Rank-73737 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498764653

General information

Number of lines	9
Number of points	29
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	18
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	3^5, 2^6, 1^{18}

Singular Points

The surface has 4 singular points:

0 : $P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
1 : $P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
2 : $P_{66} = \mathbf{P}(1,\omega^2,\omega,1) = \mathbf{P}(1,3,2,1)$
3 : $P_{78} = \mathbf{P}(1,\omega,\omega^2,1) = \mathbf{P}(1,2,3,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{11} = \mathbf{Pl}(3,0,2,0,1,0)_{42}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{14} = \mathbf{Pl}(2,0,3,0,1,0)_{48}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{129} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{129} = \mathbf{Pl}(3,2,2,3,0,1)_{173}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{149} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{149} = \mathbf{Pl}(2,3,3,2,0,1)_{166}$$

Rank of lines: (5, 11, 14, 345, 106, 355, 129, 350, 149)
Rank of points on Klein quadric: (33, 42, 48, 13, 150, 15, 173, 14, 166)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$2 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$,
$3 : P_{66} = \mathbf{P}(1,\omega^2,\omega,1) = \mathbf{P}(1,3,2,1)$,
$4 : P_{78} = \mathbf{P}(1,\omega,\omega^2,1) = \mathbf{P}(1,2,3,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{42} = (0,1,1,1) = \ell_0 \cap \ell_3$
$P_{65} = (0,3,2,1) = \ell_1 \cap \ell_5$
$P_{77} = (0,2,3,1) = \ell_2 \cap \ell_7$
$P_{11} = (0,1,1,0) = \ell_3 \cap \ell_4$
$P_{15} = (0,2,1,0) = \ell_5 \cap \ell_6$
$P_{19} = (0,3,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 :\ P_{27} = (1,1,0,1)$ lies on line ℓ_4
$1 :\ P_{31} = (1,2,0,1)$ lies on line ℓ_8
$2 :\ P_{35} = (1,3,0,1)$ lies on line ℓ_6
$3 :\ P_{39} = (1,0,1,1)$ lies on line ℓ_4
$4 :\ P_{43} = (2,1,1,1)$ lies on line ℓ_0
$5 :\ P_{44} = (3,1,1,1)$ lies on line ℓ_0

6 : $P_{45} = (0, 2, 1, 1)$ lies on line ℓ_5
7 : $P_{49} = (0, 3, 1, 1)$ lies on line ℓ_7
8 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_6
9 : $P_{57} = (0, 1, 2, 1)$ lies on line ℓ_7
10 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_3
11 : $P_{67} = (2, 3, 2, 1)$ lies on line ℓ_1
12 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_1
13 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_8
14 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_5
15 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_2
16 : $P_{80} = (3, 2, 3, 1)$ lies on line ℓ_2
17 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 0 0 1 0 1
1	1 0 1 0 1 1 0 0 1
2	1 1 0 0 1 0 1 1 0
3	1 0 0 0 1 1 0 1 0
4	0 1 1 1 0 0 1 0 1
5	0 1 0 1 0 0 1 1 0
6	1 0 1 0 1 1 0 0 1
7	0 0 1 1 0 1 0 0 1
8	1 1 0 0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_0	P_0	P_{42}	P_4	P_4

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_0	P_0	P_{66}	P_{65}	P_{66}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_0	P_0	P_{78}	P_{78}	P_{77}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7
in point	P_{42}	P_{11}	P_3	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_8
in point	P_{66}	P_{78}	P_{11}	P_{78}	P_{66}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7
in point	P_{65}	P_3	P_{15}	P_3

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_4	P_{78}	P_{78}	P_{15}	P_4

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_8
in point	P_{77}	P_3	P_3	P_{19}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_4	P_{66}	P_{66}	P_4	P_{19}

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_3 = (0,0,0,1)$
$2: P_4 = (1,1,1,1)$
$3: P_{11} = (0,1,1,0)$
$4: P_{15} = (0,2,1,0)$
$5: P_{19} = (0,3,1,0)$
$6: P_{27} = (1,1,0,1)$
$7: P_{31} = (1,2,0,1)$
$8: P_{35} = (1,3,0,1)$
$9: P_{39} = (1,0,1,1)$
$10: P_{42} = (0,1,1,1)$
$11: P_{43} = (2,1,1,1)$
$12: P_{44} = (3,1,1,1)$
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$26: P_{79} = (2,2,3,1)$
$27: P_{80} = (3,2,3,1)$
$28: P_{81} = (0,3,3,1)$