## Cheat Sheet GF(64)

January 16, 2021

polynomial:  $X^6 + X^5 + 1 = 97$   $Z_i = \log_{\alpha}(1 + \alpha^i)$  Subfields:

Subfield	Polynomial	Numerical Rank
$\mathbb{F}_4$	$X_2 + X_1 + 1$	7
$\mathbb{F}_8$	$X_3 + X_1 + 1$	11

i	$\gamma_i$	$-\gamma_i$	$\gamma_i^{-1}$	$\log_{\alpha}(\gamma_i)$	$\alpha^i$	$Z_i$	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$	1
0	0 = 0	0	DNE	DNE	1	DNE	0	0	0	0	0	0	
1	1 = 1	1	1	63	2	58	1	0	1	0	1	1	
2	$\alpha = \epsilon$	2	48	1	4	53	4	1	1	37	47	56	
3	$\alpha + 1 = \epsilon^{58}$	3	32	58	8	34	5	1	1	37	11	57	
4	$\alpha^2 = \epsilon^2$	4	24	2	16	43	16	1	1	46	11	57	
5	$\alpha^2 + 1 = \epsilon^{53}$	5	63	53	32	6	17	1	1	46	36	56	
6	$\alpha^2 + \alpha = \epsilon^{59}$	6	16	59	33	5	20	0	1	11	46	1	
7	$\alpha^2 + \alpha + 1 = \epsilon^{39}$	7	45	39	35	44	21	0	1	11	36	0	
8	$\alpha^3 = \epsilon^3$	8	12	3	39	23	33	1	1	37	46	57	
9	$\alpha^3 + 1 = \epsilon^{34}$	9	27	34	47	27	32	1	1	37	10	56	
10	$\alpha^3 + \alpha = \epsilon^{54}$	10	47	54	63	12	37	0	1	0	37	1	
11	$\alpha^3 + \alpha + 1 = \epsilon^{18}$	11	37	18	31	49	36	0	1	0	36	0	
12	$\alpha^3 + \alpha^2 = \epsilon^{60}$	12	8	60	62	10	49	0	1	11	36	0	
13	$\alpha^3 + \alpha^2 + 1 = \epsilon^{31}$	13	26	31	29	41	48	0	1	11	46	1	
14	$\alpha^3 + \alpha^2 + \alpha = \epsilon^{40}$	14	38	40	58	25	53	1	1	46	37	56	
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^{48}$	15	21	48	21	55	52	1	1	46	10	57	
16	$\alpha^4 = \epsilon^4$	16	6	4	42	46	39	1	1	10	36	56	
17	$\alpha^4 + 1 = \epsilon^{43}$	17	44	43	53	33	38	1	1	10	47	57	
18	$\alpha^4 + \alpha = \epsilon^{35}$	18	61	35	11	54	35	0	1	47	1	0	
19	$\alpha^4 + \alpha + 1 = \epsilon^{22}$	19	28	22	22	26	34	0	1	47	47	1	
20	$\alpha^4 + \alpha^2 = \epsilon^{55}$	20	39	55	44	24	55	0	1	36	10	1	
21	$\alpha^4 + \alpha^2 + 1 = \epsilon^{15}$	21	15	15	57	42	54	0	1	36	47	0	
22	$\alpha^4 + \alpha^2 + \alpha = \epsilon^{19}$	22	34	19	19	35	51	1	1	1	37	57	
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \epsilon^{26}$	23	41	26	38	8	50	1	1	1	37	56	
24	$\alpha^4 + \alpha^3 = \epsilon^{61}$	24	4	61	45	20	6	0	1	47	37	1	
25	$\alpha^4 + \alpha^3 + 1 = \epsilon^{51}$	25	62	51	59	14	7	0	1	47	11	0	

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c c} T_3(\gamma_i) & \\ \hline 57 & 56 \\ 56 & 56 \\ 57 & 0 \\ 1 & 57 \\ 56 & 1 \\ 0 & 0 \\ 1 & 56 \\ \end{array}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	56 56 57 0 1 57 56 1 0
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$ \begin{vmatrix} 36 \\ 37 \\ 38 \end{vmatrix} \qquad \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$ \begin{vmatrix} 37 \\ 38 \end{vmatrix} \qquad \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1
$\begin{vmatrix} 38 \end{vmatrix}$ $\alpha^5 + \alpha^2 + \alpha = \epsilon^{23} \begin{vmatrix} 38 \end{vmatrix} 14 \begin{vmatrix} 23   51   52   43   1   1   37   11  $	
	56
	30
	57
$ \begin{vmatrix} 40 & \alpha^5 + \alpha^3 = \epsilon^{56} & 40 & 35 \end{vmatrix}                                 $	0
$\begin{vmatrix} 41 \end{vmatrix}$ $\alpha^5 + \alpha^3 + 1 = \epsilon^{37} \begin{vmatrix} 41 \end{vmatrix} 23 \begin{vmatrix} 37 \end{vmatrix} 28 \begin{vmatrix} 13 \end{vmatrix} 31 \begin{vmatrix} 0 \end{vmatrix} 1 \begin{vmatrix} 11 \end{vmatrix} 11 \begin{vmatrix} 11 \end{vmatrix}$	1
