

Rank-76307 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1503221337

General information

Number of lines	16
Number of points	37
Number of singular points	2
Number of Eckardt points	3
Number of double points	29
Number of single points	3
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{16}
Type of lines on points	$5^2, 3^3, 2^{29}, 1^3$

Singular Points

The surface has 2 singular points:

$$0 : P_9 = \mathbf{P}(\omega, 0, 1, 0) = \mathbf{P}(2, 0, 1, 0)$$

$$1 : P_{10} = \mathbf{P}(\omega^2, 0, 1, 0) = \mathbf{P}(3, 0, 1, 0)$$

The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{42} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{42} = \mathbf{Pl}(3, 0, 0, 0, 0, 1)_{104} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{63} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{63} = \mathbf{Pl}(2, 0, 0, 0, 0, 1)_{103} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{68} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{68} = \mathbf{Pl}(1, 1, 1, 0, 2, 1)_{240} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{77} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{77} = \mathbf{Pl}(2, 3, 3, 0, 1, 1)_{187} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{74} = \mathbf{Pl}(3, 2, 2, 0, 3, 1)_{305} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{47} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{47} = \mathbf{Pl}(1, 1, 1, 0, 3, 1)_{300} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{56} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{56} = \mathbf{Pl}(2, 3, 3, 0, 2, 1)_{247} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{53} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{53} = \mathbf{Pl}(3, 2, 2, 0, 1, 1)_{185} \\
\ell_{10} &= \begin{bmatrix} 1 & \omega^2 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{165} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{165} = \mathbf{Pl}(0, 2, 3, 2, 3, 1)_{337} \\
\ell_{11} &= \begin{bmatrix} 1 & \omega & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{145} = \mathbf{Pl}(0, 3, 2, 3, 2, 1)_{271} \\
\ell_{12} &= \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(0, 3, 1, 1, 1, 1)_{204} \\
\ell_{13} &= \begin{bmatrix} 1 & \omega^2 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{333} = \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{333} = \mathbf{Pl}(0, 1, 3, 2, 3, 1)_{336} \\
\ell_{14} &= \begin{bmatrix} 1 & \omega & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{229} = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{229} = \mathbf{Pl}(0, 1, 2, 3, 2, 1)_{269} \\
\ell_{15} &= \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{206} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{206} = \mathbf{Pl}(0, 2, 1, 1, 1, 1)_{203}
\end{aligned}$$

Rank of lines: (16, 42, 63, 356, 68, 77, 74, 47, 56, 53, 165, 145, 290, 333, 229, 206)

Rank of points on Klein quadric: (2, 104, 103, 1, 240, 187, 305, 300, 247, 185, 337, 271, 204, 336, 269, 203)

Eckardt Points

The surface has 3 Eckardt points:

$$0 : P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1),$$

$$1 : P_{53} = \mathbf{P}(0, 0, \omega, 1) = \mathbf{P}(0, 0, 2, 1),$$

$$2 : P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1).$$

Double Points

The surface has 29 Double points:

The double points on the surface are:

$$\begin{aligned}
P_2 &= (0, 0, 1, 0) = \ell_0 \cap \ell_3 \\
P_1 &= (0, 1, 0, 0) = \ell_1 \cap \ell_2 \\
P_{14} &= (3, 1, 1, 0) = \ell_1 \cap \ell_{11} \\
P_{18} &= (3, 2, 1, 0) = \ell_1 \cap \ell_{13} \\
P_{22} &= (3, 3, 1, 0) = \ell_1 \cap \ell_{15} \\
P_{13} &= (2, 1, 1, 0) = \ell_2 \cap \ell_{10} \\
P_{17} &= (2, 2, 1, 0) = \ell_2 \cap \ell_{12} \\
P_{21} &= (2, 3, 1, 0) = \ell_2 \cap \ell_{14} \\
P_{42} &= (0, 1, 1, 1) = \ell_4 \cap \ell_7 \\
P_{76} &= (3, 1, 3, 1) = \ell_4 \cap \ell_{11} \\
P_{58} &= (1, 1, 2, 1) = \ell_4 \cap \ell_{12} \\
P_{28} &= (2, 1, 0, 1) = \ell_4 \cap \ell_{13} \\
P_{77} &= (0, 2, 3, 1) = \ell_5 \cap \ell_8 \\
P_{48} &= (3, 2, 1, 1) = \ell_5 \cap \ell_{10} \\
P_{31} &= (1, 2, 0, 1) = \ell_5 \cap \ell_{11}
\end{aligned}$$

$$\begin{aligned}
P_{63} &= (2, 2, 2, 1) = \ell_5 \cap \ell_{15} \\
P_{65} &= (0, 3, 2, 1) = \ell_6 \cap \ell_9 \\
P_{50} &= (1, 3, 1, 1) = \ell_6 \cap \ell_{13} \\
P_{83} &= (2, 3, 3, 1) = \ell_6 \cap \ell_{14} \\
P_{37} &= (3, 3, 0, 1) = \ell_6 \cap \ell_{15} \\
P_{59} &= (2, 1, 2, 1) = \ell_7 \cap \ell_{10} \\
P_{29} &= (3, 1, 0, 1) = \ell_7 \cap \ell_{14} \\
P_{74} &= (1, 1, 3, 1) = \ell_7 \cap \ell_{15} \\
P_{32} &= (2, 2, 0, 1) = \ell_8 \cap \ell_{12} \\
P_{64} &= (3, 2, 2, 1) = \ell_8 \cap \ell_{13} \\
P_{46} &= (1, 2, 1, 1) = \ell_8 \cap \ell_{14} \\
P_{35} &= (1, 3, 0, 1) = \ell_9 \cap \ell_{10} \\
P_{51} &= (2, 3, 1, 1) = \ell_9 \cap \ell_{11} \\
P_{84} &= (3, 3, 3, 1) = \ell_9 \cap \ell_{12}
\end{aligned}$$

