Rank-74296 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_2^3 + X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over ${\rm GF}(8)$ is 1361388174

General information

Number of lines	1
Number of points	73
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

Singular Points

The surface has 1 singular points:

$$0: P_{139} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{1323}$$

Rank of lines: (666)

Rank of points on Klein quadric: (1323)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

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\begin{array}{lll} 0: \ P_5 = (1,1,0,0) \ \text{lies on line} \ \ell_0 \\ 1: \ P_{139} = (1,0,1,1) \ \text{lies on line} \ \ell_0 \\ 2: \ P_{146} = (0,1,1,1) \ \text{lies on line} \ \ell_0 \\ 3: \ P_{156} = (3,2,1,1) \ \text{lies on line} \ \ell_0 \\ 4: \ P_{163} = (2,3,1,1) \ \text{lies on line} \ \ell_0 \end{array}
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The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line: The points on the surface but not on lines are:

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0: P_1 = (0, 1, 0, 0)
                                                                  18: P_{239} = (6, 4, 2, 1)
1: P_3 = (0, 0, 0, 1)
                                                                  19: P_{254} = (5, 6, 2, 1)
2: P_{13} = (2,0,1,0)
                                                                  20: P_{255} = (6, 6, 2, 1)
3: P_{15} = (4,0,1,0)
                                                                  21: P_{256} = (7, 6, 2, 1)
4: P_{18} = (7, 0, 1, 0)
                                                                  22: P_{262} = (5,7,2,1)
5: P_{22} = (3, 1, 1, 0)
                                                                  23: P_{264} = (7,7,2,1)
6: P_{24} = (5, 1, 1, 0)
                                                                  24: P_{272} = (7,0,3,1)
7: P_{25} = (6, 1, 1, 0)
                                                                  25: P_{276} = (3, 1, 3, 1)
8: P_{75} = (1, 0, 0, 1)
                                                                  26: P_{294} = (5, 3, 3, 1)
9: P_{83} = (1, 1, 0, 1)
                                                                  27: P_{306} = (1, 5, 3, 1)
10: P_{159} = (6, 2, 1, 1)
                                                                  28: P_{320} = (7,6,3,1)
                                                                  29: P_{321} = (0, 7, 3, 1)
11: P_{168} = (7, 3, 1, 1)
12: P_{172} = (3, 4, 1, 1)
                                                                  30: P_{322} = (1,7,3,1)
13: P_{179} = (2, 5, 1, 1)
                                                                  31: P_{326} = (5,7,3,1)
14: P_{189} = (4, 6, 1, 1)
                                                                  32: P_{347} = (2, 2, 4, 1)
15: P_{198} = (5, 7, 1, 1)
                                                                  33: P_{351} = (6, 2, 4, 1)
16: P_{225} = (0, 3, 2, 1)
                                                                  34: P_{355} = (2, 3, 4, 1)
17: P_{235} = (2, 4, 2, 1)
                                                                  35: P_{356} = (3, 3, 4, 1)
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36: P_{359} = (6, 3, 4, 1)
                                                                  51: P_{489} = (0, 4, 6, 1)
37: P_{369} = (0, 5, 4, 1)
                                                                  52: P_{490} = (1, 4, 6, 1)
                                                                  53: P_{492} = (3, 4, 6, 1)
38: P_{388} = (3,7,4,1)
39: P_{389} = (4,7,4,1)
                                                                  54: P_{501} = (4, 5, 6, 1)
40: P_{395} = (2,0,5,1)
                                                                  55: P_{508} = (3, 6, 6, 1)
                                                                  56: P_{542} = (5, 2, 7, 1)
41: P_{406} = (5, 1, 5, 1)
42: P_{409} = (0, 2, 5, 1)
                                                                  57: P_{544} = (7, 2, 7, 1)
                                                                  58: P_{556} = (3, 4, 7, 1)
43: P_{410} = (1, 2, 5, 1)
44: P_{415} = (6, 2, 5, 1)
                                                                  59: P_{557} = (4, 4, 7, 1)
45: P_{419} = (2, 3, 5, 1)
                                                                  60: P_{564} = (3, 5, 7, 1)
46: P_{439} = (6, 5, 5, 1)
                                                                  61: P_{565} = (4, 5, 7, 1)
                                                                  62: P_{566} = (5, 5, 7, 1)
47: P_{442} = (1, 6, 5, 1)
48: P_{461} = (4, 0, 6, 1)
                                                                  63: P_{569} = (0, 6, 7, 1)
49: P_{471} = (6, 1, 6, 1)
50: P_{482} = (1, 3, 6, 1)
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Line Intersection Graph

