Rank-74500 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is 1090785350

General information

Number of lines	7
Number of points	4289
Number of singular points	2
Number of Eckardt points	1
Number of double points	5
Number of single points	438
Number of points off lines	3844
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65 ⁷
Type of lines on points	$4, 3, 2^5, 1^{438}, 0^{3844}$

Singular Points

The surface has 2 singular points:

$$\begin{aligned} 0: \ P_2 &= \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0) \\ 1: \ P_{4163} &= \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1) \end{aligned}$$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17043457} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{278529}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_{1}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{270400} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{66}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & \epsilon^{21} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15187585} = \begin{bmatrix} 1 & 1 & 0 & 57 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{15187585} = \mathbf{Pl}(0, 57, 1, 0, 0, 1)_{270648}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & \epsilon^{42} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{14921281} = \begin{bmatrix} 1 & 1 & 0 & 56 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{14921281} = \mathbf{Pl}(0, 56, 1, 0, 0, 1)_{270647}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{21} & \epsilon^{21} \end{bmatrix}_{270009} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 57 & 57 \end{bmatrix}_{270009} = \mathbf{Pl}(1, 1, 1, 56, 1, 0)_{238649}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{42} & \epsilon^{42} \end{bmatrix}_{269944} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 56 & 56 \end{bmatrix}_{269944} = \mathbf{Pl}(1, 1, 1, 57, 1, 0)_{242618}$$

Rank of lines: (17043457, 17047616, 270400, 15187585, 14921281, 270009, 269944) Rank of points on Klein quadric: (278529, 1, 66, 270648, 270647, 238649, 242618)

Eckardt Points

The surface has 1 Eckardt points: $0: P_{4163} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1).$

Double Points

The surface has 5 Double points: The double points on the surface are:

$$P_{8258} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

$$P_{11841} = (0, 56, 1, 1) = \ell_0 \cap \ell_5$$

$$P_{11905} = (0, 57, 1, 1) = \ell_0 \cap \ell_6$$

 $P_{241273} = (56, 56, 57, 1) = \ell_3 \cap \ell_6$ $P_{237242} = (57, 57, 56, 1) = \ell_4 \cap \ell_5$

Single Points

The surface has 438 single points: The single points on the surface are:

 $\begin{array}{l} 0: \ P_1 = (0,1,0,0) \ \text{lies on line} \ \ell_0 \\ 1: \ P_3 = (0,0,0,1) \ \text{lies on line} \ \ell_1 \\ 2: \ P_{3652} = (1,56,1,0) \ \text{lies on line} \ \ell_5 \\ 3: \ P_{3716} = (1,57,1,0) \ \text{lies on line} \ \ell_6 \\ 4: \ P_{7802} = (56,56,0,1) \ \text{lies on line} \ \ell_3 \\ 5: \ P_{7867} = (57,57,0,1) \ \text{lies on line} \ \ell_4 \\ 6: \ P_{8259} = (1,0,1,1) \ \text{lies on line} \ \ell_2 \\ 7: \ P_{8322} = (0,1,1,1) \ \text{lies on line} \ \ell_0 \\ 8: \ P_{8385} = (0,2,1,1) \ \text{lies on line} \ \ell_0 \\ 9: \ P_{8449} = (0,3,1,1) \ \text{lies on line} \ \ell_0 \\ 10: \ P_{8513} = (0,4,1,1) \ \text{lies on line} \ \ell_0 \\ 11: \ P_{8577} = (0,5,1,1) \ \text{lies on line} \ \ell_0 \\ 12: \ P_{8641} = (0,6,1,1) \ \text{lies on line} \ \ell_0 \end{array}$

13: $P_{8705} = (0, 7, 1, 1)$ lies on line ℓ_0 14: $P_{8769} = (0, 8, 1, 1)$ lies on line ℓ_0 15: $P_{8833} = (0, 9, 1, 1)$ lies on line ℓ_0 16: $P_{8897} = (0, 10, 1, 1)$ lies on line ℓ_0 17: $P_{8961} = (0, 11, 1, 1)$ lies on line ℓ_0 18: $P_{9025} = (0, 12, 1, 1)$ lies on line ℓ_0 19: $P_{9089} = (0, 13, 1, 1)$ lies on line ℓ_0 20: $P_{9153} = (0, 14, 1, 1)$ lies on line ℓ_0 21: $P_{9217} = (0, 15, 1, 1)$ lies on line ℓ_0 22: $P_{9281} = (0, 16, 1, 1)$ lies on line ℓ_0 23: $P_{9345} = (0, 17, 1, 1)$ lies on line ℓ_0 24: $P_{9409} = (0, 18, 1, 1)$ lies on line ℓ_0

