Rank-76051 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_1 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286396693

General information

Number of lines	9
Number of points	305
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	126
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^5, 2^6, 1^{126}, 0^{168}

Singular Points

The surface has 4 singular points:

$0:\ P_{29} = \mathbf{P}(\delta^{10}, 0, 1, 0) = \mathbf{P}(10, 0, 1, 0)$
$1:\ P_{30} = \mathbf{P}(\delta^{5}, 0, 1, 0) = \mathbf{P}(11, 0, 1, 0)$
$2:\ P_{3020} = \mathbf{P}(\delta^{5}, \delta^{5}, \delta^{10}, 1) = \mathbf{P}(11, 11, 10, 1)$
$3:\ P_{3259} = \mathbf{P}(\delta^{10}, \delta^{10}, \delta^{5}, 1) = \mathbf{P}(10, 10, 11, 1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \delta^5 & 0 \\ 0 & 1 & 1 & \delta^5 \end{bmatrix}_{3180} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 1 & 1 & 11 \end{bmatrix}_{3180} = \mathbf{Pl}(10,11,10,0,1,1)_{9120}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^5 & 0 \\ 0 & 1 & 0 & \delta^{10} \end{bmatrix}_{3163} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 1 & 0 & 10 \end{bmatrix}_{3163} = \mathbf{Pl}(11,10,0,0,11,1)_{49771}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \delta^{10} & 0 \\ 0 & 1 & 0 & \delta^5 \end{bmatrix}_{2906} = \begin{bmatrix} 1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 11 \end{bmatrix}_{2906} = \mathbf{Pl}(10,11,0,0,10,1)_{45690}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^{10} & 0 \\ 0 & 1 & 1 & \delta^{10} \end{bmatrix}_{2891} = \begin{bmatrix} 1 & 0 & 10 & 0 \\ 0 & 1 & 1 & 10 \end{bmatrix}_{2891} = \mathbf{Pl}(11,10,11,0,1,1)_{9136}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \mathbf{Pl}(0,1,1,1,1,1)_{9442}$$

Rank of lines: (256, 69904, 70160, 3180, 3163, 2906, 2891, 69921, 4898)
Rank of points on Klein quadric: (2, 33, 1, 9120, 49771, 45690, 9136, 49, 9442)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_{29} = \mathbf{P}(\delta^{10},0,1,0) = \mathbf{P}(10,0,1,0)$,
$2 : P_{30} = \mathbf{P}(\delta^5,0,1,0) = \mathbf{P}(11,0,1,0)$,
$3 : P_{3020} = \mathbf{P}(\delta^5,\delta^5,\delta^{10},1) = \mathbf{P}(11,11,10,1)$,
$4 : P_{3259} = \mathbf{P}(\delta^{10},\delta^{10},\delta^5,1) = \mathbf{P}(10,10,11,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$
$P_{450} = (0,11,0,1) = \ell_1 \cap \ell_4$
$P_{434} = (0,10,0,1) = \ell_1 \cap \ell_5$
$P_{530} = (0,0,1,1) = \ell_2 \cap \ell_8$
$P_{2993} = (0,10,10,1) = \ell_3 \cap \ell_7$
$P_{3265} = (0,11,11,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 126 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_1 = (0,1,0,0)$ lies on line ℓ_1
$2 :\ P_{20} = (1,0,1,0)$ lies on line ℓ_0
$3 :\ P_{21} = (2,0,1,0)$ lies on line ℓ_0
$4 :\ P_{22} = (3,0,1,0)$ lies on line ℓ_0
$5 :\ P_{23} = (4,0,1,0)$ lies on line ℓ_0

