Rank-65899 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 304161045

General information

Number of lines	8
Number of points	305
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	114
Number of points off lines	182
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^8
Type of lines on points	4, 3^2, 2^6, 1^{114}, 0^{182}

Singular Points

The surface has 3 singular points:

$0:\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1:\ P_{435} = \mathbf{P}(1,\delta^{10},0,1) = \mathbf{P}(1,10,0,1)$
$2:\ P_{451} = \mathbf{P}(1,\delta^{5},0,1) = \mathbf{P}(1,11,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2986} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2986} = \mathbf{Pl}(0,0,11,0,0,1)_{4966}$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^{5} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3259} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3259} = \mathbf{Pl}(0,0,10,0,0,1)_{4935}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1,0,0,1,0,0)_{34}$$

$$\ell_4 = \begin{bmatrix} 1 & \delta^{10} & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{7354} = \begin{bmatrix} 1 & 10 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{7354} = \mathbf{Pl}(0,11,11,0,0,1)_{4992}$$

$$\ell_5 = \begin{bmatrix} 1 & \delta^{5} & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{7627} = \begin{bmatrix} 1 & 11 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{7627} = \mathbf{Pl}(0,10,10,0,0,1)_{4960}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^{5} & \delta^{10} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46700} = \begin{bmatrix} 1 & 0 & 11 & 10 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{46700} = \mathbf{Pl}(11,10,10,11,10,1)_{48646}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^{10} & \delta^{5} \\ 0 & 1 & 1 & 1 \end{bmatrix}_{50795} = \begin{bmatrix} 1 & 0 & 10 & 11 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{50795} = \mathbf{Pl}(10,11,11,10,11,1)_{52920}$$

Rank of lines: (0, 2986, 3259, 4368, 7354, 7627, 46700, 50795)
Rank of points on Klein quadric: (0, 4966, 4935, 34, 4992, 4960, 48646, 52920)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{435} = \mathbf{P}(1, \delta^{10}, 0, 1) = \mathbf{P}(1, 10, 0, 1)$,
$1 : P_{451} = \mathbf{P}(1, \delta^{5}, 0, 1) = \mathbf{P}(1, 11, 0, 1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{15} = (11,1,0,0) = \ell_0 \cap \ell_1$
$P_{14} = (10,1,0,0) = \ell_0 \cap \ell_2$
$P_{1} = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_{180} = (1,10,1,0) = \ell_1 \cap \ell_6$
$P_{196} = (1,11,1,0) = \ell_2 \cap \ell_7$
$P_{546} = (0,1,1,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 114 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$5 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$6 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$7 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$8 : P_{12} = (8,1,0,0)$ lies on line ℓ_0
$9 : P_{13} = (9,1,0,0)$ lies on line ℓ_0
$10 : P_{16} = (12,1,0,0)$ lies on line ℓ_0
$11 : P_{17} = (13,1,0,0)$ lies on line ℓ_0
