Rank-487 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_3 + X_0X_1^2 + X_1^2X_2 = 0$$

(0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 571613725

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	6
Number of double points	0
Number of single points	135
Number of points off lines	180
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^6, 1^{135}, 0^{180}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \delta^6 & \delta^7 \\ 0 & 1 & \delta^{13} & \delta^9 \end{bmatrix}_{34757} = \begin{bmatrix} 1 & 0 & 15 & 7 \\ 0 & 1 & 6 & 5 \end{bmatrix}_{34757} = \mathbf{Pl}(13,9,12,2,8,1)_{40908}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & \delta^{12} & \delta^{14} \\ 0 & 1 & \delta^{11} & \delta^{3} \end{array}\right]_{53376} = \left[\begin{array}{cccc} 1 & 0 & 3 & 12 \\ 0 & 1 & 13 & 8 \end{array}\right]_{53376} = \mathbf{Pl}(7,14,6,4,15,1)_{68187}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & \delta^{14} & \delta^{13} \\ 0 & 1 & \delta^{7} & \delta^{11} \end{array}\right]_{29699} = \left[\begin{array}{cccc} 1 & 0 & 12 & 6 \\ 0 & 1 & 7 & 13 \end{array}\right]_{29699} = \mathbf{Pl}(12,2,2,12,3,1)_{18392}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & \delta^{11} & \delta^{7} \\ 0 & 1 & \delta^{13} & \delta^{14} \end{array}\right]_{34323} = \left[\begin{array}{cccc} 1 & 0 & 13 & 7 \\ 0 & 1 & 6 & 12 \end{array}\right]_{34323} = \mathbf{Pl}(13,9,9,13,8,1)_{40263}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & \delta^{9} & \delta^{13} \\ 0 & 1 & \delta^{7} & \delta^{6} \end{array}\right]_{27820} = \left[\begin{array}{cccc} 1 & 0 & 5 & 6 \\ 0 & 1 & 7 & 15 \end{array}\right]_{27820} = \mathbf{Pl}(12,2,13,9,3,1)_{20717}$$

$$\ell_6 = \left[\begin{array}{cccc} 1 & 0 & \delta^{7} & \delta^{14} \\ 0 & 1 & \delta^{11} & \delta^{13} \end{array}\right]_{54436} = \left[\begin{array}{cccc} 1 & 0 & 7 & 12 \\ 0 & 1 & 13 & 6 \end{array}\right]_{54436} = \mathbf{Pl}(7,14,14,7,15,1)_{69882}$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 0 & \delta^{13} & \delta^{11} \\ 0 & 1 & \delta^{14} & \delta^{7} \end{array}\right]_{58546} = \left[\begin{array}{cccc} 1 & 0 & 6 & 13 \\ 0 & 1 & 12 & 7 \end{array}\right]_{58546} = \mathbf{Pl}(6,4,4,6,5,1)_{26981}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & \delta^{3} & \delta^{11} \\ 0 & 1 & \delta^{14} & \delta^{12} \end{array}\right]_{59028} = \left[\begin{array}{cccc} 1 & 0 & 8 & 13 \\ 0 & 1 & 12 & 3 \end{array}\right]_{59028} = \mathbf{Pl}(6,4,7,14,5,1)_{27596}$$

Rank of lines: (530, 34757, 53376, 29699, 34323, 27820, 54436, 58546, 59028)
Rank of points on Klein quadric: (9426, 40908, 68187, 18392, 40263, 20717, 69882, 26981, 27596)

Eckardt Points

The surface has 6 Eckardt points:
$0 : P_{699} = \mathbf{P}(\delta^{10}, \delta^{10}, 1, 1) = \mathbf{P}(10, 10, 1, 1)$,
$1 : P_{716} = \mathbf{P}(\delta^{5}, \delta^{5}, 1, 1) = \mathbf{P}(11, 11, 1, 1)$,
$2 : P_{1174} = \mathbf{P}(\delta^{9}, \delta^{3}, \delta^{12}, 1) = \mathbf{P}(5, 8, 3, 1)$,
$3 : P_{1801} = \mathbf{P}(\delta^{3}, \delta^{6}, \delta^{9}, 1) = \mathbf{P}(8, 15, 5, 1)$,
$4 : P_{2384} = \mathbf{P}(\delta^{6}, \delta^{12}, \delta^{3}, 1) = \mathbf{P}(15, 3, 8, 1)$,
$5 : P_{4196} = \mathbf{P}(\delta^{12}, \delta^{9}, \delta^{6}, 1) = \mathbf{P}(3, 5, 15, 1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 135 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_0
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_{79} = (12,3,1,0)$ lies on line ℓ_1
$3 : P_{105} = (6,5,1,0)$ lies on line ℓ_2
$4 : P_{128} = (13,6,1,0)$ lies on line ℓ_3
$5 : P_{143} = (12,7,1,0)$ lies on line ℓ_4
$6 : P_{160} = (13,8,1,0)$ lies on line ℓ_5
$7 : P_{217} = (6,12,1,0)$ lies on line ℓ_6
$8 : P_{234} = (7,13,1,0)$ lies on line ℓ_7
$9 : P_{266} = (7,15,1,0)$ lies on line ℓ_8
$10 : P_{375} = (5,6,0,1)$ lies on line ℓ_8
$11 : P_{382} = (12,6,0,1)$ lies on line ℓ_7
$12 : P_{399} = (13,7,0,1)$ lies on line ℓ_6
$13 : P_{401} = (15,7,0,1)$ lies on line ℓ_2
$14 : P_{469} = (3,12,0,1)$ lies on line ℓ_5
$15 : P_{473} = (7,12,0,1)$ lies on line ℓ_3