25: $P_{9473} = (0, 19, 1, 1)$ lies on line ℓ_0

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80: P_{20089} = (56, 56, 3, 1) lies on line \ell_3
26: P_{9537} = (0, 20, 1, 1) lies on line \ell_0
27: P_{9601} = (0, 21, 1, 1) lies on line \ell_0
                                                                     81: P_{20154} = (57, 57, 3, 1) lies on line \ell_4
                                                                     82: P_{20545} = (0,0,4,1) lies on line \ell_1
28: P_{9665} = (0, 22, 1, 1) lies on line \ell_0
29: P_{9729} = (0, 23, 1, 1) lies on line \ell_0
                                                                     83: P_{20546} = (1, 0, 4, 1) lies on line \ell_2
30: P_{9793} = (0, 24, 1, 1) lies on line \ell_0
                                                                     84: P_{22726} = (5, 34, 4, 1) lies on line \ell_5
                                                                     85: P_{22982} = (5, 38, 4, 1) lies on line \ell_6
31: P_{9857} = (0, 25, 1, 1) lies on line \ell_0
32: P_{9921} = (0, 26, 1, 1) lies on line \ell_0
                                                                     86: P_{24185} = (56, 56, 4, 1) lies on line \ell_3
                                                                     87: P_{24250} = (57, 57, 4, 1) lies on line \ell_4
33: P_{9985} = (0, 27, 1, 1) lies on line \ell_0
34: P_{10049} = (0, 28, 1, 1) lies on line \ell_0
                                                                     88: P_{24641} = (0, 0, 5, 1) lies on line \ell_1
35: P_{10113} = (0, 29, 1, 1) lies on line \ell_0
                                                                     89: P_{24642} = (1, 0, 5, 1) lies on line \ell_2
36: P_{10177} = (0, 30, 1, 1) lies on line \ell_0
                                                                     90: P_{26309} = (4, 26, 5, 1) lies on line \ell_5
37: P_{10241} = (0, 31, 1, 1) lies on line \ell_0
                                                                     91: P_{26629} = (4, 31, 5, 1) lies on line \ell_6
38: P_{10305} = (0, 32, 1, 1) lies on line \ell_0
                                                                     92: P_{28281} = (56, 56, 5, 1) lies on line \ell_3
39: P_{10369} = (0, 33, 1, 1) lies on line \ell_0
                                                                     93: P_{28346} = (57, 57, 5, 1) lies on line \ell_4
40: P_{10433} = (0, 34, 1, 1) lies on line \ell_0
                                                                     94: P_{28737} = (0,0,6,1) lies on line \ell_1
41: P_{10497} = (0, 35, 1, 1) lies on line \ell_0
                                                                     95: P_{28738} = (1, 0, 6, 1) lies on line \ell_2
42: P_{10561} = (0, 36, 1, 1) lies on line \ell_0
                                                                     96: P_{32008} = (7, 51, 6, 1) lies on line \ell_5
43: P_{10625} = (0, 37, 1, 1) lies on line \ell_0
                                                                     97: P_{32136} = (7, 53, 6, 1) lies on line \ell_6
44 : P_{10689} = (0, 38, 1, 1) lies on line \ell_0
                                                                     98: P_{32377} = (56, 56, 6, 1) lies on line \ell_3
45: P_{10753} = (0, 39, 1, 1) lies on line \ell_0
                                                                     99: P_{32442} = (57, 57, 6, 1) lies on line \ell_4
46: P_{10817} = (0, 40, 1, 1) lies on line \ell_0
                                                                     100: P_{32833} = (0, 0, 7, 1) lies on line \ell_1
47: P_{10881} = (0, 41, 1, 1) lies on line \ell_0
                                                                     101: P_{32834} = (1, 0, 7, 1) lies on line \ell_2
48: P_{10945} = (0, 42, 1, 1) lies on line \ell_0
                                                                     102: P_{33543} = (6, 11, 7, 1) lies on line \ell_5
                                                                     103 : P_{33607} = (6, 12, 7, 1) lies on line \ell_6
49: P_{11009} = (0, 43, 1, 1) lies on line \ell_0
50: P_{11073} = (0, 44, 1, 1) lies on line \ell_0
                                                                     104: P_{36473} = (56, 56, 7, 1) lies on line \ell_3
51: P_{11137} = (0, 45, 1, 1) lies on line \ell_0
                                                                     105: P_{36538} = (57, 57, 7, 1) lies on line \ell_4
52: P_{11201} = (0, 46, 1, 1) lies on line \ell_0
                                                                     106: P_{36929} = (0, 0, 8, 1) lies on line \ell_1
53: P_{11265} = (0, 47, 1, 1) lies on line \ell_0
                                                                     107: P_{36930} = (1,0,8,1) lies on line \ell_2
54: P_{11329} = (0, 48, 1, 1) lies on line \ell_0
                                                                     108: P_{39306} = (9, 37, 8, 1) lies on line \ell_5
55: P_{11393} = (0, 49, 1, 1) lies on line \ell_0
                                                                     109: P_{39818} = (9, 45, 8, 1) lies on line \ell_6
56: P_{11457} = (0, 50, 1, 1) lies on line \ell_0
                                                                     110: P_{40569} = (56, 56, 8, 1) lies on line \ell_3
