

Rank-65634 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431660206

General information

Number of lines	2
Number of points	25
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	10
Number of points off lines	15
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^2
Type of lines on points	$1^{10}, 0^{15}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{71} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{71} = \mathbf{Pl}(3, 2, 0, 0, 3, 1)_{299} \\ \ell_1 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{54} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{54} = \mathbf{Pl}(2, 3, 0, 0, 2, 1)_{238}\end{aligned}$$

Rank of lines: (71, 54)

Rank of points on Klein quadric: (299, 238)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 10 single points:

The single points on the surface are:

0 : $P_9 = (2, 0, 1, 0)$ lies on line ℓ_0
 1 : $P_{10} = (3, 0, 1, 0)$ lies on line ℓ_1
 2 : $P_{30} = (0, 2, 0, 1)$ lies on line ℓ_1
 3 : $P_{34} = (0, 3, 0, 1)$ lies on line ℓ_0
 4 : $P_{48} = (3, 2, 1, 1)$ lies on line ℓ_1
 5 : $P_{51} = (2, 3, 1, 1)$ lies on line ℓ_0

6 : $P_{62} = (1, 2, 2, 1)$ lies on line ℓ_1
 7 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_0
 8 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_1
 9 : $P_{82} = (1, 3, 3, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 15 points not on any line:

The points on the surface but not on lines are:

0 : $P_8 = (1, 0, 1, 0)$
 1 : $P_{11} = (0, 1, 1, 0)$
 2 : $P_{13} = (2, 1, 1, 0)$
 3 : $P_{14} = (3, 1, 1, 0)$
 4 : $P_{15} = (0, 2, 1, 0)$
 5 : $P_{19} = (0, 3, 1, 0)$
 6 : $P_{26} = (0, 1, 0, 1)$
 7 : $P_{33} = (3, 2, 0, 1)$

8 : $P_{36} = (2, 3, 0, 1)$
 9 : $P_{38} = (0, 0, 1, 1)$
 10 : $P_{39} = (1, 0, 1, 1)$
 11 : $P_{53} = (0, 0, 2, 1)$
 12 : $P_{54} = (1, 0, 2, 1)$
 13 : $P_{69} = (0, 0, 3, 1)$
 14 : $P_{70} = (1, 0, 3, 1)$

Line Intersection Graph

	0	1
0	0	0
1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 25 points:

The points on the surface are:

0 : $P_8 = (1, 0, 1, 0)$
 1 : $P_9 = (2, 0, 1, 0)$
 2 : $P_{10} = (3, 0, 1, 0)$
 3 : $P_{11} = (0, 1, 1, 0)$
 4 : $P_{13} = (2, 1, 1, 0)$
 5 : $P_{14} = (3, 1, 1, 0)$
 6 : $P_{15} = (0, 2, 1, 0)$
 7 : $P_{19} = (0, 3, 1, 0)$
 8 : $P_{26} = (0, 1, 0, 1)$

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 12 : $P_{36} = (2, 3, 0, 1)$
 13 : $P_{38} = (0, 0, 1, 1)$
 14 : $P_{39} = (1, 0, 1, 1)$
 15 : $P_{48} = (3, 2, 1, 1)$
 16 : $P_{51} = (2, 3, 1, 1)$
 17 : $P_{53} = (0, 0, 2, 1)$

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 20 : $P_{68} = (3, 3, 2, 1)$
 21 : $P_{69} = (0, 0, 3, 1)$
 22 : $P_{70} = (1, 0, 3, 1)$
 23 : $P_{79} = (2, 2, 3, 1)$
 24 : $P_{82} = (1, 3, 3, 1)$