# Rank-65869 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

( 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(4) is 1431725469

## General information

| | |
|---|---|
| Number of lines | 2 |
| Number of points | 21 |
| Number of singular points | 2 |
| Number of Eckardt points | 0 |
| Number of double points | 1 |
| Number of single points | 8 |
| Number of points off lines | 12 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^2$ |
| Type of lines on points | 2, $1^8$, $0^{12}$ |

### Singular Points

The surface has 2 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
1 : $P_{23} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

Rank of lines: ( 16, 100 )
Rank of points on Klein quadric: ( 2, 6 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

### Single Points

The surface has 8 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line $\ell_0$
1 : $P_8 = (1,0,1,0)$ lies on line $\ell_0$
2 : $P_9 = (2,0,1,0)$ lies on line $\ell_0$
3 : $P_{10} = (3,0,1,0)$ lies on line $\ell_0$
4 : $P_{23} = (1,0,0,1)$ lies on line $\ell_1$
5 : $P_{39} = (1,0,1,1)$ lies on line $\ell_1$
6 : $P_{54} = (1,0,2,1)$ lies on line $\ell_1$
7 : $P_{70} = (1,0,3,1)$ lies on line $\ell_1$

The single points on the surface are:

### Points on surface but on no line

The surface has 12 points not on any line:
The points on the surface but not on lines are:

0 : $P_{11} = (0,1,1,0)$
1 : $P_{16} = (1,2,1,0)$
2 : $P_{20} = (1,3,1,0)$
3 : $P_{26} = (0,1,0,1)$
4 : $P_{30} = (0,2,0,1)$
5 : $P_{34} = (0,3,0,1)$
6 : $P_{43} = (2,1,1,1)$
7 : $P_{44} = (3,1,1,1)$
8 : $P_{46} = (1,2,1,1)$
9 : $P_{48} = (3,2,1,1)$
10 : $P_{50} = (1,3,1,1)$
11 : $P_{51} = (2,3,1,1)$

## Line Intersection Graph

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ |
|---|---|
| in point | $P_2$ |

Line 1 intersects

| Line | $\ell_0$ |
|---|---|
| in point | $P_2$ |

The surface has 21 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_8 = (1, 0, 1, 0)$
3 : $P_9 = (2, 0, 1, 0)$
4 : $P_{10} = (3, 0, 1, 0)$
5 : $P_{11} = (0, 1, 1, 0)$
6 : $P_{16} = (1, 2, 1, 0)$
7 : $P_{20} = (1, 3, 1, 0)$
8 : $P_{23} = (1, 0, 0, 1)$
9 : $P_{26} = (0, 1, 0, 1)$
10 : $P_{30} = (0, 2, 0, 1)$
11 : $P_{34} = (0, 3, 0, 1)$
12 : $P_{39} = (1, 0, 1, 1)$
13 : $P_{43} = (2, 1, 1, 1)$
14 : $P_{44} = (3, 1, 1, 1)$
15 : $P_{46} = (1, 2, 1, 1)$
16 : $P_{48} = (3, 2, 1, 1)$
17 : $P_{50} = (1, 3, 1, 1)$
18 : $P_{51} = (2, 3, 1, 1)$
19 : $P_{54} = (1, 0, 2, 1)$
20 : $P_{70} = (1, 0, 3, 1)$