Rank-76051 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_1 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1503024729

General information

Number of lines	9
Number of points	29
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	18
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	3^5, 2^6, 1^{18}

Singular Points

The surface has 4 singular points:

0 : $P_9 = \mathbf{P}(\omega, 0, 1, 0) = \mathbf{P}(2, 0, 1, 0)$
1 : $P_{10} = \mathbf{P}(\omega^2, 0, 1, 0) = \mathbf{P}(3, 0, 1, 0)$
2 : $P_{68} = \mathbf{P}(\omega^2, \omega^2, \omega, 1) = \mathbf{P}(3, 3, 2, 1)$
3 : $P_{79} = \mathbf{P}(\omega, \omega, \omega^2, 1) = \mathbf{P}(2, 2, 3, 1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{76} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{76} = \mathbf{Pl}(2,3,2,0,1,1)_{184}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{71} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{71} = \mathbf{Pl}(3,2,0,0,3,1)_{299}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{54} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{54} = \mathbf{Pl}(2,3,0,0,2,1)_{238}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{51} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{51} = \mathbf{Pl}(3,2,3,0,1,1)_{188}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \mathbf{Pl}(0,1,1,1,1,1)_{202}$$

Rank of lines: (16, 340, 356, 76, 71, 54, 51, 345, 122)
Rank of points on Klein quadric: (2, 9, 1, 184, 299, 238, 188, 13, 202)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_9 = \mathbf{P}(\omega,0,1,0) = \mathbf{P}(2,0,1,0)$,
$2 : P_{10} = \mathbf{P}(\omega^2,0,1,0) = \mathbf{P}(3,0,1,0)$,
$3 : P_{68} = \mathbf{P}(\omega^2,\omega^2,\omega,1) = \mathbf{P}(3,3,2,1)$,
$4 : P_{79} = \mathbf{P}(\omega,\omega,\omega^2,1) = \mathbf{P}(2,2,3,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$
$P_{34} = (0,3,0,1) = \ell_1 \cap \ell_4$
$P_{30} = (0,2,0,1) = \ell_1 \cap \ell_5$
$P_{38} = (0,0,1,1) = \ell_2 \cap \ell_8$
$P_{61} = (0,2,2,1) = \ell_3 \cap \ell_7$
$P_{81} = (0,3,3,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_1 = (0,1,0,0)$ lies on line ℓ_1
$2 :\ P_8 = (1,0,1,0)$ lies on line ℓ_0
$3 :\ P_{11} = (0,1,1,0)$ lies on line ℓ_7
$4 :\ P_{12} = (1,1,1,0)$ lies on line ℓ_8
$5 :\ P_{26} = (0,1,0,1)$ lies on line ℓ_1

6 : $P_{27} = (1, 1, 0, 1)$ lies on line ℓ_8
7 : $P_{33} = (3, 2, 0, 1)$ lies on line ℓ_3
8 : $P_{36} = (2, 3, 0, 1)$ lies on line ℓ_6
9 : $P_{42} = (0, 1, 1, 1)$ lies on line ℓ_7
10 : $P_{46} = (1, 2, 1, 1)$ lies on line ℓ_3
11 : $P_{48} = (3, 2, 1, 1)$ lies on line ℓ_5
12 : $P_{50} = (1, 3, 1, 1)$ lies on line ℓ_6
13 : $P_{51} = (2, 3, 1, 1)$ lies on line ℓ_4
14 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_2
15 : $P_{62} = (1, 2, 2, 1)$ lies on line ℓ_5
16 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_2
17 : $P_{82} = (1, 3, 3, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 0 1 1 1 1 1 0 0
1	0 0 1 0 1 1 0 1 0
2	1 1 0 0 0 0 0 1 1
3	1 0 0 0 1 1 0 1 1
4	1 1 0 1 0 0 1 0 1
5	1 1 0 1 0 0 1 0 1
6	1 0 0 0 1 1 0 1 1
7	0 1 1 1 0 0 1 0 0
8	0 0 1 1 1 1 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_9	P_9	P_{10}	P_{10}

Line 1 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_{34}	P_{30}	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_2	P_3	P_3	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_9	P_9	P_{79}	P_{61}	P_{79}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_9	P_{34}	P_9	P_{68}	P_{68}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_{10}	P_{30}	P_{79}	P_{10}	P_{79}

Line 6 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{10}	P_{68}	P_{10}	P_{81}	P_{68}

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6
in point	P_3	P_3	P_{61}	P_{81}

Line 8 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{38}	P_{79}	P_{68}	P_{79}	P_{68}

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_8 = (1,0,1,0)$
$5: P_9 = (2,0,1,0)$
$6: P_{10} = (3,0,1,0)$
$7: P_{11} = (0,1,1,0)$
$8: P_{12} = (1,1,1,0)$
$9: P_{26} = (0,1,0,1)$
$10: P_{27} = (1,1,0,1)$
$11: P_{30} = (0,2,0,1)$
$12: P_{33} = (3,2,0,1)$
$13: P_{34} = (0,3,0,1)$
$14: P_{36} = (2,3,0,1)$
$15: P_{38} = (0,0,1,1)$
$16: P_{42} = (0,1,1,1)$
$17: P_{46} = (1,2,1,1)$
$18: P_{48} = (3,2,1,1)$
$19: P_{50} = (1,3,1,1)$
$20: P_{51} = (2,3,1,1)$
$21: P_{53} = (0,0,2,1)$
$22: P_{61} = (0,2,2,1)$
$23: P_{62} = (1,2,2,1)$
$24: P_{68} = (3,3,2,1)$
$25: P_{69} = (0,0,3,1)$
$26: P_{79} = (2,2,3,1)$
$27: P_{81} = (0,3,3,1)$
$28: P_{82} = (1,3,3,1)$