, 11, 7, 1)_{34185} \\
\ell_4 &= \begin{bmatrix} 1 & \delta^{11} & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3815} = \begin{bmatrix} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3815} = \mathbf{Pl}(0, 0, 11, 10, 12, 1)_{54616} \\
\ell_5 &= \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \mathbf{Pl}(0, 0, 1, 1, 10, 1)_{46146} \\
\ell_6 &= \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \mathbf{Pl}(0, 0, 1, 1, 11, 1)_{50226} \\
\ell_7 &= \begin{bmatrix} 1 & \delta^7 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{2178} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{2178} = \mathbf{Pl}(0, 0, 10, 11, 6, 1)_{30105} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{14} & \delta^7 \end{bmatrix}_{4492} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 12 & 7 \end{bmatrix}_{4492} = \mathbf{Pl}(14, 7, 7, 14, 1, 0)_{4278} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{13} & \delta^{14} \end{bmatrix}_{4566} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 6 & 12 \end{bmatrix}_{4566} = \mathbf{Pl}(2, 12, 12, 2, 1, 0)_{1641} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \delta^{13} & \delta^{11} \\ 0 & 1 & \delta^{13} & \delta^{14} \end{bmatrix}_{58620} = \begin{bmatrix} 1 & 0 & 6 & 13 \\ 0 & 1 & 6 & 12 \end{bmatrix}_{58620} = \mathbf{Pl}(1, 1, 4, 6, 2, 1)_{14616} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \delta^7 & \delta^{14} \\ 0 & 1 & \delta^7 & \delta^{11} \end{bmatrix}_{54542} = \begin{bmatrix} 1 & 0 & 7 & 12 \\ 0 & 1 & 7 & 13 \end{bmatrix}_{54542} = \mathbf{Pl}(1, 1, 14, 7, 9, 1)_{45351} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{11} & \delta^{13} \end{bmatrix}_{4477} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 13 & 6 \end{bmatrix}_{4477} = \mathbf{Pl}(4, 6, 6, 4, 1, 0)_{2003} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & \delta^{14} & \delta^{13} \\ 0 & 1 & \delta^{14} & \delta^7 \end{bmatrix}_{29608} = \begin{bmatrix} 1 & 0 & 12 & 6 \\ 0 & 1 & 12 & 7 \end{bmatrix}_{29608} = \mathbf{Pl}(1, 1, 2, 12, 14, 1)_{63336} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & \delta^{11} & \delta^7 \\ 0 & 1 & \delta^{11} & \delta^{13} \end{bmatrix}_{34234} = \begin{bmatrix} 1 & 0 & 13 & 7 \\ 0 & 1 & 13 & 6 \end{bmatrix}_{34234} = \mathbf{Pl}(1, 1, 9, 13, 4, 1)_{23856} \\
\ell_{15} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^7 & \delta^{11} \end{bmatrix}_{4583} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 7 & 13 \end{bmatrix}_{4583} = \mathbf{Pl}(9, 13, 13, 9, 1, 0)_{3238}
\end{aligned}$$

Rank of lines: ( 0, 4368, 3542, 1905, 3815, 3260, 2987, 2178, 4492, 4566, 58620, 54542, 4477, 29608, 34234, 4583 )

Rank of points on Klein quadric: ( 0, 34, 58696, 34185, 54616, 46146, 50226, 30105, 4278, 1641, 14616, 45351, 2003, 63336, 23856, 3238 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 38 Double points:

The double points on the surface are:

$$\begin{aligned}
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_1 \\
P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_2 \\
P_8 &= (4, 1, 0, 0) = \ell_0 \cap \ell_3 \\
P_{13} &= (9, 1, 0, 0) = \ell_0 \cap \ell_4 \\
P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_5 \\
P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_6 \\
P_{18} &= (14, 1, 0, 0) = \ell_0 \cap \ell_7 \\
P_{3089} &= (0, 0, 11, 1) = \ell_2 \cap \ell_4 \\
P_{3336} &= (7, 15, 11, 1) = \ell_2 \cap \ell_8 \\
P_{3215} &= (14, 7, 11, 1) = \ell_2 \cap \ell_9 \\
P_{3125} &= (4, 2, 11, 1) = \ell_2 \cap \ell_{11} \\
P_{3143} &= (6, 3, 11, 1) = \ell_2 \cap \ell_{14} \\
P_{2833} &= (0, 0, 10, 1) = \ell_3 \cap \ell_7 \\
P_{2893} &= (12, 3, 10, 1) = \ell_3 \cap \ell_9 \\
P_{2926} &= (13, 5, 10, 1) = \ell_3 \cap \ell_{11} \\
P_{3027} &= (2, 12, 10, 1) = \ell_3 \cap \ell_{12} \\
P_{2906} &= (9, 4, 10, 1) = \ell_3 \cap \ell_{13} \\
P_{3247} &= (14, 9, 11, 1) = \ell_4 \cap \ell_{10} \\
P_{3175} &= (6, 5, 11, 1) = \ell_4 \cap \ell_{12} \\
P_{3224} &= (7, 8, 11, 1) = \ell_4 \cap \ell_{13}
\end{aligned}$$

$$\begin{aligned}
P_{3189} &= (4, 6, 11, 1) = \ell_4 \cap \ell_{15} \\
P_{530} &= (0, 0, 1, 1) = \ell_5 \cap \ell_6 \\
P_{596} &= (3, 4, 1, 1) = \ell_5 \cap \ell_9 \\
P_{752} &= (15, 13, 1, 1) = \ell_5 \cap \ell_{13} \\
P_{726} &= (5, 12, 1, 1) = \ell_5 \cap \ell_{14} \\
P_{761} &= (8, 14, 1, 1) = \ell_5 \cap \ell_{15} \\
P_{576} &= (15, 2, 1, 1) = \ell_6 \cap \ell_8 \\
P_{644} &= (3, 7, 1, 1) = \ell_6 \cap \ell_{10} \\
P_{633} &= (8, 6, 1, 1) = \ell_6 \cap \ell_{11} \\
P_{678} &= (5, 9, 1, 1) = \ell_6 \cap \ell_{12} \\
P_{3050} &= (9, 13, 10, 1) = \ell_7 \cap \ell_8 \\
P_{3085} &= (12, 15, 10, 1) = \ell_7 \cap \ell_{10} \\
P_{3059} &= (2, 14, 10, 1) = \ell_7 \cap \ell_{14} \\
P_{2974} &= (13, 8, 10, 1) = \ell_7 \cap \ell_{15} \\
P_{2289} &= (0, 14, 7, 1) = \ell_8 \cap \ell_{13} \\
P_{3377} &= (0, 2, 12, 1) = \ell_9 \cap \ell_{10} \\
P_{3745} &= (0, 9, 13, 1) = \ell_{11} \cap \ell_{15} \\
P_{1873} &= (0, 4, 6, 1) = \ell_{12} \cap \ell_{14}
\end{aligned}$$

