Rank-355 over GF(64)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_1 + X_0^2X_3 + X_1^2X_2 = 0$$

(0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(64) is 1107824717

General information

Number of lines	3
Number of points	4225
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	191
Number of points off lines	4032
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	2^2, 1^{191}, 0^{4032}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4096} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(1,0,1,0,1,0)_{4353}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \mathbf{Pl}(0,1,1,0,0,0)_{66}$$

Rank of lines: (4096, 65, 270400)
Rank of points on Klein quadric: (2, 4353, 66)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_2 = (0,0,1,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 191 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_1
1 : $P_{68} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{69} = (2,0,1,0)$ lies on line ℓ_0
3 : $P_{70} = (3,0,1,0)$ lies on line ℓ_0
4 : $P_{71} = (4,0,1,0)$ lies on line ℓ_0
5 : $P_{72} = (5,0,1,0)$ lies on line ℓ_0
6 : $P_{73} = (6,0,1,0)$ lies on line ℓ_0
7 : $P_{74} = (7,0,1,0)$ lies on line ℓ_0
8 : $P_{75} = (8,0,1,0)$ lies on line ℓ_0
9 : $P_{76} = (9,0,1,0)$ lies on line ℓ_0
10 : $P_{77} = (10,0,1,0)$ lies on line ℓ_0
11 : $P_{78} = (11,0,1,0)$ lies on line ℓ_0
12 : $P_{79} = (12,0,1,0)$ lies on line ℓ_0
13 : $P_{80} = (13,0,1,0)$ lies on line ℓ_0
14 : $P_{81} = (14,0,1,0)$ lies on line ℓ_0
15 : $P_{82} = (15,0,1,0)$ lies on line ℓ_0
16 : $P_{83} = (16,0,1,0)$ lies on line ℓ_0
17 : $P_{84} = (17,0,1,0)$ lies on line ℓ_0
18 : $P_{85} = (18,0,1,0)$ lies on line ℓ_0
19 : $P_{86} = (19,0,1,0)$ lies on line ℓ_0
20 : $P_{87} = (20,0,1,0)$ lies on line ℓ_0
21 : $P_{88} = (21,0,1,0)$ lies on line ℓ_0
22 : $P_{89} = (22,0,1,0)$ lies on line ℓ_0
23 : $P_{90} = (23,0,1,0)$ lies on line ℓ_0
24 : $P_{91} = (24,0,1,0)$ lies on line ℓ_0
25 : $P_{92} = (25,0,1,0)$ lies on line ℓ_0
26 : $P_{93} = (26,0,1,0)$ lies on line ℓ_0
27 : $P_{94} = (27,0,1,0)$ lies on line ℓ_0
28 : $P_{95} = (28,0,1,0)$ lies on line ℓ_0
29 : $P_{96} = (29,0,1,0)$ lies on line ℓ_0
30 : $P_{97} = (30,0,1,0)$ lies on line ℓ_0
31 : $P_{98} = (31,0,1,0)$ lies on line ℓ_0
32 : $P_{99} = (32,0,1,0)$ lies on line ℓ_0
33 : $P_{100} = (33,0,1,0)$ lies on line ℓ_0
34 : $P_{101} = (34,0,1,0)$ lies on line ℓ_0
35 : $P_{102} = (35,0,1,0)$ lies on line ℓ_0
36 : $P_{103} = (36,0,1,0)$ lies on line ℓ_0
37 : $P_{104} = (37,0,1,0)$ lies on line ℓ_0
38 : $P_{105} = (38,0,1,0)$ lies on line ℓ_0
39 : $P_{106} = (39,0,1,0)$ lies on line ℓ_0
40 : $P_{107} = (40,0,1,0)$ lies on line ℓ_0
41 : $P_{108} = (41,0,1,0)$ lies on line ℓ_0
42 : $P_{109} = (42,0,1,0)$ lies on line ℓ_0
43 : $P_{110} = (43,0,1,0)$ lies on line ℓ_0
44 : $P_{111} = (44,0,1,0)$ lies on line ℓ_0
45 : $P_{112} = (45,0,1,0)$ lies on line ℓ_0
46 : $P_{113} = (46,0,1,0)$ lies on line ℓ_0
47 : $P_{114} = (47,0,1,0)$ lies on line ℓ_0
48 : $P_{115} = (48,0,1,0)$ lies on line ℓ_0
49 : $P_{116} = (49,0,1,0)$ lies on line ℓ_0
50 : $P_{117} = (50,0,1,0)$ lies on line ℓ_0
51 : $P_{118} = (51,0,1,0)$ lies on line ℓ_0

