Rank-76292 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is 1090785350

General information

Number of lines	4
Number of points	4161
Number of singular points	2
Number of Eckardt points	1
Number of double points	1
Number of single points	255
Number of points off lines	3904
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{4}
Type of lines on points	$3, 2, 1^{255}, 0^{3904}$

Singular Points

The surface has 2 singular points:

$$\begin{array}{l} 0: \ P_{4163} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1) \\ 1: \ P_{8259} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1) \end{array}$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\ell_{1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{66}$$

$$\ell_{2} = \begin{bmatrix} 1 & 1 & 0 & \epsilon^{21} \\ 0 & 0 & 1 & 1 \end{bmatrix}_{15187586} = \begin{bmatrix} 1 & 1 & 0 & 57 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{15187586} = \mathbf{Pl}(0, 57, 1, 1, 1, 1)_{544698}$$

$$\ell_{3} = \begin{bmatrix} 1 & 1 & 0 & \epsilon^{42} \\ 0 & 0 & 1 & 1 \end{bmatrix}_{14921282} = \begin{bmatrix} 1 & 1 & 0 & 56 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{14921282} = \mathbf{Pl}(0, 56, 1, 1, 1, 1)_{544697}$$

Rank of lines: (17047616, 270400, 15187586, 14921282) Rank of points on Klein quadric: (1, 66, 544698, 544697)

Eckardt Points

The surface has 1 Eckardt points: $0: P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1).$

Double Points

The surface has 1 Double points: The double points on the surface are:

$$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$$

Single Points

The surface has 255 single points: The single points on the surface are:

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0: P_3 = (0,0,0,1) lies on line \ell_0
                                                                     23: P_{28737} = (0, 0, 6, 1) lies on line \ell_0
1: P_{3707} = (56, 56, 1, 0) lies on line \ell_2
                                                                     24: P_{28738} = (1, 0, 6, 1) lies on line \ell_1
2: P_{3772} = (57, 57, 1, 0) lies on line \ell_3
                                                                     25: P_{29452} = (11, 11, 6, 1) lies on line \ell_2
3: P_{4163} = (1,0,0,1) lies on line \ell_1
                                                                     26: P_{29517} = (12, 12, 6, 1) lies on line \ell_3
4: P_{7802} = (56, 56, 0, 1) lies on line \ell_2
                                                                     27: P_{32833} = (0, 0, 7, 1) lies on line \ell_0
5: P_{7867} = (57, 57, 0, 1) lies on line \ell_3
                                                                     28: P_{32834} = (1,0,7,1) lies on line \ell_1
6: P_{8259} = (1, 0, 1, 1) lies on line \ell_1
                                                                     29: P_{36148} = (51, 51, 7, 1) lies on line \ell_2
7: P_{12353} = (0,0,2,1) lies on line \ell_0
                                                                     30: P_{36278} = (53, 53, 7, 1) lies on line \ell_3
8: P_{12354} = (1,0,2,1) lies on line \ell_1
                                                                     31: P_{36929} = (0,0,8,1) lies on line \ell_0
9: P_{15018} = (41, 41, 2, 1) lies on line \ell_2
                                                                     32: P_{36930} = (1,0,8,1) lies on line \ell_1
                                                                     33: P_{38229} = (20, 20, 8, 1) lies on line \ell_3
10: P_{15083} = (42, 42, 2, 1) lies on line \ell_3
                                                                     34 : P_{38814} = (29, 29, 8, 1) lies on line \ell_2
11: P_{16449} = (0,0,3,1) lies on line \ell_0
                                                                     35: P_{41025} = (0,0,9,1) lies on line \ell_0
12: P_{16450} = (1, 0, 3, 1) lies on line \ell_1
13: P_{17554} = (17, 17, 3, 1) lies on line \ell_2
                                                                     36: P_{41026} = (1,0,9,1) lies on line \ell_1
14: P_{17684} = (19, 19, 3, 1) lies on line \ell_3
                                                                     37: P_{43430} = (37, 37, 9, 1) lies on line \ell_2
15: P_{20545} = (0, 0, 4, 1) lies on line \ell_0
                                                                     38: P_{43950} = (45, 45, 9, 1) lies on line \ell_3
16: P_{20546} = (1, 0, 4, 1) lies on line \ell_1
                                                                     39: P_{45121} = (0,0,10,1) lies on line \ell_0
17: P_{22235} = (26, 26, 4, 1) lies on line \ell_2
                                                                     40: P_{45122} = (1,0,10,1) lies on line \ell_1
18: P_{22560} = (31, 31, 4, 1) lies on line \ell_3
                                                                     41: P_{45576} = (7, 7, 10, 1) lies on line \ell_3
19 : P_{24641} = (0, 0, 5, 1) lies on line \ell_0
                                                                     42: P_{45901} = (12, 12, 10, 1) lies on line \ell_2
20: P_{24642} = (1, 0, 5, 1) lies on line \ell_1
                                                                     43: P_{49217} = (0, 0, 11, 1) lies on line \ell_0
21: P_{26851} = (34, 34, 5, 1) lies on line \ell_2
                                                                     44: P_{49218} = (1, 0, 11, 1) lies on line \ell_1
22: P_{27111} = (38, 38, 5, 1) lies on line \ell_3
                                                                     45: P_{52597} = (52, 52, 11, 1) lies on line \ell_2
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46 : P_{53247} = (62, 62, 11, 1) lies on line \ell_3
                                                                     100: P_{106562} = (1, 0, 25, 1) lies on line \ell_1
47: P_{53313} = (0, 0, 12, 1) lies on line \ell_0
                                                                     101: P_{107471} = (14, 14, 25, 1) lies on line \ell_2
                                                                     102 : P_{107991} = (22, 22, 25, 1) lies on line \ell_3
48: P_{53314} = (1, 0, 12, 1) lies on line \ell_1
49: P_{56563} = (50, 50, 12, 1) lies on line \ell_3
