

Rank-65603 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431659865

General information

Number of lines	29
Number of points	37
Number of singular points	9
Number of Eckardt points	0
Number of double points	16
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{29}
Type of lines on points	$6^8, 5^{13}, 2^{16}$

Singular Points

The surface has 9 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$3 : P_{26} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$$

$$4 : P_{30} = \mathbf{P}(0, \omega, 0, 1) = \mathbf{P}(0, 2, 0, 1)$$

$$5 : P_{34} = \mathbf{P}(0, \omega^2, 0, 1) = \mathbf{P}(0, 3, 0, 1)$$

$$6 : P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$7 : P_{53} = \mathbf{P}(0, 0, \omega, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$8 : P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1)$$

The 29 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{101} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_9 \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{129} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0, 0, 0, 3, 0, 1)_{143} \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0, 0, 0, 2, 0, 1)_{136} \\
\ell_7 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_8 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{105} \\
\ell_9 &= \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0, 3, 0, 0, 0, 1)_{107} \\
\ell_{10} &= \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0, 2, 0, 0, 0, 1)_{106} \\
\ell_{11} &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{198} \\
\ell_{12} &= \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{82} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{82} = \mathbf{Pl}(0, 0, 2, 3, 1, 1)_{205} \\
\ell_{13} &= \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{60} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{60} = \mathbf{Pl}(0, 0, 3, 2, 1, 1)_{212} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1, 1, 0, 0, 1, 1)_{177} \\
\ell_{15} &= \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & 0 & \omega^2 \end{bmatrix}_{75} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{75} = \mathbf{Pl}(2, 3, 0, 0, 1, 1)_{178} \\
\ell_{16} &= \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & 0 & \omega \end{bmatrix}_{50} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}_{50} = \mathbf{Pl}(3, 2, 0, 0, 1, 1)_{179} \\
\ell_{17} &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{13} \\
\ell_{18} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{133} \\
\ell_{19} &= \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{354} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{354} = \mathbf{Pl}(0, 3, 0, 3, 0, 1)_{149}
\end{aligned}$$

$$\begin{aligned}
\ell_{20} &= \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{bmatrix}_{348} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{348} = \mathbf{Pl}(0, 2, 0, 2, 0, 1)_{141} \\
\ell_{21} &= \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0, 3, 0, 1, 0, 0)_{15} \\
\ell_{22} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{343} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{343} = \mathbf{Pl}(0, 1, 0, 2, 0, 1)_{140} \\
\ell_{23} &= \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \mathbf{Pl}(0, 3, 0, 1, 0, 1)_{135} \\
\ell_{24} &= \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{349} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{349} = \mathbf{Pl}(0, 2, 0, 3, 0, 1)_{148} \\
\ell_{25} &= \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0, 2, 0, 1, 0, 0)_{14} \\
\ell_{26} &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{344} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{344} = \mathbf{Pl}(0, 1, 0, 3, 0, 1)_{147} \\
\ell_{27} &= \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{353} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{353} = \mathbf{Pl}(0, 3, 0, 2, 0, 1)_{142} \\
\ell_{28} &= \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \mathbf{Pl}(0, 2, 0, 1, 0, 1)_{134}
\end{aligned}$$

Rank of lines: (0, 16, 336, 340, 337, 339, 338, 356, 341, 351, 346, 38, 82, 60, 25, 75, 50, 345, 342, 354, 348, 355, 343, 352, 349, 350, 344, 353, 347)

Rank of points on Klein quadric: (0, 2, 101, 9, 129, 143, 136, 1, 105, 107, 106, 198, 205, 212, 177, 178, 179, 13, 133, 149, 141, 15, 140, 135, 148, 14, 147, 142, 134)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 16 Double points:

The double points on the surface are:

$$\begin{aligned}
P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 & P_{52} &= (3, 3, 1, 1) = \ell_{11} \cap \ell_{16} \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_{11} & P_{59} &= (2, 1, 2, 1) = \ell_{12} \cap \ell_{14} \\
P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_{12} & P_{64} &= (3, 2, 2, 1) = \ell_{12} \cap \ell_{15} \\
P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_{13} & P_{66} &= (1, 3, 2, 1) = \ell_{12} \cap \ell_{16} \\
P_8 &= (1, 0, 1, 0) = \ell_1 \cap \ell_{14} & P_{76} &= (3, 1, 3, 1) = \ell_{13} \cap \ell_{14} \\
P_9 &= (2, 0, 1, 0) = \ell_1 \cap \ell_{15} & P_{78} &= (1, 2, 3, 1) = \ell_{13} \cap \ell_{15} \\
P_{10} &= (3, 0, 1, 0) = \ell_1 \cap \ell_{16} & P_{83} &= (2, 3, 3, 1) = \ell_{13} \cap \ell_{16} \\
P_4 &= (1, 1, 1, 1) = \ell_{11} \cap \ell_{14} \\
P_{47} &= (2, 2, 1, 1) = \ell_{11} \cap \ell_{15}
\end{aligned}$$

Single Points

The surface has 0 single points:

The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
0	0	1	1	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	1	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
3	1	0	1	0	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
5	1	0	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
6	1	0	1	1	1	1	0	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
7	0	1	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
8	0	1	1	1	1	1	1	0	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1
10	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	0	0	0	1	0	0	1	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	0	0	0	0	1
12	1	0	0	0	0	1	0	1	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	1	0	1	0	0	0
13	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	1	0	1	0	1	0	0	0	1	0
14	0	1	0	1	0	0	0	0	1	0	1	1	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
15	0	1	0	1	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0
16	0	1	0	1	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
17	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
18	0	0	1	1	1	1	1	1	1	1	1	0	0	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1
19	0	0	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1	1
20	0	0	1	1	1	1	1	1	1	1	1	0	0	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1
21	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1
22	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1
23	0	0	1	1	1	1	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1
24	0	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	1
25	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1
26	0	0	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1
27	0	0	1	1	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
28	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_0	P_1	P_1	P_1	P_1	P_1	P_5	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_0	P_2	P_2	P_2	P_2	P_2	P_8	P_9	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_1	P_2	P_1	P_1	P_1	P_1	P_2	P_2	P_2	P_2	P_{11}	P_{11}	P_{11}	P_{11}	P_{15}	P_{15}	P_{15}	P_{15}	P_{19}	P_{19}	P_{19}

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_1	P_1	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_1	P_1	P_1	P_1	P_1	P_{38}	P_{42}	P_{45}	P_{49}	P_{38}	P_{42}	P_{38}	P_{49}	P_{45}	P_{45}	P_{49}	P_{38}	P_{42}	P_{49}	P_{45}	P_{45}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_1	P_1	P_1	P_1	P_1	P_{53}	P_{57}	P_{61}	P_{65}	P_{53}	P_{61}	P_{65}	P_{53}	P_{57}	P_{65}	P_{61}	P_{57}	P_{53}	P_{57}	P_{53}	P_{57}

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_1	P_1	P_1	P_1	P_1	P_{69}	P_{73}	P_{77}	P_{81}	P_{69}	P_{81}	P_{77}	P_{73}	P_{69}	P_{73}	P_{69}	P_{81}	P_{77}	P_{77}	P_{81}	P_{77}

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_2	P_2	P_2	P_{38}	P_{53}	P_{69}	P_3	P_{38}	P_{53}	P_{69}	P_3	P_{69}	P_{38}	P_{53}	P_3

Line 8 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_2	P_2	P_{26}	P_{42}	P_{57}	P_{73}	P_2	P_2	P_2	P_{26}	P_{42}	P_{26}	P_{73}	P_{57}	P_{73}	P_{26}	P_{57}	P_{42}	P_{57}	P_{26}	P_{26}

