Rank-74055 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108629

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	141
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	5, 3^2, 2^9, 1^{141}, 0^{168}

Singular Points

The surface has 1 singular points:

$0: \ P_{546} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1,0,0,1,0,0)_{34}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \delta^5 & \delta^5 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{51068} = \begin{bmatrix} 1 & 0 & 11 & 11 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{51068} = \mathbf{Pl}(1,0,1,1,10,1)_{46147}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{10} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46427} = \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46427} = \mathbf{Pl}(1,0,1,1,11,1)_{50227}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & \delta^{10} \\ 0 & 1 & 0 & 1 \end{bmatrix}_{43969} = \begin{bmatrix} 1 & 0 & 1 & 10 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{43969} = \mathbf{Pl}(1,1,0,10,1,1)_{9336}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^5 & \delta^{10} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46700} = \begin{bmatrix} 1 & 0 & 11 & 10 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46700} = \mathbf{Pl}(11,10,10,11,10,1)_{48646}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & \delta^5 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{48337} = \begin{bmatrix} 1 & 0 & 1 & 11 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{48337} = \mathbf{Pl}(1,1,0,11,1,1)_{9351}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^5 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{50795} = \begin{bmatrix} 1 & 0 & 10 & 11 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{50795} = \mathbf{Pl}(10,11,11,10,11,1)_{52920}$$

Rank of lines: (0, 69904, 4368, 51068, 46427, 69921, 43969, 46700, 48337, 50795)
Rank of points on Klein quadric: (0, 33, 34, 46147, 50227, 49, 9336, 48646, 9351, 52920)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{290} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_{14} = (10,1,0,0) = \ell_0 \cap \ell_3$
$P_{15} = (11,1,0,0) = \ell_0 \cap \ell_4$
$P_3 = (0,0,0,1) = \ell_1 \cap \ell_5$
$P_{451} = (1,11,0,1) = \ell_2 \cap \ell_7$
$P_{435} = (1,10,0,1) = \ell_2 \cap \ell_9$
$P_{690} = (1,10,1,1) = \ell_3 \cap \ell_8$
$P_{706} = (1,11,1,1) = \ell_4 \cap \ell_6$
$P_{180} = (1,10,1,0) = \ell_6 \cap \ell_7$
$P_{196} = (1,11,1,0) = \ell_8 \cap \ell_9$

Single Points

The surface has 141 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
4 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
5 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
6 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
7 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
8 : $P_{12} = (8,1,0,0)$ lies on line ℓ_0
9 : $P_{13} = (9,1,0,0)$ lies on line ℓ_0
10 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0
11 : $P_{17} = (13,1,0,0)$ lies on line ℓ_0
12 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0
13 : $P_{19} = (15,1,0,0)$ lies on line ℓ_0
14 : $P_{35} = (0,1,1,0)$ lies on line ℓ_5
