# Rank-140 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 = 0$$

( 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )
The point rank of the equation over GF(8) is 74912

## General information

| | |
|---|---|
| Number of lines | 15 |
| Number of points | 105 |
| Number of singular points | 0 |
| Number of Eckardt points | 15 |
| Number of double points | 0 |
| Number of single points | 90 |
| Number of points off lines | 0 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^{15}$ |
| Type of lines on points | $3^{15}$, $1^{90}$ |

### Singular Points

The surface has 0 singular points:

## The 15 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{503} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{503} = \mathbf{Pl}(0,0,1,1,2,1)_{1826}$$

$$\ell_1 = \begin{bmatrix} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{284} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{284} = \mathbf{Pl}(0,0,1,1,4,1)_{2834}$$

$$\ell_2 = \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{430} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{430} = \mathbf{Pl}(0,0,1,1,7,1)_{4346}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{446} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{446} = \mathbf{Pl}(1,1,0,0,2,1)_{1721}$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{227} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{227} = \mathbf{Pl}(1,1,0,0,4,1)_{2729}$$
$$\ell_5 = \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{373} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{373} = \mathbf{Pl}(1,1,0,0,7,1)_{4241}$$
$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3505} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{3505} = \mathbf{Pl}(1,1,2,1,0,0)_{39}$$
$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1753} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1753} = \mathbf{Pl}(1,1,4,1,0,0)_{53}$$
$$\ell_8 = \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2921} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2921} = \mathbf{Pl}(1,1,7,1,0,0)_{74}$$
$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & \gamma^5 \\ 0 & 1 & \gamma^4 & \gamma^5 \end{bmatrix}_{1856} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 7 & 3 \end{bmatrix}_{1856} = \mathbf{Pl}(5,7,4,3,3,1)_{2600}$$
$$\ell_{10} = \begin{bmatrix} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & \gamma^6 & \gamma^2 \end{bmatrix}_{1060} = \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 6 & 4 \end{bmatrix}_{1060} = \mathbf{Pl}(7,5,6,2,5,1)_{3666}$$
$$\ell_{11} = \begin{bmatrix} 1 & 0 & 1 & \gamma^3 \\ 0 & 1 & \gamma & \gamma^3 \end{bmatrix}_{3035} = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 2 & 5 \end{bmatrix}_{3035} = \mathbf{Pl}(6,2,7,5,5,1)_{3700}$$
$$\ell_{12} = \begin{bmatrix} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & \gamma^5 & \gamma^4 \end{bmatrix}_{862} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 3 & 7 \end{bmatrix}_{862} = \mathbf{Pl}(2,6,3,4,6,1)_{4053}$$
$$\ell_{13} = \begin{bmatrix} 1 & 0 & \gamma^3 & 1 \\ 0 & 1 & \gamma^3 & \gamma \end{bmatrix}_{970} = \begin{bmatrix} 1 & 0 & 5 & 1 \\ 0 & 1 & 5 & 2 \end{bmatrix}_{970} = \mathbf{Pl}(4,3,5,7,3,1)_{2634}$$
$$\ell_{14} = \begin{bmatrix} 1 & 0 & 1 & \gamma^6 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{bmatrix}_{3629} = \begin{bmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & 4 & 6 \end{bmatrix}_{3629} = \mathbf{Pl}(3,4,2,6,6,1)_{4005}$$

Rank of lines: ( 503, 284, 430, 446, 227, 373, 3505, 1753, 2921, 1856, 1060, 3035, 862, 970, 3629 )
Rank of points on Klein quadric: ( 1826, 2834, 4346, 1721, 2729, 4241, 39, 53, 74, 2600, 3666, 3700, 4053, 2634, 4005 )

