Rank-76099 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172877

General information

Number of lines	27
Number of points	121
Number of singular points	0
Number of Eckardt points	13
Number of double points	96
Number of single points	12
Number of points off lines	0
Number of Hesse planes	0
Number of axes	16
Type of points on lines	9^{27}
Type of lines on points	3^{13}, 2^{96}, 1^{12}

Singular Points

The surface has 0 singular points:

The 27 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = a_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = a_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_2 = a_3 = \left[\begin{array}{cccc} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 1 & \gamma^3 \end{array}\right]_{507} = \left[\begin{array}{cccc} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{array}\right]_{507} = \mathbf{Pl}(0,0,7,5,7,1)_{4436}$$

$$\ell_3 = a_4 = \left[\begin{array}{cccc} 1 & 0 & \gamma^2 & 1 \\ 0 & 1 & 1 & \gamma^5 \end{array}\right]_{901} = \left[\begin{array}{cccc} 1 & 0 & 4 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right]_{901} = \mathbf{Pl}(4,0,4,3,5,1)_{3387}$$

$$\ell_4 = a_5 = \left[\begin{array}{cccc} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 1 & \gamma^6 \end{array}\right]_{289} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{array}\right]_{289} = \mathbf{Pl}(0,0,2,6,2,1)_{1841}$$

$$\ell_5 = a_6 = \left[\begin{array}{cccc} 1 & \gamma^3 & 0 & 1 \\ 0 & 0 & 1 & \gamma^5 \end{array}\right]_{1016} = \left[\begin{array}{cccc} 1 & 5 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]_{1016} = \mathbf{Pl}(0,7,4,3,4,1)_{2893}$$

$$\ell_6 = b_1 = \left[\begin{array}{cccc} 1 & 0 & \gamma^3 & 1 \\ 0 & 1 & 0 & \gamma^5 \end{array}\right]_{973} = \left[\begin{array}{cccc} 1 & 0 & 5 & 1 \\ 0 & 1 & 0 & 3 \end{array}\right]_{973} = \mathbf{Pl}(4,3,0,7,4,1)_{2830}$$

$$\ell_7 = b_2 = \left[\begin{array}{cccc} 1 & 0 & \gamma^2 & 0 \\ 0 & 1 & 1 & \gamma^5 \end{array}\right]_{317} = \left[\begin{array}{cccc} 1 & 0 & 4 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right]_{317} = \mathbf{Pl}(4,3,4,0,5,1)_{3264}$$

$$\ell_8 = b_3 = \left[\begin{array}{cccc} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & 0 & \gamma^6 \end{array}\right]_{267} = \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 6 \end{array}\right]_{267} = \mathbf{Pl}(2,6,0,0,2,1)_{1722}$$

$$\ell_9 = b_4 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_{10} = b_5 = \left[\begin{array}{cccc} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & 0 & \gamma^3 \end{array}\right]_{478} = \left[\begin{array}{cccc} 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & 5 \end{array}\right]_{478} = \mathbf{Pl}(7,5,0,0,7,1)_{4247}$$

$$\ell_{11} = b_6 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_{0} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_{0} = \mathbf{Pl}(1,0,0,0,0,0)_{0}$$

$$\ell_{12} = c_{12} = \left[\begin{array}{cccc} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & 0 & \gamma^5 \end{array}\right]_{389} = \left[\begin{array}{cccc} 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right]_{389} = \mathbf{Pl}(4,3,0,0,4,1)_{2732}$$

$$\ell_{13} = c_{13} = \left[\begin{array}{cccc} 1 & 0 & \gamma & 0 \\ 0 & 1 & 1 & \gamma^6 \end{array}\right]_{195} = \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 6 \end{array}\right]_{195} = \mathbf{Pl}(2,6,2,0,3,1)_{2240}$$

$$\ell_{14} = c_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{648} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

$$\ell_{15} = c_{15} = \left[\begin{array}{cccc} 1 & 0 & \gamma^4 & 0 \\ 0 & 1 & 1 & \gamma^3 \end{array}\right]_{552} = \left[\begin{array}{cccc} 1 & 0 & 7 & 0 \\ 0 & 1 & 1 & 5 \end{array}\right]_{552} = \mathbf{Pl}(7,5,7,0,6,1)_{3792}$$

