Rank-74276 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(16)$ is 287379734

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	90
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17 ⁶
Type of lines on points	$4, 2^4, 1^{90}, 0^{210}$

Singular Points

The surface has 1 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{69888} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{4625}$$

$$\ell_{1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_{1}$$

$$\ell_{2} = \begin{bmatrix} 1 & 1 & 0 & \delta^{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \begin{bmatrix} 1 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \mathbf{Pl}(0, 11, 1, 0, 0, 1)_{4682}$$

$$\ell_{3} = \begin{bmatrix} 1 & 1 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \begin{bmatrix} 1 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \mathbf{Pl}(0, 10, 1, 0, 0, 1)_{4681}$$

$$\ell_{4} = \begin{bmatrix} 1 & 0 & \delta^{5} & \delta^{5} \\ 0 & 1 & \delta^{5} & 0 \end{bmatrix}_{51062} = \begin{bmatrix} 1 & 0 & 11 & 11 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{51062} = \mathbf{Pl}(10, 11, 1, 1, 0, 1)_{5595}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{10} \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{46420} = \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{46420} = \mathbf{Pl}(11, 10, 1, 1, 0, 1)_{5596}$$

Rank of lines: (69888, 70160, 48577, 44209, 51062, 46420) Rank of points on Klein quadric: (4625, 1, 4682, 4681, 5595, 5596)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points: The double points on the surface are:

$$P_{179} = (0, 10, 1, 0) = \ell_0 \cap \ell_4$$

$$P_{195} = (0, 11, 1, 0) = \ell_0 \cap \ell_5$$

$$P_{444} = (10, 10, 0, 1) = \ell_2 \cap \ell_4$$

 $P_{461} = (11, 11, 0, 1) = \ell_3 \cap \ell_5$

Single Points

The surface has 90 single points: The single points on the surface are:

 $0: P_1 = (0, 1, 0, 0)$ lies on line ℓ_0 1: $P_3 = (0,0,0,1)$ lies on line ℓ_1 2: $P_{35} = (0, 1, 1, 0)$ lies on line ℓ_0 $3: P_{51} = (0, 2, 1, 0)$ lies on line ℓ_0 4: $P_{67} = (0, 3, 1, 0)$ lies on line ℓ_0 5: $P_{83} = (0, 4, 1, 0)$ lies on line ℓ_0 6: $P_{99} = (0, 5, 1, 0)$ lies on line ℓ_0 7: $P_{115} = (0, 6, 1, 0)$ lies on line ℓ_0 8: $P_{131} = (0,7,1,0)$ lies on line ℓ_0 9: $P_{147} = (0, 8, 1, 0)$ lies on line ℓ_0 10: $P_{163} = (0, 9, 1, 0)$ lies on line ℓ_0 11: $P_{211} = (0, 12, 1, 0)$ lies on line ℓ_0 12 : $P_{227} = (0, 13, 1, 0)$ lies on line ℓ_0 13: $P_{243} = (0, 14, 1, 0)$ lies on line ℓ_0 14: $P_{259} = (0, 15, 1, 0)$ lies on line ℓ_0 15: $P_{530} = (0,0,1,1)$ lies on line ℓ_1 16: $P_{540} = (10, 0, 1, 1)$ lies on line ℓ_4

