Rank-77 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 = 0$$

(1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 139825

General information

Number of lines	3
Number of points	241
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	48
Number of points off lines	192
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	3, 1^{48}, 0^{192}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{4369} = \mathbf{Pl}(1,1,1,1,0,0)_{64}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \delta^5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{48049} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{48049} = \mathbf{Pl}(1,1,10,1,0,0)_{199}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \delta^{10} \\ 0 & 1 & 1 & 0 \end{bmatrix}_{43681} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{43681} = \mathbf{Pl}(1,1,11,1,0,0)_{214}$$

Rank of lines: (4369, 48049, 43681)
Rank of points on Klein quadric: (64, 199, 214)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{35} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 48 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_0
$1 : P_{275} = (1,0,0,1)$ lies on line ℓ_0
$2 : P_{284} = (10,0,0,1)$ lies on line ℓ_1
$3 : P_{285} = (11,0,0,1)$ lies on line ℓ_2
$4 : P_{555} = (10,1,1,1)$ lies on line ℓ_1
$5 : P_{556} = (11,1,1,1)$ lies on line ℓ_2
$6 : P_{818} = (1,2,2,1)$ lies on line ℓ_0
$7 : P_{827} = (10,2,2,1)$ lies on line ℓ_1
$8 : P_{828} = (11,2,2,1)$ lies on line ℓ_2
$9 : P_{1090} = (1,3,3,1)$ lies on line ℓ_0
$10 : P_{1099} = (10,3,3,1)$ lies on line ℓ_1
$11 : P_{1100} = (11,3,3,1)$ lies on line ℓ_2
$12 : P_{1362} = (1,4,4,1)$ lies on line ℓ_0
$13 : P_{1371} = (10,4,4,1)$ lies on line ℓ_1
$14 : P_{1372} = (11,4,4,1)$ lies on line ℓ_2
$15 : P_{1634} = (1,5,5,1)$ lies on line ℓ_0
$16 : P_{1643} = (10,5,5,1)$ lies on line ℓ_1
$17 : P_{1644} = (11,5,5,1)$ lies on line ℓ_2
$18 : P_{1906} = (1,6,6,1)$ lies on line ℓ_0
$19 : P_{1915} = (10,6,6,1)$ lies on line ℓ_1
$20 : P_{1916} = (11,6,6,1)$ lies on line ℓ_2
$21 : P_{2178} = (1,7,7,1)$ lies on line ℓ_0
$22 : P_{2187} = (10,7,7,1)$ lies on line ℓ_1
$23 : P_{2188} = (11,7,7,1)$ lies on line ℓ_2
$24 : P_{2450} = (1,8,8,1)$ lies on line ℓ_0
