Rank-67243 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_0X_1^2 + X_1^2X_3 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 555819285

General information

Number of lines	4
Number of points	289
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	60
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^4
Type of lines on points	2^4, 1^{60}, 0^{225}

Singular Points

The surface has 2 singular points:

0 : $P_{2849} = \mathbf{P}(0, 1, \delta^{10}, 1) = \mathbf{P}(0, 1, 10, 1)$
1 : $P_{3105} = \mathbf{P}(0, 1, \delta^{5}, 1) = \mathbf{P}(0, 1, 11, 1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{4625}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69905} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69905} = \mathbf{Pl}(0,1,0,0,0,1)_{4641}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^{5} & 1 \end{array}\right]_{7125} = \left[\begin{array}{cccc} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 1 \end{array}\right]_{7125} = \mathbf{Pl}(10,11,10,11,11,1)_{52710}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & \delta^{5} & 1 \\ 0 & 1 & \delta^{10} & 1 \end{array}\right]_{7397} = \left[\begin{array}{cccc} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 1 \end{array}\right]_{7397} = \mathbf{Pl}(11,10,11,10,10,1)_{48856}$$

Rank of lines: (69888, 69905, 7125, 7397)
Rank of points on Klein quadric: (4625, 4641, 52710, 48856)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$
$P_{3105} = (0,1,11,1) = \ell_1 \cap \ell_2$
$P_{2849} = (0,1,10,1) = \ell_1 \cap \ell_3$
$P_{36} = (1,1,1,0) = \ell_2 \cap \ell_3$

Single Points

The surface has 60 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_{35} = (0,1,1,0)$ lies on line ℓ_0
2 : $P_{51} = (0,2,1,0)$ lies on line ℓ_0
3 : $P_{67} = (0,3,1,0)$ lies on line ℓ_0
4 : $P_{83} = (0,4,1,0)$ lies on line ℓ_0
5 : $P_{99} = (0,5,1,0)$ lies on line ℓ_0
6 : $P_{115} = (0,6,1,0)$ lies on line ℓ_0
7 : $P_{131} = (0,7,1,0)$ lies on line ℓ_0
8 : $P_{147} = (0,8,1,0)$ lies on line ℓ_0
9 : $P_{163} = (0,9,1,0)$ lies on line ℓ_0
10 : $P_{179} = (0,10,1,0)$ lies on line ℓ_0
11 : $P_{195} = (0,11,1,0)$ lies on line ℓ_0
12 : $P_{211} = (0,12,1,0)$ lies on line ℓ_0
13 : $P_{227} = (0,13,1,0)$ lies on line ℓ_0
14 : $P_{243} = (0,14,1,0)$ lies on line ℓ_0
15 : $P_{259} = (0,15,1,0)$ lies on line ℓ_0
16 : $P_{290} = (0,1,0,1)$ lies on line ℓ_1
17 : $P_{445} = (11,10,0,1)$ lies on line ℓ_2
18 : $P_{460} = (10,11,0,1)$ lies on line ℓ_3
19 : $P_{546} = (0,1,1,1)$ lies on line ℓ_1
20 : $P_{700} = (11,10,1,1)$ lies on line ℓ_3
21 : $P_{715} = (10,11,1,1)$ lies on line ℓ_2