6 : $P_{24} = (5, 0, 1, 0)$ lies on line ℓ_0
7 : $P_{25} = (6, 0, 1, 0)$ lies on line ℓ_0
8 : $P_{26} = (7, 0, 1, 0)$ lies on line ℓ_0
9 : $P_{27} = (8, 0, 1, 0)$ lies on line ℓ_0
10 : $P_{28} = (9, 0, 1, 0)$ lies on line ℓ_0
11 : $P_{31} = (12, 0, 1, 0)$ lies on line ℓ_0
12 : $P_{32} = (13, 0, 1, 0)$ lies on line ℓ_0
13 : $P_{33} = (14, 0, 1, 0)$ lies on line ℓ_0
14 : $P_{34} = (15, 0, 1, 0)$ lies on line ℓ_0
15 : $P_{35} = (0, 1, 1, 0)$ lies on line ℓ_7
16 : $P_{36} = (1, 1, 1, 0)$ lies on line ℓ_8
17 : $P_{290} = (0, 1, 0, 1)$ lies on line ℓ_1
18 : $P_{291} = (1, 1, 0, 1)$ lies on line ℓ_8
19 : $P_{306} = (0, 2, 0, 1)$ lies on line ℓ_1
20 : $P_{322} = (0, 3, 0, 1)$ lies on line ℓ_1
21 : $P_{338} = (0, 4, 0, 1)$ lies on line ℓ_1
22 : $P_{354} = (0, 5, 0, 1)$ lies on line ℓ_1
23 : $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_1
24 : $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_1
25 : $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_1
26 : $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_1
27 : $P_{445} = (11, 10, 0, 1)$ lies on line ℓ_3
28 : $P_{460} = (10, 11, 0, 1)$ lies on line ℓ_6
29 : $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_1
30 : $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_1
31 : $P_{498} = (0, 14, 0, 1)$ lies on line ℓ_1
32 : $P_{514} = (0, 15, 0, 1)$ lies on line ℓ_1
33 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_7
34 : $P_{690} = (1, 10, 1, 1)$ lies on line ℓ_3
35 : $P_{700} = (11, 10, 1, 1)$ lies on line ℓ_5
36 : $P_{706} = (1, 11, 1, 1)$ lies on line ℓ_6
37 : $P_{715} = (10, 11, 1, 1)$ lies on line ℓ_4
38 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_2
39 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_7
40 : $P_{836} = (3, 3, 2, 1)$ lies on line ℓ_8
41 : $P_{951} = (6, 10, 2, 1)$ lies on line ℓ_3
42 : $P_{960} = (15, 10, 2, 1)$ lies on line ℓ_5
43 : $P_{966} = (5, 11, 2, 1)$ lies on line ℓ_6
44 : $P_{974} = (13, 11, 2, 1)$ lies on line ℓ_4
45 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_2
46 : $P_{1075} = (2, 2, 3, 1)$ lies on line ℓ_8
47 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_7
48 : $P_{1205} = (4, 10, 3, 1)$ lies on line ℓ_5
49 : $P_{1213} = (12, 10, 3, 1)$ lies on line ℓ_3
50 : $P_{1224} = (7, 11, 3, 1)$ lies on line ℓ_4
51 : $P_{1231} = (14, 11, 3, 1)$ lies on line ℓ_6
52 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_2
53 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_7
54 : $P_{1382} = (5, 5, 4, 1)$ lies on line ℓ_8
55 : $P_{1464} = (7, 10, 4, 1)$ lies on line ℓ_5
56 : $P_{1465} = (8, 10, 4, 1)$ lies on line ℓ_3
57 : $P_{1476} = (3, 11, 4, 1)$ lies on line ℓ_4
58 : $P_{1486} = (13, 11, 4, 1)$ lies on line ℓ_6
59 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_2
60 : $P_{1621} = (4, 4, 5, 1)$ lies on line ℓ_8
61 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_7
62 : $P_{1715} = (2, 10, 5, 1)$ lies on line ℓ_3
63 : $P_{1725} = (12, 10, 5, 1)$ lies on line ℓ_5
64 : $P_{1735} = (6, 11, 5, 1)$ lies on line ℓ_6
65 : $P_{1738} = (9, 11, 5, 1)$ lies on line ℓ_4
66 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_2
67 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_7
68 : $P_{1928} = (7, 7, 6, 1)$ lies on line ℓ_8
69 : $P_{1974} = (5, 10, 6, 1)$ lies on line ℓ_3
70 : $P_{1977} = (8, 10, 6, 1)$ lies on line ℓ_5
71 : $P_{1987} = (2, 11, 6, 1)$ lies on line ℓ_6
72 : $P_{1999} = (14, 11, 6, 1)$ lies on line ℓ_4
73 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_2
74 : $P_{2167} = (6, 6, 7, 1)$ lies on line ℓ_8
75 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_7
76 : $P_{2228} = (3, 10, 7, 1)$ lies on line ℓ_5
77 : $P_{2240} = (15, 10, 7, 1)$ lies on line ℓ_3
78 : $P_{2245} = (4, 11, 7, 1)$ lies on line ℓ_4
79 : $P_{2250} = (9, 11, 7, 1)$ lies on line ℓ_6
80 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_2
81 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_7
82 : $P_{2474} = (9, 9, 8, 1)$ lies on line ℓ_8
83 : $P_{2494} = (13, 10, 8, 1)$ lies on line ℓ_3
84 : $P_{2495} = (14, 10, 8, 1)$ lies on line ℓ_5
85 : $P_{2501} = (4, 11, 8, 1)$ lies on line ℓ_6
86 : $P_{2503} = (6, 11, 8, 1)$ lies on line ℓ_4
87 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_2
88 : $P_{2713} = (8, 8, 9, 1)$ lies on line ℓ_8
89 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_7
90 : $P_{2742} = (5, 10, 9, 1)$ lies on line ℓ_5
91 : $P_{2744} = (7, 10, 9, 1)$ lies on line ℓ_3
92 : $P_{2765} = (12, 11, 9, 1)$ lies on line ℓ_4
93 : $P_{2768} = (15, 11, 9, 1)$ lies on line ℓ_6
94 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_2
95 : $P_{2994} = (1, 10, 10, 1)$ lies on line ℓ_5
96 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_2
97 : $P_{3266} = (1, 11, 11, 1)$ lies on line ℓ_4
98 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_2
99 : $P_{3514} = (9, 10, 12, 1)$ lies on line ℓ_5
100 : $P_{3519} = (14, 10, 12, 1)$ lies on line ℓ_3
101 : $P_{3524} = (3, 11, 12, 1)$ lies on line ℓ_6
102 : $P_{3526} = (5, 11, 12, 1)$ lies on line ℓ_4
103 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_7
104 : $P_{3566} = (13, 13, 12, 1)$ lies on line ℓ_8
105 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_2
106 : $P_{3763} = (2, 10, 13, 1)$ lies on line ℓ_5
107 : $P_{3765} = (4, 10, 13, 1)$ lies on line ℓ_3
108 : $P_{3785} = (8, 11, 13, 1)$ lies on line ℓ_6
109 : $P_{3792} = (15, 11, 13, 1)$ lies on line ℓ_4
110 : $P_{3805} = (12, 12, 13, 1)$ lies on line ℓ_8
111 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_7
112 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_2
113 : $P_{4020} = (3, 10, 14, 1)$ lies on line ℓ_3

