Rank-35 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 117

General information

Number of lines	5
Number of points	21
Number of singular points	5
Number of Eckardt points	0
Number of double points	0
Number of single points	20
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$5, 1^{20}$

Singular Points

The surface has 5 singular points:

0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$
1 : $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$
2 : $P_{26} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$

3 : $P_{30} = \mathbf{P}(0, \omega, 0, 1) = \mathbf{P}(0, 2, 0, 1)$
4 : $P_{34} = \mathbf{P}(0, \omega^2, 0, 1) = \mathbf{P}(0, 3, 0, 1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_{25}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{293} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{293} = \mathbf{Pl}(0,3,0,1,1,0)_{59}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209} = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209} = \mathbf{Pl}(0,2,0,1,1,0)_{58}$$

Rank of lines: (20, 340, 125, 293, 209)
Rank of points on Klein quadric: (25, 9, 57, 59, 58)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 20 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_4 = (1,1,1,1)$ lies on line ℓ_2
3 : $P_{12} = (1,1,1,0)$ lies on line ℓ_2
4 : $P_{17} = (2,2,1,0)$ lies on line ℓ_3
5 : $P_{22} = (3,3,1,0)$ lies on line ℓ_4
6 : $P_{23} = (1,0,0,1)$ lies on line ℓ_0
7 : $P_{24} = (2,0,0,1)$ lies on line ℓ_0
8 : $P_{25} = (3,0,0,1)$ lies on line ℓ_0
9 : $P_{26} = (0,1,0,1)$ lies on line ℓ_1
10 : $P_{30} = (0,2,0,1)$ lies on line ℓ_1

11 : $P_{34} = (0,3,0,1)$ lies on line ℓ_1
12 : $P_{47} = (2,2,1,1)$ lies on line ℓ_3
13 : $P_{52} = (3,3,1,1)$ lies on line ℓ_4
14 : $P_{58} = (1,1,2,1)$ lies on line ℓ_4
15 : $P_{63} = (2,2,2,1)$ lies on line ℓ_2
16 : $P_{68} = (3,3,2,1)$ lies on line ℓ_3
17 : $P_{74} = (1,1,3,1)$ lies on line ℓ_3
18 : $P_{79} = (2,2,3,1)$ lies on line ℓ_4
19 : $P_{84} = (3,3,3,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	1	1	1
2	1	1	0	1	1
3	1	1	1	0	1
4	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3	P_3

The surface has 21 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$

$1: P_1 = (0,1,0,0)$

$2: P_3 = (0,0,0,1)$

$3: P_4 = (1,1,1,1)$

$4: P_{12} = (1,1,1,0)$

$5: P_{17} = (2,2,1,0)$

$6: P_{22} = (3,3,1,0)$

$7: P_{23} = (1,0,0,1)$

$8: P_{24} = (2,0,0,1)$

$9: P_{25} = (3,0,0,1)$

$10: P_{26} = (0,1,0,1)$

$11: P_{30} = (0,2,0,1)$

$12: P_{34} = (0,3,0,1)$

$13: P_{47} = (2,2,1,1)$

$14: P_{52} = (3,3,1,1)$

$15: P_{58} = (1,1,2,1)$

$16: P_{63} = (2,2,2,1)$

$17: P_{68} = (3,3,2,1)$

$18: P_{74} = (1,1,3,1)$

$19: P_{79} = (2,2,3,1)$

$20: P_{84} = (3,3,3,1)$