Rank-68 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

(0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 632

General information

Number of lines	25
Number of points	37
Number of singular points	5
Number of Eckardt points	0
Number of double points	0
Number of single points	16
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{25}
Type of lines on points	9, 5^{20}, 1^{16}

Singular Points

The surface has 5 singular points:

0 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
1 : $P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
2 : $P_{12} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
3 : $P_{63} = \mathbf{P}(\omega,\omega,\omega,1) = \mathbf{P}(2,2,2,1)$
4 : $P_{84} = \mathbf{P}(\omega^2,\omega^2,\omega^2,1) = \mathbf{P}(3,3,3,1)$

The 25 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \mathbf{Pl}(0,0,0,0,1,0)_{25}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{5} = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \omega^2 \end{array}\right]_{13} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right]_{13} = \mathbf{Pl}(2,0,2,0,1,0)_{41}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \omega \end{array}\right]_{9} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \end{array}\right]_{9} = \mathbf{Pl}(3,0,3,0,1,0)_{49}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{41} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_6 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{104} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_7 = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{85} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{85} = \mathbf{Pl}(1,1,1,1,0,0)_{16}$$

$$\ell_9 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & 0 \end{array}\right]_{253} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \end{array}\right]_{253} = \mathbf{Pl}(1,1,2,1,0,0)_{19}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & 0 \end{array}\right]_{169} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right]_{169} = \mathbf{Pl}(1,1,3,1,0,0)_{22}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{125} = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{89} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & \omega^2 \end{array}\right]_{265} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \end{array}\right]_{265} = \mathbf{Pl}(1,1,2,1,1,0)_{77}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & \omega \end{array}\right]_{177} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 \end{array}\right]_{177} = \mathbf{Pl}(1,1,3,1,1,0)_{80}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & \omega^2 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{335} = \left[\begin{array}{cccc} 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{335} = \mathbf{Pl}(0,3,0,3,1,0)_{73}$$

$$\ell_{16} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & \omega^2 \end{array}\right]_{181} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right]_{181} = \mathbf{Pl}(2,3,2,3,1,0)_{96}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & \omega \end{array}\right]_{93} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right]_{93} = \mathbf{Pl}(2,3,3,3,1,0)_{99}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & 1 \end{array}\right]_{257} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right]_{257} = \mathbf{Pl}(2,3,1,3,1,0)_{93}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & \omega & \omega & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{230} = \left[\begin{array}{cccc} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{230} = \mathbf{Pl}(0,2,0,2,1,0)_{65}$$

$$\ell_{20} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & \omega \end{array}\right]_{261} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 2 \end{array}\right]_{261} = \mathbf{Pl}(3,2,3,2,1,0)_{91}$$

$$\ell_{21} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & \omega^2 \end{array}\right]_{97} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right]_{97} = \mathbf{Pl}(3,2,2,2,1,0)_{88}$$

$$\ell_{22} = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{173} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{173} = \mathbf{Pl}(3,2,1,2,1,0)_{85}$$

$$\ell_{23} = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_{24} = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