### Single Points

The surface has 186 single points:  
The single points on the surface are:

$$\begin{aligned}
0 : P_0 &= (1, 0, 0, 0) \text{ lies on line } \ell_0 \\
1 : P_5 &= (1, 1, 0, 0) \text{ lies on line } \ell_0 \\
2 : P_7 &= (3, 1, 0, 0) \text{ lies on line } \ell_0 \\
3 : P_9 &= (5, 1, 0, 0) \text{ lies on line } \ell_0 \\
4 : P_{10} &= (6, 1, 0, 0) \text{ lies on line } \ell_0 \\
5 : P_{11} &= (7, 1, 0, 0) \text{ lies on line } \ell_0 \\
6 : P_{12} &= (8, 1, 0, 0) \text{ lies on line } \ell_0 \\
7 : P_{16} &= (12, 1, 0, 0) \text{ lies on line } \ell_0 \\
8 : P_{17} &= (13, 1, 0, 0) \text{ lies on line } \ell_0 \\
9 : P_{19} &= (15, 1, 0, 0) \text{ lies on line } \ell_0 \\
10 : P_{65} &= (14, 2, 1, 0) \text{ lies on line } \ell_8 \\
11 : P_{85} &= (2, 4, 1, 0) \text{ lies on line } \ell_9 \\
12 : P_{117} &= (2, 6, 1, 0) \text{ lies on line } \ell_{10} \\
13 : P_{140} &= (9, 7, 1, 0) \text{ lies on line } \ell_{11} \\
14 : P_{167} &= (4, 9, 1, 0) \text{ lies on line } \ell_{12} \\
15 : P_{225} &= (14, 12, 1, 0) \text{ lies on line } \ell_{13} \\
16 : P_{231} &= (4, 13, 1, 0) \text{ lies on line } \ell_{14} \\
17 : P_{252} &= (9, 14, 1, 0) \text{ lies on line } \ell_{15} \\
18 : P_{307} &= (1, 2, 0, 1) \text{ lies on line } \ell_1 \\
19 : P_{323} &= (1, 3, 0, 1) \text{ lies on line } \ell_1 \\
20 : P_{339} &= (1, 4, 0, 1) \text{ lies on line } \ell_1 \\
21 : P_{355} &= (1, 5, 0, 1) \text{ lies on line } \ell_1 \\
22 : P_{371} &= (1, 6, 0, 1) \text{ lies on line } \ell_1 \\
23 : P_{387} &= (1, 7, 0, 1) \text{ lies on line } \ell_1 \\
24 : P_{403} &= (1, 8, 0, 1) \text{ lies on line } \ell_1 \\
25 : P_{419} &= (1, 9, 0, 1) \text{ lies on line } \ell_1 \\
26 : P_{435} &= (1, 10, 0, 1) \text{ lies on line } \ell_1
\end{aligned}$$

$$\begin{aligned}
27 : P_{451} &= (1, 11, 0, 1) \text{ lies on line } \ell_1 \\
28 : P_{467} &= (1, 12, 0, 1) \text{ lies on line } \ell_1 \\
29 : P_{483} &= (1, 13, 0, 1) \text{ lies on line } \ell_1 \\
30 : P_{499} &= (1, 14, 0, 1) \text{ lies on line } \ell_1 \\
31 : P_{515} &= (1, 15, 0, 1) \text{ lies on line } \ell_1 \\
32 : P_{555} &= (10, 1, 1, 1) \text{ lies on line } \ell_5 \\
33 : P_{556} &= (11, 1, 1, 1) \text{ lies on line } \ell_6 \\
34 : P_{574} &= (13, 2, 1, 1) \text{ lies on line } \ell_5 \\
35 : P_{581} &= (4, 3, 1, 1) \text{ lies on line } \ell_6 \\
36 : P_{584} &= (7, 3, 1, 1) \text{ lies on line } \ell_5 \\
37 : P_{600} &= (7, 4, 1, 1) \text{ lies on line } \ell_6 \\
38 : P_{618} &= (9, 5, 1, 1) \text{ lies on line } \ell_5 \\
39 : P_{621} &= (12, 5, 1, 1) \text{ lies on line } \ell_6 \\
40 : P_{639} &= (14, 6, 1, 1) \text{ lies on line } \ell_5 \\
41 : P_{645} &= (4, 7, 1, 1) \text{ lies on line } \ell_5 \\
42 : P_{663} &= (6, 8, 1, 1) \text{ lies on line } \ell_5 \\
43 : P_{671} &= (14, 8, 1, 1) \text{ lies on line } \ell_6 \\
44 : P_{685} &= (12, 9, 1, 1) \text{ lies on line } \ell_5 \\
45 : P_{690} &= (1, 10, 1, 1) \text{ lies on line } \ell_6 \\
46 : P_{700} &= (11, 10, 1, 1) \text{ lies on line } \ell_5 \\
47 : P_{706} &= (1, 11, 1, 1) \text{ lies on line } \ell_5 \\
48 : P_{715} &= (10, 11, 1, 1) \text{ lies on line } \ell_6 \\
49 : P_{730} &= (9, 12, 1, 1) \text{ lies on line } \ell_6 \\
50 : P_{739} &= (2, 13, 1, 1) \text{ lies on line } \ell_6 \\
51 : P_{759} &= (6, 14, 1, 1) \text{ lies on line } \ell_6 \\
52 : P_{771} &= (2, 15, 1, 1) \text{ lies on line } \ell_5 \\
53 : P_{782} &= (13, 15, 1, 1) \text{ lies on line } \ell_6
\end{aligned}$$

