

Rank-74279 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1499027817

General information

Number of lines	3
Number of points	29
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	9
Number of points off lines	17
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	$2^3, 1^9, 0^{17}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \mathbf{Pl}(1, 0, 1, 1, 2, 1)_{259}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \mathbf{Pl}(1, 0, 1, 1, 3, 1)_{319}$$

Rank of lines: (0, 320, 215)

Rank of points on Klein quadric: (0, 259, 319)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_6 = (2, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_7 = (3, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{42} = (0, 1, 1, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 9 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

2 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0

3 : $P_{40} = (2, 0, 1, 1)$ lies on line ℓ_1

4 : $P_{41} = (3, 0, 1, 1)$ lies on line ℓ_2

5 : $P_{46} = (1, 2, 1, 1)$ lies on line ℓ_1

6 : $P_{47} = (2, 2, 1, 1)$ lies on line ℓ_2

7 : $P_{50} = (1, 3, 1, 1)$ lies on line ℓ_2

8 : $P_{52} = (3, 3, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 17 points not on any line:

The points on the surface but not on lines are:

0 : $P_3 = (0, 0, 0, 1)$

1 : $P_8 = (1, 0, 1, 0)$

2 : $P_{13} = (2, 1, 1, 0)$

3 : $P_{14} = (3, 1, 1, 0)$

4 : $P_{27} = (1, 1, 0, 1)$

5 : $P_{33} = (3, 2, 0, 1)$

6 : $P_{36} = (2, 3, 0, 1)$

7 : $P_{57} = (0, 1, 2, 1)$

8 : $P_{59} = (2, 1, 2, 1)$

9 : $P_{63} = (2, 2, 2, 1)$

10 : $P_{64} = (3, 2, 2, 1)$

11 : $P_{68} = (3, 3, 2, 1)$

12 : $P_{73} = (0, 1, 3, 1)$

13 : $P_{76} = (3, 1, 3, 1)$

14 : $P_{79} = (2, 2, 3, 1)$

15 : $P_{83} = (2, 3, 3, 1)$

16 : $P_{84} = (3, 3, 3, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_6	P_{42}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_7	P_{42}

The surface has 29 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_3 = (0, 0, 0, 1)$
 3 : $P_5 = (1, 1, 0, 0)$
 4 : $P_6 = (2, 1, 0, 0)$
 5 : $P_7 = (3, 1, 0, 0)$
 6 : $P_8 = (1, 0, 1, 0)$
 7 : $P_{13} = (2, 1, 1, 0)$
 8 : $P_{14} = (3, 1, 1, 0)$
 9 : $P_{27} = (1, 1, 0, 1)$

10 : $P_{33} = (3, 2, 0, 1)$
 11 : $P_{36} = (2, 3, 0, 1)$
 12 : $P_{40} = (2, 0, 1, 1)$
 13 : $P_{41} = (3, 0, 1, 1)$
 14 : $P_{42} = (0, 1, 1, 1)$
 15 : $P_{46} = (1, 2, 1, 1)$
 16 : $P_{47} = (2, 2, 1, 1)$
 17 : $P_{50} = (1, 3, 1, 1)$
 18 : $P_{52} = (3, 3, 1, 1)$
 19 : $P_{57} = (0, 1, 2, 1)$

20 : $P_{59} = (2, 1, 2, 1)$
 21 : $P_{63} = (2, 2, 2, 1)$
 22 : $P_{64} = (3, 2, 2, 1)$
 23 : $P_{68} = (3, 3, 2, 1)$
 24 : $P_{73} = (0, 1, 3, 1)$
 25 : $P_{76} = (3, 1, 3, 1)$
 26 : $P_{79} = (2, 2, 3, 1)$
 27 : $P_{83} = (2, 3, 3, 1)$
 28 : $P_{84} = (3, 3, 3, 1)$