Rank-74499 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is 1090785349

General information

Number of lines	7
Number of points	4289
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	438
Number of points off lines	3844
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 ⁷
Type of lines on points	$4, 3, 2^5, 1^{438}, 0^{3844}$

Singular Points

The surface has 2 singular points:

$$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0) 1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_{2}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{270528}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{4353}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{278529}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_{1}$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{274561} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{274561} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{270592}$$

Rank of lines: (0, 4096, 8257, 65, 17043457, 17047616, 274561) Rank of points on Klein quadric: (0, 2, 270528, 4353, 278529, 1, 270592)

Eckardt Points

The surface has 1 Eckardt points: $0: P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0).$

Double Points

The surface has 5 Double points: The double points on the surface are:

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_4$$

$$P_{8322} = (0, 1, 1, 1) = \ell_3 \cap \ell_4$$

$$P_4 = (1, 1, 1, 1) = \ell_3 \cap \ell_6$$

$$P_{8258} = (0, 0, 1, 1) = \ell_4 \cap \ell_5$$

Single Points

The surface has 438 single points: The single points on the surface are:

 $0: P_3 = (0,0,0,1)$ lies on line ℓ_5 13: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 14: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 1: $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 2: $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0 15: $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0 3: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 16: $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0 4: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 17: $P_{22} = (18, 1, 0, 0)$ lies on line ℓ_0 18: $P_{23} = (19, 1, 0, 0)$ lies on line ℓ_0 5: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 6: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 19: $P_{24} = (20, 1, 0, 0)$ lies on line ℓ_0 7: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 20: $P_{25} = (21, 1, 0, 0)$ lies on line ℓ_0 8: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 21: $P_{26} = (22, 1, 0, 0)$ lies on line ℓ_0 9: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 $22: P_{27} = (23, 1, 0, 0)$ lies on line ℓ_0 10: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 23: $P_{28} = (24, 1, 0, 0)$ lies on line ℓ_0 11 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0 24 : $P_{29} = (25,1,0,0)$ lies on line ℓ_0 12 : $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 25: $P_{30} = (26, 1, 0, 0)$ lies on line ℓ_0

