

Rank-67150 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is -1859611562

General information

Number of lines	3
Number of points	89
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$3, 1^{24}, 0^{64}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 0 & 1 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{array} \right]_{4726} = \left[\begin{array}{cccc} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{array} \right]_{4726} = \mathbf{Pl}(0, 6, 0, 0, 0, 1)_{662} \\ \ell_1 &= \left[\begin{array}{cccc} 0 & 1 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{array} \right]_{4699} = \left[\begin{array}{cccc} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array} \right]_{4699} = \mathbf{Pl}(0, 3, 0, 0, 0, 1)_{659}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4717} = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4717} = \mathbf{PI}(0, 5, 0, 0, 0, 1)_{661}$$

Rank of lines: (4726, 4699, 4717)

Rank of points on Klein quadric: (662, 659, 661)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 24 single points:

The single points on the surface are:

- | | |
|---|---|
| 0 : $P_{90} = (0, 2, 0, 1)$ lies on line ℓ_0 | 13 : $P_{361} = (0, 4, 4, 1)$ lies on line ℓ_1 |
| 1 : $P_{106} = (0, 4, 0, 1)$ lies on line ℓ_1 | 14 : $P_{385} = (0, 7, 4, 1)$ lies on line ℓ_2 |
| 2 : $P_{130} = (0, 7, 0, 1)$ lies on line ℓ_2 | 15 : $P_{409} = (0, 2, 5, 1)$ lies on line ℓ_0 |
| 3 : $P_{153} = (0, 2, 1, 1)$ lies on line ℓ_0 | 16 : $P_{425} = (0, 4, 5, 1)$ lies on line ℓ_1 |
| 4 : $P_{169} = (0, 4, 1, 1)$ lies on line ℓ_1 | 17 : $P_{449} = (0, 7, 5, 1)$ lies on line ℓ_2 |
| 5 : $P_{193} = (0, 7, 1, 1)$ lies on line ℓ_2 | 18 : $P_{473} = (0, 2, 6, 1)$ lies on line ℓ_0 |
| 6 : $P_{217} = (0, 2, 2, 1)$ lies on line ℓ_0 | 19 : $P_{489} = (0, 4, 6, 1)$ lies on line ℓ_1 |
| 7 : $P_{233} = (0, 4, 2, 1)$ lies on line ℓ_1 | 20 : $P_{513} = (0, 7, 6, 1)$ lies on line ℓ_2 |
| 8 : $P_{257} = (0, 7, 2, 1)$ lies on line ℓ_2 | 21 : $P_{537} = (0, 2, 7, 1)$ lies on line ℓ_0 |
| 9 : $P_{281} = (0, 2, 3, 1)$ lies on line ℓ_0 | 22 : $P_{553} = (0, 4, 7, 1)$ lies on line ℓ_1 |
| 10 : $P_{297} = (0, 4, 3, 1)$ lies on line ℓ_1 | 23 : $P_{577} = (0, 7, 7, 1)$ lies on line ℓ_2 |
| 11 : $P_{321} = (0, 7, 3, 1)$ lies on line ℓ_2 | |
| 12 : $P_{345} = (0, 2, 4, 1)$ lies on line ℓ_0 | |

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:

The points on the surface but not on lines are:

- | | |
|-----------------------------|-------------------------------|
| 0 : $P_5 = (1, 1, 0, 0)$ | 6 : $P_{84} = (2, 1, 0, 1)$ |
| 1 : $P_{12} = (1, 0, 1, 0)$ | 7 : $P_{86} = (4, 1, 0, 1)$ |
| 2 : $P_{20} = (1, 1, 1, 0)$ | 8 : $P_{89} = (7, 1, 0, 1)$ |
| 3 : $P_{76} = (2, 0, 0, 1)$ | 9 : $P_{91} = (1, 2, 0, 1)$ |
| 4 : $P_{78} = (4, 0, 0, 1)$ | 10 : $P_{107} = (1, 4, 0, 1)$ |
| 5 : $P_{81} = (7, 0, 0, 1)$ | 11 : $P_{131} = (1, 7, 0, 1)$ |

