

Rank-65562 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227137686

General information

Number of lines	14
Number of points	113
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	112
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{14}
Type of lines on points	$14, 1^{112}$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 14 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{510} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{510} = \mathbf{Pl}(0, 0, 0, 6, 1, 0)_{276}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{291} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{291} = \mathbf{Pl}(0, 0, 0, 3, 1, 0)_{231} \\
\ell_2 &= \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{437} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{437} = \mathbf{Pl}(0, 0, 0, 5, 1, 0)_{261} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{89} \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25} \\
\ell_5 &= \begin{bmatrix} 1 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3649} = \begin{bmatrix} 1 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3649} = \mathbf{Pl}(0, 6, 0, 1, 1, 0)_{214} \\
\ell_6 &= \begin{bmatrix} 1 & \gamma^4 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3503} = \begin{bmatrix} 1 & 7 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3503} = \mathbf{Pl}(0, 5, 0, 7, 1, 0)_{303} \\
\ell_7 &= \begin{bmatrix} 1 & \gamma^3 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2189} = \begin{bmatrix} 1 & 5 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2189} = \mathbf{Pl}(0, 3, 0, 5, 1, 0)_{271} \\
\ell_8 &= \begin{bmatrix} 1 & \gamma & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3722} = \begin{bmatrix} 1 & 2 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3722} = \mathbf{Pl}(0, 6, 0, 2, 1, 0)_{229} \\
\ell_9 &= \begin{bmatrix} 1 & 1 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1897} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1897} = \mathbf{Pl}(0, 3, 0, 1, 1, 0)_{211} \\
\ell_{10} &= \begin{bmatrix} 1 & \gamma^6 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3430} = \begin{bmatrix} 1 & 6 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3430} = \mathbf{Pl}(0, 5, 0, 6, 1, 0)_{288} \\
\ell_{11} &= \begin{bmatrix} 1 & \gamma^5 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3795} = \begin{bmatrix} 1 & 3 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3795} = \mathbf{Pl}(0, 6, 0, 3, 1, 0)_{244} \\
\ell_{12} &= \begin{bmatrix} 1 & \gamma^2 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2116} = \begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2116} = \mathbf{Pl}(0, 3, 0, 4, 1, 0)_{256} \\
\ell_{13} &= \begin{bmatrix} 1 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3065} = \begin{bmatrix} 1 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3065} = \mathbf{Pl}(0, 5, 0, 1, 1, 0)_{213}
\end{aligned}$$

Rank of lines: (510, 291, 437, 656, 4689, 3649, 3503, 2189, 3722, 1897, 3430, 3795, 2116, 3065)

Rank of points on Klein quadric: (276, 231, 261, 89, 25, 214, 303, 271, 229, 211, 288, 244, 256, 213)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 112 single points:

The single points on the surface are:

0 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0
 1 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_1
 2 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_2
 3 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_3
 4 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_4
 5 : $P_{29} = (2, 2, 1, 0)$ lies on line ℓ_5
 6 : $P_{34} = (7, 2, 1, 0)$ lies on line ℓ_6
 7 : $P_{39} = (4, 3, 1, 0)$ lies on line ℓ_7
 8 : $P_{45} = (2, 4, 1, 0)$ lies on line ℓ_8
 9 : $P_{47} = (4, 4, 1, 0)$ lies on line ℓ_9
 10 : $P_{58} = (7, 5, 1, 0)$ lies on line ℓ_{10}
 11 : $P_{61} = (2, 6, 1, 0)$ lies on line ℓ_{11}
 12 : $P_{71} = (4, 7, 1, 0)$ lies on line ℓ_{12}
 13 : $P_{74} = (7, 7, 1, 0)$ lies on line ℓ_{13}
 14 : $P_{84} = (2, 1, 0, 1)$ lies on line ℓ_0
 15 : $P_{86} = (4, 1, 0, 1)$ lies on line ℓ_1
 16 : $P_{89} = (7, 1, 0, 1)$ lies on line ℓ_2
 17 : $P_{93} = (3, 2, 0, 1)$ lies on line ℓ_2
 18 : $P_{94} = (4, 2, 0, 1)$ lies on line ℓ_0
 19 : $P_{95} = (5, 2, 0, 1)$ lies on line ℓ_1
 20 : $P_{99} = (1, 3, 0, 1)$ lies on line ℓ_1
 21 : $P_{102} = (4, 3, 0, 1)$ lies on line ℓ_2
 22 : $P_{104} = (6, 3, 0, 1)$ lies on line ℓ_0
 23 : $P_{111} = (5, 4, 0, 1)$ lies on line ℓ_0
 24 : $P_{112} = (6, 4, 0, 1)$ lies on line ℓ_2
 25 : $P_{113} = (7, 4, 0, 1)$ lies on line ℓ_1
 26 : $P_{115} = (1, 5, 0, 1)$ lies on line ℓ_2
 27 : $P_{117} = (3, 5, 0, 1)$ lies on line ℓ_1
 28 : $P_{121} = (7, 5, 0, 1)$ lies on line ℓ_0
 29 : $P_{123} = (1, 6, 0, 1)$ lies on line ℓ_0
 30 : $P_{124} = (2, 6, 0, 1)$ lies on line ℓ_1
 31 : $P_{127} = (5, 6, 0, 1)$ lies on line ℓ_2
 32 : $P_{132} = (2, 7, 0, 1)$ lies on line ℓ_2
 33 : $P_{133} = (3, 7, 0, 1)$ lies on line ℓ_0
 34 : $P_{136} = (6, 7, 0, 1)$ lies on line ℓ_1
 35 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_3
 36 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_4
 37 : $P_{155} = (2, 2, 1, 1)$ lies on line ℓ_5
 38 : $P_{160} = (7, 2, 1, 1)$ lies on line ℓ_6
 39 : $P_{165} = (4, 3, 1, 1)$ lies on line ℓ_7
 40 : $P_{171} = (2, 4, 1, 1)$ lies on line ℓ_8
 41 : $P_{173} = (4, 4, 1, 1)$ lies on line ℓ_9
 42 : $P_{184} = (7, 5, 1, 1)$ lies on line ℓ_{10}
 43 : $P_{187} = (2, 6, 1, 1)$ lies on line ℓ_{11}
 44 : $P_{197} = (4, 7, 1, 1)$ lies on line ℓ_{12}
 45 : $P_{200} = (7, 7, 1, 1)$ lies on line ℓ_{13}
 46 : $P_{203} = (2, 0, 2, 1)$ lies on line ℓ_3
 47 : $P_{213} = (4, 1, 2, 1)$ lies on line ℓ_{11}
 48 : $P_{217} = (0, 2, 2, 1)$ lies on line ℓ_4
 49 : $P_{228} = (3, 3, 2, 1)$ lies on line ℓ_{13}
 50 : $P_{230} = (5, 3, 2, 1)$ lies on line ℓ_{12}
 51 : $P_{236} = (3, 4, 2, 1)$ lies on line ℓ_6
 52 : $P_{237} = (4, 4, 2, 1)$ lies on line ℓ_5
 53 : $P_{245} = (4, 5, 2, 1)$ lies on line ℓ_8

