# Rank-65874 over GF(16)

January 15, 2021

# The equation

The equation of the surface is:

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0 \\$$

( 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 ) The point rank of the equation over  $\mathrm{GF}(16)$  is 303112742

# General information

Number of lines	3
Number of points	241
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	45
Number of points off lines	193
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$17^{3}$
Type of lines on points	$2^3, 1^{45}, 0^{193}$

## Singular Points

The surface has 0 singular points:

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \mathbf{Pl}(1,0,1,1,1,1)_{9427}$$

$$\ell_1 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3260} = \mathbf{Pl}(0,0,1,1,10,1)_{46146}$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2987} = \mathbf{Pl}(0, 0, 1, 1, 11, 1)_{50226}$$

Rank of lines: (4658, 3260, 2987)

Rank of points on Klein quadric: (9427, 46146, 50226)

#### Eckardt Points

The surface has 0 Eckardt points:

#### **Double Points**

The surface has 3 Double points: The double points on the surface are:

$$P_{700} = (11, 10, 1, 1) = \ell_0 \cap \ell_1$$
  

$$P_{715} = (10, 11, 1, 1) = \ell_0 \cap \ell_2$$

 $P_{530} = (0,0,1,1) = \ell_1 \cap \ell_2$ 

## Single Points

The surface has 45 single points: The single points on the surface are:

 $0: P_5 = (1, 1, 0, 0)$  lies on line  $\ell_0$ 1:  $P_{14} = (10, 1, 0, 0)$  lies on line  $\ell_1$ 2:  $P_{15} = (11, 1, 0, 0)$  lies on line  $\ell_2$ 3 :  $P_{531} = (1,0,1,1)$  lies on line  $\ell_0$ 4:  $P_{546} = (0, 1, 1, 1)$  lies on line  $\ell_0$ 5:  $P_{555} = (10, 1, 1, 1)$  lies on line  $\ell_1$ 6:  $P_{556} = (11, 1, 1, 1)$  lies on line  $\ell_2$ 7:  $P_{564} = (3, 2, 1, 1)$  lies on line  $\ell_0$ 8:  $P_{574} = (13, 2, 1, 1)$  lies on line  $\ell_1$ 9:  $P_{576} = (15, 2, 1, 1)$  lies on line  $\ell_2$ 10:  $P_{579} = (2, 3, 1, 1)$  lies on line  $\ell_0$ 11:  $P_{581} = (4, 3, 1, 1)$  lies on line  $\ell_2$ 12:  $P_{584} = (7, 3, 1, 1)$  lies on line  $\ell_1$ 13:  $P_{596} = (3, 4, 1, 1)$  lies on line  $\ell_1$ 14:  $P_{598} = (5, 4, 1, 1)$  lies on line  $\ell_0$ 15:  $P_{600} = (7, 4, 1, 1)$  lies on line  $\ell_2$ 16:  $P_{613} = (4, 5, 1, 1)$  lies on line  $\ell_0$ 17:  $P_{618} = (9, 5, 1, 1)$  lies on line  $\ell_1$ 18:  $P_{621} = (12, 5, 1, 1)$  lies on line  $\ell_2$ 19:  $P_{632} = (7, 6, 1, 1)$  lies on line  $\ell_0$ 20:  $P_{633} = (8, 6, 1, 1)$  lies on line  $\ell_2$ 21:  $P_{639} = (14, 6, 1, 1)$  lies on line  $\ell_1$ 22:  $P_{644} = (3,7,1,1)$  lies on line  $\ell_2$ 

