Rank-65609 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)The point rank of the equation over GF(16) is 303108645

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	87
Number of points off lines	211
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 ⁶
Type of lines on points	$3, 2^6, 1^{87}, 0^{211}$

Singular Points

The surface has 1 singular points:

$$0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{49}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{5586}$$

$$\ell_{2} = \begin{bmatrix} 0 & 1 & \delta^{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \mathbf{Pl}(0, 11, 0, 1, 0, 0)_{59}$$

$$\ell_{3} = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^{5} & 0 \end{bmatrix}_{7109} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{7109} = \mathbf{Pl}(11, 10, 10, 11, 0, 1)_{7981}$$

$$\ell_{4} = \begin{bmatrix} 0 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \begin{bmatrix} 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \mathbf{Pl}(0, 10, 0, 1, 0, 0)_{58}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & \delta^{5} & 1 \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{7381} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{7381} = \mathbf{Pl}(10, 11, 11, 10, 0, 1)_{7770}$$

Rank of lines: (69921, 4642, 70091, 7109, 70074, 7381) Rank of points on Klein quadric: (49, 5586, 59, 7981, 58, 7770)

Eckardt Points

The surface has 1 Eckardt points: $0: P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1).$

Double Points

The surface has 6 Double points: The double points on the surface are:

$$P_{35} = (0, 1, 1, 0) = \ell_0 \cap \ell_1$$

$$P_{3250} = (1, 10, 11, 1) = \ell_1 \cap \ell_3$$

$$P_{3010} = (1, 11, 10, 1) = \ell_1 \cap \ell_5$$

$$P_{179} = (0, 10, 1, 0) = \ell_2 \cap \ell_3$$

$P_4 = (1, 1, 1, 1) = \ell_3 \cap \ell_5$ $P_{195} = (0, 11, 1, 0) = \ell_4 \cap \ell_5$

Single Points

The surface has 87 single points: The single points on the surface are:

 $0: P_{291} = (1, 1, 0, 1)$ lies on line ℓ_1 1: $P_{435} = (1, 10, 0, 1)$ lies on line ℓ_5 2: $P_{451} = (1, 11, 0, 1)$ lies on line ℓ_3 3: $P_{531} = (1,0,1,1)$ lies on line ℓ_1 4: $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_0 5: $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_2 6: $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_4 7: $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_0 8: $P_{834} = (1, 3, 2, 1)$ lies on line ℓ_1 9: $P_{866} = (1, 5, 2, 1)$ lies on line ℓ_5 10: $P_{882} = (1, 6, 2, 1)$ lies on line ℓ_3 11: $P_{993} = (0, 13, 2, 1)$ lies on line ℓ_2 12: $P_{1025} = (0, 15, 2, 1)$ lies on line ℓ_4 13: $P_{1074} = (1, 2, 3, 1)$ lies on line ℓ_1 14: $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_0 15: $P_{1105} = (0, 4, 3, 1)$ lies on line ℓ_4 16: $P_{1153} = (0,7,3,1)$ lies on line ℓ_2

17: $P_{1234} = (1, 12, 3, 1)$ lies on line ℓ_3 18: $P_{1266} = (1, 14, 3, 1)$ lies on line ℓ_5 19: $P_{1345} = (0, 3, 4, 1)$ lies on line ℓ_2 20: $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_0 21: $P_{1378} = (1, 5, 4, 1)$ lies on line ℓ_1 22: $P_{1409} = (0, 7, 4, 1)$ lies on line ℓ_4 23: $P_{1426} = (1, 8, 4, 1)$ lies on line ℓ_3 24: $P_{1506} = (1, 13, 4, 1)$ lies on line ℓ_5 25: $P_{1586} = (1, 2, 5, 1)$ lies on line ℓ_3 26: $P_{1618} = (1, 4, 5, 1)$ lies on line ℓ_1 27: $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_0 28: $P_{1650} = (1, 6, 5, 1)$ lies on line ℓ_5 29 : $P_{1697} = (0, 9, 5, 1)$ lies on line ℓ_2 $30: P_{1745} = (0, 12, 5, 1)$ lies on line ℓ_4 31 : $P_{1842} = (1, 2, 6, 1)$ lies on line ℓ_5 $32: P_{1890} = (1, 5, 6, 1)$ lies on line ℓ_3 33: $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_0

