Rank-66763 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(8)$ is -1991732147

General information

Number of lines	1
Number of points	65
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{56}$

Singular Points

The surface has 1 singular points:

$$0: P_{75} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{649}$$

Rank of lines: (4672)

Rank of points on Klein quadric: (649)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

$0: P_1 = (0, 1, 0, 0)$ lies on line ℓ_0	5: $P_{43} = (0, 4, 1, 0)$ lies on line ℓ_0
$1: P_2 = (0,0,1,0)$ lies on line ℓ_0	6: $P_{51} = (0, 5, 1, 0)$ lies on line ℓ_0
$2: P_{19} = (0, 1, 1, 0)$ lies on line ℓ_0	7: $P_{59} = (0, 6, 1, 0)$ lies on line ℓ_0
$3: P_{27} = (0, 2, 1, 0)$ lies on line ℓ_0	8: $P_{67} = (0, 7, 1, 0)$ lies on line ℓ_0
4: $P_{35} = (0, 3, 1, 0)$ lies on line ℓ_0	

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line: The points on the surface but not on lines are:

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0: P_0 = (1,0,0,0)
                                                                  18: P_{195} = (2,7,1,1)
1: P_{75} = (1, 0, 0, 1)
                                                                  19: P_{199} = (6,7,1,1)
2: P_{92} = (2, 2, 0, 1)
                                                                  20: P_{203} = (2, 0, 2, 1)
3: P_{96} = (6, 2, 0, 1)
                                                                  21: P_{207} = (6,0,2,1)
4: P_{109} = (3, 4, 0, 1)
                                                                  22: P_{212} = (3, 1, 2, 1)
5: P_{110} = (4, 4, 0, 1)
                                                                  23: P_{213} = (4,1,2,1)
6: P_{135} = (5,7,0,1)
                                                                  24: P_{219} = (2, 2, 2, 1)
7: P_{137} = (7, 7, 0, 1)
                                                                  25: P_{223} = (6, 2, 2, 1)
8: P_{156} = (3, 2, 1, 1)
                                                                  26: P_{228} = (3, 3, 2, 1)
9: P_{157} = (4, 2, 1, 1)
                                                                  27: P_{229} = (4, 3, 2, 1)
10: P_{164} = (3, 3, 1, 1)
                                                                  28: P_{276} = (3, 1, 3, 1)
                                                                  29: P_{277} = (4, 1, 3, 1)
11: P_{165} = (4, 3, 1, 1)
12: P_{174} = (5, 4, 1, 1)
                                                                  30: P_{284} = (3, 2, 3, 1)
13: P_{176} = (7, 4, 1, 1)
                                                                  31: P_{285} = (4, 2, 3, 1)
14: P_{182} = (5, 5, 1, 1)
                                                                  32: P_{332} = (3,0,4,1)
15: P_{184} = (7, 5, 1, 1)
                                                                  33: P_{333} = (4,0,4,1)
16: P_{187} = (2, 6, 1, 1)
                                                                  34: P_{342} = (5, 1, 4, 1)
17: P_{191} = (6, 6, 1, 1)
                                                                  35: P_{344} = (7, 1, 4, 1)
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36: P_{364} = (3, 4, 4, 1)
                                                                  47: P_{519} = (6,7,6,1)
37: P_{365} = (4, 4, 4, 1)
                                                                  48: P_{526} = (5, 0, 7, 1)
38: P_{374} = (5, 5, 4, 1)
                                                                  49: P_{528} = (7, 0, 7, 1)
39: P_{376} = (7, 5, 4, 1)
                                                                  50: P_{531} = (2, 1, 7, 1)
                                                                  51: P_{535} = (6, 1, 7, 1)
40: P_{406} = (5, 1, 5, 1)
41: P_{408} = (7, 1, 5, 1)
                                                                  52: P_{571} = (2, 6, 7, 1)
42: P_{430} = (5, 4, 5, 1)
                                                                  53: P_{575} = (6, 6, 7, 1)
43: P_{432} = (7, 4, 5, 1)
                                                                  54: P_{582} = (5, 7, 7, 1)
44: P_{467} = (2, 1, 6, 1)
                                                                  55: P_{584} = (7,7,7,1)
45: P_{471} = (6, 1, 6, 1)
46: P_{515} = (2, 7, 6, 1)
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Line Intersection Graph

