Rank-73797 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108389

General information

Number of lines	3
Number of points	289
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	48
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^3
Type of lines on points	3, 1^{48}, 0^{240}

Singular Points

The surface has 1 singular points:

$0 : \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$
$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4624} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4624} = \mathbf{Pl}(0,1,1,0,0,0)_{18}$$

Rank of lines: (256, 70160, 4624)
Rank of points on Klein quadric: (2, 1, 18)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 48 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_3 = (0,0,0,1)$ lies on line ℓ_1
$2 : P_{20} = (1,0,1,0)$ lies on line ℓ_0
$3 : P_{21} = (2,0,1,0)$ lies on line ℓ_0
$4 : P_{22} = (3,0,1,0)$ lies on line ℓ_0
$5 : P_{23} = (4,0,1,0)$ lies on line ℓ_0
$6 : P_{24} = (5,0,1,0)$ lies on line ℓ_0
$7 : P_{25} = (6,0,1,0)$ lies on line ℓ_0
$8 : P_{26} = (7,0,1,0)$ lies on line ℓ_0
$9 : P_{27} = (8,0,1,0)$ lies on line ℓ_0
$10 : P_{28} = (9,0,1,0)$ lies on line ℓ_0
$11 : P_{29} = (10,0,1,0)$ lies on line ℓ_0
$12 : P_{30} = (11,0,1,0)$ lies on line ℓ_0
$13 : P_{31} = (12,0,1,0)$ lies on line ℓ_0
$14 : P_{32} = (13,0,1,0)$ lies on line ℓ_0
$15 : P_{33} = (14,0,1,0)$ lies on line ℓ_0
$16 : P_{34} = (15,0,1,0)$ lies on line ℓ_0
$17 : P_{275} = (1,0,0,1)$ lies on line ℓ_2
$18 : P_{530} = (0,0,1,1)$ lies on line ℓ_1
$19 : P_{531} = (1,0,1,1)$ lies on line ℓ_2
$20 : P_{785} = (0,0,2,1)$ lies on line ℓ_1
$21 : P_{786} = (1,0,2,1)$ lies on line ℓ_2
$22 : P_{1041} = (0,0,3,1)$ lies on line ℓ_1
$23 : P_{1042} = (1,0,3,1)$ lies on line ℓ_2
$24 : P_{1297} = (0,0,4,1)$ lies on line ℓ_1
$25 : P_{1298} = (1,0,4,1)$ lies on line ℓ_2
$26 : P_{1553} = (0,0,5,1)$ lies on line ℓ_1
$27 : P_{1554} = (1,0,5,1)$ lies on line ℓ_2
$28 : P_{1809} = (0,0,6,1)$ lies on line ℓ_1
$29 : P_{1810} = (1,0,6,1)$ lies on line ℓ_2
$30 : P_{2065} = (0,0,7,1)$ lies on line ℓ_1
$31 : P_{2066} = (1,0,7,1)$ lies on line ℓ_2
$32 : P_{2321} = (0,0,8,1)$ lies on line ℓ_1
$33 : P_{2322} = (1,0,8,1)$ lies on line ℓ_2
$34 : P_{2577} = (0,0,9,1)$ lies on line ℓ_1
$35 : P_{2578} = (1,0,9,1)$ lies on line ℓ_2
$36 : P_{2833} = (0,0,10,1)$ lies on line ℓ_1
$37 : P_{2834} = (1,0,10,1)$ lies on line ℓ_2
$38 : P_{3089} = (0,0,11,1)$ lies on line ℓ_1
$39 : P_{3090} = (1,0,11,1)$ lies on line ℓ_2
