Rank-73733 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286331173

General information

Number of lines	2
Number of points	273
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	32
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^2
Type of lines on points	2, 1^{32}, 0^{240}

Singular Points

The surface has 2 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (256, 70160)
Rank of points on Klein quadric: (2, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 32 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
2 : $P_{20} = (1,0,1,0)$ lies on line ℓ_0
3 : $P_{21} = (2,0,1,0)$ lies on line ℓ_0
4 : $P_{22} = (3,0,1,0)$ lies on line ℓ_0
5 : $P_{23} = (4,0,1,0)$ lies on line ℓ_0
6 : $P_{24} = (5,0,1,0)$ lies on line ℓ_0
7 : $P_{25} = (6,0,1,0)$ lies on line ℓ_0
8 : $P_{26} = (7,0,1,0)$ lies on line ℓ_0
9 : $P_{27} = (8,0,1,0)$ lies on line ℓ_0
10 : $P_{28} = (9,0,1,0)$ lies on line ℓ_0
11 : $P_{29} = (10,0,1,0)$ lies on line ℓ_0
12 : $P_{30} = (11,0,1,0)$ lies on line ℓ_0
13 : $P_{31} = (12,0,1,0)$ lies on line ℓ_0
14 : $P_{32} = (13,0,1,0)$ lies on line ℓ_0
15 : $P_{33} = (14,0,1,0)$ lies on line ℓ_0
16 : $P_{34} = (15,0,1,0)$ lies on line ℓ_0
17 : $P_{530} = (0,0,1,1)$ lies on line ℓ_1
18 : $P_{785} = (0,0,2,1)$ lies on line ℓ_1
19 : $P_{1041} = (0,0,3,1)$ lies on line ℓ_1
20 : $P_{1297} = (0,0,4,1)$ lies on line ℓ_1
21 : $P_{1553} = (0,0,5,1)$ lies on line ℓ_1
22 : $P_{1809} = (0,0,6,1)$ lies on line ℓ_1
23 : $P_{2065} = (0,0,7,1)$ lies on line ℓ_1
24 : $P_{2321} = (0,0,8,1)$ lies on line ℓ_1
25 : $P_{2577} = (0,0,9,1)$ lies on line ℓ_1
26 : $P_{2833} = (0,0,10,1)$ lies on line ℓ_1
27 : $P_{3089} = (0,0,11,1)$ lies on line ℓ_1
28 : $P_{3345} = (0,0,12,1)$ lies on line ℓ_1
29 : $P_{3601} = (0,0,13,1)$ lies on line ℓ_1
30 : $P_{3857} = (0,0,14,1)$ lies on line ℓ_1
31 : $P_{4113} = (0,0,15,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line:
The points on the surface but not on lines are:

$0: P_{36} = (1, 1, 1, 0)$
$1: P_{55} = (4, 2, 1, 0)$
$2: P_{72} = (5, 3, 1, 0)$
$3: P_{92} = (9, 4, 1, 0)$
$4: P_{107} = (8, 5, 1, 0)$
$5: P_{128} = (13, 6, 1, 0)$
$6: P_{143} = (12, 7, 1, 0)$
$7: P_{162} = (15, 8, 1, 0)$
$8: P_{177} = (14, 9, 1, 0)$
$9: P_{190} = (11, 10, 1, 0)$
$10: P_{205} = (10, 11, 1, 0)$
$11: P_{217} = (6, 12, 1, 0)$