54 :  $P_{789} = (4, 0, 2, 1)$  lies on line  $\ell_{13}$   
 55 :  $P_{826} = (9, 2, 2, 1)$  lies on line  $\ell_{14}$   
 56 :  $P_{853} = (4, 4, 2, 1)$  lies on line  $\ell_8$   
 57 :  $P_{875} = (10, 5, 2, 1)$  lies on line  $\ell_{15}$   
 58 :  $P_{918} = (5, 8, 2, 1)$  lies on line  $\ell_9$   
 59 :  $P_{970} = (9, 11, 2, 1)$  lies on line  $\ell_{12}$   
 60 :  $P_{998} = (5, 13, 2, 1)$  lies on line  $\ell_{10}$   
 61 :  $P_{1035} = (10, 15, 2, 1)$  lies on line  $\ell_{11}$   
 62 :  $P_{1086} = (13, 2, 3, 1)$  lies on line  $\ell_{12}$   
 63 :  $P_{1147} = (10, 6, 3, 1)$  lies on line  $\ell_8$   
 64 :  $P_{1172} = (3, 8, 3, 1)$  lies on line  $\ell_{11}$   
 65 :  $P_{1220} = (3, 11, 3, 1)$  lies on line  $\ell_{15}$   
 66 :  $P_{1224} = (7, 11, 3, 1)$  lies on line  $\ell_{10}$   
 67 :  $P_{1240} = (7, 12, 3, 1)$  lies on line  $\ell_9$   
 68 :  $P_{1243} = (10, 12, 3, 1)$  lies on line  $\ell_{13}$   
 69 :  $P_{1294} = (13, 15, 3, 1)$  lies on line  $\ell_{14}$   
 70 :  $P_{1306} = (9, 0, 4, 1)$  lies on line  $\ell_{10}$   
 71 :  $P_{1356} = (11, 3, 4, 1)$  lies on line  $\ell_{13}$   
 72 :  $P_{1375} = (14, 4, 4, 1)$  lies on line  $\ell_{11}$   
 73 :  $P_{1417} = (8, 7, 4, 1)$  lies on line  $\ell_{14}$   
 74 :  $P_{1436} = (11, 8, 4, 1)$  lies on line  $\ell_8$   
 75 :  $P_{1450} = (9, 9, 4, 1)$  lies on line  $\ell_9$   
 76 :  $P_{1471} = (14, 10, 4, 1)$  lies on line  $\ell_{15}$   
 77 :  $P_{1545} = (8, 15, 4, 1)$  lies on line  $\ell_{12}$   
 78 :  $P_{1608} = (7, 3, 5, 1)$  lies on line  $\ell_{11}$   
 79 :  $P_{1624} = (7, 4, 5, 1)$  lies on line  $\ell_{15}$   
 80 :  $P_{1660} = (11, 6, 5, 1)$  lies on line  $\ell_{10}$   
 81 :  $P_{1661} = (12, 6, 5, 1)$  lies on line  $\ell_{12}$   
 82 :  $P_{1718} = (5, 10, 5, 1)$  lies on line  $\ell_8$   
 83 :  $P_{1725} = (12, 10, 5, 1)$  lies on line  $\ell_{14}$   
 84 :  $P_{1772} = (11, 13, 5, 1)$  lies on line  $\ell_9$   
 85 :  $P_{1798} = (5, 15, 5, 1)$  lies on line  $\ell_{13}$   
 86 :  $P_{1838} = (13, 1, 6, 1)$  lies on line  $\ell_9$   
 87 :  $P_{1855} = (14, 2, 6, 1)$  lies on line  $\ell_{13}$   
 88 :  $P_{1974} = (5, 10, 6, 1)$  lies on line  $\ell_{11}$   
 89 :  $P_{2014} = (13, 12, 6, 1)$  lies on line  $\ell_{10}$   
 90 :  $P_{2015} = (14, 12, 6, 1)$  lies on line  $\ell_8$   
 91 :  $P_{2054} = (5, 15, 6, 1)$  lies on line  $\ell_{15}$   
 92 :  $P_{2093} = (12, 1, 7, 1)$  lies on line  $\ell_{15}$   
 93 :  $P_{2160} = (15, 5, 7, 1)$  lies on line  $\ell_9$   
 94 :  $P_{2213} = (4, 9, 7, 1)$  lies on line  $\ell_{14}$   
 95 :  $P_{2240} = (15, 10, 7, 1)$  lies on line  $\ell_{10}$   
 96 :  $P_{2277} = (4, 13, 7, 1)$  lies on line  $\ell_{12}$   
 97 :  $P_{2285} = (12, 13, 7, 1)$  lies on line  $\ell_{11}$   
 98 :  $P_{2377} = (8, 3, 8, 1)$  lies on line  $\ell_{10}$   
 99 :  $P_{2413} = (12, 5, 8, 1)$  lies on line  $\ell_{13}$   
 100 :  $P_{2443} = (10, 7, 8, 1)$  lies on line  $\ell_{12}$   
 101 :  $P_{2477} = (12, 9, 8, 1)$  lies on line  $\ell_8$   
 102 :  $P_{2503} = (6, 11, 8, 1)$  lies on line  $\ell_{11}$   
 103 :  $P_{2505} = (8, 11, 8, 1)$  lies on line  $\ell_9$   
 104 :  $P_{2535} = (6, 13, 8, 1)$  lies on line  $\ell_{15}$   
 105 :  $P_{2539} = (10, 13, 8, 1)$  lies on line  $\ell_{14}$   
 106 :  $P_{2591} = (14, 0, 9, 1)$  lies on line  $\ell_{14}$   
 107 :  $P_{2640} = (15, 3, 9, 1)$  lies on line  $\ell_{15}$

