Rank-35 over GF(32)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_0^2 X_1 = 0$$

General information

Number of lines	33
Number of points	1057
Number of singular points	33
Number of Eckardt points	0
Number of double points	0
Number of single points	1056
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	33^{33}
Type of lines on points	$33, 1^{1056}$

Singular Points

The surface has 33 singular points:

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\begin{array}{lll} 0: \ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0) & 9: \ P_{1314} = \mathbf{P}(0,\eta^3,0,1) = \mathbf{P}(0,8,0,1) \\ 1: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1) & 10: \ P_{1346} = \mathbf{P}(0,\eta^{29},0,1) = \mathbf{P}(0,9,0,1) \\ 2: \ P_{1090} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1) & 11: \ P_{1378} = \mathbf{P}(0,\eta^6,0,1) = \mathbf{P}(0,10,0,1) \\ 3: \ P_{1122} = \mathbf{P}(0,\eta,0,1) = \mathbf{P}(0,2,0,1) & 12: \ P_{1410} = \mathbf{P}(0,\eta^{27},0,1) = \mathbf{P}(0,11,0,1) \\ 4: \ P_{1154} = \mathbf{P}(0,\eta^{18},0,1) = \mathbf{P}(0,3,0,1) & 13: \ P_{1442} = \mathbf{P}(0,\eta^{20},0,1) = \mathbf{P}(0,12,0,1) \\ 5: \ P_{1186} = \mathbf{P}(0,\eta^2,0,1) = \mathbf{P}(0,4,0,1) & 14: \ P_{1474} = \mathbf{P}(0,\eta^8,0,1) = \mathbf{P}(0,13,0,1) \\ 6: \ P_{1218} = \mathbf{P}(0,\eta^5,0,1) = \mathbf{P}(0,5,0,1) & 15: \ P_{1506} = \mathbf{P}(0,\eta^{12},0,1) = \mathbf{P}(0,14,0,1) \\ 7: \ P_{1250} = \mathbf{P}(0,\eta^{19},0,1) = \mathbf{P}(0,6,0,1) & 16: \ P_{1538} = \mathbf{P}(0,\eta^{23},0,1) = \mathbf{P}(0,15,0,1) \\ 8: \ P_{1282} = \mathbf{P}(0,\eta^{11},0,1) = \mathbf{P}(0,7,0,1) & 17: \ P_{1570} = \mathbf{P}(0,\eta^4,0,1) = \mathbf{P}(0,16,0,1) \end{array}
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\begin{array}{lll} 18: \ P_{1602} = \mathbf{P}(0,\eta^{10},0,1) = \mathbf{P}(0,17,0,1) & 26: \ P_{1858} = \mathbf{P}(0,\eta^{25},0,1) = \mathbf{P}(0,25,0,1) \\ 19: \ P_{1634} = \mathbf{P}(0,\eta^{30},0,1) = \mathbf{P}(0,18,0,1) & 27: \ P_{1890} = \mathbf{P}(0,\eta^{9},0,1) = \mathbf{P}(0,26,0,1) \\ 20: \ P_{1666} = \mathbf{P}(0,\eta^{17},0,1) = \mathbf{P}(0,19,0,1) & 28: \ P_{1922} = \mathbf{P}(0,\eta^{16},0,1) = \mathbf{P}(0,27,0,1) \\ 21: \ P_{1698} = \mathbf{P}(0,\eta^{7},0,1) = \mathbf{P}(0,20,0,1) & 29: \ P_{1954} = \mathbf{P}(0,\eta^{13},0,1) = \mathbf{P}(0,28,0,1) \\ 22: \ P_{1730} = \mathbf{P}(0,\eta^{22},0,1) = \mathbf{P}(0,21,0,1) & 30: \ P_{1986} = \mathbf{P}(0,\eta^{14},0,1) = \mathbf{P}(0,29,0,1) \\ 23: \ P_{1762} = \mathbf{P}(0,\eta^{28},0,1) = \mathbf{P}(0,22,0,1) & 31: \ P_{2018} = \mathbf{P}(0,\eta^{24},0,1) = \mathbf{P}(0,30,0,1) \\ 24: \ P_{1794} = \mathbf{P}(0,\eta^{26},0,1) = \mathbf{P}(0,23,0,1) & 32: \ P_{2050} = \mathbf{P}(0,\eta^{15},0,1) = \mathbf{P}(0,31,0,1) \end{array}
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The 33 Lines

