Rank-65863 over GF(4)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431725417

General information

Number of lines	3
Number of points	25
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	11
Number of points off lines	12
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	2^2, 1^{11}, 0^{12}

Singular Points

The surface has 1 singular points:

$0:\ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

Rank of lines: (0, 340, 345)
Rank of points on Klein quadric: (0, 9, 13)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_1$
$P_3 = (0,0,0,1) = \ell_1 \cap \ell_2$

Single Points

The surface has 11 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
2 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
3 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
4 : $P_{11} = (0,1,1,0)$ lies on line ℓ_2
5 : $P_{26} = (0,1,0,1)$ lies on line ℓ_1
6 : $P_{30} = (0,2,0,1)$ lies on line ℓ_1
7 : $P_{34} = (0,3,0,1)$ lies on line ℓ_1
8 : $P_{42} = (0,1,1,1)$ lies on line ℓ_2
9 : $P_{61} = (0,2,2,1)$ lies on line ℓ_2
10 : $P_{81} = (0,3,3,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 12 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$
1 : $P_{16} = (1,2,1,0)$
2 : $P_{20} = (1,3,1,0)$
3 : $P_{39} = (1,0,1,1)$
4 : $P_{54} = (1,0,2,1)$
5 : $P_{58} = (1,1,2,1)$
6 : $P_{60} = (3,1,2,1)$
7 : $P_{64} = (3,2,2,1)$
8 : $P_{70} = (1,0,3,1)$
9 : $P_{74} = (1,1,3,1)$
10 : $P_{75} = (2,1,3,1)$
11 : $P_{83} = (2,3,3,1)$

Line Intersection Graph

$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
2 & 0 & 1 & 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_3

Line 2 intersects

Line	ℓ_1
in point	P_3

The surface has 25 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_1 = (0,1,0,0)$

$2 : P_3 = (0,0,0,1)$

$3 : P_4 = (1,1,1,1)$

$4 : P_5 = (1,1,0,0)$

$5 : P_6 = (2,1,0,0)$

$6 : P_7 = (3,1,0,0)$

$7 : P_{11} = (0,1,1,0)$

$8 : P_{16} = (1,2,1,0)$

$9 : P_{20} = (1,3,1,0)$

$10 : P_{26} = (0,1,0,1)$

$11 : P_{30} = (0,2,0,1)$

$12 : P_{34} = (0,3,0,1)$

$13 : P_{39} = (1,0,1,1)$

$14 : P_{42} = (0,1,1,1)$

$15 : P_{54} = (1,0,2,1)$

$16 : P_{58} = (1,1,2,1)$

$17 : P_{60} = (3,1,2,1)$

$18 : P_{61} = (0,2,2,1)$

$19 : P_{64} = (3,2,2,1)$

$20 : P_{70} = (1,0,3,1)$

$21 : P_{74} = (1,1,3,1)$

$22 : P_{75} = (2,1,3,1)$

$23 : P_{81} = (0,3,3,1)$

$24 : P_{83} = (2,3,3,1)$