

Rank-74532 over GF(32)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_2 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(32) is 1141933094

General information

Number of lines	2
Number of points	1089
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	64
Number of points off lines	1024
Number of Hesse planes	0
Number of axes	0
Type of points on lines	33^2
Type of lines on points	$2, 1^{64}, 0^{1024}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{1082369} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{1082369} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{36865}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (1082369, 1083424)

Rank of points on Klein quadric: (36865, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_{2082} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

Single Points

The surface has 64 single points:

The single points on the surface are:

- | | |
|---|--|
| 0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0 | 29 : $P_{2945} = (0, 27, 1, 1)$ lies on line ℓ_0 |
| 1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_1 | 30 : $P_{2977} = (0, 28, 1, 1)$ lies on line ℓ_0 |
| 2 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1 | 31 : $P_{3009} = (0, 29, 1, 1)$ lies on line ℓ_0 |
| 3 : $P_{2114} = (0, 1, 1, 1)$ lies on line ℓ_0 | 32 : $P_{3041} = (0, 30, 1, 1)$ lies on line ℓ_0 |
| 4 : $P_{2145} = (0, 2, 1, 1)$ lies on line ℓ_0 | 33 : $P_{3073} = (0, 31, 1, 1)$ lies on line ℓ_0 |
| 5 : $P_{2177} = (0, 3, 1, 1)$ lies on line ℓ_0 | 34 : $P_{3105} = (0, 0, 2, 1)$ lies on line ℓ_1 |
| 6 : $P_{2209} = (0, 4, 1, 1)$ lies on line ℓ_0 | 35 : $P_{4129} = (0, 0, 3, 1)$ lies on line ℓ_1 |
| 7 : $P_{2241} = (0, 5, 1, 1)$ lies on line ℓ_0 | 36 : $P_{5153} = (0, 0, 4, 1)$ lies on line ℓ_1 |
| 8 : $P_{2273} = (0, 6, 1, 1)$ lies on line ℓ_0 | 37 : $P_{6177} = (0, 0, 5, 1)$ lies on line ℓ_1 |
| 9 : $P_{2305} = (0, 7, 1, 1)$ lies on line ℓ_0 | 38 : $P_{7201} = (0, 0, 6, 1)$ lies on line ℓ_1 |
| 10 : $P_{2337} = (0, 8, 1, 1)$ lies on line ℓ_0 | 39 : $P_{8225} = (0, 0, 7, 1)$ lies on line ℓ_1 |
| 11 : $P_{2369} = (0, 9, 1, 1)$ lies on line ℓ_0 | 40 : $P_{9249} = (0, 0, 8, 1)$ lies on line ℓ_1 |
| 12 : $P_{2401} = (0, 10, 1, 1)$ lies on line ℓ_0 | 41 : $P_{10273} = (0, 0, 9, 1)$ lies on line ℓ_1 |
| 13 : $P_{2433} = (0, 11, 1, 1)$ lies on line ℓ_0 | 42 : $P_{11297} = (0, 0, 10, 1)$ lies on line ℓ_1 |
| 14 : $P_{2465} = (0, 12, 1, 1)$ lies on line ℓ_0 | 43 : $P_{12321} = (0, 0, 11, 1)$ lies on line ℓ_1 |
| 15 : $P_{2497} = (0, 13, 1, 1)$ lies on line ℓ_0 | 44 : $P_{13345} = (0, 0, 12, 1)$ lies on line ℓ_1 |
| 16 : $P_{2529} = (0, 14, 1, 1)$ lies on line ℓ_0 | 45 : $P_{14369} = (0, 0, 13, 1)$ lies on line ℓ_1 |
| 17 : $P_{2561} = (0, 15, 1, 1)$ lies on line ℓ_0 | 46 : $P_{15393} = (0, 0, 14, 1)$ lies on line ℓ_1 |
| 18 : $P_{2593} = (0, 16, 1, 1)$ lies on line ℓ_0 | 47 : $P_{16417} = (0, 0, 15, 1)$ lies on line ℓ_1 |
| 19 : $P_{2625} = (0, 17, 1, 1)$ lies on line ℓ_0 | 48 : $P_{17441} = (0, 0, 16, 1)$ lies on line ℓ_1 |
| 20 : $P_{2657} = (0, 18, 1, 1)$ lies on line ℓ_0 | 49 : $P_{18465} = (0, 0, 17, 1)$ lies on line ℓ_1 |
| 21 : $P_{2689} = (0, 19, 1, 1)$ lies on line ℓ_0 | 50 : $P_{19489} = (0, 0, 18, 1)$ lies on line ℓ_1 |
| 22 : $P_{2721} = (0, 20, 1, 1)$ lies on line ℓ_0 | 51 : $P_{20513} = (0, 0, 19, 1)$ lies on line ℓ_1 |
| 23 : $P_{2753} = (0, 21, 1, 1)$ lies on line ℓ_0 | 52 : $P_{21537} = (0, 0, 20, 1)$ lies on line ℓ_1 |
| 24 : $P_{2785} = (0, 22, 1, 1)$ lies on line ℓ_0 | 53 : $P_{22561} = (0, 0, 21, 1)$ lies on line ℓ_1 |
| 25 : $P_{2817} = (0, 23, 1, 1)$ lies on line ℓ_0 | 54 : $P_{23585} = (0, 0, 22, 1)$ lies on line ℓ_1 |
| 26 : $P_{2849} = (0, 24, 1, 1)$ lies on line ℓ_0 | 55 : $P_{24609} = (0, 0, 23, 1)$ lies on line ℓ_1 |
| 27 : $P_{2881} = (0, 25, 1, 1)$ lies on line ℓ_0 | 56 : $P_{25633} = (0, 0, 24, 1)$ lies on line ℓ_1 |
| 28 : $P_{2913} = (0, 26, 1, 1)$ lies on line ℓ_0 | 57 : $P_{26657} = (0, 0, 25, 1)$ lies on line ℓ_1 |

58 : $P_{27681} = (0, 0, 26, 1)$ lies on line ℓ_1
 59 : $P_{28705} = (0, 0, 27, 1)$ lies on line ℓ_1
 60 : $P_{29729} = (0, 0, 28, 1)$ lies on line ℓ_1
 61 : $P_{30753} = (0, 0, 29, 1)$ lies on line ℓ_1

62 : $P_{31777} = (0, 0, 30, 1)$ lies on line ℓ_1
 63 : $P_{32801} = (0, 0, 31, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 1024 points not on any line:
 Too many to print.

Line Intersection Graph

	0 1
0	0 1
1	1 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_{2082}

Line 1 intersects

Line	ℓ_0
in point	P_{2082}

The surface has 1089 points:
 Too many to print.