

Rank-66763 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is 1091047493

General information

Number of lines	5
Number of points	4225
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	315
Number of points off lines	3906
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^5
Type of lines on points	$3^2, 2^2, 1^{315}, 0^{3906}$

Singular Points

The surface has 3 singular points:

$$\begin{aligned} 0 : P_{3651} &= \mathbf{P}(0, \epsilon^{42}, 1, 0) = \mathbf{P}(0, 56, 1, 0) \\ 1 : P_{3715} &= \mathbf{P}(0, \epsilon^{21}, 1, 0) = \mathbf{P}(0, 57, 1, 0) \end{aligned}$$

$$2 : P_{4163} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{56} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{56} = \mathbf{Pl}(57, 0, 1, 0, 0, 0)_{59}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{57} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{57} = \mathbf{Pl}(56, 0, 1, 0, 0, 0)_{58} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17043456} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{270401} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{21} & 0 \end{bmatrix}_{266361} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 57 & 0 \end{bmatrix}_{266361} = \mathbf{Pl}(56, 57, 57, 1, 0, 0)_{3839} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{42} & 0 \end{bmatrix}_{266360} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 56 & 0 \end{bmatrix}_{266360} = \mathbf{Pl}(57, 56, 56, 1, 0, 0)_{3777}
\end{aligned}$$

Rank of lines: (56, 57, 17043456, 266361, 266360)

Rank of points on Klein quadric: (59, 58, 270401, 3839, 3777)

Eckardt Points

The surface has 2 Eckardt points:

$$0 : P_{3651} = \mathbf{P}(0, \epsilon^{42}, 1, 0) = \mathbf{P}(0, 56, 1, 0),$$

$$1 : P_{3715} = \mathbf{P}(0, \epsilon^{21}, 1, 0) = \mathbf{P}(0, 57, 1, 0).$$

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{4163} = (1, 0, 0, 1) = \ell_3 \cap \ell_4$$

Single Points

The surface has 315 single points:

The single points on the surface are:

$$0 : P_1 = (0, 1, 0, 0) \text{ lies on line } \ell_2$$

$$1 : P_2 = (0, 0, 1, 0) \text{ lies on line } \ell_2$$

$$2 : P_{131} = (0, 1, 1, 0) \text{ lies on line } \ell_2$$

$$3 : P_{195} = (0, 2, 1, 0) \text{ lies on line } \ell_2$$

$$4 : P_{259} = (0, 3, 1, 0) \text{ lies on line } \ell_2$$

$$5 : P_{323} = (0, 4, 1, 0) \text{ lies on line } \ell_2$$

$$6 : P_{387} = (0, 5, 1, 0) \text{ lies on line } \ell_2$$

$$7 : P_{451} = (0, 6, 1, 0) \text{ lies on line } \ell_2$$

$$8 : P_{515} = (0, 7, 1, 0) \text{ lies on line } \ell_2$$

$$9 : P_{579} = (0, 8, 1, 0) \text{ lies on line } \ell_2$$

$$10 : P_{643} = (0, 9, 1, 0) \text{ lies on line } \ell_2$$

$$11 : P_{707} = (0, 10, 1, 0) \text{ lies on line } \ell_2$$

$$12 : P_{771} = (0, 11, 1, 0) \text{ lies on line } \ell_2$$

$$13 : P_{835} = (0, 12, 1, 0) \text{ lies on line } \ell_2$$

$$14 : P_{899} = (0, 13, 1, 0) \text{ lies on line } \ell_2$$

$$15 : P_{963} = (0, 14, 1, 0) \text{ lies on line } \ell_2$$

$$16 : P_{1027} = (0, 15, 1, 0) \text{ lies on line } \ell_2$$

$$17 : P_{1091} = (0, 16, 1, 0) \text{ lies on line } \ell_2$$

$$18 : P_{1155} = (0, 17, 1, 0) \text{ lies on line } \ell_2$$

$$19 : P_{1219} = (0, 18, 1, 0) \text{ lies on line } \ell_2$$

$$20 : P_{1283} = (0, 19, 1, 0) \text{ lies on line } \ell_2$$

$$21 : P_{1347} = (0, 20, 1, 0) \text{ lies on line } \ell_2$$

$$22 : P_{1411} = (0, 21, 1, 0) \text{ lies on line } \ell_2$$

$$23 : P_{1475} = (0, 22, 1, 0) \text{ lies on line } \ell_2$$

$$24 : P_{1539} = (0, 23, 1, 0) \text{ lies on line } \ell_2$$

$$25 : P_{1603} = (0, 24, 1, 0) \text{ lies on line } \ell_2$$

$$26 : P_{1667} = (0, 25, 1, 0) \text{ lies on line } \ell_2$$

$$27 : P_{1731} = (0, 26, 1, 0) \text{ lies on line } \ell_2$$

$$28 : P_{1795} = (0, 27, 1, 0) \text{ lies on line } \ell_2$$

$$29 : P_{1859} = (0, 28, 1, 0) \text{ lies on line } \ell_2$$

$$30 : P_{1923} = (0, 29, 1, 0) \text{ lies on line } \ell_2$$

$$31 : P_{1987} = (0, 30, 1, 0) \text{ lies on line } \ell_2$$

$$32 : P_{2051} = (0, 31, 1, 0) \text{ lies on line } \ell_2$$

$$33 : P_{2115} = (0, 32, 1, 0) \text{ lies on line } \ell_2$$

$$34 : P_{2179} = (0, 33, 1, 0) \text{ lies on line } \ell_2$$

$$35 : P_{2243} = (0, 34, 1, 0) \text{ lies on line } \ell_2$$

144 : $P_{3740} = (25, 57, 1, 0)$ lies on line ℓ_0
145 : $P_{3741} = (26, 57, 1, 0)$ lies on line ℓ_0
146 : $P_{3742} = (27, 57, 1, 0)$ lies on line ℓ_0
147 : $P_{3743} = (28, 57, 1, 0)$ lies on line ℓ_0
148 : $P_{3744} = (29, 57, 1, 0)$ lies on line ℓ_0
149 : $P_{3745} = (30, 57, 1, 0)$ lies on line ℓ_0
150 : $P_{3746} = (31, 57, 1, 0)$ lies on line ℓ_0
