

# Rank-355 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_1^2 X_2 = 0$$

( 0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0 )  
The point rank of the equation over GF(4) is 26273

## General information

Number of lines	3
Number of points	25
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	11
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^3$
Type of lines on points	$2^2, 1^{11}, 0^{12}$

## Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{33}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_6$$

Rank of lines: ( 16, 5, 100 )

Rank of points on Klein quadric: ( 2, 33, 6 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_2$$

### Single Points

The surface has 11 single points:

The single points on the surface are:

$$0 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_1$$

$$1 : P_8 = (1, 0, 1, 0) \text{ lies on line } \ell_0$$

$$2 : P_9 = (2, 0, 1, 0) \text{ lies on line } \ell_0$$

$$3 : P_{10} = (3, 0, 1, 0) \text{ lies on line } \ell_0$$

$$4 : P_{23} = (1, 0, 0, 1) \text{ lies on line } \ell_2$$

$$5 : P_{39} = (1, 0, 1, 1) \text{ lies on line } \ell_2$$

$$6 : P_{42} = (0, 1, 1, 1) \text{ lies on line } \ell_1$$

$$7 : P_{43} = (2, 1, 1, 1) \text{ lies on line } \ell_1$$

$$8 : P_{44} = (3, 1, 1, 1) \text{ lies on line } \ell_1$$

$$9 : P_{54} = (1, 0, 2, 1) \text{ lies on line } \ell_2$$

$$10 : P_{70} = (1, 0, 3, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

### Points on surface but on no line

The surface has 12 points not on any line:

The points on the surface but not on lines are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_{12} = (1, 1, 1, 0)$$

$$2 : P_{18} = (3, 2, 1, 0)$$

$$3 : P_{21} = (2, 3, 1, 0)$$

$$4 : P_{33} = (3, 2, 0, 1)$$

$$5 : P_{36} = (2, 3, 0, 1)$$

$$6 : P_{47} = (2, 2, 1, 1)$$

$$7 : P_{52} = (3, 3, 1, 1)$$

$$8 : P_{61} = (0, 2, 2, 1)$$

$$9 : P_{66} = (1, 3, 2, 1)$$

$$10 : P_{78} = (1, 2, 3, 1)$$

$$11 : P_{81} = (0, 3, 3, 1)$$

## Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 0
2	1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_0$	$P_2$

Line 1 intersects

Line	$\ell_0$
in point	$P_0$

Line 2 intersects

Line	$\ell_0$
in point	$P_2$

The surface has 25 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_8 = (1, 0, 1, 0)$$

$$5 : P_9 = (2, 0, 1, 0)$$

$$6 : P_{10} = (3, 0, 1, 0)$$

$$7 : P_{12} = (1, 1, 1, 0)$$

$$8 : P_{18} = (3, 2, 1, 0)$$

$$9 : P_{21} = (2, 3, 1, 0)$$

$$10 : P_{23} = (1, 0, 0, 1)$$

$$11 : P_{33} = (3, 2, 0, 1)$$

$$12 : P_{36} = (2, 3, 0, 1)$$

$$13 : P_{39} = (1, 0, 1, 1)$$

$$14 : P_{42} = (0, 1, 1, 1)$$

$$15 : P_{43} = (2, 1, 1, 1)$$

$$16 : P_{44} = (3, 1, 1, 1)$$

$$17 : P_{47} = (2, 2, 1, 1)$$

$$18 : P_{52} = (3, 3, 1, 1)$$

$$19 : P_{54} = (1, 0, 2, 1)$$

$$20 : P_{61} = (0, 2, 2, 1)$$

$$21 : P_{66} = (1, 3, 2, 1)$$

$$22 : P_{70} = (1, 0, 3, 1)$$

$$23 : P_{78} = (1, 2, 3, 1)$$

$$24 : P_{81} = (0, 3, 3, 1)$$