22 : $P_{801} = (0,1,2,1)$ lies on line ℓ_1
23 : $P_{922} = (9,8,2,1)$ lies on line ℓ_2
24 : $P_{937} = (8,9,2,1)$ lies on line ℓ_3
25 : $P_{1057} = (0,1,3,1)$ lies on line ℓ_1
26 : $P_{1178} = (9,8,3,1)$ lies on line ℓ_3
27 : $P_{1193} = (8,9,3,1)$ lies on line ℓ_2
28 : $P_{1313} = (0,1,4,1)$ lies on line ℓ_1
29 : $P_{1536} = (15,14,4,1)$ lies on line ℓ_2
30 : $P_{1551} = (14,15,4,1)$ lies on line ℓ_3
31 : $P_{1569} = (0,1,5,1)$ lies on line ℓ_1
32 : $P_{1792} = (15,14,5,1)$ lies on line ℓ_3
33 : $P_{1807} = (14,15,5,1)$ lies on line ℓ_2
34 : $P_{1825} = (0,1,6,1)$ lies on line ℓ_1
35 : $P_{2014} = (13,12,6,1)$ lies on line ℓ_2
36 : $P_{2029} = (12,13,6,1)$ lies on line ℓ_3
37 : $P_{2081} = (0,1,7,1)$ lies on line ℓ_1
38 : $P_{2270} = (13,12,7,1)$ lies on line ℓ_3
39 : $P_{2285} = (12,13,7,1)$ lies on line ℓ_2
40 : $P_{2337} = (0,1,8,1)$ lies on line ℓ_1
41 : $P_{2356} = (3,2,8,1)$ lies on line ℓ_2
42 : $P_{2371} = (2,3,8,1)$ lies on line ℓ_3
43 : $P_{2593} = (0,1,9,1)$ lies on line ℓ_1

44 : $P_{2612} = (3, 2, 9, 1)$ lies on line ℓ_3
45 : $P_{2627} = (2, 3, 9, 1)$ lies on line ℓ_2
46 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_2
47 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_3
48 : $P_{3361} = (0, 1, 12, 1)$ lies on line ℓ_1
49 : $P_{3448} = (7, 6, 12, 1)$ lies on line ℓ_2
50 : $P_{3463} = (6, 7, 12, 1)$ lies on line ℓ_3
51 : $P_{3617} = (0, 1, 13, 1)$ lies on line ℓ_1
52 : $P_{3704} = (7, 6, 13, 1)$ lies on line ℓ_3
53 : $P_{3719} = (6, 7, 13, 1)$ lies on line ℓ_2
54 : $P_{3873} = (0, 1, 14, 1)$ lies on line ℓ_1
55 : $P_{3926} = (5, 4, 14, 1)$ lies on line ℓ_2
56 : $P_{3941} = (4, 5, 14, 1)$ lies on line ℓ_3
57 : $P_{4129} = (0, 1, 15, 1)$ lies on line ℓ_1
58 : $P_{4182} = (5, 4, 15, 1)$ lies on line ℓ_3
59 : $P_{4197} = (4, 5, 15, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_{20} = (1, 0, 1, 0)$
3 : $P_{58} = (7, 2, 1, 0)$
4 : $P_{74} = (7, 3, 1, 0)$
5 : $P_{95} = (12, 4, 1, 0)$
6 : $P_{111} = (12, 5, 1, 0)$
7 : $P_{125} = (10, 6, 1, 0)$
8 : $P_{141} = (10, 7, 1, 0)$
9 : $P_{153} = (6, 8, 1, 0)$
10 : $P_{169} = (6, 9, 1, 0)$
11 : $P_{222} = (11, 12, 1, 0)$
12 : $P_{238} = (11, 13, 1, 0)$
13 : $P_{256} = (13, 14, 1, 0)$
14 : $P_{272} = (13, 15, 1, 0)$
15 : $P_{313} = (7, 2, 0, 1)$
16 : $P_{336} = (14, 3, 0, 1)$
17 : $P_{350} = (12, 4, 0, 1)$
18 : $P_{356} = (2, 5, 0, 1)$
19 : $P_{378} = (8, 6, 0, 1)$
20 : $P_{389} = (3, 7, 0, 1)$
21 : $P_{406} = (4, 8, 0, 1)$
22 : $P_{424} = (6, 9, 0, 1)$
23 : $P_{471} = (5, 12, 0, 1)$
24 : $P_{497} = (15, 13, 0, 1)$
25 : $P_{511} = (13, 14, 0, 1)$
