

Rank-65867 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244173389

General information

Number of lines	8
Number of points	89
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	50
Number of points off lines	30
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^8
Type of lines on points	$4, 3^2, 2^6, 1^{50}, 0^{30}$

Singular Points

The surface has 3 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$2 : P_{83} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{137} = \mathbf{Pl}(0, 0, 1, 0, 0, 1)_{664} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{584} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{18} \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{721} = \mathbf{Pl}(0, 1, 1, 0, 0, 1)_{672} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{10} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{82} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{1224} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{593} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{306}
\end{aligned}$$

Rank of lines: (0, 64, 137, 584, 721, 648, 82, 593)

Rank of points on Klein quadric: (0, 2, 664, 18, 672, 10, 1224, 306)

Eckardt Points

The surface has 2 Eckardt points:

$$0 : P_{75} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1),$$

$$1 : P_{83} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1).$$

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{12} = (1, 0, 1, 0) = \ell_1 \cap \ell_6$$

$$P_{20} = (1, 1, 1, 0) = \ell_2 \cap \ell_7$$

$$P_{146} = (0, 1, 1, 1) = \ell_6 \cap \ell_7$$

Single Points

The surface has 50 single points:

The single points on the surface are:

$$0 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_4$$

$$1 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_8 = (4, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_9 = (5, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{10} = (6, 1, 0, 0) \text{ lies on line } \ell_0$$

$$6 : P_{11} = (7, 1, 0, 0) \text{ lies on line } \ell_0$$

$$7 : P_{13} = (2, 0, 1, 0) \text{ lies on line } \ell_1$$

$$8 : P_{14} = (3, 0, 1, 0) \text{ lies on line } \ell_1$$

$$9 : P_{15} = (4, 0, 1, 0) \text{ lies on line } \ell_1$$

$$10 : P_{16} = (5, 0, 1, 0) \text{ lies on line } \ell_1$$

$$11 : P_{17} = (6, 0, 1, 0) \text{ lies on line } \ell_1$$

$$12 : P_{18} = (7, 0, 1, 0) \text{ lies on line } \ell_1$$

$$13 : P_{29} = (2, 2, 1, 0) \text{ lies on line } \ell_2$$

$$14 : P_{38} = (3, 3, 1, 0) \text{ lies on line } \ell_2$$

$$15 : P_{47} = (4, 4, 1, 0) \text{ lies on line } \ell_2$$

$$16 : P_{56} = (5, 5, 1, 0) \text{ lies on line } \ell_2$$

$$17 : P_{65} = (6, 6, 1, 0) \text{ lies on line } \ell_2$$

18 : $P_{74} = (7, 7, 1, 0)$ lies on line ℓ_2
 19 : $P_{91} = (1, 2, 0, 1)$ lies on line ℓ_3
 20 : $P_{99} = (1, 3, 0, 1)$ lies on line ℓ_3
 21 : $P_{107} = (1, 4, 0, 1)$ lies on line ℓ_3
 22 : $P_{115} = (1, 5, 0, 1)$ lies on line ℓ_3
 23 : $P_{123} = (1, 6, 0, 1)$ lies on line ℓ_3
 24 : $P_{131} = (1, 7, 0, 1)$ lies on line ℓ_3
 25 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_5
 26 : $P_{202} = (1, 0, 2, 1)$ lies on line ℓ_5
 27 : $P_{210} = (1, 1, 2, 1)$ lies on line ℓ_4
 28 : $P_{212} = (3, 1, 2, 1)$ lies on line ℓ_6
 29 : $P_{220} = (3, 2, 2, 1)$ lies on line ℓ_7
 30 : $P_{266} = (1, 0, 3, 1)$ lies on line ℓ_5
 31 : $P_{274} = (1, 1, 3, 1)$ lies on line ℓ_4
 32 : $P_{275} = (2, 1, 3, 1)$ lies on line ℓ_6
 33 : $P_{291} = (2, 3, 3, 1)$ lies on line ℓ_7
 34 : $P_{330} = (1, 0, 4, 1)$ lies on line ℓ_5

