Rank-76356 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108374

General information

Number of lines	3
Number of points	241
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	48
Number of points off lines	192
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	3, 1^{48}, 0^{192}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \delta^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \mathbf{Pl}(0,11,1,0,0,0)_{28}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \mathbf{Pl}(0,10,1,0,0,0)_{27}$$

Rank of lines: (70160, 48304, 43936)
Rank of points on Klein quadric: (1, 28, 27)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 48 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_0
$1 : P_{284} = (10,0,0,1)$ lies on line ℓ_1
$2 : P_{285} = (11,0,0,1)$ lies on line ℓ_2
$3 : P_{530} = (0,0,1,1)$ lies on line ℓ_0
$4 : P_{540} = (10,0,1,1)$ lies on line ℓ_1
$5 : P_{541} = (11,0,1,1)$ lies on line ℓ_2
$6 : P_{785} = (0,0,2,1)$ lies on line ℓ_0
$7 : P_{795} = (10,0,2,1)$ lies on line ℓ_1
$8 : P_{796} = (11,0,2,1)$ lies on line ℓ_2
$9 : P_{1041} = (0,0,3,1)$ lies on line ℓ_0
$10 : P_{1051} = (10,0,3,1)$ lies on line ℓ_1
$11 : P_{1052} = (11,0,3,1)$ lies on line ℓ_2
$12 : P_{1297} = (0,0,4,1)$ lies on line ℓ_0
$13 : P_{1307} = (10,0,4,1)$ lies on line ℓ_1
$14 : P_{1308} = (11,0,4,1)$ lies on line ℓ_2
$15 : P_{1553} = (0,0,5,1)$ lies on line ℓ_0
$16 : P_{1563} = (10,0,5,1)$ lies on line ℓ_1
$17 : P_{1564} = (11,0,5,1)$ lies on line ℓ_2
$18 : P_{1809} = (0,0,6,1)$ lies on line ℓ_0
$19 : P_{1819} = (10,0,6,1)$ lies on line ℓ_1
$20 : P_{1820} = (11,0,6,1)$ lies on line ℓ_2
$21 : P_{2065} = (0,0,7,1)$ lies on line ℓ_0
$22 : P_{2075} = (10,0,7,1)$ lies on line ℓ_1
$23 : P_{2076} = (11,0,7,1)$ lies on line ℓ_2
$24 : P_{2321} = (0,0,8,1)$ lies on line ℓ_0
$25 : P_{2331} = (10,0,8,1)$ lies on line ℓ_1
$26 : P_{2332} = (11,0,8,1)$ lies on line ℓ_2
$27 : P_{2577} = (0,0,9,1)$ lies on line ℓ_0
$28 : P_{2587} = (10,0,9,1)$ lies on line ℓ_1
$29 : P_{2588} = (11,0,9,1)$ lies on line ℓ_2
$30 : P_{2833} = (0,0,10,1)$ lies on line ℓ_0
$31 : P_{2843} = (10,0,10,1)$ lies on line ℓ_1
$32 : P_{2844} = (11,0,10,1)$ lies on line ℓ_2
$33 : P_{3089} = (0,0,11,1)$ lies on line ℓ_0
$34 : P_{3099} = (10,0,11,1)$ lies on line ℓ_1
$35 : P_{3100} = (11,0,11,1)$ lies on line ℓ_2
$36 : P_{3345} = (0,0,12,1)$ lies on line ℓ_0
$37 : P_{3355} = (10,0,12,1)$ lies on line ℓ_1
