Rank-65867 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303112469

General information

Number of lines	8
Number of points	305
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	114
Number of points off lines	182
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^8
Type of lines on points	$4, 3^2, 2^6, 1^{114}, 0^{182}$

Singular Points

The surface has 3 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$ 2 : $P_{291} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
1 : $P_{275} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \mathbf{Pl}(0,0,1,0,0,1)_{4656}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1,0,0,1,0,0)_{34}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \mathbf{Pl}(0,1,1,0,0,1)_{4672}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0,1,1,0,0,0)_{18}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(1,1,1,0,1,1)_{8976}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4385} = \mathbf{Pl}(1,1,1,1,1,0)_{1250}$$

Rank of lines: (0, 256, 529, 4368, 4897, 4624, 290, 4385)
Rank of points on Klein quadric: (0, 2, 4656, 34, 4672, 18, 8976, 1250)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{275} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$,
$1 : P_{291} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_{20} = (1,0,1,0) = \ell_1 \cap \ell_6$

$P_{36} = (1,1,1,0) = \ell_2 \cap \ell_7$
$P_{546} = (0,1,1,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 114 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_4
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$3 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$4 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$5 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$6 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$7 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$8 : P_{13} = (9,1,0,0)$ lies on line ℓ_0

$9 : P_{14} = (10,1,0,0)$ lies on line ℓ_0
$10 : P_{15} = (11,1,0,0)$ lies on line ℓ_0
$11 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$12 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$13 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$14 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$15 : P_{21} = (2,0,1,0)$ lies on line ℓ_1
$16 : P_{22} = (3,0,1,0)$ lies on line ℓ_1
$17 : P_{23} = (4,0,1,0)$ lies on line ℓ_1