12 : $P_{139} = (1, 0, 1, 1)$
 13 : $P_{147} = (2, 1, 1, 1)$
 14 : $P_{149} = (4, 1, 1, 1)$
 15 : $P_{152} = (7, 1, 1, 1)$
 16 : $P_{157} = (4, 2, 1, 1)$
 17 : $P_{158} = (5, 2, 1, 1)$
 18 : $P_{175} = (6, 4, 1, 1)$
 19 : $P_{176} = (7, 4, 1, 1)$
 20 : $P_{195} = (2, 7, 1, 1)$
 21 : $P_{196} = (3, 7, 1, 1)$
 22 : $P_{206} = (5, 0, 2, 1)$
 23 : $P_{218} = (1, 2, 2, 1)$
 24 : $P_{242} = (1, 5, 2, 1)$
 25 : $P_{247} = (6, 5, 2, 1)$
 26 : $P_{250} = (1, 6, 2, 1)$
 27 : $P_{252} = (3, 6, 2, 1)$
 28 : $P_{285} = (4, 2, 3, 1)$
 29 : $P_{286} = (5, 2, 3, 1)$
 30 : $P_{308} = (3, 5, 3, 1)$
 31 : $P_{310} = (5, 5, 3, 1)$
 32 : $P_{312} = (7, 5, 3, 1)$
 33 : $P_{315} = (2, 6, 3, 1)$
 34 : $P_{318} = (5, 6, 3, 1)$
 35 : $P_{319} = (6, 6, 3, 1)$
 36 : $P_{335} = (6, 0, 4, 1)$
 37 : $P_{354} = (1, 3, 4, 1)$
 38 : $P_{358} = (5, 3, 4, 1)$

39 : $P_{362} = (1, 4, 4, 1)$
 40 : $P_{378} = (1, 6, 4, 1)$
 41 : $P_{380} = (3, 6, 4, 1)$
 42 : $P_{420} = (3, 3, 5, 1)$
 43 : $P_{421} = (4, 3, 5, 1)$
 44 : $P_{423} = (6, 3, 5, 1)$
 45 : $P_{431} = (6, 4, 5, 1)$
 46 : $P_{432} = (7, 4, 5, 1)$
 47 : $P_{443} = (2, 6, 5, 1)$
 48 : $P_{446} = (5, 6, 5, 1)$
 49 : $P_{447} = (6, 6, 5, 1)$
 50 : $P_{484} = (3, 3, 6, 1)$
 51 : $P_{485} = (4, 3, 6, 1)$
 52 : $P_{487} = (6, 3, 6, 1)$
 53 : $P_{500} = (3, 5, 6, 1)$
 54 : $P_{502} = (5, 5, 6, 1)$
 55 : $P_{504} = (7, 5, 6, 1)$
 56 : $P_{515} = (2, 7, 6, 1)$
 57 : $P_{516} = (3, 7, 6, 1)$
 58 : $P_{524} = (3, 0, 7, 1)$
 59 : $P_{546} = (1, 3, 7, 1)$
 60 : $P_{550} = (5, 3, 7, 1)$
 61 : $P_{562} = (1, 5, 7, 1)$
 62 : $P_{567} = (6, 5, 7, 1)$
 63 : $P_{578} = (1, 7, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 89 points:

The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$
 1 : $P_5 = (1, 1, 0, 0)$
 2 : $P_{12} = (1, 0, 1, 0)$
 3 : $P_{20} = (1, 1, 1, 0)$
 4 : $P_{76} = (2, 0, 0, 1)$

5 : $P_{78} = (4, 0, 0, 1)$
 6 : $P_{81} = (7, 0, 0, 1)$
 7 : $P_{84} = (2, 1, 0, 1)$
 8 : $P_{86} = (4, 1, 0, 1)$
 9 : $P_{89} = (7, 1, 0, 1)$

10 : $P_{90} = (0, 2, 0, 1)$
 11 : $P_{91} = (1, 2, 0, 1)$
 12 : $P_{106} = (0, 4, 0, 1)$
 13 : $P_{107} = (1, 4, 0, 1)$
 14 : $P_{130} = (0, 7, 0, 1)$

