Rank-65899 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_3^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(64)$ is -2130177979

General information

Number of lines	8
Number of points	4289
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	498
Number of points off lines	3782
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 ⁸
Type of lines on points	$4, 3^2, 2^6, 1^{498}, 0^{3782}$

Singular Points

The surface has 3 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0) \\ 1: P_{7747} = \mathbf{P}(1,\epsilon^{42},0,1) = \mathbf{P}(1,56,0,1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_{1} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{237112} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{237112} = \mathbf{PI}(0,0,57,0,0,1)_{277640}$$

$$\ell_{2} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{241273} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{241273} = \mathbf{PI}(0,0,56,0,0,1)_{277513}$$

$$\ell_{3} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \mathbf{PI}(1,0,0,1,0,0)_{130}$$

$$\ell_{4} = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{503416} = \begin{bmatrix} 1 & 56 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{503416} = \mathbf{PI}(0,57,57,0,0,1)_{277760}$$

$$\ell_{5} = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{507577} = \begin{bmatrix} 1 & 57 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{507577} = \mathbf{PI}(0,56,56,0,0,1)_{277632}$$

$$\ell_{6} = \begin{bmatrix} 1 & 0 & \epsilon^{21} & \epsilon^{42} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15150266} = \begin{bmatrix} 1 & 0 & 57 & 56 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15150266} = \mathbf{PI}(57,56,56,57,56,1)_{15185330}$$

$$\ell_{7} = \begin{bmatrix} 1 & 0 & \epsilon^{42} & \epsilon^{21} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15412409} = \begin{bmatrix} 1 & 0 & 56 & 57 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15412409} = \mathbf{PI}(56,57,57,56,57,1)_{15451252}$$

Rank of lines: (0, 237112, 241273, 266304, 503416, 507577, 15150266, 15412409) Rank of points on Klein quadric: (0, 277640, 277513, 130, 277760, 277632, 15185330, 15451252)

Eckardt Points

The surface has 2 Eckardt points:

$$0: P_{7747} = \mathbf{P}(1, \epsilon^{42}, 0, 1) = \mathbf{P}(1, 56, 0, 1),$$

1:
$$P_{7811} = \mathbf{P}(1, \epsilon^{21}, 0, 1) = \mathbf{P}(1, 57, 0, 1).$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_{61} = (57, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{60} = (56, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{3652} = (1, 56, 1, 0) = \ell_1 \cap \ell_6$$

$$P_{3716} = (1, 57, 1, 0) = \ell_2 \cap \ell_7$$

$$P_{8322} = (0, 1, 1, 1) = \ell_6 \cap \ell_7$$

Single Points

The surface has 498 single points: The single points on the surface are:

 $0: P_0 = (1, 0, 0, 0)$ lies on line ℓ_0 9: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 1 : $P_5 = (1,1,0,0)$ lies on line ℓ_0 10: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 11 : $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 2: $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 3: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 12: $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 4: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 13: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 5: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 14: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 6: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 15: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 7: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 16: $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0 8: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 17: $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0

