Rank-73987 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286331157

General information

Number of lines	5
Number of points	289
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	74
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^5
Type of lines on points	3, 2^4, 1^{74}, 0^{210}

Singular Points

The surface has 3 singular points:

$0:\ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$2:\ P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \mathbf{Pl}(0,0,1,0,0,1)_{4656}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 256, 529, 69904, 70160)
Rank of points on Klein quadric: (0, 2, 4656, 33, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 74 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$1 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$2 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$3 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$4 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$5 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$6 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$7 : P_{13} = (9,1,0,0)$ lies on line ℓ_0
$8 : P_{14} = (10,1,0,0)$ lies on line ℓ_0
$9 : P_{15} = (11,1,0,0)$ lies on line ℓ_0
$10 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$11 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$12 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$13 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$14 : P_{20} = (1,0,1,0)$ lies on line ℓ_1
$15 : P_{21} = (2,0,1,0)$ lies on line ℓ_1
$16 : P_{22} = (3,0,1,0)$ lies on line ℓ_1
$17 : P_{23} = (4,0,1,0)$ lies on line ℓ_1
$18 : P_{24} = (5,0,1,0)$ lies on line ℓ_1
$19 : P_{25} = (6,0,1,0)$ lies on line ℓ_1
$20 : P_{26} = (7,0,1,0)$ lies on line ℓ_1
$21 : P_{27} = (8,0,1,0)$ lies on line ℓ_1
$22 : P_{28} = (9,0,1,0)$ lies on line ℓ_1
$23 : P_{29} = (10,0,1,0)$ lies on line ℓ_1
$24 : P_{30} = (11,0,1,0)$ lies on line ℓ_1
$25 : P_{31} = (12,0,1,0)$ lies on line ℓ_1
$26 : P_{32} = (13,0,1,0)$ lies on line ℓ_1
$27 : P_{33} = (14,0,1,0)$ lies on line ℓ_1
$28 : P_{34} = (15,0,1,0)$ lies on line ℓ_1
$29 : P_{36} = (1,1,1,0)$ lies on line ℓ_2
$30 : P_{53} = (2,2,1,0)$ lies on line ℓ_2
$31 : P_{70} = (3,3,1,0)$ lies on line ℓ_2
$32 : P_{87} = (4,4,1,0)$ lies on line ℓ_2
$33 : P_{104} = (5,5,1,0)$ lies on line ℓ_2
$34 : P_{121} = (6,6,1,0)$ lies on line ℓ_2
$35 : P_{138} = (7,7,1,0)$ lies on line ℓ_2

36 : $P_{155} = (8, 8, 1, 0)$ lies on line ℓ_2
37 : $P_{172} = (9, 9, 1, 0)$ lies on line ℓ_2
