Rank-31 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 = 0$$

(1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 59

General information

Number of lines	27
Number of points	45
Number of singular points	0
Number of Eckardt points	45
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	40
Number of axes	240
Type of points on lines	5^{27}
Type of lines on points	3^{45}

Singular Points

The surface has 0 singular points:

The 27 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = a_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{40} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{40} = \mathbf{Pl}(0,0,2,3,3,1)_{325}$$

$$\ell_1 = a_2 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \mathbf{Pl}(0,0,1,1,2,1)_{258}$$

$$\ell_2 = a_3 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{25} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_3 = a_4 = \left[\begin{array}{cccc} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{75} = \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right]_{75} = \mathbf{Pl}(2,3,0,0,1,1)_{178}$$

$$\ell_4 = a_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & \omega & 0 \end{array}\right]_{170} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{array}\right]_{170} = \mathbf{Pl}(3,2,1,1,0,0)_{18}$$

$$\ell_5 = a_6 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega^2 & 0 \end{array}\right]_{87} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 \end{array}\right]_{87} = \mathbf{Pl}(2,3,3,1,0,0)_{23}$$

$$\ell_6 = b_1 = \left[\begin{array}{cccc} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{67} = \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{67} = \mathbf{Pl}(1,1,0,0,2,1)_{237}$$

$$\ell_7 = b_2 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{33} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right]_{33} = \mathbf{Pl}(2,3,0,0,3,1)_{298}$$

$$\ell_8 = b_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega & 0 \end{array}\right]_{86} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right]_{86} = \mathbf{Pl}(3,2,2,1,0,0)_{21}$$

$$\ell_9 = b_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & \omega^2 & 0 \end{array}\right]_{171} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 \end{array}\right]_{171} = \mathbf{Pl}(2,3,2,1,0,0)_{20}$$

$$\ell_{10} = b_5 = \left[\begin{array}{cccc} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{82} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{82} = \mathbf{Pl}(0,0,2,3,1,1)_{205}$$

$$\ell_{11} = b_6 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{38} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_{12} = c_{12} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & 0 \end{array}\right]_{253} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \end{array}\right]_{253} = \mathbf{Pl}(1,1,2,1,0,0)_{19}$$

$$\ell_{13} = c_{13} = \left[\begin{array}{cccc} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{46} = \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{46} = \mathbf{Pl}(1,1,0,0,3,1)_{297}$$

$$\ell_{14} = c_{14} = \left[\begin{array}{cccc} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega \end{array}\right]_{71} = \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]_{71} = \mathbf{Pl}(3,2,0,0,3,1)_{299}$$

$$\ell_{15} = c_{15} = \left[\begin{array}{cccc} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{61} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{61} = \mathbf{Pl}(0,0,2,3,2,1)_{265}$$

$$\ell_{16} = c_{16} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{39} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{39} = \mathbf{Pl}(0,0,3,2,2,1)_{272}$$

$$\ell_{17} = c_{23} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \omega \end{array}\right]_{29} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]_{29} = \mathbf{Pl}(3,2,0,0,2,1)_{239}$$

$$\ell_{18} = c_{24} = \left[\begin{array}{cccc} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega^2 \end{array}\right]_{54} = \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right]_{54} = \mathbf{Pl}(2,3,0,0,2,1)_{238}$$

$$\ell_{19} = c_{25} = \left[\begin{array}{cccc} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{81} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{81} = \mathbf{Pl}(0,0,3,2,3,1)_{332}$$

$$\ell_{20} = c_{26} = \left[\begin{array}{cccc} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{59} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{59} = \mathbf{Pl}(0,0,1,1,3,1)_{318}$$

$$\ell_{21} = c_{34} = \left[\begin{array}{cccc} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{60} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{60} = \mathbf{Pl}(0,0,3,2,1,1)_{212}$$

$$\ell_{22} = c_{35} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & \omega & 0 \end{array}\right]_{254} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 0 \end{array}\right]_{254} = \mathbf{Pl}(3,2,3,1,0,0)_{24}$$

$$\ell_{23} = c_{36} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{85} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{85} = \mathbf{Pl}(1,1,1,1,0,0)_{16}$$

$$\ell_{24} = c_{45} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & 0 \end{array}\right]_{169} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right]_{169} = \mathbf{Pl}(1,1,3,1,0,0)_{22}$$

$$\ell_{25} = c_{46} = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & \omega^2 & 0 \end{array}\right]_{255} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{array}\right]_{255} = \mathbf{Pl}(2,3,1,1,0,0)_{17}$$

$$\ell_{26} = c_{56} = \left[\begin{array}{cccc} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega \end{array}\right]_{50} = \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]_{50} = \mathbf{Pl}(3,2,0,0,1,1)_{179}$$