$25 : P_{2459} = (10,8,8,1)$ lies on line ℓ_1
$26 : P_{2460} = (11,8,8,1)$ lies on line ℓ_2
$27 : P_{2722} = (1,9,9,1)$ lies on line ℓ_0
$28 : P_{2731} = (10,9,9,1)$ lies on line ℓ_1
$29 : P_{2732} = (11,9,9,1)$ lies on line ℓ_2
$30 : P_{2994} = (1,10,10,1)$ lies on line ℓ_0
$31 : P_{3003} = (10,10,10,1)$ lies on line ℓ_1
$32 : P_{3004} = (11,10,10,1)$ lies on line ℓ_2
$33 : P_{3266} = (1,11,11,1)$ lies on line ℓ_0
$34 : P_{3275} = (10,11,11,1)$ lies on line ℓ_1
$35 : P_{3276} = (11,11,11,1)$ lies on line ℓ_2
$36 : P_{3538} = (1,12,12,1)$ lies on line ℓ_0
$37 : P_{3547} = (10,12,12,1)$ lies on line ℓ_1
$38 : P_{3548} = (11,12,12,1)$ lies on line ℓ_2
$39 : P_{3810} = (1,13,13,1)$ lies on line ℓ_0
$40 : P_{3819} = (10,13,13,1)$ lies on line ℓ_1
$41 : P_{3820} = (11,13,13,1)$ lies on line ℓ_2
$42 : P_{4082} = (1,14,14,1)$ lies on line ℓ_0
$43 : P_{4091} = (10,14,14,1)$ lies on line ℓ_1
$44 : P_{4092} = (11,14,14,1)$ lies on line ℓ_2
$45 : P_{4354} = (1,15,15,1)$ lies on line ℓ_0
$46 : P_{4363} = (10,15,15,1)$ lies on line ℓ_1
$47 : P_{4364} = (11,15,15,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 192 points not on any line:
The points on the surface but not on lines are:

$0: P_{63} = (12, 2, 1, 0)$
$1: P_{74} = (7, 3, 1, 0)$
$2: P_{77} = (10, 3, 1, 0)$
$3: P_{82} = (15, 3, 1, 0)$
$4: P_{89} = (6, 4, 1, 0)$
$5: P_{102} = (3, 5, 1, 0)$
$6: P_{110} = (11, 5, 1, 0)$
$7: P_{111} = (12, 5, 1, 0)$
$8: P_{128} = (13, 6, 1, 0)$
$9: P_{143} = (12, 7, 1, 0)$
$10: P_{152} = (5, 8, 1, 0)$
$11: P_{153} = (6, 8, 1, 0)$
$12: P_{157} = (10, 8, 1, 0)$
$13: P_{176} = (13, 9, 1, 0)$
$14: P_{179} = (0, 10, 1, 0)$
$15: P_{190} = (11, 10, 1, 0)$
$16: P_{195} = (0, 11, 1, 0)$
$17: P_{205} = (10, 11, 1, 0)$
$18: P_{217} = (6, 12, 1, 0)$
$19: P_{234} = (7, 13, 1, 0)$
$20: P_{250} = (7, 14, 1, 0)$
$21: P_{267} = (8, 15, 1, 0)$
$22: P_{270} = (11, 15, 1, 0)$
$23: P_{272} = (13, 15, 1, 0)$
$24: P_{290} = (0, 1, 0, 1)$
$25: P_{291} = (1, 1, 0, 1)$
$26: P_{434} = (0, 10, 0, 1)$
$27: P_{444} = (10, 10, 0, 1)$
$28: P_{450} = (0, 11, 0, 1)$
$29: P_{461} = (11, 11, 0, 1)$
$30: P_{530} = (0, 0, 1, 1)$
$31: P_{531} = (1, 0, 1, 1)$
$32: P_{570} = (9, 2, 1, 1)$
$33: P_{589} = (12, 3, 1, 1)$