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34: P_{1922} = (1,7,6,1) lies on line \ell_1
                                                                      61: P_{3249} = (0, 10, 11, 1) lies on line \ell_4
                                                                      62: P_{3265} = (0, 11, 11, 1) lies on line \ell_0
35: P_{1937} = (0, 8, 6, 1) lies on line \ell_4
                                                                      63: P_{3394} = (1, 3, 12, 1) lies on line \ell_5
36: P_{2033} = (0, 14, 6, 1) lies on line \ell_2
37: P_{2113} = (0, 3, 7, 1) lies on line \ell_4
                                                                      64: P_{3425} = (0, 5, 12, 1) lies on line \ell_2
38: P_{2129} = (0, 4, 7, 1) lies on line \ell_2
                                                                      65: P_{3489} = (0, 9, 12, 1) lies on line \ell_4
                                                                      66: P_{3537} = (0, 12, 12, 1) lies on line \ell_0
39: P_{2162} = (1, 6, 7, 1) lies on line \ell_1
                                                                      67 : P_{3554} = (1, 13, 12, 1) lies on line \ell_1
40: P_{2177} = (0,7,7,1) lies on line \ell_0
41: P_{2210} = (1, 9, 7, 1) lies on line \ell_5
                                                                      68: P_{3570} = (1, 14, 12, 1) lies on line \ell_3
42: P_{2306} = (1, 15, 7, 1) lies on line \ell_3
                                                                      69: P_{3633} = (0, 2, 13, 1) lies on line \ell_4
43: P_{2386} = (1, 4, 8, 1) lies on line \ell_5
                                                                      70: P_{3666} = (1, 4, 13, 1) lies on line \ell_3
44: P_{2417} = (0, 6, 8, 1) lies on line \ell_2
                                                                      71: P_{3730} = (1, 8, 13, 1) lies on line \ell_5
45: P_{2449} = (0, 8, 8, 1) lies on line \ell_0
                                                                      72: P_{3794} = (1, 12, 13, 1) lies on line \ell_1
46: P_{2466} = (1, 9, 8, 1) lies on line \ell_1
                                                                      73: P_{3809} = (0, 13, 13, 1) lies on line \ell_0
47: P_{2530} = (1, 13, 8, 1) lies on line \ell_3
                                                                      74: P_{3841} = (0, 15, 13, 1) lies on line \ell_2
48: P_{2545} = (0, 14, 8, 1) lies on line \ell_4
                                                                      75: P_{3906} = (1, 3, 14, 1) lies on line \ell_3
49: P_{2657} = (0, 5, 9, 1) lies on line \ell_4
                                                                      76: P_{3953} = (0, 6, 14, 1) lies on line \ell_4
                                                                      77: P_{3985} = (0, 8, 14, 1) lies on line \ell_2
50: P_{2690} = (1,7,9,1) lies on line \ell_3
51: P_{2706} = (1, 8, 9, 1) lies on line \ell_1
                                                                      78: P_{4050} = (1, 12, 14, 1) lies on line \ell_5
                                                                      79 : P_{4081} = (0, 14, 14, 1) lies on line \ell_0
52: P_{2721} = (0, 9, 9, 1) lies on line \ell_0
53: P_{2769} = (0, 12, 9, 1) lies on line \ell_2
                                                                      80: P_{4098} = (1, 15, 14, 1) lies on line \ell_1
54: P_{2818} = (1, 15, 9, 1) lies on line \ell_5
                                                                      81: P_{4145} = (0, 2, 15, 1) lies on line \ell_2
55: P_{2834} = (1, 0, 10, 1) lies on line \ell_3
                                                                      82: P_{4226} = (1, 7, 15, 1) lies on line \ell_5
56: P_{2849} = (0, 1, 10, 1) lies on line \ell_4
                                                                      83: P_{4258} = (1, 9, 15, 1) lies on line \ell_3
                                                                      84 : P_{4321} = (0, 13, 15, 1) lies on line \ell_4
57: P_{2993} = (0, 10, 10, 1) lies on line \ell_0
58: P_{3009} = (0, 11, 10, 1) lies on line \ell_2
                                                                      85: P_{4338} = (1, 14, 15, 1) lies on line \ell_1
59: P_{3090} = (1, 0, 11, 1) lies on line \ell_5
                                                                      86: P_{4353} = (0, 15, 15, 1) lies on line \ell_0
60: P_{3105} = (0, 1, 11, 1) lies on line \ell_2
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The single points on the surface are:

Points on surface but on no line

The surface has 211 points not on any line: The points on the surface but not on lines are:

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0: P_0 = (1, 0, 0, 0)
                                                                  17: P_{373} = (3, 6, 0, 1)
1: P_{59} = (8, 2, 1, 0)
                                                                  18: P_{394} = (8,7,0,1)
2: P_{80} = (13, 3, 1, 0)
                                                                  19: P_{405} = (3, 8, 0, 1)
3: P_{98} = (15, 4, 1, 0)
                                                                  20: P_{433} = (15, 9, 0, 1)
4: P_{106} = (7, 5, 1, 0)
                                                                  21: P_{481} = (15, 12, 0, 1)
5: P_{124} = (9, 6, 1, 0)
                                                                  22: P_{487} = (5, 13, 0, 1)
6: P_{133} = (2,7,1,0)
                                                                  23: P_{501} = (3, 14, 0, 1)
7: P_{159} = (12, 8, 1, 0)
                                                                  24: P_{519} = (5, 15, 0, 1)
8: P_{166} = (3, 9, 1, 0)
                                                                  25: P_{585} = (8, 3, 1, 1)
9: P_{215} = (4, 12, 1, 0)
                                                                  26: P_{588} = (11, 3, 1, 1)
10: P_{241} = (14, 13, 1, 0)
                                                                  27: P_{619} = (10, 5, 1, 1)
11: P_{248} = (5, 14, 1, 0)
                                                                  28: P_{624} = (15, 5, 1, 1)
12: P_{265} = (6, 15, 1, 0)
                                                                  29: P_{660} = (3, 8, 1, 1)
13: P_{311} = (5, 2, 0, 1)
                                                                  30: P_{668} = (11, 8, 1, 1)
14: P_{330} = (8, 3, 0, 1)
                                                                  31: P_{699} = (10, 10, 1, 1)
15: P_{346} = (8, 4, 0, 1)
                                                                  32: P_{716} = (11, 11, 1, 1)
16: P_{369} = (15, 5, 0, 1)
                                                                  33: P_{774} = (5, 15, 1, 1)
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88: P_{1854} = (13, 2, 6, 1)
34: P_{779} = (10, 15, 1, 1)
35: P_{790} = (5,0,2,1)
                                                                  89: P_{1861} = (4, 3, 6, 1)
36: P_{821} = (4, 2, 2, 1)
                                                                  90: P_{1871} = (14, 3, 6, 1)
37: P_{840} = (7, 3, 2, 1)
                                                                  91: P_{1885} = (12, 4, 6, 1)
38: P_{876} = (11, 5, 2, 1)
                                                                  92: P_{1886} = (13, 4, 6, 1)
39: P_{894} = (13, 6, 2, 1)
                                                                  93: P_{1895} = (6, 5, 6, 1)
40: P_{931} = (2, 9, 2, 1)
                                                                  94: P_{1918} = (13, 6, 6, 1)
41: P_{938} = (9, 9, 2, 1)
                                                                  95: P_{1931} = (10, 7, 6, 1)
42: P_{966} = (5, 11, 2, 1)
                                                                  96: P_{1939} = (2, 8, 6, 1)
43: P_{971} = (10, 11, 2, 1)
                                                                  97: P_{1988} = (3, 11, 6, 1)
44: P_{983} = (6, 12, 2, 1)
                                                                  98: P_{1996} = (11, 11, 6, 1)
45: P_{984} = (7, 12, 2, 1)
                                                                  99: P_{2048} = (15, 14, 6, 1)
46: P_{996} = (3, 13, 2, 1)
                                                                  100: P_{2073} = (8, 0, 7, 1)
47: P_{1032} = (7, 15, 2, 1)
                                                                  101: P_{2122} = (9, 3, 7, 1)
                                                                  102: P_{2134} = (5, 4, 7, 1)
48: P_{1049} = (8, 0, 3, 1)
49: P_{1065} = (8, 1, 3, 1)
                                                                  103: P_{2171} = (10, 6, 7, 1)
                                                                  104: P_{2189} = (12, 7, 7, 1)
50: P_{1068} = (11, 1, 3, 1)
51: P_{1080} = (7, 2, 3, 1)
                                                                  105: P_{2197} = (4, 8, 7, 1)
                                                                  106: P_{2207} = (14, 8, 7, 1)
52: P_{1094} = (5, 3, 3, 1)
53: P_{1117} = (12, 4, 3, 1)
                                                                  107: P_{2221} = (12, 9, 7, 1)
54: P_{1141} = (4, 6, 3, 1)
                                                                  108: P_{2249} = (8, 11, 7, 1)
                                                                  109: P_{2252} = (11, 11, 7, 1)
55: P_{1151} = (14, 6, 3, 1)
56: P_{1162} = (9,7,3,1)
                                                                  110: P_{2301} = (12, 14, 7, 1)
57: P_{1181} = (12, 8, 3, 1)
                                                                  111: P_{2302} = (13, 14, 7, 1)
58: P_{1182} = (13, 8, 3, 1)
                                                                  112: P_{2312} = (7, 15, 7, 1)
59: P_{1245} = (12, 12, 3, 1)
                                                                  113: P_{2324} = (3, 0, 8, 1)
