

Rank-65743 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(16) is 571548181

General information

Number of lines	21
Number of points	353
Number of singular points	1
Number of Eckardt points	1
Number of double points	72
Number of single points	204
Number of points off lines	75
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{21}
Type of lines on points	$6, 3, 2^{72}, 1^{204}, 0^{75}$

Singular Points

The surface has 1 singular points:

$$0 : P_{275} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{5121}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{69899} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{69899} = \mathbf{Pl}(0, 0, 0, 11, 0, 1)_{5431} \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{69898} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{69898} = \mathbf{Pl}(0, 0, 0, 10, 0, 1)_{5400} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & \delta & \delta^7 \\ 0 & 1 & \delta^3 & \delta^8 \end{bmatrix}_{31354} = \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 8 & 14 \end{bmatrix}_{31354} = \mathbf{Pl}(10, 11, 5, 15, 7, 1)_{35220} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \delta^{12} & \delta^{14} \\ 0 & 1 & \delta^{14} & \delta^4 \end{bmatrix}_{53391} = \begin{bmatrix} 1 & 0 & 3 & 12 \\ 0 & 1 & 12 & 9 \end{bmatrix}_{53391} = \mathbf{Pl}(10, 11, 6, 4, 7, 1)_{35445} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \delta^2 & \delta^{14} \\ 0 & 1 & \delta^6 & \delta \end{bmatrix}_{53555} = \begin{bmatrix} 1 & 0 & 4 & 12 \\ 0 & 1 & 15 & 2 \end{bmatrix}_{53555} = \mathbf{Pl}(11, 10, 8, 3, 12, 1)_{56281} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \delta^9 & \delta^{13} \\ 0 & 1 & \delta^{13} & \delta^8 \end{bmatrix}_{27803} = \begin{bmatrix} 1 & 0 & 5 & 6 \\ 0 & 1 & 6 & 14 \end{bmatrix}_{27803} = \mathbf{Pl}(11, 10, 13, 9, 12, 1)_{57346} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^2 & \delta^{12} \end{bmatrix}_{4420} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{bmatrix}_{4420} = \mathbf{Pl}(10, 11, 11, 8, 1, 0)_{2984} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & \delta^{13} & \delta^{11} \\ 0 & 1 & \delta^8 & \delta^8 \end{bmatrix}_{58660} = \begin{bmatrix} 1 & 0 & 6 & 13 \\ 0 & 1 & 14 & 14 \end{bmatrix}_{58660} = \mathbf{Pl}(1, 1, 4, 6, 10, 1)_{47301} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^8 & \delta^3 \end{bmatrix}_{4510} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 14 & 8 \end{bmatrix}_{4510} = \mathbf{Pl}(10, 11, 11, 3, 1, 0)_{1859} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \delta^7 & \delta^{14} \\ 0 & 1 & \delta^2 & \delta^2 \end{bmatrix}_{54395} = \begin{bmatrix} 1 & 0 & 7 & 12 \\ 0 & 1 & 4 & 4 \end{bmatrix}_{54395} = \mathbf{Pl}(1, 1, 14, 7, 10, 1)_{49416} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \delta^3 & \delta^{11} \\ 0 & 1 & \delta^{11} & \delta \end{bmatrix}_{59013} = \begin{bmatrix} 1 & 0 & 8 & 13 \\ 0 & 1 & 13 & 2 \end{bmatrix}_{59013} = \mathbf{Pl}(10, 11, 7, 14, 6, 1)_{31650} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & \delta^4 & \delta^{13} \\ 0 & 1 & \delta^{12} & \delta^2 \end{bmatrix}_{28732} = \begin{bmatrix} 1 & 0 & 9 & 6 \\ 0 & 1 & 3 & 4 \end{bmatrix}_{28732} = \mathbf{Pl}(10, 11, 15, 5, 6, 1)_{33345} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^5 & \delta^5 \end{bmatrix}_{4555} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 11 & 11 \end{bmatrix}_{4555} = \mathbf{Pl}(1, 1, 1, 10, 1, 0)_{3275} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{10} & \delta^{10} \end{bmatrix}_{4538} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 10 & 10 \end{bmatrix}_{4538} = \mathbf{Pl}(1, 1, 1, 11, 1, 0)_{3500} \\
\ell_{15} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta & \delta^6 \end{bmatrix}_{4610} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 15 \end{bmatrix}_{4610} = \mathbf{Pl}(11, 10, 10, 5, 1, 0)_{2295} \\
\ell_{16} &= \begin{bmatrix} 1 & 0 & \delta^{14} & \delta^{13} \\ 0 & 1 & \delta^4 & \delta^4 \end{bmatrix}_{29637} = \begin{bmatrix} 1 & 0 & 12 & 6 \\ 0 & 1 & 9 & 9 \end{bmatrix}_{29637} = \mathbf{Pl}(1, 1, 2, 12, 11, 1)_{51051} \\
\ell_{17} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^4 & \delta^9 \end{bmatrix}_{4457} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 9 & 5 \end{bmatrix}_{4457} = \mathbf{Pl}(11, 10, 10, 15, 1, 0)_{4545} \\
\ell_{18} &= \begin{bmatrix} 1 & 0 & \delta^{11} & \delta^7 \\ 0 & 1 & \delta & \delta \end{bmatrix}_{34159} = \begin{bmatrix} 1 & 0 & 13 & 7 \\ 0 & 1 & 2 & 2 \end{bmatrix}_{34159} = \mathbf{Pl}(1, 1, 9, 13, 11, 1)_{52536} \\
\ell_{19} &= \begin{bmatrix} 1 & 0 & \delta^8 & \delta^{11} \\ 0 & 1 & \delta^9 & \delta^4 \end{bmatrix}_{60755} = \begin{bmatrix} 1 & 0 & 14 & 13 \\ 0 & 1 & 5 & 9 \end{bmatrix}_{60755} = \mathbf{Pl}(11, 10, 3, 8, 13, 1)_{59461} \\
\ell_{20} &= \begin{bmatrix} 1 & 0 & \delta^6 & \delta^7 \\ 0 & 1 & \delta^7 & \delta^2 \end{bmatrix}_{34742} = \begin{bmatrix} 1 & 0 & 15 & 7 \\ 0 & 1 & 7 & 4 \end{bmatrix}_{34742} = \mathbf{Pl}(11, 10, 12, 2, 13, 1)_{61366}
\end{aligned}$$

Rank of lines: (69889, 69899, 69898, 31354, 53391, 53555, 27803, 4420, 58660, 4510, 54395, 59013, 28732, 4555, 4538, 4610, 29637, 4457, 34159, 60755, 34742)

Rank of points on Klein quadric: (5121, 5431, 5400, 35220, 35445, 56281, 57346, 2984, 47301, 1859, 49416, 31650, 33345, 3275, 3500, 2295, 51051, 4545, 52536, 59461, 61366)

Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0).$$

Double Points

The surface has 72 Double points:

The double points on the surface are:

$$\begin{aligned} P_{641} &= (0, 7, 1, 1) = \ell_0 \cap \ell_8 \\ P_{625} &= (0, 6, 1, 1) = \ell_0 \cap \ell_{10} \\ P_{689} &= (0, 10, 1, 1) = \ell_0 \cap \ell_{13} \\ P_{705} &= (0, 11, 1, 1) = \ell_0 \cap \ell_{14} \\ P_{737} &= (0, 13, 1, 1) = \ell_0 \cap \ell_{16} \\ P_{721} &= (0, 12, 1, 1) = \ell_0 \cap \ell_{18} \\ P_{2945} &= (0, 7, 10, 1) = \ell_1 \cap \ell_3 \\ P_{3041} &= (0, 13, 10, 1) = \ell_1 \cap \ell_4 \\ P_{3025} &= (0, 12, 10, 1) = \ell_1 \cap \ell_{11} \\ P_{2929} &= (0, 6, 10, 1) = \ell_1 \cap \ell_{12} \\ P_{2913} &= (0, 5, 10, 1) = \ell_1 \cap \ell_{15} \\ P_{3073} &= (0, 15, 10, 1) = \ell_1 \cap \ell_{17} \\ P_{3281} &= (0, 12, 11, 1) = \ell_2 \cap \ell_5 \\ P_{3201} &= (0, 7, 11, 1) = \ell_2 \cap \ell_6 \\ P_{3217} &= (0, 8, 11, 1) = \ell_2 \cap \ell_7 \\ P_{3137} &= (0, 3, 11, 1) = \ell_2 \cap \ell_9 \\ P_{3297} &= (0, 13, 11, 1) = \ell_2 \cap \ell_{19} \\ P_{3185} &= (0, 6, 11, 1) = \ell_2 \cap \ell_{20} \\ P_{437} &= (3, 10, 0, 1) = \ell_3 \cap \ell_4 \\ P_{1339} &= (10, 2, 4, 1) = \ell_3 \cap \ell_5 \\ P_{4106} &= (9, 15, 14, 1) = \ell_3 \cap \ell_7 \\ P_{3750} &= (5, 9, 13, 1) = \ell_3 \cap \ell_{10} \\ P_{2135} &= (6, 4, 7, 1) = \ell_3 \cap \ell_{13} \\ P_{1248} &= (15, 12, 3, 1) = \ell_3 \cap \ell_{16} \\ P_{1020} &= (11, 14, 2, 1) = \ell_3 \cap \ell_{19} \\ P_{1403} &= (10, 6, 4, 1) = \ell_4 \cap \ell_6 \\ P_{3670} &= (5, 4, 13, 1) = \ell_4 \cap \ell_8 \\ P_{3882} &= (9, 1, 14, 1) = \ell_4 \cap \ell_9 \\ P_{2119} &= (6, 3, 7, 1) = \ell_4 \cap \ell_{14} \\ P_{1296} &= (15, 15, 3, 1) = \ell_4 \cap \ell_{18} \\ P_{988} &= (11, 12, 2, 1) = \ell_4 \cap \ell_{20} \\ P_{455} &= (5, 11, 0, 1) = \ell_5 \cap \ell_6 \\ P_{1652} &= (3, 6, 5, 1) = \ell_5 \cap \ell_8 \\ P_{2652} &= (11, 4, 9, 1) = \ell_5 \cap \ell_{12} \\ P_{3502} &= (13, 9, 12, 1) = \ell_5 \cap \ell_{14} \\ P_{2297} &= (8, 14, 7, 1) = \ell_5 \cap \ell_{16} \\ P_{847} &= (14, 3, 2, 1) = \ell_5 \cap \ell_{17} \end{aligned}$$

$$\begin{aligned} P_{1604} &= (3, 3, 5, 1) = \ell_6 \cap \ell_{10} \\ P_{2796} &= (11, 13, 9, 1) = \ell_6 \cap \ell_{11} \\ P_{3438} &= (13, 5, 12, 1) = \ell_6 \cap \ell_{13} \\ P_{815} &= (14, 1, 2, 1) = \ell_6 \cap \ell_{15} \\ P_{2217} &= (8, 9, 7, 1) = \ell_6 \cap \ell_{18} \\ P_{125} &= (10, 6, 1, 0) = \ell_7 \cap \ell_8 \\ P_{1315} &= (2, 1, 4, 1) = \ell_7 \cap \ell_{11} \\ P_{3067} &= (10, 14, 10, 1) = \ell_7 \cap \ell_{18} \\ P_{300} &= (10, 1, 0, 1) = \ell_8 \cap \ell_{10} \\ P_{3392} &= (15, 2, 12, 1) = \ell_8 \cap \ell_{12} \\ P_{3244} &= (11, 9, 11, 1) = \ell_8 \cap \ell_{15} \\ P_{4249} &= (8, 8, 15, 1) = \ell_8 \cap \ell_{20} \\ P_{141} &= (10, 7, 1, 0) = \ell_9 \cap \ell_{10} \\ P_{1379} &= (2, 5, 4, 1) = \ell_9 \cap \ell_{12} \\ P_{2907} &= (10, 4, 10, 1) = \ell_9 \cap \ell_{16} \\ P_{3584} &= (15, 14, 12, 1) = \ell_{10} \cap \ell_{11} \\ P_{3132} &= (11, 2, 11, 1) = \ell_{10} \cap \ell_{17} \\ P_{4233} &= (8, 7, 15, 1) = \ell_{10} \cap \ell_{19} \\ P_{442} &= (8, 10, 0, 1) = \ell_{11} \cap \ell_{12} \\ P_{1944} &= (7, 8, 6, 1) = \ell_{11} \cap \ell_{14} \\ P_{2406} &= (5, 5, 8, 1) = \ell_{11} \cap \ell_{16} \\ P_{3979} &= (10, 7, 14, 1) = \ell_{11} \cap \ell_{20} \\ P_{2040} &= (7, 14, 6, 1) = \ell_{12} \cap \ell_{13} \\ P_{2534} &= (5, 13, 8, 1) = \ell_{12} \cap \ell_{18} \\ P_{4011} &= (10, 9, 14, 1) = \ell_{12} \cap \ell_{19} \\ P_{3853} &= (12, 15, 13, 1) = \ell_{13} \cap \ell_{20} \\ P_{3645} &= (12, 2, 13, 1) = \ell_{14} \cap \ell_{19} \\ P_{222} &= (11, 12, 1, 0) = \ell_{15} \cap \ell_{16} \\ P_{2709} &= (4, 8, 9, 1) = \ell_{15} \cap \ell_{19} \\ P_{301} &= (11, 1, 0, 1) = \ell_{16} \cap \ell_{18} \\ P_{1844} &= (3, 2, 6, 1) = \ell_{16} \cap \ell_{20} \\ P_{238} &= (11, 13, 1, 0) = \ell_{17} \cap \ell_{18} \\ P_{2597} &= (4, 1, 9, 1) = \ell_{17} \cap \ell_{20} \\ P_{1876} &= (3, 4, 6, 1) = \ell_{18} \cap \ell_{19} \\ P_{465} &= (15, 11, 0, 1) = \ell_{19} \cap \ell_{20} \end{aligned}$$

Single Points

The surface has 204 single points:

The single points on the surface are:

0 : $P_{55} = (4, 2, 1, 0)$ lies on line ℓ_3
 1 : $P_{71} = (4, 3, 1, 0)$ lies on line ℓ_4
 2 : $P_{92} = (9, 4, 1, 0)$ lies on line ℓ_5
 3 : $P_{108} = (9, 5, 1, 0)$ lies on line ℓ_6
 4 : $P_{161} = (14, 8, 1, 0)$ lies on line ℓ_{11}
 5 : $P_{177} = (14, 9, 1, 0)$ lies on line ℓ_{12}
 6 : $P_{180} = (1, 10, 1, 0)$ lies on line ℓ_{13}
 7 : $P_{196} = (1, 11, 1, 0)$ lies on line ℓ_{14}
 8 : $P_{245} = (2, 14, 1, 0)$ lies on line ℓ_{19}
 9 : $P_{261} = (2, 15, 1, 0)$ lies on line ℓ_{20}
 10 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_0
 11 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_0
 12 : $P_{561} = (0, 2, 1, 1)$ lies on line ℓ_0
 13 : $P_{567} = (6, 2, 1, 1)$ lies on line ℓ_{11}
 14 : $P_{577} = (0, 3, 1, 1)$ lies on line ℓ_0
 15 : $P_{583} = (6, 3, 1, 1)$ lies on line ℓ_{12}
 16 : $P_{593} = (0, 4, 1, 1)$ lies on line ℓ_0
 17 : $P_{606} = (13, 4, 1, 1)$ lies on line ℓ_{20}
 18 : $P_{609} = (0, 5, 1, 1)$ lies on line ℓ_0
 19 : $P_{622} = (13, 5, 1, 1)$ lies on line ℓ_{19}
 20 : $P_{636} = (11, 6, 1, 1)$ lies on line ℓ_7
 21 : $P_{652} = (11, 7, 1, 1)$ lies on line ℓ_9
 22 : $P_{657} = (0, 8, 1, 1)$ lies on line ℓ_0
 23 : $P_{664} = (7, 8, 1, 1)$ lies on line ℓ_3
 24 : $P_{673} = (0, 9, 1, 1)$ lies on line ℓ_0
 25 : $P_{680} = (7, 9, 1, 1)$ lies on line ℓ_4
 26 : $P_{731} = (10, 12, 1, 1)$ lies on line ℓ_{15}
 27 : $P_{747} = (10, 13, 1, 1)$ lies on line ℓ_{17}
 28 : $P_{753} = (0, 14, 1, 1)$ lies on line ℓ_0
 29 : $P_{765} = (12, 14, 1, 1)$ lies on line ℓ_6
 30 : $P_{769} = (0, 15, 1, 1)$ lies on line ℓ_0
 31 : $P_{781} = (12, 15, 1, 1)$ lies on line ℓ_5
 32 : $P_{789} = (4, 0, 2, 1)$ lies on line ℓ_{16}
 33 : $P_{814} = (13, 1, 2, 1)$ lies on line ℓ_{12}
 34 : $P_{821} = (4, 2, 2, 1)$ lies on line ℓ_{18}
 35 : $P_{846} = (13, 3, 2, 1)$ lies on line ℓ_{11}
 36 : $P_{989} = (12, 12, 2, 1)$ lies on line ℓ_7
 37 : $P_{996} = (3, 13, 2, 1)$ lies on line ℓ_{13}
 38 : $P_{1000} = (7, 13, 2, 1)$ lies on line ℓ_8
 39 : $P_{1021} = (12, 14, 2, 1)$ lies on line ℓ_9
 40 : $P_{1028} = (3, 15, 2, 1)$ lies on line ℓ_{14}
 41 : $P_{1032} = (7, 15, 2, 1)$ lies on line ℓ_{10}
 42 : $P_{1050} = (9, 0, 3, 1)$ lies on line ℓ_{19}
 43 : $P_{1098} = (9, 3, 3, 1)$ lies on line ℓ_{20}
 44 : $P_{1107} = (2, 4, 3, 1)$ lies on line ℓ_{14}
 45 : $P_{1112} = (7, 4, 3, 1)$ lies on line ℓ_6
 46 : $P_{1155} = (2, 7, 3, 1)$ lies on line ℓ_{13}
 47 : $P_{1160} = (7, 7, 3, 1)$ lies on line ℓ_5
 48 : $P_{1172} = (3, 8, 3, 1)$ lies on line ℓ_{12}
 49 : $P_{1182} = (13, 8, 3, 1)$ lies on line ℓ_{10}
 50 : $P_{1191} = (6, 9, 3, 1)$ lies on line ℓ_9
 51 : $P_{1207} = (6, 10, 3, 1)$ lies on line ℓ_7
 52 : $P_{1220} = (3, 11, 3, 1)$ lies on line ℓ_{11}
 53 : $P_{1230} = (13, 11, 3, 1)$ lies on line ℓ_8

54 : $P_{1254} = (5, 13, 3, 1)$ lies on line ℓ_{15}
 55 : $P_{1270} = (5, 14, 3, 1)$ lies on line ℓ_{17}
 56 : $P_{1306} = (9, 0, 4, 1)$ lies on line ℓ_8
 57 : $P_{1320} = (7, 1, 4, 1)$ lies on line ℓ_{19}
 58 : $P_{1335} = (6, 2, 4, 1)$ lies on line ℓ_{15}
 59 : $P_{1350} = (5, 3, 4, 1)$ lies on line ℓ_{13}
 60 : $P_{1357} = (12, 3, 4, 1)$ lies on line ℓ_{16}
 61 : $P_{1370} = (9, 4, 4, 1)$ lies on line ℓ_{10}
 62 : $P_{1384} = (7, 5, 4, 1)$ lies on line ℓ_{20}
 63 : $P_{1399} = (6, 6, 4, 1)$ lies on line ℓ_{17}
 64 : $P_{1414} = (5, 7, 4, 1)$ lies on line ℓ_{14}
 65 : $P_{1421} = (12, 7, 4, 1)$ lies on line ℓ_{18}
 66 : $P_{1567} = (14, 0, 5, 1)$ lies on line ℓ_3
 67 : $P_{1593} = (8, 2, 5, 1)$ lies on line ℓ_9
 68 : $P_{1647} = (14, 5, 5, 1)$ lies on line ℓ_4
 69 : $P_{1673} = (8, 7, 5, 1)$ lies on line ℓ_7
 70 : $P_{1701} = (4, 9, 5, 1)$ lies on line ℓ_{13}
 71 : $P_{1709} = (12, 9, 5, 1)$ lies on line ℓ_{11}
 72 : $P_{1718} = (5, 10, 5, 1)$ lies on line ℓ_{20}
 73 : $P_{1720} = (7, 10, 5, 1)$ lies on line ℓ_{18}
 74 : $P_{1742} = (13, 11, 5, 1)$ lies on line ℓ_{17}
 75 : $P_{1749} = (4, 12, 5, 1)$ lies on line ℓ_{14}
 76 : $P_{1757} = (12, 12, 5, 1)$ lies on line ℓ_{12}
 77 : $P_{1790} = (13, 14, 5, 1)$ lies on line ℓ_{15}
 78 : $P_{1798} = (5, 15, 5, 1)$ lies on line ℓ_{19}
 79 : $P_{1800} = (7, 15, 5, 1)$ lies on line ℓ_{16}
 80 : $P_{1811} = (2, 0, 6, 1)$ lies on line ℓ_4
 81 : $P_{1866} = (9, 3, 6, 1)$ lies on line ℓ_{15}
 82 : $P_{1898} = (9, 5, 6, 1)$ lies on line ℓ_{17}
 83 : $P_{1907} = (2, 6, 6, 1)$ lies on line ℓ_3
 84 : $P_{1970} = (1, 10, 6, 1)$ lies on line ℓ_5
 85 : $P_{1973} = (4, 10, 6, 1)$ lies on line ℓ_{10}
 86 : $P_{2000} = (15, 11, 6, 1)$ lies on line ℓ_9
 87 : $P_{2002} = (1, 12, 6, 1)$ lies on line ℓ_6
 88 : $P_{2005} = (4, 12, 6, 1)$ lies on line ℓ_8
 89 : $P_{2032} = (15, 13, 6, 1)$ lies on line ℓ_7
 90 : $P_{2074} = (9, 0, 7, 1)$ lies on line ℓ_{11}
 91 : $P_{2186} = (9, 7, 7, 1)$ lies on line ℓ_{12}
 92 : $P_{2195} = (2, 8, 7, 1)$ lies on line ℓ_{17}
 93 : $P_{2226} = (1, 10, 7, 1)$ lies on line ℓ_{19}
 94 : $P_{2239} = (14, 10, 7, 1)$ lies on line ℓ_8
 95 : $P_{2246} = (5, 11, 7, 1)$ lies on line ℓ_7
 96 : $P_{2262} = (5, 12, 7, 1)$ lies on line ℓ_9
 97 : $P_{2274} = (1, 13, 7, 1)$ lies on line ℓ_{20}
 98 : $P_{2287} = (14, 13, 7, 1)$ lies on line ℓ_{10}
 99 : $P_{2307} = (2, 15, 7, 1)$ lies on line ℓ_{15}
 100 : $P_{2323} = (2, 0, 8, 1)$ lies on line ℓ_5
 101 : $P_{2360} = (7, 2, 8, 1)$ lies on line ℓ_7
 102 : $P_{2377} = (8, 3, 8, 1)$ lies on line ℓ_3
 103 : $P_{2381} = (12, 3, 8, 1)$ lies on line ℓ_8
 104 : $P_{2400} = (15, 4, 8, 1)$ lies on line ℓ_{15}
 105 : $P_{2423} = (6, 6, 8, 1)$ lies on line ℓ_{19}
 106 : $P_{2426} = (9, 6, 8, 1)$ lies on line ℓ_{13}
 107 : $P_{2451} = (2, 8, 8, 1)$ lies on line ℓ_6

