Rank-74007 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286396949

General information

Number of lines	5
Number of points	289
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	74
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^5
Type of lines on points	3, 2^4, 1^{74}, 0^{210}

Singular Points

The surface has 3 singular points:

$0:\ P_{546} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$
$1:\ P_{3019} = \mathbf{P}(\delta^{10},\delta^5,\delta^{10},1) = \mathbf{P}(10,11,10,1)$
$2:\ P_{3260} = \mathbf{P}(\delta^5,\delta^{10},\delta^5,1) = \mathbf{P}(11,10,11,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69904} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \mathbf{Pl}(1,1,0,1,1,1)_{9201}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{7115} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{7115} = \mathbf{Pl}(11,10,10,11,11,1)_{52726}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^{5} & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{7388} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{7388} = \mathbf{Pl}(10,11,11,10,10,1)_{48840}$$

Rank of lines: (69904, 69921, 4657, 7115, 7388)
Rank of points on Klein quadric: (33, 49, 9201, 52726, 48840)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{546} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$
$P_{290} = (0,1,0,1) = \ell_0 \cap \ell_2$
$P_{3260} = (11,10,11,1) = \ell_2 \cap \ell_3$
$P_{3019} = (10,11,10,1) = \ell_2 \cap \ell_4$

Single Points

The surface has 74 single points:
The single points on the surface are:

$0 : P_1 = (0,1,0,0)$ lies on line ℓ_0
$1 : P_{35} = (0,1,1,0)$ lies on line ℓ_1
$2 : P_{36} = (1,1,1,0)$ lies on line ℓ_2
$3 : P_{189} = (10,10,1,0)$ lies on line ℓ_3
$4 : P_{206} = (11,11,1,0)$ lies on line ℓ_4
$5 : P_{306} = (0,2,0,1)$ lies on line ℓ_0
$6 : P_{322} = (0,3,0,1)$ lies on line ℓ_0
$7 : P_{338} = (0,4,0,1)$ lies on line ℓ_0
$8 : P_{354} = (0,5,0,1)$ lies on line ℓ_0
$9 : P_{370} = (0,6,0,1)$ lies on line ℓ_0
$10 : P_{386} = (0,7,0,1)$ lies on line ℓ_0
$11 : P_{402} = (0,8,0,1)$ lies on line ℓ_0
$12 : P_{418} = (0,9,0,1)$ lies on line ℓ_0
$13 : P_{434} = (0,10,0,1)$ lies on line ℓ_0
$14 : P_{445} = (11,10,0,1)$ lies on line ℓ_4
$15 : P_{450} = (0,11,0,1)$ lies on line ℓ_0
$16 : P_{460} = (10,11,0,1)$ lies on line ℓ_3
$17 : P_{466} = (0,12,0,1)$ lies on line ℓ_0
$18 : P_{482} = (0,13,0,1)$ lies on line ℓ_0
$19 : P_{498} = (0,14,0,1)$ lies on line ℓ_0
$20 : P_{514} = (0,15,0,1)$ lies on line ℓ_0
$21 : P_{531} = (1,0,1,1)$ lies on line ℓ_2
$22 : P_{817} = (0,2,2,1)$ lies on line ℓ_1
$23 : P_{835} = (2,3,2,1)$ lies on line ℓ_2
