Rank-73987 over GF(2)

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The equation

The equation of the surface is :

$$X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 73987

General information

Number of lines	5
Number of points	9
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	4
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	3, 2^4, 1^4

Singular Points

The surface has 3 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$2: P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0,0,1,0,0,1)_{22}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 4, 11, 30, 34)
Rank of points on Klein quadric: (0, 2, 22, 5, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).\ T = -1$

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0 :\ P_6 = (1,0,1,0)$ lies on line ℓ_1
$1 :\ P_8 = (1,1,1,0)$ lies on line ℓ_2
$2 :\ P_{10} = (0,1,0,1)$ lies on line ℓ_3
$3 :\ P_{12} = (0,0,1,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4
0	0 1 1 1 0
1	1 0 1 0 1
2	1 1 0 0 1
3	1 0 0 0 1
4	0 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$
$1: P_1 = (0, 1, 0, 0)$
$2: P_2 = (0, 0, 1, 0)$
$3: P_3 = (0, 0, 0, 1)$
$4: P_5 = (1, 1, 0, 0)$
$5: P_6 = (1, 0, 1, 0)$
$6: P_8 = (1, 1, 1, 0)$
$7: P_{10} = (0, 1, 0, 1)$
$8: P_{12} = (0, 0, 1, 1)$