                                                                  114: P_{2340} = (3, 1, 8, 1)
60: P_{1275} = (10, 14, 3, 1)
61: P_{1305} = (8, 0, 4, 1)
                                                                  115: P_{2348} = (11, 1, 8, 1)
62: P_{1357} = (12, 3, 4, 1)
                                                                  116: P_{2381} = (12, 3, 8, 1)
63: P_{1370} = (9, 4, 4, 1)
                                                                  117: P_{2382} = (13, 3, 8, 1)
64: P_{1389} = (12, 5, 4, 1)
                                                                  118: P_{2395} = (10, 4, 8, 1)
65: P_{1405} = (12, 6, 4, 1)
                                                                  119: P_{2419} = (2, 6, 8, 1)
66: P_{1406} = (13, 6, 4, 1)
                                                                  120: P_{2437} = (4, 7, 8, 1)
67: P_{1414} = (5,7,4,1)
                                                                  121: P_{2447} = (14, 7, 8, 1)
                                                                  122: P_{2464} = (15, 8, 8, 1)
68: P_{1435} = (10, 8, 4, 1)
69: P_{1465} = (8, 10, 4, 1)
                                                                  123: P_{2471} = (6, 9, 8, 1)
70: P_{1468} = (11, 10, 4, 1)
                                                                  124: P_{2542} = (13, 13, 8, 1)
                                                                  125: P_{2558} = (13, 14, 8, 1)
71: P_{1512} = (7, 13, 4, 1)
72: P_{1525} = (4, 14, 4, 1)
                                                                  126: P_{2592} = (15, 0, 9, 1)
73: P_{1535} = (14, 14, 4, 1)
                                                                  127: P_{2611} = (2, 2, 9, 1)
74: P_{1568} = (15, 0, 5, 1)
                                                                  128: P_{2618} = (9, 2, 9, 1)
                                                                  129: P_{2663} = (6, 5, 9, 1)
75: P_{1579} = (10, 1, 5, 1)
76: P_{1584} = (15, 1, 5, 1)
                                                                  130: P_{2701} = (12, 7, 9, 1)
77: P_{1596} = (11, 2, 5, 1)
                                                                  131: P_{2711} = (6, 8, 9, 1)
78: P_{1629} = (12, 4, 5, 1)
                                                                  132: P_{2735} = (14, 9, 9, 1)
79: P_{1641} = (8, 5, 5, 1)
                                                                  133: P_{2763} = (10, 11, 9, 1)
80: P_{1655} = (6, 6, 5, 1)
                                                                  134: P_{2768} = (15, 11, 9, 1)
81: P_{1703} = (6, 9, 5, 1)
                                                                  135: P_{2777} = (8, 12, 9, 1)
                                                                  136: P_{2791} = (6, 13, 9, 1)
82: P_{1759} = (14, 12, 5, 1)
83: P_{1763} = (2, 13, 5, 1)
                                                                  137: P_{2792} = (7, 13, 9, 1)
84: P_{1770} = (9, 13, 5, 1)
                                                                  138: P_{2828} = (11, 15, 9, 1)
85: P_{1799} = (6, 15, 5, 1)
                                                                  139: P_{2859} = (10, 1, 10, 1)
86: P_{1800} = (7, 15, 5, 1)
                                                                  140: P_{2905} = (8, 4, 10, 1)
87: P_{1812} = (3,0,6,1)
                                                                  141: P_{2908} = (11, 4, 10, 1)
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142: P_{3004} = (11, 10, 10, 1)
                                                                177: P_{3742} = (13, 8, 13, 1)
                                                                178: P_{3751} = (6, 9, 13, 1)
143: P_{3035} = (10, 12, 10, 1)
144: P_{3040} = (15, 12, 10, 1)
                                                                179: P_{3752} = (7, 9, 13, 1)
                                                                180: P_{3766} = (5, 10, 13, 1)
145: P_{3046} = (5, 13, 10, 1)
146: P_{3051} = (10, 13, 10, 1)
                                                                181: P_{3771} = (10, 10, 13, 1)
147: P_{3060} = (3, 14, 10, 1)
                                                                182: P_{3804} = (11, 12, 13, 1)
148: P_{3068} = (11, 14, 10, 1)
                                                                183: P_{3816} = (7, 13, 13, 1)
149: P_{3116} = (11, 1, 11, 1)
                                                                184: P_{3845} = (4, 15, 13, 1)
150: P_{3126} = (5, 2, 11, 1)
                                                                185: P_{3860} = (3, 0, 14, 1)
151: P_{3131} = (10, 2, 11, 1)
                                                                186: P_{3915} = (10, 3, 14, 1)
152: P_{3188} = (3, 6, 11, 1)
                                                                187: P_{3925} = (4, 4, 14, 1)
153: P_{3196} = (11, 6, 11, 1)
                                                                188: P_{3935} = (14, 4, 14, 1)
154: P_{3209} = (8, 7, 11, 1)
                                                                189: P_{3968} = (15, 6, 14, 1)
155: P_{3212} = (11, 7, 11, 1)