$\begin{vmatrix} 42 \end{vmatrix}$ $\alpha^5 + \alpha^3 + \alpha = \epsilon^{16} \begin{vmatrix} 42 \end{vmatrix}   55 \end{vmatrix}$ $\begin{vmatrix} 16 \end{vmatrix}   56 \end{vmatrix}   21 \end{vmatrix}   26 \end{vmatrix}$ $\begin{vmatrix} 1 \end{vmatrix}$ $\begin{vmatrix} 1 \end{vmatrix}$ $\begin{vmatrix} 46 \end{vmatrix}$ $\begin{vmatrix} 11 \end{vmatrix}$	56
$\begin{vmatrix} 43 \end{vmatrix}$ $\alpha^5 + \alpha^3 + \alpha + 1 = \epsilon^{46} \begin{vmatrix} 43 \end{vmatrix}$ $\begin{vmatrix} 53 \end{vmatrix}$ $\begin{vmatrix} 46 \end{vmatrix}$ $\begin{vmatrix} 17 \end{vmatrix}$ $\begin{vmatrix} 4 \end{vmatrix}$ $\begin{vmatrix} 27 \end{vmatrix}$ $\begin{vmatrix} 1 \end{vmatrix}$ $\begin{vmatrix} 46 \end{vmatrix}$ $\begin{vmatrix} 36 \end{vmatrix}$	57
$\begin{vmatrix} 44 \end{vmatrix}$ $\alpha^5 + \alpha^3 + \alpha^2 = \epsilon^{20} \begin{vmatrix} 44 \end{vmatrix} 17 \begin{vmatrix} 20   34   7   14   1   1   37   10  $	57
$\begin{vmatrix} 45 \end{vmatrix}$ $\alpha^5 + \alpha^3 + \alpha^2 + 1 = \epsilon^{24} \begin{vmatrix} 45 \end{vmatrix}$ $\begin{vmatrix} 7 \end{vmatrix}$ $\begin{vmatrix} 24 \begin{vmatrix} 37 \end{vmatrix} \begin{vmatrix} 36 \end{vmatrix}$ $\begin{vmatrix} 15 \end{vmatrix}$ $\begin{vmatrix} 1 \end{vmatrix}$ $\begin{vmatrix} 1 \end{vmatrix}$ $\begin{vmatrix} 37 \end{vmatrix}$ $\begin{vmatrix} 46 \end{vmatrix}$	56
$\begin{vmatrix} 46 \end{vmatrix}$ $\alpha^5 + \alpha^3 + \alpha^2 + \alpha = \epsilon^{27} \begin{vmatrix} 46 \end{vmatrix} 36 \begin{vmatrix} 27   43   16 \end{vmatrix} 10 \begin{vmatrix} 0 \end{vmatrix} 1 \begin{vmatrix} 0 \end{vmatrix}$	1
$\begin{vmatrix} 47 & \alpha^5 + \alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^9 & 47 & 10 \end{vmatrix}$ 9 55 30 11 0 1 0 11	0
$\begin{vmatrix} 48 \end{vmatrix}$ $\alpha^5 + \alpha^4 = \epsilon^{62} \begin{vmatrix} 48 \end{vmatrix} 2 \begin{vmatrix} 62   15   40   24   0   1   36   10  $	1
$\begin{vmatrix} 49 \end{vmatrix}$ $\alpha^5 + \alpha^4 + 1 = \epsilon^{57} \begin{vmatrix} 49 \end{vmatrix} 33 \begin{vmatrix} 57 \end{vmatrix} 30 \begin{vmatrix} 11 \end{vmatrix} 25 \begin{vmatrix} 0 \end{vmatrix} 1 \begin{vmatrix} 36 \end{vmatrix} 47 \begin{vmatrix} 47 \end{vmatrix}$	0
$  50  $ $\alpha^5 + \alpha^4 + \alpha = \epsilon^{52}   50   31  $ $  52   60   28  $ $  28  $ $  1  $ $  1  $ $  46  $	57
	<b>N</b> T ( )
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$N_2(\gamma_i)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	46
	37
$\begin{vmatrix} 53 \\ 64 \end{vmatrix} = \begin{vmatrix} \alpha^5 + \alpha^4 + \alpha^2 + 1 = \epsilon^{17} \\ 54 \end{vmatrix} = \begin{vmatrix} 53 \\ 64 \end{vmatrix} = \begin{vmatrix} 43 \\ 64 \end{vmatrix} = \begin{vmatrix} 17 \\ 54 \end{vmatrix} = \begin{vmatrix} 53 \\ 20 \end{vmatrix} = \begin{vmatrix} 16 \\ 12 \end{vmatrix} = \begin{vmatrix} 16 \\ 12 \end{vmatrix} = \begin{vmatrix} 16 \\ 14 \end{vmatrix} = \begin{vmatrix} $	46
$\begin{bmatrix} 54 \\ & \alpha^5 + \alpha^4 + \alpha^2 + \alpha = \epsilon^{30} \\ 5 + 4 + 2 + 1 + 47 \end{bmatrix} \begin{bmatrix} 54 \\ 57 \end{bmatrix} \begin{bmatrix} 54 \\ 47 \end{bmatrix} \begin{bmatrix} 30 \\ 47 \end{bmatrix} \begin{bmatrix} 10 \\ 47 \end{bmatrix} \begin{bmatrix} 18 \\ 47 \end{bmatrix} \begin{bmatrix} 12 \\ 47 \end{bmatrix}$	11
$\begin{vmatrix} 55 & \alpha^5 + \alpha^4 + \alpha^2 + \alpha + 1 = \epsilon^{47} & 55 & 42 & 47 & 20 & 15 & 13 & 0 & 1 & 47 \\ 55 & 5 & 4 & 3 & 42 & 56 & 57 & 49 & 49 & 49 & 49 & 49 & 49 & 49 & 4$	37
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1
