Rank-66763 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_3 + X_0X_1^2 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 571547925

General information

Number of lines	5
Number of points	289
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	75
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^5
Type of lines on points	3^2, 2^2, 1^{75}, 0^{210}

Singular Points

The surface has 3 singular points:

$0:\ P_{179} = \mathbf{P}(0, \delta^{10}, 1, 0) = \mathbf{P}(0, 10, 1, 0)$
$1:\ P_{195} = \mathbf{P}(0, \delta^{5}, 1, 0) = \mathbf{P}(0, 11, 1, 0)$
$2:\ P_{275} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{10} & 0 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 10 & 0 \end{bmatrix}_{10} = \mathbf{Pl}(11, 0, 1, 0, 0, 0)_{13}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^5 & 0 \end{array}\right]_{11} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 11 & 0 \end{array}\right]_{11} = \mathbf{Pl}(10,0,1,0,0,0)_{12}$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{69888} = \mathbf{Pl}(0,0,0,0,0,1)_{4625}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^5 & 0 \end{array}\right]_{4379} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 11 & 0 \end{array}\right]_{4379} = \mathbf{Pl}(10,11,11,1,0,0)_{223}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{10} & 0 \end{array}\right]_{4378} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 10 & 0 \end{array}\right]_{4378} = \mathbf{Pl}(11,10,10,1,0,0)_{209}$$

Rank of lines: (10, 11, 69888, 4379, 4378)
Rank of points on Klein quadric: (13, 12, 4625, 223, 209)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{179} = \mathbf{P}(0, \delta^{10}, 1, 0) = \mathbf{P}(0, 10, 1, 0)$,
$1 : P_{195} = \mathbf{P}(0, \delta^5, 1, 0) = \mathbf{P}(0, 11, 1, 0)$.

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{275} = (1,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 75 single points:
The single points on the surface are:

$0 : P_1 = (0,1,0,0)$ lies on line ℓ_2
$1 : P_2 = (0,0,1,0)$ lies on line ℓ_2
$2 : P_{35} = (0,1,1,0)$ lies on line ℓ_2
$3 : P_{51} = (0,2,1,0)$ lies on line ℓ_2
$4 : P_{67} = (0,3,1,0)$ lies on line ℓ_2
$5 : P_{83} = (0,4,1,0)$ lies on line ℓ_2
$6 : P_{99} = (0,5,1,0)$ lies on line ℓ_2
$7 : P_{115} = (0,6,1,0)$ lies on line ℓ_2
$8 : P_{131} = (0,7,1,0)$ lies on line ℓ_2
$9 : P_{147} = (0,8,1,0)$ lies on line ℓ_2
$10 : P_{163} = (0,9,1,0)$ lies on line ℓ_2
$11 : P_{180} = (1,10,1,0)$ lies on line ℓ_1
$12 : P_{181} = (2,10,1,0)$ lies on line ℓ_1
$13 : P_{182} = (3,10,1,0)$ lies on line ℓ_1
$14 : P_{183} = (4,10,1,0)$ lies on line ℓ_1
$15 : P_{184} = (5,10,1,0)$ lies on line ℓ_1
$16 : P_{185} = (6,10,1,0)$ lies on line ℓ_1
$17 : P_{186} = (7,10,1,0)$ lies on line ℓ_1
