Rank-38 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 4658

General information

Number of lines	9
Number of points	145
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	144
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	$9, 1^{144}$

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4640} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4640} = \mathbf{Pl}(0,1,0,0,1,0)_{305}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \delta^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{48320} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{48320} = \mathbf{Pl}(0,11,0,0,1,0)_{315}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{43952} = \begin{bmatrix} 1 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{43952} = \mathbf{Pl}(0,10,0,0,1,0)_{314}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4913} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4913} = \mathbf{Pl}(0,1,0,1,1,0)_{801}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \delta^{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \mathbf{Pl}(0,11,0,1,0,0)_{59}$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & \delta^{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{48593} = \begin{bmatrix} 1 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{48593} = \mathbf{Pl}(0,11,0,1,1,0)_{811}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \begin{bmatrix} 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \mathbf{Pl}(0,10,0,1,0,0)_{58}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{44225} = \begin{bmatrix} 1 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{44225} = \mathbf{Pl}(0,10,0,1,1,0)_{810}$$

Rank of lines: (4640, 48320, 43952, 69921, 4913, 70091, 48593, 70074, 44225)
Rank of points on Klein quadric: (305, 315, 314, 49, 801, 59, 811, 58, 810)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 144 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_4

$1 : P_{20} = (1,0,1,0)$ lies on line ℓ_0

$2 : P_{29} = (10,0,1,0)$ lies on line ℓ_1

$3 : P_{30} = (11,0,1,0)$ lies on line ℓ_2

$4 : P_{35} = (0,1,1,0)$ lies on line ℓ_3

$5 : P_{36} = (1,1,1,0)$ lies on line ℓ_4

$6 : P_{179} = (0,10,1,0)$ lies on line ℓ_5

$7 : P_{189} = (10,10,1,0)$ lies on line ℓ_6

$8 : P_{195} = (0,11,1,0)$ lies on line ℓ_7

$9 : P_{206} = (11,11,1,0)$ lies on line ℓ_8

$10 : P_{531} = (1,0,1,1)$ lies on line ℓ_0

$11 : P_{540} = (10,0,1,1)$ lies on line ℓ_1

$12 : P_{541} = (11,0,1,1)$ lies on line ℓ_2

$13 : P_{546} = (0,1,1,1)$ lies on line ℓ_3

$14 : P_{689} = (0,10,1,1)$ lies on line ℓ_5

$15 : P_{699} = (10,10,1,1)$ lies on line ℓ_6

$16 : P_{705} = (0,11,1,1)$ lies on line ℓ_7

$17 : P_{716} = (11,11,1,1)$ lies on line ℓ_8

$18 : P_{787} = (2,0,2,1)$ lies on line ℓ_0

$19 : P_{798} = (13,0,2,1)$ lies on line ℓ_1

$20 : P_{800} = (15,0,2,1)$ lies on line ℓ_2

$21 : P_{817} = (0,2,2,1)$ lies on line ℓ_3

$22 : P_{819} = (2,2,2,1)$ lies on line ℓ_4

