Rank-65550 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431655838

General information

Number of lines	15
Number of points	37
Number of singular points	1
Number of Eckardt points	3
Number of double points	27
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{15}
Type of lines on points	6, 3^3, 2^{27}, 1^6

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 15 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \mathbf{Pl}(0,3,1,0,0,0)_8$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \mathbf{Pl}(0,2,1,0,0,0)_7$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0,3,0,0,0,1)_{107}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0,2,0,0,0,1)_{106}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1,0,1,1,1,1)_{199}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & \omega^2 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{286} = \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{286} = \mathbf{Pl}(2,0,2,3,3,1)_{327}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & \omega \\ 0 & 1 & 1 & \omega \end{bmatrix}_{198} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{198} = \mathbf{Pl}(3,0,3,2,2,1)_{275}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{225} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{225} = \mathbf{Pl}(2,0,1,1,2,1)_{260}$$

$$\ell_{10} = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{137} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{137} = \mathbf{Pl}(3,0,2,3,1,1)_{208}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{301} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{301} = \mathbf{Pl}(1,0,3,2,3,1)_{333}$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & \omega & \omega \end{bmatrix}_{325} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{325} = \mathbf{Pl}(3,0,1,1,3,1)_{321}$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & \omega & 1 \end{bmatrix}_{237} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{237} = \mathbf{Pl}(1,0,2,3,2,1)_{266}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{161} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{161} = \mathbf{Pl}(2,0,3,2,1,1)_{214}$$

Rank of lines: (100, 268, 184, 341, 351, 346, 110, 286, 198, 225, 137, 301, 325, 237, 161)
Rank of points on Klein quadric: (6, 8, 7, 105, 107, 106, 199, 327, 275, 260, 208, 333, 321, 266, 214)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$,
$1 : P_6 = \mathbf{P}(\omega,1,0,0) = \mathbf{P}(2,1,0,0)$,
$2 : P_7 = \mathbf{P}(\omega^2,1,0,0) = \mathbf{P}(3,1,0,0)$.

Double Points

The surface has 27 Double points:
The double points on the surface are:

$P_{39} = (1,0,1,1) = \ell_0 \cap \ell_6$
$P_{54} = (1,0,2,1) = \ell_0 \cap \ell_{10}$
$P_{70} = (1,0,3,1) = \ell_0 \cap \ell_{14}$
$P_{55} = (2,0,2,1) = \ell_1 \cap \ell_7$
$P_{71} = (2,0,3,1) = \ell_1 \cap \ell_{11}$
$P_{40} = (2,0,1,1) = \ell_1 \cap \ell_{12}$
$P_{72} = (3,0,3,1) = \ell_2 \cap \ell_8$
$P_{41} = (3,0,1,1) = \ell_2 \cap \ell_9$
$P_{56} = (3,0,2,1) = \ell_2 \cap \ell_{13}$
$P_{42} = (0,1,1,1) = \ell_3 \cap \ell_6$
$P_{73} = (0,1,3,1) = \ell_3 \cap \ell_{11}$
$P_{57} = (0,1,2,1) = \ell_3 \cap \ell_{13}$
$P_{61} = (0,2,2,1) = \ell_4 \cap \ell_7$
$P_{45} = (0,2,1,1) = \ell_4 \cap \ell_9$
$P_{77} = (0,2,3,1) = \ell_4 \cap \ell_{14}$
$P_{81} = (0,3,3,1) = \ell_5 \cap \ell_8$
$P_{65} = (0,3,2,1) = \ell_5 \cap \ell_{10}$
$P_{49} = (0,3,1,1) = \ell_5 \cap \ell_{12}$
$P_{51} = (2,3,1,1) = \ell_6 \cap \ell_9$
$P_{48} = (3,2,1,1) = \ell_6 \cap \ell_{12}$
$P_{60} = (3,1,2,1) = \ell_7 \cap \ell_{10}$
$P_{66} = (1,3,2,1) = \ell_7 \cap \ell_{13}$
$P_{78} = (1,2,3,1) = \ell_8 \cap \ell_{11}$
$P_{75} = (2,1,3,1) = \ell_8 \cap \ell_{14}$
$P_4 = (1,1,1,1) = \ell_9 \cap \ell_{12}$
$P_{63} = (2,2,2,1) = \ell_{10} \cap \ell_{13}$
$P_{84} = (3,3,3,1) = \ell_{11} \cap \ell_{14}$

Single Points

The surface has 6 single points:
The single points on the surface are:

$0:\ P_{23} = (1,0,0,1)$ lies on line ℓ_0
$1:\ P_{24} = (2,0,0,1)$ lies on line ℓ_1
$2:\ P_{25} = (3,0,0,1)$ lies on line ℓ_2
$3:\ P_{26} = (0,1,0,1)$ lies on line ℓ_3
$4:\ P_{30} = (0,2,0,1)$ lies on line ℓ_4
$5:\ P_{34} = (0,3,0,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	1	1	1	1	1	0	0	0	1	0	0	0	1
1	1	0	1	1	1	1	0	1	0	0	0	1	1	0	0
2	1	1	0	1	1	1	0	0	1	1	0	0	0	1	0
3	1	1	1	0	1	1	1	0	0	0	0	1	0	1	0
4	1	1	1	1	0	1	0	1	0	1	0	0	0	0	1
5	1	1	1	1	1	0	0	0	1	0	1	0	1	0	0
6	1	0	0	1	0	0	0	1	1	1	0	0	1	0	0
7	0	1	0	0	1	0	1	0	1	0	1	0	0	1	0
8	0	0	1	0	0	1	1	1	0	0	0	1	0	0	1
9	0	0	1	0	1	0	1	0	0	0	1	1	1	0	0
10	1	0	0	0	0	1	0	1	0	1	0	1	0	1	0
11	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1
12	0	1	0	0	0	1	1	0	0	1	0	0	0	1	1
13	0	0	1	1	0	0	0	1	0	0	1	0	1	0	1
14	1	0	0	0	1	0	0	0	1	0	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{14}
in point	P_2	P_2	P_2	P_2	P_2	P_{39}	P_{54}	P_{70}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{12}
in point	P_2	P_2	P_2	P_2	P_2	P_{55}	P_{71}	P_{40}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_9	ℓ_{13}
in point	P_2	P_2	P_2	P_2	P_2	P_{72}	P_{41}	P_{56}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{13}
in point	P_2	P_2	P_2	P_2	P_2	P_{42}	P_{73}	P_{57}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{14}
in point	P_2	P_2	P_2	P_2	P_2	P_{61}	P_{45}	P_{77}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}
in point	P_2	P_2	P_2	P_2	P_2	P_{81}	P_{65}	P_{49}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8	ℓ_9	ℓ_{12}
in point	P_{39}	P_{42}	P_5	P_5	P_{51}	P_{48}

Line 7 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{13}
in point	P_{55}	P_{61}	P_5	P_5	P_{60}	P_{66}

Line 8 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{14}
in point	P_{72}	P_{81}	P_5	P_5	P_{78}	P_{75}

Line 9 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_{41}	P_{45}	P_{51}	P_6	P_6	P_4

Line 10 intersects

Line	ℓ_0	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}
in point	P_{54}	P_{65}	P_{60}	P_6	P_6	P_{63}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}
in point	P_{71}	P_{73}	P_{78}	P_6	P_6	P_{84}

Line 12 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_9	ℓ_{13}	ℓ_{14}
in point	P_{40}	P_{49}	P_{48}	P_4	P_7	P_7

Line 13 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{56}	P_{57}	P_{66}	P_{63}	P_7	P_7

Line 14 intersects

Line	ℓ_0	ℓ_4	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_{70}	P_{77}	P_{75}	P_{84}	P_7	P_7

The surface has 37 points:

The points on the surface are:

$0:\ P_2 = (0,0,1,0)$
$1:\ P_4 = (1,1,1,1)$
$2:\ P_5 = (1,1,0,0)$
$3:\ P_6 = (2,1,0,0)$
$4:\ P_7 = (3,1,0,0)$
$5:\ P_{23} = (1,0,0,1)$
$6:\ P_{24} = (2,0,0,1)$
$7:\ P_{25} = (3,0,0,1)$
$8:\ P_{26} = (0,1,0,1)$
$9:\ P_{30} = (0,2,0,1)$
$10:\ P_{34} = (0,3,0,1)$
$11:\ P_{39} = (1,0,1,1)$
$12:\ P_{40} = (2,0,1,1)$
$13:\ P_{41} = (3,0,1,1)$
$14:\ P_{42} = (0,1,1,1)$
$15:\ P_{45} = (0,2,1,1)$
$16:\ P_{48} = (3,2,1,1)$
$17:\ P_{49} = (0,3,1,1)$
$18:\ P_{51} = (2,3,1,1)$
$19:\ P_{54} = (1,0,2,1)$
$20:\ P_{55} = (2,0,2,1)$
$21:\ P_{56} = (3,0,2,1)$
$22:\ P_{57} = (0,1,2,1)$
$23:\ P_{60} = (3,1,2,1)$
$24:\ P_{61} = (0,2,2,1)$
$25:\ P_{63} = (2,2,2,1)$
$26:\ P_{65} = (0,3,2,1)$
$27:\ P_{66} = (1,3,2,1)$
$28:\ P_{70} = (1,0,3,1)$
$29:\ P_{71} = (2,0,3,1)$
$30:\ P_{72} = (3,0,3,1)$
$31:\ P_{73} = (0,1,3,1)$
$32:\ P_{75} = (2,1,3,1)$
$33:\ P_{77} = (0,2,3,1)$
$34:\ P_{78} = (1,2,3,1)$
$35:\ P_{81} = (0,3,3,1)$
$36:\ P_{84} = (3,3,3,1)$