

# Rank-65921 over GF(64)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

( 0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(64) is -2113396603

## General information

Number of lines	21
Number of points	4481
Number of singular points	1
Number of Eckardt points	3
Number of double points	66
Number of single points	1218
Number of points off lines	3193
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^{21}$
Type of lines on points	$6, 3^3, 2^{66}, 1^{1218}, 0^{3193}$

## Singular Points

The surface has 1 singular points:

$$0 : P_{8259} = \mathbf{P}(1, 0, 1, 1) = \mathbf{P}(1, 0, 1, 1)$$

## The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{64} = \mathbf{Pl}(1, 0, 0, 0, 1, 0)_{4226}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \mathbf{Pl}(0, 0, 1, 0, 1, 0)_{4352} \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{270530} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{270530} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{544579} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{266369} = \mathbf{Pl}(1, 1, 1, 1, 1, 0)_{20354} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{270529} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{270529} = \mathbf{Pl}(1, 1, 0, 1, 1, 1)_{540609} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^9 \\ 0 & 1 & \epsilon^9 & 0 \end{bmatrix}_{12562106} = \begin{bmatrix} 1 & 0 & 11 & 47 \\ 0 & 1 & 47 & 0 \end{bmatrix}_{12562106} = \mathbf{Pl}(1, 1, 10, 10, 0, 1)_{322818} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^{18} \\ 0 & 1 & \epsilon^9 & 0 \end{bmatrix}_{2975162} = \begin{bmatrix} 1 & 0 & 11 & 11 \\ 0 & 1 & 47 & 0 \end{bmatrix}_{2975162} = \mathbf{Pl}(10, 47, 10, 1, 0, 1)_{287106} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^{36} \\ 0 & 1 & \epsilon^9 & 0 \end{bmatrix}_{9632762} = \begin{bmatrix} 1 & 0 & 11 & 36 \\ 0 & 1 & 47 & 0 \end{bmatrix}_{9632762} = \mathbf{Pl}(36, 46, 10, 11, 0, 1)_{326822} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & \epsilon^{18} & \epsilon^9 \\ 0 & 1 & \epsilon^{27} & \epsilon^{54} \end{bmatrix}_{12562745} = \begin{bmatrix} 1 & 0 & 11 & 47 \\ 0 & 1 & 46 & 10 \end{bmatrix}_{12562745} = \mathbf{Pl}(1, 1, 47, 10, 36, 1)_{9907827} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \epsilon^{54} & \epsilon^{45} \end{bmatrix}_{272843} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 10 & 37 \end{bmatrix}_{272843} = \mathbf{Pl}(10, 47, 1, 1, 37, 1)_{9990303} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \epsilon^{36} & \epsilon^{18} \\ 0 & 1 & \epsilon^{18} & 0 \end{bmatrix}_{3079151} = \begin{bmatrix} 1 & 0 & 36 & 11 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{3079151} = \mathbf{Pl}(1, 1, 37, 37, 0, 1)_{431682} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & \epsilon^{36} & \epsilon^{36} \\ 0 & 1 & \epsilon^{18} & 0 \end{bmatrix}_{9736751} = \begin{bmatrix} 1 & 0 & 36 & 36 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{9736751} = \mathbf{Pl}(37, 11, 37, 1, 0, 1)_{288834} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & \epsilon^{36} & \epsilon^9 \\ 0 & 1 & \epsilon^{18} & 0 \end{bmatrix}_{12666095} = \begin{bmatrix} 1 & 0 & 36 & 47 \\ 0 & 1 & 11 & 0 \end{bmatrix}_{12666095} = \mathbf{Pl}(47, 10, 37, 36, 0, 1)_{427759} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & \epsilon^{36} & \epsilon^{18} \\ 0 & 1 & \epsilon^{54} & \epsilon^{45} \end{bmatrix}_{3081518} = \begin{bmatrix} 1 & 0 & 36 & 11 \\ 0 & 1 & 10 & 37 \end{bmatrix}_{3081518} = \mathbf{Pl}(1, 1, 11, 37, 47, 1)_{12647823} \\
\ell_{15} &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \epsilon^{45} & \epsilon^{27} \end{bmatrix}_{273446} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 37 & 46 \end{bmatrix}_{273446} = \mathbf{Pl}(37, 11, 1, 1, 46, 1)_{12346782} \\
\ell_{16} &= \begin{bmatrix} 1 & 0 & \epsilon^9 & \epsilon^{36} \\ 0 & 1 & \epsilon^{36} & 0 \end{bmatrix}_{9782547} = \begin{bmatrix} 1 & 0 & 47 & 36 \\ 0 & 1 & 36 & 0 \end{bmatrix}_{9782547} = \mathbf{Pl}(1, 1, 46, 46, 0, 1)_{467970} \\
\ell_{17} &= \begin{bmatrix} 1 & 0 & \epsilon^9 & \epsilon^{18} \\ 0 & 1 & \epsilon^{36} & 0 \end{bmatrix}_{3124947} = \begin{bmatrix} 1 & 0 & 47 & 11 \\ 0 & 1 & 36 & 0 \end{bmatrix}_{3124947} = \mathbf{Pl}(11, 37, 46, 47, 0, 1)_{471949} \\
\ell_{18} &= \begin{bmatrix} 1 & 0 & \epsilon^9 & \epsilon^9 \\ 0 & 1 & \epsilon^{36} & 0 \end{bmatrix}_{12711891} = \begin{bmatrix} 1 & 0 & 47 & 47 \\ 0 & 1 & 36 & 0 \end{bmatrix}_{12711891} = \mathbf{Pl}(46, 36, 46, 1, 0, 1)_{289410} \\
\ell_{19} &= \begin{bmatrix} 1 & 0 & \epsilon^9 & \epsilon^{36} \\ 0 & 1 & \epsilon^{45} & \epsilon^{27} \end{bmatrix}_{9785492} = \begin{bmatrix} 1 & 0 & 47 & 36 \\ 0 & 1 & 37 & 46 \end{bmatrix}_{9785492} = \mathbf{Pl}(1, 1, 36, 46, 11, 1)_{3312294} \\
\ell_{20} &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \epsilon^{27} & \epsilon^{54} \end{bmatrix}_{271151} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 46 & 10 \end{bmatrix}_{271151} = \mathbf{Pl}(46, 36, 1, 1, 10, 1)_{2913486}
\end{aligned}$$