Rank of lines: (1, 20, 5, 13, 9, 41, 104, 345, 85, 253, 169, 125, 89, 265, 177, 335, 181, 93, 257, 230, 261, 97, 173, 355, 350)
Rank of points on Klein quadric: (3, 25, 33, 41, 49, 53, 29, 13, 16, 19, 22, 57, 74, 77, 80, 73, 96, 99, 93, 65, 91, 88, 85, 15, 14)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_5
1 : $P_8 = (1,0,1,0)$ lies on line ℓ_6
2 : $P_{15} = (0,2,1,0)$ lies on line ℓ_{23}
3 : $P_{19} = (0,3,1,0)$ lies on line ℓ_{24}
4 : $P_{27} = (1,1,0,1)$ lies on line ℓ_5
5 : $P_{32} = (2,2,0,1)$ lies on line ℓ_5
6 : $P_{37} = (3,3,0,1)$ lies on line ℓ_5
7 : $P_{39} = (1,0,1,1)$ lies on line ℓ_6
8 : $P_{45} = (0,2,1,1)$ lies on line ℓ_{23}
9 : $P_{49} = (0,3,1,1)$ lies on line ℓ_{24}
10 : $P_{55} = (2,0,2,1)$ lies on line ℓ_6
11 : $P_{57} = (0,1,2,1)$ lies on line ℓ_{24}
12 : $P_{65} = (0,3,2,1)$ lies on line ℓ_{23}
13 : $P_{72} = (3,0,3,1)$ lies on line ℓ_6
14 : $P_{73} = (0,1,3,1)$ lies on line ℓ_{23}
15 : $P_{77} = (0,2,3,1)$ lies on line ℓ_{24}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
3	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
5	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1
6	0	1	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1
7	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
10	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
11	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0
13	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
14	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0
17	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0
18	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
20	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0
21	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0
22	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
23	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	1
24	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_0	P_0	P_0	P_{11}	P_{11}	P_{11}	P_{11}	P_{12}	P_{12}	P_{12}	P_{12}	P_{13}	P_{13}	P_{13}	P_{13}	P_{14}	P_{14}	P_{14}	P_{14}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_0	P_0	P_0	P_0	P_3	P_3	P_3	P_{23}	P_{24}	P_{25}	P_3	P_{23}	P_{24}	P_{25}	P_3	P_{25}	P_{23}	P_{24}	P_3	P_{24}	P_{23}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_0	P_0	P_0	P_{42}	P_4	P_{43}	P_{44}	P_4	P_{42}	P_{44}	P_{43}	P_{43}	P_4	P_{44}	P_{42}	P_{44}	P_4	P_{43}	P_{42}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_0	P_0	P_0	P_{61}	P_{62}	P_{63}	P_{64}	P_{63}	P_{64}	P_{61}	P_{62}	P_{64}	P_{61}	P_{63}	P_{62}	P_{62}	P_{64}	P_{61}	P_{63}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_0	P_0	P_0	P_{81}	P_{82}	P_{83}	P_{84}	P_{84}	P_{83}	P_{82}	P_{81}	P_{82}	P_{83}	P_{81}	P_{84}	P_{83}	P_{81}	P_{84}	P_{82}

Line 5 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{23}	ℓ_{24}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{23}	ℓ_{24}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_3	P_{11}	P_{11}	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{61}	P_{81}	P_{42}	P_3	P_{81}	P

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{11}	P_{23}	P_4	P_{62}	P_{82}	P_{11}	P_{11}	P_{11}	P_4	P_{23}	P_{82}	P_{62}	P_{82}	P_4	P_{23}	P_{62}	P_{62}	P_4	P_{23}	P_{82}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{11}	P_{24}	P_{43}	P_{63}	P_{83}	P_{11}	P_{11}	P_{11}	P_{63}	P_{83}	P_{24}	P_{43}	P_{43}	P_{83}	P_{63}	P_{24}	P_{83}	P_{24}	P_{43}	P_{63}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{11}	P_{25}	P_{44}	P_{64}	P_{84}	P_{11}	P_{11}	P_{11}	P_{84}	P_{64}	P_{44}	P_{25}	P_{64}	P_{25}	P_{44}	P_{84}	P_{44}	P_{64}	P_{84}	P_{2}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_{12}	P_3	P_4	P_{63}	P_{84}	P_3	P_3	P_3	P_4	P_{63}	P_{84}	P_{12}	P_{12}	P_{12}	P_3	P_4	P_{63}	P_{84}	P_3	P_4	P_{84}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{12}	P_{23}	P_{42}	P_{64}	P_{83}	P_{42}	P_{23}	P_{83}	P_{64}	P_{12}	P_{12}	P_{12}	P_{64}	P_{83}	P_{23}	P_{42}	P_{83}	P_{64}	P_{23}	P_{4}

Line 13 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{12}	P_{24}	P_{44}	P_{61}	P_{82}	P_{61}	P_{82}	P_{24}	P_{44}	P_{12}	P_{12}	P_{12}	P_{82}	P_{61}	P_{44}	P_{24}	P_{44}	P_{24}	P_{61}	P_{8}

Line 14 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{12}	P_{25}	P_{43}	P_{62}	P_{81}	P_{81}	P_{62}	P_{43}	P_{25}	P_{12}	P_{12}	P_{12}	P_{43}	P_{25}	P_{81}	P_{62}	P_{62}	P_{81}	P_{43}	P_{2}

