

Rank-65867 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is 1091047493

General information

Number of lines	8
Number of points	4289
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	498
Number of points off lines	3782
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^8
Type of lines on points	$4, 3^2, 2^6, 1^{498}, 0^{3782}$

Singular Points

The surface has 3 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_{4227} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$$

$$1 : P_{4163} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{8257} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{270528} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{130} \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{274561} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{274561} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{270592} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{66} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4226} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4226} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{536640} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{20354}
\end{aligned}$$

Rank of lines: (0, 4096, 8257, 266304, 274561, 270400, 4226, 266369)

Rank of points on Klein quadric: (0, 2, 270528, 130, 270592, 66, 536640, 20354)

Eckardt Points

The surface has 2 Eckardt points:

$$0 : P_{4163} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1),$$

$$1 : P_{4227} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1).$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{68} = (1, 0, 1, 0) = \ell_1 \cap \ell_6$$

$$P_{132} = (1, 1, 1, 0) = \ell_2 \cap \ell_7$$

$$P_{8322} = (0, 1, 1, 1) = \ell_6 \cap \ell_7$$

Single Points

The surface has 498 single points:

The single points on the surface are:

$$0 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_4$$

$$1 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_8 = (4, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_9 = (5, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{10} = (6, 1, 0, 0) \text{ lies on line } \ell_0$$