                                                                190: P_{3981} = (12, 7, 14, 1)
                                                                191: P_{3982} = (13, 7, 14, 1)
156: P_{3243} = (10, 9, 11, 1)
157: P_{3248} = (15, 9, 11, 1)
                                                                192: P_{3998} = (13, 8, 14, 1)
158: P_{3275} = (10, 11, 11, 1)
                                                                193: P_{4020} = (3, 10, 14, 1)
                                                                194: P_{4028} = (11, 10, 14, 1)
159: P_{3360} = (15, 0, 12, 1)
160: P_{3383} = (6, 2, 12, 1)
                                                                195: P_{4055} = (6, 12, 14, 1)
161: P_{3384} = (7, 2, 12, 1)
                                                                196: P_{4083} = (2, 14, 14, 1)
162: P_{3405} = (12, 3, 12, 1)
                                                                197: P_{4110} = (13, 15, 14, 1)
                                                                198: P_{4118} = (5, 0, 15, 1)
163: P_{3439} = (14, 5, 12, 1)
164: P_{3497} = (8, 9, 12, 1)
                                                                199: P_{4134} = (5, 1, 15, 1)
165: P_{3515} = (10, 10, 12, 1)
                                                                200: P_{4139} = (10, 1, 15, 1)
166: P_{3520} = (15, 10, 12, 1)
                                                                201: P_{4152} = (7, 2, 15, 1)
167: P_{3543} = (6, 12, 12, 1)
                                                                202: P_{4199} = (6, 5, 15, 1)
168: P_{3564} = (11, 13, 12, 1)
                                                                203: P_{4200} = (7, 5, 15, 1)
169: P_{3575} = (6, 14, 12, 1)
                                                                204: P_{4232} = (7, 7, 15, 1)
170: P_{3587} = (2, 15, 12, 1)
                                                                205: P_{4268} = (11, 9, 15, 1)
171: P_{3594} = (9, 15, 12, 1)
                                                                206: P_{4307} = (2, 12, 15, 1)
172: P_{3606} = (5, 0, 13, 1)
                                                                207: P_{4314} = (9, 12, 15, 1)
173: P_{3636} = (3, 2, 13, 1)
                                                                208: P_{4325} = (4, 13, 15, 1)
174: P_{3672} = (7, 4, 13, 1)
                                                                209: P_{4350} = (13, 14, 15, 1)
175: P_{3683} = (2, 5, 13, 1)
                                                                210: P_{4356} = (3, 15, 15, 1)
176: P_{3690} = (9, 5, 13, 1)
```

Line Intersection Graph

	012345
0	011010
1	100101
2	100110
3	011001
4	101001
5	$\begin{matrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{matrix}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{35}	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_{35}	P_{3250}	P_{3010}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_3	P_{179}	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_{3250}	P_{179}	P_4

${\bf Line~4~intersects}$

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_3	P_3	P_{195}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{3010}	P_4	P_{195}

The surface has 305 points: The points on the surface are:

(,)	(
$0: P_0 = (1, 0, 0, 0)$	$37: P_{619} = (10, 5, 1, 1)$	$74: P_{1117} = (12, 4, 3, 1)$
$1: P_3 = (0,0,0,1)$	$38: P_{624} = (15, 5, 1, 1)$	$75: P_{1141} = (4, 6, 3, 1)$
$2: P_4 = (1, 1, 1, 1)$	$39: P_{660} = (3, 8, 1, 1)$	$76: P_{1151} = (14, 6, 3, 1)$
$3: P_{35} = (0, 1, 1, 0)$	$40: P_{668} = (11, 8, 1, 1)$	$77: P_{1153} = (0,7,3,1)$
$4: P_{59} = (8, 2, 1, 0)$	$41: P_{689} = (0, 10, 1, 1)$	$78: P_{1162} = (9, 7, 3, 1)$
$5: P_{80} = (13, 3, 1, 0)$	$42: P_{699} = (10, 10, 1, 1)$	$79: P_{1181} = (12, 8, 3, 1)$
$6: P_{98} = (15, 4, 1, 0)$	$43: P_{705} = (0, 11, 1, 1)$	$80: P_{1182} = (13, 8, 3, 1)$
$7: P_{106} = (7, 5, 1, 0)$	$44: P_{716} = (11, 11, 1, 1)$	$81: P_{1234} = (1, 12, 3, 1)$
$8: P_{124} = (9, 6, 1, 0)$	$45: P_{774} = (5, 15, 1, 1)$	$82: P_{1245} = (12, 12, 3, 1)$
$9: P_{133} = (2,7,1,0)$	$46: P_{779} = (10, 15, 1, 1)$	$83: P_{1266} = (1, 14, 3, 1)$
$10: P_{159} = (12, 8, 1, 0)$	$47: P_{790} = (5, 0, 2, 1)$	$84: P_{1275} = (10, 14, 3, 1)$
$11: P_{166} = (3, 9, 1, 0)$	$48: P_{817} = (0, 2, 2, 1)$	$85: P_{1305} = (8, 0, 4, 1)$
$12: P_{179} = (0, 10, 1, 0)$	$49: P_{821} = (4, 2, 2, 1)$	$86: P_{1345} = (0, 3, 4, 1)$
$13: P_{195} = (0, 11, 1, 0)$	$50: P_{834} = (1, 3, 2, 1)$	$87: P_{1357} = (12, 3, 4, 1)$
$14: P_{215} = (4, 12, 1, 0)$	$51: P_{840} = (7, 3, 2, 1)$	$88: P_{1361} = (0, 4, 4, 1)$
$15: P_{241} = (14, 13, 1, 0)$	$52: P_{866} = (1, 5, 2, 1)$	$89: P_{1370} = (9, 4, 4, 1)$
$16: P_{248} = (5, 14, 1, 0)$	$53: P_{876} = (11, 5, 2, 1)$	90: $P_{1378} = (1, 5, 4, 1)$
17: $P_{265} = (6, 15, 1, 0)$	$54: P_{882} = (1, 6, 2, 1)$	$91: P_{1389} = (12, 5, 4, 1)$
$18: P_{291} = (1, 1, 0, 1)$	$55: P_{894} = (13, 6, 2, 1)$	$92: P_{1405} = (12, 6, 4, 1)$
$19: P_{311} = (5, 2, 0, 1)$	$56: P_{931} = (2, 9, 2, 1)$	93: $P_{1406} = (13, 6, 4, 1)$
$20: P_{330} = (8, 3, 0, 1)$	$57: P_{938} = (9, 9, 2, 1)$	$94: P_{1409} = (0,7,4,1)$
$21: P_{346} = (8, 4, 0, 1)$	$58: P_{966} = (5, 11, 2, 1)$	$95: P_{1414} = (5,7,4,1)$