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26: P_{31} = (27, 1, 0, 0) lies on line \ell_0
                                                                      80: P_{85} = (18, 0, 1, 0) lies on line \ell_1
27: P_{32} = (28, 1, 0, 0) lies on line \ell_0
                                                                      81: P_{86} = (19, 0, 1, 0) lies on line \ell_1
28: P_{33} = (29, 1, 0, 0) lies on line \ell_0
                                                                      82: P_{87} = (20, 0, 1, 0) lies on line \ell_1
29: P_{34} = (30, 1, 0, 0) lies on line \ell_0
                                                                      83: P_{88} = (21, 0, 1, 0) lies on line \ell_1
30: P_{35} = (31, 1, 0, 0) lies on line \ell_0
                                                                      84: P_{89} = (22, 0, 1, 0) lies on line \ell_1
31: P_{36} = (32, 1, 0, 0) lies on line \ell_0
                                                                      85: P_{90} = (23, 0, 1, 0) lies on line \ell_1
32 : P_{37} = (33,1,0,0) lies on line \ell_0
                                                                      86: P_{91} = (24, 0, 1, 0) lies on line \ell_1
33: P_{38} = (34, 1, 0, 0) lies on line \ell_0
                                                                      87: P_{92} = (25, 0, 1, 0) lies on line \ell_1
34: P_{39} = (35, 1, 0, 0) lies on line \ell_0
                                                                      88: P_{93} = (26, 0, 1, 0) lies on line \ell_1
35: P_{40} = (36, 1, 0, 0) lies on line \ell_0
                                                                      89: P_{94} = (27, 0, 1, 0) lies on line \ell_1
36: P_{41} = (37, 1, 0, 0) lies on line \ell_0
                                                                      90: P_{95} = (28, 0, 1, 0) lies on line \ell_1
37: P_{42} = (38, 1, 0, 0) lies on line \ell_0
                                                                      91: P_{96} = (29, 0, 1, 0) lies on line \ell_1
38: P_{43} = (39, 1, 0, 0) lies on line \ell_0
                                                                      92: P_{97} = (30, 0, 1, 0) lies on line \ell_1
39: P_{44} = (40, 1, 0, 0) lies on line \ell_0
                                                                      93: P_{98} = (31, 0, 1, 0) lies on line \ell_1
40: P_{45} = (41, 1, 0, 0) lies on line \ell_0
                                                                      94: P_{99} = (32, 0, 1, 0) lies on line \ell_1
41: P_{46} = (42, 1, 0, 0) lies on line \ell_0
                                                                      95: P_{100} = (33, 0, 1, 0) lies on line \ell_1
                                                                      96: P_{101} = (34, 0, 1, 0) lies on line \ell_1
42: P_{47} = (43, 1, 0, 0) lies on line \ell_0
43: P_{48} = (44, 1, 0, 0) lies on line \ell_0
                                                                      97: P_{102} = (35, 0, 1, 0) lies on line \ell_1
44: P_{49} = (45, 1, 0, 0) lies on line \ell_0
                                                                      98: P_{103} = (36, 0, 1, 0) lies on line \ell_1
45: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                      99: P_{104} = (37, 0, 1, 0) lies on line \ell_1
46: P_{51} = (47, 1, 0, 0) lies on line \ell_0
                                                                      100: P_{105} = (38, 0, 1, 0) lies on line \ell_1
47: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                      101: P_{106} = (39, 0, 1, 0) lies on line \ell_1
48: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                      102: P_{107} = (40, 0, 1, 0) lies on line \ell_1
49: P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                      103: P_{108} = (41, 0, 1, 0) lies on line \ell_1
50: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                      104: P_{109} = (42, 0, 1, 0) lies on line \ell_1
51: P_{56} = (52, 1, 0, 0) lies on line \ell_0
                                                                      105: P_{110} = (43, 0, 1, 0) lies on line \ell_1
52: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                      106: P_{111} = (44, 0, 1, 0) lies on line \ell_1
53: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                      107: P_{112} = (45, 0, 1, 0) lies on line \ell_1
54: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                      108: P_{113} = (46, 0, 1, 0) lies on line \ell_1
55: P_{60} = (56, 1, 0, 0) lies on line \ell_0
                                                                      109: P_{114} = (47, 0, 1, 0) lies on line \ell_1
56: P_{61} = (57, 1, 0, 0) lies on line \ell_0
                                                                      110: P_{115} = (48, 0, 1, 0) lies on line \ell_1
57: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                      111: P_{116} = (49, 0, 1, 0) lies on line \ell_1
