Rank-264 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_0 X_1^2 = 0$$

(0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over $\mathrm{GF}(64)$ is -2113396658

General information

Number of lines	7
Number of points	4289
Number of singular points	0
Number of Eckardt points	3
Number of double points	0
Number of single points	446
Number of points off lines	3840
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 ⁷
Type of lines on points	$3^3, 1^{446}, 0^{3840}$

Singular Points

The surface has 0 singular points:

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{4352}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{278529}$$

$$\ell_{2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{21} \end{bmatrix}_{17043513} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 57 \end{bmatrix}_{17043513} = \mathbf{Pl}(0, 0, 0, 57, 0, 1)_{285641}$$

$$\ell_{3} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon^{42} \end{bmatrix}_{17043512} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 56 \end{bmatrix}_{17043512} = \mathbf{Pl}(0, 0, 0, 56, 0, 1)_{285514}$$

$$\ell_{4} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4226} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4226} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{536640}$$

$$\ell_{6} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \mathbf{Pl}(1, 1, 1, 1, 1, 1, 0)_{20354}$$

Rank of lines: (4097, 17043457, 17043513, 17043512, 8258, 4226, 266369) Rank of points on Klein quadric: (4352, 278529, 285641, 285514, 544578, 536640, 20354)

Eckardt Points

The surface has 3 Eckardt points:

 $0: P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$

1: $P_{8258} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1),$

 $2: P_{8322} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1).$

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 446 single points: The single points on the surface are:

```
0: P_0 = (1,0,0,0) lies on line \ell_0
                                                                     17: P_{8269} = (11, 0, 1, 1) lies on line \ell_0
1: P_4 = (1, 1, 1, 1) lies on line \ell_4
                                                                     18: P_{8270} = (12, 0, 1, 1) lies on line \ell_0
2: P_5 = (1, 1, 0, 0) lies on line \ell_4
                                                                     19: P_{8271} = (13, 0, 1, 1) lies on line \ell_0
3: P_{68} = (1,0,1,0) lies on line \ell_5
                                                                     20: P_{8272} = (14, 0, 1, 1) lies on line \ell_0
                                                                     21 : P_{8273} = (15, 0, 1, 1) lies on line \ell_0
4: P_{132} = (1, 1, 1, 0) lies on line \ell_6
5: P_{4163} = (1,0,0,1) lies on line \ell_6
                                                                     22: P_{8274} = (16, 0, 1, 1) lies on line \ell_0
6: P_{4227} = (1, 1, 0, 1) lies on line \ell_5
                                                                     23: P_{8275} = (17, 0, 1, 1) lies on line \ell_0
7: P_{8259} = (1, 0, 1, 1) lies on line \ell_0
                                                                     24: P_{8276} = (18, 0, 1, 1) lies on line \ell_0
8: P_{8260} = (2, 0, 1, 1) lies on line \ell_0
                                                                     25: P_{8277} = (19, 0, 1, 1) lies on line \ell_0
9: P_{8261} = (3,0,1,1) lies on line \ell_0
                                                                     26: P_{8278} = (20, 0, 1, 1) lies on line \ell_0
10: P_{8262} = (4, 0, 1, 1) lies on line \ell_0
                                                                     27: P_{8279} = (21, 0, 1, 1) lies on line \ell_0
11: P_{8263} = (5, 0, 1, 1) lies on line \ell_0
                                                                     28: P_{8280} = (22, 0, 1, 1) lies on line \ell_0
12: P_{8264} = (6,0,1,1) lies on line \ell_0
                                                                     29: P_{8281} = (23, 0, 1, 1) lies on line \ell_0
13: P_{8265} = (7,0,1,1) lies on line \ell_0
                                                                     30: P_{8282} = (24, 0, 1, 1) lies on line \ell_0
14: P_{8266} = (8,0,1,1) lies on line \ell_0
                                                                     31: P_{8283} = (25, 0, 1, 1) lies on line \ell_0
15: P_{8267} = (9, 0, 1, 1) lies on line \ell_0
                                                                     32: P_{8284} = (26, 0, 1, 1) lies on line \ell_0
16: P_{8268} = (10, 0, 1, 1) lies on line \ell_0
                                                                     33: P_{8285} = (27, 0, 1, 1) lies on line \ell_0
```

```
34: P_{8286} = (28, 0, 1, 1) lies on line \ell_0
                                                                      88: P_{8961} = (0, 11, 1, 1) lies on line \ell_1
35: P_{8287} = (29, 0, 1, 1) lies on line \ell_0
                                                                     89: P_{8972} = (11, 11, 1, 1) lies on line \ell_4
36: P_{8288} = (30, 0, 1, 1) lies on line \ell_0
                                                                     90: P_{9025} = (0, 12, 1, 1) lies on line \ell_1
37: P_{8289} = (31, 0, 1, 1) lies on line \ell_0
                                                                     91: P_{9037} = (12, 12, 1, 1) lies on line \ell_4
38: P_{8290} = (32, 0, 1, 1) lies on line \ell_0
                                                                     92: P_{9089} = (0, 13, 1, 1) lies on line \ell_1
39: P_{8291} = (33, 0, 1, 1) lies on line \ell_0
                                                                     93: P_{9102} = (13, 13, 1, 1) lies on line \ell_4
40: P_{8292} = (34, 0, 1, 1) lies on line \ell_0
                                                                     94: P_{9153} = (0, 14, 1, 1) lies on line \ell_1
41: P_{8293} = (35, 0, 1, 1) lies on line \ell_0
                                                                     95: P_{9167} = (14, 14, 1, 1) lies on line \ell_4
42: P_{8294} = (36, 0, 1, 1) lies on line \ell_0
                                                                     96: P_{9217} = (0, 15, 1, 1) lies on line \ell_1
43: P_{8295} = (37, 0, 1, 1) lies on line \ell_0
                                                                     97: P_{9232} = (15, 15, 1, 1) lies on line \ell_4
44: P_{8296} = (38, 0, 1, 1) lies on line \ell_0
                                                                     98: P_{9281} = (0, 16, 1, 1) lies on line \ell_1
45: P_{8297} = (39, 0, 1, 1) lies on line \ell_0
                                                                     99: P_{9297} = (16, 16, 1, 1) lies on line \ell_4
46: P_{8298} = (40, 0, 1, 1) lies on line \ell_0
                                                                     100: P_{9345} = (0, 17, 1, 1) lies on line \ell_1
                                                                     101: P_{9362} = (17, 17, 1, 1) lies on line \ell_4
47: P_{8299} = (41, 0, 1, 1) lies on line \ell_0
48: P_{8300} = (42, 0, 1, 1) lies on line \ell_0
                                                                     102: P_{9409} = (0, 18, 1, 1) lies on line \ell_1