$12: P_{234} = (7, 13, 1, 0)$
$13: P_{245} = (2, 14, 1, 0)$
$14: P_{262} = (3, 15, 1, 0)$
$15: P_{291} = (1, 1, 0, 1)$
$16: P_{314} = (8, 2, 0, 1)$
$17: P_{337} = (15, 3, 0, 1)$
$18: P_{353} = (15, 4, 0, 1)$
$19: P_{357} = (3, 5, 0, 1)$
$20: P_{375} = (5, 6, 0, 1)$
$21: P_{401} = (15, 7, 0, 1)$
$22: P_{407} = (5, 8, 0, 1)$
$23: P_{421} = (3, 9, 0, 1)$
$24: P_{435} = (1, 10, 0, 1)$
$25: P_{451} = (1, 11, 0, 1)$
$26: P_{469} = (3, 12, 0, 1)$
$27: P_{490} = (8, 13, 0, 1)$
$28: P_{503} = (5, 14, 0, 1)$
$29: P_{522} = (8, 15, 0, 1)$
$30: P_{576} = (15, 2, 1, 1)$
$31: P_{588} = (11, 3, 1, 1)$
$32: P_{596} = (3, 4, 1, 1)$
$33: P_{619} = (10, 5, 1, 1)$
$34: P_{629} = (4, 6, 1, 1)$
$35: P_{655} = (14, 7, 1, 1)$
$36: P_{668} = (11, 8, 1, 1)$
$37: P_{678} = (5, 9, 1, 1)$
$38: P_{699} = (10, 10, 1, 1)$
$39: P_{716} = (11, 11, 1, 1)$
$40: P_{723} = (2, 12, 1, 1)$
$41: P_{746} = (9, 13, 1, 1)$
$42: P_{761} = (8, 14, 1, 1)$
$43: P_{779} = (10, 15, 1, 1)$
$44: P_{809} = (8, 1, 2, 1)$
$45: P_{822} = (5, 2, 2, 1)$
$46: P_{845} = (12, 3, 2, 1)$
$47: P_{853} = (4, 4, 2, 1)$
$48: P_{872} = (7, 5, 2, 1)$
$49: P_{887} = (6, 6, 2, 1)$
$50: P_{898} = (1, 7, 2, 1)$
$51: P_{928} = (15, 8, 2, 1)$
$52: P_{933} = (4, 9, 2, 1)$
$53: P_{947} = (2, 10, 2, 1)$
$54: P_{968} = (7, 11, 2, 1)$
$55: P_{997} = (4, 13, 2, 1)$
$56: P_{1016} = (7, 14, 2, 1)$
$57: P_{1036} = (11, 15, 2, 1)$
$58: P_{1069} = (12, 1, 3, 1)$
$59: P_{1086} = (13, 2, 3, 1)$
$60: P_{1098} = (9, 3, 3, 1)$
$61: P_{1115} = (10, 4, 3, 1)$
$62: P_{1130} = (9, 5, 3, 1)$
$63: P_{1146} = (9, 6, 3, 1)$
$64: P_{1161} = (8, 7, 3, 1)$
$65: P_{1186} = (1, 9, 3, 1)$
$66: P_{1205} = (4, 10, 3, 1)$
$67: P_{1232} = (15, 11, 3, 1)$
$68: P_{1239} = (6, 12, 3, 1)$
$69: P_{1252} = (3, 13, 3, 1)$
$70: P_{1277} = (12, 14, 3, 1)$
$71: P_{1293} = (12, 15, 3, 1)$
$72: P_{1328} = (15, 1, 4, 1)$
$73: P_{1341} = (12, 2, 4, 1)$
$74: P_{1355} = (10, 3, 4, 1)$
$75: P_{1369} = (8, 4, 4, 1)$
$76: P_{1383} = (6, 5, 4, 1)$
$77: P_{1418} = (9, 7, 4, 1)$
$78: P_{1437} = (12, 8, 4, 1)$
$79: P_{1450} = (9, 9, 4, 1)$
$80: P_{1469} = (12, 10, 4, 1)$
$81: P_{1477} = (4, 11, 4, 1)$
$82: P_{1490} = (1, 12, 4, 1)$
$83: P_{1518} = (13, 13, 4, 1)$
$84: P_{1530} = (9, 14, 4, 1)$
$85: P_{1540} = (3, 15, 4, 1)$
$86: P_{1575} = (6, 1, 5, 1)$
$87: P_{1591} = (6, 2, 5, 1)$