17: $P_{541} = (11, 0, 1, 1)$ lies on line ℓ_5 18: $P_{699} = (10, 10, 1, 1)$ lies on line ℓ_2 19: $P_{716} = (11, 11, 1, 1)$ lies on line ℓ_3 20: $P_{785} = (0,0,2,1)$ lies on line ℓ_1 21: $P_{860} = (11, 4, 2, 1)$ lies on line ℓ_5 22: $P_{907} = (10, 7, 2, 1)$ lies on line ℓ_4 23: $P_{955} = (10, 10, 2, 1)$ lies on line ℓ_2 24: $P_{972} = (11, 11, 2, 1)$ lies on line ℓ_3 25: $P_{1041} = (0,0,3,1)$ lies on line ℓ_1 26: $P_{1211} = (10, 10, 3, 1)$ lies on line ℓ_2 27: $P_{1228} = (11, 11, 3, 1)$ lies on line ℓ_3 28: $P_{1259} = (10, 13, 3, 1)$ lies on line ℓ_4 29: $P_{1292} = (11, 15, 3, 1)$ lies on line ℓ_5 $30: P_{1297} = (0,0,4,1)$ lies on line ℓ_1 31: $P_{1451} = (10, 9, 4, 1)$ lies on line ℓ_4 $32: P_{1467} = (10, 10, 4, 1)$ lies on line ℓ_2 33: $P_{1484} = (11, 11, 4, 1)$ lies on line ℓ_3

```
34 : P_{1500} = (11, 12, 4, 1) lies on line \ell_5
                                                                     63: P_{3004} = (11, 10, 10, 1) lies on line \ell_5
                                                                     64: P_{3020} = (11, 11, 10, 1) lies on line \ell_3
35: P_{1553} = (0,0,5,1) lies on line \ell_1
36: P_{1611} = (10, 3, 5, 1) lies on line \ell_4
                                                                     65: P_{3089} = (0, 0, 11, 1) lies on line \ell_1
37: P_{1676} = (11, 7, 5, 1) lies on line \ell_5
                                                                     66: P_{3116} = (11, 1, 11, 1) lies on line \ell_5
38: P_{1723} = (10, 10, 5, 1) lies on line \ell_2
                                                                     67: P_{3259} = (10, 10, 11, 1) lies on line \ell_2
                                                                     68: P_{3275} = (10, 11, 11, 1) lies on line \ell_4
39: P_{1740} = (11, 11, 5, 1) lies on line \ell_3
40: P_{1809} = (0,0,6,1) lies on line \ell_1
                                                                     69: P_{3276} = (11, 11, 11, 1) lies on line \ell_3
41: P_{1868} = (11, 3, 6, 1) lies on line \ell_5
                                                                     70: P_{3345} = (0, 0, 12, 1) lies on line \ell_1
42: P_{1883} = (10, 4, 6, 1) lies on line \ell_4
                                                                     71: P_{3388} = (11, 2, 12, 1) lies on line \ell_5
43: P_{1979} = (10, 10, 6, 1) lies on line \ell_2
                                                                     72: P_{3515} = (10, 10, 12, 1) lies on line \ell_2
44: P_{1996} = (11, 11, 6, 1) lies on line \ell_3
                                                                     73: P_{3532} = (11, 11, 12, 1) lies on line \ell_3
45: P_{2065} = (0,0,7,1) lies on line \ell_1
                                                                     74: P_{3595} = (10, 15, 12, 1) lies on line \ell_4
46: P_{2204} = (11, 8, 7, 1) lies on line \ell_5
                                                                     75: P_{3601} = (0, 0, 13, 1) lies on line \ell_1
                                                                     76: P_{3691} = (10, 5, 13, 1) lies on line \ell_4
47: P_{2235} = (10, 10, 7, 1) lies on line \ell_2
48: P_{2252} = (11, 11, 7, 1) lies on line \ell_3
                                                                     77: P_{3756} = (11, 9, 13, 1) lies on line \ell_5
49: P_{2299} = (10, 14, 7, 1) lies on line \ell_4
                                                                     78: P_{3771} = (10, 10, 13, 1) lies on line \ell_2
50: P_{2321} = (0,0,8,1) lies on line \ell_1
                                                                     79: P_{3788} = (11, 11, 13, 1) lies on line \ell_3
51: P_{2412} = (11, 5, 8, 1) lies on line \ell_5
                                                                     80: P_{3857} = (0, 0, 14, 1) lies on line \ell_1
52: P_{2491} = (10, 10, 8, 1) lies on line \ell_2
                                                                     81: P_{3899} = (10, 2, 14, 1) lies on line \ell_4
                                                                     82: P_{4027} = (10, 10, 14, 1) lies on line \ell_2
53: P_{2508} = (11, 11, 8, 1) lies on line \ell_3
54: P_{2523} = (10, 12, 8, 1) lies on line \ell_4
                                                                     83: P_{4044} = (11, 11, 14, 1) lies on line \ell_3
55: P_{2577} = (0,0,9,1) lies on line \ell_1
                                                                     84: P_{4076} = (11, 13, 14, 1) lies on line \ell_5
56: P_{2683} = (10, 6, 9, 1) lies on line \ell_4
                                                                     85: P_{4113} = (0, 0, 15, 1) lies on line \ell_1
                                                                     86 : P_{4220} = (11, 6, 15, 1) lies on line \ell_5
57: P_{2747} = (10, 10, 9, 1) lies on line \ell_2
58: P_{2764} = (11, 11, 9, 1) lies on line \ell_3
                                                                     87: P_{4251} = (10, 8, 15, 1) lies on line \ell_4
59: P_{2812} = (11, 14, 9, 1) lies on line \ell_5
                                                                     88: P_{4283} = (10, 10, 15, 1) lies on line \ell_2
60: P_{2833} = (0, 0, 10, 1) lies on line \ell_1
                                                                     89: P_{4300} = (11, 11, 15, 1) lies on line \ell_3
61: P_{2859} = (10, 1, 10, 1) lies on line \ell_4
```

The single points on the surface are:

62: $P_{3003} = (10, 10, 10, 1)$ lies on line ℓ_2

Points on surface but on no line

The surface has 210 points not on any line: The points on the surface but not on lines are:

```
15: P_{275} = (1, 0, 0, 1)
0: P_{20} = (1,0,1,0)
1: P_{45} = (10, 1, 1, 0)
                                                                  16: P_{335} = (13, 3, 0, 1)
2: P_{46} = (11, 1, 1, 0)
                                                                  17: P_{361} = (7, 5, 0, 1)
3: P_{117} = (2, 6, 1, 0)
                                                                  18: P_{378} = (8, 6, 0, 1)
4: P_{118} = (3, 6, 1, 0)
                                                                  19: P_{389} = (3,7,0,1)
5: P_{139} = (8,7,1,0)
                                                                  20: P_{414} = (12, 8, 0, 1)
6: P_{140} = (9, 7, 1, 0)
                                                                  21: P_{438} = (4, 10, 0, 1)
7: P_{191} = (12, 10, 1, 0)
                                                                  22: P_{448} = (14, 10, 0, 1)
8: P_{192} = (13, 10, 1, 0)
                                                                  23: P_{452} = (2, 11, 0, 1)
9: P_{201} = (6, 11, 1, 0)
                                                                  24: P_{459} = (9, 11, 0, 1)
10: P_{202} = (7, 11, 1, 0)
                                                                  25: P_{471} = (5, 12, 0, 1)
11: P_{225} = (14, 12, 1, 0)
                                                                  26: P_{497} = (15, 13, 0, 1)
12: P_{226} = (15, 12, 1, 0)
                                                                  27: P_{520} = (6, 15, 0, 1)
13: P_{231} = (4, 13, 1, 0)
                                                                  28: P_{566} = (5, 2, 1, 1)
14: P_{232} = (5, 13, 1, 0)
                                                                  29: P_{601} = (8, 4, 1, 1)
```

```
84: P_{1939} = (2, 8, 6, 1)
30: P_{640} = (15, 6, 1, 1)
                                                                 85: P_{1960} = (7, 9, 6, 1)
31: P_{646} = (5,7,1,1)
32: P_{688} = (15, 9, 1, 1)
                                                                 86: P_{1988} = (3, 11, 6, 1)
33: P_{690} = (1, 10, 1, 1)
                                                                 87: P_{1999} = (14, 11, 6, 1)
34: P_{706} = (1, 11, 1, 1)
                                                                 88: P_{2025} = (8, 13, 6, 1)
35: P_{729} = (8, 12, 1, 1)
                                                                 89: P_{2057} = (8, 15, 6, 1)
36: P_{740} = (3, 13, 1, 1)
                                                                 90: P_{2083} = (2, 1, 7, 1)
37: P_{756} = (3, 14, 1, 1)
                                                                 91: P_{2089} = (8, 1, 7, 1)
38: P_{789} = (4,0,2,1)
                                                                 92: P_{2094} = (13, 1, 7, 1)
39: P_{791} = (6,0,2,1)
                                                                 93: P_{2103} = (6, 2, 7, 1)
40: P_{866} = (1, 5, 2, 1)
                                                                 94: P_{2122} = (9, 3, 7, 1)
41: P_{898} = (1,7,2,1)
                                                                 95: P_{2148} = (3, 5, 7, 1)
42: P_{906} = (9,7,2,1)
                                                                 96: P_{2173} = (12, 6, 7, 1)
43: P_{917} = (4, 8, 2, 1)
                                                                 97: P_{2215} = (6, 9, 7, 1)
                                                                 98: P_{2221} = (12, 9, 7, 1)
44: P_{983} = (6, 12, 2, 1)
45: P_{986} = (9, 12, 2, 1)
                                                                 99: P_{2222} = (13, 9, 7, 1)
46: P_{990} = (13, 12, 2, 1)
                                                                 100: P_{2245} = (4, 11, 7, 1)
47: P_{1038} = (13, 15, 2, 1)
                                                                 101: P_{2249} = (8, 11, 7, 1)
                                                                 102: P_{2260} = (3, 12, 7, 1)
48: P_{1110} = (5,4,3,1)
49: P_{1114} = (9, 4, 3, 1)
                                                                 103: P_{2293} = (4, 14, 7, 1)
50: P_{1120} = (15, 4, 3, 1)
                                                                 104: P_{2298} = (9, 14, 7, 1)
                                                                 105: P_{2307} = (2, 15, 7, 1)
51: P_{1134} = (13, 5, 3, 1)
                                                                 106: P_{2435} = (2,7,8,1)
52: P_{1146} = (9, 6, 3, 1)
53: P_{1222} = (5, 11, 3, 1)
                                                                 107: P_{2509} = (12, 11, 8, 1)
54: P_{1230} = (13, 11, 3, 1)
                                                                 108: P_{2512} = (15, 11, 8, 1)
55: P_{1248} = (15, 12, 3, 1)
                                                                 109: P_{2534} = (5, 13, 8, 1)
                                                                 110: P_{2547} = (2, 14, 8, 1)
56: P_{1306} = (9, 0, 4, 1)
57: P_{1310} = (13, 0, 4, 1)
                                                                 111: P_{2550} = (5, 14, 8, 1)
58: P_{1352} = (7, 3, 4, 1)
                                                                 112: P_{2560} = (15, 14, 8, 1)
59: P_{1400} = (7, 6, 4, 1)
                                                                 113: P_{2573} = (12, 15, 8, 1)
60: P_{1406} = (13, 6, 4, 1)
                                                                 114: P_{2584} = (7, 0, 9, 1)
61: P_{1407} = (14, 6, 4, 1)
                                                                 115: P_{2591} = (14, 0, 9, 1)
62: P_{1426} = (1, 8, 4, 1)
                                                                 116: P_{2639} = (14, 3, 9, 1)
63: P_{1490} = (1, 12, 4, 1)
                                                                 117: P_{2669} = (12, 5, 9, 1)
64: P_{1503} = (14, 12, 4, 1)
                                                                 118: P_{2674} = (1, 6, 9, 1)
65: P_{1546} = (9, 15, 4, 1)
                                                                 119: P_{2675} = (2, 6, 9, 1)
66: P_{1652} = (3, 6, 5, 1)
                                                                 120: P_{2787} = (2, 13, 9, 1)
                                                                 121: P_{2792} = (7, 13, 9, 1)
67: P_{1688} = (7, 8, 5, 1)