114 : $P_{4023} = (6, 10, 14, 1)$ lies on line ℓ_5
115 : $P_{4041} = (8, 11, 14, 1)$ lies on line ℓ_4
116 : $P_{4045} = (12, 11, 14, 1)$ lies on line ℓ_6
117 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_7
118 : $P_{4112} = (15, 15, 14, 1)$ lies on line ℓ_8
119 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_2
120 : $P_{4282} = (9, 10, 15, 1)$ lies on line ℓ_3
121 : $P_{4286} = (13, 10, 15, 1)$ lies on line ℓ_5
122 : $P_{4291} = (2, 11, 15, 1)$ lies on line ℓ_4
123 : $P_{4296} = (7, 11, 15, 1)$ lies on line ℓ_6
124 : $P_{4351} = (14, 14, 15, 1)$ lies on line ℓ_8
125 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_{123} = (8, 6, 1, 0)$
1 : $P_{124} = (9, 6, 1, 0)$
2 : $P_{133} = (2, 7, 1, 0)$
3 : $P_{134} = (3, 7, 1, 0)$
4 : $P_{185} = (6, 10, 1, 0)$
5 : $P_{186} = (7, 10, 1, 0)$
6 : $P_{207} = (12, 11, 1, 0)$
7 : $P_{208} = (13, 11, 1, 0)$
8 : $P_{215} = (4, 12, 1, 0)$
9 : $P_{216} = (5, 12, 1, 0)$
10 : $P_{241} = (14, 13, 1, 0)$
11 : $P_{242} = (15, 13, 1, 0)$
12 : $P_{318} = (12, 2, 0, 1)$
13 : $P_{330} = (8, 3, 0, 1)$
14 : $P_{344} = (6, 4, 0, 1)$
15 : $P_{369} = (15, 5, 0, 1)$
16 : $P_{374} = (4, 6, 0, 1)$
17 : $P_{400} = (14, 7, 0, 1)$
18 : $P_{405} = (3, 8, 0, 1)$
19 : $P_{431} = (13, 9, 0, 1)$
20 : $P_{468} = (2, 12, 0, 1)$
21 : $P_{491} = (9, 13, 0, 1)$
22 : $P_{505} = (7, 14, 0, 1)$
23 : $P_{519} = (5, 15, 0, 1)$
24 : $P_{566} = (5, 2, 1, 1)$
25 : $P_{569} = (8, 2, 1, 1)$
26 : $P_{601} = (8, 4, 1, 1)$
27 : $P_{608} = (15, 4, 1, 1)$
28 : $P_{676} = (3, 9, 1, 1)$
29 : $P_{688} = (15, 9, 1, 1)$
30 : $P_{756} = (3, 14, 1, 1)$
31 : $P_{758} = (5, 14, 1, 1)$
32 : $P_{831} = (14, 2, 2, 1)$
33 : $P_{842} = (9, 3, 2, 1)$
34 : $P_{851} = (2, 4, 2, 1)$
35 : $P_{855} = (6, 4, 2, 1)$
36 : $P_{883} = (2, 6, 2, 1)$
37 : $P_{885} = (4, 6, 2, 1)$
38 : $P_{917} = (4, 8, 2, 1)$
39 : $P_{918} = (5, 8, 2, 1)$
40 : $P_{980} = (3, 12, 2, 1)$
41 : $P_{1034} = (9, 15, 2, 1)$
42 : $P_{1039} = (14, 15, 2, 1)$
43 : $P_{1065} = (8, 1, 3, 1)$
44 : $P_{1067} = (10, 1, 3, 1)$
45 : $P_{1086} = (13, 2, 3, 1)$
46 : $P_{1100} = (11, 3, 3, 1)$
47 : $P_{1147} = (10, 6, 3, 1)$
48 : $P_{1150} = (13, 6, 3, 1)$
49 : $P_{1154} = (1, 7, 3, 1)$
50 : $P_{1165} = (12, 7, 3, 1)$
51 : $P_{1171} = (2, 8, 3, 1)$
52 : $P_{1250} = (1, 13, 3, 1)$
53 : $P_{1260} = (11, 13, 3, 1)$
54 : $P_{1289} = (8, 15, 3, 1)$
55 : $P_{1295} = (14, 15, 3, 1)$
56 : $P_{1347} = (2, 3, 4, 1)$
57 : $P_{1359} = (14, 3, 4, 1)$
58 : $P_{1363} = (2, 4, 4, 1)$
59 : $P_{1391} = (14, 5, 4, 1)$
60 : $P_{1398} = (5, 6, 4, 1)$
61 : $P_{1445} = (4, 9, 4, 1)$
62 : $P_{1454} = (13, 9, 4, 1)$
63 : $P_{1509} = (4, 13, 4, 1)$
64 : $P_{1514} = (9, 13, 4, 1)$
65 : $P_{1545} = (8, 15, 4, 1)$
66 : $P_{1546} = (9, 15, 4, 1)$
67 : $P_{1580} = (11, 1, 5, 1)$
68 : $P_{1584} = (15, 1, 5, 1)$
69 : $P_{1603} = (2, 3, 5, 1)$
70 : $P_{1616} = (15, 3, 5, 1)$
71 : $P_{1624} = (7, 4, 5, 1)$
72 : $P_{1643} = (10, 5, 5, 1)$
73 : $P_{1666} = (1, 7, 5, 1)$

