# Rank-74499 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(8) is 1378128461

## General information

| | |
|---|---|
| Number of lines | 7 |
| Number of points | 89 |
| Number of singular points | 2 |
| Number of Eckardt points | 1 |
| Number of double points | 5 |
| Number of single points | 46 |
| Number of points off lines | 36 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^7$ |
| Type of lines on points | 4, 3, $2^5$, $1^{46}$, $0^{36}$ |

### Singular Points

The surface has 2 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

## The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{64} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{137} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{137} = \mathbf{Pl}(0,0,1,0,0,1)_{664}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{9} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{9} = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_4 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{4673} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

$$\ell_5 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{721} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{721} = \mathbf{Pl}(0,1,1,0,0,1)_{672}$$

Rank of lines: ( 0, 64, 137, 9, 4673, 4744, 721 )
Rank of points on Klein quadric: ( 0, 2, 664, 97, 769, 1, 672 )

### Eckardt Points

The surface has 1 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$.

### Double Points

The surface has 5 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_4$
$P_{146} = (0,1,1,1) = \ell_3 \cap \ell_4$
$P_4 = (1,1,1,1) = \ell_3 \cap \ell_6$
$P_{138} = (0,0,1,1) = \ell_4 \cap \ell_5$

### Single Points

The surface has 46 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line $\ell_5$
$1 : P_6 = (2,1,0,0)$ lies on line $\ell_0$
$2 : P_7 = (3,1,0,0)$ lies on line $\ell_0$
$3 : P_8 = (4,1,0,0)$ lies on line $\ell_0$
$4 : P_9 = (5,1,0,0)$ lies on line $\ell_0$
$5 : P_{10} = (6,1,0,0)$ lies on line $\ell_0$
$6 : P_{11} = (7,1,0,0)$ lies on line $\ell_0$
$7 : P_{12} = (1,0,1,0)$ lies on line $\ell_1$
$8 : P_{13} = (2,0,1,0)$ lies on line $\ell_1$
$9 : P_{14} = (3,0,1,0)$ lies on line $\ell_1$
$10 : P_{15} = (4,0,1,0)$ lies on line $\ell_1$
$11 : P_{16} = (5,0,1,0)$ lies on line $\ell_1$
$12 : P_{17} = (6,0,1,0)$ lies on line $\ell_1$
$13 : P_{18} = (7,0,1,0)$ lies on line $\ell_1$
$14 : P_{20} = (1,1,1,0)$ lies on line $\ell_2$
$15 : P_{29} = (2,2,1,0)$ lies on line $\ell_2$
$16 : P_{38} = (3,3,1,0)$ lies on line $\ell_2$
$17 : P_{47} = (4,4,1,0)$ lies on line $\ell_2$
$18 : P_{56} = (5,5,1,0)$ lies on line $\ell_2$
$19 : P_{65} = (6,6,1,0)$ lies on line $\ell_2$
$20 : P_{74} = (7,7,1,0)$ lies on line $\ell_2$
$21 : P_{83} = (1,1,0,1)$ lies on line $\ell_6$
$22 : P_{147} = (2,1,1,1)$ lies on line $\ell_3$
$23 : P_{148} = (3,1,1,1)$ lies on line $\ell_3$
$24 : P_{149} = (4,1,1,1)$ lies on line $\ell_3$
$25 : P_{150} = (5,1,1,1)$ lies on line $\ell_3$

26 : $P_{151} = (6,1,1,1)$ lies on line $\ell_3$
27 : $P_{152} = (7,1,1,1)$ lies on line $\ell_3$
28 : $P_{153} = (0,2,1,1)$ lies on line $\ell_4$
29 : $P_{161} = (0,3,1,1)$ lies on line $\ell_4$
30 : $P_{169} = (0,4,1,1)$ lies on line $\ell_4$
31 : $P_{177} = (0,5,1,1)$ lies on line $\ell_4$
32 : $P_{185} = (0,6,1,1)$ lies on line $\ell_4$
33 : $P_{193} = (0,7,1,1)$ lies on line $\ell_4$
34 : $P_{201} = (0,0,2,1)$ lies on line $\ell_5$
35 : $P_{210} = (1,1,2,1)$ lies on line $\ell_6$
36 : $P_{265} = (0,0,3,1)$ lies on line $\ell_5$
37 : $P_{274} = (1,1,3,1)$ lies on line $\ell_6$
38 : $P_{329} = (0,0,4,1)$ lies on line $\ell_5$
39 : $P_{338} = (1,1,4,1)$ lies on line $\ell_6$
40 : $P_{393} = (0,0,5,1)$ lies on line $\ell_5$
41 : $P_{402} = (1,1,5,1)$ lies on line $\ell_6$
42 : $P_{457} = (0,0,6,1)$ lies on line $\ell_5$
43 : $P_{466} = (1,1,6,1)$ lies on line $\ell_6$
44 : $P_{521} = (0,0,7,1)$ lies on line $\ell_5$
45 : $P_{530} = (1,1,7,1)$ lies on line $\ell_6$

