

Rank-76292 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1503221082

General information

Number of lines	4
Number of points	21
Number of singular points	2
Number of Eckardt points	1
Number of double points	1
Number of single points	15
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	$3, 2, 1^{15}, 0^4$

Singular Points

The surface has 2 singular points:

$$0 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$1 : P_{39} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_6 \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(0, 3, 1, 1, 1, 1)_{204} \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{206} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{206} = \mathbf{Pl}(0, 2, 1, 1, 1, 1)_{203}\end{aligned}$$

Rank of lines: (356, 100, 290, 206)

Rank of points on Klein quadric: (1, 6, 204, 203)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1).$$

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

Single Points

The surface has 15 single points:

The single points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0	8 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_1
1 : $P_{17} = (2, 2, 1, 0)$ lies on line ℓ_2	9 : $P_{58} = (1, 1, 2, 1)$ lies on line ℓ_2
2 : $P_{22} = (3, 3, 1, 0)$ lies on line ℓ_3	10 : $P_{63} = (2, 2, 2, 1)$ lies on line ℓ_3
3 : $P_{23} = (1, 0, 0, 1)$ lies on line ℓ_1	11 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_0
4 : $P_{32} = (2, 2, 0, 1)$ lies on line ℓ_2	12 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_1
5 : $P_{37} = (3, 3, 0, 1)$ lies on line ℓ_3	13 : $P_{74} = (1, 1, 3, 1)$ lies on line ℓ_3
6 : $P_{39} = (1, 0, 1, 1)$ lies on line ℓ_1	14 : $P_{84} = (3, 3, 3, 1)$ lies on line ℓ_2
7 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_0	

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:

The points on the surface but not on lines are:

0 : $P_1 = (0, 1, 0, 0)$	3 : $P_{77} = (0, 2, 3, 1)$
1 : $P_{42} = (0, 1, 1, 1)$	
2 : $P_{65} = (0, 3, 2, 1)$	

Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	0	0
2	1	0	0	1
3	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_{38}	P_{38}

Line 1 intersects

Line	ℓ_0
in point	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{38}	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{38}	P_{38}

The surface has 21 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
 1 : $P_2 = (0, 0, 1, 0)$
 2 : $P_3 = (0, 0, 0, 1)$
 3 : $P_{17} = (2, 2, 1, 0)$
 4 : $P_{22} = (3, 3, 1, 0)$
 5 : $P_{23} = (1, 0, 0, 1)$
 6 : $P_{32} = (2, 2, 0, 1)$
 7 : $P_{37} = (3, 3, 0, 1)$

8 : $P_{38} = (0, 0, 1, 1)$
 9 : $P_{39} = (1, 0, 1, 1)$
 10 : $P_{42} = (0, 1, 1, 1)$
 11 : $P_{53} = (0, 0, 2, 1)$
 12 : $P_{54} = (1, 0, 2, 1)$
 13 : $P_{58} = (1, 1, 2, 1)$
 14 : $P_{63} = (2, 2, 2, 1)$
 15 : $P_{65} = (0, 3, 2, 1)$

16 : $P_{69} = (0, 0, 3, 1)$
 17 : $P_{70} = (1, 0, 3, 1)$
 18 : $P_{74} = (1, 1, 3, 1)$
 19 : $P_{77} = (0, 2, 3, 1)$
 20 : $P_{84} = (3, 3, 3, 1)$