26 : $P_{523} = (9, 15, 0, 1)$
27 : $P_{540} = (10, 0, 1, 1)$
28 : $P_{541} = (11, 0, 1, 1)$
29 : $P_{569} = (8, 2, 1, 1)$
30 : $P_{576} = (15, 2, 1, 1)$
31 : $P_{596} = (3, 4, 1, 1)$
32 : $P_{608} = (15, 4, 1, 1)$
33 : $P_{676} = (3, 9, 1, 1)$
34 : $P_{678} = (5, 9, 1, 1)$
35 : $P_{758} = (5, 14, 1, 1)$
36 : $P_{761} = (8, 14, 1, 1)$
37 : $P_{816} = (15, 1, 2, 1)$
38 : $P_{921} = (8, 8, 2, 1)$
39 : $P_{933} = (4, 9, 2, 1)$
40 : $P_{981} = (4, 12, 2, 1)$
41 : $P_{986} = (9, 12, 2, 1)$
42 : $P_{1008} = (15, 13, 2, 1)$
43 : $P_{1026} = (1, 15, 2, 1)$
44 : $P_{1067} = (10, 1, 3, 1)$
45 : $P_{1079} = (6, 2, 3, 1)$
46 : $P_{1085} = (12, 2, 3, 1)$
47 : $P_{1101} = (12, 3, 3, 1)$
48 : $P_{1104} = (15, 3, 3, 1)$
49 : $P_{1113} = (8, 4, 3, 1)$
50 : $P_{1124} = (3, 5, 3, 1)$
51 : $P_{1131} = (10, 5, 3, 1)$
52 : $P_{1160} = (7, 7, 3, 1)$
53 : $P_{1176} = (7, 8, 3, 1)$
54 : $P_{1200} = (15, 9, 3, 1)$
55 : $P_{1202} = (1, 10, 3, 1)$
56 : $P_{1207} = (6, 10, 3, 1)$
57 : $P_{1220} = (3, 11, 3, 1)$
58 : $P_{1230} = (13, 11, 3, 1)$
59 : $P_{1274} = (9, 14, 3, 1)$
60 : $P_{1278} = (13, 14, 3, 1)$
61 : $P_{1316} = (3, 1, 4, 1)$
62 : $P_{1346} = (1, 3, 4, 1)$
63 : $P_{1402} = (9, 6, 4, 1)$
64 : $P_{1407} = (14, 6, 4, 1)$
65 : $P_{1412} = (3, 7, 4, 1)$
66 : $P_{1530} = (9, 14, 4, 1)$
67 : $P_{1552} = (15, 15, 4, 1)$
68 : $P_{1580} = (11, 1, 5, 1)$
69 : $P_{1592} = (7, 2, 5, 1)$

70 : $P_{1599} = (14, 2, 5, 1)$
71 : $P_{1623} = (6, 4, 5, 1)$
72 : $P_{1630} = (13, 4, 5, 1)$
73 : $P_{1636} = (3, 5, 5, 1)$
74 : $P_{1639} = (6, 5, 5, 1)$
75 : $P_{1686} = (5, 8, 5, 1)$
76 : $P_{1692} = (11, 8, 5, 1)$
77 : $P_{1712} = (15, 9, 5, 1)$
78 : $P_{1718} = (5, 10, 5, 1)$
79 : $P_{1720} = (7, 10, 5, 1)$
80 : $P_{1730} = (1, 11, 5, 1)$
81 : $P_{1742} = (13, 11, 5, 1)$
82 : $P_{1757} = (12, 12, 5, 1)$
83 : $P_{1780} = (3, 14, 5, 1)$
84 : $P_{1805} = (12, 15, 5, 1)$
85 : $P_{1828} = (3, 1, 6, 1)$
86 : $P_{1858} = (1, 3, 6, 1)$
87 : $P_{1866} = (9, 3, 6, 1)$
88 : $P_{1876} = (3, 4, 6, 1)$
89 : $P_{1887} = (14, 4, 6, 1)$
90 : $P_{1895} = (6, 5, 6, 1)$
91 : $P_{1903} = (14, 5, 6, 1)$
92 : $P_{1916} = (11, 6, 6, 1)$
93 : $P_{1918} = (13, 6, 6, 1)$
94 : $P_{1948} = (11, 8, 6, 1)$
95 : $P_{1955} = (2, 9, 6, 1)$
96 : $P_{1960} = (7, 9, 6, 1)$
97 : $P_{2007} = (6, 12, 6, 1)$
98 : $P_{2019} = (2, 13, 6, 1)$
99 : $P_{2040} = (7, 14, 6, 1)$
100 : $P_{2058} = (9, 15, 6, 1)$
101 : $P_{2061} = (12, 15, 6, 1)$
102 : $P_{2089} = (8, 1, 7, 1)$
103 : $P_{2103} = (6, 2, 7, 1)$
104 : $P_{2106} = (9, 2, 7, 1)$
105 : $P_{2124} = (11, 3, 7, 1)$
106 : $P_{2135} = (6, 4, 7, 1)$
