Rank-65618 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227396246

General information

Number of lines	4
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	28
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	$2^4, 1^{28}, 0^{49}$

Singular Points

The surface has 1 singular points:

$0 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \mathbf{Pl}(1,1,0,0,1,1)_{1217}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \mathbf{Pl}(1,1,1,1,0,1)_{874}$$

Rank of lines: (74, 138, 81, 658)
Rank of points on Klein quadric: (665, 1322, 1217, 874)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$ $P_4 = (1,1,1,1) = \ell_1 \cap \ell_2$
$P_{12} = (1,0,1,0) = \ell_0 \cap \ell_2$
$P_{19} = (0,1,1,0) = \ell_0 \cap \ell_3$

Single Points

The surface has 28 single points:
The single points on the surface are:

$0: P_{30} = (3,2,1,0)$ lies on line ℓ_0 $15: P_{200} = (7,7,1,1)$ lies on line ℓ_1
$1: P_{37} = (2,3,1,0)$ lies on line ℓ_0 $16: P_{211} = (2,1,2,1)$ lies on line ℓ_2
$2: P_{48} = (5,4,1,0)$ lies on line ℓ_0 $17: P_{226} = (1,3,2,1)$ lies on line ℓ_3
$3: P_{55} = (4,5,1,0)$ lies on line ℓ_0 $18: P_{276} = (3,1,3,1)$ lies on line ℓ_2
$4: P_{66} = (7,6,1,0)$ lies on line ℓ_0 $19: P_{282} = (1,2,3,1)$ lies on line ℓ_3
$5: P_{73} = (6,7,1,0)$ lies on line ℓ_0 $20: P_{341} = (4,1,4,1)$ lies on line ℓ_2
$6: P_{82} = (0,1,0,1)$ lies on line ℓ_2 $21: P_{370} = (1,5,4,1)$ lies on line ℓ_3
$7: P_{83} = (1,1,0,1)$ lies on line ℓ_3 $22: P_{406} = (5,1,5,1)$ lies on line ℓ_2
$8: P_{138} = (0,0,1,1)$ lies on line ℓ_1 $23: P_{426} = (1,4,5,1)$ lies on line ℓ_3
$9: P_{139} = (1,0,1,1)$ lies on line ℓ_3 $24: P_{471} = (6,1,6,1)$ lies on line ℓ_2
$10: P_{155} = (2,2,1,1)$ lies on line ℓ_1 $25: P_{514} = (1,7,6,1)$ lies on line ℓ_3
$11: P_{164} = (3,3,1,1)$ lies on line ℓ_1 $26: P_{536} = (7,1,7,1)$ lies on line ℓ_2
$12: P_{173} = (4,4,1,1)$ lies on line ℓ_1 $27: P_{570} = (1,6,7,1)$ lies on line ℓ_3
$13: P_{182} = (5,5,1,1)$ lies on line ℓ_1
$14: P_{191} = (6,6,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:
The points on the surface but not on lines are:

$0 : P_{20} = (1, 1, 1, 0)$

$1 : P_{76} = (2, 0, 0, 1)$

$2 : P_{78} = (4, 0, 0, 1)$

$3 : P_{81} = (7, 0, 0, 1)$

$4 : P_{96} = (6, 2, 0, 1)$

$5 : P_{109} = (3, 4, 0, 1)$

$6 : P_{135} = (5, 7, 0, 1)$

$7 : P_{158} = (5, 2, 1, 1)$

$8 : P_{159} = (6, 2, 1, 1)$

$9 : P_{172} = (3, 4, 1, 1)$

$10 : P_{175} = (6, 4, 1, 1)$

$11 : P_{196} = (3, 7, 1, 1)$

$12 : P_{198} = (5, 7, 1, 1)$

$13 : P_{207} = (6, 0, 2, 1)$

$14 : P_{214} = (5, 1, 2, 1)$

$15 : P_{215} = (6, 1, 2, 1)$

$16 : P_{222} = (5, 2, 2, 1)$

$17 : P_{230} = (5, 3, 2, 1)$

$18 : P_{237} = (4, 4, 2, 1)$

$19 : P_{241} = (0, 5, 2, 1)$

$20 : P_{251} = (2, 6, 2, 1)$

$21 : P_{259} = (2, 7, 2, 1)$

$22 : P_{286} = (5, 2, 3, 1)$

$23 : P_{301} = (4, 4, 3, 1)$

$24 : P_{321} = (0, 7, 3, 1)$

$25 : P_{332} = (3, 0, 4, 1)$

$26 : P_{340} = (3, 1, 4, 1)$

$27 : P_{343} = (6, 1, 4, 1)$

$28 : P_{349} = (4, 2, 4, 1)$

$29 : P_{357} = (4, 3, 4, 1)$

$30 : P_{367} = (6, 4, 4, 1)$

$31 : P_{375} = (6, 5, 4, 1)$

$32 : P_{377} = (0, 6, 4, 1)$

$33 : P_{392} = (7, 7, 4, 1)$

$34 : P_{409} = (0, 2, 5, 1)$

$35 : P_{431} = (6, 4, 5, 1)$

$36 : P_{456} = (7, 7, 5, 1)$

$37 : P_{475} = (2, 2, 6, 1)$

$38 : P_{489} = (0, 4, 6, 1)$

$39 : P_{516} = (3, 7, 6, 1)$

$40 : P_{526} = (5, 0, 7, 1)$

$41 : P_{532} = (3, 1, 7, 1)$

$42 : P_{534} = (5, 1, 7, 1)$

$43 : P_{539} = (2, 2, 7, 1)$

$44 : P_{545} = (0, 3, 7, 1)$

$45 : P_{560} = (7, 4, 7, 1)$

$46 : P_{568} = (7, 5, 7, 1)$

$47 : P_{572} = (3, 6, 7, 1)$

$48 : P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

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 |0 1 2 3
─┼───────
0|0 1 1 1
1|1 0 1 0
2|1 1 0 0
3|1 0 0 0
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Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_{12}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_5	P_4

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{12}	P_4

Line 3 intersects

Line	ℓ_0
in point	P_{19}

The surface has 81 points:

The points on the surface are:

$0 : P_4 = (1,1,1,1)$

$1 : P_5 = (1,1,0,0)$

$2 : P_{12} = (1,0,1,0)$

$3 : P_{19} = (0,1,1,0)$

$4 : P_{20} = (1,1,1,0)$

$5 : P_{30} = (3,2,1,0)$

$6 : P_{37} = (2,3,1,0)$

$7 : P_{48} = (5,4,1,0)$

$8 : P_{55} = (4,5,1,0)$

$9 : P_{66} = (7,6,1,0)$

$10 : P_{73} = (6,7,1,0)$

$11 : P_{76} = (2,0,0,1)$

$12 : P_{78} = (4,0,0,1)$

$13 : P_{81} = (7,0,0,1)$

$14 : P_{82} = (0,1,0,1)$

$15 : P_{83} = (1,1,0,1)$

$16 : P_{96} = (6,2,0,1)$

$17 : P_{109} = (3,4,0,1)$

$18 : P_{135} = (5,7,0,1)$

$19 : P_{138} = (0,0,1,1)$

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$31 : P_{198} = (5,7,1,1)$

$32 : P_{200} = (7,7,1,1)$

$33 : P_{207} = (6,0,2,1)$

$34 : P_{211} = (2,1,2,1)$

$35 : P_{214} = (5,1,2,1)$

$36 : P_{215} = (6,1,2,1)$

$37 : P_{222} = (5,2,2,1)$

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$80 : P_{580} = (3,7,7,1)$