$40 : P_{3345} = (0,0,12,1)$ lies on line ℓ_1
$41 : P_{3346} = (1,0,12,1)$ lies on line ℓ_2
$42 : P_{3601} = (0,0,13,1)$ lies on line ℓ_1
$43 : P_{3602} = (1,0,13,1)$ lies on line ℓ_2
$44 : P_{3857} = (0,0,14,1)$ lies on line ℓ_1
$45 : P_{3858} = (1,0,14,1)$ lies on line ℓ_2
$46 : P_{4113} = (0,0,15,1)$ lies on line ℓ_1
$47 : P_{4114} = (1,0,15,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line:
The points on the surface but not on lines are:

$0: P_4 = (1,1,1,1)$
$1: P_{36} = (1,1,1,0)$
$2: P_{55} = (4,2,1,0)$
$3: P_{72} = (5,3,1,0)$
$4: P_{92} = (9,4,1,0)$
$5: P_{107} = (8,5,1,0)$
$6: P_{128} = (13,6,1,0)$
$7: P_{143} = (12,7,1,0)$
$8: P_{162} = (15,8,1,0)$
$9: P_{177} = (14,9,1,0)$
$10: P_{190} = (11,10,1,0)$
$11: P_{205} = (10,11,1,0)$
$12: P_{217} = (6,12,1,0)$
$13: P_{234} = (7,13,1,0)$
$14: P_{245} = (2,14,1,0)$
$15: P_{262} = (3,15,1,0)$
$16: P_{300} = (10,1,0,1)$
$17: P_{301} = (11,1,0,1)$
$18: P_{444} = (10,10,0,1)$
$19: P_{445} = (11,10,0,1)$
$20: P_{460} = (10,11,0,1)$
$21: P_{461} = (11,11,0,1)$
$22: P_{633} = (8,6,1,1)$
$23: P_{640} = (15,6,1,1)$
$24: P_{644} = (3,7,1,1)$
$25: P_{646} = (5,7,1,1)$
$26: P_{692} = (3,10,1,1)$
$27: P_{697} = (8,10,1,1)$
$28: P_{710} = (5,11,1,1)$
$29: P_{720} = (15,11,1,1)$
$30: P_{726} = (5,12,1,1)$
$31: P_{729} = (8,12,1,1)$
$32: P_{740} = (3,13,1,1)$
$33: P_{752} = (15,13,1,1)$
$34: P_{826} = (9,2,2,1)$
$35: P_{829} = (12,2,2,1)$
$36: P_{843} = (10,3,2,1)$
$37: P_{846} = (13,3,2,1)$
$38: P_{851} = (2,4,2,1)$
$39: P_{860} = (11,4,2,1)$
$40: P_{935} = (6,9,2,1)$
$41: P_{941} = (12,9,2,1)$
$42: P_{963} = (2,11,2,1)$
$43: P_{973} = (12,11,2,1)$
$44: P_{992} = (15,12,2,1)$
$45: P_{1010} = (1,14,2,1)$
$46: P_{1014} = (5,14,2,1)$
$47: P_{1027} = (2,15,2,1)$
$48: P_{1029} = (4,15,2,1)$
$49: P_{1061} = (4,1,3,1)$
$50: P_{1063} = (6,1,3,1)$
$51: P_{1109} = (4,4,3,1)$
$52: P_{1114} = (9,4,3,1)$
$53: P_{1125} = (4,5,3,1)$
$54: P_{1131} = (10,5,3,1)$
$55: P_{1143} = (6,6,3,1)$
$56: P_{1150} = (13,6,3,1)$
$57: P_{1159} = (6,7,3,1)$
$58: P_{1167} = (14,7,3,1)$
$59: P_{1172} = (3,8,3,1)$
$60: P_{1240} = (7,12,3,1)$
$61: P_{1244} = (11,12,3,1)$
$62: P_{1282} = (1,15,3,1)$
$63: P_{1289} = (8,15,3,1)$
$64: P_{1330} = (1,2,4,1)$
$65: P_{1337} = (8,2,4,1)$
$66: P_{1349} = (4,3,4,1)$
$67: P_{1354} = (9,3,4,1)$
$68: P_{1367} = (6,4,4,1)$
$69: P_{1375} = (14,4,4,1)$
$70: P_{1384} = (7,5,4,1)$
$71: P_{1388} = (11,5,4,1)$
$72: P_{1396} = (3,6,4,1)$
$73: P_{1445} = (4,9,4,1)$
