

Rank-73797 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498768733

General information

Number of lines	3
Number of points	25
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	12
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	$3, 1^{12}, 0^{12}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0)_6$$

Rank of lines: (16, 356, 100)

Rank of points on Klein quadric: (2, 1, 6)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 12 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1

2 : $P_8 = (1, 0, 1, 0)$ lies on line ℓ_0

3 : $P_9 = (2, 0, 1, 0)$ lies on line ℓ_0

4 : $P_{10} = (3, 0, 1, 0)$ lies on line ℓ_0

5 : $P_{23} = (1, 0, 0, 1)$ lies on line ℓ_2

6 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_1

7 : $P_{39} = (1, 0, 1, 1)$ lies on line ℓ_2

8 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_1

9 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_2

10 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_1

11 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:

The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$

1 : $P_{12} = (1, 1, 1, 0)$

2 : $P_{18} = (3, 2, 1, 0)$

3 : $P_{21} = (2, 3, 1, 0)$

4 : $P_{28} = (2, 1, 0, 1)$

5 : $P_{29} = (3, 1, 0, 1)$

6 : $P_{32} = (2, 2, 0, 1)$

7 : $P_{33} = (3, 2, 0, 1)$

8 : $P_{36} = (2, 3, 0, 1)$

9 : $P_{37} = (3, 3, 0, 1)$

10 : $P_{66} = (1, 3, 2, 1)$

11 : $P_{78} = (1, 2, 3, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 25 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_8 = (1, 0, 1, 0)$$

$$5 : P_9 = (2, 0, 1, 0)$$

$$6 : P_{10} = (3, 0, 1, 0)$$

$$7 : P_{12} = (1, 1, 1, 0)$$

$$8 : P_{18} = (3, 2, 1, 0)$$

$$9 : P_{21} = (2, 3, 1, 0)$$

$$10 : P_{23} = (1, 0, 0, 1)$$

$$11 : P_{28} = (2, 1, 0, 1)$$

$$12 : P_{29} = (3, 1, 0, 1)$$

$$13 : P_{32} = (2, 2, 0, 1)$$

$$14 : P_{33} = (3, 2, 0, 1)$$

$$15 : P_{36} = (2, 3, 0, 1)$$

$$16 : P_{37} = (3, 3, 0, 1)$$

$$17 : P_{38} = (0, 0, 1, 1)$$

$$18 : P_{39} = (1, 0, 1, 1)$$

$$19 : P_{53} = (0, 0, 2, 1)$$

$$20 : P_{54} = (1, 0, 2, 1)$$

$$21 : P_{66} = (1, 3, 2, 1)$$

$$22 : P_{69} = (0, 0, 3, 1)$$

$$23 : P_{70} = (1, 0, 3, 1)$$

$$24 : P_{78} = (1, 2, 3, 1)$$