Rank-65547 over GF(2)

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The equation

The equation of the surface is :
$$X_3^3 + X_0 X_1 X_2 = 0$$
(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(2)$ is 65547

General information

Number of lines	3
Number of points	7
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	3
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	2^3, 1^3, 0

Singular Points

The surface has 3 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$ 2 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

1 : $P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

Rank of lines: (0, 4, 28)
Rank of points on Klein quadric: (0, 2, 19)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_2 = (0,0,1,0) = \ell_1 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 3 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_0 $2 : P_7 = (0,1,1,0)$ lies on line ℓ_2
$1 : P_6 = (1,0,1,0)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1,1,1,1)$

Line Intersection Graph

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_2

2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_2

The surface has 7 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$

$1 : P_1 = (0, 1, 0, 0)$

$2 : P_2 = (0, 0, 1, 0)$

$3 : P_4 = (1, 1, 1, 1)$

$4 : P_5 = (1, 1, 0, 0)$

$5 : P_6 = (1, 0, 1, 0)$

$6 : P_7 = (0, 1, 1, 0)$