$$\ell_{16} = c_{16} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_{74} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_{17} = c_{23} = \left[\begin{array}{cccc} 1 & 0 & \gamma^5 & 1 \\ 0 & 1 & 0 & \gamma^6 \end{array}\right]_{851} = \left[\begin{array}{cccc} 1 & 0 & 3 & 1 \\ 0 & 1 & 0 & 6 \end{array}\right]_{851} = \mathbf{Pl}(2,6,0,4,2,1)_{1799}$$

$$\ell_{18} = c_{24} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4689} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_{19} = c_{25} = \left[\begin{array}{cccc} 1 & 0 & \gamma^6 & 1 \\ 0 & 1 & 0 & \gamma^3 \end{array}\right]_{1062} = \left[\begin{array}{cccc} 1 & 0 & 6 & 1 \\ 0 & 1 & 0 & 5 \end{array}\right]_{1062} = \mathbf{Pl}(7,5,0,2,7,1)_{4310}$$

$$\ell_{20} = c_{26} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{584} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{584} = \mathbf{Pl}(1,0,0,1,0,0)_{18}$$

$$\ell_{21} = c_{34} = \left[\begin{array}{cccc} 1 & \gamma^6 & 0 & 1 \\ 0 & 0 & 1 & \gamma^3 \end{array}\right]_{1091} = \left[\begin{array}{cccc} 1 & 6 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{array}\right]_{1091} = \mathbf{Pl}(0,2,7,5,7,1)_{4445}$$

$$\ell_{22} = c_{35} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{658} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{658} = \mathbf{Pl}(1,1,1,1,0,1)_{874}$$

$$\ell_{23} = c_{36} = \left[\begin{array}{cccc} 1 & 0 & \gamma^4 & 1 \\ 0 & 1 & 1 & \gamma^3 \end{array}\right]_{1136} = \left[\begin{array}{cccc} 1 & 0 & 7 & 1 \\ 0 & 1 & 1 & 5 \end{array}\right]_{1136} = \mathbf{Pl}(7,0,7,5,6,1)_{3939}$$

$$\ell_{24} = c_{45} = \left[\begin{array}{cccc} 1 & \gamma^5 & 0 & 1 \\ 0 & 0 & 1 & \gamma^6 \end{array}\right]_{873} = \left[\begin{array}{cccc} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 6 \end{array}\right]_{873} = \mathbf{Pl}(0,4,2,6,2,1)_{1852}$$

$$\ell_{25} = c_{46} = \left[\begin{array}{cccc} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 1 & \gamma^5 \end{array}\right]_{432} = \left[\begin{array}{cccc} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{432} = \mathbf{Pl}(0,0,4,3,4,1)_{2879}$$

$$\ell_{26} = c_{56} = \left[\begin{array}{cccc} 1 & 0 & \gamma & 1 \\ 0 & 1 & 1 & \gamma^6 \end{array}\right]_{779} = \left[\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 6 \end{array}\right]_{779} = \mathbf{Pl}(2,0,2,6,3,1)_{2347}$$

Rank of lines: (64, 4680, 507, 901, 289, 1016, 973, 317, 267, 4744, 478, 0, 389, 195, 648, 552, 74, 851, 4689, 1062, 584, 1091, 658, 1136, 873, 432, 779)
Rank of points on Klein quadric: (2, 17, 4436, 3387, 1841, 2893, 2830, 3264, 1722, 1, 4247, 0, 2732, 2240, 10, 3792, 665, 1799, 25, 4310, 18, 4445, 874, 3939, 1852, 2879, 2347)

