# Rank-139 over GF(64)

January 15, 2021

# The equation

The equation of the surface is:

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 = 0$$

( 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) The point rank of the equation over  $\mathrm{GF}(64)$  is -2113396593

## General information

Number of lines	6
Number of points	4289
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	378
Number of points off lines	3906
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^{6}$
Type of lines on points	$3^2, 2^3, 1^{378}, 0^{3906}$

# Singular Points

The surface has 1 singular points:

$$0: P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

## The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1, 0, 1, 0, 0, 0)_3$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{64} = \mathbf{Pl}(1,0,0,0,1,0)_{4226}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4097} = \mathbf{Pl}(0,0,1,0,1,0)_{4352}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{8258} = \mathbf{Pl}(0,0,1,1,1,1)_{544578}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4225} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4225} = \mathbf{Pl}(1,1,0,0,1,1)_{536577}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{266305} = \mathbf{Pl}(1,1,1,0,0)_{256}$$

Rank of lines: (1, 64, 4097, 8258, 4225, 266305)

Rank of points on Klein quadric: (3, 4226, 4352, 544578, 536577, 256)

#### **Eckardt Points**

The surface has 2 Eckardt points:

$$0: P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0),$$

$$1: P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1).$$

## **Double Points**

The surface has 3 Double points:

The double points on the surface are:

$$P_{131} = (0, 1, 1, 0) = \ell_0 \cap \ell_5$$
  
 $P_{4226} = (0, 1, 0, 1) = \ell_1 \cap \ell_4$ 

$$P_{8258} = (0, 0, 1, 1) = \ell_2 \cap \ell_3$$

## Single Points

The surface has 378 single points:

The single points on the surface are:

 $0: P_5 = (1, 1, 0, 0)$  lies on line  $\ell_3$ 16:  $P_{146} = (15, 1, 1, 0)$  lies on line  $\ell_0$ 17:  $P_{147} = (16, 1, 1, 0)$  lies on line  $\ell_0$ 1:  $P_{68} = (1, 0, 1, 0)$  lies on line  $\ell_4$ 2:  $P_{132} = (1, 1, 1, 0)$  lies on line  $\ell_0$ 18:  $P_{148} = (17, 1, 1, 0)$  lies on line  $\ell_0$  $3: P_{133} = (2, 1, 1, 0)$  lies on line  $\ell_0$ 19:  $P_{149} = (18, 1, 1, 0)$  lies on line  $\ell_0$ 4:  $P_{134} = (3, 1, 1, 0)$  lies on line  $\ell_0$ 20:  $P_{150} = (19, 1, 1, 0)$  lies on line  $\ell_0$ 5:  $P_{135} = (4, 1, 1, 0)$  lies on line  $\ell_0$ 21:  $P_{151} = (20, 1, 1, 0)$  lies on line  $\ell_0$ 6:  $P_{136} = (5, 1, 1, 0)$  lies on line  $\ell_0$ 22:  $P_{152} = (21, 1, 1, 0)$  lies on line  $\ell_0$ 7:  $P_{137} = (6, 1, 1, 0)$  lies on line  $\ell_0$ 23:  $P_{153} = (22, 1, 1, 0)$  lies on line  $\ell_0$ 8:  $P_{138} = (7, 1, 1, 0)$  lies on line  $\ell_0$  $24: P_{154} = (23, 1, 1, 0)$  lies on line  $\ell_0$ 9:  $P_{139} = (8, 1, 1, 0)$  lies on line  $\ell_0$ 25:  $P_{155} = (24, 1, 1, 0)$  lies on line  $\ell_0$ 10 :  $P_{140} = (9, 1, 1, 0)$  lies on line  $\ell_0$ 26:  $P_{156} = (25, 1, 1, 0)$  lies on line  $\ell_0$ 11:  $P_{141} = (10, 1, 1, 0)$  lies on line  $\ell_0$ 27:  $P_{157} = (26, 1, 1, 0)$  lies on line  $\ell_0$ 12:  $P_{142} = (11, 1, 1, 0)$  lies on line  $\ell_0$ 28:  $P_{158} = (27, 1, 1, 0)$  lies on line  $\ell_0$ 13:  $P_{143} = (12, 1, 1, 0)$  lies on line  $\ell_0$ 29:  $P_{159} = (28, 1, 1, 0)$  lies on line  $\ell_0$ 14:  $P_{144} = (13, 1, 1, 0)$  lies on line  $\ell_0$  $30: P_{160} = (29, 1, 1, 0)$  lies on line  $\ell_0$ 15:  $P_{145} = (14, 1, 1, 0)$  lies on line  $\ell_0$  $31: P_{161} = (30, 1, 1, 0)$  lies on line  $\ell_0$ 

