Rank-73802 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108646

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	129
Number of points off lines	182
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^4, 2^6, 1^{129}, 0^{182}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{274} = \mathbf{Pl}(1,0,1,0,0,1)_{4657}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \delta^{10} & 0 \\ 0 & 1 & \delta^5 & 0 \end{bmatrix}_{2741} = \begin{bmatrix} 1 & 0 & 10 & 0 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{2741} = \mathbf{Pl}(11,0,10,0,0,1)_{4946}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \delta^5 & 0 \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{3013} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{3013} = \mathbf{Pl}(10,0,11,0,0,1)_{4976}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1,1,1,1,0,1)_{5586}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \delta^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \mathbf{Pl}(0,11,0,1,0,0)_{59}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^5 & 0 \end{bmatrix}_{7109} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{7109} = \mathbf{Pl}(11,10,10,11,0,1)_{7981}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \begin{bmatrix} 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \mathbf{Pl}(0,10,0,1,0,0)_{58}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^5 & 1 \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{7381} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{7381} = \mathbf{Pl}(10,11,11,10,0,1)_{7770}$$

Rank of lines: (274, 2741, 3013, 69921, 4642, 70091, 7109, 70074, 7381)
Rank of points on Klein quadric: (4657, 4946, 4976, 49, 5586, 59, 7981, 58, 7770)

Eckardt Points

The surface has 4 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_{35} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$2 : P_{179} = \mathbf{P}(0,\delta^{10},1,0) = \mathbf{P}(0,10,1,0)$,
$3 : P_{195} = \mathbf{P}(0,\delta^5,1,0) = \mathbf{P}(0,11,1,0)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{205} = (10,11,1,0) = \ell_0 \cap \ell_1$
$P_{190} = (11,10,1,0) = \ell_0 \cap \ell_2$
$P_{36} = (1,1,1,0) = \ell_1 \cap \ell_2$
$P_{3250} = (1,10,11,1) = \ell_4 \cap \ell_6$
$P_{3010} = (1,11,10,1) = \ell_4 \cap \ell_8$
$P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$

Single Points

The surface has 129 single points:
The single points on the surface are:

$0 :\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$1 :\ P_{14} = (10,1,0,0)$ lies on line ℓ_1
$2 :\ P_{15} = (11,1,0,0)$ lies on line ℓ_2
$3 :\ P_{20} = (1,0,1,0)$ lies on line ℓ_0
$4 :\ P_{29} = (10,0,1,0)$ lies on line ℓ_2
$5 :\ P_{30} = (11,0,1,0)$ lies on line ℓ_1
$6 :\ P_{54} = (3,2,1,0)$ lies on line ℓ_0
$7 :\ P_{56} = (5,2,1,0)$ lies on line ℓ_2
$8 :\ P_{57} = (6,2,1,0)$ lies on line ℓ_1
$9 :\ P_{69} = (2,3,1,0)$ lies on line ℓ_0
$10 :\ P_{79} = (12,3,1,0)$ lies on line ℓ_1
$11 :\ P_{81} = (14,3,1,0)$ lies on line ℓ_2
$12 :\ P_{88} = (5,4,1,0)$ lies on line ℓ_0
$13 :\ P_{91} = (8,4,1,0)$ lies on line ℓ_1

