

Rank-65904 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244206222

General information

Number of lines	15
Number of points	105
Number of singular points	0
Number of Eckardt points	1
Number of double points	42
Number of single points	48
Number of points off lines	14
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{15}
Type of lines on points	$3, 2^{42}, 1^{48}, 0^{14}$

Singular Points

The surface has 0 singular points:

The 15 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \left[\begin{array}{cccc} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \mathbf{Pl}(2, 0, 0, 1, 0, 0)_{19} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 0 & 0 \end{array} \right]_{1752} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{array} \right]_{1752} = \mathbf{Pl}(4, 0, 0, 1, 0, 0)_{21}\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \mathbf{Pl}(7, 0, 0, 1, 0, 0)_{24} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^3 & 1 \end{bmatrix}_{451} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 5 & 1 \end{bmatrix}_{451} = \mathbf{Pl}(1, 1, 5, 0, 2, 1)_{1756} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^4 & \gamma^3 \end{bmatrix}_{485} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 7 & 5 \end{bmatrix}_{485} = \mathbf{Pl}(7, 5, 2, 0, 7, 1)_{4261} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^6 & 1 \end{bmatrix}_{233} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 6 & 1 \end{bmatrix}_{233} = \mathbf{Pl}(1, 1, 6, 0, 4, 1)_{2771} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma & \gamma^6 \end{bmatrix}_{269} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 6 \end{bmatrix}_{269} = \mathbf{Pl}(2, 6, 4, 0, 2, 1)_{1750} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^5 & 1 \end{bmatrix}_{376} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{376} = \mathbf{Pl}(1, 1, 3, 0, 7, 1)_{4262} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^2 & \gamma^5 \end{bmatrix}_{393} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 4 & 3 \end{bmatrix}_{393} = \mathbf{Pl}(4, 3, 7, 0, 4, 1)_{2781} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 1 & \gamma^3 \\ 0 & 1 & \gamma^4 & \gamma^3 \end{bmatrix}_{3040} = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 7 & 5 \end{bmatrix}_{3040} = \mathbf{Pl}(3, 4, 7, 5, 5, 1)_{3732} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & \gamma^3 & 1 \end{bmatrix}_{816} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 1 \end{bmatrix}_{816} = \mathbf{Pl}(6, 2, 3, 4, 4, 1)_{3035} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \gamma^3 & 1 \\ 0 & 1 & \gamma^6 & 1 \end{bmatrix}_{963} = \begin{bmatrix} 1 & 0 & 5 & 1 \\ 0 & 1 & 6 & 1 \end{bmatrix}_{963} = \mathbf{Pl}(3, 4, 5, 7, 7, 1)_{4642} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & 1 & \gamma^6 \\ 0 & 1 & \gamma & \gamma^6 \end{bmatrix}_{3627} = \begin{bmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & 2 & 6 \end{bmatrix}_{3627} = \mathbf{Pl}(5, 7, 2, 6, 6, 1)_{3993} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^5 & 1 \end{bmatrix}_{1033} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{1033} = \mathbf{Pl}(5, 7, 6, 2, 2, 1)_{2173} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & 1 & \gamma^5 \\ 0 & 1 & \gamma^2 & \gamma^5 \end{bmatrix}_{1853} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 4 & 3 \end{bmatrix}_{1853} = \mathbf{Pl}(6, 2, 4, 3, 3, 1)_{2580}
\end{aligned}$$

Rank of lines: (3504, 1752, 2920, 451, 485, 233, 269, 376, 393, 3040, 816, 963, 3627, 1033, 1853)

Rank of points on Klein quadric: (19, 21, 24, 1756, 4261, 2771, 1750, 4262, 2781, 3732, 3035, 4642, 3993, 2173, 2580)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0).$$

