Rank-73795 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498768729

General information

Number of lines	31
Number of points	41
Number of singular points	5
Number of Eckardt points	0
Number of double points	20
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{31}
Type of lines on points	7^5, 5^{16}, 2^{20}

Singular Points

The surface has 5 singular points:

0 : $P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$
1 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
2 : $P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$
3 : $P_{65} = \mathbf{P}(0,\omega^2,\omega,1) = \mathbf{P}(0,3,2,1)$
4 : $P_{77} = \mathbf{P}(0,\omega,\omega^2,1) = \mathbf{P}(0,2,3,1)$

The 31 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{337} = \mathbf{Pl}(0,0,0,1,0,1)_{129}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{339} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{339} = \mathbf{Pl}(0,0,0,3,0,1)_{143}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{338} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{338} = \mathbf{Pl}(0,0,0,2,0,1)_{136}$$

$$\ell_8 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0,1,1,0,0,0)_6$$

$$\ell_{10} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_{11} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{351} = \mathbf{Pl}(0,3,0,0,0,1)_{107}$$

$$\ell_{12} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{346} = \mathbf{Pl}(0,2,0,0,0,1)_{106}$$

$$\ell_{13} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1,0,1,1,1,1)_{199}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{137} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{137} = \mathbf{Pl}(3,0,2,3,1,1)_{208}$$

$$\ell_{15} = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{161} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{161} = \mathbf{Pl}(2,0,3,2,1,1)_{214}$$

$$\ell_{16} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_{17} = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 1 & \omega & \omega^2 \end{bmatrix}_{77} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{77} = \mathbf{Pl}(2,3,3,0,1,1)_{187}$$

$$\ell_{18} = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 1 & \omega^2 & \omega \end{bmatrix}_{53} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix}_{53} = \mathbf{Pl}(3,2,2,0,1,1)_{185}$$

$$\ell_{19} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_{20} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{342} = \mathbf{Pl}(0,1,0,1,0,1)_{133}$$

$$\ell_{21} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{354} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{354} = \mathbf{Pl}(0,3,0,3,0,1)_{149}$$

$$\ell_{22} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{bmatrix}_{348} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{348} = \mathbf{Pl}(0,2,0,2,0,1)_{141}$$
$$\ell_{23} = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$
$$\ell_{24} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{343} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{343} = \mathbf{Pl}(0,1,0,2,0,1)_{140}$$
$$\ell_{25} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{352} = \mathbf{Pl}(0,3,0,1,0,1)_{135}$$
$$\ell_{26} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{349} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{349} = \mathbf{Pl}(0,2,0,3,0,1)_{148}$$
$$\ell_{27} = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$
$$\ell_{28} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{344} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{344} = \mathbf{Pl}(0,1,0,3,0,1)_{147}$$
$$\ell_{29} = \begin{bmatrix} 0 & 1 & 0 & \omega^2 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{353} = \begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{353} = \mathbf{Pl}(0,3,0,2,0,1)_{142}$$
$$\ell_{30} = \begin{bmatrix} 0 & 1 & 0 & \omega \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{347} = \mathbf{Pl}(0,2,0,1,0,1)_{134}$$

Rank of lines: (0, 16, 336, 340, 84, 337, 339, 338, 356, 100, 341, 351, 346, 110, 137, 161, 26, 77, 53, 345, 342, 354, 348, 355, 343, 352, 349, 350, 344, 353, 347)
Rank of points on Klein quadric: (0, 2, 101, 9, 10, 129, 143, 136, 1, 6, 105, 107, 106, 199, 208, 214, 180, 187, 185, 13, 133, 149, 141, 15, 140, 135, 148, 14, 147, 142, 134)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 20 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_{13}$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_{14}$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_{15}$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_{16}$
$P_9 = (2,0,1,0) = \ell_1 \cap \ell_{17}$
$P_{10} = (3,0,1,0) = \ell_1 \cap \ell_{18}$
$P_{23} = (1,0,0,1) = \ell_4 \cap \ell_9$
$P_{27} = (1,1,0,1) = \ell_4 \cap \ell_{16}$
$P_{31} = (1,2,0,1) = \ell_4 \cap \ell_{17}$
$P_{35} = (1,3,0,1) = \ell_4 \cap \ell_{18}$
$P_{39} = (1,0,1,1) = \ell_9 \cap \ell_{13}$
$P_{54} = (1,0,2,1) = \ell_9 \cap \ell_{14}$
$P_{70} = (1,0,3,1) = \ell_9 \cap \ell_{15}$
$P_{48} = (3,2,1,1) = \ell_{13} \cap \ell_{17}$
$P_{51} = (2,3,1,1) = \ell_{13} \cap \ell_{18}$
$P_{60} = (3,1,2,1) = \ell_{14} \cap \ell_{16}$
$P_{63} = (2,2,2,1) = \ell_{14} \cap \ell_{17}$
$P_{75} = (2,1,3,1) = \ell_{15} \cap \ell_{16}$
$P_{84} = (3,3,3,1) = \ell_{15} \cap \ell_{18}$

