

Rank-65843 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1243947597

General information

Number of lines	10
Number of points	81
Number of singular points	9
Number of Eckardt points	0
Number of double points	0
Number of single points	80
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{10}
Type of lines on points	$10, 1^{80}$

Singular Points

The surface has 9 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{138} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{201} = \mathbf{P}(0, 0, \gamma, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{265} = \mathbf{P}(0, 0, \gamma^5, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{329} = \mathbf{P}(0, 0, \gamma^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{393} = \mathbf{P}(0, 0, \gamma^3, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{457} = \mathbf{P}(0, 0, \gamma^6, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{521} = \mathbf{P}(0, 0, \gamma^4, 1) = \mathbf{P}(0, 0, 7, 1)$$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_{81} \\
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{17} \\
\ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{209} \\
\ell_4 &= \begin{bmatrix} 1 & \gamma^3 & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2773} = \begin{bmatrix} 1 & 5 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2773} = \mathbf{Pl}(0, 4, 0, 5, 1, 0)_{272} \\
\ell_5 &= \begin{bmatrix} 1 & \gamma^2 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1532} = \begin{bmatrix} 1 & 4 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1532} = \mathbf{Pl}(0, 2, 0, 4, 1, 0)_{255} \\
\ell_6 &= \begin{bmatrix} 1 & \gamma^4 & \gamma^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2919} = \begin{bmatrix} 1 & 7 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2919} = \mathbf{Pl}(0, 4, 0, 7, 1, 0)_{302} \\
\ell_7 &= \begin{bmatrix} 1 & \gamma^6 & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4598} = \begin{bmatrix} 1 & 6 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4598} = \mathbf{Pl}(0, 7, 0, 6, 1, 0)_{290} \\
\ell_8 &= \begin{bmatrix} 1 & \gamma & \gamma^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4306} = \begin{bmatrix} 1 & 2 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4306} = \mathbf{Pl}(0, 7, 0, 2, 1, 0)_{230} \\
\ell_9 &= \begin{bmatrix} 1 & \gamma^5 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1459} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1459} = \mathbf{Pl}(0, 2, 0, 3, 1, 0)_{240}
\end{aligned}$$

Rank of lines: (72, 4680, 4744, 729, 2773, 1532, 2919, 4598, 4306, 1459)

Rank of points on Klein quadric: (81, 17, 1, 209, 272, 255, 302, 290, 230, 240)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 80 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_1

