Rank-74051 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172877

General information

Number of lines	10
Number of points	97
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	61
Number of points off lines	24
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{10}
Type of lines on points	$5, 3^2, 2^9, 1^{61}, 0^{24}$

Singular Points

The surface has 1 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \mathbf{Pl}(0,0,1,0,0,1)_{664}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1,0,0,1,0,0)_{18}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \mathbf{Pl}(0,1,1,0,0,1)_{672}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \mathbf{Pl}(1,1,0,0,1,1)_{1217}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \mathbf{Pl}(1,1,0,1,1,1)_{1273}$$

Rank of lines: (0, 64, 137, 4680, 584, 4744, 721, 648, 81, 665)
Rank of points on Klein quadric: (0, 2, 664, 17, 18, 1, 672, 10, 1217, 1273)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{82} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $\qquad$ $P_{83} = (1,1,0,1) = \ell_4 \cap \ell_6$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$ $\qquad$ $P_{75} = (1,0,0,1) = \ell_4 \cap \ell_7$
$P_{12} = (1,0,1,0) = \ell_1 \cap \ell_8$ $\qquad$ $P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$
$P_{20} = (1,1,1,0) = \ell_2 \cap \ell_9$ $\qquad$ $P_{139} = (1,0,1,1) = \ell_7 \cap \ell_9$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_5$

Single Points

The surface has 61 single points:
The single points on the surface are:

2

$0: P_6 = (2,1,0,0)$ lies on line ℓ_0

$1: P_7 = (3,1,0,0)$ lies on line ℓ_0

$2: P_8 = (4,1,0,0)$ lies on line ℓ_0

$3: P_9 = (5,1,0,0)$ lies on line ℓ_0

$4: P_{10} = (6,1,0,0)$ lies on line ℓ_0

$5: P_{11} = (7,1,0,0)$ lies on line ℓ_0

$6: P_{13} = (2,0,1,0)$ lies on line ℓ_1

$7: P_{14} = (3,0,1,0)$ lies on line ℓ_1

$8: P_{15} = (4,0,1,0)$ lies on line ℓ_1

$9: P_{16} = (5,0,1,0)$ lies on line ℓ_1

$10: P_{17} = (6,0,1,0)$ lies on line ℓ_1

$11: P_{18} = (7,0,1,0)$ lies on line ℓ_1

$12: P_{29} = (2,2,1,0)$ lies on line ℓ_2

$13: P_{38} = (3,3,1,0)$ lies on line ℓ_2

$14: P_{47} = (4,4,1,0)$ lies on line ℓ_2

$15: P_{56} = (5,5,1,0)$ lies on line ℓ_2

$16: P_{65} = (6,6,1,0)$ lies on line ℓ_2

$17: P_{74} = (7,7,1,0)$ lies on line ℓ_2

$18: P_{90} = (0,2,0,1)$ lies on line ℓ_3

$19: P_{91} = (1,2,0,1)$ lies on line ℓ_4

$20: P_{98} = (0,3,0,1)$ lies on line ℓ_3

$21: P_{99} = (1,3,0,1)$ lies on line ℓ_4

$22: P_{106} = (0,4,0,1)$ lies on line ℓ_3

$23: P_{107} = (1,4,0,1)$ lies on line ℓ_4

$24: P_{114} = (0,5,0,1)$ lies on line ℓ_3

$25: P_{115} = (1,5,0,1)$ lies on line ℓ_4

$26: P_{122} = (0,6,0,1)$ lies on line ℓ_3

$27: P_{123} = (1,6,0,1)$ lies on line ℓ_4

$28: P_{130} = (0,7,0,1)$ lies on line ℓ_3

$29: P_{131} = (1,7,0,1)$ lies on line ℓ_4

$30: P_{138} = (0,0,1,1)$ lies on line ℓ_5

$31: P_{201} = (0,0,2,1)$ lies on line ℓ_5

$32: P_{202} = (1,0,2,1)$ lies on line ℓ_7

$33: P_{210} = (1,1,2,1)$ lies on line ℓ_6

$34: P_{211} = (2,1,2,1)$ lies on line ℓ_8

$35: P_{227} = (2,3,2,1)$ lies on line ℓ_9

$36: P_{265} = (0,0,3,1)$ lies on line ℓ_5

$37: P_{266} = (1,0,3,1)$ lies on line ℓ_7

$38: P_{274} = (1,1,3,1)$ lies on line ℓ_6

$39: P_{276} = (3,1,3,1)$ lies on line ℓ_8

$40: P_{284} = (3,2,3,1)$ lies on line ℓ_9

$41: P_{329} = (0,0,4,1)$ lies on line ℓ_5

$42: P_{330} = (1,0,4,1)$ lies on line ℓ_7

$43: P_{338} = (1,1,4,1)$ lies on line ℓ_6

$44: P_{341} = (4,1,4,1)$ lies on line ℓ_8

$45: P_{373} = (4,5,4,1)$ lies on line ℓ_9

$46: P_{393} = (0,0,5,1)$ lies on line ℓ_5

$47: P_{394} = (1,0,5,1)$ lies on line ℓ_7

$48: P_{402} = (1,1,5,1)$ lies on line ℓ_6

$49: P_{406} = (5,1,5,1)$ lies on line ℓ_8

$50: P_{430} = (5,4,5,1)$ lies on line ℓ_9

$51: P_{457} = (0,0,6,1)$ lies on line ℓ_5

$52: P_{458} = (1,0,6,1)$ lies on line ℓ_7

$53: P_{466} = (1,1,6,1)$ lies on line ℓ_6

$54: P_{471} = (6,1,6,1)$ lies on line ℓ_8

$55: P_{519} = (6,7,6,1)$ lies on line ℓ_9

$56: P_{521} = (0,0,7,1)$ lies on line ℓ_5

$57: P_{522} = (1,0,7,1)$ lies on line ℓ_7

$58: P_{530} = (1,1,7,1)$ lies on line ℓ_6

$59: P_{536} = (7,1,7,1)$ lies on line ℓ_8

$60: P_{576} = (7,6,7,1)$ lies on line ℓ_9

The single points on the surface are:

Points on surface but on no line

The surface has 24 points not on any line:
The points on the surface but not on lines are:

$0: P_{158} = (5,2,1,1)$

$1: P_{159} = (6,2,1,1)$

$2: P_{172} = (3,4,1,1)$

$3: P_{175} = (6,4,1,1)$

$4: P_{196} = (3,7,1,1)$

$5: P_{198} = (5,7,1,1)$

$6: P_{230} = (5,3,2,1)$

$7: P_{254} = (5,6,2,1)$

$8: P_{285} = (4,2,3,1)$

$9: P_{299} = (2,4,3,1)$

$10: P_{315} = (2,6,3,1)$

$11: P_{317} = (4,6,3,1)$

$12: P_{359} = (6,3,4,1)$

$13: P_{375} = (6,5,4,1)$

$14: P_{421} = (4,3,5,1)$

$15: P_{424} = (7,3,5,1)$

$16: P_{432} = (7,4,5,1)$

$17: P_{453} = (4,7,5,1)$

$18: P_{480} = (7,2,6,1)$

$19: P_{499} = (2,5,6,1)$

$20: P_{504} = (7,5,6,1)$

$21: P_{515} = (2,7,6,1)$

$22: P_{564} = (3,5,7,1)$

$23: P_{572} = (3,6,7,1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 1 1 1 0 0 0 0 0
1	1 0 1 0 0 1 1 1 1 0
2	1 1 0 0 0 1 1 1 0 1
3	1 0 0 0 1 1 0 0 1 1
4	1 0 0 1 0 0 1 1 0 0
5	0 1 1 1 0 0 1 1 0 0
6	0 1 1 0 1 1 0 1 1 0
7	0 1 1 0 1 1 1 0 0 1
8	0 1 0 1 0 0 1 0 0 1
9	0 0 1 1 0 0 0 1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_2	P_2	P_2	P_2	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_5	P_2	P_2	P_2	P_2	P_{20}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{82}	P_{82}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7
in point	P_1	P_1	P_{83}	P_{75}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_2	P_3	P_2	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_{83}	P_2	P_2	P_4

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_2	P_2	P_{75}	P_2	P_2	P_{139}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_9
in point	P_{12}	P_{82}	P_4	P_{82}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_{20}	P_{82}	P_{139}	P_{82}

The surface has 97 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (4,1,0,0)$
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$23: P_{56} = (5,5,1,0)$
$24: P_{65} = (6,6,1,0)$
$25: P_{74} = (7,7,1,0)$
$26: P_{75} = (1,0,0,1)$
$27: P_{82} = (0,1,0,1)$
$28: P_{83} = (1,1,0,1)$
$29: P_{90} = (0,2,0,1)$
$30: P_{91} = (1,2,0,1)$
$31: P_{98} = (0,3,0,1)$
$32: P_{99} = (1,3,0,1)$

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