Rank-74263 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361355405

General information

Number of lines	3
Number of points	89
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	65
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	2^3, 1^{21}, 0^{65}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & \gamma & \gamma^5 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{bmatrix}_{1935} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{1935} = \mathbf{Pl}(7,5,5,7,2,1)_{2112}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^3 \\ 0 & 1 & \gamma^6 & \gamma^4 \end{bmatrix}_{3274} = \begin{bmatrix} 1 & 0 & 4 & 5 \\ 0 & 1 & 6 & 7 \end{bmatrix}_{3274} = \mathbf{Pl}(2,6,6,2,4,1)_{3171}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^5 & \gamma \end{bmatrix}_{4034} = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{4034} = \mathbf{Pl}(4,3,3,4,7,1)_{4566}$$

Rank of lines: (1935, 3274, 4034)
Rank of points on Klein quadric: (2112, 3171, 4566)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{555} = (2,4,7,1) = \ell_0 \cap \ell_1$
$P_{352} = (7,2,4,1) = \ell_0 \cap \ell_2$
$P_{261} = (4,7,2,1) = \ell_1 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_{30} = (3,2,1,0)$ lies on line ℓ_0
1 : $P_{48} = (5,4,1,0)$ lies on line ℓ_1
2 : $P_{73} = (6,7,1,0)$ lies on line ℓ_2
3 : $P_{93} = (3,2,0,1)$ lies on line ℓ_1
4 : $P_{111} = (5,4,0,1)$ lies on line ℓ_2
5 : $P_{136} = (6,7,0,1)$ lies on line ℓ_0
6 : $P_{164} = (3,3,1,1)$ lies on line ℓ_2
7 : $P_{182} = (5,5,1,1)$ lies on line ℓ_0
8 : $P_{191} = (6,6,1,1)$ lies on line ℓ_1
9 : $P_{225} = (0,3,2,1)$ lies on line ℓ_0
10 : $P_{267} = (2,0,3,1)$ lies on line ℓ_2
11 : $P_{276} = (3,1,3,1)$ lies on line ℓ_0
12 : $P_{290} = (1,3,3,1)$ lies on line ℓ_1
13 : $P_{369} = (0,5,4,1)$ lies on line ℓ_1
14 : $P_{397} = (4,0,5,1)$ lies on line ℓ_0
15 : $P_{406} = (5,1,5,1)$ lies on line ℓ_1
16 : $P_{434} = (1,5,5,1)$ lies on line ℓ_2
17 : $P_{464} = (7,0,6,1)$ lies on line ℓ_1
18 : $P_{471} = (6,1,6,1)$ lies on line ℓ_2
19 : $P_{506} = (1,6,6,1)$ lies on line ℓ_0
20 : $P_{569} = (0,6,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 65 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_3 = (0,0,0,1)$
3 : $P_{29} = (2,2,1,0)$
4 : $P_{47} = (4,4,1,0)$
5 : $P_{74} = (7,7,1,0)$
6 : $P_{95} = (5,2,0,1)$
7 : $P_{112} = (6,4,0,1)$
8 : $P_{133} = (3,7,0,1)$
9 : $P_{139} = (1,0,1,1)$
10 : $P_{146} = (0,1,1,1)$
11 : $P_{167} = (6,3,1,1)$
12 : $P_{180} = (3,5,1,1)$
13 : $P_{190} = (5,6,1,1)$
14 : $P_{206} = (5,0,2,1)$
15 : $P_{214} = (5,1,2,1)$

16 : $P_{215} = (6, 1, 2, 1)$
17 : $P_{231} = (6, 3, 2, 1)$
18 : $P_{235} = (2, 4, 2, 1)$
19 : $P_{236} = (3, 4, 2, 1)$
20 : $P_{242} = (1, 5, 2, 1)$
21 : $P_{245} = (4, 5, 2, 1)$
22 : $P_{250} = (1, 6, 2, 1)$
23 : $P_{260} = (3, 7, 2, 1)$
24 : $P_{274} = (1, 1, 3, 1)$
25 : $P_{283} = (2, 2, 3, 1)$
26 : $P_{288} = (7, 2, 3, 1)$
27 : $P_{295} = (6, 3, 3, 1)$
28 : $P_{304} = (7, 4, 3, 1)$
29 : $P_{321} = (0, 7, 3, 1)$
30 : $P_{327} = (6, 7, 3, 1)$
31 : $P_{335} = (6, 0, 4, 1)$
32 : $P_{340} = (3, 1, 4, 1)$
33 : $P_{343} = (6, 1, 4, 1)$
34 : $P_{350} = (5, 2, 4, 1)$
35 : $P_{354} = (1, 3, 4, 1)$
36 : $P_{372} = (3, 5, 4, 1)$
37 : $P_{378} = (1, 6, 4, 1)$
38 : $P_{384} = (7, 6, 4, 1)$
39 : $P_{389} = (4, 7, 4, 1)$
40 : $P_{390} = (5, 7, 4, 1)$
41 : $P_{402} = (1, 1, 5, 1)$
42 : $P_{409} = (0, 2, 5, 1)$
43 : $P_{412} = (3, 2, 5, 1)$
44 : $P_{427} = (2, 4, 5, 1)$
45 : $P_{429} = (4, 4, 5, 1)$
46 : $P_{436} = (3, 5, 5, 1)$
47 : $P_{451} = (2, 7, 5, 1)$
48 : $P_{466} = (1, 1, 6, 1)$
49 : $P_{477} = (4, 2, 6, 1)$
50 : $P_{489} = (0, 4, 6, 1)$
51 : $P_{494} = (5, 4, 6, 1)$
52 : $P_{510} = (5, 6, 6, 1)$
53 : $P_{517} = (4, 7, 6, 1)$
54 : $P_{520} = (7, 7, 6, 1)$
55 : $P_{524} = (3, 0, 7, 1)$
56 : $P_{532} = (3, 1, 7, 1)$
57 : $P_{534} = (5, 1, 7, 1)$
58 : $P_{543} = (6, 2, 7, 1)$
59 : $P_{544} = (7, 2, 7, 1)$
60 : $P_{546} = (1, 3, 7, 1)$
61 : $P_{547} = (2, 3, 7, 1)$
62 : $P_{559} = (6, 4, 7, 1)$
63 : $P_{562} = (1, 5, 7, 1)$
64 : $P_{574} = (5, 6, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{555}	P_{352}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{555}	P_{261}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{352}	P_{261}

The surface has 89 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_{29} = (2, 2, 1, 0)$
4 : $P_{30} = (3, 2, 1, 0)$
5 : $P_{47} = (4, 4, 1, 0)$
6 : $P_{48} = (5, 4, 1, 0)$
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