Rank-65569 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1107828869

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4096
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	3, 1^{192}, 0^{4096}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \mathbf{Pl}(0,0,1,0,1,0)_{4352}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{4153} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{4153} = \mathbf{Pl}(0,0,56,0,1,0)_{11337}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{array}\right]_{4152} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{array}\right]_{4152} = \mathbf{Pl}(0,0,57,0,1,0)_{11464}$$

Rank of lines: (4097, 4153, 4152)
Rank of points on Klein quadric: (4352, 11337, 11464)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_{8258} = (0,0,1,1)$ lies on line ℓ_0
$1 : P_{8259} = (1,0,1,1)$ lies on line ℓ_0
$2 : P_{8260} = (2,0,1,1)$ lies on line ℓ_0
$3 : P_{8261} = (3,0,1,1)$ lies on line ℓ_0
$4 : P_{8262} = (4,0,1,1)$ lies on line ℓ_0
$5 : P_{8263} = (5,0,1,1)$ lies on line ℓ_0
$6 : P_{8264} = (6,0,1,1)$ lies on line ℓ_0
$7 : P_{8265} = (7,0,1,1)$ lies on line ℓ_0
$8 : P_{8266} = (8,0,1,1)$ lies on line ℓ_0
$9 : P_{8267} = (9,0,1,1)$ lies on line ℓ_0
$10 : P_{8268} = (10,0,1,1)$ lies on line ℓ_0
$11 : P_{8269} = (11,0,1,1)$ lies on line ℓ_0
$12 : P_{8270} = (12,0,1,1)$ lies on line ℓ_0
$13 : P_{8271} = (13,0,1,1)$ lies on line ℓ_0
$14 : P_{8272} = (14,0,1,1)$ lies on line ℓ_0
$15 : P_{8273} = (15,0,1,1)$ lies on line ℓ_0
$16 : P_{8274} = (16,0,1,1)$ lies on line ℓ_0
$17 : P_{8275} = (17,0,1,1)$ lies on line ℓ_0
$18 : P_{8276} = (18,0,1,1)$ lies on line ℓ_0
$19 : P_{8277} = (19,0,1,1)$ lies on line ℓ_0
$20 : P_{8278} = (20,0,1,1)$ lies on line ℓ_0
$21 : P_{8279} = (21,0,1,1)$ lies on line ℓ_0
$22 : P_{8280} = (22,0,1,1)$ lies on line ℓ_0
$23 : P_{8281} = (23,0,1,1)$ lies on line ℓ_0
$24 : P_{8282} = (24,0,1,1)$ lies on line ℓ_0
$25 : P_{8283} = (25,0,1,1)$ lies on line ℓ_0
$26 : P_{8284} = (26,0,1,1)$ lies on line ℓ_0
$27 : P_{8285} = (27,0,1,1)$ lies on line ℓ_0
$28 : P_{8286} = (28,0,1,1)$ lies on line ℓ_0
$29 : P_{8287} = (29,0,1,1)$ lies on line ℓ_0
$30 : P_{8288} = (30,0,1,1)$ lies on line ℓ_0
$31 : P_{8289} = (31,0,1,1)$ lies on line ℓ_0
$32 : P_{8290} = (32,0,1,1)$ lies on line ℓ_0
$33 : P_{8291} = (33,0,1,1)$ lies on line ℓ_0
$34 : P_{8292} = (34,0,1,1)$ lies on line ℓ_0
$35 : P_{8293} = (35,0,1,1)$ lies on line ℓ_0
$36 : P_{8294} = (36,0,1,1)$ lies on line ℓ_0
$37 : P_{8295} = (37,0,1,1)$ lies on line ℓ_0
$38 : P_{8296} = (38,0,1,1)$ lies on line ℓ_0
$39 : P_{8297} = (39,0,1,1)$ lies on line ℓ_0
$40 : P_{8298} = (40,0,1,1)$ lies on line ℓ_0
$41 : P_{8299} = (41,0,1,1)$ lies on line ℓ_0
