

Rank-46 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(64) is 532626

General information

Number of lines	27
Number of points	4545
Number of singular points	0
Number of Eckardt points	45
Number of double points	0
Number of single points	1620
Number of points off lines	2880
Number of Hesse planes	40
Number of axes	240
Type of points on lines	65^{27}
Type of lines on points	$3^{45}, 1^{1620}, 0^{2880}$

Singular Points

The surface has 0 singular points:

The 27 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned} \ell_0 = a_1 &= \left[\begin{array}{cccc} 1 & \epsilon^9 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{array} \right]_{199720} = \left[\begin{array}{cccc} 1 & 47 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{array} \right]_{199720} = \mathbf{Pl}(0, 0, 56, 57, 62, 1)_{16538443} \\ \ell_1 = a_2 &= \left[\begin{array}{cccc} 1 & \epsilon^{18} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{49868} = \left[\begin{array}{cccc} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{49868} = \mathbf{Pl}(0, 0, 1, 1, 37, 1)_{9979458} \end{aligned}$$

$$\begin{aligned}
\ell_2 = a_3 &= \begin{bmatrix} 1 & 0 & \epsilon^{28} & \epsilon^{14} \\ 0 & 1 & \epsilon^{49} & \epsilon^{56} \end{bmatrix}_{15702043} = \begin{bmatrix} 1 & 0 & 61 & 58 \\ 0 & 1 & 30 & 40 \end{bmatrix}_{15702043} = \mathbf{Pl}(30, 58, 58, 30, 61, 1)_{16503452} \\
\ell_3 = a_4 &= \begin{bmatrix} 1 & 0 & \epsilon^{56} & \epsilon^7 \\ 0 & 1 & \epsilon^{35} & \epsilon^{28} \end{bmatrix}_{9491002} = \begin{bmatrix} 1 & 0 & 40 & 35 \\ 0 & 1 & 18 & 61 \end{bmatrix}_{9491002} = \mathbf{Pl}(40, 35, 30, 58, 18, 1)_{5124717} \\
\ell_4 = a_5 &= \begin{bmatrix} 1 & 0 & \epsilon^{49} & \epsilon^{35} \\ 0 & 1 & \epsilon^7 & \epsilon^{14} \end{bmatrix}_{4922049} = \begin{bmatrix} 1 & 0 & 30 & 18 \\ 0 & 1 & 35 & 58 \end{bmatrix}_{4922049} = \mathbf{Pl}(61, 18, 58, 30, 61, 1)_{16503483} \\
\ell_5 = a_6 &= \begin{bmatrix} 1 & 0 & \epsilon^{35} & \epsilon^{28} \\ 0 & 1 & \epsilon^{14} & \epsilon^{49} \end{bmatrix}_{16321420} = \begin{bmatrix} 1 & 0 & 18 & 61 \\ 0 & 1 & 58 & 30 \end{bmatrix}_{16321420} = \mathbf{Pl}(18, 61, 35, 40, 58, 1)_{15627425} \\
\ell_6 = b_1 &= \begin{bmatrix} 1 & 0 & \epsilon^{49} & \epsilon^{14} \\ 0 & 1 & \epsilon^7 & \epsilon^{56} \end{bmatrix}_{15573057} = \begin{bmatrix} 1 & 0 & 30 & 58 \\ 0 & 1 & 35 & 40 \end{bmatrix}_{15573057} = \mathbf{Pl}(30, 58, 18, 61, 35, 1)_{9533132} \\
\ell_7 = b_2 &= \begin{bmatrix} 1 & 0 & \epsilon^{28} & \epsilon^{35} \\ 0 & 1 & \epsilon^{49} & \epsilon^{14} \end{bmatrix}_{5051035} = \begin{bmatrix} 1 & 0 & 61 & 18 \\ 0 & 1 & 30 & 58 \end{bmatrix}_{5051035} = \mathbf{Pl}(61, 18, 40, 35, 30, 1)_{8308695} \\
\ell_8 = b_3 &= \begin{bmatrix} 1 & 0 & \epsilon^{56} & \epsilon^{28} \\ 0 & 1 & \epsilon^{35} & \epsilon^{49} \end{bmatrix}_{16412922} = \begin{bmatrix} 1 & 0 & 40 & 61 \\ 0 & 1 & 18 & 30 \end{bmatrix}_{16412922} = \mathbf{Pl}(18, 61, 61, 18, 40, 1)_{11011541} \\
\ell_9 = b_4 &= \begin{bmatrix} 1 & \epsilon^{18} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{49924} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{49924} = \mathbf{Pl}(0, 0, 56, 57, 8, 1)_{2386123} \\