49: P_{8301} = (43, 0, 1, 1) lies on line \ell_0
                                                                     103: P_{9427} = (18, 18, 1, 1) lies on line \ell_4
50: P_{8302} = (44, 0, 1, 1) lies on line \ell_0
                                                                     104: P_{9473} = (0, 19, 1, 1) lies on line \ell_1
51: P_{8303} = (45, 0, 1, 1) lies on line \ell_0
                                                                     105: P_{9492} = (19, 19, 1, 1) lies on line \ell_4
52: P_{8304} = (46, 0, 1, 1) lies on line \ell_0
                                                                     106: P_{9537} = (0, 20, 1, 1) lies on line \ell_1
53: P_{8305} = (47, 0, 1, 1) lies on line \ell_0
                                                                     107: P_{9557} = (20, 20, 1, 1) lies on line \ell_4
54: P_{8306} = (48, 0, 1, 1) lies on line \ell_0
                                                                     108: P_{9601} = (0, 21, 1, 1) lies on line \ell_1
                                                                     109: P_{9622} = (21, 21, 1, 1) lies on line \ell_4
55: P_{8307} = (49, 0, 1, 1) lies on line \ell_0
                                                                     110: P_{9665} = (0, 22, 1, 1) lies on line \ell_1
56: P_{8308} = (50, 0, 1, 1) lies on line \ell_0
                                                                     111 : P_{9687} = (22, 22, 1, 1) lies on line \ell_4
57: P_{8309} = (51, 0, 1, 1) lies on line \ell_0
58: P_{8310} = (52, 0, 1, 1) lies on line \ell_0
                                                                     112: P_{9729} = (0, 23, 1, 1) lies on line \ell_1
59: P_{8311} = (53, 0, 1, 1) lies on line \ell_0
                                                                     113: P_{9752} = (23, 23, 1, 1) lies on line \ell_4
60: P_{8312} = (54, 0, 1, 1) lies on line \ell_0
                                                                     114: P_{9793} = (0, 24, 1, 1) lies on line \ell_1
61: P_{8313} = (55, 0, 1, 1) lies on line \ell_0
                                                                     115: P_{9817} = (24, 24, 1, 1) lies on line \ell_4
62: P_{8314} = (56, 0, 1, 1) lies on line \ell_0
                                                                     116: P_{9857} = (0, 25, 1, 1) lies on line \ell_1
63: P_{8315} = (57, 0, 1, 1) lies on line \ell_0
                                                                     117: P_{9882} = (25, 25, 1, 1) lies on line \ell_4
64: P_{8316} = (58, 0, 1, 1) lies on line \ell_0
                                                                     118: P_{9921} = (0, 26, 1, 1) lies on line \ell_1
65: P_{8317} = (59, 0, 1, 1) lies on line \ell_0
                                                                     119: P_{9947} = (26, 26, 1, 1) lies on line \ell_4
66: P_{8318} = (60, 0, 1, 1) lies on line \ell_0
                                                                     120: P_{9985} = (0, 27, 1, 1) lies on line \ell_1
                                                                     121 : P_{10012} = (27, 27, 1, 1) lies on line \ell_4
67: P_{8319} = (61, 0, 1, 1) lies on line \ell_0
68: P_{8320} = (62, 0, 1, 1) lies on line \ell_0
                                                                     122: P_{10049} = (0, 28, 1, 1) lies on line \ell_1
69: P_{8321} = (63, 0, 1, 1) lies on line \ell_0
                                                                     123: P_{10077} = (28, 28, 1, 1) lies on line \ell_4
70: P_{8385} = (0, 2, 1, 1) lies on line \ell_1
                                                                     124: P_{10113} = (0, 29, 1, 1) lies on line \ell_1
71: P_{8387} = (2, 2, 1, 1) lies on line \ell_4
                                                                     125: P_{10142} = (29, 29, 1, 1) lies on line \ell_4
72: P_{8449} = (0, 3, 1, 1) lies on line \ell_1
                                                                     126: P_{10177} = (0, 30, 1, 1) lies on line \ell_1
                                                                     127: P_{10207} = (30, 30, 1, 1) lies on line \ell_4
73: P_{8452} = (3, 3, 1, 1) lies on line \ell_4
74: P_{8513} = (0, 4, 1, 1) lies on line \ell_1
                                                                     128: P_{10241} = (0, 31, 1, 1) lies on line \ell_1
75: P_{8517} = (4, 4, 1, 1) lies on line \ell_4
                                                                     129: P_{10272} = (31, 31, 1, 1) lies on line \ell_4
76: P_{8577} = (0, 5, 1, 1) lies on line \ell_1
                                                                     130: P_{10305} = (0, 32, 1, 1) lies on line \ell_1
77: P_{8582} = (5, 5, 1, 1) lies on line \ell_4
                                                                     131: P_{10337} = (32, 32, 1, 1) lies on line \ell_4
78: P_{8641} = (0, 6, 1, 1) lies on line \ell_1
                                                                     132: P_{10369} = (0, 33, 1, 1) lies on line \ell_1
79: P_{8647} = (6, 6, 1, 1) lies on line \ell_4
                                                                     133: P_{10402} = (33, 33, 1, 1) lies on line \ell_4
80: P_{8705} = (0,7,1,1) lies on line \ell_1
                                                                     134: P_{10433} = (0, 34, 1, 1) lies on line \ell_1
81: P_{8712} = (7,7,1,1) lies on line \ell_4
                                                                     135: P_{10467} = (34, 34, 1, 1) lies on line \ell_4
