# Rank-74531 over GF(2)

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## The equation

The equation of the surface is :

$$X_0^2X_2 + X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(2) is 74531

## General information

| | |
|---|---|
| Number of lines | 4 |
| Number of points | 9 |
| Number of singular points | 1 |
| Number of Eckardt points | 0 |
| Number of double points | 4 |
| Number of single points | 4 |
| Number of points off lines | 1 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $3^4$ |
| Type of lines on points | $2^4$, $1^4$, 0 |

### Singular Points

The surface has 1 singular points:

$0:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$
$$\ell_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1,0,1,1,1,1)_{33}$$

Rank of lines: ( 0, 29, 34, 24 )
Rank of points on Klein quadric: ( 0, 25, 1, 33 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$
$P_{12} = (0,0,1,1) = \ell_1 \cap \ell_2$
$P_{14} = (0,1,1,1) = \ell_1 \cap \ell_3$

### Single Points

The surface has 4 single points:
The single points on the surface are:

$0:\ P_0 = (1,0,0,0)$ lies on line $\ell_0$
$1:\ P_2 = (0,0,1,0)$ lies on line $\ell_2$
$2:\ P_3 = (0,0,0,1)$ lies on line $\ell_2$
$3:\ P_{13} = (1,0,1,1)$ lies on line $\ell_3$

The single points on the surface are:

### Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0:\ P_{11} = (1,1,0,1)$

## Line Intersection Graph

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 1 | 1 | 0 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_3$ |
|---|---|---|
| in point | $P_1$ | $P_5$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_3$ |
|---|---|---|---|
| in point | $P_1$ | $P_{12}$ | $P_{14}$ |

Line 2 intersects

| Line | $\ell_1$ |
|---|---|
| in point | $P_{12}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_1$ |
|---|---|---|
| in point | $P_5$ | $P_{14}$ |

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_{11} = (1,1,0,1)$
$6: P_{12} = (0,0,1,1)$
$7: P_{13} = (1,0,1,1)$
$8: P_{14} = (0,1,1,1)$