$22: P_{369} = (15, 5, 0, 1)$	$59: P_{971} = (10, 11, 2, 1)$	$96: P_{1426} = (1, 8, 4, 1)$
$23: P_{373} = (3, 6, 0, 1)$	$60: P_{983} = (6, 12, 2, 1)$	97: $P_{1435} = (10, 8, 4, 1)$
$24: P_{394} = (8,7,0,1)$	$61: P_{984} = (7, 12, 2, 1)$	$98: P_{1465} = (8, 10, 4, 1)$
$25: P_{405} = (3, 8, 0, 1)$	$62: P_{993} = (0, 13, 2, 1)$	99: $P_{1468} = (11, 10, 4, 1)$
$26: P_{433} = (15, 9, 0, 1)$	$63: P_{996} = (3, 13, 2, 1)$	$100: P_{1506} = (1, 13, 4, 1)$
$27: P_{435} = (1, 10, 0, 1)$	$64: P_{1025} = (0, 15, 2, 1)$	$101: P_{1512} = (7, 13, 4, 1)$
$28: P_{451} = (1, 11, 0, 1)$	$65: P_{1032} = (7, 15, 2, 1)$	$102: P_{1525} = (4, 14, 4, 1)$
$29: P_{481} = (15, 12, 0, 1)$	$66: P_{1049} = (8, 0, 3, 1)$	$103: P_{1535} = (14, 14, 4, 1)$
$30: P_{487} = (5, 13, 0, 1)$	$67: P_{1065} = (8, 1, 3, 1)$	$104: P_{1568} = (15, 0, 5, 1)$
$31: P_{501} = (3, 14, 0, 1)$	$68: P_{1068} = (11, 1, 3, 1)$	$105: P_{1579} = (10, 1, 5, 1)$
$32: P_{519} = (5, 15, 0, 1)$	$69: P_{1074} = (1, 2, 3, 1)$	$106: P_{1584} = (15, 1, 5, 1)$
$33: P_{531} = (1,0,1,1)$	$70: P_{1080} = (7, 2, 3, 1)$	$107: P_{1586} = (1, 2, 5, 1)$
$34: P_{546} = (0, 1, 1, 1)$	$71: P_{1089} = (0, 3, 3, 1)$	$108: P_{1596} = (11, 2, 5, 1)$
$35: P_{585} = (8, 3, 1, 1)$	$72: P_{1094} = (5, 3, 3, 1)$	$109: P_{1618} = (1, 4, 5, 1)$
$36: P_{588} = (11, 3, 1, 1)$	$73: P_{1105} = (0,4,3,1)$	$110: P_{1629} = (12, 4, 5, 1)$

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111: P_{1633} = (0, 5, 5, 1)
                                           165: P_{2382} = (13, 3, 8, 1)
                                                                                       219: P_{3188} = (3, 6, 11, 1)
112: P_{1641} = (8, 5, 5, 1)
                                           166: P_{2386} = (1,4,8,1)
                                                                                       220: P_{3196} = (11, 6, 11, 1)
                                                                                       221: P_{3209} = (8, 7, 11, 1)
113: P_{1650} = (1, 6, 5, 1)
                                           167: P_{2395} = (10, 4, 8, 1)
                                           168: P_{2417} = (0, 6, 8, 1)
                                                                                       222: P_{3212} = (11, 7, 11, 1)
114: P_{1655} = (6, 6, 5, 1)
115: P_{1697} = (0, 9, 5, 1)
                                           169: P_{2419} = (2, 6, 8, 1)
                                                                                       223: P_{3243} = (10, 9, 11, 1)
116: P_{1703} = (6, 9, 5, 1)
                                           170: P_{2437} = (4,7,8,1)
                                                                                       224: P_{3248} = (15, 9, 11, 1)
117: P_{1745} = (0, 12, 5, 1)
                                           171: P_{2447} = (14, 7, 8, 1)
                                                                                       225: P_{3249} = (0, 10, 11, 1)
118: P_{1759} = (14, 12, 5, 1)
                                           172: P_{2449} = (0, 8, 8, 1)
                                                                                       226: P_{3250} = (1, 10, 11, 1)
119: P_{1763} = (2, 13, 5, 1)
                                                                                       227: P_{3265} = (0, 11, 11, 1)
                                           173: P_{2464} = (15, 8, 8, 1)
120: P_{1770} = (9, 13, 5, 1)
                                           174: P_{2466} = (1, 9, 8, 1)
                                                                                       228: P_{3275} = (10, 11, 11, 1)
121: P_{1799} = (6, 15, 5, 1)
                                           175: P_{2471} = (6, 9, 8, 1)
                                                                                       229: P_{3360} = (15, 0, 12, 1)
122: P_{1800} = (7, 15, 5, 1)
                                           176: P_{2530} = (1, 13, 8, 1)
                                                                                       230: P_{3383} = (6, 2, 12, 1)
123: P_{1812} = (3, 0, 6, 1)
                                           177: P_{2542} = (13, 13, 8, 1)
                                                                                       231: P_{3384} = (7, 2, 12, 1)
124: P_{1842} = (1, 2, 6, 1)
                                                                                       232: P_{3394} = (1, 3, 12, 1)
                                           178: P_{2545} = (0, 14, 8, 1)
                                                                                       233: P_{3405} = (12, 3, 12, 1)
125: P_{1854} = (13, 2, 6, 1)
                                           179: P_{2558} = (13, 14, 8, 1)
126: P_{1861} = (4, 3, 6, 1)
                                           180: P_{2592} = (15, 0, 9, 1)
                                                                                       234: P_{3425} = (0, 5, 12, 1)
                                           181 : P_{2611} = (2, 2, 9, 1)
                                                                                       235: P_{3439} = (14, 5, 12, 1)
127: P_{1871} = (14, 3, 6, 1)
                                           182: P_{2618} = (9, 2, 9, 1)
128: P_{1885} = (12, 4, 6, 1)
