Rank-20 over GF(16)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1^2 + X_1^2X_2 + X_1^2X_3$$
$$+X_0X_2^2 + X_1X_2^2 + X_2^2X_3 + X_0X_3^2 + X_1X_3^2 + X_2X_3^2 + X_0X_1X_2 + X_0X_1X_3 + X_0X_2X_3 + X_1X_2X_3 = 0$$

(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
The point rank of the equation over GF(16) is 20

General information

Number of lines	9
Number of points	305
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	126
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^5, 2^6, 1^{126}, 0^{168}

Singular Points

The surface has 4 singular points:

0 : $P_{36} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
1 : $P_{291} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
2 : $P_{531} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$
3 : $P_{546} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \mathbf{Pl}(1,0,1,1,1,1)_{9427}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \mathbf{Pl}(1,1,0,0,1,1)_{8961}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(1,1,1,0,1,1)_{8976}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \mathbf{Pl}(1,1,1,1,0,0)_{64}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4642} = \mathbf{Pl}(1,1,1,1,0,1)_{5586}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \mathbf{Pl}(1,1,1,1,1,0)_{1250}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \mathbf{Pl}(1,1,0,1,1,1)_{9201}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \mathbf{Pl}(0,1,1,1,1,1)_{9442}$$

Rank of lines: (530, 4658, 289, 290, 4369, 4642, 4385, 4657, 4898)
Rank of points on Klein quadric: (9426, 9427, 8961, 8976, 64, 5586, 1250, 9201, 9442)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$,
$1 : P_{36} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$,
$2 : P_{291} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$,
$3 : P_{531} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$,
$4 : P_{546} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$
$P_{530} = (0,0,1,1) = \ell_0 \cap \ell_8$
$P_{20} = (1,0,1,0) = \ell_2 \cap \ell_3$
$P_{290} = (0,1,0,1) = \ell_2 \cap \ell_7$
$P_{35} = (0,1,1,0) = \ell_4 \cap \ell_5$
$P_{275} = (1,0,0,1) = \ell_4 \cap \ell_6$

Single Points

The surface has 126 single points:
The single points on the surface are:

0 : $P_{563} = (2,2,1,1)$ lies on line ℓ_0
1 : $P_{564} = (3,2,1,1)$ lies on line ℓ_1
2 : $P_{579} = (2,3,1,1)$ lies on line ℓ_1
3 : $P_{580} = (3,3,1,1)$ lies on line ℓ_0
4 : $P_{597} = (4,4,1,1)$ lies on line ℓ_0
5 : $P_{598} = (5,4,1,1)$ lies on line ℓ_1
6 : $P_{613} = (4,5,1,1)$ lies on line ℓ_1
7 : $P_{614} = (5,5,1,1)$ lies on line ℓ_0
8 : $P_{631} = (6,6,1,1)$ lies on line ℓ_0
9 : $P_{632} = (7,6,1,1)$ lies on line ℓ_1
10 : $P_{647} = (6,7,1,1)$ lies on line ℓ_1
11 : $P_{648} = (7,7,1,1)$ lies on line ℓ_0
12 : $P_{665} = (8,8,1,1)$ lies on line ℓ_0
13 : $P_{666} = (9,8,1,1)$ lies on line ℓ_1
14 : $P_{681} = (8,9,1,1)$ lies on line ℓ_1
15 : $P_{682} = (9,9,1,1)$ lies on line ℓ_0
16 : $P_{699} = (10,10,1,1)$ lies on line ℓ_0
17 : $P_{700} = (11,10,1,1)$ lies on line ℓ_1
18 : $P_{715} = (10,11,1,1)$ lies on line ℓ_1
19 : $P_{716} = (11,11,1,1)$ lies on line ℓ_0
20 : $P_{733} = (12,12,1,1)$ lies on line ℓ_0
21 : $P_{734} = (13,12,1,1)$ lies on line ℓ_1
22 : $P_{749} = (12,13,1,1)$ lies on line ℓ_1
23 : $P_{750} = (13,13,1,1)$ lies on line ℓ_0
24 : $P_{767} = (14,14,1,1)$ lies on line ℓ_0
25 : $P_{768} = (15,14,1,1)$ lies on line ℓ_1
26 : $P_{783} = (14,15,1,1)$ lies on line ℓ_1
27 : $P_{784} = (15,15,1,1)$ lies on line ℓ_0
28 : $P_{803} = (2,1,2,1)$ lies on line ℓ_2
29 : $P_{804} = (3,1,2,1)$ lies on line ℓ_3
30 : $P_{818} = (1,2,2,1)$ lies on line ℓ_4
31 : $P_{820} = (3,2,2,1)$ lies on line ℓ_6
32 : $P_{834} = (1,3,2,1)$ lies on line ℓ_5
33 : $P_{835} = (2,3,2,1)$ lies on line ℓ_7
34 : $P_{836} = (3,3,2,1)$ lies on line ℓ_8
35 : $P_{1059} = (2,1,3,1)$ lies on line ℓ_3
36 : $P_{1060} = (3,1,3,1)$ lies on line ℓ_2
37 : $P_{1074} = (1,2,3,1)$ lies on line ℓ_5
38 : $P_{1075} = (2,2,3,1)$ lies on line ℓ_8
39 : $P_{1076} = (3,2,3,1)$ lies on line ℓ_7
40 : $P_{1090} = (1,3,3,1)$ lies on line ℓ_4
41 : $P_{1091} = (2,3,3,1)$ lies on line ℓ_6
42 : $P_{1317} = (4,1,4,1)$ lies on line ℓ_2
43 : $P_{1318} = (5,1,4,1)$ lies on line ℓ_3
44 : $P_{1362} = (1,4,4,1)$ lies on line ℓ_4
45 : $P_{1366} = (5,4,4,1)$ lies on line ℓ_6
46 : $P_{1378} = (1,5,4,1)$ lies on line ℓ_5
47 : $P_{1381} = (4,5,4,1)$ lies on line ℓ_7
48 : $P_{1382} = (5,5,4,1)$ lies on line ℓ_8
49 : $P_{1573} = (4,1,5,1)$ lies on line ℓ_3
50 : $P_{1574} = (5,1,5,1)$ lies on line ℓ_2
51 : $P_{1618} = (1,4,5,1)$ lies on line ℓ_5
52 : $P_{1621} = (4,4,5,1)$ lies on line ℓ_8
53 : $P_{1622} = (5,4,5,1)$ lies on line ℓ_7
54 : $P_{1634} = (1,5,5,1)$ lies on line ℓ_4
55 : $P_{1637} = (4,5,5,1)$ lies on line ℓ_6
56 : $P_{1831} = (6,1,6,1)$ lies on line ℓ_2
57 : $P_{1832} = (7,1,6,1)$ lies on line ℓ_3
58 : $P_{1906} = (1,6,6,1)$ lies on line ℓ_4
59 : $P_{1912} = (7,6,6,1)$ lies on line ℓ_6
60 : $P_{1922} = (1,7,6,1)$ lies on line ℓ_5
61 : $P_{1927} = (6,7,6,1)$ lies on line ℓ_7
62 : $P_{1928} = (7,7,6,1)$ lies on line ℓ_8
63 : $P_{2087} = (6,1,7,1)$ lies on line ℓ_3
64 : $P_{2088} = (7,1,7,1)$ lies on line ℓ_2
65 : $P_{2162} = (1,6,7,1)$ lies on line ℓ_5
66 : $P_{2167} = (6,6,7,1)$ lies on line ℓ_8
67 : $P_{2168} = (7,6,7,1)$ lies on line ℓ_7
68 : $P_{2178} = (1,7,7,1)$ lies on line ℓ_4
69 : $P_{2183} = (6,7,7,1)$ lies on line ℓ_6
70 : $P_{2345} = (8,1,8,1)$ lies on line ℓ_2
71 : $P_{2346} = (9,1,8,1)$ lies on line ℓ_3
72 : $P_{2450} = (1,8,8,1)$ lies on line ℓ_4
73 : $P_{2458} = (9,8,8,1)$ lies on line ℓ_6
74 : $P_{2466} = (1,9,8,1)$ lies on line ℓ_5
75 : $P_{2473} = (8,9,8,1)$ lies on line ℓ_7
76 : $P_{2474} = (9,9,8,1)$ lies on line ℓ_8
77 : $P_{2601} = (8,1,9,1)$ lies on line ℓ_3
78 : $P_{2602} = (9,1,9,1)$ lies on line ℓ_2
79 : $P_{2706} = (1,8,9,1)$ lies on line ℓ_5
80 : $P_{2713} = (8,8,9,1)$ lies on line ℓ_8
81 : $P_{2714} = (9,8,9,1)$ lies on line ℓ_7
82 : $P_{2722} = (1,9,9,1)$ lies on line ℓ_4
83 : $P_{2729} = (8,9,9,1)$ lies on line ℓ_6
84 : $P_{2859} = (10,1,10,1)$ lies on line ℓ_2
85 : $P_{2860} = (11,1,10,1)$ lies on line ℓ_3
86 : $P_{2994} = (1,10,10,1)$ lies on line ℓ_4
87 : $P_{3004} = (11,10,10,1)$ lies on line ℓ_6
88 : $P_{3010} = (1,11,10,1)$ lies on line ℓ_5
89 : $P_{3019} = (10,11,10,1)$ lies on line ℓ_7
90 : $P_{3020} = (11,11,10,1)$ lies on line ℓ_8
91 : $P_{3115} = (10,1,11,1)$ lies on line ℓ_3
92 : $P_{3116} = (11,1,11,1)$ lies on line ℓ_2
93 : $P_{3250} = (1,10,11,1)$ lies on line ℓ_5
94 : $P_{3259} = (10,10,11,1)$ lies on line ℓ_8
95 : $P_{3260} = (11,10,11,1)$ lies on line ℓ_7
96 : $P_{3266} = (1,11,11,1)$ lies on line ℓ_4
97 : $P_{3275} = (10,11,11,1)$ lies on line ℓ_6
98 : $P_{3373} = (12,1,12,1)$ lies on line ℓ_2
99 : $P_{3374} = (13,1,12,1)$ lies on line ℓ_3
100 : $P_{3538} = (1,12,12,1)$ lies on line ℓ_4
101 : $P_{3550} = (13,12,12,1)$ lies on line ℓ_6
102 : $P_{3554} = (1,13,12,1)$ lies on line ℓ_5
103 : $P_{3565} = (12,13,12,1)$ lies on line ℓ_7
104 : $P_{3566} = (13,13,12,1)$ lies on line ℓ_8
105 : $P_{3629} = (12,1,13,1)$ lies on line ℓ_3
106 : $P_{3630} = (13,1,13,1)$ lies on line ℓ_2
107 : $P_{3794} = (1,12,13,1)$ lies on line ℓ_5