151 : $P_{3747} = (32, 57, 1, 0)$ lies on line ℓ_0
152 : $P_{3748} = (33, 57, 1, 0)$ lies on line ℓ_0
153 : $P_{3749} = (34, 57, 1, 0)$ lies on line ℓ_0
154 : $P_{3750} = (35, 57, 1, 0)$ lies on line ℓ_0
155 : $P_{3751} = (36, 57, 1, 0)$ lies on line ℓ_0
156 : $P_{3752} = (37, 57, 1, 0)$ lies on line ℓ_0
157 : $P_{3753} = (38, 57, 1, 0)$ lies on line ℓ_0
158 : $P_{3754} = (39, 57, 1, 0)$ lies on line ℓ_0
159 : $P_{3755} = (40, 57, 1, 0)$ lies on line ℓ_0
160 : $P_{3756} = (41, 57, 1, 0)$ lies on line ℓ_0
161 : $P_{3757} = (42, 57, 1, 0)$ lies on line ℓ_0
162 : $P_{3758} = (43, 57, 1, 0)$ lies on line ℓ_0
163 : $P_{3759} = (44, 57, 1, 0)$ lies on line ℓ_0
164 : $P_{3760} = (45, 57, 1, 0)$ lies on line ℓ_0
165 : $P_{3761} = (46, 57, 1, 0)$ lies on line ℓ_0
166 : $P_{3762} = (47, 57, 1, 0)$ lies on line ℓ_0
167 : $P_{3763} = (48, 57, 1, 0)$ lies on line ℓ_0
168 : $P_{3764} = (49, 57, 1, 0)$ lies on line ℓ_0
169 : $P_{3765} = (50, 57, 1, 0)$ lies on line ℓ_0
170 : $P_{3766} = (51, 57, 1, 0)$ lies on line ℓ_0
171 : $P_{3767} = (52, 57, 1, 0)$ lies on line ℓ_0
172 : $P_{3768} = (53, 57, 1, 0)$ lies on line ℓ_0
173 : $P_{3769} = (54, 57, 1, 0)$ lies on line ℓ_0
174 : $P_{3770} = (55, 57, 1, 0)$ lies on line ℓ_0
175 : $P_{3771} = (56, 57, 1, 0)$ lies on line ℓ_0
176 : $P_{3772} = (57, 57, 1, 0)$ lies on line ℓ_0
177 : $P_{3773} = (58, 57, 1, 0)$ lies on line ℓ_0
178 : $P_{3774} = (59, 57, 1, 0)$ lies on line ℓ_0
179 : $P_{3775} = (60, 57, 1, 0)$ lies on line ℓ_0
180 : $P_{3776} = (61, 57, 1, 0)$ lies on line ℓ_0
181 : $P_{3777} = (62, 57, 1, 0)$ lies on line ℓ_0
182 : $P_{3778} = (63, 57, 1, 0)$ lies on line ℓ_0
183 : $P_{3779} = (0, 58, 1, 0)$ lies on line ℓ_2
184 : $P_{3843} = (0, 59, 1, 0)$ lies on line ℓ_2
185 : $P_{3907} = (0, 60, 1, 0)$ lies on line ℓ_2
186 : $P_{3971} = (0, 61, 1, 0)$ lies on line ℓ_2
187 : $P_{4035} = (0, 62, 1, 0)$ lies on line ℓ_2
188 : $P_{4099} = (0, 63, 1, 0)$ lies on line ℓ_2
189 : $P_{11842} = (1, 56, 1, 1)$ lies on line ℓ_3
190 : $P_{11906} = (1, 57, 1, 1)$ lies on line ℓ_4
191 : $P_{13442} = (1, 17, 2, 1)$ lies on line ℓ_3
192 : $P_{13570} = (1, 19, 2, 1)$ lies on line ℓ_4
193 : $P_{19074} = (1, 41, 3, 1)$ lies on line ℓ_3
194 : $P_{19138} = (1, 42, 3, 1)$ lies on line ℓ_4
195 : $P_{22722} = (1, 34, 4, 1)$ lies on line ℓ_3
196 : $P_{22978} = (1, 38, 4, 1)$ lies on line ℓ_4
197 : $P_{26306} = (1, 26, 5, 1)$ lies on line ℓ_3
198 : $P_{26626} = (1, 31, 5, 1)$ lies on line ℓ_4
199 : $P_{32002} = (1, 51, 6, 1)$ lies on line ℓ_3
200 : $P_{32130} = (1, 53, 6, 1)$ lies on line ℓ_4
201 : $P_{33538} = (1, 11, 7, 1)$ lies on line ℓ_3
