Rank-35 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 665

General information

Number of lines	9
Number of points	73
Number of singular points	9
Number of Eckardt points	0
Number of double points	0
Number of single points	72
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^9
Type of lines on points	$9, 1^{72}$

Singular Points

The surface has 9 singular points:

$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$2 : P_{82} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$
$3 : P_{90} = \mathbf{P}(0,\gamma,0,1) = \mathbf{P}(0,2,0,1)$
$4 : P_{98} = \mathbf{P}(0,\gamma^5,0,1) = \mathbf{P}(0,3,0,1)$

$5 : P_{106} = \mathbf{P}(0,\gamma^2,0,1) = \mathbf{P}(0,4,0,1)$
$6 : P_{114} = \mathbf{P}(0,\gamma^3,0,1) = \mathbf{P}(0,5,0,1)$
$7 : P_{122} = \mathbf{P}(0,\gamma^6,0,1) = \mathbf{P}(0,6,0,1)$
$8 : P_{130} = \mathbf{P}(0,\gamma^4,0,1) = \mathbf{P}(0,7,0,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \mathbf{Pl}(0,0,0,0,1,0)_{81}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0,1,0,1,1,0)_{209}$$

$$\ell_3 = \begin{bmatrix} 1 & \gamma^5 & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4379} = \begin{bmatrix} 1 & 3 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4379} = \mathbf{Pl}(0,7,0,3,1,0)_{245}$$

$$\ell_4 = \begin{bmatrix} 1 & \gamma^4 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4087} = \begin{bmatrix} 1 & 7 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4087} = \mathbf{Pl}(0,6,0,7,1,0)_{304}$$

$$\ell_5 = \begin{bmatrix} 1 & \gamma^3 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1605} = \begin{bmatrix} 1 & 5 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1605} = \mathbf{Pl}(0,2,0,5,1,0)_{270}$$

$$\ell_6 = \begin{bmatrix} 1 & \gamma & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1970} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1970} = \mathbf{Pl}(0,3,0,2,1,0)_{226}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^2 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3284} = \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3284} = \mathbf{Pl}(0,5,0,4,1,0)_{258}$$

$$\ell_8 = \begin{bmatrix} 1 & \gamma^6 & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2846} = \begin{bmatrix} 1 & 6 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2846} = \mathbf{Pl}(0,4,0,6,1,0)_{287}$$

Rank of lines: (72, 4680, 729, 4379, 4087, 1605, 1970, 3284, 2846)
Rank of points on Klein quadric: (81, 17, 209, 245, 304, 270, 226, 258, 287)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 72 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_4 = (1,1,1,1)$ lies on line ℓ_2
3 : $P_{20} = (1,1,1,0)$ lies on line ℓ_2
4 : $P_{32} = (5,2,1,0)$ lies on line ℓ_3
5 : $P_{37} = (2,3,1,0)$ lies on line ℓ_4
6 : $P_{49} = (6,4,1,0)$ lies on line ℓ_5
7 : $P_{55} = (4,5,1,0)$ lies on line ℓ_6
8 : $P_{66} = (7,6,1,0)$ lies on line ℓ_7
9 : $P_{70} = (3,7,1,0)$ lies on line ℓ_8
10 : $P_{75} = (1,0,0,1)$ lies on line ℓ_0
11 : $P_{76} = (2,0,0,1)$ lies on line ℓ_0

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$12 : P_{77} = (3,0,0,1)$ lies on line ℓ_0

