Rank-65744 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_3^3 + X_0^2X_3 + X_0X_1^2 + X_0X_1X_2 = 0$$

(1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 571548182

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	1
Number of double points	15
Number of single points	120
Number of points off lines	185
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3, 2^{15}, 1^{120}, 0^{185}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{69889} = \mathbf{Pl}(0,0,0,1,0,1)_{5121}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{69899} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{69899} = \mathbf{Pl}(0,0,0,11,0,1)_{5431}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{69898} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{69898} = \mathbf{Pl}(0,0,0,10,0,1)_{5400}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{4658} = \mathbf{Pl}(1,0,1,1,1,1)_{9427}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^{10} & \delta^{5} \end{bmatrix}_{459} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & 11 \end{bmatrix}_{459} = \mathbf{Pl}(10,11,11,0,11,1)_{49935}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^{5} & \delta^{10} \end{bmatrix}_{444} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{444} = \mathbf{Pl}(11,10,10,0,10,1)_{45841}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & \delta^{5} & 1 \end{bmatrix}_{7125} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 11 & 1 \end{bmatrix}_{7125} = \mathbf{Pl}(10,11,10,11,11,1)_{52710}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^{5} & 1 \\ 0 & 1 & \delta^{10} & 1 \end{bmatrix}_{7397} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 10 & 1 \end{bmatrix}_{7397} = \mathbf{Pl}(11,10,11,10,10,1)_{48856}$$

Rank of lines: (69889, 69899, 69898, 530, 4658, 459, 444, 7125, 7397)
Rank of points on Klein quadric: (5121, 5431, 5400, 9426, 9427, 49935, 45841, 52710, 48856)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 15 Double points:
The double points on the surface are:

$P_{530} = (0,0,1,1) = \ell_0 \cap \ell_3$
$P_{546} = (0,1,1,1) = \ell_0 \cap \ell_4$
$P_{3009} = (0,11,10,1) = \ell_1 \cap \ell_6$
$P_{2849} = (0,1,10,1) = \ell_1 \cap \ell_8$
$P_{3249} = (0,10,11,1) = \ell_2 \cap \ell_5$
$P_{3105} = (0,1,11,1) = \ell_2 \cap \ell_7$
$P_5 = (1,1,0,0) = \ell_3 \cap \ell_4$
$P_{699} = (10,10,1,1) = \ell_3 \cap \ell_5$
$P_{716} = (11,11,1,1) = \ell_3 \cap \ell_6$
$P_{715} = (10,11,1,1) = \ell_4 \cap \ell_7$
$P_{700} = (11,10,1,1) = \ell_4 \cap \ell_8$
$P_{20} = (1,0,1,0) = \ell_5 \cap \ell_6$
$P_{445} = (11,10,0,1) = \ell_5 \cap \ell_7$
$P_{460} = (10,11,0,1) = \ell_6 \cap \ell_8$
$P_{36} = (1,1,1,0) = \ell_7 \cap \ell_8$

Single Points

The surface has 120 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_3
$1 : P_{531} = (1,0,1,1)$ lies on line ℓ_4
$2 : P_{561} = (0,2,1,1)$ lies on line ℓ_0
$3 : P_{563} = (2,2,1,1)$ lies on line ℓ_3
$4 : P_{564} = (3,2,1,1)$ lies on line ℓ_4
$5 : P_{577} = (0,3,1,1)$ lies on line ℓ_0
$6 : P_{579} = (2,3,1,1)$ lies on line ℓ_4
$7 : P_{580} = (3,3,1,1)$ lies on line ℓ_3
$8 : P_{593} = (0,4,1,1)$ lies on line ℓ_0
$9 : P_{597} = (4,4,1,1)$ lies on line ℓ_3
$10 : P_{598} = (5,4,1,1)$ lies on line ℓ_4
$11 : P_{609} = (0,5,1,1)$ lies on line ℓ_0