                                                                 122: P_{2797} = (12, 13, 9, 1)
68: P_{1700} = (3, 9, 5, 1)
69: P_{1705} = (8, 9, 5, 1)
                                                                 123: P_{2818} = (1, 15, 9, 1)
70: P_{1711} = (14, 9, 5, 1)
                                                                 124: P_{2838} = (5, 0, 10, 1)
                                                                 125: P_{2848} = (15, 0, 10, 1)
71: P_{1720} = (7, 10, 5, 1)
72: P_{1721} = (8, 10, 5, 1)
                                                                 126: P_{2880} = (15, 2, 10, 1)
73: P_{1775} = (14, 13, 5, 1)
                                                                 127: P_{2889} = (8, 3, 10, 1)
74: P_{1828} = (3, 1, 6, 1)
                                                                 128: P_{2898} = (1, 4, 10, 1)
                                                                 129: P_{2917} = (4, 5, 10, 1)
75: P_{1834} = (9, 1, 6, 1)
76: P_{1837} = (12, 1, 6, 1)
                                                                 130: P_{2919} = (6, 5, 10, 1)
77: P_{1848} = (7, 2, 6, 1)
                                                                 131: P_{2921} = (8, 5, 10, 1)
78: P_{1853} = (12, 2, 6, 1)
                                                                 132: P_{2933} = (4, 6, 10, 1)
79: P_{1854} = (13, 2, 6, 1)
                                                                 133: P_{2959} = (14, 7, 10, 1)
80: P_{1875} = (2,4,6,1)
                                                                 134: P_{2964} = (3, 8, 10, 1)
81: P_{1887} = (14, 4, 6, 1)
                                                                 135: P_{2982} = (5, 9, 10, 1)
82: P_{1898} = (9, 5, 6, 1)
                                                                 136: P_{3015} = (6, 11, 10, 1)
83: P_{1934} = (13, 7, 6, 1)
                                                                 137: P_{3016} = (7, 11, 10, 1)
```

```
138: P_{3058} = (1, 14, 10, 1)
                                                                 175: P_{3582} = (13, 14, 12, 1)
139: P_{3076} = (3, 15, 10, 1)
                                                                 176: P_{3622} = (5, 1, 13, 1)
140: P_{3080} = (7, 15, 10, 1)
                                                                 177: P_{3623} = (6, 1, 13, 1)
141: P_{3087} = (14, 15, 10, 1)
                                                                 178: P_{3631} = (14, 1, 13, 1)
142: P_{3092} = (3, 0, 11, 1)
                                                                 179: P_{3664} = (15, 3, 13, 1)
143: P_{3097} = (8, 0, 11, 1)
                                                                 180: P_{3671} = (6, 4, 13, 1)
144: P_{3122} = (1, 2, 11, 1)
                                                                 181: P_{3672} = (7, 4, 13, 1)
145: P_{3139} = (2, 3, 11, 1)
                                                                 182: P_{3677} = (12, 4, 13, 1)
146: P_{3142} = (5, 3, 11, 1)
                                                                 183: P_{3728} = (15, 7, 13, 1)
147: P_{3149} = (12, 3, 11, 1)
                                                                 184: P_{3743} = (14, 8, 13, 1)
148: P_{3156} = (3, 4, 11, 1)
                                                                 185 : P_{3747} = (2, 9, 13, 1)
                                                                 186: P_{3749} = (4, 9, 13, 1)
149: P_{3184} = (15, 5, 11, 1)
150: P_{3226} = (9, 8, 11, 1)
                                                                 187: P_{3763} = (2, 10, 13, 1)
151: P_{3230} = (13, 8, 11, 1)
                                                                 188: P_{3766} = (5, 10, 13, 1)
                                                                 189: P_{3800} = (7, 12, 13, 1)
152: P_{3232} = (15, 8, 11, 1)
153: P_{3234} = (1, 9, 11, 1)
                                                                 190: P_{3837} = (12, 14, 13, 1)
154: P_{3261} = (12, 10, 11, 1)
                                                                 191: P_{3845} = (4, 15, 13, 1)
155: P_{3262} = (13, 10, 11, 1)
                                                                 192: P_{3859} = (2, 0, 14, 1)
156: P_{3283} = (2, 12, 11, 1)
                                                                 193: P_{3869} = (12, 0, 14, 1)
                                                                 194: P_{3906} = (1, 3, 14, 1)
157: P_{3306} = (9, 13, 11, 1)
158: P_{3321} = (8, 14, 11, 1)
                                                                 195: P_{3939} = (2, 5, 14, 1)
                                                                 196: P_{3973} = (4, 7, 14, 1)
159: P_{3334} = (5, 15, 11, 1)
                                                                 197: P_{3975} = (6, 7, 14, 1)
160: P_{3365} = (4, 1, 12, 1)
161: P_{3368} = (7, 1, 12, 1)
                                                                 198: P_{3981} = (12, 7, 14, 1)
162: P_{3376} = (15, 1, 12, 1)
                                                                 199: P_{3991} = (6, 8, 14, 1)
163: P_{3386} = (9, 2, 12, 1)
                                                                 200: P_{4066} = (1, 13, 14, 1)
164: P_{3391} = (14, 2, 12, 1)
                                                                 201: P_{4069} = (4, 13, 14, 1)
165: P_{3397} = (4, 3, 12, 1)
                                                                 202: P_{4148} = (3, 2, 15, 1)
                                                                 203: P_{4149} = (4, 2, 15, 1)
166: P_{3422} = (13, 4, 12, 1)
167: P_{3439} = (14, 5, 12, 1)
                                                                 204: P_{4153} = (8, 2, 15, 1)
168: P_{3446} = (5, 6, 12, 1)
                                                                 205: P_{4167} = (6, 3, 15, 1)
169: P_{3478} = (5, 8, 12, 1)
                                                                 206: P_{4233} = (8, 7, 15, 1)
170: P_{3514} = (9, 10, 12, 1)
                                                                 207: P_{4276} = (3, 10, 15, 1)
171: P_{3520} = (15, 10, 12, 1)
                                                                 208: P_{4279} = (6, 10, 15, 1)
172: P_{3559} = (6, 13, 12, 1)
                                                                 209: P_{4309} = (4, 12, 15, 1)
173: P_{3575} = (6, 14, 12, 1)
174: P_{3576} = (7, 14, 12, 1)
```

