

Rank-74279 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is -2130436027

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	4097
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$2^3, 1^{189}, 0^{4097}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\ \ell_1 &= \begin{bmatrix} 1 & 0 & \epsilon^{21} & \epsilon^{21} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15416570} = \begin{bmatrix} 1 & 0 & 57 & 57 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15416570} = \mathbf{Pl}(1, 0, 1, 1, 56, 1)_{14958979}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \epsilon^{42} & \epsilon^{42} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15146105} = \begin{bmatrix} 1 & 0 & 56 & 56 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{15146105} = \mathbf{PI}(1, 0, 1, 1, 57, 1)_{15221059}$$

Rank of lines: (0, 15416570, 15146105)

Rank of points on Klein quadric: (0, 14958979, 15221059)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_{60} = (56, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{61} = (57, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{8322} = (0, 1, 1, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 189 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0
2 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0
3 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0
4 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0
5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0
6 : $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0
7 : $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0
8 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0
9 : $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0
10 : $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0
11 : $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0
12 : $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0
13 : $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0
14 : $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0
15 : $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0
16 : $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0
17 : $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0
18 : $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0
19 : $P_{22} = (18, 1, 0, 0)$ lies on line ℓ_0
20 : $P_{23} = (19, 1, 0, 0)$ lies on line ℓ_0
21 : $P_{24} = (20, 1, 0, 0)$ lies on line ℓ_0
22 : $P_{25} = (21, 1, 0, 0)$ lies on line ℓ_0
23 : $P_{26} = (22, 1, 0, 0)$ lies on line ℓ_0
24 : $P_{27} = (23, 1, 0, 0)$ lies on line ℓ_0
25 : $P_{28} = (24, 1, 0, 0)$ lies on line ℓ_0
26 : $P_{29} = (25, 1, 0, 0)$ lies on line ℓ_0
27 : $P_{30} = (26, 1, 0, 0)$ lies on line ℓ_0

28 : $P_{31} = (27, 1, 0, 0)$ lies on line ℓ_0
29 : $P_{32} = (28, 1, 0, 0)$ lies on line ℓ_0
30 : $P_{33} = (29, 1, 0, 0)$ lies on line ℓ_0
31 : $P_{34} = (30, 1, 0, 0)$ lies on line ℓ_0
32 : $P_{35} = (31, 1, 0, 0)$ lies on line ℓ_0
33 : $P_{36} = (32, 1, 0, 0)$ lies on line ℓ_0
34 : $P_{37} = (33, 1, 0, 0)$ lies on line ℓ_0
35 : $P_{38} = (34, 1, 0, 0)$ lies on line ℓ_0
36 : $P_{39} = (35, 1, 0, 0)$ lies on line ℓ_0
37 : $P_{40} = (36, 1, 0, 0)$ lies on line ℓ_0
38 : $P_{41} = (37, 1, 0, 0)$ lies on line ℓ_0
39 : $P_{42} = (38, 1, 0, 0)$ lies on line ℓ_0
40 : $P_{43} = (39, 1, 0, 0)$ lies on line ℓ_0
41 : $P_{44} = (40, 1, 0, 0)$ lies on line ℓ_0
42 : $P_{45} = (41, 1, 0, 0)$ lies on line ℓ_0
43 : $P_{46} = (42, 1, 0, 0)$ lies on line ℓ_0
44 : $P_{47} = (43, 1, 0, 0)$ lies on line ℓ_0
45 : $P_{48} = (44, 1, 0, 0)$ lies on line ℓ_0
46 : $P_{49} = (45, 1, 0, 0)$ lies on line ℓ_0
47 : $P_{50} = (46, 1, 0, 0)$ lies on line ℓ_0
48 : $P_{51} = (47, 1, 0, 0)$ lies on line ℓ_0
49 : $P_{52} = (48, 1, 0, 0)$ lies on line ℓ_0
50 : $P_{53} = (49, 1, 0, 0)$ lies on line ℓ_0
51 : $P_{54} = (50, 1, 0, 0)$ lies on line ℓ_0
52 : $P_{55} = (51, 1, 0, 0)$ lies on line ℓ_0
53 : $P_{56} = (52, 1, 0, 0)$ lies on line ℓ_0
54 : $P_{57} = (53, 1, 0, 0)$ lies on line ℓ_0
55 : $P_{58} = (54, 1, 0, 0)$ lies on line ℓ_0

