Rank-73733 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1090785413

General information

Number of lines	2
Number of points	4161
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	128
Number of points off lines	4032
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^2
Type of lines on points	2, 1^{128}, 0^{4032}

Singular Points

The surface has 2 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (4096, 17047616)
Rank of points on Klein quadric: (2, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 128 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_1
2 : $P_{68} = (1,0,1,0)$ lies on line ℓ_0
3 : $P_{69} = (2,0,1,0)$ lies on line ℓ_0
4 : $P_{70} = (3,0,1,0)$ lies on line ℓ_0
5 : $P_{71} = (4,0,1,0)$ lies on line ℓ_0
6 : $P_{72} = (5,0,1,0)$ lies on line ℓ_0
7 : $P_{73} = (6,0,1,0)$ lies on line ℓ_0
8 : $P_{74} = (7,0,1,0)$ lies on line ℓ_0
9 : $P_{75} = (8,0,1,0)$ lies on line ℓ_0
10 : $P_{76} = (9,0,1,0)$ lies on line ℓ_0
11 : $P_{77} = (10,0,1,0)$ lies on line ℓ_0
12 : $P_{78} = (11,0,1,0)$ lies on line ℓ_0
13 : $P_{79} = (12,0,1,0)$ lies on line ℓ_0
14 : $P_{80} = (13,0,1,0)$ lies on line ℓ_0
15 : $P_{81} = (14,0,1,0)$ lies on line ℓ_0
16 : $P_{82} = (15,0,1,0)$ lies on line ℓ_0
17 : $P_{83} = (16,0,1,0)$ lies on line ℓ_0
18 : $P_{84} = (17,0,1,0)$ lies on line ℓ_0
19 : $P_{85} = (18,0,1,0)$ lies on line ℓ_0
20 : $P_{86} = (19,0,1,0)$ lies on line ℓ_0
21 : $P_{87} = (20,0,1,0)$ lies on line ℓ_0
22 : $P_{88} = (21,0,1,0)$ lies on line ℓ_0
23 : $P_{89} = (22,0,1,0)$ lies on line ℓ_0
24 : $P_{90} = (23,0,1,0)$ lies on line ℓ_0
25 : $P_{91} = (24,0,1,0)$ lies on line ℓ_0
26 : $P_{92} = (25,0,1,0)$ lies on line ℓ_0
27 : $P_{93} = (26,0,1,0)$ lies on line ℓ_0
28 : $P_{94} = (27,0,1,0)$ lies on line ℓ_0
29 : $P_{95} = (28,0,1,0)$ lies on line ℓ_0
30 : $P_{96} = (29,0,1,0)$ lies on line ℓ_0
31 : $P_{97} = (30,0,1,0)$ lies on line ℓ_0
32 : $P_{98} = (31,0,1,0)$ lies on line ℓ_0
33 : $P_{99} = (32,0,1,0)$ lies on line ℓ_0
34 : $P_{100} = (33,0,1,0)$ lies on line ℓ_0
35 : $P_{101} = (34,0,1,0)$ lies on line ℓ_0
36 : $P_{102} = (35,0,1,0)$ lies on line ℓ_0
37 : $P_{103} = (36,0,1,0)$ lies on line ℓ_0
38 : $P_{104} = (37,0,1,0)$ lies on line ℓ_0
39 : $P_{105} = (38,0,1,0)$ lies on line ℓ_0
40 : $P_{106} = (39,0,1,0)$ lies on line ℓ_0
41 : $P_{107} = (40,0,1,0)$ lies on line ℓ_0
42 : $P_{108} = (41,0,1,0)$ lies on line ℓ_0
43 : $P_{109} = (42,0,1,0)$ lies on line ℓ_0
44 : $P_{110} = (43,0,1,0)$ lies on line ℓ_0
45 : $P_{111} = (44,0,1,0)$ lies on line ℓ_0
46 : $P_{112} = (45,0,1,0)$ lies on line ℓ_0
47 : $P_{113} = (46,0,1,0)$ lies on line ℓ_0
48 : $P_{114} = (47,0,1,0)$ lies on line ℓ_0
49 : $P_{115} = (48,0,1,0)$ lies on line ℓ_0
50 : $P_{116} = (49,0,1,0)$ lies on line ℓ_0
51 : $P_{117} = (50,0,1,0)$ lies on line ℓ_0
52 : $P_{118} = (51,0,1,0)$ lies on line ℓ_0
53 : $P_{119} = (52,0,1,0)$ lies on line ℓ_0
54 : $P_{120} = (53,0,1,0)$ lies on line ℓ_0
55 : $P_{121} = (54,0,1,0)$ lies on line ℓ_0
56 : $P_{122} = (55,0,1,0)$ lies on line ℓ_0
57 : $P_{123} = (56,0,1,0)$ lies on line ℓ_0

