Rank-74247 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286331413

General information

Number of lines	21
Number of points	353
Number of singular points	1
Number of Eckardt points	0
Number of double points	75
Number of single points	201
Number of points off lines	76
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{21}
Type of lines on points	6, 2^{75}, 1^{201}, 0^{76}

Singular Points

The surface has 1 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_{1} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{17} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{17} = \mathbf{Pl}(1,0,1,0,1,0)_{321}$$

$$\ell_{2} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{9} & \delta^{12} \end{array}\right]_{53} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 3 \end{array}\right]_{53} = \mathbf{Pl}(8,0,3,0,1,0)_{390}$$

$$\ell_{3} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{3} & \delta^{9} \end{array}\right]_{88} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 8 & 5 \end{array}\right]_{88} = \mathbf{Pl}(15,0,5,0,1,0)_{459}$$

$$\ell_{4} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{6} & \delta^{3} \end{array}\right]_{143} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 15 & 8 \end{array}\right]_{143} = \mathbf{Pl}(3,0,8,0,1,0)_{540}$$

$$\ell_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{12} & \delta^{6} \end{array}\right]_{243} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 15 \end{array}\right]_{243} = \mathbf{Pl}(5,0,15,0,1,0)_{759}$$

$$\ell_{6} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{4658} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{4658} = \mathbf{Pl}(1,0,1,1,1,1)_{9427}$$

$$\ell_{7} = \left[\begin{array}{cccc} 1 & 0 & \delta^{9} & \delta^{6} \\ 0 & 1 & \delta^{6} & \delta^{3} \end{array}\right]_{67028} = \left[\begin{array}{cccc} 1 & 0 & 5 & 15 \\ 0 & 1 & 15 & 8 \end{array}\right]_{67028} = \mathbf{Pl}(3,0,8,3,5,1)_{25966}$$

$$\ell_{8} = \left[\begin{array}{cccc} 1 & 0 & \delta^{3} & \delta^{12} \\ 0 & 1 & \delta^{12} & \delta^{6} \end{array}\right]_{15531} = \left[\begin{array}{cccc} 1 & 0 & 8 & 3 \\ 0 & 1 & 3 & 15 \end{array}\right]_{15531} = \mathbf{Pl}(5,0,15,5,8,1)_{38425}$$

$$\ell_{9} = \left[\begin{array}{cccc} 1 & 0 & \delta^{6} & \delta^{9} \\ 0 & 1 & \delta^{9} & \delta^{12} \end{array}\right]_{25988} = \left[\begin{array}{cccc} 1 & 0 & 15 & 5 \\ 0 & 1 & 5 & 3 \end{array}\right]_{25988} = \mathbf{Pl}(8,0,3,8,15,1)_{66616}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & \delta^{12} & \delta^{3} \\ 0 & 1 & \delta^{3} & \delta^{9} \end{array}\right]_{35851} = \left[\begin{array}{cccc} 1 & 0 & 3 & 8 \\ 0 & 1 & 8 & 5 \end{array}\right]_{35851} = \mathbf{Pl}(15,0,5,15,3,1)_{17725}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^{5} & 1 \end{array}\right]_{7125} = \left[\begin{array}{cccc} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 1 \end{array}\right]_{7125} = \mathbf{Pl}(10,11,10,11,11,1)_{52710}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 0 & \delta^{5} & 1 \\ 0 & 1 & \delta^{10} & 1 \end{array}\right]_{7397} = \left[\begin{array}{cccc} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 1 \end{array}\right]_{7397} = \mathbf{Pl}(11,10,11,10,10,1)_{48856}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & 0 & \delta^{7} & \delta^{3} \\ 0 & 1 & \delta^{8} & \delta^{9} \end{array}\right]_{36949} = \left[\begin{array}{cccc} 1 & 0 & 7 & 8 \\ 0 & 1 & 14 & 5 \end{array}\right]_{36949} = \mathbf{Pl}(2,12,9,13,4,1)_{23932}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & 0 & \delta^{2} & \delta^{3} \\ 0 & 1 & \delta^{13} & \delta^{9} \end{array}\right]_{36122} = \left[\begin{array}{cccc} 1 & 0 & 4 & 8 \\ 0 & 1 & 6 & 5 \end{array}\right]_{36122} = \mathbf{Pl}(13,9,12,2,7,1)_{36828}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & 0 & \delta^{14} & \delta^{6} \\ 0 & 1 & \delta & \delta^{3} \end{array}\right]_{68926} = \left[\begin{array}{cccc} 1 & 0 & 12 & 15 \\ 0 & 1 & 2 & 8 \end{array}\right]_{68926} = \mathbf{Pl}(4,6,14,7,9,1)_{45399}$$

$$\ell_{16} = \left[\begin{array}{cccc} 1 & 0 & \delta^{4} & \delta^{6} \\ 0 & 1 & \delta^{11} & \delta^{3} \end{array}\right]_{68118} = \left[\begin{array}{cccc} 1 & 0 & 9 & 15 \\ 0 & 1 & 13 & 8 \end{array}\right]_{68118} = \mathbf{Pl}(7,14,6,4,12,1)_{55947}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & 0 & \delta^{13} & \delta^{12} \\ 0 & 1 & \delta^{2} & \delta^{6} \end{array}\right]_{14986} = \left[\begin{array}{cccc} 1 & 0 & 6 & 3 \\ 0 & 1 & 4 & 15 \end{array}\right]_{14986} = \mathbf{Pl}(9,13,2,12,14,1)_{63269}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & 0 & \delta^{8} & \delta^{12} \\ 0 & 1 & \delta^{7} & \delta^{6} \end{array}\right]_{17173} = \left[\begin{array}{cccc} 1 & 0 & 14 & 3 \\ 0 & 1 & 7 & 15 \end{array}\right]_{17173} = \mathbf{Pl}(12,2,13,9,6,1)_{32957}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & 0 & \delta & \delta^{9} \\ 0 & 1 & \delta^{14} & \delta^{12} \end{array}\right]_{22446} = \left[\begin{array}{cccc} 1 & 0 & 2 & 5 \\ 0 & 1 & 12 & 3 \end{array}\right]_{22446} = \mathbf{Pl}(6,4,7,14,13,1)_{60236}$$

$$\ell_{20} = \left[\begin{array}{cccc} 1 & 0 & \delta^{11} & \delta^{9} \\ 0 & 1 & \delta^{4} & \delta^{12} \end{array}\right]_{25446} = \left[\begin{array}{cccc} 1 & 0 & 13 & 5 \\ 0 & 1 & 9 & 3 \end{array}\right]_{25446} = \mathbf{Pl}(14,7,4,6,2,1)_{14749}$$

Rank of lines: (0, 17, 53, 88, 143, 243, 4658, 67028, 15531, 25988, 35851, 7125, 7397, 36949, 36122, 68926, 68118, 14986, 17173, 22446, 25446)

Rank of points on Klein quadric: (0, 321, 390, 459, 540, 759, 9427, 25966, 38425, 66616, 17725, 52710, 48856, 23932, 36828, 45399, 55947, 63269, 32957, 60236, 14749)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 75 Double points:
The double points on the surface are:

$P_{5} = (1,1,0,0) = \ell_{0} \cap \ell_{6}$
$P_{7} = (3,1,0,0) = \ell_{0} \cap \ell_{7}$
$P_{9} = (5,1,0,0) = \ell_{0} \cap \ell_{8}$
$P_{12} = (8,1,0,0) = \ell_{0} \cap \ell_{9}$
$P_{19} = (15,1,0,0) = \ell_{0} \cap \ell_{10}$
$P_{546} = (0,1,1,1) = \ell_{1} \cap \ell_{6}$
$P_{557} = (12,1,1,1) = \ell_{1} \cap \ell_{13}$
$P_{551} = (6,1,1,1) = \ell_{1} \cap \ell_{15}$
$P_{558} = (13,1,1,1) = \ell_{1} \cap \ell_{17}$
$P_{552} = (7,1,1,1) = \ell_{1} \cap \ell_{20}$
$P_{1169} = (0,8,3,1) = \ell_{2} \cap \ell_{9}$
$P_{1178} = (9,8,3,1) = \ell_{2} \cap \ell_{12}$
$P_{1175} = (6,8,3,1) = \ell_{2} \cap \ell_{14}$
$P_{1173} = (4,8,3,1) = \ell_{2} \cap \ell_{16}$
$P_{1180} = (11,8,3,1) = \ell_{2} \cap \ell_{17}$
$P_{1793} = (0,15,5,1) = \ell_{3} \cap \ell_{10}$
$P_{1807} = (14,15,5,1) = \ell_{3} \cap \ell_{11}$
$P_{1806} = (13,15,5,1) = \ell_{3} \cap \ell_{16}$
$P_{1802} = (9,15,5,1) = \ell_{3} \cap \ell_{18}$
$P_{1803} = (10,15,5,1) = \ell_{3} \cap \ell_{20}$
$P_{2369} = (0,3,8,1) = \ell_{4} \cap \ell_{7}$
$P_{2371} = (2,3,8,1) = \ell_{4} \cap \ell_{12}$
$P_{2380} = (11,3,8,1) = \ell_{4} \cap \ell_{13}$
$P_{2376} = (7,3,8,1) = \ell_{4} \cap \ell_{18}$
$P_{2383} = (14,3,8,1) = \ell_{4} \cap \ell_{19}$
$P_{4193} = (0,5,15,1) = \ell_{5} \cap \ell_{8}$
$P_{4197} = (4,5,15,1) = \ell_{5} \cap \ell_{11}$
$P_{4195} = (2,5,15,1) = \ell_{5} \cap \ell_{14}$
$P_{4203} = (10,5,15,1) = \ell_{5} \cap \ell_{15}$
$P_{4205} = (12,5,15,1) = \ell_{5} \cap \ell_{19}$
$P_{715} = (10,11,1,1) = \ell_{6} \cap \ell_{11}$
$P_{700} = (11,10,1,1) = \ell_{6} \cap \ell_{12}$
$P_{768} = (15,14,1,1) = \ell_{6} \cap \ell_{14}$
$P_{564} = (3,2,1,1) = \ell_{6} \cap \ell_{16}$
$P_{598} = (5,4,1,1) = \ell_{6} \cap \ell_{18}$
$P_{681} = (8,9,1,1) = \ell_{6} \cap \ell_{19}$
$P_{2356} = (3,2,8,1) = \ell_{7} \cap \ell_{11}$
$P_{2521} = (8,12,8,1) = \ell_{7} \cap \ell_{14}$
$P_{2445} = (12,7,8,1) = \ell_{7} \cap \ell_{15}$
$P_{2394} = (9,4,8,1) = \ell_{7} \cap \ell_{16}$
$P_{2432} = (15,6,8,1) = \ell_{7} \cap \ell_{17}$
$P_{2498} = (1,11,8,1) = \ell_{7} \cap \ell_{20}$
$P_{4182} = (5,4,15,1) = \ell_{8} \cap \ell_{12}$
$P_{4274} = (1,10,15,1) = \ell_{8} \cap \ell_{13}$
$P_{4224} = (15,6,15,1) = \ell_{8} \cap \ell_{16}$
$P_{4311} = (6,12,15,1) = \ell_{8} \cap \ell_{17}$
$P_{4271} = (14,9,15,1) = \ell_{8} \cap \ell_{18}$
$P_{4324} = (3,13,15,1) = \ell_{8} \cap \ell_{20}$
$P_{1193} = (8,9,3,1) = \ell_{9} \cap \ell_{11}$
$P_{1158} = (5,7,3,1) = \ell_{9} \cap \ell_{13}$
$P_{1218} = (1,11,3,1) = \ell_{9} \cap \ell_{15}$
$P_{1252} = (3,13,3,1) = \ell_{9} \cap \ell_{18}$
$P_{1267} = (2,14,3,1) = \ell_{9} \cap \ell_{19}$
$P_{1150} = (13,6,3,1) = \ell_{9} \cap \ell_{20}$
$P_{1792} = (15,14,5,1) = \ell_{10} \cap \ell_{12}$
$P_{1768} = (7,13,5,1) = \ell_{10} \cap \ell_{13}$
$P_{1589} = (4,2,5,1) = \ell_{10} \cap \ell_{14}$
$P_{1753} = (8,12,5,1) = \ell_{10} \cap \ell_{15}$
$P_{1714} = (1,10,5,1) = \ell_{10} \cap \ell_{17}$
$P_{1670} = (5,7,5,1) = \ell_{10} \cap \ell_{19}$
$P_{36} = (1,1,1,0) = \ell_{11} \cap \ell_{12}$
$P_{1536} = (15,14,4,1) = \ell_{11} \cap \ell_{13}$
$P_{3926} = (5,4,14,1) = \ell_{11} \cap \ell_{17}$
$P_{2612} = (3,2,9,1) = \ell_{12} \cap \ell_{15}$
$P_{937} = (8,9,2,1) = \ell_{12} \cap \ell_{20}$
$P_{75} = (8,3,1,0) = \ell_{13} \cap \ell_{14}$
$P_{3801} = (8,12,13,1) = \ell_{13} \cap \ell_{16}$
$P_{2994} = (1,10,10,1) = \ell_{14} \cap \ell_{18}$
$P_{1926} = (5,7,6,1) = \ell_{14} \cap \ell_{20}$
$P_{114} = (15,5,1,0) = \ell_{15} \cap \ell_{16}$
$P_{2176} = (15,6,7,1) = \ell_{15} \cap \ell_{18}$
$P_{3266} = (1,11,11,1) = \ell_{16} \cap \ell_{19}$
$P_{150} = (3,8,1,0) = \ell_{17} \cap \ell_{18}$
$P_{3556} = (3,13,12,1) = \ell_{17} \cap \ell_{19}$
$P_{264} = (5,15,1,0) = \ell_{19} \cap \ell_{20}$

Single Points

The surface has 201 single points:
The single points on the surface are:

0 : $P_{1} = (0,1,0,0)$ lies on line ℓ_{0}
1 : $P_{4} = (1,1,1,1)$ lies on line ℓ_{1}
2 : $P_{6} = (2,1,0,0)$ lies on line ℓ_{0}
3 : $P_{8} = (4,1,0,0)$ lies on line ℓ_{0}
4 : $P_{10} = (6,1,0,0)$ lies on line ℓ_{0}
5 : $P_{11} = (7,1,0,0)$ lies on line ℓ_{0}
6 : $P_{13} = (9,1,0,0)$ lies on line ℓ_{0}
7 : $P_{14} = (10,1,0,0)$ lies on line ℓ_{0}
8 : $P_{15} = (11,1,0,0)$ lies on line ℓ_{0}
9 : $P_{16} = (12,1,0,0)$ lies on line ℓ_{0}
10 : $P_{17} = (13,1,0,0)$ lies on line ℓ_{0}
11 : $P_{18} = (14,1,0,0)$ lies on line ℓ_{0}
12 : $P_{310} = (4,2,0,1)$ lies on line ℓ_{13}
13 : $P_{347} = (9,4,0,1)$ lies on line ℓ_{15}
14 : $P_{383} = (13,6,0,1)$ lies on line ℓ_{19}
15 : $P_{398} = (12,7,0,1)$ lies on line ℓ_{16}
16 : $P_{432} = (14,9,0,1)$ lies on line ℓ_{17}
17 : $P_{445} = (11,10,0,1)$ lies on line ℓ_{11}
18 : $P_{460} = (10,11,0,1)$ lies on line ℓ_{12}
19 : $P_{472} = (6,12,0,1)$ lies on line ℓ_{18}
20 : $P_{489} = (7,13,0,1)$ lies on line ℓ_{14}
21 : $P_{500} = (2,14,0,1)$ lies on line ℓ_{20}
22 : $P_{531} = (1,0,1,1)$ lies on line ℓ_{6}
23 : $P_{547} = (2,1,1,1)$ lies on line ℓ_{1}
24 : $P_{548} = (3,1,1,1)$ lies on line ℓ_{1}
25 : $P_{549} = (4,1,1,1)$ lies on line ℓ_{1}
26 : $P_{550} = (5,1,1,1)$ lies on line ℓ_{1}
27 : $P_{553} = (8,1,1,1)$ lies on line ℓ_{1}
28 : $P_{554} = (9,1,1,1)$ lies on line ℓ_{1}
29 : $P_{555} = (10,1,1,1)$ lies on line ℓ_{1}
30 : $P_{556} = (11,1,1,1)$ lies on line ℓ_{1}
31 : $P_{559} = (14,1,1,1)$ lies on line ℓ_{1}
32 : $P_{560} = (15,1,1,1)$ lies on line ℓ_{1}
33 : $P_{579} = (2,3,1,1)$ lies on line ℓ_{6}
34 : $P_{613} = (4,5,1,1)$ lies on line ℓ_{6}
35 : $P_{632} = (7,6,1,1)$ lies on line ℓ_{6}
36 : $P_{647} = (6,7,1,1)$ lies on line ℓ_{6}
37 : $P_{666} = (9,8,1,1)$ lies on line ℓ_{6}
38 : $P_{734} = (13,12,1,1)$ lies on line ℓ_{6}
39 : $P_{749} = (12,13,1,1)$ lies on line ℓ_{6}
40 : $P_{783} = (14,15,1,1)$ lies on line ℓ_{6}
41 : $P_{793} = (8,0,2,1)$ lies on line ℓ_{17}
42 : $P_{808} = (7,1,2,1)$ lies on line ℓ_{19}
43 : $P_{862} = (13,4,2,1)$ lies on line ℓ_{13}
44 : $P_{865} = (0,5,2,1)$ lies on line ℓ_{18}
45 : $P_{922} = (9,8,2,1)$ lies on line ℓ_{11}
46 : $P_{975} = (14,11,2,1)$ lies on line ℓ_{14}
47 : $P_{1004} = (11,13,2,1)$ lies on line ℓ_{16}
48 : $P_{1023} = (14,14,2,1)$ lies on line ℓ_{15}
49 : $P_{1056} = (15,0,3,1)$ lies on line ℓ_{9}
50 : $P_{1064} = (7,1,3,1)$ lies on line ℓ_{9}
51 : $P_{1079} = (6,2,3,1)$ lies on line ℓ_{9}
52 : $P_{1103} = (14,3,3,1)$ lies on line ℓ_{9}
53 : $P_{1109} = (4,4,3,1)$ lies on line ℓ_{9}
54 : $P_{1133} = (12,5,3,1)$ lies on line ℓ_{9}
55 : $P_{1170} = (1,8,3,1)$ lies on line ℓ_{2}
56 : $P_{1171} = (2,8,3,1)$ lies on line ℓ_{2}
57 : $P_{1172} = (3,8,3,1)$ lies on line ℓ_{2}
58 : $P_{1174} = (5,8,3,1)$ lies on line ℓ_{2}
59 : $P_{1176} = (7,8,3,1)$ lies on line ℓ_{2}
60 : $P_{1177} = (8,8,3,1)$ lies on line ℓ_{2}
61 : $P_{1179} = (10,8,3,1)$ lies on line ℓ_{2}
62 : $P_{1181} = (12,8,3,1)$ lies on line ℓ_{2}
63 : $P_{1182} = (13,8,3,1)$ lies on line ℓ_{2}
64 : $P_{1183} = (14,8,3,1)$ lies on line ℓ_{2}
65 : $P_{1184} = (15,8,3,1)$ lies on line ℓ_{2}
66 : $P_{1210} = (9,10,3,1)$ lies on line ℓ_{9}
67 : $P_{1244} = (11,12,3,1)$ lies on line ℓ_{9}
68 : $P_{1291} = (10,15,3,1)$ lies on line ℓ_{9}
69 : $P_{1312} = (15,0,4,1)$ lies on line ℓ_{20}
70 : $P_{1325} = (12,1,4,1)$ lies on line ℓ_{14}
71 : $P_{1331} = (2,2,4,1)$ lies on line ℓ_{17}
72 : $P_{1419} = (10,7,4,1)$ lies on line ℓ_{18}
73 : $P_{1425} = (0,8,4,1)$ lies on line ℓ_{19}
74 : $P_{1448} = (7,9,4,1)$ lies on line ℓ_{15}
75 : $P_{1459} = (2,10,4,1)$ lies on line ℓ_{16}
76 : $P_{1551} = (14,15,4,1)$ lies on line ℓ_{12}
77 : $P_{1556} = (3,0,5,1)$ lies on line ℓ_{10}
78 : $P_{1581} = (12,1,5,1)$ lies on line ℓ_{10}
79 : $P_{1612} = (11,3,5,1)$ lies on line ℓ_{10}
80 : $P_{1630} = (13,4,5,1)$ lies on line ℓ_{10}
81 : $P_{1635} = (2,5,5,1)$ lies on line ℓ_{10}
82 : $P_{1659} = (10,6,5,1)$ lies on line ℓ_{10}
83 : $P_{1687} = (6,8,5,1)$ lies on line ℓ_{10}
84 : $P_{1706} = (9,9,5,1)$ lies on line ℓ_{10}
85 : $P_{1743} = (14,11,5,1)$ lies on line ℓ_{10}
86 : $P_{1794} = (1,15,5,1)$ lies on line ℓ_{3}
87 : $P_{1795} = (2,15,5,1)$ lies on line ℓ_{3}
88 : $P_{1796} = (3,15,5,1)$ lies on line ℓ_{3}
89 : $P_{1797} = (4,15,5,1)$ lies on line ℓ_{3}
90 : $P_{1798} = (5,15,5,1)$ lies on line ℓ_{3}
91 : $P_{1799} = (6,15,5,1)$ lies on line ℓ_{3}
92 : $P_{1800} = (7,15,5,1)$ lies on line ℓ_{3}
93 : $P_{1801} = (8,15,5,1)$ lies on line ℓ_{3}
94 : $P_{1804} = (11,15,5,1)$ lies on line ℓ_{3}
95 : $P_{1805} = (12,15,5,1)$ lies on line ℓ_{3}
96 : $P_{1808} = (15,15,5,1)$ lies on line ℓ_{3}
97 : $P_{1814} = (5,0,6,1)$ lies on line ℓ_{16}
98 : $P_{1857} = (0,3,6,1)$ lies on line ℓ_{15}
99 : $P_{1943} = (6,8,6,1)$ lies on line ℓ_{13}
100 : $P_{1989} = (4,11,6,1)$ lies on line ℓ_{17}
101 : $P_{2014} = (13,12,6,1)$ lies on line ℓ_{11}
102 : $P_{2029} = (12,13,6,1)$ lies on line ℓ_{12}
103 : $P_{2045} = (12,14,6,1)$ lies on line ℓ_{18}
104 : $P_{2059} = (10,15,6,1)$ lies on line ℓ_{19}
105 : $P_{2080} = (15,0,7,1)$ lies on line ℓ_{19}
106 : $P_{2120} = (7,3,7,1)$ lies on line ℓ_{17}
107 : $P_{2142} = (13,4,7,1)$ lies on line ℓ_{14}

