Rank-74531 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_2 + X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 287379733

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	0
Number of double points	15
Number of single points	135
Number of points off lines	170
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	5, 2^{15}, 1^{135}, 0^{170}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2986} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2986} = \mathbf{Pl}(0,0,11,0,0,1)_{4966}$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^{5} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3259} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3259} = \mathbf{Pl}(0,0,10,0,0,1)_{4935}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \mathbf{Pl}(0,0,0,1,0,1)_{5121}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_5 = \begin{bmatrix} 1 & \delta^{5} & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{46939} = \begin{bmatrix} 1 & 11 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{46939} = \mathbf{Pl}(0,11,10,0,0,1)_{4961}$$

$$\ell_6 = \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{51034} = \begin{bmatrix} 1 & 10 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{51034} = \mathbf{Pl}(0,10,11,0,0,1)_{4991}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \mathbf{Pl}(1,0,1,1,1,1)_{9427}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{5} \\ 0 & 1 & \delta^{10} & \delta^{10} \end{bmatrix}_{50948} = \begin{bmatrix} 1 & 0 & 10 & 11 \\ 0 & 1 & 10 & 10 \end{bmatrix}_{50948} = \mathbf{Pl}(1,1,11,10,1,1)_{12111}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \delta^{5} & \delta^{10} \\ 0 & 1 & \delta^{5} & \delta^{5} \end{bmatrix}_{46870} = \begin{bmatrix} 1 & 0 & 11 & 10 \\ 0 & 1 & 11 & 11 \end{bmatrix}_{46870} = \mathbf{Pl}(1,1,10,11,1,1)_{11916}$$

Rank of lines: (0, 2986, 3259, 69889, 70160, 46939, 51034, 4658, 50948, 46870)
Rank of points on Klein quadric: (0, 4966, 4935, 5121, 1, 4961, 4991, 9427, 12111, 11916)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{15} = (11,1,0,0) = \ell_0 \cap \ell_1$
$P_{14} = (10,1,0,0) = \ell_0 \cap \ell_2$
$P_{1} = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_{5} = (1,1,0,0) = \ell_0 \cap \ell_7$
$P_{180} = (1,10,1,0) = \ell_1 \cap \ell_8$
$P_{196} = (1,11,1,0) = \ell_2 \cap \ell_9$
$P_{530} = (0,0,1,1) = \ell_3 \cap \ell_4$
$P_{546} = (0,1,1,1) = \ell_3 \cap \ell_7$
$P_{705} = (0,11,1,1) = \ell_3 \cap \ell_8$
$P_{689} = (0,10,1,1) = \ell_3 \cap \ell_9$
$P_{700} = (11,10,1,1) = \ell_5 \cap \ell_7$
$P_{3004} = (11,10,10,1) = \ell_5 \cap \ell_8$
$P_{715} = (10,11,1,1) = \ell_6 \cap \ell_7$
$P_{3275} = (10,11,11,1) = \ell_6 \cap \ell_9$
$P_{291} = (1,1,0,1) = \ell_8 \cap \ell_9$