18 : $P_{24} = (5, 0, 1, 0)$ lies on line ℓ_1

19 : $P_{25} = (6, 0, 1, 0)$ lies on line ℓ_1

20 : $P_{26} = (7, 0, 1, 0)$ lies on line ℓ_1

21 : $P_{27} = (8, 0, 1, 0)$ lies on line ℓ_1

22 : $P_{28} = (9, 0, 1, 0)$ lies on line ℓ_1

23 : $P_{29} = (10, 0, 1, 0)$ lies on line ℓ_1

24 : $P_{30} = (11, 0, 1, 0)$ lies on line ℓ_1

25 : $P_{31} = (12, 0, 1, 0)$ lies on line ℓ_1

26 : $P_{32} = (13, 0, 1, 0)$ lies on line ℓ_1

27 : $P_{33} = (14, 0, 1, 0)$ lies on line ℓ_1

28 : $P_{34} = (15, 0, 1, 0)$ lies on line ℓ_1

29 : $P_{53} = (2, 2, 1, 0)$ lies on line ℓ_2

30 : $P_{70} = (3, 3, 1, 0)$ lies on line ℓ_2

31 : $P_{87} = (4, 4, 1, 0)$ lies on line ℓ_2

32 : $P_{104} = (5, 5, 1, 0)$ lies on line ℓ_2

33 : $P_{121} = (6, 6, 1, 0)$ lies on line ℓ_2

34 : $P_{138} = (7, 7, 1, 0)$ lies on line ℓ_2

35 : $P_{155} = (8, 8, 1, 0)$ lies on line ℓ_2

36 : $P_{172} = (9, 9, 1, 0)$ lies on line ℓ_2

37 : $P_{189} = (10, 10, 1, 0)$ lies on line ℓ_2

38 : $P_{206} = (11, 11, 1, 0)$ lies on line ℓ_2

39 : $P_{223} = (12, 12, 1, 0)$ lies on line ℓ_2

40 : $P_{240} = (13, 13, 1, 0)$ lies on line ℓ_2

41 : $P_{257} = (14, 14, 1, 0)$ lies on line ℓ_2

42 : $P_{274} = (15, 15, 1, 0)$ lies on line ℓ_2

43 : $P_{307} = (1, 2, 0, 1)$ lies on line ℓ_3

44 : $P_{323} = (1, 3, 0, 1)$ lies on line ℓ_3

45 : $P_{339} = (1, 4, 0, 1)$ lies on line ℓ_3

46 : $P_{355} = (1, 5, 0, 1)$ lies on line ℓ_3

47 : $P_{371} = (1, 6, 0, 1)$ lies on line ℓ_3

48 : $P_{387} = (1, 7, 0, 1)$ lies on line ℓ_3

49 : $P_{403} = (1, 8, 0, 1)$ lies on line ℓ_3

50 : $P_{419} = (1, 9, 0, 1)$ lies on line ℓ_3

51 : $P_{435} = (1, 10, 0, 1)$ lies on line ℓ_3

52 : $P_{451} = (1, 11, 0, 1)$ lies on line ℓ_3

53 : $P_{467} = (1, 12, 0, 1)$ lies on line ℓ_3

54 : $P_{483} = (1, 13, 0, 1)$ lies on line ℓ_3

55 : $P_{499} = (1, 14, 0, 1)$ lies on line ℓ_3

56 : $P_{515} = (1, 15, 0, 1)$ lies on line ℓ_3

57 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_5

58 : $P_{786} = (1, 0, 2, 1)$ lies on line ℓ_5

59 : $P_{802} = (1, 1, 2, 1)$ lies on line ℓ_4

60 : $P_{804} = (3, 1, 2, 1)$ lies on line ℓ_6

61 : $P_{820} = (3, 2, 2, 1)$ lies on line ℓ_7

62 : $P_{1042} = (1, 0, 3, 1)$ lies on line ℓ_5

63 : $P_{1058} = (1, 1, 3, 1)$ lies on line ℓ_4

64 : $P_{1059} = (2, 1, 3, 1)$ lies on line ℓ_6

65 : $P_{1091} = (2, 3, 3, 1)$ lies on line ℓ_7

66 : $P_{1298} = (1, 0, 4, 1)$ lies on line ℓ_5

67 : $P_{1314} = (1, 1, 4, 1)$ lies on line ℓ_4

68 : $P_{1318} = (5, 1, 4, 1)$ lies on line ℓ_6

69 : $P_{1366} = (5, 4, 4, 1)$ lies on line ℓ_7

70 : $P_{1554} = (1, 0, 5, 1)$ lies on line ℓ_5

71 : $P_{1570} = (1, 1, 5, 1)$ lies on line ℓ_4

72 : $P_{1573} = (4, 1, 5, 1)$ lies on line ℓ_6

73 : $P_{1637} = (4, 5, 5, 1)$ lies on line ℓ_7

74 : $P_{1810} = (1, 0, 6, 1)$ lies on line ℓ_5

75 : $P_{1826} = (1, 1, 6, 1)$ lies on line ℓ_4

76 : $P_{1832} = (7, 1, 6, 1)$ lies on line ℓ_6

77 : $P_{1912} = (7, 6, 6, 1)$ lies on line ℓ_7

78 : $P_{2066} = (1, 0, 7, 1)$ lies on line ℓ_5

79 : $P_{2082} = (1, 1, 7, 1)$ lies on line ℓ_4

80 : $P_{2087} = (6, 1, 7, 1)$ lies on line ℓ_6

81 : $P_{2183} = (6, 7, 7, 1)$ lies on line ℓ_7

82 : $P_{2322} = (1, 0, 8, 1)$ lies on line ℓ_5

83 : $P_{2338} = (1, 1, 8, 1)$ lies on line ℓ_4

84 : $P_{2346} = (9, 1, 8, 1)$ lies on line ℓ_6

85 : $P_{2458} = (9, 8, 8, 1)$ lies on line ℓ_7

86 : $P_{2578} = (1, 0, 9, 1)$ lies on line ℓ_5

87 : $P_{2594} = (1, 1, 9, 1)$ lies on line ℓ_4

88 : $P_{2601} = (8, 1, 9, 1)$ lies on line ℓ_6

89 : $P_{2729} = (8, 9, 9, 1)$ lies on line ℓ_7

90 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_5

91 : $P_{2850} = (1, 1, 10, 1)$ lies on line ℓ_4

92 : $P_{2860} = (11, 1, 10, 1)$ lies on line ℓ_6

93 : $P_{3004} = (11, 10, 10, 1)$ lies on line ℓ_7

94 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_5

95 : $P_{3106} = (1, 1, 11, 1)$ lies on line ℓ_4

96 : $P_{3115} = (10, 1, 11, 1)$ lies on line ℓ_6

97 : $P_{3275} = (10, 11, 11, 1)$ lies on line ℓ_7

98 : $P_{3346} = (1, 0, 12, 1)$ lies on line ℓ_5

99 : $P_{3362} = (1, 1, 12, 1)$ lies on line ℓ_4

100 : $P_{3374} = (13, 1, 12, 1)$ lies on line ℓ_6

101 : $P_{3550} = (13, 12, 12, 1)$ lies on line ℓ_7

102 : $P_{3602} = (1, 0, 13, 1)$ lies on line ℓ_5

103 : $P_{3618} = (1, 1, 13, 1)$ lies on line ℓ_4

104 : $P_{3629} = (12, 1, 13, 1)$ lies on line ℓ_6

105 : $P_{3821} = (12, 13, 13, 1)$ lies on line ℓ_7

106 : $P_{3858} = (1, 0, 14, 1)$ lies on line ℓ_5

107 : $P_{3874} = (1, 1, 14, 1)$ lies on line ℓ_4

108 : $P_{3888} = (15, 1, 14, 1)$ lies on line ℓ_6

109 : $P_{4096} = (15, 14, 14, 1)$ lies on line ℓ_7

110 : $P_{4114} = (1, 0, 15, 1)$ lies on line ℓ_5

111 : $P_{4130} = (1, 1, 15, 1)$ lies on line ℓ_4

112 : $P_{4143} = (14, 1, 15, 1)$ lies on line ℓ_6

113 : $P_{4367} = (14, 15, 15, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 182 points not on any line:
The points on the surface but not on lines are:

$0: P_{570} = (9, 2, 1, 1)$

$1: P_{572} = (11, 2, 1, 1)$

$2: P_{603} = (10, 4, 1, 1)$

$3: P_{607} = (14, 4, 1, 1)$

$4: P_{675} = (2, 9, 1, 1)$

$5: P_{684} = (11, 9, 1, 1)$

$6: P_{693} = (4, 10, 1, 1)$

$7: P_{703} = (14, 10, 1, 1)$

$8: P_{707} = (2, 11, 1, 1)$

$9: P_{714} = (9, 11, 1, 1)$

$10: P_{757} = (4, 14, 1, 1)$

$11: P_{763} = (10, 14, 1, 1)$

$12: P_{824} = (7, 2, 2, 1)$

$13: P_{872} = (7, 5, 2, 1)$

$14: P_{878} = (13, 5, 2, 1)$

$15: P_{897} = (0, 7, 2, 1)$

$16: P_{911} = (14, 7, 2, 1)$

$17: P_{917} = (4, 8, 2, 1)$

$18: P_{926} = (13, 8, 2, 1)$

$19: P_{934} = (5, 9, 2, 1)$

$20: P_{943} = (14, 9, 2, 1)$

$21: P_{981} = (4, 12, 2, 1)$

$22: P_{982} = (5, 12, 2, 1)$

$23: P_{1081} = (8, 2, 3, 1)$

$24: P_{1087} = (14, 2, 3, 1)$

$25: P_{1096} = (7, 3, 3, 1)$

$26: P_{1112} = (7, 4, 3, 1)$

$27: P_{1116} = (11, 4, 3, 1)$

$28: P_{1121} = (0, 5, 3, 1)$

$29: P_{1136} = (15, 5, 3, 1)$

$30: P_{1209} = (8, 10, 3, 1)$

$31: P_{1216} = (15, 10, 3, 1)$

$32: P_{1236} = (3, 12, 3, 1)$

$33: P_{1247} = (14, 12, 3, 1)$

$34: P_{1284} = (3, 15, 3, 1)$

$35: P_{1292} = (11, 15, 3, 1)$

$36: P_{1373} = (12, 4, 4, 1)$

$37: P_{1401} = (8, 6, 4, 1)$

$38: P_{1402} = (9, 6, 4, 1)$

$39: P_{1432} = (7, 8, 4, 1)$

$40: P_{1437} = (12, 8, 4, 1)$

$41: P_{1489} = (0, 12, 4, 1)$

$42: P_{1491} = (2, 12, 4, 1)$

$43: P_{1523} = (2, 14, 4, 1)$

$44: P_{1529} = (8, 14, 4, 1)$

$45: P_{1544} = (7, 15, 4, 1)$

$46: P_{1546} = (9, 15, 4, 1)$

$47: P_{1606} = (5, 3, 5, 1)$

$48: P_{1611} = (10, 3, 5, 1)$

$49: P_{1619} = (2, 4, 5, 1)$

$50: P_{1632} = (15, 4, 5, 1)$

$51: P_{1645} = (12, 5, 5, 1)$

$52: P_{1651} = (2, 6, 5, 1)$

$53: P_{1654} = (5, 6, 5, 1)$

$54: P_{1681} = (0, 8, 5, 1)$

$55: P_{1684} = (3, 8, 5, 1)$

$56: P_{1707} = (10, 9, 5, 1)$

$57: P_{1709} = (12, 9, 5, 1)$

$58: P_{1732} = (3, 11, 5, 1)$

$59: P_{1744} = (15, 11, 5, 1)$

$60: P_{1841} = (0, 2, 6, 1)$

$61: P_{1853} = (12, 2, 6, 1)$

$62: P_{1898} = (9, 5, 6, 1)$

$63: P_{1903} = (14, 5, 6, 1)$

$64: P_{1915} = (10, 6, 6, 1)$

$65: P_{1991} = (6, 11, 6, 1)$

$66: P_{1999} = (14, 11, 6, 1)$

$67: P_{2010} = (9, 12, 6, 1)$

$68: P_{2011} = (10, 12, 6, 1)$

$69: P_{2020} = (3, 13, 6, 1)$

$70: P_{2023} = (6, 13, 6, 1)$

$71: P_{2036} = (3, 14, 6, 1)$

$72: P_{2045} = (12, 14, 6, 1)$

$73: P_{2137} = (8, 4, 7, 1)$

$74: P_{2142} = (13, 4, 7, 1)$

$75: P_{2187} = (10, 7, 7, 1)$

$76: P_{2209} = (0, 9, 7, 1)$

$77: P_{2222} = (13, 9, 7, 1)$

$78: P_{2245} = (4, 11, 7, 1)$

$79: P_{2248} = (7, 11, 7, 1)$

$80: P_{2264} = (7, 12, 7, 1)$

$81: P_{2265} = (8, 12, 7, 1)$

$82: P_{2275} = (2, 13, 7, 1)$

$83: P_{2283} = (10, 13, 7, 1)$

$84: P_{2307} = (2, 15, 7, 1)$

$85: P_{2309} = (4, 15, 7, 1)$

$86: P_{2409} = (8, 5, 8, 1)$

$87: P_{2412} = (11, 5, 8, 1)$

$88: P_{2455} = (6, 8, 8, 1)$

$89: P_{2468} = (3, 9, 8, 1)$

$90: P_{2469} = (4, 9, 8, 1)$

$91: P_{2484} = (3, 10, 8, 1)$

$92: P_{2486} = (5, 10, 8, 1)$