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32: P_{162} = (31, 1, 1, 0) lies on line \ell_0
                                                                      86: P_{4247} = (21, 1, 0, 1) lies on line \ell_1
33: P_{163} = (32, 1, 1, 0) lies on line \ell_0
                                                                      87: P_{4248} = (22, 1, 0, 1) lies on line \ell_1
                                                                      88: P_{4249} = (23, 1, 0, 1) lies on line \ell_1
34: P_{164} = (33, 1, 1, 0) lies on line \ell_0
35: P_{165} = (34, 1, 1, 0) lies on line \ell_0
                                                                      89: P_{4250} = (24, 1, 0, 1) lies on line \ell_1
36: P_{166} = (35, 1, 1, 0) lies on line \ell_0
                                                                      90: P_{4251} = (25, 1, 0, 1) lies on line \ell_1
37: P_{167} = (36, 1, 1, 0) lies on line \ell_0
                                                                      91: P_{4252} = (26, 1, 0, 1) lies on line \ell_1
                                                                      92: P_{4253} = (27, 1, 0, 1) lies on line \ell_1
38: P_{168} = (37, 1, 1, 0) lies on line \ell_0
39: P_{169} = (38, 1, 1, 0) lies on line \ell_0
                                                                      93: P_{4254} = (28, 1, 0, 1) lies on line \ell_1
40: P_{170} = (39, 1, 1, 0) lies on line \ell_0
                                                                      94: P_{4255} = (29, 1, 0, 1) lies on line \ell_1
41: P_{171} = (40, 1, 1, 0) lies on line \ell_0
                                                                      95: P_{4256} = (30, 1, 0, 1) lies on line \ell_1
42: P_{172} = (41, 1, 1, 0) lies on line \ell_0
                                                                      96: P_{4257} = (31, 1, 0, 1) lies on line \ell_1
                                                                      97: P_{4258} = (32, 1, 0, 1) lies on line \ell_1
43: P_{173} = (42, 1, 1, 0) lies on line \ell_0
44: P_{174} = (43, 1, 1, 0) lies on line \ell_0
                                                                      98: P_{4259} = (33, 1, 0, 1) lies on line \ell_1
45: P_{175} = (44, 1, 1, 0) lies on line \ell_0
                                                                      99: P_{4260} = (34, 1, 0, 1) lies on line \ell_1
46: P_{176} = (45, 1, 1, 0) lies on line \ell_0
                                                                      100: P_{4261} = (35, 1, 0, 1) lies on line \ell_1
47: P_{177} = (46, 1, 1, 0) lies on line \ell_0
                                                                      101: P_{4262} = (36, 1, 0, 1) lies on line \ell_1
48: P_{178} = (47, 1, 1, 0) lies on line \ell_0
                                                                      102: P_{4263} = (37, 1, 0, 1) lies on line \ell_1
49: P_{179} = (48, 1, 1, 0) lies on line \ell_0
                                                                      103: P_{4264} = (38, 1, 0, 1) lies on line \ell_1
50: P_{180} = (49, 1, 1, 0) lies on line \ell_0
                                                                      104: P_{4265} = (39, 1, 0, 1) lies on line \ell_1
51: P_{181} = (50, 1, 1, 0) lies on line \ell_0
                                                                      105: P_{4266} = (40, 1, 0, 1) lies on line \ell_1
52: P_{182} = (51, 1, 1, 0) lies on line \ell_0
                                                                      106: P_{4267} = (41, 1, 0, 1) lies on line \ell_1
53: P_{183} = (52, 1, 1, 0) lies on line \ell_0
                                                                      107: P_{4268} = (42, 1, 0, 1) lies on line \ell_1
                                                                      108: P_{4269} = (43, 1, 0, 1) lies on line \ell_1
54: P_{184} = (53, 1, 1, 0) lies on line \ell_0
55 : P_{185} = (54, 1, 1, 0) lies on line \ell_0
                                                                      109 : P_{4270} = (44, 1, 0, 1) lies on line \ell_1
56: P_{186} = (55, 1, 1, 0) lies on line \ell_0
                                                                      110: P_{4271} = (45, 1, 0, 1) lies on line \ell_1
57: P_{187} = (56, 1, 1, 0) lies on line \ell_0
                                                                      111: P_{4272} = (46, 1, 0, 1) lies on line \ell_1
