Rank-65760 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 571613718

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	1
Number of double points	15
Number of single points	120
Number of points off lines	185
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3, 2^{15}, 1^{120}, 0^{185}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \mathbf{Pl}(0,0,0,1,0,1)_{5121}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{69899} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{69899} = \mathbf{Pl}(0,0,0,11,0,1)_{5431}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{69898} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{69898} = \mathbf{Pl}(0,0,0,10,0,1)_{5400}$$

$$\ell_3 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{3270} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{3270} = \mathbf{Pl}(0,0,10,11,1,1)_{9705}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^5 & 1 \\ 0 & 1 & 1 & \delta^{10} \end{bmatrix}_{7532} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 1 & 10 \end{bmatrix}_{7532} = \mathbf{Pl}(11,0,11,10,11,1)_{50547}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & 1 & \delta^5 \end{bmatrix}_{7275} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 1 & 11 \end{bmatrix}_{7275} = \mathbf{Pl}(10,0,10,11,10,1)_{46435}$$

$$\ell_6 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{2996} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{2996} = \mathbf{Pl}(0,0,11,10,1,1)_{9736}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^5 & \delta^5 \end{bmatrix}_{460} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 11 & 11 \end{bmatrix}_{460} = \mathbf{Pl}(10,11,1,0,11,1)_{49785}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^{10} & \delta^{10} \end{bmatrix}_{443} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & 10 \end{bmatrix}_{443} = \mathbf{Pl}(11,10,1,0,10,1)_{45706}$$

Rank of lines: (69889, 69899, 69898, 3270, 7532, 7275, 2996, 460, 443)
Rank of points on Klein quadric: (5121, 5431, 5400, 9705, 50547, 46435, 9736, 49785, 45706)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{689} = (0,10,1,1) = \ell_0 \cap \ell_7$
$P_{705} = (0,11,1,1) = \ell_0 \cap \ell_8$
$P_{2833} = (0,0,10,1) = \ell_1 \cap \ell_3$
$P_{2993} = (0,10,10,1) = \ell_1 \cap \ell_5$
$P_{3265} = (0,11,11,1) = \ell_2 \cap \ell_4$
$P_{3089} = (0,0,11,1) = \ell_2 \cap \ell_6$
$P_{14} = (10,1,0,0) = \ell_3 \cap \ell_4$
$P_{2859} = (10,1,10,1) = \ell_3 \cap \ell_5$
$P_{3004} = (11,10,10,1) = \ell_3 \cap \ell_7$
$P_{3116} = (11,1,11,1) = \ell_4 \cap \ell_6$
$P_{3259} = (10,10,11,1) = \ell_4 \cap \ell_7$
$P_{15} = (11,1,0,0) = \ell_5 \cap \ell_6$
$P_{3020} = (11,11,10,1) = \ell_5 \cap \ell_8$
$P_{3275} = (10,11,11,1) = \ell_6 \cap \ell_8$
$P_{20} = (1,0,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 120 single points:
The single points on the surface are:

$0 : P_{435} = (1,10,0,1)$ lies on line ℓ_7
$1 : P_{451} = (1,11,0,1)$ lies on line ℓ_8
$2 : P_{530} = (0,0,1,1)$ lies on line ℓ_0
$3 : P_{546} = (0,1,1,1)$ lies on line ℓ_0
$4 : P_{561} = (0,2,1,1)$ lies on line ℓ_0
$5 : P_{577} = (0,3,1,1)$ lies on line ℓ_0
$6 : P_{593} = (0,4,1,1)$ lies on line ℓ_0
$7 : P_{609} = (0,5,1,1)$ lies on line ℓ_0
$8 : P_{625} = (0,6,1,1)$ lies on line ℓ_0
$9 : P_{641} = (0,7,1,1)$ lies on line ℓ_0
$10 : P_{657} = (0,8,1,1)$ lies on line ℓ_0
$11 : P_{673} = (0,9,1,1)$ lies on line ℓ_0