12 : $P_{613} = (4, 5, 1, 1)$ lies on line ℓ_4
13 : $P_{614} = (5, 5, 1, 1)$ lies on line ℓ_3
14 : $P_{625} = (0, 6, 1, 1)$ lies on line ℓ_0
15 : $P_{631} = (6, 6, 1, 1)$ lies on line ℓ_3
16 : $P_{632} = (7, 6, 1, 1)$ lies on line ℓ_4
17 : $P_{641} = (0, 7, 1, 1)$ lies on line ℓ_0
18 : $P_{647} = (6, 7, 1, 1)$ lies on line ℓ_4
19 : $P_{648} = (7, 7, 1, 1)$ lies on line ℓ_3
20 : $P_{657} = (0, 8, 1, 1)$ lies on line ℓ_0
21 : $P_{665} = (8, 8, 1, 1)$ lies on line ℓ_3
22 : $P_{666} = (9, 8, 1, 1)$ lies on line ℓ_4
23 : $P_{673} = (0, 9, 1, 1)$ lies on line ℓ_0
24 : $P_{681} = (8, 9, 1, 1)$ lies on line ℓ_4
25 : $P_{682} = (9, 9, 1, 1)$ lies on line ℓ_3
26 : $P_{689} = (0, 10, 1, 1)$ lies on line ℓ_0
27 : $P_{705} = (0, 11, 1, 1)$ lies on line ℓ_0
28 : $P_{721} = (0, 12, 1, 1)$ lies on line ℓ_0
29 : $P_{733} = (12, 12, 1, 1)$ lies on line ℓ_3
30 : $P_{734} = (13, 12, 1, 1)$ lies on line ℓ_4
31 : $P_{737} = (0, 13, 1, 1)$ lies on line ℓ_0
32 : $P_{749} = (12, 13, 1, 1)$ lies on line ℓ_4
33 : $P_{750} = (13, 13, 1, 1)$ lies on line ℓ_3
34 : $P_{753} = (0, 14, 1, 1)$ lies on line ℓ_0
35 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_3
36 : $P_{768} = (15, 14, 1, 1)$ lies on line ℓ_4
37 : $P_{769} = (0, 15, 1, 1)$ lies on line ℓ_0
38 : $P_{783} = (14, 15, 1, 1)$ lies on line ℓ_4
39 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_3
40 : $P_{922} = (9, 8, 2, 1)$ lies on line ℓ_7
41 : $P_{937} = (8, 9, 2, 1)$ lies on line ℓ_8
42 : $P_{954} = (9, 10, 2, 1)$ lies on line ℓ_5
43 : $P_{969} = (8, 11, 2, 1)$ lies on line ℓ_6
44 : $P_{1178} = (9, 8, 3, 1)$ lies on line ℓ_8
45 : $P_{1193} = (8, 9, 3, 1)$ lies on line ℓ_7
46 : $P_{1209} = (8, 10, 3, 1)$ lies on line ℓ_5
47 : $P_{1226} = (9, 11, 3, 1)$ lies on line ℓ_6
48 : $P_{1472} = (15, 10, 4, 1)$ lies on line ℓ_5
49 : $P_{1487} = (14, 11, 4, 1)$ lies on line ℓ_6
50 : $P_{1536} = (15, 14, 4, 1)$ lies on line ℓ_7
51 : $P_{1551} = (14, 15, 4, 1)$ lies on line ℓ_8
52 : $P_{1727} = (14, 10, 5, 1)$ lies on line ℓ_5
53 : $P_{1744} = (15, 11, 5, 1)$ lies on line ℓ_6
54 : $P_{1792} = (15, 14, 5, 1)$ lies on line ℓ_8
55 : $P_{1807} = (14, 15, 5, 1)$ lies on line ℓ_7
56 : $P_{1982} = (13, 10, 6, 1)$ lies on line ℓ_5
57 : $P_{1997} = (12, 11, 6, 1)$ lies on line ℓ_6
58 : $P_{2014} = (13, 12, 6, 1)$ lies on line ℓ_7
59 : $P_{2029} = (12, 13, 6, 1)$ lies on line ℓ_8
60 : $P_{2237} = (12, 10, 7, 1)$ lies on line ℓ_5
61 : $P_{2254} = (13, 11, 7, 1)$ lies on line ℓ_6
62 : $P_{2270} = (13, 12, 7, 1)$ lies on line ℓ_8
63 : $P_{2285} = (12, 13, 7, 1)$ lies on line ℓ_7
64 : $P_{2356} = (3, 2, 8, 1)$ lies on line ℓ_7
65 : $P_{2371} = (2, 3, 8, 1)$ lies on line ℓ_8
66 : $P_{2484} = (3, 10, 8, 1)$ lies on line ℓ_5
67 : $P_{2499} = (2, 11, 8, 1)$ lies on line ℓ_6
68 : $P_{2612} = (3, 2, 9, 1)$ lies on line ℓ_8
69 : $P_{2627} = (2, 3, 9, 1)$ lies on line ℓ_7
70 : $P_{2739} = (2, 10, 9, 1)$ lies on line ℓ_5
71 : $P_{2756} = (3, 11, 9, 1)$ lies on line ℓ_6
72 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_1
73 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_7
74 : $P_{2865} = (0, 2, 10, 1)$ lies on line ℓ_1
75 : $P_{2881} = (0, 3, 10, 1)$ lies on line ℓ_1
76 : $P_{2897} = (0, 4, 10, 1)$ lies on line ℓ_1
77 : $P_{2913} = (0, 5, 10, 1)$ lies on line ℓ_1
