Rank-77 over GF(64)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 = 0$$

(1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(64) is 34087057

General information

Number of lines	3
Number of points	4289
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	4096
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	3, 1^{192}, 0^{4096}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \mathbf{Pl}(1,1,1,1,0,0)_{256}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \epsilon^{21} \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15179329} = \begin{bmatrix} 1 & 0 & 0 & 57 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15179329} = \mathbf{Pl}(1,1,56,1,0,0)_{3721}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & \epsilon^{42} \\ 0 & 1 & 1 & 0 \end{array}\right]_{14913025} = \left[\begin{array}{cccc} 1 & 0 & 0 & 56 \\ 0 & 1 & 1 & 0 \end{array}\right]_{14913025} = \mathbf{Pl}(1,1,57,1,0,0)_{3784}$$

Rank of lines: (266305, 15179329, 14913025)
Rank of points on Klein quadric: (256, 3721, 3784)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{131} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_0
$1 : P_{4163} = (1,0,0,1)$ lies on line ℓ_0
$2 : P_{4218} = (56,0,0,1)$ lies on line ℓ_1
$3 : P_{4219} = (57,0,0,1)$ lies on line ℓ_2
$4 : P_{8377} = (56,1,1,1)$ lies on line ℓ_1
$5 : P_{8378} = (57,1,1,1)$ lies on line ℓ_2
$6 : P_{12482} = (1,2,2,1)$ lies on line ℓ_0
$7 : P_{12537} = (56,2,2,1)$ lies on line ℓ_1
$8 : P_{12538} = (57,2,2,1)$ lies on line ℓ_2
$9 : P_{16642} = (1,3,3,1)$ lies on line ℓ_0
$10 : P_{16697} = (56,3,3,1)$ lies on line ℓ_1
$11 : P_{16698} = (57,3,3,1)$ lies on line ℓ_2
$12 : P_{20802} = (1,4,4,1)$ lies on line ℓ_0
$13 : P_{20857} = (56,4,4,1)$ lies on line ℓ_1
$14 : P_{20858} = (57,4,4,1)$ lies on line ℓ_2
$15 : P_{24962} = (1,5,5,1)$ lies on line ℓ_0
$16 : P_{25017} = (56,5,5,1)$ lies on line ℓ_1
$17 : P_{25018} = (57,5,5,1)$ lies on line ℓ_2
$18 : P_{29122} = (1,6,6,1)$ lies on line ℓ_0
$19 : P_{29177} = (56,6,6,1)$ lies on line ℓ_1
$20 : P_{29178} = (57,6,6,1)$ lies on line ℓ_2
$21 : P_{33282} = (1,7,7,1)$ lies on line ℓ_0
$22 : P_{33337} = (56,7,7,1)$ lies on line ℓ_1
$23 : P_{33338} = (57,7,7,1)$ lies on line ℓ_2
$24 : P_{37442} = (1,8,8,1)$ lies on line ℓ_0
$25 : P_{37497} = (56,8,8,1)$ lies on line ℓ_1
$26 : P_{37498} = (57,8,8,1)$ lies on line ℓ_2
$27 : P_{41602} = (1,9,9,1)$ lies on line ℓ_0
$28 : P_{41657} = (56,9,9,1)$ lies on line ℓ_1
$29 : P_{41658} = (57,9,9,1)$ lies on line ℓ_2
$30 : P_{45762} = (1,10,10,1)$ lies on line ℓ_0
$31 : P_{45817} = (56,10,10,1)$ lies on line ℓ_1
$32 : P_{45818} = (57,10,10,1)$ lies on line ℓ_2
$33 : P_{49922} = (1,11,11,1)$ lies on line ℓ_0
$34 : P_{49977} = (56,11,11,1)$ lies on line ℓ_1
$35 : P_{49978} = (57,11,11,1)$ lies on line ℓ_2
$36 : P_{54082} = (1,12,12,1)$ lies on line ℓ_0
$37 : P_{54137} = (56,12,12,1)$ lies on line ℓ_1
$38 : P_{54138} = (57,12,12,1)$ lies on line ℓ_2
$39 : P_{58242} = (1,13,13,1)$ lies on line ℓ_0
$40 : P_{58297} = (56,13,13,1)$ lies on line ℓ_1
$41 : P_{58298} = (57,13,13,1)$ lies on line ℓ_2
$42 : P_{62402} = (1,14,14,1)$ lies on line ℓ_0
$43 : P_{62457} = (56,14,14,1)$ lies on line ℓ_1
$44 : P_{62458} = (57,14,14,1)$ lies on line ℓ_2
$45 : P_{66562} = (1,15,15,1)$ lies on line ℓ_0
$46 : P_{66617} = (56,15,15,1)$ lies on line ℓ_1
$47 : P_{66618} = (57,15,15,1)$ lies on line ℓ_2
$48 : P_{70722} = (1,16,16,1)$ lies on line ℓ_0
$49 : P_{70777} = (56,16,16,1)$ lies on line ℓ_1
$50 : P_{70778} = (57,16,16,1)$ lies on line ℓ_2
$51 : P_{74882} = (1,17,17,1)$ lies on line ℓ_0
$52 : P_{74937} = (56,17,17,1)$ lies on line ℓ_1
$53 : P_{74938} = (57,17,17,1)$ lies on line ℓ_2
$54 : P_{79042} = (1,18,18,1)$ lies on line ℓ_0
$55 : P_{79097} = (56,18,18,1)$ lies on line ℓ_1
$56 : P_{79098} = (57,18,18,1)$ lies on line ℓ_2
$57 : P_{83202} = (1,19,19,1)$ lies on line ℓ_0