12 : $P_{721} = (0, 12, 1, 1)$ lies on line ℓ_0
13 : $P_{737} = (0, 13, 1, 1)$ lies on line ℓ_0
14 : $P_{753} = (0, 14, 1, 1)$ lies on line ℓ_0
15 : $P_{769} = (0, 15, 1, 1)$ lies on line ℓ_0
16 : $P_{948} = (3, 10, 2, 1)$ lies on line ℓ_7
17 : $P_{964} = (3, 11, 2, 1)$ lies on line ℓ_8
18 : $P_{1203} = (2, 10, 3, 1)$ lies on line ℓ_7
19 : $P_{1219} = (2, 11, 3, 1)$ lies on line ℓ_8
20 : $P_{1462} = (5, 10, 4, 1)$ lies on line ℓ_7
21 : $P_{1478} = (5, 11, 4, 1)$ lies on line ℓ_8
22 : $P_{1717} = (4, 10, 5, 1)$ lies on line ℓ_7
23 : $P_{1733} = (4, 11, 5, 1)$ lies on line ℓ_8
24 : $P_{1976} = (7, 10, 6, 1)$ lies on line ℓ_7
25 : $P_{1992} = (7, 11, 6, 1)$ lies on line ℓ_8
26 : $P_{2231} = (6, 10, 7, 1)$ lies on line ℓ_7
27 : $P_{2247} = (6, 11, 7, 1)$ lies on line ℓ_8
28 : $P_{2490} = (9, 10, 8, 1)$ lies on line ℓ_7
29 : $P_{2506} = (9, 11, 8, 1)$ lies on line ℓ_8
30 : $P_{2745} = (8, 10, 9, 1)$ lies on line ℓ_7
31 : $P_{2761} = (8, 11, 9, 1)$ lies on line ℓ_8
32 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_5
33 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_1
34 : $P_{2865} = (0, 2, 10, 1)$ lies on line ℓ_1
35 : $P_{2878} = (13, 2, 10, 1)$ lies on line ℓ_3
36 : $P_{2879} = (14, 2, 10, 1)$ lies on line ℓ_5
37 : $P_{2881} = (0, 3, 10, 1)$ lies on line ℓ_1
38 : $P_{2886} = (5, 3, 10, 1)$ lies on line ℓ_5
39 : $P_{2888} = (7, 3, 10, 1)$ lies on line ℓ_3
40 : $P_{2897} = (0, 4, 10, 1)$ lies on line ℓ_1
41 : $P_{2900} = (3, 4, 10, 1)$ lies on line ℓ_3
42 : $P_{2903} = (6, 4, 10, 1)$ lies on line ℓ_5
43 : $P_{2913} = (0, 5, 10, 1)$ lies on line ℓ_1
44 : $P_{2922} = (9, 5, 10, 1)$ lies on line ℓ_3
45 : $P_{2926} = (13, 5, 10, 1)$ lies on line ℓ_5
46 : $P_{2929} = (0, 6, 10, 1)$ lies on line ℓ_1
47 : $P_{2938} = (9, 6, 10, 1)$ lies on line ℓ_5
48 : $P_{2943} = (14, 6, 10, 1)$ lies on line ℓ_3
49 : $P_{2945} = (0, 7, 10, 1)$ lies on line ℓ_1
50 : $P_{2947} = (2, 7, 10, 1)$ lies on line ℓ_5
51 : $P_{2949} = (4, 7, 10, 1)$ lies on line ℓ_3
52 : $P_{2961} = (0, 8, 10, 1)$ lies on line ℓ_1
53 : $P_{2967} = (6, 8, 10, 1)$ lies on line ℓ_3
54 : $P_{2976} = (15, 8, 10, 1)$ lies on line ℓ_5
55 : $P_{2977} = (0, 9, 10, 1)$ lies on line ℓ_1
56 : $P_{2981} = (4, 9, 10, 1)$ lies on line ℓ_5
57 : $P_{2989} = (12, 9, 10, 1)$ lies on line ℓ_3
58 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_1
59 : $P_{3010} = (1, 11, 10, 1)$ lies on line ℓ_3
60 : $P_{3025} = (0, 12, 10, 1)$ lies on line ℓ_1
61 : $P_{3030} = (5, 12, 10, 1)$ lies on line ℓ_3
62 : $P_{3033} = (8, 12, 10, 1)$ lies on line ℓ_5
63 : $P_{3041} = (0, 13, 10, 1)$ lies on line ℓ_1
64 : $P_{3044} = (3, 13, 10, 1)$ lies on line ℓ_5
65 : $P_{3056} = (15, 13, 10, 1)$ lies on line ℓ_3
66 : $P_{3057} = (0, 14, 10, 1)$ lies on line ℓ_1
67 : $P_{3064} = (7, 14, 10, 1)$ lies on line ℓ_5
68 : $P_{3065} = (8, 14, 10, 1)$ lies on line ℓ_3
69 : $P_{3073} = (0, 15, 10, 1)$ lies on line ℓ_1
70 : $P_{3075} = (2, 15, 10, 1)$ lies on line ℓ_3
71 : $P_{3085} = (12, 15, 10, 1)$ lies on line ℓ_5
72 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_4
73 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_2
74 : $P_{3121} = (0, 2, 11, 1)$ lies on line ℓ_2
75 : $P_{3133} = (12, 2, 11, 1)$ lies on line ℓ_4
76 : $P_{3136} = (15, 2, 11, 1)$ lies on line ℓ_6
77 : $P_{3137} = (0, 3, 11, 1)$ lies on line ℓ_2
78 : $P_{3141} = (4, 3, 11, 1)$ lies on line ℓ_6
79 : $P_{3143} = (6, 3, 11, 1)$ lies on line ℓ_4
80 : $P_{3153} = (0, 4, 11, 1)$ lies on line ℓ_2
81 : $P_{3155} = (2, 4, 11, 1)$ lies on line ℓ_4
82 : $P_{3160} = (7, 4, 11, 1)$ lies on line ℓ_6
83 : $P_{3169} = (0, 5, 11, 1)$ lies on line ℓ_2
84 : $P_{3177} = (8, 5, 11, 1)$ lies on line ℓ_4
85 : $P_{3181} = (12, 5, 11, 1)$ lies on line ℓ_6
86 : $P_{3185} = (0, 6, 11, 1)$ lies on line ℓ_2
87 : $P_{3193} = (8, 6, 11, 1)$ lies on line ℓ_6
88 : $P_{3200} = (15, 6, 11, 1)$ lies on line ℓ_4
89 : $P_{3201} = (0, 7, 11, 1)$ lies on line ℓ_2
90 : $P_{3204} = (3, 7, 11, 1)$ lies on line ℓ_6
91 : $P_{3206} = (5, 7, 11, 1)$ lies on line ℓ_4
92 : $P_{3217} = (0, 8, 11, 1)$ lies on line ℓ_2
93 : $P_{3224} = (7, 8, 11, 1)$ lies on line ℓ_4
94 : $P_{3231} = (14, 8, 11, 1)$ lies on line ℓ_6
95 : $P_{3233} = (0, 9, 11, 1)$ lies on line ℓ_2
96 : $P_{3238} = (5, 9, 11, 1)$ lies on line ℓ_6
97 : $P_{3246} = (13, 9, 11, 1)$ lies on line ℓ_4
98 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_2
99 : $P_{3250} = (1, 10, 11, 1)$ lies on line ℓ_6
100 : $P_{3281} = (0, 12, 11, 1)$ lies on line ℓ_2
101 : $P_{3285} = (4, 12, 11, 1)$ lies on line ℓ_4
102 : $P_{3290} = (9, 12, 11, 1)$ lies on line ℓ_6
103 : $P_{3297} = (0, 13, 11, 1)$ lies on line ℓ_2
104 : $P_{3299} = (2, 13, 11, 1)$ lies on line ℓ_6
105 : $P_{3311} = (14, 13, 11, 1)$ lies on line ℓ_4
106 : $P_{3313} = (0, 14, 11, 1)$ lies on line ℓ_2
107 : $P_{3319} = (6, 14, 11, 1)$ lies on line ℓ_6
108 : $P_{3322} = (9, 14, 11, 1)$ lies on line ℓ_4
109 : $P_{3329} = (0, 15, 11, 1)$ lies on line ℓ_2
110 : $P_{3332} = (3, 15, 11, 1)$ lies on line ℓ_4
111 : $P_{3342} = (13, 15, 11, 1)$ lies on line ℓ_6
112 : $P_{3518} = (13, 10, 12, 1)$ lies on line ℓ_7
113 : $P_{3534} = (13, 11, 12, 1)$ lies on line ℓ_8
114 : $P_{3773} = (12, 10, 13, 1)$ lies on line ℓ_7
115 : $P_{3789} = (12, 11, 13, 1)$ lies on line ℓ_8
116 : $P_{4032} = (15, 10, 14, 1)$ lies on line ℓ_7
117 : $P_{4048} = (15, 11, 14, 1)$ lies on line ℓ_8
118 : $P_{4287} = (14, 10, 15, 1)$ lies on line ℓ_7
119 : $P_{4303} = (14, 11, 15, 1)$ lies on line ℓ_8

