

# Rank-31 over GF(64)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 = 0$$

( 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(64) is 8339

## General information

Number of lines	27
Number of points	4545
Number of singular points	0
Number of Eckardt points	45
Number of double points	0
Number of single points	1620
Number of points off lines	2880
Number of Hesse planes	40
Number of axes	240
Type of points on lines	$65^{27}$
Type of lines on points	$3^{45}, 1^{1620}, 0^{2880}$

## Singular Points

The surface has 0 singular points:

## The 27 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned} \ell_0 = a_1 &= \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{array} \right]_{8314} = \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{array} \right]_{8314} = \mathbf{Pl}(0, 0, 56, 57, 57, 1)_{15228043} \\ \ell_1 = a_2 &= \left[ \begin{array}{cccc} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{241274} = \left[ \begin{array}{cccc} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{241274} = \mathbf{Pl}(0, 0, 1, 1, 56, 1)_{14958978} \end{aligned}$$

$$\begin{aligned}
\ell_2 = a_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4225} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4225} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{536577} \\
\ell_3 = a_4 &= \begin{bmatrix} 1 & 0 & \epsilon^{21} & 0 \\ 0 & 1 & 0 & \epsilon^{21} \end{bmatrix}_{240825} = \begin{bmatrix} 1 & 0 & 57 & 0 \\ 0 & 1 & 0 & 57 \end{bmatrix}_{240825} = \mathbf{Pl}(56, 57, 0, 0, 1, 1)_{536632} \\
\ell_4 = a_5 &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{14913080} = \begin{bmatrix} 1 & 0 & 0 & 56 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{14913080} = \mathbf{Pl}(57, 56, 1, 1, 0, 0)_{312} \\
\ell_5 = a_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{266361} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{266361} = \mathbf{Pl}(56, 57, 57, 1, 0, 0)_{3839} \\
\ell_6 = b_1 &= \begin{bmatrix} 1 & 0 & \epsilon^{21} & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{237241} = \begin{bmatrix} 1 & 0 & 57 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{237241} = \mathbf{Pl}(1, 1, 0, 0, 56, 1)_{14950977} \\
\ell_7 = b_2 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \epsilon^{21} \end{bmatrix}_{7809} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 57 \end{bmatrix}_{7809} = \mathbf{Pl}(56, 57, 0, 0, 57, 1)_{15213112} \\
\ell_8 = b_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{266360} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{266360} = \mathbf{Pl}(57, 56, 56, 1, 0, 0)_{3777} \\
\ell_9 = b_4 &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{14913081} = \begin{bmatrix} 1 & 0 & 0 & 56 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{14913081} = \mathbf{Pl}(56, 57, 56, 1, 0, 0)_{3776} \\
\ell_{10} = b_5 &= \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{241330} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{241330} = \mathbf{Pl}(0, 0, 56, 57, 1, 1)_{551563} \\
\ell_{11} = b_6 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578} \\
\ell_{12} = c_{12} &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15179329} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15179329} = \mathbf{Pl}(1, 1, 56, 1, 0, 0)_{3721} \\
\ell_{13} = c_{13} &= \begin{bmatrix} 1 & 0 & \epsilon^{42} & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{233080} = \begin{bmatrix} 1 & 0 & 56 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{233080} = \mathbf{Pl}(1, 1, 0, 0, 57, 1)_{15213057} \\
\ell_{14} = c_{14} &= \begin{bmatrix} 1 & 0 & \epsilon^{21} & 0 \\ 0 & 1 & 0 & \epsilon^{42} \end{bmatrix}_{240761} = \begin{bmatrix} 1 & 0 & 57 & 0 \\ 0 & 1 & 0 & 56 \end{bmatrix}_{240761} = \mathbf{Pl}(57, 56, 0, 0, 57, 1)_{15213113} \\
\ell_{15} = c_{15} &= \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{237169} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{237169} = \mathbf{Pl}(0, 0, 56, 57, 56, 1)_{14965963} \\
\ell_{16} = c_{16} &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{8313} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{8313} = \mathbf{Pl}(0, 0, 57, 56, 56, 1)_{14966090} \\
\ell_{17} = c_{23} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \epsilon^{42} \end{bmatrix}_{7745} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 56 \end{bmatrix}_{7745} = \mathbf{Pl}(57, 56, 0, 0, 56, 1)_{14951033} \\
\ell_{18} = c_{24} &= \begin{bmatrix} 1 & 0 & \epsilon^{42} & 0 \\ 0 & 1 & 0 & \epsilon^{21} \end{bmatrix}_{236664} = \begin{bmatrix} 1 & 0 & 56 & 0 \\ 0 & 1 & 0 & 57 \end{bmatrix}_{236664} = \mathbf{Pl}(56, 57, 0, 0, 56, 1)_{14951032} \\
\ell_{19} = c_{25} &= \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{241329} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{241329} = \mathbf{Pl}(0, 0, 57, 56, 57, 1)_{15228170} \\
\ell_{20} = c_{26} &= \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \mathbf{Pl}(0, 0, 1, 1, 57, 1)_{15221058} \\
\ell_{21} = c_{34} &= \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{237168} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{237168} = \mathbf{Pl}(0, 0, 57, 56, 1, 1)_{551690} \\
\ell_{22} = c_{35} &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{15179384} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{15179384} = \mathbf{Pl}(57, 56, 57, 1, 0, 0)_{3840}
\end{aligned}$$