$42 : P_{8300} = (42,0,1,1)$ lies on line ℓ_0
$43 : P_{8301} = (43,0,1,1)$ lies on line ℓ_0
$44 : P_{8302} = (44,0,1,1)$ lies on line ℓ_0
$45 : P_{8303} = (45,0,1,1)$ lies on line ℓ_0
$46 : P_{8304} = (46,0,1,1)$ lies on line ℓ_0
$47 : P_{8305} = (47,0,1,1)$ lies on line ℓ_0
$48 : P_{8306} = (48,0,1,1)$ lies on line ℓ_0
$49 : P_{8307} = (49,0,1,1)$ lies on line ℓ_0
$50 : P_{8308} = (50,0,1,1)$ lies on line ℓ_0
$51 : P_{8309} = (51,0,1,1)$ lies on line ℓ_0
$52 : P_{8310} = (52,0,1,1)$ lies on line ℓ_0
$53 : P_{8311} = (53,0,1,1)$ lies on line ℓ_0
$54 : P_{8312} = (54,0,1,1)$ lies on line ℓ_0
$55 : P_{8313} = (55,0,1,1)$ lies on line ℓ_0
$56 : P_{8314} = (56,0,1,1)$ lies on line ℓ_0
$57 : P_{8315} = (57,0,1,1)$ lies on line ℓ_0

58 : $P_{8316} = (58, 0, 1, 1)$ lies on line ℓ_0
59 : $P_{8317} = (59, 0, 1, 1)$ lies on line ℓ_0
60 : $P_{8318} = (60, 0, 1, 1)$ lies on line ℓ_0
61 : $P_{8319} = (61, 0, 1, 1)$ lies on line ℓ_0
62 : $P_{8320} = (62, 0, 1, 1)$ lies on line ℓ_0
63 : $P_{8321} = (63, 0, 1, 1)$ lies on line ℓ_0
64 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_1
65 : $P_{233538} = (1, 0, 56, 1)$ lies on line ℓ_1
66 : $P_{233539} = (2, 0, 56, 1)$ lies on line ℓ_1
67 : $P_{233540} = (3, 0, 56, 1)$ lies on line ℓ_1
68 : $P_{233541} = (4, 0, 56, 1)$ lies on line ℓ_1
69 : $P_{233542} = (5, 0, 56, 1)$ lies on line ℓ_1
70 : $P_{233543} = (6, 0, 56, 1)$ lies on line ℓ_1
71 : $P_{233544} = (7, 0, 56, 1)$ lies on line ℓ_1
72 : $P_{233545} = (8, 0, 56, 1)$ lies on line ℓ_1
73 : $P_{233546} = (9, 0, 56, 1)$ lies on line ℓ_1
74 : $P_{233547} = (10, 0, 56, 1)$ lies on line ℓ_1
75 : $P_{233548} = (11, 0, 56, 1)$ lies on line ℓ_1
76 : $P_{233549} = (12, 0, 56, 1)$ lies on line ℓ_1
77 : $P_{233550} = (13, 0, 56, 1)$ lies on line ℓ_1
78 : $P_{233551} = (14, 0, 56, 1)$ lies on line ℓ_1
79 : $P_{233552} = (15, 0, 56, 1)$ lies on line ℓ_1
80 : $P_{233553} = (16, 0, 56, 1)$ lies on line ℓ_1
81 : $P_{233554} = (17, 0, 56, 1)$ lies on line ℓ_1
82 : $P_{233555} = (18, 0, 56, 1)$ lies on line ℓ_1
83 : $P_{233556} = (19, 0, 56, 1)$ lies on line ℓ_1
84 : $P_{233557} = (20, 0, 56, 1)$ lies on line ℓ_1
85 : $P_{233558} = (21, 0, 56, 1)$ lies on line ℓ_1
86 : $P_{233559} = (22, 0, 56, 1)$ lies on line ℓ_1
87 : $P_{233560} = (23, 0, 56, 1)$ lies on line ℓ_1
88 : $P_{233561} = (24, 0, 56, 1)$ lies on line ℓ_1
89 : $P_{233562} = (25, 0, 56, 1)$ lies on line ℓ_1
90 : $P_{233563} = (26, 0, 56, 1)$ lies on line ℓ_1
91 : $P_{233564} = (27, 0, 56, 1)$ lies on line ℓ_1
92 : $P_{233565} = (28, 0, 56, 1)$ lies on line ℓ_1
93 : $P_{233566} = (29, 0, 56, 1)$ lies on line ℓ_1
94 : $P_{233567} = (30, 0, 56, 1)$ lies on line ℓ_1