$23 : P_{993} = (0,13,2,1)$ lies on line ℓ_5

$24 : P_{1006} = (13,13,2,1)$ lies on line ℓ_6

$25 : P_{1025} = (0,15,2,1)$ lies on line ℓ_7

$26 : P_{1040} = (15,15,2,1)$ lies on line ℓ_8

$27 : P_{1044} = (3,0,3,1)$ lies on line ℓ_0

28 : $P_{1045} = (4, 0, 3, 1)$ lies on line ℓ_2
29 : $P_{1048} = (7, 0, 3, 1)$ lies on line ℓ_1
30 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_3
31 : $P_{1092} = (3, 3, 3, 1)$ lies on line ℓ_4
32 : $P_{1105} = (0, 4, 3, 1)$ lies on line ℓ_7
33 : $P_{1109} = (4, 4, 3, 1)$ lies on line ℓ_8
34 : $P_{1153} = (0, 7, 3, 1)$ lies on line ℓ_5
35 : $P_{1160} = (7, 7, 3, 1)$ lies on line ℓ_6
36 : $P_{1300} = (3, 0, 4, 1)$ lies on line ℓ_1
37 : $P_{1301} = (4, 0, 4, 1)$ lies on line ℓ_0
38 : $P_{1304} = (7, 0, 4, 1)$ lies on line ℓ_2
39 : $P_{1345} = (0, 3, 4, 1)$ lies on line ℓ_5
40 : $P_{1348} = (3, 3, 4, 1)$ lies on line ℓ_6
41 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_3
42 : $P_{1365} = (4, 4, 4, 1)$ lies on line ℓ_4
43 : $P_{1409} = (0, 7, 4, 1)$ lies on line ℓ_7
44 : $P_{1416} = (7, 7, 4, 1)$ lies on line ℓ_8
45 : $P_{1558} = (5, 0, 5, 1)$ lies on line ℓ_0
46 : $P_{1562} = (9, 0, 5, 1)$ lies on line ℓ_1
47 : $P_{1565} = (12, 0, 5, 1)$ lies on line ℓ_2
48 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_3
49 : $P_{1638} = (5, 5, 5, 1)$ lies on line ℓ_4
50 : $P_{1697} = (0, 9, 5, 1)$ lies on line ℓ_5
51 : $P_{1706} = (9, 9, 5, 1)$ lies on line ℓ_6
52 : $P_{1745} = (0, 12, 5, 1)$ lies on line ℓ_7
53 : $P_{1757} = (12, 12, 5, 1)$ lies on line ℓ_8
54 : $P_{1815} = (6, 0, 6, 1)$ lies on line ℓ_0
55 : $P_{1817} = (8, 0, 6, 1)$ lies on line ℓ_2
56 : $P_{1823} = (14, 0, 6, 1)$ lies on line ℓ_1
57 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_3
58 : $P_{1911} = (6, 6, 6, 1)$ lies on line ℓ_4
59 : $P_{1937} = (0, 8, 6, 1)$ lies on line ℓ_7
60 : $P_{1945} = (8, 8, 6, 1)$ lies on line ℓ_8
61 : $P_{2033} = (0, 14, 6, 1)$ lies on line ℓ_5
62 : $P_{2047} = (14, 14, 6, 1)$ lies on line ℓ_6
63 : $P_{2068} = (3, 0, 7, 1)$ lies on line ℓ_2
64 : $P_{2069} = (4, 0, 7, 1)$ lies on line ℓ_1
65 : $P_{2072} = (7, 0, 7, 1)$ lies on line ℓ_0
66 : $P_{2113} = (0, 3, 7, 1)$ lies on line ℓ_7
67 : $P_{2116} = (3, 3, 7, 1)$ lies on line ℓ_8
68 : $P_{2129} = (0, 4, 7, 1)$ lies on line ℓ_5
69 : $P_{2133} = (4, 4, 7, 1)$ lies on line ℓ_6
70 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_3
71 : $P_{2184} = (7, 7, 7, 1)$ lies on line ℓ_4
72 : $P_{2327} = (6, 0, 8, 1)$ lies on line ℓ_1
73 : $P_{2329} = (8, 0, 8, 1)$ lies on line ℓ_0
74 : $P_{2335} = (14, 0, 8, 1)$ lies on line ℓ_2
75 : $P_{2417} = (0, 6, 8, 1)$ lies on line ℓ_5
76 : $P_{2423} = (6, 6, 8, 1)$ lies on line ℓ_6
77 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_3
78 : $P_{2457} = (8, 8, 8, 1)$ lies on line ℓ_4
79 : $P_{2545} = (0, 14, 8, 1)$ lies on line ℓ_7
80 : $P_{2559} = (14, 14, 8, 1)$ lies on line ℓ_8
81 : $P_{2582} = (5, 0, 9, 1)$ lies on line ℓ_2