202 : $P_{33602} = (1, 12, 7, 1)$ lies on line ℓ_4
203 : $P_{39298} = (1, 37, 8, 1)$ lies on line ℓ_3
204 : $P_{39810} = (1, 45, 8, 1)$ lies on line ℓ_4
205 : $P_{42306} = (1, 20, 9, 1)$ lies on line ℓ_4
206 : $P_{42882} = (1, 29, 9, 1)$ lies on line ℓ_3
207 : $P_{48450} = (1, 52, 10, 1)$ lies on line ℓ_3
208 : $P_{49090} = (1, 62, 10, 1)$ lies on line ℓ_4
209 : $P_{49666} = (1, 7, 11, 1)$ lies on line ℓ_4
210 : $P_{49986} = (1, 12, 11, 1)$ lies on line ℓ_3
211 : $P_{53762} = (1, 7, 12, 1)$ lies on line ℓ_3
212 : $P_{54018} = (1, 11, 12, 1)$ lies on line ℓ_4
213 : $P_{60610} = (1, 50, 13, 1)$ lies on line ℓ_4
214 : $P_{61442} = (1, 63, 13, 1)$ lies on line ℓ_3
215 : $P_{62914} = (1, 22, 14, 1)$ lies on line ℓ_3
216 : $P_{63042} = (1, 24, 14, 1)$ lies on line ℓ_4
217 : $P_{67714} = (1, 33, 15, 1)$ lies on line ℓ_4
218 : $P_{68546} = (1, 46, 15, 1)$ lies on line ℓ_3
219 : $P_{72450} = (1, 43, 16, 1)$ lies on line ℓ_3
220 : $P_{73474} = (1, 59, 16, 1)$ lies on line ℓ_4
221 : $P_{73922} = (1, 2, 17, 1)$ lies on line ℓ_4
222 : $P_{75010} = (1, 19, 17, 1)$ lies on line ℓ_3
223 : $P_{80450} = (1, 40, 18, 1)$ lies on line ℓ_4
224 : $P_{81602} = (1, 58, 18, 1)$ lies on line ℓ_3
225 : $P_{82114} = (1, 2, 19, 1)$ lies on line ℓ_3
226 : $P_{83074} = (1, 17, 19, 1)$ lies on line ℓ_4
227 : $P_{86658} = (1, 9, 20, 1)$ lies on line ℓ_3
228 : $P_{87938} = (1, 29, 20, 1)$ lies on line ℓ_4
229 : $P_{92482} = (1, 36, 21, 1)$ lies on line ℓ_4
230 : $P_{93314} = (1, 49, 21, 1)$ lies on line ℓ_3
231 : $P_{95170} = (1, 14, 22, 1)$ lies on line ℓ_4
232 : $P_{95810} = (1, 24, 22, 1)$ lies on line ℓ_3
233 : $P_{100418} = (1, 32, 23, 1)$ lies on line ℓ_3
234 : $P_{101890} = (1, 55, 23, 1)$ lies on line ℓ_4
235 : $P_{103362} = (1, 14, 24, 1)$ lies on line ℓ_3
236 : $P_{103874} = (1, 22, 24, 1)$ lies on line ℓ_4
237 : $P_{109570} = (1, 47, 25, 1)$ lies on line ℓ_4
238 : $P_{110018} = (1, 54, 25, 1)$ lies on line ℓ_3
239 : $P_{110978} = (1, 5, 26, 1)$ lies on line ℓ_4
240 : $P_{112642} = (1, 31, 26, 1)$ lies on line ℓ_3
241 : $P_{117250} = (1, 39, 27, 1)$ lies on line ℓ_3
242 : $P_{118594} = (1, 60, 27, 1)$ lies on line ℓ_4
243 : $P_{121666} = (1, 44, 28, 1)$ lies on line ℓ_3
244 : $P_{121922} = (1, 48, 28, 1)$ lies on line ℓ_4
245 : $P_{123522} = (1, 9, 29, 1)$ lies on line ℓ_4
246 : $P_{124226} = (1, 20, 29, 1)$ lies on line ℓ_3
247 : $P_{129282} = (1, 35, 30, 1)$ lies on line ℓ_4
248 : $P_{130946} = (1, 61, 30, 1)$ lies on line ℓ_3
249 : $P_{131458} = (1, 5, 31, 1)$ lies on line ℓ_3
250 : $P_{132802} = (1, 26, 31, 1)$ lies on line ℓ_4
251 : $P_{136706} = (1, 23, 32, 1)$ lies on line ℓ_4