15 : $P_{275} = (1,0,0,1)$ lies on line ℓ_2
16 : $P_{291} = (1,1,0,1)$ lies on line ℓ_2
17 : $P_{306} = (0,2,0,1)$ lies on line ℓ_1
18 : $P_{307} = (1,2,0,1)$ lies on line ℓ_2
19 : $P_{322} = (0,3,0,1)$ lies on line ℓ_1
20 : $P_{323} = (1,3,0,1)$ lies on line ℓ_2
21 : $P_{338} = (0,4,0,1)$ lies on line ℓ_1
22 : $P_{339} = (1,4,0,1)$ lies on line ℓ_2
23 : $P_{354} = (0,5,0,1)$ lies on line ℓ_1
24 : $P_{355} = (1,5,0,1)$ lies on line ℓ_2
25 : $P_{370} = (0,6,0,1)$ lies on line ℓ_1
26 : $P_{371} = (1,6,0,1)$ lies on line ℓ_2
27 : $P_{386} = (0,7,0,1)$ lies on line ℓ_1
28 : $P_{387} = (1,7,0,1)$ lies on line ℓ_2
29 : $P_{402} = (0,8,0,1)$ lies on line ℓ_1
30 : $P_{403} = (1,8,0,1)$ lies on line ℓ_2
31 : $P_{418} = (0,9,0,1)$ lies on line ℓ_1
32 : $P_{419} = (1,9,0,1)$ lies on line ℓ_2
33 : $P_{434} = (0,10,0,1)$ lies on line ℓ_1
34 : $P_{450} = (0,11,0,1)$ lies on line ℓ_1
35 : $P_{466} = (0,12,0,1)$ lies on line ℓ_1
36 : $P_{467} = (1,12,0,1)$ lies on line ℓ_2
37 : $P_{482} = (0,13,0,1)$ lies on line ℓ_1
38 : $P_{483} = (1,13,0,1)$ lies on line ℓ_2
39 : $P_{498} = (0,14,0,1)$ lies on line ℓ_1
40 : $P_{499} = (1,14,0,1)$ lies on line ℓ_2
41 : $P_{514} = (0,15,0,1)$ lies on line ℓ_1
42 : $P_{515} = (1,15,0,1)$ lies on line ℓ_2
43 : $P_{540} = (10,0,1,1)$ lies on line ℓ_3
44 : $P_{541} = (11,0,1,1)$ lies on line ℓ_4
45 : $P_{565} = (4,2,1,1)$ lies on line ℓ_4
46 : $P_{568} = (7,2,1,1)$ lies on line ℓ_3
47 : $P_{590} = (13,3,1,1)$ lies on line ℓ_3
48 : $P_{592} = (15,3,1,1)$ lies on line ℓ_4
49 : $P_{602} = (9,4,1,1)$ lies on line ℓ_3
50 : $P_{605} = (12,4,1,1)$ lies on line ℓ_4
51 : $P_{612} = (3,5,1,1)$ lies on line ℓ_3
52 : $P_{616} = (7,5,1,1)$ lies on line ℓ_4
53 : $P_{628} = (3,6,1,1)$ lies on line ℓ_4
54 : $P_{629} = (4,6,1,1)$ lies on line ℓ_3
55 : $P_{649} = (8,7,1,1)$ lies on line ℓ_4
56 : $P_{655} = (14,7,1,1)$ lies on line ℓ_3
57 : $P_{662} = (5,8,1,1)$ lies on line ℓ_4
58 : $P_{669} = (12,8,1,1)$ lies on line ℓ_3
59 : $P_{679} = (6,9,1,1)$ lies on line ℓ_3
60 : $P_{687} = (14,9,1,1)$ lies on line ℓ_4
61 : $P_{699} = (10,10,1,1)$ lies on line ℓ_4
62 : $P_{716} = (11,11,1,1)$ lies on line ℓ_3
63 : $P_{723} = (2,12,1,1)$ lies on line ℓ_4
64 : $P_{736} = (15,12,1,1)$ lies on line ℓ_3
65 : $P_{742} = (5,13,1,1)$ lies on line ℓ_3
66 : $P_{746} = (9,13,1,1)$ lies on line ℓ_4
67 : $P_{755} = (2,14,1,1)$ lies on line ℓ_3
68 : $P_{766} = (13,14,1,1)$ lies on line ℓ_4
69 : $P_{775} = (6,15,1,1)$ lies on line ℓ_4
70 : $P_{777} = (8,15,1,1)$ lies on line ℓ_3
71 : $P_{817} = (0,2,2,1)$ lies on line ℓ_5
72 : $P_{868} = (3,5,2,1)$ lies on line ℓ_9
73 : $P_{884} = (3,6,2,1)$ lies on line ℓ_7
74 : $P_{979} = (2,12,2,1)$ lies on line ℓ_6
75 : $P_{1011} = (2,14,2,1)$ lies on line ℓ_8
76 : $P_{1089} = (0,3,3,1)$ lies on line ℓ_5
77 : $P_{1124} = (3,5,3,1)$ lies on line ℓ_8
78 : $P_{1140} = (3,6,3,1)$ lies on line ℓ_6
79 : $P_{1235} = (2,12,3,1)$ lies on line ℓ_7
80 : $P_{1267} = (2,14,3,1)$ lies on line ℓ_9
81 : $P_{1333} = (4,2,4,1)$ lies on line ℓ_6
82 : $P_{1361} = (0,4,4,1)$ lies on line ℓ_5
83 : $P_{1397} = (4,6,4,1)$ lies on line ℓ_8
84 : $P_{1430} = (5,8,4,1)$ lies on line ℓ_7