                                                                                       236: P_{3489} = (0, 9, 12, 1)
129: P_{1886} = (13, 4, 6, 1)
                                                                                       237: P_{3497} = (8, 9, 12, 1)
                                           183: P_{2657} = (0, 5, 9, 1)
130: P_{1890} = (1, 5, 6, 1)
                                           184: P_{2663} = (6, 5, 9, 1)
                                                                                       238: P_{3515} = (10, 10, 12, 1)
131: P_{1895} = (6, 5, 6, 1)
                                           185: P_{2690} = (1,7,9,1)
                                                                                       239: P_{3520} = (15, 10, 12, 1)
132: P_{1905} = (0, 6, 6, 1)
                                           186: P_{2701} = (12, 7, 9, 1)
                                                                                       240: P_{3537} = (0, 12, 12, 1)
                                           187: P_{2706} = (1, 8, 9, 1)
                                                                                       241: P_{3543} = (6, 12, 12, 1)
133: P_{1918} = (13, 6, 6, 1)
134: P_{1922} = (1,7,6,1)
                                           188: P_{2711} = (6, 8, 9, 1)
                                                                                       242: P_{3554} = (1, 13, 12, 1)
135: P_{1931} = (10, 7, 6, 1)
                                           189: P_{2721} = (0, 9, 9, 1)
                                                                                       243: P_{3564} = (11, 13, 12, 1)
136: P_{1937} = (0, 8, 6, 1)
                                           190: P_{2735} = (14, 9, 9, 1)
                                                                                       244: P_{3570} = (1, 14, 12, 1)
137: P_{1939} = (2, 8, 6, 1)
                                           191: P_{2763} = (10, 11, 9, 1)
                                                                                       245: P_{3575} = (6, 14, 12, 1)
138: P_{1988} = (3, 11, 6, 1)
                                           192: P_{2768} = (15, 11, 9, 1)
                                                                                       246: P_{3587} = (2, 15, 12, 1)
                                                                                       247: P_{3594} = (9, 15, 12, 1)
139: P_{1996} = (11, 11, 6, 1)
                                           193: P_{2769} = (0, 12, 9, 1)
                                           194 : P_{2777} = (8, 12, 9, 1)
                                                                                       248: P_{3606} = (5, 0, 13, 1)
140: P_{2033} = (0, 14, 6, 1)
141: P_{2048} = (15, 14, 6, 1)
                                           195: P_{2791} = (6, 13, 9, 1)
                                                                                       249: P_{3633} = (0, 2, 13, 1)
142: P_{2073} = (8, 0, 7, 1)
                                           196: P_{2792} = (7, 13, 9, 1)
                                                                                       250: P_{3636} = (3, 2, 13, 1)
                                                                                       251: P_{3666} = (1, 4, 13, 1)
143: P_{2113} = (0, 3, 7, 1)
                                           197: P_{2818} = (1, 15, 9, 1)
144: P_{2122} = (9, 3, 7, 1)
                                           198: P_{2828} = (11, 15, 9, 1)
                                                                                       252: P_{3672} = (7, 4, 13, 1)
145: P_{2129} = (0, 4, 7, 1)
                                           199: P_{2834} = (1, 0, 10, 1)
                                                                                       253: P_{3683} = (2, 5, 13, 1)
                                           200: P_{2849} = (0, 1, 10, 1)
146: P_{2134} = (5, 4, 7, 1)
                                                                                       254: P_{3690} = (9, 5, 13, 1)
147: P_{2162} = (1, 6, 7, 1)
                                           201: P_{2859} = (10, 1, 10, 1)
                                                                                       255: P_{3730} = (1, 8, 13, 1)
                                                                                       256: P_{3742} = (13, 8, 13, 1)
148: P_{2171} = (10, 6, 7, 1)
                                           202: P_{2905} = (8, 4, 10, 1)
149: P_{2177} = (0, 7, 7, 1)
                                           203: P_{2908} = (11, 4, 10, 1)
                                                                                       257: P_{3751} = (6, 9, 13, 1)
                                                                                       258: P_{3752} = (7, 9, 13, 1)
150: P_{2189} = (12, 7, 7, 1)
                                           204: P_{2993} = (0, 10, 10, 1)
151: P_{2197} = (4, 8, 7, 1)
                                           205: P_{3004} = (11, 10, 10, 1)
                                                                                       259: P_{3766} = (5, 10, 13, 1)
152: P_{2207} = (14, 8, 7, 1)