- 108 : $P_{2488} = (7, 10, 8, 1)$ lies on line ℓ_9
 109 : $P_{2505} = (8, 11, 8, 1)$ lies on line ℓ_4
 110 : $P_{2509} = (12, 11, 8, 1)$ lies on line ℓ_{10}
 111 : $P_{2528} = (15, 12, 8, 1)$ lies on line ℓ_{17}
 112 : $P_{2551} = (6, 14, 8, 1)$ lies on line ℓ_{20}
 113 : $P_{2554} = (9, 14, 8, 1)$ lies on line ℓ_{14}
 114 : $P_{2591} = (14, 0, 9, 1)$ lies on line ℓ_{18}
 115 : $P_{2605} = (12, 1, 9, 1)$ lies on line ℓ_3
 116 : $P_{2654} = (13, 4, 9, 1)$ lies on line ℓ_7
 117 : $P_{2663} = (6, 5, 9, 1)$ lies on line ℓ_8
 118 : $P_{2665} = (8, 5, 9, 1)$ lies on line ℓ_{14}
 119 : $P_{2717} = (12, 8, 9, 1)$ lies on line ℓ_4
 120 : $P_{2735} = (14, 9, 9, 1)$ lies on line ℓ_{16}
 121 : $P_{2775} = (6, 12, 9, 1)$ lies on line ℓ_{10}
 122 : $P_{2777} = (8, 12, 9, 1)$ lies on line ℓ_{13}
 123 : $P_{2798} = (13, 13, 9, 1)$ lies on line ℓ_9
 124 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_1
 125 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_1
 126 : $P_{2860} = (11, 1, 10, 1)$ lies on line ℓ_{14}
 127 : $P_{2865} = (0, 2, 10, 1)$ lies on line ℓ_1
 128 : $P_{2874} = (9, 2, 10, 1)$ lies on line ℓ_6
 129 : $P_{2881} = (0, 3, 10, 1)$ lies on line ℓ_1
 130 : $P_{2883} = (2, 3, 10, 1)$ lies on line ℓ_{19}
 131 : $P_{2897} = (0, 4, 10, 1)$ lies on line ℓ_1
 132 : $P_{2914} = (1, 5, 10, 1)$ lies on line ℓ_{10}
 133 : $P_{2961} = (0, 8, 10, 1)$ lies on line ℓ_1
 134 : $P_{2970} = (9, 8, 10, 1)$ lies on line ℓ_5
 135 : $P_{2977} = (0, 9, 10, 1)$ lies on line ℓ_1
 136 : $P_{2979} = (2, 9, 10, 1)$ lies on line ℓ_{20}
 137 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_1
 138 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_1
 139 : $P_{3020} = (11, 11, 10, 1)$ lies on line ℓ_{13}
 140 : $P_{3057} = (0, 14, 10, 1)$ lies on line ℓ_1
 141 : $P_{3074} = (1, 15, 10, 1)$ lies on line ℓ_8
 142 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_2
 143 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_2
 144 : $P_{3115} = (10, 1, 11, 1)$ lies on line ℓ_{13}
 145 : $P_{3121} = (0, 2, 11, 1)$ lies on line ℓ_2
 146 : $P_{3138} = (1, 3, 11, 1)$ lies on line ℓ_{18}
 147 : $P_{3153} = (0, 4, 11, 1)$ lies on line ℓ_2
 148 : $P_{3167} = (14, 4, 11, 1)$ lies on line ℓ_{11}
 149 : $P_{3169} = (0, 5, 11, 1)$ lies on line ℓ_2
 150 : $P_{3173} = (4, 5, 11, 1)$ lies on line ℓ_3
 151 : $P_{3218} = (1, 8, 11, 1)$ lies on line ℓ_{16}
 152 : $P_{3233} = (0, 9, 11, 1)$ lies on line ℓ_2
 153 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_2
 154 : $P_{3259} = (10, 10, 11, 1)$ lies on line ℓ_{14}
 155 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_2
 156 : $P_{3313} = (0, 14, 11, 1)$ lies on line ℓ_2
 157 : $P_{3317} = (4, 14, 11, 1)$ lies on line ℓ_4
 158 : $P_{3329} = (0, 15, 11, 1)$ lies on line ℓ_2
 159 : $P_{3343} = (14, 15, 11, 1)$ lies on line ℓ_{12}
 160 : $P_{3359} = (14, 0, 12, 1)$ lies on line ℓ_{20}
 161 : $P_{3397} = (4, 3, 12, 1)$ lies on line ℓ_7
 162 : $P_{3449} = (8, 6, 12, 1)$ lies on line ℓ_{15}
 163 : $P_{3458} = (1, 7, 12, 1)$ lies on line ℓ_4
 164 : $P_{3459} = (2, 7, 12, 1)$ lies on line ℓ_{16}
 165 : $P_{3513} = (8, 10, 12, 1)$ lies on line ℓ_{17}
 166 : $P_{3522} = (1, 11, 12, 1)$ lies on line ℓ_3
 167 : $P_{3523} = (2, 11, 12, 1)$ lies on line ℓ_{18}
 168 : $P_{3551} = (14, 12, 12, 1)$ lies on line ℓ_{19}
 169 : $P_{3589} = (4, 15, 12, 1)$ lies on line ℓ_9
 170 : $P_{3605} = (4, 0, 13, 1)$ lies on line ℓ_6
 171 : $P_{3695} = (14, 5, 13, 1)$ lies on line ℓ_7
 172 : $P_{3698} = (1, 6, 13, 1)$ lies on line ℓ_{11}
 173 : $P_{3706} = (9, 6, 13, 1)$ lies on line ℓ_{18}
 174 : $P_{3716} = (3, 7, 13, 1)$ lies on line ℓ_{17}
 175 : $P_{3743} = (14, 8, 13, 1)$ lies on line ℓ_9
 176 : $P_{3764} = (3, 10, 13, 1)$ lies on line ℓ_{15}
 177 : $P_{3778} = (1, 11, 13, 1)$ lies on line ℓ_{12}
 178 : $P_{3786} = (9, 11, 13, 1)$ lies on line ℓ_{16}
 179 : $P_{3813} = (4, 13, 13, 1)$ lies on line ℓ_5
 180 : $P_{3859} = (2, 0, 14, 1)$ lies on line ℓ_{10}
 181 : $P_{3879} = (6, 1, 14, 1)$ lies on line ℓ_5
 182 : $P_{3966} = (13, 6, 14, 1)$ lies on line ℓ_{16}
 183 : $P_{3968} = (15, 6, 14, 1)$ lies on line ℓ_{14}
 184 : $P_{3976} = (7, 7, 14, 1)$ lies on line ℓ_{15}
 185 : $P_{3998} = (13, 8, 14, 1)$ lies on line ℓ_{18}
 186 : $P_{4000} = (15, 8, 14, 1)$ lies on line ℓ_{13}
 187 : $P_{4008} = (7, 9, 14, 1)$ lies on line ℓ_{17}
 188 : $P_{4083} = (2, 14, 14, 1)$ lies on line ℓ_8
 189 : $P_{4103} = (6, 15, 14, 1)$ lies on line ℓ_6
 190 : $P_{4117} = (4, 0, 15, 1)$ lies on line ℓ_{12}
 191 : $P_{4158} = (13, 2, 15, 1)$ lies on line ℓ_4
 192 : $P_{4159} = (14, 2, 15, 1)$ lies on line ℓ_{13}
 193 : $P_{4189} = (12, 4, 15, 1)$ lies on line ℓ_{17}
 194 : $P_{4199} = (6, 5, 15, 1)$ lies on line ℓ_{18}
 195 : $P_{4208} = (15, 5, 15, 1)$ lies on line ℓ_5
 196 : $P_{4212} = (3, 6, 15, 1)$ lies on line ℓ_9
 197 : $P_{4260} = (3, 9, 15, 1)$ lies on line ℓ_7
 198 : $P_{4279} = (6, 10, 15, 1)$ lies on line ℓ_{16}
 199 : $P_{4288} = (15, 10, 15, 1)$ lies on line ℓ_6
 200 : $P_{4301} = (12, 11, 15, 1)$ lies on line ℓ_{15}
 201 : $P_{4334} = (13, 13, 15, 1)$ lies on line ℓ_3
 202 : $P_{4335} = (14, 13, 15, 1)$ lies on line ℓ_{14}
 203 : $P_{4357} = (4, 15, 15, 1)$ lies on line ℓ_{11}

The single points on the surface are:

Points on surface but on no line

The surface has 75 points not on any line:

The points on the surface but not on lines are:

- | | |
|----------------------------------|-----------------------------------|
| 0 : $P_0 = (1, 0, 0, 0)$ | 38 : $P_{2312} = (7, 15, 7, 1)$ |
| 1 : $P_{315} = (9, 2, 0, 1)$ | 39 : $P_{2363} = (10, 2, 8, 1)$ |
| 2 : $P_{319} = (13, 2, 0, 1)$ | 40 : $P_{2398} = (13, 4, 8, 1)$ |
| 3 : $P_{345} = (7, 4, 0, 1)$ | 41 : $P_{2415} = (14, 5, 8, 1)$ |
| 4 : $P_{352} = (14, 4, 0, 1)$ | 42 : $P_{2491} = (10, 10, 8, 1)$ |
| 5 : $P_{420} = (2, 9, 0, 1)$ | 43 : $P_{2526} = (13, 12, 8, 1)$ |
| 6 : $P_{430} = (12, 9, 0, 1)$ | 44 : $P_{2543} = (14, 13, 8, 1)$ |
| 7 : $P_{502} = (4, 14, 0, 1)$ | 45 : $P_{2932} = (3, 6, 10, 1)$ |
| 8 : $P_{504} = (6, 14, 0, 1)$ | 46 : $P_{2953} = (8, 7, 10, 1)$ |
| 9 : $P_{690} = (1, 10, 1, 1)$ | 47 : $P_{3028} = (3, 12, 10, 1)$ |
| 10 : $P_{706} = (1, 11, 1, 1)$ | 48 : $P_{3049} = (8, 13, 10, 1)$ |
| 11 : $P_{1195} = (10, 9, 3, 1)$ | 49 : $P_{3190} = (5, 6, 11, 1)$ |
| 12 : $P_{1211} = (10, 10, 3, 1)$ | 50 : $P_{3216} = (15, 7, 11, 1)$ |
| 13 : $P_{1237} = (4, 12, 3, 1)$ | 51 : $P_{3296} = (15, 12, 11, 1)$ |
| 14 : $P_{1261} = (12, 13, 3, 1)$ | 52 : $P_{3302} = (5, 13, 11, 1)$ |
| 15 : $P_{1277} = (12, 14, 3, 1)$ | 53 : $P_{3387} = (10, 2, 12, 1)$ |
| 16 : $P_{1285} = (4, 15, 3, 1)$ | 54 : $P_{3405} = (12, 3, 12, 1)$ |
| 17 : $P_{1591} = (6, 2, 5, 1)$ | 55 : $P_{3436} = (11, 5, 12, 1)$ |
| 18 : $P_{1610} = (9, 3, 5, 1)$ | 56 : $P_{3447} = (6, 6, 12, 1)$ |
| 19 : $P_{1658} = (9, 6, 5, 1)$ | 57 : $P_{3500} = (11, 9, 12, 1)$ |
| 20 : $P_{1671} = (6, 7, 5, 1)$ | 58 : $P_{3511} = (6, 10, 12, 1)$ |
| 21 : $P_{1740} = (11, 11, 5, 1)$ | 59 : $P_{3579} = (10, 14, 12, 1)$ |
| 22 : $P_{1788} = (11, 14, 5, 1)$ | 60 : $P_{3597} = (12, 15, 12, 1)$ |
| 23 : $P_{1852} = (11, 2, 6, 1)$ | 61 : $P_{3644} = (11, 2, 13, 1)$ |
| 24 : $P_{1863} = (6, 3, 6, 1)$ | 62 : $P_{3675} = (10, 4, 13, 1)$ |
| 25 : $P_{1884} = (11, 4, 6, 1)$ | 63 : $P_{3694} = (13, 5, 13, 1)$ |
| 26 : $P_{1895} = (6, 5, 6, 1)$ | 64 : $P_{3720} = (7, 7, 13, 1)$ |
| 27 : $P_{1947} = (10, 8, 6, 1)$ | 65 : $P_{3742} = (13, 8, 13, 1)$ |
| 28 : $P_{1998} = (13, 11, 6, 1)$ | 66 : $P_{3755} = (10, 9, 13, 1)$ |
| 29 : $P_{2030} = (13, 13, 6, 1)$ | 67 : $P_{3768} = (7, 10, 13, 1)$ |
| 30 : $P_{2043} = (10, 14, 6, 1)$ | 68 : $P_{3852} = (11, 15, 13, 1)$ |
| 31 : $P_{2123} = (10, 3, 7, 1)$ | 69 : $P_{4188} = (11, 4, 15, 1)$ |
| 32 : $P_{2139} = (10, 4, 7, 1)$ | 70 : $P_{4216} = (7, 6, 15, 1)$ |
| 33 : $P_{2200} = (7, 8, 7, 1)$ | 71 : $P_{4227} = (2, 7, 15, 1)$ |
| 34 : $P_{2220} = (11, 9, 7, 1)$ | 72 : $P_{4243} = (2, 8, 15, 1)$ |
| 35 : $P_{2253} = (12, 11, 7, 1)$ | 73 : $P_{4264} = (7, 9, 15, 1)$ |
| 36 : $P_{2269} = (12, 12, 7, 1)$ | 74 : $P_{4300} = (11, 11, 15, 1)$ |
| 37 : $P_{2300} = (11, 14, 7, 1)$ | |

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	0	0	0
1	1	0	1	1	1	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0
2	1	1	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	1	1
3	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
4	0	1	0	1	0	0	1	0	1	1	0	0	0	0	1	0	0	0	1	0	1
5	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	1	1	0	0	0	0
6	0	0	1	0	1	1	0	0	0	0	1	1	0	1	0	1	0	0	1	0	0
7	0	0	1	1	0	0	0	0	1	1	0	1	0	1	1	0	1	1	0	0	0
8	1	0	0	0	1	1	0	1	0	0	1	0	1	0	0	1	0	0	0	0	1
9	0	0	1	0	1	0	0	1	0	0	1	0	1	1	1	1	1	1	0	0	0
10	1	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	1	0	1	0	0
11	0	1	0	0	0	0	1	1	0	0	1	0	1	0	1	0	1	0	0	0	1
12	0	1	0	0	0	1	0	0	1	0	1	0	1	0	0	0	0	1	1	0	0
13	1	0	0	1	0	0	1	1	0	1	0	1	1	0	1	0	1	0	0	1	0
14	1	0	0	0	1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
15	0	1	0	0	0	0	1	1	1	1	0	1	1	0	1	1	0	1	0	1	0
16	1	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1
17	0	1	0	0	0	1	0	1	0	1	1	1	0	0	1	0	0	1	0	1	0
18	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	1	1	0	1	0	0
19	0	0	1	1	0	0	0	0	0	1	0	1	0	1	1	0	0	1	0	1	0
20	0	0	1	0	1	0	0	0	1	0	0	1	0	1	0	0	1	1	0	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_8	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{18}
in point	P_1	P_1	P_{641}	P_{625}	P_{689}	P_{705}	P_{737}	P_{721}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_{11}	ℓ_{12}	ℓ_{15}	ℓ_{17}
in point	P_1	P_1	P_{2945}	P_{3041}	P_{3025}	P_{2929}	P_{2913}	P_{3073}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{19}	ℓ_{20}
in point	P_1	P_1	P_{3281}	P_{3201}	P_{3217}	P_{3137}	P_{3297}	P_{3185}

Line 3 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{13}	ℓ_{16}	ℓ_{19}
in point	P_{2945}	P_{437}	P_{1339}	P_{4106}	P_{3750}	P_{2135}	P_{1248}	P_{1020}

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{14}	ℓ_{18}	ℓ_{20}
in point	P_{3041}	P_{437}	P_{1403}	P_{3670}	P_{3882}	P_{2119}	P_{1296}	P_{988}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_8	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{17}
in point	P_{3281}	P_{1339}	P_{455}	P_{1652}	P_{2652}	P_{3502}	P_{2297}	P_{847}

Line 6 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{18}
in point	P_{3201}	P_{1403}	P_{455}	P_{1604}	P_{2796}	P_{3438}	P_{815}	P_{2217}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}
in point	P_{3217}	P_{4106}	P_{125}	P_{275}	P_{1315}	P_{275}	P_{275}	P_{275}	P_{275}	P_{3067}