14 : $P_{96} = (13, 4, 1, 0)$ lies on line ℓ_2
15 : $P_{101} = (2, 5, 1, 0)$ lies on line ℓ_1
16 : $P_{103} = (4, 5, 1, 0)$ lies on line ℓ_0
17 : $P_{105} = (6, 5, 1, 0)$ lies on line ℓ_2
18 : $P_{117} = (2, 6, 1, 0)$ lies on line ℓ_2
19 : $P_{120} = (5, 6, 1, 0)$ lies on line ℓ_1
20 : $P_{122} = (7, 6, 1, 0)$ lies on line ℓ_0
21 : $P_{137} = (6, 7, 1, 0)$ lies on line ℓ_0
22 : $P_{140} = (9, 7, 1, 0)$ lies on line ℓ_2
23 : $P_{146} = (15, 7, 1, 0)$ lies on line ℓ_1
24 : $P_{151} = (4, 8, 1, 0)$ lies on line ℓ_2
25 : $P_{156} = (9, 8, 1, 0)$ lies on line ℓ_0
26 : $P_{160} = (13, 8, 1, 0)$ lies on line ℓ_1
27 : $P_{170} = (7, 9, 1, 0)$ lies on line ℓ_1
28 : $P_{171} = (8, 9, 1, 0)$ lies on line ℓ_0
29 : $P_{178} = (15, 9, 1, 0)$ lies on line ℓ_2
30 : $P_{214} = (3, 12, 1, 0)$ lies on line ℓ_2
31 : $P_{224} = (13, 12, 1, 0)$ lies on line ℓ_0
32 : $P_{225} = (14, 12, 1, 0)$ lies on line ℓ_1
33 : $P_{231} = (4, 13, 1, 0)$ lies on line ℓ_1
34 : $P_{235} = (8, 13, 1, 0)$ lies on line ℓ_2
35 : $P_{239} = (12, 13, 1, 0)$ lies on line ℓ_0
36 : $P_{246} = (3, 14, 1, 0)$ lies on line ℓ_1
37 : $P_{255} = (12, 14, 1, 0)$ lies on line ℓ_2
38 : $P_{258} = (15, 14, 1, 0)$ lies on line ℓ_0
39 : $P_{266} = (7, 15, 1, 0)$ lies on line ℓ_2
40 : $P_{268} = (9, 15, 1, 0)$ lies on line ℓ_1
41 : $P_{273} = (14, 15, 1, 0)$ lies on line ℓ_0
42 : $P_{291} = (1, 1, 0, 1)$ lies on line ℓ_4
43 : $P_{435} = (1, 10, 0, 1)$ lies on line ℓ_8
44 : $P_{451} = (1, 11, 0, 1)$ lies on line ℓ_6
45 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_4
46 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_3
47 : $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_5
48 : $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_7
49 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_3
50 : $P_{834} = (1, 3, 2, 1)$ lies on line ℓ_4
51 : $P_{866} = (1, 5, 2, 1)$ lies on line ℓ_8
52 : $P_{882} = (1, 6, 2, 1)$ lies on line ℓ_6
53 : $P_{993} = (0, 13, 2, 1)$ lies on line ℓ_5
54 : $P_{1025} = (0, 15, 2, 1)$ lies on line ℓ_7
55 : $P_{1074} = (1, 2, 3, 1)$ lies on line ℓ_4
56 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_3
57 : $P_{1105} = (0, 4, 3, 1)$ lies on line ℓ_7
58 : $P_{1153} = (0, 7, 3, 1)$ lies on line ℓ_5
59 : $P_{1234} = (1, 12, 3, 1)$ lies on line ℓ_6
60 : $P_{1266} = (1, 14, 3, 1)$ lies on line ℓ_8
61 : $P_{1345} = (0, 3, 4, 1)$ lies on line ℓ_5
62 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_3
63 : $P_{1378} = (1, 5, 4, 1)$ lies on line ℓ_4
64 : $P_{1409} = (0, 7, 4, 1)$ lies on line ℓ_7
65 : $P_{1426} = (1, 8, 4, 1)$ lies on line ℓ_6
66 : $P_{1506} = (1, 13, 4, 1)$ lies on line ℓ_8
67 : $P_{1586} = (1, 2, 5, 1)$ lies on line ℓ_6
68 : $P_{1618} = (1, 4, 5, 1)$ lies on line ℓ_4
69 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_3
70 : $P_{1650} = (1, 6, 5, 1)$ lies on line ℓ_8
71 : $P_{1697} = (0, 9, 5, 1)$ lies on line ℓ_5
72 : $P_{1745} = (0, 12, 5, 1)$ lies on line ℓ_7
73 : $P_{1842} = (1, 2, 6, 1)$ lies on line ℓ_8
74 : $P_{1890} = (1, 5, 6, 1)$ lies on line ℓ_6
75 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_3
76 : $P_{1922} = (1, 7, 6, 1)$ lies on line ℓ_4
77 : $P_{1937} = (0, 8, 6, 1)$ lies on line ℓ_7
78 : $P_{2033} = (0, 14, 6, 1)$ lies on line ℓ_5
79 : $P_{2113} = (0, 3, 7, 1)$ lies on line ℓ_7
80 : $P_{2129} = (0, 4, 7, 1)$ lies on line ℓ_5
81 : $P_{2162} = (1, 6, 7, 1)$ lies on line ℓ_4
82 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_3
83 : $P_{2210} = (1, 9, 7, 1)$ lies on line ℓ_8
84 : $P_{2306} = (1, 15, 7, 1)$ lies on line ℓ_6
85 : $P_{2386} = (1, 4, 8, 1)$ lies on line ℓ_8
86 : $P_{2417} = (0, 6, 8, 1)$ lies on line ℓ_5
87 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_3
88 : $P_{2466} = (1, 9, 8, 1)$ lies on line ℓ_4
89 : $P_{2530} = (1, 13, 8, 1)$ lies on line ℓ_6
90 : $P_{2545} = (0, 14, 8, 1)$ lies on line ℓ_7
91 : $P_{2657} = (0, 5, 9, 1)$ lies on line ℓ_7
92 : $P_{2690} = (1, 7, 9, 1)$ lies on line ℓ_6
93 : $P_{2706} = (1, 8, 9, 1)$ lies on line ℓ_4
94 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_3
95 : $P_{2769} = (0, 12, 9, 1)$ lies on line ℓ_5
96 : $P_{2818} = (1, 15, 9, 1)$ lies on line ℓ_8
97 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_6
98 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_7
99 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_3
100 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_5
101 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_8
102 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_5
103 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_7
104 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_3
105 : $P_{3394} = (1, 3, 12, 1)$ lies on line ℓ_8
106 : $P_{3425} = (0, 5, 12, 1)$ lies on line ℓ_5
107 : $P_{3489} = (0, 9, 12, 1)$ lies on line ℓ_7
108 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_3
109 : $P_{3554} = (1, 13, 12, 1)$ lies on line ℓ_4
110 : $P_{3570} = (1, 14, 12, 1)$ lies on line ℓ_6
111 : $P_{3633} = (0, 2, 13, 1)$ lies on line ℓ_7
112 : $P_{3666} = (1, 4, 13, 1)$ lies on line ℓ_6
113 : $P_{3730} = (1, 8, 13, 1)$ lies on line ℓ_8
114 : $P_{3794} = (1, 12, 13, 1)$ lies on line ℓ_4
115 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_3
116 : $P_{3841} = (0, 15, 13, 1)$ lies on line ℓ_5
117 : $P_{3906} = (1, 3, 14, 1)$ lies on line ℓ_6
118 : $P_{3953} = (0, 6, 14, 1)$ lies on line ℓ_7
119 : $P_{3985} = (0, 8, 14, 1)$ lies on line ℓ_5
120 : $P_{4050} = (1, 12, 14, 1)$ lies on line ℓ_8
121 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_3