$24 : P_{869} = (4,5,2,1)$ lies on line ℓ_4
$25 : P_{888} = (7,6,2,1)$ lies on line ℓ_3
$26 : P_{1076} = (3,2,3,1)$ lies on line ℓ_2
$27 : P_{1089} = (0,3,3,1)$ lies on line ℓ_1
$28 : P_{1246} = (13,12,3,1)$ lies on line ℓ_3
$29 : P_{1280} = (15,14,3,1)$ lies on line ℓ_4
$30 : P_{1361} = (0,4,4,1)$ lies on line ℓ_1
$31 : P_{1381} = (4,5,4,1)$ lies on line ℓ_2
$32 : P_{1434} = (9,8,4,1)$ lies on line ℓ_3
$33 : P_{1517} = (12,13,4,1)$ lies on line ℓ_4
$34 : P_{1588} = (3,2,5,1)$ lies on line ℓ_3
$35 : P_{1622} = (5,4,5,1)$ lies on line ℓ_2

36 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_1
37 : $P_{1656} = (7, 6, 5, 1)$ lies on line ℓ_4
38 : $P_{1844} = (3, 2, 6, 1)$ lies on line ℓ_4
39 : $P_{1893} = (4, 5, 6, 1)$ lies on line ℓ_3
40 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_1
41 : $P_{1927} = (6, 7, 6, 1)$ lies on line ℓ_2
42 : $P_{2168} = (7, 6, 7, 1)$ lies on line ℓ_2
43 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_1
44 : $P_{2217} = (8, 9, 7, 1)$ lies on line ℓ_4
45 : $P_{2319} = (14, 15, 7, 1)$ lies on line ℓ_3
46 : $P_{2390} = (5, 4, 8, 1)$ lies on line ℓ_4
47 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_1
48 : $P_{2473} = (8, 9, 8, 1)$ lies on line ℓ_2
49 : $P_{2541} = (12, 13, 8, 1)$ lies on line ℓ_3
50 : $P_{2695} = (6, 7, 9, 1)$ lies on line ℓ_3
51 : $P_{2714} = (9, 8, 9, 1)$ lies on line ℓ_2
52 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_1
53 : $P_{2831} = (14, 15, 9, 1)$ lies on line ℓ_4
54 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_3
55 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_1
56 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_4
57 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_1
58 : $P_{3395} = (2, 3, 12, 1)$ lies on line ℓ_4
59 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_1
60 : $P_{3565} = (12, 13, 12, 1)$ lies on line ℓ_2
61 : $P_{3584} = (15, 14, 12, 1)$ lies on line ℓ_3
62 : $P_{3670} = (5, 4, 13, 1)$ lies on line ℓ_3
63 : $P_{3738} = (9, 8, 13, 1)$ lies on line ℓ_4
64 : $P_{3806} = (13, 12, 13, 1)$ lies on line ℓ_2
65 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_1
66 : $P_{3907} = (2, 3, 14, 1)$ lies on line ℓ_3
67 : $P_{4062} = (13, 12, 14, 1)$ lies on line ℓ_4
68 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_1
69 : $P_{4111} = (14, 15, 14, 1)$ lies on line ℓ_2
70 : $P_{4231} = (6, 7, 15, 1)$ lies on line ℓ_4
71 : $P_{4265} = (8, 9, 15, 1)$ lies on line ℓ_3
72 : $P_{4352} = (15, 14, 15, 1)$ lies on line ℓ_2
73 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{73} = (6, 3, 1, 0)$
2 : $P_{74} = (7, 3, 1, 0)$
3 : $P_{111} = (12, 5, 1, 0)$
4 : $P_{112} = (13, 5, 1, 0)$