58 : $P_{83257} = (56, 19, 19, 1)$ lies on line ℓ_1
59 : $P_{83258} = (57, 19, 19, 1)$ lies on line ℓ_2
60 : $P_{87362} = (1, 20, 20, 1)$ lies on line ℓ_0
61 : $P_{87417} = (56, 20, 20, 1)$ lies on line ℓ_1
62 : $P_{87418} = (57, 20, 20, 1)$ lies on line ℓ_2
63 : $P_{91522} = (1, 21, 21, 1)$ lies on line ℓ_0
64 : $P_{91577} = (56, 21, 21, 1)$ lies on line ℓ_1
65 : $P_{91578} = (57, 21, 21, 1)$ lies on line ℓ_2
66 : $P_{95682} = (1, 22, 22, 1)$ lies on line ℓ_0
67 : $P_{95737} = (56, 22, 22, 1)$ lies on line ℓ_1
68 : $P_{95738} = (57, 22, 22, 1)$ lies on line ℓ_2
69 : $P_{99842} = (1, 23, 23, 1)$ lies on line ℓ_0
70 : $P_{99897} = (56, 23, 23, 1)$ lies on line ℓ_1
71 : $P_{99898} = (57, 23, 23, 1)$ lies on line ℓ_2
72 : $P_{104002} = (1, 24, 24, 1)$ lies on line ℓ_0
73 : $P_{104057} = (56, 24, 24, 1)$ lies on line ℓ_1
74 : $P_{104058} = (57, 24, 24, 1)$ lies on line ℓ_2
75 : $P_{108162} = (1, 25, 25, 1)$ lies on line ℓ_0
76 : $P_{108217} = (56, 25, 25, 1)$ lies on line ℓ_1
77 : $P_{108218} = (57, 25, 25, 1)$ lies on line ℓ_2
78 : $P_{112322} = (1, 26, 26, 1)$ lies on line ℓ_0
79 : $P_{112377} = (56, 26, 26, 1)$ lies on line ℓ_1
80 : $P_{112378} = (57, 26, 26, 1)$ lies on line ℓ_2
81 : $P_{116482} = (1, 27, 27, 1)$ lies on line ℓ_0
82 : $P_{116537} = (56, 27, 27, 1)$ lies on line ℓ_1
83 : $P_{116538} = (57, 27, 27, 1)$ lies on line ℓ_2
84 : $P_{120642} = (1, 28, 28, 1)$ lies on line ℓ_0
85 : $P_{120697} = (56, 28, 28, 1)$ lies on line ℓ_1
86 : $P_{120698} = (57, 28, 28, 1)$ lies on line ℓ_2
87 : $P_{124802} = (1, 29, 29, 1)$ lies on line ℓ_0
88 : $P_{124857} = (56, 29, 29, 1)$ lies on line ℓ_1
89 : $P_{124858} = (57, 29, 29, 1)$ lies on line ℓ_2
90 : $P_{128962} = (1, 30, 30, 1)$ lies on line ℓ_0
91 : $P_{129017} = (56, 30, 30, 1)$ lies on line ℓ_1
92 : $P_{129018} = (57, 30, 30, 1)$ lies on line ℓ_2
93 : $P_{133122} = (1, 31, 31, 1)$ lies on line ℓ_0
94 : $P_{133177} = (56, 31, 31, 1)$ lies on line ℓ_1
95 : $P_{133178} = (57, 31, 31, 1)$ lies on line ℓ_2
96 : $P_{137282} = (1, 32, 32, 1)$ lies on line ℓ_0
97 : $P_{137337} = (56, 32, 32, 1)$ lies on line ℓ_1
98 : $P_{137338} = (57, 32, 32, 1)$ lies on line ℓ_2
99 : $P_{141442} = (1, 33, 33, 1)$ lies on line ℓ_0
100 : $P_{141497} = (56, 33, 33, 1)$ lies on line ℓ_1
101 : $P_{141498} = (57, 33, 33, 1)$ lies on line ℓ_2
102 : $P_{145602} = (1, 34, 34, 1)$ lies on line ℓ_0
103 : $P_{145657} = (56, 34, 34, 1)$ lies on line ℓ_1
104 : $P_{145658} = (57, 34, 34, 1)$ lies on line ℓ_2
105 : $P_{149762} = (1, 35, 35, 1)$ lies on line ℓ_0
106 : $P_{149817} = (56, 35, 35, 1)$ lies on line ℓ_1
107 : $P_{149818} = (57, 35, 35, 1)$ lies on line ℓ_2
108 : $P_{153922} = (1, 36, 36, 1)$ lies on line ℓ_0
109 : $P_{153977} = (56, 36, 36, 1)$ lies on line ℓ_1
110 : $P_{153978} = (57, 36, 36, 1)$ lies on line ℓ_2
111 : $P_{158082} = (1, 37, 37, 1)$ lies on line ℓ_0
112 : $P_{158137} = (56, 37, 37, 1)$ lies on line ℓ_1