$18 : P_{187} = (8,10,1,0)$ lies on line ℓ_1
$19 : P_{188} = (9,10,1,0)$ lies on line ℓ_1
$20 : P_{189} = (10,10,1,0)$ lies on line ℓ_1
$21 : P_{190} = (11,10,1,0)$ lies on line ℓ_1
$22 : P_{191} = (12,10,1,0)$ lies on line ℓ_1
$23 : P_{192} = (13,10,1,0)$ lies on line ℓ_1
$24 : P_{193} = (14,10,1,0)$ lies on line ℓ_1
$25 : P_{194} = (15,10,1,0)$ lies on line ℓ_1
$26 : P_{196} = (1,11,1,0)$ lies on line ℓ_0
$27 : P_{197} = (2,11,1,0)$ lies on line ℓ_0
$28 : P_{198} = (3,11,1,0)$ lies on line ℓ_0
$29 : P_{199} = (4,11,1,0)$ lies on line ℓ_0
$30 : P_{200} = (5,11,1,0)$ lies on line ℓ_0
$31 : P_{201} = (6,11,1,0)$ lies on line ℓ_0
$32 : P_{202} = (7,11,1,0)$ lies on line ℓ_0
$33 : P_{203} = (8,11,1,0)$ lies on line ℓ_0
$34 : P_{204} = (9,11,1,0)$ lies on line ℓ_0
$35 : P_{205} = (10,11,1,0)$ lies on line ℓ_0

36 : $P_{206} = (11, 11, 1, 0)$ lies on line ℓ_0
37 : $P_{207} = (12, 11, 1, 0)$ lies on line ℓ_0
38 : $P_{208} = (13, 11, 1, 0)$ lies on line ℓ_0
39 : $P_{209} = (14, 11, 1, 0)$ lies on line ℓ_0
40 : $P_{210} = (15, 11, 1, 0)$ lies on line ℓ_0
41 : $P_{211} = (0, 12, 1, 0)$ lies on line ℓ_2
42 : $P_{227} = (0, 13, 1, 0)$ lies on line ℓ_2
43 : $P_{243} = (0, 14, 1, 0)$ lies on line ℓ_2
44 : $P_{259} = (0, 15, 1, 0)$ lies on line ℓ_2
45 : $P_{690} = (1, 10, 1, 1)$ lies on line ℓ_3
46 : $P_{706} = (1, 11, 1, 1)$ lies on line ℓ_4
47 : $P_{994} = (1, 13, 2, 1)$ lies on line ℓ_3
48 : $P_{1026} = (1, 15, 2, 1)$ lies on line ℓ_4
49 : $P_{1106} = (1, 4, 3, 1)$ lies on line ℓ_4
50 : $P_{1154} = (1, 7, 3, 1)$ lies on line ℓ_3
51 : $P_{1346} = (1, 3, 4, 1)$ lies on line ℓ_3
52 : $P_{1410} = (1, 7, 4, 1)$ lies on line ℓ_4
53 : $P_{1698} = (1, 9, 5, 1)$ lies on line ℓ_3
54 : $P_{1746} = (1, 12, 5, 1)$ lies on line ℓ_4
55 : $P_{1938} = (1, 8, 6, 1)$ lies on line ℓ_4
56 : $P_{2034} = (1, 14, 6, 1)$ lies on line ℓ_3
57 : $P_{2114} = (1, 3, 7, 1)$ lies on line ℓ_4
58 : $P_{2130} = (1, 4, 7, 1)$ lies on line ℓ_3
59 : $P_{2418} = (1, 6, 8, 1)$ lies on line ℓ_3
60 : $P_{2546} = (1, 14, 8, 1)$ lies on line ℓ_4
61 : $P_{2658} = (1, 5, 9, 1)$ lies on line ℓ_4
62 : $P_{2770} = (1, 12, 9, 1)$ lies on line ℓ_3
63 : $P_{2850} = (1, 1, 10, 1)$ lies on line ℓ_4
64 : $P_{3010} = (1, 11, 10, 1)$ lies on line ℓ_3
65 : $P_{3106} = (1, 1, 11, 1)$ lies on line ℓ_3
66 : $P_{3250} = (1, 10, 11, 1)$ lies on line ℓ_4
67 : $P_{3426} = (1, 5, 12, 1)$ lies on line ℓ_3
68 : $P_{3490} = (1, 9, 12, 1)$ lies on line ℓ_4
69 : $P_{3634} = (1, 2, 13, 1)$ lies on line ℓ_4
70 : $P_{3842} = (1, 15, 13, 1)$ lies on line ℓ_3
71 : $P_{3954} = (1, 6, 14, 1)$ lies on line ℓ_4
72 : $P_{3986} = (1, 8, 14, 1)$ lies on line ℓ_3
73 : $P_{4146} = (1, 2, 15, 1)$ lies on line ℓ_3