16 : $P_{488} = (6, 13, 0, 1)$ lies on line ℓ_4
17 : $P_{490} = (8, 13, 0, 1)$ lies on line ℓ_1
18 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_0
19 : $P_{563} = (2, 2, 1, 1)$ lies on line ℓ_0
20 : $P_{570} = (9, 2, 1, 1)$ lies on line ℓ_2
21 : $P_{580} = (3, 3, 1, 1)$ lies on line ℓ_0
22 : $P_{597} = (4, 4, 1, 1)$ lies on line ℓ_0
23 : $P_{607} = (14, 4, 1, 1)$ lies on line ℓ_5
24 : $P_{614} = (5, 5, 1, 1)$ lies on line ℓ_0
25 : $P_{631} = (6, 6, 1, 1)$ lies on line ℓ_0
26 : $P_{648} = (7, 7, 1, 1)$ lies on line ℓ_0
27 : $P_{665} = (8, 8, 1, 1)$ lies on line ℓ_0
28 : $P_{675} = (2, 9, 1, 1)$ lies on line ℓ_8
29 : $P_{682} = (9, 9, 1, 1)$ lies on line ℓ_0
30 : $P_{733} = (12, 12, 1, 1)$ lies on line ℓ_0
31 : $P_{750} = (13, 13, 1, 1)$ lies on line ℓ_0
32 : $P_{757} = (4, 14, 1, 1)$ lies on line ℓ_1
33 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_0
34 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_0
35 : $P_{789} = (4, 0, 2, 1)$ lies on line ℓ_3
36 : $P_{812} = (11, 1, 2, 1)$ lies on line ℓ_8
37 : $P_{840} = (7, 3, 2, 1)$ lies on line ℓ_4
38 : $P_{865} = (0, 5, 2, 1)$ lies on line ℓ_5
39 : $P_{867} = (2, 5, 2, 1)$ lies on line ℓ_7
40 : $P_{882} = (1, 6, 2, 1)$ lies on line ℓ_6
41 : $P_{970} = (9, 11, 2, 1)$ lies on line ℓ_1
42 : $P_{996} = (3, 13, 2, 1)$ lies on line ℓ_2
43 : $P_{1116} = (11, 4, 3, 1)$ lies on line ℓ_4
44 : $P_{1146} = (9, 6, 3, 1)$ lies on line ℓ_3
45 : $P_{1208} = (7, 10, 3, 1)$ lies on line ℓ_6
46 : $P_{1262} = (13, 13, 3, 1)$ lies on line ℓ_5
47 : $P_{1277} = (12, 14, 3, 1)$ lies on line ℓ_8
48 : $P_{1306} = (9, 0, 4, 1)$ lies on line ℓ_7
49 : $P_{1323} = (10, 1, 4, 1)$ lies on line ℓ_1
50 : $P_{1389} = (12, 5, 4, 1)$ lies on line ℓ_6
51 : $P_{1414} = (5, 7, 4, 1)$ lies on line ℓ_5
52 : $P_{1425} = (0, 8, 4, 1)$ lies on line ℓ_8
53 : $P_{1429} = (4, 8, 4, 1)$ lies on line ℓ_4
54 : $P_{1471} = (14, 10, 4, 1)$ lies on line ℓ_2
55 : $P_{1506} = (1, 13, 4, 1)$ lies on line ℓ_3
56 : $P_{1591} = (6, 2, 5, 1)$ lies on line ℓ_1
57 : $P_{1672} = (7, 7, 5, 1)$ lies on line ℓ_8
58 : $P_{1707} = (10, 9, 5, 1)$ lies on line ℓ_6
59 : $P_{1741} = (12, 11, 5, 1)$ lies on line ℓ_3
60 : $P_{1775} = (14, 13, 5, 1)$ lies on line ℓ_7
61 : $P_{1811} = (2, 0, 6, 1)$ lies on line ℓ_2
62 : $P_{1827} = (2, 1, 6, 1)$ lies on line ℓ_3
63 : $P_{1864} = (7, 3, 6, 1)$ lies on line ℓ_7
64 : $P_{1873} = (0, 4, 6, 1)$ lies on line ℓ_6
65 : $P_{1910} = (5, 6, 6, 1)$ lies on line ℓ_4
66 : $P_{1932} = (11, 7, 6, 1)$ lies on line ℓ_1
67 : $P_{2039} = (6, 14, 6, 1)$ lies on line ℓ_5
68 : $P_{2063} = (14, 15, 6, 1)$ lies on line ℓ_8
69 : $P_{2074} = (9, 0, 7, 1)$ lies on line ℓ_8
70 : $P_{2090} = (9, 1, 7, 1)$ lies on line ℓ_4
71 : $P_{2136} = (7, 4, 7, 1)$ lies on line ℓ_1
72 : $P_{2149} = (4, 5, 7, 1)$ lies on line ℓ_2
73 : $P_{2172} = (11, 6, 7, 1)$ lies on line ℓ_5
74 : $P_{2192} = (15, 7, 7, 1)$ lies on line ℓ_3
75 : $P_{2199} = (6, 8, 7, 1)$ lies on line ℓ_6
76 : $P_{2289} = (0, 14, 7, 1)$ lies on line ℓ_7
77 : $P_{2398} = (13, 4, 8, 1)$ lies on line ℓ_2
78 : $P_{2435} = (2, 7, 8, 1)$ lies on line ℓ_4
79 : $P_{2487} = (6, 10, 8, 1)$ lies on line ℓ_7
80 : $P_{2525} = (12, 12, 8, 1)$ lies on line ℓ_1
81 : $P_{2556} = (11, 14, 8, 1)$ lies on line ℓ_3
82 : $P_{2591} = (14, 0, 9, 1)$ lies on line ℓ_4
83 : $P_{2604} = (11, 1, 9, 1)$ lies on line ℓ_2
84 : $P_{2690} = (1, 7, 9, 1)$ lies on line ℓ_7
85 : $P_{2711} = (6, 8, 9, 1)$ lies on line ℓ_3
86 : $P_{2755} = (2, 11, 9, 1)$ lies on line ℓ_5
87 : $P_{2777} = (8, 12, 9, 1)$ lies on line ℓ_8
88 : $P_{2817} = (0, 15, 9, 1)$ lies on line ℓ_1
89 : $P_{2826} = (9, 15, 9, 1)$ lies on line ℓ_6
90 : $P_{2868} = (3, 2, 10, 1)$ lies on line ℓ_6
91 : $P_{2873} = (8, 2, 10, 1)$ lies on line ℓ_3
92 : $P_{2898} = (1, 4, 10, 1)$ lies on line ℓ_8
93 : $P_{2980} = (3, 9, 10, 1)$ lies on line ℓ_4
94 : $P_{2985} = (8, 9, 10, 1)$ lies on line ℓ_7
95 : $P_{3005} = (12, 10, 10, 1)$ lies on line ℓ_5
96 : $P_{3006} = (13, 10, 10, 1)$ lies on line ℓ_1
97 : $P_{3058} = (1, 14, 10, 1)$ lies on line ℓ_2
98 : $P_{3122} = (1, 2, 11, 1)$ lies on line ℓ_5
99 : $P_{3158} = (5, 4, 11, 1)$ lies on line ℓ_3
100 : $P_{3168} = (15, 4, 11, 1)$ lies on line ℓ_7
101 : $P_{3234} = (1, 9, 11, 1)$ lies on line ℓ_1
102 : $P_{3271} = (6, 11, 11, 1)$ lies on line ℓ_8
103 : $P_{3272} = (7, 11, 11, 1)$ lies on line ℓ_2
104 : $P_{3318} = (5, 14, 11, 1)$ lies on line ℓ_6
105 : $P_{3328} = (15, 14, 11, 1)$ lies on line ℓ_4
106 : $P_{3359} = (14, 0, 12, 1)$ lies on line ℓ_1
107 : $P_{3375} = (14, 1, 12, 1)$ lies on line ℓ_6
108 : $P_{3377} = (0, 2, 12, 1)$ lies on line ℓ_4
109 : $P_{3482} = (9, 8, 12, 1)$ lies on line ℓ_5
110 : $P_{3501} = (12, 9, 12, 1)$ lies on line ℓ_2
111 : $P_{3540} = (3, 12, 12, 1)$ lies on line ℓ_7
112 : $P_{3563} = (10, 13, 12, 1)$ lies on line ℓ_8
113 : $P_{3598} = (13, 15, 12, 1)$ lies on line ℓ_3
114 : $P_{3605} = (4, 0, 13, 1)$ lies on line ℓ_5
115 : $P_{3621} = (4, 1, 13, 1)$ lies on line ℓ_7
116 : $P_{3646} = (13, 2, 13, 1)$ lies on line ℓ_8
117 : $P_{3651} = (2, 3, 13, 1)$ lies on line ℓ_1
118 : $P_{3693} = (12, 5, 13, 1)$ lies on line ℓ_4
119 : $P_{3745} = (0, 9, 13, 1)$ lies on line ℓ_3
120 : $P_{3803} = (10, 12, 13, 1)$ lies on line ℓ_2
121 : $P_{3817} = (8, 13, 13, 1)$ lies on line ℓ_6
122 : $P_{3859} = (2, 0, 14, 1)$ lies on line ℓ_6
123 : $P_{3883} = (10, 1, 14, 1)$ lies on line ℓ_5

