Rank-73797 over GF(64)

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The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1090785413

General information

Number of lines	3
Number of points	4225
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4032
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	3, 1^{192}, 0^{4032}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{17047616} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{270400} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]_{270400} = \mathbf{Pl}(0,1,1,0,0,0)_{66}$$

Rank of lines: (4096, 17047616, 270400)
Rank of points on Klein quadric: (2, 1, 66)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_3 = (0,0,0,1)$ lies on line ℓ_1
$2 : P_{68} = (1,0,1,0)$ lies on line ℓ_0
$3 : P_{69} = (2,0,1,0)$ lies on line ℓ_0
$4 : P_{70} = (3,0,1,0)$ lies on line ℓ_0
$5 : P_{71} = (4,0,1,0)$ lies on line ℓ_0
$6 : P_{72} = (5,0,1,0)$ lies on line ℓ_0
$7 : P_{73} = (6,0,1,0)$ lies on line ℓ_0
$8 : P_{74} = (7,0,1,0)$ lies on line ℓ_0
$9 : P_{75} = (8,0,1,0)$ lies on line ℓ_0
$10 : P_{76} = (9,0,1,0)$ lies on line ℓ_0
$11 : P_{77} = (10,0,1,0)$ lies on line ℓ_0
$12 : P_{78} = (11,0,1,0)$ lies on line ℓ_0
$13 : P_{79} = (12,0,1,0)$ lies on line ℓ_0
$14 : P_{80} = (13,0,1,0)$ lies on line ℓ_0
$15 : P_{81} = (14,0,1,0)$ lies on line ℓ_0
$16 : P_{82} = (15,0,1,0)$ lies on line ℓ_0
$17 : P_{83} = (16,0,1,0)$ lies on line ℓ_0
$18 : P_{84} = (17,0,1,0)$ lies on line ℓ_0
$19 : P_{85} = (18,0,1,0)$ lies on line ℓ_0
$20 : P_{86} = (19,0,1,0)$ lies on line ℓ_0
$21 : P_{87} = (20,0,1,0)$ lies on line ℓ_0
$22 : P_{88} = (21,0,1,0)$ lies on line ℓ_0
$23 : P_{89} = (22,0,1,0)$ lies on line ℓ_0
$24 : P_{90} = (23,0,1,0)$ lies on line ℓ_0
$25 : P_{91} = (24,0,1,0)$ lies on line ℓ_0
$26 : P_{92} = (25,0,1,0)$ lies on line ℓ_0
$27 : P_{93} = (26,0,1,0)$ lies on line ℓ_0
$28 : P_{94} = (27,0,1,0)$ lies on line ℓ_0
$29 : P_{95} = (28,0,1,0)$ lies on line ℓ_0
$30 : P_{96} = (29,0,1,0)$ lies on line ℓ_0
$31 : P_{97} = (30,0,1,0)$ lies on line ℓ_0
$32 : P_{98} = (31,0,1,0)$ lies on line ℓ_0
$33 : P_{99} = (32,0,1,0)$ lies on line ℓ_0
$34 : P_{100} = (33,0,1,0)$ lies on line ℓ_0
$35 : P_{101} = (34,0,1,0)$ lies on line ℓ_0
$36 : P_{102} = (35,0,1,0)$ lies on line ℓ_0
$37 : P_{103} = (36,0,1,0)$ lies on line ℓ_0
$38 : P_{104} = (37,0,1,0)$ lies on line ℓ_0
$39 : P_{105} = (38,0,1,0)$ lies on line ℓ_0
$40 : P_{106} = (39,0,1,0)$ lies on line ℓ_0
$41 : P_{107} = (40,0,1,0)$ lies on line ℓ_0
$42 : P_{108} = (41,0,1,0)$ lies on line ℓ_0
