Rank-65843 over GF(32)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^2 X_1 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)The point rank of the equation over GF(32) is 1142981669

General information

34
1089
33
0
0
1088
0
0
0
33^{34}
$34, 1^{1088}$

Singular Points

The surface has 33 singular points:

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\begin{array}{lll} 0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0) & 8: \ P_{8225} = \mathbf{P}(0,0,\eta^{11},1) = \mathbf{P}(0,0,7,1) \\ 1: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1) & 9: \ P_{9249} = \mathbf{P}(0,0,\eta^3,1) = \mathbf{P}(0,0,8,1) \\ 2: \ P_{2082} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1) & 10: \ P_{10273} = \mathbf{P}(0,0,\eta^{29},1) = \mathbf{P}(0,0,9,1) \\ 3: \ P_{3105} = \mathbf{P}(0,0,\eta,1) = \mathbf{P}(0,0,2,1) & 11: \ P_{11297} = \mathbf{P}(0,0,\eta^6,1) = \mathbf{P}(0,0,10,1) \\ 4: \ P_{4129} = \mathbf{P}(0,0,\eta^{18},1) = \mathbf{P}(0,0,3,1) & 12: \ P_{12321} = \mathbf{P}(0,0,\eta^{27},1) = \mathbf{P}(0,0,11,1) \\ 5: \ P_{5153} = \mathbf{P}(0,0,\eta^2,1) = \mathbf{P}(0,0,4,1) & 13: \ P_{13345} = \mathbf{P}(0,0,\eta^{20},1) = \mathbf{P}(0,0,12,1) \\ 6: \ P_{6177} = \mathbf{P}(0,0,\eta^5,1) = \mathbf{P}(0,0,5,1) & 14: \ P_{14369} = \mathbf{P}(0,0,\eta^8,1) = \mathbf{P}(0,0,13,1) \\ 7: \ P_{7201} = \mathbf{P}(0,0,\eta^{19},1) = \mathbf{P}(0,0,6,1) & 15: \ P_{15393} = \mathbf{P}(0,0,\eta^{12},1) = \mathbf{P}(0,0,14,1) \end{array}
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\begin{array}{lll} 16: \ P_{16417} = \mathbf{P}(0,0,\eta^{23},1) = \mathbf{P}(0,0,15,1) & 25: \ P_{25633} = \mathbf{P}(0,0,\eta^{21},1) = \mathbf{P}(0,0,24,1) \\ 17: \ P_{17441} = \mathbf{P}(0,0,\eta^4,1) = \mathbf{P}(0,0,16,1) & 26: \ P_{26657} = \mathbf{P}(0,0,\eta^{25},1) = \mathbf{P}(0,0,25,1) \\ 18: \ P_{18465} = \mathbf{P}(0,0,\eta^{10},1) = \mathbf{P}(0,0,17,1) & 27: \ P_{27681} = \mathbf{P}(0,0,\eta^9,1) = \mathbf{P}(0,0,26,1) \\ 19: \ P_{19489} = \mathbf{P}(0,0,\eta^{30},1) = \mathbf{P}(0,0,18,1) & 28: \ P_{28705} = \mathbf{P}(0,0,\eta^{16},1) = \mathbf{P}(0,0,27,1) \\ 20: \ P_{20513} = \mathbf{P}(0,0,\eta^7,1) = \mathbf{P}(0,0,19,1) & 29: \ P_{29729} = \mathbf{P}(0,0,\eta^{13},1) = \mathbf{P}(0,0,28,1) \\ 21: \ P_{21537} = \mathbf{P}(0,0,\eta^{27},1) = \mathbf{P}(0,0,20,1) & 30: \ P_{30753} = \mathbf{P}(0,0,\eta^{14},1) = \mathbf{P}(0,0,29,1) \\ 22: \ P_{22561} = \mathbf{P}(0,0,\eta^{22},1) = \mathbf{P}(0,0,21,1) & 31: \ P_{31777} = \mathbf{P}(0,0,\eta^{24},1) = \mathbf{P}(0,0,30,1) \\ 23: \ P_{23585} = \mathbf{P}(0,0,\eta^{26},1) = \mathbf{P}(0,0,23,1) & 32: \ P_{32801} = \mathbf{P}(0,0,\eta^{15},1) = \mathbf{P}(0,0,31,1) \end{array}
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The 34 Lines