The lines and their Pluecker coordinates are:

$$\begin{split} \ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \mathbf{PI}(0,0,0,1,0)_{1089} \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{PI}(0,0,0,1,0,0)_{65} \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \mathbf{PI}(0,1,0,1,1,0)_{3137} \\ \ell_3 &= \begin{bmatrix} 1 & \eta^{17} & \eta^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{934387} = \begin{bmatrix} 1 & 19 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{934387} = \mathbf{PI}(0,27,0,19,1,0)_{4297} \\ \ell_4 &= \begin{bmatrix} 1 & \eta^{27} & \eta^9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{892107} = \begin{bmatrix} 1 & 11 & 26 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{394387} = \mathbf{PI}(0,27,0,19,1,0)_{4297} \\ \ell_5 &= \begin{bmatrix} 1 & \eta^3 & \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{77160} = \begin{bmatrix} 1 & 8 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{77160} = \mathbf{PI}(0,2,0,8,1,0)_{3579} \\ \ell_6 &= \begin{bmatrix} 1 & \eta^{23} & \eta^{18} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{118383} = \begin{bmatrix} 1 & 15 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{118383} = \mathbf{PI}(0,3,0,15,1,0)_{4021} \\ \ell_7 &= \begin{bmatrix} 1 & \eta^{13} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{876252} = \begin{bmatrix} 1 & 28 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{876252} = \mathbf{PI}(0,24,0,2,1,0)_{3223} \\ \ell_9 &= \begin{bmatrix} 1 & \eta^{20} & \eta^{17} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656396} = \begin{bmatrix} 1 & 12 & 19 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656396} = \mathbf{PI}(0,19,0,12,1,0)_{3848} \\ \ell_{10} &= \begin{bmatrix} 1 & \eta^{28} & \eta^{30} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{299130} = \begin{bmatrix} 1 & 26 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{299130} = \mathbf{PI}(0,8,0,26,1,0)_{4719} \\ \ell_{12} &= \begin{bmatrix} 1 & \eta^{25} & \eta^{29} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{331897} = \begin{bmatrix} 1 & 25 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{331897} = \mathbf{PI}(0,17,0,18,1,0)_{4224} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^{30} & \eta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{557038} = \begin{bmatrix} 1 & 18 & 17 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{342467} = \mathbf{PI}(0,10,0,3,1,0)_{3272} \end{aligned}$$