74 : $P_{4322} = (1, 13, 15, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_{300} = (10, 1, 0, 1)$
1 : $P_{301} = (11, 1, 0, 1)$
2 : $P_{315} = (9, 2, 0, 1)$
3 : $P_{319} = (13, 2, 0, 1)$
4 : $P_{345} = (7, 4, 0, 1)$
5 : $P_{352} = (14, 4, 0, 1)$
6 : $P_{420} = (2, 9, 0, 1)$
7 : $P_{430} = (12, 9, 0, 1)$
8 : $P_{437} = (3, 10, 0, 1)$
9 : $P_{442} = (8, 10, 0, 1)$
10 : $P_{455} = (5, 11, 0, 1)$
11 : $P_{465} = (15, 11, 0, 1)$
12 : $P_{502} = (4, 14, 0, 1)$
13 : $P_{504} = (6, 14, 0, 1)$
14 : $P_{540} = (10, 0, 1, 1)$
15 : $P_{541} = (11, 0, 1, 1)$
16 : $P_{555} = (10, 1, 1, 1)$
17 : $P_{556} = (11, 1, 1, 1)$
18 : $P_{630} = (5, 6, 1, 1)$
19 : $P_{640} = (15, 6, 1, 1)$
20 : $P_{646} = (5, 7, 1, 1)$
21 : $P_{656} = (15, 7, 1, 1)$
22 : $P_{724} = (3, 12, 1, 1)$
23 : $P_{729} = (8, 12, 1, 1)$
24 : $P_{740} = (3, 13, 1, 1)$
25 : $P_{745} = (8, 13, 1, 1)$
26 : $P_{794} = (9, 0, 2, 1)$
27 : $P_{798} = (13, 0, 2, 1)$
28 : $P_{826} = (9, 2, 2, 1)$
29 : $P_{830} = (13, 2, 2, 1)$
30 : $P_{917} = (4, 8, 2, 1)$
31 : $P_{919} = (6, 8, 2, 1)$
32 : $P_{939} = (10, 9, 2, 1)$
33 : $P_{940} = (11, 9, 2, 1)$
34 : $P_{949} = (4, 10, 2, 1)$
35 : $P_{951} = (6, 10, 2, 1)$
36 : $P_{971} = (10, 11, 2, 1)$
37 : $P_{972} = (11, 11, 2, 1)$
38 : $P_{1125} = (4, 5, 3, 1)$
39 : $P_{1127} = (6, 5, 3, 1)$
40 : $P_{1141} = (4, 6, 3, 1)$
41 : $P_{1143} = (6, 6, 3, 1)$
42 : $P_{1172} = (3, 8, 3, 1)$
43 : $P_{1177} = (8, 8, 3, 1)$
44 : $P_{1192} = (7, 9, 3, 1)$
45 : $P_{1199} = (14, 9, 3, 1)$
46 : $P_{1208} = (7, 10, 3, 1)$
47 : $P_{1215} = (14, 10, 3, 1)$

48 : $P_{1220} = (3, 11, 3, 1)$
49 : $P_{1225} = (8, 11, 3, 1)$
50 : $P_{1235} = (2, 12, 3, 1)$
51 : $P_{1245} = (12, 12, 3, 1)$
52 : $P_{1283} = (2, 15, 3, 1)$
53 : $P_{1293} = (12, 15, 3, 1)$
54 : $P_{1304} = (7, 0, 4, 1)$
55 : $P_{1311} = (14, 0, 4, 1)$
56 : $P_{1368} = (7, 4, 4, 1)$
57 : $P_{1375} = (14, 4, 4, 1)$
58 : $P_{1467} = (10, 10, 4, 1)$
59 : $P_{1468} = (11, 10, 4, 1)$
60 : $P_{1482} = (9, 11, 4, 1)$
61 : $P_{1486} = (13, 11, 4, 1)$
62 : $P_{1531} = (10, 14, 4, 1)$
63 : $P_{1532} = (11, 14, 4, 1)$
64 : $P_{1546} = (9, 15, 4, 1)$
65 : $P_{1550} = (13, 15, 4, 1)$
66 : $P_{1605} = (4, 3, 5, 1)$
67 : $P_{1607} = (6, 3, 5, 1)$
68 : $P_{1653} = (4, 6, 5, 1)$
69 : $P_{1655} = (6, 6, 5, 1)$
70 : $P_{1690} = (9, 8, 5, 1)$
71 : $P_{1694} = (13, 8, 5, 1)$
72 : $P_{1718} = (5, 10, 5, 1)$
73 : $P_{1728} = (15, 10, 5, 1)$
74 : $P_{1731} = (2, 11, 5, 1)$
75 : $P_{1741} = (12, 11, 5, 1)$
76 : $P_{1770} = (9, 13, 5, 1)$
77 : $P_{1774} = (13, 13, 5, 1)$
78 : $P_{1779} = (2, 14, 5, 1)$
79 : $P_{1789} = (12, 14, 5, 1)$
80 : $P_{1798} = (5, 15, 5, 1)$
81 : $P_{1808} = (15, 15, 5, 1)$
82 : $P_{1830} = (5, 1, 6, 1)$
83 : $P_{1840} = (15, 1, 6, 1)$
84 : $P_{1861} = (4, 3, 6, 1)$