108 : $P_{2155} = (10, 5, 7, 1)$ lies on line ℓ_{16}
109 : $P_{2193} = (0, 8, 7, 1)$ lies on line ℓ_{20}
110 : $P_{2255} = (14, 11, 7, 1)$ lies on line ℓ_{13}
111 : $P_{2270} = (13, 12, 7, 1)$ lies on line ℓ_{12}
112 : $P_{2285} = (12, 13, 7, 1)$ lies on line ℓ_{11}
113 : $P_{2326} = (5, 0, 8, 1)$ lies on line ℓ_{7}
114 : $P_{2343} = (6, 1, 8, 1)$ lies on line ℓ_{7}
115 : $P_{2370} = (1, 3, 8, 1)$ lies on line ℓ_{4}
116 : $P_{2372} = (3, 3, 8, 1)$ lies on line ℓ_{4}
117 : $P_{2373} = (4, 3, 8, 1)$ lies on line ℓ_{4}
118 : $P_{2374} = (5, 3, 8, 1)$ lies on line ℓ_{4}
119 : $P_{2375} = (6, 3, 8, 1)$ lies on line ℓ_{4}
120 : $P_{2377} = (8, 3, 8, 1)$ lies on line ℓ_{4}
121 : $P_{2378} = (9, 3, 8, 1)$ lies on line ℓ_{4}
122 : $P_{2379} = (10, 3, 8, 1)$ lies on line ℓ_{4}
123 : $P_{2381} = (12, 3, 8, 1)$ lies on line ℓ_{4}
124 : $P_{2382} = (13, 3, 8, 1)$ lies on line ℓ_{4}
125 : $P_{2384} = (15, 3, 8, 1)$ lies on line ℓ_{4}
126 : $P_{2411} = (10, 5, 8, 1)$ lies on line ℓ_{7}
127 : $P_{2453} = (4, 8, 8, 1)$ lies on line ℓ_{7}
128 : $P_{2472} = (7, 9, 8, 1)$ lies on line ℓ_{7}
129 : $P_{2483} = (2, 10, 8, 1)$ lies on line ℓ_{7}
130 : $P_{2540} = (11, 13, 8, 1)$ lies on line ℓ_{7}
131 : $P_{2559} = (14, 14, 8, 1)$ lies on line ℓ_{7}
132 : $P_{2574} = (13, 15, 8, 1)$ lies on line ℓ_{7}
133 : $P_{2580} = (3, 0, 9, 1)$ lies on line ℓ_{13}
134 : $P_{2599} = (6, 1, 9, 1)$ lies on line ℓ_{16}
135 : $P_{2627} = (2, 3, 9, 1)$ lies on line ℓ_{11}
136 : $P_{2645} = (4, 4, 9, 1)$ lies on line ℓ_{20}
137 : $P_{2757} = (4, 11, 9, 1)$ lies on line ℓ_{18}
138 : $P_{2780} = (11, 12, 9, 1)$ lies on line ℓ_{19}
139 : $P_{2813} = (12, 14, 9, 1)$ lies on line ℓ_{17}
140 : $P_{2817} = (0, 15, 9, 1)$ lies on line ℓ_{14}
141 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_{11}
142 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_{12}
143 : $P_{2901} = (4, 4, 10, 1)$ lies on line ℓ_{19}
144 : $P_{2915} = (2, 5, 10, 1)$ lies on line ℓ_{13}
145 : $P_{3036} = (11, 12, 10, 1)$ lies on line ℓ_{20}
146 : $P_{3052} = (11, 13, 10, 1)$ lies on line ℓ_{15}
147 : $P_{3071} = (14, 14, 10, 1)$ lies on line ℓ_{16}
148 : $P_{3082} = (9, 15, 10, 1)$ lies on line ℓ_{17}
149 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_{12}
150 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_{11}
151 : $P_{3123} = (2, 2, 11, 1)$ lies on line ℓ_{18}
152 : $P_{3151} = (14, 3, 11, 1)$ lies on line ℓ_{20}
153 : $P_{3195} = (10, 6, 11, 1)$ lies on line ℓ_{13}
154 : $P_{3211} = (10, 7, 11, 1)$ lies on line ℓ_{17}
155 : $P_{3221} = (4, 8, 11, 1)$ lies on line ℓ_{15}
156 : $P_{3242} = (9, 9, 11, 1)$ lies on line ℓ_{14}
157 : $P_{3348} = (3, 0, 12, 1)$ lies on line ℓ_{14}
158 : $P_{3437} = (12, 5, 12, 1)$ lies on line ℓ_{20}
159 : $P_{3448} = (7, 6, 12, 1)$ lies on line ℓ_{11}
160 : $P_{3463} = (6, 7, 12, 1)$ lies on line ℓ_{12}
161 : $P_{3484} = (11, 8, 12, 1)$ lies on line ℓ_{18}
162 : $P_{3496} = (7, 9, 12, 1)$ lies on line ℓ_{16}
163 : $P_{3507} = (2, 10, 12, 1)$ lies on line ℓ_{15}
164 : $P_{3585} = (0, 15, 12, 1)$ lies on line ℓ_{13}
165 : $P_{3609} = (8, 0, 13, 1)$ lies on line ℓ_{18}
166 : $P_{3639} = (6, 2, 13, 1)$ lies on line ℓ_{19}
167 : $P_{3660} = (11, 3, 13, 1)$ lies on line ℓ_{14}
168 : $P_{3681} = (0, 5, 13, 1)$ lies on line ℓ_{17}
169 : $P_{3704} = (7, 6, 13, 1)$ lies on line ℓ_{12}
170 : $P_{3719} = (6, 7, 13, 1)$ lies on line ℓ_{11}
171 : $P_{3770} = (9, 10, 13, 1)$ lies on line ℓ_{20}
172 : $P_{3854} = (13, 15, 13, 1)$ lies on line ℓ_{15}
173 : $P_{3862} = (5, 0, 14, 1)$ lies on line ℓ_{15}
174 : $P_{3886} = (13, 1, 14, 1)$ lies on line ℓ_{18}
175 : $P_{3895} = (6, 2, 14, 1)$ lies on line ℓ_{20}
176 : $P_{3905} = (0, 3, 14, 1)$ lies on line ℓ_{16}
177 : $P_{3941} = (4, 5, 14, 1)$ lies on line ℓ_{12}
178 : $P_{3963} = (10, 6, 14, 1)$ lies on line ℓ_{14}
179 : $P_{4010} = (9, 9, 14, 1)$ lies on line ℓ_{13}
180 : $P_{4026} = (9, 10, 14, 1)$ lies on line ℓ_{19}
181 : $P_{4121} = (8, 0, 15, 1)$ lies on line ℓ_{8}
182 : $P_{4142} = (13, 1, 15, 1)$ lies on line ℓ_{8}
183 : $P_{4147} = (2, 2, 15, 1)$ lies on line ℓ_{8}
184 : $P_{4168} = (7, 3, 15, 1)$ lies on line ℓ_{8}
185 : $P_{4194} = (1, 5, 15, 1)$ lies on line ℓ_{5}
186 : $P_{4196} = (3, 5, 15, 1)$ lies on line ℓ_{5}
187 : $P_{4198} = (5, 5, 15, 1)$ lies on line ℓ_{5}
188 : $P_{4199} = (6, 5, 15, 1)$ lies on line ℓ_{5}
189 : $P_{4200} = (7, 5, 15, 1)$ lies on line ℓ_{5}
190 : $P_{4201} = (8, 5, 15, 1)$ lies on line ℓ_{5}
191 : $P_{4202} = (9, 5, 15, 1)$ lies on line ℓ_{5}
192 : $P_{4204} = (11, 5, 15, 1)$ lies on line ℓ_{5}
193 : $P_{4206} = (13, 5, 15, 1)$ lies on line ℓ_{5}
194 : $P_{4207} = (14, 5, 15, 1)$ lies on line ℓ_{5}
195 : $P_{4208} = (15, 5, 15, 1)$ lies on line ℓ_{5}
196 : $P_{4235} = (10, 7, 15, 1)$ lies on line ℓ_{8}
197 : $P_{4252} = (11, 8, 15, 1)$ lies on line ℓ_{8}
198 : $P_{4293} = (4, 11, 15, 1)$ lies on line ℓ_{8}
199 : $P_{4349} = (12, 14, 15, 1)$ lies on line ℓ_{8}
200 : $P_{4362} = (9, 15, 15, 1)$ lies on line ℓ_{8}