$$6 : P_{11} = (7, 1, 0, 0) \text{ lies on line } \ell_0$$

$$7 : P_{12} = (8, 1, 0, 0) \text{ lies on line } \ell_0$$

$$8 : P_{13} = (9, 1, 0, 0) \text{ lies on line } \ell_0$$

$$9 : P_{14} = (10, 1, 0, 0) \text{ lies on line } \ell_0$$

$$10 : P_{15} = (11, 1, 0, 0) \text{ lies on line } \ell_0$$

$$11 : P_{16} = (12, 1, 0, 0) \text{ lies on line } \ell_0$$

$$12 : P_{17} = (13, 1, 0, 0) \text{ lies on line } \ell_0$$

$$13 : P_{18} = (14, 1, 0, 0) \text{ lies on line } \ell_0$$

$$14 : P_{19} = (15, 1, 0, 0) \text{ lies on line } \ell_0$$

$$15 : P_{20} = (16, 1, 0, 0) \text{ lies on line } \ell_0$$

$$16 : P_{21} = (17, 1, 0, 0) \text{ lies on line } \ell_0$$

$$17 : P_{22} = (18, 1, 0, 0) \text{ lies on line } \ell_0$$

126 : $P_{262} = (3, 3, 1, 0)$ lies on line ℓ_2
 127 : $P_{327} = (4, 4, 1, 0)$ lies on line ℓ_2
 128 : $P_{392} = (5, 5, 1, 0)$ lies on line ℓ_2
 129 : $P_{457} = (6, 6, 1, 0)$ lies on line ℓ_2
 130 : $P_{522} = (7, 7, 1, 0)$ lies on line ℓ_2
 131 : $P_{587} = (8, 8, 1, 0)$ lies on line ℓ_2
 132 : $P_{652} = (9, 9, 1, 0)$ lies on line ℓ_2
 133 : $P_{717} = (10, 10, 1, 0)$ lies on line ℓ_2
 134 : $P_{782} = (11, 11, 1, 0)$ lies on line ℓ_2
 135 : $P_{847} = (12, 12, 1, 0)$ lies on line ℓ_2
 136 : $P_{912} = (13, 13, 1, 0)$ lies on line ℓ_2
 137 : $P_{977} = (14, 14, 1, 0)$ lies on line ℓ_2
 138 : $P_{1042} = (15, 15, 1, 0)$ lies on line ℓ_2
 139 : $P_{1107} = (16, 16, 1, 0)$ lies on line ℓ_2
 140 : $P_{1172} = (17, 17, 1, 0)$ lies on line ℓ_2
 141 : $P_{1237} = (18, 18, 1, 0)$ lies on line ℓ_2
 142 : $P_{1302} = (19, 19, 1, 0)$ lies on line ℓ_2
 143 : $P_{1367} = (20, 20, 1, 0)$ lies on line ℓ_2
 144 : $P_{1432} = (21, 21, 1, 0)$ lies on line ℓ_2
 145 : $P_{1497} = (22, 22, 1, 0)$ lies on line ℓ_2
 146 : $P_{1562} = (23, 23, 1, 0)$ lies on line ℓ_2
 147 : $P_{1627} = (24, 24, 1, 0)$ lies on line ℓ_2
 148 : $P_{1692} = (25, 25, 1, 0)$ lies on line ℓ_2
 149 : $P_{1757} = (26, 26, 1, 0)$ lies on line ℓ_2
 150 : $P_{1822} = (27, 27, 1, 0)$ lies on line ℓ_2
 151 : $P_{1887} = (28, 28, 1, 0)$ lies on line ℓ_2
 152 : $P_{1952} = (29, 29, 1, 0)$ lies on line ℓ_2
 153 : $P_{2017} = (30, 30, 1, 0)$ lies on line ℓ_2
 154 : $P_{2082} = (31, 31, 1, 0)$ lies on line ℓ_2
 155 : $P_{2147} = (32, 32, 1, 0)$ lies on line ℓ_2
 156 : $P_{2212} = (33, 33, 1, 0)$ lies on line ℓ_2
 157 : $P_{2277} = (34, 34, 1, 0)$ lies on line ℓ_2
 158 : $P_{2342} = (35, 35, 1, 0)$ lies on line ℓ_2
 159 : $P_{2407} = (36, 36, 1, 0)$ lies on line ℓ_2
 160 : $P_{2472} = (37, 37, 1, 0)$ lies on line ℓ_2
 161 : $P_{2537} = (38, 38, 1, 0)$ lies on line ℓ_2
 162 : $P_{2602} = (39, 39, 1, 0)$ lies on line ℓ_2
 163 : $P_{2667} = (40, 40, 1, 0)$ lies on line ℓ_2
 164 : $P_{2732} = (41, 41, 1, 0)$ lies on line ℓ_2
 165 : $P_{2797} = (42, 42, 1, 0)$ lies on line ℓ_2
 166 : $P_{2862} = (43, 43, 1, 0)$ lies on line ℓ_2
 167 : $P_{2927} = (44, 44, 1, 0)$ lies on line ℓ_2
 168 : $P_{2992} = (45, 45, 1, 0)$ lies on line ℓ_2
 169 : $P_{3057} = (46, 46, 1, 0)$ lies on line ℓ_2
 170 : $P_{3122} = (47, 47, 1, 0)$ lies on line ℓ_2
 171 : $P_{3187} = (48, 48, 1, 0)$ lies on line ℓ_2
 172 : $P_{3252} = (49, 49, 1, 0)$ lies on line ℓ_2
 173 : $P_{3317} = (50, 50, 1, 0)$ lies on line ℓ_2
 174 : $P_{3382} = (51, 51, 1, 0)$ lies on line ℓ_2
 175 : $P_{3447} = (52, 52, 1, 0)$ lies on line ℓ_2
 176 : $P_{3512} = (53, 53, 1, 0)$ lies on line ℓ_2
 177 : $P_{3577} = (54, 54, 1, 0)$ lies on line ℓ_2
 178 : $P_{3642} = (55, 55, 1, 0)$ lies on line ℓ_2
 179 : $P_{3707} = (56, 56, 1, 0)$ lies on line ℓ_2