$38 : P_{3356} = (11,0,12,1)$ lies on line ℓ_2
$39 : P_{3601} = (0,0,13,1)$ lies on line ℓ_0
$40 : P_{3611} = (10,0,13,1)$ lies on line ℓ_1
$41 : P_{3612} = (11,0,13,1)$ lies on line ℓ_2
$42 : P_{3857} = (0,0,14,1)$ lies on line ℓ_0
$43 : P_{3867} = (10,0,14,1)$ lies on line ℓ_1
$44 : P_{3868} = (11,0,14,1)$ lies on line ℓ_2
$45 : P_{4113} = (0,0,15,1)$ lies on line ℓ_0
$46 : P_{4123} = (10,0,15,1)$ lies on line ℓ_1
$47 : P_{4124} = (11,0,15,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 192 points not on any line:
The points on the surface but not on lines are:

0 : $P_{1} = (0, 1, 0, 0)$
1 : $P_{4} = (1, 1, 1, 1)$
2 : $P_{63} = (12, 2, 1, 0)$
3 : $P_{71} = (4, 3, 1, 0)$
4 : $P_{89} = (6, 4, 1, 0)$
5 : $P_{108} = (9, 5, 1, 0)$
6 : $P_{161} = (14, 8, 1, 0)$
7 : $P_{176} = (13, 9, 1, 0)$
8 : $P_{184} = (5, 10, 1, 0)$
9 : $P_{189} = (10, 10, 1, 0)$
10 : $P_{194} = (15, 10, 1, 0)$
11 : $P_{198} = (3, 11, 1, 0)$
12 : $P_{203} = (8, 11, 1, 0)$
13 : $P_{206} = (11, 11, 1, 0)$
14 : $P_{250} = (7, 14, 1, 0)$
15 : $P_{261} = (2, 15, 1, 0)$
16 : $P_{291} = (1, 1, 0, 1)$
17 : $P_{313} = (7, 2, 0, 1)$
18 : $P_{315} = (9, 2, 0, 1)$
19 : $P_{321} = (15, 2, 0, 1)$
20 : $P_{341} = (3, 4, 0, 1)$
21 : $P_{350} = (12, 4, 0, 1)$
22 : $P_{352} = (14, 4, 0, 1)$
23 : $P_{420} = (2, 9, 0, 1)$
24 : $P_{423} = (5, 9, 0, 1)$
25 : $P_{424} = (6, 9, 0, 1)$
26 : $P_{502} = (4, 14, 0, 1)$
27 : $P_{506} = (8, 14, 0, 1)$
28 : $P_{511} = (13, 14, 0, 1)$
29 : $P_{546} = (0, 1, 1, 1)$
30 : $P_{586} = (9, 3, 1, 1)$
31 : $P_{623} = (14, 5, 1, 1)$
32 : $P_{659} = (2, 8, 1, 1)$
33 : $P_{693} = (4, 10, 1, 1)$
34 : $P_{700} = (11, 10, 1, 1)$
35 : $P_{703} = (14, 10, 1, 1)$
36 : $P_{707} = (2, 11, 1, 1)$
37 : $P_{714} = (9, 11, 1, 1)$
38 : $P_{715} = (10, 11, 1, 1)$
39 : $P_{773} = (4, 15, 1, 1)$
40 : $P_{838} = (5, 3, 2, 1)$
41 : $P_{849} = (0, 4, 2, 1)$
42 : $P_{890} = (9, 6, 2, 1)$
43 : $P_{931} = (2, 9, 2, 1)$
44 : $P_{938} = (9, 9, 2, 1)$
45 : $P_{939} = (10, 9, 2, 1)$
46 : $P_{972} = (11, 11, 2, 1)$
47 : $P_{998} = (5, 13, 2, 1)$
48 : $P_{1063} = (6, 1, 3, 1)$
49 : $P_{1068} = (11, 1, 3, 1)$
50 : $P_{1069} = (12, 1, 3, 1)$
51 : $P_{1083} = (10, 2, 3, 1)$
52 : $P_{1108} = (3, 4, 3, 1)$
53 : $P_{1121} = (0, 5, 3, 1)$
54 : $P_{1168} = (15, 7, 3, 1)$
55 : $P_{1171} = (2, 8, 3, 1)$
56 : $P_{1197} = (12, 9, 3, 1)$
57 : $P_{1216} = (15, 10, 3, 1)$
58 : $P_{1219} = (2, 11, 3, 1)$
59 : $P_{1271} = (6, 14, 3, 1)$
