Rank-74099 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is -2113662907

General information

Number of lines	2
Number of points	4097
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	128
Number of points off lines	3968
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^2
Type of lines on points	2, 1^{128}, 0^{3968}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{17043520} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{17043520} = \mathbf{Pl}(0,0,0,1,0,0)_{129}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (17043520, 17047616)
Rank of points on Klein quadric: (129, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 128 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_2 = (0,0,1,0)$ lies on line ℓ_1
2 : $P_{4226} = (0,1,0,1)$ lies on line ℓ_0
3 : $P_{4290} = (0,2,0,1)$ lies on line ℓ_0
4 : $P_{4354} = (0,3,0,1)$ lies on line ℓ_0
5 : $P_{4418} = (0,4,0,1)$ lies on line ℓ_0
6 : $P_{4482} = (0,5,0,1)$ lies on line ℓ_0
7 : $P_{4546} = (0,6,0,1)$ lies on line ℓ_0
8 : $P_{4610} = (0,7,0,1)$ lies on line ℓ_0
9 : $P_{4674} = (0,8,0,1)$ lies on line ℓ_0
10 : $P_{4738} = (0,9,0,1)$ lies on line ℓ_0
11 : $P_{4802} = (0,10,0,1)$ lies on line ℓ_0
12 : $P_{4866} = (0,11,0,1)$ lies on line ℓ_0
13 : $P_{4930} = (0,12,0,1)$ lies on line ℓ_0
14 : $P_{4994} = (0,13,0,1)$ lies on line ℓ_0
15 : $P_{5058} = (0,14,0,1)$ lies on line ℓ_0
16 : $P_{5122} = (0,15,0,1)$ lies on line ℓ_0
17 : $P_{5186} = (0,16,0,1)$ lies on line ℓ_0
18 : $P_{5250} = (0,17,0,1)$ lies on line ℓ_0
19 : $P_{5314} = (0,18,0,1)$ lies on line ℓ_0
20 : $P_{5378} = (0,19,0,1)$ lies on line ℓ_0
21 : $P_{5442} = (0,20,0,1)$ lies on line ℓ_0
22 : $P_{5506} = (0,21,0,1)$ lies on line ℓ_0
23 : $P_{5570} = (0,22,0,1)$ lies on line ℓ_0
24 : $P_{5634} = (0,23,0,1)$ lies on line ℓ_0
25 : $P_{5698} = (0,24,0,1)$ lies on line ℓ_0
26 : $P_{5762} = (0,25,0,1)$ lies on line ℓ_0
27 : $P_{5826} = (0,26,0,1)$ lies on line ℓ_0
28 : $P_{5890} = (0,27,0,1)$ lies on line ℓ_0
29 : $P_{5954} = (0,28,0,1)$ lies on line ℓ_0
30 : $P_{6018} = (0,29,0,1)$ lies on line ℓ_0
31 : $P_{6082} = (0,30,0,1)$ lies on line ℓ_0
32 : $P_{6146} = (0,31,0,1)$ lies on line ℓ_0
33 : $P_{6210} = (0,32,0,1)$ lies on line ℓ_0
34 : $P_{6274} = (0,33,0,1)$ lies on line ℓ_0
35 : $P_{6338} = (0,34,0,1)$ lies on line ℓ_0
36 : $P_{6402} = (0,35,0,1)$ lies on line ℓ_0
37 : $P_{6466} = (0,36,0,1)$ lies on line ℓ_0
38 : $P_{6530} = (0,37,0,1)$ lies on line ℓ_0
39 : $P_{6594} = (0,38,0,1)$ lies on line ℓ_0
40 : $P_{6658} = (0,39,0,1)$ lies on line ℓ_0
41 : $P_{6722} = (0,40,0,1)$ lies on line ℓ_0
42 : $P_{6786} = (0,41,0,1)$ lies on line ℓ_0
43 : $P_{6850} = (0,42,0,1)$ lies on line ℓ_0
44 : $P_{6914} = (0,43,0,1)$ lies on line ℓ_0
45 : $P_{6978} = (0,44,0,1)$ lies on line ℓ_0
46 : $P_{7042} = (0,45,0,1)$ lies on line ℓ_0
47 : $P_{7106} = (0,46,0,1)$ lies on line ℓ_0
48 : $P_{7170} = (0,47,0,1)$ lies on line ℓ_0
49 : $P_{7234} = (0,48,0,1)$ lies on line ℓ_0
50 : $P_{7298} = (0,49,0,1)$ lies on line ℓ_0
51 : $P_{7362} = (0,50,0,1)$ lies on line ℓ_0
52 : $P_{7426} = (0,51,0,1)$ lies on line ℓ_0
53 : $P_{7490} = (0,52,0,1)$ lies on line ℓ_0
54 : $P_{7554} = (0,53,0,1)$ lies on line ℓ_0
55 : $P_{7618} = (0,54,0,1)$ lies on line ℓ_0
56 : $P_{7682} = (0,55,0,1)$ lies on line ℓ_0
57 : $P_{7746} = (0,56,0,1)$ lies on line ℓ_0