Double Points

The surface has 42 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{84} &= (2, 1, 0, 1) = \ell_0 \cap \ell_5 \\
P_{108} &= (2, 4, 0, 1) = \ell_0 \cap \ell_8 \\
P_{100} &= (2, 3, 0, 1) = \ell_0 \cap \ell_{11} \\
P_{124} &= (2, 6, 0, 1) = \ell_0 \cap \ell_{14}
\end{aligned}$$

$$\begin{aligned}
P_{134} &= (4, 7, 0, 1) = \ell_1 \cap \ell_4 \\
P_{86} &= (4, 1, 0, 1) = \ell_1 \cap \ell_7 \\
P_{102} &= (4, 3, 0, 1) = \ell_1 \cap \ell_9 \\
P_{118} &= (4, 5, 0, 1) = \ell_1 \cap \ell_{13}
\end{aligned}$$

$$\begin{aligned}
P_{89} &= (7, 1, 0, 1) = \ell_2 \cap \ell_3 \\
P_{97} &= (7, 2, 0, 1) = \ell_2 \cap \ell_6 \\
P_{129} &= (7, 6, 0, 1) = \ell_2 \cap \ell_{10} \\
P_{121} &= (7, 5, 0, 1) = \ell_2 \cap \ell_{12} \\
P_{13} &= (2, 0, 1, 0) = \ell_3 \cap \ell_4 \\
P_{533} &= (4, 1, 7, 1) = \ell_3 \cap \ell_5 \\
P_{339} &= (2, 1, 4, 1) = \ell_3 \cap \ell_7 \\
P_{401} &= (0, 1, 5, 1) = \ell_3 \cap \ell_{10} \\
P_{150} &= (5, 1, 1, 1) = \ell_3 \cap \ell_{14} \\
P_{257} &= (0, 7, 2, 1) = \ell_4 \cap \ell_9 \\
P_{199} &= (6, 7, 1, 1) = \ell_4 \cap \ell_{11} \\
P_{518} &= (5, 7, 6, 1) = \ell_4 \cap \ell_{12} \\
P_{452} &= (3, 7, 5, 1) = \ell_4 \cap \ell_{14} \\
P_{15} &= (4, 0, 1, 0) = \ell_5 \cap \ell_6 \\
P_{216} &= (7, 1, 2, 1) = \ell_5 \cap \ell_7 \\
P_{151} &= (6, 1, 1, 1) = \ell_5 \cap \ell_9 \\
P_{465} &= (0, 1, 6, 1) = \ell_5 \cap \ell_{11} \\
P_{478} &= (5, 2, 6, 1) = \ell_6 \cap \ell_9
\end{aligned}$$

$$\begin{aligned}
P_{345} &= (0, 2, 4, 1) = \ell_6 \cap \ell_{12} \\
P_{156} &= (3, 2, 1, 1) = \ell_6 \cap \ell_{13} \\
P_{287} &= (6, 2, 3, 1) = \ell_6 \cap \ell_{14} \\
P_{18} &= (7, 0, 1, 0) = \ell_7 \cap \ell_8 \\
P_{148} &= (3, 1, 1, 1) = \ell_7 \cap \ell_{12} \\
P_{273} &= (0, 1, 3, 1) = \ell_7 \cap \ell_{13} \\
P_{428} &= (3, 4, 5, 1) = \ell_8 \cap \ell_9 \\
P_{174} &= (5, 4, 1, 1) = \ell_8 \cap \ell_{10} \\
P_{303} &= (6, 4, 3, 1) = \ell_8 \cap \ell_{12} \\
P_{553} &= (0, 4, 7, 1) = \ell_8 \cap \ell_{14} \\
P_{29} &= (2, 2, 1, 0) = \ell_9 \cap \ell_{10} \\
P_{565} &= (4, 5, 7, 1) = \ell_{10} \cap \ell_{11} \\
P_{355} &= (2, 3, 4, 1) = \ell_{10} \cap \ell_{13} \\
P_{47} &= (4, 4, 1, 0) = \ell_{11} \cap \ell_{12} \\
P_{256} &= (7, 6, 2, 1) = \ell_{11} \cap \ell_{13} \\
P_{74} &= (7, 7, 1, 0) = \ell_{13} \cap \ell_{14}
\end{aligned}$$

