Rank-346 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 346

General information

Number of lines	3
Number of points	9
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	1^9

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1,1,1,0,1,1)_{30}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

Rank of lines: (12, 10, 23)
Rank of points on Klein quadric: (32, 30, 31)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 9 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_0
1 : $P_5 = (1,1,0,0)$ lies on line ℓ_0
2 : $P_6 = (1,0,1,0)$ lies on line ℓ_1
3 : $P_8 = (1,1,1,0)$ lies on line ℓ_2
4 : $P_{10} = (0,1,0,1)$ lies on line ℓ_2
5 : $P_{11} = (1,1,0,1)$ lies on line ℓ_1
6 : $P_{12} = (0,0,1,1)$ lies on line ℓ_0
7 : $P_{13} = (1,0,1,1)$ lies on line ℓ_2
8 : $P_{14} = (0,1,1,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2
0	0 0 0
1	0 0 0
2	0 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

Line 2 intersects

Line
in point

The surface has 9 points:
The points on the surface are:

$0: P_4 = (1,1,1,1)$
$1: P_5 = (1,1,0,0)$
$2: P_6 = (1,0,1,0)$
$3: P_8 = (1,1,1,0)$
$4: P_{10} = (0,1,0,1)$
$5: P_{11} = (1,1,0,1)$
$6: P_{12} = (0,0,1,1)$
$7: P_{13} = (1,0,1,1)$
$8: P_{14} = (0,1,1,1)$