Rank-73753 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2X_1 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286396965

General information

Number of lines	9
Number of points	305
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	126
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^5, 2^6, 1^{126}, 0^{168}

Singular Points

The surface has 4 singular points:

$0:\ P_{985} = \mathbf{P}(\delta^3, \delta^{14}, \delta, 1) = \mathbf{P}(8, 12, 2, 1)$
$1:\ P_{1408} = \mathbf{P}(\delta^6, \delta^{13}, \delta^2, 1) = \mathbf{P}(15, 6, 4, 1)$
$2:\ P_{2788} = \mathbf{P}(\delta^{12}, \delta^{11}, \delta^4, 1) = \mathbf{P}(3, 13, 9, 1)$
$3:\ P_{3974} = \mathbf{P}(\delta^9, \delta^7, \delta^8, 1) = \mathbf{P}(5, 7, 14, 1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69921} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{49}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \delta^4 & \delta^6 \\ 0 & 1 & \delta^{10} & \delta^3 \end{bmatrix}_{68115} = \begin{bmatrix} 1 & 0 & 9 & 15 \\ 0 & 1 & 10 & 8 \end{bmatrix}_{68115} = \mathbf{Pl}(10,11,6,4,12,1)_{55875}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \delta^8 & \delta^{12} \\ 0 & 1 & \delta^5 & \delta^6 \end{bmatrix}_{17177} = \begin{bmatrix} 1 & 0 & 14 & 3 \\ 0 & 1 & 11 & 15 \end{bmatrix}_{17177} = \mathbf{Pl}(11,10,13,9,6,1)_{32911}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \delta & \delta^9 \\ 0 & 1 & \delta^{10} & \delta^{12} \end{bmatrix}_{22444} = \begin{bmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & 10 & 3 \end{bmatrix}_{22444} = \mathbf{Pl}(10,11,7,14,13,1)_{60315}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & \delta^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \begin{bmatrix} 0 & 1 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70091} = \mathbf{Pl}(0,11,0,1,0,0)_{59}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \delta^5 & 1 \\ 0 & 1 & 1 & \delta^5 \end{bmatrix}_{7548} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 1 & 11 \end{bmatrix}_{7548} = \mathbf{Pl}(1,1,11,10,1,1)_{12126}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & \delta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \begin{bmatrix} 0 & 1 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70074} = \mathbf{Pl}(0,10,0,1,0,0)_{58}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & 1 & \delta^{10} \end{bmatrix}_{7259} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 1 & 10 \end{bmatrix}_{7259} = \mathbf{Pl}(1,1,10,11,1,1)_{11901}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^2 & \delta^3 \\ 0 & 1 & \delta^5 & \delta^9 \end{bmatrix}_{36127} = \begin{bmatrix} 1 & 0 & 4 & 8 \\ 0 & 1 & 11 & 5 \end{bmatrix}_{36127} = \mathbf{Pl}(11,10,12,2,7,1)_{36706}$$

Rank of lines: (69921, 68115, 17177, 22444, 70091, 7548, 70074, 7259, 36127)
Rank of points on Klein quadric: (49, 55875, 32911, 60315, 59, 12126, 58, 11901, 36706)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$1 : P_{985} = \mathbf{P}(\delta^3, \delta^{14}, \delta, 1) = \mathbf{P}(8,12,2,1)$,
$2 : P_{1408} = \mathbf{P}(\delta^6, \delta^{13}, \delta^2, 1) = \mathbf{P}(15,6,4,1)$,
$3 : P_{2788} = \mathbf{P}(\delta^{12}, \delta^{11}, \delta^4, 1) = \mathbf{P}(3,13,9,1)$,
$4 : P_{3974} = \mathbf{P}(\delta^9, \delta^7, \delta^8, 1) = \mathbf{P}(5,7,14,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{2993} = (0,10,10,1) = \ell_0 \cap \ell_5$
$P_{3265} = (0,11,11,1) = \ell_0 \cap \ell_7$
$P_{2113} = (0,3,7,1) = \ell_1 \cap \ell_6$
$P_{3425} = (0,5,12,1) = \ell_2 \cap \ell_4$
$P_{1937} = (0,8,6,1) = \ell_3 \cap \ell_6$
$P_{3841} = (0,15,13,1) = \ell_4 \cap \ell_8$

Single Points

The surface has 126 single points:
The single points on the surface are:

$0 :\ P_{35} = (0,1,1,0)$ lies on line ℓ_0
$1 :\ P_{72} = (5,3,1,0)$ lies on line ℓ_1
$2 :\ P_{107} = (8,5,1,0)$ lies on line ℓ_2
$3 :\ P_{162} = (15,8,1,0)$ lies on line ℓ_3
$4 :\ P_{179} = (0,10,1,0)$ lies on line ℓ_4
$5 :\ P_{180} = (1,10,1,0)$ lies on line ℓ_5