108 :  $P_{2667} = (10, 5, 9, 1)$  lies on line  $\ell_{10}$   
 109 :  $P_{2723} = (2, 9, 9, 1)$  lies on line  $\ell_{13}$   
 110 :  $P_{2755} = (2, 11, 9, 1)$  lies on line  $\ell_8$   
 111 :  $P_{2784} = (15, 12, 9, 1)$  lies on line  $\ell_{11}$   
 112 :  $P_{2815} = (14, 14, 9, 1)$  lies on line  $\ell_{12}$   
 113 :  $P_{2827} = (10, 15, 9, 1)$  lies on line  $\ell_9$   
 114 :  $P_{2853} = (4, 1, 10, 1)$  lies on line  $\ell_3$   
 115 :  $P_{2863} = (14, 1, 10, 1)$  lies on line  $\ell_7$   
 116 :  $P_{2870} = (5, 2, 10, 1)$  lies on line  $\ell_7$   
 117 :  $P_{2873} = (8, 2, 10, 1)$  lies on line  $\ell_3$   
 118 :  $P_{2892} = (11, 3, 10, 1)$  lies on line  $\ell_7$   
 119 :  $P_{2907} = (10, 4, 10, 1)$  lies on line  $\ell_7$   
 120 :  $P_{2917} = (4, 5, 10, 1)$  lies on line  $\ell_7$   
 121 :  $P_{2930} = (1, 6, 10, 1)$  lies on line  $\ell_3$   
 122 :  $P_{2944} = (15, 6, 10, 1)$  lies on line  $\ell_7$   
 123 :  $P_{2946} = (1, 7, 10, 1)$  lies on line  $\ell_7$   
 124 :  $P_{2950} = (5, 7, 10, 1)$  lies on line  $\ell_3$   
 125 :  $P_{2972} = (11, 8, 10, 1)$  lies on line  $\ell_3$   
 126 :  $P_{2980} = (3, 9, 10, 1)$  lies on line  $\ell_7$   
 127 :  $P_{2992} = (15, 9, 10, 1)$  lies on line  $\ell_3$   
 128 :  $P_{2996} = (3, 10, 10, 1)$  lies on line  $\ell_3$   
 129 :  $P_{3001} = (8, 10, 10, 1)$  lies on line  $\ell_7$   
 130 :  $P_{3015} = (6, 11, 10, 1)$  lies on line  $\ell_7$   
 131 :  $P_{3016} = (7, 11, 10, 1)$  lies on line  $\ell_3$   
 132 :  $P_{3032} = (7, 12, 10, 1)$  lies on line  $\ell_7$   
 133 :  $P_{3047} = (6, 13, 10, 1)$  lies on line  $\ell_3$   
 134 :  $P_{3067} = (10, 14, 10, 1)$  lies on line  $\ell_3$   
 135 :  $P_{3087} = (14, 15, 10, 1)$  lies on line  $\ell_3$   
 136 :  $P_{3107} = (2, 1, 11, 1)$  lies on line  $\ell_2$   
 137 :  $P_{3114} = (9, 1, 11, 1)$  lies on line  $\ell_4$   
 138 :  $P_{3132} = (11, 2, 11, 1)$  lies on line  $\ell_4$   
 139 :  $P_{3139} = (2, 3, 11, 1)$  lies on line  $\ell_4$   
 140 :  $P_{3161} = (8, 4, 11, 1)$  lies on line  $\ell_2$   
 141 :  $P_{3168} = (15, 4, 11, 1)$  lies on line  $\ell_4$   
 142 :  $P_{3179} = (10, 5, 11, 1)$  lies on line  $\ell_2$   
 143 :  $P_{3197} = (12, 6, 11, 1)$  lies on line  $\ell_2$   
 144 :  $P_{3214} = (13, 7, 11, 1)$  lies on line  $\ell_4$   
 145 :  $P_{3226} = (9, 8, 11, 1)$  lies on line  $\ell_2$   
 146 :  $P_{3244} = (11, 9, 11, 1)$  lies on line  $\ell_2$   
 147 :  $P_{3261} = (12, 10, 11, 1)$  lies on line  $\ell_4$   
 148 :  $P_{3262} = (13, 10, 11, 1)$  lies on line  $\ell_2$   
 149 :  $P_{3270} = (5, 11, 11, 1)$  lies on line  $\ell_4$   
 150 :  $P_{3280} = (15, 11, 11, 1)$  lies on line  $\ell_2$   
 151 :  $P_{3282} = (1, 12, 11, 1)$  lies on line  $\ell_2$   
 152 :  $P_{3289} = (8, 12, 11, 1)$  lies on line  $\ell_4$   
 153 :  $P_{3298} = (1, 13, 11, 1)$  lies on line  $\ell_4$   
 154 :  $P_{3300} = (3, 13, 11, 1)$  lies on line  $\ell_2$   
 155 :  $P_{3316} = (3, 14, 11, 1)$  lies on line  $\ell_4$   
 156 :  $P_{3318} = (5, 14, 11, 1)$  lies on line  $\ell_2$   
 157 :  $P_{3339} = (10, 15, 11, 1)$  lies on line  $\ell_4$   
 158 :  $P_{3367} = (6, 1, 12, 1)$  lies on line  $\ell_8$   
 159 :  $P_{3463} = (6, 7, 12, 1)$  lies on line  $\ell_{13}$   
 160 :  $P_{3466} = (9, 7, 12, 1)$  lies on line  $\ell_{15}$   
 161 :  $P_{3476} = (3, 8, 12, 1)$  lies on line  $\ell_{12}$

