Rank-74007 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74007

General information

Number of lines	3
Number of points	9
Number of singular points	1
Number of Eckardt points	0
Number of double points	2
Number of single points	5
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^3
Type of lines on points	$2^2,\ 1^5,\ 0^2$

Singular Points

The surface has 1 singular points:

$$0: P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{30} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

Rank of lines: (30, 33, 23)
Rank of points on Klein quadric: (5, 7, 31)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 2 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$
$P_{10} = (0,1,0,1) = \ell_0 \cap \ell_2$

Single Points

The surface has 5 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0 3 : $P_{13} = (1,0,1,1)$ lies on line ℓ_2
1 : $P_7 = (0,1,1,0)$ lies on line ℓ_1 4 : $P_{14} = (0,1,1,1)$ lies on line ℓ_1
2 : $P_8 = (1,1,1,0)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1,0,0,0)$
1 : $P_{11} = (1,1,0,1)$

Line Intersection Graph

```
 |0 1 2
0|0 1 1
1|1 0 0
2|1 0 0
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Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_3	P_{10}

Line 1 intersects

Line	ℓ_0
in point	P_3

Line 2 intersects

Line	ℓ_0
in point	P_{10}

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$

$1: P_1 = (0,1,0,0)$

$2: P_3 = (0,0,0,1)$

$3: P_7 = (0,1,1,0)$

$4: P_8 = (1,1,1,0)$

$5: P_{10} = (0,1,0,1)$

$6: P_{11} = (1,1,0,1)$

$7: P_{13} = (1,0,1,1)$

$8: P_{14} = (0,1,1,1)$