58: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                      112: P_{117} = (50, 0, 1, 0) lies on line \ell_1
59 : P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                      113: P_{118} = (51, 0, 1, 0) lies on line \ell_1
60: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                      114: P_{119} = (52, 0, 1, 0) lies on line \ell_1
                                                                      115 : P_{120} = (53, 0, 1, 0) lies on line \ell_1
61: P_{66} = (62, 1, 0, 0) lies on line \ell_0
62: P_{67} = (63, 1, 0, 0) lies on line \ell_0
                                                                      116: P_{121} = (54, 0, 1, 0) lies on line \ell_1
63: P_{68} = (1, 0, 1, 0) lies on line \ell_1
                                                                      117: P_{122} = (55, 0, 1, 0) lies on line \ell_1
64: P_{69} = (2, 0, 1, 0) lies on line \ell_1
                                                                      118: P_{123} = (56, 0, 1, 0) lies on line \ell_1
65: P_{70} = (3,0,1,0) lies on line \ell_1
                                                                      119: P_{124} = (57, 0, 1, 0) lies on line \ell_1
66: P_{71} = (4, 0, 1, 0) lies on line \ell_1
                                                                      120: P_{125} = (58, 0, 1, 0) lies on line \ell_1
67: P_{72} = (5, 0, 1, 0) lies on line \ell_1
                                                                      121: P_{126} = (59, 0, 1, 0) lies on line \ell_1
68: P_{73} = (6,0,1,0) lies on line \ell_1
                                                                      122: P_{127} = (60, 0, 1, 0) lies on line \ell_1
69: P_{74} = (7, 0, 1, 0) lies on line \ell_1
                                                                      123: P_{128} = (61, 0, 1, 0) lies on line \ell_1
70: P_{75} = (8, 0, 1, 0) lies on line \ell_1
                                                                      124: P_{129} = (62, 0, 1, 0) lies on line \ell_1
71: P_{76} = (9, 0, 1, 0) lies on line \ell_1
                                                                      125: P_{130} = (63, 0, 1, 0) lies on line \ell_1
72: P_{77} = (10, 0, 1, 0) lies on line \ell_1
                                                                      126: P_{132} = (1, 1, 1, 0) lies on line \ell_2
73: P_{78} = (11, 0, 1, 0) lies on line \ell_1
                                                                      127: P_{197} = (2, 2, 1, 0) lies on line \ell_2
74: P_{79} = (12, 0, 1, 0) lies on line \ell_1
                                                                      128: P_{262} = (3, 3, 1, 0) lies on line \ell_2
75: P_{80} = (13, 0, 1, 0) lies on line \ell_1
                                                                      129: P_{327} = (4, 4, 1, 0) lies on line \ell_2
                                                                      130 : P_{392} = (5, 5, 1, 0) lies on line \ell_2
76: P_{81} = (14, 0, 1, 0) lies on line \ell_1
77: P_{82} = (15, 0, 1, 0) lies on line \ell_1
                                                                      131: P_{457} = (6, 6, 1, 0) lies on line \ell_2
78 : P_{83} = (16, 0, 1, 0) lies on line \ell_1
                                                                      132: P_{522} = (7, 7, 1, 0) lies on line \ell_2
79 : P_{84} = (17, 0, 1, 0) lies on line \ell_1
                                                                      133: P_{587} = (8, 8, 1, 0) lies on line \ell_2
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134: P_{652} = (9, 9, 1, 0) lies on line \ell_2
                                                                     188: P_{4162} = (63, 63, 1, 0) lies on line \ell_2
                                                                     189 : P_{4227} = (1, 1, 0, 1) lies on line \ell_6
135: P_{717} = (10, 10, 1, 0) lies on line \ell_2
136: P_{782} = (11, 11, 1, 0) lies on line \ell_2
                                                                     190: P_{8323} = (2, 1, 1, 1) lies on line \ell_3
137: P_{847} = (12, 12, 1, 0) lies on line \ell_2
                                                                     191 : P_{8324} = (3, 1, 1, 1) lies on line \ell_3
138: P_{912} = (13, 13, 1, 0) lies on line \ell_2
                                                                     192 : P_{8325} = (4, 1, 1, 1) lies on line \ell_3
139: P_{977} = (14, 14, 1, 0) lies on line \ell_2
                                                                     193: P_{8326} = (5, 1, 1, 1) lies on line \ell_3
140: P_{1042} = (15, 15, 1, 0) lies on line \ell_2
                                                                     194: P_{8327} = (6, 1, 1, 1) lies on line \ell_3
141: P_{1107} = (16, 16, 1, 0) lies on line \ell_2
                                                                     195: P_{8328} = (7, 1, 1, 1) lies on line \ell_3
142 : P_{1172} = (17, 17, 1, 0) lies on line \ell_2
                                                                     196: P_{8329} = (8, 1, 1, 1) lies on line \ell_3