252 : $P_{138754} = (1, 55, 32, 1)$ lies on line ℓ_3
 253 : $P_{140290} = (1, 15, 33, 1)$ lies on line ℓ_3
 254 : $P_{142274} = (1, 46, 33, 1)$ lies on line ℓ_4
 255 : $P_{143682} = (1, 4, 34, 1)$ lies on line ℓ_4
 256 : $P_{145858} = (1, 38, 34, 1)$ lies on line ℓ_3
 257 : $P_{149442} = (1, 30, 35, 1)$ lies on line ℓ_3
 258 : $P_{151426} = (1, 61, 35, 1)$ lies on line ℓ_4
 259 : $P_{152962} = (1, 21, 36, 1)$ lies on line ℓ_3
 260 : $P_{154754} = (1, 49, 36, 1)$ lies on line ℓ_4
 261 : $P_{156226} = (1, 8, 37, 1)$ lies on line ℓ_4
 262 : $P_{158594} = (1, 45, 37, 1)$ lies on line ℓ_3
 263 : $P_{160066} = (1, 4, 38, 1)$ lies on line ℓ_3
 264 : $P_{161986} = (1, 34, 38, 1)$ lies on line ℓ_4
 265 : $P_{165634} = (1, 27, 39, 1)$ lies on line ℓ_4
 266 : $P_{167746} = (1, 60, 39, 1)$ lies on line ℓ_3
 267 : $P_{169154} = (1, 18, 40, 1)$ lies on line ℓ_3
 268 : $P_{171714} = (1, 58, 40, 1)$ lies on line ℓ_4
 269 : $P_{172290} = (1, 3, 41, 1)$ lies on line ℓ_4
 270 : $P_{174786} = (1, 42, 41, 1)$ lies on line ℓ_3
 271 : $P_{176386} = (1, 3, 42, 1)$ lies on line ℓ_3
 272 : $P_{178818} = (1, 41, 42, 1)$ lies on line ℓ_4
 273 : $P_{181314} = (1, 16, 43, 1)$ lies on line ℓ_4
 274 : $P_{184066} = (1, 59, 43, 1)$ lies on line ℓ_3
 275 : $P_{186178} = (1, 28, 44, 1)$ lies on line ℓ_4
 276 : $P_{187458} = (1, 48, 44, 1)$ lies on line ℓ_3
 277 : $P_{188994} = (1, 8, 45, 1)$ lies on line ℓ_3
 278 : $P_{190850} = (1, 37, 45, 1)$ lies on line ℓ_4
 279 : $P_{193538} = (1, 15, 46, 1)$ lies on line ℓ_4
 280 : $P_{194690} = (1, 33, 46, 1)$ lies on line ℓ_3
 281 : $P_{198274} = (1, 25, 47, 1)$ lies on line ℓ_3
 282 : $P_{200130} = (1, 54, 47, 1)$ lies on line ℓ_4
 283 : $P_{202562} = (1, 28, 48, 1)$ lies on line ℓ_3
 284 : $P_{203586} = (1, 44, 48, 1)$ lies on line ℓ_4
 285 : $P_{206210} = (1, 21, 49, 1)$ lies on line ℓ_4
 286 : $P_{207170} = (1, 36, 49, 1)$ lies on line ℓ_3
 287 : $P_{209794} = (1, 13, 50, 1)$ lies on line ℓ_3
 288 : $P_{212994} = (1, 63, 50, 1)$ lies on line ℓ_4
 289 : $P_{213442} = (1, 6, 51, 1)$ lies on line ℓ_4
 290 : $P_{216450} = (1, 53, 51, 1)$ lies on line ℓ_3
 291 : $P_{217794} = (1, 10, 52, 1)$ lies on line ℓ_4
 292 : $P_{221122} = (1, 62, 52, 1)$ lies on line ℓ_3
 293 : $P_{221634} = (1, 6, 53, 1)$ lies on line ℓ_3
 294 : $P_{224514} = (1, 51, 53, 1)$ lies on line ℓ_4
 295 : $P_{226946} = (1, 25, 54, 1)$ lies on line ℓ_4
 296 : $P_{228354} = (1, 47, 54, 1)$ lies on line ℓ_3
 297 : $P_{230914} = (1, 23, 55, 1)$ lies on line ℓ_3
 298 : $P_{231490} = (1, 32, 55, 1)$ lies on line ℓ_4
 299 : $P_{233602} = (1, 1, 56, 1)$ lies on line ℓ_4
 300 : $P_{237186} = (1, 57, 56, 1)$ lies on line ℓ_3
 301 : $P_{237698} = (1, 1, 57, 1)$ lies on line ℓ_3
 302 : $P_{241218} = (1, 56, 57, 1)$ lies on line ℓ_4
 303 : $P_{242882} = (1, 18, 58, 1)$ lies on line ℓ_4
 304 : $P_{244290} = (1, 40, 58, 1)$ lies on line ℓ_3
 305 : $P_{246850} = (1, 16, 59, 1)$ lies on line ℓ_3
 306 : $P_{248578} = (1, 43, 59, 1)$ lies on line ℓ_4
 307 : $P_{251650} = (1, 27, 60, 1)$ lies on line ℓ_3
 308 : $P_{252418} = (1, 39, 60, 1)$ lies on line ℓ_4
 309 : $P_{255938} = (1, 30, 61, 1)$ lies on line ℓ_4
 310 : $P_{256258} = (1, 35, 61, 1)$ lies on line ℓ_3
 311 : $P_{258754} = (1, 10, 62, 1)$ lies on line ℓ_3
 312 : $P_{261442} = (1, 52, 62, 1)$ lies on line ℓ_4
 313 : $P_{263042} = (1, 13, 63, 1)$ lies on line ℓ_4
 314 : $P_{265410} = (1, 50, 63, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 3906 points not on any line:
Too many to print.

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	1
1	1	0	1	1	0
2	1	1	0	1	1
3	0	1	1	0	1
4	1	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_0	P_{3715}	P_{3715}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_{3651}	P_{3651}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{3715}	P_{3651}	P_{3651}	P_{3715}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{3651}	P_{3651}	P_{4163}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_{3715}	P_{3715}	P_{4163}

The surface has 4225 points:
Too many to print.