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	1	1	1
1	1	0	1	1	0	0
2	1	1	0	1	1	0
3	1	1	1	0	0	1
4	1	0	1	0	0	0
0 1 2 3 4 5	1	0	0	1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_2	P_2	P_2	P_{179}	P_{195}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_2	P_2	P_2	P_{444}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5
in point	P_2	P_2	P_2	P_{461}

Line 4 intersects

Line	ℓ_0	ℓ_2
in point	P_{179}	P_{444}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_{195}	P_{461}

The surface has 305 points: The points on the surface are:

$0: P_1 = (0, 1, 0, 0)$	$37: P_{389} = (3,7,0,1)$	$74: P_{983} = (6, 12, 2, 1)$
$1: P_2 = (0,0,1,0)$	$38: P_{414} = (12, 8, 0, 1)$	75: $P_{986} = (9, 12, 2, 1)$
$2: P_3 = (0,0,0,1)$	$39: P_{438} = (4, 10, 0, 1)$	$76: P_{990} = (13, 12, 2, 1)$
$3: P_{20} = (1,0,1,0)$	$40: P_{444} = (10, 10, 0, 1)$	77: $P_{1038} = (13, 15, 2, 1)$
$4: P_{35} = (0, 1, 1, 0)$	$41: P_{448} = (14, 10, 0, 1)$	$78: P_{1041} = (0,0,3,1)$
$5: P_{45} = (10, 1, 1, 0)$	$42: P_{452} = (2, 11, 0, 1)$	$79: P_{1110} = (5, 4, 3, 1)$
$6: P_{46} = (11, 1, 1, 0)$	$43: P_{459} = (9, 11, 0, 1)$	$80: P_{1114} = (9,4,3,1)$
$7: P_{51} = (0, 2, 1, 0)$	$44: P_{461} = (11, 11, 0, 1)$	$81: P_{1120} = (15, 4, 3, 1)$
$8: P_{67} = (0, 3, 1, 0)$	$45: P_{471} = (5, 12, 0, 1)$	$82: P_{1134} = (13, 5, 3, 1)$
$9: P_{83} = (0, 4, 1, 0)$	$46: P_{497} = (15, 13, 0, 1)$	$83: P_{1146} = (9, 6, 3, 1)$
$10: P_{99} = (0, 5, 1, 0)$	$47: P_{520} = (6, 15, 0, 1)$	$84: P_{1211} = (10, 10, 3, 1)$
$11: P_{115} = (0, 6, 1, 0)$	$48: P_{530} = (0, 0, 1, 1)$	$85: P_{1222} = (5, 11, 3, 1)$
$12: P_{117} = (2, 6, 1, 0)$	$49: P_{540} = (10, 0, 1, 1)$	$86: P_{1228} = (11, 11, 3, 1)$
$13: P_{118} = (3, 6, 1, 0)$	$50: P_{541} = (11, 0, 1, 1)$	$87: P_{1230} = (13, 11, 3, 1)$
$14: P_{131} = (0,7,1,0)$	$51: P_{566} = (5, 2, 1, 1)$	$88: P_{1248} = (15, 12, 3, 1)$
$15: P_{139} = (8,7,1,0)$	$52: P_{601} = (8, 4, 1, 1)$	$89: P_{1259} = (10, 13, 3, 1)$
$16: P_{140} = (9, 7, 1, 0)$	$53: P_{640} = (15, 6, 1, 1)$	$90: P_{1292} = (11, 15, 3, 1)$
$17: P_{147} = (0, 8, 1, 0)$	$54: P_{646} = (5, 7, 1, 1)$	$91: P_{1297} = (0,0,4,1)$
$18: P_{163} = (0, 9, 1, 0)$	$55: P_{688} = (15, 9, 1, 1)$	$92: P_{1306} = (9, 0, 4, 1)$
$19: P_{179} = (0, 10, 1, 0)$	$56: P_{690} = (1, 10, 1, 1)$	93: $P_{1310} = (13, 0, 4, 1)$
$20: P_{191} = (12, 10, 1, 0)$	$57: P_{699} = (10, 10, 1, 1)$	$94: P_{1352} = (7, 3, 4, 1)$
$21: P_{192} = (13, 10, 1, 0)$	$58: P_{706} = (1, 11, 1, 1)$	95: $P_{1400} = (7, 6, 4, 1)$
$22: P_{195} = (0, 11, 1, 0)$	$59: P_{716} = (11, 11, 1, 1)$	$96: P_{1406} = (13, 6, 4, 1)$
$23: P_{201} = (6, 11, 1, 0)$	$60: P_{729} = (8, 12, 1, 1)$	$97: P_{1407} = (14, 6, 4, 1)$
$24: P_{202} = (7, 11, 1, 0)$	$61: P_{740} = (3, 13, 1, 1)$	$98: P_{1426} = (1, 8, 4, 1)$
$25: P_{211} = (0, 12, 1, 0)$	$62: P_{756} = (3, 14, 1, 1)$	$99: P_{1451} = (10, 9, 4, 1)$
$26: P_{225} = (14, 12, 1, 0)$	$63: P_{785} = (0, 0, 2, 1)$	$100: P_{1467} = (10, 10, 4, 1)$
$27: P_{226} = (15, 12, 1, 0)$	$64: P_{789} = (4, 0, 2, 1)$	$101: P_{1484} = (11, 11, 4, 1)$
$28: P_{227} = (0, 13, 1, 0)$	$65: P_{791} = (6, 0, 2, 1)$	$102: P_{1490} = (1, 12, 4, 1)$
$29: P_{231} = (4, 13, 1, 0)$	$66: P_{860} = (11, 4, 2, 1)$	$103: P_{1500} = (11, 12, 4, 1)$
$30: P_{232} = (5, 13, 1, 0)$	$67: P_{866} = (1, 5, 2, 1)$	$104: P_{1503} = (14, 12, 4, 1)$
$31: P_{243} = (0, 14, 1, 0)$	$68: P_{898} = (1, 7, 2, 1)$	$105: P_{1546} = (9, 15, 4, 1)$
$32: P_{259} = (0, 15, 1, 0)$	$69: P_{906} = (9, 7, 2, 1)$	$106: P_{1553} = (0, 0, 5, 1)$
$33: P_{275} = (1,0,0,1)$	$70: P_{907} = (10, 7, 2, 1)$	$107: P_{1611} = (10, 3, 5, 1)$
$34: P_{335} = (13, 3, 0, 1)$	$71: P_{917} = (4, 8, 2, 1)$	$108: P_{1652} = (3, 6, 5, 1)$
$35: P_{361} = (7, 5, 0, 1)$	$72: P_{955} = (10, 10, 2, 1)$	$109: P_{1676} = (11, 7, 5, 1)$
$36: P_{378} = (8, 6, 0, 1)$	$73: P_{972} = (11, 11, 2, 1)$	$110: P_{1688} = (7, 8, 5, 1)$

```
111: P_{1700} = (3, 9, 5, 1)
                                           165: P_{2508} = (11, 11, 8, 1)
                                                                                       219: P_{3149} = (12, 3, 11, 1)
112: P_{1705} = (8, 9, 5, 1)
                                           166: P_{2509} = (12, 11, 8, 1)
                                                                                       220: P_{3156} = (3, 4, 11, 1)
                                                                                       221: P_{3184} = (15, 5, 11, 1)
113: P_{1711} = (14, 9, 5, 1)
                                           167: P_{2512} = (15, 11, 8, 1)
