

Rank-331 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(16) is 303108381

General information

Number of lines	19
Number of points	289
Number of singular points	17
Number of Eckardt points	0
Number of double points	34
Number of single points	255
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{19}
Type of lines on points	$2^{34}, 1^{255}$

Singular Points

The surface has 17 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{530} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{785} = \mathbf{P}(0, 0, \delta, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{1041} = \mathbf{P}(0, 0, \delta^{12}, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{1297} = \mathbf{P}(0, 0, \delta^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{1553} = \mathbf{P}(0, 0, \delta^9, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{1809} = \mathbf{P}(0, 0, \delta^{13}, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{2065} = \mathbf{P}(0, 0, \delta^7, 1) = \mathbf{P}(0, 0, 7, 1)$$

$$9 : P_{2321} = \mathbf{P}(0, 0, \delta^3, 1) = \mathbf{P}(0, 0, 8, 1)$$

$$10 : P_{2577} = \mathbf{P}(0, 0, \delta^4, 1) = \mathbf{P}(0, 0, 9, 1)$$

$$11 : P_{2833} = \mathbf{P}(0, 0, \delta^{10}, 1) = \mathbf{P}(0, 0, 10, 1)$$

$$12 : P_{3089} = \mathbf{P}(0, 0, \delta^5, 1) = \mathbf{P}(0, 0, 11, 1)$$

$$13 : P_{3345} = \mathbf{P}(0, 0, \delta^{14}, 1) = \mathbf{P}(0, 0, 12, 1)$$

$$14 : P_{3601} = \mathbf{P}(0, 0, \delta^{11}, 1) = \mathbf{P}(0, 0, 13, 1)$$

$$15 : P_{3857} = \mathbf{P}(0, 0, \delta^8, 1) = \mathbf{P}(0, 0, 14, 1)$$

$$16 : P_{4113} = \mathbf{P}(0, 0, \delta^6, 1) = \mathbf{P}(0, 0, 15, 1)$$

The 19 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{33} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{9426} \\
\ell_5 &= \begin{bmatrix} 1 & \delta^{14} & 0 & 0 \\ 0 & 0 & 1 & \delta^{13} \end{bmatrix}_{3538} = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{3538} = \mathbf{Pl}(0, 0, 4, 6, 12, 1)_{54399} \\
\ell_6 &= \begin{bmatrix} 1 & \delta^3 & 0 & 0 \\ 0 & 0 & 1 & \delta^6 \end{bmatrix}_{2455} = \begin{bmatrix} 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 15 \end{bmatrix}_{2455} = \mathbf{Pl}(0, 0, 5, 15, 8, 1)_{38110} \\
\ell_7 &= \begin{bmatrix} 1 & \delta^{13} & 0 & 0 \\ 0 & 0 & 1 & \delta^{11} \end{bmatrix}_{1907} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{1907} = \mathbf{Pl}(0, 0, 9, 13, 6, 1)_{30074} \\
\ell_8 &= \begin{bmatrix} 1 & \delta^6 & 0 & 0 \\ 0 & 0 & 1 & \delta^{12} \end{bmatrix}_{4354} = \begin{bmatrix} 1 & 15 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{4354} = \mathbf{Pl}(0, 0, 8, 3, 15, 1)_{66763} \\
\ell_9 &= \begin{bmatrix} 1 & \delta^2 & 0 & 0 \\ 0 & 0 & 1 & \delta^4 \end{bmatrix}_{1357} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{bmatrix}_{1357} = \mathbf{Pl}(0, 0, 13, 9, 4, 1)_{22038} \\
\ell_{10} &= \begin{bmatrix} 1 & \delta^8 & 0 & 0 \\ 0 & 0 & 1 & \delta \end{bmatrix}_{4080} = \begin{bmatrix} 1 & 14 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{4080} = \mathbf{Pl}(0, 0, 12, 2, 14, 1)_{62807} \\
\ell_{11} &= \begin{bmatrix} 1 & \delta^{12} & 0 & 0 \\ 0 & 0 & 1 & \delta^9 \end{bmatrix}_{1080} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{1080} = \mathbf{Pl}(0, 0, 15, 5, 3, 1)_{18020} \\
\ell_{12} &= \begin{bmatrix} 1 & \delta^{11} & 0 & 0 \\ 0 & 0 & 1 & \delta^7 \end{bmatrix}_{3812} = \begin{bmatrix} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{3812} = \mathbf{Pl}(0, 0, 14, 7, 13, 1)_{58789} \\
\ell_{13} &= \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3269} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3269} = \mathbf{Pl}(0, 0, 11, 10, 11, 1)_{50536} \\
\ell_{14} &= \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{2997} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{2997} = \mathbf{Pl}(0, 0, 10, 11, 10, 1)_{46425} \\
\ell_{15} &= \begin{bmatrix} 1 & \delta & 0 & 0 \\ 0 & 0 & 1 & \delta^2 \end{bmatrix}_{806} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{806} = \mathbf{Pl}(0, 0, 6, 4, 2, 1)_{13661} \\
\ell_{16} &= \begin{bmatrix} 1 & \delta^4 & 0 & 0 \\ 0 & 0 & 1 & \delta^8 \end{bmatrix}_{2727} = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 14 \end{bmatrix}_{2727} = \mathbf{Pl}(0, 0, 7, 14, 9, 1)_{42252} \\
\ell_{17} &= \begin{bmatrix} 1 & \delta^7 & 0 & 0 \\ 0 & 0 & 1 & \delta^{14} \end{bmatrix}_{2179} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{2179} = \mathbf{Pl}(0, 0, 2, 12, 7, 1)_{33937} \\
\ell_{18} &= \begin{bmatrix} 1 & \delta^9 & 0 & 0 \\ 0 & 0 & 1 & \delta^3 \end{bmatrix}_{1629} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{bmatrix}_{1629} = \mathbf{Pl}(0, 0, 3, 8, 5, 1)_{25808}
\end{aligned}$$

Rank of lines: (0, 256, 69904, 70160, 530, 3538, 2455, 1907, 4354, 1357, 4080, 1080, 3812, 3269, 2997, 806, 2727, 2179, 1629)

Rank of points on Klein quadric: (0, 2, 33, 1, 9426, 54399, 38110, 30074, 66763, 22038, 62807, 18020, 58789, 50536, 46425, 13661, 42252, 33937, 25808)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 34 Double points:

The double points on the surface are:

$$\begin{aligned} P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 \\ P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_2 \\ P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_4 \\ P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_5 \\ P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_6 \\ P_8 &= (4, 1, 0, 0) = \ell_0 \cap \ell_7 \\ P_9 &= (5, 1, 0, 0) = \ell_0 \cap \ell_8 \\ P_{10} &= (6, 1, 0, 0) = \ell_0 \cap \ell_9 \\ P_{11} &= (7, 1, 0, 0) = \ell_0 \cap \ell_{10} \\ P_{12} &= (8, 1, 0, 0) = \ell_0 \cap \ell_{11} \\ P_{13} &= (9, 1, 0, 0) = \ell_0 \cap \ell_{12} \\ P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_{13} \\ P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_{14} \\ P_{16} &= (12, 1, 0, 0) = \ell_0 \cap \ell_{15} \\ P_{17} &= (13, 1, 0, 0) = \ell_0 \cap \ell_{16} \\ P_{18} &= (14, 1, 0, 0) = \ell_0 \cap \ell_{17} \\ P_{19} &= (15, 1, 0, 0) = \ell_0 \cap \ell_{18} \\ P_2 &= (0, 0, 1, 0) = \ell_1 \cap \ell_3 \end{aligned}$$

$$\begin{aligned} P_3 &= (0, 0, 0, 1) = \ell_2 \cap \ell_3 \\ P_{530} &= (0, 0, 1, 1) = \ell_3 \cap \ell_4 \\ P_{1297} &= (0, 0, 4, 1) = \ell_3 \cap \ell_5 \\ P_{1553} &= (0, 0, 5, 1) = \ell_3 \cap \ell_6 \\ P_{2577} &= (0, 0, 9, 1) = \ell_3 \cap \ell_7 \\ P_{2321} &= (0, 0, 8, 1) = \ell_3 \cap \ell_8 \\ P_{3601} &= (0, 0, 13, 1) = \ell_3 \cap \ell_9 \\ P_{3345} &= (0, 0, 12, 1) = \ell_3 \cap \ell_{10} \\ P_{4113} &= (0, 0, 15, 1) = \ell_3 \cap \ell_{11} \\ P_{3857} &= (0, 0, 14, 1) = \ell_3 \cap \ell_{12} \\ P_{3089} &= (0, 0, 11, 1) = \ell_3 \cap \ell_{13} \\ P_{2833} &= (0, 0, 10, 1) = \ell_3 \cap \ell_{14} \\ P_{1809} &= (0, 0, 6, 1) = \ell_3 \cap \ell_{15} \\ P_{2065} &= (0, 0, 7, 1) = \ell_3 \cap \ell_{16} \\ P_{785} &= (0, 0, 2, 1) = \ell_3 \cap \ell_{17} \\ P_{1041} &= (0, 0, 3, 1) = \ell_3 \cap \ell_{18} \end{aligned}$$

Single Points

The surface has 255 single points:

The single points on the surface are:

$$\begin{aligned} 0 : P_4 &= (1, 1, 1, 1) \text{ lies on line } \ell_4 \\ 1 : P_{20} &= (1, 0, 1, 0) \text{ lies on line } \ell_1 \\ 2 : P_{21} &= (2, 0, 1, 0) \text{ lies on line } \ell_1 \\ 3 : P_{22} &= (3, 0, 1, 0) \text{ lies on line } \ell_1 \\ 4 : P_{23} &= (4, 0, 1, 0) \text{ lies on line } \ell_1 \\ 5 : P_{24} &= (5, 0, 1, 0) \text{ lies on line } \ell_1 \\ 6 : P_{25} &= (6, 0, 1, 0) \text{ lies on line } \ell_1 \\ 7 : P_{26} &= (7, 0, 1, 0) \text{ lies on line } \ell_1 \\ 8 : P_{27} &= (8, 0, 1, 0) \text{ lies on line } \ell_1 \\ 9 : P_{28} &= (9, 0, 1, 0) \text{ lies on line } \ell_1 \\ 10 : P_{29} &= (10, 0, 1, 0) \text{ lies on line } \ell_1 \\ 11 : P_{30} &= (11, 0, 1, 0) \text{ lies on line } \ell_1 \\ 12 : P_{31} &= (12, 0, 1, 0) \text{ lies on line } \ell_1 \\ 13 : P_{32} &= (13, 0, 1, 0) \text{ lies on line } \ell_1 \\ 14 : P_{33} &= (14, 0, 1, 0) \text{ lies on line } \ell_1 \\ 15 : P_{34} &= (15, 0, 1, 0) \text{ lies on line } \ell_1 \\ 16 : P_{290} &= (0, 1, 0, 1) \text{ lies on line } \ell_2 \\ 17 : P_{306} &= (0, 2, 0, 1) \text{ lies on line } \ell_2 \\ 18 : P_{322} &= (0, 3, 0, 1) \text{ lies on line } \ell_2 \end{aligned}$$

$$\begin{aligned} 19 : P_{338} &= (0, 4, 0, 1) \text{ lies on line } \ell_2 \\ 20 : P_{354} &= (0, 5, 0, 1) \text{ lies on line } \ell_2 \\ 21 : P_{370} &= (0, 6, 0, 1) \text{ lies on line } \ell_2 \\ 22 : P_{386} &= (0, 7, 0, 1) \text{ lies on line } \ell_2 \\ 23 : P_{402} &= (0, 8, 0, 1) \text{ lies on line } \ell_2 \\ 24 : P_{418} &= (0, 9, 0, 1) \text{ lies on line } \ell_2 \\ 25 : P_{434} &= (0, 10, 0, 1) \text{ lies on line } \ell_2 \\ 26 : P_{450} &= (0, 11, 0, 1) \text{ lies on line } \ell_2 \\ 27 : P_{466} &= (0, 12, 0, 1) \text{ lies on line } \ell_2 \\ 28 : P_{482} &= (0, 13, 0, 1) \text{ lies on line } \ell_2 \\ 29 : P_{498} &= (0, 14, 0, 1) \text{ lies on line } \ell_2 \\ 30 : P_{514} &= (0, 15, 0, 1) \text{ lies on line } \ell_2 \\ 31 : P_{563} &= (2, 2, 1, 1) \text{ lies on line } \ell_4 \\ 32 : P_{580} &= (3, 3, 1, 1) \text{ lies on line } \ell_4 \\ 33 : P_{597} &= (4, 4, 1, 1) \text{ lies on line } \ell_4 \\ 34 : P_{614} &= (5, 5, 1, 1) \text{ lies on line } \ell_4 \\ 35 : P_{631} &= (6, 6, 1, 1) \text{ lies on line } \ell_4 \\ 36 : P_{648} &= (7, 7, 1, 1) \text{ lies on line } \ell_4 \\ 37 : P_{665} &= (8, 8, 1, 1) \text{ lies on line } \ell_4 \end{aligned}$$