38 : $P_{189} = (10, 10, 1, 0)$ lies on line ℓ_2
39 : $P_{206} = (11, 11, 1, 0)$ lies on line ℓ_2
40 : $P_{223} = (12, 12, 1, 0)$ lies on line ℓ_2
41 : $P_{240} = (13, 13, 1, 0)$ lies on line ℓ_2
42 : $P_{257} = (14, 14, 1, 0)$ lies on line ℓ_2
43 : $P_{274} = (15, 15, 1, 0)$ lies on line ℓ_2
44 : $P_{290} = (0, 1, 0, 1)$ lies on line ℓ_3
45 : $P_{306} = (0, 2, 0, 1)$ lies on line ℓ_3
46 : $P_{322} = (0, 3, 0, 1)$ lies on line ℓ_3
47 : $P_{338} = (0, 4, 0, 1)$ lies on line ℓ_3
48 : $P_{354} = (0, 5, 0, 1)$ lies on line ℓ_3
49 : $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_3
50 : $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_3
51 : $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_3
52 : $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_3
53 : $P_{434} = (0, 10, 0, 1)$ lies on line ℓ_3
54 : $P_{450} = (0, 11, 0, 1)$ lies on line ℓ_3
55 : $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_3
56 : $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_3
57 : $P_{498} = (0, 14, 0, 1)$ lies on line ℓ_3
58 : $P_{514} = (0, 15, 0, 1)$ lies on line ℓ_3
59 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_4
60 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_4
61 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_4
62 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_4
63 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_4
64 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_4
65 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_4
66 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_4
67 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_4
68 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_4
69 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_4
70 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_4
71 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_4
72 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_4
73 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_{572} = (11, 2, 1, 1)$
1 : $P_{591} = (14, 3, 1, 1)$
2 : $P_{603} = (10, 4, 1, 1)$
3 : $P_{611} = (2, 5, 1, 1)$
4 : $P_{634} = (9, 6, 1, 1)$
5 : $P_{643} = (2, 7, 1, 1)$
6 : $P_{661} = (4, 8, 1, 1)$
7 : $P_{684} = (11, 9, 1, 1)$
8 : $P_{690} = (1, 10, 1, 1)$
9 : $P_{706} = (1, 11, 1, 1)$
10 : $P_{725} = (4, 12, 1, 1)$
11 : $P_{751} = (14, 13, 1, 1)$
12 : $P_{763} = (10, 14, 1, 1)$
13 : $P_{778} = (9, 15, 1, 1)$
14 : $P_{810} = (9, 1, 2, 1)$
15 : $P_{822} = (5, 2, 2, 1)$
16 : $P_{841} = (8, 3, 2, 1)$