82: P_{8769} = (0, 8, 1, 1) lies on line \ell_1
                                                                     136: P_{10497} = (0, 35, 1, 1) lies on line \ell_1
83: P_{8777} = (8, 8, 1, 1) lies on line \ell_4
                                                                     137: P_{10532} = (35, 35, 1, 1) lies on line \ell_4
84: P_{8833} = (0, 9, 1, 1) lies on line \ell_1
                                                                     138: P_{10561} = (0, 36, 1, 1) lies on line \ell_1
85: P_{8842} = (9, 9, 1, 1) lies on line \ell_4
                                                                     139: P_{10597} = (36, 36, 1, 1) lies on line \ell_4
86: P_{8897} = (0, 10, 1, 1) lies on line \ell_1
                                                                     140: P_{10625} = (0, 37, 1, 1) lies on line \ell_1
87: P_{8907} = (10, 10, 1, 1) lies on line \ell_4
                                                                     141: P_{10662} = (37, 37, 1, 1) lies on line \ell_4
```

```
142: P_{10689} = (0, 38, 1, 1) lies on line \ell_1
                                                                     196: P_{16515} = (2, 1, 3, 1) lies on line \ell_5
143 : P_{10727} = (38, 38, 1, 1) lies on line \ell_4
                                                                     197: P_{16643} = (2, 3, 3, 1) lies on line \ell_6
                                                                     198: P_{20614} = (5, 1, 4, 1) lies on line \ell_5
144: P_{10753} = (0, 39, 1, 1) lies on line \ell_1
145: P_{10792} = (39, 39, 1, 1) lies on line \ell_4
                                                                     199: P_{20806} = (5, 4, 4, 1) lies on line \ell_6
146 : P_{10817} = (0, 40, 1, 1) lies on line \ell_1
                                                                     200: P_{24709} = (4, 1, 5, 1) lies on line \ell_5
147: P_{10857} = (40, 40, 1, 1) lies on line \ell_4
                                                                     201: P_{24965} = (4, 5, 5, 1) lies on line \ell_6
148: P_{10881} = (0, 41, 1, 1) lies on line \ell_1
                                                                     202: P_{28808} = (7, 1, 6, 1) lies on line \ell_5
149: P_{10922} = (41, 41, 1, 1) lies on line \ell_4
                                                                     203: P_{29128} = (7, 6, 6, 1) lies on line \ell_6
150: P_{10945} = (0, 42, 1, 1) lies on line \ell_1
                                                                     204 : P_{32903} = (6, 1, 7, 1) lies on line \ell_5
151: P_{10987} = (42, 42, 1, 1) lies on line \ell_4
                                                                     205: P_{33287} = (6,7,7,1) lies on line \ell_6
152: P_{11009} = (0, 43, 1, 1) lies on line \ell_1
                                                                     206: P_{37002} = (9, 1, 8, 1) lies on line \ell_5
                                                                     207: P_{37450} = (9, 8, 8, 1) lies on line \ell_6
153: P_{11052} = (43, 43, 1, 1) lies on line \ell_4
154: P_{11073} = (0, 44, 1, 1) lies on line \ell_1
                                                                     208: P_{41097} = (8, 1, 9, 1) lies on line \ell_5
                                                                     209: P_{41609} = (8, 9, 9, 1) lies on line \ell_6
155: P_{11117} = (44, 44, 1, 1) lies on line \ell_4
156: P_{11137} = (0, 45, 1, 1) lies on line \ell_1
                                                                     210: P_{45196} = (11, 1, 10, 1) lies on line \ell_5
157: P_{11182} = (45, 45, 1, 1) lies on line \ell_4
                                                                     211: P_{45772} = (11, 10, 10, 1) lies on line \ell_6
158: P_{11201} = (0, 46, 1, 1) lies on line \ell_1
                                                                     212: P_{49291} = (10, 1, 11, 1) lies on line \ell_5
159: P_{11247} = (46, 46, 1, 1) lies on line \ell_4
                                                                     213: P_{49931} = (10, 11, 11, 1) lies on line \ell_6
160: P_{11265} = (0, 47, 1, 1) lies on line \ell_1
                                                                     214: P_{53390} = (13, 1, 12, 1) lies on line \ell_5
161: P_{11312} = (47, 47, 1, 1) lies on line \ell_4
                                                                     215: P_{54094} = (13, 12, 12, 1) lies on line \ell_6
162: P_{11329} = (0, 48, 1, 1) lies on line \ell_1
                                                                     216: P_{57485} = (12, 1, 13, 1) lies on line \ell_5
163: P_{11377} = (48, 48, 1, 1) lies on line \ell_4
                                                                     217: P_{58253} = (12, 13, 13, 1) lies on line \ell_6
                                                                     218: P_{61584} = (15, 1, 14, 1) lies on line \ell_5
164: P_{11393} = (0, 49, 1, 1) lies on line \ell_1
165: P_{11442} = (49, 49, 1, 1) lies on line \ell_4
                                                                     219: P_{62416} = (15, 14, 14, 1) lies on line \ell_6
166: P_{11457} = (0, 50, 1, 1) lies on line \ell_1
                                                                     220: P_{65679} = (14, 1, 15, 1) lies on line \ell_5
167: P_{11507} = (50, 50, 1, 1) lies on line \ell_4
                                                                     221: P_{66575} = (14, 15, 15, 1) lies on line \ell_6
168: P_{11521} = (0, 51, 1, 1) lies on line \ell_1
                                                                     222: P_{69778} = (17, 1, 16, 1) lies on line \ell_5
