Rank-74295 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is -2113658811

General information

Number of lines	21
Number of points	4481
Number of singular points	1
Number of Eckardt points	0
Number of double points	75
Number of single points	1209
Number of points off lines	3196
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{21}
Type of lines on points	$6, 2^{75}, 1^{1209}, 0^{3196}$

Singular Points

The surface has 1 singular points:

$$0: P_{68} = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4161} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4161} = \mathbf{Pl}(1, 0, 0, 0, 0, 1)_{270402}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \mathbf{PI}(1,0,1,0,1,0)_{4353}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \epsilon^{42} & 1 \end{bmatrix}_{4281} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 56 & 1 \end{bmatrix}_{4281} = \mathbf{PI}(1,1,56,0,1,1)_{540105}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \epsilon^{42} & 1 \end{bmatrix}_{4282} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 57 & 1 \end{bmatrix}_{4282} = \mathbf{PI}(1,1,57,0,1,1)_{540108}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \epsilon^{45} & \epsilon^9 \end{bmatrix}_{7206} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 37 & 47 \end{bmatrix}_{7206} = \mathbf{PI}(10,47,36,0,47,1)_{12594534}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \epsilon^{54} & \epsilon^{9} \end{bmatrix}_{4911} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & 36 \end{bmatrix}_{6475} = \mathbf{PI}(10,47,36,0,47,1)_{12594534}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \epsilon^{54} & \epsilon^{36} \end{bmatrix}_{6475} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & 36 \end{bmatrix}_{6475} = \mathbf{PI}(46,36,11,0,36,1)_{9710115}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \epsilon^{49} & \epsilon^4 \\ 0 & 1 & \epsilon^{54} & \epsilon^{36} \end{bmatrix}_{4389200} = \begin{bmatrix} 1 & 0 & 30 & 16 \\ 0 & 1 & 50 & 54 \end{bmatrix}_{4389200} = \mathbf{PI}(15,21,37,11,34,1)_{3342605}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \epsilon^{26} & \epsilon^4 \\ 0 & 1 & \epsilon^{54} & \epsilon^{36} \end{bmatrix}_{2850700} = \begin{bmatrix} 1 & 0 & 36 & 1 \\ 0 & 1 & 10 & 36 \end{bmatrix}_{2850700} = \mathbf{PI}(10,47,36,46,47,1)_{12747183}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \epsilon^{28} & \epsilon^6 \\ 0 & 1 & \epsilon^{13} & \epsilon^{36} \end{bmatrix}_{146751} = \begin{bmatrix} 1 & 0 & 36 & 1 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{416751} = \mathbf{PI}(10,47,36,46,11,1)_{3310602}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \epsilon^{35} & \epsilon^8 \\ 0 & 1 & \epsilon^{13} & \epsilon^{36} \end{bmatrix}_{10461550} = \begin{bmatrix} 1 & 0 & 18 & 39 \\ 0 & 1 & 28 & 12 \end{bmatrix}_{10461550} = \mathbf{PI}(62,25,46,36,31,1)_{859453}$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & \epsilon^{35} & \epsilon^8 \\ 0 & 1 & \epsilon^{35} & \epsilon^8 \end{bmatrix}_{12404991} = \begin{bmatrix} 1 & 0 & 13 & 37 \\ 0 & 1 & 37 & 47 \end{bmatrix}_{987903} = \mathbf{PI}(37,11,47,10,11,1)_{3355863}$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^{34} \\ 0 & 1 & \epsilon^{38} & \epsilon^{51} \end{bmatrix}_{1260491} = \begin{bmatrix} 1 & 0 & 37 & 46 \\ 0 & 1 & 36 & 37 \end{bmatrix}_{461275} = \mathbf{PI}(46,36,11,37,36,1)_{9907296}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^{34} \\ 0 & 1 & \epsilon^{36} & \epsilon^{45} \end{bmatrix}_{12404991} = \begin{bmatrix} 1 & 0 & 37 & 46 \\ 0 & 1 & 46 & 11 \end{bmatrix}_{1240491} = \mathbf{PI}(46,36,11,37,47,1)_{12050136}$$

$$\ell_{15} = \begin{bmatrix} 1 & 0 & \epsilon^{45} & \epsilon^{27} \\ 0 & 1 & \epsilon^{27} & \epsilon^{18} \end{bmatrix}_{13404991} = \begin{bmatrix} 1 & 0 & 37 & 46 \\ 0 & 1 & 46 & 11 \end{bmatrix}_{12404991} = \mathbf{PI}(46,36,11,37,47,1)_{1205$$