23:  $P_{645} = (4,7,1,1)$  lies on line  $\ell_1$ 24:  $P_{647} = (6,7,1,1)$  lies on line  $\ell_0$ 25:  $P_{663} = (6, 8, 1, 1)$  lies on line  $\ell_1$ 26:  $P_{666} = (9, 8, 1, 1)$  lies on line  $\ell_0$ 27:  $P_{671} = (14, 8, 1, 1)$  lies on line  $\ell_2$ 28:  $P_{678} = (5, 9, 1, 1)$  lies on line  $\ell_2$ 29:  $P_{681} = (8, 9, 1, 1)$  lies on line  $\ell_0$  $30: P_{685} = (12, 9, 1, 1)$  lies on line  $\ell_1$  $31: P_{690} = (1, 10, 1, 1)$  lies on line  $\ell_2$  $32: P_{706} = (1, 11, 1, 1)$  lies on line  $\ell_1$ 33:  $P_{726} = (5, 12, 1, 1)$  lies on line  $\ell_1$  $34: P_{730} = (9, 12, 1, 1)$  lies on line  $\ell_2$  $35: P_{734} = (13, 12, 1, 1)$  lies on line  $\ell_0$  $36: P_{739} = (2, 13, 1, 1)$  lies on line  $\ell_2$  $37: P_{749} = (12, 13, 1, 1)$  lies on line  $\ell_0$  $38: P_{752} = (15, 13, 1, 1)$  lies on line  $\ell_1$  $39: P_{759} = (6, 14, 1, 1)$  lies on line  $\ell_2$  $40: P_{761} = (8, 14, 1, 1)$  lies on line  $\ell_1$ 41:  $P_{768} = (15, 14, 1, 1)$  lies on line  $\ell_0$  $42: P_{771} = (2, 15, 1, 1)$  lies on line  $\ell_1$ 43 :  $P_{782} = (13, 15, 1, 1)$  lies on line  $\ell_2$  $44: P_{783} = (14, 15, 1, 1)$  lies on line  $\ell_0$ 

The single points on the surface are:

#### Points on surface but on no line

The surface has 193 points not on any line: The points on the surface but not on lines are:

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0: P_{20} = (1,0,1,0)
                                                                  48: P_{1385} = (8, 5, 4, 1)
1: P_{29} = (10, 0, 1, 0)
                                                                  49: P_{1414} = (5, 7, 4, 1)
2: P_{30} = (11, 0, 1, 0)
                                                                  50: P_{1419} = (10, 7, 4, 1)
                                                                  51: P_{1423} = (14, 7, 4, 1)
3: P_{126} = (11, 6, 1, 0)
4: P_{142} = (11, 7, 1, 0)
                                                                  52: P_{1450} = (9, 9, 4, 1)
5: P_{180} = (1, 10, 1, 0)
                                                                  53: P_{1469} = (12, 10, 4, 1)
6: P_{185} = (6, 10, 1, 0)
                                                                  54: P_{1492} = (3, 12, 4, 1)
7: P_{186} = (7, 10, 1, 0)
                                                                  55: P_{1502} = (13, 12, 4, 1)
8: P_{196} = (1, 11, 1, 0)
                                                                  56: P_{1504} = (15, 12, 4, 1)
9: P_{207} = (12, 11, 1, 0)
                                                                  57: P_{1516} = (11, 13, 4, 1)
10: P_{208} = (13, 11, 1, 0)
                                                                  58: P_{1571} = (2, 1, 5, 1)
11: P_{221} = (10, 12, 1, 0)
                                                                  59: P_{1592} = (7, 2, 5, 1)
12: P_{237} = (10, 13, 1, 0)
                                                                  60: P_{1608} = (7, 3, 5, 1)
13: P_{290} = (0, 1, 0, 1)
                                                                  61: P_{1624} = (7, 4, 5, 1)
                                                                  62: P_{1673} = (8, 7, 5, 1)
14: P_{291} = (1, 1, 0, 1)
15: P_{434} = (0, 10, 0, 1)
                                                                  63: P_{1703} = (6, 9, 5, 1)
16: P_{435} = (1, 10, 0, 1)
                                                                  64: P_{1758} = (13, 12, 5, 1)
17: P_{450} = (0, 11, 0, 1)
                                                                  65: P_{1770} = (9, 13, 5, 1)
18: P_{451} = (1, 11, 0, 1)
                                                                  66: P_{1791} = (14, 14, 5, 1)
19: P_{823} = (6, 2, 2, 1)
                                                                  67: P_{1836} = (11, 1, 6, 1)
20: P_{827} = (10, 2, 2, 1)
                                                                  68: P_{1843} = (2, 2, 6, 1)
21: P_{830} = (13, 2, 2, 1)
                                                                  69: P_{1865} = (8, 3, 6, 1)
22: P_{838} = (5, 3, 2, 1)
                                                                  70: P_{1908} = (3, 6, 6, 1)
23: P_{853} = (4,4,2,1)
                                                                  71: P_{1930} = (9, 7, 6, 1)
24: P_{891} = (10, 6, 2, 1)
                                                                  72: P_{1939} = (2, 8, 6, 1)
25: P_{903} = (6,7,2,1)
                                                                  73: P_{1980} = (11, 10, 6, 1)
26: P_{905} = (8,7,2,1)
                                                                  74: P_{1988} = (3, 11, 6, 1)
27: P_{912} = (15, 7, 2, 1)
                                                                  75: P_{2003} = (2, 12, 6, 1)
28: P_{968} = (7, 11, 2, 1)
                                                                  76: P_{2005} = (4, 12, 6, 1)
29: P_{996} = (3, 13, 2, 1)
                                                                  77: P_{2008} = (7, 12, 6, 1)
30: P_{1002} = (9, 13, 2, 1)
                                                                  78: P_{2028} = (11, 13, 6, 1)
31: P_{1004} = (11, 13, 2, 1)
                                                                  79: P_{2059} = (10, 15, 6, 1)
32: P_{1026} = (1, 15, 2, 1)