169: P_{11572} = (51, 51, 1, 1) lies on line \ell_4
                                                                     223: P_{70738} = (17, 16, 16, 1) lies on line \ell_6
170: P_{11585} = (0, 52, 1, 1) lies on line \ell_1
                                                                     224: P_{73873} = (16, 1, 17, 1) lies on line \ell_5
171: P_{11637} = (52, 52, 1, 1) lies on line \ell_4
                                                                     225: P_{74897} = (16, 17, 17, 1) lies on line \ell_6
172: P_{11649} = (0, 53, 1, 1) lies on line \ell_1
                                                                     226: P_{77972} = (19, 1, 18, 1) lies on line \ell_5
173: P_{11702} = (53, 53, 1, 1) lies on line \ell_4
                                                                     227: P_{79060} = (19, 18, 18, 1) lies on line \ell_6
174: P_{11713} = (0, 54, 1, 1) lies on line \ell_1
                                                                     228: P_{82067} = (18, 1, 19, 1) lies on line \ell_5
175: P_{11767} = (54, 54, 1, 1) lies on line \ell_4
                                                                     229: P_{83219} = (18, 19, 19, 1) lies on line \ell_6
176: P_{11777} = (0, 55, 1, 1) lies on line \ell_1
                                                                     230: P_{86166} = (21, 1, 20, 1) lies on line \ell_5
                                                                     231: P_{87382} = (21, 20, 20, 1) lies on line \ell_6
177: P_{11832} = (55, 55, 1, 1) lies on line \ell_4
178: P_{11841} = (0, 56, 1, 1) lies on line \ell_1
                                                                     232: P_{90261} = (20, 1, 21, 1) lies on line \ell_5
179: P_{11897} = (56, 56, 1, 1) lies on line \ell_4
                                                                     233: P_{91541} = (20, 21, 21, 1) lies on line \ell_6
180: P_{11905} = (0, 57, 1, 1) lies on line \ell_1
                                                                     234 : P_{94360} = (23, 1, 22, 1) lies on line \ell_5
181: P_{11962} = (57, 57, 1, 1) lies on line \ell_4
                                                                     235: P_{95704} = (23, 22, 22, 1) lies on line \ell_6
                                                                     236: P_{98455} = (22, 1, 23, 1) lies on line \ell_5
182: P_{11969} = (0, 58, 1, 1) lies on line \ell_1
183: P_{12027} = (58, 58, 1, 1) lies on line \ell_4
                                                                     237: P_{99863} = (22, 23, 23, 1) lies on line \ell_6
184: P_{12033} = (0, 59, 1, 1) lies on line \ell_1
                                                                     238: P_{102554} = (25, 1, 24, 1) lies on line \ell_5
185: P_{12092} = (59, 59, 1, 1) lies on line \ell_4
                                                                     239: P_{104026} = (25, 24, 24, 1) lies on line \ell_6
186: P_{12097} = (0, 60, 1, 1) lies on line \ell_1
                                                                     240: P_{106649} = (24, 1, 25, 1) lies on line \ell_5
187: P_{12157} = (60, 60, 1, 1) lies on line \ell_4
                                                                     241: P_{108185} = (24, 25, 25, 1) lies on line \ell_6
188: P_{12161} = (0, 61, 1, 1) lies on line \ell_1
                                                                     242: P_{110748} = (27, 1, 26, 1) lies on line \ell_5
189: P_{12222} = (61, 61, 1, 1) lies on line \ell_4
                                                                     243: P_{112348} = (27, 26, 26, 1) lies on line \ell_6
                                                                     244 : P_{114843} = (26, 1, 27, 1) lies on line \ell_5
190: P_{12225} = (0, 62, 1, 1) lies on line \ell_1
191: P_{12287} = (62, 62, 1, 1) lies on line \ell_4
                                                                     245: P_{116507} = (26, 27, 27, 1) lies on line \ell_6
192: P_{12289} = (0, 63, 1, 1) lies on line \ell_1
                                                                     246: P_{118942} = (29, 1, 28, 1) lies on line \ell_5
193: P_{12352} = (63, 63, 1, 1) lies on line \ell_4
                                                                     247: P_{120670} = (29, 28, 28, 1) lies on line \ell_6
194: P_{12420} = (3, 1, 2, 1) lies on line \ell_5
                                                                     248: P_{123037} = (28, 1, 29, 1) lies on line \ell_5
195: P_{12484} = (3, 2, 2, 1) lies on line \ell_6
                                                                     249: P_{124829} = (28, 29, 29, 1) lies on line \ell_6
```

```
304: P_{233658} = (57, 1, 56, 1) lies on line \ell_5
250: P_{127136} = (31, 1, 30, 1) lies on line \ell_5
                                                                    305: P_{233665} = (0, 2, 56, 1) lies on line \ell_2
251: P_{128992} = (31, 30, 30, 1) lies on line \ell_6
                                                                    306: P_{233729} = (0, 3, 56, 1) lies on line \ell_2
252: P_{131231} = (30, 1, 31, 1) lies on line \ell_5
253: P_{133151} = (30, 31, 31, 1) lies on line \ell_6
                                                                    307: P_{233793} = (0, 4, 56, 1) lies on line \ell_2
254: P_{135330} = (33, 1, 32, 1) lies on line \ell_5
                                                                    308: P_{233857} = (0, 5, 56, 1) lies on line \ell_2
255: P_{137314} = (33, 32, 32, 1) lies on line \ell_6