17 : $P_{851} = (2, 4, 2, 1)$
18 : $P_{877} = (12, 5, 2, 1)$
19 : $P_{883} = (2, 6, 2, 1)$
20 : $P_{902} = (5, 7, 2, 1)$
21 : $P_{922} = (9, 8, 2, 1)$
22 : $P_{941} = (12, 9, 2, 1)$
23 : $P_{952} = (7, 10, 2, 1)$
24 : $P_{969} = (8, 11, 2, 1)$
25 : $P_{1000} = (7, 13, 2, 1)$
26 : $P_{1010} = (1, 14, 2, 1)$
27 : $P_{1026} = (1, 15, 2, 1)$
28 : $P_{1070} = (13, 1, 3, 1)$
29 : $P_{1080} = (7, 2, 3, 1)$
30 : $P_{1098} = (9, 3, 3, 1)$
31 : $P_{1116} = (11, 4, 3, 1)$
32 : $P_{1128} = (7, 5, 3, 1)$
33 : $P_{1145} = (8, 6, 3, 1)$
34 : $P_{1167} = (14, 7, 3, 1)$
35 : $P_{1199} = (14, 9, 3, 1)$
36 : $P_{1210} = (9, 10, 3, 1)$
37 : $P_{1223} = (6, 11, 3, 1)$
38 : $P_{1246} = (13, 12, 3, 1)$
39 : $P_{1255} = (6, 13, 3, 1)$
40 : $P_{1273} = (8, 14, 3, 1)$
41 : $P_{1292} = (11, 15, 3, 1)$
42 : $P_{1327} = (14, 1, 4, 1)$
43 : $P_{1330} = (1, 2, 4, 1)$
44 : $P_{1346} = (1, 3, 4, 1)$
45 : $P_{1369} = (8, 4, 4, 1)$
46 : $P_{1392} = (15, 5, 4, 1)$
47 : $P_{1421} = (12, 7, 4, 1)$

$48: P_{1431} = (6, 8, 4, 1)$
$49: P_{1445} = (4, 9, 4, 1)$
$50: P_{1472} = (15, 10, 4, 1)$
$51: P_{1485} = (12, 11, 4, 1)$
$52: P_{1497} = (8, 12, 4, 1)$
$53: P_{1509} = (4, 13, 4, 1)$
$54: P_{1527} = (6, 14, 4, 1)$
$55: P_{1551} = (14, 15, 4, 1)$
$56: P_{1576} = (7, 1, 5, 1)$
$57: P_{1600} = (15, 2, 5, 1)$
$58: P_{1611} = (10, 3, 5, 1)$
$59: P_{1629} = (12, 4, 5, 1)$
$60: P_{1647} = (14, 5, 5, 1)$
$61: P_{1656} = (7, 6, 5, 1)$
$62: P_{1678} = (13, 7, 5, 1)$
$63: P_{1693} = (12, 8, 5, 1)$
$64: P_{1707} = (10, 9, 5, 1)$
$65: P_{1726} = (13, 10, 5, 1)$
$66: P_{1743} = (14, 11, 5, 1)$
$67: P_{1747} = (2, 12, 5, 1)$
$68: P_{1776} = (15, 13, 5, 1)$
$69: P_{1779} = (2, 14, 5, 1)$
$70: P_{1840} = (15, 1, 6, 1)$
$71: P_{1850} = (9, 2, 6, 1)$
$72: P_{1861} = (4, 3, 6, 1)$
$73: P_{1897} = (8, 5, 6, 1)$
$74: P_{1915} = (10, 6, 6, 1)$
$75: P_{1925} = (4, 7, 6, 1)$
$76: P_{1944} = (7, 8, 6, 1)$
$77: P_{1956} = (3, 9, 6, 1)$
$78: P_{1972} = (3, 10, 6, 1)$
$79: P_{1994} = (9, 11, 6, 1)$
$80: P_{2011} = (10, 12, 6, 1)$
$81: P_{2025} = (8, 13, 6, 1)$
$82: P_{2048} = (15, 14, 6, 1)$
$83: P_{2056} = (7, 15, 6, 1)$
$84: P_{2086} = (5, 1, 7, 1)$
$85: P_{2105} = (8, 2, 7, 1)$
$86: P_{2119} = (6, 3, 7, 1)$
$87: P_{2134} = (5, 4, 7, 1)$
$88: P_{2151} = (6, 5, 7, 1)$
$89: P_{2175} = (14, 6, 7, 1)$
$90: P_{2187} = (10, 7, 7, 1)$
$91: P_{2207} = (14, 8, 7, 1)$
$92: P_{2211} = (2, 9, 7, 1)$
$93: P_{2233} = (8, 10, 7, 1)$
$94: P_{2243} = (2, 11, 7, 1)$
$95: P_{2260} = (3, 12, 7, 1)$
$96: P_{2283} = (10, 13, 7, 1)$