$12 : P_{18} = (14,1,0,0)$ lies on line ℓ_0
$13 : P_{19} = (15,1,0,0)$ lies on line ℓ_0
$14 : P_{45} = (10,1,1,0)$ lies on line ℓ_2
$15 : P_{46} = (11,1,1,0)$ lies on line ℓ_1
$16 : P_{64} = (13,2,1,0)$ lies on line ℓ_2
$17 : P_{66} = (15,2,1,0)$ lies on line ℓ_1

18 : $P_{71} = (4,3,1,0)$ lies on line ℓ_1
19 : $P_{74} = (7,3,1,0)$ lies on line ℓ_2
20 : $P_{86} = (3,4,1,0)$ lies on line ℓ_2
21 : $P_{90} = (7,4,1,0)$ lies on line ℓ_1
22 : $P_{108} = (9,5,1,0)$ lies on line ℓ_2
23 : $P_{111} = (12,5,1,0)$ lies on line ℓ_1
24 : $P_{123} = (8,6,1,0)$ lies on line ℓ_1
25 : $P_{129} = (14,6,1,0)$ lies on line ℓ_2
26 : $P_{134} = (3,7,1,0)$ lies on line ℓ_1
27 : $P_{135} = (4,7,1,0)$ lies on line ℓ_2
28 : $P_{153} = (6,8,1,0)$ lies on line ℓ_2
29 : $P_{161} = (14,8,1,0)$ lies on line ℓ_1
30 : $P_{168} = (5,9,1,0)$ lies on line ℓ_1
31 : $P_{175} = (12,9,1,0)$ lies on line ℓ_2
32 : $P_{190} = (11,10,1,0)$ lies on line ℓ_2
33 : $P_{205} = (10,11,1,0)$ lies on line ℓ_1
34 : $P_{216} = (5,12,1,0)$ lies on line ℓ_2
35 : $P_{220} = (9,12,1,0)$ lies on line ℓ_1
36 : $P_{229} = (2,13,1,0)$ lies on line ℓ_1
37 : $P_{242} = (15,13,1,0)$ lies on line ℓ_2
38 : $P_{249} = (6,14,1,0)$ lies on line ℓ_1
39 : $P_{251} = (8,14,1,0)$ lies on line ℓ_2
40 : $P_{261} = (2,15,1,0)$ lies on line ℓ_2
41 : $P_{272} = (13,15,1,0)$ lies on line ℓ_1
42 : $P_{275} = (1,0,0,1)$ lies on line ℓ_3
43 : $P_{291} = (1,1,0,1)$ lies on line ℓ_3
44 : $P_{307} = (1,2,0,1)$ lies on line ℓ_3
45 : $P_{323} = (1,3,0,1)$ lies on line ℓ_3
46 : $P_{339} = (1,4,0,1)$ lies on line ℓ_3
47 : $P_{355} = (1,5,0,1)$ lies on line ℓ_3
48 : $P_{371} = (1,6,0,1)$ lies on line ℓ_3
49 : $P_{387} = (1,7,0,1)$ lies on line ℓ_3
50 : $P_{403} = (1,8,0,1)$ lies on line ℓ_3
51 : $P_{419} = (1,9,0,1)$ lies on line ℓ_3
52 : $P_{467} = (1,12,0,1)$ lies on line ℓ_3
53 : $P_{483} = (1,13,0,1)$ lies on line ℓ_3
54 : $P_{499} = (1,14,0,1)$ lies on line ℓ_3
55 : $P_{515} = (1,15,0,1)$ lies on line ℓ_3
56 : $P_{690} = (1,10,1,1)$ lies on line ℓ_4
57 : $P_{706} = (1,11,1,1)$ lies on line ℓ_5
58 : $P_{868} = (3,5,2,1)$ lies on line ℓ_7
59 : $P_{884} = (3,6,2,1)$ lies on line ℓ_6
60 : $P_{946} = (1,10,2,1)$ lies on line ℓ_4
61 : $P_{962} = (1,11,2,1)$ lies on line ℓ_5
62 : $P_{1202} = (1,10,3,1)$ lies on line ℓ_4
63 : $P_{1218} = (1,11,3,1)$ lies on line ℓ_5
64 : $P_{1235} = (2,12,3,1)$ lies on line ℓ_6
65 : $P_{1267} = (2,14,3,1)$ lies on line ℓ_7
66 : $P_{1430} = (5,8,4,1)$ lies on line ℓ_6
67 : $P_{1458} = (1,10,4,1)$ lies on line ℓ_4
68 : $P_{1474} = (1,11,4,1)$ lies on line ℓ_5
69 : $P_{1510} = (5,13,4,1)$ lies on line ℓ_7
70 : $P_{1589} = (4,2,5,1)$ lies on line ℓ_6
71 : $P_{1653} = (4,6,5,1)$ lies on line ℓ_7
72 : $P_{1714} = (1,10,5,1)$ lies on line ℓ_4
73 : $P_{1730} = (1,11,5,1)$ lies on line ℓ_5
74 : $P_{1848} = (7,2,6,1)$ lies on line ℓ_7
75 : $P_{1896} = (7,5,6,1)$ lies on line ℓ_6
76 : $P_{1970} = (1,10,6,1)$ lies on line ℓ_4
77 : $P_{1986} = (1,11,6,1)$ lies on line ℓ_5
78 : $P_{2215} = (6,9,7,1)$ lies on line ℓ_7
79 : $P_{2226} = (1,10,7,1)$ lies on line ℓ_4