82 : $P_{2586} = (9, 0, 9, 1)$ lies on line ℓ_0
83 : $P_{2589} = (12, 0, 9, 1)$ lies on line ℓ_1
84 : $P_{2657} = (0, 5, 9, 1)$ lies on line ℓ_7
85 : $P_{2662} = (5, 5, 9, 1)$ lies on line ℓ_8
86 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_3
87 : $P_{2730} = (9, 9, 9, 1)$ lies on line ℓ_4
88 : $P_{2769} = (0, 12, 9, 1)$ lies on line ℓ_5
89 : $P_{2781} = (12, 12, 9, 1)$ lies on line ℓ_6
90 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_2
91 : $P_{2843} = (10, 0, 10, 1)$ lies on line ℓ_0
92 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_1
93 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_7
94 : $P_{2850} = (1, 1, 10, 1)$ lies on line ℓ_8
95 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_3
96 : $P_{3003} = (10, 10, 10, 1)$ lies on line ℓ_4
97 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_5
98 : $P_{3020} = (11, 11, 10, 1)$ lies on line ℓ_6
99 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_1
100 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_2
101 : $P_{3100} = (11, 0, 11, 1)$ lies on line ℓ_0
102 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_5
103 : $P_{3106} = (1, 1, 11, 1)$ lies on line ℓ_6
104 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_7
105 : $P_{3259} = (10, 10, 11, 1)$ lies on line ℓ_8
106 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_3
107 : $P_{3276} = (11, 11, 11, 1)$ lies on line ℓ_4
108 : $P_{3350} = (5, 0, 12, 1)$ lies on line ℓ_1
109 : $P_{3354} = (9, 0, 12, 1)$ lies on line ℓ_2
110 : $P_{3357} = (12, 0, 12, 1)$ lies on line ℓ_0
111 : $P_{3425} = (0, 5, 12, 1)$ lies on line ℓ_5
112 : $P_{3430} = (5, 5, 12, 1)$ lies on line ℓ_6
113 : $P_{3489} = (0, 9, 12, 1)$ lies on line ℓ_7
114 : $P_{3498} = (9, 9, 12, 1)$ lies on line ℓ_8
115 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_3
116 : $P_{3549} = (12, 12, 12, 1)$ lies on line ℓ_4
117 : $P_{3603} = (2, 0, 13, 1)$ lies on line ℓ_2
118 : $P_{3614} = (13, 0, 13, 1)$ lies on line ℓ_0
119 : $P_{3616} = (15, 0, 13, 1)$ lies on line ℓ_1
120 : $P_{3633} = (0, 2, 13, 1)$ lies on line ℓ_7
121 : $P_{3635} = (2, 2, 13, 1)$ lies on line ℓ_8
122 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_3
123 : $P_{3822} = (13, 13, 13, 1)$ lies on line ℓ_4
124 : $P_{3841} = (0, 15, 13, 1)$ lies on line ℓ_5
125 : $P_{3856} = (15, 15, 13, 1)$ lies on line ℓ_6
126 : $P_{3863} = (6, 0, 14, 1)$ lies on line ℓ_2
127 : $P_{3865} = (8, 0, 14, 1)$ lies on line ℓ_1
128 : $P_{3871} = (14, 0, 14, 1)$ lies on line ℓ_0
129 : $P_{3953} = (0, 6, 14, 1)$ lies on line ℓ_7
130 : $P_{3959} = (6, 6, 14, 1)$ lies on line ℓ_8
131 : $P_{3985} = (0, 8, 14, 1)$ lies on line ℓ_5
132 : $P_{3993} = (8, 8, 14, 1)$ lies on line ℓ_6
133 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_3
134 : $P_{4095} = (14, 14, 14, 1)$ lies on line ℓ_4
135 : $P_{4115} = (2, 0, 15, 1)$ lies on line ℓ_1

136 : $P_{4126} = (13, 0, 15, 1)$ lies on line ℓ_2
137 : $P_{4128} = (15, 0, 15, 1)$ lies on line ℓ_0
138 : $P_{4145} = (0, 2, 15, 1)$ lies on line ℓ_5
139 : $P_{4147} = (2, 2, 15, 1)$ lies on line ℓ_6
140 : $P_{4321} = (0, 13, 15, 1)$ lies on line ℓ_7

141 : $P_{4334} = (13, 13, 15, 1)$ lies on line ℓ_8
142 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_3
143 : $P_{4368} = (15, 15, 15, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
 |0 1 2 3 4 5 6 7 8
0|0 1 1 1 1 1 1 1 1
1|1 0 1 1 1 1 1 1 1
2|1 1 0 1 1 1 1 1 1
3|1 1 1 0 1 1 1 1 1
4|1 1 1 1 0 1 1 1 1
5|1 1 1 1 1 0 1 1 1
6|1 1 1 1 1 1 0 1 1
7|1 1 1 1 1 1 1 0 1
8|1 1 1 1 1 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 145 points:
The points on the surface are:

$0: P_3 = (0, 0, 0, 1)$
$1: P_4 = (1, 1, 1, 1)$
$2: P_{20} = (1, 0, 1, 0)$
$3: P_{29} = (10, 0, 1, 0)$
$4: P_{30} = (11, 0, 1, 0)$
$5: P_{35} = (0, 1, 1, 0)$
$6: P_{36} = (1, 1, 1, 0)$
$7: P_{179} = (0, 10, 1, 0)$
$8: P_{189} = (10, 10, 1, 0)$
$9: P_{195} = (0, 11, 1, 0)$
$10: P_{206} = (11, 11, 1, 0)$
$11: P_{531} = (1, 0, 1, 1)$
$12: P_{540} = (10, 0, 1, 1)$
$13: P_{541} = (11, 0, 1, 1)$
$14: P_{546} = (0, 1, 1, 1)$
$15: P_{689} = (0, 10, 1, 1)$
$16: P_{699} = (10, 10, 1, 1)$
$17: P_{705} = (0, 11, 1, 1)$
$18: P_{716} = (11, 11, 1, 1)$
$19: P_{787} = (2, 0, 2, 1)$
$20: P_{798} = (13, 0, 2, 1)$
$21: P_{800} = (15, 0, 2, 1)$
$22: P_{817} = (0, 2, 2, 1)$
$23: P_{819} = (2, 2, 2, 1)$
$24: P_{993} = (0, 13, 2, 1)$
$25: P_{1006} = (13, 13, 2, 1)$
$26: P_{1025} = (0, 15, 2, 1)$
$27: P_{1040} = (15, 15, 2, 1)$
$28: P_{1044} = (3, 0, 3, 1)$
$29: P_{1045} = (4, 0, 3, 1)$
$30: P_{1048} = (7, 0, 3, 1)$
$31: P_{1089} = (0, 3, 3, 1)$
$32: P_{1092} = (3, 3, 3, 1)$
$33: P_{1105} = (0, 4, 3, 1)$
$34: P_{1109} = (4, 4, 3, 1)$
$35: P_{1153} = (0, 7, 3, 1)$
$36: P_{1160} = (7, 7, 3, 1)$
$37: P_{1300} = (3, 0, 4, 1)$
$38: P_{1301} = (4, 0, 4, 1)$
$39: P_{1304} = (7, 0, 4, 1)$
$40: P_{1345} = (0, 3, 4, 1)$
$41: P_{1348} = (3, 3, 4, 1)$
$42: P_{1361} = (0, 4, 4, 1)$
$43: P_{1365} = (4, 4, 4, 1)$
$44: P_{1409} = (0, 7, 4, 1)$
$45: P_{1416} = (7, 7, 4, 1)$
$46: P_{1558} = (5, 0, 5, 1)$

$47: P_{1562} = (9, 0, 5, 1)$
$48: P_{1565} = (12, 0, 5, 1)$
$49: P_{1633} = (0, 5, 5, 1)$
$50: P_{1638} = (5, 5, 5, 1)$
$51: P_{1697} = (0, 9, 5, 1)$
$52: P_{1706} = (9, 9, 5, 1)$
$53: P_{1745} = (0, 12, 5, 1)$
$54: P_{1757} = (12, 12, 5, 1)$
$55: P_{1815} = (6, 0, 6, 1)$
$56: P_{1817} = (8, 0, 6, 1)$
$57: P_{1823} = (14, 0, 6, 1)$
$58: P_{1905} = (0, 6, 6, 1)$
$59: P_{1911} = (6, 6, 6, 1)$
$60: P_{1937} = (0, 8, 6, 1)$
$61: P_{1945} = (8, 8, 6, 1)$
$62: P_{2033} = (0, 14, 6, 1)$
$63: P_{2047} = (14, 14, 6, 1)$
$64: P_{2068} = (3, 0, 7, 1)$
$65: P_{2069} = (4, 0, 7, 1)$
$66: P_{2072} = (7, 0, 7, 1)$
$67: P_{2113} = (0, 3, 7, 1)$
$68: P_{2116} = (3, 3, 7, 1)$
$69: P_{2129} = (0, 4, 7, 1)$
$70: P_{2133} = (4, 4, 7, 1)$
$71: P_{2177} = (0, 7, 7, 1)$
$72: P_{2184} = (7, 7, 7, 1)$
$73: P_{2327} = (6, 0, 8, 1)$
$74: P_{2329} = (8, 0, 8, 1)$
$75: P_{2335} = (14, 0, 8, 1)$
$76: P_{2417} = (0, 6, 8, 1)$
$77: P_{2423} = (6, 6, 8, 1)$
$78: P_{2449} = (0, 8, 8, 1)$
$79: P_{2457} = (8, 8, 8, 1)$
$80: P_{2545} = (0, 14, 8, 1)$
$81: P_{2559} = (14, 14, 8, 1)$
$82: P_{2582} = (5, 0, 9, 1)$
$83: P_{2586} = (9, 0, 9, 1)$
$84: P_{2589} = (12, 0, 9, 1)$
$85: P_{2657} = (0, 5, 9, 1)$
$86: P_{2662} = (5, 5, 9, 1)$
$87: P_{2721} = (0, 9, 9, 1)$
$88: P_{2730} = (9, 9, 9, 1)$
$89: P_{2769} = (0, 12, 9, 1)$
$90: P_{2781} = (12, 12, 9, 1)$
$91: P_{2834} = (1, 0, 10, 1)$
$92: P_{2843} = (10, 0, 10, 1)$
$93: P_{2844} = (11, 0, 10, 1)$