78 : $P_{2929} = (0, 6, 10, 1)$ lies on line ℓ_1
79 : $P_{2945} = (0, 7, 10, 1)$ lies on line ℓ_1
80 : $P_{2961} = (0, 8, 10, 1)$ lies on line ℓ_1
81 : $P_{2977} = (0, 9, 10, 1)$ lies on line ℓ_1
82 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_1
83 : $P_{2994} = (1, 10, 10, 1)$ lies on line ℓ_5
84 : $P_{3025} = (0, 12, 10, 1)$ lies on line ℓ_1
85 : $P_{3041} = (0, 13, 10, 1)$ lies on line ℓ_1
86 : $P_{3057} = (0, 14, 10, 1)$ lies on line ℓ_1
87 : $P_{3073} = (0, 15, 10, 1)$ lies on line ℓ_1
88 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_2
89 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_8
90 : $P_{3121} = (0, 2, 11, 1)$ lies on line ℓ_2
91 : $P_{3137} = (0, 3, 11, 1)$ lies on line ℓ_2
92 : $P_{3153} = (0, 4, 11, 1)$ lies on line ℓ_2
93 : $P_{3169} = (0, 5, 11, 1)$ lies on line ℓ_2
94 : $P_{3185} = (0, 6, 11, 1)$ lies on line ℓ_2
95 : $P_{3201} = (0, 7, 11, 1)$ lies on line ℓ_2
96 : $P_{3217} = (0, 8, 11, 1)$ lies on line ℓ_2
97 : $P_{3233} = (0, 9, 11, 1)$ lies on line ℓ_2
98 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_2
99 : $P_{3266} = (1, 11, 11, 1)$ lies on line ℓ_6
100 : $P_{3281} = (0, 12, 11, 1)$ lies on line ℓ_2
101 : $P_{3297} = (0, 13, 11, 1)$ lies on line ℓ_2
102 : $P_{3313} = (0, 14, 11, 1)$ lies on line ℓ_2
103 : $P_{3329} = (0, 15, 11, 1)$ lies on line ℓ_2
104 : $P_{3448} = (7, 6, 12, 1)$ lies on line ℓ_7
105 : $P_{3463} = (6, 7, 12, 1)$ lies on line ℓ_8
106 : $P_{3512} = (7, 10, 12, 1)$ lies on line ℓ_5
107 : $P_{3527} = (6, 11, 12, 1)$ lies on line ℓ_6
108 : $P_{3704} = (7, 6, 13, 1)$ lies on line ℓ_8
109 : $P_{3719} = (6, 7, 13, 1)$ lies on line ℓ_7
110 : $P_{3767} = (6, 10, 13, 1)$ lies on line ℓ_5
111 : $P_{3784} = (7, 11, 13, 1)$ lies on line ℓ_6
112 : $P_{3926} = (5, 4, 14, 1)$ lies on line ℓ_7
113 : $P_{3941} = (4, 5, 14, 1)$ lies on line ℓ_8
114 : $P_{4022} = (5, 10, 14, 1)$ lies on line ℓ_5
115 : $P_{4037} = (4, 11, 14, 1)$ lies on line ℓ_6
116 : $P_{4182} = (5, 4, 15, 1)$ lies on line ℓ_8
117 : $P_{4197} = (4, 5, 15, 1)$ lies on line ℓ_7
118 : $P_{4277} = (4, 10, 15, 1)$ lies on line ℓ_5
119 : $P_{4294} = (5, 11, 15, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 185 points not on any line:
The points on the surface but not on lines are:

$0: P_{29} = (10, 0, 1, 0)$
$1: P_{30} = (11, 0, 1, 0)$
$2: P_{45} = (10, 1, 1, 0)$
$3: P_{46} = (11, 1, 1, 0)$
$4: P_{66} = (15, 2, 1, 0)$
$5: P_{82} = (15, 3, 1, 0)$
$6: P_{86} = (3, 4, 1, 0)$
$7: P_{102} = (3, 5, 1, 0)$
$8: P_{152} = (5, 8, 1, 0)$
$9: P_{168} = (5, 9, 1, 0)$
$10: P_{251} = (8, 14, 1, 0)$
$11: P_{267} = (8, 15, 1, 0)$
$12: P_{291} = (1, 1, 0, 1)$
$13: P_{314} = (8, 2, 0, 1)$
$14: P_{329} = (7, 3, 0, 1)$
$15: P_{353} = (15, 4, 0, 1)$
$16: P_{366} = (12, 5, 0, 1)$
$17: P_{408} = (6, 8, 0, 1)$
$18: P_{421} = (3, 9, 0, 1)$
$19: P_{438} = (4, 10, 0, 1)$
$20: P_{448} = (14, 10, 0, 1)$
$21: P_{452} = (2, 11, 0, 1)$
$22: P_{459} = (9, 11, 0, 1)$
$23: P_{503} = (5, 14, 0, 1)$
$24: P_{527} = (13, 15, 0, 1)$
$25: P_{852} = (3, 4, 2, 1)$
$26: P_{861} = (12, 4, 2, 1)$
$27: P_{863} = (14, 4, 2, 1)$
$28: P_{884} = (3, 6, 2, 1)$
$29: P_{893} = (12, 6, 2, 1)$
$30: P_{895} = (14, 6, 2, 1)$
$31: P_{982} = (5, 12, 2, 1)$
$32: P_{999} = (6, 13, 2, 1)$
$33: P_{1003} = (10, 13, 2, 1)$
$34: P_{1006} = (13, 13, 2, 1)$
