Rank-0 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 0

General information

Number of lines	21
Number of points	21
Number of singular points	21
Number of Eckardt points	0
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{21}
Type of lines on points	5^{21}

Singular Points

The surface has 21 singular points:

0 : $P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
2 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
3 : $P_{11} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$
4 : $P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0)$
5 : $P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0)$
6 : $P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$
7 : $P_{30} = \mathbf{P}(0,\omega,0,1) = \mathbf{P}(0,2,0,1)$
8 : $P_{34} = \mathbf{P}(0,\omega^2,0,1) = \mathbf{P}(0,3,0,1)$
9 : $P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
10 : $P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$
11 : $P_{45} = \mathbf{P}(0,\omega,1,1) = \mathbf{P}(0,2,1,1)$
12 : $P_{49} = \mathbf{P}(0,\omega^2,1,1) = \mathbf{P}(0,3,1,1)$
13 : $P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$
14 : $P_{57} = \mathbf{P}(0,1,\omega,1) = \mathbf{P}(0,1,2,1)$
15 : $P_{61} = \mathbf{P}(0,\omega,\omega,1) = \mathbf{P}(0,2,2,1)$
16 : $P_{65} = \mathbf{P}(0,\omega^2,\omega,1) = \mathbf{P}(0,3,2,1)$
17 : $P_{69} = \mathbf{P}(0,0,\omega^2,1) = \mathbf{P}(0,0,3,1)$

18 : $P_{73} = \mathbf{P}(0,1,\omega^2,1) = \mathbf{P}(0,1,3,1)$
19 : $P_{77} = \mathbf{P}(0,\omega,\omega^2,1) = \mathbf{P}(0,2,3,1)$
20 : $P_{81} = \mathbf{P}(0,\omega^2,\omega^2,1) = \mathbf{P}(0,3,3,1)$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_{9}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0,3,0,0,0,1)_{107}$$

$$\ell_8 = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0,2,0,0,0,1)_{106}$$

$$\ell_9 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_{10} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \mathbf{Pl}(0,1,0,1,0,1)_{133}$$

$$\ell_{11} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{354} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{354} = \mathbf{Pl}(0,3,0,3,0,1)_{149}$$

$$\ell_{12} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{bmatrix}_{348} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{348} = \mathbf{Pl}(0,2,0,2,0,1)_{141}$$

$$\ell_{13} = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_{14} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{343} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{343} = \mathbf{Pl}(0,1,0,2,0,1)_{140}$$

$$\ell_{15} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \mathbf{Pl}(0,3,0,1,0,1)_{135}$$

$$\ell_{16} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{349} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{349} = \mathbf{Pl}(0,2,0,3,0,1)_{148}$$

$$\ell_{17} = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_{18} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{344} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{344} = \mathbf{Pl}(0,1,0,3,0,1)_{147}$$

$$\ell_{19} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{353} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{353} = \mathbf{Pl}(0,3,0,2,0,1)_{142}$$

$$\ell_{20} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \mathbf{Pl}(0,2,0,1,0,1)_{134}$$

Rank of lines: (336, 340, 337, 339, 338, 356, 341, 351, 346, 345, 342, 354, 348, 355, 343, 352, 349, 350, 344, 353, 347)
Rank of points on Klein quadric: (101, 9, 129, 143, 136, 1, 105, 107, 106, 13, 133, 149, 141, 15, 140, 135, 148, 14, 147, 142, 134)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_1	P_1	P_2	P_2	P_2	P_2	P_{11}	P_{11}	P_{11}	P_{11}	P_{15}	P_{15}	P_{15}	P_{15}	P_{19}	P_{19}	P_{19}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_1	P_1	P_{38}	P_{42}	P_{45}	P_{49}	P_{42}	P_{38}	P_{49}	P_{45}	P_{45}	P_{49}	P_{38}	P_{42}	P_{49}	P_{45}	P_{42}	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_1	P_1	P_{53}	P_{57}	P_{61}	P_{65}	P_{61}	P_{65}	P_{53}	P_{57}	P_{65}	P_{61}	P_{57}	P_{53}	P_{57}	P_{53}	P_{65}	P_{61}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_1	P_1	P_{69}	P_{73}	P_{77}	P_{81}	P_{81}	P_{77}	P_{73}	P_{69}	P_{73}	P_{69}	P_{81}	P_{77}	P_{77}	P_{81}	P_{69}	P_{73}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_2	P_3	P_{38}	P_{53}	P_{69}	P_2	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_3	P_{69}	P_{38}	P_{53}	P_3	P_{53}	P_{69}	P_{38}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_2	P_{26}	P_{42}	P_{57}	P_{73}	P_2	P_2	P_2	P_{42}	P_{26}	P_{73}	P_{57}	P_{73}	P_{26}	P_{57}	P_{42}	P_{57}	P_{26}	P_{42}	P_{73}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_2	P_{30}	P_{45}	P_{61}	P_{77}	P_2	P_2	P_2	P_{61}	P_{77}	P_{30}	P_{45}	P_{45}	P_{61}	P_{30}	P_{77}	P_{77}	P_{45}	P_{30}	P_{61}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_2	P_{34}	P_{49}	P_{65}	P_{81}	P_2	P_2	P_2	P_{81}	P_{65}	P_{49}	P_{34}	P_{65}	P_{49}	P_{81}	P_{34}	P_{49}	P_{81}	P_{65}	P_{34}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{42}	P_{61}	P_{81}	P_{11}	P_{11}	P_{11}	P_3	P_{61}	P_{81}	P_{42}	P_3	P_{81}	P_{42}	P_{61}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{11}	P_{26}	P_{38}	P_{65}	P_{77}	P_{38}	P_{26}	P_{77}	P_{65}	P_{11}	P_{11}	P_{11}	P_{65}	P_{26}	P_{38}	P_{77}	P_{77}	P_{26}	P_{65}	P_{38}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{11}	P_{30}	P_{49}	P_{53}	P_{73}	P_{53}	P_{73}	P_{30}	P_{49}	P_{11}	P_{11}	P_{11}	P_{73}	P_{49}	P_{30}	P_{53}	P_{49}	P_{53}	P_{30}	P_{73}

