Rank-76355 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(8)$ is 1361613389

General information

Number of lines	5
Number of points	89
Number of singular points	0
Number of Eckardt points	1
Number of double points	3
Number of single points	36
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^5
Type of lines on points	$3, 2^3, 1^{36}, 0^{49}$

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{73} = \mathbf{Pl}(1,0,0,0,0,1)_{650}$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

Rank of lines: (0, 64, 73, 4744, 648)
Rank of points on Klein quadric: (0, 2, 650, 1, 10)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_{12} = (1,0,1,0) = \ell_1 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 36 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line ℓ_3
$1 : P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 : P_6 = (2,1,0,0)$ lies on line ℓ_0
$3 : P_7 = (3,1,0,0)$ lies on line ℓ_0
$4 : P_8 = (4,1,0,0)$ lies on line ℓ_0
$5 : P_9 = (5,1,0,0)$ lies on line ℓ_0
$6 : P_{10} = (6,1,0,0)$ lies on line ℓ_0
$7 : P_{11} = (7,1,0,0)$ lies on line ℓ_0
$8 : P_{13} = (2,0,1,0)$ lies on line ℓ_1
$9 : P_{14} = (3,0,1,0)$ lies on line ℓ_1
$10 : P_{15} = (4,0,1,0)$ lies on line ℓ_1
$11 : P_{16} = (5,0,1,0)$ lies on line ℓ_1
$12 : P_{17} = (6,0,1,0)$ lies on line ℓ_1
$13 : P_{18} = (7,0,1,0)$ lies on line ℓ_1
$14 : P_{20} = (1,1,1,0)$ lies on line ℓ_2
$15 : P_{28} = (1,2,1,0)$ lies on line ℓ_2
$16 : P_{36} = (1,3,1,0)$ lies on line ℓ_2
$17 : P_{44} = (1,4,1,0)$ lies on line ℓ_2
$18 : P_{52} = (1,5,1,0)$ lies on line ℓ_2

$19 : P_{60} = (1,6,1,0)$ lies on line ℓ_2
$20 : P_{68} = (1,7,1,0)$ lies on line ℓ_2
$21 : P_{75} = (1,0,0,1)$ lies on line ℓ_4
$22 : P_{138} = (0,0,1,1)$ lies on line ℓ_3
$23 : P_{139} = (1,0,1,1)$ lies on line ℓ_4
$24 : P_{201} = (0,0,2,1)$ lies on line ℓ_3
$25 : P_{202} = (1,0,2,1)$ lies on line ℓ_4
$26 : P_{265} = (0,0,3,1)$ lies on line ℓ_3
$27 : P_{266} = (1,0,3,1)$ lies on line ℓ_4
$28 : P_{329} = (0,0,4,1)$ lies on line ℓ_3
$29 : P_{330} = (1,0,4,1)$ lies on line ℓ_4
$30 : P_{393} = (0,0,5,1)$ lies on line ℓ_3
$31 : P_{394} = (1,0,5,1)$ lies on line ℓ_4
$32 : P_{457} = (0,0,6,1)$ lies on line ℓ_3
$33 : P_{458} = (1,0,6,1)$ lies on line ℓ_4
$34 : P_{521} = (0,0,7,1)$ lies on line ℓ_3
$35 : P_{522} = (1,0,7,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:
The points on the surface but not on lines are:

$0: P_{104} = (6,3,0,1)$
$1: P_{105} = (7,3,0,1)$
$2: P_{116} = (2,5,0,1)$
$3: P_{117} = (3,5,0,1)$
$4: P_{126} = (4,6,0,1)$
$5: P_{127} = (5,6,0,1)$
$6: P_{146} = (0,1,1,1)$
$7: P_{157} = (4,2,1,1)$
$8: P_{160} = (7,2,1,1)$
$9: P_{171} = (2,4,1,1)$
$10: P_{176} = (7,4,1,1)$
$11: P_{195} = (2,7,1,1)$
$12: P_{197} = (4,7,1,1)$
$13: P_{233} = (0,4,2,1)$
$14: P_{237} = (4,4,2,1)$
$15: P_{243} = (2,5,2,1)$
$16: P_{245} = (4,5,2,1)$
$17: P_{250} = (1,6,2,1)$
$18: P_{299} = (2,4,3,1)$
$19: P_{305} = (0,5,3,1)$
$20: P_{308} = (3,5,3,1)$
$21: P_{314} = (1,6,3,1)$
$22: P_{320} = (7,6,3,1)$
$23: P_{323} = (2,7,3,1)$
$24: P_{328} = (7,7,3,1)$

$25: P_{354} = (1,3,4,1)$
$26: P_{381} = (4,6,4,1)$
$27: P_{384} = (7,6,4,1)$
$28: P_{385} = (0,7,4,1)$
$29: P_{392} = (7,7,4,1)$
$30: P_{411} = (2,2,5,1)$
$31: P_{413} = (4,2,5,1)$
$32: P_{418} = (1,3,5,1)$
$33: P_{419} = (2,3,5,1)$
$34: P_{441} = (0,6,5,1)$
$35: P_{446} = (5,6,5,1)$
$36: P_{453} = (4,7,5,1)$
$37: P_{480} = (7,2,6,1)$
$38: P_{481} = (0,3,6,1)$
$39: P_{487} = (6,3,6,1)$
$40: P_{493} = (4,4,6,1)$
$41: P_{496} = (7,4,6,1)$
$42: P_{498} = (1,5,6,1)$
$43: P_{501} = (4,5,6,1)$
$44: P_{537} = (0,2,7,1)$
$45: P_{539} = (2,2,7,1)$
$46: P_{547} = (2,3,7,1)$
$47: P_{552} = (7,3,7,1)$
$48: P_{562} = (1,5,7,1)$