$35: P_{1014} = (5, 14, 2, 1)$
$36: P_{1031} = (6, 15, 2, 1)$
$37: P_{1035} = (10, 15, 2, 1)$
$38: P_{1038} = (13, 15, 2, 1)$
$39: P_{1061} = (4, 1, 3, 1)$
$40: P_{1077} = (4, 2, 3, 1)$
$41: P_{1131} = (10, 5, 3, 1)$
$42: P_{1147} = (10, 6, 3, 1)$
$43: P_{1170} = (1, 8, 3, 1)$
$44: P_{1218} = (1, 11, 3, 1)$
$45: P_{1236} = (3, 12, 3, 1)$
$46: P_{1256} = (7, 13, 3, 1)$
$47: P_{1260} = (11, 13, 3, 1)$
$48: P_{1262} = (13, 13, 3, 1)$
$49: P_{1272} = (7, 14, 3, 1)$
$50: P_{1276} = (11, 14, 3, 1)$
$51: P_{1278} = (13, 14, 3, 1)$
$52: P_{1284} = (3, 15, 3, 1)$
$53: P_{1337} = (8, 2, 4, 1)$
$54: P_{1352} = (7, 3, 4, 1)$
$55: P_{1356} = (11, 3, 4, 1)$
$56: P_{1358} = (13, 3, 4, 1)$
$57: P_{1401} = (8, 6, 4, 1)$
$58: P_{1416} = (7, 7, 4, 1)$
$59: P_{1420} = (11, 7, 4, 1)$
$60: P_{1422} = (13, 7, 4, 1)$
$61: P_{1443} = (2, 9, 4, 1)$
$62: P_{1446} = (5, 9, 4, 1)$
$63: P_{1447} = (6, 9, 4, 1)$
$64: P_{1507} = (2, 13, 4, 1)$
$65: P_{1510} = (5, 13, 4, 1)$
$66: P_{1511} = (6, 13, 4, 1)$
$67: P_{1578} = (9, 1, 5, 1)$
$68: P_{1592} = (7, 2, 5, 1)$
$69: P_{1595} = (10, 2, 5, 1)$
$70: P_{1597} = (12, 2, 5, 1)$
$71: P_{1606} = (5, 3, 5, 1)$
$72: P_{1626} = (9, 4, 5, 1)$
$73: P_{1654} = (5, 6, 5, 1)$
$74: P_{1672} = (7, 7, 5, 1)$
$75: P_{1675} = (10, 7, 5, 1)$
$76: P_{1677} = (12, 7, 5, 1)$
$77: P_{1692} = (11, 8, 5, 1)$
$78: P_{1714} = (1, 10, 5, 1)$
$79: P_{1772} = (11, 13, 5, 1)$
$80: P_{1794} = (1, 15, 5, 1)$
$81: P_{1940} = (3, 8, 6, 1)$
$82: P_{1991} = (6, 11, 6, 1)$
$83: P_{1996} = (11, 11, 6, 1)$
$84: P_{2023} = (6, 13, 6, 1)$
$85: P_{2028} = (11, 13, 6, 1)$
$86: P_{2036} = (3, 14, 6, 1)$
$87: P_{2121} = (8, 3, 7, 1)$
$88: P_{2137} = (8, 4, 7, 1)$
$89: P_{2248} = (7, 11, 7, 1)$

$90: P_{2252} = (11, 11, 7, 1)$
$91: P_{2264} = (7, 12, 7, 1)$
$92: P_{2268} = (11, 12, 7, 1)$
$93: P_{2351} = (14, 1, 8, 1)$
$94: P_{2370} = (1, 3, 8, 1)$
$95: P_{2391} = (6, 4, 8, 1)$
$96: P_{2396} = (11, 4, 8, 1)$
$97: P_{2397} = (12, 4, 8, 1)$
$98: P_{2409} = (8, 5, 8, 1)$
$99: P_{2443} = (10, 7, 8, 1)$
$100: P_{2479} = (14, 9, 8, 1)$
$101: P_{2498} = (1, 11, 8, 1)$
$102: P_{2519} = (6, 12, 8, 1)$
$103: P_{2524} = (11, 12, 8, 1)$
$104: P_{2525} = (12, 12, 8, 1)$
$105: P_{2537} = (8, 13, 8, 1)$
$106: P_{2571} = (10, 15, 8, 1)$
$107: P_{2656} = (15, 4, 9, 1)$
$108: P_{2664} = (7, 5, 9, 1)$
$109: P_{2667} = (10, 5, 9, 1)$
$110: P_{2669} = (12, 5, 9, 1)$
$111: P_{2693} = (4, 7, 9, 1)$
$112: P_{2697} = (8, 7, 9, 1)$
$113: P_{2702} = (13, 7, 9, 1)$
$114: P_{2776} = (7, 12, 9, 1)$
$115: P_{2779} = (10, 12, 9, 1)$
$116: P_{2781} = (12, 12, 9, 1)$
$117: P_{2800} = (15, 13, 9, 1)$
$118: P_{2805} = (4, 14, 9, 1)$
$119: P_{2809} = (8, 14, 9, 1)$
$120: P_{2814} = (13, 14, 9, 1)$
$121: P_{2855} = (6, 1, 10, 1)$
$122: P_{2856} = (7, 1, 10, 1)$
$123: P_{2909} = (12, 4, 10, 1)$
$124: P_{2910} = (13, 4, 10, 1)$
$125: P_{2923} = (10, 5, 10, 1)$
$126: P_{2924} = (11, 5, 10, 1)$
$127: P_{3015} = (6, 11, 10, 1)$
$128: P_{3016} = (7, 11, 10, 1)$
$129: P_{3069} = (12, 14, 10, 1)$
$130: P_{3070} = (13, 14, 10, 1)$
$131: P_{3083} = (10, 15, 10, 1)$
$132: P_{3084} = (11, 15, 10, 1)$
$133: P_{3117} = (12, 1, 11, 1)$
$134: P_{3118} = (13, 1, 11, 1)$
$135: P_{3127} = (6, 2, 11, 1)$
$136: P_{3128} = (7, 2, 11, 1)$
$137: P_{3147} = (10, 3, 11, 1)$