85 : $P_{1863} = (6, 3, 6, 1)$
86 : $P_{1893} = (4, 5, 6, 1)$
87 : $P_{1895} = (6, 5, 6, 1)$
88 : $P_{1926} = (5, 7, 6, 1)$
89 : $P_{1936} = (15, 7, 6, 1)$
90 : $P_{2086} = (5, 1, 7, 1)$
91 : $P_{2096} = (15, 1, 7, 1)$
92 : $P_{2166} = (5, 6, 7, 1)$
93 : $P_{2176} = (15, 6, 7, 1)$
94 : $P_{2200} = (7, 8, 7, 1)$
95 : $P_{2207} = (14, 8, 7, 1)$
96 : $P_{2312} = (7, 15, 7, 1)$
97 : $P_{2319} = (14, 15, 7, 1)$
98 : $P_{2357} = (4, 2, 8, 1)$
99 : $P_{2359} = (6, 2, 8, 1)$
100 : $P_{2372} = (3, 3, 8, 1)$
101 : $P_{2377} = (8, 3, 8, 1)$
102 : $P_{2410} = (9, 5, 8, 1)$
103 : $P_{2414} = (13, 5, 8, 1)$
104 : $P_{2440} = (7, 7, 8, 1)$
105 : $P_{2447} = (14, 7, 8, 1)$
106 : $P_{2485} = (4, 10, 8, 1)$
107 : $P_{2487} = (6, 10, 8, 1)$
108 : $P_{2500} = (3, 11, 8, 1)$
109 : $P_{2505} = (8, 11, 8, 1)$
110 : $P_{2538} = (9, 13, 8, 1)$
111 : $P_{2542} = (13, 13, 8, 1)$
112 : $P_{2568} = (7, 15, 8, 1)$
113 : $P_{2575} = (14, 15, 8, 1)$
114 : $P_{2579} = (2, 0, 9, 1)$
115 : $P_{2589} = (12, 0, 9, 1)$
116 : $P_{2619} = (10, 2, 9, 1)$
117 : $P_{2620} = (11, 2, 9, 1)$
118 : $P_{2632} = (7, 3, 9, 1)$
119 : $P_{2639} = (14, 3, 9, 1)$
120 : $P_{2723} = (2, 9, 9, 1)$
121 : $P_{2733} = (12, 9, 9, 1)$
122 : $P_{2744} = (7, 10, 9, 1)$
123 : $P_{2751} = (14, 10, 9, 1)$
124 : $P_{2763} = (10, 11, 9, 1)$
125 : $P_{2764} = (11, 11, 9, 1)$
126 : $P_{2836} = (3, 0, 10, 1)$
127 : $P_{2841} = (8, 0, 10, 1)$
128 : $P_{2869} = (4, 2, 10, 1)$
129 : $P_{2871} = (6, 2, 10, 1)$
130 : $P_{2888} = (7, 3, 10, 1)$
131 : $P_{2895} = (14, 3, 10, 1)$
132 : $P_{2907} = (10, 4, 10, 1)$
133 : $P_{2908} = (11, 4, 10, 1)$
134 : $P_{2918} = (5, 5, 10, 1)$
135 : $P_{2928} = (15, 5, 10, 1)$
136 : $P_{2965} = (4, 8, 10, 1)$
137 : $P_{2967} = (6, 8, 10, 1)$
138 : $P_{2984} = (7, 9, 10, 1)$
139 : $P_{2991} = (14, 9, 10, 1)$
140 : $P_{2996} = (3, 10, 10, 1)$
141 : $P_{3001} = (8, 10, 10, 1)$
142 : $P_{3067} = (10, 14, 10, 1)$
143 : $P_{3068} = (11, 14, 10, 1)$
144 : $P_{3078} = (5, 15, 10, 1)$
145 : $P_{3088} = (15, 15, 10, 1)$
146 : $P_{3094} = (5, 0, 11, 1)$
147 : $P_{3104} = (15, 0, 11, 1)$
148 : $P_{3131} = (10, 2, 11, 1)$
149 : $P_{3132} = (11, 2, 11, 1)$
150 : $P_{3140} = (3, 3, 11, 1)$
151 : $P_{3145} = (8, 3, 11, 1)$
152 : $P_{3162} = (9, 4, 11, 1)$
153 : $P_{3166} = (13, 4, 11, 1)$
154 : $P_{3171} = (2, 5, 11, 1)$
155 : $P_{3181} = (12, 5, 11, 1)$

156 : $P_{3220} = (3, 8, 11, 1)$
157 : $P_{3225} = (8, 8, 11, 1)$
158 : $P_{3243} = (10, 9, 11, 1)$
159 : $P_{3244} = (11, 9, 11, 1)$
160 : $P_{3270} = (5, 11, 11, 1)$
161 : $P_{3280} = (15, 11, 11, 1)$
162 : $P_{3315} = (2, 14, 11, 1)$
163 : $P_{3325} = (12, 14, 11, 1)$
164 : $P_{3338} = (9, 15, 11, 1)$
165 : $P_{3342} = (13, 15, 11, 1)$