Single Points

The surface has 48 single points:
The single points on the surface are:

$$\begin{aligned}
0 : P_{76} &= (2, 0, 0, 1) \text{ lies on line } \ell_0 \\
1 : P_{78} &= (4, 0, 0, 1) \text{ lies on line } \ell_1 \\
2 : P_{81} &= (7, 0, 0, 1) \text{ lies on line } \ell_2 \\
3 : P_{92} &= (2, 2, 0, 1) \text{ lies on line } \ell_0 \\
4 : P_{94} &= (4, 2, 0, 1) \text{ lies on line } \ell_1 \\
5 : P_{105} &= (7, 3, 0, 1) \text{ lies on line } \ell_2 \\
6 : P_{110} &= (4, 4, 0, 1) \text{ lies on line } \ell_1 \\
7 : P_{113} &= (7, 4, 0, 1) \text{ lies on line } \ell_2 \\
8 : P_{116} &= (2, 5, 0, 1) \text{ lies on line } \ell_0 \\
9 : P_{126} &= (4, 6, 0, 1) \text{ lies on line } \ell_1 \\
10 : P_{132} &= (2, 7, 0, 1) \text{ lies on line } \ell_0 \\
11 : P_{137} &= (7, 7, 0, 1) \text{ lies on line } \ell_2 \\
12 : P_{203} &= (2, 0, 2, 1) \text{ lies on line } \ell_{12} \\
13 : P_{212} &= (3, 1, 2, 1) \text{ lies on line } \ell_3 \\
14 : P_{219} &= (2, 2, 2, 1) \text{ lies on line } \ell_6 \\
15 : P_{220} &= (3, 2, 2, 1) \text{ lies on line } \ell_{10} \\
16 : P_{234} &= (1, 4, 2, 1) \text{ lies on line } \ell_8 \\
17 : P_{242} &= (1, 5, 2, 1) \text{ lies on line } \ell_{14} \\
18 : P_{266} &= (1, 0, 3, 1) \text{ lies on line } \ell_{10} \\
19 : P_{274} &= (1, 1, 3, 1) \text{ lies on line } \ell_3 \\
20 : P_{276} &= (3, 1, 3, 1) \text{ lies on line } \ell_5 \\
21 : P_{284} &= (3, 2, 3, 1) \text{ lies on line } \ell_{11} \\
22 : P_{307} &= (2, 5, 3, 1) \text{ lies on line } \ell_9 \\
23 : P_{323} &= (2, 7, 3, 1) \text{ lies on line } \ell_4 \\
24 : P_{333} &= (4, 0, 4, 1) \text{ lies on line } \ell_{14}
\end{aligned}$$

