Rank-74057 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 1244172949

General information

Number of lines	3
Number of points	89
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$3, 1^{24}, 0^{64}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \mathbf{Pl}(0,6,0,1,0,0)_{30}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \mathbf{Pl}(0,3,0,1,0,0)_{27}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \mathbf{Pl}(0,5,0,1,0,0)_{29}$$

Rank of lines: (4734, 4707, 4725)
Rank of points on Klein quadric: (30, 27, 29)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_{27} = (0,2,1,0)$ lies on line ℓ_0
$1 : P_{43} = (0,4,1,0)$ lies on line ℓ_1
$2 : P_{67} = (0,7,1,0)$ lies on line ℓ_2
$3 : P_{153} = (0,2,1,1)$ lies on line ℓ_0
$4 : P_{169} = (0,4,1,1)$ lies on line ℓ_1
$5 : P_{193} = (0,7,1,1)$ lies on line ℓ_2
$6 : P_{225} = (0,3,2,1)$ lies on line ℓ_2
$7 : P_{233} = (0,4,2,1)$ lies on line ℓ_0
$8 : P_{241} = (0,5,2,1)$ lies on line ℓ_1
$9 : P_{273} = (0,1,3,1)$ lies on line ℓ_1
$10 : P_{297} = (0,4,3,1)$ lies on line ℓ_2
$11 : P_{313} = (0,6,3,1)$ lies on line ℓ_0
$12 : P_{369} = (0,5,4,1)$ lies on line ℓ_0

$13 : P_{377} = (0,6,4,1)$ lies on line ℓ_2
$14 : P_{385} = (0,7,4,1)$ lies on line ℓ_1
$15 : P_{401} = (0,1,5,1)$ lies on line ℓ_2
$16 : P_{417} = (0,3,5,1)$ lies on line ℓ_1
$17 : P_{449} = (0,7,5,1)$ lies on line ℓ_0
$18 : P_{465} = (0,1,6,1)$ lies on line ℓ_0
$19 : P_{473} = (0,2,6,1)$ lies on line ℓ_1
$20 : P_{497} = (0,5,6,1)$ lies on line ℓ_2
$21 : P_{537} = (0,2,7,1)$ lies on line ℓ_2
$22 : P_{545} = (0,3,7,1)$ lies on line ℓ_0
$23 : P_{569} = (0,6,7,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:
The points on the surface but not on lines are:

$0 : P_0 = (1,0,0,0)$
$1 : P_4 = (1,1,1,1)$
$2 : P_{20} = (1,1,1,0)$
$3 : P_{37} = (2,3,1,0)$
$4 : P_{55} = (4,5,1,0)$
$5 : P_{66} = (7,6,1,0)$

$6 : P_{75} = (1,0,0,1)$
$7 : P_{96} = (6,2,0,1)$
$8 : P_{97} = (7,2,0,1)$
$9 : P_{108} = (2,4,0,1)$
$10 : P_{109} = (3,4,0,1)$
$11 : P_{134} = (4,7,0,1)$

$12 : P_{135} = (5, 7, 0, 1)$
$13 : P_{156} = (3, 2, 1, 1)$
$14 : P_{174} = (5, 4, 1, 1)$
$15 : P_{199} = (6, 7, 1, 1)$
$16 : P_{207} = (6, 0, 2, 1)$
$17 : P_{208} = (7, 0, 2, 1)$
$18 : P_{213} = (4, 1, 2, 1)$
$19 : P_{216} = (7, 1, 2, 1)$
$20 : P_{232} = (7, 3, 2, 1)$
$21 : P_{237} = (4, 4, 2, 1)$
$22 : P_{247} = (6, 5, 2, 1)$
$23 : P_{251} = (2, 6, 2, 1)$
$24 : P_{261} = (4, 7, 2, 1)$
$25 : P_{263} = (6, 7, 2, 1)$
$26 : P_{275} = (2, 1, 3, 1)$
$27 : P_{282} = (1, 2, 3, 1)$
$28 : P_{287} = (6, 2, 3, 1)$
$29 : P_{319} = (6, 6, 3, 1)$
$30 : P_{324} = (3, 7, 3, 1)$
$31 : P_{327} = (6, 7, 3, 1)$
$32 : P_{331} = (2, 0, 4, 1)$
$33 : P_{332} = (3, 0, 4, 1)$
$34 : P_{339} = (2, 1, 4, 1)$
$35 : P_{344} = (7, 1, 4, 1)$
$36 : P_{348} = (3, 2, 4, 1)$
$37 : P_{352} = (7, 2, 4, 1)$
$38 : P_{357} = (4, 3, 4, 1)$

