

Rank-65903 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244206221

General information

Number of lines	4
Number of points	89
Number of singular points	0
Number of Eckardt points	0
Number of double points	4
Number of single points	28
Number of points off lines	57
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	$2^4, 1^{28}, 0^{57}$

Singular Points

The surface has 0 singular points:

The 4 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{96}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{18}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{1322}$$

Rank of lines: (0, 65, 584, 138)

Rank of points on Klein quadric: (0, 96, 18, 1322)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{138} = (0, 0, 1, 1) = \ell_1 \cap \ell_3$$

Single Points

The surface has 28 single points:

The single points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_3

1 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0

2 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0

3 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0

4 : $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0

5 : $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0

6 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0

7 : $P_{75} = (1, 0, 0, 1)$ lies on line ℓ_2

8 : $P_{83} = (1, 1, 0, 1)$ lies on line ℓ_2

9 : $P_{91} = (1, 2, 0, 1)$ lies on line ℓ_2

10 : $P_{99} = (1, 3, 0, 1)$ lies on line ℓ_2

11 : $P_{107} = (1, 4, 0, 1)$ lies on line ℓ_2

12 : $P_{115} = (1, 5, 0, 1)$ lies on line ℓ_2

13 : $P_{123} = (1, 6, 0, 1)$ lies on line ℓ_2

14 : $P_{131} = (1, 7, 0, 1)$ lies on line ℓ_2

15 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_1

16 : $P_{140} = (2, 0, 1, 1)$ lies on line ℓ_1

17 : $P_{141} = (3, 0, 1, 1)$ lies on line ℓ_1

18 : $P_{142} = (4, 0, 1, 1)$ lies on line ℓ_1

19 : $P_{143} = (5, 0, 1, 1)$ lies on line ℓ_1

20 : $P_{144} = (6, 0, 1, 1)$ lies on line ℓ_1

21 : $P_{145} = (7, 0, 1, 1)$ lies on line ℓ_1

22 : $P_{155} = (2, 2, 1, 1)$ lies on line ℓ_3

23 : $P_{164} = (3, 3, 1, 1)$ lies on line ℓ_3

24 : $P_{173} = (4, 4, 1, 1)$ lies on line ℓ_3

25 : $P_{182} = (5, 5, 1, 1)$ lies on line ℓ_3

26 : $P_{191} = (6, 6, 1, 1)$ lies on line ℓ_3

27 : $P_{200} = (7, 7, 1, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 57 points not on any line:

The points on the surface but not on lines are:

0 : $P_{12} = (1, 0, 1, 0)$	29 : $P_{359} = (6, 3, 4, 1)$
1 : $P_{19} = (0, 1, 1, 0)$	30 : $P_{371} = (2, 5, 4, 1)$
2 : $P_{20} = (1, 1, 1, 0)$	31 : $P_{375} = (6, 5, 4, 1)$
3 : $P_{40} = (5, 3, 1, 0)$	32 : $P_{387} = (2, 7, 4, 1)$
4 : $P_{41} = (6, 3, 1, 0)$	33 : $P_{392} = (7, 7, 4, 1)$
5 : $P_{54} = (3, 5, 1, 0)$	34 : $P_{400} = (7, 0, 5, 1)$
6 : $P_{57} = (6, 5, 1, 0)$	35 : $P_{401} = (0, 1, 5, 1)$
7 : $P_{62} = (3, 6, 1, 0)$	36 : $P_{403} = (2, 1, 5, 1)$
8 : $P_{64} = (5, 6, 1, 0)$	37 : $P_{419} = (2, 3, 5, 1)$
9 : $P_{205} = (4, 0, 2, 1)$	38 : $P_{421} = (4, 3, 5, 1)$
10 : $P_{230} = (5, 3, 2, 1)$	39 : $P_{453} = (4, 7, 5, 1)$
11 : $P_{232} = (7, 3, 2, 1)$	40 : $P_{456} = (7, 7, 5, 1)$
12 : $P_{237} = (4, 4, 2, 1)$	41 : $P_{459} = (2, 0, 6, 1)$
13 : $P_{240} = (7, 4, 2, 1)$	42 : $P_{465} = (0, 1, 6, 1)$
14 : $P_{250} = (1, 6, 2, 1)$	43 : $P_{469} = (4, 1, 6, 1)$
15 : $P_{254} = (5, 6, 2, 1)$	44 : $P_{475} = (2, 2, 6, 1)$
16 : $P_{257} = (0, 7, 2, 1)$	45 : $P_{480} = (7, 2, 6, 1)$
17 : $P_{258} = (1, 7, 2, 1)$	46 : $P_{501} = (4, 5, 6, 1)$
18 : $P_{269} = (4, 0, 3, 1)$	47 : $P_{504} = (7, 5, 6, 1)$
19 : $P_{273} = (0, 1, 3, 1)$	48 : $P_{523} = (2, 0, 7, 1)$
20 : $P_{280} = (7, 1, 3, 1)$	49 : $P_{539} = (2, 2, 7, 1)$
21 : $P_{299} = (2, 4, 3, 1)$	50 : $P_{541} = (4, 2, 7, 1)$
22 : $P_{301} = (4, 4, 3, 1)$	51 : $P_{553} = (0, 4, 7, 1)$
23 : $P_{315} = (2, 6, 3, 1)$	52 : $P_{554} = (1, 4, 7, 1)$
24 : $P_{320} = (7, 6, 3, 1)$	53 : $P_{562} = (1, 5, 7, 1)$
25 : $P_{336} = (7, 0, 4, 1)$	54 : $P_{564} = (3, 5, 7, 1)$
26 : $P_{345} = (0, 2, 4, 1)$	55 : $P_{572} = (3, 6, 7, 1)$
27 : $P_{346} = (1, 2, 4, 1)$	56 : $P_{573} = (4, 6, 7, 1)$
28 : $P_{354} = (1, 3, 4, 1)$	

Line Intersection Graph

	0 1 2 3
0	0 1 1 1
1	1 0 0 1
2	1 0 0 0
3	1 1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_{138}

Line 2 intersects

Line	ℓ_0
in point	P_1

Line 3 intersects

Line	ℓ_0	ℓ_1
in point	P_5	P_{138}

The surface has 89 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_4 = (1, 1, 1, 1)$
 3 : $P_5 = (1, 1, 0, 0)$
 4 : $P_6 = (2, 1, 0, 0)$
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