Rank-43 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_1 = 0$$

(0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 1177

General information

Number of lines	2
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	2, 1^{16}, 0^{56}

Singular Points

The surface has 1 singular points:

$0: \ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0,0,1,0,1,0)_{96}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

Rank of lines: (65, 4673)
Rank of points on Klein quadric: (96, 769)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_{138} = (0,0,1,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_{139} = (1,0,1,1)$ lies on line ℓ_0
3 : $P_{140} = (2,0,1,1)$ lies on line ℓ_0
4 : $P_{141} = (3,0,1,1)$ lies on line ℓ_0
5 : $P_{142} = (4,0,1,1)$ lies on line ℓ_0
6 : $P_{143} = (5,0,1,1)$ lies on line ℓ_0
7 : $P_{144} = (6,0,1,1)$ lies on line ℓ_0
8 : $P_{145} = (7,0,1,1)$ lies on line ℓ_0
9 : $P_{146} = (0,1,1,1)$ lies on line ℓ_1
10 : $P_{153} = (0,2,1,1)$ lies on line ℓ_1
11 : $P_{161} = (0,3,1,1)$ lies on line ℓ_1
12 : $P_{169} = (0,4,1,1)$ lies on line ℓ_1
13 : $P_{177} = (0,5,1,1)$ lies on line ℓ_1
14 : $P_{185} = (0,6,1,1)$ lies on line ℓ_1
15 : $P_{193} = (0,7,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_{20} = (1,1,1,0)$
1 : $P_{32} = (5,2,1,0)$
2 : $P_{37} = (2,3,1,0)$
3 : $P_{49} = (6,4,1,0)$
4 : $P_{55} = (4,5,1,0)$
5 : $P_{66} = (7,6,1,0)$
6 : $P_{70} = (3,7,1,0)$
7 : $P_{83} = (1,1,0,1)$
8 : $P_{95} = (5,2,0,1)$
9 : $P_{100} = (2,3,0,1)$
10 : $P_{112} = (6,4,0,1)$
11 : $P_{118} = (4,5,0,1)$
12 : $P_{129} = (7,6,0,1)$
13 : $P_{133} = (3,7,0,1)$
14 : $P_{211} = (2,1,2,1)$
15 : $P_{224} = (7,2,2,1)$
16 : $P_{229} = (4,3,2,1)$
17 : $P_{234} = (1,4,2,1)$
18 : $P_{246} = (5,5,2,1)$
19 : $P_{252} = (3,6,2,1)$
20 : $P_{263} = (6,7,2,1)$
21 : $P_{279} = (6,1,3,1)$
22 : $P_{285} = (4,2,3,1)$
23 : $P_{290} = (1,3,3,1)$
24 : $P_{300} = (3,4,3,1)$
25 : $P_{307} = (2,5,3,1)$

26 : $P_{318} = (5, 6, 3, 1)$
27 : $P_{328} = (7, 7, 3, 1)$
28 : $P_{341} = (4, 1, 4, 1)$
29 : $P_{348} = (3, 2, 4, 1)$
30 : $P_{358} = (5, 3, 4, 1)$
31 : $P_{363} = (2, 4, 4, 1)$
32 : $P_{376} = (7, 5, 4, 1)$
33 : $P_{383} = (6, 6, 4, 1)$
34 : $P_{386} = (1, 7, 4, 1)$
35 : $P_{404} = (3, 1, 5, 1)$
36 : $P_{411} = (2, 2, 5, 1)$
37 : $P_{423} = (6, 3, 5, 1)$
38 : $P_{432} = (7, 4, 5, 1)$
39 : $P_{434} = (1, 5, 5, 1)$
40 : $P_{445} = (4, 6, 5, 1)$
41 : $P_{454} = (5, 7, 5, 1)$
42 : $P_{470} = (5, 1, 6, 1)$
43 : $P_{479} = (6, 2, 6, 1)$
44 : $P_{488} = (7, 3, 6, 1)$
45 : $P_{493} = (4, 4, 6, 1)$
46 : $P_{500} = (3, 5, 6, 1)$
47 : $P_{506} = (1, 6, 6, 1)$
48 : $P_{515} = (2, 7, 6, 1)$
49 : $P_{536} = (7, 1, 7, 1)$
50 : $P_{538} = (1, 2, 7, 1)$
51 : $P_{548} = (3, 3, 7, 1)$
52 : $P_{558} = (5, 4, 7, 1)$
53 : $P_{567} = (6, 5, 7, 1)$
54 : $P_{571} = (2, 6, 7, 1)$
55 : $P_{581} = (4, 7, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{138}

Line 1 intersects

Line	ℓ_0
in point	P_{138}

The surface has 73 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
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