Rank-65921 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244210325

General information

Number of lines	21
Number of points	113
Number of singular points	1
Number of Eckardt points	3
Number of double points	66
Number of single points	42
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{21}
Type of lines on points	6, 3^3, 2^{66}, 1^{42}, 0

Singular Points

The surface has 1 singular points:

$0: \ P_{139} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \mathbf{Pl}(1,0,0,0,1,0)_{82}$$

$$\ell_{1} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{65} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{65} = \mathbf{Pl}(0,0,1,0,1,0)_{96}$$

$$\ell_{2} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{138} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_{3} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{666} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{666} = \mathbf{Pl}(1,0,1,1,1,1)_{1323}$$

$$\ell_{4} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{593} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]_{593} = \mathbf{Pl}(1,1,1,1,1,0)_{306}$$

$$\ell_{5} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{665} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right]_{665} = \mathbf{Pl}(1,1,0,1,1,1)_{1273}$$

$$\ell_{6} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{5} & \gamma^{6} \\ 0 & 1 & \gamma^{6} & 0 \end{array}\right]_{3729} = \left[\begin{array}{cccc} 1 & 0 & 3 & 6 \\ 0 & 1 & 6 & 0 \end{array}\right]_{3729} = \mathbf{Pl}(1,1,2,2,0,1)_{930}$$

$$\ell_{7} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{5} & \gamma^{5} \\ 0 & 1 & \gamma^{6} & 0 \end{array}\right]_{1977} = \left[\begin{array}{cccc} 1 & 0 & 3 & 3 \\ 0 & 1 & 6 & 0 \end{array}\right]_{1977} = \mathbf{Pl}(2,6,2,1,0,1)_{882}$$

$$\ell_{8} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{5} & \gamma^{3} \\ 0 & 1 & \gamma^{6} & 0 \end{array}\right]_{3145} = \left[\begin{array}{cccc} 1 & 0 & 3 & 5 \\ 0 & 1 & 6 & 0 \end{array}\right]_{3145} = \mathbf{Pl}(5,7,2,3,0,1)_{983}$$

$$\ell_{9} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{5} & \gamma^{6} \\ 0 & 1 & \gamma^{4} & \gamma \end{array}\right]_{3746} = \left[\begin{array}{cccc} 1 & 0 & 3 & 6 \\ 0 & 1 & 7 & 2 \end{array}\right]_{3746} = \mathbf{Pl}(1,1,6,2,5,1)_{3688}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma & \gamma^{2} \end{array}\right]_{691} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 4 \end{array}\right]_{691} = \mathbf{Pl}(2,6,1,1,4,1)_{2968}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{3} & \gamma^{5} \\ 0 & 1 & \gamma^{5} & 0 \end{array}\right]_{2120} = \left[\begin{array}{cccc} 1 & 0 & 5 & 3 \\ 0 & 1 & 3 & 0 \end{array}\right]_{2120} = \mathbf{Pl}(1,1,4,4,0,1)_{1042}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{3} & \gamma^{3} \\ 0 & 1 & \gamma^{5} & 0 \end{array}\right]_{3288} = \left[\begin{array}{cccc} 1 & 0 & 5 & 5 \\ 0 & 1 & 3 & 0 \end{array}\right]_{3288} = \mathbf{Pl}(4,3,4,1,0,1)_{898}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{3} & \gamma^{6} \\ 0 & 1 & \gamma^{5} & 0 \end{array}\right]_{3872} = \left[\begin{array}{cccc} 1 & 0 & 5 & 6 \\ 0 & 1 & 3 & 0 \end{array}\right]_{3872} = \mathbf{Pl}(6,2,4,5,0,1)_{1096}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{3} & \gamma^{5} \\ 0 & 1 & \gamma & \gamma^{2} \end{array}\right]_{2151} = \left[\begin{array}{cccc} 1 & 0 & 5 & 3 \\ 0 & 1 & 2 & 4 \end{array}\right]_{2151} = \mathbf{Pl}(1,1,3,4,6,1)_{4031}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma^{2} & \gamma^{4} \end{array}\right]_{717} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 4 & 7 \end{array}\right]_{717} = \mathbf{Pl}(4,3,1,1,7,1)_{4461}$$

