Rank-65869 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 65869

General information

Number of lines	2
Number of points	7
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	4
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^2
Type of lines on points	2, 1^4, 0^2

Singular Points

The surface has 2 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1: \ P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0,1,1,0,0,0)_4$$

Rank of lines: (4, 18)
Rank of points on Klein quadric: (2, 4)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0:\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1:\ P_6 = (1,0,1,0)$ lies on line ℓ_0
$2:\ P_9 = (1,0,0,1)$ lies on line ℓ_1
$3:\ P_{13} = (1,0,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

$0:\ P_7 = (0,1,1,0)$
$1:\ P_{10} = (0,1,0,1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 7 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_2 = (0,0,1,0)$
$2: P_6 = (1,0,1,0)$
$3: P_7 = (0,1,1,0)$
$4: P_9 = (1,0,0,1)$
$5: P_{10} = (0,1,0,1)$
$6: P_{13} = (1,0,1,1)$