Rank-65609 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227395733

General information

Number of lines	2
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	2, 1^{16}, 0^{56}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0,1,0,1,0,0)_{25}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \mathbf{Pl}(1,1,1,1,0,1)_{874}$$

Rank of lines: (4689, 658)
Rank of points on Klein quadric: (25, 874)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_{19} = (0,1,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_3 = (0,0,0,1)$ lies on line ℓ_0
1 : $P_{83} = (1,1,0,1)$ lies on line ℓ_1
2 : $P_{139} = (1,0,1,1)$ lies on line ℓ_1
3 : $P_{146} = (0,1,1,1)$ lies on line ℓ_0
4 : $P_{217} = (0,2,2,1)$ lies on line ℓ_0
5 : $P_{226} = (1,3,2,1)$ lies on line ℓ_1
6 : $P_{282} = (1,2,3,1)$ lies on line ℓ_1
7 : $P_{289} = (0,3,3,1)$ lies on line ℓ_0
8 : $P_{361} = (0,4,4,1)$ lies on line ℓ_0
9 : $P_{370} = (1,5,4,1)$ lies on line ℓ_1
10 : $P_{426} = (1,4,5,1)$ lies on line ℓ_1
11 : $P_{433} = (0,5,5,1)$ lies on line ℓ_0
12 : $P_{505} = (0,6,6,1)$ lies on line ℓ_0
13 : $P_{514} = (1,7,6,1)$ lies on line ℓ_1
14 : $P_{570} = (1,6,7,1)$ lies on line ℓ_1
15 : $P_{577} = (0,7,7,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_{29} = (2,2,1,0)$
3 : $P_{36} = (1,3,1,0)$
4 : $P_{47} = (4,4,1,0)$
5 : $P_{52} = (1,5,1,0)$
6 : $P_{60} = (1,6,1,0)$
7 : $P_{74} = (7,7,1,0)$
8 : $P_{93} = (3,2,0,1)$
9 : $P_{105} = (7,3,0,1)$
10 : $P_{111} = (5,4,0,1)$
11 : $P_{116} = (2,5,0,1)$
12 : $P_{126} = (4,6,0,1)$
13 : $P_{136} = (6,7,0,1)$
14 : $P_{204} = (3,0,2,1)$
15 : $P_{221} = (4,2,2,1)$
16 : $P_{232} = (7,3,2,1)$
17 : $P_{235} = (2,4,2,1)$
18 : $P_{240} = (7,4,2,1)$
19 : $P_{244} = (3,5,2,1)$
20 : $P_{245} = (4,5,2,1)$
21 : $P_{261} = (4,7,2,1)$
22 : $P_{264} = (7,7,2,1)$
23 : $P_{272} = (7,0,3,1)$
24 : $P_{288} = (7,2,3,1)$
25 : $P_{294} = (5,3,3,1)$

26 : $P_{323} = (2, 7, 3, 1)$
27 : $P_{327} = (6, 7, 3, 1)$
28 : $P_{334} = (5, 0, 4, 1)$
29 : $P_{347} = (2, 2, 4, 1)$
30 : $P_{352} = (7, 2, 4, 1)$
31 : $P_{368} = (7, 4, 4, 1)$
32 : $P_{371} = (2, 5, 4, 1)$
33 : $P_{382} = (5, 6, 4, 1)$
34 : $P_{384} = (7, 6, 4, 1)$
35 : $P_{387} = (2, 7, 4, 1)$
36 : $P_{389} = (4, 7, 4, 1)$
37 : $P_{395} = (2, 0, 5, 1)$
38 : $P_{412} = (3, 2, 5, 1)$
39 : $P_{413} = (4, 2, 5, 1)$
40 : $P_{427} = (2, 4, 5, 1)$
41 : $P_{439} = (6, 5, 5, 1)$
42 : $P_{461} = (4, 0, 6, 1)$
43 : $P_{494} = (5, 4, 6, 1)$
44 : $P_{496} = (7, 4, 6, 1)$
45 : $P_{508} = (3, 6, 6, 1)$
46 : $P_{517} = (4, 7, 6, 1)$
47 : $P_{527} = (6, 0, 7, 1)$
48 : $P_{541} = (4, 2, 7, 1)$
49 : $P_{544} = (7, 2, 7, 1)$
50 : $P_{547} = (2, 3, 7, 1)$
51 : $P_{551} = (6, 3, 7, 1)$
52 : $P_{555} = (2, 4, 7, 1)$
53 : $P_{557} = (4, 4, 7, 1)$
54 : $P_{573} = (4, 6, 7, 1)$
55 : $P_{579} = (2, 7, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{19}

Line 1 intersects

Line	ℓ_0
in point	P_{19}

The surface has 73 points:
The points on the surface are:

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1 : $P_3 = (0, 0, 0, 1)$
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