Rank-65547 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1091047493

General information

Number of lines	3
Number of points	4161
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	189
Number of points off lines	3969
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^3, 1^{189}, 0^{3969}

Singular Points

The surface has 3 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
$2: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{4096} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{17043456} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{17043456} = \mathbf{Pl}(0,0,0,0,0,1)_{270401}$$

Rank of lines: (0, 4096, 17043456)
Rank of points on Klein quadric: (0, 2, 270401)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 189 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
2 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
3 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
4 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
5 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
6 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
7 : $P_{12} = (8,1,0,0)$ lies on line ℓ_0
8 : $P_{13} = (9,1,0,0)$ lies on line ℓ_0
9 : $P_{14} = (10,1,0,0)$ lies on line ℓ_0
10 : $P_{15} = (11,1,0,0)$ lies on line ℓ_0
11 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0
12 : $P_{17} = (13,1,0,0)$ lies on line ℓ_0
13 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0
14 : $P_{19} = (15,1,0,0)$ lies on line ℓ_0
15 : $P_{20} = (16,1,0,0)$ lies on line ℓ_0
16 : $P_{21} = (17,1,0,0)$ lies on line ℓ_0
17 : $P_{22} = (18,1,0,0)$ lies on line ℓ_0
18 : $P_{23} = (19,1,0,0)$ lies on line ℓ_0
19 : $P_{24} = (20,1,0,0)$ lies on line ℓ_0
20 : $P_{25} = (21,1,0,0)$ lies on line ℓ_0
21 : $P_{26} = (22,1,0,0)$ lies on line ℓ_0
22 : $P_{27} = (23,1,0,0)$ lies on line ℓ_0
23 : $P_{28} = (24,1,0,0)$ lies on line ℓ_0
24 : $P_{29} = (25,1,0,0)$ lies on line ℓ_0
25 : $P_{30} = (26,1,0,0)$ lies on line ℓ_0
26 : $P_{31} = (27,1,0,0)$ lies on line ℓ_0
27 : $P_{32} = (28,1,0,0)$ lies on line ℓ_0
28 : $P_{33} = (29,1,0,0)$ lies on line ℓ_0
29 : $P_{34} = (30,1,0,0)$ lies on line ℓ_0
30 : $P_{35} = (31,1,0,0)$ lies on line ℓ_0
31 : $P_{36} = (32,1,0,0)$ lies on line ℓ_0
32 : $P_{37} = (33,1,0,0)$ lies on line ℓ_0
33 : $P_{38} = (34,1,0,0)$ lies on line ℓ_0
34 : $P_{39} = (35,1,0,0)$ lies on line ℓ_0
35 : $P_{40} = (36,1,0,0)$ lies on line ℓ_0
36 : $P_{41} = (37,1,0,0)$ lies on line ℓ_0
37 : $P_{42} = (38,1,0,0)$ lies on line ℓ_0
38 : $P_{43} = (39,1,0,0)$ lies on line ℓ_0
39 : $P_{44} = (40,1,0,0)$ lies on line ℓ_0
40 : $P_{45} = (41,1,0,0)$ lies on line ℓ_0
41 : $P_{46} = (42,1,0,0)$ lies on line ℓ_0
42 : $P_{47} = (43,1,0,0)$ lies on line ℓ_0
43 : $P_{48} = (44,1,0,0)$ lies on line ℓ_0
44 : $P_{49} = (45,1,0,0)$ lies on line ℓ_0
45 : $P_{50} = (46,1,0,0)$ lies on line ℓ_0
46 : $P_{51} = (47,1,0,0)$ lies on line ℓ_0
47 : $P_{52} = (48,1,0,0)$ lies on line ℓ_0
48 : $P_{53} = (49,1,0,0)$ lies on line ℓ_0
49 : $P_{54} = (50,1,0,0)$ lies on line ℓ_0
50 : $P_{55} = (51,1,0,0)$ lies on line ℓ_0
51 : $P_{56} = (52,1,0,0)$ lies on line ℓ_0