\ell_{10} = b_5 &= \begin{bmatrix} 1 & 0 & \epsilon^{35} & \epsilon^7 \\ 0 & 1 & \epsilon^{14} & \epsilon^{28} \end{bmatrix}_{9399500} = \begin{bmatrix} 1 & 0 & 18 & 35 \\ 0 & 1 & 58 & 61 \end{bmatrix}_{9399500} = \mathbf{Pl}(40, 35, 61, 18, 40, 1)_{11011563} \\
\ell_{11} = b_6 &= \begin{bmatrix} 1 & \epsilon^9 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{199664} = \begin{bmatrix} 1 & 47 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{199664} = \mathbf{Pl}(0, 0, 1, 1, 10, 1)_{2903298} \\
\ell_{12} = c_{12} &= \begin{bmatrix} 1 & 0 & \epsilon^{14} & \epsilon^{49} \\ 0 & 1 & \epsilon^{56} & \epsilon^7 \end{bmatrix}_{8232738} = \begin{bmatrix} 1 & 0 & 58 & 30 \\ 0 & 1 & 40 & 35 \end{bmatrix}_{8232738} = \mathbf{Pl}(58, 30, 61, 18, 40, 1)_{11011581} \\
\ell_{13} = c_{13} &= \begin{bmatrix} 1 & 0 & \epsilon^7 & \epsilon^{14} \\ 0 & 1 & \epsilon^{28} & \epsilon^{56} \end{bmatrix}_{15593888} = \begin{bmatrix} 1 & 0 & 35 & 58 \\ 0 & 1 & 61 & 40 \end{bmatrix}_{15593888} = \mathbf{Pl}(30, 58, 40, 35, 30, 1)_{8308664} \\
\ell_{14} = c_{14} &= \begin{bmatrix} 1 & \epsilon^{36} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{153949} = \begin{bmatrix} 1 & 36 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{153949} = \mathbf{Pl}(0, 0, 56, 57, 15, 1)_{4220683} \\
\ell_{15} = c_{15} &= \begin{bmatrix} 1 & 0 & \epsilon^{49} & \epsilon^{56} \\ 0 & 1 & \epsilon^7 & \epsilon^{35} \end{bmatrix}_{10778177} = \begin{bmatrix} 1 & 0 & 30 & 40 \\ 0 & 1 & 35 & 18 \end{bmatrix}_{10778177} = \mathbf{Pl}(35, 40, 40, 35, 30, 1)_{8308669} \\
\ell_{16} = c_{16} &= \begin{bmatrix} 1 & \epsilon^9 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{199719} = \begin{bmatrix} 1 & 47 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{199719} = \mathbf{Pl}(0, 0, 57, 56, 52, 1)_{13917770} \\
\ell_{17} = c_{23} &= \begin{bmatrix} 1 & 0 & \epsilon^{28} & \epsilon^{56} \\ 0 & 1 & \epsilon^{49} & \epsilon^{35} \end{bmatrix}_{10907163} = \begin{bmatrix} 1 & 0 & 61 & 40 \\ 0 & 1 & 30 & 18 \end{bmatrix}_{10907163} = \mathbf{Pl}(35, 40, 18, 61, 35, 1)_{9533137} \\
\ell_{18} = c_{24} &= \begin{bmatrix} 1 & \epsilon^{18} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{49923} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{49923} = \mathbf{Pl}(0, 0, 57, 56, 45, 1)_{12083210} \\
\ell_{19} = c_{25} &= \begin{bmatrix} 1 & 0 & \epsilon^7 & \epsilon^{35} \\ 0 & 1 & \epsilon^{28} & \epsilon^{14} \end{bmatrix}_{4942880} = \begin{bmatrix} 1 & 0 & 35 & 18 \\ 0 & 1 & 61 & 58 \end{bmatrix}_{4942880} = \mathbf{Pl}(61, 18, 18, 61, 35, 1)_{9533163} \\
\ell_{20} = c_{26} &= \begin{bmatrix} 1 & \epsilon^{36} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{153893} = \begin{bmatrix} 1 & 36 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{153893} = \mathbf{Pl}(0, 0, 1, 1, 46, 1)_{12338178} \\
\ell_{21} = c_{34} &= \begin{bmatrix} 1 & 0 & \epsilon^{56} & \epsilon^{49} \\ 0 & 1 & \epsilon^{35} & \epsilon^7 \end{bmatrix}_{8157818} = \begin{bmatrix} 1 & 0 & 40 & 30 \\ 0 & 1 & 18 & 35 \end{bmatrix}_{8157818} = \mathbf{Pl}(58, 30, 35, 40, 58, 1)_{15627465} \\
\ell_{22} = c_{35} &= \begin{bmatrix} 1 & \epsilon^{36} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{153948} = \begin{bmatrix} 1 & 36 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{153948} = \mathbf{Pl}(0, 0, 57, 56, 33, 1)_{8938250}
\end{aligned}$$