38 : $P_{682} = (9, 9, 1, 1)$ lies on line ℓ_4
 39 : $P_{699} = (10, 10, 1, 1)$ lies on line ℓ_4
 40 : $P_{716} = (11, 11, 1, 1)$ lies on line ℓ_4
 41 : $P_{733} = (12, 12, 1, 1)$ lies on line ℓ_4
 42 : $P_{750} = (13, 13, 1, 1)$ lies on line ℓ_4
 43 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_4
 44 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_4
 45 : $P_{815} = (14, 1, 2, 1)$ lies on line ℓ_{17}
 46 : $P_{822} = (5, 2, 2, 1)$ lies on line ℓ_{17}
 47 : $P_{844} = (11, 3, 2, 1)$ lies on line ℓ_{17}
 48 : $P_{859} = (10, 4, 2, 1)$ lies on line ℓ_{17}
 49 : $P_{869} = (4, 5, 2, 1)$ lies on line ℓ_{17}
 50 : $P_{896} = (15, 6, 2, 1)$ lies on line ℓ_{17}
 51 : $P_{898} = (1, 7, 2, 1)$ lies on line ℓ_{17}
 52 : $P_{926} = (13, 8, 2, 1)$ lies on line ℓ_{17}
 53 : $P_{932} = (3, 9, 2, 1)$ lies on line ℓ_{17}
 54 : $P_{953} = (8, 10, 2, 1)$ lies on line ℓ_{17}
 55 : $P_{967} = (6, 11, 2, 1)$ lies on line ℓ_{17}
 56 : $P_{984} = (7, 12, 2, 1)$ lies on line ℓ_{17}
 57 : $P_{1002} = (9, 13, 2, 1)$ lies on line ℓ_{17}
 58 : $P_{1011} = (2, 14, 2, 1)$ lies on line ℓ_{17}
 59 : $P_{1037} = (12, 15, 2, 1)$ lies on line ℓ_{17}
 60 : $P_{1072} = (15, 1, 3, 1)$ lies on line ℓ_{18}
 61 : $P_{1080} = (7, 2, 3, 1)$ lies on line ℓ_{18}
 62 : $P_{1097} = (8, 3, 3, 1)$ lies on line ℓ_{18}
 63 : $P_{1119} = (14, 4, 3, 1)$ lies on line ℓ_{18}
 64 : $P_{1122} = (1, 5, 3, 1)$ lies on line ℓ_{18}
 65 : $P_{1146} = (9, 6, 3, 1)$ lies on line ℓ_{18}
 66 : $P_{1159} = (6, 7, 3, 1)$ lies on line ℓ_{18}
 67 : $P_{1174} = (5, 8, 3, 1)$ lies on line ℓ_{18}
 68 : $P_{1195} = (10, 9, 3, 1)$ lies on line ℓ_{18}
 69 : $P_{1203} = (2, 10, 3, 1)$ lies on line ℓ_{18}
 70 : $P_{1230} = (13, 11, 3, 1)$ lies on line ℓ_{18}
 71 : $P_{1244} = (11, 12, 3, 1)$ lies on line ℓ_{18}
 72 : $P_{1253} = (4, 13, 3, 1)$ lies on line ℓ_{18}
 73 : $P_{1277} = (12, 14, 3, 1)$ lies on line ℓ_{18}
 74 : $P_{1284} = (3, 15, 3, 1)$ lies on line ℓ_{18}
 75 : $P_{1315} = (2, 1, 4, 1)$ lies on line ℓ_5
 76 : $P_{1333} = (4, 2, 4, 1)$ lies on line ℓ_5
 77 : $P_{1351} = (6, 3, 4, 1)$ lies on line ℓ_5
 78 : $P_{1369} = (8, 4, 4, 1)$ lies on line ℓ_5
 79 : $P_{1387} = (10, 5, 4, 1)$ lies on line ℓ_5
 80 : $P_{1405} = (12, 6, 4, 1)$ lies on line ℓ_5
 81 : $P_{1423} = (14, 7, 4, 1)$ lies on line ℓ_5
 82 : $P_{1434} = (9, 8, 4, 1)$ lies on line ℓ_5
 83 : $P_{1452} = (11, 9, 4, 1)$ lies on line ℓ_5
 84 : $P_{1470} = (13, 10, 4, 1)$ lies on line ℓ_5
 85 : $P_{1488} = (15, 11, 4, 1)$ lies on line ℓ_5
 86 : $P_{1490} = (1, 12, 4, 1)$ lies on line ℓ_5
 87 : $P_{1508} = (3, 13, 4, 1)$ lies on line ℓ_5
 88 : $P_{1526} = (5, 14, 4, 1)$ lies on line ℓ_5
 89 : $P_{1544} = (7, 15, 4, 1)$ lies on line ℓ_5
 90 : $P_{1572} = (3, 1, 5, 1)$ lies on line ℓ_6
 91 : $P_{1591} = (6, 2, 5, 1)$ lies on line ℓ_6