The single points on the surface are:

Points on surface but on no line

The surface has 76 points not on any line:
The points on the surface but not on lines are:

$0: P_3 = (0,0,0,1)$
$1: P_{63} = (12,2,1,0)$
$2: P_{89} = (6,4,1,0)$
$3: P_{119} = (4,6,1,0)$
$4: P_{145} = (14,7,1,0)$
$5: P_{176} = (13,9,1,0)$
$6: P_{190} = (11,10,1,0)$
$7: P_{205} = (10,11,1,0)$
$8: P_{213} = (2,12,1,0)$
$9: P_{236} = (9,13,1,0)$
$10: P_{250} = (7,14,1,0)$
$11: P_{291} = (1,1,0,1)$
$12: P_{327} = (5,3,0,1)$
$13: P_{362} = (8,5,0,1)$
$14: P_{417} = (15,8,0,1)$
$15: P_{517} = (3,15,0,1)$
$16: P_{828} = (11,2,2,1)$
$17: P_{842} = (9,3,2,1)$
$18: P_{887} = (6,6,2,1)$
$19: P_{910} = (13,7,2,1)$
$20: P_{951} = (6,10,2,1)$
$21: P_{1032} = (7,15,2,1)$
$22: P_{1357} = (12,3,4,1)$
$23: P_{1371} = (10,4,4,1)$
$24: P_{1391} = (14,5,4,1)$
$25: P_{1486} = (13,11,4,1)$
$26: P_{1496} = (7,12,4,1)$
$27: P_{1518} = (13,13,4,1)$
$28: P_{1835} = (10,1,6,1)$
$29: P_{1850} = (9,2,6,1)$
$30: P_{1895} = (6,5,6,1)$
$31: P_{1914} = (9,6,6,1)$
$32: P_{1966} = (13,9,6,1)$
$33: P_{1973} = (4,10,6,1)$
$34: P_{2091} = (10,1,7,1)$
$35: P_{2109} = (12,2,7,1)$
$36: P_{2179} = (2,7,7,1)$
$37: P_{2211} = (2,9,7,1)$
$38: P_{2239} = (14,10,7,1)$
$39: P_{2312} = (7,15,7,1)$
$40: P_{2663} = (6,5,9,1)$
$41: P_{2685} = (12,6,9,1)$
$42: P_{2696} = (7,7,9,1)$
$43: P_{2707} = (2,8,9,1)$
$44: P_{2732} = (11,9,9,1)$
$45: P_{2744} = (7,10,9,1)$
$46: P_{2875} = (10,2,10,1)$
$47: P_{2890} = (9,3,10,1)$
$48: P_{2943} = (14,6,10,1)$
$49: P_{2949} = (4,7,10,1)$
$50: P_{2963} = (2,8,10,1)$
$51: P_{2987} = (10,9,10,1)$
$52: P_{3164} = (11,4,11,1)$
$53: P_{3183} = (14,5,11,1)$
$54: P_{3290} = (9,12,11,1)$
$55: P_{3299} = (2,13,11,1)$
$56: P_{3324} = (11,14,11,1)$
$57: P_{3333} = (4,15,11,1)$
$58: P_{3372} = (11,1,12,1)$
$59: P_{3405} = (12,3,12,1)$
$60: P_{3415} = (6,4,12,1)$
$61: P_{3523} = (2,11,12,1)$
$62: P_{3541} = (4,12,12,1)$
$63: P_{3573} = (4,14,12,1)$
$64: P_{3628} = (11,1,13,1)$
$65: P_{3679} = (14,4,13,1)$
$66: P_{3742} = (13,8,13,1)$
$67: P_{3786} = (9,11,13,1)$
$68: P_{3823} = (14,13,13,1)$
$69: P_{3832} = (7,14,13,1)$
$70: P_{3998} = (13,8,14,1)$
$71: P_{4045} = (12,11,14,1)$
$72: P_{4061} = (12,12,14,1)$
$73: P_{4071} = (6,13,14,1)$
$74: P_{4091} = (10,14,14,1)$
$75: P_{4101} = (4,15,14,1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0	0	1
2	1	1	0	1	1	1	0	0	0	1	0	0	1	0	1	0	1	1	0	0	0
3	1	1	1	0	1	1	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1
4	1	1	1	1	0	1	0	1	0	0	0	0	1	1	0	0	0	0	1	1	0
5	1	1	1	1	1	0	0	0	1	0	0	1	0	0	1	1	0	0	0	1	0
6	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1	1	0
7	1	0	0	0	1	0	0	0	0	0	0	1	0	0	1	1	1	1	0	0	1
8	1	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	1	1	0	1
9	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	1	1
10	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	1	0
11	0	0	0	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1	0	0	0
12	0	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1	0	0	0	0	1
13	0	1	0	0	1	0	0	0	1	1	1	1	0	0	1	0	1	0	0	0	0
14	0	0	1	0	0	1	1	1	0	0	1	0	0	1	0	0	0	0	1	0	1
15	0	1	0	0	0	1	0	1	0	1	1	0	1	0	0	0	1	0	1	0	0
16	0	0	1	1	0	0	1	1	1	0	0	0	0	1	0	1	0	0	0	1	0
17	0	1	1	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	1	1	0
18	0	0	0	1	1	0	1	0	1	1	0	0	0	0	1	1	0	1	0	0	0
19	0	0	0	0	1	1	1	0	0	1	1	0	0	0	0	0	1	1	0	0	1
20	0	1	0	1	0	0	0	1	1	1	0	0	1	0	1	0	0	0	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}
in point	P_0	P_0	P_0	P_0	P_0	P_5	P_7	P_9	P_{12}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_0	P_{546}	P_{557}	P_{551}	P_{558}	P_{552}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_9	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{17}
in point	P_0	P_0	P_0	P_0	P_0	P_{1169}	P_{1178}	P_{1175}	P_{1173}	P_{1180}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{16}	ℓ_{18}	ℓ_{20}
in point	P_0	P_0	P_0	P_0	P_0	P_{1793}	P_{1807}	P_{1806}	P_{1802}	P_{1803}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_{12}	ℓ_{13}	ℓ_{18}	ℓ_{19}
in point	P_0	P_0	P_0	P_0	P_0	P_{2369}	P_{2371}	P_{2380}	P_{2376}	P_{2383}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_8	ℓ_{11}	ℓ_{14}	ℓ_{15}	ℓ_{19}
in point	P_0	P_0	P_0	P_0	P_0	P_{4193}	P_{4197}	P_{4195}	P_{4203}	P_{4205}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{19}
in point	P_5	P_{546}	P_{715}	P_{700}	P_{768}	P_{564}	P_{598}	P_{681}

