Rank-74532 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_2 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1378161230

General information

Number of lines	2
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	$2, 1^{16}, 0^{64}$

Singular Points

The surface has 1 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (4673, 4744)
Rank of points on Klein quadric: (769, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_{138} = (0,0,1,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

$0: P_1 = (0,1,0,0)$ lies on line ℓ_0
$1: P_2 = (0,0,1,0)$ lies on line ℓ_1
$2: P_3 = (0,0,0,1)$ lies on line ℓ_1
$3: P_{146} = (0,1,1,1)$ lies on line ℓ_0
$4: P_{153} = (0,2,1,1)$ lies on line ℓ_0
$5: P_{161} = (0,3,1,1)$ lies on line ℓ_0
$6: P_{169} = (0,4,1,1)$ lies on line ℓ_0
$7: P_{177} = (0,5,1,1)$ lies on line ℓ_0
$8: P_{185} = (0,6,1,1)$ lies on line ℓ_0

$9: P_{193} = (0,7,1,1)$ lies on line ℓ_0
$10: P_{201} = (0,0,2,1)$ lies on line ℓ_1
$11: P_{265} = (0,0,3,1)$ lies on line ℓ_1
$12: P_{329} = (0,0,4,1)$ lies on line ℓ_1
$13: P_{393} = (0,0,5,1)$ lies on line ℓ_1
$14: P_{457} = (0,0,6,1)$ lies on line ℓ_1
$15: P_{521} = (0,0,7,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:
The points on the surface but not on lines are:

$0: P_4 = (1,1,1,1)$
$1: P_{12} = (1,0,1,0)$
$2: P_{20} = (1,1,1,0)$
$3: P_{31} = (4,2,1,0)$
$4: P_{42} = (7,3,1,0)$
$5: P_{50} = (7,4,1,0)$
$6: P_{53} = (2,5,1,0)$
$7: P_{63} = (4,6,1,0)$
$8: P_{69} = (2,7,1,0)$
$9: P_{75} = (1,0,0,1)$
$10: P_{85} = (3,1,0,1)$

$11: P_{87} = (5,1,0,1)$
$12: P_{88} = (6,1,0,1)$
$13: P_{97} = (7,2,0,1)$
$14: P_{108} = (2,4,0,1)$
$15: P_{134} = (4,7,0,1)$
$16: P_{157} = (4,2,1,1)$
$17: P_{158} = (5,2,1,1)$
$18: P_{175} = (6,4,1,1)$
$19: P_{176} = (7,4,1,1)$
$20: P_{195} = (2,7,1,1)$
$21: P_{196} = (3,7,1,1)$

22 : $P_{206} = (5, 0, 2, 1)$
23 : $P_{208} = (7, 0, 2, 1)$
24 : $P_{216} = (7, 1, 2, 1)$
25 : $P_{236} = (3, 4, 2, 1)$
26 : $P_{245} = (4, 5, 2, 1)$
27 : $P_{252} = (3, 6, 2, 1)$
28 : $P_{253} = (4, 6, 2, 1)$
29 : $P_{254} = (5, 6, 2, 1)$
30 : $P_{282} = (1, 2, 3, 1)$
31 : $P_{302} = (5, 4, 3, 1)$
32 : $P_{306} = (1, 5, 3, 1)$
33 : $P_{310} = (5, 5, 3, 1)$
34 : $P_{312} = (7, 5, 3, 1)$
35 : $P_{328} = (7, 7, 3, 1)$
36 : $P_{331} = (2, 0, 4, 1)$
37 : $P_{335} = (6, 0, 4, 1)$
38 : $P_{339} = (2, 1, 4, 1)$
39 : $P_{358} = (5, 3, 4, 1)$
40 : $P_{359} = (6, 3, 4, 1)$
41 : $P_{360} = (7, 3, 4, 1)$
42 : $P_{384} = (7, 6, 4, 1)$
43 : $P_{390} = (5, 7, 4, 1)$

44 : $P_{411} = (2, 2, 5, 1)$
45 : $P_{426} = (1, 4, 5, 1)$
46 : $P_{442} = (1, 6, 5, 1)$
47 : $P_{443} = (2, 6, 5, 1)$
48 : $P_{447} = (6, 6, 5, 1)$
49 : $P_{455} = (6, 7, 5, 1)$
50 : $P_{476} = (3, 2, 6, 1)$
51 : $P_{482} = (1, 3, 6, 1)$
52 : $P_{484} = (3, 3, 6, 1)$
53 : $P_{485} = (4, 3, 6, 1)$
54 : $P_{493} = (4, 4, 6, 1)$
55 : $P_{514} = (1, 7, 6, 1)$
56 : $P_{524} = (3, 0, 7, 1)$
57 : $P_{525} = (4, 0, 7, 1)$
58 : $P_{533} = (4, 1, 7, 1)$
59 : $P_{543} = (6, 2, 7, 1)$
60 : $P_{547} = (2, 3, 7, 1)$
61 : $P_{563} = (2, 5, 7, 1)$
62 : $P_{564} = (3, 5, 7, 1)$
63 : $P_{567} = (6, 5, 7, 1)$

