Rank-73987 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1243910733

General information

Number of lines	5
Number of points	81
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	34
Number of points off lines	42
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	3, 2^4, 1^{34}, 0^{42}

Singular Points

The surface has 3 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$2: \ P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \mathbf{Pl}(0,0,1,0,0,1)_{664}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 64, 137, 4680, 4744)
Rank of points on Klein quadric: (0, 2, 664, 17, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 34 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$1 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$2 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$3 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$4 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$5 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$6 : P_{12} = (1,0,1,0)$ lies on line ℓ_1
$7 : P_{13} = (2,0,1,0)$ lies on line ℓ_1
$8 : P_{14} = (3,0,1,0)$ lies on line ℓ_1
$9 : P_{15} = (4,0,1,0)$ lies on line ℓ_1
$10 : P_{16} = (5,0,1,0)$ lies on line ℓ_1
$11 : P_{17} = (6,0,1,0)$ lies on line ℓ_1
$12 : P_{18} = (7,0,1,0)$ lies on line ℓ_1
$13 : P_{20} = (1,1,1,0)$ lies on line ℓ_2
$14 : P_{29} = (2,2,1,0)$ lies on line ℓ_2
$15 : P_{38} = (3,3,1,0)$ lies on line ℓ_2
$16 : P_{47} = (4,4,1,0)$ lies on line ℓ_2
$17 : P_{56} = (5,5,1,0)$ lies on line ℓ_2
$18 : P_{65} = (6,6,1,0)$ lies on line ℓ_2
$19 : P_{74} = (7,7,1,0)$ lies on line ℓ_2
$20 : P_{82} = (0,1,0,1)$ lies on line ℓ_3
$21 : P_{90} = (0,2,0,1)$ lies on line ℓ_3
$22 : P_{98} = (0,3,0,1)$ lies on line ℓ_3
$23 : P_{106} = (0,4,0,1)$ lies on line ℓ_3
$24 : P_{114} = (0,5,0,1)$ lies on line ℓ_3
$25 : P_{122} = (0,6,0,1)$ lies on line ℓ_3
$26 : P_{130} = (0,7,0,1)$ lies on line ℓ_3
$27 : P_{138} = (0,0,1,1)$ lies on line ℓ_4
$28 : P_{201} = (0,0,2,1)$ lies on line ℓ_4
$29 : P_{265} = (0,0,3,1)$ lies on line ℓ_4
$30 : P_{329} = (0,0,4,1)$ lies on line ℓ_4
$31 : P_{393} = (0,0,5,1)$ lies on line ℓ_4
$32 : P_{457} = (0,0,6,1)$ lies on line ℓ_4
$33 : P_{521} = (0,0,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 42 points not on any line:
The points on the surface but not on lines are:

$0: P_{160} = (7,2,1,1)$
$1: P_{165} = (4,3,1,1)$
$2: P_{171} = (2,4,1,1)$
$3: P_{184} = (7,5,1,1)$
$4: P_{187} = (2,6,1,1)$
$5: P_{197} = (4,7,1,1)$
$6: P_{214} = (5,1,2,1)$
$7: P_{218} = (1,2,2,1)$
$8: P_{226} = (1,3,2,1)$
$9: P_{238} = (5,4,2,1)$
$10: P_{243} = (2,5,2,1)$
$11: P_{259} = (2,7,2,1)$
$12: P_{280} = (7,1,3,1)$
$13: P_{286} = (5,2,3,1)$
$14: P_{295} = (6,3,3,1)$
$15: P_{311} = (6,5,3,1)$
$16: P_{320} = (7,6,3,1)$
$17: P_{326} = (5,7,3,1)$
$18: P_{343} = (6,1,4,1)$
$19: P_{349} = (4,2,4,1)$
$20: P_{362} = (1,4,4,1)$
$21: P_{370} = (1,5,4,1)$
$22: P_{381} = (4,6,4,1)$
$23: P_{391} = (6,7,4,1)$
$24: P_{403} = (2,1,5,1)$
$25: P_{415} = (6,2,5,1)$
$26: P_{419} = (2,3,5,1)$
$27: P_{431} = (6,4,5,1)$
$28: P_{436} = (3,5,5,1)$
$29: P_{444} = (3,6,5,1)$
$30: P_{469} = (4,1,6,1)$
$31: P_{486} = (5,3,6,1)$
$32: P_{492} = (3,4,6,1)$
$33: P_{501} = (4,5,6,1)$
$34: P_{510} = (5,6,6,1)$
$35: P_{516} = (3,7,6,1)$
$36: P_{532} = (3,1,7,1)$
$37: P_{540} = (3,2,7,1)$
$38: P_{552} = (7,3,7,1)$
$39: P_{560} = (7,4,7,1)$
$40: P_{570} = (1,6,7,1)$
$41: P_{578} = (1,7,7,1)$

Line Intersection Graph

	0 1 2 3 4
0	0 1 1 1 0
1	1 0 1 0 1
2	1 1 0 0 1
3	1 0 0 0 1
4	0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

The surface has 81 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_5 = (1,1,0,0)$
5 : $P_6 = (2,1,0,0)$
6 : $P_7 = (3,1,0,0)$
7 : $P_8 = (4,1,0,0)$
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