Rank-65548 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65548

General information

Number of lines	3
Number of points	7
Number of singular points	2
Number of Eckardt points	0
Number of double points	3
Number of single points	3
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	2^3, 1^3, 0

Singular Points

The surface has 2 singular points:

$0: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{28} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{28} = \mathbf{Pl}(0,0,0,0,0,1)_{19}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$
$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

Rank of lines: (28, 14, 18)
Rank of points on Klein quadric: (19, 6, 4)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_1$
$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$
$P_9 = (1,0,0,1) = \ell_1 \cap \ell_2$

Single Points

The surface has 3 single points:
The single points on the surface are:

$0:\ P_7 = (0,1,1,0)$ lies on line ℓ_0
$1:\ P_{11} = (1,1,0,1)$ lies on line ℓ_1
$2:\ P_{13} = (1,0,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:
The points on the surface but not on lines are:

$0:\ P_8 = (1,1,1,0)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_9

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_9

The surface has 7 points:
The points on the surface are:

$0: P_1 = (0,1,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_7 = (0,1,1,0)$
$3: P_8 = (1,1,1,0)$
$4: P_9 = (1,0,0,1)$
$5: P_{11} = (1,1,0,1)$
$6: P_{13} = (1,0,1,1)$