

Rank-74099 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1498835545

General information

| | |
|----------------------------|---------------|
| Number of lines | 2 |
| Number of points | 17 |
| Number of singular points | 1 |
| Number of Eckardt points | 0 |
| Number of double points | 1 |
| Number of single points | 8 |
| Number of points off lines | 8 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | 5^2 |
| Type of lines on points | $2, 1^8, 0^8$ |

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_9$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (340, 356)

Rank of points on Klein quadric: (9, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_3 = (0, 0, 0, 1) = \ell_0 \cap \ell_1$$

Single Points

The surface has 8 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1

2 : $P_{26} = (0, 1, 0, 1)$ lies on line ℓ_0

3 : $P_{30} = (0, 2, 0, 1)$ lies on line ℓ_0

4 : $P_{34} = (0, 3, 0, 1)$ lies on line ℓ_0

5 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_1

6 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_1

7 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 8 points not on any line:

The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$

1 : $P_4 = (1, 1, 1, 1)$

2 : $P_{12} = (1, 1, 1, 0)$

3 : $P_{23} = (1, 0, 0, 1)$

4 : $P_{32} = (2, 2, 0, 1)$

5 : $P_{37} = (3, 3, 0, 1)$

6 : $P_{55} = (2, 0, 2, 1)$

7 : $P_{72} = (3, 0, 3, 1)$

Line Intersection Graph

$$\begin{array}{c|c} & 0 \ 1 \\ \hline 0 & 0 \ 1 \\ 1 & 1 \ 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

| | |
|----------|----------|
| Line | ℓ_1 |
| in point | P_3 |

Line 1 intersects

| | |
|----------|----------|
| Line | ℓ_0 |
| in point | P_3 |

The surface has 17 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_{12} = (1, 1, 1, 0)$

6 : $P_{23} = (1, 0, 0, 1)$
 7 : $P_{26} = (0, 1, 0, 1)$
 8 : $P_{30} = (0, 2, 0, 1)$
 9 : $P_{32} = (2, 2, 0, 1)$
 10 : $P_{34} = (0, 3, 0, 1)$
 11 : $P_{37} = (3, 3, 0, 1)$

12 : $P_{38} = (0, 0, 1, 1)$
 13 : $P_{53} = (0, 0, 2, 1)$
 14 : $P_{55} = (2, 0, 2, 1)$
 15 : $P_{69} = (0, 0, 3, 1)$
 16 : $P_{72} = (3, 0, 3, 1)$