Rank-76 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 = 0$$

(0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 9376

General information

Number of lines	1
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	1^9, 0^{64}

Singular Points

The surface has 1 singular points:

$0: \ P_{20} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

Rank of lines: (1)
Rank of points on Klein quadric: (3)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 9 single points:
The single points on the surface are:

$0: P_0 = (1,0,0,0)$ lies on line ℓ_0
$1: P_{19} = (0,1,1,0)$ lies on line ℓ_0
$2: P_{20} = (1,1,1,0)$ lies on line ℓ_0
$3: P_{21} = (2,1,1,0)$ lies on line ℓ_0
$4: P_{22} = (3,1,1,0)$ lies on line ℓ_0
$5: P_{23} = (4,1,1,0)$ lies on line ℓ_0
$6: P_{24} = (5,1,1,0)$ lies on line ℓ_0
$7: P_{25} = (6,1,1,0)$ lies on line ℓ_0
$8: P_{26} = (7,1,1,0)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:
The points on the surface but not on lines are:

$0: P_5 = (1,1,0,0)$
$1: P_{12} = (1,0,1,0)$
$2: P_{31} = (4,2,1,0)$
$3: P_{39} = (4,3,1,0)$
$4: P_{50} = (7,4,1,0)$
$5: P_{58} = (7,5,1,0)$
$6: P_{61} = (2,6,1,0)$
$7: P_{69} = (2,7,1,0)$
$8: P_{82} = (0,1,0,1)$
$9: P_{97} = (7,2,0,1)$
$10: P_{99} = (1,3,0,1)$
$11: P_{108} = (2,4,0,1)$
$12: P_{115} = (1,5,0,1)$
$13: P_{123} = (1,6,0,1)$
$14: P_{134} = (4,7,0,1)$
$15: P_{138} = (0,0,1,1)$
$16: P_{159} = (6,2,1,1)$
$17: P_{162} = (1,3,1,1)$
$18: P_{172} = (3,4,1,1)$
$19: P_{178} = (1,5,1,1)$
$20: P_{186} = (1,6,1,1)$
$21: P_{198} = (5,7,1,1)$
$22: P_{208} = (7,0,2,1)$
$23: P_{215} = (6,1,2,1)$
$24: P_{230} = (5,3,2,1)$
$25: P_{235} = (2,4,2,1)$
$26: P_{241} = (0,5,2,1)$
$27: P_{252} = (3,6,2,1)$
$28: P_{264} = (7,7,2,1)$
$29: P_{266} = (1,0,3,1)$
$30: P_{274} = (1,1,3,1)$
$31: P_{286} = (5,2,3,1)$
$32: P_{302} = (5,4,3,1)$
$33: P_{311} = (6,5,3,1)$
$34: P_{318} = (5,6,3,1)$
$35: P_{321} = (0,7,3,1)$

36 : $P_{331} = (2, 0, 4, 1)$
37 : $P_{340} = (3, 1, 4, 1)$
38 : $P_{347} = (2, 2, 4, 1)$
39 : $P_{358} = (5, 3, 4, 1)$
40 : $P_{375} = (6, 5, 4, 1)$
41 : $P_{377} = (0, 6, 4, 1)$
42 : $P_{389} = (4, 7, 4, 1)$
43 : $P_{394} = (1, 0, 5, 1)$
44 : $P_{402} = (1, 1, 5, 1)$
45 : $P_{409} = (0, 2, 5, 1)$
46 : $P_{423} = (6, 3, 5, 1)$
47 : $P_{431} = (6, 4, 5, 1)$
48 : $P_{444} = (3, 6, 5, 1)$
49 : $P_{455} = (6, 7, 5, 1)$
50 : $P_{458} = (1, 0, 6, 1)$
51 : $P_{466} = (1, 1, 6, 1)$
52 : $P_{476} = (3, 2, 6, 1)$
53 : $P_{486} = (5, 3, 6, 1)$
54 : $P_{489} = (0, 4, 6, 1)$
55 : $P_{500} = (3, 5, 6, 1)$
56 : $P_{516} = (3, 7, 6, 1)$
57 : $P_{525} = (4, 0, 7, 1)$
58 : $P_{534} = (5, 1, 7, 1)$
59 : $P_{544} = (7, 2, 7, 1)$
60 : $P_{545} = (0, 3, 7, 1)$
61 : $P_{557} = (4, 4, 7, 1)$
62 : $P_{567} = (6, 5, 7, 1)$
63 : $P_{572} = (3, 6, 7, 1)$

Line Intersection Graph

	0
0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 73 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{12} = (1, 0, 1, 0)$
3 : $P_{19} = (0, 1, 1, 0)$
4 : $P_{20} = (1, 1, 1, 0)$
5 : $P_{21} = (2, 1, 1, 0)$
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19 : $P_{99} = (1, 3, 0, 1)$
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21 : $P_{115} = (1, 5, 0, 1)$
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