80 : $P_{2242} = (1,11,7,1)$ lies on line ℓ_5
81 : $P_{2311} = (6,15,7,1)$ lies on line ℓ_6
82 : $P_{2394} = (9,4,8,1)$ lies on line ℓ_7
83 : $P_{2482} = (1,10,8,1)$ lies on line ℓ_4
84 : $P_{2498} = (1,11,8,1)$ lies on line ℓ_5
85 : $P_{2538} = (9,13,8,1)$ lies on line ℓ_6
86 : $P_{2697} = (8,7,9,1)$ lies on line ℓ_6
87 : $P_{2738} = (1,10,9,1)$ lies on line ℓ_4
88 : $P_{2754} = (1,11,9,1)$ lies on line ℓ_5
89 : $P_{2825} = (8,15,9,1)$ lies on line ℓ_7
90 : $P_{2844} = (11,0,10,1)$ lies on line ℓ_6
91 : $P_{2994} = (1,10,10,1)$ lies on line ℓ_4
92 : $P_{3010} = (1,11,10,1)$ lies on line ℓ_5
93 : $P_{3020} = (11,11,10,1)$ lies on line ℓ_7
94 : $P_{3099} = (10,0,11,1)$ lies on line ℓ_7
95 : $P_{3250} = (1,10,11,1)$ lies on line ℓ_4
96 : $P_{3259} = (10,10,11,1)$ lies on line ℓ_6
97 : $P_{3266} = (1,11,11,1)$ lies on line ℓ_5
98 : $P_{3406} = (13,3,12,1)$ lies on line ℓ_7
99 : $P_{3506} = (1,10,12,1)$ lies on line ℓ_4
100 : $P_{3522} = (1,11,12,1)$ lies on line ℓ_5
101 : $P_{3582} = (13,14,12,1)$ lies on line ℓ_6
102 : $P_{3677} = (12,4,13,1)$ lies on line ℓ_6
103 : $P_{3741} = (12,8,13,1)$ lies on line ℓ_7
104 : $P_{3762} = (1,10,13,1)$ lies on line ℓ_4
105 : $P_{3778} = (1,11,13,1)$ lies on line ℓ_5
106 : $P_{3920} = (15,3,14,1)$ lies on line ℓ_6
107 : $P_{4018} = (1,10,14,1)$ lies on line ℓ_4
108 : $P_{4034} = (1,11,14,1)$ lies on line ℓ_5
109 : $P_{4064} = (15,12,14,1)$ lies on line ℓ_7
110 : $P_{4239} = (14,7,15,1)$ lies on line ℓ_7
111 : $P_{4271} = (14,9,15,1)$ lies on line ℓ_6
112 : $P_{4274} = (1,10,15,1)$ lies on line ℓ_4
113 : $P_{4290} = (1,11,15,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 182 points not on any line:
The points on the surface but not on lines are:

0 : $P_{575} = (14, 2, 1, 1)$
1 : $P_{588} = (11, 3, 1, 1)$
2 : $P_{595} = (2, 4, 1, 1)$
3 : $P_{619} = (10, 5, 1, 1)$
4 : $P_{627} = (2, 6, 1, 1)$
5 : $P_{650} = (9, 7, 1, 1)$
6 : $P_{668} = (11, 8, 1, 1)$
7 : $P_{677} = (4, 9, 1, 1)$
8 : $P_{735} = (14, 12, 1, 1)$
9 : $P_{741} = (4, 13, 1, 1)$
10 : $P_{762} = (9, 14, 1, 1)$
11 : $P_{779} = (10, 15, 1, 1)$
12 : $P_{790} = (5, 0, 2, 1)$
13 : $P_{808} = (7, 1, 2, 1)$
14 : $P_{815} = (14, 1, 2, 1)$
15 : $P_{870} = (5, 5, 2, 1)$
16 : $P_{888} = (7, 6, 2, 1)$
17 : $P_{897} = (0, 7, 2, 1)$
18 : $P_{910} = (13, 7, 2, 1)$
19 : $P_{958} = (13, 10, 2, 1)$
20 : $P_{965} = (4, 11, 2, 1)$
21 : $P_{1029} = (4, 15, 2, 1)$
22 : $P_{1039} = (14, 15, 2, 1)$
23 : $P_{1048} = (7, 0, 3, 1)$
24 : $P_{1113} = (8, 4, 3, 1)$
25 : $P_{1119} = (14, 4, 3, 1)$
26 : $P_{1121} = (0, 5, 3, 1)$
27 : $P_{1132} = (11, 5, 3, 1)$
28 : $P_{1160} = (7, 7, 3, 1)$
29 : $P_{1168} = (15, 7, 3, 1)$
30 : $P_{1172} = (3, 8, 3, 1)$
31 : $P_{1184} = (15, 8, 3, 1)$
32 : $P_{1215} = (14, 10, 3, 1)$
33 : $P_{1220} = (3, 11, 3, 1)$
34 : $P_{1241} = (8, 12, 3, 1)$
35 : $P_{1276} = (11, 14, 3, 1)$
36 : $P_{1305} = (8, 0, 4, 1)$
37 : $P_{1315} = (2, 1, 4, 1)$
38 : $P_{1325} = (12, 1, 4, 1)$
39 : $P_{1347} = (2, 3, 4, 1)$
40 : $P_{1354} = (9, 3, 4, 1)$
41 : $P_{1433} = (8, 8, 4, 1)$