Rank of lines: ( 64, 4097, 8258, 270530, 266369, 270529, 12562106, 2975162, 9632762, 12562745, 272843, 3079151, 9736751, 12666095, 3081518, 273446, 9782547, 3124947, 12711891, 9785492, 271151 )

Rank of points on Klein quadric: ( 4226, 4352, 544578, 544579, 20354, 540609, 322818, 287106, 326822, 9907827, 9990303, 431682, 288834, 427759, 12647823, 12346782, 467970, 471949, 289410, 3312294, 2913486 )

## Eckardt Points

The surface has 3 Eckardt points:

$$\begin{aligned} 0 : P_{707} &= \mathbf{P}(0, \epsilon^{54}, 1, 0) = \mathbf{P}(0, 10, 1, 0), \\ 1 : P_{2435} &= \mathbf{P}(0, \epsilon^{45}, 1, 0) = \mathbf{P}(0, 37, 1, 0), \\ 2 : P_{3011} &= \mathbf{P}(0, \epsilon^{27}, 1, 0) = \mathbf{P}(0, 46, 1, 0). \end{aligned}$$

## Double Points

The surface has 66 Double points:

The double points on the surface are:

$$\begin{aligned} P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 \\ P_{4226} &= (0, 1, 0, 1) = \ell_0 \cap \ell_5 \\ P_{4236} &= (10, 1, 0, 1) = \ell_0 \cap \ell_6 \\ P_{4273} &= (47, 1, 0, 1) = \ell_0 \cap \ell_9 \\ P_{4263} &= (37, 1, 0, 1) = \ell_0 \cap \ell_{11} \\ P_{4237} &= (11, 1, 0, 1) = \ell_0 \cap \ell_{14} \\ P_{4272} &= (46, 1, 0, 1) = \ell_0 \cap \ell_{16} \\ P_{4262} &= (36, 1, 0, 1) = \ell_0 \cap \ell_{19} \\ P_{8258} &= (0, 0, 1, 1) = \ell_1 \cap \ell_2 \\ P_{8295} &= (37, 0, 1, 1) = \ell_1 \cap \ell_7 \\ P_{8304} &= (46, 0, 1, 1) = \ell_1 \cap \ell_{12} \\ P_{8268} &= (10, 0, 1, 1) = \ell_1 \cap \ell_{18} \\ P_5 &= (1, 1, 0, 0) = \ell_2 \cap \ell_3 \\ P_{11247} &= (46, 46, 1, 1) = \ell_2 \cap \ell_8 \\ P_{8972} &= (11, 11, 1, 1) = \ell_2 \cap \ell_9 \\ P_{8907} &= (10, 10, 1, 1) = \ell_2 \cap \ell_{13} \\ P_{10597} &= (36, 36, 1, 1) = \ell_2 \cap \ell_{14} \\ P_{10662} &= (37, 37, 1, 1) = \ell_2 \cap \ell_{17} \\ P_{11312} &= (47, 47, 1, 1) = \ell_2 \cap \ell_{19} \\ P_{8322} &= (0, 1, 1, 1) = \ell_3 \cap \ell_4 \\ P_{8971} &= (10, 11, 1, 1) = \ell_3 \cap \ell_6 \\ P_{10598} &= (37, 36, 1, 1) = \ell_3 \cap \ell_{11} \\ P_{11311} &= (46, 47, 1, 1) = \ell_3 \cap \ell_{16} \\ P_{132} &= (1, 1, 1, 0) = \ell_4 \cap \ell_5 \\ P_{153958} &= (37, 36, 36, 1) = \ell_4 \cap \ell_7 \\ P_{158117} &= (36, 37, 37, 1) = \ell_4 \cap \ell_9 \\ P_{199727} &= (46, 47, 47, 1) = \ell_4 \cap \ell_{12} \\ P_{195568} &= (47, 46, 46, 