Rank-74499 over GF(2)

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The equation

The equation of the surface is :

$$X_1^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74499

General information

Number of lines	7
Number of points	11
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	4
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^7
Type of lines on points	4, 3, 2^5, 1^4

Singular Points

The surface has 2 singular points:

$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{11} = \mathbf{Pl}(0,0,1,0,0,1)_{22}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \mathbf{Pl}(1,0,1,0,1,0)_{13}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{25} = \mathbf{Pl}(0,1,1,0,0,1)_{24}$$

Rank of lines: (0, 4, 11, 3, 29, 34, 25)
Rank of points on Klein quadric: (0, 2, 22, 13, 25, 1, 24)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0).\ T = -1$

Double Points

The surface has 5 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_4$
$P_{14} = (0,1,1,1) = \ell_3 \cap \ell_4$
$P_4 = (1,1,1,1) = \ell_3 \cap \ell_6$
$P_{12} = (0,0,1,1) = \ell_4 \cap \ell_5$

Single Points

The surface has 4 single points:
The single points on the surface are:

$0 :\ P_3 = (0,0,0,1)$ lies on line ℓ_5
$1 :\ P_6 = (1,0,1,0)$ lies on line ℓ_1
$2 :\ P_8 = (1,1,1,0)$ lies on line ℓ_2
$3 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6
0	0 1 1 1 1 0 0
1	1 0 1 1 0 1 1
2	1 1 0 0 0 1 1
3	1 1 0 0 1 0 1
4	1 0 0 1 0 1 0
5	0 1 1 0 1 0 1
6	0 1 1 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_0	P_2	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6
in point	P_5	P_2	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_6
in point	P_0	P_0	P_{14}	P_4

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_5
in point	P_1	P_{14}	P_{12}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_2	P_{12}	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_4	P_2

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (1,0,1,0)$
$7: P_8 = (1,1,1,0)$
$8: P_{11} = (1,1,0,1)$
$9: P_{12} = (0,0,1,1)$
$10: P_{14} = (0,1,1,1)$