$$\ell_{16} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{6} & \gamma^{3} \\ 0 & 1 & \gamma^{3} & 0 \end{array}\right]_{3363} = \left[\begin{array}{cccc} 1 & 0 & 6 & 5 \\ 0 & 1 & 5 & 0 \end{array}\right]_{3363} = \mathbf{Pl}(1,1,7,7,0,1)_{1210}$$

$$\ell_{17} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{6} & \gamma^{5} \\ 0 & 1 & \gamma^{3} & 0 \end{array}\right]_{2195} = \left[\begin{array}{cccc} 1 & 0 & 6 & 3 \\ 0 & 1 & 5 & 0 \end{array}\right]_{2195} = \mathbf{Pl}(3,4,7,6,0,1)_{1163}$$

$$\ell_{18} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{6} & \gamma^{6} \\ 0 & 1 & \gamma^{3} & 0 \end{array}\right]_{3947} = \left[\begin{array}{cccc} 1 & 0 & 6 & 6 \\ 0 & 1 & 5 & 0 \end{array}\right]_{3947} = \mathbf{Pl}(7,5,7,1,0,1)_{922}$$

$$\ell_{19} = \left[\begin{array}{cccc} 1 & 0 & \gamma^{6} & \gamma^{3} \\ 0 & 1 & \gamma^{2} & \gamma^{4} \end{array}\right]_{3418} = \left[\begin{array}{cccc} 1 & 0 & 6 & 5 \\ 0 & 1 & 4 & 7 \end{array}\right]_{3418} = \mathbf{Pl}(1,1,5,7,3,1)_{2617}$$

$$\ell_{20} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma^{4} & \gamma \end{array}\right]_{680} = \left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & 7 & 2 \end{array}\right]_{680} = \mathbf{Pl}(7,5,1,1,2,1)_{1958}$$

Rank of lines: (8, 65, 138, 666, 593, 665, 3729, 1977, 3145, 3746, 691, 2120, 3288, 3872, 2151, 717, 3363, 2195, 3947, 3418, 680)

Rank of points on Klein quadric: (82, 96, 1322, 1323, 306, 1273, 930, 882, 983, 3688, 2968, 1042, 898, 1096, 4031, 4461, 1210, 1163, 922, 2617, 1958)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_{27} = \mathbf{P}(0, \gamma, 1, 0) = \mathbf{P}(0, 2, 1, 0)$,
$1 : P_{43} = \mathbf{P}(0, \gamma^2, 1, 0) = \mathbf{P}(0, 4, 1, 0)$,
$2 : P_{67} = \mathbf{P}(0, \gamma^4, 1, 0) = \mathbf{P}(0, 7, 1, 0)$.