122 : $P_{4098} = (1, 15, 14, 1)$ lies on line ℓ_4
123 : $P_{4145} = (0, 2, 15, 1)$ lies on line ℓ_5
124 : $P_{4226} = (1, 7, 15, 1)$ lies on line ℓ_8
125 : $P_{4258} = (1, 9, 15, 1)$ lies on line ℓ_6
126 : $P_{4321} = (0, 13, 15, 1)$ lies on line ℓ_7
127 : $P_{4338} = (1, 14, 15, 1)$ lies on line ℓ_4
128 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 182 points not on any line:
The points on the surface but not on lines are:

0 : $P_{284} = (10, 0, 0, 1)$
1 : $P_{285} = (11, 0, 0, 1)$
2 : $P_{581} = (4, 3, 1, 1)$
3 : $P_{618} = (9, 5, 1, 1)$
4 : $P_{671} = (14, 8, 1, 1)$
5 : $P_{695} = (6, 10, 1, 1)$
6 : $P_{696} = (7, 10, 1, 1)$
7 : $P_{717} = (12, 11, 1, 1)$
8 : $P_{718} = (13, 11, 1, 1)$
9 : $P_{771} = (2, 15, 1, 1)$
10 : $P_{846} = (13, 3, 2, 1)$
11 : $P_{851} = (2, 4, 2, 1)$
12 : $P_{875} = (10, 5, 2, 1)$
13 : $P_{887} = (6, 6, 2, 1)$
14 : $P_{906} = (9, 7, 2, 1)$
15 : $P_{917} = (4, 8, 2, 1)$
16 : $P_{963} = (2, 11, 2, 1)$
17 : $P_{982} = (5, 12, 2, 1)$
18 : $P_{988} = (11, 12, 2, 1)$
19 : $P_{992} = (15, 12, 2, 1)$
20 : $P_{1023} = (14, 14, 2, 1)$
21 : $P_{1027} = (2, 15, 2, 1)$
22 : $P_{1028} = (3, 15, 2, 1)$
23 : $P_{1061} = (4, 1, 3, 1)$
24 : $P_{1086} = (13, 2, 3, 1)$
25 : $P_{1109} = (4, 4, 3, 1)$
26 : $P_{1110} = (5, 4, 3, 1)$
27 : $P_{1125} = (4, 5, 3, 1)$
28 : $P_{1172} = (3, 8, 3, 1)$
29 : $P_{1177} = (8, 8, 3, 1)$
30 : $P_{1179} = (10, 8, 3, 1)$
31 : $P_{1199} = (14, 9, 3, 1)$
32 : $P_{1240} = (7, 12, 3, 1)$
33 : $P_{1258} = (9, 13, 3, 1)$
34 : $P_{1276} = (11, 14, 3, 1)$
35 : $P_{1283} = (2, 15, 3, 1)$
36 : $P_{1331} = (2, 2, 4, 1)$
37 : $P_{1349} = (4, 3, 4, 1)$
38 : $P_{1350} = (5, 3, 4, 1)$
39 : $P_{1384} = (7, 5, 4, 1)$
40 : $P_{1396} = (3, 6, 4, 1)$
41 : $P_{1401} = (8, 6, 4, 1)$
42 : $P_{1403} = (10, 6, 4, 1)$
43 : $P_{1436} = (11, 8, 4, 1)$
44 : $P_{1445} = (4, 9, 4, 1)$
45 : $P_{1461} = (4, 10, 4, 1)$
46 : $P_{1503} = (14, 12, 4, 1)$
47 : $P_{1518} = (13, 13, 4, 1)$
48 : $P_{1546} = (9, 15, 4, 1)$
49 : $P_{1578} = (9, 1, 5, 1)$
50 : $P_{1595} = (10, 2, 5, 1)$
51 : $P_{1605} = (4, 3, 5, 1)$
52 : $P_{1624} = (7, 4, 5, 1)$
53 : $P_{1661} = (12, 6, 5, 1)$
54 : $P_{1679} = (14, 7, 5, 1)$
55 : $P_{1690} = (9, 8, 5, 1)$
56 : $P_{1705} = (8, 9, 5, 1)$
57 : $P_{1706} = (9, 9, 5, 1)$
58 : $P_{1779} = (2, 14, 5, 1)$
59 : $P_{1798} = (5, 15, 5, 1)$
60 : $P_{1804} = (11, 15, 5, 1)$
61 : $P_{1808} = (15, 15, 5, 1)$
62 : $P_{1847} = (6, 2, 6, 1)$
63 : $P_{1876} = (3, 4, 6, 1)$
64 : $P_{1881} = (8, 4, 6, 1)$
65 : $P_{1883} = (10, 4, 6, 1)$
66 : $P_{1901} = (12, 5, 6, 1)$
67 : $P_{1919} = (14, 6, 6, 1)$
68 : $P_{1920} = (15, 6, 6, 1)$
69 : $P_{1932} = (11, 7, 6, 1)$
70 : $P_{1955} = (2, 9, 6, 1)$
71 : $P_{1999} = (14, 11, 6, 1)$
72 : $P_{2010} = (9, 12, 6, 1)$
73 : $P_{2031} = (14, 13, 6, 1)$
74 : $P_{2053} = (4, 15, 6, 1)$
75 : $P_{2106} = (9, 2, 7, 1)$
76 : $P_{2159} = (14, 5, 7, 1)$
77 : $P_{2172} = (11, 6, 7, 1)$
78 : $P_{2181} = (4, 7, 7, 1)$
79 : $P_{2182} = (5, 7, 7, 1)$