74 : $P_{1675} = (10, 7, 5, 1)$
75 : $P_{1746} = (1, 12, 5, 1)$
76 : $P_{1751} = (6, 12, 5, 1)$
77 : $P_{1768} = (7, 13, 5, 1)$
78 : $P_{1772} = (11, 13, 5, 1)$
79 : $P_{1797} = (4, 15, 5, 1)$
80 : $P_{1846} = (5, 2, 6, 1)$
81 : $P_{1856} = (15, 2, 6, 1)$
82 : $P_{1880} = (7, 4, 6, 1)$
83 : $P_{1907} = (2, 6, 6, 1)$
84 : $P_{1936} = (15, 7, 6, 1)$
85 : $P_{2166} = (5, 6, 7, 1)$
86 : $P_{2186} = (9, 7, 7, 1)$
87 : $P_{2214} = (5, 9, 7, 1)$
88 : $P_{2224} = (15, 9, 7, 1)$
89 : $P_{2295} = (6, 14, 7, 1)$
90 : $P_{2340} = (3, 1, 8, 1)$
91 : $P_{2347} = (10, 1, 8, 1)$
92 : $P_{2378} = (9, 3, 8, 1)$
93 : $P_{2404} = (3, 5, 8, 1)$
94 : $P_{2405} = (4, 5, 8, 1)$
95 : $P_{2418} = (1, 6, 8, 1)$
96 : $P_{2430} = (13, 6, 8, 1)$
97 : $P_{2443} = (10, 7, 8, 1)$
98 : $P_{2445} = (12, 7, 8, 1)$
99 : $P_{2460} = (11, 8, 8, 1)$
100 : $P_{2477} = (12, 9, 8, 1)$
101 : $P_{2514} = (1, 12, 8, 1)$
102 : $P_{2524} = (11, 12, 8, 1)$
103 : $P_{2639} = (14, 3, 9, 1)$
104 : $P_{2640} = (15, 3, 9, 1)$
105 : $P_{2659} = (2, 5, 9, 1)$
106 : $P_{2661} = (4, 5, 9, 1)$
107 : $P_{2698} = (9, 7, 9, 1)$
108 : $P_{2703} = (14, 7, 9, 1)$
109 : $P_{2707} = (2, 8, 9, 1)$
110 : $P_{2725} = (4, 9, 9, 1)$
111 : $P_{2793} = (8, 13, 9, 1)$
112 : $P_{2808} = (7, 14, 9, 1)$
113 : $P_{2810} = (9, 14, 9, 1)$
114 : $P_{2852} = (3, 1, 10, 1)$
115 : $P_{2857} = (8, 1, 10, 1)$
116 : $P_{2900} = (3, 4, 10, 1)$
117 : $P_{2912} = (15, 4, 10, 1)$
118 : $P_{2923} = (10, 5, 10, 1)$
119 : $P_{2928} = (15, 5, 10, 1)$
120 : $P_{3062} = (5, 14, 10, 1)$
121 : $P_{3065} = (8, 14, 10, 1)$
122 : $P_{3078} = (5, 15, 10, 1)$
123 : $P_{3083} = (10, 15, 10, 1)$
124 : $P_{3110} = (5, 1, 11, 1)$
125 : $P_{3120} = (15, 1, 11, 1)$
126 : $P_{3129} = (8, 2, 11, 1)$
127 : $P_{3136} = (15, 2, 11, 1)$
128 : $P_{3145} = (8, 3, 11, 1)$
129 : $P_{3148} = (11, 3, 11, 1)$
130 : $P_{3220} = (3, 8, 11, 1)$
131 : $P_{3228} = (11, 8, 11, 1)$
132 : $P_{3236} = (3, 9, 11, 1)$
133 : $P_{3238} = (5, 9, 11, 1)$
134 : $P_{3390} = (13, 2, 12, 1)$
135 : $P_{3551} = (14, 12, 12, 1)$
136 : $P_{3561} = (8, 13, 12, 1)$
137 : $P_{3572} = (3, 14, 12, 1)$
138 : $P_{3577} = (8, 14, 12, 1)$
139 : $P_{3668} = (3, 4, 13, 1)$
140 : $P_{3673} = (8, 4, 13, 1)$
141 : $P_{3757} = (12, 9, 13, 1)$
142 : $P_{3796} = (3, 12, 13, 1)$
143 : $P_{3813} = (4, 13, 13, 1)$
144 : $P_{3901} = (12, 2, 14, 1)$
145 : $P_{3903} = (14, 2, 14, 1)$
146 : $P_{3939} = (2, 5, 14, 1)$
147 : $P_{3940} = (3, 5, 14, 1)$
148 : $P_{3984} = (15, 7, 14, 1)$
149 : $P_{3989} = (4, 8, 14, 1)$
150 : $P_{3994} = (9, 8, 14, 1)$
151 : $P_{4051} = (2, 12, 14, 1)$
152 : $P_{4063} = (14, 12, 14, 1)$
153 : $P_{4090} = (9, 14, 14, 1)$
154 : $P_{4101} = (4, 15, 14, 1)$
155 : $P_{4134} = (5, 1, 15, 1)$
156 : $P_{4140} = (11, 1, 15, 1)$
157 : $P_{4207} = (14, 5, 15, 1)$
158 : $P_{4210} = (1, 6, 15, 1)$
159 : $P_{4219} = (10, 6, 15, 1)$
160 : $P_{4246} = (5, 8, 15, 1)$
161 : $P_{4250} = (9, 8, 15, 1)$
162 : $P_{4311} = (6, 12, 15, 1)$
163 : $P_{4316} = (11, 12, 15, 1)$
164 : $P_{4322} = (1, 13, 15, 1)$
165 : $P_{4328} = (7, 13, 15, 1)$
166 : $P_{4343} = (6, 14, 15, 1)$
167 : $P_{4363} = (10, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 0 1 1 1 1 1 0 0
1	0 0 1 0 1 1 0 1 0
2	1 1 0 0 0 0 0 1 1
3	1 0 0 0 1 1 0 1 1
4	1 1 0 1 0 0 1 0 1
5	1 1 0 1 0 0 1 0 1
6	1 0 0 0 1 1 0 1 1
7	0 1 1 1 0 0 1 0 0
8	0 0 1 1 1 1 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_{29}	P_{29}	P_{30}	P_{30}