107 : $P_{2147} = (2, 5, 7, 1)$
108 : $P_{2158} = (13, 5, 7, 1)$
109 : $P_{2188} = (11, 7, 7, 1)$
110 : $P_{2189} = (12, 7, 7, 1)$
111 : $P_{2194} = (1, 8, 7, 1)$
112 : $P_{2195} = (2, 8, 7, 1)$
113 : $P_{2266} = (9, 12, 7, 1)$
114 : $P_{2280} = (7, 13, 7, 1)$
115 : $P_{2293} = (4, 14, 7, 1)$
116 : $P_{2297} = (8, 14, 7, 1)$
117 : $P_{2309} = (4, 15, 7, 1)$
118 : $P_{2312} = (7, 15, 7, 1)$
119 : $P_{2347} = (10, 1, 8, 1)$
120 : $P_{2358} = (5, 2, 8, 1)$
121 : $P_{2375} = (6, 3, 8, 1)$
122 : $P_{2387} = (2, 4, 8, 1)$
123 : $P_{2397} = (12, 4, 8, 1)$
124 : $P_{2423} = (6, 6, 8, 1)$
125 : $P_{2454} = (5, 8, 8, 1)$
126 : $P_{2462} = (13, 8, 8, 1)$
127 : $P_{2472} = (7, 9, 8, 1)$
128 : $P_{2478} = (13, 9, 8, 1)$
129 : $P_{2482} = (1, 10, 8, 1)$
130 : $P_{2488} = (7, 10, 8, 1)$
131 : $P_{2505} = (8, 11, 8, 1)$
132 : $P_{2509} = (12, 11, 8, 1)$
133 : $P_{2548} = (3, 14, 8, 1)$
134 : $P_{2569} = (8, 15, 8, 1)$
135 : $P_{2571} = (10, 15, 8, 1)$
136 : $P_{2598} = (5, 1, 9, 1)$
137 : $P_{2623} = (14, 2, 9, 1)$
138 : $P_{2628} = (3, 3, 9, 1)$
139 : $P_{2658} = (1, 5, 9, 1)$
140 : $P_{2774} = (5, 12, 9, 1)$
141 : $P_{2787} = (2, 13, 9, 1)$
142 : $P_{2799} = (14, 13, 9, 1)$
143 : $P_{2844} = (11, 0, 10, 1)$
144 : $P_{2901} = (4, 4, 10, 1)$
145 : $P_{2912} = (15, 4, 10, 1)$
146 : $P_{2998} = (5, 10, 10, 1)$
147 : $P_{3008} = (15, 10, 10, 1)$
148 : $P_{3020} = (11, 11, 10, 1)$
149 : $P_{3029} = (4, 12, 10, 1)$
150 : $P_{3035} = (10, 12, 10, 1)$
151 : $P_{3051} = (10, 13, 10, 1)$
152 : $P_{3055} = (14, 13, 10, 1)$
153 : $P_{3062} = (5, 14, 10, 1)$
154 : $P_{3071} = (14, 14, 10, 1)$
155 : $P_{3099} = (10, 0, 11, 1)$
156 : $P_{3123} = (2, 2, 11, 1)$
157 : $P_{3129} = (8, 2, 11, 1)$
158 : $P_{3194} = (9, 6, 11, 1)$
159 : $P_{3196} = (11, 6, 11, 1)$
160 : $P_{3203} = (2, 7, 11, 1)$
161 : $P_{3212} = (11, 7, 11, 1)$
162 : $P_{3236} = (3, 9, 11, 1)$
163 : $P_{3242} = (9, 9, 11, 1)$
164 : $P_{3259} = (10, 10, 11, 1)$
165 : $P_{3268} = (3, 11, 11, 1)$
166 : $P_{3273} = (8, 11, 11, 1)$
167 : $P_{3376} = (15, 1, 12, 1)$
168 : $P_{3386} = (9, 2, 12, 1)$
169 : $P_{3392} = (15, 2, 12, 1)$
170 : $P_{3402} = (9, 3, 12, 1)$
171 : $P_{3405} = (12, 3, 12, 1)$
172 : $P_{3422} = (13, 4, 12, 1)$
173 : $P_{3423} = (14, 4, 12, 1)$
174 : $P_{3435} = (10, 5, 12, 1)$
175 : $P_{3455} = (14, 6, 12, 1)$
176 : $P_{3469} = (12, 7, 12, 1)$
177 : $P_{3477} = (4, 8, 12, 1)$

178 : $P_{3480} = (7, 8, 12, 1)$
179 : $P_{3502} = (13, 9, 12, 1)$
180 : $P_{3543} = (6, 12, 12, 1)$
181 : $P_{3547} = (10, 12, 12, 1)$
182 : $P_{3586} = (1, 15, 12, 1)$
183 : $P_{3589} = (4, 15, 12, 1)$