85 : $P_{1510} = (5,13,4,1)$ lies on line ℓ_9
86 : $P_{1589} = (4,2,5,1)$ lies on line ℓ_7
87 : $P_{1633} = (0,5,5,1)$ lies on line ℓ_5
88 : $P_{1653} = (4,6,5,1)$ lies on line ℓ_9
89 : $P_{1686} = (5,8,5,1)$ lies on line ℓ_6
90 : $P_{1766} = (5,13,5,1)$ lies on line ℓ_8
91 : $P_{1848} = (7,2,6,1)$ lies on line ℓ_9
92 : $P_{1896} = (7,5,6,1)$ lies on line ℓ_7
93 : $P_{1905} = (0,6,6,1)$ lies on line ℓ_5
94 : $P_{1959} = (6,9,6,1)$ lies on line ℓ_8
95 : $P_{2055} = (6,15,6,1)$ lies on line ℓ_6
96 : $P_{2104} = (7,2,7,1)$ lies on line ℓ_8
97 : $P_{2152} = (7,5,7,1)$ lies on line ℓ_6
98 : $P_{2177} = (0,7,7,1)$ lies on line ℓ_5
99 : $P_{2215} = (6,9,7,1)$ lies on line ℓ_9
100 : $P_{2311} = (6,15,7,1)$ lies on line ℓ_7
101 : $P_{2394} = (9,4,8,1)$ lies on line ℓ_9
102 : $P_{2441} = (8,7,8,1)$ lies on line ℓ_6
103 : $P_{2449} = (0,8,8,1)$ lies on line ℓ_5
104 : $P_{2538} = (9,13,8,1)$ lies on line ℓ_7
105 : $P_{2569} = (8,15,8,1)$ lies on line ℓ_8
106 : $P_{2650} = (9,4,9,1)$ lies on line ℓ_8
107 : $P_{2697} = (8,7,9,1)$ lies on line ℓ_7

108 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_5
109 : $P_{2794} = (9, 13, 9, 1)$ lies on line ℓ_6
110 : $P_{2825} = (8, 15, 9, 1)$ lies on line ℓ_9
111 : $P_{2843} = (10, 0, 10, 1)$ lies on line ℓ_8
112 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_7
113 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_5
114 : $P_{3003} = (10, 10, 10, 1)$ lies on line ℓ_6
115 : $P_{3020} = (11, 11, 10, 1)$ lies on line ℓ_9
116 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_9
117 : $P_{3100} = (11, 0, 11, 1)$ lies on line ℓ_6
118 : $P_{3259} = (10, 10, 11, 1)$ lies on line ℓ_7
119 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_5
120 : $P_{3276} = (11, 11, 11, 1)$ lies on line ℓ_8
121 : $P_{3406} = (13, 3, 12, 1)$ lies on line ℓ_9
122 : $P_{3421} = (12, 4, 12, 1)$ lies on line ℓ_6
123 : $P_{3485} = (12, 8, 12, 1)$ lies on line ℓ_8
124 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_5
125 : $P_{3582} = (13, 14, 12, 1)$ lies on line ℓ_7
126 : $P_{3662} = (13, 3, 13, 1)$ lies on line ℓ_8
127 : $P_{3677} = (12, 4, 13, 1)$ lies on line ℓ_7
128 : $P_{3741} = (12, 8, 13, 1)$ lies on line ℓ_9
129 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_5
130 : $P_{3838} = (13, 14, 13, 1)$ lies on line ℓ_6
131 : $P_{3920} = (15, 3, 14, 1)$ lies on line ℓ_7
132 : $P_{3983} = (14, 7, 14, 1)$ lies on line ℓ_8
133 : $P_{4015} = (14, 9, 14, 1)$ lies on line ℓ_6
134 : $P_{4064} = (15, 12, 14, 1)$ lies on line ℓ_9
135 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_5
136 : $P_{4176} = (15, 3, 15, 1)$ lies on line ℓ_6
137 : $P_{4239} = (14, 7, 15, 1)$ lies on line ℓ_9
138 : $P_{4271} = (14, 9, 15, 1)$ lies on line ℓ_7
139 : $P_{4320} = (15, 12, 15, 1)$ lies on line ℓ_8
140 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_{65} = (14, 2, 1, 0)$
1 : $P_{78} = (11, 3, 1, 0)$
2 : $P_{85} = (2, 4, 1, 0)$
3 : $P_{109} = (10, 5, 1, 0)$
4 : $P_{117} = (2, 6, 1, 0)$
5 : $P_{140} = (9, 7, 1, 0)$
6 : $P_{158} = (11, 8, 1, 0)$
7 : $P_{167} = (4, 9, 1, 0)$
8 : $P_{225} = (14, 12, 1, 0)$
9 : $P_{231} = (4, 13, 1, 0)$
10 : $P_{252} = (9, 14, 1, 0)$