143: P_{1237} = (18, 18, 1, 0) lies on line \ell_2
                                                                     197: P_{8330} = (9, 1, 1, 1) lies on line \ell_3
144: P_{1302} = (19, 19, 1, 0) lies on line \ell_2
                                                                     198: P_{8331} = (10, 1, 1, 1) lies on line \ell_3
145: P_{1367} = (20, 20, 1, 0) lies on line \ell_2
                                                                     199: P_{8332} = (11, 1, 1, 1) lies on line \ell_3
146: P_{1432} = (21, 21, 1, 0) lies on line \ell_2
                                                                     200: P_{8333} = (12, 1, 1, 1) lies on line \ell_3
147: P_{1497} = (22, 22, 1, 0) lies on line \ell_2
                                                                     201: P_{8334} = (13, 1, 1, 1) lies on line \ell_3
148: P_{1562} = (23, 23, 1, 0) lies on line \ell_2
                                                                     202: P_{8335} = (14, 1, 1, 1) lies on line \ell_3
149: P_{1627} = (24, 24, 1, 0) lies on line \ell_2
                                                                     203: P_{8336} = (15, 1, 1, 1) lies on line \ell_3
150: P_{1692} = (25, 25, 1, 0) lies on line \ell_2
                                                                     204: P_{8337} = (16, 1, 1, 1) lies on line \ell_3
151: P_{1757} = (26, 26, 1, 0) lies on line \ell_2
                                                                     205: P_{8338} = (17, 1, 1, 1) lies on line \ell_3
152: P_{1822} = (27, 27, 1, 0) lies on line \ell_2
                                                                     206: P_{8339} = (18, 1, 1, 1) lies on line \ell_3
153: P_{1887} = (28, 28, 1, 0) lies on line \ell_2
                                                                     207: P_{8340} = (19, 1, 1, 1) lies on line \ell_3
154: P_{1952} = (29, 29, 1, 0) lies on line \ell_2
                                                                     208: P_{8341} = (20, 1, 1, 1) lies on line \ell_3
155: P_{2017} = (30, 30, 1, 0) lies on line \ell_2
                                                                     209: P_{8342} = (21, 1, 1, 1) lies on line \ell_3
                                                                     210: P_{8343} = (22, 1, 1, 1) lies on line \ell_3
156: P_{2082} = (31, 31, 1, 0) lies on line \ell_2
157: P_{2147} = (32, 32, 1, 0) lies on line \ell_2
                                                                     211: P_{8344} = (23, 1, 1, 1) lies on line \ell_3
158: P_{2212} = (33, 33, 1, 0) lies on line \ell_2
                                                                     212: P_{8345} = (24, 1, 1, 1) lies on line \ell_3
159: P_{2277} = (34, 34, 1, 0) lies on line \ell_2
                                                                     213: P_{8346} = (25, 1, 1, 1) lies on line \ell_3
160: P_{2342} = (35, 35, 1, 0) lies on line \ell_2
                                                                     214: P_{8347} = (26, 1, 1, 1) lies on line \ell_3
161: P_{2407} = (36, 36, 1, 0) lies on line \ell_2
                                                                     215: P_{8348} = (27, 1, 1, 1) lies on line \ell_3
162: P_{2472} = (37, 37, 1, 0) lies on line \ell_2
                                                                     216: P_{8349} = (28, 1, 1, 1) lies on line \ell_3
163: P_{2537} = (38, 38, 1, 0) lies on line \ell_2
                                                                     217: P_{8350} = (29, 1, 1, 1) lies on line \ell_3
164: P_{2602} = (39, 39, 1, 0) lies on line \ell_2
                                                                     218: P_{8351} = (30, 1, 1, 1) lies on line \ell_3
165: P_{2667} = (40, 40, 1, 0) lies on line \ell_2
                                                                     219: P_{8352} = (31, 1, 1, 1) lies on line \ell_3
166: P_{2732} = (41, 41, 1, 0) lies on line \ell_2
                                                                     220: P_{8353} = (32, 1, 1, 1) lies on line \ell_3
167: P_{2797} = (42, 42, 1, 0) lies on line \ell_2
                                                                     221: P_{8354} = (33, 1, 1, 1) lies on line \ell_3
168: P_{2862} = (43, 43, 1, 0) lies on line \ell_2
                                                                     222: P_{8355} = (34, 1, 1, 1) lies on line \ell_3
169: P_{2927} = (44, 44, 1, 0) lies on line \ell_2
                                                                     223: P_{8356} = (35, 1, 1, 1) lies on line \ell_3
170: P_{2992} = (45, 45, 1, 0) lies on line \ell_2
                                                                     224: P_{8357} = (36, 1, 1, 1) lies on line \ell_3
171: P_{3057} = (46, 46, 1, 0) lies on line \ell_2
                                                                     225: P_{8358} = (37, 1, 1, 1) lies on line \ell_3
172: P_{3122} = (47, 47, 1, 0) lies on line \ell_2
                                                                     226: P_{8359} = (38, 1, 1, 1) lies on line \ell_3
