

# Rank-74500 over GF(16)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(16) is 286331158

## General information

Number of lines	7
Number of points	305
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	102
Number of points off lines	196
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$17^7$
Type of lines on points	$4, 3, 2^5, 1^{102}, 0^{196}$

## Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{275} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

## The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{69889} = \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{69889} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{5121}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{18} \\
\ell_3 &= \begin{bmatrix} 1 & 1 & 0 & \delta^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \begin{bmatrix} 1 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \mathbf{Pl}(0, 11, 1, 0, 0, 1)_{4682} \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \begin{bmatrix} 1 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \mathbf{Pl}(0, 10, 1, 0, 0, 1)_{4681} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^5 & \delta^5 \end{bmatrix}_{4555} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 11 & 11 \end{bmatrix}_{4555} = \mathbf{Pl}(1, 1, 1, 10, 1, 0)_{3275} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{10} & \delta^{10} \end{bmatrix}_{4538} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 10 & 10 \end{bmatrix}_{4538} = \mathbf{Pl}(1, 1, 1, 11, 1, 0)_{3500}
\end{aligned}$$

Rank of lines: ( 69889, 70160, 4624, 48577, 44209, 4555, 4538 )

Rank of points on Klein quadric: ( 5121, 1, 18, 4682, 4681, 3275, 3500 )

### Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_{275} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1).$$

### Double Points

The surface has 5 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{530} &= (0, 0, 1, 1) = \ell_0 \cap \ell_1 \\
P_{689} &= (0, 10, 1, 1) = \ell_0 \cap \ell_5 \\
P_{705} &= (0, 11, 1, 1) = \ell_0 \cap \ell_6
\end{aligned}$$

$$\begin{aligned}
P_{3259} &= (10, 10, 11, 1) = \ell_3 \cap \ell_6 \\
P_{3020} &= (11, 11, 10, 1) = \ell_4 \cap \ell_5
\end{aligned}$$

### Single Points

The surface has 102 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line $\ell_0$	13 : $P_{641} = (0, 7, 1, 1)$ lies on line $\ell_0$
1 : $P_3 = (0, 0, 0, 1)$ lies on line $\ell_1$	14 : $P_{657} = (0, 8, 1, 1)$ lies on line $\ell_0$
2 : $P_{180} = (1, 10, 1, 0)$ lies on line $\ell_5$	15 : $P_{673} = (0, 9, 1, 1)$ lies on line $\ell_0$
3 : $P_{196} = (1, 11, 1, 0)$ lies on line $\ell_6$	16 : $P_{699} = (10, 10, 1, 1)$ lies on line $\ell_3$
4 : $P_{444} = (10, 10, 0, 1)$ lies on line $\ell_3$	17 : $P_{716} = (11, 11, 1, 1)$ lies on line $\ell_4$
5 : $P_{461} = (11, 11, 0, 1)$ lies on line $\ell_4$	18 : $P_{721} = (0, 12, 1, 1)$ lies on line $\ell_0$
6 : $P_{531} = (1, 0, 1, 1)$ lies on line $\ell_2$	19 : $P_{737} = (0, 13, 1, 1)$ lies on line $\ell_0$
7 : $P_{546} = (0, 1, 1, 1)$ lies on line $\ell_0$	20 : $P_{753} = (0, 14, 1, 1)$ lies on line $\ell_0$
8 : $P_{561} = (0, 2, 1, 1)$ lies on line $\ell_0$	21 : $P_{769} = (0, 15, 1, 1)$ lies on line $\ell_0$
9 : $P_{577} = (0, 3, 1, 1)$ lies on line $\ell_0$	22 : $P_{785} = (0, 0, 2, 1)$ lies on line $\ell_1$
10 : $P_{593} = (0, 4, 1, 1)$ lies on line $\ell_0$	23 : $P_{786} = (1, 0, 2, 1)$ lies on line $\ell_2$
11 : $P_{609} = (0, 5, 1, 1)$ lies on line $\ell_0$	24 : $P_{955} = (10, 10, 2, 1)$ lies on line $\ell_3$
12 : $P_{625} = (0, 6, 1, 1)$ lies on line $\ell_0$	25 : $P_{972} = (11, 11, 2, 1)$ lies on line $\ell_4$