58 : $P_{7810} = (0, 57, 0, 1)$ lies on line ℓ_0
59 : $P_{7874} = (0, 58, 0, 1)$ lies on line ℓ_0
60 : $P_{7938} = (0, 59, 0, 1)$ lies on line ℓ_0
61 : $P_{8002} = (0, 60, 0, 1)$ lies on line ℓ_0
62 : $P_{8066} = (0, 61, 0, 1)$ lies on line ℓ_0
63 : $P_{8130} = (0, 62, 0, 1)$ lies on line ℓ_0
64 : $P_{8194} = (0, 63, 0, 1)$ lies on line ℓ_0
65 : $P_{8258} = (0, 0, 1, 1)$ lies on line ℓ_1
66 : $P_{12353} = (0, 0, 2, 1)$ lies on line ℓ_1
67 : $P_{16449} = (0, 0, 3, 1)$ lies on line ℓ_1
68 : $P_{20545} = (0, 0, 4, 1)$ lies on line ℓ_1
69 : $P_{24641} = (0, 0, 5, 1)$ lies on line ℓ_1
70 : $P_{28737} = (0, 0, 6, 1)$ lies on line ℓ_1
71 : $P_{32833} = (0, 0, 7, 1)$ lies on line ℓ_1
72 : $P_{36929} = (0, 0, 8, 1)$ lies on line ℓ_1
73 : $P_{41025} = (0, 0, 9, 1)$ lies on line ℓ_1
74 : $P_{45121} = (0, 0, 10, 1)$ lies on line ℓ_1
75 : $P_{49217} = (0, 0, 11, 1)$ lies on line ℓ_1
76 : $P_{53313} = (0, 0, 12, 1)$ lies on line ℓ_1
77 : $P_{57409} = (0, 0, 13, 1)$ lies on line ℓ_1
78 : $P_{61505} = (0, 0, 14, 1)$ lies on line ℓ_1
79 : $P_{65601} = (0, 0, 15, 1)$ lies on line ℓ_1
80 : $P_{69697} = (0, 0, 16, 1)$ lies on line ℓ_1
81 : $P_{73793} = (0, 0, 17, 1)$ lies on line ℓ_1
82 : $P_{77889} = (0, 0, 18, 1)$ lies on line ℓ_1
83 : $P_{81985} = (0, 0, 19, 1)$ lies on line ℓ_1
84 : $P_{86081} = (0, 0, 20, 1)$ lies on line ℓ_1
85 : $P_{90177} = (0, 0, 21, 1)$ lies on line ℓ_1
86 : $P_{94273} = (0, 0, 22, 1)$ lies on line ℓ_1
87 : $P_{98369} = (0, 0, 23, 1)$ lies on line ℓ_1
88 : $P_{102465} = (0, 0, 24, 1)$ lies on line ℓ_1
89 : $P_{106561} = (0, 0, 25, 1)$ lies on line ℓ_1
90 : $P_{110657} = (0, 0, 26, 1)$ lies on line ℓ_1
91 : $P_{114753} = (0, 0, 27, 1)$ lies on line ℓ_1
92 : $P_{118849} = (0, 0, 28, 1)$ lies on line ℓ_1
93 : $P_{122945} = (0, 0, 29, 1)$ lies on line ℓ_1
94 : $P_{127041} = (0, 0, 30, 1)$ lies on line ℓ_1
95 : $P_{131137} = (0, 0, 31, 1)$ lies on line ℓ_1
96 : $P_{135233} = (0, 0, 32, 1)$ lies on line ℓ_1
97 : $P_{139329} = (0, 0, 33, 1)$ lies on line ℓ_1
98 : $P_{143425} = (0, 0, 34, 1)$ lies on line ℓ_1
99 : $P_{147521} = (0, 0, 35, 1)$ lies on line ℓ_1
100 : $P_{151617} = (0, 0, 36, 1)$ lies on line ℓ_1
101 : $P_{155713} = (0, 0, 37, 1)$ lies on line ℓ_1
102 : $P_{159809} = (0, 0, 38, 1)$ lies on line ℓ_1
103 : $P_{163905} = (0, 0, 39, 1)$ lies on line ℓ_1
104 : $P_{168001} = (0, 0, 40, 1)$ lies on line ℓ_1
105 : $P_{172097} = (0, 0, 41, 1)$ lies on line ℓ_1
106 : $P_{176193} = (0, 0, 42, 1)$ lies on line ℓ_1
107 : $P_{180289} = (0, 0, 43, 1)$ lies on line ℓ_1
108 : $P_{184385} = (0, 0, 44, 1)$ lies on line ℓ_1
109 : $P_{188481} = (0, 0, 45, 1)$ lies on line ℓ_1
110 : $P_{192577} = (0, 0, 46, 1)$ lies on line ℓ_1
111 : $P_{196673} = (0, 0, 47, 1)$ lies on line ℓ_1
112 : $P_{200769} = (0, 0, 48, 1)$ lies on line ℓ_1
113 : $P_{204865} = (0, 0, 49, 1)$ lies on line ℓ_1
114 : $P_{208961} = (0, 0, 50, 1)$ lies on line ℓ_1
115 : $P_{213057} = (0, 0, 51, 1)$ lies on line ℓ_1
116 : $P_{217153} = (0, 0, 52, 1)$ lies on line ℓ_1
117 : $P_{221249} = (0, 0, 53, 1)$ lies on line ℓ_1
118 : $P_{225345} = (0, 0, 54, 1)$ lies on line ℓ_1
119 : $P_{229441} = (0, 0, 55, 1)$ lies on line ℓ_1
120 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_1
121 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_1
122 : $P_{241729} = (0, 0, 58, 1)$ lies on line ℓ_1
123 : $P_{245825} = (0, 0, 59, 1)$ lies on line ℓ_1
124 : $P_{249921} = (0, 0, 60, 1)$ lies on line ℓ_1
125 : $P_{254017} = (0, 0, 61, 1)$ lies on line ℓ_1
126 : $P_{258113} = (0, 0, 62, 1)$ lies on line ℓ_1
127 : $P_{262209} = (0, 0, 63, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 3968 points not on any line:
Too many to print.

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_3

Line 1 intersects

Line	ℓ_0
in point	P_3

The surface has 4097 points:
Too many to print.