Rank-65570 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431656110

General information

Number of lines	3
Number of points	29
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	12
Number of points off lines	16
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	3, 1^{12}, 0^{16}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{33} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{33} = \mathbf{Pl}(2,3,0,0,3,1)_{298}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{29} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{29} = \mathbf{Pl}(3,2,0,0,2,1)_{239}$$

Rank of lines: (25, 33, 29)
Rank of points on Klein quadric: (177, 298, 239)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_8 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 12 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_0
$1 : P_{26} = (0,1,0,1)$ lies on line ℓ_0
$2 : P_{30} = (0,2,0,1)$ lies on line ℓ_1
$3 : P_{34} = (0,3,0,1)$ lies on line ℓ_2
$4 : P_{46} = (1,2,1,1)$ lies on line ℓ_1
$5 : P_{50} = (1,3,1,1)$ lies on line ℓ_2
$6 : P_{59} = (2,1,2,1)$ lies on line ℓ_0
$7 : P_{63} = (2,2,2,1)$ lies on line ℓ_1
$8 : P_{67} = (2,3,2,1)$ lies on line ℓ_2
$9 : P_{76} = (3,1,3,1)$ lies on line ℓ_0
$10 : P_{80} = (3,2,3,1)$ lies on line ℓ_1
$11 : P_{84} = (3,3,3,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 16 points not on any line:
The points on the surface but not on lines are:

$0 : P_9 = (2,0,1,0)$
$1 : P_{10} = (3,0,1,0)$
$2 : P_{11} = (0,1,1,0)$
$3 : P_{13} = (2,1,1,0)$
$4 : P_{14} = (3,1,1,0)$
$5 : P_{15} = (0,2,1,0)$
$6 : P_{19} = (0,3,1,0)$
$7 : P_{23} = (1,0,0,1)$
$8 : P_{24} = (2,0,0,1)$
$9 : P_{25} = (3,0,0,1)$
$10 : P_{27} = (1,1,0,1)$
$11 : P_{32} = (2,2,0,1)$
$12 : P_{37} = (3,3,0,1)$
$13 : P_{38} = (0,0,1,1)$
$14 : P_{53} = (0,0,2,1)$
$15 : P_{69} = (0,0,3,1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_8	P_8

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_8	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_8	P_8

The surface has 29 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_8 = (1, 0, 1, 0)$
2 : $P_9 = (2, 0, 1, 0)$
3 : $P_{10} = (3, 0, 1, 0)$
4 : $P_{11} = (0, 1, 1, 0)$
5 : $P_{13} = (2, 1, 1, 0)$
6 : $P_{14} = (3, 1, 1, 0)$
7 : $P_{15} = (0, 2, 1, 0)$
8 : $P_{19} = (0, 3, 1, 0)$
9 : $P_{23} = (1, 0, 0, 1)$
10 : $P_{24} = (2, 0, 0, 1)$
11 : $P_{25} = (3, 0, 0, 1)$
12 : $P_{26} = (0, 1, 0, 1)$
13 : $P_{27} = (1, 1, 0, 1)$
14 : $P_{30} = (0, 2, 0, 1)$
15 : $P_{32} = (2, 2, 0, 1)$
16 : $P_{34} = (0, 3, 0, 1)$
17 : $P_{37} = (3, 3, 0, 1)$
18 : $P_{38} = (0, 0, 1, 1)$
19 : $P_{46} = (1, 2, 1, 1)$
20 : $P_{50} = (1, 3, 1, 1)$
21 : $P_{53} = (0, 0, 2, 1)$
22 : $P_{59} = (2, 1, 2, 1)$
23 : $P_{63} = (2, 2, 2, 1)$
24 : $P_{67} = (2, 3, 2, 1)$
25 : $P_{69} = (0, 0, 3, 1)$
26 : $P_{76} = (3, 1, 3, 1)$
27 : $P_{80} = (3, 2, 3, 1)$
28 : $P_{84} = (3, 3, 3, 1)$