

Rank-65613 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1227396181

General information

Number of lines	5
Number of points	81
Number of singular points	2
Number of Eckardt points	1
Number of double points	4
Number of single points	34
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	$3, 2^4, 1^{34}, 0^{42}$

Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{10} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4681} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{657} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{81} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{1217} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{306}
\end{aligned}$$

Rank of lines: (64, 648, 4681, 81, 593)

Rank of points on Klein quadric: (2, 10, 657, 1217, 306)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0).$$

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{12} &= (1, 0, 1, 0) = \ell_0 \cap \ell_3 \\
P_{75} &= (1, 0, 0, 1) = \ell_1 \cap \ell_4 \\
P_{82} &= (0, 1, 0, 1) = \ell_2 \cap \ell_3
\end{aligned}$$

$$P_{146} = (0, 1, 1, 1) = \ell_2 \cap \ell_4$$

Single Points

The surface has 34 single points:

The single points on the surface are:

- 0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
- 1 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_3
- 2 : $P_{13} = (2, 0, 1, 0)$ lies on line ℓ_0
- 3 : $P_{14} = (3, 0, 1, 0)$ lies on line ℓ_0
- 4 : $P_{15} = (4, 0, 1, 0)$ lies on line ℓ_0
- 5 : $P_{16} = (5, 0, 1, 0)$ lies on line ℓ_0
- 6 : $P_{17} = (6, 0, 1, 0)$ lies on line ℓ_0
- 7 : $P_{18} = (7, 0, 1, 0)$ lies on line ℓ_0
- 8 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_4
- 9 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_1
- 10 : $P_{202} = (1, 0, 2, 1)$ lies on line ℓ_1
- 11 : $P_{209} = (0, 1, 2, 1)$ lies on line ℓ_2
- 12 : $P_{211} = (2, 1, 2, 1)$ lies on line ℓ_3
- 13 : $P_{220} = (3, 2, 2, 1)$ lies on line ℓ_4
- 14 : $P_{266} = (1, 0, 3, 1)$ lies on line ℓ_1
- 15 : $P_{273} = (0, 1, 3, 1)$ lies on line ℓ_2
- 16 : $P_{276} = (3, 1, 3, 1)$ lies on line ℓ_3
- 17 : $P_{291} = (2, 3, 3, 1)$ lies on line ℓ_4

- 18 : $P_{330} = (1, 0, 4, 1)$ lies on line ℓ_1
- 19 : $P_{337} = (0, 1, 4, 1)$ lies on line ℓ_2
- 20 : $P_{341} = (4, 1, 4, 1)$ lies on line ℓ_3
- 21 : $P_{366} = (5, 4, 4, 1)$ lies on line ℓ_4
- 22 : $P_{394} = (1, 0, 5, 1)$ lies on line ℓ_1
- 23 : $P_{401} = (0, 1, 5, 1)$ lies on line ℓ_2
- 24 : $P_{406} = (5, 1, 5, 1)$ lies on line ℓ_3
- 25 : $P_{437} = (4, 5, 5, 1)$ lies on line ℓ_4
- 26 : $P_{458} = (1, 0, 6, 1)$ lies on line ℓ_1
- 27 : $P_{465} = (0, 1, 6, 1)$ lies on line ℓ_2
- 28 : $P_{471} = (6, 1, 6, 1)$ lies on line ℓ_3
- 29 : $P_{512} = (7, 6, 6, 1)$ lies on line ℓ_4
- 30 : $P_{522} = (1, 0, 7, 1)$ lies on line ℓ_1
- 31 : $P_{529} = (0, 1, 7, 1)$ lies on line ℓ_2
- 32 : $P_{536} = (7, 1, 7, 1)$ lies on line ℓ_3
- 33 : $P_{583} = (6, 7, 7, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:

The points on the surface but not on lines are:

0 : $P_{31} = (4, 2, 1, 0)$	22 : $P_{346} = (1, 2, 4, 1)$
1 : $P_{40} = (5, 3, 1, 0)$	23 : $P_{349} = (4, 2, 4, 1)$
2 : $P_{50} = (7, 4, 1, 0)$	24 : $P_{357} = (4, 3, 4, 1)$
3 : $P_{57} = (6, 5, 1, 0)$	25 : $P_{358} = (5, 3, 4, 1)$
4 : $P_{62} = (3, 6, 1, 0)$	26 : $P_{363} = (2, 4, 4, 1)$
5 : $P_{69} = (2, 7, 1, 0)$	27 : $P_{388} = (3, 7, 4, 1)$
6 : $P_{92} = (2, 2, 0, 1)$	28 : $P_{390} = (5, 7, 4, 1)$
7 : $P_{104} = (6, 3, 0, 1)$	29 : $P_{418} = (1, 3, 5, 1)$
8 : $P_{110} = (4, 4, 0, 1)$	30 : $P_{420} = (3, 3, 5, 1)$
9 : $P_{117} = (3, 5, 0, 1)$	31 : $P_{435} = (2, 5, 5, 1)$
10 : $P_{127} = (5, 6, 0, 1)$	32 : $P_{498} = (1, 5, 6, 1)$
11 : $P_{137} = (7, 7, 0, 1)$	33 : $P_{502} = (5, 5, 6, 1)$
12 : $P_{224} = (7, 2, 2, 1)$	34 : $P_{509} = (4, 6, 6, 1)$
13 : $P_{236} = (3, 4, 2, 1)$	35 : $P_{542} = (5, 2, 7, 1)$
14 : $P_{239} = (6, 4, 2, 1)$	36 : $P_{543} = (6, 2, 7, 1)$
15 : $P_{251} = (2, 6, 2, 1)$	37 : $P_{554} = (1, 4, 7, 1)$
16 : $P_{252} = (3, 6, 2, 1)$	38 : $P_{560} = (7, 4, 7, 1)$
17 : $P_{258} = (1, 7, 2, 1)$	39 : $P_{567} = (6, 5, 7, 1)$
18 : $P_{259} = (2, 7, 2, 1)$	40 : $P_{568} = (7, 5, 7, 1)$
19 : $P_{296} = (7, 3, 3, 1)$	41 : $P_{581} = (4, 7, 7, 1)$
20 : $P_{314} = (1, 6, 3, 1)$	
21 : $P_{319} = (6, 6, 3, 1)$	