                                                                  80: P_{2092} = (11, 1, 7, 1)
33: P_{1038} = (13, 15, 2, 1)
                                                                  81: P_{2122} = (9, 3, 7, 1)
                                                                  82: P_{2155} = (10, 5, 7, 1)
34: P_{1071} = (14, 1, 3, 1)
35: P_{1086} = (13, 2, 3, 1)
                                                                  83: P_{2163} = (2, 6, 7, 1)
36: P_{1117} = (12, 4, 3, 1)
                                                                  84: P_{2185} = (8,7,7,1)
37: P_{1141} = (4, 6, 3, 1)
                                                                  85: P_{2196} = (3, 8, 7, 1)
38: P_{1159} = (6,7,3,1)
                                                                  86: P_{2218} = (9, 9, 7, 1)
39: P_{1194} = (9, 9, 3, 1)
                                                                  87: P_{2236} = (11, 10, 7, 1)
40: P_{1254} = (5, 13, 3, 1)
                                                                  88: P_{2249} = (8, 11, 7, 1)
41: P_{1278} = (13, 14, 3, 1)
                                                                  89: P_{2268} = (11, 12, 7, 1)
42: P_{1294} = (13, 15, 3, 1)
                                                                  90: P_{2279} = (6, 13, 7, 1)
43: P_{1346} = (1, 3, 4, 1)
                                                                  91: P_{2282} = (9, 13, 7, 1)
                                                                  92: P_{2287} = (14, 13, 7, 1)
44: P_{1352} = (7, 3, 4, 1)
45: P_{1368} = (7, 4, 4, 1)
                                                                  93: P_{2341} = (4, 1, 8, 1)
46: P_{1372} = (11, 4, 4, 1)
                                                                  94: P_{2355} = (2, 2, 8, 1)
47: P_{1374} = (13, 4, 4, 1)
                                                                  95: P_{2397} = (12, 4, 8, 1)
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145: P_{3451} = (10, 6, 12, 1)
96: P_{2413} = (12, 5, 8, 1)
97: P_{2424} = (7,6,8,1)
                                                                 146: P_{3459} = (2, 7, 12, 1)
98: P_{2447} = (14, 7, 8, 1)
                                                                 147: P_{3470} = (13, 7, 12, 1)
99: P_{2477} = (12, 9, 8, 1)
                                                                 148: P_{3471} = (14, 7, 12, 1)
100: P_{2528} = (15, 12, 8, 1)
                                                                 149: P_{3484} = (11, 8, 12, 1)
101: P_{2558} = (13, 14, 8, 1)
                                                                 150: P_{3520} = (15, 10, 12, 1)
102: P_{2658} = (1, 5, 9, 1)
                                                                 151: P_{3531} = (10, 11, 12, 1)
103: P_{2669} = (12, 5, 9, 1)
                                                                 152: P_{3552} = (15, 12, 12, 1)
104: P_{2676} = (3, 6, 9, 1)
                                                                 153: P_{3557} = (4, 13, 12, 1)
105: P_{2678} = (5, 6, 9, 1)
                                                                 154: P_{3583} = (14, 14, 12, 1)
106: P_{2680} = (7, 6, 9, 1)
                                                                 155: P_{3590} = (5, 15, 12, 1)
107: P_{2699} = (10, 7, 9, 1)
                                                                 156: P_{3627} = (10, 1, 13, 1)
108: P_{2720} = (15, 8, 9, 1)
                                                                 157: P_{3660} = (11, 3, 13, 1)
109: P_{2728} = (7, 9, 9, 1)
                                                                 158: P_{3669} = (4, 4, 13, 1)
110: P_{2731} = (10, 9, 9, 1)
                                                                 159: P_{3696} = (15, 5, 13, 1)
111: P_{2733} = (12, 9, 9, 1)
                                                                 160: P_{3701} = (4, 6, 13, 1)
                                                                 161: P_{3706} = (9, 6, 13, 1)
112: P_{2759} = (6, 11, 9, 1)