$$\begin{aligned}
\ell_{23} = c_{36} &= \begin{bmatrix} 1 & 0 & \epsilon^{14} & \epsilon^{28} \\ 0 & 1 & \epsilon^{56} & \epsilon^{49} \end{bmatrix}_{16487842} = \begin{bmatrix} 1 & 0 & 58 & 61 \\ 0 & 1 & 40 & 30 \end{bmatrix}_{16487842} = \mathbf{Pl}(18, 61, 30, 58, 18, 1)_{5124695} \\
\ell_{24} = c_{45} &= \begin{bmatrix} 1 & 0 & \epsilon^{14} & \epsilon^7 \\ 0 & 1 & \epsilon^{56} & \epsilon^{28} \end{bmatrix}_{9565922} = \begin{bmatrix} 1 & 0 & 58 & 35 \\ 0 & 1 & 40 & 61 \end{bmatrix}_{9565922} = \mathbf{Pl}(40, 35, 35, 40, 58, 1)_{15627447} \\
\ell_{25} = c_{46} &= \begin{bmatrix} 1 & 0 & \epsilon^7 & \epsilon^{56} \\ 0 & 1 & \epsilon^{28} & \epsilon^{35} \end{bmatrix}_{10799008} = \begin{bmatrix} 1 & 0 & 35 & 40 \\ 0 & 1 & 61 & 18 \end{bmatrix}_{10799008} = \mathbf{Pl}(35, 40, 58, 30, 61, 1)_{16503457} \\
\ell_{26} = c_{56} &= \begin{bmatrix} 1 & 0 & \epsilon^{35} & \epsilon^{49} \\ 0 & 1 & \epsilon^{14} & \epsilon^7 \end{bmatrix}_{8066316} = \begin{bmatrix} 1 & 0 & 18 & 30 \\ 0 & 1 & 58 & 35 \end{bmatrix}_{8066316} = \mathbf{Pl}(58, 30, 30, 58, 18, 1)_{5124735}
\end{aligned}$$

Rank of lines: (199720, 49868, 15702043, 9491002, 4922049, 16321420, 15573057, 5051035, 16412922, 49924, 9399500, 199664, 8232738, 15593888, 153949, 10778177, 199719, 10907163, 49923, 4942880, 153893, 8157818, 153948, 16487842, 9565922, 10799008, 8066316)

Rank of points on Klein quadric: (16538443, 9979458, 16503452, 5124717, 16503483, 15627425, 9533132, 8308695, 11011541, 2386123, 11011563, 2903298, 11011581, 8308664, 4220683, 8308669, 13917770, 9533137, 12083210, 9533163, 12338178, 15627465, 8938250, 5124695, 15627447, 16503457, 5124735)

Eckardt Points

The surface has 45 Eckardt points:

- 0 : $E_{16} = a_1 \cap b_6 \cap c_{16} = P_{14} = \mathbf{P}(\epsilon^{54}, 1, 0, 0) = \mathbf{P}(10, 1, 0, 0)$,
- 1 : $E_{24} = a_2 \cap b_4 \cap c_{24} = P_{41} = \mathbf{P}(\epsilon^{45}, 1, 0, 0) = \mathbf{P}(37, 1, 0, 0)$,
- 2 : $E_{14,26,35} = c_{14} \cap c_{26} \cap c_{35} = P_{50} = \mathbf{P}(\epsilon^{27}, 1, 0, 0) = \mathbf{P}(46, 1, 0, 0)$,
- 3 : $E_{32} = a_3 \cap b_2 \cap c_{23} = P_{1277} = \mathbf{P}(\epsilon^{14}, \epsilon^{35}, 1, 0) = \mathbf{P}(58, 18, 1, 0)$,
- 4 : $E_{12,36,45} = c_{12} \cap c_{36} \cap c_{45} = P_{2022} = \mathbf{P}(\epsilon^7, \epsilon^{49}, 1, 0) = \mathbf{P}(35, 30, 1, 0)$,
- 5 : $E_{43} = a_4 \cap b_3 \cap c_{34} = P_{2368} = \mathbf{P}(\epsilon^{28}, \epsilon^7, 1, 0) = \mathbf{P}(61, 35, 1, 0)$,
- 6 : $E_{13,25,46} = c_{13} \cap c_{25} \cap c_{46} = P_{2645} = \mathbf{P}(\epsilon^{35}, \epsilon^{56}, 1, 0) = \mathbf{P}(18, 40, 1, 0)$,
- 7 : $E_{51} = a_5 \cap b_1 \cap c_{15} = P_{3819} = \mathbf{P}(\epsilon^{56}, \epsilon^{14}, 1, 0) = \mathbf{P}(40, 58, 1, 0)$,
- 8 : $E_{65} = a_6 \cap b_5 \cap c_{56} = P_{4001} = \mathbf{P}(\epsilon^{49}, \epsilon^{28}, 1, 0) = \mathbf{P}(30, 61, 1, 0)$,
- 9 : $E_{63} = a_6 \cap b_3 \cap c_{36} = P_{5372} = \mathbf{P}(\epsilon^{14}, \epsilon^{35}, 0, 1) = \mathbf{P}(58, 18, 0, 1)$,
- 10 : $E_{31} = a_3 \cap b_1 \cap c_{13} = P_{6117} = \mathbf{P}(\epsilon^7, \epsilon^{49}, 0, 1) = \mathbf{P}(35, 30, 0, 1)$,
- 11 : $E_{15,23,46} = c_{15} \cap c_{23} \cap c_{46} = P_{6463} = \mathbf{P}(\epsilon^{28}, \epsilon^7, 0, 1) = \mathbf{P}(61, 35, 0, 1)$,
- 12 : $E_{45} = a_4 \cap b_5 \cap c_{45} = P_{6740} = \mathbf{P}(\epsilon^{35}, \epsilon^{56}, 0, 1) = \mathbf{P}(18, 40, 0, 1)$,
- 13 : $E_{12,34,56} = c_{12} \cap c_{34} \cap c_{56} = P_{7914} = \mathbf{P}(\epsilon^{56}, \epsilon^{14}, 0, 1) = \mathbf{P}(40, 58, 0, 1)$,
- 14 : $E_{52} = a_5 \cap b_2 \cap c_{25} = P_{8096} = \mathbf{P}(\epsilon^{49}, \epsilon^{28}, 0, 1) = \mathbf{P}(30, 61, 0, 1)$,
- 15 : $E_{26} = a_2 \cap b_6 \cap c_{26} = P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$,
- 16 : $E_{56} = a_5 \cap b_6 \cap c_{56} = P_{8759} = \mathbf{P}(\epsilon^{30}, \epsilon^{39}, 1, 1) = \mathbf{P}(54, 7, 1, 1)$,
- 17 : $E_{46} = a_4 \cap b_6 \cap c_{46} = P_{9008} = \mathbf{P}(\epsilon^9, \epsilon^{18}, 1, 1) = \mathbf{P}(47, 11, 1, 1)$,
- 18 : $E_{36} = a_3 \cap b_6 \cap c_{36} = P_{9050} = \mathbf{P}(\epsilon^{51}, \epsilon^{60}, 1, 1) = \mathbf{P}(25, 12, 1, 1)$,
- 19 : $E_{25} = a_2 \cap b_5 \cap c_{25} = P_{9613} = \mathbf{P}(\epsilon^{60}, \epsilon^{15}, 1, 1) = \mathbf{P}(12, 21, 1, 1)$,
- 20 : $E_{15,26,34} = c_{15} \cap c_{26} \cap c_{34} = P_{9878} = \mathbf{P}(\epsilon^{15}, \epsilon^{51}, 1, 1) = \mathbf{P}(21, 25, 1, 1)$,
- 21 : $E_{21} = a_2 \cap b_1 \cap c_{12} = P_{10572} = \mathbf{P}(\epsilon^{18}, \epsilon^{36}, 1, 1) = \mathbf{P}(11, 36, 1, 1)$,
- 22 : $E_{62} = a_6 \cap b_2 \cap c_{26} = P_{11301} = \mathbf{P}(\epsilon^{36}, \epsilon^9, 1, 1) = \mathbf{P}(36, 47, 1, 1)$,
- 23 : $E_{23} = a_2 \cap b_3 \cap c_{23} = P_{11400} = \mathbf{P}(\epsilon^{39}, \epsilon^{57}, 1, 1) = \mathbf{P}(7, 49, 1, 1)$,
- 24 : $E_{13,26,45} = c_{13} \cap c_{26} \cap c_{45} = P_{11762} = \mathbf{P}(\epsilon^{57}, \epsilon^{30}, 1, 1) = \mathbf{P}(49, 54, 1, 1)$,
- 25 : $E_{14} = a_1 \cap b_4 \cap c_{14} = P_{233537} = \mathbf{P}(0, 0, \epsilon^{42}, 1) = \mathbf{P}(0, 0, 56, 1)$,
- 26 : $E_{12} = a_1 \cap b_2 \cap c_{12} = P_{234039} = \mathbf{P}(\epsilon^{30}, \epsilon^{39}, \epsilon^{42}, 1) = \mathbf{P}(54, 7, 56, 1)$,
- 27 : $E_{15} = a_1 \cap b_5 \cap c_{15} = P_{234288} = \mathbf{P}(\epsilon^9, \epsilon^{18}, \epsilon^{42}, 1) = \mathbf{P}(47, 11, 56, 1)$,
- 28 : $E_{13} = a_1 \cap b_3 \cap c_{13} = P_{234330} = \mathbf{P}(\epsilon^{51}, \epsilon^{60}, \epsilon^{42}, 1) = \mathbf{P}(25, 12, 56, 1)$,
- 29 : $E_{54} = a_5 \cap b_4 \cap c_{45} = P_{234893} = \mathbf{P}(\epsilon^{60}, \epsilon^{15}, \epsilon^{42}, 1) = \mathbf{P}(12, 21, 56, 1)$,
- 30 : $E_{14,23,56} = c_{14} \cap c_{23} \cap c_{56} = P_{235158} = \mathbf{P}(\epsilon^{15}, \epsilon^{51}, \epsilon^{42}, 1) = \mathbf{P}(21, 25, 56, 1)$,
- 31 : $E_{34} = a_3 \cap b_4 \cap c_{34} = P_{235852} = \mathbf{P}(\epsilon^{18}, \epsilon^{36}, \epsilon^{42}, 1) = \mathbf{P}(11, 36, 56, 1)$,
- 32 : $E_{14,25,36} = c_{14} \cap c_{25} \cap c_{36} = P_{236581} = \mathbf{P}(\epsilon^{36}, \epsilon^9, \epsilon^{42}, 1) = \mathbf{P}(36, 47, 56, 1)$,

