

Rank-10566 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0)

The point rank of the equation over GF(4) is 26633564

General information

Number of lines	27
Number of points	45
Number of singular points	0
Number of Eckardt points	45
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	40
Number of axes	240
Type of points on lines	5^{27}
Type of lines on points	3^{45}

Singular Points

The surface has 0 singular points:

The 27 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 = a_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1, 0, 1, 0, 0, 0)_3 \\ \ell_1 = a_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_9\end{aligned}$$

$$\begin{aligned}
\ell_2 = a_3 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{198} \\
\ell_3 = a_4 &= \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{81} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{81} = \mathbf{Pl}(0, 0, 3, 2, 3, 1)_{332} \\
\ell_4 = a_5 &= \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{61} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{61} = \mathbf{Pl}(0, 0, 2, 3, 2, 1)_{265} \\
\ell_5 = a_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_6 \\
\ell_6 = b_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_7 = b_2 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \mathbf{Pl}(1, 1, 1, 1, 0, 0)_{16} \\
\ell_8 = b_3 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1, 1, 0, 1, 1, 1)_{189} \\
\ell_9 = b_4 &= \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{155} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{155} = \mathbf{Pl}(3, 2, 0, 2, 3, 1)_{314} \\
\ell_{10} = b_5 &= \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{138} = \mathbf{Pl}(2, 3, 0, 3, 2, 1)_{256} \\
\ell_{11} = b_6 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_{12} = c_{12} &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{13} \\
\ell_{13} = c_{13} &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{202} \\
\ell_{14} = c_{14} &= \begin{bmatrix} 1 & \omega^2 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{165} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{165} = \mathbf{Pl}(0, 2, 3, 2, 3, 1)_{337} \\
\ell_{15} = c_{15} &= \begin{bmatrix} 1 & \omega & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{145} = \mathbf{Pl}(0, 3, 2, 3, 2, 1)_{271} \\
\ell_{16} = c_{16} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_{17} = c_{23} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{177} \\
\ell_{18} = c_{24} &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{71} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{71} = \mathbf{Pl}(3, 2, 0, 0, 3, 1)_{299} \\
\ell_{19} = c_{25} &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{54} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{54} = \mathbf{Pl}(2, 3, 0, 0, 2, 1)_{238} \\
\ell_{20} = c_{26} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{10} \\
\ell_{21} = c_{34} &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{55} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{55} = \mathbf{Pl}(2, 3, 2, 0, 2, 1)_{244} \\
\ell_{22} = c_{35} &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{72} = \mathbf{Pl}(3, 2, 3, 0, 3, 1)_{308}
\end{aligned}$$

$$\begin{aligned}
\ell_{23} = c_{36} &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1, 0, 1, 1, 1)_{199} \\
\ell_{24} = c_{45} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1, 1, 1, 0, 1)_{180} \\
\ell_{25} = c_{46} &= \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{156} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{156} = \mathbf{Pl}(3, 0, 3, 2, 3)_{335} \\
\ell_{26} = c_{56} &= \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{139} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{139} = \mathbf{Pl}(2, 0, 2, 3, 2)_{267}
\end{aligned}$$

Rank of lines: (1, 340, 38, 81, 61, 100, 356, 85, 109, 155, 138, 0, 345, 122, 165, 145, 16, 25, 71, 54, 84, 55, 72, 110, 26, 156, 139)

Rank of points on Klein quadric: (3, 9, 198, 332, 265, 6, 1, 16, 189, 314, 256, 0, 13, 202, 337, 271, 2, 177, 299, 238, 10, 244, 308, 199, 180, 335, 267)

Eckardt Points

The surface has 45 Eckardt points:

- 0 : $E_{16} = a_1 \cap b_6 \cap c_{16} = P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0)$,
- 1 : $E_{26} = a_2 \cap b_6 \cap c_{26} = P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$,
- 2 : $E_{61} = a_6 \cap b_1 \cap c_{16} = P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$,
- 3 : $E_{21} = a_2 \cap b_1 \cap c_{12} = P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$,
- 4 : $E_{32} = a_3 \cap b_2 \cap c_{23} = P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$,
- 5 : $E_{36} = a_3 \cap b_6 \cap c_{36} = P_5 = \mathbf{P}(1, 1, 0, 0) = \mathbf{P}(1, 1, 0, 0)$,
- 6 : $E_{46} = a_4 \cap b_6 \cap c_{46} = P_6 = \mathbf{P}(\omega, 1, 0, 0) = \mathbf{P}(2, 1, 0, 0)$,
- 7 : $E_{56} = a_5 \cap b_6 \cap c_{56} = P_7 = \mathbf{P}(\omega^2, 1, 0, 0) = \mathbf{P}(3, 1, 0, 0)$,
- 8 : $E_{16,23,45} = c_{16} \cap c_{23} \cap c_{45} = P_8 = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0)$,
- 9 : $E_{16,24,35} = c_{16} \cap c_{24} \cap c_{35} = P_9 = \mathbf{P}(\omega, 0, 1, 0) = \mathbf{P}(2, 0, 1, 0)$,
- 10 : $E_{16,25,34} = c_{16} \cap c_{25} \cap c_{34} = P_{10} = \mathbf{P}(\omega^2, 0, 1, 0) = \mathbf{P}(3, 0, 1, 0)$,
- 11 : $E_{12} = a_1 \cap b_2 \cap c_{12} = P_{11} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$,
- 12 : $E_{13} = a_1 \cap b_3 \cap c_{13} = P_{12} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$,
- 13 : $E_{14} = a_1 \cap b_4 \cap c_{14} = P_{13} = \mathbf{P}(\omega, 1, 1, 0) = \mathbf{P}(2, 1, 1, 0)$,
- 14 : $E_{15} = a_1 \cap b_5 \cap c_{15} = P_{14} = \mathbf{P}(\omega^2, 1, 1, 0) = \mathbf{P}(3, 1, 1, 0)$,
- 15 : $E_{62} = a_6 \cap b_2 \cap c_{26} = P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$,
- 16 : $E_{23} = a_2 \cap b_3 \cap c_{23} = P_{26} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$,
- 17 : $E_{13,26,45} = c_{13} \cap c_{26} \cap c_{45} = P_{27} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$,
- 18 : $E_{25} = a_2 \cap b_5 \cap c_{25} = P_{30} = \mathbf{P}(0, \omega, 0, 1) = \mathbf{P}(0, 2, 0, 1)$,
- 19 : $E_{15,26,34} = c_{15} \cap c_{26} \cap c_{34} = P_{31} = \mathbf{P}(1, \omega, 0, 1) = \mathbf{P}(1, 2, 0, 1)$,
- 20 : $E_{24} = a_2 \cap b_4 \cap c_{24} = P_{34} = \mathbf{P}(0, \omega^2, 0, 1) = \mathbf{P}(0, 3, 0, 1)$,
- 21 : $E_{14,26,35} = c_{14} \cap c_{26} \cap c_{35} = P_{35} = \mathbf{P}(1, \omega^2, 0, 1) = \mathbf{P}(1, 3, 0, 1)$,
- 22 : $E_{31} = a_3 \cap b_1 \cap c_{13} = P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$,
- 23 : $E_{63} = a_6 \cap b_3 \cap c_{36} = P_{39} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1)$,
- 24 : $E_{12,36,45} = c_{12} \cap c_{36} \cap c_{45} = P_{42} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$,
- 25 : $E_{34} = a_3 \cap b_4 \cap c_{34} = P_{47} = \mathbf{P}(\omega, \omega, 1, 1) = \mathbf{P}(2, 2, 1, 1)$,
- 26 : $E_{14,25,36} = c_{14} \cap c_{25} \cap c_{36} = P_{48} = \mathbf{P}(\omega^2, \omega, 1, 1) = \mathbf{P}(3, 2, 1, 1)$,
- 27 : $E_{15,24,36} = c_{15} \cap c_{24} \cap c_{36} = P_{51} = \mathbf{P}(\omega, \omega^2, 1, 1) = \mathbf{P}(2, 3, 1, 1)$,
- 28 : $E_{35} = a_3 \cap b_5 \cap c_{35} = P_{52} = \mathbf{P}(\omega^2, \omega^2, 1, 1) = \mathbf{P}(3, 3, 1, 1)$,
- 29 : $E_{51} = a_5 \cap b_1 \cap c_{15} = P_{53} = \mathbf{P}(0, 0, \omega, 1) = \mathbf{P}(0, 0, 2, 1)$,
- 30 : $E_{65} = a_6 \cap b_5 \cap c_{56} = P_{54} = \mathbf{P}(1, 0, \omega, 1) = \mathbf{P}(1, 0, 2, 1)$,
- 31 : $E_{14,23,56} = c_{14} \cap c_{23} \cap c_{56} = P_{59} = \mathbf{P}(\omega, 1, \omega, 1) = \mathbf{P}(2, 1, 2, 1)$,
- 32 : $E_{54} = a_5 \cap b_4 \cap c_{45} = P_{60} = \mathbf{P}(\omega^2, 1, \omega, 1) = \mathbf{P}(3, 1, 2, 1)$,
- 33 : $E_{12,34,56} = c_{12} \cap c_{34} \cap c_{56} = P_{61} = \mathbf{P}(0, \omega, \omega, 1) = \mathbf{P}(0, 2, 2, 1)$,
- 34 : $E_{52} = a_5 \cap b_2 \cap c_{25} = P_{62} = \mathbf{P}(1, \omega, \omega, 1) = \mathbf{P}(1, 2, 2, 1)$,

