Rank-73993 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1243910805

General information

Number of lines	9
Number of points	89
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	54
Number of points off lines	24
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^9
Type of lines on points	$3^5, 2^6, 1^{54}, 0^{24}$

Singular Points

The surface has 4 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$ $3: P_{478} = \mathbf{P}(\gamma^3, \gamma, \gamma^6, 1) = \mathbf{P}(5,2,6,1)$
$1: P_{303} = \mathbf{P}(\gamma^6, \gamma^2, \gamma^5, 1) = \mathbf{P}(6,4,3,1)$
$2: P_{452} = \mathbf{P}(\gamma^5, \gamma^4, \gamma^3, 1) = \mathbf{P}(3,7,5,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^3 & \gamma^5 \end{bmatrix}_{29} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 3 \end{bmatrix}_{29} = \mathbf{Pl}(4,0,3,0,1,0)_{130}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^6 & \gamma^3 \end{bmatrix}_{46} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & 5 \end{bmatrix}_{46} = \mathbf{Pl}(7,0,5,0,1,0)_{163}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \gamma^5 & \gamma^6 \end{bmatrix}_{51} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 6 \end{bmatrix}_{51} = \mathbf{Pl}(2,0,6,0,1,0)_{173}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4734} = \mathbf{Pl}(0,6,0,1,0,0)_{30}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^3 & \gamma^2 \end{bmatrix}_{4052} = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{4052} = \mathbf{Pl}(5,7,3,4,3,1)_{2544}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4707} = \mathbf{Pl}(0,3,0,1,0,0)_{27}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \gamma & \gamma^5 \\ 0 & 1 & \gamma^6 & \gamma^4 \end{bmatrix}_{1960} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 6 & 7 \end{bmatrix}_{1960} = \mathbf{Pl}(6,2,5,7,5,1)_{3644}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^3 \\ 0 & 1 & \gamma^5 & \gamma \end{bmatrix}_{3231} = \begin{bmatrix} 1 & 0 & 4 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{3231} = \mathbf{Pl}(3,4,6,2,6,1)_{4166}$$

$$\ell_8 = \begin{bmatrix} 0 & 1 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4725} = \mathbf{Pl}(0,5,0,1,0,0)_{29}$$

Rank of lines: (29, 46, 51, 4734, 4052, 4707, 1960, 3231, 4725)
Rank of points on Klein quadric: (130, 163, 173, 30, 2544, 27, 3644, 4166, 29)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$2 : P_{303} = \mathbf{P}(\gamma^6,\gamma^2,\gamma^5,1) = \mathbf{P}(6,4,3,1)$,
$3 : P_{452} = \mathbf{P}(\gamma^5,\gamma^4,\gamma^3,1) = \mathbf{P}(3,7,5,1)$,
$4 : P_{478} = \mathbf{P}(\gamma^3,\gamma,\gamma^6,1) = \mathbf{P}(5,2,6,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{297} = (0,4,3,1) = \ell_0 \cap \ell_8$ $P_{225} = (0,3,2,1) = \ell_4 \cap \ell_8$
$P_{449} = (0,7,5,1) = \ell_1 \cap \ell_3$ $P_{569} = (0,6,7,1) = \ell_5 \cap \ell_7$
$P_{473} = (0,2,6,1) = \ell_2 \cap \ell_5$
$P_{369} = (0,5,4,1) = \ell_3 \cap \ell_6$

Single Points

The surface has 54 single points:
The single points on the surface are:

$0 : P_{27} = (0,2,1,0)$ lies on line ℓ_3 $3 : P_{55} = (4,5,1,0)$ lies on line ℓ_6
$1 : P_{37} = (2,3,1,0)$ lies on line ℓ_4 $4 : P_{66} = (7,6,1,0)$ lies on line ℓ_7
$2 : P_{43} = (0,4,1,0)$ lies on line ℓ_5 $5 : P_{67} = (0,7,1,0)$ lies on line ℓ_8