$$\begin{aligned}
\ell_{23} = c_{36} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \mathbf{Pl}(1, 1, 1, 1, 0, 0)_{256} \\
\ell_{24} = c_{45} &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 1 & 1 & 0 \end{bmatrix}_{14913025} = \begin{bmatrix} 1 & 0 & 0 & 56 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{14913025} = \mathbf{Pl}(1, 1, 57, 1, 0, 0)_{3784} \\
\ell_{25} = c_{46} &= \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{15179385} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{15179385} = \mathbf{Pl}(56, 57, 1, 1, 0, 0)_{311} \\
\ell_{26} = c_{56} &= \begin{bmatrix} 1 & 0 & \epsilon^{42} & 0 \\ 0 & 1 & 0 & \epsilon^{42} \end{bmatrix}_{236600} = \begin{bmatrix} 1 & 0 & 56 & 0 \\ 0 & 1 & 0 & 56 \end{bmatrix}_{236600} = \mathbf{Pl}(57, 56, 0, 0, 1, 1)_{536633}
\end{aligned}$$

Rank of lines: ( 8314, 241274, 4225, 240825, 14913080, 266361, 237241, 7809, 266360, 14913081, 241330, 8258, 15179329, 233080, 240761, 237169, 8313, 7745, 236664, 241329, 237113, 237168, 15179384, 266305, 14913025, 15179385, 236600 )

Rank of points on Klein quadric: ( 15228043, 14958978, 536577, 536632, 312, 3839, 14950977, 15213112, 3777, 3776, 551563, 544578, 3721, 15213057, 15213113, 14965963, 14966090, 14951033, 14951032, 15228170, 15221058, 551690, 3840, 256, 3784, 311, 536633 )

## Eckardt Points

The surface has 45 Eckardt points:

- 0 :  $E_{36} = a_3 \cap b_6 \cap c_{36} = P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$ ,
- 1 :  $E_{16} = a_1 \cap b_6 \cap c_{16} = P_5 = \mathbf{P}(1, 1, 0, 0) = \mathbf{P}(1, 1, 0, 0)$ ,
- 2 :  $E_{25} = a_2 \cap b_5 \cap c_{25} = P_{60} = \mathbf{P}(\epsilon^{42}, 1, 0, 0) = \mathbf{P}(56, 1, 0, 0)$ ,
- 3 :  $E_{15,26,34} = c_{15} \cap c_{26} \cap c_{34} = P_{61} = \mathbf{P}(\epsilon^{21}, 1, 0, 0) = \mathbf{P}(57, 1, 0, 0)$ ,
- 4 :  $E_{32} = a_3 \cap b_2 \cap c_{23} = P_{68} = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0)$ ,
- 5 :  $E_{41} = a_4 \cap b_1 \cap c_{14} = P_{123} = \mathbf{P}(\epsilon^{42}, 0, 1, 0) = \mathbf{P}(56, 0, 1, 0)$ ,
- 6 :  $E_{13,24,56} = c_{13} \cap c_{24} \cap c_{56} = P_{124} = \mathbf{P}(\epsilon^{21}, 0, 1, 0) = \mathbf{P}(57, 0, 1, 0)$ ,
- 7 :  $E_{12,36,45} = c_{12} \cap c_{36} \cap c_{45} = P_{131} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$ ,
- 8 :  $E_{64} = a_6 \cap b_4 \cap c_{46} = P_{3651} = \mathbf{P}(0, \epsilon^{42}, 1, 0) = \mathbf{P}(0, 56, 1, 0)$ ,
- 9 :  $E_{53} = a_5 \cap b_3 \cap c_{35} = P_{3715} = \mathbf{P}(0, \epsilon^{21}, 1, 0) = \mathbf{P}(0, 57, 1, 0)$ ,
- 10 :  $E_{63} = a_6 \cap b_3 \cap c_{36} = P_{4163} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$ ,
- 11 :  $E_{12,35,46} = c_{12} \cap c_{35} \cap c_{46} = P_{4218} = \mathbf{P}(\epsilon^{42}, 0, 0, 1) = \mathbf{P}(56, 0, 0, 1)$ ,
- 12 :  $E_{54} = a_5 \cap b_4 \cap c_{45} = P_{4219} = \mathbf{P}(\epsilon^{21}, 0, 0, 1) = \mathbf{P}(57, 0, 0, 1)$ ,
- 13 :  $E_{31} = a_3 \cap b_1 \cap c_{13} = P_{4226} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$ ,
- 14 :  $E_{42} = a_4 \cap b_2 \cap c_{24} = P_{7746} = \mathbf{P}(0, \epsilon^{42}, 0, 1) = \mathbf{P}(0, 56, 0, 1)$ ,
- 15 :  $E_{14,23,56} = c_{14} \cap c_{23} \cap c_{56} = P_{7810} = \mathbf{P}(0, \epsilon^{21}, 0, 1) = \mathbf{P}(0, 57, 0, 1)$ ,
- 16 :  $E_{26} = a_2 \cap b_6 \cap c_{26} = P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$ ,
- 17 :  $E_{21} = a_2 \cap b_1 \cap c_{12} = P_{8377} = \mathbf{P}(\epsilon^{42}, 1, 1, 1) = \mathbf{P}(56, 1, 1, 1)$ ,
- 18 :  $E_{13,26,45} = c_{13} \cap c_{26} \cap c_{45} = P_{8378} = \mathbf{P}(\epsilon^{21}, 1, 1, 1) = \mathbf{P}(57, 1, 1, 1)$ ,
- 19 :  $E_{62} = a_6 \cap b_2 \cap c_{26} = P_{11842} = \mathbf{P}(1, \epsilon^{42}, 1, 1) = \mathbf{P}(1, 56, 1, 1)$ ,
- 20 :  $E_{46} = a_4 \cap b_6 \cap c_{46} = P_{11897} = \mathbf{P}(\epsilon^{42}, \epsilon^{42}, 1, 1) = \mathbf{P}(56, 56, 1, 1)$ ,
- 21 :  $E_{24} = a_2 \cap b_4 \cap c_{24} = P_{11898} = \mathbf{P}(\epsilon^{21}, \epsilon^{42}, 1, 1) = \mathbf{P}(57, 56, 1, 1)$ ,
- 22 :  $E_{23} = a_2 \cap b_3 \cap c_{23} = P_{11906} = \mathbf{P}(1, \epsilon^{21}, 1, 1) = \mathbf{P}(1, 57, 1, 1)$ ,
- 23 :  $E_{14,26,35} = c_{14} \cap c_{26} \cap c_{35} = P_{11961} = \mathbf{P}(\epsilon^{42}, \epsilon^{21}, 1, 1) = \mathbf{P}(56, 57, 1, 1)$ ,
- 24 :  $E_{56} = a_5 \cap b_6 \cap c_{56} = P_{11962} = \mathbf{P}(\epsilon^{21}, \epsilon^{21}, 1, 1) = \mathbf{P}(57, 57, 1, 1)$ ,
- 25 :  $E_{15} = a_1 \cap b_5 \cap c_{15} = P_{233537} = \mathbf{P}(0, 0, \epsilon^{42}, 1) = \mathbf{P}(0, 0, 56, 1)$ ,
- 26 :  $E_{13} = a_1 \cap b_3 \cap c_{13} = P_{233602} = \mathbf{P}(1, 1, \epsilon^{42}, 1) = \mathbf{P}(1, 1, 56, 1)$ ,
- 27 :  $E_{35} = a_3 \cap b_5 \cap c_{35} = P_{233657} = \mathbf{P}(\epsilon^{42}, 1, \epsilon^{42}, 1) = \mathbf{P}(56, 1, 56, 1)$ ,
- 28 :  $E_{51} = a_5 \cap b_1 \cap c_{15} = P_{233658} = \mathbf{P}(\epsilon^{21}, 1, \epsilon^{42}, 1) = \mathbf{P}(57, 1, 56, 1)$ ,
- 29 :  $E_{15,24,36} = c_{15} \cap c_{24} \cap c_{36} = P_{237122} = \mathbf{P}(1, \epsilon^{42}, \epsilon^{42}, 1) = \mathbf{P}(1, 56, 56, 1)$ ,
- 30 :  $E_{12} = a_1 \cap b_2 \cap c_{12} = P_{237177} = \mathbf{P}(\epsilon^{42}, \epsilon^{42}, \epsilon^{42}, 1) = \mathbf{P}(56, 56, 56, 1)$ ,
- 31 :  $E_{45} = a_4 \cap b_5 \cap c_{45} = P_{237178} = \mathbf{P}(\epsilon^{21}, \epsilon^{42}, \epsilon^{42}, 1) = \mathbf{P}(57, 56, 56, 1)$ ,
- 32 :  $E_{65} = a_6 \cap b_5 \cap c_{56} = P_{237186} = \mathbf{P}(1, \epsilon^{21}, \epsilon^{42}, 1) = \mathbf{P}(1, 57, 56, 1)$ ,