Line 8 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{20}
in point	P_{641}	P_{3670}	P_{1652}	P_{125}	P_{300}	P_{3392}	P_{3244}	P_{4249}

Line 9 intersects

Line	ℓ_2	ℓ_4	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}
in point	P_{3137}	P_{3882}	P_{275}	P_{141}	P_{1379}	P_{275}	P_{275}	P_{275}	P_{2907}	P_{275}

Line 10 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{17}	ℓ_{19}
in point	P_{625}	P_{3750}	P_{1604}	P_{300}	P_{141}	P_{3584}	P_{3132}	P_{4233}

Line 11 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{16}	ℓ_{20}
in point	P_{3025}	P_{2796}	P_{1315}	P_{3584}	P_{442}	P_{1944}	P_{2406}	P_{3979}

Line 12 intersects

Line	ℓ_1	ℓ_5	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{18}	ℓ_{19}
in point	P_{2929}	P_{2652}	P_{3392}	P_{1379}	P_{442}	P_{2040}	P_{2534}	P_{4011}

Line 13 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7	ℓ_9	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{20}
in point	P_{689}	P_{2135}	P_{3438}	P_{275}	P_{275}	P_{2040}	P_{275}	P_{275}	P_{275}	P_{3853}

Line 14 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}
in point	P_{705}	P_{2119}	P_{3502}	P_{275}	P_{275}	P_{1944}	P_{275}	P_{275}	P_{275}	P_{3645}

Line 15 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{17}	ℓ_{19}
in point	P_{2913}	P_{815}	P_{275}	P_{3244}	P_{275}	P_{275}	P_{275}	P_{222}	P_{275}	P_{2709}

Line 16 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_9	ℓ_{11}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_{737}	P_{1248}	P_{2297}	P_{2907}	P_{2406}	P_{222}	P_{301}	P_{1844}

Line 17 intersects

Line	ℓ_1	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_{3073}	P_{847}	P_{275}	P_{275}	P_{3132}	P_{275}	P_{275}	P_{275}	P_{238}	P_{2597}

Line 18 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_7	ℓ_{12}	ℓ_{16}	ℓ_{17}	ℓ_{19}
in point	P_{721}	P_{1296}	P_{2217}	P_{3067}	P_{2534}	P_{301}	P_{238}	P_{1876}

Line 19 intersects

Line	ℓ_2	ℓ_3	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{18}	ℓ_{20}
in point	P_{3297}	P_{1020}	P_{4233}	P_{4011}	P_{3645}	P_{2709}	P_{1876}	P_{465}

Line 20 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{11}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{19}
in point	P_{3185}	P_{988}	P_{4249}	P_{3979}	P_{3853}	P_{1844}	P_{2597}	P_{465}

The surface has 353 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	45 : $P_{657} = (0, 8, 1, 1)$	90 : $P_{1248} = (15, 12, 3, 1)$
1 : $P_1 = (0, 1, 0, 0)$	46 : $P_{664} = (7, 8, 1, 1)$	91 : $P_{1254} = (5, 13, 3, 1)$
2 : $P_{55} = (4, 2, 1, 0)$	47 : $P_{673} = (0, 9, 1, 1)$	92 : $P_{1261} = (12, 13, 3, 1)$
3 : $P_{71} = (4, 3, 1, 0)$	48 : $P_{680} = (7, 9, 1, 1)$	93 : $P_{1270} = (5, 14, 3, 1)$
4 : $P_{92} = (9, 4, 1, 0)$	49 : $P_{689} = (0, 10, 1, 1)$	94 : $P_{1277} = (12, 14, 3, 1)$
5 : $P_{108} = (9, 5, 1, 0)$	50 : $P_{690} = (1, 10, 1, 1)$	95 : $P_{1285} = (4, 15, 3, 1)$
6 : $P_{125} = (10, 6, 1, 0)$	51 : $P_{705} = (0, 11, 1, 1)$	96 : $P_{1296} = (15, 15, 3, 1)$
7 : $P_{141} = (10, 7, 1, 0)$	52 : $P_{706} = (1, 11, 1, 1)$	97 : $P_{1306} = (9, 0, 4, 1)$
8 : $P_{161} = (14, 8, 1, 0)$	53 : $P_{721} = (0, 12, 1, 1)$	98 : $P_{1315} = (2, 1, 4, 1)$
9 : $P_{177} = (14, 9, 1, 0)$	54 : $P_{731} = (10, 12, 1, 1)$	99 : $P_{1320} = (7, 1, 4, 1)$
10 : $P_{180} = (1, 10, 1, 0)$	55 : $P_{737} = (0, 13, 1, 1)$	100 : $P_{1335} = (6, 2, 4, 1)$
11 : $P_{196} = (1, 11, 1, 0)$	56 : $P_{747} = (10, 13, 1, 1)$	101 : $P_{1339} = (10, 2, 4, 1)$
12 : $P_{222} = (11, 12, 1, 0)$	57 : $P_{753} = (0, 14, 1, 1)$	102 : $P_{1350} = (5, 3, 4, 1)$
13 : $P_{238} = (11, 13, 1, 0)$	58 : $P_{765} = (12, 14, 1, 1)$	103 : $P_{1357} = (12, 3, 4, 1)$
14 : $P_{245} = (2, 14, 1, 0)$	59 : $P_{769} = (0, 15, 1, 1)$	104 : $P_{1370} = (9, 4, 4, 1)$
15 : $P_{261} = (2, 15, 1, 0)$	60 : $P_{781} = (12, 15, 1, 1)$	105 : $P_{1379} = (2, 5, 4, 1)$
16 : $P_{275} = (1, 0, 0, 1)$	61 : $P_{789} = (4, 0, 2, 1)$	106 : $P_{1384} = (7, 5, 4, 1)$
17 : $P_{300} = (10, 1, 0, 1)$	62 : $P_{814} = (13, 1, 2, 1)$	107 : $P_{1399} = (6, 6, 4, 1)$
18 : $P_{301} = (11, 1, 0, 1)$	63 : $P_{815} = (14, 1, 2, 1)$	108 : $P_{1403} = (10, 6, 4, 1)$
19 : $P_{315} = (9, 2, 0, 1)$	64 : $P_{821} = (4, 2, 2, 1)$	109 : $P_{1414} = (5, 7, 4, 1)$
20 : $P_{319} = (13, 2, 0, 1)$	65 : $P_{846} = (13, 3, 2, 1)$	110 : $P_{1421} = (12, 7, 4, 1)$
21 : $P_{345} = (7, 4, 0, 1)$	66 : $P_{847} = (14, 3, 2, 1)$	111 : $P_{1567} = (14, 0, 5, 1)$
22 : $P_{352} = (14, 4, 0, 1)$	67 : $P_{988} = (11, 12, 2, 1)$	112 : $P_{1591} = (6, 2, 5, 1)$
23 : $P_{420} = (2, 9, 0, 1)$	68 : $P_{989} = (12, 12, 2, 1)$	113 : $P_{1593} = (8, 2, 5, 1)$
24 : $P_{430} = (12, 9, 0, 1)$	69 : $P_{996} = (3, 13, 2, 1)$	114 : $P_{1604} = (3, 3, 5, 1)$
25 : $P_{437} = (3, 10, 0, 1)$	70 : $P_{1000} = (7, 13, 2, 1)$	115 : $P_{1610} = (9, 3, 5, 1)$
26 : $P_{442} = (8, 10, 0, 1)$	71 : $P_{1020} = (11, 14, 2, 1)$	116 : $P_{1647} = (14, 5, 5, 1)$
27 : $P_{455} = (5, 11, 0, 1)$	72 : $P_{1021} = (12, 14, 2, 1)$	117 : $P_{1652} = (3, 6, 5, 1)$
28 : $P_{465} = (15, 11, 0, 1)$	73 : $P_{1028} = (3, 15, 2, 1)$	118 : $P_{1658} = (9, 6, 5, 1)$
29 : $P_{502} = (4, 14, 0, 1)$	74 : $P_{1032} = (7, 15, 2, 1)$	119 : $P_{1671} = (6, 7, 5, 1)$
30 : $P_{504} = (6, 14, 0, 1)$	75 : $P_{1050} = (9, 0, 3, 1)$	120 : $P_{1673} = (8, 7, 5, 1)$
31 : $P_{530} = (0, 0, 1, 1)$	76 : $P_{1098} = (9, 3, 3, 1)$	121 : $P_{1701} = (4, 9, 5, 1)$
32 : $P_{546} = (0, 1, 1, 1)$	77 : $P_{1107} = (2, 4, 3, 1)$	122 : $P_{1709} = (12, 9, 5, 1)$
33 : $P_{561} = (0, 2, 1, 1)$	78 : $P_{1112} = (7, 4, 3, 1)$	123 : $P_{1718} = (5, 10, 5, 1)$
34 : $P_{567} = (6, 2, 1, 1)$	79 : $P_{1155} = (2, 7, 3, 1)$	124 : $P_{1720} = (7, 10, 5, 1)$
35 : $P_{577} = (0, 3, 1, 1)$	80 : $P_{1160} = (7, 7, 3, 1)$	125 : $P_{1740} = (11, 11, 5, 1)$
36 : $P_{583} = (6, 3, 1, 1)$	81 : $P_{1172} = (3, 8, 3, 1)$	126 : $P_{1742} = (13, 11, 5, 1)$
37 : $P_{593} = (0, 4, 1, 1)$	82 : $P_{1182} = (13, 8, 3, 1)$	127 : $P_{1749} = (4, 12, 5, 1)$
38 : $P_{606} = (13, 4, 1, 1)$	83 : $P_{1191} = (6, 9, 3, 1)$	128 : $P_{1757} = (12, 12, 5, 1)$
39 : $P_{609} = (0, 5, 1, 1)$	84 : $P_{1195} = (10, 9, 3, 1)$	129 : $P_{1788} = (11, 14, 5, 1)$
40 : $P_{622} = (13, 5, 1, 1)$	85 : $P_{1207} = (6, 10, 3, 1)$	130 : $P_{1790} = (13, 14, 5, 1)$
41 : $P_{625} = (0, 6, 1, 1)$	86 : $P_{1211} = (10, 10, 3, 1)$	131 : $P_{1798} = (5, 15, 5, 1)$
42 : $P_{636} = (11, 6, 1, 1)$	87 : $P_{1220} = (3, 11, 3, 1)$	132 : $P_{1800} = (7, 15, 5, 1)$
43 : $P_{641} = (0, 7, 1, 1)$	88 : $P_{1230} = (13, 11, 3, 1)$	133 : $P_{1811} = (2, 0, 6, 1)$
44 : $P_{652} = (11, 7, 1, 1)$	89 : $P_{1237} = (4, 12, 3, 1)$	134 : $P_{1844} = (3, 2, 6, 1)$

