Rank-65858 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_1^2X_2 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is -2113396602

General information

Number of lines	3
Number of points	4161
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	3969
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^3, 1^{189}, 0^{3969}

Singular Points

The surface has 3 singular points:

$0:\ P_{818} = \mathbf{P}(\epsilon^9, \epsilon^{18}, 1, 0) = \mathbf{P}(47, 11, 1, 0)$
$1:\ P_{2382} = \mathbf{P}(\epsilon^{18}, \epsilon^{36}, 1, 0) = \mathbf{P}(11, 36, 1, 0)$
$2:\ P_{3111} = \mathbf{P}(\epsilon^{36}, \epsilon^9, 1, 0) = \mathbf{P}(36, 47, 1, 0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & \epsilon^{18} & 0 \\ 0 & 1 & \epsilon^9 & 0 \end{bmatrix}_{45818} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 1 & 47 & 0 \end{bmatrix}_{45818} = \mathbf{Pl}(37, 0, 10, 0, 0, 1)_{271708}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \epsilon^{36} & 0 \\ 0 & 1 & \epsilon^{18} & 0 \end{bmatrix}_{149807} = \begin{bmatrix} 1 & 0 & 36 & 0 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{149807} = \mathbf{Pl}(46,0,37,0,0,1)_{275146}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \epsilon^{9} & 0 \\ 0 & 1 & \epsilon^{36} & 0 \end{bmatrix}_{195603} = \begin{bmatrix} 1 & 0 & 47 & 0 \\ 0 & 1 & 36 & 0 \end{bmatrix}_{195603} = \mathbf{Pl}(10,0,46,0,0,1)_{276253}$$

Rank of lines: (45818, 149807, 195603)
Rank of points on Klein quadric: (271708, 275146, 276253)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{3111} = (36,47,1,0) = \ell_0 \cap \ell_1$
$P_{2382} = (11,36,1,0) = \ell_0 \cap \ell_2$
$P_{818} = (47,11,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 189 single points:
The single points on the surface are:

0 : $P_{14} = (10,1,0,0)$ lies on line ℓ_0
1 : $P_{41} = (37,1,0,0)$ lies on line ℓ_1
2 : $P_{50} = (46,1,0,0)$ lies on line ℓ_2
3 : $P_{77} = (10,0,1,0)$ lies on line ℓ_2
4 : $P_{104} = (37,0,1,0)$ lies on line ℓ_0
5 : $P_{113} = (46,0,1,0)$ lies on line ℓ_1
6 : $P_{142} = (11,1,1,0)$ lies on line ℓ_1
7 : $P_{167} = (36,1,1,0)$ lies on line ℓ_2
8 : $P_{178} = (47,1,1,0)$ lies on line ℓ_0
9 : $P_{200} = (5,2,1,0)$ lies on line ℓ_1
10 : $P_{244} = (49,2,1,0)$ lies on line ℓ_0
11 : $P_{250} = (55,2,1,0)$ lies on line ℓ_2
12 : $P_{284} = (25,3,1,0)$ lies on line ℓ_2
13 : $P_{291} = (32,3,1,0)$ lies on line ℓ_1
14 : $P_{318} = (59,3,1,0)$ lies on line ℓ_0
15 : $P_{336} = (13,4,1,0)$ lies on line ℓ_0
16 : $P_{340} = (17,4,1,0)$ lies on line ℓ_2
17 : $P_{348} = (25,4,1,0)$ lies on line ℓ_1
18 : $P_{394} = (7,5,1,0)$ lies on line ℓ_0
19 : $P_{447} = (60,5,1,0)$ lies on line ℓ_1
20 : $P_{450} = (63,5,1,0)$ lies on line ℓ_2
21 : $P_{476} = (25,6,1,0)$ lies on line ℓ_0
22 : $P_{495} = (44,6,1,0)$ lies on line ℓ_2
23 : $P_{501} = (50,6,1,0)$ lies on line ℓ_1
24 : $P_{517} = (2,7,1,0)$ lies on line ℓ_2
25 : $P_{534} = (19,7,1,0)$ lies on line ℓ_0
26 : $P_{538} = (23,7,1,0)$ lies on line ℓ_1
27 : $P_{599} = (20,8,1,0)$ lies on line ℓ_0
28 : $P_{612} = (33,8,1,0)$ lies on line ℓ_1
29 : $P_{639} = (60,8,1,0)$ lies on line ℓ_2
30 : $P_{647} = (4,9,1,0)$ lies on line ℓ_1
31 : $P_{661} = (18,9,1,0)$ lies on line ℓ_2
32 : $P_{673} = (30,9,1,0)$ lies on line ℓ_0
33 : $P_{707} = (0,10,1,0)$ lies on line ℓ_0
34 : $P_{708} = (1,10,1,0)$ lies on line ℓ_2
35 : $P_{717} = (10,10,1,0)$ lies on line ℓ_1
36 : $P_{781} = (10,11,1,0)$ lies on line ℓ_0
37 : $P_{857} = (22,12,1,0)$ lies on line ℓ_1
38 : $P_{874} = (39,12,1,0)$ lies on line ℓ_2
39 : $P_{895} = (60,12,1,0)$ lies on line ℓ_0
40 : $P_{908} = (9,13,1,0)$ lies on line ℓ_2
41 : $P_{950} = (51,13,1,0)$ lies on line ℓ_1
42 : $P_{953} = (54,13,1,0)$ lies on line ℓ_0
43 : $P_{989} = (26,14,1,0)$ lies on line ℓ_2
44 : $P_{1003} = (40,14,1,0)$ lies on line ℓ_0
45 : $P_{1024} = (61,14,1,0)$ lies on line ℓ_1
46 : $P_{1051} = (24,15,1,0)$ lies on line ℓ_1
47 : $P_{1061} = (34,15,1,0)$ lies on line ℓ_0
48 : $P_{1079} = (52,15,1,0)$ lies on line ℓ_2
49 : $P_{1098} = (7,16,1,0)$ lies on line ℓ_2
50 : $P_{1129} = (38,16,1,0)$ lies on line ℓ_0
51 : $P_{1139} = (48,16,1,0)$ lies on line ℓ_1

