Rank-73801 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227395733

General information

Number of lines	7
Number of points	89
Number of singular points	0
Number of Eckardt points	3
Number of double points	0
Number of single points	54
Number of points off lines	32
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^7
Type of lines on points	3^3, 1^{54}, 0^{32}

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^3 \\ 0 & 1 & 1 & \gamma^3 \end{bmatrix}_{3472} = \begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 1 & 5 \end{bmatrix}_{3472} = \mathbf{Pl}(4,3,2,6,6,1)_{4013}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \gamma & \gamma^4 \\ 0 & 1 & 1 & \gamma^6 \end{bmatrix}_{4283} = \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 1 & 6 \end{bmatrix}_{4283} = \mathbf{Pl}(4,3,7,5,3,1)_{2725}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \gamma & \gamma^6 \\ 0 & 1 & 1 & \gamma^6 \end{bmatrix}_{3699} = \begin{bmatrix} 1 & 0 & 2 & 6 \\ 0 & 1 & 1 & 6 \end{bmatrix}_{3699} = \mathbf{Pl}(7,5,4,3,3,1)_{2595}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma \\ 0 & 1 & 1 & \gamma^5 \end{bmatrix}_{1485} = \begin{bmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{1485} = \mathbf{Pl}(7,5,2,6,5,1)_{3491}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^2 \\ 0 & 1 & 1 & \gamma^3 \end{bmatrix}_{2888} = \begin{bmatrix} 1 & 0 & 7 & 4 \\ 0 & 1 & 1 & 5 \end{bmatrix}_{2888} = \mathbf{Pl}(2,6,4,3,6,1)_{4088}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^5 \\ 0 & 1 & 1 & \gamma^5 \end{bmatrix}_{2069} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{2069} = \mathbf{Pl}(2,6,7,5,5,1)_{3703}$$

Rank of lines: (4689, 3472, 4283, 3699, 1485, 2888, 2069)
Rank of points on Klein quadric: (25, 4013, 2725, 2595, 3491, 4088, 3703)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_{217} = \mathbf{P}(0,\gamma,\gamma,1) = \mathbf{P}(0,2,2,1)$,
$1 : P_{361} = \mathbf{P}(0,\gamma^2,\gamma^2,1) = \mathbf{P}(0,4,4,1)$,
$2 : P_{577} = \mathbf{P}(0,\gamma^4,\gamma^4,1) = \mathbf{P}(0,7,7,1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 54 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_0
$1 : P_{19} = (0,1,1,0)$ lies on line ℓ_0
$2 : P_{29} = (2,2,1,0)$ lies on line ℓ_1
$3 : P_{36} = (1,3,1,0)$ lies on line ℓ_2
$4 : P_{47} = (4,4,1,0)$ lies on line ℓ_3
$5 : P_{52} = (1,5,1,0)$ lies on line ℓ_4
$6 : P_{60} = (1,6,1,0)$ lies on line ℓ_5
$7 : P_{74} = (7,7,1,0)$ lies on line ℓ_6
$8 : P_{96} = (6,2,0,1)$ lies on line ℓ_6
$9 : P_{97} = (7,2,0,1)$ lies on line ℓ_5
$10 : P_{108} = (2,4,0,1)$ lies on line ℓ_2
$11 : P_{109} = (3,4,0,1)$ lies on line ℓ_1
$12 : P_{134} = (4,7,0,1)$ lies on line ℓ_4
$13 : P_{135} = (5,7,0,1)$ lies on line ℓ_3
$14 : P_{146} = (0,1,1,1)$ lies on line ℓ_0
$15 : P_{158} = (5,2,1,1)$ lies on line ℓ_4
$16 : P_{162} = (1,3,1,1)$ lies on line ℓ_3
$17 : P_{175} = (6,4,1,1)$ lies on line ℓ_5
$18 : P_{178} = (1,5,1,1)$ lies on line ℓ_6
$19 : P_{186} = (1,6,1,1)$ lies on line ℓ_1
$20 : P_{196} = (3,7,1,1)$ lies on line ℓ_2
$21 : P_{207} = (6,0,2,1)$ lies on line ℓ_4
$22 : P_{208} = (7,0,2,1)$ lies on line ℓ_1
$23 : P_{214} = (5,1,2,1)$ lies on line ℓ_6
$24 : P_{230} = (5,3,2,1)$ lies on line ℓ_5
$25 : P_{274} = (1,1,3,1)$ lies on line ℓ_2
$26 : P_{286} = (5,2,3,1)$ lies on line ℓ_1
$27 : P_{289} = (0,3,3,1)$ lies on line ℓ_0
$28 : P_{309} = (4,5,3,1)$ lies on line ℓ_5
$29 : P_{312} = (7,5,3,1)$ lies on line ℓ_4
$30 : P_{315} = (2,6,3,1)$ lies on line ℓ_6
$31 : P_{317} = (4,6,3,1)$ lies on line ℓ_3
$32 : P_{331} = (2,0,4,1)$ lies on line ℓ_3
$33 : P_{332} = (3,0,4,1)$ lies on line ℓ_5

