

# Rank-31 over GF(2)

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## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 = 0$$

( 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(2) is 31

## General information

Number of lines	3
Number of points	7
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$3^3$
Type of lines on points	$3, 1^6$

## Singular Points

The surface has 0 singular points:

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{32} \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{29}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \mathbf{Pl}(1, 1, 1, 1, 0, 0)_8$$

Rank of lines: ( 12, 9, 15 )

Rank of points on Klein quadric: ( 32, 29, 8 )

### Eckardt Points

The surface has 1 Eckardt points:

0 :  $P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$ .  $T = 11$

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 6 single points:

The single points on the surface are:

0 :  $P_5 = (1, 1, 0, 0)$  lies on line  $\ell_0$

1 :  $P_6 = (1, 0, 1, 0)$  lies on line  $\ell_1$

2 :  $P_7 = (0, 1, 1, 0)$  lies on line  $\ell_2$

3 :  $P_9 = (1, 0, 0, 1)$  lies on line  $\ell_2$

4 :  $P_{10} = (0, 1, 0, 1)$  lies on line  $\ell_1$

5 :  $P_{12} = (0, 0, 1, 1)$  lies on line  $\ell_0$

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

### Line Intersection Graph

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_4$	$P_4$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_4$	$P_4$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_4$	$P_4$

The surface has 7 points:

The points on the surface are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_5 = (1, 1, 0, 0)$$

$$2 : P_6 = (1, 0, 1, 0)$$

$$3 : P_7 = (0, 1, 1, 0)$$

$$4 : P_9 = (1, 0, 0, 1)$$

$$5 : P_{10} = (0, 1, 0, 1)$$

$$6 : P_{12} = (0, 0, 1, 1)$$