$ \begin{vmatrix} 57 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	36
$ \begin{vmatrix} 60 \\ 0 \end{vmatrix} \qquad \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 = \epsilon^{50} \begin{vmatrix} 60 \\ 29 \end{vmatrix} \qquad 50 \begin{vmatrix} 12 \\ 31 \end{vmatrix} \qquad 41 \begin{vmatrix} 0 \\ 1 \end{vmatrix} \qquad 47 \begin{vmatrix} 47 \\ 47 \end{vmatrix} $	47
$ \begin{vmatrix} 61 \\ \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + 1 = \epsilon^{28} \\ 5 + 4 + 3 + 2 + 12 \\ 62 \end{vmatrix} \begin{vmatrix} 61 \\ 18 \\ 28 \end{vmatrix} \begin{vmatrix} 24 \\ 51 \\ 40 \end{vmatrix} \begin{vmatrix} 61 \\ 0 \\ 1 \end{vmatrix} \begin{vmatrix} 47 \\ 47 \\ 49 \end{vmatrix} $	1
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	37
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	46

	+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	$\frac{1}{2}$	1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22	25
	$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	$\frac{2}{3}$	$\frac{3}{2}$	0 $1$	$\frac{1}{0}$	6 7	7 6	4	5 4	10 11	11 10	8	9	14 15	15 14	12 13	13 12	18 19	19 18	16 17	17 16	22 23	23 22	20 21	21 20	26 27
	$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$	3 4	5	6	7	0	1	$\frac{5}{2}$	$\frac{4}{3}$	12	13	9 14	15	8	9	10	11	20	21	22	23	23 16	17	18	19	28
	5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10	21	20	23	22	17	16	19	18	29
	6	6	7	4	5	$\overline{2}$	3	0	1	14	15	12	13	10	11	8	9	$\frac{1}{22}$	23	20	21	18	19	16	17	30
	7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8	23	22	21	20	19	18	17	16	31
	8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16
	9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6	25	24	27	26	29	28	31	30	17
	10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5	26	27	24	25	30	31	28	29	18
	$\begin{vmatrix} 11 \\ 12 \end{vmatrix}$	11	10 13	9	8	15	14 9	13	12	3	2	1 6	0 7	7	6	$\frac{5}{2}$	$\frac{4}{3}$	27 28	26 29	25 30	24 31	31	30	29	28	19
	$\begin{vmatrix} 12 \\ 13 \end{vmatrix}$	12 13	13 12	14 15	15 14	8 9	8	10 11	11 10	$\frac{4}{5}$	$\frac{5}{4}$	7	6	0 1	$\frac{1}{0}$	3	2	29	28	31	30	$\frac{24}{25}$	$\frac{25}{24}$	$\frac{26}{27}$	27 26	$\frac{20}{21}$
	$\frac{13}{14}$	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1	30	31	28	29	26	27	24	25	22
	15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	31	30	29	28	27	26	25	24	23
	16	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8
	17	17	16	19	18	21	20	23	22	25	24	27	26	29	28	31	30	1	0	3	2	5	4	7	6	9
	18	18	19	16	17	22	23	20	21	26	27	24	25	30	31	28	29	2	3	0	1	6	7	4	5	10
	19	19	18	17	16	23	22	21	20	27	26	25	24	31	30	29	28	3	2	1	0	7	6	5	4	11
	$\frac{20}{21}$	20	21	22	23	16	17	18	19	28	29	30	31	24	25	26	27	4	5	6	7	0	1	2	3	12
	$\begin{vmatrix} 21 \\ 22 \end{vmatrix}$	21 22	20 23	23 20	22 21	17 18	16 19	19 16	18 17	29 30	28 31	31 28	$\frac{30}{29}$	25 26	$\frac{24}{27}$	27 24	$\frac{26}{25}$	5 6	$\frac{4}{7}$	7 $4$	6 5	$\frac{1}{2}$	$0 \\ 3$	$\frac{3}{0}$	2	13 $14$
	$\begin{vmatrix} 22 \\ 23 \end{vmatrix}$	23	$\frac{23}{22}$	21	20	19	18	17	16	31	30	29	28	$\frac{20}{27}$	26	$\frac{24}{25}$	$\frac{25}{24}$	7	6	5	4	3	2	1	0	15
	$\frac{23}{24}$	$\frac{23}{24}$	$\frac{22}{25}$	26	27	28	29	30	31	16	17	18	19	20	21	$\frac{25}{22}$	23	8	9	10	11	12	13	14	15	0
	25	25	24	$\frac{1}{27}$	26	29	28	31	30	17	16	19	18	21	20	23	$\frac{1}{2}$	9	8	11	10	13	12	15	14	1