58: P_{188} = (57, 1, 1, 0) lies on line \ell_0
                                                                      112: P_{4273} = (47, 1, 0, 1) lies on line \ell_1
59: P_{189} = (58, 1, 1, 0) lies on line \ell_0
                                                                      113: P_{4274} = (48, 1, 0, 1) lies on line \ell_1
60: P_{190} = (59, 1, 1, 0) lies on line \ell_0
                                                                      114: P_{4275} = (49, 1, 0, 1) lies on line \ell_1
61: P_{191} = (60, 1, 1, 0) lies on line \ell_0
                                                                      115: P_{4276} = (50, 1, 0, 1) lies on line \ell_1
62: P_{192} = (61, 1, 1, 0) lies on line \ell_0
                                                                      116: P_{4277} = (51, 1, 0, 1) lies on line \ell_1
63: P_{193} = (62, 1, 1, 0) lies on line \ell_0
                                                                      117: P_{4278} = (52, 1, 0, 1) lies on line \ell_1
64: P_{194} = (63, 1, 1, 0) lies on line \ell_0
                                                                      118: P_{4279} = (53, 1, 0, 1) lies on line \ell_1
65: P_{4163} = (1,0,0,1) lies on line \ell_5
                                                                      119: P_{4280} = (54, 1, 0, 1) lies on line \ell_1
66: P_{4227} = (1, 1, 0, 1) lies on line \ell_1
                                                                      120: P_{4281} = (55, 1, 0, 1) lies on line \ell_1
67: P_{4228} = (2, 1, 0, 1) lies on line \ell_1
                                                                      121: P_{4282} = (56, 1, 0, 1) lies on line \ell_1
68: P_{4229} = (3, 1, 0, 1) lies on line \ell_1
                                                                      122: P_{4283} = (57, 1, 0, 1) lies on line \ell_1
69: P_{4230} = (4, 1, 0, 1) lies on line \ell_1
                                                                      123: P_{4284} = (58, 1, 0, 1) lies on line \ell_1
70: P_{4231} = (5, 1, 0, 1) lies on line \ell_1
                                                                      124: P_{4285} = (59, 1, 0, 1) lies on line \ell_1
                                                                      125: P_{4286} = (60, 1, 0, 1) lies on line \ell_1
71: P_{4232} = (6, 1, 0, 1) lies on line \ell_1
                                                                      126 : P_{4287} = (61, 1, 0, 1) lies on line \ell_1
72: P_{4233} = (7, 1, 0, 1) lies on line \ell_1
73: P_{4234} = (8, 1, 0, 1) lies on line \ell_1
                                                                      127: P_{4288} = (62, 1, 0, 1) lies on line \ell_1
74: P_{4235} = (9, 1, 0, 1) lies on line \ell_1
                                                                      128: P_{4289} = (63, 1, 0, 1) lies on line \ell_1
75: P_{4236} = (10, 1, 0, 1) lies on line \ell_1
                                                                      129: P_{8259} = (1, 0, 1, 1) lies on line \ell_2
76: P_{4237} = (11, 1, 0, 1) lies on line \ell_1
                                                                      130: P_{8260} = (2, 0, 1, 1) lies on line \ell_2
77: P_{4238} = (12, 1, 0, 1) lies on line \ell_1
                                                                      131: P_{8261} = (3,0,1,1) lies on line \ell_2
78: P_{4239} = (13, 1, 0, 1) lies on line \ell_1
                                                                      132: P_{8262} = (4, 0, 1, 1) lies on line \ell_2
79: P_{4240} = (14, 1, 0, 1) lies on line \ell_1
                                                                      133: P_{8263} = (5, 0, 1, 1) lies on line \ell_2
80: P_{4241} = (15, 1, 0, 1) lies on line \ell_1
                                                                      134: P_{8264} = (6,0,1,1) lies on line \ell_2
81: P_{4242} = (16, 1, 0, 1) lies on line \ell_1
                                                                      135: P_{8265} = (7, 0, 1, 1) lies on line \ell_2
82: P_{4243} = (17, 1, 0, 1) lies on line \ell_1
                                                                      136: P_{8266} = (8, 0, 1, 1) lies on line \ell_2
83: P_{4244} = (18, 1, 0, 1) lies on line \ell_1
                                                                      137: P_{8267} = (9, 0, 1, 1) lies on line \ell_2