$88: P_{1607} = (6, 3, 5, 1)$
$89: P_{1624} = (7, 4, 5, 1)$
$90: P_{1647} = (14, 5, 5, 1)$
$91: P_{1662} = (13, 6, 5, 1)$
$92: P_{1670} = (5, 7, 5, 1)$
$93: P_{1695} = (14, 8, 5, 1)$
$94: P_{1708} = (11, 9, 5, 1)$
$95: P_{1716} = (3, 10, 5, 1)$
$96: P_{1738} = (9, 11, 5, 1)$
$97: P_{1760} = (15, 12, 5, 1)$
$98: P_{1775} = (14, 13, 5, 1)$
$99: P_{1778} = (1, 14, 5, 1)$
$100: P_{1839} = (14, 1, 6, 1)$
$101: P_{1848} = (7, 2, 6, 1)$
$102: P_{1859} = (2, 3, 6, 1)$
$103: P_{1901} = (12, 5, 6, 1)$
$104: P_{1915} = (10, 6, 6, 1)$
$105: P_{1934} = (13, 7, 6, 1)$
$106: P_{1940} = (3, 8, 6, 1)$
$107: P_{1961} = (8, 9, 6, 1)$

$108: P_{1974} = (5, 10, 6, 1)$
$109: P_{1998} = (13, 11, 6, 1)$
$110: P_{2014} = (13, 12, 6, 1)$
$111: P_{2019} = (2, 13, 6, 1)$
$112: P_{2035} = (2, 14, 6, 1)$
$113: P_{2050} = (1, 15, 6, 1)$
$114: P_{2085} = (4, 1, 7, 1)$
$115: P_{2100} = (3, 2, 7, 1)$
$116: P_{2121} = (8, 3, 7, 1)$
$117: P_{2138} = (9, 4, 7, 1)$
$118: P_{2146} = (1, 5, 7, 1)$
$119: P_{2173} = (12, 6, 7, 1)$
$120: P_{2187} = (10, 7, 7, 1)$
$121: P_{2202} = (9, 8, 7, 1)$
$122: P_{2215} = (6, 9, 7, 1)$
$123: P_{2240} = (15, 10, 7, 1)$
$124: P_{2253} = (12, 11, 7, 1)$
$125: P_{2266} = (9, 12, 7, 1)$
$126: P_{2285} = (12, 13, 7, 1)$
$127: P_{2318} = (13, 15, 7, 1)$
$128: P_{2350} = (13, 1, 8, 1)$
$129: P_{2354} = (1, 2, 8, 1)$
$130: P_{2398} = (13, 4, 8, 1)$
$131: P_{2414} = (13, 5, 8, 1)$
$132: P_{2420} = (3, 6, 8, 1)$
$133: P_{2435} = (2, 7, 8, 1)$
$134: P_{2451} = (2, 8, 8, 1)$
$135: P_{2477} = (12, 9, 8, 1)$
$136: P_{2495} = (14, 10, 8, 1)$
$137: P_{2502} = (5, 11, 8, 1)$
$138: P_{2521} = (8, 12, 8, 1)$
$139: P_{2536} = (7, 13, 8, 1)$
$140: P_{2555} = (10, 14, 8, 1)$
$141: P_{2563} = (2, 15, 8, 1)$
$142: P_{2596} = (3, 1, 9, 1)$
$143: P_{2623} = (14, 2, 9, 1)$
$144: P_{2630} = (5, 3, 9, 1)$
$145: P_{2647} = (6, 4, 9, 1)$
$146: P_{2668} = (11, 5, 9, 1)$
$147: P_{2674} = (1, 6, 9, 1)$
$148: P_{2696} = (7, 7, 9, 1)$
$149: P_{2718} = (13, 8, 9, 1)$
$150: P_{2736} = (15, 9, 9, 1)$
$151: P_{2746} = (9, 10, 9, 1)$
$152: P_{2759} = (6, 11, 9, 1)$
$153: P_{2783} = (14, 12, 9, 1)$
$154: P_{2815} = (14, 14, 9, 1)$
$155: P_{2823} = (6, 15, 9, 1)$
$156: P_{2859} = (10, 1, 10, 1)$
$157: P_{2874} = (9, 2, 10, 1)$
$158: P_{2895} = (14, 3, 10, 1)$
$159: P_{2908} = (11, 4, 10, 1)$