$$\begin{aligned}
35 : E_{53} &= a_5 \cap b_3 \cap c_{35} = P_{67} = \mathbf{P}(\omega, \omega^2, \omega, 1) = \mathbf{P}(2, 3, 2, 1), \\
36 : E_{13,24,56} &= c_{13} \cap c_{24} \cap c_{56} = P_{68} = \mathbf{P}(\omega^2, \omega^2, \omega, 1) = \mathbf{P}(3, 3, 2, 1), \\
37 : E_{41} &= a_4 \cap b_1 \cap c_{14} = P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1), \\
38 : E_{64} &= a_6 \cap b_4 \cap c_{46} = P_{70} = \mathbf{P}(1, 0, \omega^2, 1) = \mathbf{P}(1, 0, 3, 1), \\
39 : E_{45} &= a_4 \cap b_5 \cap c_{45} = P_{75} = \mathbf{P}(\omega, 1, \omega^2, 1) = \mathbf{P}(2, 1, 3, 1), \\
40 : E_{15,23,46} &= c_{15} \cap c_{23} \cap c_{46} = P_{76} = \mathbf{P}(\omega^2, 1, \omega^2, 1) = \mathbf{P}(3, 1, 3, 1), \\
41 : E_{13,25,46} &= c_{13} \cap c_{25} \cap c_{46} = P_{79} = \mathbf{P}(\omega, \omega, \omega^2, 1) = \mathbf{P}(2, 2, 3, 1), \\
42 : E_{43} &= a_4 \cap b_3 \cap c_{34} = P_{80} = \mathbf{P}(\omega^2, \omega, \omega^2, 1) = \mathbf{P}(3, 2, 3, 1), \\
43 : E_{12,35,46} &= c_{12} \cap c_{35} \cap c_{46} = P_{81} = \mathbf{P}(0, \omega^2, \omega^2, 1) = \mathbf{P}(0, 3, 3, 1), \\
44 : E_{42} &= a_4 \cap b_2 \cap c_{24} = P_{82} = \mathbf{P}(1, \omega^2, \omega^2, 1) = \mathbf{P}(1, 3, 3, 1).
\end{aligned}$$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
		a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{23}	c_{24}	c_{25}	c_{26}	c_{34}	c_{35}	c_{36}	c_{45}	c_{46}	c_{56}
0	a_1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	a_2	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
2	a_3	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
3	a_4	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
4	a_5	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
5	a_6	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6	b_1	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	b_2	1	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	b_3	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9	b_4	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10	b_5	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11	b_6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
12	c_{12}	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
13	c_{13}	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
14	c_{14}	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
15	c_{15}	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
16	c_{16}	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
17	c_{23}	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
18	c_{24}	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
19	c_{25}	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
20	c_{26}	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
21	c_{34}	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
22	c_{35}	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
23	c_{36}	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
24	c_{45}	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
25	c_{46}	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
26	c_{56}	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{11}	P_{12}	P_{13}	P_{14}	P_0	P_{11}	P_{12}	P_{13}	P_{14}	P_0