173: P_{3187} = (48, 48, 1, 0) lies on line \ell_2
                                                                     227: P_{8360} = (39, 1, 1, 1) lies on line \ell_3
174: P_{3252} = (49, 49, 1, 0) lies on line \ell_2
                                                                     228: P_{8361} = (40, 1, 1, 1) lies on line \ell_3
175: P_{3317} = (50, 50, 1, 0) lies on line \ell_2
                                                                     229: P_{8362} = (41, 1, 1, 1) lies on line \ell_3
176: P_{3382} = (51, 51, 1, 0) lies on line \ell_2
                                                                     230: P_{8363} = (42, 1, 1, 1) lies on line \ell_3
177: P_{3447} = (52, 52, 1, 0) lies on line \ell_2
                                                                     231: P_{8364} = (43, 1, 1, 1) lies on line \ell_3
178: P_{3512} = (53, 53, 1, 0) lies on line \ell_2
                                                                     232: P_{8365} = (44, 1, 1, 1) lies on line \ell_3
179: P_{3577} = (54, 54, 1, 0) lies on line \ell_2
                                                                     233: P_{8366} = (45, 1, 1, 1) lies on line \ell_3
180: P_{3642} = (55, 55, 1, 0) lies on line \ell_2
                                                                     234: P_{8367} = (46, 1, 1, 1) lies on line \ell_3
181: P_{3707} = (56, 56, 1, 0) lies on line \ell_2
                                                                     235: P_{8368} = (47, 1, 1, 1) lies on line \ell_3
182 : P_{3772} = (57, 57, 1, 0) lies on line \ell_2
                                                                     236: P_{8369} = (48, 1, 1, 1) lies on line \ell_3
183: P_{3837} = (58, 58, 1, 0) lies on line \ell_2
                                                                     237: P_{8370} = (49, 1, 1, 1) lies on line \ell_3
184 : P_{3902} = (59, 59, 1, 0) lies on line \ell_2
                                                                     238: P_{8371} = (50, 1, 1, 1) lies on line \ell_3
185: P_{3967} = (60, 60, 1, 0) lies on line \ell_2
                                                                     239: P_{8372} = (51, 1, 1, 1) lies on line \ell_3
                                                                     240 : P_{8373} = (52, 1, 1, 1) lies on line \ell_3
186: P_{4032} = (61, 61, 1, 0) lies on line \ell_2
187: P_{4097} = (62, 62, 1, 0) lies on line \ell_2
                                                                     241: P_{8374} = (53, 1, 1, 1) lies on line \ell_3
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242: P_{8375} = (54, 1, 1, 1) lies on line \ell_3
                                                                     296: P_{11201} = (0, 46, 1, 1) lies on line \ell_4
243: P_{8376} = (55, 1, 1, 1) lies on line \ell_3
                                                                     297: P_{11265} = (0, 47, 1, 1) lies on line \ell_4
244 : P_{8377} = (56, 1, 1, 1) lies on line \ell_3
                                                                     298: P_{11329} = (0, 48, 1, 1) lies on line \ell_4
245 : P_{8378} = (57, 1, 1, 1) lies on line \ell_3
                                                                     299: P_{11393} = (0, 49, 1, 1) lies on line \ell_4
246 : P_{8379} = (58, 1, 1, 1) lies on line \ell_3
                                                                     300: P_{11457} = (0, 50, 1, 1) lies on line \ell_4
247: P_{8380} = (59, 1, 1, 1) lies on line \ell_3
                                                                     301: P_{11521} = (0, 51, 1, 1) lies on line \ell_4
248: P_{8381} = (60, 1, 1, 1) lies on line \ell_3
                                                                     302: P_{11585} = (0, 52, 1, 1) lies on line \ell_4
249: P_{8382} = (61, 1, 1, 1) lies on line \ell_3
                                                                     303: P_{11649} = (0, 53, 1, 1) lies on line \ell_4
250: P_{8383} = (62, 1, 1, 1) lies on line \ell_3
                                                                     304: P_{11713} = (0, 54, 1, 1) lies on line \ell_4
251: P_{8384} = (63, 1, 1, 1) lies on line \ell_3
                                                                     305: P_{11777} = (0, 55, 1, 1) lies on line \ell_4
252: P_{8385} = (0, 2, 1, 1) lies on line \ell_4
                                                                     306: P_{11841} = (0, 56, 1, 1) lies on line \ell_4
                                                                     307: P_{11905} = (0, 57, 1, 1) lies on line \ell_4
253: P_{8449} = (0, 3, 1, 1) lies on line \ell_4
254: P_{8513} = (0, 4, 1, 1) lies on line \ell_4
                                                                     308: P_{11969} = (0, 58, 1, 1) lies on line \ell_4
255: P_{8577} = (0, 5, 1, 1) lies on line \ell_4
                                                                     309: P_{12033} = (0, 59, 1, 1) lies on line \ell_4