108 : $P_{3805} = (12, 12, 13, 1)$ lies on line ℓ_8
109 : $P_{3806} = (13, 12, 13, 1)$ lies on line ℓ_7
110 : $P_{3810} = (1, 13, 13, 1)$ lies on line ℓ_4
111 : $P_{3821} = (12, 13, 13, 1)$ lies on line ℓ_6
112 : $P_{3887} = (14, 1, 14, 1)$ lies on line ℓ_2
113 : $P_{3888} = (15, 1, 14, 1)$ lies on line ℓ_3
114 : $P_{4082} = (1, 14, 14, 1)$ lies on line ℓ_4
115 : $P_{4096} = (15, 14, 14, 1)$ lies on line ℓ_6
116 : $P_{4098} = (1, 15, 14, 1)$ lies on line ℓ_5
117 : $P_{4111} = (14, 15, 14, 1)$ lies on line ℓ_7
118 : $P_{4112} = (15, 15, 14, 1)$ lies on line ℓ_8
119 : $P_{4143} = (14, 1, 15, 1)$ lies on line ℓ_3
120 : $P_{4144} = (15, 1, 15, 1)$ lies on line ℓ_2
121 : $P_{4338} = (1, 14, 15, 1)$ lies on line ℓ_5
122 : $P_{4351} = (14, 14, 15, 1)$ lies on line ℓ_8
123 : $P_{4352} = (15, 14, 15, 1)$ lies on line ℓ_7
124 : $P_{4354} = (1, 15, 15, 1)$ lies on line ℓ_4
125 : $P_{4367} = (14, 15, 15, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_{72} = (5, 3, 1, 0)$
1 : $P_{75} = (8, 3, 1, 0)$
2 : $P_{82} = (15, 3, 1, 0)$
3 : $P_{102} = (3, 5, 1, 0)$
4 : $P_{107} = (8, 5, 1, 0)$
5 : $P_{114} = (15, 5, 1, 0)$
6 : $P_{150} = (3, 8, 1, 0)$
7 : $P_{152} = (5, 8, 1, 0)$
8 : $P_{162} = (15, 8, 1, 0)$
9 : $P_{262} = (3, 15, 1, 0)$
10 : $P_{264} = (5, 15, 1, 0)$
11 : $P_{267} = (8, 15, 1, 0)$
12 : $P_{327} = (5, 3, 0, 1)$
13 : $P_{330} = (8, 3, 0, 1)$
14 : $P_{337} = (15, 3, 0, 1)$
15 : $P_{357} = (3, 5, 0, 1)$
16 : $P_{362} = (8, 5, 0, 1)$
17 : $P_{369} = (15, 5, 0, 1)$
18 : $P_{405} = (3, 8, 0, 1)$
19 : $P_{407} = (5, 8, 0, 1)$
20 : $P_{417} = (15, 8, 0, 1)$
21 : $P_{517} = (3, 15, 0, 1)$
22 : $P_{519} = (5, 15, 0, 1)$
23 : $P_{522} = (8, 15, 0, 1)$
24 : $P_{862} = (13, 4, 2, 1)$
25 : $P_{876} = (11, 5, 2, 1)$
26 : $P_{895} = (14, 6, 2, 1)$
27 : $P_{957} = (12, 10, 2, 1)$
28 : $P_{966} = (5, 11, 2, 1)$
29 : $P_{987} = (10, 12, 2, 1)$
30 : $P_{997} = (4, 13, 2, 1)$
31 : $P_{1015} = (6, 14, 2, 1)$
32 : $P_{1046} = (5, 0, 3, 1)$
33 : $P_{1049} = (8, 0, 3, 1)$
34 : $P_{1056} = (15, 0, 3, 1)$
35 : $P_{1121} = (0, 5, 3, 1)$
36 : $P_{1129} = (8, 5, 3, 1)$
37 : $P_{1136} = (15, 5, 3, 1)$
38 : $P_{1165} = (12, 7, 3, 1)$
39 : $P_{1169} = (0, 8, 3, 1)$
40 : $P_{1174} = (5, 8, 3, 1)$
41 : $P_{1184} = (15, 8, 3, 1)$
42 : $P_{1215} = (14, 10, 3, 1)$
43 : $P_{1240} = (7, 12, 3, 1)$
44 : $P_{1275} = (10, 14, 3, 1)$
45 : $P_{1281} = (0, 15, 3, 1)$
46 : $P_{1286} = (5, 15, 3, 1)$
47 : $P_{1289} = (8, 15, 3, 1)$
48 : $P_{1342} = (13, 2, 4, 1)$
49 : $P_{1404} = (11, 6, 4, 1)$
50 : $P_{1418} = (9, 7, 4, 1)$
51 : $P_{1435} = (10, 8, 4, 1)$
52 : $P_{1448} = (7, 9, 4, 1)$
53 : $P_{1465} = (8, 10, 4, 1)$
54 : $P_{1479} = (6, 11, 4, 1)$
55 : $P_{1507} = (2, 13, 4, 1)$
56 : $P_{1556} = (3, 0, 5, 1)$
57 : $P_{1561} = (8, 0, 5, 1)$
58 : $P_{1568} = (15, 0, 5, 1)$
59 : $P_{1596} = (11, 2, 5, 1)$
60 : $P_{1601} = (0, 3, 5, 1)$
61 : $P_{1609} = (8, 3, 5, 1)$
62 : $P_{1616} = (15, 3, 5, 1)$
63 : $P_{1661} = (12, 6, 5, 1)$
64 : $P_{1681} = (0, 8, 5, 1)$
65 : $P_{1684} = (3, 8, 5, 1)$
66 : $P_{1696} = (15, 8, 5, 1)$
67 : $P_{1731} = (2, 11, 5, 1)$

