Rank-74263 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1107566661

General information

Number of lines	3
Number of points	4033
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	3841
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^3, 1^{189}, 0^{3841}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & \epsilon^{54} & \epsilon^{18} \\ 0 & 1 & \epsilon^{36} & \epsilon^{45} \end{array}\right]_{2973358} = \left[\begin{array}{cccc} 1 & 0 & 10 & 11 \\ 0 & 1 & 36 & 37 \end{array}\right]_{2973358} = \mathbf{Pl}(46, 36, 36, 46, 10, 1)_{3048621}$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & \epsilon^{45} & \epsilon^{36} \\ 0 & 1 & \epsilon^{9} & \epsilon^{27} \end{array}\right]_{9743892} = \left[\begin{array}{cccc} 1 & 0 & 37 & 36 \\ 0 & 1 & 47 & 46 \end{array}\right]_{9743892} = \mathbf{Pl}(10, 47, 47, 10, 37, 1)_{10169286}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \epsilon^{27} & \epsilon^{9} \\ 0 & 1 & \epsilon^{18} & \epsilon^{54} \end{bmatrix}_{12708345} = \begin{bmatrix} 1 & 0 & 46 & 47 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{12708345} = \mathbf{Pl}(37,11,11,37,46,1)_{12388110}$$

Rank of lines: (2973358, 9743892, 12708345)
Rank of points on Klein quadric: (3048621, 10169286, 12388110)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{194955} = (10,37,46,1) = \ell_0 \cap \ell_1$
$P_{156399} = (46,10,37,1) = \ell_0 \cap \ell_2$
$P_{48102} = (37,46,10,1) = \ell_1 \cap \ell_2$

Single Points

The surface has 189 single points:
The single points on the surface are:

0 : $P_{718} = (11,10,1,0)$ lies on line ℓ_0
1 : $P_{2471} = (36,37,1,0)$ lies on line ℓ_1
2 : $P_{3058} = (47,46,1,0)$ lies on line ℓ_2
3 : $P_{4813} = (11,10,0,1)$ lies on line ℓ_1
4 : $P_{6566} = (36,37,0,1)$ lies on line ℓ_2
5 : $P_{7153} = (47,46,0,1)$ lies on line ℓ_0
6 : $P_{8972} = (11,11,1,1)$ lies on line ℓ_2
7 : $P_{10597} = (36,36,1,1)$ lies on line ℓ_0
8 : $P_{11312} = (47,47,1,1)$ lies on line ℓ_1
9 : $P_{13916} = (27,24,2,1)$ lies on line ℓ_2
10 : $P_{14499} = (34,33,2,1)$ lies on line ℓ_1
11 : $P_{16122} = (57,58,2,1)$ lies on line ℓ_0
12 : $P_{16711} = (6,4,3,1)$ lies on line ℓ_1
13 : $P_{19571} = (50,48,3,1)$ lies on line ℓ_0
14 : $P_{19957} = (52,54,3,1)$ lies on line ℓ_2
15 : $P_{20932} = (3,6,4,1)$ lies on line ℓ_0
16 : $P_{24505} = (56,61,4,1)$ lies on line ℓ_1
17 : $P_{24572} = (59,62,4,1)$ lies on line ℓ_2
18 : $P_{25417} = (8,12,5,1)$ lies on line ℓ_0
19 : $P_{25685} = (20,16,5,1)$ lies on line ℓ_2
20 : $P_{26205} = (28,24,5,1)$ lies on line ℓ_1
21 : $P_{28933} = (4,3,6,1)$ lies on line ℓ_2
22 : $P_{29910} = (21,18,6,1)$ lies on line ℓ_0
23 : $P_{30162} = (17,22,6,1)$ lies on line ℓ_1
24 : $P_{34399} = (30,24,7,1)$ lies on line ℓ_0
25 : $P_{35756} = (43,45,7,1)$ lies on line ℓ_2
26 : $P_{36150} = (53,51,7,1)$ lies on line ℓ_1
27 : $P_{37261} = (12,5,8,1)$ lies on line ℓ_1
28 : $P_{38171} = (26,19,8,1)$ lies on line ℓ_2
29 : $P_{38935} = (22,31,8,1)$ lies on line ℓ_0
30 : $P_{42398} = (29,21,9,1)$ lies on line ℓ_0
31 : $P_{43113} = (40,32,9,1)$ lies on line ℓ_1
32 : $P_{44982} = (53,61,9,1)$ lies on line ℓ_2
33 : $P_{45825} = (0,11,10,1)$ lies on line ℓ_0
34 : $P_{49227} = (10,0,11,1)$ lies on line ℓ_2
35 : $P_{49292} = (11,1,11,1)$ lies on line ℓ_0
36 : $P_{49922} = (1,11,11,1)$ lies on line ℓ_1
37 : $P_{53830} = (5,8,12,1)$ lies on line ℓ_2
38 : $P_{56576} = (63,50,12,1)$ lies on line ℓ_1
39 : $P_{56891} = (58,55,12,1)$ lies on line ℓ_0
40 : $P_{58908} = (27,23,13,1)$ lies on line ℓ_1
41 : $P_{59883} = (42,38,13,1)$ lies on line ℓ_2
42 : $P_{61362} = (49,61,13,1)$ lies on line ℓ_0
43 : $P_{63127} = (22,25,14,1)$ lies on line ℓ_1
44 : $P_{63789} = (44,35,14,1)$ lies on line ℓ_0
45 : $P_{64955} = (58,53,14,1)$ lies on line ℓ_2
46 : $P_{67350} = (21,27,15,1)$ lies on line ℓ_2
47 : $P_{68264} = (39,41,15,1)$ lies on line ℓ_0
48 : $P_{69491} = (50,60,15,1)$ lies on line ℓ_1
49 : $P_{70982} = (5,20,16,1)$ lies on line ℓ_1
50 : $P_{72314} = (57,40,16,1)$ lies on line ℓ_2
51 : $P_{72637} = (60,45,16,1)$ lies on line ℓ_0
52 : $P_{74199} = (22,6,17,1)$ lies on line ℓ_2
53 : $P_{76344} = (55,39,17,1)$ lies on line ℓ_0
54 : $P_{76962} = (33,49,17,1)$ lies on line ℓ_1
55 : $P_{79239} = (6,21,18,1)$ lies on line ℓ_2

