Rank-76355 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 76355

General information

Number of lines	5
Number of points	11
Number of singular points	0
Number of Eckardt points	1
Number of double points	3
Number of single points	6
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	$3, 2^3, 1^6, 0$

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_7 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_7 = \mathbf{Pl}(1,0,0,0,0,1)_{20}$$
$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_{4}$$

Rank of lines: (0, 4, 7, 34, 18)
Rank of points on Klein quadric: (0, 2, 20, 1, 4)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$. $T = 6$

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_6 = (1,0,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 6 single points:
The single points on the surface are:

$0: \ P_3 = (0,0,0,1)$ lies on line ℓ_3
$1: \ P_5 = (1,1,0,0)$ lies on line ℓ_0
$2: \ P_8 = (1,1,1,0)$ lies on line ℓ_2
$3: \ P_9 = (1,0,0,1)$ lies on line ℓ_4
$4: \ P_{12} = (0,0,1,1)$ lies on line ℓ_3
$5: \ P_{13} = (1,0,1,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0: \ P_{14} = (0,1,1,1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	1
2	1	1	0	0	0
3	0	1	0	0	1
4	0	1	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_6	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_6

Line 3 intersects

Line	ℓ_1	ℓ_4
in point	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_2	P_2

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_1 = (0, 1, 0, 0)$
$2: P_2 = (0, 0, 1, 0)$
$3: P_3 = (0, 0, 0, 1)$
$4: P_5 = (1, 1, 0, 0)$
$5: P_6 = (1, 0, 1, 0)$
$6: P_8 = (1, 1, 1, 0)$
$7: P_9 = (1, 0, 0, 1)$
$8: P_{12} = (0, 0, 1, 1)$
$9: P_{13} = (1, 0, 1, 1)$
$10: P_{14} = (0, 1, 1, 1)$