

# Rank-77 over GF(4)

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## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

( 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 )

The point rank of the equation over GF(4) is 697

## General information

Number of lines	3
Number of points	29
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	12
Number of points off lines	16
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^3$
Type of lines on points	$3, 1^{12}, 0^{16}$

## Singular Points

The surface has 0 singular points:

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right]_{85} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right]_{85} = \mathbf{Pl}(1, 1, 1, 1, 0, 0)_{16} \\ \ell_1 &= \left[ \begin{array}{cccc} 1 & 0 & 0 & \omega^2 \\ 0 & 1 & 1 & 0 \end{array} \right]_{253} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \end{array} \right]_{253} = \mathbf{Pl}(1, 1, 2, 1, 0, 0)_{19}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 1 & 1 & 0 \end{bmatrix}_{169} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{169} = \mathbf{Pl}(1, 1, 3, 1, 0, 0)_{22}$$

Rank of lines: ( 85, 253, 169 )

Rank of points on Klein quadric: ( 16, 19, 22 )

### Eckardt Points

The surface has 1 Eckardt points:

0 :  $P_{11} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$ .

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 12 single points:

The single points on the surface are:

0 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_0$

1 :  $P_{23} = (1, 0, 0, 1)$  lies on line  $\ell_0$

2 :  $P_{24} = (2, 0, 0, 1)$  lies on line  $\ell_1$

3 :  $P_{25} = (3, 0, 0, 1)$  lies on line  $\ell_2$

4 :  $P_{43} = (2, 1, 1, 1)$  lies on line  $\ell_1$

5 :  $P_{44} = (3, 1, 1, 1)$  lies on line  $\ell_2$

6 :  $P_{62} = (1, 2, 2, 1)$  lies on line  $\ell_0$

7 :  $P_{63} = (2, 2, 2, 1)$  lies on line  $\ell_1$

8 :  $P_{64} = (3, 2, 2, 1)$  lies on line  $\ell_2$

9 :  $P_{82} = (1, 3, 3, 1)$  lies on line  $\ell_0$

10 :  $P_{83} = (2, 3, 3, 1)$  lies on line  $\ell_1$

11 :  $P_{84} = (3, 3, 3, 1)$  lies on line  $\ell_2$

The single points on the surface are:

### Points on surface but on no line

The surface has 16 points not on any line:

The points on the surface but not on lines are:

0 :  $P_{15} = (0, 2, 1, 0)$

1 :  $P_{18} = (3, 2, 1, 0)$

2 :  $P_{19} = (0, 3, 1, 0)$

3 :  $P_{21} = (2, 3, 1, 0)$

4 :  $P_{26} = (0, 1, 0, 1)$

5 :  $P_{27} = (1, 1, 0, 1)$

6 :  $P_{30} = (0, 2, 0, 1)$

7 :  $P_{32} = (2, 2, 0, 1)$

8 :  $P_{34} = (0, 3, 0, 1)$

9 :  $P_{37} = (3, 3, 0, 1)$

10 :  $P_{38} = (0, 0, 1, 1)$

11 :  $P_{39} = (1, 0, 1, 1)$

12 :  $P_{53} = (0, 0, 2, 1)$

13 :  $P_{55} = (2, 0, 2, 1)$

14 :  $P_{69} = (0, 0, 3, 1)$

15 :  $P_{72} = (3, 0, 3, 1)$

## Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_{11}$	$P_{11}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{11}$	$P_{11}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_{11}$	$P_{11}$

The surface has 29 points:

The points on the surface are:

0 :  $P_4 = (1, 1, 1, 1)$   
 1 :  $P_{11} = (0, 1, 1, 0)$   
 2 :  $P_{15} = (0, 2, 1, 0)$   
 3 :  $P_{18} = (3, 2, 1, 0)$   
 4 :  $P_{19} = (0, 3, 1, 0)$   
 5 :  $P_{21} = (2, 3, 1, 0)$   
 6 :  $P_{23} = (1, 0, 0, 1)$   
 7 :  $P_{24} = (2, 0, 0, 1)$   
 8 :  $P_{25} = (3, 0, 0, 1)$   
 9 :  $P_{26} = (0, 1, 0, 1)$

10 :  $P_{27} = (1, 1, 0, 1)$   
 11 :  $P_{30} = (0, 2, 0, 1)$   
 12 :  $P_{32} = (2, 2, 0, 1)$   
 13 :  $P_{34} = (0, 3, 0, 1)$   
 14 :  $P_{37} = (3, 3, 0, 1)$   
 15 :  $P_{38} = (0, 0, 1, 1)$   
 16 :  $P_{39} = (1, 0, 1, 1)$   
 17 :  $P_{43} = (2, 1, 1, 1)$   
 18 :  $P_{44} = (3, 1, 1, 1)$   
 19 :  $P_{53} = (0, 0, 2, 1)$

20 :  $P_{55} = (2, 0, 2, 1)$   
 21 :  $P_{62} = (1, 2, 2, 1)$   
 22 :  $P_{63} = (2, 2, 2, 1)$   
 23 :  $P_{64} = (3, 2, 2, 1)$   
 24 :  $P_{69} = (0, 0, 3, 1)$   
 25 :  $P_{72} = (3, 0, 3, 1)$   
 26 :  $P_{82} = (1, 3, 3, 1)$   
 27 :  $P_{83} = (2, 3, 3, 1)$   
 28 :  $P_{84} = (3, 3, 3, 1)$