- 26 :  $P_{996} = (3, 13, 2, 1)$  lies on line  $\ell_5$   
 27 :  $P_{1028} = (3, 15, 2, 1)$  lies on line  $\ell_6$   
 28 :  $P_{1041} = (0, 0, 3, 1)$  lies on line  $\ell_1$   
 29 :  $P_{1042} = (1, 0, 3, 1)$  lies on line  $\ell_2$   
 30 :  $P_{1107} = (2, 4, 3, 1)$  lies on line  $\ell_6$   
 31 :  $P_{1155} = (2, 7, 3, 1)$  lies on line  $\ell_5$   
 32 :  $P_{1211} = (10, 10, 3, 1)$  lies on line  $\ell_3$   
 33 :  $P_{1228} = (11, 11, 3, 1)$  lies on line  $\ell_4$   
 34 :  $P_{1297} = (0, 0, 4, 1)$  lies on line  $\ell_1$   
 35 :  $P_{1298} = (1, 0, 4, 1)$  lies on line  $\ell_2$   
 36 :  $P_{1350} = (5, 3, 4, 1)$  lies on line  $\ell_5$   
 37 :  $P_{1414} = (5, 7, 4, 1)$  lies on line  $\ell_6$   
 38 :  $P_{1467} = (10, 10, 4, 1)$  lies on line  $\ell_3$   
 39 :  $P_{1484} = (11, 11, 4, 1)$  lies on line  $\ell_4$   
 40 :  $P_{1553} = (0, 0, 5, 1)$  lies on line  $\ell_1$   
 41 :  $P_{1554} = (1, 0, 5, 1)$  lies on line  $\ell_2$   
 42 :  $P_{1701} = (4, 9, 5, 1)$  lies on line  $\ell_5$   
 43 :  $P_{1723} = (10, 10, 5, 1)$  lies on line  $\ell_3$   
 44 :  $P_{1740} = (11, 11, 5, 1)$  lies on line  $\ell_4$   
 45 :  $P_{1749} = (4, 12, 5, 1)$  lies on line  $\ell_6$   
 46 :  $P_{1809} = (0, 0, 6, 1)$  lies on line  $\ell_1$   
 47 :  $P_{1810} = (1, 0, 6, 1)$  lies on line  $\ell_2$   
 48 :  $P_{1944} = (7, 8, 6, 1)$  lies on line  $\ell_6$   
 49 :  $P_{1979} = (10, 10, 6, 1)$  lies on line  $\ell_3$   
 50 :  $P_{1996} = (11, 11, 6, 1)$  lies on line  $\ell_4$   
 51 :  $P_{2040} = (7, 14, 6, 1)$  lies on line  $\ell_5$   
 52 :  $P_{2065} = (0, 0, 7, 1)$  lies on line  $\ell_1$   
 53 :  $P_{2066} = (1, 0, 7, 1)$  lies on line  $\ell_2$   
 54 :  $P_{2119} = (6, 3, 7, 1)$  lies on line  $\ell_6$   
 55 :  $P_{2135} = (6, 4, 7, 1)$  lies on line  $\ell_5$   
 56 :  $P_{2235} = (10, 10, 7, 1)$  lies on line  $\ell_3$   
 57 :  $P_{2252} = (11, 11, 7, 1)$  lies on line  $\ell_4$   
 58 :  $P_{2321} = (0, 0, 8, 1)$  lies on line  $\ell_1$   
 59 :  $P_{2322} = (1, 0, 8, 1)$  lies on line  $\ell_2$   
 60 :  $P_{2426} = (9, 6, 8, 1)$  lies on line  $\ell_5$   
 61 :  $P_{2491} = (10, 10, 8, 1)$  lies on line  $\ell_3$   
 62 :  $P_{2508} = (11, 11, 8, 1)$  lies on line  $\ell_4$   
 63 :  $P_{2554} = (9, 14, 8, 1)$  lies on line  $\ell_6$   
 64 :  $P_{2577} = (0, 0, 9, 1)$  lies on line  $\ell_1$   
 65 :  $P_{2578} = (1, 0, 9, 1)$  lies on line  $\ell_2$   
 66 :  $P_{2665} = (8, 5, 9, 1)$  lies on line  $\ell_6$   
 67 :  $P_{2747} = (10, 10, 9, 1)$  lies on line  $\ell_3$   
 68 :  $P_{2764} = (11, 11, 9, 1)$  lies on line  $\ell_4$   
 69 :  $P_{2777} = (8, 12, 9, 1)$  lies on line  $\ell_5$   
 70 :  $P_{2833} = (0, 0, 10, 1)$  lies on line  $\ell_1$   
 71 :  $P_{2834} = (1, 0, 10, 1)$  lies on line  $\ell_2$   
 72 :  $P_{2860} = (11, 1, 10, 1)$  lies on line  $\ell_6$   
 73 :  $P_{3003} = (10, 10, 10, 1)$  lies on line  $\ell_3$   
 74 :  $P_{3089} = (0, 0, 11, 1)$  lies on line  $\ell_1$   
 75 :  $P_{3090} = (1, 0, 11, 1)$  lies on line  $\ell_2$   
 76 :  $P_{3115} = (10, 1, 11, 1)$  lies on line  $\ell_5$   
 77 :  $P_{3276} = (11, 11, 11, 1)$  lies on line  $\ell_4$   
 78 :  $P_{3345} = (0, 0, 12, 1)$  lies on line  $\ell_1$   
 79 :  $P_{3346} = (1, 0, 12, 1)$  lies on line  $\ell_2$   
 80 :  $P_{3438} = (13, 5, 12, 1)$  lies on line  $\ell_5$   
 81 :  $P_{3502} = (13, 9, 12, 1)$  lies on line  $\ell_6$   
 82 :  $P_{3515} = (10, 10, 12, 1)$  lies on line  $\ell_3$   
 83 :  $P_{3532} = (11, 11, 12, 1)$  lies on line  $\ell_4$   
 84 :  $P_{3601} = (0, 0, 13, 1)$  lies on line  $\ell_1$   
 85 :  $P_{3602} = (1, 0, 13, 1)$  lies on line  $\ell_2$   
 86 :  $P_{3645} = (12, 2, 13, 1)$  lies on line  $\ell_6$   
 87 :  $P_{3771} = (10, 10, 13, 1)$  lies on line  $\ell_3$   
 88 :  $P_{3788} = (11, 11, 13, 1)$  lies on line  $\ell_4$   
 89 :  $P_{3853} = (12, 15, 13, 1)$  lies on line  $\ell_5$   
 90 :  $P_{3857} = (0, 0, 14, 1)$  lies on line  $\ell_1$   
 91 :  $P_{3858} = (1, 0, 14, 1)$  lies on line  $\ell_2$   
 92 :  $P_{3968} = (15, 6, 14, 1)$  lies on line  $\ell_6$   
 93 :  $P_{4000} = (15, 8, 14, 1)$  lies on line  $\ell_5$   
 94 :  $P_{4027} = (10, 10, 14, 1)$  lies on line  $\ell_3$   
 95 :  $P_{4044} = (11, 11, 14, 1)$  lies on line  $\ell_4$   
 96 :  $P_{4113} = (0, 0, 15, 1)$  lies on line  $\ell_1$   
 97 :  $P_{4114} = (1, 0, 15, 1)$  lies on line  $\ell_2$   
 98 :  $P_{4159} = (14, 2, 15, 1)$  lies on line  $\ell_5$   
 99 :  $P_{4283} = (10, 10, 15, 1)$  lies on line  $\ell_3$   
 100 :  $P_{4300} = (11, 11, 15, 1)$  lies on line  $\ell_4$   
 101 :  $P_{4335} = (14, 13, 15, 1)$  lies on line  $\ell_6$

