

Rank-65614 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431659934

General information

Number of lines	3
Number of points	25
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	12
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	$3, 1^{12}, 0^{12}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{105}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0, 3, 0, 0, 0, 1)_{107}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0, 2, 0, 0, 0, 1)_{106}$$

Rank of lines: (341, 351, 346)

Rank of points on Klein quadric: (105, 107, 106)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 12 single points:

The single points on the surface are:

0 : $P_{26} = (0, 1, 0, 1)$ lies on line ℓ_0
 1 : $P_{30} = (0, 2, 0, 1)$ lies on line ℓ_1
 2 : $P_{34} = (0, 3, 0, 1)$ lies on line ℓ_2
 3 : $P_{42} = (0, 1, 1, 1)$ lies on line ℓ_0
 4 : $P_{45} = (0, 2, 1, 1)$ lies on line ℓ_1
 5 : $P_{49} = (0, 3, 1, 1)$ lies on line ℓ_2
 6 : $P_{57} = (0, 1, 2, 1)$ lies on line ℓ_0

7 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_1
 8 : $P_{65} = (0, 3, 2, 1)$ lies on line ℓ_2
 9 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_0
 10 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_1
 11 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:

The points on the surface but not on lines are:

0 : $P_5 = (1, 1, 0, 0)$
 1 : $P_6 = (2, 1, 0, 0)$
 2 : $P_7 = (3, 1, 0, 0)$
 3 : $P_{27} = (1, 1, 0, 1)$
 4 : $P_{31} = (1, 2, 0, 1)$
 5 : $P_{35} = (1, 3, 0, 1)$
 6 : $P_{43} = (2, 1, 1, 1)$

7 : $P_{44} = (3, 1, 1, 1)$
 8 : $P_{67} = (2, 3, 2, 1)$
 9 : $P_{68} = (3, 3, 2, 1)$
 10 : $P_{79} = (2, 2, 3, 1)$
 11 : $P_{80} = (3, 2, 3, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 25 points:

The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$
 1 : $P_5 = (1, 1, 0, 0)$
 2 : $P_6 = (2, 1, 0, 0)$
 3 : $P_7 = (3, 1, 0, 0)$
 4 : $P_{26} = (0, 1, 0, 1)$
 5 : $P_{27} = (1, 1, 0, 1)$
 6 : $P_{30} = (0, 2, 0, 1)$
 7 : $P_{31} = (1, 2, 0, 1)$
 8 : $P_{34} = (0, 3, 0, 1)$

9 : $P_{35} = (1, 3, 0, 1)$
 10 : $P_{42} = (0, 1, 1, 1)$
 11 : $P_{43} = (2, 1, 1, 1)$
 12 : $P_{44} = (3, 1, 1, 1)$
 13 : $P_{45} = (0, 2, 1, 1)$
 14 : $P_{49} = (0, 3, 1, 1)$
 15 : $P_{57} = (0, 1, 2, 1)$
 16 : $P_{61} = (0, 2, 2, 1)$
 17 : $P_{65} = (0, 3, 2, 1)$

18 : $P_{67} = (2, 3, 2, 1)$
 19 : $P_{68} = (3, 3, 2, 1)$
 20 : $P_{73} = (0, 1, 3, 1)$
 21 : $P_{77} = (0, 2, 3, 1)$
 22 : $P_{79} = (2, 2, 3, 1)$
 23 : $P_{80} = (3, 2, 3, 1)$
 24 : $P_{81} = (0, 3, 3, 1)$