$68: P_{1751} = (6, 12, 5, 1)$
$69: P_{1793} = (0, 15, 5, 1)$
$70: P_{1796} = (3, 15, 5, 1)$
$71: P_{1801} = (8, 15, 5, 1)$
$72: P_{1855} = (14, 2, 6, 1)$
$73: P_{1884} = (11, 4, 6, 1)$
$74: P_{1901} = (12, 5, 6, 1)$
$75: P_{1950} = (13, 8, 6, 1)$
$76: P_{1989} = (4, 11, 6, 1)$
$77: P_{2006} = (5, 12, 6, 1)$
$78: P_{2025} = (8, 13, 6, 1)$
$79: P_{2035} = (2, 14, 6, 1)$
$80: P_{2125} = (12, 3, 7, 1)$
$81: P_{2138} = (9, 4, 7, 1)$
$82: P_{2213} = (4, 9, 7, 1)$
$83: P_{2255} = (14, 11, 7, 1)$
$84: P_{2260} = (3, 12, 7, 1)$
$85: P_{2288} = (15, 13, 7, 1)$
$86: P_{2300} = (11, 14, 7, 1)$
$87: P_{2318} = (13, 15, 7, 1)$
$88: P_{2324} = (3, 0, 8, 1)$
$89: P_{2326} = (5, 0, 8, 1)$
$90: P_{2336} = (15, 0, 8, 1)$
$91: P_{2369} = (0, 3, 8, 1)$
$92: P_{2374} = (5, 3, 8, 1)$
$93: P_{2384} = (15, 3, 8, 1)$
$94: P_{2395} = (10, 4, 8, 1)$
$95: P_{2401} = (0, 5, 8, 1)$
$96: P_{2404} = (3, 5, 8, 1)$
$97: P_{2416} = (15, 5, 8, 1)$
$98: P_{2430} = (13, 6, 8, 1)$
$99: P_{2485} = (4, 10, 8, 1)$
$100: P_{2535} = (6, 13, 8, 1)$
$101: P_{2561} = (0, 15, 8, 1)$
$102: P_{2564} = (3, 15, 8, 1)$
$103: P_{2566} = (5, 15, 8, 1)$
$104: P_{2648} = (7, 4, 9, 1)$
$105: P_{2693} = (4, 7, 9, 1)$
$106: P_{2750} = (13, 10, 9, 1)$
$107: P_{2768} = (15, 11, 9, 1)$
$108: P_{2783} = (14, 12, 9, 1)$
$109: P_{2795} = (10, 13, 9, 1)$
$110: P_{2813} = (12, 14, 9, 1)$
$111: P_{2828} = (11, 15, 9, 1)$
$112: P_{2877} = (12, 2, 10, 1)$
$113: P_{2895} = (14, 3, 10, 1)$
$114: P_{2905} = (8, 4, 10, 1)$
$115: P_{2965} = (4, 8, 10, 1)$
$116: P_{2990} = (13, 9, 10, 1)$
$117: P_{3027} = (2, 12, 10, 1)$
$118: P_{3050} = (9, 13, 10, 1)$
$119: P_{3060} = (3, 14, 10, 1)$
$120: P_{3126} = (5, 2, 11, 1)$
$121: P_{3159} = (6, 4, 11, 1)$
$122: P_{3171} = (2, 5, 11, 1)$
$123: P_{3189} = (4, 6, 11, 1)$
$124: P_{3215} = (14, 7, 11, 1)$
$125: P_{3248} = (15, 9, 11, 1)$
$126: P_{3320} = (7, 14, 11, 1)$
$127: P_{3338} = (9, 15, 11, 1)$
$128: P_{3387} = (10, 2, 12, 1)$
$129: P_{3400} = (7, 3, 12, 1)$
$130: P_{3431} = (6, 5, 12, 1)$
$131: P_{3446} = (5, 6, 12, 1)$
$132: P_{3460} = (3, 7, 12, 1)$
$133: P_{3503} = (14, 9, 12, 1)$
$134: P_{3507} = (2, 10, 12, 1)$
$135: P_{3578} = (9, 14, 12, 1)$
$136: P_{3637} = (4, 2, 13, 1)$
$137: P_{3667} = (2, 4, 13, 1)$
$138: P_{3705} = (8, 6, 13, 1)$
$139: P_{3728} = (15, 7, 13, 1)$
$140: P_{3735} = (6, 8, 13, 1)$
$141: P_{3755} = (10, 9, 13, 1)$
$142: P_{3770} = (9, 10, 13, 1)$
$143: P_{3848} = (7, 15, 13, 1)$
$144: P_{3895} = (6, 2, 14, 1)$
$145: P_{3915} = (10, 3, 14, 1)$
$146: P_{3955} = (2, 6, 14, 1)$
$147: P_{3980} = (11, 7, 14, 1)$
$148: P_{4013} = (12, 9, 14, 1)$
$149: P_{4020} = (3, 10, 14, 1)$
$150: P_{4040} = (7, 11, 14, 1)$
$151: P_{4058} = (9, 12, 14, 1)$
$152: P_{4116} = (3, 0, 15, 1)$
$153: P_{4118} = (5, 0, 15, 1)$
$154: P_{4121} = (8, 0, 15, 1)$
$155: P_{4161} = (0, 3, 15, 1)$
$156: P_{4166} = (5, 3, 15, 1)$
$157: P_{4169} = (8, 3, 15, 1)$
$158: P_{4193} = (0, 5, 15, 1)$
$159: P_{4196} = (3, 5, 15, 1)$
$160: P_{4201} = (8, 5, 15, 1)$
$161: P_{4238} = (13, 7, 15, 1)$
$162: P_{4241} = (0, 8, 15, 1)$
$163: P_{4244} = (3, 8, 15, 1)$
$164: P_{4246} = (5, 8, 15, 1)$
$165: P_{4268} = (11, 9, 15, 1)$
$166: P_{4298} = (9, 11, 15, 1)$
$167: P_{4328} = (7, 13, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 0 1 0 0 0 1
1	1 0 0 1 0 1 1 1 0
2	1 0 0 1 1 0 0 1 0
3	0 1 1 0 0 1 1 0 1
4	1 0 1 0 0 1 1 0 0
5	0 1 0 1 1 0 0 1 1
6	0 1 0 1 1 0 0 1 1
7	0 1 1 0 0 1 1 0 1
8	1 0 0 1 0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_8
in point	P_5	P_4	P_4	P_{530}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_5	P_{546}	P_{531}	P_{546}	P_{531}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_7
in point	P_4	P_{20}	P_4	P_{290}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{546}	P_{20}	P_{291}	P_{546}	P_{291}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_4	P_4	P_{35}	P_{275}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{531}	P_{291}	P_{35}	P_{531}	P_{291}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{546}	P_{546}	P_{275}	P_{36}	P_{36}

