Rank-74051 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498834265

General information

Number of lines	10
Number of points	33
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	21
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{10}
Type of lines on points	5, 3^2, 2^9, 1^{21}

Singular Points

The surface has 1 singular points:

$0 :\ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{37} = \mathbf{Pl}(0,0,1,0,0,1)_{108}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{121} = \mathbf{Pl}(0,1,1,0,0,1)_{112}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1,1,0,1,1,1)_{189}$$

Rank of lines: (0, 16, 37, 340, 84, 356, 121, 100, 25, 109)
Rank of points on Klein quadric: (0, 2, 108, 9, 10, 1, 112, 6, 177, 189)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_8$
$P_{12} = (1,1,1,0) = \ell_2 \cap \ell_9$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_5$
$P_{27} = (1,1,0,1) = \ell_4 \cap \ell_6$
$P_{23} = (1,0,0,1) = \ell_4 \cap \ell_7$
$P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$
$P_{39} = (1,0,1,1) = \ell_7 \cap \ell_9$

Single Points

The surface has 21 single points:
The single points on the surface are:

0 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
1 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
2 : $P_9 = (2,0,1,0)$ lies on line ℓ_1
3 : $P_{10} = (3,0,1,0)$ lies on line ℓ_1
4 : $P_{17} = (2,2,1,0)$ lies on line ℓ_2
5 : $P_{22} = (3,3,1,0)$ lies on line ℓ_2
6 : $P_{30} = (0,2,0,1)$ lies on line ℓ_3
7 : $P_{31} = (1,2,0,1)$ lies on line ℓ_4
8 : $P_{34} = (0,3,0,1)$ lies on line ℓ_3
9 : $P_{35} = (1,3,0,1)$ lies on line ℓ_4
10 : $P_{38} = (0,0,1,1)$ lies on line ℓ_5
11 : $P_{53} = (0,0,2,1)$ lies on line ℓ_5
12 : $P_{54} = (1,0,2,1)$ lies on line ℓ_7
13 : $P_{58} = (1,1,2,1)$ lies on line ℓ_6
14 : $P_{59} = (2,1,2,1)$ lies on line ℓ_8
15 : $P_{67} = (2,3,2,1)$ lies on line ℓ_9
16 : $P_{69} = (0,0,3,1)$ lies on line ℓ_5
17 : $P_{70} = (1,0,3,1)$ lies on line ℓ_7
18 : $P_{74} = (1,1,3,1)$ lies on line ℓ_6
19 : $P_{76} = (3,1,3,1)$ lies on line ℓ_8
20 : $P_{80} = (3,2,3,1)$ lies on line ℓ_9

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	1	0	0	0	0	0
1	1	0	1	0	0	1	1	1	1	0
2	1	1	0	0	0	1	1	1	0	1
3	1	0	0	0	1	1	0	0	1	1
4	1	0	0	1	0	0	1	1	0	0
5	0	1	1	1	0	0	1	1	0	0
6	0	1	1	0	1	1	0	1	1	0
7	0	1	1	0	1	1	1	0	0	1
8	0	1	0	1	0	0	1	0	0	1
9	0	0	1	1	0	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_2	P_2	P_2	P_2	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_5	P_2	P_2	P_2	P_2	P_{12}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{26}	P_{26}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7
in point	P_1	P_1	P_{27}	P_{23}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_2	P_3	P_2	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_{27}	P_2	P_2	P_4

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_2	P_2	P_{23}	P_2	P_2	P_{39}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_9
in point	P_8	P_{26}	P_4	P_{26}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_{12}	P_{26}	P_{39}	P_{26}

The surface has 33 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (1,0,1,0)$
$9: P_9 = (2,0,1,0)$
$10: P_{10} = (3,0,1,0)$
$11: P_{12} = (1,1,1,0)$
$12: P_{17} = (2,2,1,0)$
$13: P_{22} = (3,3,1,0)$
$14: P_{23} = (1,0,0,1)$
$15: P_{26} = (0,1,0,1)$
$16: P_{27} = (1,1,0,1)$
$17: P_{30} = (0,2,0,1)$
$18: P_{31} = (1,2,0,1)$
$19: P_{34} = (0,3,0,1)$
$20: P_{35} = (1,3,0,1)$
$21: P_{38} = (0,0,1,1)$
$22: P_{39} = (1,0,1,1)$
$23: P_{53} = (0,0,2,1)$
$24: P_{54} = (1,0,2,1)$
$25: P_{58} = (1,1,2,1)$
$26: P_{59} = (2,1,2,1)$
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$28: P_{69} = (0,0,3,1)$
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$31: P_{76} = (3,1,3,1)$
$32: P_{80} = (3,2,3,1)$