$80: P_{2216} = (7, 9, 7, 1)$
$81: P_{2245} = (4, 11, 7, 1)$
$82: P_{2261} = (4, 12, 7, 1)$
$83: P_{2275} = (2, 13, 7, 1)$
$84: P_{2292} = (3, 14, 7, 1)$
$85: P_{2297} = (8, 14, 7, 1)$
$86: P_{2299} = (10, 14, 7, 1)$
$87: P_{2318} = (13, 15, 7, 1)$
$88: P_{2351} = (14, 1, 8, 1)$
$89: P_{2357} = (4, 2, 8, 1)$
$90: P_{2372} = (3, 3, 8, 1)$
$91: P_{2377} = (8, 3, 8, 1)$
$92: P_{2379} = (10, 3, 8, 1)$
$93: P_{2396} = (11, 4, 8, 1)$
$94: P_{2410} = (9, 5, 8, 1)$
$95: P_{2477} = (12, 9, 8, 1)$
$96: P_{2515} = (2, 12, 8, 1)$
$97: P_{2535} = (6, 13, 8, 1)$
$98: P_{2559} = (14, 14, 8, 1)$
$99: P_{2560} = (15, 14, 8, 1)$
$100: P_{2575} = (14, 15, 8, 1)$
$101: P_{2639} = (14, 3, 9, 1)$
$102: P_{2645} = (4, 4, 9, 1)$
$103: P_{2665} = (8, 5, 9, 1)$
$104: P_{2666} = (9, 5, 9, 1)$
$105: P_{2675} = (2, 6, 9, 1)$
$106: P_{2696} = (7, 7, 9, 1)$
$107: P_{2717} = (12, 8, 9, 1)$
$108: P_{2762} = (9, 11, 9, 1)$
$109: P_{2790} = (5, 13, 9, 1)$
$110: P_{2796} = (11, 13, 9, 1)$
$111: P_{2800} = (15, 13, 9, 1)$
$112: P_{2810} = (9, 14, 9, 1)$
$113: P_{2827} = (10, 15, 9, 1)$
$114: P_{2855} = (6, 1, 10, 1)$
$115: P_{2856} = (7, 1, 10, 1)$
$116: P_{2901} = (4, 4, 10, 1)$
$117: P_{3005} = (12, 10, 10, 1)$
$118: P_{3006} = (13, 10, 10, 1)$
$119: P_{3034} = (9, 12, 10, 1)$
$120: P_{3043} = (2, 13, 10, 1)$
$121: P_{3071} = (14, 14, 10, 1)$
$122: P_{3117} = (12, 1, 11, 1)$
$123: P_{3118} = (13, 1, 11, 1)$
$124: P_{3123} = (2, 2, 11, 1)$
$125: P_{3199} = (14, 6, 11, 1)$
$126: P_{3205} = (4, 7, 11, 1)$
$127: P_{3242} = (9, 9, 11, 1)$
$128: P_{3271} = (6, 11, 11, 1)$
$129: P_{3272} = (7, 11, 11, 1)$
$130: P_{3382} = (5, 2, 12, 1)$
$131: P_{3388} = (11, 2, 12, 1)$
$132: P_{3392} = (15, 2, 12, 1)$
$133: P_{3400} = (7, 3, 12, 1)$
$134: P_{3423} = (14, 4, 12, 1)$
$135: P_{3450} = (9, 6, 12, 1)$
$136: P_{3461} = (4, 7, 12, 1)$
$137: P_{3475} = (2, 8, 12, 1)$
$138: P_{3514} = (9, 10, 12, 1)$
$139: P_{3545} = (8, 12, 12, 1)$
$140: P_{3546} = (9, 12, 12, 1)$
$141: P_{3563} = (10, 13, 12, 1)$
$142: P_{3581} = (12, 14, 12, 1)$
$143: P_{3658} = (9, 3, 13, 1)$
$144: P_{3678} = (13, 4, 13, 1)$
$145: P_{3711} = (14, 6, 13, 1)$
$146: P_{3715} = (2, 7, 13, 1)$
$147: P_{3735} = (6, 8, 13, 1)$
$148: P_{3750} = (5, 9, 13, 1)$
$149: P_{3756} = (11, 9, 13, 1)$
$150: P_{3760} = (15, 9, 13, 1)$
$151: P_{3763} = (2, 10, 13, 1)$
$152: P_{3803} = (10, 12, 13, 1)$
$153: P_{3811} = (2, 13, 13, 1)$
$154: P_{3812} = (3, 13, 13, 1)$
$155: P_{3829} = (4, 14, 13, 1)$
$156: P_{3903} = (14, 2, 14, 1)$
$157: P_{3916} = (11, 3, 14, 1)$
$158: P_{3939} = (2, 5, 14, 1)$
$159: P_{3972} = (3, 7, 14, 1)$
$160: P_{3977} = (8, 7, 14, 1)$
$161: P_{3979} = (10, 7, 14, 1)$
$162: P_{3999} = (14, 8, 14, 1)$
$163: P_{4000} = (15, 8, 14, 1)$
$164: P_{4010} = (9, 9, 14, 1)$
$165: P_{4031} = (14, 10, 14, 1)$
$166: P_{4061} = (12, 12, 14, 1)$
$167: P_{4069} = (4, 13, 14, 1)$
$168: P_{4103} = (6, 15, 14, 1)$
$169: P_{4131} = (2, 1, 15, 1)$
$170: P_{4147} = (2, 2, 15, 1)$
$171: P_{4148} = (3, 2, 15, 1)$
$172: P_{4163} = (2, 3, 15, 1)$
$173: P_{4186} = (9, 4, 15, 1)$
$174: P_{4198} = (5, 5, 15, 1)$
$175: P_{4204} = (11, 5, 15, 1)$
$176: P_{4208} = (15, 5, 15, 1)$
$177: P_{4213} = (4, 6, 15, 1)$
$178: P_{4238} = (13, 7, 15, 1)$
$179: P_{4255} = (14, 8, 15, 1)$
$180: P_{4267} = (10, 9, 15, 1)$
$181: P_{4343} = (6, 14, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 1 0 0 0 0
1	1 0 1 0 0 1 1 0 0
2	1 1 0 0 0 0 0 1 1
3	1 0 0 0 1 1 0 1 0
4	1 0 0 1 0 0 1 0 1
5	0 1 0 1 0 0 1 1 0
6	0 1 0 0 1 1 0 0 1
7	0 0 1 1 0 1 0 0 1
8	0 0 1 0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_{205}	P_{190}	P_{35}	P_{35}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_{205}	P_{36}	P_{179}	P_{179}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_8
in point	P_{190}	P_{36}	P_{195}	P_{195}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_{35}	P_{35}	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_8
in point	P_{35}	P_{35}	P_{3250}	P_{3010}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_{179}	P_3	P_{179}	P_3

