

Rank-331 over GF(32)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(32) is -2112846803

General information

Number of lines	35
Number of points	1089
Number of singular points	33
Number of Eckardt points	0
Number of double points	66
Number of single points	1023
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	33^{35}
Type of lines on points	$2^{66}, 1^{1023}$

Singular Points

The surface has 33 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{2082} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{3105} = \mathbf{P}(0, 0, \eta, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{4129} = \mathbf{P}(0, 0, \eta^{18}, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{5153} = \mathbf{P}(0, 0, \eta^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{6177} = \mathbf{P}(0, 0, \eta^5, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{7201} = \mathbf{P}(0, 0, \eta^{19}, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{8225} = \mathbf{P}(0, 0, \eta^{11}, 1) = \mathbf{P}(0, 0, 7, 1)$$

$$9 : P_{9249} = \mathbf{P}(0, 0, \eta^3, 1) = \mathbf{P}(0, 0, 8, 1)$$

$$10 : P_{10273} = \mathbf{P}(0, 0, \eta^{29}, 1) = \mathbf{P}(0, 0, 9, 1)$$

$$11 : P_{11297} = \mathbf{P}(0, 0, \eta^6, 1) = \mathbf{P}(0, 0, 10, 1)$$

$$12 : P_{12321} = \mathbf{P}(0, 0, \eta^{27}, 1) = \mathbf{P}(0, 0, 11, 1)$$

$$13 : P_{13345} = \mathbf{P}(0, 0, \eta^{20}, 1) = \mathbf{P}(0, 0, 12, 1)$$

$$14 : P_{14369} = \mathbf{P}(0, 0, \eta^8, 1) = \mathbf{P}(0, 0, 13, 1)$$

$$15 : P_{15393} = \mathbf{P}(0, 0, \eta^{12}, 1) = \mathbf{P}(0, 0, 14, 1)$$

$$16 : P_{16417} = \mathbf{P}(0, 0, \eta^{23}, 1) = \mathbf{P}(0, 0, 15, 1)$$

$$17 : P_{17441} = \mathbf{P}(0, 0, \eta^4, 1) = \mathbf{P}(0, 0, 16, 1)$$

$$\begin{aligned}
18 : P_{18465} &= \mathbf{P}(0, 0, \eta^{10}, 1) = \mathbf{P}(0, 0, 17, 1) \\
19 : P_{19489} &= \mathbf{P}(0, 0, \eta^{30}, 1) = \mathbf{P}(0, 0, 18, 1) \\
20 : P_{20513} &= \mathbf{P}(0, 0, \eta^{17}, 1) = \mathbf{P}(0, 0, 19, 1) \\
21 : P_{21537} &= \mathbf{P}(0, 0, \eta^7, 1) = \mathbf{P}(0, 0, 20, 1) \\
22 : P_{22561} &= \mathbf{P}(0, 0, \eta^{22}, 1) = \mathbf{P}(0, 0, 21, 1) \\
23 : P_{23585} &= \mathbf{P}(0, 0, \eta^{28}, 1) = \mathbf{P}(0, 0, 22, 1) \\
24 : P_{24609} &= \mathbf{P}(0, 0, \eta^{26}, 1) = \mathbf{P}(0, 0, 23, 1) \\
25 : P_{25633} &= \mathbf{P}(0, 0, \eta^{21}, 1) = \mathbf{P}(0, 0, 24, 1)
\end{aligned}$$

$$\begin{aligned}
26 : P_{26657} &= \mathbf{P}(0, 0, \eta^{25}, 1) = \mathbf{P}(0, 0, 25, 1) \\
27 : P_{27681} &= \mathbf{P}(0, 0, \eta^9, 1) = \mathbf{P}(0, 0, 26, 1) \\
28 : P_{28705} &= \mathbf{P}(0, 0, \eta^{16}, 1) = \mathbf{P}(0, 0, 27, 1) \\
29 : P_{29729} &= \mathbf{P}(0, 0, \eta^{13}, 1) = \mathbf{P}(0, 0, 28, 1) \\
30 : P_{30753} &= \mathbf{P}(0, 0, \eta^{14}, 1) = \mathbf{P}(0, 0, 29, 1) \\
31 : P_{31777} &= \mathbf{P}(0, 0, \eta^{24}, 1) = \mathbf{P}(0, 0, 30, 1) \\
32 : P_{32801} &= \mathbf{P}(0, 0, \eta^{15}, 1) = \mathbf{P}(0, 0, 31, 1)
\end{aligned}$$

