Rank-73796 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498768730

General information

Number of lines	31
Number of points	41
Number of singular points	5
Number of Eckardt points	0
Number of double points	20
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{31}
Type of lines on points	$7^5, 5^{16}, 2^{20}$

Singular Points

The surface has 5 singular points:

0 : $P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
2 : $P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

3 : $P_{65} = \mathbf{P}(0,\omega^2,\omega,1) = \mathbf{P}(0,3,2,1)$
4 : $P_{77} = \mathbf{P}(0,\omega,\omega^2,1) = \mathbf{P}(0,2,3,1)$

The 31 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 0 & 0 \end{array}\right]_{252} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]_{252} = \mathbf{Pl}(2,0,0,1,0,0)_{11}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 1 & 0 & 0 \end{array}\right]_{168} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{array}\right]_{168} = \mathbf{Pl}(3,0,0,1,0,0)_{12}$$

$$\ell_4 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_5 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{339} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_6 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{338} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_7 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{268} = \left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]_{268} = \mathbf{Pl}(0,3,1,0,0,0)_8$$

$$\ell_9 = \left[\begin{array}{cccc} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{array}\right]_{184} = \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{184} = \mathbf{Pl}(0,2,1,0,0,0)_7$$

$$\ell_{10} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{341} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_{11} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{351} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]_{351} = \mathbf{Pl}(0,3,0,0,0,1)_{107}$$

$$\ell_{12} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{array}\right]_{346} = \left[\begin{array}{cccc} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]_{346} = \mathbf{Pl}(0,2,0,0,0,1)_{106}$$

$$\ell_{13} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_{14} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{342} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{342} = \mathbf{Pl}(0,1,0,1,0,1)_{133}$$

$$\ell_{15} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{354} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]_{354} = \mathbf{Pl}(0,3,0,3,0,1)_{149}$$

$$\ell_{16} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{array}\right]_{348} = \left[\begin{array}{cccc} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]_{348} = \mathbf{Pl}(0,2,0,2,0,1)_{141}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & 0 & \omega & \omega \\ 0 & 1 & \omega^2 & \omega \end{array}\right]_{221} = \left[\begin{array}{cccc} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 2 \end{array}\right]_{221} = \mathbf{Pl}(1,1,1,1,1,1)_{219}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & \omega & \omega^2 \end{array}\right]_{329} = \left[\begin{array}{cccc} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]_{329} = \mathbf{Pl}(1,1,1,1,1,1)_{222}$$

$$\ell_{19} = \left[\begin{array}{cccc} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{355} = \left[\begin{array}{cccc} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_{20} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{array}\right]_{343} = \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]_{343} = \mathbf{Pl}(0,1,0,2,0,1)_{140}$$

$$\ell_{21} = \left[\begin{array}{cccc} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{array}\right]_{352} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]_{352} = \mathbf{Pl}(0,3,0,1,0,1)_{135}$$

$$\ell_{22} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{349} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{349} = \mathbf{Pl}(0,2,0,3,0,1)_{148}$$

$$\ell_{23} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{245} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{245} = \mathbf{Pl}(3,2,2,3,1,1)_{227}$$

$$\ell_{24} = \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{278} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{278} = \mathbf{Pl}(3,2,2,3,1,1)_{230}$$

$$\ell_{25} = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_{26} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{344} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{344} = \mathbf{Pl}(0,1,0,3,0,1)_{147}$$

$$\ell_{27} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{353} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{353} = \mathbf{Pl}(0,3,0,2,0,1)_{142}$$

$$\ell_{28} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \mathbf{Pl}(0,2,0,1,0,1)_{134}$$

$$\ell_{29} = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{305} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{305} = \mathbf{Pl}(2,3,3,2,1,1)_{235}$$

$$\ell_{30} = \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{194} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{194} = \mathbf{Pl}(2,3,3,2,1,1)_{232}$$

Rank of lines: (336, 340, 252, 168, 337, 339, 338, 356, 268, 184, 341, 351, 346, 345, 342, 354, 348, 221, 329, 355, 343, 352, 349, 245, 278, 350, 344, 353, 347, 305, 194)

Rank of points on Klein quadric: (101, 9, 11, 12, 129, 143, 136, 1, 8, 7, 105, 107, 106, 13, 133, 149, 141, 219, 222, 15, 140, 135, 148, 227, 230, 14, 147, 142, 134, 235, 232)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 20 Double points:
The double points on the surface are:

$P_{24} = (2,0,0,1) = \ell_2 \cap \ell_8$

$P_{28} = (2,1,0,1) = \ell_2 \cap \ell_{17}$

$P_{36} = (2,3,0,1) = \ell_2 \cap \ell_{23}$

$P_{32} = (2,2,0,1) = \ell_2 \cap \ell_{30}$

$P_{25} = (3,0,0,1) = \ell_3 \cap \ell_9$

$P_{29} = (3,1,0,1) = \ell_3 \cap \ell_{18}$

$P_{37} = (3,3,0,1) = \ell_3 \cap \ell_{24}$

$P_{33} = (3,2,0,1) = \ell_3 \cap \ell_{29}$

$P_{40} = (2,0,1,1) = \ell_8 \cap \ell_{18}$

$P_{55} = (2,0,2,1) = \ell_8 \cap \ell_{24}$

$P_{71} = (2,0,3,1) = \ell_8 \cap \ell_{29}$

$P_{41} = (3,0,1,1) = \ell_9 \cap \ell_{17}$

$P_{56} = (3,0,2,1) = \ell_9 \cap \ell_{23}$

$P_{72} = (3,0,3,1) = \ell_9 \cap \ell_{30}$

$P_{12} = (1,1,1,0) = \ell_{17} \cap \ell_{18}$

$P_{78} = (1,2,3,1) = \ell_{17} \cap \ell_{24}$

$P_{66} = (1,3,2,1) = \ell_{18} \cap \ell_{30}$

$P_{18} = (3,2,1,0) = \ell_{23} \cap \ell_{24}$

$P_4 = (1,1,1,1) = \ell_{23} \cap \ell_{29}$

$P_{21} = (2,3,1,0) = \ell_{29} \cap \ell_{30}$

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
0	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0
1	1 0 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0
2	1 1 0 1 1 1 1 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1
3	1 1 1 0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0
4	1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 0 1
5	1 1 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 1 0
6	1 1 1 1 1 1 0 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 0 0
7	1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0
8	1 0 1 0 0 0 0 1 0 1 1 1 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0
9	1 0 0 1 0 0 0 1 1 0 1 1 1 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1
10	1 1 0 0 1 1 1 1 1 1 0 1 1 1 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 0 1
11	1 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 0 0
12	1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 1 1 1 1 0 0 1 1 1 1 1 0
13	1 1 0 0 1 1 1 1 0 0 1 1 1 0 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 0 1
14	1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0
15	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 0 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0
16	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 0 0 0 1 1 1 1 0 0 1 1 1 1 0 0
17	0 0 1 0 0 1 0 0 0 1 0 0 1 0 1 0 0 0 1 1 0 0 0 0 1 0 0 1 0 1 0
18	0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 0 1 0 0 0 0 1
19	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 0 0 1 1 1 1 1 0
20	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 0 1 1 0 0 1 1 1 1 0 0
21	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 0 1 0 0 1 1 1 1 0 0
22	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 0 1
23	0 0 1 0 0 0 1 0 0 1 0 1 0 0 1 0 0 0 1 0 0 0 1 0 1 1 0 0 0 1 0
24	0 0 0 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 0 0 0 1 1 0 0 0 1 0 0 1
25	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 0 1 1 1 0 0
26	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 1 0 1 1 0 0
27	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 0 1 1 1
28	1 1 0 0 1 1 1 1 0 0 1 1 1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 0 0 0
29	0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 0 0 1 0 1 0 0 0 1 0 0 0 1 0 0 1
30	0 0 1 0 1 0 0 0 0 1 1 0 0 1 0 0 0 0 1 0 0 0 1 0 1 0 0 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_2	P_2	P_2	P_2	P_2	P_2	P_{11}	P_{11}	P_{11}	P_{11}	P_{15}	P_{15}	P_{15}	P_{15}	P_{19}	

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{17}	ℓ_{23}	ℓ_{30}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{24}	P_{28}	P_{36}	P_{32}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_9	ℓ_{18}	ℓ_{24}	ℓ_{29}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{25}	P_{29}	P_{37}	P_{33}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}	ℓ_{2}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{38}	P_{42}	P_{45}	P_{49}	P_{42}	P_{38}	P_{49}	P_{45}	P_{45}	P_{49}	P_{38}	P_{42}	P_{42}	P_{49}	P_4

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{2}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{53}	P_{57}	P_{61}	P_{65}	P_{61}	P_{65}	P_{53}	P_{57}	P_{65}	P_{65}	P_{61}	P_{57}	P_{53}	P_{57}	P_5

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{2}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{69}	P_{73}	P_{77}	P_{81}	P_{81}	P_{77}	P_{73}	P_{69}	P_{77}	P_{73}	P_{69}	P_{81}	P_{77}	P_{77}	P_7

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_2	P_3	P_{38}	P_{53}	P_{69}	P_2	P_2	P_2	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_3	P_{69}	P_{38}	P_{53}	P_3	P_{53}	P_{69}