42 : $P_{1466} = (9, 10, 4, 1)$
43 : $P_{1480} = (7, 11, 4, 1)$
44 : $P_{1489} = (0, 12, 4, 1)$
45 : $P_{1496} = (7, 12, 4, 1)$
46 : $P_{1517} = (12, 13, 4, 1)$
47 : $P_{1565} = (12, 0, 5, 1)$
48 : $P_{1595} = (10, 2, 5, 1)$
49 : $P_{1664} = (15, 6, 5, 1)$
50 : $P_{1681} = (0, 8, 5, 1)$
51 : $P_{1691} = (10, 8, 5, 1)$
52 : $P_{1699} = (2, 9, 5, 1)$
53 : $P_{1712} = (15, 9, 5, 1)$
54 : $P_{1718} = (5, 10, 5, 1)$
55 : $P_{1731} = (2, 11, 5, 1)$
56 : $P_{1748} = (3, 12, 5, 1)$
57 : $P_{1757} = (12, 12, 5, 1)$
58 : $P_{1796} = (3, 15, 5, 1)$
59 : $P_{1798} = (5, 15, 5, 1)$
60 : $P_{1818} = (9, 0, 6, 1)$
61 : $P_{1841} = (0, 2, 6, 1)$
62 : $P_{1863} = (6, 3, 6, 1)$
63 : $P_{1871} = (14, 3, 6, 1)$
64 : $P_{1895} = (6, 5, 6, 1)$
65 : $P_{1931} = (10, 7, 6, 1)$
66 : $P_{1933} = (12, 7, 6, 1)$
67 : $P_{1956} = (3, 9, 6, 1)$
68 : $P_{1962} = (9, 9, 6, 1)$
69 : $P_{1972} = (3, 10, 6, 1)$
70 : $P_{1997} = (12, 11, 6, 1)$
71 : $P_{2027} = (10, 13, 6, 1)$
72 : $P_{2031} = (14, 13, 6, 1)$
73 : $P_{2067} = (2, 0, 7, 1)$
74 : $P_{2099} = (2, 2, 7, 1)$
75 : $P_{2105} = (8, 2, 7, 1)$
76 : $P_{2171} = (10, 6, 7, 1)$
77 : $P_{2174} = (13, 6, 7, 1)$
78 : $P_{2197} = (4, 8, 7, 1)$
79 : $P_{2200} = (7, 8, 7, 1)$
80 : $P_{2209} = (0, 9, 7, 1)$
81 : $P_{2233} = (8, 10, 7, 1)$
82 : $P_{2254} = (13, 11, 7, 1)$
83 : $P_{2261} = (4, 12, 7, 1)$
84 : $P_{2267} = (10, 12, 7, 1)$
85 : $P_{2312} = (7, 15, 7, 1)$
86 : $P_{2327} = (6, 0, 8, 1)$
87 : $P_{2374} = (5, 3, 8, 1)$
88 : $P_{2377} = (8, 3, 8, 1)$
89 : $P_{2396} = (11, 4, 8, 1)$
90 : $P_{2422} = (5, 6, 8, 1)$
91 : $P_{2423} = (6, 6, 8, 1)$
92 : $P_{2485} = (4, 10, 8, 1)$
93 : $P_{2505} = (8, 11, 8, 1)$
94 : $P_{2532} = (3, 13, 8, 1)$
95 : $P_{2548} = (3, 14, 8, 1)$

96 : $P_{2549} = (4, 14, 8, 1)$
97 : $P_{2561} = (0, 15, 8, 1)$
98 : $P_{2572} = (11, 15, 8, 1)$
99 : $P_{2592} = (15, 0, 9, 1)$
100 : $P_{2597} = (4, 1, 9, 1)$
101 : $P_{2599} = (6, 1, 9, 1)$
102 : $P_{2661} = (4, 5, 9, 1)$
103 : $P_{2671} = (14, 5, 9, 1)$
104 : $P_{2673} = (0, 6, 9, 1)$
105 : $P_{2685} = (12, 6, 9, 1)$
106 : $P_{2695} = (6, 7, 9, 1)$
107 : $P_{2749} = (12, 10, 9, 1)$
108 : $P_{2767} = (14, 11, 9, 1)$
109 : $P_{2832} = (15, 15, 9, 1)$
110 : $P_{2852} = (3, 1, 10, 1)$
111 : $P_{2857} = (8, 1, 10, 1)$
112 : $P_{2868} = (3, 2, 10, 1)$
113 : $P_{2877} = (12, 2, 10, 1)$
114 : $P_{2907} = (10, 4, 10, 1)$
115 : $P_{2910} = (13, 4, 10, 1)$
116 : $P_{2985} = (8, 9, 10, 1)$
117 : $P_{2990} = (13, 9, 10, 1)$
118 : $P_{2993} = (0, 10, 10, 1)$
119 : $P_{3067} = (10, 14, 10, 1)$
120 : $P_{3069} = (12, 14, 10, 1)$
121 : $P_{3110} = (5, 1, 11, 1)$
122 : $P_{3120} = (15, 1, 11, 1)$
123 : $P_{3127} = (6, 2, 11, 1)$
124 : $P_{3132} = (11, 2, 11, 1)$
125 : $P_{3158} = (5, 4, 11, 1)$
126 : $P_{3159} = (6, 4, 11, 1)$
127 : $P_{3240} = (7, 9, 11, 1)$
128 : $P_{3244} = (11, 9, 11, 1)$
129 : $P_{3265} = (0, 11, 11, 1)$
130 : $P_{3320} = (7, 14, 11, 1)$
131 : $P_{3328} = (15, 14, 11, 1)$
132 : $P_{3349} = (4, 0, 12, 1)$
133 : $P_{3405} = (12, 3, 12, 1)$
134 : $P_{3413} = (4, 4, 12, 1)$
135 : $P_{3424} = (15, 4, 12, 1)$