256: P_{8641} = (0, 6, 1, 1) lies on line \ell_4
                                                                     310 : P_{12097} = (0, 60, 1, 1) lies on line \ell_4
257: P_{8705} = (0, 7, 1, 1) lies on line \ell_4
                                                                     311: P_{12161} = (0, 61, 1, 1) lies on line \ell_4
258: P_{8769} = (0, 8, 1, 1) lies on line \ell_4
                                                                     312: P_{12225} = (0, 62, 1, 1) lies on line \ell_4
259: P_{8833} = (0, 9, 1, 1) lies on line \ell_4
                                                                     313: P_{12289} = (0, 63, 1, 1) lies on line \ell_4
                                                                     314 : P_{12353} = (0, 0, 2, 1) lies on line \ell_5
260: P_{8897} = (0, 10, 1, 1) lies on line \ell_4
261: P_{8961} = (0, 11, 1, 1) lies on line \ell_4
                                                                     315: P_{12418} = (1, 1, 2, 1) lies on line \ell_6
262: P_{9025} = (0, 12, 1, 1) lies on line \ell_4
                                                                     316: P_{16449} = (0, 0, 3, 1) lies on line \ell_5
263: P_{9089} = (0, 13, 1, 1) lies on line \ell_4
                                                                     317: P_{16514} = (1, 1, 3, 1) lies on line \ell_6
264: P_{9153} = (0, 14, 1, 1) lies on line \ell_4
                                                                     318: P_{20545} = (0, 0, 4, 1) lies on line \ell_5
265: P_{9217} = (0, 15, 1, 1) lies on line \ell_4
                                                                     319: P_{20610} = (1, 1, 4, 1) lies on line \ell_6
266: P_{9281} = (0, 16, 1, 1) lies on line \ell_4
                                                                     320: P_{24641} = (0,0,5,1) lies on line \ell_5
267: P_{9345} = (0, 17, 1, 1) lies on line \ell_4
                                                                     321: P_{24706} = (1, 1, 5, 1) lies on line \ell_6
268: P_{9409} = (0, 18, 1, 1) lies on line \ell_4
                                                                     322: P_{28737} = (0,0,6,1) lies on line \ell_5
269: P_{9473} = (0, 19, 1, 1) lies on line \ell_4
                                                                     323: P_{28802} = (1, 1, 6, 1) lies on line \ell_6
270: P_{9537} = (0, 20, 1, 1) lies on line \ell_4
                                                                     324: P_{32833} = (0,0,7,1) lies on line \ell_5
271: P_{9601} = (0, 21, 1, 1) lies on line \ell_4
                                                                     325: P_{32898} = (1, 1, 7, 1) lies on line \ell_6
272: P_{9665} = (0, 22, 1, 1) lies on line \ell_4
                                                                     326: P_{36929} = (0,0,8,1) lies on line \ell_5
273: P_{9729} = (0, 23, 1, 1) lies on line \ell_4
                                                                     327: P_{36994} = (1, 1, 8, 1) lies on line \ell_6
274: P_{9793} = (0, 24, 1, 1) lies on line \ell_4
                                                                     328: P_{41025} = (0, 0, 9, 1) lies on line \ell_5
275: P_{9857} = (0, 25, 1, 1) lies on line \ell_4
                                                                     329: P_{41090} = (1, 1, 9, 1) lies on line \ell_6
276: P_{9921} = (0, 26, 1, 1) lies on line \ell_4
                                                                     330: P_{45121} = (0, 0, 10, 1) lies on line \ell_5
                                                                     331: P_{45186} = (1, 1, 10, 1) lies on line \ell_6
277: P_{9985} = (0, 27, 1, 1) lies on line \ell_4
278: P_{10049} = (0, 28, 1, 1) lies on line \ell_4
                                                                     332: P_{49217} = (0, 0, 11, 1) lies on line \ell_5
279: P_{10113} = (0, 29, 1, 1) lies on line \ell_4
                                                                     333: P_{49282} = (1, 1, 11, 1) lies on line \ell_6
280: P_{10177} = (0, 30, 1, 1) lies on line \ell_4
                                                                     334: P_{53313} = (0, 0, 12, 1) lies on line \ell_5
                                                                     335 : P_{53378} = (1, 1, 12, 1) lies on line \ell_6
281: P_{10241} = (0, 31, 1, 1) lies on line \ell_4
                                                                     336: P_{57409} = (0, 0, 13, 1) lies on line \ell_5
282: P_{10305} = (0, 32, 1, 1) lies on line \ell_4
283: P_{10369} = (0, 33, 1, 1) lies on line \ell_4
                                                                     337: P_{57474} = (1, 1, 13, 1) lies on line \ell_6
                                                                     338 : P_{61505} = (0, 0, 14, 1) lies on line \ell_5
284: P_{10433} = (0, 34, 1, 1) lies on line \ell_4
285: P_{10497} = (0, 35, 1, 1) lies on line \ell_4
                                                                     339: P_{61570} = (1, 1, 14, 1) lies on line \ell_6