6 : $P_{195} = (0, 11, 1, 0)$ lies on line ℓ_6
7 : $P_{196} = (1, 11, 1, 0)$ lies on line ℓ_7
8 : $P_{262} = (3, 15, 1, 0)$ lies on line ℓ_8
9 : $P_{300} = (10, 1, 0, 1)$ lies on line ℓ_5
10 : $P_{301} = (11, 1, 0, 1)$ lies on line ℓ_7
11 : $P_{436} = (2, 10, 0, 1)$ lies on line ℓ_1
12 : $P_{443} = (9, 10, 0, 1)$ lies on line ℓ_3
13 : $P_{454} = (4, 11, 0, 1)$ lies on line ℓ_2
14 : $P_{464} = (14, 11, 0, 1)$ lies on line ℓ_8
15 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_0
16 : $P_{567} = (6, 2, 1, 1)$ lies on line ℓ_3
17 : $P_{606} = (13, 4, 1, 1)$ lies on line ℓ_8
18 : $P_{680} = (7, 9, 1, 1)$ lies on line ℓ_1
19 : $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_4
20 : $P_{699} = (10, 10, 1, 1)$ lies on line ℓ_7
21 : $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_6
22 : $P_{716} = (11, 11, 1, 1)$ lies on line ℓ_5
23 : $P_{765} = (12, 14, 1, 1)$ lies on line ℓ_2
24 : $P_{814} = (13, 1, 2, 1)$ lies on line ℓ_2
25 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_0
26 : $P_{847} = (14, 3, 2, 1)$ lies on line ℓ_3
27 : $P_{993} = (0, 13, 2, 1)$ lies on line ℓ_4
28 : $P_{1018} = (9, 14, 2, 1)$ lies on line ℓ_7
29 : $P_{1025} = (0, 15, 2, 1)$ lies on line ℓ_6
30 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_0
31 : $P_{1100} = (11, 3, 3, 1)$ lies on line ℓ_8
32 : $P_{1105} = (0, 4, 3, 1)$ lies on line ℓ_6
33 : $P_{1110} = (5, 4, 3, 1)$ lies on line ℓ_2
34 : $P_{1129} = (8, 5, 3, 1)$ lies on line ℓ_7
35 : $P_{1146} = (9, 6, 3, 1)$ lies on line ℓ_5
36 : $P_{1153} = (0, 7, 3, 1)$ lies on line ℓ_4
37 : $P_{1218} = (1, 11, 3, 1)$ lies on line ℓ_3
38 : $P_{1294} = (13, 15, 3, 1)$ lies on line ℓ_1
39 : $P_{1320} = (7, 1, 4, 1)$ lies on line ℓ_3
40 : $P_{1343} = (14, 2, 4, 1)$ lies on line ℓ_5
41 : $P_{1345} = (0, 3, 4, 1)$ lies on line ℓ_4
42 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_0
43 : $P_{1379} = (2, 5, 4, 1)$ lies on line ℓ_8
44 : $P_{1409} = (0, 7, 4, 1)$ lies on line ℓ_6
45 : $P_{1608} = (7, 3, 5, 1)$ lies on line ℓ_2
46 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_0
47 : $P_{1643} = (10, 5, 5, 1)$ lies on line ℓ_1
48 : $P_{1696} = (15, 8, 5, 1)$ lies on line ℓ_5
49 : $P_{1697} = (0, 9, 5, 1)$ lies on line ℓ_4
50 : $P_{1705} = (8, 9, 5, 1)$ lies on line ℓ_3
51 : $P_{1714} = (1, 10, 5, 1)$ lies on line ℓ_8
52 : $P_{1745} = (0, 12, 5, 1)$ lies on line ℓ_6
53 : $P_{1775} = (14, 13, 5, 1)$ lies on line ℓ_7
54 : $P_{1814} = (5, 0, 6, 1)$ lies on line ℓ_1
55 : $P_{1845} = (4, 2, 6, 1)$ lies on line ℓ_8
56 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_0
57 : $P_{1966} = (13, 9, 6, 1)$ lies on line ℓ_7
58 : $P_{2007} = (6, 12, 6, 1)$ lies on line ℓ_2
59 : $P_{2033} = (0, 14, 6, 1)$ lies on line ℓ_4
60 : $P_{2061} = (12, 15, 6, 1)$ lies on line ℓ_5
61 : $P_{2080} = (15, 0, 7, 1)$ lies on line ℓ_3
62 : $P_{2109} = (12, 2, 7, 1)$ lies on line ℓ_7
63 : $P_{2129} = (0, 4, 7, 1)$ lies on line ℓ_4
64 : $P_{2158} = (13, 5, 7, 1)$ lies on line ℓ_5
65 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_0
66 : $P_{2223} = (14, 9, 7, 1)$ lies on line ℓ_2
67 : $P_{2280} = (7, 13, 7, 1)$ lies on line ℓ_8
68 : $P_{2413} = (12, 5, 8, 1)$ lies on line ℓ_3
69 : $P_{2417} = (0, 6, 8, 1)$ lies on line ℓ_4
70 : $P_{2435} = (2, 7, 8, 1)$ lies on line ℓ_5
71 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_0
72 : $P_{2460} = (11, 8, 8, 1)$ lies on line ℓ_2
73 : $P_{2498} = (1, 11, 8, 1)$ lies on line ℓ_1
74 : $P_{2545} = (0, 14, 8, 1)$ lies on line ℓ_6
75 : $P_{2560} = (15, 14, 8, 1)$ lies on line ℓ_8
76 : $P_{2564} = (3, 15, 8, 1)$ lies on line ℓ_7
77 : $P_{2605} = (12, 1, 9, 1)$ lies on line ℓ_8
78 : $P_{2643} = (2, 4, 9, 1)$ lies on line ℓ_7
79 : $P_{2657} = (0, 5, 9, 1)$ lies on line ℓ_6
80 : $P_{2709} = (4, 8, 9, 1)$ lies on line ℓ_1
81 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_0
82 : $P_{2769} = (0, 12, 9, 1)$ lies on line ℓ_4
83 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_7
84 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_6
85 : $P_{2867} = (2, 2, 10, 1)$ lies on line ℓ_2
86 : $P_{2986} = (9, 9, 10, 1)$ lies on line ℓ_8
87 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_4
88 : $P_{3036} = (11, 12, 10, 1)$ lies on line ℓ_3
89 : $P_{3052} = (11, 13, 10, 1)$ lies on line ℓ_1
90 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_5
91 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_4
92 : $P_{3157} = (4, 4, 11, 1)$ lies on line ℓ_3
93 : $P_{3195} = (10, 6, 11, 1)$ lies on line ℓ_8
94 : $P_{3211} = (10, 7, 11, 1)$ lies on line ℓ_2
95 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_6
96 : $P_{3327} = (14, 14, 11, 1)$ lies on line ℓ_1
97 : $P_{3348} = (3, 0, 12, 1)$ lies on line ℓ_8
98 : $P_{3415} = (6, 4, 12, 1)$ lies on line ℓ_5
99 : $P_{3469} = (12, 7, 12, 1)$ lies on line ℓ_1
100 : $P_{3480} = (7, 8, 12, 1)$ lies on line ℓ_7
101 : $P_{3489} = (0, 9, 12, 1)$ lies on line ℓ_6
102 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_0
103 : $P_{3571} = (2, 14, 12, 1)$ lies on line ℓ_3
104 : $P_{3609} = (8, 0, 13, 1)$ lies on line ℓ_2
105 : $P_{3633} = (0, 2, 13, 1)$ lies on line ℓ_6
106 : $P_{3655} = (6, 3, 13, 1)$ lies on line ℓ_7
107 : $P_{3674} = (9, 4, 13, 1)$ lies on line ℓ_1
108 : $P_{3710} = (13, 6, 13, 1)$ lies on line ℓ_3
109 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_0
110 : $P_{3832} = (7, 14, 13, 1)$ lies on line ℓ_5
111 : $P_{3879} = (6, 1, 14, 1)$ lies on line ℓ_1
112 : $P_{3953} = (0, 6, 14, 1)$ lies on line ℓ_6
113 : $P_{3985} = (0, 8, 14, 1)$ lies on line ℓ_4