Line 12 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{11}	P_{34}	P_{45}	P_{57}	P_{69}	P_{69}	P_{57}	P_{45}	P_{34}	P_{11}	P_{11}	P_{11}	P_{45}	P_{69}	P_{57}	P_{34}	P_{57}	P_{45}	P_{69}	P_{34}

Line 13 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{15}	P_3	P_{45}	P_{65}	P_{73}	P_3	P_{73}	P_{45}	P_{65}	P_3	P_{65}	P_{73}	P_{45}	P_{15}	P_{15}	P_{15}	P_3	P_{45}	P_{65}	P_{73}

Line 14 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{15}	P_{26}	P_{49}	P_{61}	P_{69}	P_{69}	P_{26}	P_{61}	P_{49}	P_{61}	P_{26}	P_{49}	P_{69}	P_{15}	P_{15}	P_{15}	P_{49}	P_{26}	P_{69}	P_{6}

Line 15 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{15}	P_{30}	P_{38}	P_{57}	P_{81}	P_{38}	P_{57}	P_{30}	P_{81}	P_{81}	P_{38}	P_{30}	P_{57}	P_{15}	P_{15}	P_{15}	P_{57}	P_{81}	P_{30}	P_{3}

Line 16 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{15}	P_{34}	P_{42}	P_{53}	P_{77}	P_{53}	P_{42}	P_{77}	P_{34}	P_{42}	P_{77}	P_{53}	P_{34}	P_{15}	P_{15}	P_{15}	P_{77}	P_{53}	P_{42}	P_{3}

Line 17 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{19}	P_3	P_{49}	P_{57}	P_{77}	P_3	P_{57}	P_{77}	P_{49}	P_3	P_{77}	P_{49}	P_{57}	P_3	P_{49}	P_{57}	P_{77}	P_{19}	P_{19}	P_{19}

Line 18 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}
in point	P_{19}	P_{26}	P_{45}	P_{53}	P_{81}	P_{53}	P_{26}	P_{45}	P_{81}	P_{81}	P_{26}	P_{53}	P_{45}	P_{45}	P_{26}	P_{81}	P_{53}	P_{19}	P_{19}	P_{1}

Line 19 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}
in point	P_{19}	P_{30}	P_{42}	P_{65}	P_{69}	P_{69}	P_{42}	P_{30}	P_{65}	P_{42}	P_{65}	P_{30}	P_{69}	P_{65}	P_{69}	P_{30}	P_{42}	P_{19}	P_{19}	P_{1}

Line 20 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}
in point	P_{19}	P_{34}	P_{38}	P_{61}	P_{73}	P_{38}	P_{73}	P_{61}	P_{34}	P_{61}	P_{38}	P_{73}	P_{34}	P_{73}	P_{61}	P_{38}	P_{34}	P_{19}	P_{19}	P_{1}

The surface has 21 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_{11} = (0,1,1,0)$
$4: P_{15} = (0,2,1,0)$
$5: P_{19} = (0,3,1,0)$
$6: P_{26} = (0,1,0,1)$
$7: P_{30} = (0,2,0,1)$
$8: P_{34} = (0,3,0,1)$
$9: P_{38} = (0,0,1,1)$
$10: P_{42} = (0,1,1,1)$
$11: P_{45} = (0,2,1,1)$
$12: P_{49} = (0,3,1,1)$
$13: P_{53} = (0,0,2,1)$
$14: P_{57} = (0,1,2,1)$
$15: P_{61} = (0,2,2,1)$
$16: P_{65} = (0,3,2,1)$
$17: P_{69} = (0,0,3,1)$
$18: P_{73} = (0,1,3,1)$
$19: P_{77} = (0,2,3,1)$
$20: P_{81} = (0,3,3,1)$