180 : $P_{3772} = (57, 57, 1, 0)$ lies on line ℓ_2
 181 : $P_{3837} = (58, 58, 1, 0)$ lies on line ℓ_2
 182 : $P_{3902} = (59, 59, 1, 0)$ lies on line ℓ_2
 183 : $P_{3967} = (60, 60, 1, 0)$ lies on line ℓ_2
 184 : $P_{4032} = (61, 61, 1, 0)$ lies on line ℓ_2
 185 : $P_{4097} = (62, 62, 1, 0)$ lies on line ℓ_2
 186 : $P_{4162} = (63, 63, 1, 0)$ lies on line ℓ_2
 187 : $P_{4291} = (1, 2, 0, 1)$ lies on line ℓ_3
 188 : $P_{4355} = (1, 3, 0, 1)$ lies on line ℓ_3
 189 : $P_{4419} = (1, 4, 0, 1)$ lies on line ℓ_3
 190 : $P_{4483} = (1, 5, 0, 1)$ lies on line ℓ_3
 191 : $P_{4547} = (1, 6, 0, 1)$ lies on line ℓ_3
 192 : $P_{4611} = (1, 7, 0, 1)$ lies on line ℓ_3
 193 : $P_{4675} = (1, 8, 0, 1)$ lies on line ℓ_3
 194 : $P_{4739} = (1, 9, 0, 1)$ lies on line ℓ_3
 195 : $P_{4803} = (1, 10, 0, 1)$ lies on line ℓ_3
 196 : $P_{4867} = (1, 11, 0, 1)$ lies on line ℓ_3
 197 : $P_{4931} = (1, 12, 0, 1)$ lies on line ℓ_3
 198 : $P_{4995} = (1, 13, 0, 1)$ lies on line ℓ_3
 199 : $P_{5059} = (1, 14, 0, 1)$ lies on line ℓ_3
 200 : $P_{5123} = (1, 15, 0, 1)$ lies on line ℓ_3
 201 : $P_{5187} = (1, 16, 0, 1)$ lies on line ℓ_3
 202 : $P_{5251} = (1, 17, 0, 1)$ lies on line ℓ_3
 203 : $P_{5315} = (1, 18, 0, 1)$ lies on line ℓ_3
 204 : $P_{5379} = (1, 19, 0, 1)$ lies on line ℓ_3
 205 : $P_{5443} = (1, 20, 0, 1)$ lies on line ℓ_3
 206 : $P_{5507} = (1, 21, 0, 1)$ lies on line ℓ_3
 207 : $P_{5571} = (1, 22, 0, 1)$ lies on line ℓ_3
 208 : $P_{5635} = (1, 23, 0, 1)$ lies on line ℓ_3
 209 : $P_{5699} = (1, 24, 0, 1)$ lies on line ℓ_3
 210 : $P_{5763} = (1, 25, 0, 1)$ lies on line ℓ_3
 211 : $P_{5827} = (1, 26, 0, 1)$ lies on line ℓ_3
 212 : $P_{5891} = (1, 27, 0, 1)$ lies on line ℓ_3
 213 : $P_{5955} = (1, 28, 0, 1)$ lies on line ℓ_3
 214 : $P_{6019} = (1, 29, 0, 1)$ lies on line ℓ_3
 215 : $P_{6083} = (1, 30, 0, 1)$ lies on line ℓ_3
 216 : $P_{6147} = (1, 31, 0, 1)$ lies on line ℓ_3
 217 : $P_{6211} = (1, 32, 0, 1)$ lies on line ℓ_3
 218 : $P_{6275} = (1, 33, 0, 1)$ lies on line ℓ_3
 219 : $P_{6339} = (1, 34, 0, 1)$ lies on line ℓ_3
 220 : $P_{6403} = (1, 35, 0, 1)$ lies on line ℓ_3
 221 : $P_{6467} = (1, 36, 0, 1)$ lies on line ℓ_3
 222 : $P_{6531} = (1, 37, 0, 1)$ lies on line ℓ_3
 223 : $P_{6595} = (1, 38, 0, 1)$ lies on line ℓ_3
 224 : $P_{6659} = (1, 39, 0, 1)$ lies on line ℓ_3
 225 : $P_{6723} = (1, 40, 0, 1)$ lies on line ℓ_3
 226 : $P_{6787} = (1, 41, 0, 1)$ lies on line ℓ_3
 227 : $P_{6851} = (1, 42, 0, 1)$ lies on line ℓ_3
 228 : $P_{6915} = (1, 43, 0, 1)$ lies on line ℓ_3
 229 : $P_{6979} = (1, 44, 0, 1)$ lies on line ℓ_3
 230 : $P_{7043} = (1, 45, 0, 1)$ lies on line ℓ_3
 231 : $P_{7107} = (1, 46, 0, 1)$ lies on line ℓ_3
 232 : $P_{7171} = (1, 47, 0, 1)$ lies on line ℓ_3
 233 : $P_{7235} = (1, 48, 0, 1)$ lies on line ℓ_3