$39 : P_{371} = (2, 5, 4, 1)$
$40 : P_{380} = (3, 6, 4, 1)$
$41 : P_{392} = (7, 7, 4, 1)$
$42 : P_{405} = (4, 1, 5, 1)$
$43 : P_{412} = (3, 2, 5, 1)$
$44 : P_{414} = (5, 2, 5, 1)$
$45 : P_{420} = (3, 3, 5, 1)$
$46 : P_{426} = (1, 4, 5, 1)$
$47 : P_{428} = (3, 4, 5, 1)$
$48 : P_{472} = (7, 1, 6, 1)$
$49 : P_{494} = (5, 4, 6, 1)$
$50 : P_{495} = (6, 4, 6, 1)$
$51 : P_{502} = (5, 5, 6, 1)$
$52 : P_{514} = (1, 7, 6, 1)$
$53 : P_{518} = (5, 7, 6, 1)$
$54 : P_{525} = (4, 0, 7, 1)$
$55 : P_{526} = (5, 0, 7, 1)$
$56 : P_{531} = (2, 1, 7, 1)$
$57 : P_{533} = (4, 1, 7, 1)$
$58 : P_{539} = (2, 2, 7, 1)$
$59 : P_{550} = (5, 3, 7, 1)$
$60 : P_{555} = (2, 4, 7, 1)$
$61 : P_{558} = (5, 4, 7, 1)$
$62 : P_{568} = (7, 5, 7, 1)$
$63 : P_{573} = (4, 6, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 89 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
$1 : P_3 = (0, 0, 0, 1)$
$2 : P_4 = (1, 1, 1, 1)$
$3 : P_{20} = (1, 1, 1, 0)$
$4 : P_{27} = (0, 2, 1, 0)$

$5 : P_{37} = (2, 3, 1, 0)$
$6 : P_{43} = (0, 4, 1, 0)$
$7 : P_{55} = (4, 5, 1, 0)$
$8 : P_{66} = (7, 6, 1, 0)$
$9 : P_{67} = (0, 7, 1, 0)$

$10 : P_{75} = (1, 0, 0, 1)$
$11 : P_{96} = (6, 2, 0, 1)$
$12 : P_{97} = (7, 2, 0, 1)$
$13 : P_{108} = (2, 4, 0, 1)$
$14 : P_{109} = (3, 4, 0, 1)$

3

$15 : P_{134} = (4, 7, 0, 1)$

$16 : P_{135} = (5, 7, 0, 1)$

$17 : P_{153} = (0, 2, 1, 1)$

$18 : P_{156} = (3, 2, 1, 1)$

$19 : P_{169} = (0, 4, 1, 1)$

$20 : P_{174} = (5, 4, 1, 1)$

$21 : P_{193} = (0, 7, 1, 1)$

$22 : P_{199} = (6, 7, 1, 1)$

$23 : P_{207} = (6, 0, 2, 1)$

$24 : P_{208} = (7, 0, 2, 1)$

$25 : P_{213} = (4, 1, 2, 1)$

$26 : P_{216} = (7, 1, 2, 1)$

$27 : P_{225} = (0, 3, 2, 1)$

$28 : P_{232} = (7, 3, 2, 1)$

$29 : P_{233} = (0, 4, 2, 1)$

$30 : P_{237} = (4, 4, 2, 1)$

$31 : P_{241} = (0, 5, 2, 1)$

$32 : P_{247} = (6, 5, 2, 1)$

$33 : P_{251} = (2, 6, 2, 1)$

$34 : P_{261} = (4, 7, 2, 1)$

$35 : P_{263} = (6, 7, 2, 1)$

$36 : P_{273} = (0, 1, 3, 1)$

$37 : P_{275} = (2, 1, 3, 1)$

$38 : P_{282} = (1, 2, 3, 1)$

$39 : P_{287} = (6, 2, 3, 1)$

$40 : P_{297} = (0, 4, 3, 1)$

$41 : P_{313} = (0, 6, 3, 1)$

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$43 : P_{324} = (3, 7, 3, 1)$

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$45 : P_{331} = (2, 0, 4, 1)$

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$50 : P_{352} = (7, 2, 4, 1)$

$51 : P_{357} = (4, 3, 4, 1)$

$52 : P_{369} = (0, 5, 4, 1)$

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$54 : P_{377} = (0, 6, 4, 1)$

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$56 : P_{385} = (0, 7, 4, 1)$

$57 : P_{392} = (7, 7, 4, 1)$

$58 : P_{401} = (0, 1, 5, 1)$

$59 : P_{405} = (4, 1, 5, 1)$

$60 : P_{412} = (3, 2, 5, 1)$

$61 : P_{414} = (5, 2, 5, 1)$

$62 : P_{417} = (0, 3, 5, 1)$

$63 : P_{420} = (3, 3, 5, 1)$

$64 : P_{426} = (1, 4, 5, 1)$

$65 : P_{428} = (3, 4, 5, 1)$

$66 : P_{449} = (0, 7, 5, 1)$

$67 : P_{465} = (0, 1, 6, 1)$

$68 : P_{472} = (7, 1, 6, 1)$

$69 : P_{473} = (0, 2, 6, 1)$

$70 : P_{494} = (5, 4, 6, 1)$

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$72 : P_{497} = (0, 5, 6, 1)$

$73 : P_{502} = (5, 5, 6, 1)$

$74 : P_{514} = (1, 7, 6, 1)$

$75 : P_{518} = (5, 7, 6, 1)$

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$82 : P_{545} = (0, 3, 7, 1)$

$83 : P_{550} = (5, 3, 7, 1)$

$84 : P_{555} = (2, 4, 7, 1)$

$85 : P_{558} = (5, 4, 7, 1)$

$86 : P_{568} = (7, 5, 7, 1)$

$87 : P_{569} = (0, 6, 7, 1)$

$88 : P_{573} = (4, 6, 7, 1)$