Line 6 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_8
in point	P_{179}	P_{3250}	P_{179}	P_4

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_{195}	P_3	P_3	P_{195}

Line 8 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_7
in point	P_{195}	P_{3010}	P_4	P_{195}

The surface has 321 points:
The points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_{14} = (10, 1, 0, 0)$
4 : $P_{15} = (11, 1, 0, 0)$
5 : $P_{20} = (1, 0, 1, 0)$
6 : $P_{29} = (10, 0, 1, 0)$
7 : $P_{30} = (11, 0, 1, 0)$
8 : $P_{35} = (0, 1, 1, 0)$
9 : $P_{36} = (1, 1, 1, 0)$
10 : $P_{54} = (3, 2, 1, 0)$
11 : $P_{56} = (5, 2, 1, 0)$
12 : $P_{57} = (6, 2, 1, 0)$
13 : $P_{69} = (2, 3, 1, 0)$
14 : $P_{79} = (12, 3, 1, 0)$
15 : $P_{81} = (14, 3, 1, 0)$
16 : $P_{88} = (5, 4, 1, 0)$
17 : $P_{91} = (8, 4, 1, 0)$
18 : $P_{96} = (13, 4, 1, 0)$
19 : $P_{101} = (2, 5, 1, 0)$
20 : $P_{103} = (4, 5, 1, 0)$

