Rank-66755 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_0X_1^2 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 66755

General information

Number of lines	7
Number of points	11
Number of singular points	1
Number of Eckardt points	7
Number of double points	0
Number of single points	0
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^7
Type of lines on points	3^7, 0^4

Singular Points

The surface has 1 singular points:

$$0: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_{5}$$
$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$
$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$
$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0,1,0,0,0,1)_{21}$$
$$\ell_5 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_{7}$$
$$\ell_6 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0,1,0,1,0,1)_{27}$$

Rank of lines: (28, 30, 29, 34, 31, 33, 32)
Rank of points on Klein quadric: (19, 5, 25, 1, 21, 7, 27)

Eckardt Points

The surface has 7 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),\ T = 14$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0),\ T = 14$
$2 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1),\ T = -1$
$3 : P_7 = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0),\ T = 14$
$4 : P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1),\ T = 14$
$5 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1),\ T = 14$
$6 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 14$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_{11} = (1,1,0,1)$
3 : $P_{13} = (1,0,1,1)$

Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	0	1	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	1	0	1	1
5	1	1	1	1	1	0	1
6	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_2	P_2	P_7	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_3	P_{10}	P_3	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_{12}	P_{14}	P_{14}	P_{12}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_3	P_{12}	P_2	P_3	P_{12}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_{10}	P_{14}	P_2	P_{14}	P_{10}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_7	P_3	P_{14}	P_3	P_{14}	P_7

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_7	P_{10}	P_{12}	P_{12}	P_{10}	P_7

The surface has 11 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_4 = (1,1,1,1)$
5 : $P_7 = (0,1,1,0)$
6 : $P_{10} = (0,1,0,1)$
7 : $P_{11} = (1,1,0,1)$
8 : $P_{12} = (0,0,1,1)$
9 : $P_{13} = (1,0,1,1)$
10 : $P_{14} = (0,1,1,1)$