Rank-65570 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1107828870

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4096
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$3, 1^{192}, 0^{4096}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{4225} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]_{4225} = \mathbf{Pl}(1,1,0,0,1,1)_{536577}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \epsilon^{21} \end{array}\right]_{7809} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 57 \end{array}\right]_{7809} = \mathbf{Pl}(56,57,0,0,57,1)_{15213112}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \epsilon^{42} \end{bmatrix}_{7745} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 56 \end{bmatrix}_{7745} = \mathbf{Pl}(57, 56, 0, 0, 56, 1)_{14951033}$$

Rank of lines: (4225, 7809, 7745)
Rank of points on Klein quadric: (536577, 15213112, 14951033)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{68} = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_4 = (1, 1, 1, 1)$ lies on line ℓ_0
$1 : P_{4226} = (0, 1, 0, 1)$ lies on line ℓ_0
$2 : P_{7746} = (0, 56, 0, 1)$ lies on line ℓ_1
$3 : P_{7810} = (0, 57, 0, 1)$ lies on line ℓ_2
$4 : P_{11842} = (1, 56, 1, 1)$ lies on line ℓ_1
$5 : P_{11906} = (1, 57, 1, 1)$ lies on line ℓ_2
$6 : P_{12419} = (2, 1, 2, 1)$ lies on line ℓ_0
$7 : P_{15939} = (2, 56, 2, 1)$ lies on line ℓ_1
$8 : P_{16003} = (2, 57, 2, 1)$ lies on line ℓ_2
$9 : P_{16516} = (3, 1, 3, 1)$ lies on line ℓ_0
$10 : P_{20036} = (3, 56, 3, 1)$ lies on line ℓ_1
$11 : P_{20100} = (3, 57, 3, 1)$ lies on line ℓ_2
$12 : P_{20613} = (4, 1, 4, 1)$ lies on line ℓ_0
$13 : P_{24133} = (4, 56, 4, 1)$ lies on line ℓ_1
$14 : P_{24197} = (4, 57, 4, 1)$ lies on line ℓ_2
$15 : P_{24710} = (5, 1, 5, 1)$ lies on line ℓ_0
$16 : P_{28230} = (5, 56, 5, 1)$ lies on line ℓ_1
$17 : P_{28294} = (5, 57, 5, 1)$ lies on line ℓ_2
$18 : P_{28807} = (6, 1, 6, 1)$ lies on line ℓ_0
$19 : P_{32327} = (6, 56, 6, 1)$ lies on line ℓ_1
$20 : P_{32391} = (6, 57, 6, 1)$ lies on line ℓ_2
$21 : P_{32904} = (7, 1, 7, 1)$ lies on line ℓ_0
$22 : P_{36424} = (7, 56, 7, 1)$ lies on line ℓ_1
$23 : P_{36488} = (7, 57, 7, 1)$ lies on line ℓ_2
$24 : P_{37001} = (8, 1, 8, 1)$ lies on line ℓ_0
$25 : P_{40521} = (8, 56, 8, 1)$ lies on line ℓ_1
$26 : P_{40585} = (8, 57, 8, 1)$ lies on line ℓ_2
$27 : P_{41098} = (9, 1, 9, 1)$ lies on line ℓ_0
$28 : P_{44618} = (9, 56, 9, 1)$ lies on line ℓ_1
$29 : P_{44682} = (9, 57, 9, 1)$ lies on line ℓ_2
$30 : P_{45195} = (10, 1, 10, 1)$ lies on line ℓ_0
$31 : P_{48715} = (10, 56, 10, 1)$ lies on line ℓ_1
$32 : P_{48779} = (10, 57, 10, 1)$ lies on line ℓ_2
$33 : P_{49292} = (11, 1, 11, 1)$ lies on line ℓ_0
$34 : P_{52812} = (11, 56, 11, 1)$ lies on line ℓ_1
$35 : P_{52876} = (11, 57, 11, 1)$ lies on line ℓ_2
$36 : P_{53389} = (12, 1, 12, 1)$ lies on line ℓ_0