56 : $P_{81579} = (42, 57, 18, 1)$ lies on line ℓ_0
57 : $P_{81965} = (44, 63, 18, 1)$ lies on line ℓ_1
58 : $P_{83657} = (8, 26, 19, 1)$ lies on line ℓ_1
59 : $P_{85282} = (33, 51, 19, 1)$ lies on line ℓ_0
60 : $P_{85802} = (41, 59, 19, 1)$ lies on line ℓ_2
61 : $P_{86417} = (16, 5, 20, 1)$ lies on line ℓ_0
62 : $P_{88375} = (54, 35, 20, 1)$ lies on line ℓ_1
63 : $P_{89383} = (38, 51, 20, 1)$ lies on line ℓ_2
64 : $P_{90579} = (18, 6, 21, 1)$ lies on line ℓ_1
65 : $P_{91164} = (27, 15, 21, 1)$ lies on line ℓ_0
66 : $P_{92042} = (9, 29, 21, 1)$ lies on line ℓ_2
67 : $P_{94816} = (31, 8, 22, 1)$ lies on line ℓ_1
68 : $P_{95194} = (25, 14, 22, 1)$ lies on line ℓ_2
69 : $P_{95367} = (6, 17, 22, 1)$ lies on line ℓ_0
70 : $P_{100110} = (13, 27, 23, 1)$ lies on line ℓ_0
71 : $P_{100471} = (54, 32, 23, 1)$ lies on line ℓ_2
72 : $P_{101308} = (59, 45, 23, 1)$ lies on line ℓ_1
73 : $P_{104195} = (2, 27, 24, 1)$ lies on line ℓ_1
74 : $P_{104262} = (5, 28, 24, 1)$ lies on line ℓ_0
75 : $P_{104392} = (7, 30, 24, 1)$ lies on line ℓ_2
76 : $P_{107983} = (14, 22, 25, 1)$ lies on line ℓ_0
77 : $P_{109673} = (40, 48, 25, 1)$ lies on line ℓ_2
78 : $P_{110567} = (38, 62, 25, 1)$ lies on line ℓ_1
79 : $P_{111188} = (19, 8, 26, 1)$ lies on line ℓ_0
80 : $P_{112953} = (56, 35, 26, 1)$ lies on line ℓ_2
81 : $P_{113772} = (43, 48, 26, 1)$ lies on line ℓ_1
82 : $P_{114905} = (24, 2, 27, 1)$ lies on line ℓ_0
83 : $P_{115608} = (23, 13, 27, 1)$ lies on line ℓ_2
84 : $P_{116112} = (15, 21, 27, 1)$ lies on line ℓ_1
85 : $P_{119193} = (24, 5, 28, 1)$ lies on line ℓ_2
86 : $P_{121714} = (49, 44, 28, 1)$ lies on line ℓ_1
87 : $P_{122218} = (41, 52, 28, 1)$ lies on line ℓ_0
88 : $P_{123542} = (21, 9, 29, 1)$ lies on line ℓ_1
89 : $P_{125752} = (55, 43, 29, 1)$ lies on line ℓ_2
90 : $P_{126947} = (34, 62, 29, 1)$ lies on line ℓ_0
91 : $P_{127513} = (24, 7, 30, 1)$ lies on line ℓ_1
92 : $P_{129152} = (63, 32, 30, 1)$ lies on line ℓ_0
93 : $P_{130664} = (39, 56, 30, 1)$ lies on line ℓ_2
94 : $P_{132553} = (8, 22, 31, 1)$ lies on line ℓ_2
95 : $P_{133373} = (60, 34, 31, 1)$ lies on line ℓ_1
96 : $P_{133877} = (52, 42, 31, 1)$ lies on line ℓ_0
97 : $P_{137802} = (9, 40, 32, 1)$ lies on line ℓ_0
98 : $P_{138712} = (23, 54, 32, 1)$ lies on line ℓ_1
99 : $P_{139295} = (30, 63, 32, 1)$ lies on line ℓ_2
100 : $P_{140466} = (49, 17, 33, 1)$ lies on line ℓ_2
101 : $P_{140596} = (51, 19, 33, 1)$ lies on line ℓ_1
102 : $P_{141507} = (2, 34, 33, 1)$ lies on line ℓ_0
103 : $P_{143586} = (33, 2, 34, 1)$ lies on line ℓ_2
104 : $P_{145343} = (62, 29, 34, 1)$ lies on line ℓ_1
105 : $P_{147296} = (31, 60, 34, 1)$ lies on line ℓ_0
106 : $P_{150351} = (14, 44, 35, 1)$ lies on line ℓ_2
107 : $P_{150997} = (20, 54, 35, 1)$ lies on line ℓ_0
108 : $P_{151131} = (26, 56, 35, 1)$ lies on line ℓ_1
109 : $P_{151654} = (37, 0, 36, 1)$ lies on line ℓ_0
110 : $P_{151717} = (36, 1, 36, 1)$ lies on line ℓ_1