114 : $P_{4005} = (4, 9, 14, 1)$ lies on line ℓ_5
115 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_0
116 : $P_{4106} = (9, 15, 14, 1)$ lies on line ℓ_2
117 : $P_{4145} = (0, 2, 15, 1)$ lies on line ℓ_4
118 : $P_{4148} = (3, 2, 15, 1)$ lies on line ℓ_1
119 : $P_{4166} = (5, 3, 15, 1)$ lies on line ℓ_5
120 : $P_{4247} = (6, 8, 15, 1)$ lies on line ℓ_8
121 : $P_{4274} = (1, 10, 15, 1)$ lies on line ℓ_2
122 : $P_{4309} = (4, 12, 15, 1)$ lies on line ℓ_7
123 : $P_{4321} = (0, 13, 15, 1)$ lies on line ℓ_6
124 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_0
125 : $P_{4363} = (10, 15, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{36} = (1, 1, 1, 0)$
3 : $P_{71} = (4, 3, 1, 0)$
4 : $P_{108} = (9, 5, 1, 0)$
5 : $P_{161} = (14, 8, 1, 0)$
6 : $P_{261} = (2, 15, 1, 0)$
7 : $P_{381} = (11, 6, 0, 1)$
8 : $P_{385} = (15, 6, 0, 1)$
9 : $P_{391} = (5, 7, 0, 1)$
10 : $P_{397} = (11, 7, 0, 1)$
11 : $P_{474} = (8, 12, 0, 1)$
12 : $P_{476} = (10, 12, 0, 1)$
13 : $P_{485} = (3, 13, 0, 1)$
14 : $P_{492} = (10, 13, 0, 1)$
15 : $P_{531} = (1, 0, 1, 1)$
16 : $P_{572} = (11, 2, 1, 1)$
17 : $P_{603} = (10, 4, 1, 1)$
18 : $P_{684} = (11, 9, 1, 1)$
19 : $P_{763} = (10, 14, 1, 1)$
20 : $P_{793} = (8, 0, 2, 1)$
21 : $P_{815} = (14, 1, 2, 1)$
22 : $P_{831} = (14, 2, 2, 1)$
23 : $P_{837} = (4, 3, 2, 1)$
24 : $P_{1004} = (11, 13, 2, 1)$
25 : $P_{1021} = (12, 14, 2, 1)$
26 : $P_{1032} = (7, 15, 2, 1)$
27 : $P_{1056} = (15, 0, 3, 1)$
28 : $P_{1125} = (4, 5, 3, 1)$
29 : $P_{1151} = (14, 6, 3, 1)$
30 : $P_{1166} = (13, 7, 3, 1)$
31 : $P_{1182} = (13, 8, 3, 1)$
32 : $P_{1208} = (7, 10, 3, 1)$
33 : $P_{1216} = (15, 10, 3, 1)$
34 : $P_{1225} = (8, 11, 3, 1)$
35 : $P_{1243} = (10, 12, 3, 1)$
36 : $P_{1244} = (11, 12, 3, 1)$
37 : $P_{1273} = (8, 14, 3, 1)$
38 : $P_{1277} = (12, 14, 3, 1)$
39 : $P_{1292} = (11, 15, 3, 1)$
40 : $P_{1312} = (15, 0, 4, 1)$
41 : $P_{1315} = (2, 1, 4, 1)$
42 : $P_{1335} = (6, 2, 4, 1)$
43 : $P_{1357} = (12, 3, 4, 1)$
44 : $P_{1363} = (2, 4, 4, 1)$
45 : $P_{1386} = (9, 5, 4, 1)$
46 : $P_{1419} = (10, 7, 4, 1)$
47 : $P_{1556} = (3, 0, 5, 1)$
48 : $P_{1591} = (6, 2, 5, 1)$
49 : $P_{1600} = (15, 2, 5, 1)$
50 : $P_{1611} = (10, 3, 5, 1)$
51 : $P_{1659} = (10, 6, 5, 1)$
52 : $P_{1660} = (11, 6, 5, 1)$
53 : $P_{1690} = (9, 8, 5, 1)$
54 : $P_{1728} = (15, 10, 5, 1)$
55 : $P_{1732} = (3, 11, 5, 1)$
56 : $P_{1741} = (12, 11, 5, 1)$
57 : $P_{1752} = (7, 12, 5, 1)$
58 : $P_{1763} = (2, 13, 5, 1)$
59 : $P_{1800} = (7, 15, 5, 1)$
60 : $P_{1855} = (14, 2, 6, 1)$
61 : $P_{1882} = (9, 4, 6, 1)$
62 : $P_{1895} = (6, 5, 6, 1)$
63 : $P_{1904} = (15, 5, 6, 1)$
64 : $P_{1907} = (2, 6, 6, 1)$
65 : $P_{1942} = (5, 8, 6, 1)$
66 : $P_{1959} = (6, 9, 6, 1)$
67 : $P_{1971} = (2, 10, 6, 1)$
68 : $P_{1984} = (15, 10, 6, 1)$
69 : $P_{1992} = (7, 11, 6, 1)$
70 : $P_{1996} = (11, 11, 6, 1)$
71 : $P_{2003} = (2, 12, 6, 1)$
72 : $P_{2034} = (1, 14, 6, 1)$
73 : $P_{2064} = (15, 15, 6, 1)$