57: P_{11521} = (0, 51, 1, 1) lies on line \ell_0
                                                                     111: P_{40634} = (57, 57, 8, 1) lies on line \ell_4
58: P_{11585} = (0, 52, 1, 1) lies on line \ell_0
                                                                     112: P_{41025} = (0, 0, 9, 1) lies on line \ell_1
59: P_{11649} = (0, 53, 1, 1) lies on line \ell_0
                                                                     113: P_{41026} = (1, 0, 9, 1) lies on line \ell_2
60: P_{11713} = (0, 54, 1, 1) lies on line \ell_0
                                                                     114: P_{42313} = (8, 20, 9, 1) lies on line \ell_6
61 : P_{11777} = (0, 55, 1, 1) lies on line \ell_0
                                                                     115 : P_{42889} = (8, 29, 9, 1) lies on line \ell_5
62: P_{11897} = (56, 56, 1, 1) lies on line \ell_3
                                                                     116: P_{44665} = (56, 56, 9, 1) lies on line \ell_3
63: P_{11962} = (57, 57, 1, 1) lies on line \ell_4
                                                                     117: P_{44730} = (57, 57, 9, 1) lies on line \ell_4
64: P_{11969} = (0, 58, 1, 1) lies on line \ell_0
                                                                     118: P_{45121} = (0, 0, 10, 1) lies on line \ell_1
                                                                     119: P_{45122} = (1, 0, 10, 1) lies on line \ell_2
65: P_{12033} = (0, 59, 1, 1) lies on line \ell_0
66: P_{12097} = (0, 60, 1, 1) lies on line \ell_0
                                                                     120: P_{48460} = (11, 52, 10, 1) lies on line \ell_5
67: P_{12161} = (0, 61, 1, 1) lies on line \ell_0
                                                                     121: P_{48761} = (56, 56, 10, 1) lies on line \ell_3
68: P_{12225} = (0, 62, 1, 1) lies on line \ell_0
                                                                     122: P_{48826} = (57, 57, 10, 1) lies on line \ell_4
69: P_{12289} = (0, 63, 1, 1) lies on line \ell_0
                                                                     123: P_{49100} = (11, 62, 10, 1) lies on line \ell_6
70: P_{12353} = (0, 0, 2, 1) lies on line \ell_1
                                                                     124: P_{49217} = (0, 0, 11, 1) lies on line \ell_1
71: P_{12354} = (1, 0, 2, 1) lies on line \ell_2
                                                                     125: P_{49218} = (1, 0, 11, 1) lies on line \ell_2
72: P_{13444} = (3, 17, 2, 1) lies on line \ell_5
                                                                     126: P_{49675} = (10, 7, 11, 1) lies on line \ell_6
73: P_{13572} = (3, 19, 2, 1) lies on line \ell_6
                                                                     127: P_{49995} = (10, 12, 11, 1) lies on line \ell_5
74: P_{15993} = (56, 56, 2, 1) lies on line \ell_3
                                                                     128: P_{52857} = (56, 56, 11, 1) lies on line \ell_3
                                                                     129 : P_{52922} = (57, 57, 11, 1) lies on line \ell_4
75: P_{16058} = (57, 57, 2, 1) lies on line \ell_4
76: P_{16449} = (0,0,3,1) lies on line \ell_1
                                                                     130: P_{53313} = (0, 0, 12, 1) lies on line \ell_1
77: P_{16450} = (1, 0, 3, 1) lies on line \ell_2
                                                                     131: P_{53314} = (1, 0, 12, 1) lies on line \ell_2
                                                                     132 : P_{53774} = (13, 7, 12, 1) lies on line \ell_5
78: P_{19075} = (2, 41, 3, 1) lies on line \ell_5
79: P_{19139} = (2, 42, 3, 1) lies on line \ell_6
                                                                     133: P_{54030} = (13, 11, 12, 1) lies on line \ell_6
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134 : P_{56953} = (56, 56, 12, 1) lies on line \ell_3
                                                                    188 : P_{93817} = (56, 56, 21, 1) lies on line \ell_3
135: P_{57018} = (57, 57, 12, 1) lies on line \ell_4
                                                                     189 : P_{93882} = (57, 57, 21, 1) lies on line \ell_4
                                                                     190: P_{94273} = (0, 0, 22, 1) lies on line \ell_1
136: P_{57409} = (0, 0, 13, 1) lies on line \ell_1
137: P_{57410} = (1, 0, 13, 1) lies on line \ell_2
                                                                    191: P_{94274} = (1, 0, 22, 1) lies on line \ell_2
138: P_{60621} = (12, 50, 13, 1) lies on line \ell_6
                                                                    192: P_{95192} = (23, 14, 22, 1) lies on line \ell_6
                                                                    193 : P_{95832} = (23, 24, 22, 1) lies on line \ell_5
139: P_{61049} = (56, 56, 13, 1) lies on line \ell_3
140 : P_{61114} = (57, 57, 13, 1) lies on line \ell_4
                                                                     194: P_{97913} = (56, 56, 22, 1) lies on line \ell_3
