Rank-65665 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227433109

General information

Number of lines	16
Number of points	105
Number of singular points	2
Number of Eckardt points	0
Number of double points	38
Number of single points	58
Number of points off lines	7
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{16}
Type of lines on points	5^2, 2^{38}, 1^{58}, 0^7

Singular Points

The surface has 2 singular points:

0 : $P_{83} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
1 : $P_{139} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$

The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_8 = \mathbf{Pl}(1,0,0,0,1,0)_{82}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{65} = \mathbf{Pl}(0,0,1,0,1,0)_{96}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{658} = \mathbf{Pl}(1,1,1,1,0,1)_{874}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^6 \\ 0 & 1 & \gamma & \gamma^3 \end{bmatrix}_{3765} = \begin{bmatrix} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 5 \end{bmatrix}_{3765} = \mathbf{Pl}(1,1,6,2,3,1)_{2673}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^6 \\ 0 & 1 & \gamma^5 & \gamma^4 \end{bmatrix}_{3782} = \begin{bmatrix} 1 & 0 & 3 & 6 \\ 0 & 1 & 3 & 7 \end{bmatrix}_{3782} = \mathbf{Pl}(1,1,6,2,6,1)_{4171}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \gamma & \gamma \\ 0 & 1 & \gamma^5 & \gamma^4 \end{bmatrix}_{1373} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 7 \end{bmatrix}_{1373} = \mathbf{Pl}(3,4,1,1,5,1)_{3445}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma^4 & \gamma^5 \end{bmatrix}_{688} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 7 & 3 \end{bmatrix}_{688} = \mathbf{Pl}(3,4,1,1,3,1)_{2465}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^5 \\ 0 & 1 & \gamma^3 & \gamma \end{bmatrix}_{2138} = \begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 5 & 2 \end{bmatrix}_{2138} = \mathbf{Pl}(1,1,3,4,3,1)_{2554}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^5 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{bmatrix}_{2169} = \begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 4 & 6 \end{bmatrix}_{2169} = \mathbf{Pl}(1,1,3,4,5,1)_{3534}$$

$$\ell_{10} = \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^2 \\ 0 & 1 & \gamma^3 & \gamma \end{bmatrix}_{2649} = \begin{bmatrix} 1 & 0 & 4 & 4 \\ 0 & 1 & 5 & 2 \end{bmatrix}_{2649} = \mathbf{Pl}(5,7,1,1,6,1)_{3965}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma & \gamma^3 \end{bmatrix}_{699} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 5 \end{bmatrix}_{699} = \mathbf{Pl}(5,7,1,1,5,1)_{3454}$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \gamma^2 & \gamma^6 \end{bmatrix}_{709} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 4 & 6 \end{bmatrix}_{709} = \mathbf{Pl}(6,2,1,1,6,1)_{3973}$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^4 \\ 0 & 1 & \gamma^6 & \gamma^2 \end{bmatrix}_{4637} = \begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 1 & 6 & 4 \end{bmatrix}_{4637} = \mathbf{Pl}(6,2,1,1,3,1)_{2475}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^3 \\ 0 & 1 & \gamma^6 & \gamma^2 \end{bmatrix}_{3396} = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & 6 & 4 \end{bmatrix}_{3396} = \mathbf{Pl}(1,1,5,7,5,1)_{3611}$$

$$\ell_{15} = \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^3 \\ 0 & 1 & \gamma^4 & \gamma^5 \end{bmatrix}_{3389} = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & 7 & 3 \end{bmatrix}_{3389} = \mathbf{Pl}(1,1,5,7,6,1)_{4136}$$

Rank of lines: (74, 8, 65, 658, 3765, 3782, 1373, 688, 2138, 2169, 2649, 699, 709, 4637, 3396, 3389)
Rank of points on Klein quadric: (665, 82, 96, 874, 2673, 4171, 3445, 2465, 2554, 3534, 3965, 3454, 3973, 2475, 3611, 4136)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 38 Double points:
The double points on the surface are:

$P_{19} = (0,1,1,0) = \ell_0 \cap \ell_3$
$P_{30} = (3,2,1,0) = \ell_0 \cap \ell_4$
$P_{37} = (2,3,1,0) = \ell_0 \cap \ell_6$
$P_{48} = (5,4,1,0) = \ell_0 \cap \ell_9$
$P_{55} = (4,5,1,0) = \ell_0 \cap \ell_{10}$
$P_{66} = (7,6,1,0) = \ell_0 \cap \ell_{13}$
$P_{73} = (6,7,1,0) = \ell_0 \cap \ell_{15}$
$P_0 = (1,0,0,0) = \ell_1 \cap \ell_2$
$P_{87} = (5,1,0,1) = \ell_1 \cap \ell_4$
$P_{88} = (6,1,0,1) = \ell_1 \cap \ell_9$
$P_{85} = (3,1,0,1) = \ell_1 \cap \ell_{15}$
$P_{144} = (6,0,1,1) = \ell_2 \cap \ell_6$
$P_{141} = (3,0,1,1) = \ell_2 \cap \ell_{10}$
$P_{143} = (5,0,1,1) = \ell_2 \cap \ell_{13}$
$P_{459} = (2,0,6,1) = \ell_4 \cap \ell_5$
$P_{244} = (3,5,2,1) = \ell_4 \cap \ell_7$
$P_{321} = (0,7,3,1) = \ell_4 \cap \ell_{11}$
$P_{365} = (4,4,4,1) = \ell_4 \cap \ell_{13}$
$P_{241} = (0,5,2,1) = \ell_5 \cap \ell_6$
$P_{327} = (6,7,3,1) = \ell_5 \cap \ell_{10}$
$P_{364} = (3,4,4,1) = \ell_5 \cap \ell_{12}$
$P_{102} = (4,3,0,1) = \ell_6 \cap \ell_7$
$P_{584} = (7,7,7,1) = \ell_6 \cap \ell_9$
$P_{494} = (5,4,6,1) = \ell_6 \cap \ell_{14}$
$P_{582} = (5,7,7,1) = \ell_7 \cap \ell_8$
$P_{489} = (0,4,6,1) = \ell_7 \cap \ell_{15}$
$P_{269} = (4,0,3,1) = \ell_8 \cap \ell_9$
$P_{377} = (0,6,4,1) = \ell_8 \cap \ell_{10}$
$P_{412} = (3,2,5,1) = \ell_8 \cap \ell_{13}$
$P_{382} = (5,6,4,1) = \ell_9 \cap \ell_{11}$
$P_{409} = (0,2,5,1) = \ell_9 \cap \ell_{12}$
$P_{121} = (7,5,0,1) = \ell_{10} \cap \ell_{11}$
$P_{219} = (2,2,2,1) = \ell_{10} \cap \ell_{15}$
$P_{223} = (6,2,2,1) = \ell_{11} \cap \ell_{14}$
$P_{124} = (2,6,0,1) = \ell_{12} \cap \ell_{13}$
$P_{551} = (6,3,7,1) = \ell_{12} \cap \ell_{15}$
$P_{545} = (0,3,7,1) = \ell_{13} \cap \ell_{14}$
$P_{400} = (7,0,5,1) = \ell_{14} \cap \ell_{15}$