136 : $P_{3450} = (9, 6, 12, 1)$
137 : $P_{3452} = (11, 6, 12, 1)$
138 : $P_{3512} = (7, 10, 12, 1)$
139 : $P_{3536} = (15, 11, 12, 1)$
140 : $P_{3560} = (7, 13, 12, 1)$
141 : $P_{3564} = (11, 13, 12, 1)$
142 : $P_{3569} = (0, 14, 12, 1)$
143 : $P_{3594} = (9, 15, 12, 1)$
144 : $P_{3597} = (12, 15, 12, 1)$
145 : $P_{3615} = (14, 0, 13, 1)$
146 : $P_{3665} = (0, 4, 13, 1)$
147 : $P_{3683} = (2, 5, 13, 1)$
148 : $P_{3694} = (13, 5, 13, 1)$
149 : $P_{3715} = (2, 7, 13, 1)$
150 : $P_{3724} = (11, 7, 13, 1)$
151 : $P_{3742} = (13, 8, 13, 1)$
152 : $P_{3767} = (6, 10, 13, 1)$
153 : $P_{3782} = (5, 11, 13, 1)$
154 : $P_{3799} = (6, 12, 13, 1)$
155 : $P_{3804} = (11, 12, 13, 1)$
156 : $P_{3830} = (5, 14, 13, 1)$
157 : $P_{3839} = (14, 14, 13, 1)$
158 : $P_{3860} = (3, 0, 14, 1)$
159 : $P_{3882} = (9, 1, 14, 1)$
160 : $P_{3886} = (13, 1, 14, 1)$
161 : $P_{3908} = (3, 3, 14, 1)$
162 : $P_{3987} = (2, 8, 14, 1)$
163 : $P_{3994} = (9, 8, 14, 1)$
164 : $P_{4019} = (2, 10, 14, 1)$
165 : $P_{4039} = (6, 11, 14, 1)$
166 : $P_{4062} = (13, 12, 14, 1)$
167 : $P_{4065} = (0, 13, 14, 1)$
168 : $P_{4071} = (6, 13, 14, 1)$
169 : $P_{4126} = (13, 0, 15, 1)$
170 : $P_{4150} = (5, 2, 15, 1)$
171 : $P_{4154} = (9, 2, 15, 1)$
172 : $P_{4161} = (0, 3, 15, 1)$
173 : $P_{4171} = (10, 3, 15, 1)$
174 : $P_{4201} = (8, 5, 15, 1)$
175 : $P_{4208} = (15, 5, 15, 1)$
176 : $P_{4230} = (5, 7, 15, 1)$
177 : $P_{4267} = (10, 9, 15, 1)$
178 : $P_{4288} = (15, 10, 15, 1)$
179 : $P_{4298} = (9, 11, 15, 1)$
180 : $P_{4329} = (8, 13, 15, 1)$
181 : $P_{4334} = (13, 13, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7
0	0 1 1 1 0 0 0 0
1	1 0 1 0 1 1 1 0
2	1 1 0 0 1 1 0 1
3	1 0 0 0 1 1 1 1
4	0 1 1 1 0 1 0 1
5	0 1 1 1 1 0 1 0
6	0 1 0 1 0 1 0 1
7	0 0 1 1 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_{15}	P_{14}	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_{15}	P_2	P_2	P_2	P_{180}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_{14}	P_2	P_2	P_2	P_{196}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{435}	P_{451}	P_{451}	P_{435}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7
in point	P_2	P_2	P_{435}	P_2	P_{435}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_2	P_2	P_{451}	P_2	P_{451}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7
in point	P_{180}	P_{451}	P_{451}	P_{546}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_{196}	P_{435}	P_{435}	P_{546}

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_6 = (2, 1, 0, 0)$
5 : $P_7 = (3, 1, 0, 0)$
6 : $P_8 = (4, 1, 0, 0)$
7 : $P_9 = (5, 1, 0, 0)$
8 : $P_{10} = (6, 1, 0, 0)$
9 : $P_{11} = (7, 1, 0, 0)$
10 : $P_{12} = (8, 1, 0, 0)$
11 : $P_{13} = (9, 1, 0, 0)$
12 : $P_{14} = (10, 1, 0, 0)$
13 : $P_{15} = (11, 1, 0, 0)$
14 : $P_{16} = (12, 1, 0, 0)$
15 : $P_{17} = (13, 1, 0, 0)$
16 : $P_{18} = (14, 1, 0, 0)$
17 : $P_{19} = (15, 1, 0, 0)$
18 : $P_{45} = (10, 1, 1, 0)$
19 : $P_{46} = (11, 1, 1, 0)$
20 : $P_{64} = (13, 2, 1, 0)$
21 : $P_{66} = (15, 2, 1, 0)$
22 : $P_{71} = (4, 3, 1, 0)$
23 : $P_{74} = (7, 3, 1, 0)$
24 : $P_{86} = (3, 4, 1, 0)$