95 : $P_{233568} = (31, 0, 56, 1)$ lies on line ℓ_1
96 : $P_{233569} = (32, 0, 56, 1)$ lies on line ℓ_1
97 : $P_{233570} = (33, 0, 56, 1)$ lies on line ℓ_1
98 : $P_{233571} = (34, 0, 56, 1)$ lies on line ℓ_1
99 : $P_{233572} = (35, 0, 56, 1)$ lies on line ℓ_1
100 : $P_{233573} = (36, 0, 56, 1)$ lies on line ℓ_1
101 : $P_{233574} = (37, 0, 56, 1)$ lies on line ℓ_1
102 : $P_{233575} = (38, 0, 56, 1)$ lies on line ℓ_1
103 : $P_{233576} = (39, 0, 56, 1)$ lies on line ℓ_1
104 : $P_{233577} = (40, 0, 56, 1)$ lies on line ℓ_1
105 : $P_{233578} = (41, 0, 56, 1)$ lies on line ℓ_1
106 : $P_{233579} = (42, 0, 56, 1)$ lies on line ℓ_1
107 : $P_{233580} = (43, 0, 56, 1)$ lies on line ℓ_1
108 : $P_{233581} = (44, 0, 56, 1)$ lies on line ℓ_1
109 : $P_{233582} = (45, 0, 56, 1)$ lies on line ℓ_1
110 : $P_{233583} = (46, 0, 56, 1)$ lies on line ℓ_1
111 : $P_{233584} = (47, 0, 56, 1)$ lies on line ℓ_1
112 : $P_{233585} = (48, 0, 56, 1)$ lies on line ℓ_1
113 : $P_{233586} = (49, 0, 56, 1)$ lies on line ℓ_1
114 : $P_{233587} = (50, 0, 56, 1)$ lies on line ℓ_1
115 : $P_{233588} = (51, 0, 56, 1)$ lies on line ℓ_1
116 : $P_{233589} = (52, 0, 56, 1)$ lies on line ℓ_1
117 : $P_{233590} = (53, 0, 56, 1)$ lies on line ℓ_1
118 : $P_{233591} = (54, 0, 56, 1)$ lies on line ℓ_1
119 : $P_{233592} = (55, 0, 56, 1)$ lies on line ℓ_1
120 : $P_{233593} = (56, 0, 56, 1)$ lies on line ℓ_1
121 : $P_{233594} = (57, 0, 56, 1)$ lies on line ℓ_1
122 : $P_{233595} = (58, 0, 56, 1)$ lies on line ℓ_1
123 : $P_{233596} = (59, 0, 56, 1)$ lies on line ℓ_1
124 : $P_{233597} = (60, 0, 56, 1)$ lies on line ℓ_1
125 : $P_{233598} = (61, 0, 56, 1)$ lies on line ℓ_1
126 : $P_{233599} = (62, 0, 56, 1)$ lies on line ℓ_1
127 : $P_{233600} = (63, 0, 56, 1)$ lies on line ℓ_1
128 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_2
129 : $P_{237634} = (1, 0, 57, 1)$ lies on line ℓ_2
130 : $P_{237635} = (2, 0, 57, 1)$ lies on line ℓ_2
131 : $P_{237636} = (3, 0, 57, 1)$ lies on line ℓ_2
132 : $P_{237637} = (4, 0, 57, 1)$ lies on line ℓ_2
133 : $P_{237638} = (5, 0, 57, 1)$ lies on line ℓ_2
134 : $P_{237639} = (6, 0, 57, 1)$ lies on line ℓ_2
135 : $P_{237640} = (7, 0, 57, 1)$ lies on line ℓ_2
136 : $P_{237641} = (8, 0, 57, 1)$ lies on line ℓ_2
137 : $P_{237642} = (9, 0, 57, 1)$ lies on line ℓ_2
138 : $P_{237643} = (10, 0, 57, 1)$ lies on line ℓ_2
139 : $P_{237644} = (11, 0, 57, 1)$ lies on line ℓ_2
140 : $P_{237645} = (12, 0, 57, 1)$ lies on line ℓ_2
141 : $P_{237646} = (13, 0, 57, 1)$ lies on line ℓ_2
142 : $P_{237647} = (14, 0, 57, 1)$ lies on line ℓ_2
143 : $P_{237648} = (15, 0, 57, 1)$ lies on line ℓ_2
144 : $P_{237649} = (16, 0, 57, 1)$ lies on line ℓ_2
145 : $P_{237650} = (17, 0, 57, 1)$ lies on line ℓ_2
