Rank-65919 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over GF(8) is 1244210317

General information

Number of lines	1
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

Singular Points

The surface has 1 singular points:

$$0: P_{139} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{96}$$

Rank of lines: (65)

Rank of points on Klein quadric: (96)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

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\begin{array}{lll} 0: \ P_0 = (1,0,0,0) \ \text{lies on line} \ \ell_0 \\ 1: \ P_{138} = (0,0,1,1) \ \text{lies on line} \ \ell_0 \\ 2: \ P_{139} = (1,0,1,1) \ \text{lies on line} \ \ell_0 \\ 3: \ P_{140} = (2,0,1,1) \ \text{lies on line} \ \ell_0 \\ 4: \ P_{141} = (3,0,1,1) \ \text{lies on line} \ \ell_0 \\ \end{array}
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The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line: The points on the surface but not on lines are:

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0: P_1 = (0, 1, 0, 0)
                                                                  18: P_{183} = (6, 5, 1, 1)
1: P_{12} = (1,0,1,0)
                                                                  19: P_{184} = (7, 5, 1, 1)
2: P_{19} = (0, 1, 1, 0)
                                                                  20: P_{187} = (2, 6, 1, 1)
3: P_{29} = (2, 2, 1, 0)
                                                                  21: P_{188} = (3, 6, 1, 1)
4: P_{34} = (7, 2, 1, 0)
                                                                  22: P_{205} = (4,0,2,1)
5: P_{45} = (2, 4, 1, 0)
                                                                  23: P_{213} = (4,1,2,1)
6: P_{47} = (4, 4, 1, 0)
                                                                  24: P_{214} = (5, 1, 2, 1)
7: P_{71} = (4, 7, 1, 0)
                                                                  25: P_{219} = (2, 2, 2, 1)
8: P_{74} = (7,7,1,0)
                                                                  26: P_{223} = (6, 2, 2, 1)
9: P_{75} = (1, 0, 0, 1)
                                                                  27: P_{231} = (6, 3, 2, 1)
10: P_{92} = (2, 2, 0, 1)
                                                                  28: P_{257} = (0,7,2,1)
                                                                  29: P_{262} = (5, 7, 2, 1)
11: P_{103} = (5, 3, 0, 1)
12: P_{110} = (4, 4, 0, 1)
                                                                  30: P_{269} = (4,0,3,1)
13: P_{120} = (6, 5, 0, 1)
                                                                  31: P_{273} = (0, 1, 3, 1)
14: P_{125} = (3, 6, 0, 1)
                                                                  32: P_{274} = (1, 1, 3, 1)
                                                                  33: P_{285} = (4, 2, 3, 1)
15: P_{137} = (7,7,0,1)
16: P_{165} = (4, 3, 1, 1)
                                                                  34: P_{298} = (1,4,3,1)
17: P_{166} = (5, 3, 1, 1)
                                                                  35: P_{300} = (3,4,3,1)
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36: P_{336} = (7,0,4,1)
                                                                  51: P_{465} = (0, 1, 6, 1)
37: P_{343} = (6, 1, 4, 1)
                                                                  52: P_{466} = (1, 1, 6, 1)
38: P_{344} = (7, 1, 4, 1)
                                                                  53: P_{474} = (1, 2, 6, 1)
                                                                  54: P_{479} = (6, 2, 6, 1)
39: P_{345} = (0, 2, 4, 1)
40: P_{351} = (6, 2, 4, 1)
                                                                  55: P_{515} = (2,7,6,1)
41: P_{364} = (3, 4, 4, 1)
                                                                  56: P_{523} = (2, 0, 7, 1)
42: P_{365} = (4, 4, 4, 1)
                                                                  57: P_{531} = (2, 1, 7, 1)
43: P_{372} = (3, 5, 4, 1)
                                                                  58: P_{532} = (3, 1, 7, 1)
44: P_{400} = (7, 0, 5, 1)
                                                                  59: P_{553} = (0,4,7,1)
45: P_{401} = (0, 1, 5, 1)
                                                                  60: P_{556} = (3, 4, 7, 1)
46: P_{402} = (1, 1, 5, 1)
                                                                  61: P_{574} = (5, 6, 7, 1)
                                                                  62: P_{582} = (5, 7, 7, 1)
47: P_{432} = (7, 4, 5, 1)
                                                                  63: P_{584} = (7, 7, 7, 1)
48: P_{450} = (1, 7, 5, 1)
49: P_{454} = (5, 7, 5, 1)
50: P_{459} = (2, 0, 6, 1)
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Line Intersection Graph