$$\begin{aligned}
25 : P_{342} &= (5, 1, 4, 1) \text{ lies on line } \ell_5 \\
26 : P_{365} &= (4, 4, 4, 1) \text{ lies on line } \ell_8 \\
27 : P_{366} &= (5, 4, 4, 1) \text{ lies on line } \ell_{11} \\
28 : P_{378} &= (1, 6, 4, 1) \text{ lies on line } \ell_9 \\
29 : P_{386} &= (1, 7, 4, 1) \text{ lies on line } \ell_4 \\
30 : P_{394} &= (1, 0, 5, 1) \text{ lies on line } \ell_{11} \\
31 : P_{402} &= (1, 1, 5, 1) \text{ lies on line } \ell_5 \\
32 : P_{406} &= (5, 1, 5, 1) \text{ lies on line } \ell_7 \\
33 : P_{413} &= (4, 2, 5, 1) \text{ lies on line } \ell_6 \\
34 : P_{430} &= (5, 4, 5, 1) \text{ lies on line } \ell_{13} \\
35 : P_{445} &= (4, 6, 5, 1) \text{ lies on line } \ell_{12} \\
36 : P_{458} &= (1, 0, 6, 1) \text{ lies on line } \ell_{13} \\
37 : P_{466} &= (1, 1, 6, 1) \text{ lies on line } \ell_7 \\
38 : P_{471} &= (6, 1, 6, 1) \text{ lies on line } \ell_3 \\
39 : P_{488} &= (7, 3, 6, 1) \text{ lies on line } \ell_{14} \\
40 : P_{496} &= (7, 4, 6, 1) \text{ lies on line } \ell_8 \\
41 : P_{519} &= (6, 7, 6, 1) \text{ lies on line } \ell_{10} \\
42 : P_{528} &= (7, 0, 7, 1) \text{ lies on line } \ell_9 \\
43 : P_{535} &= (6, 1, 7, 1) \text{ lies on line } \ell_7 \\
44 : P_{538} &= (1, 2, 7, 1) \text{ lies on line } \ell_6 \\
45 : P_{546} &= (1, 3, 7, 1) \text{ lies on line } \ell_{12} \\
46 : P_{583} &= (6, 7, 7, 1) \text{ lies on line } \ell_{13} \\
47 : P_{584} &= (7, 7, 7, 1) \text{ lies on line } \ell_4
\end{aligned}$$

The single points on the surface are:

Points on surface but on no line

The surface has 14 points not on any line:

The points on the surface but not on lines are:

0 : $P_{19} = (0, 1, 1, 0)$	8 : $P_{288} = (7, 2, 3, 1)$
1 : $P_{32} = (5, 2, 1, 0)$	9 : $P_{312} = (7, 5, 3, 1)$
2 : $P_{33} = (6, 2, 1, 0)$	10 : $P_{427} = (2, 4, 5, 1)$
3 : $P_{46} = (3, 4, 1, 0)$	11 : $P_{443} = (2, 6, 5, 1)$
4 : $P_{49} = (6, 4, 1, 0)$	12 : $P_{485} = (4, 3, 6, 1)$
5 : $P_{70} = (3, 7, 1, 0)$	13 : $P_{517} = (4, 7, 6, 1)$
6 : $P_{72} = (5, 7, 1, 0)$	
7 : $P_{138} = (0, 0, 1, 1)$	

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1
1	1	0	1	0	1	0	0	1	0	1	0	0	0	1	0
2	1	1	0	1	0	0	1	0	0	0	1	0	1	0	0
3	0	0	1	0	1	1	0	1	0	0	1	0	0	0	1
4	0	1	0	1	0	0	0	0	0	1	0	1	1	0	1
5	1	0	0	1	0	0	1	1	0	1	0	0	0	0	0
6	0	0	1	0	0	1	0	0	0	1	0	0	1	1	1
7	0	1	0	1	0	1	0	0	1	0	0	1	1	0	0
8	1	0	0	0	0	0	1	0	1	1	0	1	0	1	0
9	0	1	0	0	1	1	1	0	1	0	0	0	0	0	0
10	0	0	1	1	0	0	0	0	1	1	0	1	0	1	0
11	1	0	0	0	1	1	0	0	0	0	1	0	1	1	0
12	0	0	1	0	1	0	1	1	1	0	0	1	0	0	0
13	0	1	0	0	0	0	1	1	0	0	1	1	0	0	1
14	1	0	0	1	1	0	1	0	1	0	0	0	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{14}
in point	P_1	P_1	P_{84}	P_{108}	P_{100}	P_{124}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7	ℓ_9	ℓ_{13}
in point	P_1	P_1	P_{134}	P_{86}	P_{102}	P_{118}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_6	ℓ_{10}	ℓ_{12}
in point	P_1	P_1	P_{89}	P_{97}	P_{129}	P_{121}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{14}
in point	P_{89}	P_{13}	P_{533}	P_{339}	P_{401}	P_{150}