$37 : P_{56909} = (12, 56, 12, 1)$ lies on line ℓ_1
$38 : P_{56973} = (12, 57, 12, 1)$ lies on line ℓ_2
$39 : P_{57486} = (13, 1, 13, 1)$ lies on line ℓ_0
$40 : P_{61006} = (13, 56, 13, 1)$ lies on line ℓ_1
$41 : P_{61070} = (13, 57, 13, 1)$ lies on line ℓ_2
$42 : P_{61583} = (14, 1, 14, 1)$ lies on line ℓ_0
$43 : P_{65103} = (14, 56, 14, 1)$ lies on line ℓ_1
$44 : P_{65167} = (14, 57, 14, 1)$ lies on line ℓ_2
$45 : P_{65680} = (15, 1, 15, 1)$ lies on line ℓ_0
$46 : P_{69200} = (15, 56, 15, 1)$ lies on line ℓ_1
$47 : P_{69264} = (15, 57, 15, 1)$ lies on line ℓ_2
$48 : P_{69777} = (16, 1, 16, 1)$ lies on line ℓ_0
$49 : P_{73297} = (16, 56, 16, 1)$ lies on line ℓ_1
$50 : P_{73361} = (16, 57, 16, 1)$ lies on line ℓ_2
$51 : P_{73874} = (17, 1, 17, 1)$ lies on line ℓ_0
$52 : P_{77394} = (17, 56, 17, 1)$ lies on line ℓ_1
$53 : P_{77458} = (17, 57, 17, 1)$ lies on line ℓ_2
$54 : P_{77971} = (18, 1, 18, 1)$ lies on line ℓ_0
$55 : P_{81491} = (18, 56, 18, 1)$ lies on line ℓ_1
$56 : P_{81555} = (18, 57, 18, 1)$ lies on line ℓ_2
$57 : P_{82068} = (19, 1, 19, 1)$ lies on line ℓ_0

58 : $P_{85588} = (19, 56, 19, 1)$ lies on line ℓ_1
59 : $P_{85652} = (19, 57, 19, 1)$ lies on line ℓ_2
60 : $P_{86165} = (20, 1, 20, 1)$ lies on line ℓ_0
61 : $P_{89685} = (20, 56, 20, 1)$ lies on line ℓ_1
62 : $P_{89749} = (20, 57, 20, 1)$ lies on line ℓ_2
63 : $P_{90262} = (21, 1, 21, 1)$ lies on line ℓ_0
64 : $P_{93782} = (21, 56, 21, 1)$ lies on line ℓ_1
65 : $P_{93846} = (21, 57, 21, 1)$ lies on line ℓ_2
66 : $P_{94359} = (22, 1, 22, 1)$ lies on line ℓ_0
67 : $P_{97879} = (22, 56, 22, 1)$ lies on line ℓ_1
68 : $P_{97943} = (22, 57, 22, 1)$ lies on line ℓ_2
69 : $P_{98456} = (23, 1, 23, 1)$ lies on line ℓ_0
70 : $P_{101976} = (23, 56, 23, 1)$ lies on line ℓ_1
71 : $P_{102040} = (23, 57, 23, 1)$ lies on line ℓ_2
72 : $P_{102553} = (24, 1, 24, 1)$ lies on line ℓ_0
73 : $P_{106073} = (24, 56, 24, 1)$ lies on line ℓ_1
74 : $P_{106137} = (24, 57, 24, 1)$ lies on line ℓ_2
75 : $P_{106650} = (25, 1, 25, 1)$ lies on line ℓ_0
76 : $P_{110170} = (25, 56, 25, 1)$ lies on line ℓ_1
77 : $P_{110234} = (25, 57, 25, 1)$ lies on line ℓ_2
78 : $P_{110747} = (26, 1, 26, 1)$ lies on line ℓ_0
79 : $P_{114267} = (26, 56, 26, 1)$ lies on line ℓ_1
80 : $P_{114331} = (26, 57, 26, 1)$ lies on line ℓ_2
81 : $P_{114844} = (27, 1, 27, 1)$ lies on line ℓ_0
82 : $P_{118364} = (27, 56, 27, 1)$ lies on line ℓ_1
83 : $P_{118428} = (27, 57, 27, 1)$ lies on line ℓ_2
84 : $P_{118941} = (28, 1, 28, 1)$ lies on line ℓ_0
85 : $P_{122461} = (28, 56, 28, 1)$ lies on line ℓ_1
86 : $P_{122525} = (28, 57, 28, 1)$ lies on line ℓ_2
87 : $P_{123038} = (29, 1, 29, 1)$ lies on line ℓ_0
88 : $P_{126558} = (29, 56, 29, 1)$ lies on line ℓ_1
89 : $P_{126622} = (29, 57, 29, 1)$ lies on line ℓ_2
90 : $P_{127135} = (30, 1, 30, 1)$ lies on line ℓ_0