5 : $P_{153} = (6, 8, 1, 0)$
6 : $P_{154} = (7, 8, 1, 0)$
7 : $P_{190} = (11, 10, 1, 0)$
8 : $P_{205} = (10, 11, 1, 0)$
9 : $P_{271} = (12, 15, 1, 0)$
10 : $P_{272} = (13, 15, 1, 0)$
11 : $P_{291} = (1, 1, 0, 1)$
12 : $P_{318} = (12, 2, 0, 1)$
13 : $P_{330} = (8, 3, 0, 1)$
14 : $P_{344} = (6, 4, 0, 1)$
15 : $P_{369} = (15, 5, 0, 1)$
16 : $P_{374} = (4, 6, 0, 1)$
17 : $P_{400} = (14, 7, 0, 1)$
18 : $P_{405} = (3, 8, 0, 1)$
19 : $P_{431} = (13, 9, 0, 1)$
20 : $P_{468} = (2, 12, 0, 1)$
21 : $P_{491} = (9, 13, 0, 1)$
22 : $P_{505} = (7, 14, 0, 1)$
23 : $P_{519} = (5, 15, 0, 1)$
24 : $P_{635} = (10, 6, 1, 1)$
25 : $P_{640} = (15, 6, 1, 1)$
26 : $P_{646} = (5, 7, 1, 1)$
27 : $P_{651} = (10, 7, 1, 1)$
28 : $P_{694} = (5, 10, 1, 1)$
29 : $P_{704} = (15, 10, 1, 1)$
30 : $P_{708} = (3, 11, 1, 1)$
31 : $P_{713} = (8, 11, 1, 1)$
32 : $P_{729} = (8, 12, 1, 1)$
33 : $P_{732} = (11, 12, 1, 1)$
34 : $P_{740} = (3, 13, 1, 1)$
35 : $P_{748} = (11, 13, 1, 1)$
36 : $P_{793} = (8, 0, 2, 1)$
37 : $P_{831} = (14, 2, 2, 1)$
38 : $P_{841} = (8, 3, 2, 1)$
39 : $P_{874} = (9, 5, 2, 1)$
40 : $P_{882} = (1, 6, 2, 1)$
41 : $P_{904} = (7, 7, 2, 1)$
42 : $P_{908} = (11, 7, 2, 1)$
43 : $P_{933} = (4, 9, 2, 1)$
44 : $P_{940} = (11, 9, 2, 1)$
45 : $P_{962} = (1, 11, 2, 1)$
46 : $P_{970} = (9, 11, 2, 1)$
47 : $P_{982} = (5, 12, 2, 1)$

$48: P_{998} = (5, 13, 2, 1)$
$49: P_{1007} = (14, 13, 2, 1)$
$50: P_{1056} = (15, 0, 3, 1)$
$51: P_{1085} = (12, 2, 3, 1)$
$52: P_{1100} = (11, 3, 3, 1)$
$53: P_{1173} = (4, 8, 3, 1)$
$54: P_{1221} = (4, 11, 3, 1)$
$55: P_{1230} = (13, 11, 3, 1)$
$56: P_{1245} = (12, 12, 3, 1)$
$57: P_{1276} = (11, 14, 3, 1)$
$58: P_{1312} = (15, 0, 4, 1)$
$59: P_{1363} = (2, 4, 4, 1)$
$60: P_{1392} = (15, 5, 4, 1)$
$61: P_{1401} = (8, 6, 4, 1)$
$62: P_{1411} = (2, 7, 4, 1)$
$63: P_{1417} = (8, 7, 4, 1)$
$64: P_{1439} = (14, 8, 4, 1)$
$65: P_{1458} = (1, 10, 4, 1)$
$66: P_{1471} = (14, 10, 4, 1)$
$67: P_{1499} = (10, 12, 4, 1)$
$68: P_{1501} = (12, 12, 4, 1)$
$69: P_{1506} = (1, 13, 4, 1)$
$70: P_{1530} = (9, 14, 4, 1)$
$71: P_{1531} = (10, 14, 4, 1)$
$72: P_{1556} = (3, 0, 5, 1)$
$73: P_{1595} = (10, 2, 5, 1)$
$74: P_{1623} = (6, 4, 5, 1)$
$75: P_{1643} = (10, 5, 5, 1)$
$76: P_{1655} = (6, 6, 5, 1)$
$77: P_{1720} = (7, 10, 5, 1)$
$78: P_{1722} = (9, 10, 5, 1)$
$79: P_{1802} = (9, 15, 5, 1)$
$80: P_{1814} = (5, 0, 6, 1)$
$81: P_{1834} = (9, 1, 6, 1)$
$82: P_{1839} = (14, 1, 6, 1)$
$83: P_{1850} = (9, 2, 6, 1)$
$84: P_{1885} = (12, 4, 6, 1)$
$85: P_{1902} = (13, 5, 6, 1)$
$86: P_{1907} = (2, 6, 6, 1)$
$87: P_{1935} = (14, 7, 6, 1)$
$88: P_{1960} = (7, 9, 6, 1)$
$89: P_{1965} = (12, 9, 6, 1)$