Double Points

The surface has 66 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{82} = (0,1,0,1) = \ell_0 \cap \ell_5$
$P_{84} = (2,1,0,1) = \ell_0 \cap \ell_6$
$P_{88} = (6,1,0,1) = \ell_0 \cap \ell_9$
$P_{86} = (4,1,0,1) = \ell_0 \cap \ell_{11}$
$P_{85} = (3,1,0,1) = \ell_0 \cap \ell_{14}$
$P_{89} = (7,1,0,1) = \ell_0 \cap \ell_{16}$
$P_{87} = (5,1,0,1) = \ell_0 \cap \ell_{19}$
$P_{138} = (0,0,1,1) = \ell_1 \cap \ell_2$
$P_{142} = (4,0,1,1) = \ell_1 \cap \ell_7$
$P_{145} = (7,0,1,1) = \ell_1 \cap \ell_{12}$
$P_{140} = (2,0,1,1) = \ell_1 \cap \ell_{18}$
$P_5 = (1,1,0,0) = \ell_2 \cap \ell_3$
$P_{200} = (7,7,1,1) = \ell_2 \cap \ell_8$
$P_{164} = (3,3,1,1) = \ell_2 \cap \ell_9$
$P_{155} = (2,2,1,1) = \ell_2 \cap \ell_{13}$
$P_{182} = (5,5,1,1) = \ell_2 \cap \ell_{14}$
$P_{173} = (4,4,1,1) = \ell_2 \cap \ell_{17}$
$P_{191} = (6,6,1,1) = \ell_2 \cap \ell_{19}$
$P_{146} = (0,1,1,1) = \ell_3 \cap \ell_4$
$P_{163} = (2,3,1,1) = \ell_3 \cap \ell_6$
$P_{181} = (4,5,1,1) = \ell_3 \cap \ell_{11}$
$P_{192} = (7,6,1,1) = \ell_3 \cap \ell_{16}$
$P_{20} = (1,1,1,0) = \ell_4 \cap \ell_5$
$P_{437} = (4,5,5,1) = \ell_4 \cap \ell_7$
$P_{366} = (5,4,4,1) = \ell_4 \cap \ell_9$
$P_{512} = (7,6,6,1) = \ell_4 \cap \ell_{12}$
$P_{583} = (6,7,7,1) = \ell_4 \cap \ell_{14}$
$P_{291} = (2,3,3,1) = \ell_4 \cap \ell_{18}$
$P_{220} = (3,2,2,1) = \ell_4 \cap \ell_{19}$
$P_{576} = (7,6,7,1) = \ell_5 \cap \ell_8$
$P_{227} = (2,3,2,1) = \ell_5 \cap \ell_{13}$
$P_{373} = (4,5,4,1) = \ell_5 \cap \ell_{17}$
$P_{459} = (2,0,6,1) = \ell_6 \cap \ell_9$
$P_{323} = (2,7,3,1) = \ell_6 \cap \ell_{13}$
$P_{539} = (2,2,7,1) = \ell_6 \cap \ell_{15}$
$P_{443} = (2,6,5,1) = \ell_6 \cap \ell_{18}$
$P_{94} = (4,2,0,1) = \ell_7 \cap \ell_{10}$
$P_{485} = (4,3,6,1) = \ell_7 \cap \ell_{11}$
$P_{533} = (4,1,7,1) = \ell_7 \cap \ell_{17}$
$P_{389} = (4,7,4,1) = \ell_7 \cap \ell_{19}$
$P_{216} = (7,1,2,1) = \ell_8 \cap \ell_{12}$
$P_{416} = (7,2,5,1) = \ell_8 \cap \ell_{14}$
$P_{496} = (7,4,6,1) = \ell_8 \cap \ell_{16}$
$P_{296} = (7,3,3,1) = \ell_8 \cap \ell_{20}$
$P_{32} = (5,2,1,0) = \ell_9 \cap \ell_{10}$
$P_{544} = (7,2,7,1) = \ell_9 \cap \ell_{12}$
$P_{325} = (4,7,3,1) = \ell_9 \cap \ell_{17}$
$P_{441} = (0,6,5,1) = \ell_9 \cap \ell_{20}$
$P_{435} = (2,5,5,1) = \ell_{10} \cap \ell_{13}$
$P_{481} = (0,3,6,1) = \ell_{10} \cap \ell_{14}$
$P_{392} = (7,7,4,1) = \ell_{10} \cap \ell_{16}$
$P_{269} = (4,0,3,1) = \ell_{11} \cap \ell_{14}$
$P_{413} = (4,2,5,1) = \ell_{11} \cap \ell_{17}$
$P_{237} = (4,4,2,1) = \ell_{11} \cap \ell_{20}$
$P_{113} = (7,4,0,1) = \ell_{12} \cap \ell_{15}$
$P_{312} = (7,5,3,1) = \ell_{12} \cap \ell_{16}$
$P_{339} = (2,1,4,1) = \ell_{13} \cap \ell_{18}$
$P_{491} = (2,4,6,1) = \ell_{13} \cap \ell_{19}$
$P_{49} = (6,4,1,0) = \ell_{14} \cap \ell_{15}$
$P_{235} = (2,4,2,1) = \ell_{14} \cap \ell_{18}$
$P_{509} = (4,6,6,1) = \ell_{15} \cap \ell_{17}$
$P_{305} = (0,5,3,1) = \ell_{15} \cap \ell_{19}$
$P_{400} = (7,0,5,1) = \ell_{16} \cap \ell_{19}$
$P_{132} = (2,7,0,1) = \ell_{18} \cap \ell_{20}$
$P_{70} = (3,7,1,0) = \ell_{19} \cap \ell_{20}$