162 :  $P_{3524} = (3, 11, 12, 1)$  lies on line  $\ell_{14}$   
 163 :  $P_{3578} = (9, 14, 12, 1)$  lies on line  $\ell_{11}$   
 164 :  $P_{3624} = (7, 1, 13, 1)$  lies on line  $\ell_{12}$   
 165 :  $P_{3657} = (8, 3, 13, 1)$  lies on line  $\ell_8$   
 166 :  $P_{3667} = (2, 4, 13, 1)$  lies on line  $\ell_{10}$   
 167 :  $P_{3699} = (2, 6, 13, 1)$  lies on line  $\ell_9$   
 168 :  $P_{3704} = (7, 6, 13, 1)$  lies on line  $\ell_{14}$   
 169 :  $P_{3785} = (8, 11, 13, 1)$  lies on line  $\ell_{13}$   
 170 :  $P_{3859} = (2, 0, 14, 1)$  lies on line  $\ell_{11}$   
 171 :  $P_{3891} = (2, 2, 14, 1)$  lies on line  $\ell_{15}$   
 172 :  $P_{3916} = (11, 3, 14, 1)$  lies on line  $\ell_{12}$   
 173 :  $P_{3940} = (3, 5, 14, 1)$  lies on line  $\ell_8$   
 174 :  $P_{3956} = (3, 6, 14, 1)$  lies on line  $\ell_{13}$

175 :  $P_{3996} = (11, 8, 14, 1)$  lies on line  $\ell_{14}$   
 176 :  $P_{4021} = (4, 10, 14, 1)$  lies on line  $\ell_9$   
 177 :  $P_{4085} = (4, 14, 14, 1)$  lies on line  $\ell_{10}$   
 178 :  $P_{4208} = (15, 5, 15, 1)$  lies on line  $\ell_{14}$   
 179 :  $P_{4236} = (11, 7, 15, 1)$  lies on line  $\ell_{11}$   
 180 :  $P_{4238} = (13, 7, 15, 1)$  lies on line  $\ell_8$   
 181 :  $P_{4247} = (6, 8, 15, 1)$  lies on line  $\ell_{10}$   
 182 :  $P_{4286} = (13, 10, 15, 1)$  lies on line  $\ell_{13}$   
 183 :  $P_{4288} = (15, 10, 15, 1)$  lies on line  $\ell_{12}$   
 184 :  $P_{4316} = (11, 12, 15, 1)$  lies on line  $\ell_{15}$   
 185 :  $P_{4343} = (6, 14, 15, 1)$  lies on line  $\ell_9$

The single points on the surface are:

### Points on surface but on no line

The surface has 111 points not on any line:

The points on the surface but not on lines are:

0 : $P_{35} = (0, 1, 1, 0)$	31 : $P_{1609} = (8, 3, 5, 1)$
1 : $P_{78} = (11, 3, 1, 0)$	32 : $P_{1627} = (10, 4, 5, 1)$
2 : $P_{109} = (10, 5, 1, 0)$	33 : $P_{1733} = (4, 11, 5, 1)$
3 : $P_{158} = (11, 8, 1, 0)$	34 : $P_{1737} = (8, 11, 5, 1)$
4 : $P_{180} = (1, 10, 1, 0)$	35 : $P_{1761} = (0, 13, 5, 1)$
5 : $P_{196} = (1, 11, 1, 0)$	36 : $P_{1787} = (10, 14, 5, 1)$
6 : $P_{269} = (10, 15, 1, 0)$	37 : $P_{1791} = (14, 14, 5, 1)$
7 : $P_{830} = (13, 2, 2, 1)$	38 : $P_{1797} = (4, 15, 5, 1)$
8 : $P_{861} = (12, 4, 2, 1)$	39 : $P_{1811} = (2, 0, 6, 1)$
9 : $P_{865} = (0, 5, 2, 1)$	40 : $P_{1836} = (11, 1, 6, 1)$
10 : $P_{925} = (12, 8, 2, 1)$	41 : $P_{1843} = (2, 2, 6, 1)$
11 : $P_{967} = (6, 11, 2, 1)$	42 : $P_{1863} = (6, 3, 6, 1)$
12 : $P_{999} = (6, 13, 2, 1)$	43 : $P_{1869} = (12, 3, 6, 1)$
13 : $P_{1038} = (13, 15, 2, 1)$	44 : $P_{1874} = (1, 4, 6, 1)$
14 : $P_{1050} = (9, 0, 3, 1)$	45 : $P_{1890} = (1, 5, 6, 1)$
15 : $P_{1084} = (11, 2, 3, 1)$	46 : $P_{1895} = (6, 5, 6, 1)$
16 : $P_{1137} = (0, 6, 3, 1)$	47 : $P_{1980} = (11, 10, 6, 1)$
17 : $P_{1171} = (2, 8, 3, 1)$	48 : $P_{2061} = (12, 15, 6, 1)$
18 : $P_{1194} = (9, 9, 3, 1)$	49 : $P_{2074} = (9, 0, 7, 1)$
19 : $P_{1196} = (11, 9, 3, 1)$	50 : $P_{2092} = (11, 1, 7, 1)$
20 : $P_{1203} = (2, 10, 3, 1)$	51 : $P_{2158} = (13, 5, 7, 1)$
21 : $P_{1206} = (5, 10, 3, 1)$	52 : $P_{2200} = (7, 8, 7, 1)$
22 : $P_{1286} = (5, 15, 3, 1)$	53 : $P_{2206} = (13, 8, 7, 1)$
23 : $P_{1352} = (7, 3, 4, 1)$	54 : $P_{2218} = (9, 9, 7, 1)$
24 : $P_{1368} = (7, 4, 4, 1)$	55 : $P_{2236} = (11, 10, 7, 1)$
25 : $P_{1422} = (13, 7, 4, 1)$	56 : $P_{2290} = (1, 14, 7, 1)$
26 : $P_{1425} = (0, 8, 4, 1)$	57 : $P_{2306} = (1, 15, 7, 1)$
27 : $P_{1447} = (6, 9, 4, 1)$	58 : $P_{2312} = (7, 15, 7, 1)$
28 : $P_{1470} = (13, 10, 4, 1)$	59 : $P_{2323} = (2, 0, 8, 1)$
29 : $P_{1543} = (6, 15, 4, 1)$	60 : $P_{2355} = (2, 2, 8, 1)$
30 : $P_{1567} = (14, 0, 5, 1)$	61 : $P_{2364} = (11, 2, 8, 1)$