$6 : P_{100} = (2, 3, 0, 1)$ lies on line ℓ_7

$7 : P_{118} = (4, 5, 0, 1)$ lies on line ℓ_4

$8 : P_{129} = (7, 6, 0, 1)$ lies on line ℓ_6

$9 : P_{153} = (0, 2, 1, 1)$ lies on line ℓ_3

$10 : P_{164} = (3, 3, 1, 1)$ lies on line ℓ_6

$11 : P_{169} = (0, 4, 1, 1)$ lies on line ℓ_5

$12 : P_{182} = (5, 5, 1, 1)$ lies on line ℓ_7

$13 : P_{191} = (6, 6, 1, 1)$ lies on line ℓ_4

$14 : P_{193} = (0, 7, 1, 1)$ lies on line ℓ_8

$15 : P_{211} = (2, 1, 2, 1)$ lies on line ℓ_6

$16 : P_{218} = (1, 2, 2, 1)$ lies on line ℓ_7

$17 : P_{233} = (0, 4, 2, 1)$ lies on line ℓ_3

$18 : P_{241} = (0, 5, 2, 1)$ lies on line ℓ_5

$19 : P_{267} = (2, 0, 3, 1)$ lies on line ℓ_4

$20 : P_{273} = (0, 1, 3, 1)$ lies on line ℓ_5

$21 : P_{298} = (1, 4, 3, 1)$ lies on line ℓ_0

$22 : P_{299} = (2, 4, 3, 1)$ lies on line ℓ_0

$23 : P_{300} = (3, 4, 3, 1)$ lies on line ℓ_0

$24 : P_{301} = (4, 4, 3, 1)$ lies on line ℓ_0

$25 : P_{302} = (5, 4, 3, 1)$ lies on line ℓ_0

$26 : P_{304} = (7, 4, 3, 1)$ lies on line ℓ_0

$27 : P_{313} = (0, 6, 3, 1)$ lies on line ℓ_3

$28 : P_{341} = (4, 1, 4, 1)$ lies on line ℓ_7

$29 : P_{362} = (1, 4, 4, 1)$ lies on line ℓ_4

$30 : P_{377} = (0, 6, 4, 1)$ lies on line ℓ_8

$31 : P_{385} = (0, 7, 4, 1)$ lies on line ℓ_5

$32 : P_{397} = (4, 0, 5, 1)$ lies on line ℓ_6

$33 : P_{401} = (0, 1, 5, 1)$ lies on line ℓ_8

$34 : P_{417} = (0, 3, 5, 1)$ lies on line ℓ_5

$35 : P_{450} = (1, 7, 5, 1)$ lies on line ℓ_1

$36 : P_{451} = (2, 7, 5, 1)$ lies on line ℓ_1

$37 : P_{453} = (4, 7, 5, 1)$ lies on line ℓ_1

$38 : P_{454} = (5, 7, 5, 1)$ lies on line ℓ_1

$39 : P_{455} = (6, 7, 5, 1)$ lies on line ℓ_1

$40 : P_{456} = (7, 7, 5, 1)$ lies on line ℓ_1

$41 : P_{464} = (7, 0, 6, 1)$ lies on line ℓ_7

$42 : P_{465} = (0, 1, 6, 1)$ lies on line ℓ_3

$43 : P_{474} = (1, 2, 6, 1)$ lies on line ℓ_2

$44 : P_{475} = (2, 2, 6, 1)$ lies on line ℓ_2

$45 : P_{476} = (3, 2, 6, 1)$ lies on line ℓ_2

$46 : P_{477} = (4, 2, 6, 1)$ lies on line ℓ_2

$47 : P_{479} = (6, 2, 6, 1)$ lies on line ℓ_2

$48 : P_{480} = (7, 2, 6, 1)$ lies on line ℓ_2

$49 : P_{497} = (0, 5, 6, 1)$ lies on line ℓ_8

$50 : P_{536} = (7, 1, 7, 1)$ lies on line ℓ_4

$51 : P_{537} = (0, 2, 7, 1)$ lies on line ℓ_8

$52 : P_{545} = (0, 3, 7, 1)$ lies on line ℓ_3

$53 : P_{578} = (1, 7, 7, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 24 points not on any line:
The points on the surface but not on lines are:

$0 : P_{20} = (1, 1, 1, 0)$

$1 : P_{83} = (1, 1, 0, 1)$

$2 : P_{95} = (5, 2, 0, 1)$

$3 : P_{112} = (6, 4, 0, 1)$

$4 : P_{133} = (3, 7, 0, 1)$

$5 : P_{139} = (1, 0, 1, 1)$

$6 : P_{206} = (5, 0, 2, 1)$

$7 : P_{258} = (1, 7, 2, 1)$

$8 : P_{285} = (4, 2, 3, 1)$

$9 : P_{295} = (6, 3, 3, 1)$

$10 : P_{309} = (4, 5, 3, 1)$

$11 : P_{323} = (2, 7, 3, 1)$

$12 : P_{335} = (6, 0, 4, 1)$

$13 : P_{346} = (1, 2, 4, 1)$

$14 : P_{413} = (4, 2, 5, 1)$

$15 : P_{432} = (7, 4, 5, 1)$

$16 : P_{436} = (3, 5, 5, 1)$

$17 : P_{448} = (7, 6, 5, 1)$

$18 : P_{483} = (2, 3, 6, 1)$

$19 : P_{496} = (7, 4, 6, 1)$

$20 : P_{510} = (5, 6, 6, 1)$

$21 : P_{515} = (2, 7, 6, 1)$

$22 : P_{524} = (3, 0, 7, 1)$

$23 : P_{554} = (1, 4, 7, 1)$

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7 8
--+------------------
0 |0 1 1 0 0 0 1 1 1
1 |1 0 1 1 1 0 0 1 0
2 |1 1 0 0 1 1 1 0 0
3 |0 1 0 0 0 1 1 0 1
4 |0 1 1 0 0 0 1 1 1
5 |0 0 1 1 0 0 0 1 1
6 |1 0 1 1 1 0 0 1 0
7 |1 1 0 0 1 1 1 0 0
8 |1 0 0 1 1 1 0 0 0
```

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_0	P_{303}	P_{303}	P_{297}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_0	P_0	P_{449}	P_{452}	P_{452}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_0	P_{478}	P_{473}	P_{478}

Line 3 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_8
in point	P_{449}	P_3	P_{369}	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_6	ℓ_7	ℓ_8
in point	P_{452}	P_{478}	P_{478}	P_{452}	P_{225}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_{473}	P_3	P_{569}	P_3

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_7
in point	P_{303}	P_{478}	P_{369}	P_{478}	P_{303}

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_6
in point	P_{303}	P_{452}	P_{452}	P_{569}	P_{303}

Line 8 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5
in point	P_{297}	P_3	P_{225}	P_3

The surface has 89 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$
$1 : P_3 = (0,0,0,1)$
$2 : P_{20} = (1,1,1,0)$
$3 : P_{27} = (0,2,1,0)$
$4 : P_{37} = (2,3,1,0)$
$5 : P_{43} = (0,4,1,0)$
$6 : P_{55} = (4,5,1,0)$

$7 : P_{66} = (7,6,1,0)$
$8 : P_{67} = (0,7,1,0)$
$9 : P_{83} = (1,1,0,1)$
$10 : P_{95} = (5,2,0,1)$
$11 : P_{100} = (2,3,0,1)$
$12 : P_{112} = (6,4,0,1)$
$13 : P_{118} = (4,5,0,1)$