Single Points

The surface has 42 single points:
The single points on the surface are:

0 : $P_{4} = (1, 1, 1, 1)$ lies on line ℓ_{2}
1 : $P_{75} = (1, 0, 0, 1)$ lies on line ℓ_{4}
2 : $P_{83} = (1, 1, 0, 1)$ lies on line ℓ_{0}
3 : $P_{102} = (4, 3, 0, 1)$ lies on line ℓ_{17}
4 : $P_{121} = (7, 5, 0, 1)$ lies on line ℓ_{8}
5 : $P_{124} = (2, 6, 0, 1)$ lies on line ℓ_{13}
6 : $P_{141} = (3, 0, 1, 1)$ lies on line ℓ_{1}
7 : $P_{143} = (5, 0, 1, 1)$ lies on line ℓ_{1}
8 : $P_{144} = (6, 0, 1, 1)$ lies on line ℓ_{1}
9 : $P_{156} = (3, 2, 1, 1)$ lies on line ℓ_{3}
10 : $P_{174} = (5, 4, 1, 1)$ lies on line ℓ_{3}
11 : $P_{199} = (6, 7, 1, 1)$ lies on line ℓ_{3}
12 : $P_{205} = (4, 0, 2, 1)$ lies on line ℓ_{17}
13 : $P_{215} = (6, 1, 2, 1)$ lies on line ℓ_{15}
14 : $P_{224} = (7, 2, 2, 1)$ lies on line ℓ_{16}
15 : $P_{242} = (1, 5, 2, 1)$ lies on line ℓ_{9}
16 : $P_{243} = (2, 5, 2, 1)$ lies on line ℓ_{6}
17 : $P_{252} = (3, 6, 2, 1)$ lies on line ℓ_{10}
18 : $P_{253} = (4, 6, 2, 1)$ lies on line ℓ_{7}
19 : $P_{284} = (3, 2, 3, 1)$ lies on line ℓ_{5}
20 : $P_{301} = (4, 4, 3, 1)$ lies on line ℓ_{7}
21 : $P_{303} = (6, 4, 3, 1)$ lies on line ℓ_{10}
22 : $P_{336} = (7, 0, 4, 1)$ lies on line ℓ_{8}
23 : $P_{340} = (3, 1, 4, 1)$ lies on line ℓ_{20}
24 : $P_{358} = (5, 3, 4, 1)$ lies on line ℓ_{15}
25 : $P_{360} = (7, 3, 4, 1)$ lies on line ℓ_{12}
26 : $P_{363} = (2, 4, 4, 1)$ lies on line ℓ_{6}
27 : $P_{378} = (1, 6, 4, 1)$ lies on line ℓ_{14}
28 : $P_{381} = (4, 6, 4, 1)$ lies on line ℓ_{11}
29 : $P_{430} = (5, 4, 5, 1)$ lies on line ℓ_{5}
30 : $P_{452} = (3, 7, 5, 1)$ lies on line ℓ_{15}
31 : $P_{456} = (7, 7, 5, 1)$ lies on line ℓ_{12}
32 : $P_{475} = (2, 2, 6, 1)$ lies on line ℓ_{18}
33 : $P_{478} = (5, 2, 6, 1)$ lies on line ℓ_{20}
34 : $P_{519} = (6, 7, 6, 1)$ lies on line ℓ_{5}
35 : $P_{523} = (2, 0, 7, 1)$ lies on line ℓ_{13}
36 : $P_{534} = (5, 1, 7, 1)$ lies on line ℓ_{10}
37 : $P_{546} = (1, 3, 7, 1)$ lies on line ℓ_{19}
38 : $P_{552} = (7, 3, 7, 1)$ lies on line ℓ_{16}
39 : $P_{563} = (2, 5, 7, 1)$ lies on line ℓ_{18}
40 : $P_{567} = (6, 5, 7, 1)$ lies on line ℓ_{20}
41 : $P_{581} = (4, 7, 7, 1)$ lies on line ℓ_{11}