Line 1 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_{450}	P_{434}	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_2	P_3	P_3	P_{530}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{29}	P_{29}	P_{3259}	P_{2993}	P_{3259}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_{29}	P_{450}	P_{29}	P_{3020}	P_{3020}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_{30}	P_{434}	P_{3259}	P_{30}	P_{3259}

Line 6 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{30}	P_{3020}	P_{30}	P_{3265}	P_{3020}

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6
in point	P_3	P_3	P_{2993}	P_{3265}

Line 8 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{530}	P_{3259}	P_{3020}	P_{3259}	P_{3020}

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_{20} = (1, 0, 1, 0)$
5 : $P_{21} = (2, 0, 1, 0)$
6 : $P_{22} = (3, 0, 1, 0)$
7 : $P_{23} = (4, 0, 1, 0)$
8 : $P_{24} = (5, 0, 1, 0)$
9 : $P_{25} = (6, 0, 1, 0)$
10 : $P_{26} = (7, 0, 1, 0)$
11 : $P_{27} = (8, 0, 1, 0)$
12 : $P_{28} = (9, 0, 1, 0)$
13 : $P_{29} = (10, 0, 1, 0)$
14 : $P_{30} = (11, 0, 1, 0)$
15 : $P_{31} = (12, 0, 1, 0)$
16 : $P_{32} = (13, 0, 1, 0)$
17 : $P_{33} = (14, 0, 1, 0)$
18 : $P_{34} = (15, 0, 1, 0)$
19 : $P_{35} = (0, 1, 1, 0)$
20 : $P_{36} = (1, 1, 1, 0)$