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18: P_{22} = (18, 1, 0, 0) lies on line \ell_0
                                                                      72: P_{502} = (51, 6, 1, 0) lies on line \ell_2
19: P_{23} = (19, 1, 0, 0) lies on line \ell_0
                                                                      73: P_{504} = (53, 6, 1, 0) lies on line \ell_1
20: P_{24} = (20, 1, 0, 0) lies on line \ell_0
                                                                      74: P_{526} = (11, 7, 1, 0) lies on line \ell_2
21: P_{25} = (21, 1, 0, 0) lies on line \ell_0
                                                                      75: P_{527} = (12, 7, 1, 0) lies on line \ell_1
22: P_{26} = (22, 1, 0, 0) lies on line \ell_0
                                                                      76: P_{616} = (37, 8, 1, 0) lies on line \ell_2
23: P_{27} = (23, 1, 0, 0) lies on line \ell_0
                                                                      77: P_{624} = (45, 8, 1, 0) lies on line \ell_1
                                                                      78 : P_{663} = (20, 9, 1, 0) lies on line \ell_1
24: P_{28} = (24, 1, 0, 0) lies on line \ell_0
25: P_{29} = (25, 1, 0, 0) lies on line \ell_0
                                                                      79: P_{672} = (29, 9, 1, 0) lies on line \ell_2
                                                                      80 : P_{759} = (52, 10, 1, 0) lies on line \ell_2
26: P_{30} = (26, 1, 0, 0) lies on line \ell_0
27: P_{31} = (27, 1, 0, 0) lies on line \ell_0
                                                                      81: P_{769} = (62, 10, 1, 0) lies on line \ell_1
28: P_{32} = (28, 1, 0, 0) lies on line \ell_0
                                                                      82: P_{778} = (7, 11, 1, 0) lies on line \ell_1
29: P_{33} = (29, 1, 0, 0) lies on line \ell_0
                                                                      83: P_{783} = (12, 11, 1, 0) lies on line \ell_2
30: P_{34} = (30, 1, 0, 0) lies on line \ell_0
                                                                      84: P_{842} = (7, 12, 1, 0) lies on line \ell_2
31 : P_{35} = (31, 1, 0, 0) lies on line \ell_0
                                                                      85: P_{846} = (11, 12, 1, 0) lies on line \ell_1
32: P_{36} = (32, 1, 0, 0) lies on line \ell_0
                                                                      86: P_{949} = (50, 13, 1, 0) lies on line \ell_1
33: P_{37} = (33, 1, 0, 0) lies on line \ell_0
                                                                      87: P_{962} = (63, 13, 1, 0) lies on line \ell_2
34: P_{38} = (34, 1, 0, 0) lies on line \ell_0
                                                                      88: P_{985} = (22, 14, 1, 0) lies on line \ell_2
35: P_{39} = (35, 1, 0, 0) lies on line \ell_0
                                                                      89: P_{987} = (24, 14, 1, 0) lies on line \ell_1
36: P_{40} = (36, 1, 0, 0) lies on line \ell_0
                                                                      90: P_{1060} = (33, 15, 1, 0) lies on line \ell_1
37: P_{41} = (37, 1, 0, 0) lies on line \ell_0
                                                                      91: P_{1073} = (46, 15, 1, 0) lies on line \ell_2
38 : P_{42} = (38,1,0,0) lies on line \ell_0
                                                                      92: P_{1134} = (43, 16, 1, 0) lies on line \ell_2
39: P_{43} = (39, 1, 0, 0) lies on line \ell_0
                                                                      93: P_{1150} = (59, 16, 1, 0) lies on line \ell_1
40: P_{44} = (40, 1, 0, 0) lies on line \ell_0
                                                                      94: P_{1157} = (2, 17, 1, 0) lies on line \ell_1
41: P_{45} = (41, 1, 0, 0) lies on line \ell_0
                                                                      95: P_{1174} = (19, 17, 1, 0) lies on line \ell_2
42: P_{46} = (42, 1, 0, 0) lies on line \ell_0
                                                                      96: P_{1259} = (40, 18, 1, 0) lies on line \ell_1
43: P_{47} = (43, 1, 0, 0) lies on line \ell_0
                                                                      97: P_{1277} = (58, 18, 1, 0) lies on line \ell_2
44: P_{48} = (44, 1, 0, 0) lies on line \ell_0
                                                                      98: P_{1285} = (2, 19, 1, 0) lies on line \ell_2
45: P_{49} = (45, 1, 0, 0) lies on line \ell_0
                                                                      99: P_{1300} = (17, 19, 1, 0) lies on line \ell_1
46: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                      100: P_{1356} = (9, 20, 1, 0) lies on line \ell_2
47: P_{51} = (47, 1, 0, 0) lies on line \ell_0
                                                                      101: P_{1376} = (29, 20, 1, 0) lies on line \ell_1
48: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                      102: P_{1447} = (36, 21, 1, 0) lies on line \ell_1
49: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                      103: P_{1460} = (49, 21, 1, 0) lies on line \ell_2
50: P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                      104: P_{1489} = (14, 22, 1, 0) lies on line \ell_1
51: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                      105: P_{1499} = (24, 22, 1, 0) lies on line \ell_2
52: P_{56} = (52, 1, 0, 0) lies on line \ell_0
                                                                      106: P_{1571} = (32, 23, 1, 0) lies on line \ell_2
53: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                      107: P_{1594} = (55, 23, 1, 0) lies on line \ell_1
54: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                      108: P_{1617} = (14, 24, 1, 0) lies on line \ell_2