166 : $P_{3364} = (3, 1, 12, 1)$
167 : $P_{3369} = (8, 1, 12, 1)$
168 : $P_{3395} = (2, 3, 12, 1)$
169 : $P_{3405} = (12, 3, 12, 1)$
170 : $P_{3556} = (3, 13, 12, 1)$
171 : $P_{3561} = (8, 13, 12, 1)$
172 : $P_{3587} = (2, 15, 12, 1)$
173 : $P_{3597} = (12, 15, 12, 1)$
174 : $P_{3620} = (3, 1, 13, 1)$
175 : $P_{3625} = (8, 1, 13, 1)$
176 : $P_{3690} = (9, 5, 13, 1)$
177 : $P_{3694} = (13, 5, 13, 1)$
178 : $P_{3738} = (9, 8, 13, 1)$
179 : $P_{3742} = (13, 8, 13, 1)$
180 : $P_{3796} = (3, 12, 13, 1)$
181 : $P_{3801} = (8, 12, 13, 1)$
182 : $P_{3861} = (4, 0, 14, 1)$
183 : $P_{3863} = (6, 0, 14, 1)$
184 : $P_{3931} = (10, 4, 14, 1)$
185 : $P_{3932} = (11, 4, 14, 1)$
186 : $P_{3939} = (2, 5, 14, 1)$
187 : $P_{3949} = (12, 5, 14, 1)$
188 : $P_{4027} = (10, 10, 14, 1)$
189 : $P_{4028} = (11, 10, 14, 1)$
190 : $P_{4035} = (2, 11, 14, 1)$
191 : $P_{4045} = (12, 11, 14, 1)$
192 : $P_{4085} = (4, 14, 14, 1)$
193 : $P_{4087} = (6, 14, 14, 1)$
194 : $P_{4163} = (2, 3, 15, 1)$
195 : $P_{4173} = (12, 3, 15, 1)$
196 : $P_{4186} = (9, 4, 15, 1)$
197 : $P_{4190} = (13, 4, 15, 1)$
198 : $P_{4198} = (5, 5, 15, 1)$
199 : $P_{4208} = (15, 5, 15, 1)$
200 : $P_{4232} = (7, 7, 15, 1)$
201 : $P_{4239} = (14, 7, 15, 1)$
202 : $P_{4248} = (7, 8, 15, 1)$
203 : $P_{4255} = (14, 8, 15, 1)$
204 : $P_{4278} = (5, 10, 15, 1)$
205 : $P_{4288} = (15, 10, 15, 1)$
206 : $P_{4298} = (9, 11, 15, 1)$
207 : $P_{4302} = (13, 11, 15, 1)$
208 : $P_{4307} = (2, 12, 15, 1)$
209 : $P_{4317} = (12, 12, 15, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	1
1	1	0	1	1	0
2	1	1	0	1	1
3	0	1	1	0	1
4	1	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_0	P_{195}	P_{195}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_{179}	P_{179}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{195}	P_{179}	P_{179}	P_{195}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{179}	P_{179}	P_{275}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_{195}	P_{195}	P_{275}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_{35} = (0,1,1,0)$
4 : $P_{51} = (0,2,1,0)$
5 : $P_{67} = (0,3,1,0)$
6 : $P_{83} = (0,4,1,0)$
7 : $P_{99} = (0,5,1,0)$
8 : $P_{115} = (0,6,1,0)$
9 : $P_{131} = (0,7,1,0)$
10 : $P_{147} = (0,8,1,0)$
11 : $P_{163} = (0,9,1,0)$
12 : $P_{179} = (0,10,1,0)$
13 : $P_{180} = (1,10,1,0)$
14 : $P_{181} = (2,10,1,0)$
15 : $P_{182} = (3,10,1,0)$
16 : $P_{183} = (4,10,1,0)$
17 : $P_{184} = (5,10,1,0)$
18 : $P_{185} = (6,10,1,0)$
19 : $P_{186} = (7,10,1,0)$
20 : $P_{187} = (8,10,1,0)$
21 : $P_{188} = (9,10,1,0)$
22 : $P_{189} = (10,10,1,0)$
23 : $P_{190} = (11,10,1,0)$
24 : $P_{191} = (12,10,1,0)$
25 : $P_{192} = (13,10,1,0)$
26 : $P_{193} = (14,10,1,0)$
27 : $P_{194} = (15,10,1,0)$
28 : $P_{195} = (0,11,1,0)$
29 : $P_{196} = (1,11,1,0)$