60 : $P_{1385} = (8, 5, 4, 1)$
61 : $P_{1417} = (8, 7, 4, 1)$
62 : $P_{1441} = (0, 9, 4, 1)$
63 : $P_{1467} = (10, 10, 4, 1)$
64 : $P_{1519} = (14, 13, 4, 1)$
65 : $P_{1525} = (4, 14, 4, 1)$
66 : $P_{1532} = (11, 14, 4, 1)$
67 : $P_{1535} = (14, 14, 4, 1)$
68 : $P_{1575} = (6, 1, 5, 1)$
69 : $P_{1579} = (10, 1, 5, 1)$
70 : $P_{1582} = (13, 1, 5, 1)$
71 : $P_{1598} = (13, 2, 5, 1)$
72 : $P_{1628} = (11, 4, 5, 1)$
73 : $P_{1681} = (0, 8, 5, 1)$
74 : $P_{1702} = (5, 9, 5, 1)$
75 : $P_{1717} = (4, 10, 5, 1)$
76 : $P_{1732} = (3, 11, 5, 1)$
77 : $P_{1748} = (3, 12, 5, 1)$
78 : $P_{1783} = (6, 14, 5, 1)$
79 : $P_{1797} = (4, 15, 5, 1)$
80 : $P_{1858} = (1, 3, 6, 1)$
81 : $P_{1867} = (10, 3, 6, 1)$
82 : $P_{1886} = (13, 4, 6, 1)$
83 : $P_{1900} = (11, 5, 6, 1)$
84 : $P_{1938} = (1, 8, 6, 1)$
85 : $P_{1952} = (15, 8, 6, 1)$
86 : $P_{1959} = (6, 9, 6, 1)$
87 : $P_{1988} = (3, 11, 6, 1)$
88 : $P_{2004} = (3, 12, 6, 1)$
89 : $P_{2014} = (13, 12, 6, 1)$
90 : $P_{2016} = (15, 12, 6, 1)$
91 : $P_{2017} = (0, 13, 6, 1)$
92 : $P_{2104} = (7, 2, 7, 1)$
93 : $P_{2114} = (1, 3, 7, 1)$
94 : $P_{2118} = (5, 3, 7, 1)$
95 : $P_{2194} = (1, 8, 7, 1)$

96 : $P_{2203} = (10, 8, 7, 1)$
97 : $P_{2249} = (8, 11, 7, 1)$
98 : $P_{2257} = (0, 12, 7, 1)$
99 : $P_{2278} = (5, 13, 7, 1)$
100 : $P_{2281} = (8, 13, 7, 1)$
101 : $P_{2285} = (12, 13, 7, 1)$
102 : $P_{2301} = (12, 14, 7, 1)$
103 : $P_{2316} = (11, 15, 7, 1)$
104 : $P_{2344} = (7, 1, 8, 1)$
105 : $P_{2348} = (11, 1, 8, 1)$
106 : $P_{2350} = (13, 1, 8, 1)$
107 : $P_{2366} = (13, 2, 8, 1)$
108 : $P_{2378} = (9, 3, 8, 1)$
109 : $P_{2392} = (7, 4, 8, 1)$
110 : $P_{2422} = (5, 6, 8, 1)$
111 : $P_{2475} = (10, 9, 8, 1)$
112 : $P_{2486} = (5, 10, 8, 1)$
113 : $P_{2506} = (9, 11, 8, 1)$
114 : $P_{2553} = (8, 14, 8, 1)$
115 : $P_{2561} = (0, 15, 8, 1)$
116 : $P_{2611} = (2, 2, 9, 1)$
117 : $P_{2618} = (9, 2, 9, 1)$
118 : $P_{2619} = (10, 2, 9, 1)$
119 : $P_{2691} = (2, 7, 9, 1)$
120 : $P_{2720} = (15, 8, 9, 1)$
121 : $P_{2764} = (11, 11, 9, 1)$
122 : $P_{2784} = (15, 12, 9, 1)$
123 : $P_{2801} = (0, 14, 9, 1)$
124 : $P_{2871} = (6, 2, 10, 1)$
125 : $P_{2901} = (4, 4, 10, 1)$
126 : $P_{2936} = (7, 6, 10, 1)$
127 : $P_{2951} = (6, 7, 10, 1)$
128 : $P_{2984} = (7, 9, 10, 1)$
129 : $P_{2994} = (1, 10, 10, 1)$
130 : $P_{3004} = (11, 10, 10, 1)$
131 : $P_{3009} = (0, 11, 10, 1)$
132 : $P_{3010} = (1, 11, 10, 1)$
133 : $P_{3029} = (4, 12, 10, 1)$