$138: P_{3148} = (11, 3, 11, 1)$
$139: P_{3227} = (10, 8, 11, 1)$
$140: P_{3228} = (11, 8, 11, 1)$
$141: P_{3239} = (6, 9, 11, 1)$
$142: P_{3240} = (7, 9, 11, 1)$
$143: P_{3261} = (12, 10, 11, 1)$
$144: P_{3262} = (13, 10, 11, 1)$
$145: P_{3440} = (15, 5, 12, 1)$
$146: P_{3451} = (10, 6, 12, 1)$
$147: P_{3453} = (12, 6, 12, 1)$
$148: P_{3504} = (15, 9, 12, 1)$
$149: P_{3515} = (10, 10, 12, 1)$
$150: P_{3517} = (12, 10, 12, 1)$
$151: P_{3638} = (5, 2, 13, 1)$
$152: P_{3723} = (10, 7, 13, 1)$
$153: P_{3726} = (13, 7, 13, 1)$
$154: P_{3771} = (10, 10, 13, 1)$
$155: P_{3774} = (13, 10, 13, 1)$
$156: P_{3846} = (5, 15, 13, 1)$
$157: P_{3896} = (7, 2, 14, 1)$
$158: P_{3898} = (9, 2, 14, 1)$
$159: P_{3904} = (15, 2, 14, 1)$
$160: P_{3959} = (6, 6, 14, 1)$
$161: P_{3964} = (11, 6, 14, 1)$
$162: P_{3965} = (12, 6, 14, 1)$
$163: P_{3972} = (3, 7, 14, 1)$
$164: P_{3991} = (6, 8, 14, 1)$
$165: P_{3996} = (11, 8, 14, 1)$
$166: P_{3997} = (12, 8, 14, 1)$
$167: P_{4004} = (3, 9, 14, 1)$
$168: P_{4056} = (7, 12, 14, 1)$
$169: P_{4058} = (9, 12, 14, 1)$
$170: P_{4064} = (15, 12, 14, 1)$
$171: P_{4131} = (2, 1, 15, 1)$
$172: P_{4172} = (11, 3, 15, 1)$
$173: P_{4194} = (1, 5, 15, 1)$
$174: P_{4215} = (6, 6, 15, 1)$
$175: P_{4219} = (10, 6, 15, 1)$
$176: P_{4222} = (13, 6, 15, 1)$
$177: P_{4240} = (15, 7, 15, 1)$
$178: P_{4256} = (15, 8, 15, 1)$
$179: P_{4263} = (6, 9, 15, 1)$
$180: P_{4267} = (10, 9, 15, 1)$
$181: P_{4270} = (13, 9, 15, 1)$
$182: P_{4274} = (1, 10, 15, 1)$
$183: P_{4316} = (11, 12, 15, 1)$
$184: P_{4339} = (2, 14, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 1 0 0 0 0
1	1 0 1 0 0 0 1 0 1
2	1 1 0 0 0 1 0 1 0
3	1 0 0 0 1 1 1 0 0
4	1 0 0 1 0 0 0 1 1
5	0 0 1 1 0 0 1 1 0
6	0 1 0 1 0 1 0 0 1
7	0 0 1 0 1 1 0 0 1
8	0 1 0 0 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_1	P_1	P_{530}	P_{546}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8
in point	P_1	P_1	P_{3009}	P_{2849}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_1	P_1	P_{3249}	P_{3105}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6
in point	P_{530}	P_5	P_{699}	P_{716}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8
in point	P_{546}	P_5	P_{715}	P_{700}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{3249}	P_{699}	P_{20}	P_{445}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8
in point	P_{3009}	P_{716}	P_{20}	P_{460}

Line 7 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_8
in point	P_{3105}	P_{715}	P_{445}	P_{36}

Line 8 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_7
in point	P_{2849}	P_{700}	P_{460}	P_{36}

The surface has 321 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_{20} = (1, 0, 1, 0)$
4 : $P_{29} = (10, 0, 1, 0)$
5 : $P_{30} = (11, 0, 1, 0)$
6 : $P_{36} = (1, 1, 1, 0)$
7 : $P_{45} = (10, 1, 1, 0)$
8 : $P_{46} = (11, 1, 1, 0)$
9 : $P_{66} = (15, 2, 1, 0)$
10 : $P_{82} = (15, 3, 1, 0)$
11 : $P_{86} = (3, 4, 1, 0)$
12 : $P_{102} = (3, 5, 1, 0)$
13 : $P_{152} = (5, 8, 1, 0)$
14 : $P_{168} = (5, 9, 1, 0)$
15 : $P_{251} = (8, 14, 1, 0)$
16 : $P_{267} = (8, 15, 1, 0)$
17 : $P_{291} = (1, 1, 0, 1)$
18 : $P_{314} = (8, 2, 0, 1)$
19 : $P_{329} = (7, 3, 0, 1)$
20 : $P_{353} = (15, 4, 0, 1)$

21 : $P_{366} = (12, 5, 0, 1)$
22 : $P_{408} = (6, 8, 0, 1)$
23 : $P_{421} = (3, 9, 0, 1)$