                                                                     195: P_{97978} = (57, 57, 22, 1) lies on line \ell_4
141: P_{61453} = (12, 63, 13, 1) lies on line \ell_5
142: P_{61505} = (0, 0, 14, 1) lies on line \ell_1
                                                                    196: P_{98369} = (0, 0, 23, 1) lies on line \ell_1
143: P_{61506} = (1, 0, 14, 1) lies on line \ell_2
                                                                    197: P_{98370} = (1, 0, 23, 1) lies on line \ell_2
144: P_{62928} = (15, 22, 14, 1) lies on line \ell_5
                                                                    198: P_{100439} = (22, 32, 23, 1) lies on line \ell_5
145: P_{63056} = (15, 24, 14, 1) lies on line \ell_6
                                                                    199: P_{101911} = (22, 55, 23, 1) lies on line \ell_6
146: P_{65145} = (56, 56, 14, 1) lies on line \ell_3
                                                                    200: P_{102009} = (56, 56, 23, 1) lies on line \ell_3
147: P_{65210} = (57, 57, 14, 1) lies on line \ell_4
                                                                    201: P_{102074} = (57, 57, 23, 1) lies on line \ell_4
148: P_{65601} = (0, 0, 15, 1) lies on line \ell_1
                                                                    202: P_{102465} = (0, 0, 24, 1) lies on line \ell_1
149: P_{65602} = (1, 0, 15, 1) lies on line \ell_2
                                                                    203: P_{102466} = (1, 0, 24, 1) lies on line \ell_2
150: P_{67727} = (14, 33, 15, 1) lies on line \ell_6
                                                                    204: P_{103386} = (25, 14, 24, 1) lies on line \ell_5
151: P_{68559} = (14, 46, 15, 1) lies on line \ell_5
                                                                    205: P_{103898} = (25, 22, 24, 1) lies on line \ell_6
152 : P_{69241} = (56, 56, 15, 1) lies on line \ell_3
                                                                    206 : P_{106105} = (56, 56, 24, 1) lies on line \ell_3
153: P_{69306} = (57, 57, 15, 1) lies on line \ell_4
                                                                    207: P_{106170} = (57, 57, 24, 1) lies on line \ell_4
154: P_{69697} = (0, 0, 16, 1) lies on line \ell_1
                                                                    208: P_{106561} = (0, 0, 25, 1) lies on line \ell_1
155: P_{69698} = (1, 0, 16, 1) lies on line \ell_2
                                                                    209 : P_{106562} = (1, 0, 25, 1) lies on line \ell_2
156: P_{72466} = (17, 43, 16, 1) lies on line \ell_5
                                                                    210: P_{109593} = (24, 47, 25, 1) lies on line \ell_6
157: P_{73337} = (56, 56, 16, 1) lies on line \ell_3
                                                                    211: P_{110041} = (24, 54, 25, 1) lies on line \ell_5
158: P_{73402} = (57, 57, 16, 1) lies on line \ell_4
                                                                    212: P_{110201} = (56, 56, 25, 1) lies on line \ell_3
159: P_{73490} = (17, 59, 16, 1) lies on line \ell_6
                                                                    213: P_{110266} = (57, 57, 25, 1) lies on line \ell_4
160: P_{73793} = (0, 0, 17, 1) lies on line \ell_1
                                                                    214: P_{110657} = (0, 0, 26, 1) lies on line \ell_1
161: P_{73794} = (1, 0, 17, 1) lies on line \ell_2
                                                                    215: P_{110658} = (1, 0, 26, 1) lies on line \ell_2
162: P_{73937} = (16, 2, 17, 1) lies on line \ell_6
                                                                    216: P_{111004} = (27, 5, 26, 1) lies on line \ell_6
163: P_{75025} = (16, 19, 17, 1) lies on line \ell_5
                                                                    217: P_{112668} = (27, 31, 26, 1) lies on line \ell_5
164: P_{77433} = (56, 56, 17, 1) lies on line \ell_3
                                                                    218: P_{114297} = (56, 56, 26, 1) lies on line \ell_3
165: P_{77498} = (57, 57, 17, 1) lies on line \ell_4
                                                                    219: P_{114362} = (57, 57, 26, 1) lies on line \ell_4
166: P_{77889} = (0, 0, 18, 1) lies on line \ell_1
                                                                     220: P_{114753} = (0, 0, 27, 1) lies on line \ell_1
                                                                     221 : P_{114754} = (1, 0, 27, 1) lies on line \ell_2
167: P_{77890} = (1, 0, 18, 1) lies on line \ell_2
168: P_{80468} = (19, 40, 18, 1) lies on line \ell_6
                                                                    222: P_{117275} = (26, 39, 27, 1) lies on line \ell_5
                                                                    223: P_{118393} = (56, 56, 27, 1) lies on line \ell_3
169: P_{81529} = (56, 56, 18, 1) lies on line \ell_3
170: P_{81594} = (57, 57, 18, 1) lies on line \ell_4
                                                                     224: P_{118458} = (57, 57, 27, 1) lies on line \ell_4