84 : P_{4245} = (19, 1, 0, 1) lies on line \ell_1
                                                                      138 : P_{8268} = (10, 0, 1, 1) lies on line \ell_2
85: P_{4246} = (20, 1, 0, 1) lies on line \ell_1
                                                                      139: P_{8269} = (11, 0, 1, 1) lies on line \ell_2
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140: P_{8270} = (12, 0, 1, 1) lies on line \ell_2
                                                                     194: P_{8517} = (4, 4, 1, 1) lies on line \ell_3
141: P_{8271} = (13, 0, 1, 1) lies on line \ell_2
                                                                     195: P_{8582} = (5, 5, 1, 1) lies on line \ell_3
142: P_{8272} = (14, 0, 1, 1) lies on line \ell_2
                                                                     196: P_{8647} = (6, 6, 1, 1) lies on line \ell_3
                                                                     197: P_{8712} = (7, 7, 1, 1) lies on line \ell_3
143: P_{8273} = (15, 0, 1, 1) lies on line \ell_2
144: P_{8274} = (16, 0, 1, 1) lies on line \ell_2
                                                                     198 : P_{8777} = (8, 8, 1, 1) lies on line \ell_3
145: P_{8275} = (17, 0, 1, 1) lies on line \ell_2
                                                                     199: P_{8842} = (9, 9, 1, 1) lies on line \ell_3
146: P_{8276} = (18, 0, 1, 1) lies on line \ell_2
                                                                     200: P_{8907} = (10, 10, 1, 1) lies on line \ell_3
147: P_{8277} = (19, 0, 1, 1) lies on line \ell_2
                                                                     201: P_{8972} = (11, 11, 1, 1) lies on line \ell_3
148: P_{8278} = (20, 0, 1, 1) lies on line \ell_2
                                                                     202 : P_{9037} = (12, 12, 1, 1) lies on line \ell_3
149: P_{8279} = (21, 0, 1, 1) lies on line \ell_2
                                                                     203: P_{9102} = (13, 13, 1, 1) lies on line \ell_3
150: P_{8280} = (22, 0, 1, 1) lies on line \ell_2
                                                                     204: P_{9167} = (14, 14, 1, 1) lies on line \ell_3
151: P_{8281} = (23, 0, 1, 1) lies on line \ell_2
                                                                     205: P_{9232} = (15, 15, 1, 1) lies on line \ell_3
152: P_{8282} = (24, 0, 1, 1) lies on line \ell_2
                                                                     206: P_{9297} = (16, 16, 1, 1) lies on line \ell_3
153: P_{8283} = (25, 0, 1, 1) lies on line \ell_2
                                                                     207: P_{9362} = (17, 17, 1, 1) lies on line \ell_3
154: P_{8284} = (26, 0, 1, 1) lies on line \ell_2
                                                                     208: P_{9427} = (18, 18, 1, 1) lies on line \ell_3
155: P_{8285} = (27, 0, 1, 1) lies on line \ell_2
                                                                     209: P_{9492} = (19, 19, 1, 1) lies on line \ell_3
156: P_{8286} = (28, 0, 1, 1) lies on line \ell_2
                                                                     210: P_{9557} = (20, 20, 1, 1) lies on line \ell_3
157: P_{8287} = (29, 0, 1, 1) lies on line \ell_2
                                                                     211: P_{9622} = (21, 21, 1, 1) lies on line \ell_3
                                                                     212: P_{9687} = (22, 22, 1, 1) lies on line \ell_3
158: P_{8288} = (30, 0, 1, 1) lies on line \ell_2
159: P_{8289} = (31, 0, 1, 1) lies on line \ell_2
                                                                     213: P_{9752} = (23, 23, 1, 1) lies on line \ell_3
160: P_{8290} = (32, 0, 1, 1) lies on line \ell_2
                                                                     214 : P_{9817} = (24, 24, 1, 1) lies on line \ell_3
                                                                     215: P_{9882} = (25, 25, 1, 1) lies on line \ell_3
161: P_{8291} = (33, 0, 1, 1) lies on line \ell_2
162: P_{8292} = (34, 0, 1, 1) lies on line \ell_2
                                                                     216: P_{9947} = (26, 26, 1, 1) lies on line \ell_3