Single Points

The surface has 135 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_4
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
4 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
5 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
6 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
7 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
8 : $P_{12} = (8,1,0,0)$ lies on line ℓ_0
9 : $P_{13} = (9,1,0,0)$ lies on line ℓ_0
10 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0
11 : $P_{17} = (13,1,0,0)$ lies on line ℓ_0
12 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0
13 : $P_{19} = (15,1,0,0)$ lies on line ℓ_0
14 : $P_{45} = (10,1,1,0)$ lies on line ℓ_2
15 : $P_{46} = (11,1,1,0)$ lies on line ℓ_1
16 : $P_{64} = (13,2,1,0)$ lies on line ℓ_2
17 : $P_{66} = (15,2,1,0)$ lies on line ℓ_1
18 : $P_{71} = (4,3,1,0)$ lies on line ℓ_1
19 : $P_{74} = (7,3,1,0)$ lies on line ℓ_2
20 : $P_{86} = (3,4,1,0)$ lies on line ℓ_2
21 : $P_{90} = (7,4,1,0)$ lies on line ℓ_1
22 : $P_{108} = (9,5,1,0)$ lies on line ℓ_2
23 : $P_{111} = (12,5,1,0)$ lies on line ℓ_1
24 : $P_{123} = (8,6,1,0)$ lies on line ℓ_1
25 : $P_{129} = (14,6,1,0)$ lies on line ℓ_2
26 : $P_{134} = (3,7,1,0)$ lies on line ℓ_1
27 : $P_{135} = (4,7,1,0)$ lies on line ℓ_2
28 : $P_{153} = (6,8,1,0)$ lies on line ℓ_2
29 : $P_{161} = (14,8,1,0)$ lies on line ℓ_1
30 : $P_{168} = (5,9,1,0)$ lies on line ℓ_1
31 : $P_{175} = (12,9,1,0)$ lies on line ℓ_2
32 : $P_{190} = (11,10,1,0)$ lies on line ℓ_2
33 : $P_{205} = (10,11,1,0)$ lies on line ℓ_1
34 : $P_{216} = (5,12,1,0)$ lies on line ℓ_2
35 : $P_{220} = (9,12,1,0)$ lies on line ℓ_1
36 : $P_{229} = (2,13,1,0)$ lies on line ℓ_1
37 : $P_{242} = (15,13,1,0)$ lies on line ℓ_2
38 : $P_{249} = (6,14,1,0)$ lies on line ℓ_1
39 : $P_{251} = (8,14,1,0)$ lies on line ℓ_2
40 : $P_{261} = (2,15,1,0)$ lies on line ℓ_2
41 : $P_{272} = (13,15,1,0)$ lies on line ℓ_1
42 : $P_{445} = (11,10,0,1)$ lies on line ℓ_5
43 : $P_{460} = (10,11,0,1)$ lies on line ℓ_6
44 : $P_{531} = (1,0,1,1)$ lies on line ℓ_7
45 : $P_{561} = (0,2,1,1)$ lies on line ℓ_3
46 : $P_{564} = (3,2,1,1)$ lies on line ℓ_7
47 : $P_{577} = (0,3,1,1)$ lies on line ℓ_3
48 : $P_{579} = (2,3,1,1)$ lies on line ℓ_7
49 : $P_{593} = (0,4,1,1)$ lies on line ℓ_3
50 : $P_{598} = (5,4,1,1)$ lies on line ℓ_7
51 : $P_{609} = (0,5,1,1)$ lies on line ℓ_3
52 : $P_{613} = (4,5,1,1)$ lies on line ℓ_7
53 : $P_{625} = (0,6,1,1)$ lies on line ℓ_3
54 : $P_{632} = (7,6,1,1)$ lies on line ℓ_7
55 : $P_{641} = (0,7,1,1)$ lies on line ℓ_3
56 : $P_{647} = (6,7,1,1)$ lies on line ℓ_7
57 : $P_{657} = (0,8,1,1)$ lies on line ℓ_3
58 : $P_{666} = (9,8,1,1)$ lies on line ℓ_7
59 : $P_{673} = (0,9,1,1)$ lies on line ℓ_3
60 : $P_{681} = (8,9,1,1)$ lies on line ℓ_7
61 : $P_{721} = (0,12,1,1)$ lies on line ℓ_3
62 : $P_{734} = (13,12,1,1)$ lies on line ℓ_7
63 : $P_{737} = (0,13,1,1)$ lies on line ℓ_3
64 : $P_{749} = (12,13,1,1)$ lies on line ℓ_7
65 : $P_{753} = (0,14,1,1)$ lies on line ℓ_3
66 : $P_{768} = (15,14,1,1)$ lies on line ℓ_7
67 : $P_{769} = (0,15,1,1)$ lies on line ℓ_3
68 : $P_{783} = (14,15,1,1)$ lies on line ℓ_7
69 : $P_{785} = (0,0,2,1)$ lies on line ℓ_4
70 : $P_{956} = (11,10,2,1)$ lies on line ℓ_5
71 : $P_{971} = (10,11,2,1)$ lies on line ℓ_6
72 : $P_{980} = (3,12,2,1)$ lies on line ℓ_8
73 : $P_{1012} = (3,14,2,1)$ lies on line ℓ_9
74 : $P_{1041} = (0,0,3,1)$ lies on line ℓ_4
75 : $P_{1123} = (2,5,3,1)$ lies on line ℓ_9
76 : $P_{1139} = (2,6,3,1)$ lies on line ℓ_8
77 : $P_{1212} = (11,10,3,1)$ lies on line ℓ_5
78 : $P_{1227} = (10,11,3,1)$ lies on line ℓ_6
79 : $P_{1297} = (0,0,4,1)$ lies on line ℓ_4
80 : $P_{1334} = (5,2,4,1)$ lies on line ℓ_8
81 : $P_{1398} = (5,6,4,1)$ lies on line ℓ_9
82 : $P_{1468} = (11,10,4,1)$ lies on line ℓ_5
83 : $P_{1483} = (10,11,4,1)$ lies on line ℓ_6
84 : $P_{1553} = (0,0,5,1)$ lies on line ℓ_4
85 : $P_{1685} = (4,8,5,1)$ lies on line ℓ_8
86 : $P_{1724} = (11,10,5,1)$ lies on line ℓ_5
87 : $P_{1739} = (10,11,5,1)$ lies on line ℓ_6
88 : $P_{1765} = (4,13,5,1)$ lies on line ℓ_9
89 : $P_{1809} = (0,0,6,1)$ lies on line ℓ_4
90 : $P_{1960} = (7,9,6,1)$ lies on line ℓ_9
91 : $P_{1980} = (11,10,6,1)$ lies on line ℓ_5
92 : $P_{1995} = (10,11,6,1)$ lies on line ℓ_6
93 : $P_{2056} = (7,15,6,1)$ lies on line ℓ_8
94 : $P_{2065} = (0,0,7,1)$ lies on line ℓ_4
95 : $P_{2103} = (6,2,7,1)$ lies on line ℓ_9
96 : $P_{2151} = (6,5,7,1)$ lies on line ℓ_8
97 : $P_{2236} = (11,10,7,1)$ lies on line ℓ_5
98 : $P_{2251} = (10,11,7,1)$ lies on line ℓ_6
99 : $P_{2321} = (0,0,8,1)$ lies on line ℓ_4
100 : $P_{2442} = (9,7,8,1)$ lies on line ℓ_8
101 : $P_{2492} = (11,10,8,1)$ lies on line ℓ_5
102 : $P_{2507} = (10,11,8,1)$ lies on line ℓ_6
103 : $P_{2570} = (9,15,8,1)$ lies on line ℓ_9
104 : $P_{2577} = (0,0,9,1)$ lies on line ℓ_4
105 : $P_{2649} = (8,4,9,1)$ lies on line ℓ_9
106 : $P_{2748} = (11,10,9,1)$ lies on line ℓ_5
107 : $P_{2763} = (10,11,9,1)$ lies on line ℓ_6