                                                                     103: P_{110657} = (0, 0, 26, 1) lies on line \ell_0
50: P_{57408} = (63, 63, 12, 1) lies on line \ell_2
                                                                     104: P_{110658} = (1, 0, 26, 1) lies on line \ell_1
51: P_{57409} = (0, 0, 13, 1) lies on line \ell_0
                                                                     105: P_{113192} = (39, 39, 26, 1) lies on line \ell_2
                                                                     106: P_{114557} = (60, 60, 26, 1) lies on line \ell_3
52: P_{57410} = (1, 0, 13, 1) lies on line \ell_1
53: P_{57864} = (7, 7, 13, 1) lies on line \ell_2
                                                                     107: P_{114753} = (0, 0, 27, 1) lies on line \ell_0
54: P_{58124} = (11, 11, 13, 1) lies on line \ell_3
                                                                     108: P_{114754} = (1, 0, 27, 1) lies on line \ell_1
55: P_{61505} = (0, 0, 14, 1) lies on line \ell_0
                                                                     109: P_{115078} = (5, 5, 27, 1) lies on line \ell_3
56: P_{61506} = (1, 0, 14, 1) lies on line \ell_1
                                                                     110: P_{116768} = (31, 31, 27, 1) lies on line \ell_2
                                                                     111 : P_{118849} = (0, 0, 28, 1) lies on line \ell_0
57: P_{63650} = (33, 33, 14, 1) lies on line \ell_3
58: P_{64495} = (46, 46, 14, 1) lies on line \ell_2
                                                                     112: P_{118850} = (1, 0, 28, 1) lies on line \ell_1
59: P_{65601} = (0, 0, 15, 1) lies on line \ell_0
                                                                     113: P_{119434} = (9, 9, 28, 1) lies on line \ell_3
                                                                     114: P_{120149} = (20, 20, 28, 1) lies on line \ell_2
60: P_{65602} = (1, 0, 15, 1) lies on line \ell_1
61: P_{67031} = (22, 22, 15, 1) lies on line \ell_2
                                                                     115: P_{122945} = (0, 0, 29, 1) lies on line \ell_0
                                                                     116: P_{122946} = (1, 0, 29, 1) lies on line \ell_1
62: P_{67161} = (24, 24, 15, 1) lies on line \ell_3
63: P_{69697} = (0, 0, 16, 1) lies on line \ell_0
                                                                     117: P_{125805} = (44, 44, 29, 1) lies on line \ell_2
64 : P_{69698} = (1, 0, 16, 1) lies on line \ell_1
                                                                     118 : P_{126065} = (48, 48, 29, 1) lies on line \ell_3
65: P_{69827} = (2, 2, 16, 1) lies on line \ell_3
                                                                     119: P_{127041} = (0, 0, 30, 1) lies on line \ell_0
66: P_{70932} = (19, 19, 16, 1) lies on line \ell_2
                                                                     120: P_{127042} = (1, 0, 30, 1) lies on line \ell_1
67: P_{73793} = (0, 0, 17, 1) lies on line \ell_0
                                                                     121: P_{127366} = (5, 5, 30, 1) lies on line \ell_2
68: P_{73794} = (1, 0, 17, 1) lies on line \ell_1
                                                                     122: P_{128731} = (26, 26, 30, 1) lies on line \ell_3
                                                                     123 : P_{131137} = (0, 0, 31, 1) lies on line \ell_0
69: P_{76588} = (43, 43, 17, 1) lies on line \ell_2
70: P_{77628} = (59, 59, 17, 1) lies on line \ell_3
                                                                     124: P_{131138} = (1, 0, 31, 1) lies on line \ell_1
71: P_{77889} = (0, 0, 18, 1) lies on line \ell_0
                                                                     125: P_{133412} = (35, 35, 31, 1) lies on line \ell_3
72: P_{77890} = (1, 0, 18, 1) lies on line \ell_1
                                                                     126: P_{135102} = (61, 61, 31, 1) lies on line \ell_2
73: P_{78019} = (2, 2, 18, 1) lies on line \ell_2
                                                                     127: P_{135233} = (0, 0, 32, 1) lies on line \ell_0
74: P_{78994} = (17, 17, 18, 1) lies on line \ell_3
                                                                     128: P_{135234} = (1, 0, 32, 1) lies on line \ell_1
75: P_{81985} = (0, 0, 19, 1) lies on line \ell_0