55: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                      109: P_{1625} = (22, 24, 1, 0) lies on line \ell_1
56: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                      110: P_{1714} = (47, 25, 1, 0) lies on line \ell_1
57: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                      111: P_{1721} = (54, 25, 1, 0) lies on line \ell_2
58: P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                      112: P_{1736} = (5, 26, 1, 0) lies on line \ell_1
59: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                      113: P_{1762} = (31, 26, 1, 0) lies on line \ell_2
                                                                      114: P_{1834} = (39, 27, 1, 0) lies on line \ell_2
60: P_{66} = (62, 1, 0, 0) lies on line \ell_0
61: P_{67} = (63, 1, 0, 0) lies on line \ell_0
                                                                      115: P_{1855} = (60, 27, 1, 0) lies on line \ell_1
62: P_{187} = (56, 1, 1, 0) lies on line \ell_2
                                                                      116: P_{1903} = (44, 28, 1, 0) lies on line \ell_2
63: P_{188} = (57, 1, 1, 0) lies on line \ell_1
                                                                      117: P_{1907} = (48, 28, 1, 0) lies on line \ell_1
64: P_{212} = (17, 2, 1, 0) lies on line \ell_2
                                                                      118: P_{1932} = (9, 29, 1, 0) lies on line \ell_1
65: P_{214} = (19, 2, 1, 0) lies on line \ell_1
                                                                      119: P_{1943} = (20, 29, 1, 0) lies on line \ell_2
66: P_{300} = (41, 3, 1, 0) lies on line \ell_2
                                                                      120: P_{2022} = (35, 30, 1, 0) lies on line \ell_1
67: P_{301} = (42, 3, 1, 0) lies on line \ell_1
                                                                      121: P_{2048} = (61, 30, 1, 0) lies on line \ell_2
68: P_{357} = (34, 4, 1, 0) lies on line \ell_2
                                                                      122: P_{2056} = (5, 31, 1, 0) lies on line \ell_2
69: P_{361} = (38, 4, 1, 0) lies on line \ell_1
                                                                      123: P_{2077} = (26, 31, 1, 0) lies on line \ell_1
70: P_{413} = (26, 5, 1, 0) lies on line \ell_2
                                                                      124: P_{2138} = (23, 32, 1, 0) lies on line \ell_1
71 : P_{418} = (31, 5, 1, 0) lies on line \ell_1
                                                                      125: P_{2170} = (55, 32, 1, 0) lies on line \ell_2
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126: P_{2194} = (15, 33, 1, 0) lies on line \ell_2
                                                                     180: P_{4001} = (30, 61, 1, 0) lies on line \ell_1
127: P_{2225} = (46, 33, 1, 0) lies on line \ell_1
                                                                     181: P_{4006} = (35, 61, 1, 0) lies on line \ell_2
128: P_{2247} = (4, 34, 1, 0) lies on line \ell_1
                                                                     182 : P_{4045} = (10, 62, 1, 0) lies on line \ell_2
129: P_{2281} = (38, 34, 1, 0) lies on line \ell_2
                                                                     183 : P_{4087} = (52, 62, 1, 0) lies on line \ell_1
130: P_{2337} = (30, 35, 1, 0) lies on line \ell_2
                                                                     184: P_{4112} = (13, 63, 1, 0) lies on line \ell_1
131: P_{2368} = (61, 35, 1, 0) lies on line \ell_1
                                                                     185: P_{4149} = (50, 63, 1, 0) lies on line \ell_2
132: P_{2392} = (21, 36, 1, 0) lies on line \ell_2
                                                                     186: P_{4163} = (1,0,0,1) lies on line \ell_3
133: P_{2420} = (49, 36, 1, 0) lies on line \ell_1
                                                                     187: P_{4227} = (1, 1, 0, 1) lies on line \ell_3
134 : P_{2443} = (8, 37, 1, 0) lies on line \ell_1
                                                                     188 : P_{4291} = (1, 2, 0, 1) lies on line \ell_3
135: P_{2480} = (45, 37, 1, 0) lies on line \ell_2
                                                                     189: P_{4355} = (1, 3, 0, 1) lies on line \ell_3
136: P_{2503} = (4, 38, 1, 0) lies on line \ell_2
                                                                     190: P_{4419} = (1, 4, 0, 1) lies on line \ell_3
137: P_{2533} = (34, 38, 1, 0) lies on line \ell_1
                                                                     191: P_{4483} = (1, 5, 0, 1) lies on line \ell_3
138: P_{2590} = (27, 39, 1, 0) lies on line \ell_1
                                                                     192: P_{4547} = (1, 6, 0, 1) lies on line \ell_3
139: P_{2623} = (60, 39, 1, 0) lies on line \ell_2
                                                                     193: P_{4611} = (1,7,0,1) lies on line \ell_3
                                                                     194 : P_{4675} = (1, 8, 0, 1) lies on line \ell_3
140: P_{2645} = (18, 40, 1, 0) lies on line \ell_2
141: P_{2685} = (58, 40, 1, 0) lies on line \ell_1
                                                                     195: P_{4739} = (1, 9, 0, 1) lies on line \ell_3
142: P_{2694} = (3, 41, 1, 0) lies on line \ell_1
                                                                     196: P_{4803} = (1, 10, 0, 1) lies on line \ell_3
143: P_{2733} = (42, 41, 1, 0) lies on line \ell_2
                                                                     197: P_{4867} = (1, 11, 0, 1) lies on line \ell_3
144: P_{2758} = (3, 42, 1, 0) lies on line \ell_2
                                                                     198: P_{4931} = (1, 12, 0, 1) lies on line \ell_3
145: P_{2796} = (41, 42, 1, 0) lies on line \ell_1
                                                                     199: P_{4995} = (1, 13, 0, 1) lies on line \ell_3