$97: P_{2308} = (3, 15, 7, 1)$
$98: P_{2349} = (12, 1, 8, 1)$
$99: P_{2357} = (4, 2, 8, 1)$
$100: P_{2388} = (3, 4, 8, 1)$
$101: P_{2412} = (11, 5, 8, 1)$
$102: P_{2421} = (4, 6, 8, 1)$
$103: P_{2436} = (3, 7, 8, 1)$
$104: P_{2451} = (2, 8, 8, 1)$
$105: P_{2471} = (6, 9, 8, 1)$
$106: P_{2483} = (2, 10, 8, 1)$
$107: P_{2504} = (7, 11, 8, 1)$
$108: P_{2520} = (7, 12, 8, 1)$
$109: P_{2541} = (12, 13, 8, 1)$
$110: P_{2556} = (11, 14, 8, 1)$
$111: P_{2567} = (6, 15, 8, 1)$
$112: P_{2595} = (2, 1, 9, 1)$
$113: P_{2622} = (13, 2, 9, 1)$
$114: P_{2627} = (2, 3, 9, 1)$
$115: P_{2642} = (1, 4, 9, 1)$
$116: P_{2658} = (1, 5, 9, 1)$
$117: P_{2688} = (15, 6, 9, 1)$
$118: P_{2698} = (9, 7, 9, 1)$
$119: P_{2708} = (3, 8, 9, 1)$
$120: P_{2736} = (15, 9, 9, 1)$
$121: P_{2743} = (6, 10, 9, 1)$
$122: P_{2756} = (3, 11, 9, 1)$
$123: P_{2775} = (6, 12, 9, 1)$
$124: P_{2810} = (9, 14, 9, 1)$
$125: P_{2830} = (13, 15, 9, 1)$
$126: P_{2860} = (11, 1, 10, 1)$
$127: P_{2871} = (6, 2, 10, 1)$
$128: P_{2896} = (15, 3, 10, 1)$
$129: P_{2903} = (6, 4, 10, 1)$
$130: P_{2923} = (10, 5, 10, 1)$
$131: P_{2930} = (1, 6, 10, 1)$
$132: P_{2946} = (1, 7, 10, 1)$
$133: P_{2966} = (5, 8, 10, 1)$
$134: P_{2984} = (7, 9, 10, 1)$
$135: P_{3004} = (11, 10, 10, 1)$
$136: P_{3040} = (15, 12, 10, 1)$
$137: P_{3046} = (5, 13, 10, 1)$
$138: P_{3064} = (7, 14, 10, 1)$
$139: P_{3083} = (10, 15, 10, 1)$
$140: P_{3115} = (10, 1, 11, 1)$
$141: P_{3133} = (12, 2, 11, 1)$
$142: P_{3148} = (11, 3, 11, 1)$
$143: P_{3166} = (13, 4, 11, 1)$
$144: P_{3172} = (3, 5, 11, 1)$
$145: P_{3188} = (3, 6, 11, 1)$
$146: P_{3209} = (8, 7, 11, 1)$
$147: P_{3228} = (11, 8, 11, 1)$
$148: P_{3246} = (13, 9, 11, 1)$
$149: P_{3275} = (10, 11, 11, 1)$
$150: P_{3282} = (1, 12, 11, 1)$
$151: P_{3298} = (1, 13, 11, 1)$
$152: P_{3325} = (12, 14, 11, 1)$
$153: P_{3337} = (8, 15, 11, 1)$
$154: P_{3369} = (8, 1, 12, 1)$
$155: P_{3398} = (5, 3, 12, 1)$

156 : $P_{3424} = (15, 4, 12, 1)$
157 : $P_{3438} = (13, 5, 12, 1)$
158 : $P_{3446} = (5, 6, 12, 1)$
159 : $P_{3468} = (11, 7, 12, 1)$
160 : $P_{3486} = (13, 8, 12, 1)$
161 : $P_{3497} = (8, 9, 12, 1)$
162 : $P_{3509} = (4, 10, 12, 1)$
163 : $P_{3536} = (15, 11, 12, 1)$
164 : $P_{3548} = (11, 12, 12, 1)$
165 : $P_{3555} = (2, 13, 12, 1)$
166 : $P_{3573} = (4, 14, 12, 1)$
167 : $P_{3587} = (2, 15, 12, 1)$
168 : $P_{3620} = (3, 1, 13, 1)$
169 : $P_{3636} = (3, 2, 13, 1)$
170 : $P_{3661} = (12, 3, 13, 1)$
171 : $P_{3679} = (14, 4, 13, 1)$
172 : $P_{3690} = (9, 5, 13, 1)$
173 : $P_{3708} = (11, 6, 13, 1)$