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	1	1	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
3	1	0	1	0	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	1	1	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	1	1	1	0	1	1	1	0	1	1	1	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
6	1	0	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	0	1	1	1	1	1	0	1	0	1	1	1	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
8	0	1	1	1	0	1	1	1	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
9	0	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	1	1	1	0	1	1	1	1	1	0	1	1	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
11	0	1	1	1	0	1	1	1	1	1	1	0	1	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1
12	0	1	1	1	0	1	1	1	1	1	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0
14	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	1	1	0	1	0	0	1	0	0	0	0	0	1	0
15	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	0
16	0	1	0	0	1	1	0	0	0	0	1	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0
17	0	1	0	0	1	0	0	1	0	0	1	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0
18	0	1	0	0	1	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0
19	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
20	0	0	1	1	0	1	1	1	1	0	1	1	1	0	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1
21	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1
22	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1	1	1	1
23	0	0	1	1	0	1	1	1	1	0	1	1	1	0	1	0	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1
24	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1
25	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	0	1	1	1	1
26	0	0	1	1	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1
27	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	1	0	1	0	1	1	1	1	1	1	1	1	0	1	1	1
28	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	1
29	0	0	1	1	0	1	1	1	1	0	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
30	0	0	1	1	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_0	P_1	P_1	P_1	P_1	P_1	P_1	P_5	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{16}	ℓ_{17}	ℓ_{18}
in point	P_0	P_2	P_2	P_2	P_2	P_2	P_2	P_8	P_9	P_{10}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ
in point	P_1	P_2	P_1	P_1	P_1	P_1	P_1	P_2	P_2	P_2	P_2	P_2	P_{11}	P_{11}	P_{11}	P_{11}	P_{15}	P_{15}	P_{15}	P_{15}	P_{19}	P

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}	P_{34}	P_3	P_{26}	P_{30}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{16}	ℓ_{17}	ℓ_{18}
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{23}	P_{27}	P_{31}	P_{35}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_2
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{38}	P_{42}	P_{45}	P_{49}	P_{42}	P_{42}	P_{42}	P_{38}	P_{49}	P_{45}	P_{45}	P_{49}	P_{38}	P_{42}	P_4

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_2
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{53}	P_{57}	P_{61}	P_{65}	P_{65}	P_{65}	P_{61}	P_{65}	P_{53}	P_{57}	P_{65}	P_{61}	P_{57}	P_{53}	P_5

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_2
in point	P_1	P_1	P_1	P_1	P_1	P_1	P_{69}	P_{73}	P_{77}	P_{81}	P_{77}	P_{77}	P_{81}	P_{77}	P_{73}	P_{69}	P_{73}	P_{69}	P_{81}	P_{77}	P_7

Line 8 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}
in point	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_2	P_2	P_2	P_2	P_3	P_{38}	P_{53}	P_{69}	P_3	P_{69}	P_{38}	P_{53}	P_3	P_{53}	P_{69}

Line 9 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_2	P_2	P_{23}	P_2	P_2	P_2	P_2	P_{39}	P_{54}	P_{70}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	$\ell_$
in point	P_2	P_2	P_{26}	P_{42}	P_{57}	P_{73}	P_2	P_2	P_2	P_2	P_{42}	P_{42}	P_{42}	P_{26}	P_{73}	P_{57}	P_{73}	P_{26}	P_{57}	P_{42}	P