Single Points

The surface has 3 single points:
The single points on the surface are:

$$\begin{aligned}
0 : P_0 &= (1, 0, 0, 0) \text{ lies on line } \ell_0 \\
1 : P_3 &= (0, 0, 0, 1) \text{ lies on line } \ell_3
\end{aligned}$$

$$2 : P_8 = (1, 0, 1, 0) \text{ lies on line } \ell_0$$

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0	1	1	1	0	1	0	1	0	1
2	1	1	0	0	1	1	1	0	0	0	1	0	1	0	1	0
3	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
4	1	0	1	0	0	1	1	1	0	0	0	1	1	1	0	0
5	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1
6	1	0	1	0	1	1	0	0	0	1	0	0	0	1	1	1
7	1	1	0	0	1	0	0	0	1	1	1	0	0	0	1	1
8	1	1	0	0	0	1	0	1	0	1	0	0	1	1	1	0
9	1	1	0	0	0	0	1	1	1	0	1	1	0	0	0	0
10	0	0	1	1	0	1	0	1	0	1	0	0	0	1	0	0
11	0	1	0	1	1	1	0	0	0	1	0	0	0	0	1	0
12	0	0	1	1	1	0	0	0	1	1	0	0	0	0	0	1
13	0	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0
14	0	0	1	1	0	0	1	1	1	0	0	1	0	0	0	0
15	0	1	0	1	0	1	1	1	0	0	0	0	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_{10}	P_9	P_2	P_9	P_9	P_9	P_{10}	P_{10}	P_{10}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}
in point	P_{10}	P_1	P_{10}	P_{10}	P_{10}	P_{14}	P_{18}	P_{22}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_9	P_1	P_9	P_9	P_9	P_{13}	P_{17}	P_{21}

Line 3 intersects

Line	ℓ_0	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_2	P_{69}	P_{53}	P_{38}	P_{69}	P_{53}	P_{38}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_9	P_9	P_9	P_9	P_{42}	P_{76}	P_{58}	P_{28}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{15}
in point	P_9	P_9	P_9	P_9	P_{77}	P_{48}	P_{31}	P_{63}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_9	P_9	P_9	P_9	P_{65}	P_{50}	P_{83}	P_{37}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{15}
in point	P_{10}	P_{10}	P_{42}	P_{10}	P_{10}	P_{59}	P_{29}	P_{74}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{14}
in point	P_{10}	P_{10}	P_{77}	P_{10}	P_{10}	P_{32}	P_{64}	P_{46}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_{10}	P_{10}	P_{65}	P_{10}	P_{10}	P_{35}	P_{51}	P_{84}

Line 10 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{13}
in point	P_{13}	P_{69}	P_{48}	P_{59}	P_{35}	P_{69}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_9	ℓ_{14}
in point	P_{14}	P_{53}	P_{76}	P_{31}	P_{51}	P_{53}

Line 12 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_8	ℓ_9	ℓ_{15}
in point	P_{17}	P_{38}	P_{58}	P_{32}	P_{84}	P_{38}

Line 13 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}
in point	P_{18}	P_{69}	P_{28}	P_{50}	P_{64}	P_{69}

Line 14 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}
in point	P_{21}	P_{53}	P_{83}	P_{29}	P_{46}	P_{53}

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_{12}
in point	P_{22}	P_{38}	P_{63}	P_{37}	P_{74}	P_{38}

The surface has 37 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_8 = (1, 0, 1, 0)$
 5 : $P_9 = (2, 0, 1, 0)$
 6 : $P_{10} = (3, 0, 1, 0)$
 7 : $P_{13} = (2, 1, 1, 0)$
 8 : $P_{14} = (3, 1, 1, 0)$
 9 : $P_{17} = (2, 2, 1, 0)$
 10 : $P_{18} = (3, 2, 1, 0)$
 11 : $P_{21} = (2, 3, 1, 0)$
 12 : $P_{22} = (3, 3, 1, 0)$

13 : $P_{28} = (2, 1, 0, 1)$
 14 : $P_{29} = (3, 1, 0, 1)$
 15 : $P_{31} = (1, 2, 0, 1)$
 16 : $P_{32} = (2, 2, 0, 1)$
 17 : $P_{35} = (1, 3, 0, 1)$
 18 : $P_{37} = (3, 3, 0, 1)$
 19 : $P_{38} = (0, 0, 1, 1)$
 20 : $P_{42} = (0, 1, 1, 1)$
 21 : $P_{46} = (1, 2, 1, 1)$
 22 : $P_{48} = (3, 2, 1, 1)$
 23 : $P_{50} = (1, 3, 1, 1)$
 24 : $P_{51} = (2, 3, 1, 1)$
 25 : $P_{53} = (0, 0, 2, 1)$

26 : $P_{58} = (1, 1, 2, 1)$
 27 : $P_{59} = (2, 1, 2, 1)$
 28 : $P_{63} = (2, 2, 2, 1)$
 29 : $P_{64} = (3, 2, 2, 1)$
 30 : $P_{65} = (0, 3, 2, 1)$
 31 : $P_{69} = (0, 0, 3, 1)$
 32 : $P_{74} = (1, 1, 3, 1)$
 33 : $P_{76} = (3, 1, 3, 1)$
 34 : $P_{77} = (0, 2, 3, 1)$
 35 : $P_{83} = (2, 3, 3, 1)$
 36 : $P_{84} = (3, 3, 3, 1)$