52 : $P_{119} = (52, 0, 1, 0)$ lies on line ℓ_0
53 : $P_{120} = (53, 0, 1, 0)$ lies on line ℓ_0
54 : $P_{121} = (54, 0, 1, 0)$ lies on line ℓ_0
55 : $P_{122} = (55, 0, 1, 0)$ lies on line ℓ_0
56 : $P_{123} = (56, 0, 1, 0)$ lies on line ℓ_0
57 : $P_{124} = (57, 0, 1, 0)$ lies on line ℓ_0
58 : $P_{125} = (58, 0, 1, 0)$ lies on line ℓ_0
59 : $P_{126} = (59, 0, 1, 0)$ lies on line ℓ_0
60 : $P_{127} = (60, 0, 1, 0)$ lies on line ℓ_0
61 : $P_{128} = (61, 0, 1, 0)$ lies on line ℓ_0
62 : $P_{129} = (62, 0, 1, 0)$ lies on line ℓ_0
63 : $P_{130} = (63, 0, 1, 0)$ lies on line ℓ_0
64 : $P_{4163} = (1, 0, 0, 1)$ lies on line ℓ_2
65 : $P_{8259} = (1, 0, 1, 1)$ lies on line ℓ_2
66 : $P_{8322} = (0, 1, 1, 1)$ lies on line ℓ_1
67 : $P_{8323} = (2, 1, 1, 1)$ lies on line ℓ_1
68 : $P_{8324} = (3, 1, 1, 1)$ lies on line ℓ_1
69 : $P_{8325} = (4, 1, 1, 1)$ lies on line ℓ_1
70 : $P_{8326} = (5, 1, 1, 1)$ lies on line ℓ_1
71 : $P_{8327} = (6, 1, 1, 1)$ lies on line ℓ_1
72 : $P_{8328} = (7, 1, 1, 1)$ lies on line ℓ_1
73 : $P_{8329} = (8, 1, 1, 1)$ lies on line ℓ_1
74 : $P_{8330} = (9, 1, 1, 1)$ lies on line ℓ_1
75 : $P_{8331} = (10, 1, 1, 1)$ lies on line ℓ_1
76 : $P_{8332} = (11, 1, 1, 1)$ lies on line ℓ_1
77 : $P_{8333} = (12, 1, 1, 1)$ lies on line ℓ_1
78 : $P_{8334} = (13, 1, 1, 1)$ lies on line ℓ_1
79 : $P_{8335} = (14, 1, 1, 1)$ lies on line ℓ_1
80 : $P_{8336} = (15, 1, 1, 1)$ lies on line ℓ_1
81 : $P_{8337} = (16, 1, 1, 1)$ lies on line ℓ_1
82 : $P_{8338} = (17, 1, 1, 1)$ lies on line ℓ_1
83 : $P_{8339} = (18, 1, 1, 1)$ lies on line ℓ_1
84 : $P_{8340} = (19, 1, 1, 1)$ lies on line ℓ_1
85 : $P_{8341} = (20, 1, 1, 1)$ lies on line ℓ_1
86 : $P_{8342} = (21, 1, 1, 1)$ lies on line ℓ_1
87 : $P_{8343} = (22, 1, 1, 1)$ lies on line ℓ_1
88 : $P_{8344} = (23, 1, 1, 1)$ lies on line ℓ_1
89 : $P_{8345} = (24, 1, 1, 1)$ lies on line ℓ_1
90 : $P_{8346} = (25, 1, 1, 1)$ lies on line ℓ_1
91 : $P_{8347} = (26, 1, 1, 1)$ lies on line ℓ_1
92 : $P_{8348} = (27, 1, 1, 1)$ lies on line ℓ_1
93 : $P_{8349} = (28, 1, 1, 1)$ lies on line ℓ_1
94 : $P_{8350} = (29, 1, 1, 1)$ lies on line ℓ_1
95 : $P_{8351} = (30, 1, 1, 1)$ lies on line ℓ_1
96 : $P_{8352} = (31, 1, 1, 1)$ lies on line ℓ_1
97 : $P_{8353} = (32, 1, 1, 1)$ lies on line ℓ_1
98 : $P_{8354} = (33, 1, 1, 1)$ lies on line ℓ_1
99 : $P_{8355} = (34, 1, 1, 1)$ lies on line ℓ_1
100 : $P_{8356} = (35, 1, 1, 1)$ lies on line ℓ_1
101 : $P_{8357} = (36, 1, 1, 1)$ lies on line ℓ_1
102 : $P_{8358} = (37, 1, 1, 1)$ lies on line ℓ_1
103 : $P_{8359} = (38, 1, 1, 1)$ lies on line ℓ_1
104 : $P_{8360} = (39, 1, 1, 1)$ lies on line ℓ_1
105 : $P_{8361} = (40, 1, 1, 1)$ lies on line ℓ_1
106 : $P_{8362} = (41, 1, 1, 1)$ lies on line ℓ_1