56 : $P_{59} = (55, 1, 0, 0)$ lies on line ℓ_0
 57 : $P_{62} = (58, 1, 0, 0)$ lies on line ℓ_0
 58 : $P_{63} = (59, 1, 0, 0)$ lies on line ℓ_0
 59 : $P_{64} = (60, 1, 0, 0)$ lies on line ℓ_0
 60 : $P_{65} = (61, 1, 0, 0)$ lies on line ℓ_0
 61 : $P_{66} = (62, 1, 0, 0)$ lies on line ℓ_0
 62 : $P_{67} = (63, 1, 0, 0)$ lies on line ℓ_0
 63 : $P_{8314} = (56, 0, 1, 1)$ lies on line ℓ_1
 64 : $P_{8315} = (57, 0, 1, 1)$ lies on line ℓ_2
 65 : $P_{8426} = (41, 2, 1, 1)$ lies on line ℓ_1
 66 : $P_{8427} = (42, 2, 1, 1)$ lies on line ℓ_2
 67 : $P_{8466} = (17, 3, 1, 1)$ lies on line ℓ_1
 68 : $P_{8468} = (19, 3, 1, 1)$ lies on line ℓ_2
 69 : $P_{8539} = (26, 4, 1, 1)$ lies on line ℓ_1
 70 : $P_{8544} = (31, 4, 1, 1)$ lies on line ℓ_2
 71 : $P_{8611} = (34, 5, 1, 1)$ lies on line ℓ_1
 72 : $P_{8615} = (38, 5, 1, 1)$ lies on line ℓ_2
 73 : $P_{8652} = (11, 6, 1, 1)$ lies on line ℓ_1
 74 : $P_{8653} = (12, 6, 1, 1)$ lies on line ℓ_2
 75 : $P_{8756} = (51, 7, 1, 1)$ lies on line ℓ_1
 76 : $P_{8758} = (53, 7, 1, 1)$ lies on line ℓ_2
 77 : $P_{8789} = (20, 8, 1, 1)$ lies on line ℓ_2
 78 : $P_{8798} = (29, 8, 1, 1)$ lies on line ℓ_1
 79 : $P_{8870} = (37, 9, 1, 1)$ lies on line ℓ_1
 80 : $P_{8878} = (45, 9, 1, 1)$ lies on line ℓ_2
 81 : $P_{8904} = (7, 10, 1, 1)$ lies on line ℓ_2
 82 : $P_{8909} = (12, 10, 1, 1)$ lies on line ℓ_1
 83 : $P_{9013} = (52, 11, 1, 1)$ lies on line ℓ_1
 84 : $P_{9023} = (62, 11, 1, 1)$ lies on line ℓ_2
 85 : $P_{9075} = (50, 12, 1, 1)$ lies on line ℓ_2
 86 : $P_{9088} = (63, 12, 1, 1)$ lies on line ℓ_1
 87 : $P_{9096} = (7, 13, 1, 1)$ lies on line ℓ_1
 88 : $P_{9100} = (11, 13, 1, 1)$ lies on line ℓ_2
 89 : $P_{9186} = (33, 14, 1, 1)$ lies on line ℓ_2
 90 : $P_{9199} = (46, 14, 1, 1)$ lies on line ℓ_1
 91 : $P_{9239} = (22, 15, 1, 1)$ lies on line ℓ_1
 92 : $P_{9241} = (24, 15, 1, 1)$ lies on line ℓ_2
 93 : $P_{9283} = (2, 16, 1, 1)$ lies on line ℓ_2
 94 : $P_{9300} = (19, 16, 1, 1)$ lies on line ℓ_1
 95 : $P_{9388} = (43, 17, 1, 1)$ lies on line ℓ_1
 96 : $P_{9404} = (59, 17, 1, 1)$ lies on line ℓ_2
 97 : $P_{9411} = (2, 18, 1, 1)$ lies on line ℓ_1
 98 : $P_{9426} = (17, 18, 1, 1)$ lies on line ℓ_2
 99 : $P_{9513} = (40, 19, 1, 1)$ lies on line ℓ_2
 100 : $P_{9531} = (58, 19, 1, 1)$ lies on line ℓ_1
 101 : $P_{9573} = (36, 20, 1, 1)$ lies on line ℓ_2
 102 : $P_{9586} = (49, 20, 1, 1)$ lies on line ℓ_1
 103 : $P_{9610} = (9, 21, 1, 1)$ lies on line ℓ_1
 104 : $P_{9630} = (29, 21, 1, 1)$ lies on line ℓ_2
 105 : $P_{9697} = (32, 22, 1, 1)$ lies on line ℓ_1
 106 : $P_{9720} = (55, 22, 1, 1)$ lies on line ℓ_2
 107 : $P_{9743} = (14, 23, 1, 1)$ lies on line ℓ_2
 108 : $P_{9753} = (24, 23, 1, 1)$ lies on line ℓ_1
 109 : $P_{9840} = (47, 24, 1, 1)$ lies on line ℓ_2

