Rank-65613 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65613

General information

Number of lines	5
Number of points	9
Number of singular points	2
Number of Eckardt points	1
Number of double points	4
Number of single points	4
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	3, 2^4, 1^4

Singular Points

The surface has 2 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1 : P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0,1,0,0,0,1)_{21}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{9} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{9} = \mathbf{Pl}(1,1,0,0,1,1)_{29}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,1,1,1,1,0)_{18}$$

Rank of lines: (4, 18, 31, 9, 17)
Rank of points on Klein quadric: (2, 4, 21, 29, 18)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).\ T = -1$

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_6 = (1,0,1,0) = \ell_0 \cap \ell_3$
$P_9 = (1,0,0,1) = \ell_1 \cap \ell_4$
$P_{10} = (0,1,0,1) = \ell_2 \cap \ell_3$
$P_{14} = (0,1,1,1) = \ell_2 \cap \ell_4$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_4 = (1,1,1,1)$ lies on line ℓ_3
$2 :\ P_8 = (1,1,1,0)$ lies on line ℓ_4
$3 :\ P_{13} = (1,0,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4
0	0 1 1 1 0
1	1 0 1 0 1
2	1 1 0 1 1
3	1 0 1 0 0
4	0 1 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_6

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_2	P_2	P_9

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_2	P_{10}	P_{14}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_6	P_{10}

Line 4 intersects

Line	ℓ_1	ℓ_2
in point	P_9	P_{14}

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_2 = (0, 0, 1, 0)$
$2: P_4 = (1, 1, 1, 1)$
$3: P_6 = (1, 0, 1, 0)$
$4: P_8 = (1, 1, 1, 0)$
$5: P_9 = (1, 0, 0, 1)$
$6: P_{10} = (0, 1, 0, 1)$
$7: P_{13} = (1, 0, 1, 1)$
$8: P_{14} = (0, 1, 1, 1)$