Rank-74532 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_2 + X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1499093338

General information

Number of lines	2
Number of points	17
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	8
Number of points off lines	8
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^2
Type of lines on points	2, 1^8, 0^8

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (337, 356)
Rank of points on Klein quadric: (129, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_{38} = (0,0,1,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 8 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_2 = (0,0,1,0)$ lies on line ℓ_1
2 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
3 : $P_{42} = (0,1,1,1)$ lies on line ℓ_0
4 : $P_{45} = (0,2,1,1)$ lies on line ℓ_0
5 : $P_{49} = (0,3,1,1)$ lies on line ℓ_0
6 : $P_{53} = (0,0,2,1)$ lies on line ℓ_1
7 : $P_{69} = (0,0,3,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 8 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$
1 : $P_8 = (1,0,1,0)$
2 : $P_{12} = (1,1,1,0)$
3 : $P_{23} = (1,0,0,1)$
4 : $P_{32} = (2,2,0,1)$
5 : $P_{37} = (3,3,0,1)$
6 : $P_{40} = (2,0,1,1)$
7 : $P_{41} = (3,0,1,1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{38}

Line 1 intersects

Line	ℓ_0
in point	P_{38}

The surface has 17 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_8 = (1,0,1,0)$
$5: P_{12} = (1,1,1,0)$
$6: P_{23} = (1,0,0,1)$
$7: P_{32} = (2,2,0,1)$
$8: P_{37} = (3,3,0,1)$
$9: P_{38} = (0,0,1,1)$
$10: P_{40} = (2,0,1,1)$
$11: P_{41} = (3,0,1,1)$
$12: P_{42} = (0,1,1,1)$
$13: P_{45} = (0,2,1,1)$
$14: P_{49} = (0,3,1,1)$
$15: P_{53} = (0,0,2,1)$
$16: P_{69} = (0,0,3,1)$