Rank-67243 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_2 + X_0 X_1^2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is -1857743795

General information

Number of lines	2
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	40
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	$2, 1^{16}, 0^{40}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \mathbf{Pl}(0,1,0,0,0,1)_{657}$$

Rank of lines: (4672, 4681)
Rank of points on Klein quadric: (649, 657)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0
1 : $P_{19} = (0, 1, 1, 0)$ lies on line ℓ_0
2 : $P_{27} = (0, 2, 1, 0)$ lies on line ℓ_0
3 : $P_{35} = (0, 3, 1, 0)$ lies on line ℓ_0
4 : $P_{43} = (0, 4, 1, 0)$ lies on line ℓ_0
5 : $P_{51} = (0, 5, 1, 0)$ lies on line ℓ_0
6 : $P_{59} = (0, 6, 1, 0)$ lies on line ℓ_0
7 : $P_{67} = (0, 7, 1, 0)$ lies on line ℓ_0
8 : $P_{82} = (0, 1, 0, 1)$ lies on line ℓ_1

9 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_1
10 : $P_{209} = (0, 1, 2, 1)$ lies on line ℓ_1
11 : $P_{273} = (0, 1, 3, 1)$ lies on line ℓ_1
12 : $P_{337} = (0, 1, 4, 1)$ lies on line ℓ_1
13 : $P_{401} = (0, 1, 5, 1)$ lies on line ℓ_1
14 : $P_{465} = (0, 1, 6, 1)$ lies on line ℓ_1
15 : $P_{529} = (0, 1, 7, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 40 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_{12} = (1, 0, 1, 0)$
3 : $P_{20} = (1, 1, 1, 0)$
4 : $P_{34} = (7, 2, 1, 0)$
5 : $P_{42} = (7, 3, 1, 0)$
6 : $P_{45} = (2, 4, 1, 0)$
7 : $P_{53} = (2, 5, 1, 0)$
8 : $P_{63} = (4, 6, 1, 0)$
9 : $P_{71} = (4, 7, 1, 0)$
10 : $P_{92} = (2, 2, 0, 1)$
11 : $P_{104} = (6, 3, 0, 1)$
12 : $P_{110} = (4, 4, 0, 1)$

13 : $P_{117} = (3, 5, 0, 1)$
14 : $P_{127} = (5, 6, 0, 1)$
15 : $P_{137} = (7, 7, 0, 1)$
16 : $P_{214} = (5, 1, 2, 1)$
17 : $P_{242} = (1, 5, 2, 1)$
18 : $P_{246} = (5, 5, 2, 1)$
19 : $P_{270} = (5, 0, 3, 1)$
20 : $P_{271} = (6, 0, 3, 1)$
21 : $P_{279} = (6, 1, 3, 1)$
22 : $P_{314} = (1, 6, 3, 1)$
23 : $P_{318} = (5, 6, 3, 1)$
24 : $P_{343} = (6, 1, 4, 1)$
25 : $P_{378} = (1, 6, 4, 1)$

$26: P_{383} = (6, 6, 4, 1)$
$27: P_{396} = (3, 0, 5, 1)$
$28: P_{399} = (6, 0, 5, 1)$
$29: P_{404} = (3, 1, 5, 1)$
$30: P_{418} = (1, 3, 5, 1)$
$31: P_{423} = (6, 3, 5, 1)$
$32: P_{460} = (3, 0, 6, 1)$
$33: P_{462} = (5, 0, 6, 1)$

$34: P_{470} = (5, 1, 6, 1)$
$35: P_{498} = (1, 5, 6, 1)$
$36: P_{500} = (3, 5, 6, 1)$
$37: P_{532} = (3, 1, 7, 1)$
$38: P_{546} = (1, 3, 7, 1)$
$39: P_{548} = (3, 3, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 57 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_1 = (0, 1, 0, 0)$
$2: P_2 = (0, 0, 1, 0)$
$3: P_4 = (1, 1, 1, 1)$
$4: P_{12} = (1, 0, 1, 0)$
$5: P_{19} = (0, 1, 1, 0)$
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$56: P_{548} = (3, 3, 7, 1)$