Line 8 intersects

Line	ℓ_0	ℓ_2	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{18}	ℓ_{24}	ℓ_{29}
in point	P_2	P_{24}	P_2	P_2	P_2	P_2	P_2	P_{40}	P_{55}	P_{71}

Line 9 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{17}	ℓ_{23}	ℓ_{30}
in point	P_2	P_{25}	P_2	P_2	P_2	P_2	P_2	P_{41}	P_{56}	P_{72}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}	$\ell_{}$
in point	P_2	P_{26}	P_{42}	P_{57}	P_{73}	P_2	P_2	P_2	P_2	P_2	P_{42}	P_{26}	P_{73}	P_{57}	P_{73}	P_{26}	P_{57}	P_{42}	P_{42}	P_{57}	P

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	$\ell_{}$
in point	P_2	P_{30}	P_{45}	P_{61}	P_{77}	P_2	P_2	P_2	P_2	P_2	P_{61}	P_{77}	P_{30}	P_{45}	P_{77}	P_{45}	P_{61}	P_{30}	P_{77}	P_{77}	P

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	$\ell_{}$
in point	P_2	P_{34}	P_{49}	P_{65}	P_{81}	P_2	P_2	P_2	P_2	P_2	P_{81}	P_{65}	P_{49}	P_{34}	P_{65}	P_{65}	P_{49}	P_{81}	P_{34}	P_{49}	P

Line 13 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{42}	P_{61}	P_{81}	P_{11}	P_{11}	P_{11}	P_3	P_{61}	P_{81}	P_{42}	P_{42}	P_3	P_{81}	P_{42}

Line 14 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{25}
in point	P_{11}	P_{26}	P_{38}	P_{65}	P_{77}	P_{38}	P_{26}	P_{77}	P_{65}	P_{11}	P_{11}	P_{11}	P_{65}	P_{77}	P_{65}	P_{26}	P_{38}	P_{77}	P_{77}	P_7

Line 15 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{11}	P_{30}	P_{49}	P_{53}	P_{73}	P_{53}	P_{73}	P_{30}	P_{49}	P_{11}	P_{11}	P_{11}	P_{73}	P_{49}	P_{30}	P_{53}	P_{49}	P_{53}	P_{30}	P_{73}

Line 16 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{11}	P_{34}	P_{45}	P_{57}	P_{69}	P_{69}	P_{57}	P_{45}	P_{34}	P_{11}	P_{11}	P_{11}	P_{45}	P_{69}	P_{57}	P_{34}	P_{57}	P_{45}	P_{69}	P_{34}

Line 17 intersects

Line	ℓ_2	ℓ_5	ℓ_9	ℓ_{12}	ℓ_{14}	ℓ_{18}	ℓ_{19}	ℓ_{24}	ℓ_{27}	ℓ_{29}
in point	P_{28}	P_{65}	P_{41}	P_{65}	P_{65}	P_{12}	P_{65}	P_{78}	P_{65}	P_{65}

Line 18 intersects

Line	ℓ_3	ℓ_6	ℓ_8	ℓ_{11}	ℓ_{14}	ℓ_{17}	ℓ_{22}	ℓ_{23}	ℓ_{25}	ℓ_{30}
in point	P_{29}	P_{77}	P_{40}	P_{77}	P_{77}	P_{12}	P_{77}	P_{77}	P_{77}	P_{66}

Line 19 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_{15}	P_3	P_{45}	P_{65}	P_{73}	P_3	P_{73}	P_{45}	P_{65}	P_3	P_{65}	P_{73}	P_{45}	P_{65}	P_{15}	P_{15}	P_{15}	P_3	P_{45}	P_{65}

Line 20 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{15}	P_{26}	P_{49}	P_{61}	P_{69}	P_{69}	P_{26}	P_{61}	P_{49}	P_{61}	P_{26}	P_{49}	P_{69}	P_{15}	P_{15}	P_{15}	P_{49}	P_{26}	P_{69}	P_{61}

Line 21 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{15}	P_{30}	P_{38}	P_{57}	P_{81}	P_{38}	P_{57}	P_{30}	P_{81}	P_{81}	P_{38}	P_{30}	P_{57}	P_{15}	P_{15}	P_{15}	P_{57}	P_{81}	P_{30}	P_{38}

Line 22 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{15}	P_{34}	P_{42}	P_{53}	P_{77}	P_{53}	P_{42}	P_{77}	P_{34}	P_{42}	P_{77}	P_{53}	P_{34}	P_{77}	P_{15}	P_{15}	P_{15}	P_{77}	P_{42}	P_{77}

Line 23 intersects

Line	ℓ_2	ℓ_6	ℓ_9	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{22}	ℓ_{24}	ℓ_{25}	ℓ_{29}
in point	P_{36}	P_{77}	P_{56}	P_{77}	P_{77}	P_{77}	P_{77}	P_{18}	P_{77}	P_4