11 : $P_{269} = (10, 15, 1, 0)$
12 : $P_{822} = (5, 2, 2, 1)$
13 : $P_{873} = (8, 5, 2, 1)$
14 : $P_{895} = (14, 6, 2, 1)$
15 : $P_{902} = (5, 7, 2, 1)$
16 : $P_{907} = (10, 7, 2, 1)$
17 : $P_{919} = (6, 8, 2, 1)$
18 : $P_{927} = (14, 8, 2, 1)$
19 : $P_{1001} = (8, 13, 2, 1)$
20 : $P_{1003} = (10, 13, 2, 1)$
21 : $P_{1015} = (6, 14, 2, 1)$
22 : $P_{1082} = (9, 2, 3, 1)$
23 : $P_{1087} = (14, 2, 3, 1)$
24 : $P_{1093} = (4, 3, 3, 1)$
25 : $P_{1134} = (13, 5, 3, 1)$
26 : $P_{1145} = (8, 6, 3, 1)$
27 : $P_{1157} = (4, 7, 3, 1)$
28 : $P_{1165} = (12, 7, 3, 1)$
29 : $P_{1182} = (13, 8, 3, 1)$
30 : $P_{1226} = (9, 11, 3, 1)$
31 : $P_{1229} = (12, 11, 3, 1)$
32 : $P_{1247} = (14, 12, 3, 1)$
33 : $P_{1273} = (8, 14, 3, 1)$
34 : $P_{1342} = (13, 2, 4, 1)$
35 : $P_{1369} = (8, 4, 4, 1)$
36 : $P_{1420} = (11, 7, 4, 1)$
37 : $P_{1424} = (15, 7, 4, 1)$
38 : $P_{1440} = (15, 8, 4, 1)$
39 : $P_{1497} = (8, 12, 4, 1)$
40 : $P_{1500} = (11, 12, 4, 1)$
41 : $P_{1507} = (2, 13, 4, 1)$
42 : $P_{1539} = (2, 15, 4, 1)$
43 : $P_{1550} = (13, 15, 4, 1)$
44 : $P_{1600} = (15, 2, 5, 1)$
45 : $P_{1619} = (2, 4, 5, 1)$
46 : $P_{1631} = (14, 4, 5, 1)$
47 : $P_{1642} = (9, 5, 5, 1)$
48 : $P_{1651} = (2, 6, 5, 1)$
49 : $P_{1688} = (7, 8, 5, 1)$
50 : $P_{1719} = (6, 10, 5, 1)$
51 : $P_{1727} = (14, 10, 5, 1)$
52 : $P_{1751} = (6, 12, 5, 1)$
53 : $P_{1754} = (9, 12, 5, 1)$

$54: P_{1776} = (15, 13, 5, 1)$
$55: P_{1800} = (7, 15, 5, 1)$
$56: P_{1834} = (9, 1, 6, 1)$
$57: P_{1839} = (14, 1, 6, 1)$
$58: P_{1851} = (10, 2, 6, 1)$
$59: P_{1874} = (1, 4, 6, 1)$
$60: P_{1890} = (1, 5, 6, 1)$
$61: P_{1917} = (12, 6, 6, 1)$
$62: P_{1946} = (9, 8, 6, 1)$
$63: P_{1947} = (10, 8, 6, 1)$
$64: P_{1956} = (3, 9, 6, 1)$
$65: P_{1972} = (3, 10, 6, 1)$
$66: P_{1981} = (12, 10, 6, 1)$
$67: P_{2063} = (14, 15, 6, 1)$
$68: P_{2083} = (2, 1, 7, 1)$
$69: P_{2085} = (4, 1, 7, 1)$
$70: P_{2105} = (8, 2, 7, 1)$
$71: P_{2115} = (2, 3, 7, 1)$
$72: P_{2123} = (10, 3, 7, 1)$
$73: P_{2149} = (4, 5, 7, 1)$
$74: P_{2190} = (13, 7, 7, 1)$
$75: P_{2219} = (10, 9, 7, 1)$
$76: P_{2233} = (8, 10, 7, 1)$
$77: P_{2238} = (13, 10, 7, 1)$
$78: P_{2290} = (1, 14, 7, 1)$
$79: P_{2306} = (1, 15, 7, 1)$
$80: P_{2381} = (12, 3, 8, 1)$
$81: P_{2388} = (3, 4, 8, 1)$
$82: P_{2430} = (13, 6, 8, 1)$
$83: P_{2431} = (14, 6, 8, 1)$
$84: P_{2436} = (3, 7, 8, 1)$
$85: P_{2463} = (14, 8, 8, 1)$
$86: P_{2467} = (2, 9, 8, 1)$
$87: P_{2469} = (4, 9, 8, 1)$
$88: P_{2499} = (2, 11, 8, 1)$
$89: P_{2510} = (13, 11, 8, 1)$
$90: P_{2533} = (4, 13, 8, 1)$
$91: P_{2573} = (12, 15, 8, 1)$
$92: P_{2629} = (4, 3, 9, 1)$
$93: P_{2632} = (7, 3, 9, 1)$
$94: P_{2648} = (7, 4, 9, 1)$
$95: P_{2683} = (10, 6, 9, 1)$
$96: P_{2688} = (15, 6, 9, 1)$
$97: P_{2693} = (4, 7, 9, 1)$
$98: P_{2736} = (15, 9, 9, 1)$
$99: P_{2772} = (3, 12, 9, 1)$
$100: P_{2779} = (10, 12, 9, 1)$
$101: P_{2820} = (3, 15, 9, 1)$
$102: P_{2851} = (2, 1, 10, 1)$
$103: P_{2858} = (9, 1, 10, 1)$
$104: P_{2883} = (2, 3, 10, 1)$
$105: P_{2885} = (4, 3, 10, 1)$
$106: P_{2930} = (1, 6, 10, 1)$
$107: P_{2943} = (14, 6, 10, 1)$
$108: P_{2946} = (1, 7, 10, 1)$
$109: P_{2949} = (4, 7, 10, 1)$
$110: P_{2970} = (9, 8, 10, 1)$