135 : $P_{1852} = (11, 2, 6, 1)$	189 : $P_{2488} = (7, 10, 8, 1)$	243 : $P_{3089} = (0, 0, 11, 1)$
136 : $P_{1863} = (6, 3, 6, 1)$	190 : $P_{2491} = (10, 10, 8, 1)$	244 : $P_{3105} = (0, 1, 11, 1)$
137 : $P_{1866} = (9, 3, 6, 1)$	191 : $P_{2505} = (8, 11, 8, 1)$	245 : $P_{3115} = (10, 1, 11, 1)$
138 : $P_{1876} = (3, 4, 6, 1)$	192 : $P_{2509} = (12, 11, 8, 1)$	246 : $P_{3121} = (0, 2, 11, 1)$
139 : $P_{1884} = (11, 4, 6, 1)$	193 : $P_{2526} = (13, 12, 8, 1)$	247 : $P_{3132} = (11, 2, 11, 1)$
140 : $P_{1895} = (6, 5, 6, 1)$	194 : $P_{2528} = (15, 12, 8, 1)$	248 : $P_{3137} = (0, 3, 11, 1)$
141 : $P_{1898} = (9, 5, 6, 1)$	195 : $P_{2534} = (5, 13, 8, 1)$	249 : $P_{3138} = (1, 3, 11, 1)$
142 : $P_{1907} = (2, 6, 6, 1)$	196 : $P_{2543} = (14, 13, 8, 1)$	250 : $P_{3153} = (0, 4, 11, 1)$
143 : $P_{1944} = (7, 8, 6, 1)$	197 : $P_{2551} = (6, 14, 8, 1)$	251 : $P_{3167} = (14, 4, 11, 1)$
144 : $P_{1947} = (10, 8, 6, 1)$	198 : $P_{2554} = (9, 14, 8, 1)$	252 : $P_{3169} = (0, 5, 11, 1)$
145 : $P_{1970} = (1, 10, 6, 1)$	199 : $P_{2591} = (14, 0, 9, 1)$	253 : $P_{3173} = (4, 5, 11, 1)$
146 : $P_{1973} = (4, 10, 6, 1)$	200 : $P_{2597} = (4, 1, 9, 1)$	254 : $P_{3185} = (0, 6, 11, 1)$
147 : $P_{1998} = (13, 11, 6, 1)$	201 : $P_{2605} = (12, 1, 9, 1)$	255 : $P_{3190} = (5, 6, 11, 1)$
148 : $P_{2000} = (15, 11, 6, 1)$	202 : $P_{2652} = (11, 4, 9, 1)$	256 : $P_{3201} = (0, 7, 11, 1)$
149 : $P_{2002} = (1, 12, 6, 1)$	203 : $P_{2654} = (13, 4, 9, 1)$	257 : $P_{3216} = (15, 7, 11, 1)$
150 : $P_{2005} = (4, 12, 6, 1)$	204 : $P_{2663} = (6, 5, 9, 1)$	258 : $P_{3217} = (0, 8, 11, 1)$
151 : $P_{2030} = (13, 13, 6, 1)$	205 : $P_{2665} = (8, 5, 9, 1)$	259 : $P_{3218} = (1, 8, 11, 1)$
152 : $P_{2032} = (15, 13, 6, 1)$	206 : $P_{2709} = (4, 8, 9, 1)$	260 : $P_{3233} = (0, 9, 11, 1)$
153 : $P_{2040} = (7, 14, 6, 1)$	207 : $P_{2717} = (12, 8, 9, 1)$	261 : $P_{3244} = (11, 9, 11, 1)$
154 : $P_{2043} = (10, 14, 6, 1)$	208 : $P_{2735} = (14, 9, 9, 1)$	262 : $P_{3249} = (0, 10, 11, 1)$
155 : $P_{2074} = (9, 0, 7, 1)$	209 : $P_{2775} = (6, 12, 9, 1)$	263 : $P_{3259} = (10, 10, 11, 1)$
156 : $P_{2119} = (6, 3, 7, 1)$	210 : $P_{2777} = (8, 12, 9, 1)$	264 : $P_{3265} = (0, 11, 11, 1)$
157 : $P_{2123} = (10, 3, 7, 1)$	211 : $P_{2796} = (11, 13, 9, 1)$	265 : $P_{3281} = (0, 12, 11, 1)$
158 : $P_{2135} = (6, 4, 7, 1)$	212 : $P_{2798} = (13, 13, 9, 1)$	266 : $P_{3296} = (15, 12, 11, 1)$
159 : $P_{2139} = (10, 4, 7, 1)$	213 : $P_{2833} = (0, 0, 10, 1)$	267 : $P_{3297} = (0, 13, 11, 1)$
160 : $P_{2186} = (9, 7, 7, 1)$	214 : $P_{2849} = (0, 1, 10, 1)$	268 : $P_{3302} = (5, 13, 11, 1)$
161 : $P_{2195} = (2, 8, 7, 1)$	215 : $P_{2860} = (11, 1, 10, 1)$	269 : $P_{3313} = (0, 14, 11, 1)$
162 : $P_{2200} = (7, 8, 7, 1)$	216 : $P_{2865} = (0, 2, 10, 1)$	270 : $P_{3317} = (4, 14, 11, 1)$
163 : $P_{2217} = (8, 9, 7, 1)$	217 : $P_{2874} = (9, 2, 10, 1)$	271 : $P_{3329} = (0, 15, 11, 1)$
164 : $P_{2220} = (11, 9, 7, 1)$	218 : $P_{2881} = (0, 3, 10, 1)$	272 : $P_{3343} = (14, 15, 11, 1)$
165 : $P_{2226} = (1, 10, 7, 1)$	219 : $P_{2883} = (2, 3, 10, 1)$	273 : $P_{3359} = (14, 0, 12, 1)$
166 : $P_{2239} = (14, 10, 7, 1)$	220 : $P_{2897} = (0, 4, 10, 1)$	274 : $P_{3387} = (10, 2, 12, 1)$
167 : $P_{2246} = (5, 11, 7, 1)$	221 : $P_{2907} = (10, 4, 10, 1)$	275 : $P_{3392} = (15, 2, 12, 1)$
168 : $P_{2253} = (12, 11, 7, 1)$	222 : $P_{2913} = (0, 5, 10, 1)$	276 : $P_{3397} = (4, 3, 12, 1)$
169 : $P_{2262} = (5, 12, 7, 1)$	223 : $P_{2914} = (1, 5, 10, 1)$	277 : $P_{3405} = (12, 3, 12, 1)$
170 : $P_{2269} = (12, 12, 7, 1)$	224 : $P_{2929} = (0, 6, 10, 1)$	278 : $P_{3436} = (11, 5, 12, 1)$
171 : $P_{2274} = (1, 13, 7, 1)$	225 : $P_{2932} = (3, 6, 10, 1)$	279 : $P_{3438} = (13, 5, 12, 1)$
172 : $P_{2287} = (14, 13, 7, 1)$	226 : $P_{2945} = (0, 7, 10, 1)$	280 : $P_{3447} = (6, 6, 12, 1)$
173 : $P_{2297} = (8, 14, 7, 1)$	227 : $P_{2953} = (8, 7, 10, 1)$	281 : $P_{3449} = (8, 6, 12, 1)$
174 : $P_{2300} = (11, 14, 7, 1)$	228 : $P_{2961} = (0, 8, 10, 1)$	282 : $P_{3458} = (1, 7, 12, 1)$
175 : $P_{2307} = (2, 15, 7, 1)$	229 : $P_{2970} = (9, 8, 10, 1)$	283 : $P_{3459} = (2, 7, 12, 1)$
176 : $P_{2312} = (7, 15, 7, 1)$	230 : $P_{2977} = (0, 9, 10, 1)$	284 : $P_{3500} = (11, 9, 12, 1)$
177 : $P_{2323} = (2, 0, 8, 1)$	231 : $P_{2979} = (2, 9, 10, 1)$	285 : $P_{3502} = (13, 9, 12, 1)$
178 : $P_{2360} = (7, 2, 8, 1)$	232 : $P_{2993} = (0, 10, 10, 1)$	286 : $P_{3511} = (6, 10, 12, 1)$
179 : $P_{2363} = (10, 2, 8, 1)$	233 : $P_{3009} = (0, 11, 10, 1)$	287 : $P_{3513} = (8, 10, 12, 1)$
180 : $P_{2377} = (8, 3, 8, 1)$	234 : $P_{3020} = (11, 11, 10, 1)$	288 : $P_{3522} = (1, 11, 12, 1)$
181 : $P_{2381} = (12, 3, 8, 1)$	235 : $P_{3025} = (0, 12, 10, 1)$	289 : $P_{3523} = (2, 11, 12, 1)$
182 : $P_{2398} = (13, 4, 8, 1)$	236 : $P_{3028} = (3, 12, 10, 1)$	290 : $P_{3551} = (14, 12, 12, 1)$
183 : $P_{2400} = (15, 4, 8, 1)$	237 : $P_{3041} = (0, 13, 10, 1)$	291 : $P_{3579} = (10, 14, 12, 1)$
184 : $P_{2406} = (5, 5, 8, 1)$	238 : $P_{3049} = (8, 13, 10, 1)$	292 : $P_{3584} = (15, 14, 12, 1)$
185 : $P_{2415} = (14, 5, 8, 1)$	239 : $P_{3057} = (0, 14, 10, 1)$	293 : $P_{3589} = (4, 15, 12, 1)$
186 : $P_{2423} = (6, 6, 8, 1)$	240 : $P_{3067} = (10, 14, 10, 1)$	294 : $P_{3597} = (12, 15, 12, 1)$
187 : $P_{2426} = (9, 6, 8, 1)$	241 : $P_{3073} = (0, 15, 10, 1)$	295 : $P_{3605} = (4, 0, 13, 1)$
188 : $P_{2451} = (2, 8, 8, 1)$	242 : $P_{3074} = (1, 15, 10, 1)$	296 : $P_{3644} = (11, 2, 13, 1)$

