

Rank-76099 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(16) is 303108373

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	129
Number of points off lines	182
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	$3^4, 2^6, 1^{129}, 0^{182}$

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(1, 0, 1, 0, 0, 1)_{4657} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{33} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{34} \\
\ell_5 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{18} \\
\ell_7 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{49} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1, 1, 1, 1, 0, 1)_{5586}
\end{aligned}$$

Rank of lines: (0, 256, 274, 69904, 4368, 70160, 4624, 69921, 4642)

Rank of points on Klein quadric: (0, 2, 4657, 33, 34, 1, 18, 49, 5586)

Eckardt Points

The surface has 4 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0),$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1),$$

$$3 : P_{35} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0).$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{20} = (1, 0, 1, 0) = \ell_1 \cap \ell_2$$

$$P_{275} = (1, 0, 0, 1) = \ell_4 \cap \ell_6$$

$$P_{291} = (1, 1, 0, 1) = \ell_4 \cap \ell_8$$

$$P_{531} = (1, 0, 1, 1) = \ell_6 \cap \ell_8$$

Single Points

The surface has 129 single points:

The single points on the surface are:

$$0 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_8 = (4, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_9 = (5, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_{10} = (6, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{11} = (7, 1, 0, 0) \text{ lies on line } \ell_0$$

$$6 : P_{12} = (8, 1, 0, 0) \text{ lies on line } \ell_0$$

$$7 : P_{13} = (9, 1, 0, 0) \text{ lies on line } \ell_0$$

$$8 : P_{14} = (10, 1, 0, 0) \text{ lies on line } \ell_0$$

$$9 : P_{15} = (11, 1, 0, 0) \text{ lies on line } \ell_0$$

$$10 : P_{16} = (12, 1, 0, 0) \text{ lies on line } \ell_0$$

$$11 : P_{17} = (13, 1, 0, 0) \text{ lies on line } \ell_0$$

$$12 : P_{18} = (14, 1, 0, 0) \text{ lies on line } \ell_0$$

$$13 : P_{19} = (15, 1, 0, 0) \text{ lies on line } \ell_0$$

14 : $P_{21} = (2, 0, 1, 0)$ lies on line ℓ_1
 15 : $P_{22} = (3, 0, 1, 0)$ lies on line ℓ_1
 16 : $P_{23} = (4, 0, 1, 0)$ lies on line ℓ_1
 17 : $P_{24} = (5, 0, 1, 0)$ lies on line ℓ_1
 18 : $P_{25} = (6, 0, 1, 0)$ lies on line ℓ_1
 19 : $P_{26} = (7, 0, 1, 0)$ lies on line ℓ_1
 20 : $P_{27} = (8, 0, 1, 0)$ lies on line ℓ_1
 21 : $P_{28} = (9, 0, 1, 0)$ lies on line ℓ_1
 22 : $P_{29} = (10, 0, 1, 0)$ lies on line ℓ_1
 23 : $P_{30} = (11, 0, 1, 0)$ lies on line ℓ_1
 24 : $P_{31} = (12, 0, 1, 0)$ lies on line ℓ_1
 25 : $P_{32} = (13, 0, 1, 0)$ lies on line ℓ_1
 26 : $P_{33} = (14, 0, 1, 0)$ lies on line ℓ_1
 27 : $P_{34} = (15, 0, 1, 0)$ lies on line ℓ_1
 28 : $P_{54} = (3, 2, 1, 0)$ lies on line ℓ_2
 29 : $P_{69} = (2, 3, 1, 0)$ lies on line ℓ_2
 30 : $P_{88} = (5, 4, 1, 0)$ lies on line ℓ_2
 31 : $P_{103} = (4, 5, 1, 0)$ lies on line ℓ_2
 32 : $P_{122} = (7, 6, 1, 0)$ lies on line ℓ_2
 33 : $P_{137} = (6, 7, 1, 0)$ lies on line ℓ_2
 34 : $P_{156} = (9, 8, 1, 0)$ lies on line ℓ_2
 35 : $P_{171} = (8, 9, 1, 0)$ lies on line ℓ_2
 36 : $P_{190} = (11, 10, 1, 0)$ lies on line ℓ_2
 37 : $P_{205} = (10, 11, 1, 0)$ lies on line ℓ_2
 38 : $P_{224} = (13, 12, 1, 0)$ lies on line ℓ_2
 39 : $P_{239} = (12, 13, 1, 0)$ lies on line ℓ_2
 40 : $P_{258} = (15, 14, 1, 0)$ lies on line ℓ_2
 41 : $P_{273} = (14, 15, 1, 0)$ lies on line ℓ_2
 42 : $P_{290} = (0, 1, 0, 1)$ lies on line ℓ_3
 43 : $P_{306} = (0, 2, 0, 1)$ lies on line ℓ_3
 44 : $P_{307} = (1, 2, 0, 1)$ lies on line ℓ_4
 45 : $P_{322} = (0, 3, 0, 1)$ lies on line ℓ_3
 46 : $P_{323} = (1, 3, 0, 1)$ lies on line ℓ_4
 47 : $P_{338} = (0, 4, 0, 1)$ lies on line ℓ_3
 48 : $P_{339} = (1, 4, 0, 1)$ lies on line ℓ_4
 49 : $P_{354} = (0, 5, 0, 1)$ lies on line ℓ_3
 50 : $P_{355} = (1, 5, 0, 1)$ lies on line ℓ_4
 51 : $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_3
 52 : $P_{371} = (1, 6, 0, 1)$ lies on line ℓ_4
 53 : $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_3
 54 : $P_{387} = (1, 7, 0, 1)$ lies on line ℓ_4
 55 : $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_3
 56 : $P_{403} = (1, 8, 0, 1)$ lies on line ℓ_4
 57 : $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_3
 58 : $P_{419} = (1, 9, 0, 1)$ lies on line ℓ_4
 59 : $P_{434} = (0, 10, 0, 1)$ lies on line ℓ_3
 60 : $P_{435} = (1, 10, 0, 1)$ lies on line ℓ_4
 61 : $P_{450} = (0, 11, 0, 1)$ lies on line ℓ_3
 62 : $P_{451} = (1, 11, 0, 1)$ lies on line ℓ_4
 63 : $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_3
 64 : $P_{467} = (1, 12, 0, 1)$ lies on line ℓ_4
 65 : $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_3
 66 : $P_{483} = (1, 13, 0, 1)$ lies on line ℓ_4
 67 : $P_{498} = (0, 14, 0, 1)$ lies on line ℓ_3