113: P_{2771} = (2, 12, 9, 1)
                                                                 162: P_{3709} = (12, 6, 13, 1)
114: P_{2777} = (8, 12, 9, 1)
                                                                 163: P_{3723} = (10, 7, 13, 1)
115: P_{2780} = (11, 12, 9, 1)
                                                                 164: P_{3766} = (5, 10, 13, 1)
116: P_{2815} = (14, 14, 9, 1)
                                                                 165: P_{3787} = (10, 11, 13, 1)
117: P_{2833} = (0, 0, 10, 1)
                                                                 166: P_{3807} = (14, 12, 13, 1)
118: P_{2834} = (1, 0, 10, 1)
                                                                 167: P_{3814} = (5, 13, 13, 1)
119: P_{2872} = (7, 2, 10, 1)
                                                                 168: P_{3845} = (4, 15, 13, 1)
120: P_{2895} = (14, 3, 10, 1)
                                                                 169: P_{3891} = (2, 2, 14, 1)
121: P_{2898} = (1, 4, 10, 1)
                                                                 170: P_{3957} = (4, 6, 14, 1)
122: P_{2912} = (15, 4, 10, 1)
                                                                 171: P_{3963} = (10, 6, 14, 1)
123: P_{2965} = (4, 8, 10, 1)
                                                                 172: P_{3968} = (15, 6, 14, 1)
124: P_{2983} = (6, 9, 10, 1)
                                                                 173: P_{3986} = (1, 8, 14, 1)
125: P_{2993} = (0, 10, 10, 1)
                                                                 174: P_{3991} = (6, 8, 14, 1)
126: P_{2999} = (6, 10, 10, 1)
                                                                 175: P_{4030} = (13, 10, 14, 1)
127: P_{3000} = (7, 10, 10, 1)
                                                                 176: P_{4060} = (11, 12, 14, 1)
128: P_{3058} = (1, 14, 10, 1)
                                                                 177: P_{4070} = (5, 13, 14, 1)
129: P_{3062} = (5, 14, 10, 1)
                                                                 178: P_{4073} = (8, 13, 14, 1)
130: P_{3089} = (0, 0, 11, 1)
                                                                 179: P_{4077} = (12, 13, 14, 1)
131: P_{3090} = (1, 0, 11, 1)
                                                                 180: P_{4087} = (6, 14, 14, 1)
132: P_{3122} = (1, 2, 11, 1)
                                                                 181: P_{4092} = (11, 14, 14, 1)
133: P_{3129} = (8, 2, 11, 1)
                                                                 182: P_{4093} = (12, 14, 14, 1)
134: P_{3165} = (12, 4, 11, 1)
                                                                 183: P_{4100} = (3, 15, 14, 1)
135: P_{3171} = (2, 5, 11, 1)
                                                                 184: P_{4138} = (9, 1, 15, 1)
136: P_{3234} = (1, 9, 11, 1)
                                                                 185: P_{4152} = (7, 2, 15, 1)
                                                                 186: P_{4181} = (4, 4, 15, 1)
137: P_{3236} = (3, 9, 11, 1)
138: P_{3265} = (0, 11, 11, 1)
                                                                 187: P_{4212} = (3, 6, 15, 1)
                                                                 188: P_{4247} = (6, 8, 15, 1)
139: P_{3277} = (12, 11, 11, 1)
140: P_{3278} = (13, 11, 11, 1)
                                                                 189: P_{4263} = (6, 9, 15, 1)
141: P_{3326} = (13, 14, 11, 1)
                                                                 190: P_{4307} = (2, 12, 15, 1)
142: P_{3338} = (9, 15, 11, 1)
                                                                 191: P_{4333} = (12, 13, 15, 1)
143: P_{3371} = (10, 1, 12, 1)
                                                                 192: P_{4343} = (6, 14, 15, 1)
144: P_{3439} = (14, 5, 12, 1)
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# Line Intersection Graph