108 : $P_{2793} = (8, 13, 9, 1)$ lies on line ℓ_8
109 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_4
110 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_9
111 : $P_{3019} = (10, 11, 10, 1)$ lies on line ℓ_6
112 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_4
113 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_8
114 : $P_{3260} = (11, 10, 11, 1)$ lies on line ℓ_5
115 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_4
116 : $P_{3422} = (13, 4, 12, 1)$ lies on line ℓ_8
117 : $P_{3486} = (13, 8, 12, 1)$ lies on line ℓ_9
118 : $P_{3516} = (11, 10, 12, 1)$ lies on line ℓ_5
119 : $P_{3531} = (10, 11, 12, 1)$ lies on line ℓ_6
120 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_4
121 : $P_{3661} = (12, 3, 13, 1)$ lies on line ℓ_9
122 : $P_{3772} = (11, 10, 13, 1)$ lies on line ℓ_5
123 : $P_{3787} = (10, 11, 13, 1)$ lies on line ℓ_6
124 : $P_{3837} = (12, 14, 13, 1)$ lies on line ℓ_8
125 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_4
126 : $P_{3984} = (15, 7, 14, 1)$ lies on line ℓ_9
127 : $P_{4016} = (15, 9, 14, 1)$ lies on line ℓ_8
128 : $P_{4028} = (11, 10, 14, 1)$ lies on line ℓ_5
129 : $P_{4043} = (10, 11, 14, 1)$ lies on line ℓ_6
130 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_4
131 : $P_{4175} = (14, 3, 15, 1)$ lies on line ℓ_8
132 : $P_{4284} = (11, 10, 15, 1)$ lies on line ℓ_5
133 : $P_{4299} = (10, 11, 15, 1)$ lies on line ℓ_6
134 : $P_{4319} = (14, 12, 15, 1)$ lies on line ℓ_9

The single points on the surface are:

Points on surface but on no line

The surface has 170 points not on any line:
The points on the surface but not on lines are:

0 : $P_{310} = (4, 2, 0, 1)$
1 : $P_{327} = (5, 3, 0, 1)$
2 : $P_{347} = (9, 4, 0, 1)$
3 : $P_{362} = (8, 5, 0, 1)$
4 : $P_{383} = (13, 6, 0, 1)$
5 : $P_{398} = (12, 7, 0, 1)$
6 : $P_{417} = (15, 8, 0, 1)$
7 : $P_{432} = (14, 9, 0, 1)$
8 : $P_{472} = (6, 12, 0, 1)$
9 : $P_{489} = (7, 13, 0, 1)$
10 : $P_{500} = (2, 14, 0, 1)$
11 : $P_{517} = (3, 15, 0, 1)$
12 : $P_{797} = (12, 0, 2, 1)$
13 : $P_{898} = (1, 7, 2, 1)$
14 : $P_{907} = (10, 7, 2, 1)$
15 : $P_{921} = (8, 8, 2, 1)$
16 : $P_{925} = (12, 8, 2, 1)$
17 : $P_{958} = (13, 10, 2, 1)$
18 : $P_{974} = (13, 11, 2, 1)$
19 : $P_{1010} = (1, 14, 2, 1)$
20 : $P_{1033} = (8, 15, 2, 1)$
21 : $P_{1036} = (11, 15, 2, 1)$
22 : $P_{1049} = (8, 0, 3, 1)$
23 : $P_{1063} = (6, 1, 3, 1)$
24 : $P_{1072} = (15, 1, 3, 1)$
25 : $P_{1111} = (6, 4, 3, 1)$
26 : $P_{1115} = (10, 4, 3, 1)$
27 : $P_{1136} = (15, 5, 3, 1)$
28 : $P_{1149} = (12, 6, 3, 1)$
29 : $P_{1180} = (11, 8, 3, 1)$
30 : $P_{1210} = (9, 10, 3, 1)$
31 : $P_{1226} = (9, 11, 3, 1)$
32 : $P_{1241} = (8, 12, 3, 1)$
33 : $P_{1245} = (12, 12, 3, 1)$
34 : $P_{1303} = (6, 0, 4, 1)$
35 : $P_{1330} = (1, 2, 4, 1)$
36 : $P_{1355} = (10, 3, 4, 1)$
37 : $P_{1360} = (15, 3, 4, 1)$
38 : $P_{1464} = (7, 10, 4, 1)$
39 : $P_{1480} = (7, 11, 4, 1)$
40 : $P_{1490} = (1, 12, 4, 1)$
41 : $P_{1500} = (11, 12, 4, 1)$
42 : $P_{1543} = (6, 15, 4, 1)$
43 : $P_{1552} = (15, 15, 4, 1)$
44 : $P_{1568} = (15, 0, 5, 1)$
45 : $P_{1572} = (3, 1, 5, 1)$
46 : $P_{1582} = (13, 1, 5, 1)$
47 : $P_{1655} = (6, 6, 5, 1)$
48 : $P_{1664} = (15, 6, 5, 1)$
49 : $P_{1684} = (3, 8, 5, 1)$
50 : $P_{1708} = (11, 9, 5, 1)$
51 : $P_{1710} = (13, 9, 5, 1)$
52 : $P_{1727} = (14, 10, 5, 1)$
53 : $P_{1743} = (14, 11, 5, 1)$
54 : $P_{1767} = (6, 13, 5, 1)$
55 : $P_{1803} = (10, 15, 5, 1)$
56 : $P_{1813} = (4, 0, 6, 1)$
57 : $P_{1849} = (8, 2, 6, 1)$
58 : $P_{1855} = (14, 2, 6, 1)$
59 : $P_{1885} = (12, 4, 6, 1)$