Line 9 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_2	P_2	P_{30}	P_{45}	P_{61}	P_{77}	P_2	P_2	P_2	P_{30}	P_{61}	P_{77}	P_{30}	P_{45}	P_{45}	P_{61}	P_{30}	P_{77}	P_{77}	P_{45}	P_{45}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}
in point	P_2	P_2	P_{34}	P_{49}	P_{65}	P_{81}	P_2	P_2	P_2	P_{34}	P_{81}	P_{65}	P_{49}	P_{34}	P_{65}	P_{49}	P_{81}	P_{34}	P_{49}	P_{81}	P_{34}

Line 11 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}	ℓ_{23}	ℓ_{28}
in point	P_5	P_{38}	P_{38}	P_4	P_{47}	P_{52}	P_{38}	P_{38}	P_{38}

Line 12 intersects

Line	ℓ_0	ℓ_5	ℓ_7	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{24}	ℓ_{26}
in point	P_6	P_{53}	P_{53}	P_{59}	P_{64}	P_{66}	P_{53}	P_{53}	P_{53}

Line 13 intersects

Line	ℓ_0	ℓ_6	ℓ_7	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{20}	ℓ_{22}	ℓ_{27}
in point	P_7	P_{69}	P_{69}	P_{76}	P_{78}	P_{83}	P_{69}	P_{69}	P_{69}

Line 14 intersects

Line	ℓ_1	ℓ_3	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{18}	ℓ_{22}	ℓ_{26}
in point	P_8	P_{26}	P_{26}	P_4	P_{59}	P_{76}	P_{26}	P_{26}	P_{26}

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{19}	ℓ_{23}	ℓ_{27}
in point	P_9	P_{30}	P_{30}	P_{47}	P_{64}	P_{78}	P_{30}	P_{30}	P_{30}

Line 16 intersects

Line	ℓ_1	ℓ_3	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{20}	ℓ_{24}	ℓ_{28}
in point	P_{10}	P_{34}	P_{34}	P_{52}	P_{66}	P_{83}	P_{34}	P_{34}	P_{34}

Line 17 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{42}	P_{61}	P_{81}	P_{11}	P_{11}	P_{11}	P_3	P_{61}	P_{81}	P_{42}	P_3	P_{81}	P_{42}	P_{61}

Line 18 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11}	P_{26}	P_{38}	P_{65}	P_{77}	P_{38}	P_{26}	P_{77}	P_{65}	P_{38}	P_{26}	P_{11}	P_{11}	P_{11}	P_{65}	P_{26}	P_{38}	P_{77}	P_{77}	P_{26}

Line 19 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11}	P_{30}	P_{49}	P_{53}	P_{73}	P_{53}	P_{73}	P_{30}	P_{49}	P_{53}	P_{30}	P_{11}	P_{11}	P_{11}	P_{73}	P_{49}	P_{30}	P_{53}	P_{49}	P_{53}

Line 20 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11}	P_{34}	P_{45}	P_{57}	P_{69}	P_{69}	P_{57}	P_{45}	P_{34}	P_{69}	P_{34}	P_{11}	P_{11}	P_{11}	P_{45}	P_{69}	P_{57}	P_{34}	P_{57}	P_{45}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{15}	P_3	P_{45}	P_{65}	P_{73}	P_3	P_{73}	P_{45}	P_{65}	P_3	P_{65}	P_{73}	P_{45}	P_{15}	P_{15}	P_{15}	P_3	P_{45}	P_{65}	P_{73}

Line 22 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{15}	P_{26}	P_{49}	P_{61}	P_{69}	P_{69}	P_{26}	P_{61}	P_{49}	P_{69}	P_{26}	P_{61}	P_{26}	P_{49}	P_{69}	P_{15}	P_{15}	P_{15}	P_{49}	P_{26}

