Rank-74499 over GF(4)

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The equation

The equation of the surface is :

$$X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1499092313

General information

Number of lines	7
Number of points	29
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	18
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	4, 3, 2^5, 1^{18}, 0^4

Singular Points

The surface has 2 singular points:

$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

1

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \mathbf{Pl}(0,0,1,0,0,1)_{108}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{5} = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \mathbf{Pl}(0,1,1,0,0,1)_{112}$$

Rank of lines: (0, 16, 37, 5, 337, 356, 121)
Rank of points on Klein quadric: (0, 2, 108, 33, 129, 1, 112)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$.

Double Points

The surface has 5 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$ $P_4 = (1,1,1,1) = \ell_3 \cap \ell_6$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_4$ $P_{38} = (0,0,1,1) = \ell_4 \cap \ell_5$
$P_{42} = (0,1,1,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_5 $10 : P_{43} = (2,1,1,1)$ lies on line ℓ_3
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_0 $11 : P_{44} = (3,1,1,1)$ lies on line ℓ_3
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_0 $12 : P_{45} = (0,2,1,1)$ lies on line ℓ_4
$3 : P_8 = (1,0,1,0)$ lies on line ℓ_1 $13 : P_{49} = (0,3,1,1)$ lies on line ℓ_4
$4 : P_9 = (2,0,1,0)$ lies on line ℓ_1 $14 : P_{53} = (0,0,2,1)$ lies on line ℓ_5
$5 : P_{10} = (3,0,1,0)$ lies on line ℓ_1 $15 : P_{58} = (1,1,2,1)$ lies on line ℓ_6
$6 : P_{12} = (1,1,1,0)$ lies on line ℓ_2 $16 : P_{69} = (0,0,3,1)$ lies on line ℓ_5
$7 : P_{17} = (2,2,1,0)$ lies on line ℓ_2 $17 : P_{74} = (1,1,3,1)$ lies on line ℓ_6
$8 : P_{22} = (3,3,1,0)$ lies on line ℓ_2
$9 : P_{27} = (1,1,0,1)$ lies on line ℓ_6

     The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

$0: P_{33} = (3, 2, 0, 1)$
$1: P_{36} = (2, 3, 0, 1)$
$2: P_{62} = (1, 2, 2, 1)$

$3: P_{82} = (1, 3, 3, 1)$

Line Intersection Graph

```
  |0 1 2 3 4 5 6
0 |0 1 1 1 1 0 0
1 |1 0 1 1 0 1 1
2 |1 1 0 0 0 1 1
3 |1 1 0 0 1 0 1
4 |1 0 0 1 0 1 0
5 |0 1 1 0 1 0 1
6 |0 1 1 1 0 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_0	P_2	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_5	P_2	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_0	P_0	P_{42}	P_4

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_1	P_{42}	P_{38}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_2	P_{38}	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_4	P_2

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_1 = (0, 1, 0, 0)$
$2: P_2 = (0, 0, 1, 0)$
$3: P_3 = (0, 0, 0, 1)$
$4: P_4 = (1, 1, 1, 1)$
$5: P_5 = (1, 1, 0, 0)$

$6: P_6 = (2, 1, 0, 0)$
$7: P_7 = (3, 1, 0, 0)$
$8: P_8 = (1, 0, 1, 0)$
$9: P_9 = (2, 0, 1, 0)$
$10: P_{10} = (3, 0, 1, 0)$
$11: P_{12} = (1, 1, 1, 0)$

$12: P_{17} = (2, 2, 1, 0)$
$13: P_{22} = (3, 3, 1, 0)$
$14: P_{27} = (1, 1, 0, 1)$
$15: P_{33} = (3, 2, 0, 1)$
$16: P_{36} = (2, 3, 0, 1)$
$17: P_{38} = (0, 0, 1, 1)$

$18: P_{42} = (0, 1, 1, 1)$ $22: P_{49} = (0, 3, 1, 1)$ $26: P_{69} = (0, 0, 3, 1)$

$19: P_{43} = (2, 1, 1, 1)$ $23: P_{53} = (0, 0, 2, 1)$ $27: P_{74} = (1, 1, 3, 1)$

$20: P_{44} = (3, 1, 1, 1)$ $24: P_{58} = (1, 1, 2, 1)$ $28: P_{82} = (1, 3, 3, 1)$

$21: P_{45} = (0, 2, 1, 1)$ $25: P_{62} = (1, 2, 2, 1)$