33 : $E_{64} = a_6 \cap b_4 \cap c_{46} = P_{236680} = \mathbf{P}(\epsilon^{39}, \epsilon^{57}, \epsilon^{42}, 1) = \mathbf{P}(7, 49, 56, 1)$,
 34 : $E_{41} = a_4 \cap b_1 \cap c_{14} = P_{237042} = \mathbf{P}(\epsilon^{57}, \epsilon^{30}, \epsilon^{42}, 1) = \mathbf{P}(49, 54, 56, 1)$,
 35 : $E_{16,24,35} = c_{16} \cap c_{24} \cap c_{35} = P_{237633} = \mathbf{P}(0, 0, \epsilon^{21}, 1) = \mathbf{P}(0, 0, 57, 1)$,
 36 : $E_{16,25,34} = c_{16} \cap c_{25} \cap c_{34} = P_{238135} = \mathbf{P}(\epsilon^{30}, \epsilon^{39}, \epsilon^{21}, 1) = \mathbf{P}(54, 7, 57, 1)$,
 37 : $E_{16,23,45} = c_{16} \cap c_{23} \cap c_{45} = P_{238384} = \mathbf{P}(\epsilon^9, \epsilon^{18}, \epsilon^{21}, 1) = \mathbf{P}(47, 11, 57, 1)$,
 38 : $E_{61} = a_6 \cap b_1 \cap c_{16} = P_{238426} = \mathbf{P}(\epsilon^{51}, \epsilon^{60}, \epsilon^{21}, 1) = \mathbf{P}(25, 12, 57, 1)$,
 39 : $E_{42} = a_4 \cap b_2 \cap c_{24} = P_{238989} = \mathbf{P}(\epsilon^{60}, \epsilon^{15}, \epsilon^{21}, 1) = \mathbf{P}(12, 21, 57, 1)$,
 40 : $E_{12,35,46} = c_{12} \cap c_{35} \cap c_{46} = P_{239254} = \mathbf{P}(\epsilon^{15}, \epsilon^{51}, \epsilon^{21}, 1) = \mathbf{P}(21, 25, 57, 1)$,
 41 : $E_{13,24,56} = c_{13} \cap c_{24} \cap c_{56} = P_{239948} = \mathbf{P}(\epsilon^{18}, \epsilon^{36}, \epsilon^{21}, 1) = \mathbf{P}(11, 36, 57, 1)$,
 42 : $E_{53} = a_5 \cap b_3 \cap c_{35} = P_{240677} = \mathbf{P}(\epsilon^{36}, \epsilon^9, \epsilon^{21}, 1) = \mathbf{P}(36, 47, 57, 1)$,
 43 : $E_{15,24,36} = c_{15} \cap c_{24} \cap c_{36} = P_{240776} = \mathbf{P}(\epsilon^{39}, \epsilon^{57}, \epsilon^{21}, 1) = \mathbf{P}(7, 49, 57, 1)$,
 44 : $E_{35} = a_3 \cap b_5 \cap c_{35} = P_{241138} = \mathbf{P}(\epsilon^{57}, \epsilon^{30}, \epsilon^{21}, 1) = \mathbf{P}(49, 54, 57, 1)$.