124 : $P_{3905} = (0, 3, 14, 1)$ lies on line ℓ_2
125 : $P_{3919} = (14, 3, 14, 1)$ lies on line ℓ_3
126 : $P_{3968} = (15, 6, 14, 1)$ lies on line ℓ_1
127 : $P_{4021} = (4, 10, 14, 1)$ lies on line ℓ_8
128 : $P_{4050} = (1, 12, 14, 1)$ lies on line ℓ_4
129 : $P_{4110} = (13, 15, 14, 1)$ lies on line ℓ_7
130 : $P_{4155} = (10, 2, 15, 1)$ lies on line ℓ_7
131 : $P_{4215} = (6, 6, 15, 1)$ lies on line ℓ_2
132 : $P_{4264} = (7, 9, 15, 1)$ lies on line ℓ_5
133 : $P_{4302} = (13, 11, 15, 1)$ lies on line ℓ_4
134 : $P_{4309} = (4, 12, 15, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 180 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_{35} = (0, 1, 1, 0)$
3 : $P_{36} = (1, 1, 1, 0)$
4 : $P_{82} = (15, 3, 1, 0)$
5 : $P_{102} = (3, 5, 1, 0)$
6 : $P_{126} = (11, 6, 1, 0)$
7 : $P_{142} = (11, 7, 1, 0)$
8 : $P_{152} = (5, 8, 1, 0)$
9 : $P_{184} = (5, 10, 1, 0)$
10 : $P_{194} = (15, 10, 1, 0)$
11 : $P_{198} = (3, 11, 1, 0)$
12 : $P_{203} = (8, 11, 1, 0)$
13 : $P_{221} = (10, 12, 1, 0)$
14 : $P_{237} = (10, 13, 1, 0)$
15 : $P_{267} = (8, 15, 1, 0)$
16 : $P_{275} = (1, 0, 0, 1)$
17 : $P_{291} = (1, 1, 0, 1)$
18 : $P_{327} = (5, 3, 0, 1)$
19 : $P_{333} = (11, 3, 0, 1)$
20 : $P_{362} = (8, 5, 0, 1)$
21 : $P_{364} = (10, 5, 0, 1)$
22 : $P_{413} = (11, 8, 0, 1)$
23 : $P_{417} = (15, 8, 0, 1)$
24 : $P_{446} = (12, 10, 0, 1)$
25 : $P_{447} = (13, 10, 0, 1)$
26 : $P_{456} = (6, 11, 0, 1)$
27 : $P_{457} = (7, 11, 0, 1)$
28 : $P_{517} = (3, 15, 0, 1)$
29 : $P_{524} = (10, 15, 0, 1)$
30 : $P_{590} = (13, 3, 1, 1)$
31 : $P_{616} = (7, 5, 1, 1)$
32 : $P_{640} = (15, 6, 1, 1)$
33 : $P_{646} = (5, 7, 1, 1)$
34 : $P_{669} = (12, 8, 1, 1)$
35 : $P_{700} = (11, 10, 1, 1)$
36 : $P_{715} = (10, 11, 1, 1)$
37 : $P_{729} = (8, 12, 1, 1)$
38 : $P_{740} = (3, 13, 1, 1)$
39 : $P_{775} = (6, 15, 1, 1)$
40 : $P_{842} = (9, 3, 2, 1)$
41 : $P_{889} = (8, 6, 2, 1)$
42 : $P_{916} = (3, 8, 2, 1)$
43 : $P_{920} = (7, 8, 2, 1)$
44 : $P_{969} = (8, 11, 2, 1)$
45 : $P_{1006} = (13, 13, 2, 1)$
46 : $P_{1010} = (1, 14, 2, 1)$
47 : $P_{1020} = (11, 14, 2, 1)$
48 : $P_{1029} = (4, 15, 2, 1)$
49 : $P_{1038} = (13, 15, 2, 1)$
50 : $P_{1050} = (9, 0, 3, 1)$
51 : $P_{1070} = (13, 1, 3, 1)$
52 : $P_{1094} = (5, 3, 3, 1)$
53 : $P_{1100} = (11, 3, 3, 1)$
54 : $P_{1106} = (1, 4, 3, 1)$
55 : $P_{1137} = (0, 6, 3, 1)$
56 : $P_{1170} = (1, 8, 3, 1)$
57 : $P_{1192} = (7, 9, 3, 1)$
58 : $P_{1197} = (12, 9, 3, 1)$
59 : $P_{1207} = (6, 10, 3, 1)$
60 : $P_{1252} = (3, 13, 3, 1)$
61 : $P_{1271} = (6, 14, 3, 1)$
62 : $P_{1330} = (1, 2, 4, 1)$
63 : $P_{1339} = (10, 2, 4, 1)$
64 : $P_{1352} = (7, 3, 4, 1)$
65 : $P_{1354} = (9, 3, 4, 1)$
66 : $P_{1391} = (14, 5, 4, 1)$
67 : $P_{1416} = (7, 7, 4, 1)$
68 : $P_{1472} = (15, 10, 4, 1)$
69 : $P_{1520} = (15, 13, 4, 1)$
70 : $P_{1542} = (5, 15, 4, 1)$
71 : $P_{1549} = (12, 15, 4, 1)$
72 : $P_{1567} = (14, 0, 5, 1)$
73 : $P_{1576} = (7, 1, 5, 1)$
74 : $P_{1598} = (13, 2, 5, 1)$
75 : $P_{1641} = (8, 5, 5, 1)$

