

Rank-65605 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227395669

General information

Number of lines	9
Number of points	73
Number of singular points	9
Number of Eckardt points	0
Number of double points	8
Number of single points	65
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^9
Type of lines on points	$2^8, 1^{65}$

Singular Points

The surface has 9 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{138} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{201} = \mathbf{P}(0, 0, \gamma, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{265} = \mathbf{P}(0, 0, \gamma^5, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{329} = \mathbf{P}(0, 0, \gamma^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{393} = \mathbf{P}(0, 0, \gamma^3, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{457} = \mathbf{P}(0, 0, \gamma^6, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{521} = \mathbf{P}(0, 0, \gamma^4, 1) = \mathbf{P}(0, 0, 7, 1)$$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{1330} \\
\ell_3 &= \begin{bmatrix} 1 & \gamma^6 & 0 & \gamma^4 \\ 0 & 0 & 1 & \gamma^6 \end{bmatrix}_{4596} = \begin{bmatrix} 1 & 6 & 0 & 7 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{4596} = \mathbf{Pl}(0, 3, 2, 6, 1, 1)_{1347} \\
\ell_4 &= \begin{bmatrix} 1 & \gamma^2 & 0 & \gamma^6 \\ 0 & 0 & 1 & \gamma^2 \end{bmatrix}_{3864} = \begin{bmatrix} 1 & 4 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{3864} = \mathbf{Pl}(0, 7, 3, 4, 1, 1)_{1366} \\
\ell_5 &= \begin{bmatrix} 1 & \gamma^5 & 0 & \gamma \\ 0 & 0 & 1 & \gamma^5 \end{bmatrix}_{1454} = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{1454} = \mathbf{Pl}(0, 5, 4, 3, 1, 1)_{1379} \\
\ell_6 &= \begin{bmatrix} 1 & \gamma^4 & 0 & \gamma^5 \\ 0 & 0 & 1 & \gamma^4 \end{bmatrix}_{2334} = \begin{bmatrix} 1 & 7 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{2334} = \mathbf{Pl}(0, 2, 5, 7, 1, 1)_{1391} \\
\ell_7 &= \begin{bmatrix} 1 & \gamma & 0 & \gamma^3 \\ 0 & 0 & 1 & \gamma \end{bmatrix}_{3132} = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{3132} = \mathbf{Pl}(0, 4, 6, 2, 1, 1)_{1408} \\
\ell_8 &= \begin{bmatrix} 1 & \gamma^3 & 0 & \gamma^2 \\ 0 & 0 & 1 & \gamma^3 \end{bmatrix}_{2770} = \begin{bmatrix} 1 & 5 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{2770} = \mathbf{Pl}(0, 6, 7, 5, 1, 1)_{1425}
\end{aligned}$$

Rank of lines: (64, 4744, 722, 4596, 3864, 1454, 2334, 3132, 2770)

Rank of points on Klein quadric: (2, 1, 1330, 1347, 1366, 1379, 1391, 1408, 1425)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 8 Double points:

The double points on the surface are:

$$\begin{aligned}
P_2 &= (0, 0, 1, 0) = \ell_0 \cap \ell_1 & P_{393} &= (0, 0, 5, 1) = \ell_1 \cap \ell_6 \\
P_{138} &= (0, 0, 1, 1) = \ell_1 \cap \ell_2 & P_{457} &= (0, 0, 6, 1) = \ell_1 \cap \ell_7 \\
P_{201} &= (0, 0, 2, 1) = \ell_1 \cap \ell_3 & P_{521} &= (0, 0, 7, 1) = \ell_1 \cap \ell_8 \\
P_{265} &= (0, 0, 3, 1) = \ell_1 \cap \ell_4 \\
P_{329} &= (0, 0, 4, 1) = \ell_1 \cap \ell_5
\end{aligned}$$

