Rank-73753 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

 $(\ 0,\ 1,\ 1,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0)$

The point rank of the equation over GF(8) is 1227137685

General information

Number of lines	1
Number of points	73
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	64
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{64}$

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4689} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{25}$$

Rank of lines: (4689)

Rank of points on Klein quadric: (25)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

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0: P_3 = (0, 0, 0, 1) lies on line \ell_0

1: P_{19} = (0, 1, 1, 0) lies on line \ell_0

2: P_{146} = (0, 1, 1, 1) lies on line \ell_0

3: P_{217} = (0, 2, 2, 1) lies on line \ell_0

4: P_{289} = (0, 3, 3, 1) lies on line \ell_0
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The single points on the surface are:

Points on surface but on no line

The surface has 64 points not on any line: The points on the surface but not on lines are:

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0: P_0 = (1,0,0,0)
1: P_5 = (1, 1, 0, 0)
2: P_{20} = (1, 1, 1, 0)
3: P_{99} = (1, 3, 0, 1)
4: P_{103} = (5, 3, 0, 1)
5: P_{115} = (1, 5, 0, 1)
6: P_{120} = (6, 5, 0, 1)
7: P_{123} = (1, 6, 0, 1)
8: P_{125} = (3, 6, 0, 1)
9: P_{139} = (1,0,1,1)
10: P_{206} = (5, 0, 2, 1)
11: P_{214} = (5, 1, 2, 1)
12: P_{215} = (6, 1, 2, 1)
13: P_{221} = (4, 2, 2, 1)
14: P_{239} = (6, 4, 2, 1)
15: P_{240} = (7, 4, 2, 1)
16: P_{244} = (3, 5, 2, 1)
17: P_{247} = (6, 5, 2, 1)
18: P_{251} = (2, 6, 2, 1)
19: P_{267} = (2, 0, 3, 1)
20: P_{274} = (1, 1, 3, 1)
21: P_{276} = (3, 1, 3, 1)
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22: P_{296} = (7,3,3,1)
23: P_{298} = (1,4,3,1)
24: P_{306} = (1, 5, 3, 1)
25: P_{310} = (5, 5, 3, 1)
26: P_{323} = (2,7,3,1)
27: P_{325} = (4,7,3,1)
28: P_{335} = (6,0,4,1)
29: P_{340} = (3, 1, 4, 1)
30: P_{343} = (6, 1, 4, 1)
31: P_{357} = (4, 3, 4, 1)
32: P_{368} = (7, 4, 4, 1)
33: P_{380} = (3, 6, 4, 1)
34: P_{382} = (5, 6, 4, 1)
35: P_{387} = (2,7,4,1)
36: P_{388} = (3,7,4,1)
37: P_{397} = (4,0,5,1)
38: P_{402} = (1, 1, 5, 1)
39: P_{406} = (5, 1, 5, 1)
40: P_{413} = (4, 2, 5, 1)
41: P_{416} = (7, 2, 5, 1)
42: P_{435} = (2, 5, 5, 1)
43: P_{442} = (1, 6, 5, 1)
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5: $P_{361} = (0, 4, 4, 1)$ lies on line ℓ_0

6: $P_{433} = (0, 5, 5, 1)$ lies on line ℓ_0

7: $P_{505} = (0, 6, 6, 1)$ lies on line ℓ_0

8: $P_{577} = (0,7,7,1)$ lies on line ℓ_0

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44: P_{447} = (6, 6, 5, 1)
                                                                   55: P_{524} = (3, 0, 7, 1)
45 : P_{450} = (1, 7, 5, 1)
                                                                   56: P_{532} = (3, 1, 7, 1)
46: P_{464} = (7, 0, 6, 1)
                                                                   57: P_{534} = (5, 1, 7, 1)
47: P_{466} = (1, 1, 6, 1)
                                                                   58: P_{541} = (4, 2, 7, 1)
                                                                   59: P_{542} = (5, 2, 7, 1)
48: P_{471} = (6, 1, 6, 1)
49: P_{474} = (1, 2, 6, 1)
                                                                   60: P_{550} = (5, 3, 7, 1)
50: P_{482} = (1, 3, 6, 1)
                                                                   61: P_{551} = (6, 3, 7, 1)
51: P_{484} = (3, 3, 6, 1)
                                                                   62: P_{568} = (7, 5, 7, 1)
52: P_{491} = (2, 4, 6, 1)
                                                                   63: P_{579} = (2, 7, 7, 1)
53: P_{496} = (7, 4, 6, 1)
54: P_{509} = (4, 6, 6, 1)
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Line Intersection Graph

