# Rank-76099 over GF(4)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(4) is 1503028569

## General information

| | |
|---|---|
| Number of lines | 9 |
| Number of points | 33 |
| Number of singular points | 0 |
| Number of Eckardt points | 4 |
| Number of double points | 6 |
| Number of single points | 21 |
| Number of points off lines | 2 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^9$ |
| Type of lines on points | $3^4, 2^6, 1^{21}, 0^2$ |

### Singular Points

The surface has 0 singular points:

## The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

1

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_{9}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_{6}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

Rank of lines: ( 0, 16, 22, 340, 84, 356, 100, 345, 106 )
Rank of points on Klein quadric: ( 0, 2, 109, 9, 10, 1, 6, 13, 150 )

## Eckardt Points

The surface has 4 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0),$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0),$
$2 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1),$
$3 : P_{11} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0).$

## Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_2$
$P_{23} = (1,0,0,1) = \ell_4 \cap \ell_6$

$P_{27} = (1,1,0,1) = \ell_4 \cap \ell_8$
$P_{39} = (1,0,1,1) = \ell_6 \cap \ell_8$

## Single Points

The surface has 21 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line $\ell_0$
$1 : P_7 = (3,1,0,0)$ lies on line $\ell_0$
$2 : P_9 = (2,0,1,0)$ lies on line $\ell_1$
$3 : P_{10} = (3,0,1,0)$ lies on line $\ell_1$
$4 : P_{18} = (3,2,1,0)$ lies on line $\ell_2$
$5 : P_{21} = (2,3,1,0)$ lies on line $\ell_2$
$6 : P_{26} = (0,1,0,1)$ lies on line $\ell_3$

$7 : P_{30} = (0,2,0,1)$ lies on line $\ell_3$
$8 : P_{31} = (1,2,0,1)$ lies on line $\ell_4$
$9 : P_{34} = (0,3,0,1)$ lies on line $\ell_3$
$10 : P_{35} = (1,3,0,1)$ lies on line $\ell_4$
$11 : P_{38} = (0,0,1,1)$ lies on line $\ell_5$
$12 : P_{42} = (0,1,1,1)$ lies on line $\ell_7$
$13 : P_{53} = (0,0,2,1)$ lies on line $\ell_5$

14 : $P_{54} = (1,0,2,1)$ lies on line $\ell_6$  
15 : $P_{61} = (0,2,2,1)$ lies on line $\ell_7$  
16 : $P_{66} = (1,3,2,1)$ lies on line $\ell_8$  
17 : $P_{69} = (0,0,3,1)$ lies on line $\ell_5$  

18 : $P_{70} = (1,0,3,1)$ lies on line $\ell_6$  
19 : $P_{78} = (1,2,3,1)$ lies on line $\ell_8$  
20 : $P_{81} = (0,3,3,1)$ lies on line $\ell_7$  

The single points on the surface are:

## Points on surface but on no line

The surface has 2 points not on any line:  
The points on the surface but not on lines are:

0 : $P_{63} = (2,2,2,1)$  
1 : $P_{84} = (3,3,3,1)$  

## Line Intersection Graph

```
 |0 1 2 3 4 5 6 7 8
0|0 1 1 1 1 0 0 0 0
1|1 0 1 0 0 1 1 0 0
2|1 1 0 0 0 0 0 1 1
3|1 0 0 0 1 1 0 1 0
4|1 0 0 1 0 0 1 0 1
5|0 1 0 1 0 0 1 1 0
6|0 1 0 0 1 1 0 0 1
7|0 0 1 1 0 1 0 0 1
8|0 0 1 0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:  
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|---|
| in point | $P_0$ | $P_5$ | $P_1$ | $P_1$ |

Line 1 intersects

| Line | $\ell_0$ | $\ell_2$ | $\ell_5$ | $\ell_6$ |
|---|---|---|---|---|
| in point | $P_0$ | $P_8$ | $P_2$ | $P_2$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_7$ | $\ell_8$ |
|---|---|---|---|---|
| in point | $P_5$ | $P_8$ | $P_{11}$ | $P_{11}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_4$ | $\ell_5$ | $\ell_7$ |
|---|---|---|---|---|
| in point | $P_1$ | $P_1$ | $P_3$ | $P_3$ |

Line 4 intersects

| Line | $\ell_0$ | $\ell_3$ | $\ell_6$ | $\ell_8$ |
|---|---|---|---|---|
| in point | $P_1$ | $P_1$ | $P_{23}$ | $P_{27}$ |

Line 5 intersects

| Line | $\ell_1$ | $\ell_3$ | $\ell_6$ | $\ell_7$ |
|---|---|---|---|---|
| in point | $P_2$ | $P_3$ | $P_2$ | $P_3$ |

Line 6 intersects

| Line | $\ell_1$ | $\ell_4$ | $\ell_5$ | $\ell_8$ |
|---|---|---|---|---|
| in point | $P_2$ | $P_{23}$ | $P_2$ | $P_{39}$ |

Line 7 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_5$ | $\ell_8$ |
|---|---|---|---|---|
| in point | $P_{11}$ | $P_3$ | $P_3$ | $P_{11}$ |

Line 8 intersects

| Line | $\ell_2$ | $\ell_4$ | $\ell_6$ | $\ell_7$ |
|---|---|---|---|---|
| in point | $P_{11}$ | $P_{27}$ | $P_{39}$ | $P_{11}$ |

The surface has **33** points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$

$1 : P_1 = (0, 1, 0, 0)$

$2 : P_2 = (0, 0, 1, 0)$

$3 : P_3 = (0, 0, 0, 1)$

$4 : P_5 = (1, 1, 0, 0)$

$5 : P_6 = (2, 1, 0, 0)$

$6 : P_7 = (3, 1, 0, 0)$

$7 : P_8 = (1, 0, 1, 0)$

$8 : P_9 = (2, 0, 1, 0)$

$9 : P_{10} = (3, 0, 1, 0)$

$10 : P_{11} = (0, 1, 1, 0)$

$11 : P_{18} = (3, 2, 1, 0)$

$12 : P_{21} = (2, 3, 1, 0)$

$13 : P_{23} = (1, 0, 0, 1)$

$14 : P_{26} = (0, 1, 0, 1)$

$15 : P_{27} = (1, 1, 0, 1)$

$16 : P_{30} = (0, 2, 0, 1)$

$17 : P_{31} = (1, 2, 0, 1)$

$18 : P_{34} = (0, 3, 0, 1)$

$19 : P_{35} = (1, 3, 0, 1)$

$20 : P_{38} = (0, 0, 1, 1)$

$21 : P_{39} = (1, 0, 1, 1)$

$22 : P_{42} = (0, 1, 1, 1)$

$23 : P_{53} = (0, 0, 2, 1)$

$24 : P_{54} = (1, 0, 2, 1)$

$25 : P_{61} = (0, 2, 2, 1)$

$26 : P_{63} = (2, 2, 2, 1)$

$27 : P_{66} = (1, 3, 2, 1)$

$28 : P_{69} = (0, 0, 3, 1)$

$29 : P_{70} = (1, 0, 3, 1)$

$30 : P_{78} = (1, 2, 3, 1)$

$31 : P_{81} = (0, 3, 3, 1)$

$32 : P_{84} = (3, 3, 3, 1)$