Rank of lines: (40, 80, 25, 75, 170, 87, 67, 33, 86, 171, 82, 38, 253, 46, 71, 61, 39, 29, 54, 81, 59, 60, 254, 85, 169, 255, 50)
Rank of points on Klein quadric: (325, 258, 177, 178, 18, 23, 237, 298, 21, 20, 205, 198, 19, 297, 299, 265, 272, 239, 238, 332, 318, 212, 24, 16, 22, 17, 179)

Eckardt Points

The surface has 45 Eckardt points:
$0 : E_{36} = a_3 \cap b_6 \cap c_{36} = P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$,
$1 : E_{16} = a_1 \cap b_6 \cap c_{16} = P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$,
$2 : E_{25} = a_2 \cap b_5 \cap c_{25} = P_6 = \mathbf{P}(\omega,1,0,0) = \mathbf{P}(2,1,0,0)$,
$3 : E_{15,26,34} = c_{15} \cap c_{26} \cap c_{34} = P_7 = \mathbf{P}(\omega^2,1,0,0) = \mathbf{P}(3,1,0,0)$,
$4 : E_{32} = a_3 \cap b_2 \cap c_{23} = P_8 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$,
$5 : E_{41} = a_4 \cap b_1 \cap c_{14} = P_9 = \mathbf{P}(\omega,0,1,0) = \mathbf{P}(2,0,1,0)$,
$6 : E_{13,24,56} = c_{13} \cap c_{24} \cap c_{56} = P_{10} = \mathbf{P}(\omega^2,0,1,0) = \mathbf{P}(3,0,1,0)$,
$7 : E_{12,36,45} = c_{12} \cap c_{36} \cap c_{45} = P_{11} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$,
$8 : E_{64} = a_6 \cap b_4 \cap c_{46} = P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0)$,
$9 : E_{53} = a_5 \cap b_3 \cap c_{35} = P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0)$,
$10 : E_{63} = a_6 \cap b_3 \cap c_{36} = P_{23} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$,
$11 : E_{12,35,46} = c_{12} \cap c_{35} \cap c_{46} = P_{24} = \mathbf{P}(\omega,0,0,1) = \mathbf{P}(2,0,0,1)$,
$12 : E_{54} = a_5 \cap b_4 \cap c_{45} = P_{25} = \mathbf{P}(\omega^2,0,0,1) = \mathbf{P}(3,0,0,1)$,
$13 : E_{31} = a_3 \cap b_1 \cap c_{13} = P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$,
$14 : E_{42} = a_4 \cap b_2 \cap c_{24} = P_{30} = \mathbf{P}(0,\omega,0,1) = \mathbf{P}(0,2,0,1)$,
$15 : E_{14,23,56} = c_{14} \cap c_{23} \cap c_{56} = P_{34} = \mathbf{P}(0,\omega^2,0,1) = \mathbf{P}(0,3,0,1)$,
$16 : E_{26} = a_2 \cap b_6 \cap c_{26} = P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$,
$17 : E_{21} = a_2 \cap b_1 \cap c_{12} = P_{43} = \mathbf{P}(\omega,1,1,1) = \mathbf{P}(2,1,1,1)$,
$18 : E_{13,26,45} = c_{13} \cap c_{26} \cap c_{45} = P_{44} = \mathbf{P}(\omega^2,1,1,1) = \mathbf{P}(3,1,1,1)$,
$19 : E_{62} = a_6 \cap b_2 \cap c_{26} = P_{46} = \mathbf{P}(1,\omega,1,1) = \mathbf{P}(1,2,1,1)$,
$20 : E_{46} = a_4 \cap b_6 \cap c_{46} = P_{47} = \mathbf{P}(\omega,\omega,1,1) = \mathbf{P}(2,2,1,1)$,
$21 : E_{24} = a_2 \cap b_4 \cap c_{24} = P_{48} = \mathbf{P}(\omega^2,\omega,1,1) = \mathbf{P}(3,2,1,1)$,
$22 : E_{23} = a_2 \cap b_3 \cap c_{23} = P_{50} = \mathbf{P}(1,\omega^2,1,1) = \mathbf{P}(1,3,1,1)$,
$23 : E_{14,26,35} = c_{14} \cap c_{26} \cap c_{35} = P_{51} = \mathbf{P}(\omega,\omega^2,1,1) = \mathbf{P}(2,3,1,1)$,
$24 : E_{56} = a_5 \cap b_6 \cap c_{56} = P_{52} = \mathbf{P}(\omega^2,\omega^2,1,1) = \mathbf{P}(3,3,1,1)$,
$25 : E_{15} = a_1 \cap b_5 \cap c_{15} = P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$,
$26 : E_{13} = a_1 \cap b_3 \cap c_{13} = P_{58} = \mathbf{P}(1,1,\omega,1) = \mathbf{P}(1,1,2,1)$,
$27 : E_{35} = a_3 \cap b_5 \cap c_{35} = P_{59} = \mathbf{P}(\omega,1,\omega,1) = \mathbf{P}(2,1,2,1)$,
$28 : E_{51} = a_5 \cap b_1 \cap c_{15} = P_{60} = \mathbf{P}(\omega^2,1,\omega,1) = \mathbf{P}(3,1,2,1)$,
$29 : E_{15,24,36} = c_{15} \cap c_{24} \cap c_{36} = P_{62} = \mathbf{P}(1,\omega,\omega,1) = \mathbf{P}(1,2,2,1)$,
$30 : E_{12} = a_1 \cap b_2 \cap c_{12} = P_{63} = \mathbf{P}(\omega,\omega,\omega,1) = \mathbf{P}(2,2,2,1)$,
$31 : E_{45} = a_4 \cap b_5 \cap c_{45} = P_{64} = \mathbf{P}(\omega^2,\omega,\omega,1) = \mathbf{P}(3,2,2,1)$,
$32 : E_{65} = a_6 \cap b_5 \cap c_{56} = P_{66} = \mathbf{P}(1,\omega^2,\omega,1) = \mathbf{P}(1,3,2,1)$,
$33 : E_{15,23,46} = c_{15} \cap c_{23} \cap c_{46} = P_{67} = \mathbf{P}(\omega,\omega^2,\omega,1) = \mathbf{P}(2,3,2,1)$,
$34 : E_{14} = a_1 \cap b_4 \cap c_{14} = P_{68} = \mathbf{P}(\omega^2,\omega^2,\omega,1) = \mathbf{P}(3,3,2,1)$,

