

# Rank-65743 over GF(4)

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## The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(4) is 1431676329

## General information

Number of lines	5
Number of points	25
Number of singular points	1
Number of Eckardt points	1
Number of double points	3
Number of single points	16
Number of points off lines	5
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^5$
Type of lines on points	$3, 2^3, 1^{16}, 0^5$

## Singular Points

The surface has 1 singular points:

$$0 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

## The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{129}$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0, 0, 0, 3, 0, 1)_{143} \\ \ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0, 0, 0, 2, 0, 1)_{136} \\ \ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega^2 & \omega^2 \end{bmatrix}_{99} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{99} = \mathbf{Pl}(1, 1, 1, 2, 1, 0)_{83} \\ \ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega & \omega \end{bmatrix}_{94} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{94} = \mathbf{Pl}(1, 1, 1, 3, 1, 0)_{92}\end{aligned}$$

Rank of lines: ( 337, 339, 338, 99, 94 )

Rank of points on Klein quadric: ( 129, 143, 136, 83, 92 )

### Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0).$$

### Double Points

The surface has 3 Double points:

The double points on the surface are:

$$\begin{aligned}P_{45} &= (0, 2, 1, 1) = \ell_0 \cap \ell_3 \\ P_{49} &= (0, 3, 1, 1) = \ell_0 \cap \ell_4\end{aligned}$$

$$P_{23} = (1, 0, 0, 1) = \ell_3 \cap \ell_4$$

### Single Points

The surface has 16 single points:

The single points on the surface are:

- 0 :  $P_{16} = (1, 2, 1, 0)$  lies on line  $\ell_3$
- 1 :  $P_{20} = (1, 3, 1, 0)$  lies on line  $\ell_4$
- 2 :  $P_{38} = (0, 0, 1, 1)$  lies on line  $\ell_0$
- 3 :  $P_{42} = (0, 1, 1, 1)$  lies on line  $\ell_0$
- 4 :  $P_{53} = (0, 0, 2, 1)$  lies on line  $\ell_1$
- 5 :  $P_{57} = (0, 1, 2, 1)$  lies on line  $\ell_1$
- 6 :  $P_{60} = (3, 1, 2, 1)$  lies on line  $\ell_4$
- 7 :  $P_{61} = (0, 2, 2, 1)$  lies on line  $\ell_1$
- 8 :  $P_{65} = (0, 3, 2, 1)$  lies on line  $\ell_1$

- 9 :  $P_{68} = (3, 3, 2, 1)$  lies on line  $\ell_3$
- 10 :  $P_{69} = (0, 0, 3, 1)$  lies on line  $\ell_2$
- 11 :  $P_{73} = (0, 1, 3, 1)$  lies on line  $\ell_2$
- 12 :  $P_{75} = (2, 1, 3, 1)$  lies on line  $\ell_3$
- 13 :  $P_{77} = (0, 2, 3, 1)$  lies on line  $\ell_2$
- 14 :  $P_{79} = (2, 2, 3, 1)$  lies on line  $\ell_4$
- 15 :  $P_{81} = (0, 3, 3, 1)$  lies on line  $\ell_2$

The single points on the surface are:

### Points on surface but on no line

The surface has 5 points not on any line:

The points on the surface but not on lines are:

$0 : P_0 = (1, 0, 0, 0)$   
 $1 : P_{28} = (2, 1, 0, 1)$   
 $2 : P_{29} = (3, 1, 0, 1)$

$3 : P_{46} = (1, 2, 1, 1)$   
 $4 : P_{50} = (1, 3, 1, 1)$

## Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	1	0	0
2	1	1	0	0	0
3	1	0	0	0	1
4	1	0	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_1$	$P_1$	$P_{45}$	$P_{49}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$
in point	$P_1$	$P_1$

Line 2 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_1$	$P_1$

Line 3 intersects

Line	$\ell_0$	$\ell_4$
in point	$P_{45}$	$P_{23}$

Line 4 intersects

Line	$\ell_0$	$\ell_3$
in point	$P_{49}$	$P_{23}$

The surface has 25 points:

The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$   
 $1 : P_1 = (0, 1, 0, 0)$   
 $2 : P_{16} = (1, 2, 1, 0)$   
 $3 : P_{20} = (1, 3, 1, 0)$   
 $4 : P_{23} = (1, 0, 0, 1)$   
 $5 : P_{28} = (2, 1, 0, 1)$   
 $6 : P_{29} = (3, 1, 0, 1)$   
 $7 : P_{38} = (0, 0, 1, 1)$   
 $8 : P_{42} = (0, 1, 1, 1)$

$9 : P_{45} = (0, 2, 1, 1)$   
 $10 : P_{46} = (1, 2, 1, 1)$   
 $11 : P_{49} = (0, 3, 1, 1)$   
 $12 : P_{50} = (1, 3, 1, 1)$   
 $13 : P_{53} = (0, 0, 2, 1)$   
 $14 : P_{57} = (0, 1, 2, 1)$   
 $15 : P_{60} = (3, 1, 2, 1)$   
 $16 : P_{61} = (0, 2, 2, 1)$   
 $17 : P_{65} = (0, 3, 2, 1)$

$18 : P_{68} = (3, 3, 2, 1)$   
 $19 : P_{69} = (0, 0, 3, 1)$   
 $20 : P_{73} = (0, 1, 3, 1)$   
 $21 : P_{75} = (2, 1, 3, 1)$   
 $22 : P_{77} = (0, 2, 3, 1)$   
 $23 : P_{79} = (2, 2, 3, 1)$   
 $24 : P_{81} = (0, 3, 3, 1)$