Cheat Sheet GF(32)

January 16, 2021

polynomial: $X^5 + X^2 + 1 = 37$ $Z_i = \log_{\alpha}(1 + \alpha^i)$

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i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_{\alpha}(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	0 = 0	0	DNE	DNE	1	DNE	0	0	0
1	1 = 1	1	1	31	2	18	1	1	1
2	$\alpha = \eta$	2	18	1	4	5	4	0	1
3	$\alpha + 1 = \eta^{18}$	3	28	18	8	29	5	1	1
4	$\alpha^2 = \eta^2$	4	9	2	16	10	16	0	1
5	$\alpha^2 + 1 = \eta^5$	5	23	5	5	2	17	1	1
6	$\alpha^2 + \alpha = \eta^{19}$	6	14	19	10	27	20	0	1
7	$\alpha^2 + \alpha + 1 = \eta^{11}$	7	12	11	20	22	21	1	1
8	$\alpha^3 = \eta^3$	8	22	3	13	20	10	1	1
9	$\alpha^3 + 1 = \eta^{29}$	9	4	29	26	16	11	0	1
10	$\alpha^3 + \alpha = \eta^6$	10	25	6	17	4	14	1	1
11	$\alpha^3 + \alpha + 1 = \eta^{27}$	11	16	27	7	19	15	0	1
12	$\alpha^3 + \alpha^2 = \eta^{20}$	12	7	20	14	23	26	1	1
13	$\alpha^3 + \alpha^2 + 1 = \eta^8$	13	15	8	28	14	27	0	1
14	$\alpha^3 + \alpha^2 + \alpha = \eta^{12}$	14	6	12	29	13	30	1	1
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \eta^{23}$	15	13	23	31	24	31	0	1
16	$\alpha^4 = \eta^4$	16	11	4	27	9	13	0	1
17	$\alpha^4 + 1 = \eta^{10}$	17	24	10	19	30	12	1	1
18	$\alpha^4 + \alpha = \eta^{30}$	18	2	30	3	1	9	0	1
19	$\alpha^4 + \alpha + 1 = \eta^{17}$	19	29	17	6	11	8	1	1
20	$\alpha^4 + \alpha^2 = \eta^7$	20	30	7	12	8	29	0	1
21	$\alpha^4 + \alpha^2 + 1 = \eta^{22}$	21	26	22	24	25	28	1	1
22	$\alpha^4 + \alpha^2 + \alpha = \eta^{28}$	22	8	28	21	7	25	0	1
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \eta^{26}$	23	5	26	15	12	24	1	1
24	$\alpha^4 + \alpha^3 = \eta^{21}$	24	17	21	30	15	7	1	1
25	$\alpha^4 + \alpha^3 + 1 = \eta^{25}$	25	10	25	25	21	6	0	1
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i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_{\alpha}(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
26	$\alpha^4 + \alpha^3 + \alpha = \eta^9$	26	21	9	23	28	3	1	1
27	$\alpha^4 + \alpha^3 + \alpha + 1 = \eta^{16}$	27	31	16	11	6	2	0	1
28	$\alpha^4 + \alpha^3 + \alpha^2 = \eta^{13}$	28	3	13	22	26	23	1	1
29	$\alpha^4 + \alpha^3 + \alpha^2 + 1 = \eta^{14}$	29	19	14	9	3	22	0	1
30	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha = \eta^{24}$	30	20	24	18	17	19	1	1
31	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = \eta^{15}$	31	27	15	1	DNE	18	0	1