62 : $P_{2378} = (9, 3, 8, 1)$	87 : $P_{3655} = (6, 3, 13, 1)$
63 : $P_{2416} = (15, 5, 8, 1)$	88 : $P_{3669} = (4, 4, 13, 1)$
64 : $P_{2433} = (0, 7, 8, 1)$	89 : $P_{3687} = (6, 5, 13, 1)$
65 : $P_{2476} = (11, 9, 8, 1)$	90 : $P_{3694} = (13, 5, 13, 1)$
66 : $P_{2490} = (9, 10, 8, 1)$	91 : $P_{3730} = (1, 8, 13, 1)$
67 : $P_{2496} = (15, 10, 8, 1)$	92 : $P_{3742} = (13, 8, 13, 1)$
68 : $P_{2638} = (13, 3, 9, 1)$	93 : $P_{3746} = (1, 9, 13, 1)$
69 : $P_{2669} = (12, 5, 9, 1)$	94 : $P_{3787} = (10, 11, 13, 1)$
70 : $P_{2733} = (12, 9, 9, 1)$	95 : $P_{3896} = (7, 2, 14, 1)$
71 : $P_{2760} = (7, 11, 9, 1)$	96 : $P_{3905} = (0, 3, 14, 1)$
72 : $P_{2776} = (7, 12, 9, 1)$	97 : $P_{3944} = (7, 5, 14, 1)$
73 : $P_{2814} = (13, 14, 9, 1)$	98 : $P_{3965} = (12, 6, 14, 1)$
74 : $P_{2817} = (0, 15, 9, 1)$	99 : $P_{3991} = (6, 8, 14, 1)$
75 : $P_{3359} = (14, 0, 12, 1)$	100 : $P_{4029} = (12, 10, 14, 1)$
76 : $P_{3371} = (10, 1, 12, 1)$	101 : $P_{4087} = (6, 14, 14, 1)$
77 : $P_{3378} = (1, 2, 12, 1)$	102 : $P_{4117} = (4, 0, 15, 1)$
78 : $P_{3394} = (1, 3, 12, 1)$	103 : $P_{4181} = (4, 4, 15, 1)$
79 : $P_{3405} = (12, 3, 12, 1)$	104 : $P_{4187} = (10, 4, 15, 1)$
80 : $P_{3480} = (7, 8, 12, 1)$	105 : $P_{4207} = (14, 5, 15, 1)$
81 : $P_{3531} = (10, 11, 12, 1)$	106 : $P_{4244} = (3, 8, 15, 1)$
82 : $P_{3583} = (14, 14, 12, 1)$	107 : $P_{4292} = (3, 11, 15, 1)$
83 : $P_{3592} = (7, 15, 12, 1)$	108 : $P_{4303} = (14, 11, 15, 1)$
84 : $P_{3597} = (12, 15, 12, 1)$	109 : $P_{4305} = (0, 12, 15, 1)$
85 : $P_{3605} = (4, 0, 13, 1)$	110 : $P_{4347} = (10, 14, 15, 1)$
86 : $P_{3627} = (10, 1, 13, 1)$	

## Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
2	1	0	0	0	1	0	0	1	1	0	1	0	0	1	0	0
3	1	0	0	0	0	0	1	0	1	0	1	1	1	0	0	0
4	1	0	1	0	0	0	0	0	0	1	0	1	1	0	1	1
5	1	0	0	0	0	0	1	0	0	1	0	0	1	1	1	1
6	1	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0
7	1	0	0	1	0	0	0	0	1	0	1	0	0	1	1	1
8	0	1	1	0	0	0	1	1	0	1	0	0	1	1	0	1
9	0	1	1	1	0	1	0	0	1	0	1	0	0	0	1	1
10	0	1	0	0	1	0	1	1	0	1	0	1	0	1	1	0
11	0	1	1	1	0	0	1	0	0	1	0	0	1	1	1	1
12	0	1	0	1	1	0	1	0	1	1	0	0	0	0	1	1
13	0	1	0	1	1	1	0	0	1	0	1	1	0	0	1	0
14	0	1	1	0	0	1	0	1	0	0	1	1	1	1	0	0
15	0	1	0	0	1	1	0	1	1	1	0	1	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$
in point	$P_1$	$P_6$	$P_8$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{18}$

Line 1 intersects

Line	$\ell_0$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_1$	$P_{275}$	$P_{275}$	$P_{291}$	$P_{291}$	$P_{275}$	$P_{291}$	$P_{291}$	$P_{275}$

Line 2 intersects

Line	$\ell_0$	$\ell_4$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{14}$
in point	$P_6$	$P_{3089}$	$P_{3336}$	$P_{3215}$	$P_{3125}$	$P_{3143}$

Line 3 intersects

Line	$\ell_0$	$\ell_7$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$
in point	$P_8$	$P_{2833}$	$P_{2893}$	$P_{2926}$	$P_{3027}$	$P_{2906}$

Line 4 intersects

Line	$\ell_0$	$\ell_2$	$\ell_{10}$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$
in point	$P_{13}$	$P_{3089}$	$P_{3247}$	$P_{3175}$	$P_{3224}$	$P_{3189}$

Line 5 intersects

Line	$\ell_0$	$\ell_6$	$\ell_9$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_{14}$	$P_{530}$	$P_{596}$	$P_{752}$	$P_{726}$	$P_{761}$

Line 6 intersects

Line	$\ell_0$	$\ell_5$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$
in point	$P_{15}$	$P_{530}$	$P_{576}$	$P_{644}$	$P_{633}$	$P_{678}$

Line 7 intersects

Line	$\ell_0$	$\ell_3$	$\ell_8$	$\ell_{10}$	$\ell_{14}$	$\ell_{15}$
in point	$P_{18}$	$P_{2833}$	$P_{3050}$	$P_{3085}$	$P_{3059}$	$P_{2974}$

Line 8 intersects

Line	$\ell_1$	$\ell_2$	$\ell_6$	$\ell_7$	$\ell_9$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$
in point	$P_{275}$	$P_{3336}$	$P_{576}$	$P_{3050}$	$P_{275}$	$P_{275}$	$P_{2289}$	$P_{275}$

Line 9 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_8$	$\ell_{10}$	$\ell_{12}$	$\ell_{15}$
in point	$P_{275}$	$P_{3215}$	$P_{2893}$	$P_{596}$	$P_{275}$	$P_{3377}$	$P_{275}$	$P_{275}$

Line 10 intersects

Line	$\ell_1$	$\ell_4$	$\ell_6$	$\ell_7$	$\ell_9$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$
in point	$P_{291}$	$P_{3247}$	$P_{644}$	$P_{3085}$	$P_{3377}$	$P_{291}$	$P_{291}$	$P_{291}$