$35 : E_{16,25,34} = c_{16} \cap c_{25} \cap c_{34} = P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1),$
$36 : E_{61} = a_6 \cap b_1 \cap c_{16} = P_{74} = \mathbf{P}(1, 1, \omega^2, 1) = \mathbf{P}(1, 1, 3, 1),$
$37 : E_{13,25,46} = c_{13} \cap c_{25} \cap c_{46} = P_{75} = \mathbf{P}(\omega, 1, \omega^2, 1) = \mathbf{P}(2, 1, 3, 1),$
$38 : E_{34} = a_3 \cap b_4 \cap c_{34} = P_{76} = \mathbf{P}(\omega^2, 1, \omega^2, 1) = \mathbf{P}(3, 1, 3, 1),$
$39 : E_{43} = a_4 \cap b_3 \cap c_{34} = P_{78} = \mathbf{P}(1, \omega, \omega^2, 1) = \mathbf{P}(1, 2, 3, 1),$
$40 : E_{16,24,35} = c_{16} \cap c_{24} \cap c_{35} = P_{79} = \mathbf{P}(\omega, \omega, \omega^2, 1) = \mathbf{P}(2, 2, 3, 1),$
$41 : E_{52} = a_5 \cap b_2 \cap c_{25} = P_{80} = \mathbf{P}(\omega^2, \omega, \omega^2, 1) = \mathbf{P}(3, 2, 3, 1),$
$42 : E_{14,25,36} = c_{14} \cap c_{25} \cap c_{36} = P_{82} = \mathbf{P}(1, \omega^2, \omega^2, 1) = \mathbf{P}(1, 3, 3, 1),$
$43 : E_{12,34,56} = c_{12} \cap c_{34} \cap c_{56} = P_{83} = \mathbf{P}(\omega, \omega^2, \omega^2, 1) = \mathbf{P}(2, 3, 3, 1),$
$44 : E_{16,23,45} = c_{16} \cap c_{23} \cap c_{45} = P_{84} = \mathbf{P}(\omega^2, \omega^2, \omega^2, 1) = \mathbf{P}(3, 3, 3, 1).$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
		a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{23}	c_{24}	c_{25}	c_{26}	c_{34}	c_{35}	c_{36}	c_{45}	c_{46}	c_{56}
0	a_1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	a_2	0	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
2	a_3	0	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
3	a_4	0	0	0	0	0	0	1	1	1	0	1	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
4	a_5	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
5	a_6	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
6	b_1	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	b_2	1	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
8	b_3	1	1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
9	b_4	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0
10	b_5	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
11	b_6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1
12	c_{12}	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
13	c_{13}	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1
14	c_{14}	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0	0	1
15	c_{15}	1	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0	1	0	1	0
16	c_{16}	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	1	1	0	1	0	0
17	c_{23}	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	1
18	c_{24}	0	1	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	0	1
19	c_{25}	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0	1	0	1	0
20	c_{26}	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0
21	c_{34}	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	1
22	c_{35}	0	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	1	0
23	c_{36}	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	1	1	0	0	0	0	1	0	0
24	c_{45}	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0
25	c_{46}	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0	0	1	0	0	0	0
26	c_{56}	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	0	0	0	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{63}	P_{58}	P_{68}	P_{53}	P_5	P_{63}	P_{58}	P_{68}	P_{53}	P_5

