

Rank-65867 over GF(2)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65867

General information

Number of lines	8
Number of points	11
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	2
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^8
Type of lines on points	$4, 3^2, 2^6, 1^2$

Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0) \\ 1 : P_9 &= \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1) \end{aligned}$$

$$2 : P_{11} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{22} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_6 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{24} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4 \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{30} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{18}
\end{aligned}$$

Rank of lines: (0, 4, 11, 14, 25, 18, 10, 17)

Rank of points on Klein quadric: (0, 2, 22, 6, 24, 4, 30, 18)

Eckardt Points

The surface has 2 Eckardt points:

0 : $P_9 = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$, $T = -1$

1 : $P_{11} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$. $T = -1$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_6 = (1, 0, 1, 0) = \ell_1 \cap \ell_6$$

$$P_8 = (1, 1, 1, 0) = \ell_2 \cap \ell_7$$

$$P_{14} = (0, 1, 1, 1) = \ell_6 \cap \ell_7$$

Single Points

The surface has 2 single points:

The single points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_4

1 : $P_{13} = (1, 0, 1, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	0	0	0	0
1	1	0	1	0	1	1	1	0
2	1	1	0	0	1	1	0	1
3	1	0	0	0	1	1	1	1
4	0	1	1	1	0	1	1	0
5	0	1	1	1	1	0	0	1
6	0	1	0	1	1	0	0	1
7	0	0	1	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_2	P_2	P_2	P_6

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_5	P_2	P_2	P_2	P_8

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{11}	P_9	P_{11}	P_9

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{11}	P_2	P_{11}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_9	P_2	P_9

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_6	P_{11}	P_{11}	P_{14}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_8	P_9	P_9	P_{14}

The surface has 11 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_2 = (0, 0, 1, 0)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_5 = (1, 1, 0, 0)$$

$$5 : P_6 = (1, 0, 1, 0)$$

$$6 : P_8 = (1, 1, 1, 0)$$

$$7 : P_9 = (1, 0, 0, 1)$$

$$8 : P_{11} = (1, 1, 0, 1)$$

$$9 : P_{13} = (1, 0, 1, 1)$$

$$10 : P_{14} = (0, 1, 1, 1)$$