# Rank-73733 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

( 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(8) is 1227133525

## General information

| | |
|---|---|
| Number of lines | 2 |
| Number of points | 73 |
| Number of singular points | 2 |
| Number of Eckardt points | 0 |
| Number of double points | 1 |
| Number of single points | 16 |
| Number of points off lines | 56 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^2$ |
| Type of lines on points | 2, $1^{16}$, $0^{56}$ |

### Singular Points

The surface has 2 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: ( 64, 4744 )
Rank of points on Klein quadric: ( 2, 1 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

### Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line $\ell_0$
1 : $P_3 = (0,0,0,1)$ lies on line $\ell_1$
2 : $P_{12} = (1,0,1,0)$ lies on line $\ell_0$
3 : $P_{13} = (2,0,1,0)$ lies on line $\ell_0$
4 : $P_{14} = (3,0,1,0)$ lies on line $\ell_0$
5 : $P_{15} = (4,0,1,0)$ lies on line $\ell_0$
6 : $P_{16} = (5,0,1,0)$ lies on line $\ell_0$
7 : $P_{17} = (6,0,1,0)$ lies on line $\ell_0$
8 : $P_{18} = (7,0,1,0)$ lies on line $\ell_0$
9 : $P_{138} = (0,0,1,1)$ lies on line $\ell_1$
10 : $P_{201} = (0,0,2,1)$ lies on line $\ell_1$
11 : $P_{265} = (0,0,3,1)$ lies on line $\ell_1$
12 : $P_{329} = (0,0,4,1)$ lies on line $\ell_1$
13 : $P_{393} = (0,0,5,1)$ lies on line $\ell_1$
14 : $P_{457} = (0,0,6,1)$ lies on line $\ell_1$
15 : $P_{521} = (0,0,7,1)$ lies on line $\ell_1$

The single points on the surface are:

### Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_{20} = (1,1,1,0)$
1 : $P_{31} = (4,2,1,0)$
2 : $P_{40} = (5,3,1,0)$
3 : $P_{50} = (7,4,1,0)$
4 : $P_{57} = (6,5,1,0)$
5 : $P_{62} = (3,6,1,0)$
6 : $P_{69} = (2,7,1,0)$
7 : $P_{83} = (1,1,0,1)$
8 : $P_{95} = (5,2,0,1)$
9 : $P_{100} = (2,3,0,1)$
10 : $P_{112} = (6,4,0,1)$
11 : $P_{118} = (4,5,0,1)$
12 : $P_{129} = (7,6,0,1)$
13 : $P_{133} = (3,7,0,1)$
14 : $P_{156} = (3,2,1,1)$
15 : $P_{162} = (1,3,1,1)$
16 : $P_{174} = (5,4,1,1)$
17 : $P_{178} = (1,5,1,1)$
18 : $P_{186} = (1,6,1,1)$
19 : $P_{199} = (6,7,1,1)$
20 : $P_{213} = (4,1,2,1)$
21 : $P_{218} = (1,2,2,1)$

22 : $P_{232} = (7, 3, 2, 1)$
23 : $P_{240} = (7, 4, 2, 1)$
24 : $P_{246} = (5, 5, 2, 1)$
25 : $P_{264} = (7, 7, 2, 1)$
26 : $P_{279} = (6, 1, 3, 1)$
27 : $P_{287} = (6, 2, 3, 1)$
28 : $P_{295} = (6, 3, 3, 1)$
29 : $P_{312} = (7, 5, 3, 1)$
30 : $P_{316} = (3, 6, 3, 1)$
31 : $P_{325} = (4, 7, 3, 1)$
32 : $P_{344} = (7, 1, 4, 1)$
33 : $P_{347} = (2, 2, 4, 1)$
34 : $P_{362} = (1, 4, 4, 1)$
35 : $P_{371} = (2, 5, 4, 1)$
36 : $P_{383} = (6, 6, 4, 1)$
37 : $P_{387} = (2, 7, 4, 1)$
38 : $P_{404} = (3, 1, 5, 1)$
39 : $P_{416} = (7, 2, 5, 1)$
40 : $P_{422} = (5, 3, 5, 1)$
41 : $P_{428} = (3, 4, 5, 1)$
42 : $P_{436} = (3, 5, 5, 1)$
43 : $P_{443} = (2, 6, 5, 1)$
44 : $P_{470} = (5, 1, 6, 1)$
45 : $P_{485} = (4, 3, 6, 1)$
46 : $P_{491} = (2, 4, 6, 1)$
47 : $P_{503} = (6, 5, 6, 1)$
48 : $P_{510} = (5, 6, 6, 1)$
49 : $P_{518} = (5, 7, 6, 1)$
50 : $P_{531} = (2, 1, 7, 1)$
51 : $P_{541} = (4, 2, 7, 1)$
52 : $P_{548} = (3, 3, 7, 1)$
53 : $P_{557} = (4, 4, 7, 1)$
54 : $P_{573} = (4, 6, 7, 1)$
55 : $P_{578} = (1, 7, 7, 1)$

## Line Intersection Graph

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ |
|---|---|
| in point | $P_2$ |

Line 1 intersects

| Line | $\ell_0$ |
|---|---|
| in point | $P_2$ |

The surface has 73 points:
The points on the surface are:

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