

# Rank-77 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

( 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(8) is 9377

## General information

Number of lines	1
Number of points	73
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

## Singular Points

The surface has 0 singular points:

## The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \mathbf{Pl}(1, 1, 1, 1, 0, 0)_{32}$$

Rank of lines: ( 585 )

Rank of points on Klein quadric: ( 32 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 9 single points:

The single points on the surface are:

- 0 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_0$
- 1 :  $P_{19} = (0, 1, 1, 0)$  lies on line  $\ell_0$
- 2 :  $P_{75} = (1, 0, 0, 1)$  lies on line  $\ell_0$
- 3 :  $P_{218} = (1, 2, 2, 1)$  lies on line  $\ell_0$
- 4 :  $P_{290} = (1, 3, 3, 1)$  lies on line  $\ell_0$

- 5 :  $P_{362} = (1, 4, 4, 1)$  lies on line  $\ell_0$
- 6 :  $P_{434} = (1, 5, 5, 1)$  lies on line  $\ell_0$
- 7 :  $P_{506} = (1, 6, 6, 1)$  lies on line  $\ell_0$
- 8 :  $P_{578} = (1, 7, 7, 1)$  lies on line  $\ell_0$

The single points on the surface are:

### Points on surface but on no line

The surface has 64 points not on any line:

The points on the surface but not on lines are:

- |                               |                               |
|-------------------------------|-------------------------------|
| 0 : $P_6 = (2, 1, 0, 0)$      | 22 : $P_{216} = (7, 1, 2, 1)$ |
| 1 : $P_8 = (4, 1, 0, 0)$      | 23 : $P_{240} = (7, 4, 2, 1)$ |
| 2 : $P_{11} = (7, 1, 0, 0)$   | 24 : $P_{241} = (0, 5, 2, 1)$ |
| 3 : $P_{13} = (2, 0, 1, 0)$   | 25 : $P_{248} = (7, 5, 2, 1)$ |
| 4 : $P_{15} = (4, 0, 1, 0)$   | 26 : $P_{261} = (4, 7, 2, 1)$ |
| 5 : $P_{18} = (7, 0, 1, 0)$   | 27 : $P_{270} = (5, 0, 3, 1)$ |
| 6 : $P_{29} = (2, 2, 1, 0)$   | 28 : $P_{306} = (1, 5, 3, 1)$ |
| 7 : $P_{36} = (1, 3, 1, 0)$   | 29 : $P_{307} = (2, 5, 3, 1)$ |
| 8 : $P_{47} = (4, 4, 1, 0)$   | 30 : $P_{310} = (5, 5, 3, 1)$ |
| 9 : $P_{52} = (1, 5, 1, 0)$   | 31 : $P_{314} = (1, 6, 3, 1)$ |
| 10 : $P_{60} = (1, 6, 1, 0)$  | 32 : $P_{316} = (3, 6, 3, 1)$ |
| 11 : $P_{74} = (7, 7, 1, 0)$  | 33 : $P_{320} = (7, 6, 3, 1)$ |
| 12 : $P_{82} = (0, 1, 0, 1)$  | 34 : $P_{321} = (0, 7, 3, 1)$ |
| 13 : $P_{83} = (1, 1, 0, 1)$  | 35 : $P_{325} = (4, 7, 3, 1)$ |
| 14 : $P_{103} = (5, 3, 0, 1)$ | 36 : $P_{339} = (2, 1, 4, 1)$ |
| 15 : $P_{120} = (6, 5, 0, 1)$ | 37 : $P_{352} = (7, 2, 4, 1)$ |
| 16 : $P_{125} = (3, 6, 0, 1)$ | 38 : $P_{377} = (0, 6, 4, 1)$ |
| 17 : $P_{138} = (0, 0, 1, 1)$ | 39 : $P_{379} = (2, 6, 4, 1)$ |
| 18 : $P_{139} = (1, 0, 1, 1)$ | 40 : $P_{387} = (2, 7, 4, 1)$ |
| 19 : $P_{160} = (7, 2, 1, 1)$ | 41 : $P_{399} = (6, 0, 5, 1)$ |
| 20 : $P_{171} = (2, 4, 1, 1)$ | 42 : $P_{409} = (0, 2, 5, 1)$ |
| 21 : $P_{197} = (4, 7, 1, 1)$ | 43 : $P_{416} = (7, 2, 5, 1)$ |

44 :  $P_{418} = (1, 3, 5, 1)$   
 45 :  $P_{419} = (2, 3, 5, 1)$   
 46 :  $P_{422} = (5, 3, 5, 1)$   
 47 :  $P_{442} = (1, 6, 5, 1)$   
 48 :  $P_{445} = (4, 6, 5, 1)$   
 49 :  $P_{447} = (6, 6, 5, 1)$   
 50 :  $P_{460} = (3, 0, 6, 1)$   
 51 :  $P_{482} = (1, 3, 6, 1)$   
 52 :  $P_{484} = (3, 3, 6, 1)$   
 53 :  $P_{488} = (7, 3, 6, 1)$   
 54 :  $P_{489} = (0, 4, 6, 1)$