Line 11 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	$\ell_$
in point	P_2	P_2	P_{30}	P_{45}	P_{61}	P_{77}	P_2	P_2	P_2	P_2	P_{77}	P_{77}	P_{61}	P_{77}	P_{30}	P_{45}	P_{45}	P_{61}	P_{30}	P_{77}	P

Line 12 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{14}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	$\ell_$
in point	P_2	P_2	P_{34}	P_{49}	P_{65}	P_{81}	P_2	P_2	P_2	P_2	P_{65}	P_{65}	P_{81}	P_{65}	P_{49}	P_{34}	P_{65}	P_{49}	P_{81}	P_{34}	P

Line 13 intersects

Line	ℓ_0	ℓ_5	ℓ_9	ℓ_{10}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{26}	ℓ_{29}
in point	P_5	P_{42}	P_{39}	P_{42}	P_{42}	P_{48}	P_{51}	P_{42}	P_{42}	P_{42}

Line 14 intersects

Line	ℓ_0	ℓ_6	ℓ_9	ℓ_{12}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{23}	ℓ_{29}
in point	P_6	P_{65}	P_{54}	P_{65}	P_{60}	P_{63}	P_{65}	P_{65}	P_{65}	P_{65}

Line 15 intersects

Line	ℓ_0	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{20}	ℓ_{26}	ℓ_{27}
in point	P_7	P_{77}	P_{70}	P_{77}	P_{75}	P_{77}	P_{84}	P_{77}	P_{77}	P_{77}

Line 16 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_{10}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{19}	ℓ_{26}	ℓ_{29}
in point	P_8	P_{27}	P_{42}	P_{42}	P_{42}	P_{60}	P_{75}	P_{42}	P_{42}	P_{42}

Line 17 intersects

Line	ℓ_1	ℓ_4	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{20}	ℓ_{26}	ℓ_{27}
in point	P_9	P_{31}	P_{77}	P_{77}	P_{48}	P_{63}	P_{77}	P_{77}	P_{77}	P_{77}

Line 18 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{20}	ℓ_{23}	ℓ_{29}
in point	P_{10}	P_{35}	P_{65}	P_{65}	P_{51}	P_{65}	P_{84}	P_{65}	P_{65}	P_{65}

Line 19 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{16}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{11}	P_3	P_{42}	P_{61}	P_{81}	P_3	P_{42}	P_{61}	P_{81}	P_{42}	P_{42}	P_{11}	P_{11}	P_{11}	P_3	P_{61}	P_{81}	P_{42}	P_3	P_{81}

Line 20 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}
in point	P_{11}	P_{26}	P_{38}	P_{65}	P_{77}	P_{38}	P_{26}	P_{77}	P_{65}	P_{65}	P_{77}	P_{77}	P_{65}	P_{11}	P_{11}	P_{11}	P_{65}	P_{26}	P_{38}	P_{77}

Line 21 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}	ℓ_{30}
in point	P_{11}	P_{30}	P_{49}	P_{53}	P_{73}	P_{53}	P_{73}	P_{30}	P_{49}	P_{11}	P_{11}	P_{11}	P_{73}	P_{49}	P_{30}	P_{53}	P_{49}	P_{53}	P_{30}	P_{73}

Line 22 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}	ℓ_{30}
in point	P_{11}	P_{34}	P_{45}	P_{57}	P_{69}	P_{69}	P_{57}	P_{45}	P_{34}	P_{11}	P_{11}	P_{11}	P_{45}	P_{69}	P_{57}	P_{34}	P_{57}	P_{45}	P_{69}	P_{34}

Line 23 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{14}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}
in point	P_{15}	P_3	P_{45}	P_{65}	P_{73}	P_3	P_{73}	P_{45}	P_{65}	P_{65}	P_{65}	P_3	P_{65}	P_{73}	P_{45}	P_{15}	P_{15}	P_{15}	P_3	P_{45}

Line 24 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}	ℓ_{30}
in point	P_{15}	P_{26}	P_{49}	P_{61}	P_{69}	P_{69}	P_{26}	P_{61}	P_{49}	P_{61}	P_{26}	P_{49}	P_{69}	P_{15}	P_{15}	P_{15}	P_{49}	P_{26}	P_{69}	P_6