The single points on the surface are:

### Points on surface but on no line

The surface has 196 points not on any line:

The points on the surface but not on lines are:

- 0 :  $P_{78} = (11, 3, 1, 0)$   
 1 :  $P_{109} = (10, 5, 1, 0)$   
 2 :  $P_{128} = (13, 6, 1, 0)$   
 3 :  $P_{143} = (12, 7, 1, 0)$   
 4 :  $P_{158} = (11, 8, 1, 0)$   
 5 :  $P_{185} = (6, 10, 1, 0)$   
 6 :  $P_{186} = (7, 10, 1, 0)$   
 7 :  $P_{207} = (12, 11, 1, 0)$   
 8 :  $P_{208} = (13, 11, 1, 0)$   
 9 :  $P_{217} = (6, 12, 1, 0)$

10 : $P_{234} = (7, 13, 1, 0)$	64 : $P_{1338} = (9, 2, 4, 1)$
11 : $P_{269} = (10, 15, 1, 0)$	65 : $P_{1339} = (10, 2, 4, 1)$
12 : $P_{335} = (13, 3, 0, 1)$	66 : $P_{1392} = (15, 5, 4, 1)$
13 : $P_{361} = (7, 5, 0, 1)$	67 : $P_{1417} = (8, 7, 4, 1)$
14 : $P_{378} = (8, 6, 0, 1)$	68 : $P_{1422} = (13, 7, 4, 1)$
15 : $P_{389} = (3, 7, 0, 1)$	69 : $P_{1444} = (3, 9, 4, 1)$
16 : $P_{414} = (12, 8, 0, 1)$	70 : $P_{1453} = (12, 9, 4, 1)$
17 : $P_{438} = (4, 10, 0, 1)$	71 : $P_{1456} = (15, 9, 4, 1)$
18 : $P_{448} = (14, 10, 0, 1)$	72 : $P_{1459} = (2, 10, 4, 1)$
19 : $P_{452} = (2, 11, 0, 1)$	73 : $P_{1465} = (8, 10, 4, 1)$
20 : $P_{459} = (9, 11, 0, 1)$	74 : $P_{1516} = (11, 13, 4, 1)$
21 : $P_{471} = (5, 12, 0, 1)$	75 : $P_{1522} = (1, 14, 4, 1)$
22 : $P_{497} = (15, 13, 0, 1)$	76 : $P_{1533} = (12, 14, 4, 1)$
23 : $P_{520} = (6, 15, 0, 1)$	77 : $P_{1534} = (13, 14, 4, 1)$
24 : $P_{576} = (15, 2, 1, 1)$	78 : $P_{1539} = (2, 15, 4, 1)$
25 : $P_{591} = (14, 3, 1, 1)$	79 : $P_{1575} = (6, 1, 5, 1)$
26 : $P_{596} = (3, 4, 1, 1)$	80 : $P_{1603} = (2, 3, 5, 1)$
27 : $P_{611} = (2, 5, 1, 1)$	81 : $P_{1645} = (12, 5, 5, 1)$
28 : $P_{638} = (13, 6, 1, 1)$	82 : $P_{1666} = (1, 7, 5, 1)$
29 : $P_{653} = (12, 7, 1, 1)$	83 : $P_{1692} = (11, 8, 5, 1)$
30 : $P_{661} = (4, 8, 1, 1)$	84 : $P_{1719} = (6, 10, 5, 1)$
31 : $P_{678} = (5, 9, 1, 1)$	85 : $P_{1725} = (12, 10, 5, 1)$
32 : $P_{727} = (6, 12, 1, 1)$	86 : $P_{1763} = (2, 13, 5, 1)$
33 : $P_{744} = (7, 13, 1, 1)$	87 : $P_{1787} = (10, 14, 5, 1)$
34 : $P_{761} = (8, 14, 1, 1)$	88 : $P_{1903} = (14, 5, 6, 1)$
35 : $P_{778} = (9, 15, 1, 1)$	89 : $P_{1916} = (11, 6, 6, 1)$
36 : $P_{805} = (4, 1, 2, 1)$	90 : $P_{1930} = (9, 7, 6, 1)$
37 : $P_{841} = (8, 3, 2, 1)$	91 : $P_{1947} = (10, 8, 6, 1)$
38 : $P_{856} = (7, 4, 2, 1)$	92 : $P_{1950} = (13, 8, 6, 1)$
39 : $P_{857} = (8, 4, 2, 1)$	93 : $P_{1967} = (14, 9, 6, 1)$
40 : $P_{864} = (15, 4, 2, 1)$	94 : $P_{2014} = (13, 12, 6, 1)$
41 : $P_{891} = (10, 6, 2, 1)$	95 : $P_{2026} = (9, 13, 6, 1)$
42 : $P_{927} = (14, 8, 2, 1)$	96 : $P_{2050} = (1, 15, 6, 1)$
43 : $P_{930} = (1, 9, 2, 1)$	97 : $P_{2101} = (4, 2, 7, 1)$
44 : $P_{935} = (6, 9, 2, 1)$	98 : $P_{2123} = (10, 3, 7, 1)$
45 : $P_{936} = (7, 9, 2, 1)$	99 : $P_{2125} = (12, 3, 7, 1)$
46 : $P_{966} = (5, 11, 2, 1)$	100 : $P_{2146} = (1, 5, 7, 1)$
47 : $P_{975} = (14, 11, 2, 1)$	101 : $P_{2163} = (2, 6, 7, 1)$
48 : $P_{998} = (5, 13, 2, 1)$	102 : $P_{2188} = (11, 7, 7, 1)$
49 : $P_{999} = (6, 13, 2, 1)$	103 : $P_{2259} = (2, 12, 7, 1)$
50 : $P_{1013} = (4, 14, 2, 1)$	104 : $P_{2285} = (12, 13, 7, 1)$
51 : $P_{1020} = (11, 14, 2, 1)$	105 : $P_{2309} = (4, 15, 7, 1)$
52 : $P_{1024} = (15, 14, 2, 1)$	106 : $P_{2350} = (13, 1, 8, 1)$
53 : $P_{1069} = (12, 1, 3, 1)$	107 : $P_{2364} = (11, 2, 8, 1)$
54 : $P_{1096} = (7, 3, 3, 1)$	108 : $P_{2405} = (4, 5, 8, 1)$
55 : $P_{1131} = (10, 5, 3, 1)$	109 : $P_{2437} = (4, 7, 8, 1)$
56 : $P_{1151} = (14, 6, 3, 1)$	110 : $P_{2455} = (6, 8, 8, 1)$
57 : $P_{1196} = (11, 9, 3, 1)$	111 : $P_{2503} = (6, 11, 8, 1)$
58 : $P_{1224} = (7, 11, 3, 1)$	112 : $P_{2510} = (13, 11, 8, 1)$
59 : $P_{1229} = (12, 11, 3, 1)$	113 : $P_{2514} = (1, 12, 8, 1)$
60 : $P_{1250} = (1, 13, 3, 1)$	114 : $P_{2571} = (10, 15, 8, 1)$
61 : $P_{1295} = (14, 15, 3, 1)$	115 : $P_{2607} = (14, 1, 9, 1)$
62 : $P_{1322} = (9, 1, 4, 1)$	116 : $P_{2610} = (1, 2, 9, 1)$
63 : $P_{1332} = (3, 2, 4, 1)$	117 : $P_{2615} = (6, 2, 9, 1)$