**Eckardt Points**

The surface has 15 Eckardt points:
$0 : P_{19} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$1 : P_{82} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$,
$2 : P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$,
$3 : P_{147} = \mathbf{P}(\gamma,1,1,1) = \mathbf{P}(2,1,1,1)$,
$4 : P_{149} = \mathbf{P}(\gamma^2,1,1,1) = \mathbf{P}(4,1,1,1)$,
$5 : P_{152} = \mathbf{P}(\gamma^4,1,1,1) = \mathbf{P}(7,1,1,1)$,
$6 : P_{158} = \mathbf{P}(\gamma^3,\gamma,1,1) = \mathbf{P}(5,2,1,1)$,
$7 : P_{175} = \mathbf{P}(\gamma^6,\gamma^2,1,1) = \mathbf{P}(6,4,1,1)$,
$8 : P_{196} = \mathbf{P}(\gamma^5,\gamma^4,1,1) = \mathbf{P}(3,7,1,1)$,
$9 : P_{214} = \mathbf{P}(\gamma^3,1,\gamma,1) = \mathbf{P}(5,1,2,1)$,
$10 : P_{296} = \mathbf{P}(\gamma^4,\gamma^5,\gamma^5,1) = \mathbf{P}(7,3,3,1)$,
$11 : P_{343} = \mathbf{P}(\gamma^6,1,\gamma^2,1) = \mathbf{P}(6,1,4,1)$,
$12 : P_{435} = \mathbf{P}(\gamma,\gamma^3,\gamma^3,1) = \mathbf{P}(2,5,5,1)$,
$13 : P_{509} = \mathbf{P}(\gamma^2,\gamma^6,\gamma^6,1) = \mathbf{P}(4,6,6,1)$,
$14 : P_{532} = \mathbf{P}(\gamma^5,1,\gamma^4,1) = \mathbf{P}(3,1,7,1)$.

### Double Points

The surface has 0 Double points:
The double points on the surface are:

### Single Points

The surface has 90 single points:
The single points on the surface are:

0 : $P_6 = (2,1,0,0)$ lies on line $\ell_0$
1 : $P_8 = (4,1,0,0)$ lies on line $\ell_1$
2 : $P_{11} = (7,1,0,0)$ lies on line $\ell_2$
3 : $P_{13} = (2,0,1,0)$ lies on line $\ell_3$
4 : $P_{15} = (4,0,1,0)$ lies on line $\ell_4$
5 : $P_{18} = (7,0,1,0)$ lies on line $\ell_5$
6 : $P_{29} = (2,2,1,0)$ lies on line $\ell_9$
7 : $P_{36} = (1,3,1,0)$ lies on line $\ell_{10}$
8 : $P_{47} = (4,4,1,0)$ lies on line $\ell_{11}$
9 : $P_{52} = (1,5,1,0)$ lies on line $\ell_{12}$
10 : $P_{60} = (1,6,1,0)$ lies on line $\ell_{13}$
11 : $P_{74} = (7,7,1,0)$ lies on line $\ell_{14}$
12 : $P_{76} = (2,0,0,1)$ lies on line $\ell_6$
13 : $P_{78} = (4,0,0,1)$ lies on line $\ell_7$
14 : $P_{81} = (7,0,0,1)$ lies on line $\ell_8$
15 : $P_{92} = (2,2,0,1)$ lies on line $\ell_{12}$
16 : $P_{99} = (1,3,0,1)$ lies on line $\ell_{14}$
17 : $P_{110} = (4,4,0,1)$ lies on line $\ell_{13}$
18 : $P_{115} = (1,5,0,1)$ lies on line $\ell_9$
19 : $P_{123} = (1,6,0,1)$ lies on line $\ell_{11}$
20 : $P_{137} = (7,7,0,1)$ lies on line $\ell_{10}$
21 : $P_{156} = (3,2,1,1)$ lies on line $\ell_2$
22 : $P_{157} = (4,2,1,1)$ lies on line $\ell_0$
23 : $P_{162} = (1,3,1,1)$ lies on line $\ell_1$
24 : $P_{165} = (4,3,1,1)$ lies on line $\ell_2$
25 : $P_{167} = (6,3,1,1)$ lies on line $\ell_0$
26 : $P_{174} = (5,4,1,1)$ lies on line $\ell_0$
27 : $P_{176} = (7,4,1,1)$ lies on line $\ell_1$
28 : $P_{178} = (1,5,1,1)$ lies on line $\ell_2$
29 : $P_{180} = (3,5,1,1)$ lies on line $\ell_1$
30 : $P_{184} = (7,5,1,1)$ lies on line $\ell_0$
31 : $P_{186} = (1,6,1,1)$ lies on line $\ell_0$
32 : $P_{187} = (2,6,1,1)$ lies on line $\ell_1$
33 : $P_{190} = (5,6,1,1)$ lies on line $\ell_2$
34 : $P_{195} = (2,7,1,1)$ lies on line $\ell_2$
35 : $P_{199} = (6,7,1,1)$ lies on line $\ell_1$
36 : $P_{203} = (2,0,2,1)$ lies on line $\ell_{14}$
37 : $P_{212} = (3,1,2,1)$ lies on line $\ell_5$
38 : $P_{213} = (4,1,2,1)$ lies on line $\ell_3$
39 : $P_{219} = (2,2,2,1)$ lies on line $\ell_6$
40 : $P_{221} = (4,2,2,1)$ lies on line $\ell_7$
41 : $P_{224} = (7,2,2,1)$ lies on line $\ell_8$
42 : $P_{229} = (4,3,2,1)$ lies on line $\ell_{11}$
43 : $P_{241} = (0,5,2,1)$ lies on line $\ell_{12}$
44 : $P_{247} = (6,5,2,1)$ lies on line $\ell_{13}$
45 : $P_{266} = (1,0,3,1)$ lies on line $\ell_{12}$
46 : $P_{274} = (1,1,3,1)$ lies on line $\ell_4$
47 : $P_{277} = (4,1,3,1)$ lies on line $\ell_5$
48 : $P_{279} = (6,1,3,1)$ lies on line $\ell_3$
49 : $P_{285} = (4,2,3,1)$ lies on line $\ell_{10}$
50 : $P_{291} = (2,3,3,1)$ lies on line $\ell_6$
51 : $P_{293} = (4,3,3,1)$ lies on line $\ell_7$
52 : $P_{321} = (0,7,3,1)$ lies on line $\ell_{11}$
53 : $P_{326} = (5,7,3,1)$ lies on line $\ell_{14}$
54 : $P_{333} = (4,0,4,1)$ lies on line $\ell_9$
55 : $P_{342} = (5,1,4,1)$ lies on line $\ell_3$
56 : $P_{344} = (7,1,4,1)$ lies on line $\ell_4$
57 : $P_{363} = (2,4,4,1)$ lies on line $\ell_6$
58 : $P_{365} = (4,4,4,1)$ lies on line $\ell_7$
59 : $P_{368} = (7,4,4,1)$ lies on line $\ell_8$
60 : $P_{376} = (7,5,4,1)$ lies on line $\ell_{14}$
61 : $P_{377} = (0,6,4,1)$ lies on line $\ell_{13}$
62 : $P_{380} = (3,6,4,1)$ lies on line $\ell_{10}$
63 : $P_{394} = (1,0,5,1)$ lies on line $\ell_{13}$
64 : $P_{402} = (1,1,5,1)$ lies on line $\ell_5$
65 : $P_{404} = (3,1,5,1)$ lies on line $\ell_4$
66 : $P_{408} = (7,1,5,1)$ lies on line $\ell_3$
67 : $P_{409} = (0,2,5,1)$ lies on line $\ell_{14}$
68 : $P_{415} = (6,2,5,1)$ lies on line $\ell_9$
69 : $P_{432} = (7,4,5,1)$ lies on line $\ell_{12}$
70 : $P_{437} = (4,5,5,1)$ lies on line $\ell_7$
71 : $P_{440} = (7,5,5,1)$ lies on line $\ell_8$
72 : $P_{458} = (1,0,6,1)$ lies on line $\ell_{10}$
73 : $P_{466} = (1,1,6,1)$ lies on line $\ell_3$
74 : $P_{467} = (2,1,6,1)$ lies on line $\ell_4$
75 : $P_{470} = (5,1,6,1)$ lies on line $\ell_5$
76 : $P_{489} = (0,4,6,1)$ lies on line $\ell_9$
77 : $P_{492} = (3,4,6,1)$ lies on line $\ell_{11}$
78 : $P_{507} = (2,6,6,1)$ lies on line $\ell_6$
79 : $P_{512} = (7,6,6,1)$ lies on line $\ell_8$
80 : $P_{515} = (2,7,6,1)$ lies on line $\ell_{13}$
81 : $P_{528} = (7,0,7,1)$ lies on line $\ell_{11}$

