

# Rank-76389 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(8) is 1361646165

## General information

Number of lines	2
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	40
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$9^2$
Type of lines on points	$2, 1^{16}, 0^{40}$

## Singular Points

The surface has 0 singular points:

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4744} = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_1 &= \left[ \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]_{722} = \left[ \begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]_{722} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{1330}\end{aligned}$$

Rank of lines: ( 4744, 722 )  
Rank of points on Klein quadric: ( 1, 1330 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 1 Double points:  
The double points on the surface are:

$$P_{138} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

### Single Points

The surface has 16 single points:  
The single points on the surface are:

- |  |   |
|--|---|
| 0 : $P_2 = (0, 0, 1, 0)$ lies on line $\ell_0$     | 9 : $P_{374} = (5, 5, 4, 1)$ lies on line $\ell_1$  |
| 1 : $P_3 = (0, 0, 0, 1)$ lies on line $\ell_0$     | 10 : $P_{393} = (0, 0, 5, 1)$ lies on line $\ell_0$ |
| 2 : $P_{20} = (1, 1, 1, 0)$ lies on line $\ell_1$  | 11 : $P_{429} = (4, 4, 5, 1)$ lies on line $\ell_1$ |
| 3 : $P_{83} = (1, 1, 0, 1)$ lies on line $\ell_1$  | 12 : $P_{457} = (0, 0, 6, 1)$ lies on line $\ell_0$ |
| 4 : $P_{201} = (0, 0, 2, 1)$ lies on line $\ell_0$ | 13 : $P_{520} = (7, 7, 6, 1)$ lies on line $\ell_1$ |
| 5 : $P_{228} = (3, 3, 2, 1)$ lies on line $\ell_1$ | 14 : $P_{521} = (0, 0, 7, 1)$ lies on line $\ell_0$ |
| 6 : $P_{265} = (0, 0, 3, 1)$ lies on line $\ell_0$ | 15 : $P_{575} = (6, 6, 7, 1)$ lies on line $\ell_1$ |
| 7 : $P_{283} = (2, 2, 3, 1)$ lies on line $\ell_1$ |   |
| 8 : $P_{329} = (0, 0, 4, 1)$ lies on line $\ell_0$ |   |

The single points on the surface are:

### Points on surface but on no line

The surface has 40 points not on any line:  
The points on the surface but not on lines are:

- |                               |                               |
|-------------------------------|-------------------------------|
| 0 : $P_0 = (1, 0, 0, 0)$      | 13 : $P_{271} = (6, 0, 3, 1)$ |
| 1 : $P_{19} = (0, 1, 1, 0)$   | 14 : $P_{288} = (7, 2, 3, 1)$ |
| 2 : $P_{75} = (1, 0, 0, 1)$   | 15 : $P_{298} = (1, 4, 3, 1)$ |
| 3 : $P_{82} = (0, 1, 0, 1)$   | 16 : $P_{313} = (0, 6, 3, 1)$ |
| 4 : $P_{154} = (1, 2, 1, 1)$  | 17 : $P_{316} = (3, 6, 3, 1)$ |
| 5 : $P_{163} = (2, 3, 1, 1)$  | 18 : $P_{321} = (0, 7, 3, 1)$ |
| 6 : $P_{170} = (1, 4, 1, 1)$  | 19 : $P_{325} = (4, 7, 3, 1)$ |
| 7 : $P_{181} = (4, 5, 1, 1)$  | 20 : $P_{336} = (7, 0, 4, 1)$ |
| 8 : $P_{192} = (7, 6, 1, 1)$  | 21 : $P_{355} = (2, 3, 4, 1)$ |
| 9 : $P_{194} = (1, 7, 1, 1)$  | 22 : $P_{375} = (6, 5, 4, 1)$ |
| 10 : $P_{205} = (4, 0, 2, 1)$ | 23 : $P_{396} = (3, 0, 5, 1)$ |
| 11 : $P_{230} = (5, 3, 2, 1)$ | 24 : $P_{409} = (0, 2, 5, 1)$ |
| 12 : $P_{256} = (7, 6, 2, 1)$ | 25 : $P_{416} = (7, 2, 5, 1)$ |

26 :  $P_{417} = (0, 3, 5, 1)$   
 27 :  $P_{422} = (5, 3, 5, 1)$   
 28 :  $P_{427} = (2, 4, 5, 1)$   
 29 :  $P_{450} = (1, 7, 5, 1)$   
 30 :  $P_{462} = (5, 0, 6, 1)$   
 31 :  $P_{474} = (1, 2, 6, 1)$   
 32 :  $P_{489} = (0, 4, 6, 1)$   
 33 :  $P_{491} = (2, 4, 6, 1)$

34 :  $P_{497} = (0, 5, 6, 1)$   
 35 :  $P_{503} = (6, 5, 6, 1)$   
 36 :  $P_{517} = (4, 7, 6, 1)$   
 37 :  $P_{523} = (2, 0, 7, 1)$   
 38 :  $P_{565} = (4, 5, 7, 1)$   
 39 :  $P_{572} = (3, 6, 7, 1)$

## Line Intersection Graph

	0 1
0	0 1
1	1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$
in point	$P_{138}$

Line 1 intersects

Line	$\ell_0$
in point	$P_{138}$

The surface has 57 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_2 = (0, 0, 1, 0)$   
 2 :  $P_3 = (0, 0, 0, 1)$   
 3 :  $P_{19} = (0, 1, 1, 0)$   
 4 :  $P_{20} = (1, 1, 1, 0)$   
 5 :  $P_{75} = (1, 0, 0, 1)$   
 6 :  $P_{82} = (0, 1, 0, 1)$   
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 56 :  $P_{575} = (6, 6, 7, 1)$