$74: P_{1451} = (10,9,4,1)$
$75: P_{1461} = (4,10,4,1)$
$76: P_{1463} = (6,10,4,1)$
$77: P_{1527} = (6,14,4,1)$
$78: P_{1534} = (13,14,4,1)$
$79: P_{1578} = (9,1,5,1)$
$80: P_{1582} = (13,1,5,1)$
$81: P_{1602} = (1,3,5,1)$
$82: P_{1616} = (15,3,5,1)$
$83: P_{1659} = (10,6,5,1)$
$84: P_{1661} = (12,6,5,1)$
$85: P_{1690} = (9,8,5,1)$
$86: P_{1692} = (11,8,5,1)$
$87: P_{1706} = (9,9,5,1)$
$88: P_{1711} = (14,9,5,1)$
$89: P_{1747} = (2,12,5,1)$
$90: P_{1758} = (13,12,5,1)$
$91: P_{1768} = (7,13,5,1)$
$92: P_{1774} = (13,13,5,1)$
$93: P_{1798} = (5,15,5,1)$
$94: P_{1847} = (6,2,6,1)$
$95: P_{1852} = (11,2,6,1)$

96 : $P_{1864} = (7, 3, 6, 1)$
97 : $P_{1869} = (12, 3, 6, 1)$
98 : $P_{1881} = (8, 4, 6, 1)$
99 : $P_{1907} = (2, 6, 6, 1)$
100 : $P_{1919} = (14, 6, 6, 1)$
101 : $P_{1941} = (4, 8, 6, 1)$
102 : $P_{1944} = (7, 8, 6, 1)$
103 : $P_{1992} = (7, 11, 6, 1)$
104 : $P_{1999} = (14, 11, 6, 1)$
105 : $P_{2002} = (1, 12, 6, 1)$
106 : $P_{2004} = (3, 12, 6, 1)$
107 : $P_{2027} = (10, 13, 6, 1)$
108 : $P_{2031} = (14, 13, 6, 1)$
109 : $P_{2119} = (6, 3, 7, 1)$
110 : $P_{2127} = (14, 3, 7, 1)$
111 : $P_{2181} = (4, 7, 7, 1)$
112 : $P_{2186} = (9, 7, 7, 1)$
113 : $P_{2199} = (6, 8, 7, 1)$
114 : $P_{2206} = (13, 8, 7, 1)$
115 : $P_{2216} = (7, 9, 7, 1)$
116 : $P_{2220} = (11, 9, 7, 1)$
117 : $P_{2245} = (4, 11, 7, 1)$
118 : $P_{2247} = (6, 11, 7, 1)$
119 : $P_{2261} = (4, 12, 7, 1)$
120 : $P_{2267} = (10, 12, 7, 1)$
121 : $P_{2274} = (1, 13, 7, 1)$
122 : $P_{2281} = (8, 13, 7, 1)$
123 : $P_{2292} = (3, 14, 7, 1)$
124 : $P_{2344} = (7, 1, 8, 1)$
125 : $P_{2351} = (14, 1, 8, 1)$
126 : $P_{2377} = (8, 3, 8, 1)$
127 : $P_{2402} = (1, 5, 8, 1)$
128 : $P_{2404} = (3, 5, 8, 1)$
129 : $P_{2421} = (4, 6, 8, 1)$
130 : $P_{2424} = (7, 6, 8, 1)$
131 : $P_{2440} = (7, 7, 8, 1)$
132 : $P_{2445} = (12, 7, 8, 1)$
133 : $P_{2535} = (6, 13, 8, 1)$
134 : $P_{2540} = (11, 13, 8, 1)$
135 : $P_{2547} = (2, 14, 8, 1)$
136 : $P_{2559} = (14, 14, 8, 1)$
137 : $P_{2571} = (10, 15, 8, 1)$
138 : $P_{2575} = (14, 15, 8, 1)$
139 : $P_{2616} = (7, 2, 9, 1)$
140 : $P_{2622} = (13, 2, 9, 1)$
141 : $P_{2642} = (1, 4, 9, 1)$
142 : $P_{2656} = (15, 4, 9, 1)$
143 : $P_{2666} = (9, 5, 9, 1)$
144 : $P_{2671} = (14, 5, 9, 1)$
145 : $P_{2715} = (10, 8, 9, 1)$
146 : $P_{2717} = (12, 8, 9, 1)$
147 : $P_{2723} = (2, 9, 9, 1)$
148 : $P_{2734} = (13, 9, 9, 1)$
149 : $P_{2762} = (9, 11, 9, 1)$
150 : $P_{2766} = (13, 11, 9, 1)$
151 : $P_{2790} = (5, 13, 9, 1)$