Line 7 intersects

Line	ℓ_0	ℓ_4	ℓ_{11}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{20}
in point	P_7	P_{2369}	P_{2356}	P_{2521}	P_{2445}	P_{2394}	P_{2432}	P_{2498}

Line 8 intersects

Line	ℓ_0	ℓ_5	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}
in point	P_9	P_{4193}	P_{4182}	P_{4274}	P_{4224}	P_{4311}	P_{4271}	P_{4324}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{12}	P_{1169}	P_{1193}	P_{1158}	P_{1218}	P_{1252}	P_{1267}	P_{1150}

Line 10 intersects

Line	ℓ_0	ℓ_3	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{19}
in point	P_{19}	P_{1793}	P_{1792}	P_{1768}	P_{1589}	P_{1753}	P_{1714}	P_{1670}

Line 11 intersects

Line	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{17}
in point	P_{1807}	P_{4197}	P_{715}	P_{2356}	P_{1193}	P_{36}	P_{1536}	P_{3926}

Line 12 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{15}	ℓ_{20}
in point	P_{1178}	P_{2371}	P_{700}	P_{4182}	P_{1792}	P_{36}	P_{2612}	P_{937}

Line 13 intersects

Line	ℓ_1	ℓ_4	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{16}
in point	P_{557}	P_{2380}	P_{4274}	P_{1158}	P_{1768}	P_{1536}	P_{75}	P_{3801}

Line 14 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{18}	ℓ_{20}
in point	P_{1175}	P_{4195}	P_{768}	P_{2521}	P_{1589}	P_{75}	P_{2994}	P_{1926}

Line 15 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{16}	ℓ_{18}
in point	P_{551}	P_{4203}	P_{2445}	P_{1218}	P_{1753}	P_{2612}	P_{114}	P_{2176}

Line 16 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_8	ℓ_{13}	ℓ_{15}	ℓ_{19}
in point	P_{1173}	P_{1806}	P_{564}	P_{2394}	P_{4224}	P_{3801}	P_{114}	P_{3266}

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{18}	ℓ_{19}
in point	P_{558}	P_{1180}	P_{2432}	P_{4311}	P_{1714}	P_{3926}	P_{150}	P_{3556}

Line 18 intersects

Line	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_9	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_{1802}	P_{2376}	P_{598}	P_{4271}	P_{1252}	P_{2994}	P_{2176}	P_{150}

Line 19 intersects

Line	ℓ_4	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{17}	ℓ_{20}
in point	P_{2383}	P_{4205}	P_{681}	P_{1267}	P_{1670}	P_{3266}	P_{3556}	P_{264}

Line 20 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{14}	ℓ_{19}
in point	P_{552}	P_{1803}	P_{2498}	P_{4324}	P_{1150}	P_{937}	P_{1926}	P_{264}

The surface has 353 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_3 = (0,0,0,1)$
3 : $P_4 = (1,1,1,1)$
4 : $P_5 = (1,1,0,0)$
5 : $P_6 = (2,1,0,0)$
6 : $P_7 = (3,1,0,0)$
7 : $P_8 = (4,1,0,0)$
8 : $P_9 = (5,1,0,0)$
9 : $P_{10} = (6,1,0,0)$
10 : $P_{11} = (7,1,0,0)$
11 : $P_{12} = (8,1,0,0)$
12 : $P_{13} = (9,1,0,0)$
13 : $P_{14} = (10,1,0,0)$
14 : $P_{15} = (11,1,0,0)$
15 : $P_{16} = (12,1,0,0)$
16 : $P_{17} = (13,1,0,0)$
17 : $P_{18} = (14,1,0,0)$
18 : $P_{19} = (15,1,0,0)$
19 : $P_{36} = (1,1,1,0)$
20 : $P_{63} = (12,2,1,0)$
21 : $P_{75} = (8,3,1,0)$
22 : $P_{89} = (6,4,1,0)$
23 : $P_{114} = (15,5,1,0)$
24 : $P_{119} = (4,6,1,0)$
25 : $P_{145} = (14,7,1,0)$
26 : $P_{150} = (3,8,1,0)$
27 : $P_{176} = (13,9,1,0)$
28 : $P_{190} = (11,10,1,0)$
29 : $P_{205} = (10,11,1,0)$
30 : $P_{213} = (2,12,1,0)$
31 : $P_{236} = (9,13,1,0)$
32 : $P_{250} = (7,14,1,0)$
33 : $P_{264} = (5,15,1,0)$
34 : $P_{291} = (1,1,0,1)$
35 : $P_{310} = (4,2,0,1)$
36 : $P_{327} = (5,3,0,1)$
37 : $P_{347} = (9,4,0,1)$
38 : $P_{362} = (8,5,0,1)$
39 : $P_{383} = (13,6,0,1)$
40 : $P_{398} = (12,7,0,1)$
41 : $P_{417} = (15,8,0,1)$
42 : $P_{432} = (14,9,0,1)$
43 : $P_{445} = (11,10,0,1)$
44 : $P_{460} = (10,11,0,1)$
45 : $P_{472} = (6,12,0,1)$
46 : $P_{489} = (7,13,0,1)$
47 : $P_{500} = (2,14,0,1)$
48 : $P_{517} = (3,15,0,1)$
49 : $P_{531} = (1,0,1,1)$
50 : $P_{546} = (0,1,1,1)$
51 : $P_{547} = (2,1,1,1)$
52 : $P_{548} = (3,1,1,1)$
53 : $P_{549} = (4,1,1,1)$
54 : $P_{550} = (5,1,1,1)$
55 : $P_{551} = (6,1,1,1)$
56 : $P_{552} = (7,1,1,1)$
57 : $P_{553} = (8,1,1,1)$
58 : $P_{554} = (9,1,1,1)$
59 : $P_{555} = (10,1,1,1)$
60 : $P_{556} = (11,1,1,1)$
61 : $P_{557} = (12,1,1,1)$
62 : $P_{558} = (13,1,1,1)$
63 : $P_{559} = (14,1,1,1)$
64 : $P_{560} = (15,1,1,1)$
65 : $P_{564} = (3,2,1,1)$
66 : $P_{579} = (2,3,1,1)$
67 : $P_{598} = (5,4,1,1)$
68 : $P_{613} = (4,5,1,1)$
69 : $P_{632} = (7,6,1,1)$
70 : $P_{647} = (6,7,1,1)$
71 : $P_{666} = (9,8,1,1)$
72 : $P_{681} = (8,9,1,1)$
73 : $P_{700} = (11,10,1,1)$
74 : $P_{715} = (10,11,1,1)$
75 : $P_{734} = (13,12,1,1)$
76 : $P_{749} = (12,13,1,1)$
77 : $P_{768} = (15,14,1,1)$
78 : $P_{783} = (14,15,1,1)$
79 : $P_{793} = (8,0,2,1)$
80 : $P_{808} = (7,1,2,1)$
81 : $P_{828} = (11,2,2,1)$
82 : $P_{842} = (9,3,2,1)$
83 : $P_{862} = (13,4,2,1)$
84 : $P_{865} = (0,5,2,1)$
85 : $P_{887} = (6,6,2,1)$
86 : $P_{910} = (13,7,2,1)$
87 : $P_{922} = (9,8,2,1)$
88 : $P_{937} = (8,9,2,1)$
89 : $P_{951} = (6,10,2,1)$
90 : $P_{975} = (14,11,2,1)$
91 : $P_{1004} = (11,13,2,1)$
92 : $P_{1023} = (14,14,2,1)$
93 : $P_{1032} = (7,15,2,1)$
94 : $P_{1056} = (15,0,3,1)$
95 : $P_{1064} = (7,1,3,1)$
96 : $P_{1079} = (6,2,3,1)$
97 : $P_{1103} = (14,3,3,1)$
98 : $P_{1109} = (4,4,3,1)$
99 : $P_{1133} = (12,5,3,1)$
100 : $P_{1150} = (13,6,3,1)$
101 : $P_{1158} = (5,7,3,1)$
102 : $P_{1169} = (0,8,3,1)$
103 : $P_{1170} = (1,8,3,1)$
104 : $P_{1171} = (2,8,3,1)$
105 : $P_{1172} = (3,8,3,1)$
106 : $P_{1173} = (4,8,3,1)$
107 : $P_{1174} = (5,8,3,1)$
108 : $P_{1175} = (6,8,3,1)$
109 : $P_{1176} = (7,8,3,1)$
110 : $P_{1177} = (8,8,3,1)$
111 : $P_{1178} = (9,8,3,1)$
112 : $P_{1179} = (10,8,3,1)$
113 : $P_{1180} = (11,8,3,1)$
114 : $P_{1181} = (12,8,3,1)$
115 : $P_{1182} = (13,8,3,1)$
116 : $P_{1183} = (14,8,3,1)$
117 : $P_{1184} = (15,8,3,1)$
118 : $P_{1193} = (8,9,3,1)$
119 : $P_{1210} = (9,10,3,1)$
120 : $P_{1218} = (1,11,3,1)$
121 : $P_{1244} = (11,12,3,1)$
122 : $P_{1252} = (3,13,3,1)$
123 : $P_{1267} = (2,14,3,1)$
124 : $P_{1291} = (10,15,3,1)$
125 : $P_{1312} = (15,0,4,1)$
126 : $P_{1325} = (12,1,4,1)$
127 : $P_{1331} = (2,2,4,1)$
128 : $P_{1357} = (12,3,4,1)$
129 : $P_{1371} = (10,4,4,1)$
130 : $P_{1391} = (14,5,4,1)$
131 : $P_{1419} = (10,7,4,1)$
132 : $P_{1425} = (0,8,4,1)$
133 : $P_{1448} = (7,9,4,1)$
134 : $P_{1459} = (2,10,4,1)$