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	+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22	25
	2	2	3	0	1	6	7	4	5	10	11	8	9	14	15	12	13	18	19	16	17	22	23	20	21	26
	3	3	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12	19	18	17	16	23	22	21	20	27
	4	4	5	6	7	0	1	2	3	12	13	14	15	8	9	10	11	20	21	22	23	16	17	18	19	28
	5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10	21	20	23	22	17	16	19	18	29
	6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30
	7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8	23	22	21	20	19	18	17	16	31
	8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16
	9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6	25	24	27	26	29	28	31	30	17
1	10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5	26	27	24	25	30	31	28	29	18
- 1		11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4	27	26	25	24	31	30	29	28	19
		12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3	28	29	30	31	24	25	26	27	20
1	- 1	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2	29	28	31	30	25	24	27	26	21
1	14	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1	30	31	28	29	26	27	24	25	22
1	15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	31	30	29	28	27	26	25	24	23
	16	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8
1	17	17	16	19	18	21	20	23	22	25	24	27	26	29	28	31	30	1	0	3	2	5	4	7	6	9
		18	19	16	17	22	23	20	21	26	27	24	25	30	31	28	29	2	3	0	1	6	7	4	5	10
- 1		19	18	17	16	23	22	21	20	27	26	25	24	31	30	29	28	3	2	1	0	7	6	5	4	11
		20	21	22	23	16	17	18	19	28	29	30	31	24	25	26	27	4	5	6	7	0	1	2	3	12
	- 1	21	20	23	22	17	16	19	18	29	28	31	30	25	24	27	26	5	4	7	6	1	0	3	2	13
		22	23	20	21	18	19	16	17	30	31	28	29	26	27	24	25	6	7	4	5	2	3	0	1	14
		23	22	21	20	19	18	17	16	31	30	29	28	27	26	25	24	7	6	5	4	3	2	1	0	15
	- 1	24	25	26	27	28	29	30	31	16	17	18	19	20	21	22	23	8	9	10	11	12	13	14	15	0
ŀ		25	24	27	26	29	28	31	30	17	16	19	18	21	20	23	22	9	8	11	10	13	12	15	14	1
		26	27	24	25	30	31	28	29	18	19	16	17	22	23	20	21	10	11	8	9	14	15	12	13	2
		27	26	25	24	31	30	29	28	19	18	17	16	23	22	21	20	11	10	9	8	15	14	13	12	3
		28	29	30	31	24	25	26	27	20	21	22	23	16	17	18	19	12	13	14	15	8	9	10	11	4
		29	28	31	30	25	24	27	26	21	20	23	22	17	16	19	18	13	12	15	14	9	8	11	10	5
		30	31	28	29	26	27	24	25	22	23	20	21	18	19	16	17	14	15	12	13	10	11	8	9	6
	31	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7