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1, 0, 1, 0)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	0	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0
1	1	0	1	1	0	1	0	1	0	0	1	0	1	0	0	1	0	0	1	0	1
2	0	1	0	1	0	0	0	0	1	1	0	0	0	1	1	0	0	1	0	1	0
3	0	1	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1
4	0	0	0	1	0	1	0	1	0	1	0	0	1	0	1	0	0	0	1	1	0
5	1	1	0	1	1	0	0	0	1	0	1	0	0	1	0	1	0	1	0	0	1
6	1	0	0	1	0	0	0	1	1	1	0	0	0	1	0	1	0	0	1	0	0
7	0	1	0	0	1	0	1	0	1	0	1	1	0	0	0	0	0	1	0	1	0
8	0	0	1	0	0	1	1	1	0	0	0	0	1	0	1	0	1	0	0	0	1
9	1	0	1	0	1	0	1	0	0	0	1	0	1	0	0	0	0	1	0	0	1
10	0	1	0	1	0	1	0	1	0	1	0	0	0	1	1	1	1	0	0	0	1
11	1	0	0	1	0	0	0	1	0	0	0	0	1	1	1	0	0	1	0	0	1
12	0	1	0	0	1	0	0	0	1	1	0	1	0	1	0	1	1	0	0	0	0
13	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	1	1	0
14	1	0	1	0	1	0	0	0	1	0	1	1	0	0	0	1	0	0	1	0	0
15	0	1	0	1	0	1	1	0	0	0	1	0	1	0	1	0	0	1	0	1	1
16	1	0	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	1	1	1	0
17	0	0	1	0	0	1	0	1	0	1	0	1	0	0	0	1	1	0	1	0	0
18	0	1	0	0	1	0	1	0	0	0	0	0	1	1	0	1	1	0	0	0	1
19	1	0	1	0	1	0	0	1	0	0	0	0	1	0	1	1	0	0	0	0	1
20	0	1	0	1	0	1	0	0	1	1	1	1	0	0	0	1	0	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_9	ℓ_{11}	ℓ_{14}	ℓ_{16}	ℓ_{19}
in point	P_0	P_{82}	P_{84}	P_{88}	P_{86}	P_{85}	P_{89}	P_{87}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_0	P_{138}	P_{139}	P_{139}	P_{142}	P_{139}	P_{145}	P_{139}	P_{140}	P_{139}

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{19}
in point	P_{138}	P_5	P_{200}	P_{164}	P_{155}	P_{182}	P_{173}	P_{191}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{15}	ℓ_{16}	ℓ_{20}
in point	P_{139}	P_5	P_{146}	P_{139}	P_{163}	P_{139}	P_{181}	P_{139}	P_{192}	P_{139}

Line 4 intersects

Line	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{14}	ℓ_{18}	ℓ_{19}
in point	P_{146}	P_{20}	P_{437}	P_{366}	P_{512}	P_{583}	P_{291}	P_{220}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{20}
in point	P_{82}	P_{139}	P_{139}	P_{20}	P_{576}	P_{139}	P_{227}	P_{139}	P_{373}	P_{139}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{15}	ℓ_{18}
in point	P_{84}	P_{163}	P_{27}	P_{27}	P_{459}	P_{323}	P_{539}	P_{443}

Line 7 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{17}	ℓ_{19}
in point	P_{142}	P_{437}	P_{27}	P_{27}	P_{94}	P_{485}	P_{533}	P_{389}

Line 8 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{20}
in point	P_{200}	P_{576}	P_{27}	P_{27}	P_{216}	P_{416}	P_{496}	P_{296}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{17}	ℓ_{20}
in point	P_{88}	P_{164}	P_{366}	P_{459}	P_{32}	P_{544}	P_{325}	P_{441}

Line 10 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{20}
in point	P_{139}	P_{139}	P_{139}	P_{94}	P_{32}	P_{435}	P_{481}	P_{139}	P_{392}	P_{139}