171: P_{81620} = (19, 58, 18, 1) lies on line \ell_5
                                                                     225: P_{118619} = (26, 60, 27, 1) lies on line \ell_6
172: P_{81985} = (0, 0, 19, 1) lies on line \ell_1
                                                                    226: P_{118849} = (0, 0, 28, 1) lies on line \ell_1
173: P_{81986} = (1, 0, 19, 1) lies on line \ell_2
                                                                    227: P_{118850} = (1, 0, 28, 1) lies on line \ell_2
                                                                    228: P_{121694} = (29, 44, 28, 1) lies on line \ell_5
174: P_{82131} = (18, 2, 19, 1) lies on line \ell_5
175: P_{83091} = (18, 17, 19, 1) lies on line \ell_6
                                                                    229: P_{121950} = (29, 48, 28, 1) lies on line \ell_6
176: P_{85625} = (56, 56, 19, 1) lies on line \ell_3
                                                                    230: P_{122489} = (56, 56, 28, 1) lies on line \ell_3
177: P_{85690} = (57, 57, 19, 1) lies on line \ell_4
                                                                    231: P_{122554} = (57, 57, 28, 1) lies on line \ell_4
178: P_{86081} = (0, 0, 20, 1) lies on line \ell_1
                                                                    232 : P_{122945} = (0, 0, 29, 1) lies on line \ell_1
179: P_{86082} = (1, 0, 20, 1) lies on line \ell_2
                                                                     233: P_{122946} = (1, 0, 29, 1) lies on line \ell_2
180: P_{86678} = (21, 9, 20, 1) lies on line \ell_5
                                                                    234: P_{123549} = (28, 9, 29, 1) lies on line \ell_6
181: P_{87958} = (21, 29, 20, 1) lies on line \ell_6
                                                                    235: P_{124253} = (28, 20, 29, 1) lies on line \ell_5
182: P_{89721} = (56, 56, 20, 1) lies on line \ell_3
                                                                    236: P_{126585} = (56, 56, 29, 1) lies on line \ell_3
183: P_{89786} = (57, 57, 20, 1) lies on line \ell_4
                                                                    237: P_{126650} = (57, 57, 29, 1) lies on line \ell_4
                                                                    238: P_{127041} = (0, 0, 30, 1) lies on line \ell_1
184: P_{90177} = (0, 0, 21, 1) lies on line \ell_1
185: P_{90178} = (1, 0, 21, 1) lies on line \ell_2
                                                                    239: P_{127042} = (1, 0, 30, 1) lies on line \ell_2
186 : P_{92501} = (20, 36, 21, 1) lies on line \ell_6
                                                                    240: P_{129312} = (31, 35, 30, 1) lies on line \ell_6
187: P_{93333} = (20, 49, 21, 1) lies on line \ell_5
                                                                    241: P_{130681} = (56, 56, 30, 1) lies on line \ell_3
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296: P_{167610} = (57, 57, 39, 1) lies on line \ell_4
242: P_{130746} = (57, 57, 30, 1) lies on line \ell_4
243 : P_{130976} = (31, 61, 30, 1) lies on line \ell_5
                                                                    297: P_{167783} = (38, 60, 39, 1) lies on line \ell_5
                                                                    298 : P_{168001} = (0, 0, 40, 1) lies on line \ell_1
244: P_{131137} = (0, 0, 31, 1) lies on line \ell_1
245 : P_{131138} = (1, 0, 31, 1) lies on line \ell_2
                                                                    299: P_{168002} = (1, 0, 40, 1) lies on line \ell_2
246: P_{131487} = (30, 5, 31, 1) lies on line \ell_5
                                                                    300: P_{169194} = (41, 18, 40, 1) lies on line \ell_5
247: P_{132831} = (30, 26, 31, 1) lies on line \ell_6
                                                                    301: P_{171641} = (56, 56, 40, 1) lies on line \ell_3
248: P_{134777} = (56, 56, 31, 1) lies on line \ell_3
                                                                    302: P_{171706} = (57, 57, 40, 1) lies on line \ell_4
249: P_{134842} = (57, 57, 31, 1) lies on line \ell_4
                                                                    303: P_{171754} = (41, 58, 40, 1) lies on line \ell_6
250: P_{135233} = (0, 0, 32, 1) lies on line \ell_1
                                                                    304: P_{172097} = (0,0,41,1) lies on line \ell_1
251: P_{135234} = (1, 0, 32, 1) lies on line \ell_2
                                                                    305: P_{172098} = (1, 0, 41, 1) lies on line \ell_2
252: P_{136738} = (33, 23, 32, 1) lies on line \ell_6
                                                                    306: P_{172329} = (40, 3, 41, 1) lies on line \ell_6
253 : P_{138786} = (33, 55, 32, 1) lies on line \ell_5
                                                                    307: P_{174825} = (40, 42, 41, 1) lies on line \ell_5
254: P_{138873} = (56, 56, 32, 1) lies on line \ell_3
                                                                    308: P_{175737} = (56, 56, 41, 1) lies on line \ell_3