$93: P_{2533} = (4, 13, 8, 1)$

$94: P_{2537} = (8, 13, 8, 1)$

$95: P_{2551} = (6, 14, 8, 1)$

$96: P_{2556} = (11, 14, 8, 1)$
$97: P_{2561} = (0, 15, 8, 1)$
$98: P_{2566} = (5, 15, 8, 1)$
$99: P_{2613} = (4, 2, 9, 1)$
$100: P_{2624} = (15, 2, 9, 1)$
$101: P_{2637} = (12, 3, 9, 1)$
$102: P_{2639} = (14, 3, 9, 1)$
$103: P_{2673} = (0, 6, 9, 1)$
$104: P_{2677} = (4, 6, 9, 1)$
$105: P_{2727} = (6, 9, 9, 1)$
$106: P_{2799} = (14, 13, 9, 1)$
$107: P_{2800} = (15, 13, 9, 1)$
$108: P_{2823} = (6, 15, 9, 1)$
$109: P_{2829} = (12, 15, 9, 1)$
$110: P_{2916} = (3, 5, 10, 1)$
$111: P_{2923} = (10, 5, 10, 1)$
$112: P_{2932} = (3, 6, 10, 1)$
$113: P_{2942} = (13, 6, 10, 1)$
$114: P_{2953} = (8, 7, 10, 1)$
$115: P_{2957} = (12, 7, 10, 1)$
$116: P_{2993} = (0, 10, 10, 1)$
$117: P_{3021} = (12, 11, 10, 1)$
$118: P_{3022} = (13, 11, 10, 1)$
$119: P_{3081} = (8, 15, 10, 1)$
$120: P_{3083} = (10, 15, 10, 1)$
$121: P_{3148} = (11, 3, 11, 1)$
$122: P_{3152} = (15, 3, 11, 1)$
$123: P_{3222} = (5, 8, 11, 1)$
$124: P_{3228} = (11, 8, 11, 1)$
$125: P_{3255} = (6, 10, 11, 1)$
$126: P_{3256} = (7, 10, 11, 1)$
$127: P_{3265} = (0, 11, 11, 1)$
$128: P_{3287} = (6, 12, 11, 1)$
$129: P_{3296} = (15, 12, 11, 1)$
$130: P_{3302} = (5, 13, 11, 1)$
$131: P_{3304} = (7, 13, 11, 1)$
$132: P_{3397} = (4, 3, 12, 1)$
$133: P_{3402} = (9, 3, 12, 1)$
$134: P_{3453} = (12, 6, 12, 1)$
$135: P_{3456} = (15, 6, 12, 1)$
$136: P_{3461} = (4, 7, 12, 1)$
$137: P_{3468} = (11, 7, 12, 1)$
$138: P_{3496} = (7, 9, 12, 1)$
$139: P_{3504} = (15, 9, 12, 1)$

$140: P_{3514} = (9, 10, 12, 1)$
$141: P_{3517} = (12, 10, 12, 1)$
$142: P_{3548} = (11, 12, 12, 1)$
$143: P_{3569} = (0, 14, 12, 1)$
$144: P_{3576} = (7, 14, 12, 1)$
$145: P_{3638} = (5, 2, 13, 1)$
$146: P_{3639} = (6, 2, 13, 1)$
$147: P_{3665} = (0, 4, 13, 1)$
$148: P_{3671} = (6, 4, 13, 1)$
$149: P_{3708} = (11, 6, 13, 1)$
$150: P_{3711} = (14, 6, 13, 1)$
$151: P_{3718} = (5, 7, 13, 1)$
$152: P_{3726} = (13, 7, 13, 1)$
$153: P_{3731} = (2, 8, 13, 1)$
$154: P_{3743} = (14, 8, 13, 1)$
$155: P_{3763} = (2, 10, 13, 1)$
$156: P_{3774} = (13, 10, 13, 1)$
$157: P_{3820} = (11, 13, 13, 1)$
$158: P_{3911} = (6, 3, 14, 1)$
$159: P_{3918} = (13, 3, 14, 1)$
$160: P_{3924} = (3, 4, 14, 1)$
$161: P_{3930} = (9, 4, 14, 1)$
$162: P_{3939} = (2, 5, 14, 1)$
$163: P_{3943} = (6, 5, 14, 1)$
$164: P_{3971} = (2, 7, 14, 1)$
$165: P_{3972} = (3, 7, 14, 1)$
$166: P_{4065} = (0, 13, 14, 1)$
$167: P_{4074} = (9, 13, 14, 1)$
$168: P_{4094} = (13, 14, 14, 1)$
$169: P_{4155} = (10, 2, 15, 1)$
$170: P_{4158} = (13, 2, 15, 1)$
$171: P_{4161} = (0, 3, 15, 1)$
$172: P_{4169} = (8, 3, 15, 1)$
$173: P_{4234} = (9, 7, 15, 1)$
$174: P_{4240} = (15, 7, 15, 1)$
$175: P_{4251} = (10, 8, 15, 1)$
$176: P_{4256} = (15, 8, 15, 1)$
$177: P_{4294} = (5, 11, 15, 1)$
$178: P_{4297} = (8, 11, 15, 1)$
$179: P_{4342} = (5, 14, 15, 1)$
$180: P_{4346} = (9, 14, 15, 1)$
$181: P_{4366} = (13, 15, 15, 1)$

