Rank-73802 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227395734

General information

Number of lines	3
Number of points	57
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	32
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	3, 1^{24}, 0^{32}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \mathbf{Pl}(1,1,1,1,0,1)_{874}$$

Rank of lines: (74, 4689, 658)
Rank of points on Klein quadric: (665, 25, 874)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{19} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_1
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_{12} = (1,0,1,0)$ lies on line ℓ_0
$3 : P_{30} = (3,2,1,0)$ lies on line ℓ_0
$4 : P_{37} = (2,3,1,0)$ lies on line ℓ_0
$5 : P_{48} = (5,4,1,0)$ lies on line ℓ_0
$6 : P_{55} = (4,5,1,0)$ lies on line ℓ_0
$7 : P_{66} = (7,6,1,0)$ lies on line ℓ_0
$8 : P_{73} = (6,7,1,0)$ lies on line ℓ_0
$9 : P_{83} = (1,1,0,1)$ lies on line ℓ_2
$10 : P_{139} = (1,0,1,1)$ lies on line ℓ_2
$11 : P_{146} = (0,1,1,1)$ lies on line ℓ_1
$12 : P_{217} = (0,2,2,1)$ lies on line ℓ_1
$13 : P_{226} = (1,3,2,1)$ lies on line ℓ_2
$14 : P_{282} = (1,2,3,1)$ lies on line ℓ_2
$15 : P_{289} = (0,3,3,1)$ lies on line ℓ_1
$16 : P_{361} = (0,4,4,1)$ lies on line ℓ_1
$17 : P_{370} = (1,5,4,1)$ lies on line ℓ_2
$18 : P_{426} = (1,4,5,1)$ lies on line ℓ_2
$19 : P_{433} = (0,5,5,1)$ lies on line ℓ_1
$20 : P_{505} = (0,6,6,1)$ lies on line ℓ_1
$21 : P_{514} = (1,7,6,1)$ lies on line ℓ_2
$22 : P_{570} = (1,6,7,1)$ lies on line ℓ_2
$23 : P_{577} = (0,7,7,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 32 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1,1,1,1)$
$1 : P_{20} = (1,1,1,0)$
$2 : P_{94} = (4,2,0,1)$
$3 : P_{104} = (6,3,0,1)$
$4 : P_{113} = (7,4,0,1)$
$5 : P_{117} = (3,5,0,1)$
$6 : P_{127} = (5,6,0,1)$
$7 : P_{132} = (2,7,0,1)$
$8 : P_{205} = (4,0,2,1)$
$9 : P_{223} = (6,2,2,1)$
$10 : P_{224} = (7,2,2,1)$
$11 : P_{229} = (4,3,2,1)$

12 : $P_{254} = (5, 6, 2, 1)$
13 : $P_{271} = (6, 0, 3, 1)$
14 : $P_{285} = (4, 2, 3, 1)$
15 : $P_{303} = (6, 4, 3, 1)$
16 : $P_{336} = (7, 0, 4, 1)$
17 : $P_{359} = (6, 3, 4, 1)$
18 : $P_{363} = (2, 4, 4, 1)$
19 : $P_{364} = (3, 4, 4, 1)$
20 : $P_{376} = (7, 5, 4, 1)$
21 : $P_{396} = (3, 0, 5, 1)$
22 : $P_{432} = (7, 4, 5, 1)$
23 : $P_{452} = (3, 7, 5, 1)$
24 : $P_{462} = (5, 0, 6, 1)$
25 : $P_{478} = (5, 2, 6, 1)$
26 : $P_{515} = (2, 7, 6, 1)$
27 : $P_{523} = (2, 0, 7, 1)$
28 : $P_{564} = (3, 5, 7, 1)$
29 : $P_{571} = (2, 6, 7, 1)$
30 : $P_{581} = (4, 7, 7, 1)$
31 : $P_{582} = (5, 7, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{19}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{19}	P_{19}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{19}	P_{19}

The surface has 57 points:
The points on the surface are:

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1 : $P_4 = (1, 1, 1, 1)$
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3 : $P_{12} = (1, 0, 1, 0)$
4 : $P_{19} = (0, 1, 1, 0)$
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54 : $P_{577} = (0, 7, 7, 1)$
55 : $P_{581} = (4, 7, 7, 1)$
56 : $P_{582} = (5, 7, 7, 1)$