

# Rank-65634 over GF(64)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

( 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(64) is 1107828870

## General information

Number of lines	2
Number of points	4225
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	130
Number of points off lines	4095
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^2$
Type of lines on points	$1^{130}, 0^{4095}$

## Singular Points

The surface has 0 singular points:

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 1 & 0 & \epsilon^{21} & 0 \\ 0 & 1 & 0 & \epsilon^{42} \end{array} \right]_{240761} = \left[ \begin{array}{cccc} 1 & 0 & 57 & 0 \\ 0 & 1 & 0 & 56 \end{array} \right]_{240761} = \mathbf{Pl}(57, 56, 0, 0, 57, 1)_{15213113} \\ \ell_1 &= \left[ \begin{array}{cccc} 1 & 0 & \epsilon^{42} & 0 \\ 0 & 1 & 0 & \epsilon^{21} \end{array} \right]_{236664} = \left[ \begin{array}{cccc} 1 & 0 & 56 & 0 \\ 0 & 1 & 0 & 57 \end{array} \right]_{236664} = \mathbf{Pl}(56, 57, 0, 0, 56, 1)_{14951032}\end{aligned}$$

Rank of lines: ( 240761, 236664 )  
Rank of points on Klein quadric: ( 15213113, 14951032 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:  
The double points on the surface are:

### Single Points

The surface has 130 single points:  
The single points on the surface are:

- |  |   |
|--|---|
| 0 : $P_{123} = (56, 0, 1, 0)$ lies on line $\ell_0$      | 33 : $P_{69295} = (46, 57, 15, 1)$ lies on line $\ell_0$  |
| 1 : $P_{124} = (57, 0, 1, 0)$ lies on line $\ell_1$      | 34 : $P_{73340} = (59, 56, 16, 1)$ lies on line $\ell_1$  |
| 2 : $P_{7746} = (0, 56, 0, 1)$ lies on line $\ell_1$     | 35 : $P_{73388} = (43, 57, 16, 1)$ lies on line $\ell_0$  |
| 3 : $P_{7810} = (0, 57, 0, 1)$ lies on line $\ell_0$     | 36 : $P_{77379} = (2, 56, 17, 1)$ lies on line $\ell_1$   |
| 4 : $P_{11898} = (57, 56, 1, 1)$ lies on line $\ell_1$   | 37 : $P_{77460} = (19, 57, 17, 1)$ lies on line $\ell_0$  |
| 5 : $P_{11961} = (56, 57, 1, 1)$ lies on line $\ell_0$   | 38 : $P_{81513} = (40, 56, 18, 1)$ lies on line $\ell_1$  |
| 6 : $P_{15956} = (19, 56, 2, 1)$ lies on line $\ell_1$   | 39 : $P_{81595} = (58, 57, 18, 1)$ lies on line $\ell_0$  |
| 7 : $P_{16018} = (17, 57, 2, 1)$ lies on line $\ell_0$   | 40 : $P_{85586} = (17, 56, 19, 1)$ lies on line $\ell_1$  |
| 8 : $P_{20075} = (42, 56, 3, 1)$ lies on line $\ell_1$   | 41 : $P_{85635} = (2, 57, 19, 1)$ lies on line $\ell_0$   |
| 9 : $P_{20138} = (41, 57, 3, 1)$ lies on line $\ell_0$   | 42 : $P_{89694} = (29, 56, 20, 1)$ lies on line $\ell_1$  |
| 10 : $P_{24167} = (38, 56, 4, 1)$ lies on line $\ell_1$  | 43 : $P_{89738} = (9, 57, 20, 1)$ lies on line $\ell_0$   |
| 11 : $P_{24227} = (34, 57, 4, 1)$ lies on line $\ell_0$  | 44 : $P_{93797} = (36, 56, 21, 1)$ lies on line $\ell_1$  |
| 12 : $P_{28256} = (31, 56, 5, 1)$ lies on line $\ell_1$  | 45 : $P_{93874} = (49, 57, 21, 1)$ lies on line $\ell_0$  |
| 13 : $P_{28315} = (26, 57, 5, 1)$ lies on line $\ell_0$  | 46 : $P_{97871} = (14, 56, 22, 1)$ lies on line $\ell_1$  |
| 14 : $P_{32374} = (53, 56, 6, 1)$ lies on line $\ell_1$  | 47 : $P_{97945} = (24, 57, 22, 1)$ lies on line $\ell_0$  |
| 15 : $P_{32436} = (51, 57, 6, 1)$ lies on line $\ell_0$  | 48 : $P_{102008} = (55, 56, 23, 1)$ lies on line $\ell_1$ |
| 16 : $P_{36429} = (12, 56, 7, 1)$ lies on line $\ell_1$  | 49 : $P_{102049} = (32, 57, 23, 1)$ lies on line $\ell_0$ |
| 17 : $P_{36492} = (11, 57, 7, 1)$ lies on line $\ell_0$  | 50 : $P_{106071} = (22, 56, 24, 1)$ lies on line $\ell_1$ |
| 18 : $P_{40558} = (45, 56, 8, 1)$ lies on line $\ell_1$  | 51 : $P_{106127} = (14, 57, 24, 1)$ lies on line $\ell_0$ |
| 19 : $P_{40614} = (37, 57, 8, 1)$ lies on line $\ell_0$  | 52 : $P_{110192} = (47, 56, 25, 1)$ lies on line $\ell_1$ |
| 20 : $P_{44629} = (20, 56, 9, 1)$ lies on line $\ell_1$  | 53 : $P_{110263} = (54, 57, 25, 1)$ lies on line $\ell_0$ |
| 21 : $P_{44702} = (29, 57, 9, 1)$ lies on line $\ell_0$  | 54 : $P_{114246} = (5, 56, 26, 1)$ lies on line $\ell_1$  |
| 22 : $P_{48767} = (62, 56, 10, 1)$ lies on line $\ell_1$ | 55 : $P_{114336} = (31, 57, 26, 1)$ lies on line $\ell_0$ |
| 23 : $P_{48821} = (52, 57, 10, 1)$ lies on line $\ell_0$ | 56 : $P_{118397} = (60, 56, 27, 1)$ lies on line $\ell_1$ |
| 24 : $P_{52808} = (7, 56, 11, 1)$ lies on line $\ell_1$  | 57 : $P_{118440} = (39, 57, 27, 1)$ lies on line $\ell_0$ |
| 25 : $P_{52877} = (12, 57, 11, 1)$ lies on line $\ell_0$ | 58 : $P_{122481} = (48, 56, 28, 1)$ lies on line $\ell_1$ |
| 26 : $P_{56908} = (11, 56, 12, 1)$ lies on line $\ell_1$ | 59 : $P_{122541} = (44, 57, 28, 1)$ lies on line $\ell_0$ |
| 27 : $P_{56968} = (7, 57, 12, 1)$ lies on line $\ell_0$  | 60 : $P_{126538} = (9, 56, 29, 1)$ lies on line $\ell_1$  |
| 28 : $P_{61043} = (50, 56, 13, 1)$ lies on line $\ell_1$ | 61 : $P_{126613} = (20, 57, 29, 1)$ lies on line $\ell_0$ |
| 29 : $P_{61120} = (63, 57, 13, 1)$ lies on line $\ell_0$ | 62 : $P_{130660} = (35, 56, 30, 1)$ lies on line $\ell_1$ |
| 30 : $P_{65113} = (24, 56, 14, 1)$ lies on line $\ell_1$ | 63 : $P_{130750} = (61, 57, 30, 1)$ lies on line $\ell_0$ |
| 31 : $P_{65175} = (22, 57, 14, 1)$ lies on line $\ell_0$ | 64 : $P_{134747} = (26, 56, 31, 1)$ lies on line $\ell_1$ |
| 32 : $P_{69218} = (33, 56, 15, 1)$ lies on line $\ell_1$ | 65 : $P_{134790} = (5, 57, 31, 1)$ lies on line $\ell_0$  |