54 : $P_{246} = (5, 5, 2, 1)$ lies on line ℓ_9
 55 : $P_{254} = (5, 6, 2, 1)$ lies on line ℓ_7
 56 : $P_{260} = (3, 7, 2, 1)$ lies on line ℓ_{10}
 57 : $P_{268} = (3, 0, 3, 1)$ lies on line ℓ_3
 58 : $P_{274} = (1, 1, 3, 1)$ lies on line ℓ_9
 59 : $P_{279} = (6, 1, 3, 1)$ lies on line ℓ_8
 60 : $P_{285} = (4, 2, 3, 1)$ lies on line ℓ_{10}
 61 : $P_{289} = (0, 3, 3, 1)$ lies on line ℓ_4
 62 : $P_{298} = (1, 4, 3, 1)$ lies on line ℓ_{12}
 63 : $P_{301} = (4, 4, 3, 1)$ lies on line ℓ_{13}
 64 : $P_{306} = (1, 5, 3, 1)$ lies on line ℓ_7
 65 : $P_{317} = (4, 6, 3, 1)$ lies on line ℓ_6
 66 : $P_{319} = (6, 6, 3, 1)$ lies on line ℓ_5
 67 : $P_{327} = (6, 7, 3, 1)$ lies on line ℓ_{11}
 68 : $P_{333} = (4, 0, 4, 1)$ lies on line ℓ_3
 69 : $P_{344} = (7, 1, 4, 1)$ lies on line ℓ_7
 70 : $P_{350} = (5, 2, 4, 1)$ lies on line ℓ_{11}
 71 : $P_{359} = (6, 3, 4, 1)$ lies on line ℓ_{10}
 72 : $P_{361} = (0, 4, 4, 1)$ lies on line ℓ_4
 73 : $P_{374} = (5, 5, 4, 1)$ lies on line ℓ_5
 74 : $P_{375} = (6, 5, 4, 1)$ lies on line ℓ_6
 75 : $P_{383} = (6, 6, 4, 1)$ lies on line ℓ_{13}
 76 : $P_{384} = (7, 6, 4, 1)$ lies on line ℓ_{12}
 77 : $P_{390} = (5, 7, 4, 1)$ lies on line ℓ_8
 78 : $P_{392} = (7, 7, 4, 1)$ lies on line ℓ_9
 79 : $P_{398} = (5, 0, 5, 1)$ lies on line ℓ_3
 80 : $P_{402} = (1, 1, 5, 1)$ lies on line ℓ_{13}
 81 : $P_{404} = (3, 1, 5, 1)$ lies on line ℓ_{12}
 82 : $P_{412} = (3, 2, 5, 1)$ lies on line ℓ_7
 83 : $P_{420} = (3, 3, 5, 1)$ lies on line ℓ_9
 84 : $P_{424} = (7, 3, 5, 1)$ lies on line ℓ_8
 85 : $P_{432} = (7, 4, 5, 1)$ lies on line ℓ_{11}
 86 : $P_{433} = (0, 5, 5, 1)$ lies on line ℓ_4
 87 : $P_{442} = (1, 6, 5, 1)$ lies on line ℓ_{10}
 88 : $P_{450} = (1, 7, 5, 1)$ lies on line ℓ_6
 89 : $P_{456} = (7, 7, 5, 1)$ lies on line ℓ_5
 90 : $P_{463} = (6, 0, 6, 1)$ lies on line ℓ_3
 91 : $P_{466} = (1, 1, 6, 1)$ lies on line ℓ_5
 92 : $P_{470} = (5, 1, 6, 1)$ lies on line ℓ_6
 93 : $P_{474} = (1, 2, 6, 1)$ lies on line ℓ_8
 94 : $P_{475} = (2, 2, 6, 1)$ lies on line ℓ_9
 95 : $P_{482} = (1, 3, 6, 1)$ lies on line ℓ_{11}
 96 : $P_{494} = (5, 4, 6, 1)$ lies on line ℓ_{10}
 97 : $P_{499} = (2, 5, 6, 1)$ lies on line ℓ_{12}
 98 : $P_{502} = (5, 5, 6, 1)$ lies on line ℓ_{13}
 99 : $P_{505} = (0, 6, 6, 1)$ lies on line ℓ_4
 100 : $P_{515} = (2, 7, 6, 1)$ lies on line ℓ_7
 101 : $P_{528} = (7, 0, 7, 1)$ lies on line ℓ_3
 102 : $P_{531} = (2, 1, 7, 1)$ lies on line ℓ_{10}
 103 : $P_{539} = (2, 2, 7, 1)$ lies on line ℓ_{13}
 104 : $P_{543} = (6, 2, 7, 1)$ lies on line ℓ_{12}
 105 : $P_{547} = (2, 3, 7, 1)$ lies on line ℓ_6
 106 : $P_{548} = (3, 3, 7, 1)$ lies on line ℓ_5
 107 : $P_{559} = (6, 4, 7, 1)$ lies on line ℓ_7