74 : $P_{2104} = (7, 2, 7, 1)$
75 : $P_{2128} = (15, 3, 7, 1)$
76 : $P_{2130} = (1, 4, 7, 1)$
77 : $P_{2150} = (5, 5, 7, 1)$
78 : $P_{2186} = (9, 7, 7, 1)$
79 : $P_{2213} = (4, 9, 7, 1)$
80 : $P_{2230} = (5, 10, 7, 1)$
81 : $P_{2234} = (9, 10, 7, 1)$
82 : $P_{2247} = (6, 11, 7, 1)$
83 : $P_{2252} = (11, 11, 7, 1)$
84 : $P_{2282} = (9, 13, 7, 1)$
85 : $P_{2291} = (2, 14, 7, 1)$
86 : $P_{2310} = (5, 15, 7, 1)$
87 : $P_{2312} = (7, 15, 7, 1)$
88 : $P_{2326} = (5, 0, 8, 1)$
89 : $P_{2381} = (12, 3, 8, 1)$
90 : $P_{2388} = (3, 4, 8, 1)$
91 : $P_{2398} = (13, 4, 8, 1)$
92 : $P_{2412} = (11, 5, 8, 1)$
93 : $P_{2429} = (12, 6, 8, 1)$
94 : $P_{2437} = (4, 7, 8, 1)$
95 : $P_{2486} = (5, 10, 8, 1)$
96 : $P_{2487} = (6, 10, 8, 1)$
97 : $P_{2500} = (3, 11, 8, 1)$
98 : $P_{2539} = (10, 13, 8, 1)$
99 : $P_{2540} = (11, 13, 8, 1)$
100 : $P_{2575} = (14, 15, 8, 1)$
101 : $P_{2580} = (3, 0, 9, 1)$
102 : $P_{2597} = (4, 1, 9, 1)$
103 : $P_{2654} = (13, 4, 9, 1)$
104 : $P_{2663} = (6, 5, 9, 1)$
105 : $P_{2719} = (14, 8, 9, 1)$
106 : $P_{2725} = (4, 9, 9, 1)$
107 : $P_{2780} = (11, 12, 9, 1)$
108 : $P_{2860} = (11, 1, 10, 1)$
109 : $P_{2869} = (4, 2, 10, 1)$
110 : $P_{2991} = (14, 9, 10, 1)$
111 : $P_{2994} = (1, 10, 10, 1)$
112 : $P_{3028} = (3, 12, 10, 1)$
113 : $P_{3049} = (8, 13, 10, 1)$
114 : $P_{3115} = (10, 1, 11, 1)$
115 : $P_{3162} = (9, 4, 11, 1)$
116 : $P_{3190} = (5, 6, 11, 1)$
117 : $P_{3216} = (15, 7, 11, 1)$
118 : $P_{3266} = (1, 11, 11, 1)$
119 : $P_{3315} = (2, 14, 11, 1)$
120 : $P_{3381} = (4, 2, 12, 1)$
121 : $P_{3401} = (8, 3, 12, 1)$
122 : $P_{3405} = (12, 3, 12, 1)$
123 : $P_{3421} = (12, 4, 12, 1)$
124 : $P_{3428} = (3, 5, 12, 1)$
125 : $P_{3471} = (14, 7, 12, 1)$
126 : $P_{3481} = (8, 8, 12, 1)$
127 : $P_{3490} = (1, 9, 12, 1)$
128 : $P_{3515} = (10, 10, 12, 1)$
129 : $P_{3518} = (13, 10, 12, 1)$
130 : $P_{3529} = (8, 11, 12, 1)$
131 : $P_{3535} = (14, 11, 12, 1)$
132 : $P_{3551} = (14, 12, 12, 1)$
133 : $P_{3578} = (9, 14, 12, 1)$
134 : $P_{3634} = (1, 2, 13, 1)$
135 : $P_{3652} = (3, 3, 13, 1)$
136 : $P_{3667} = (2, 4, 13, 1)$
137 : $P_{3701} = (4, 6, 13, 1)$
138 : $P_{3732} = (3, 8, 13, 1)$
139 : $P_{3742} = (13, 8, 13, 1)$
140 : $P_{3759} = (14, 9, 13, 1)$
141 : $P_{3771} = (10, 10, 13, 1)$
142 : $P_{3773} = (12, 10, 13, 1)$
143 : $P_{3780} = (3, 11, 13, 1)$
144 : $P_{3781} = (4, 11, 13, 1)$
145 : $P_{3813} = (4, 13, 13, 1)$
146 : $P_{3838} = (13, 14, 13, 1)$
147 : $P_{3849} = (8, 15, 13, 1)$
148 : $P_{3862} = (5, 0, 14, 1)$
149 : $P_{3882} = (9, 1, 14, 1)$
150 : $P_{3963} = (10, 6, 14, 1)$
151 : $P_{3998} = (13, 8, 14, 1)$
152 : $P_{4008} = (7, 9, 14, 1)$
153 : $P_{4090} = (9, 14, 14, 1)$
154 : $P_{4099} = (2, 15, 14, 1)$
155 : $P_{4121} = (8, 0, 15, 1)$
156 : $P_{4163} = (2, 3, 15, 1)$
157 : $P_{4199} = (6, 5, 15, 1)$
158 : $P_{4235} = (10, 7, 15, 1)$
159 : $P_{4236} = (11, 7, 15, 1)$
160 : $P_{4251} = (10, 8, 15, 1)$
161 : $P_{4262} = (5, 9, 15, 1)$
162 : $P_{4264} = (7, 9, 15, 1)$
163 : $P_{4278} = (5, 10, 15, 1)$
164 : $P_{4297} = (8, 11, 15, 1)$
165 : $P_{4302} = (13, 11, 15, 1)$
166 : $P_{4314} = (9, 12, 15, 1)$
167 : $P_{4327} = (6, 13, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 0 0 0 1 1 1 1 0
1	0 0 1 0 0 1 1 1 1
2	0 1 0 1 1 1 0 1 0
3	0 0 1 0 0 1 1 1 1
4	1 0 1 0 0 0 1 0 1
5	1 1 1 1 0 0 0 0 1
6	1 1 0 1 1 0 0 0 0
7	1 1 1 1 0 0 0 0 1
8	0 1 0 1 1 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_{2993}	P_3	P_{3265}