52 : $P_{57} = (53, 1, 0, 0)$ lies on line ℓ_0
53 : $P_{58} = (54, 1, 0, 0)$ lies on line ℓ_0
54 : $P_{59} = (55, 1, 0, 0)$ lies on line ℓ_0
55 : $P_{60} = (56, 1, 0, 0)$ lies on line ℓ_0
56 : $P_{61} = (57, 1, 0, 0)$ lies on line ℓ_0
57 : $P_{62} = (58, 1, 0, 0)$ lies on line ℓ_0
58 : $P_{63} = (59, 1, 0, 0)$ lies on line ℓ_0
59 : $P_{64} = (60, 1, 0, 0)$ lies on line ℓ_0
60 : $P_{65} = (61, 1, 0, 0)$ lies on line ℓ_0
61 : $P_{66} = (62, 1, 0, 0)$ lies on line ℓ_0
62 : $P_{67} = (63, 1, 0, 0)$ lies on line ℓ_0
63 : $P_{68} = (1, 0, 1, 0)$ lies on line ℓ_1
64 : $P_{69} = (2, 0, 1, 0)$ lies on line ℓ_1
65 : $P_{70} = (3, 0, 1, 0)$ lies on line ℓ_1
66 : $P_{71} = (4, 0, 1, 0)$ lies on line ℓ_1
67 : $P_{72} = (5, 0, 1, 0)$ lies on line ℓ_1
68 : $P_{73} = (6, 0, 1, 0)$ lies on line ℓ_1
69 : $P_{74} = (7, 0, 1, 0)$ lies on line ℓ_1
70 : $P_{75} = (8, 0, 1, 0)$ lies on line ℓ_1
71 : $P_{76} = (9, 0, 1, 0)$ lies on line ℓ_1
72 : $P_{77} = (10, 0, 1, 0)$ lies on line ℓ_1
73 : $P_{78} = (11, 0, 1, 0)$ lies on line ℓ_1
74 : $P_{79} = (12, 0, 1, 0)$ lies on line ℓ_1
75 : $P_{80} = (13, 0, 1, 0)$ lies on line ℓ_1
76 : $P_{81} = (14, 0, 1, 0)$ lies on line ℓ_1
77 : $P_{82} = (15, 0, 1, 0)$ lies on line ℓ_1
78 : $P_{83} = (16, 0, 1, 0)$ lies on line ℓ_1
79 : $P_{84} = (17, 0, 1, 0)$ lies on line ℓ_1
80 : $P_{85} = (18, 0, 1, 0)$ lies on line ℓ_1
81 : $P_{86} = (19, 0, 1, 0)$ lies on line ℓ_1
82 : $P_{87} = (20, 0, 1, 0)$ lies on line ℓ_1
83 : $P_{88} = (21, 0, 1, 0)$ lies on line ℓ_1
84 : $P_{89} = (22, 0, 1, 0)$ lies on line ℓ_1
85 : $P_{90} = (23, 0, 1, 0)$ lies on line ℓ_1
86 : $P_{91} = (24, 0, 1, 0)$ lies on line ℓ_1
87 : $P_{92} = (25, 0, 1, 0)$ lies on line ℓ_1
88 : $P_{93} = (26, 0, 1, 0)$ lies on line ℓ_1
89 : $P_{94} = (27, 0, 1, 0)$ lies on line ℓ_1
90 : $P_{95} = (28, 0, 1, 0)$ lies on line ℓ_1
91 : $P_{96} = (29, 0, 1, 0)$ lies on line ℓ_1
92 : $P_{97} = (30, 0, 1, 0)$ lies on line ℓ_1
93 : $P_{98} = (31, 0, 1, 0)$ lies on line ℓ_1
94 : $P_{99} = (32, 0, 1, 0)$ lies on line ℓ_1
95 : $P_{100} = (33, 0, 1, 0)$ lies on line ℓ_1
96 : $P_{101} = (34, 0, 1, 0)$ lies on line ℓ_1
97 : $P_{102} = (35, 0, 1, 0)$ lies on line ℓ_1
98 : $P_{103} = (36, 0, 1, 0)$ lies on line ℓ_1
99 : $P_{104} = (37, 0, 1, 0)$ lies on line ℓ_1
100 : $P_{105} = (38, 0, 1, 0)$ lies on line ℓ_1
101 : $P_{106} = (39, 0, 1, 0)$ lies on line ℓ_1
102 : $P_{107} = (40, 0, 1, 0)$ lies on line ℓ_1
103 : $P_{108} = (41, 0, 1, 0)$ lies on line ℓ_1
104 : $P_{109} = (42, 0, 1, 0)$ lies on line ℓ_1
105 : $P_{110} = (43, 0, 1, 0)$ lies on line ℓ_1
106 : $P_{111} = (44, 0, 1, 0)$ lies on line ℓ_1