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7
--+----------------
0 |0 1 1 1 0 0 0 0
1 |1 0 1 0 1 1 1 0
2 |1 1 0 0 1 1 0 1
3 |1 0 0 0 1 1 1 1
4 |0 1 1 1 0 1 1 0
5 |0 1 1 1 1 0 0 1
6 |0 1 0 1 1 0 0 1
7 |0 0 1 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_2	P_2	P_2	P_{20}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_5	P_2	P_2	P_2	P_{36}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{291}	P_{275}	P_{291}	P_{275}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{291}	P_2	P_{291}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_{275}	P_2	P_{275}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_{20}	P_{291}	P_{291}	P_{546}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_{36}	P_{275}	P_{275}	P_{546}

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_4 = (1,1,1,1)$
4 : $P_5 = (1,1,0,0)$
5 : $P_6 = (2,1,0,0)$
6 : $P_7 = (3,1,0,0)$
7 : $P_8 = (4,1,0,0)$
8 : $P_9 = (5,1,0,0)$
9 : $P_{10} = (6,1,0,0)$
10 : $P_{11} = (7,1,0,0)$

11 : $P_{12} = (8,1,0,0)$
12 : $P_{13} = (9,1,0,0)$
13 : $P_{14} = (10,1,0,0)$
14 : $P_{15} = (11,1,0,0)$
15 : $P_{16} = (12,1,0,0)$
16 : $P_{17} = (13,1,0,0)$
17 : $P_{18} = (14,1,0,0)$
18 : $P_{19} = (15,1,0,0)$
19 : $P_{20} = (1,0,1,0)$
20 : $P_{21} = (2,0,1,0)$
21 : $P_{22} = (3,0,1,0)$

22 : $P_{23} = (4,0,1,0)$
23 : $P_{24} = (5,0,1,0)$
24 : $P_{25} = (6,0,1,0)$
25 : $P_{26} = (7,0,1,0)$
26 : $P_{27} = (8,0,1,0)$
27 : $P_{28} = (9,0,1,0)$
28 : $P_{29} = (10,0,1,0)$
29 : $P_{30} = (11,0,1,0)$
30 : $P_{31} = (12,0,1,0)$
31 : $P_{32} = (13,0,1,0)$
32 : $P_{33} = (14,0,1,0)$

