Rank-264 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1^2 = 0$$

(0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(8) is 599190

General information

Number of lines	5
Number of points	89
Number of singular points	0
Number of Eckardt points	2
Number of double points	0
Number of single points	39
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	3^2, 1^{39}, 0^{48}

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0,0,1,0,1,0)_{96}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \mathbf{Pl}(1,1,1,0,1,1)_{1224}$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \mathbf{Pl}(1,1,1,1,1,0)_{306}$$

Rank of lines: (65, 4673, 138, 82, 593)
Rank of points on Klein quadric: (96, 769, 1322, 1224, 306)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$,
$1 : P_{146} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 39 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_1 = (0,1,0,0)$ lies on line ℓ_1
$2 : P_4 = (1,1,1,1)$ lies on line ℓ_2
$3 : P_5 = (1,1,0,0)$ lies on line ℓ_2
$4 : P_{12} = (1,0,1,0)$ lies on line ℓ_3
$5 : P_{20} = (1,1,1,0)$ lies on line ℓ_4
$6 : P_{75} = (1,0,0,1)$ lies on line ℓ_4
$7 : P_{83} = (1,1,0,1)$ lies on line ℓ_3
$8 : P_{139} = (1,0,1,1)$ lies on line ℓ_0
$9 : P_{140} = (2,0,1,1)$ lies on line ℓ_0
$10 : P_{141} = (3,0,1,1)$ lies on line ℓ_0
$11 : P_{142} = (4,0,1,1)$ lies on line ℓ_0
$12 : P_{143} = (5,0,1,1)$ lies on line ℓ_0
$13 : P_{144} = (6,0,1,1)$ lies on line ℓ_0
$14 : P_{145} = (7,0,1,1)$ lies on line ℓ_0
$15 : P_{153} = (0,2,1,1)$ lies on line ℓ_1
$16 : P_{155} = (2,2,1,1)$ lies on line ℓ_2
$17 : P_{161} = (0,3,1,1)$ lies on line ℓ_1
$18 : P_{164} = (3,3,1,1)$ lies on line ℓ_2
$19 : P_{169} = (0,4,1,1)$ lies on line ℓ_1
$20 : P_{173} = (4,4,1,1)$ lies on line ℓ_2
$21 : P_{177} = (0,5,1,1)$ lies on line ℓ_1
$22 : P_{182} = (5,5,1,1)$ lies on line ℓ_2
$23 : P_{185} = (0,6,1,1)$ lies on line ℓ_1
$24 : P_{191} = (6,6,1,1)$ lies on line ℓ_2
$25 : P_{193} = (0,7,1,1)$ lies on line ℓ_1
$26 : P_{200} = (7,7,1,1)$ lies on line ℓ_2
$27 : P_{212} = (3,1,2,1)$ lies on line ℓ_3
$28 : P_{220} = (3,2,2,1)$ lies on line ℓ_4
$29 : P_{275} = (2,1,3,1)$ lies on line ℓ_3
$30 : P_{291} = (2,3,3,1)$ lies on line ℓ_4
$31 : P_{342} = (5,1,4,1)$ lies on line ℓ_3
$32 : P_{366} = (5,4,4,1)$ lies on line ℓ_4
$33 : P_{405} = (4,1,5,1)$ lies on line ℓ_3
$34 : P_{437} = (4,5,5,1)$ lies on line ℓ_4
$35 : P_{472} = (7,1,6,1)$ lies on line ℓ_3
$36 : P_{512} = (7,6,6,1)$ lies on line ℓ_4
$37 : P_{535} = (6,1,7,1)$ lies on line ℓ_3
$38 : P_{583} = (6,7,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:
The points on the surface but not on lines are:

0 : $P_{205} = (4, 0, 2, 1)$
1 : $P_{214} = (5, 1, 2, 1)$
2 : $P_{224} = (7, 2, 2, 1)$
3 : $P_{231} = (6, 3, 2, 1)$
4 : $P_{237} = (4, 4, 2, 1)$
5 : $P_{238} = (5, 4, 2, 1)$
6 : $P_{247} = (6, 5, 2, 1)$
7 : $P_{248} = (7, 5, 2, 1)$
8 : $P_{269} = (4, 0, 3, 1)$
9 : $P_{279} = (6, 1, 3, 1)$
10 : $P_{286} = (5, 2, 3, 1)$
11 : $P_{296} = (7, 3, 3, 1)$
12 : $P_{301} = (4, 4, 3, 1)$
13 : $P_{304} = (7, 4, 3, 1)$
14 : $P_{326} = (5, 7, 3, 1)$
15 : $P_{327} = (6, 7, 3, 1)$
16 : $P_{336} = (7, 0, 4, 1)$
17 : $P_{343} = (6, 1, 4, 1)$
18 : $P_{363} = (2, 4, 4, 1)$
19 : $P_{372} = (3, 5, 4, 1)$
20 : $P_{379} = (2, 6, 4, 1)$
21 : $P_{380} = (3, 6, 4, 1)$
22 : $P_{391} = (6, 7, 4, 1)$
23 : $P_{392} = (7, 7, 4, 1)$
24 : $P_{400} = (7, 0, 5, 1)$
25 : $P_{404} = (3, 1, 5, 1)$
26 : $P_{412} = (3, 2, 5, 1)$
27 : $P_{415} = (6, 2, 5, 1)$
28 : $P_{431} = (6, 4, 5, 1)$
29 : $P_{435} = (2, 5, 5, 1)$
30 : $P_{451} = (2, 7, 5, 1)$
31 : $P_{456} = (7, 7, 5, 1)$
32 : $P_{459} = (2, 0, 6, 1)$
33 : $P_{470} = (5, 1, 6, 1)$
34 : $P_{475} = (2, 2, 6, 1)$
35 : $P_{477} = (4, 2, 6, 1)$
36 : $P_{492} = (3, 4, 6, 1)$
37 : $P_{494} = (5, 4, 6, 1)$
38 : $P_{509} = (4, 6, 6, 1)$
39 : $P_{516} = (3, 7, 6, 1)$
40 : $P_{523} = (2, 0, 7, 1)$
41 : $P_{532} = (3, 1, 7, 1)$
42 : $P_{539} = (2, 2, 7, 1)$
43 : $P_{540} = (3, 2, 7, 1)$
44 : $P_{549} = (4, 3, 7, 1)$
45 : $P_{550} = (5, 3, 7, 1)$
46 : $P_{574} = (5, 6, 7, 1)$
47 : $P_{581} = (4, 7, 7, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	1
2	1	1	0	0	0
3	0	1	0	0	1
4	0	1	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{138}	P_{138}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{138}	P_{138}	P_{146}	P_{146}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{138}	P_{138}

Line 3 intersects

Line	ℓ_1	ℓ_4
in point	P_{146}	P_{146}

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_{146}	P_{146}

The surface has 89 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_4 = (1,1,1,1)$
3 : $P_5 = (1,1,0,0)$
4 : $P_{12} = (1,0,1,0)$
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