152 : $P_{2810} = (9, 14, 9, 1)$
153 : $P_{2812} = (11, 14, 9, 1)$
154 : $P_{2852} = (3, 1, 10, 1)$
155 : $P_{2857} = (8, 1, 10, 1)$
156 : $P_{2868} = (3, 2, 10, 1)$
157 : $P_{2880} = (15, 2, 10, 1)$
158 : $P_{2884} = (3, 3, 10, 1)$
159 : $P_{2886} = (5, 3, 10, 1)$
160 : $P_{2969} = (8, 8, 10, 1)$
161 : $P_{2976} = (15, 8, 10, 1)$
162 : $P_{2982} = (5, 9, 10, 1)$
163 : $P_{2985} = (8, 9, 10, 1)$
164 : $P_{2998} = (5, 10, 10, 1)$
165 : $P_{3008} = (15, 10, 10, 1)$
166 : $P_{3010} = (1, 11, 10, 1)$
167 : $P_{3110} = (5, 1, 11, 1)$
168 : $P_{3120} = (15, 1, 11, 1)$
169 : $P_{3156} = (3, 4, 11, 1)$
170 : $P_{3158} = (5, 4, 11, 1)$
171 : $P_{3174} = (5, 5, 11, 1)$
172 : $P_{3177} = (8, 5, 11, 1)$
173 : $P_{3250} = (1, 10, 11, 1)$
174 : $P_{3268} = (3, 11, 11, 1)$
175 : $P_{3273} = (8, 11, 11, 1)$
176 : $P_{3321} = (8, 14, 11, 1)$
177 : $P_{3328} = (15, 14, 11, 1)$
178 : $P_{3332} = (3, 15, 11, 1)$
179 : $P_{3344} = (15, 15, 11, 1)$
180 : $P_{3382} = (5, 2, 12, 1)$
181 : $P_{3427} = (2, 5, 12, 1)$
182 : $P_{3438} = (13, 5, 12, 1)$
183 : $P_{3450} = (9, 6, 12, 1)$
184 : $P_{3452} = (11, 6, 12, 1)$
185 : $P_{3458} = (1, 7, 12, 1)$
186 : $P_{3472} = (15, 7, 12, 1)$
187 : $P_{3514} = (9, 10, 12, 1)$
188 : $P_{3518} = (13, 10, 12, 1)$
189 : $P_{3546} = (9, 12, 12, 1)$
190 : $P_{3551} = (14, 12, 12, 1)$
191 : $P_{3579} = (10, 14, 12, 1)$
192 : $P_{3581} = (12, 14, 12, 1)$
193 : $P_{3592} = (7, 15, 12, 1)$
194 : $P_{3598} = (13, 15, 12, 1)$
195 : $P_{3675} = (10, 4, 13, 1)$
196 : $P_{3678} = (13, 4, 13, 1)$
197 : $P_{3687} = (6, 5, 13, 1)$
198 : $P_{3693} = (12, 5, 13, 1)$
199 : $P_{3698} = (1, 6, 13, 1)$
200 : $P_{3702} = (5, 6, 13, 1)$
201 : $P_{3715} = (2, 7, 13, 1)$
202 : $P_{3724} = (11, 7, 13, 1)$
203 : $P_{3760} = (15, 9, 13, 1)$

204 : $P_{3763} = (2, 10, 13, 1)$
205 : $P_{3773} = (12, 10, 13, 1)$
206 : $P_{3811} = (2, 13, 13, 1)$
207 : $P_{3813} = (4, 13, 13, 1)$
208 : $P_{3850} = (9, 15, 13, 1)$
209 : $P_{3853} = (12, 15, 13, 1)$
210 : $P_{3899} = (10, 2, 14, 1)$
211 : $P_{3903} = (14, 2, 14, 1)$
212 : $P_{3928} = (7, 4, 14, 1)$
213 : $P_{3933} = (12, 4, 14, 1)$
214 : $P_{3977} = (8, 7, 14, 1)$
215 : $P_{3987} = (2, 8, 14, 1)$
216 : $P_{3999} = (14, 8, 14, 1)$
217 : $P_{4002} = (1, 9, 14, 1)$
218 : $P_{4004} = (3, 9, 14, 1)$
219 : $P_{4024} = (7, 10, 14, 1)$
220 : $P_{4031} = (14, 10, 14, 1)$
221 : $P_{4085} = (4, 14, 14, 1)$
222 : $P_{4088} = (7, 14, 14, 1)$
223 : $P_{4103} = (6, 15, 14, 1)$
