Rank-65759 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1229497485

General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	57
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	2^3, 1^{21}, 0^{57}

Singular Points

The surface has 1 singular points:

$0: \ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

Rank of lines: (9, 4673, 138)
Rank of points on Klein quadric: (97, 769, 1322)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{146} = (0,1,1,1) = \ell_0 \cap \ell_1$
$P_{138} = (0,0,1,1) = \ell_1 \cap \ell_2$
$P_4 = (1,1,1,1) = \ell_0 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_5 = (1,1,0,0)$ lies on line ℓ_2
3 : $P_{147} = (2,1,1,1)$ lies on line ℓ_0
4 : $P_{148} = (3,1,1,1)$ lies on line ℓ_0
5 : $P_{149} = (4,1,1,1)$ lies on line ℓ_0
6 : $P_{150} = (5,1,1,1)$ lies on line ℓ_0
7 : $P_{151} = (6,1,1,1)$ lies on line ℓ_0
8 : $P_{152} = (7,1,1,1)$ lies on line ℓ_0
9 : $P_{153} = (0,2,1,1)$ lies on line ℓ_1
10 : $P_{155} = (2,2,1,1)$ lies on line ℓ_2
11 : $P_{161} = (0,3,1,1)$ lies on line ℓ_1
12 : $P_{164} = (3,3,1,1)$ lies on line ℓ_2
13 : $P_{169} = (0,4,1,1)$ lies on line ℓ_1
14 : $P_{173} = (4,4,1,1)$ lies on line ℓ_2
15 : $P_{177} = (0,5,1,1)$ lies on line ℓ_1
16 : $P_{182} = (5,5,1,1)$ lies on line ℓ_2
17 : $P_{185} = (0,6,1,1)$ lies on line ℓ_1
18 : $P_{191} = (6,6,1,1)$ lies on line ℓ_2
19 : $P_{193} = (0,7,1,1)$ lies on line ℓ_1
20 : $P_{200} = (7,7,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 57 points not on any line:
The points on the surface but not on lines are:

0 : $P_{20} = (1,1,1,0)$
1 : $P_{31} = (4,2,1,0)$
2 : $P_{34} = (7,2,1,0)$
3 : $P_{45} = (2,4,1,0)$
4 : $P_{50} = (7,4,1,0)$
5 : $P_{69} = (2,7,1,0)$
6 : $P_{71} = (4,7,1,0)$
7 : $P_{75} = (1,0,0,1)$
8 : $P_{83} = (1,1,0,1)$
9 : $P_{203} = (2,0,2,1)$

10 : $P_{216} = (7, 1, 2, 1)$
11 : $P_{221} = (4, 2, 2, 1)$
12 : $P_{237} = (4, 4, 2, 1)$
13 : $P_{240} = (7, 4, 2, 1)$
14 : $P_{271} = (6, 0, 3, 1)$
15 : $P_{280} = (7, 1, 3, 1)$
16 : $P_{293} = (4, 3, 3, 1)$
17 : $P_{298} = (1, 4, 3, 1)$
18 : $P_{301} = (4, 4, 3, 1)$
19 : $P_{311} = (6, 5, 3, 1)$
20 : $P_{312} = (7, 5, 3, 1)$
21 : $P_{314} = (1, 6, 3, 1)$
22 : $P_{315} = (2, 6, 3, 1)$
23 : $P_{323} = (2, 7, 3, 1)$
24 : $P_{324} = (3, 7, 3, 1)$
25 : $P_{333} = (4, 0, 4, 1)$
26 : $P_{339} = (2, 1, 4, 1)$
27 : $P_{368} = (7, 4, 4, 1)$
28 : $P_{387} = (2, 7, 4, 1)$
29 : $P_{392} = (7, 7, 4, 1)$
30 : $P_{396} = (3, 0, 5, 1)$
31 : $P_{403} = (2, 1, 5, 1)$
32 : $P_{413} = (4, 2, 5, 1)$
33 : $P_{414} = (5, 2, 5, 1)$
34 : $P_{418} = (1, 3, 5, 1)$
35 : $P_{421} = (4, 3, 5, 1)$
36 : $P_{440} = (7, 5, 5, 1)$
37 : $P_{443} = (2, 6, 5, 1)$
38 : $P_{444} = (3, 6, 5, 1)$
39 : $P_{450} = (1, 7, 5, 1)$
40 : $P_{456} = (7, 7, 5, 1)$
41 : $P_{462} = (5, 0, 6, 1)$
42 : $P_{469} = (4, 1, 6, 1)$
43 : $P_{474} = (1, 2, 6, 1)$
44 : $P_{475} = (2, 2, 6, 1)$
45 : $P_{485} = (4, 3, 6, 1)$
46 : $P_{486} = (5, 3, 6, 1)$
47 : $P_{495} = (6, 4, 6, 1)$
48 : $P_{496} = (7, 4, 6, 1)$
49 : $P_{498} = (1, 5, 6, 1)$
50 : $P_{504} = (7, 5, 6, 1)$
51 : $P_{507} = (2, 6, 6, 1)$
52 : $P_{528} = (7, 0, 7, 1)$
53 : $P_{533} = (4, 1, 7, 1)$
54 : $P_{539} = (2, 2, 7, 1)$
55 : $P_{541} = (4, 2, 7, 1)$
56 : $P_{579} = (2, 7, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{146}	P_4

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{146}	P_{138}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_4	P_{138}

The surface has 81 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_{20} = (1, 1, 1, 0)$
5 : $P_{31} = (4, 2, 1, 0)$
6 : $P_{34} = (7, 2, 1, 0)$
7 : $P_{45} = (2, 4, 1, 0)$
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