35 : $P_{338} = (1, 1, 4, 1)$ lies on line ℓ_4
 36 : $P_{342} = (5, 1, 4, 1)$ lies on line ℓ_6
 37 : $P_{366} = (5, 4, 4, 1)$ lies on line ℓ_7
 38 : $P_{394} = (1, 0, 5, 1)$ lies on line ℓ_5
 39 : $P_{402} = (1, 1, 5, 1)$ lies on line ℓ_4
 40 : $P_{405} = (4, 1, 5, 1)$ lies on line ℓ_6
 41 : $P_{437} = (4, 5, 5, 1)$ lies on line ℓ_7
 42 : $P_{458} = (1, 0, 6, 1)$ lies on line ℓ_5
 43 : $P_{466} = (1, 1, 6, 1)$ lies on line ℓ_4
 44 : $P_{472} = (7, 1, 6, 1)$ lies on line ℓ_6
 45 : $P_{512} = (7, 6, 6, 1)$ lies on line ℓ_7
 46 : $P_{522} = (1, 0, 7, 1)$ lies on line ℓ_5
 47 : $P_{530} = (1, 1, 7, 1)$ lies on line ℓ_4
 48 : $P_{535} = (6, 1, 7, 1)$ lies on line ℓ_6
 49 : $P_{583} = (6, 7, 7, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 30 points not on any line:

The points on the surface but not on lines are:

0 : $P_{166} = (5, 3, 1, 1)$	16 : $P_{379} = (2, 6, 4, 1)$
1 : $P_{167} = (6, 3, 1, 1)$	17 : $P_{425} = (0, 4, 5, 1)$
2 : $P_{180} = (3, 5, 1, 1)$	18 : $P_{428} = (3, 4, 5, 1)$
3 : $P_{183} = (6, 5, 1, 1)$	19 : $P_{435} = (2, 5, 5, 1)$
4 : $P_{188} = (3, 6, 1, 1)$	20 : $P_{451} = (2, 7, 5, 1)$
5 : $P_{190} = (5, 6, 1, 1)$	21 : $P_{452} = (3, 7, 5, 1)$
6 : $P_{224} = (7, 2, 2, 1)$	22 : $P_{477} = (4, 2, 6, 1)$
7 : $P_{241} = (0, 5, 2, 1)$	23 : $P_{478} = (5, 2, 6, 1)$
8 : $P_{248} = (7, 5, 2, 1)$	24 : $P_{509} = (4, 6, 6, 1)$
9 : $P_{281} = (0, 2, 3, 1)$	25 : $P_{513} = (0, 7, 6, 1)$
10 : $P_{287} = (6, 2, 3, 1)$	26 : $P_{518} = (5, 7, 6, 1)$
11 : $P_{296} = (7, 3, 3, 1)$	27 : $P_{545} = (0, 3, 7, 1)$
12 : $P_{303} = (6, 4, 3, 1)$	28 : $P_{549} = (4, 3, 7, 1)$
13 : $P_{304} = (7, 4, 3, 1)$	29 : $P_{581} = (4, 7, 7, 1)$
14 : $P_{363} = (2, 4, 4, 1)$	
15 : $P_{377} = (0, 6, 4, 1)$	

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	0	0	0	0
1	1	0	1	0	1	1	1	0
2	1	1	0	0	1	1	0	1
3	1	0	0	0	1	1	1	1
4	0	1	1	1	0	1	1	0
5	0	1	1	1	1	0	0	1
6	0	1	0	1	1	0	0	1
7	0	0	1	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_2	P_2	P_2	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_5	P_2	P_2	P_2	P_{20}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{83}	P_{75}	P_{83}	P_{75}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{83}	P_2	P_{83}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_{75}	P_2	P_{75}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_{12}	P_{83}	P_{83}	P_{146}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_{20}	P_{75}	P_{75}	P_{146}