82 : $P_{531} = (2, 1, 7, 1)$ lies on line $\ell_5$
83 : $P_{535} = (6, 1, 7, 1)$ lies on line $\ell_4$
84 : $P_{545} = (0, 3, 7, 1)$ lies on line $\ell_{10}$
85 : $P_{550} = (5, 3, 7, 1)$ lies on line $\ell_{12}$
86 : $P_{571} = (2, 6, 7, 1)$ lies on line $\ell_9$
87 : $P_{579} = (2, 7, 7, 1)$ lies on line $\ell_6$
88 : $P_{581} = (4, 7, 7, 1)$ lies on line $\ell_7$
89 : $P_{584} = (7, 7, 7, 1)$ lies on line $\ell_8$

The single points on the surface are:

**Points on surface but on no line**

The surface has 0 points not on any line:
The points on the surface but not on lines are:

## Line Intersection Graph

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 6 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 7 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 8 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 9 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 11 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 13 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 14 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ | $\ell_6$ | $\ell_9$ | $\ell_{12}$ |
|---|---|---|---|---|---|---|
| in point | $P_{138}$ | $P_{138}$ | $P_{147}$ | $P_{147}$ | $P_{196}$ | $P_{196}$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_4$ | $\ell_7$ | $\ell_{11}$ | $\ell_{13}$ |
|---|---|---|---|---|---|---|
| in point | $P_{138}$ | $P_{138}$ | $P_{149}$ | $P_{149}$ | $P_{158}$ | $P_{158}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_5$ | $\ell_8$ | $\ell_{10}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{138}$ | $P_{138}$ | $P_{152}$ | $P_{152}$ | $P_{175}$ | $P_{175}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_4$ | $\ell_5$ | $\ell_6$ | $\ell_{13}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{147}$ | $P_{82}$ | $P_{82}$ | $P_{147}$ | $P_{532}$ | $P_{532}$ |

Line 4 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_5$ | $\ell_7$ | $\ell_9$ | $\ell_{10}$ |
|---|---|---|---|---|---|---|
| in point | $P_{149}$ | $P_{82}$ | $P_{82}$ | $P_{149}$ | $P_{214}$ | $P_{214}$ |

Line 5 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_4$ | $\ell_8$ | $\ell_{11}$ | $\ell_{12}$ |
|---|---|---|---|---|---|---|
| in point | $P_{152}$ | $P_{82}$ | $P_{82}$ | $P_{152}$ | $P_{343}$ | $P_{343}$ |

Line 6 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_7$ | $\ell_8$ | $\ell_{10}$ | $\ell_{11}$ |
|---|---|---|---|---|---|---|
| in point | $P_{147}$ | $P_{147}$ | $P_{19}$ | $P_{19}$ | $P_{435}$ | $P_{435}$ |

Line 7 intersects

| Line | $\ell_1$ | $\ell_4$ | $\ell_6$ | $\ell_8$ | $\ell_{12}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{149}$ | $P_{149}$ | $P_{19}$ | $P_{19}$ | $P_{509}$ | $P_{509}$ |

Line 8 intersects

| Line | $\ell_2$ | $\ell_5$ | $\ell_6$ | $\ell_7$ | $\ell_9$ | $\ell_{13}$ |
|---|---|---|---|---|---|---|
| in point | $P_{152}$ | $P_{152}$ | $P_{19}$ | $P_{19}$ | $P_{296}$ | $P_{296}$ |