$111: P_{2975} = (14, 8, 10, 1)$
$112: P_{3109} = (4, 1, 11, 1)$
$113: P_{3119} = (14, 1, 11, 1)$
$114: P_{3173} = (4, 5, 11, 1)$
$115: P_{3178} = (9, 5, 11, 1)$
$116: P_{3282} = (1, 12, 11, 1)$
$117: P_{3290} = (9, 12, 11, 1)$
$118: P_{3298} = (1, 13, 11, 1)$
$119: P_{3299} = (2, 13, 11, 1)$
$120: P_{3331} = (2, 15, 11, 1)$
$121: P_{3343} = (14, 15, 11, 1)$
$122: P_{3365} = (4, 1, 12, 1)$
$123: P_{3370} = (9, 1, 12, 1)$
$124: P_{3378} = (1, 2, 12, 1)$
$125: P_{3394} = (1, 3, 12, 1)$
$126: P_{3424} = (15, 4, 12, 1)$
$127: P_{3429} = (4, 5, 12, 1)$
$128: P_{3436} = (11, 5, 12, 1)$
$129: P_{3482} = (9, 8, 12, 1)$
$130: P_{3528} = (7, 11, 12, 1)$
$131: P_{3536} = (15, 11, 12, 1)$
$132: P_{3544} = (7, 12, 12, 1)$
$133: P_{3580} = (11, 14, 12, 1)$
$134: P_{3619} = (2, 1, 13, 1)$
$135: P_{3631} = (14, 1, 13, 1)$
$136: P_{3651} = (2, 3, 13, 1)$
$137: P_{3676} = (11, 4, 13, 1)$
$138: P_{3730} = (1, 8, 13, 1)$
$139: P_{3746} = (1, 9, 13, 1)$
$140: P_{3782} = (5, 11, 13, 1)$
$141: P_{3783} = (6, 11, 13, 1)$
$142: P_{3815} = (6, 13, 13, 1)$
$143: P_{3830} = (5, 14, 13, 1)$
$144: P_{3852} = (11, 15, 13, 1)$
$145: P_{3855} = (14, 15, 13, 1)$
$146: P_{3910} = (5, 3, 14, 1)$
$147: P_{3946} = (9, 5, 14, 1)$
$148: P_{3949} = (12, 5, 14, 1)$
$149: P_{3958} = (5, 6, 14, 1)$
$150: P_{3964} = (11, 6, 14, 1)$
$151: P_{4013} = (12, 9, 14, 1)$
$152: P_{4058} = (9, 12, 14, 1)$
$153: P_{4068} = (3, 13, 14, 1)$
$154: P_{4076} = (11, 13, 14, 1)$
$155: P_{4084} = (3, 14, 14, 1)$
$156: P_{4167} = (6, 3, 15, 1)$
$157: P_{4199} = (6, 5, 15, 1)$
$158: P_{4234} = (9, 7, 15, 1)$
$159: P_{4262} = (5, 9, 15, 1)$
$160: P_{4277} = (4, 10, 15, 1)$
$161: P_{4280} = (7, 10, 15, 1)$

162 : $P_{4310} = (5, 12, 15, 1)$
163 : $P_{4323} = (2, 13, 15, 1)$
164 : $P_{4328} = (7, 13, 15, 1)$
165 : $P_{4341} = (4, 14, 15, 1)$
166 : $P_{4346} = (9, 14, 15, 1)$
167 : $P_{4355} = (2, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 1 1 1 0 0 0 0 0
1	1 0 1 0 0 1 1 0 1 0
2	1 1 0 0 0 0 0 1 0 1
3	1 0 0 0 1 1 0 1 1 1
4	1 0 0 1 0 1 1 1 0 1
5	0 1 0 1 1 0 0 1 0 1
6	0 1 0 0 1 0 0 1 1 0
7	0 0 1 1 1 1 1 0 0 1
8	0 1 0 1 0 0 1 0 0 1
9	0 0 1 1 1 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_1	P_1	P_{14}	P_{15}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_1	P_1	P_3	P_{290}	P_{290}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_9
in point	P_1	P_1	P_{451}	P_{435}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9
in point	P_{14}	P_{546}	P_{546}	P_{546}	P_{690}	P_{546}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_{15}	P_{546}	P_{546}	P_{706}	P_{546}	P_{546}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_9
in point	P_3	P_{546}	P_{546}	P_{546}	P_{546}

Line 6 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_8
in point	P_{290}	P_{706}	P_{180}	P_{290}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_{451}	P_{546}	P_{546}	P_{546}	P_{180}	P_{546}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_9
in point	P_{290}	P_{690}	P_{290}	P_{196}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{435}	P_{546}	P_{546}	P_{546}	P_{546}	P_{196}

The surface has 321 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (2,1,0,0)$