Line Intersection Graph

	0 1 2 3 4
0	0 1 1 1 0
1	1 0 1 0 1
2	1 1 0 1 1
3	1 0 1 0 0
4	0 1 1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_{12}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_2	P_2	P_{75}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_2	P_{82}	P_{146}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{12}	P_{82}

Line 4 intersects

Line	ℓ_1	ℓ_2
in point	P_{75}	P_{146}

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	28 : $P_{209} = (0, 1, 2, 1)$	56 : $P_{394} = (1, 0, 5, 1)$
1 : $P_2 = (0, 0, 1, 0)$	29 : $P_{211} = (2, 1, 2, 1)$	57 : $P_{401} = (0, 1, 5, 1)$
2 : $P_4 = (1, 1, 1, 1)$	30 : $P_{220} = (3, 2, 2, 1)$	58 : $P_{406} = (5, 1, 5, 1)$
3 : $P_{12} = (1, 0, 1, 0)$	31 : $P_{224} = (7, 2, 2, 1)$	59 : $P_{418} = (1, 3, 5, 1)$
4 : $P_{13} = (2, 0, 1, 0)$	32 : $P_{236} = (3, 4, 2, 1)$	60 : $P_{420} = (3, 3, 5, 1)$
5 : $P_{14} = (3, 0, 1, 0)$	33 : $P_{239} = (6, 4, 2, 1)$	61 : $P_{435} = (2, 5, 5, 1)$
6 : $P_{15} = (4, 0, 1, 0)$	34 : $P_{251} = (2, 6, 2, 1)$	62 : $P_{437} = (4, 5, 5, 1)$
7 : $P_{16} = (5, 0, 1, 0)$	35 : $P_{252} = (3, 6, 2, 1)$	63 : $P_{458} = (1, 0, 6, 1)$
8 : $P_{17} = (6, 0, 1, 0)$	36 : $P_{258} = (1, 7, 2, 1)$	64 : $P_{465} = (0, 1, 6, 1)$
9 : $P_{18} = (7, 0, 1, 0)$	37 : $P_{259} = (2, 7, 2, 1)$	65 : $P_{471} = (6, 1, 6, 1)$
10 : $P_{20} = (1, 1, 1, 0)$	38 : $P_{266} = (1, 0, 3, 1)$	66 : $P_{498} = (1, 5, 6, 1)$
11 : $P_{31} = (4, 2, 1, 0)$	39 : $P_{273} = (0, 1, 3, 1)$	67 : $P_{502} = (5, 5, 6, 1)$
12 : $P_{40} = (5, 3, 1, 0)$	40 : $P_{276} = (3, 1, 3, 1)$	68 : $P_{509} = (4, 6, 6, 1)$
13 : $P_{50} = (7, 4, 1, 0)$	41 : $P_{291} = (2, 3, 3, 1)$	69 : $P_{512} = (7, 6, 6, 1)$
14 : $P_{57} = (6, 5, 1, 0)$	42 : $P_{296} = (7, 3, 3, 1)$	70 : $P_{522} = (1, 0, 7, 1)$
15 : $P_{62} = (3, 6, 1, 0)$	43 : $P_{314} = (1, 6, 3, 1)$	71 : $P_{529} = (0, 1, 7, 1)$
16 : $P_{69} = (2, 7, 1, 0)$	44 : $P_{319} = (6, 6, 3, 1)$	72 : $P_{536} = (7, 1, 7, 1)$
17 : $P_{75} = (1, 0, 0, 1)$	45 : $P_{330} = (1, 0, 4, 1)$	73 : $P_{542} = (5, 2, 7, 1)$
18 : $P_{82} = (0, 1, 0, 1)$	46 : $P_{337} = (0, 1, 4, 1)$	74 : $P_{543} = (6, 2, 7, 1)$
19 : $P_{92} = (2, 2, 0, 1)$	47 : $P_{341} = (4, 1, 4, 1)$	75 : $P_{554} = (1, 4, 7, 1)$
20 : $P_{104} = (6, 3, 0, 1)$	48 : $P_{346} = (1, 2, 4, 1)$	76 : $P_{560} = (7, 4, 7, 1)$
21 : $P_{110} = (4, 4, 0, 1)$	49 : $P_{349} = (4, 2, 4, 1)$	77 : $P_{567} = (6, 5, 7, 1)$
22 : $P_{117} = (3, 5, 0, 1)$	50 : $P_{357} = (4, 3, 4, 1)$	78 : $P_{568} = (7, 5, 7, 1)$
23 : $P_{127} = (5, 6, 0, 1)$	51 : $P_{358} = (5, 3, 4, 1)$	79 : $P_{581} = (4, 7, 7, 1)$
24 : $P_{137} = (7, 7, 0, 1)$	52 : $P_{363} = (2, 4, 4, 1)$	80 : $P_{583} = (6, 7, 7, 1)$
25 : $P_{139} = (1, 0, 1, 1)$	53 : $P_{366} = (5, 4, 4, 1)$	
26 : $P_{146} = (0, 1, 1, 1)$	54 : $P_{388} = (3, 7, 4, 1)$	
27 : $P_{202} = (1, 0, 2, 1)$	55 : $P_{390} = (5, 7, 4, 1)$	