Rank-65612 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 1227396174

General information

Number of lines	7
Number of points	89
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	49
Number of points off lines	35
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^7
Type of lines on points	$4^2, 2^3, 1^{49}, 0^{35}$

Singular Points

The surface has 2 singular points:

$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{3504} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{3504} = \mathbf{Pl}(2,0,0,1,0,0)_{19}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \mathbf{Pl}(4,0,0,1,0,0)_{21}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \mathbf{Pl}(7,0,0,1,0,0)_{24}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \mathbf{Pl}(0,6,1,0,0,0)_{15}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \mathbf{Pl}(0,3,1,0,0,0)_{12}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \mathbf{Pl}(0,5,1,0,0,0)_{14}$$

Rank of lines: (4672, 3504, 1752, 2920, 3568, 1816, 2984)
Rank of points on Klein quadric: (649, 19, 21, 24, 15, 12, 14)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{76} = (2,0,0,1) = \ell_1 \cap \ell_4$ $P_{81} = (7,0,0,1) = \ell_3 \cap \ell_6$
$P_{78} = (4,0,0,1) = \ell_2 \cap \ell_5$

Single Points

The surface has 49 single points:
The single points on the surface are:

$0 : P_{19} = (0,1,1,0)$ lies on line ℓ_0 $15 : P_{105} = (7,3,0,1)$ lies on line ℓ_3
$1 : P_{27} = (0,2,1,0)$ lies on line ℓ_0 $16 : P_{108} = (2,4,0,1)$ lies on line ℓ_1
$2 : P_{35} = (0,3,1,0)$ lies on line ℓ_0 $17 : P_{110} = (4,4,0,1)$ lies on line ℓ_2
$3 : P_{43} = (0,4,1,0)$ lies on line ℓ_0 $18 : P_{113} = (7,4,0,1)$ lies on line ℓ_3
$4 : P_{51} = (0,5,1,0)$ lies on line ℓ_0 $19 : P_{116} = (2,5,0,1)$ lies on line ℓ_1
$5 : P_{59} = (0,6,1,0)$ lies on line ℓ_0 $20 : P_{118} = (4,5,0,1)$ lies on line ℓ_2
$6 : P_{67} = (0,7,1,0)$ lies on line ℓ_0 $21 : P_{121} = (7,5,0,1)$ lies on line ℓ_3
$7 : P_{84} = (2,1,0,1)$ lies on line ℓ_1 $22 : P_{124} = (2,6,0,1)$ lies on line ℓ_1
$8 : P_{86} = (4,1,0,1)$ lies on line ℓ_2 $23 : P_{126} = (4,6,0,1)$ lies on line ℓ_2
$9 : P_{89} = (7,1,0,1)$ lies on line ℓ_3 $24 : P_{129} = (7,6,0,1)$ lies on line ℓ_3
$10 : P_{92} = (2,2,0,1)$ lies on line ℓ_1 $25 : P_{132} = (2,7,0,1)$ lies on line ℓ_1
$11 : P_{94} = (4,2,0,1)$ lies on line ℓ_2 $26 : P_{134} = (4,7,0,1)$ lies on line ℓ_2
$12 : P_{97} = (7,2,0,1)$ lies on line ℓ_3 $27 : P_{137} = (7,7,0,1)$ lies on line ℓ_3
$13 : P_{100} = (2,3,0,1)$ lies on line ℓ_1 $28 : P_{140} = (2,0,1,1)$ lies on line ℓ_4
$14 : P_{102} = (4,3,0,1)$ lies on line ℓ_2 $29 : P_{142} = (4,0,1,1)$ lies on line ℓ_5