 $\frac{0}{0 \mid 0}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 73 points: The points on the surface are:

$0: P_1 = (0, 1, 0, 0)$	$25: P_{225} = (0, 3, 2, 1)$	$50: P_{406} = (5, 1, 5, 1)$
$1: P_3 = (0,0,0,1)$	$26: P_{235} = (2, 4, 2, 1)$	$51: P_{409} = (0, 2, 5, 1)$
$2: P_5 = (1, 1, 0, 0)$	$27: P_{239} = (6, 4, 2, 1)$	$52: P_{410} = (1, 2, 5, 1)$
$3: P_{13} = (2,0,1,0)$	$28: P_{254} = (5, 6, 2, 1)$	$53: P_{415} = (6, 2, 5, 1)$
$4: P_{15} = (4,0,1,0)$	$29: P_{255} = (6, 6, 2, 1)$	$54: P_{419} = (2, 3, 5, 1)$
$5: P_{18} = (7,0,1,0)$	$30: P_{256} = (7, 6, 2, 1)$	$55: P_{439} = (6, 5, 5, 1)$
$6: P_{22} = (3, 1, 1, 0)$	$31: P_{262} = (5,7,2,1)$	$56: P_{442} = (1, 6, 5, 1)$
$7: P_{24} = (5, 1, 1, 0)$	$32: P_{264} = (7,7,2,1)$	$57: P_{461} = (4, 0, 6, 1)$
$8: P_{25} = (6, 1, 1, 0)$	$33: P_{272} = (7,0,3,1)$	$58: P_{471} = (6, 1, 6, 1)$
9: $P_{75} = (1,0,0,1)$	$34: P_{276} = (3, 1, 3, 1)$	$59: P_{482} = (1, 3, 6, 1)$
$10: P_{83} = (1, 1, 0, 1)$	$35: P_{294} = (5, 3, 3, 1)$	$60: P_{489} = (0,4,6,1)$
$11: P_{139} = (1, 0, 1, 1)$	$36: P_{306} = (1, 5, 3, 1)$	$61: P_{490} = (1, 4, 6, 1)$
$12: P_{146} = (0, 1, 1, 1)$	$37: P_{320} = (7, 6, 3, 1)$	$62: P_{492} = (3,4,6,1)$
$13: P_{156} = (3, 2, 1, 1)$	$38: P_{321} = (0,7,3,1)$	$63: P_{501} = (4, 5, 6, 1)$
$14: P_{159} = (6, 2, 1, 1)$	$39: P_{322} = (1,7,3,1)$	$64: P_{508} = (3, 6, 6, 1)$
$15: P_{163} = (2, 3, 1, 1)$	$40: P_{326} = (5, 7, 3, 1)$	$65: P_{542} = (5, 2, 7, 1)$
$16: P_{168} = (7, 3, 1, 1)$	$41: P_{347} = (2, 2, 4, 1)$	$66: P_{544} = (7, 2, 7, 1)$
$17: P_{172} = (3, 4, 1, 1)$	$42: P_{351} = (6, 2, 4, 1)$	$67: P_{556} = (3, 4, 7, 1)$
$18: P_{174} = (5, 4, 1, 1)$	$43: P_{355} = (2, 3, 4, 1)$	$68: P_{557} = (4, 4, 7, 1)$
$19: P_{179} = (2, 5, 1, 1)$	$44: P_{356} = (3, 3, 4, 1)$	$69: P_{564} = (3, 5, 7, 1)$
$20: P_{181} = (4, 5, 1, 1)$	$45: P_{359} = (6, 3, 4, 1)$	$70: P_{565} = (4, 5, 7, 1)$
$21: P_{189} = (4, 6, 1, 1)$	$46: P_{369} = (0, 5, 4, 1)$	$71: P_{566} = (5, 5, 7, 1)$
$22: P_{192} = (7, 6, 1, 1)$	$47: P_{388} = (3, 7, 4, 1)$	$72: P_{569} = (0, 6, 7, 1)$
$23: P_{198} = (5,7,1,1)$	$48: P_{389} = (4,7,4,1)$	
$24: P_{199} = (6,7,1,1)$	$49: P_{395} = (2, 0, 5, 1)$	