21 : $P_{123} = (8, 6, 1, 0)$
22 : $P_{124} = (9, 6, 1, 0)$
23 : $P_{133} = (2, 7, 1, 0)$
24 : $P_{134} = (3, 7, 1, 0)$
25 : $P_{185} = (6, 10, 1, 0)$
26 : $P_{186} = (7, 10, 1, 0)$
27 : $P_{207} = (12, 11, 1, 0)$
28 : $P_{208} = (13, 11, 1, 0)$
29 : $P_{215} = (4, 12, 1, 0)$
30 : $P_{216} = (5, 12, 1, 0)$
31 : $P_{241} = (14, 13, 1, 0)$
32 : $P_{242} = (15, 13, 1, 0)$
33 : $P_{290} = (0, 1, 0, 1)$
34 : $P_{291} = (1, 1, 0, 1)$
35 : $P_{306} = (0, 2, 0, 1)$
36 : $P_{318} = (12, 2, 0, 1)$
37 : $P_{322} = (0, 3, 0, 1)$
38 : $P_{330} = (8, 3, 0, 1)$
39 : $P_{338} = (0, 4, 0, 1)$
40 : $P_{344} = (6, 4, 0, 1)$
41 : $P_{354} = (0, 5, 0, 1)$
42 : $P_{369} = (15, 5, 0, 1)$
43 : $P_{370} = (0, 6, 0, 1)$
44 : $P_{374} = (4, 6, 0, 1)$
45 : $P_{386} = (0, 7, 0, 1)$
46 : $P_{400} = (14, 7, 0, 1)$
47 : $P_{402} = (0, 8, 0, 1)$
48 : $P_{405} = (3, 8, 0, 1)$
49 : $P_{418} = (0, 9, 0, 1)$
50 : $P_{431} = (13, 9, 0, 1)$
51 : $P_{434} = (0, 10, 0, 1)$
52 : $P_{445} = (11, 10, 0, 1)$
53 : $P_{450} = (0, 11, 0, 1)$
54 : $P_{460} = (10, 11, 0, 1)$
55 : $P_{466} = (0, 12, 0, 1)$
56 : $P_{468} = (2, 12, 0, 1)$
57 : $P_{482} = (0, 13, 0, 1)$
58 : $P_{491} = (9, 13, 0, 1)$
59 : $P_{498} = (0, 14, 0, 1)$
60 : $P_{505} = (7, 14, 0, 1)$
61 : $P_{514} = (0, 15, 0, 1)$
62 : $P_{519} = (5, 15, 0, 1)$
63 : $P_{530} = (0, 0, 1, 1)$
64 : $P_{546} = (0, 1, 1, 1)$
65 : $P_{566} = (5, 2, 1, 1)$
66 : $P_{569} = (8, 2, 1, 1)$
67 : $P_{601} = (8, 4, 1, 1)$
68 : $P_{608} = (15, 4, 1, 1)$
69 : $P_{676} = (3, 9, 1, 1)$
70 : $P_{688} = (15, 9, 1, 1)$
71 : $P_{690} = (1, 10, 1, 1)$
72 : $P_{700} = (11, 10, 1, 1)$
73 : $P_{706} = (1, 11, 1, 1)$
74 : $P_{715} = (10, 11, 1, 1)$
75 : $P_{756} = (3, 14, 1, 1)$
76 : $P_{758} = (5, 14, 1, 1)$
77 : $P_{785} = (0, 0, 2, 1)$
78 : $P_{817} = (0, 2, 2, 1)$
79 : $P_{831} = (14, 2, 2, 1)$
80 : $P_{836} = (3, 3, 2, 1)$
81 : $P_{842} = (9, 3, 2, 1)$
82 : $P_{851} = (2, 4, 2, 1)$
83 : $P_{855} = (6, 4, 2, 1)$
84 : $P_{883} = (2, 6, 2, 1)$
85 : $P_{885} = (4, 6, 2, 1)$
86 : $P_{917} = (4, 8, 2, 1)$
87 : $P_{918} = (5, 8, 2, 1)$
88 : $P_{951} = (6, 10, 2, 1)$
89 : $P_{960} = (15, 10, 2, 1)$
90 : $P_{966} = (5, 11, 2, 1)$
91 : $P_{974} = (13, 11, 2, 1)$
92 : $P_{980} = (3, 12, 2, 1)$
93 : $P_{1034} = (9, 15, 2, 1)$
94 : $P_{1039} = (14, 15, 2, 1)$
95 : $P_{1041} = (0, 0, 3, 1)$
96 : $P_{1065} = (8, 1, 3, 1)$
97 : $P_{1067} = (10, 1, 3, 1)$
98 : $P_{1075} = (2, 2, 3, 1)$
99 : $P_{1086} = (13, 2, 3, 1)$
100 : $P_{1089} = (0, 3, 3, 1)$
101 : $P_{1100} = (11, 3, 3, 1)$
102 : $P_{1147} = (10, 6, 3, 1)$
103 : $P_{1150} = (13, 6, 3, 1)$