108 : $P_{564} = (3, 5, 7, 1)$ lies on line ℓ_{11}
 109 : $P_{572} = (3, 6, 7, 1)$ lies on line ℓ_8
 110 : $P_{575} = (6, 6, 7, 1)$ lies on line ℓ_9

111 : $P_{577} = (0, 7, 7, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	0	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	0	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	0	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	0	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	0	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	0	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	0	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	0	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 13 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 113 points:

The points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$	9 : $P_{45} = (2, 4, 1, 0)$	18 : $P_{93} = (3, 2, 0, 1)$
1 : $P_6 = (2, 1, 0, 0)$	10 : $P_{47} = (4, 4, 1, 0)$	19 : $P_{94} = (4, 2, 0, 1)$
2 : $P_8 = (4, 1, 0, 0)$	11 : $P_{58} = (7, 5, 1, 0)$	20 : $P_{95} = (5, 2, 0, 1)$
3 : $P_{11} = (7, 1, 0, 0)$	12 : $P_{61} = (2, 6, 1, 0)$	21 : $P_{99} = (1, 3, 0, 1)$
4 : $P_{12} = (1, 0, 1, 0)$	13 : $P_{71} = (4, 7, 1, 0)$	22 : $P_{102} = (4, 3, 0, 1)$
5 : $P_{19} = (0, 1, 1, 0)$	14 : $P_{74} = (7, 7, 1, 0)$	23 : $P_{104} = (6, 3, 0, 1)$
6 : $P_{29} = (2, 2, 1, 0)$	15 : $P_{84} = (2, 1, 0, 1)$	24 : $P_{111} = (5, 4, 0, 1)$
7 : $P_{34} = (7, 2, 1, 0)$	16 : $P_{86} = (4, 1, 0, 1)$	25 : $P_{112} = (6, 4, 0, 1)$
8 : $P_{39} = (4, 3, 1, 0)$	17 : $P_{89} = (7, 1, 0, 1)$	26 : $P_{113} = (7, 4, 0, 1)$

27 : $P_{115} = (1, 5, 0, 1)$
 28 : $P_{117} = (3, 5, 0, 1)$
 29 : $P_{121} = (7, 5, 0, 1)$
 30 : $P_{123} = (1, 6, 0, 1)$
 31 : $P_{124} = (2, 6, 0, 1)$
 32 : $P_{127} = (5, 6, 0, 1)$
 33 : $P_{132} = (2, 7, 0, 1)$
 34 : $P_{133} = (3, 7, 0, 1)$
 35 : $P_{136} = (6, 7, 0, 1)$
 36 : $P_{139} = (1, 0, 1, 1)$
 37 : $P_{146} = (0, 1, 1, 1)$
 38 : $P_{155} = (2, 2, 1, 1)$
 39 : $P_{160} = (7, 2, 1, 1)$
 40 : $P_{165} = (4, 3, 1, 1)$
 41 : $P_{171} = (2, 4, 1, 1)$
 42 : $P_{173} = (4, 4, 1, 1)$
 43 : $P_{184} = (7, 5, 1, 1)$
 44 : $P_{187} = (2, 6, 1, 1)$
 45 : $P_{197} = (4, 7, 1, 1)$
 46 : $P_{200} = (7, 7, 1, 1)$
 47 : $P_{203} = (2, 0, 2, 1)$
 48 : $P_{213} = (4, 1, 2, 1)$
 49 : $P_{217} = (0, 2, 2, 1)$
 50 : $P_{228} = (3, 3, 2, 1)$
 51 : $P_{230} = (5, 3, 2, 1)$
 52 : $P_{236} = (3, 4, 2, 1)$
 53 : $P_{237} = (4, 4, 2, 1)$
 54 : $P_{245} = (4, 5, 2, 1)$
 55 : $P_{246} = (5, 5, 2, 1)$