                                                                                       222: P_{3226} = (9, 8, 11, 1)
114: P_{1720} = (7, 10, 5, 1)
                                           168: P_{2523} = (10, 12, 8, 1)
115: P_{1721} = (8, 10, 5, 1)
                                           169: P_{2534} = (5, 13, 8, 1)
                                                                                      223: P_{3230} = (13, 8, 11, 1)
116: P_{1723} = (10, 10, 5, 1)
                                           170: P_{2547} = (2, 14, 8, 1)
                                                                                       224: P_{3232} = (15, 8, 11, 1)
117: P_{1740} = (11, 11, 5, 1)
                                           171: P_{2550} = (5, 14, 8, 1)
                                                                                       225: P_{3234} = (1, 9, 11, 1)
118: P_{1775} = (14, 13, 5, 1)
                                           172: P_{2560} = (15, 14, 8, 1)
                                                                                       226: P_{3259} = (10, 10, 11, 1)
119: P_{1809} = (0, 0, 6, 1)
                                           173: P_{2573} = (12, 15, 8, 1)
                                                                                      227: P_{3261} = (12, 10, 11, 1)
120: P_{1828} = (3, 1, 6, 1)
                                           174: P_{2577} = (0,0,9,1)
                                                                                       228: P_{3262} = (13, 10, 11, 1)
121: P_{1834} = (9, 1, 6, 1)
                                           175: P_{2584} = (7,0,9,1)
                                                                                       229: P_{3275} = (10, 11, 11, 1)
122: P_{1837} = (12, 1, 6, 1)
                                           176: P_{2591} = (14, 0, 9, 1)
                                                                                       230: P_{3276} = (11, 11, 11, 1)
123: P_{1848} = (7, 2, 6, 1)
                                           177: P_{2639} = (14, 3, 9, 1)
                                                                                       231: P_{3283} = (2, 12, 11, 1)
124: P_{1853} = (12, 2, 6, 1)
                                           178: P_{2669} = (12, 5, 9, 1)
                                                                                       232: P_{3306} = (9, 13, 11, 1)
                                                                                       233: P_{3321} = (8, 14, 11, 1)
125: P_{1854} = (13, 2, 6, 1)
                                           179: P_{2674} = (1, 6, 9, 1)
126: P_{1868} = (11, 3, 6, 1)
                                           180: P_{2675} = (2, 6, 9, 1)
                                                                                       234: P_{3334} = (5, 15, 11, 1)
                                           181: P_{2683} = (10, 6, 9, 1)
                                                                                       235: P_{3345} = (0, 0, 12, 1)
127: P_{1875} = (2,4,6,1)
                                           182: P_{2747} = (10, 10, 9, 1)
128: P_{1883} = (10, 4, 6, 1)
                                                                                       236: P_{3365} = (4, 1, 12, 1)
129: P_{1887} = (14, 4, 6, 1)
                                                                                       237: P_{3368} = (7, 1, 12, 1)
                                           183: P_{2764} = (11, 11, 9, 1)
130: P_{1898} = (9, 5, 6, 1)
                                           184: P_{2787} = (2, 13, 9, 1)
                                                                                       238: P_{3376} = (15, 1, 12, 1)
131: P_{1934} = (13, 7, 6, 1)
                                           185: P_{2792} = (7, 13, 9, 1)
                                                                                       239: P_{3386} = (9, 2, 12, 1)
132: P_{1939} = (2, 8, 6, 1)
                                           186: P_{2797} = (12, 13, 9, 1)
                                                                                       240: P_{3388} = (11, 2, 12, 1)
                                                                                       241: P_{3391} = (14, 2, 12, 1)
133: P_{1960} = (7, 9, 6, 1)
                                           187: P_{2812} = (11, 14, 9, 1)
134: P_{1979} = (10, 10, 6, 1)
                                           188: P_{2818} = (1, 15, 9, 1)