110 : $P_{9847} = (54, 24, 1, 1)$ lies on line ℓ_1
 111 : $P_{9871} = (14, 25, 1, 1)$ lies on line ℓ_1
 112 : $P_{9879} = (22, 25, 1, 1)$ lies on line ℓ_2
 113 : $P_{9960} = (39, 26, 1, 1)$ lies on line ℓ_1
 114 : $P_{9981} = (60, 26, 1, 1)$ lies on line ℓ_2
 115 : $P_{9990} = (5, 27, 1, 1)$ lies on line ℓ_2
 116 : $P_{10016} = (31, 27, 1, 1)$ lies on line ℓ_1
 117 : $P_{10058} = (9, 28, 1, 1)$ lies on line ℓ_2
 118 : $P_{10069} = (20, 28, 1, 1)$ lies on line ℓ_1
 119 : $P_{10157} = (44, 29, 1, 1)$ lies on line ℓ_1
 120 : $P_{10161} = (48, 29, 1, 1)$ lies on line ℓ_2
 121 : $P_{10182} = (5, 30, 1, 1)$ lies on line ℓ_1
 122 : $P_{10203} = (26, 30, 1, 1)$ lies on line ℓ_2
 123 : $P_{10276} = (35, 31, 1, 1)$ lies on line ℓ_2
 124 : $P_{10302} = (61, 31, 1, 1)$ lies on line ℓ_1
 125 : $P_{10320} = (15, 32, 1, 1)$ lies on line ℓ_1
 126 : $P_{10351} = (46, 32, 1, 1)$ lies on line ℓ_2
 127 : $P_{10392} = (23, 33, 1, 1)$ lies on line ℓ_2
 128 : $P_{10424} = (55, 33, 1, 1)$ lies on line ℓ_1
 129 : $P_{10463} = (30, 34, 1, 1)$ lies on line ℓ_1
 130 : $P_{10494} = (61, 34, 1, 1)$ lies on line ℓ_2
 131 : $P_{10501} = (4, 35, 1, 1)$ lies on line ℓ_2
 132 : $P_{10535} = (38, 35, 1, 1)$ lies on line ℓ_1
 133 : $P_{10569} = (8, 36, 1, 1)$ lies on line ℓ_2
 134 : $P_{10606} = (45, 36, 1, 1)$ lies on line ℓ_1
 135 : $P_{10646} = (21, 37, 1, 1)$ lies on line ℓ_1
 136 : $P_{10674} = (49, 37, 1, 1)$ lies on line ℓ_2
 137 : $P_{10716} = (27, 38, 1, 1)$ lies on line ℓ_2
 138 : $P_{10749} = (60, 38, 1, 1)$ lies on line ℓ_1
 139 : $P_{10757} = (4, 39, 1, 1)$ lies on line ℓ_1
 140 : $P_{10787} = (34, 39, 1, 1)$ lies on line ℓ_2
 141 : $P_{10820} = (3, 40, 1, 1)$ lies on line ℓ_2
 142 : $P_{10859} = (42, 40, 1, 1)$ lies on line ℓ_1
 143 : $P_{10899} = (18, 41, 1, 1)$ lies on line ℓ_1
 144 : $P_{10939} = (58, 41, 1, 1)$ lies on line ℓ_2
 145 : $P_{10961} = (16, 42, 1, 1)$ lies on line ℓ_2
 146 : $P_{11004} = (59, 42, 1, 1)$ lies on line ℓ_1
 147 : $P_{11012} = (3, 43, 1, 1)$ lies on line ℓ_1
 148 : $P_{11050} = (41, 43, 1, 1)$ lies on line ℓ_2
 149 : $P_{11081} = (8, 44, 1, 1)$ lies on line ℓ_1
 150 : $P_{11110} = (37, 44, 1, 1)$ lies on line ℓ_2
 151 : $P_{11165} = (28, 45, 1, 1)$ lies on line ℓ_2
 152 : $P_{11185} = (48, 45, 1, 1)$ lies on line ℓ_1
 153 : $P_{11226} = (25, 46, 1, 1)$ lies on line ℓ_1
 154 : $P_{11255} = (54, 46, 1, 1)$ lies on line ℓ_2
 155 : $P_{11280} = (15, 47, 1, 1)$ lies on line ℓ_2
 156 : $P_{11298} = (33, 47, 1, 1)$ lies on line ℓ_1
 157 : $P_{11350} = (21, 48, 1, 1)$ lies on line ℓ_2
 158 : $P_{11365} = (36, 48, 1, 1)$ lies on line ℓ_1
 159 : $P_{11421} = (28, 49, 1, 1)$ lies on line ℓ_1
 160 : $P_{11437} = (44, 49, 1, 1)$ lies on line ℓ_2
 161 : $P_{11463} = (6, 50, 1, 1)$ lies on line ℓ_2
 162 : $P_{11510} = (53, 50, 1, 1)$ lies on line ℓ_1
 163 : $P_{11534} = (13, 51, 1, 1)$ lies on line ℓ_1