Line 1 intersects

Line	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_3	P_{26}	P_{34}	P_{30}	P_1	P_3	P_{26}	P_{34}	P_{30}	P_1

Line 2 intersects

Line	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{38}	P_4	P_{47}	P_{52}	P_5	P_{38}	P_4	P_{47}	P_{52}	P_5

Line 3 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{69}	P_{82}	P_{80}	P_{75}	P_6	P_{69}	P_{82}	P_{80}	P_{75}	P_6

Line 4 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{53}	P_{62}	P_{67}	P_{60}	P_7	P_{53}	P_{62}	P_{67}	P_{60}	P_7

Line 5 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_2	P_{23}	P_{39}	P_{70}	P_{54}	P_2	P_{23}	P_{39}	P_{70}	P_{54}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_3	P_{38}	P_{69}	P_{53}	P_2	P_3	P_{38}	P_{69}	P_{53}	P_2

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{11}	P_4	P_{82}	P_{62}	P_{23}	P_{11}	P_4	P_{82}	P_{62}	P_{23}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{12}	P_{26}	P_{80}	P_{67}	P_{39}	P_{12}	P_{26}	P_{80}	P_{67}	P_{39}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{13}	P_{34}	P_{47}	P_{60}	P_{70}	P_{13}	P_{34}	P_{47}	P_{60}	P_{70}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{14}	P_{30}	P_{52}	P_{75}	P_{54}	P_{14}	P_{30}	P_{52}	P_{75}	P_{54}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_0	P_1	P_5	P_6	P_7	P_0	P_1	P_5	P_6	P_7

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11}	P_3	P_3	P_{11}	P_{61}	P_{81}	P_{42}	P_{42}	P_{81}	P_{61}

Line 13 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{12}	P_{38}	P_{38}	P_{12}	P_{68}	P_{79}	P_{27}	P_{27}	P_{79}	P_{68}

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_9	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{13}	P_{69}	P_{69}	P_{13}	P_{59}	P_{48}	P_{35}	P_{35}	P_{48}	P_{59}

Line 15 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{14}	P_{53}	P_{53}	P_{14}	P_{76}	P_{51}	P_{31}	P_{31}	P_{51}	P_{76}

Line 16 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_0	P_2	P_2	P_0	P_8	P_9	P_{10}	P_{10}	P_9	P_8

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{26}	P_4	P_4	P_{26}	P_{59}	P_{76}	P_8	P_8	P_{76}	P_{59}

Line 18 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{34}	P_{82}	P_{82}	P_{34}	P_{68}	P_{51}	P_9	P_9	P_{51}	P_{68}

Line 19 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{30}	P_{62}	P_{62}	P_{30}	P_{79}	P_{48}	P_{10}	P_{10}	P_{48}	P_{79}

Line 20 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_1	P_{23}	P_{23}	P_1	P_{27}	P_{35}	P_{31}	P_{31}	P_{35}	P_{27}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{26}
in point	P_{47}	P_{80}	P_{80}	P_{47}	P_{61}	P_{31}	P_{10}	P_{10}	P_{31}	P_{61}