$90: P_{2004} = (3, 12, 6, 1)$
$91: P_{2008} = (7, 12, 6, 1)$
$92: P_{2019} = (2, 13, 6, 1)$
$93: P_{2030} = (13, 13, 6, 1)$
$94: P_{2037} = (4, 14, 6, 1)$
$95: P_{2038} = (5, 14, 6, 1)$
$96: P_{2080} = (15, 0, 7, 1)$
$97: P_{2083} = (2, 1, 7, 1)$
$98: P_{2085} = (4, 1, 7, 1)$
$99: P_{2103} = (6, 2, 7, 1)$
$100: P_{2110} = (13, 2, 7, 1)$
$101: P_{2143} = (14, 4, 7, 1)$
$102: P_{2144} = (15, 4, 7, 1)$
$103: P_{2165} = (4, 6, 7, 1)$
$104: P_{2186} = (9, 7, 7, 1)$
$105: P_{2211} = (2, 9, 7, 1)$
$106: P_{2266} = (9, 12, 7, 1)$
$107: P_{2269} = (12, 12, 7, 1)$
$108: P_{2279} = (6, 13, 7, 1)$
$109: P_{2281} = (8, 13, 7, 1)$
$110: P_{2302} = (13, 14, 7, 1)$
$111: P_{2317} = (12, 15, 7, 1)$
$112: P_{2326} = (5, 0, 8, 1)$
$113: P_{2383} = (14, 3, 8, 1)$
$114: P_{2396} = (11, 4, 8, 1)$
$115: P_{2460} = (11, 8, 8, 1)$
$116: P_{2478} = (13, 9, 8, 1)$
$117: P_{2509} = (12, 11, 8, 1)$
$118: P_{2511} = (14, 11, 8, 1)$
$119: P_{2542} = (13, 13, 8, 1)$
$120: P_{2580} = (3, 0, 9, 1)$
$121: P_{2620} = (11, 2, 9, 1)$
$122: P_{2623} = (14, 2, 9, 1)$
$123: P_{2679} = (6, 6, 9, 1)$
$124: P_{2684} = (11, 6, 9, 1)$
$125: P_{2690} = (1, 7, 9, 1)$
$126: P_{2708} = (3, 8, 9, 1)$
$127: P_{2725} = (4, 9, 9, 1)$
$128: P_{2754} = (1, 11, 9, 1)$
$129: P_{2755} = (2, 11, 9, 1)$
$130: P_{2773} = (4, 12, 9, 1)$
$131: P_{2784} = (15, 12, 9, 1)$
$132: P_{2800} = (15, 13, 9, 1)$
$133: P_{2819} = (2, 15, 9, 1)$
$134: P_{2851} = (2, 1, 10, 1)$
$135: P_{2858} = (9, 1, 10, 1)$
$136: P_{2899} = (2, 4, 10, 1)$
$137: P_{2911} = (14, 4, 10, 1)$
$138: P_{2924} = (11, 5, 10, 1)$
$139: P_{2927} = (14, 5, 10, 1)$
$140: P_{2994} = (1, 10, 10, 1)$
$141: P_{3061} = (4, 14, 10, 1)$
$142: P_{3066} = (9, 14, 10, 1)$
$143: P_{3077} = (4, 15, 10, 1)$
$144: P_{3084} = (11, 15, 10, 1)$
$145: P_{3109} = (4, 1, 11, 1)$
$146: P_{3119} = (14, 1, 11, 1)$
$147: P_{3130} = (9, 2, 11, 1)$
$148: P_{3135} = (14, 2, 11, 1)$
$149: P_{3146} = (9, 3, 11, 1)$
$150: P_{3147} = (10, 3, 11, 1)$
$151: P_{3219} = (2, 8, 11, 1)$
$152: P_{3227} = (10, 8, 11, 1)$
$153: P_{3235} = (2, 9, 11, 1)$
$154: P_{3237} = (4, 9, 11, 1)$
$155: P_{3266} = (1, 11, 11, 1)$

156 : $P_{3348} = (3, 0, 12, 1)$
157 : $P_{3365} = (4, 1, 12, 1)$
158 : $P_{3370} = (9, 1, 12, 1)$
159 : $P_{3384} = (7, 2, 12, 1)$
160 : $P_{3399} = (6, 3, 12, 1)$
161 : $P_{3416} = (7, 4, 12, 1)$
162 : $P_{3422} = (13, 4, 12, 1)$
163 : $P_{3447} = (6, 6, 12, 1)$
164 : $P_{3455} = (14, 6, 12, 1)$
165 : $P_{3470} = (13, 7, 12, 1)$
166 : $P_{3472} = (15, 7, 12, 1)$
167 : $P_{3491} = (2, 9, 12, 1)$
168 : $P_{3492} = (3, 9, 12, 1)$
169 : $P_{3551} = (14, 12, 12, 1)$
170 : $P_{3562} = (9, 13, 12, 1)$