174 : $P_{3728} = (15, 7, 13, 1)$
175 : $P_{3744} = (15, 8, 13, 1)$
176 : $P_{3775} = (14, 10, 13, 1)$
177 : $P_{3782} = (5, 11, 13, 1)$
178 : $P_{3802} = (9, 12, 13, 1)$
179 : $P_{3820} = (11, 13, 13, 1)$
180 : $P_{3830} = (5, 14, 13, 1)$
181 : $P_{3853} = (12, 15, 13, 1)$
182 : $P_{3877} = (4, 1, 14, 1)$
183 : $P_{3903} = (14, 2, 14, 1)$
184 : $P_{3912} = (7, 3, 14, 1)$
185 : $P_{3928} = (7, 4, 14, 1)$
186 : $P_{3941} = (4, 5, 14, 1)$
187 : $P_{3966} = (13, 6, 14, 1)$
188 : $P_{3986} = (1, 8, 14, 1)$
189 : $P_{4002} = (1, 9, 14, 1)$
190 : $P_{4022} = (5, 10, 14, 1)$
191 : $P_{4046} = (13, 11, 14, 1)$
192 : $P_{4063} = (14, 12, 14, 1)$
193 : $P_{4068} = (3, 13, 14, 1)$
194 : $P_{4084} = (3, 14, 14, 1)$
195 : $P_{4102} = (5, 15, 14, 1)$
196 : $P_{4135} = (6, 1, 15, 1)$
197 : $P_{4155} = (10, 2, 15, 1)$
198 : $P_{4174} = (13, 3, 15, 1)$
199 : $P_{4186} = (9, 4, 15, 1)$
200 : $P_{4221} = (12, 6, 15, 1)$
201 : $P_{4231} = (6, 7, 15, 1)$
202 : $P_{4251} = (10, 8, 15, 1)$
203 : $P_{4262} = (5, 9, 15, 1)$
204 : $P_{4285} = (12, 10, 15, 1)$
205 : $P_{4293} = (4, 11, 15, 1)$
206 : $P_{4310} = (5, 12, 15, 1)$
207 : $P_{4330} = (9, 13, 15, 1)$
208 : $P_{4350} = (13, 14, 15, 1)$
209 : $P_{4357} = (4, 15, 15, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	0	1
3	1	0	0	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_5 = (1,1,0,0)$
5 : $P_6 = (2,1,0,0)$
6 : $P_7 = (3,1,0,0)$
7 : $P_8 = (4,1,0,0)$
8 : $P_9 = (5,1,0,0)$
9 : $P_{10} = (6,1,0,0)$
10 : $P_{11} = (7,1,0,0)$
11 : $P_{12} = (8,1,0,0)$
12 : $P_{13} = (9,1,0,0)$
13 : $P_{14} = (10,1,0,0)$
14 : $P_{15} = (11,1,0,0)$
15 : $P_{16} = (12,1,0,0)$
16 : $P_{17} = (13,1,0,0)$
17 : $P_{18} = (14,1,0,0)$
18 : $P_{19} = (15,1,0,0)$
19 : $P_{20} = (1,0,1,0)$
20 : $P_{21} = (2,0,1,0)$
21 : $P_{22} = (3,0,1,0)$
22 : $P_{23} = (4,0,1,0)$
23 : $P_{24} = (5,0,1,0)$
24 : $P_{25} = (6,0,1,0)$
25 : $P_{26} = (7,0,1,0)$
26 : $P_{27} = (8,0,1,0)$
27 : $P_{28} = (9,0,1,0)$
28 : $P_{29} = (10,0,1,0)$
29 : $P_{30} = (11,0,1,0)$
30 : $P_{31} = (12,0,1,0)$
31 : $P_{32} = (13,0,1,0)$
32 : $P_{33} = (14,0,1,0)$
33 : $P_{34} = (15,0,1,0)$
34 : $P_{36} = (1,1,1,0)$
35 : $P_{53} = (2,2,1,0)$
36 : $P_{70} = (3,3,1,0)$
37 : $P_{87} = (4,4,1,0)$
38 : $P_{104} = (5,5,1,0)$
39 : $P_{121} = (6,6,1,0)$
40 : $P_{138} = (7,7,1,0)$
41 : $P_{155} = (8,8,1,0)$
42 : $P_{172} = (9,9,1,0)$
43 : $P_{189} = (10,10,1,0)$
44 : $P_{206} = (11,11,1,0)$
45 : $P_{223} = (12,12,1,0)$
46 : $P_{240} = (13,13,1,0)$