60 : $P_{1929} = (8, 7, 6, 1)$
61 : $P_{1932} = (11, 7, 6, 1)$
62 : $P_{1963} = (10, 9, 6, 1)$
63 : $P_{1974} = (5, 10, 6, 1)$
64 : $P_{1990} = (5, 11, 6, 1)$
65 : $P_{2037} = (4, 14, 6, 1)$
66 : $P_{2047} = (14, 14, 6, 1)$
67 : $P_{2061} = (12, 15, 6, 1)$
68 : $P_{2079} = (14, 0, 7, 1)$
69 : $P_{2107} = (10, 2, 7, 1)$
70 : $P_{2133} = (4, 4, 7, 1)$
71 : $P_{2143} = (14, 4, 7, 1)$
72 : $P_{2158} = (13, 5, 7, 1)$
73 : $P_{2164} = (3, 6, 7, 1)$
74 : $P_{2172} = (11, 6, 7, 1)$
75 : $P_{2212} = (3, 9, 7, 1)$
76 : $P_{2213} = (4, 9, 7, 1)$
77 : $P_{2240} = (15, 10, 7, 1)$
78 : $P_{2256} = (15, 11, 7, 1)$
79 : $P_{2302} = (13, 14, 7, 1)$
80 : $P_{2324} = (3, 0, 8, 1)$
81 : $P_{2342} = (5, 1, 8, 1)$
82 : $P_{2344} = (7, 1, 8, 1)$
83 : $P_{2380} = (11, 3, 8, 1)$
84 : $P_{2446} = (13, 7, 8, 1)$
85 : $P_{2483} = (2, 10, 8, 1)$
86 : $P_{2499} = (2, 11, 8, 1)$
87 : $P_{2532} = (3, 13, 8, 1)$
88 : $P_{2542} = (13, 13, 8, 1)$
89 : $P_{2552} = (7, 14, 8, 1)$
90 : $P_{2555} = (10, 14, 8, 1)$
91 : $P_{2566} = (5, 15, 8, 1)$
92 : $P_{2590} = (13, 0, 9, 1)$
93 : $P_{2628} = (3, 3, 9, 1)$
94 : $P_{2638} = (13, 3, 9, 1)$
95 : $P_{2642} = (1, 4, 9, 1)$
96 : $P_{2660} = (3, 5, 9, 1)$
97 : $P_{2668} = (11, 5, 9, 1)$
98 : $P_{2674} = (1, 6, 9, 1)$
99 : $P_{2683} = (10, 6, 9, 1)$
100 : $P_{2749} = (12, 10, 9, 1)$
101 : $P_{2765} = (12, 11, 9, 1)$
102 : $P_{2853} = (4, 1, 10, 1)$
103 : $P_{2863} = (14, 1, 10, 1)$
104 : $P_{2882} = (1, 3, 10, 1)$
105 : $P_{2890} = (9, 3, 10, 1)$
106 : $P_{2933} = (4, 6, 10, 1)$
107 : $P_{2938} = (9, 6, 10, 1)$
108 : $P_{2947} = (2, 7, 10, 1)$
109 : $P_{2959} = (14, 7, 10, 1)$
110 : $P_{2962} = (1, 8, 10, 1)$
111 : $P_{2963} = (2, 8, 10, 1)$
112 : $P_{3003} = (10, 10, 10, 1)$
113 : $P_{3107} = (2, 1, 11, 1)$
114 : $P_{3114} = (9, 1, 11, 1)$
115 : $P_{3170} = (1, 5, 11, 1)$
116 : $P_{3183} = (14, 5, 11, 1)$
117 : $P_{3276} = (11, 11, 11, 1)$
118 : $P_{3283} = (2, 12, 11, 1)$
119 : $P_{3285} = (4, 12, 11, 1)$
120 : $P_{3306} = (9, 13, 11, 1)$
121 : $P_{3311} = (14, 13, 11, 1)$
122 : $P_{3330} = (1, 15, 11, 1)$
123 : $P_{3333} = (4, 15, 11, 1)$
124 : $P_{3347} = (2, 0, 12, 1)$
125 : $P_{3384} = (7, 2, 12, 1)$
126 : $P_{3420} = (11, 4, 12, 1)$
127 : $P_{3480} = (7, 8, 12, 1)$
128 : $P_{3491} = (2, 9, 12, 1)$
129 : $P_{3498} = (9, 9, 12, 1)$
130 : $P_{3508} = (3, 10, 12, 1)$
131 : $P_{3524} = (3, 11, 12, 1)$
132 : $P_{3558} = (5, 13, 12, 1)$
133 : $P_{3563} = (10, 13, 12, 1)$
134 : $P_{3574} = (5, 14, 12, 1)$
135 : $P_{3578} = (9, 14, 12, 1)$
136 : $P_{3610} = (9, 0, 13, 1)$
137 : $P_{3635} = (2, 2, 13, 1)$
138 : $P_{3642} = (9, 2, 13, 1)$
139 : $P_{3655} = (6, 3, 13, 1)$
140 : $P_{3667} = (2, 4, 13, 1)$
141 : $P_{3680} = (15, 4, 13, 1)$
142 : $P_{3751} = (6, 9, 13, 1)$
143 : $P_{3769} = (8, 10, 13, 1)$
144 : $P_{3785} = (8, 11, 13, 1)$
145 : $P_{3803} = (10, 12, 13, 1)$
146 : $P_{3808} = (15, 12, 13, 1)$
147 : $P_{3836} = (11, 14, 13, 1)$
148 : $P_{3864} = (7, 0, 14, 1)$
149 : $P_{3942} = (5, 5, 14, 1)$
150 : $P_{3944} = (7, 5, 14, 1)$
151 : $P_{3990} = (5, 8, 14, 1)$
152 : $P_{3995} = (10, 8, 14, 1)$
153 : $P_{4002} = (1, 9, 14, 1)$
154 : $P_{4023} = (6, 10, 14, 1)$
155 : $P_{4039} = (6, 11, 14, 1)$
156 : $P_{4066} = (1, 13, 14, 1)$
157 : $P_{4076} = (11, 13, 14, 1)$
158 : $P_{4118} = (5, 0, 15, 1)$
159 : $P_{4137} = (8, 1, 15, 1)$
160 : $P_{4141} = (12, 1, 15, 1)$
161 : $P_{4156} = (11, 2, 15, 1)$
162 : $P_{4157} = (12, 2, 15, 1)$
163 : $P_{4169} = (8, 3, 15, 1)$
164 : $P_{4203} = (10, 5, 15, 1)$
165 : $P_{4230} = (5, 7, 15, 1)$
166 : $P_{4232} = (7, 7, 15, 1)$
167 : $P_{4277} = (4, 10, 15, 1)$

