Rank-66763 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1432724889

General information

Number of lines	5
Number of points	25
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	15
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$3^2, 2^2, 1^{15}, 0^6$

Singular Points

The surface has 3 singular points:

0 : $P_{15} = \mathbf{P}(0, \omega, 1, 0) = \mathbf{P}(0, 2, 1, 0)$ 2 : $P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$
1 : $P_{19} = \mathbf{P}(0, \omega^2, 1, 0) = \mathbf{P}(0, 3, 1, 0)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega & 0 \end{bmatrix}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}_2 = \mathbf{Pl}(3, 0, 1, 0, 0, 0)_5$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}_3 = \mathbf{Pl}(2,0,1,0,0,0)_4$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{87} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{87} = \mathbf{Pl}(2,3,3,1,0,0)_{23}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{86} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{86} = \mathbf{Pl}(3,2,2,1,0,0)_{21}$$

Rank of lines: (2, 3, 336, 87, 86)
Rank of points on Klein quadric: (5, 4, 101, 23, 21)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{15} = \mathbf{P}(0,\omega,1,0) = \mathbf{P}(0,2,1,0),$
$1 : P_{19} = \mathbf{P}(0,\omega^2,1,0) = \mathbf{P}(0,3,1,0).$

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{23} = (1,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 15 single points:
The single points on the surface are:

$0 : P_1 = (0,1,0,0)$ lies on line ℓ_2
$1 : P_2 = (0,0,1,0)$ lies on line ℓ_2
$2 : P_{11} = (0,1,1,0)$ lies on line ℓ_2
$3 : P_{16} = (1,2,1,0)$ lies on line ℓ_1
$4 : P_{17} = (2,2,1,0)$ lies on line ℓ_1
$5 : P_{18} = (3,2,1,0)$ lies on line ℓ_1
$6 : P_{20} = (1,3,1,0)$ lies on line ℓ_0
$7 : P_{21} = (2,3,1,0)$ lies on line ℓ_0

$8 : P_{22} = (3,3,1,0)$ lies on line ℓ_0
$9 : P_{46} = (1,2,1,1)$ lies on line ℓ_3
$10 : P_{50} = (1,3,1,1)$ lies on line ℓ_4
$11 : P_{58} = (1,1,2,1)$ lies on line ℓ_4
$12 : P_{66} = (1,3,2,1)$ lies on line ℓ_3
$13 : P_{74} = (1,1,3,1)$ lies on line ℓ_3
$14 : P_{78} = (1,2,3,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:
The points on the surface but not on lines are:

0 : $P_{28} = (2, 1, 0, 1)$ 4 : $P_{43} = (2, 1, 1, 1)$
1 : $P_{29} = (3, 1, 0, 1)$ 5 : $P_{44} = (3, 1, 1, 1)$
2 : $P_{40} = (2, 0, 1, 1)$
3 : $P_{41} = (3, 0, 1, 1)$

Line Intersection Graph

```
  |0 1 2 3 4
--+---------
0 |0 1 1 0 1
1 |1 0 1 1 0
2 |1 1 0 1 1
3 |0 1 1 0 1
4 |1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_0	P_{19}	P_{19}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_0	P_{15}	P_{15}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{19}	P_{15}	P_{15}	P_{19}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_{15}	P_{15}	P_{23}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_{19}	P_{19}	P_{23}

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ 9 : $P_{20} = (1, 3, 1, 0)$ 18 : $P_{44} = (3, 1, 1, 1)$
1 : $P_1 = (0, 1, 0, 0)$ 10 : $P_{21} = (2, 3, 1, 0)$ 19 : $P_{46} = (1, 2, 1, 1)$
2 : $P_2 = (0, 0, 1, 0)$ 11 : $P_{22} = (3, 3, 1, 0)$ 20 : $P_{50} = (1, 3, 1, 1)$
3 : $P_{11} = (0, 1, 1, 0)$ 12 : $P_{23} = (1, 0, 0, 1)$ 21 : $P_{58} = (1, 1, 2, 1)$
4 : $P_{15} = (0, 2, 1, 0)$ 13 : $P_{28} = (2, 1, 0, 1)$ 22 : $P_{66} = (1, 3, 2, 1)$
5 : $P_{16} = (1, 2, 1, 0)$ 14 : $P_{29} = (3, 1, 0, 1)$ 23 : $P_{74} = (1, 1, 3, 1)$
6 : $P_{17} = (2, 2, 1, 0)$ 15 : $P_{40} = (2, 0, 1, 1)$ 24 : $P_{78} = (1, 2, 3, 1)$
7 : $P_{18} = (3, 2, 1, 0)$ 16 : $P_{41} = (3, 0, 1, 1)$
8 : $P_{19} = (0, 3, 1, 0)$ 17 : $P_{43} = (2, 1, 1, 1)$