297 : $P_{3645} = (12, 2, 13, 1)$	316 : $P_{3853} = (12, 15, 13, 1)$	335 : $P_{4189} = (12, 4, 15, 1)$
298 : $P_{3670} = (5, 4, 13, 1)$	317 : $P_{3859} = (2, 0, 14, 1)$	336 : $P_{4199} = (6, 5, 15, 1)$
299 : $P_{3675} = (10, 4, 13, 1)$	318 : $P_{3879} = (6, 1, 14, 1)$	337 : $P_{4208} = (15, 5, 15, 1)$
300 : $P_{3694} = (13, 5, 13, 1)$	319 : $P_{3882} = (9, 1, 14, 1)$	338 : $P_{4212} = (3, 6, 15, 1)$
301 : $P_{3695} = (14, 5, 13, 1)$	320 : $P_{3966} = (13, 6, 14, 1)$	339 : $P_{4216} = (7, 6, 15, 1)$
302 : $P_{3698} = (1, 6, 13, 1)$	321 : $P_{3968} = (15, 6, 14, 1)$	340 : $P_{4227} = (2, 7, 15, 1)$
303 : $P_{3706} = (9, 6, 13, 1)$	322 : $P_{3976} = (7, 7, 14, 1)$	341 : $P_{4233} = (8, 7, 15, 1)$
304 : $P_{3716} = (3, 7, 13, 1)$	323 : $P_{3979} = (10, 7, 14, 1)$	342 : $P_{4243} = (2, 8, 15, 1)$
305 : $P_{3720} = (7, 7, 13, 1)$	324 : $P_{3998} = (13, 8, 14, 1)$	343 : $P_{4249} = (8, 8, 15, 1)$
306 : $P_{3742} = (13, 8, 13, 1)$	325 : $P_{4000} = (15, 8, 14, 1)$	344 : $P_{4260} = (3, 9, 15, 1)$
307 : $P_{3743} = (14, 8, 13, 1)$	326 : $P_{4008} = (7, 9, 14, 1)$	345 : $P_{4264} = (7, 9, 15, 1)$
308 : $P_{3750} = (5, 9, 13, 1)$	327 : $P_{4011} = (10, 9, 14, 1)$	346 : $P_{4279} = (6, 10, 15, 1)$
309 : $P_{3755} = (10, 9, 13, 1)$	328 : $P_{4083} = (2, 14, 14, 1)$	347 : $P_{4288} = (15, 10, 15, 1)$
310 : $P_{3764} = (3, 10, 13, 1)$	329 : $P_{4103} = (6, 15, 14, 1)$	348 : $P_{4300} = (11, 11, 15, 1)$
311 : $P_{3768} = (7, 10, 13, 1)$	330 : $P_{4106} = (9, 15, 14, 1)$	349 : $P_{4301} = (12, 11, 15, 1)$
312 : $P_{3778} = (1, 11, 13, 1)$	331 : $P_{4117} = (4, 0, 15, 1)$	350 : $P_{4334} = (13, 13, 15, 1)$
313 : $P_{3786} = (9, 11, 13, 1)$	332 : $P_{4158} = (13, 2, 15, 1)$	351 : $P_{4335} = (14, 13, 15, 1)$
314 : $P_{3813} = (4, 13, 13, 1)$	333 : $P_{4159} = (14, 2, 15, 1)$	352 : $P_{4357} = (4, 15, 15, 1)$
315 : $P_{3852} = (11, 15, 13, 1)$	334 : $P_{4188} = (11, 4, 15, 1)$	