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•	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	5	7	1	3	13	15	9	11	21	23
3	3	6	5	12	15	10	9	24	27	30	29	20	23	18	17	21	22	19	16	25	26	31	28	13	14
4	4	8	12	16	20	24	28	5	1	13	9	21	17	29	25	10	14	2	6	26	30	18	22	15	11
5	5	10	15	20	17	30	27	13	8	7	2	25	28	19	22	26	31	16	21	14	11	4	1	23	18
6	6	12	10	24	30	20	18	21	19	25	31	13	11	1	7	15	9	3	5	23	17	27	29	26	28
7	7	14	9	28	27	18	21	29	26	19	20	1	6	15	8	31	24	17	22	3	4	13	10	2	5
8	8	16	24	5	13	21	29	10	2	26	18	15	7	31	23	20	28	4	12	17	25	1	9	30	22
9	9	18	27	1	8	19	26	2	11	16	25	3	10	17	24	4	13	22	31	5	12	23	30	6	15
10	10	20	30	13	7	25	19	26	16	14	4	23	29	3	9	17	27	5	15	28	22	8	2	11	1
11	11	22	29	9	2	31	20	18	25	4	15	27	16	13	6	1	10	23	28	8	3	30	21	19	24
12	12	24	20	21	25	13	1	15	3	23	27	26	22	2	14	30	18	6	10	11	7	19	31	17	29
13	13	26	23	17	28	11	6	7	10	29	16	22	27	12	1	14	3	20	25	31	18	5	8	9	4
14	14	28	18	29	19	1	15	31	17	3	13	2	12	30	16	27	21	7	9	6	8	26	20	4	10
15	15	30	17	25	22	7	8	23	24	9	6	14	1	16	31	11	4	21	26	18	29	12	3	28	19
16	16	5	21	10	26	15	31	20	4	17	1	30	14	27	11	13	29	8	24	7	23	2	18	25	9
17	17	7	22	14	31	9	24	28	13	27	10	18	3	21	4	29	12	26	11	19	2	20	5	1	16
18	18	1	19	2	16	3	17	4	22	5	23	6	20	7	21	8	26	9	27	10	24	11	25	12	30
19	19	3	16	6	21	5	22	12	31	15	28	10	25	9	26	24	11	27	8	30	13	29	14	20	7
20	20	13	25	26	14	23	3	17	5	28	8	11	31	6	18	7	19	10	30	29	9	16	4	22	2
21	21	15	26	30	11	17	4	25	12	22	3	7	18	8	29	23	2	24	13	9	28	6	19	14	27
22	22	9	31	18	4	27	13	1	23	8	30	19	5	26	12	2	20	11	29	16	6	25	15	3	21
23	23	11	28	22	1	29 20	10	9	30	2	21	31	8	20	3	18	5	25	14	4	19	15	24	27	12
24	24	21	13	15	23	26	2	30	6	11	19	17	9	4	28	25	1	12	20	22	14	3	27	7	31
25	25	23	14	11	18	28	5	22	15	1	24	29	4	10	19	9	16	30	7	2	27	21	12	31	6
26	26	17	11	7	29	22	12	14	20	31	5	9	19	24	2	28	6	13	23	27	1	10	16	18	8
27	27	19	8	3 31	24	16	11	6	29	21	14	5	30	22	13	12	23	31	4	15 12	20	28	7 19	10	17
28	28	29	1		3	2	30	27	7	$\frac{6}{12}$	26	4	24	$\frac{25}{22}$	5	19	15	14	18	$\frac{12}{24}$	16	$\frac{17}{7}$	13 26	8	20
29	29	31	$\frac{2}{7}$	27	6	4 14	25 16	19	14		17 12	8 28	21	23	10	$\frac{3}{22}$	30	28	1		5 31	7 24		16 29	13
30	30	$\frac{25}{27}$	7	23	9 12	14	16	11	21	18			2 15	5 11	27		8 25	15 20	17	1			6		3
31	31	27	4	19	12	8	23	3	28	24	7	16	15	11	20	6	25	29	2	21	10	14	17	5	26