224 : $P_{4108} = (11, 15, 14, 1)$
225 : $P_{4131} = (2, 1, 15, 1)$
226 : $P_{4141} = (12, 1, 15, 1)$
227 : $P_{4147} = (2, 2, 15, 1)$
228 : $P_{4149} = (4, 2, 15, 1)$
229 : $P_{4163} = (2, 3, 15, 1)$
230 : $P_{4172} = (11, 3, 15, 1)$
231 : $P_{4208} = (15, 5, 15, 1)$
232 : $P_{4235} = (10, 7, 15, 1)$
233 : $P_{4238} = (13, 7, 15, 1)$
234 : $P_{4242} = (1, 8, 15, 1)$
235 : $P_{4246} = (5, 8, 15, 1)$
236 : $P_{4311} = (6, 12, 15, 1)$
237 : $P_{4317} = (12, 12, 15, 1)$
238 : $P_{4330} = (9, 13, 15, 1)$
239 : $P_{4333} = (12, 13, 15, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{20} = (1, 0, 1, 0)$
5 : $P_{21} = (2, 0, 1, 0)$
6 : $P_{22} = (3, 0, 1, 0)$
7 : $P_{23} = (4, 0, 1, 0)$
8 : $P_{24} = (5, 0, 1, 0)$
9 : $P_{25} = (6, 0, 1, 0)$
10 : $P_{26} = (7, 0, 1, 0)$
11 : $P_{27} = (8, 0, 1, 0)$
12 : $P_{28} = (9, 0, 1, 0)$
13 : $P_{29} = (10, 0, 1, 0)$
14 : $P_{30} = (11, 0, 1, 0)$
15 : $P_{31} = (12, 0, 1, 0)$
16 : $P_{32} = (13, 0, 1, 0)$
17 : $P_{33} = (14, 0, 1, 0)$
18 : $P_{34} = (15, 0, 1, 0)$
19 : $P_{36} = (1, 1, 1, 0)$
20 : $P_{55} = (4, 2, 1, 0)$
21 : $P_{72} = (5, 3, 1, 0)$
22 : $P_{92} = (9, 4, 1, 0)$
23 : $P_{107} = (8, 5, 1, 0)$
24 : $P_{128} = (13, 6, 1, 0)$
25 : $P_{143} = (12, 7, 1, 0)$
26 : $P_{162} = (15, 8, 1, 0)$
27 : $P_{177} = (14, 9, 1, 0)$
28 : $P_{190} = (11, 10, 1, 0)$
29 : $P_{205} = (10, 11, 1, 0)$
30 : $P_{217} = (6, 12, 1, 0)$
31 : $P_{234} = (7, 13, 1, 0)$
32 : $P_{245} = (2, 14, 1, 0)$
33 : $P_{262} = (3, 15, 1, 0)$
34 : $P_{275} = (1, 0, 0, 1)$
35 : $P_{300} = (10, 1, 0, 1)$
36 : $P_{301} = (11, 1, 0, 1)$
37 : $P_{444} = (10, 10, 0, 1)$
38 : $P_{445} = (11, 10, 0, 1)$

39 : $P_{460} = (10, 11, 0, 1)$
40 : $P_{461} = (11, 11, 0, 1)$
41 : $P_{530} = (0, 0, 1, 1)$
42 : $P_{531} = (1, 0, 1, 1)$
43 : $P_{633} = (8, 6, 1, 1)$
44 : $P_{640} = (15, 6, 1, 1)$
45 : $P_{644} = (3, 7, 1, 1)$
46 : $P_{646} = (5, 7, 1, 1)$
47 : $P_{692} = (3, 10, 1, 1)$
48 : $P_{697} = (8, 10, 1, 1)$
49 : $P_{710} = (5, 11, 1, 1)$
50 : $P_{720} = (15, 11, 1, 1)$
51 : $P_{726} = (5, 12, 1, 1)$
52 : $P_{729} = (8, 12, 1, 1)$
53 : $P_{740} = (3, 13, 1, 1)$
54 : $P_{752} = (15, 13, 1, 1)$
55 : $P_{785} = (0, 0, 2, 1)$
56 : $P_{786} = (1, 0, 2, 1)$
57 : $P_{826} = (9, 2, 2, 1)$
58 : $P_{829} = (12, 2, 2, 1)$
59 : $P_{843} = (10, 3, 2, 1)$
60 : $P_{846} = (13, 3, 2, 1)$
61 : $P_{851} = (2, 4, 2, 1)$
62 : $P_{860} = (11, 4, 2, 1)$
63 : $P_{935} = (6, 9, 2, 1)$
64 : $P_{941} = (12, 9, 2, 1)$
65 : $P_{963} = (2, 11, 2, 1)$