$76: P_{1643} = (10, 5, 5, 1)$
$77: P_{1670} = (5, 7, 5, 1)$
$78: P_{1698} = (1, 9, 5, 1)$
$79: P_{1742} = (13, 11, 5, 1)$
$80: P_{1761} = (0, 13, 5, 1)$
$81: P_{1783} = (6, 14, 5, 1)$
$82: P_{1789} = (12, 14, 5, 1)$
$83: P_{1794} = (1, 15, 5, 1)$
$84: P_{1866} = (9, 3, 6, 1)$
$85: P_{1883} = (10, 4, 6, 1)$
$86: P_{1917} = (12, 6, 6, 1)$
$87: P_{1930} = (9, 7, 6, 1)$
$88: P_{1958} = (5, 9, 6, 1)$
$89: P_{1967} = (14, 9, 6, 1)$
$90: P_{1995} = (10, 11, 6, 1)$
$91: P_{1996} = (11, 11, 6, 1)$
$92: P_{2045} = (12, 14, 6, 1)$
$93: P_{2056} = (7, 15, 6, 1)$
$94: P_{2101} = (4, 2, 7, 1)$
$95: P_{2112} = (15, 2, 7, 1)$
$96: P_{2142} = (13, 4, 7, 1)$
$97: P_{2151} = (6, 5, 7, 1)$
$98: P_{2163} = (2, 6, 7, 1)$
$99: P_{2190} = (13, 7, 7, 1)$
$100: P_{2195} = (2, 8, 7, 1)$
$101: P_{2251} = (10, 11, 7, 1)$
$102: P_{2252} = (11, 11, 7, 1)$
$103: P_{2299} = (10, 14, 7, 1)$
$104: P_{2323} = (2, 0, 8, 1)$
$105: P_{2349} = (12, 1, 8, 1)$
$106: P_{2359} = (6, 2, 8, 1)$
$107: P_{2366} = (13, 2, 8, 1)$
$108: P_{2370} = (1, 3, 8, 1)$
$109: P_{2392} = (7, 4, 8, 1)$
$110: P_{2433} = (0, 7, 8, 1)$
$111: P_{2460} = (11, 8, 8, 1)$
$112: P_{2464} = (15, 8, 8, 1)$
$113: P_{2488} = (7, 10, 8, 1)$
$114: P_{2521} = (8, 12, 8, 1)$
$115: P_{2546} = (1, 14, 8, 1)$
$116: P_{2631} = (6, 3, 9, 1)$
$117: P_{2633} = (8, 3, 9, 1)$
$118: P_{2642} = (1, 4, 9, 1)$
$119: P_{2652} = (11, 4, 9, 1)$
$120: P_{2669} = (12, 5, 9, 1)$
$121: P_{2671} = (14, 5, 9, 1)$
$122: P_{2692} = (3, 7, 9, 1)$
$123: P_{2707} = (2, 8, 9, 1)$
$124: P_{2756} = (3, 11, 9, 1)$
$125: P_{2781} = (12, 12, 9, 1)$
$126: P_{2833} = (0, 0, 10, 1)$
$127: P_{2859} = (10, 1, 10, 1)$
$128: P_{2908} = (11, 4, 10, 1)$
$129: P_{3021} = (12, 11, 10, 1)$
$130: P_{3022} = (13, 11, 10, 1)$
$131: P_{3068} = (11, 14, 10, 1)$
$132: P_{3089} = (0, 0, 11, 1)$
$133: P_{3116} = (11, 1, 11, 1)$
$134: P_{3131} = (10, 2, 11, 1)$
$135: P_{3243} = (10, 9, 11, 1)$
$136: P_{3255} = (6, 10, 11, 1)$
$137: P_{3256} = (7, 10, 11, 1)$
$138: P_{3388} = (11, 2, 12, 1)$
$139: P_{3412} = (3, 4, 12, 1)$
$140: P_{3418} = (9, 4, 12, 1)$
$141: P_{3486} = (13, 8, 12, 1)$
$142: P_{3496} = (7, 9, 12, 1)$
$143: P_{3515} = (10, 10, 12, 1)$
$144: P_{3516} = (11, 10, 12, 1)$
$145: P_{3544} = (7, 12, 12, 1)$
$146: P_{3557} = (4, 13, 12, 1)$
$147: P_{3589} = (4, 15, 12, 1)$
$148: P_{3639} = (6, 2, 13, 1)$
$149: P_{3661} = (12, 3, 13, 1)$
$150: P_{3695} = (14, 5, 13, 1)$
$151: P_{3756} = (11, 9, 13, 1)$
$152: P_{3771} = (10, 10, 13, 1)$
$153: P_{3772} = (11, 10, 13, 1)$
$154: P_{3807} = (14, 12, 13, 1)$
$155: P_{3815} = (6, 13, 13, 1)$
$156: P_{3827} = (2, 14, 13, 1)$
$157: P_{3833} = (8, 14, 13, 1)$
$158: P_{3950} = (13, 5, 14, 1)$
$159: P_{3952} = (15, 5, 14, 1)$
$160: P_{3959} = (6, 6, 14, 1)$
$161: P_{3987} = (2, 8, 14, 1)$
$162: P_{3991} = (6, 8, 14, 1)$
$163: P_{4002} = (1, 9, 14, 1)$
$164: P_{4011} = (10, 9, 14, 1)$
$165: P_{4022} = (5, 10, 14, 1)$
$166: P_{4054} = (5, 12, 14, 1)$
$167: P_{4101} = (4, 15, 14, 1)$
$168: P_{4117} = (4, 0, 15, 1)$
$169: P_{4135} = (6, 1, 15, 1)$
$170: P_{4146} = (1, 2, 15, 1)$
$171: P_{4184} = (7, 4, 15, 1)$
$172: P_{4190} = (13, 4, 15, 1)$
$173: P_{4194} = (1, 5, 15, 1)$
$174: P_{4224} = (15, 6, 15, 1)$
$175: P_{4269} = (12, 9, 15, 1)$
$176: P_{4301} = (12, 11, 15, 1)$
$177: P_{4305} = (0, 12, 15, 1)$
$178: P_{4356} = (3, 15, 15, 1)$
$179: P_{4363} = (10, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 0 0 1 1 0 1 1 0
1	0 0 1 1 0 0 0 1 1
2	0 1 0 0 1 1 0 1 0
3	1 1 0 0 1 0 0 0 1
4	1 0 1 1 0 1 0 0 0
5	0 0 1 0 1 0 1 0 1
6	1 0 0 0 0 1 0 1 1
7	1 1 1 0 0 0 1 0 0
8	0 1 0 1 0 1 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_{699}	P_{699}	P_{716}	P_{716}