	26	26	27	24	25	30	31	28	29	18	19	16	17	22	23	20	21	10	11	8	9	14	15	12	13	2
	27	27	26	25	24	31	30	29	28	19	18	17	16	23	22	21	20	11	10	9	8	15	14	13	12	3
	28	28	29	30	31	24	25	26	27	20	21	22	23	16	17	18	19	12	13	14	15	8	9	10	11	4
	29	29	28	31	30	25	24	27	26	21	20	23	22	17	16	19	18	13	12	15	14	9	8	11	10	5
	30											20										10		8		6
	$\frac{31}{32}$	31 32	30 33	29 34	28 35	27 36	26 37	$\frac{25}{38}$	24 39	23 40	22 41	21 42	20 43	19 44	18 45	17 46	16 47	15 48	14 49	13 50	12 51	11 52	10 53	9 54	8 55	7 56
	$\begin{vmatrix} 32 \\ 33 \end{vmatrix}$	$\frac{32}{33}$	32	35	34	37	36	39	38	41	40	43	42	45	44	47	46	49	48	51	50	52	52	55	54	57
	$\frac{34}{34}$	34	35	32	33	38	39	36	37	42	43	40	41	46	47	44	45	50	51	48	49	54	55	52	53	58
	35	35	34	33	32	39	38	37	36	43	42	41	40	47	46	45	44	51	50	49	48	55	54	53	52	59
	36	36	37	38	39	32	33	34	35	44	45	46	47	40	41	42	43	52	53	54	55	48	49	50	51	60
	37	37	36	39	38	33	32	35	34	45	44	47	46	41	40	43	42	53	52	55	54	49	48	51	50	61
	38	38	39	36	37	34	35	32	33	46	47	44	45	42	43	40	41	54	55	52	53	50	51	48	49	62
	39	39	38	37	36	35	34	33	32	47	46	45	44	43	42	41	40	55	54	53	52	51	50	49	48	63
	40	40	41	42	43	44	45	46	47	32	33	34	35	36	37	38	39	56	57 56	58	59 E0	60	61	62	63	48
	$\begin{vmatrix} 41 \\ 42 \end{vmatrix}$	41 42	40 43	43 40	42 41	45 46	$\frac{44}{47}$	47 $44$	46 45	33 34	$\frac{32}{35}$	35	34 33	$\frac{37}{38}$	36 39	39 36	$\frac{38}{37}$	57 58	56 59	59 56	58 57	61 62	60 63	63 60	62 61	49 50
	$\frac{42}{43}$	43	43	40	41	$\frac{40}{47}$	46	$\frac{44}{45}$	43	$\frac{34}{35}$	34	32 33	32	39	38	37	37 36	59	58	57	56	63	62	61	60	50 51
	$\begin{bmatrix} 45 \\ 44 \end{bmatrix}$	44	45	46	47	40	41	42	43	36	37	38	39	32	33	34	35	60	61	62	63	56	57	58	59	52
	45	45	44	47	46	41	40	43	42	37	36	39	38	33	32	35	34	61	60	63	62	57	56	59	58	53
	46	46	47	44	45	42	43	40	41	38	39	36	37	34	35	32	33	62	63	60	61	58	59	56	57	54
	47	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	63	62	61	60	59	58	57	56	55
	48	48	49	50	51	52	53	54	55	56		358	59	60	61	62	63	32	33	34	35	36	37	38	39	40
	49	49	48	51	50	53	52	55	54	57	56	59	58	61	60	63	62	33	32	35	34	37	36	39	38	41
	50	50	51	48	49	54 55	55	52 52	53	58	59	56 57	57 50	62	63	60	61	34	35	32	33	38	39	36 27	37	42
- 1	h !	h 1	h 1 1	/111	/I 💭	ما ما	h /1	h.,	h. • )	h ( )	L U	L 7	h 6.	6.,	6.1	61	611	・ノに	- <i>)</i> /l	٠,٠,	-9.1	·J11	· , ()	- <b>)</b> '7	16	71.7

51 51 50 49 48 55 54 53 52 59 58 57 56 63 62 61 60 35 34 33 32 39 38 37 36 43

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$\frac{1}{2}$	$\begin{vmatrix} 1 \\ 2 \end{vmatrix}$	2	3	4	5	6	7	8	9	10	11 22	12	13	14	15	16 32	17 24	18	19	20	21	22	23	24	25
$\begin{vmatrix} 2\\3 \end{vmatrix}$	$\begin{vmatrix} 2\\3 \end{vmatrix}$	$\frac{4}{6}$	6 5	8 12	10 15	12 10	14 9	16 24	18 27	20 30	29	24 20	26 23	28 18	30 17	32 48	34 51	36 54	38 53	40 60	42 63	44 58	46 57	48 40	50 43
$\frac{3}{4}$	$\begin{vmatrix} 3 \\ 4 \end{vmatrix}$	8	12	16	20	24	28	32	36	40	$\frac{23}{44}$	48	52	56	60	33	37	41	45	49	53	57	61	1	5
5	5	10	15	20	$\frac{17}{17}$	30	27	40	45	34	39	60	57	54	51	49	52	59	62	37	32	47	42	25	28
6	6	12	10	24	30	20	18	48	54	60	58	40	46	36	34	1	7	13	11	25	31	21	19	49	55
7	7	14	9	28	27	18	21	56	63	54	49	36	35	42	45	17	22	31	24	13	10	3	4	41	46
8	8	16	24	32	40	48	56	33	41	49	57	1	9	17	25	35	43	51	59	3	11	19	27	2	10
9	9	18	27	36	45	54	63	41	32	59	50	13	4	31	22	51	58	33	40	23	30	5	12	26	19
10	10	20 22	30 29	40	34	60	54	49 57	59 50	$\frac{37}{47}$	47	25 21	19 30	13	7	3	9 24	23	29	43 63	33 52	63	53 34	50 42	56 33