66 :  $P_{138840} = (23, 56, 32, 1)$  lies on line  $\ell_1$   
 67 :  $P_{138936} = (55, 57, 32, 1)$  lies on line  $\ell_0$   
 68 :  $P_{142959} = (46, 56, 33, 1)$  lies on line  $\ell_1$   
 69 :  $P_{142992} = (15, 57, 33, 1)$  lies on line  $\ell_0$   
 70 :  $P_{147013} = (4, 56, 34, 1)$  lies on line  $\ell_1$   
 71 :  $P_{147111} = (38, 57, 34, 1)$  lies on line  $\ell_0$   
 72 :  $P_{151166} = (61, 56, 35, 1)$  lies on line  $\ell_1$   
 73 :  $P_{151199} = (30, 57, 35, 1)$  lies on line  $\ell_0$   
 74 :  $P_{155250} = (49, 56, 36, 1)$  lies on line  $\ell_1$   
 75 :  $P_{155286} = (21, 57, 36, 1)$  lies on line  $\ell_0$   
 76 :  $P_{159305} = (8, 56, 37, 1)$  lies on line  $\ell_1$   
 77 :  $P_{159406} = (45, 57, 37, 1)$  lies on line  $\ell_0$   
 78 :  $P_{163427} = (34, 56, 38, 1)$  lies on line  $\ell_1$   
 79 :  $P_{163461} = (4, 57, 38, 1)$  lies on line  $\ell_0$   
 80 :  $P_{167516} = (27, 56, 39, 1)$  lies on line  $\ell_1$   
 81 :  $P_{167613} = (60, 57, 39, 1)$  lies on line  $\ell_0$   
 82 :  $P_{171643} = (58, 56, 40, 1)$  lies on line  $\ell_1$   
 83 :  $P_{171667} = (18, 57, 40, 1)$  lies on line  $\ell_0$   
 84 :  $P_{175684} = (3, 56, 41, 1)$  lies on line  $\ell_1$   
 85 :  $P_{175787} = (42, 57, 41, 1)$  lies on line  $\ell_0$   
 86 :  $P_{179818} = (41, 56, 42, 1)$  lies on line  $\ell_1$   
 87 :  $P_{179844} = (3, 57, 42, 1)$  lies on line  $\ell_0$   
 88 :  $P_{183889} = (16, 56, 43, 1)$  lies on line  $\ell_1$   
 89 :  $P_{183996} = (59, 57, 43, 1)$  lies on line  $\ell_0$   
 90 :  $P_{187997} = (28, 56, 44, 1)$  lies on line  $\ell_1$   
 91 :  $P_{188081} = (48, 57, 44, 1)$  lies on line  $\ell_0$   
 92 :  $P_{192102} = (37, 56, 45, 1)$  lies on line  $\ell_1$   
 93 :  $P_{192137} = (8, 57, 45, 1)$  lies on line  $\ell_0$   
 94 :  $P_{196176} = (15, 56, 46, 1)$  lies on line  $\ell_1$   
 95 :  $P_{196258} = (33, 57, 46, 1)$  lies on line  $\ell_0$   
 96 :  $P_{200311} = (54, 56, 47, 1)$  lies on line  $\ell_1$   
 97 :  $P_{200346} = (25, 57, 47, 1)$  lies on line  $\ell_0$   
 98 :  $P_{204397} = (44, 56, 48, 1)$  lies on line  $\ell_1$