The 35 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0 \\
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1024} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1082400} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{65} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1083424} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2082} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{2082} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{70562} \\
\ell_5 &= \begin{bmatrix} 1 & \eta^{30} & 0 & 0 \\ 0 & 0 & 1 & \eta^{29} \end{bmatrix}_{20059} = \begin{bmatrix} 1 & 18 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{bmatrix}_{20059} = \mathbf{Pl}(0, 0, 4, 9, 18, 1)_{627263} \\
\ell_6 &= \begin{bmatrix} 1 & \eta^{13} & 0 & 0 \\ 0 & 0 & 1 & \eta^{26} \end{bmatrix}_{30643} = \begin{bmatrix} 1 & 28 & 0 & 0 \\ 0 & 0 & 1 & 23 \end{bmatrix}_{30643} = \mathbf{Pl}(0, 0, 5, 23, 28, 1)_{954686} \\
\ell_7 &= \begin{bmatrix} 1 & \eta^{29} & 0 & 0 \\ 0 & 0 & 1 & \eta^{27} \end{bmatrix}_{10548} = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{10548} = \mathbf{Pl}(0, 0, 16, 11, 9, 1)_{333395} \\
\ell_8 &= \begin{bmatrix} 1 & \eta^{26} & 0 & 0 \\ 0 & 0 & 1 & \eta^{21} \end{bmatrix}_{25359} = \begin{bmatrix} 1 & 23 & 0 & 0 \\ 0 & 0 & 1 & 24 \end{bmatrix}_{25359} = \mathbf{Pl}(0, 0, 17, 24, 23, 1)_{791762} \\
\ell_9 &= \begin{bmatrix} 1 & \eta^{12} & 0 & 0 \\ 0 & 0 & 1 & \eta^{24} \end{bmatrix}_{15852} = \begin{bmatrix} 1 & 14 & 0 & 0 \\ 0 & 0 & 1 & 30 \end{bmatrix}_{15852} = \mathbf{Pl}(0, 0, 20, 30, 14, 1)_{497327} \\
\ell_{10} &= \begin{bmatrix} 1 & \eta^{20} & 0 & 0 \\ 0 & 0 & 1 & \eta^9 \end{bmatrix}_{13734} = \begin{bmatrix} 1 & 12 & 0 & 0 \\ 0 & 0 & 1 & 26 \end{bmatrix}_{13734} = \mathbf{Pl}(0, 0, 21, 26, 12, 1)_{431918} \\
\ell_{11} &= \begin{bmatrix} 1 & \eta^{28} & 0 & 0 \\ 0 & 0 & 1 & \eta^{25} \end{bmatrix}_{24303} = \begin{bmatrix} 1 & 22 & 0 & 0 \\ 0 & 0 & 1 & 25 \end{bmatrix}_{24303} = \mathbf{Pl}(0, 0, 10, 25, 22, 1)_{758585} \\
\ell_{12} &= \begin{bmatrix} 1 & \eta^2 & 0 & 0 \\ 0 & 0 & 1 & \eta^4 \end{bmatrix}_{5268} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}_{5268} = \mathbf{Pl}(0, 0, 11, 16, 4, 1)_{169400} \\
\ell_{13} &= \begin{bmatrix} 1 & \eta^{25} & 0 & 0 \\ 0 & 0 & 1 & \eta^{19} \end{bmatrix}_{27455} = \begin{bmatrix} 1 & 25 & 0 & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{27455} = \mathbf{Pl}(0, 0, 14, 6, 25, 1)_{857045} \\
\ell_{14} &= \begin{bmatrix} 1 & \eta^4 & 0 & 0 \\ 0 & 0 & 1 & \eta^8 \end{bmatrix}_{17949} = \begin{bmatrix} 1 & 16 & 0 & 0 \\ 0 & 0 & 1 & 13 \end{bmatrix}_{17949} = \mathbf{Pl}(0, 0, 15, 13, 16, 1)_{562484} \\
\ell_{15} &= \begin{bmatrix} 1 & \eta^{11} & 0 & 0 \\ 0 & 0 & 1 & \eta^{22} \end{bmatrix}_{8444} = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 21 \end{bmatrix}_{8444} = \mathbf{Pl}(0, 0, 26, 21, 7, 1)_{268553}
\end{aligned}$$

