# Rank-65542 over GF(32)

January 15, 2021

## The equation

The equation of the surface is:

$$X_0^3 + X_1^3 + X_0 X_1 X_2 = 0$$

### General information

Number of lines	32
Number of points	1025
Number of singular points	33
Number of Eckardt points	0
Number of double points	0
Number of single points	1024
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$33^{32}$
Type of lines on points	$32, 1^{1024}$

#### Singular Points

The surface has 33 singular points:

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\begin{array}{lll} 0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0) & 9: \ P_{9249} = \mathbf{P}(0,0,\eta^3,1) = \mathbf{P}(0,0,8,1) \\ 1: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1) & 10: \ P_{10273} = \mathbf{P}(0,0,\eta^{29},1) = \mathbf{P}(0,0,9,1) \\ 2: \ P_{2082} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1) & 11: \ P_{11297} = \mathbf{P}(0,0,\eta^6,1) = \mathbf{P}(0,0,10,1) \\ 3: \ P_{3105} = \mathbf{P}(0,0,\eta,1) = \mathbf{P}(0,0,2,1) & 12: \ P_{12321} = \mathbf{P}(0,0,\eta^{27},1) = \mathbf{P}(0,0,11,1) \\ 4: \ P_{4129} = \mathbf{P}(0,0,\eta^{18},1) = \mathbf{P}(0,0,3,1) & 13: \ P_{13345} = \mathbf{P}(0,0,\eta^{20},1) = \mathbf{P}(0,0,12,1) \\ 5: \ P_{5153} = \mathbf{P}(0,0,\eta^2,1) = \mathbf{P}(0,0,4,1) & 14: \ P_{14369} = \mathbf{P}(0,0,\eta^8,1) = \mathbf{P}(0,0,13,1) \\ 6: \ P_{6177} = \mathbf{P}(0,0,\eta^5,1) = \mathbf{P}(0,0,5,1) & 15: \ P_{15393} = \mathbf{P}(0,0,\eta^{12},1) = \mathbf{P}(0,0,14,1) \\ 7: \ P_{7201} = \mathbf{P}(0,0,\eta^{19},1) = \mathbf{P}(0,0,6,1) & 16: \ P_{16417} = \mathbf{P}(0,0,\eta^{23},1) = \mathbf{P}(0,0,15,1) \\ 8: \ P_{8225} = \mathbf{P}(0,0,\eta^{11},1) = \mathbf{P}(0,0,7,1) & 17: \ P_{17441} = \mathbf{P}(0,0,\eta^4,1) = \mathbf{P}(0,0,16,1) \end{array}
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\begin{array}{lll} 18: \ P_{18465} = \mathbf{P}(0,0,\eta^{10},1) = \mathbf{P}(0,0,17,1) \\ 19: \ P_{19489} = \mathbf{P}(0,0,\eta^{30},1) = \mathbf{P}(0,0,18,1) \\ 20: \ P_{20513} = \mathbf{P}(0,0,\eta^{17},1) = \mathbf{P}(0,0,19,1) \\ 21: \ P_{21537} = \mathbf{P}(0,0,\eta^{7},1) = \mathbf{P}(0,0,20,1) \\ 22: \ P_{22561} = \mathbf{P}(0,0,\eta^{22},1) = \mathbf{P}(0,0,21,1) \\ 23: \ P_{23585} = \mathbf{P}(0,0,\eta^{28},1) = \mathbf{P}(0,0,22,1) \\ 24: \ P_{24609} = \mathbf{P}(0,0,\eta^{26},1) = \mathbf{P}(0,0,24,1) \\ 25: \ P_{25633} = \mathbf{P}(0,0,\eta^{21},1) = \mathbf{P}(0,0,24,1) \\ 26: \ P_{26657} = \mathbf{P}(0,0,\eta^{25},1) = \mathbf{P}(0,0,26,1) \\ 27: \ P_{27681} = \mathbf{P}(0,0,\eta^{16},1) = \mathbf{P}(0,0,27,1) \\ 28: \ P_{28705} = \mathbf{P}(0,0,\eta^{16},1) = \mathbf{P}(0,0,27,1) \\ 29: \ P_{29729} = \mathbf{P}(0,0,\eta^{13},1) = \mathbf{P}(0,0,28,1) \\ 30: \ P_{30753} = \mathbf{P}(0,0,\eta^{14},1) = \mathbf{P}(0,0,29,1) \\ 31: \ P_{31777} = \mathbf{P}(0,0,\eta^{24},1) = \mathbf{P}(0,0,30,1) \\ 32: \ P_{32801} = \mathbf{P}(0,0,\eta^{15},1) = \mathbf{P}(0,0,31,1) \end{array}
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#### The 32 Lines