$$\begin{aligned}
33 : E_{15,23,46} &= c_{15} \cap c_{23} \cap c_{46} = P_{237241} = \mathbf{P}(\epsilon^{42}, \epsilon^{21}, \epsilon^{42}, 1) = \mathbf{P}(56, 57, 56, 1), \\
34 : E_{14} &= a_1 \cap b_4 \cap c_{14} = P_{237242} = \mathbf{P}(\epsilon^{21}, \epsilon^{21}, \epsilon^{42}, 1) = \mathbf{P}(57, 57, 56, 1), \\
35 : E_{16,25,34} &= c_{16} \cap c_{25} \cap c_{34} = P_{237633} = \mathbf{P}(0, 0, \epsilon^{21}, 1) = \mathbf{P}(0, 0, 57, 1), \\
36 : E_{61} &= a_6 \cap b_1 \cap c_{16} = P_{237698} = \mathbf{P}(1, 1, \epsilon^{21}, 1) = \mathbf{P}(1, 1, 57, 1), \\
37 : E_{13,25,46} &= c_{13} \cap c_{25} \cap c_{46} = P_{237753} = \mathbf{P}(\epsilon^{42}, 1, \epsilon^{21}, 1) = \mathbf{P}(56, 1, 57, 1), \\
38 : E_{34} &= a_3 \cap b_4 \cap c_{34} = P_{237754} = \mathbf{P}(\epsilon^{21}, 1, \epsilon^{21}, 1) = \mathbf{P}(57, 1, 57, 1), \\
39 : E_{43} &= a_4 \cap b_3 \cap c_{34} = P_{241218} = \mathbf{P}(1, \epsilon^{42}, \epsilon^{21}, 1) = \mathbf{P}(1, 56, 57, 1), \\
40 : E_{16,24,35} &= c_{16} \cap c_{24} \cap c_{35} = P_{241273} = \mathbf{P}(\epsilon^{42}, \epsilon^{42}, \epsilon^{21}, 1) = \mathbf{P}(56, 56, 57, 1), \\
41 : E_{52} &= a_5 \cap b_2 \cap c_{25} = P_{241274} = \mathbf{P}(\epsilon^{21}, \epsilon^{42}, \epsilon^{21}, 1) = \mathbf{P}(57, 56, 57, 1), \\
42 : E_{14,25,36} &= c_{14} \cap c_{25} \cap c_{36} = P_{241282} = \mathbf{P}(1, \epsilon^{21}, \epsilon^{21}, 1) = \mathbf{P}(1, 57, 57, 1), \\
43 : E_{12,34,56} &= c_{12} \cap c_{34} \cap c_{56} = P_{241337} = \mathbf{P}(\epsilon^{42}, \epsilon^{21}, \epsilon^{21}, 1) = \mathbf{P}(56, 57, 57, 1), \\
44 : E_{16,23,45} &= c_{16} \cap c_{23} \cap c_{45} = P_{241338} = \mathbf{P}(\epsilon^{21}, \epsilon^{21}, \epsilon^{21}, 1) = \mathbf{P}(57, 57, 57, 1).
\end{aligned}$$