$34: P_{607} = (14, 4, 1, 1)$
$35: P_{615} = (6, 5, 1, 1)$
$36: P_{636} = (11, 6, 1, 1)$
$37: P_{652} = (11, 7, 1, 1)$
$38: P_{670} = (13, 8, 1, 1)$
$39: P_{675} = (2, 9, 1, 1)$
$40: P_{731} = (10, 12, 1, 1)$
$41: P_{747} = (10, 13, 1, 1)$
$42: P_{757} = (4, 14, 1, 1)$
$43: P_{776} = (7, 15, 1, 1)$
$44: P_{810} = (9, 1, 2, 1)$
$45: P_{837} = (4, 3, 2, 1)$
$46: P_{926} = (13, 8, 2, 1)$
$47: P_{930} = (1, 9, 2, 1)$
$48: P_{933} = (4, 9, 2, 1)$
$49: P_{943} = (14, 9, 2, 1)$
$50: P_{949} = (4, 10, 2, 1)$
$51: P_{962} = (1, 11, 2, 1)$
$52: P_{999} = (6, 13, 2, 1)$
$53: P_{1033} = (8, 15, 2, 1)$
$54: P_{1069} = (12, 1, 3, 1)$
$55: P_{1077} = (4, 2, 3, 1)$
$56: P_{1120} = (15, 4, 3, 1)$
$57: P_{1155} = (2, 7, 3, 1)$
$58: P_{1179} = (10, 8, 3, 1)$
$59: P_{1181} = (12, 8, 3, 1)$
$60: P_{1182} = (13, 8, 3, 1)$
$61: P_{1197} = (12, 9, 3, 1)$
$62: P_{1209} = (8, 10, 3, 1)$
$63: P_{1227} = (10, 11, 3, 1)$
$64: P_{1327} = (14, 1, 4, 1)$
$65: P_{1360} = (15, 3, 4, 1)$
$66: P_{1386} = (9, 5, 4, 1)$
$67: P_{1422} = (13, 7, 4, 1)$
$68: P_{1458} = (1, 10, 4, 1)$
$69: P_{1482} = (9, 11, 4, 1)$
$70: P_{1522} = (1, 14, 4, 1)$
$71: P_{1523} = (2, 14, 4, 1)$
$72: P_{1530} = (9, 14, 4, 1)$
$73: P_{1544} = (7, 15, 4, 1)$
$74: P_{1575} = (6, 1, 5, 1)$
$75: P_{1626} = (9, 4, 5, 1)$
$76: P_{1700} = (3, 9, 5, 1)$
$77: P_{1724} = (11, 10, 5, 1)$
$78: P_{1744} = (15, 11, 5, 1)$
$79: P_{1749} = (4, 12, 5, 1)$
$80: P_{1783} = (6, 14, 5, 1)$
$81: P_{1799} = (6, 15, 5, 1)$
$82: P_{1800} = (7, 15, 5, 1)$
$83: P_{1804} = (11, 15, 5, 1)$
$84: P_{1836} = (11, 1, 6, 1)$
$85: P_{1926} = (5, 7, 6, 1)$
$86: P_{1932} = (11, 7, 6, 1)$
$87: P_{1936} = (15, 7, 6, 1)$
$88: P_{1946} = (9, 8, 6, 1)$
$89: P_{1984} = (15, 10, 6, 1)$
$90: P_{1992} = (7, 11, 6, 1)$
$91: P_{2009} = (8, 12, 6, 1)$
$92: P_{2032} = (15, 13, 6, 1)$
$93: P_{2045} = (12, 14, 6, 1)$
$94: P_{2092} = (11, 1, 7, 1)$
$95: P_{2115} = (2, 3, 7, 1)$

96 : $P_{2142} = (13, 4, 7, 1)$
97 : $P_{2166} = (5, 6, 7, 1)$
98 : $P_{2172} = (11, 6, 7, 1)$
99 : $P_{2176} = (15, 6, 7, 1)$
100 : $P_{2230} = (5, 10, 7, 1)$
101 : $P_{2247} = (6, 11, 7, 1)$
102 : $P_{2262} = (5, 12, 7, 1)$
103 : $P_{2276} = (3, 13, 7, 1)$
104 : $P_{2350} = (13, 1, 8, 1)$