286: P_{10561} = (0, 36, 1, 1) lies on line \ell_4
                                                                     340: P_{65601} = (0, 0, 15, 1) lies on line \ell_5
                                                                     341: P_{65666} = (1, 1, 15, 1) lies on line \ell_6
287: P_{10625} = (0, 37, 1, 1) lies on line \ell_4
288: P_{10689} = (0, 38, 1, 1) lies on line \ell_4
                                                                     342: P_{69697} = (0, 0, 16, 1) lies on line \ell_5
289: P_{10753} = (0, 39, 1, 1) lies on line \ell_4
                                                                     343: P_{69762} = (1, 1, 16, 1) lies on line \ell_6
290: P_{10817} = (0, 40, 1, 1) lies on line \ell_4
                                                                     344: P_{73793} = (0, 0, 17, 1) lies on line \ell_5
291: P_{10881} = (0, 41, 1, 1) lies on line \ell_4
                                                                     345: P_{73858} = (1, 1, 17, 1) lies on line \ell_6
292: P_{10945} = (0, 42, 1, 1) lies on line \ell_4
                                                                     346: P_{77889} = (0, 0, 18, 1) lies on line \ell_5
293: P_{11009} = (0, 43, 1, 1) lies on line \ell_4
                                                                     347: P_{77954} = (1, 1, 18, 1) lies on line \ell_6
294 : P_{11073} = (0, 44, 1, 1) lies on line \ell_4
                                                                     348 : P_{81985} = (0,0,19,1) lies on line \ell_5
295: P_{11137} = (0, 45, 1, 1) lies on line \ell_4
                                                                     349: P_{82050} = (1, 1, 19, 1) lies on line \ell_6
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350: P_{86081} = (0, 0, 20, 1) lies on line \ell_5
                                                                     395: P_{176258} = (1, 1, 42, 1) lies on line \ell_6
351: P_{86146} = (1, 1, 20, 1) lies on line \ell_6
                                                                     396: P_{180289} = (0, 0, 43, 1) lies on line \ell_5
352 : P_{90177} = (0, 0, 21, 1) lies on line \ell_5
                                                                     397: P_{180354} = (1, 1, 43, 1) lies on line \ell_6
353: P_{90242} = (1, 1, 21, 1) lies on line \ell_6
                                                                     398: P_{184385} = (0, 0, 44, 1) lies on line \ell_5
354: P_{94273} = (0, 0, 22, 1) lies on line \ell_5
                                                                     399: P_{184450} = (1, 1, 44, 1) lies on line \ell_6
355: P_{94338} = (1, 1, 22, 1) lies on line \ell_6
                                                                     400: P_{188481} = (0, 0, 45, 1) lies on line \ell_5
356: P_{98369} = (0, 0, 23, 1) lies on line \ell_5
                                                                     401: P_{188546} = (1, 1, 45, 1) lies on line \ell_6
357: P_{98434} = (1, 1, 23, 1) lies on line \ell_6
                                                                     402: P_{192577} = (0, 0, 46, 1) lies on line \ell_5
358 : P_{102465} = (0, 0, 24, 1) lies on line \ell_5
                                                                     403: P_{192642} = (1, 1, 46, 1) lies on line \ell_6
359: P_{102530} = (1, 1, 24, 1) lies on line \ell_6
                                                                     404: P_{196673} = (0, 0, 47, 1) lies on line \ell_5
360: P_{106561} = (0, 0, 25, 1) lies on line \ell_5
                                                                     405: P_{196738} = (1, 1, 47, 1) lies on line \ell_6
361: P_{106626} = (1, 1, 25, 1) lies on line \ell_6
                                                                     406: P_{200769} = (0, 0, 48, 1) lies on line \ell_5
362: P_{110657} = (0, 0, 26, 1) lies on line \ell_5
                                                                     407: P_{200834} = (1, 1, 48, 1) lies on line \ell_6
363: P_{110722} = (1, 1, 26, 1) lies on line \ell_6
                                                                     408: P_{204865} = (0, 0, 49, 1) lies on line \ell_5
364 : P_{114753} = (0, 0, 27, 1) lies on line \ell_5
                                                                     409: P_{204930} = (1, 1, 49, 1) lies on line \ell_6
365: P_{114818} = (1, 1, 27, 1) lies on line \ell_6
                                                                     410: P_{208961} = (0, 0, 50, 1) lies on line \ell_5
366: P_{118849} = (0, 0, 28, 1) lies on line \ell_5
                                                                     411: P_{209026} = (1, 1, 50, 1) lies on line \ell_6
367: P_{118914} = (1, 1, 28, 1) lies on line \ell_6
                                                                     412: P_{213057} = (0, 0, 51, 1) lies on line \ell_5
                                                                     413 : P_{213122} = (1, 1, 51, 1) lies on line \ell_6