234 : $P_{7299} = (1, 49, 0, 1)$ lies on line ℓ_3
 235 : $P_{7363} = (1, 50, 0, 1)$ lies on line ℓ_3
 236 : $P_{7427} = (1, 51, 0, 1)$ lies on line ℓ_3
 237 : $P_{7491} = (1, 52, 0, 1)$ lies on line ℓ_3
 238 : $P_{7555} = (1, 53, 0, 1)$ lies on line ℓ_3
 239 : $P_{7619} = (1, 54, 0, 1)$ lies on line ℓ_3
 240 : $P_{7683} = (1, 55, 0, 1)$ lies on line ℓ_3
 241 : $P_{7747} = (1, 56, 0, 1)$ lies on line ℓ_3
 242 : $P_{7811} = (1, 57, 0, 1)$ lies on line ℓ_3
 243 : $P_{7875} = (1, 58, 0, 1)$ lies on line ℓ_3
 244 : $P_{7939} = (1, 59, 0, 1)$ lies on line ℓ_3
 245 : $P_{8003} = (1, 60, 0, 1)$ lies on line ℓ_3
 246 : $P_{8067} = (1, 61, 0, 1)$ lies on line ℓ_3
 247 : $P_{8131} = (1, 62, 0, 1)$ lies on line ℓ_3
 248 : $P_{8195} = (1, 63, 0, 1)$ lies on line ℓ_3
 249 : $P_{8259} = (1, 0, 1, 1)$ lies on line ℓ_5
 250 : $P_{12354} = (1, 0, 2, 1)$ lies on line ℓ_5
 251 : $P_{12418} = (1, 1, 2, 1)$ lies on line ℓ_4
 252 : $P_{12420} = (3, 1, 2, 1)$ lies on line ℓ_6
 253 : $P_{12484} = (3, 2, 2, 1)$ lies on line ℓ_7
 254 : $P_{16450} = (1, 0, 3, 1)$ lies on line ℓ_5
 255 : $P_{16514} = (1, 1, 3, 1)$ lies on line ℓ_4
 256 : $P_{16515} = (2, 1, 3, 1)$ lies on line ℓ_6
 257 : $P_{16643} = (2, 3, 3, 1)$ lies on line ℓ_7
 258 : $P_{20546} = (1, 0, 4, 1)$ lies on line ℓ_5
 259 : $P_{20610} = (1, 1, 4, 1)$ lies on line ℓ_4
 260 : $P_{20614} = (5, 1, 4, 1)$ lies on line ℓ_6
 261 : $P_{20806} = (5, 4, 4, 1)$ lies on line ℓ_7
 262 : $P_{24642} = (1, 0, 5, 1)$ lies on line ℓ_5
 263 : $P_{24706} = (1, 1, 5, 1)$ lies on line ℓ_4
 264 : $P_{24709} = (4, 1, 5, 1)$ lies on line ℓ_6
 265 : $P_{24965} = (4, 5, 5, 1)$ lies on line ℓ_7
 266 : $P_{28738} = (1, 0, 6, 1)$ lies on line ℓ_5
 267 : $P_{28802} = (1, 1, 6, 1)$ lies on line ℓ_4
 268 : $P_{28808} = (7, 1, 6, 1)$ lies on line ℓ_6
 269 : $P_{29128} = (7, 6, 6, 1)$ lies on line ℓ_7
 270 : $P_{32834} = (1, 0, 7, 1)$ lies on line ℓ_5
 271 : $P_{32898} = (1, 1, 7, 1)$ lies on line ℓ_4
 272 : $P_{32903} = (6, 1, 7, 1)$ lies on line ℓ_6
 273 : $P_{33287} = (6, 7, 7, 1)$ lies on line ℓ_7
 274 : $P_{36930} = (1, 0, 8, 1)$ lies on line ℓ_5
 275 : $P_{36994} = (1, 1, 8, 1)$ lies on line ℓ_4
 276 : $P_{37002} = (9, 1, 8, 1)$ lies on line ℓ_6
 277 : $P_{37450} = (9, 8, 8, 1)$ lies on line ℓ_7
 278 : $P_{41026} = (1, 0, 9, 1)$ lies on line ℓ_5
 279 : $P_{41090} = (1, 1, 9, 1)$ lies on line ℓ_4
 280 : $P_{41097} = (8, 1, 9, 1)$ lies on line ℓ_6
 281 : $P_{41609} = (8, 9, 9, 1)$ lies on line ℓ_7
 282 : $P_{45122} = (1, 0, 10, 1)$ lies on line ℓ_5
 283 : $P_{45186} = (1, 1, 10, 1)$ lies on line ℓ_4
 284 : $P_{45196} = (11, 1, 10, 1)$ lies on line ℓ_6
 285 : $P_{45772} = (11, 10, 10, 1)$ lies on line ℓ_7
 286 : $P_{49218} = (1, 0, 11, 1)$ lies on line ℓ_5
 287 : $P_{49282} = (1, 1, 11, 1)$ lies on line ℓ_4