118 : $P_{2616} = (7, 2, 9, 1)$	158 : $P_{3474} = (1, 8, 12, 1)$
119 : $P_{2629} = (4, 3, 9, 1)$	159 : $P_{3547} = (10, 12, 12, 1)$
120 : $P_{2646} = (5, 4, 9, 1)$	160 : $P_{3557} = (4, 13, 12, 1)$
121 : $P_{2652} = (11, 4, 9, 1)$	161 : $P_{3650} = (1, 3, 13, 1)$
122 : $P_{2655} = (14, 4, 9, 1)$	162 : $P_{3704} = (7, 6, 13, 1)$
123 : $P_{2699} = (10, 7, 9, 1)$	163 : $P_{3727} = (14, 7, 13, 1)$
124 : $P_{2708} = (3, 8, 9, 1)$	164 : $P_{3731} = (2, 8, 13, 1)$
125 : $P_{2757} = (4, 11, 9, 1)$	165 : $P_{3807} = (14, 12, 13, 1)$
126 : $P_{2768} = (15, 11, 9, 1)$	166 : $P_{3819} = (10, 13, 13, 1)$
127 : $P_{2776} = (7, 12, 9, 1)$	167 : $P_{3827} = (2, 14, 13, 1)$
128 : $P_{2784} = (15, 12, 9, 1)$	168 : $P_{3848} = (7, 15, 13, 1)$
129 : $P_{2804} = (3, 14, 9, 1)$	169 : $P_{3852} = (11, 15, 13, 1)$
130 : $P_{2806} = (5, 14, 9, 1)$	170 : $P_{3875} = (2, 1, 14, 1)$
131 : $P_{2807} = (6, 14, 9, 1)$	171 : $P_{3894} = (5, 2, 14, 1)$
132 : $P_{2852} = (3, 1, 10, 1)$	172 : $P_{3897} = (8, 2, 14, 1)$
133 : $P_{2857} = (8, 1, 10, 1)$	173 : $P_{3902} = (13, 2, 14, 1)$
134 : $P_{2916} = (3, 5, 10, 1)$	174 : $P_{3922} = (1, 4, 14, 1)$
135 : $P_{2934} = (5, 6, 10, 1)$	175 : $P_{3933} = (12, 4, 14, 1)$
136 : $P_{2960} = (15, 7, 10, 1)$	176 : $P_{3934} = (13, 4, 14, 1)$
137 : $P_{2998} = (5, 10, 10, 1)$	177 : $P_{3946} = (9, 5, 14, 1)$
138 : $P_{3008} = (15, 10, 10, 1)$	178 : $P_{3956} = (3, 6, 14, 1)$
139 : $P_{3010} = (1, 11, 10, 1)$	179 : $P_{3965} = (12, 6, 14, 1)$
140 : $P_{3019} = (10, 11, 10, 1)$	180 : $P_{4003} = (2, 9, 14, 1)$
141 : $P_{3081} = (8, 15, 10, 1)$	181 : $P_{4009} = (8, 9, 14, 1)$
142 : $P_{3110} = (5, 1, 11, 1)$	182 : $P_{4011} = (10, 9, 14, 1)$
143 : $P_{3120} = (15, 1, 11, 1)$	183 : $P_{4020} = (3, 10, 14, 1)$
144 : $P_{3152} = (15, 3, 11, 1)$	184 : $P_{4026} = (9, 10, 14, 1)$
145 : $P_{3222} = (5, 8, 11, 1)$	185 : $P_{4060} = (11, 12, 14, 1)$
146 : $P_{3250} = (1, 10, 11, 1)$	186 : $P_{4102} = (5, 15, 14, 1)$
147 : $P_{3260} = (11, 10, 11, 1)$	187 : $P_{4136} = (7, 1, 15, 1)$
148 : $P_{3268} = (3, 11, 11, 1)$	188 : $P_{4172} = (11, 3, 15, 1)$
149 : $P_{3273} = (8, 11, 11, 1)$	189 : $P_{4187} = (10, 4, 15, 1)$
150 : $P_{3284} = (3, 12, 11, 1)$	190 : $P_{4210} = (1, 6, 15, 1)$
151 : $P_{3305} = (8, 13, 11, 1)$	191 : $P_{4250} = (9, 8, 15, 1)$
152 : $P_{3402} = (9, 3, 12, 1)$	192 : $P_{4280} = (7, 10, 15, 1)$
153 : $P_{3418} = (9, 4, 12, 1)$	193 : $P_{4286} = (13, 10, 15, 1)$
154 : $P_{3431} = (6, 5, 12, 1)$	194 : $P_{4314} = (9, 12, 15, 1)$
155 : $P_{3436} = (11, 5, 12, 1)$	195 : $P_{4366} = (13, 15, 15, 1)$
156 : $P_{3445} = (4, 6, 12, 1)$	
157 : $P_{3463} = (6, 7, 12, 1)$	