105 : $P_{2366} = (13, 2, 8, 1)$
106 : $P_{2379} = (10, 3, 8, 1)$
107 : $P_{2381} = (12, 3, 8, 1)$
108 : $P_{2382} = (13, 3, 8, 1)$
109 : $P_{2426} = (9, 6, 8, 1)$
110 : $P_{2479} = (14, 9, 8, 1)$
111 : $P_{2484} = (3, 10, 8, 1)$
112 : $P_{2507} = (10, 11, 8, 1)$
113 : $P_{2550} = (5, 14, 8, 1)$
114 : $P_{2595} = (2, 1, 9, 1)$
115 : $P_{2610} = (1, 2, 9, 1)$
116 : $P_{2613} = (4, 2, 9, 1)$
117 : $P_{2623} = (14, 2, 9, 1)$
118 : $P_{2637} = (12, 3, 9, 1)$
119 : $P_{2660} = (3, 5, 9, 1)$
120 : $P_{2719} = (14, 8, 9, 1)$
121 : $P_{2751} = (14, 10, 9, 1)$
122 : $P_{2754} = (1, 11, 9, 1)$
123 : $P_{2776} = (7, 12, 9, 1)$
124 : $P_{2833} = (0, 0, 10, 1)$
125 : $P_{2843} = (10, 0, 10, 1)$
126 : $P_{2869} = (4, 2, 10, 1)$
127 : $P_{2889} = (8, 3, 10, 1)$
128 : $P_{2898} = (1, 4, 10, 1)$
129 : $P_{2924} = (11, 5, 10, 1)$
130 : $P_{2944} = (15, 6, 10, 1)$
131 : $P_{2950} = (5, 7, 10, 1)$
132 : $P_{2964} = (3, 8, 10, 1)$
133 : $P_{2991} = (14, 9, 10, 1)$
134 : $P_{3038} = (13, 12, 10, 1)$
135 : $P_{3053} = (12, 13, 10, 1)$
136 : $P_{3058} = (1, 14, 10, 1)$
137 : $P_{3084} = (11, 15, 10, 1)$
138 : $P_{3089} = (0, 0, 11, 1)$
139 : $P_{3100} = (11, 0, 11, 1)$
140 : $P_{3122} = (1, 2, 11, 1)$
141 : $P_{3147} = (10, 3, 11, 1)$
142 : $P_{3162} = (9, 4, 11, 1)$
143 : $P_{3184} = (15, 5, 11, 1)$
144 : $P_{3192} = (7, 6, 11, 1)$
145 : $P_{3207} = (6, 7, 11, 1)$
146 : $P_{3227} = (10, 8, 11, 1)$
147 : $P_{3234} = (1, 9, 11, 1)$
148 : $P_{3289} = (8, 12, 11, 1)$
149 : $P_{3300} = (3, 13, 11, 1)$
150 : $P_{3315} = (2, 14, 11, 1)$
151 : $P_{3334} = (5, 15, 11, 1)$
152 : $P_{3371} = (10, 1, 12, 1)$
153 : $P_{3429} = (4, 5, 12, 1)$
154 : $P_{3449} = (8, 6, 12, 1)$
155 : $P_{3462} = (5, 7, 12, 1)$
156 : $P_{3496} = (7, 9, 12, 1)$
157 : $P_{3518} = (13, 10, 12, 1)$
158 : $P_{3529} = (8, 11, 12, 1)$
159 : $P_{3556} = (3, 13, 12, 1)$
160 : $P_{3561} = (8, 13, 12, 1)$
161 : $P_{3563} = (10, 13, 12, 1)$
162 : $P_{3627} = (10, 1, 13, 1)$
163 : $P_{3639} = (6, 2, 13, 1)$
164 : $P_{3712} = (15, 6, 13, 1)$
165 : $P_{3716} = (3, 7, 13, 1)$
166 : $P_{3773} = (12, 10, 13, 1)$
167 : $P_{3780} = (3, 11, 13, 1)$
168 : $P_{3796} = (3, 12, 13, 1)$
169 : $P_{3801} = (8, 12, 13, 1)$
170 : $P_{3803} = (10, 12, 13, 1)$