113 : $P_{158138} = (57, 37, 37, 1)$ lies on line ℓ_2
114 : $P_{162242} = (1, 38, 38, 1)$ lies on line ℓ_0
115 : $P_{162297} = (56, 38, 38, 1)$ lies on line ℓ_1
116 : $P_{162298} = (57, 38, 38, 1)$ lies on line ℓ_2
117 : $P_{166402} = (1, 39, 39, 1)$ lies on line ℓ_0
118 : $P_{166457} = (56, 39, 39, 1)$ lies on line ℓ_1
119 : $P_{166458} = (57, 39, 39, 1)$ lies on line ℓ_2
120 : $P_{170562} = (1, 40, 40, 1)$ lies on line ℓ_0
121 : $P_{170617} = (56, 40, 40, 1)$ lies on line ℓ_1
122 : $P_{170618} = (57, 40, 40, 1)$ lies on line ℓ_2
123 : $P_{174722} = (1, 41, 41, 1)$ lies on line ℓ_0
124 : $P_{174777} = (56, 41, 41, 1)$ lies on line ℓ_1
125 : $P_{174778} = (57, 41, 41, 1)$ lies on line ℓ_2
126 : $P_{178882} = (1, 42, 42, 1)$ lies on line ℓ_0
127 : $P_{178937} = (56, 42, 42, 1)$ lies on line ℓ_1
128 : $P_{178938} = (57, 42, 42, 1)$ lies on line ℓ_2
129 : $P_{183042} = (1, 43, 43, 1)$ lies on line ℓ_0
130 : $P_{183097} = (56, 43, 43, 1)$ lies on line ℓ_1
131 : $P_{183098} = (57, 43, 43, 1)$ lies on line ℓ_2
132 : $P_{187202} = (1, 44, 44, 1)$ lies on line ℓ_0
133 : $P_{187257} = (56, 44, 44, 1)$ lies on line ℓ_1
134 : $P_{187258} = (57, 44, 44, 1)$ lies on line ℓ_2
135 : $P_{191362} = (1, 45, 45, 1)$ lies on line ℓ_0
136 : $P_{191417} = (56, 45, 45, 1)$ lies on line ℓ_1
137 : $P_{191418} = (57, 45, 45, 1)$ lies on line ℓ_2
138 : $P_{195522} = (1, 46, 46, 1)$ lies on line ℓ_0
139 : $P_{195577} = (56, 46, 46, 1)$ lies on line ℓ_1
140 : $P_{195578} = (57, 46, 46, 1)$ lies on line ℓ_2
141 : $P_{199682} = (1, 47, 47, 1)$ lies on line ℓ_0
142 : $P_{199737} = (56, 47, 47, 1)$ lies on line ℓ_1
143 : $P_{199738} = (57, 47, 47, 1)$ lies on line ℓ_2
144 : $P_{203842} = (1, 48, 48, 1)$ lies on line ℓ_0
145 : $P_{203897} = (56, 48, 48, 1)$ lies on line ℓ_1
146 : $P_{203898} = (57, 48, 48, 1)$ lies on line ℓ_2
147 : $P_{208002} = (1, 49, 49, 1)$ lies on line ℓ_0
148 : $P_{208057} = (56, 49, 49, 1)$ lies on line ℓ_1
149 : $P_{208058} = (57, 49, 49, 1)$ lies on line ℓ_2
150 : $P_{212162} = (1, 50, 50, 1)$ lies on line ℓ_0
151 : $P_{212217} = (56, 50, 50, 1)$ lies on line ℓ_1
152 : $P_{212218} = (57, 50, 50, 1)$ lies on line ℓ_2
153 : $P_{216322} = (1, 51, 51, 1)$ lies on line ℓ_0
154 : $P_{216377} = (56, 51, 51, 1)$ lies on line ℓ_1
155 : $P_{216378} = (57, 51, 51, 1)$ lies on line ℓ_2
156 : $P_{220482} = (1, 52, 52, 1)$ lies on line ℓ_0
157 : $P_{220537} = (56, 52, 52, 1)$ lies on line ℓ_1
158 : $P_{220538} = (57, 52, 52, 1)$ lies on line ℓ_2
159 : $P_{224642} = (1, 53, 53, 1)$ lies on line ℓ_0
160 : $P_{224697} = (56, 53, 53, 1)$ lies on line ℓ_1
161 : $P_{224698} = (57, 53, 53, 1)$ lies on line ℓ_2
162 : $P_{228802} = (1, 54, 54, 1)$ lies on line ℓ_0
163 : $P_{228857} = (56, 54, 54, 1)$ lies on line ℓ_1
164 : $P_{228858} = (57, 54, 54, 1)$ lies on line ℓ_2
165 : $P_{232962} = (1, 55, 55, 1)$ lies on line ℓ_0