163: P_{8293} = (35, 0, 1, 1) lies on line \ell_2
                                                                     217: P_{10012} = (27, 27, 1, 1) lies on line \ell_3
164: P_{8294} = (36, 0, 1, 1) lies on line \ell_2
                                                                     218: P_{10077} = (28, 28, 1, 1) lies on line \ell_3
165: P_{8295} = (37, 0, 1, 1) lies on line \ell_2
                                                                     219: P_{10142} = (29, 29, 1, 1) lies on line \ell_3
166: P_{8296} = (38, 0, 1, 1) lies on line \ell_2
                                                                     220: P_{10207} = (30, 30, 1, 1) lies on line \ell_3
167: P_{8297} = (39, 0, 1, 1) lies on line \ell_2
                                                                     221: P_{10272} = (31, 31, 1, 1) lies on line \ell_3
                                                                     222: P_{10337} = (32, 32, 1, 1) lies on line \ell_3
168: P_{8298} = (40, 0, 1, 1) lies on line \ell_2
169: P_{8299} = (41, 0, 1, 1) lies on line \ell_2
                                                                     223: P_{10402} = (33, 33, 1, 1) lies on line \ell_3
170: P_{8300} = (42, 0, 1, 1) lies on line \ell_2
                                                                     224: P_{10467} = (34, 34, 1, 1) lies on line \ell_3
171: P_{8301} = (43, 0, 1, 1) lies on line \ell_2
                                                                     225: P_{10532} = (35, 35, 1, 1) lies on line \ell_3
172: P_{8302} = (44, 0, 1, 1) lies on line \ell_2
                                                                     226: P_{10597} = (36, 36, 1, 1) lies on line \ell_3
                                                                     227 : P_{10662} = (37, 37, 1, 1) lies on line \ell_3
173: P_{8303} = (45, 0, 1, 1) lies on line \ell_2
174: P_{8304} = (46, 0, 1, 1) lies on line \ell_2
                                                                     228: P_{10727} = (38, 38, 1, 1) lies on line \ell_3
175: P_{8305} = (47, 0, 1, 1) lies on line \ell_2
                                                                     229: P_{10792} = (39, 39, 1, 1) lies on line \ell_3
176: P_{8306} = (48, 0, 1, 1) lies on line \ell_2
                                                                     230: P_{10857} = (40, 40, 1, 1) lies on line \ell_3
177: P_{8307} = (49, 0, 1, 1) lies on line \ell_2
                                                                     231 : P_{10922} = (41, 41, 1, 1) lies on line \ell_3
178: P_{8308} = (50, 0, 1, 1) lies on line \ell_2
                                                                     232 : P_{10987} = (42, 42, 1, 1) lies on line \ell_3
179: P_{8309} = (51, 0, 1, 1) lies on line \ell_2
                                                                     233: P_{11052} = (43, 43, 1, 1) lies on line \ell_3
180: P_{8310} = (52, 0, 1, 1) lies on line \ell_2
                                                                     234: P_{11117} = (44, 44, 1, 1) lies on line \ell_3
181: P_{8311} = (53, 0, 1, 1) lies on line \ell_2
                                                                     235: P_{11182} = (45, 45, 1, 1) lies on line \ell_3
182: P_{8312} = (54, 0, 1, 1) lies on line \ell_2
                                                                     236: P_{11247} = (46, 46, 1, 1) lies on line \ell_3
183: P_{8313} = (55, 0, 1, 1) lies on line \ell_2
                                                                     237: P_{11312} = (47, 47, 1, 1) lies on line \ell_3
184 : P_{8314} = (56, 0, 1, 1) lies on line \ell_2
                                                                     238: P_{11377} = (48, 48, 1, 1) lies on line \ell_3
                                                                     239 : P_{11442} = (49, 49, 1, 1) lies on line \ell_3
185: P_{8315} = (57, 0, 1, 1) lies on line \ell_2
186: P_{8316} = (58, 0, 1, 1) lies on line \ell_2
                                                                     240: P_{11507} = (50, 50, 1, 1) lies on line \ell_3
187: P_{8317} = (59, 0, 1, 1) lies on line \ell_2
                                                                     241: P_{11572} = (51, 51, 1, 1) lies on line \ell_3
188 : P_{8318} = (60, 0, 1, 1) lies on line \ell_2