Single Points

The surface has 65 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
 1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1
 2 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_0
 3 : $P_{13} = (2, 0, 1, 0)$ lies on line ℓ_0
 4 : $P_{14} = (3, 0, 1, 0)$ lies on line ℓ_0
 5 : $P_{15} = (4, 0, 1, 0)$ lies on line ℓ_0
 6 : $P_{16} = (5, 0, 1, 0)$ lies on line ℓ_0
 7 : $P_{17} = (6, 0, 1, 0)$ lies on line ℓ_0
 8 : $P_{18} = (7, 0, 1, 0)$ lies on line ℓ_0
 9 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_2
 10 : $P_{31} = (4, 2, 1, 0)$ lies on line ℓ_3
 11 : $P_{40} = (5, 3, 1, 0)$ lies on line ℓ_4
 12 : $P_{50} = (7, 4, 1, 0)$ lies on line ℓ_5
 13 : $P_{57} = (6, 5, 1, 0)$ lies on line ℓ_6
 14 : $P_{62} = (3, 6, 1, 0)$ lies on line ℓ_7
 15 : $P_{69} = (2, 7, 1, 0)$ lies on line ℓ_8
 16 : $P_{83} = (1, 1, 0, 1)$ lies on line ℓ_2
 17 : $P_{93} = (3, 2, 0, 1)$ lies on line ℓ_8
 18 : $P_{105} = (7, 3, 0, 1)$ lies on line ℓ_7
 19 : $P_{111} = (5, 4, 0, 1)$ lies on line ℓ_3
 20 : $P_{116} = (2, 5, 0, 1)$ lies on line ℓ_4
 21 : $P_{126} = (4, 6, 0, 1)$ lies on line ℓ_6
 22 : $P_{136} = (6, 7, 0, 1)$ lies on line ℓ_5
 23 : $P_{162} = (1, 3, 1, 1)$ lies on line ℓ_5
 24 : $P_{163} = (2, 3, 1, 1)$ lies on line ℓ_6
 25 : $P_{178} = (1, 5, 1, 1)$ lies on line ℓ_8
 26 : $P_{181} = (4, 5, 1, 1)$ lies on line ℓ_7
 27 : $P_{186} = (1, 6, 1, 1)$ lies on line ℓ_3
 28 : $P_{192} = (7, 6, 1, 1)$ lies on line ℓ_4
 29 : $P_{214} = (5, 1, 2, 1)$ lies on line ℓ_6
 30 : $P_{216} = (7, 1, 2, 1)$ lies on line ℓ_8
 31 : $P_{218} = (1, 2, 2, 1)$ lies on line ℓ_7
 32 : $P_{222} = (5, 2, 2, 1)$ lies on line ℓ_5

33 : $P_{228} = (3, 3, 2, 1)$ lies on line ℓ_2
 34 : $P_{230} = (5, 3, 2, 1)$ lies on line ℓ_4
 35 : $P_{283} = (2, 2, 3, 1)$ lies on line ℓ_2
 36 : $P_{285} = (4, 2, 3, 1)$ lies on line ℓ_3
 37 : $P_{299} = (2, 4, 3, 1)$ lies on line ℓ_7
 38 : $P_{300} = (3, 4, 3, 1)$ lies on line ℓ_6
 39 : $P_{315} = (2, 6, 3, 1)$ lies on line ℓ_5
 40 : $P_{318} = (5, 6, 3, 1)$ lies on line ℓ_8
 41 : $P_{339} = (2, 1, 4, 1)$ lies on line ℓ_3
 42 : $P_{343} = (6, 1, 4, 1)$ lies on line ℓ_7
 43 : $P_{362} = (1, 4, 4, 1)$ lies on line ℓ_4
 44 : $P_{367} = (6, 4, 4, 1)$ lies on line ℓ_8
 45 : $P_{374} = (5, 5, 4, 1)$ lies on line ℓ_2
 46 : $P_{375} = (6, 5, 4, 1)$ lies on line ℓ_6
 47 : $P_{421} = (4, 3, 5, 1)$ lies on line ℓ_8
 48 : $P_{423} = (6, 3, 5, 1)$ lies on line ℓ_3
 49 : $P_{429} = (4, 4, 5, 1)$ lies on line ℓ_2
 50 : $P_{432} = (7, 4, 5, 1)$ lies on line ℓ_5
 51 : $P_{453} = (4, 7, 5, 1)$ lies on line ℓ_4
 52 : $P_{454} = (5, 7, 5, 1)$ lies on line ℓ_7
 53 : $P_{479} = (6, 2, 6, 1)$ lies on line ℓ_4
 54 : $P_{480} = (7, 2, 6, 1)$ lies on line ℓ_6
 55 : $P_{500} = (3, 5, 6, 1)$ lies on line ℓ_5
 56 : $P_{504} = (7, 5, 6, 1)$ lies on line ℓ_3
 57 : $P_{515} = (2, 7, 6, 1)$ lies on line ℓ_8
 58 : $P_{520} = (7, 7, 6, 1)$ lies on line ℓ_2
 59 : $P_{532} = (3, 1, 7, 1)$ lies on line ℓ_4
 60 : $P_{533} = (4, 1, 7, 1)$ lies on line ℓ_5
 61 : $P_{572} = (3, 6, 7, 1)$ lies on line ℓ_7
 62 : $P_{575} = (6, 6, 7, 1)$ lies on line ℓ_2
 63 : $P_{578} = (1, 7, 7, 1)$ lies on line ℓ_6
 64 : $P_{580} = (3, 7, 7, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	0	0	0	0	0	0	0
1	1	0	1	1	1	1	1	1	1
2	0	1	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	0	0
8	0	1	0	0	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_2	P_{138}	P_{201}	P_{265}	P_{329}	P_{393}	P_{457}	P_{521}