92 : $P_{1606} = (5, 3, 5, 1)$ lies on line ℓ_6
 93 : $P_{1629} = (12, 4, 5, 1)$ lies on line ℓ_6
 94 : $P_{1648} = (15, 5, 5, 1)$ lies on line ℓ_6
 95 : $P_{1659} = (10, 6, 5, 1)$ lies on line ℓ_6
 96 : $P_{1674} = (9, 7, 5, 1)$ lies on line ℓ_6
 97 : $P_{1682} = (1, 8, 5, 1)$ lies on line ℓ_6
 98 : $P_{1699} = (2, 9, 5, 1)$ lies on line ℓ_6
 99 : $P_{1720} = (7, 10, 5, 1)$ lies on line ℓ_6
 100 : $P_{1733} = (4, 11, 5, 1)$ lies on line ℓ_6
 101 : $P_{1758} = (13, 12, 5, 1)$ lies on line ℓ_6
 102 : $P_{1775} = (14, 13, 5, 1)$ lies on line ℓ_6
 103 : $P_{1788} = (11, 14, 5, 1)$ lies on line ℓ_6
 104 : $P_{1801} = (8, 15, 5, 1)$ lies on line ℓ_6
 105 : $P_{1837} = (12, 1, 6, 1)$ lies on line ℓ_{15}
 106 : $P_{1842} = (1, 2, 6, 1)$ lies on line ℓ_{15}
 107 : $P_{1870} = (13, 3, 6, 1)$ lies on line ℓ_{15}
 108 : $P_{1875} = (2, 4, 6, 1)$ lies on line ℓ_{15}
 109 : $P_{1903} = (14, 5, 6, 1)$ lies on line ℓ_{15}
 110 : $P_{1908} = (3, 6, 6, 1)$ lies on line ℓ_{15}
 111 : $P_{1936} = (15, 7, 6, 1)$ lies on line ℓ_{15}
 112 : $P_{1941} = (4, 8, 6, 1)$ lies on line ℓ_{15}
 113 : $P_{1961} = (8, 9, 6, 1)$ lies on line ℓ_{15}
 114 : $P_{1974} = (5, 10, 6, 1)$ lies on line ℓ_{15}
 115 : $P_{1994} = (9, 11, 6, 1)$ lies on line ℓ_{15}
 116 : $P_{2007} = (6, 12, 6, 1)$ lies on line ℓ_{15}
 117 : $P_{2027} = (10, 13, 6, 1)$ lies on line ℓ_{15}
 118 : $P_{2040} = (7, 14, 6, 1)$ lies on line ℓ_{15}
 119 : $P_{2060} = (11, 15, 6, 1)$ lies on line ℓ_{15}
 120 : $P_{2094} = (13, 1, 7, 1)$ lies on line ℓ_{16}
 121 : $P_{2100} = (3, 2, 7, 1)$ lies on line ℓ_{16}
 122 : $P_{2127} = (14, 3, 7, 1)$ lies on line ℓ_{16}
 123 : $P_{2135} = (6, 4, 7, 1)$ lies on line ℓ_{16}
 124 : $P_{2156} = (11, 5, 7, 1)$ lies on line ℓ_{16}
 125 : $P_{2166} = (5, 6, 7, 1)$ lies on line ℓ_{16}
 126 : $P_{2185} = (8, 7, 7, 1)$ lies on line ℓ_{16}
 127 : $P_{2205} = (12, 8, 7, 1)$ lies on line ℓ_{16}
 128 : $P_{2210} = (1, 9, 7, 1)$ lies on line ℓ_{16}
 129 : $P_{2240} = (15, 10, 7, 1)$ lies on line ℓ_{16}
 130 : $P_{2243} = (2, 11, 7, 1)$ lies on line ℓ_{16}
 131 : $P_{2267} = (10, 12, 7, 1)$ lies on line ℓ_{16}
 132 : $P_{2280} = (7, 13, 7, 1)$ lies on line ℓ_{16}
 133 : $P_{2298} = (9, 14, 7, 1)$ lies on line ℓ_{16}
 134 : $P_{2309} = (4, 15, 7, 1)$ lies on line ℓ_{16}
 135 : $P_{2342} = (5, 1, 8, 1)$ lies on line ℓ_8
 136 : $P_{2363} = (10, 2, 8, 1)$ lies on line ℓ_8
 137 : $P_{2384} = (15, 3, 8, 1)$ lies on line ℓ_8
 138 : $P_{2398} = (13, 4, 8, 1)$ lies on line ℓ_8
 139 : $P_{2409} = (8, 5, 8, 1)$ lies on line ℓ_8
 140 : $P_{2424} = (7, 6, 8, 1)$ lies on line ℓ_8
 141 : $P_{2435} = (2, 7, 8, 1)$ lies on line ℓ_8
 142 : $P_{2452} = (3, 8, 8, 1)$ lies on line ℓ_8
 143 : $P_{2471} = (6, 9, 8, 1)$ lies on line ℓ_8
 144 : $P_{2490} = (9, 10, 8, 1)$ lies on line ℓ_8
 145 : $P_{2509} = (12, 11, 8, 1)$ lies on line ℓ_8

