

Rank-65868 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244173390

General information

Number of lines	15
Number of points	105
Number of singular points	1
Number of Eckardt points	1
Number of double points	33
Number of single points	60
Number of points off lines	10
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{15}
Type of lines on points	$6, 3, 2^{33}, 1^{60}, 0^{10}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 15 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{3504} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{3504} = \mathbf{Pl}(2, 0, 0, 1, 0, 0)_{19}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \mathbf{Pl}(4, 0, 0, 1, 0, 0)_{21} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \mathbf{Pl}(7, 0, 0, 1, 0, 0)_{24} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3568} = \mathbf{Pl}(0, 6, 1, 0, 0, 0)_{15} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1816} = \mathbf{Pl}(0, 3, 1, 0, 0, 0)_{12} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2984} = \mathbf{Pl}(0, 5, 1, 0, 0, 0)_{14} \\
\ell_6 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3641} = \begin{bmatrix} 1 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3641} = \mathbf{Pl}(0, 6, 1, 0, 0, 1)_{677} \\
\ell_7 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1889} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1889} = \mathbf{Pl}(0, 3, 1, 0, 0, 1)_{674} \\
\ell_8 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3057} = \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3057} = \mathbf{Pl}(0, 5, 1, 0, 0, 1)_{676} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & \gamma^5 & \gamma^5 \\ 0 & 1 & \gamma^2 & \gamma^3 \end{bmatrix}_{2015} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 4 & 5 \end{bmatrix}_{2015} = \mathbf{Pl}(1, 1, 1, 1, 3, 1)_{2463} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \gamma^3 & \gamma^3 \\ 0 & 1 & \gamma^4 & \gamma^6 \end{bmatrix}_{3340} = \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & 7 & 6 \end{bmatrix}_{3340} = \mathbf{Pl}(1, 1, 1, 1, 5, 1)_{3450} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \gamma^6 & \gamma^6 \\ 0 & 1 & \gamma & \gamma^5 \end{bmatrix}_{3968} = \begin{bmatrix} 1 & 0 & 6 & 6 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{3968} = \mathbf{Pl}(1, 1, 1, 1, 6, 1)_{3968} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^6 \\ 0 & 1 & \gamma^2 & \gamma^3 \end{bmatrix}_{4059} = \begin{bmatrix} 1 & 0 & 7 & 6 \\ 0 & 1 & 4 & 5 \end{bmatrix}_{4059} = \mathbf{Pl}(6, 2, 3, 4, 6, 1)_{4036} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^5 \\ 0 & 1 & \gamma^4 & \gamma^6 \end{bmatrix}_{1953} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 7 & 6 \end{bmatrix}_{1953} = \mathbf{Pl}(3, 4, 5, 7, 3, 1)_{2619} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma^3 \\ 0 & 1 & \gamma & \gamma^5 \end{bmatrix}_{3238} = \begin{bmatrix} 1 & 0 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{3238} = \mathbf{Pl}(5, 7, 6, 2, 5, 1)_{3692}
\end{aligned}$$

Rank of lines: (3504, 1752, 2920, 3568, 1816, 2984, 3641, 1889, 3057, 2015, 3340, 3968, 4059, 1953, 3238)
Rank of points on Klein quadric: (19, 21, 24, 15, 12, 14, 677, 674, 676, 2463, 3450, 3968, 4036, 2619, 3692)

Eckardt Points

The surface has 1 Eckardt points:
 $0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0).$