Line 2 intersects

Line	ℓ_1
in point	P_{138}

Line 3 intersects

Line	ℓ_1
in point	P_{201}

Line 4 intersects

Line	ℓ_1
in point	P_{265}

Line 5 intersects

Line	ℓ_1
in point	P_{329}

Line 6 intersects

Line	ℓ_1
in point	P_{393}

Line 7 intersects

Line	ℓ_1
in point	P_{457}

Line 8 intersects

Line	ℓ_1
in point	P_{521}

The surface has 73 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_2 = (0, 0, 1, 0)$
 2 : $P_3 = (0, 0, 0, 1)$
 3 : $P_{12} = (1, 0, 1, 0)$
 4 : $P_{13} = (2, 0, 1, 0)$
 5 : $P_{14} = (3, 0, 1, 0)$
 6 : $P_{15} = (4, 0, 1, 0)$

7 : $P_{16} = (5, 0, 1, 0)$
 8 : $P_{17} = (6, 0, 1, 0)$
 9 : $P_{18} = (7, 0, 1, 0)$
 10 : $P_{20} = (1, 1, 1, 0)$
 11 : $P_{31} = (4, 2, 1, 0)$
 12 : $P_{40} = (5, 3, 1, 0)$
 13 : $P_{50} = (7, 4, 1, 0)$

14 : $P_{57} = (6, 5, 1, 0)$
 15 : $P_{62} = (3, 6, 1, 0)$
 16 : $P_{69} = (2, 7, 1, 0)$
 17 : $P_{83} = (1, 1, 0, 1)$
 18 : $P_{93} = (3, 2, 0, 1)$
 19 : $P_{105} = (7, 3, 0, 1)$
 20 : $P_{111} = (5, 4, 0, 1)$

21 : $P_{116} = (2, 5, 0, 1)$	39 : $P_{283} = (2, 2, 3, 1)$	57 : $P_{453} = (4, 7, 5, 1)$
22 : $P_{126} = (4, 6, 0, 1)$	40 : $P_{285} = (4, 2, 3, 1)$	58 : $P_{454} = (5, 7, 5, 1)$
23 : $P_{136} = (6, 7, 0, 1)$	41 : $P_{299} = (2, 4, 3, 1)$	59 : $P_{457} = (0, 0, 6, 1)$
24 : $P_{138} = (0, 0, 1, 1)$	42 : $P_{300} = (3, 4, 3, 1)$	60 : $P_{479} = (6, 2, 6, 1)$
25 : $P_{162} = (1, 3, 1, 1)$	43 : $P_{315} = (2, 6, 3, 1)$	61 : $P_{480} = (7, 2, 6, 1)$
26 : $P_{163} = (2, 3, 1, 1)$	44 : $P_{318} = (5, 6, 3, 1)$	62 : $P_{500} = (3, 5, 6, 1)$
27 : $P_{178} = (1, 5, 1, 1)$	45 : $P_{329} = (0, 0, 4, 1)$	63 : $P_{504} = (7, 5, 6, 1)$
28 : $P_{181} = (4, 5, 1, 1)$	46 : $P_{339} = (2, 1, 4, 1)$	64 : $P_{515} = (2, 7, 6, 1)$
29 : $P_{186} = (1, 6, 1, 1)$	47 : $P_{343} = (6, 1, 4, 1)$	65 : $P_{520} = (7, 7, 6, 1)$
30 : $P_{192} = (7, 6, 1, 1)$	48 : $P_{362} = (1, 4, 4, 1)$	66 : $P_{521} = (0, 0, 7, 1)$
31 : $P_{201} = (0, 0, 2, 1)$	49 : $P_{367} = (6, 4, 4, 1)$	67 : $P_{532} = (3, 1, 7, 1)$
32 : $P_{214} = (5, 1, 2, 1)$	50 : $P_{374} = (5, 5, 4, 1)$	68 : $P_{533} = (4, 1, 7, 1)$
33 : $P_{216} = (7, 1, 2, 1)$	51 : $P_{375} = (6, 5, 4, 1)$	69 : $P_{572} = (3, 6, 7, 1)$
34 : $P_{218} = (1, 2, 2, 1)$	52 : $P_{393} = (0, 0, 5, 1)$	70 : $P_{575} = (6, 6, 7, 1)$
35 : $P_{222} = (5, 2, 2, 1)$	53 : $P_{421} = (4, 3, 5, 1)$	71 : $P_{578} = (1, 7, 7, 1)$
36 : $P_{228} = (3, 3, 2, 1)$	54 : $P_{423} = (6, 3, 5, 1)$	72 : $P_{580} = (3, 7, 7, 1)$
37 : $P_{230} = (5, 3, 2, 1)$	55 : $P_{429} = (4, 4, 5, 1)$	
38 : $P_{265} = (0, 0, 3, 1)$	56 : $P_{432} = (7, 4, 5, 1)$	