58 : $P_{124} = (57, 0, 1, 0)$ lies on line ℓ_0
59 : $P_{125} = (58, 0, 1, 0)$ lies on line ℓ_0
60 : $P_{126} = (59, 0, 1, 0)$ lies on line ℓ_0
61 : $P_{127} = (60, 0, 1, 0)$ lies on line ℓ_0
62 : $P_{128} = (61, 0, 1, 0)$ lies on line ℓ_0
63 : $P_{129} = (62, 0, 1, 0)$ lies on line ℓ_0
64 : $P_{130} = (63, 0, 1, 0)$ lies on line ℓ_0
65 : $P_{8258} = (0, 0, 1, 1)$ lies on line ℓ_1
66 : $P_{12353} = (0, 0, 2, 1)$ lies on line ℓ_1
67 : $P_{16449} = (0, 0, 3, 1)$ lies on line ℓ_1
68 : $P_{20545} = (0, 0, 4, 1)$ lies on line ℓ_1
69 : $P_{24641} = (0, 0, 5, 1)$ lies on line ℓ_1
70 : $P_{28737} = (0, 0, 6, 1)$ lies on line ℓ_1
71 : $P_{32833} = (0, 0, 7, 1)$ lies on line ℓ_1
72 : $P_{36929} = (0, 0, 8, 1)$ lies on line ℓ_1
73 : $P_{41025} = (0, 0, 9, 1)$ lies on line ℓ_1
74 : $P_{45121} = (0, 0, 10, 1)$ lies on line ℓ_1
75 : $P_{49217} = (0, 0, 11, 1)$ lies on line ℓ_1
76 : $P_{53313} = (0, 0, 12, 1)$ lies on line ℓ_1
77 : $P_{57409} = (0, 0, 13, 1)$ lies on line ℓ_1
78 : $P_{61505} = (0, 0, 14, 1)$ lies on line ℓ_1
79 : $P_{65601} = (0, 0, 15, 1)$ lies on line ℓ_1
80 : $P_{69697} = (0, 0, 16, 1)$ lies on line ℓ_1
81 : $P_{73793} = (0, 0, 17, 1)$ lies on line ℓ_1
82 : $P_{77889} = (0, 0, 18, 1)$ lies on line ℓ_1
83 : $P_{81985} = (0, 0, 19, 1)$ lies on line ℓ_1
84 : $P_{86081} = (0, 0, 20, 1)$ lies on line ℓ_1
85 : $P_{90177} = (0, 0, 21, 1)$ lies on line ℓ_1
86 : $P_{94273} = (0, 0, 22, 1)$ lies on line ℓ_1
87 : $P_{98369} = (0, 0, 23, 1)$ lies on line ℓ_1
88 : $P_{102465} = (0, 0, 24, 1)$ lies on line ℓ_1
89 : $P_{106561} = (0, 0, 25, 1)$ lies on line ℓ_1
90 : $P_{110657} = (0, 0, 26, 1)$ lies on line ℓ_1
91 : $P_{114753} = (0, 0, 27, 1)$ lies on line ℓ_1
92 : $P_{118849} = (0, 0, 28, 1)$ lies on line ℓ_1
93 : $P_{122945} = (0, 0, 29, 1)$ lies on line ℓ_1
94 : $P_{127041} = (0, 0, 30, 1)$ lies on line ℓ_1
95 : $P_{131137} = (0, 0, 31, 1)$ lies on line ℓ_1
96 : $P_{135233} = (0, 0, 32, 1)$ lies on line ℓ_1
97 : $P_{139329} = (0, 0, 33, 1)$ lies on line ℓ_1
98 : $P_{143425} = (0, 0, 34, 1)$ lies on line ℓ_1
99 : $P_{147521} = (0, 0, 35, 1)$ lies on line ℓ_1
100 : $P_{151617} = (0, 0, 36, 1)$ lies on line ℓ_1
101 : $P_{155713} = (0, 0, 37, 1)$ lies on line ℓ_1
102 : $P_{159809} = (0, 0, 38, 1)$ lies on line ℓ_1
103 : $P_{163905} = (0, 0, 39, 1)$ lies on line ℓ_1
104 : $P_{168001} = (0, 0, 40, 1)$ lies on line ℓ_1
105 : $P_{172097} = (0, 0, 41, 1)$ lies on line ℓ_1
106 : $P_{176193} = (0, 0, 42, 1)$ lies on line ℓ_1
107 : $P_{180289} = (0, 0, 43, 1)$ lies on line ℓ_1
108 : $P_{184385} = (0, 0, 44, 1)$ lies on line ℓ_1
109 : $P_{188481} = (0, 0, 45, 1)$ lies on line ℓ_1
110 : $P_{192577} = (0, 0, 46, 1)$ lies on line ℓ_1
111 : $P_{196673} = (0, 0, 47, 1)$ lies on line ℓ_1
112 : $P_{200769} = (0, 0, 48, 1)$ lies on line ℓ_1
113 : $P_{204865} = (0, 0, 49, 1)$ lies on line ℓ_1
114 : $P_{208961} = (0, 0, 50, 1)$ lies on line ℓ_1
115 : $P_{213057} = (0, 0, 51, 1)$ lies on line ℓ_1
116 : $P_{217153} = (0, 0, 52, 1)$ lies on line ℓ_1
117 : $P_{221249} = (0, 0, 53, 1)$ lies on line ℓ_1
118 : $P_{225345} = (0, 0, 54, 1)$ lies on line ℓ_1
119 : $P_{229441} = (0, 0, 55, 1)$ lies on line ℓ_1
120 : $P_{233537} = (0, 0, 56, 1)$ lies on line ℓ_1
121 : $P_{237633} = (0, 0, 57, 1)$ lies on line ℓ_1
122 : $P_{241729} = (0, 0, 58, 1)$ lies on line ℓ_1
123 : $P_{245825} = (0, 0, 59, 1)$ lies on line ℓ_1
124 : $P_{249921} = (0, 0, 60, 1)$ lies on line ℓ_1
125 : $P_{254017} = (0, 0, 61, 1)$ lies on line ℓ_1
126 : $P_{258113} = (0, 0, 62, 1)$ lies on line ℓ_1
127 : $P_{262209} = (0, 0, 63, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 4032 points not on any line:
Too many to print.

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 4161 points:
Too many to print.