Rank-73733 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498764637

General information

Number of lines	2
Number of points	21
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	8
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^2
Type of lines on points	2, 1^8, 0^{12}

Singular Points

The surface has 2 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (16, 356)
Rank of points on Klein quadric: (2, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 8 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
2 : $P_8 = (1,0,1,0)$ lies on line ℓ_0
3 : $P_9 = (2,0,1,0)$ lies on line ℓ_0
4 : $P_{10} = (3,0,1,0)$ lies on line ℓ_0
5 : $P_{38} = (0,0,1,1)$ lies on line ℓ_1
6 : $P_{53} = (0,0,2,1)$ lies on line ℓ_1
7 : $P_{69} = (0,0,3,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1,1,1,0)$
1 : $P_{18} = (3,2,1,0)$
2 : $P_{21} = (2,3,1,0)$
3 : $P_{27} = (1,1,0,1)$
4 : $P_{31} = (1,2,0,1)$
5 : $P_{35} = (1,3,0,1)$
6 : $P_{47} = (2,2,1,1)$
7 : $P_{52} = (3,3,1,1)$
8 : $P_{59} = (2,1,2,1)$
9 : $P_{64} = (3,2,2,1)$
10 : $P_{76} = (3,1,3,1)$
11 : $P_{83} = (2,3,3,1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 21 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_8 = (1, 0, 1, 0)$
4 : $P_9 = (2, 0, 1, 0)$
5 : $P_{10} = (3, 0, 1, 0)$
6 : $P_{12} = (1, 1, 1, 0)$
7 : $P_{18} = (3, 2, 1, 0)$
8 : $P_{21} = (2, 3, 1, 0)$
9 : $P_{27} = (1, 1, 0, 1)$
10 : $P_{31} = (1, 2, 0, 1)$
11 : $P_{35} = (1, 3, 0, 1)$
12 : $P_{38} = (0, 0, 1, 1)$
13 : $P_{47} = (2, 2, 1, 1)$
14 : $P_{52} = (3, 3, 1, 1)$
15 : $P_{53} = (0, 0, 2, 1)$
16 : $P_{59} = (2, 1, 2, 1)$
17 : $P_{64} = (3, 2, 2, 1)$
18 : $P_{69} = (0, 0, 3, 1)$
19 : $P_{76} = (3, 1, 3, 1)$
20 : $P_{83} = (2, 3, 3, 1)$