Rank-65867 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431725465

General information

Number of lines	8
Number of points	29
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	18
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^8
Type of lines on points	4, 3^2, 2^6, 1^{18}, 0^2

Singular Points

The surface has 3 singular points:

$0:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1:\ P_{23} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$
$2:\ P_{27} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \mathbf{Pl}(0,0,1,0,0,1)_{108}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \mathbf{Pl}(0,1,1,0,0,1)_{112}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$

Rank of lines: (0, 16, 37, 84, 121, 100, 26, 89)
Rank of points on Klein quadric: (0, 2, 108, 10, 112, 6, 180, 74)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{23} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$,
$1 : P_{27} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_6$
$P_{12} = (1,1,1,0) = \ell_2 \cap \ell_7$
$P_{42} = (0,1,1,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_4
$1 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$2 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$3 : P_9 = (2,0,1,0)$ lies on line ℓ_1
$4 : P_{10} = (3,0,1,0)$ lies on line ℓ_1
$5 : P_{17} = (2,2,1,0)$ lies on line ℓ_2
$6 : P_{22} = (3,3,1,0)$ lies on line ℓ_2
$7 : P_{31} = (1,2,0,1)$ lies on line ℓ_3
$8 : P_{35} = (1,3,0,1)$ lies on line ℓ_3
$9 : P_{39} = (1,0,1,1)$ lies on line ℓ_5
$10 : P_{54} = (1,0,2,1)$ lies on line ℓ_5
$11 : P_{58} = (1,1,2,1)$ lies on line ℓ_4
$12 : P_{60} = (3,1,2,1)$ lies on line ℓ_6
$13 : P_{64} = (3,2,2,1)$ lies on line ℓ_7
$14 : P_{70} = (1,0,3,1)$ lies on line ℓ_5
$15 : P_{74} = (1,1,3,1)$ lies on line ℓ_4
$16 : P_{75} = (2,1,3,1)$ lies on line ℓ_6
$17 : P_{83} = (2,3,3,1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

0 : $P_{61} = (0, 2, 2, 1)$
1 : $P_{81} = (0, 3, 3, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7
0	0 1 1 1 0 0 0 0
1	1 0 1 0 1 1 1 0
2	1 1 0 0 1 1 0 1
3	1 0 0 0 1 1 1 1
4	0 1 1 1 0 1 1 0
5	0 1 1 1 1 0 0 1
6	0 1 0 1 1 0 0 1
7	0 0 1 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_2	P_2	P_2	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_5	P_2	P_2	P_2	P_{12}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{27}	P_{23}	P_{27}	P_{23}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_2	P_2	P_{27}	P_2	P_{27}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_2	P_2	P_{23}	P_2	P_{23}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7
in point	P_8	P_{27}	P_{27}	P_{42}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_{12}	P_{23}	P_{23}	P_{42}

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_4 = (1,1,1,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (1,0,1,0)$
$8: P_9 = (2,0,1,0)$
$9: P_{10} = (3,0,1,0)$
$10: P_{12} = (1,1,1,0)$
$11: P_{17} = (2,2,1,0)$
$12: P_{22} = (3,3,1,0)$
$13: P_{23} = (1,0,0,1)$
$14: P_{27} = (1,1,0,1)$
$15: P_{31} = (1,2,0,1)$
$16: P_{35} = (1,3,0,1)$
$17: P_{39} = (1,0,1,1)$
$18: P_{42} = (0,1,1,1)$
$19: P_{54} = (1,0,2,1)$
$20: P_{58} = (1,1,2,1)$
$21: P_{60} = (3,1,2,1)$
$22: P_{61} = (0,2,2,1)$
$23: P_{64} = (3,2,2,1)$
$24: P_{70} = (1,0,3,1)$
$25: P_{74} = (1,1,3,1)$
$26: P_{75} = (2,1,3,1)$
$27: P_{81} = (0,3,3,1)$
$28: P_{83} = (2,3,3,1)$