$33: P_{34} = (15, 0, 1, 0)$

$34: P_{36} = (1, 1, 1, 0)$

$35: P_{53} = (2, 2, 1, 0)$

$36: P_{70} = (3, 3, 1, 0)$

$37: P_{87} = (4, 4, 1, 0)$

$38: P_{104} = (5, 5, 1, 0)$

$39: P_{121} = (6, 6, 1, 0)$

$40: P_{138} = (7, 7, 1, 0)$

$41: P_{155} = (8, 8, 1, 0)$

$42: P_{172} = (9, 9, 1, 0)$

$43: P_{189} = (10, 10, 1, 0)$

$44: P_{206} = (11, 11, 1, 0)$

$45: P_{223} = (12, 12, 1, 0)$

$46: P_{240} = (13, 13, 1, 0)$

$47: P_{257} = (14, 14, 1, 0)$

$48: P_{274} = (15, 15, 1, 0)$

$49: P_{275} = (1, 0, 0, 1)$

$50: P_{291} = (1, 1, 0, 1)$

$51: P_{307} = (1, 2, 0, 1)$

$52: P_{323} = (1, 3, 0, 1)$

$53: P_{339} = (1, 4, 0, 1)$

$54: P_{355} = (1, 5, 0, 1)$

$55: P_{371} = (1, 6, 0, 1)$

$56: P_{387} = (1, 7, 0, 1)$

$57: P_{403} = (1, 8, 0, 1)$

$58: P_{419} = (1, 9, 0, 1)$

$59: P_{435} = (1, 10, 0, 1)$

$60: P_{451} = (1, 11, 0, 1)$

$61: P_{467} = (1, 12, 0, 1)$

$62: P_{483} = (1, 13, 0, 1)$

$63: P_{499} = (1, 14, 0, 1)$

$64: P_{515} = (1, 15, 0, 1)$

$65: P_{531} = (1, 0, 1, 1)$

$66: P_{546} = (0, 1, 1, 1)$

$67: P_{570} = (9, 2, 1, 1)$

$68: P_{572} = (11, 2, 1, 1)$

$69: P_{603} = (10, 4, 1, 1)$

$70: P_{607} = (14, 4, 1, 1)$

$71: P_{675} = (2, 9, 1, 1)$

$72: P_{684} = (11, 9, 1, 1)$

$73: P_{693} = (4, 10, 1, 1)$

$74: P_{703} = (14, 10, 1, 1)$

$75: P_{707} = (2, 11, 1, 1)$

$76: P_{714} = (9, 11, 1, 1)$

$77: P_{757} = (4, 14, 1, 1)$

$78: P_{763} = (10, 14, 1, 1)$

$79: P_{786} = (1, 0, 2, 1)$

$80: P_{802} = (1, 1, 2, 1)$

$81: P_{804} = (3, 1, 2, 1)$

$82: P_{820} = (3, 2, 2, 1)$

$83: P_{824} = (7, 2, 2, 1)$

$84: P_{872} = (7, 5, 2, 1)$

$85: P_{878} = (13, 5, 2, 1)$

$86: P_{897} = (0, 7, 2, 1)$

$87: P_{911} = (14, 7, 2, 1)$

$88: P_{917} = (4, 8, 2, 1)$

$89: P_{926} = (13, 8, 2, 1)$

$90: P_{934} = (5, 9, 2, 1)$

$91: P_{943} = (14, 9, 2, 1)$

$92: P_{981} = (4, 12, 2, 1)$

$93: P_{982} = (5, 12, 2, 1)$

$94: P_{1042} = (1, 0, 3, 1)$

$95: P_{1058} = (1, 1, 3, 1)$

$96: P_{1059} = (2, 1, 3, 1)$

$97: P_{1081} = (8, 2, 3, 1)$

$98: P_{1087} = (14, 2, 3, 1)$

$99: P_{1091} = (2, 3, 3, 1)$

$100: P_{1096} = (7, 3, 3, 1)$

$101: P_{1112} = (7, 4, 3, 1)$

$102: P_{1116} = (11, 4, 3, 1)$

$103: P_{1121} = (0, 5, 3, 1)$

$104: P_{1136} = (15, 5, 3, 1)$

$105: P_{1209} = (8, 10, 3, 1)$

$106: P_{1216} = (15, 10, 3, 1)$

$107: P_{1236} = (3, 12, 3, 1)$

$108: P_{1247} = (14, 12, 3, 1)$

$109: P_{1284} = (3, 15, 3, 1)$

$110: P_{1292} = (11, 15, 3, 1)$

$111: P_{1298} = (1, 0, 4, 1)$

$112: P_{1314} = (1, 1, 4, 1)$

$113: P_{1318} = (5, 1, 4, 1)$

$114: P_{1366} = (5, 4, 4, 1)$

$115: P_{1373} = (12, 4, 4, 1)$

$116: P_{1401} = (8, 6, 4, 1)$

$117: P_{1402} = (9, 6, 4, 1)$

$118: P_{1432} = (7, 8, 4, 1)$

$119: P_{1437} = (12, 8, 4, 1)$

$120: P_{1489} = (0, 12, 4, 1)$

$121: P_{1491} = (2, 12, 4, 1)$

$122: P_{1523} = (2, 14, 4, 1)$

$123: P_{1529} = (8, 14, 4, 1)$

$124: P_{1544} = (7, 15, 4, 1)$

$125: P_{1546} = (9, 15, 4, 1)$

$126: P_{1554} = (1, 0, 5, 1)$

$127: P_{1570} = (1, 1, 5, 1)$

$128: P_{1573} = (4, 1, 5, 1)$

$129: P_{1606} = (5, 3, 5, 1)$

$130: P_{1611} = (10, 3, 5, 1)$

$131: P_{1619} = (2, 4, 5, 1)$

$132: P_{1632} = (15, 4, 5, 1)$

$133: P_{1637} = (4, 5, 5, 1)$

$134: P_{1645} = (12, 5, 5, 1)$