104 : $P_{1154} = (1, 7, 3, 1)$
105 : $P_{1165} = (12, 7, 3, 1)$
106 : $P_{1171} = (2, 8, 3, 1)$
107 : $P_{1205} = (4, 10, 3, 1)$
108 : $P_{1213} = (12, 10, 3, 1)$
109 : $P_{1224} = (7, 11, 3, 1)$
110 : $P_{1231} = (14, 11, 3, 1)$
111 : $P_{1250} = (1, 13, 3, 1)$
112 : $P_{1260} = (11, 13, 3, 1)$
113 : $P_{1289} = (8, 15, 3, 1)$
114 : $P_{1295} = (14, 15, 3, 1)$
115 : $P_{1297} = (0, 0, 4, 1)$
116 : $P_{1347} = (2, 3, 4, 1)$
117 : $P_{1359} = (14, 3, 4, 1)$
118 : $P_{1361} = (0, 4, 4, 1)$
119 : $P_{1363} = (2, 4, 4, 1)$
120 : $P_{1382} = (5, 5, 4, 1)$
121 : $P_{1391} = (14, 5, 4, 1)$
122 : $P_{1398} = (5, 6, 4, 1)$
123 : $P_{1445} = (4, 9, 4, 1)$
124 : $P_{1454} = (13, 9, 4, 1)$
125 : $P_{1464} = (7, 10, 4, 1)$
126 : $P_{1465} = (8, 10, 4, 1)$
127 : $P_{1476} = (3, 11, 4, 1)$
128 : $P_{1486} = (13, 11, 4, 1)$
129 : $P_{1509} = (4, 13, 4, 1)$
130 : $P_{1514} = (9, 13, 4, 1)$
131 : $P_{1545} = (8, 15, 4, 1)$
132 : $P_{1546} = (9, 15, 4, 1)$
133 : $P_{1553} = (0, 0, 5, 1)$
134 : $P_{1580} = (11, 1, 5, 1)$
135 : $P_{1584} = (15, 1, 5, 1)$
136 : $P_{1603} = (2, 3, 5, 1)$
137 : $P_{1616} = (15, 3, 5, 1)$
138 : $P_{1621} = (4, 4, 5, 1)$
139 : $P_{1624} = (7, 4, 5, 1)$
140 : $P_{1633} = (0, 5, 5, 1)$
141 : $P_{1643} = (10, 5, 5, 1)$
142 : $P_{1666} = (1, 7, 5, 1)$
143 : $P_{1675} = (10, 7, 5, 1)$
144 : $P_{1715} = (2, 10, 5, 1)$
145 : $P_{1725} = (12, 10, 5, 1)$
146 : $P_{1735} = (6, 11, 5, 1)$
147 : $P_{1738} = (9, 11, 5, 1)$
148 : $P_{1746} = (1, 12, 5, 1)$
149 : $P_{1751} = (6, 12, 5, 1)$
150 : $P_{1768} = (7, 13, 5, 1)$
151 : $P_{1772} = (11, 13, 5, 1)$
152 : $P_{1797} = (4, 15, 5, 1)$
153 : $P_{1809} = (0, 0, 6, 1)$
154 : $P_{1846} = (5, 2, 6, 1)$
155 : $P_{1856} = (15, 2, 6, 1)$
156 : $P_{1880} = (7, 4, 6, 1)$
157 : $P_{1905} = (0, 6, 6, 1)$
158 : $P_{1907} = (2, 6, 6, 1)$
159 : $P_{1928} = (7, 7, 6, 1)$
160 : $P_{1936} = (15, 7, 6, 1)$
161 : $P_{1974} = (5, 10, 6, 1)$
162 : $P_{1977} = (8, 10, 6, 1)$
163 : $P_{1987} = (2, 11, 6, 1)$
164 : $P_{1999} = (14, 11, 6, 1)$
165 : $P_{2065} = (0, 0, 7, 1)$
166 : $P_{2166} = (5, 6, 7, 1)$
167 : $P_{2167} = (6, 6, 7, 1)$
168 : $P_{2177} = (0, 7, 7, 1)$
169 : $P_{2186} = (9, 7, 7, 1)$
170 : $P_{2214} = (5, 9, 7, 1)$
171 : $P_{2224} = (15, 9, 7, 1)$
172 : $P_{2228} = (3, 10, 7, 1)$
173 : $P_{2240} = (15, 10, 7, 1)$
174 : $P_{2245} = (4, 11, 7, 1)$
175 : $P_{2250} = (9, 11, 7, 1)$
176 : $P_{2295} = (6, 14, 7, 1)$
177 : $P_{2321} = (0, 0, 8, 1)$
178 : $P_{2340} = (3, 1, 8, 1)$
179 : $P_{2347} = (10, 1, 8, 1)$
180 : $P_{2378} = (9, 3, 8, 1)$
181 : $P_{2404} = (3, 5, 8, 1)$
182 : $P_{2405} = (4, 5, 8, 1)$