171 : $P_{3573} = (4, 14, 12, 1)$
172 : $P_{3609} = (8, 0, 13, 1)$
173 : $P_{3619} = (2, 1, 13, 1)$
174 : $P_{3631} = (14, 1, 13, 1)$
175 : $P_{3641} = (8, 2, 13, 1)$
176 : $P_{3642} = (9, 2, 13, 1)$
177 : $P_{3679} = (14, 4, 13, 1)$
178 : $P_{3702} = (5, 6, 13, 1)$
179 : $P_{3709} = (12, 6, 13, 1)$
180 : $P_{3717} = (4, 7, 13, 1)$
181 : $P_{3720} = (7, 7, 13, 1)$
182 : $P_{3736} = (7, 8, 13, 1)$
183 : $P_{3751} = (6, 9, 13, 1)$
184 : $P_{3795} = (2, 12, 13, 1)$
185 : $P_{3813} = (4, 13, 13, 1)$
186 : $P_{3831} = (6, 14, 13, 1)$
187 : $P_{3837} = (12, 14, 13, 1)$
188 : $P_{3862} = (5, 0, 14, 1)$
189 : $P_{3909} = (4, 3, 14, 1)$
190 : $P_{3923} = (2, 4, 14, 1)$
191 : $P_{3931} = (10, 4, 14, 1)$
192 : $P_{3956} = (3, 6, 14, 1)$
193 : $P_{3962} = (9, 6, 14, 1)$
194 : $P_{3972} = (3, 7, 14, 1)$
195 : $P_{4018} = (1, 10, 14, 1)$
196 : $P_{4021} = (4, 10, 14, 1)$
197 : $P_{4050} = (1, 12, 14, 1)$
198 : $P_{4075} = (10, 13, 14, 1)$
199 : $P_{4078} = (13, 13, 14, 1)$
200 : $P_{4090} = (9, 14, 14, 1)$
201 : $P_{4102} = (5, 15, 14, 1)$
202 : $P_{4121} = (8, 0, 15, 1)$
203 : $P_{4195} = (2, 5, 15, 1)$
204 : $P_{4232} = (7, 7, 15, 1)$
205 : $P_{4267} = (10, 9, 15, 1)$
206 : $P_{4275} = (2, 10, 15, 1)$
207 : $P_{4279} = (6, 10, 15, 1)$
208 : $P_{4344} = (7, 14, 15, 1)$
209 : $P_{4363} = (10, 15, 15, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	0	1	1
2	1	0	0	1	1
3	0	1	1	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_{290}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_3	P_{546}	P_{546}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_{290}	P_{3260}	P_{3019}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{546}	P_{3260}	P_{546}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_{546}	P_{3019}	P_{546}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_3 = (0,0,0,1)$
3 : $P_{35} = (0,1,1,0)$
4 : $P_{36} = (1,1,1,0)$
5 : $P_{73} = (6,3,1,0)$
6 : $P_{74} = (7,3,1,0)$
7 : $P_{111} = (12,5,1,0)$
8 : $P_{112} = (13,5,1,0)$
9 : $P_{153} = (6,8,1,0)$
10 : $P_{154} = (7,8,1,0)$
11 : $P_{189} = (10,10,1,0)$
12 : $P_{190} = (11,10,1,0)$
13 : $P_{205} = (10,11,1,0)$
14 : $P_{206} = (11,11,1,0)$
15 : $P_{271} = (12,15,1,0)$
16 : $P_{272} = (13,15,1,0)$
17 : $P_{290} = (0,1,0,1)$
18 : $P_{291} = (1,1,0,1)$
19 : $P_{306} = (0,2,0,1)$
20 : $P_{318} = (12,2,0,1)$
21 : $P_{322} = (0,3,0,1)$
22 : $P_{330} = (8,3,0,1)$
23 : $P_{338} = (0,4,0,1)$
24 : $P_{344} = (6,4,0,1)$
25 : $P_{354} = (0,5,0,1)$
26 : $P_{369} = (15,5,0,1)$
27 : $P_{370} = (0,6,0,1)$
28 : $P_{374} = (4,6,0,1)$
29 : $P_{386} = (0,7,0,1)$
30 : $P_{400} = (14,7,0,1)$
31 : $P_{402} = (0,8,0,1)$
32 : $P_{405} = (3,8,0,1)$