15 : $P_{131} = (1, 7, 0, 1)$	40 : $P_{286} = (5, 2, 3, 1)$	65 : $P_{443} = (2, 6, 5, 1)$
16 : $P_{139} = (1, 0, 1, 1)$	41 : $P_{297} = (0, 4, 3, 1)$	66 : $P_{446} = (5, 6, 5, 1)$
17 : $P_{147} = (2, 1, 1, 1)$	42 : $P_{308} = (3, 5, 3, 1)$	67 : $P_{447} = (6, 6, 5, 1)$
18 : $P_{149} = (4, 1, 1, 1)$	43 : $P_{310} = (5, 5, 3, 1)$	68 : $P_{449} = (0, 7, 5, 1)$
19 : $P_{152} = (7, 1, 1, 1)$	44 : $P_{312} = (7, 5, 3, 1)$	69 : $P_{473} = (0, 2, 6, 1)$
20 : $P_{153} = (0, 2, 1, 1)$	45 : $P_{315} = (2, 6, 3, 1)$	70 : $P_{484} = (3, 3, 6, 1)$
21 : $P_{157} = (4, 2, 1, 1)$	46 : $P_{318} = (5, 6, 3, 1)$	71 : $P_{485} = (4, 3, 6, 1)$
22 : $P_{158} = (5, 2, 1, 1)$	47 : $P_{319} = (6, 6, 3, 1)$	72 : $P_{487} = (6, 3, 6, 1)$
23 : $P_{169} = (0, 4, 1, 1)$	48 : $P_{321} = (0, 7, 3, 1)$	73 : $P_{489} = (0, 4, 6, 1)$
24 : $P_{175} = (6, 4, 1, 1)$	49 : $P_{335} = (6, 0, 4, 1)$	74 : $P_{500} = (3, 5, 6, 1)$
25 : $P_{176} = (7, 4, 1, 1)$	50 : $P_{345} = (0, 2, 4, 1)$	75 : $P_{502} = (5, 5, 6, 1)$
26 : $P_{193} = (0, 7, 1, 1)$	51 : $P_{354} = (1, 3, 4, 1)$	76 : $P_{504} = (7, 5, 6, 1)$
27 : $P_{195} = (2, 7, 1, 1)$	52 : $P_{358} = (5, 3, 4, 1)$	77 : $P_{513} = (0, 7, 6, 1)$
28 : $P_{196} = (3, 7, 1, 1)$	53 : $P_{361} = (0, 4, 4, 1)$	78 : $P_{515} = (2, 7, 6, 1)$
29 : $P_{206} = (5, 0, 2, 1)$	54 : $P_{362} = (1, 4, 4, 1)$	79 : $P_{516} = (3, 7, 6, 1)$
30 : $P_{217} = (0, 2, 2, 1)$	55 : $P_{378} = (1, 6, 4, 1)$	80 : $P_{524} = (3, 0, 7, 1)$
31 : $P_{218} = (1, 2, 2, 1)$	56 : $P_{380} = (3, 6, 4, 1)$	81 : $P_{537} = (0, 2, 7, 1)$
32 : $P_{233} = (0, 4, 2, 1)$	57 : $P_{385} = (0, 7, 4, 1)$	82 : $P_{546} = (1, 3, 7, 1)$
33 : $P_{242} = (1, 5, 2, 1)$	58 : $P_{409} = (0, 2, 5, 1)$	83 : $P_{550} = (5, 3, 7, 1)$
34 : $P_{247} = (6, 5, 2, 1)$	59 : $P_{420} = (3, 3, 5, 1)$	84 : $P_{553} = (0, 4, 7, 1)$
35 : $P_{250} = (1, 6, 2, 1)$	60 : $P_{421} = (4, 3, 5, 1)$	85 : $P_{562} = (1, 5, 7, 1)$
36 : $P_{252} = (3, 6, 2, 1)$	61 : $P_{423} = (6, 3, 5, 1)$	86 : $P_{567} = (6, 5, 7, 1)$
37 : $P_{257} = (0, 7, 2, 1)$	62 : $P_{425} = (0, 4, 5, 1)$	87 : $P_{577} = (0, 7, 7, 1)$
38 : $P_{281} = (0, 2, 3, 1)$	63 : $P_{431} = (6, 4, 5, 1)$	88 : $P_{578} = (1, 7, 7, 1)$
39 : $P_{285} = (4, 2, 3, 1)$	64 : $P_{432} = (7, 4, 5, 1)$	