                                           206: P_{3009} = (0, 11, 10, 1)
                                                                                       260: P_{3771} = (10, 10, 13, 1)
153: P_{2210} = (1, 9, 7, 1)
                                           207: P_{3010} = (1, 11, 10, 1)
                                                                                       261: P_{3794} = (1, 12, 13, 1)
                                           208: P_{3035} = (10, 12, 10, 1)
                                                                                       262: P_{3804} = (11, 12, 13, 1)
154: P_{2221} = (12, 9, 7, 1)
155: P_{2249} = (8, 11, 7, 1)
                                           209: P_{3040} = (15, 12, 10, 1)
                                                                                       263: P_{3809} = (0, 13, 13, 1)
                                                                                       264: P_{3816} = (7, 13, 13, 1)
                                           210: P_{3046} = (5, 13, 10, 1)
156: P_{2252} = (11, 11, 7, 1)
                                                                                       265: P_{3841} = (0, 15, 13, 1)
157: P_{2301} = (12, 14, 7, 1)
                                           211: P_{3051} = (10, 13, 10, 1)
158: P_{2302} = (13, 14, 7, 1)
                                           212: P_{3060} = (3, 14, 10, 1)
                                                                                       266: P_{3845} = (4, 15, 13, 1)
                                                                                       267: P_{3860} = (3, 0, 14, 1)
159: P_{2306} = (1, 15, 7, 1)
                                           213: P_{3068} = (11, 14, 10, 1)
                                           214: P_{3090} = (1, 0, 11, 1)
160: P_{2312} = (7, 15, 7, 1)
                                                                                       268: P_{3906} = (1, 3, 14, 1)
161: P_{2324} = (3, 0, 8, 1)
                                           215: P_{3105} = (0, 1, 11, 1)
                                                                                       269: P_{3915} = (10, 3, 14, 1)
162: P_{2340} = (3, 1, 8, 1)
                                           216: P_{3116} = (11, 1, 11, 1)
                                                                                       270: P_{3925} = (4, 4, 14, 1)
163: P_{2348} = (11, 1, 8, 1)
                                           217: P_{3126} = (5, 2, 11, 1)
                                                                                       271: P_{3935} = (14, 4, 14, 1)
164: P_{2381} = (12, 3, 8, 1)
                                           218: P_{3131} = (10, 2, 11, 1)
                                                                                       272: P_{3953} = (0, 6, 14, 1)
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273: P_{3968} = (15, 6, 14, 1)
                                           284: P_{4098} = (1, 15, 14, 1)
                                                                                      295: P_{4258} = (1, 9, 15, 1)
                                                                                      296: P_{4268} = (11, 9, 15, 1)
274: P_{3981} = (12, 7, 14, 1)
                                           285: P_{4110} = (13, 15, 14, 1)
                                                                                      297: P_{4307} = (2, 12, 15, 1)
275: P_{3982} = (13, 7, 14, 1)
                                           286: P_{4118} = (5, 0, 15, 1)
276: P_{3985} = (0, 8, 14, 1)
                                           287: P_{4134} = (5, 1, 15, 1)
                                                                                      298 : P_{4314} = (9, 12, 15, 1)
277: P_{3998} = (13, 8, 14, 1)
                                           288: P_{4139} = (10, 1, 15, 1)
                                                                                      299: P_{4321} = (0, 13, 15, 1)
278: P_{4020} = (3, 10, 14, 1)
                                           289 : P_{4145} = (0, 2, 15, 1)
                                                                                      300: P_{4325} = (4, 13, 15, 1)
279: P_{4028} = (11, 10, 14, 1)
                                           290: P_{4152} = (7, 2, 15, 1)
                                                                                      301: P_{4338} = (1, 14, 15, 1)
280: P_{4050} = (1, 12, 14, 1)
                                           291: P_{4199} = (6, 5, 15, 1)
                                                                                      302: P_{4350} = (13, 14, 15, 1)
281: P_{4055} = (6, 12, 14, 1)
                                           292: P_{4200} = (7, 5, 15, 1)
                                                                                      303: P_{4353} = (0, 15, 15, 1)
                                                                                      304: P_{4356} = (3, 15, 15, 1)
282: P_{4081} = (0, 14, 14, 1)
                                           293: P_{4226} = (1, 7, 15, 1)
283: P_{4083} = (2, 14, 14, 1)
                                           294: P_{4232} = (7, 7, 15, 1)
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