The single points on the surface are:

Points on surface but on no line

The surface has 185 points not on any line:
The points on the surface but not on lines are:

$0: P_{29} = (10, 0, 1, 0)$
$1: P_{30} = (11, 0, 1, 0)$
$2: P_{128} = (13, 6, 1, 0)$
$3: P_{143} = (12, 7, 1, 0)$
$4: P_{190} = (11, 10, 1, 0)$
$5: P_{191} = (12, 10, 1, 0)$
$6: P_{192} = (13, 10, 1, 0)$
$7: P_{201} = (6, 11, 1, 0)$
$8: P_{202} = (7, 11, 1, 0)$
$9: P_{205} = (10, 11, 1, 0)$
$10: P_{217} = (6, 12, 1, 0)$
$11: P_{234} = (7, 13, 1, 0)$
$12: P_{317} = (11, 2, 0, 1)$
$13: P_{328} = (6, 3, 0, 1)$
$14: P_{330} = (8, 3, 0, 1)$
$15: P_{334} = (12, 3, 0, 1)$
$16: P_{348} = (10, 4, 0, 1)$
$17: P_{360} = (6, 5, 0, 1)$
$18: P_{367} = (13, 5, 0, 1)$
$19: P_{369} = (15, 5, 0, 1)$
$20: P_{405} = (3, 8, 0, 1)$
$21: P_{409} = (7, 8, 0, 1)$
$22: P_{415} = (13, 8, 0, 1)$
$23: P_{429} = (11, 9, 0, 1)$
$24: P_{439} = (5, 10, 0, 1)$
$25: P_{449} = (15, 10, 0, 1)$
$26: P_{453} = (3, 11, 0, 1)$
$27: P_{458} = (8, 11, 0, 1)$
$28: P_{508} = (10, 14, 0, 1)$
$29: P_{519} = (5, 15, 0, 1)$
$30: P_{521} = (7, 15, 0, 1)$
$31: P_{526} = (12, 15, 0, 1)$
$32: P_{531} = (1, 0, 1, 1)$
$33: P_{585} = (8, 3, 1, 1)$
$34: P_{587} = (10, 3, 1, 1)$
$35: P_{620} = (11, 5, 1, 1)$
$36: P_{624} = (15, 5, 1, 1)$
$37: P_{660} = (3, 8, 1, 1)$
$38: P_{667} = (10, 8, 1, 1)$
$39: P_{692} = (3, 10, 1, 1)$
$40: P_{697} = (8, 10, 1, 1)$
$41: P_{710} = (5, 11, 1, 1)$
$42: P_{720} = (15, 11, 1, 1)$
$43: P_{774} = (5, 15, 1, 1)$
$44: P_{780} = (11, 15, 1, 1)$
$45: P_{829} = (12, 2, 2, 1)$
$46: P_{843} = (10, 3, 2, 1)$
$47: P_{894} = (13, 6, 2, 1)$
$48: P_{919} = (6, 8, 2, 1)$
$49: P_{931} = (2, 9, 2, 1)$
$50: P_{936} = (7, 9, 2, 1)$
$51: P_{942} = (13, 9, 2, 1)$
$52: P_{971} = (10, 11, 2, 1)$
$53: P_{983} = (6, 12, 2, 1)$
$54: P_{984} = (7, 12, 2, 1)$
$55: P_{989} = (12, 12, 2, 1)$
$56: P_{994} = (1, 13, 2, 1)$
$57: P_{1010} = (1, 14, 2, 1)$
$58: P_{1068} = (11, 1, 3, 1)$
$59: P_{1096} = (7, 3, 3, 1)$
$60: P_{1099} = (10, 3, 3, 1)$
$61: P_{1104} = (15, 3, 3, 1)$
$62: P_{1160} = (7, 7, 3, 1)$
$63: P_{1196} = (11, 9, 3, 1)$
$64: P_{1204} = (3, 10, 3, 1)$
$65: P_{1211} = (10, 10, 3, 1)$
$66: P_{1296} = (15, 15, 3, 1)$
$67: P_{1330} = (1, 2, 4, 1)$
$68: P_{1367} = (6, 4, 4, 1)$
$69: P_{1388} = (11, 5, 4, 1)$
$70: P_{1399} = (6, 6, 4, 1)$
$71: P_{1405} = (12, 6, 4, 1)$
$72: P_{1406} = (13, 6, 4, 1)$
$73: P_{1410} = (1, 7, 4, 1)$
$74: P_{1468} = (11, 10, 4, 1)$
$75: P_{1512} = (7, 13, 4, 1)$
$76: P_{1525} = (4, 14, 4, 1)$
$77: P_{1528} = (7, 14, 4, 1)$
$78: P_{1533} = (12, 14, 4, 1)$
$79: P_{1550} = (13, 15, 4, 1)$
$80: P_{1579} = (10, 1, 5, 1)$
$81: P_{1604} = (3, 3, 5, 1)$
$82: P_{1636} = (3, 5, 5, 1)$
$83: P_{1644} = (11, 5, 5, 1)$
$84: P_{1645} = (12, 5, 5, 1)$
$85: P_{1734} = (5, 11, 5, 1)$
$86: P_{1740} = (11, 11, 5, 1)$
$87: P_{1757} = (12, 12, 5, 1)$
$88: P_{1787} = (10, 14, 5, 1)$
$89: P_{1839} = (14, 1, 6, 1)$