168 : $P_{4293} = (4, 11, 15, 1)$
169 : $P_{4312} = (7, 12, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 1 1 0 0 0 1 0 0
1	1 0 1 0 1 1 1 0 1 0
2	1 1 0 0 1 1 1 0 0 1
3	1 0 0 0 1 0 0 1 1 1
4	0 1 1 1 0 1 1 0 0 0
5	0 1 1 0 1 0 1 1 1 0
6	0 1 1 0 1 1 0 1 0 1
7	1 0 0 1 0 1 1 0 0 0
8	0 1 0 1 0 1 0 0 0 1
9	0 0 1 1 0 0 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_7
in point	P_{15}	P_{14}	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_{15}	P_2	P_2	P_2	P_2	P_{180}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_{14}	P_2	P_2	P_2	P_2	P_{196}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_8	ℓ_9
in point	P_1	P_{530}	P_{546}	P_{705}	P_{689}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{530}	P_2	P_2

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{700}	P_{3004}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{715}	P_{3275}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6
in point	P_5	P_{546}	P_{700}	P_{715}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_9
in point	P_{180}	P_{705}	P_{3004}	P_{291}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_{196}	P_{689}	P_{3275}	P_{291}

The surface has 321 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (4,1,0,0)$
$8: P_9 = (5,1,0,0)$
$9: P_{10} = (6,1,0,0)$
$10: P_{11} = (7,1,0,0)$
$11: P_{12} = (8,1,0,0)$
$12: P_{13} = (9,1,0,0)$
$13: P_{14} = (10,1,0,0)$
$14: P_{15} = (11,1,0,0)$
$15: P_{16} = (12,1,0,0)$
$16: P_{17} = (13,1,0,0)$
$17: P_{18} = (14,1,0,0)$
$18: P_{19} = (15,1,0,0)$
$19: P_{45} = (10,1,1,0)$
$20: P_{46} = (11,1,1,0)$
$21: P_{64} = (13,2,1,0)$
$22: P_{66} = (15,2,1,0)$
$23: P_{71} = (4,3,1,0)$
$24: P_{74} = (7,3,1,0)$
$25: P_{86} = (3,4,1,0)$
$26: P_{90} = (7,4,1,0)$
$27: P_{108} = (9,5,1,0)$
$28: P_{111} = (12,5,1,0)$
$29: P_{123} = (8,6,1,0)$
$30: P_{129} = (14,6,1,0)$
$31: P_{134} = (3,7,1,0)$
$32: P_{135} = (4,7,1,0)$
$33: P_{153} = (6,8,1,0)$
$34: P_{161} = (14,8,1,0)$
$35: P_{168} = (5,9,1,0)$
$36: P_{175} = (12,9,1,0)$
$37: P_{180} = (1,10,1,0)$
$38: P_{190} = (11,10,1,0)$
$39: P_{196} = (1,11,1,0)$
$40: P_{205} = (10,11,1,0)$
$41: P_{216} = (5,12,1,0)$
$42: P_{220} = (9,12,1,0)$
$43: P_{229} = (2,13,1,0)$
$44: P_{242} = (15,13,1,0)$
$45: P_{249} = (6,14,1,0)$
$46: P_{251} = (8,14,1,0)$
$47: P_{261} = (2,15,1,0)$
$48: P_{272} = (13,15,1,0)$
$49: P_{291} = (1,1,0,1)$
$50: P_{310} = (4,2,0,1)$
$51: P_{327} = (5,3,0,1)$
$52: P_{347} = (9,4,0,1)$
$53: P_{362} = (8,5,0,1)$
$54: P_{383} = (13,6,0,1)$
$55: P_{398} = (12,7,0,1)$
$56: P_{417} = (15,8,0,1)$
$57: P_{432} = (14,9,0,1)$
$58: P_{445} = (11,10,0,1)$
$59: P_{460} = (10,11,0,1)$
$60: P_{472} = (6,12,0,1)$
$61: P_{489} = (7,13,0,1)$
$62: P_{500} = (2,14,0,1)$
$63: P_{517} = (3,15,0,1)$
$64: P_{530} = (0,0,1,1)$