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{531}	P_{290}	P_{531}	P_{36}	P_{36}

Line 8 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_{530}	P_{291}	P_{291}	P_{36}	P_{36}

The surface has 305 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{20} = (1, 0, 1, 0)$
3 : $P_{35} = (0, 1, 1, 0)$
4 : $P_{36} = (1, 1, 1, 0)$
5 : $P_{72} = (5, 3, 1, 0)$
6 : $P_{75} = (8, 3, 1, 0)$
7 : $P_{82} = (15, 3, 1, 0)$
8 : $P_{102} = (3, 5, 1, 0)$
9 : $P_{107} = (8, 5, 1, 0)$
10 : $P_{114} = (15, 5, 1, 0)$
11 : $P_{150} = (3, 8, 1, 0)$
12 : $P_{152} = (5, 8, 1, 0)$
13 : $P_{162} = (15, 8, 1, 0)$
14 : $P_{262} = (3, 15, 1, 0)$
15 : $P_{264} = (5, 15, 1, 0)$
16 : $P_{267} = (8, 15, 1, 0)$
17 : $P_{275} = (1, 0, 0, 1)$
18 : $P_{290} = (0, 1, 0, 1)$
19 : $P_{291} = (1, 1, 0, 1)$
20 : $P_{327} = (5, 3, 0, 1)$

