Rank-73797 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1227395669

General information

Number of lines	3
Number of points	81
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	3, 1^{24}, 0^{56}

Singular Points

The surface has 1 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

Rank of lines: (64, 4744, 648)
Rank of points on Klein quadric: (2, 1, 10)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0).$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 24 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 : P_3 = (0,0,0,1)$ lies on line ℓ_1
$2 : P_{12} = (1,0,1,0)$ lies on line ℓ_0
$3 : P_{13} = (2,0,1,0)$ lies on line ℓ_0
$4 : P_{14} = (3,0,1,0)$ lies on line ℓ_0
$5 : P_{15} = (4,0,1,0)$ lies on line ℓ_0
$6 : P_{16} = (5,0,1,0)$ lies on line ℓ_0
$7 : P_{17} = (6,0,1,0)$ lies on line ℓ_0
$8 : P_{18} = (7,0,1,0)$ lies on line ℓ_0
$9 : P_{75} = (1,0,0,1)$ lies on line ℓ_2
$10 : P_{138} = (0,0,1,1)$ lies on line ℓ_1
$11 : P_{139} = (1,0,1,1)$ lies on line ℓ_2
$12 : P_{201} = (0,0,2,1)$ lies on line ℓ_1
$13 : P_{202} = (1,0,2,1)$ lies on line ℓ_2
$14 : P_{265} = (0,0,3,1)$ lies on line ℓ_1
$15 : P_{266} = (1,0,3,1)$ lies on line ℓ_2
$16 : P_{329} = (0,0,4,1)$ lies on line ℓ_1
$17 : P_{330} = (1,0,4,1)$ lies on line ℓ_2
$18 : P_{393} = (0,0,5,1)$ lies on line ℓ_1
$19 : P_{394} = (1,0,5,1)$ lies on line ℓ_2
$20 : P_{457} = (0,0,6,1)$ lies on line ℓ_1
$21 : P_{458} = (1,0,6,1)$ lies on line ℓ_2
$22 : P_{521} = (0,0,7,1)$ lies on line ℓ_1
$23 : P_{522} = (1,0,7,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line:
The points on the surface but not on lines are:

$0 : P_4 = (1,1,1,1)$
$1 : P_{20} = (1,1,1,0)$
$2 : P_{31} = (4,2,1,0)$
$3 : P_{40} = (5,3,1,0)$
$4 : P_{50} = (7,4,1,0)$
$5 : P_{57} = (6,5,1,0)$
$6 : P_{62} = (3,6,1,0)$
$7 : P_{69} = (2,7,1,0)$

8 : $P_{96} = (6, 2, 0, 1)$
9 : $P_{97} = (7, 2, 0, 1)$
10 : $P_{108} = (2, 4, 0, 1)$
11 : $P_{109} = (3, 4, 0, 1)$
12 : $P_{134} = (4, 7, 0, 1)$
13 : $P_{135} = (5, 7, 0, 1)$
14 : $P_{165} = (4, 3, 1, 1)$
15 : $P_{167} = (6, 3, 1, 1)$
16 : $P_{180} = (3, 5, 1, 1)$
17 : $P_{184} = (7, 5, 1, 1)$
18 : $P_{187} = (2, 6, 1, 1)$
19 : $P_{190} = (5, 6, 1, 1)$
20 : $P_{253} = (4, 6, 2, 1)$
21 : $P_{258} = (1, 7, 2, 1)$
22 : $P_{260} = (3, 7, 2, 1)$
23 : $P_{278} = (5, 1, 3, 1)$
24 : $P_{280} = (7, 1, 3, 1)$
25 : $P_{302} = (5, 4, 3, 1)$
26 : $P_{310} = (5, 5, 3, 1)$
27 : $P_{311} = (6, 5, 3, 1)$
28 : $P_{314} = (1, 6, 3, 1)$
29 : $P_{320} = (7, 6, 3, 1)$
30 : $P_{323} = (2, 7, 3, 1)$
31 : $P_{328} = (7, 7, 3, 1)$
32 : $P_{346} = (1, 2, 4, 1)$

33 : $P_{350} = (5, 2, 4, 1)$
34 : $P_{360} = (7, 3, 4, 1)$
35 : $P_{403} = (2, 1, 5, 1)$
36 : $P_{407} = (6, 1, 5, 1)$
37 : $P_{411} = (2, 2, 5, 1)$
38 : $P_{413} = (4, 2, 5, 1)$
39 : $P_{418} = (1, 3, 5, 1)$
40 : $P_{419} = (2, 3, 5, 1)$
41 : $P_{444} = (3, 6, 5, 1)$
42 : $P_{447} = (6, 6, 5, 1)$
43 : $P_{455} = (6, 7, 5, 1)$
44 : $P_{468} = (3, 1, 6, 1)$
45 : $P_{469} = (4, 1, 6, 1)$
46 : $P_{476} = (3, 2, 6, 1)$
47 : $P_{484} = (3, 3, 6, 1)$
48 : $P_{486} = (5, 3, 6, 1)$
49 : $P_{493} = (4, 4, 6, 1)$
50 : $P_{496} = (7, 4, 6, 1)$
51 : $P_{498} = (1, 5, 6, 1)$
52 : $P_{501} = (4, 5, 6, 1)$
53 : $P_{554} = (1, 4, 7, 1)$
54 : $P_{559} = (6, 4, 7, 1)$
55 : $P_{563} = (2, 5, 7, 1)$

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 81 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{12} = (1, 0, 1, 0)$
5 : $P_{13} = (2, 0, 1, 0)$
6 : $P_{14} = (3, 0, 1, 0)$

7 : $P_{15} = (4, 0, 1, 0)$
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