## Line Intersection Graph

	0	1	2	3	4	5	6
0	0	1	0	0	0	1	1
1	1	0	1	1	1	0	0
2	0	1	0	1	1	1	1
3	0	1	1	0	1	0	1
4	0	1	1	1	0	1	0
5	1	0	1	0	1	0	1
6	1	0	1	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_5$	$\ell_6$
in point	$P_{530}$	$P_{689}$	$P_{705}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_{530}$	$P_2$	$P_2$	$P_2$

Line 2 intersects

Line	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$
in point	$P_2$	$P_2$	$P_2$	$P_{275}$	$P_{275}$

Line 3 intersects

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_6$
in point	$P_2$	$P_2$	$P_2$	$P_{3259}$

Line 4 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_5$
in point	$P_2$	$P_2$	$P_2$	$P_{3020}$

Line 5 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_6$
in point	$P_{689}$	$P_{275}$	$P_{3020}$	$P_{275}$

Line 6 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_5$
in point	$P_{705}$	$P_{275}$	$P_{3259}$	$P_{275}$

The surface has 305 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	25 : $P_{448} = (14, 10, 0, 1)$	50 : $P_{678} = (5, 9, 1, 1)$
1 : $P_2 = (0, 0, 1, 0)$	26 : $P_{452} = (2, 11, 0, 1)$	51 : $P_{689} = (0, 10, 1, 1)$
2 : $P_3 = (0, 0, 0, 1)$	27 : $P_{459} = (9, 11, 0, 1)$	52 : $P_{699} = (10, 10, 1, 1)$
3 : $P_{78} = (11, 3, 1, 0)$	28 : $P_{461} = (11, 11, 0, 1)$	53 : $P_{705} = (0, 11, 1, 1)$
4 : $P_{109} = (10, 5, 1, 0)$	29 : $P_{471} = (5, 12, 0, 1)$	54 : $P_{716} = (11, 11, 1, 1)$
5 : $P_{128} = (13, 6, 1, 0)$	30 : $P_{497} = (15, 13, 0, 1)$	55 : $P_{721} = (0, 12, 1, 1)$
6 : $P_{143} = (12, 7, 1, 0)$	31 : $P_{520} = (6, 15, 0, 1)$	56 : $P_{727} = (6, 12, 1, 1)$
7 : $P_{158} = (11, 8, 1, 0)$	32 : $P_{530} = (0, 0, 1, 1)$	57 : $P_{737} = (0, 13, 1, 1)$
8 : $P_{180} = (1, 10, 1, 0)$	33 : $P_{531} = (1, 0, 1, 1)$	58 : $P_{744} = (7, 13, 1, 1)$
9 : $P_{185} = (6, 10, 1, 0)$	34 : $P_{546} = (0, 1, 1, 1)$	59 : $P_{753} = (0, 14, 1, 1)$
10 : $P_{186} = (7, 10, 1, 0)$	35 : $P_{561} = (0, 2, 1, 1)$	60 : $P_{761} = (8, 14, 1, 1)$
11 : $P_{196} = (1, 11, 1, 0)$	36 : $P_{576} = (15, 2, 1, 1)$	61 : $P_{769} = (0, 15, 1, 1)$
12 : $P_{207} = (12, 11, 1, 0)$	37 : $P_{577} = (0, 3, 1, 1)$	62 : $P_{778} = (9, 15, 1, 1)$
13 : $P_{208} = (13, 11, 1, 0)$	38 : $P_{591} = (14, 3, 1, 1)$	63 : $P_{785} = (0, 0, 2, 1)$
14 : $P_{217} = (6, 12, 1, 0)$	39 : $P_{593} = (0, 4, 1, 1)$	64 : $P_{786} = (1, 0, 2, 1)$
15 : $P_{234} = (7, 13, 1, 0)$	40 : $P_{596} = (3, 4, 1, 1)$	65 : $P_{805} = (4, 1, 2, 1)$
16 : $P_{269} = (10, 15, 1, 0)$	41 : $P_{609} = (0, 5, 1, 1)$	66 : $P_{841} = (8, 3, 2, 1)$
17 : $P_{275} = (1, 0, 0, 1)$	42 : $P_{611} = (2, 5, 1, 1)$	67 : $P_{856} = (7, 4, 2, 1)$
18 : $P_{335} = (13, 3, 0, 1)$	43 : $P_{625} = (0, 6, 1, 1)$	68 : $P_{857} = (8, 4, 2, 1)$
19 : $P_{361} = (7, 5, 0, 1)$	44 : $P_{638} = (13, 6, 1, 1)$	69 : $P_{864} = (15, 4, 2, 1)$
20 : $P_{378} = (8, 6, 0, 1)$	45 : $P_{641} = (0, 7, 1, 1)$	70 : $P_{891} = (10, 6, 2, 1)$
21 : $P_{389} = (3, 7, 0, 1)$	46 : $P_{653} = (12, 7, 1, 1)$	71 : $P_{927} = (14, 8, 2, 1)$
22 : $P_{414} = (12, 8, 0, 1)$	47 : $P_{657} = (0, 8, 1, 1)$	72 : $P_{930} = (1, 9, 2, 1)$
23 : $P_{438} = (4, 10, 0, 1)$	48 : $P_{661} = (4, 8, 1, 1)$	73 : $P_{935} = (6, 9, 2, 1)$
24 : $P_{444} = (10, 10, 0, 1)$	49 : $P_{673} = (0, 9, 1, 1)$	74 : $P_{936} = (7, 9, 2, 1)$