146 : $P_{2527} = (14, 12, 8, 1)$ lies on line ℓ_8
 147 : $P_{2540} = (11, 13, 8, 1)$ lies on line ℓ_8
 148 : $P_{2549} = (4, 14, 8, 1)$ lies on line ℓ_8
 149 : $P_{2562} = (1, 15, 8, 1)$ lies on line ℓ_8
 150 : $P_{2597} = (4, 1, 9, 1)$ lies on line ℓ_7
 151 : $P_{2617} = (8, 2, 9, 1)$ lies on line ℓ_7
 152 : $P_{2637} = (12, 3, 9, 1)$ lies on line ℓ_7
 153 : $P_{2650} = (9, 4, 9, 1)$ lies on line ℓ_7
 154 : $P_{2670} = (13, 5, 9, 1)$ lies on line ℓ_7
 155 : $P_{2674} = (1, 6, 9, 1)$ lies on line ℓ_7
 156 : $P_{2694} = (5, 7, 9, 1)$ lies on line ℓ_7
 157 : $P_{2716} = (11, 8, 9, 1)$ lies on line ℓ_7
 158 : $P_{2736} = (15, 9, 9, 1)$ lies on line ℓ_7
 159 : $P_{2740} = (3, 10, 9, 1)$ lies on line ℓ_7
 160 : $P_{2760} = (7, 11, 9, 1)$ lies on line ℓ_7
 161 : $P_{2771} = (2, 12, 9, 1)$ lies on line ℓ_7
 162 : $P_{2791} = (6, 13, 9, 1)$ lies on line ℓ_7
 163 : $P_{2811} = (10, 14, 9, 1)$ lies on line ℓ_7
 164 : $P_{2831} = (14, 15, 9, 1)$ lies on line ℓ_7
 165 : $P_{2860} = (11, 1, 10, 1)$ lies on line ℓ_{14}
 166 : $P_{2880} = (15, 2, 10, 1)$ lies on line ℓ_{14}
 167 : $P_{2885} = (4, 3, 10, 1)$ lies on line ℓ_{14}
 168 : $P_{2904} = (7, 4, 10, 1)$ lies on line ℓ_{14}
 169 : $P_{2925} = (12, 5, 10, 1)$ lies on line ℓ_{14}
 170 : $P_{2937} = (8, 6, 10, 1)$ lies on line ℓ_{14}
 171 : $P_{2948} = (3, 7, 10, 1)$ lies on line ℓ_{14}
 172 : $P_{2975} = (14, 8, 10, 1)$ lies on line ℓ_{14}
 173 : $P_{2982} = (5, 9, 10, 1)$ lies on line ℓ_{14}
 174 : $P_{2994} = (1, 10, 10, 1)$ lies on line ℓ_{14}
 175 : $P_{3019} = (10, 11, 10, 1)$ lies on line ℓ_{14}
 176 : $P_{3034} = (9, 12, 10, 1)$ lies on line ℓ_{14}
 177 : $P_{3043} = (2, 13, 10, 1)$ lies on line ℓ_{14}
 178 : $P_{3063} = (6, 14, 10, 1)$ lies on line ℓ_{14}
 179 : $P_{3086} = (13, 15, 10, 1)$ lies on line ℓ_{14}
 180 : $P_{3115} = (10, 1, 11, 1)$ lies on line ℓ_{13}
 181 : $P_{3134} = (13, 2, 11, 1)$ lies on line ℓ_{13}
 182 : $P_{3144} = (7, 3, 11, 1)$ lies on line ℓ_{13}
 183 : $P_{3156} = (3, 4, 11, 1)$ lies on line ℓ_{13}
 184 : $P_{3178} = (9, 5, 11, 1)$ lies on line ℓ_{13}
 185 : $P_{3199} = (14, 6, 11, 1)$ lies on line ℓ_{13}
 186 : $P_{3205} = (4, 7, 11, 1)$ lies on line ℓ_{13}
 187 : $P_{3223} = (6, 8, 11, 1)$ lies on line ℓ_{13}
 188 : $P_{3245} = (12, 9, 11, 1)$ lies on line ℓ_{13}
 189 : $P_{3260} = (11, 10, 11, 1)$ lies on line ℓ_{13}
 190 : $P_{3266} = (1, 11, 11, 1)$ lies on line ℓ_{13}
 191 : $P_{3286} = (5, 12, 11, 1)$ lies on line ℓ_{13}
 192 : $P_{3312} = (15, 13, 11, 1)$ lies on line ℓ_{13}
 193 : $P_{3321} = (8, 14, 11, 1)$ lies on line ℓ_{13}
 194 : $P_{3331} = (2, 15, 11, 1)$ lies on line ℓ_{13}
 195 : $P_{3368} = (7, 1, 12, 1)$ lies on line ℓ_{10}
 196 : $P_{3391} = (14, 2, 12, 1)$ lies on line ℓ_{10}
 197 : $P_{3402} = (9, 3, 12, 1)$ lies on line ℓ_{10}
 198 : $P_{3414} = (5, 4, 12, 1)$ lies on line ℓ_{10}
 199 : $P_{3427} = (2, 5, 12, 1)$ lies on line ℓ_{10}
 200 : $P_{3452} = (11, 6, 12, 1)$ lies on line ℓ_{10}
 201 : $P_{3469} = (12, 7, 12, 1)$ lies on line ℓ_{10}
 202 : $P_{3483} = (10, 8, 12, 1)$ lies on line ℓ_{10}
 203 : $P_{3502} = (13, 9, 12, 1)$ lies on line ℓ_{10}
 204 : $P_{3509} = (4, 10, 12, 1)$ lies on line ℓ_{10}
 205 : $P_{3524} = (3, 11, 12, 1)$ lies on line ℓ_{10}
 206 : $P_{3552} = (15, 12, 12, 1)$ lies on line ℓ_{10}
 207 : $P_{3561} = (8, 13, 12, 1)$ lies on line ℓ_{10}
 208 : $P_{3570} = (1, 14, 12, 1)$ lies on line ℓ_{10}
 209 : $P_{3591} = (6, 15, 12, 1)$ lies on line ℓ_{10}
 210 : $P_{3623} = (6, 1, 13, 1)$ lies on line ℓ_9
 211 : $P_{3645} = (12, 2, 13, 1)$ lies on line ℓ_9
 212 : $P_{3659} = (10, 3, 13, 1)$ lies on line ℓ_9
 213 : $P_{3666} = (1, 4, 13, 1)$ lies on line ℓ_9
 214 : $P_{3688} = (7, 5, 13, 1)$ lies on line ℓ_9
 215 : $P_{3710} = (13, 6, 13, 1)$ lies on line ℓ_9
 216 : $P_{3724} = (11, 7, 13, 1)$ lies on line ℓ_9
 217 : $P_{3731} = (2, 8, 13, 1)$ lies on line ℓ_9
 218 : $P_{3749} = (4, 9, 13, 1)$ lies on line ℓ_9
 219 : $P_{3775} = (14, 10, 13, 1)$ lies on line ℓ_9
 220 : $P_{3785} = (8, 11, 13, 1)$ lies on line ℓ_9
 221 : $P_{3796} = (3, 12, 13, 1)$ lies on line ℓ_9
 222 : $P_{3814} = (5, 13, 13, 1)$ lies on line ℓ_9
 223 : $P_{3840} = (15, 14, 13, 1)$ lies on line ℓ_9
 224 : $P_{3850} = (9, 15, 13, 1)$ lies on line ℓ_9
 225 : $P_{3882} = (9, 1, 14, 1)$ lies on line ℓ_{12}
 226 : $P_{3900} = (11, 2, 14, 1)$ lies on line ℓ_{12}
 227 : $P_{3907} = (2, 3, 14, 1)$ lies on line ℓ_{12}
 228 : $P_{3936} = (15, 4, 14, 1)$ lies on line ℓ_{12}
 229 : $P_{3943} = (6, 5, 14, 1)$ lies on line ℓ_{12}
 230 : $P_{3957} = (4, 6, 14, 1)$ lies on line ℓ_{12}
 231 : $P_{3982} = (13, 7, 14, 1)$ lies on line ℓ_{12}
 232 : $P_{3992} = (7, 8, 14, 1)$ lies on line ℓ_{12}
 233 : $P_{4015} = (14, 9, 14, 1)$ lies on line ℓ_{12}
 234 : $P_{4029} = (12, 10, 14, 1)$ lies on line ℓ_{12}
 235 : $P_{4038} = (5, 11, 14, 1)$ lies on line ℓ_{12}
 236 : $P_{4057} = (8, 12, 14, 1)$ lies on line ℓ_{12}
 237 : $P_{4066} = (1, 13, 14, 1)$ lies on line ℓ_{12}
 238 : $P_{4084} = (3, 14, 14, 1)$ lies on line ℓ_{12}
 239 : $P_{4107} = (10, 15, 14, 1)$ lies on line ℓ_{12}
 240 : $P_{4137} = (8, 1, 15, 1)$ lies on line ℓ_{11}
 241 : $P_{4154} = (9, 2, 15, 1)$ lies on line ℓ_{11}
 242 : $P_{4162} = (1, 3, 15, 1)$ lies on line ℓ_{11}
 243 : $P_{4188} = (11, 4, 15, 1)$ lies on line ℓ_{11}
 244 : $P_{4196} = (3, 5, 15, 1)$ lies on line ℓ_{11}
 245 : $P_{4211} = (2, 6, 15, 1)$ lies on line ℓ_{11}
 246 : $P_{4235} = (10, 7, 15, 1)$ lies on line ℓ_{11}
 247 : $P_{4256} = (15, 8, 15, 1)$ lies on line ℓ_{11}
 248 : $P_{4264} = (7, 9, 15, 1)$ lies on line ℓ_{11}
 249 : $P_{4279} = (6, 10, 15, 1)$ lies on line ℓ_{11}
 250 : $P_{4303} = (14, 11, 15, 1)$ lies on line ℓ_{11}
 251 : $P_{4309} = (4, 12, 15, 1)$ lies on line ℓ_{11}
 252 : $P_{4333} = (12, 13, 15, 1)$ lies on line ℓ_{11}
 253 : $P_{4350} = (13, 14, 15, 1)$ lies on line ℓ_{11}
 254 : $P_{4358} = (5, 15, 15, 1)$ lies on line ℓ_{11}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}
in point	P_0	P_1	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}
in point	P_2	P_3	P_{530}	P_{1297}	P_{1553}	P_{2577}	P_{2321}	P_{3601}	P_{3345}	P_{4113}	P_{3857}	P_{3089}	P_{2833}	P_{1809}	P_{2065}	P_{785}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{530}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{1297}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_7	P_{1553}

