Cheat Sheet GF(16)

January 16, 2021

polynomial: $X^4 + X^3 + 1 = 25$
$Z_i = \log_\alpha(1 + \alpha^i)$
Subfields:

Subfield	Polynomial	Numerical Rank
$\mathbb{F}_4$	$X_2 + X_1 + 1$	7

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0	0	0
1	$1 = 1$	1	1	15	2	12	1	0	1	0	1
2	$\alpha = \delta$	2	12	1	4	9	4	1	1	11	11
3	$\alpha + 1 = \delta^{12}$	3	8	12	8	4	5	1	1	11	1
4	$\alpha^2 = \delta^2$	4	6	2	9	3	9	1	1	10	10
5	$\alpha^2 + 1 = \delta^9$	5	15	9	11	10	8	1	1	10	1
6	$\alpha^2 + \alpha = \delta^{13}$	6	4	13	15	8	13	0	1	1	11
7	$\alpha^2 + \alpha + 1 = \delta^7$	7	14	7	7	13	12	0	1	1	11
8	$\alpha^3 = \delta^3$	8	3	3	14	6	15	1	1	11	1
9	$\alpha^3 + 1 = \delta^4$	9	13	4	5	2	14	1	1	11	11
10	$\alpha^3 + \alpha = \delta^{10}$	10	11	10	10	5	11	0	1	0	11
11	$\alpha^3 + \alpha + 1 = \delta^5$	11	10	5	13	14	10	0	1	0	10
12	$\alpha^3 + \alpha^2 = \delta^{14}$	12	2	14	3	1	6	0	1	1	10
13	$\alpha^3 + \alpha^2 + 1 = \delta^{11}$	13	9	11	6	7	7	0	1	1	10
14	$\alpha^3 + \alpha^2 + \alpha = \delta^8$	14	7	8	12	11	2	1	1	10	10
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \delta^6$	15	5	6	1	DNE	3	1	1	10	1

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14
2	2	3	0	1	6	7	4	5	10	11	8	9	14	15	12	13
3	3	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12
4	4	5	6	7	0	1	2	3	12	13	14	15	8	9	10	11
5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10
6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9
7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8
8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6
10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5
11	11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3
13	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2
14	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1
15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

·	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	9	11	13	15	1	3	5	7
3	3	6	5	12	15	10	9	1	2	7	4	13	14	11	8
4	4	8	12	9	13	1	5	11	15	3	7	2	6	10	14
5	5	10	15	13	8	7	2	3	6	9	12	14	11	4	1
6	6	12	10	1	7	13	11	2	4	14	8	3	5	15	9
7	7	14	9	5	2	11	12	10	13	4	3	15	8	1	6
8	8	9	1	11	3	2	10	15	7	6	14	4	12	13	5
9	9	11	2	15	6	4	13	7	14	12	5	8	1	3	10
10	10	13	7	3	9	14	4	6	12	11	1	5	15	8	2
11	11	15	4	7	12	8	3	14	5	1	10	9	2	6	13
12	12	1	13	2	14	3	15	4	8	5	9	6	10	7	11
13	13	3	14	6	11	5	8	12	1	15	2	10	7	9	4
14	14	5	11	10	4	15	1	13	3	8	6	7	9	2	12
15	15	7	8	14	1	9	6	5	10	2	13	11	4	12	3

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 9$
$2^5 = 11$
$2^6 = 15$
$2^7 = 7$
$2^8 = 14$
$2^9 = 5$
$2^{10} = 10$
$2^{11} = 13$
$2^{12} = 3$
$2^{13} = 6$
$2^{14} = 12$
$2^{15} = 1$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$	$T_2(\gamma_i)$	$N_2(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0	0	0
1	$1 = 1$	1	1	15	2	12	1	0	1	0	1
2	$\alpha = \delta$	2	12	1	4	9	4	1	1	11	11
3	$\alpha + 1 = \delta^{12}$	3	8	12	8	4	5	1	1	11	1
4	$\alpha^2 = \delta^2$	4	6	2	9	3	9	1	1	10	10
5	$\alpha^2 + 1 = \delta^9$	5	15	9	11	10	8	1	1	10	1
6	$\alpha^2 + \alpha = \delta^{13}$	6	4	13	15	8	13	0	1	1	11
7	$\alpha^2 + \alpha + 1 = \delta^7$	7	14	7	7	13	12	0	1	1	11
8	$\alpha^3 = \delta^3$	8	3	3	14	6	15	1	1	11	1
9	$\alpha^3 + 1 = \delta^4$	9	13	4	5	2	14	1	1	11	11
10	$\alpha^3 + \alpha = \delta^{10}$	10	11	10	10	5	11	0	1	0	11
11	$\alpha^3 + \alpha + 1 = \delta^5$	11	10	5	13	14	10	0	1	0	10
12	$\alpha^3 + \alpha^2 = \delta^{14}$	12	2	14	3	1	6	0	1	1	10
13	$\alpha^3 + \alpha^2 + 1 = \delta^{11}$	13	9	11	6	7	7	0	1	1	10
14	$\alpha^3 + \alpha^2 + \alpha = \delta^8$	14	7	8	12	11	2	1	1	10	10
15	$\alpha^3 + \alpha^2 + \alpha + 1 = \delta^6$	15	5	6	1	DNE	3	1	1	10	1

$+$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	3	2	5	4	7	6	9	8	11	10	13	12	15	14
2	2	3	0	1	6	7	4	5	10	11	8	9	14	15	12	13
3	3	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12
4	4	5	6	7	0	1	2	3	12	13	14	15	8	9	10	11
5	5	4	7	6	1	0	3	2	13	12	15	14	9	8	11	10
6	6	7	4	5	2	3	0	1	14	15	12	13	10	11	8	9
7	7	6	5	4	3	2	1	0	15	14	13	12	11	10	9	8
8	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
9	9	8	11	10	13	12	15	14	1	0	3	2	5	4	7	6
10	10	11	8	9	14	15	12	13	2	3	0	1	6	7	4	5
11	11	10	9	8	15	14	13	12	3	2	1	0	7	6	5	4
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3
13	13	12	15	14	9	8	11	10	5	4	7	6	1	0	3	2
14	14	15	12	13	10	11	8	9	6	7	4	5	2	3	0	1
15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

$\cdot$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	9	11	13	15	1	3	5	7
3	3	6	5	12	15	10	9	1	2	7	4	13	14	11	8
4	4	8	12	9	13	1	5	11	15	3	7	2	6	10	14
5	5	10	15	13	8	7	2	3	6	9	12	14	11	4	1
6	6	12	10	1	7	13	11	2	4	14	8	3	5	15	9
7	7	14	9	5	2	11	12	10	13	4	3	15	8	1	6
8	8	9	1	11	3	2	10	15	7	6	14	4	12	13	5
9	9	11	2	15	6	4	13	7	14	12	5	8	1	3	10
10	10	13	7	3	9	14	4	6	12	11	1	5	15	8	2
11	11	15	4	7	12	8	3	14	5	1	10	9	2	6	13
12	12	1	13	2	14	3	15	4	8	5	9	6	10	7	11
13	13	3	14	6	11	5	8	12	1	15	2	10	7	9	4
14	14	5	11	10	4	15	1	13	3	8	6	7	9	2	12
15	15	7	8	14	1	9	6	5	10	2	13	11	4	12	3

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$2^{11} = 13$
$2^{12} = 3$
$2^{13} = 6$
$2^{14} = 12$
$2^{15} = 1$