                                                                     242: P_{11637} = (52, 52, 1, 1) lies on line \ell_3
189: P_{8319} = (61, 0, 1, 1) lies on line \ell_2
                                                                     243: P_{11702} = (53, 53, 1, 1) lies on line \ell_3
190: P_{8320} = (62, 0, 1, 1) lies on line \ell_2
                                                                     244: P_{11767} = (54, 54, 1, 1) lies on line \ell_3
191: P_{8321} = (63, 0, 1, 1) lies on line \ell_2
                                                                     245: P_{11832} = (55, 55, 1, 1) lies on line \ell_3
192: P_{8387} = (2, 2, 1, 1) lies on line \ell_3
                                                                     246: P_{11897} = (56, 56, 1, 1) lies on line \ell_3
193: P_{8452} = (3, 3, 1, 1) lies on line \ell_3
                                                                     247: P_{11962} = (57, 57, 1, 1) lies on line \ell_3
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302: P_{110747} = (26, 1, 26, 1) lies on line \ell_4
248: P_{12027} = (58, 58, 1, 1) lies on line \ell_3
249 : P_{12092} = (59, 59, 1, 1) lies on line \ell_3
                                                                     303: P_{112322} = (1, 26, 26, 1) lies on line \ell_5
250: P_{12157} = (60, 60, 1, 1) lies on line \ell_3
                                                                     304: P_{114844} = (27, 1, 27, 1) lies on line \ell_4
251 : P_{12222} = (61, 61, 1, 1) lies on line \ell_3
                                                                     305: P_{116482} = (1, 27, 27, 1) lies on line \ell_5
252 : P_{12287} = (62, 62, 1, 1) lies on line \ell_3
                                                                     306: P_{118941} = (28, 1, 28, 1) lies on line \ell_4
253: P_{12352} = (63, 63, 1, 1) lies on line \ell_3
                                                                     307: P_{120642} = (1, 28, 28, 1) lies on line \ell_5
254 : P_{12419} = (2, 1, 2, 1) lies on line \ell_4
                                                                     308: P_{123038} = (29, 1, 29, 1) lies on line \ell_4
255: P_{12482} = (1, 2, 2, 1) lies on line \ell_5
                                                                     309: P_{124802} = (1, 29, 29, 1) lies on line \ell_5
256: P_{16516} = (3, 1, 3, 1) lies on line \ell_4
                                                                     310: P_{127135} = (30, 1, 30, 1) lies on line \ell_4
257: P_{16642} = (1, 3, 3, 1) lies on line \ell_5
                                                                     311: P_{128962} = (1, 30, 30, 1) lies on line \ell_5
258: P_{20613} = (4, 1, 4, 1) lies on line \ell_4
                                                                     312: P_{131232} = (31, 1, 31, 1) lies on line \ell_4
259 : P_{20802} = (1, 4, 4, 1) lies on line \ell_5
                                                                     313: P_{133122} = (1, 31, 31, 1) lies on line \ell_5
260: P_{24710} = (5, 1, 5, 1) lies on line \ell_4
                                                                     314: P_{135329} = (32, 1, 32, 1) lies on line \ell_4
261: P_{24962} = (1, 5, 5, 1) lies on line \ell_5
                                                                     315: P_{137282} = (1, 32, 32, 1) lies on line \ell_5
262 : P_{28807} = (6, 1, 6, 1) lies on line \ell_4
                                                                     316: P_{139426} = (33, 1, 33, 1) lies on line \ell_4
263: P_{29122} = (1, 6, 6, 1) lies on line \ell_5
                                                                     317: P_{141442} = (1, 33, 33, 1) lies on line \ell_5
264: P_{32904} = (7, 1, 7, 1) lies on line \ell_4
                                                                     318: P_{143523} = (34, 1, 34, 1) lies on line \ell_4
265: P_{33282} = (1, 7, 7, 1) lies on line \ell_5
                                                                     319: P_{145602} = (1, 34, 34, 1) lies on line \ell_5
266: P_{37001} = (8, 1, 8, 1) lies on line \ell_4
                                                                     320: P_{147620} = (35, 1, 35, 1) lies on line \ell_4
                                                                     321: P_{149762} = (1, 35, 35, 1) lies on line \ell_5
267: P_{37442} = (1, 8, 8, 1) lies on line \ell_5