Line 7 intersects

Line	ℓ_0	ℓ_3
in point	P_8	P_{2577}

Line 8 intersects

Line	ℓ_0	ℓ_3
in point	P_9	P_{2321}

Line 9 intersects

Line	ℓ_0	ℓ_3
in point	P_{10}	P_{3601}

Line 10 intersects

Line	ℓ_0	ℓ_3
in point	P_{11}	P_{3345}

Line 11 intersects

Line	ℓ_0	ℓ_3
in point	P_{12}	P_{4113}

Line 12 intersects

Line	ℓ_0	ℓ_3
in point	P_{13}	P_{3857}

Line 13 intersects

Line	ℓ_0	ℓ_3
in point	P_{14}	P_{3089}

Line 14 intersects

Line	ℓ_0	ℓ_3
in point	P_{15}	P_{2833}

Line 15 intersects

Line	ℓ_0	ℓ_3
in point	P_{16}	P_{1809}

Line 16 intersects

Line	ℓ_0	ℓ_3
in point	P_{17}	P_{2065}

Line 17 intersects

Line	ℓ_0	ℓ_3
in point	P_{18}	P_{785}

Line 18 intersects

Line	ℓ_0	ℓ_3
in point	P_{19}	P_{1041}

The surface has 289 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_4 = (1, 1, 1, 1)$
5 : $P_5 = (1, 1, 0, 0)$

6 : $P_6 = (2, 1, 0, 0)$
7 : $P_7 = (3, 1, 0, 0)$
8 : $P_8 = (4, 1, 0, 0)$
9 : $P_9 = (5, 1, 0, 0)$
10 : $P_{10} = (6, 1, 0, 0)$
11 : $P_{11} = (7, 1, 0, 0)$

12 : $P_{12} = (8, 1, 0, 0)$
13 : $P_{13} = (9, 1, 0, 0)$
14 : $P_{14} = (10, 1, 0, 0)$
15 : $P_{15} = (11, 1, 0, 0)$
16 : $P_{16} = (12, 1, 0, 0)$
17 : $P_{17} = (13, 1, 0, 0)$

