Rank-76356 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1090785350

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4096
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$3, 1^{192}, 0^{4096}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15183424} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15183424} = \mathbf{Pl}(0,57,1,0,0,0)_{122}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 0 & 1 & 0 \end{array}\right]_{14917120} = \left[\begin{array}{cccc} 1 & 0 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{array}\right]_{14917120} = \mathbf{Pl}(0, 56, 1, 0, 0, 0)_{121}$$

Rank of lines: (17047616, 15183424, 14917120)
Rank of points on Klein quadric: (1, 122, 121)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_3 = (0, 0, 0, 1)$ lies on line ℓ_0
$1 : P_{4218} = (56, 0, 0, 1)$ lies on line ℓ_1
$2 : P_{4219} = (57, 0, 0, 1)$ lies on line ℓ_2
$3 : P_{8258} = (0, 0, 1, 1)$ lies on line ℓ_0
$4 : P_{8314} = (56, 0, 1, 1)$ lies on line ℓ_1
$5 : P_{8315} = (57, 0, 1, 1)$ lies on line ℓ_2
$6 : P_{12353} = (0, 0, 2, 1)$ lies on line ℓ_0
$7 : P_{12409} = (56, 0, 2, 1)$ lies on line ℓ_1
$8 : P_{12410} = (57, 0, 2, 1)$ lies on line ℓ_2
$9 : P_{16449} = (0, 0, 3, 1)$ lies on line ℓ_0
$10 : P_{16505} = (56, 0, 3, 1)$ lies on line ℓ_1
$11 : P_{16506} = (57, 0, 3, 1)$ lies on line ℓ_2
$12 : P_{20545} = (0, 0, 4, 1)$ lies on line ℓ_0
$13 : P_{20601} = (56, 0, 4, 1)$ lies on line ℓ_1
$14 : P_{20602} = (57, 0, 4, 1)$ lies on line ℓ_2
$15 : P_{24641} = (0, 0, 5, 1)$ lies on line ℓ_0
$16 : P_{24697} = (56, 0, 5, 1)$ lies on line ℓ_1
$17 : P_{24698} = (57, 0, 5, 1)$ lies on line ℓ_2
$18 : P_{28737} = (0, 0, 6, 1)$ lies on line ℓ_0
$19 : P_{28793} = (56, 0, 6, 1)$ lies on line ℓ_1
$20 : P_{28794} = (57, 0, 6, 1)$ lies on line ℓ_2
$21 : P_{32833} = (0, 0, 7, 1)$ lies on line ℓ_0
$22 : P_{32889} = (56, 0, 7, 1)$ lies on line ℓ_1
$23 : P_{32890} = (57, 0, 7, 1)$ lies on line ℓ_2
$24 : P_{36929} = (0, 0, 8, 1)$ lies on line ℓ_0
$25 : P_{36985} = (56, 0, 8, 1)$ lies on line ℓ_1
$26 : P_{36986} = (57, 0, 8, 1)$ lies on line ℓ_2
$27 : P_{41025} = (0, 0, 9, 1)$ lies on line ℓ_0
$28 : P_{41081} = (56, 0, 9, 1)$ lies on line ℓ_1

$29 : P_{41082} = (57, 0, 9, 1)$ lies on line ℓ_2
$30 : P_{45121} = (0, 0, 10, 1)$ lies on line ℓ_0
$31 : P_{45177} = (56, 0, 10, 1)$ lies on line ℓ_1
$32 : P_{45178} = (57, 0, 10, 1)$ lies on line ℓ_2
$33 : P_{49217} = (0, 0, 11, 1)$ lies on line ℓ_0
$34 : P_{49273} = (56, 0, 11, 1)$ lies on line ℓ_1
$35 : P_{49274} = (57, 0, 11, 1)$ lies on line ℓ_2
$36 : P_{53313} = (0, 0, 12, 1)$ lies on line ℓ_0
$37 : P_{53369} = (56, 0, 12, 1)$ lies on line ℓ_1
$38 : P_{53370} = (57, 0, 12, 1)$ lies on line ℓ_2
$39 : P_{57409} = (0, 0, 13, 1)$ lies on line ℓ_0
$40 : P_{57465} = (56, 0, 13, 1)$ lies on line ℓ_1
$41 : P_{57466} = (57, 0, 13, 1)$ lies on line ℓ_2
$42 : P_{61505} = (0, 0, 14, 1)$ lies on line ℓ_0
$43 : P_{61561} = (56, 0, 14, 1)$ lies on line ℓ_1
$44 : P_{61562} = (57, 0, 14, 1)$ lies on line ℓ_2
$45 : P_{65601} = (0, 0, 15, 1)$ lies on line ℓ_0
$46 : P_{65657} = (56, 0, 15, 1)$ lies on line ℓ_1
$47 : P_{65658} = (57, 0, 15, 1)$ lies on line ℓ_2
$48 : P_{69697} = (0, 0, 16, 1)$ lies on line ℓ_0
$49 : P_{69753} = (56, 0, 16, 1)$ lies on line ℓ_1
$50 : P_{69754} = (57, 0, 16, 1)$ lies on line ℓ_2
$51 : P_{73793} = (0, 0, 17, 1)$ lies on line ℓ_0
$52 : P_{73849} = (56, 0, 17, 1)$ lies on line ℓ_1
$53 : P_{73850} = (57, 0, 17, 1)$ lies on line ℓ_2
$54 : P_{77889} = (0, 0, 18, 1)$ lies on line ℓ_0
$55 : P_{77945} = (56, 0, 18, 1)$ lies on line ℓ_1
$56 : P_{77946} = (57, 0, 18, 1)$ lies on line ℓ_2
$57 : P_{81985} = (0, 0, 19, 1)$ lies on line ℓ_0