Line 9 intersects

| Line | $\ell_0$ | $\ell_4$ | $\ell_8$ | $\ell_{10}$ | $\ell_{12}$ | $\ell_{13}$ |
|---|---|---|---|---|---|---|
| in point | $P_{196}$ | $P_{214}$ | $P_{296}$ | $P_{214}$ | $P_{196}$ | $P_{296}$ |

Line 10 intersects

| Line | $\ell_2$ | $\ell_4$ | $\ell_6$ | $\ell_9$ | $\ell_{11}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{175}$ | $P_{214}$ | $P_{435}$ | $P_{214}$ | $P_{435}$ | $P_{175}$ |

Line 11 intersects

| Line | $\ell_1$ | $\ell_5$ | $\ell_6$ | $\ell_{10}$ | $\ell_{12}$ | $\ell_{13}$ |
|---|---|---|---|---|---|---|
| in point | $P_{158}$ | $P_{343}$ | $P_{435}$ | $P_{435}$ | $P_{343}$ | $P_{158}$ |

Line 12 intersects

| Line | $\ell_0$ | $\ell_5$ | $\ell_7$ | $\ell_9$ | $\ell_{11}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{196}$ | $P_{343}$ | $P_{509}$ | $P_{196}$ | $P_{343}$ | $P_{509}$ |

Line 13 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_8$ | $\ell_9$ | $\ell_{11}$ | $\ell_{14}$ |
|---|---|---|---|---|---|---|
| in point | $P_{158}$ | $P_{532}$ | $P_{296}$ | $P_{296}$ | $P_{158}$ | $P_{532}$ |

Line 14 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_7$ | $\ell_{10}$ | $\ell_{12}$ | $\ell_{13}$ |
|---|---|---|---|---|---|---|
| in point | $P_{175}$ | $P_{532}$ | $P_{509}$ | $P_{175}$ | $P_{509}$ | $P_{532}$ |

The surface has 105 points:
The points on the surface are:

0 : $P_{6} = (2, 1, 0, 0)$
1 : $P_{8} = (4, 1, 0, 0)$
2 : $P_{11} = (7, 1, 0, 0)$
3 : $P_{13} = (2, 0, 1, 0)$
4 : $P_{15} = (4, 0, 1, 0)$
5 : $P_{18} = (7, 0, 1, 0)$
6 : $P_{19} = (0, 1, 1, 0)$
7 : $P_{29} = (2, 2, 1, 0)$
8 : $P_{36} = (1, 3, 1, 0)$
9 : $P_{47} = (4, 4, 1, 0)$
10 : $P_{52} = (1, 5, 1, 0)$
11 : $P_{60} = (1, 6, 1, 0)$
12 : $P_{74} = (7, 7, 1, 0)$
13 : $P_{76} = (2, 0, 0, 1)$
14 : $P_{78} = (4, 0, 0, 1)$
15 : $P_{81} = (7, 0, 0, 1)$
16 : $P_{82} = (0, 1, 0, 1)$
17 : $P_{92} = (2, 2, 0, 1)$
18 : $P_{99} = (1, 3, 0, 1)$
19 : $P_{110} = (4, 4, 0, 1)$
20 : $P_{115} = (1, 5, 0, 1)$
21 : $P_{123} = (1, 6, 0, 1)$
22 : $P_{137} = (7, 7, 0, 1)$
23 : $P_{138} = (0, 0, 1, 1)$
24 : $P_{147} = (2, 1, 1, 1)$
25 : $P_{149} = (4, 1, 1, 1)$
26 : $P_{152} = (7, 1, 1, 1)$
27 : $P_{156} = (3, 2, 1, 1)$
28 : $P_{157} = (4, 2, 1, 1)$
29 : $P_{158} = (5, 2, 1, 1)$
30 : $P_{162} = (1, 3, 1, 1)$
31 : $P_{165} = (4, 3, 1, 1)$
32 : $P_{167} = (6, 3, 1, 1)$
33 : $P_{174} = (5, 4, 1, 1)$
34 : $P_{175} = (6, 4, 1, 1)$
35 : $P_{176} = (7, 4, 1, 1)$
36 : $P_{178} = (1, 5, 1, 1)$
37 : $P_{180} = (3, 5, 1, 1)$
38 : $P_{184} = (7, 5, 1, 1)$
39 : $P_{186} = (1, 6, 1, 1)$
40 : $P_{187} = (2, 6, 1, 1)$
41 : $P_{190} = (5, 6, 1, 1)$
42 : $P_{195} = (2, 7, 1, 1)$
43 : $P_{196} = (3, 7, 1, 1)$
44 : $P_{199} = (6, 7, 1, 1)$
45 : $P_{203} = (2, 0, 2, 1)$
46 : $P_{212} = (3, 1, 2, 1)$
47 : $P_{213} = (4, 1, 2, 1)$
48 : $P_{214} = (5, 1, 2, 1)$
49 : $P_{219} = (2, 2, 2, 1)$
50 : $P_{221} = (4, 2, 2, 1)$
51 : $P_{224} = (7, 2, 2, 1)$
52 : $P_{229} = (4, 3, 2, 1)$
53 : $P_{241} = (0, 5, 2, 1)$
54 : $P_{247} = (6, 5, 2, 1)$
55 : $P_{266} = (1, 0, 3, 1)$
56 : $P_{274} = (1, 1, 3, 1)$
57 : $P_{277} = (4, 1, 3, 1)$
58 : $P_{279} = (6, 1, 3, 1)$
59 : $P_{285} = (4, 2, 3, 1)$