$94: P_{2849} = (0, 1, 10, 1)$
$95: P_{2850} = (1, 1, 10, 1)$
$96: P_{2993} = (0, 10, 10, 1)$
$97: P_{3003} = (10, 10, 10, 1)$
$98: P_{3009} = (0, 11, 10, 1)$
$99: P_{3020} = (11, 11, 10, 1)$
$100: P_{3090} = (1, 0, 11, 1)$
$101: P_{3099} = (10, 0, 11, 1)$
$102: P_{3100} = (11, 0, 11, 1)$
$103: P_{3105} = (0, 1, 11, 1)$
$104: P_{3106} = (1, 1, 11, 1)$
$105: P_{3249} = (0, 10, 11, 1)$
$106: P_{3259} = (10, 10, 11, 1)$
$107: P_{3265} = (0, 11, 11, 1)$
$108: P_{3276} = (11, 11, 11, 1)$
$109: P_{3350} = (5, 0, 12, 1)$
$110: P_{3354} = (9, 0, 12, 1)$
$111: P_{3357} = (12, 0, 12, 1)$
$112: P_{3425} = (0, 5, 12, 1)$
$113: P_{3430} = (5, 5, 12, 1)$
$114: P_{3489} = (0, 9, 12, 1)$
$115: P_{3498} = (9, 9, 12, 1)$
$116: P_{3537} = (0, 12, 12, 1)$
$117: P_{3549} = (12, 12, 12, 1)$
$118: P_{3603} = (2, 0, 13, 1)$
$119: P_{3614} = (13, 0, 13, 1)$
$120: P_{3616} = (15, 0, 13, 1)$
$121: P_{3633} = (0, 2, 13, 1)$
$122: P_{3635} = (2, 2, 13, 1)$
$123: P_{3809} = (0, 13, 13, 1)$
$124: P_{3822} = (13, 13, 13, 1)$
$125: P_{3841} = (0, 15, 13, 1)$
$126: P_{3856} = (15, 15, 13, 1)$
$127: P_{3863} = (6, 0, 14, 1)$
$128: P_{3865} = (8, 0, 14, 1)$
$129: P_{3871} = (14, 0, 14, 1)$
$130: P_{3953} = (0, 6, 14, 1)$
$131: P_{3959} = (6, 6, 14, 1)$
$132: P_{3985} = (0, 8, 14, 1)$
$133: P_{3993} = (8, 8, 14, 1)$
$134: P_{4081} = (0, 14, 14, 1)$
$135: P_{4095} = (14, 14, 14, 1)$
$136: P_{4115} = (2, 0, 15, 1)$
$137: P_{4126} = (13, 0, 15, 1)$
$138: P_{4128} = (15, 0, 15, 1)$
$139: P_{4145} = (0, 2, 15, 1)$
$140: P_{4147} = (2, 2, 15, 1)$

$141: P_{4321} = (0, 13, 15, 1)$ $143: P_{4353} = (0, 15, 15, 1)$
$142: P_{4334} = (13, 13, 15, 1)$ $144: P_{4368} = (15, 15, 15, 1)$