$13 : P_{78} = (4,0,0,1)$ lies on line ℓ_0

$14 : P_{79} = (5,0,0,1)$ lies on line ℓ_0

$15 : P_{80} = (6,0,0,1)$ lies on line ℓ_0

$16 : P_{81} = (7,0,0,1)$ lies on line ℓ_0

$17 : P_{82} = (0,1,0,1)$ lies on line ℓ_1

$18 : P_{90} = (0,2,0,1)$ lies on line ℓ_1

$19 : P_{98} = (0,3,0,1)$ lies on line ℓ_1

$20 : P_{106} = (0,4,0,1)$ lies on line ℓ_1

$21 : P_{114} = (0,5,0,1)$ lies on line ℓ_1

$22 : P_{122} = (0,6,0,1)$ lies on line ℓ_1

$23 : P_{130} = (0,7,0,1)$ lies on line ℓ_1

$24 : P_{158} = (5,2,1,1)$ lies on line ℓ_3

$25 : P_{163} = (2,3,1,1)$ lies on line ℓ_4

$26 : P_{175} = (6,4,1,1)$ lies on line ℓ_5

$27 : P_{181} = (4,5,1,1)$ lies on line ℓ_6

$28 : P_{192} = (7,6,1,1)$ lies on line ℓ_7

$29 : P_{196} = (3,7,1,1)$ lies on line ℓ_8

$30 : P_{212} = (3,1,2,1)$ lies on line ℓ_7

$31 : P_{219} = (2,2,2,1)$ lies on line ℓ_2

$32 : P_{231} = (6,3,2,1)$ lies on line ℓ_8

$33 : P_{240} = (7,4,2,1)$ lies on line ℓ_3

$34 : P_{242} = (1,5,2,1)$ lies on line ℓ_5

$35 : P_{253} = (4,6,2,1)$ lies on line ℓ_4

$36 : P_{262} = (5,7,2,1)$ lies on line ℓ_6

$37 : P_{280} = (7,1,3,1)$ lies on line ℓ_5

$38 : P_{282} = (1,2,3,1)$ lies on line ℓ_6

$39 : P_{292} = (3,3,3,1)$ lies on line ℓ_2

$40 : P_{302} = (5,4,3,1)$ lies on line ℓ_8

$41 : P_{311} = (6,5,3,1)$ lies on line ℓ_4

$42 : P_{315} = (2,6,3,1)$ lies on line ℓ_3

$43 : P_{325} = (4,7,3,1)$ lies on line ℓ_7

$44 : P_{342} = (5,1,4,1)$ lies on line ℓ_4

$45 : P_{351} = (6,2,4,1)$ lies on line ℓ_7

$46 : P_{360} = (7,3,4,1)$ lies on line ℓ_6

$47 : P_{365} = (4,4,4,1)$ lies on line ℓ_2

$48 : P_{372} = (3,5,4,1)$ lies on line ℓ_3

$49 : P_{378} = (1,6,4,1)$ lies on line ℓ_8

$50 : P_{387} = (2,7,4,1)$ lies on line ℓ_5

$51 : P_{403} = (2,1,5,1)$ lies on line ℓ_8

$52 : P_{416} = (7,2,5,1)$ lies on line ℓ_4

$53 : P_{421} = (4,3,5,1)$ lies on line ℓ_5

$54 : P_{426} = (1,4,5,1)$ lies on line ℓ_7

$55 : P_{438} = (5,5,5,1)$ lies on line ℓ_2

$56 : P_{444} = (3,6,5,1)$ lies on line ℓ_6

$57 : P_{455} = (6,7,5,1)$ lies on line ℓ_3

$58 : P_{469} = (4,1,6,1)$ lies on line ℓ_3

$59 : P_{476} = (3,2,6,1)$ lies on line ℓ_5

$60 : P_{486} = (5,3,6,1)$ lies on line ℓ_7

$61 : P_{491} = (2,4,6,1)$ lies on line ℓ_6

$62 : P_{504} = (7,5,6,1)$ lies on line ℓ_8

$63 : P_{511} = (6,6,6,1)$ lies on line ℓ_2

$64 : P_{514} = (1,7,6,1)$ lies on line ℓ_4

$65 : P_{535} = (6,1,7,1)$ lies on line ℓ_6

$66 : P_{541} = (4,2,7,1)$ lies on line ℓ_8

$67 : P_{546} = (1,3,7,1)$ lies on line ℓ_3

$68 : P_{556} = (3,4,7,1)$ lies on line ℓ_4

$69 : P_{563} = (2,5,7,1)$ lies on line ℓ_7

$70 : P_{574} = (5,6,7,1)$ lies on line ℓ_5

$71 : P_{584} = (7,7,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1
5	1	1	1	1	1	0	1	1	1
6	1	1	1	1	1	1	0	1	1
7	1	1	1	1	1	1	1	0	1
8	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 73 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_{20} = (1,1,1,0)$
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$8: P_{55} = (4,5,1,0)$
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$10: P_{70} = (3,7,1,0)$
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$12: P_{76} = (2,0,0,1)$
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$17: P_{81} = (7,0,0,1)$
$18: P_{82} = (0,1,0,1)$

$19: P_{90} = (0,2,0,1)$
$20: P_{98} = (0,3,0,1)$
$21: P_{106} = (0,4,0,1)$
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$71: P_{574} = (5, 6, 7, 1)$

$72: P_{584} = (7, 7, 7, 1)$