$$\begin{array}{c|c} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$
in point	$P_{700}$	$P_{715}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_{700}$	$P_{530}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_{715}$	$P_{530}$

The surface has 241 points: The points on the surface are:

$0: P_5 = (1, 1, 0, 0)$	$33: P_{596} = (3, 4, 1, 1)$	$66: P_{783} = (14, 15, 1, 1)$
$1: P_{14} = (10, 1, 0, 0)$	$34: P_{598} = (5, 4, 1, 1)$	$67: P_{823} = (6, 2, 2, 1)$
$2: P_{15} = (11, 1, 0, 0)$	$35: P_{600} = (7, 4, 1, 1)$	$68: P_{827} = (10, 2, 2, 1)$
$3: P_{20} = (1,0,1,0)$	$36: P_{613} = (4, 5, 1, 1)$	$69: P_{830} = (13, 2, 2, 1)$
$4: P_{29} = (10, 0, 1, 0)$	$37: P_{618} = (9, 5, 1, 1)$	$70: P_{838} = (5, 3, 2, 1)$
$5: P_{30} = (11, 0, 1, 0)$	$38: P_{621} = (12, 5, 1, 1)$	$71: P_{853} = (4, 4, 2, 1)$
$6: P_{126} = (11, 6, 1, 0)$	$39: P_{632} = (7, 6, 1, 1)$	$72: P_{891} = (10, 6, 2, 1)$
$7: P_{142} = (11, 7, 1, 0)$	$40: P_{633} = (8, 6, 1, 1)$	$73: P_{903} = (6,7,2,1)$
$8: P_{180} = (1, 10, 1, 0)$	$41: P_{639} = (14, 6, 1, 1)$	$74: P_{905} = (8, 7, 2, 1)$
$9: P_{185} = (6, 10, 1, 0)$	$42: P_{644} = (3,7,1,1)$	$75: P_{912} = (15, 7, 2, 1)$
$10: P_{186} = (7, 10, 1, 0)$	$43: P_{645} = (4,7,1,1)$	$76: P_{968} = (7, 11, 2, 1)$
$11: P_{196} = (1, 11, 1, 0)$	$44: P_{647} = (6, 7, 1, 1)$	$77: P_{996} = (3, 13, 2, 1)$
$12: P_{207} = (12, 11, 1, 0)$	$45: P_{663} = (6, 8, 1, 1)$	$78: P_{1002} = (9, 13, 2, 1)$
$13: P_{208} = (13, 11, 1, 0)$	$46: P_{666} = (9, 8, 1, 1)$	$79: P_{1004} = (11, 13, 2, 1)$
$14: P_{221} = (10, 12, 1, 0)$	$47: P_{671} = (14, 8, 1, 1)$	$80: P_{1026} = (1, 15, 2, 1)$
$15: P_{237} = (10, 13, 1, 0)$	$48: P_{678} = (5, 9, 1, 1)$	$81: P_{1038} = (13, 15, 2, 1)$
$16: P_{290} = (0, 1, 0, 1)$	$49: P_{681} = (8, 9, 1, 1)$	$82: P_{1071} = (14, 1, 3, 1)$
$17: P_{291} = (1, 1, 0, 1)$	$50: P_{685} = (12, 9, 1, 1)$	$83: P_{1086} = (13, 2, 3, 1)$
$18: P_{434} = (0, 10, 0, 1)$	$51: P_{690} = (1, 10, 1, 1)$	$84: P_{1117} = (12, 4, 3, 1)$
$19: P_{435} = (1, 10, 0, 1)$	$52: P_{700} = (11, 10, 1, 1)$	$85: P_{1141} = (4, 6, 3, 1)$
$20: P_{450} = (0, 11, 0, 1)$	$53: P_{706} = (1, 11, 1, 1)$	$86: P_{1159} = (6,7,3,1)$
$21: P_{451} = (1, 11, 0, 1)$	$54: P_{715} = (10, 11, 1, 1)$	$87: P_{1194} = (9, 9, 3, 1)$
$22: P_{530} = (0, 0, 1, 1)$	$55: P_{726} = (5, 12, 1, 1)$	$88: P_{1254} = (5, 13, 3, 1)$
$23: P_{531} = (1,0,1,1)$	$56: P_{730} = (9, 12, 1, 1)$	$89: P_{1278} = (13, 14, 3, 1)$
$24: P_{546} = (0, 1, 1, 1)$	$57: P_{734} = (13, 12, 1, 1)$	90: $P_{1294} = (13, 15, 3, 1)$
$25: P_{555} = (10, 1, 1, 1)$	$58: P_{739} = (2, 13, 1, 1)$	$91: P_{1346} = (1, 3, 4, 1)$
$26: P_{556} = (11, 1, 1, 1)$	$59: P_{749} = (12, 13, 1, 1)$	92: $P_{1352} = (7, 3, 4, 1)$
$27: P_{564} = (3, 2, 1, 1)$	$60: P_{752} = (15, 13, 1, 1)$	93: $P_{1368} = (7, 4, 4, 1)$
$28: P_{574} = (13, 2, 1, 1)$	$61: P_{759} = (6, 14, 1, 1)$	$94: P_{1372} = (11, 4, 4, 1)$
$29: P_{576} = (15, 2, 1, 1)$	$62: P_{761} = (8, 14, 1, 1)$	$95: P_{1374} = (13, 4, 4, 1)$
$30: P_{579} = (2, 3, 1, 1)$	$63: P_{768} = (15, 14, 1, 1)$	$96: P_{1385} = (8, 5, 4, 1)$
$31: P_{581} = (4, 3, 1, 1)$	$64: P_{771} = (2, 15, 1, 1)$	$97: P_{1414} = (5, 7, 4, 1)$
$32: P_{584} = (7, 3, 1, 1)$	$65: P_{782} = (13, 15, 1, 1)$	98: $P_{1419} = (10, 7, 4, 1)$
331 ( / / / /	.52 ( ) / /	1110 ( ) ) )