184 : $P_{3622} = (5, 1, 13, 1)$
185 : $P_{3645} = (12, 2, 13, 1)$
186 : $P_{3655} = (6, 3, 13, 1)$
187 : $P_{3663} = (14, 3, 13, 1)$
188 : $P_{3682} = (1, 5, 13, 1)$
189 : $P_{3695} = (14, 5, 13, 1)$
190 : $P_{3710} = (13, 6, 13, 1)$
191 : $P_{3717} = (4, 7, 13, 1)$
192 : $P_{3731} = (2, 8, 13, 1)$
193 : $P_{3742} = (13, 8, 13, 1)$
194 : $P_{3747} = (2, 9, 13, 1)$
195 : $P_{3750} = (5, 9, 13, 1)$
196 : $P_{3816} = (7, 13, 13, 1)$
197 : $P_{3819} = (10, 13, 13, 1)$
198 : $P_{3829} = (4, 14, 13, 1)$
199 : $P_{3837} = (12, 14, 13, 1)$
200 : $P_{3851} = (10, 15, 13, 1)$
201 : $P_{3881} = (8, 1, 14, 1)$
202 : $P_{3923} = (2, 4, 14, 1)$
203 : $P_{3942} = (5, 5, 14, 1)$
204 : $P_{3961} = (8, 6, 14, 1)$
205 : $P_{3971} = (2, 7, 14, 1)$
206 : $P_{3973} = (4, 7, 14, 1)$
207 : $P_{3986} = (1, 8, 14, 1)$
208 : $P_{4140} = (11, 1, 15, 1)$
209 : $P_{4150} = (5, 2, 15, 1)$
210 : $P_{4172} = (11, 3, 15, 1)$
211 : $P_{4176} = (15, 3, 15, 1)$
212 : $P_{4185} = (8, 4, 15, 1)$
213 : $P_{4206} = (13, 5, 15, 1)$
214 : $P_{4261} = (4, 9, 15, 1)$
215 : $P_{4263} = (6, 9, 15, 1)$
216 : $P_{4279} = (6, 10, 15, 1)$
217 : $P_{4288} = (15, 10, 15, 1)$
218 : $P_{4290} = (1, 11, 15, 1)$
219 : $P_{4301} = (12, 11, 15, 1)$
220 : $P_{4334} = (13, 13, 15, 1)$
221 : $P_{4344} = (7, 14, 15, 1)$
222 : $P_{4349} = (12, 14, 15, 1)$
223 : $P_{4360} = (7, 15, 15, 1)$
224 : $P_{4361} = (8, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3
0	0 1 0 0
1	1 0 1 1
2	0 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_{3105}	P_{2849}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{3105}	P_{36}

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_{2849}	P_{36}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{20} = (1, 0, 1, 0)$
5 : $P_{35} = (0, 1, 1, 0)$
6 : $P_{36} = (1, 1, 1, 0)$
7 : $P_{51} = (0, 2, 1, 0)$
8 : $P_{58} = (7, 2, 1, 0)$

$9: P_{67} = (0,3,1,0)$
$10: P_{74} = (7,3,1,0)$
$11: P_{83} = (0,4,1,0)$
$12: P_{95} = (12,4,1,0)$
$13: P_{99} = (0,5,1,0)$
$14: P_{111} = (12,5,1,0)$
$15: P_{115} = (0,6,1,0)$
$16: P_{125} = (10,6,1,0)$
$17: P_{131} = (0,7,1,0)$
$18: P_{141} = (10,7,1,0)$
$19: P_{147} = (0,8,1,0)$
$20: P_{153} = (6,8,1,0)$
$21: P_{163} = (0,9,1,0)$
$22: P_{169} = (6,9,1,0)$
$23: P_{179} = (0,10,1,0)$
$24: P_{195} = (0,11,1,0)$
$25: P_{211} = (0,12,1,0)$
$26: P_{222} = (11,12,1,0)$
$27: P_{227} = (0,13,1,0)$
$28: P_{238} = (11,13,1,0)$
$29: P_{243} = (0,14,1,0)$
$30: P_{256} = (13,14,1,0)$
$31: P_{259} = (0,15,1,0)$
$32: P_{272} = (13,15,1,0)$
$33: P_{290} = (0,1,0,1)$
$34: P_{313} = (7,2,0,1)$
$35: P_{336} = (14,3,0,1)$
$36: P_{350} = (12,4,0,1)$