Rank of lines: (4161, 65, 4281, 4282, 7206, 4911, 6475, 4389200, 2856760, 416751, 786906, 10461550, 9897903, 464275, 7166893, 12404691, 315066, 1233023, 15146160, 15416626, 11333561)Rank of points on Klein quadric: (270402, 4353, 540105, 540168, 12594534, 3160374, 9710115, 9342605, 12747183, 3310602, 8594653, 15932253, 3355863, 9907296, 11178317, 9764988, 12650136, 5307863, 15232516, 14970500, 16050901)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 75 Double points: The double points on the surface are:

$P_{772} = (1, 11, 1, 0) = \ell_0 \cap \ell_8$
$P_{2372} = (1, 36, 1, 0) = \ell_0 \cap \ell_{12}$
$P_{3076} = (1, 47, 1, 0) = \ell_0 \cap \ell_{15}$
$P_{3652} = (1, 56, 1, 0) = \ell_0 \cap \ell_{18}$
$P_{3716} = (1, 57, 1, 0) = \ell_0 \cap \ell_{19}$
$P_{8378} = (57, 1, 1, 1) = \ell_1 \cap \ell_2$
$P_{8377} = (56, 1, 1, 1) = \ell_1 \cap \ell_3$
$P_{8331} = (10, 1, 1, 1) = \ell_1 \cap \ell_8$
$P_{8332} = (11, 1, 1, 1) = \ell_1 \cap \ell_9$
$P_{8358} = (37, 1, 1, 1) = \ell_1 \cap \ell_{12}$
$P_{8357} = (36, 1, 1, 1) = \ell_1 \cap \ell_{13}$
$P_{8367} = (46, 1, 1, 1) = \ell_1 \cap \ell_{15}$
$P_{8368} = (47, 1, 1, 1) = \ell_1 \cap \ell_{16}$
$P_{172178} = (17, 1, 41, 1) = \ell_2 \cap \ell_7$
$P_{245892} = (3, 1, 59, 1) = \ell_2 \cap \ell_{10}$
$P_{233601} = (0, 1, 56, 1) = \ell_2 \cap \ell_{18}$
$P_{82092} = (43, 1, 19, 1) = \ell_2 \cap \ell_{20}$
$P_{131239} = (38, 1, 31, 1) = \ell_3 \cap \ell_{11}$
$P_{143516} = (27, 1, 34, 1) = \ell_3 \cap \ell_{14}$
$P_{249990} = (5, 1, 60, 1) = \ell_3 \cap \ell_{17}$
$P_{237697} = (0, 1, 57, 1) = \ell_3 \cap \ell_{19}$
$P_{4838} = (36, 10, 0, 1) = \ell_4 \cap \ell_9$
$P_{66284} = (43, 10, 15, 1) = \ell_4 \cap \ell_{10}$
$P_{139974} = (5, 10, 33, 1) = \ell_4 \cap \ell_{11}$
$P_{152257} = (0, 10, 36, 1) = \ell_4 \cap \ell_{12}$
$P_{193227} = (0, 10, 46, 1) = \ell_4 \cap \ell_{16}$ $P_{193227} = (10, 10, 46, 1) = \ell_4 \cap \ell_{16}$
$P_{47526} = (37, 37, 10, 1) = \ell_5 \cap \ell_9$
$P_{6577} = (47, 37, 0, 1) = \ell_5 \cap \ell_{13}$
$P_{199041} = (0, 37, 47, 1) = \ell_5 \cap \ell_{15}$
$F_{199041} = (0, 37, 47, 1) = \ell_5 + \ell_{15}$
$P_{219548} = (27, 37, 52, 1) = \ell_5 \cap \ell_{17}$
$P_{260498} = (17, 37, 62, 1) = \ell_5 \cap \ell_{20}$
$P_{39876} = (3, 46, 8, 1) = \ell_6 \cap \ell_7$
$P_{52161} = (0, 46, 11, 1) = \ell_6 \cap \ell_8$
$P_{158703} = (46, 46, 37, 1) = \ell_6 \cap \ell_{13}$
$P_{191463} = (38, 46, 45, 1) = \ell_6 \cap \ell_{14}$
$P_{7117} = (11, 46, 0, 1) = \ell_6 \cap \ell_{16}$
$P_{17973} = (52, 23, 3, 1) = \ell_7 \cap \ell_9$
$P_{106925} = (44, 5, 25, 1) = \ell_7 \cap \ell_{11}$

