

Rank-65612 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431659930

General information

Number of lines	1
Number of points	17
Number of singular points	2
Number of Eckardt points	0
Number of double points	0
Number of single points	5
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5
Type of lines on points	$1^5, 0^{12}$

Singular Points

The surface has 2 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{101}$$

Rank of lines: (336)

Rank of points on Klein quadric: (101)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 5 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_0

2 : $P_{11} = (0, 1, 1, 0)$ lies on line ℓ_0

3 : $P_{15} = (0, 2, 1, 0)$ lies on line ℓ_0

4 : $P_{19} = (0, 3, 1, 0)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:

The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$

1 : $P_{12} = (1, 1, 1, 0)$

2 : $P_{18} = (3, 2, 1, 0)$

3 : $P_{21} = (2, 3, 1, 0)$

4 : $P_{47} = (2, 2, 1, 1)$

5 : $P_{52} = (3, 3, 1, 1)$

6 : $P_{59} = (2, 1, 2, 1)$

7 : $P_{64} = (3, 2, 2, 1)$

8 : $P_{66} = (1, 3, 2, 1)$

9 : $P_{76} = (3, 1, 3, 1)$

10 : $P_{78} = (1, 2, 3, 1)$

11 : $P_{83} = (2, 3, 3, 1)$

Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 17 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
 1 : $P_2 = (0, 0, 1, 0)$
 2 : $P_4 = (1, 1, 1, 1)$
 3 : $P_{11} = (0, 1, 1, 0)$
 4 : $P_{12} = (1, 1, 1, 0)$
 5 : $P_{15} = (0, 2, 1, 0)$

6 : $P_{18} = (3, 2, 1, 0)$
 7 : $P_{19} = (0, 3, 1, 0)$
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12 : $P_{64} = (3, 2, 2, 1)$
 13 : $P_{66} = (1, 3, 2, 1)$
 14 : $P_{76} = (3, 1, 3, 1)$
 15 : $P_{78} = (1, 2, 3, 1)$
 16 : $P_{83} = (2, 3, 3, 1)$