$90: P_{1869} = (12, 3, 6, 1)$
$91: P_{1899} = (10, 5, 6, 1)$
$92: P_{1918} = (13, 6, 6, 1)$
$93: P_{1936} = (15, 7, 6, 1)$
$94: P_{1939} = (2, 8, 6, 1)$
$95: P_{1963} = (10, 9, 6, 1)$
$96: P_{1965} = (12, 9, 6, 1)$
$97: P_{1967} = (14, 9, 6, 1)$
$98: P_{2003} = (2, 12, 6, 1)$
$99: P_{2030} = (13, 13, 6, 1)$
$100: P_{2039} = (6, 14, 6, 1)$
$101: P_{2048} = (15, 14, 6, 1)$
$102: P_{2085} = (4, 1, 7, 1)$
$103: P_{2101} = (4, 2, 7, 1)$
$104: P_{2107} = (10, 2, 7, 1)$
$105: P_{2110} = (13, 2, 7, 1)$
$106: P_{2122} = (9, 3, 7, 1)$
$107: P_{2134} = (5, 4, 7, 1)$
$108: P_{2136} = (7, 4, 7, 1)$
$109: P_{2166} = (5, 6, 7, 1)$
$110: P_{2189} = (12, 7, 7, 1)$
$111: P_{2206} = (13, 8, 7, 1)$
$112: P_{2269} = (12, 12, 7, 1)$
$113: P_{2282} = (9, 13, 7, 1)$
$114: P_{2315} = (10, 15, 7, 1)$
$115: P_{2348} = (11, 1, 8, 1)$
$116: P_{2364} = (11, 2, 8, 1)$
$117: P_{2406} = (5, 5, 8, 1)$
$118: P_{2423} = (6, 6, 8, 1)$
$119: P_{2454} = (5, 8, 8, 1)$
$120: P_{2455} = (6, 8, 8, 1)$
$121: P_{2459} = (10, 8, 8, 1)$
$122: P_{2489} = (8, 10, 8, 1)$
$123: P_{2491} = (10, 10, 8, 1)$
$124: P_{2615} = (6, 2, 9, 1)$
$125: P_{2618} = (9, 2, 9, 1)$
$126: P_{2621} = (12, 2, 9, 1)$
$127: P_{2632} = (7, 3, 9, 1)$
$128: P_{2642} = (1, 4, 9, 1)$
$129: P_{2701} = (12, 7, 9, 1)$
$130: P_{2715} = (10, 8, 9, 1)$
$131: P_{2734} = (13, 9, 9, 1)$
$132: P_{2763} = (10, 11, 9, 1)$
$133: P_{2770} = (1, 12, 9, 1)$
$134: P_{2791} = (6, 13, 9, 1)$
$135: P_{2792} = (7, 13, 9, 1)$
$136: P_{2798} = (13, 13, 9, 1)$
$137: P_{3370} = (9, 1, 12, 1)$
$138: P_{3404} = (11, 3, 12, 1)$
$139: P_{3416} = (7, 4, 12, 1)$
$140: P_{3418} = (9, 4, 12, 1)$
$141: P_{3420} = (11, 4, 12, 1)$
$142: P_{3439} = (14, 5, 12, 1)$
$143: P_{3447} = (6, 6, 12, 1)$
$144: P_{3471} = (14, 7, 12, 1)$
$145: P_{3497} = (8, 9, 12, 1)$
$146: P_{3501} = (12, 9, 12, 1)$
$147: P_{3543} = (6, 12, 12, 1)$
$148: P_{3561} = (8, 13, 12, 1)$
$149: P_{3592} = (7, 15, 12, 1)$
$150: P_{3619} = (2, 1, 13, 1)$
$151: P_{3636} = (3, 2, 13, 1)$
$152: P_{3646} = (13, 2, 13, 1)$
$153: P_{3687} = (6, 5, 13, 1)$
$154: P_{3701} = (4, 6, 13, 1)$
$155: P_{3720} = (7, 7, 13, 1)$
$156: P_{3740} = (11, 8, 13, 1)$
$157: P_{3796} = (3, 12, 13, 1)$
$158: P_{3816} = (7, 13, 13, 1)$
$159: P_{3827} = (2, 14, 13, 1)$
$160: P_{3831} = (6, 14, 13, 1)$
$161: P_{3836} = (11, 14, 13, 1)$
$162: P_{3845} = (4, 15, 13, 1)$
$163: P_{3927} = (6, 4, 14, 1)$
$164: P_{3934} = (13, 4, 14, 1)$
$165: P_{3935} = (14, 4, 14, 1)$
$166: P_{3949} = (12, 5, 14, 1)$
$167: P_{3954} = (1, 6, 14, 1)$
$168: P_{3976} = (7, 7, 14, 1)$
$169: P_{3981} = (12, 7, 14, 1)$
$170: P_{3982} = (13, 7, 14, 1)$
$171: P_{4002} = (1, 9, 14, 1)$
$172: P_{4028} = (11, 10, 14, 1)$
$173: P_{4055} = (6, 12, 14, 1)$
$174: P_{4088} = (7, 14, 14, 1)$
$175: P_{4108} = (11, 15, 14, 1)$
$176: P_{4139} = (10, 1, 15, 1)$
$177: P_{4187} = (10, 4, 15, 1)$
$178: P_{4249} = (8, 8, 15, 1)$
$179: P_{4300} = (11, 11, 15, 1)$
$180: P_{4304} = (15, 11, 15, 1)$
$181: P_{4334} = (13, 13, 15, 1)$
$182: P_{4361} = (8, 15, 15, 1)$
$183: P_{4364} = (11, 15, 15, 1)$
$184: P_{4366} = (13, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 0 0 0 0 1 1
1	1 0 1 1 0 1 0 0 0
2	1 1 0 0 1 0 1 0 0
3	0 1 0 0 1 1 0 1 0
4	0 0 1 1 0 0 1 1 0
5	0 1 0 1 0 0 1 0 1
6	0 0 1 0 1 1 0 0 1
7	1 0 0 1 1 0 0 0 1
8	1 0 0 0 0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8
in point	P_1	P_1	P_{689}	P_{705}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_1	P_1	P_{2833}	P_{2993}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_1	P_1	P_{3265}	P_{3089}

