Rank-34 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_1 = 0$$

(1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 106

General information

Number of lines	1
Number of points	5
Number of singular points	5
Number of Eckardt points	0
Number of double points	0
Number of single points	5
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5
Type of lines on points	1^5

Singular Points

The surface has 5 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$2: P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
$3: P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$
$4: P_{69} = \mathbf{P}(0,0,\omega^2,1) = \mathbf{P}(0,0,3,1)$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (356)
Rank of points on Klein quadric: (1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 5 single points:
The single points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_0
1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0
2 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_0
3 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_0
4 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0
0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 5 points:
The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_{38} = (0, 0, 1, 1)$
3 : $P_{53} = (0, 0, 2, 1)$
4 : $P_{69} = (0, 0, 3, 1)$