

# Rank-65687 over GF(2)

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## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(2) is 65687

## General information

Number of lines	4
Number of points	9
Number of singular points	3
Number of Eckardt points	0
Number of double points	0
Number of single points	8
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$3^4$
Type of lines on points	4, $1^8$

## Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_3 &= \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1) \\ 1 : P_4 &= \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1) \end{aligned}$$

$$2 : P_8 = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_9$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{15} \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}\end{aligned}$$

Rank of lines: ( 6, 30, 13, 27 )

Rank of points on Klein quadric: ( 9, 5, 15, 17 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 8 single points:

The single points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$  lies on line  $\ell_0$

1 :  $P_1 = (0, 1, 0, 0)$  lies on line  $\ell_1$

2 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_3$

3 :  $P_5 = (1, 1, 0, 0)$  lies on line  $\ell_2$

4 :  $P_8 = (1, 1, 1, 0)$  lies on line  $\ell_3$

5 :  $P_9 = (1, 0, 0, 1)$  lies on line  $\ell_0$

6 :  $P_{10} = (0, 1, 0, 1)$  lies on line  $\ell_1$

7 :  $P_{11} = (1, 1, 0, 1)$  lies on line  $\ell_2$

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

### Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	1	0	1
2	1	0	1	1
3	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$
in point	$P_3$	$P_3$	$P_3$

The surface has 9 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_5 = (1, 1, 0, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_9 = (1, 0, 0, 1)$$

$$7 : P_{10} = (0, 1, 0, 1)$$

$$8 : P_{11} = (1, 1, 0, 1)$$