

Rank-65550 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0X_1X_2 = 0$$

(1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65550

General information

Number of lines	3
Number of points	7
Number of singular points	1
Number of Eckardt points	0
Number of double points	3
Number of single points	3
Number of points off lines	1
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	$2^3, 1^3, 0$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{18} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_4 \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21}\end{aligned}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{33}$$

Rank of lines: (18, 31, 24)

Rank of points on Klein quadric: (4, 21, 33)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

$$P_{13} = (1, 0, 1, 1) = \ell_0 \cap \ell_2$$

$$P_{14} = (0, 1, 1, 1) = \ell_1 \cap \ell_2$$

Single Points

The surface has 3 single points:

The single points on the surface are:

$$0 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_2$$

$$1 : P_9 = (1, 0, 0, 1) \text{ lies on line } \ell_0$$

$$2 : P_{10} = (0, 1, 0, 1) \text{ lies on line } \ell_1$$

The single points on the surface are:

Points on surface but on no line

The surface has 1 points not on any line:

The points on the surface but not on lines are:

$$0 : P_4 = (1, 1, 1, 1)$$

Line Intersection Graph

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_{13}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_{14}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{13}	P_{14}

The surface has 7 points:

The points on the surface are:

$$0 : P_2 = (0, 0, 1, 0)$$

$$1 : P_4 = (1, 1, 1, 1)$$

$$2 : P_5 = (1, 1, 0, 0)$$

$$3 : P_9 = (1, 0, 0, 1)$$

$$4 : P_{10} = (0, 1, 0, 1)$$

$$5 : P_{13} = (1, 0, 1, 1)$$

$$6 : P_{14} = (0, 1, 1, 1)$$