Line 3 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_{2833}	P_{14}	P_{2859}	P_{3004}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{3265}	P_{14}	P_{3116}	P_{3259}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8
in point	P_{2993}	P_{2859}	P_{15}	P_{3020}

Line 6 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_{3089}	P_{3116}	P_{15}	P_{3275}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_8
in point	P_{689}	P_{3004}	P_{3259}	P_{20}

Line 8 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_7
in point	P_{705}	P_{3020}	P_{3275}	P_{20}

The surface has 321 points:
The points on the surface are:

$0: P_1 = (0, 1, 0, 0)$
$1: P_{14} = (10, 1, 0, 0)$
$2: P_{15} = (11, 1, 0, 0)$
$3: P_{20} = (1, 0, 1, 0)$
$4: P_{29} = (10, 0, 1, 0)$
$5: P_{30} = (11, 0, 1, 0)$
$6: P_{128} = (13, 6, 1, 0)$
$7: P_{143} = (12, 7, 1, 0)$
$8: P_{190} = (11, 10, 1, 0)$
$9: P_{191} = (12, 10, 1, 0)$
$10: P_{192} = (13, 10, 1, 0)$
$11: P_{201} = (6, 11, 1, 0)$
$12: P_{202} = (7, 11, 1, 0)$
$13: P_{205} = (10, 11, 1, 0)$
$14: P_{217} = (6, 12, 1, 0)$
$15: P_{234} = (7, 13, 1, 0)$
$16: P_{317} = (11, 2, 0, 1)$
$17: P_{328} = (6, 3, 0, 1)$
$18: P_{330} = (8, 3, 0, 1)$
$19: P_{334} = (12, 3, 0, 1)$
$20: P_{348} = (10, 4, 0, 1)$