75 : $P_{955} = (10, 10, 2, 1)$	129 : $P_{1666} = (1, 7, 5, 1)$	183 : $P_{2571} = (10, 15, 8, 1)$
76 : $P_{966} = (5, 11, 2, 1)$	130 : $P_{1692} = (11, 8, 5, 1)$	184 : $P_{2577} = (0, 0, 9, 1)$
77 : $P_{972} = (11, 11, 2, 1)$	131 : $P_{1701} = (4, 9, 5, 1)$	185 : $P_{2578} = (1, 0, 9, 1)$
78 : $P_{975} = (14, 11, 2, 1)$	132 : $P_{1719} = (6, 10, 5, 1)$	186 : $P_{2607} = (14, 1, 9, 1)$
79 : $P_{996} = (3, 13, 2, 1)$	133 : $P_{1723} = (10, 10, 5, 1)$	187 : $P_{2610} = (1, 2, 9, 1)$
80 : $P_{998} = (5, 13, 2, 1)$	134 : $P_{1725} = (12, 10, 5, 1)$	188 : $P_{2615} = (6, 2, 9, 1)$
81 : $P_{999} = (6, 13, 2, 1)$	135 : $P_{1740} = (11, 11, 5, 1)$	189 : $P_{2616} = (7, 2, 9, 1)$
82 : $P_{1013} = (4, 14, 2, 1)$	136 : $P_{1749} = (4, 12, 5, 1)$	190 : $P_{2629} = (4, 3, 9, 1)$
83 : $P_{1020} = (11, 14, 2, 1)$	137 : $P_{1763} = (2, 13, 5, 1)$	191 : $P_{2646} = (5, 4, 9, 1)$
84 : $P_{1024} = (15, 14, 2, 1)$	138 : $P_{1787} = (10, 14, 5, 1)$	192 : $P_{2652} = (11, 4, 9, 1)$
85 : $P_{1028} = (3, 15, 2, 1)$	139 : $P_{1809} = (0, 0, 6, 1)$	193 : $P_{2655} = (14, 4, 9, 1)$
86 : $P_{1041} = (0, 0, 3, 1)$	140 : $P_{1810} = (1, 0, 6, 1)$	194 : $P_{2665} = (8, 5, 9, 1)$
87 : $P_{1042} = (1, 0, 3, 1)$	141 : $P_{1903} = (14, 5, 6, 1)$	195 : $P_{2699} = (10, 7, 9, 1)$
88 : $P_{1069} = (12, 1, 3, 1)$	142 : $P_{1916} = (11, 6, 6, 1)$	196 : $P_{2708} = (3, 8, 9, 1)$
89 : $P_{1096} = (7, 3, 3, 1)$	143 : $P_{1930} = (9, 7, 6, 1)$	197 : $P_{2747} = (10, 10, 9, 1)$
90 : $P_{1107} = (2, 4, 3, 1)$	144 : $P_{1944} = (7, 8, 6, 1)$	198 : $P_{2757} = (4, 11, 9, 1)$
91 : $P_{1131} = (10, 5, 3, 1)$	145 : $P_{1947} = (10, 8, 6, 1)$	199 : $P_{2764} = (11, 11, 9, 1)$
92 : $P_{1151} = (14, 6, 3, 1)$	146 : $P_{1950} = (13, 8, 6, 1)$	200 : $P_{2768} = (15, 11, 9, 1)$
93 : $P_{1155} = (2, 7, 3, 1)$	147 : $P_{1967} = (14, 9, 6, 1)$	201 : $P_{2776} = (7, 12, 9, 1)$
94 : $P_{1196} = (11, 9, 3, 1)$	148 : $P_{1979} = (10, 10, 6, 1)$	202 : $P_{2777} = (8, 12, 9, 1)$
95 : $P_{1211} = (10, 10, 3, 1)$	149 : $P_{1996} = (11, 11, 6, 1)$	203 : $P_{2784} = (15, 12, 9, 1)$
96 : $P_{1224} = (7, 11, 3, 1)$	150 : $P_{2014} = (13, 12, 6, 1)$	204 : $P_{2804} = (3, 14, 9, 1)$
97 : $P_{1228} = (11, 11, 3, 1)$	151 : $P_{2026} = (9, 13, 6, 1)$	205 : $P_{2806} = (5, 14, 9, 1)$
98 : $P_{1229} = (12, 11, 3, 1)$	152 : $P_{2040} = (7, 14, 6, 1)$	206 : $P_{2807} = (6, 14, 9, 1)$
99 : $P_{1250} = (1, 13, 3, 1)$	153 : $P_{2050} = (1, 15, 6, 1)$	207 : $P_{2833} = (0, 0, 10, 1)$
100 : $P_{1295} = (14, 15, 3, 1)$	154 : $P_{2065} = (0, 0, 7, 1)$	208 : $P_{2834} = (1, 0, 10, 1)$
101 : $P_{1297} = (0, 0, 4, 1)$	155 : $P_{2066} = (1, 0, 7, 1)$	209 : $P_{2852} = (3, 1, 10, 1)$