The single points on the surface are:

### Points on surface but on no line

The surface has 36 points not on any line:
The points on the surface but not on lines are:

0 : $P_{94} = (4,2,0,1)$
1 : $P_{103} = (5,3,0,1)$
2 : $P_{113} = (7,4,0,1)$
3 : $P_{120} = (6,5,0,1)$
4 : $P_{125} = (3,6,0,1)$
5 : $P_{132} = (2,7,0,1)$
6 : $P_{224} = (7,2,2,1)$
7 : $P_{232} = (7,3,2,1)$
8 : $P_{234} = (1,4,2,1)$
9 : $P_{244} = (3,5,2,1)$
10 : $P_{260} = (3,7,2,1)$
11 : $P_{287} = (6,2,3,1)$
12 : $P_{293} = (4,3,3,1)$
13 : $P_{309} = (4,5,3,1)$
14 : $P_{314} = (1,6,3,1)$
15 : $P_{327} = (6,7,3,1)$
16 : $P_{350} = (5,2,4,1)$
17 : $P_{363} = (2,4,4,1)$
18 : $P_{371} = (2,5,4,1)$
19 : $P_{382} = (5,6,4,1)$
20 : $P_{386} = (1,7,4,1)$
21 : $P_{412} = (3,2,5,1)$
22 : $P_{418} = (1,3,5,1)$
23 : $P_{428} = (3,4,5,1)$
24 : $P_{440} = (7,5,5,1)$
25 : $P_{448} = (7,6,5,1)$
26 : $P_{483} = (2,3,6,1)$
27 : $P_{494} = (5,4,6,1)$
28 : $P_{498} = (1,5,6,1)$
29 : $P_{507} = (2,6,6,1)$
30 : $P_{518} = (5,7,6,1)$
31 : $P_{538} = (1,2,7,1)$
32 : $P_{551} = (6,3,7,1)$
33 : $P_{559} = (6,4,7,1)$
34 : $P_{573} = (4,6,7,1)$
35 : $P_{581} = (4,7,7,1)$

## Line Intersection Graph

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 2 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 3 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|---|
| in point | $P_0$ | $P_5$ | $P_0$ | $P_1$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_3$ | $\ell_5$ | $\ell_6$ |
|---|---|---|---|---|---|
| in point | $P_0$ | $P_2$ | $P_0$ | $P_2$ | $P_2$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_5$ | $\ell_6$ |
|---|---|---|---|---|
| in point | $P_5$ | $P_2$ | $P_2$ | $P_2$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_4$ | $\ell_6$ |
|---|---|---|---|---|
| in point | $P_0$ | $P_0$ | $P_{146}$ | $P_4$ |

Line 4 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_5$ |
|---|---|---|---|
| in point | $P_1$ | $P_{146}$ | $P_{138}$ |

Line 5 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_4$ | $\ell_6$ |
|---|---|---|---|---|
| in point | $P_2$ | $P_2$ | $P_{138}$ | $P_2$ |

Line 6 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ | $\ell_5$ |
|---|---|---|---|---|
| in point | $P_2$ | $P_2$ | $P_4$ | $P_2$ |

The surface has 89 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (4,1,0,0)$
$9: P_9 = (5,1,0,0)$
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$24: P_{65} = (6,6,1,0)$
$25: P_{74} = (7,7,1,0)$
$26: P_{83} = (1,1,0,1)$
$27: P_{94} = (4,2,0,1)$
$28: P_{103} = (5,3,0,1)$
$29: P_{113} = (7,4,0,1)$
$30: P_{120} = (6,5,0,1)$
$31: P_{125} = (3,6,0,1)$
$32: P_{132} = (2,7,0,1)$
$33: P_{138} = (0,0,1,1)$
$34: P_{146} = (0,1,1,1)$
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