

# Rank-31 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 = 0$$

( 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(8) is 163

## General information

Number of lines	3
Number of points	73
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^3$
Type of lines on points	$3, 1^{24}, 0^{48}$

## Singular Points

The surface has 0 singular points:

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{1322} \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{1217}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{585} = \mathbf{PI}(1, 1, 1, 1, 0, 0)_{32}$$

Rank of lines: ( 138, 81, 585 )

Rank of points on Klein quadric: ( 1322, 1217, 32 )

### Eckardt Points

The surface has 1 Eckardt points:

0 :  $P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$ .

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 24 single points:

The single points on the surface are:

- |   |   |
|---|---|
| 0 : $P_5 = (1, 1, 0, 0)$ lies on line $\ell_0$      | 13 : $P_{218} = (1, 2, 2, 1)$ lies on line $\ell_2$ |
| 1 : $P_{12} = (1, 0, 1, 0)$ lies on line $\ell_1$   | 14 : $P_{276} = (3, 1, 3, 1)$ lies on line $\ell_1$ |
| 2 : $P_{19} = (0, 1, 1, 0)$ lies on line $\ell_2$   | 15 : $P_{290} = (1, 3, 3, 1)$ lies on line $\ell_2$ |
| 3 : $P_{75} = (1, 0, 0, 1)$ lies on line $\ell_2$   | 16 : $P_{341} = (4, 1, 4, 1)$ lies on line $\ell_1$ |
| 4 : $P_{82} = (0, 1, 0, 1)$ lies on line $\ell_1$   | 17 : $P_{362} = (1, 4, 4, 1)$ lies on line $\ell_2$ |
| 5 : $P_{138} = (0, 0, 1, 1)$ lies on line $\ell_0$  | 18 : $P_{406} = (5, 1, 5, 1)$ lies on line $\ell_1$ |
| 6 : $P_{155} = (2, 2, 1, 1)$ lies on line $\ell_0$  | 19 : $P_{434} = (1, 5, 5, 1)$ lies on line $\ell_2$ |
| 7 : $P_{164} = (3, 3, 1, 1)$ lies on line $\ell_0$  | 20 : $P_{471} = (6, 1, 6, 1)$ lies on line $\ell_1$ |
| 8 : $P_{173} = (4, 4, 1, 1)$ lies on line $\ell_0$  | 21 : $P_{506} = (1, 6, 6, 1)$ lies on line $\ell_2$ |
| 9 : $P_{182} = (5, 5, 1, 1)$ lies on line $\ell_0$  | 22 : $P_{536} = (7, 1, 7, 1)$ lies on line $\ell_1$ |
| 10 : $P_{191} = (6, 6, 1, 1)$ lies on line $\ell_0$ | 23 : $P_{578} = (1, 7, 7, 1)$ lies on line $\ell_2$ |
| 11 : $P_{200} = (7, 7, 1, 1)$ lies on line $\ell_0$ |   |
| 12 : $P_{211} = (2, 1, 2, 1)$ lies on line $\ell_1$ |   |

The single points on the surface are:

### Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

- |                             |                               |
|-----------------------------|-------------------------------|
| 0 : $P_{32} = (5, 2, 1, 0)$ | 6 : $P_{95} = (5, 2, 0, 1)$   |
| 1 : $P_{42} = (7, 3, 1, 0)$ | 7 : $P_{105} = (7, 3, 0, 1)$  |
| 2 : $P_{49} = (6, 4, 1, 0)$ | 8 : $P_{112} = (6, 4, 0, 1)$  |
| 3 : $P_{53} = (2, 5, 1, 0)$ | 9 : $P_{116} = (2, 5, 0, 1)$  |
| 4 : $P_{63} = (4, 6, 1, 0)$ | 10 : $P_{126} = (4, 6, 0, 1)$ |
| 5 : $P_{70} = (3, 7, 1, 0)$ | 11 : $P_{133} = (3, 7, 0, 1)$ |

12 :  $P_{206} = (5, 0, 2, 1)$   
 13 :  $P_{229} = (4, 3, 2, 1)$   
 14 :  $P_{236} = (3, 4, 2, 1)$   
 15 :  $P_{241} = (0, 5, 2, 1)$   
 16 :  $P_{256} = (7, 6, 2, 1)$   
 17 :  $P_{263} = (6, 7, 2, 1)$   
 18 :  $P_{272} = (7, 0, 3, 1)$   
 19 :  $P_{285} = (4, 2, 3, 1)$   
 20 :  $P_{299} = (2, 4, 3, 1)$   
 21 :  $P_{311} = (6, 5, 3, 1)$   
 22 :  $P_{318} = (5, 6, 3, 1)$   
 23 :  $P_{321} = (0, 7, 3, 1)$   
 24 :  $P_{335} = (6, 0, 4, 1)$   
 25 :  $P_{348} = (3, 2, 4, 1)$   
 26 :  $P_{355} = (2, 3, 4, 1)$   
 27 :  $P_{376} = (7, 5, 4, 1)$   
 28 :  $P_{377} = (0, 6, 4, 1)$   
 29 :  $P_{390} = (5, 7, 4, 1)$   
 30 :  $P_{395} = (2, 0, 5, 1)$

31 :  $P_{409} = (0, 2, 5, 1)$   
 32 :  $P_{423} = (6, 3, 5, 1)$   
 33 :  $P_{432} = (7, 4, 5, 1)$   
 34 :  $P_{444} = (3, 6, 5, 1)$   
 35 :  $P_{453} = (4, 7, 5, 1)$   
 36 :  $P_{461} = (4, 0, 6, 1)$   
 37 :  $P_{480} = (7, 2, 6, 1)$   
 38 :  $P_{486} = (5, 3, 6, 1)$   
 39 :  $P_{489} = (0, 4, 6, 1)$   
 40 :  $P_{500} = (3, 5, 6, 1)$   
 41 :  $P_{515} = (2, 7, 6, 1)$   
 42 :  $P_{524} = (3, 0, 7, 1)$   
 43 :  $P_{543} = (6, 2, 7, 1)$   
 44 :  $P_{545} = (0, 3, 7, 1)$   
 45 :  $P_{558} = (5, 4, 7, 1)$   
 46 :  $P_{565} = (4, 5, 7, 1)$   
 47 :  $P_{571} = (2, 6, 7, 1)$

## Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_4$	$P_4$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_4$	$P_4$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_4$	$P_4$

The surface has 73 points:

The points on the surface are:

0 :  $P_4 = (1, 1, 1, 1)$   
 1 :  $P_5 = (1, 1, 0, 0)$   
 2 :  $P_{12} = (1, 0, 1, 0)$   
 3 :  $P_{19} = (0, 1, 1, 0)$   
 4 :  $P_{32} = (5, 2, 1, 0)$   
 5 :  $P_{42} = (7, 3, 1, 0)$   
 6 :  $P_{49} = (6, 4, 1, 0)$   
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 9 :  $P_{70} = (3, 7, 1, 0)$   
 10 :  $P_{75} = (1, 0, 0, 1)$   
 11 :  $P_{82} = (0, 1, 0, 1)$   
 12 :  $P_{95} = (5, 2, 0, 1)$

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 17 :  $P_{133} = (3, 7, 0, 1)$   
 18 :  $P_{138} = (0, 0, 1, 1)$   
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 72 :  $P_{578} = (1, 7, 7, 1)$