$$\begin{split} \ell_{16} &= \begin{bmatrix} 1 & \eta^{19} & \eta^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{379462} = \begin{bmatrix} 1 & 6 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{379462} = \mathbf{Pl}(0,11,0,6,1,0)_{3462} \\ \ell_{17} &= \begin{bmatrix} 1 & \eta^6 & \eta^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{146922} = \begin{bmatrix} 1 & 10 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{146922} = \mathbf{Pl}(0,4,0,10,1,0)_{3707} \\ \ell_{18} &= \begin{bmatrix} 1 & \eta^{15} & \eta^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{202943} = \begin{bmatrix} 1 & 31 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{202943} = \mathbf{Pl}(0,5,0,31,1,0)_{5031} \\ \ell_{19} &= \begin{bmatrix} 1 & \eta^{14} & \eta^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1080253} = \begin{bmatrix} 1 & 29 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1080253} = \mathbf{Pl}(0,31,0,29,1,0)_{4931} \\ \ell_{20} &= \begin{bmatrix} 1 & \eta^{10} & \eta^{24} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1033745} = \begin{bmatrix} 1 & 27 & 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1033745} = \mathbf{Pl}(0,30,0,17,1,0)_{4174} \\ \ell_{21} &= \begin{bmatrix} 1 & \eta^{26} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{228311} = \begin{bmatrix} 1 & 23 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{228311} = \mathbf{Pl}(0,6,0,23,1,0)_{4528} \\ \ell_{22} &= \begin{bmatrix} 1 & \eta^{2} & \eta^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{242052} = \begin{bmatrix} 1 & 4 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{242052} = \mathbf{Pl}(0,7,0,4,1,0)_{3332} \\ \ell_{23} &= \begin{bmatrix} 1 & \eta^{11} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{961869} = \begin{bmatrix} 1 & 13 & 28 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{989351} = \mathbf{Pl}(0,29,0,7,1,0)_{3543} \\ \ell_{24} &= \begin{bmatrix} 1 & \eta^{8} & \eta^{13} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{961869} = \begin{bmatrix} 1 & 27 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{807547} = \mathbf{Pl}(0,23,0,27,1,0)_{4797} \\ \ell_{26} &= \begin{bmatrix} 1 & \eta^{16} & \eta^{26} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{767381} = \begin{bmatrix} 1 & 27 & 23 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{367547} = \mathbf{Pl}(0,23,0,27,1,0)_{4797} \\ \ell_{28} &= \begin{bmatrix} 1 & \eta^{22} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{472478} = \begin{bmatrix} 1 & 9 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{472478} = \mathbf{Pl}(0,12,0,9,1,0)_{3652} \\ \ell_{29} &= \begin{bmatrix} 1 & \eta^4 & \eta^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{728272} = \begin{bmatrix} 1 & 16 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{728272} = \mathbf{Pl}(0,21,0,16,1,0)_{4102} \\ \ell_{30} &= \begin{bmatrix} 1 & \eta^2 & \eta^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{728272} = \begin{bmatrix} 1 & 5 & 14 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{702904} = \mathbf{Pl}(0,14,0,5,1,0)_{3402} \\ \ell_{31} &= \begin{bmatrix} 1 & \eta^7 & \eta^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{479877} = \begin{bmatrix} 1 & 20 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{479877} = \mathbf{Pl}(0,14,0,5,1,0)_{4408} \\ \ell_{32} &= \begin{bmatrix} 1 & \eta^7 & \eta^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{479877} = \begin{bmatrix} 1 & 20 & 15 & 0 \\ 0 & 0 & 0 &$$

Rank of lines: (1056, 1082400, 35937, 934387, 892107, 77160, 118383, 876252, 814946, 656396, 633142, 299130, 331897, 595090, 557038, 342467, 379462, 146922, 202943, 1080253, 1033745, 228311, 242052, 989351, 961869, 807547, 767381, 416457, 472478, 728272, 702904, 479877, 529556)

Rank of points on Klein quadric: (1089, 65, 3137, 4297, 3792, 3579, 4021, 4862, 3223, 3848, 4477, 4719, 4657, 4224, 3971, 3272, 3462, 3707, 5031, 4931, 4174, 4528, 3332, 3543, 3920, 4797, 4418, 3652, 4976, 4102, 4605, 3402, 4348)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 1056 single points: Too many to print.

Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

Line Intersection Graph

	0	1 2	2 :	3 4	Ę	6	; 7	7 8	3 9	9	10	11	1	2	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
0	0	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0 :	1 1	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1 ()]	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1 :	1 (1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1 :	1 1	(1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1 :	1 1	1	. (1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1 :	1 1	1	. 1	. ()]	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1 :	1 1	1	. 1	. 1	. () [1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1 :	1 1	1	. 1	. 1	. 1	1 () :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 (0	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1	1	0	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	0		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
22	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
23	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
24	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
25	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
26	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
27	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
28	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
29	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
30	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
31	1	1 :	1 1	1	. 1	. 1	. 1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
32	1	1 :	1 1	1	. 1	. 1	. 1	1 :	1 :	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	Τ									

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 2 intersects

ſ	т.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_
	Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	
	in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	Π									

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 4 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	22
in point P ₃ P ₃	P_3
Line 5 intersects	
	22
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 6 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	222
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 7 intersects	
	22
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 8 intersects	
	222
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 9 intersects	
	22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 10 intersects	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 11 intersects	
	22
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	P ₃
Line 12 intersects	
	22
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 13 intersects	
	22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 14 intersects	
	22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P ₃
Line 15 intersects	
	22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	P_3
Line 16 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	22
	P_3

Line 17 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 18 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 19 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 20 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 21 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 22 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 23 int		ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 24 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 25 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 26 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 27 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 28 int		ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 29 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										

Line 30 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	-
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	l										

Line 31 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	Ī										

${\bf Line~32~intersects}$

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	Γ.										

The surface has 1057 points:

Too many to print.