$60: P_{291} = (2,3,3,1)$
$61: P_{293} = (4,3,3,1)$
$62: P_{296} = (7,3,3,1)$
$63: P_{321} = (0,7,3,1)$
$64: P_{326} = (5,7,3,1)$
$65: P_{333} = (4,0,4,1)$
$66: P_{342} = (5,1,4,1)$
$67: P_{343} = (6,1,4,1)$
$68: P_{344} = (7,1,4,1)$
$69: P_{363} = (2,4,4,1)$
$70: P_{365} = (4,4,4,1)$
$71: P_{368} = (7,4,4,1)$
$72: P_{376} = (7,5,4,1)$
$73: P_{377} = (0,6,4,1)$
$74: P_{380} = (3,6,4,1)$
$75: P_{394} = (1,0,5,1)$
$76: P_{402} = (1,1,5,1)$
$77: P_{404} = (3,1,5,1)$
$78: P_{408} = (7,1,5,1)$
$79: P_{409} = (0,2,5,1)$
$80: P_{415} = (6,2,5,1)$
$81: P_{432} = (7,4,5,1)$
$82: P_{435} = (2,5,5,1)$
$83: P_{437} = (4,5,5,1)$
$84: P_{440} = (7,5,5,1)$
$85: P_{458} = (1,0,6,1)$
$86: P_{466} = (1,1,6,1)$
$87: P_{467} = (2,1,6,1)$
$88: P_{470} = (5,1,6,1)$
$89: P_{489} = (0,4,6,1)$
$90: P_{492} = (3,4,6,1)$
$91: P_{507} = (2,6,6,1)$
$92: P_{509} = (4,6,6,1)$
$93: P_{512} = (7,6,6,1)$
$94: P_{515} = (2,7,6,1)$
$95: P_{528} = (7,0,7,1)$
$96: P_{531} = (2,1,7,1)$
$97: P_{532} = (3,1,7,1)$
$98: P_{535} = (6,1,7,1)$
$99: P_{545} = (0,3,7,1)$
$100: P_{550} = (5,3,7,1)$
$101: P_{571} = (2,6,7,1)$
$102: P_{579} = (2,7,7,1)$
$103: P_{581} = (4,7,7,1)$
$104: P_{584} = (7,7,7,1)$