

Rank-65874 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244173462

General information

Number of lines	1
Number of points	73
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{1323}$$

Rank of lines: (666)

Rank of points on Klein quadric: (1323)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 9 single points:

The single points on the surface are:

- 0 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0
- 1 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_0
- 2 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_0
- 3 : $P_{156} = (3, 2, 1, 1)$ lies on line ℓ_0
- 4 : $P_{163} = (2, 3, 1, 1)$ lies on line ℓ_0

- 5 : $P_{174} = (5, 4, 1, 1)$ lies on line ℓ_0
- 6 : $P_{181} = (4, 5, 1, 1)$ lies on line ℓ_0
- 7 : $P_{192} = (7, 6, 1, 1)$ lies on line ℓ_0
- 8 : $P_{199} = (6, 7, 1, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line:

The points on the surface but not on lines are:

- | | |
|-------------------------------|-------------------------------|
| 0 : $P_{12} = (1, 0, 1, 0)$ | 22 : $P_{207} = (6, 0, 2, 1)$ |
| 1 : $P_{22} = (3, 1, 1, 0)$ | 23 : $P_{218} = (1, 2, 2, 1)$ |
| 2 : $P_{24} = (5, 1, 1, 0)$ | 24 : $P_{219} = (2, 2, 2, 1)$ |
| 3 : $P_{25} = (6, 1, 1, 0)$ | 25 : $P_{247} = (6, 5, 2, 1)$ |
| 4 : $P_{27} = (0, 2, 1, 0)$ | 26 : $P_{253} = (4, 6, 2, 1)$ |
| 5 : $P_{34} = (7, 2, 1, 0)$ | 27 : $P_{258} = (1, 7, 2, 1)$ |
| 6 : $P_{40} = (5, 3, 1, 0)$ | 28 : $P_{263} = (6, 7, 2, 1)$ |
| 7 : $P_{43} = (0, 4, 1, 0)$ | 29 : $P_{287} = (6, 2, 3, 1)$ |
| 8 : $P_{45} = (2, 4, 1, 0)$ | 30 : $P_{298} = (1, 4, 3, 1)$ |
| 9 : $P_{57} = (6, 5, 1, 0)$ | 31 : $P_{305} = (0, 5, 3, 1)$ |
| 10 : $P_{62} = (3, 6, 1, 0)$ | 32 : $P_{319} = (6, 6, 3, 1)$ |
| 11 : $P_{67} = (0, 7, 1, 0)$ | 33 : $P_{324} = (3, 7, 3, 1)$ |
| 12 : $P_{71} = (4, 7, 1, 0)$ | 34 : $P_{325} = (4, 7, 3, 1)$ |
| 13 : $P_{76} = (2, 0, 0, 1)$ | 35 : $P_{327} = (6, 7, 3, 1)$ |
| 14 : $P_{78} = (4, 0, 0, 1)$ | 36 : $P_{332} = (3, 0, 4, 1)$ |
| 15 : $P_{81} = (7, 0, 0, 1)$ | 37 : $P_{346} = (1, 2, 4, 1)$ |
| 16 : $P_{82} = (0, 1, 0, 1)$ | 38 : $P_{348} = (3, 2, 4, 1)$ |
| 17 : $P_{83} = (1, 1, 0, 1)$ | 39 : $P_{360} = (7, 3, 4, 1)$ |
| 18 : $P_{96} = (6, 2, 0, 1)$ | 40 : $P_{362} = (1, 4, 4, 1)$ |
| 19 : $P_{109} = (3, 4, 0, 1)$ | 41 : $P_{365} = (4, 4, 4, 1)$ |
| 20 : $P_{135} = (5, 7, 0, 1)$ | 42 : $P_{380} = (3, 6, 4, 1)$ |
| 21 : $P_{138} = (0, 0, 1, 1)$ | 43 : $P_{412} = (3, 2, 5, 1)$ |

44 : $P_{414} = (5, 2, 5, 1)$
 45 : $P_{416} = (7, 2, 5, 1)$
 46 : $P_{420} = (3, 3, 5, 1)$
 47 : $P_{428} = (3, 4, 5, 1)$
 48 : $P_{441} = (0, 6, 5, 1)$
 49 : $P_{450} = (1, 7, 5, 1)$
 50 : $P_{474} = (1, 2, 6, 1)$
 51 : $P_{481} = (0, 3, 6, 1)$
 52 : $P_{491} = (2, 4, 6, 1)$
 53 : $P_{494} = (5, 4, 6, 1)$
 54 : $P_{495} = (6, 4, 6, 1)$