21 : $P_{330} = (8, 3, 0, 1)$
22 : $P_{337} = (15, 3, 0, 1)$
23 : $P_{357} = (3, 5, 0, 1)$
24 : $P_{362} = (8, 5, 0, 1)$
25 : $P_{369} = (15, 5, 0, 1)$
26 : $P_{405} = (3, 8, 0, 1)$
27 : $P_{407} = (5, 8, 0, 1)$
28 : $P_{417} = (15, 8, 0, 1)$
29 : $P_{517} = (3, 15, 0, 1)$
30 : $P_{519} = (5, 15, 0, 1)$
31 : $P_{522} = (8, 15, 0, 1)$
32 : $P_{530} = (0, 0, 1, 1)$
33 : $P_{531} = (1, 0, 1, 1)$
34 : $P_{546} = (0, 1, 1, 1)$
35 : $P_{563} = (2, 2, 1, 1)$
36 : $P_{564} = (3, 2, 1, 1)$
37 : $P_{579} = (2, 3, 1, 1)$
38 : $P_{580} = (3, 3, 1, 1)$
39 : $P_{597} = (4, 4, 1, 1)$
40 : $P_{598} = (5, 4, 1, 1)$
41 : $P_{613} = (4, 5, 1, 1)$
42 : $P_{614} = (5, 5, 1, 1)$
43 : $P_{631} = (6, 6, 1, 1)$
44 : $P_{632} = (7, 6, 1, 1)$
45 : $P_{647} = (6, 7, 1, 1)$
46 : $P_{648} = (7, 7, 1, 1)$
47 : $P_{665} = (8, 8, 1, 1)$
48 : $P_{666} = (9, 8, 1, 1)$
49 : $P_{681} = (8, 9, 1, 1)$
50 : $P_{682} = (9, 9, 1, 1)$
51 : $P_{699} = (10, 10, 1, 1)$
52 : $P_{700} = (11, 10, 1, 1)$
53 : $P_{715} = (10, 11, 1, 1)$
54 : $P_{716} = (11, 11, 1, 1)$
55 : $P_{733} = (12, 12, 1, 1)$
56 : $P_{734} = (13, 12, 1, 1)$
57 : $P_{749} = (12, 13, 1, 1)$
58 : $P_{750} = (13, 13, 1, 1)$
59 : $P_{767} = (14, 14, 1, 1)$
60 : $P_{768} = (15, 14, 1, 1)$
61 : $P_{783} = (14, 15, 1, 1)$
62 : $P_{784} = (15, 15, 1, 1)$
63 : $P_{803} = (2, 1, 2, 1)$
64 : $P_{804} = (3, 1, 2, 1)$
65 : $P_{818} = (1, 2, 2, 1)$
66 : $P_{820} = (3, 2, 2, 1)$
67 : $P_{834} = (1, 3, 2, 1)$
68 : $P_{835} = (2, 3, 2, 1)$
69 : $P_{836} = (3, 3, 2, 1)$
70 : $P_{862} = (13, 4, 2, 1)$
71 : $P_{876} = (11, 5, 2, 1)$
72 : $P_{895} = (14, 6, 2, 1)$
73 : $P_{957} = (12, 10, 2, 1)$
74 : $P_{966} = (5, 11, 2, 1)$
75 : $P_{987} = (10, 12, 2, 1)$
76 : $P_{997} = (4, 13, 2, 1)$
77 : $P_{1015} = (6, 14, 2, 1)$
78 : $P_{1046} = (5, 0, 3, 1)$
79 : $P_{1049} = (8, 0, 3, 1)$
80 : $P_{1056} = (15, 0, 3, 1)$
81 : $P_{1059} = (2, 1, 3, 1)$
82 : $P_{1060} = (3, 1, 3, 1)$
83 : $P_{1074} = (1, 2, 3, 1)$
84 : $P_{1075} = (2, 2, 3, 1)$
85 : $P_{1076} = (3, 2, 3, 1)$
86 : $P_{1090} = (1, 3, 3, 1)$
87 : $P_{1091} = (2, 3, 3, 1)$
88 : $P_{1121} = (0, 5, 3, 1)$
89 : $P_{1129} = (8, 5, 3, 1)$
90 : $P_{1136} = (15, 5, 3, 1)$
91 : $P_{1165} = (12, 7, 3, 1)$
92 : $P_{1169} = (0, 8, 3, 1)$
93 : $P_{1174} = (5, 8, 3, 1)$
94 : $P_{1184} = (15, 8, 3, 1)$
95 : $P_{1215} = (14, 10, 3, 1)$
96 : $P_{1240} = (7, 12, 3, 1)$
97 : $P_{1275} = (10, 14, 3, 1)$
98 : $P_{1281} = (0, 15, 3, 1)$
99 : $P_{1286} = (5, 15, 3, 1)$
100 : $P_{1289} = (8, 15, 3, 1)$
101 : $P_{1317} = (4, 1, 4, 1)$
102 : $P_{1318} = (5, 1, 4, 1)$
103 : $P_{1342} = (13, 2, 4, 1)$