Line 1 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_{1174}	P_{4196}	P_{1174}	P_{4196}

Line 2 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_{1174}	P_{1801}	P_{1801}	P_{1174}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_8
in point	P_{699}	P_{4196}	P_{699}	P_{4196}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_{699}	P_{1801}	P_{699}	P_{1801}

Line 5 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_8
in point	P_{1801}	P_{1801}	P_{2384}	P_{2384}

Line 6 intersects

Line	ℓ_0	ℓ_5	ℓ_7	ℓ_8
in point	P_{716}	P_{2384}	P_{716}	P_{2384}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_6
in point	P_{716}	P_{1174}	P_{1174}	P_{716}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6
in point	P_{4196}	P_{4196}	P_{2384}	P_{2384}

The surface has 321 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_{35} = (0, 1, 1, 0)$
5 : $P_{36} = (1, 1, 1, 0)$
6 : $P_{79} = (12, 3, 1, 0)$
7 : $P_{82} = (15, 3, 1, 0)$
8 : $P_{102} = (3, 5, 1, 0)$
9 : $P_{105} = (6, 5, 1, 0)$
10 : $P_{126} = (11, 6, 1, 0)$
11 : $P_{128} = (13, 6, 1, 0)$
12 : $P_{142} = (11, 7, 1, 0)$
13 : $P_{143} = (12, 7, 1, 0)$
14 : $P_{152} = (5, 8, 1, 0)$
15 : $P_{160} = (13, 8, 1, 0)$
16 : $P_{184} = (5, 10, 1, 0)$
17 : $P_{194} = (15, 10, 1, 0)$
18 : $P_{198} = (3, 11, 1, 0)$
19 : $P_{203} = (8, 11, 1, 0)$
20 : $P_{217} = (6, 12, 1, 0)$