Eckardt Points

The surface has 13 Eckardt points:
$0 : E_{26} = a_2 \cap b_6 \cap c_{26} = P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : E_{14} = a_1 \cap b_4 \cap c_{14} = P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$,
$2 : E_{24} = a_2 \cap b_4 \cap c_{24} = P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$3 : E_{16,24,35} = c_{16} \cap c_{24} \cap c_{35} = P_{19} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$4 : E_{23} = a_2 \cap b_3 \cap c_{23} = P_{90} = \mathbf{P}(0,\gamma,0,1) = \mathbf{P}(0,2,0,1)$,
$5 : E_{21} = a_2 \cap b_1 \cap c_{12} = P_{106} = \mathbf{P}(0,\gamma^2,0,1) = \mathbf{P}(0,4,0,1)$,
$6 : E_{25} = a_2 \cap b_5 \cap c_{25} = P_{130} = \mathbf{P}(0,\gamma^4,0,1) = \mathbf{P}(0,7,0,1)$,
$7 : E_{54} = a_5 \cap b_4 \cap c_{45} = P_{201} = \mathbf{P}(0,0,\gamma,1) = \mathbf{P}(0,0,2,1)$,
$8 : E_{13,24,56} = c_{13} \cap c_{24} \cap c_{56} = P_{217} = \mathbf{P}(0,\gamma,\gamma,1) = \mathbf{P}(0,2,2,1)$,
$9 : E_{64} = a_6 \cap b_4 \cap c_{46} = P_{329} = \mathbf{P}(0,0,\gamma^2,1) = \mathbf{P}(0,0,4,1)$,
$10 : E_{42} = a_4 \cap b_2 \cap c_{24} = P_{361} = \mathbf{P}(0,\gamma^2,\gamma^2,1) = \mathbf{P}(0,4,4,1)$,
$11 : E_{34} = a_3 \cap b_4 \cap c_{34} = P_{521} = \mathbf{P}(0,0,\gamma^4,1) = \mathbf{P}(0,0,7,1)$,
$12 : E_{15,24,36} = c_{15} \cap c_{24} \cap c_{36} = P_{577} = \mathbf{P}(0,\gamma^4,\gamma^4,1) = \mathbf{P}(0,7,7,1)$.

Double Points

The surface has 96 Double points:
The double points on the surface are:

$P_{14} = (3,0,1,0) = \ell_0 \cap \ell_7 = a_1 \cap b_2$
$P_{15} = (4,0,1,0) = \ell_0 \cap \ell_8 = a_1 \cap b_3$
$P_{13} = (2,0,1,0) = \ell_0 \cap \ell_{10} = a_1 \cap b_5$
$P_0 = (1,0,0,0) = \ell_0 \cap \ell_{11} = a_1 \cap b_6$
$P_{18} = (7,0,1,0) = \ell_0 \cap \ell_{12} = a_1 \cap c_{12}$
$P_{17} = (6,0,1,0) = \ell_0 \cap \ell_{13} = a_1 \cap c_{13}$
$P_{16} = (5,0,1,0) = \ell_0 \cap \ell_{15} = a_1 \cap c_{15}$
$P_{12} = (1,0,1,0) = \ell_0 \cap \ell_{16} = a_1 \cap c_{16}$
$P_{531} = (2,1,7,1) = \ell_2 \cap \ell_6 = a_3 \cap b_1$
$P_{558} = (5,4,7,1) = \ell_2 \cap \ell_7 = a_3 \cap b_2$
$P_{580} = (3,7,7,1) = \ell_2 \cap \ell_{10} = a_3 \cap b_5$
$P_6 = (2,1,0,0) = \ell_2 \cap \ell_{11} = a_3 \cap b_6$
$P_{541} = (4,2,7,1) = \ell_2 \cap \ell_{13} = a_3 \cap c_{13}$
$P_{551} = (6,3,7,1) = \ell_2 \cap \ell_{17} = a_3 \cap c_{23}$
$P_{570} = (1,6,7,1) = \ell_2 \cap \ell_{22} = a_3 \cap c_{35}$
$P_{568} = (7,5,7,1) = \ell_2 \cap \ell_{23} = a_3 \cap c_{36}$
$P_{383} = (6,6,4,1) = \ell_3 \cap \ell_6 = a_4 \cap b_1$
$P_{352} = (7,2,4,1) = \ell_3 \cap \ell_8 = a_4 \cap b_3$
$P_{390} = (5,7,4,1) = \ell_3 \cap \ell_{10} = a_4 \cap b_5$
$P_7 = (3,1,0,0) = \ell_3 \cap \ell_{11} = a_4 \cap b_6$
$P_{330} = (1,0,4,1) = \ell_3 \cap \ell_{14} = a_4 \cap c_{14}$
$P_{339} = (2,1,4,1) = \ell_3 \cap \ell_{21} = a_4 \cap c_{34}$
$P_{372} = (3,5,4,1) = \ell_3 \cap \ell_{24} = a_4 \cap c_{45}$
$P_{357} = (4,3,4,1) = \ell_3 \cap \ell_{25} = a_4 \cap c_{46}$
$P_{244} = (3,5,2,1) = \ell_4 \cap \ell_6 = a_5 \cap b_1$
$P_{240} = (7,4,2,1) = \ell_4 \cap \ell_7 = a_5 \cap b_2$
$P_{222} = (5,2,2,1) = \ell_4 \cap \ell_8 = a_5 \cap b_3$
$P_8 = (4,1,0,0) = \ell_4 \cap \ell_{11} = a_5 \cap b_6$

