

Rank-76051 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_1 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 76051

General information

Number of lines	5
Number of points	11
Number of singular points	0
Number of Eckardt points	1
Number of double points	2
Number of single points	8
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	$3, 2^2, 1^8$

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5\end{aligned}$$

$$\begin{aligned}\ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_3 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\ \ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{34}\end{aligned}$$

Rank of lines: (4, 30, 34, 33, 26)

Rank of points on Klein quadric: (2, 5, 1, 7, 34)

Eckardt Points

The surface has 1 Eckardt points:

$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$. $T = 14$

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_2$$

$$P_{12} = (0, 0, 1, 1) = \ell_2 \cap \ell_4$$

Single Points

The surface has 8 single points:

The single points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

$1 : P_1 = (0, 1, 0, 0)$ lies on line ℓ_1

$2 : P_6 = (1, 0, 1, 0)$ lies on line ℓ_0

$3 : P_7 = (0, 1, 1, 0)$ lies on line ℓ_3

$4 : P_8 = (1, 1, 1, 0)$ lies on line ℓ_4

$5 : P_{10} = (0, 1, 0, 1)$ lies on line ℓ_1

$6 : P_{11} = (1, 1, 0, 1)$ lies on line ℓ_4

$7 : P_{14} = (0, 1, 1, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	0	1	0	0
1	0	0	1	1	0
2	1	1	0	1	1
3	0	1	1	0	0
4	0	0	1	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_2
in point	P_2

Line 1 intersects

Line	ℓ_2	ℓ_3
in point	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_3	P_3	P_{12}

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_3

Line 4 intersects

Line	ℓ_2
in point	P_{12}

The surface has 11 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$

4 : $P_6 = (1, 0, 1, 0)$
5 : $P_7 = (0, 1, 1, 0)$
6 : $P_8 = (1, 1, 1, 0)$
7 : $P_{10} = (0, 1, 0, 1)$

8 : $P_{11} = (1, 1, 0, 1)$
9 : $P_{12} = (0, 0, 1, 1)$
10 : $P_{14} = (0, 1, 1, 1)$