Rank-65687 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_0X_1^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1229234829

General information

Number of lines	10
Number of points	81
Number of singular points	9
Number of Eckardt points	0
Number of double points	0
Number of single points	80
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{10}
Type of lines on points	10, 1^{80}

Singular Points

The surface has 9 singular points:

0 : $P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
1 : $P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
2 : $P_{20} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
3 : $P_{219} = \mathbf{P}(\gamma,\gamma,\gamma,1) = \mathbf{P}(2,2,2,1)$
4 : $P_{292} = \mathbf{P}(\gamma^5,\gamma^5,\gamma^5,1) = \mathbf{P}(3,3,3,1)$
5 : $P_{365} = \mathbf{P}(\gamma^2,\gamma^2,\gamma^2,1) = \mathbf{P}(4,4,4,1)$
6 : $P_{438} = \mathbf{P}(\gamma^3,\gamma^3,\gamma^3,1) = \mathbf{P}(5,5,5,1)$
7 : $P_{511} = \mathbf{P}(\gamma^6,\gamma^6,\gamma^6,1) = \mathbf{P}(6,6,6,1)$
8 : $P_{584} = \mathbf{P}(\gamma^4,\gamma^4,\gamma^4,1) = \mathbf{P}(7,7,7,1)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \mathbf{Pl}(0,0,0,0,1,0)_{81}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{145} = \mathbf{Pl}(0,0,0,1,1,0)_{201}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{729} = \mathbf{Pl}(0,1,0,1,1,0)_{209}$$

$$\ell_4 = \begin{bmatrix} 1 & \gamma^6 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2262} = \begin{bmatrix} 1 & 6 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2262} = \mathbf{Pl}(0,3,0,6,1,0)_{286}$$

$$\ell_5 = \begin{bmatrix} 1 & \gamma^4 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3503} = \begin{bmatrix} 1 & 7 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3503} = \mathbf{Pl}(0,5,0,7,1,0)_{303}$$

$$\ell_6 = \begin{bmatrix} 1 & \gamma & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3722} = \begin{bmatrix} 1 & 2 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3722} = \mathbf{Pl}(0,6,0,2,1,0)_{229}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^5 & \gamma^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3211} = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3211} = \mathbf{Pl}(0,5,0,3,1,0)_{243}$$

$$\ell_8 = \begin{bmatrix} 1 & \gamma^3 & \gamma^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3941} = \begin{bmatrix} 1 & 5 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3941} = \mathbf{Pl}(0,6,0,5,1,0)_{274}$$

$$\ell_9 = \begin{bmatrix} 1 & \gamma^2 & \gamma^5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2116} = \begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2116} = \mathbf{Pl}(0,3,0,4,1,0)_{256}$$

Rank of lines: (72, 4680, 145, 729, 2262, 3503, 3722, 3211, 3941, 2116)
Rank of points on Klein quadric: (81, 17, 201, 209, 286, 303, 229, 243, 274, 256)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 80 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_1 = (0,1,0,0)$ lies on line ℓ_1
2 : $P_4 = (1,1,1,1)$ lies on line ℓ_3
3 : $P_5 = (1,1,0,0)$ lies on line ℓ_2
4 : $P_{20} = (1,1,1,0)$ lies on line ℓ_3
5 : $P_{31} = (4,2,1,0)$ lies on line ℓ_4

