

Rank-73798 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1498768734

General information

Number of lines	3
Number of points	25
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	12
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	$3, 1^{12}, 0^{12}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{268} = \mathbf{Pl}(0, 3, 1, 0, 0, 0)_8$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{184} = \mathbf{Pl}(0, 2, 1, 0, 0, 0)_7$$

Rank of lines: (356, 268, 184)

Rank of points on Klein quadric: (1, 8, 7)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 12 single points:

The single points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0
1 : $P_{24} = (2, 0, 0, 1)$ lies on line ℓ_1
2 : $P_{25} = (3, 0, 0, 1)$ lies on line ℓ_2
3 : $P_{38} = (0, 0, 1, 1)$ lies on line ℓ_0
4 : $P_{40} = (2, 0, 1, 1)$ lies on line ℓ_1
5 : $P_{41} = (3, 0, 1, 1)$ lies on line ℓ_2
6 : $P_{53} = (0, 0, 2, 1)$ lies on line ℓ_0

7 : $P_{55} = (2, 0, 2, 1)$ lies on line ℓ_1
8 : $P_{56} = (3, 0, 2, 1)$ lies on line ℓ_2
9 : $P_{69} = (0, 0, 3, 1)$ lies on line ℓ_0
10 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_1
11 : $P_{72} = (3, 0, 3, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:

The points on the surface but not on lines are:

0 : $P_5 = (1, 1, 0, 0)$
1 : $P_6 = (2, 1, 0, 0)$
2 : $P_7 = (3, 1, 0, 0)$
3 : $P_{27} = (1, 1, 0, 1)$
4 : $P_{31} = (1, 2, 0, 1)$
5 : $P_{35} = (1, 3, 0, 1)$
6 : $P_{48} = (3, 2, 1, 1)$

7 : $P_{51} = (2, 3, 1, 1)$
8 : $P_{60} = (3, 1, 2, 1)$
9 : $P_{63} = (2, 2, 2, 1)$
10 : $P_{75} = (2, 1, 3, 1)$
11 : $P_{84} = (3, 3, 3, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 25 points:

The points on the surface are:

$$0 : P_2 = (0, 0, 1, 0)$$

$$1 : P_3 = (0, 0, 0, 1)$$

$$2 : P_5 = (1, 1, 0, 0)$$

$$3 : P_6 = (2, 1, 0, 0)$$

$$4 : P_7 = (3, 1, 0, 0)$$

$$5 : P_{24} = (2, 0, 0, 1)$$

$$6 : P_{25} = (3, 0, 0, 1)$$

$$7 : P_{27} = (1, 1, 0, 1)$$

$$8 : P_{31} = (1, 2, 0, 1)$$

$$9 : P_{35} = (1, 3, 0, 1)$$

$$10 : P_{38} = (0, 0, 1, 1)$$

$$11 : P_{40} = (2, 0, 1, 1)$$

$$12 : P_{41} = (3, 0, 1, 1)$$

$$13 : P_{48} = (3, 2, 1, 1)$$

$$14 : P_{51} = (2, 3, 1, 1)$$

$$15 : P_{53} = (0, 0, 2, 1)$$

$$16 : P_{55} = (2, 0, 2, 1)$$

$$17 : P_{56} = (3, 0, 2, 1)$$

$$18 : P_{60} = (3, 1, 2, 1)$$

$$19 : P_{63} = (2, 2, 2, 1)$$

$$20 : P_{69} = (0, 0, 3, 1)$$

$$21 : P_{71} = (2, 0, 3, 1)$$

$$22 : P_{72} = (3, 0, 3, 1)$$

$$23 : P_{75} = (2, 1, 3, 1)$$

$$24 : P_{84} = (3, 3, 3, 1)$$