25 : $P_{90} = (7, 4, 1, 0)$
26 : $P_{108} = (9, 5, 1, 0)$
27 : $P_{111} = (12, 5, 1, 0)$
28 : $P_{123} = (8, 6, 1, 0)$
29 : $P_{129} = (14, 6, 1, 0)$
30 : $P_{134} = (3, 7, 1, 0)$
31 : $P_{135} = (4, 7, 1, 0)$
32 : $P_{153} = (6, 8, 1, 0)$

33 : $P_{161} = (14, 8, 1, 0)$
34 : $P_{168} = (5, 9, 1, 0)$
35 : $P_{175} = (12, 9, 1, 0)$
36 : $P_{180} = (1, 10, 1, 0)$
37 : $P_{190} = (11, 10, 1, 0)$
38 : $P_{196} = (1, 11, 1, 0)$
39 : $P_{205} = (10, 11, 1, 0)$
40 : $P_{216} = (5, 12, 1, 0)$
41 : $P_{220} = (9, 12, 1, 0)$
42 : $P_{229} = (2, 13, 1, 0)$
43 : $P_{242} = (15, 13, 1, 0)$
44 : $P_{249} = (6, 14, 1, 0)$
45 : $P_{251} = (8, 14, 1, 0)$
46 : $P_{261} = (2, 15, 1, 0)$
47 : $P_{272} = (13, 15, 1, 0)$
48 : $P_{275} = (1, 0, 0, 1)$
49 : $P_{291} = (1, 1, 0, 1)$
50 : $P_{307} = (1, 2, 0, 1)$
51 : $P_{323} = (1, 3, 0, 1)$
52 : $P_{339} = (1, 4, 0, 1)$
53 : $P_{355} = (1, 5, 0, 1)$
54 : $P_{371} = (1, 6, 0, 1)$
55 : $P_{387} = (1, 7, 0, 1)$
56 : $P_{403} = (1, 8, 0, 1)$
57 : $P_{419} = (1, 9, 0, 1)$
58 : $P_{435} = (1, 10, 0, 1)$
59 : $P_{451} = (1, 11, 0, 1)$
60 : $P_{467} = (1, 12, 0, 1)$
61 : $P_{483} = (1, 13, 0, 1)$
62 : $P_{499} = (1, 14, 0, 1)$
63 : $P_{515} = (1, 15, 0, 1)$
64 : $P_{546} = (0, 1, 1, 1)$
65 : $P_{575} = (14, 2, 1, 1)$
66 : $P_{588} = (11, 3, 1, 1)$
67 : $P_{595} = (2, 4, 1, 1)$
68 : $P_{619} = (10, 5, 1, 1)$
69 : $P_{627} = (2, 6, 1, 1)$
70 : $P_{650} = (9, 7, 1, 1)$
71 : $P_{668} = (11, 8, 1, 1)$
72 : $P_{677} = (4, 9, 1, 1)$
73 : $P_{690} = (1, 10, 1, 1)$
74 : $P_{706} = (1, 11, 1, 1)$
75 : $P_{735} = (14, 12, 1, 1)$
76 : $P_{741} = (4, 13, 1, 1)$
77 : $P_{762} = (9, 14, 1, 1)$
78 : $P_{779} = (10, 15, 1, 1)$
79 : $P_{790} = (5, 0, 2, 1)$
80 : $P_{808} = (7, 1, 2, 1)$
81 : $P_{815} = (14, 1, 2, 1)$
82 : $P_{868} = (3, 5, 2, 1)$
83 : $P_{870} = (5, 5, 2, 1)$
84 : $P_{884} = (3, 6, 2, 1)$
85 : $P_{888} = (7, 6, 2, 1)$
86 : $P_{897} = (0, 7, 2, 1)$
87 : $P_{910} = (13, 7, 2, 1)$
88 : $P_{946} = (1, 10, 2, 1)$
89 : $P_{958} = (13, 10, 2, 1)$
90 : $P_{962} = (1, 11, 2, 1)$
91 : $P_{965} = (4, 11, 2, 1)$
92 : $P_{1029} = (4, 15, 2, 1)$
93 : $P_{1039} = (14, 15, 2, 1)$
94 : $P_{1048} = (7, 0, 3, 1)$
95 : $P_{1113} = (8, 4, 3, 1)$
96 : $P_{1119} = (14, 4, 3, 1)$
97 : $P_{1121} = (0, 5, 3, 1)$
98 : $P_{1132} = (11, 5, 3, 1)$
99 : $P_{1160} = (7, 7, 3, 1)$
100 : $P_{1168} = (15, 7, 3, 1)$
101 : $P_{1172} = (3, 8, 3, 1)$
102 : $P_{1184} = (15, 8, 3, 1)$
103 : $P_{1202} = (1, 10, 3, 1)$
104 : $P_{1215} = (14, 10, 3, 1)$
105 : $P_{1218} = (1, 11, 3, 1)$
106 : $P_{1220} = (3, 11, 3, 1)$
107 : $P_{1235} = (2, 12, 3, 1)$
108 : $P_{1241} = (8, 12, 3, 1)$
109 : $P_{1267} = (2, 14, 3, 1)$
110 : $P_{1276} = (11, 14, 3, 1)$
111 : $P_{1305} = (8, 0, 4, 1)$
112 : $P_{1315} = (2, 1, 4, 1)$
113 : $P_{1325} = (12, 1, 4, 1)$
114 : $P_{1347} = (2, 3, 4, 1)$
115 : $P_{1354} = (9, 3, 4, 1)$
116 : $P_{1430} = (5, 8, 4, 1)$
117 : $P_{1433} = (8, 8, 4, 1)$
118 : $P_{1458} = (1, 10, 4, 1)$
119 : $P_{1466} = (9, 10, 4, 1)$
120 : $P_{1474} = (1, 11, 4, 1)$
121 : $P_{1480} = (7, 11, 4, 1)$
122 : $P_{1489} = (0, 12, 4, 1)$