68 : $P_{499} = (1, 14, 0, 1)$ lies on line ℓ_4
 69 : $P_{514} = (0, 15, 0, 1)$ lies on line ℓ_3
 70 : $P_{515} = (1, 15, 0, 1)$ lies on line ℓ_4
 71 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_5
 72 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_7
 73 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_5
 74 : $P_{786} = (1, 0, 2, 1)$ lies on line ℓ_6
 75 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_7
 76 : $P_{834} = (1, 3, 2, 1)$ lies on line ℓ_8
 77 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_5
 78 : $P_{1042} = (1, 0, 3, 1)$ lies on line ℓ_6
 79 : $P_{1074} = (1, 2, 3, 1)$ lies on line ℓ_8
 80 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_7
 81 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_5
 82 : $P_{1298} = (1, 0, 4, 1)$ lies on line ℓ_6
 83 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_7
 84 : $P_{1378} = (1, 5, 4, 1)$ lies on line ℓ_8
 85 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_5
 86 : $P_{1554} = (1, 0, 5, 1)$ lies on line ℓ_6
 87 : $P_{1618} = (1, 4, 5, 1)$ lies on line ℓ_8
 88 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_7
 89 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_5
 90 : $P_{1810} = (1, 0, 6, 1)$ lies on line ℓ_6
 91 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_7
 92 : $P_{1922} = (1, 7, 6, 1)$ lies on line ℓ_8
 93 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_5
 94 : $P_{2066} = (1, 0, 7, 1)$ lies on line ℓ_6
 95 : $P_{2162} = (1, 6, 7, 1)$ lies on line ℓ_8
 96 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_7
 97 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_5
 98 : $P_{2322} = (1, 0, 8, 1)$ lies on line ℓ_6
 99 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_7
 100 : $P_{2466} = (1, 9, 8, 1)$ lies on line ℓ_8
 101 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_5
 102 : $P_{2578} = (1, 0, 9, 1)$ lies on line ℓ_6
 103 : $P_{2706} = (1, 8, 9, 1)$ lies on line ℓ_8
 104 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_7
 105 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_5
 106 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_6
 107 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_7
 108 : $P_{3010} = (1, 11, 10, 1)$ lies on line ℓ_8
 109 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_5
 110 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_6
 111 : $P_{3250} = (1, 10, 11, 1)$ lies on line ℓ_8
 112 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_7
 113 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_5
 114 : $P_{3346} = (1, 0, 12, 1)$ lies on line ℓ_6
 115 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_7
 116 : $P_{3554} = (1, 13, 12, 1)$ lies on line ℓ_8
 117 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_5
 118 : $P_{3602} = (1, 0, 13, 1)$ lies on line ℓ_6
 119 : $P_{3794} = (1, 12, 13, 1)$ lies on line ℓ_8
 120 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_7
 121 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_5

122 : $P_{3858} = (1, 0, 14, 1)$ lies on line ℓ_6
123 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_7
124 : $P_{4098} = (1, 15, 14, 1)$ lies on line ℓ_8
125 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_5

126 : $P_{4114} = (1, 0, 15, 1)$ lies on line ℓ_6
127 : $P_{4338} = (1, 14, 15, 1)$ lies on line ℓ_8
128 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 182 points not on any line:

The points on the surface but not on lines are:

0 : $P_{585} = (8, 3, 1, 1)$	40 : $P_{1407} = (14, 6, 4, 1)$
1 : $P_{587} = (10, 3, 1, 1)$	41 : $P_{1427} = (2, 8, 4, 1)$
2 : $P_{620} = (11, 5, 1, 1)$	42 : $P_{1433} = (8, 8, 4, 1)$
3 : $P_{624} = (15, 5, 1, 1)$	43 : $P_{1502} = (13, 12, 4, 1)$
4 : $P_{660} = (3, 8, 1, 1)$	44 : $P_{1503} = (14, 12, 4, 1)$
5 : $P_{667} = (10, 8, 1, 1)$	45 : $P_{1523} = (2, 14, 4, 1)$
6 : $P_{692} = (3, 10, 1, 1)$	46 : $P_{1530} = (9, 14, 4, 1)$
7 : $P_{697} = (8, 10, 1, 1)$	47 : $P_{1580} = (11, 1, 5, 1)$
8 : $P_{710} = (5, 11, 1, 1)$	48 : $P_{1584} = (15, 1, 5, 1)$
9 : $P_{720} = (15, 11, 1, 1)$	49 : $P_{1590} = (5, 2, 5, 1)$
10 : $P_{774} = (5, 15, 1, 1)$	50 : $P_{1599} = (14, 2, 5, 1)$
11 : $P_{780} = (11, 15, 1, 1)$	51 : $P_{1630} = (13, 4, 5, 1)$
12 : $P_{822} = (5, 2, 2, 1)$	52 : $P_{1642} = (9, 5, 5, 1)$
13 : $P_{839} = (6, 3, 2, 1)$	53 : $P_{1706} = (9, 9, 5, 1)$
14 : $P_{870} = (5, 5, 2, 1)$	54 : $P_{1711} = (14, 9, 5, 1)$
15 : $P_{879} = (14, 5, 2, 1)$	55 : $P_{1731} = (2, 11, 5, 1)$
16 : $P_{903} = (6, 7, 2, 1)$	56 : $P_{1744} = (15, 11, 5, 1)$
17 : $P_{906} = (9, 7, 2, 1)$	57 : $P_{1747} = (2, 12, 5, 1)$
18 : $P_{933} = (4, 9, 2, 1)$	58 : $P_{1758} = (13, 12, 5, 1)$
19 : $P_{943} = (14, 9, 2, 1)$	59 : $P_{1804} = (11, 15, 5, 1)$
20 : $P_{986} = (9, 12, 2, 1)$	60 : $P_{1887} = (14, 4, 6, 1)$
21 : $P_{1027} = (2, 15, 2, 1)$	61 : $P_{1917} = (12, 6, 6, 1)$
22 : $P_{1029} = (4, 15, 2, 1)$	62 : $P_{1932} = (11, 7, 6, 1)$
23 : $P_{1065} = (8, 1, 3, 1)$	63 : $P_{1941} = (4, 8, 6, 1)$
24 : $P_{1067} = (10, 1, 3, 1)$	64 : $P_{1944} = (7, 8, 6, 1)$
25 : $P_{1079} = (6, 2, 3, 1)$	65 : $P_{1955} = (2, 9, 6, 1)$
26 : $P_{1093} = (4, 3, 3, 1)$	66 : $P_{1960} = (7, 9, 6, 1)$
27 : $P_{1109} = (4, 4, 3, 1)$	67 : $P_{1973} = (4, 10, 6, 1)$
28 : $P_{1114} = (9, 4, 3, 1)$	68 : $P_{1980} = (11, 10, 6, 1)$
29 : $P_{1159} = (6, 7, 3, 1)$	69 : $P_{2013} = (12, 12, 6, 1)$
30 : $P_{1167} = (14, 7, 3, 1)$	70 : $P_{2015} = (14, 12, 6, 1)$
31 : $P_{1179} = (10, 8, 3, 1)$	71 : $P_{2019} = (2, 13, 6, 1)$
32 : $P_{1209} = (8, 10, 3, 1)$	72 : $P_{2023} = (6, 13, 6, 1)$
33 : $P_{1215} = (14, 10, 3, 1)$	73 : $P_{2103} = (6, 2, 7, 1)$
34 : $P_{1268} = (3, 14, 3, 1)$	74 : $P_{2106} = (9, 2, 7, 1)$
35 : $P_{1274} = (9, 14, 3, 1)$	75 : $P_{2119} = (6, 3, 7, 1)$
36 : $P_{1349} = (4, 3, 4, 1)$	76 : $P_{2127} = (14, 3, 7, 1)$
37 : $P_{1354} = (9, 3, 4, 1)$	77 : $P_{2172} = (11, 6, 7, 1)$
38 : $P_{1369} = (8, 4, 4, 1)$	78 : $P_{2190} = (13, 7, 7, 1)$
39 : $P_{1390} = (13, 5, 4, 1)$	79 : $P_{2236} = (11, 10, 7, 1)$