166 : $P_{233017} = (56, 55, 55, 1)$ lies on line ℓ_1
167 : $P_{233018} = (57, 55, 55, 1)$ lies on line ℓ_2
168 : $P_{237122} = (1, 56, 56, 1)$ lies on line ℓ_0
169 : $P_{237177} = (56, 56, 56, 1)$ lies on line ℓ_1
170 : $P_{237178} = (57, 56, 56, 1)$ lies on line ℓ_2
171 : $P_{241282} = (1, 57, 57, 1)$ lies on line ℓ_0
172 : $P_{241337} = (56, 57, 57, 1)$ lies on line ℓ_1
173 : $P_{241338} = (57, 57, 57, 1)$ lies on line ℓ_2
174 : $P_{245442} = (1, 58, 58, 1)$ lies on line ℓ_0
175 : $P_{245497} = (56, 58, 58, 1)$ lies on line ℓ_1
176 : $P_{245498} = (57, 58, 58, 1)$ lies on line ℓ_2
177 : $P_{249602} = (1, 59, 59, 1)$ lies on line ℓ_0
178 : $P_{249657} = (56, 59, 59, 1)$ lies on line ℓ_1
179 : $P_{249658} = (57, 59, 59, 1)$ lies on line ℓ_2
180 : $P_{253762} = (1, 60, 60, 1)$ lies on line ℓ_0
181 : $P_{253817} = (56, 60, 60, 1)$ lies on line ℓ_1
182 : $P_{253818} = (57, 60, 60, 1)$ lies on line ℓ_2
183 : $P_{257922} = (1, 61, 61, 1)$ lies on line ℓ_0
184 : $P_{257977} = (56, 61, 61, 1)$ lies on line ℓ_1
185 : $P_{257978} = (57, 61, 61, 1)$ lies on line ℓ_2
186 : $P_{262082} = (1, 62, 62, 1)$ lies on line ℓ_0
187 : $P_{262137} = (56, 62, 62, 1)$ lies on line ℓ_1
188 : $P_{262138} = (57, 62, 62, 1)$ lies on line ℓ_2
189 : $P_{266242} = (1, 63, 63, 1)$ lies on line ℓ_0
190 : $P_{266297} = (56, 63, 63, 1)$ lies on line ℓ_1
191 : $P_{266298} = (57, 63, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4096 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{131}	P_{131}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{131}	P_{131}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{131}	P_{131}

The surface has 4289 points:
Too many to print.