123 : $P_{1496} = (7, 12, 4, 1)$
124 : $P_{1510} = (5, 13, 4, 1)$
125 : $P_{1517} = (12, 13, 4, 1)$
126 : $P_{1565} = (12, 0, 5, 1)$
127 : $P_{1589} = (4, 2, 5, 1)$
128 : $P_{1595} = (10, 2, 5, 1)$
129 : $P_{1653} = (4, 6, 5, 1)$
130 : $P_{1664} = (15, 6, 5, 1)$
131 : $P_{1681} = (0, 8, 5, 1)$
132 : $P_{1691} = (10, 8, 5, 1)$
133 : $P_{1699} = (2, 9, 5, 1)$
134 : $P_{1712} = (15, 9, 5, 1)$
135 : $P_{1714} = (1, 10, 5, 1)$
136 : $P_{1718} = (5, 10, 5, 1)$
137 : $P_{1730} = (1, 11, 5, 1)$
138 : $P_{1731} = (2, 11, 5, 1)$
139 : $P_{1748} = (3, 12, 5, 1)$
140 : $P_{1757} = (12, 12, 5, 1)$
141 : $P_{1796} = (3, 15, 5, 1)$
142 : $P_{1798} = (5, 15, 5, 1)$
143 : $P_{1818} = (9, 0, 6, 1)$
144 : $P_{1841} = (0, 2, 6, 1)$
145 : $P_{1848} = (7, 2, 6, 1)$
146 : $P_{1863} = (6, 3, 6, 1)$
147 : $P_{1871} = (14, 3, 6, 1)$
148 : $P_{1895} = (6, 5, 6, 1)$
149 : $P_{1896} = (7, 5, 6, 1)$
150 : $P_{1931} = (10, 7, 6, 1)$
151 : $P_{1933} = (12, 7, 6, 1)$
152 : $P_{1956} = (3, 9, 6, 1)$
153 : $P_{1962} = (9, 9, 6, 1)$
154 : $P_{1970} = (1, 10, 6, 1)$
155 : $P_{1972} = (3, 10, 6, 1)$
156 : $P_{1986} = (1, 11, 6, 1)$
157 : $P_{1997} = (12, 11, 6, 1)$
158 : $P_{2027} = (10, 13, 6, 1)$
159 : $P_{2031} = (14, 13, 6, 1)$
160 : $P_{2067} = (2, 0, 7, 1)$
161 : $P_{2099} = (2, 2, 7, 1)$
162 : $P_{2105} = (8, 2, 7, 1)$
163 : $P_{2171} = (10, 6, 7, 1)$
164 : $P_{2174} = (13, 6, 7, 1)$
165 : $P_{2197} = (4, 8, 7, 1)$
166 : $P_{2200} = (7, 8, 7, 1)$
167 : $P_{2209} = (0, 9, 7, 1)$
168 : $P_{2215} = (6, 9, 7, 1)$
169 : $P_{2226} = (1, 10, 7, 1)$
170 : $P_{2233} = (8, 10, 7, 1)$
171 : $P_{2242} = (1, 11, 7, 1)$
172 : $P_{2254} = (13, 11, 7, 1)$
173 : $P_{2261} = (4, 12, 7, 1)$
174 : $P_{2267} = (10, 12, 7, 1)$
175 : $P_{2311} = (6, 15, 7, 1)$
176 : $P_{2312} = (7, 15, 7, 1)$
177 : $P_{2327} = (6, 0, 8, 1)$
178 : $P_{2374} = (5, 3, 8, 1)$
179 : $P_{2377} = (8, 3, 8, 1)$
180 : $P_{2394} = (9, 4, 8, 1)$
181 : $P_{2396} = (11, 4, 8, 1)$
182 : $P_{2422} = (5, 6, 8, 1)$
183 : $P_{2423} = (6, 6, 8, 1)$
184 : $P_{2482} = (1, 10, 8, 1)$
185 : $P_{2485} = (4, 10, 8, 1)$
186 : $P_{2498} = (1, 11, 8, 1)$
187 : $P_{2505} = (8, 11, 8, 1)$
188 : $P_{2532} = (3, 13, 8, 1)$
189 : $P_{2538} = (9, 13, 8, 1)$
190 : $P_{2548} = (3, 14, 8, 1)$
191 : $P_{2549} = (4, 14, 8, 1)$
192 : $P_{2561} = (0, 15, 8, 1)$
193 : $P_{2572} = (11, 15, 8, 1)$
194 : $P_{2592} = (15, 0, 9, 1)$

195 : $P_{2597} = (4, 1, 9, 1)$
196 : $P_{2599} = (6, 1, 9, 1)$
197 : $P_{2661} = (4, 5, 9, 1)$
198 : $P_{2671} = (14, 5, 9, 1)$
199 : $P_{2673} = (0, 6, 9, 1)$
200 : $P_{2685} = (12, 6, 9, 1)$
201 : $P_{2695} = (6, 7, 9, 1)$
202 : $P_{2697} = (8, 7, 9, 1)$
203 : $P_{2738} = (1, 10, 9, 1)$
204 : $P_{2749} = (12, 10, 9, 1)$
205 : $P_{2754} = (1, 11, 9, 1)$
206 : $P_{2767} = (14, 11, 9, 1)$
207 : $P_{2825} = (8, 15, 9, 1)$
208 : $P_{2832} = (15, 15, 9, 1)$
209 : $P_{2844} = (11, 0, 10, 1)$
210 : $P_{2852} = (3, 1, 10, 1)$
211 : $P_{2857} = (8, 1, 10, 1)$
212 : $P_{2868} = (3, 2, 10, 1)$
213 : $P_{2877} = (12, 2, 10, 1)$
214 : $P_{2907} = (10, 4, 10, 1)$