134 : $P_{3055} = (14, 13, 10, 1)$
135 : $P_{3071} = (14, 14, 10, 1)$
136 : $P_{3123} = (2, 2, 11, 1)$
137 : $P_{3166} = (13, 4, 11, 1)$
138 : $P_{3194} = (9, 6, 11, 1)$
139 : $P_{3203} = (2, 7, 11, 1)$
140 : $P_{3242} = (9, 9, 11, 1)$
141 : $P_{3249} = (0, 10, 11, 1)$
142 : $P_{3250} = (1, 10, 11, 1)$
143 : $P_{3266} = (1, 11, 11, 1)$
144 : $P_{3275} = (10, 11, 11, 1)$
145 : $P_{3294} = (13, 12, 11, 1)$
146 : $P_{3309} = (12, 13, 11, 1)$
147 : $P_{3325} = (12, 14, 11, 1)$
148 : $P_{3383} = (6, 2, 12, 1)$
149 : $P_{3403} = (10, 3, 12, 1)$
150 : $P_{3421} = (12, 4, 12, 1)$
151 : $P_{3426} = (1, 5, 12, 1)$
152 : $P_{3433} = (8, 5, 12, 1)$
153 : $P_{3441} = (0, 6, 12, 1)$
154 : $P_{3463} = (6, 7, 12, 1)$
155 : $P_{3465} = (8, 7, 12, 1)$
156 : $P_{3472} = (15, 7, 12, 1)$
157 : $P_{3520} = (15, 10, 12, 1)$
158 : $P_{3586} = (1, 15, 12, 1)$
159 : $P_{3596} = (11, 15, 12, 1)$
160 : $P_{3682} = (1, 5, 13, 1)$
161 : $P_{3692} = (11, 5, 13, 1)$
162 : $P_{3700} = (3, 6, 13, 1)$
163 : $P_{3702} = (5, 6, 13, 1)$
164 : $P_{3704} = (7, 6, 13, 1)$
165 : $P_{3713} = (0, 7, 13, 1)$
166 : $P_{3739} = (10, 8, 13, 1)$
167 : $P_{3752} = (7, 9, 13, 1)$
168 : $P_{3766} = (5, 10, 13, 1)$
169 : $P_{3838} = (13, 14, 13, 1)$
170 : $P_{3842} = (1, 15, 13, 1)$
171 : $P_{3844} = (3, 15, 13, 1)$
172 : $P_{3889} = (0, 2, 14, 1)$
173 : $P_{3925} = (4, 4, 14, 1)$
174 : $P_{3932} = (11, 4, 14, 1)$
175 : $P_{3935} = (14, 4, 14, 1)$
176 : $P_{3956} = (3, 6, 14, 1)$
177 : $P_{4027} = (10, 10, 14, 1)$
178 : $P_{4053} = (4, 12, 14, 1)$
179 : $P_{4100} = (3, 15, 14, 1)$
180 : $P_{4136} = (7, 1, 15, 1)$
181 : $P_{4139} = (10, 1, 15, 1)$
182 : $P_{4141} = (12, 1, 15, 1)$
183 : $P_{4160} = (15, 2, 15, 1)$
184 : $P_{4161} = (0, 3, 15, 1)$
185 : $P_{4184} = (7, 4, 15, 1)$
186 : $P_{4207} = (14, 5, 15, 1)$
187 : $P_{4269} = (12, 9, 15, 1)$
188 : $P_{4287} = (14, 10, 15, 1)$
189 : $P_{4297} = (8, 11, 15, 1)$
190 : $P_{4329} = (8, 13, 15, 1)$
191 : $P_{4348} = (11, 14, 15, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 241 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{63} = (12, 2, 1, 0)$
5 : $P_{71} = (4, 3, 1, 0)$
6 : $P_{89} = (6, 4, 1, 0)$
7 : $P_{108} = (9, 5, 1, 0)$
8 : $P_{161} = (14, 8, 1, 0)$
9 : $P_{176} = (13, 9, 1, 0)$
10 : $P_{184} = (5, 10, 1, 0)$
11 : $P_{189} = (10, 10, 1, 0)$
12 : $P_{194} = (15, 10, 1, 0)$
13 : $P_{198} = (3, 11, 1, 0)$
14 : $P_{203} = (8, 11, 1, 0)$