80 : $P_{2239} = (14, 10, 7, 1)$	132 : $P_{3386} = (9, 2, 12, 1)$
81 : $P_{2264} = (7, 12, 7, 1)$	133 : $P_{3422} = (13, 4, 12, 1)$
82 : $P_{2266} = (9, 12, 7, 1)$	134 : $P_{3423} = (14, 4, 12, 1)$
83 : $P_{2277} = (4, 13, 7, 1)$	135 : $P_{3427} = (2, 5, 12, 1)$
84 : $P_{2286} = (13, 13, 7, 1)$	136 : $P_{3438} = (13, 5, 12, 1)$
85 : $P_{2293} = (4, 14, 7, 1)$	137 : $P_{3453} = (12, 6, 12, 1)$
86 : $P_{2340} = (3, 1, 8, 1)$	138 : $P_{3455} = (14, 6, 12, 1)$
87 : $P_{2347} = (10, 1, 8, 1)$	139 : $P_{3464} = (7, 7, 12, 1)$
88 : $P_{2379} = (10, 3, 8, 1)$	140 : $P_{3466} = (9, 7, 12, 1)$
89 : $P_{2387} = (2, 4, 8, 1)$	141 : $P_{3523} = (2, 11, 12, 1)$
90 : $P_{2393} = (8, 4, 8, 1)$	142 : $P_{3531} = (10, 11, 12, 1)$
91 : $P_{2421} = (4, 6, 8, 1)$	143 : $P_{3544} = (7, 12, 12, 1)$
92 : $P_{2424} = (7, 6, 8, 1)$	144 : $P_{3563} = (10, 13, 12, 1)$
93 : $P_{2463} = (14, 8, 8, 1)$	145 : $P_{3699} = (2, 6, 13, 1)$
94 : $P_{2472} = (7, 9, 8, 1)$	146 : $P_{3703} = (6, 6, 13, 1)$
95 : $P_{2484} = (3, 10, 8, 1)$	147 : $P_{3717} = (4, 7, 13, 1)$
96 : $P_{2485} = (4, 10, 8, 1)$	148 : $P_{3726} = (13, 7, 13, 1)$
97 : $P_{2547} = (2, 14, 8, 1)$	149 : $P_{3747} = (2, 9, 13, 1)$
98 : $P_{2559} = (14, 14, 8, 1)$	150 : $P_{3786} = (9, 11, 13, 1)$
99 : $P_{2613} = (4, 2, 9, 1)$	151 : $P_{3787} = (10, 11, 13, 1)$
100 : $P_{2623} = (14, 2, 9, 1)$	152 : $P_{3803} = (10, 12, 13, 1)$
101 : $P_{2666} = (9, 5, 9, 1)$	153 : $P_{3815} = (6, 13, 13, 1)$
102 : $P_{2671} = (14, 5, 9, 1)$	154 : $P_{3829} = (4, 14, 13, 1)$
103 : $P_{2675} = (2, 6, 9, 1)$	155 : $P_{3837} = (12, 14, 13, 1)$
104 : $P_{2680} = (7, 6, 9, 1)$	156 : $P_{3850} = (9, 15, 13, 1)$
105 : $P_{2712} = (7, 8, 9, 1)$	157 : $P_{3853} = (12, 15, 13, 1)$
106 : $P_{2736} = (15, 9, 9, 1)$	158 : $P_{3908} = (3, 3, 14, 1)$
107 : $P_{2787} = (2, 13, 9, 1)$	159 : $P_{3914} = (9, 3, 14, 1)$
108 : $P_{2821} = (4, 15, 9, 1)$	160 : $P_{3923} = (2, 4, 14, 1)$
109 : $P_{2832} = (15, 15, 9, 1)$	161 : $P_{3930} = (9, 4, 14, 1)$
110 : $P_{2852} = (3, 1, 10, 1)$	162 : $P_{3973} = (4, 7, 14, 1)$
111 : $P_{2857} = (8, 1, 10, 1)$	163 : $P_{3987} = (2, 8, 14, 1)$
112 : $P_{2889} = (8, 3, 10, 1)$	164 : $P_{3999} = (14, 8, 14, 1)$
113 : $P_{2895} = (14, 3, 10, 1)$	165 : $P_{4069} = (4, 13, 14, 1)$
114 : $P_{2933} = (4, 6, 10, 1)$	166 : $P_{4077} = (12, 13, 14, 1)$
115 : $P_{2940} = (11, 6, 10, 1)$	167 : $P_{4084} = (3, 14, 14, 1)$
116 : $P_{2956} = (11, 7, 10, 1)$	168 : $P_{4109} = (12, 15, 14, 1)$
117 : $P_{2959} = (14, 7, 10, 1)$	169 : $P_{4134} = (5, 1, 15, 1)$
118 : $P_{2964} = (3, 8, 10, 1)$	170 : $P_{4140} = (11, 1, 15, 1)$
119 : $P_{2965} = (4, 8, 10, 1)$	171 : $P_{4147} = (2, 2, 15, 1)$
120 : $P_{3003} = (10, 10, 10, 1)$	172 : $P_{4149} = (4, 2, 15, 1)$
121 : $P_{3110} = (5, 1, 11, 1)$	173 : $P_{4204} = (11, 5, 15, 1)$
122 : $P_{3120} = (15, 1, 11, 1)$	174 : $P_{4261} = (4, 9, 15, 1)$
123 : $P_{3171} = (2, 5, 11, 1)$	175 : $P_{4272} = (15, 9, 15, 1)$
124 : $P_{3184} = (15, 5, 11, 1)$	176 : $P_{4294} = (5, 11, 15, 1)$
125 : $P_{3276} = (11, 11, 11, 1)$	177 : $P_{4298} = (9, 11, 15, 1)$
126 : $P_{3283} = (2, 12, 11, 1)$	178 : $P_{4330} = (9, 13, 15, 1)$
127 : $P_{3291} = (10, 12, 11, 1)$	179 : $P_{4333} = (12, 13, 15, 1)$
128 : $P_{3306} = (9, 13, 11, 1)$	180 : $P_{4349} = (12, 14, 15, 1)$
129 : $P_{3307} = (10, 13, 11, 1)$	181 : $P_{4355} = (2, 15, 15, 1)$
130 : $P_{3334} = (5, 15, 11, 1)$	
131 : $P_{3338} = (9, 15, 11, 1)$	