24 : $P_{438} = (4, 10, 0, 1)$
25 : $P_{445} = (11, 10, 0, 1)$
26 : $P_{448} = (14, 10, 0, 1)$
27 : $P_{452} = (2, 11, 0, 1)$
28 : $P_{459} = (9, 11, 0, 1)$
29 : $P_{460} = (10, 11, 0, 1)$
30 : $P_{503} = (5, 14, 0, 1)$
31 : $P_{527} = (13, 15, 0, 1)$
32 : $P_{530} = (0, 0, 1, 1)$
33 : $P_{531} = (1, 0, 1, 1)$
34 : $P_{546} = (0, 1, 1, 1)$
35 : $P_{561} = (0, 2, 1, 1)$
36 : $P_{563} = (2, 2, 1, 1)$
37 : $P_{564} = (3, 2, 1, 1)$
38 : $P_{577} = (0, 3, 1, 1)$
39 : $P_{579} = (2, 3, 1, 1)$
40 : $P_{580} = (3, 3, 1, 1)$
41 : $P_{593} = (0, 4, 1, 1)$
42 : $P_{597} = (4, 4, 1, 1)$
43 : $P_{598} = (5, 4, 1, 1)$
44 : $P_{609} = (0, 5, 1, 1)$
45 : $P_{613} = (4, 5, 1, 1)$
46 : $P_{614} = (5, 5, 1, 1)$
47 : $P_{625} = (0, 6, 1, 1)$
48 : $P_{631} = (6, 6, 1, 1)$
49 : $P_{632} = (7, 6, 1, 1)$
50 : $P_{641} = (0, 7, 1, 1)$
51 : $P_{647} = (6, 7, 1, 1)$
52 : $P_{648} = (7, 7, 1, 1)$
53 : $P_{657} = (0, 8, 1, 1)$
54 : $P_{665} = (8, 8, 1, 1)$
55 : $P_{666} = (9, 8, 1, 1)$
56 : $P_{673} = (0, 9, 1, 1)$
57 : $P_{681} = (8, 9, 1, 1)$
58 : $P_{682} = (9, 9, 1, 1)$
59 : $P_{689} = (0, 10, 1, 1)$
60 : $P_{699} = (10, 10, 1, 1)$
61 : $P_{700} = (11, 10, 1, 1)$
62 : $P_{705} = (0, 11, 1, 1)$
63 : $P_{715} = (10, 11, 1, 1)$
64 : $P_{716} = (11, 11, 1, 1)$
65 : $P_{721} = (0, 12, 1, 1)$
66 : $P_{733} = (12, 12, 1, 1)$
67 : $P_{734} = (13, 12, 1, 1)$
68 : $P_{737} = (0, 13, 1, 1)$
69 : $P_{749} = (12, 13, 1, 1)$
70 : $P_{750} = (13, 13, 1, 1)$
71 : $P_{753} = (0, 14, 1, 1)$
72 : $P_{767} = (14, 14, 1, 1)$
73 : $P_{768} = (15, 14, 1, 1)$
74 : $P_{769} = (0, 15, 1, 1)$
75 : $P_{783} = (14, 15, 1, 1)$
76 : $P_{784} = (15, 15, 1, 1)$
77 : $P_{852} = (3, 4, 2, 1)$
78 : $P_{861} = (12, 4, 2, 1)$
79 : $P_{863} = (14, 4, 2, 1)$
80 : $P_{884} = (3, 6, 2, 1)$
81 : $P_{893} = (12, 6, 2, 1)$
82 : $P_{895} = (14, 6, 2, 1)$
83 : $P_{922} = (9, 8, 2, 1)$
84 : $P_{937} = (8, 9, 2, 1)$
85 : $P_{954} = (9, 10, 2, 1)$
86 : $P_{969} = (8, 11, 2, 1)$
87 : $P_{982} = (5, 12, 2, 1)$
88 : $P_{999} = (6, 13, 2, 1)$
89 : $P_{1003} = (10, 13, 2, 1)$
90 : $P_{1006} = (13, 13, 2, 1)$
91 : $P_{1014} = (5, 14, 2, 1)$
92 : $P_{1031} = (6, 15, 2, 1)$
93 : $P_{1035} = (10, 15, 2, 1)$
94 : $P_{1038} = (13, 15, 2, 1)$
95 : $P_{1061} = (4, 1, 3, 1)$
96 : $P_{1077} = (4, 2, 3, 1)$
97 : $P_{1131} = (10, 5, 3, 1)$
98 : $P_{1147} = (10, 6, 3, 1)$
99 : $P_{1170} = (1, 8, 3, 1)$
100 : $P_{1178} = (9, 8, 3, 1)$
101 : $P_{1193} = (8, 9, 3, 1)$
102 : $P_{1209} = (8, 10, 3, 1)$
103 : $P_{1218} = (1, 11, 3, 1)$
104 : $P_{1226} = (9, 11, 3, 1)$
105 : $P_{1236} = (3, 12, 3, 1)$
106 : $P_{1256} = (7, 13, 3, 1)$
107 : $P_{1260} = (11, 13, 3, 1)$
108 : $P_{1262} = (13, 13, 3, 1)$
109 : $P_{1272} = (7, 14, 3, 1)$
110 : $P_{1276} = (11, 14, 3, 1)$
111 : $P_{1278} = (13, 14, 3, 1)$
112 : $P_{1284} = (3, 15, 3, 1)$
113 : $P_{1337} = (8, 2, 4, 1)$
114 : $P_{1352} = (7, 3, 4, 1)$
115 : $P_{1356} = (11, 3, 4, 1)$
116 : $P_{1358} = (13, 3, 4, 1)$
117 : $P_{1401} = (8, 6, 4, 1)$
118 : $P_{1416} = (7, 7, 4, 1)$
119 : $P_{1420} = (11, 7, 4, 1)$
120 : $P_{1422} = (13, 7, 4, 1)$
121 : $P_{1443} = (2, 9, 4, 1)$
122 : $P_{1446} = (5, 9, 4, 1)$
123 : $P_{1447} = (6, 9, 4, 1)$
124 : $P_{1472} = (15, 10, 4, 1)$
125 : $P_{1487} = (14, 11, 4, 1)$
126 : $P_{1507} = (2, 13, 4, 1)$