                                                                     129: P_{136208} = (15, 15, 32, 1) lies on line \ell_2
76: P_{81986} = (1, 0, 19, 1) lies on line \ell_1
                                                                     130: P_{138223} = (46, 46, 32, 1) lies on line \ell_3
77: P_{84585} = (40, 40, 19, 1) lies on line \ell_3
                                                                     131: P_{139329} = (0, 0, 33, 1) lies on line \ell_0
78: P_{85755} = (58, 58, 19, 1) lies on line \ell_2
                                                                     132: P_{139330} = (1, 0, 33, 1) lies on line \ell_1
79: P_{86081} = (0, 0, 20, 1) lies on line \ell_0
                                                                     133 : P_{140824} = (23, 23, 33, 1) lies on line \ell_3
80: P_{86082} = (1, 0, 20, 1) lies on line \ell_1
                                                                     134: P_{142904} = (55, 55, 33, 1) lies on line \ell_2
                                                                     135 : P_{143425} = (0, 0, 34, 1) lies on line \ell_0
81: P_{88421} = (36, 36, 20, 1) lies on line \ell_3
82: P_{89266} = (49, 49, 20, 1) lies on line \ell_2
                                                                     136: P_{143426} = (1, 0, 34, 1) lies on line \ell_1
83: P_{90177} = (0, 0, 21, 1) lies on line \ell_0
                                                                     137: P_{145375} = (30, 30, 34, 1) lies on line \ell_2
                                                                     138: P_{147390} = (61, 61, 34, 1) lies on line \ell_3
84: P_{90178} = (1, 0, 21, 1) lies on line \ell_1
85: P_{90762} = (9, 9, 21, 1) lies on line \ell_2
                                                                     139: P_{147521} = (0, 0, 35, 1) lies on line \ell_0
86: P_{92062} = (29, 29, 21, 1) lies on line \ell_3
                                                                     140: P_{147522} = (1, 0, 35, 1) lies on line \ell_1
87: P_{94273} = (0, 0, 22, 1) lies on line \ell_0
                                                                     141: P_{147781} = (4, 4, 35, 1) lies on line \ell_3
88 : P_{94274} = (1, 0, 22, 1) lies on line \ell_1
                                                                     142: P_{149991} = (38, 38, 35, 1) lies on line \ell_2
89: P_{96353} = (32, 32, 22, 1) lies on line \ell_2
                                                                     143: P_{151617} = (0, 0, 36, 1) lies on line \ell_0
90: P_{97848} = (55, 55, 22, 1) lies on line \ell_3
                                                                     144: P_{151618} = (1, 0, 36, 1) lies on line \ell_1
                                                                     145 : P_{152137} = (8, 8, 36, 1) lies on line \ell_3
91: P_{98369} = (0, 0, 23, 1) lies on line \ell_0
92: P_{98370} = (1, 0, 23, 1) lies on line \ell_1
                                                                     146: P_{154542} = (45, 45, 36, 1) lies on line \ell_2
93: P_{99279} = (14, 14, 23, 1) lies on line \ell_3
                                                                     147: P_{155713} = (0, 0, 37, 1) lies on line \ell_0
94: P_{99929} = (24, 24, 23, 1) lies on line \ell_2
                                                                     148: P_{155714} = (1, 0, 37, 1) lies on line \ell_1
                                                                     149 : P_{157078} = (21, 21, 37, 1) lies on line \ell_2
95: P_{102465} = (0, 0, 24, 1) lies on line \ell_0
                                                                     150: P_{158898} = (49, 49, 37, 1) lies on line \ell_3
96: P_{102466} = (1, 0, 24, 1) lies on line \ell_1
97: P_{105520} = (47, 47, 24, 1) lies on line \ell_3
                                                                     151: P_{159809} = (0, 0, 38, 1) lies on line \ell_0
98: P_{105975} = (54, 54, 24, 1) lies on line \ell_2
                                                                     152: P_{159810} = (1, 0, 38, 1) lies on line \ell_1
99: P_{106561} = (0, 0, 25, 1) lies on line \ell_0
                                                                     153: P_{161564} = (27, 27, 38, 1) lies on line \ell_3
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154 : P_{163709} = (60, 60, 38, 1) lies on line \ell_2
                                                                    205 : P_{213902} = (13, 13, 51, 1) lies on line \ell_2
155: P_{163905} = (0, 0, 39, 1) lies on line \ell_0
