

Rank-362 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(4) is 26294

General information

| | |
|----------------------------|------------------|
| Number of lines | 2 |
| Number of points | 25 |
| Number of singular points | 0 |
| Number of Eckardt points | 0 |
| Number of double points | 0 |
| Number of single points | 10 |
| Number of points off lines | 15 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | 5^2 |
| Type of lines on points | $1^{10}, 0^{15}$ |

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{76} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{76} = \mathbf{Pl}(2, 3, 2, 0, 1, 1)_{184} \\ \ell_1 &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{51} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{51} = \mathbf{Pl}(3, 2, 3, 0, 1, 1)_{188}\end{aligned}$$

Rank of lines: (76, 51)

Rank of points on Klein quadric: (184, 188)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 10 single points:

The single points on the surface are:

0 : $P_9 = (2, 0, 1, 0)$ lies on line ℓ_0
 1 : $P_{10} = (3, 0, 1, 0)$ lies on line ℓ_1
 2 : $P_{33} = (3, 2, 0, 1)$ lies on line ℓ_0
 3 : $P_{36} = (2, 3, 0, 1)$ lies on line ℓ_1
 4 : $P_{46} = (1, 2, 1, 1)$ lies on line ℓ_0
 5 : $P_{50} = (1, 3, 1, 1)$ lies on line ℓ_1

6 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_0
 7 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_1
 8 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_0
 9 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 15 points not on any line:

The points on the surface but not on lines are:

0 : $P_8 = (1, 0, 1, 0)$
 1 : $P_{16} = (1, 2, 1, 0)$
 2 : $P_{20} = (1, 3, 1, 0)$
 3 : $P_{26} = (0, 1, 0, 1)$
 4 : $P_{30} = (0, 2, 0, 1)$
 5 : $P_{34} = (0, 3, 0, 1)$
 6 : $P_{38} = (0, 0, 1, 1)$
 7 : $P_{39} = (1, 0, 1, 1)$

8 : $P_{42} = (0, 1, 1, 1)$
 9 : $P_{53} = (0, 0, 2, 1)$
 10 : $P_{54} = (1, 0, 2, 1)$
 11 : $P_{64} = (3, 2, 2, 1)$
 12 : $P_{69} = (0, 0, 3, 1)$
 13 : $P_{70} = (1, 0, 3, 1)$
 14 : $P_{83} = (2, 3, 3, 1)$

Line Intersection Graph

| | | |
|---|---|---|
| | 0 | 1 |
| 0 | 0 | 0 |
| 1 | 0 | 0 |

Neighbor sets in the line intersection graph:

Line 0 intersects

| |
|----------|
| Line |
| in point |

Line 1 intersects

| Line |
|----------|
| in point |

The surface has 25 points:

The points on the surface are:

0 : $P_8 = (1, 0, 1, 0)$
1 : $P_9 = (2, 0, 1, 0)$
2 : $P_{10} = (3, 0, 1, 0)$
3 : $P_{16} = (1, 2, 1, 0)$
4 : $P_{20} = (1, 3, 1, 0)$
5 : $P_{26} = (0, 1, 0, 1)$
6 : $P_{30} = (0, 2, 0, 1)$
7 : $P_{33} = (3, 2, 0, 1)$
8 : $P_{34} = (0, 3, 0, 1)$

9 : $P_{36} = (2, 3, 0, 1)$
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13 : $P_{46} = (1, 2, 1, 1)$
14 : $P_{50} = (1, 3, 1, 1)$
15 : $P_{53} = (0, 0, 2, 1)$
16 : $P_{54} = (1, 0, 2, 1)$
17 : $P_{61} = (0, 2, 2, 1)$

18 : $P_{64} = (3, 2, 2, 1)$
19 : $P_{68} = (3, 3, 2, 1)$
20 : $P_{69} = (0, 0, 3, 1)$
21 : $P_{70} = (1, 0, 3, 1)$
22 : $P_{79} = (2, 2, 3, 1)$
23 : $P_{81} = (0, 3, 3, 1)$
24 : $P_{83} = (2, 3, 3, 1)$