$37: P_{356} = (2,5,0,1)$
$38: P_{378} = (8,6,0,1)$
$39: P_{389} = (3,7,0,1)$
$40: P_{406} = (4,8,0,1)$
$41: P_{424} = (6,9,0,1)$
$42: P_{445} = (11,10,0,1)$
$43: P_{460} = (10,11,0,1)$
$44: P_{471} = (5,12,0,1)$
$45: P_{497} = (15,13,0,1)$
$46: P_{511} = (13,14,0,1)$
$47: P_{523} = (9,15,0,1)$
$48: P_{540} = (10,0,1,1)$
$49: P_{541} = (11,0,1,1)$
$50: P_{546} = (0,1,1,1)$
$51: P_{569} = (8,2,1,1)$
$52: P_{576} = (15,2,1,1)$
$53: P_{596} = (3,4,1,1)$
$54: P_{608} = (15,4,1,1)$
$55: P_{676} = (3,9,1,1)$
$56: P_{678} = (5,9,1,1)$
$57: P_{700} = (11,10,1,1)$
$58: P_{715} = (10,11,1,1)$
$59: P_{758} = (5,14,1,1)$
$60: P_{761} = (8,14,1,1)$
$61: P_{801} = (0,1,2,1)$
$62: P_{816} = (15,1,2,1)$
$63: P_{921} = (8,8,2,1)$
$64: P_{922} = (9,8,2,1)$
$65: P_{933} = (4,9,2,1)$
$66: P_{937} = (8,9,2,1)$
$67: P_{981} = (4,12,2,1)$
$68: P_{986} = (9,12,2,1)$
$69: P_{1008} = (15,13,2,1)$
$70: P_{1026} = (1,15,2,1)$
$71: P_{1057} = (0,1,3,1)$
$72: P_{1067} = (10,1,3,1)$
$73: P_{1079} = (6,2,3,1)$
$74: P_{1085} = (12,2,3,1)$
$75: P_{1101} = (12,3,3,1)$
$76: P_{1104} = (15,3,3,1)$
$77: P_{1113} = (8,4,3,1)$
$78: P_{1124} = (3,5,3,1)$
$79: P_{1131} = (10,5,3,1)$
$80: P_{1160} = (7,7,3,1)$
$81: P_{1176} = (7,8,3,1)$
$82: P_{1178} = (9,8,3,1)$
$83: P_{1193} = (8,9,3,1)$
$84: P_{1200} = (15,9,3,1)$
$85: P_{1202} = (1,10,3,1)$
$86: P_{1207} = (6,10,3,1)$
$87: P_{1220} = (3,11,3,1)$
$88: P_{1230} = (13,11,3,1)$
$89: P_{1274} = (9,14,3,1)$
$90: P_{1278} = (13,14,3,1)$
$91: P_{1313} = (0,1,4,1)$
$92: P_{1316} = (3,1,4,1)$
$93: P_{1346} = (1,3,4,1)$
$94: P_{1402} = (9,6,4,1)$
$95: P_{1407} = (14,6,4,1)$
$96: P_{1412} = (3,7,4,1)$
$97: P_{1530} = (9,14,4,1)$
$98: P_{1536} = (15,14,4,1)$
$99: P_{1551} = (14,15,4,1)$
$100: P_{1552} = (15,15,4,1)$
$101: P_{1569} = (0,1,5,1)$
$102: P_{1580} = (11,1,5,1)$
$103: P_{1592} = (7,2,5,1)$
$104: P_{1599} = (14,2,5,1)$
$105: P_{1623} = (6,4,5,1)$
$106: P_{1630} = (13,4,5,1)$
$107: P_{1636} = (3,5,5,1)$
$108: P_{1639} = (6,5,5,1)$
$109: P_{1686} = (5,8,5,1)$
$110: P_{1692} = (11,8,5,1)$
$111: P_{1712} = (15,9,5,1)$
$112: P_{1718} = (5,10,5,1)$
$113: P_{1720} = (7,10,5,1)$
$114: P_{1730} = (1,11,5,1)$
$115: P_{1742} = (13,11,5,1)$
$116: P_{1757} = (12,12,5,1)$
$117: P_{1780} = (3,14,5,1)$
$118: P_{1792} = (15,14,5,1)$
$119: P_{1805} = (12,15,5,1)$
$120: P_{1807} = (14,15,5,1)$
$121: P_{1825} = (0,1,6,1)$
$122: P_{1828} = (3,1,6,1)$
$123: P_{1858} = (1,3,6,1)$
$124: P_{1866} = (9,3,6,1)$
$125: P_{1876} = (3,4,6,1)$
$126: P_{1887} = (14,4,6,1)$
$127: P_{1895} = (6,5,6,1)$
$128: P_{1903} = (14,5,6,1)$
$129: P_{1916} = (11,6,6,1)$