$5: P_7 = (3,1,0,0)$
$6: P_8 = (4,1,0,0)$
$7: P_9 = (5,1,0,0)$
$8: P_{10} = (6,1,0,0)$
$9: P_{11} = (7,1,0,0)$
$10: P_{12} = (8,1,0,0)$
$11: P_{13} = (9,1,0,0)$
$12: P_{14} = (10,1,0,0)$
$13: P_{15} = (11,1,0,0)$
$14: P_{16} = (12,1,0,0)$
$15: P_{17} = (13,1,0,0)$
$16: P_{18} = (14,1,0,0)$
$17: P_{19} = (15,1,0,0)$
$18: P_{35} = (0,1,1,0)$
$19: P_{65} = (14,2,1,0)$
$20: P_{78} = (11,3,1,0)$
$21: P_{85} = (2,4,1,0)$
$22: P_{109} = (10,5,1,0)$
$23: P_{117} = (2,6,1,0)$
$24: P_{140} = (9,7,1,0)$
$25: P_{158} = (11,8,1,0)$
$26: P_{167} = (4,9,1,0)$
$27: P_{180} = (1,10,1,0)$
$28: P_{196} = (1,11,1,0)$
$29: P_{225} = (14,12,1,0)$
$30: P_{231} = (4,13,1,0)$
$31: P_{252} = (9,14,1,0)$
$32: P_{269} = (10,15,1,0)$
$33: P_{275} = (1,0,0,1)$
$34: P_{290} = (0,1,0,1)$
$35: P_{291} = (1,1,0,1)$
$36: P_{306} = (0,2,0,1)$
$37: P_{307} = (1,2,0,1)$
$38: P_{322} = (0,3,0,1)$
$39: P_{323} = (1,3,0,1)$
$40: P_{338} = (0,4,0,1)$
$41: P_{339} = (1,4,0,1)$
$42: P_{354} = (0,5,0,1)$
$43: P_{355} = (1,5,0,1)$
$44: P_{370} = (0,6,0,1)$
$45: P_{371} = (1,6,0,1)$
$46: P_{386} = (0,7,0,1)$
$47: P_{387} = (1,7,0,1)$
$48: P_{402} = (0,8,0,1)$
$49: P_{403} = (1,8,0,1)$
$50: P_{418} = (0,9,0,1)$
$51: P_{419} = (1,9,0,1)$
$52: P_{434} = (0,10,0,1)$
$53: P_{435} = (1,10,0,1)$
$54: P_{450} = (0,11,0,1)$
$55: P_{451} = (1,11,0,1)$
$56: P_{466} = (0,12,0,1)$
$57: P_{467} = (1,12,0,1)$
$58: P_{482} = (0,13,0,1)$
$59: P_{483} = (1,13,0,1)$
$60: P_{498} = (0,14,0,1)$
$61: P_{499} = (1,14,0,1)$
$62: P_{514} = (0,15,0,1)$
$63: P_{515} = (1,15,0,1)$
$64: P_{540} = (10,0,1,1)$
$65: P_{541} = (11,0,1,1)$
$66: P_{546} = (0,1,1,1)$
$67: P_{565} = (4,2,1,1)$
$68: P_{568} = (7,2,1,1)$
$69: P_{590} = (13,3,1,1)$
$70: P_{592} = (15,3,1,1)$
$71: P_{602} = (9,4,1,1)$
$72: P_{605} = (12,4,1,1)$
$73: P_{612} = (3,5,1,1)$
$74: P_{616} = (7,5,1,1)$
$75: P_{628} = (3,6,1,1)$
$76: P_{629} = (4,6,1,1)$
$77: P_{649} = (8,7,1,1)$
$78: P_{655} = (14,7,1,1)$
$79: P_{662} = (5,8,1,1)$
$80: P_{669} = (12,8,1,1)$
$81: P_{679} = (6,9,1,1)$
$82: P_{687} = (14,9,1,1)$
$83: P_{690} = (1,10,1,1)$
$84: P_{699} = (10,10,1,1)$
$85: P_{706} = (1,11,1,1)$
$86: P_{716} = (11,11,1,1)$
$87: P_{723} = (2,12,1,1)$
$88: P_{736} = (15,12,1,1)$
$89: P_{742} = (5,13,1,1)$
$90: P_{746} = (9,13,1,1)$
$91: P_{755} = (2,14,1,1)$
$92: P_{766} = (13,14,1,1)$
$93: P_{775} = (6,15,1,1)$
$94: P_{777} = (8,15,1,1)$
$95: P_{817} = (0,2,2,1)$
$96: P_{822} = (5,2,2,1)$
$97: P_{868} = (3,5,2,1)$
$98: P_{873} = (8,5,2,1)$
$99: P_{884} = (3,6,2,1)$
$100: P_{895} = (14,6,2,1)$
$101: P_{902} = (5,7,2,1)$
$102: P_{907} = (10,7,2,1)$
$103: P_{919} = (6,8,2,1)$
$104: P_{927} = (14,8,2,1)$
$105: P_{979} = (2,12,2,1)$
$106: P_{1001} = (8,13,2,1)$
$107: P_{1003} = (10,13,2,1)$
$108: P_{1011} = (2,14,2,1)$
$109: P_{1015} = (6,14,2,1)$
$110: P_{1082} = (9,2,3,1)$
$111: P_{1087} = (14,2,3,1)$
$112: P_{1089} = (0,3,3,1)$
$113: P_{1093} = (4,3,3,1)$
$114: P_{1124} = (3,5,3,1)$
$115: P_{1134} = (13,5,3,1)$
$116: P_{1140} = (3,6,3,1)$
$117: P_{1145} = (8,6,3,1)$
$118: P_{1157} = (4,7,3,1)$
$119: P_{1165} = (12,7,3,1)$
$120: P_{1182} = (13,8,3,1)$
$121: P_{1226} = (9,11,3,1)$
$122: P_{1229} = (12,11,3,1)$