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	0	0	0	0
1	1	0	1	0	0	1	1	0	0
2	1	1	0	0	0	0	0	1	1
3	1	0	0	0	1	1	0	1	0
4	1	0	0	1	0	0	1	0	1
5	0	1	0	1	0	0	1	1	0
6	0	1	0	0	1	1	0	0	1
7	0	0	1	1	0	1	0	0	1
8	0	0	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_0	P_{20}	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_5	P_{20}	P_{35}	P_{35}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_1	P_1	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_8
in point	P_1	P_1	P_{275}	P_{291}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_3	P_2	P_3

Line 6 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_8
in point	P_2	P_{275}	P_2	P_{531}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_{35}	P_3	P_3	P_{35}

Line 8 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_7
in point	P_{35}	P_{291}	P_{531}	P_{35}

The surface has 321 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_5 = (1, 1, 0, 0)$
 5 : $P_6 = (2, 1, 0, 0)$
 6 : $P_7 = (3, 1, 0, 0)$

7 : $P_8 = (4, 1, 0, 0)$
 8 : $P_9 = (5, 1, 0, 0)$
 9 : $P_{10} = (6, 1, 0, 0)$
 10 : $P_{11} = (7, 1, 0, 0)$
 11 : $P_{12} = (8, 1, 0, 0)$
 12 : $P_{13} = (9, 1, 0, 0)$
 13 : $P_{14} = (10, 1, 0, 0)$

14 : $P_{15} = (11, 1, 0, 0)$
 15 : $P_{16} = (12, 1, 0, 0)$
 16 : $P_{17} = (13, 1, 0, 0)$
 17 : $P_{18} = (14, 1, 0, 0)$
 18 : $P_{19} = (15, 1, 0, 0)$
 19 : $P_{20} = (1, 0, 1, 0)$
 20 : $P_{21} = (2, 0, 1, 0)$