55 :  $P_{491} = (2, 4, 6, 1)$   
 56 :  $P_{498} = (1, 5, 6, 1)$   
 57 :  $P_{501} = (4, 5, 6, 1)$   
 58 :  $P_{503} = (6, 5, 6, 1)$   
 59 :  $P_{533} = (4, 1, 7, 1)$   
 60 :  $P_{541} = (4, 2, 7, 1)$   
 61 :  $P_{545} = (0, 3, 7, 1)$   
 62 :  $P_{549} = (4, 3, 7, 1)$   
 63 :  $P_{555} = (2, 4, 7, 1)$

## Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 73 points:

The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$	25 : $P_{216} = (7, 1, 2, 1)$	50 : $P_{418} = (1, 3, 5, 1)$
1 : $P_6 = (2, 1, 0, 0)$	26 : $P_{218} = (1, 2, 2, 1)$	51 : $P_{419} = (2, 3, 5, 1)$
2 : $P_8 = (4, 1, 0, 0)$	27 : $P_{240} = (7, 4, 2, 1)$	52 : $P_{422} = (5, 3, 5, 1)$
3 : $P_{11} = (7, 1, 0, 0)$	28 : $P_{241} = (0, 5, 2, 1)$	53 : $P_{434} = (1, 5, 5, 1)$
4 : $P_{13} = (2, 0, 1, 0)$	29 : $P_{248} = (7, 5, 2, 1)$	54 : $P_{442} = (1, 6, 5, 1)$
5 : $P_{15} = (4, 0, 1, 0)$	30 : $P_{261} = (4, 7, 2, 1)$	55 : $P_{445} = (4, 6, 5, 1)$
6 : $P_{18} = (7, 0, 1, 0)$	31 : $P_{270} = (5, 0, 3, 1)$	56 : $P_{447} = (6, 6, 5, 1)$
7 : $P_{19} = (0, 1, 1, 0)$	32 : $P_{290} = (1, 3, 3, 1)$	57 : $P_{460} = (3, 0, 6, 1)$
8 : $P_{29} = (2, 2, 1, 0)$	33 : $P_{306} = (1, 5, 3, 1)$	58 : $P_{482} = (1, 3, 6, 1)$
9 : $P_{36} = (1, 3, 1, 0)$	34 : $P_{307} = (2, 5, 3, 1)$	59 : $P_{484} = (3, 3, 6, 1)$
10 : $P_{47} = (4, 4, 1, 0)$	35 : $P_{310} = (5, 5, 3, 1)$	60 : $P_{488} = (7, 3, 6, 1)$
11 : $P_{52} = (1, 5, 1, 0)$	36 : $P_{314} = (1, 6, 3, 1)$	61 : $P_{489} = (0, 4, 6, 1)$
12 : $P_{60} = (1, 6, 1, 0)$	37 : $P_{316} = (3, 6, 3, 1)$	62 : $P_{491} = (2, 4, 6, 1)$
13 : $P_{74} = (7, 7, 1, 0)$	38 : $P_{320} = (7, 6, 3, 1)$	63 : $P_{498} = (1, 5, 6, 1)$
14 : $P_{75} = (1, 0, 0, 1)$	39 : $P_{321} = (0, 7, 3, 1)$	64 : $P_{501} = (4, 5, 6, 1)$
15 : $P_{82} = (0, 1, 0, 1)$	40 : $P_{325} = (4, 7, 3, 1)$	65 : $P_{503} = (6, 5, 6, 1)$
16 : $P_{83} = (1, 1, 0, 1)$	41 : $P_{339} = (2, 1, 4, 1)$	66 : $P_{506} = (1, 6, 6, 1)$
17 : $P_{103} = (5, 3, 0, 1)$	42 : $P_{352} = (7, 2, 4, 1)$	67 : $P_{533} = (4, 1, 7, 1)$
18 : $P_{120} = (6, 5, 0, 1)$	43 : $P_{362} = (1, 4, 4, 1)$	68 : $P_{541} = (4, 2, 7, 1)$
19 : $P_{125} = (3, 6, 0, 1)$	44 : $P_{377} = (0, 6, 4, 1)$	69 : $P_{545} = (0, 3, 7, 1)$
20 : $P_{138} = (0, 0, 1, 1)$	45 : $P_{379} = (2, 6, 4, 1)$	70 : $P_{549} = (4, 3, 7, 1)$
21 : $P_{139} = (1, 0, 1, 1)$	46 : $P_{387} = (2, 7, 4, 1)$	71 : $P_{555} = (2, 4, 7, 1)$
22 : $P_{160} = (7, 2, 1, 1)$	47 : $P_{399} = (6, 0, 5, 1)$	72 : $P_{578} = (1, 7, 7, 1)$
23 : $P_{171} = (2, 4, 1, 1)$	48 : $P_{409} = (0, 2, 5, 1)$	
24 : $P_{197} = (4, 7, 1, 1)$	49 : $P_{416} = (7, 2, 5, 1)$	