Double Points

The surface has 33 Double points:
The double points on the surface are:

$$\begin{aligned}
P_{76} &= (2, 0, 0, 1) = \ell_0 \cap \ell_3 \\
P_{92} &= (2, 2, 0, 1) = \ell_0 \cap \ell_6 \\
P_{84} &= (2, 1, 0, 1) = \ell_0 \cap \ell_{10} \\
P_{100} &= (2, 3, 0, 1) = \ell_0 \cap \ell_{13} \\
P_{78} &= (4, 0, 0, 1) = \ell_1 \cap \ell_4 \\
P_{110} &= (4, 4, 0, 1) = \ell_1 \cap \ell_7 \\
P_{86} &= (4, 1, 0, 1) = \ell_1 \cap \ell_{11} \\
P_{118} &= (4, 5, 0, 1) = \ell_1 \cap \ell_{14} \\
P_{81} &= (7, 0, 0, 1) = \ell_2 \cap \ell_5 \\
P_{137} &= (7, 7, 0, 1) = \ell_2 \cap \ell_8 \\
P_{89} &= (7, 1, 0, 1) = \ell_2 \cap \ell_9 \\
P_{129} &= (7, 6, 0, 1) = \ell_2 \cap \ell_{12} \\
P_{140} &= (2, 0, 1, 1) = \ell_3 \cap \ell_{11} \\
P_{267} &= (2, 0, 3, 1) = \ell_3 \cap \ell_{12} \\
P_{142} &= (4, 0, 1, 1) = \ell_4 \cap \ell_9 \\
P_{397} &= (4, 0, 5, 1) = \ell_4 \cap \ell_{13} \\
P_{145} &= (7, 0, 1, 1) = \ell_5 \cap \ell_{10}
\end{aligned}$$

$$\begin{aligned}
P_{464} &= (7, 0, 6, 1) = \ell_5 \cap \ell_{14} \\
P_{283} &= (2, 2, 3, 1) = \ell_6 \cap \ell_9 \\
P_{155} &= (2, 2, 1, 1) = \ell_6 \cap \ell_{14} \\
P_{429} &= (4, 4, 5, 1) = \ell_7 \cap \ell_{10} \\
P_{173} &= (4, 4, 1, 1) = \ell_7 \cap \ell_{12} \\
P_{520} &= (7, 7, 6, 1) = \ell_8 \cap \ell_{11} \\
P_{200} &= (7, 7, 1, 1) = \ell_8 \cap \ell_{13} \\
P_{572} &= (3, 6, 7, 1) = \ell_9 \cap \ell_{10} \\
P_{375} &= (6, 5, 4, 1) = \ell_9 \cap \ell_{11} \\
P_{513} &= (0, 7, 6, 1) = \ell_9 \cap \ell_{12} \\
P_{230} &= (5, 3, 2, 1) = \ell_{10} \cap \ell_{11} \\
P_{281} &= (0, 2, 3, 1) = \ell_{10} \cap \ell_{13} \\
P_{425} &= (0, 4, 5, 1) = \ell_{11} \cap \ell_{14} \\
P_{564} &= (3, 5, 7, 1) = \ell_{12} \cap \ell_{13} \\
P_{359} &= (6, 3, 4, 1) = \ell_{12} \cap \ell_{14} \\
P_{254} &= (5, 6, 2, 1) = \ell_{13} \cap \ell_{14}
\end{aligned}$$

Single Points

The surface has 60 single points:
The single points on the surface are:

$$\begin{aligned}
0 : P_{22} &= (3, 1, 1, 0) \text{ lies on line } \ell_9 \\
1 : P_{24} &= (5, 1, 1, 0) \text{ lies on line } \ell_{10} \\
2 : P_{25} &= (6, 1, 1, 0) \text{ lies on line } \ell_{11} \\
3 : P_{30} &= (3, 2, 1, 0) \text{ lies on line } \ell_{12} \\
4 : P_{48} &= (5, 4, 1, 0) \text{ lies on line } \ell_{13} \\
5 : P_{73} &= (6, 7, 1, 0) \text{ lies on line } \ell_{14} \\
6 : P_{94} &= (4, 2, 0, 1) \text{ lies on line } \ell_1 \\
7 : P_{97} &= (7, 2, 0, 1) \text{ lies on line } \ell_2 \\
8 : P_{102} &= (4, 3, 0, 1) \text{ lies on line } \ell_1 \\
9 : P_{105} &= (7, 3, 0, 1) \text{ lies on line } \ell_2 \\
10 : P_{108} &= (2, 4, 0, 1) \text{ lies on line } \ell_0 \\
11 : P_{113} &= (7, 4, 0, 1) \text{ lies on line } \ell_2 \\
12 : P_{116} &= (2, 5, 0, 1) \text{ lies on line } \ell_0 \\
13 : P_{121} &= (7, 5, 0, 1) \text{ lies on line } \ell_2 \\
14 : P_{124} &= (2, 6, 0, 1) \text{ lies on line } \ell_0 \\
15 : P_{126} &= (4, 6, 0, 1) \text{ lies on line } \ell_1 \\
16 : P_{132} &= (2, 7, 0, 1) \text{ lies on line } \ell_0 \\
17 : P_{134} &= (4, 7, 0, 1) \text{ lies on line } \ell_1 \\
18 : P_{203} &= (2, 0, 2, 1) \text{ lies on line } \ell_3 \\
19 : P_{205} &= (4, 0, 2, 1) \text{ lies on line } \ell_4 \\
20 : P_{208} &= (7, 0, 2, 1) \text{ lies on line } \ell_5 \\
21 : P_{218} &= (1, 2, 2, 1) \text{ lies on line } \ell_{12} \\
22 : P_{219} &= (2, 2, 2, 1) \text{ lies on line } \ell_6 \\
23 : P_{226} &= (1, 3, 2, 1) \text{ lies on line } \ell_9 \\
24 : P_{237} &= (4, 4, 2, 1) \text{ lies on line } \ell_7 \\
25 : P_{264} &= (7, 7, 2, 1) \text{ lies on line } \ell_8 \\
26 : P_{269} &= (4, 0, 3, 1) \text{ lies on line } \ell_4 \\
27 : P_{272} &= (7, 0, 3, 1) \text{ lies on line } \ell_5 \\
28 : P_{276} &= (3, 1, 3, 1) \text{ lies on line } \ell_{14} \\
29 : P_{284} &= (3, 2, 3, 1) \text{ lies on line } \ell_{11} \\
30 : P_{301} &= (4, 4, 3, 1) \text{ lies on line } \ell_7
\end{aligned}$$

$$\begin{aligned}
31 : P_{328} &= (7, 7, 3, 1) \text{ lies on line } \ell_8 \\
32 : P_{331} &= (2, 0, 4, 1) \text{ lies on line } \ell_3 \\
33 : P_{333} &= (4, 0, 4, 1) \text{ lies on line } \ell_4 \\
34 : P_{336} &= (7, 0, 4, 1) \text{ lies on line } \ell_5 \\
35 : P_{347} &= (2, 2, 4, 1) \text{ lies on line } \ell_6 \\
36 : P_{362} &= (1, 4, 4, 1) \text{ lies on line } \ell_{13} \\
37 : P_{365} &= (4, 4, 4, 1) \text{ lies on line } \ell_7 \\
38 : P_{370} &= (1, 5, 4, 1) \text{ lies on line } \ell_{10} \\
39 : P_{392} &= (7, 7, 4, 1) \text{ lies on line } \ell_8 \\
40 : P_{395} &= (2, 0, 5, 1) \text{ lies on line } \ell_3 \\
41 : P_{400} &= (7, 0, 5, 1) \text{ lies on line } \ell_5 \\
42 : P_{406} &= (5, 1, 5, 1) \text{ lies on line } \ell_{12} \\
43 : P_{411} &= (2, 2, 5, 1) \text{ lies on line } \ell_6 \\
44 : P_{430} &= (5, 4, 5, 1) \text{ lies on line } \ell_9 \\
45 : P_{456} &= (7, 7, 5, 1) \text{ lies on line } \ell_8 \\
46 : P_{459} &= (2, 0, 6, 1) \text{ lies on line } \ell_3 \\
47 : P_{461} &= (4, 0, 6, 1) \text{ lies on line } \ell_4 \\
48 : P_{471} &= (6, 1, 6, 1) \text{ lies on line } \ell_{13} \\
49 : P_{475} &= (2, 2, 6, 1) \text{ lies on line } \ell_6 \\
50 : P_{493} &= (4, 4, 6, 1) \text{ lies on line } \ell_7 \\
51 : P_{519} &= (6, 7, 6, 1) \text{ lies on line } \ell_{10} \\
52 : P_{523} &= (2, 0, 7, 1) \text{ lies on line } \ell_3 \\
53 : P_{525} &= (4, 0, 7, 1) \text{ lies on line } \ell_4 \\
54 : P_{528} &= (7, 0, 7, 1) \text{ lies on line } \ell_5 \\
55 : P_{539} &= (2, 2, 7, 1) \text{ lies on line } \ell_6 \\
56 : P_{557} &= (4, 4, 7, 1) \text{ lies on line } \ell_7 \\
57 : P_{570} &= (1, 6, 7, 1) \text{ lies on line } \ell_{11} \\
58 : P_{578} &= (1, 7, 7, 1) \text{ lies on line } \ell_{14} \\
59 : P_{584} &= (7, 7, 7, 1) \text{ lies on line } \ell_8
\end{aligned}$$