Double Points

The surface has 0 Double points:
 The double points on the surface are:

Single Points

The surface has 1620 single points:
 Too many to print.

Points on surface but on no line

The surface has 2880 points not on any line:
 Too many to print.

Line Intersection Graph

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
		a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{23}	c_{24}	c_{25}	c_{26}	c_{34}	c_{35}	c_{36}	c_{45}	c_{46}	c_{56}
0	a_1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	a_2	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
2	a_3	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
3	a_4	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
4	a_5	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
5	a_6	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6	b_1	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	b_2	1	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	b_3	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9	b_4	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10	b_5	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11	b_6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
12	c_{12}	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
13	c_{13}	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
14	c_{14}	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
15	c_{15}	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
16	c_{16}	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
17	c_{23}	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
18	c_{24}	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
19	c_{25}	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
20	c_{26}	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
21	c_{34}	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
22	c_{35}	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
23	c_{36}	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
24	c_{45}	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
25	c_{46}	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
26	c_{56}	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{234039}	P_{234330}	P_{233537}	P_{234288}	P_{14}	P_{234039}	P_{234330}	P_{233537}	P_{234288}	P_{14}

Line 1 intersects

Line	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{10572}	P_{11400}	P_{41}	P_{9613}	P_{8258}	P_{10572}	P_{11400}	P_{41}	P_{9613}	P_{8258}

Line 2 intersects

Line	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{6117}	P_{1277}	P_{235852}	P_{241138}	P_{9050}	P_{6117}	P_{1277}	P_{235852}	P_{241138}	P_{9050}

Line 3 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{237042}	P_{238989}	P_{2368}	P_{6740}	P_{9008}	P_{237042}	P_{238989}	P_{2368}	P_{6740}	P_{9008}

Line 4 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{3819}	P_{8096}	P_{240677}	P_{234893}	P_{8759}	P_{3819}	P_{8096}	P_{240677}	P_{234893}	P_{8759}