                                                                    309: P_{175802} = (57, 57, 41, 1) lies on line \ell_4
255: P_{138938} = (57, 57, 32, 1) lies on line \ell_4
256: P_{139329} = (0, 0, 33, 1) lies on line \ell_1
                                                                    310: P_{176193} = (0, 0, 42, 1) lies on line \ell_1
257: P_{139330} = (1, 0, 33, 1) lies on line \ell_2
                                                                    311: P_{176194} = (1, 0, 42, 1) lies on line \ell_2
258: P_{140321} = (32, 15, 33, 1) lies on line \ell_5
                                                                    312: P_{176428} = (43, 3, 42, 1) lies on line \ell_5
259: P_{142305} = (32, 46, 33, 1) lies on line \ell_6
                                                                    313: P_{178860} = (43, 41, 42, 1) lies on line \ell_6
260 : P_{142969} = (56, 56, 33, 1) lies on line \ell_3
                                                                    314: P_{179833} = (56, 56, 42, 1) lies on line \ell_3
                                                                    315: P_{179898} = (57, 57, 42, 1) lies on line \ell_4
261: P_{143034} = (57, 57, 33, 1) lies on line \ell_4
262 : P_{143425} = (0, 0, 34, 1) lies on line \ell_1
                                                                    316: P_{180289} = (0, 0, 43, 1) lies on line \ell_1
263: P_{143426} = (1, 0, 34, 1) lies on line \ell_2
                                                                    317: P_{180290} = (1, 0, 43, 1) lies on line \ell_2
                                                                    318: P_{181355} = (42, 16, 43, 1) lies on line \ell_6
264: P_{143716} = (35, 4, 34, 1) lies on line \ell_6
                                                                    319 : P_{183929} = (56, 56, 43, 1) lies on line \ell_3
265: P_{145892} = (35, 38, 34, 1) lies on line \ell_5
266: P_{147065} = (56, 56, 34, 1) lies on line \ell_3
                                                                    320: P_{183994} = (57, 57, 43, 1) lies on line \ell_4
267: P_{147130} = (57, 57, 34, 1) lies on line \ell_4
                                                                    321: P_{184107} = (42, 59, 43, 1) lies on line \ell_5
268: P_{147521} = (0, 0, 35, 1) lies on line \ell_1
                                                                    322: P_{184385} = (0, 0, 44, 1) lies on line \ell_1
269: P_{147522} = (1, 0, 35, 1) lies on line \ell_2
                                                                    323: P_{184386} = (1, 0, 44, 1) lies on line \ell_2
270: P_{149475} = (34, 30, 35, 1) lies on line \ell_5
                                                                    324: P_{186222} = (45, 28, 44, 1) lies on line \ell_6
271: P_{151161} = (56, 56, 35, 1) lies on line \ell_3
                                                                    325: P_{187502} = (45, 48, 44, 1) lies on line \ell_5
272: P_{151226} = (57, 57, 35, 1) lies on line \ell_4
                                                                    326: P_{188025} = (56, 56, 44, 1) lies on line \ell_3
273 : P_{151459} = (34, 61, 35, 1) lies on line \ell_6
                                                                    327: P_{188090} = (57, 57, 44, 1) lies on line \ell_4
274: P_{151617} = (0, 0, 36, 1) lies on line \ell_1
                                                                    328: P_{188481} = (0, 0, 45, 1) lies on line \ell_1
275: P_{151618} = (1, 0, 36, 1) lies on line \ell_2
                                                                    329: P_{188482} = (1, 0, 45, 1) lies on line \ell_2
276: P_{152998} = (37, 21, 36, 1) lies on line \ell_5
                                                                    330: P_{189037} = (44, 8, 45, 1) lies on line \ell_5
277: P_{154790} = (37, 49, 36, 1) lies on line \ell_6
                                                                    331: P_{190893} = (44, 37, 45, 1) lies on line \ell_6
278: P_{155257} = (56, 56, 36, 1) lies on line \ell_3
                                                                    332: P_{192121} = (56, 56, 45, 1) lies on line \ell_3
279: P_{155322} = (57, 57, 36, 1) lies on line \ell_4
                                                                    333 : P_{192186} = (57, 57, 45, 1) lies on line \ell_4
280 : P_{155713} = (0, 0, 37, 1) lies on line \ell_1
                                                                    334 : P_{192577} = (0, 0, 46, 1) lies on line \ell_1
281 : P_{155714} = (1, 0, 37, 1) lies on line \ell_2
                                                                    335: P_{192578} = (1, 0, 46, 1) lies on line \ell_2
282 : P_{156261} = (36, 8, 37, 1) lies on line \ell_6
                                                                    336: P_{193584} = (47, 15, 46, 1) lies on line \ell_6
283: P_{158629} = (36, 45, 37, 1) lies on line \ell_5
                                                                    337: P_{194736} = (47, 33, 46, 1) lies on line \ell_5
284: P_{159353} = (56, 56, 37, 1) lies on line \ell_3
                                                                    338: P_{196217} = (56, 56, 46, 1) lies on line \ell_3
285: P_{159418} = (57, 57, 37, 1) lies on line \ell_4