288 : $P_{49291} = (10, 1, 11, 1)$ lies on line ℓ_6
 289 : $P_{49931} = (10, 11, 11, 1)$ lies on line ℓ_7
 290 : $P_{53314} = (1, 0, 12, 1)$ lies on line ℓ_5
 291 : $P_{53378} = (1, 1, 12, 1)$ lies on line ℓ_4
 292 : $P_{53390} = (13, 1, 12, 1)$ lies on line ℓ_6
 293 : $P_{54094} = (13, 12, 12, 1)$ lies on line ℓ_7
 294 : $P_{57410} = (1, 0, 13, 1)$ lies on line ℓ_5
 295 : $P_{57474} = (1, 1, 13, 1)$ lies on line ℓ_4
 296 : $P_{57485} = (12, 1, 13, 1)$ lies on line ℓ_6
 297 : $P_{58253} = (12, 13, 13, 1)$ lies on line ℓ_7
 298 : $P_{61506} = (1, 0, 14, 1)$ lies on line ℓ_5
 299 : $P_{61570} = (1, 1, 14, 1)$ lies on line ℓ_4
 300 : $P_{61584} = (15, 1, 14, 1)$ lies on line ℓ_6
 301 : $P_{62416} = (15, 14, 14, 1)$ lies on line ℓ_7
 302 : $P_{65602} = (1, 0, 15, 1)$ lies on line ℓ_5
 303 : $P_{65666} = (1, 1, 15, 1)$ lies on line ℓ_4
 304 : $P_{65679} = (14, 1, 15, 1)$ lies on line ℓ_6
 305 : $P_{66575} = (14, 15, 15, 1)$ lies on line ℓ_7
 306 : $P_{69698} = (1, 0, 16, 1)$ lies on line ℓ_5
 307 : $P_{69762} = (1, 1, 16, 1)$ lies on line ℓ_4
 308 : $P_{69778} = (17, 1, 16, 1)$ lies on line ℓ_6
 309 : $P_{70738} = (17, 16, 16, 1)$ lies on line ℓ_7
 310 : $P_{73794} = (1, 0, 17, 1)$ lies on line ℓ_5
 311 : $P_{73858} = (1, 1, 17, 1)$ lies on line ℓ_4
 312 : $P_{73873} = (16, 1, 17, 1)$ lies on line ℓ_6
 313 : $P_{74897} = (16, 17, 17, 1)$ lies on line ℓ_7
 314 : $P_{77890} = (1, 0, 18, 1)$ lies on line ℓ_5
 315 : $P_{77954} = (1, 1, 18, 1)$ lies on line ℓ_4
 316 : $P_{77972} = (19, 1, 18, 1)$ lies on line ℓ_6
 317 : $P_{79060} = (19, 18, 18, 1)$ lies on line ℓ_7
 318 : $P_{81986} = (1, 0, 19, 1)$ lies on line ℓ_5
 319 : $P_{82050} = (1, 1, 19, 1)$ lies on line ℓ_4
 320 : $P_{82067} = (18, 1, 19, 1)$ lies on line ℓ_6
 321 : $P_{83219} = (18, 19, 19, 1)$ lies on line ℓ_7
 322 : $P_{86082} = (1, 0, 20, 1)$ lies on line ℓ_5
 323 : $P_{86146} = (1, 1, 20, 1)$ lies on line ℓ_4
 324 : $P_{86166} = (21, 1, 20, 1)$ lies on line ℓ_6
 325 : $P_{87382} = (21, 20, 20, 1)$ lies on line ℓ_7
 326 : $P_{90178} = (1, 0, 21, 1)$ lies on line ℓ_5
 327 : $P_{90242} = (1, 1, 21, 1)$ lies on line ℓ_4
 328 : $P_{90261} = (20, 1, 21, 1)$ lies on line ℓ_6
 329 : $P_{91541} = (20, 21, 21, 1)$ lies on line ℓ_7
 330 : $P_{94274} = (1, 0, 22, 1)$ lies on line ℓ_5
 331 : $P_{94338} = (1, 1, 22, 1)$ lies on line ℓ_4
 332 : $P_{94360} = (23, 1, 22, 1)$ lies on line ℓ_6
 333 : $P_{95704} = (23, 22, 22, 1)$ lies on line ℓ_7
 334 : $P_{98370} = (1, 0, 23, 1)$ lies on line ℓ_5
 335 : $P_{98434} = (1, 1, 23, 1)$ lies on line ℓ_4
 336 : $P_{98455} = (22, 1, 23, 1)$ lies on line ℓ_6
 337 : $P_{99863} = (22, 23, 23, 1)$ lies on line ℓ_7
 338 : $P_{102466} = (1, 0, 24, 1)$ lies on line ℓ_5
 339 : $P_{102530} = (1, 1, 24, 1)$ lies on line ℓ_4
 340 : $P_{102554} = (25, 1, 24, 1)$ lies on line ℓ_6
 341 : $P_{104026} = (25, 24, 24, 1)$ lies on line ℓ_7