$P_{263} = (6,7,2,1) = \ell_4 \cap \ell_{15} = a_5 \cap c_{15}$
$P_{213} = (4,1,2,1) = \ell_4 \cap \ell_{19} = a_5 \cap c_{25}$
$P_{226} = (1,3,2,1) = \ell_4 \cap \ell_{22} = a_5 \cap c_{35}$
$P_{251} = (2,6,2,1) = \ell_4 \cap \ell_{26} = a_5 \cap c_{56}$
$P_{293} = (4,3,3,1) = \ell_5 \cap \ell_6 = a_6 \cap b_1$
$P_{495} = (6,4,6,1) = \ell_5 \cap \ell_7 = a_6 \cap b_2$
$P_{412} = (3,2,5,1) = \ell_5 \cap \ell_8 = a_6 \cap b_3$
$P_{195} = (2,7,1,1) = \ell_5 \cap \ell_{10} = a_6 \cap b_5$
$P_{30} = (3,2,1,0) = \ell_5 \cap \ell_{16} = a_6 \cap c_{16}$
$P_{115} = (1,5,0,1) = \ell_5 \cap \ell_{20} = a_6 \cap c_{26}$
$P_{574} = (5,6,7,1) = \ell_5 \cap \ell_{23} = a_6 \cap c_{36}$
$P_{216} = (7,1,2,1) = \ell_5 \cap \ell_{26} = a_6 \cap c_{56}$
$P_{160} = (7,2,1,1) = \ell_6 \cap \ell_{13} = b_1 \cap c_{13}$
$P_{394} = (1,0,5,1) = \ell_6 \cap \ell_{14} = b_1 \cap c_{14}$
$P_{518} = (5,7,6,1) = \ell_6 \cap \ell_{15} = b_1 \cap c_{15}$
$P_{66} = (7,6,1,0) = \ell_6 \cap \ell_{16} = b_1 \cap c_{16}$
$P_{301} = (4,4,3,1) = \ell_7 \cap \ell_{12} = b_2 \cap c_{12}$
$P_{428} = (3,4,5,1) = \ell_7 \cap \ell_{17} = b_2 \cap c_{23}$
$P_{171} = (2,4,1,1) = \ell_7 \cap \ell_{19} = b_2 \cap c_{25}$
$P_{107} = (1,4,0,1) = \ell_7 \cap \ell_{20} = b_2 \cap c_{26}$
$P_{475} = (2,2,6,1) = \ell_8 \cap \ell_{13} = b_3 \cap c_{13}$
$P_{157} = (4,2,1,1) = \ell_8 \cap \ell_{21} = b_3 \cap c_{34}$
$P_{282} = (1,2,3,1) = \ell_8 \cap \ell_{22} = b_3 \cap c_{35}$
$P_{543} = (6,2,7,1) = \ell_8 \cap \ell_{23} = b_3 \cap c_{36}$
$P_{456} = (7,7,5,1) = \ell_{10} \cap \ell_{15} = b_5 \cap c_{15}$
$P_{514} = (1,7,6,1) = \ell_{10} \cap \ell_{22} = b_5 \cap c_{35}$
$P_{327} = (6,7,3,1) = \ell_{10} \cap \ell_{24} = b_5 \cap c_{45}$
$P_{261} = (4,7,2,1) = \ell_{10} \cap \ell_{26} = b_5 \cap c_{56}$
$P_5 = (1,1,0,0) = \ell_{11} \cap \ell_{16} = b_6 \cap c_{16}$
$P_9 = (5,1,0,0) = \ell_{11} \cap \ell_{23} = b_6 \cap c_{36}$
$P_{11} = (7,1,0,0) = \ell_{11} \cap \ell_{25} = b_6 \cap c_{46}$
$P_{10} = (6,1,0,0) = \ell_{11} \cap \ell_{26} = b_6 \cap c_{56}$
$P_{494} = (5,4,6,1) = \ell_{12} \cap \ell_{21} = c_{12} \cap c_{34}$
$P_{426} = (1,4,5,1) = \ell_{12} \cap \ell_{22} = c_{12} \cap c_{35}$
$P_{555} = (2,4,7,1) = \ell_{12} \cap \ell_{23} = c_{12} \cap c_{36}$