The lines and their Pluecker coordinates are:

$$\begin{split} \ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1056} = \mathbf{PI}(0,0,0,1,0)_{1089} \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{PI}(0,0,0,1,0,0)_{65} \\ \ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{PI}(0,1,0,0,0,0)_{1} \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{35937} = \mathbf{PI}(0,1,0,1,1,0)_{3137} \\ \ell_4 &= \begin{bmatrix} 1 & \eta^{29} & \eta^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{247337} &= \begin{bmatrix} 1 & 9 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{247337} = \mathbf{PI}(0,7,0,9,1,0)_{3647} \\ \ell_5 &= \begin{bmatrix} 1 & \eta^3 & \eta^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{922760} &= \begin{bmatrix} 1 & 8 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{922760} &= \mathbf{PI}(0,27,0,8,1,0)_{3604} \\ \ell_6 &= \begin{bmatrix} 1 & \eta^6 & \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{722987} &= \begin{bmatrix} 1 & 10 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{722987} &= \mathbf{PI}(0,2,0,10,1,0)_{3705} \\ \ell_7 &= \begin{bmatrix} 1 & \eta^{27} & \eta^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{722987} &= \begin{bmatrix} 1 & 11 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{722987} &= \mathbf{PI}(0,24,0,2,1,0)_{3223} \\ \ell_9 &= \begin{bmatrix} 1 & \eta^{18} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{814946} &= \begin{bmatrix} 1 & 2 & 24 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{814946} &= \mathbf{PI}(0,24,0,2,1,0)_{3223} \\ \ell_{10} &= \begin{bmatrix} 1 & \eta^{18} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{263780} &= \begin{bmatrix} 1 & 20 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{563380} &= \mathbf{PI}(0,16,0,20,1,0)_{4349} \\ \ell_{11} &= \begin{bmatrix} 1 & \eta^{22} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27132} &= \begin{bmatrix} 1 & 21 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{226197} &= \mathbf{PI}(0,16,0,21,1,0)_{4402} \\ \ell_{12} &= \begin{bmatrix} 1 & \eta^{14} & \eta^8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{471421} &= \begin{bmatrix} 1 & 29 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{707132} &= \mathbf{PI}(0,20,0,28,1,0)_{4857} \\ \ell_{14} &= \begin{bmatrix} 1 & \eta^{13} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{811775} &= \begin{bmatrix} 1 & 31 & 23 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{811775} &= \mathbf{PI}(0,20,0,28,1,0)_{4979} \\ \ell_{15} &= \begin{bmatrix} 1 & \eta^{13} & \eta^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{811775} &= \begin{bmatrix} 1 & 30 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{811775} &= \mathbf{PI}(0,16,0,30,1,0)_{4979} \end{aligned}$$