52 : $P_{1176} = (21, 17, 1, 0)$ lies on line ℓ_1
53 : $P_{1196} = (41, 17, 1, 0)$ lies on line ℓ_2
54 : $P_{1199} = (44, 17, 1, 0)$ lies on line ℓ_0
55 : $P_{1246} = (27, 18, 1, 0)$ lies on line ℓ_1
56 : $P_{1269} = (50, 18, 1, 0)$ lies on line ℓ_0
57 : $P_{1277} = (58, 18, 1, 0)$ lies on line ℓ_2
58 : $P_{1303} = (20, 19, 1, 0)$ lies on line ℓ_2
59 : $P_{1339} = (56, 19, 1, 0)$ lies on line ℓ_0
60 : $P_{1345} = (62, 19, 1, 0)$ lies on line ℓ_1
61 : $P_{1354} = (7, 20, 1, 0)$ lies on line ℓ_1
62 : $P_{1361} = (14, 20, 1, 0)$ lies on line ℓ_0
63 : $P_{1375} = (28, 20, 1, 0)$ lies on line ℓ_2
64 : $P_{1415} = (4, 21, 1, 0)$ lies on line ℓ_0
65 : $P_{1445} = (34, 21, 1, 0)$ lies on line ℓ_1
66 : $P_{1461} = (50, 21, 1, 0)$ lies on line ℓ_2
67 : $P_{1501} = (26, 22, 1, 0)$ lies on line ℓ_0
68 : $P_{1508} = (33, 22, 1, 0)$ lies on line ℓ_2
69 : $P_{1519} = (44, 22, 1, 0)$ lies on line ℓ_1
70 : $P_{1548} = (9, 23, 1, 0)$ lies on line ℓ_1
71 : $P_{1554} = (15, 23, 1, 0)$ lies on line ℓ_2
72 : $P_{1555} = (16, 23, 1, 0)$ lies on line ℓ_0
73 : $P_{1626} = (23, 24, 1, 0)$ lies on line ℓ_0
74 : $P_{1652} = (49, 24, 1, 0)$ lies on line ℓ_2
75 : $P_{1666} = (63, 24, 1, 0)$ lies on line ℓ_1
76 : $P_{1693} = (26, 25, 1, 0)$ lies on line ℓ_1
77 : $P_{1696} = (29, 25, 1, 0)$ lies on line ℓ_0
78 : $P_{1698} = (31, 25, 1, 0)$ lies on line ℓ_2
79 : $P_{1734} = (3, 26, 1, 0)$ lies on line ℓ_0
80 : $P_{1743} = (12, 26, 1, 0)$ lies on line ℓ_2
81 : $P_{1751} = (20, 26, 1, 0)$ lies on line ℓ_1
82 : $P_{1804} = (9, 27, 1, 0)$ lies on line ℓ_0
83 : $P_{1829} = (34, 27, 1, 0)$ lies on line ℓ_2
84 : $P_{1844} = (49, 27, 1, 0)$ lies on line ℓ_1
85 : $P_{1867} = (8, 28, 1, 0)$ lies on line ℓ_1
86 : $P_{1901} = (42, 28, 1, 0)$ lies on line ℓ_2
87 : $P_{1922} = (63, 28, 1, 0)$ lies on line ℓ_0
88 : $P_{1927} = (4, 29, 1, 0)$ lies on line ℓ_2
89 : $P_{1968} = (45, 29, 1, 0)$ lies on line ℓ_1
90 : $P_{1976} = (53, 29, 1, 0)$ lies on line ℓ_0
91 : $P_{2010} = (23, 30, 1, 0)$ lies on line ℓ_2
92 : $P_{2022} = (35, 30, 1, 0)$ lies on line ℓ_1
93 : $P_{2030} = (43, 30, 1, 0)$ lies on line ℓ_0
94 : $P_{2057} = (6, 31, 1, 0)$ lies on line ℓ_1
95 : $P_{2084} = (33, 31, 1, 0)$ lies on line ℓ_0
96 : $P_{2108} = (57, 31, 1, 0)$ lies on line ℓ_2
97 : $P_{2131} = (16, 32, 1, 0)$ lies on line ℓ_2
98 : $P_{2133} = (18, 32, 1, 0)$ lies on line ℓ_1
99 : $P_{2150} = (35, 32, 1, 0)$ lies on line ℓ_0
100 : $P_{2220} = (41, 33, 1, 0)$ lies on line ℓ_0
101 : $P_{2234} = (55, 33, 1, 0)$ lies on line ℓ_1
102 : $P_{2241} = (62, 33, 1, 0)$ lies on line ℓ_2
103 : $P_{2288} = (45, 34, 1, 0)$ lies on line ℓ_2
104 : $P_{2298} = (55, 34, 1, 0)$ lies on line ℓ_0
105 : $P_{2300} = (57, 34, 1, 0)$ lies on line ℓ_1