Line 11 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{20}
in point	P_{86}	P_{181}	P_{485}	P_{43}	P_{43}	P_{269}	P_{413}	P_{237}

Line 12 intersects

Line	ℓ_1	ℓ_4	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{16}
in point	P_{145}	P_{512}	P_{216}	P_{544}	P_{43}	P_{43}	P_{113}	P_{312}

Line 13 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{18}	ℓ_{19}
in point	P_{155}	P_{227}	P_{323}	P_{435}	P_{43}	P_{43}	P_{339}	P_{491}

Line 14 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{15}	ℓ_{18}
in point	P_{85}	P_{182}	P_{583}	P_{416}	P_{481}	P_{269}	P_{49}	P_{235}

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{17}	ℓ_{19}	ℓ_{20}
in point	P_{139}	P_{139}	P_{139}	P_{539}	P_{139}	P_{113}	P_{49}	P_{509}	P_{305}	P_{139}

Line 16 intersects

Line	ℓ_0	ℓ_3	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}
in point	P_{89}	P_{192}	P_{496}	P_{392}	P_{312}	P_{67}	P_{67}	P_{400}

Line 17 intersects

Line	ℓ_2	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{16}	ℓ_{18}
in point	P_{173}	P_{373}	P_{533}	P_{325}	P_{413}	P_{509}	P_{67}	P_{67}

Line 18 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{20}
in point	P_{140}	P_{291}	P_{443}	P_{339}	P_{235}	P_{67}	P_{67}	P_{132}

Line 19 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{20}
in point	P_{87}	P_{191}	P_{220}	P_{389}	P_{491}	P_{305}	P_{400}	P_{70}

Line 20 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{15}	ℓ_{18}	ℓ_{19}
in point	P_{139}	P_{139}	P_{139}	P_{296}	P_{441}	P_{139}	P_{237}	P_{139}	P_{132}	P_{70}

The surface has 113 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_4 = (1, 1, 1, 1)$
$2: P_5 = (1, 1, 0, 0)$
$3: P_{12} = (1, 0, 1, 0)$
$4: P_{20} = (1, 1, 1, 0)$
$5: P_{27} = (0, 2, 1, 0)$
$6: P_{32} = (5, 2, 1, 0)$
$7: P_{43} = (0, 4, 1, 0)$
$8: P_{49} = (6, 4, 1, 0)$

