Rank-73798 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1090785414

General information

Number of lines	3
Number of points	4225
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4032
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$3, 1^{192}, 0^{4032}$

Singular Points

The surface has 1 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15183424} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15183424} = \mathbf{Pl}(0,57,1,0,0,0)_{122}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{14917120} = \begin{bmatrix} 1 & 0 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{14917120} = \mathbf{Pl}(0,56,1,0,0,0)_{121}$$

Rank of lines: (17047616, 15183424, 14917120)
Rank of points on Klein quadric: (1, 122, 121)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_0
$1 : P_{4218} = (56,0,0,1)$ lies on line ℓ_1
$2 : P_{4219} = (57,0,0,1)$ lies on line ℓ_2
$3 : P_{8258} = (0,0,1,1)$ lies on line ℓ_0
$4 : P_{8314} = (56,0,1,1)$ lies on line ℓ_1
$5 : P_{8315} = (57,0,1,1)$ lies on line ℓ_2
$6 : P_{12353} = (0,0,2,1)$ lies on line ℓ_0
$7 : P_{12409} = (56,0,2,1)$ lies on line ℓ_1
$8 : P_{12410} = (57,0,2,1)$ lies on line ℓ_2
$9 : P_{16449} = (0,0,3,1)$ lies on line ℓ_0
$10 : P_{16505} = (56,0,3,1)$ lies on line ℓ_1
$11 : P_{16506} = (57,0,3,1)$ lies on line ℓ_2
$12 : P_{20545} = (0,0,4,1)$ lies on line ℓ_0
$13 : P_{20601} = (56,0,4,1)$ lies on line ℓ_1
$14 : P_{20602} = (57,0,4,1)$ lies on line ℓ_2
$15 : P_{24641} = (0,0,5,1)$ lies on line ℓ_0
$16 : P_{24697} = (56,0,5,1)$ lies on line ℓ_1
$17 : P_{24698} = (57,0,5,1)$ lies on line ℓ_2
$18 : P_{28737} = (0,0,6,1)$ lies on line ℓ_0
$19 : P_{28793} = (56,0,6,1)$ lies on line ℓ_1
$20 : P_{28794} = (57,0,6,1)$ lies on line ℓ_2
$21 : P_{32833} = (0,0,7,1)$ lies on line ℓ_0
$22 : P_{32889} = (56,0,7,1)$ lies on line ℓ_1
$23 : P_{32890} = (57,0,7,1)$ lies on line ℓ_2
$24 : P_{36929} = (0,0,8,1)$ lies on line ℓ_0
$25 : P_{36985} = (56,0,8,1)$ lies on line ℓ_1
$26 : P_{36986} = (57,0,8,1)$ lies on line ℓ_2

$27 : P_{41025} = (0,0,9,1)$ lies on line ℓ_0
$28 : P_{41081} = (56,0,9,1)$ lies on line ℓ_1
$29 : P_{41082} = (57,0,9,1)$ lies on line ℓ_2
$30 : P_{45121} = (0,0,10,1)$ lies on line ℓ_0
$31 : P_{45177} = (56,0,10,1)$ lies on line ℓ_1
$32 : P_{45178} = (57,0,10,1)$ lies on line ℓ_2
$33 : P_{49217} = (0,0,11,1)$ lies on line ℓ_0
$34 : P_{49273} = (56,0,11,1)$ lies on line ℓ_1
$35 : P_{49274} = (57,0,11,1)$ lies on line ℓ_2
$36 : P_{53313} = (0,0,12,1)$ lies on line ℓ_0
$37 : P_{53369} = (56,0,12,1)$ lies on line ℓ_1
$38 : P_{53370} = (57,0,12,1)$ lies on line ℓ_2
$39 : P_{57409} = (0,0,13,1)$ lies on line ℓ_0
$40 : P_{57465} = (56,0,13,1)$ lies on line ℓ_1
$41 : P_{57466} = (57,0,13,1)$ lies on line ℓ_2
$42 : P_{61505} = (0,0,14,1)$ lies on line ℓ_0
$43 : P_{61561} = (56,0,14,1)$ lies on line ℓ_1
$44 : P_{61562} = (57,0,14,1)$ lies on line ℓ_2
$45 : P_{65601} = (0,0,15,1)$ lies on line ℓ_0
$46 : P_{65657} = (56,0,15,1)$ lies on line ℓ_1
$47 : P_{65658} = (57,0,15,1)$ lies on line ℓ_2
$48 : P_{69697} = (0,0,16,1)$ lies on line ℓ_0
$49 : P_{69753} = (56,0,16,1)$ lies on line ℓ_1
$50 : P_{69754} = (57,0,16,1)$ lies on line ℓ_2
$51 : P_{73793} = (0,0,17,1)$ lies on line ℓ_0
$52 : P_{73849} = (56,0,17,1)$ lies on line ℓ_1
$53 : P_{73850} = (57,0,17,1)$ lies on line ℓ_2