The lines and their Pluecker coordinates are:

$$\begin{split} \ell_0 &= \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{PI}(0,1,0,0,0,0)_1 \\ \ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2113} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2113} = \mathbf{PI}(0,0,0,1,1,0)_{3105} \\ \ell_2 &= \begin{bmatrix} 1 & \eta^8 & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{691277} = \begin{bmatrix} 1 & 13 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{691277} = \mathbf{PI}(0,20,0,13,1,0)_{3912} \\ \ell_3 &= \begin{bmatrix} 1 & \eta^{28} & \eta^{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{599318} = \begin{bmatrix} 1 & 22 & 17 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{599318} = \mathbf{PI}(0,17,0,22,1,0)_{4476} \\ \ell_4 &= \begin{bmatrix} 1 & \eta^{16} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1010491} = \begin{bmatrix} 1 & 27 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1010491} = \mathbf{PI}(0,29,0,27,1,0)_{4803} \\ \ell_5 &= \begin{bmatrix} 1 & \eta^{25} & \eta^{20} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{433369} = \begin{bmatrix} 1 & 25 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{333369} = \mathbf{PI}(0,12,0,25,1,0)_{4660} \\ \ell_6 &= \begin{bmatrix} 1 & \eta^6 & \eta^{26} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{789578} = \begin{bmatrix} 1 & 10 & 23 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{789578} = \mathbf{PI}(0,23,0,10,1,0)_{3726} \\ \ell_7 &= \begin{bmatrix} 1 & \eta^{22} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{226197} = \begin{bmatrix} 1 & 21 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{226197} = \mathbf{PI}(0,6,0,21,1,0)_{4402} \\ \ell_8 &= \begin{bmatrix} 1 & \eta^{30} & \eta^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{392146} = \begin{bmatrix} 1 & 18 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{392146} = \mathbf{PI}(0,11,0,18,1,0)_{4218} \\ \ell_9 &= \begin{bmatrix} 1 & \eta^{10} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{695505} = \begin{bmatrix} 1 & 17 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{517929} = \mathbf{PI}(0,20,0,17,1,0)_{4164} \\ \ell_{10} &= \begin{bmatrix} 1 & \eta^{29} & \eta^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{994636} = \begin{bmatrix} 1 & 12 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{994636} = \mathbf{PI}(0,29,0,12,1,0)_{3858} \\ \ell_{11} &= \begin{bmatrix} 1 & \eta^{20} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{707132} = \begin{bmatrix} 1 & 28 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{123668} = \mathbf{PI}(0,29,0,28,1,0)_{4357} \\ \ell_{13} &= \begin{bmatrix} 1 & \eta^7 & \eta^{18} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{123668} = \begin{bmatrix} 1 & 20 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1772666} = \mathbf{PI}(0,29,0,28,1,0)_{4335} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^9 & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{772666} = \begin{bmatrix} 1 & 26 & 22 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{772666} = \mathbf{PI}(0,22,0,26,1,0)_{4335} \end{aligned}$$