21 : $P_{105} = (6, 5, 1, 0)$
22 : $P_{117} = (2, 6, 1, 0)$
23 : $P_{120} = (5, 6, 1, 0)$
24 : $P_{122} = (7, 6, 1, 0)$
25 : $P_{137} = (6, 7, 1, 0)$
26 : $P_{140} = (9, 7, 1, 0)$
27 : $P_{146} = (15, 7, 1, 0)$
28 : $P_{151} = (4, 8, 1, 0)$
29 : $P_{156} = (9, 8, 1, 0)$
30 : $P_{160} = (13, 8, 1, 0)$
31 : $P_{170} = (7, 9, 1, 0)$
32 : $P_{171} = (8, 9, 1, 0)$
33 : $P_{178} = (15, 9, 1, 0)$
34 : $P_{179} = (0, 10, 1, 0)$
35 : $P_{190} = (11, 10, 1, 0)$
36 : $P_{195} = (0, 11, 1, 0)$
37 : $P_{205} = (10, 11, 1, 0)$
38 : $P_{214} = (3, 12, 1, 0)$
39 : $P_{224} = (13, 12, 1, 0)$
40 : $P_{225} = (14, 12, 1, 0)$
41 : $P_{231} = (4, 13, 1, 0)$
42 : $P_{235} = (8, 13, 1, 0)$
43 : $P_{239} = (12, 13, 1, 0)$
44 : $P_{246} = (3, 14, 1, 0)$
45 : $P_{255} = (12, 14, 1, 0)$
46 : $P_{258} = (15, 14, 1, 0)$
47 : $P_{266} = (7, 15, 1, 0)$
48 : $P_{268} = (9, 15, 1, 0)$
49 : $P_{273} = (14, 15, 1, 0)$
50 : $P_{284} = (10, 0, 0, 1)$
51 : $P_{285} = (11, 0, 0, 1)$
52 : $P_{291} = (1, 1, 0, 1)$
53 : $P_{435} = (1, 10, 0, 1)$
54 : $P_{451} = (1, 11, 0, 1)$
55 : $P_{531} = (1, 0, 1, 1)$
56 : $P_{546} = (0, 1, 1, 1)$
57 : $P_{581} = (4, 3, 1, 1)$
58 : $P_{618} = (9, 5, 1, 1)$
59 : $P_{671} = (14, 8, 1, 1)$
60 : $P_{689} = (0, 10, 1, 1)$
61 : $P_{695} = (6, 10, 1, 1)$
62 : $P_{696} = (7, 10, 1, 1)$
63 : $P_{705} = (0, 11, 1, 1)$
64 : $P_{717} = (12, 11, 1, 1)$
65 : $P_{718} = (13, 11, 1, 1)$
66 : $P_{771} = (2, 15, 1, 1)$
67 : $P_{817} = (0, 2, 2, 1)$
68 : $P_{834} = (1, 3, 2, 1)$
69 : $P_{846} = (13, 3, 2, 1)$
70 : $P_{851} = (2, 4, 2, 1)$
71 : $P_{866} = (1, 5, 2, 1)$
72 : $P_{875} = (10, 5, 2, 1)$
73 : $P_{882} = (1, 6, 2, 1)$
74 : $P_{887} = (6, 6, 2, 1)$
75 : $P_{906} = (9, 7, 2, 1)$
76 : $P_{917} = (4, 8, 2, 1)$
77 : $P_{963} = (2, 11, 2, 1)$
78 : $P_{982} = (5, 12, 2, 1)$
79 : $P_{988} = (11, 12, 2, 1)$
80 : $P_{992} = (15, 12, 2, 1)$
81 : $P_{993} = (0, 13, 2, 1)$
82 : $P_{1023} = (14, 14, 2, 1)$
83 : $P_{1025} = (0, 15, 2, 1)$
84 : $P_{1027} = (2, 15, 2, 1)$
85 : $P_{1028} = (3, 15, 2, 1)$
86 : $P_{1061} = (4, 1, 3, 1)$
87 : $P_{1074} = (1, 2, 3, 1)$
88 : $P_{1086} = (13, 2, 3, 1)$
89 : $P_{1089} = (0, 3, 3, 1)$
90 : $P_{1105} = (0, 4, 3, 1)$
91 : $P_{1109} = (4, 4, 3, 1)$
92 : $P_{1110} = (5, 4, 3, 1)$
93 : $P_{1125} = (4, 5, 3, 1)$
94 : $P_{1153} = (0, 7, 3, 1)$
95 : $P_{1172} = (3, 8, 3, 1)$
96 : $P_{1177} = (8, 8, 3, 1)$
97 : $P_{1179} = (10, 8, 3, 1)$
98 : $P_{1199} = (14, 9, 3, 1)$
99 : $P_{1234} = (1, 12, 3, 1)$
100 : $P_{1240} = (7, 12, 3, 1)$
101 : $P_{1258} = (9, 13, 3, 1)$
102 : $P_{1266} = (1, 14, 3, 1)$
103 : $P_{1276} = (11, 14, 3, 1)$