$65: P_{531} = (1,0,1,1)$
$66: P_{546} = (0,1,1,1)$
$67: P_{561} = (0,2,1,1)$
$68: P_{564} = (3,2,1,1)$
$69: P_{577} = (0,3,1,1)$
$70: P_{579} = (2,3,1,1)$
$71: P_{593} = (0,4,1,1)$
$72: P_{598} = (5,4,1,1)$
$73: P_{609} = (0,5,1,1)$
$74: P_{613} = (4,5,1,1)$
$75: P_{625} = (0,6,1,1)$
$76: P_{632} = (7,6,1,1)$
$77: P_{641} = (0,7,1,1)$
$78: P_{647} = (6,7,1,1)$
$79: P_{657} = (0,8,1,1)$
$80: P_{666} = (9,8,1,1)$
$81: P_{673} = (0,9,1,1)$
$82: P_{681} = (8,9,1,1)$
$83: P_{689} = (0,10,1,1)$
$84: P_{700} = (11,10,1,1)$
$85: P_{705} = (0,11,1,1)$
$86: P_{715} = (10,11,1,1)$
$87: P_{721} = (0,12,1,1)$
$88: P_{734} = (13,12,1,1)$
$89: P_{737} = (0,13,1,1)$
$90: P_{749} = (12,13,1,1)$
$91: P_{753} = (0,14,1,1)$
$92: P_{768} = (15,14,1,1)$
$93: P_{769} = (0,15,1,1)$
$94: P_{783} = (14,15,1,1)$
$95: P_{785} = (0,0,2,1)$
$96: P_{797} = (12,0,2,1)$
$97: P_{898} = (1,7,2,1)$
$98: P_{907} = (10,7,2,1)$
$99: P_{921} = (8,8,2,1)$
$100: P_{925} = (12,8,2,1)$
$101: P_{956} = (11,10,2,1)$
$102: P_{958} = (13,10,2,1)$
$103: P_{971} = (10,11,2,1)$
$104: P_{974} = (13,11,2,1)$
$105: P_{980} = (3,12,2,1)$
$106: P_{1010} = (1,14,2,1)$
$107: P_{1012} = (3,14,2,1)$
$108: P_{1033} = (8,15,2,1)$
$109: P_{1036} = (11,15,2,1)$
$110: P_{1041} = (0,0,3,1)$
$111: P_{1049} = (8,0,3,1)$
$112: P_{1063} = (6,1,3,1)$
$113: P_{1072} = (15,1,3,1)$
$114: P_{1111} = (6,4,3,1)$
$115: P_{1115} = (10,4,3,1)$
$116: P_{1123} = (2,5,3,1)$
$117: P_{1136} = (15,5,3,1)$
$118: P_{1139} = (2,6,3,1)$
$119: P_{1149} = (12,6,3,1)$
$120: P_{1180} = (11,8,3,1)$
$121: P_{1210} = (9,10,3,1)$
$122: P_{1212} = (11,10,3,1)$
$123: P_{1226} = (9,11,3,1)$
$124: P_{1227} = (10,11,3,1)$
$125: P_{1241} = (8,12,3,1)$
$126: P_{1245} = (12,12,3,1)$
$127: P_{1297} = (0,0,4,1)$
$128: P_{1303} = (6,0,4,1)$
$129: P_{1330} = (1,2,4,1)$
$130: P_{1334} = (5,2,4,1)$
$131: P_{1355} = (10,3,4,1)$
$132: P_{1360} = (15,3,4,1)$
$133: P_{1398} = (5,6,4,1)$
$134: P_{1464} = (7,10,4,1)$
$135: P_{1468} = (11,10,4,1)$
$136: P_{1480} = (7,11,4,1)$
$137: P_{1483} = (10,11,4,1)$
$138: P_{1490} = (1,12,4,1)$
$139: P_{1500} = (11,12,4,1)$
$140: P_{1543} = (6,15,4,1)$
$141: P_{1552} = (15,15,4,1)$
$142: P_{1553} = (0,0,5,1)$
$143: P_{1568} = (15,0,5,1)$
$144: P_{1572} = (3,1,5,1)$
$145: P_{1582} = (13,1,5,1)$
$146: P_{1655} = (6,6,5,1)$
$147: P_{1664} = (15,6,5,1)$
$148: P_{1684} = (3,8,5,1)$
$149: P_{1685} = (4,8,5,1)$
$150: P_{1708} = (11,9,5,1)$
$151: P_{1710} = (13,9,5,1)$
$152: P_{1724} = (11,10,5,1)$
$153: P_{1727} = (14,10,5,1)$
$154: P_{1739} = (10,11,5,1)$
$155: P_{1743} = (14,11,5,1)$
$156: P_{1765} = (4,13,5,1)$
$157: P_{1767} = (6,13,5,1)$
$158: P_{1803} = (10,15,5,1)$
$159: P_{1809} = (0,0,6,1)$
$160: P_{1813} = (4,0,6,1)$
$161: P_{1849} = (8,2,6,1)$