 $\begin{array}{c|c} 0 \\ \hline 0 & 0 \end{array}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 73 points:

The points on the surface are:

$0: P_0 = (1, 0, 0, 0)$	$25: P_{276} = (3, 1, 3, 1)$	$50: P_{442} = (1, 6, 5, 1)$
$1: P_3 = (0,0,0,1)$	$26: P_{289} = (0, 3, 3, 1)$	$51: P_{447} = (6, 6, 5, 1)$
$2: P_5 = (1, 1, 0, 0)$	$27: P_{296} = (7, 3, 3, 1)$	$52: P_{450} = (1,7,5,1)$
$3: P_{19} = (0, 1, 1, 0)$	$28: P_{298} = (1,4,3,1)$	$53: P_{464} = (7,0,6,1)$
$4: P_{20} = (1, 1, 1, 0)$	$29: P_{306} = (1, 5, 3, 1)$	$54: P_{466} = (1, 1, 6, 1)$
$5: P_{99} = (1, 3, 0, 1)$	$30: P_{310} = (5, 5, 3, 1)$	$55: P_{471} = (6, 1, 6, 1)$
$6: P_{103} = (5, 3, 0, 1)$	$31: P_{323} = (2,7,3,1)$	$56: P_{474} = (1, 2, 6, 1)$
$7: P_{115} = (1, 5, 0, 1)$	$32: P_{325} = (4,7,3,1)$	$57: P_{482} = (1, 3, 6, 1)$
$8: P_{120} = (6, 5, 0, 1)$	$33: P_{335} = (6,0,4,1)$	$58: P_{484} = (3, 3, 6, 1)$
$9: P_{123} = (1, 6, 0, 1)$	$34: P_{340} = (3, 1, 4, 1)$	$59: P_{491} = (2,4,6,1)$
$10: P_{125} = (3, 6, 0, 1)$	$35: P_{343} = (6, 1, 4, 1)$	$60: P_{496} = (7, 4, 6, 1)$
$11: P_{139} = (1,0,1,1)$	$36: P_{357} = (4, 3, 4, 1)$	$61: P_{505} = (0, 6, 6, 1)$
$12: P_{146} = (0, 1, 1, 1)$	$37: P_{361} = (0, 4, 4, 1)$	$62: P_{509} = (4, 6, 6, 1)$
13: $P_{206} = (5, 0, 2, 1)$	$38: P_{368} = (7, 4, 4, 1)$	$63: P_{524} = (3, 0, 7, 1)$
$14: P_{214} = (5, 1, 2, 1)$	$39: P_{380} = (3, 6, 4, 1)$	$64: P_{532} = (3, 1, 7, 1)$
$15: P_{215} = (6, 1, 2, 1)$	$40: P_{382} = (5, 6, 4, 1)$	$65: P_{534} = (5, 1, 7, 1)$
$16: P_{217} = (0, 2, 2, 1)$	$41: P_{387} = (2,7,4,1)$	$66: P_{541} = (4, 2, 7, 1)$
$17: P_{221} = (4, 2, 2, 1)$	$42: P_{388} = (3,7,4,1)$	$67: P_{542} = (5, 2, 7, 1)$
$18: P_{239} = (6, 4, 2, 1)$	$43: P_{397} = (4, 0, 5, 1)$	$68: P_{550} = (5, 3, 7, 1)$
$19: P_{240} = (7, 4, 2, 1)$	$44: P_{402} = (1, 1, 5, 1)$	$69: P_{551} = (6, 3, 7, 1)$
$20: P_{244} = (3, 5, 2, 1)$	$45: P_{406} = (5, 1, 5, 1)$	$70: P_{568} = (7, 5, 7, 1)$
$21: P_{247} = (6, 5, 2, 1)$	$46: P_{413} = (4, 2, 5, 1)$	$71: P_{577} = (0,7,7,1)$
$22: P_{251} = (2, 6, 2, 1)$	$47: P_{416} = (7, 2, 5, 1)$	$72: P_{579} = (2, 7, 7, 1)$
$23: P_{267} = (2, 0, 3, 1)$	$48: P_{433} = (0, 5, 5, 1)$	
$24: P_{274} = (1, 1, 3, 1)$	$49: P_{435} = (2, 5, 5, 1)$	