Rank-192 over GF(4)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_0 X_1^2 = 0$$

(0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 6770

General information

Number of lines	5
Number of points	21
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	20
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$5, 1^{20}$

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0,0,0,0,1,0)_{25}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

Rank of lines: (20, 340, 41, 104, 125)
Rank of points on Klein quadric: (25, 9, 53, 29, 57)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 20 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_4 = (1,1,1,1)$ lies on line ℓ_4
3 : $P_5 = (1,1,0,0)$ lies on line ℓ_2
4 : $P_8 = (1,0,1,0)$ lies on line ℓ_3
5 : $P_{12} = (1,1,1,0)$ lies on line ℓ_4
6 : $P_{23} = (1,0,0,1)$ lies on line ℓ_0
7 : $P_{24} = (2,0,0,1)$ lies on line ℓ_0
8 : $P_{25} = (3,0,0,1)$ lies on line ℓ_0
9 : $P_{26} = (0,1,0,1)$ lies on line ℓ_1
10 : $P_{27} = (1,1,0,1)$ lies on line ℓ_2
11 : $P_{30} = (0,2,0,1)$ lies on line ℓ_1
12 : $P_{32} = (2,2,0,1)$ lies on line ℓ_2
13 : $P_{34} = (0,3,0,1)$ lies on line ℓ_1
14 : $P_{37} = (3,3,0,1)$ lies on line ℓ_2
15 : $P_{39} = (1,0,1,1)$ lies on line ℓ_3
16 : $P_{55} = (2,0,2,1)$ lies on line ℓ_3
17 : $P_{63} = (2,2,2,1)$ lies on line ℓ_4
18 : $P_{72} = (3,0,3,1)$ lies on line ℓ_3
19 : $P_{84} = (3,3,3,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 & 1 & 1 \\ 3 & 1 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3	P_3

The surface has 21 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_1 = (0,1,0,0)$

$2 : P_3 = (0,0,0,1)$

$3 : P_4 = (1,1,1,1)$

$4 : P_5 = (1,1,0,0)$

$5 : P_8 = (1,0,1,0)$

$6 : P_{12} = (1,1,1,0)$

$7 : P_{23} = (1,0,0,1)$

$8 : P_{24} = (2,0,0,1)$

$9 : P_{25} = (3,0,0,1)$

$10 : P_{26} = (0,1,0,1)$

$11 : P_{27} = (1,1,0,1)$

$12 : P_{30} = (0,2,0,1)$

$13 : P_{32} = (2,2,0,1)$

$14 : P_{34} = (0,3,0,1)$

$15 : P_{37} = (3,3,0,1)$

$16 : P_{39} = (1,0,1,1)$

$17 : P_{55} = (2,0,2,1)$

$18 : P_{63} = (2,2,2,1)$

$19 : P_{72} = (3,0,3,1)$

$20 : P_{84} = (3,3,3,1)$