

# Rank-65687 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )  
The point rank of the equation over GF(4) is 1431672425

## General information

Number of lines	4
Number of points	17
Number of singular points	5
Number of Eckardt points	0
Number of double points	0
Number of single points	16
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^4$
Type of lines on points	$4, 1^{16}$

## Singular Points

The surface has 5 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$1 : P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

$$2 : P_{12} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

$$3 : P_{63} = \mathbf{P}(\omega, \omega, \omega, 1) = \mathbf{P}(2, 2, 2, 1)$$

$$4 : P_{84} = \mathbf{P}(\omega^2, \omega^2, \omega^2, 1) = \mathbf{P}(3, 3, 3, 1)$$

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_{25}$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_9 \\ \ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{53} \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{57}\end{aligned}$$

Rank of lines: ( 20, 340, 41, 125 )

Rank of points on Klein quadric: ( 25, 9, 53, 57 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:

The double points on the surface are:

### Single Points

The surface has 16 single points:

The single points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$  lies on line  $\ell_0$   
1 :  $P_1 = (0, 1, 0, 0)$  lies on line  $\ell_1$   
2 :  $P_4 = (1, 1, 1, 1)$  lies on line  $\ell_3$   
3 :  $P_5 = (1, 1, 0, 0)$  lies on line  $\ell_2$   
4 :  $P_{12} = (1, 1, 1, 0)$  lies on line  $\ell_3$   
5 :  $P_{23} = (1, 0, 0, 1)$  lies on line  $\ell_0$   
6 :  $P_{24} = (2, 0, 0, 1)$  lies on line  $\ell_0$   
7 :  $P_{25} = (3, 0, 0, 1)$  lies on line  $\ell_0$   
8 :  $P_{26} = (0, 1, 0, 1)$  lies on line  $\ell_1$

9 :  $P_{27} = (1, 1, 0, 1)$  lies on line  $\ell_2$   
10 :  $P_{30} = (0, 2, 0, 1)$  lies on line  $\ell_1$   
11 :  $P_{32} = (2, 2, 0, 1)$  lies on line  $\ell_2$   
12 :  $P_{34} = (0, 3, 0, 1)$  lies on line  $\ell_1$   
13 :  $P_{37} = (3, 3, 0, 1)$  lies on line  $\ell_2$   
14 :  $P_{63} = (2, 2, 2, 1)$  lies on line  $\ell_3$   
15 :  $P_{84} = (3, 3, 3, 1)$  lies on line  $\ell_3$

The single points on the surface are:

### Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

## Line Intersection Graph

	0	1	2	3
0	0	1	1	1
1	1	0	1	1
2	1	1	0	1
3	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$
in point	$P_3$	$P_3$	$P_3$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_2$
in point	$P_3$	$P_3$	$P_3$

The surface has 17 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_1 = (0, 1, 0, 0)$   
 2 :  $P_3 = (0, 0, 0, 1)$   
 3 :  $P_4 = (1, 1, 1, 1)$   
 4 :  $P_5 = (1, 1, 0, 0)$   
 5 :  $P_{12} = (1, 1, 1, 0)$

6 :  $P_{23} = (1, 0, 0, 1)$   
 7 :  $P_{24} = (2, 0, 0, 1)$   
 8 :  $P_{25} = (3, 0, 0, 1)$   
 9 :  $P_{26} = (0, 1, 0, 1)$   
 10 :  $P_{27} = (1, 1, 0, 1)$   
 11 :  $P_{30} = (0, 2, 0, 1)$

12 :  $P_{32} = (2, 2, 0, 1)$   
 13 :  $P_{34} = (0, 3, 0, 1)$   
 14 :  $P_{37} = (3, 3, 0, 1)$   
 15 :  $P_{63} = (2, 2, 2, 1)$   
 16 :  $P_{84} = (3, 3, 3, 1)$