171 : $P_{3855} = (14, 15, 13, 1)$
172 : $P_{3877} = (4, 1, 14, 1)$
173 : $P_{3922} = (1, 4, 14, 1)$
174 : $P_{3923} = (2, 4, 14, 1)$
175 : $P_{3930} = (9, 4, 14, 1)$
176 : $P_{3943} = (6, 5, 14, 1)$
177 : $P_{3965} = (12, 6, 14, 1)$
178 : $P_{3990} = (5, 8, 14, 1)$
179 : $P_{4018} = (1, 10, 14, 1)$
180 : $P_{4035} = (2, 11, 14, 1)$
181 : $P_{4099} = (2, 15, 14, 1)$
182 : $P_{4136} = (7, 1, 15, 1)$
183 : $P_{4153} = (8, 2, 15, 1)$
184 : $P_{4184} = (7, 4, 15, 1)$
185 : $P_{4199} = (6, 5, 15, 1)$
186 : $P_{4200} = (7, 5, 15, 1)$
187 : $P_{4204} = (11, 5, 15, 1)$
188 : $P_{4284} = (11, 10, 15, 1)$
189 : $P_{4294} = (5, 11, 15, 1)$
190 : $P_{4335} = (14, 13, 15, 1)$
191 : $P_{4339} = (2, 14, 15, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{35}	P_{35}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{35}	P_{35}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{35}	P_{35}

The surface has 241 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_{35} = (0, 1, 1, 0)$
2 : $P_{63} = (12, 2, 1, 0)$
3 : $P_{74} = (7, 3, 1, 0)$
4 : $P_{77} = (10, 3, 1, 0)$
5 : $P_{82} = (15, 3, 1, 0)$
6 : $P_{89} = (6, 4, 1, 0)$
7 : $P_{102} = (3, 5, 1, 0)$
8 : $P_{110} = (11, 5, 1, 0)$
9 : $P_{111} = (12, 5, 1, 0)$
10 : $P_{128} = (13, 6, 1, 0)$
11 : $P_{143} = (12, 7, 1, 0)$
12 : $P_{152} = (5, 8, 1, 0)$
13 : $P_{153} = (6, 8, 1, 0)$
14 : $P_{157} = (10, 8, 1, 0)$
15 : $P_{176} = (13, 9, 1, 0)$
16 : $P_{179} = (0, 10, 1, 0)$
17 : $P_{190} = (11, 10, 1, 0)$
18 : $P_{195} = (0, 11, 1, 0)$
19 : $P_{205} = (10, 11, 1, 0)$
20 : $P_{217} = (6, 12, 1, 0)$
21 : $P_{234} = (7, 13, 1, 0)$
22 : $P_{250} = (7, 14, 1, 0)$
23 : $P_{267} = (8, 15, 1, 0)$
24 : $P_{270} = (11, 15, 1, 0)$
25 : $P_{272} = (13, 15, 1, 0)$
26 : $P_{275} = (1, 0, 0, 1)$
27 : $P_{284} = (10, 0, 0, 1)$
28 : $P_{285} = (11, 0, 0, 1)$
29 : $P_{290} = (0, 1, 0, 1)$
30 : $P_{291} = (1, 1, 0, 1)$
31 : $P_{434} = (0, 10, 0, 1)$
32 : $P_{444} = (10, 10, 0, 1)$
33 : $P_{450} = (0, 11, 0, 1)$
34 : $P_{461} = (11, 11, 0, 1)$
35 : $P_{530} = (0, 0, 1, 1)$
36 : $P_{531} = (1, 0, 1, 1)$
37 : $P_{555} = (10, 1, 1, 1)$
38 : $P_{556} = (11, 1, 1, 1)$
39 : $P_{570} = (9, 2, 1, 1)$
40 : $P_{589} = (12, 3, 1, 1)$