107 : $P_{112} = (45, 0, 1, 0)$ lies on line ℓ_1
108 : $P_{113} = (46, 0, 1, 0)$ lies on line ℓ_1
109 : $P_{114} = (47, 0, 1, 0)$ lies on line ℓ_1
110 : $P_{115} = (48, 0, 1, 0)$ lies on line ℓ_1
111 : $P_{116} = (49, 0, 1, 0)$ lies on line ℓ_1
112 : $P_{117} = (50, 0, 1, 0)$ lies on line ℓ_1
113 : $P_{118} = (51, 0, 1, 0)$ lies on line ℓ_1
114 : $P_{119} = (52, 0, 1, 0)$ lies on line ℓ_1
115 : $P_{120} = (53, 0, 1, 0)$ lies on line ℓ_1
116 : $P_{121} = (54, 0, 1, 0)$ lies on line ℓ_1
117 : $P_{122} = (55, 0, 1, 0)$ lies on line ℓ_1
118 : $P_{123} = (56, 0, 1, 0)$ lies on line ℓ_1
119 : $P_{124} = (57, 0, 1, 0)$ lies on line ℓ_1
120 : $P_{125} = (58, 0, 1, 0)$ lies on line ℓ_1
121 : $P_{126} = (59, 0, 1, 0)$ lies on line ℓ_1
122 : $P_{127} = (60, 0, 1, 0)$ lies on line ℓ_1
123 : $P_{128} = (61, 0, 1, 0)$ lies on line ℓ_1
124 : $P_{129} = (62, 0, 1, 0)$ lies on line ℓ_1
125 : $P_{130} = (63, 0, 1, 0)$ lies on line ℓ_1
126 : $P_{131} = (0, 1, 1, 0)$ lies on line ℓ_2
127 : $P_{195} = (0, 2, 1, 0)$ lies on line ℓ_2
128 : $P_{259} = (0, 3, 1, 0)$ lies on line ℓ_2
129 : $P_{323} = (0, 4, 1, 0)$ lies on line ℓ_2
130 : $P_{387} = (0, 5, 1, 0)$ lies on line ℓ_2
131 : $P_{451} = (0, 6, 1, 0)$ lies on line ℓ_2
132 : $P_{515} = (0, 7, 1, 0)$ lies on line ℓ_2
133 : $P_{579} = (0, 8, 1, 0)$ lies on line ℓ_2
134 : $P_{643} = (0, 9, 1, 0)$ lies on line ℓ_2
135 : $P_{707} = (0, 10, 1, 0)$ lies on line ℓ_2
136 : $P_{771} = (0, 11, 1, 0)$ lies on line ℓ_2
137 : $P_{835} = (0, 12, 1, 0)$ lies on line ℓ_2
138 : $P_{899} = (0, 13, 1, 0)$ lies on line ℓ_2
139 : $P_{963} = (0, 14, 1, 0)$ lies on line ℓ_2
140 : $P_{1027} = (0, 15, 1, 0)$ lies on line ℓ_2
141 : $P_{1091} = (0, 16, 1, 0)$ lies on line ℓ_2
142 : $P_{1155} = (0, 17, 1, 0)$ lies on line ℓ_2
143 : $P_{1219} = (0, 18, 1, 0)$ lies on line ℓ_2
144 : $P_{1283} = (0, 19, 1, 0)$ lies on line ℓ_2
145 : $P_{1347} = (0, 20, 1, 0)$ lies on line ℓ_2
146 : $P_{1411} = (0, 21, 1, 0)$ lies on line ℓ_2
147 : $P_{1475} = (0, 22, 1, 0)$ lies on line ℓ_2
148 : $P_{1539} = (0, 23, 1, 0)$ lies on line ℓ_2
149 : $P_{1603} = (0, 24, 1, 0)$ lies on line ℓ_2
150 : $P_{1667} = (0, 25, 1, 0)$ lies on line ℓ_2
151 : $P_{1731} = (0, 26, 1, 0)$ lies on line ℓ_2
152 : $P_{1795} = (0, 27, 1, 0)$ lies on line ℓ_2
153 : $P_{1859} = (0, 28, 1, 0)$ lies on line ℓ_2
154 : $P_{1923} = (0, 29, 1, 0)$ lies on line ℓ_2
155 : $P_{1987} = (0, 30, 1, 0)$ lies on line ℓ_2
156 : $P_{2051} = (0, 31, 1, 0)$ lies on line ℓ_2
157 : $P_{2115} = (0, 32, 1, 0)$ lies on line ℓ_2
158 : $P_{2179} = (0, 33, 1, 0)$ lies on line ℓ_2
159 : $P_{2243} = (0, 34, 1, 0)$ lies on line ℓ_2