55 : $P_{502} = (5, 5, 6, 1)$
 56 : $P_{518} = (5, 7, 6, 1)$
 57 : $P_{526} = (5, 0, 7, 1)$
 58 : $P_{550} = (5, 3, 7, 1)$
 59 : $P_{554} = (1, 4, 7, 1)$
 60 : $P_{558} = (5, 4, 7, 1)$
 61 : $P_{563} = (2, 5, 7, 1)$
 62 : $P_{578} = (1, 7, 7, 1)$
 63 : $P_{584} = (7, 7, 7, 1)$

Line Intersection Graph

$$\begin{array}{c|c} 0 & \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 73 points:

The points on the surface are:

0 : $P_5 = (1, 1, 0, 0)$	25 : $P_{156} = (3, 2, 1, 1)$	50 : $P_{365} = (4, 4, 4, 1)$
1 : $P_{12} = (1, 0, 1, 0)$	26 : $P_{163} = (2, 3, 1, 1)$	51 : $P_{380} = (3, 6, 4, 1)$
2 : $P_{22} = (3, 1, 1, 0)$	27 : $P_{174} = (5, 4, 1, 1)$	52 : $P_{412} = (3, 2, 5, 1)$
3 : $P_{24} = (5, 1, 1, 0)$	28 : $P_{181} = (4, 5, 1, 1)$	53 : $P_{414} = (5, 2, 5, 1)$
4 : $P_{25} = (6, 1, 1, 0)$	29 : $P_{192} = (7, 6, 1, 1)$	54 : $P_{416} = (7, 2, 5, 1)$
5 : $P_{27} = (0, 2, 1, 0)$	30 : $P_{199} = (6, 7, 1, 1)$	55 : $P_{420} = (3, 3, 5, 1)$
6 : $P_{34} = (7, 2, 1, 0)$	31 : $P_{207} = (6, 0, 2, 1)$	56 : $P_{428} = (3, 4, 5, 1)$
7 : $P_{40} = (5, 3, 1, 0)$	32 : $P_{218} = (1, 2, 2, 1)$	57 : $P_{441} = (0, 6, 5, 1)$
8 : $P_{43} = (0, 4, 1, 0)$	33 : $P_{219} = (2, 2, 2, 1)$	58 : $P_{450} = (1, 7, 5, 1)$
9 : $P_{45} = (2, 4, 1, 0)$	34 : $P_{247} = (6, 5, 2, 1)$	59 : $P_{474} = (1, 2, 6, 1)$
10 : $P_{57} = (6, 5, 1, 0)$	35 : $P_{253} = (4, 6, 2, 1)$	60 : $P_{481} = (0, 3, 6, 1)$
11 : $P_{62} = (3, 6, 1, 0)$	36 : $P_{258} = (1, 7, 2, 1)$	61 : $P_{491} = (2, 4, 6, 1)$
12 : $P_{67} = (0, 7, 1, 0)$	37 : $P_{263} = (6, 7, 2, 1)$	62 : $P_{494} = (5, 4, 6, 1)$
13 : $P_{71} = (4, 7, 1, 0)$	38 : $P_{287} = (6, 2, 3, 1)$	63 : $P_{495} = (6, 4, 6, 1)$
14 : $P_{76} = (2, 0, 0, 1)$	39 : $P_{298} = (1, 4, 3, 1)$	64 : $P_{502} = (5, 5, 6, 1)$
15 : $P_{78} = (4, 0, 0, 1)$	40 : $P_{305} = (0, 5, 3, 1)$	65 : $P_{518} = (5, 7, 6, 1)$
16 : $P_{81} = (7, 0, 0, 1)$	41 : $P_{319} = (6, 6, 3, 1)$	66 : $P_{526} = (5, 0, 7, 1)$
17 : $P_{82} = (0, 1, 0, 1)$	42 : $P_{324} = (3, 7, 3, 1)$	67 : $P_{550} = (5, 3, 7, 1)$
18 : $P_{83} = (1, 1, 0, 1)$	43 : $P_{325} = (4, 7, 3, 1)$	68 : $P_{554} = (1, 4, 7, 1)$
19 : $P_{96} = (6, 2, 0, 1)$	44 : $P_{327} = (6, 7, 3, 1)$	69 : $P_{558} = (5, 4, 7, 1)$
20 : $P_{109} = (3, 4, 0, 1)$	45 : $P_{332} = (3, 0, 4, 1)$	70 : $P_{563} = (2, 5, 7, 1)$
21 : $P_{135} = (5, 7, 0, 1)$	46 : $P_{346} = (1, 2, 4, 1)$	71 : $P_{578} = (1, 7, 7, 1)$
22 : $P_{138} = (0, 0, 1, 1)$	47 : $P_{348} = (3, 2, 4, 1)$	72 : $P_{584} = (7, 7, 7, 1)$
23 : $P_{139} = (1, 0, 1, 1)$	48 : $P_{360} = (7, 3, 4, 1)$	
24 : $P_{146} = (0, 1, 1, 1)$	49 : $P_{362} = (1, 4, 4, 1)$	