# Rank-65633 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_3 + X_0X_1X_2 = 0$$

( 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(8) is 1227400341

## General information

| | |
|---|---|
| Number of lines | 2 |
| Number of points | 81 |
| Number of singular points | 0 |
| Number of Eckardt points | 0 |
| Number of double points | 0 |
| Number of single points | 18 |
| Number of points off lines | 63 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^2$ |
| Type of lines on points | $1^{18}$, $0^{63}$ |

### Singular Points

The surface has 0 singular points:

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \mathbf{Pl}(1,0,0,0,1,0)_{82}$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

Rank of lines: ( 8, 138 )
Rank of points on Klein quadric: ( 82, 1322 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:
The double points on the surface are:

### Single Points

The surface has 18 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line $\ell_0$
1 : $P_4 = (1,1,1,1)$ lies on line $\ell_1$
2 : $P_5 = (1,1,0,0)$ lies on line $\ell_1$
3 : $P_{82} = (0,1,0,1)$ lies on line $\ell_0$
4 : $P_{83} = (1,1,0,1)$ lies on line $\ell_0$
5 : $P_{84} = (2,1,0,1)$ lies on line $\ell_0$
6 : $P_{85} = (3,1,0,1)$ lies on line $\ell_0$
7 : $P_{86} = (4,1,0,1)$ lies on line $\ell_0$
8 : $P_{87} = (5,1,0,1)$ lies on line $\ell_0$
9 : $P_{88} = (6,1,0,1)$ lies on line $\ell_0$
10 : $P_{89} = (7,1,0,1)$ lies on line $\ell_0$
11 : $P_{138} = (0,0,1,1)$ lies on line $\ell_1$
12 : $P_{155} = (2,2,1,1)$ lies on line $\ell_1$
13 : $P_{164} = (3,3,1,1)$ lies on line $\ell_1$
14 : $P_{173} = (4,4,1,1)$ lies on line $\ell_1$
15 : $P_{182} = (5,5,1,1)$ lies on line $\ell_1$
16 : $P_{191} = (6,6,1,1)$ lies on line $\ell_1$
17 : $P_{200} = (7,7,1,1)$ lies on line $\ell_1$

The single points on the surface are:

### Points on surface but on no line

The surface has 63 points not on any line:
The points on the surface but not on lines are:

0 : $P_{19} = (0,1,1,0)$
1 : $P_{20} = (1,1,1,0)$
2 : $P_{75} = (1,0,0,1)$
3 : $P_{94} = (4,2,0,1)$
4 : $P_{102} = (4,3,0,1)$
5 : $P_{113} = (7,4,0,1)$
6 : $P_{121} = (7,5,0,1)$
7 : $P_{124} = (2,6,0,1)$
8 : $P_{132} = (2,7,0,1)$
9 : $P_{160} = (7,2,1,1)$
10 : $P_{165} = (4,3,1,1)$
11 : $P_{171} = (2,4,1,1)$
12 : $P_{184} = (7,5,1,1)$
13 : $P_{187} = (2,6,1,1)$
14 : $P_{197} = (4,7,1,1)$
15 : $P_{203} = (2,0,2,1)$
16 : $P_{213} = (4,1,2,1)$
17 : $P_{237} = (4,4,2,1)$
18 : $P_{238} = (5,4,2,1)$
19 : $P_{241} = (0,5,2,1)$
20 : $P_{245} = (4,5,2,1)$
21 : $P_{271} = (6,0,3,1)$
22 : $P_{278} = (5,1,3,1)$
23 : $P_{285} = (4,2,3,1)$
24 : $P_{287} = (6,2,3,1)$
25 : $P_{300} = (3,4,3,1)$

26 : $P_{301} = (4, 4, 3, 1)$
27 : $P_{317} = (4, 6, 3, 1)$
28 : $P_{318} = (5, 6, 3, 1)$
29 : $P_{321} = (0, 7, 3, 1)$
30 : $P_{326} = (5, 7, 3, 1)$
31 : $P_{333} = (4, 0, 4, 1)$
32 : $P_{344} = (7, 1, 4, 1)$
33 : $P_{377} = (0, 6, 4, 1)$
34 : $P_{384} = (7, 6, 4, 1)$
35 : $P_{391} = (6, 7, 4, 1)$
36 : $P_{392} = (7, 7, 4, 1)$
37 : $P_{396} = (3, 0, 5, 1)$
38 : $P_{407} = (6, 1, 5, 1)$
39 : $P_{409} = (0, 2, 5, 1)$
40 : $P_{415} = (6, 2, 5, 1)$
41 : $P_{423} = (6, 3, 5, 1)$
42 : $P_{424} = (7, 3, 5, 1)$
43 : $P_{428} = (3, 4, 5, 1)$
44 : $P_{432} = (7, 4, 5, 1)$
45 : $P_{454} = (5, 7, 5, 1)$
46 : $P_{456} = (7, 7, 5, 1)$
47 : $P_{462} = (5, 0, 6, 1)$
48 : $P_{468} = (3, 1, 6, 1)$
49 : $P_{475} = (2, 2, 6, 1)$
50 : $P_{479} = (6, 2, 6, 1)$
51 : $P_{489} = (0, 4, 6, 1)$
52 : $P_{492} = (3, 4, 6, 1)$
53 : $P_{499} = (2, 5, 6, 1)$
54 : $P_{500} = (3, 5, 6, 1)$
55 : $P_{515} = (2, 7, 6, 1)$
56 : $P_{518} = (5, 7, 6, 1)$
57 : $P_{528} = (7, 0, 7, 1)$
58 : $P_{531} = (2, 1, 7, 1)$
59 : $P_{539} = (2, 2, 7, 1)$
60 : $P_{540} = (3, 2, 7, 1)$
61 : $P_{545} = (0, 3, 7, 1)$
62 : $P_{547} = (2, 3, 7, 1)$

## Line Intersection Graph

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line |
|---|
| in point |

Line 1 intersects

| Line |
|---|
| in point |

The surface has 81 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
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2 : $P_5 = (1, 1, 0, 0)$
3 : $P_{19} = (0, 1, 1, 0)$
4 : $P_{20} = (1, 1, 1, 0)$
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