6 : $P_{34} = (7, 2, 1, 0)$ lies on line ℓ_5
7 : $P_{45} = (2, 4, 1, 0)$ lies on line ℓ_6
8 : $P_{50} = (7, 4, 1, 0)$ lies on line ℓ_7
9 : $P_{69} = (2, 7, 1, 0)$ lies on line ℓ_8
10 : $P_{71} = (4, 7, 1, 0)$ lies on line ℓ_9
11 : $P_{75} = (1, 0, 0, 1)$ lies on line ℓ_0
12 : $P_{76} = (2, 0, 0, 1)$ lies on line ℓ_0
13 : $P_{77} = (3, 0, 0, 1)$ lies on line ℓ_0
14 : $P_{78} = (4, 0, 0, 1)$ lies on line ℓ_0
15 : $P_{79} = (5, 0, 0, 1)$ lies on line ℓ_0
16 : $P_{80} = (6, 0, 0, 1)$ lies on line ℓ_0
17 : $P_{81} = (7, 0, 0, 1)$ lies on line ℓ_0
18 : $P_{82} = (0, 1, 0, 1)$ lies on line ℓ_1
19 : $P_{83} = (1, 1, 0, 1)$ lies on line ℓ_2
20 : $P_{90} = (0, 2, 0, 1)$ lies on line ℓ_1
21 : $P_{92} = (2, 2, 0, 1)$ lies on line ℓ_2
22 : $P_{98} = (0, 3, 0, 1)$ lies on line ℓ_1
23 : $P_{101} = (3, 3, 0, 1)$ lies on line ℓ_2
24 : $P_{106} = (0, 4, 0, 1)$ lies on line ℓ_1
25 : $P_{110} = (4, 4, 0, 1)$ lies on line ℓ_2
26 : $P_{114} = (0, 5, 0, 1)$ lies on line ℓ_1
27 : $P_{119} = (5, 5, 0, 1)$ lies on line ℓ_2
28 : $P_{122} = (0, 6, 0, 1)$ lies on line ℓ_1
29 : $P_{128} = (6, 6, 0, 1)$ lies on line ℓ_2
30 : $P_{130} = (0, 7, 0, 1)$ lies on line ℓ_1
31 : $P_{137} = (7, 7, 0, 1)$ lies on line ℓ_2
32 : $P_{157} = (4, 2, 1, 1)$ lies on line ℓ_4
33 : $P_{160} = (7, 2, 1, 1)$ lies on line ℓ_5
34 : $P_{171} = (2, 4, 1, 1)$ lies on line ℓ_6
35 : $P_{176} = (7, 4, 1, 1)$ lies on line ℓ_7
36 : $P_{195} = (2, 7, 1, 1)$ lies on line ℓ_8
37 : $P_{197} = (4, 7, 1, 1)$ lies on line ℓ_9
38 : $P_{219} = (2, 2, 2, 1)$ lies on line ℓ_3
39 : $P_{229} = (4, 3, 2, 1)$ lies on line ℓ_8
40 : $P_{230} = (5, 3, 2, 1)$ lies on line ℓ_9
41 : $P_{236} = (3, 4, 2, 1)$ lies on line ℓ_5
42 : $P_{238} = (5, 4, 2, 1)$ lies on line ℓ_4
43 : $P_{244} = (3, 5, 2, 1)$ lies on line ℓ_7
44 : $P_{245} = (4, 5, 2, 1)$ lies on line ℓ_6
45 : $P_{277} = (4, 1, 3, 1)$ lies on line ℓ_7
46 : $P_{279} = (6, 1, 3, 1)$ lies on line ℓ_6
47 : $P_{292} = (3, 3, 3, 1)$ lies on line ℓ_3
48 : $P_{298} = (1, 4, 3, 1)$ lies on line ℓ_9
49 : $P_{303} = (6, 4, 3, 1)$ lies on line ℓ_8
50 : $P_{314} = (1, 6, 3, 1)$ lies on line ℓ_4
51 : $P_{317} = (4, 6, 3, 1)$ lies on line ℓ_5
52 : $P_{365} = (4, 4, 4, 1)$ lies on line ℓ_3
53 : $P_{375} = (6, 5, 4, 1)$ lies on line ℓ_5
54 : $P_{376} = (7, 5, 4, 1)$ lies on line ℓ_4
55 : $P_{382} = (5, 6, 4, 1)$ lies on line ℓ_8
56 : $P_{384} = (7, 6, 4, 1)$ lies on line ℓ_9
57 : $P_{390} = (5, 7, 4, 1)$ lies on line ℓ_6
58 : $P_{391} = (6, 7, 4, 1)$ lies on line ℓ_7
59 : $P_{404} = (3, 1, 5, 1)$ lies on line ℓ_9
60 : $P_{408} = (7, 1, 5, 1)$ lies on line ℓ_8
61 : $P_{418} = (1, 3, 5, 1)$ lies on line ℓ_7
62 : $P_{424} = (7, 3, 5, 1)$ lies on line ℓ_6
63 : $P_{438} = (5, 5, 5, 1)$ lies on line ℓ_3
64 : $P_{450} = (1, 7, 5, 1)$ lies on line ℓ_5
65 : $P_{452} = (3, 7, 5, 1)$ lies on line ℓ_4
66 : $P_{467} = (2, 1, 6, 1)$ lies on line ℓ_4
67 : $P_{470} = (5, 1, 6, 1)$ lies on line ℓ_5
68 : $P_{474} = (1, 2, 6, 1)$ lies on line ℓ_6
69 : $P_{478} = (5, 2, 6, 1)$ lies on line ℓ_7
70 : $P_{498} = (1, 5, 6, 1)$ lies on line ℓ_8
71 : $P_{499} = (2, 5, 6, 1)$ lies on line ℓ_9
72 : $P_{511} = (6, 6, 6, 1)$ lies on line ℓ_3
73 : $P_{540} = (3, 2, 7, 1)$ lies on line ℓ_8
74 : $P_{543} = (6, 2, 7, 1)$ lies on line ℓ_9
75 : $P_{547} = (2, 3, 7, 1)$ lies on line ℓ_5
76 : $P_{551} = (6, 3, 7, 1)$ lies on line ℓ_4
77 : $P_{571} = (2, 6, 7, 1)$ lies on line ℓ_7
78 : $P_{572} = (3, 6, 7, 1)$ lies on line ℓ_6
79 : $P_{584} = (7, 7, 7, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1
3	1	1	1	0	1	1	1	1	1	1
4	1	1	1	1	0	1	1	1	1	1
5	1	1	1	1	1	0	1	1	1	1
6	1	1	1	1	1	1	0	1	1	1
7	1	1	1	1	1	1	1	0	1	1
8	1	1	1	1	1	1	1	1	0	1
9	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 9 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 81 points:

The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_3 = (0,0,0,1)$
$3: P_4 = (1,1,1,1)$
$4: P_5 = (1,1,0,0)$
$5: P_{20} = (1,1,1,0)$
$6: P_{31} = (4,2,1,0)$
$7: P_{34} = (7,2,1,0)$
$8: P_{45} = (2,4,1,0)$
$9: P_{50} = (7,4,1,0)$
$10: P_{69} = (2,7,1,0)$
$11: P_{71} = (4,7,1,0)$
$12: P_{75} = (1,0,0,1)$
$13: P_{76} = (2,0,0,1)$
$14: P_{77} = (3,0,0,1)$
$15: P_{78} = (4,0,0,1)$
$16: P_{79} = (5,0,0,1)$
$17: P_{80} = (6,0,0,1)$
$18: P_{81} = (7,0,0,1)$
$19: P_{82} = (0,1,0,1)$
$20: P_{83} = (1,1,0,1)$
$21: P_{90} = (0,2,0,1)$
$22: P_{92} = (2,2,0,1)$
$23: P_{98} = (0,3,0,1)$
$24: P_{101} = (3,3,0,1)$
$25: P_{106} = (0,4,0,1)$
$26: P_{110} = (4,4,0,1)$
$27: P_{114} = (0,5,0,1)$
$28: P_{119} = (5,5,0,1)$
$29: P_{122} = (0,6,0,1)$
$30: P_{128} = (6,6,0,1)$
$31: P_{130} = (0,7,0,1)$
$32: P_{137} = (7,7,0,1)$
$33: P_{157} = (4,2,1,1)$
$34: P_{160} = (7,2,1,1)$
$35: P_{171} = (2,4,1,1)$
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$80: P_{584} = (7,7,7,1)$