41 : $P_{607} = (14, 4, 1, 1)$
42 : $P_{615} = (6, 5, 1, 1)$
43 : $P_{636} = (11, 6, 1, 1)$
44 : $P_{652} = (11, 7, 1, 1)$
45 : $P_{670} = (13, 8, 1, 1)$
46 : $P_{675} = (2, 9, 1, 1)$
47 : $P_{731} = (10, 12, 1, 1)$
48 : $P_{747} = (10, 13, 1, 1)$
49 : $P_{757} = (4, 14, 1, 1)$
50 : $P_{776} = (7, 15, 1, 1)$
51 : $P_{810} = (9, 1, 2, 1)$
52 : $P_{818} = (1, 2, 2, 1)$
53 : $P_{827} = (10, 2, 2, 1)$
54 : $P_{828} = (11, 2, 2, 1)$
55 : $P_{837} = (4, 3, 2, 1)$
56 : $P_{926} = (13, 8, 2, 1)$
57 : $P_{930} = (1, 9, 2, 1)$
58 : $P_{933} = (4, 9, 2, 1)$
59 : $P_{943} = (14, 9, 2, 1)$
60 : $P_{949} = (4, 10, 2, 1)$
61 : $P_{962} = (1, 11, 2, 1)$
62 : $P_{999} = (6, 13, 2, 1)$
63 : $P_{1033} = (8, 15, 2, 1)$
64 : $P_{1069} = (12, 1, 3, 1)$
65 : $P_{1077} = (4, 2, 3, 1)$
66 : $P_{1090} = (1, 3, 3, 1)$
67 : $P_{1099} = (10, 3, 3, 1)$
68 : $P_{1100} = (11, 3, 3, 1)$
69 : $P_{1120} = (15, 4, 3, 1)$
70 : $P_{1155} = (2, 7, 3, 1)$
71 : $P_{1179} = (10, 8, 3, 1)$
72 : $P_{1181} = (12, 8, 3, 1)$
73 : $P_{1182} = (13, 8, 3, 1)$
74 : $P_{1197} = (12, 9, 3, 1)$
75 : $P_{1209} = (8, 10, 3, 1)$
76 : $P_{1227} = (10, 11, 3, 1)$
77 : $P_{1327} = (14, 1, 4, 1)$
78 : $P_{1360} = (15, 3, 4, 1)$
79 : $P_{1362} = (1, 4, 4, 1)$
80 : $P_{1371} = (10, 4, 4, 1)$
81 : $P_{1372} = (11, 4, 4, 1)$
82 : $P_{1386} = (9, 5, 4, 1)$
83 : $P_{1422} = (13, 7, 4, 1)$
84 : $P_{1458} = (1, 10, 4, 1)$
85 : $P_{1482} = (9, 11, 4, 1)$
86 : $P_{1522} = (1, 14, 4, 1)$
87 : $P_{1523} = (2, 14, 4, 1)$
88 : $P_{1530} = (9, 14, 4, 1)$
89 : $P_{1544} = (7, 15, 4, 1)$
90 : $P_{1575} = (6, 1, 5, 1)$
91 : $P_{1626} = (9, 4, 5, 1)$
92 : $P_{1634} = (1, 5, 5, 1)$
93 : $P_{1643} = (10, 5, 5, 1)$
94 : $P_{1644} = (11, 5, 5, 1)$
95 : $P_{1700} = (3, 9, 5, 1)$
96 : $P_{1724} = (11, 10, 5, 1)$
97 : $P_{1744} = (15, 11, 5, 1)$
98 : $P_{1749} = (4, 12, 5, 1)$

99 : $P_{1783} = (6, 14, 5, 1)$
100 : $P_{1799} = (6, 15, 5, 1)$
101 : $P_{1800} = (7, 15, 5, 1)$
102 : $P_{1804} = (11, 15, 5, 1)$
103 : $P_{1836} = (11, 1, 6, 1)$
104 : $P_{1906} = (1, 6, 6, 1)$
105 : $P_{1915} = (10, 6, 6, 1)$
106 : $P_{1916} = (11, 6, 6, 1)$
107 : $P_{1926} = (5, 7, 6, 1)$
108 : $P_{1932} = (11, 7, 6, 1)$
109 : $P_{1936} = (15, 7, 6, 1)$