146: P_{2835} = (16, 43, 1, 0) lies on line \ell_1
                                                                     200: P_{5059} = (1, 14, 0, 1) lies on line \ell_3
147 : P_{2878} = (59, 43, 1, 0) lies on line \ell_2
                                                                     201: P_{5123} = (1, 15, 0, 1) lies on line \ell_3
148: P_{2911} = (28, 44, 1, 0) lies on line \ell_1
                                                                     202: P_{5187} = (1, 16, 0, 1) lies on line \ell_3
149: P_{2931} = (48, 44, 1, 0) lies on line \ell_2
                                                                     203: P_{5251} = (1, 17, 0, 1) lies on line \ell_3
150: P_{2955} = (8, 45, 1, 0) lies on line \ell_2
                                                                     204: P_{5315} = (1, 18, 0, 1) lies on line \ell_3
151: P_{2984} = (37, 45, 1, 0) lies on line \ell_1
                                                                     205 : P_{5379} = (1, 19, 0, 1) lies on line \ell_3
152: P_{3026} = (15, 46, 1, 0) lies on line \ell_1
                                                                     206: P_{5443} = (1, 20, 0, 1) lies on line \ell_3
153: P_{3044} = (33, 46, 1, 0) lies on line \ell_2
                                                                     207: P_{5507} = (1, 21, 0, 1) lies on line \ell_3
154: P_{3100} = (25, 47, 1, 0) lies on line \ell_2
                                                                     208: P_{5571} = (1, 22, 0, 1) lies on line \ell_3
155: P_{3129} = (54, 47, 1, 0) lies on line \ell_1
                                                                     209: P_{5635} = (1, 23, 0, 1) lies on line \ell_3
156: P_{3167} = (28, 48, 1, 0) lies on line \ell_2
                                                                     210: P_{5699} = (1, 24, 0, 1) lies on line \ell_3
157: P_{3183} = (44, 48, 1, 0) lies on line \ell_1
                                                                     211: P_{5763} = (1, 25, 0, 1) lies on line \ell_3
158: P_{3224} = (21, 49, 1, 0) lies on line \ell_1
                                                                     212: P_{5827} = (1, 26, 0, 1) lies on line \ell_3
159: P_{3239} = (36, 49, 1, 0) lies on line \ell_2
                                                                     213: P_{5891} = (1, 27, 0, 1) lies on line \ell_3
160: P_{3280} = (13, 50, 1, 0) lies on line \ell_2
                                                                     214: P_{5955} = (1, 28, 0, 1) lies on line \ell_3
161: P_{3330} = (63, 50, 1, 0) lies on line \ell_1
                                                                     215: P_{6019} = (1, 29, 0, 1) lies on line \ell_3
162: P_{3337} = (6, 51, 1, 0) lies on line \ell_1
                                                                     216: P_{6083} = (1, 30, 0, 1) lies on line \ell_3
163: P_{3384} = (53, 51, 1, 0) lies on line \ell_2
                                                                     217: P_{6147} = (1, 31, 0, 1) lies on line \ell_3
164: P_{3405} = (10, 52, 1, 0) lies on line \ell_1
                                                                     218: P_{6211} = (1, 32, 0, 1) lies on line \ell_3
165: P_{3457} = (62, 52, 1, 0) lies on line \ell_2
                                                                     219: P_{6275} = (1, 33, 0, 1) lies on line \ell_3
166: P_{3465} = (6, 53, 1, 0) lies on line \ell_2
                                                                     220: P_{6339} = (1, 34, 0, 1) lies on line \ell_3
167: P_{3510} = (51, 53, 1, 0) lies on line \ell_1
                                                                     221: P_{6403} = (1, 35, 0, 1) lies on line \ell_3
168: P_{3548} = (25, 54, 1, 0) lies on line \ell_1
                                                                     222: P_{6467} = (1, 36, 0, 1) lies on line \ell_3
169: P_{3570} = (47, 54, 1, 0) lies on line \ell_2
                                                                     223: P_{6531} = (1, 37, 0, 1) lies on line \ell_3
170: P_{3610} = (23, 55, 1, 0) lies on line \ell_2
                                                                     224: P_{6595} = (1, 38, 0, 1) lies on line \ell_3
171: P_{3619} = (32, 55, 1, 0) lies on line \ell_1
                                                                     225: P_{6659} = (1, 39, 0, 1) lies on line \ell_3
172: P_{3708} = (57, 56, 1, 0) lies on line \ell_2
                                                                     226: P_{6723} = (1, 40, 0, 1) lies on line \ell_3
173: P_{3771} = (56, 57, 1, 0) lies on line \ell_1
                                                                     227: P_{6787} = (1, 41, 0, 1) lies on line \ell_3
174: P_{3797} = (18, 58, 1, 0) lies on line \ell_1
                                                                     228: P_{6851} = (1, 42, 0, 1) lies on line \ell_3
175: P_{3819} = (40, 58, 1, 0) lies on line \ell_2
                                                                     229: P_{6915} = (1, 43, 0, 1) lies on line \ell_3
176: P_{3859} = (16, 59, 1, 0) lies on line \ell_2
                                                                     230: P_{6979} = (1, 44, 0, 1) lies on line \ell_3
177: P_{3886} = (43, 59, 1, 0) lies on line \ell_1
                                                                     231: P_{7043} = (1, 45, 0, 1) lies on line \ell_3
                                                                     232 : P_{7107} = (1, 46, 0, 1) lies on line \ell_3
178: P_{3934} = (27, 60, 1, 0) lies on line \ell_2
179: P_{3946} = (39, 60, 1, 0) lies on line \ell_1
                                                                     233: P_{7171} = (1, 47, 0, 1) lies on line \ell_3
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288 : P_{52866} = (1, 57, 11, 1) lies on line \ell_5
234: P_{7235} = (1, 48, 0, 1) lies on line \ell_3