160 : $P_{2307} = (0, 35, 1, 0)$ lies on line ℓ_2
161 : $P_{2371} = (0, 36, 1, 0)$ lies on line ℓ_2
162 : $P_{2435} = (0, 37, 1, 0)$ lies on line ℓ_2
163 : $P_{2499} = (0, 38, 1, 0)$ lies on line ℓ_2
164 : $P_{2563} = (0, 39, 1, 0)$ lies on line ℓ_2
165 : $P_{2627} = (0, 40, 1, 0)$ lies on line ℓ_2
166 : $P_{2691} = (0, 41, 1, 0)$ lies on line ℓ_2
167 : $P_{2755} = (0, 42, 1, 0)$ lies on line ℓ_2
168 : $P_{2819} = (0, 43, 1, 0)$ lies on line ℓ_2
169 : $P_{2883} = (0, 44, 1, 0)$ lies on line ℓ_2
170 : $P_{2947} = (0, 45, 1, 0)$ lies on line ℓ_2
171 : $P_{3011} = (0, 46, 1, 0)$ lies on line ℓ_2
172 : $P_{3075} = (0, 47, 1, 0)$ lies on line ℓ_2
173 : $P_{3139} = (0, 48, 1, 0)$ lies on line ℓ_2
174 : $P_{3203} = (0, 49, 1, 0)$ lies on line ℓ_2
175 : $P_{3267} = (0, 50, 1, 0)$ lies on line ℓ_2
176 : $P_{3331} = (0, 51, 1, 0)$ lies on line ℓ_2
177 : $P_{3395} = (0, 52, 1, 0)$ lies on line ℓ_2
178 : $P_{3459} = (0, 53, 1, 0)$ lies on line ℓ_2
179 : $P_{3523} = (0, 54, 1, 0)$ lies on line ℓ_2
180 : $P_{3587} = (0, 55, 1, 0)$ lies on line ℓ_2
181 : $P_{3651} = (0, 56, 1, 0)$ lies on line ℓ_2
182 : $P_{3715} = (0, 57, 1, 0)$ lies on line ℓ_2
183 : $P_{3779} = (0, 58, 1, 0)$ lies on line ℓ_2
184 : $P_{3843} = (0, 59, 1, 0)$ lies on line ℓ_2
185 : $P_{3907} = (0, 60, 1, 0)$ lies on line ℓ_2
186 : $P_{3971} = (0, 61, 1, 0)$ lies on line ℓ_2
187 : $P_{4035} = (0, 62, 1, 0)$ lies on line ℓ_2
188 : $P_{4099} = (0, 63, 1, 0)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 3969 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_2

The surface has 4161 points:
Too many to print.