Rank-65617 over GF(8)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227396245

General information

Number of lines	3
Number of points	65
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	40
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	3, 1^{24}, 0^{40}

Singular Points

The surface has 1 singular points:

$0: \ P_{75} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & \gamma^6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3578} = \begin{bmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3578} = \mathbf{Pl}(2,6,1,6,0,1)_{1120}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & \gamma^5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1826} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1826} = \mathbf{Pl}(4,3,1,3,0,1)_{975}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & \gamma^3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2994} = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2994} = \mathbf{Pl}(7,5,1,5,0,1)_{1076}$$

Rank of lines: (3578, 1826, 2994)
Rank of points on Klein quadric: (1120, 975, 1076)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{19} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_{92} = (2,2,0,1)$ lies on line ℓ_0
$1 : P_{110} = (4,4,0,1)$ lies on line ℓ_1
$2 : P_{137} = (7,7,0,1)$ lies on line ℓ_2
$3 : P_{163} = (2,3,1,1)$ lies on line ℓ_0
$4 : P_{181} = (4,5,1,1)$ lies on line ℓ_1
$5 : P_{192} = (7,6,1,1)$ lies on line ℓ_2
$6 : P_{203} = (2,0,2,1)$ lies on line ℓ_0
$7 : P_{248} = (7,5,2,1)$ lies on line ℓ_2
$8 : P_{253} = (4,6,2,1)$ lies on line ℓ_1
$9 : P_{275} = (2,1,3,1)$ lies on line ℓ_0
$10 : P_{304} = (7,4,3,1)$ lies on line ℓ_2
$11 : P_{325} = (4,7,3,1)$ lies on line ℓ_1
$12 : P_{333} = (4,0,4,1)$ lies on line ℓ_1
$13 : P_{360} = (7,3,4,1)$ lies on line ℓ_2
$14 : P_{379} = (2,6,4,1)$ lies on line ℓ_0
$15 : P_{405} = (4,1,5,1)$ lies on line ℓ_1
$16 : P_{416} = (7,2,5,1)$ lies on line ℓ_2
$17 : P_{451} = (2,7,5,1)$ lies on line ℓ_0
$18 : P_{472} = (7,1,6,1)$ lies on line ℓ_2
$19 : P_{477} = (4,2,6,1)$ lies on line ℓ_1
$20 : P_{491} = (2,4,6,1)$ lies on line ℓ_0
$21 : P_{528} = (7,0,7,1)$ lies on line ℓ_2
$22 : P_{549} = (4,3,7,1)$ lies on line ℓ_1
$23 : P_{563} = (2,5,7,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 40 points not on any line:
The points on the surface but not on lines are:

$0 : P_0 = (1,0,0,0)$
$1 : P_{29} = (2,2,1,0)$
$2 : P_{36} = (1,3,1,0)$
$3 : P_{47} = (4,4,1,0)$
$4 : P_{52} = (1,5,1,0)$
$5 : P_{60} = (1,6,1,0)$
$6 : P_{74} = (7,7,1,0)$
$7 : P_{75} = (1,0,0,1)$

8 : $P_{82} = (0, 1, 0, 1)$
9 : $P_{104} = (6, 3, 0, 1)$
10 : $P_{117} = (3, 5, 0, 1)$
11 : $P_{127} = (5, 6, 0, 1)$
12 : $P_{138} = (0, 0, 1, 1)$
13 : $P_{162} = (1, 3, 1, 1)$
14 : $P_{178} = (1, 5, 1, 1)$
15 : $P_{186} = (1, 6, 1, 1)$
16 : $P_{219} = (2, 2, 2, 1)$
17 : $P_{223} = (6, 2, 2, 1)$
18 : $P_{241} = (0, 5, 2, 1)$
19 : $P_{254} = (5, 6, 2, 1)$
20 : $P_{271} = (6, 0, 3, 1)$
21 : $P_{274} = (1, 1, 3, 1)$
22 : $P_{303} = (6, 4, 3, 1)$
23 : $P_{321} = (0, 7, 3, 1)$
24 : $P_{359} = (6, 3, 4, 1)$
25 : $P_{364} = (3, 4, 4, 1)$
26 : $P_{365} = (4, 4, 4, 1)$
27 : $P_{377} = (0, 6, 4, 1)$
28 : $P_{396} = (3, 0, 5, 1)$
29 : $P_{402} = (1, 1, 5, 1)$
30 : $P_{409} = (0, 2, 5, 1)$
31 : $P_{452} = (3, 7, 5, 1)$
32 : $P_{462} = (5, 0, 6, 1)$
33 : $P_{466} = (1, 1, 6, 1)$
34 : $P_{478} = (5, 2, 6, 1)$
35 : $P_{489} = (0, 4, 6, 1)$
36 : $P_{545} = (0, 3, 7, 1)$
37 : $P_{564} = (3, 5, 7, 1)$
38 : $P_{582} = (5, 7, 7, 1)$
39 : $P_{584} = (7, 7, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{19}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{19}	P_{19}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{19}	P_{19}

The surface has 65 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{19} = (0, 1, 1, 0)$
2 : $P_{29} = (2, 2, 1, 0)$
3 : $P_{36} = (1, 3, 1, 0)$
4 : $P_{47} = (4, 4, 1, 0)$
5 : $P_{52} = (1, 5, 1, 0)$
6 : $P_{60} = (1, 6, 1, 0)$
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$48: P_{416} = (7, 2, 5, 1)$
$49: P_{451} = (2, 7, 5, 1)$
$50: P_{452} = (3, 7, 5, 1)$
$51: P_{462} = (5, 0, 6, 1)$
$52: P_{466} = (1, 1, 6, 1)$
$53: P_{472} = (7, 1, 6, 1)$
$54: P_{477} = (4, 2, 6, 1)$
$55: P_{478} = (5, 2, 6, 1)$
$56: P_{489} = (0, 4, 6, 1)$
$57: P_{491} = (2, 4, 6, 1)$
$58: P_{528} = (7, 0, 7, 1)$
$59: P_{545} = (0, 3, 7, 1)$
$60: P_{549} = (4, 3, 7, 1)$
$61: P_{563} = (2, 5, 7, 1)$
$62: P_{564} = (3, 5, 7, 1)$
$63: P_{582} = (5, 7, 7, 1)$
$64: P_{584} = (7, 7, 7, 1)$