Rank-65744 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431676330

General information

Number of lines	9
Number of points	33
Number of singular points	0
Number of Eckardt points	1
Number of double points	15
Number of single points	12
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	3, 2^{15}, 1^{12}, 0^5

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1,0,1,1,1,1)_{199}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{35} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{35} = \mathbf{Pl}(2,3,3,0,3,1)_{307}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{32} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{32} = \mathbf{Pl}(3,2,2,0,2,1)_{245}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{133} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{133} = \mathbf{Pl}(2,3,2,3,3,1)_{346}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 1 \end{bmatrix}_{153} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{153} = \mathbf{Pl}(3,2,3,2,2,1)_{296}$$

Rank of lines: (337, 339, 338, 38, 110, 35, 32, 133, 153)
Rank of points on Klein quadric: (129, 143, 136, 198, 199, 307, 245, 346, 296)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{38} = (0,0,1,1) = \ell_0 \cap \ell_3$
$P_{42} = (0,1,1,1) = \ell_0 \cap \ell_4$
$P_{65} = (0,3,2,1) = \ell_1 \cap \ell_6$
$P_{57} = (0,1,2,1) = \ell_1 \cap \ell_8$
$P_{77} = (0,2,3,1) = \ell_2 \cap \ell_5$
$P_{73} = (0,1,3,1) = \ell_2 \cap \ell_7$
$P_5 = (1,1,0,0) = \ell_3 \cap \ell_4$
$P_{47} = (2,2,1,1) = \ell_3 \cap \ell_5$
$P_{52} = (3,3,1,1) = \ell_3 \cap \ell_6$
$P_{51} = (2,3,1,1) = \ell_4 \cap \ell_7$
$P_{48} = (3,2,1,1) = \ell_4 \cap \ell_8$
$P_8 = (1,0,1,0) = \ell_5 \cap \ell_6$
$P_{33} = (3,2,0,1) = \ell_5 \cap \ell_7$
$P_{36} = (2,3,0,1) = \ell_6 \cap \ell_8$
$P_{12} = (1,1,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 12 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_3
$1 : P_{39} = (1,0,1,1)$ lies on line ℓ_4
$2 : P_{45} = (0,2,1,1)$ lies on line ℓ_0
$3 : P_{49} = (0,3,1,1)$ lies on line ℓ_0
$4 : P_{53} = (0,0,2,1)$ lies on line ℓ_1
$5 : P_{54} = (1,0,2,1)$ lies on line ℓ_7
$6 : P_{61} = (0,2,2,1)$ lies on line ℓ_1
$7 : P_{62} = (1,2,2,1)$ lies on line ℓ_5
$8 : P_{69} = (0,0,3,1)$ lies on line ℓ_2
$9 : P_{70} = (1,0,3,1)$ lies on line ℓ_8
$10 : P_{81} = (0,3,3,1)$ lies on line ℓ_2
$11 : P_{82} = (1,3,3,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 5 points not on any line:
The points on the surface but not on lines are:

$0:\ P_9 = (2,0,1,0)$
$1:\ P_{10} = (3,0,1,0)$
$2:\ P_{13} = (2,1,1,0)$
$3:\ P_{14} = (3,1,1,0)$
$4:\ P_{27} = (1,1,0,1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	1	0	0	0	0
1	1	0	1	0	0	0	1	0	1
2	1	1	0	0	0	1	0	1	0
3	1	0	0	0	1	1	1	0	0
4	1	0	0	1	0	0	0	1	1
5	0	0	1	1	0	0	1	1	0
6	0	1	0	1	0	1	0	0	1
7	0	0	1	0	1	1	0	0	1
8	0	1	0	0	1	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_1	P_1	P_{38}	P_{42}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8
in point	P_1	P_1	P_{65}	P_{57}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_1	P_1	P_{77}	P_{73}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6
in point	P_{38}	P_5	P_{47}	P_{52}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8
in point	P_{42}	P_5	P_{51}	P_{48}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{77}	P_{47}	P_8	P_{33}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8
in point	P_{65}	P_{52}	P_8	P_{36}

Line 7 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_{73}	P_{51}	P_{33}	P_{12}

Line 8 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_{57}	P_{48}	P_{36}	P_{12}

The surface has 33 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_4 = (1,1,1,1)$
$2: P_5 = (1,1,0,0)$
$3: P_8 = (1,0,1,0)$
$4: P_9 = (2,0,1,0)$
$5: P_{10} = (3,0,1,0)$
$6: P_{12} = (1,1,1,0)$
$7: P_{13} = (2,1,1,0)$
$8: P_{14} = (3,1,1,0)$
$9: P_{27} = (1,1,0,1)$
$10: P_{33} = (3,2,0,1)$
$11: P_{36} = (2,3,0,1)$
$12: P_{38} = (0,0,1,1)$
$13: P_{39} = (1,0,1,1)$
$14: P_{42} = (0,1,1,1)$
$15: P_{45} = (0,2,1,1)$
$16: P_{47} = (2,2,1,1)$
$17: P_{48} = (3,2,1,1)$
$18: P_{49} = (0,3,1,1)$
$19: P_{51} = (2,3,1,1)$
$20: P_{52} = (3,3,1,1)$
$21: P_{53} = (0,0,2,1)$
$22: P_{54} = (1,0,2,1)$
$23: P_{57} = (0,1,2,1)$
$24: P_{61} = (0,2,2,1)$
$25: P_{62} = (1,2,2,1)$
$26: P_{65} = (0,3,2,1)$
$27: P_{69} = (0,0,3,1)$
$28: P_{70} = (1,0,3,1)$
$29: P_{73} = (0,1,3,1)$
$30: P_{77} = (0,2,3,1)$
$31: P_{81} = (0,3,3,1)$
$32: P_{82} = (1,3,3,1)$