30 : $P_{145} = (7,0,1,1)$ lies on line ℓ_6
31 : $P_{203} = (2,0,2,1)$ lies on line ℓ_4
32 : $P_{205} = (4,0,2,1)$ lies on line ℓ_5
33 : $P_{208} = (7,0,2,1)$ lies on line ℓ_6
34 : $P_{267} = (2,0,3,1)$ lies on line ℓ_4
35 : $P_{269} = (4,0,3,1)$ lies on line ℓ_5
36 : $P_{272} = (7,0,3,1)$ lies on line ℓ_6
37 : $P_{331} = (2,0,4,1)$ lies on line ℓ_4
38 : $P_{333} = (4,0,4,1)$ lies on line ℓ_5
39 : $P_{336} = (7,0,4,1)$ lies on line ℓ_6

40 : $P_{395} = (2,0,5,1)$ lies on line ℓ_4
41 : $P_{397} = (4,0,5,1)$ lies on line ℓ_5
42 : $P_{400} = (7,0,5,1)$ lies on line ℓ_6
43 : $P_{459} = (2,0,6,1)$ lies on line ℓ_4
44 : $P_{461} = (4,0,6,1)$ lies on line ℓ_5
45 : $P_{464} = (7,0,6,1)$ lies on line ℓ_6
46 : $P_{523} = (2,0,7,1)$ lies on line ℓ_4
47 : $P_{525} = (4,0,7,1)$ lies on line ℓ_5
48 : $P_{528} = (7,0,7,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 35 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$
1 : $P_{20} = (1,1,1,0)$
2 : $P_{34} = (7,2,1,0)$
3 : $P_{41} = (6,3,1,0)$
4 : $P_{45} = (2,4,1,0)$
5 : $P_{54} = (3,5,1,0)$
6 : $P_{64} = (5,6,1,0)$
7 : $P_{71} = (4,7,1,0)$
8 : $P_{156} = (3,2,1,1)$
9 : $P_{174} = (5,4,1,1)$
10 : $P_{199} = (6,7,1,1)$
11 : $P_{212} = (3,1,2,1)$
12 : $P_{222} = (5,2,2,1)$
13 : $P_{247} = (6,5,2,1)$
14 : $P_{250} = (1,6,2,1)$
15 : $P_{298} = (1,4,3,1)$
16 : $P_{308} = (3,5,3,1)$
17 : $P_{319} = (6,6,3,1)$

18 : $P_{326} = (5,7,3,1)$
19 : $P_{342} = (5,1,4,1)$
20 : $P_{354} = (1,3,4,1)$
21 : $P_{367} = (6,4,4,1)$
22 : $P_{380} = (3,6,4,1)$
23 : $P_{415} = (6,2,5,1)$
24 : $P_{420} = (3,3,5,1)$
25 : $P_{446} = (5,6,5,1)$
26 : $P_{450} = (1,7,5,1)$
27 : $P_{474} = (1,2,6,1)$
28 : $P_{487} = (6,3,6,1)$
29 : $P_{492} = (3,4,6,1)$
30 : $P_{502} = (5,5,6,1)$
31 : $P_{535} = (6,1,7,1)$
32 : $P_{550} = (5,3,7,1)$
33 : $P_{562} = (1,5,7,1)$
34 : $P_{580} = (3,7,7,1)$

