Rank-67243 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_0X_1^2 + X_1^2X_3 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1432983961

General information

Number of lines	4
Number of points	25
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	12
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	2^4, 1^{12}, 0^9

Singular Points

The surface has 2 singular points:

$0:\ P_{57} = \mathbf{P}(0,1,\omega,1) = \mathbf{P}(0,1,2,1)$
$1:\ P_{73} = \mathbf{P}(0,1,\omega^2,1) = \mathbf{P}(0,1,3,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{133} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{133} = \mathbf{Pl}(2,3,2,3,3,1)_{346}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 1 \end{bmatrix}_{153} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{153} = \mathbf{Pl}(3,2,3,2,2,1)_{296}$$

Rank of lines: (336, 341, 133, 153)
Rank of points on Klein quadric: (101, 105, 346, 296)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$
$P_{12} = (1,1,1,0) = \ell_2 \cap \ell_3$
$P_{73} = (0,1,3,1) = \ell_1 \cap \ell_2$
$P_{57} = (0,1,2,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 12 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_{11} = (0,1,1,0)$ lies on line ℓ_0
2 : $P_{15} = (0,2,1,0)$ lies on line ℓ_0
3 : $P_{19} = (0,3,1,0)$ lies on line ℓ_0
4 : $P_{26} = (0,1,0,1)$ lies on line ℓ_1
5 : $P_{33} = (3,2,0,1)$ lies on line ℓ_2
6 : $P_{36} = (2,3,0,1)$ lies on line ℓ_3
7 : $P_{42} = (0,1,1,1)$ lies on line ℓ_1
8 : $P_{48} = (3,2,1,1)$ lies on line ℓ_3
9 : $P_{51} = (2,3,1,1)$ lies on line ℓ_2
10 : $P_{54} = (1,0,2,1)$ lies on line ℓ_2
11 : $P_{70} = (1,0,3,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_8 = (1,0,1,0)$
3 : $P_{40} = (2,0,1,1)$
4 : $P_{41} = (3,0,1,1)$
5 : $P_{56} = (3,0,2,1)$
6 : $P_{68} = (3,3,2,1)$
7 : $P_{71} = (2,0,3,1)$
8 : $P_{79} = (2,2,3,1)$

Line Intersection Graph

	0 1 2 3
0	0 1 0 0
1	1 0 1 1
2	0 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_2	P_{73}	P_{57}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{73}	P_{12}

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_{57}	P_{12}

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_8 = (1, 0, 1, 0)$
5 : $P_{11} = (0, 1, 1, 0)$
6 : $P_{12} = (1, 1, 1, 0)$
7 : $P_{15} = (0, 2, 1, 0)$
8 : $P_{19} = (0, 3, 1, 0)$
9 : $P_{26} = (0, 1, 0, 1)$
10 : $P_{33} = (3, 2, 0, 1)$
11 : $P_{36} = (2, 3, 0, 1)$
12 : $P_{40} = (2, 0, 1, 1)$
13 : $P_{41} = (3, 0, 1, 1)$
14 : $P_{42} = (0, 1, 1, 1)$
15 : $P_{48} = (3, 2, 1, 1)$
16 : $P_{51} = (2, 3, 1, 1)$
17 : $P_{54} = (1, 0, 2, 1)$
18 : $P_{56} = (3, 0, 2, 1)$
19 : $P_{57} = (0, 1, 2, 1)$
20 : $P_{68} = (3, 3, 2, 1)$
21 : $P_{70} = (1, 0, 3, 1)$
22 : $P_{71} = (2, 0, 3, 1)$
23 : $P_{73} = (0, 1, 3, 1)$
24 : $P_{79} = (2, 2, 3, 1)$