                                                                    309: P_{233921} = (0, 6, 56, 1) lies on line \ell_2
256 : P_{139425} = (32, 1, 33, 1) lies on line \ell_5
                                                                    310 : P_{233985} = (0, 7, 56, 1) lies on line \ell_2
257: P_{141473} = (32, 33, 33, 1) lies on line \ell_6
                                                                    311: P_{234049} = (0, 8, 56, 1) lies on line \ell_2
258: P_{143524} = (35, 1, 34, 1) lies on line \ell_5
                                                                    312: P_{234113} = (0, 9, 56, 1) lies on line \ell_2
259: P_{145636} = (35, 34, 34, 1) lies on line \ell_6
                                                                    313: P_{234177} = (0, 10, 56, 1) lies on line \ell_2
260: P_{147619} = (34, 1, 35, 1) lies on line \ell_5
                                                                    314: P_{234241} = (0, 11, 56, 1) lies on line \ell_2
261 : P_{149795} = (34, 35, 35, 1) lies on line \ell_6
                                                                    315: P_{234305} = (0, 12, 56, 1) lies on line \ell_2
262: P_{151718} = (37, 1, 36, 1) lies on line \ell_5
                                                                    316: P_{234369} = (0, 13, 56, 1) lies on line \ell_2
263: P_{153958} = (37, 36, 36, 1) lies on line \ell_6
                                                                    317: P_{234433} = (0, 14, 56, 1) lies on line \ell_2
264: P_{155813} = (36, 1, 37, 1) lies on line \ell_5
                                                                    318: P_{234497} = (0, 15, 56, 1) lies on line \ell_2
265: P_{158117} = (36, 37, 37, 1) lies on line \ell_6
                                                                    319: P_{234561} = (0, 16, 56, 1) lies on line \ell_2
266: P_{159912} = (39, 1, 38, 1) lies on line \ell_5
                                                                    320: P_{234625} = (0, 17, 56, 1) lies on line \ell_2
267 : P_{162280} = (39, 38, 38, 1) lies on line \ell_6
                                                                    321: P_{234689} = (0, 18, 56, 1) lies on line \ell_2
268: P_{164007} = (38, 1, 39, 1) lies on line \ell_5
                                                                    322: P_{234753} = (0, 19, 56, 1) lies on line \ell_2
                                                                    323: P_{234817} = (0, 20, 56, 1) lies on line \ell_2
269: P_{166439} = (38, 39, 39, 1) lies on line \ell_6
270: P_{168106} = (41, 1, 40, 1) lies on line \ell_5
                                                                    324: P_{234881} = (0, 21, 56, 1) lies on line \ell_2
271: P_{170602} = (41, 40, 40, 1) lies on line \ell_6
                                                                    325: P_{234945} = (0, 22, 56, 1) lies on line \ell_2
272: P_{172201} = (40, 1, 41, 1) lies on line \ell_5
                                                                    326: P_{235009} = (0, 23, 56, 1) lies on line \ell_2
273 : P_{174761} = (40, 41, 41, 1) lies on line \ell_6
                                                                    327: P_{235073} = (0, 24, 56, 1) lies on line \ell_2
274: P_{176300} = (43, 1, 42, 1) lies on line \ell_5
                                                                    328: P_{235137} = (0, 25, 56, 1) lies on line \ell_2
275: P_{178924} = (43, 42, 42, 1) lies on line \ell_6
                                                                    329: P_{235201} = (0, 26, 56, 1) lies on line \ell_2
276: P_{180395} = (42, 1, 43, 1) lies on line \ell_5
                                                                    330: P_{235265} = (0, 27, 56, 1) lies on line \ell_2
277: P_{183083} = (42, 43, 43, 1) lies on line \ell_6
                                                                    331: P_{235329} = (0, 28, 56, 1) lies on line \ell_2
278: P_{184494} = (45, 1, 44, 1) lies on line \ell_5
                                                                    332: P_{235393} = (0, 29, 56, 1) lies on line \ell_2
279: P_{187246} = (45, 44, 44, 1) lies on line \ell_6
                                                                    333: P_{235457} = (0, 30, 56, 1) lies on line \ell_2
280: P_{188589} = (44, 1, 45, 1) lies on line \ell_5
                                                                    334: P_{235521} = (0, 31, 56, 1) lies on line \ell_2
281: P_{191405} = (44, 45, 45, 1) lies on line \ell_6
                                                                    335: P_{235585} = (0, 32, 56, 1) lies on line \ell_2
282 : P_{192688} = (47, 1, 46, 1) lies on line \ell_5
                                                                    336: P_{235649} = (0, 33, 56, 1) lies on line \ell_2
283 : P_{195568} = (47, 46, 46, 1) lies on line \ell_6
                                                                    337: P_{235713} = (0, 34, 56, 1) lies on line \ell_2
284: P_{196783} = (46, 1, 47, 1) lies on line \ell_5
                                                                    338: P_{235777} = (0, 35, 56, 1) lies on line \ell_2
285 : P_{199727} = (46, 47, 47, 1) lies on line \ell_6
                                                                    339: P_{235841} = (0, 36, 56, 1) lies on line \ell_2
286: P_{200882} = (49, 1, 48, 1) lies on line \ell_5
                                                                    340: P_{235905} = (0, 37, 56, 1) lies on line \ell_2
287: P_{203890} = (49, 48, 48, 1) lies on line \ell_6