15 : $P_{206} = (11, 11, 1, 0)$
16 : $P_{250} = (7, 14, 1, 0)$
17 : $P_{261} = (2, 15, 1, 0)$
18 : $P_{284} = (10, 0, 0, 1)$
19 : $P_{285} = (11, 0, 0, 1)$
20 : $P_{291} = (1, 1, 0, 1)$
21 : $P_{313} = (7, 2, 0, 1)$
22 : $P_{315} = (9, 2, 0, 1)$
23 : $P_{321} = (15, 2, 0, 1)$
24 : $P_{341} = (3, 4, 0, 1)$
25 : $P_{350} = (12, 4, 0, 1)$
26 : $P_{352} = (14, 4, 0, 1)$
27 : $P_{420} = (2, 9, 0, 1)$
28 : $P_{423} = (5, 9, 0, 1)$
29 : $P_{424} = (6, 9, 0, 1)$
30 : $P_{502} = (4, 14, 0, 1)$
31 : $P_{506} = (8, 14, 0, 1)$
32 : $P_{511} = (13, 14, 0, 1)$
33 : $P_{530} = (0, 0, 1, 1)$
34 : $P_{540} = (10, 0, 1, 1)$
35 : $P_{541} = (11, 0, 1, 1)$
36 : $P_{546} = (0, 1, 1, 1)$
37 : $P_{586} = (9, 3, 1, 1)$
38 : $P_{623} = (14, 5, 1, 1)$
39 : $P_{659} = (2, 8, 1, 1)$
40 : $P_{693} = (4, 10, 1, 1)$
41 : $P_{700} = (11, 10, 1, 1)$
42 : $P_{703} = (14, 10, 1, 1)$
43 : $P_{707} = (2, 11, 1, 1)$
44 : $P_{714} = (9, 11, 1, 1)$
45 : $P_{715} = (10, 11, 1, 1)$
46 : $P_{773} = (4, 15, 1, 1)$
47 : $P_{785} = (0, 0, 2, 1)$
48 : $P_{795} = (10, 0, 2, 1)$
49 : $P_{796} = (11, 0, 2, 1)$
50 : $P_{838} = (5, 3, 2, 1)$
51 : $P_{849} = (0, 4, 2, 1)$
52 : $P_{890} = (9, 6, 2, 1)$
53 : $P_{931} = (2, 9, 2, 1)$
54 : $P_{938} = (9, 9, 2, 1)$
55 : $P_{939} = (10, 9, 2, 1)$
56 : $P_{972} = (11, 11, 2, 1)$
57 : $P_{998} = (5, 13, 2, 1)$
58 : $P_{1041} = (0, 0, 3, 1)$
59 : $P_{1051} = (10, 0, 3, 1)$
60 : $P_{1052} = (11, 0, 3, 1)$
61 : $P_{1063} = (6, 1, 3, 1)$
62 : $P_{1068} = (11, 1, 3, 1)$
63 : $P_{1069} = (12, 1, 3, 1)$
64 : $P_{1083} = (10, 2, 3, 1)$
65 : $P_{1108} = (3, 4, 3, 1)$
66 : $P_{1121} = (0, 5, 3, 1)$
67 : $P_{1168} = (15, 7, 3, 1)$
68 : $P_{1171} = (2, 8, 3, 1)$
69 : $P_{1197} = (12, 9, 3, 1)$
70 : $P_{1216} = (15, 10, 3, 1)$
71 : $P_{1219} = (2, 11, 3, 1)$
72 : $P_{1271} = (6, 14, 3, 1)$
73 : $P_{1297} = (0, 0, 4, 1)$
74 : $P_{1307} = (10, 0, 4, 1)$
75 : $P_{1308} = (11, 0, 4, 1)$
76 : $P_{1385} = (8, 5, 4, 1)$
77 : $P_{1417} = (8, 7, 4, 1)$
78 : $P_{1441} = (0, 9, 4, 1)$
79 : $P_{1467} = (10, 10, 4, 1)$
80 : $P_{1519} = (14, 13, 4, 1)$
81 : $P_{1525} = (4, 14, 4, 1)$
82 : $P_{1532} = (11, 14, 4, 1)$
83 : $P_{1535} = (14, 14, 4, 1)$
84 : $P_{1553} = (0, 0, 5, 1)$
85 : $P_{1563} = (10, 0, 5, 1)$
86 : $P_{1564} = (11, 0, 5, 1)$
87 : $P_{1575} = (6, 1, 5, 1)$
88 : $P_{1579} = (10, 1, 5, 1)$
89 : $P_{1582} = (13, 1, 5, 1)$
90 : $P_{1598} = (13, 2, 5, 1)$
91 : $P_{1628} = (11, 4, 5, 1)$