215 : $P_{2910} = (13, 4, 10, 1)$
216 : $P_{2985} = (8, 9, 10, 1)$
217 : $P_{2990} = (13, 9, 10, 1)$
218 : $P_{2993} = (0, 10, 10, 1)$
219 : $P_{2994} = (1, 10, 10, 1)$
220 : $P_{3010} = (1, 11, 10, 1)$
221 : $P_{3020} = (11, 11, 10, 1)$
222 : $P_{3067} = (10, 14, 10, 1)$
223 : $P_{3069} = (12, 14, 10, 1)$
224 : $P_{3099} = (10, 0, 11, 1)$
225 : $P_{3110} = (5, 1, 11, 1)$
226 : $P_{3120} = (15, 1, 11, 1)$
227 : $P_{3127} = (6, 2, 11, 1)$
228 : $P_{3132} = (11, 2, 11, 1)$
229 : $P_{3158} = (5, 4, 11, 1)$
230 : $P_{3159} = (6, 4, 11, 1)$
231 : $P_{3240} = (7, 9, 11, 1)$
232 : $P_{3244} = (11, 9, 11, 1)$
233 : $P_{3250} = (1, 10, 11, 1)$
234 : $P_{3259} = (10, 10, 11, 1)$
235 : $P_{3265} = (0, 11, 11, 1)$
236 : $P_{3266} = (1, 11, 11, 1)$
237 : $P_{3320} = (7, 14, 11, 1)$
238 : $P_{3328} = (15, 14, 11, 1)$
239 : $P_{3349} = (4, 0, 12, 1)$
240 : $P_{3405} = (12, 3, 12, 1)$
241 : $P_{3406} = (13, 3, 12, 1)$
242 : $P_{3413} = (4, 4, 12, 1)$
243 : $P_{3424} = (15, 4, 12, 1)$
244 : $P_{3450} = (9, 6, 12, 1)$
245 : $P_{3452} = (11, 6, 12, 1)$
246 : $P_{3506} = (1, 10, 12, 1)$
247 : $P_{3512} = (7, 10, 12, 1)$
248 : $P_{3522} = (1, 11, 12, 1)$
249 : $P_{3536} = (15, 11, 12, 1)$
250 : $P_{3560} = (7, 13, 12, 1)$
251 : $P_{3564} = (11, 13, 12, 1)$
252 : $P_{3569} = (0, 14, 12, 1)$
253 : $P_{3582} = (13, 14, 12, 1)$
254 : $P_{3594} = (9, 15, 12, 1)$
255 : $P_{3597} = (12, 15, 12, 1)$
256 : $P_{3615} = (14, 0, 13, 1)$
257 : $P_{3665} = (0, 4, 13, 1)$
258 : $P_{3677} = (12, 4, 13, 1)$
259 : $P_{3683} = (2, 5, 13, 1)$
260 : $P_{3694} = (13, 5, 13, 1)$
261 : $P_{3715} = (2, 7, 13, 1)$
262 : $P_{3724} = (11, 7, 13, 1)$
263 : $P_{3741} = (12, 8, 13, 1)$
264 : $P_{3742} = (13, 8, 13, 1)$
265 : $P_{3762} = (1, 10, 13, 1)$
266 : $P_{3767} = (6, 10, 13, 1)$
267 : $P_{3778} = (1, 11, 13, 1)$
268 : $P_{3782} = (5, 11, 13, 1)$
269 : $P_{3799} = (6, 12, 13, 1)$
270 : $P_{3804} = (11, 12, 13, 1)$
271 : $P_{3830} = (5, 14, 13, 1)$
272 : $P_{3839} = (14, 14, 13, 1)$
273 : $P_{3860} = (3, 0, 14, 1)$
274 : $P_{3882} = (9, 1, 14, 1)$
275 : $P_{3886} = (13, 1, 14, 1)$
276 : $P_{3908} = (3, 3, 14, 1)$
277 : $P_{3920} = (15, 3, 14, 1)$
278 : $P_{3987} = (2, 8, 14, 1)$
279 : $P_{3994} = (9, 8, 14, 1)$
280 : $P_{4018} = (1, 10, 14, 1)$
281 : $P_{4019} = (2, 10, 14, 1)$
282 : $P_{4034} = (1, 11, 14, 1)$
283 : $P_{4039} = (6, 11, 14, 1)$
284 : $P_{4062} = (13, 12, 14, 1)$
285 : $P_{4064} = (15, 12, 14, 1)$
286 : $P_{4065} = (0, 13, 14, 1)$
287 : $P_{4071} = (6, 13, 14, 1)$
288 : $P_{4126} = (13, 0, 15, 1)$
289 : $P_{4150} = (5, 2, 15, 1)$
290 : $P_{4154} = (9, 2, 15, 1)$
291 : $P_{4161} = (0, 3, 15, 1)$
292 : $P_{4171} = (10, 3, 15, 1)$
293 : $P_{4201} = (8, 5, 15, 1)$
294 : $P_{4208} = (15, 5, 15, 1)$
295 : $P_{4230} = (5, 7, 15, 1)$
296 : $P_{4239} = (14, 7, 15, 1)$
297 : $P_{4267} = (10, 9, 15, 1)$
298 : $P_{4271} = (14, 9, 15, 1)$
299 : $P_{4274} = (1, 10, 15, 1)$
300 : $P_{4288} = (15, 10, 15, 1)$
301 : $P_{4290} = (1, 11, 15, 1)$
302 : $P_{4298} = (9, 11, 15, 1)$
303 : $P_{4329} = (8, 13, 15, 1)$
304 : $P_{4334} = (13, 13, 15, 1)$