$P_{32803} = (34, 63, 6, 1) = \ell_7 \cap \ell_{12}$ $P_{103352} = (55, 13, 24, 1) = \ell_7 \cap \ell_{15}$ $P_{205120} = (63, 3, 49, 1) = \ell_7 \cap \ell_{17}$ $P_{152158} = (29, 8, 36, 1) = \ell_7 \cap \ell_{19}$ $P_{156463} = (46, 11, 37, 1) = \ell_8 \cap \ell_9$ $P_{58695} = (6, 20, 13, 1) = \ell_8 \cap \ell_{10}$ $P_{31822} = (13, 48, 6, 1) = \ell_8 \cap \ell_{11}$ $P_{104532} = (19, 32, 24, 1) = \ell_8 \cap \ell_{17}$ $P_{230397} = (60, 14, 55, 1) = \ell_8 \cap \ell_{20}$ $P_{161279} = (62, 22, 38, 1) = \ell_9 \cap \ell_{14}$ $P_{242755} = (2, 16, 58, 1) = \ell_9 \cap \ell_{18}$ $P_{128744} = (39, 26, 30, 1) = \ell_9 \cap \ell_{19}$ $P_{183598} = (45, 51, 43, 1) = \ell_{10} \cap \ell_{13}$ $P_{228106} = (9, 43, 54, 1) = \ell_{10} \cap \ell_{14}$ $P_{201376} = (31, 9, 48, 1) = \ell_{10} \cap \ell_{15}$ $P_{55073} = (32, 27, 12, 1) = \ell_{10} \cap \ell_{17}$ $P_{197655} = (22, 15, 47, 1) = \ell_{10} \cap \ell_{19}$ $P_{27849} = (8, 50, 5, 1) = \ell_{11} \cap \ell_{13}$ $P_{88956} = (59, 44, 20, 1) = \ell_{11} \cap \ell_{15}$ $P_{198808} = (23, 33, 47, 1) = \ell_{11} \cap \ell_{18}$ $P_{33935} = (14, 17, 7, 1) = \ell_{11} \cap \ell_{20}$ $P_{194891} = (10, 36, 46, 1) = \ell_{12} \cap \ell_{13}$ $P_{60842} = (41, 53, 13, 1) = \ell_{12} \cap \ell_{14}$ $P_{204309} = (20, 55, 48, 1) = \ell_{12} \cap \ell_{17}$ $P_{87665} = (48, 24, 20, 1) = \ell_{12} \cap \ell_{20}$ $P_{78059} = (42, 2, 18, 1) = \ell_{13} \cap \ell_{18}$ $P_{256517} = (4, 39, 61, 1) = \ell_{13} \cap \ell_{19}$ $P_{229849} = (24, 6, 55, 1) = \ell_{14} \cap \ell_{15}$ $P_{154525} = (28, 45, 36, 1) = \ell_{14} \cap \ell_{18}$ $P_{92662} = (53, 38, 21, 1) = \ell_{14} \cap \ell_{20}$ $P_{48166} = (37, 47, 10, 1) = \ell_{15} \cap \ell_{16}$ $P_{116624} = (15, 29, 27, 1) = \ell_{16} \cap \ell_{17}$ $P_{170705} = (16, 42, 40, 1) = \ell_{16} \cap \ell_{18}$ $P_{147803} = (26, 4, 35, 1) = \ell_{16} \cap \ell_{19}$ $P_{75618} = (33, 28, 17, 1) = \ell_{16} \cap \ell_{20}$ $P_{52596} = (51, 52, 11, 1) = \ell_{17} \cap \ell_{18}$ $P_{53235} = (50, 62, 11, 1) = \ell_{19} \cap \ell_{20}$

Single Points

The surface has 1209 single points: Too many to print.

Points on surface but on no line

The surface has 3196 points not on any line: Too many to print.

Line Intersection Graph

	0123456789	10	11	12	13	14	15	16	17	18	19	20
0	0011111010	0	0	1	0	0	1	0	0	1	1	0
1	0011000011	0	0	1	1	0	1	1	0	0	0	0
2	1101111100	1	0	0	0	0	0	0	0	1	0	1
3	1110111000	0	1	0	0	1	0	0	1	0	1	0
4	1011011001	1	1	1	0	0	0	1	0	0	0	0
5	1011101001	0	0	0	1	0	1	0	1	0	0	1
6	1011110110	0	0	0	1	1	0	1	0	0	0	0
7	0010001001	0	1	1	0	0	1	0	1	0	1	0
8	1100001001	1	1	0	0	0	0	0	1	0	0	1
9	0100110110	0	0	0	0	1	0	0	0	1	1	0
10	0010100010	0	0	0	1	1	1	0	1	0	1	0
11	0001100110	0	0	0	1	0	1	0	0	1	0	1
12	1100100100	0	0	0	1	1	0	0	1	0	0	1
13	0100011000	1	1	1	0	0	0	0	0	1	1	0
14	0001001001	1	0	1	0	0	1	0	0	1	0	1
15	1100010100	1	1	0	0	1	0	1	0	0	0	0
16	0100101000	0	0	0	0	0	1	0	1	1	1	1
17	0001010110	1	0	1	0	0	0	1	0	1	0	0
18	1010000001	0	1	0	1	1	0	1	1	0	0	0
19	1001000101	1	0	0	1	0	0	1	0	0	0	1
20	0010010010	0	1	1	0	1	0	1	0	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{12}	ℓ_{15}	ℓ_{18}	ℓ_{19}
in point	P_{68}	P_{68}	P_{68}	P_{68}	P_{68}	P_{772}	P_{2372}	P_{3076}	P_{3652}	P_{3716}