$$\begin{split} \ell_{16} &= \begin{bmatrix} 1 & \eta^{26} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1006263} = \begin{bmatrix} 1 & 23 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1006263} = \mathbf{PI}(0, 29, 0, 23, 1, 0)_{4551} \\ \ell_{17} &= \begin{bmatrix} 1 & \eta^{27} & \eta^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1061227} = \begin{bmatrix} 1 & 11 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1061227} = \mathbf{PI}(0, 31, 0, 11, 1, 0)_{3797} \\ \ell_{18} &= \begin{bmatrix} 1 & \eta & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{747298} = \begin{bmatrix} 1 & 2 & 22 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{747298} = \mathbf{PI}(0, 22, 0, 2, 1, 0)_{3221} \\ \ell_{19} &= \begin{bmatrix} 1 & \eta^{19} & \eta^{9} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{886822} = \begin{bmatrix} 1 & 6 & 26 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{886822} = \mathbf{PI}(0, 26, 0, 6, 1, 0)_{3477} \\ \ell_{20} &= \begin{bmatrix} 1 & \eta^{15} & \eta^{29} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{338239} = \begin{bmatrix} 1 & 31 & 9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{338239} = \mathbf{PI}(0, 9, 0, 31, 1, 0)_{5035} \\ \ell_{21} &= \begin{bmatrix} 1 & \eta^{11} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{354055} = \begin{bmatrix} 1 & 7 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{354055} = \mathbf{PI}(0, 25, 0, 7, 1, 0)_{3539} \\ \ell_{22} &= \begin{bmatrix} 1 & \eta^{5} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209285} = \begin{bmatrix} 1 & 5 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{329285} = \mathbf{PI}(0, 6, 0, 5, 1, 0)_{3394} \\ \ell_{23} &= \begin{bmatrix} 1 & \eta^{12} & \eta^{21} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209285} = \begin{bmatrix} 1 & 14 & 24 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3209285} = \mathbf{PI}(0, 24, 0, 14, 1, 0)_{3979} \\ \ell_{24} &= \begin{bmatrix} 1 & \eta^{77} & \eta^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{31443} = \begin{bmatrix} 1 & 19 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{37443} = \mathbf{PI}(0, 21, 0, 19, 1, 0)_{4291} \\ \ell_{25} &= \begin{bmatrix} 1 & \eta^{3} & \eta^{13} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{200829} = \begin{bmatrix} 1 & 29 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{200829} = \mathbf{PI}(0, 5, 0, 29, 1, 0)_{4905} \\ \ell_{27} &= \begin{bmatrix} 1 & \eta^{4} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{200829} = \begin{bmatrix} 1 & 16 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{200829} = \mathbf{PI}(0, 6, 0, 16, 1, 0)_{4087} \\ \ell_{28} &= \begin{bmatrix} 1 & \eta^{24} & \eta^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209534} = \begin{bmatrix} 1 & 16 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209524} = \mathbf{PI}(0, 18, 0, 15, 1, 0)_{4036} \\ \ell_{29} &= \begin{bmatrix} 1 & \eta^{23} & \eta^{30} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{625743} = \begin{bmatrix} 1 & 15 & 18 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{849827} = \mathbf{PI}(0, 22, 0, 24, 1, 0)_{4007} \\ \ell_{29} &= \begin{bmatrix} 1 & \eta^{23} & \eta^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{849827} = \begin{bmatrix} 1 & 30 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{849827} = \mathbf{PI}(0, 25, 0, 3, 1, 0)_{3287} \end{aligned}$$

 $\begin{array}{l} \text{Rank of lines:} \ (\ 1083424,\ 2113,\ 691277,\ 599318,\ 1010491,\ 433369,\ 789578,\ 226197,\ 392146,\ 695505,\ 517929,\ 994636,\ 707132,\ 123668,\ 850884,\ 772666,\ 1006263,\ 1061227,\ 747298,\ 886822,\ 338239,\ 854055,\ 209285,\ 827630,\ 731443,\ 956584,\ 200829,\ 220912,\ 269534,\ 625743,\ 770552,\ 849827\ ) \end{array}$ 

Rank of points on Klein quadric: (  $1,\,3105,\,3912,\,4476,\,4803,\,4660,\,3726,\,4402,\,4218,\,4164,\,3655,\,3858,\,4857,\,4336,\,3350,\,4733,\,4551,\,3797,\,3221,\,3477,\,5035,\,3539,\,3394,\,3979,\,4291,\,3605,\,4905,\,4087,\,4970,\,4036,\,4607,\,3287$  )

### **Eckardt Points**

The surface has 0 Eckardt points:

## **Double Points**

The surface has 0 Double points: The double points on the surface are:

# Single Points

The surface has 1024 single points: Too many to print.

## Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

# Line Intersection Graph

	0 1	2	3 4	15	6	7	8 9	9 1	0	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27 '	28 '	29 :	30 :	31
0	0 1	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$^{2}$	$ 1 \ 1$	0	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	$ 1 \ 1$	. 1	0 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	$ 1 \ 1$	. 1	1 (	1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	$ 1 \ 1$	. 1	1 :	10	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1 1	. 1	1 :	1 1	0	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	$ 1 \ 1$	. 1	1 :	1 1	1	0	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	$ 1 \ 1$	. 1	1 :	1 1	1	1	0 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 (	)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1 1	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
19	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
20	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
22	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
23	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
24		_			_	_		_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
25	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
26	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
27	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
28	1	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
29	1	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
30							1 1	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
31	$ 1 \ 1$	. 1	1 :	1 1	1	1	1 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

