

Rank-65872 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244173454

General information

Number of lines	3
Number of points	73
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$3, 1^{24}, 0^{48}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \left[\begin{array}{cccc} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \mathbf{Pl}(2, 0, 0, 1, 0, 0)_{19} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 0 & 0 \end{array} \right]_{1752} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{array} \right]_{1752} = \mathbf{Pl}(4, 0, 0, 1, 0, 0)_{21}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \mathbf{PI}(7, 0, 0, 1, 0, 0)_{24}$$

Rank of lines: (3504, 1752, 2920)

Rank of points on Klein quadric: (19, 21, 24)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 24 single points:

The single points on the surface are:

0 : $P_{76} = (2, 0, 0, 1)$ lies on line ℓ_0
1 : $P_{78} = (4, 0, 0, 1)$ lies on line ℓ_1
2 : $P_{81} = (7, 0, 0, 1)$ lies on line ℓ_2
3 : $P_{84} = (2, 1, 0, 1)$ lies on line ℓ_0
4 : $P_{86} = (4, 1, 0, 1)$ lies on line ℓ_1
5 : $P_{89} = (7, 1, 0, 1)$ lies on line ℓ_2
6 : $P_{92} = (2, 2, 0, 1)$ lies on line ℓ_0
7 : $P_{94} = (4, 2, 0, 1)$ lies on line ℓ_1
8 : $P_{97} = (7, 2, 0, 1)$ lies on line ℓ_2
9 : $P_{100} = (2, 3, 0, 1)$ lies on line ℓ_0
10 : $P_{102} = (4, 3, 0, 1)$ lies on line ℓ_1
11 : $P_{105} = (7, 3, 0, 1)$ lies on line ℓ_2
12 : $P_{108} = (2, 4, 0, 1)$ lies on line ℓ_0

13 : $P_{110} = (4, 4, 0, 1)$ lies on line ℓ_1
14 : $P_{113} = (7, 4, 0, 1)$ lies on line ℓ_2
15 : $P_{116} = (2, 5, 0, 1)$ lies on line ℓ_0
16 : $P_{118} = (4, 5, 0, 1)$ lies on line ℓ_1
17 : $P_{121} = (7, 5, 0, 1)$ lies on line ℓ_2
18 : $P_{124} = (2, 6, 0, 1)$ lies on line ℓ_0
19 : $P_{126} = (4, 6, 0, 1)$ lies on line ℓ_1
20 : $P_{129} = (7, 6, 0, 1)$ lies on line ℓ_2
21 : $P_{132} = (2, 7, 0, 1)$ lies on line ℓ_0
22 : $P_{134} = (4, 7, 0, 1)$ lies on line ℓ_1
23 : $P_{137} = (7, 7, 0, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_{12} = (1, 0, 1, 0)$
2 : $P_{19} = (0, 1, 1, 0)$
3 : $P_{20} = (1, 1, 1, 0)$
4 : $P_{138} = (0, 0, 1, 1)$
5 : $P_{139} = (1, 0, 1, 1)$
6 : $P_{157} = (4, 2, 1, 1)$
7 : $P_{168} = (7, 3, 1, 1)$
8 : $P_{176} = (7, 4, 1, 1)$
9 : $P_{179} = (2, 5, 1, 1)$
10 : $P_{189} = (4, 6, 1, 1)$
11 : $P_{195} = (2, 7, 1, 1)$

12 : $P_{207} = (6, 0, 2, 1)$
 13 : $P_{222} = (5, 2, 2, 1)$
 14 : $P_{229} = (4, 3, 2, 1)$
 15 : $P_{255} = (6, 6, 2, 1)$
 16 : $P_{257} = (0, 7, 2, 1)$
 17 : $P_{261} = (4, 7, 2, 1)$
 18 : $P_{262} = (5, 7, 2, 1)$
 19 : $P_{273} = (0, 1, 3, 1)$
 20 : $P_{277} = (4, 1, 3, 1)$
 21 : $P_{278} = (5, 1, 3, 1)$
 22 : $P_{302} = (5, 4, 3, 1)$
 23 : $P_{309} = (4, 5, 3, 1)$
 24 : $P_{332} = (3, 0, 4, 1)$
 25 : $P_{345} = (0, 2, 4, 1)$
 26 : $P_{351} = (6, 2, 4, 1)$
 27 : $P_{352} = (7, 2, 4, 1)$
 28 : $P_{356} = (3, 3, 4, 1)$
 29 : $P_{367} = (6, 4, 4, 1)$
 30 : $P_{376} = (7, 5, 4, 1)$

31 : $P_{401} = (0, 1, 5, 1)$
 32 : $P_{407} = (6, 1, 5, 1)$
 33 : $P_{408} = (7, 1, 5, 1)$
 34 : $P_{448} = (7, 6, 5, 1)$
 35 : $P_{455} = (6, 7, 5, 1)$
 36 : $P_{465} = (0, 1, 6, 1)$
 37 : $P_{467} = (2, 1, 6, 1)$
 38 : $P_{468} = (3, 1, 6, 1)$
 39 : $P_{476} = (3, 2, 6, 1)$
 40 : $P_{483} = (2, 3, 6, 1)$
 41 : $P_{526} = (5, 0, 7, 1)$
 42 : $P_{553} = (0, 4, 7, 1)$
 43 : $P_{555} = (2, 4, 7, 1)$
 44 : $P_{556} = (3, 4, 7, 1)$
 45 : $P_{566} = (5, 5, 7, 1)$
 46 : $P_{571} = (2, 6, 7, 1)$
 47 : $P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_1

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_1

The surface has 73 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
 1 : $P_4 = (1, 1, 1, 1)$
 2 : $P_{12} = (1, 0, 1, 0)$
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