

# Rank-65843 over GF(2)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(2) is 65843

## General information

|                            |          |
|----------------------------|----------|
| Number of lines            | 4        |
| Number of points           | 9        |
| Number of singular points  | 3        |
| Number of Eckardt points   | 0        |
| Number of double points    | 0        |
| Number of single points    | 8        |
| Number of points off lines | 0        |
| Number of Hesse planes     | 0        |
| Number of axes             | 0        |
| Type of points on lines    | $3^4$    |
| Type of lines on points    | 4, $1^8$ |

## Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0) \\ 1 : P_3 &= \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1) \end{aligned}$$

$$2 : P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_9$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\ \ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}\end{aligned}$$

Rank of lines: ( 6, 30, 34, 27 )

Rank of points on Klein quadric: ( 9, 5, 1, 17 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 8 single points:

The single points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$  lies on line  $\ell_0$

1 :  $P_1 = (0, 1, 0, 0)$  lies on line  $\ell_1$

2 :  $P_2 = (0, 0, 1, 0)$  lies on line  $\ell_2$

3 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_3$

4 :  $P_8 = (1, 1, 1, 0)$  lies on line  $\ell_3$

5 :  $P_9 = (1, 0, 0, 1)$  lies on line  $\ell_0$

6 :  $P_{10} = (0, 1, 0, 1)$  lies on line  $\ell_1$

7 :  $P_{12} = (0, 0, 1, 1)$  lies on line  $\ell_2$

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

### Line Intersection Graph

|   | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 2 | 1 | 0 | 1 | 0 |
| 3 | 1 | 1 | 0 | 1 |

Neighbor sets in the line intersection graph:

Line 0 intersects

| Line     | $\ell_1$ | $\ell_2$ | $\ell_3$ |
|----------|----------|----------|----------|
| in point | $P_3$    | $P_3$    | $P_3$    |

Line 1 intersects

| Line     | $\ell_0$ | $\ell_2$ | $\ell_3$ |
|----------|----------|----------|----------|
| in point | $P_3$    | $P_3$    | $P_3$    |

Line 2 intersects

| Line     | $\ell_0$ | $\ell_1$ | $\ell_3$ |
|----------|----------|----------|----------|
| in point | $P_3$    | $P_3$    | $P_3$    |

Line 3 intersects

| Line     | $\ell_0$ | $\ell_1$ | $\ell_2$ |
|----------|----------|----------|----------|
| in point | $P_3$    | $P_3$    | $P_3$    |

The surface has 9 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_3 = (0, 0, 0, 1)$$

$$4 : P_4 = (1, 1, 1, 1)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_9 = (1, 0, 0, 1)$$

$$7 : P_{10} = (0, 1, 0, 1)$$

$$8 : P_{12} = (0, 0, 1, 1)$$