The single points on the surface are:

Points on surface but on no line

The surface has 10 points not on any line:

The points on the surface but not on lines are:

0 : $P_{146} = (0, 1, 1, 1)$	6 : $P_{198} = (5, 7, 1, 1)$
1 : $P_{158} = (5, 2, 1, 1)$	7 : $P_{241} = (0, 5, 2, 1)$
2 : $P_{159} = (6, 2, 1, 1)$	8 : $P_{377} = (0, 6, 4, 1)$
3 : $P_{172} = (3, 4, 1, 1)$	9 : $P_{545} = (0, 3, 7, 1)$
4 : $P_{175} = (6, 4, 1, 1)$	
5 : $P_{196} = (3, 7, 1, 1)$	

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	1	1	0	0	1	0	0	0	1	0	0	1	0
1	1	0	1	0	1	0	0	1	0	0	0	1	0	0	1
2	1	1	0	0	0	1	0	0	1	1	0	0	1	0	0
3	1	0	0	0	1	1	1	1	1	0	0	1	1	0	0
4	0	1	0	1	0	1	1	1	1	1	0	0	0	1	0
5	0	0	1	1	1	0	1	1	1	0	1	0	0	0	1
6	1	0	0	1	1	1	0	1	1	1	0	0	0	0	1
7	0	1	0	1	1	1	1	0	1	0	1	0	1	0	0
8	0	0	1	1	1	1	1	1	0	0	0	1	0	1	0
9	0	0	1	0	1	0	1	0	0	0	1	1	1	0	0
10	1	0	0	0	0	1	0	1	0	1	0	1	0	1	0
11	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1
12	0	0	1	1	0	0	0	1	0	1	0	0	0	1	1
13	1	0	0	0	1	0	0	0	1	0	1	0	1	0	1
14	0	1	0	0	0	1	1	0	0	0	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_{10}	ℓ_{13}
in point	P_1	P_1	P_{76}	P_{92}	P_{84}	P_{100}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7	ℓ_{11}	ℓ_{14}
in point	P_1	P_1	P_{78}	P_{110}	P_{86}	P_{118}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_8	ℓ_9	ℓ_{12}
in point	P_1	P_1	P_{81}	P_{137}	P_{89}	P_{129}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_{11}	ℓ_{12}
in point	P_{76}	P_2	P_2	P_2	P_2	P_2	P_{140}	P_{267}