18 : $P_{18} = (14, 1, 0, 0)$	72 : $P_{898} = (1, 7, 2, 1)$	126 : $P_{1775} = (14, 13, 5, 1)$
19 : $P_{19} = (15, 1, 0, 0)$	73 : $P_{926} = (13, 8, 2, 1)$	127 : $P_{1788} = (11, 14, 5, 1)$
20 : $P_{20} = (1, 0, 1, 0)$	74 : $P_{932} = (3, 9, 2, 1)$	128 : $P_{1801} = (8, 15, 5, 1)$
21 : $P_{21} = (2, 0, 1, 0)$	75 : $P_{953} = (8, 10, 2, 1)$	129 : $P_{1809} = (0, 0, 6, 1)$
22 : $P_{22} = (3, 0, 1, 0)$	76 : $P_{967} = (6, 11, 2, 1)$	130 : $P_{1837} = (12, 1, 6, 1)$
23 : $P_{23} = (4, 0, 1, 0)$	77 : $P_{984} = (7, 12, 2, 1)$	131 : $P_{1842} = (1, 2, 6, 1)$
24 : $P_{24} = (5, 0, 1, 0)$	78 : $P_{1002} = (9, 13, 2, 1)$	132 : $P_{1870} = (13, 3, 6, 1)$
25 : $P_{25} = (6, 0, 1, 0)$	79 : $P_{1011} = (2, 14, 2, 1)$	133 : $P_{1875} = (2, 4, 6, 1)$
26 : $P_{26} = (7, 0, 1, 0)$	80 : $P_{1037} = (12, 15, 2, 1)$	134 : $P_{1903} = (14, 5, 6, 1)$
27 : $P_{27} = (8, 0, 1, 0)$	81 : $P_{1041} = (0, 0, 3, 1)$	135 : $P_{1908} = (3, 6, 6, 1)$
28 : $P_{28} = (9, 0, 1, 0)$	82 : $P_{1072} = (15, 1, 3, 1)$	136 : $P_{1936} = (15, 7, 6, 1)$
29 : $P_{29} = (10, 0, 1, 0)$	83 : $P_{1080} = (7, 2, 3, 1)$	137 : $P_{1941} = (4, 8, 6, 1)$
30 : $P_{30} = (11, 0, 1, 0)$	84 : $P_{1097} = (8, 3, 3, 1)$	138 : $P_{1961} = (8, 9, 6, 1)$
31 : $P_{31} = (12, 0, 1, 0)$	85 : $P_{1119} = (14, 4, 3, 1)$	139 : $P_{1974} = (5, 10, 6, 1)$
32 : $P_{32} = (13, 0, 1, 0)$	86 : $P_{1122} = (1, 5, 3, 1)$	140 : $P_{1994} = (9, 11, 6, 1)$
33 : $P_{33} = (14, 0, 1, 0)$	87 : $P_{1146} = (9, 6, 3, 1)$	141 : $P_{2007} = (6, 12, 6, 1)$
34 : $P_{34} = (15, 0, 1, 0)$	88 : $P_{1159} = (6, 7, 3, 1)$	142 : $P_{2027} = (10, 13, 6, 1)$
35 : $P_{290} = (0, 1, 0, 1)$	89 : $P_{1174} = (5, 8, 3, 1)$	143 : $P_{2040} = (7, 14, 6, 1)$
36 : $P_{306} = (0, 2, 0, 1)$	90 : $P_{1195} = (10, 9, 3, 1)$	144 : $P_{2060} = (11, 15, 6, 1)$
37 : $P_{322} = (0, 3, 0, 1)$	91 : $P_{1203} = (2, 10, 3, 1)$	145 : $P_{2065} = (0, 0, 7, 1)$
38 : $P_{338} = (0, 4, 0, 1)$	92 : $P_{1230} = (13, 11, 3, 1)$	146 : $P_{2094} = (13, 1, 7, 1)$
39 : $P_{354} = (0, 5, 0, 1)$	93 : $P_{1244} = (11, 12, 3, 1)$	147 : $P_{2100} = (3, 2, 7, 1)$
40 : $P_{370} = (0, 6, 0, 1)$	94 : $P_{1253} = (4, 13, 3, 1)$	148 : $P_{2127} = (14, 3, 7, 1)$
41 : $P_{386} = (0, 7, 0, 1)$	95 : $P_{1277} = (12, 14, 3, 1)$	149 : $P_{2135} = (6, 4, 7, 1)$
42 : $P_{402} = (0, 8, 0, 1)$	96 : $P_{1284} = (3, 15, 3, 1)$	150 : $P_{2156} = (11, 5, 7, 1)$
43 : $P_{418} = (0, 9, 0, 1)$	97 : $P_{1297} = (0, 0, 4, 1)$	151 : $P_{2166} = (5, 6, 7, 1)$
44 : $P_{434} = (0, 10, 0, 1)$	98 : $P_{1315} = (2, 1, 4, 1)$	152 : $P_{2185} = (8, 7, 7, 1)$
45 : $P_{450} = (0, 11, 0, 1)$	99 : $P_{1333} = (4, 2, 4, 1)$	153 : $P_{2205} = (12, 8, 7, 1)$
46 : $P_{466} = (0, 12, 0, 1)$	100 : $P_{1351} = (6, 3, 4, 1)$	154 : $P_{2210} = (1, 9, 7, 1)$
47 : $P_{482} = (0, 13, 0, 1)$	101 : $P_{1369} = (8, 4, 4, 1)$	155 : $P_{2240} = (15, 10, 7, 1)$
48 : $P_{498} = (0, 14, 0, 1)$	102 : $P_{1387} = (10, 5, 4, 1)$	156 : $P_{2243} = (2, 11, 7, 1)$
49 : $P_{514} = (0, 15, 0, 1)$	103 : $P_{1405} = (12, 6, 4, 1)$	157 : $P_{2267} = (10, 12, 7, 1)$
50 : $P_{530} = (0, 0, 1, 1)$	104 : $P_{1423} = (14, 7, 4, 1)$	158 : $P_{2280} = (7, 13, 7, 1)$
51 : $P_{563} = (2, 2, 1, 1)$	105 : $P_{1434} = (9, 8, 4, 1)$	159 : $P_{2298} = (9, 14, 7, 1)$
52 : $P_{580} = (3, 3, 1, 1)$	106 : $P_{1452} = (11, 9, 4, 1)$	160 : $P_{2309} = (4, 15, 7, 1)$
53 : $P_{597} = (4, 4, 1, 1)$	107 : $P_{1470} = (13, 10, 4, 1)$	161 : $P_{2321} = (0, 0, 8, 1)$
54 : $P_{614} = (5, 5, 1, 1)$	108 : $P_{1488} = (15, 11, 4, 1)$	162 : $P_{2342} = (5, 1, 8, 1)$
55 : $P_{631} = (6, 6, 1, 1)$	109 : $P_{1490} = (1, 12, 4, 1)$	163 : $P_{2363} = (10, 2, 8, 1)$
56 : $P_{648} = (7, 7, 1, 1)$	110 : $P_{1508} = (3, 13, 4, 1)$	164 : $P_{2384} = (15, 3, 8, 1)$
57 : $P_{665} = (8, 8, 1, 1)$	111 : $P_{1526} = (5, 14, 4, 1)$	165 : $P_{2398} = (13, 4, 8, 1)$
58 : $P_{682} = (9, 9, 1, 1)$	112 : $P_{1544} = (7, 15, 4, 1)$	166 : $P_{2409} = (8, 5, 8, 1)$
59 : $P_{699} = (10, 10, 1, 1)$	113 : $P_{1553} = (0, 0, 5, 1)$	167 : $P_{2424} = (7, 6, 8, 1)$
60 : $P_{716} = (11, 11, 1, 1)$	114 : $P_{1572} = (3, 1, 5, 1)$	168 : $P_{2435} = (2, 7, 8, 1)$
61 : $P_{733} = (12, 12, 1, 1)$	115 : $P_{1591} = (6, 2, 5, 1)$	169 : $P_{2452} = (3, 8, 8, 1)$
62 : $P_{750} = (13, 13, 1, 1)$	116 : $P_{1606} = (5, 3, 5, 1)$	170 : $P_{2471} = (6, 9, 8, 1)$
63 : $P_{767} = (14, 14, 1, 1)$	117 : $P_{1629} = (12, 4, 5, 1)$	171 : $P_{2490} = (9, 10, 8, 1)$
64 : $P_{784} = (15, 15, 1, 1)$	118 : $P_{1648} = (15, 5, 5, 1)$	172 : $P_{2509} = (12, 11, 8, 1)$
65 : $P_{785} = (0, 0, 2, 1)$	119 : $P_{1659} = (10, 6, 5, 1)$	173 : $P_{2527} = (14, 12, 8, 1)$
66 : $P_{815} = (14, 1, 2, 1)$	120 : $P_{1674} = (9, 7, 5, 1)$	174 : $P_{2540} = (11, 13, 8, 1)$
67 : $P_{822} = (5, 2, 2, 1)$	121 : $P_{1682} = (1, 8, 5, 1)$	175 : $P_{2549} = (4, 14, 8, 1)$
68 : $P_{844} = (11, 3, 2, 1)$	122 : $P_{1699} = (2, 9, 5, 1)$	176 : $P_{2562} = (1, 15, 8, 1)$
69 : $P_{859} = (10, 4, 2, 1)$	123 : $P_{1720} = (7, 10, 5, 1)$	177 : $P_{2577} = (0, 0, 9, 1)$
70 : $P_{869} = (4, 5, 2, 1)$	124 : $P_{1733} = (4, 11, 5, 1)$	178 : $P_{2597} = (4, 1, 9, 1)$
71 : $P_{896} = (15, 6, 2, 1)$	125 : $P_{1758} = (13, 12, 5, 1)$	179 : $P_{2617} = (8, 2, 9, 1)$