106 : $P_{2310} = (3, 35, 1, 0)$ lies on line ℓ_2
107 : $P_{2335} = (28, 35, 1, 0)$ lies on line ℓ_1
108 : $P_{2368} = (61, 35, 1, 0)$ lies on line ℓ_0
109 : $P_{2408} = (37, 36, 1, 0)$ lies on line ℓ_1
110 : $P_{2435} = (0, 37, 1, 0)$ lies on line ℓ_1
111 : $P_{2436} = (1, 37, 1, 0)$ lies on line ℓ_0
112 : $P_{2472} = (37, 37, 1, 0)$ lies on line ℓ_2
113 : $P_{2513} = (14, 38, 1, 0)$ lies on line ℓ_1
114 : $P_{2530} = (31, 38, 1, 0)$ lies on line ℓ_0
115 : $P_{2553} = (54, 38, 1, 0)$ lies on line ℓ_2
116 : $P_{2584} = (21, 39, 1, 0)$ lies on line ℓ_0
117 : $P_{2587} = (24, 39, 1, 0)$ lies on line ℓ_2
118 : $P_{2606} = (43, 39, 1, 0)$ lies on line ℓ_1
119 : $P_{2645} = (18, 40, 1, 0)$ lies on line ℓ_0
120 : $P_{2656} = (29, 40, 1, 0)$ lies on line ℓ_1
121 : $P_{2665} = (38, 40, 1, 0)$ lies on line ℓ_2
122 : $P_{2699} = (8, 41, 1, 0)$ lies on line ℓ_2
123 : $P_{2715} = (24, 41, 1, 0)$ lies on line ℓ_0
124 : $P_{2747} = (56, 41, 1, 0)$ lies on line ℓ_1
125 : $P_{2761} = (6, 42, 1, 0)$ lies on line ℓ_0
126 : $P_{2782} = (27, 42, 1, 0)$ lies on line ℓ_2
127 : $P_{2809} = (54, 42, 1, 0)$ lies on line ℓ_1
128 : $P_{2831} = (12, 43, 1, 0)$ lies on line ℓ_0
129 : $P_{2838} = (19, 43, 1, 0)$ lies on line ℓ_1
130 : $P_{2872} = (53, 43, 1, 0)$ lies on line ℓ_2
131 : $P_{2925} = (42, 44, 1, 0)$ lies on line ℓ_1
132 : $P_{2941} = (58, 44, 1, 0)$ lies on line ℓ_0
133 : $P_{2944} = (61, 44, 1, 0)$ lies on line ℓ_2
134 : $P_{2962} = (15, 45, 1, 0)$ lies on line ℓ_1
135 : $P_{2966} = (19, 45, 1, 0)$ lies on line ℓ_2
136 : $P_{2995} = (48, 45, 1, 0)$ lies on line ℓ_0
137 : $P_{3011} = (0, 46, 1, 0)$ lies on line ℓ_2
138 : $P_{3012} = (1, 46, 1, 0)$ lies on line ℓ_1
139 : $P_{3057} = (46, 46, 1, 0)$ lies on line ℓ_0
140 : $P_{3121} = (46, 47, 1, 0)$ lies on line ℓ_2
141 : $P_{3151} = (12, 48, 1, 0)$ lies on line ℓ_1
142 : $P_{3168} = (29, 48, 1, 0)$ lies on line ℓ_2
143 : $P_{3171} = (32, 48, 1, 0)$ lies on line ℓ_0
144 : $P_{3244} = (41, 49, 1, 0)$ lies on line ℓ_1
145 : $P_{3245} = (42, 49, 1, 0)$ lies on line ℓ_0
146 : $P_{3254} = (51, 49, 1, 0)$ lies on line ℓ_2
147 : $P_{3299} = (32, 50, 1, 0)$ lies on line ℓ_2
148 : $P_{3306} = (39, 50, 1, 0)$ lies on line ℓ_1
149 : $P_{3319} = (52, 50, 1, 0)$ lies on line ℓ_0
150 : $P_{3333} = (2, 51, 1, 0)$ lies on line ℓ_1
151 : $P_{3345} = (14, 51, 1, 0)$ lies on line ℓ_2
152 : $P_{3393} = (62, 51, 1, 0)$ lies on line ℓ_0
153 : $P_{3401} = (6, 52, 1, 0)$ lies on line ℓ_2
154 : $P_{3403} = (8, 52, 1, 0)$ lies on line ℓ_0
155 : $P_{3454} = (59, 52, 1, 0)$ lies on line ℓ_1
156 : $P_{3461} = (2, 53, 1, 0)$ lies on line ℓ_0
157 : $P_{3489} = (30, 53, 1, 0)$ lies on line ℓ_1
158 : $P_{3499} = (40, 53, 1, 0)$ lies on line ℓ_2
159 : $P_{3539} = (16, 54, 1, 0)$ lies on line ℓ_1