Line Intersection Graph

```
 |0 1 2 3 4 5 6
-+-------------
0|0 1 1 1 1 1 1
1|1 0 1 1 1 0 0
2|1 1 0 1 0 1 0
3|1 1 1 0 0 0 1
4|1 1 0 0 0 1 1
5|1 0 1 0 1 0 1
6|1 0 0 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_1	P_2	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_1	P_1	P_1	P_{76}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5
in point	P_1	P_1	P_1	P_{78}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_1	P_1	P_1	P_{81}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_2	P_{76}	P_2	P_2

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_{78}	P_2	P_2

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_2	P_{81}	P_2	P_2

The surface has 89 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$

$1: P_2 = (0,0,1,0)$

$2: P_4 = (1,1,1,1)$

$3: P_{19} = (0,1,1,0)$

$4: P_{20} = (1,1,1,0)$

$5: P_{27} = (0,2,1,0)$

$6: P_{34} = (7,2,1,0)$

$7: P_{35} = (0,3,1,0)$

$8: P_{41} = (6,3,1,0)$

$9: P_{43} = (0,4,1,0)$

$10: P_{45} = (2,4,1,0)$

$11: P_{51} = (0,5,1,0)$

$12: P_{54} = (3,5,1,0)$

$13: P_{59} = (0,6,1,0)$

$14: P_{64} = (5,6,1,0)$

$15: P_{67} = (0,7,1,0)$

$16: P_{71} = (4,7,1,0)$

$17: P_{76} = (2,0,0,1)$

$18: P_{78} = (4,0,0,1)$

$19: P_{81} = (7,0,0,1)$

$20: P_{84} = (2,1,0,1)$

$21: P_{86} = (4,1,0,1)$

$22: P_{89} = (7,1,0,1)$

$23: P_{92} = (2,2,0,1)$

$24: P_{94} = (4,2,0,1)$

$25: P_{97} = (7,2,0,1)$

$26: P_{100} = (2,3,0,1)$

$27: P_{102} = (4,3,0,1)$

$28: P_{105} = (7,3,0,1)$

$29: P_{108} = (2,4,0,1)$

$30: P_{110} = (4,4,0,1)$

$31: P_{113} = (7,4,0,1)$

$32: P_{116} = (2,5,0,1)$

$33: P_{118} = (4,5,0,1)$

$34: P_{121} = (7,5,0,1)$

$35: P_{124} = (2,6,0,1)$

$36: P_{126} = (4,6,0,1)$

$37: P_{129} = (7,6,0,1)$

$38: P_{132} = (2,7,0,1)$

$39: P_{134} = (4,7,0,1)$

$40: P_{137} = (7,7,0,1)$

$41: P_{140} = (2,0,1,1)$

$42: P_{142} = (4,0,1,1)$

$43: P_{145} = (7,0,1,1)$

$44: P_{156} = (3,2,1,1)$

$45: P_{174} = (5,4,1,1)$

$46: P_{199} = (6,7,1,1)$

$47: P_{203} = (2,0,2,1)$

$48: P_{205} = (4,0,2,1)$

$49: P_{208} = (7,0,2,1)$

$50: P_{212} = (3,1,2,1)$

$51: P_{222} = (5,2,2,1)$

$52: P_{247} = (6,5,2,1)$

$53: P_{250} = (1,6,2,1)$

$54: P_{267} = (2,0,3,1)$

$55: P_{269} = (4,0,3,1)$

$56: P_{272} = (7,0,3,1)$

$57: P_{298} = (1,4,3,1)$

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$61: P_{331} = (2,0,4,1)$

$62: P_{333} = (4,0,4,1)$

$63: P_{336} = (7,0,4,1)$

$64: P_{342} = (5,1,4,1)$

$65: P_{354} = (1,3,4,1)$

$66: P_{367} = (6,4,4,1)$

$67: P_{380} = (3,6,4,1)$

$68: P_{395} = (2,0,5,1)$

$69: P_{397} = (4,0,5,1)$

$70: P_{400} = (7,0,5,1)$

$71: P_{415} = (6,2,5,1)$

$72: P_{420} = (3,3,5,1)$

$73: P_{446} = (5,6,5,1)$

$74: P_{450} = (1,7,5,1)$

$75: P_{459} = (2,0,6,1)$

$76: P_{461} = (4,0,6,1)$

$77: P_{464} = (7,0,6,1)$

$78: P_{474} = (1,2,6,1)$

$79: P_{487} = (6,3,6,1)$

$80: P_{492} = (3,4,6,1)$

$81: P_{502} = (5,5,6,1)$

$82: P_{523} = (2,0,7,1)$

$83: P_{525} = (4,0,7,1)$

$84: P_{528} = (7,0,7,1)$

$85: P_{535} = (6,1,7,1)$

$86: P_{550} = (5,3,7,1)$

$87: P_{562} = (1,5,7,1)$

$88: P_{580} = (3,7,7,1)$