$P_{176} = (7,4,1,1) = \ell_{12} \cap \ell_{24} = c_{12} \cap c_{45}$
$P_{367} = (6,4,4,1) = \ell_{12} \cap \ell_{25} = c_{12} \cap c_{46}$
$P_{236} = (3,4,2,1) = \ell_{12} \cap \ell_{26} = c_{12} \cap c_{56}$
$P_{287} = (6,2,3,1) = \ell_{13} \cap \ell_{19} = c_{13} \cap c_{25}$
$P_{91} = (1,2,0,1) = \ell_{13} \cap \ell_{20} = c_{13} \cap c_{26}$
$P_{414} = (5,2,5,1) = \ell_{13} \cap \ell_{24} = c_{13} \cap c_{45}$
$P_{348} = (3,2,4,1) = \ell_{13} \cap \ell_{25} = c_{13} \cap c_{46}$
$P_{266} = (1,0,3,1) = \ell_{14} \cap \ell_{17} = c_{14} \cap c_{23}$
$P_{458} = (1,0,6,1) = \ell_{14} \cap \ell_{19} = c_{14} \cap c_{25}$
$P_{75} = (1,0,0,1) = \ell_{14} \cap \ell_{20} = c_{14} \cap c_{26}$
$P_{139} = (1,0,1,1) = \ell_{14} \cap \ell_{22} = c_{14} \cap c_{35}$
$P_{522} = (1,0,7,1) = \ell_{14} \cap \ell_{23} = c_{14} \cap c_{36}$
$P_{202} = (1,0,2,1) = \ell_{14} \cap \ell_{26} = c_{14} \cap c_{56}$
$P_{197} = (4,7,1,1) = \ell_{15} \cap \ell_{17} = c_{15} \cap c_{23}$
$P_{131} = (1,7,0,1) = \ell_{15} \cap \ell_{20} = c_{15} \cap c_{26}$
$P_{324} = (3,7,3,1) = \ell_{15} \cap \ell_{21} = c_{15} \cap c_{34}$
$P_{387} = (2,7,4,1) = \ell_{15} \cap \ell_{25} = c_{15} \cap c_{46}$
$P_{55} = (4,5,1,0) = \ell_{16} \cap \ell_{17} = c_{16} \cap c_{23}$
$P_{37} = (2,3,1,0) = \ell_{16} \cap \ell_{19} = c_{16} \cap c_{25}$
$P_{48} = (5,4,1,0) = \ell_{16} \cap \ell_{21} = c_{16} \cap c_{34}$
$P_{73} = (6,7,1,0) = \ell_{16} \cap \ell_{24} = c_{16} \cap c_{45}$
$P_{507} = (2,6,6,1) = \ell_{17} \cap \ell_{24} = c_{23} \cap c_{45}$
$P_{344} = (7,1,4,1) = \ell_{17} \cap \ell_{25} = c_{23} \cap c_{46}$
$P_{246} = (5,5,2,1) = \ell_{17} \cap \ell_{26} = c_{23} \cap c_{56}$
$P_{440} = (7,5,5,1) = \ell_{19} \cap \ell_{21} = c_{25} \cap c_{34}$
$P_{548} = (3,3,7,1) = \ell_{19} \cap \ell_{23} = c_{25} \cap c_{36}$
$P_{382} = (5,6,4,1) = \ell_{19} \cap \ell_{25} = c_{25} \cap c_{46}$
$P_{123} = (1,6,0,1) = \ell_{20} \cap \ell_{21} = c_{26} \cap c_{34}$
$P_{83} = (1,1,0,1) = \ell_{20} \cap \ell_{22} = c_{26} \cap c_{35}$
$P_{99} = (1,3,0,1) = \ell_{20} \cap \ell_{24} = c_{26} \cap c_{45}$
$P_{231} = (6,3,2,1) = \ell_{21} \cap \ell_{26} = c_{34} \cap c_{56}$
$P_{370} = (1,5,4,1) = \ell_{22} \cap \ell_{25} = c_{35} \cap c_{46}$
$P_{533} = (4,1,7,1) = \ell_{23} \cap \ell_{24} = c_{36} \cap c_{45}$