91 : $P_{130655} = (30, 56, 30, 1)$ lies on line ℓ_1
92 : $P_{130719} = (30, 57, 30, 1)$ lies on line ℓ_2
93 : $P_{131232} = (31, 1, 31, 1)$ lies on line ℓ_0
94 : $P_{134752} = (31, 56, 31, 1)$ lies on line ℓ_1
95 : $P_{134816} = (31, 57, 31, 1)$ lies on line ℓ_2
96 : $P_{135329} = (32, 1, 32, 1)$ lies on line ℓ_0
97 : $P_{138849} = (32, 56, 32, 1)$ lies on line ℓ_1
98 : $P_{138913} = (32, 57, 32, 1)$ lies on line ℓ_2
99 : $P_{139426} = (33, 1, 33, 1)$ lies on line ℓ_0
100 : $P_{142946} = (33, 56, 33, 1)$ lies on line ℓ_1
101 : $P_{143010} = (33, 57, 33, 1)$ lies on line ℓ_2
102 : $P_{143523} = (34, 1, 34, 1)$ lies on line ℓ_0
103 : $P_{147043} = (34, 56, 34, 1)$ lies on line ℓ_1
104 : $P_{147107} = (34, 57, 34, 1)$ lies on line ℓ_2
105 : $P_{147620} = (35, 1, 35, 1)$ lies on line ℓ_0
106 : $P_{151140} = (35, 56, 35, 1)$ lies on line ℓ_1
107 : $P_{151204} = (35, 57, 35, 1)$ lies on line ℓ_2
108 : $P_{151717} = (36, 1, 36, 1)$ lies on line ℓ_0
109 : $P_{155237} = (36, 56, 36, 1)$ lies on line ℓ_1
110 : $P_{155301} = (36, 57, 36, 1)$ lies on line ℓ_2
111 : $P_{155814} = (37, 1, 37, 1)$ lies on line ℓ_0
112 : $P_{159334} = (37, 56, 37, 1)$ lies on line ℓ_1
113 : $P_{159398} = (37, 57, 37, 1)$ lies on line ℓ_2
114 : $P_{159911} = (38, 1, 38, 1)$ lies on line ℓ_0
115 : $P_{163431} = (38, 56, 38, 1)$ lies on line ℓ_1
116 : $P_{163495} = (38, 57, 38, 1)$ lies on line ℓ_2
117 : $P_{164008} = (39, 1, 39, 1)$ lies on line ℓ_0
118 : $P_{167528} = (39, 56, 39, 1)$ lies on line ℓ_1
119 : $P_{167592} = (39, 57, 39, 1)$ lies on line ℓ_2
120 : $P_{168105} = (40, 1, 40, 1)$ lies on line ℓ_0
121 : $P_{171625} = (40, 56, 40, 1)$ lies on line ℓ_1
122 : $P_{171689} = (40, 57, 40, 1)$ lies on line ℓ_2
123 : $P_{172202} = (41, 1, 41, 1)$ lies on line ℓ_0
124 : $P_{175722} = (41, 56, 41, 1)$ lies on line ℓ_1
125 : $P_{175786} = (41, 57, 41, 1)$ lies on line ℓ_2
126 : $P_{176299} = (42, 1, 42, 1)$ lies on line ℓ_0
127 : $P_{179819} = (42, 56, 42, 1)$ lies on line ℓ_1
128 : $P_{179883} = (42, 57, 42, 1)$ lies on line ℓ_2
129 : $P_{180396} = (43, 1, 43, 1)$ lies on line ℓ_0
130 : $P_{183916} = (43, 56, 43, 1)$ lies on line ℓ_1
131 : $P_{183980} = (43, 57, 43, 1)$ lies on line ℓ_2
132 : $P_{184493} = (44, 1, 44, 1)$ lies on line ℓ_0
133 : $P_{188013} = (44, 56, 44, 1)$ lies on line ℓ_1
134 : $P_{188077} = (44, 57, 44, 1)$ lies on line ℓ_2
135 : $P_{188590} = (45, 1, 45, 1)$ lies on line ℓ_0
136 : $P_{192110} = (45, 56, 45, 1)$ lies on line ℓ_1
137 : $P_{192174} = (45, 57, 45, 1)$ lies on line ℓ_2
138 : $P_{192687} = (46, 1, 46, 1)$ lies on line ℓ_0
139 : $P_{196207} = (46, 56, 46, 1)$ lies on line ℓ_1
140 : $P_{196271} = (46, 57, 46, 1)$ lies on line ℓ_2
141 : $P_{196784} = (47, 1, 47, 1)$ lies on line ℓ_0
142 : $P_{200304} = (47, 56, 47, 1)$ lies on line ℓ_1
143 : $P_{200368} = (47, 57, 47, 1)$ lies on line ℓ_2