Line 5 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_{238426}	P_{11301}	P_{5372}	P_{236680}	P_{4001}	P_{238426}	P_{11301}	P_{5372}	P_{236680}	P_{4001}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{10572}	P_{6117}	P_{237042}	P_{3819}	P_{238426}	P_{10572}	P_{6117}	P_{237042}	P_{3819}	P_{238426}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{234039}	P_{1277}	P_{238989}	P_{8096}	P_{11301}	P_{234039}	P_{1277}	P_{238989}	P_{8096}	P_{11301}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{234330}	P_{11400}	P_{2368}	P_{240677}	P_{5372}	P_{234330}	P_{11400}	P_{2368}	P_{240677}	P_{5372}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{233537}	P_{41}	P_{235852}	P_{234893}	P_{236680}	P_{233537}	P_{41}	P_{235852}	P_{234893}	P_{236680}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{234288}	P_{9613}	P_{241138}	P_{6740}	P_{4001}	P_{234288}	P_{9613}	P_{241138}	P_{6740}	P_{4001}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_{14}	P_{8258}	P_{9050}	P_{9008}	P_{8759}	P_{14}	P_{8258}	P_{9050}	P_{9008}	P_{8759}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{234039}	P_{10572}	P_{10572}	P_{234039}	P_{7914}	P_{239254}	P_{2022}	P_{2022}	P_{239254}	P_{7914}

Line 13 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{234330}	P_{6117}	P_{6117}	P_{234330}	P_{239948}	P_{2645}	P_{11762}	P_{11762}	P_{2645}	P_{239948}

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_9	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{233537}	P_{237042}	P_{237042}	P_{233537}	P_{235158}	P_{236581}	P_{50}	P_{50}	P_{236581}	P_{235158}

Line 15 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{234288}	P_{3819}	P_{3819}	P_{234288}	P_{6463}	P_{240776}	P_{9878}	P_{9878}	P_{240776}	P_{6463}

Line 16 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_{14}	P_{238426}	P_{238426}	P_{14}	P_{238384}	P_{237633}	P_{238135}	P_{238135}	P_{237633}	P_{238384}

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11400}	P_{1277}	P_{1277}	P_{11400}	P_{235158}	P_{6463}	P_{238384}	P_{238384}	P_{6463}	P_{235158}

Line 18 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{41}	P_{238989}	P_{238989}	P_{41}	P_{239948}	P_{240776}	P_{237633}	P_{237633}	P_{240776}	P_{239948}

Line 19 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{9613}	P_{8096}	P_{8096}	P_{9613}	P_{2645}	P_{236581}	P_{238135}	P_{238135}	P_{236581}	P_{2645}

Line 20 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_{8258}	P_{11301}	P_{11301}	P_{8258}	P_{11762}	P_{50}	P_{9878}	P_{9878}	P_{50}	P_{11762}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{26}
in point	P_{235852}	P_{2368}	P_{2368}	P_{235852}	P_{7914}	P_{9878}	P_{238135}	P_{238135}	P_{9878}	P_{7914}

Line 22 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{20}	ℓ_{25}
in point	P_{241138}	P_{240677}	P_{240677}	P_{241138}	P_{239254}	P_{50}	P_{237633}	P_{237633}	P_{50}	P_{239254}

Line 23 intersects

Line	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{19}	ℓ_{24}
in point	P_{9050}	P_{5372}	P_{5372}	P_{9050}	P_{2022}	P_{236581}	P_{240776}	P_{240776}	P_{236581}	P_{2022}

Line 24 intersects

Line	ℓ_3	ℓ_4	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{20}	ℓ_{23}
in point	P_{6740}	P_{234893}	P_{234893}	P_{6740}	P_{2022}	P_{11762}	P_{238384}	P_{238384}	P_{11762}	P_{2022}

Line 25 intersects

Line	ℓ_3	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{22}
in point	P_{9008}	P_{236680}	P_{236680}	P_{9008}	P_{239254}	P_{2645}	P_{6463}	P_{6463}	P_{2645}	P_{239254}

Line 26 intersects

Line	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{21}
in point	P_{8759}	P_{4001}	P_{4001}	P_{8759}	P_{7914}	P_{239948}	P_{235158}	P_{235158}	P_{239948}	P_{7914}

The surface has 4545 points:

Too many to print.