107 : $P_{8363} = (42, 1, 1, 1)$ lies on line ℓ_1
108 : $P_{8364} = (43, 1, 1, 1)$ lies on line ℓ_1
109 : $P_{8365} = (44, 1, 1, 1)$ lies on line ℓ_1
110 : $P_{8366} = (45, 1, 1, 1)$ lies on line ℓ_1
111 : $P_{8367} = (46, 1, 1, 1)$ lies on line ℓ_1
112 : $P_{8368} = (47, 1, 1, 1)$ lies on line ℓ_1
113 : $P_{8369} = (48, 1, 1, 1)$ lies on line ℓ_1
114 : $P_{8370} = (49, 1, 1, 1)$ lies on line ℓ_1
115 : $P_{8371} = (50, 1, 1, 1)$ lies on line ℓ_1
116 : $P_{8372} = (51, 1, 1, 1)$ lies on line ℓ_1
117 : $P_{8373} = (52, 1, 1, 1)$ lies on line ℓ_1
118 : $P_{8374} = (53, 1, 1, 1)$ lies on line ℓ_1
119 : $P_{8375} = (54, 1, 1, 1)$ lies on line ℓ_1
120 : $P_{8376} = (55, 1, 1, 1)$ lies on line ℓ_1
121 : $P_{8377} = (56, 1, 1, 1)$ lies on line ℓ_1
122 : $P_{8378} = (57, 1, 1, 1)$ lies on line ℓ_1
123 : $P_{8379} = (58, 1, 1, 1)$ lies on line ℓ_1
124 : $P_{8380} = (59, 1, 1, 1)$ lies on line ℓ_1
125 : $P_{8381} = (60, 1, 1, 1)$ lies on line ℓ_1
126 : $P_{8382} = (61, 1, 1, 1)$ lies on line ℓ_1
127 : $P_{8383} = (62, 1, 1, 1)$ lies on line ℓ_1
128 : $P_{8384} = (63, 1, 1, 1)$ lies on line ℓ_1
129 : $P_{12354} = (1, 0, 2, 1)$ lies on line ℓ_2
130 : $P_{16450} = (1, 0, 3, 1)$ lies on line ℓ_2
131 : $P_{20546} = (1, 0, 4, 1)$ lies on line ℓ_2
132 : $P_{24642} = (1, 0, 5, 1)$ lies on line ℓ_2
133 : $P_{28738} = (1, 0, 6, 1)$ lies on line ℓ_2
134 : $P_{32834} = (1, 0, 7, 1)$ lies on line ℓ_2
135 : $P_{36930} = (1, 0, 8, 1)$ lies on line ℓ_2
136 : $P_{41026} = (1, 0, 9, 1)$ lies on line ℓ_2
137 : $P_{45122} = (1, 0, 10, 1)$ lies on line ℓ_2
138 : $P_{49218} = (1, 0, 11, 1)$ lies on line ℓ_2
139 : $P_{53314} = (1, 0, 12, 1)$ lies on line ℓ_2
140 : $P_{57410} = (1, 0, 13, 1)$ lies on line ℓ_2
141 : $P_{61506} = (1, 0, 14, 1)$ lies on line ℓ_2
142 : $P_{65602} = (1, 0, 15, 1)$ lies on line ℓ_2
143 : $P_{69698} = (1, 0, 16, 1)$ lies on line ℓ_2
144 : $P_{73794} = (1, 0, 17, 1)$ lies on line ℓ_2
145 : $P_{77890} = (1, 0, 18, 1)$ lies on line ℓ_2
146 : $P_{81986} = (1, 0, 19, 1)$ lies on line ℓ_2
147 : $P_{86082} = (1, 0, 20, 1)$ lies on line ℓ_2
148 : $P_{90178} = (1, 0, 21, 1)$ lies on line ℓ_2
149 : $P_{94274} = (1, 0, 22, 1)$ lies on line ℓ_2
150 : $P_{98370} = (1, 0, 23, 1)$ lies on line ℓ_2
151 : $P_{102466} = (1, 0, 24, 1)$ lies on line ℓ_2
152 : $P_{106562} = (1, 0, 25, 1)$ lies on line ℓ_2
153 : $P_{110658} = (1, 0, 26, 1)$ lies on line ℓ_2
154 : $P_{114754} = (1, 0, 27, 1)$ lies on line ℓ_2
155 : $P_{118850} = (1, 0, 28, 1)$ lies on line ℓ_2
156 : $P_{122946} = (1, 0, 29, 1)$ lies on line ℓ_2
157 : $P_{127042} = (1, 0, 30, 1)$ lies on line ℓ_2
158 : $P_{131138} = (1, 0, 31, 1)$ lies on line ℓ_2
159 : $P_{135234} = (1, 0, 32, 1)$ lies on line ℓ_2