21 : $P_{221} = (10, 12, 1, 0)$
22 : $P_{234} = (7, 13, 1, 0)$
23 : $P_{237} = (10, 13, 1, 0)$
24 : $P_{266} = (7, 15, 1, 0)$
25 : $P_{267} = (8, 15, 1, 0)$
26 : $P_{275} = (1, 0, 0, 1)$
27 : $P_{291} = (1, 1, 0, 1)$
28 : $P_{327} = (5, 3, 0, 1)$
29 : $P_{333} = (11, 3, 0, 1)$
30 : $P_{362} = (8, 5, 0, 1)$
31 : $P_{364} = (10, 5, 0, 1)$
32 : $P_{375} = (5, 6, 0, 1)$
33 : $P_{382} = (12, 6, 0, 1)$
34 : $P_{399} = (13, 7, 0, 1)$
35 : $P_{401} = (15, 7, 0, 1)$
36 : $P_{413} = (11, 8, 0, 1)$
37 : $P_{417} = (15, 8, 0, 1)$
38 : $P_{446} = (12, 10, 0, 1)$
39 : $P_{447} = (13, 10, 0, 1)$
40 : $P_{456} = (6, 11, 0, 1)$
41 : $P_{457} = (7, 11, 0, 1)$
42 : $P_{469} = (3, 12, 0, 1)$
43 : $P_{473} = (7, 12, 0, 1)$
44 : $P_{488} = (6, 13, 0, 1)$
45 : $P_{490} = (8, 13, 0, 1)$
46 : $P_{517} = (3, 15, 0, 1)$
47 : $P_{524} = (10, 15, 0, 1)$
48 : $P_{530} = (0, 0, 1, 1)$
49 : $P_{563} = (2, 2, 1, 1)$
50 : $P_{570} = (9, 2, 1, 1)$
51 : $P_{580} = (3, 3, 1, 1)$
52 : $P_{590} = (13, 3, 1, 1)$
53 : $P_{597} = (4, 4, 1, 1)$
54 : $P_{607} = (14, 4, 1, 1)$
55 : $P_{614} = (5, 5, 1, 1)$
56 : $P_{616} = (7, 5, 1, 1)$
57 : $P_{631} = (6, 6, 1, 1)$
58 : $P_{640} = (15, 6, 1, 1)$
59 : $P_{646} = (5, 7, 1, 1)$
60 : $P_{648} = (7, 7, 1, 1)$
61 : $P_{665} = (8, 8, 1, 1)$
62 : $P_{669} = (12, 8, 1, 1)$
63 : $P_{675} = (2, 9, 1, 1)$
64 : $P_{682} = (9, 9, 1, 1)$
65 : $P_{699} = (10, 10, 1, 1)$
66 : $P_{700} = (11, 10, 1, 1)$
67 : $P_{715} = (10, 11, 1, 1)$
68 : $P_{716} = (11, 11, 1, 1)$
69 : $P_{729} = (8, 12, 1, 1)$
70 : $P_{733} = (12, 12, 1, 1)$
71 : $P_{740} = (3, 13, 1, 1)$
72 : $P_{750} = (13, 13, 1, 1)$
73 : $P_{757} = (4, 14, 1, 1)$
74 : $P_{767} = (14, 14, 1, 1)$
75 : $P_{775} = (6, 15, 1, 1)$
76 : $P_{784} = (15, 15, 1, 1)$
77 : $P_{789} = (4, 0, 2, 1)$
78 : $P_{812} = (11, 1, 2, 1)$
79 : $P_{840} = (7, 3, 2, 1)$
80 : $P_{842} = (9, 3, 2, 1)$
81 : $P_{865} = (0, 5, 2, 1)$
82 : $P_{867} = (2, 5, 2, 1)$
83 : $P_{882} = (1, 6, 2, 1)$
84 : $P_{889} = (8, 6, 2, 1)$
85 : $P_{916} = (3, 8, 2, 1)$
86 : $P_{920} = (7, 8, 2, 1)$
87 : $P_{969} = (8, 11, 2, 1)$
88 : $P_{970} = (9, 11, 2, 1)$
89 : $P_{996} = (3, 13, 2, 1)$
90 : $P_{1006} = (13, 13, 2, 1)$
91 : $P_{1010} = (1, 14, 2, 1)$
92 : $P_{1020} = (11, 14, 2, 1)$
93 : $P_{1029} = (4, 15, 2, 1)$
94 : $P_{1038} = (13, 15, 2, 1)$
95 : $P_{1050} = (9, 0, 3, 1)$
96 : $P_{1070} = (13, 1, 3, 1)$
97 : $P_{1094} = (5, 3, 3, 1)$
98 : $P_{1100} = (11, 3, 3, 1)$
99 : $P_{1106} = (1, 4, 3, 1)$
100 : $P_{1116} = (11, 4, 3, 1)$
101 : $P_{1137} = (0, 6, 3, 1)$
102 : $P_{1146} = (9, 6, 3, 1)$
103 : $P_{1170} = (1, 8, 3, 1)$