                                                                    341: P_{235969} = (0, 38, 56, 1) lies on line \ell_2
                                                                    342: P_{236033} = (0, 39, 56, 1) lies on line \ell_2
288: P_{204977} = (48, 1, 49, 1) lies on line \ell_5
289 : P_{208049} = (48, 49, 49, 1) lies on line \ell_6
                                                                    343: P_{236097} = (0, 40, 56, 1) lies on line \ell_2
290: P_{209076} = (51, 1, 50, 1) lies on line \ell_5
                                                                    344: P_{236161} = (0, 41, 56, 1) lies on line \ell_2
291: P_{212212} = (51, 50, 50, 1) lies on line \ell_6
                                                                    345: P_{236225} = (0, 42, 56, 1) lies on line \ell_2
292 : P_{213171} = (50, 1, 51, 1) lies on line \ell_5
                                                                    346: P_{236289} = (0, 43, 56, 1) lies on line \ell_2
293: P_{216371} = (50, 51, 51, 1) lies on line \ell_6
                                                                    347: P_{236353} = (0, 44, 56, 1) lies on line \ell_2
294: P_{217270} = (53, 1, 52, 1) lies on line \ell_5
                                                                    348: P_{236417} = (0, 45, 56, 1) lies on line \ell_2
295: P_{220534} = (53, 52, 52, 1) lies on line \ell_6
                                                                    349: P_{236481} = (0, 46, 56, 1) lies on line \ell_2
296: P_{221365} = (52, 1, 53, 1) lies on line \ell_5
                                                                    350: P_{236545} = (0, 47, 56, 1) lies on line \ell_2
297: P_{224693} = (52, 53, 53, 1) lies on line \ell_6
                                                                    351: P_{236609} = (0, 48, 56, 1) lies on line \ell_2
298: P_{225464} = (55, 1, 54, 1) lies on line \ell_5
                                                                    352: P_{236673} = (0, 49, 56, 1) lies on line \ell_2
299: P_{228856} = (55, 54, 54, 1) lies on line \ell_6
                                                                    353: P_{236737} = (0, 50, 56, 1) lies on line \ell_2
300: P_{229559} = (54, 1, 55, 1) lies on line \ell_5
                                                                    354: P_{236801} = (0, 51, 56, 1) lies on line \ell_2
301: P_{233015} = (54, 55, 55, 1) lies on line \ell_6
                                                                    355: P_{236865} = (0, 52, 56, 1) lies on line \ell_2
302 : P_{233537} = (0, 0, 56, 1) lies on line \ell_2
                                                                    356: P_{236929} = (0, 53, 56, 1) lies on line \ell_2
303: P_{233601} = (0, 1, 56, 1) lies on line \ell_2
                                                                    357: P_{236993} = (0, 54, 56, 1) lies on line \ell_2
```

```
403: P_{239809} = (0, 34, 57, 1) lies on line \ell_3
358: P_{237057} = (0, 55, 56, 1) lies on line \ell_2
                                                                    404: P_{239873} = (0, 35, 57, 1) lies on line \ell_3
359: P_{237121} = (0, 56, 56, 1) lies on line \ell_2
360: P_{237178} = (57, 56, 56, 1) lies on line \ell_6
                                                                    405: P_{239937} = (0, 36, 57, 1) lies on line \ell_3
361: P_{237185} = (0, 57, 56, 1) lies on line \ell_2
                                                                    406: P_{240001} = (0, 37, 57, 1) lies on line \ell_3
362: P_{237249} = (0, 58, 56, 1) lies on line \ell_2
                                                                    407: P_{240065} = (0, 38, 57, 1) lies on line \ell_3
363: P_{237313} = (0, 59, 56, 1) lies on line \ell_2
                                                                    408: P_{240129} = (0, 39, 57, 1) lies on line \ell_3
364: P_{237377} = (0, 60, 56, 1) lies on line \ell_2
                                                                    409: P_{240193} = (0, 40, 57, 1) lies on line \ell_3
365: P_{237441} = (0, 61, 56, 1) lies on line \ell_2
                                                                    410: P_{240257} = (0, 41, 57, 1) lies on line \ell_3
366: P_{237505} = (0, 62, 56, 1) lies on line \ell_2
                                                                    411: P_{240321} = (0, 42, 57, 1) lies on line \ell_3
367: P_{237569} = (0, 63, 56, 1) lies on line \ell_2
                                                                    412: P_{240385} = (0, 43, 57, 1) lies on line \ell_3
368: P_{237633} = (0, 0, 57, 1) lies on line \ell_3
                                                                    413: P_{240449} = (0, 44, 57, 1) lies on line \ell_3
369: P_{237697} = (0, 1, 57, 1) lies on line \ell_3
                                                                    414: P_{240513} = (0, 45, 57, 1) lies on line \ell_3
370: P_{237753} = (56, 1, 57, 1) lies on line \ell_5
                                                                    415: P_{240577} = (0, 46, 57, 1) lies on line \ell_3
                                                                    416: P_{240641} = (0, 47, 57, 1) lies on line \ell_3
371: P_{237761} = (0, 2, 57, 1) lies on line \ell_3
372: P_{237825} = (0, 3, 57, 1) lies on line \ell_3
                                                                    417: P_{240705} = (0, 48, 57, 1) lies on line \ell_3
373: P_{237889} = (0, 4, 57, 1) lies on line \ell_3
                                                                    418: P_{240769} = (0, 49, 57, 1) lies on line \ell_3