Line 11 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_6$	$\ell_{10}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_{291}$	$P_{3125}$	$P_{2926}$	$P_{633}$	$P_{291}$	$P_{291}$	$P_{291}$	$P_{3745}$

Line 12 intersects

Line	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_6$	$\ell_8$	$\ell_9$	$\ell_{14}$	$\ell_{15}$
in point	$P_{275}$	$P_{3027}$	$P_{3175}$	$P_{678}$	$P_{275}$	$P_{275}$	$P_{1873}$	$P_{275}$

Line 13 intersects

Line	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$
in point	$P_{291}$	$P_{2906}$	$P_{3224}$	$P_{752}$	$P_{2289}$	$P_{291}$	$P_{291}$	$P_{291}$

Line 14 intersects

Line	$\ell_1$	$\ell_2$	$\ell_5$	$\ell_7$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$
in point	$P_{291}$	$P_{3143}$	$P_{726}$	$P_{3059}$	$P_{291}$	$P_{291}$	$P_{1873}$	$P_{291}$

Line 15 intersects

Line	$\ell_1$	$\ell_4$	$\ell_5$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{12}$
in point	$P_{275}$	$P_{3189}$	$P_{761}$	$P_{2974}$	$P_{275}$	$P_{275}$	$P_{3745}$	$P_{275}$

The surface has 337 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	54 : $P_{584} = (7, 3, 1, 1)$	108 : $P_{1243} = (10, 12, 3, 1)$
1 : $P_1 = (0, 1, 0, 0)$	55 : $P_{596} = (3, 4, 1, 1)$	109 : $P_{1286} = (5, 15, 3, 1)$
2 : $P_5 = (1, 1, 0, 0)$	56 : $P_{600} = (7, 4, 1, 1)$	110 : $P_{1294} = (13, 15, 3, 1)$
3 : $P_6 = (2, 1, 0, 0)$	57 : $P_{618} = (9, 5, 1, 1)$	111 : $P_{1306} = (9, 0, 4, 1)$
4 : $P_7 = (3, 1, 0, 0)$	58 : $P_{621} = (12, 5, 1, 1)$	112 : $P_{1352} = (7, 3, 4, 1)$
5 : $P_8 = (4, 1, 0, 0)$	59 : $P_{633} = (8, 6, 1, 1)$	113 : $P_{1356} = (11, 3, 4, 1)$
6 : $P_9 = (5, 1, 0, 0)$	60 : $P_{639} = (14, 6, 1, 1)$	114 : $P_{1368} = (7, 4, 4, 1)$
7 : $P_{10} = (6, 1, 0, 0)$	61 : $P_{644} = (3, 7, 1, 1)$	115 : $P_{1375} = (14, 4, 4, 1)$
8 : $P_{11} = (7, 1, 0, 0)$	62 : $P_{645} = (4, 7, 1, 1)$	116 : $P_{1417} = (8, 7, 4, 1)$
9 : $P_{12} = (8, 1, 0, 0)$	63 : $P_{663} = (6, 8, 1, 1)$	117 : $P_{1422} = (13, 7, 4, 1)$
10 : $P_{13} = (9, 1, 0, 0)$	64 : $P_{671} = (14, 8, 1, 1)$	118 : $P_{1425} = (0, 8, 4, 1)$
11 : $P_{14} = (10, 1, 0, 0)$	65 : $P_{678} = (5, 9, 1, 1)$	119 : $P_{1436} = (11, 8, 4, 1)$
12 : $P_{15} = (11, 1, 0, 0)$	66 : $P_{685} = (12, 9, 1, 1)$	120 : $P_{1447} = (6, 9, 4, 1)$
13 : $P_{16} = (12, 1, 0, 0)$	67 : $P_{690} = (1, 10, 1, 1)$	121 : $P_{1450} = (9, 9, 4, 1)$
14 : $P_{17} = (13, 1, 0, 0)$	68 : $P_{700} = (11, 10, 1, 1)$	122 : $P_{1470} = (13, 10, 4, 1)$
15 : $P_{18} = (14, 1, 0, 0)$	69 : $P_{706} = (1, 11, 1, 1)$	123 : $P_{1471} = (14, 10, 4, 1)$
16 : $P_{19} = (15, 1, 0, 0)$	70 : $P_{715} = (10, 11, 1, 1)$	124 : $P_{1543} = (6, 15, 4, 1)$
17 : $P_{35} = (0, 1, 1, 0)$	71 : $P_{726} = (5, 12, 1, 1)$	125 : $P_{1545} = (8, 15, 4, 1)$
18 : $P_{65} = (14, 2, 1, 0)$	72 : $P_{730} = (9, 12, 1, 1)$	126 : $P_{1567} = (14, 0, 5, 1)$
19 : $P_{78} = (11, 3, 1, 0)$	73 : $P_{739} = (2, 13, 1, 1)$	127 : $P_{1608} = (7, 3, 5, 1)$
20 : $P_{85} = (2, 4, 1, 0)$	74 : $P_{752} = (15, 13, 1, 1)$	128 : $P_{1609} = (8, 3, 5, 1)$
21 : $P_{109} = (10, 5, 1, 0)$	75 : $P_{759} = (6, 14, 1, 1)$	129 : $P_{1624} = (7, 4, 5, 1)$
22 : $P_{117} = (2, 6, 1, 0)$	76 : $P_{761} = (8, 14, 1, 1)$	130 : $P_{1627} = (10, 4, 5, 1)$
23 : $P_{140} = (9, 7, 1, 0)$	77 : $P_{771} = (2, 15, 1, 1)$	131 : $P_{1660} = (11, 6, 5, 1)$
24 : $P_{158} = (11, 8, 1, 0)$	78 : $P_{782} = (13, 15, 1, 1)$	132 : $P_{1661} = (12, 6, 5, 1)$
25 : $P_{167} = (4, 9, 1, 0)$	79 : $P_{789} = (4, 0, 2, 1)$	133 : $P_{1718} = (5, 10, 5, 1)$
26 : $P_{180} = (1, 10, 1, 0)$	80 : $P_{826} = (9, 2, 2, 1)$	134 : $P_{1725} = (12, 10, 5, 1)$
27 : $P_{196} = (1, 11, 1, 0)$	81 : $P_{830} = (13, 2, 2, 1)$	135 : $P_{1733} = (4, 11, 5, 1)$
28 : $P_{225} = (14, 12, 1, 0)$	82 : $P_{853} = (4, 4, 2, 1)$	136 : $P_{1737} = (8, 11, 5, 1)$
29 : $P_{231} = (4, 13, 1, 0)$	83 : $P_{861} = (12, 4, 2, 1)$	137 : $P_{1761} = (0, 13, 5, 1)$
30 : $P_{252} = (9, 14, 1, 0)$	84 : $P_{865} = (0, 5, 2, 1)$	138 : $P_{1772} = (11, 13, 5, 1)$
31 : $P_{269} = (10, 15, 1, 0)$	85 : $P_{875} = (10, 5, 2, 1)$	139 : $P_{1787} = (10, 14, 5, 1)$
32 : $P_{275} = (1, 0, 0, 1)$	86 : $P_{918} = (5, 8, 2, 1)$	140 : $P_{1791} = (14, 14, 5, 1)$
33 : $P_{291} = (1, 1, 0, 1)$	87 : $P_{925} = (12, 8, 2, 1)$	141 : $P_{1797} = (4, 15, 5, 1)$
34 : $P_{307} = (1, 2, 0, 1)$	88 : $P_{967} = (6, 11, 2, 1)$	142 : $P_{1798} = (5, 15, 5, 1)$
35 : $P_{323} = (1, 3, 0, 1)$	89 : $P_{970} = (9, 11, 2, 1)$	143 : $P_{1811} = (2, 0, 6, 1)$
36 : $P_{339} = (1, 4, 0, 1)$	90 : $P_{998} = (5, 13, 2, 1)$	144 : $P_{1836} = (11, 1, 6, 1)$
37 : $P_{355} = (1, 5, 0, 1)$	91 : $P_{999} = (6, 13, 2, 1)$	145 : $P_{1838} = (13, 1, 6, 1)$
38 : $P_{371} = (1, 6, 0, 1)$	92 : $P_{1035} = (10, 15, 2, 1)$	146 : $P_{1843} = (2, 2, 6, 1)$
39 : $P_{387} = (1, 7, 0, 1)$	93 : $P_{1038} = (13, 15, 2, 1)$	147 : $P_{1855} = (14, 2, 6, 1)$
40 : $P_{403} = (1, 8, 0, 1)$	94 : $P_{1050} = (9, 0, 3, 1)$	148 : $P_{1863} = (6, 3, 6, 1)$
41 : $P_{419} = (1, 9, 0, 1)$	95 : $P_{1084} = (11, 2, 3, 1)$	149 : $P_{1869} = (12, 3, 6, 1)$
42 : $P_{435} = (1, 10, 0, 1)$	96 : $P_{1086} = (13, 2, 3, 1)$	150 : $P_{1873} = (0, 4, 6, 1)$
43 : $P_{451} = (1, 11, 0, 1)$	97 : $P_{1137} = (0, 6, 3, 1)$	151 : $P_{1874} = (1, 4, 6, 1)$
44 : $P_{467} = (1, 12, 0, 1)$	98 : $P_{1147} = (10, 6, 3, 1)$	152 : $P_{1890} = (1, 5, 6, 1)$
45 : $P_{483} = (1, 13, 0, 1)$	99 : $P_{1171} = (2, 8, 3, 1)$	153 : $P_{1895} = (6, 5, 6, 1)$
46 : $P_{499} = (1, 14, 0, 1)$	100 : $P_{1172} = (3, 8, 3, 1)$	154 : $P_{1974} = (5, 10, 6, 1)$
47 : $P_{515} = (1, 15, 0, 1)$	101 : $P_{1194} = (9, 9, 3, 1)$	155 : $P_{1980} = (11, 10, 6, 1)$
48 : $P_{530} = (0, 0, 1, 1)$	102 : $P_{1196} = (11, 9, 3, 1)$	156 : $P_{2014} = (13, 12, 6, 1)$
49 : $P_{555} = (10, 1, 1, 1)$	103 : $P_{1203} = (2, 10, 3, 1)$	157 : $P_{2015} = (14, 12, 6, 1)$
50 : $P_{556} = (11, 1, 1, 1)$	104 : $P_{1206} = (5, 10, 3, 1)$	158 : $P_{2054} = (5, 15, 6, 1)$
51 : $P_{574} = (13, 2, 1, 1)$	105 : $P_{1220} = (3, 11, 3, 1)$	159 : $P_{2061} = (12, 15, 6, 1)$
52 : $P_{576} = (15, 2, 1, 1)$	106 : $P_{1224} = (7, 11, 3, 1)$	160 : $P_{2074} = (9, 0, 7, 1)$
53 : $P_{581} = (4, 3, 1, 1)$	107 : $P_{1240} = (7, 12, 3, 1)$	161 : $P_{2092} = (11, 1, 7, 1)$