342 : $P_{106562} = (1, 0, 25, 1)$ lies on line ℓ_5
 343 : $P_{106626} = (1, 1, 25, 1)$ lies on line ℓ_4
 344 : $P_{106649} = (24, 1, 25, 1)$ lies on line ℓ_6
 345 : $P_{108185} = (24, 25, 25, 1)$ lies on line ℓ_7
 346 : $P_{110658} = (1, 0, 26, 1)$ lies on line ℓ_5
 347 : $P_{110722} = (1, 1, 26, 1)$ lies on line ℓ_4
 348 : $P_{110748} = (27, 1, 26, 1)$ lies on line ℓ_6
 349 : $P_{112348} = (27, 26, 26, 1)$ lies on line ℓ_7
 350 : $P_{114754} = (1, 0, 27, 1)$ lies on line ℓ_5
 351 : $P_{114818} = (1, 1, 27, 1)$ lies on line ℓ_4
 352 : $P_{114843} = (26, 1, 27, 1)$ lies on line ℓ_6
 353 : $P_{116507} = (26, 27, 27, 1)$ lies on line ℓ_7
 354 : $P_{118850} = (1, 0, 28, 1)$ lies on line ℓ_5
 355 : $P_{118914} = (1, 1, 28, 1)$ lies on line ℓ_4
 356 : $P_{118942} = (29, 1, 28, 1)$ lies on line ℓ_6
 357 : $P_{120670} = (29, 28, 28, 1)$ lies on line ℓ_7
 358 : $P_{122946} = (1, 0, 29, 1)$ lies on line ℓ_5
 359 : $P_{123010} = (1, 1, 29, 1)$ lies on line ℓ_4
 360 : $P_{123037} = (28, 1, 29, 1)$ lies on line ℓ_6
 361 : $P_{124829} = (28, 29, 29, 1)$ lies on line ℓ_7
 362 : $P_{127042} = (1, 0, 30, 1)$ lies on line ℓ_5
 363 : $P_{127106} = (1, 1, 30, 1)$ lies on line ℓ_4
 364 : $P_{127136} = (31, 1, 30, 1)$ lies on line ℓ_6
 365 : $P_{128992} = (31, 30, 30, 1)$ lies on line ℓ_7
 366 : $P_{131138} = (1, 0, 31, 1)$ lies on line ℓ_5
 367 : $P_{131202} = (1, 1, 31, 1)$ lies on line ℓ_4
 368 : $P_{131231} = (30, 1, 31, 1)$ lies on line ℓ_6
 369 : $P_{133151} = (30, 31, 31, 1)$ lies on line ℓ_7
 370 : $P_{135234} = (1, 0, 32, 1)$ lies on line ℓ_5
 371 : $P_{135298} = (1, 1, 32, 1)$ lies on line ℓ_4
 372 : $P_{135330} = (33, 1, 32, 1)$ lies on line ℓ_6
 373 : $P_{137314} = (33, 32, 32, 1)$ lies on line ℓ_7
 374 : $P_{139330} = (1, 0, 33, 1)$ lies on line ℓ_5
 375 : $P_{139394} = (1, 1, 33, 1)$ lies on line ℓ_4
 376 : $P_{139425} = (32, 1, 33, 1)$ lies on line ℓ_6
 377 : $P_{141473} = (32, 33, 33, 1)$ lies on line ℓ_7
 378 : $P_{143426} = (1, 0, 34, 1)$ lies on line ℓ_5
 379 : $P_{143490} = (1, 1, 34, 1)$ lies on line ℓ_4
 380 : $P_{143524} = (35, 1, 34, 1)$ lies on line ℓ_6
 381 : $P_{145636} = (35, 34, 34, 1)$ lies on line ℓ_7
 382 : $P_{147522} = (1, 0, 35, 1)$ lies on line ℓ_5
 383 : $P_{147586} = (1, 1, 35, 1)$ lies on line ℓ_4
 384 : $P_{147619} = (34, 1, 35, 1)$ lies on line ℓ_6
 385 : $P_{149795} = (34, 35, 35, 1)$ lies on line ℓ_7
 386 : $P_{151618} = (1, 0, 36, 1)$ lies on line ℓ_5
 387 : $P_{151682} = (1, 1, 36, 1)$ lies on line ℓ_4
 388 : $P_{151718} = (37, 1, 36, 1)$ lies on line ℓ_6
 389 : $P_{153958} = (37, 36, 36, 1)$ lies on line ℓ_7
 390 : $P_{155714} = (1, 0, 37, 1)$ lies on line ℓ_5
 391 : $P_{155778} = (1, 1, 37, 1)$ lies on line ℓ_4
 392 : $P_{155813} = (36, 1, 37, 1)$ lies on line ℓ_6
 393 : $P_{158117} = (36, 37, 37, 1)$ lies on line ℓ_7
 394 : $P_{159810} = (1, 0, 38, 1)$ lies on line ℓ_5
 395 : $P_{159874} = (1, 1, 38, 1)$ lies on line ℓ_4