 $\frac{0}{0 \mid 0}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 73 points: The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	$25: P_{165} = (4, 3, 1, 1)$	$50: P_{364} = (3, 4, 4, 1)$
$1: P_1 = (0, 1, 0, 0)$	$26: P_{166} = (5, 3, 1, 1)$	$51: P_{365} = (4, 4, 4, 1)$
$2: P_{12} = (1,0,1,0)$	$27: P_{183} = (6, 5, 1, 1)$	$52: P_{372} = (3, 5, 4, 1)$
$3: P_{19} = (0, 1, 1, 0)$	$28: P_{184} = (7, 5, 1, 1)$	$53: P_{400} = (7, 0, 5, 1)$
$4: P_{29} = (2, 2, 1, 0)$	$29: P_{187} = (2, 6, 1, 1)$	$54: P_{401} = (0, 1, 5, 1)$
$5: P_{34} = (7, 2, 1, 0)$	$30: P_{188} = (3, 6, 1, 1)$	$55: P_{402} = (1, 1, 5, 1)$
$6: P_{45} = (2, 4, 1, 0)$	$31: P_{205} = (4, 0, 2, 1)$	$56: P_{432} = (7, 4, 5, 1)$
$7: P_{47} = (4, 4, 1, 0)$	$32: P_{213} = (4, 1, 2, 1)$	$57: P_{450} = (1,7,5,1)$
$8: P_{71} = (4,7,1,0)$	$33: P_{214} = (5, 1, 2, 1)$	$58: P_{454} = (5,7,5,1)$
$9: P_{74} = (7,7,1,0)$	$34: P_{219} = (2, 2, 2, 1)$	$59: P_{459} = (2,0,6,1)$
$10: P_{75} = (1,0,0,1)$	$35: P_{223} = (6, 2, 2, 1)$	$60: P_{465} = (0, 1, 6, 1)$
$11: P_{92} = (2, 2, 0, 1)$	$36: P_{231} = (6, 3, 2, 1)$	$61: P_{466} = (1, 1, 6, 1)$
$12: P_{103} = (5, 3, 0, 1)$	$37: P_{257} = (0, 7, 2, 1)$	$62: P_{474} = (1, 2, 6, 1)$
13: $P_{110} = (4, 4, 0, 1)$	$38: P_{262} = (5, 7, 2, 1)$	$63: P_{479} = (6, 2, 6, 1)$
$14: P_{120} = (6, 5, 0, 1)$	$39: P_{269} = (4, 0, 3, 1)$	$64: P_{515} = (2,7,6,1)$
$15: P_{125} = (3, 6, 0, 1)$	$40: P_{273} = (0, 1, 3, 1)$	$65: P_{523} = (2,0,7,1)$
$16: P_{137} = (7,7,0,1)$	$41: P_{274} = (1, 1, 3, 1)$	$66: P_{531} = (2, 1, 7, 1)$
17: $P_{138} = (0, 0, 1, 1)$	$42: P_{285} = (4, 2, 3, 1)$	$67: P_{532} = (3, 1, 7, 1)$
$18: P_{139} = (1, 0, 1, 1)$	$43: P_{298} = (1,4,3,1)$	$68: P_{553} = (0,4,7,1)$
$19: P_{140} = (2, 0, 1, 1)$	$44: P_{300} = (3,4,3,1)$	$69: P_{556} = (3,4,7,1)$
$20: P_{141} = (3,0,1,1)$	$45: P_{336} = (7, 0, 4, 1)$	$70: P_{574} = (5, 6, 7, 1)$
$21: P_{142} = (4,0,1,1)$	$46: P_{343} = (6, 1, 4, 1)$	$71: P_{582} = (5, 7, 7, 1)$
$22: P_{143} = (5, 0, 1, 1)$	$47: P_{344} = (7, 1, 4, 1)$	$72: P_{584} = (7,7,7,1)$
$23: P_{144} = (6, 0, 1, 1)$	$48: P_{345} = (0, 2, 4, 1)$	
$24: P_{145} = (7, 0, 1, 1)$	$49: P_{351} = (6, 2, 4, 1)$	