47 : $P_{257} = (14,14,1,0)$
48 : $P_{274} = (15,15,1,0)$
49 : $P_{290} = (0,1,0,1)$
50 : $P_{306} = (0,2,0,1)$
51 : $P_{322} = (0,3,0,1)$
52 : $P_{338} = (0,4,0,1)$
53 : $P_{354} = (0,5,0,1)$
54 : $P_{370} = (0,6,0,1)$
55 : $P_{386} = (0,7,0,1)$
56 : $P_{402} = (0,8,0,1)$
57 : $P_{418} = (0,9,0,1)$
58 : $P_{434} = (0,10,0,1)$
59 : $P_{450} = (0,11,0,1)$
60 : $P_{466} = (0,12,0,1)$
61 : $P_{482} = (0,13,0,1)$
62 : $P_{498} = (0,14,0,1)$
63 : $P_{514} = (0,15,0,1)$
64 : $P_{530} = (0,0,1,1)$
65 : $P_{572} = (11,2,1,1)$
66 : $P_{591} = (14,3,1,1)$
67 : $P_{603} = (10,4,1,1)$
68 : $P_{611} = (2,5,1,1)$
69 : $P_{634} = (9,6,1,1)$
70 : $P_{643} = (2,7,1,1)$
71 : $P_{661} = (4,8,1,1)$
72 : $P_{684} = (11,9,1,1)$
73 : $P_{690} = (1,10,1,1)$
74 : $P_{706} = (1,11,1,1)$
75 : $P_{725} = (4,12,1,1)$
76 : $P_{751} = (14,13,1,1)$
77 : $P_{763} = (10,14,1,1)$
78 : $P_{778} = (9,15,1,1)$
79 : $P_{785} = (0,0,2,1)$
80 : $P_{810} = (9,1,2,1)$
81 : $P_{822} = (5,2,2,1)$
82 : $P_{841} = (8,3,2,1)$
83 : $P_{851} = (2,4,2,1)$
84 : $P_{877} = (12,5,2,1)$
85 : $P_{883} = (2,6,2,1)$
86 : $P_{902} = (5,7,2,1)$
87 : $P_{922} = (9,8,2,1)$
88 : $P_{941} = (12,9,2,1)$
89 : $P_{952} = (7,10,2,1)$
90 : $P_{969} = (8,11,2,1)$
91 : $P_{1000} = (7,13,2,1)$
92 : $P_{1010} = (1,14,2,1)$
93 : $P_{1026} = (1,15,2,1)$
94 : $P_{1041} = (0,0,3,1)$
95 : $P_{1070} = (13,1,3,1)$
96 : $P_{1080} = (7,2,3,1)$
97 : $P_{1098} = (9,3,3,1)$
98 : $P_{1116} = (11,4,3,1)$
99 : $P_{1128} = (7,5,3,1)$
100 : $P_{1145} = (8,6,3,1)$
101 : $P_{1167} = (14,7,3,1)$
102 : $P_{1199} = (14,9,3,1)$
103 : $P_{1210} = (9,10,3,1)$
104 : $P_{1223} = (6,11,3,1)$
105 : $P_{1246} = (13,12,3,1)$
106 : $P_{1255} = (6,13,3,1)$
107 : $P_{1273} = (8,14,3,1)$
108 : $P_{1292} = (11,15,3,1)$
109 : $P_{1297} = (0,0,4,1)$
110 : $P_{1327} = (14,1,4,1)$
111 : $P_{1330} = (1,2,4,1)$
112 : $P_{1346} = (1,3,4,1)$
113 : $P_{1369} = (8,4,4,1)$
114 : $P_{1392} = (15,5,4,1)$
115 : $P_{1421} = (12,7,4,1)$
116 : $P_{1431} = (6,8,4,1)$
117 : $P_{1445} = (4,9,4,1)$
118 : $P_{1472} = (15,10,4,1)$
119 : $P_{1485} = (12,11,4,1)$
120 : $P_{1497} = (8,12,4,1)$
121 : $P_{1509} = (4,13,4,1)$
122 : $P_{1527} = (6,14,4,1)$
123 : $P_{1551} = (14,15,4,1)$
124 : $P_{1553} = (0,0,5,1)$
125 : $P_{1576} = (7,1,5,1)$
126 : $P_{1600} = (15,2,5,1)$
127 : $P_{1611} = (10,3,5,1)$
128 : $P_{1629} = (12,4,5,1)$
129 : $P_{1647} = (14,5,5,1)$
130 : $P_{1656} = (7,6,5,1)$
131 : $P_{1678} = (13,7,5,1)$
132 : $P_{1693} = (12,8,5,1)$
133 : $P_{1707} = (10,9,5,1)$