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195: P_{3470} = (13, 7, 12, 1)
99: P_{1423} = (14, 7, 4, 1)
                                           147: P_{2477} = (12, 9, 8, 1)
                                                                                       196: P_{3471} = (14, 7, 12, 1)
100: P_{1450} = (9, 9, 4, 1)
                                           148: P_{2528} = (15, 12, 8, 1)
101: P_{1469} = (12, 10, 4, 1)
                                           149: P_{2558} = (13, 14, 8, 1)
                                                                                       197: P_{3484} = (11, 8, 12, 1)
                                                                                       198: P_{3520} = (15, 10, 12, 1)
102: P_{1492} = (3, 12, 4, 1)
                                           150: P_{2658} = (1, 5, 9, 1)
103: P_{1502} = (13, 12, 4, 1)
                                           151: P_{2669} = (12, 5, 9, 1)
                                                                                       199: P_{3531} = (10, 11, 12, 1)
104: P_{1504} = (15, 12, 4, 1)
                                           152: P_{2676} = (3, 6, 9, 1)
                                                                                       200: P_{3552} = (15, 12, 12, 1)
105: P_{1516} = (11, 13, 4, 1)
                                           153: P_{2678} = (5, 6, 9, 1)
                                                                                       201: P_{3557} = (4, 13, 12, 1)
106: P_{1571} = (2, 1, 5, 1)
                                           154: P_{2680} = (7, 6, 9, 1)
                                                                                       202: P_{3583} = (14, 14, 12, 1)
107: P_{1592} = (7, 2, 5, 1)
                                           155: P_{2699} = (10, 7, 9, 1)
                                                                                       203: P_{3590} = (5, 15, 12, 1)
108: P_{1608} = (7, 3, 5, 1)
                                           156: P_{2720} = (15, 8, 9, 1)
                                                                                       204: P_{3627} = (10, 1, 13, 1)
109: P_{1624} = (7, 4, 5, 1)
                                           157: P_{2728} = (7, 9, 9, 1)
                                                                                       205: P_{3660} = (11, 3, 13, 1)
110: P_{1673} = (8, 7, 5, 1)
                                           158: P_{2731} = (10, 9, 9, 1)
                                                                                       206: P_{3669} = (4, 4, 13, 1)
111: P_{1703} = (6, 9, 5, 1)
                                           159: P_{2733} = (12, 9, 9, 1)
                                                                                       207: P_{3696} = (15, 5, 13, 1)
112: P_{1758} = (13, 12, 5, 1)
                                                                                       208: P_{3701} = (4, 6, 13, 1)
                                           160: P_{2759} = (6, 11, 9, 1)
                                                                                       209: P_{3706} = (9, 6, 13, 1)
113: P_{1770} = (9, 13, 5, 1)
                                           161: P_{2771} = (2, 12, 9, 1)
114: P_{1791} = (14, 14, 5, 1)
                                           162: P_{2777} = (8, 12, 9, 1)
                                                                                       210: P_{3709} = (12, 6, 13, 1)
                                                                                       211: P_{3723} = (10, 7, 13, 1)
115: P_{1836} = (11, 1, 6, 1)
                                           163: P_{2780} = (11, 12, 9, 1)
116: P_{1843} = (2, 2, 6, 1)
                                           164: P_{2815} = (14, 14, 9, 1)
                                                                                       212: P_{3766} = (5, 10, 13, 1)
117: P_{1865} = (8, 3, 6, 1)
                                           165: P_{2833} = (0, 0, 10, 1)
                                                                                       213: P_{3787} = (10, 11, 13, 1)
118: P_{1908} = (3, 6, 6, 1)
                                           166: P_{2834} = (1, 0, 10, 1)
                                                                                       214: P_{3807} = (14, 12, 13, 1)
119: P_{1930} = (9, 7, 6, 1)
                                           167: P_{2872} = (7, 2, 10, 1)
                                                                                       215: P_{3814} = (5, 13, 13, 1)
120: P_{1939} = (2, 8, 6, 1)
                                           168: P_{2895} = (14, 3, 10, 1)
                                                                                       216: P_{3845} = (4, 15, 13, 1)
121: P_{1980} = (11, 10, 6, 1)
                                           169: P_{2898} = (1, 4, 10, 1)
                                                                                       217: P_{3891} = (2, 2, 14, 1)
122: P_{1988} = (3, 11, 6, 1)
                                           170: P_{2912} = (15, 4, 10, 1)