66 : $P_{973} = (12, 11, 2, 1)$
67 : $P_{992} = (15, 12, 2, 1)$
68 : $P_{1010} = (1, 14, 2, 1)$
69 : $P_{1014} = (5, 14, 2, 1)$
70 : $P_{1027} = (2, 15, 2, 1)$
71 : $P_{1029} = (4, 15, 2, 1)$
72 : $P_{1041} = (0, 0, 3, 1)$
73 : $P_{1042} = (1, 0, 3, 1)$
74 : $P_{1061} = (4, 1, 3, 1)$
75 : $P_{1063} = (6, 1, 3, 1)$
76 : $P_{1109} = (4, 4, 3, 1)$
77 : $P_{1114} = (9, 4, 3, 1)$
78 : $P_{1125} = (4, 5, 3, 1)$
79 : $P_{1131} = (10, 5, 3, 1)$
80 : $P_{1143} = (6, 6, 3, 1)$
81 : $P_{1150} = (13, 6, 3, 1)$
82 : $P_{1159} = (6, 7, 3, 1)$
83 : $P_{1167} = (14, 7, 3, 1)$
84 : $P_{1172} = (3, 8, 3, 1)$
85 : $P_{1240} = (7, 12, 3, 1)$
86 : $P_{1244} = (11, 12, 3, 1)$
87 : $P_{1282} = (1, 15, 3, 1)$
88 : $P_{1289} = (8, 15, 3, 1)$
89 : $P_{1297} = (0, 0, 4, 1)$
90 : $P_{1298} = (1, 0, 4, 1)$
91 : $P_{1330} = (1, 2, 4, 1)$
92 : $P_{1337} = (8, 2, 4, 1)$
93 : $P_{1349} = (4, 3, 4, 1)$
94 : $P_{1354} = (9, 3, 4, 1)$
95 : $P_{1367} = (6, 4, 4, 1)$
96 : $P_{1375} = (14, 4, 4, 1)$
97 : $P_{1384} = (7, 5, 4, 1)$
98 : $P_{1388} = (11, 5, 4, 1)$
99 : $P_{1396} = (3, 6, 4, 1)$
100 : $P_{1445} = (4, 9, 4, 1)$
101 : $P_{1451} = (10, 9, 4, 1)$
102 : $P_{1461} = (4, 10, 4, 1)$
103 : $P_{1463} = (6, 10, 4, 1)$
104 : $P_{1527} = (6, 14, 4, 1)$
105 : $P_{1534} = (13, 14, 4, 1)$
106 : $P_{1553} = (0, 0, 5, 1)$
107 : $P_{1554} = (1, 0, 5, 1)$
108 : $P_{1578} = (9, 1, 5, 1)$
109 : $P_{1582} = (13, 1, 5, 1)$
110 : $P_{1602} = (1, 3, 5, 1)$
111 : $P_{1616} = (15, 3, 5, 1)$
112 : $P_{1659} = (10, 6, 5, 1)$
113 : $P_{1661} = (12, 6, 5, 1)$
114 : $P_{1690} = (9, 8, 5, 1)$
115 : $P_{1692} = (11, 8, 5, 1)$
116 : $P_{1706} = (9, 9, 5, 1)$
117 : $P_{1711} = (14, 9, 5, 1)$
118 : $P_{1747} = (2, 12, 5, 1)$
119 : $P_{1758} = (13, 12, 5, 1)$
120 : $P_{1768} = (7, 13, 5, 1)$
121 : $P_{1774} = (13, 13, 5, 1)$
122 : $P_{1798} = (5, 15, 5, 1)$
123 : $P_{1809} = (0, 0, 6, 1)$
124 : $P_{1810} = (1, 0, 6, 1)$
125 : $P_{1847} = (6, 2, 6, 1)$
126 : $P_{1852} = (11, 2, 6, 1)$
127 : $P_{1864} = (7, 3, 6, 1)$
128 : $P_{1869} = (12, 3, 6, 1)$
129 : $P_{1881} = (8, 4, 6, 1)$
130 : $P_{1907} = (2, 6, 6, 1)$
131 : $P_{1919} = (14, 6, 6, 1)$
132 : $P_{1941} = (4, 8, 6, 1)$
133 : $P_{1944} = (7, 8, 6, 1)$
134 : $P_{1992} = (7, 11, 6, 1)$
135 : $P_{1999} = (14, 11, 6, 1)$
136 : $P_{2002} = (1, 12, 6, 1)$
137 : $P_{2004} = (3, 12, 6, 1)$
138 : $P_{2027} = (10, 13, 6, 1)$
139 : $P_{2031} = (14, 13, 6, 1)$
140 : $P_{2065} = (0, 0, 7, 1)$
141 : $P_{2066} = (1, 0, 7, 1)$