 $2^{0} = 1$ $2^{1} = 2$ $2^{2} = 4$ $2^{3} = 8$ $2^{4} = 16$ $2^{5} = 5$ $2^{6} = 10$ $2^{7} = 20$ $2^{8} = 13$ $2^{9} = 26$ $2^{10} = 17$ $2^{11} = 7$ $2^{12} = 14$ $2^{13} = 28$ $2^{14} = 29$

 $2^{15} = 31$ $2^{16} = 27$ $2^{17} = 19$ $2^{18} = 3$ $2^{19} = 6$ $2^{20} = 12$ $2^{21} = 24$ $2^{22} = 21$ $2^{23} = 15$ $2^{24} = 30$ $2^{25} = 25$ $2^{26} = 23$ $2^{27} = 11$ $2^{28} = 22$ $2^{29} = 9$

$$2^{30} = 18$$
 $2^{31} = 1$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_{\alpha}(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	0 = 0	0	DNE	DNE	1	DNE	0	0	0
1	1 = 1	1	1	31	2	18	1	1	1
2	$\alpha = \eta$	2	18	1	4	5	4	0	1
3	$\alpha + 1 = \eta^{18}$	3	28	18	8	29	5	1	1
4	$\alpha^2 = \eta^2$	4	9	2	16	10	16	0	1
5	$\alpha^2 + 1 = \eta^5$	5	23	5	5	2	17	1	1
6	$\alpha^2 + \alpha = \eta^{19}$	6	14	19	10	27	20	0	$\mid \qquad 1 \mid$
7	$\alpha^2 + \alpha + 1 = \eta^{11}$	7	12	11	20	22	21	1	1
8	$\alpha^3 = \eta^3$	8	22	3	13	20	10	1	1
9	$\alpha^3 + 1 = \eta^{29}$	9	4	29	26	16	11	0	1
10	$\alpha^3 + \alpha = \eta^6$	10	25	6	17	4	14	1	1
11	$\alpha^3 + \alpha + 1 = \eta^{27}$	11	16	27	7	19	15	0	1
12	$\alpha^3 + \alpha^2 = \eta^{20}$	12	7	20	14	23	26	1	1
13	$\alpha^3 + \alpha^2 + 1 = \eta^8$	13	15	8	28	14	27	0	1
14	$\alpha^3 + \alpha^2 + \alpha = \eta^{12}$	14	6	12	29	13	30	1	1
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \eta^{23}$	15	13	23	31	24	31	0	1
16	$\alpha^4 = \eta^4$	16	11	4	27	9	13	0	1
17	$\alpha^4 + 1 = \eta^{10}$	17	24	10	19	30	12	1	1
18	$\alpha^4 + \alpha = \eta^{30}$	18	2	30	3	1	9	0	1
19	$\alpha^4 + \alpha + 1 = \eta^{17}$	19	29	17	6	11	8	1	1
20	$\alpha^4 + \alpha^2 = \eta^7$	20	30	7	12	8	29	0	1
21	$\alpha^4 + \alpha^2 + 1 = \eta^{22}$	21	26	22	24	25	28	1	1
22	$\alpha^4 + \alpha^2 + \alpha = \eta^{28}$	22	8	28	21	7	25	0	1
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \eta^{26}$	23	5	26	15	12	24	1	1
24	$\alpha^4 + \alpha^3 = \eta^{21}$	24	17	21	30	15	7	1	1
25	$\alpha^4 + \alpha^3 + 1 = \eta^{25}$	25	10	25	25	21	6	0	1

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_{\alpha}(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
26	$\alpha^4 + \alpha^3 + \alpha = \eta^9$	26	21	9	23	28	3	1	1
27	$\alpha^4 + \alpha^3 + \alpha + 1 = \eta^{16}$	27	31	16	11	6	2	0	1
28	$\alpha^4 + \alpha^3 + \alpha^2 = \eta^{13}$	28	3	13	22	26	23	1	1
29	$\alpha^4 + \alpha^3 + \alpha^2 + 1 = \eta^{14}$	29	19	14	9	3	22	0	1
30	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha = \eta^{24}$	30	20	24	18	17	19	1	1
31	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = \eta^{15}$	31	27	15	1	DNE	18	0	1