164 : $P_{11584} = (63, 51, 1, 1)$ lies on line ℓ_2
 165 : $P_{11591} = (6, 52, 1, 1)$ lies on line ℓ_1
 166 : $P_{11636} = (51, 52, 1, 1)$ lies on line ℓ_2
 167 : $P_{11659} = (10, 53, 1, 1)$ lies on line ℓ_2
 168 : $P_{11711} = (62, 53, 1, 1)$ lies on line ℓ_1
 169 : $P_{11736} = (23, 54, 1, 1)$ lies on line ℓ_1
 170 : $P_{11745} = (32, 54, 1, 1)$ lies on line ℓ_2
 171 : $P_{11802} = (25, 55, 1, 1)$ lies on line ℓ_2
 172 : $P_{11824} = (47, 55, 1, 1)$ lies on line ℓ_1
 173 : $P_{11842} = (1, 56, 1, 1)$ lies on line ℓ_1
 174 : $P_{11897} = (56, 56, 1, 1)$ lies on line ℓ_2
 175 : $P_{11906} = (1, 57, 1, 1)$ lies on line ℓ_2
 176 : $P_{11962} = (57, 57, 1, 1)$ lies on line ℓ_1

177 : $P_{11985} = (16, 58, 1, 1)$ lies on line ℓ_1
 178 : $P_{12012} = (43, 58, 1, 1)$ lies on line ℓ_2
 179 : $P_{12051} = (18, 59, 1, 1)$ lies on line ℓ_2
 180 : $P_{12073} = (40, 59, 1, 1)$ lies on line ℓ_1
 181 : $P_{12127} = (30, 60, 1, 1)$ lies on line ℓ_2
 182 : $P_{12132} = (35, 60, 1, 1)$ lies on line ℓ_1
 183 : $P_{12188} = (27, 61, 1, 1)$ lies on line ℓ_1
 184 : $P_{12200} = (39, 61, 1, 1)$ lies on line ℓ_2
 185 : $P_{12238} = (13, 62, 1, 1)$ lies on line ℓ_2
 186 : $P_{12275} = (50, 62, 1, 1)$ lies on line ℓ_1
 187 : $P_{12299} = (10, 63, 1, 1)$ lies on line ℓ_1
 188 : $P_{12341} = (52, 63, 1, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4097 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{60}	P_{61}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{60}	P_{8322}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{61}	P_{8322}

The surface has 4289 points:

Too many to print.