                                                                                      242: P_{3397} = (4, 3, 12, 1)
135: P_{1988} = (3, 11, 6, 1)
                                           189: P_{2833} = (0, 0, 10, 1)
                                                                                       243: P_{3422} = (13, 4, 12, 1)
136: P_{1996} = (11, 11, 6, 1)
                                           190: P_{2838} = (5, 0, 10, 1)
                                                                                       244: P_{3439} = (14, 5, 12, 1)
                                                                                       245: P_{3446} = (5, 6, 12, 1)
137: P_{1999} = (14, 11, 6, 1)
                                           191: P_{2848} = (15, 0, 10, 1)
138: P_{2025} = (8, 13, 6, 1)
                                           192: P_{2859} = (10, 1, 10, 1)
                                                                                      246: P_{3478} = (5, 8, 12, 1)
                                                                                       247: P_{3514} = (9, 10, 12, 1)
139: P_{2057} = (8, 15, 6, 1)
                                           193: P_{2880} = (15, 2, 10, 1)
140: P_{2065} = (0, 0, 7, 1)
                                           194: P_{2889} = (8, 3, 10, 1)
                                                                                       248: P_{3515} = (10, 10, 12, 1)
141: P_{2083} = (2, 1, 7, 1)
                                           195: P_{2898} = (1, 4, 10, 1)
                                                                                       249: P_{3520} = (15, 10, 12, 1)
142: P_{2089} = (8, 1, 7, 1)
                                           196: P_{2917} = (4, 5, 10, 1)
                                                                                       250: P_{3532} = (11, 11, 12, 1)
143: P_{2094} = (13, 1, 7, 1)
                                           197: P_{2919} = (6, 5, 10, 1)
                                                                                       251: P_{3559} = (6, 13, 12, 1)
144: P_{2103} = (6, 2, 7, 1)
                                           198: P_{2921} = (8, 5, 10, 1)
                                                                                       252: P_{3575} = (6, 14, 12, 1)
145: P_{2122} = (9, 3, 7, 1)
                                                                                       253: P_{3576} = (7, 14, 12, 1)
                                           199: P_{2933} = (4, 6, 10, 1)
                                           200: P_{2959} = (14, 7, 10, 1)
                                                                                       254: P_{3582} = (13, 14, 12, 1)
146: P_{2148} = (3, 5, 7, 1)
147: P_{2173} = (12, 6, 7, 1)
                                           201: P_{2964} = (3, 8, 10, 1)
                                                                                       255: P_{3595} = (10, 15, 12, 1)
                                                                                       256: P_{3601} = (0, 0, 13, 1)
148: P_{2204} = (11, 8, 7, 1)
                                           202: P_{2982} = (5, 9, 10, 1)
149: P_{2215} = (6, 9, 7, 1)
                                           203: P_{3003} = (10, 10, 10, 1)
                                                                                      257: P_{3622} = (5, 1, 13, 1)
150: P_{2221} = (12, 9, 7, 1)
                                           204: P_{3004} = (11, 10, 10, 1)
                                                                                       258: P_{3623} = (6, 1, 13, 1)
151: P_{2222} = (13, 9, 7, 1)
                                           205: P_{3015} = (6, 11, 10, 1)
                                                                                       259: P_{3631} = (14, 1, 13, 1)
152: P_{2235} = (10, 10, 7, 1)
                                           206: P_{3016} = (7, 11, 10, 1)
                                                                                       260: P_{3664} = (15, 3, 13, 1)
153: P_{2245} = (4, 11, 7, 1)
                                           207: P_{3020} = (11, 11, 10, 1)
                                                                                      261: P_{3671} = (6, 4, 13, 1)
                                           208: P_{3058} = (1, 14, 10, 1)
                                                                                       262: P_{3672} = (7, 4, 13, 1)
