Rank-65609 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65609

General information

Number of lines	2
Number of points	7
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	4
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^2
Type of lines on points	2, 1^4, 0^2

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,1,1,1,0,1)_{28}$$

Rank of lines: (33, 22)
Rank of points on Klein quadric: (7, 28)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_7 = (0, 1, 1, 0) = \ell_0 \cap \ell_1$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0: P_3 = (0, 0, 0, 1)$ lies on line ℓ_0
$1: P_{11} = (1, 1, 0, 1)$ lies on line ℓ_1
$2: P_{13} = (1, 0, 1, 1)$ lies on line ℓ_1
$3: P_{14} = (0, 1, 1, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_4 = (1, 1, 1, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_7

Line 1 intersects

Line	ℓ_0
in point	P_7

The surface has 7 points:
The points on the surface are:

$0:\ P_0 = (1,0,0,0)$
$1:\ P_3 = (0,0,0,1)$
$2:\ P_4 = (1,1,1,1)$
$3:\ P_7 = (0,1,1,0)$
$4:\ P_{11} = (1,1,0,1)$
$5:\ P_{13} = (1,0,1,1)$
$6:\ P_{14} = (0,1,1,1)$