30 : $P_{197} = (2,11,1,0)$
31 : $P_{198} = (3,11,1,0)$
32 : $P_{199} = (4,11,1,0)$
33 : $P_{200} = (5,11,1,0)$
34 : $P_{201} = (6,11,1,0)$
35 : $P_{202} = (7,11,1,0)$
36 : $P_{203} = (8,11,1,0)$
37 : $P_{204} = (9,11,1,0)$
38 : $P_{205} = (10,11,1,0)$
39 : $P_{206} = (11,11,1,0)$
40 : $P_{207} = (12,11,1,0)$
41 : $P_{208} = (13,11,1,0)$
42 : $P_{209} = (14,11,1,0)$
43 : $P_{210} = (15,11,1,0)$
44 : $P_{211} = (0,12,1,0)$
45 : $P_{227} = (0,13,1,0)$
46 : $P_{243} = (0,14,1,0)$
47 : $P_{259} = (0,15,1,0)$
48 : $P_{275} = (1,0,0,1)$
49 : $P_{300} = (10,1,0,1)$
50 : $P_{301} = (11,1,0,1)$
51 : $P_{315} = (9,2,0,1)$
52 : $P_{319} = (13,2,0,1)$
53 : $P_{345} = (7,4,0,1)$
54 : $P_{352} = (14,4,0,1)$
55 : $P_{420} = (2,9,0,1)$
56 : $P_{430} = (12,9,0,1)$
57 : $P_{437} = (3,10,0,1)$
58 : $P_{442} = (8,10,0,1)$
59 : $P_{455} = (5,11,0,1)$
60 : $P_{465} = (15,11,0,1)$
61 : $P_{502} = (4,14,0,1)$
62 : $P_{504} = (6,14,0,1)$
63 : $P_{540} = (10,0,1,1)$
64 : $P_{541} = (11,0,1,1)$
65 : $P_{555} = (10,1,1,1)$
66 : $P_{556} = (11,1,1,1)$
67 : $P_{630} = (5,6,1,1)$
68 : $P_{640} = (15,6,1,1)$
69 : $P_{646} = (5,7,1,1)$
70 : $P_{656} = (15,7,1,1)$
71 : $P_{690} = (1,10,1,1)$
72 : $P_{706} = (1,11,1,1)$
73 : $P_{724} = (3,12,1,1)$
74 : $P_{729} = (8,12,1,1)$
75 : $P_{740} = (3,13,1,1)$
76 : $P_{745} = (8,13,1,1)$
77 : $P_{794} = (9,0,2,1)$
78 : $P_{798} = (13,0,2,1)$
79 : $P_{826} = (9,2,2,1)$
80 : $P_{830} = (13,2,2,1)$
81 : $P_{917} = (4,8,2,1)$
82 : $P_{919} = (6,8,2,1)$
83 : $P_{939} = (10,9,2,1)$
84 : $P_{940} = (11,9,2,1)$
85 : $P_{949} = (4,10,2,1)$
86 : $P_{951} = (6,10,2,1)$
87 : $P_{971} = (10,11,2,1)$
88 : $P_{972} = (11,11,2,1)$
89 : $P_{994} = (1,13,2,1)$
90 : $P_{1026} = (1,15,2,1)$
91 : $P_{1106} = (1,4,3,1)$
92 : $P_{1125} = (4,5,3,1)$
93 : $P_{1127} = (6,5,3,1)$
94 : $P_{1141} = (4,6,3,1)$
95 : $P_{1143} = (6,6,3,1)$
96 : $P_{1154} = (1,7,3,1)$
97 : $P_{1172} = (3,8,3,1)$
98 : $P_{1177} = (8,8,3,1)$
99 : $P_{1192} = (7,9,3,1)$
100 : $P_{1199} = (14,9,3,1)$
101 : $P_{1208} = (7,10,3,1)$
102 : $P_{1215} = (14,10,3,1)$
103 : $P_{1220} = (3,11,3,1)$
104 : $P_{1225} = (8,11,3,1)$
105 : $P_{1235} = (2,12,3,1)$
106 : $P_{1245} = (12,12,3,1)$
107 : $P_{1283} = (2,15,3,1)$
108 : $P_{1293} = (12,15,3,1)$
109 : $P_{1304} = (7,0,4,1)$
110 : $P_{1311} = (14,0,4,1)$
111 : $P_{1346} = (1,3,4,1)$
112 : $P_{1368} = (7,4,4,1)$
113 : $P_{1375} = (14,4,4,1)$
114 : $P_{1410} = (1,7,4,1)$
115 : $P_{1467} = (10,10,4,1)$
116 : $P_{1468} = (11,10,4,1)$
117 : $P_{1482} = (9,11,4,1)$
118 : $P_{1486} = (13,11,4,1)$
119 : $P_{1531} = (10,14,4,1)$
120 : $P_{1532} = (11,14,4,1)$
121 : $P_{1546} = (9,15,4,1)$
122 : $P_{1550} = (13,15,4,1)$
123 : $P_{1605} = (4,3,5,1)$