146 : $P_{237651} = (18, 0, 57, 1)$ lies on line ℓ_2
147 : $P_{237652} = (19, 0, 57, 1)$ lies on line ℓ_2
148 : $P_{237653} = (20, 0, 57, 1)$ lies on line ℓ_2
149 : $P_{237654} = (21, 0, 57, 1)$ lies on line ℓ_2
150 : $P_{237655} = (22, 0, 57, 1)$ lies on line ℓ_2
151 : $P_{237656} = (23, 0, 57, 1)$ lies on line ℓ_2
152 : $P_{237657} = (24, 0, 57, 1)$ lies on line ℓ_2
153 : $P_{237658} = (25, 0, 57, 1)$ lies on line ℓ_2
154 : $P_{237659} = (26, 0, 57, 1)$ lies on line ℓ_2
155 : $P_{237660} = (27, 0, 57, 1)$ lies on line ℓ_2
156 : $P_{237661} = (28, 0, 57, 1)$ lies on line ℓ_2
157 : $P_{237662} = (29, 0, 57, 1)$ lies on line ℓ_2
158 : $P_{237663} = (30, 0, 57, 1)$ lies on line ℓ_2
159 : $P_{237664} = (31, 0, 57, 1)$ lies on line ℓ_2
160 : $P_{237665} = (32, 0, 57, 1)$ lies on line ℓ_2
161 : $P_{237666} = (33, 0, 57, 1)$ lies on line ℓ_2
162 : $P_{237667} = (34, 0, 57, 1)$ lies on line ℓ_2
163 : $P_{237668} = (35, 0, 57, 1)$ lies on line ℓ_2
164 : $P_{237669} = (36, 0, 57, 1)$ lies on line ℓ_2
165 : $P_{237670} = (37, 0, 57, 1)$ lies on line ℓ_2

166 : $P_{237671} = (38, 0, 57, 1)$ lies on line ℓ_2
167 : $P_{237672} = (39, 0, 57, 1)$ lies on line ℓ_2
168 : $P_{237673} = (40, 0, 57, 1)$ lies on line ℓ_2
169 : $P_{237674} = (41, 0, 57, 1)$ lies on line ℓ_2
170 : $P_{237675} = (42, 0, 57, 1)$ lies on line ℓ_2
171 : $P_{237676} = (43, 0, 57, 1)$ lies on line ℓ_2
172 : $P_{237677} = (44, 0, 57, 1)$ lies on line ℓ_2
173 : $P_{237678} = (45, 0, 57, 1)$ lies on line ℓ_2
174 : $P_{237679} = (46, 0, 57, 1)$ lies on line ℓ_2
175 : $P_{237680} = (47, 0, 57, 1)$ lies on line ℓ_2
176 : $P_{237681} = (48, 0, 57, 1)$ lies on line ℓ_2
177 : $P_{237682} = (49, 0, 57, 1)$ lies on line ℓ_2
178 : $P_{237683} = (50, 0, 57, 1)$ lies on line ℓ_2
179 : $P_{237684} = (51, 0, 57, 1)$ lies on line ℓ_2
180 : $P_{237685} = (52, 0, 57, 1)$ lies on line ℓ_2
181 : $P_{237686} = (53, 0, 57, 1)$ lies on line ℓ_2
182 : $P_{237687} = (54, 0, 57, 1)$ lies on line ℓ_2
183 : $P_{237688} = (55, 0, 57, 1)$ lies on line ℓ_2
184 : $P_{237689} = (56, 0, 57, 1)$ lies on line ℓ_2
185 : $P_{237690} = (57, 0, 57, 1)$ lies on line ℓ_2
186 : $P_{237691} = (58, 0, 57, 1)$ lies on line ℓ_2
187 : $P_{237692} = (59, 0, 57, 1)$ lies on line ℓ_2
188 : $P_{237693} = (60, 0, 57, 1)$ lies on line ℓ_2
189 : $P_{237694} = (61, 0, 57, 1)$ lies on line ℓ_2
190 : $P_{237695} = (62, 0, 57, 1)$ lies on line ℓ_2
191 : $P_{237696} = (63, 0, 57, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4096 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_0

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_0	P_0

The surface has 4289 points:
Too many to print.