Rank-76387 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_2 + X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1361646157

General information

Number of lines	16
Number of points	105
Number of singular points	2
Number of Eckardt points	3
Number of double points	29
Number of single points	67
Number of points off lines	4
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{16}
Type of lines on points	5^2, 3^3, 2^{29}, 1^{67}, 0^4

Singular Points

The surface has 2 singular points:

$0:\ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
$1:\ P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_9 = \mathbf{Pl}(1,0,1,0,1,0)_{97}$$

$$\ell_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^5 \\ 0 & 1 & \gamma^6 & \gamma^5 \end{bmatrix}_{2220} = \begin{bmatrix} 1 & 0 & 6 & 3 \\ 0 & 1 & 6 & 3 \end{bmatrix}_{2220} = \mathbf{Pl}(4,0,2,6,6,1)_{3861}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^3 \\ 0 & 1 & \gamma^5 & \gamma^3 \end{bmatrix}_{3182} = \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{bmatrix}_{3182} = \mathbf{Pl}(7,0,4,3,3,1)_{2382}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^6 \\ 0 & 1 & \gamma^3 & \gamma^6 \end{bmatrix}_{3922} = \begin{bmatrix} 1 & 0 & 5 & 6 \\ 0 & 1 & 5 & 6 \end{bmatrix}_{3922} = \mathbf{Pl}(2,0,7,5,5,1)_{3430}$$

$$\ell_7 = \begin{bmatrix} 1 & \gamma^6 & 0 & 0 \\ 0 & 0 & 1 & \gamma^3 \end{bmatrix}_{507} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{507} = \mathbf{Pl}(0,0,7,5,7,1)_{4436}$$

$$\ell_8 = \begin{bmatrix} 1 & \gamma^5 & 0 & 0 \\ 0 & 0 & 1 & \gamma^6 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{289} = \mathbf{Pl}(0,0,2,6,2,1)_{1841}$$

$$\ell_9 = \begin{bmatrix} 1 & \gamma^3 & 0 & 0 \\ 0 & 0 & 1 & \gamma^5 \end{bmatrix}_{432} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{432} = \mathbf{Pl}(0,0,4,3,4,1)_{2879}$$

$$\ell_{10} = \begin{bmatrix} 1 & \gamma^6 & 0 & \gamma \\ 0 & 0 & 1 & \gamma^3 \end{bmatrix}_{1675} = \begin{bmatrix} 1 & 6 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{1675} = \mathbf{Pl}(0,4,7,5,7,1)_{4447}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^6 \\ 0 & 1 & \gamma^2 & \gamma^4 \end{bmatrix}_{3929} = \begin{bmatrix} 1 & 0 & 5 & 6 \\ 0 & 1 & 4 & 7 \end{bmatrix}_{3929} = \mathbf{Pl}(3,4,7,5,2,1)_{2185}$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^5 \\ 0 & 1 & \gamma^4 & \gamma \end{bmatrix}_{2213} = \begin{bmatrix} 1 & 0 & 6 & 3 \\ 0 & 1 & 7 & 2 \end{bmatrix}_{2213} = \mathbf{Pl}(5,7,2,6,4,1)_{2999}$$

$$\ell_{13} = \begin{bmatrix} 1 & \gamma^5 & 0 & \gamma^2 \\ 0 & 0 & 1 & \gamma^6 \end{bmatrix}_{2625} = \begin{bmatrix} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{2625} = \mathbf{Pl}(0,7,2,6,2,1)_{1855}$$

$$\ell_{14} = \begin{bmatrix} 1 & \gamma^3 & 0 & \gamma^4 \\ 0 & 0 & 1 & \gamma^5 \end{bmatrix}_{4520} = \begin{bmatrix} 1 & 5 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{4520} = \mathbf{Pl}(0,2,4,3,4,1)_{2888}$$

$$\ell_{15} = \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^3 \\ 0 & 1 & \gamma & \gamma^2 \end{bmatrix}_{3173} = \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 2 & 4 \end{bmatrix}_{3173} = \mathbf{Pl}(6,2,4,3,7,1)_{4617}$$

Rank of lines: (0, 9, 4744, 138, 2220, 3182, 3922, 507, 289, 432, 1675, 3929, 2213, 2625, 4520, 3173)
Rank of points on Klein quadric: (0, 97, 1, 1322, 3861, 2382, 3430, 4436, 1841, 2879, 4447, 2185, 2999, 1855, 2888, 4617)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_{201} = \mathbf{P}(0,0,\gamma,1) = \mathbf{P}(0,0,2,1)$,
$1 : P_{329} = \mathbf{P}(0,0,\gamma^2,1) = \mathbf{P}(0,0,4,1)$,
$2 : P_{521} = \mathbf{P}(0,0,\gamma^4,1) = \mathbf{P}(0,0,7,1)$.