 $\begin{array}{c|c} 0 \\ \hline 0 & 0 \end{array}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 65 points:

The points on the surface are:

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$0: P_0 = (1, 0, 0, 0)$	$22: P_{176} = (7, 4, 1, 1)$	$44: P_{344} = (7, 1, 4, 1)$
$1: P_1 = (0, 1, 0, 0)$	$23: P_{182} = (5, 5, 1, 1)$	$45: P_{364} = (3,4,4,1)$
$2: P_2 = (0, 0, 1, 0)$	$24: P_{184} = (7, 5, 1, 1)$	$46: P_{365} = (4, 4, 4, 1)$
$3: P_{19} = (0, 1, 1, 0)$	$25: P_{187} = (2, 6, 1, 1)$	$47: P_{374} = (5, 5, 4, 1)$
$4: P_{27} = (0, 2, 1, 0)$	$26: P_{191} = (6, 6, 1, 1)$	$48: P_{376} = (7, 5, 4, 1)$
$5: P_{35} = (0, 3, 1, 0)$	$27: P_{195} = (2, 7, 1, 1)$	$49: P_{406} = (5, 1, 5, 1)$
$6: P_{43} = (0, 4, 1, 0)$	$28: P_{199} = (6, 7, 1, 1)$	$50: P_{408} = (7, 1, 5, 1)$
$7: P_{51} = (0, 5, 1, 0)$	$29: P_{203} = (2,0,2,1)$	$51: P_{430} = (5, 4, 5, 1)$
$8: P_{59} = (0, 6, 1, 0)$	$30: P_{207} = (6,0,2,1)$	$52: P_{432} = (7, 4, 5, 1)$
$9: P_{67} = (0, 7, 1, 0)$	$31: P_{212} = (3, 1, 2, 1)$	$53: P_{467} = (2, 1, 6, 1)$
$10: P_{75} = (1,0,0,1)$	$32: P_{213} = (4, 1, 2, 1)$	$54: P_{471} = (6, 1, 6, 1)$
$11: P_{92} = (2, 2, 0, 1)$	$33: P_{219} = (2, 2, 2, 1)$	$55: P_{515} = (2, 7, 6, 1)$
$12: P_{96} = (6, 2, 0, 1)$	$34: P_{223} = (6, 2, 2, 1)$	$56: P_{519} = (6, 7, 6, 1)$
$13: P_{109} = (3, 4, 0, 1)$	$35: P_{228} = (3, 3, 2, 1)$	$57: P_{526} = (5, 0, 7, 1)$
$14: P_{110} = (4, 4, 0, 1)$	$36: P_{229} = (4, 3, 2, 1)$	$58: P_{528} = (7, 0, 7, 1)$
$15: P_{135} = (5,7,0,1)$	$37: P_{276} = (3, 1, 3, 1)$	$59: P_{531} = (2, 1, 7, 1)$
16: $P_{137} = (7, 7, 0, 1)$	$38: P_{277} = (4, 1, 3, 1)$	$60: P_{535} = (6, 1, 7, 1)$
$17: P_{156} = (3, 2, 1, 1)$	$39: P_{284} = (3, 2, 3, 1)$	$61: P_{571} = (2, 6, 7, 1)$
$18: P_{157} = (4, 2, 1, 1)$	$40: P_{285} = (4, 2, 3, 1)$	$62: P_{575} = (6, 6, 7, 1)$
19: $P_{164} = (3, 3, 1, 1)$	$41: P_{332} = (3,0,4,1)$	$63: P_{582} = (5, 7, 7, 1)$
$20: P_{165} = (4, 3, 1, 1)$	$42: P_{333} = (4,0,4,1)$	$64: P_{584} = (7, 7, 7, 1)$
$21: P_{174} = (5, 4, 1, 1)$	$43: P_{342} = (5, 1, 4, 1)$	(