Rank-74247 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361351309

General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	57
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	2^3, 1^{21}, 0^{57}

Singular Points

The surface has 1 singular points:

$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1,0,1,1,1,1)_{1323}$$

Rank of lines: (0, 9, 666)
Rank of points on Klein quadric: (0, 97, 1323)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $\qquad\qquad\qquad P_{146} = (0,1,1,1) = \ell_1 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0 $\qquad$ 11 : $P_{149} = (4,1,1,1)$ lies on line ℓ_1
1 : $P_4 = (1,1,1,1)$ lies on line ℓ_1 $\qquad$ 12 : $P_{150} = (5,1,1,1)$ lies on line ℓ_1
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_0 $\qquad$ 13 : $P_{151} = (6,1,1,1)$ lies on line ℓ_1
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_0 $\qquad$ 14 : $P_{152} = (7,1,1,1)$ lies on line ℓ_1
4 : $P_8 = (4,1,0,0)$ lies on line ℓ_0 $\qquad$ 15 : $P_{156} = (3,2,1,1)$ lies on line ℓ_2
5 : $P_9 = (5,1,0,0)$ lies on line ℓ_0 $\qquad$ 16 : $P_{163} = (2,3,1,1)$ lies on line ℓ_2
6 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0 $\qquad$ 17 : $P_{174} = (5,4,1,1)$ lies on line ℓ_2
7 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0 $\qquad$ 18 : $P_{181} = (4,5,1,1)$ lies on line ℓ_2
8 : $P_{139} = (1,0,1,1)$ lies on line ℓ_2 $\qquad$ 19 : $P_{192} = (7,6,1,1)$ lies on line ℓ_2
9 : $P_{147} = (2,1,1,1)$ lies on line ℓ_1 $\qquad$ 20 : $P_{199} = (6,7,1,1)$ lies on line ℓ_2
10 : $P_{148} = (3,1,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 57 points not on any line:
The points on the surface but not on lines are:

0 : $P_3 = (0,0,0,1)$ $\qquad\qquad\qquad$ 5 : $P_{58} = (7,5,1,0)$
1 : $P_{20} = (1,1,1,0)$ $\qquad\qquad\qquad$ 6 : $P_{61} = (2,6,1,0)$
2 : $P_{33} = (6,2,1,0)$ $\qquad\qquad\qquad$ 7 : $P_{72} = (5,7,1,0)$
3 : $P_{39} = (4,3,1,0)$ $\qquad\qquad\qquad$ 8 : $P_{83} = (1,1,0,1)$
4 : $P_{46} = (3,4,1,0)$ $\qquad\qquad\qquad$ 9 : $P_{94} = (4,2,0,1)$

10 : $P_{103} = (5, 3, 0, 1)$
11 : $P_{113} = (7, 4, 0, 1)$
12 : $P_{120} = (6, 5, 0, 1)$
13 : $P_{125} = (3, 6, 0, 1)$
14 : $P_{132} = (2, 7, 0, 1)$
15 : $P_{206} = (5, 0, 2, 1)$
16 : $P_{216} = (7, 1, 2, 1)$
17 : $P_{224} = (7, 2, 2, 1)$
18 : $P_{225} = (0, 3, 2, 1)$
19 : $P_{239} = (6, 4, 2, 1)$
20 : $P_{247} = (6, 5, 2, 1)$
21 : $P_{262} = (5, 7, 2, 1)$
22 : $P_{267} = (2, 0, 3, 1)$
23 : $P_{280} = (7, 1, 3, 1)$
24 : $P_{285} = (4, 2, 3, 1)$
25 : $P_{293} = (4, 3, 3, 1)$
26 : $P_{312} = (7, 5, 3, 1)$
27 : $P_{315} = (2, 6, 3, 1)$
28 : $P_{321} = (0, 7, 3, 1)$
29 : $P_{335} = (6, 0, 4, 1)$
30 : $P_{339} = (2, 1, 4, 1)$
31 : $P_{351} = (6, 2, 4, 1)$
32 : $P_{363} = (2, 4, 4, 1)$
33 : $P_{369} = (0, 5, 4, 1)$

34 : $P_{380} = (3, 6, 4, 1)$
35 : $P_{388} = (3, 7, 4, 1)$
36 : $P_{397} = (4, 0, 5, 1)$
37 : $P_{403} = (2, 1, 5, 1)$
38 : $P_{409} = (0, 2, 5, 1)$
39 : $P_{421} = (4, 3, 5, 1)$
40 : $P_{432} = (7, 4, 5, 1)$
41 : $P_{440} = (7, 5, 5, 1)$
42 : $P_{443} = (2, 6, 5, 1)$
43 : $P_{464} = (7, 0, 6, 1)$
44 : $P_{469} = (4, 1, 6, 1)$
45 : $P_{485} = (4, 3, 6, 1)$
46 : $P_{489} = (0, 4, 6, 1)$
47 : $P_{504} = (7, 5, 6, 1)$
48 : $P_{507} = (2, 6, 6, 1)$
49 : $P_{515} = (2, 7, 6, 1)$
50 : $P_{524} = (3, 0, 7, 1)$
51 : $P_{533} = (4, 1, 7, 1)$
52 : $P_{542} = (5, 2, 7, 1)$
53 : $P_{550} = (5, 3, 7, 1)$
54 : $P_{556} = (3, 4, 7, 1)$
55 : $P_{569} = (0, 6, 7, 1)$
56 : $P_{581} = (4, 7, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_{146}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_5	P_{146}

The surface has 81 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_5 = (1, 1, 0, 0)$
5 : $P_6 = (2, 1, 0, 0)$
6 : $P_7 = (3, 1, 0, 0)$
7 : $P_8 = (4, 1, 0, 0)$

8 : $P_9 = (5, 1, 0, 0)$
9 : $P_{10} = (6, 1, 0, 0)$
10 : $P_{11} = (7, 1, 0, 0)$
11 : $P_{20} = (1, 1, 1, 0)$
12 : $P_{33} = (6, 2, 1, 0)$
13 : $P_{39} = (4, 3, 1, 0)$
14 : $P_{46} = (3, 4, 1, 0)$
15 : $P_{58} = (7, 5, 1, 0)$

16 : $P_{61} = (2, 6, 1, 0)$
17 : $P_{72} = (5, 7, 1, 0)$
18 : $P_{83} = (1, 1, 0, 1)$
19 : $P_{94} = (4, 2, 0, 1)$
20 : $P_{103} = (5, 3, 0, 1)$
21 : $P_{113} = (7, 4, 0, 1)$
22 : $P_{120} = (6, 5, 0, 1)$
23 : $P_{125} = (3, 6, 0, 1)$

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$28: P_{148} = (3, 1, 1, 1)$

$29: P_{149} = (4, 1, 1, 1)$

$30: P_{150} = (5, 1, 1, 1)$

$31: P_{151} = (6, 1, 1, 1)$

$32: P_{152} = (7, 1, 1, 1)$

$33: P_{156} = (3, 2, 1, 1)$

$34: P_{163} = (2, 3, 1, 1)$

$35: P_{174} = (5, 4, 1, 1)$

$36: P_{181} = (4, 5, 1, 1)$

$37: P_{192} = (7, 6, 1, 1)$

$38: P_{199} = (6, 7, 1, 1)$

$39: P_{206} = (5, 0, 2, 1)$

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$80: P_{581} = (4, 7, 7, 1)$