34 : $P_{343} = (6, 1, 4, 1)$ lies on line ℓ_1
35 : $P_{375} = (6, 5, 4, 1)$ lies on line ℓ_2
36 : $P_{402} = (1, 1, 5, 1)$ lies on line ℓ_4
37 : $P_{421} = (4, 3, 5, 1)$ lies on line ℓ_1
38 : $P_{424} = (7, 3, 5, 1)$ lies on line ℓ_6
39 : $P_{431} = (6, 4, 5, 1)$ lies on line ℓ_3
40 : $P_{433} = (0, 5, 5, 1)$ lies on line ℓ_0
41 : $P_{443} = (2, 6, 5, 1)$ lies on line ℓ_5
42 : $P_{448} = (7, 6, 5, 1)$ lies on line ℓ_2
43 : $P_{466} = (1, 1, 6, 1)$ lies on line ℓ_5
44 : $P_{483} = (2, 3, 6, 1)$ lies on line ℓ_4
45 : $P_{485} = (4, 3, 6, 1)$ lies on line ℓ_2
46 : $P_{499} = (2, 5, 6, 1)$ lies on line ℓ_1
47 : $P_{504} = (7, 5, 6, 1)$ lies on line ℓ_3
48 : $P_{505} = (0, 6, 6, 1)$ lies on line ℓ_0
49 : $P_{516} = (3, 7, 6, 1)$ lies on line ℓ_6
50 : $P_{525} = (4, 0, 7, 1)$ lies on line ℓ_6
51 : $P_{526} = (5, 0, 7, 1)$ lies on line ℓ_2
52 : $P_{532} = (3, 1, 7, 1)$ lies on line ℓ_3
53 : $P_{572} = (3, 6, 7, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 32 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{75} = (1, 0, 0, 1)$
2 : $P_{159} = (6, 2, 1, 1)$
3 : $P_{164} = (3, 3, 1, 1)$
4 : $P_{172} = (3, 4, 1, 1)$
5 : $P_{182} = (5, 5, 1, 1)$
6 : $P_{191} = (6, 6, 1, 1)$
7 : $P_{198} = (5, 7, 1, 1)$
8 : $P_{215} = (6, 1, 2, 1)$
9 : $P_{222} = (5, 2, 2, 1)$
10 : $P_{227} = (2, 3, 2, 1)$
11 : $P_{256} = (7, 6, 2, 1)$
12 : $P_{276} = (3, 1, 3, 1)$
13 : $P_{283} = (2, 2, 3, 1)$
14 : $P_{293} = (4, 3, 3, 1)$
15 : $P_{299} = (2, 4, 3, 1)$
16 : $P_{340} = (3, 1, 4, 1)$
17 : $P_{355} = (2, 3, 4, 1)$
18 : $P_{367} = (6, 4, 4, 1)$
19 : $P_{373} = (4, 5, 4, 1)$
20 : $P_{406} = (5, 1, 5, 1)$
21 : $P_{429} = (4, 4, 5, 1)$
22 : $P_{440} = (7, 5, 5, 1)$
23 : $P_{453} = (4, 7, 5, 1)$
24 : $P_{471} = (6, 1, 6, 1)$
25 : $P_{480} = (7, 2, 6, 1)$
26 : $P_{507} = (2, 6, 6, 1)$
27 : $P_{520} = (7, 7, 6, 1)$
28 : $P_{534} = (5, 1, 7, 1)$
29 : $P_{565} = (4, 5, 7, 1)$
30 : $P_{576} = (7, 6, 7, 1)$
31 : $P_{580} = (3, 7, 7, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 1 1 1 1
1	1 0 0 0 0 1 0
2	1 0 0 1 0 0 0
3	1 0 1 0 0 0 0
4	1 0 0 0 0 0 1
5	1 1 0 0 0 0 0
6	1 0 0 0 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{577}	P_{217}	P_{217}	P_{361}	P_{577}	P_{361}

Line 1 intersects

Line	ℓ_0	ℓ_5
in point	P_{577}	P_{577}

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{217}	P_{217}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{217}	P_{217}

Line 4 intersects

Line	ℓ_0	ℓ_6
in point	P_{361}	P_{361}

Line 5 intersects

Line	ℓ_0	ℓ_1
in point	P_{577}	P_{577}

Line 6 intersects

Line	ℓ_0	ℓ_4
in point	P_{361}	P_{361}

The surface has 89 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_3 = (0,0,0,1)$
$2: P_{19} = (0,1,1,0)$
$3: P_{29} = (2,2,1,0)$
$4: P_{36} = (1,3,1,0)$
$5: P_{47} = (4,4,1,0)$
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$86: P_{576} = (7,6,7,1)$
$87: P_{577} = (0,7,7,1)$
$88: P_{580} = (3,7,7,1)$