104 : $P_{1174} = (5, 8, 3, 1)$
105 : $P_{1192} = (7, 9, 3, 1)$
106 : $P_{1197} = (12, 9, 3, 1)$
107 : $P_{1207} = (6, 10, 3, 1)$
108 : $P_{1208} = (7, 10, 3, 1)$
109 : $P_{1252} = (3, 13, 3, 1)$
110 : $P_{1262} = (13, 13, 3, 1)$
111 : $P_{1271} = (6, 14, 3, 1)$
112 : $P_{1277} = (12, 14, 3, 1)$
113 : $P_{1306} = (9, 0, 4, 1)$
114 : $P_{1323} = (10, 1, 4, 1)$
115 : $P_{1330} = (1, 2, 4, 1)$
116 : $P_{1339} = (10, 2, 4, 1)$
117 : $P_{1352} = (7, 3, 4, 1)$
118 : $P_{1354} = (9, 3, 4, 1)$
119 : $P_{1389} = (12, 5, 4, 1)$
120 : $P_{1391} = (14, 5, 4, 1)$
121 : $P_{1414} = (5, 7, 4, 1)$
122 : $P_{1416} = (7, 7, 4, 1)$
123 : $P_{1425} = (0, 8, 4, 1)$
124 : $P_{1429} = (4, 8, 4, 1)$
125 : $P_{1471} = (14, 10, 4, 1)$
126 : $P_{1472} = (15, 10, 4, 1)$
127 : $P_{1506} = (1, 13, 4, 1)$
128 : $P_{1520} = (15, 13, 4, 1)$
129 : $P_{1542} = (5, 15, 4, 1)$
130 : $P_{1549} = (12, 15, 4, 1)$
131 : $P_{1567} = (14, 0, 5, 1)$
132 : $P_{1576} = (7, 1, 5, 1)$
133 : $P_{1591} = (6, 2, 5, 1)$
134 : $P_{1598} = (13, 2, 5, 1)$
135 : $P_{1641} = (8, 5, 5, 1)$
136 : $P_{1643} = (10, 5, 5, 1)$
137 : $P_{1670} = (5, 7, 5, 1)$
138 : $P_{1672} = (7, 7, 5, 1)$
139 : $P_{1698} = (1, 9, 5, 1)$
140 : $P_{1707} = (10, 9, 5, 1)$
141 : $P_{1741} = (12, 11, 5, 1)$
142 : $P_{1742} = (13, 11, 5, 1)$
143 : $P_{1761} = (0, 13, 5, 1)$
144 : $P_{1775} = (14, 13, 5, 1)$
145 : $P_{1783} = (6, 14, 5, 1)$
146 : $P_{1789} = (12, 14, 5, 1)$
147 : $P_{1794} = (1, 15, 5, 1)$
148 : $P_{1801} = (8, 15, 5, 1)$
149 : $P_{1811} = (2, 0, 6, 1)$
150 : $P_{1827} = (2, 1, 6, 1)$
151 : $P_{1864} = (7, 3, 6, 1)$
152 : $P_{1866} = (9, 3, 6, 1)$
153 : $P_{1873} = (0, 4, 6, 1)$
154 : $P_{1883} = (10, 4, 6, 1)$
155 : $P_{1910} = (5, 6, 6, 1)$
156 : $P_{1917} = (12, 6, 6, 1)$
157 : $P_{1930} = (9, 7, 6, 1)$
158 : $P_{1932} = (11, 7, 6, 1)$
159 : $P_{1958} = (5, 9, 6, 1)$
160 : $P_{1967} = (14, 9, 6, 1)$
161 : $P_{1995} = (10, 11, 6, 1)$
162 : $P_{1996} = (11, 11, 6, 1)$
163 : $P_{2039} = (6, 14, 6, 1)$
164 : $P_{2045} = (12, 14, 6, 1)$
165 : $P_{2056} = (7, 15, 6, 1)$
166 : $P_{2063} = (14, 15, 6, 1)$
167 : $P_{2074} = (9, 0, 7, 1)$
168 : $P_{2090} = (9, 1, 7, 1)$
169 : $P_{2101} = (4, 2, 7, 1)$
170 : $P_{2112} = (15, 2, 7, 1)$
171 : $P_{2136} = (7, 4, 7, 1)$
172 : $P_{2142} = (13, 4, 7, 1)$
173 : $P_{2149} = (4, 5, 7, 1)$
174 : $P_{2151} = (6, 5, 7, 1)$
175 : $P_{2163} = (2, 6, 7, 1)$
176 : $P_{2172} = (11, 6, 7, 1)$
177 : $P_{2190} = (13, 7, 7, 1)$
178 : $P_{2192} = (15, 7, 7, 1)$
179 : $P_{2195} = (2, 8, 7, 1)$
180 : $P_{2199} = (6, 8, 7, 1)$
181 : $P_{2251} = (10, 11, 7, 1)$
182 : $P_{2252} = (11, 11, 7, 1)$