Line 22 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{20}	ℓ_{25}
in point	P_{52}	P_{67}	P_{67}	P_{52}	P_{81}	P_{35}	P_9	P_9	P_{35}	P_{81}

Line 23 intersects

Line	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{19}	ℓ_{24}
in point	P_5	P_{39}	P_{39}	P_5	P_{42}	P_{48}	P_{51}	P_{51}	P_{48}	P_{42}

Line 24 intersects

Line	ℓ_3	ℓ_4	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{20}	ℓ_{23}
in point	P_{75}	P_{60}	P_{60}	P_{75}	P_{42}	P_{27}	P_8	P_8	P_{27}	P_{42}

Line 25 intersects

Line	ℓ_3	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{22}
in point	P_6	P_{70}	P_{70}	P_6	P_{81}	P_{79}	P_{76}	P_{76}	P_{79}	P_{81}

Line 26 intersects

Line	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{21}
in point	P_7	P_{54}	P_{54}	P_7	P_{61}	P_{68}	P_{59}	P_{59}	P_{68}	P_{61}

The surface has 45 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	9 : $P_9 = (2, 0, 1, 0)$	18 : $P_{30} = (0, 2, 0, 1)$
1 : $P_1 = (0, 1, 0, 0)$	10 : $P_{10} = (3, 0, 1, 0)$	19 : $P_{31} = (1, 2, 0, 1)$
2 : $P_2 = (0, 0, 1, 0)$	11 : $P_{11} = (0, 1, 1, 0)$	20 : $P_{34} = (0, 3, 0, 1)$
3 : $P_3 = (0, 0, 0, 1)$	12 : $P_{12} = (1, 1, 1, 0)$	21 : $P_{35} = (1, 3, 0, 1)$
4 : $P_4 = (1, 1, 1, 1)$	13 : $P_{13} = (2, 1, 1, 0)$	22 : $P_{38} = (0, 0, 1, 1)$
5 : $P_5 = (1, 1, 0, 0)$	14 : $P_{14} = (3, 1, 1, 0)$	23 : $P_{39} = (1, 0, 1, 1)$
6 : $P_6 = (2, 1, 0, 0)$	15 : $P_{23} = (1, 0, 0, 1)$	24 : $P_{42} = (0, 1, 1, 1)$
7 : $P_7 = (3, 1, 0, 0)$	16 : $P_{26} = (0, 1, 0, 1)$	25 : $P_{47} = (2, 2, 1, 1)$
8 : $P_8 = (1, 0, 1, 0)$	17 : $P_{27} = (1, 1, 0, 1)$	26 : $P_{48} = (3, 2, 1, 1)$

27 : $P_{51} = (2, 3, 1, 1)$
 28 : $P_{52} = (3, 3, 1, 1)$
 29 : $P_{53} = (0, 0, 2, 1)$
 30 : $P_{54} = (1, 0, 2, 1)$
 31 : $P_{59} = (2, 1, 2, 1)$
 32 : $P_{60} = (3, 1, 2, 1)$
 33 : $P_{61} = (0, 2, 2, 1)$

34 : $P_{62} = (1, 2, 2, 1)$
 35 : $P_{67} = (2, 3, 2, 1)$
 36 : $P_{68} = (3, 3, 2, 1)$
 37 : $P_{69} = (0, 0, 3, 1)$
 38 : $P_{70} = (1, 0, 3, 1)$
 39 : $P_{75} = (2, 1, 3, 1)$
 40 : $P_{76} = (3, 1, 3, 1)$

41 : $P_{79} = (2, 2, 3, 1)$
 42 : $P_{80} = (3, 2, 3, 1)$
 43 : $P_{81} = (0, 3, 3, 1)$
 44 : $P_{82} = (1, 3, 3, 1)$