Line 1 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_{1408}	P_{985}	P_{2113}	P_{1408}	P_{985}

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_7
in point	P_{1408}	P_{2788}	P_{3425}	P_{2788}	P_{1408}

Line 3 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_{2788}	P_{2788}	P_{1937}	P_{3974}	P_{3974}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8
in point	P_3	P_{3425}	P_3	P_{3841}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_8
in point	P_{2993}	P_{985}	P_{2788}	P_{2788}	P_{985}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_3	P_{2113}	P_{1937}	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_8
in point	P_{3265}	P_{1408}	P_{1408}	P_{3974}	P_{3974}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_7
in point	P_{985}	P_{3974}	P_{3841}	P_{985}	P_{3974}

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_{35} = (0, 1, 1, 0)$
4 : $P_{36} = (1, 1, 1, 0)$
5 : $P_{71} = (4, 3, 1, 0)$
6 : $P_{72} = (5, 3, 1, 0)$
7 : $P_{107} = (8, 5, 1, 0)$
8 : $P_{108} = (9, 5, 1, 0)$
9 : $P_{161} = (14, 8, 1, 0)$
10 : $P_{162} = (15, 8, 1, 0)$
11 : $P_{179} = (0, 10, 1, 0)$
12 : $P_{180} = (1, 10, 1, 0)$
13 : $P_{195} = (0, 11, 1, 0)$
14 : $P_{196} = (1, 11, 1, 0)$
15 : $P_{261} = (2, 15, 1, 0)$
16 : $P_{262} = (3, 15, 1, 0)$
17 : $P_{300} = (10, 1, 0, 1)$
18 : $P_{301} = (11, 1, 0, 1)$
19 : $P_{381} = (11, 6, 0, 1)$
20 : $P_{385} = (15, 6, 0, 1)$