21 : $P_{22} = (3, 0, 1, 0)$	75 : $P_{483} = (1, 13, 0, 1)$	129 : $P_{1349} = (4, 3, 4, 1)$
22 : $P_{23} = (4, 0, 1, 0)$	76 : $P_{498} = (0, 14, 0, 1)$	130 : $P_{1354} = (9, 3, 4, 1)$
23 : $P_{24} = (5, 0, 1, 0)$	77 : $P_{499} = (1, 14, 0, 1)$	131 : $P_{1361} = (0, 4, 4, 1)$
24 : $P_{25} = (6, 0, 1, 0)$	78 : $P_{514} = (0, 15, 0, 1)$	132 : $P_{1369} = (8, 4, 4, 1)$
25 : $P_{26} = (7, 0, 1, 0)$	79 : $P_{515} = (1, 15, 0, 1)$	133 : $P_{1378} = (1, 5, 4, 1)$
26 : $P_{27} = (8, 0, 1, 0)$	80 : $P_{530} = (0, 0, 1, 1)$	134 : $P_{1390} = (13, 5, 4, 1)$
27 : $P_{28} = (9, 0, 1, 0)$	81 : $P_{531} = (1, 0, 1, 1)$	135 : $P_{1407} = (14, 6, 4, 1)$
28 : $P_{29} = (10, 0, 1, 0)$	82 : $P_{546} = (0, 1, 1, 1)$	136 : $P_{1427} = (2, 8, 4, 1)$
29 : $P_{30} = (11, 0, 1, 0)$	83 : $P_{585} = (8, 3, 1, 1)$	137 : $P_{1433} = (8, 8, 4, 1)$
30 : $P_{31} = (12, 0, 1, 0)$	84 : $P_{587} = (10, 3, 1, 1)$	138 : $P_{1502} = (13, 12, 4, 1)$
31 : $P_{32} = (13, 0, 1, 0)$	85 : $P_{620} = (11, 5, 1, 1)$	139 : $P_{1503} = (14, 12, 4, 1)$
32 : $P_{33} = (14, 0, 1, 0)$	86 : $P_{624} = (15, 5, 1, 1)$	140 : $P_{1523} = (2, 14, 4, 1)$
33 : $P_{34} = (15, 0, 1, 0)$	87 : $P_{660} = (3, 8, 1, 1)$	141 : $P_{1530} = (9, 14, 4, 1)$
34 : $P_{35} = (0, 1, 1, 0)$	88 : $P_{667} = (10, 8, 1, 1)$	142 : $P_{1553} = (0, 0, 5, 1)$
35 : $P_{54} = (3, 2, 1, 0)$	89 : $P_{692} = (3, 10, 1, 1)$	143 : $P_{1554} = (1, 0, 5, 1)$
36 : $P_{69} = (2, 3, 1, 0)$	90 : $P_{697} = (8, 10, 1, 1)$	144 : $P_{1580} = (11, 1, 5, 1)$
37 : $P_{88} = (5, 4, 1, 0)$	91 : $P_{710} = (5, 11, 1, 1)$	145 : $P_{1584} = (15, 1, 5, 1)$
38 : $P_{103} = (4, 5, 1, 0)$	92 : $P_{720} = (15, 11, 1, 1)$	146 : $P_{1590} = (5, 2, 5, 1)$
39 : $P_{122} = (7, 6, 1, 0)$	93 : $P_{774} = (5, 15, 1, 1)$	147 : $P_{1599} = (14, 2, 5, 1)$
40 : $P_{137} = (6, 7, 1, 0)$	94 : $P_{780} = (11, 15, 1, 1)$	148 : $P_{1618} = (1, 4, 5, 1)$
41 : $P_{156} = (9, 8, 1, 0)$	95 : $P_{785} = (0, 0, 2, 1)$	149 : $P_{1630} = (13, 4, 5, 1)$
42 : $P_{171} = (8, 9, 1, 0)$	96 : $P_{786} = (1, 0, 2, 1)$	150 : $P_{1633} = (0, 5, 5, 1)$
43 : $P_{190} = (11, 10, 1, 0)$	97 : $P_{817} = (0, 2, 2, 1)$	151 : $P_{1642} = (9, 5, 5, 1)$
44 : $P_{205} = (10, 11, 1, 0)$	98 : $P_{822} = (5, 2, 2, 1)$	152 : $P_{1706} = (9, 9, 5, 1)$
45 : $P_{224} = (13, 12, 1, 0)$	99 : $P_{834} = (1, 3, 2, 1)$	153 : $P_{1711} = (14, 9, 5, 1)$
46 : $P_{239} = (12, 13, 1, 0)$	100 : $P_{839} = (6, 3, 2, 1)$	154 : $P_{1731} = (2, 11, 5, 1)$
47 : $P_{258} = (15, 14, 1, 0)$	101 : $P_{870} = (5, 5, 2, 1)$	155 : $P_{1744} = (15, 11, 5, 1)$