In point   R3   R3   R3   R3   R3   R3   R3   R	Neighbor s Line 0 inte			line	inter	secti	on gr	aph:															
									$\ell_8$			$\ell_{11}$	$\ell_{12}$										$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	${ m Line~1~inte}$	ersect	S																				
																							$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	${ m Line}~2~{ m inte}$	ersect	S																				
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line																			$\ell_{19}$			$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	${ m Line} \ 3 \ { m int} \epsilon$	ersect	s																				
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	in point	$P_3$	$P_3$	$P_3$	$P_3$					$P_3$	$P_3$	$P_3$				$P_3$	$P_3$	$P_3$	$P_3$		$P_3$		$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	${\rm Line}  4  {\rm int} \epsilon$	ersect	S																				
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	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 5 inte	ersect	s																				
Line 6 intersects  \[ \begin{array}{c c c c c c c c c c c c c c c c c c c					$\ell_3$						$\ell_{10}$									$\ell_{19}$			$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 6 inte	ersect	S																				
Line 7 intersects																							$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 7 inte	ersect	S																				
Line 8 intersects  Line $\ell_0$ $\ell_1$ $\ell_2$ $\ell_3$ $\ell_4$ $\ell_5$ $\ell_6$ $\ell_7$ $\ell_9$ $\ell_{10}$ $\ell_{11}$ $\ell_{12}$ $\ell_{13}$ $\ell_{14}$ $\ell_{15}$ $\ell_{16}$ $\ell_{17}$ $\ell_{18}$ $\ell_{19}$ $\ell_{20}$ $\ell_{21}$ $\ell_{21}$ in point $P_3$									$\ell_8$									$\ell_{17}$			$\ell_{20}$		$\ell_{22}$
Line $\ell_0$ $\ell_1$ $\ell_2$ $\ell_3$ $\ell_4$ $\ell_5$ $\ell_6$ $\ell_7$ $\ell_9$ $\ell_{10}$ $\ell_{11}$ $\ell_{12}$ $\ell_{13}$ $\ell_{14}$ $\ell_{15}$ $\ell_{16}$ $\ell_{17}$ $\ell_{18}$ $\ell_{19}$ $\ell_{20}$ $\ell_{21}$ $\ell_{21}$ in point $P_3$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	${\rm Line~8~inte}$	ersect	S																				
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line		$\ell_1$		$\ell_3$										$\ell_{14}$		$\ell_{16}$	$\ell_{17}$	$\ell_{18}$				$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$\overline{P_3}$	$P_3$	$P_3$	$P_3$	$\overline{P_3}$	$\overline{P_3}$	$P_3$	$P_3$	$P_3$	$\overline{P_3}$	$P_3$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	${ m Line} \ 9 \ { m inter}$	ersect	S																				
in point $ P_3  P_3 $	Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point																						$P_3$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 10 int	erse	${ m cts}$			_	_		_	_					_		_			_			
in point   Pa   Pa   Pa   Pa   Pa   Pa   Pa   P	Line	$\ell_0$	$\ell_1$	$\ell_2$		$\ell_4$		$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$
In point   13   13   13   13   13   13   13   1	in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$

 $\ell_8$ 

 $P_3$ 

 $\ell_9$ 

 $P_3$ 

 $\ell_{10}$ 

 $P_3$ 

 $\ell_{12} \mid \ell_{13}$ 

 $P_3$ 

 $P_3$ 

 $\ell_{14}$ 

 $P_3$ 

 $\ell_{15}$ 

 $P_3$ 

 $\ell_{16}$ 

 $P_3$ 

 $\ell_{17}$ 

 $P_3$ 

 $\ell_{18}$ 

 $P_3$ 

 $\ell_{20}$ 

 $P_3$ 

 $\ell_{21}$ 

 $P_3$ 

 $P_3$ 

 $\ell_{19}$ 

 $P_3$ 

 $\ell_7$ 

 $P_3$ 

 $\ell_6$ 

 $P_3$ 

 $P_3$ 

Line 11 intersects

 $P_3$ 

 $P_3$ 

 $P_3$   $P_3$ 

Line

in point

Line 12 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 13 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 14 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 15 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$ .										
Line 16 in	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$ .										
Line 17 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 18 int		ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 19 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{20}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 20 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{21}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 21 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{22}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 22 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 23 int		ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										
Line 24 int	ersec	ets																				
Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$ $\ell$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

## Line 25 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$											

# Line 26 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	Г
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$											

## Line 27 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	Ī										

### Line 28 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$	_
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$											

# Line 29 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

# Line 30 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

# Line 31 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$	$\ell_{20}$	$\ell_{21}$
in point	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$	$P_3$										

The surface has 1025 points:

Too many to print.