92 : $P_{1681} = (0, 8, 5, 1)$
93 : $P_{1702} = (5, 9, 5, 1)$
94 : $P_{1717} = (4, 10, 5, 1)$
95 : $P_{1732} = (3, 11, 5, 1)$
96 : $P_{1748} = (3, 12, 5, 1)$
97 : $P_{1783} = (6, 14, 5, 1)$
98 : $P_{1797} = (4, 15, 5, 1)$

$99: P_{1809} = (0, 0, 6, 1)$
$100: P_{1819} = (10, 0, 6, 1)$
$101: P_{1820} = (11, 0, 6, 1)$
$102: P_{1858} = (1, 3, 6, 1)$
$103: P_{1867} = (10, 3, 6, 1)$
$104: P_{1886} = (13, 4, 6, 1)$
$105: P_{1900} = (11, 5, 6, 1)$
$106: P_{1938} = (1, 8, 6, 1)$
$107: P_{1952} = (15, 8, 6, 1)$
$108: P_{1959} = (6, 9, 6, 1)$
$109: P_{1988} = (3, 11, 6, 1)$
$110: P_{2004} = (3, 12, 6, 1)$
$111: P_{2014} = (13, 12, 6, 1)$
$112: P_{2016} = (15, 12, 6, 1)$
$113: P_{2017} = (0, 13, 6, 1)$
$114: P_{2065} = (0, 0, 7, 1)$
$115: P_{2075} = (10, 0, 7, 1)$
$116: P_{2076} = (11, 0, 7, 1)$
$117: P_{2104} = (7, 2, 7, 1)$
$118: P_{2114} = (1, 3, 7, 1)$
$119: P_{2118} = (5, 3, 7, 1)$
$120: P_{2194} = (1, 8, 7, 1)$
$121: P_{2203} = (10, 8, 7, 1)$
$122: P_{2249} = (8, 11, 7, 1)$
$123: P_{2257} = (0, 12, 7, 1)$
$124: P_{2278} = (5, 13, 7, 1)$
$125: P_{2281} = (8, 13, 7, 1)$
$126: P_{2285} = (12, 13, 7, 1)$
$127: P_{2301} = (12, 14, 7, 1)$
$128: P_{2316} = (11, 15, 7, 1)$
$129: P_{2321} = (0, 0, 8, 1)$
$130: P_{2331} = (10, 0, 8, 1)$
$131: P_{2332} = (11, 0, 8, 1)$
$132: P_{2344} = (7, 1, 8, 1)$
$133: P_{2348} = (11, 1, 8, 1)$
$134: P_{2350} = (13, 1, 8, 1)$
$135: P_{2366} = (13, 2, 8, 1)$
$136: P_{2378} = (9, 3, 8, 1)$
$137: P_{2392} = (7, 4, 8, 1)$
$138: P_{2422} = (5, 6, 8, 1)$
$139: P_{2475} = (10, 9, 8, 1)$
$140: P_{2486} = (5, 10, 8, 1)$
$141: P_{2506} = (9, 11, 8, 1)$
$142: P_{2553} = (8, 14, 8, 1)$
$143: P_{2561} = (0, 15, 8, 1)$
$144: P_{2577} = (0, 0, 9, 1)$
$145: P_{2587} = (10, 0, 9, 1)$
$146: P_{2588} = (11, 0, 9, 1)$
$147: P_{2611} = (2, 2, 9, 1)$
$148: P_{2618} = (9, 2, 9, 1)$
$149: P_{2619} = (10, 2, 9, 1)$
$150: P_{2691} = (2, 7, 9, 1)$
$151: P_{2720} = (15, 8, 9, 1)$
$152: P_{2764} = (11, 11, 9, 1)$
$153: P_{2784} = (15, 12, 9, 1)$
$154: P_{2801} = (0, 14, 9, 1)$
$155: P_{2833} = (0, 0, 10, 1)$
$156: P_{2843} = (10, 0, 10, 1)$
$157: P_{2844} = (11, 0, 10, 1)$
$158: P_{2871} = (6, 2, 10, 1)$
$159: P_{2901} = (4, 4, 10, 1)$
$160: P_{2936} = (7, 6, 10, 1)$
$161: P_{2951} = (6, 7, 10, 1)$
$162: P_{2984} = (7, 9, 10, 1)$
$163: P_{2994} = (1, 10, 10, 1)$
$164: P_{3004} = (11, 10, 10, 1)$
$165: P_{3009} = (0, 11, 10, 1)$
$166: P_{3010} = (1, 11, 10, 1)$
$167: P_{3029} = (4, 12, 10, 1)$
