Rank-74276 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1499027802

General information

Number of lines	6
Number of points	29
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	18
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^6
Type of lines on points	$4, 2^4, 1^{18}, 0^6$

Singular Points

The surface has 1 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \mathbf{Pl}(0,3,1,0,0,1)_{114}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \mathbf{Pl}(0,2,1,0,0,1)_{113}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{318} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{318} = \mathbf{Pl}(2,3,1,1,0,1)_{151}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & \omega & 0 \end{bmatrix}_{212} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{212} = \mathbf{Pl}(3,2,1,1,0,1)_{152}$$

Rank of lines: (336, 356, 289, 205, 318, 212)
Rank of points on Klein quadric: (101, 1, 114, 113, 151, 152)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_{15} = (0,2,1,0) = \ell_0 \cap \ell_4$ $P_{37} = (3,3,0,1) = \ell_3 \cap \ell_5$
$P_{19} = (0,3,1,0) = \ell_0 \cap \ell_5$
$P_{32} = (2,2,0,1) = \ell_2 \cap \ell_4$

Single Points

The surface has 18 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0 10 : $P_{63} = (2,2,2,1)$ lies on line ℓ_2
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1 11 : $P_{64} = (3,2,2,1)$ lies on line ℓ_5
2 : $P_{11} = (0,1,1,0)$ lies on line ℓ_0 12 : $P_{68} = (3,3,2,1)$ lies on line ℓ_3
3 : $P_{38} = (0,0,1,1)$ lies on line ℓ_1 13 : $P_{69} = (0,0,3,1)$ lies on line ℓ_1
4 : $P_{40} = (2,0,1,1)$ lies on line ℓ_4 14 : $P_{76} = (3,1,3,1)$ lies on line ℓ_5
5 : $P_{41} = (3,0,1,1)$ lies on line ℓ_5 15 : $P_{79} = (2,2,3,1)$ lies on line ℓ_2
6 : $P_{47} = (2,2,1,1)$ lies on line ℓ_2 16 : $P_{83} = (2,3,3,1)$ lies on line ℓ_4
7 : $P_{52} = (3,3,1,1)$ lies on line ℓ_3 17 : $P_{84} = (3,3,3,1)$ lies on line ℓ_3
8 : $P_{53} = (0,0,2,1)$ lies on line ℓ_1
9 : $P_{59} = (2,1,2,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:
The points on the surface but not on lines are:

$0 : P_8 = (1, 0, 1, 0)$
$1 : P_{13} = (2, 1, 1, 0)$
$2 : P_{14} = (3, 1, 1, 0)$
$3 : P_{23} = (1, 0, 0, 1)$

$4 : P_{46} = (1, 2, 1, 1)$
$5 : P_{50} = (1, 3, 1, 1)$

Line Intersection Graph

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 |0 1 2 3 4 5
─┼───────────
0|0 1 1 1 1 1
1|1 0 1 1 0 0
2|1 1 0 1 1 0
3|1 1 1 0 0 1
4|1 0 1 0 0 0
5|1 0 0 1 0 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_2	P_2	P_2	P_{15}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_2	P_2	P_{32}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5
in point	P_2	P_2	P_2	P_{37}

Line 4 intersects

Line	ℓ_0	ℓ_2
in point	P_{15}	P_{32}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_{19}	P_{37}

The surface has 29 points:
The points on the surface are:

$0 : P_1 = (0, 1, 0, 0)$
$1 : P_2 = (0, 0, 1, 0)$
$2 : P_3 = (0, 0, 0, 1)$
$3 : P_8 = (1, 0, 1, 0)$
$4 : P_{11} = (0, 1, 1, 0)$
$5 : P_{13} = (2, 1, 1, 0)$
$6 : P_{14} = (3, 1, 1, 0)$
$7 : P_{15} = (0, 2, 1, 0)$
$8 : P_{19} = (0, 3, 1, 0)$
$9 : P_{23} = (1, 0, 0, 1)$

$10 : P_{32} = (2, 2, 0, 1)$
$11 : P_{37} = (3, 3, 0, 1)$
$12 : P_{38} = (0, 0, 1, 1)$
$13 : P_{40} = (2, 0, 1, 1)$
$14 : P_{41} = (3, 0, 1, 1)$
$15 : P_{46} = (1, 2, 1, 1)$
$16 : P_{47} = (2, 2, 1, 1)$
$17 : P_{50} = (1, 3, 1, 1)$
$18 : P_{52} = (3, 3, 1, 1)$
$19 : P_{53} = (0, 0, 2, 1)$

$20 : P_{59} = (2, 1, 2, 1)$
$21 : P_{63} = (2, 2, 2, 1)$
$22 : P_{64} = (3, 2, 2, 1)$
$23 : P_{68} = (3, 3, 2, 1)$
$24 : P_{69} = (0, 0, 3, 1)$
$25 : P_{76} = (3, 1, 3, 1)$
$26 : P_{79} = (2, 2, 3, 1)$
$27 : P_{83} = (2, 3, 3, 1)$
$28 : P_{84} = (3, 3, 3, 1)$