Line 1 intersects

	Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}
in	point	P_{8378}	P_{8377}	P_{8331}	P_{8332}	P_{8358}	P_{8357}	P_{8367}	P_{8368}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{18}	ℓ_{20}
in point	P_{68}	P_{8378}	P_{68}	P_{68}	P_{68}	P_{68}	P_{172178}	P_{245892}	P_{233601}	P_{82092}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{14}	ℓ_{17}	ℓ_{19}
in point	P_{68}	P_{8377}	P_{68}	P_{68}	P_{68}	P_{68}	P_{131239}	P_{143516}	P_{249990}	P_{237697}

Line 4 intersects

	Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{16}
Ī	in point	P_{68}	P_{68}	P_{68}	P_{68}	P_{68}	P_{4838}	P_{66284}	P_{139974}	P_{152257}	P_{193227}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{20}
in point	P_{68}	P_{68}	P_{68}	P_{68}	P_{68}	P_{47526}	P_{6577}	P_{199041}	P_{219548}	P_{260498}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_{13}	ℓ_{14}	ℓ_{16}
in point	P_{68}	P_{68}	P_{68}	P_{68}	P_{68}	P_{39876}	P_{52161}	P_{158703}	P_{191463}	P_{7117}

Line 7 intersects

Line	ℓ_2	ℓ_6	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{19}
in point	P_{172178}	P_{39876}	P_{17973}	P_{106925}	P_{32803}	P_{103352}	P_{205120}	P_{152158}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{17}	ℓ_{20}
in point	P_{772}	P_{8331}	P_{52161}	P_{156463}	P_{58695}	P_{31822}	P_{104532}	P_{230397}

${\bf Line~9~intersects}$

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_{14}	ℓ_{18}	ℓ_{19}
in point	P_{8332}	P_{4838}	P_{47526}	P_{17973}	P_{156463}	P_{161279}	P_{242755}	P_{128744}

Line 10 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{19}
in point	P_{245892}	P_{66284}	P_{58695}	P_{183598}	P_{228106}	P_{201376}	P_{55073}	P_{197655}

Line 11 intersects

Line	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_{13}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_{131239}	P_{139974}	P_{106925}	P_{31822}	P_{27849}	P_{88956}	P_{198808}	P_{33935}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_7	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{20}
in point	P_{2372}	P_{8358}	P_{152257}	P_{32803}	P_{194891}	P_{60842}	P_{204309}	P_{87665}

${\rm Line}\ 13\ {\rm intersects}$

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{18}	ℓ_{19}
in point	P_{8357}	P_{6577}	P_{158703}	P_{183598}	P_{27849}	P_{194891}	P_{78059}	P_{256517}

Line 14 intersects

Line	ℓ_3	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_{143516}	P_{191463}	P_{161279}	P_{228106}	P_{60842}	P_{229849}	P_{154525}	P_{92662}

${\bf Line~15~intersects}$

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{16}
in point	P_{3076}	P_{8367}	P_{199041}	P_{103352}	P_{201376}	P_{88956}	P_{229849}	P_{48166}

Line 16 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{8368}	P_{193227}	P_{7117}	P_{48166}	P_{116624}	P_{170705}	P_{147803}	P_{75618}

Line 17 intersects

Line	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{16}	ℓ_{18}
in point	P_{249990}	P_{219548}	P_{205120}	P_{104532}	P_{55073}	P_{204309}	P_{116624}	P_{52596}

${\bf Line~18~intersects}$

Line	ℓ_0	ℓ_2	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}
in point	P_{3652}	P_{233601}	P_{242755}	P_{198808}	P_{78059}	P_{154525}	P_{170705}	P_{52596}

Line 19 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{16}	ℓ_{20}
in point	P_{3716}	P_{237697}	P_{152158}	P_{128744}	P_{197655}	P_{256517}	P_{147803}	P_{53235}

Line 20 intersects

Line	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{19}
in point	P_{82092}	P_{260498}	P_{230397}	P_{33935}	P_{87665}	P_{92662}	P_{75618}	P_{53235}

The surface has 4481 points:

Too many to print.