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 1620 single points:

Too many to print.

### Points on surface but on no line

The surface has 2880 points not on any line:

Too many to print.

## Line Intersection Graph

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
		$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$c_{12}$	$c_{13}$	$c_{14}$	$c_{15}$	$c_{16}$	$c_{23}$	$c_{24}$	$c_{25}$	$c_{26}$	$c_{34}$	$c_{35}$	$c_{36}$	$c_{45}$	$c_{46}$	$c_{56}$
0	$a_1$	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	$a_2$	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
2	$a_3$	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
3	$a_4$	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
4	$a_5$	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
5	$a_6$	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6	$b_1$	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	$b_2$	1	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	$b_3$	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9	$b_4$	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10	$b_5$	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11	$b_6$	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
12	$c_{12}$	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
13	$c_{13}$	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
14	$c_{14}$	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
15	$c_{15}$	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
16	$c_{16}$	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
17	$c_{23}$	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
18	$c_{24}$	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
19	$c_{25}$	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
20	$c_{26}$	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
21	$c_{34}$	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
22	$c_{35}$	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
23	$c_{36}$	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
24	$c_{45}$	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
25	$c_{46}$	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
26	$c_{56}$	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$
in point	$P_{237177}$	$P_{233602}$	$P_{237242}$	$P_{233537}$	$P_5$	$P_{237177}$	$P_{233602}$	$P_{237242}$	$P_{233537}$	$P_5$

Line 1 intersects

Line	$\ell_6$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{8377}$	$P_{11906}$	$P_{11898}$	$P_{60}$	$P_{8258}$	$P_{8377}$	$P_{11906}$	$P_{11898}$	$P_{60}$	$P_{8258}$

Line 2 intersects

Line	$\ell_6$	$\ell_7$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{17}$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$
in point	$P_{4226}$	$P_{68}$	$P_{237754}$	$P_{233657}$	$P_4$	$P_{4226}$	$P_{68}$	$P_{237754}$	$P_{233657}$	$P_4$

Line 3 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$	$\ell_{18}$	$\ell_{21}$	$\ell_{24}$	$\ell_{25}$
in point	$P_{123}$	$P_{7746}$	$P_{241218}$	$P_{237178}$	$P_{11897}$	$P_{123}$	$P_{7746}$	$P_{241218}$	$P_{237178}$	$P_{11897}$

Line 4 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{15}$	$\ell_{19}$	$\ell_{22}$	$\ell_{24}$	$\ell_{26}$
in point	$P_{233658}$	$P_{241274}$	$P_{3715}$	$P_{4219}$	$P_{11962}$	$P_{233658}$	$P_{241274}$	$P_{3715}$	$P_{4219}$	$P_{11962}$

Line 5 intersects

Line	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{16}$	$\ell_{20}$	$\ell_{23}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{237698}$	$P_{11842}$	$P_{4163}$	$P_{3651}$	$P_{237186}$	$P_{237698}$	$P_{11842}$	$P_{4163}$	$P_{3651}$	$P_{237186}$