Single Points

The surface has 58 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
1 : $P_{12} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{33} = (6,2,1,0)$ lies on line ℓ_5
3 : $P_{40} = (5,3,1,0)$ lies on line ℓ_7
4 : $P_{46} = (3,4,1,0)$ lies on line ℓ_8
5 : $P_{57} = (6,5,1,0)$ lies on line ℓ_{11}
6 : $P_{62} = (3,6,1,0)$ lies on line ℓ_{12}
7 : $P_{72} = (5,7,1,0)$ lies on line ℓ_{14}
8 : $P_{82} = (0,1,0,1)$ lies on line ℓ_1
9 : $P_{84} = (2,1,0,1)$ lies on line ℓ_1
10 : $P_{86} = (4,1,0,1)$ lies on line ℓ_1
11 : $P_{89} = (7,1,0,1)$ lies on line ℓ_1
12 : $P_{138} = (0,0,1,1)$ lies on line ℓ_2
13 : $P_{140} = (2,0,1,1)$ lies on line ℓ_2
14 : $P_{142} = (4,0,1,1)$ lies on line ℓ_2
15 : $P_{145} = (7,0,1,1)$ lies on line ℓ_2
16 : $P_{167} = (6,3,1,1)$ lies on line ℓ_4
17 : $P_{168} = (7,3,1,1)$ lies on line ℓ_5
18 : $P_{179} = (2,5,1,1)$ lies on line ℓ_8
19 : $P_{180} = (3,5,1,1)$ lies on line ℓ_9
20 : $P_{189} = (4,6,1,1)$ lies on line ℓ_{14}
21 : $P_{190} = (5,6,1,1)$ lies on line ℓ_{15}
22 : $P_{226} = (1,3,2,1)$ lies on line ℓ_3
23 : $P_{234} = (1,4,2,1)$ lies on line ℓ_9
24 : $P_{240} = (7,4,2,1)$ lies on line ℓ_8
25 : $P_{258} = (1,7,2,1)$ lies on line ℓ_{13}
26 : $P_{261} = (4,7,2,1)$ lies on line ℓ_{12}
27 : $P_{279} = (6,1,3,1)$ lies on line ℓ_{13}
28 : $P_{280} = (7,1,3,1)$ lies on line ℓ_{12}
29 : $P_{282} = (1,2,3,1)$ lies on line ℓ_3
30 : $P_{308} = (3,5,3,1)$ lies on line ℓ_{14}
31 : $P_{309} = (4,5,3,1)$ lies on line ℓ_{15}
32 : $P_{315} = (2,6,3,1)$ lies on line ℓ_6
33 : $P_{319} = (6,6,3,1)$ lies on line ℓ_7
34 : $P_{346} = (1,2,4,1)$ lies on line ℓ_6
35 : $P_{352} = (7,2,4,1)$ lies on line ℓ_7
36 : $P_{370} = (1,5,4,1)$ lies on line ℓ_3
37 : $P_{386} = (1,7,4,1)$ lies on line ℓ_{15}
38 : $P_{387} = (2,7,4,1)$ lies on line ℓ_{14}
39 : $P_{403} = (2,1,5,1)$ lies on line ℓ_7
40 : $P_{404} = (3,1,5,1)$ lies on line ℓ_6
41 : $P_{420} = (3,3,5,1)$ lies on line ℓ_{11}
42 : $P_{421} = (4,3,5,1)$ lies on line ℓ_{10}
43 : $P_{426} = (1,4,5,1)$ lies on line ℓ_3
44 : $P_{446} = (5,6,5,1)$ lies on line ℓ_5
45 : $P_{448} = (7,6,5,1)$ lies on line ℓ_4
46 : $P_{469} = (4,1,6,1)$ lies on line ℓ_{11}
47 : $P_{470} = (5,1,6,1)$ lies on line ℓ_{10}
48 : $P_{483} = (2,3,6,1)$ lies on line ℓ_9
49 : $P_{487} = (6,3,6,1)$ lies on line ℓ_8
50 : $P_{502} = (5,5,6,1)$ lies on line ℓ_{12}
51 : $P_{504} = (7,5,6,1)$ lies on line ℓ_{13}
52 : $P_{514} = (1,7,6,1)$ lies on line ℓ_3
53 : $P_{538} = (1,2,7,1)$ lies on line ℓ_4

54 : $P_{541} = (4, 2, 7, 1)$ lies on line ℓ_5
55 : $P_{554} = (1, 4, 7, 1)$ lies on line ℓ_{10}
56 : $P_{555} = (2, 4, 7, 1)$ lies on line ℓ_{11}
57 : $P_{570} = (1, 6, 7, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 7 points not on any line:
The points on the surface but not on lines are:

0 : $P_{75} = (1, 0, 0, 1)$
1 : $P_{94} = (4, 2, 0, 1)$
2 : $P_{113} = (7, 4, 0, 1)$
3 : $P_{132} = (2, 7, 0, 1)$
4 : $P_{205} = (4, 0, 2, 1)$
5 : $P_{336} = (7, 0, 4, 1)$
6 : $P_{523} = (2, 0, 7, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	1	1	0	1	0	0	1	1	0	0	1	0	1
1	0	0	1	1	1	1	0	0	1	1	0	0	0	0	1	1
2	0	1	0	1	0	0	1	1	0	0	1	1	1	1	0	0
3	1	1	1	0	0	1	0	1	1	0	0	1	1	0	1	0
4	1	1	0	0	0	1	0	1	0	0	0	1	0	1	0	0
5	0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0
6	1	0	1	0	0	1	0	1	0	1	0	0	0	0	1	0
7	0	0	1	1	1	0	1	0	1	0	0	1	1	0	0	1
8	0	1	0	1	0	1	0	1	0	1	1	0	0	1	1	0
9	1	1	0	0	0	0	1	0	1	0	0	1	1	0	0	0
10	1	0	1	0	0	1	0	0	1	0	0	1	0	0	0	1
11	0	0	1	1	1	0	0	1	0	1	1	0	1	0	1	0
12	0	0	1	1	0	1	0	1	0	1	0	1	0	1	0	1
13	1	0	1	0	1	0	0	0	1	0	0	0	1	0	1	0
14	0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1
15	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_3	ℓ_4	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{15}
in point	P_{19}	P_{30}	P_{37}	P_{48}	P_{55}	P_{66}	P_{73}