                                                                    339: P_{196282} = (57, 57, 46, 1) lies on line \ell_4
286: P_{159809} = (0, 0, 38, 1) lies on line \ell_1
                                                                    340: P_{196673} = (0, 0, 47, 1) lies on line \ell_1
                                                                    341 : P_{196674} = (1, 0, 47, 1) lies on line \ell_2
287: P_{159810} = (1, 0, 38, 1) lies on line \ell_2
288: P_{160104} = (39, 4, 38, 1) lies on line \ell_5
                                                                    342: P_{198319} = (46, 25, 47, 1) lies on line \ell_5
289: P_{162024} = (39, 34, 38, 1) lies on line \ell_6
                                                                    343: P_{200175} = (46, 54, 47, 1) lies on line \ell_6
290: P_{163449} = (56, 56, 38, 1) lies on line \ell_3
                                                                    344: P_{200313} = (56, 56, 47, 1) lies on line \ell_3
291: P_{163514} = (57, 57, 38, 1) lies on line \ell_4
                                                                    345: P_{200378} = (57, 57, 47, 1) lies on line \ell_4
                                                                    346: P_{200769} = (0, 0, 48, 1) lies on line \ell_1
292: P_{163905} = (0, 0, 39, 1) lies on line \ell_1
293: P_{163906} = (1, 0, 39, 1) lies on line \ell_2
                                                                    347: P_{200770} = (1, 0, 48, 1) lies on line \ell_2
294 : P_{165671} = (38, 27, 39, 1) lies on line \ell_6
                                                                    348 : P_{202610} = (49, 28, 48, 1) lies on line \ell_5
295: P_{167545} = (56, 56, 39, 1) lies on line \ell_3
                                                                    349: P_{203634} = (49, 44, 48, 1) lies on line \ell_6
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395: P_{233538} = (1, 0, 56, 1) lies on line \ell_2
350: P_{204409} = (56, 56, 48, 1) lies on line \ell_3
                                                                    396: P_{233658} = (57, 1, 56, 1) lies on line \ell_6
351: P_{204474} = (57, 57, 48, 1) lies on line \ell_4
                                                                    397 : P_{237177} = (56, 56, 56, 1) lies on line \ell_3
352: P_{204865} = (0, 0, 49, 1) lies on line \ell_1
353 : P_{204866} = (1, 0, 49, 1) lies on line \ell_2
                                                                    398: P_{237633} = (0, 0, 57, 1) lies on line \ell_1
354 : P_{206257} = (48, 21, 49, 1) lies on line \ell_6
                                                                    399: P_{237634} = (1, 0, 57, 1) lies on line \ell_2
355: P_{207217} = (48, 36, 49, 1) lies on line \ell_5
                                                                    400: P_{237753} = (56, 1, 57, 1) lies on line \ell_5
356 : P_{208505} = (56, 56, 49, 1) lies on line \ell_3
                                                                    401: P_{241338} = (57, 57, 57, 1) lies on line \ell_4
357: P_{208570} = (57, 57, 49, 1) lies on line \ell_4
                                                                    402: P_{241729} = (0, 0, 58, 1) lies on line \ell_1
358: P_{208961} = (0, 0, 50, 1) lies on line \ell_1
                                                                    403: P_{241730} = (1, 0, 58, 1) lies on line \ell_2
359: P_{208962} = (1, 0, 50, 1) lies on line \ell_2
                                                                    404: P_{242940} = (59, 18, 58, 1) lies on line \ell_6
360: P_{209844} = (51, 13, 50, 1) lies on line \ell_5
                                                                    405: P_{244348} = (59, 40, 58, 1) lies on line \ell_5
361: P_{212601} = (56, 56, 50, 1) lies on line \ell_3
                                                                    406: P_{245369} = (56, 56, 58, 1) lies on line \ell_3
362: P_{212666} = (57, 57, 50, 1) lies on line \ell_4
                                                                    407: P_{245434} = (57, 57, 58, 1) lies on line \ell_4
363: P_{213044} = (51, 63, 50, 1) lies on line \ell_6
                                                                    408: P_{245825} = (0, 0, 59, 1) lies on line \ell_1
364 : P_{213057} = (0, 0, 51, 1) lies on line \ell_1
                                                                    409: P_{245826} = (1, 0, 59, 1) lies on line \ell_2
365: P_{213058} = (1, 0, 51, 1) lies on line \ell_2
                                                                    410: P_{246907} = (58, 16, 59, 1) lies on line \ell_5
366: P_{213491} = (50, 6, 51, 1) lies on line \ell_6
                                                                    411: P_{248635} = (58, 43, 59, 1) lies on line \ell_6
367: P_{216499} = (50, 53, 51, 1) lies on line \ell_5
                                                                    412: P_{249465} = (56, 56, 59, 1) lies on line \ell_3
368: P_{216697} = (56, 56, 51, 1) lies on line \ell_3
                                                                    413: P_{249530} = (57, 57, 59, 1) lies on line \ell_4