396 : $P_{159912} = (39, 1, 38, 1)$ lies on line ℓ_6
 397 : $P_{162280} = (39, 38, 38, 1)$ lies on line ℓ_7
 398 : $P_{163906} = (1, 0, 39, 1)$ lies on line ℓ_5
 399 : $P_{163970} = (1, 1, 39, 1)$ lies on line ℓ_4
 400 : $P_{164007} = (38, 1, 39, 1)$ lies on line ℓ_6
 401 : $P_{166439} = (38, 39, 39, 1)$ lies on line ℓ_7
 402 : $P_{168002} = (1, 0, 40, 1)$ lies on line ℓ_5
 403 : $P_{168066} = (1, 1, 40, 1)$ lies on line ℓ_4
 404 : $P_{168106} = (41, 1, 40, 1)$ lies on line ℓ_6
 405 : $P_{170602} = (41, 40, 40, 1)$ lies on line ℓ_7
 406 : $P_{172098} = (1, 0, 41, 1)$ lies on line ℓ_5
 407 : $P_{172162} = (1, 1, 41, 1)$ lies on line ℓ_4
 408 : $P_{172201} = (40, 1, 41, 1)$ lies on line ℓ_6
 409 : $P_{174761} = (40, 41, 41, 1)$ lies on line ℓ_7
 410 : $P_{176194} = (1, 0, 42, 1)$ lies on line ℓ_5
 411 : $P_{176258} = (1, 1, 42, 1)$ lies on line ℓ_4
 412 : $P_{176300} = (43, 1, 42, 1)$ lies on line ℓ_6
 413 : $P_{178924} = (43, 42, 42, 1)$ lies on line ℓ_7
 414 : $P_{180290} = (1, 0, 43, 1)$ lies on line ℓ_5
 415 : $P_{180354} = (1, 1, 43, 1)$ lies on line ℓ_4
 416 : $P_{180395} = (42, 1, 43, 1)$ lies on line ℓ_6
 417 : $P_{183083} = (42, 43, 43, 1)$ lies on line ℓ_7
 418 : $P_{184386} = (1, 0, 44, 1)$ lies on line ℓ_5
 419 : $P_{184450} = (1, 1, 44, 1)$ lies on line ℓ_4
 420 : $P_{184494} = (45, 1, 44, 1)$ lies on line ℓ_6
 421 : $P_{187246} = (45, 44, 44, 1)$ lies on line ℓ_7
 422 : $P_{188482} = (1, 0, 45, 1)$ lies on line ℓ_5
 423 : $P_{188546} = (1, 1, 45, 1)$ lies on line ℓ_4
 424 : $P_{188589} = (44, 1, 45, 1)$ lies on line ℓ_6
 425 : $P_{191405} = (44, 45, 45, 1)$ lies on line ℓ_7
 426 : $P_{192578} = (1, 0, 46, 1)$ lies on line ℓ_5
 427 : $P_{192642} = (1, 1, 46, 1)$ lies on line ℓ_4
 428 : $P_{192688} = (47, 1, 46, 1)$ lies on line ℓ_6
 429 : $P_{195568} = (47, 46, 46, 1)$ lies on line ℓ_7
 430 : $P_{196674} = (1, 0, 47, 1)$ lies on line ℓ_5
 431 : $P_{196738} = (1, 1, 47, 1)$ lies on line ℓ_4
 432 : $P_{196783} = (46, 1, 47, 1)$ lies on line ℓ_6
 433 : $P_{199727} = (46, 47, 47, 1)$ lies on line ℓ_7
 434 : $P_{200770} = (1, 0, 48, 1)$ lies on line ℓ_5
 435 : $P_{200834} = (1, 1, 48, 1)$ lies on line ℓ_4
 436 : $P_{200882} = (49, 1, 48, 1)$ lies on line ℓ_6
 437 : $P_{203890} = (49, 48, 48, 1)$ lies on line ℓ_7
 438 : $P_{204866} = (1, 0, 49, 1)$ lies on line ℓ_5
 439 : $P_{204930} = (1, 1, 49, 1)$ lies on line ℓ_4
 440 : $P_{204977} = (48, 1, 49, 1)$ lies on line ℓ_6
 441 : $P_{208049} = (48, 49, 49, 1)$ lies on line ℓ_7
 442 : $P_{208962} = (1, 0, 50, 1)$ lies on line ℓ_5
 443 : $P_{209026} = (1, 1, 50, 1)$ lies on line ℓ_4
 444 : $P_{209076} = (51, 1, 50, 1)$ lies on line ℓ_6
 445 : $P_{212212} = (51, 50, 50, 1)$ lies on line ℓ_7
 446 : $P_{213058} = (1, 0, 51, 1)$ lies on line ℓ_5
 447 : $P_{213122} = (1, 1, 51, 1)$ lies on line ℓ_4
 448 : $P_{213171} = (50, 1, 51, 1)$ lies on line ℓ_6
 449 : $P_{216371} = (50, 51, 51, 1)$ lies on line ℓ_7