48 : $P_{273} = (14, 15, 1, 0)$	102 : $P_{879} = (14, 5, 2, 1)$	156 : $P_{1747} = (2, 12, 5, 1)$
49 : $P_{275} = (1, 0, 0, 1)$	103 : $P_{903} = (6, 7, 2, 1)$	157 : $P_{1758} = (13, 12, 5, 1)$
50 : $P_{290} = (0, 1, 0, 1)$	104 : $P_{906} = (9, 7, 2, 1)$	158 : $P_{1804} = (11, 15, 5, 1)$
51 : $P_{291} = (1, 1, 0, 1)$	105 : $P_{933} = (4, 9, 2, 1)$	159 : $P_{1809} = (0, 0, 6, 1)$
52 : $P_{306} = (0, 2, 0, 1)$	106 : $P_{943} = (14, 9, 2, 1)$	160 : $P_{1810} = (1, 0, 6, 1)$
53 : $P_{307} = (1, 2, 0, 1)$	107 : $P_{986} = (9, 12, 2, 1)$	161 : $P_{1887} = (14, 4, 6, 1)$
54 : $P_{322} = (0, 3, 0, 1)$	108 : $P_{1027} = (2, 15, 2, 1)$	162 : $P_{1905} = (0, 6, 6, 1)$
55 : $P_{323} = (1, 3, 0, 1)$	109 : $P_{1029} = (4, 15, 2, 1)$	163 : $P_{1917} = (12, 6, 6, 1)$
56 : $P_{338} = (0, 4, 0, 1)$	110 : $P_{1041} = (0, 0, 3, 1)$	164 : $P_{1922} = (1, 7, 6, 1)$
57 : $P_{339} = (1, 4, 0, 1)$	111 : $P_{1042} = (1, 0, 3, 1)$	165 : $P_{1932} = (11, 7, 6, 1)$
58 : $P_{354} = (0, 5, 0, 1)$	112 : $P_{1065} = (8, 1, 3, 1)$	166 : $P_{1941} = (4, 8, 6, 1)$
59 : $P_{355} = (1, 5, 0, 1)$	113 : $P_{1067} = (10, 1, 3, 1)$	167 : $P_{1944} = (7, 8, 6, 1)$
60 : $P_{370} = (0, 6, 0, 1)$	114 : $P_{1074} = (1, 2, 3, 1)$	168 : $P_{1955} = (2, 9, 6, 1)$
61 : $P_{371} = (1, 6, 0, 1)$	115 : $P_{1079} = (6, 2, 3, 1)$	169 : $P_{1960} = (7, 9, 6, 1)$
62 : $P_{386} = (0, 7, 0, 1)$	116 : $P_{1089} = (0, 3, 3, 1)$	170 : $P_{1973} = (4, 10, 6, 1)$
63 : $P_{387} = (1, 7, 0, 1)$	117 : $P_{1093} = (4, 3, 3, 1)$	171 : $P_{1980} = (11, 10, 6, 1)$
64 : $P_{402} = (0, 8, 0, 1)$	118 : $P_{1109} = (4, 4, 3, 1)$	172 : $P_{2013} = (12, 12, 6, 1)$
65 : $P_{403} = (1, 8, 0, 1)$	119 : $P_{1114} = (9, 4, 3, 1)$	173 : $P_{2015} = (14, 12, 6, 1)$
66 : $P_{418} = (0, 9, 0, 1)$	120 : $P_{1159} = (6, 7, 3, 1)$	174 : $P_{2019} = (2, 13, 6, 1)$
67 : $P_{419} = (1, 9, 0, 1)$	121 : $P_{1167} = (14, 7, 3, 1)$	175 : $P_{2023} = (6, 13, 6, 1)$
68 : $P_{434} = (0, 10, 0, 1)$	122 : $P_{1179} = (10, 8, 3, 1)$	176 : $P_{2065} = (0, 0, 7, 1)$
69 : $P_{435} = (1, 10, 0, 1)$	123 : $P_{1209} = (8, 10, 3, 1)$	177 : $P_{2066} = (1, 0, 7, 1)$
70 : $P_{450} = (0, 11, 0, 1)$	124 : $P_{1215} = (14, 10, 3, 1)$	178 : $P_{2103} = (6, 2, 7, 1)$
71 : $P_{451} = (1, 11, 0, 1)$	125 : $P_{1268} = (3, 14, 3, 1)$	179 : $P_{2106} = (9, 2, 7, 1)$
72 : $P_{466} = (0, 12, 0, 1)$	126 : $P_{1274} = (9, 14, 3, 1)$	180 : $P_{2119} = (6, 3, 7, 1)$
73 : $P_{467} = (1, 12, 0, 1)$	127 : $P_{1297} = (0, 0, 4, 1)$	181 : $P_{2127} = (14, 3, 7, 1)$
74 : $P_{482} = (0, 13, 0, 1)$	128 : $P_{1298} = (1, 0, 4, 1)$	182 : $P_{2162} = (1, 6, 7, 1)$