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{13}
in point	P_{78}	P_2	P_2	P_2	P_2	P_2	P_{142}	P_{397}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{14}
in point	P_{81}	P_2	P_2	P_2	P_2	P_2	P_{145}	P_{464}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9	ℓ_{14}
in point	P_{92}	P_2	P_2	P_2	P_2	P_2	P_{283}	P_{155}

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{12}
in point	P_{110}	P_2	P_2	P_2	P_2	P_2	P_{429}	P_{173}

Line 8 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{11}	ℓ_{13}
in point	P_{137}	P_2	P_2	P_2	P_2	P_2	P_{520}	P_{200}

Line 9 intersects

Line	ℓ_2	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_{89}	P_{142}	P_{283}	P_{572}	P_{375}	P_{513}

Line 10 intersects

Line	ℓ_0	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}
in point	P_{84}	P_{145}	P_{429}	P_{572}	P_{230}	P_{281}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{14}
in point	P_{86}	P_{140}	P_{520}	P_{375}	P_{230}	P_{425}

Line 12 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{14}
in point	P_{129}	P_{267}	P_{173}	P_{513}	P_{564}	P_{359}

Line 13 intersects

Line	ℓ_0	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{100}	P_{397}	P_{200}	P_{281}	P_{564}	P_{254}

Line 14 intersects

Line	ℓ_1	ℓ_5	ℓ_6	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_{118}	P_{464}	P_{155}	P_{425}	P_{359}	P_{254}

The surface has 105 points:

The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$	20 : $P_{108} = (2, 4, 0, 1)$	40 : $P_{173} = (4, 4, 1, 1)$
1 : $P_2 = (0, 0, 1, 0)$	21 : $P_{110} = (4, 4, 0, 1)$	41 : $P_{175} = (6, 4, 1, 1)$
2 : $P_{22} = (3, 1, 1, 0)$	22 : $P_{113} = (7, 4, 0, 1)$	42 : $P_{196} = (3, 7, 1, 1)$
3 : $P_{24} = (5, 1, 1, 0)$	23 : $P_{116} = (2, 5, 0, 1)$	43 : $P_{198} = (5, 7, 1, 1)$
4 : $P_{25} = (6, 1, 1, 0)$	24 : $P_{118} = (4, 5, 0, 1)$	44 : $P_{200} = (7, 7, 1, 1)$
5 : $P_{30} = (3, 2, 1, 0)$	25 : $P_{121} = (7, 5, 0, 1)$	45 : $P_{203} = (2, 0, 2, 1)$
6 : $P_{48} = (5, 4, 1, 0)$	26 : $P_{124} = (2, 6, 0, 1)$	46 : $P_{205} = (4, 0, 2, 1)$
7 : $P_{73} = (6, 7, 1, 0)$	27 : $P_{126} = (4, 6, 0, 1)$	47 : $P_{208} = (7, 0, 2, 1)$
8 : $P_{76} = (2, 0, 0, 1)$	28 : $P_{129} = (7, 6, 0, 1)$	48 : $P_{218} = (1, 2, 2, 1)$
9 : $P_{78} = (4, 0, 0, 1)$	29 : $P_{132} = (2, 7, 0, 1)$	49 : $P_{219} = (2, 2, 2, 1)$
10 : $P_{81} = (7, 0, 0, 1)$	30 : $P_{134} = (4, 7, 0, 1)$	50 : $P_{226} = (1, 3, 2, 1)$
11 : $P_{84} = (2, 1, 0, 1)$	31 : $P_{137} = (7, 7, 0, 1)$	51 : $P_{230} = (5, 3, 2, 1)$
12 : $P_{86} = (4, 1, 0, 1)$	32 : $P_{140} = (2, 0, 1, 1)$	52 : $P_{237} = (4, 4, 2, 1)$
13 : $P_{89} = (7, 1, 0, 1)$	33 : $P_{142} = (4, 0, 1, 1)$	53 : $P_{241} = (0, 5, 2, 1)$
14 : $P_{92} = (2, 2, 0, 1)$	34 : $P_{145} = (7, 0, 1, 1)$	54 : $P_{254} = (5, 6, 2, 1)$
15 : $P_{94} = (4, 2, 0, 1)$	35 : $P_{146} = (0, 1, 1, 1)$	55 : $P_{264} = (7, 7, 2, 1)$
16 : $P_{97} = (7, 2, 0, 1)$	36 : $P_{155} = (2, 2, 1, 1)$	56 : $P_{267} = (2, 0, 3, 1)$
17 : $P_{100} = (2, 3, 0, 1)$	37 : $P_{158} = (5, 2, 1, 1)$	57 : $P_{269} = (4, 0, 3, 1)$
18 : $P_{102} = (4, 3, 0, 1)$	38 : $P_{159} = (6, 2, 1, 1)$	58 : $P_{272} = (7, 0, 3, 1)$
19 : $P_{105} = (7, 3, 0, 1)$	39 : $P_{172} = (3, 4, 1, 1)$	59 : $P_{276} = (3, 1, 3, 1)$