33 : $P_{418} = (0,9,0,1)$
34 : $P_{431} = (13,9,0,1)$
35 : $P_{434} = (0,10,0,1)$
36 : $P_{445} = (11,10,0,1)$
37 : $P_{450} = (0,11,0,1)$
38 : $P_{460} = (10,11,0,1)$
39 : $P_{466} = (0,12,0,1)$
40 : $P_{468} = (2,12,0,1)$
41 : $P_{482} = (0,13,0,1)$
42 : $P_{491} = (9,13,0,1)$
43 : $P_{498} = (0,14,0,1)$
44 : $P_{505} = (7,14,0,1)$
45 : $P_{514} = (0,15,0,1)$
46 : $P_{519} = (5,15,0,1)$
47 : $P_{531} = (1,0,1,1)$
48 : $P_{546} = (0,1,1,1)$
49 : $P_{635} = (10,6,1,1)$
50 : $P_{640} = (15,6,1,1)$
51 : $P_{646} = (5,7,1,1)$
52 : $P_{651} = (10,7,1,1)$
53 : $P_{694} = (5,10,1,1)$
54 : $P_{704} = (15,10,1,1)$
55 : $P_{708} = (3,11,1,1)$
56 : $P_{713} = (8,11,1,1)$
57 : $P_{729} = (8,12,1,1)$
58 : $P_{732} = (11,12,1,1)$
59 : $P_{740} = (3,13,1,1)$
60 : $P_{748} = (11,13,1,1)$
61 : $P_{793} = (8,0,2,1)$
62 : $P_{817} = (0,2,2,1)$
63 : $P_{831} = (14,2,2,1)$
64 : $P_{835} = (2,3,2,1)$
65 : $P_{841} = (8,3,2,1)$
66 : $P_{869} = (4,5,2,1)$
67 : $P_{874} = (9,5,2,1)$
68 : $P_{882} = (1,6,2,1)$
69 : $P_{888} = (7,6,2,1)$
70 : $P_{904} = (7,7,2,1)$
71 : $P_{908} = (11,7,2,1)$
72 : $P_{933} = (4,9,2,1)$
73 : $P_{940} = (11,9,2,1)$
74 : $P_{962} = (1,11,2,1)$
75 : $P_{970} = (9,11,2,1)$
76 : $P_{982} = (5,12,2,1)$
77 : $P_{998} = (5,13,2,1)$
78 : $P_{1007} = (14,13,2,1)$
79 : $P_{1056} = (15,0,3,1)$
80 : $P_{1076} = (3,2,3,1)$
81 : $P_{1085} = (12,2,3,1)$
82 : $P_{1089} = (0,3,3,1)$
83 : $P_{1100} = (11,3,3,1)$
84 : $P_{1173} = (4,8,3,1)$
85 : $P_{1221} = (4,11,3,1)$
86 : $P_{1230} = (13,11,3,1)$
87 : $P_{1245} = (12,12,3,1)$
88 : $P_{1246} = (13,12,3,1)$
89 : $P_{1276} = (11,14,3,1)$
90 : $P_{1280} = (15,14,3,1)$
91 : $P_{1312} = (15,0,4,1)$
92 : $P_{1361} = (0,4,4,1)$
93 : $P_{1363} = (2,4,4,1)$
94 : $P_{1381} = (4,5,4,1)$
95 : $P_{1392} = (15,5,4,1)$
96 : $P_{1401} = (8,6,4,1)$
97 : $P_{1411} = (2,7,4,1)$
98 : $P_{1417} = (8,7,4,1)$
99 : $P_{1434} = (9,8,4,1)$
100 : $P_{1439} = (14,8,4,1)$
101 : $P_{1458} = (1,10,4,1)$
102 : $P_{1471} = (14,10,4,1)$
103 : $P_{1499} = (10,12,4,1)$
104 : $P_{1501} = (12,12,4,1)$
105 : $P_{1506} = (1,13,4,1)$
106 : $P_{1517} = (12,13,4,1)$
107 : $P_{1530} = (9,14,4,1)$
108 : $P_{1531} = (10,14,4,1)$
109 : $P_{1556} = (3,0,5,1)$
110 : $P_{1588} = (3,2,5,1)$
111 : $P_{1595} = (10,2,5,1)$
112 : $P_{1622} = (5,4,5,1)$
113 : $P_{1623} = (6,4,5,1)$
114 : $P_{1633} = (0,5,5,1)$
115 : $P_{1643} = (10,5,5,1)$
116 : $P_{1655} = (6,6,5,1)$
117 : $P_{1656} = (7,6,5,1)$
118 : $P_{1720} = (7,10,5,1)$
119 : $P_{1722} = (9,10,5,1)$
120 : $P_{1802} = (9,15,5,1)$
121 : $P_{1814} = (5,0,6,1)$
122 : $P_{1834} = (9,1,6,1)$
123 : $P_{1839} = (14,1,6,1)$
124 : $P_{1844} = (3,2,6,1)$