183 : $P_{2172} = (11, 6, 7, 1)$	230 : $P_{2895} = (14, 3, 10, 1)$	277 : $P_{3726} = (13, 7, 13, 1)$
184 : $P_{2177} = (0, 7, 7, 1)$	231 : $P_{2933} = (4, 6, 10, 1)$	278 : $P_{3747} = (2, 9, 13, 1)$
185 : $P_{2190} = (13, 7, 7, 1)$	232 : $P_{2940} = (11, 6, 10, 1)$	279 : $P_{3786} = (9, 11, 13, 1)$
186 : $P_{2236} = (11, 10, 7, 1)$	233 : $P_{2956} = (11, 7, 10, 1)$	280 : $P_{3787} = (10, 11, 13, 1)$
187 : $P_{2239} = (14, 10, 7, 1)$	234 : $P_{2959} = (14, 7, 10, 1)$	281 : $P_{3794} = (1, 12, 13, 1)$
188 : $P_{2264} = (7, 12, 7, 1)$	235 : $P_{2964} = (3, 8, 10, 1)$	282 : $P_{3803} = (10, 12, 13, 1)$
189 : $P_{2266} = (9, 12, 7, 1)$	236 : $P_{2965} = (4, 8, 10, 1)$	283 : $P_{3809} = (0, 13, 13, 1)$
190 : $P_{2277} = (4, 13, 7, 1)$	237 : $P_{2993} = (0, 10, 10, 1)$	284 : $P_{3815} = (6, 13, 13, 1)$
191 : $P_{2286} = (13, 13, 7, 1)$	238 : $P_{3003} = (10, 10, 10, 1)$	285 : $P_{3829} = (4, 14, 13, 1)$
192 : $P_{2293} = (4, 14, 7, 1)$	239 : $P_{3010} = (1, 11, 10, 1)$	286 : $P_{3837} = (12, 14, 13, 1)$
193 : $P_{2321} = (0, 0, 8, 1)$	240 : $P_{3089} = (0, 0, 11, 1)$	287 : $P_{3850} = (9, 15, 13, 1)$
194 : $P_{2322} = (1, 0, 8, 1)$	241 : $P_{3090} = (1, 0, 11, 1)$	288 : $P_{3853} = (12, 15, 13, 1)$
195 : $P_{2340} = (3, 1, 8, 1)$	242 : $P_{3110} = (5, 1, 11, 1)$	289 : $P_{3857} = (0, 0, 14, 1)$
196 : $P_{2347} = (10, 1, 8, 1)$	243 : $P_{3120} = (15, 1, 11, 1)$	290 : $P_{3858} = (1, 0, 14, 1)$
197 : $P_{2379} = (10, 3, 8, 1)$	244 : $P_{3171} = (2, 5, 11, 1)$	291 : $P_{3908} = (3, 3, 14, 1)$
198 : $P_{2387} = (2, 4, 8, 1)$	245 : $P_{3184} = (15, 5, 11, 1)$	292 : $P_{3914} = (9, 3, 14, 1)$
199 : $P_{2393} = (8, 4, 8, 1)$	246 : $P_{3250} = (1, 10, 11, 1)$	293 : $P_{3923} = (2, 4, 14, 1)$
200 : $P_{2421} = (4, 6, 8, 1)$	247 : $P_{3265} = (0, 11, 11, 1)$	294 : $P_{3930} = (9, 4, 14, 1)$
201 : $P_{2424} = (7, 6, 8, 1)$	248 : $P_{3276} = (11, 11, 11, 1)$	295 : $P_{3973} = (4, 7, 14, 1)$
202 : $P_{2449} = (0, 8, 8, 1)$	249 : $P_{3283} = (2, 12, 11, 1)$	296 : $P_{3987} = (2, 8, 14, 1)$
203 : $P_{2463} = (14, 8, 8, 1)$	250 : $P_{3291} = (10, 12, 11, 1)$	297 : $P_{3999} = (14, 8, 14, 1)$
204 : $P_{2466} = (1, 9, 8, 1)$	251 : $P_{3306} = (9, 13, 11, 1)$	298 : $P_{4069} = (4, 13, 14, 1)$
205 : $P_{2472} = (7, 9, 8, 1)$	252 : $P_{3307} = (10, 13, 11, 1)$	299 : $P_{4077} = (12, 13, 14, 1)$
206 : $P_{2484} = (3, 10, 8, 1)$	253 : $P_{3334} = (5, 15, 11, 1)$	300 : $P_{4081} = (0, 14, 14, 1)$
207 : $P_{2485} = (4, 10, 8, 1)$	254 : $P_{3338} = (9, 15, 11, 1)$	301 : $P_{4084} = (3, 14, 14, 1)$
208 : $P_{2547} = (2, 14, 8, 1)$	255 : $P_{3345} = (0, 0, 12, 1)$	302 : $P_{4098} = (1, 15, 14, 1)$
209 : $P_{2559} = (14, 14, 8, 1)$	256 : $P_{3346} = (1, 0, 12, 1)$	303 : $P_{4109} = (12, 15, 14, 1)$
210 : $P_{2577} = (0, 0, 9, 1)$	257 : $P_{3386} = (9, 2, 12, 1)$	304 : $P_{4113} = (0, 0, 15, 1)$
211 : $P_{2578} = (1, 0, 9, 1)$	258 : $P_{3422} = (13, 4, 12, 1)$	305 : $P_{4114} = (1, 0, 15, 1)$
212 : $P_{2613} = (4, 2, 9, 1)$	259 : $P_{3423} = (14, 4, 12, 1)$	306 : $P_{4134} = (5, 1, 15, 1)$
213 : $P_{2623} = (14, 2, 9, 1)$	260 : $P_{3427} = (2, 5, 12, 1)$	307 : $P_{4140} = (11, 1, 15, 1)$
214 : $P_{2666} = (9, 5, 9, 1)$	261 : $P_{3438} = (13, 5, 12, 1)$	308 : $P_{4147} = (2, 2, 15, 1)$
215 : $P_{2671} = (14, 5, 9, 1)$	262 : $P_{3453} = (12, 6, 12, 1)$	309 : $P_{4149} = (4, 2, 15, 1)$
216 : $P_{2675} = (2, 6, 9, 1)$	263 : $P_{3455} = (14, 6, 12, 1)$	310 : $P_{4204} = (11, 5, 15, 1)$
217 : $P_{2680} = (7, 6, 9, 1)$	264 : $P_{3464} = (7, 7, 12, 1)$	311 : $P_{4261} = (4, 9, 15, 1)$
218 : $P_{2706} = (1, 8, 9, 1)$	265 : $P_{3466} = (9, 7, 12, 1)$	312 : $P_{4272} = (15, 9, 15, 1)$
219 : $P_{2712} = (7, 8, 9, 1)$	266 : $P_{3523} = (2, 11, 12, 1)$	313 : $P_{4294} = (5, 11, 15, 1)$
220 : $P_{2721} = (0, 9, 9, 1)$	267 : $P_{3531} = (10, 11, 12, 1)$	314 : $P_{4298} = (9, 11, 15, 1)$
221 : $P_{2736} = (15, 9, 9, 1)$	268 : $P_{3537} = (0, 12, 12, 1)$	315 : $P_{4330} = (9, 13, 15, 1)$
222 : $P_{2787} = (2, 13, 9, 1)$	269 : $P_{3544} = (7, 12, 12, 1)$	316 : $P_{4333} = (12, 13, 15, 1)$
223 : $P_{2821} = (4, 15, 9, 1)$	270 : $P_{3554} = (1, 13, 12, 1)$	317 : $P_{4338} = (1, 14, 15, 1)$
224 : $P_{2832} = (15, 15, 9, 1)$	271 : $P_{3563} = (10, 13, 12, 1)$	318 : $P_{4349} = (12, 14, 15, 1)$
225 : $P_{2833} = (0, 0, 10, 1)$	272 : $P_{3601} = (0, 0, 13, 1)$	319 : $P_{4353} = (0, 15, 15, 1)$
226 : $P_{2834} = (1, 0, 10, 1)$	273 : $P_{3602} = (1, 0, 13, 1)$	320 : $P_{4355} = (2, 15, 15, 1)$
227 : $P_{2852} = (3, 1, 10, 1)$	274 : $P_{3699} = (2, 6, 13, 1)$	
228 : $P_{2857} = (8, 1, 10, 1)$	275 : $P_{3703} = (6, 6, 13, 1)$	
229 : $P_{2889} = (8, 3, 10, 1)$	276 : $P_{3717} = (4, 7, 13, 1)$	