110 : $P_{1946} = (9, 8, 6, 1)$
111 : $P_{1984} = (15, 10, 6, 1)$
112 : $P_{1992} = (7, 11, 6, 1)$
113 : $P_{2009} = (8, 12, 6, 1)$
114 : $P_{2032} = (15, 13, 6, 1)$
115 : $P_{2045} = (12, 14, 6, 1)$
116 : $P_{2092} = (11, 1, 7, 1)$
117 : $P_{2115} = (2, 3, 7, 1)$
118 : $P_{2142} = (13, 4, 7, 1)$
119 : $P_{2166} = (5, 6, 7, 1)$
120 : $P_{2172} = (11, 6, 7, 1)$
121 : $P_{2176} = (15, 6, 7, 1)$
122 : $P_{2178} = (1, 7, 7, 1)$
123 : $P_{2187} = (10, 7, 7, 1)$
124 : $P_{2188} = (11, 7, 7, 1)$
125 : $P_{2230} = (5, 10, 7, 1)$
126 : $P_{2247} = (6, 11, 7, 1)$
127 : $P_{2262} = (5, 12, 7, 1)$
128 : $P_{2276} = (3, 13, 7, 1)$
129 : $P_{2350} = (13, 1, 8, 1)$
130 : $P_{2366} = (13, 2, 8, 1)$
131 : $P_{2379} = (10, 3, 8, 1)$
132 : $P_{2381} = (12, 3, 8, 1)$
133 : $P_{2382} = (13, 3, 8, 1)$
134 : $P_{2426} = (9, 6, 8, 1)$
135 : $P_{2450} = (1, 8, 8, 1)$
136 : $P_{2459} = (10, 8, 8, 1)$
137 : $P_{2460} = (11, 8, 8, 1)$
138 : $P_{2479} = (14, 9, 8, 1)$
139 : $P_{2484} = (3, 10, 8, 1)$
140 : $P_{2507} = (10, 11, 8, 1)$
141 : $P_{2550} = (5, 14, 8, 1)$
142 : $P_{2595} = (2, 1, 9, 1)$
143 : $P_{2610} = (1, 2, 9, 1)$
144 : $P_{2613} = (4, 2, 9, 1)$
145 : $P_{2623} = (14, 2, 9, 1)$
146 : $P_{2637} = (12, 3, 9, 1)$
147 : $P_{2660} = (3, 5, 9, 1)$
148 : $P_{2719} = (14, 8, 9, 1)$
149 : $P_{2722} = (1, 9, 9, 1)$
150 : $P_{2731} = (10, 9, 9, 1)$
151 : $P_{2732} = (11, 9, 9, 1)$
152 : $P_{2751} = (14, 10, 9, 1)$
153 : $P_{2754} = (1, 11, 9, 1)$
154 : $P_{2776} = (7, 12, 9, 1)$
155 : $P_{2833} = (0, 0, 10, 1)$
156 : $P_{2843} = (10, 0, 10, 1)$
157 : $P_{2869} = (4, 2, 10, 1)$
158 : $P_{2889} = (8, 3, 10, 1)$
159 : $P_{2898} = (1, 4, 10, 1)$
160 : $P_{2924} = (11, 5, 10, 1)$
161 : $P_{2944} = (15, 6, 10, 1)$
162 : $P_{2950} = (5, 7, 10, 1)$
163 : $P_{2964} = (3, 8, 10, 1)$
164 : $P_{2991} = (14, 9, 10, 1)$
165 : $P_{2994} = (1, 10, 10, 1)$
166 : $P_{3003} = (10, 10, 10, 1)$
167 : $P_{3004} = (11, 10, 10, 1)$
168 : $P_{3038} = (13, 12, 10, 1)$
169 : $P_{3053} = (12, 13, 10, 1)$
170 : $P_{3058} = (1, 14, 10, 1)$
171 : $P_{3084} = (11, 15, 10, 1)$
172 : $P_{3089} = (0, 0, 11, 1)$
173 : $P_{3100} = (11, 0, 11, 1)$
174 : $P_{3122} = (1, 2, 11, 1)$
175 : $P_{3147} = (10, 3, 11, 1)$