111 : $P_{153922} = (1, 36, 36, 1)$ lies on line ℓ_2
112 : $P_{158017} = (0, 36, 37, 1)$ lies on line ℓ_1
113 : $P_{161140} = (51, 20, 38, 1)$ lies on line ℓ_0
114 : $P_{161471} = (62, 25, 38, 1)$ lies on line ℓ_2
115 : $P_{162510} = (13, 42, 38, 1)$ lies on line ℓ_1
116 : $P_{164906} = (41, 15, 39, 1)$ lies on line ℓ_1
117 : $P_{165881} = (56, 30, 39, 1)$ lies on line ℓ_0
118 : $P_{167442} = (17, 55, 39, 1)$ lies on line ℓ_2
119 : $P_{168609} = (32, 9, 40, 1)$ lies on line ℓ_2
120 : $P_{169649} = (48, 25, 40, 1)$ lies on line ℓ_0
121 : $P_{171665} = (16, 57, 40, 1)$ lies on line ℓ_1
122 : $P_{173372} = (59, 19, 41, 1)$ lies on line ℓ_0
123 : $P_{173941} = (52, 28, 41, 1)$ lies on line ℓ_1
124 : $P_{174608} = (15, 39, 41, 1)$ lies on line ℓ_2
125 : $P_{177063} = (38, 13, 42, 1)$ lies on line ℓ_0
126 : $P_{177402} = (57, 18, 42, 1)$ lies on line ℓ_1
127 : $P_{179552} = (31, 52, 42, 1)$ lies on line ℓ_2
128 : $P_{180782} = (45, 7, 43, 1)$ lies on line ℓ_0
129 : $P_{182001} = (48, 26, 43, 1)$ lies on line ℓ_2
130 : $P_{183838} = (29, 55, 43, 1)$ lies on line ℓ_1
131 : $P_{185316} = (35, 14, 44, 1)$ lies on line ℓ_1
132 : $P_{185600} = (63, 18, 44, 1)$ lies on line ℓ_2
133 : $P_{187549} = (28, 49, 44, 1)$ lies on line ℓ_0
134 : $P_{191240} = (7, 43, 45, 1)$ lies on line ℓ_1
135 : $P_{192280} = (23, 59, 45, 1)$ lies on line ℓ_0
136 : $P_{192337} = (16, 60, 45, 1)$ lies on line ℓ_2
137 : $P_{195585} = (0, 47, 46, 1)$ lies on line ℓ_2
138 : $P_{196719} = (46, 0, 47, 1)$ lies on line ℓ_1
139 : $P_{196784} = (47, 1, 47, 1)$ lies on line ℓ_2
140 : $P_{199682} = (1, 47, 47, 1)$ lies on line ℓ_0
141 : $P_{203354} = (25, 40, 48, 1)$ lies on line ℓ_1
142 : $P_{203547} = (26, 43, 48, 1)$ lies on line ℓ_0
143 : $P_{203972} = (3, 50, 48, 1)$ lies on line ℓ_2
144 : $P_{205758} = (61, 13, 49, 1)$ lies on line ℓ_1
145 : $P_{206701} = (44, 28, 49, 1)$ lies on line ℓ_2
146 : $P_{206994} = (17, 33, 49, 1)$ lies on line ℓ_0
147 : $P_{209201} = (48, 3, 50, 1)$ lies on line ℓ_1
148 : $P_{209981} = (60, 15, 50, 1)$ lies on line ℓ_2
149 : $P_{213005} = (12, 63, 50, 1)$ lies on line ℓ_0
150 : $P_{215188} = (19, 33, 51, 1)$ lies on line ℓ_2
151 : $P_{215509} = (20, 38, 51, 1)$ lies on line ℓ_1
152 : $P_{216456} = (7, 53, 51, 1)$ lies on line ℓ_0
153 : $P_{217399} = (54, 3, 52, 1)$ lies on line ℓ_0
154 : $P_{219179} = (42, 31, 52, 1)$ lies on line ℓ_1
155 : $P_{219805} = (28, 41, 52, 1)$ lies on line ℓ_2
156 : $P_{221748} = (51, 7, 53, 1)$ lies on line ℓ_2
157 : $P_{221886} = (61, 9, 53, 1)$ lies on line ℓ_0
158 : $P_{224975} = (14, 58, 53, 1)$ lies on line ℓ_1
159 : $P_{226660} = (35, 20, 54, 1)$ lies on line ℓ_2
160 : $P_{226849} = (32, 23, 54, 1)$ lies on line ℓ_0
161 : $P_{228676} = (3, 52, 54, 1)$ lies on line ℓ_1
162 : $P_{230568} = (39, 17, 55, 1)$ lies on line ℓ_1
163 : $P_{231340} = (43, 29, 55, 1)$ lies on line ℓ_0