99 :  $P_{204445} = (28, 57, 48, 1)$  lies on line  $\ell_0$   
 100 :  $P_{208470} = (21, 56, 49, 1)$  lies on line  $\ell_1$   
 101 :  $P_{208549} = (36, 57, 49, 1)$  lies on line  $\ell_0$   
 102 :  $P_{212608} = (63, 56, 50, 1)$  lies on line  $\ell_1$   
 103 :  $P_{212622} = (13, 57, 50, 1)$  lies on line  $\ell_0$   
 104 :  $P_{216647} = (6, 56, 51, 1)$  lies on line  $\ell_1$   
 105 :  $P_{216758} = (53, 57, 51, 1)$  lies on line  $\ell_0$   
 106 :  $P_{220747} = (10, 56, 52, 1)$  lies on line  $\ell_1$   
 107 :  $P_{220863} = (62, 57, 52, 1)$  lies on line  $\ell_0$   
 108 :  $P_{224884} = (51, 56, 53, 1)$  lies on line  $\ell_1$   
 109 :  $P_{224903} = (6, 57, 53, 1)$  lies on line  $\ell_0$   
 110 :  $P_{228954} = (25, 56, 54, 1)$  lies on line  $\ell_1$   
 111 :  $P_{229040} = (47, 57, 54, 1)$  lies on line  $\ell_0$   
 112 :  $P_{233057} = (32, 56, 55, 1)$  lies on line  $\ell_1$   
 113 :  $P_{233112} = (23, 57, 55, 1)$  lies on line  $\ell_0$   
 114 :  $P_{237122} = (1, 56, 56, 1)$  lies on line  $\ell_1$   
 115 :  $P_{237242} = (57, 57, 56, 1)$  lies on line  $\ell_0$   
 116 :  $P_{241273} = (56, 56, 57, 1)$  lies on line  $\ell_1$   
 117 :  $P_{241282} = (1, 57, 57, 1)$  lies on line  $\ell_0$   
 118 :  $P_{245331} = (18, 56, 58, 1)$  lies on line  $\ell_1$   
 119 :  $P_{245417} = (40, 57, 58, 1)$  lies on line  $\ell_0$   
 120 :  $P_{249452} = (43, 56, 59, 1)$  lies on line  $\ell_1$   
 121 :  $P_{249489} = (16, 57, 59, 1)$  lies on line  $\ell_0$   
 122 :  $P_{253544} = (39, 56, 60, 1)$  lies on line  $\ell_1$   
 123 :  $P_{253596} = (27, 57, 60, 1)$  lies on line  $\ell_0$   
 124 :  $P_{257631} = (30, 56, 61, 1)$  lies on line  $\ell_1$   
 125 :  $P_{257700} = (35, 57, 61, 1)$  lies on line  $\ell_0$   
 126 :  $P_{261749} = (52, 56, 62, 1)$  lies on line  $\ell_1$   
 127 :  $P_{261771} = (10, 57, 62, 1)$  lies on line  $\ell_0$   
 128 :  $P_{265806} = (13, 56, 63, 1)$  lies on line  $\ell_1$   
 129 :  $P_{265907} = (50, 57, 63, 1)$  lies on line  $\ell_0$

The single points on the surface are:

### Points on surface but on no line

The surface has 4095 points not on any line:  
Too many to print.

### Line Intersection Graph

	0 1
0	0 0
1	0 0

Neighbor sets in the line intersection graph:  
Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 4225 points:  
Too many to print.