$\begin{vmatrix} 11 \\ 12 \end{vmatrix}$	$\begin{vmatrix} 11 \\ 12 \end{vmatrix}$	$\frac{22}{24}$	29	44 48	39 60	58 40	49 36	1	50 13	25	36 21	49	61	$\frac{3}{41}$	8 37	19 2	$\frac{24}{14}$	5 26	14 22	50	62	41 42	38	3	აა 15
13	$\begin{vmatrix} 12 \\ 13 \end{vmatrix}$	26	23	52	57	46	35	9	4	19	30	61	48	39	42	18	31	8	5	38	43	60	49	27	22
14	14	28	18	56	54	36	42	17	31	13	3	41	39	53	59	34	44	62	48	26	20	6	8	51	61
15	15	30	17	60	51	34	45	25	22	7	8	37	42	59	52	50	61	44	35	14	1	16	31	43	36
16	16	32	48	33	49	1	17	35	51	3	19	2	18	34	50	39	55	7	23	6	22	38	54	4	20
17	17	34	51	37	52	7	22	43	58	9	24	14	31	44	61	55	38	21	4	18	3	48	33	28	13
18	18	36	54	41	59	13	31	51	33	23	5	26	8	62	44	7	21	35	49	46	60	10	24	52	38
19	19	38	53	45	62	11	24	59	40	29	14	22	5	48	35	23	4	49	34	58	41	28 31	15	44	63
$\begin{array}{c c} 20 \\ 21 \end{array}$	$\begin{vmatrix} 20\\21 \end{vmatrix}$	40 42	60 63	49 53	$\frac{37}{32}$	25 31	13 10	3 11	23 30	43 33	63 52	50 62	38 43	26 20	14 1	$\frac{6}{22}$	18 3	46 60	58 41	55 35	$\frac{35}{54}$	9	11 28	5 29	17 8
$\frac{21}{22}$		44	58	57	47	21	3	19	5	63	41	42	60	6	16	38	48	10	28	31	9	51	$\frac{20}{37}$	53	35
23		46	57	61	42	19	4	27	12	53	34	38	49	8	31	54	33	24	15	11	28	37	50	45	58
24	24	48	40	1	25	49	41	2	26	50	42	3	27	51	43	4	28	52	44	5	29	53	45	6	30
25	25	50	43	5	28	55	46	10	19	56	33	15	22	61	36	20	13	38	63	17	8	35	58	30	7
26		52	46	9	19	61	39	18	8	38	60	27	1	47	53	36	62	16	10	45	55	25	3	54	44
27	27	54	45	13	22	59	32	26	1	44	55	23	12	33	58	52	47	2	25	57	34	15	20	46	53
28	$\begin{vmatrix} 28 \\ 20 \end{vmatrix}$	56	36	17	13	41	53	34	62	26	6	51	47	11	23	37	57	29	1	52	40	12	16	7 31	27
$\begin{vmatrix} 29 \\ 30 \end{vmatrix}$	30	58 60	39 34	21 25	8 7	$\frac{47}{37}$	50 59	42 50	55 44	16 14	13 16	63 43	34 53	5 23	24 9	53 5	$\frac{40}{27}$	15 57	18 39	32 28	61	26 32	7 62	55	$\frac{2}{41}$
31	1	62	33	29	2	35	60	58	37	4	27	39	56	$\frac{25}{25}$	6	21	10	43	52	8		54		47	48
32	32	33	1	35	3	2	34	39	7	6	38	4	36	37	5	47	15	14	46	12	44	45	13	8	40
33		35	2	39	6	4	37	47	14	12	45	8	41	43	10	63	30	28	61	24	57	59	26	16	49
34		37	7	43	9	14	44	55	21	18	48	28	62	57	27	15	45	42	8	36	6	1	35	56	26
35		39	4	47	12	8	43	63	28	24	59	16	51	55	20	31	60	56	27	48	19	23	52	32	3
36		41	13	51	23	26	62	7	35	46	10	52 50	16	29	57	14	42	39	3	61	25	20	48	9	45
$\begin{array}{c c} 37 \\ 38 \end{array}$		43 45	14 11	55 59	18 29	$\frac{28}{22}$	57 48	15 23	42 49	36 58	1 28	56 44	29 10	19	54 39	30 46	59 8	53	16 37	41 21	12 51	2 56	39 30	17 57	52 31
39		$\frac{45}{47}$	8	63	$\frac{29}{24}$	22 16	48 55	23 31	49 56	58 48	28 23	32	7	1 15	39 40	40 62	$\frac{8}{25}$	3 17	54	21 1	38	50 46	30 9	33	31 6
40		49	$\frac{3}{25}$	3	43	50	26	6	46	55	31	5	45	52	28	$\frac{02}{12}$	$\frac{25}{36}$	61	21	15	39	62	$\frac{9}{22}$	10	34
41	41	51	26	7	46	52	29	14	39	61	20	9	32	58	19	28	53	47	6	27	50	40	1	18	59
42	42	53	31	11	33	62	20	22	60	35	9	29	55	40	2	44	6	25	51	39	13	18	56	58	16
43	43	55	28	15	36	56	19	30	53	41	2	17	58	38	13	60	23	11	32	51	24	4	47	34	9
44		57	21	19	63	42	6	38	10	31	51	53	25	12	32	45	1	20	56	62	18	7	43	11	39
45		59	22	23	58	44	1	46	3	21	56	57	20	2	47	61	16	6	43	42	7	17	60	19	62
46		61 62	19 16	27 21	53	38	8 15	54 62	24 17	11	37 46	45	3	16	62	13	35	48	30	22	56 45	43 61	5 19	59 35	21
47		63 1	16 49	$\begin{array}{c} 31 \\ 2 \end{array}$	48 50	$\frac{32}{3}$	15 51	62 4	17 52	1 5	46 53	33 6	14 54	30 7	49 55	29 8	50 56	34 9	13 57	2 10	45 58	61 11	18 59	$\frac{35}{12}$	12 60