1) = \ell_4 \cap \ell_{14} \\ P_{49931} &= (10, 11, 11, 1) = \ell_4 \cap \ell_{18} \\ P_{45772} &= (11, 10, 10, 1) = \ell_4 \cap \ell_{19} \\ P_{195631} &= (46, 47, 46, 1) = \ell_5 \cap \ell_8 \\ P_{45835} &= (10, 11, 10, 1) = \ell_5 \cap \ell_{13} \\ P_{158054} &= (37, 36, 37, 1) = \ell_5 \cap \ell_{17} \\ P_{196683} &= (10, 0, 47, 1) = \ell_6 \cap \ell_9 \\ P_{52171} &= (10, 46, 11, 1) = \ell_6 \cap \ell_{13} \\ P_{193227} &= (10, 10, 46, 1) = \ell_6 \cap \ell_{15} \\ P_{154635} &= (10, 47, 36, 1) = \ell_6 \cap \ell_{18} \\ P_{4839} &= (37, 10, 0, 1) = \ell_7 \cap \ell_{10} \\ P_{197414} &= (37, 11, 47, 1) = \ell_7 \cap \ell_{11} \\ P_{192678} &= (37, 1, 46, 1) = \ell_7 \cap \ell_{17} \\ P_{158694} &= (37, 46, 37, 1) = \ell_7 \cap \ell_{19} \\ P_{45231} &= (46, 1, 10, 1) = \ell_8 \cap \ell_{12} \\ P_{152303} &= (46, 10, 36, 1) = \ell_8 \cap \ell_{14} \\ P_{199087} &= (46, 37, 47, 1) = \ell_8 \cap \ell_{16} \\ P_{49967} &= (46, 11, 11, 1) = \ell_8 \cap \ell_{20} \\ P_{743} &= (36, 10, 1, 0) = \ell_9 \cap \ell_{10} \\ P_{193263} &= (46, 10, 46, 1) = \ell_9 \cap \ell_{12} \\ P_{52198} &= (37, 46, 11, 1) = \ell_9 \cap \ell_{17} \\ P_{154625} &= (0, 47, 36, 1) = \ell_9 \cap \ell_{20} \\ P_{153931} &= (10, 36, 36, 1) = \ell_{10} \cap \ell_{13} \\ P_{197377} &= (0, 11, 47, 1) = \ell_{10} \cap \ell_{14} \\ P_{158703} &= (46, 46, 37, 1) = \ell_{10} \cap \ell_{16} \\ P_{49254} &= (37, 0, 11, 1) = \ell_{11} \cap \ell_{14} \\ P_{152294} &= (37, 10, 36, 1) = \ell_{11} \cap \ell_{17} \\ P_{47526} &= (37, 37, 10, 1) = \ell_{11} \cap \ell_{20} \\ P_{6576} &= (46, 37, 0, 1) = \ell_{12} \cap \ell_{15} \\ P_{51567} &= (46, 36, 11, 1) = \ell_{12} \cap \ell_{16} \\ P_{155787} &= (10, 1, 37, 1) = \ell_{13} \cap \ell_{18} \\ P_{199051} &= (10, 37, 47, 1) = \ell_{13} \cap \ell_{19} \\ P_{2482} &= (47, 37, 1, 0) = \ell_{14} \cap \ell_{15} \\ P_{47499} &= (10, 37, 10, 1) = \ell_{14} \cap \ell_{18} \\ P_{199718} &= (37, 47, 47, 1) = \ell_{15} \cap \ell_{17} \\ P_{51521} &= (0, 36, 11, 1) = \ell_{15} \cap \ell_{19} \\ P_{151663} &= (46, 0, 36, 1) = \ell_{16} \cap \ell_{19} \\ P_{7116} &= (10, 46, 0, 1) = \ell_{18} \cap \ell_{20} \\ P_{3022} &= (11, 46, 1, 0) = \ell_{19} \cap \ell_{20} \end{aligned}$$

## Single Points

The surface has 1218 single points:

Too many to print.