374: P_{237953} = (0, 5, 57, 1) lies on line \ell_3
                                                                    419: P_{240833} = (0, 50, 57, 1) lies on line \ell_3
375: P_{238017} = (0, 6, 57, 1) lies on line \ell_3
                                                                    420: P_{240897} = (0, 51, 57, 1) lies on line \ell_3
376: P_{238081} = (0, 7, 57, 1) lies on line \ell_3
                                                                    421: P_{240961} = (0, 52, 57, 1) lies on line \ell_3
377: P_{238145} = (0, 8, 57, 1) lies on line \ell_3
                                                                    422: P_{241025} = (0, 53, 57, 1) lies on line \ell_3
378: P_{238209} = (0, 9, 57, 1) lies on line \ell_3
                                                                    423: P_{241089} = (0, 54, 57, 1) lies on line \ell_3
379: P_{238273} = (0, 10, 57, 1) lies on line \ell_3
                                                                    424: P_{241153} = (0, 55, 57, 1) lies on line \ell_3
380: P_{238337} = (0, 11, 57, 1) lies on line \ell_3
                                                                    425: P_{241217} = (0, 56, 57, 1) lies on line \ell_3
381 : P_{238401} = (0, 12, 57, 1) lies on line \ell_3
                                                                    426 : P_{241281} = (0, 57, 57, 1) lies on line \ell_3
382: P_{238465} = (0, 13, 57, 1) lies on line \ell_3
                                                                    427: P_{241337} = (56, 57, 57, 1) lies on line \ell_6
383: P_{238529} = (0, 14, 57, 1) lies on line \ell_3
                                                                    428: P_{241345} = (0, 58, 57, 1) lies on line \ell_3
384: P_{238593} = (0, 15, 57, 1) lies on line \ell_3
                                                                    429: P_{241409} = (0, 59, 57, 1) lies on line \ell_3
385: P_{238657} = (0, 16, 57, 1) lies on line \ell_3
                                                                    430: P_{241473} = (0, 60, 57, 1) lies on line \ell_3
386: P_{238721} = (0, 17, 57, 1) lies on line \ell_3
                                                                    431: P_{241537} = (0, 61, 57, 1) lies on line \ell_3
387: P_{238785} = (0, 18, 57, 1) lies on line \ell_3
                                                                    432: P_{241601} = (0, 62, 57, 1) lies on line \ell_3
388: P_{238849} = (0, 19, 57, 1) lies on line \ell_3
                                                                    433: P_{241665} = (0, 63, 57, 1) lies on line \ell_3
389: P_{238913} = (0, 20, 57, 1) lies on line \ell_3
                                                                    434: P_{241852} = (59, 1, 58, 1) lies on line \ell_5
390: P_{238977} = (0, 21, 57, 1) lies on line \ell_3
                                                                    435: P_{245500} = (59, 58, 58, 1) lies on line \ell_6
391 : P_{239041} = (0, 22, 57, 1) lies on line \ell_3
                                                                    436: P_{245947} = (58, 1, 59, 1) lies on line \ell_5
392: P_{239105} = (0, 23, 57, 1) lies on line \ell_3
                                                                    437: P_{249659} = (58, 59, 59, 1) lies on line \ell_6
393: P_{239169} = (0, 24, 57, 1) lies on line \ell_3
                                                                    438: P_{250046} = (61, 1, 60, 1) lies on line \ell_5
394: P_{239233} = (0, 25, 57, 1) lies on line \ell_3
                                                                    439: P_{253822} = (61, 60, 60, 1) lies on line \ell_6
395: P_{239297} = (0, 26, 57, 1) lies on line \ell_3
                                                                    440: P_{254141} = (60, 1, 61, 1) lies on line \ell_5
396: P_{239361} = (0, 27, 57, 1) lies on line \ell_3
                                                                    441: P_{257981} = (60, 61, 61, 1) lies on line \ell_6
397 : P_{239425} = (0, 28, 57, 1) lies on line \ell_3
                                                                    442 : P_{258240} = (63, 1, 62, 1) lies on line \ell_5
398: P_{239489} = (0, 29, 57, 1) lies on line \ell_3
                                                                    443: P_{262144} = (63, 62, 62, 1) lies on line \ell_6
399: P_{239553} = (0, 30, 57, 1) lies on line \ell_3
                                                                    444: P_{262335} = (62, 1, 63, 1) lies on line \ell_5
400 : P_{239617} = (0, 31, 57, 1) lies on line \ell_3
                                                                    445: P_{266303} = (62, 63, 63, 1) lies on line \ell_6
401: P_{239681} = (0, 32, 57, 1) lies on line \ell_3
402: P_{239745} = (0, 33, 57, 1) lies on line \ell_3
```

The single points on the surface are:

Points on surface but on no line

The surface has 3840 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c|c} 0123456 \\ \hline 0 & 0100100 \\ 1 & 1011111 \\ 2 & 0101000 \\ 3 & 0110000 \\ 4 & 1100000 \\ 5 & 0100001 \\ 6 & 0100010 \\ \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_4
in point	P_{8258}	P_{8258}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_{8258}	P_1	P_1	P_{8258}	P_{8322}	P_{8322}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_1	P_1

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 4 intersects

Line	ℓ_0	ℓ_1
in point	P_{8258}	P_{8258}

Line 5 intersects

Line	ℓ_1	ℓ_6
in point	P_{8322}	P_{8322}

Line 6 intersects

Line	ℓ_1	ℓ_5
in point	P_{8322}	P_{8322}

The surface has 4289 points:

Too many to print.