Single Points

The surface has 12 single points:
The single points on the surface are:

0 : $P_{82} = (0,1,0,1)$ lies on line a_2
1 : $P_{98} = (0,3,0,1)$ lies on line a_2
2 : $P_{114} = (0,5,0,1)$ lies on line a_2
3 : $P_{122} = (0,6,0,1)$ lies on line a_2
4 : $P_{138} = (0,0,1,1)$ lies on line b_4
5 : $P_{146} = (0,1,1,1)$ lies on line c_{24}
6 : $P_{265} = (0,0,3,1)$ lies on line b_4
7 : $P_{289} = (0,3,3,1)$ lies on line c_{24}
8 : $P_{393} = (0,0,5,1)$ lies on line b_4
9 : $P_{433} = (0,5,5,1)$ lies on line c_{24}
10 : $P_{457} = (0,0,6,1)$ lies on line b_4
11 : $P_{505} = (0,6,6,1)$ lies on line c_{24}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
		a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{23}	c_{24}	c_{25}	c_{26}	c_{34}	c_{35}	c_{36}	c_{45}	c_{46}	c_{56}
0	a_1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	a_2	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
2	a_3	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
3	a_4	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
4	a_5	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
5	a_6	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6	b_1	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	b_2	1	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	b_3	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9	b_4	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10	b_5	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11	b_6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
12	c_{12}	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
13	c_{13}	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
14	c_{14}	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
15	c_{15}	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
16	c_{16}	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
17	c_{23}	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
18	c_{24}	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
19	c_{25}	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
20	c_{26}	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
21	c_{34}	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
22	c_{35}	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
23	c_{36}	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
24	c_{45}	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
25	c_{46}	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
26	c_{56}	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{14}	P_{15}	P_2	P_{13}	P_0	P_{18}	P_{17}	P_2	P_{16}	P_{12}

Line 1 intersects

Line	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{106}	P_{90}	P_3	P_{130}	P_1	P_{106}	P_{90}	P_3	P_{130}	P_1

Line 2 intersects

Line	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{531}	P_{558}	P_{521}	P_{580}	P_6	P_{541}	P_{551}	P_{521}	P_{570}	P_{568}

Line 3 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{383}	P_{361}	P_{352}	P_{390}	P_7	P_{330}	P_{361}	P_{339}	P_{372}	P_{357}

Line 4 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{244}	P_{240}	P_{222}	P_{201}	P_8	P_{263}	P_{213}	P_{226}	P_{201}	P_{251}

Line 5 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_{293}	P_{495}	P_{412}	P_{329}	P_{195}	P_{30}	P_{115}	P_{574}	P_{329}	P_{216}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{106}	P_{531}	P_{383}	P_{244}	P_{293}	P_{106}	P_{160}	P_{394}	P_{518}	P_{66}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{14}	P_{558}	P_{361}	P_{240}	P_{495}	P_{301}	P_{428}	P_{361}	P_{171}	P_{107}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{15}	P_{90}	P_{352}	P_{222}	P_{412}	P_{475}	P_{90}	P_{157}	P_{282}	P_{543}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_2	P_3	P_{521}	P_{201}	P_{329}	P_2	P_3	P_{521}	P_{201}	P_{329}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{13}	P_{130}	P_{580}	P_{390}	P_{195}	P_{456}	P_{130}	P_{514}	P_{327}	P_{261}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_0	P_1	P_6	P_7	P_8	P_5	P_1	P_9	P_{11}	P_{10}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{18}	P_{106}	P_{106}	P_{301}	P_{494}	P_{426}	P_{555}	P_{176}	P_{367}	P_{236}

Line 13 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{17}	P_{541}	P_{160}	P_{475}	P_{217}	P_{287}	P_{91}	P_{414}	P_{348}	P_{217}

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_9	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_2	P_{330}	P_{394}	P_2	P_{266}	P_{458}	P_{75}	P_{139}	P_{522}	P_{202}

Line 15 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{16}	P_{263}	P_{518}	P_{456}	P_{197}	P_{577}	P_{131}	P_{324}	P_{577}	P_{387}

Line 16 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_{12}	P_{30}	P_{66}	P_5	P_{55}	P_{19}	P_{37}	P_{48}	P_{19}	P_{73}