21 : $P_{391} = (5, 7, 0, 1)$
22 : $P_{397} = (11, 7, 0, 1)$
23 : $P_{436} = (2, 10, 0, 1)$
24 : $P_{443} = (9, 10, 0, 1)$
25 : $P_{454} = (4, 11, 0, 1)$
26 : $P_{464} = (14, 11, 0, 1)$
27 : $P_{474} = (8, 12, 0, 1)$
28 : $P_{476} = (10, 12, 0, 1)$
29 : $P_{485} = (3, 13, 0, 1)$
30 : $P_{492} = (10, 13, 0, 1)$
31 : $P_{531} = (1, 0, 1, 1)$
32 : $P_{546} = (0, 1, 1, 1)$
33 : $P_{567} = (6, 2, 1, 1)$
34 : $P_{572} = (11, 2, 1, 1)$
35 : $P_{603} = (10, 4, 1, 1)$
36 : $P_{606} = (13, 4, 1, 1)$
37 : $P_{680} = (7, 9, 1, 1)$
38 : $P_{684} = (11, 9, 1, 1)$
39 : $P_{689} = (0, 10, 1, 1)$
40 : $P_{699} = (10, 10, 1, 1)$
41 : $P_{705} = (0, 11, 1, 1)$
42 : $P_{716} = (11, 11, 1, 1)$
43 : $P_{763} = (10, 14, 1, 1)$
44 : $P_{765} = (12, 14, 1, 1)$
45 : $P_{793} = (8, 0, 2, 1)$
46 : $P_{814} = (13, 1, 2, 1)$
47 : $P_{815} = (14, 1, 2, 1)$
48 : $P_{817} = (0, 2, 2, 1)$
49 : $P_{831} = (14, 2, 2, 1)$
50 : $P_{837} = (4, 3, 2, 1)$
51 : $P_{847} = (14, 3, 2, 1)$
52 : $P_{985} = (8, 12, 2, 1)$
53 : $P_{993} = (0, 13, 2, 1)$
54 : $P_{1004} = (11, 13, 2, 1)$
55 : $P_{1018} = (9, 14, 2, 1)$
56 : $P_{1021} = (12, 14, 2, 1)$
57 : $P_{1025} = (0, 15, 2, 1)$
58 : $P_{1032} = (7, 15, 2, 1)$
59 : $P_{1056} = (15, 0, 3, 1)$
60 : $P_{1089} = (0, 3, 3, 1)$
61 : $P_{1100} = (11, 3, 3, 1)$
62 : $P_{1105} = (0, 4, 3, 1)$
63 : $P_{1110} = (5, 4, 3, 1)$
64 : $P_{1125} = (4, 5, 3, 1)$
65 : $P_{1129} = (8, 5, 3, 1)$
66 : $P_{1146} = (9, 6, 3, 1)$
67 : $P_{1151} = (14, 6, 3, 1)$
68 : $P_{1153} = (0, 7, 3, 1)$
69 : $P_{1166} = (13, 7, 3, 1)$
70 : $P_{1182} = (13, 8, 3, 1)$
71 : $P_{1208} = (7, 10, 3, 1)$
72 : $P_{1216} = (15, 10, 3, 1)$
73 : $P_{1218} = (1, 11, 3, 1)$
74 : $P_{1225} = (8, 11, 3, 1)$
75 : $P_{1243} = (10, 12, 3, 1)$
76 : $P_{1244} = (11, 12, 3, 1)$
77 : $P_{1273} = (8, 14, 3, 1)$
78 : $P_{1277} = (12, 14, 3, 1)$
79 : $P_{1292} = (11, 15, 3, 1)$
80 : $P_{1294} = (13, 15, 3, 1)$
81 : $P_{1312} = (15, 0, 4, 1)$
82 : $P_{1315} = (2, 1, 4, 1)$
83 : $P_{1320} = (7, 1, 4, 1)$
84 : $P_{1335} = (6, 2, 4, 1)$
85 : $P_{1343} = (14, 2, 4, 1)$
86 : $P_{1345} = (0, 3, 4, 1)$
87 : $P_{1357} = (12, 3, 4, 1)$
88 : $P_{1361} = (0, 4, 4, 1)$
89 : $P_{1363} = (2, 4, 4, 1)$
90 : $P_{1379} = (2, 5, 4, 1)$
91 : $P_{1386} = (9, 5, 4, 1)$
92 : $P_{1408} = (15, 6, 4, 1)$
93 : $P_{1409} = (0, 7, 4, 1)$
94 : $P_{1419} = (10, 7, 4, 1)$
95 : $P_{1556} = (3, 0, 5, 1)$
96 : $P_{1591} = (6, 2, 5, 1)$
97 : $P_{1600} = (15, 2, 5, 1)$
98 : $P_{1608} = (7, 3, 5, 1)$
99 : $P_{1611} = (10, 3, 5, 1)$
100 : $P_{1633} = (0, 5, 5, 1)$
101 : $P_{1643} = (10, 5, 5, 1)$
102 : $P_{1659} = (10, 6, 5, 1)$
103 : $P_{1660} = (11, 6, 5, 1)$