58 : $P_{82041} = (56, 0, 19, 1)$ lies on line ℓ_1
59 : $P_{82042} = (57, 0, 19, 1)$ lies on line ℓ_2
60 : $P_{86081} = (0, 0, 20, 1)$ lies on line ℓ_0
61 : $P_{86137} = (56, 0, 20, 1)$ lies on line ℓ_1
62 : $P_{86138} = (57, 0, 20, 1)$ lies on line ℓ_2
63 : $P_{90177} = (0, 0, 21, 1)$ lies on line ℓ_0
64 : $P_{90233} = (56, 0, 21, 1)$ lies on line ℓ_1
65 : $P_{90234} = (57, 0, 21, 1)$ lies on line ℓ_2
66 : $P_{94273} = (0, 0, 22, 1)$ lies on line ℓ_0
67 : $P_{94329} = (56, 0, 22, 1)$ lies on line ℓ_1
68 : $P_{94330} = (57, 0, 22, 1)$ lies on line ℓ_2
69 : $P_{98369} = (0, 0, 23, 1)$ lies on line ℓ_0
70 : $P_{98425} = (56, 0, 23, 1)$ lies on line ℓ_1
71 : $P_{98426} = (57, 0, 23, 1)$ lies on line ℓ_2
72 : $P_{102465} = (0, 0, 24, 1)$ lies on line ℓ_0
73 : $P_{102521} = (56, 0, 24, 1)$ lies on line ℓ_1
74 : $P_{102522} = (57, 0, 24, 1)$ lies on line ℓ_2
75 : $P_{106561} = (0, 0, 25, 1)$ lies on line ℓ_0
76 : $P_{106617} = (56, 0, 25, 1)$ lies on line ℓ_1
77 : $P_{106618} = (57, 0, 25, 1)$ lies on line ℓ_2
78 : $P_{110657} = (0, 0, 26, 1)$ lies on line ℓ_0
79 : $P_{110713} = (56, 0, 26, 1)$ lies on line ℓ_1
80 : $P_{110714} = (57, 0, 26, 1)$ lies on line ℓ_2
81 : $P_{114753} = (0, 0, 27, 1)$ lies on line ℓ_0
82 : $P_{114809} = (56, 0, 27, 1)$ lies on line ℓ_1
83 : $P_{114810} = (57, 0, 27, 1)$ lies on line ℓ_2
84 : $P_{118849} = (0, 0, 28, 1)$ lies on line ℓ_0
85 : $P_{118905} = (56, 0, 28, 1)$ lies on line ℓ_1
86 : $P_{118906} = (57, 0, 28, 1)$ lies on line ℓ_2
87 : $P_{122945} = (0, 0, 29, 1)$ lies on line ℓ_0
88 : $P_{123001} = (56, 0, 29, 1)$ lies on line ℓ_1
89 : $P_{123002} = (57, 0, 29, 1)$ lies on line ℓ_2
90 : $P_{127041} = (0, 0, 30, 1)$ lies on line ℓ_0
91 : $P_{127097} = (56, 0, 30, 1)$ lies on line ℓ_1
92 : $P_{127098} = (57, 0, 30, 1)$ lies on line ℓ_2
93 : $P_{131137} = (0, 0, 31, 1)$ lies on line ℓ_0
94 : $P_{131193} = (56, 0, 31, 1)$ lies on line ℓ_1
95 : $P_{131194} = (57, 0, 31, 1)$ lies on line ℓ_2
96 : $P_{135233} = (0, 0, 32, 1)$ lies on line ℓ_0
97 : $P_{135289} = (56, 0, 32, 1)$ lies on line ℓ_1
98 : $P_{135290} = (57, 0, 32, 1)$ lies on line ℓ_2
99 : $P_{139329} = (0, 0, 33, 1)$ lies on line ℓ_0
100 : $P_{139385} = (56, 0, 33, 1)$ lies on line ℓ_1
101 : $P_{139386} = (57, 0, 33, 1)$ lies on line ℓ_2
102 : $P_{143425} = (0, 0, 34, 1)$ lies on line ℓ_0
103 : $P_{143481} = (56, 0, 34, 1)$ lies on line ℓ_1
104 : $P_{143482} = (57, 0, 34, 1)$ lies on line ℓ_2
105 : $P_{147521} = (0, 0, 35, 1)$ lies on line ℓ_0
106 : $P_{147577} = (56, 0, 35, 1)$ lies on line ℓ_1
107 : $P_{147578} = (57, 0, 35, 1)$ lies on line ℓ_2
108 : $P_{151617} = (0, 0, 36, 1)$ lies on line ℓ_0
109 : $P_{151673} = (56, 0, 36, 1)$ lies on line ℓ_1
110 : $P_{151674} = (57, 0, 36, 1)$ lies on line ℓ_2
111 : $P_{155713} = (0, 0, 37, 1)$ lies on line ℓ_0