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{90}	P_{551}	P_{428}	P_{90}	P_{266}	P_{197}	P_{55}	P_{507}	P_{344}	P_{246}

Line 18 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_3	P_{361}	P_{361}	P_3	P_{217}	P_{577}	P_{19}	P_{19}	P_{577}	P_{217}

Line 19 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{130}	P_{213}	P_{171}	P_{130}	P_{287}	P_{458}	P_{37}	P_{440}	P_{548}	P_{382}

Line 20 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_1	P_{115}	P_{107}	P_1	P_{91}	P_{75}	P_{131}	P_{123}	P_{83}	P_{99}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{26}
in point	P_{521}	P_{339}	P_{157}	P_{521}	P_{494}	P_{324}	P_{48}	P_{440}	P_{123}	P_{231}

Line 22 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{20}	ℓ_{25}
in point	P_{570}	P_{226}	P_{282}	P_{514}	P_{426}	P_{139}	P_{19}	P_{19}	P_{83}	P_{370}

Line 23 intersects

Line	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{19}	ℓ_{24}
in point	P_{568}	P_{574}	P_{543}	P_9	P_{555}	P_{522}	P_{577}	P_{577}	P_{548}	P_{533}

Line 24 intersects

Line	ℓ_3	ℓ_4	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{20}	ℓ_{23}
in point	P_{372}	P_{201}	P_{201}	P_{327}	P_{176}	P_{414}	P_{73}	P_{507}	P_{99}	P_{533}

Line 25 intersects

Line	ℓ_3	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{22}
in point	P_{357}	P_{329}	P_{329}	P_{11}	P_{367}	P_{348}	P_{387}	P_{344}	P_{382}	P_{370}

Line 26 intersects

Line	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{21}
in point	P_{251}	P_{216}	P_{261}	P_{10}	P_{236}	P_{217}	P_{202}	P_{246}	P_{217}	P_{231}

The surface has 121 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (4,1,0,0)$
$8: P_9 = (5,1,0,0)$
$9: P_{10} = (6,1,0,0)$
$10: P_{11} = (7,1,0,0)$
$11: P_{12} = (1,0,1,0)$
$12: P_{13} = (2,0,1,0)$
$13: P_{14} = (3,0,1,0)$
$14: P_{15} = (4,0,1,0)$