127 : $P_{1510} = (5, 13, 4, 1)$
128 : $P_{1511} = (6, 13, 4, 1)$
129 : $P_{1536} = (15, 14, 4, 1)$
130 : $P_{1551} = (14, 15, 4, 1)$
131 : $P_{1578} = (9, 1, 5, 1)$
132 : $P_{1592} = (7, 2, 5, 1)$
133 : $P_{1595} = (10, 2, 5, 1)$
134 : $P_{1597} = (12, 2, 5, 1)$
135 : $P_{1606} = (5, 3, 5, 1)$
136 : $P_{1626} = (9, 4, 5, 1)$
137 : $P_{1654} = (5, 6, 5, 1)$
138 : $P_{1672} = (7, 7, 5, 1)$
139 : $P_{1675} = (10, 7, 5, 1)$
140 : $P_{1677} = (12, 7, 5, 1)$
141 : $P_{1692} = (11, 8, 5, 1)$
142 : $P_{1714} = (1, 10, 5, 1)$
143 : $P_{1727} = (14, 10, 5, 1)$
144 : $P_{1744} = (15, 11, 5, 1)$
145 : $P_{1772} = (11, 13, 5, 1)$
146 : $P_{1792} = (15, 14, 5, 1)$
147 : $P_{1794} = (1, 15, 5, 1)$
148 : $P_{1807} = (14, 15, 5, 1)$
149 : $P_{1940} = (3, 8, 6, 1)$
150 : $P_{1982} = (13, 10, 6, 1)$
151 : $P_{1991} = (6, 11, 6, 1)$
152 : $P_{1996} = (11, 11, 6, 1)$
153 : $P_{1997} = (12, 11, 6, 1)$
154 : $P_{2014} = (13, 12, 6, 1)$
155 : $P_{2023} = (6, 13, 6, 1)$
156 : $P_{2028} = (11, 13, 6, 1)$
157 : $P_{2029} = (12, 13, 6, 1)$
158 : $P_{2036} = (3, 14, 6, 1)$
159 : $P_{2121} = (8, 3, 7, 1)$
160 : $P_{2137} = (8, 4, 7, 1)$
161 : $P_{2237} = (12, 10, 7, 1)$
162 : $P_{2248} = (7, 11, 7, 1)$
163 : $P_{2252} = (11, 11, 7, 1)$
164 : $P_{2254} = (13, 11, 7, 1)$
165 : $P_{2264} = (7, 12, 7, 1)$
166 : $P_{2268} = (11, 12, 7, 1)$
167 : $P_{2270} = (13, 12, 7, 1)$
168 : $P_{2285} = (12, 13, 7, 1)$
169 : $P_{2351} = (14, 1, 8, 1)$
170 : $P_{2356} = (3, 2, 8, 1)$
171 : $P_{2370} = (1, 3, 8, 1)$
172 : $P_{2371} = (2, 3, 8, 1)$
173 : $P_{2391} = (6, 4, 8, 1)$
174 : $P_{2396} = (11, 4, 8, 1)$
175 : $P_{2397} = (12, 4, 8, 1)$
176 : $P_{2409} = (8, 5, 8, 1)$
177 : $P_{2443} = (10, 7, 8, 1)$
178 : $P_{2479} = (14, 9, 8, 1)$
179 : $P_{2484} = (3, 10, 8, 1)$
180 : $P_{2498} = (1, 11, 8, 1)$
181 : $P_{2499} = (2, 11, 8, 1)$
182 : $P_{2519} = (6, 12, 8, 1)$

183 : $P_{2524} = (11, 12, 8, 1)$
184 : $P_{2525} = (12, 12, 8, 1)$
185 : $P_{2537} = (8, 13, 8, 1)$
186 : $P_{2571} = (10, 15, 8, 1)$
187 : $P_{2612} = (3, 2, 9, 1)$
188 : $P_{2627} = (2, 3, 9, 1)$
189 : $P_{2656} = (15, 4, 9, 1)$
190 : $P_{2664} = (7, 5, 9, 1)$
191 : $P_{2667} = (10, 5, 9, 1)$
192 : $P_{2669} = (12, 5, 9, 1)$
193 : $P_{2693} = (4, 7, 9, 1)$
194 : $P_{2697} = (8, 7, 9, 1)$
195 : $P_{2702} = (13, 7, 9, 1)$
196 : $P_{2739} = (2, 10, 9, 1)$
197 : $P_{2756} = (3, 11, 9, 1)$
198 : $P_{2776} = (7, 12, 9, 1)$
199 : $P_{2779} = (10, 12, 9, 1)$
200 : $P_{2781} = (12, 12, 9, 1)$
201 : $P_{2800} = (15, 13, 9, 1)$
202 : $P_{2805} = (4, 14, 9, 1)$
203 : $P_{2809} = (8, 14, 9, 1)$
204 : $P_{2814} = (13, 14, 9, 1)$
205 : $P_{2833} = (0, 0, 10, 1)$
206 : $P_{2834} = (1, 0, 10, 1)$
207 : $P_{2849} = (0, 1, 10, 1)$
208 : $P_{2855} = (6, 1, 10, 1)$
209 : $P_{2856} = (7, 1, 10, 1)$
210 : $P_{2865} = (0, 2, 10, 1)$
211 : $P_{2881} = (0, 3, 10, 1)$
212 : $P_{2897} = (0, 4, 10, 1)$
213 : $P_{2909} = (12, 4, 10, 1)$
214 : $P_{2910} = (13, 4, 10, 1)$
215 : $P_{2913} = (0, 5, 10, 1)$
216 : $P_{2923} = (10, 5, 10, 1)$
217 : $P_{2924} = (11, 5, 10, 1)$
218 : $P_{2929} = (0, 6, 10, 1)$
219 : $P_{2945} = (0, 7, 10, 1)$
220 : $P_{2961} = (0, 8, 10, 1)$
221 : $P_{2977} = (0, 9, 10, 1)$
222 : $P_{2993} = (0, 10, 10, 1)$
223 : $P_{2994} = (1, 10, 10, 1)$
224 : $P_{3009} = (0, 11, 10, 1)$