                                                                     289: P_{53259} = (10, 63, 11, 1) lies on line \ell_7
235: P_{7299} = (1, 49, 0, 1) lies on line \ell_3
236: P_{7363} = (1, 50, 0, 1) lies on line \ell_3
                                                                     290: P_{56590} = (13, 51, 12, 1) lies on line \ell_7
237: P_{7427} = (1, 51, 0, 1) lies on line \ell_3
                                                                     291 : P_{56898} = (1, 56, 12, 1) lies on line \ell_4
                                                                     292 : P_{56962} = (1, 57, 12, 1) lies on line \ell_5
238 : P_{7491} = (1, 52, 0, 1) lies on line \ell_3
239: P_{7555} = (1, 53, 0, 1) lies on line \ell_3
                                                                     293: P_{57294} = (13, 62, 12, 1) lies on line \ell_6
240: P_{7619} = (1, 54, 0, 1) lies on line \ell_3
                                                                     294: P_{57805} = (12, 6, 13, 1) lies on line \ell_6
                                                                     295: P_{58061} = (12, 10, 13, 1) lies on line \ell_7
241: P_{7683} = (1, 55, 0, 1) lies on line \ell_3
242 : P_{7875} = (1, 58, 0, 1) lies on line \ell_3
                                                                     296: P_{60994} = (1, 56, 13, 1) lies on line \ell_4
243: P_{7939} = (1, 59, 0, 1) lies on line \ell_3
                                                                     297: P_{61058} = (1, 57, 13, 1) lies on line \ell_5
244: P_{8003} = (1, 60, 0, 1) lies on line \ell_3
                                                                     298: P_{63568} = (15, 32, 14, 1) lies on line \ell_7
245: P_{8067} = (1, 61, 0, 1) lies on line \ell_3
                                                                     299: P_{64528} = (15, 47, 14, 1) lies on line \ell_6
246: P_{8131} = (1, 62, 0, 1) lies on line \ell_3
                                                                     300: P_{65090} = (1, 56, 14, 1) lies on line \ell_4
247: P_{8195} = (1, 63, 0, 1) lies on line \ell_3
                                                                     301: P_{65154} = (1, 57, 14, 1) lies on line \ell_5
248: P_{11842} = (1, 56, 1, 1) lies on line \ell_4
                                                                     302: P_{67087} = (14, 23, 15, 1) lies on line \ell_6
249: P_{11906} = (1, 57, 1, 1) lies on line \ell_5
                                                                     303: P_{67215} = (14, 25, 15, 1) lies on line \ell_7
250: P_{14916} = (3, 40, 2, 1) lies on line \ell_6
                                                                     304: P_{69186} = (1, 56, 15, 1) lies on line \ell_4
251: P_{15108} = (3, 43, 2, 1) lies on line \ell_7
                                                                     305: P_{69250} = (1, 57, 15, 1) lies on line \ell_5
252 : P_{15938} = (1, 56, 2, 1) lies on line \ell_4
                                                                     306: P_{69906} = (17, 3, 16, 1) lies on line \ell_7
253: P_{16002} = (1, 57, 2, 1) lies on line \ell_5
                                                                     307: P_{70866} = (17, 18, 16, 1) lies on line \ell_6
254: P_{17475} = (2, 16, 3, 1) lies on line \ell_6
                                                                     308: P_{73282} = (1, 56, 16, 1) lies on line \ell_4
255: P_{17603} = (2, 18, 3, 1) lies on line \ell_7
                                                                     309: P_{73346} = (1, 57, 16, 1) lies on line \ell_5
                                                                     310: P_{76497} = (16, 42, 17, 1) lies on line \ell_6
256: P_{20034} = (1, 56, 3, 1) lies on line \ell_4
257 : P_{20098} = (1, 57, 3, 1) lies on line \ell_5
                                                                     311: P_{77378} = (1, 56, 17, 1) lies on line \ell_4
258: P_{22278} = (5, 27, 4, 1) lies on line \ell_6
                                                                     312: P_{77442} = (1, 57, 17, 1) lies on line \ell_5
259: P_{22470} = (5, 30, 4, 1) lies on line \ell_7
                                                                     313: P_{77521} = (16, 58, 17, 1) lies on line \ell_7
260: P_{24130} = (1, 56, 4, 1) lies on line \ell_4
                                                                     314: P_{78100} = (19, 3, 18, 1) lies on line \ell_6
261: P_{24194} = (1, 57, 4, 1) lies on line \ell_5
                                                                     315: P_{78932} = (19, 16, 18, 1) lies on line \ell_7
262: P_{26885} = (4, 35, 5, 1) lies on line \ell_6
                                                                     316: P_{81474} = (1, 56, 18, 1) lies on line \ell_4
263: P_{27141} = (4, 39, 5, 1) lies on line \ell_7
                                                                     317: P_{81538} = (1, 57, 18, 1) lies on line \ell_5
264: P_{28226} = (1, 56, 5, 1) lies on line \ell_4
                                                                     318: P_{84627} = (18, 41, 19, 1) lies on line \ell_7
265: P_{28290} = (1, 57, 5, 1) lies on line \ell_5
                                                                     319: P_{85570} = (1, 56, 19, 1) lies on line \ell_4
266: P_{29384} = (7, 10, 6, 1) lies on line \ell_6
                                                                     320: P_{85634} = (1, 57, 19, 1) lies on line \ell_5
267: P_{29576} = (7, 13, 6, 1) lies on line \ell_7
                                                                     321: P_{85779} = (18, 59, 19, 1) lies on line \ell_6
268: P_{32322} = (1, 56, 6, 1) lies on line \ell_4
                                                                     322: P_{88470} = (21, 37, 20, 1) lies on line \ell_7
                                                                     323: P_{89174} = (21, 48, 20, 1) lies on line \ell_6
269: P_{32386} = (1, 57, 6, 1) lies on line \ell_5