142 : $P_{2119} = (6, 3, 7, 1)$
143 : $P_{2127} = (14, 3, 7, 1)$
144 : $P_{2181} = (4, 7, 7, 1)$
145 : $P_{2186} = (9, 7, 7, 1)$
146 : $P_{2199} = (6, 8, 7, 1)$
147 : $P_{2206} = (13, 8, 7, 1)$
148 : $P_{2216} = (7, 9, 7, 1)$
149 : $P_{2220} = (11, 9, 7, 1)$
150 : $P_{2245} = (4, 11, 7, 1)$
151 : $P_{2247} = (6, 11, 7, 1)$
152 : $P_{2261} = (4, 12, 7, 1)$
153 : $P_{2267} = (10, 12, 7, 1)$
154 : $P_{2274} = (1, 13, 7, 1)$
155 : $P_{2281} = (8, 13, 7, 1)$
156 : $P_{2292} = (3, 14, 7, 1)$
157 : $P_{2321} = (0, 0, 8, 1)$
158 : $P_{2322} = (1, 0, 8, 1)$
159 : $P_{2344} = (7, 1, 8, 1)$
160 : $P_{2351} = (14, 1, 8, 1)$
161 : $P_{2377} = (8, 3, 8, 1)$
162 : $P_{2402} = (1, 5, 8, 1)$
163 : $P_{2404} = (3, 5, 8, 1)$
164 : $P_{2421} = (4, 6, 8, 1)$
165 : $P_{2424} = (7, 6, 8, 1)$
166 : $P_{2440} = (7, 7, 8, 1)$
167 : $P_{2445} = (12, 7, 8, 1)$
168 : $P_{2535} = (6, 13, 8, 1)$
169 : $P_{2540} = (11, 13, 8, 1)$
170 : $P_{2547} = (2, 14, 8, 1)$
171 : $P_{2559} = (14, 14, 8, 1)$
172 : $P_{2571} = (10, 15, 8, 1)$
173 : $P_{2575} = (14, 15, 8, 1)$
174 : $P_{2577} = (0, 0, 9, 1)$
175 : $P_{2578} = (1, 0, 9, 1)$
176 : $P_{2616} = (7, 2, 9, 1)$
177 : $P_{2622} = (13, 2, 9, 1)$
178 : $P_{2642} = (1, 4, 9, 1)$
179 : $P_{2656} = (15, 4, 9, 1)$
180 : $P_{2666} = (9, 5, 9, 1)$
181 : $P_{2671} = (14, 5, 9, 1)$
182 : $P_{2715} = (10, 8, 9, 1)$
183 : $P_{2717} = (12, 8, 9, 1)$
184 : $P_{2723} = (2, 9, 9, 1)$
185 : $P_{2734} = (13, 9, 9, 1)$
186 : $P_{2762} = (9, 11, 9, 1)$
187 : $P_{2766} = (13, 11, 9, 1)$
188 : $P_{2790} = (5, 13, 9, 1)$
189 : $P_{2810} = (9, 14, 9, 1)$
190 : $P_{2812} = (11, 14, 9, 1)$
191 : $P_{2833} = (0, 0, 10, 1)$
192 : $P_{2834} = (1, 0, 10, 1)$
193 : $P_{2852} = (3, 1, 10, 1)$
194 : $P_{2857} = (8, 1, 10, 1)$
195 : $P_{2868} = (3, 2, 10, 1)$
196 : $P_{2880} = (15, 2, 10, 1)$
197 : $P_{2884} = (3, 3, 10, 1)$
198 : $P_{2886} = (5, 3, 10, 1)$
199 : $P_{2969} = (8, 8, 10, 1)$
200 : $P_{2976} = (15, 8, 10, 1)$

201 : $P_{2982} = (5, 9, 10, 1)$
202 : $P_{2985} = (8, 9, 10, 1)$
203 : $P_{2998} = (5, 10, 10, 1)$
204 : $P_{3008} = (15, 10, 10, 1)$
205 : $P_{3010} = (1, 11, 10, 1)$
206 : $P_{3089} = (0, 0, 11, 1)$
207 : $P_{3090} = (1, 0, 11, 1)$
208 : $P_{3110} = (5, 1, 11, 1)$
209 : $P_{3120} = (15, 1, 11, 1)$
210 : $P_{3156} = (3, 4, 11, 1)$
211 : $P_{3158} = (5, 4, 11, 1)$
212 : $P_{3174} = (5, 5, 11, 1)$
213 : $P_{3177} = (8, 5, 11, 1)$
214 : $P_{3250} = (1, 10, 11, 1)$
215 : $P_{3268} = (3, 11, 11, 1)$
216 : $P_{3273} = (8, 11, 11, 1)$