Line 23 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{15}	P_{30}	P_{38}	P_{57}	P_{81}	P_{38}	P_{57}	P_{30}	P_{81}	P_{38}	P_{30}	P_{81}	P_{38}	P_{30}	P_{57}	P_{15}	P_{15}	P_{15}	P_{57}	P_{81}

Line 24 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{25}	ℓ_{26}
in point	P_{15}	P_{34}	P_{42}	P_{53}	P_{77}	P_{53}	P_{42}	P_{77}	P_{34}	P_{53}	P_{34}	P_{42}	P_{77}	P_{53}	P_{34}	P_{15}	P_{15}	P_{15}	P_{77}	P_{53}

Line 25 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{19}	P_3	P_{49}	P_{57}	P_{77}	P_3	P_{57}	P_{77}	P_{49}	P_3	P_{77}	P_{49}	P_{57}	P_3	P_{49}	P_{57}	P_{77}	P_{19}	P_{19}	P_{19}

Line 26 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{14}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{19}	P_{26}	P_{45}	P_{53}	P_{81}	P_{53}	P_{26}	P_{45}	P_{81}	P_{53}	P_{26}	P_{81}	P_{26}	P_{53}	P_{45}	P_{45}	P_{26}	P_{81}	P_{53}	P_{19}

Line 27 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{19}	P_{30}	P_{42}	P_{65}	P_{69}	P_{69}	P_{42}	P_{30}	P_{65}	P_{69}	P_{30}	P_{42}	P_{65}	P_{30}	P_{69}	P_{65}	P_{69}	P_{30}	P_{42}	P_{19}

Line 28 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{19}	P_{34}	P_{38}	P_{61}	P_{73}	P_{38}	P_{73}	P_{61}	P_{34}	P_{38}	P_{34}	P_{61}	P_{38}	P_{73}	P_{34}	P_{73}	P_{61}	P_{38}	P_{34}	P_{19}

The surface has 37 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_5 = (1, 1, 0, 0)$
 6 : $P_6 = (2, 1, 0, 0)$
 7 : $P_7 = (3, 1, 0, 0)$
 8 : $P_8 = (1, 0, 1, 0)$
 9 : $P_9 = (2, 0, 1, 0)$
 10 : $P_{10} = (3, 0, 1, 0)$
 11 : $P_{11} = (0, 1, 1, 0)$
 12 : $P_{15} = (0, 2, 1, 0)$

13 : $P_{19} = (0, 3, 1, 0)$
 14 : $P_{26} = (0, 1, 0, 1)$
 15 : $P_{30} = (0, 2, 0, 1)$
 16 : $P_{34} = (0, 3, 0, 1)$
 17 : $P_{38} = (0, 0, 1, 1)$
 18 : $P_{42} = (0, 1, 1, 1)$
 19 : $P_{45} = (0, 2, 1, 1)$
 20 : $P_{47} = (2, 2, 1, 1)$
 21 : $P_{49} = (0, 3, 1, 1)$
 22 : $P_{52} = (3, 3, 1, 1)$
 23 : $P_{53} = (0, 0, 2, 1)$
 24 : $P_{57} = (0, 1, 2, 1)$
 25 : $P_{59} = (2, 1, 2, 1)$

26 : $P_{61} = (0, 2, 2, 1)$
 27 : $P_{64} = (3, 2, 2, 1)$
 28 : $P_{65} = (0, 3, 2, 1)$
 29 : $P_{66} = (1, 3, 2, 1)$
 30 : $P_{69} = (0, 0, 3, 1)$
 31 : $P_{73} = (0, 1, 3, 1)$
 32 : $P_{76} = (3, 1, 3, 1)$
 33 : $P_{77} = (0, 2, 3, 1)$
 34 : $P_{78} = (1, 2, 3, 1)$
 35 : $P_{81} = (0, 3, 3, 1)$
 36 : $P_{83} = (2, 3, 3, 1)$