134 : $P_{1726} = (13,10,5,1)$
135 : $P_{1743} = (14,11,5,1)$
136 : $P_{1747} = (2,12,5,1)$
137 : $P_{1776} = (15,13,5,1)$
138 : $P_{1779} = (2,14,5,1)$
139 : $P_{1809} = (0,0,6,1)$
140 : $P_{1840} = (15,1,6,1)$
141 : $P_{1850} = (9,2,6,1)$
142 : $P_{1861} = (4,3,6,1)$
143 : $P_{1897} = (8,5,6,1)$
144 : $P_{1915} = (10,6,6,1)$
145 : $P_{1925} = (4,7,6,1)$
146 : $P_{1944} = (7,8,6,1)$
147 : $P_{1956} = (3,9,6,1)$
148 : $P_{1972} = (3,10,6,1)$
149 : $P_{1994} = (9,11,6,1)$

$150: P_{2011} = (10, 12, 6, 1)$
$151: P_{2025} = (8, 13, 6, 1)$
$152: P_{2048} = (15, 14, 6, 1)$
$153: P_{2056} = (7, 15, 6, 1)$
$154: P_{2065} = (0, 0, 7, 1)$
$155: P_{2086} = (5, 1, 7, 1)$
$156: P_{2105} = (8, 2, 7, 1)$
$157: P_{2119} = (6, 3, 7, 1)$
$158: P_{2134} = (5, 4, 7, 1)$
$159: P_{2151} = (6, 5, 7, 1)$
$160: P_{2175} = (14, 6, 7, 1)$
$161: P_{2187} = (10, 7, 7, 1)$
$162: P_{2207} = (14, 8, 7, 1)$
$163: P_{2211} = (2, 9, 7, 1)$
$164: P_{2233} = (8, 10, 7, 1)$
$165: P_{2243} = (2, 11, 7, 1)$
$166: P_{2260} = (3, 12, 7, 1)$
$167: P_{2283} = (10, 13, 7, 1)$
$168: P_{2308} = (3, 15, 7, 1)$
$169: P_{2321} = (0, 0, 8, 1)$
$170: P_{2349} = (12, 1, 8, 1)$
$171: P_{2357} = (4, 2, 8, 1)$
$172: P_{2388} = (3, 4, 8, 1)$
$173: P_{2412} = (11, 5, 8, 1)$
$174: P_{2421} = (4, 6, 8, 1)$
$175: P_{2436} = (3, 7, 8, 1)$
$176: P_{2451} = (2, 8, 8, 1)$
$177: P_{2471} = (6, 9, 8, 1)$
$178: P_{2483} = (2, 10, 8, 1)$
$179: P_{2504} = (7, 11, 8, 1)$
$180: P_{2520} = (7, 12, 8, 1)$
$181: P_{2541} = (12, 13, 8, 1)$
$182: P_{2556} = (11, 14, 8, 1)$
$183: P_{2567} = (6, 15, 8, 1)$
$184: P_{2577} = (0, 0, 9, 1)$
$185: P_{2595} = (2, 1, 9, 1)$
$186: P_{2622} = (13, 2, 9, 1)$
$187: P_{2627} = (2, 3, 9, 1)$
$188: P_{2642} = (1, 4, 9, 1)$
$189: P_{2658} = (1, 5, 9, 1)$
$190: P_{2688} = (15, 6, 9, 1)$
$191: P_{2698} = (9, 7, 9, 1)$
$192: P_{2708} = (3, 8, 9, 1)$
$193: P_{2736} = (15, 9, 9, 1)$
$194: P_{2743} = (6, 10, 9, 1)$
$195: P_{2756} = (3, 11, 9, 1)$
$196: P_{2775} = (6, 12, 9, 1)$
$197: P_{2810} = (9, 14, 9, 1)$
$198: P_{2830} = (13, 15, 9, 1)$
$199: P_{2833} = (0, 0, 10, 1)$
$200: P_{2860} = (11, 1, 10, 1)$
$201: P_{2871} = (6, 2, 10, 1)$
$202: P_{2896} = (15, 3, 10, 1)$
$203: P_{2903} = (6, 4, 10, 1)$
$204: P_{2923} = (10, 5, 10, 1)$
$205: P_{2930} = (1, 6, 10, 1)$
$206: P_{2946} = (1, 7, 10, 1)$
$207: P_{2966} = (5, 8, 10, 1)$
$208: P_{2984} = (7, 9, 10, 1)$
$209: P_{3004} = (11, 10, 10, 1)$
$210: P_{3040} = (15, 12, 10, 1)$
$211: P_{3046} = (5, 13, 10, 1)$
$212: P_{3064} = (7, 14, 10, 1)$
