Rank-65561 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431656045

General information

Number of lines	8
Number of points	33
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	32
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^8
Type of lines on points	8, 1^{32}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0,0,0,0,1,0)_{25}$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{41} = \mathbf{Pl}(0,0,0,1,1,0)_{53}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{146} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{146} = \mathbf{Pl}(0,1,0,2,1,0)_{64}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_7 = \begin{bmatrix} 1 & \omega^2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{167} = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{167} = \mathbf{Pl}(0,1,0,3,1,0)_{71}$$

Rank of lines: (20, 41, 345, 125, 355, 146, 350, 167)
Rank of points on Klein quadric: (25, 53, 13, 57, 15, 64, 14, 71)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 32 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_4 = (1,1,1,1)$ lies on line ℓ_3
2 : $P_5 = (1,1,0,0)$ lies on line ℓ_1
3 : $P_{11} = (0,1,1,0)$ lies on line ℓ_2
4 : $P_{12} = (1,1,1,0)$ lies on line ℓ_3
5 : $P_{15} = (0,2,1,0)$ lies on line ℓ_4
6 : $P_{16} = (1,2,1,0)$ lies on line ℓ_5
7 : $P_{19} = (0,3,1,0)$ lies on line ℓ_6
8 : $P_{20} = (1,3,1,0)$ lies on line ℓ_7
9 : $P_{23} = (1,0,0,1)$ lies on line ℓ_0
10 : $P_{24} = (2,0,0,1)$ lies on line ℓ_0
11 : $P_{25} = (3,0,0,1)$ lies on line ℓ_0
12 : $P_{27} = (1,1,0,1)$ lies on line ℓ_1
13 : $P_{32} = (2,2,0,1)$ lies on line ℓ_1
14 : $P_{37} = (3,3,0,1)$ lies on line ℓ_1
15 : $P_{42} = (0,1,1,1)$ lies on line ℓ_2
16 : $P_{45} = (0,2,1,1)$ lies on line ℓ_4
17 : $P_{46} = (1,2,1,1)$ lies on line ℓ_5
18 : $P_{49} = (0,3,1,1)$ lies on line ℓ_6
19 : $P_{50} = (1,3,1,1)$ lies on line ℓ_7
20 : $P_{57} = (0,1,2,1)$ lies on line ℓ_6
21 : $P_{59} = (2,1,2,1)$ lies on line ℓ_7
22 : $P_{61} = (0,2,2,1)$ lies on line ℓ_2
23 : $P_{63} = (2,2,2,1)$ lies on line ℓ_3
24 : $P_{65} = (0,3,2,1)$ lies on line ℓ_4
25 : $P_{67} = (2,3,2,1)$ lies on line ℓ_5
26 : $P_{73} = (0,1,3,1)$ lies on line ℓ_4
27 : $P_{76} = (3,1,3,1)$ lies on line ℓ_5
28 : $P_{77} = (0,2,3,1)$ lies on line ℓ_6
29 : $P_{80} = (3,2,3,1)$ lies on line ℓ_7
30 : $P_{81} = (0,3,3,1)$ lies on line ℓ_2
31 : $P_{84} = (3,3,3,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1
3	1	1	1	0	1	1	1	1
4	1	1	1	1	0	1	1	1
5	1	1	1	1	1	0	1	1
6	1	1	1	1	1	1	0	1
7	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 33 points:
The points on the surface are:

$0:\ P_0 = (1,0,0,0)$
$1:\ P_3 = (0,0,0,1)$
$2:\ P_4 = (1,1,1,1)$
$3:\ P_5 = (1,1,0,0)$
$4:\ P_{11} = (0,1,1,0)$
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$32:\ P_{84} = (3,3,3,1)$