                                                                     206: P_{217152} = (63, 63, 51, 1) lies on line \ell_3
                                                                     207: P_{217153} = (0, 0, 52, 1) lies on line \ell_0
156: P_{163906} = (1, 0, 39, 1) lies on line \ell_1
157: P_{164165} = (4, 4, 39, 1) lies on line \ell_2
                                                                     208: P_{217154} = (1, 0, 52, 1) lies on line \ell_1
158: P_{166115} = (34, 34, 39, 1) lies on line \ell_3
                                                                    209: P_{217543} = (6, 6, 52, 1) lies on line \ell_2
159: P_{168001} = (0, 0, 40, 1) lies on line \ell_0
                                                                    210: P_{220468} = (51, 51, 52, 1) lies on line \ell_3
160: P_{168002} = (1, 0, 40, 1) lies on line \ell_1
                                                                    211: P_{221249} = (0, 0, 53, 1) lies on line \ell_0
161: P_{168196} = (3, 3, 40, 1) lies on line \ell_3
                                                                    212: P_{221250} = (1, 0, 53, 1) lies on line \ell_1
162: P_{170731} = (42, 42, 40, 1) lies on line \ell_2
                                                                    213: P_{221899} = (10, 10, 53, 1) lies on line \ell_3
163: P_{172097} = (0, 0, 41, 1) lies on line \ell_0
                                                                    214: P_{225279} = (62, 62, 53, 1) lies on line \ell_2
164: P_{172098} = (1, 0, 41, 1) lies on line \ell_1
                                                                    215: P_{225345} = (0, 0, 54, 1) lies on line \ell_0
165: P_{173267} = (18, 18, 41, 1) lies on line \ell_2
                                                                    216: P_{225346} = (1, 0, 54, 1) lies on line \ell_1
166: P_{175867} = (58, 58, 41, 1) lies on line \ell_3
                                                                    217: P_{226840} = (23, 23, 54, 1) lies on line \ell_2
167: P_{176193} = (0, 0, 42, 1) lies on line \ell_0
                                                                    218: P_{227425} = (32, 32, 54, 1) lies on line \ell_3
168: P_{176194} = (1, 0, 42, 1) lies on line \ell_1
                                                                     219: P_{229441} = (0, 0, 55, 1) lies on line \ell_0
169: P_{177233} = (16, 16, 42, 1) lies on line \ell_3
                                                                     220: P_{229442} = (1, 0, 55, 1) lies on line \ell_1
170: P_{180028} = (59, 59, 42, 1) lies on line \ell_2
                                                                     221: P_{231066} = (25, 25, 55, 1) lies on line \ell_3
171: P_{180289} = (0, 0, 43, 1) lies on line \ell_0
                                                                    222: P_{232496} = (47, 47, 55, 1) lies on line \ell_2
172: P_{180290} = (1, 0, 43, 1) lies on line \ell_1
                                                                    223: P_{233537} = (0, 0, 56, 1) lies on line \ell_0
173: P_{180484} = (3, 3, 43, 1) lies on line \ell_2
                                                                    224: P_{233538} = (1,0,56,1) lies on line \ell_1
174: P_{182954} = (41, 41, 43, 1) lies on line \ell_3
                                                                    225: P_{233602} = (1, 1, 56, 1) lies on line \ell_2
175: P_{184385} = (0, 0, 44, 1) lies on line \ell_0
                                                                    226 : P_{237177} = (56, 56, 56, 1) lies on line \ell_3
                                                                     227: P_{237633} = (0, 0, 57, 1) lies on line \ell_0
176: P_{184386} = (1, 0, 44, 1) lies on line \ell_1
177 : P_{184905} = (8, 8, 44, 1) lies on line \ell_2
                                                                    228: P_{237634} = (1, 0, 57, 1) lies on line \ell_1
178: P_{186790} = (37, 37, 44, 1) lies on line \ell_3
                                                                    229: P_{237698} = (1, 1, 57, 1) lies on line \ell_3
179: P_{188481} = (0, 0, 45, 1) lies on line \ell_0
                                                                    230: P_{241338} = (57, 57, 57, 1) lies on line \ell_2
180: P_{188482} = (1, 0, 45, 1) lies on line \ell_1
                                                                     231: P_{241729} = (0, 0, 58, 1) lies on line \ell_0
181: P_{190301} = (28, 28, 45, 1) lies on line \ell_3