125 : $P_{1850} = (9,2,6,1)$
126 : $P_{1885} = (12,4,6,1)$
127 : $P_{1893} = (4,5,6,1)$
128 : $P_{1902} = (13,5,6,1)$
129 : $P_{1905} = (0,6,6,1)$
130 : $P_{1907} = (2,6,6,1)$
131 : $P_{1927} = (6,7,6,1)$
132 : $P_{1935} = (14,7,6,1)$
133 : $P_{1960} = (7,9,6,1)$
134 : $P_{1965} = (12,9,6,1)$
135 : $P_{2004} = (3,12,6,1)$
136 : $P_{2008} = (7,12,6,1)$
137 : $P_{2019} = (2,13,6,1)$
138 : $P_{2030} = (13,13,6,1)$
139 : $P_{2037} = (4,14,6,1)$
140 : $P_{2038} = (5,14,6,1)$
141 : $P_{2080} = (15,0,7,1)$
142 : $P_{2083} = (2,1,7,1)$
143 : $P_{2085} = (4,1,7,1)$
144 : $P_{2103} = (6,2,7,1)$
145 : $P_{2110} = (13,2,7,1)$
146 : $P_{2143} = (14,4,7,1)$
147 : $P_{2144} = (15,4,7,1)$
148 : $P_{2165} = (4,6,7,1)$
149 : $P_{2168} = (7,6,7,1)$

$150: P_{2177} = (0, 7, 7, 1)$
$151: P_{2186} = (9, 7, 7, 1)$
$152: P_{2211} = (2, 9, 7, 1)$
$153: P_{2217} = (8, 9, 7, 1)$
$154: P_{2266} = (9, 12, 7, 1)$
$155: P_{2269} = (12, 12, 7, 1)$
$156: P_{2279} = (6, 13, 7, 1)$
$157: P_{2281} = (8, 13, 7, 1)$
$158: P_{2302} = (13, 14, 7, 1)$
$159: P_{2317} = (12, 15, 7, 1)$
$160: P_{2319} = (14, 15, 7, 1)$
$161: P_{2326} = (5, 0, 8, 1)$
$162: P_{2383} = (14, 3, 8, 1)$
$163: P_{2390} = (5, 4, 8, 1)$
$164: P_{2396} = (11, 4, 8, 1)$
$165: P_{2449} = (0, 8, 8, 1)$
$166: P_{2460} = (11, 8, 8, 1)$
$167: P_{2473} = (8, 9, 8, 1)$
$168: P_{2478} = (13, 9, 8, 1)$
$169: P_{2509} = (12, 11, 8, 1)$
$170: P_{2511} = (14, 11, 8, 1)$
$171: P_{2541} = (12, 13, 8, 1)$
$172: P_{2542} = (13, 13, 8, 1)$
$173: P_{2580} = (3, 0, 9, 1)$
$174: P_{2620} = (11, 2, 9, 1)$
$175: P_{2623} = (14, 2, 9, 1)$
$176: P_{2679} = (6, 6, 9, 1)$
$177: P_{2684} = (11, 6, 9, 1)$
$178: P_{2690} = (1, 7, 9, 1)$
$179: P_{2695} = (6, 7, 9, 1)$
$180: P_{2708} = (3, 8, 9, 1)$
$181: P_{2714} = (9, 8, 9, 1)$
$182: P_{2721} = (0, 9, 9, 1)$
$183: P_{2725} = (4, 9, 9, 1)$
$184: P_{2754} = (1, 11, 9, 1)$
$185: P_{2755} = (2, 11, 9, 1)$
$186: P_{2773} = (4, 12, 9, 1)$
$187: P_{2784} = (15, 12, 9, 1)$
$188: P_{2800} = (15, 13, 9, 1)$
$189: P_{2819} = (2, 15, 9, 1)$
$190: P_{2831} = (14, 15, 9, 1)$
$191: P_{2834} = (1, 0, 10, 1)$
$192: P_{2851} = (2, 1, 10, 1)$
$193: P_{2858} = (9, 1, 10, 1)$
$194: P_{2899} = (2, 4, 10, 1)$
$195: P_{2911} = (14, 4, 10, 1)$
$196: P_{2924} = (11, 5, 10, 1)$
$197: P_{2927} = (14, 5, 10, 1)$
$198: P_{2993} = (0, 10, 10, 1)$
$199: P_{2994} = (1, 10, 10, 1)$
$200: P_{3019} = (10, 11, 10, 1)$
$201: P_{3061} = (4, 14, 10, 1)$
$202: P_{3066} = (9, 14, 10, 1)$
$203: P_{3077} = (4, 15, 10, 1)$
$204: P_{3084} = (11, 15, 10, 1)$
$205: P_{3090} = (1, 0, 11, 1)$
$206: P_{3109} = (4, 1, 11, 1)$
$207: P_{3119} = (14, 1, 11, 1)$
$208: P_{3130} = (9, 2, 11, 1)$