$130: P_{1918} = (13,6,6,1)$
$131: P_{1948} = (11,8,6,1)$
$132: P_{1955} = (2,9,6,1)$
$133: P_{1960} = (7,9,6,1)$
$134: P_{2007} = (6,12,6,1)$
$135: P_{2014} = (13,12,6,1)$
$136: P_{2019} = (2,13,6,1)$
$137: P_{2029} = (12,13,6,1)$
$138: P_{2040} = (7,14,6,1)$
$139: P_{2058} = (9,15,6,1)$
$140: P_{2061} = (12,15,6,1)$
$141: P_{2081} = (0,1,7,1)$
$142: P_{2089} = (8,1,7,1)$
$143: P_{2103} = (6,2,7,1)$
$144: P_{2106} = (9,2,7,1)$
$145: P_{2124} = (11,3,7,1)$
$146: P_{2135} = (6,4,7,1)$
$147: P_{2147} = (2,5,7,1)$
$148: P_{2158} = (13,5,7,1)$
$149: P_{2188} = (11,7,7,1)$
$150: P_{2189} = (12,7,7,1)$
$151: P_{2194} = (1,8,7,1)$
$152: P_{2195} = (2,8,7,1)$
$153: P_{2266} = (9,12,7,1)$
$154: P_{2270} = (13,12,7,1)$
$155: P_{2280} = (7,13,7,1)$
$156: P_{2285} = (12,13,7,1)$
$157: P_{2293} = (4,14,7,1)$
$158: P_{2297} = (8,14,7,1)$
$159: P_{2309} = (4,15,7,1)$
$160: P_{2312} = (7,15,7,1)$
$161: P_{2337} = (0,1,8,1)$
$162: P_{2347} = (10,1,8,1)$
$163: P_{2356} = (3,2,8,1)$
$164: P_{2358} = (5,2,8,1)$
$165: P_{2371} = (2,3,8,1)$
$166: P_{2375} = (6,3,8,1)$
$167: P_{2387} = (2,4,8,1)$
$168: P_{2397} = (12,4,8,1)$
$169: P_{2423} = (6,6,8,1)$
$170: P_{2454} = (5,8,8,1)$

171 : $P_{2462} = (13, 8, 8, 1)$
172 : $P_{2472} = (7, 9, 8, 1)$
173 : $P_{2478} = (13, 9, 8, 1)$
174 : $P_{2482} = (1, 10, 8, 1)$
175 : $P_{2488} = (7, 10, 8, 1)$
176 : $P_{2505} = (8, 11, 8, 1)$
177 : $P_{2509} = (12, 11, 8, 1)$
178 : $P_{2548} = (3, 14, 8, 1)$
179 : $P_{2569} = (8, 15, 8, 1)$
180 : $P_{2571} = (10, 15, 8, 1)$
181 : $P_{2593} = (0, 1, 9, 1)$
182 : $P_{2598} = (5, 1, 9, 1)$
183 : $P_{2612} = (3, 2, 9, 1)$
184 : $P_{2623} = (14, 2, 9, 1)$
185 : $P_{2627} = (2, 3, 9, 1)$
186 : $P_{2628} = (3, 3, 9, 1)$
187 : $P_{2658} = (1, 5, 9, 1)$
188 : $P_{2774} = (5, 12, 9, 1)$
189 : $P_{2787} = (2, 13, 9, 1)$
190 : $P_{2799} = (14, 13, 9, 1)$
191 : $P_{2834} = (1, 0, 10, 1)$
192 : $P_{2844} = (11, 0, 10, 1)$
193 : $P_{2849} = (0, 1, 10, 1)$
194 : $P_{2901} = (4, 4, 10, 1)$
195 : $P_{2912} = (15, 4, 10, 1)$
196 : $P_{2998} = (5, 10, 10, 1)$
197 : $P_{3008} = (15, 10, 10, 1)$
198 : $P_{3020} = (11, 11, 10, 1)$
199 : $P_{3029} = (4, 12, 10, 1)$
200 : $P_{3035} = (10, 12, 10, 1)$
201 : $P_{3051} = (10, 13, 10, 1)$
202 : $P_{3055} = (14, 13, 10, 1)$
203 : $P_{3062} = (5, 14, 10, 1)$
204 : $P_{3071} = (14, 14, 10, 1)$
205 : $P_{3090} = (1, 0, 11, 1)$
206 : $P_{3099} = (10, 0, 11, 1)$
207 : $P_{3105} = (0, 1, 11, 1)$
208 : $P_{3123} = (2, 2, 11, 1)$
209 : $P_{3129} = (8, 2, 11, 1)$
210 : $P_{3194} = (9, 6, 11, 1)$
211 : $P_{3196} = (11, 6, 11, 1)$
212 : $P_{3203} = (2, 7, 11, 1)$