Line 1 intersects

Line	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{43}	P_{50}	P_{48}	P_6	P_{38}	P_{43}	P_{50}	P_{48}	P_6	P_{38}

Line 2 intersects

Line	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{26}	P_8	P_{76}	P_{59}	P_4	P_{26}	P_8	P_{76}	P_{59}	P_4

Line 3 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_9	P_{30}	P_{78}	P_{64}	P_{47}	P_9	P_{30}	P_{78}	P_{64}	P_{47}

Line 4 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{60}	P_{80}	P_{19}	P_{25}	P_{52}	P_{60}	P_{80}	P_{19}	P_{25}	P_{52}

Line 5 intersects

Line	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_{74}	P_{46}	P_{23}	P_{15}	P_{66}	P_{74}	P_{46}	P_{23}	P_{15}	P_{66}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{43}	P_{26}	P_9	P_{60}	P_{74}	P_{43}	P_{26}	P_9	P_{60}	P_{74}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{63}	P_8	P_{30}	P_{80}	P_{46}	P_{63}	P_8	P_{30}	P_{80}	P_{46}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_{13}	ℓ_{17}	ℓ_{21}	ℓ_{22}	ℓ_{23}
in point	P_{58}	P_{50}	P_{78}	P_{19}	P_{23}	P_{58}	P_{50}	P_{78}	P_{19}	P_{23}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_{14}	ℓ_{18}	ℓ_{21}	ℓ_{24}	ℓ_{25}
in point	P_{68}	P_{48}	P_{76}	P_{25}	P_{15}	P_{68}	P_{48}	P_{76}	P_{25}	P_{15}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_{15}	ℓ_{19}	ℓ_{22}	ℓ_{24}	ℓ_{26}
in point	P_{53}	P_6	P_{59}	P_{64}	P_{66}	P_{53}	P_6	P_{59}	P_{64}	P_{66}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_{16}	ℓ_{20}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_5	P_{38}	P_4	P_{47}	P_{52}	P_5	P_{38}	P_4	P_{47}	P_{52}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{63}	P_{43}	P_{43}	P_{63}	P_{83}	P_{24}	P_{11}	P_{11}	P_{24}	P_{83}

Line 13 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{58}	P_{26}	P_{26}	P_{58}	P_{10}	P_{75}	P_{44}	P_{44}	P_{75}	P_{10}

Line 14 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_9	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{68}	P_9	P_9	P_{68}	P_{34}	P_{82}	P_{51}	P_{51}	P_{82}	P_{34}

Line 15 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_{53}	P_{60}	P_{60}	P_{53}	P_{67}	P_{62}	P_7	P_7	P_{62}	P_{67}

Line 16 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_5	P_{74}	P_{74}	P_5	P_{84}	P_{79}	P_{69}	P_{69}	P_{79}	P_{84}