183 : $P_{2289} = (0, 14, 7, 1)$
184 : $P_{2299} = (10, 14, 7, 1)$
185 : $P_{2323} = (2, 0, 8, 1)$
186 : $P_{2349} = (12, 1, 8, 1)$
187 : $P_{2359} = (6, 2, 8, 1)$
188 : $P_{2366} = (13, 2, 8, 1)$
189 : $P_{2370} = (1, 3, 8, 1)$
190 : $P_{2384} = (15, 3, 8, 1)$
191 : $P_{2392} = (7, 4, 8, 1)$
192 : $P_{2398} = (13, 4, 8, 1)$
193 : $P_{2433} = (0, 7, 8, 1)$
194 : $P_{2435} = (2, 7, 8, 1)$
195 : $P_{2460} = (11, 8, 8, 1)$
196 : $P_{2464} = (15, 8, 8, 1)$
197 : $P_{2487} = (6, 10, 8, 1)$
198 : $P_{2488} = (7, 10, 8, 1)$
199 : $P_{2521} = (8, 12, 8, 1)$
200 : $P_{2525} = (12, 12, 8, 1)$
201 : $P_{2546} = (1, 14, 8, 1)$
202 : $P_{2556} = (11, 14, 8, 1)$
203 : $P_{2591} = (14, 0, 9, 1)$
204 : $P_{2604} = (11, 1, 9, 1)$
205 : $P_{2631} = (6, 3, 9, 1)$
206 : $P_{2633} = (8, 3, 9, 1)$
207 : $P_{2642} = (1, 4, 9, 1)$
208 : $P_{2652} = (11, 4, 9, 1)$
209 : $P_{2669} = (12, 5, 9, 1)$
210 : $P_{2671} = (14, 5, 9, 1)$
211 : $P_{2690} = (1, 7, 9, 1)$
212 : $P_{2692} = (3, 7, 9, 1)$
213 : $P_{2707} = (2, 8, 9, 1)$
214 : $P_{2711} = (6, 8, 9, 1)$
215 : $P_{2755} = (2, 11, 9, 1)$
216 : $P_{2756} = (3, 11, 9, 1)$
217 : $P_{2777} = (8, 12, 9, 1)$
218 : $P_{2781} = (12, 12, 9, 1)$
219 : $P_{2817} = (0, 15, 9, 1)$
220 : $P_{2826} = (9, 15, 9, 1)$
221 : $P_{2833} = (0, 0, 10, 1)$
222 : $P_{2859} = (10, 1, 10, 1)$
223 : $P_{2868} = (3, 2, 10, 1)$
224 : $P_{2873} = (8, 2, 10, 1)$
225 : $P_{2898} = (1, 4, 10, 1)$
226 : $P_{2908} = (11, 4, 10, 1)$
227 : $P_{2980} = (3, 9, 10, 1)$
228 : $P_{2985} = (8, 9, 10, 1)$
229 : $P_{3005} = (12, 10, 10, 1)$
230 : $P_{3006} = (13, 10, 10, 1)$
231 : $P_{3021} = (12, 11, 10, 1)$
232 : $P_{3022} = (13, 11, 10, 1)$
233 : $P_{3058} = (1, 14, 10, 1)$
234 : $P_{3068} = (11, 14, 10, 1)$
235 : $P_{3089} = (0, 0, 11, 1)$
236 : $P_{3116} = (11, 1, 11, 1)$
237 : $P_{3122} = (1, 2, 11, 1)$
238 : $P_{3131} = (10, 2, 11, 1)$
239 : $P_{3158} = (5, 4, 11, 1)$
240 : $P_{3168} = (15, 4, 11, 1)$
241 : $P_{3234} = (1, 9, 11, 1)$
242 : $P_{3243} = (10, 9, 11, 1)$
243 : $P_{3255} = (6, 10, 11, 1)$
244 : $P_{3256} = (7, 10, 11, 1)$
245 : $P_{3271} = (6, 11, 11, 1)$
246 : $P_{3272} = (7, 11, 11, 1)$
247 : $P_{3318} = (5, 14, 11, 1)$
248 : $P_{3328} = (15, 14, 11, 1)$
249 : $P_{3359} = (14, 0, 12, 1)$
250 : $P_{3375} = (14, 1, 12, 1)$
251 : $P_{3377} = (0, 2, 12, 1)$
252 : $P_{3388} = (11, 2, 12, 1)$
253 : $P_{3412} = (3, 4, 12, 1)$
254 : $P_{3418} = (9, 4, 12, 1)$
255 : $P_{3482} = (9, 8, 12, 1)$
256 : $P_{3486} = (13, 8, 12, 1)$
257 : $P_{3496} = (7, 9, 12, 1)$
258 : $P_{3501} = (12, 9, 12, 1)$
259 : $P_{3515} = (10, 10, 12, 1)$
260 : $P_{3516} = (11, 10, 12, 1)$
261 : $P_{3540} = (3, 12, 12, 1)$
262 : $P_{3544} = (7, 12, 12, 1)$
263 : $P_{3557} = (4, 13, 12, 1)$
264 : $P_{3563} = (10, 13, 12, 1)$
265 : $P_{3589} = (4, 15, 12, 1)$
266 : $P_{3598} = (13, 15, 12, 1)$
267 : $P_{3605} = (4, 0, 13, 1)$
268 : $P_{3621} = (4, 1, 13, 1)$
269 : $P_{3639} = (6, 2, 13, 1)$
270 : $P_{3646} = (13, 2, 13, 1)$
271 : $P_{3651} = (2, 3, 13, 1)$
272 : $P_{3661} = (12, 3, 13, 1)$
273 : $P_{3693} = (12, 5, 13, 1)$
274 : $P_{3695} = (14, 5, 13, 1)$
275 : $P_{3745} = (0, 9, 13, 1)$
276 : $P_{3756} = (11, 9, 13, 1)$
277 : $P_{3771} = (10, 10, 13, 1)$
278 : $P_{3772} = (11, 10, 13, 1)$
279 : $P_{3803} = (10, 12, 13, 1)$
280 : $P_{3807} = (14, 12, 13, 1)$
281 : $P_{3815} = (6, 13, 13, 1)$
282 : $P_{3817} = (8, 13, 13, 1)$
283 : $P_{3827} = (2, 14, 13, 1)$
284 : $P_{3833} = (8, 14, 13, 1)$
285 : $P_{3859} = (2, 0, 14, 1)$
286 : $P_{3883} = (10, 1, 14, 1)$
287 : $P_{3905} = (0, 3, 14, 1)$
288 : $P_{3919} = (14, 3, 14, 1)$
289 : $P_{3950} = (13, 5, 14, 1)$
290 : $P_{3952} = (15, 5, 14, 1)$
291 : $P_{3959} = (6, 6, 14, 1)$
292 : $P_{3968} = (15, 6, 14, 1)$
293 : $P_{3987} = (2, 8, 14, 1)$
294 : $P_{3991} = (6, 8, 14, 1)$
295 : $P_{4002} = (1, 9, 14, 1)$
296 : $P_{4011} = (10, 9, 14, 1)$
297 : $P_{4021} = (4, 10, 14, 1)$
298 : $P_{4022} = (5, 10, 14, 1)$
299 : $P_{4050} = (1, 12, 14, 1)$
300 : $P_{4054} = (5, 12, 14, 1)$
301 : $P_{4101} = (4, 15, 14, 1)$
302 : $P_{4110} = (13, 15, 14, 1)$
303 : $P_{4117} = (4, 0, 15, 1)$
304 : $P_{4135} = (6, 1, 15, 1)$
305 : $P_{4146} = (1, 2, 15, 1)$
306 : $P_{4155} = (10, 2, 15, 1)$
307 : $P_{4184} = (7, 4, 15, 1)$
308 : $P_{4190} = (13, 4, 15, 1)$
309 : $P_{4194} = (1, 5, 15, 1)$
310 : $P_{4196} = (3, 5, 15, 1)$
311 : $P_{4215} = (6, 6, 15, 1)$
312 : $P_{4224} = (15, 6, 15, 1)$
313 : $P_{4264} = (7, 9, 15, 1)$
314 : $P_{4269} = (12, 9, 15, 1)$
315 : $P_{4301} = (12, 11, 15, 1)$
316 : $P_{4302} = (13, 11, 15, 1)$
317 : $P_{4305} = (0, 12, 15, 1)$
318 : $P_{4309} = (4, 12, 15, 1)$
319 : $P_{4356} = (3, 15, 15, 1)$
320 : $P_{4363} = (10, 15, 15, 1)$