112 : $P_{155769} = (56, 0, 37, 1)$ lies on line ℓ_1
113 : $P_{155770} = (57, 0, 37, 1)$ lies on line ℓ_2
114 : $P_{159809} = (0, 0, 38, 1)$ lies on line ℓ_0
115 : $P_{159865} = (56, 0, 38, 1)$ lies on line ℓ_1
116 : $P_{159866} = (57, 0, 38, 1)$ lies on line ℓ_2
117 : $P_{163905} = (0, 0, 39, 1)$ lies on line ℓ_0
118 : $P_{163961} = (56, 0, 39, 1)$ lies on line ℓ_1
119 : $P_{163962} = (57, 0, 39, 1)$ lies on line ℓ_2
120 : $P_{168001} = (0, 0, 40, 1)$ lies on line ℓ_0
121 : $P_{168057} = (56, 0, 40, 1)$ lies on line ℓ_1
122 : $P_{168058} = (57, 0, 40, 1)$ lies on line ℓ_2
123 : $P_{172097} = (0, 0, 41, 1)$ lies on line ℓ_0
124 : $P_{172153} = (56, 0, 41, 1)$ lies on line ℓ_1
125 : $P_{172154} = (57, 0, 41, 1)$ lies on line ℓ_2
126 : $P_{176193} = (0, 0, 42, 1)$ lies on line ℓ_0
127 : $P_{176249} = (56, 0, 42, 1)$ lies on line ℓ_1
128 : $P_{176250} = (57, 0, 42, 1)$ lies on line ℓ_2
129 : $P_{180289} = (0, 0, 43, 1)$ lies on line ℓ_0
130 : $P_{180345} = (56, 0, 43, 1)$ lies on line ℓ_1
131 : $P_{180346} = (57, 0, 43, 1)$ lies on line ℓ_2
132 : $P_{184385} = (0, 0, 44, 1)$ lies on line ℓ_0
133 : $P_{184441} = (56, 0, 44, 1)$ lies on line ℓ_1
134 : $P_{184442} = (57, 0, 44, 1)$ lies on line ℓ_2
135 : $P_{188481} = (0, 0, 45, 1)$ lies on line ℓ_0
136 : $P_{188537} = (56, 0, 45, 1)$ lies on line ℓ_1
137 : $P_{188538} = (57, 0, 45, 1)$ lies on line ℓ_2
138 : $P_{192577} = (0, 0, 46, 1)$ lies on line ℓ_0
139 : $P_{192633} = (56, 0, 46, 1)$ lies on line ℓ_1
140 : $P_{192634} = (57, 0, 46, 1)$ lies on line ℓ_2
141 : $P_{196673} = (0, 0, 47, 1)$ lies on line ℓ_0
142 : $P_{196729} = (56, 0, 47, 1)$ lies on line ℓ_1
143 : $P_{196730} = (57, 0, 47, 1)$ lies on line ℓ_2
144 : $P_{200769} = (0, 0, 48, 1)$ lies on line ℓ_0
145 : $P_{200825} = (56, 0, 48, 1)$ lies on line ℓ_1
146 : $P_{200826} = (57, 0, 48, 1)$ lies on line ℓ_2
147 : $P_{204865} = (0, 0, 49, 1)$ lies on line ℓ_0
148 : $P_{204921} = (56, 0, 49, 1)$ lies on line ℓ_1
149 : $P_{204922} = (57, 0, 49, 1)$ lies on line ℓ_2
150 : $P_{208961} = (0, 0, 50, 1)$ lies on line ℓ_0
151 : $P_{209017} = (56, 0, 50, 1)$ lies on line ℓ_1
152 : $P_{209018} = (57, 0, 50, 1)$ lies on line ℓ_2
153 : $P_{213057} = (0, 0, 51, 1)$ lies on line ℓ_0
154 : $P_{213113} = (56, 0, 51, 1)$ lies on line ℓ_1
155 : $P_{213114} = (57, 0, 51, 1)$ lies on line ℓ_2
156 : $P_{217153} = (0, 0, 52, 1)$ lies on line ℓ_0
157 : $P_{217209} = (56, 0, 52, 1)$ lies on line ℓ_1
158 : $P_{217210} = (57, 0, 52, 1)$ lies on line ℓ_2
159 : $P_{221249} = (0, 0, 53, 1)$ lies on line ℓ_0
160 : $P_{221305} = (56, 0, 53, 1)$ lies on line ℓ_1
161 : $P_{221306} = (57, 0, 53, 1)$ lies on line ℓ_2
162 : $P_{225345} = (0, 0, 54, 1)$ lies on line ℓ_0
163 : $P_{225401} = (56, 0, 54, 1)$ lies on line ℓ_1
164 : $P_{225402} = (57, 0, 54, 1)$ lies on line ℓ_2
165 : $P_{229441} = (0, 0, 55, 1)$ lies on line ℓ_0