$54 : P_{77889} = (0, 0, 18, 1)$ lies on line ℓ_0

$55 : P_{77945} = (56, 0, 18, 1)$ lies on line ℓ_1

$56 : P_{77946} = (57, 0, 18, 1)$ lies on line ℓ_2

$57 : P_{81985} = (0, 0, 19, 1)$ lies on line ℓ_0

$58 : P_{82041} = (56, 0, 19, 1)$ lies on line ℓ_1

$59 : P_{82042} = (57, 0, 19, 1)$ lies on line ℓ_2

$60 : P_{86081} = (0, 0, 20, 1)$ lies on line ℓ_0

$61 : P_{86137} = (56, 0, 20, 1)$ lies on line ℓ_1

$62 : P_{86138} = (57, 0, 20, 1)$ lies on line ℓ_2

$63 : P_{90177} = (0, 0, 21, 1)$ lies on line ℓ_0

$64 : P_{90233} = (56, 0, 21, 1)$ lies on line ℓ_1

$65 : P_{90234} = (57, 0, 21, 1)$ lies on line ℓ_2

$66 : P_{94273} = (0, 0, 22, 1)$ lies on line ℓ_0

$67 : P_{94329} = (56, 0, 22, 1)$ lies on line ℓ_1

$68 : P_{94330} = (57, 0, 22, 1)$ lies on line ℓ_2

$69 : P_{98369} = (0, 0, 23, 1)$ lies on line ℓ_0

$70 : P_{98425} = (56, 0, 23, 1)$ lies on line ℓ_1

$71 : P_{98426} = (57, 0, 23, 1)$ lies on line ℓ_2

$72 : P_{102465} = (0, 0, 24, 1)$ lies on line ℓ_0

$73 : P_{102521} = (56, 0, 24, 1)$ lies on line ℓ_1

$74 : P_{102522} = (57, 0, 24, 1)$ lies on line ℓ_2

$75 : P_{106561} = (0, 0, 25, 1)$ lies on line ℓ_0

$76 : P_{106617} = (56, 0, 25, 1)$ lies on line ℓ_1

$77 : P_{106618} = (57, 0, 25, 1)$ lies on line ℓ_2

$78 : P_{110657} = (0, 0, 26, 1)$ lies on line ℓ_0

$79 : P_{110713} = (56, 0, 26, 1)$ lies on line ℓ_1

$80 : P_{110714} = (57, 0, 26, 1)$ lies on line ℓ_2

$81 : P_{114753} = (0, 0, 27, 1)$ lies on line ℓ_0

$82 : P_{114809} = (56, 0, 27, 1)$ lies on line ℓ_1

$83 : P_{114810} = (57, 0, 27, 1)$ lies on line ℓ_2

$84 : P_{118849} = (0, 0, 28, 1)$ lies on line ℓ_0

$85 : P_{118905} = (56, 0, 28, 1)$ lies on line ℓ_1

$86 : P_{118906} = (57, 0, 28, 1)$ lies on line ℓ_2

$87 : P_{122945} = (0, 0, 29, 1)$ lies on line ℓ_0

$88 : P_{123001} = (56, 0, 29, 1)$ lies on line ℓ_1

$89 : P_{123002} = (57, 0, 29, 1)$ lies on line ℓ_2

$90 : P_{127041} = (0, 0, 30, 1)$ lies on line ℓ_0

$91 : P_{127097} = (56, 0, 30, 1)$ lies on line ℓ_1

$92 : P_{127098} = (57, 0, 30, 1)$ lies on line ℓ_2

$93 : P_{131137} = (0, 0, 31, 1)$ lies on line ℓ_0

$94 : P_{131193} = (56, 0, 31, 1)$ lies on line ℓ_1

$95 : P_{131194} = (57, 0, 31, 1)$ lies on line ℓ_2

$96 : P_{135233} = (0, 0, 32, 1)$ lies on line ℓ_0

$97 : P_{135289} = (56, 0, 32, 1)$ lies on line ℓ_1

$98 : P_{135290} = (57, 0, 32, 1)$ lies on line ℓ_2

$99 : P_{139329} = (0, 0, 33, 1)$ lies on line ℓ_0

$100 : P_{139385} = (56, 0, 33, 1)$ lies on line ℓ_1

$101 : P_{139386} = (57, 0, 33, 1)$ lies on line ℓ_2