Line 15 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{1}
in point	P_{13}	P_3	P_{43}	P_{64}	P_{82}	P_3	P_3	P_3	P_{82}	P_{43}	P_{64}	P_3	P_{64}	P_{82}	P_{43}	P_{13}	P_{13}	P_{13}	P_3	P_{64}	P

Line 16 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{13}	P_{25}	P_4	P_{61}	P_{83}	P_{61}	P_4	P_{83}	P_{25}	P_4	P_{83}	P_{61}	P_{25}	P_{13}	P_{13}	P_{13}	P_{83}	P_4	P_{61}	P_{25}

Line 17 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{13}	P_{23}	P_{44}	P_{63}	P_{81}	P_{81}	P_{23}	P_{63}	P_{44}	P_{63}	P_{23}	P_{44}	P_{81}	P_{13}	P_{13}	P_{13}	P_{44}	P_{81}	P_{23}	P_{6}

Line 18 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_{13}	P_{24}	P_{42}	P_{62}	P_{84}	P_{42}	P_{62}	P_{24}	P_{84}	P_{84}	P_{42}	P_{24}	P_{62}	P_{13}	P_{13}	P_{13}	P_{62}	P_{24}	P_{84}	P_{4}

Line 19 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ
in point	P_{14}	P_3	P_{44}	P_{62}	P_{83}	P_3	P_3	P_3	P_{62}	P_{83}	P_{44}	P_3	P_{83}	P_{44}	P_{62}	P_3	P_{83}	P_{44}	P_{62}	P_{14}	P

Line 20 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}
in point	P_{14}	P_{24}	P_4	P_{64}	P_{81}	P_{81}	P_4	P_{24}	P_{64}	P_4	P_{64}	P_{24}	P_{81}	P_{64}	P_4	P_{81}	P_{24}	P_{14}	P_{14}	P_{14}

Line 21 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{22}
in point	P_{14}	P_{23}	P_{43}	P_{61}	P_{84}	P_{61}	P_{23}	P_{43}	P_{84}	P_{84}	P_{23}	P_{61}	P_{43}	P_{43}	P_{61}	P_{23}	P_{84}	P_{14}	P_{14}	P_1

Line 22 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_{14}	P_{25}	P_{42}	P_{63}	P_{82}	P_{42}	P_{82}	P_{63}	P_{25}	P_{63}	P_{42}	P_{82}	P_{25}	P_{82}	P_{25}	P_{63}	P_{42}	P_{14}	P_{14}	P_1

Line 23 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{24}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 24 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{23}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 37 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_3 = (0,0,0,1)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_8 = (1,0,1,0)$
$5: P_{11} = (0,1,1,0)$
$6: P_{12} = (1,1,1,0)$
$7: P_{13} = (2,1,1,0)$
$8: P_{14} = (3,1,1,0)$
$9: P_{15} = (0,2,1,0)$
$10: P_{19} = (0,3,1,0)$
$11: P_{23} = (1,0,0,1)$
$12: P_{24} = (2,0,0,1)$
$13: P_{25} = (3,0,0,1)$
$14: P_{27} = (1,1,0,1)$
$15: P_{32} = (2,2,0,1)$
$16: P_{37} = (3,3,0,1)$
$17: P_{39} = (1,0,1,1)$
$18: P_{42} = (0,1,1,1)$
$19: P_{43} = (2,1,1,1)$
$20: P_{44} = (3,1,1,1)$
$21: P_{45} = (0,2,1,1)$
$22: P_{49} = (0,3,1,1)$
$23: P_{55} = (2,0,2,1)$
$24: P_{57} = (0,1,2,1)$
$25: P_{61} = (0,2,2,1)$
$26: P_{62} = (1,2,2,1)$
$27: P_{63} = (2,2,2,1)$
$28: P_{64} = (3,2,2,1)$
$29: P_{65} = (0,3,2,1)$
$30: P_{72} = (3,0,3,1)$
$31: P_{73} = (0,1,3,1)$
$32: P_{77} = (0,2,3,1)$
$33: P_{81} = (0,3,3,1)$
$34: P_{82} = (1,3,3,1)$
$35: P_{83} = (2,3,3,1)$
$36: P_{84} = (3,3,3,1)$