Rank-76100 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_3 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172878

General information

Number of lines	3
Number of points	57
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	32
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	3, 1^{24}, 0^{32}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

Rank of lines: (4680, 4744, 4689)
Rank of points on Klein quadric: (17, 1, 25)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_1 = (0,1,0,0)$ lies on line ℓ_0
$1 : P_2 = (0,0,1,0)$ lies on line ℓ_1
$2 : P_{19} = (0,1,1,0)$ lies on line ℓ_2
$3 : P_{82} = (0,1,0,1)$ lies on line ℓ_0
$4 : P_{90} = (0,2,0,1)$ lies on line ℓ_0
$5 : P_{98} = (0,3,0,1)$ lies on line ℓ_0
$6 : P_{106} = (0,4,0,1)$ lies on line ℓ_0
$7 : P_{114} = (0,5,0,1)$ lies on line ℓ_0
$8 : P_{122} = (0,6,0,1)$ lies on line ℓ_0
$9 : P_{130} = (0,7,0,1)$ lies on line ℓ_0
$10 : P_{138} = (0,0,1,1)$ lies on line ℓ_1
$11 : P_{146} = (0,1,1,1)$ lies on line ℓ_2
$12 : P_{201} = (0,0,2,1)$ lies on line ℓ_1
$13 : P_{217} = (0,2,2,1)$ lies on line ℓ_2
$14 : P_{265} = (0,0,3,1)$ lies on line ℓ_1
$15 : P_{289} = (0,3,3,1)$ lies on line ℓ_2
$16 : P_{329} = (0,0,4,1)$ lies on line ℓ_1
$17 : P_{361} = (0,4,4,1)$ lies on line ℓ_2
$18 : P_{393} = (0,0,5,1)$ lies on line ℓ_1
$19 : P_{433} = (0,5,5,1)$ lies on line ℓ_2
$20 : P_{457} = (0,0,6,1)$ lies on line ℓ_1
$21 : P_{505} = (0,6,6,1)$ lies on line ℓ_2
$22 : P_{521} = (0,0,7,1)$ lies on line ℓ_1
$23 : P_{577} = (0,7,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 32 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1,1,1,1)$
$1 : P_{20} = (1,1,1,0)$
$2 : P_{33} = (6,2,1,0)$
$3 : P_{40} = (5,3,1,0)$
$4 : P_{46} = (3,4,1,0)$
$5 : P_{57} = (6,5,1,0)$
$6 : P_{62} = (3,6,1,0)$
$7 : P_{72} = (5,7,1,0)$
$8 : P_{223} = (6,2,2,1)$
$9 : P_{224} = (7,2,2,1)$
$10 : P_{234} = (1,4,2,1)$
$11 : P_{235} = (2,4,2,1)$

12 : $P_{255} = (6, 6, 2, 1)$
13 : $P_{258} = (1, 7, 2, 1)$
14 : $P_{264} = (7, 7, 2, 1)$
15 : $P_{300} = (3, 4, 3, 1)$
16 : $P_{346} = (1, 2, 4, 1)$
17 : $P_{347} = (2, 2, 4, 1)$
18 : $P_{356} = (3, 3, 4, 1)$
19 : $P_{363} = (2, 4, 4, 1)$
20 : $P_{364} = (3, 4, 4, 1)$
21 : $P_{386} = (1, 7, 4, 1)$
22 : $P_{389} = (4, 7, 4, 1)$
23 : $P_{454} = (5, 7, 5, 1)$
24 : $P_{479} = (6, 2, 6, 1)$
25 : $P_{538} = (1, 2, 7, 1)$
26 : $P_{544} = (7, 2, 7, 1)$
27 : $P_{554} = (1, 4, 7, 1)$
28 : $P_{557} = (4, 4, 7, 1)$
29 : $P_{566} = (5, 5, 7, 1)$
30 : $P_{581} = (4, 7, 7, 1)$
31 : $P_{582} = (5, 7, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_3	P_3

The surface has 57 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{19} = (0, 1, 1, 0)$
5 : $P_{20} = (1, 1, 1, 0)$
6 : $P_{33} = (6, 2, 1, 0)$
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15 : $P_{106} = (0, 4, 0, 1)$
16 : $P_{114} = (0, 5, 0, 1)$
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50 : $P_{544} = (7, 2, 7, 1)$
51 : $P_{554} = (1, 4, 7, 1)$
52 : $P_{557} = (4, 4, 7, 1)$
53 : $P_{566} = (5, 5, 7, 1)$
54 : $P_{577} = (0, 7, 7, 1)$
55 : $P_{581} = (4, 7, 7, 1)$
56 : $P_{582} = (5, 7, 7, 1)$