104 : $P_{1690} = (9, 8, 5, 1)$
105 : $P_{1696} = (15, 8, 5, 1)$
106 : $P_{1697} = (0, 9, 5, 1)$
107 : $P_{1705} = (8, 9, 5, 1)$
108 : $P_{1714} = (1, 10, 5, 1)$
109 : $P_{1728} = (15, 10, 5, 1)$
110 : $P_{1732} = (3, 11, 5, 1)$
111 : $P_{1741} = (12, 11, 5, 1)$
112 : $P_{1745} = (0, 12, 5, 1)$
113 : $P_{1752} = (7, 12, 5, 1)$
114 : $P_{1763} = (2, 13, 5, 1)$
115 : $P_{1775} = (14, 13, 5, 1)$
116 : $P_{1800} = (7, 15, 5, 1)$
117 : $P_{1814} = (5, 0, 6, 1)$
118 : $P_{1845} = (4, 2, 6, 1)$
119 : $P_{1855} = (14, 2, 6, 1)$
120 : $P_{1882} = (9, 4, 6, 1)$
121 : $P_{1895} = (6, 5, 6, 1)$
122 : $P_{1904} = (15, 5, 6, 1)$
123 : $P_{1905} = (0, 6, 6, 1)$
124 : $P_{1907} = (2, 6, 6, 1)$
125 : $P_{1937} = (0, 8, 6, 1)$
126 : $P_{1942} = (5, 8, 6, 1)$
127 : $P_{1959} = (6, 9, 6, 1)$
128 : $P_{1966} = (13, 9, 6, 1)$
129 : $P_{1971} = (2, 10, 6, 1)$
130 : $P_{1984} = (15, 10, 6, 1)$
131 : $P_{1992} = (7, 11, 6, 1)$
132 : $P_{1996} = (11, 11, 6, 1)$
133 : $P_{2003} = (2, 12, 6, 1)$
134 : $P_{2007} = (6, 12, 6, 1)$
135 : $P_{2033} = (0, 14, 6, 1)$
136 : $P_{2034} = (1, 14, 6, 1)$
137 : $P_{2061} = (12, 15, 6, 1)$
138 : $P_{2064} = (15, 15, 6, 1)$
139 : $P_{2080} = (15, 0, 7, 1)$
140 : $P_{2104} = (7, 2, 7, 1)$
141 : $P_{2109} = (12, 2, 7, 1)$
142 : $P_{2113} = (0, 3, 7, 1)$
143 : $P_{2128} = (15, 3, 7, 1)$
144 : $P_{2129} = (0, 4, 7, 1)$
145 : $P_{2130} = (1, 4, 7, 1)$
146 : $P_{2150} = (5, 5, 7, 1)$
147 : $P_{2158} = (13, 5, 7, 1)$
148 : $P_{2177} = (0, 7, 7, 1)$
149 : $P_{2186} = (9, 7, 7, 1)$
150 : $P_{2213} = (4, 9, 7, 1)$
151 : $P_{2223} = (14, 9, 7, 1)$
152 : $P_{2230} = (5, 10, 7, 1)$
153 : $P_{2234} = (9, 10, 7, 1)$
154 : $P_{2247} = (6, 11, 7, 1)$
155 : $P_{2252} = (11, 11, 7, 1)$
156 : $P_{2280} = (7, 13, 7, 1)$
157 : $P_{2282} = (9, 13, 7, 1)$
158 : $P_{2291} = (2, 14, 7, 1)$
159 : $P_{2310} = (5, 15, 7, 1)$
160 : $P_{2312} = (7, 15, 7, 1)$
161 : $P_{2326} = (5, 0, 8, 1)$
162 : $P_{2381} = (12, 3, 8, 1)$
163 : $P_{2388} = (3, 4, 8, 1)$
164 : $P_{2398} = (13, 4, 8, 1)$
165 : $P_{2412} = (11, 5, 8, 1)$
166 : $P_{2413} = (12, 5, 8, 1)$
167 : $P_{2417} = (0, 6, 8, 1)$
168 : $P_{2429} = (12, 6, 8, 1)$
169 : $P_{2435} = (2, 7, 8, 1)$
170 : $P_{2437} = (4, 7, 8, 1)$
171 : $P_{2449} = (0, 8, 8, 1)$
172 : $P_{2460} = (11, 8, 8, 1)$
173 : $P_{2486} = (5, 10, 8, 1)$
174 : $P_{2487} = (6, 10, 8, 1)$
175 : $P_{2498} = (1, 11, 8, 1)$
176 : $P_{2500} = (3, 11, 8, 1)$
177 : $P_{2539} = (10, 13, 8, 1)$
178 : $P_{2540} = (11, 13, 8, 1)$
179 : $P_{2545} = (0, 14, 8, 1)$
180 : $P_{2560} = (15, 14, 8, 1)$
181 : $P_{2564} = (3, 15, 8, 1)$
182 : $P_{2575} = (14, 15, 8, 1)$

