# Rank-65873 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

( 0, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(8) is 1244173461

## General information

| | |
|---|---|
| Number of lines | 1 |
| Number of points | 73 |
| Number of singular points | 1 |
| Number of Eckardt points | 0 |
| Number of double points | 0 |
| Number of single points | 9 |
| Number of points off lines | 64 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | 9 |
| Type of lines on points | $1^9$, $0^{64}$ |

### Singular Points

The surface has 1 singular points:

$0: \ P_{75} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

## The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \mathbf{Pl}(1,1,1,1,1,0)_{306}$$

Rank of lines: ( 593 )
Rank of points on Klein quadric: ( 306 )

**Eckardt Points**

The surface has 0 Eckardt points:

**Double Points**

The surface has 0 Double points:
The double points on the surface are:

**Single Points**

The surface has 9 single points:
The single points on the surface are:

$0: P_{20} = (1,1,1,0)$ lies on line $\ell_0$
$1: P_{75} = (1,0,0,1)$ lies on line $\ell_0$
$2: P_{146} = (0,1,1,1)$ lies on line $\ell_0$
$3: P_{220} = (3,2,2,1)$ lies on line $\ell_0$
$4: P_{291} = (2,3,3,1)$ lies on line $\ell_0$
$5: P_{366} = (5,4,4,1)$ lies on line $\ell_0$
$6: P_{437} = (4,5,5,1)$ lies on line $\ell_0$
$7: P_{512} = (7,6,6,1)$ lies on line $\ell_0$
$8: P_{583} = (6,7,7,1)$ lies on line $\ell_0$

The single points on the surface are:

**Points on surface but on no line**

The surface has 64 points not on any line:
The points on the surface but not on lines are:

$0: P_0 = (1,0,0,0)$
$1: P_4 = (1,1,1,1)$
$2: P_{27} = (0,2,1,0)$
$3: P_{37} = (2,3,1,0)$
$4: P_{43} = (0,4,1,0)$
$5: P_{55} = (4,5,1,0)$
$6: P_{66} = (7,6,1,0)$
$7: P_{67} = (0,7,1,0)$
$8: P_{82} = (0,1,0,1)$
$9: P_{92} = (2,2,0,1)$
$10: P_{104} = (6,3,0,1)$
$11: P_{110} = (4,4,0,1)$
$12: P_{117} = (3,5,0,1)$
$13: P_{127} = (5,6,0,1)$
$14: P_{137} = (7,7,0,1)$
$15: P_{138} = (0,0,1,1)$
$16: P_{158} = (5,2,1,1)$
$17: P_{160} = (7,2,1,1)$
$18: P_{171} = (2,4,1,1)$
$19: P_{175} = (6,4,1,1)$
$20: P_{196} = (3,7,1,1)$
$21: P_{197} = (4,7,1,1)$
$22: P_{203} = (2,0,2,1)$
$23: P_{224} = (7,2,2,1)$
$24: P_{244} = (3,5,2,1)$
$25: P_{245} = (4,5,2,1)$
$26: P_{255} = (6,6,2,1)$
$27: P_{256} = (7,6,2,1)$
$28: P_{271} = (6,0,3,1)$
$29: P_{282} = (1,2,3,1)$
$30: P_{288} = (7,2,3,1)$
$31: P_{296} = (7,3,3,1)$
$32: P_{305} = (0,5,3,1)$
$33: P_{307} = (2,5,3,1)$
$34: P_{316} = (3,6,3,1)$
$35: P_{317} = (4,6,3,1)$

36 : $P_{333} = (4, 0, 4, 1)$
37 : $P_{355} = (2, 3, 4, 1)$
38 : $P_{356} = (3, 3, 4, 1)$
39 : $P_{363} = (2, 4, 4, 1)$
40 : $P_{382} = (5, 6, 4, 1)$
41 : $P_{384} = (7, 6, 4, 1)$
42 : $P_{396} = (3, 0, 5, 1)$
43 : $P_{422} = (5, 3, 5, 1)$
44 : $P_{424} = (7, 3, 5, 1)$
45 : $P_{426} = (1, 4, 5, 1)$
46 : $P_{427} = (2, 4, 5, 1)$
47 : $P_{435} = (2, 5, 5, 1)$
48 : $P_{441} = (0, 6, 5, 1)$
49 : $P_{445} = (4, 6, 5, 1)$
50 : $P_{462} = (5, 0, 6, 1)$
51 : $P_{481} = (0, 3, 6, 1)$
52 : $P_{488} = (7, 3, 6, 1)$
53 : $P_{499} = (2, 5, 6, 1)$
54 : $P_{503} = (6, 5, 6, 1)$
55 : $P_{509} = (4, 6, 6, 1)$
56 : $P_{514} = (1, 7, 6, 1)$
57 : $P_{517} = (4, 7, 6, 1)$
58 : $P_{528} = (7, 0, 7, 1)$
59 : $P_{547} = (2, 3, 7, 1)$
60 : $P_{551} = (6, 3, 7, 1)$
61 : $P_{565} = (4, 5, 7, 1)$
62 : $P_{566} = (5, 5, 7, 1)$
63 : $P_{581} = (4, 7, 7, 1)$

## Line Intersection Graph

| | 0 |
|---|---|
| 0 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line |
|---|
| in point |

The surface has 73 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
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