104 : $P_{1362} = (1, 4, 4, 1)$
105 : $P_{1366} = (5, 4, 4, 1)$
106 : $P_{1378} = (1, 5, 4, 1)$
107 : $P_{1381} = (4, 5, 4, 1)$
108 : $P_{1382} = (5, 5, 4, 1)$
109 : $P_{1404} = (11, 6, 4, 1)$
110 : $P_{1418} = (9, 7, 4, 1)$
111 : $P_{1435} = (10, 8, 4, 1)$
112 : $P_{1448} = (7, 9, 4, 1)$
113 : $P_{1465} = (8, 10, 4, 1)$
114 : $P_{1479} = (6, 11, 4, 1)$
115 : $P_{1507} = (2, 13, 4, 1)$
116 : $P_{1556} = (3, 0, 5, 1)$
117 : $P_{1561} = (8, 0, 5, 1)$
118 : $P_{1568} = (15, 0, 5, 1)$
119 : $P_{1573} = (4, 1, 5, 1)$
120 : $P_{1574} = (5, 1, 5, 1)$
121 : $P_{1596} = (11, 2, 5, 1)$
122 : $P_{1601} = (0, 3, 5, 1)$
123 : $P_{1609} = (8, 3, 5, 1)$
124 : $P_{1616} = (15, 3, 5, 1)$
125 : $P_{1618} = (1, 4, 5, 1)$
126 : $P_{1621} = (4, 4, 5, 1)$
127 : $P_{1622} = (5, 4, 5, 1)$
128 : $P_{1634} = (1, 5, 5, 1)$
129 : $P_{1637} = (4, 5, 5, 1)$
130 : $P_{1661} = (12, 6, 5, 1)$
131 : $P_{1681} = (0, 8, 5, 1)$
132 : $P_{1684} = (3, 8, 5, 1)$
133 : $P_{1696} = (15, 8, 5, 1)$
134 : $P_{1731} = (2, 11, 5, 1)$
135 : $P_{1751} = (6, 12, 5, 1)$
136 : $P_{1793} = (0, 15, 5, 1)$
137 : $P_{1796} = (3, 15, 5, 1)$
138 : $P_{1801} = (8, 15, 5, 1)$
139 : $P_{1831} = (6, 1, 6, 1)$
140 : $P_{1832} = (7, 1, 6, 1)$
141 : $P_{1855} = (14, 2, 6, 1)$
142 : $P_{1884} = (11, 4, 6, 1)$
143 : $P_{1901} = (12, 5, 6, 1)$
144 : $P_{1906} = (1, 6, 6, 1)$
145 : $P_{1912} = (7, 6, 6, 1)$
146 : $P_{1922} = (1, 7, 6, 1)$
147 : $P_{1927} = (6, 7, 6, 1)$
148 : $P_{1928} = (7, 7, 6, 1)$
149 : $P_{1950} = (13, 8, 6, 1)$
150 : $P_{1989} = (4, 11, 6, 1)$
151 : $P_{2006} = (5, 12, 6, 1)$
152 : $P_{2025} = (8, 13, 6, 1)$
153 : $P_{2035} = (2, 14, 6, 1)$
154 : $P_{2087} = (6, 1, 7, 1)$
155 : $P_{2088} = (7, 1, 7, 1)$
156 : $P_{2125} = (12, 3, 7, 1)$
157 : $P_{2138} = (9, 4, 7, 1)$
158 : $P_{2162} = (1, 6, 7, 1)$
159 : $P_{2167} = (6, 6, 7, 1)$
160 : $P_{2168} = (7, 6, 7, 1)$
161 : $P_{2178} = (1, 7, 7, 1)$
162 : $P_{2183} = (6, 7, 7, 1)$
163 : $P_{2213} = (4, 9, 7, 1)$
164 : $P_{2255} = (14, 11, 7, 1)$
165 : $P_{2260} = (3, 12, 7, 1)$
166 : $P_{2288} = (15, 13, 7, 1)$
167 : $P_{2300} = (11, 14, 7, 1)$
168 : $P_{2318} = (13, 15, 7, 1)$
169 : $P_{2324} = (3, 0, 8, 1)$
170 : $P_{2326} = (5, 0, 8, 1)$
171 : $P_{2336} = (15, 0, 8, 1)$
172 : $P_{2345} = (8, 1, 8, 1)$
173 : $P_{2346} = (9, 1, 8, 1)$
174 : $P_{2369} = (0, 3, 8, 1)$
175 : $P_{2374} = (5, 3, 8, 1)$
176 : $P_{2384} = (15, 3, 8, 1)$
177 : $P_{2395} = (10, 4, 8, 1)$
178 : $P_{2401} = (0, 5, 8, 1)$
179 : $P_{2404} = (3, 5, 8, 1)$
180 : $P_{2416} = (15, 5, 8, 1)$
181 : $P_{2430} = (13, 6, 8, 1)$
182 : $P_{2450} = (1, 8, 8, 1)$

