Rank-65542 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0X_1X_2 = 0$$

(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431655774

General information

Number of lines	4
Number of points	17
Number of singular points	5
Number of Eckardt points	0
Number of double points	0
Number of single points	16
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	4, 1^{16}

Singular Points

The surface has 5 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
1 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
2 : $P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
3 : $P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$
4 : $P_{69} = \mathbf{P}(0,0,\omega^2,1) = \mathbf{P}(0,0,3,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_2 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{83} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{83} = \mathbf{Pl}(0,0,0,3,1,0)_{67}$$

$$\ell_3 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{62} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{62} = \mathbf{Pl}(0,0,0,2,1,0)_{60}$$

Rank of lines: (356, 41, 83, 62)
Rank of points on Klein quadric: (1, 53, 67, 60)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_2 = (0,0,1,0)$ lies on line ℓ_0
1 : $P_5 = (1,1,0,0)$ lies on line ℓ_1
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_2
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_3
4 : $P_{27} = (1,1,0,1)$ lies on line ℓ_1
5 : $P_{28} = (2,1,0,1)$ lies on line ℓ_2
6 : $P_{29} = (3,1,0,1)$ lies on line ℓ_3
7 : $P_{31} = (1,2,0,1)$ lies on line ℓ_3
8 : $P_{32} = (2,2,0,1)$ lies on line ℓ_1
9 : $P_{33} = (3,2,0,1)$ lies on line ℓ_2
10 : $P_{35} = (1,3,0,1)$ lies on line ℓ_2
11 : $P_{36} = (2,3,0,1)$ lies on line ℓ_3
12 : $P_{37} = (3,3,0,1)$ lies on line ℓ_1
13 : $P_{38} = (0,0,1,1)$ lies on line ℓ_0
14 : $P_{53} = (0,0,2,1)$ lies on line ℓ_0
15 : $P_{69} = (0,0,3,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3
0	0 1 1 1
1	1 0 1 1
2	1 1 0 1
3	1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_3	P_3	P_3

The surface has 17 points:
The points on the surface are:

0 : $P_2 = (0, 0, 1, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_6 = (2, 1, 0, 0)$
4 : $P_7 = (3, 1, 0, 0)$
5 : $P_{27} = (1, 1, 0, 1)$
6 : $P_{28} = (2, 1, 0, 1)$
7 : $P_{29} = (3, 1, 0, 1)$
8 : $P_{31} = (1, 2, 0, 1)$
9 : $P_{32} = (2, 2, 0, 1)$
10 : $P_{33} = (3, 2, 0, 1)$
11 : $P_{35} = (1, 3, 0, 1)$
12 : $P_{36} = (2, 3, 0, 1)$
13 : $P_{37} = (3, 3, 0, 1)$
14 : $P_{38} = (0, 0, 1, 1)$
15 : $P_{53} = (0, 0, 2, 1)$
16 : $P_{69} = (0, 0, 3, 1)$