$135: P_{1651} = (2, 6, 5, 1)$

$136: P_{1654} = (5, 6, 5, 1)$

$137: P_{1681} = (0, 8, 5, 1)$

$138: P_{1684} = (3, 8, 5, 1)$

$139: P_{1707} = (10, 9, 5, 1)$

$140: P_{1709} = (12, 9, 5, 1)$

$141: P_{1732} = (3, 11, 5, 1)$

$142: P_{1744} = (15, 11, 5, 1)$

$143: P_{1810} = (1, 0, 6, 1)$

$144: P_{1826} = (1, 1, 6, 1)$

$145: P_{1832} = (7, 1, 6, 1)$

$146: P_{1841} = (0, 2, 6, 1)$

$147: P_{1853} = (12, 2, 6, 1)$

$148: P_{1898} = (9, 5, 6, 1)$

$149: P_{1903} = (14, 5, 6, 1)$

$150: P_{1912} = (7, 6, 6, 1)$

$151: P_{1915} = (10, 6, 6, 1)$

$152: P_{1991} = (6, 11, 6, 1)$

$153: P_{1999} = (14, 11, 6, 1)$

$154: P_{2010} = (9, 12, 6, 1)$

$155: P_{2011} = (10, 12, 6, 1)$

$156: P_{2020} = (3, 13, 6, 1)$

$157: P_{2023} = (6, 13, 6, 1)$

$158: P_{2036} = (3, 14, 6, 1)$

$159: P_{2045} = (12, 14, 6, 1)$

$160: P_{2066} = (1, 0, 7, 1)$

$161: P_{2082} = (1, 1, 7, 1)$

$162: P_{2087} = (6, 1, 7, 1)$

$163: P_{2137} = (8, 4, 7, 1)$

$164: P_{2142} = (13, 4, 7, 1)$

$165: P_{2183} = (6, 7, 7, 1)$

$166: P_{2187} = (10, 7, 7, 1)$

$167: P_{2209} = (0, 9, 7, 1)$

$168: P_{2222} = (13, 9, 7, 1)$

$169: P_{2245} = (4, 11, 7, 1)$

$170: P_{2248} = (7, 11, 7, 1)$

$171: P_{2264} = (7, 12, 7, 1)$

$172: P_{2265} = (8, 12, 7, 1)$

$173: P_{2275} = (2, 13, 7, 1)$

$174: P_{2283} = (10, 13, 7, 1)$

$175: P_{2307} = (2, 15, 7, 1)$

$176: P_{2309} = (4, 15, 7, 1)$

$177: P_{2322} = (1, 0, 8, 1)$

$178: P_{2338} = (1, 1, 8, 1)$

$179: P_{2346} = (9, 1, 8, 1)$

$180: P_{2409} = (8, 5, 8, 1)$

$181: P_{2412} = (11, 5, 8, 1)$

$182: P_{2455} = (6, 8, 8, 1)$

$183: P_{2458} = (9, 8, 8, 1)$

$184: P_{2468} = (3, 9, 8, 1)$

$185: P_{2469} = (4, 9, 8, 1)$

$186: P_{2484} = (3, 10, 8, 1)$

$187: P_{2486} = (5, 10, 8, 1)$

$188: P_{2533} = (4, 13, 8, 1)$

$189: P_{2537} = (8, 13, 8, 1)$

$190: P_{2551} = (6, 14, 8, 1)$

$191: P_{2556} = (11, 14, 8, 1)$

$192: P_{2561} = (0, 15, 8, 1)$

$193: P_{2566} = (5, 15, 8, 1)$

$194: P_{2578} = (1, 0, 9, 1)$

$195: P_{2594} = (1, 1, 9, 1)$
$196: P_{2601} = (8, 1, 9, 1)$
$197: P_{2613} = (4, 2, 9, 1)$
$198: P_{2624} = (15, 2, 9, 1)$
$199: P_{2637} = (12, 3, 9, 1)$
$200: P_{2639} = (14, 3, 9, 1)$
$201: P_{2673} = (0, 6, 9, 1)$
$202: P_{2677} = (4, 6, 9, 1)$
$203: P_{2727} = (6, 9, 9, 1)$
$204: P_{2729} = (8, 9, 9, 1)$
$205: P_{2799} = (14, 13, 9, 1)$
$206: P_{2800} = (15, 13, 9, 1)$
$207: P_{2823} = (6, 15, 9, 1)$
$208: P_{2829} = (12, 15, 9, 1)$
$209: P_{2834} = (1, 0, 10, 1)$
$210: P_{2850} = (1, 1, 10, 1)$
$211: P_{2860} = (11, 1, 10, 1)$
$212: P_{2916} = (3, 5, 10, 1)$
$213: P_{2923} = (10, 5, 10, 1)$
$214: P_{2932} = (3, 6, 10, 1)$
$215: P_{2942} = (13, 6, 10, 1)$
$216: P_{2953} = (8, 7, 10, 1)$
$217: P_{2957} = (12, 7, 10, 1)$
$218: P_{2993} = (0, 10, 10, 1)$
$219: P_{3004} = (11, 10, 10, 1)$
$220: P_{3021} = (12, 11, 10, 1)$
$221: P_{3022} = (13, 11, 10, 1)$
$222: P_{3081} = (8, 15, 10, 1)$
$223: P_{3083} = (10, 15, 10, 1)$
$224: P_{3090} = (1, 0, 11, 1)$
$225: P_{3106} = (1, 1, 11, 1)$
$226: P_{3115} = (10, 1, 11, 1)$
$227: P_{3148} = (11, 3, 11, 1)$
$228: P_{3152} = (15, 3, 11, 1)$
$229: P_{3222} = (5, 8, 11, 1)$
$230: P_{3228} = (11, 8, 11, 1)$
$231: P_{3255} = (6, 10, 11, 1)$