144 : $P_{200881} = (48, 1, 48, 1)$ lies on line ℓ_0
145 : $P_{204401} = (48, 56, 48, 1)$ lies on line ℓ_1
146 : $P_{204465} = (48, 57, 48, 1)$ lies on line ℓ_2
147 : $P_{204978} = (49, 1, 49, 1)$ lies on line ℓ_0
148 : $P_{208498} = (49, 56, 49, 1)$ lies on line ℓ_1
149 : $P_{208562} = (49, 57, 49, 1)$ lies on line ℓ_2
150 : $P_{209075} = (50, 1, 50, 1)$ lies on line ℓ_0
151 : $P_{212595} = (50, 56, 50, 1)$ lies on line ℓ_1
152 : $P_{212659} = (50, 57, 50, 1)$ lies on line ℓ_2
153 : $P_{213172} = (51, 1, 51, 1)$ lies on line ℓ_0
154 : $P_{216692} = (51, 56, 51, 1)$ lies on line ℓ_1
155 : $P_{216756} = (51, 57, 51, 1)$ lies on line ℓ_2
156 : $P_{217269} = (52, 1, 52, 1)$ lies on line ℓ_0
157 : $P_{220789} = (52, 56, 52, 1)$ lies on line ℓ_1
158 : $P_{220853} = (52, 57, 52, 1)$ lies on line ℓ_2
159 : $P_{221366} = (53, 1, 53, 1)$ lies on line ℓ_0
160 : $P_{224886} = (53, 56, 53, 1)$ lies on line ℓ_1
161 : $P_{224950} = (53, 57, 53, 1)$ lies on line ℓ_2
162 : $P_{225463} = (54, 1, 54, 1)$ lies on line ℓ_0
163 : $P_{228983} = (54, 56, 54, 1)$ lies on line ℓ_1
164 : $P_{229047} = (54, 57, 54, 1)$ lies on line ℓ_2
165 : $P_{229560} = (55, 1, 55, 1)$ lies on line ℓ_0

166 : $P_{233080} = (55, 56, 55, 1)$ lies on line ℓ_1
167 : $P_{233144} = (55, 57, 55, 1)$ lies on line ℓ_2
168 : $P_{233657} = (56, 1, 56, 1)$ lies on line ℓ_0
169 : $P_{237177} = (56, 56, 56, 1)$ lies on line ℓ_1
170 : $P_{237241} = (56, 57, 56, 1)$ lies on line ℓ_2
171 : $P_{237754} = (57, 1, 57, 1)$ lies on line ℓ_0
172 : $P_{241274} = (57, 56, 57, 1)$ lies on line ℓ_1
173 : $P_{241338} = (57, 57, 57, 1)$ lies on line ℓ_2
174 : $P_{241851} = (58, 1, 58, 1)$ lies on line ℓ_0
175 : $P_{245371} = (58, 56, 58, 1)$ lies on line ℓ_1
176 : $P_{245435} = (58, 57, 58, 1)$ lies on line ℓ_2
177 : $P_{245948} = (59, 1, 59, 1)$ lies on line ℓ_0
178 : $P_{249468} = (59, 56, 59, 1)$ lies on line ℓ_1
179 : $P_{249532} = (59, 57, 59, 1)$ lies on line ℓ_2
180 : $P_{250045} = (60, 1, 60, 1)$ lies on line ℓ_0
181 : $P_{253565} = (60, 56, 60, 1)$ lies on line ℓ_1
182 : $P_{253629} = (60, 57, 60, 1)$ lies on line ℓ_2
183 : $P_{254142} = (61, 1, 61, 1)$ lies on line ℓ_0
184 : $P_{257662} = (61, 56, 61, 1)$ lies on line ℓ_1
185 : $P_{257726} = (61, 57, 61, 1)$ lies on line ℓ_2
186 : $P_{258239} = (62, 1, 62, 1)$ lies on line ℓ_0
187 : $P_{261759} = (62, 56, 62, 1)$ lies on line ℓ_1
188 : $P_{261823} = (62, 57, 62, 1)$ lies on line ℓ_2
189 : $P_{262336} = (63, 1, 63, 1)$ lies on line ℓ_0
190 : $P_{265856} = (63, 56, 63, 1)$ lies on line ℓ_1
191 : $P_{265920} = (63, 57, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4096 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{68}	P_{68}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{68}	P_{68}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{68}	P_{68}

The surface has 4289 points:
Too many to print.