$$\begin{aligned}
\ell_{16} &= \begin{bmatrix} 1 & \eta^{23} & 0 & 0 \\ 0 & 0 & 1 & \eta^{15} \end{bmatrix}_{16910} = \begin{bmatrix} 1 & 15 & 0 & 0 \\ 0 & 0 & 1 & 31 \end{bmatrix}_{16910} = \mathbf{Pl}(0, 0, 27, 31, 15, 1)_{530504} \\
\ell_{17} &= \begin{bmatrix} 1 & \eta^{19} & 0 & 0 \\ 0 & 0 & 1 & \eta^7 \end{bmatrix}_{7386} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{bmatrix}_{7386} = \mathbf{Pl}(0, 0, 30, 20, 6, 1)_{236069} \\
\ell_{18} &= \begin{bmatrix} 1 & \eta^8 & 0 & 0 \\ 0 & 0 & 1 & \eta^{16} \end{bmatrix}_{14792} = \begin{bmatrix} 1 & 13 & 0 & 0 \\ 0 & 0 & 1 & 27 \end{bmatrix}_{14792} = \mathbf{Pl}(0, 0, 31, 27, 13, 1)_{465284} \\
\ell_{19} &= \begin{bmatrix} 1 & \eta^{27} & 0 & 0 \\ 0 & 0 & 1 & \eta^{23} \end{bmatrix}_{12666} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 15 \end{bmatrix}_{12666} = \mathbf{Pl}(0, 0, 13, 15, 11, 1)_{398678} \\
\ell_{20} &= \begin{bmatrix} 1 & \eta^{21} & 0 & 0 \\ 0 & 0 & 1 & \eta^{11} \end{bmatrix}_{26399} = \begin{bmatrix} 1 & 24 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}_{26399} = \mathbf{Pl}(0, 0, 12, 7, 24, 1)_{824183} \\
\ell_{21} &= \begin{bmatrix} 1 & \eta & 0 & 0 \\ 0 & 0 & 1 & \eta^2 \end{bmatrix}_{3142} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{3142} = \mathbf{Pl}(0, 0, 9, 4, 2, 1)_{103802} \\
\ell_{22} &= \begin{bmatrix} 1 & \eta^{14} & 0 & 0 \\ 0 & 0 & 1 & \eta^{28} \end{bmatrix}_{31699} = \begin{bmatrix} 1 & 29 & 0 & 0 \\ 0 & 0 & 1 & 22 \end{bmatrix}_{31699} = \mathbf{Pl}(0, 0, 8, 22, 29, 1)_{987611} \\
\ell_{23} &= \begin{bmatrix} 1 & \eta^{24} & 0 & 0 \\ 0 & 0 & 1 & \eta^{17} \end{bmatrix}_{32753} = \begin{bmatrix} 1 & 30 & 0 & 0 \\ 0 & 0 & 1 & 19 \end{bmatrix}_{32753} = \mathbf{Pl}(0, 0, 29, 19, 30, 1)_{1021670} \\
\ell_{24} &= \begin{bmatrix} 1 & \eta^9 & 0 & 0 \\ 0 & 0 & 1 & \eta^{18} \end{bmatrix}_{28509} = \begin{bmatrix} 1 & 26 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{28509} = \mathbf{Pl}(0, 0, 28, 3, 26, 1)_{890663} \\
\ell_{25} &= \begin{bmatrix} 1 & \eta^3 & 0 & 0 \\ 0 & 0 & 1 & \eta^6 \end{bmatrix}_{9490} = \begin{bmatrix} 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{9490} = \mathbf{Pl}(0, 0, 25, 10, 8, 1)_{301226} \\
\ell_{26} &= \begin{bmatrix} 1 & \eta^5 & 0 & 0 \\ 0 & 0 & 1 & \eta^{10} \end{bmatrix}_{6326} = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 17 \end{bmatrix}_{6326} = \mathbf{Pl}(0, 0, 24, 17, 5, 1)_{202955} \\
\ell_{27} &= \begin{bmatrix} 1 & \eta^{10} & 0 & 0 \\ 0 & 0 & 1 & \eta^{20} \end{bmatrix}_{19005} = \begin{bmatrix} 1 & 17 & 0 & 0 \\ 0 & 0 & 1 & 12 \end{bmatrix}_{19005} = \mathbf{Pl}(0, 0, 7, 12, 17, 1)_{594716} \\
\ell_{28} &= \begin{bmatrix} 1 & \eta^6 & 0 & 0 \\ 0 & 0 & 1 & \eta^{12} \end{bmatrix}_{11608} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 14 \end{bmatrix}_{11608} = \mathbf{Pl}(0, 0, 6, 14, 10, 1)_{365501} \\
\ell_{29} &= \begin{bmatrix} 1 & \eta^{22} & 0 & 0 \\ 0 & 0 & 1 & \eta^{13} \end{bmatrix}_{23249} = \begin{bmatrix} 1 & 21 & 0 & 0 \\ 0 & 0 & 1 & 28 \end{bmatrix}_{23249} = \mathbf{Pl}(0, 0, 3, 28, 21, 1)_{725408} \\
\ell_{30} &= \begin{bmatrix} 1 & \eta^{15} & 0 & 0 \\ 0 & 0 & 1 & \eta^{30} \end{bmatrix}_{33809} = \begin{bmatrix} 1 & 31 & 0 & 0 \\ 0 & 0 & 1 & 18 \end{bmatrix}_{33809} = \mathbf{Pl}(0, 0, 2, 18, 31, 1)_{1052705} \\
\ell_{31} &= \begin{bmatrix} 1 & \eta^{18} & 0 & 0 \\ 0 & 0 & 1 & \eta^5 \end{bmatrix}_{4200} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{4200} = \mathbf{Pl}(0, 0, 23, 5, 3, 1)_{137420} \\
\ell_{32} &= \begin{bmatrix} 1 & \eta^{17} & 0 & 0 \\ 0 & 0 & 1 & \eta^3 \end{bmatrix}_{21115} = \begin{bmatrix} 1 & 19 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{bmatrix}_{21115} = \mathbf{Pl}(0, 0, 22, 8, 19, 1)_{661133} \\
\ell_{33} &= \begin{bmatrix} 1 & \eta^7 & 0 & 0 \\ 0 & 0 & 1 & \eta^{14} \end{bmatrix}_{22193} = \begin{bmatrix} 1 & 20 & 0 & 0 \\ 0 & 0 & 1 & 29 \end{bmatrix}_{22193} = \mathbf{Pl}(0, 0, 19, 29, 20, 1)_{693680} \\
\ell_{34} &= \begin{bmatrix} 1 & \eta^{16} & 0 & 0 \\ 0 & 0 & 1 & \eta \end{bmatrix}_{29565} = \begin{bmatrix} 1 & 27 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{29565} = \mathbf{Pl}(0, 0, 18, 2, 27, 1)_{922769}
\end{aligned}$$