$$\begin{split} &\ell_{16} = \begin{bmatrix} 1 & \eta^{26} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1006263} = \begin{bmatrix} 1 & 23 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1006263} = \mathbf{PI}(0,29,0,23,1,0)_{4551} \\ &\ell_{17} = \begin{bmatrix} 1 & \eta^{28} & \eta^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{937558} = \begin{bmatrix} 1 & 22 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{937558} = \mathbf{PI}(0,27,0,22,1,0)_{4486} \\ &\ell_{18} = \begin{bmatrix} 1 & \eta^{23} & \eta^{13} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{963983} = \begin{bmatrix} 1 & 15 & 28 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{963983} = \mathbf{PI}(0,28,0,15,1,0)_{4046} \\ &\ell_{19} = \begin{bmatrix} 1 & \eta^{12} & \eta^{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{151150} = \begin{bmatrix} 1 & 14 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{151150} = \mathbf{PI}(0,4,0,14,1,0)_{3959} \\ &\ell_{20} = \begin{bmatrix} 1 & \eta^{19} & \eta^{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{142694} = \begin{bmatrix} 1 & 6 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{142694} = \mathbf{PI}(0,4,0,6,1,0)_{3455} \\ &\ell_{21} = \begin{bmatrix} 1 & \eta^{11} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{242052} = \begin{bmatrix} 1 & 7 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{854055} = \mathbf{PI}(0,25,0,7,1,0)_{3539} \\ &\ell_{22} = \begin{bmatrix} 1 & \eta^{2} & \eta^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{242052} = \begin{bmatrix} 1 & 5 & 29 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{987237} = \mathbf{PI}(0,29,0,5,1,0)_{3417} \\ &\ell_{24} = \begin{bmatrix} 1 & \eta^{5} & \eta^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{961869} = \begin{bmatrix} 1 & 13 & 28 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{961869} = \mathbf{PI}(0,28,0,13,1,0)_{3920} \\ &\ell_{25} = \begin{bmatrix} 1 & \eta^{20} & \eta^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{857347} = \begin{bmatrix} 1 & 12 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{859340} = \mathbf{PI}(0,28,0,13,1,0)_{3854} \\ &\ell_{27} = \begin{bmatrix} 1 & \eta^{16} & \eta^{26} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{807547} = \begin{bmatrix} 1 & 27 & 23 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{307547} = \mathbf{PI}(0,23,0,27,1,0)_{4797} \\ &\ell_{27} = \begin{bmatrix} 1 & \eta^{9} & \eta^{19} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{231482} = \begin{bmatrix} 1 & 26 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{331558} = \mathbf{PI}(0,24,0,18,1,0)_{4283} \\ &\ell_{29} = \begin{bmatrix} 1 & \eta^{17} & \eta^{8} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{763153} = \begin{bmatrix} 1 & 18 & 24 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{763153} = \mathbf{PI}(0,24,0,18,1,0)_{4283} \\ &\ell_{31} = \begin{bmatrix} 1 & \eta^{4} & \eta^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{763153} = \begin{bmatrix} 1 & 16 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{763153} = \mathbf{PI}(0,22,0,24,1,0)_{4607} \\ &0 & 0 & 0 & 1 \end{bmatrix}_{96129} = \mathbf{PI}(0,22,0,25,1,0)_{4607} \\ &\ell_{33} = \begin{bmatrix} 1 & \eta^{25} & \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{95129} = \begin{bmatrix} 1 & 162 & 22 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{96129} = \mathbf{PI}(0,22,0,25,1,0)_{4650} \\ &0 & 0 & 0 & 1 \end{bmatrix}_{9$$

Rank of lines: (1056, 1082400, 1083424, 35937, 247337, 922760, 79274, 722987, 814946, 680707, 563380, 226197, 471421, 707132, 811775, 573950, 1006263, 937558, 963983, 151150, 142694, 854055, 242052, 987237, 961869, 859340, 807547, 231482, 460851, 831858, 763153, 728272, 770552, 95129) Rank of points on Klein quadric: (1089, 65, 1, 3137, 3647, 3604, 3705, 3787, 3223, 3282, 4349, 4402, 4913, 4857, 5049, 4979, 4551, 4486, 4046, 3959, 3455, 3539, 3332, 3417, 3920, 3854, 4797, 4717, 4283, 4231, 4166, 4102, 4607, 4650)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 1088 single points: Too many to print.

Points on surface but on no line

The surface has 0 points not on any line: The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	1 :	5 (6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24:	25	26	27	28	29	30	31	32	33
0	0	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	0	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1 () [1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1 ()	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1 :	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1 :	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1 :	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1 :	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1 :	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1 :	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1 :	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1 :	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1 :	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	Ι-	1	_	_		_	_	_	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20		1							1	_	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
21		1									1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
22		1									1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
23	-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
24	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
25	-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
27	-	1	_	_		_	_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
	Ι-	1	_	_				_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
	-	1	_	_			_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
30	-	1	_	_		_	_	_	1	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
31	-	1	_	_	-	-	_	1	1	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
32	-	1	_	_		_	_	_	_	_	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
33	1	1	1	1	1 .	L	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 3 intersects $\frac{1}{2}$