$102 : P_{143425} = (0, 0, 34, 1)$ lies on line ℓ_0

$103 : P_{143481} = (56, 0, 34, 1)$ lies on line ℓ_1

$104 : P_{143482} = (57, 0, 34, 1)$ lies on line ℓ_2

$105 : P_{147521} = (0, 0, 35, 1)$ lies on line ℓ_0

$106 : P_{147577} = (56, 0, 35, 1)$ lies on line ℓ_1

$107 : P_{147578} = (57, 0, 35, 1)$ lies on line ℓ_2

$108 : P_{151617} = (0, 0, 36, 1)$ lies on line ℓ_0

$109 : P_{151673} = (56, 0, 36, 1)$ lies on line ℓ_1

$110 : P_{151674} = (57, 0, 36, 1)$ lies on line ℓ_2

$111 : P_{155713} = (0, 0, 37, 1)$ lies on line ℓ_0

$112 : P_{155769} = (56, 0, 37, 1)$ lies on line ℓ_1

$113 : P_{155770} = (57, 0, 37, 1)$ lies on line ℓ_2

$114 : P_{159809} = (0, 0, 38, 1)$ lies on line ℓ_0

$115 : P_{159865} = (56, 0, 38, 1)$ lies on line ℓ_1

$116 : P_{159866} = (57, 0, 38, 1)$ lies on line ℓ_2

$117 : P_{163905} = (0, 0, 39, 1)$ lies on line ℓ_0

$118 : P_{163961} = (56, 0, 39, 1)$ lies on line ℓ_1

$119 : P_{163962} = (57, 0, 39, 1)$ lies on line ℓ_2

$120 : P_{168001} = (0, 0, 40, 1)$ lies on line ℓ_0

$121 : P_{168057} = (56, 0, 40, 1)$ lies on line ℓ_1

$122 : P_{168058} = (57, 0, 40, 1)$ lies on line ℓ_2

$123 : P_{172097} = (0, 0, 41, 1)$ lies on line ℓ_0

$124 : P_{172153} = (56, 0, 41, 1)$ lies on line ℓ_1

$125 : P_{172154} = (57, 0, 41, 1)$ lies on line ℓ_2

$126 : P_{176193} = (0, 0, 42, 1)$ lies on line ℓ_0

$127 : P_{176249} = (56, 0, 42, 1)$ lies on line ℓ_1

$128 : P_{176250} = (57, 0, 42, 1)$ lies on line ℓ_2

$129 : P_{180289} = (0, 0, 43, 1)$ lies on line ℓ_0

$130 : P_{180345} = (56, 0, 43, 1)$ lies on line ℓ_1

$131 : P_{180346} = (57, 0, 43, 1)$ lies on line ℓ_2

$132 : P_{184385} = (0, 0, 44, 1)$ lies on line ℓ_0

$133 : P_{184441} = (56, 0, 44, 1)$ lies on line ℓ_1

$134 : P_{184442} = (57, 0, 44, 1)$ lies on line ℓ_2

$135 : P_{188481} = (0, 0, 45, 1)$ lies on line ℓ_0

$136 : P_{188537} = (56, 0, 45, 1)$ lies on line ℓ_1

$137 : P_{188538} = (57, 0, 45, 1)$ lies on line ℓ_2

$138 : P_{192577} = (0, 0, 46, 1)$ lies on line ℓ_0

$139 : P_{192633} = (56, 0, 46, 1)$ lies on line ℓ_1

$140 : P_{192634} = (57, 0, 46, 1)$ lies on line ℓ_2

$141 : P_{196673} = (0, 0, 47, 1)$ lies on line ℓ_0

$142 : P_{196729} = (56, 0, 47, 1)$ lies on line ℓ_1

$143 : P_{196730} = (57, 0, 47, 1)$ lies on line ℓ_2

$144 : P_{200769} = (0, 0, 48, 1)$ lies on line ℓ_0

$145 : P_{200825} = (56, 0, 48, 1)$ lies on line ℓ_1

$146 : P_{200826} = (57, 0, 48, 1)$ lies on line ℓ_2

$147 : P_{204865} = (0, 0, 49, 1)$ lies on line ℓ_0

$148 : P_{204921} = (56, 0, 49, 1)$ lies on line ℓ_1