166 : $P_{229497} = (56, 0, 55, 1)$ lies on line ℓ_1
167 : $P_{229498} = (57, 0, 55, 1)$ lies on line ℓ_2
168 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_0
169 : $P_{233593} = (56, 0, 56, 1)$ lies on line ℓ_1
170 : $P_{233594} = (57, 0, 56, 1)$ lies on line ℓ_2
171 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_0
172 : $P_{237689} = (56, 0, 57, 1)$ lies on line ℓ_1
173 : $P_{237690} = (57, 0, 57, 1)$ lies on line ℓ_2
174 : $P_{241729} = (0, 0, 58, 1)$ lies on line ℓ_0
175 : $P_{241785} = (56, 0, 58, 1)$ lies on line ℓ_1
176 : $P_{241786} = (57, 0, 58, 1)$ lies on line ℓ_2
177 : $P_{245825} = (0, 0, 59, 1)$ lies on line ℓ_0
178 : $P_{245881} = (56, 0, 59, 1)$ lies on line ℓ_1
179 : $P_{245882} = (57, 0, 59, 1)$ lies on line ℓ_2

180 : $P_{249921} = (0, 0, 60, 1)$ lies on line ℓ_0
181 : $P_{249977} = (56, 0, 60, 1)$ lies on line ℓ_1
182 : $P_{249978} = (57, 0, 60, 1)$ lies on line ℓ_2
183 : $P_{254017} = (0, 0, 61, 1)$ lies on line ℓ_0
184 : $P_{254073} = (56, 0, 61, 1)$ lies on line ℓ_1
185 : $P_{254074} = (57, 0, 61, 1)$ lies on line ℓ_2
186 : $P_{258113} = (0, 0, 62, 1)$ lies on line ℓ_0
187 : $P_{258169} = (56, 0, 62, 1)$ lies on line ℓ_1
188 : $P_{258170} = (57, 0, 62, 1)$ lies on line ℓ_2
189 : $P_{262209} = (0, 0, 63, 1)$ lies on line ℓ_0
190 : $P_{262265} = (56, 0, 63, 1)$ lies on line ℓ_1
191 : $P_{262266} = (57, 0, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4096 points not on any line:
Too many to print.

Line Intersection Graph

```
 |0 1 2
0|0 1 1
1|1 0 1
2|1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 4289 points:
Too many to print.