                                                                    232: P_{241730} = (1, 0, 58, 1) lies on line \ell_1
182: P_{191601} = (48, 48, 45, 1) lies on line \ell_2
                                                                    233: P_{242769} = (16, 16, 58, 1) lies on line \ell_2
183: P_{192577} = (0, 0, 46, 1) lies on line \ell_0
                                                                    234: P_{244524} = (43, 43, 58, 1) lies on line \ell_3
184: P_{192578} = (1, 0, 46, 1) lies on line \ell_1
                                                                    235: P_{245825} = (0, 0, 59, 1) lies on line \ell_0
185: P_{194202} = (25, 25, 46, 1) lies on line \ell_2
                                                                    236: P_{245826} = (1, 0, 59, 1) lies on line \ell_1
186: P_{196087} = (54, 54, 46, 1) lies on line \ell_3
                                                                     237: P_{246995} = (18, 18, 59, 1) lies on line \ell_3
187 : P_{196673} = (0, 0, 47, 1) lies on line \ell_0
                                                                     238: P_{248425} = (40, 40, 59, 1) lies on line \ell_2
188: P_{196674} = (1, 0, 47, 1) lies on line \ell_1
                                                                    239: P_{249921} = (0, 0, 60, 1) lies on line \ell_0
189: P_{197648} = (15, 15, 47, 1) lies on line \ell_3
                                                                    240: P_{249922} = (1, 0, 60, 1) lies on line \ell_1
190: P_{198818} = (33, 33, 47, 1) lies on line \ell_2
                                                                    241: P_{251871} = (30, 30, 60, 1) lies on line \ell_3
191 : P_{200769} = (0, 0, 48, 1) lies on line \ell_0
                                                                     242: P_{252196} = (35, 35, 60, 1) lies on line \ell_2
                                                                    243 : P_{254017} = (0, 0, 61, 1) lies on line \ell_0
192 : P_{200770} = (1, 0, 48, 1) lies on line \ell_1
193: P_{202134} = (21, 21, 48, 1) lies on line \ell_3
                                                                    244: P_{254018} = (1, 0, 61, 1) lies on line \ell_1
                                                                    245: P_{255772} = (27, 27, 61, 1) lies on line \ell_2
194: P_{203109} = (36, 36, 48, 1) lies on line \ell_2
195: P_{204865} = (0, 0, 49, 1) lies on line \ell_0
                                                                    246: P_{256552} = (39, 39, 61, 1) lies on line \ell_3
196: P_{204866} = (1, 0, 49, 1) lies on line \ell_1
                                                                    247: P_{258113} = (0, 0, 62, 1) lies on line \ell_0
197: P_{206685} = (28, 28, 49, 1) lies on line \ell_2
                                                                    248: P_{258114} = (1, 0, 62, 1) lies on line \ell_1
198: P_{207725} = (44, 44, 49, 1) lies on line \ell_3
                                                                     249: P_{258958} = (13, 13, 62, 1) lies on line \ell_3
199 : P_{208961} = (0, 0, 50, 1) lies on line \ell_0
                                                                     250: P_{261363} = (50, 50, 62, 1) lies on line \ell_2
200: P_{208962} = (1, 0, 50, 1) lies on line \ell_1
                                                                    251: P_{262209} = (0, 0, 63, 1) lies on line \ell_0
201: P_{209351} = (6, 6, 50, 1) lies on line \ell_3
                                                                    252: P_{262210} = (1, 0, 63, 1) lies on line \ell_1
202: P_{212406} = (53, 53, 50, 1) lies on line \ell_2
                                                                    253: P_{262859} = (10, 10, 63, 1) lies on line \ell_2
203: P_{213057} = (0, 0, 51, 1) lies on line \ell_0
                                                                    254: P_{265589} = (52, 52, 63, 1) lies on line \ell_3
204: P_{213058} = (1, 0, 51, 1) lies on line \ell_1
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The single points on the surface are:

Points on surface but on no line

The surface has 3904 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c|c} 0123 \\ \hline 0 & 0111 \\ 1 & 1000 \\ 2 & 1001 \\ 3 & 1010 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_{8258}	P_{8258}

 ${\bf Line~1~intersects}$

Line	ℓ_0
in point	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{8258}	P_{8258}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{8258}	P_{8258}

The surface has 4161 points:

Too many to print.