Rank-65744 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1229493390

General information

Number of lines	3
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	3
Number of single points	21
Number of points off lines	33
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	2^3, 1^{21}, 0^{33}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0,0,0,1,0,1)_{769}$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1,0,1,1,1,1)_{1323}$$

Rank of lines: (4673, 138, 666)
Rank of points on Klein quadric: (769, 1322, 1323)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{138} = (0,0,1,1) = \ell_0 \cap \ell_1$
$P_{146} = (0,1,1,1) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_4 = (1,1,1,1)$ lies on line ℓ_1
2 : $P_{139} = (1,0,1,1)$ lies on line ℓ_2
3 : $P_{153} = (0,2,1,1)$ lies on line ℓ_0
4 : $P_{155} = (2,2,1,1)$ lies on line ℓ_1
5 : $P_{156} = (3,2,1,1)$ lies on line ℓ_2
6 : $P_{161} = (0,3,1,1)$ lies on line ℓ_0
7 : $P_{163} = (2,3,1,1)$ lies on line ℓ_2
8 : $P_{164} = (3,3,1,1)$ lies on line ℓ_1
9 : $P_{169} = (0,4,1,1)$ lies on line ℓ_0
10 : $P_{173} = (4,4,1,1)$ lies on line ℓ_1
11 : $P_{174} = (5,4,1,1)$ lies on line ℓ_2
12 : $P_{177} = (0,5,1,1)$ lies on line ℓ_0
13 : $P_{181} = (4,5,1,1)$ lies on line ℓ_2
14 : $P_{182} = (5,5,1,1)$ lies on line ℓ_1
15 : $P_{185} = (0,6,1,1)$ lies on line ℓ_0
16 : $P_{191} = (6,6,1,1)$ lies on line ℓ_1
17 : $P_{192} = (7,6,1,1)$ lies on line ℓ_2
18 : $P_{193} = (0,7,1,1)$ lies on line ℓ_0
19 : $P_{199} = (6,7,1,1)$ lies on line ℓ_2
20 : $P_{200} = (7,7,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 33 points not on any line:
The points on the surface but not on lines are:

0 : $P_{12} = (1,0,1,0)$
1 : $P_{20} = (1,1,1,0)$
2 : $P_{76} = (2,0,0,1)$
3 : $P_{78} = (4,0,0,1)$
4 : $P_{81} = (7,0,0,1)$
5 : $P_{83} = (1,1,0,1)$
6 : $P_{95} = (5,2,0,1)$
7 : $P_{112} = (6,4,0,1)$
8 : $P_{133} = (3,7,0,1)$
9 : $P_{207} = (6,0,2,1)$
10 : $P_{223} = (6,2,2,1)$
11 : $P_{237} = (4,4,2,1)$
12 : $P_{244} = (3,5,2,1)$
13 : $P_{253} = (4,6,2,1)$
14 : $P_{260} = (3,7,2,1)$
15 : $P_{301} = (4,4,3,1)$

16 : $P_{325} = (4, 7, 3, 1)$
17 : $P_{332} = (3, 0, 4, 1)$
18 : $P_{350} = (5, 2, 4, 1)$
19 : $P_{360} = (7, 3, 4, 1)$
20 : $P_{364} = (3, 4, 4, 1)$
21 : $P_{382} = (5, 6, 4, 1)$
22 : $P_{392} = (7, 7, 4, 1)$
23 : $P_{416} = (7, 2, 5, 1)$
24 : $P_{456} = (7, 7, 5, 1)$
25 : $P_{475} = (2, 2, 6, 1)$
26 : $P_{491} = (2, 4, 6, 1)$
27 : $P_{526} = (5, 0, 7, 1)$
28 : $P_{539} = (2, 2, 7, 1)$
29 : $P_{551} = (6, 3, 7, 1)$
30 : $P_{559} = (6, 4, 7, 1)$
31 : $P_{563} = (2, 5, 7, 1)$
32 : $P_{582} = (5, 7, 7, 1)$

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{138}	P_{146}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{138}	P_5

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{146}	P_5

The surface has 57 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_{12} = (1, 0, 1, 0)$
4 : $P_{20} = (1, 1, 1, 0)$
5 : $P_{76} = (2, 0, 0, 1)$
6 : $P_{78} = (4, 0, 0, 1)$
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53 : $P_{551} = (6, 3, 7, 1)$
54 : $P_{559} = (6, 4, 7, 1)$
55 : $P_{563} = (2, 5, 7, 1)$
56 : $P_{582} = (5, 7, 7, 1)$