Double Points

The surface has 29 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_7$
$P_8 = (4,1,0,0) = \ell_0 \cap \ell_8$
$P_{11} = (7,1,0,0) = \ell_0 \cap \ell_9$
$P_{147} = (2,1,1,1) = \ell_1 \cap \ell_{10}$
$P_{149} = (4,1,1,1) = \ell_1 \cap \ell_{13}$
$P_{152} = (7,1,1,1) = \ell_1 \cap \ell_{14}$
$P_{138} = (0,0,1,1) = \ell_2 \cap \ell_3$
$P_{251} = (2,6,2,1) = \ell_4 \cap \ell_8$
$P_{260} = (3,7,2,1) = \ell_4 \cap \ell_{10}$
$P_{205} = (4,0,2,1) = \ell_4 \cap \ell_{12}$
$P_{242} = (1,5,2,1) = \ell_4 \cap \ell_{14}$
$P_{357} = (4,3,4,1) = \ell_5 \cap \ell_9$
$P_{378} = (1,6,4,1) = \ell_5 \cap \ell_{10}$
$P_{350} = (5,2,4,1) = \ell_5 \cap \ell_{13}$
$P_{336} = (7,0,4,1) = \ell_5 \cap \ell_{15}$
$P_{568} = (7,5,7,1) = \ell_6 \cap \ell_7$
$P_{523} = (2,0,7,1) = \ell_6 \cap \ell_{11}$
$P_{546} = (1,3,7,1) = \ell_6 \cap \ell_{13}$
$P_{559} = (6,4,7,1) = \ell_6 \cap \ell_{14}$
$P_{551} = (6,3,7,1) = \ell_7 \cap \ell_{12}$
$P_{558} = (5,4,7,1) = \ell_7 \cap \ell_{15}$
$P_{263} = (6,7,2,1) = \ell_8 \cap \ell_{11}$
$P_{244} = (3,5,2,1) = \ell_8 \cap \ell_{15}$
$P_{382} = (5,6,4,1) = \ell_9 \cap \ell_{11}$
$P_{348} = (3,2,4,1) = \ell_9 \cap \ell_{12}$
$P_{477} = (4,2,6,1) = \ell_{10} \cap \ell_{11}$
$P_{304} = (7,4,3,1) = \ell_{12} \cap \ell_{13}$
$P_{451} = (2,7,5,1) = \ell_{14} \cap \ell_{15}$

Single Points

The surface has 67 single points:
The single points on the surface are:

$0:\ P_1 = (0,1,0,0)$ lies on line ℓ_0
$1:\ P_2 = (0,0,1,0)$ lies on line ℓ_2
$2:\ P_3 = (0,0,0,1)$ lies on line ℓ_2
$3:\ P_7 = (3,1,0,0)$ lies on line ℓ_0
$4:\ P_9 = (5,1,0,0)$ lies on line ℓ_0
$5:\ P_{10} = (6,1,0,0)$ lies on line ℓ_0
$6:\ P_{31} = (4,2,1,0)$ lies on line ℓ_{10}
$7:\ P_{33} = (6,2,1,0)$ lies on line ℓ_{11}
$8:\ P_{46} = (3,4,1,0)$ lies on line ℓ_{12}
$9:\ P_{50} = (7,4,1,0)$ lies on line ℓ_{13}
$10:\ P_{69} = (2,7,1,0)$ lies on line ℓ_{14}
$11:\ P_{72} = (5,7,1,0)$ lies on line ℓ_{15}
$12:\ P_{104} = (6,3,0,1)$ lies on line ℓ_{10}
$13:\ P_{105} = (7,3,0,1)$ lies on line ℓ_{11}
$14:\ P_{116} = (2,5,0,1)$ lies on line ℓ_{12}
$15:\ P_{117} = (3,5,0,1)$ lies on line ℓ_{13}
$16:\ P_{126} = (4,6,0,1)$ lies on line ℓ_{15}
$17:\ P_{127} = (5,6,0,1)$ lies on line ℓ_{14}
$18:\ P_{146} = (0,1,1,1)$ lies on line ℓ_1
$19:\ P_{148} = (3,1,1,1)$ lies on line ℓ_1
$20:\ P_{150} = (5,1,1,1)$ lies on line ℓ_1
$21:\ P_{151} = (6,1,1,1)$ lies on line ℓ_1
$22:\ P_{155} = (2,2,1,1)$ lies on line ℓ_3
$23:\ P_{164} = (3,3,1,1)$ lies on line ℓ_3
$24:\ P_{173} = (4,4,1,1)$ lies on line ℓ_3
$25:\ P_{182} = (5,5,1,1)$ lies on line ℓ_3
$26:\ P_{191} = (6,6,1,1)$ lies on line ℓ_3
$27:\ P_{200} = (7,7,1,1)$ lies on line ℓ_3
$28:\ P_{213} = (4,1,2,1)$ lies on line ℓ_8
$29:\ P_{214} = (5,1,2,1)$ lies on line ℓ_4
$30:\ P_{222} = (5,2,2,1)$ lies on line ℓ_8
$31:\ P_{223} = (6,2,2,1)$ lies on line ℓ_4
$32:\ P_{226} = (1,3,2,1)$ lies on line ℓ_8
$33:\ P_{232} = (7,3,2,1)$ lies on line ℓ_4
$34:\ P_{233} = (0,4,2,1)$ lies on line ℓ_4
$35:\ P_{240} = (7,4,2,1)$ lies on line ℓ_8
$36:\ P_{265} = (0,0,3,1)$ lies on line ℓ_2
$37:\ P_{284} = (3,2,3,1)$ lies on line ℓ_{14}
$38:\ P_{287} = (6,2,3,1)$ lies on line ℓ_{15}
$39:\ P_{305} = (0,5,3,1)$ lies on line ℓ_{11}
$40:\ P_{312} = (7,5,3,1)$ lies on line ℓ_{10}
$41:\ P_{343} = (6,1,4,1)$ lies on line ℓ_5
$42:\ P_{344} = (7,1,4,1)$ lies on line ℓ_9
$43:\ P_{364} = (3,4,4,1)$ lies on line ℓ_5
$44:\ P_{367} = (6,4,4,1)$ lies on line ℓ_9
$45:\ P_{370} = (1,5,4,1)$ lies on line ℓ_9
$46:\ P_{371} = (2,5,4,1)$ lies on line ℓ_5
$47:\ P_{385} = (0,7,4,1)$ lies on line ℓ_5
$48:\ P_{387} = (2,7,4,1)$ lies on line ℓ_9
$49:\ P_{393} = (0,0,5,1)$ lies on line ℓ_2
$50:\ P_{428} = (3,4,5,1)$ lies on line ℓ_{11}
$51:\ P_{430} = (5,4,5,1)$ lies on line ℓ_{10}
$52:\ P_{441} = (0,6,5,1)$ lies on line ℓ_{12}
$53:\ P_{443} = (2,6,5,1)$ lies on line ℓ_{13}
$54:\ P_{457} = (0,0,6,1)$ lies on line ℓ_2
$55:\ P_{481} = (0,3,6,1)$ lies on line ℓ_{15}
$56:\ P_{485} = (4,3,6,1)$ lies on line ℓ_{14}
$57:\ P_{518} = (5,7,6,1)$ lies on line ℓ_{12}
$58:\ P_{519} = (6,7,6,1)$ lies on line ℓ_{13}
$59:\ P_{531} = (2,1,7,1)$ lies on line ℓ_7
$60:\ P_{532} = (3,1,7,1)$ lies on line ℓ_6
$61:\ P_{537} = (0,2,7,1)$ lies on line ℓ_6
$62:\ P_{541} = (4,2,7,1)$ lies on line ℓ_7
$63:\ P_{570} = (1,6,7,1)$ lies on line ℓ_7

64 : $P_{573} = (4, 6, 7, 1)$ lies on line ℓ_6
65 : $P_{580} = (3, 7, 7, 1)$ lies on line ℓ_7
66 : $P_{582} = (5, 7, 7, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:
The points on the surface but not on lines are:

0 : $P_{75} = (1, 0, 0, 1)$
1 : $P_{271} = (6, 0, 3, 1)$
2 : $P_{396} = (3, 0, 5, 1)$
3 : $P_{462} = (5, 0, 6, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0	1	1	1	1	1	1
2	0	0	0	1	0	0	0	1	1	1	1	0	0	1	1	0
3	1	1	1	0	1	1	1	0	0	0	0	1	1	0	0	1
4	1	0	0	1	0	1	1	0	1	0	1	0	1	0	1	0
5	1	0	0	1	1	0	1	0	0	1	1	0	0	1	0	1
6	1	0	0	1	1	1	0	1	0	0	0	1	0	1	1	0
7	1	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
8	1	0	1	0	1	0	0	0	0	0	0	1	0	1	0	1
9	1	0	1	0	0	1	0	0	0	0	0	1	1	0	1	0
10	0	1	1	0	1	1	0	1	0	0	0	1	0	0	0	0
11	0	1	0	1	0	0	1	0	1	1	1	0	1	0	0	1
12	0	1	0	1	1	0	0	1	0	1	0	1	0	1	0	1
13	0	1	1	0	0	1	1	0	1	0	0	0	1	0	0	0
14	0	1	1	0	1	0	1	0	0	1	0	0	0	0	0	1
15	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_0	P_5	P_5	P_5	P_5	P_6	P_8	P_{11}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_0	P_4	P_{147}	P_4	P_4	P_{149}	P_{152}	P_4

Line 2 intersects

Line	ℓ_3	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{14}
in point	P_{138}	P_{521}	P_{201}	P_{329}	P_{521}	P_{201}	P_{329}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{12}	ℓ_{15}
in point	P_5	P_4	P_{138}	P_5	P_5	P_5	P_4	P_4	P_4

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_5	P_5	P_5	P_5	P_{251}	P_{260}	P_{205}	P_{242}

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{15}
in point	P_5	P_5	P_5	P_5	P_{357}	P_{378}	P_{350}	P_{336}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}
in point	P_5	P_5	P_5	P_5	P_{568}	P_{523}	P_{546}	P_{559}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{15}
in point	P_6	P_{521}	P_{568}	P_{521}	P_{551}	P_{558}

Line 8 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_{11}	ℓ_{13}	ℓ_{15}
in point	P_8	P_{201}	P_{251}	P_{263}	P_{201}	P_{244}

Line 9 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_{11}	ℓ_{12}	ℓ_{14}
in point	P_{11}	P_{329}	P_{357}	P_{382}	P_{348}	P_{329}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_{11}
in point	P_{147}	P_{521}	P_{260}	P_{378}	P_{521}	P_{477}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}
in point	P_4	P_4	P_{523}	P_{263}	P_{382}	P_{477}	P_4	P_4

Line 12 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{15}
in point	P_4	P_4	P_{205}	P_{551}	P_{348}	P_4	P_{304}	P_4

Line 13 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8	ℓ_{12}
in point	P_{149}	P_{201}	P_{350}	P_{546}	P_{201}	P_{304}

Line 14 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6	ℓ_9	ℓ_{15}
in point	P_{152}	P_{329}	P_{242}	P_{559}	P_{329}	P_{451}

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{14}
in point	P_4	P_4	P_{336}	P_{558}	P_{244}	P_4	P_4	P_{451}

The surface has 105 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (4,1,0,0)$
$9: P_9 = (5,1,0,0)$
$10: P_{10} = (6,1,0,0)$
$11: P_{11} = (7,1,0,0)$
$12: P_{31} = (4,2,1,0)$
$13: P_{33} = (6,2,1,0)$
$14: P_{46} = (3,4,1,0)$
$15: P_{50} = (7,4,1,0)$
$16: P_{69} = (2,7,1,0)$
$17: P_{72} = (5,7,1,0)$
$18: P_{75} = (1,0,0,1)$
$19: P_{104} = (6,3,0,1)$
$20: P_{105} = (7,3,0,1)$
$21: P_{116} = (2,5,0,1)$
$22: P_{117} = (3,5,0,1)$
$23: P_{126} = (4,6,0,1)$
$24: P_{127} = (5,6,0,1)$
$25: P_{138} = (0,0,1,1)$
$26: P_{146} = (0,1,1,1)$
$27: P_{147} = (2,1,1,1)$
$28: P_{148} = (3,1,1,1)$
$29: P_{149} = (4,1,1,1)$
$30: P_{150} = (5,1,1,1)$
$31: P_{151} = (6,1,1,1)$
$32: P_{152} = (7,1,1,1)$
$33: P_{155} = (2,2,1,1)$
$34: P_{164} = (3,3,1,1)$
$35: P_{173} = (4,4,1,1)$
$36: P_{182} = (5,5,1,1)$
$37: P_{191} = (6,6,1,1)$
$38: P_{200} = (7,7,1,1)$