2 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_2

3 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_3

4 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_3

5 : $P_{30} = (3, 2, 1, 0)$ lies on line ℓ_4

6 : $P_{33} = (6, 2, 1, 0)$ lies on line ℓ_5
 7 : $P_{46} = (3, 4, 1, 0)$ lies on line ℓ_6
 8 : $P_{48} = (5, 4, 1, 0)$ lies on line ℓ_7
 9 : $P_{72} = (5, 7, 1, 0)$ lies on line ℓ_8
 10 : $P_{73} = (6, 7, 1, 0)$ lies on line ℓ_9
 11 : $P_{75} = (1, 0, 0, 1)$ lies on line ℓ_0
 12 : $P_{76} = (2, 0, 0, 1)$ lies on line ℓ_0
 13 : $P_{77} = (3, 0, 0, 1)$ lies on line ℓ_0
 14 : $P_{78} = (4, 0, 0, 1)$ lies on line ℓ_0
 15 : $P_{79} = (5, 0, 0, 1)$ lies on line ℓ_0
 16 : $P_{80} = (6, 0, 0, 1)$ lies on line ℓ_0
 17 : $P_{81} = (7, 0, 0, 1)$ lies on line ℓ_0
 18 : $P_{82} = (0, 1, 0, 1)$ lies on line ℓ_1
 19 : $P_{90} = (0, 2, 0, 1)$ lies on line ℓ_1
 20 : $P_{98} = (0, 3, 0, 1)$ lies on line ℓ_1
 21 : $P_{106} = (0, 4, 0, 1)$ lies on line ℓ_1
 22 : $P_{114} = (0, 5, 0, 1)$ lies on line ℓ_1
 23 : $P_{122} = (0, 6, 0, 1)$ lies on line ℓ_1
 24 : $P_{130} = (0, 7, 0, 1)$ lies on line ℓ_1
 25 : $P_{138} = (0, 0, 1, 1)$ lies on line ℓ_2
 26 : $P_{156} = (3, 2, 1, 1)$ lies on line ℓ_4
 27 : $P_{159} = (6, 2, 1, 1)$ lies on line ℓ_5
 28 : $P_{172} = (3, 4, 1, 1)$ lies on line ℓ_6
 29 : $P_{174} = (5, 4, 1, 1)$ lies on line ℓ_7
 30 : $P_{198} = (5, 7, 1, 1)$ lies on line ℓ_8
 31 : $P_{199} = (6, 7, 1, 1)$ lies on line ℓ_9
 32 : $P_{201} = (0, 0, 2, 1)$ lies on line ℓ_2
 33 : $P_{219} = (2, 2, 2, 1)$ lies on line ℓ_3
 34 : $P_{226} = (1, 3, 2, 1)$ lies on line ℓ_9
 35 : $P_{232} = (7, 3, 2, 1)$ lies on line ℓ_8
 36 : $P_{234} = (1, 4, 2, 1)$ lies on line ℓ_5
 37 : $P_{239} = (6, 4, 2, 1)$ lies on line ℓ_4
 38 : $P_{247} = (6, 5, 2, 1)$ lies on line ℓ_6
 39 : $P_{248} = (7, 5, 2, 1)$ lies on line ℓ_7
 40 : $P_{265} = (0, 0, 3, 1)$ lies on line ℓ_2
 41 : $P_{275} = (2, 1, 3, 1)$ lies on line ℓ_7
 42 : $P_{278} = (5, 1, 3, 1)$ lies on line ℓ_6
 43 : $P_{292} = (3, 3, 3, 1)$ lies on line ℓ_3