$123: P_{1235} = (2,12,3,1)$
$124: P_{1247} = (14,12,3,1)$
$125: P_{1267} = (2,14,3,1)$
$126: P_{1273} = (8,14,3,1)$
$127: P_{1333} = (4,2,4,1)$
$128: P_{1342} = (13,2,4,1)$
$129: P_{1361} = (0,4,4,1)$
$130: P_{1369} = (8,4,4,1)$
$131: P_{1397} = (4,6,4,1)$
$132: P_{1420} = (11,7,4,1)$
$133: P_{1424} = (15,7,4,1)$
$134: P_{1430} = (5,8,4,1)$
$135: P_{1440} = (15,8,4,1)$
$136: P_{1497} = (8,12,4,1)$
$137: P_{1500} = (11,12,4,1)$
$138: P_{1507} = (2,13,4,1)$
$139: P_{1510} = (5,13,4,1)$
$140: P_{1539} = (2,15,4,1)$
$141: P_{1550} = (13,15,4,1)$
$142: P_{1589} = (4,2,5,1)$
$143: P_{1600} = (15,2,5,1)$
$144: P_{1619} = (2,4,5,1)$
$145: P_{1631} = (14,4,5,1)$
$146: P_{1633} = (0,5,5,1)$
$147: P_{1642} = (9,5,5,1)$
$148: P_{1651} = (2,6,5,1)$
$149: P_{1653} = (4,6,5,1)$
$150: P_{1686} = (5,8,5,1)$
$151: P_{1688} = (7,8,5,1)$
$152: P_{1719} = (6,10,5,1)$
$153: P_{1727} = (14,10,5,1)$
$154: P_{1751} = (6,12,5,1)$
$155: P_{1754} = (9,12,5,1)$
$156: P_{1766} = (5,13,5,1)$
$157: P_{1776} = (15,13,5,1)$
$158: P_{1800} = (7,15,5,1)$
$159: P_{1834} = (9,1,6,1)$
$160: P_{1839} = (14,1,6,1)$
$161: P_{1848} = (7,2,6,1)$

$162: P_{1851} = (10, 2, 6, 1)$
$163: P_{1874} = (1, 4, 6, 1)$
$164: P_{1890} = (1, 5, 6, 1)$
$165: P_{1896} = (7, 5, 6, 1)$
$166: P_{1905} = (0, 6, 6, 1)$
$167: P_{1917} = (12, 6, 6, 1)$
$168: P_{1946} = (9, 8, 6, 1)$
$169: P_{1947} = (10, 8, 6, 1)$
$170: P_{1956} = (3, 9, 6, 1)$
$171: P_{1959} = (6, 9, 6, 1)$
$172: P_{1972} = (3, 10, 6, 1)$
$173: P_{1981} = (12, 10, 6, 1)$
$174: P_{2055} = (6, 15, 6, 1)$
$175: P_{2063} = (14, 15, 6, 1)$
$176: P_{2083} = (2, 1, 7, 1)$
$177: P_{2085} = (4, 1, 7, 1)$
$178: P_{2104} = (7, 2, 7, 1)$
$179: P_{2105} = (8, 2, 7, 1)$
$180: P_{2115} = (2, 3, 7, 1)$
$181: P_{2123} = (10, 3, 7, 1)$
$182: P_{2149} = (4, 5, 7, 1)$
$183: P_{2152} = (7, 5, 7, 1)$
$184: P_{2177} = (0, 7, 7, 1)$
$185: P_{2190} = (13, 7, 7, 1)$
$186: P_{2215} = (6, 9, 7, 1)$
$187: P_{2219} = (10, 9, 7, 1)$
$188: P_{2233} = (8, 10, 7, 1)$
$189: P_{2238} = (13, 10, 7, 1)$
$190: P_{2290} = (1, 14, 7, 1)$
$191: P_{2306} = (1, 15, 7, 1)$
$192: P_{2311} = (6, 15, 7, 1)$
$193: P_{2381} = (12, 3, 8, 1)$
$194: P_{2388} = (3, 4, 8, 1)$
$195: P_{2394} = (9, 4, 8, 1)$
$196: P_{2430} = (13, 6, 8, 1)$
$197: P_{2431} = (14, 6, 8, 1)$
$198: P_{2436} = (3, 7, 8, 1)$
$199: P_{2441} = (8, 7, 8, 1)$
$200: P_{2449} = (0, 8, 8, 1)$
$201: P_{2463} = (14, 8, 8, 1)$
$202: P_{2467} = (2, 9, 8, 1)$
$203: P_{2469} = (4, 9, 8, 1)$
$204: P_{2499} = (2, 11, 8, 1)$
$205: P_{2510} = (13, 11, 8, 1)$
$206: P_{2533} = (4, 13, 8, 1)$
$207: P_{2538} = (9, 13, 8, 1)$
$208: P_{2569} = (8, 15, 8, 1)$
$209: P_{2573} = (12, 15, 8, 1)$
$210: P_{2629} = (4, 3, 9, 1)$
$211: P_{2632} = (7, 3, 9, 1)$
$212: P_{2648} = (7, 4, 9, 1)$
$213: P_{2650} = (9, 4, 9, 1)$
$214: P_{2683} = (10, 6, 9, 1)$
$215: P_{2688} = (15, 6, 9, 1)$
$216: P_{2693} = (4, 7, 9, 1)$
$217: P_{2697} = (8, 7, 9, 1)$
$218: P_{2721} = (0, 9, 9, 1)$
$219: P_{2736} = (15, 9, 9, 1)$
$220: P_{2772} = (3, 12, 9, 1)$
$221: P_{2779} = (10, 12, 9, 1)$
$222: P_{2794} = (9, 13, 9, 1)$
$223: P_{2820} = (3, 15, 9, 1)$
$224: P_{2825} = (8, 15, 9, 1)$