### Points on surface but on no line

The surface has 3193 points not on any line:  
Too many to print.

### Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	0	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	0	1	0
1	1	0	1	1	0	1	0	1	0	0	1	0	0	1	0	0	1	0	1	0	1
2	0	1	0	1	0	0	0	0	1	1	0	0	0	1	1	0	0	1	0	1	0
3	0	1	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1
4	0	0	0	1	0	1	0	1	0	1	0	0	1	0	0	0	0	1	1	0	0
5	1	1	0	1	1	0	0	0	1	0	0	1	0	1	0	1	0	1	0	0	1
6	1	0	0	1	0	0	0	1	1	1	0	0	0	1	0	1	0	0	1	0	0
7	0	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1	0	1	0	0
8	0	0	1	0	0	1	1	1	0	0	0	1	0	1	0	1	0	0	0	0	1
9	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	1
10	0	1	0	1	0	1	0	1	0	0	0	1	1	1	1	1	0	0	0	0	1
11	1	0	0	1	0	0	0	1	0	0	1	1	1	0	0	1	0	0	1	0	0
12	0	1	0	0	1	0	0	0	1	1	0	1	0	1	1	0	0	0	0	0	0
13	0	0	1	0	0	1	1	0	0	0	1	1	0	0	0	0	0	1	1	0	0
14	1	0	1	0	1	0	0	0	1	1	0	0	0	1	0	0	1	0	0	0	0
15	0	1	0	1	0	1	1	0	0	0	1	0	1	0	0	1	0	1	1	0	1
16	1	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	1	1	1	0	0
17	0	0	1	0	0	1	0	1	0	1	0	0	0	1	1	0	1	0	0	0	0
18	0	1	0	0	1	0	1	0	0	0	1	1	0	1	1	0	1	0	0	1	0
19	1	0	1	0	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	1	0
20	0	1	0	1	0	1	0	0	1	1	1	0	0	0	1	0	0	1	1	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_5$	$\ell_6$	$\ell_9$	$\ell_{11}$	$\ell_{14}$	$\ell_{16}$	$\ell_{19}$
in point	$P_0$	$P_{4226}$	$P_{4236}$	$P_{4273}$	$P_{4263}$	$P_{4237}$	$P_{4272}$	$P_{4262}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_7$	$\ell_{10}$	$\ell_{12}$	$\ell_{15}$	$\ell_{18}$	$\ell_{20}$
in point	$P_0$	$P_{8258}$	$P_{8259}$	$P_{8259}$	$P_{8295}$	$P_{8259}$	$P_{8304}$	$P_{8259}$	$P_{8268}$	$P_{8259}$

Line 2 intersects

Line	$\ell_1$	$\ell_3$	$\ell_8$	$\ell_9$	$\ell_{13}$	$\ell_{14}$	$\ell_{17}$	$\ell_{19}$
in point	$P_{8258}$	$P_5$	$P_{11247}$	$P_{8972}$	$P_{8907}$	$P_{10597}$	$P_{10662}$	$P_{11312}$

Line 3 intersects

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{11}$	$\ell_{15}$	$\ell_{16}$	$\ell_{20}$
in point	$P_{8259}$	$P_5$	$P_{8322}$	$P_{8259}$	$P_{8971}$	$P_{8259}$	$P_{10598}$	$P_{8259}$	$P_{11311}$	$P_{8259}$

Line 4 intersects

Line	$\ell_3$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{12}$	$\ell_{14}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{8322}$	$P_{132}$	$P_{153958}$	$P_{158117}$	$P_{199727}$	$P_{195568}$	$P_{49931}$	$P_{45772}$

Line 5 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_8$	$\ell_{10}$	$\ell_{13}$	$\ell_{15}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{4226}$	$P_{8259}$	$P_{8259}$	$P_{132}$	$P_{195631}$	$P_{8259}$	$P_{45835}$	$P_{8259}$	$P_{158054}$	$P_{8259}$