44 : $P_{299} = (2, 4, 3, 1)$ lies on line ℓ_8
 45 : $P_{304} = (7, 4, 3, 1)$ lies on line ℓ_9
 46 : $P_{318} = (5, 6, 3, 1)$ lies on line ℓ_4
 47 : $P_{320} = (7, 6, 3, 1)$ lies on line ℓ_5
 48 : $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_2
 49 : $P_{365} = (4, 4, 4, 1)$ lies on line ℓ_3
 50 : $P_{370} = (1, 5, 4, 1)$ lies on line ℓ_4
 51 : $P_{371} = (2, 5, 4, 1)$ lies on line ℓ_5
 52 : $P_{379} = (2, 6, 4, 1)$ lies on line ℓ_9
 53 : $P_{380} = (3, 6, 4, 1)$ lies on line ℓ_8
 54 : $P_{386} = (1, 7, 4, 1)$ lies on line ℓ_6
 55 : $P_{388} = (3, 7, 4, 1)$ lies on line ℓ_7
 56 : $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_2
 57 : $P_{405} = (4, 1, 5, 1)$ lies on line ℓ_9
 58 : $P_{407} = (6, 1, 5, 1)$ lies on line ℓ_8
 59 : $P_{419} = (2, 3, 5, 1)$ lies on line ℓ_6
 60 : $P_{423} = (6, 3, 5, 1)$ lies on line ℓ_7
 61 : $P_{438} = (5, 5, 5, 1)$ lies on line ℓ_3
 62 : $P_{451} = (2, 7, 5, 1)$ lies on line ℓ_4
 63 : $P_{453} = (4, 7, 5, 1)$ lies on line ℓ_5
 64 : $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_2
 65 : $P_{468} = (3, 1, 6, 1)$ lies on line ℓ_5
 66 : $P_{472} = (7, 1, 6, 1)$ lies on line ℓ_4
 67 : $P_{477} = (4, 2, 6, 1)$ lies on line ℓ_7
 68 : $P_{480} = (7, 2, 6, 1)$ lies on line ℓ_6
 69 : $P_{500} = (3, 5, 6, 1)$ lies on line ℓ_9
 70 : $P_{501} = (4, 5, 6, 1)$ lies on line ℓ_8
 71 : $P_{511} = (6, 6, 6, 1)$ lies on line ℓ_3
 72 : $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_2
 73 : $P_{538} = (1, 2, 7, 1)$ lies on line ℓ_8
 74 : $P_{542} = (5, 2, 7, 1)$ lies on line ℓ_9
 75 : $P_{549} = (4, 3, 7, 1)$ lies on line ℓ_4
 76 : $P_{550} = (5, 3, 7, 1)$ lies on line ℓ_5
 77 : $P_{570} = (1, 6, 7, 1)$ lies on line ℓ_7
 78 : $P_{573} = (4, 6, 7, 1)$ lies on line ℓ_6
 79 : $P_{584} = (7, 7, 7, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1	1
5	1	1	1	1	1	0	1	1	1	1
6	1	1	1	1	1	1	0	1	1	1
7	1	1	1	1	1	1	1	0	1	1
8	1	1	1	1	1	1	1	1	0	1
9	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_{20} = (1, 1, 1, 0)$
 6 : $P_{30} = (3, 2, 1, 0)$
 7 : $P_{33} = (6, 2, 1, 0)$
 8 : $P_{46} = (3, 4, 1, 0)$
 9 : $P_{48} = (5, 4, 1, 0)$
 10 : $P_{72} = (5, 7, 1, 0)$
 11 : $P_{73} = (6, 7, 1, 0)$
 12 : $P_{75} = (1, 0, 0, 1)$
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 14 : $P_{77} = (3, 0, 0, 1)$
 15 : $P_{78} = (4, 0, 0, 1)$
 16 : $P_{79} = (5, 0, 0, 1)$
 17 : $P_{80} = (6, 0, 0, 1)$
 18 : $P_{81} = (7, 0, 0, 1)$
 19 : $P_{82} = (0, 1, 0, 1)$
 20 : $P_{90} = (0, 2, 0, 1)$
 21 : $P_{98} = (0, 3, 0, 1)$
 22 : $P_{106} = (0, 4, 0, 1)$
 23 : $P_{114} = (0, 5, 0, 1)$
 24 : $P_{122} = (0, 6, 0, 1)$
 25 : $P_{130} = (0, 7, 0, 1)$
 26 : $P_{138} = (0, 0, 1, 1)$
 27 : $P_{156} = (3, 2, 1, 1)$

28 : $P_{159} = (6, 2, 1, 1)$
 29 : $P_{172} = (3, 4, 1, 1)$
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 40 : $P_{248} = (7, 5, 2, 1)$
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 44 : $P_{292} = (3, 3, 3, 1)$
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 47 : $P_{318} = (5, 6, 3, 1)$
 48 : $P_{320} = (7, 6, 3, 1)$
 49 : $P_{329} = (0, 0, 4, 1)$
 50 : $P_{365} = (4, 4, 4, 1)$
 51 : $P_{370} = (1, 5, 4, 1)$
 52 : $P_{371} = (2, 5, 4, 1)$
 53 : $P_{379} = (2, 6, 4, 1)$
 54 : $P_{380} = (3, 6, 4, 1)$
 55 : $P_{386} = (1, 7, 4, 1)$

56 : $P_{388} = (3, 7, 4, 1)$
 57 : $P_{393} = (0, 0, 5, 1)$
 58 : $P_{405} = (4, 1, 5, 1)$
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 77 : $P_{550} = (5, 3, 7, 1)$
 78 : $P_{570} = (1, 6, 7, 1)$
 79 : $P_{573} = (4, 6, 7, 1)$
 80 : $P_{584} = (7, 7, 7, 1)$