Line 6 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$
in point	$P_{8377}$	$P_{4226}$	$P_{123}$	$P_{233658}$	$P_{237698}$	$P_{8377}$	$P_{4226}$	$P_{123}$	$P_{233658}$	$P_{237698}$

Line 7 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{12}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{237177}$	$P_{68}$	$P_{7746}$	$P_{241274}$	$P_{11842}$	$P_{237177}$	$P_{68}$	$P_{7746}$	$P_{241274}$	$P_{11842}$

Line 8 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_{13}$	$\ell_{17}$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$
in point	$P_{233602}$	$P_{11906}$	$P_{241218}$	$P_{3715}$	$P_{4163}$	$P_{233602}$	$P_{11906}$	$P_{241218}$	$P_{3715}$	$P_{4163}$

Line 9 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_{14}$	$\ell_{18}$	$\ell_{21}$	$\ell_{24}$	$\ell_{25}$
in point	$P_{237242}$	$P_{11898}$	$P_{237754}$	$P_{4219}$	$P_{3651}$	$P_{237242}$	$P_{11898}$	$P_{237754}$	$P_{4219}$	$P_{3651}$

Line 10 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_{15}$	$\ell_{19}$	$\ell_{22}$	$\ell_{24}$	$\ell_{26}$
in point	$P_{233537}$	$P_{60}$	$P_{233657}$	$P_{237178}$	$P_{237186}$	$P_{233537}$	$P_{60}$	$P_{233657}$	$P_{237178}$	$P_{237186}$

Line 11 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_{16}$	$\ell_{20}$	$\ell_{23}$	$\ell_{25}$	$\ell_{26}$
in point	$P_5$	$P_{8258}$	$P_4$	$P_{11897}$	$P_{11962}$	$P_5$	$P_{8258}$	$P_4$	$P_{11897}$	$P_{11962}$

Line 12 intersects

Line	$\ell_0$	$\ell_1$	$\ell_6$	$\ell_7$	$\ell_{21}$	$\ell_{22}$	$\ell_{23}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{237177}$	$P_{8377}$	$P_{8377}$	$P_{237177}$	$P_{241337}$	$P_{4218}$	$P_{131}$	$P_{131}$	$P_{4218}$	$P_{241337}$

Line 13 intersects

Line	$\ell_0$	$\ell_2$	$\ell_6$	$\ell_8$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{233602}$	$P_{4226}$	$P_{4226}$	$P_{233602}$	$P_{124}$	$P_{237753}$	$P_{8378}$	$P_{8378}$	$P_{237753}$	$P_{124}$

Line 14 intersects

Line	$\ell_0$	$\ell_3$	$\ell_6$	$\ell_9$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$	$\ell_{22}$	$\ell_{23}$	$\ell_{26}$
in point	$P_{237242}$	$P_{123}$	$P_{123}$	$P_{237242}$	$P_{7810}$	$P_{241282}$	$P_{11961}$	$P_{11961}$	$P_{241282}$	$P_{7810}$

Line 15 intersects

Line	$\ell_0$	$\ell_4$	$\ell_6$	$\ell_{10}$	$\ell_{17}$	$\ell_{18}$	$\ell_{20}$	$\ell_{21}$	$\ell_{23}$	$\ell_{25}$
in point	$P_{233537}$	$P_{233658}$	$P_{233658}$	$P_{233537}$	$P_{237241}$	$P_{237122}$	$P_{61}$	$P_{61}$	$P_{237122}$	$P_{237241}$

Line 16 intersects

Line	$\ell_0$	$\ell_5$	$\ell_6$	$\ell_{11}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{21}$	$\ell_{22}$	$\ell_{24}$
in point	$P_5$	$P_{237698}$	$P_{237698}$	$P_5$	$P_{241338}$	$P_{241273}$	$P_{237633}$	$P_{237633}$	$P_{241273}$	$P_{241338}$

Line 17 intersects

Line	$\ell_1$	$\ell_2$	$\ell_7$	$\ell_8$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{24}$	$\ell_{25}$	$\ell_{26}$
in point	$P_{11906}$	$P_{68}$	$P_{68}$	$P_{11906}$	$P_{7810}$	$P_{237241}$	$P_{241338}$	$P_{241338}$	$P_{237241}$	$P_{7810}$