217 : $P_{3321} = (8, 14, 11, 1)$
218 : $P_{3328} = (15, 14, 11, 1)$
219 : $P_{3332} = (3, 15, 11, 1)$
220 : $P_{3344} = (15, 15, 11, 1)$
221 : $P_{3345} = (0, 0, 12, 1)$
222 : $P_{3346} = (1, 0, 12, 1)$
223 : $P_{3382} = (5, 2, 12, 1)$
224 : $P_{3427} = (2, 5, 12, 1)$
225 : $P_{3438} = (13, 5, 12, 1)$
226 : $P_{3450} = (9, 6, 12, 1)$
227 : $P_{3452} = (11, 6, 12, 1)$
228 : $P_{3458} = (1, 7, 12, 1)$
229 : $P_{3472} = (15, 7, 12, 1)$
230 : $P_{3514} = (9, 10, 12, 1)$
231 : $P_{3518} = (13, 10, 12, 1)$
232 : $P_{3546} = (9, 12, 12, 1)$
233 : $P_{3551} = (14, 12, 12, 1)$
234 : $P_{3579} = (10, 14, 12, 1)$
235 : $P_{3581} = (12, 14, 12, 1)$
236 : $P_{3592} = (7, 15, 12, 1)$
237 : $P_{3598} = (13, 15, 12, 1)$
238 : $P_{3601} = (0, 0, 13, 1)$
239 : $P_{3602} = (1, 0, 13, 1)$
240 : $P_{3675} = (10, 4, 13, 1)$
241 : $P_{3678} = (13, 4, 13, 1)$
242 : $P_{3687} = (6, 5, 13, 1)$
243 : $P_{3693} = (12, 5, 13, 1)$
244 : $P_{3698} = (1, 6, 13, 1)$
245 : $P_{3702} = (5, 6, 13, 1)$
246 : $P_{3715} = (2, 7, 13, 1)$
247 : $P_{3724} = (11, 7, 13, 1)$
248 : $P_{3760} = (15, 9, 13, 1)$
249 : $P_{3763} = (2, 10, 13, 1)$
250 : $P_{3773} = (12, 10, 13, 1)$
251 : $P_{3811} = (2, 13, 13, 1)$
252 : $P_{3813} = (4, 13, 13, 1)$
253 : $P_{3850} = (9, 15, 13, 1)$
254 : $P_{3853} = (12, 15, 13, 1)$
255 : $P_{3857} = (0, 0, 14, 1)$
256 : $P_{3858} = (1, 0, 14, 1)$
257 : $P_{3899} = (10, 2, 14, 1)$
258 : $P_{3903} = (14, 2, 14, 1)$
259 : $P_{3928} = (7, 4, 14, 1)$
260 : $P_{3933} = (12, 4, 14, 1)$
261 : $P_{3977} = (8, 7, 14, 1)$
262 : $P_{3987} = (2, 8, 14, 1)$
263 : $P_{3999} = (14, 8, 14, 1)$
264 : $P_{4002} = (1, 9, 14, 1)$
265 : $P_{4004} = (3, 9, 14, 1)$
266 : $P_{4024} = (7, 10, 14, 1)$
267 : $P_{4031} = (14, 10, 14, 1)$
268 : $P_{4085} = (4, 14, 14, 1)$
269 : $P_{4088} = (7, 14, 14, 1)$
270 : $P_{4103} = (6, 15, 14, 1)$
271 : $P_{4108} = (11, 15, 14, 1)$
272 : $P_{4113} = (0, 0, 15, 1)$
273 : $P_{4114} = (1, 0, 15, 1)$
274 : $P_{4131} = (2, 1, 15, 1)$
275 : $P_{4141} = (12, 1, 15, 1)$
276 : $P_{4147} = (2, 2, 15, 1)$
277 : $P_{4149} = (4, 2, 15, 1)$
278 : $P_{4163} = (2, 3, 15, 1)$
279 : $P_{4172} = (11, 3, 15, 1)$
280 : $P_{4208} = (15, 5, 15, 1)$
281 : $P_{4235} = (10, 7, 15, 1)$
282 : $P_{4238} = (13, 7, 15, 1)$
283 : $P_{4242} = (1, 8, 15, 1)$
284 : $P_{4246} = (5, 8, 15, 1)$
285 : $P_{4311} = (6, 12, 15, 1)$
286 : $P_{4317} = (12, 12, 15, 1)$
287 : $P_{4330} = (9, 13, 15, 1)$
288 : $P_{4333} = (12, 13, 15, 1)$