160 : $P_{139330} = (1, 0, 33, 1)$ lies on line ℓ_2
161 : $P_{143426} = (1, 0, 34, 1)$ lies on line ℓ_2
162 : $P_{147522} = (1, 0, 35, 1)$ lies on line ℓ_2
163 : $P_{151618} = (1, 0, 36, 1)$ lies on line ℓ_2
164 : $P_{155714} = (1, 0, 37, 1)$ lies on line ℓ_2
165 : $P_{159810} = (1, 0, 38, 1)$ lies on line ℓ_2
166 : $P_{163906} = (1, 0, 39, 1)$ lies on line ℓ_2
167 : $P_{168002} = (1, 0, 40, 1)$ lies on line ℓ_2
168 : $P_{172098} = (1, 0, 41, 1)$ lies on line ℓ_2
169 : $P_{176194} = (1, 0, 42, 1)$ lies on line ℓ_2
170 : $P_{180290} = (1, 0, 43, 1)$ lies on line ℓ_2
171 : $P_{184386} = (1, 0, 44, 1)$ lies on line ℓ_2
172 : $P_{188482} = (1, 0, 45, 1)$ lies on line ℓ_2
173 : $P_{192578} = (1, 0, 46, 1)$ lies on line ℓ_2
174 : $P_{196674} = (1, 0, 47, 1)$ lies on line ℓ_2
175 : $P_{200770} = (1, 0, 48, 1)$ lies on line ℓ_2
176 : $P_{204866} = (1, 0, 49, 1)$ lies on line ℓ_2
177 : $P_{208962} = (1, 0, 50, 1)$ lies on line ℓ_2
178 : $P_{213058} = (1, 0, 51, 1)$ lies on line ℓ_2
179 : $P_{217154} = (1, 0, 52, 1)$ lies on line ℓ_2
180 : $P_{221250} = (1, 0, 53, 1)$ lies on line ℓ_2
181 : $P_{225346} = (1, 0, 54, 1)$ lies on line ℓ_2
182 : $P_{229442} = (1, 0, 55, 1)$ lies on line ℓ_2
183 : $P_{233538} = (1, 0, 56, 1)$ lies on line ℓ_2
184 : $P_{237634} = (1, 0, 57, 1)$ lies on line ℓ_2
185 : $P_{241730} = (1, 0, 58, 1)$ lies on line ℓ_2
186 : $P_{245826} = (1, 0, 59, 1)$ lies on line ℓ_2
187 : $P_{249922} = (1, 0, 60, 1)$ lies on line ℓ_2
188 : $P_{254018} = (1, 0, 61, 1)$ lies on line ℓ_2
189 : $P_{258114} = (1, 0, 62, 1)$ lies on line ℓ_2
190 : $P_{262210} = (1, 0, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 4032 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 0
2	1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_2

Line 1 intersects

Line	ℓ_0
in point	P_0

Line 2 intersects

Line	ℓ_0
in point	P_2

The surface has 4225 points:
Too many to print.