Rank-265 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_0 X_1^2 = 0$$

(1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(4)$ is 10931

General information

Number of lines	7
Number of points	29
Number of singular points	0
Number of Eckardt points	3
Number of double points	0
Number of single points	26
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	3^3, 1^{26}

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_3 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{80} = \mathbf{Pl}(0,0,1,1,2,1)_{258}$$

$$\ell_4 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{59} = \mathbf{Pl}(0,0,1,1,3,1)_{318}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \mathbf{Pl}(3,2,2,3,2,1)_{290}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \mathbf{Pl}(2,3,3,2,3,1)_{352}$$

Rank of lines: (337, 339, 338, 80, 59, 236, 299)
Rank of points on Klein quadric: (129, 143, 136, 258, 318, 290, 352)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),$
$1 : P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1),$
$2 : P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 26 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line ℓ_3
$1 : P_7 = (3,1,0,0)$ lies on line ℓ_4
$2 : P_{16} = (1,2,1,0)$ lies on line ℓ_5
$3 : P_{20} = (1,3,1,0)$ lies on line ℓ_6
$4 : P_{31} = (1,2,0,1)$ lies on line ℓ_6
$5 : P_{35} = (1,3,0,1)$ lies on line ℓ_5
$6 : P_{43} = (2,1,1,1)$ lies on line ℓ_3
$7 : P_{44} = (3,1,1,1)$ lies on line ℓ_4
$8 : P_{45} = (0,2,1,1)$ lies on line ℓ_0
$9 : P_{46} = (1,2,1,1)$ lies on line ℓ_4
$10 : P_{48} = (3,2,1,1)$ lies on line ℓ_3
$11 : P_{49} = (0,3,1,1)$ lies on line ℓ_0
$12 : P_{50} = (1,3,1,1)$ lies on line ℓ_3
$13 : P_{51} = (2,3,1,1)$ lies on line ℓ_4

$14 : P_{53} = (0,0,2,1)$ lies on line ℓ_1
$15 : P_{56} = (3,0,2,1)$ lies on line ℓ_5
$16 : P_{57} = (0,1,2,1)$ lies on line ℓ_1
$17 : P_{61} = (0,2,2,1)$ lies on line ℓ_1
$18 : P_{65} = (0,3,2,1)$ lies on line ℓ_1
$19 : P_{68} = (3,3,2,1)$ lies on line ℓ_6
$20 : P_{69} = (0,0,3,1)$ lies on line ℓ_2
$21 : P_{71} = (2,0,3,1)$ lies on line ℓ_6
$22 : P_{73} = (0,1,3,1)$ lies on line ℓ_2
$23 : P_{77} = (0,2,3,1)$ lies on line ℓ_2
$24 : P_{79} = (2,2,3,1)$ lies on line ℓ_5
$25 : P_{81} = (0,3,3,1)$ lies on line ℓ_2

The single points on the surface are:

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Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 1 1 1 1
1	1 0 1 0 0 0 0
2	1 1 0 0 0 0 0
3	1 0 0 0 1 0 0
4	1 0 0 1 0 0 0
5	1 0 0 0 0 0 1
6	1 0 0 0 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_1	P_1	P_{38}	P_{38}	P_{42}	P_{42}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_1

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_1

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_{38}	P_{38}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_{38}	P_{38}

Line 5 intersects

Line	ℓ_0	ℓ_6
in point	P_{42}	P_{42}

Line 6 intersects

Line	ℓ_0	ℓ_5
in point	P_{42}	P_{42}

The surface has 29 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_6 = (2, 1, 0, 0)$
2 : $P_7 = (3, 1, 0, 0)$
3 : $P_{16} = (1, 2, 1, 0)$
4 : $P_{20} = (1, 3, 1, 0)$
5 : $P_{31} = (1, 2, 0, 1)$
6 : $P_{35} = (1, 3, 0, 1)$
7 : $P_{38} = (0, 0, 1, 1)$

8 : $P_{42} = (0, 1, 1, 1)$
9 : $P_{43} = (2, 1, 1, 1)$
10 : $P_{44} = (3, 1, 1, 1)$
11 : $P_{45} = (0, 2, 1, 1)$
12 : $P_{46} = (1, 2, 1, 1)$
13 : $P_{48} = (3, 2, 1, 1)$
14 : $P_{49} = (0, 3, 1, 1)$
15 : $P_{50} = (1, 3, 1, 1)$

16 : $P_{51} = (2, 3, 1, 1)$
17 : $P_{53} = (0, 0, 2, 1)$
18 : $P_{56} = (3, 0, 2, 1)$
19 : $P_{57} = (0, 1, 2, 1)$
20 : $P_{61} = (0, 2, 2, 1)$
21 : $P_{65} = (0, 3, 2, 1)$
22 : $P_{68} = (3, 3, 2, 1)$
23 : $P_{69} = (0, 0, 3, 1)$

$24:\ P_{71} = (2,0,3,1)$

$25:\ P_{73} = (0,1,3,1)$

$26:\ P_{77} = (0,2,3,1)$

$27:\ P_{79} = (2,2,3,1)$

$28:\ P_{81} = (0,3,3,1)$