Rank of lines: (0, 1024, 1082400, 1083424, 2082, 20059, 30643, 10548, 25359, 15852, 13734, 24303, 5268, 27455, 17949, 8444, 16910, 7386, 14792, 12666, 26399, 3142, 31699, 32753, 28509, 9490, 6326, 19005, 11608, 23249, 33809, 4200, 21115, 22193, 29565)

Rank of points on Klein quadric: (0, 2, 65, 1, 70562, 627263, 954686, 333395, 791762, 497327, 431918, 758585, 169400, 857045, 562484, 268553, 530504, 236069, 465284, 398678, 824183, 103802, 987611, 1021670, 890663, 301226, 202955, 594716, 365501, 725408, 1052705, 137420, 661133, 693680, 922769)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 66 Double points:

The double points on the surface are:

$$\begin{aligned} P_0 &= (1, 0, 0, 0) = \ell_0 \cap \ell_1 \\ P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_2 \\ P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_4 \\ P_6 &= (2, 1, 0, 0) = \ell_0 \cap \ell_5 \\ P_7 &= (3, 1, 0, 0) = \ell_0 \cap \ell_6 \\ P_8 &= (4, 1, 0, 0) = \ell_0 \cap \ell_7 \\ P_9 &= (5, 1, 0, 0) = \ell_0 \cap \ell_8 \\ P_{10} &= (6, 1, 0, 0) = \ell_0 \cap \ell_9 \\ P_{11} &= (7, 1, 0, 0) = \ell_0 \cap \ell_{10} \\ P_{12} &= (8, 1, 0, 0) = \ell_0 \cap \ell_{11} \\ P_{13} &= (9, 1, 0, 0) = \ell_0 \cap \ell_{12} \\ P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_{13} \\ P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_{14} \\ P_{16} &= (12, 1, 0, 0) = \ell_0 \cap \ell_{15} \\ P_{17} &= (13, 1, 0, 0) = \ell_0 \cap \ell_{16} \\ P_{18} &= (14, 1, 0, 0) = \ell_0 \cap \ell_{17} \\ P_{19} &= (15, 1, 0, 0) = \ell_0 \cap \ell_{18} \\ P_{20} &= (16, 1, 0, 0) = \ell_0 \cap \ell_{19} \\ P_{21} &= (17, 1, 0, 0) = \ell_0 \cap \ell_{20} \\ P_{22} &= (18, 1, 0, 0) = \ell_0 \cap \ell_{21} \\ P_{23} &= (19, 1, 0, 0) = \ell_0 \cap \ell_{22} \\ P_{24} &= (20, 1, 0, 0) = \ell_0 \cap \ell_{23} \\ P_{25} &= (21, 1, 0, 0) = \ell_0 \cap \ell_{24} \\ P_{26} &= (22, 1, 0, 0) = \ell_0 \cap \ell_{25} \\ P_{27} &= (23, 1, 0, 0) = \ell_0 \cap \ell_{26} \\ P_{28} &= (24, 1, 0, 0) = \ell_0 \cap \ell_{27} \\ P_{29} &= (25, 1, 0, 0) = \ell_0 \cap \ell_{28} \\ P_{30} &= (26, 