$162: P_{1855} = (14, 2, 6, 1)$
$163: P_{1885} = (12, 4, 6, 1)$
$164: P_{1929} = (8, 7, 6, 1)$
$165: P_{1932} = (11, 7, 6, 1)$
$166: P_{1960} = (7, 9, 6, 1)$
$167: P_{1963} = (10, 9, 6, 1)$
$168: P_{1974} = (5, 10, 6, 1)$
$169: P_{1980} = (11, 10, 6, 1)$
$170: P_{1990} = (5, 11, 6, 1)$
$171: P_{1995} = (10, 11, 6, 1)$
$172: P_{2037} = (4, 14, 6, 1)$
$173: P_{2047} = (14, 14, 6, 1)$
$174: P_{2056} = (7, 15, 6, 1)$
$175: P_{2061} = (12, 15, 6, 1)$
$176: P_{2065} = (0, 0, 7, 1)$
$177: P_{2079} = (14, 0, 7, 1)$
$178: P_{2103} = (6, 2, 7, 1)$
$179: P_{2107} = (10, 2, 7, 1)$
$180: P_{2133} = (4, 4, 7, 1)$
$181: P_{2143} = (14, 4, 7, 1)$
$182: P_{2151} = (6, 5, 7, 1)$
$183: P_{2158} = (13, 5, 7, 1)$
$184: P_{2164} = (3, 6, 7, 1)$
$185: P_{2172} = (11, 6, 7, 1)$
$186: P_{2212} = (3, 9, 7, 1)$
$187: P_{2213} = (4, 9, 7, 1)$
$188: P_{2236} = (11, 10, 7, 1)$
$189: P_{2240} = (15, 10, 7, 1)$
$190: P_{2251} = (10, 11, 7, 1)$
$191: P_{2256} = (15, 11, 7, 1)$
$192: P_{2302} = (13, 14, 7, 1)$
$193: P_{2321} = (0, 0, 8, 1)$
$194: P_{2324} = (3, 0, 8, 1)$
$195: P_{2342} = (5, 1, 8, 1)$
$196: P_{2344} = (7, 1, 8, 1)$
$197: P_{2380} = (11, 3, 8, 1)$
$198: P_{2442} = (9, 7, 8, 1)$
$199: P_{2446} = (13, 7, 8, 1)$
$200: P_{2483} = (2, 10, 8, 1)$
$201: P_{2492} = (11, 10, 8, 1)$
$202: P_{2499} = (2, 11, 8, 1)$
$203: P_{2507} = (10, 11, 8, 1)$
$204: P_{2532} = (3, 13, 8, 1)$
$205: P_{2542} = (13, 13, 8, 1)$
$206: P_{2552} = (7, 14, 8, 1)$
$207: P_{2555} = (10, 14, 8, 1)$
$208: P_{2566} = (5, 15, 8, 1)$
$209: P_{2570} = (9, 15, 8, 1)$
$210: P_{2577} = (0, 0, 9, 1)$
$211: P_{2590} = (13, 0, 9, 1)$
$212: P_{2628} = (3, 3, 9, 1)$
$213: P_{2638} = (13, 3, 9, 1)$
$214: P_{2642} = (1, 4, 9, 1)$
$215: P_{2649} = (8, 4, 9, 1)$
$216: P_{2660} = (3, 5, 9, 1)$
$217: P_{2668} = (11, 5, 9, 1)$
$218: P_{2674} = (1, 6, 9, 1)$
$219: P_{2683} = (10, 6, 9, 1)$
$220: P_{2748} = (11, 10, 9, 1)$
$221: P_{2749} = (12, 10, 9, 1)$
$222: P_{2763} = (10, 11, 9, 1)$
$223: P_{2765} = (12, 11, 9, 1)$
$224: P_{2793} = (8, 13, 9, 1)$
$225: P_{2833} = (0, 0, 10, 1)$
$226: P_{2844} = (11, 0, 10, 1)$
$227: P_{2853} = (4, 1, 10, 1)$
$228: P_{2863} = (14, 1, 10, 1)$
$229: P_{2882} = (1, 3, 10, 1)$
$230: P_{2890} = (9, 3, 10, 1)$
$231: P_{2933} = (4, 6, 10, 1)$
$232: P_{2938} = (9, 6, 10, 1)$
$233: P_{2947} = (2, 7, 10, 1)$
$234: P_{2959} = (14, 7, 10, 1)$
$235: P_{2962} = (1, 8, 10, 1)$
$236: P_{2963} = (2, 8, 10, 1)$
$237: P_{3003} = (10, 10, 10, 1)$
$238: P_{3004} = (11, 10, 10, 1)$
$239: P_{3019} = (10, 11, 10, 1)$
$240: P_{3089} = (0, 0, 11, 1)$
$241: P_{3099} = (10, 0, 11, 1)$