$135: P_{1486} = (13, 11, 4, 1)$
$136: P_{1496} = (7, 12, 4, 1)$
$137: P_{1518} = (13, 13, 4, 1)$
$138: P_{1536} = (15, 14, 4, 1)$
$139: P_{1551} = (14, 15, 4, 1)$
$140: P_{1556} = (3, 0, 5, 1)$
$141: P_{1581} = (12, 1, 5, 1)$
$142: P_{1589} = (4, 2, 5, 1)$
$143: P_{1612} = (11, 3, 5, 1)$
$144: P_{1630} = (13, 4, 5, 1)$
$145: P_{1635} = (2, 5, 5, 1)$
$146: P_{1659} = (10, 6, 5, 1)$
$147: P_{1670} = (5, 7, 5, 1)$
$148: P_{1687} = (6, 8, 5, 1)$
$149: P_{1706} = (9, 9, 5, 1)$
$150: P_{1714} = (1, 10, 5, 1)$
$151: P_{1743} = (14, 11, 5, 1)$
$152: P_{1753} = (8, 12, 5, 1)$
$153: P_{1768} = (7, 13, 5, 1)$
$154: P_{1792} = (15, 14, 5, 1)$
$155: P_{1793} = (0, 15, 5, 1)$
$156: P_{1794} = (1, 15, 5, 1)$
$157: P_{1795} = (2, 15, 5, 1)$
$158: P_{1796} = (3, 15, 5, 1)$
$159: P_{1797} = (4, 15, 5, 1)$
$160: P_{1798} = (5, 15, 5, 1)$
$161: P_{1799} = (6, 15, 5, 1)$
$162: P_{1800} = (7, 15, 5, 1)$
$163: P_{1801} = (8, 15, 5, 1)$
$164: P_{1802} = (9, 15, 5, 1)$
$165: P_{1803} = (10, 15, 5, 1)$
$166: P_{1804} = (11, 15, 5, 1)$
$167: P_{1805} = (12, 15, 5, 1)$
$168: P_{1806} = (13, 15, 5, 1)$
$169: P_{1807} = (14, 15, 5, 1)$
$170: P_{1808} = (15, 15, 5, 1)$
$171: P_{1814} = (5, 0, 6, 1)$
$172: P_{1835} = (10, 1, 6, 1)$
$173: P_{1850} = (9, 2, 6, 1)$
$174: P_{1857} = (0, 3, 6, 1)$
$175: P_{1895} = (6, 5, 6, 1)$
$176: P_{1914} = (9, 6, 6, 1)$
$177: P_{1926} = (5, 7, 6, 1)$
$178: P_{1943} = (6, 8, 6, 1)$
$179: P_{1966} = (13, 9, 6, 1)$
$180: P_{1973} = (4, 10, 6, 1)$
$181: P_{1989} = (4, 11, 6, 1)$
$182: P_{2014} = (13, 12, 6, 1)$
$183: P_{2029} = (12, 13, 6, 1)$
$184: P_{2045} = (12, 14, 6, 1)$
$185: P_{2059} = (10, 15, 6, 1)$
$186: P_{2080} = (15, 0, 7, 1)$
$187: P_{2091} = (10, 1, 7, 1)$
$188: P_{2109} = (12, 2, 7, 1)$
$189: P_{2120} = (7, 3, 7, 1)$
$190: P_{2142} = (13, 4, 7, 1)$
$191: P_{2155} = (10, 5, 7, 1)$
$192: P_{2176} = (15, 6, 7, 1)$
$193: P_{2179} = (2, 7, 7, 1)$
$194: P_{2193} = (0, 8, 7, 1)$
$195: P_{2211} = (2, 9, 7, 1)$
$196: P_{2239} = (14, 10, 7, 1)$
$197: P_{2255} = (14, 11, 7, 1)$
$198: P_{2270} = (13, 12, 7, 1)$
$199: P_{2285} = (12, 13, 7, 1)$
$200: P_{2312} = (7, 15, 7, 1)$
$201: P_{2326} = (5, 0, 8, 1)$
$202: P_{2343} = (6, 1, 8, 1)$
$203: P_{2356} = (3, 2, 8, 1)$
$204: P_{2369} = (0, 3, 8, 1)$
$205: P_{2370} = (1, 3, 8, 1)$
$206: P_{2371} = (2, 3, 8, 1)$
$207: P_{2372} = (3, 3, 8, 1)$
$208: P_{2373} = (4, 3, 8, 1)$
$209: P_{2374} = (5, 3, 8, 1)$
$210: P_{2375} = (6, 3, 8, 1)$
$211: P_{2376} = (7, 3, 8, 1)$
$212: P_{2377} = (8, 3, 8, 1)$
$213: P_{2378} = (9, 3, 8, 1)$
$214: P_{2379} = (10, 3, 8, 1)$
$215: P_{2380} = (11, 3, 8, 1)$
$216: P_{2381} = (12, 3, 8, 1)$
$217: P_{2382} = (13, 3, 8, 1)$
$218: P_{2383} = (14, 3, 8, 1)$
$219: P_{2384} = (15, 3, 8, 1)$
$220: P_{2394} = (9, 4, 8, 1)$
$221: P_{2411} = (10, 5, 8, 1)$
$222: P_{2432} = (15, 6, 8, 1)$
$223: P_{2445} = (12, 7, 8, 1)$
$224: P_{2453} = (4, 8, 8, 1)$
$225: P_{2472} = (7, 9, 8, 1)$
$226: P_{2483} = (2, 10, 8, 1)$
$227: P_{2498} = (1, 11, 8, 1)$
$228: P_{2521} = (8, 12, 8, 1)$
$229: P_{2540} = (11, 13, 8, 1)$
$230: P_{2559} = (14, 14, 8, 1)$
$231: P_{2574} = (13, 15, 8, 1)$
$232: P_{2580} = (3, 0, 9, 1)$
$233: P_{2599} = (6, 1, 9, 1)$
$234: P_{2612} = (3, 2, 9, 1)$
$235: P_{2627} = (2, 3, 9, 1)$
$236: P_{2645} = (4, 4, 9, 1)$
$237: P_{2663} = (6, 5, 9, 1)$
$238: P_{2685} = (12, 6, 9, 1)$
$239: P_{2696} = (7, 7, 9, 1)$
$240: P_{2707} = (2, 8, 9, 1)$
$241: P_{2732} = (11, 9, 9, 1)$
$242: P_{2744} = (7, 10, 9, 1)$
$243: P_{2757} = (4, 11, 9, 1)$
$244: P_{2780} = (11, 12, 9, 1)$
$245: P_{2813} = (12, 14, 9, 1)$
$246: P_{2817} = (0, 15, 9, 1)$
$247: P_{2834} = (1, 0, 10, 1)$
$248: P_{2849} = (0, 1, 10, 1)$
$249: P_{2875} = (10, 2, 10, 1)$
$250: P_{2890} = (9, 3, 10, 1)$
$251: P_{2901} = (4, 4, 10, 1)$
$252: P_{2915} = (2, 5, 10, 1)$
$253: P_{2943} = (14, 6, 10, 1)$
$254: P_{2949} = (4, 7, 10, 1)$
$255: P_{2963} = (2, 8, 10, 1)$
$256: P_{2987} = (10, 9, 10, 1)$
$257: P_{2994} = (1, 10, 10, 1)$
$258: P_{3036} = (11, 12, 10, 1)$
$259: P_{3052} = (11, 13, 10, 1)$
$260: P_{3071} = (14, 14, 10, 1)$
$261: P_{3082} = (9, 15, 10, 1)$
$262: P_{3090} = (1, 0, 11, 1)$
$263: P_{3105} = (0, 1, 11, 1)$
$264: P_{3123} = (2, 2, 11, 1)$
$265: P_{3151} = (14, 3, 11, 1)$
$266: P_{3164} = (11, 4, 11, 1)$
$267: P_{3183} = (14, 5, 11, 1)$
$268: P_{3195} = (10, 6, 11, 1)$
$269: P_{3211} = (10, 7, 11, 1)$
$270: P_{3221} = (4, 8, 11, 1)$
$271: P_{3242} = (9, 9, 11, 1)$
$272: P_{3266} = (1, 11, 11, 1)$
$273: P_{3290} = (9, 12, 11, 1)$
$274: P_{3299} = (2, 13, 11, 1)$
$275: P_{3324} = (11, 14, 11, 1)$
$276: P_{3333} = (4, 15, 11, 1)$
$277: P_{3348} = (3, 0, 12, 1)$
$278: P_{3372} = (11, 1, 12, 1)$
$279: P_{3405} = (12, 3, 12, 1)$
$280: P_{3415} = (6, 4, 12, 1)$
$281: P_{3437} = (12, 5, 12, 1)$
$282: P_{3448} = (7, 6, 12, 1)$
$283: P_{3463} = (6, 7, 12, 1)$
$284: P_{3484} = (11, 8, 12, 1)$
$285: P_{3496} = (7, 9, 12, 1)$
$286: P_{3507} = (2, 10, 12, 1)$
$287: P_{3523} = (2, 11, 12, 1)$
$288: P_{3541} = (4, 12, 12, 1)$
$289: P_{3556} = (3, 13, 12, 1)$
$290: P_{3573} = (4, 14, 12, 1)$
$291: P_{3585} = (0, 15, 12, 1)$
$292: P_{3609} = (8, 0, 13, 1)$
$293: P_{3628} = (11, 1, 13, 1)$
$294: P_{3639} = (6, 2, 13, 1)$
$295: P_{3660} = (11, 3, 13, 1)$
$296: P_{3679} = (14, 4, 13, 1)$