$43 : P_{109} = (42,0,1,0)$ lies on line ℓ_0
$44 : P_{110} = (43,0,1,0)$ lies on line ℓ_0
$45 : P_{111} = (44,0,1,0)$ lies on line ℓ_0
$46 : P_{112} = (45,0,1,0)$ lies on line ℓ_0
$47 : P_{113} = (46,0,1,0)$ lies on line ℓ_0
$48 : P_{114} = (47,0,1,0)$ lies on line ℓ_0
$49 : P_{115} = (48,0,1,0)$ lies on line ℓ_0
$50 : P_{116} = (49,0,1,0)$ lies on line ℓ_0
$51 : P_{117} = (50,0,1,0)$ lies on line ℓ_0
$52 : P_{118} = (51,0,1,0)$ lies on line ℓ_0
$53 : P_{119} = (52,0,1,0)$ lies on line ℓ_0

54 : $P_{120} = (53, 0, 1, 0)$ lies on line ℓ_0
55 : $P_{121} = (54, 0, 1, 0)$ lies on line ℓ_0
56 : $P_{122} = (55, 0, 1, 0)$ lies on line ℓ_0
57 : $P_{123} = (56, 0, 1, 0)$ lies on line ℓ_0
58 : $P_{124} = (57, 0, 1, 0)$ lies on line ℓ_0
59 : $P_{125} = (58, 0, 1, 0)$ lies on line ℓ_0
60 : $P_{126} = (59, 0, 1, 0)$ lies on line ℓ_0
61 : $P_{127} = (60, 0, 1, 0)$ lies on line ℓ_0
62 : $P_{128} = (61, 0, 1, 0)$ lies on line ℓ_0
63 : $P_{129} = (62, 0, 1, 0)$ lies on line ℓ_0
64 : $P_{130} = (63, 0, 1, 0)$ lies on line ℓ_0
65 : $P_{4163} = (1, 0, 0, 1)$ lies on line ℓ_2
66 : $P_{8258} = (0, 0, 1, 1)$ lies on line ℓ_1
67 : $P_{8259} = (1, 0, 1, 1)$ lies on line ℓ_2
68 : $P_{12353} = (0, 0, 2, 1)$ lies on line ℓ_1
69 : $P_{12354} = (1, 0, 2, 1)$ lies on line ℓ_2
70 : $P_{16449} = (0, 0, 3, 1)$ lies on line ℓ_1
71 : $P_{16450} = (1, 0, 3, 1)$ lies on line ℓ_2
72 : $P_{20545} = (0, 0, 4, 1)$ lies on line ℓ_1
73 : $P_{20546} = (1, 0, 4, 1)$ lies on line ℓ_2
74 : $P_{24641} = (0, 0, 5, 1)$ lies on line ℓ_1
75 : $P_{24642} = (1, 0, 5, 1)$ lies on line ℓ_2
76 : $P_{28737} = (0, 0, 6, 1)$ lies on line ℓ_1
77 : $P_{28738} = (1, 0, 6, 1)$ lies on line ℓ_2
78 : $P_{32833} = (0, 0, 7, 1)$ lies on line ℓ_1
79 : $P_{32834} = (1, 0, 7, 1)$ lies on line ℓ_2
80 : $P_{36929} = (0, 0, 8, 1)$ lies on line ℓ_1
81 : $P_{36930} = (1, 0, 8, 1)$ lies on line ℓ_2
82 : $P_{41025} = (0, 0, 9, 1)$ lies on line ℓ_1
83 : $P_{41026} = (1, 0, 9, 1)$ lies on line ℓ_2
84 : $P_{45121} = (0, 0, 10, 1)$ lies on line ℓ_1
85 : $P_{45122} = (1, 0, 10, 1)$ lies on line ℓ_2
86 : $P_{49217} = (0, 0, 11, 1)$ lies on line ℓ_1
87 : $P_{49218} = (1, 0, 11, 1)$ lies on line ℓ_2
88 : $P_{53313} = (0, 0, 12, 1)$ lies on line ℓ_1
89 : $P_{53314} = (1, 0, 12, 1)$ lies on line ℓ_2
90 : $P_{57409} = (0, 0, 13, 1)$ lies on line ℓ_1
91 : $P_{57410} = (1, 0, 13, 1)$ lies on line ℓ_2
92 : $P_{61505} = (0, 0, 14, 1)$ lies on line ℓ_1
93 : $P_{61506} = (1, 0, 14, 1)$ lies on line ℓ_2
94 : $P_{65601} = (0, 0, 15, 1)$ lies on line ℓ_1
95 : $P_{65602} = (1, 0, 15, 1)$ lies on line ℓ_2