154: P_{2249} = (8, 11, 7, 1)
155: P_{2252} = (11, 11, 7, 1)
                                           209: P_{3076} = (3, 15, 10, 1)
                                                                                       263: P_{3677} = (12, 4, 13, 1)
                                           210: P_{3080} = (7, 15, 10, 1)
                                                                                       264: P_{3691} = (10, 5, 13, 1)
156: P_{2260} = (3, 12, 7, 1)
157: P_{2293} = (4, 14, 7, 1)
                                           211: P_{3087} = (14, 15, 10, 1)
                                                                                       265: P_{3728} = (15, 7, 13, 1)
158: P_{2298} = (9, 14, 7, 1)
                                           212: P_{3089} = (0, 0, 11, 1)
                                                                                       266: P_{3743} = (14, 8, 13, 1)
                                                                                       267: P_{3747} = (2, 9, 13, 1)
159: P_{2299} = (10, 14, 7, 1)
                                           213: P_{3092} = (3, 0, 11, 1)
160: P_{2307} = (2, 15, 7, 1)
                                           214: P_{3097} = (8, 0, 11, 1)
                                                                                       268: P_{3749} = (4, 9, 13, 1)
                                           215: P_{3116} = (11, 1, 11, 1)
161: P_{2321} = (0, 0, 8, 1)
                                                                                       269: P_{3756} = (11, 9, 13, 1)
162: P_{2412} = (11, 5, 8, 1)
                                           216: P_{3122} = (1, 2, 11, 1)
                                                                                       270: P_{3763} = (2, 10, 13, 1)
163: P_{2435} = (2,7,8,1)
                                           217: P_{3139} = (2, 3, 11, 1)
                                                                                       271: P_{3766} = (5, 10, 13, 1)
164: P_{2491} = (10, 10, 8, 1)
                                           218: P_{3142} = (5, 3, 11, 1)
                                                                                       272: P_{3771} = (10, 10, 13, 1)
```

```
295: P_{4153} = (8, 2, 15, 1)
273: P_{3788} = (11, 11, 13, 1)
                                           284: P_{3975} = (6, 7, 14, 1)
                                                                                     296: P_{4167} = (6, 3, 15, 1)
274: P_{3800} = (7, 12, 13, 1)
                                           285: P_{3981} = (12, 7, 14, 1)
275: P_{3837} = (12, 14, 13, 1)
                                           286: P_{3991} = (6, 8, 14, 1)
                                                                                     297: P_{4220} = (11, 6, 15, 1)
276: P_{3845} = (4, 15, 13, 1)
                                           287:\,P_{4027}=(10,10,14,1)
                                                                                     298: P_{4233} = (8, 7, 15, 1)
277: P_{3857} = (0, 0, 14, 1)
                                           288: P_{4044} = (11, 11, 14, 1)
                                                                                     299: P_{4251} = (10, 8, 15, 1)
278: P_{3859} = (2, 0, 14, 1)
                                           289: P_{4066} = (1, 13, 14, 1)
                                                                                     300: P_{4276} = (3, 10, 15, 1)
279: P_{3869} = (12, 0, 14, 1)
                                           290: P_{4069} = (4, 13, 14, 1)
                                                                                     301: P_{4279} = (6, 10, 15, 1)
280: P_{3899} = (10, 2, 14, 1)
                                           291: P_{4076} = (11, 13, 14, 1)
                                                                                     302: P_{4283} = (10, 10, 15, 1)
281: P_{3906} = (1, 3, 14, 1)
                                           292: P_{4113} = (0, 0, 15, 1)
                                                                                     303: P_{4300} = (11, 11, 15, 1)
                                                                                     304: P_{4309} = (4, 12, 15, 1)
282: P_{3939} = (2, 5, 14, 1)
                                           293: P_{4148} = (3, 2, 15, 1)
283: P_{3973} = (4, 7, 14, 1)
                                           294: P_{4149} = (4, 2, 15, 1)
```