Rank-74053 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498834269

General information

Number of lines	10
Number of points	33
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	21
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{10}
Type of lines on points	5, 3^2, 2^9, 1^{21}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{289} = \mathbf{Pl}(0,3,1,0,0,1)_{114}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{205} = \mathbf{Pl}(0,2,1,0,0,1)_{113}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{76} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{76} = \mathbf{Pl}(2,3,2,0,1,1)_{184}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{51} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{51} = \mathbf{Pl}(3,2,3,0,1,1)_{188}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{160} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{160} = \mathbf{Pl}(1,1,3,2,1,1)_{234}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{135} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{135} = \mathbf{Pl}(1,1,2,3,1,1)_{225}$$

Rank of lines: (16, 356, 100, 289, 205, 76, 51, 345, 160, 135)
Rank of points on Klein quadric: (2, 1, 6, 114, 113, 184, 188, 13, 234, 225)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{61} = \mathbf{P}(0,\omega,\omega,1) = \mathbf{P}(0,2,2,1)$,
$1 : P_{81} = \mathbf{P}(0,\omega^2,\omega^2,1) = \mathbf{P}(0,3,3,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_9 = (2,0,1,0) = \ell_0 \cap \ell_5$
$P_{10} = (3,0,1,0) = \ell_0 \cap \ell_6$
$P_3 = (0,0,0,1) = \ell_1 \cap \ell_7$
$P_{70} = (1,0,3,1) = \ell_2 \cap \ell_8$
$P_{54} = (1,0,2,1) = \ell_2 \cap \ell_9$
$P_{79} = (2,2,3,1) = \ell_3 \cap \ell_5$
$P_{47} = (2,2,1,1) = \ell_3 \cap \ell_9$
$P_{68} = (3,3,2,1) = \ell_4 \cap \ell_6$
$P_{52} = (3,3,1,1) = \ell_4 \cap \ell_8$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_8 = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{11} = (0,1,1,0)$ lies on line ℓ_7
3 : $P_{16} = (1,2,1,0)$ lies on line ℓ_8
4 : $P_{20} = (1,3,1,0)$ lies on line ℓ_9
5 : $P_{23} = (1,0,0,1)$ lies on line ℓ_2
6 : $P_{28} = (2,1,0,1)$ lies on line ℓ_8
7 : $P_{29} = (3,1,0,1)$ lies on line ℓ_9
8 : $P_{32} = (2,2,0,1)$ lies on line ℓ_3
9 : $P_{33} = (3,2,0,1)$ lies on line ℓ_5
10 : $P_{36} = (2,3,0,1)$ lies on line ℓ_6
11 : $P_{37} = (3,3,0,1)$ lies on line ℓ_4
12 : $P_{38} = (0,0,1,1)$ lies on line ℓ_1
13 : $P_{39} = (1,0,1,1)$ lies on line ℓ_2
14 : $P_{42} = (0,1,1,1)$ lies on line ℓ_7
15 : $P_{46} = (1,2,1,1)$ lies on line ℓ_5
16 : $P_{50} = (1,3,1,1)$ lies on line ℓ_6
17 : $P_{53} = (0,0,2,1)$ lies on line ℓ_1
18 : $P_{63} = (2,2,2,1)$ lies on line ℓ_3
19 : $P_{69} = (0,0,3,1)$ lies on line ℓ_1
20 : $P_{84} = (3,3,3,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 1 1 1 1 1 0 0 0
1	1 0 1 1 1 0 0 1 0 0
2	1 1 0 1 1 0 0 0 1 1
3	1 1 1 0 1 1 0 0 0 1
4	1 1 1 1 0 0 1 0 1 0
5	1 0 0 1 0 0 0 1 1 0
6	1 0 0 0 1 0 0 1 0 1
7	0 1 0 0 0 1 1 0 1 1
8	0 0 1 0 1 1 0 1 0 0
9	0 0 1 1 0 0 1 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_2	P_2	P_9	P_{10}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_2	P_2	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_8	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{70}	P_{54}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{79}	P_{47}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{68}	P_{52}

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8
in point	P_9	P_{79}	P_{61}	P_{61}

Line 6 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_9
in point	P_{10}	P_{68}	P_{81}	P_{81}

Line 7 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_8	ℓ_9
in point	P_3	P_{61}	P_{81}	P_{61}	P_{81}

Line 8 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_7
in point	P_{70}	P_{52}	P_{61}	P_{61}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{54}	P_{47}	P_{81}	P_{81}

The surface has 33 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_3 = (0,0,0,1)$
$3: P_8 = (1,0,1,0)$
$4: P_9 = (2,0,1,0)$
$5: P_{10} = (3,0,1,0)$
$6: P_{11} = (0,1,1,0)$
$7: P_{16} = (1,2,1,0)$
$8: P_{20} = (1,3,1,0)$
$9: P_{23} = (1,0,0,1)$
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$19: P_{46} = (1,2,1,1)$
$20: P_{47} = (2,2,1,1)$
$21: P_{50} = (1,3,1,1)$
$22: P_{52} = (3,3,1,1)$
$23: P_{53} = (0,0,2,1)$
$24: P_{54} = (1,0,2,1)$
$25: P_{61} = (0,2,2,1)$
$26: P_{63} = (2,2,2,1)$
$27: P_{68} = (3,3,2,1)$
$28: P_{69} = (0,0,3,1)$
$29: P_{70} = (1,0,3,1)$
$30: P_{79} = (2,2,3,1)$
$31: P_{81} = (0,3,3,1)$
$32: P_{84} = (3,3,3,1)$