104 : $P_{1283} = (2, 15, 3, 1)$
105 : $P_{1331} = (2, 2, 4, 1)$
106 : $P_{1345} = (0, 3, 4, 1)$
107 : $P_{1349} = (4, 3, 4, 1)$
108 : $P_{1350} = (5, 3, 4, 1)$
109 : $P_{1361} = (0, 4, 4, 1)$
110 : $P_{1378} = (1, 5, 4, 1)$
111 : $P_{1384} = (7, 5, 4, 1)$
112 : $P_{1396} = (3, 6, 4, 1)$
113 : $P_{1401} = (8, 6, 4, 1)$
114 : $P_{1403} = (10, 6, 4, 1)$
115 : $P_{1409} = (0, 7, 4, 1)$
116 : $P_{1426} = (1, 8, 4, 1)$
117 : $P_{1436} = (11, 8, 4, 1)$
118 : $P_{1445} = (4, 9, 4, 1)$
119 : $P_{1461} = (4, 10, 4, 1)$
120 : $P_{1503} = (14, 12, 4, 1)$
121 : $P_{1506} = (1, 13, 4, 1)$
122 : $P_{1518} = (13, 13, 4, 1)$
123 : $P_{1546} = (9, 15, 4, 1)$
124 : $P_{1578} = (9, 1, 5, 1)$
125 : $P_{1586} = (1, 2, 5, 1)$
126 : $P_{1595} = (10, 2, 5, 1)$
127 : $P_{1605} = (4, 3, 5, 1)$
128 : $P_{1618} = (1, 4, 5, 1)$
129 : $P_{1624} = (7, 4, 5, 1)$
130 : $P_{1633} = (0, 5, 5, 1)$
131 : $P_{1650} = (1, 6, 5, 1)$
132 : $P_{1661} = (12, 6, 5, 1)$
133 : $P_{1679} = (14, 7, 5, 1)$
134 : $P_{1690} = (9, 8, 5, 1)$
135 : $P_{1697} = (0, 9, 5, 1)$
136 : $P_{1705} = (8, 9, 5, 1)$
137 : $P_{1706} = (9, 9, 5, 1)$
138 : $P_{1745} = (0, 12, 5, 1)$
139 : $P_{1779} = (2, 14, 5, 1)$
140 : $P_{1798} = (5, 15, 5, 1)$
141 : $P_{1804} = (11, 15, 5, 1)$
142 : $P_{1808} = (15, 15, 5, 1)$
143 : $P_{1842} = (1, 2, 6, 1)$
144 : $P_{1847} = (6, 2, 6, 1)$
145 : $P_{1876} = (3, 4, 6, 1)$
146 : $P_{1881} = (8, 4, 6, 1)$
147 : $P_{1883} = (10, 4, 6, 1)$
148 : $P_{1890} = (1, 5, 6, 1)$
149 : $P_{1901} = (12, 5, 6, 1)$
150 : $P_{1905} = (0, 6, 6, 1)$
151 : $P_{1919} = (14, 6, 6, 1)$
152 : $P_{1920} = (15, 6, 6, 1)$
153 : $P_{1922} = (1, 7, 6, 1)$
154 : $P_{1932} = (11, 7, 6, 1)$
155 : $P_{1937} = (0, 8, 6, 1)$
156 : $P_{1955} = (2, 9, 6, 1)$
157 : $P_{1999} = (14, 11, 6, 1)$
158 : $P_{2010} = (9, 12, 6, 1)$
159 : $P_{2031} = (14, 13, 6, 1)$
160 : $P_{2033} = (0, 14, 6, 1)$
161 : $P_{2053} = (4, 15, 6, 1)$
162 : $P_{2106} = (9, 2, 7, 1)$
163 : $P_{2113} = (0, 3, 7, 1)$
164 : $P_{2129} = (0, 4, 7, 1)$
165 : $P_{2159} = (14, 5, 7, 1)$
166 : $P_{2162} = (1, 6, 7, 1)$
167 : $P_{2172} = (11, 6, 7, 1)$
168 : $P_{2177} = (0, 7, 7, 1)$
169 : $P_{2181} = (4, 7, 7, 1)$
170 : $P_{2182} = (5, 7, 7, 1)$
171 : $P_{2210} = (1, 9, 7, 1)$
172 : $P_{2216} = (7, 9, 7, 1)$
173 : $P_{2245} = (4, 11, 7, 1)$
174 : $P_{2261} = (4, 12, 7, 1)$
175 : $P_{2275} = (2, 13, 7, 1)$
176 : $P_{2292} = (3, 14, 7, 1)$
177 : $P_{2297} = (8, 14, 7, 1)$
178 : $P_{2299} = (10, 14, 7, 1)$
179 : $P_{2306} = (1, 15, 7, 1)$
180 : $P_{2318} = (13, 15, 7, 1)$
181 : $P_{2351} = (14, 1, 8, 1)$
182 : $P_{2357} = (4, 2, 8, 1)$