Line 24 intersects

Line	ℓ_3	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{13}	ℓ_{17}	ℓ_{22}	ℓ_{23}	ℓ_{27}	ℓ_{30}
in point	P_{37}	P_{42}	P_{55}	P_{42}	P_{42}	P_{78}	P_{42}	P_{18}	P_{42}	P_{42}

Line 25 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{26}
in point	P_{19}	P_3	P_{49}	P_{57}	P_{77}	P_3	P_{57}	P_{77}	P_{49}	P_3	P_{77}	P_{49}	P_{57}	P_{77}	P_3	P_{49}	P_{57}	P_{77}	P_{77}	P_{19}

Line 26 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{27}	ℓ_{28}
in point	P_{19}	P_{26}	P_{45}	P_{53}	P_{81}	P_{53}	P_{26}	P_{45}	P_{81}	P_{81}	P_{26}	P_{53}	P_{45}	P_{45}	P_{26}	P_{81}	P_{53}	P_{19}	P_{19}	P_{19}

Line 27 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}
in point	P_{19}	P_{30}	P_{42}	P_{65}	P_{69}	P_{69}	P_{42}	P_{30}	P_{65}	P_{42}	P_{65}	P_{30}	P_{69}	P_{65}	P_{65}	P_{69}	P_{30}	P_{42}	P_{42}	P_{19}

Line 28 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_{19}	P_{34}	P_{38}	P_{61}	P_{73}	P_{38}	P_{73}	P_{61}	P_{34}	P_{61}	P_{38}	P_{73}	P_{34}	P_{73}	P_{61}	P_{38}	P_{34}	P_{19}	P_{19}	P_{1}

Line 29 intersects

Line	ℓ_3	ℓ_5	ℓ_8	ℓ_{12}	ℓ_{14}	ℓ_{17}	ℓ_{19}	ℓ_{23}	ℓ_{27}	ℓ_{30}
in point	P_{33}	P_{65}	P_{71}	P_{65}	P_{65}	P_{65}	P_{65}	P_{4}	P_{65}	P_{21}

Line 30 intersects

Line	ℓ_2	ℓ_4	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{18}	ℓ_{22}	ℓ_{24}	ℓ_{27}	ℓ_{29}
in point	P_{32}	P_{42}	P_{72}	P_{42}	P_{42}	P_{66}	P_{42}	P_{42}	P_{42}	P_{21}

The surface has 41 points:
The points on the surface are:

$0 : P_1 = (0,1,0,0)$
$1 : P_2 = (0,0,1,0)$
$2 : P_3 = (0,0,0,1)$
$3 : P_4 = (1,1,1,1)$
$4 : P_{11} = (0,1,1,0)$
$5 : P_{12} = (1,1,1,0)$
$6 : P_{15} = (0,2,1,0)$
$7 : P_{18} = (3,2,1,0)$
$8 : P_{19} = (0,3,1,0)$
$9 : P_{21} = (2,3,1,0)$
$10 : P_{24} = (2,0,0,1)$
$11 : P_{25} = (3,0,0,1)$
$12 : P_{26} = (0,1,0,1)$
$13 : P_{28} = (2,1,0,1)$

$14 : P_{29} = (3,1,0,1)$
$15 : P_{30} = (0,2,0,1)$
$16 : P_{32} = (2,2,0,1)$
$17 : P_{33} = (3,2,0,1)$
$18 : P_{34} = (0,3,0,1)$
$19 : P_{36} = (2,3,0,1)$
$20 : P_{37} = (3,3,0,1)$
$21 : P_{38} = (0,0,1,1)$
$22 : P_{40} = (2,0,1,1)$
$23 : P_{41} = (3,0,1,1)$
$24 : P_{42} = (0,1,1,1)$
$25 : P_{45} = (0,2,1,1)$
$26 : P_{49} = (0,3,1,1)$
$27 : P_{53} = (0,0,2,1)$

$28 : P_{55} = (2,0,2,1)$
$29 : P_{56} = (3,0,2,1)$
$30 : P_{57} = (0,1,2,1)$
$31 : P_{61} = (0,2,2,1)$
$32 : P_{65} = (0,3,2,1)$
$33 : P_{66} = (1,3,2,1)$
$34 : P_{69} = (0,0,3,1)$
$35 : P_{71} = (2,0,3,1)$
$36 : P_{72} = (3,0,3,1)$
$37 : P_{73} = (0,1,3,1)$
$38 : P_{77} = (0,2,3,1)$
$39 : P_{78} = (1,2,3,1)$
$40 : P_{81} = (0,3,3,1)$