124 : $P_{1607} = (6,3,5,1)$
125 : $P_{1653} = (4,6,5,1)$
126 : $P_{1655} = (6,6,5,1)$
127 : $P_{1690} = (9,8,5,1)$
128 : $P_{1694} = (13,8,5,1)$
129 : $P_{1698} = (1,9,5,1)$
130 : $P_{1718} = (5,10,5,1)$
131 : $P_{1728} = (15,10,5,1)$
132 : $P_{1731} = (2,11,5,1)$
133 : $P_{1741} = (12,11,5,1)$
134 : $P_{1746} = (1,12,5,1)$
135 : $P_{1770} = (9,13,5,1)$
136 : $P_{1774} = (13,13,5,1)$
137 : $P_{1779} = (2,14,5,1)$
138 : $P_{1789} = (12,14,5,1)$
139 : $P_{1798} = (5,15,5,1)$
140 : $P_{1808} = (15,15,5,1)$
141 : $P_{1830} = (5,1,6,1)$
142 : $P_{1840} = (15,1,6,1)$
143 : $P_{1861} = (4,3,6,1)$
144 : $P_{1863} = (6,3,6,1)$
145 : $P_{1893} = (4,5,6,1)$
146 : $P_{1895} = (6,5,6,1)$
147 : $P_{1926} = (5,7,6,1)$
148 : $P_{1936} = (15,7,6,1)$
149 : $P_{1938} = (1,8,6,1)$

150 : $P_{2034} = (1, 14, 6, 1)$
151 : $P_{2086} = (5, 1, 7, 1)$
152 : $P_{2096} = (15, 1, 7, 1)$
153 : $P_{2114} = (1, 3, 7, 1)$
154 : $P_{2130} = (1, 4, 7, 1)$
155 : $P_{2166} = (5, 6, 7, 1)$
156 : $P_{2176} = (15, 6, 7, 1)$
157 : $P_{2200} = (7, 8, 7, 1)$
158 : $P_{2207} = (14, 8, 7, 1)$
159 : $P_{2312} = (7, 15, 7, 1)$
160 : $P_{2319} = (14, 15, 7, 1)$
161 : $P_{2357} = (4, 2, 8, 1)$
162 : $P_{2359} = (6, 2, 8, 1)$
163 : $P_{2372} = (3, 3, 8, 1)$
164 : $P_{2377} = (8, 3, 8, 1)$
165 : $P_{2410} = (9, 5, 8, 1)$
166 : $P_{2414} = (13, 5, 8, 1)$
167 : $P_{2418} = (1, 6, 8, 1)$
168 : $P_{2440} = (7, 7, 8, 1)$
169 : $P_{2447} = (14, 7, 8, 1)$
170 : $P_{2485} = (4, 10, 8, 1)$
171 : $P_{2487} = (6, 10, 8, 1)$
172 : $P_{2500} = (3, 11, 8, 1)$
173 : $P_{2505} = (8, 11, 8, 1)$
174 : $P_{2538} = (9, 13, 8, 1)$
175 : $P_{2542} = (13, 13, 8, 1)$
176 : $P_{2546} = (1, 14, 8, 1)$
177 : $P_{2568} = (7, 15, 8, 1)$
178 : $P_{2575} = (14, 15, 8, 1)$
179 : $P_{2579} = (2, 0, 9, 1)$
180 : $P_{2589} = (12, 0, 9, 1)$
181 : $P_{2619} = (10, 2, 9, 1)$
182 : $P_{2620} = (11, 2, 9, 1)$
183 : $P_{2632} = (7, 3, 9, 1)$
184 : $P_{2639} = (14, 3, 9, 1)$
185 : $P_{2658} = (1, 5, 9, 1)$
186 : $P_{2723} = (2, 9, 9, 1)$
187 : $P_{2733} = (12, 9, 9, 1)$
188 : $P_{2744} = (7, 10, 9, 1)$
189 : $P_{2751} = (14, 10, 9, 1)$
190 : $P_{2763} = (10, 11, 9, 1)$
191 : $P_{2764} = (11, 11, 9, 1)$
192 : $P_{2770} = (1, 12, 9, 1)$
193 : $P_{2836} = (3, 0, 10, 1)$
194 : $P_{2841} = (8, 0, 10, 1)$
195 : $P_{2850} = (1, 1, 10, 1)$
196 : $P_{2869} = (4, 2, 10, 1)$
197 : $P_{2871} = (6, 2, 10, 1)$
198 : $P_{2888} = (7, 3, 10, 1)$
199 : $P_{2895} = (14, 3, 10, 1)$
200 : $P_{2907} = (10, 4, 10, 1)$
201 : $P_{2908} = (11, 4, 10, 1)$
202 : $P_{2918} = (5, 5, 10, 1)$
203 : $P_{2928} = (15, 5, 10, 1)$
204 : $P_{2965} = (4, 8, 10, 1)$
205 : $P_{2967} = (6, 8, 10, 1)$
206 : $P_{2984} = (7, 9, 10, 1)$
207 : $P_{2991} = (14, 9, 10, 1)$
208 : $P_{2996} = (3, 10, 10, 1)$