162 : $P_{2093} = (12, 1, 7, 1)$	216 : $P_{2906} = (9, 4, 10, 1)$	270 : $P_{3339} = (10, 15, 11, 1)$
163 : $P_{2158} = (13, 5, 7, 1)$	217 : $P_{2907} = (10, 4, 10, 1)$	271 : $P_{3359} = (14, 0, 12, 1)$
164 : $P_{2160} = (15, 5, 7, 1)$	218 : $P_{2917} = (4, 5, 10, 1)$	272 : $P_{3367} = (6, 1, 12, 1)$
165 : $P_{2200} = (7, 8, 7, 1)$	219 : $P_{2926} = (13, 5, 10, 1)$	273 : $P_{3371} = (10, 1, 12, 1)$
166 : $P_{2206} = (13, 8, 7, 1)$	220 : $P_{2930} = (1, 6, 10, 1)$	274 : $P_{3377} = (0, 2, 12, 1)$
167 : $P_{2213} = (4, 9, 7, 1)$	221 : $P_{2944} = (15, 6, 10, 1)$	275 : $P_{3378} = (1, 2, 12, 1)$
168 : $P_{2218} = (9, 9, 7, 1)$	222 : $P_{2946} = (1, 7, 10, 1)$	276 : $P_{3394} = (1, 3, 12, 1)$
169 : $P_{2236} = (11, 10, 7, 1)$	223 : $P_{2950} = (5, 7, 10, 1)$	277 : $P_{3405} = (12, 3, 12, 1)$
170 : $P_{2240} = (15, 10, 7, 1)$	224 : $P_{2972} = (11, 8, 10, 1)$	278 : $P_{3463} = (6, 7, 12, 1)$
171 : $P_{2277} = (4, 13, 7, 1)$	225 : $P_{2974} = (13, 8, 10, 1)$	279 : $P_{3466} = (9, 7, 12, 1)$
172 : $P_{2285} = (12, 13, 7, 1)$	226 : $P_{2980} = (3, 9, 10, 1)$	280 : $P_{3476} = (3, 8, 12, 1)$
173 : $P_{2289} = (0, 14, 7, 1)$	227 : $P_{2992} = (15, 9, 10, 1)$	281 : $P_{3480} = (7, 8, 12, 1)$
174 : $P_{2290} = (1, 14, 7, 1)$	228 : $P_{2996} = (3, 10, 10, 1)$	282 : $P_{3524} = (3, 11, 12, 1)$
175 : $P_{2306} = (1, 15, 7, 1)$	229 : $P_{3001} = (8, 10, 10, 1)$	283 : $P_{3531} = (10, 11, 12, 1)$
176 : $P_{2312} = (7, 15, 7, 1)$	230 : $P_{3015} = (6, 11, 10, 1)$	284 : $P_{3578} = (9, 14, 12, 1)$
177 : $P_{2323} = (2, 0, 8, 1)$	231 : $P_{3016} = (7, 11, 10, 1)$	285 : $P_{3583} = (14, 14, 12, 1)$
178 : $P_{2355} = (2, 2, 8, 1)$	232 : $P_{3027} = (2, 12, 10, 1)$	286 : $P_{3592} = (7, 15, 12, 1)$
179 : $P_{2364} = (11, 2, 8, 1)$	233 : $P_{3032} = (7, 12, 10, 1)$	287 : $P_{3597} = (12, 15, 12, 1)$
180 : $P_{2377} = (8, 3, 8, 1)$	234 : $P_{3047} = (6, 13, 10, 1)$	288 : $P_{3605} = (4, 0, 13, 1)$
181 : $P_{2378} = (9, 3, 8, 1)$	235 : $P_{3050} = (9, 13, 10, 1)$	289 : $P_{3624} = (7, 1, 13, 1)$
182 : $P_{2413} = (12, 5, 8, 1)$	236 : $P_{3059} = (2, 14, 10, 1)$	290 : $P_{3627} = (10, 1, 13, 1)$
183 : $P_{2416} = (15, 5, 8, 1)$	237 : $P_{3067} = (10, 14, 10, 1)$	291 : $P_{3655} = (6, 3, 13, 1)$
184 : $P_{2433} = (0, 7, 8, 1)$	238 : $P_{3085} = (12, 15, 10, 1)$	292 : $P_{3657} = (8, 3, 13, 1)$
185 : $P_{2443} = (10, 7, 8, 1)$	239 : $P_{3087} = (14, 15, 10, 1)$	293 : $P_{3667} = (2, 4, 13, 1)$
186 : $P_{2476} = (11, 9, 8, 1)$	240 : $P_{3089} = (0, 0, 11, 1)$	294 : $P_{3669} = (4, 4, 13, 1)$
187 : $P_{2477} = (12, 9, 8, 1)$	241 : $P_{3107} = (2, 1, 11, 1)$	295 : $P_{3687} = (6, 5, 13, 1)$
188 : $P_{2490} = (9, 10, 8, 1)$	242 : $P_{3114} = (9, 1, 11, 1)$	296 : $P_{3694} = (13, 5, 13, 1)$
189 : $P_{2496} = (15, 10, 8, 1)$	243 : $P_{3125} = (4, 2, 11, 1)$	297 : $P_{3699} = (2, 6, 13, 1)$
190 : $P_{2503} = (6, 11, 8, 1)$	244 : $P_{3132} = (11, 2, 11, 1)$	298 : $P_{3704} = (7, 6, 13, 1)$
191 : $P_{2505} = (8, 11, 8, 1)$	245 : $P_{3139} = (2, 3, 11, 1)$	299 : $P_{3730} = (1, 8, 13, 1)$
192 : $P_{2535} = (6, 13, 8, 1)$	246 : $P_{3143} = (6, 3, 11, 1)$	300 : $P_{3742} = (13, 8, 13, 1)$
193 : $P_{2539} = (10, 13, 8, 1)$	247 : $P_{3161} = (8, 4, 11, 1)$	301 : $P_{3745} = (0, 9, 13, 1)$
194 : $P_{2591} = (14, 0, 9, 1)$	248 : $P_{3168} = (15, 4, 11, 1)$	302 : $P_{3746} = (1, 9, 13, 1)$
195 : $P_{2638} = (13, 3, 9, 1)$	249 : $P_{3175} = (6, 5, 11, 1)$	303 : $P_{3785} = (8, 11, 13, 1)$
196 : $P_{2640} = (15, 3, 9, 1)$	250 : $P_{3179} = (10, 5, 11, 1)$	304 : $P_{3787} = (10, 11, 13, 1)$
197 : $P_{2667} = (10, 5, 9, 1)$	251 : $P_{3189} = (4, 6, 11, 1)$	305 : $P_{3859} = (2, 0, 14, 1)$
198 : $P_{2669} = (12, 5, 9, 1)$	252 : $P_{3197} = (12, 6, 11, 1)$	306 : $P_{3891} = (2, 2, 14, 1)$
199 : $P_{2723} = (2, 9, 9, 1)$	253 : $P_{3214} = (13, 7, 11, 1)$	307 : $P_{3896} = (7, 2, 14, 1)$
200 : $P_{2733} = (12, 9, 9, 1)$	254 : $P_{3215} = (14, 7, 11, 1)$	308 : $P_{3905} = (0, 3, 14, 1)$
201 : $P_{2755} = (2, 11, 9, 1)$	255 : $P_{3224} = (7, 8, 11, 1)$	309 : $P_{3916} = (11, 3, 14, 1)$
202 : $P_{2760} = (7, 11, 9, 1)$	256 : $P_{3226} = (9, 8, 11, 1)$	310 : $P_{3940} = (3, 5, 14, 1)$
203 : $P_{2776} = (7, 12, 9, 1)$	257 : $P_{3244} = (11, 9, 11, 1)$	311 : $P_{3944} = (7, 5, 14, 1)$
204 : $P_{2784} = (15, 12, 9, 1)$	258 : $P_{3247} = (14, 9, 11, 1)$	312 : $P_{3956} = (3, 6, 14, 1)$
205 : $P_{2814} = (13, 14, 9, 1)$	259 : $P_{3261} = (12, 10, 11, 1)$	313 : $P_{3965} = (12, 6, 14, 1)$
206 : $P_{2815} = (14, 14, 9, 1)$	260 : $P_{3262} = (13, 10, 11, 1)$	314 : $P_{3991} = (6, 8, 14, 1)$
207 : $P_{2817} = (0, 15, 9, 1)$	261 : $P_{3270} = (5, 11, 11, 1)$	315 : $P_{3996} = (11, 8, 14, 1)$
208 : $P_{2827} = (10, 15, 9, 1)$	262 : $P_{3280} = (15, 11, 11, 1)$	316 : $P_{4021} = (4, 10, 14, 1)$
209 : $P_{2833} = (0, 0, 10, 1)$	263 : $P_{3282} = (1, 12, 11, 1)$	317 : $P_{4029} = (12, 10, 14, 1)$
210 : $P_{2853} = (4, 1, 10, 1)$	264 : $P_{3289} = (8, 12, 11, 1)$	318 : $P_{4085} = (4, 14, 14, 1)$
211 : $P_{2863} = (14, 1, 10, 1)$	265 : $P_{3298} = (1, 13, 11, 1)$	319 : $P_{4087} = (6, 14, 14, 1)$
212 : $P_{2870} = (5, 2, 10, 1)$	266 : $P_{3300} = (3, 13, 11, 1)$	320 : $P_{4117} = (4, 0, 15, 1)$
213 : $P_{2873} = (8, 2, 10, 1)$	267 : $P_{3316} = (3, 14, 11, 1)$	321 : $P_{4181} = (4, 4, 15, 1)$
214 : $P_{2892} = (11, 3, 10, 1)$	268 : $P_{3318} = (5, 14, 11, 1)$	322 : $P_{4187} = (10, 4, 15, 1)$
215 : $P_{2893} = (12, 3, 10, 1)$	269 : $P_{3336} = (7, 15, 11, 1)$	323 : $P_{4207} = (14, 5, 15, 1)$

324 :  $P_{4208} = (15, 5, 15, 1)$   
 325 :  $P_{4236} = (11, 7, 15, 1)$   
 326 :  $P_{4238} = (13, 7, 15, 1)$   
 327 :  $P_{4244} = (3, 8, 15, 1)$   
 328 :  $P_{4247} = (6, 8, 15, 1)$

329 :  $P_{4286} = (13, 10, 15, 1)$   
 330 :  $P_{4288} = (15, 10, 15, 1)$   
 331 :  $P_{4292} = (3, 11, 15, 1)$   
 332 :  $P_{4303} = (14, 11, 15, 1)$   
 333 :  $P_{4305} = (0, 12, 15, 1)$

334 :  $P_{4316} = (11, 12, 15, 1)$   
 335 :  $P_{4343} = (6, 14, 15, 1)$   
 336 :  $P_{4347} = (10, 14, 15, 1)$