225 : $P_{3015} = (6, 11, 10, 1)$
226 : $P_{3016} = (7, 11, 10, 1)$
227 : $P_{3025} = (0, 12, 10, 1)$
228 : $P_{3041} = (0, 13, 10, 1)$
229 : $P_{3057} = (0, 14, 10, 1)$
230 : $P_{3069} = (12, 14, 10, 1)$
231 : $P_{3070} = (13, 14, 10, 1)$
232 : $P_{3073} = (0, 15, 10, 1)$
233 : $P_{3083} = (10, 15, 10, 1)$
234 : $P_{3084} = (11, 15, 10, 1)$
235 : $P_{3089} = (0, 0, 11, 1)$
236 : $P_{3090} = (1, 0, 11, 1)$
237 : $P_{3105} = (0, 1, 11, 1)$
238 : $P_{3117} = (12, 1, 11, 1)$
239 : $P_{3118} = (13, 1, 11, 1)$
240 : $P_{3121} = (0, 2, 11, 1)$
241 : $P_{3127} = (6, 2, 11, 1)$
242 : $P_{3128} = (7, 2, 11, 1)$
243 : $P_{3137} = (0, 3, 11, 1)$
244 : $P_{3147} = (10, 3, 11, 1)$
245 : $P_{3148} = (11, 3, 11, 1)$
246 : $P_{3153} = (0, 4, 11, 1)$
247 : $P_{3169} = (0, 5, 11, 1)$
248 : $P_{3185} = (0, 6, 11, 1)$
249 : $P_{3201} = (0, 7, 11, 1)$
250 : $P_{3217} = (0, 8, 11, 1)$
251 : $P_{3227} = (10, 8, 11, 1)$
252 : $P_{3228} = (11, 8, 11, 1)$
253 : $P_{3233} = (0, 9, 11, 1)$
254 : $P_{3239} = (6, 9, 11, 1)$
255 : $P_{3240} = (7, 9, 11, 1)$
256 : $P_{3249} = (0, 10, 11, 1)$
257 : $P_{3261} = (12, 10, 11, 1)$
258 : $P_{3262} = (13, 10, 11, 1)$
259 : $P_{3265} = (0, 11, 11, 1)$
260 : $P_{3266} = (1, 11, 11, 1)$
261 : $P_{3281} = (0, 12, 11, 1)$
262 : $P_{3297} = (0, 13, 11, 1)$
263 : $P_{3313} = (0, 14, 11, 1)$
264 : $P_{3329} = (0, 15, 11, 1)$
265 : $P_{3440} = (15, 5, 12, 1)$
266 : $P_{3448} = (7, 6, 12, 1)$
267 : $P_{3451} = (10, 6, 12, 1)$
268 : $P_{3453} = (12, 6, 12, 1)$
269 : $P_{3463} = (6, 7, 12, 1)$
270 : $P_{3504} = (15, 9, 12, 1)$
271 : $P_{3512} = (7, 10, 12, 1)$
272 : $P_{3515} = (10, 10, 12, 1)$
273 : $P_{3517} = (12, 10, 12, 1)$
274 : $P_{3527} = (6, 11, 12, 1)$
275 : $P_{3638} = (5, 2, 13, 1)$
276 : $P_{3704} = (7, 6, 13, 1)$
277 : $P_{3719} = (6, 7, 13, 1)$
278 : $P_{3723} = (10, 7, 13, 1)$
279 : $P_{3726} = (13, 7, 13, 1)$
280 : $P_{3767} = (6, 10, 13, 1)$
281 : $P_{3771} = (10, 10, 13, 1)$
282 : $P_{3774} = (13, 10, 13, 1)$
283 : $P_{3784} = (7, 11, 13, 1)$
284 : $P_{3846} = (5, 15, 13, 1)$
285 : $P_{3896} = (7, 2, 14, 1)$
286 : $P_{3898} = (9, 2, 14, 1)$
287 : $P_{3904} = (15, 2, 14, 1)$
288 : $P_{3926} = (5, 4, 14, 1)$
289 : $P_{3941} = (4, 5, 14, 1)$
290 : $P_{3959} = (6, 6, 14, 1)$
291 : $P_{3964} = (11, 6, 14, 1)$
292 : $P_{3965} = (12, 6, 14, 1)$
293 : $P_{3972} = (3, 7, 14, 1)$
294 : $P_{3991} = (6, 8, 14, 1)$
295 : $P_{3996} = (11, 8, 14, 1)$
296 : $P_{3997} = (12, 8, 14, 1)$
297 : $P_{4004} = (3, 9, 14, 1)$
298 : $P_{4022} = (5, 10, 14, 1)$
299 : $P_{4037} = (4, 11, 14, 1)$
300 : $P_{4056} = (7, 12, 14, 1)$
301 : $P_{4058} = (9, 12, 14, 1)$
302 : $P_{4064} = (15, 12, 14, 1)$
303 : $P_{4131} = (2, 1, 15, 1)$
304 : $P_{4172} = (11, 3, 15, 1)$
305 : $P_{4182} = (5, 4, 15, 1)$
306 : $P_{4194} = (1, 5, 15, 1)$
307 : $P_{4197} = (4, 5, 15, 1)$
308 : $P_{4215} = (6, 6, 15, 1)$
309 : $P_{4219} = (10, 6, 15, 1)$
310 : $P_{4222} = (13, 6, 15, 1)$
311 : $P_{4240} = (15, 7, 15, 1)$
312 : $P_{4256} = (15, 8, 15, 1)$
313 : $P_{4263} = (6, 9, 15, 1)$
314 : $P_{4267} = (10, 9, 15, 1)$
315 : $P_{4270} = (13, 9, 15, 1)$
316 : $P_{4274} = (1, 10, 15, 1)$
317 : $P_{4277} = (4, 10, 15, 1)$
318 : $P_{4294} = (5, 11, 15, 1)$
319 : $P_{4316} = (11, 12, 15, 1)$
320 : $P_{4339} = (2, 14, 15, 1)$