56 : $P_{254} = (5, 6, 2, 1)$
 57 : $P_{260} = (3, 7, 2, 1)$
 58 : $P_{268} = (3, 0, 3, 1)$
 59 : $P_{274} = (1, 1, 3, 1)$
 60 : $P_{279} = (6, 1, 3, 1)$
 61 : $P_{285} = (4, 2, 3, 1)$
 62 : $P_{289} = (0, 3, 3, 1)$
 63 : $P_{298} = (1, 4, 3, 1)$
 64 : $P_{301} = (4, 4, 3, 1)$
 65 : $P_{306} = (1, 5, 3, 1)$
 66 : $P_{317} = (4, 6, 3, 1)$
 67 : $P_{319} = (6, 6, 3, 1)$
 68 : $P_{327} = (6, 7, 3, 1)$
 69 : $P_{333} = (4, 0, 4, 1)$
 70 : $P_{344} = (7, 1, 4, 1)$
 71 : $P_{350} = (5, 2, 4, 1)$
 72 : $P_{359} = (6, 3, 4, 1)$
 73 : $P_{361} = (0, 4, 4, 1)$
 74 : $P_{374} = (5, 5, 4, 1)$
 75 : $P_{375} = (6, 5, 4, 1)$
 76 : $P_{383} = (6, 6, 4, 1)$
 77 : $P_{384} = (7, 6, 4, 1)$
 78 : $P_{390} = (5, 7, 4, 1)$
 79 : $P_{392} = (7, 7, 4, 1)$
 80 : $P_{398} = (5, 0, 5, 1)$
 81 : $P_{402} = (1, 1, 5, 1)$
 82 : $P_{404} = (3, 1, 5, 1)$
 83 : $P_{412} = (3, 2, 5, 1)$
 84 : $P_{420} = (3, 3, 5, 1)$

85 : $P_{424} = (7, 3, 5, 1)$
 86 : $P_{432} = (7, 4, 5, 1)$
 87 : $P_{433} = (0, 5, 5, 1)$
 88 : $P_{442} = (1, 6, 5, 1)$
 89 : $P_{450} = (1, 7, 5, 1)$
 90 : $P_{456} = (7, 7, 5, 1)$
 91 : $P_{463} = (6, 0, 6, 1)$
 92 : $P_{466} = (1, 1, 6, 1)$
 93 : $P_{470} = (5, 1, 6, 1)$
 94 : $P_{474} = (1, 2, 6, 1)$
 95 : $P_{475} = (2, 2, 6, 1)$
 96 : $P_{482} = (1, 3, 6, 1)$
 97 : $P_{494} = (5, 4, 6, 1)$
 98 : $P_{499} = (2, 5, 6, 1)$
 99 : $P_{502} = (5, 5, 6, 1)$
 100 : $P_{505} = (0, 6, 6, 1)$
 101 : $P_{515} = (2, 7, 6, 1)$
 102 : $P_{528} = (7, 0, 7, 1)$
 103 : $P_{531} = (2, 1, 7, 1)$
 104 : $P_{539} = (2, 2, 7, 1)$
 105 : $P_{543} = (6, 2, 7, 1)$
 106 : $P_{547} = (2, 3, 7, 1)$
 107 : $P_{548} = (3, 3, 7, 1)$
 108 : $P_{559} = (6, 4, 7, 1)$
 109 : $P_{564} = (3, 5, 7, 1)$
 110 : $P_{572} = (3, 6, 7, 1)$
 111 : $P_{575} = (6, 6, 7, 1)$
 112 : $P_{577} = (0, 7, 7, 1)$