$213: P_{3083} = (10, 15, 10, 1)$
$214: P_{3089} = (0, 0, 11, 1)$
$215: P_{3115} = (10, 1, 11, 1)$
$216: P_{3133} = (12, 2, 11, 1)$
$217: P_{3148} = (11, 3, 11, 1)$
$218: P_{3166} = (13, 4, 11, 1)$
$219: P_{3172} = (3, 5, 11, 1)$
$220: P_{3188} = (3, 6, 11, 1)$
$221: P_{3209} = (8, 7, 11, 1)$
$222: P_{3228} = (11, 8, 11, 1)$
$223: P_{3246} = (13, 9, 11, 1)$
$224: P_{3275} = (10, 11, 11, 1)$
$225: P_{3282} = (1, 12, 11, 1)$
$226: P_{3298} = (1, 13, 11, 1)$
$227: P_{3325} = (12, 14, 11, 1)$
$228: P_{3337} = (8, 15, 11, 1)$
$229: P_{3345} = (0, 0, 12, 1)$
$230: P_{3369} = (8, 1, 12, 1)$
$231: P_{3398} = (5, 3, 12, 1)$
$232: P_{3424} = (15, 4, 12, 1)$
$233: P_{3438} = (13, 5, 12, 1)$
$234: P_{3446} = (5, 6, 12, 1)$
$235: P_{3468} = (11, 7, 12, 1)$
$236: P_{3486} = (13, 8, 12, 1)$
$237: P_{3497} = (8, 9, 12, 1)$
$238: P_{3509} = (4, 10, 12, 1)$
$239: P_{3536} = (15, 11, 12, 1)$
$240: P_{3548} = (11, 12, 12, 1)$
$241: P_{3555} = (2, 13, 12, 1)$
$242: P_{3573} = (4, 14, 12, 1)$
$243: P_{3587} = (2, 15, 12, 1)$
$244: P_{3601} = (0, 0, 13, 1)$
$245: P_{3620} = (3, 1, 13, 1)$
$246: P_{3636} = (3, 2, 13, 1)$
$247: P_{3661} = (12, 3, 13, 1)$
$248: P_{3679} = (14, 4, 13, 1)$
$249: P_{3690} = (9, 5, 13, 1)$
$250: P_{3708} = (11, 6, 13, 1)$
$251: P_{3728} = (15, 7, 13, 1)$
$252: P_{3744} = (15, 8, 13, 1)$
$253: P_{3775} = (14, 10, 13, 1)$
$254: P_{3782} = (5, 11, 13, 1)$
$255: P_{3802} = (9, 12, 13, 1)$
$256: P_{3820} = (11, 13, 13, 1)$
$257: P_{3830} = (5, 14, 13, 1)$
$258: P_{3853} = (12, 15, 13, 1)$
$259: P_{3857} = (0, 0, 14, 1)$
$260: P_{3877} = (4, 1, 14, 1)$
$261: P_{3903} = (14, 2, 14, 1)$
$262: P_{3912} = (7, 3, 14, 1)$
$263: P_{3928} = (7, 4, 14, 1)$
$264: P_{3941} = (4, 5, 14, 1)$
$265: P_{3966} = (13, 6, 14, 1)$
$266: P_{3986} = (1, 8, 14, 1)$
$267: P_{4002} = (1, 9, 14, 1)$
$268: P_{4022} = (5, 10, 14, 1)$
$269: P_{4046} = (13, 11, 14, 1)$
$270: P_{4063} = (14, 12, 14, 1)$
$271: P_{4068} = (3, 13, 14, 1)$
$272: P_{4084} = (3, 14, 14, 1)$
$273: P_{4102} = (5, 15, 14, 1)$
$274: P_{4113} = (0, 0, 15, 1)$
$275: P_{4135} = (6, 1, 15, 1)$
$276: P_{4155} = (10, 2, 15, 1)$
$277: P_{4174} = (13, 3, 15, 1)$
$278: P_{4186} = (9, 4, 15, 1)$
$279: P_{4221} = (12, 6, 15, 1)$
$280: P_{4231} = (6, 7, 15, 1)$
$281: P_{4251} = (10, 8, 15, 1)$
$282: P_{4262} = (5, 9, 15, 1)$
$283: P_{4285} = (12, 10, 15, 1)$
$284: P_{4293} = (4, 11, 15, 1)$
$285: P_{4310} = (5, 12, 15, 1)$
$286: P_{4330} = (9, 13, 15, 1)$
$287: P_{4350} = (13, 14, 15, 1)$
$288: P_{4357} = (4, 15, 15, 1)$