164 : $P_{233165} = (12, 58, 55, 1)$ lies on line ℓ_2
165 : $P_{233854} = (61, 4, 56, 1)$ lies on line ℓ_2
166 : $P_{235236} = (35, 26, 56, 1)$ lies on line ℓ_0
167 : $P_{236063} = (30, 39, 56, 1)$ lies on line ℓ_1
168 : $P_{237819} = (58, 2, 57, 1)$ lies on line ℓ_1
169 : $P_{238697} = (40, 16, 57, 1)$ lies on line ℓ_0
170 : $P_{240339} = (18, 42, 57, 1)$ lies on line ℓ_2
171 : $P_{242552} = (55, 12, 58, 1)$ lies on line ℓ_1
172 : $P_{242678} = (53, 14, 58, 1)$ lies on line ℓ_0
173 : $P_{245379} = (2, 57, 58, 1)$ lies on line ℓ_2
174 : $P_{246143} = (62, 4, 59, 1)$ lies on line ℓ_0
175 : $P_{247342} = (45, 23, 59, 1)$ lies on line ℓ_2
176 : $P_{248468} = (19, 41, 59, 1)$ lies on line ℓ_1
177 : $P_{250990} = (45, 16, 60, 1)$ lies on line ℓ_1
178 : $P_{251939} = (34, 31, 60, 1)$ lies on line ℓ_2
179 : $P_{253136} = (15, 50, 60, 1)$ lies on line ℓ_0
180 : $P_{257166} = (13, 49, 61, 1)$ lies on line ℓ_2
181 : $P_{257418} = (9, 53, 61, 1)$ lies on line ℓ_1
182 : $P_{257605} = (4, 56, 61, 1)$ lies on line ℓ_0
183 : $P_{260318} = (29, 34, 62, 1)$ lies on line ℓ_2
184 : $P_{260570} = (25, 38, 62, 1)$ lies on line ℓ_0
185 : $P_{261893} = (4, 59, 62, 1)$ lies on line ℓ_1
186 : $P_{263027} = (50, 12, 63, 1)$ lies on line ℓ_2
187 : $P_{264161} = (32, 30, 63, 1)$ lies on line ℓ_1
188 : $P_{265043} = (18, 44, 63, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 3841 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{194955}	P_{156399}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{194955}	P_{48102}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{156399}	P_{48102}

The surface has 4033 points:
Too many to print.