

Rank-65613 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431659933

General information

Number of lines	11
Number of points	33
Number of singular points	2
Number of Eckardt points	0
Number of double points	15
Number of single points	16
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{11}
Type of lines on points	$5, 4, 2^{15}, 1^{16}$

Singular Points

The surface has 2 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 11 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_6 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{105} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0, 3, 0, 0, 0, 1)_{107} \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0, 2, 0, 0, 0, 1)_{106} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{177} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{75} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{75} = \mathbf{Pl}(2, 3, 0, 0, 1, 1)_{178} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{50} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{50} = \mathbf{Pl}(3, 2, 0, 0, 1, 1)_{179} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{74} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{95} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{95} = \mathbf{Pl}(3, 2, 2, 3, 1, 0)_{97} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{98} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{98} = \mathbf{Pl}(2, 3, 3, 2, 1, 0)_{90}
\end{aligned}$$

Rank of lines: (16, 100, 341, 351, 346, 25, 75, 50, 89, 95, 98)

Rank of points on Klein quadric: (2, 6, 105, 107, 106, 177, 178, 179, 74, 97, 90)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 15 Double points:

The double points on the surface are:

$$\begin{aligned}
P_8 &= (1, 0, 1, 0) = \ell_0 \cap \ell_5 & P_{65} &= (0, 3, 2, 1) = \ell_4 \cap \ell_9 \\
P_9 &= (2, 0, 1, 0) = \ell_0 \cap \ell_6 & P_{76} &= (3, 1, 3, 1) = \ell_5 \cap \ell_9 \\
P_{10} &= (3, 0, 1, 0) = \ell_0 \cap \ell_7 & P_{59} &= (2, 1, 2, 1) = \ell_5 \cap \ell_{10} \\
P_{26} &= (0, 1, 0, 1) = \ell_2 \cap \ell_5 & P_{64} &= (3, 2, 2, 1) = \ell_6 \cap \ell_8 \\
P_{42} &= (0, 1, 1, 1) = \ell_2 \cap \ell_8 & P_{47} &= (2, 2, 1, 1) = \ell_6 \cap \ell_9 \\
P_{30} &= (0, 2, 0, 1) = \ell_3 \cap \ell_6 & P_{83} &= (2, 3, 3, 1) = \ell_7 \cap \ell_8 \\
P_{77} &= (0, 2, 3, 1) = \ell_3 \cap \ell_{10} & P_{52} &= (3, 3, 1, 1) = \ell_7 \cap \ell_{10} \\
P_{34} &= (0, 3, 0, 1) = \ell_4 \cap \ell_7
\end{aligned}$$

Single Points

The surface has 16 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
 1 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_5
 2 : $P_{12} = (1, 1, 1, 0)$ lies on line ℓ_8
 3 : $P_{18} = (3, 2, 1, 0)$ lies on line ℓ_9
 4 : $P_{21} = (2, 3, 1, 0)$ lies on line ℓ_{10}
 5 : $P_{39} = (1, 0, 1, 1)$ lies on line ℓ_1
 6 : $P_{45} = (0, 2, 1, 1)$ lies on line ℓ_3
 7 : $P_{49} = (0, 3, 1, 1)$ lies on line ℓ_4
 8 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_1

9 : $P_{57} = (0, 1, 2, 1)$ lies on line ℓ_2
 10 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_3
 11 : $P_{66} = (1, 3, 2, 1)$ lies on line ℓ_7
 12 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_1
 13 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_2
 14 : $P_{78} = (1, 2, 3, 1)$ lies on line ℓ_6
 15 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10
0	0	1	1	1	1	1	1	1	0	0	0
1	1	0	1	1	1	0	0	0	1	1	1
2	1	1	0	1	1	1	0	0	1	0	0
3	1	1	1	0	1	0	1	0	0	0	1
4	1	1	1	1	0	0	0	1	0	1	0
5	1	0	1	0	0	0	0	0	0	1	1
6	1	0	0	1	0	0	0	0	1	1	0
7	1	0	0	0	1	0	0	0	1	0	1
8	0	1	1	0	0	0	1	1	0	1	1
9	0	1	0	0	1	1	1	0	1	0	1
10	0	1	0	1	0	1	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_2	P_2	P_2	P_2	P_8	P_9	P_{10}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_2	P_2	P_2	P_{23}	P_{23}	P_{23}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{26}	P_{42}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_6	ℓ_{10}
in point	P_2	P_2	P_2	P_2	P_{30}	P_{77}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_7	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{34}	P_{65}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_9	ℓ_{10}
in point	P_8	P_{26}	P_{76}	P_{59}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_8	ℓ_9
in point	P_9	P_{30}	P_{64}	P_{47}

Line 7 intersects

Line	ℓ_0	ℓ_4	ℓ_8	ℓ_{10}
in point	P_{10}	P_{34}	P_{83}	P_{52}

Line 8 intersects

Line	ℓ_1	ℓ_2	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}
in point	P_{23}	P_{42}	P_{64}	P_{83}	P_{23}	P_{23}

Line 9 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}
in point	P_{23}	P_{65}	P_{76}	P_{47}	P_{23}	P_{23}

Line 10 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_9
in point	P_{23}	P_{77}	P_{59}	P_{52}	P_{23}	P_{23}

The surface has 33 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	12 : $P_{34} = (0, 3, 0, 1)$	24 : $P_{65} = (0, 3, 2, 1)$
1 : $P_2 = (0, 0, 1, 0)$	13 : $P_{39} = (1, 0, 1, 1)$	25 : $P_{66} = (1, 3, 2, 1)$
2 : $P_4 = (1, 1, 1, 1)$	14 : $P_{42} = (0, 1, 1, 1)$	26 : $P_{70} = (1, 0, 3, 1)$
3 : $P_8 = (1, 0, 1, 0)$	15 : $P_{45} = (0, 2, 1, 1)$	27 : $P_{73} = (0, 1, 3, 1)$
4 : $P_9 = (2, 0, 1, 0)$	16 : $P_{47} = (2, 2, 1, 1)$	28 : $P_{76} = (3, 1, 3, 1)$
5 : $P_{10} = (3, 0, 1, 0)$	17 : $P_{49} = (0, 3, 1, 1)$	29 : $P_{77} = (0, 2, 3, 1)$
6 : $P_{12} = (1, 1, 1, 0)$	18 : $P_{52} = (3, 3, 1, 1)$	30 : $P_{78} = (1, 2, 3, 1)$
7 : $P_{18} = (3, 2, 1, 0)$	19 : $P_{54} = (1, 0, 2, 1)$	31 : $P_{81} = (0, 3, 3, 1)$
8 : $P_{21} = (2, 3, 1, 0)$	20 : $P_{57} = (0, 1, 2, 1)$	32 : $P_{83} = (2, 3, 3, 1)$
9 : $P_{23} = (1, 0, 0, 1)$	21 : $P_{59} = (2, 1, 2, 1)$	
10 : $P_{26} = (0, 1, 0, 1)$	22 : $P_{61} = (0, 2, 2, 1)$	
11 : $P_{30} = (0, 2, 0, 1)$	23 : $P_{64} = (3, 2, 2, 1)$	