$242: P_{3107} = (2, 1, 11, 1)$
$243: P_{3114} = (9, 1, 11, 1)$
$244: P_{3170} = (1, 5, 11, 1)$
$245: P_{3183} = (14, 5, 11, 1)$
$246: P_{3260} = (11, 10, 11, 1)$
$247: P_{3275} = (10, 11, 11, 1)$
$248: P_{3276} = (11, 11, 11, 1)$
$249: P_{3283} = (2, 12, 11, 1)$
$250: P_{3285} = (4, 12, 11, 1)$
$251: P_{3306} = (9, 13, 11, 1)$
$252: P_{3311} = (14, 13, 11, 1)$
$253: P_{3330} = (1, 15, 11, 1)$
$254: P_{3333} = (4, 15, 11, 1)$
$255: P_{3345} = (0, 0, 12, 1)$
$256: P_{3347} = (2, 0, 12, 1)$
$257: P_{3384} = (7, 2, 12, 1)$
$258: P_{3420} = (11, 4, 12, 1)$
$259: P_{3422} = (13, 4, 12, 1)$
$260: P_{3480} = (7, 8, 12, 1)$
$261: P_{3486} = (13, 8, 12, 1)$
$262: P_{3491} = (2, 9, 12, 1)$
$263: P_{3498} = (9, 9, 12, 1)$
$264: P_{3508} = (3, 10, 12, 1)$
$265: P_{3516} = (11, 10, 12, 1)$
$266: P_{3524} = (3, 11, 12, 1)$
$267: P_{3531} = (10, 11, 12, 1)$
$268: P_{3558} = (5, 13, 12, 1)$
$269: P_{3563} = (10, 13, 12, 1)$
$270: P_{3574} = (5, 14, 12, 1)$
$271: P_{3578} = (9, 14, 12, 1)$
$272: P_{3601} = (0, 0, 13, 1)$
$273: P_{3610} = (9, 0, 13, 1)$
$274: P_{3635} = (2, 2, 13, 1)$
$275: P_{3642} = (9, 2, 13, 1)$
$276: P_{3655} = (6, 3, 13, 1)$
$277: P_{3661} = (12, 3, 13, 1)$
$278: P_{3667} = (2, 4, 13, 1)$
$279: P_{3680} = (15, 4, 13, 1)$
$280: P_{3751} = (6, 9, 13, 1)$
$281: P_{3769} = (8, 10, 13, 1)$
$282: P_{3772} = (11, 10, 13, 1)$
$283: P_{3785} = (8, 11, 13, 1)$
$284: P_{3787} = (10, 11, 13, 1)$
$285: P_{3803} = (10, 12, 13, 1)$
$286: P_{3808} = (15, 12, 13, 1)$
$287: P_{3836} = (11, 14, 13, 1)$
$288: P_{3837} = (12, 14, 13, 1)$
$289: P_{3857} = (0, 0, 14, 1)$
$290: P_{3864} = (7, 0, 14, 1)$
$291: P_{3942} = (5, 5, 14, 1)$
$292: P_{3944} = (7, 5, 14, 1)$
$293: P_{3984} = (15, 7, 14, 1)$
$294: P_{3990} = (5, 8, 14, 1)$
$295: P_{3995} = (10, 8, 14, 1)$
$296: P_{4002} = (1, 9, 14, 1)$
$297: P_{4016} = (15, 9, 14, 1)$
$298: P_{4023} = (6, 10, 14, 1)$
$299: P_{4028} = (11, 10, 14, 1)$
$300: P_{4039} = (6, 11, 14, 1)$
$301: P_{4043} = (10, 11, 14, 1)$
$302: P_{4066} = (1, 13, 14, 1)$
$303: P_{4076} = (11, 13, 14, 1)$
$304: P_{4113} = (0, 0, 15, 1)$
$305: P_{4118} = (5, 0, 15, 1)$
$306: P_{4137} = (8, 1, 15, 1)$
$307: P_{4141} = (12, 1, 15, 1)$
$308: P_{4156} = (11, 2, 15, 1)$
$309: P_{4157} = (12, 2, 15, 1)$
$310: P_{4169} = (8, 3, 15, 1)$
$311: P_{4175} = (14, 3, 15, 1)$
$312: P_{4203} = (10, 5, 15, 1)$
$313: P_{4230} = (5, 7, 15, 1)$
$314: P_{4232} = (7, 7, 15, 1)$
$315: P_{4277} = (4, 10, 15, 1)$
$316: P_{4284} = (11, 10, 15, 1)$
$317: P_{4293} = (4, 11, 15, 1)$
$318: P_{4299} = (10, 11, 15, 1)$
$319: P_{4312} = (7, 12, 15, 1)$
$320: P_{4319} = (14, 12, 15, 1)$