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{14}
in point	P_{134}	P_{13}	P_{257}	P_{199}	P_{518}	P_{452}

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7	ℓ_9	ℓ_{11}
in point	P_{84}	P_{533}	P_{15}	P_{216}	P_{151}	P_{465}

Line 6 intersects

Line	ℓ_2	ℓ_5	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{14}
in point	P_{97}	P_{15}	P_{478}	P_{345}	P_{156}	P_{287}

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8	ℓ_{12}	ℓ_{13}
in point	P_{86}	P_{339}	P_{216}	P_{18}	P_{148}	P_{273}

Line 8 intersects

Line	ℓ_0	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{108}	P_{18}	P_{428}	P_{174}	P_{303}	P_{553}

Line 9 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}
in point	P_{102}	P_{257}	P_{151}	P_{478}	P_{428}	P_{29}

Line 10 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}
in point	P_{129}	P_{401}	P_{174}	P_{29}	P_{565}	P_{355}

Line 11 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{12}	ℓ_{13}
in point	P_{100}	P_{199}	P_{465}	P_{565}	P_{47}	P_{256}

Line 12 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}
in point	P_{121}	P_{518}	P_{345}	P_{148}	P_{303}	P_{47}

Line 13 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{14}
in point	P_{118}	P_{156}	P_{273}	P_{355}	P_{256}	P_{74}

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{13}
in point	P_{124}	P_{150}	P_{452}	P_{287}	P_{553}	P_{74}

The surface has 105 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	20 : $P_{92} = (2, 2, 0, 1)$	40 : $P_{150} = (5, 1, 1, 1)$
1 : $P_{13} = (2, 0, 1, 0)$	21 : $P_{94} = (4, 2, 0, 1)$	41 : $P_{151} = (6, 1, 1, 1)$
2 : $P_{15} = (4, 0, 1, 0)$	22 : $P_{97} = (7, 2, 0, 1)$	42 : $P_{156} = (3, 2, 1, 1)$
3 : $P_{18} = (7, 0, 1, 0)$	23 : $P_{100} = (2, 3, 0, 1)$	43 : $P_{174} = (5, 4, 1, 1)$
4 : $P_{19} = (0, 1, 1, 0)$	24 : $P_{102} = (4, 3, 0, 1)$	44 : $P_{199} = (6, 7, 1, 1)$
5 : $P_{29} = (2, 2, 1, 0)$	25 : $P_{105} = (7, 3, 0, 1)$	45 : $P_{203} = (2, 0, 2, 1)$
6 : $P_{32} = (5, 2, 1, 0)$	26 : $P_{108} = (2, 4, 0, 1)$	46 : $P_{212} = (3, 1, 2, 1)$
7 : $P_{33} = (6, 2, 1, 0)$	27 : $P_{110} = (4, 4, 0, 1)$	47 : $P_{216} = (7, 1, 2, 1)$
8 : $P_{46} = (3, 4, 1, 0)$	28 : $P_{113} = (7, 4, 0, 1)$	48 : $P_{219} = (2, 2, 2, 1)$
9 : $P_{47} = (4, 4, 1, 0)$	29 : $P_{116} = (2, 5, 0, 1)$	49 : $P_{220} = (3, 2, 2, 1)$
10 : $P_{49} = (6, 4, 1, 0)$	30 : $P_{118} = (4, 5, 0, 1)$	50 : $P_{234} = (1, 4, 2, 1)$
11 : $P_{70} = (3, 7, 1, 0)$	31 : $P_{121} = (7, 5, 0, 1)$	51 : $P_{242} = (1, 5, 2, 1)$
12 : $P_{72} = (5, 7, 1, 0)$	32 : $P_{124} = (2, 6, 0, 1)$	52 : $P_{256} = (7, 6, 2, 1)$
13 : $P_{74} = (7, 7, 1, 0)$	33 : $P_{126} = (4, 6, 0, 1)$	53 : $P_{257} = (0, 7, 2, 1)$
14 : $P_{76} = (2, 0, 0, 1)$	34 : $P_{129} = (7, 6, 0, 1)$	54 : $P_{266} = (1, 0, 3, 1)$
15 : $P_{78} = (4, 0, 0, 1)$	35 : $P_{132} = (2, 7, 0, 1)$	55 : $P_{273} = (0, 1, 3, 1)$
16 : $P_{81} = (7, 0, 0, 1)$	36 : $P_{134} = (4, 7, 0, 1)$	56 : $P_{274} = (1, 1, 3, 1)$
17 : $P_{84} = (2, 1, 0, 1)$	37 : $P_{137} = (7, 7, 0, 1)$	57 : $P_{276} = (3, 1, 3, 1)$
18 : $P_{86} = (4, 1, 0, 1)$	38 : $P_{138} = (0, 0, 1, 1)$	58 : $P_{284} = (3, 2, 3, 1)$
19 : $P_{89} = (7, 1, 0, 1)$	39 : $P_{148} = (3, 1, 1, 1)$	59 : $P_{287} = (6, 2, 3, 1)$