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	0	$\frac{1}{1}$	$\frac{2}{2}$	3	4	$\frac{5}{5}$	6	$\frac{7}{7}$	8	$\frac{9}{9}$	10	11	$\frac{12}{12}$	13	$\frac{14}{14}$	$\frac{15}{15}$	16	$\frac{17}{17}$	18	$\frac{19}{19}$	$\frac{20}{20}$	$\frac{21}{21}$	$\frac{22}{22}$	$\frac{23}{23}$	$\frac{24}{24}$
1	1	0	3	3 2	5	3 4	7	6	9	8	11	10	13	13 12	14 15	$\frac{15}{14}$	17	16	19	18	$\frac{20}{21}$	$\frac{21}{20}$	23	23 22	$\frac{24}{25}$
$\frac{1}{2}$	$\begin{vmatrix} 1 \\ 2 \end{vmatrix}$	3	о 0	1	6	7	4	5	10	11	8	9	13 14	$\frac{12}{15}$	12	13	18	19	16	17	$\frac{21}{22}$	$\frac{20}{23}$	23 20	21	26 26
$\frac{2}{3}$	$\begin{vmatrix} 2 \\ 3 \end{vmatrix}$	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12	19	18	17	16	23	$\frac{23}{22}$	$\frac{20}{21}$	20	$\frac{20}{27}$
4	$\begin{vmatrix} 3 \\ 4 \end{vmatrix}$	5	6	7	0	1	2	3	12	13	$\frac{3}{14}$	15	8	9	10	11	20	21	22	23	16	17	18	19	28
5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10	$\frac{20}{21}$	20	23	22	17	16	19	18	29
6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30
7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8	$\frac{-}{23}$	$\frac{1}{2}$	$\frac{1}{21}$	20	19	18	17	16	31
8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16
9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6	25	24	27	26	29	28	31	30	17
10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5	26	27	24	25	30	31	28	29	18
11	11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4	27	26	25	24	31	30	29	28	19
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3	28	29	30	31	24	25	26	27	20
13	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2	29	28	31	30	25	24	27	26	21
14	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1	30	31	28	29	26	27	24	25	22
15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	31	30	29	28	27	26	25	24	23
16	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8
17	17	16	19	18	21	20	23	22	25	24	27	26	29	28	31	30	1	0	3	2	5	4	7	6	9
18	18	19	16	17	22	23	20	21	26	27	24	25	30	31	28	29	2	3	0	1	6	7	4	5	10
19	19	18	17	16	23	22	21	20	27	26	25	24	31	30	29	28	3	2	1	0	7	6	5	4	11
20	20	21	22	23	16	17	18	19	28	29	30	31	24	25	26	27	4	5	6	7	0	1	2	3	12
21	$\begin{vmatrix} 21 \\ 22 \end{vmatrix}$	20	23	22	17	16	19	18	29	28	31	30	25	24	27	26	5	4	7	6	1	0	3	2	13
22	22	23	20	21	18	19	16	17	30	31	28	29	26	27	24	25	6	7	4	5	2	3	0	1	14
23	23	22 25	21 26	20 27	19 28	18 29	17 30	16 31	31	30	29	28	27 20	26	$\frac{25}{22}$	24	7	6	5	4	3 12	2 13	1	0 15	$\begin{array}{c} 15 \\ 0 \end{array}$
$\begin{array}{ c c } 24 \\ 25 \end{array}$	$\begin{vmatrix} 24 \\ 25 \end{vmatrix}$	$\frac{23}{24}$	20 27	26	29	29 28	31	30	16 17	17 16	18 19	19 18	20	21 20	23	23 22	8	9	10 11	11 10	13	13 12	14 15	13 14	1
$\frac{25}{26}$	$\begin{vmatrix} 25\\26\end{vmatrix}$	$\frac{24}{27}$	24	$\frac{20}{25}$	30	31	28	30 29	18	19	16	17	$\frac{21}{22}$	23	20	21	10	11	8	9	13 14	$\frac{12}{15}$	12	13	2
$\frac{20}{27}$	$\begin{vmatrix} 20\\27\end{vmatrix}$	26	$\frac{24}{25}$	$\frac{25}{24}$	31	30	29	28	19	18	17	16	23	$\frac{23}{22}$	21	20	11	10	9	8	15	14	13	12	$\frac{2}{3}$
28	$\begin{vmatrix} 21 \\ 28 \end{vmatrix}$	29	30	31	$\frac{31}{24}$	25	26	$\frac{20}{27}$	20	21	22	23	16	17	18	19	12	13	14	15	8	9	10	11	4
29	$\begin{vmatrix} 20 \\ 29 \end{vmatrix}$	28	31	30	$\frac{24}{25}$	$\frac{26}{24}$	27	26	21	20	23	22	17	16	19	18	13	12	15	14	9	8	11	10	5
30	30	31	28	29	26	27	24	$\frac{25}{25}$	22	23	20	21	18	19	16	17	14	15	12	13	10	11	8	9	6
31	$\begin{vmatrix} 30 \\ 31 \end{vmatrix}$	30	29	28	27	26	25	$\frac{-3}{24}$	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7

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