$232: P_{3256} = (7, 10, 11, 1)$
$233: P_{3265} = (0, 11, 11, 1)$
$234: P_{3275} = (10, 11, 11, 1)$
$235: P_{3287} = (6, 12, 11, 1)$
$236: P_{3296} = (15, 12, 11, 1)$
$237: P_{3302} = (5, 13, 11, 1)$
$238: P_{3304} = (7, 13, 11, 1)$
$239: P_{3346} = (1, 0, 12, 1)$
$240: P_{3362} = (1, 1, 12, 1)$
$241: P_{3374} = (13, 1, 12, 1)$
$242: P_{3397} = (4, 3, 12, 1)$
$243: P_{3402} = (9, 3, 12, 1)$
$244: P_{3453} = (12, 6, 12, 1)$
$245: P_{3456} = (15, 6, 12, 1)$
$246: P_{3461} = (4, 7, 12, 1)$
$247: P_{3468} = (11, 7, 12, 1)$
$248: P_{3496} = (7, 9, 12, 1)$
$249: P_{3504} = (15, 9, 12, 1)$
$250: P_{3514} = (9, 10, 12, 1)$
$251: P_{3517} = (12, 10, 12, 1)$
$252: P_{3548} = (11, 12, 12, 1)$
$253: P_{3550} = (13, 12, 12, 1)$
$254: P_{3569} = (0, 14, 12, 1)$
$255: P_{3576} = (7, 14, 12, 1)$
$256: P_{3602} = (1, 0, 13, 1)$
$257: P_{3618} = (1, 1, 13, 1)$
$258: P_{3629} = (12, 1, 13, 1)$
$259: P_{3638} = (5, 2, 13, 1)$
$260: P_{3639} = (6, 2, 13, 1)$
$261: P_{3665} = (0, 4, 13, 1)$
$262: P_{3671} = (6, 4, 13, 1)$
$263: P_{3708} = (11, 6, 13, 1)$
$264: P_{3711} = (14, 6, 13, 1)$
$265: P_{3718} = (5, 7, 13, 1)$
$266: P_{3726} = (13, 7, 13, 1)$
$267: P_{3731} = (2, 8, 13, 1)$
$268: P_{3743} = (14, 8, 13, 1)$

$269: P_{3763} = (2, 10, 13, 1)$
$270: P_{3774} = (13, 10, 13, 1)$
$271: P_{3820} = (11, 13, 13, 1)$
$272: P_{3821} = (12, 13, 13, 1)$
$273: P_{3858} = (1, 0, 14, 1)$
$274: P_{3874} = (1, 1, 14, 1)$
$275: P_{3888} = (15, 1, 14, 1)$
$276: P_{3911} = (6, 3, 14, 1)$
$277: P_{3918} = (13, 3, 14, 1)$
$278: P_{3924} = (3, 4, 14, 1)$
$279: P_{3930} = (9, 4, 14, 1)$
$280: P_{3939} = (2, 5, 14, 1)$
$281: P_{3943} = (6, 5, 14, 1)$
$282: P_{3971} = (2, 7, 14, 1)$
$283: P_{3972} = (3, 7, 14, 1)$
$284: P_{4065} = (0, 13, 14, 1)$
$285: P_{4074} = (9, 13, 14, 1)$
$286: P_{4094} = (13, 14, 14, 1)$
$287: P_{4096} = (15, 14, 14, 1)$
$288: P_{4114} = (1, 0, 15, 1)$
$289: P_{4130} = (1, 1, 15, 1)$
$290: P_{4143} = (14, 1, 15, 1)$
$291: P_{4155} = (10, 2, 15, 1)$
$292: P_{4158} = (13, 2, 15, 1)$
$293: P_{4161} = (0, 3, 15, 1)$
$294: P_{4169} = (8, 3, 15, 1)$
$295: P_{4234} = (9, 7, 15, 1)$
$296: P_{4240} = (15, 7, 15, 1)$
$297: P_{4251} = (10, 8, 15, 1)$
$298: P_{4256} = (15, 8, 15, 1)$
$299: P_{4294} = (5, 11, 15, 1)$
$300: P_{4297} = (8, 11, 15, 1)$
$301: P_{4342} = (5, 14, 15, 1)$
$302: P_{4346} = (9, 14, 15, 1)$
$303: P_{4366} = (13, 15, 15, 1)$
$304: P_{4367} = (14, 15, 15, 1)$