Line Intersection Graph

```
 |0 1 2 3 4
0|0 1 1 0 0
1|1 0 1 1 1
2|1 1 0 0 0
3|0 1 0 0 1
4|0 1 0 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_{12}	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_{12}

Line 3 intersects

Line	ℓ_1	ℓ_4
in point	P_2	P_2

3

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_2	P_2

The surface has 89 points:
The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$

$1 : P_1 = (0, 1, 0, 0)$

$2 : P_2 = (0, 0, 1, 0)$

$3 : P_3 = (0, 0, 0, 1)$

$4 : P_5 = (1, 1, 0, 0)$

$5 : P_6 = (2, 1, 0, 0)$

$6 : P_7 = (3, 1, 0, 0)$

$7 : P_8 = (4, 1, 0, 0)$

$8 : P_9 = (5, 1, 0, 0)$

$9 : P_{10} = (6, 1, 0, 0)$

$10 : P_{11} = (7, 1, 0, 0)$

$11 : P_{12} = (1, 0, 1, 0)$

$12 : P_{13} = (2, 0, 1, 0)$

$13 : P_{14} = (3, 0, 1, 0)$

$14 : P_{15} = (4, 0, 1, 0)$

$15 : P_{16} = (5, 0, 1, 0)$

$16 : P_{17} = (6, 0, 1, 0)$

$17 : P_{18} = (7, 0, 1, 0)$

$18 : P_{20} = (1, 1, 1, 0)$

$19 : P_{28} = (1, 2, 1, 0)$

$20 : P_{36} = (1, 3, 1, 0)$

$21 : P_{44} = (1, 4, 1, 0)$

$22 : P_{52} = (1, 5, 1, 0)$

$23 : P_{60} = (1, 6, 1, 0)$

$24 : P_{68} = (1, 7, 1, 0)$

$25 : P_{75} = (1, 0, 0, 1)$

$26 : P_{104} = (6, 3, 0, 1)$

$27 : P_{105} = (7, 3, 0, 1)$

$28 : P_{116} = (2, 5, 0, 1)$

$29 : P_{117} = (3, 5, 0, 1)$

$30 : P_{126} = (4, 6, 0, 1)$

$31 : P_{127} = (5, 6, 0, 1)$

$32 : P_{138} = (0, 0, 1, 1)$

$33 : P_{139} = (1, 0, 1, 1)$

$34 : P_{146} = (0, 1, 1, 1)$

$35 : P_{157} = (4, 2, 1, 1)$

$36 : P_{160} = (7, 2, 1, 1)$

$37 : P_{171} = (2, 4, 1, 1)$

$38 : P_{176} = (7, 4, 1, 1)$

$39 : P_{195} = (2, 7, 1, 1)$

$40 : P_{197} = (4, 7, 1, 1)$

$41 : P_{201} = (0, 0, 2, 1)$

$42 : P_{202} = (1, 0, 2, 1)$

$43 : P_{233} = (0, 4, 2, 1)$

$44 : P_{237} = (4, 4, 2, 1)$

$45 : P_{243} = (2, 5, 2, 1)$

$46 : P_{245} = (4, 5, 2, 1)$

$47 : P_{250} = (1, 6, 2, 1)$

$48 : P_{265} = (0, 0, 3, 1)$

$49 : P_{266} = (1, 0, 3, 1)$

$50 : P_{299} = (2, 4, 3, 1)$

$51 : P_{305} = (0, 5, 3, 1)$

$52 : P_{308} = (3, 5, 3, 1)$

$53 : P_{314} = (1, 6, 3, 1)$

$54 : P_{320} = (7, 6, 3, 1)$

$55 : P_{323} = (2, 7, 3, 1)$

$56 : P_{328} = (7, 7, 3, 1)$

$57 : P_{329} = (0, 0, 4, 1)$

$58 : P_{330} = (1, 0, 4, 1)$

$59 : P_{354} = (1, 3, 4, 1)$

$60 : P_{381} = (4, 6, 4, 1)$

$61 : P_{384} = (7, 6, 4, 1)$

$62 : P_{385} = (0, 7, 4, 1)$

$63 : P_{392} = (7, 7, 4, 1)$

$64 : P_{393} = (0, 0, 5, 1)$

$65 : P_{394} = (1, 0, 5, 1)$

$66 : P_{411} = (2, 2, 5, 1)$

$67 : P_{413} = (4, 2, 5, 1)$

$68 : P_{418} = (1, 3, 5, 1)$

$69 : P_{419} = (2, 3, 5, 1)$

$70 : P_{441} = (0, 6, 5, 1)$

$71 : P_{446} = (5, 6, 5, 1)$

$72 : P_{453} = (4, 7, 5, 1)$

$73 : P_{457} = (0, 0, 6, 1)$

$74 : P_{458} = (1, 0, 6, 1)$

$75 : P_{480} = (7, 2, 6, 1)$

$76 : P_{481} = (0, 3, 6, 1)$

$77 : P_{487} = (6, 3, 6, 1)$

$78 : P_{493} = (4, 4, 6, 1)$

$79 : P_{496} = (7, 4, 6, 1)$

$80 : P_{498} = (1, 5, 6, 1)$

$81 : P_{501} = (4, 5, 6, 1)$

$82 : P_{521} = (0, 0, 7, 1)$

$83 : P_{522} = (1, 0, 7, 1)$

$84 : P_{537} = (0, 2, 7, 1)$

$85 : P_{539} = (2, 2, 7, 1)$

$86 : P_{547} = (2, 3, 7, 1)$

$87 : P_{552} = (7, 3, 7, 1)$

$88 : P_{562} = (1, 5, 7, 1)$