160 : $P_{3551} = (28, 54, 1, 0)$ lies on line ℓ_0
161 : $P_{3582} = (59, 54, 1, 0)$ lies on line ℓ_2
162 : $P_{3608} = (21, 55, 1, 0)$ lies on line ℓ_2
163 : $P_{3609} = (22, 55, 1, 0)$ lies on line ℓ_0
164 : $P_{3640} = (53, 55, 1, 0)$ lies on line ℓ_1
165 : $P_{3654} = (3, 56, 1, 0)$ lies on line ℓ_1
166 : $P_{3668} = (17, 56, 1, 0)$ lies on line ℓ_0
167 : $P_{3694} = (43, 56, 1, 0)$ lies on line ℓ_2
168 : $P_{3720} = (5, 57, 1, 0)$ lies on line ℓ_2
169 : $P_{3742} = (27, 57, 1, 0)$ lies on line ℓ_0
170 : $P_{3753} = (38, 57, 1, 0)$ lies on line ℓ_1
171 : $P_{3784} = (5, 58, 1, 0)$ lies on line ℓ_0
172 : $P_{3801} = (22, 58, 1, 0)$ lies on line ℓ_2
173 : $P_{3819} = (40, 58, 1, 0)$ lies on line ℓ_1
174 : $P_{3856} = (13, 59, 1, 0)$ lies on line ℓ_1
175 : $P_{3858} = (15, 59, 1, 0)$ lies on line ℓ_0
176 : $P_{3899} = (56, 59, 1, 0)$ lies on line ℓ_2
177 : $P_{3955} = (48, 60, 1, 0)$ lies on line ℓ_2
178 : $P_{3959} = (52, 60, 1, 0)$ lies on line ℓ_1
179 : $P_{3964} = (57, 60, 1, 0)$ lies on line ℓ_0
180 : $P_{3988} = (17, 61, 1, 0)$ lies on line ℓ_1
181 : $P_{4001} = (30, 61, 1, 0)$ lies on line ℓ_2
182 : $P_{4022} = (51, 61, 1, 0)$ lies on line ℓ_0
183 : $P_{4048} = (13, 62, 1, 0)$ lies on line ℓ_2
184 : $P_{4066} = (31, 62, 1, 0)$ lies on line ℓ_1
185 : $P_{4080} = (45, 62, 1, 0)$ lies on line ℓ_0
186 : $P_{4134} = (35, 63, 1, 0)$ lies on line ℓ_2
187 : $P_{4138} = (39, 63, 1, 0)$ lies on line ℓ_0
188 : $P_{4157} = (58, 63, 1, 0)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 3969 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{3111}	P_{2382}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{3111}	P_{818}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{2382}	P_{818}

The surface has 4161 points:
Too many to print.