Rank-24 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 = 0$$

(1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 28

General information

Number of lines	9
Number of points	37
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	36
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	9, 1^{36}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_1 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{83} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{83} = \mathbf{Pl}(0,0,0,3,1,0)_{67}$$

$$\ell_2 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{62} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{62} = \mathbf{Pl}(0,0,0,2,1,0)_{60}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \mathbf{Pl}(0,3,0,0,1,0)_{31}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \mathbf{Pl}(0,2,0,0,1,0)_{30}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_8 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

Rank of lines: (41, 83, 62, 104, 272, 188, 345, 355, 350)
Rank of points on Klein quadric: (53, 67, 60, 29, 31, 30, 13, 15, 14)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 36 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_6 = (2,1,0,0)$ lies on line ℓ_1
2 : $P_7 = (3,1,0,0)$ lies on line ℓ_2
3 : $P_8 = (1,0,1,0)$ lies on line ℓ_3
4 : $P_9 = (2,0,1,0)$ lies on line ℓ_4
5 : $P_{10} = (3,0,1,0)$ lies on line ℓ_5
6 : $P_{11} = (0,1,1,0)$ lies on line ℓ_6
7 : $P_{15} = (0,2,1,0)$ lies on line ℓ_7
8 : $P_{19} = (0,3,1,0)$ lies on line ℓ_8
9 : $P_{27} = (1,1,0,1)$ lies on line ℓ_0
10 : $P_{28} = (2,1,0,1)$ lies on line ℓ_1
11 : $P_{29} = (3,1,0,1)$ lies on line ℓ_2
12 : $P_{31} = (1,2,0,1)$ lies on line ℓ_2
13 : $P_{32} = (2,2,0,1)$ lies on line ℓ_0
14 : $P_{33} = (3,2,0,1)$ lies on line ℓ_1
15 : $P_{35} = (1,3,0,1)$ lies on line ℓ_1
16 : $P_{36} = (2,3,0,1)$ lies on line ℓ_2
17 : $P_{37} = (3,3,0,1)$ lies on line ℓ_0
18 : $P_{39} = (1,0,1,1)$ lies on line ℓ_3
19 : $P_{40} = (2,0,1,1)$ lies on line ℓ_4
20 : $P_{41} = (3,0,1,1)$ lies on line ℓ_5
21 : $P_{42} = (0,1,1,1)$ lies on line ℓ_6
22 : $P_{45} = (0,2,1,1)$ lies on line ℓ_7
23 : $P_{49} = (0,3,1,1)$ lies on line ℓ_8
24 : $P_{54} = (1,0,2,1)$ lies on line ℓ_5
25 : $P_{55} = (2,0,2,1)$ lies on line ℓ_3
26 : $P_{56} = (3,0,2,1)$ lies on line ℓ_4
27 : $P_{57} = (0,1,2,1)$ lies on line ℓ_8

28 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_6
29 : $P_{65} = (0, 3, 2, 1)$ lies on line ℓ_7
30 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_4
31 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_5
32 : $P_{72} = (3, 0, 3, 1)$ lies on line ℓ_3
33 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_7
34 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_8
35 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 1 1 1 1 1
1	1 0 1 1 1 1 1 1 1
2	1 1 0 1 1 1 1 1 1
3	1 1 1 0 1 1 1 1 1
4	1 1 1 1 0 1 1 1 1
5	1 1 1 1 1 0 1 1 1
6	1 1 1 1 1 1 0 1 1
7	1 1 1 1 1 1 1 0 1
8	1 1 1 1 1 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 37 points:
The points on the surface are:

0 : $P_3 = (0,0,0,1)$
1 : $P_5 = (1,1,0,0)$
2 : $P_6 = (2,1,0,0)$
3 : $P_7 = (3,1,0,0)$
4 : $P_8 = (1,0,1,0)$
5 : $P_9 = (2,0,1,0)$
6 : $P_{10} = (3,0,1,0)$
7 : $P_{11} = (0,1,1,0)$
8 : $P_{15} = (0,2,1,0)$
9 : $P_{19} = (0,3,1,0)$
10 : $P_{27} = (1,1,0,1)$
11 : $P_{28} = (2,1,0,1)$
12 : $P_{29} = (3,1,0,1)$
13 : $P_{31} = (1,2,0,1)$
14 : $P_{32} = (2,2,0,1)$
15 : $P_{33} = (3,2,0,1)$
16 : $P_{35} = (1,3,0,1)$
17 : $P_{36} = (2,3,0,1)$
18 : $P_{37} = (3,3,0,1)$
19 : $P_{39} = (1,0,1,1)$
20 : $P_{40} = (2,0,1,1)$
21 : $P_{41} = (3,0,1,1)$
22 : $P_{42} = (0,1,1,1)$
23 : $P_{45} = (0,2,1,1)$
24 : $P_{49} = (0,3,1,1)$
25 : $P_{54} = (1,0,2,1)$
26 : $P_{55} = (2,0,2,1)$
27 : $P_{56} = (3,0,2,1)$
28 : $P_{57} = (0,1,2,1)$
29 : $P_{61} = (0,2,2,1)$
30 : $P_{65} = (0,3,2,1)$
31 : $P_{70} = (1,0,3,1)$
32 : $P_{71} = (2,0,3,1)$
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34 : $P_{73} = (0,1,3,1)$
35 : $P_{77} = (0,2,3,1)$
36 : $P_{81} = (0,3,3,1)$