268: P_{41098} = (9, 1, 9, 1) lies on line \ell_4
                                                                     322: P_{151717} = (36, 1, 36, 1) lies on line \ell_4
269 : P_{41602} = (1, 9, 9, 1) lies on line \ell_5
                                                                     323: P_{153922} = (1, 36, 36, 1) lies on line \ell_5
270: P_{45195} = (10, 1, 10, 1) lies on line \ell_4
                                                                     324: P_{155814} = (37, 1, 37, 1) lies on line \ell_4
271: P_{45762} = (1, 10, 10, 1) lies on line \ell_5
                                                                     325: P_{158082} = (1, 37, 37, 1) lies on line \ell_5
272: P_{49292} = (11, 1, 11, 1) lies on line \ell_4
                                                                     326: P_{159911} = (38, 1, 38, 1) lies on line \ell_4
273: P_{49922} = (1, 11, 11, 1) lies on line \ell_5
                                                                     327: P_{162242} = (1, 38, 38, 1) lies on line \ell_5
274: P_{53389} = (12, 1, 12, 1) lies on line \ell_4
                                                                     328: P_{164008} = (39, 1, 39, 1) lies on line \ell_4
275: P_{54082} = (1, 12, 12, 1) lies on line \ell_5
                                                                     329: P_{166402} = (1, 39, 39, 1) lies on line \ell_5
276: P_{57486} = (13, 1, 13, 1) lies on line \ell_4
                                                                     330: P_{168105} = (40, 1, 40, 1) lies on line \ell_4
277: P_{58242} = (1, 13, 13, 1) lies on line \ell_5
                                                                     331: P_{170562} = (1, 40, 40, 1) lies on line \ell_5
278: P_{61583} = (14, 1, 14, 1) lies on line \ell_4
                                                                     332: P_{172202} = (41, 1, 41, 1) lies on line \ell_4
279: P_{62402} = (1, 14, 14, 1) lies on line \ell_5
                                                                     333: P_{174722} = (1, 41, 41, 1) lies on line \ell_5
280: P_{65680} = (15, 1, 15, 1) lies on line \ell_4
                                                                     334: P_{176299} = (42, 1, 42, 1) lies on line \ell_4
                                                                     335 : P_{178882} = (1, 42, 42, 1) lies on line \ell_5
281: P_{66562} = (1, 15, 15, 1) lies on line \ell_5
282: P_{69777} = (16, 1, 16, 1) lies on line \ell_4
                                                                     336: P_{180396} = (43, 1, 43, 1) lies on line \ell_4
283: P_{70722} = (1, 16, 16, 1) lies on line \ell_5
                                                                     337: P_{183042} = (1, 43, 43, 1) lies on line \ell_5
284: P_{73874} = (17, 1, 17, 1) lies on line \ell_4
                                                                     338: P_{184493} = (44, 1, 44, 1) lies on line \ell_4
285 : P_{74882} = (1, 17, 17, 1) lies on line \ell_5
                                                                     339 : P_{187202} = (1, 44, 44, 1) lies on line \ell_5
286: P_{77971} = (18, 1, 18, 1) lies on line \ell_4
                                                                     340: P_{188590} = (45, 1, 45, 1) lies on line \ell_4
287 : P_{79042} = (1, 18, 18, 1) lies on line \ell_5
                                                                     341: P_{191362} = (1, 45, 45, 1) lies on line \ell_5
288: P_{82068} = (19, 1, 19, 1) lies on line \ell_4
                                                                     342: P_{192687} = (46, 1, 46, 1) lies on line \ell_4
289: P_{83202} = (1, 19, 19, 1) lies on line \ell_5
                                                                     343: P_{195522} = (1, 46, 46, 1) lies on line \ell_5
                                                                     344 : P_{196784} = (47, 1, 47, 1) lies on line \ell_4
290: P_{86165} = (20, 1, 20, 1) lies on line \ell_4
291: P_{87362} = (1, 20, 20, 1) lies on line \ell_5
                                                                     345: P_{199682} = (1, 47, 47, 1) lies on line \ell_5
292: P_{90262} = (21, 1, 21, 1) lies on line \ell_4
                                                                     346: P_{200881} = (48, 1, 48, 1) lies on line \ell_4