$160: P_{2918} = (5, 5, 10, 1)$
$161: P_{2937} = (8, 6, 10, 1)$
$162: P_{2948} = (3, 7, 10, 1)$
$163: P_{2965} = (4, 8, 10, 1)$
$164: P_{2979} = (2, 9, 10, 1)$
$165: P_{3004} = (11, 10, 10, 1)$
$166: P_{3035} = (10, 12, 10, 1)$
$167: P_{3051} = (10, 13, 10, 1)$
$168: P_{3068} = (11, 14, 10, 1)$
$169: P_{3088} = (15, 15, 10, 1)$
$170: P_{3116} = (11, 1, 11, 1)$
$171: P_{3131} = (10, 2, 11, 1)$
$172: P_{3140} = (3, 3, 11, 1)$
$173: P_{3167} = (14, 4, 11, 1)$
$174: P_{3171} = (2, 5, 11, 1)$
$175: P_{3196} = (11, 6, 11, 1)$
$176: P_{3212} = (11, 7, 11, 1)$
$177: P_{3225} = (8, 8, 11, 1)$
$178: P_{3243} = (10, 9, 11, 1)$
$179: P_{3275} = (10, 11, 11, 1)$
$180: P_{3286} = (5, 12, 11, 1)$
$181: P_{3312} = (15, 13, 11, 1)$
$182: P_{3317} = (4, 14, 11, 1)$
$183: P_{3338} = (9, 15, 11, 1)$
$184: P_{3370} = (9, 1, 12, 1)$
$185: P_{3400} = (7, 3, 12, 1)$
$186: P_{3414} = (5, 4, 12, 1)$
$187: P_{3440} = (15, 5, 12, 1)$
$188: P_{3455} = (14, 6, 12, 1)$
$189: P_{3463} = (6, 7, 12, 1)$
$190: P_{3474} = (1, 8, 12, 1)$
$191: P_{3503} = (14, 9, 12, 1)$
$192: P_{3511} = (6, 10, 12, 1)$
$193: P_{3524} = (3, 11, 12, 1)$
$194: P_{3548} = (11, 12, 12, 1)$
$195: P_{3559} = (6, 13, 12, 1)$
$196: P_{3582} = (13, 14, 12, 1)$
$197: P_{3599} = (14, 15, 12, 1)$
$198: P_{3619} = (2, 1, 13, 1)$
$199: P_{3637} = (4, 2, 13, 1)$
$200: P_{3650} = (1, 3, 13, 1)$
$201: P_{3677} = (12, 4, 13, 1)$
$202: P_{3685} = (4, 5, 13, 1)$
$203: P_{3704} = (7, 6, 13, 1)$
$204: P_{3717} = (4, 7, 13, 1)$
$205: P_{3735} = (6, 8, 13, 1)$
$206: P_{3768} = (7, 10, 13, 1)$
$207: P_{3785} = (8, 11, 13, 1)$
$208: P_{3800} = (7, 12, 13, 1)$
$209: P_{3820} = (11, 13, 13, 1)$
$210: P_{3840} = (15, 14, 13, 1)$
$211: P_{3846} = (5, 15, 13, 1)$
$212: P_{3878} = (5, 1, 14, 1)$
$213: P_{3891} = (2, 2, 14, 1)$
$214: P_{3918} = (13, 3, 14, 1)$
$215: P_{3923} = (2, 4, 14, 1)$

216 : $P_{3945} = (8, 5, 14, 1)$
217 : $P_{3955} = (2, 6, 14, 1)$
218 : $P_{3995} = (10, 8, 14, 1)$
219 : $P_{4014} = (13, 9, 14, 1)$
220 : $P_{4030} = (13, 10, 14, 1)$
221 : $P_{4047} = (14, 11, 14, 1)$
222 : $P_{4061} = (12, 12, 14, 1)$
223 : $P_{4066} = (1, 13, 14, 1)$
224 : $P_{4084} = (3, 14, 14, 1)$
225 : $P_{4104} = (7, 15, 14, 1)$
226 : $P_{4136} = (7, 1, 15, 1)$
227 : $P_{4156} = (11, 2, 15, 1)$
228 : $P_{4165} = (4, 3, 15, 1)$
229 : $P_{4178} = (1, 4, 15, 1)$