183 : $P_{2580} = (3, 0, 9, 1)$
184 : $P_{2597} = (4, 1, 9, 1)$
185 : $P_{2605} = (12, 1, 9, 1)$
186 : $P_{2643} = (2, 4, 9, 1)$
187 : $P_{2654} = (13, 4, 9, 1)$
188 : $P_{2657} = (0, 5, 9, 1)$
189 : $P_{2663} = (6, 5, 9, 1)$
190 : $P_{2709} = (4, 8, 9, 1)$
191 : $P_{2719} = (14, 8, 9, 1)$
192 : $P_{2721} = (0, 9, 9, 1)$
193 : $P_{2725} = (4, 9, 9, 1)$
194 : $P_{2769} = (0, 12, 9, 1)$
195 : $P_{2780} = (11, 12, 9, 1)$
196 : $P_{2788} = (3, 13, 9, 1)$
197 : $P_{2834} = (1, 0, 10, 1)$
198 : $P_{2849} = (0, 1, 10, 1)$
199 : $P_{2860} = (11, 1, 10, 1)$
200 : $P_{2867} = (2, 2, 10, 1)$
201 : $P_{2869} = (4, 2, 10, 1)$
202 : $P_{2986} = (9, 9, 10, 1)$
203 : $P_{2991} = (14, 9, 10, 1)$
204 : $P_{2993} = (0, 10, 10, 1)$
205 : $P_{2994} = (1, 10, 10, 1)$
206 : $P_{3009} = (0, 11, 10, 1)$
207 : $P_{3028} = (3, 12, 10, 1)$
208 : $P_{3036} = (11, 12, 10, 1)$
209 : $P_{3049} = (8, 13, 10, 1)$
210 : $P_{3052} = (11, 13, 10, 1)$
211 : $P_{3090} = (1, 0, 11, 1)$
212 : $P_{3105} = (0, 1, 11, 1)$
213 : $P_{3115} = (10, 1, 11, 1)$
214 : $P_{3157} = (4, 4, 11, 1)$
215 : $P_{3162} = (9, 4, 11, 1)$
216 : $P_{3190} = (5, 6, 11, 1)$
217 : $P_{3195} = (10, 6, 11, 1)$
218 : $P_{3211} = (10, 7, 11, 1)$
219 : $P_{3216} = (15, 7, 11, 1)$
220 : $P_{3249} = (0, 10, 11, 1)$
221 : $P_{3265} = (0, 11, 11, 1)$
222 : $P_{3266} = (1, 11, 11, 1)$
223 : $P_{3315} = (2, 14, 11, 1)$
224 : $P_{3327} = (14, 14, 11, 1)$
225 : $P_{3348} = (3, 0, 12, 1)$
226 : $P_{3381} = (4, 2, 12, 1)$
227 : $P_{3401} = (8, 3, 12, 1)$
228 : $P_{3405} = (12, 3, 12, 1)$
229 : $P_{3415} = (6, 4, 12, 1)$
230 : $P_{3421} = (12, 4, 12, 1)$
231 : $P_{3425} = (0, 5, 12, 1)$
232 : $P_{3428} = (3, 5, 12, 1)$
233 : $P_{3469} = (12, 7, 12, 1)$
234 : $P_{3471} = (14, 7, 12, 1)$
235 : $P_{3480} = (7, 8, 12, 1)$
236 : $P_{3481} = (8, 8, 12, 1)$
237 : $P_{3489} = (0, 9, 12, 1)$
238 : $P_{3490} = (1, 9, 12, 1)$
239 : $P_{3515} = (10, 10, 12, 1)$
240 : $P_{3518} = (13, 10, 12, 1)$
241 : $P_{3529} = (8, 11, 12, 1)$
242 : $P_{3535} = (14, 11, 12, 1)$
243 : $P_{3537} = (0, 12, 12, 1)$
244 : $P_{3551} = (14, 12, 12, 1)$
245 : $P_{3571} = (2, 14, 12, 1)$
246 : $P_{3578} = (9, 14, 12, 1)$
247 : $P_{3609} = (8, 0, 13, 1)$
248 : $P_{3633} = (0, 2, 13, 1)$
249 : $P_{3634} = (1, 2, 13, 1)$
250 : $P_{3652} = (3, 3, 13, 1)$
251 : $P_{3655} = (6, 3, 13, 1)$
252 : $P_{3667} = (2, 4, 13, 1)$
253 : $P_{3674} = (9, 4, 13, 1)$
254 : $P_{3701} = (4, 6, 13, 1)$
255 : $P_{3710} = (13, 6, 13, 1)$
256 : $P_{3732} = (3, 8, 13, 1)$
257 : $P_{3742} = (13, 8, 13, 1)$
258 : $P_{3759} = (14, 9, 13, 1)$
259 : $P_{3771} = (10, 10, 13, 1)$
260 : $P_{3773} = (12, 10, 13, 1)$
261 : $P_{3780} = (3, 11, 13, 1)$
262 : $P_{3781} = (4, 11, 13, 1)$
263 : $P_{3809} = (0, 13, 13, 1)$
264 : $P_{3813} = (4, 13, 13, 1)$
265 : $P_{3832} = (7, 14, 13, 1)$
266 : $P_{3838} = (13, 14, 13, 1)$
267 : $P_{3841} = (0, 15, 13, 1)$
268 : $P_{3849} = (8, 15, 13, 1)$
269 : $P_{3862} = (5, 0, 14, 1)$
270 : $P_{3879} = (6, 1, 14, 1)$
271 : $P_{3882} = (9, 1, 14, 1)$
272 : $P_{3953} = (0, 6, 14, 1)$
273 : $P_{3963} = (10, 6, 14, 1)$
274 : $P_{3974} = (5, 7, 14, 1)$
275 : $P_{3985} = (0, 8, 14, 1)$
276 : $P_{3998} = (13, 8, 14, 1)$
277 : $P_{4005} = (4, 9, 14, 1)$
278 : $P_{4008} = (7, 9, 14, 1)$
279 : $P_{4081} = (0, 14, 14, 1)$
280 : $P_{4090} = (9, 14, 14, 1)$
281 : $P_{4099} = (2, 15, 14, 1)$
282 : $P_{4106} = (9, 15, 14, 1)$
283 : $P_{4121} = (8, 0, 15, 1)$
284 : $P_{4145} = (0, 2, 15, 1)$
285 : $P_{4148} = (3, 2, 15, 1)$
286 : $P_{4163} = (2, 3, 15, 1)$
287 : $P_{4166} = (5, 3, 15, 1)$
288 : $P_{4199} = (6, 5, 15, 1)$
289 : $P_{4235} = (10, 7, 15, 1)$
290 : $P_{4236} = (11, 7, 15, 1)$
291 : $P_{4247} = (6, 8, 15, 1)$
292 : $P_{4251} = (10, 8, 15, 1)$
293 : $P_{4262} = (5, 9, 15, 1)$
294 : $P_{4264} = (7, 9, 15, 1)$
295 : $P_{4274} = (1, 10, 15, 1)$
296 : $P_{4278} = (5, 10, 15, 1)$
297 : $P_{4297} = (8, 11, 15, 1)$
298 : $P_{4302} = (13, 11, 15, 1)$
299 : $P_{4309} = (4, 12, 15, 1)$
300 : $P_{4314} = (9, 12, 15, 1)$
301 : $P_{4321} = (0, 13, 15, 1)$
302 : $P_{4327} = (6, 13, 15, 1)$
303 : $P_{4353} = (0, 15, 15, 1)$
304 : $P_{4363} = (10, 15, 15, 1)$