Line 6 intersects

Line	$\ell_0$	$\ell_3$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{13}$	$\ell_{15}$	$\ell_{18}$
in point	$P_{4236}$	$P_{8971}$	$P_{707}$	$P_{707}$	$P_{196683}$	$P_{52171}$	$P_{193227}$	$P_{154635}$

Line 7 intersects

Line	$\ell_1$	$\ell_4$	$\ell_6$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{17}$	$\ell_{19}$
in point	$P_{8295}$	$P_{153958}$	$P_{707}$	$P_{707}$	$P_{4839}$	$P_{197414}$	$P_{192678}$	$P_{158694}$

Line 8 intersects

Line	$\ell_2$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_{12}$	$\ell_{14}$	$\ell_{16}$	$\ell_{20}$
in point	$P_{11247}$	$P_{195631}$	$P_{707}$	$P_{707}$	$P_{45231}$	$P_{152303}$	$P_{199087}$	$P_{49967}$

Line 9 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_6$	$\ell_{10}$	$\ell_{12}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{4273}$	$P_{8972}$	$P_{158117}$	$P_{196683}$	$P_{743}$	$P_{193263}$	$P_{52198}$	$P_{154625}$

Line 10 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$	$\ell_{16}$	$\ell_{20}$
in point	$P_{8259}$	$P_{8259}$	$P_{8259}$	$P_{4839}$	$P_{743}$	$P_{153931}$	$P_{197377}$	$P_{8259}$	$P_{158703}$	$P_{8259}$

Line 11 intersects

Line	$\ell_0$	$\ell_3$	$\ell_7$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{4263}$	$P_{10598}$	$P_{197414}$	$P_{2435}$	$P_{2435}$	$P_{49254}$	$P_{152294}$	$P_{47526}$

Line 12 intersects

Line	$\ell_1$	$\ell_4$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$
in point	$P_{8304}$	$P_{199727}$	$P_{45231}$	$P_{193263}$	$P_{2435}$	$P_{2435}$	$P_{6576}$	$P_{51567}$

Line 13 intersects

Line	$\ell_2$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{8907}$	$P_{45835}$	$P_{52171}$	$P_{153931}$	$P_{2435}$	$P_{2435}$	$P_{155787}$	$P_{199051}$

Line 14 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{15}$	$\ell_{18}$
in point	$P_{4237}$	$P_{10597}$	$P_{195568}$	$P_{152303}$	$P_{197377}$	$P_{49254}$	$P_{2482}$	$P_{47499}$

Line 15 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{12}$	$\ell_{14}$	$\ell_{17}$	$\ell_{19}$	$\ell_{20}$
in point	$P_{8259}$	$P_{8259}$	$P_{8259}$	$P_{193227}$	$P_{8259}$	$P_{6576}$	$P_{2482}$	$P_{199718}$	$P_{51521}$	$P_{8259}$

Line 16 intersects

Line	$\ell_0$	$\ell_3$	$\ell_8$	$\ell_{10}$	$\ell_{12}$	$\ell_{17}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{4272}$	$P_{11311}$	$P_{199087}$	$P_{158703}$	$P_{51567}$	$P_{3011}$	$P_{3011}$	$P_{151663}$

Line 17 intersects

Line	$\ell_2$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{11}$	$\ell_{15}$	$\ell_{16}$	$\ell_{18}$
in point	$P_{10662}$	$P_{158054}$	$P_{192678}$	$P_{52198}$	$P_{152294}$	$P_{199718}$	$P_{3011}$	$P_{3011}$

Line 18 intersects

Line	$\ell_1$	$\ell_4$	$\ell_6$	$\ell_{13}$	$\ell_{14}$	$\ell_{16}$	$\ell_{17}$	$\ell_{20}$
in point	$P_{8268}$	$P_{49931}$	$P_{154635}$	$P_{155787}$	$P_{47499}$	$P_{3011}$	$P_{3011}$	$P_{7116}$

Line 19 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$	$\ell_7$	$\ell_{13}$	$\ell_{15}$	$\ell_{16}$	$\ell_{20}$
in point	$P_{4262}$	$P_{11312}$	$P_{45772}$	$P_{158694}$	$P_{199051}$	$P_{51521}$	$P_{151663}$	$P_{3022}$

Line 20 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_8$	$\ell_9$	$\ell_{10}$	$\ell_{11}$	$\ell_{15}$	$\ell_{18}$	$\ell_{19}$
in point	$P_{8259}$	$P_{8259}$	$P_{8259}$	$P_{49967}$	$P_{154625}$	$P_{8259}$	$P_{47526}$	$P_{8259}$	$P_{7116}$	$P_{3022}$

The surface has 4481 points:

Too many to print.