Line 17 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{50}	P_8	P_8	P_{50}	P_{34}	P_{67}	P_{84}	P_{84}	P_{67}	P_{34}

Line 18 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{48}	P_{30}	P_{30}	P_{48}	P_{10}	P_{62}	P_{79}	P_{79}	P_{62}	P_{10}

Line 19 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{21}	ℓ_{23}	ℓ_{25}
in point	P_6	P_{80}	P_{80}	P_6	P_{75}	P_{82}	P_{69}	P_{69}	P_{82}	P_{75}

Line 20 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{21}	ℓ_{22}	ℓ_{24}
in point	P_{38}	P_{46}	P_{46}	P_{38}	P_{44}	P_{51}	P_7	P_7	P_{51}	P_{44}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{26}
in point	P_{76}	P_{78}	P_{78}	P_{76}	P_{83}	P_7	P_{69}	P_{69}	P_7	P_{83}

Line 22 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{20}	ℓ_{25}
in point	P_{59}	P_{19}	P_{19}	P_{59}	P_{24}	P_{51}	P_{79}	P_{79}	P_{51}	P_{24}

Line 23 intersects

Line	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{19}	ℓ_{24}
in point	P_4	P_{23}	P_{23}	P_4	P_{11}	P_{82}	P_{62}	P_{62}	P_{82}	P_{11}

Line 24 intersects

Line	ℓ_3	ℓ_4	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{20}	ℓ_{23}
in point	P_{64}	P_{25}	P_{25}	P_{64}	P_{11}	P_{44}	P_{84}	P_{84}	P_{44}	P_{11}

Line 25 intersects

Line	ℓ_3	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{22}
in point	P_{47}	P_{15}	P_{15}	P_{47}	P_{24}	P_{75}	P_{67}	P_{67}	P_{75}	P_{24}

Line 26 intersects

Line	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{21}
in point	P_{52}	P_{66}	P_{66}	P_{52}	P_{83}	P_{10}	P_{34}	P_{34}	P_{10}	P_{83}

The surface has 45 points:
The points on the surface are:

0 : $P_4 = (1,1,1,1)$
1 : $P_5 = (1,1,0,0)$
2 : $P_6 = (2,1,0,0)$
3 : $P_7 = (3,1,0,0)$
4 : $P_8 = (1,0,1,0)$
5 : $P_9 = (2,0,1,0)$
6 : $P_{10} = (3,0,1,0)$
7 : $P_{11} = (0,1,1,0)$
8 : $P_{15} = (0,2,1,0)$
9 : $P_{19} = (0,3,1,0)$
10 : $P_{23} = (1,0,0,1)$
11 : $P_{24} = (2,0,0,1)$
12 : $P_{25} = (3,0,0,1)$
13 : $P_{26} = (0,1,0,1)$
14 : $P_{30} = (0,2,0,1)$
15 : $P_{34} = (0,3,0,1)$
16 : $P_{38} = (0,0,1,1)$
17 : $P_{43} = (2,1,1,1)$
18 : $P_{44} = (3,1,1,1)$
19 : $P_{46} = (1,2,1,1)$
20 : $P_{47} = (2,2,1,1)$
21 : $P_{48} = (3,2,1,1)$
22 : $P_{50} = (1,3,1,1)$
23 : $P_{51} = (2,3,1,1)$
24 : $P_{52} = (3,3,1,1)$
25 : $P_{53} = (0,0,2,1)$
26 : $P_{58} = (1,1,2,1)$

$27: P_{59} = (2,1,2,1)$
$28: P_{60} = (3,1,2,1)$
$29: P_{62} = (1,2,2,1)$
$30: P_{63} = (2,2,2,1)$
$31: P_{64} = (3,2,2,1)$
$32: P_{66} = (1,3,2,1)$
$33: P_{67} = (2,3,2,1)$
$34: P_{68} = (3,3,2,1)$
$35: P_{69} = (0,0,3,1)$
$36: P_{74} = (1,1,3,1)$
$37: P_{75} = (2,1,3,1)$
$38: P_{76} = (3,1,3,1)$
$39: P_{78} = (1,2,3,1)$
$40: P_{79} = (2,2,3,1)$
$41: P_{80} = (3,2,3,1)$
$42: P_{82} = (1,3,3,1)$
$43: P_{83} = (2,3,3,1)$
$44: P_{84} = (3,3,3,1)$