209 : $P_{3001} = (8, 10, 10, 1)$
210 : $P_{3010} = (1, 11, 10, 1)$
211 : $P_{3067} = (10, 14, 10, 1)$
212 : $P_{3068} = (11, 14, 10, 1)$
213 : $P_{3078} = (5, 15, 10, 1)$
214 : $P_{3088} = (15, 15, 10, 1)$
215 : $P_{3094} = (5, 0, 11, 1)$
216 : $P_{3104} = (15, 0, 11, 1)$
217 : $P_{3106} = (1, 1, 11, 1)$
218 : $P_{3131} = (10, 2, 11, 1)$
219 : $P_{3132} = (11, 2, 11, 1)$
220 : $P_{3140} = (3, 3, 11, 1)$
221 : $P_{3145} = (8, 3, 11, 1)$
222 : $P_{3162} = (9, 4, 11, 1)$
223 : $P_{3166} = (13, 4, 11, 1)$
224 : $P_{3171} = (2, 5, 11, 1)$
225 : $P_{3181} = (12, 5, 11, 1)$
226 : $P_{3220} = (3, 8, 11, 1)$
227 : $P_{3225} = (8, 8, 11, 1)$
228 : $P_{3243} = (10, 9, 11, 1)$
229 : $P_{3244} = (11, 9, 11, 1)$
230 : $P_{3250} = (1, 10, 11, 1)$
231 : $P_{3270} = (5, 11, 11, 1)$
232 : $P_{3280} = (15, 11, 11, 1)$
233 : $P_{3315} = (2, 14, 11, 1)$
234 : $P_{3325} = (12, 14, 11, 1)$
235 : $P_{3338} = (9, 15, 11, 1)$
236 : $P_{3342} = (13, 15, 11, 1)$
237 : $P_{3364} = (3, 1, 12, 1)$
238 : $P_{3369} = (8, 1, 12, 1)$
239 : $P_{3395} = (2, 3, 12, 1)$
240 : $P_{3405} = (12, 3, 12, 1)$
241 : $P_{3426} = (1, 5, 12, 1)$
242 : $P_{3490} = (1, 9, 12, 1)$
243 : $P_{3556} = (3, 13, 12, 1)$
244 : $P_{3561} = (8, 13, 12, 1)$
245 : $P_{3587} = (2, 15, 12, 1)$
246 : $P_{3597} = (12, 15, 12, 1)$
247 : $P_{3620} = (3, 1, 13, 1)$
248 : $P_{3625} = (8, 1, 13, 1)$
249 : $P_{3634} = (1, 2, 13, 1)$
250 : $P_{3690} = (9, 5, 13, 1)$
251 : $P_{3694} = (13, 5, 13, 1)$
252 : $P_{3738} = (9, 8, 13, 1)$
253 : $P_{3742} = (13, 8, 13, 1)$
254 : $P_{3796} = (3, 12, 13, 1)$
255 : $P_{3801} = (8, 12, 13, 1)$
256 : $P_{3842} = (1, 15, 13, 1)$
257 : $P_{3861} = (4, 0, 14, 1)$
258 : $P_{3863} = (6, 0, 14, 1)$
259 : $P_{3931} = (10, 4, 14, 1)$
260 : $P_{3932} = (11, 4, 14, 1)$
261 : $P_{3939} = (2, 5, 14, 1)$
262 : $P_{3949} = (12, 5, 14, 1)$
263 : $P_{3954} = (1, 6, 14, 1)$
264 : $P_{3986} = (1, 8, 14, 1)$
265 : $P_{4027} = (10, 10, 14, 1)$
266 : $P_{4028} = (11, 10, 14, 1)$
267 : $P_{4035} = (2, 11, 14, 1)$
268 : $P_{4045} = (12, 11, 14, 1)$
269 : $P_{4085} = (4, 14, 14, 1)$
270 : $P_{4087} = (6, 14, 14, 1)$
271 : $P_{4146} = (1, 2, 15, 1)$
272 : $P_{4163} = (2, 3, 15, 1)$
273 : $P_{4173} = (12, 3, 15, 1)$
274 : $P_{4186} = (9, 4, 15, 1)$
275 : $P_{4190} = (13, 4, 15, 1)$
276 : $P_{4198} = (5, 5, 15, 1)$
277 : $P_{4208} = (15, 5, 15, 1)$
278 : $P_{4232} = (7, 7, 15, 1)$
279 : $P_{4239} = (14, 7, 15, 1)$
280 : $P_{4248} = (7, 8, 15, 1)$
281 : $P_{4255} = (14, 8, 15, 1)$
282 : $P_{4278} = (5, 10, 15, 1)$
283 : $P_{4288} = (15, 10, 15, 1)$
284 : $P_{4298} = (9, 11, 15, 1)$
285 : $P_{4302} = (13, 11, 15, 1)$
286 : $P_{4307} = (2, 12, 15, 1)$
287 : $P_{4317} = (12, 12, 15, 1)$
288 : $P_{4322} = (1, 13, 15, 1)$