49		3	50	6	55	5 5	51	12	61		462	10	5 <del>4</del>	9	56	$\frac{3}{24}$	41	27	42	30	47	29	44	20	37
50		5	55	10	56	15	61	20	38	17	$\frac{402}{35}$	30	44	27	41	40	26	45	31	34	16	39	21	60	14
51	51	7	52	14	61	9	58	28	47	27	40	18	33	21	38	56	11	63	12	54	5	49	2	36	23
52	52	9	61	18	38	27	47	36	16	45	25	54	2	63	11	41	29	32	20	59	15	50	6	13	57

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^{2} = 4$ $2^{3} = 8$ $2^{4} = 16$ $2^{5} = 32$
$2^6 = 33$
$2^7 = 35$
$2^8 = 39$
$2^9 = 47$
$2^{10} = 63$
$2^{11} = 31$
$2^{12} = 62$
$2^{13} = 29$
$2^{14} = 58$
$2^{15} = 21$
$2^{16} = 42$
$2^{17} = 53$
$2^{18} = 11$
$2^{19} = 22$
$2^{20} = 44$
$2^{21} = 57$
$2^{22} = 19$
$2^{23} = 38$
$2^{24} = 45$
$2^{25} = 59$
$2^{26} = 23$
$2^{27} = 46$
$2^{28} = 61$
$2^{29} = 27$
$2^{30} = 54$ $2^{31} = 13$
$2^{32} = 26$

$2^{33}$	=	52
$2^{34}$	=	9
$2^{35}$	=	18
$2^{36}$	=	36
$2^{37}$	=	41
$2^{38}$	=	51
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$2^{40}$	=	14
$2^{41}$	=	28
$2^{42}$	=	56
$2^{43}$	=	17
$2^{44}$	=	34
$2^{45}$	=	37
$2^{46}$	=	43
$2^{47}$	=	55
$2^{48}$	=	15
$2^{49}$	=	30
$2^{50}$	=	60
$2^{51}$	=	25
$2^{52}$	=	50
$2^{53}$	=	5
$2^{54}$	=	10
$2^{55}$	=	20
$2^{56}$	=	40
$2^{57}$	=	49
$2^{58}$	=	3
$2^{59}$	=	6
$2^{60}$	=	12
$2^{61}$	=	24
$2^{62}$	=	48
$2^{63}$	=	1

i	$\gamma_i$	$-\gamma_i$	$\gamma_i^{-1}$	$\log_{\alpha}(\gamma_i)$	$\alpha^i$	$Z_i$	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$	1
0	0 = 0	0	DNE	DNE	1	DNE	0	0	0	0	0	0	Γ
1	1 = 1	1	1	63	2	58	1	0	1	0	1	1	
2	$\alpha = \epsilon$	2	48	1	4	53	4	1	1	37	47	56	
3	$\alpha + 1 = \epsilon^{58}$	3	32	58	8	34	5	1	1	37	11	57	
4	$\alpha^2 = \epsilon^2$	4	24	2	16	43	16	1	1	46	11	57	
5	$\alpha^2 + 1 = \epsilon^{53}$	5	63	53	32	6	17	1	1	46	36	56	1
6	$\alpha^2 + \alpha = \epsilon^{59}$	6	16	59	33	5	20	0	1	11	46	1	
7	$\alpha^2 + \alpha + 1 = \epsilon^{39}$	7	45	39	35	44	21	0	1	11	36	0	
8	$\alpha^3 = \epsilon^3$	8	12	3	39	23	33	1	1	37	46	57	
9	$\alpha^3 + 1 = \epsilon^{34}$	9	27	34	47	27	32	1	1	37	10	56	1
10	$\alpha^3 + \alpha = \epsilon^{54}$	10	47	54	63	12	37	0	1	0	37	1	1
11	$\alpha^3 + \alpha + 1 = \epsilon^{18}$	11	37	18	31	49	36	0	1	0	36	0	1
12	$\alpha^3 + \alpha^2 = \epsilon^{60}$	12	8	60	62	10	49	0	1	11	36	0	1
13	$\alpha^3 + \alpha^2 + 1 = \epsilon^{31}$	13	26	31	29	41	48	0	1	11	46	1	1
14	$\alpha^3 + \alpha^2 + \alpha = \epsilon^{40}$	14	38	40	58	25	53	1	1	46	37	56	1
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^{48}$	15	21	48	21	55	52	1	1	46	10	57	1
16	$\alpha^4 = \epsilon^4$	16	6	4	42	46	39	1	1	10	36	56	1
17	$\alpha^4 + 1 = \epsilon^{43}$	17	44	43	53	33	38	1	1	10	47	57	1
18	$\alpha^4 + \alpha = \epsilon^{35}$	18	61	35	11	54	35	0	1	47	1	0	
19	$\alpha^4 + \alpha + 1 = \epsilon^{22}$	19	28	22	22	26	34	0	1	47	47	1	1
20	$\alpha^4 + \alpha^2 = \epsilon^{55}$	20	39	55	44	24	55	0	1	36	10	1	
21	$\alpha^4 + \alpha^2 + 1 = \epsilon^{15}$	21	15	15	57	42	54	0	1	36	47	0	
22	$\alpha^4 + \alpha^2 + \alpha = \epsilon^{19}$	22	34	19	19	35	51	1	1	1	37	57	
23	$\alpha^4 + \alpha^2 + \alpha + 1 = \epsilon^{26}$	23	41	26	38	8	50	1	1	1	37	56	
24	$\alpha^4 + \alpha^3 = \epsilon^{61}$	24	4	61	45	20	6	0	1	47	37	1	
25	$\alpha^4 + \alpha^3 + 1 = \epsilon^{51}$	25	62	51	59	14	7	0	1	47	11	0	

			1					— ( )	( )			
i	$\gamma_i$	$-\gamma_i$	$\gamma_i^{-1}$	$\log_{\alpha}(\gamma_i)$		$Z_i$	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$	$T_3(\gamma_i)$