15 : $P_{16} = (5, 0, 1, 0)$
16 : $P_{17} = (6, 0, 1, 0)$
17 : $P_{18} = (7, 0, 1, 0)$
18 : $P_{19} = (0, 1, 1, 0)$
19 : $P_{30} = (3, 2, 1, 0)$
20 : $P_{37} = (2, 3, 1, 0)$
21 : $P_{48} = (5, 4, 1, 0)$
22 : $P_{55} = (4, 5, 1, 0)$
23 : $P_{66} = (7, 6, 1, 0)$
24 : $P_{73} = (6, 7, 1, 0)$
25 : $P_{75} = (1, 0, 0, 1)$
26 : $P_{82} = (0, 1, 0, 1)$
27 : $P_{83} = (1, 1, 0, 1)$
28 : $P_{90} = (0, 2, 0, 1)$
29 : $P_{91} = (1, 2, 0, 1)$
30 : $P_{98} = (0, 3, 0, 1)$
31 : $P_{99} = (1, 3, 0, 1)$
32 : $P_{106} = (0, 4, 0, 1)$
33 : $P_{107} = (1, 4, 0, 1)$
34 : $P_{114} = (0, 5, 0, 1)$
35 : $P_{115} = (1, 5, 0, 1)$
36 : $P_{122} = (0, 6, 0, 1)$
37 : $P_{123} = (1, 6, 0, 1)$
38 : $P_{130} = (0, 7, 0, 1)$
39 : $P_{131} = (1, 7, 0, 1)$
40 : $P_{138} = (0, 0, 1, 1)$
41 : $P_{139} = (1, 0, 1, 1)$
42 : $P_{146} = (0, 1, 1, 1)$
43 : $P_{157} = (4, 2, 1, 1)$
44 : $P_{160} = (7, 2, 1, 1)$
45 : $P_{171} = (2, 4, 1, 1)$
46 : $P_{176} = (7, 4, 1, 1)$
47 : $P_{195} = (2, 7, 1, 1)$
48 : $P_{197} = (4, 7, 1, 1)$
49 : $P_{201} = (0, 0, 2, 1)$
50 : $P_{202} = (1, 0, 2, 1)$
51 : $P_{213} = (4, 1, 2, 1)$
52 : $P_{216} = (7, 1, 2, 1)$
53 : $P_{217} = (0, 2, 2, 1)$
54 : $P_{222} = (5, 2, 2, 1)$
55 : $P_{226} = (1, 3, 2, 1)$
56 : $P_{231} = (6, 3, 2, 1)$
57 : $P_{236} = (3, 4, 2, 1)$
58 : $P_{240} = (7, 4, 2, 1)$
59 : $P_{244} = (3, 5, 2, 1)$
60 : $P_{246} = (5, 5, 2, 1)$
61 : $P_{251} = (2, 6, 2, 1)$
62 : $P_{261} = (4, 7, 2, 1)$
63 : $P_{263} = (6, 7, 2, 1)$
64 : $P_{265} = (0, 0, 3, 1)$
65 : $P_{266} = (1, 0, 3, 1)$
66 : $P_{282} = (1, 2, 3, 1)$
67 : $P_{287} = (6, 2, 3, 1)$
68 : $P_{289} = (0, 3, 3, 1)$
69 : $P_{293} = (4, 3, 3, 1)$
70 : $P_{301} = (4, 4, 3, 1)$
71 : $P_{324} = (3, 7, 3, 1)$
72 : $P_{327} = (6, 7, 3, 1)$
73 : $P_{329} = (0, 0, 4, 1)$
74 : $P_{330} = (1, 0, 4, 1)$
75 : $P_{339} = (2, 1, 4, 1)$
76 : $P_{344} = (7, 1, 4, 1)$
77 : $P_{348} = (3, 2, 4, 1)$
78 : $P_{352} = (7, 2, 4, 1)$
79 : $P_{357} = (4, 3, 4, 1)$
80 : $P_{361} = (0, 4, 4, 1)$
81 : $P_{367} = (6, 4, 4, 1)$
82 : $P_{370} = (1, 5, 4, 1)$
83 : $P_{372} = (3, 5, 4, 1)$
84 : $P_{382} = (5, 6, 4, 1)$
85 : $P_{383} = (6, 6, 4, 1)$
86 : $P_{387} = (2, 7, 4, 1)$
87 : $P_{390} = (5, 7, 4, 1)$
88 : $P_{393} = (0, 0, 5, 1)$
89 : $P_{394} = (1, 0, 5, 1)$
90 : $P_{412} = (3, 2, 5, 1)$
91 : $P_{414} = (5, 2, 5, 1)$
92 : $P_{426} = (1, 4, 5, 1)$
93 : $P_{428} = (3, 4, 5, 1)$
94 : $P_{433} = (0, 5, 5, 1)$
95 : $P_{440} = (7, 5, 5, 1)$
96 : $P_{456} = (7, 7, 5, 1)$
97 : $P_{457} = (0, 0, 6, 1)$
98 : $P_{458} = (1, 0, 6, 1)$
99 : $P_{475} = (2, 2, 6, 1)$
100 : $P_{494} = (5, 4, 6, 1)$
101 : $P_{495} = (6, 4, 6, 1)$
102 : $P_{505} = (0, 6, 6, 1)$
103 : $P_{507} = (2, 6, 6, 1)$
104 : $P_{514} = (1, 7, 6, 1)$
105 : $P_{518} = (5, 7, 6, 1)$
106 : $P_{521} = (0, 0, 7, 1)$
107 : $P_{522} = (1, 0, 7, 1)$
108 : $P_{531} = (2, 1, 7, 1)$
109 : $P_{533} = (4, 1, 7, 1)$
110 : $P_{541} = (4, 2, 7, 1)$
111 : $P_{543} = (6, 2, 7, 1)$
112 : $P_{548} = (3, 3, 7, 1)$
113 : $P_{551} = (6, 3, 7, 1)$
114 : $P_{555} = (2, 4, 7, 1)$
115 : $P_{558} = (5, 4, 7, 1)$
116 : $P_{568} = (7, 5, 7, 1)$
117 : $P_{570} = (1, 6, 7, 1)$
118 : $P_{574} = (5, 6, 7, 1)$
119 : $P_{577} = (0, 7, 7, 1)$
120 : $P_{580} = (3, 7, 7, 1)$