213 : $P_{3212} = (11, 7, 11, 1)$
214 : $P_{3236} = (3, 9, 11, 1)$
215 : $P_{3242} = (9, 9, 11, 1)$
216 : $P_{3259} = (10, 10, 11, 1)$
217 : $P_{3268} = (3, 11, 11, 1)$
218 : $P_{3273} = (8, 11, 11, 1)$
219 : $P_{3361} = (0, 1, 12, 1)$
220 : $P_{3376} = (15, 1, 12, 1)$
221 : $P_{3386} = (9, 2, 12, 1)$
222 : $P_{3392} = (15, 2, 12, 1)$
223 : $P_{3402} = (9, 3, 12, 1)$
224 : $P_{3405} = (12, 3, 12, 1)$
225 : $P_{3422} = (13, 4, 12, 1)$
226 : $P_{3423} = (14, 4, 12, 1)$
227 : $P_{3435} = (10, 5, 12, 1)$
228 : $P_{3448} = (7, 6, 12, 1)$
229 : $P_{3455} = (14, 6, 12, 1)$
230 : $P_{3463} = (6, 7, 12, 1)$
231 : $P_{3469} = (12, 7, 12, 1)$
232 : $P_{3477} = (4, 8, 12, 1)$
233 : $P_{3480} = (7, 8, 12, 1)$
234 : $P_{3502} = (13, 9, 12, 1)$
235 : $P_{3543} = (6, 12, 12, 1)$
236 : $P_{3547} = (10, 12, 12, 1)$
237 : $P_{3586} = (1, 15, 12, 1)$
238 : $P_{3589} = (4, 15, 12, 1)$
239 : $P_{3617} = (0, 1, 13, 1)$
240 : $P_{3622} = (5, 1, 13, 1)$
241 : $P_{3645} = (12, 2, 13, 1)$
242 : $P_{3655} = (6, 3, 13, 1)$
243 : $P_{3663} = (14, 3, 13, 1)$
244 : $P_{3682} = (1, 5, 13, 1)$
245 : $P_{3695} = (14, 5, 13, 1)$
246 : $P_{3704} = (7, 6, 13, 1)$
247 : $P_{3710} = (13, 6, 13, 1)$
248 : $P_{3717} = (4, 7, 13, 1)$
249 : $P_{3719} = (6, 7, 13, 1)$
250 : $P_{3731} = (2, 8, 13, 1)$
251 : $P_{3742} = (13, 8, 13, 1)$
252 : $P_{3747} = (2, 9, 13, 1)$
253 : $P_{3750} = (5, 9, 13, 1)$
254 : $P_{3816} = (7, 13, 13, 1)$
255 : $P_{3819} = (10, 13, 13, 1)$
256 : $P_{3829} = (4, 14, 13, 1)$
257 : $P_{3837} = (12, 14, 13, 1)$
258 : $P_{3851} = (10, 15, 13, 1)$
259 : $P_{3873} = (0, 1, 14, 1)$
260 : $P_{3881} = (8, 1, 14, 1)$
261 : $P_{3923} = (2, 4, 14, 1)$
262 : $P_{3926} = (5, 4, 14, 1)$
263 : $P_{3941} = (4, 5, 14, 1)$
264 : $P_{3942} = (5, 5, 14, 1)$
265 : $P_{3961} = (8, 6, 14, 1)$
266 : $P_{3971} = (2, 7, 14, 1)$
267 : $P_{3973} = (4, 7, 14, 1)$
268 : $P_{3986} = (1, 8, 14, 1)$
269 : $P_{4129} = (0, 1, 15, 1)$
270 : $P_{4140} = (11, 1, 15, 1)$
271 : $P_{4150} = (5, 2, 15, 1)$
272 : $P_{4172} = (11, 3, 15, 1)$
273 : $P_{4176} = (15, 3, 15, 1)$
274 : $P_{4182} = (5, 4, 15, 1)$
275 : $P_{4185} = (8, 4, 15, 1)$
276 : $P_{4197} = (4, 5, 15, 1)$
277 : $P_{4206} = (13, 5, 15, 1)$
278 : $P_{4261} = (4, 9, 15, 1)$
279 : $P_{4263} = (6, 9, 15, 1)$
280 : $P_{4279} = (6, 10, 15, 1)$
281 : $P_{4288} = (15, 10, 15, 1)$
282 : $P_{4290} = (1, 11, 15, 1)$
283 : $P_{4301} = (12, 11, 15, 1)$
284 : $P_{4334} = (13, 13, 15, 1)$
285 : $P_{4344} = (7, 14, 15, 1)$
286 : $P_{4349} = (12, 14, 15, 1)$
287 : $P_{4360} = (7, 15, 15, 1)$
288 : $P_{4361} = (8, 15, 15, 1)$