1, 0, 0) = \ell_0 \cap \ell_{29} \\ P_{31} &= (27, 1, 0, 0) = \ell_0 \cap \ell_{30} \\ P_{32} &= (28, 1, 0, 0) = \ell_0 \cap \ell_{31} \\ P_{33} &= (29, 1, 0, 0) = \ell_0 \cap \ell_{32} \\ P_{34} &= (30, 1, 0, 0) = \ell_0 \cap \ell_{33} \\ P_{35} &= (31, 1, 0, 0) = \ell_0 \cap \ell_{34} \\ P_2 &= (0, 0, 1, 0) = \ell_1 \cap \ell_3 \end{aligned}$$

$$\begin{aligned} P_3 &= (0, 0, 0, 1) = \ell_2 \cap \ell_3 \\ P_{2082} &= (0, 0, 1, 1) = \ell_3 \cap \ell_4 \\ P_{5153} &= (0, 0, 4, 1) = \ell_3 \cap \ell_5 \\ P_{6177} &= (0, 0, 5, 1) = \ell_3 \cap \ell_6 \\ P_{17441} &= (0, 0, 16, 1) = \ell_3 \cap \ell_7 \\ P_{18465} &= (0, 0, 17, 1) = \ell_3 \cap \ell_8 \\ P_{21537} &= (0, 0, 20, 1) = \ell_3 \cap \ell_9 \\ P_{22561} &= (0, 0, 21, 1) = \ell_3 \cap \ell_{10} \\ P_{11297} &= (0, 0, 10, 1) = \ell_3 \cap \ell_{11} \\ P_{12321} &= (0, 0, 11, 1) = \ell_3 \cap \ell_{12} \\ P_{15393} &= (0, 0, 14, 1) = \ell_3 \cap \ell_{13} \\ P_{16417} &= (0, 0, 15, 1) = \ell_3 \cap \ell_{14} \\ P_{27681} &= (0, 0, 26, 1) = \ell_3 \cap \ell_{15} \\ P_{28705} &= (0, 0, 27, 1) = \ell_3 \cap \ell_{16} \\ P_{31777} &= (0, 0, 30, 1) = \ell_3 \cap \ell_{17} \\ P_{32801} &= (0, 0, 31, 1) = \ell_3 \cap \ell_{18} \\ P_{14369} &= (0, 0, 13, 1) = \ell_3 \cap \ell_{19} \\ P_{13345} &= (0, 0, 12, 1) = \ell_3 \cap \ell_{20} \\ P_{10273} &= (0, 0, 9, 1) = \ell_3 \cap \ell_{21} \\ P_{9249} &= (0, 0, 8, 1) = \ell_3 \cap \ell_{22} \\ P_{30753} &= (0, 0, 29, 1) = \ell_3 \cap \ell_{23} \\ P_{29729} &= (0, 0, 28, 1) = \ell_3 \cap \ell_{24} \\ P_{26657} &= (0, 0, 25, 1) = \ell_3 \cap \ell_{25} \\ P_{25633} &= (0, 0, 24, 1) = \ell_3 \cap \ell_{26} \\ P_{8225} &= (0, 0, 7, 1) = \ell_3 \cap \ell_{27} \\ P_{7201} &= (0, 0, 6, 1) = \ell_3 \cap \ell_{28} \\ P_{4129} &= (0, 0, 3, 1) = \ell_3 \cap \ell_{29} \\ P_{3105} &= (0, 0, 2, 1) = \ell_3 \cap \ell_{30} \\ P_{24609} &= (0, 0, 23, 1) = \ell_3 \cap \ell_{31} \\ P_{23585} &= (0, 0, 22, 1) = \ell_3 \cap \ell_{32} \\ P_{20513} &= (0, 0, 19, 1) = \ell_3 \cap \ell_{33} \\ P_{19489} &= (0, 0, 18, 1) = \ell_3 \cap \ell_{34} \end{aligned}$$