297 : $P_{3681} = (0, 5, 13, 1)$
298 : $P_{3704} = (7, 6, 13, 1)$
299 : $P_{3719} = (6, 7, 13, 1)$
300 : $P_{3742} = (13, 8, 13, 1)$
301 : $P_{3770} = (9, 10, 13, 1)$
302 : $P_{3786} = (9, 11, 13, 1)$
303 : $P_{3801} = (8, 12, 13, 1)$
304 : $P_{3823} = (14, 13, 13, 1)$
305 : $P_{3832} = (7, 14, 13, 1)$
306 : $P_{3854} = (13, 15, 13, 1)$
307 : $P_{3862} = (5, 0, 14, 1)$
308 : $P_{3886} = (13, 1, 14, 1)$
309 : $P_{3895} = (6, 2, 14, 1)$
310 : $P_{3905} = (0, 3, 14, 1)$
311 : $P_{3926} = (5, 4, 14, 1)$
312 : $P_{3941} = (4, 5, 14, 1)$
313 : $P_{3963} = (10, 6, 14, 1)$
314 : $P_{3998} = (13, 8, 14, 1)$
315 : $P_{4010} = (9, 9, 14, 1)$
316 : $P_{4026} = (9, 10, 14, 1)$
317 : $P_{4045} = (12, 11, 14, 1)$
318 : $P_{4061} = (12, 12, 14, 1)$
319 : $P_{4071} = (6, 13, 14, 1)$
320 : $P_{4091} = (10, 14, 14, 1)$
321 : $P_{4101} = (4, 15, 14, 1)$
322 : $P_{4121} = (8, 0, 15, 1)$
323 : $P_{4142} = (13, 1, 15, 1)$
324 : $P_{4147} = (2, 2, 15, 1)$
325 : $P_{4168} = (7, 3, 15, 1)$
326 : $P_{4182} = (5, 4, 15, 1)$
327 : $P_{4193} = (0, 5, 15, 1)$
328 : $P_{4194} = (1, 5, 15, 1)$
329 : $P_{4195} = (2, 5, 15, 1)$
330 : $P_{4196} = (3, 5, 15, 1)$
331 : $P_{4197} = (4, 5, 15, 1)$
332 : $P_{4198} = (5, 5, 15, 1)$
333 : $P_{4199} = (6, 5, 15, 1)$
334 : $P_{4200} = (7, 5, 15, 1)$
335 : $P_{4201} = (8, 5, 15, 1)$
336 : $P_{4202} = (9, 5, 15, 1)$
337 : $P_{4203} = (10, 5, 15, 1)$
338 : $P_{4204} = (11, 5, 15, 1)$
339 : $P_{4205} = (12, 5, 15, 1)$
340 : $P_{4206} = (13, 5, 15, 1)$
341 : $P_{4207} = (14, 5, 15, 1)$
342 : $P_{4208} = (15, 5, 15, 1)$
343 : $P_{4224} = (15, 6, 15, 1)$
344 : $P_{4235} = (10, 7, 15, 1)$
345 : $P_{4252} = (11, 8, 15, 1)$
346 : $P_{4271} = (14, 9, 15, 1)$
347 : $P_{4274} = (1, 10, 15, 1)$
348 : $P_{4293} = (4, 11, 15, 1)$
349 : $P_{4311} = (6, 12, 15, 1)$
350 : $P_{4324} = (3, 13, 15, 1)$
351 : $P_{4349} = (12, 14, 15, 1)$
352 : $P_{4362} = (9, 15, 15, 1)$