270: P_{36039} = (6, 50, 7, 1) lies on line \ell_6
                                                                     324: P_{89666} = (1, 56, 20, 1) lies on line \ell_4
271: P_{36167} = (6, 52, 7, 1) lies on line \ell_7
                                                                     325: P_{89730} = (1, 57, 20, 1) lies on line \ell_5
272: P_{36418} = (1, 56, 7, 1) lies on line \ell_4
                                                                     326: P_{90709} = (20, 8, 21, 1) lies on line \ell_6
273: P_{36482} = (1, 57, 7, 1) lies on line \ell_5
                                                                     327: P_{91989} = (20, 28, 21, 1) lies on line \ell_7
                                                                     328: P_{93762} = (1, 56, 21, 1) lies on line \ell_4
274: P_{38282} = (9, 21, 8, 1) lies on line \ell_7
275: P_{38730} = (9, 28, 8, 1) lies on line \ell_6
                                                                     329: P_{93826} = (1, 57, 21, 1) lies on line \ell_5
276: P_{40514} = (1, 56, 8, 1) lies on line \ell_4
                                                                     330: P_{96408} = (23, 33, 22, 1) lies on line \ell_6
277: P_{40578} = (1, 57, 8, 1) lies on line \ell_5
                                                                     331: P_{97752} = (23, 54, 22, 1) lies on line \ell_7
278: P_{43337} = (8, 36, 9, 1) lies on line \ell_6
                                                                     332: P_{97858} = (1, 56, 22, 1) lies on line \ell_4
279: P_{43849} = (8, 44, 9, 1) lies on line \ell_7
                                                                     333: P_{97922} = (1, 57, 22, 1) lies on line \ell_5
280: P_{44610} = (1, 56, 9, 1) lies on line \ell_4
                                                                     334: P_{99351} = (22, 15, 23, 1) lies on line \ell_7
281: P_{44674} = (1, 57, 9, 1) lies on line \ell_5
                                                                     335: P_{99991} = (22, 25, 23, 1) lies on line \ell_6
282 : P_{45516} = (11, 6, 10, 1) lies on line \ell_7
                                                                     336: P_{101954} = (1, 56, 23, 1) lies on line \ell_4
283: P_{45964} = (11, 13, 10, 1) lies on line \ell_6
                                                                     337: P_{102018} = (1, 57, 23, 1) lies on line \ell_5
284: P_{48706} = (1, 56, 10, 1) lies on line \ell_4
                                                                     338: P_{105434} = (25, 46, 24, 1) lies on line \ell_7
285: P_{48770} = (1, 57, 10, 1) lies on line \ell_5
                                                                     339: P_{106010} = (25, 55, 24, 1) lies on line \ell_6
286: P_{52619} = (10, 53, 11, 1) lies on line \ell_6
                                                                     340: P_{106050} = (1, 56, 24, 1) lies on line \ell_4
287: P_{52802} = (1, 56, 11, 1) lies on line \ell_4
                                                                     341: P_{106114} = (1, 57, 24, 1) lies on line \ell_5
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342: P_{107545} = (24, 15, 25, 1) lies on line \ell_6
                                                                    396: P_{163458} = (1, 57, 38, 1) lies on line \ell_5
343 : P_{108057} = (24, 23, 25, 1) lies on line \ell_7
                                                                    397: P_{163752} = (39, 61, 38, 1) lies on line \ell_6
                                                                    398: P_{164263} = (38, 5, 39, 1) lies on line \ell_6
344: P_{110146} = (1, 56, 25, 1) lies on line \ell_4
345: P_{110210} = (1, 57, 25, 1) lies on line \ell_5
                                                                    399: P_{166183} = (38, 35, 39, 1) lies on line \ell_7
                                                                    400: P_{167490} = (1, 56, 39, 1) lies on line \ell_4
346: P_{113116} = (27, 38, 26, 1) lies on line \ell_6
347: P_{114242} = (1, 56, 26, 1) lies on line \ell_4
                                                                    401: P_{167554} = (1, 57, 39, 1) lies on line \ell_5
348: P_{114306} = (1, 57, 26, 1) lies on line \ell_5
                                                                    402: P_{168170} = (41, 2, 40, 1) lies on line \ell_7
349: P_{114588} = (27, 61, 26, 1) lies on line \ell_7
                                                                    403: P_{170794} = (41, 43, 40, 1) lies on line \ell_6
350: P_{115035} = (26, 4, 27, 1) lies on line \ell_7
                                                                    404: P_{171586} = (1, 56, 40, 1) lies on line \ell_4
351: P_{116699} = (26, 30, 27, 1) lies on line \ell_6
                                                                    405: P_{171650} = (1, 57, 40, 1) lies on line \ell_5
352: P_{118338} = (1, 56, 27, 1) lies on line \ell_4
                                                                    406: P_{173353} = (40, 19, 41, 1) lies on line \ell_6
353 : P_{118402} = (1, 57, 27, 1) lies on line \ell_5
                                                                    407: P_{175682} = (1, 56, 41, 1) lies on line \ell_4
354: P_{119390} = (29, 8, 28, 1) lies on line \ell_7
                                                                    408: P_{175746} = (1, 57, 41, 1) lies on line \ell_5
355: P_{120222} = (29, 21, 28, 1) lies on line \ell_6
                                                                    409: P_{175913} = (40, 59, 41, 1) lies on line \ell_7
356: P_{122434} = (1, 56, 28, 1) lies on line \ell_4
                                                                    410: P_{177324} = (43, 17, 42, 1) lies on line \ell_7
357: P_{122498} = (1, 57, 28, 1) lies on line \ell_5
                                                                    411: P_{179778} = (1, 56, 42, 1) lies on line \ell_4