96 : $P_{69697} = (0, 0, 16, 1)$ lies on line ℓ_1
97 : $P_{69698} = (1, 0, 16, 1)$ lies on line ℓ_2
98 : $P_{73793} = (0, 0, 17, 1)$ lies on line ℓ_1
99 : $P_{73794} = (1, 0, 17, 1)$ lies on line ℓ_2
100 : $P_{77889} = (0, 0, 18, 1)$ lies on line ℓ_1
101 : $P_{77890} = (1, 0, 18, 1)$ lies on line ℓ_2
102 : $P_{81985} = (0, 0, 19, 1)$ lies on line ℓ_1
103 : $P_{81986} = (1, 0, 19, 1)$ lies on line ℓ_2
104 : $P_{86081} = (0, 0, 20, 1)$ lies on line ℓ_1
105 : $P_{86082} = (1, 0, 20, 1)$ lies on line ℓ_2
106 : $P_{90177} = (0, 0, 21, 1)$ lies on line ℓ_1
107 : $P_{90178} = (1, 0, 21, 1)$ lies on line ℓ_2
108 : $P_{94273} = (0, 0, 22, 1)$ lies on line ℓ_1
109 : $P_{94274} = (1, 0, 22, 1)$ lies on line ℓ_2
110 : $P_{98369} = (0, 0, 23, 1)$ lies on line ℓ_1
111 : $P_{98370} = (1, 0, 23, 1)$ lies on line ℓ_2
112 : $P_{102465} = (0, 0, 24, 1)$ lies on line ℓ_1
113 : $P_{102466} = (1, 0, 24, 1)$ lies on line ℓ_2
114 : $P_{106561} = (0, 0, 25, 1)$ lies on line ℓ_1
115 : $P_{106562} = (1, 0, 25, 1)$ lies on line ℓ_2
116 : $P_{110657} = (0, 0, 26, 1)$ lies on line ℓ_1
117 : $P_{110658} = (1, 0, 26, 1)$ lies on line ℓ_2
118 : $P_{114753} = (0, 0, 27, 1)$ lies on line ℓ_1
119 : $P_{114754} = (1, 0, 27, 1)$ lies on line ℓ_2
120 : $P_{118849} = (0, 0, 28, 1)$ lies on line ℓ_1
121 : $P_{118850} = (1, 0, 28, 1)$ lies on line ℓ_2
122 : $P_{122945} = (0, 0, 29, 1)$ lies on line ℓ_1
123 : $P_{122946} = (1, 0, 29, 1)$ lies on line ℓ_2
124 : $P_{127041} = (0, 0, 30, 1)$ lies on line ℓ_1
125 : $P_{127042} = (1, 0, 30, 1)$ lies on line ℓ_2
126 : $P_{131137} = (0, 0, 31, 1)$ lies on line ℓ_1
127 : $P_{131138} = (1, 0, 31, 1)$ lies on line ℓ_2
128 : $P_{135233} = (0, 0, 32, 1)$ lies on line ℓ_1
129 : $P_{135234} = (1, 0, 32, 1)$ lies on line ℓ_2
130 : $P_{139329} = (0, 0, 33, 1)$ lies on line ℓ_1
131 : $P_{139330} = (1, 0, 33, 1)$ lies on line ℓ_2
132 : $P_{143425} = (0, 0, 34, 1)$ lies on line ℓ_1
133 : $P_{143426} = (1, 0, 34, 1)$ lies on line ℓ_2
134 : $P_{147521} = (0, 0, 35, 1)$ lies on line ℓ_1
135 : $P_{147522} = (1, 0, 35, 1)$ lies on line ℓ_2
136 : $P_{151617} = (0, 0, 36, 1)$ lies on line ℓ_1
137 : $P_{151618} = (1, 0, 36, 1)$ lies on line ℓ_2
138 : $P_{155713} = (0, 0, 37, 1)$ lies on line ℓ_1
139 : $P_{155714} = (1, 0, 37, 1)$ lies on line ℓ_2
140 : $P_{159809} = (0, 0, 38, 1)$ lies on line ℓ_1
141 : $P_{159810} = (1, 0, 38, 1)$ lies on line ℓ_2
142 : $P_{163905} = (0, 0, 39, 1)$ lies on line ℓ_1
143 : $P_{163906} = (1, 0, 39, 1)$ lies on line ℓ_2
144 : $P_{168001} = (0, 0, 40, 1)$ lies on line ℓ_1
145 : $P_{168002} = (1, 0, 40, 1)$ lies on line ℓ_2