230 : $P_{4224} = (15, 6, 15, 1)$
231 : $P_{4237} = (12, 7, 15, 1)$
232 : $P_{4248} = (7, 8, 15, 1)$
233 : $P_{4264} = (7, 9, 15, 1)$
234 : $P_{4281} = (8, 10, 15, 1)$
235 : $P_{4291} = (2, 11, 15, 1)$
236 : $P_{4309} = (4, 12, 15, 1)$
237 : $P_{4326} = (5, 13, 15, 1)$
238 : $P_{4343} = (6, 14, 15, 1)$
239 : $P_{4357} = (4, 15, 15, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 273 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_{20} = (1, 0, 1, 0)$
4 : $P_{21} = (2, 0, 1, 0)$
5 : $P_{22} = (3, 0, 1, 0)$
6 : $P_{23} = (4, 0, 1, 0)$
7 : $P_{24} = (5, 0, 1, 0)$
8 : $P_{25} = (6, 0, 1, 0)$
9 : $P_{26} = (7, 0, 1, 0)$
10 : $P_{27} = (8, 0, 1, 0)$
11 : $P_{28} = (9, 0, 1, 0)$
12 : $P_{29} = (10, 0, 1, 0)$
13 : $P_{30} = (11, 0, 1, 0)$
14 : $P_{31} = (12, 0, 1, 0)$
15 : $P_{32} = (13, 0, 1, 0)$
16 : $P_{33} = (14, 0, 1, 0)$
17 : $P_{34} = (15, 0, 1, 0)$
18 : $P_{36} = (1, 1, 1, 0)$
19 : $P_{55} = (4, 2, 1, 0)$
20 : $P_{72} = (5, 3, 1, 0)$
21 : $P_{92} = (9, 4, 1, 0)$
22 : $P_{107} = (8, 5, 1, 0)$
23 : $P_{128} = (13, 6, 1, 0)$
24 : $P_{143} = (12, 7, 1, 0)$
25 : $P_{162} = (15, 8, 1, 0)$
26 : $P_{177} = (14, 9, 1, 0)$
27 : $P_{190} = (11, 10, 1, 0)$
28 : $P_{205} = (10, 11, 1, 0)$
29 : $P_{217} = (6, 12, 1, 0)$
30 : $P_{234} = (7, 13, 1, 0)$
31 : $P_{245} = (2, 14, 1, 0)$
32 : $P_{262} = (3, 15, 1, 0)$
33 : $P_{291} = (1, 1, 0, 1)$
34 : $P_{314} = (8, 2, 0, 1)$
35 : $P_{337} = (15, 3, 0, 1)$
36 : $P_{353} = (15, 4, 0, 1)$
37 : $P_{357} = (3, 5, 0, 1)$
38 : $P_{375} = (5, 6, 0, 1)$
39 : $P_{401} = (15, 7, 0, 1)$
40 : $P_{407} = (5, 8, 0, 1)$
41 : $P_{421} = (3, 9, 0, 1)$
42 : $P_{435} = (1, 10, 0, 1)$
43 : $P_{451} = (1, 11, 0, 1)$
44 : $P_{469} = (3, 12, 0, 1)$
45 : $P_{490} = (8, 13, 0, 1)$
46 : $P_{503} = (5, 14, 0, 1)$
47 : $P_{522} = (8, 15, 0, 1)$
48 : $P_{530} = (0, 0, 1, 1)$
49 : $P_{576} = (15, 2, 1, 1)$
50 : $P_{588} = (11, 3, 1, 1)$
51 : $P_{596} = (3, 4, 1, 1)$
52 : $P_{619} = (10, 5, 1, 1)$
53 : $P_{629} = (4, 6, 1, 1)$
54 : $P_{655} = (14, 7, 1, 1)$
55 : $P_{668} = (11, 8, 1, 1)$
56 : $P_{678} = (5, 9, 1, 1)$
57 : $P_{699} = (10, 10, 1, 1)$
58 : $P_{716} = (11, 11, 1, 1)$
59 : $P_{723} = (2, 12, 1, 1)$
60 : $P_{746} = (9, 13, 1, 1)$
61 : $P_{761} = (8, 14, 1, 1)$
62 : $P_{779} = (10, 15, 1, 1)$
63 : $P_{785} = (0, 0, 2, 1)$