Line 1 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_9	ℓ_{14}	ℓ_{15}
in point	P_0	P_{83}	P_{87}	P_{83}	P_{83}	P_{88}	P_{83}	P_{85}

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_0	P_{139}	P_{144}	P_{139}	P_{141}	P_{139}	P_{139}	P_{143}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}
in point	P_{19}	P_{83}	P_{139}	P_{83}	P_{139}	P_{83}	P_{139}	P_{139}	P_{83}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}
in point	P_{30}	P_{87}	P_{459}	P_{244}	P_{321}	P_{365}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{83}	P_{83}	P_{459}	P_{241}	P_{83}	P_{327}	P_{364}	P_{83}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_7	ℓ_9	ℓ_{14}
in point	P_{37}	P_{144}	P_{241}	P_{102}	P_{584}	P_{494}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{15}
in point	P_{139}	P_{139}	P_{244}	P_{102}	P_{582}	P_{139}	P_{139}	P_{489}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}
in point	P_{83}	P_{83}	P_{83}	P_{582}	P_{269}	P_{377}	P_{412}	P_{83}

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_8	ℓ_{11}	ℓ_{12}
in point	P_{48}	P_{88}	P_{584}	P_{269}	P_{382}	P_{409}

Line 10 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{15}
in point	P_{55}	P_{141}	P_{327}	P_{377}	P_{121}	P_{219}

Line 11 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{139}	P_{139}	P_{321}	P_{139}	P_{382}	P_{121}	P_{139}	P_{223}

Line 12 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}
in point	P_{139}	P_{139}	P_{364}	P_{139}	P_{409}	P_{139}	P_{124}	P_{551}

Line 13 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_8	ℓ_{12}	ℓ_{14}
in point	P_{66}	P_{143}	P_{365}	P_{412}	P_{124}	P_{545}

Line 14 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_8	ℓ_{11}	ℓ_{13}	ℓ_{15}
in point	P_{83}	P_{83}	P_{83}	P_{494}	P_{83}	P_{223}	P_{545}	P_{400}

Line 15 intersects

Line	ℓ_0	ℓ_1	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{73}	P_{85}	P_{489}	P_{219}	P_{551}	P_{400}

The surface has 105 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_5 = (1,1,0,0)$
$2: P_{12} = (1,0,1,0)$
$3: P_{19} = (0,1,1,0)$
$4: P_{30} = (3,2,1,0)$
$5: P_{33} = (6,2,1,0)$
$6: P_{37} = (2,3,1,0)$
$7: P_{40} = (5,3,1,0)$
$8: P_{46} = (3,4,1,0)$
$9: P_{48} = (5,4,1,0)$
$10: P_{55} = (4,5,1,0)$
$11: P_{57} = (6,5,1,0)$
$12: P_{62} = (3,6,1,0)$
$13: P_{66} = (7,6,1,0)$
$14: P_{72} = (5,7,1,0)$
$15: P_{73} = (6,7,1,0)$
$16: P_{75} = (1,0,0,1)$
$17: P_{82} = (0,1,0,1)$
$18: P_{83} = (1,1,0,1)$
$19: P_{84} = (2,1,0,1)$
$20: P_{85} = (3,1,0,1)$
$21: P_{86} = (4,1,0,1)$
$22: P_{87} = (5,1,0,1)$
$23: P_{88} = (6,1,0,1)$
$24: P_{89} = (7,1,0,1)$
$25: P_{94} = (4,2,0,1)$
$26: P_{102} = (4,3,0,1)$
$27: P_{113} = (7,4,0,1)$
$28: P_{121} = (7,5,0,1)$
$29: P_{124} = (2,6,0,1)$
$30: P_{132} = (2,7,0,1)$
$31: P_{138} = (0,0,1,1)$
$32: P_{139} = (1,0,1,1)$