450 : $P_{217154} = (1, 0, 52, 1)$ lies on line ℓ_5
 451 : $P_{217218} = (1, 1, 52, 1)$ lies on line ℓ_4
 452 : $P_{217270} = (53, 1, 52, 1)$ lies on line ℓ_6
 453 : $P_{220534} = (53, 52, 52, 1)$ lies on line ℓ_7
 454 : $P_{221250} = (1, 0, 53, 1)$ lies on line ℓ_5
 455 : $P_{221314} = (1, 1, 53, 1)$ lies on line ℓ_4
 456 : $P_{221365} = (52, 1, 53, 1)$ lies on line ℓ_6
 457 : $P_{224693} = (52, 53, 53, 1)$ lies on line ℓ_7
 458 : $P_{225346} = (1, 0, 54, 1)$ lies on line ℓ_5
 459 : $P_{225410} = (1, 1, 54, 1)$ lies on line ℓ_4
 460 : $P_{225464} = (55, 1, 54, 1)$ lies on line ℓ_6
 461 : $P_{228856} = (55, 54, 54, 1)$ lies on line ℓ_7
 462 : $P_{229442} = (1, 0, 55, 1)$ lies on line ℓ_5
 463 : $P_{229506} = (1, 1, 55, 1)$ lies on line ℓ_4
 464 : $P_{229559} = (54, 1, 55, 1)$ lies on line ℓ_6
 465 : $P_{233015} = (54, 55, 55, 1)$ lies on line ℓ_7
 466 : $P_{233538} = (1, 0, 56, 1)$ lies on line ℓ_5
 467 : $P_{233602} = (1, 1, 56, 1)$ lies on line ℓ_4
 468 : $P_{233658} = (57, 1, 56, 1)$ lies on line ℓ_6
 469 : $P_{237178} = (57, 56, 56, 1)$ lies on line ℓ_7
 470 : $P_{237634} = (1, 0, 57, 1)$ lies on line ℓ_5
 471 : $P_{237698} = (1, 1, 57, 1)$ lies on line ℓ_4
 472 : $P_{237753} = (56, 1, 57, 1)$ lies on line ℓ_6
 473 : $P_{241337} = (56, 57, 57, 1)$ lies on line ℓ_7
 474 : $P_{241730} = (1, 0, 58, 1)$ lies on line ℓ_5

475 : $P_{241794} = (1, 1, 58, 1)$ lies on line ℓ_4
 476 : $P_{241852} = (59, 1, 58, 1)$ lies on line ℓ_6
 477 : $P_{245500} = (59, 58, 58, 1)$ lies on line ℓ_7
 478 : $P_{245826} = (1, 0, 59, 1)$ lies on line ℓ_5
 479 : $P_{245890} = (1, 1, 59, 1)$ lies on line ℓ_4
 480 : $P_{245947} = (58, 1, 59, 1)$ lies on line ℓ_6
 481 : $P_{249659} = (58, 59, 59, 1)$ lies on line ℓ_7
 482 : $P_{249922} = (1, 0, 60, 1)$ lies on line ℓ_5
 483 : $P_{249986} = (1, 1, 60, 1)$ lies on line ℓ_4
 484 : $P_{250046} = (61, 1, 60, 1)$ lies on line ℓ_6
 485 : $P_{253822} = (61, 60, 60, 1)$ lies on line ℓ_7
 486 : $P_{254018} = (1, 0, 61, 1)$ lies on line ℓ_5
 487 : $P_{254082} = (1, 1, 61, 1)$ lies on line ℓ_4
 488 : $P_{254141} = (60, 1, 61, 1)$ lies on line ℓ_6
 489 : $P_{257981} = (60, 61, 61, 1)$ lies on line ℓ_7
 490 : $P_{258114} = (1, 0, 62, 1)$ lies on line ℓ_5
 491 : $P_{258178} = (1, 1, 62, 1)$ lies on line ℓ_4
 492 : $P_{258240} = (63, 1, 62, 1)$ lies on line ℓ_6
 493 : $P_{262144} = (63, 62, 62, 1)$ lies on line ℓ_7
 494 : $P_{262210} = (1, 0, 63, 1)$ lies on line ℓ_5
 495 : $P_{262274} = (1, 1, 63, 1)$ lies on line ℓ_4
 496 : $P_{262335} = (62, 1, 63, 1)$ lies on line ℓ_6
 497 : $P_{266303} = (62, 63, 63, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 3782 points not on any line:
Too many to print.

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	0	0	0	0
1	1	1	0	1	0	1	1	1
2	1	1	0	0	1	1	0	1
3	1	0	0	0	1	1	1	1
4	0	1	1	1	0	1	1	0
5	0	1	1	1	1	0	0	1
6	0	1	0	1	1	0	0	1
7	0	0	1	1	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_2	P_2	P_2	P_{68}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_5	P_2	P_2	P_2	P_{132}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{4227}	P_{4163}	P_{4227}	P_{4163}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{4227}	P_2	P_{4227}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_{4163}	P_2	P_{4163}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_{68}	P_{4227}	P_{4227}	P_{8322}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_{132}	P_{4163}	P_{4163}	P_{8322}

The surface has 4289 points:
Too many to print.