$168: P_{3055} = (14, 13, 10, 1)$
$169: P_{3071} = (14, 14, 10, 1)$
$170: P_{3089} = (0, 0, 11, 1)$
$171: P_{3099} = (10, 0, 11, 1)$
$172: P_{3100} = (11, 0, 11, 1)$
$173: P_{3123} = (2, 2, 11, 1)$
$174: P_{3166} = (13, 4, 11, 1)$
$175: P_{3194} = (9, 6, 11, 1)$
$176: P_{3203} = (2, 7, 11, 1)$
$177: P_{3242} = (9, 9, 11, 1)$
$178: P_{3249} = (0, 10, 11, 1)$
$179: P_{3250} = (1, 10, 11, 1)$
$180: P_{3266} = (1, 11, 11, 1)$
$181: P_{3275} = (10, 11, 11, 1)$
$182: P_{3294} = (13, 12, 11, 1)$
$183: P_{3309} = (12, 13, 11, 1)$
$184: P_{3325} = (12, 14, 11, 1)$
$185: P_{3345} = (0, 0, 12, 1)$
$186: P_{3355} = (10, 0, 12, 1)$
$187: P_{3356} = (11, 0, 12, 1)$
$188: P_{3383} = (6, 2, 12, 1)$
$189: P_{3403} = (10, 3, 12, 1)$
$190: P_{3421} = (12, 4, 12, 1)$
$191: P_{3426} = (1, 5, 12, 1)$
$192: P_{3433} = (8, 5, 12, 1)$
$193: P_{3441} = (0, 6, 12, 1)$
$194: P_{3463} = (6, 7, 12, 1)$
$195: P_{3465} = (8, 7, 12, 1)$
$196: P_{3472} = (15, 7, 12, 1)$
$197: P_{3520} = (15, 10, 12, 1)$
$198: P_{3586} = (1, 15, 12, 1)$
$199: P_{3596} = (11, 15, 12, 1)$
$200: P_{3601} = (0, 0, 13, 1)$
$201: P_{3611} = (10, 0, 13, 1)$
$202: P_{3612} = (11, 0, 13, 1)$
$203: P_{3682} = (1, 5, 13, 1)$
$204: P_{3692} = (11, 5, 13, 1)$
$205: P_{3700} = (3, 6, 13, 1)$
$206: P_{3702} = (5, 6, 13, 1)$
$207: P_{3704} = (7, 6, 13, 1)$
$208: P_{3713} = (0, 7, 13, 1)$
$209: P_{3739} = (10, 8, 13, 1)$
$210: P_{3752} = (7, 9, 13, 1)$
$211: P_{3766} = (5, 10, 13, 1)$
$212: P_{3838} = (13, 14, 13, 1)$
$213: P_{3842} = (1, 15, 13, 1)$
$214: P_{3844} = (3, 15, 13, 1)$
$215: P_{3857} = (0, 0, 14, 1)$
$216: P_{3867} = (10, 0, 14, 1)$
$217: P_{3868} = (11, 0, 14, 1)$
$218: P_{3889} = (0, 2, 14, 1)$
$219: P_{3925} = (4, 4, 14, 1)$
$220: P_{3932} = (11, 4, 14, 1)$
$221: P_{3935} = (14, 4, 14, 1)$
$222: P_{3956} = (3, 6, 14, 1)$
$223: P_{4027} = (10, 10, 14, 1)$
$224: P_{4053} = (4, 12, 14, 1)$
$225: P_{4100} = (3, 15, 14, 1)$
$226: P_{4113} = (0, 0, 15, 1)$
$227: P_{4123} = (10, 0, 15, 1)$
$228: P_{4124} = (11, 0, 15, 1)$
$229: P_{4136} = (7, 1, 15, 1)$
$230: P_{4139} = (10, 1, 15, 1)$
$231: P_{4141} = (12, 1, 15, 1)$
$232: P_{4160} = (15, 2, 15, 1)$
$233: P_{4161} = (0, 3, 15, 1)$
$234: P_{4184} = (7, 4, 15, 1)$
$235: P_{4207} = (14, 5, 15, 1)$
$236: P_{4269} = (12, 9, 15, 1)$
$237: P_{4287} = (14, 10, 15, 1)$
$238: P_{4297} = (8, 11, 15, 1)$
$239: P_{4329} = (8, 13, 15, 1)$
$240: P_{4348} = (11, 14, 15, 1)$