102 : $P_{1298} = (1, 0, 4, 1)$	156 : $P_{2101} = (4, 2, 7, 1)$	210 : $P_{2857} = (8, 1, 10, 1)$
103 : $P_{1322} = (9, 1, 4, 1)$	157 : $P_{2119} = (6, 3, 7, 1)$	211 : $P_{2860} = (11, 1, 10, 1)$
104 : $P_{1332} = (3, 2, 4, 1)$	158 : $P_{2123} = (10, 3, 7, 1)$	212 : $P_{2916} = (3, 5, 10, 1)$
105 : $P_{1338} = (9, 2, 4, 1)$	159 : $P_{2125} = (12, 3, 7, 1)$	213 : $P_{2934} = (5, 6, 10, 1)$
106 : $P_{1339} = (10, 2, 4, 1)$	160 : $P_{2135} = (6, 4, 7, 1)$	214 : $P_{2960} = (15, 7, 10, 1)$
107 : $P_{1350} = (5, 3, 4, 1)$	161 : $P_{2146} = (1, 5, 7, 1)$	215 : $P_{2998} = (5, 10, 10, 1)$
108 : $P_{1392} = (15, 5, 4, 1)$	162 : $P_{2163} = (2, 6, 7, 1)$	216 : $P_{3003} = (10, 10, 10, 1)$
109 : $P_{1414} = (5, 7, 4, 1)$	163 : $P_{2188} = (11, 7, 7, 1)$	217 : $P_{3008} = (15, 10, 10, 1)$
110 : $P_{1417} = (8, 7, 4, 1)$	164 : $P_{2235} = (10, 10, 7, 1)$	218 : $P_{3010} = (1, 11, 10, 1)$
111 : $P_{1422} = (13, 7, 4, 1)$	165 : $P_{2252} = (11, 11, 7, 1)$	219 : $P_{3019} = (10, 11, 10, 1)$
112 : $P_{1444} = (3, 9, 4, 1)$	166 : $P_{2259} = (2, 12, 7, 1)$	220 : $P_{3020} = (11, 11, 10, 1)$
113 : $P_{1453} = (12, 9, 4, 1)$	167 : $P_{2285} = (12, 13, 7, 1)$	221 : $P_{3081} = (8, 15, 10, 1)$
114 : $P_{1456} = (15, 9, 4, 1)$	168 : $P_{2309} = (4, 15, 7, 1)$	222 : $P_{3089} = (0, 0, 11, 1)$
115 : $P_{1459} = (2, 10, 4, 1)$	169 : $P_{2321} = (0, 0, 8, 1)$	223 : $P_{3090} = (1, 0, 11, 1)$
116 : $P_{1465} = (8, 10, 4, 1)$	170 : $P_{2322} = (1, 0, 8, 1)$	224 : $P_{3110} = (5, 1, 11, 1)$
117 : $P_{1467} = (10, 10, 4, 1)$	171 : $P_{2350} = (13, 1, 8, 1)$	225 : $P_{3115} = (10, 1, 11, 1)$
118 : $P_{1484} = (11, 11, 4, 1)$	172 : $P_{2364} = (11, 2, 8, 1)$	226 : $P_{3120} = (15, 1, 11, 1)$
119 : $P_{1516} = (11, 13, 4, 1)$	173 : $P_{2405} = (4, 5, 8, 1)$	227 : $P_{3152} = (15, 3, 11, 1)$
120 : $P_{1522} = (1, 14, 4, 1)$	174 : $P_{2426} = (9, 6, 8, 1)$	228 : $P_{3222} = (5, 8, 11, 1)$
121 : $P_{1533} = (12, 14, 4, 1)$	175 : $P_{2437} = (4, 7, 8, 1)$	229 : $P_{3250} = (1, 10, 11, 1)$
122 : $P_{1534} = (13, 14, 4, 1)$	176 : $P_{2455} = (6, 8, 8, 1)$	230 : $P_{3259} = (10, 10, 11, 1)$
123 : $P_{1539} = (2, 15, 4, 1)$	177 : $P_{2491} = (10, 10, 8, 1)$	231 : $P_{3260} = (11, 10, 11, 1)$
124 : $P_{1553} = (0, 0, 5, 1)$	178 : $P_{2503} = (6, 11, 8, 1)$	232 : $P_{3268} = (3, 11, 11, 1)$
125 : $P_{1554} = (1, 0, 5, 1)$	179 : $P_{2508} = (11, 11, 8, 1)$	233 : $P_{3273} = (8, 11, 11, 1)$
126 : $P_{1575} = (6, 1, 5, 1)$	180 : $P_{2510} = (13, 11, 8, 1)$	234 : $P_{3276} = (11, 11, 11, 1)$
127 : $P_{1603} = (2, 3, 5, 1)$	181 : $P_{2514} = (1, 12, 8, 1)$	235 : $P_{3284} = (3, 12, 11, 1)$
128 : $P_{1645} = (12, 5, 5, 1)$	182 : $P_{2554} = (9, 14, 8, 1)$	236 : $P_{3305} = (8, 13, 11, 1)$