Line 18 intersects

Line	$\ell_1$	$\ell_3$	$\ell_7$	$\ell_9$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$	$\ell_{22}$	$\ell_{23}$	$\ell_{26}$
in point	$P_{11898}$	$P_{7746}$	$P_{7746}$	$P_{11898}$	$P_{124}$	$P_{237122}$	$P_{241273}$	$P_{241273}$	$P_{237122}$	$P_{124}$

Line 19 intersects

Line	$\ell_1$	$\ell_4$	$\ell_7$	$\ell_{10}$	$\ell_{13}$	$\ell_{14}$	$\ell_{16}$	$\ell_{21}$	$\ell_{23}$	$\ell_{25}$
in point	$P_{60}$	$P_{241274}$	$P_{241274}$	$P_{60}$	$P_{237753}$	$P_{241282}$	$P_{237633}$	$P_{237633}$	$P_{241282}$	$P_{237753}$

Line 20 intersects

Line	$\ell_1$	$\ell_5$	$\ell_7$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{21}$	$\ell_{22}$	$\ell_{24}$
in point	$P_{8258}$	$P_{11842}$	$P_{11842}$	$P_{8258}$	$P_{8378}$	$P_{11961}$	$P_{61}$	$P_{61}$	$P_{11961}$	$P_{8378}$

Line 21 intersects

Line	$\ell_2$	$\ell_3$	$\ell_8$	$\ell_9$	$\ell_{12}$	$\ell_{15}$	$\ell_{16}$	$\ell_{19}$	$\ell_{20}$	$\ell_{26}$
in point	$P_{237754}$	$P_{241218}$	$P_{241218}$	$P_{237754}$	$P_{241337}$	$P_{61}$	$P_{237633}$	$P_{237633}$	$P_{61}$	$P_{241337}$

Line 22 intersects

Line	$\ell_2$	$\ell_4$	$\ell_8$	$\ell_{10}$	$\ell_{12}$	$\ell_{14}$	$\ell_{16}$	$\ell_{18}$	$\ell_{20}$	$\ell_{25}$
in point	$P_{233657}$	$P_{3715}$	$P_{3715}$	$P_{233657}$	$P_{4218}$	$P_{11961}$	$P_{241273}$	$P_{241273}$	$P_{11961}$	$P_{4218}$

Line 23 intersects

Line	$\ell_2$	$\ell_5$	$\ell_8$	$\ell_{11}$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{18}$	$\ell_{19}$	$\ell_{24}$
in point	$P_4$	$P_{4163}$	$P_{4163}$	$P_4$	$P_{131}$	$P_{241282}$	$P_{237122}$	$P_{237122}$	$P_{241282}$	$P_{131}$

Line 24 intersects

Line	$\ell_3$	$\ell_4$	$\ell_9$	$\ell_{10}$	$\ell_{12}$	$\ell_{13}$	$\ell_{16}$	$\ell_{17}$	$\ell_{20}$	$\ell_{23}$
in point	$P_{237178}$	$P_{4219}$	$P_{4219}$	$P_{237178}$	$P_{131}$	$P_{8378}$	$P_{241338}$	$P_{241338}$	$P_{8378}$	$P_{131}$

Line 25 intersects

Line	$\ell_3$	$\ell_5$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$	$\ell_{17}$	$\ell_{19}$	$\ell_{22}$
in point	$P_{11897}$	$P_{3651}$	$P_{3651}$	$P_{11897}$	$P_{4218}$	$P_{237753}$	$P_{237241}$	$P_{237241}$	$P_{237753}$	$P_{4218}$

Line 26 intersects

Line	$\ell_4$	$\ell_5$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{17}$	$\ell_{18}$	$\ell_{21}$
in point	$P_{11962}$	$P_{237186}$	$P_{237186}$	$P_{11962}$	$P_{241337}$	$P_{124}$	$P_{7810}$	$P_{7810}$	$P_{124}$	$P_{241337}$

The surface has 4545 points:

Too many to print.