

Rank-65605 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431659869

General information

Number of lines	5
Number of points	21
Number of singular points	5
Number of Eckardt points	0
Number of double points	4
Number of single points	17
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$2^4, 1^{17}$

Singular Points

The surface has 5 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{53} = \mathbf{P}(0, 0, \omega, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0)_1 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \mathbf{Pl}(0, 1, 1, 1, 1)_{202} \\
\ell_3 &= \begin{bmatrix} 1 & \omega^2 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{166} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{166} = \mathbf{Pl}(0, 2, 2, 3, 1)_{210} \\
\ell_4 &= \begin{bmatrix} 1 & \omega & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{144} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{144} = \mathbf{Pl}(0, 3, 3, 2, 1)_{218}
\end{aligned}$$

Rank of lines: (16, 356, 122, 166, 144)

Rank of points on Klein quadric: (2, 1, 202, 210, 218)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

$$P_{38} = (0, 0, 1, 1) = \ell_1 \cap \ell_2$$

$$P_{53} = (0, 0, 2, 1) = \ell_1 \cap \ell_3$$

$$P_{69} = (0, 0, 3, 1) = \ell_1 \cap \ell_4$$

Single Points

The surface has 17 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1

2 : $P_8 = (1, 0, 1, 0)$ lies on line ℓ_0

3 : $P_9 = (2, 0, 1, 0)$ lies on line ℓ_0

4 : $P_{10} = (3, 0, 1, 0)$ lies on line ℓ_0

5 : $P_{12} = (1, 1, 1, 0)$ lies on line ℓ_2

6 : $P_{18} = (3, 2, 1, 0)$ lies on line ℓ_3

7 : $P_{21} = (2, 3, 1, 0)$ lies on line ℓ_4

8 : $P_{27} = (1, 1, 0, 1)$ lies on line ℓ_2

9 : $P_{31} = (1, 2, 0, 1)$ lies on line ℓ_4

10 : $P_{35} = (1, 3, 0, 1)$ lies on line ℓ_3

11 : $P_{43} = (2, 1, 1, 1)$ lies on line ℓ_3

12 : $P_{44} = (3, 1, 1, 1)$ lies on line ℓ_4

13 : $P_{67} = (2, 3, 2, 1)$ lies on line ℓ_4

14 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_2

15 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_2

16 : $P_{80} = (3, 2, 3, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	0	0	0
1	1	0	1	1	1
2	0	1	0	0	0
3	0	1	0	0	0
4	0	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_2	P_{38}	P_{53}	P_{69}

Line 2 intersects

Line	ℓ_1
in point	P_{38}

Line 3 intersects

Line	ℓ_1
in point	P_{53}

Line 4 intersects

Line	ℓ_1
in point	P_{69}

The surface has 21 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_8 = (1, 0, 1, 0)$$

$$4 : P_9 = (2, 0, 1, 0)$$

$$5 : P_{10} = (3, 0, 1, 0)$$

$$6 : P_{12} = (1, 1, 1, 0)$$

$$7 : P_{18} = (3, 2, 1, 0)$$

$$8 : P_{21} = (2, 3, 1, 0)$$

$$9 : P_{27} = (1, 1, 0, 1)$$

$$10 : P_{31} = (1, 2, 0, 1)$$

$$11 : P_{35} = (1, 3, 0, 1)$$

$$12 : P_{38} = (0, 0, 1, 1)$$

$$13 : P_{43} = (2, 1, 1, 1)$$

$$14 : P_{44} = (3, 1, 1, 1)$$

$$15 : P_{53} = (0, 0, 2, 1)$$

$$16 : P_{67} = (2, 3, 2, 1)$$

$$17 : P_{68} = (3, 3, 2, 1)$$

$$18 : P_{69} = (0, 0, 3, 1)$$

$$19 : P_{79} = (2, 2, 3, 1)$$

$$20 : P_{80} = (3, 2, 3, 1)$$