$209: P_{3135} = (14, 2, 11, 1)$
$210: P_{3146} = (9, 3, 11, 1)$
$211: P_{3147} = (10, 3, 11, 1)$
$212: P_{3219} = (2, 8, 11, 1)$
$213: P_{3227} = (10, 8, 11, 1)$
$214: P_{3235} = (2, 9, 11, 1)$
$215: P_{3237} = (4, 9, 11, 1)$
$216: P_{3260} = (11, 10, 11, 1)$
$217: P_{3265} = (0, 11, 11, 1)$
$218: P_{3266} = (1, 11, 11, 1)$
$219: P_{3348} = (3, 0, 12, 1)$
$220: P_{3365} = (4, 1, 12, 1)$
$221: P_{3370} = (9, 1, 12, 1)$
$222: P_{3384} = (7, 2, 12, 1)$
$223: P_{3395} = (2, 3, 12, 1)$
$224: P_{3399} = (6, 3, 12, 1)$
$225: P_{3416} = (7, 4, 12, 1)$
$226: P_{3422} = (13, 4, 12, 1)$
$227: P_{3447} = (6, 6, 12, 1)$
$228: P_{3455} = (14, 6, 12, 1)$
$229: P_{3470} = (13, 7, 12, 1)$
$230: P_{3472} = (15, 7, 12, 1)$
$231: P_{3491} = (2, 9, 12, 1)$
$232: P_{3492} = (3, 9, 12, 1)$
$233: P_{3537} = (0, 12, 12, 1)$
$234: P_{3551} = (14, 12, 12, 1)$
$235: P_{3562} = (9, 13, 12, 1)$
$236: P_{3565} = (12, 13, 12, 1)$
$237: P_{3573} = (4, 14, 12, 1)$
$238: P_{3584} = (15, 14, 12, 1)$
$239: P_{3609} = (8, 0, 13, 1)$
$240: P_{3619} = (2, 1, 13, 1)$
$241: P_{3631} = (14, 1, 13, 1)$
$242: P_{3641} = (8, 2, 13, 1)$
$243: P_{3642} = (9, 2, 13, 1)$
$244: P_{3670} = (5, 4, 13, 1)$
$245: P_{3679} = (14, 4, 13, 1)$
$246: P_{3702} = (5, 6, 13, 1)$
$247: P_{3709} = (12, 6, 13, 1)$
$248: P_{3717} = (4, 7, 13, 1)$
$249: P_{3720} = (7, 7, 13, 1)$
$250: P_{3736} = (7, 8, 13, 1)$
$251: P_{3738} = (9, 8, 13, 1)$
$252: P_{3751} = (6, 9, 13, 1)$
$253: P_{3795} = (2, 12, 13, 1)$
$254: P_{3806} = (13, 12, 13, 1)$
$255: P_{3809} = (0, 13, 13, 1)$
$256: P_{3813} = (4, 13, 13, 1)$
$257: P_{3831} = (6, 14, 13, 1)$
$258: P_{3837} = (12, 14, 13, 1)$
$259: P_{3862} = (5, 0, 14, 1)$
$260: P_{3907} = (2, 3, 14, 1)$
$261: P_{3909} = (4, 3, 14, 1)$
$262: P_{3923} = (2, 4, 14, 1)$
$263: P_{3931} = (10, 4, 14, 1)$
$264: P_{3956} = (3, 6, 14, 1)$
$265: P_{3962} = (9, 6, 14, 1)$
$266: P_{3972} = (3, 7, 14, 1)$
$267: P_{4018} = (1, 10, 14, 1)$
$268: P_{4021} = (4, 10, 14, 1)$
$269: P_{4050} = (1, 12, 14, 1)$
$270: P_{4062} = (13, 12, 14, 1)$
$271: P_{4075} = (10, 13, 14, 1)$
$272: P_{4078} = (13, 13, 14, 1)$
$273: P_{4081} = (0, 14, 14, 1)$
$274: P_{4090} = (9, 14, 14, 1)$
$275: P_{4102} = (5, 15, 14, 1)$
$276: P_{4111} = (14, 15, 14, 1)$
$277: P_{4121} = (8, 0, 15, 1)$
$278: P_{4195} = (2, 5, 15, 1)$
$279: P_{4231} = (6, 7, 15, 1)$
$280: P_{4232} = (7, 7, 15, 1)$
$281: P_{4265} = (8, 9, 15, 1)$
$282: P_{4267} = (10, 9, 15, 1)$
$283: P_{4275} = (2, 10, 15, 1)$
$284: P_{4279} = (6, 10, 15, 1)$
$285: P_{4344} = (7, 14, 15, 1)$
$286: P_{4352} = (15, 14, 15, 1)$
$287: P_{4353} = (0, 15, 15, 1)$
$288: P_{4363} = (10, 15, 15, 1)$