176 : $P_{3162} = (9, 4, 11, 1)$
177 : $P_{3184} = (15, 5, 11, 1)$
178 : $P_{3192} = (7, 6, 11, 1)$
179 : $P_{3207} = (6, 7, 11, 1)$
180 : $P_{3227} = (10, 8, 11, 1)$
181 : $P_{3234} = (1, 9, 11, 1)$
182 : $P_{3266} = (1, 11, 11, 1)$
183 : $P_{3275} = (10, 11, 11, 1)$
184 : $P_{3276} = (11, 11, 11, 1)$
185 : $P_{3289} = (8, 12, 11, 1)$
186 : $P_{3300} = (3, 13, 11, 1)$
187 : $P_{3315} = (2, 14, 11, 1)$
188 : $P_{3334} = (5, 15, 11, 1)$
189 : $P_{3371} = (10, 1, 12, 1)$
190 : $P_{3429} = (4, 5, 12, 1)$
191 : $P_{3449} = (8, 6, 12, 1)$
192 : $P_{3462} = (5, 7, 12, 1)$
193 : $P_{3496} = (7, 9, 12, 1)$
194 : $P_{3518} = (13, 10, 12, 1)$
195 : $P_{3529} = (8, 11, 12, 1)$
196 : $P_{3538} = (1, 12, 12, 1)$
197 : $P_{3547} = (10, 12, 12, 1)$
198 : $P_{3548} = (11, 12, 12, 1)$
199 : $P_{3556} = (3, 13, 12, 1)$
200 : $P_{3561} = (8, 13, 12, 1)$
201 : $P_{3563} = (10, 13, 12, 1)$
202 : $P_{3627} = (10, 1, 13, 1)$
203 : $P_{3639} = (6, 2, 13, 1)$
204 : $P_{3712} = (15, 6, 13, 1)$
205 : $P_{3716} = (3, 7, 13, 1)$
206 : $P_{3773} = (12, 10, 13, 1)$
207 : $P_{3780} = (3, 11, 13, 1)$
208 : $P_{3796} = (3, 12, 13, 1)$
209 : $P_{3801} = (8, 12, 13, 1)$
210 : $P_{3803} = (10, 12, 13, 1)$
211 : $P_{3810} = (1, 13, 13, 1)$
212 : $P_{3819} = (10, 13, 13, 1)$
213 : $P_{3820} = (11, 13, 13, 1)$
214 : $P_{3855} = (14, 15, 13, 1)$
215 : $P_{3877} = (4, 1, 14, 1)$
216 : $P_{3922} = (1, 4, 14, 1)$
217 : $P_{3923} = (2, 4, 14, 1)$
218 : $P_{3930} = (9, 4, 14, 1)$
219 : $P_{3943} = (6, 5, 14, 1)$
220 : $P_{3965} = (12, 6, 14, 1)$
221 : $P_{3990} = (5, 8, 14, 1)$
222 : $P_{4018} = (1, 10, 14, 1)$
223 : $P_{4035} = (2, 11, 14, 1)$
224 : $P_{4082} = (1, 14, 14, 1)$
225 : $P_{4091} = (10, 14, 14, 1)$
226 : $P_{4092} = (11, 14, 14, 1)$
227 : $P_{4099} = (2, 15, 14, 1)$
228 : $P_{4136} = (7, 1, 15, 1)$
229 : $P_{4153} = (8, 2, 15, 1)$
230 : $P_{4184} = (7, 4, 15, 1)$
231 : $P_{4199} = (6, 5, 15, 1)$
232 : $P_{4200} = (7, 5, 15, 1)$
233 : $P_{4204} = (11, 5, 15, 1)$
234 : $P_{4284} = (11, 10, 15, 1)$
235 : $P_{4294} = (5, 11, 15, 1)$
236 : $P_{4335} = (14, 13, 15, 1)$
237 : $P_{4339} = (2, 14, 15, 1)$
238 : $P_{4354} = (1, 15, 15, 1)$
239 : $P_{4363} = (10, 15, 15, 1)$
240 : $P_{4364} = (11, 15, 15, 1)$