Line 25 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}	ℓ_{30}
in point	P_{15}	P_{30}	P_{38}	P_{57}	P_{81}	P_{38}	P_{57}	P_{30}	P_{81}	P_{81}	P_{38}	P_{30}	P_{57}	P_{15}	P_{15}	P_{15}	P_{57}	P_{81}	P_{30}	P_{38}

Line 26 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{15}	P_{34}	P_{42}	P_{53}	P_{77}	P_{53}	P_{42}	P_{77}	P_{34}	P_{42}	P_{77}	P_{42}	P_{77}	P_{42}	P_{77}	P_{53}	P_{34}	P_{15}	P_{15}	P_{15}

Line 27 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{28}
in point	P_{19}	P_3	P_{49}	P_{57}	P_{77}	P_3	P_{57}	P_{77}	P_{49}	P_{77}	P_{77}	P_3	P_{77}	P_{49}	P_{57}	P_3	P_{49}	P_{57}	P_{77}	P_{19}

Line 28 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{29}	ℓ_{30}
in point	P_{19}	P_{26}	P_{45}	P_{53}	P_{81}	P_{53}	P_{26}	P_{45}	P_{81}	P_{81}	P_{26}	P_{53}	P_{45}	P_{45}	P_{26}	P_{81}	P_{53}	P_{19}	P_{19}	P_{15}

Line 29 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}
in point	P_{19}	P_{30}	P_{42}	P_{65}	P_{69}	P_{69}	P_{42}	P_{30}	P_{65}	P_{42}	P_{65}	P_{42}	P_{65}	P_{42}	P_{65}	P_{30}	P_{69}	P_{65}	P_{69}	P_{3}

Line 30 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}	ℓ_{23}	ℓ_{24}	ℓ_{25}	ℓ_{26}	ℓ_{27}	ℓ_{28}	ℓ_{29}
in point	P_{19}	P_{34}	P_{38}	P_{61}	P_{73}	P_{38}	P_{73}	P_{61}	P_{34}	P_{61}	P_{38}	P_{73}	P_{34}	P_{73}	P_{61}	P_{38}	P_{34}	P_{19}	P_{19}	P_{1}

The surface has 41 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_5 = (1,1,0,0)$
$5: P_6 = (2,1,0,0)$
$6: P_7 = (3,1,0,0)$
$7: P_8 = (1,0,1,0)$
$8: P_9 = (2,0,1,0)$
$9: P_{10} = (3,0,1,0)$
$10: P_{11} = (0,1,1,0)$
$11: P_{15} = (0,2,1,0)$
$12: P_{19} = (0,3,1,0)$
$13: P_{23} = (1,0,0,1)$
$14: P_{26} = (0,1,0,1)$
$15: P_{27} = (1,1,0,1)$
$16: P_{30} = (0,2,0,1)$
$17: P_{31} = (1,2,0,1)$
$18: P_{34} = (0,3,0,1)$
$19: P_{35} = (1,3,0,1)$
$20: P_{38} = (0,0,1,1)$
$21: P_{39} = (1,0,1,1)$
$22: P_{42} = (0,1,1,1)$
$23: P_{45} = (0,2,1,1)$
$24: P_{48} = (3,2,1,1)$
$25: P_{49} = (0,3,1,1)$
$26: P_{51} = (2,3,1,1)$
$27: P_{53} = (0,0,2,1)$
$28: P_{54} = (1,0,2,1)$
$29: P_{57} = (0,1,2,1)$
$30: P_{60} = (3,1,2,1)$
$31: P_{61} = (0,2,2,1)$
$32: P_{63} = (2,2,2,1)$
$33: P_{65} = (0,3,2,1)$
$34: P_{69} = (0,0,3,1)$
$35: P_{70} = (1,0,3,1)$
$36: P_{73} = (0,1,3,1)$
$37: P_{75} = (2,1,3,1)$
$38: P_{77} = (0,2,3,1)$
$39: P_{81} = (0,3,3,1)$
$40: P_{84} = (3,3,3,1)$