183 : $P_{2458} = (9, 8, 8, 1)$
184 : $P_{2466} = (1, 9, 8, 1)$
185 : $P_{2473} = (8, 9, 8, 1)$
186 : $P_{2474} = (9, 9, 8, 1)$
187 : $P_{2485} = (4, 10, 8, 1)$
188 : $P_{2535} = (6, 13, 8, 1)$
189 : $P_{2561} = (0, 15, 8, 1)$
190 : $P_{2564} = (3, 15, 8, 1)$
191 : $P_{2566} = (5, 15, 8, 1)$
192 : $P_{2601} = (8, 1, 9, 1)$
193 : $P_{2602} = (9, 1, 9, 1)$
194 : $P_{2648} = (7, 4, 9, 1)$
195 : $P_{2693} = (4, 7, 9, 1)$
196 : $P_{2706} = (1, 8, 9, 1)$
197 : $P_{2713} = (8, 8, 9, 1)$
198 : $P_{2714} = (9, 8, 9, 1)$
199 : $P_{2722} = (1, 9, 9, 1)$
200 : $P_{2729} = (8, 9, 9, 1)$
201 : $P_{2750} = (13, 10, 9, 1)$
202 : $P_{2768} = (15, 11, 9, 1)$
203 : $P_{2783} = (14, 12, 9, 1)$
204 : $P_{2795} = (10, 13, 9, 1)$
205 : $P_{2813} = (12, 14, 9, 1)$
206 : $P_{2828} = (11, 15, 9, 1)$
207 : $P_{2859} = (10, 1, 10, 1)$
208 : $P_{2860} = (11, 1, 10, 1)$
209 : $P_{2877} = (12, 2, 10, 1)$
210 : $P_{2895} = (14, 3, 10, 1)$
211 : $P_{2905} = (8, 4, 10, 1)$
212 : $P_{2965} = (4, 8, 10, 1)$
213 : $P_{2990} = (13, 9, 10, 1)$
214 : $P_{2994} = (1, 10, 10, 1)$
215 : $P_{3004} = (11, 10, 10, 1)$
216 : $P_{3010} = (1, 11, 10, 1)$
217 : $P_{3019} = (10, 11, 10, 1)$
218 : $P_{3020} = (11, 11, 10, 1)$
219 : $P_{3027} = (2, 12, 10, 1)$
220 : $P_{3050} = (9, 13, 10, 1)$
221 : $P_{3060} = (3, 14, 10, 1)$
222 : $P_{3115} = (10, 1, 11, 1)$
223 : $P_{3116} = (11, 1, 11, 1)$
224 : $P_{3126} = (5, 2, 11, 1)$
225 : $P_{3159} = (6, 4, 11, 1)$
226 : $P_{3171} = (2, 5, 11, 1)$
227 : $P_{3189} = (4, 6, 11, 1)$
228 : $P_{3215} = (14, 7, 11, 1)$
229 : $P_{3248} = (15, 9, 11, 1)$
230 : $P_{3250} = (1, 10, 11, 1)$
231 : $P_{3259} = (10, 10, 11, 1)$
232 : $P_{3260} = (11, 10, 11, 1)$
233 : $P_{3266} = (1, 11, 11, 1)$
234 : $P_{3275} = (10, 11, 11, 1)$
235 : $P_{3320} = (7, 14, 11, 1)$
236 : $P_{3338} = (9, 15, 11, 1)$
237 : $P_{3373} = (12, 1, 12, 1)$
238 : $P_{3374} = (13, 1, 12, 1)$
239 : $P_{3387} = (10, 2, 12, 1)$
240 : $P_{3400} = (7, 3, 12, 1)$
241 : $P_{3431} = (6, 5, 12, 1)$
242 : $P_{3446} = (5, 6, 12, 1)$
243 : $P_{3460} = (3, 7, 12, 1)$
244 : $P_{3503} = (14, 9, 12, 1)$
245 : $P_{3507} = (2, 10, 12, 1)$
246 : $P_{3538} = (1, 12, 12, 1)$
247 : $P_{3550} = (13, 12, 12, 1)$
248 : $P_{3554} = (1, 13, 12, 1)$
249 : $P_{3565} = (12, 13, 12, 1)$
250 : $P_{3566} = (13, 13, 12, 1)$
251 : $P_{3578} = (9, 14, 12, 1)$
252 : $P_{3629} = (12, 1, 13, 1)$
253 : $P_{3630} = (13, 1, 13, 1)$
254 : $P_{3637} = (4, 2, 13, 1)$
255 : $P_{3667} = (2, 4, 13, 1)$
256 : $P_{3705} = (8, 6, 13, 1)$
257 : $P_{3728} = (15, 7, 13, 1)$
258 : $P_{3735} = (6, 8, 13, 1)$
259 : $P_{3755} = (10, 9, 13, 1)$
260 : $P_{3770} = (9, 10, 13, 1)$
261 : $P_{3794} = (1, 12, 13, 1)$
262 : $P_{3805} = (12, 12, 13, 1)$
263 : $P_{3806} = (13, 12, 13, 1)$
264 : $P_{3810} = (1, 13, 13, 1)$
265 : $P_{3821} = (12, 13, 13, 1)$
266 : $P_{3848} = (7, 15, 13, 1)$
267 : $P_{3887} = (14, 1, 14, 1)$
268 : $P_{3888} = (15, 1, 14, 1)$
269 : $P_{3895} = (6, 2, 14, 1)$
270 : $P_{3915} = (10, 3, 14, 1)$
271 : $P_{3955} = (2, 6, 14, 1)$
272 : $P_{3980} = (11, 7, 14, 1)$
273 : $P_{4013} = (12, 9, 14, 1)$
274 : $P_{4020} = (3, 10, 14, 1)$
275 : $P_{4040} = (7, 11, 14, 1)$
276 : $P_{4058} = (9, 12, 14, 1)$
277 : $P_{4082} = (1, 14, 14, 1)$
278 : $P_{4096} = (15, 14, 14, 1)$
279 : $P_{4098} = (1, 15, 14, 1)$
280 : $P_{4111} = (14, 15, 14, 1)$
281 : $P_{4112} = (15, 15, 14, 1)$
282 : $P_{4116} = (3, 0, 15, 1)$
283 : $P_{4118} = (5, 0, 15, 1)$
284 : $P_{4121} = (8, 0, 15, 1)$
285 : $P_{4143} = (14, 1, 15, 1)$
286 : $P_{4144} = (15, 1, 15, 1)$
287 : $P_{4161} = (0, 3, 15, 1)$
288 : $P_{4166} = (5, 3, 15, 1)$
289 : $P_{4169} = (8, 3, 15, 1)$
290 : $P_{4193} = (0, 5, 15, 1)$
291 : $P_{4196} = (3, 5, 15, 1)$
292 : $P_{4201} = (8, 5, 15, 1)$
293 : $P_{4238} = (13, 7, 15, 1)$
294 : $P_{4241} = (0, 8, 15, 1)$
295 : $P_{4244} = (3, 8, 15, 1)$
296 : $P_{4246} = (5, 8, 15, 1)$
297 : $P_{4268} = (11, 9, 15, 1)$
298 : $P_{4298} = (9, 11, 15, 1)$
299 : $P_{4328} = (7, 13, 15, 1)$
300 : $P_{4338} = (1, 14, 15, 1)$
301 : $P_{4351} = (14, 14, 15, 1)$
302 : $P_{4352} = (15, 14, 15, 1)$
303 : $P_{4354} = (1, 15, 15, 1)$
304 : $P_{4367} = (14, 15, 15, 1)$