358 : P_{125853} = (28, 45, 29, 1) lies on line \ell_6
                                                                    412: P_{179842} = (1, 57, 42, 1) lies on line \ell_5
                                                                    413 : P_{179948} = (43, 58, 42, 1) lies on line \ell_6
359: P_{126109} = (28, 49, 29, 1) lies on line \ell_7
                                                                    414 : P_{180459} = (42, 2, 43, 1) lies on line \ell_6
360: P_{126530} = (1, 56, 29, 1) lies on line \ell_4
361: P_{126594} = (1, 57, 29, 1) lies on line \ell_5
                                                                    415: P_{182891} = (42, 40, 43, 1) lies on line \ell_7
362: P_{127328} = (31, 4, 30, 1) lies on line \ell_6
                                                                    416: P_{183874} = (1, 56, 43, 1) lies on line \ell_4
363: P_{128800} = (31, 27, 30, 1) lies on line \ell_7
                                                                    417: P_{183938} = (1, 57, 43, 1) lies on line \ell_5
364: P_{130626} = (1, 56, 30, 1) lies on line \ell_4
                                                                    418: P_{185006} = (45, 9, 44, 1) lies on line \ell_6
                                                                    419 : P_{186734} = (45, 36, 44, 1) lies on line \ell_7
365: P_{130690} = (1, 57, 30, 1) lies on line \ell_5
366: P_{133343} = (30, 34, 31, 1) lies on line \ell_7
                                                                    420: P_{187970} = (1, 56, 44, 1) lies on line \ell_4
367: P_{134722} = (1, 56, 31, 1) lies on line \ell_4
                                                                    421: P_{188034} = (1, 57, 44, 1) lies on line \ell_5
368: P_{134786} = (1, 57, 31, 1) lies on line \ell_5
                                                                    422: P_{190381} = (44, 29, 45, 1) lies on line \ell_7
369: P_{135007} = (30, 60, 31, 1) lies on line \ell_6
                                                                    423: P_{191661} = (44, 49, 45, 1) lies on line \ell_6
370: P_{136162} = (33, 14, 32, 1) lies on line \ell_6
                                                                    424: P_{192066} = (1, 56, 45, 1) lies on line \ell_4
371: P_{138274} = (33, 47, 32, 1) lies on line \ell_7
                                                                    425: P_{192130} = (1, 57, 45, 1) lies on line \ell_5
372: P_{138818} = (1, 56, 32, 1) lies on line \ell_4
                                                                    426: P_{194160} = (47, 24, 46, 1) lies on line \ell_6
373: P_{138882} = (1, 57, 32, 1) lies on line \ell_5
                                                                    427: P_{196144} = (47, 55, 46, 1) lies on line \ell_7
374: P_{140769} = (32, 22, 33, 1) lies on line \ell_7
                                                                    428: P_{196162} = (1, 56, 46, 1) lies on line \ell_4
375 : P_{142817} = (32, 54, 33, 1) lies on line \ell_6
                                                                    429: P_{196226} = (1, 57, 46, 1) lies on line \ell_5
376: P_{142914} = (1, 56, 33, 1) lies on line \ell_4
                                                                    430: P_{197615} = (46, 14, 47, 1) lies on line \ell_7
                                                                    431 : P_{198767} = (46, 32, 47, 1) lies on line \ell_6
377: P_{142978} = (1, 57, 33, 1) lies on line \ell_5
378: P_{145444} = (35, 31, 34, 1) lies on line \ell_6
                                                                    432: P_{200258} = (1, 56, 47, 1) lies on line \ell_4
379: P_{147010} = (1, 56, 34, 1) lies on line \ell_4
                                                                    433: P_{200322} = (1, 57, 47, 1) lies on line \ell_5
380: P_{147074} = (1, 57, 34, 1) lies on line \ell_5
                                                                    434: P_{202098} = (49, 20, 48, 1) lies on line \ell_7
                                                                    435 : P_{203186} = (49, 37, 48, 1) lies on line \ell_6
381: P_{147300} = (35, 60, 34, 1) lies on line \ell_7
382: P_{147875} = (34, 5, 35, 1) lies on line \ell_7
                                                                    436: P_{204354} = (1, 56, 48, 1) lies on line \ell_4
383: P_{150051} = (34, 39, 35, 1) lies on line \ell_6
                                                                    437: P_{204418} = (1, 57, 48, 1) lies on line \ell_5
                                                                    438 : P_{206769} = (48, 29, 49, 1) lies on line \ell_6
384: P_{151106} = (1, 56, 35, 1) lies on line \ell_4
385: P_{151170} = (1, 57, 35, 1) lies on line \ell_5
                                                                    439: P_{207793} = (48, 45, 49, 1) lies on line \ell_7
386: P_{152230} = (37, 9, 36, 1) lies on line \ell_7
                                                                    440: P_{208450} = (1, 56, 49, 1) lies on line \ell_4
387: P_{154470} = (37, 44, 36, 1) lies on line \ell_6
                                                                    441: P_{208514} = (1, 57, 49, 1) lies on line \ell_5
388: P_{155202} = (1, 56, 36, 1) lies on line \ell_4
                                                                    442: P_{209460} = (51, 7, 50, 1) lies on line \ell_7
389: P_{155266} = (1, 57, 36, 1) lies on line \ell_5
                                                                    443: P_{212340} = (51, 52, 50, 1) lies on line \ell_6
390: P_{157029} = (36, 20, 37, 1) lies on line \ell_6
                                                                    444: P_{212546} = (1, 56, 50, 1) lies on line \ell_4
391: P_{158821} = (36, 48, 37, 1) lies on line \ell_7
                                                                    445: P_{212610} = (1, 57, 50, 1) lies on line \ell_5
392: P_{159298} = (1, 56, 37, 1) lies on line \ell_4