Line Intersection Graph

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  |0 1
 0|0 1
 1|1 0
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Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{138}

Line 1 intersects

Line	ℓ_0
in point	P_{138}

The surface has 81 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{12} = (1, 0, 1, 0)$
5 : $P_{20} = (1, 1, 1, 0)$
6 : $P_{31} = (4, 2, 1, 0)$
7 : $P_{42} = (7, 3, 1, 0)$
8 : $P_{50} = (7, 4, 1, 0)$
9 : $P_{53} = (2, 5, 1, 0)$
10 : $P_{63} = (4, 6, 1, 0)$
11 : $P_{69} = (2, 7, 1, 0)$
12 : $P_{75} = (1, 0, 0, 1)$
13 : $P_{85} = (3, 1, 0, 1)$

14 : $P_{87} = (5, 1, 0, 1)$
15 : $P_{88} = (6, 1, 0, 1)$
16 : $P_{97} = (7, 2, 0, 1)$
17 : $P_{108} = (2, 4, 0, 1)$
18 : $P_{134} = (4, 7, 0, 1)$
19 : $P_{138} = (0, 0, 1, 1)$
20 : $P_{146} = (0, 1, 1, 1)$
21 : $P_{153} = (0, 2, 1, 1)$
22 : $P_{157} = (4, 2, 1, 1)$
23 : $P_{158} = (5, 2, 1, 1)$
24 : $P_{161} = (0, 3, 1, 1)$
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26 : $P_{175} = (6, 4, 1, 1)$
27 : $P_{176} = (7, 4, 1, 1)$

28 : $P_{177} = (0, 5, 1, 1)$
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30 : $P_{193} = (0, 7, 1, 1)$
31 : $P_{195} = (2, 7, 1, 1)$
32 : $P_{196} = (3, 7, 1, 1)$
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40 : $P_{253} = (4, 6, 2, 1)$
41 : $P_{254} = (5, 6, 2, 1)$

3

$42 : P_{265} = (0, 0, 3, 1)$

$43 : P_{282} = (1, 2, 3, 1)$

$44 : P_{302} = (5, 4, 3, 1)$

$45 : P_{306} = (1, 5, 3, 1)$

$46 : P_{310} = (5, 5, 3, 1)$

$47 : P_{312} = (7, 5, 3, 1)$

$48 : P_{328} = (7, 7, 3, 1)$

$49 : P_{329} = (0, 0, 4, 1)$

$50 : P_{331} = (2, 0, 4, 1)$

$51 : P_{335} = (6, 0, 4, 1)$

$52 : P_{339} = (2, 1, 4, 1)$

$53 : P_{358} = (5, 3, 4, 1)$

$54 : P_{359} = (6, 3, 4, 1)$

$55 : P_{360} = (7, 3, 4, 1)$

$56 : P_{384} = (7, 6, 4, 1)$

$57 : P_{390} = (5, 7, 4, 1)$

$58 : P_{393} = (0, 0, 5, 1)$

$59 : P_{411} = (2, 2, 5, 1)$

$60 : P_{426} = (1, 4, 5, 1)$

$61 : P_{442} = (1, 6, 5, 1)$

$62 : P_{443} = (2, 6, 5, 1)$

$63 : P_{447} = (6, 6, 5, 1)$

$64 : P_{455} = (6, 7, 5, 1)$

$65 : P_{457} = (0, 0, 6, 1)$

$66 : P_{476} = (3, 2, 6, 1)$

$67 : P_{482} = (1, 3, 6, 1)$

$68 : P_{484} = (3, 3, 6, 1)$

$69 : P_{485} = (4, 3, 6, 1)$

$70 : P_{493} = (4, 4, 6, 1)$

$71 : P_{514} = (1, 7, 6, 1)$

$72 : P_{521} = (0, 0, 7, 1)$

$73 : P_{524} = (3, 0, 7, 1)$

$74 : P_{525} = (4, 0, 7, 1)$

$75 : P_{533} = (4, 1, 7, 1)$

$76 : P_{543} = (6, 2, 7, 1)$

$77 : P_{547} = (2, 3, 7, 1)$

$78 : P_{563} = (2, 5, 7, 1)$

$79 : P_{564} = (3, 5, 7, 1)$

$80 : P_{567} = (6, 5, 7, 1)$