$14 : P_{129} = (7,6,0,1)$
$15 : P_{133} = (3,7,0,1)$
$16 : P_{139} = (1,0,1,1)$
$17 : P_{153} = (0,2,1,1)$
$18 : P_{164} = (3,3,1,1)$
$19 : P_{169} = (0,4,1,1)$
$20 : P_{182} = (5,5,1,1)$

$21: P_{191} = (6, 6, 1, 1)$

$22: P_{193} = (0, 7, 1, 1)$

$23: P_{206} = (5, 0, 2, 1)$

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$27: P_{233} = (0, 4, 2, 1)$

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$29: P_{258} = (1, 7, 2, 1)$

$30: P_{267} = (2, 0, 3, 1)$

$31: P_{273} = (0, 1, 3, 1)$

$32: P_{285} = (4, 2, 3, 1)$

$33: P_{295} = (6, 3, 3, 1)$

$34: P_{297} = (0, 4, 3, 1)$

$35: P_{298} = (1, 4, 3, 1)$

$36: P_{299} = (2, 4, 3, 1)$

$37: P_{300} = (3, 4, 3, 1)$

$38: P_{301} = (4, 4, 3, 1)$

$39: P_{302} = (5, 4, 3, 1)$

$40: P_{303} = (6, 4, 3, 1)$

$41: P_{304} = (7, 4, 3, 1)$

$42: P_{309} = (4, 5, 3, 1)$

$43: P_{313} = (0, 6, 3, 1)$

$44: P_{323} = (2, 7, 3, 1)$

$45: P_{335} = (6, 0, 4, 1)$

$46: P_{341} = (4, 1, 4, 1)$

$47: P_{346} = (1, 2, 4, 1)$

$48: P_{362} = (1, 4, 4, 1)$

$49: P_{369} = (0, 5, 4, 1)$

$50: P_{377} = (0, 6, 4, 1)$

$51: P_{385} = (0, 7, 4, 1)$

$52: P_{397} = (4, 0, 5, 1)$

$53: P_{401} = (0, 1, 5, 1)$

$54: P_{413} = (4, 2, 5, 1)$

$55: P_{417} = (0, 3, 5, 1)$

$56: P_{432} = (7, 4, 5, 1)$

$57: P_{436} = (3, 5, 5, 1)$

$58: P_{448} = (7, 6, 5, 1)$

$59: P_{449} = (0, 7, 5, 1)$

$60: P_{450} = (1, 7, 5, 1)$

$61: P_{451} = (2, 7, 5, 1)$

$62: P_{452} = (3, 7, 5, 1)$

$63: P_{453} = (4, 7, 5, 1)$

$64: P_{454} = (5, 7, 5, 1)$

$65: P_{455} = (6, 7, 5, 1)$

$66: P_{456} = (7, 7, 5, 1)$

$67: P_{464} = (7, 0, 6, 1)$

$68: P_{465} = (0, 1, 6, 1)$

$69: P_{473} = (0, 2, 6, 1)$

$70: P_{474} = (1, 2, 6, 1)$

$71: P_{475} = (2, 2, 6, 1)$

$72: P_{476} = (3, 2, 6, 1)$

$73: P_{477} = (4, 2, 6, 1)$

$74: P_{478} = (5, 2, 6, 1)$

$75: P_{479} = (6, 2, 6, 1)$

$76: P_{480} = (7, 2, 6, 1)$

$77: P_{483} = (2, 3, 6, 1)$

$78: P_{496} = (7, 4, 6, 1)$

$79: P_{497} = (0, 5, 6, 1)$

$80: P_{510} = (5, 6, 6, 1)$

$81: P_{515} = (2, 7, 6, 1)$

$82: P_{524} = (3, 0, 7, 1)$

$83: P_{536} = (7, 1, 7, 1)$

$84: P_{537} = (0, 2, 7, 1)$

$85: P_{545} = (0, 3, 7, 1)$

$86: P_{554} = (1, 4, 7, 1)$

$87: P_{569} = (0, 6, 7, 1)$

$88: P_{578} = (1, 7, 7, 1)$