Single Points

The surface has 1023 single points:

Too many to print.

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{18}	ℓ_{19}	ℓ_{20}	ℓ_{21}	ℓ_{22}
in point	P_0	P_1	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_2	P_3	P_{2082}	P_{5153}	P_{6177}	P_{17441}	P_{18465}	P_{21537}	P_{22561}	P_{11297}	P_{12321}	P_{15393}	P_{16417}	P_{27681}	P_{28705}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{2082}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{5153}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_7	P_{6177}

Line 7 intersects

Line	ℓ_0	ℓ_3
in point	P_8	P_{17441}

Line 8 intersects

Line	ℓ_0	ℓ_3
in point	P_9	P_{18465}

Line 9 intersects

Line	ℓ_0	ℓ_3
in point	P_{10}	P_{21537}

Line 10 intersects

Line	ℓ_0	ℓ_3
in point	P_{11}	P_{22561}

Line 11 intersects

Line	ℓ_0	ℓ_3
in point	P_{12}	P_{11297}

Line 12 intersects

Line	ℓ_0	ℓ_3
in point	P_{13}	P_{12321}

Line 13 intersects

Line	ℓ_0	ℓ_3
in point	P_{14}	P_{15393}

Line 14 intersects

Line	ℓ_0	ℓ_3
in point	P_{15}	P_{16417}

Line 15 intersects

Line	ℓ_0	ℓ_3
in point	P_{16}	P_{27681}

Line 16 intersects

Line	ℓ_0	ℓ_3
in point	P_{17}	P_{28705}

Line 17 intersects

Line	ℓ_0	ℓ_3
in point	P_{18}	P_{31777}

Line 18 intersects

Line	ℓ_0	ℓ_3
in point	P_{19}	P_{32801}

Line 19 intersects

Line	ℓ_0	ℓ_3
in point	P_{20}	P_{14369}

Line 20 intersects

Line	ℓ_0	ℓ_3
in point	P_{21}	P_{13345}

Line 21 intersects

Line	ℓ_0	ℓ_3
in point	P_{22}	P_{10273}

Line 22 intersects

Line	ℓ_0	ℓ_3
in point	P_{23}	P_{9249}

Line 23 intersects

Line	ℓ_0	ℓ_3
in point	P_{24}	P_{30753}

Line 24 intersects

Line	ℓ_0	ℓ_3
in point	P_{25}	P_{29729}

Line 25 intersects

Line	ℓ_0	ℓ_3
in point	P_{26}	P_{26657}

Line 26 intersects

Line	ℓ_0	ℓ_3
in point	P_{27}	P_{25633}

Line 27 intersects

Line	ℓ_0	ℓ_3
in point	P_{28}	P_{8225}

Line 28 intersects

Line	ℓ_0	ℓ_3
in point	P_{29}	P_{7201}

Line 29 intersects

Line	ℓ_0	ℓ_3
in point	P_{30}	P_{4129}

Line 30 intersects

Line	ℓ_0	ℓ_3
in point	P_{31}	P_{3105}

Line 31 intersects

Line	ℓ_0	ℓ_3
in point	P_{32}	P_{24609}

Line 32 intersects

Line	ℓ_0	ℓ_3
in point	P_{33}	P_{23585}

Line 33 intersects

Line	ℓ_0	ℓ_3
in point	P_{34}	P_{20513}

Line 34 intersects

Line	ℓ_0	ℓ_3
in point	P_{35}	P_{19489}

The surface has 1089 points:
Too many to print.