183 : $P_{2418} = (1, 6, 8, 1)$
184 : $P_{2430} = (13, 6, 8, 1)$
185 : $P_{2443} = (10, 7, 8, 1)$
186 : $P_{2445} = (12, 7, 8, 1)$
187 : $P_{2449} = (0, 8, 8, 1)$
188 : $P_{2460} = (11, 8, 8, 1)$
189 : $P_{2474} = (9, 9, 8, 1)$
190 : $P_{2477} = (12, 9, 8, 1)$
191 : $P_{2494} = (13, 10, 8, 1)$
192 : $P_{2495} = (14, 10, 8, 1)$
193 : $P_{2501} = (4, 11, 8, 1)$
194 : $P_{2503} = (6, 11, 8, 1)$
195 : $P_{2514} = (1, 12, 8, 1)$
196 : $P_{2524} = (11, 12, 8, 1)$
197 : $P_{2577} = (0, 0, 9, 1)$
198 : $P_{2639} = (14, 3, 9, 1)$
199 : $P_{2640} = (15, 3, 9, 1)$
200 : $P_{2659} = (2, 5, 9, 1)$
201 : $P_{2661} = (4, 5, 9, 1)$
202 : $P_{2698} = (9, 7, 9, 1)$
203 : $P_{2703} = (14, 7, 9, 1)$
204 : $P_{2707} = (2, 8, 9, 1)$
205 : $P_{2713} = (8, 8, 9, 1)$
206 : $P_{2721} = (0, 9, 9, 1)$
207 : $P_{2725} = (4, 9, 9, 1)$
208 : $P_{2742} = (5, 10, 9, 1)$
209 : $P_{2744} = (7, 10, 9, 1)$
210 : $P_{2765} = (12, 11, 9, 1)$
211 : $P_{2768} = (15, 11, 9, 1)$
212 : $P_{2793} = (8, 13, 9, 1)$
213 : $P_{2808} = (7, 14, 9, 1)$
214 : $P_{2810} = (9, 14, 9, 1)$
215 : $P_{2833} = (0, 0, 10, 1)$
216 : $P_{2852} = (3, 1, 10, 1)$
217 : $P_{2857} = (8, 1, 10, 1)$
218 : $P_{2900} = (3, 4, 10, 1)$
219 : $P_{2912} = (15, 4, 10, 1)$
220 : $P_{2923} = (10, 5, 10, 1)$
221 : $P_{2928} = (15, 5, 10, 1)$
222 : $P_{2993} = (0, 10, 10, 1)$
223 : $P_{2994} = (1, 10, 10, 1)$
224 : $P_{3020} = (11, 11, 10, 1)$
225 : $P_{3062} = (5, 14, 10, 1)$
226 : $P_{3065} = (8, 14, 10, 1)$
227 : $P_{3078} = (5, 15, 10, 1)$
228 : $P_{3083} = (10, 15, 10, 1)$
229 : $P_{3089} = (0, 0, 11, 1)$
230 : $P_{3110} = (5, 1, 11, 1)$
231 : $P_{3120} = (15, 1, 11, 1)$
232 : $P_{3129} = (8, 2, 11, 1)$
233 : $P_{3136} = (15, 2, 11, 1)$
234 : $P_{3145} = (8, 3, 11, 1)$
235 : $P_{3148} = (11, 3, 11, 1)$
236 : $P_{3220} = (3, 8, 11, 1)$
237 : $P_{3228} = (11, 8, 11, 1)$
238 : $P_{3236} = (3, 9, 11, 1)$
239 : $P_{3238} = (5, 9, 11, 1)$
240 : $P_{3259} = (10, 10, 11, 1)$
241 : $P_{3265} = (0, 11, 11, 1)$
242 : $P_{3266} = (1, 11, 11, 1)$
243 : $P_{3345} = (0, 0, 12, 1)$
244 : $P_{3390} = (13, 2, 12, 1)$
245 : $P_{3514} = (9, 10, 12, 1)$
246 : $P_{3519} = (14, 10, 12, 1)$
247 : $P_{3524} = (3, 11, 12, 1)$
248 : $P_{3526} = (5, 11, 12, 1)$
249 : $P_{3537} = (0, 12, 12, 1)$
250 : $P_{3551} = (14, 12, 12, 1)$
251 : $P_{3561} = (8, 13, 12, 1)$
252 : $P_{3566} = (13, 13, 12, 1)$
253 : $P_{3572} = (3, 14, 12, 1)$
254 : $P_{3577} = (8, 14, 12, 1)$
255 : $P_{3601} = (0, 0, 13, 1)$
256 : $P_{3668} = (3, 4, 13, 1)$
257 : $P_{3673} = (8, 4, 13, 1)$
258 : $P_{3757} = (12, 9, 13, 1)$
259 : $P_{3763} = (2, 10, 13, 1)$
260 : $P_{3765} = (4, 10, 13, 1)$
261 : $P_{3785} = (8, 11, 13, 1)$
262 : $P_{3792} = (15, 11, 13, 1)$
263 : $P_{3796} = (3, 12, 13, 1)$
264 : $P_{3805} = (12, 12, 13, 1)$
265 : $P_{3809} = (0, 13, 13, 1)$
266 : $P_{3813} = (4, 13, 13, 1)$
267 : $P_{3857} = (0, 0, 14, 1)$
268 : $P_{3901} = (12, 2, 14, 1)$
269 : $P_{3903} = (14, 2, 14, 1)$
270 : $P_{3939} = (2, 5, 14, 1)$
271 : $P_{3940} = (3, 5, 14, 1)$
272 : $P_{3984} = (15, 7, 14, 1)$
273 : $P_{3989} = (4, 8, 14, 1)$
274 : $P_{3994} = (9, 8, 14, 1)$
275 : $P_{4020} = (3, 10, 14, 1)$
276 : $P_{4023} = (6, 10, 14, 1)$
277 : $P_{4041} = (8, 11, 14, 1)$
278 : $P_{4045} = (12, 11, 14, 1)$
279 : $P_{4051} = (2, 12, 14, 1)$
280 : $P_{4063} = (14, 12, 14, 1)$
281 : $P_{4081} = (0, 14, 14, 1)$
282 : $P_{4090} = (9, 14, 14, 1)$
283 : $P_{4101} = (4, 15, 14, 1)$
284 : $P_{4112} = (15, 15, 14, 1)$
285 : $P_{4113} = (0, 0, 15, 1)$
286 : $P_{4134} = (5, 1, 15, 1)$
287 : $P_{4140} = (11, 1, 15, 1)$
288 : $P_{4207} = (14, 5, 15, 1)$
289 : $P_{4210} = (1, 6, 15, 1)$
290 : $P_{4219} = (10, 6, 15, 1)$
291 : $P_{4246} = (5, 8, 15, 1)$
292 : $P_{4250} = (9, 8, 15, 1)$
293 : $P_{4282} = (9, 10, 15, 1)$
294 : $P_{4286} = (13, 10, 15, 1)$
295 : $P_{4291} = (2, 11, 15, 1)$
296 : $P_{4296} = (7, 11, 15, 1)$
297 : $P_{4311} = (6, 12, 15, 1)$
298 : $P_{4316} = (11, 12, 15, 1)$
299 : $P_{4322} = (1, 13, 15, 1)$
300 : $P_{4328} = (7, 13, 15, 1)$
301 : $P_{4343} = (6, 14, 15, 1)$
302 : $P_{4351} = (14, 14, 15, 1)$
303 : $P_{4353} = (0, 15, 15, 1)$
304 : $P_{4363} = (10, 15, 15, 1)$