60 : $P_{288} = (7, 2, 3, 1)$
 61 : $P_{303} = (6, 4, 3, 1)$
 62 : $P_{307} = (2, 5, 3, 1)$
 63 : $P_{312} = (7, 5, 3, 1)$
 64 : $P_{323} = (2, 7, 3, 1)$
 65 : $P_{333} = (4, 0, 4, 1)$
 66 : $P_{339} = (2, 1, 4, 1)$
 67 : $P_{342} = (5, 1, 4, 1)$
 68 : $P_{345} = (0, 2, 4, 1)$
 69 : $P_{355} = (2, 3, 4, 1)$
 70 : $P_{365} = (4, 4, 4, 1)$
 71 : $P_{366} = (5, 4, 4, 1)$
 72 : $P_{378} = (1, 6, 4, 1)$
 73 : $P_{386} = (1, 7, 4, 1)$
 74 : $P_{394} = (1, 0, 5, 1)$
 75 : $P_{401} = (0, 1, 5, 1)$

76 : $P_{402} = (1, 1, 5, 1)$
 77 : $P_{406} = (5, 1, 5, 1)$
 78 : $P_{413} = (4, 2, 5, 1)$
 79 : $P_{427} = (2, 4, 5, 1)$
 80 : $P_{428} = (3, 4, 5, 1)$
 81 : $P_{430} = (5, 4, 5, 1)$
 82 : $P_{443} = (2, 6, 5, 1)$
 83 : $P_{445} = (4, 6, 5, 1)$
 84 : $P_{452} = (3, 7, 5, 1)$
 85 : $P_{458} = (1, 0, 6, 1)$
 86 : $P_{465} = (0, 1, 6, 1)$
 87 : $P_{466} = (1, 1, 6, 1)$
 88 : $P_{471} = (6, 1, 6, 1)$
 89 : $P_{478} = (5, 2, 6, 1)$
 90 : $P_{485} = (4, 3, 6, 1)$
 91 : $P_{488} = (7, 3, 6, 1)$

92 : $P_{496} = (7, 4, 6, 1)$
 93 : $P_{517} = (4, 7, 6, 1)$
 94 : $P_{518} = (5, 7, 6, 1)$
 95 : $P_{519} = (6, 7, 6, 1)$
 96 : $P_{528} = (7, 0, 7, 1)$
 97 : $P_{533} = (4, 1, 7, 1)$
 98 : $P_{535} = (6, 1, 7, 1)$
 99 : $P_{538} = (1, 2, 7, 1)$
 100 : $P_{546} = (1, 3, 7, 1)$
 101 : $P_{553} = (0, 4, 7, 1)$
 102 : $P_{565} = (4, 5, 7, 1)$
 103 : $P_{583} = (6, 7, 7, 1)$
 104 : $P_{584} = (7, 7, 7, 1)$