                                                                                       218: P_{3957} = (4, 6, 14, 1)
123: P_{2003} = (2, 12, 6, 1)
                                           171: P_{2965} = (4, 8, 10, 1)
                                                                                       219: P_{3963} = (10, 6, 14, 1)
124: P_{2005} = (4, 12, 6, 1)
                                           172: P_{2983} = (6, 9, 10, 1)
                                                                                       220: P_{3968} = (15, 6, 14, 1)
125: P_{2008} = (7, 12, 6, 1)
                                           173: P_{2993} = (0, 10, 10, 1)
                                                                                       221: P_{3986} = (1, 8, 14, 1)
126: P_{2028} = (11, 13, 6, 1)
                                           174: P_{2999} = (6, 10, 10, 1)
                                                                                       222: P_{3991} = (6, 8, 14, 1)
127: P_{2059} = (10, 15, 6, 1)
                                           175: P_{3000} = (7, 10, 10, 1)
                                                                                       223: P_{4030} = (13, 10, 14, 1)
128: P_{2092} = (11, 1, 7, 1)
                                           176: P_{3058} = (1, 14, 10, 1)
                                                                                       224: P_{4060} = (11, 12, 14, 1)
129: P_{2122} = (9, 3, 7, 1)
                                           177: P_{3062} = (5, 14, 10, 1)
                                                                                       225: P_{4070} = (5, 13, 14, 1)
130: P_{2155} = (10, 5, 7, 1)
                                           178: P_{3089} = (0, 0, 11, 1)
                                                                                       226: P_{4073} = (8, 13, 14, 1)
131: P_{2163} = (2, 6, 7, 1)
                                           179: P_{3090} = (1, 0, 11, 1)
                                                                                       227: P_{4077} = (12, 13, 14, 1)
132: P_{2185} = (8,7,7,1)
                                           180: P_{3122} = (1, 2, 11, 1)
                                                                                       228: P_{4087} = (6, 14, 14, 1)
                                           181: P_{3129} = (8, 2, 11, 1)
133: P_{2196} = (3, 8, 7, 1)
                                                                                       229: P_{4092} = (11, 14, 14, 1)
                                           182: P_{3165} = (12, 4, 11, 1)
134: P_{2218} = (9, 9, 7, 1)
                                                                                       230: P_{4093} = (12, 14, 14, 1)
135: P_{2236} = (11, 10, 7, 1)
                                           183: P_{3171} = (2, 5, 11, 1)
                                                                                       231: P_{4100} = (3, 15, 14, 1)
                                           184: P_{3234} = (1, 9, 11, 1)
                                                                                       232: P_{4138} = (9, 1, 15, 1)
136: P_{2249} = (8, 11, 7, 1)
137: P_{2268} = (11, 12, 7, 1)
                                           185: P_{3236} = (3, 9, 11, 1)
                                                                                       233: P_{4152} = (7, 2, 15, 1)
138: P_{2279} = (6, 13, 7, 1)
                                           186: P_{3265} = (0, 11, 11, 1)
                                                                                       234: P_{4181} = (4, 4, 15, 1)
139: P_{2282} = (9, 13, 7, 1)
                                           187: P_{3277} = (12, 11, 11, 1)
                                                                                       235: P_{4212} = (3, 6, 15, 1)
140: P_{2287} = (14, 13, 7, 1)
                                                                                       236: P_{4247} = (6, 8, 15, 1)
                                           188: P_{3278} = (13, 11, 11, 1)
141: P_{2341} = (4, 1, 8, 1)
                                           189: P_{3326} = (13, 14, 11, 1)
                                                                                       237: P_{4263} = (6, 9, 15, 1)
                                           190: P_{3338} = (9, 15, 11, 1)
                                                                                       238: P_{4307} = (2, 12, 15, 1)
142: P_{2355} = (2, 2, 8, 1)
143: P_{2397} = (12, 4, 8, 1)
                                           191: P_{3371} = (10, 1, 12, 1)
                                                                                       239: P_{4333} = (12, 13, 15, 1)
                                                                                       240: P_{4343} = (6, 14, 15, 1)
144: P_{2413} = (12, 5, 8, 1)
                                           192: P_{3439} = (14, 5, 12, 1)
145: P_{2424} = (7, 6, 8, 1)
                                           193: P_{3451} = (10, 6, 12, 1)
146: P_{2447} = (14, 7, 8, 1)
                                           194: P_{3459} = (2,7,12,1)
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