Rank-65561 over GF(8)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227137685

General information

Number of lines	4
Number of points	33
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	32
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	4, 1^{32}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \mathbf{Pl}(0,0,0,0,1,0)_{81}$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \mathbf{Pl}(0,0,0,1,1,0)_{201}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0,1,0,1,1,0)_{209}$$

Rank of lines: (72, 145, 4689, 729)
Rank of points on Klein quadric: (81, 201, 25, 209)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 32 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_4 = (1,1,1,1)$ lies on line ℓ_3
2 : $P_5 = (1,1,0,0)$ lies on line ℓ_1
3 : $P_{19} = (0,1,1,0)$ lies on line ℓ_2
4 : $P_{20} = (1,1,1,0)$ lies on line ℓ_3
5 : $P_{75} = (1,0,0,1)$ lies on line ℓ_0
6 : $P_{76} = (2,0,0,1)$ lies on line ℓ_0
7 : $P_{77} = (3,0,0,1)$ lies on line ℓ_0
8 : $P_{78} = (4,0,0,1)$ lies on line ℓ_0
9 : $P_{79} = (5,0,0,1)$ lies on line ℓ_0
10 : $P_{80} = (6,0,0,1)$ lies on line ℓ_0
11 : $P_{81} = (7,0,0,1)$ lies on line ℓ_0
12 : $P_{83} = (1,1,0,1)$ lies on line ℓ_1
13 : $P_{92} = (2,2,0,1)$ lies on line ℓ_1
14 : $P_{101} = (3,3,0,1)$ lies on line ℓ_1
15 : $P_{110} = (4,4,0,1)$ lies on line ℓ_1
16 : $P_{119} = (5,5,0,1)$ lies on line ℓ_1
17 : $P_{128} = (6,6,0,1)$ lies on line ℓ_1
18 : $P_{137} = (7,7,0,1)$ lies on line ℓ_1
19 : $P_{146} = (0,1,1,1)$ lies on line ℓ_2
20 : $P_{217} = (0,2,2,1)$ lies on line ℓ_2
21 : $P_{219} = (2,2,2,1)$ lies on line ℓ_3
22 : $P_{289} = (0,3,3,1)$ lies on line ℓ_2
23 : $P_{292} = (3,3,3,1)$ lies on line ℓ_3
24 : $P_{361} = (0,4,4,1)$ lies on line ℓ_2
25 : $P_{365} = (4,4,4,1)$ lies on line ℓ_3
26 : $P_{433} = (0,5,5,1)$ lies on line ℓ_2
27 : $P_{438} = (5,5,5,1)$ lies on line ℓ_3
28 : $P_{505} = (0,6,6,1)$ lies on line ℓ_2
29 : $P_{511} = (6,6,6,1)$ lies on line ℓ_3
30 : $P_{577} = (0,7,7,1)$ lies on line ℓ_2
31 : $P_{584} = (7,7,7,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3
0	0 1 1 1
1	1 0 1 1
2	1 1 0 1
3	1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_3	P_3	P_3

The surface has 33 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_{19} = (0, 1, 1, 0)$
5 : $P_{20} = (1, 1, 1, 0)$
6 : $P_{75} = (1, 0, 0, 1)$
7 : $P_{76} = (2, 0, 0, 1)$
8 : $P_{77} = (3, 0, 0, 1)$
9 : $P_{78} = (4, 0, 0, 1)$
10 : $P_{79} = (5, 0, 0, 1)$
11 : $P_{80} = (6, 0, 0, 1)$
12 : $P_{81} = (7, 0, 0, 1)$
13 : $P_{83} = (1, 1, 0, 1)$
14 : $P_{92} = (2, 2, 0, 1)$
15 : $P_{101} = (3, 3, 0, 1)$
16 : $P_{110} = (4, 4, 0, 1)$
17 : $P_{119} = (5, 5, 0, 1)$
18 : $P_{128} = (6, 6, 0, 1)$
19 : $P_{137} = (7, 7, 0, 1)$
20 : $P_{146} = (0, 1, 1, 1)$
21 : $P_{217} = (0, 2, 2, 1)$
22 : $P_{219} = (2, 2, 2, 1)$
23 : $P_{289} = (0, 3, 3, 1)$
24 : $P_{292} = (3, 3, 3, 1)$
25 : $P_{361} = (0, 4, 4, 1)$
26 : $P_{365} = (4, 4, 4, 1)$
27 : $P_{433} = (0, 5, 5, 1)$
28 : $P_{438} = (5, 5, 5, 1)$
29 : $P_{505} = (0, 6, 6, 1)$
30 : $P_{511} = (6, 6, 6, 1)$
31 : $P_{577} = (0, 7, 7, 1)$
32 : $P_{584} = (7, 7, 7, 1)$