60 : $P_{281} = (0, 2, 3, 1)$
 61 : $P_{283} = (2, 2, 3, 1)$
 62 : $P_{284} = (3, 2, 3, 1)$
 63 : $P_{301} = (4, 4, 3, 1)$
 64 : $P_{328} = (7, 7, 3, 1)$
 65 : $P_{331} = (2, 0, 4, 1)$
 66 : $P_{333} = (4, 0, 4, 1)$
 67 : $P_{336} = (7, 0, 4, 1)$
 68 : $P_{347} = (2, 2, 4, 1)$
 69 : $P_{359} = (6, 3, 4, 1)$
 70 : $P_{362} = (1, 4, 4, 1)$
 71 : $P_{365} = (4, 4, 4, 1)$
 72 : $P_{370} = (1, 5, 4, 1)$
 73 : $P_{375} = (6, 5, 4, 1)$
 74 : $P_{377} = (0, 6, 4, 1)$
 75 : $P_{392} = (7, 7, 4, 1)$

76 : $P_{395} = (2, 0, 5, 1)$
 77 : $P_{397} = (4, 0, 5, 1)$
 78 : $P_{400} = (7, 0, 5, 1)$
 79 : $P_{406} = (5, 1, 5, 1)$
 80 : $P_{411} = (2, 2, 5, 1)$
 81 : $P_{425} = (0, 4, 5, 1)$
 82 : $P_{429} = (4, 4, 5, 1)$
 83 : $P_{430} = (5, 4, 5, 1)$
 84 : $P_{456} = (7, 7, 5, 1)$
 85 : $P_{459} = (2, 0, 6, 1)$
 86 : $P_{461} = (4, 0, 6, 1)$
 87 : $P_{464} = (7, 0, 6, 1)$
 88 : $P_{471} = (6, 1, 6, 1)$
 89 : $P_{475} = (2, 2, 6, 1)$
 90 : $P_{493} = (4, 4, 6, 1)$
 91 : $P_{513} = (0, 7, 6, 1)$

92 : $P_{519} = (6, 7, 6, 1)$
 93 : $P_{520} = (7, 7, 6, 1)$
 94 : $P_{523} = (2, 0, 7, 1)$
 95 : $P_{525} = (4, 0, 7, 1)$
 96 : $P_{528} = (7, 0, 7, 1)$
 97 : $P_{539} = (2, 2, 7, 1)$
 98 : $P_{545} = (0, 3, 7, 1)$
 99 : $P_{557} = (4, 4, 7, 1)$
 100 : $P_{564} = (3, 5, 7, 1)$
 101 : $P_{570} = (1, 6, 7, 1)$
 102 : $P_{572} = (3, 6, 7, 1)$
 103 : $P_{578} = (1, 7, 7, 1)$
 104 : $P_{584} = (7, 7, 7, 1)$