

Rank-65540 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65540

General information

| | |
|----------------------------|---------------|
| Number of lines | 8 |
| Number of points | 9 |
| Number of singular points | 5 |
| Number of Eckardt points | 6 |
| Number of double points | 0 |
| Number of single points | 2 |
| Number of points off lines | 0 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | 3^8 |
| Type of lines on points | $4, 3^6, 1^2$ |

Singular Points

The surface has 5 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$2 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$3 : P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$$

$$4 : P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19}$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{29} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{25} \\
\ell_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_4 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21} \\
\ell_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_6 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{32} = \mathbf{Pl}(0, 1, 0, 1, 0, 1)_{27} \\
\ell_7 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}
\end{aligned}$$

Rank of lines: (28, 30, 29, 34, 31, 33, 32, 27)

Rank of points on Klein quadric: (19, 5, 25, 1, 21, 7, 27, 17)

Eckardt Points

The surface has 6 Eckardt points:

- 0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$, $T = -1$
- 1 : $P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$, $T = -1$
- 2 : $P_7 = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$, $T = 14$
- 3 : $P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1)$, $T = -1$
- 4 : $P_{12} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$, $T = -1$
- 5 : $P_{14} = \mathbf{P}(0, 1, 1, 1) = \mathbf{P}(0, 1, 1, 1)$, $T = 14$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 2 single points:

The single points on the surface are:

- 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_7
- 1 : $P_8 = (1, 1, 1, 0)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |
| 5 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 7 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |

Neighbor sets in the line intersection graph:

Line 0 intersects

| Line | ℓ_1 | ℓ_2 | ℓ_3 | ℓ_4 | ℓ_5 | ℓ_6 |
|----------|----------|----------|----------|----------|----------|----------|
| in point | P_1 | P_1 | P_2 | P_2 | P_7 | P_7 |

Line 1 intersects

| Line | ℓ_0 | ℓ_2 | ℓ_3 | ℓ_4 | ℓ_5 | ℓ_6 | ℓ_7 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| in point | P_1 | P_1 | P_3 | P_{10} | P_3 | P_{10} | P_3 |

Line 2 intersects

| Line | ℓ_0 | ℓ_1 | ℓ_3 | ℓ_4 | ℓ_5 | ℓ_6 |
|----------|----------|----------|----------|----------|----------|----------|
| in point | P_1 | P_1 | P_{12} | P_{14} | P_{14} | P_{12} |

Line 3 intersects

| Line | ℓ_0 | ℓ_1 | ℓ_2 | ℓ_4 | ℓ_5 | ℓ_6 | ℓ_7 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| in point | P_2 | P_3 | P_{12} | P_2 | P_3 | P_{12} | P_3 |

Line 4 intersects

| Line | ℓ_0 | ℓ_1 | ℓ_2 | ℓ_3 | ℓ_5 | ℓ_6 |
|----------|----------|----------|----------|----------|----------|----------|
| in point | P_2 | P_{10} | P_{14} | P_2 | P_{14} | P_{10} |

Line 5 intersects

| Line | ℓ_0 | ℓ_1 | ℓ_2 | ℓ_3 | ℓ_4 | ℓ_6 | ℓ_7 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| in point | P_7 | P_3 | P_{14} | P_3 | P_{14} | P_7 | P_3 |

Line 6 intersects

| Line | ℓ_0 | ℓ_1 | ℓ_2 | ℓ_3 | ℓ_4 | ℓ_5 |
|----------|----------|----------|----------|----------|----------|----------|
| in point | P_7 | P_{10} | P_{12} | P_{12} | P_{10} | P_7 |

Line 7 intersects

| Line | ℓ_1 | ℓ_3 | ℓ_5 |
|----------|----------|----------|----------|
| in point | P_3 | P_3 | P_3 |

The surface has 9 points:

The points on the surface are:

$$0 : P_1 = (0, 1, 0, 0)$$

$$1 : P_2 = (0, 0, 1, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_7 = (0, 1, 1, 0)$$

$$5 : P_8 = (1, 1, 1, 0)$$

$$6 : P_{10} = (0, 1, 0, 1)$$

$$7 : P_{12} = (0, 0, 1, 1)$$

$$8 : P_{14} = (0, 1, 1, 1)$$