

# Rank-65863 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1244172941

## General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	23
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^3$
Type of lines on points	$2^2, 1^{23}, 0^{56}$

## Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

## The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25}$$

Rank of lines: ( 0, 4680, 4689 )

Rank of points on Klein quadric: ( 0, 17, 25 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_3 = (0, 0, 0, 1) = \ell_1 \cap \ell_2$$

### Single Points

The surface has 23 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_8 = (4, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_9 = (5, 1, 0, 0) \text{ lies on line } \ell_0$$

$$6 : P_{10} = (6, 1, 0, 0) \text{ lies on line } \ell_0$$

$$7 : P_{11} = (7, 1, 0, 0) \text{ lies on line } \ell_0$$

$$8 : P_{19} = (0, 1, 1, 0) \text{ lies on line } \ell_2$$

$$9 : P_{82} = (0, 1, 0, 1) \text{ lies on line } \ell_1$$

$$10 : P_{90} = (0, 2, 0, 1) \text{ lies on line } \ell_1$$

$$11 : P_{98} = (0, 3, 0, 1) \text{ lies on line } \ell_1$$

$$12 : P_{106} = (0, 4, 0, 1) \text{ lies on line } \ell_1$$

$$13 : P_{114} = (0, 5, 0, 1) \text{ lies on line } \ell_1$$

$$14 : P_{122} = (0, 6, 0, 1) \text{ lies on line } \ell_1$$

$$15 : P_{130} = (0, 7, 0, 1) \text{ lies on line } \ell_1$$

$$16 : P_{146} = (0, 1, 1, 1) \text{ lies on line } \ell_2$$

$$17 : P_{217} = (0, 2, 2, 1) \text{ lies on line } \ell_2$$

$$18 : P_{289} = (0, 3, 3, 1) \text{ lies on line } \ell_2$$

$$19 : P_{361} = (0, 4, 4, 1) \text{ lies on line } \ell_2$$

$$20 : P_{433} = (0, 5, 5, 1) \text{ lies on line } \ell_2$$

$$21 : P_{505} = (0, 6, 6, 1) \text{ lies on line } \ell_2$$

$$22 : P_{577} = (0, 7, 7, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

### Points on surface but on no line

The surface has 56 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

$$1 : P_{31} = (4, 2, 1, 0)$$

$$2 : P_{42} = (7, 3, 1, 0)$$

$$3 : P_{50} = (7, 4, 1, 0)$$

$$4 : P_{53} = (2, 5, 1, 0)$$

$$5 : P_{63} = (4, 6, 1, 0)$$

$$6 : P_{69} = (2, 7, 1, 0)$$

$$7 : P_{139} = (1, 0, 1, 1)$$

8 : $P_{166} = (5, 3, 1, 1)$	33 : $P_{375} = (6, 5, 4, 1)$
9 : $P_{167} = (6, 3, 1, 1)$	34 : $P_{395} = (2, 0, 5, 1)$
10 : $P_{180} = (3, 5, 1, 1)$	35 : $P_{411} = (2, 2, 5, 1)$
11 : $P_{183} = (6, 5, 1, 1)$	36 : $P_{414} = (5, 2, 5, 1)$
12 : $P_{188} = (3, 6, 1, 1)$	37 : $P_{421} = (4, 3, 5, 1)$
13 : $P_{190} = (5, 6, 1, 1)$	38 : $P_{423} = (6, 3, 5, 1)$
14 : $P_{204} = (3, 0, 2, 1)$	39 : $P_{439} = (6, 5, 5, 1)$
15 : $P_{221} = (4, 2, 2, 1)$	40 : $P_{453} = (4, 7, 5, 1)$
16 : $P_{228} = (3, 3, 2, 1)$	41 : $P_{454} = (5, 7, 5, 1)$
17 : $P_{230} = (5, 3, 2, 1)$	42 : $P_{461} = (4, 0, 6, 1)$
18 : $P_{253} = (4, 6, 2, 1)$	43 : $P_{479} = (6, 2, 6, 1)$
19 : $P_{254} = (5, 6, 2, 1)$	44 : $P_{480} = (7, 2, 6, 1)$
20 : $P_{272} = (7, 0, 3, 1)$	45 : $P_{493} = (4, 4, 6, 1)$
21 : $P_{294} = (5, 3, 3, 1)$	46 : $P_{495} = (6, 4, 6, 1)$
22 : $P_{299} = (2, 4, 3, 1)$	47 : $P_{500} = (3, 5, 6, 1)$
23 : $P_{300} = (3, 4, 3, 1)$	48 : $P_{504} = (7, 5, 6, 1)$
24 : $P_{315} = (2, 6, 3, 1)$	49 : $P_{508} = (3, 6, 6, 1)$
25 : $P_{318} = (5, 6, 3, 1)$	50 : $P_{527} = (6, 0, 7, 1)$
26 : $P_{324} = (3, 7, 3, 1)$	51 : $P_{563} = (2, 5, 7, 1)$
27 : $P_{328} = (7, 7, 3, 1)$	52 : $P_{564} = (3, 5, 7, 1)$
28 : $P_{334} = (5, 0, 4, 1)$	53 : $P_{572} = (3, 6, 7, 1)$
29 : $P_{359} = (6, 3, 4, 1)$	54 : $P_{575} = (6, 6, 7, 1)$
30 : $P_{360} = (7, 3, 4, 1)$	55 : $P_{579} = (2, 7, 7, 1)$
31 : $P_{368} = (7, 4, 4, 1)$	
32 : $P_{374} = (5, 5, 4, 1)$	

## Line Intersection Graph

	0 1 2
0	0 1 0
1	1 0 1
2	0 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$
in point	$P_1$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_1$	$P_3$

Line 2 intersects

Line	$\ell_1$
in point	$P_3$

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	7 : $P_8 = (4, 1, 0, 0)$	14 : $P_{50} = (7, 4, 1, 0)$
1 : $P_1 = (0, 1, 0, 0)$	8 : $P_9 = (5, 1, 0, 0)$	15 : $P_{53} = (2, 5, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$	9 : $P_{10} = (6, 1, 0, 0)$	16 : $P_{63} = (4, 6, 1, 0)$
3 : $P_4 = (1, 1, 1, 1)$	10 : $P_{11} = (7, 1, 0, 0)$	17 : $P_{69} = (2, 7, 1, 0)$
4 : $P_5 = (1, 1, 0, 0)$	11 : $P_{19} = (0, 1, 1, 0)$	18 : $P_{82} = (0, 1, 0, 1)$
5 : $P_6 = (2, 1, 0, 0)$	12 : $P_{31} = (4, 2, 1, 0)$	19 : $P_{90} = (0, 2, 0, 1)$
6 : $P_7 = (3, 1, 0, 0)$	13 : $P_{42} = (7, 3, 1, 0)$	20 : $P_{98} = (0, 3, 0, 1)$

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