

# Rank-0 over GF(2)

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## The equation

The equation of the surface is :

$$X_0^3 = 0$$

( 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(2) is 0

## General information

Number of lines	7
Number of points	7
Number of singular points	7
Number of Eckardt points	7
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$3^7$
Type of lines on points	$3^7$

## Singular Points

The surface has 7 singular points:

0 :  $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$   
1 :  $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$   
2 :  $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$   
3 :  $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$

4 :  $P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$   
5 :  $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$   
6 :  $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$

## The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{25} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{27}
\end{aligned}$$

Rank of lines: ( 28, 30, 29, 34, 31, 33, 32 )

Rank of points on Klein quadric: ( 19, 5, 25, 1, 21, 7, 27 )

### Eckardt Points

The surface has 7 Eckardt points:

- 0 :  $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$ ,  $T = -1$
- 1 :  $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$ ,  $T = -1$
- 2 :  $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$ ,  $T = -1$
- 3 :  $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$ ,  $T = -1$
- 4 :  $P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$ ,  $T = -1$
- 5 :  $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$ ,  $T = -1$
- 6 :  $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$ ,  $T = -1$

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

## Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	0	1	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	1	0	1	1
5	1	1	1	1	1	0	1
6	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_1$	$P_1$	$P_2$	$P_2$	$P_7$	$P_7$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_1$	$P_1$	$P_3$	$P_{10}$	$P_3$	$P_{10}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_1$	$P_1$	$P_{12}$	$P_{14}$	$P_{14}$	$P_{12}$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_2$	$P_3$	$P_{12}$	$P_2$	$P_3$	$P_{12}$

Line 4 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$
in point	$P_2$	$P_{10}$	$P_{14}$	$P_2$	$P_{14}$	$P_{10}$

Line 5 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_6$
in point	$P_7$	$P_3$	$P_{14}$	$P_3$	$P_{14}$	$P_7$

Line 6 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$
in point	$P_7$	$P_{10}$	$P_{12}$	$P_{12}$	$P_{10}$	$P_7$

The surface has 7 points:

The points on the surface are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_7 = (0, 1, 1, 0)$$

$$4 : P_{10} = (0, 1, 0, 1)$$

$$5 : P_{12} = (0, 0, 1, 1)$$

$$6 : P_{14} = (0, 1, 1, 1)$$