180 : $P_{2637} = (12, 3, 9, 1)$	217 : $P_{3223} = (6, 8, 11, 1)$	254 : $P_{3814} = (5, 13, 13, 1)$
181 : $P_{2650} = (9, 4, 9, 1)$	218 : $P_{3245} = (12, 9, 11, 1)$	255 : $P_{3840} = (15, 14, 13, 1)$
182 : $P_{2670} = (13, 5, 9, 1)$	219 : $P_{3260} = (11, 10, 11, 1)$	256 : $P_{3850} = (9, 15, 13, 1)$
183 : $P_{2674} = (1, 6, 9, 1)$	220 : $P_{3266} = (1, 11, 11, 1)$	257 : $P_{3857} = (0, 0, 14, 1)$
184 : $P_{2694} = (5, 7, 9, 1)$	221 : $P_{3286} = (5, 12, 11, 1)$	258 : $P_{3882} = (9, 1, 14, 1)$
185 : $P_{2716} = (11, 8, 9, 1)$	222 : $P_{3312} = (15, 13, 11, 1)$	259 : $P_{3900} = (11, 2, 14, 1)$
186 : $P_{2736} = (15, 9, 9, 1)$	223 : $P_{3321} = (8, 14, 11, 1)$	260 : $P_{3907} = (2, 3, 14, 1)$
187 : $P_{2740} = (3, 10, 9, 1)$	224 : $P_{3331} = (2, 15, 11, 1)$	261 : $P_{3936} = (15, 4, 14, 1)$
188 : $P_{2760} = (7, 11, 9, 1)$	225 : $P_{3345} = (0, 0, 12, 1)$	262 : $P_{3943} = (6, 5, 14, 1)$
189 : $P_{2771} = (2, 12, 9, 1)$	226 : $P_{3368} = (7, 1, 12, 1)$	263 : $P_{3957} = (4, 6, 14, 1)$
190 : $P_{2791} = (6, 13, 9, 1)$	227 : $P_{3391} = (14, 2, 12, 1)$	264 : $P_{3982} = (13, 7, 14, 1)$
191 : $P_{2811} = (10, 14, 9, 1)$	228 : $P_{3402} = (9, 3, 12, 1)$	265 : $P_{3992} = (7, 8, 14, 1)$
192 : $P_{2831} = (14, 15, 9, 1)$	229 : $P_{3414} = (5, 4, 12, 1)$	266 : $P_{4015} = (14, 9, 14, 1)$
193 : $P_{2833} = (0, 0, 10, 1)$	230 : $P_{3427} = (2, 5, 12, 1)$	267 : $P_{4029} = (12, 10, 14, 1)$
194 : $P_{2860} = (11, 1, 10, 1)$	231 : $P_{3452} = (11, 6, 12, 1)$	268 : $P_{4038} = (5, 11, 14, 1)$
195 : $P_{2880} = (15, 2, 10, 1)$	232 : $P_{3469} = (12, 7, 12, 1)$	269 : $P_{4057} = (8, 12, 14, 1)$
196 : $P_{2885} = (4, 3, 10, 1)$	233 : $P_{3483} = (10, 8, 12, 1)$	270 : $P_{4066} = (1, 13, 14, 1)$
197 : $P_{2904} = (7, 4, 10, 1)$	234 : $P_{3502} = (13, 9, 12, 1)$	271 : $P_{4084} = (3, 14, 14, 1)$
198 : $P_{2925} = (12, 5, 10, 1)$	235 : $P_{3509} = (4, 10, 12, 1)$	272 : $P_{4107} = (10, 15, 14, 1)$
199 : $P_{2937} = (8, 6, 10, 1)$	236 : $P_{3524} = (3, 11, 12, 1)$	273 : $P_{4113} = (0, 0, 15, 1)$
200 : $P_{2948} = (3, 7, 10, 1)$	237 : $P_{3552} = (15, 12, 12, 1)$	274 : $P_{4137} = (8, 1, 15, 1)$
201 : $P_{2975} = (14, 8, 10, 1)$	238 : $P_{3561} = (8, 13, 12, 1)$	275 : $P_{4154} = (9, 2, 15, 1)$
202 : $P_{2982} = (5, 9, 10, 1)$	239 : $P_{3570} = (1, 14, 12, 1)$	276 : $P_{4162} = (1, 3, 15, 1)$
203 : $P_{2994} = (1, 10, 10, 1)$	240 : $P_{3591} = (6, 15, 12, 1)$	277 : $P_{4188} = (11, 4, 15, 1)$
204 : $P_{3019} = (10, 11, 10, 1)$	241 : $P_{3601} = (0, 0, 13, 1)$	278 : $P_{4196} = (3, 5, 15, 1)$
205 : $P_{3034} = (9, 12, 10, 1)$	242 : $P_{3623} = (6, 1, 13, 1)$	279 : $P_{4211} = (2, 6, 15, 1)$
206 : $P_{3043} = (2, 13, 10, 1)$	243 : $P_{3645} = (12, 2, 13, 1)$	280 : $P_{4235} = (10, 7, 15, 1)$
207 : $P_{3063} = (6, 14, 10, 1)$	244 : $P_{3659} = (10, 3, 13, 1)$	281 : $P_{4256} = (15, 8, 15, 1)$
208 : $P_{3086} = (13, 15, 10, 1)$	245 : $P_{3666} = (1, 4, 13, 1)$	282 : $P_{4264} = (7, 9, 15, 1)$
209 : $P_{3089} = (0, 0, 11, 1)$	246 : $P_{3688} = (7, 5, 13, 1)$	283 : $P_{4279} = (6, 10, 15, 1)$
210 : $P_{3115} = (10, 1, 11, 1)$	247 : $P_{3710} = (13, 6, 13, 1)$	284 : $P_{4303} = (14, 11, 15, 1)$
211 : $P_{3134} = (13, 2, 11, 1)$	248 : $P_{3724} = (11, 7, 13, 1)$	285 : $P_{4309} = (4, 12, 15, 1)$
212 : $P_{3144} = (7, 3, 11, 1)$	249 : $P_{3731} = (2, 8, 13, 1)$	286 : $P_{4333} = (12, 13, 15, 1)$
213 : $P_{3156} = (3, 4, 11, 1)$	250 : $P_{3749} = (4, 9, 13, 1)$	287 : $P_{4350} = (13, 14, 15, 1)$
214 : $P_{3178} = (9, 5, 11, 1)$	251 : $P_{3775} = (14, 10, 13, 1)$	288 : $P_{4358} = (5, 15, 15, 1)$
215 : $P_{3199} = (14, 6, 11, 1)$	252 : $P_{3785} = (8, 11, 13, 1)$	
216 : $P_{3205} = (4, 7, 11, 1)$	253 : $P_{3796} = (3, 12, 13, 1)$	