$9:\ P_{67} = (0,7,1,0)$
$10:\ P_{70} = (3,7,1,0)$
$11:\ P_{75} = (1,0,0,1)$
$12:\ P_{82} = (0,1,0,1)$
$13:\ P_{83} = (1,1,0,1)$
$14:\ P_{84} = (2,1,0,1)$
$15:\ P_{85} = (3,1,0,1)$
$16:\ P_{86} = (4,1,0,1)$
$17:\ P_{87} = (5,1,0,1)$
$18:\ P_{88} = (6,1,0,1)$
$19:\ P_{89} = (7,1,0,1)$
$20:\ P_{94} = (4,2,0,1)$
$21:\ P_{102} = (4,3,0,1)$
$22:\ P_{113} = (7,4,0,1)$
$23:\ P_{121} = (7,5,0,1)$
$24:\ P_{124} = (2,6,0,1)$
$25:\ P_{132} = (2,7,0,1)$
$26:\ P_{138} = (0,0,1,1)$
$27:\ P_{139} = (1,0,1,1)$
$28:\ P_{140} = (2,0,1,1)$
$29:\ P_{141} = (3,0,1,1)$
$30:\ P_{142} = (4,0,1,1)$
$31:\ P_{143} = (5,0,1,1)$
$32:\ P_{144} = (6,0,1,1)$
$33:\ P_{145} = (7,0,1,1)$
$34:\ P_{146} = (0,1,1,1)$
$35:\ P_{155} = (2,2,1,1)$
$36:\ P_{156} = (3,2,1,1)$
$37:\ P_{163} = (2,3,1,1)$
$38:\ P_{164} = (3,3,1,1)$
$39:\ P_{173} = (4,4,1,1)$
$40:\ P_{174} = (5,4,1,1)$
$41:\ P_{181} = (4,5,1,1)$
$42:\ P_{182} = (5,5,1,1)$
$43:\ P_{191} = (6,6,1,1)$
$44:\ P_{192} = (7,6,1,1)$
$45:\ P_{199} = (6,7,1,1)$
$46:\ P_{200} = (7,7,1,1)$
$47:\ P_{205} = (4,0,2,1)$
$48:\ P_{215} = (6,1,2,1)$
$49:\ P_{216} = (7,1,2,1)$
$50:\ P_{220} = (3,2,2,1)$
$51:\ P_{224} = (7,2,2,1)$
$52:\ P_{227} = (2,3,2,1)$
$53:\ P_{235} = (2,4,2,1)$
$54:\ P_{237} = (4,4,2,1)$
$55:\ P_{242} = (1,5,2,1)$
$56:\ P_{243} = (2,5,2,1)$
$57:\ P_{252} = (3,6,2,1)$
$58:\ P_{253} = (4,6,2,1)$
$59:\ P_{269} = (4,0,3,1)$
$60:\ P_{284} = (3,2,3,1)$
$61:\ P_{291} = (2,3,3,1)$
$62:\ P_{296} = (7,3,3,1)$
$63:\ P_{301} = (4,4,3,1)$
$64:\ P_{303} = (6,4,3,1)$
$65:\ P_{305} = (0,5,3,1)$
$66:\ P_{312} = (7,5,3,1)$
$67:\ P_{323} = (2,7,3,1)$
$68:\ P_{325} = (4,7,3,1)$
$69:\ P_{336} = (7,0,4,1)$
$70:\ P_{339} = (2,1,4,1)$
$71:\ P_{340} = (3,1,4,1)$
$72:\ P_{358} = (5,3,4,1)$
$73:\ P_{360} = (7,3,4,1)$
$74:\ P_{363} = (2,4,4,1)$
$75:\ P_{366} = (5,4,4,1)$
$76:\ P_{373} = (4,5,4,1)$
$77:\ P_{378} = (1,6,4,1)$
$78:\ P_{381} = (4,6,4,1)$
$79:\ P_{389} = (4,7,4,1)$
$80:\ P_{392} = (7,7,4,1)$
$81:\ P_{400} = (7,0,5,1)$
$82:\ P_{413} = (4,2,5,1)$
$83:\ P_{416} = (7,2,5,1)$
$84:\ P_{430} = (5,4,5,1)$
$85:\ P_{435} = (2,5,5,1)$
$86:\ P_{437} = (4,5,5,1)$
$87:\ P_{441} = (0,6,5,1)$
$88:\ P_{443} = (2,6,5,1)$
$89:\ P_{452} = (3,7,5,1)$
$90:\ P_{456} = (7,7,5,1)$
$91:\ P_{459} = (2,0,6,1)$
$92:\ P_{475} = (2,2,6,1)$
$93:\ P_{478} = (5,2,6,1)$
$94:\ P_{481} = (0,3,6,1)$
$95:\ P_{485} = (4,3,6,1)$
$96:\ P_{491} = (2,4,6,1)$
$97:\ P_{496} = (7,4,6,1)$
$98:\ P_{509} = (4,6,6,1)$
$99:\ P_{512} = (7,6,6,1)$
$100:\ P_{519} = (6,7,6,1)$
$101:\ P_{523} = (2,0,7,1)$
$102:\ P_{533} = (4,1,7,1)$
$103:\ P_{534} = (5,1,7,1)$
$104:\ P_{539} = (2,2,7,1)$
$105:\ P_{544} = (7,2,7,1)$
$106:\ P_{546} = (1,3,7,1)$
$107:\ P_{552} = (7,3,7,1)$
$108:\ P_{563} = (2,5,7,1)$
$109:\ P_{567} = (6,5,7,1)$
$110:\ P_{576} = (7,6,7,1)$
$111:\ P_{581} = (4,7,7,1)$
$112:\ P_{583} = (6,7,7,1)$