21 : $P_{360} = (6, 5, 0, 1)$
22 : $P_{367} = (13, 5, 0, 1)$
23 : $P_{369} = (15, 5, 0, 1)$
24 : $P_{405} = (3, 8, 0, 1)$
25 : $P_{409} = (7, 8, 0, 1)$
26 : $P_{415} = (13, 8, 0, 1)$
27 : $P_{429} = (11, 9, 0, 1)$
28 : $P_{435} = (1, 10, 0, 1)$
29 : $P_{439} = (5, 10, 0, 1)$
30 : $P_{449} = (15, 10, 0, 1)$
31 : $P_{451} = (1, 11, 0, 1)$
32 : $P_{453} = (3, 11, 0, 1)$
33 : $P_{458} = (8, 11, 0, 1)$
34 : $P_{508} = (10, 14, 0, 1)$
35 : $P_{519} = (5, 15, 0, 1)$
36 : $P_{521} = (7, 15, 0, 1)$
37 : $P_{526} = (12, 15, 0, 1)$
38 : $P_{530} = (0, 0, 1, 1)$
39 : $P_{531} = (1, 0, 1, 1)$
40 : $P_{546} = (0, 1, 1, 1)$
41 : $P_{561} = (0, 2, 1, 1)$
42 : $P_{577} = (0, 3, 1, 1)$
43 : $P_{585} = (8, 3, 1, 1)$
44 : $P_{587} = (10, 3, 1, 1)$
45 : $P_{593} = (0, 4, 1, 1)$
46 : $P_{609} = (0, 5, 1, 1)$
47 : $P_{620} = (11, 5, 1, 1)$
48 : $P_{624} = (15, 5, 1, 1)$
49 : $P_{625} = (0, 6, 1, 1)$
50 : $P_{641} = (0, 7, 1, 1)$
51 : $P_{657} = (0, 8, 1, 1)$
52 : $P_{660} = (3, 8, 1, 1)$
53 : $P_{667} = (10, 8, 1, 1)$
54 : $P_{673} = (0, 9, 1, 1)$
55 : $P_{689} = (0, 10, 1, 1)$
56 : $P_{692} = (3, 10, 1, 1)$
57 : $P_{697} = (8, 10, 1, 1)$
58 : $P_{705} = (0, 11, 1, 1)$
59 : $P_{710} = (5, 11, 1, 1)$
60 : $P_{720} = (15, 11, 1, 1)$
61 : $P_{721} = (0, 12, 1, 1)$
62 : $P_{737} = (0, 13, 1, 1)$
63 : $P_{753} = (0, 14, 1, 1)$
64 : $P_{769} = (0, 15, 1, 1)$
65 : $P_{774} = (5, 15, 1, 1)$
66 : $P_{780} = (11, 15, 1, 1)$
67 : $P_{829} = (12, 2, 2, 1)$
68 : $P_{843} = (10, 3, 2, 1)$
69 : $P_{894} = (13, 6, 2, 1)$
70 : $P_{919} = (6, 8, 2, 1)$
71 : $P_{931} = (2, 9, 2, 1)$
72 : $P_{936} = (7, 9, 2, 1)$
73 : $P_{942} = (13, 9, 2, 1)$
74 : $P_{948} = (3, 10, 2, 1)$
75 : $P_{964} = (3, 11, 2, 1)$
76 : $P_{971} = (10, 11, 2, 1)$
77 : $P_{983} = (6, 12, 2, 1)$
78 : $P_{984} = (7, 12, 2, 1)$
79 : $P_{989} = (12, 12, 2, 1)$
80 : $P_{994} = (1, 13, 2, 1)$
81 : $P_{1010} = (1, 14, 2, 1)$
82 : $P_{1068} = (11, 1, 3, 1)$
83 : $P_{1096} = (7, 3, 3, 1)$
84 : $P_{1099} = (10, 3, 3, 1)$
85 : $P_{1104} = (15, 3, 3, 1)$
86 : $P_{1160} = (7, 7, 3, 1)$
87 : $P_{1196} = (11, 9, 3, 1)$
88 : $P_{1203} = (2, 10, 3, 1)$
89 : $P_{1204} = (3, 10, 3, 1)$
90 : $P_{1211} = (10, 10, 3, 1)$
91 : $P_{1219} = (2, 11, 3, 1)$
92 : $P_{1296} = (15, 15, 3, 1)$
93 : $P_{1330} = (1, 2, 4, 1)$
94 : $P_{1367} = (6, 4, 4, 1)$
95 : $P_{1388} = (11, 5, 4, 1)$
96 : $P_{1399} = (6, 6, 4, 1)$
97 : $P_{1405} = (12, 6, 4, 1)$
98 : $P_{1406} = (13, 6, 4, 1)$
99 : $P_{1410} = (1, 7, 4, 1)$
100 : $P_{1462} = (5, 10, 4, 1)$
101 : $P_{1468} = (11, 10, 4, 1)$
102 : $P_{1478} = (5, 11, 4, 1)$
103 : $P_{1512} = (7, 13, 4, 1)$