$149 : P_{204922} = (57, 0, 49, 1)$ lies on line ℓ_2

$150 : P_{208961} = (0, 0, 50, 1)$ lies on line ℓ_0

$151 : P_{209017} = (56, 0, 50, 1)$ lies on line ℓ_1

$152 : P_{209018} = (57, 0, 50, 1)$ lies on line ℓ_2

$153 : P_{213057} = (0, 0, 51, 1)$ lies on line ℓ_0

$154 : P_{213113} = (56, 0, 51, 1)$ lies on line ℓ_1

$155 : P_{213114} = (57, 0, 51, 1)$ lies on line ℓ_2

$156 : P_{217153} = (0, 0, 52, 1)$ lies on line ℓ_0

$157 : P_{217209} = (56, 0, 52, 1)$ lies on line ℓ_1

$158 : P_{217210} = (57, 0, 52, 1)$ lies on line ℓ_2

$159 : P_{221249} = (0, 0, 53, 1)$ lies on line ℓ_0

$160 : P_{221305} = (56, 0, 53, 1)$ lies on line ℓ_1

$161 : P_{221306} = (57, 0, 53, 1)$ lies on line ℓ_2

162 : $P_{225345} = (0, 0, 54, 1)$ lies on line ℓ_0
163 : $P_{225401} = (56, 0, 54, 1)$ lies on line ℓ_1
164 : $P_{225402} = (57, 0, 54, 1)$ lies on line ℓ_2
165 : $P_{229441} = (0, 0, 55, 1)$ lies on line ℓ_0
166 : $P_{229497} = (56, 0, 55, 1)$ lies on line ℓ_1
167 : $P_{229498} = (57, 0, 55, 1)$ lies on line ℓ_2
168 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_0
169 : $P_{233593} = (56, 0, 56, 1)$ lies on line ℓ_1
170 : $P_{233594} = (57, 0, 56, 1)$ lies on line ℓ_2
171 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_0
172 : $P_{237689} = (56, 0, 57, 1)$ lies on line ℓ_1
173 : $P_{237690} = (57, 0, 57, 1)$ lies on line ℓ_2
174 : $P_{241729} = (0, 0, 58, 1)$ lies on line ℓ_0
175 : $P_{241785} = (56, 0, 58, 1)$ lies on line ℓ_1
176 : $P_{241786} = (57, 0, 58, 1)$ lies on line ℓ_2
177 : $P_{245825} = (0, 0, 59, 1)$ lies on line ℓ_0

178 : $P_{245881} = (56, 0, 59, 1)$ lies on line ℓ_1
179 : $P_{245882} = (57, 0, 59, 1)$ lies on line ℓ_2
180 : $P_{249921} = (0, 0, 60, 1)$ lies on line ℓ_0
181 : $P_{249977} = (56, 0, 60, 1)$ lies on line ℓ_1
182 : $P_{249978} = (57, 0, 60, 1)$ lies on line ℓ_2
183 : $P_{254017} = (0, 0, 61, 1)$ lies on line ℓ_0
184 : $P_{254073} = (56, 0, 61, 1)$ lies on line ℓ_1
185 : $P_{254074} = (57, 0, 61, 1)$ lies on line ℓ_2
186 : $P_{258113} = (0, 0, 62, 1)$ lies on line ℓ_0
187 : $P_{258169} = (56, 0, 62, 1)$ lies on line ℓ_1
188 : $P_{258170} = (57, 0, 62, 1)$ lies on line ℓ_2
189 : $P_{262209} = (0, 0, 63, 1)$ lies on line ℓ_0
190 : $P_{262265} = (56, 0, 63, 1)$ lies on line ℓ_1
191 : $P_{262266} = (57, 0, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4032 points not on any line:
Too many to print.

Line Intersection Graph

$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
2 & 1 & 1 & 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 4225 points:
Too many to print.

4