64 : $P_{809} = (8, 1, 2, 1)$
65 : $P_{822} = (5, 2, 2, 1)$
66 : $P_{845} = (12, 3, 2, 1)$
67 : $P_{853} = (4, 4, 2, 1)$
68 : $P_{872} = (7, 5, 2, 1)$

69 : $P_{887} = (6, 6, 2, 1)$
70 : $P_{898} = (1, 7, 2, 1)$
71 : $P_{928} = (15, 8, 2, 1)$
72 : $P_{933} = (4, 9, 2, 1)$
73 : $P_{947} = (2, 10, 2, 1)$
74 : $P_{968} = (7, 11, 2, 1)$
75 : $P_{997} = (4, 13, 2, 1)$
76 : $P_{1016} = (7, 14, 2, 1)$
77 : $P_{1036} = (11, 15, 2, 1)$
78 : $P_{1041} = (0, 0, 3, 1)$
79 : $P_{1069} = (12, 1, 3, 1)$
80 : $P_{1086} = (13, 2, 3, 1)$
81 : $P_{1098} = (9, 3, 3, 1)$
82 : $P_{1115} = (10, 4, 3, 1)$
83 : $P_{1130} = (9, 5, 3, 1)$
84 : $P_{1146} = (9, 6, 3, 1)$
85 : $P_{1161} = (8, 7, 3, 1)$
86 : $P_{1186} = (1, 9, 3, 1)$
87 : $P_{1205} = (4, 10, 3, 1)$
88 : $P_{1232} = (15, 11, 3, 1)$
89 : $P_{1239} = (6, 12, 3, 1)$
90 : $P_{1252} = (3, 13, 3, 1)$
91 : $P_{1277} = (12, 14, 3, 1)$
92 : $P_{1293} = (12, 15, 3, 1)$
93 : $P_{1297} = (0, 0, 4, 1)$
94 : $P_{1328} = (15, 1, 4, 1)$
95 : $P_{1341} = (12, 2, 4, 1)$
96 : $P_{1355} = (10, 3, 4, 1)$
97 : $P_{1369} = (8, 4, 4, 1)$
98 : $P_{1383} = (6, 5, 4, 1)$
99 : $P_{1418} = (9, 7, 4, 1)$
100 : $P_{1437} = (12, 8, 4, 1)$
101 : $P_{1450} = (9, 9, 4, 1)$
102 : $P_{1469} = (12, 10, 4, 1)$
103 : $P_{1477} = (4, 11, 4, 1)$
104 : $P_{1490} = (1, 12, 4, 1)$
105 : $P_{1518} = (13, 13, 4, 1)$
106 : $P_{1530} = (9, 14, 4, 1)$
107 : $P_{1540} = (3, 15, 4, 1)$
108 : $P_{1553} = (0, 0, 5, 1)$
109 : $P_{1575} = (6, 1, 5, 1)$
110 : $P_{1591} = (6, 2, 5, 1)$
111 : $P_{1607} = (6, 3, 5, 1)$
112 : $P_{1624} = (7, 4, 5, 1)$
113 : $P_{1647} = (14, 5, 5, 1)$
114 : $P_{1662} = (13, 6, 5, 1)$
115 : $P_{1670} = (5, 7, 5, 1)$
116 : $P_{1695} = (14, 8, 5, 1)$
117 : $P_{1708} = (11, 9, 5, 1)$
118 : $P_{1716} = (3, 10, 5, 1)$
119 : $P_{1738} = (9, 11, 5, 1)$
120 : $P_{1760} = (15, 12, 5, 1)$
121 : $P_{1775} = (14, 13, 5, 1)$
122 : $P_{1778} = (1, 14, 5, 1)$
123 : $P_{1809} = (0, 0, 6, 1)$
124 : $P_{1839} = (14, 1, 6, 1)$
125 : $P_{1848} = (7, 2, 6, 1)$
126 : $P_{1859} = (2, 3, 6, 1)$
127 : $P_{1901} = (12, 5, 6, 1)$
128 : $P_{1915} = (10, 6, 6, 1)$
129 : $P_{1934} = (13, 7, 6, 1)$
130 : $P_{1940} = (3, 8, 6, 1)$
131 : $P_{1961} = (8, 9, 6, 1)$
132 : $P_{1974} = (5, 10, 6, 1)$
133 : $P_{1998} = (13, 11, 6, 1)$
134 : $P_{2014} = (13, 12, 6, 1)$
135 : $P_{2019} = (2, 13, 6, 1)$
