Rank-65915 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431726745

General information

Number of lines	3
Number of points	21
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	9
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	2^3, 1^9, 0^9

Singular Points

The surface has 1 singular points:

$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & \omega^2 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{247} = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{247} = \mathbf{Pl}(0, 3, 2, 0, 0, 1)_{121}$$

$$\ell_1 = \begin{bmatrix} 1 & \omega & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{310} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{310} = \mathbf{Pl}(0,2,3,0,0,1)_{127}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$

Rank of lines: (247, 310, 89)
Rank of points on Klein quadric: (121, 127, 74)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$ $P_{83} = (2,3,3,1) = \ell_1 \cap \ell_2$
$P_{64} = (3,2,2,1) = \ell_0 \cap \ell_2$

Single Points

The surface has 9 single points:
The single points on the surface are:

0 : $P_{12} = (1,1,1,0)$ lies on line ℓ_2 5 : $P_{48} = (3,2,1,1)$ lies on line ℓ_0
1 : $P_{23} = (1,0,0,1)$ lies on line ℓ_2 6 : $P_{51} = (2,3,1,1)$ lies on line ℓ_1
2 : $P_{33} = (3,2,0,1)$ lies on line ℓ_0 7 : $P_{67} = (2,3,2,1)$ lies on line ℓ_1
3 : $P_{36} = (2,3,0,1)$ lies on line ℓ_1 8 : $P_{80} = (3,2,3,1)$ lies on line ℓ_0
4 : $P_{42} = (0,1,1,1)$ lies on line ℓ_2

   The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$ 5 : $P_{56} = (3,0,2,1)$
1 : $P_1 = (0,1,0,0)$ 6 : $P_{61} = (0,2,2,1)$
2 : $P_4 = (1,1,1,1)$ 7 : $P_{71} = (2,0,3,1)$
3 : $P_{47} = (2,2,1,1)$ 8 : $P_{81} = (0,3,3,1)$
4 : $P_{52} = (3,3,1,1)$

Line Intersection Graph

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  |0 1 2
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 0|0 1 1
 1|1 0 1
 2|1 1 0
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Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_{64}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_{83}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{64}	P_{83}

The surface has 21 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
$1 : P_1 = (0, 1, 0, 0)$
$2 : P_2 = (0, 0, 1, 0)$
$3 : P_4 = (1, 1, 1, 1)$
$4 : P_{12} = (1, 1, 1, 0)$
$5 : P_{23} = (1, 0, 0, 1)$
$6 : P_{33} = (3, 2, 0, 1)$
$7 : P_{36} = (2, 3, 0, 1)$

$8 : P_{42} = (0, 1, 1, 1)$
$9 : P_{47} = (2, 2, 1, 1)$
$10 : P_{48} = (3, 2, 1, 1)$
$11 : P_{51} = (2, 3, 1, 1)$
$12 : P_{52} = (3, 3, 1, 1)$
$13 : P_{56} = (3, 0, 2, 1)$
$14 : P_{61} = (0, 2, 2, 1)$
$15 : P_{64} = (3, 2, 2, 1)$

$16 : P_{67} = (2, 3, 2, 1)$
$17 : P_{71} = (2, 0, 3, 1)$
$18 : P_{80} = (3, 2, 3, 1)$
$19 : P_{81} = (0, 3, 3, 1)$
$20 : P_{83} = (2, 3, 3, 1)$