293: P_{91522} = (1, 21, 21, 1) lies on line \ell_5
                                                                     347: P_{203842} = (1, 48, 48, 1) lies on line \ell_5
294: P_{94359} = (22, 1, 22, 1) lies on line \ell_4
                                                                     348: P_{204978} = (49, 1, 49, 1) lies on line \ell_4
295: P_{95682} = (1, 22, 22, 1) lies on line \ell_5
                                                                     349: P_{208002} = (1, 49, 49, 1) lies on line \ell_5
296: P_{98456} = (23, 1, 23, 1) lies on line \ell_4
                                                                     350: P_{209075} = (50, 1, 50, 1) lies on line \ell_4
297: P_{99842} = (1, 23, 23, 1) lies on line \ell_5
                                                                     351: P_{212162} = (1, 50, 50, 1) lies on line \ell_5
298: P_{102553} = (24, 1, 24, 1) lies on line \ell_4
                                                                     352: P_{213172} = (51, 1, 51, 1) lies on line \ell_4
299: P_{104002} = (1, 24, 24, 1) lies on line \ell_5
                                                                     353: P_{216322} = (1, 51, 51, 1) lies on line \ell_5
300 : P_{106650} = (25, 1, 25, 1) lies on line \ell_4
                                                                     354: P_{217269} = (52, 1, 52, 1) lies on line \ell_4
301: P_{108162} = (1, 25, 25, 1) lies on line \ell_5
                                                                     355: P_{220482} = (1, 52, 52, 1) lies on line \ell_5
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356: P_{221366} = (53, 1, 53, 1) lies on line \ell_4
                                                                    368: P_{245948} = (59, 1, 59, 1) lies on line \ell_4
                                                                    369 : P_{249602} = (1, 59, 59, 1) lies on line \ell_5
357: P_{224642} = (1, 53, 53, 1) lies on line \ell_5
358: P_{225463} = (54, 1, 54, 1) lies on line \ell_4
                                                                    370: P_{250045} = (60, 1, 60, 1) lies on line \ell_4
359 : P_{228802} = (1, 54, 54, 1) lies on line \ell_5
                                                                    371: P_{253762} = (1, 60, 60, 1) lies on line \ell_5
360: P_{229560} = (55, 1, 55, 1) lies on line \ell_4
                                                                    372: P_{254142} = (61, 1, 61, 1) lies on line \ell_4
361: P_{232962} = (1, 55, 55, 1) lies on line \ell_5
                                                                    373: P_{257922} = (1,61,61,1) lies on line \ell_5
362: P_{233657} = (56, 1, 56, 1) lies on line \ell_4
                                                                    374: P_{258239} = (62, 1, 62, 1) lies on line \ell_4
363: P_{237122} = (1, 56, 56, 1) lies on line \ell_5
                                                                    375: P_{262082} = (1, 62, 62, 1) lies on line \ell_5
364: P_{237754} = (57, 1, 57, 1) lies on line \ell_4
                                                                    376: P_{262336} = (63, 1, 63, 1) lies on line \ell_4
365: P_{241282} = (1, 57, 57, 1) lies on line \ell_5
                                                                    377: P_{266242} = (1, 63, 63, 1) lies on line \ell_5
366: P_{241851} = (58, 1, 58, 1) lies on line \ell_4
367: P_{245442} = (1, 58, 58, 1) lies on line \ell_5
```

The single points on the surface are:

## Points on surface but on no line

The surface has 3906 points not on any line: Too many to print.

## Line Intersection Graph

012345	)
011001	l
101010	)
110100	)
001011	L
010101	L
100110	)
	$ \begin{array}{c} 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ \end{array} $

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_5$
in point	$P_0$	$P_0$	$P_{131}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_4$
in point	$P_0$	$P_0$	$P_{4226}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$
in point	$P_0$	$P_0$	$P_{8258}$

Line 3 intersects

Line	$\ell_2$	$\ell_4$	$\ell_5$
in point	$P_{8258}$	$P_4$	$P_4$

Line 4 intersects

Line	$\ell_1$	$\ell_3$	$\ell_5$
in point	$P_{4226}$	$P_4$	$P_4$

Line 5 intersects

Line	$\ell_0$	$\ell_3$	$\ell_4$
in point	$P_{131}$	$P_4$	$P_4$

The surface has 4289 points: Too many to print.