104 : $P_{1525} = (4, 14, 4, 1)$
105 : $P_{1528} = (7, 14, 4, 1)$
106 : $P_{1533} = (12, 14, 4, 1)$
107 : $P_{1550} = (13, 15, 4, 1)$
108 : $P_{1579} = (10, 1, 5, 1)$
109 : $P_{1604} = (3, 3, 5, 1)$
110 : $P_{1636} = (3, 5, 5, 1)$
111 : $P_{1644} = (11, 5, 5, 1)$
112 : $P_{1645} = (12, 5, 5, 1)$
113 : $P_{1717} = (4, 10, 5, 1)$
114 : $P_{1733} = (4, 11, 5, 1)$
115 : $P_{1734} = (5, 11, 5, 1)$
116 : $P_{1740} = (11, 11, 5, 1)$
117 : $P_{1757} = (12, 12, 5, 1)$
118 : $P_{1787} = (10, 14, 5, 1)$
119 : $P_{1839} = (14, 1, 6, 1)$
120 : $P_{1869} = (12, 3, 6, 1)$
121 : $P_{1899} = (10, 5, 6, 1)$
122 : $P_{1918} = (13, 6, 6, 1)$
123 : $P_{1936} = (15, 7, 6, 1)$
124 : $P_{1939} = (2, 8, 6, 1)$
125 : $P_{1963} = (10, 9, 6, 1)$
126 : $P_{1965} = (12, 9, 6, 1)$
127 : $P_{1967} = (14, 9, 6, 1)$
128 : $P_{1976} = (7, 10, 6, 1)$
129 : $P_{1992} = (7, 11, 6, 1)$
130 : $P_{2003} = (2, 12, 6, 1)$
131 : $P_{2030} = (13, 13, 6, 1)$
132 : $P_{2039} = (6, 14, 6, 1)$
133 : $P_{2048} = (15, 14, 6, 1)$
134 : $P_{2085} = (4, 1, 7, 1)$
135 : $P_{2101} = (4, 2, 7, 1)$
136 : $P_{2107} = (10, 2, 7, 1)$
137 : $P_{2110} = (13, 2, 7, 1)$
138 : $P_{2122} = (9, 3, 7, 1)$
139 : $P_{2134} = (5, 4, 7, 1)$
140 : $P_{2136} = (7, 4, 7, 1)$
141 : $P_{2166} = (5, 6, 7, 1)$
142 : $P_{2189} = (12, 7, 7, 1)$
143 : $P_{2206} = (13, 8, 7, 1)$
144 : $P_{2231} = (6, 10, 7, 1)$
145 : $P_{2247} = (6, 11, 7, 1)$
146 : $P_{2269} = (12, 12, 7, 1)$
147 : $P_{2282} = (9, 13, 7, 1)$
148 : $P_{2315} = (10, 15, 7, 1)$
149 : $P_{2348} = (11, 1, 8, 1)$
150 : $P_{2364} = (11, 2, 8, 1)$
151 : $P_{2406} = (5, 5, 8, 1)$
152 : $P_{2423} = (6, 6, 8, 1)$
153 : $P_{2454} = (5, 8, 8, 1)$
154 : $P_{2455} = (6, 8, 8, 1)$
155 : $P_{2459} = (10, 8, 8, 1)$
156 : $P_{2489} = (8, 10, 8, 1)$
157 : $P_{2490} = (9, 10, 8, 1)$
158 : $P_{2491} = (10, 10, 8, 1)$
159 : $P_{2506} = (9, 11, 8, 1)$
160 : $P_{2615} = (6, 2, 9, 1)$
161 : $P_{2618} = (9, 2, 9, 1)$
162 : $P_{2621} = (12, 2, 9, 1)$
163 : $P_{2632} = (7, 3, 9, 1)$
164 : $P_{2642} = (1, 4, 9, 1)$
165 : $P_{2701} = (12, 7, 9, 1)$
166 : $P_{2715} = (10, 8, 9, 1)$
167 : $P_{2734} = (13, 9, 9, 1)$
168 : $P_{2745} = (8, 10, 9, 1)$
169 : $P_{2761} = (8, 11, 9, 1)$
170 : $P_{2763} = (10, 11, 9, 1)$
171 : $P_{2770} = (1, 12, 9, 1)$
172 : $P_{2791} = (6, 13, 9, 1)$
173 : $P_{2792} = (7, 13, 9, 1)$
174 : $P_{2798} = (13, 13, 9, 1)$
175 : $P_{2833} = (0, 0, 10, 1)$
176 : $P_{2834} = (1, 0, 10, 1)$
177 : $P_{2849} = (0, 1, 10, 1)$
178 : $P_{2859} = (10, 1, 10, 1)$
179 : $P_{2865} = (0, 2, 10, 1)$
180 : $P_{2878} = (13, 2, 10, 1)$
181 : $P_{2879} = (14, 2, 10, 1)$
182 : $P_{2881} = (0, 3, 10, 1)$

