Rank-65665 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65665

General information

Number of lines	4
Number of points	9
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	4
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^4
Type of lines on points	2^4, 1^4, 0

Singular Points

The surface has 2 singular points:

$$0 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$$
$$1 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_8 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]_8 = \mathbf{Pl}(1,0,1,0,0,1)_{23}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{2} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{2} = \mathbf{Pl}(1,0,0,0,1,0)_{10}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{22} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]_{22} = \mathbf{Pl}(1,1,1,1,0,1)_{28}$$

Rank of lines: (8, 2, 5, 22)
Rank of points on Klein quadric: (23, 10, 12, 28)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_7 = (0,1,1,0) = \ell_0 \cap \ell_3$
$P_0 = (1,0,0,0) = \ell_1 \cap \ell_2$
$P_{11} = (1,1,0,1) = \ell_1 \cap \ell_3$
$P_{13} = (1,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0: \ P_5 = (1,1,0,0)$ lies on line ℓ_0
$1: \ P_6 = (1,0,1,0)$ lies on line ℓ_0
$2: \ P_{10} = (0,1,0,1)$ lies on line ℓ_1
$3: \ P_{12} = (0,0,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0: \ P_9 = (1,0,0,1)$

Line Intersection Graph

	0	1	2	3
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_3
in point	P_7

Line 1 intersects

Line	ℓ_2	ℓ_3
in point	P_0	P_{11}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_0	P_{13}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_7	P_{11}	P_{13}

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_5 = (1,1,0,0)$
$2: P_6 = (1,0,1,0)$
$3: P_7 = (0,1,1,0)$
$4: P_9 = (1,0,0,1)$
$5: P_{10} = (0,1,0,1)$
$6: P_{11} = (1,1,0,1)$
$7: P_{12} = (0,0,1,1)$
$8: P_{13} = (1,0,1,1)$