Rank-77 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 = 0$$

(1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 77

General information

Number of lines	1
Number of points	7
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	3
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3
Type of lines on points	1^3, 0^4

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \mathbf{Pl}(1,1,1,1,0,0)_8$$

Rank of lines: (15)
Rank of points on Klein quadric: (8)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 3 single points:
The single points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_0
1 : $P_7 = (0, 1, 1, 0)$ lies on line ℓ_0
2 : $P_9 = (1, 0, 0, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

0 : $P_{10} = (0, 1, 0, 1)$
1 : $P_{11} = (1, 1, 0, 1)$
2 : $P_{12} = (0, 0, 1, 1)$
3 : $P_{13} = (1, 0, 1, 1)$

Line Intersection Graph

	0
0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 7 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_7 = (0, 1, 1, 0)$
2 : $P_9 = (1, 0, 0, 1)$
3 : $P_{10} = (0, 1, 0, 1)$
4 : $P_{11} = (1, 1, 0, 1)$
5 : $P_{12} = (0, 0, 1, 1)$
6 : $P_{13} = (1, 0, 1, 1)$