The surface has 89 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	22 : $P_{56} = (5, 5, 1, 0)$	44 : $P_{220} = (3, 2, 2, 1)$
1 : $P_1 = (0, 1, 0, 0)$	23 : $P_{65} = (6, 6, 1, 0)$	45 : $P_{224} = (7, 2, 2, 1)$
2 : $P_2 = (0, 0, 1, 0)$	24 : $P_{74} = (7, 7, 1, 0)$	46 : $P_{241} = (0, 5, 2, 1)$
3 : $P_4 = (1, 1, 1, 1)$	25 : $P_{75} = (1, 0, 0, 1)$	47 : $P_{248} = (7, 5, 2, 1)$
4 : $P_5 = (1, 1, 0, 0)$	26 : $P_{83} = (1, 1, 0, 1)$	48 : $P_{266} = (1, 0, 3, 1)$
5 : $P_6 = (2, 1, 0, 0)$	27 : $P_{91} = (1, 2, 0, 1)$	49 : $P_{274} = (1, 1, 3, 1)$
6 : $P_7 = (3, 1, 0, 0)$	28 : $P_{99} = (1, 3, 0, 1)$	50 : $P_{275} = (2, 1, 3, 1)$
7 : $P_8 = (4, 1, 0, 0)$	29 : $P_{107} = (1, 4, 0, 1)$	51 : $P_{281} = (0, 2, 3, 1)$
8 : $P_9 = (5, 1, 0, 0)$	30 : $P_{115} = (1, 5, 0, 1)$	52 : $P_{287} = (6, 2, 3, 1)$
9 : $P_{10} = (6, 1, 0, 0)$	31 : $P_{123} = (1, 6, 0, 1)$	53 : $P_{291} = (2, 3, 3, 1)$
10 : $P_{11} = (7, 1, 0, 0)$	32 : $P_{131} = (1, 7, 0, 1)$	54 : $P_{296} = (7, 3, 3, 1)$
11 : $P_{12} = (1, 0, 1, 0)$	33 : $P_{139} = (1, 0, 1, 1)$	55 : $P_{303} = (6, 4, 3, 1)$
12 : $P_{13} = (2, 0, 1, 0)$	34 : $P_{146} = (0, 1, 1, 1)$	56 : $P_{304} = (7, 4, 3, 1)$
13 : $P_{14} = (3, 0, 1, 0)$	35 : $P_{166} = (5, 3, 1, 1)$	57 : $P_{330} = (1, 0, 4, 1)$
14 : $P_{15} = (4, 0, 1, 0)$	36 : $P_{167} = (6, 3, 1, 1)$	58 : $P_{338} = (1, 1, 4, 1)$
15 : $P_{16} = (5, 0, 1, 0)$	37 : $P_{180} = (3, 5, 1, 1)$	59 : $P_{342} = (5, 1, 4, 1)$
16 : $P_{17} = (6, 0, 1, 0)$	38 : $P_{183} = (6, 5, 1, 1)$	60 : $P_{363} = (2, 4, 4, 1)$
17 : $P_{18} = (7, 0, 1, 0)$	39 : $P_{188} = (3, 6, 1, 1)$	61 : $P_{366} = (5, 4, 4, 1)$
18 : $P_{20} = (1, 1, 1, 0)$	40 : $P_{190} = (5, 6, 1, 1)$	62 : $P_{377} = (0, 6, 4, 1)$
19 : $P_{29} = (2, 2, 1, 0)$	41 : $P_{202} = (1, 0, 2, 1)$	63 : $P_{379} = (2, 6, 4, 1)$
20 : $P_{38} = (3, 3, 1, 0)$	42 : $P_{210} = (1, 1, 2, 1)$	64 : $P_{394} = (1, 0, 5, 1)$
21 : $P_{47} = (4, 4, 1, 0)$	43 : $P_{212} = (3, 1, 2, 1)$	65 : $P_{402} = (1, 1, 5, 1)$

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 84 : $P_{535} = (6, 1, 7, 1)$
 85 : $P_{545} = (0, 3, 7, 1)$
 86 : $P_{549} = (4, 3, 7, 1)$
 87 : $P_{581} = (4, 7, 7, 1)$
 88 : $P_{583} = (6, 7, 7, 1)$