Rank-192 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_0 X_1^2 = 0$$

(0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 192

General information

Number of lines	5
Number of points	11
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	10
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	$5,\ 1^{10}$

Singular Points

The surface has 1 singular points:

$$0:\ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{6} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{6} = \mathbf{Pl}(0,0,0,0,1,0)_{9}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \mathbf{Pl}(0,0,0,1,0,0)_{5}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{20} = \mathbf{Pl}(0,1,0,0,1,0)_{11}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{27} = \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{27} = \mathbf{Pl}(0,1,0,1,1,0)_{17}$$

Rank of lines: (6, 30, 13, 20, 27)
Rank of points on Klein quadric: (9, 5, 15, 11, 17)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 10 single points:
The single points on the surface are:

$0: P_0 = (1,0,0,0)$ lies on line ℓ_0
$1: P_1 = (0,1,0,0)$ lies on line ℓ_1
$2: P_4 = (1,1,1,1)$ lies on line ℓ_4
$3: P_5 = (1,1,0,0)$ lies on line ℓ_2
$4: P_6 = (1,0,1,0)$ lies on line ℓ_3
$5: P_8 = (1,1,1,0)$ lies on line ℓ_4
$6: P_9 = (1,0,0,1)$ lies on line ℓ_0
$7: P_{10} = (0,1,0,1)$ lies on line ℓ_1
$8: P_{11} = (1,1,0,1)$ lies on line ℓ_2
$9: P_{13} = (1,0,1,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	1	1	1
2	1	1	0	1	1
3	1	1	1	0	1
4	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3	P_3

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (1,0,1,0)$
$6: P_8 = (1,1,1,0)$
$7: P_9 = (1,0,0,1)$
$8: P_{10} = (0,1,0,1)$
$9: P_{11} = (1,1,0,1)$
$10: P_{13} = (1,0,1,1)$