                                                                    414 : P_{249921} = (0, 0, 60, 1) lies on line \ell_1
369: P_{216762} = (57, 57, 51, 1) lies on line \ell_4
370: P_{217153} = (0, 0, 52, 1) lies on line \ell_1
                                                                    415: P_{249922} = (1, 0, 60, 1) lies on line \ell_2
371: P_{217154} = (1, 0, 52, 1) lies on line \ell_2
                                                                    416: P_{251710} = (61, 27, 60, 1) lies on line \ell_5
                                                                    417: P_{252478} = (61, 39, 60, 1) lies on line \ell_6
372: P_{217846} = (53, 10, 52, 1) lies on line \ell_6
373 : P_{220793} = (56, 56, 52, 1) lies on line \ell_3
                                                                    418: P_{253561} = (56, 56, 60, 1) lies on line \ell_3
374: P_{220858} = (57, 57, 52, 1) lies on line \ell_4
                                                                    419: P_{253626} = (57, 57, 60, 1) lies on line \ell_4
375: P_{221174} = (53, 62, 52, 1) lies on line \ell_5
                                                                    420: P_{254017} = (0, 0, 61, 1) lies on line \ell_1
376: P_{221249} = (0, 0, 53, 1) lies on line \ell_1
                                                                    421: P_{254018} = (1, 0, 61, 1) lies on line \ell_2
377: P_{221250} = (1, 0, 53, 1) lies on line \ell_2
                                                                    422: P_{255997} = (60, 30, 61, 1) lies on line \ell_6
378: P_{221685} = (52, 6, 53, 1) lies on line \ell_5
                                                                    423: P_{256317} = (60, 35, 61, 1) lies on line \ell_5
379: P_{224565} = (52, 51, 53, 1) lies on line \ell_6
                                                                    424: P_{257657} = (56, 56, 61, 1) lies on line \ell_3
380: P_{224889} = (56, 56, 53, 1) lies on line \ell_3
                                                                    425: P_{257722} = (57, 57, 61, 1) lies on line \ell_4
381: P_{224954} = (57, 57, 53, 1) lies on line \ell_4
                                                                    426: P_{258113} = (0, 0, 62, 1) lies on line \ell_1
382: P_{225345} = (0, 0, 54, 1) lies on line \ell_1
                                                                    427: P_{258114} = (1,0,62,1) lies on line \ell_2
                                                                    428: P_{258816} = (63, 10, 62, 1) lies on line \ell_5
383: P_{225346} = (1, 0, 54, 1) lies on line \ell_2
384: P_{227000} = (55, 25, 54, 1) lies on line \ell_6
                                                                    429: P_{261504} = (63, 52, 62, 1) lies on line \ell_6
385: P_{228408} = (55, 47, 54, 1) lies on line \ell_5
                                                                    430: P_{261753} = (56, 56, 62, 1) lies on line \ell_3
386: P_{228985} = (56, 56, 54, 1) lies on line \ell_3
                                                                    431: P_{261818} = (57, 57, 62, 1) lies on line \ell_4
387: P_{229050} = (57, 57, 54, 1) lies on line \ell_4
                                                                    432 : P_{262209} = (0, 0, 63, 1) lies on line \ell_1
388: P_{229441} = (0, 0, 55, 1) lies on line \ell_1
                                                                    433: P_{262210} = (1, 0, 63, 1) lies on line \ell_2
                                                                    434: P_{263103} = (62, 13, 63, 1) lies on line \ell_6
389: P_{229442} = (1, 0, 55, 1) lies on line \ell_2
390 : P_{230967} = (54, 23, 55, 1) lies on line \ell_5
                                                                    435: P_{265471} = (62, 50, 63, 1) lies on line \ell_5
391: P_{231543} = (54, 32, 55, 1) lies on line \ell_6
                                                                    436: P_{265849} = (56, 56, 63, 1) lies on line \ell_3
392 : P_{233081} = (56, 56, 55, 1) lies on line \ell_3
                                                                    437 : P_{265914} = (57, 57, 63, 1) lies on line \ell_4
393: P_{233146} = (57, 57, 55, 1) lies on line \ell_4
394: P_{233537} = (0, 0, 56, 1) lies on line \ell_1
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The single points on the surface are:

Points on surface but on no line

The surface has 3844 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c} 0123456 \\ \hline 0 & 0100011 \\ 1 & 1011100 \\ 2 & 0101111 \\ 3 & 0110101 \\ 4 & 0111010 \\ 5 & 1010101 \\ 6 & 1011010 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_5	ℓ_6
in point	P_{8258}	P_{11841}	P_{11905}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{8258}	P_2	P_2	P_2

Line 2 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_2	P_2	P_{4163}	P_{4163}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_6
in point	P_2	P_2	P_2	P_{241273}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_2	P_2	P_{237242}

Line 5 intersects

Line	ℓ_0 ℓ_2		ℓ_4	ℓ_6	
in point	P_{11841}	P_{4163}	P_{237242}	P_{4163}	

Line 6 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_{11905}	P_{4163}	P_{241273}	P_{4163}

The surface has 4289 points: $\,$

Too many to print.