Rank-10566 over GF(2)

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The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 10566

General information

Number of lines	15
Number of points	15
Number of singular points	0
Number of Eckardt points	15
Number of double points	0
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{15}
Type of lines on points	3^{15}

Singular Points

The surface has 0 singular points:

The 15 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

1

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

$$\ell_7 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1,0,1,1,1,1)_{33}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,1,0,0,1,1)_{29}$$

$$\ell_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1,1,1,0,1,1)_{30}$$

$$\ell_{11} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \mathbf{Pl}(1,1,1,1,0,0)_8$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

$$\ell_{14} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(0,1,1,1,1,1)_{34}$$

Rank of lines: (0, 1, 4, 30, 14, 34, 18, 12, 24, 9, 10, 33, 15, 23, 26)
Rank of points on Klein quadric: (0, 3, 2, 5, 6, 1, 4, 32, 33, 29, 30, 7, 8, 31, 34)

Eckardt Points

The surface has 15 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0),\ T = 0$
$1 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),\ T = 2$
$2 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0),\ T = 6$
$3 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1),\ T = 14$
$4 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1),\ T = 11$
$5 : P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0),\ T = 1$
$6 : P_6 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0),\ T = 3$
$7 : P_7 = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0),\ T = 5$
$8 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0),\ T = 4$
$9 : P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1),\ T = 7$
$10 : P_{10} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1),\ T = 9$
$11 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1),\ T = 8$
$12 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1),\ T = 13$
$13 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1),\ T = 10$
$14 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 12$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0	0 1 1 1 1 1 0 0 1 1 0 0 0 0 0
1	1 0 1 0 0 0 0 0 0 0 0 1 1 1 1
2	1 1 0 0 0 1 1 0 0 1 1 0 0 0 0
3	1 0 0 0 1 1 0 0 0 1 0 1 0 1 0
4	1 0 0 1 0 0 1 0 0 0 1 0 1 0 1
5	0 0 1 1 0 0 1 1 0 0 0 1 0 0 1
6	0 0 1 0 1 1 0 0 1 0 0 0 1 1 0
7	1 0 0 0 0 1 0 0 1 1 0 0 1 0 1
8	1 0 0 0 0 0 1 1 0 0 1 1 0 1 0
9	0 0 1 1 0 0 0 1 0 0 1 0 1 1 0
10	0 0 1 0 1 0 0 0 1 1 0 1 0 0 1
11	0 1 0 1 0 1 0 0 1 0 1 0 1 0 0
12	0 1 0 0 0 1 0 1 1 0 1 0 1 0 0 0
13	0 1 0 1 0 0 1 0 1 0 1 1 0 0 0 0 1
14	0 1 0 0 1 1 0 1 0 1 0 0 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_0	P_0	P_1	P_1	P_5	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}
in point	P_0	P_0	P_7	P_7	P_8	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}
in point	P_0	P_0	P_2	P_2	P_6	P_6

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{13}
in point	P_1	P_1	P_3	P_{10}	P_3	P_{10}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_1	P_1	P_9	P_{11}	P_9	P_{11}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{14}
in point	P_2	P_3	P_2	P_{12}	P_3	P_{12}

Line 6 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_8	ℓ_{12}	ℓ_{13}
in point	P_2	P_9	P_2	P_{13}	P_9	P_{13}

Line 7 intersects

Line	ℓ_0	ℓ_5	ℓ_8	ℓ_9	ℓ_{12}	ℓ_{14}
in point	P_5	P_{12}	P_5	P_4	P_4	P_{12}

Line 8 intersects

Line	ℓ_0	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{13}
in point	P_5	P_{13}	P_5	P_{14}	P_{14}	P_{13}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{13}
in point	P_6	P_{10}	P_4	P_6	P_4	P_{10}

Line 10 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{14}
in point	P_6	P_{11}	P_{14}	P_6	P_{14}	P_{11}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8	ℓ_{10}	ℓ_{12}
in point	P_7	P_3	P_3	P_{14}	P_{14}	P_7

Line 12 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{11}
in point	P_7	P_9	P_9	P_4	P_4	P_7

Line 13 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{14}
in point	P_8	P_{10}	P_{13}	P_{13}	P_{10}	P_8

Line 14 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{13}
in point	P_8	P_{11}	P_{12}	P_{12}	P_{11}	P_8

The surface has 15 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$

1 : $P_1 = (0,1,0,0)$

2 : $P_2 = (0,0,1,0)$

3 : $P_3 = (0,0,0,1)$

4 : $P_4 = (1,1,1,1)$

5 : $P_5 = (1,1,0,0)$

6 : $P_6 = (1,0,1,0)$

7 : $P_7 = (0,1,1,0)$

8 : $P_8 = (1,1,1,0)$

9 : $P_9 = (1,0,0,1)$

10 : $P_{10} = (0,1,0,1)$

11 : $P_{11} = (1,1,0,1)$

12 : $P_{12} = (0,0,1,1)$

13 : $P_{13} = (1,0,1,1)$

14 : $P_{14} = (0,1,1,1)$