368: P_{122945} = (0, 0, 29, 1) lies on line \ell_5
369: P_{123010} = (1, 1, 29, 1) lies on line \ell_6
                                                                     414: P_{217153} = (0, 0, 52, 1) lies on line \ell_5
370: P_{127041} = (0, 0, 30, 1) lies on line \ell_5
                                                                     415: P_{217218} = (1, 1, 52, 1) lies on line \ell_6
371: P_{127106} = (1, 1, 30, 1) lies on line \ell_6
                                                                     416: P_{221249} = (0, 0, 53, 1) lies on line \ell_5
372: P_{131137} = (0,0,31,1) lies on line \ell_5
                                                                     417: P_{221314} = (1, 1, 53, 1) lies on line \ell_6
373 : P_{131202} = (1, 1, 31, 1) lies on line \ell_6
                                                                     418 : P_{225345} = (0, 0, 54, 1) lies on line \ell_5
374: P_{135233} = (0,0,32,1) lies on line \ell_5
                                                                     419: P_{225410} = (1, 1, 54, 1) lies on line \ell_6
375: P_{135298} = (1, 1, 32, 1) lies on line \ell_6
                                                                     420: P_{229441} = (0, 0, 55, 1) lies on line \ell_5
376: P_{139329} = (0,0,33,1) lies on line \ell_5
                                                                     421: P_{229506} = (1, 1, 55, 1) lies on line \ell_6
377: P_{139394} = (1, 1, 33, 1) lies on line \ell_6
                                                                     422: P_{233537} = (0, 0, 56, 1) lies on line \ell_5
378: P_{143425} = (0,0,34,1) lies on line \ell_5
                                                                     423: P_{233602} = (1, 1, 56, 1) lies on line \ell_6
379: P_{143490} = (1, 1, 34, 1) lies on line \ell_6
                                                                     424: P_{237633} = (0, 0, 57, 1) lies on line \ell_5
380: P_{147521} = (0, 0, 35, 1) lies on line \ell_5
                                                                     425: P_{237698} = (1, 1, 57, 1) lies on line \ell_6
381 : P_{147586} = (1, 1, 35, 1) lies on line \ell_6
                                                                     426: P_{241729} = (0, 0, 58, 1) lies on line \ell_5
382: P_{151617} = (0, 0, 36, 1) lies on line \ell_5
                                                                     427: P_{241794} = (1, 1, 58, 1) lies on line \ell_6
383 : P_{151682} = (1, 1, 36, 1) lies on line \ell_6
                                                                     428: P_{245825} = (0, 0, 59, 1) lies on line \ell_5
384: P_{155713} = (0, 0, 37, 1) lies on line \ell_5
                                                                     429: P_{245890} = (1, 1, 59, 1) lies on line \ell_6
                                                                     430 : P_{249921} = (0, 0, 60, 1) lies on line \ell_5
385: P_{155778} = (1, 1, 37, 1) lies on line \ell_6
386: P_{159809} = (0, 0, 38, 1) lies on line \ell_5
                                                                     431: P_{249986} = (1, 1, 60, 1) lies on line \ell_6
387: P_{159874} = (1, 1, 38, 1) lies on line \ell_6
                                                                     432 : P_{254017} = (0, 0, 61, 1) lies on line \ell_5
                                                                     433 : P_{254082} = (1, 1, 61, 1) lies on line \ell_6
388: P_{163905} = (0, 0, 39, 1) lies on line \ell_5
389 : P_{163970} = (1, 1, 39, 1) lies on line \ell_6
                                                                     434: P_{258113} = (0,0,62,1) lies on line \ell_5
390: P_{168001} = (0, 0, 40, 1) lies on line \ell_5
                                                                     435: P_{258178} = (1, 1, 62, 1) lies on line \ell_6
391: P_{168066} = (1, 1, 40, 1) lies on line \ell_6
                                                                     436: P_{262209} = (0, 0, 63, 1) lies on line \ell_5
392 : P_{172097} = (0, 0, 41, 1) lies on line \ell_5
                                                                     437: P_{262274} = (1, 1, 63, 1) lies on line \ell_6
393: P_{172162} = (1, 1, 41, 1) lies on line \ell_6
394 : P_{176193} = (0, 0, 42, 1) lies on line \ell_5
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The single points on the surface are:

Points on surface but on no line

The surface has 3844 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c} 0123456 \\ \hline 0 & 01111100 \\ 1 & 1011011 \\ 2 & 1100011 \\ 3 & 1100101 \\ 4 & 1001010 \\ 5 & 0110101 \\ 6 & 0111010 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_0	P_2	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_5	P_2	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_0	P_0	P_{8322}	P_4

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_1	P_{8322}	P_{8258}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_2	P_{8258}	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_4	P_2

The surface has 4289 points:

Too many to print.