Rank-74243 over GF(4)

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The equation

The equation of the surface is :

$$X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1499026777

General information

Number of lines	4
Number of points	25
Number of singular points	2
Number of Eckardt points	1
Number of double points	2
Number of single points	13
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	3, 2^2, 1^{13}, 0^9

Singular Points

The surface has 2 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 16, 336, 356)
Rank of points on Klein quadric: (0, 2, 101, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 13 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_3
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_8 = (1,0,1,0)$ lies on line ℓ_1
$5 : P_9 = (2,0,1,0)$ lies on line ℓ_1
$6 : P_{10} = (3,0,1,0)$ lies on line ℓ_1
$7 : P_{11} = (0,1,1,0)$ lies on line ℓ_2
$8 : P_{15} = (0,2,1,0)$ lies on line ℓ_2
$9 : P_{19} = (0,3,1,0)$ lies on line ℓ_2
$10 : P_{38} = (0,0,1,1)$ lies on line ℓ_3
$11 : P_{53} = (0,0,2,1)$ lies on line ℓ_3
$12 : P_{69} = (0,0,3,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

$0 : P_{27} = (1,1,0,1)$
$1 : P_{33} = (3,2,0,1)$
$2 : P_{36} = (2,3,0,1)$
$3 : P_{46} = (1,2,1,1)$
$4 : P_{50} = (1,3,1,1)$
$5 : P_{59} = (2,1,2,1)$
$6 : P_{63} = (2,2,2,1)$
$7 : P_{76} = (3,1,3,1)$
$8 : P_{84} = (3,3,3,1)$

Line Intersection Graph

	0 1 2 3
0	0 1 1 0
1	1 0 1 1
2	1 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_1	P_2	P_2

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

The surface has 25 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (1,0,1,0)$
$8: P_9 = (2,0,1,0)$
$9: P_{10} = (3,0,1,0)$
$10: P_{11} = (0,1,1,0)$
$11: P_{15} = (0,2,1,0)$
$12: P_{19} = (0,3,1,0)$
$13: P_{27} = (1,1,0,1)$
$14: P_{33} = (3,2,0,1)$
$15: P_{36} = (2,3,0,1)$
$16: P_{38} = (0,0,1,1)$
$17: P_{46} = (1,2,1,1)$
$18: P_{50} = (1,3,1,1)$
$19: P_{53} = (0,0,2,1)$
$20: P_{59} = (2,1,2,1)$
$21: P_{63} = (2,2,2,1)$
$22: P_{69} = (0,0,3,1)$
$23: P_{76} = (3,1,3,1)$
$24: P_{84} = (3,3,3,1)$