Rank-74052 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244172878

General information

Number of lines	2
Number of points	65
Number of singular points	1
Number of Eckardt points	0
Number of double points	1
Number of single points	16
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^2
Type of lines on points	2, 1^{16}, 0^{48}

Singular Points

The surface has 1 singular points:

$0: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4680} = \mathbf{Pl}(0,0,0,1,0,0)_{17}$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (4680, 4744)
Rank of points on Klein quadric: (17, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$

Single Points

The surface has 16 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_2 = (0,0,1,0)$ lies on line ℓ_1
2 : $P_{82} = (0,1,0,1)$ lies on line ℓ_0
3 : $P_{90} = (0,2,0,1)$ lies on line ℓ_0
4 : $P_{98} = (0,3,0,1)$ lies on line ℓ_0
5 : $P_{106} = (0,4,0,1)$ lies on line ℓ_0
6 : $P_{114} = (0,5,0,1)$ lies on line ℓ_0
7 : $P_{122} = (0,6,0,1)$ lies on line ℓ_0
8 : $P_{130} = (0,7,0,1)$ lies on line ℓ_0
9 : $P_{138} = (0,0,1,1)$ lies on line ℓ_1
10 : $P_{201} = (0,0,2,1)$ lies on line ℓ_1
11 : $P_{265} = (0,0,3,1)$ lies on line ℓ_1
12 : $P_{329} = (0,0,4,1)$ lies on line ℓ_1
13 : $P_{393} = (0,0,5,1)$ lies on line ℓ_1
14 : $P_{457} = (0,0,6,1)$ lies on line ℓ_1
15 : $P_{521} = (0,0,7,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:
The points on the surface but not on lines are:

0 : $P_{22} = (3,1,1,0)$
1 : $P_{24} = (5,1,1,0)$
2 : $P_{25} = (6,1,1,0)$
3 : $P_{30} = (3,2,1,0)$
4 : $P_{48} = (5,4,1,0)$
5 : $P_{73} = (6,7,1,0)$
6 : $P_{147} = (2,1,1,1)$
7 : $P_{149} = (4,1,1,1)$
8 : $P_{152} = (7,1,1,1)$
9 : $P_{163} = (2,3,1,1)$
10 : $P_{166} = (5,3,1,1)$
11 : $P_{167} = (6,3,1,1)$
12 : $P_{180} = (3,5,1,1)$
13 : $P_{181} = (4,5,1,1)$
14 : $P_{183} = (6,5,1,1)$
15 : $P_{188} = (3,6,1,1)$
16 : $P_{190} = (5,6,1,1)$
17 : $P_{192} = (7,6,1,1)$
18 : $P_{212} = (3,1,2,1)$
19 : $P_{218} = (1,2,2,1)$
20 : $P_{220} = (3,2,2,1)$
21 : $P_{226} = (1,3,2,1)$

22 : $P_{229} = (4, 3, 2, 1)$
23 : $P_{261} = (4, 7, 2, 1)$
24 : $P_{287} = (6, 2, 3, 1)$
25 : $P_{303} = (6, 4, 3, 1)$
26 : $P_{307} = (2, 5, 3, 1)$
27 : $P_{323} = (2, 7, 3, 1)$
28 : $P_{342} = (5, 1, 4, 1)$
29 : $P_{352} = (7, 2, 4, 1)$
30 : $P_{362} = (1, 4, 4, 1)$
31 : $P_{366} = (5, 4, 4, 1)$
32 : $P_{370} = (1, 5, 4, 1)$
33 : $P_{376} = (7, 5, 4, 1)$
34 : $P_{413} = (4, 2, 5, 1)$
35 : $P_{428} = (3, 4, 5, 1)$
36 : $P_{445} = (4, 6, 5, 1)$
37 : $P_{452} = (3, 7, 5, 1)$
38 : $P_{478} = (5, 2, 6, 1)$
39 : $P_{488} = (7, 3, 6, 1)$
40 : $P_{496} = (7, 4, 6, 1)$
41 : $P_{518} = (5, 7, 6, 1)$
42 : $P_{535} = (6, 1, 7, 1)$
43 : $P_{555} = (2, 4, 7, 1)$
44 : $P_{570} = (1, 6, 7, 1)$
45 : $P_{571} = (2, 6, 7, 1)$
46 : $P_{578} = (1, 7, 7, 1)$
47 : $P_{583} = (6, 7, 7, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_3

Line 1 intersects

Line	ℓ_0
in point	P_3

The surface has 65 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_{22} = (3, 1, 1, 0)$
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64 : $P_{583} = (6, 7, 7, 1)$