183 : $P_{2372} = (3, 3, 8, 1)$
184 : $P_{2377} = (8, 3, 8, 1)$
185 : $P_{2379} = (10, 3, 8, 1)$
186 : $P_{2386} = (1, 4, 8, 1)$
187 : $P_{2396} = (11, 4, 8, 1)$
188 : $P_{2410} = (9, 5, 8, 1)$
189 : $P_{2417} = (0, 6, 8, 1)$
190 : $P_{2449} = (0, 8, 8, 1)$
191 : $P_{2466} = (1, 9, 8, 1)$
192 : $P_{2477} = (12, 9, 8, 1)$
193 : $P_{2515} = (2, 12, 8, 1)$
194 : $P_{2530} = (1, 13, 8, 1)$
195 : $P_{2535} = (6, 13, 8, 1)$
196 : $P_{2545} = (0, 14, 8, 1)$
197 : $P_{2559} = (14, 14, 8, 1)$
198 : $P_{2560} = (15, 14, 8, 1)$
199 : $P_{2575} = (14, 15, 8, 1)$
200 : $P_{2639} = (14, 3, 9, 1)$
201 : $P_{2645} = (4, 4, 9, 1)$
202 : $P_{2657} = (0, 5, 9, 1)$
203 : $P_{2665} = (8, 5, 9, 1)$
204 : $P_{2666} = (9, 5, 9, 1)$
205 : $P_{2675} = (2, 6, 9, 1)$
206 : $P_{2690} = (1, 7, 9, 1)$
207 : $P_{2696} = (7, 7, 9, 1)$
208 : $P_{2706} = (1, 8, 9, 1)$
209 : $P_{2717} = (12, 8, 9, 1)$
210 : $P_{2721} = (0, 9, 9, 1)$
211 : $P_{2762} = (9, 11, 9, 1)$
212 : $P_{2769} = (0, 12, 9, 1)$
213 : $P_{2790} = (5, 13, 9, 1)$
214 : $P_{2796} = (11, 13, 9, 1)$
215 : $P_{2800} = (15, 13, 9, 1)$
216 : $P_{2810} = (9, 14, 9, 1)$
217 : $P_{2818} = (1, 15, 9, 1)$
218 : $P_{2827} = (10, 15, 9, 1)$
219 : $P_{2834} = (1, 0, 10, 1)$
220 : $P_{2849} = (0, 1, 10, 1)$
221 : $P_{2855} = (6, 1, 10, 1)$
222 : $P_{2856} = (7, 1, 10, 1)$
223 : $P_{2901} = (4, 4, 10, 1)$
224 : $P_{2993} = (0, 10, 10, 1)$
225 : $P_{3005} = (12, 10, 10, 1)$
226 : $P_{3006} = (13, 10, 10, 1)$
227 : $P_{3009} = (0, 11, 10, 1)$
228 : $P_{3010} = (1, 11, 10, 1)$
229 : $P_{3034} = (9, 12, 10, 1)$
230 : $P_{3043} = (2, 13, 10, 1)$
231 : $P_{3071} = (14, 14, 10, 1)$
232 : $P_{3090} = (1, 0, 11, 1)$
233 : $P_{3105} = (0, 1, 11, 1)$
234 : $P_{3117} = (12, 1, 11, 1)$
235 : $P_{3118} = (13, 1, 11, 1)$
236 : $P_{3123} = (2, 2, 11, 1)$
237 : $P_{3199} = (14, 6, 11, 1)$
238 : $P_{3205} = (4, 7, 11, 1)$
239 : $P_{3242} = (9, 9, 11, 1)$
240 : $P_{3249} = (0, 10, 11, 1)$
241 : $P_{3250} = (1, 10, 11, 1)$
242 : $P_{3265} = (0, 11, 11, 1)$
243 : $P_{3271} = (6, 11, 11, 1)$
244 : $P_{3272} = (7, 11, 11, 1)$
245 : $P_{3382} = (5, 2, 12, 1)$
246 : $P_{3388} = (11, 2, 12, 1)$
247 : $P_{3392} = (15, 2, 12, 1)$
248 : $P_{3394} = (1, 3, 12, 1)$
249 : $P_{3400} = (7, 3, 12, 1)$
250 : $P_{3423} = (14, 4, 12, 1)$
251 : $P_{3425} = (0, 5, 12, 1)$
252 : $P_{3450} = (9, 6, 12, 1)$
253 : $P_{3461} = (4, 7, 12, 1)$
254 : $P_{3475} = (2, 8, 12, 1)$
255 : $P_{3489} = (0, 9, 12, 1)$
256 : $P_{3514} = (9, 10, 12, 1)$
257 : $P_{3537} = (0, 12, 12, 1)$
258 : $P_{3545} = (8, 12, 12, 1)$
259 : $P_{3546} = (9, 12, 12, 1)$
260 : $P_{3554} = (1, 13, 12, 1)$
261 : $P_{3563} = (10, 13, 12, 1)$
262 : $P_{3570} = (1, 14, 12, 1)$
263 : $P_{3581} = (12, 14, 12, 1)$
264 : $P_{3633} = (0, 2, 13, 1)$
265 : $P_{3658} = (9, 3, 13, 1)$
266 : $P_{3666} = (1, 4, 13, 1)$
267 : $P_{3678} = (13, 4, 13, 1)$
268 : $P_{3711} = (14, 6, 13, 1)$
269 : $P_{3715} = (2, 7, 13, 1)$
270 : $P_{3730} = (1, 8, 13, 1)$
271 : $P_{3735} = (6, 8, 13, 1)$
272 : $P_{3750} = (5, 9, 13, 1)$
273 : $P_{3756} = (11, 9, 13, 1)$
274 : $P_{3760} = (15, 9, 13, 1)$
275 : $P_{3763} = (2, 10, 13, 1)$
276 : $P_{3794} = (1, 12, 13, 1)$
277 : $P_{3803} = (10, 12, 13, 1)$
278 : $P_{3809} = (0, 13, 13, 1)$
279 : $P_{3811} = (2, 13, 13, 1)$
280 : $P_{3812} = (3, 13, 13, 1)$
281 : $P_{3829} = (4, 14, 13, 1)$
282 : $P_{3841} = (0, 15, 13, 1)$
283 : $P_{3903} = (14, 2, 14, 1)$
284 : $P_{3906} = (1, 3, 14, 1)$
285 : $P_{3916} = (11, 3, 14, 1)$
286 : $P_{3939} = (2, 5, 14, 1)$
287 : $P_{3953} = (0, 6, 14, 1)$
288 : $P_{3972} = (3, 7, 14, 1)$
289 : $P_{3977} = (8, 7, 14, 1)$
290 : $P_{3979} = (10, 7, 14, 1)$
291 : $P_{3985} = (0, 8, 14, 1)$
292 : $P_{3999} = (14, 8, 14, 1)$
293 : $P_{4000} = (15, 8, 14, 1)$
294 : $P_{4010} = (9, 9, 14, 1)$
295 : $P_{4031} = (14, 10, 14, 1)$
296 : $P_{4050} = (1, 12, 14, 1)$
297 : $P_{4061} = (12, 12, 14, 1)$
298 : $P_{4069} = (4, 13, 14, 1)$
299 : $P_{4081} = (0, 14, 14, 1)$
300 : $P_{4098} = (1, 15, 14, 1)$
301 : $P_{4103} = (6, 15, 14, 1)$
302 : $P_{4131} = (2, 1, 15, 1)$
303 : $P_{4145} = (0, 2, 15, 1)$
304 : $P_{4147} = (2, 2, 15, 1)$
305 : $P_{4148} = (3, 2, 15, 1)$
306 : $P_{4163} = (2, 3, 15, 1)$
307 : $P_{4186} = (9, 4, 15, 1)$
308 : $P_{4198} = (5, 5, 15, 1)$
309 : $P_{4204} = (11, 5, 15, 1)$
310 : $P_{4208} = (15, 5, 15, 1)$
311 : $P_{4213} = (4, 6, 15, 1)$
312 : $P_{4226} = (1, 7, 15, 1)$
313 : $P_{4238} = (13, 7, 15, 1)$
314 : $P_{4255} = (14, 8, 15, 1)$
315 : $P_{4258} = (1, 9, 15, 1)$
316 : $P_{4267} = (10, 9, 15, 1)$
317 : $P_{4321} = (0, 13, 15, 1)$
318 : $P_{4338} = (1, 14, 15, 1)$
319 : $P_{4343} = (6, 14, 15, 1)$
320 : $P_{4353} = (0, 15, 15, 1)$