146 : $P_{172097} = (0, 0, 41, 1)$ lies on line ℓ_1
147 : $P_{172098} = (1, 0, 41, 1)$ lies on line ℓ_2
148 : $P_{176193} = (0, 0, 42, 1)$ lies on line ℓ_1
149 : $P_{176194} = (1, 0, 42, 1)$ lies on line ℓ_2
150 : $P_{180289} = (0, 0, 43, 1)$ lies on line ℓ_1
151 : $P_{180290} = (1, 0, 43, 1)$ lies on line ℓ_2
152 : $P_{184385} = (0, 0, 44, 1)$ lies on line ℓ_1
153 : $P_{184386} = (1, 0, 44, 1)$ lies on line ℓ_2
154 : $P_{188481} = (0, 0, 45, 1)$ lies on line ℓ_1
155 : $P_{188482} = (1, 0, 45, 1)$ lies on line ℓ_2
156 : $P_{192577} = (0, 0, 46, 1)$ lies on line ℓ_1
157 : $P_{192578} = (1, 0, 46, 1)$ lies on line ℓ_2
158 : $P_{196673} = (0, 0, 47, 1)$ lies on line ℓ_1
159 : $P_{196674} = (1, 0, 47, 1)$ lies on line ℓ_2
160 : $P_{200769} = (0, 0, 48, 1)$ lies on line ℓ_1
161 : $P_{200770} = (1, 0, 48, 1)$ lies on line ℓ_2

162 : $P_{204865} = (0, 0, 49, 1)$ lies on line ℓ_1
163 : $P_{204866} = (1, 0, 49, 1)$ lies on line ℓ_2
164 : $P_{208961} = (0, 0, 50, 1)$ lies on line ℓ_1
165 : $P_{208962} = (1, 0, 50, 1)$ lies on line ℓ_2
166 : $P_{213057} = (0, 0, 51, 1)$ lies on line ℓ_1
167 : $P_{213058} = (1, 0, 51, 1)$ lies on line ℓ_2
168 : $P_{217153} = (0, 0, 52, 1)$ lies on line ℓ_1
169 : $P_{217154} = (1, 0, 52, 1)$ lies on line ℓ_2
170 : $P_{221249} = (0, 0, 53, 1)$ lies on line ℓ_1
171 : $P_{221250} = (1, 0, 53, 1)$ lies on line ℓ_2
172 : $P_{225345} = (0, 0, 54, 1)$ lies on line ℓ_1
173 : $P_{225346} = (1, 0, 54, 1)$ lies on line ℓ_2
174 : $P_{229441} = (0, 0, 55, 1)$ lies on line ℓ_1
175 : $P_{229442} = (1, 0, 55, 1)$ lies on line ℓ_2
176 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_1
177 : $P_{233538} = (1, 0, 56, 1)$ lies on line ℓ_2
178 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_1
179 : $P_{237634} = (1, 0, 57, 1)$ lies on line ℓ_2
180 : $P_{241729} = (0, 0, 58, 1)$ lies on line ℓ_1
181 : $P_{241730} = (1, 0, 58, 1)$ lies on line ℓ_2
182 : $P_{245825} = (0, 0, 59, 1)$ lies on line ℓ_1
183 : $P_{245826} = (1, 0, 59, 1)$ lies on line ℓ_2
184 : $P_{249921} = (0, 0, 60, 1)$ lies on line ℓ_1
185 : $P_{249922} = (1, 0, 60, 1)$ lies on line ℓ_2
186 : $P_{254017} = (0, 0, 61, 1)$ lies on line ℓ_1
187 : $P_{254018} = (1, 0, 61, 1)$ lies on line ℓ_2
188 : $P_{258113} = (0, 0, 62, 1)$ lies on line ℓ_1
189 : $P_{258114} = (1, 0, 62, 1)$ lies on line ℓ_2
190 : $P_{262209} = (0, 0, 63, 1)$ lies on line ℓ_1
191 : $P_{262210} = (1, 0, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4032 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 4225 points:
Too many to print.