39 : $P_{201} = (0, 0, 2, 1)$
40 : $P_{205} = (4, 0, 2, 1)$
41 : $P_{213} = (4, 1, 2, 1)$
42 : $P_{214} = (5, 1, 2, 1)$
43 : $P_{222} = (5, 2, 2, 1)$
44 : $P_{223} = (6, 2, 2, 1)$
45 : $P_{226} = (1, 3, 2, 1)$
46 : $P_{232} = (7, 3, 2, 1)$
47 : $P_{233} = (0, 4, 2, 1)$
48 : $P_{240} = (7, 4, 2, 1)$
49 : $P_{242} = (1, 5, 2, 1)$
50 : $P_{244} = (3, 5, 2, 1)$
51 : $P_{251} = (2, 6, 2, 1)$
52 : $P_{260} = (3, 7, 2, 1)$
53 : $P_{263} = (6, 7, 2, 1)$
54 : $P_{265} = (0, 0, 3, 1)$
55 : $P_{271} = (6, 0, 3, 1)$
56 : $P_{284} = (3, 2, 3, 1)$
57 : $P_{287} = (6, 2, 3, 1)$
58 : $P_{304} = (7, 4, 3, 1)$
59 : $P_{305} = (0, 5, 3, 1)$
60 : $P_{312} = (7, 5, 3, 1)$
61 : $P_{329} = (0, 0, 4, 1)$
62 : $P_{336} = (7, 0, 4, 1)$
63 : $P_{343} = (6, 1, 4, 1)$
64 : $P_{344} = (7, 1, 4, 1)$
65 : $P_{348} = (3, 2, 4, 1)$
66 : $P_{350} = (5, 2, 4, 1)$
67 : $P_{357} = (4, 3, 4, 1)$
68 : $P_{364} = (3, 4, 4, 1)$
69 : $P_{367} = (6, 4, 4, 1)$
70 : $P_{370} = (1, 5, 4, 1)$
71 : $P_{371} = (2, 5, 4, 1)$
72 : $P_{378} = (1, 6, 4, 1)$
73 : $P_{382} = (5, 6, 4, 1)$
74 : $P_{385} = (0, 7, 4, 1)$
75 : $P_{387} = (2, 7, 4, 1)$
76 : $P_{393} = (0, 0, 5, 1)$
77 : $P_{396} = (3, 0, 5, 1)$
78 : $P_{428} = (3, 4, 5, 1)$
79 : $P_{430} = (5, 4, 5, 1)$
80 : $P_{441} = (0, 6, 5, 1)$
81 : $P_{443} = (2, 6, 5, 1)$
82 : $P_{451} = (2, 7, 5, 1)$
83 : $P_{457} = (0, 0, 6, 1)$
84 : $P_{462} = (5, 0, 6, 1)$
85 : $P_{477} = (4, 2, 6, 1)$
86 : $P_{481} = (0, 3, 6, 1)$
87 : $P_{485} = (4, 3, 6, 1)$
88 : $P_{518} = (5, 7, 6, 1)$
89 : $P_{519} = (6, 7, 6, 1)$
90 : $P_{521} = (0, 0, 7, 1)$
91 : $P_{523} = (2, 0, 7, 1)$
92 : $P_{531} = (2, 1, 7, 1)$
93 : $P_{532} = (3, 1, 7, 1)$
94 : $P_{537} = (0, 2, 7, 1)$
95 : $P_{541} = (4, 2, 7, 1)$
96 : $P_{546} = (1, 3, 7, 1)$
97 : $P_{551} = (6, 3, 7, 1)$
98 : $P_{558} = (5, 4, 7, 1)$
99 : $P_{559} = (6, 4, 7, 1)$
100 : $P_{568} = (7, 5, 7, 1)$
101 : $P_{570} = (1, 6, 7, 1)$
102 : $P_{573} = (4, 6, 7, 1)$
103 : $P_{580} = (3, 7, 7, 1)$
104 : $P_{582} = (5, 7, 7, 1)$