

Rank-74053 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 74053

General information

Number of lines	4
Number of points	9
Number of singular points	1
Number of Eckardt points	1
Number of double points	1
Number of single points	7
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^4
Type of lines on points	$3, 2, 1^7$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4 \\ \ell_3 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7\end{aligned}$$

Rank of lines: (4, 34, 18, 33)

Rank of points on Klein quadric: (2, 1, 4, 7)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0). T = -1$$

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_3 = (0, 0, 0, 1) = \ell_1 \cap \ell_3$$

Single Points

The surface has 7 single points:

The single points on the surface are:

$$0 : P_0 = (1, 0, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_6 = (1, 0, 1, 0) \text{ lies on line } \ell_0$$

$$2 : P_7 = (0, 1, 1, 0) \text{ lies on line } \ell_3$$

$$3 : P_9 = (1, 0, 0, 1) \text{ lies on line } \ell_2$$

$$4 : P_{12} = (0, 0, 1, 1) \text{ lies on line } \ell_1$$

$$5 : P_{13} = (1, 0, 1, 1) \text{ lies on line } \ell_2$$

$$6 : P_{14} = (0, 1, 1, 1) \text{ lies on line } \ell_3$$

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3
0	0	1	1	0
1	1	0	1	1
2	1	1	0	0
3	0	1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

Line 3 intersects

Line	ℓ_1
in point	P_3

The surface has 9 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_6 = (1, 0, 1, 0)$$

$$4 : P_7 = (0, 1, 1, 0)$$

$$5 : P_9 = (1, 0, 0, 1)$$

$$6 : P_{12} = (0, 0, 1, 1)$$

$$7 : P_{13} = (1, 0, 1, 1)$$

$$8 : P_{14} = (0, 1, 1, 1)$$