237 : $P_{3345} = (0, 0, 12, 1)$	260 : $P_{3788} = (11, 11, 13, 1)$	283 : $P_{4011} = (10, 9, 14, 1)$
238 : $P_{3346} = (1, 0, 12, 1)$	261 : $P_{3807} = (14, 12, 13, 1)$	284 : $P_{4020} = (3, 10, 14, 1)$
239 : $P_{3402} = (9, 3, 12, 1)$	262 : $P_{3819} = (10, 13, 13, 1)$	285 : $P_{4026} = (9, 10, 14, 1)$
240 : $P_{3418} = (9, 4, 12, 1)$	263 : $P_{3827} = (2, 14, 13, 1)$	286 : $P_{4027} = (10, 10, 14, 1)$
241 : $P_{3431} = (6, 5, 12, 1)$	264 : $P_{3848} = (7, 15, 13, 1)$	287 : $P_{4044} = (11, 11, 14, 1)$
242 : $P_{3436} = (11, 5, 12, 1)$	265 : $P_{3852} = (11, 15, 13, 1)$	288 : $P_{4060} = (11, 12, 14, 1)$
243 : $P_{3438} = (13, 5, 12, 1)$	266 : $P_{3853} = (12, 15, 13, 1)$	289 : $P_{4102} = (5, 15, 14, 1)$
244 : $P_{3445} = (4, 6, 12, 1)$	267 : $P_{3857} = (0, 0, 14, 1)$	290 : $P_{4113} = (0, 0, 15, 1)$
245 : $P_{3463} = (6, 7, 12, 1)$	268 : $P_{3858} = (1, 0, 14, 1)$	291 : $P_{4114} = (1, 0, 15, 1)$
246 : $P_{3474} = (1, 8, 12, 1)$	269 : $P_{3875} = (2, 1, 14, 1)$	292 : $P_{4136} = (7, 1, 15, 1)$
247 : $P_{3502} = (13, 9, 12, 1)$	270 : $P_{3894} = (5, 2, 14, 1)$	293 : $P_{4159} = (14, 2, 15, 1)$
248 : $P_{3515} = (10, 10, 12, 1)$	271 : $P_{3897} = (8, 2, 14, 1)$	294 : $P_{4172} = (11, 3, 15, 1)$
249 : $P_{3532} = (11, 11, 12, 1)$	272 : $P_{3902} = (13, 2, 14, 1)$	295 : $P_{4187} = (10, 4, 15, 1)$
250 : $P_{3547} = (10, 12, 12, 1)$	273 : $P_{3922} = (1, 4, 14, 1)$	296 : $P_{4210} = (1, 6, 15, 1)$
251 : $P_{3557} = (4, 13, 12, 1)$	274 : $P_{3933} = (12, 4, 14, 1)$	297 : $P_{4250} = (9, 8, 15, 1)$
252 : $P_{3601} = (0, 0, 13, 1)$	275 : $P_{3934} = (13, 4, 14, 1)$	298 : $P_{4280} = (7, 10, 15, 1)$
253 : $P_{3602} = (1, 0, 13, 1)$	276 : $P_{3946} = (9, 5, 14, 1)$	299 : $P_{4283} = (10, 10, 15, 1)$
254 : $P_{3645} = (12, 2, 13, 1)$	277 : $P_{3956} = (3, 6, 14, 1)$	300 : $P_{4286} = (13, 10, 15, 1)$
255 : $P_{3650} = (1, 3, 13, 1)$	278 : $P_{3965} = (12, 6, 14, 1)$	301 : $P_{4300} = (11, 11, 15, 1)$
256 : $P_{3704} = (7, 6, 13, 1)$	279 : $P_{3968} = (15, 6, 14, 1)$	302 : $P_{4314} = (9, 12, 15, 1)$
257 : $P_{3727} = (14, 7, 13, 1)$	280 : $P_{4000} = (15, 8, 14, 1)$	303 : $P_{4335} = (14, 13, 15, 1)$
258 : $P_{3731} = (2, 8, 13, 1)$	281 : $P_{4003} = (2, 9, 14, 1)$	304 : $P_{4366} = (13, 15, 15, 1)$
259 : $P_{3771} = (10, 10, 13, 1)$	282 : $P_{4009} = (8, 9, 14, 1)$	