                                                                    446: P_{213875} = (50, 12, 51, 1) lies on line \ell_6
393: P_{159362} = (1, 57, 37, 1) lies on line \ell_5
                                                                    447: P_{216642} = (1, 56, 51, 1) lies on line \ell_4
394 : P_{161512} = (39, 26, 38, 1) lies on line \ell_7
                                                                    448: P_{216706} = (1, 57, 51, 1) lies on line \ell_5
395: P_{163394} = (1, 56, 38, 1) lies on line \ell_4
                                                                    449: P_{217075} = (50, 62, 51, 1) lies on line \ell_7
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475: P_{244476} = (59, 42, 58, 1) lies on line \ell_7
450: P_{217654} = (53, 7, 52, 1) lies on line \ell_6
                                                                    476: P_{245314} = (1, 56, 58, 1) lies on line \ell_4
451: P_{220406} = (53, 50, 52, 1) lies on line \ell_7
452: P_{220738} = (1, 56, 52, 1) lies on line \ell_4
                                                                    477: P_{245378} = (1, 57, 58, 1) lies on line \ell_5
453: P_{220802} = (1, 57, 52, 1) lies on line \ell_5
                                                                    478: P_{247099} = (58, 19, 59, 1) lies on line \ell_7
                                                                    479: P_{248507} = (58, 41, 59, 1) lies on line \ell_6
454: P_{222005} = (52, 11, 53, 1) lies on line \ell_7
455: P_{224834} = (1, 56, 53, 1) lies on line \ell_4
                                                                    480: P_{249410} = (1, 56, 59, 1) lies on line \ell_4
456 : P_{224898} = (1, 57, 53, 1) lies on line \ell_5
                                                                    481: P_{249474} = (1, 57, 59, 1) lies on line \ell_5
457: P_{225333} = (52, 63, 53, 1) lies on line \ell_6
                                                                    482: P_{251966} = (61, 31, 60, 1) lies on line \ell_7
458: P_{226808} = (55, 22, 54, 1) lies on line \ell_6
                                                                    483: P_{252158} = (61, 34, 60, 1) lies on line \ell_6
459: P_{227512} = (55, 33, 54, 1) lies on line \ell_7
                                                                    484: P_{253506} = (1, 56, 60, 1) lies on line \ell_4
460: P_{228930} = (1, 56, 54, 1) lies on line \ell_4
                                                                    485: P_{253570} = (1, 57, 60, 1) lies on line \ell_5
                                                                    486: P_{255741} = (60, 26, 61, 1) lies on line \ell_6
461: P_{228994} = (1, 57, 54, 1) lies on line \ell_5
462: P_{231031} = (54, 24, 55, 1) lies on line \ell_7
                                                                    487: P_{256509} = (60, 38, 61, 1) lies on line \ell_7
463: P_{232439} = (54, 46, 55, 1) lies on line \ell_6
                                                                    488: P_{257602} = (1, 56, 61, 1) lies on line \ell_4
464: P_{233026} = (1, 56, 55, 1) lies on line \ell_4
                                                                    489: P_{257666} = (1, 57, 61, 1) lies on line \ell_5
465: P_{233090} = (1, 57, 55, 1) lies on line \ell_5
                                                                    490: P_{258944} = (63, 12, 62, 1) lies on line \ell_7
466: P_{233594} = (57, 0, 56, 1) lies on line \ell_6
                                                                    491: P_{261440} = (63, 51, 62, 1) lies on line \ell_6
467: P_{237122} = (1, 56, 56, 1) lies on line \ell_4
                                                                    492: P_{261698} = (1, 56, 62, 1) lies on line \ell_4
468: P_{237186} = (1, 57, 56, 1) lies on line \ell_5
                                                                    493: P_{261762} = (1, 57, 62, 1) lies on line \ell_5
                                                                    494: P_{262975} = (62, 11, 63, 1) lies on line \ell_6
469: P_{237242} = (57, 57, 56, 1) lies on line \ell_7
470: P_{237689} = (56, 0, 57, 1) lies on line \ell_7
                                                                    495: P_{265663} = (62, 53, 63, 1) lies on line \ell_7
471: P_{241218} = (1, 56, 57, 1) lies on line \ell_4
                                                                    496: P_{265794} = (1, 56, 63, 1) lies on line \ell_4
472: P_{241273} = (56, 56, 57, 1) lies on line \ell_6
                                                                    497: P_{265858} = (1, 57, 63, 1) lies on line \ell_5
473 : P_{241282} = (1, 57, 57, 1) lies on line \ell_5
474: P_{242876} = (59, 17, 58, 1) lies on line \ell_6
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The single points on the surface are:

Points on surface but on no line

The surface has 3782 points not on any line: Too many to print.

Line Intersection Graph

$ \begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} $	0	1	2	3	4	5	6	7
0	0	1	1	1	0	0	0	0
1	1	0	1	0	1	1	1	0
2	1	1	0	0	1	1	0	1
3	1	0	0	0	1	1	1	1
4	0	1	1	1	0	1	0	1
5	0	1	1	1	1	0	1	0
6	0	1	0	1	0	1	0	1
7	0	0	1	1	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_{61}	P_{60}	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_{61}	P_2	P_2	P_2	P_{3652}

Line 2 is	ntersects
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Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_{60}	P_2	P_2	P_2	P_{3716}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{7747}	P_{7811}	P_{7811}	P_{7747}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7
in point	P_2	P_2	P_{7747}	P_2	P_{7747}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_2	P_2	P_{7811}	P_2	P_{7811}

Line 6 intersects

I	Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7
	in point	P_{3652}	P_{7811}	P_{7811}	P_{8322}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_{3716}	P_{7747}	P_{7747}	P_{8322}

The surface has 4289 points: Too many to print.