$225: P_{2843} = (10, 0, 10, 1)$
$226: P_{2844} = (11, 0, 10, 1)$
$227: P_{2851} = (2, 1, 10, 1)$
$228: P_{2858} = (9, 1, 10, 1)$
$229: P_{2883} = (2, 3, 10, 1)$
$230: P_{2885} = (4, 3, 10, 1)$
$231: P_{2930} = (1, 6, 10, 1)$
$232: P_{2943} = (14, 6, 10, 1)$
$233: P_{2946} = (1, 7, 10, 1)$
$234: P_{2949} = (4, 7, 10, 1)$
$235: P_{2970} = (9, 8, 10, 1)$
$236: P_{2975} = (14, 8, 10, 1)$
$237: P_{2993} = (0, 10, 10, 1)$
$238: P_{3003} = (10, 10, 10, 1)$
$239: P_{3020} = (11, 11, 10, 1)$
$240: P_{3099} = (10, 0, 11, 1)$
$241: P_{3100} = (11, 0, 11, 1)$
$242: P_{3109} = (4, 1, 11, 1)$
$243: P_{3119} = (14, 1, 11, 1)$
$244: P_{3173} = (4, 5, 11, 1)$
$245: P_{3178} = (9, 5, 11, 1)$
$246: P_{3259} = (10, 10, 11, 1)$
$247: P_{3265} = (0, 11, 11, 1)$
$248: P_{3276} = (11, 11, 11, 1)$
$249: P_{3282} = (1, 12, 11, 1)$
$250: P_{3290} = (9, 12, 11, 1)$
$251: P_{3298} = (1, 13, 11, 1)$
$252: P_{3299} = (2, 13, 11, 1)$
$253: P_{3331} = (2, 15, 11, 1)$
$254: P_{3343} = (14, 15, 11, 1)$
$255: P_{3365} = (4, 1, 12, 1)$
$256: P_{3370} = (9, 1, 12, 1)$
$257: P_{3378} = (1, 2, 12, 1)$
$258: P_{3394} = (1, 3, 12, 1)$
$259: P_{3406} = (13, 3, 12, 1)$
$260: P_{3421} = (12, 4, 12, 1)$
$261: P_{3424} = (15, 4, 12, 1)$
$262: P_{3429} = (4, 5, 12, 1)$
$263: P_{3436} = (11, 5, 12, 1)$
$264: P_{3482} = (9, 8, 12, 1)$
$265: P_{3485} = (12, 8, 12, 1)$
$266: P_{3528} = (7, 11, 12, 1)$
$267: P_{3536} = (15, 11, 12, 1)$
$268: P_{3537} = (0, 12, 12, 1)$
$269: P_{3544} = (7, 12, 12, 1)$
$270: P_{3580} = (11, 14, 12, 1)$
$271: P_{3582} = (13, 14, 12, 1)$
$272: P_{3619} = (2, 1, 13, 1)$
$273: P_{3631} = (14, 1, 13, 1)$
$274: P_{3651} = (2, 3, 13, 1)$
$275: P_{3662} = (13, 3, 13, 1)$
$276: P_{3676} = (11, 4, 13, 1)$
$277: P_{3677} = (12, 4, 13, 1)$
$278: P_{3730} = (1, 8, 13, 1)$
$279: P_{3741} = (12, 8, 13, 1)$
$280: P_{3746} = (1, 9, 13, 1)$
$281: P_{3782} = (5, 11, 13, 1)$
$282: P_{3783} = (6, 11, 13, 1)$
$283: P_{3809} = (0, 13, 13, 1)$
$284: P_{3815} = (6, 13, 13, 1)$
$285: P_{3830} = (5, 14, 13, 1)$
$286: P_{3838} = (13, 14, 13, 1)$
$287: P_{3852} = (11, 15, 13, 1)$
$288: P_{3855} = (14, 15, 13, 1)$
$289: P_{3910} = (5, 3, 14, 1)$
$290: P_{3920} = (15, 3, 14, 1)$
$291: P_{3946} = (9, 5, 14, 1)$
$292: P_{3949} = (12, 5, 14, 1)$
$293: P_{3958} = (5, 6, 14, 1)$
$294: P_{3964} = (11, 6, 14, 1)$
$295: P_{3983} = (14, 7, 14, 1)$
$296: P_{4013} = (12, 9, 14, 1)$
$297: P_{4015} = (14, 9, 14, 1)$
$298: P_{4058} = (9, 12, 14, 1)$
$299: P_{4064} = (15, 12, 14, 1)$
$300: P_{4068} = (3, 13, 14, 1)$
$301: P_{4076} = (11, 13, 14, 1)$
$302: P_{4081} = (0, 14, 14, 1)$
$303: P_{4084} = (3, 14, 14, 1)$
$304: P_{4167} = (6, 3, 15, 1)$
$305: P_{4176} = (15, 3, 15, 1)$
$306: P_{4199} = (6, 5, 15, 1)$
$307: P_{4234} = (9, 7, 15, 1)$
$308: P_{4239} = (14, 7, 15, 1)$
$309: P_{4262} = (5, 9, 15, 1)$
$310: P_{4271} = (14, 9, 15, 1)$
$311: P_{4277} = (4, 10, 15, 1)$
$312: P_{4280} = (7, 10, 15, 1)$
$313: P_{4310} = (5, 12, 15, 1)$
$314: P_{4320} = (15, 12, 15, 1)$
$315: P_{4323} = (2, 13, 15, 1)$
$316: P_{4328} = (7, 13, 15, 1)$
$317: P_{4341} = (4, 14, 15, 1)$
$318: P_{4346} = (9, 14, 15, 1)$
$319: P_{4353} = (0, 15, 15, 1)$
$320: P_{4355} = (2, 15, 15, 1)$