183 : $P_{2886} = (5, 3, 10, 1)$
184 : $P_{2888} = (7, 3, 10, 1)$
185 : $P_{2897} = (0, 4, 10, 1)$
186 : $P_{2900} = (3, 4, 10, 1)$
187 : $P_{2903} = (6, 4, 10, 1)$
188 : $P_{2913} = (0, 5, 10, 1)$
189 : $P_{2922} = (9, 5, 10, 1)$
190 : $P_{2926} = (13, 5, 10, 1)$
191 : $P_{2929} = (0, 6, 10, 1)$
192 : $P_{2938} = (9, 6, 10, 1)$
193 : $P_{2943} = (14, 6, 10, 1)$
194 : $P_{2945} = (0, 7, 10, 1)$
195 : $P_{2947} = (2, 7, 10, 1)$
196 : $P_{2949} = (4, 7, 10, 1)$
197 : $P_{2961} = (0, 8, 10, 1)$
198 : $P_{2967} = (6, 8, 10, 1)$
199 : $P_{2976} = (15, 8, 10, 1)$
200 : $P_{2977} = (0, 9, 10, 1)$
201 : $P_{2981} = (4, 9, 10, 1)$
202 : $P_{2989} = (12, 9, 10, 1)$
203 : $P_{2993} = (0, 10, 10, 1)$
204 : $P_{3004} = (11, 10, 10, 1)$
205 : $P_{3009} = (0, 11, 10, 1)$
206 : $P_{3010} = (1, 11, 10, 1)$
207 : $P_{3020} = (11, 11, 10, 1)$
208 : $P_{3025} = (0, 12, 10, 1)$
209 : $P_{3030} = (5, 12, 10, 1)$
210 : $P_{3033} = (8, 12, 10, 1)$
211 : $P_{3041} = (0, 13, 10, 1)$
212 : $P_{3044} = (3, 13, 10, 1)$
213 : $P_{3056} = (15, 13, 10, 1)$
214 : $P_{3057} = (0, 14, 10, 1)$
215 : $P_{3064} = (7, 14, 10, 1)$
216 : $P_{3065} = (8, 14, 10, 1)$
217 : $P_{3073} = (0, 15, 10, 1)$
218 : $P_{3075} = (2, 15, 10, 1)$
219 : $P_{3085} = (12, 15, 10, 1)$
220 : $P_{3089} = (0, 0, 11, 1)$
221 : $P_{3090} = (1, 0, 11, 1)$
222 : $P_{3105} = (0, 1, 11, 1)$
223 : $P_{3116} = (11, 1, 11, 1)$
224 : $P_{3121} = (0, 2, 11, 1)$
225 : $P_{3133} = (12, 2, 11, 1)$
226 : $P_{3136} = (15, 2, 11, 1)$
227 : $P_{3137} = (0, 3, 11, 1)$
228 : $P_{3141} = (4, 3, 11, 1)$
229 : $P_{3143} = (6, 3, 11, 1)$
230 : $P_{3153} = (0, 4, 11, 1)$
231 : $P_{3155} = (2, 4, 11, 1)$
232 : $P_{3160} = (7, 4, 11, 1)$
233 : $P_{3169} = (0, 5, 11, 1)$
234 : $P_{3177} = (8, 5, 11, 1)$
235 : $P_{3181} = (12, 5, 11, 1)$
236 : $P_{3185} = (0, 6, 11, 1)$
237 : $P_{3193} = (8, 6, 11, 1)$
238 : $P_{3200} = (15, 6, 11, 1)$
239 : $P_{3201} = (0, 7, 11, 1)$
240 : $P_{3204} = (3, 7, 11, 1)$
241 : $P_{3206} = (5, 7, 11, 1)$
242 : $P_{3217} = (0, 8, 11, 1)$
243 : $P_{3224} = (7, 8, 11, 1)$
244 : $P_{3231} = (14, 8, 11, 1)$
245 : $P_{3233} = (0, 9, 11, 1)$
246 : $P_{3238} = (5, 9, 11, 1)$
247 : $P_{3246} = (13, 9, 11, 1)$
248 : $P_{3249} = (0, 10, 11, 1)$
249 : $P_{3250} = (1, 10, 11, 1)$
250 : $P_{3259} = (10, 10, 11, 1)$
251 : $P_{3265} = (0, 11, 11, 1)$
252 : $P_{3275} = (10, 11, 11, 1)$
253 : $P_{3281} = (0, 12, 11, 1)$
254 : $P_{3285} = (4, 12, 11, 1)$
255 : $P_{3290} = (9, 12, 11, 1)$
256 : $P_{3297} = (0, 13, 11, 1)$
257 : $P_{3299} = (2, 13, 11, 1)$
258 : $P_{3311} = (14, 13, 11, 1)$
259 : $P_{3313} = (0, 14, 11, 1)$
260 : $P_{3319} = (6, 14, 11, 1)$
261 : $P_{3322} = (9, 14, 11, 1)$
262 : $P_{3329} = (0, 15, 11, 1)$
263 : $P_{3332} = (3, 15, 11, 1)$
264 : $P_{3342} = (13, 15, 11, 1)$
265 : $P_{3370} = (9, 1, 12, 1)$
266 : $P_{3404} = (11, 3, 12, 1)$
267 : $P_{3416} = (7, 4, 12, 1)$
268 : $P_{3418} = (9, 4, 12, 1)$
269 : $P_{3420} = (11, 4, 12, 1)$
270 : $P_{3439} = (14, 5, 12, 1)$
271 : $P_{3447} = (6, 6, 12, 1)$
272 : $P_{3471} = (14, 7, 12, 1)$
273 : $P_{3497} = (8, 9, 12, 1)$
274 : $P_{3501} = (12, 9, 12, 1)$
275 : $P_{3518} = (13, 10, 12, 1)$
276 : $P_{3534} = (13, 11, 12, 1)$
277 : $P_{3543} = (6, 12, 12, 1)$
278 : $P_{3561} = (8, 13, 12, 1)$
279 : $P_{3592} = (7, 15, 12, 1)$
280 : $P_{3619} = (2, 1, 13, 1)$
281 : $P_{3636} = (3, 2, 13, 1)$
282 : $P_{3646} = (13, 2, 13, 1)$
283 : $P_{3687} = (6, 5, 13, 1)$
284 : $P_{3701} = (4, 6, 13, 1)$
285 : $P_{3720} = (7, 7, 13, 1)$
286 : $P_{3740} = (11, 8, 13, 1)$
287 : $P_{3773} = (12, 10, 13, 1)$
288 : $P_{3789} = (12, 11, 13, 1)$
289 : $P_{3796} = (3, 12, 13, 1)$
290 : $P_{3816} = (7, 13, 13, 1)$
291 : $P_{3827} = (2, 14, 13, 1)$
292 : $P_{3831} = (6, 14, 13, 1)$
293 : $P_{3836} = (11, 14, 13, 1)$
294 : $P_{3845} = (4, 15, 13, 1)$
295 : $P_{3927} = (6, 4, 14, 1)$
296 : $P_{3934} = (13, 4, 14, 1)$
297 : $P_{3935} = (14, 4, 14, 1)$
298 : $P_{3949} = (12, 5, 14, 1)$
299 : $P_{3954} = (1, 6, 14, 1)$
300 : $P_{3976} = (7, 7, 14, 1)$
301 : $P_{3981} = (12, 7, 14, 1)$
302 : $P_{3982} = (13, 7, 14, 1)$
303 : $P_{4002} = (1, 9, 14, 1)$
304 : $P_{4028} = (11, 10, 14, 1)$
305 : $P_{4032} = (15, 10, 14, 1)$
306 : $P_{4048} = (15, 11, 14, 1)$
307 : $P_{4055} = (6, 12, 14, 1)$
308 : $P_{4088} = (7, 14, 14, 1)$
309 : $P_{4108} = (11, 15, 14, 1)$
310 : $P_{4139} = (10, 1, 15, 1)$
311 : $P_{4187} = (10, 4, 15, 1)$
312 : $P_{4249} = (8, 8, 15, 1)$
313 : $P_{4287} = (14, 10, 15, 1)$
314 : $P_{4300} = (11, 11, 15, 1)$
315 : $P_{4303} = (14, 11, 15, 1)$
316 : $P_{4304} = (15, 11, 15, 1)$
317 : $P_{4334} = (13, 13, 15, 1)$
318 : $P_{4361} = (8, 15, 15, 1)$
319 : $P_{4364} = (11, 15, 15, 1)$
320 : $P_{4366} = (13, 15, 15, 1)$