

Rank-74247 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1499026793

General information

Number of lines	5
Number of points	25
Number of singular points	1
Number of Eckardt points	0
Number of double points	6
Number of single points	13
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$2^6, 1^{13}, 0^6$

Singular Points

The surface has 1 singular points:

$$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{33} \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{199} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{133} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{133} = \mathbf{Pl}(2, 3, 2, 3, 3, 1)_{346} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega & 1 \end{bmatrix}_{153} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{153} = \mathbf{Pl}(3, 2, 3, 2, 2, 1)_{296}
\end{aligned}$$

Rank of lines: (0, 5, 110, 133, 153)

Rank of points on Klein quadric: (0, 33, 199, 346, 296)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 6 Double points:

The double points on the surface are:

$$\begin{aligned}
P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 & P_{48} &= (3, 2, 1, 1) = \ell_2 \cap \ell_4 \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_2 & P_{12} &= (1, 1, 1, 0) = \ell_3 \cap \ell_4 \\
P_{42} &= (0, 1, 1, 1) = \ell_1 \cap \ell_2 \\
P_{51} &= (2, 3, 1, 1) = \ell_2 \cap \ell_3
\end{aligned}$$

Single Points

The surface has 13 single points:

The single points on the surface are:

$$\begin{aligned}
0 : P_1 &= (0, 1, 0, 0) \text{ lies on line } \ell_0 & 7 : P_{43} &= (2, 1, 1, 1) \text{ lies on line } \ell_1 \\
1 : P_4 &= (1, 1, 1, 1) \text{ lies on line } \ell_1 & 8 : P_{44} &= (3, 1, 1, 1) \text{ lies on line } \ell_1 \\
2 : P_6 &= (2, 1, 0, 0) \text{ lies on line } \ell_0 & 9 : P_{54} &= (1, 0, 2, 1) \text{ lies on line } \ell_3 \\
3 : P_7 &= (3, 1, 0, 0) \text{ lies on line } \ell_0 & 10 : P_{57} &= (0, 1, 2, 1) \text{ lies on line } \ell_4 \\
4 : P_{33} &= (3, 2, 0, 1) \text{ lies on line } \ell_3 & 11 : P_{70} &= (1, 0, 3, 1) \text{ lies on line } \ell_4 \\
5 : P_{36} &= (2, 3, 0, 1) \text{ lies on line } \ell_4 & 12 : P_{73} &= (0, 1, 3, 1) \text{ lies on line } \ell_3 \\
6 : P_{39} &= (1, 0, 1, 1) \text{ lies on line } \ell_2
\end{aligned}$$

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:

The points on the surface but not on lines are:

$$\begin{aligned}
0 : P_3 &= (0, 0, 0, 1) \\
1 : P_{18} &= (3, 2, 1, 0) \\
2 : P_{21} &= (2, 3, 1, 0) \\
3 : P_{27} &= (1, 1, 0, 1)
\end{aligned}$$

$$\begin{aligned}
4 : P_{62} &= (1, 2, 2, 1) \\
5 : P_{82} &= (1, 3, 3, 1)
\end{aligned}$$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	0	0
2	1	1	0	1	1
3	0	0	1	0	1
4	0	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_{42}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_5	P_{42}	P_{51}	P_{48}

Line 3 intersects

Line	ℓ_2	ℓ_4
in point	P_{51}	P_{12}

Line 4 intersects

Line	ℓ_2	ℓ_3
in point	P_{48}	P_{12}

The surface has 25 points:

The points on the surface are:

$$\begin{aligned}
0 : P_0 &= (1, 0, 0, 0) \\
1 : P_1 &= (0, 1, 0, 0) \\
2 : P_3 &= (0, 0, 0, 1) \\
3 : P_4 &= (1, 1, 1, 1) \\
4 : P_5 &= (1, 1, 0, 0) \\
5 : P_6 &= (2, 1, 0, 0) \\
6 : P_7 &= (3, 1, 0, 0) \\
7 : P_{12} &= (1, 1, 1, 0) \\
8 : P_{18} &= (3, 2, 1, 0)
\end{aligned}$$

$$\begin{aligned}
9 : P_{21} &= (2, 3, 1, 0) \\
10 : P_{27} &= (1, 1, 0, 1) \\
11 : P_{33} &= (3, 2, 0, 1) \\
12 : P_{36} &= (2, 3, 0, 1) \\
13 : P_{39} &= (1, 0, 1, 1) \\
14 : P_{42} &= (0, 1, 1, 1) \\
15 : P_{43} &= (2, 1, 1, 1) \\
16 : P_{44} &= (3, 1, 1, 1) \\
17 : P_{48} &= (3, 2, 1, 1)
\end{aligned}$$

$$\begin{aligned}
18 : P_{51} &= (2, 3, 1, 1) \\
19 : P_{54} &= (1, 0, 2, 1) \\
20 : P_{57} &= (0, 1, 2, 1) \\
21 : P_{62} &= (1, 2, 2, 1) \\
22 : P_{70} &= (1, 0, 3, 1) \\
23 : P_{73} &= (0, 1, 3, 1) \\
24 : P_{82} &= (1, 3, 3, 1)
\end{aligned}$$