26	$\alpha^4 + \alpha^3 + \alpha = \epsilon^{32}$	26	13	3		19	2	1	1	10	36	57
27	$\alpha^4 + \alpha^3 + \alpha + 1 = \epsilon^{29}$	27	9	2		9	3	1	1	10	47	56
28	$\alpha^4 + \alpha^3 + \alpha^2 = \epsilon^{41}$	28	19	4		50	22	1	1	1	10	56
29	$\alpha^4 + \alpha^3 + \alpha^2 + 1 = \epsilon^{13}$	29	60	1		32	23	1	1	1	10	57
30	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha = \epsilon^{49}$	30	58	4		47	18	0	1	36	1	0
31	$\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^{11}$	31	50	1		60	19	0	1	36	36	1
32	$\alpha^5 = \epsilon^5$	32	3		$5 \mid 26$	29	63	1	1	46	37	57
33	$\alpha^5 + 1 = \epsilon^6$	33	49		$6 \mid 52$	17	62	1	1	46	10	56
34	$\alpha^5 + \alpha = \epsilon^{44}$	34	22	4		3	59	0	1	11	11	1
35	$\alpha^5 + \alpha + 1 = \epsilon^7$	35	40		7   18	22	58	0	1	11	1	0
36	$\alpha^5 + \alpha^2 = \epsilon^{36}$	36	46	3		45	47	0	1	0	47	0
37	$\alpha^5 + \alpha^2 + 1 = \epsilon^{45}$	37	11	4		56	46	0	1	0	46	1
38	$\alpha^5 + \alpha^2 + \alpha = \epsilon^{23}$	38	14	2		52	43	1	1	37	11	56
39	$\alpha^5 + \alpha^2 + \alpha + 1 = \epsilon^8$	39	20		8 7	59	42	1	1	37	47	57
40	$\alpha^5 + \alpha^3 = \epsilon^{56}$	40	35	5		48	30	0	1	11	1	0
41	$\alpha^5 + \alpha^3 + 1 = \epsilon^{37}$	41	23	3		13	31	0	1	11	11	1
42	$\alpha^5 + \alpha^3 + \alpha = \epsilon^{16}$	42	55	1		21	26	1	1	46	11	56
43	$\alpha^5 + \alpha^3 + \alpha + 1 = \epsilon^{46}$	43	53	4		4	27	1	1	46	36	57
44	$\alpha^5 + \alpha^3 + \alpha^2 = \epsilon^{20}$	44	17	2		7	14	1	1	37	10	57
45	$\alpha^5 + \alpha^3 + \alpha^2 + 1 = \epsilon^{24}$	45	7	2		36	15	1	1	37	46	56
46	$\alpha^5 + \alpha^3 + \alpha^2 + \alpha = \epsilon^{27}$	46	36	2		16	10	0	1	0	10	1
47	$\alpha^5 + \alpha^3 + \alpha^2 + \alpha + 1 = \epsilon^9$	47	10		$9 \mid 55$	30	11	0	1	0	11	0
48	$\alpha^5 + \alpha^4 = \epsilon^{62}$	48	2	6		40	24	0	1	36	10	1
49	$\alpha^5 + \alpha^4 + 1 = \epsilon^{57}$	49	33	5		11	25	0	1	36	47	0
50	$\alpha^5 + \alpha^4 + \alpha = \epsilon^{52}$	50	31	5	$2 \mid 60$	28	28	1	1	1	46	57
i		$\gamma_i$	$-\gamma_i$	$\gamma_i^{-1} \mid lo$	$g_{\alpha}(\gamma_i)$	$\alpha^i$	$Z_i$	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$
51	$\alpha^5 + \alpha^4 + \alpha + 1 =$		51	59	$\frac{38}{38}$		$\frac{-\iota}{61}$	29	1	1	1	46
52	$\alpha^5 + \alpha^4 + \alpha^2 =$		52	54	33		38	8	1	1	10	37
53	$\alpha^5 + \alpha^4 + \alpha^2 + 1 =$	$= \epsilon^{17}$	53	43	17		2	9	1	1	10	46
54	$\alpha^5 + \alpha^4 + \alpha^2 + \alpha =$		54	52	30		18	12	0	1	47	11
55	$\alpha^5 + \alpha^4 + \alpha^2 + \alpha + 1 =$		55	42	47		15	13	0	1	47	37
56	$\alpha^5 + \alpha^4 + \alpha^3 =$	$= \epsilon^{42}$	56	57	42		37	57	1	1	$\begin{vmatrix} & 1 & 1 & 1 \end{vmatrix}$	1
57	$\alpha^5 + \alpha^4 + \alpha^3 + 1 =$	$= \epsilon^{21}$	57	56	21		62	56	1	1	1	1
58	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha =$	$= \epsilon^{14}$	58	30	14		1	61	0	1	36	1
59	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha + 1 =$	- 1	59	51	25		39	60	0	1	36	36
60	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 =$	- 1	60	29	50		31	41	0	1	47	47
61	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + 1 =$	~	61	18	28		51	40	0	1	47	1
62	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha =$		62	$\frac{15}{25}$	12		57	45		1	10	37
63	$\alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 =$		63	5	10			44	1	1	10	46
55		-	00				121,1				10	10

	1 ^			-	4	<u> </u>					10		10	10			4.0		10	4.0	22	23		22	2 .