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3									

Line 5 intersects Line L	Line 4 intersects	
Line 5 intersects Line L	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point P ₃ P ₃	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 5 intersects	
Line 6 intersects Line Li		
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
		2
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 7 intersects	
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 8 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	3
Line 10 intersects Line \$\lambda_0 \begin{array}{c c c c c c c c c c c c c c c c c c c	Line 9 intersects	
Line 10 intersects Line $\begin{pmatrix} l_0 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & l_8 & l_9 & l_{11} & l_{12} & l_{13} & l_{14} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{19} & l_{19} & l_{19} & l_{19} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{20} & l_{21} & l_{22} & l_{19} & l_{1$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
	in point $\mid P_3 \mid P_3 \mid$	3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
Line 11 intersects Line $ l_0 l_1 l_2 l_3 l_4 l_5 l_6 l_7 l_8 l_9 l_{10} l_{12} l_{13} l_{14} l_{15} l_{16} l_{17} l_{18} l_{19} l_{20} l_{21} l_{22} l_{10} l_{10} l_{10} l_{11} l_{13} l_{14} l_{15} l_{16} l_{17} l_{18} l_{19} l_{20} l_{21} l_{22} l_{10} l_{10} l_{10} l_{11} l_{13} l_{14} l_{15} l_{16} l_{17} l_{18} l_{19} l_{20} l_{21} l_{22} l_{21} l_{22} l_{10} l_{10} l_{11} l_{13} l_{14} l_{15} l_{16} l_{17} l_{18} l_{19} l_{20} l_{21} l_{22} l_{21} l_{22} l_{21} l_{22} l_{22} l_{23} l_{24} l_{25} l_{2$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	2
	in point $ P_3 $ $ P_$;
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		
Line 12 intersects Line $ \ell_0 \ell_1 \ell_2 \ell_3 \ell_4 \ell_5 \ell_6 \ell_7 \ell_8 \ell_9 \ell_{10} \ell_{11} \ell_{13} \ell_{14} \ell_{15} \ell_{16} \ell_{17} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{10} \ell_{10} \ell_{11} \ell_{13} \ell_{14} \ell_{15} \ell_{16} \ell_{17} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{10} \ell_{10} \ell_{11} \ell_{13} \ell_{14} \ell_{15} \ell_{16} \ell_{17} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{22} \ell_{23} \ell_{24} \ell_{2$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $;
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 12 intersects	
Line 13 intersects Line ℓ_0 ℓ_1 ℓ_2 ℓ_3 ℓ_4 ℓ_5 ℓ_6 ℓ_7 ℓ_8 ℓ_9 ℓ_{10} ℓ_{11} ℓ_{12} ℓ_{14} ℓ_{15} ℓ_{16} ℓ_{17} ℓ_{18} ℓ_{19} ℓ_{20} ℓ_{21} ℓ_{22} in point P_3		_
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	<u>:</u>
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 13 intersects	
Line 14 intersects Line ℓ_0 ℓ_1 ℓ_2 ℓ_3 ℓ_4 ℓ_5 ℓ_6 ℓ_7 ℓ_8 ℓ_9 ℓ_{10} ℓ_{11} ℓ_{12} ℓ_{13} ℓ_{15} ℓ_{16} ℓ_{17} ℓ_{18} ℓ_{19} ℓ_{20} ℓ_{21} ℓ_{22} in point P_3	V 2 2 3 V V V V V 2 2 2 2 2 2 2 2 2 2 2	_
	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	<u>. </u>
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 14 intersects	
Line 15 intersects Line ℓ_0 ℓ_1 ℓ_2 ℓ_3 ℓ_4 ℓ_5 ℓ_6 ℓ_7 ℓ_8 ℓ_9 ℓ_{10} ℓ_{11} ℓ_{12} ℓ_{13} ℓ_{14} ℓ_{16} ℓ_{17} ℓ_{18} ℓ_{19} ℓ_{20} ℓ_{21} ℓ_{22} in point P_3	0 1 2 0 1 0 0 1 0 1 1 12 10 10 11 12 12	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $;
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Line 15 intersects	
Line 16 intersects Line $ \ell_0 \ell_1 \ell_2 \ell_3 \ell_4 \ell_5 \ell_6 \ell_7 \ell_8 \ell_9 \ell_{10} \ell_{11} \ell_{12} \ell_{13} \ell_{14} \ell_{15} \ell_{17} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{18} \ell_{19} \ell_{20} \ell_{21} \ell_{22} \ell_{22} \ell_{21} \ell_{22} \ell_{22$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $,
	Line 16 intersects	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$;

Line 17 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 18 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 19 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 20 int	erse	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 21 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{22} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 22 int	ersec	ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 23 int		ets																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 24 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 25 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 26 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 27 int																						
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 28 int		cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										
Line 29 int	ersec	cts																				
Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										

Line 30 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	,
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3											

Line 31 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										

${\bf Line~32~intersects}$

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	1
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3											

Line 33 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21} ℓ
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3										

The surface has 1089 points:

Too many to print.