136 : $P_{2035} = (2, 14, 6, 1)$
137 : $P_{2050} = (1, 15, 6, 1)$
138 : $P_{2065} = (0, 0, 7, 1)$
139 : $P_{2085} = (4, 1, 7, 1)$
140 : $P_{2100} = (3, 2, 7, 1)$
141 : $P_{2121} = (8, 3, 7, 1)$
142 : $P_{2138} = (9, 4, 7, 1)$
143 : $P_{2146} = (1, 5, 7, 1)$
144 : $P_{2173} = (12, 6, 7, 1)$
145 : $P_{2187} = (10, 7, 7, 1)$
146 : $P_{2202} = (9, 8, 7, 1)$
147 : $P_{2215} = (6, 9, 7, 1)$
148 : $P_{2240} = (15, 10, 7, 1)$
149 : $P_{2253} = (12, 11, 7, 1)$
150 : $P_{2266} = (9, 12, 7, 1)$
151 : $P_{2285} = (12, 13, 7, 1)$
152 : $P_{2318} = (13, 15, 7, 1)$
153 : $P_{2321} = (0, 0, 8, 1)$
154 : $P_{2350} = (13, 1, 8, 1)$
155 : $P_{2354} = (1, 2, 8, 1)$
156 : $P_{2398} = (13, 4, 8, 1)$
157 : $P_{2414} = (13, 5, 8, 1)$
158 : $P_{2420} = (3, 6, 8, 1)$
159 : $P_{2435} = (2, 7, 8, 1)$
160 : $P_{2451} = (2, 8, 8, 1)$
161 : $P_{2477} = (12, 9, 8, 1)$
162 : $P_{2495} = (14, 10, 8, 1)$
163 : $P_{2502} = (5, 11, 8, 1)$
164 : $P_{2521} = (8, 12, 8, 1)$
165 : $P_{2536} = (7, 13, 8, 1)$
166 : $P_{2555} = (10, 14, 8, 1)$
167 : $P_{2563} = (2, 15, 8, 1)$
168 : $P_{2577} = (0, 0, 9, 1)$
169 : $P_{2596} = (3, 1, 9, 1)$
170 : $P_{2623} = (14, 2, 9, 1)$
171 : $P_{2630} = (5, 3, 9, 1)$
172 : $P_{2647} = (6, 4, 9, 1)$
173 : $P_{2668} = (11, 5, 9, 1)$
174 : $P_{2674} = (1, 6, 9, 1)$
175 : $P_{2696} = (7, 7, 9, 1)$
176 : $P_{2718} = (13, 8, 9, 1)$
177 : $P_{2736} = (15, 9, 9, 1)$
178 : $P_{2746} = (9, 10, 9, 1)$
179 : $P_{2759} = (6, 11, 9, 1)$
180 : $P_{2783} = (14, 12, 9, 1)$
181 : $P_{2815} = (14, 14, 9, 1)$
182 : $P_{2823} = (6, 15, 9, 1)$
183 : $P_{2833} = (0, 0, 10, 1)$
184 : $P_{2859} = (10, 1, 10, 1)$
185 : $P_{2874} = (9, 2, 10, 1)$
186 : $P_{2895} = (14, 3, 10, 1)$
187 : $P_{2908} = (11, 4, 10, 1)$
188 : $P_{2918} = (5, 5, 10, 1)$
189 : $P_{2937} = (8, 6, 10, 1)$
190 : $P_{2948} = (3, 7, 10, 1)$
191 : $P_{2965} = (4, 8, 10, 1)$
192 : $P_{2979} = (2, 9, 10, 1)$
193 : $P_{3004} = (11, 10, 10, 1)$
194 : $P_{3035} = (10, 12, 10, 1)$
195 : $P_{3051} = (10, 13, 10, 1)$
196 : $P_{3068} = (11, 14, 10, 1)$
197 : $P_{3088} = (15, 15, 10, 1)$
198 : $P_{3089} = (0, 0, 11, 1)$
199 : $P_{3116} = (11, 1, 11, 1)$
200 : $P_{3131} = (10, 2, 11, 1)$
201 : $P_{3140} = (3, 3, 11, 1)$
202 : $P_{3167} = (14, 4, 11, 1)$
203 : $P_{3171} = (2, 5, 11, 1)$
204 : $P_{3196} = (11, 6, 11, 1)$