+	0	1	2	3	4	<u>5</u>	6	7	8	$\frac{9}{2}$	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	$\frac{1}{2}$	0	3	2	5	4	1	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22	25
$\frac{1}{2}$	$\begin{vmatrix} 2 \\ 2 \end{vmatrix}$	3	0	1	6	7 6	4	5 4	10	11	8	9	14	15	12 13	13 12	18	19 18	16	17 16	22 23	23 22	20 21	21 20	$\frac{26}{27}$
$\begin{vmatrix} 3\\4 \end{vmatrix}$	$\begin{vmatrix} 3 \\ 4 \end{vmatrix}$	$\frac{2}{5}$	1 6	$0 \\ 7$	$\frac{7}{0}$	6 1	$\frac{5}{2}$	$\frac{4}{3}$	11 12	10 13	$\frac{9}{14}$	8 15	15 8	14 9	10	11	19 20	21	$\frac{17}{22}$	16 23	23 16	$\frac{22}{17}$	18	19	28
5	$\begin{array}{ c c c c }\hline & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & $	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10	21	20	23	$\frac{23}{22}$	17	16	19	18	$\frac{20}{29}$
6	$\begin{vmatrix} 6 \\ 6 \end{vmatrix}$	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30
7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8	23	22	21	20	19	18	17	16	31
8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16
9	9	8	11	10	13	12	15	14	1	0	3	$\overline{2}$	5	4	7	6	25	24	27	26	29	28	31	30	17
10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5	26	27	24	25	30	31	28	29	18
11	11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4	27	26	25	24	31	30	29	28	19
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3	28	29	30	31	24	25	26	27	20
13	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2	29	28	31	30	25	24	27	26	21
14		15	12	13	10	11	8	9	6	7	4	5	2	3	0	1	30	31	28	29	26	27	24	25	22
15		14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	31	30	29	28	27	26	25	24	23
16	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	0	1	2	3	4	5	6	7	8
17	17	16	19	18	21	20	23	22	25 26	24	27	26	29	28	31	30	1	0	3	2	5	4	7	6	9
18		19	16 17	17 16	22 23	23 22	20 21	21	26 27	27	24 25	$\frac{25}{24}$	30 31	31	28	29 28	2	3	0	1	6	7 6	4	5 4	10
$\begin{vmatrix} 19 \\ 20 \end{vmatrix}$	$\begin{vmatrix} 19 \\ 20 \end{vmatrix}$	$\frac{18}{21}$	17 22	16 23	25 16	$\frac{22}{17}$	18	20 19	27 28	26 29	$\frac{25}{30}$	31	$\frac{31}{24}$	$\frac{30}{25}$	29 26	28 27	$\frac{3}{4}$	2	1	$0 \\ 7$	$\frac{7}{0}$	6 1	$\frac{5}{2}$	$\frac{4}{3}$	11 12
20	$\begin{vmatrix} 20\\21 \end{vmatrix}$	20	23	23 22	17	16	19	18	29	28	31	$\frac{31}{30}$	$\frac{24}{25}$	$\frac{25}{24}$	27	26	5	$\frac{5}{4}$	6 7	6	1	0	3	2	13
$\begin{vmatrix} 21\\22\end{vmatrix}$		23	20	21	18	19	16	17	$\frac{29}{30}$	31	28	29	26	27	24	25	6	7	4	5	2	3	0	1	14
23		22	21	20	19	18	17	16	31	30	29	28	27	26	25	24	7	6	5	4	3	2	1	0	15
24		25	26	27	28	29	30	31	16	17	18	19	20	21	$\frac{20}{22}$	23	8	9	10	11	12	13	14	15	0
25		$\frac{1}{24}$	$\frac{1}{27}$	26	29	28	31	30	17	16	19	18	21	20	23	$\frac{1}{2}$	9	8	11	10	13	12	15	14	1
26	26	27	24	25	30	31	28	29	18	19	16	17	22	23	20	21	10	11	8	9	14	15	12	13	2
27	27	26	25	24	31	30	29	28	19	18	17	16	23	22	21	20	11	10	9	8	15	14	13	12	3
28	28	29	30	31	24	25	26	27	20	21	22	23	16	17	18	19	12	13	14	15	8	9	10	11	4
29		28	31	30	25	24	27	26	21	20	23	22	17	16	19	18	13	12	15	14	9	8	11	10	5
30		31	28	29	26	27	24	25	22	23	20												8		6
31	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7
32	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
33		32	35	34	37	36	39	38	41	40	43	42	45 46	44	47	46	49	48	51	50	53 54	52 55	55 50	54 52	57
34 35		$\frac{35}{34}$	32 33	$\frac{33}{32}$	38 39	39 38	36 37	37 36	42 43	43 42	40 41	41 40	46 47	47 46	44 45	$\frac{45}{44}$	50 51	51 50	48 49	49 48	54 55	55 54	52 53	53 52	58 59
36		$\frac{34}{37}$	38	$\frac{32}{39}$	39 32	33	34	35	43 44	$\frac{42}{45}$	46	$\frac{40}{47}$	40	40	42	43	$\frac{51}{52}$	53	49 54	48 55	48	49	50	$\frac{52}{51}$	60
37		36	39	38	$\frac{32}{33}$	32	35	34	45	44	47	46	41	40	43	42	$\frac{52}{53}$	52	55	54	49	48	51	50	61
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$\begin{array}{c c} 1 \\ 1 \end{array}$			20 22	30 29	40 44	34 39	60 58	54 49	49 57	59 50	$\frac{37}{47}$	$\frac{47}{36}$	23 21	19 30	13 3	7 8	3 19	9 24	23 5	29 14	43 63	33 52	63 41	34	50 42	33
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$\frac{4}{5}$			$\frac{3}{5}$	55	10	56	о 15	52 61	$\frac{12}{20}$	61 38	15 17	$962 \\ 35$	10 30	59 44	9 27	90 41	40	$\frac{41}{26}$	$\frac{27}{45}$	31	$\frac{30}{34}$	47 16	39	21	60	37 14
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