33 : $P_{140} = (2, 0, 1, 1)$
34 : $P_{141} = (3, 0, 1, 1)$
35 : $P_{142} = (4, 0, 1, 1)$
36 : $P_{143} = (5, 0, 1, 1)$
37 : $P_{144} = (6, 0, 1, 1)$
38 : $P_{145} = (7, 0, 1, 1)$
39 : $P_{167} = (6, 3, 1, 1)$
40 : $P_{168} = (7, 3, 1, 1)$
41 : $P_{179} = (2, 5, 1, 1)$
42 : $P_{180} = (3, 5, 1, 1)$
43 : $P_{189} = (4, 6, 1, 1)$
44 : $P_{190} = (5, 6, 1, 1)$
45 : $P_{205} = (4, 0, 2, 1)$
46 : $P_{219} = (2, 2, 2, 1)$
47 : $P_{223} = (6, 2, 2, 1)$
48 : $P_{226} = (1, 3, 2, 1)$
49 : $P_{234} = (1, 4, 2, 1)$
50 : $P_{240} = (7, 4, 2, 1)$
51 : $P_{241} = (0, 5, 2, 1)$
52 : $P_{244} = (3, 5, 2, 1)$
53 : $P_{258} = (1, 7, 2, 1)$
54 : $P_{261} = (4, 7, 2, 1)$
55 : $P_{269} = (4, 0, 3, 1)$
56 : $P_{279} = (6, 1, 3, 1)$
57 : $P_{280} = (7, 1, 3, 1)$
58 : $P_{282} = (1, 2, 3, 1)$
59 : $P_{308} = (3, 5, 3, 1)$
60 : $P_{309} = (4, 5, 3, 1)$
61 : $P_{315} = (2, 6, 3, 1)$
62 : $P_{319} = (6, 6, 3, 1)$
63 : $P_{321} = (0, 7, 3, 1)$
64 : $P_{327} = (6, 7, 3, 1)$
65 : $P_{336} = (7, 0, 4, 1)$
66 : $P_{346} = (1, 2, 4, 1)$
67 : $P_{352} = (7, 2, 4, 1)$
68 : $P_{364} = (3, 4, 4, 1)$
69 : $P_{365} = (4, 4, 4, 1)$
70 : $P_{370} = (1, 5, 4, 1)$
71 : $P_{377} = (0, 6, 4, 1)$
72 : $P_{382} = (5, 6, 4, 1)$
73 : $P_{386} = (1, 7, 4, 1)$
74 : $P_{387} = (2, 7, 4, 1)$
75 : $P_{400} = (7, 0, 5, 1)$
76 : $P_{403} = (2, 1, 5, 1)$
77 : $P_{404} = (3, 1, 5, 1)$
78 : $P_{409} = (0, 2, 5, 1)$
79 : $P_{412} = (3, 2, 5, 1)$
80 : $P_{420} = (3, 3, 5, 1)$
81 : $P_{421} = (4, 3, 5, 1)$
82 : $P_{426} = (1, 4, 5, 1)$
83 : $P_{446} = (5, 6, 5, 1)$
84 : $P_{448} = (7, 6, 5, 1)$
85 : $P_{459} = (2, 0, 6, 1)$
86 : $P_{469} = (4, 1, 6, 1)$
87 : $P_{470} = (5, 1, 6, 1)$
88 : $P_{483} = (2, 3, 6, 1)$
89 : $P_{487} = (6, 3, 6, 1)$
90 : $P_{489} = (0, 4, 6, 1)$
91 : $P_{494} = (5, 4, 6, 1)$
92 : $P_{502} = (5, 5, 6, 1)$
93 : $P_{504} = (7, 5, 6, 1)$
94 : $P_{514} = (1, 7, 6, 1)$
95 : $P_{523} = (2, 0, 7, 1)$
96 : $P_{538} = (1, 2, 7, 1)$
97 : $P_{541} = (4, 2, 7, 1)$
98 : $P_{545} = (0, 3, 7, 1)$
99 : $P_{551} = (6, 3, 7, 1)$
100 : $P_{554} = (1, 4, 7, 1)$
101 : $P_{555} = (2, 4, 7, 1)$
102 : $P_{570} = (1, 6, 7, 1)$
103 : $P_{582} = (5, 7, 7, 1)$
104 : $P_{584} = (7, 7, 7, 1)$