Rank-65542 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0X_1X_2 = 0$$

(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 1227133526

General information

Number of lines	8
Number of points	65
Number of singular points	9
Number of Eckardt points	0
Number of double points	0
Number of single points	64
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^8
Type of lines on points	$8, 1^{64}$

Singular Points

The surface has 9 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$2 : P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
$3 : P_{201} = \mathbf{P}(0,0,\gamma,1) = \mathbf{P}(0,0,2,1)$
$4 : P_{265} = \mathbf{P}(0,0,\gamma^5,1) = \mathbf{P}(0,0,3,1)$

$5 : P_{329} = \mathbf{P}(0,0,\gamma^2,1) = \mathbf{P}(0,0,4,1)$
$6 : P_{393} = \mathbf{P}(0,0,\gamma^3,1) = \mathbf{P}(0,0,5,1)$
$7 : P_{457} = \mathbf{P}(0,0,\gamma^6,1) = \mathbf{P}(0,0,6,1)$
$8 : P_{521} = \mathbf{P}(0,0,\gamma^4,1) = \mathbf{P}(0,0,7,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \mathbf{Pl}(0,0,0,1,1,0)_{201}$$

$$\ell_2 = \begin{bmatrix} 1 & \gamma^2 & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2700} = \begin{bmatrix} 1 & 4 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2700} = \mathbf{Pl}(0,4,0,4,1,0)_{257}$$

$$\ell_3 = \begin{bmatrix} 1 & \gamma^4 & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4671} = \begin{bmatrix} 1 & 7 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4671} = \mathbf{Pl}(0,7,0,7,1,0)_{305}$$

$$\ell_4 = \begin{bmatrix} 1 & \gamma & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1386} = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1386} = \mathbf{Pl}(0,2,0,2,1,0)_{225}$$

$$\ell_5 = \begin{bmatrix} 1 & \gamma^5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{875} = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{875} = \mathbf{Pl}(0,1,0,3,1,0)_{239}$$

$$\ell_6 = \begin{bmatrix} 1 & \gamma^3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1021} = \begin{bmatrix} 1 & 5 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1021} = \mathbf{Pl}(0,1,0,5,1,0)_{269}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^6 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1094} = \begin{bmatrix} 1 & 6 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1094} = \mathbf{Pl}(0,1,0,6,1,0)_{284}$$

Rank of lines: (4744, 145, 2700, 4671, 1386, 875, 1021, 1094)
Rank of points on Klein quadric: (1, 201, 257, 305, 225, 239, 269, 284)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 64 single points:
The single points on the surface are:

0 : $P_2 = (0,0,1,0)$ lies on line ℓ_0

1 : $P_5 = (1,1,0,0)$ lies on line ℓ_1

2 : $P_{22} = (3,1,1,0)$ lies on line ℓ_2

3 : $P_{24} = (5,1,1,0)$ lies on line ℓ_3

4 : $P_{25} = (6,1,1,0)$ lies on line ℓ_4

5 : $P_{36} = (1,3,1,0)$ lies on line ℓ_5

6 : $P_{52} = (1,5,1,0)$ lies on line ℓ_6

7 : $P_{60} = (1,6,1,0)$ lies on line ℓ_7

8 : $P_{83} = (1,1,0,1)$ lies on line ℓ_1

9 : $P_{92} = (2,2,0,1)$ lies on line ℓ_1

10 : $P_{101} = (3,3,0,1)$ lies on line ℓ_1

11 : $P_{110} = (4,4,0,1)$ lies on line ℓ_1

12 : $P_{119} = (5,5,0,1)$ lies on line ℓ_1

13 : $P_{128} = (6,6,0,1)$ lies on line ℓ_1

14 : $P_{137} = (7,7,0,1)$ lies on line ℓ_1

15 : $P_{138} = (0,0,1,1)$ lies on line ℓ_0

16 : $P_{148} = (3,1,1,1)$ lies on line ℓ_2
17 : $P_{150} = (5,1,1,1)$ lies on line ℓ_3
18 : $P_{151} = (6,1,1,1)$ lies on line ℓ_4
19 : $P_{162} = (1,3,1,1)$ lies on line ℓ_5
20 : $P_{178} = (1,5,1,1)$ lies on line ℓ_6
21 : $P_{186} = (1,6,1,1)$ lies on line ℓ_7
22 : $P_{201} = (0,0,2,1)$ lies on line ℓ_0
23 : $P_{211} = (2,1,2,1)$ lies on line ℓ_7
24 : $P_{218} = (1,2,2,1)$ lies on line ℓ_4
25 : $P_{223} = (6,2,2,1)$ lies on line ℓ_2
26 : $P_{224} = (7,2,2,1)$ lies on line ℓ_3
27 : $P_{251} = (2,6,2,1)$ lies on line ℓ_5
28 : $P_{259} = (2,7,2,1)$ lies on line ℓ_6
29 : $P_{265} = (0,0,3,1)$ lies on line ℓ_0
30 : $P_{284} = (3,2,3,1)$ lies on line ℓ_6
31 : $P_{291} = (2,3,3,1)$ lies on line ℓ_3
32 : $P_{294} = (5,3,3,1)$ lies on line ℓ_2
33 : $P_{296} = (7,3,3,1)$ lies on line ℓ_4
34 : $P_{308} = (3,5,3,1)$ lies on line ℓ_5
35 : $P_{324} = (3,7,3,1)$ lies on line ℓ_7
36 : $P_{329} = (0,0,4,1)$ lies on line ℓ_0
37 : $P_{341} = (4,1,4,1)$ lies on line ℓ_5
38 : $P_{349} = (4,2,4,1)$ lies on line ℓ_7
39 : $P_{357} = (4,3,4,1)$ lies on line ℓ_6
40 : $P_{362} = (1,4,4,1)$ lies on line ℓ_2

41 : $P_{363} = (2,4,4,1)$ lies on line ℓ_4
42 : $P_{364} = (3,4,4,1)$ lies on line ℓ_3
43 : $P_{393} = (0,0,5,1)$ lies on line ℓ_0
44 : $P_{414} = (5,2,5,1)$ lies on line ℓ_5
45 : $P_{430} = (5,4,5,1)$ lies on line ℓ_7
46 : $P_{435} = (2,5,5,1)$ lies on line ℓ_2
47 : $P_{437} = (4,5,5,1)$ lies on line ℓ_4
48 : $P_{439} = (6,5,5,1)$ lies on line ℓ_3
49 : $P_{446} = (5,6,5,1)$ lies on line ℓ_6
50 : $P_{457} = (0,0,6,1)$ lies on line ℓ_0
51 : $P_{487} = (6,3,6,1)$ lies on line ℓ_7
52 : $P_{495} = (6,4,6,1)$ lies on line ℓ_6
53 : $P_{508} = (3,6,6,1)$ lies on line ℓ_4
54 : $P_{509} = (4,6,6,1)$ lies on line ℓ_3
55 : $P_{512} = (7,6,6,1)$ lies on line ℓ_2
56 : $P_{519} = (6,7,6,1)$ lies on line ℓ_5
57 : $P_{521} = (0,0,7,1)$ lies on line ℓ_0
58 : $P_{536} = (7,1,7,1)$ lies on line ℓ_6
59 : $P_{560} = (7,4,7,1)$ lies on line ℓ_5
60 : $P_{568} = (7,5,7,1)$ lies on line ℓ_7
61 : $P_{578} = (1,7,7,1)$ lies on line ℓ_3
62 : $P_{581} = (4,7,7,1)$ lies on line ℓ_2
63 : $P_{582} = (5,7,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7
 0|0 1 1 1 1 1 1 1
 1|1 0 1 1 1 1 1 1
 2|1 1 0 1 1 1 1 1
 3|1 1 1 0 1 1 1 1
 4|1 1 1 1 0 1 1 1
 5|1 1 1 1 1 0 1 1
 6|1 1 1 1 1 1 0 1
 7|1 1 1 1 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 65 points:
The points on the surface are:

$0:\ P_2 = (0,0,1,0)$

$1:\ P_3 = (0,0,0,1)$

$2:\ P_5 = (1,1,0,0)$

$3:\ P_{22} = (3,1,1,0)$

$4:\ P_{24} = (5,1,1,0)$

$5:\ P_{25} = (6,1,1,0)$

$6:\ P_{36} = (1,3,1,0)$

$7:\ P_{52} = (1,5,1,0)$

$8:\ P_{60} = (1,6,1,0)$

$9:\ P_{83} = (1,1,0,1)$

$10:\ P_{92} = (2,2,0,1)$

$11:\ P_{101} = (3,3,0,1)$

$12:\ P_{110} = (4,4,0,1)$

$13:\ P_{119} = (5,5,0,1)$

$14:\ P_{128} = (6,6,0,1)$

$15:\ P_{137} = (7,7,0,1)$

$16:\ P_{138} = (0,0,1,1)$

$17:\ P_{148} = (3,1,1,1)$

$18:\ P_{150} = (5,1,1,1)$

$19:\ P_{151} = (6,1,1,1)$

$20:\ P_{162} = (1,3,1,1)$

$21:\ P_{178} = (1,5,1,1)$

$22:\ P_{186} = (1,6,1,1)$

$23:\ P_{201} = (0,0,2,1)$

$24:\ P_{211} = (2,1,2,1)$

$25:\ P_{218} = (1,2,2,1)$

$26:\ P_{223} = (6,2,2,1)$

$27:\ P_{224} = (7,2,2,1)$

$28:\ P_{251} = (2,6,2,1)$

$29:\ P_{259} = (2,7,2,1)$

$30:\ P_{265} = (0,0,3,1)$

$31:\ P_{284} = (3,2,3,1)$

$32:\ P_{291} = (2,3,3,1)$

$33:\ P_{294} = (5,3,3,1)$

$34:\ P_{296} = (7,3,3,1)$

$35:\ P_{308} = (3,5,3,1)$

$36:\ P_{324} = (3,7,3,1)$

$37:\ P_{329} = (0,0,4,1)$

$38:\ P_{341} = (4,1,4,1)$

$39:\ P_{349} = (4,2,4,1)$

$40:\ P_{357} = (4,3,4,1)$

$41:\ P_{362} = (1,4,4,1)$

$42:\ P_{363} = (2,4,4,1)$

$43:\ P_{364} = (3,4,4,1)$

$44:\ P_{393} = (0,0,5,1)$

$45:\ P_{414} = (5,2,5,1)$

$46:\ P_{430} = (5,4,5,1)$

$47:\ P_{435} = (2,5,5,1)$

$48:\ P_{437} = (4,5,5,1)$

$49:\ P_{439} = (6,5,5,1)$

$50:\ P_{446} = (5,6,5,1)$

$51:\ P_{457} = (0,0,6,1)$

$52:\ P_{487} = (6,3,6,1)$

$53:\ P_{495} = (6,4,6,1)$

$54:\ P_{508} = (3,6,6,1)$

$55:\ P_{509} = (4,6,6,1)$

$56:\ P_{512} = (7,6,6,1)$

$57:\ P_{519} = (6,7,6,1)$

$58:\ P_{521} = (0,0,7,1)$

$59:\ P_{536} = (7,1,7,1)$

$60:\ P_{560} = (7,4,7,1)$

$61:\ P_{568} = (7,5,7,1)$

$62:\ P_{578} = (1,7,7,1)$

$63:\ P_{581} = (4,7,7,1)$

$64:\ P_{582} = (5,7,7,1)$