205 : $P_{3212} = (11, 7, 11, 1)$
206 : $P_{3225} = (8, 8, 11, 1)$
207 : $P_{3243} = (10, 9, 11, 1)$
208 : $P_{3275} = (10, 11, 11, 1)$
209 : $P_{3286} = (5, 12, 11, 1)$
210 : $P_{3312} = (15, 13, 11, 1)$
211 : $P_{3317} = (4, 14, 11, 1)$
212 : $P_{3338} = (9, 15, 11, 1)$
213 : $P_{3345} = (0, 0, 12, 1)$
214 : $P_{3370} = (9, 1, 12, 1)$
215 : $P_{3400} = (7, 3, 12, 1)$
216 : $P_{3414} = (5, 4, 12, 1)$
217 : $P_{3440} = (15, 5, 12, 1)$
218 : $P_{3455} = (14, 6, 12, 1)$
219 : $P_{3463} = (6, 7, 12, 1)$
220 : $P_{3474} = (1, 8, 12, 1)$
221 : $P_{3503} = (14, 9, 12, 1)$
222 : $P_{3511} = (6, 10, 12, 1)$
223 : $P_{3524} = (3, 11, 12, 1)$
224 : $P_{3548} = (11, 12, 12, 1)$
225 : $P_{3559} = (6, 13, 12, 1)$
226 : $P_{3582} = (13, 14, 12, 1)$
227 : $P_{3599} = (14, 15, 12, 1)$
228 : $P_{3601} = (0, 0, 13, 1)$
229 : $P_{3619} = (2, 1, 13, 1)$
230 : $P_{3637} = (4, 2, 13, 1)$

231 : $P_{3650} = (1, 3, 13, 1)$
232 : $P_{3677} = (12, 4, 13, 1)$
233 : $P_{3685} = (4, 5, 13, 1)$
234 : $P_{3704} = (7, 6, 13, 1)$
235 : $P_{3717} = (4, 7, 13, 1)$
236 : $P_{3735} = (6, 8, 13, 1)$
237 : $P_{3768} = (7, 10, 13, 1)$
238 : $P_{3785} = (8, 11, 13, 1)$
239 : $P_{3800} = (7, 12, 13, 1)$
240 : $P_{3820} = (11, 13, 13, 1)$
241 : $P_{3840} = (15, 14, 13, 1)$
242 : $P_{3846} = (5, 15, 13, 1)$
243 : $P_{3857} = (0, 0, 14, 1)$
244 : $P_{3878} = (5, 1, 14, 1)$
245 : $P_{3891} = (2, 2, 14, 1)$
246 : $P_{3918} = (13, 3, 14, 1)$
247 : $P_{3923} = (2, 4, 14, 1)$
248 : $P_{3945} = (8, 5, 14, 1)$
249 : $P_{3955} = (2, 6, 14, 1)$
250 : $P_{3995} = (10, 8, 14, 1)$
251 : $P_{4014} = (13, 9, 14, 1)$
252 : $P_{4030} = (13, 10, 14, 1)$
253 : $P_{4047} = (14, 11, 14, 1)$
254 : $P_{4061} = (12, 12, 14, 1)$
255 : $P_{4066} = (1, 13, 14, 1)$
256 : $P_{4084} = (3, 14, 14, 1)$
257 : $P_{4104} = (7, 15, 14, 1)$
258 : $P_{4113} = (0, 0, 15, 1)$
259 : $P_{4136} = (7, 1, 15, 1)$
260 : $P_{4156} = (11, 2, 15, 1)$
261 : $P_{4165} = (4, 3, 15, 1)$
262 : $P_{4178} = (1, 4, 15, 1)$
263 : $P_{4224} = (15, 6, 15, 1)$
264 : $P_{4237} = (12, 7, 15, 1)$
265 : $P_{4248} = (7, 8, 15, 1)$
266 : $P_{4264} = (7, 9, 15, 1)$
267 : $P_{4281} = (8, 10, 15, 1)$
268 : $P_{4291} = (2, 11, 15, 1)$
269 : $P_{4309} = (4, 12, 15, 1)$
270 : $P_{4326} = (5, 13, 15, 1)$
271 : $P_{4343} = (6, 14, 15, 1)$
272 : $P_{4357} = (4, 15, 15, 1)$