

Rank-487 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1^2 + X_1^2 X_2 = 0$$

(0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(8) is 4760725

General information

Number of lines	1
Number of points	57
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{48}$

Singular Points

The surface has 0 singular points:

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{1322}$$

Rank of lines: (138)

Rank of points on Klein quadric: (1322)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 9 single points:

The single points on the surface are:

- 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_0
- 1 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0
- 2 : $P_{138} = (0, 0, 1, 1)$ lies on line ℓ_0
- 3 : $P_{155} = (2, 2, 1, 1)$ lies on line ℓ_0
- 4 : $P_{164} = (3, 3, 1, 1)$ lies on line ℓ_0

- 5 : $P_{173} = (4, 4, 1, 1)$ lies on line ℓ_0
- 6 : $P_{182} = (5, 5, 1, 1)$ lies on line ℓ_0
- 7 : $P_{191} = (6, 6, 1, 1)$ lies on line ℓ_0
- 8 : $P_{200} = (7, 7, 1, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

- | | |
|-------------------------------|-------------------------------|
| 0 : $P_0 = (1, 0, 0, 0)$ | 22 : $P_{308} = (3, 5, 3, 1)$ |
| 1 : $P_1 = (0, 1, 0, 0)$ | 23 : $P_{309} = (4, 5, 3, 1)$ |
| 2 : $P_{19} = (0, 1, 1, 0)$ | 24 : $P_{333} = (4, 0, 4, 1)$ |
| 3 : $P_{20} = (1, 1, 1, 0)$ | 25 : $P_{340} = (3, 1, 4, 1)$ |
| 4 : $P_{75} = (1, 0, 0, 1)$ | 26 : $P_{345} = (0, 2, 4, 1)$ |
| 5 : $P_{83} = (1, 1, 0, 1)$ | 27 : $P_{352} = (7, 2, 4, 1)$ |
| 6 : $P_{158} = (5, 2, 1, 1)$ | 28 : $P_{388} = (3, 7, 4, 1)$ |
| 7 : $P_{168} = (7, 3, 1, 1)$ | 29 : $P_{392} = (7, 7, 4, 1)$ |
| 8 : $P_{175} = (6, 4, 1, 1)$ | 30 : $P_{396} = (3, 0, 5, 1)$ |
| 9 : $P_{179} = (2, 5, 1, 1)$ | 31 : $P_{401} = (0, 1, 5, 1)$ |
| 10 : $P_{189} = (4, 6, 1, 1)$ | 32 : $P_{446} = (5, 6, 5, 1)$ |
| 11 : $P_{196} = (3, 7, 1, 1)$ | 33 : $P_{448} = (7, 6, 5, 1)$ |
| 12 : $P_{203} = (2, 0, 2, 1)$ | 34 : $P_{452} = (3, 7, 5, 1)$ |
| 13 : $P_{215} = (6, 1, 2, 1)$ | 35 : $P_{456} = (7, 7, 5, 1)$ |
| 14 : $P_{237} = (4, 4, 2, 1)$ | 36 : $P_{462} = (5, 0, 6, 1)$ |
| 15 : $P_{239} = (6, 4, 2, 1)$ | 37 : $P_{465} = (0, 1, 6, 1)$ |
| 16 : $P_{257} = (0, 7, 2, 1)$ | 38 : $P_{475} = (2, 2, 6, 1)$ |
| 17 : $P_{261} = (4, 7, 2, 1)$ | 39 : $P_{478} = (5, 2, 6, 1)$ |
| 18 : $P_{271} = (6, 0, 3, 1)$ | 40 : $P_{483} = (2, 3, 6, 1)$ |
| 19 : $P_{273} = (0, 1, 3, 1)$ | 41 : $P_{487} = (6, 3, 6, 1)$ |
| 20 : $P_{301} = (4, 4, 3, 1)$ | 42 : $P_{528} = (7, 0, 7, 1)$ |
| 21 : $P_{303} = (6, 4, 3, 1)$ | 43 : $P_{534} = (5, 1, 7, 1)$ |

44 : $P_{539} = (2, 2, 7, 1)$
 45 : $P_{542} = (5, 2, 7, 1)$
 46 : $P_{553} = (0, 4, 7, 1)$

47 : $P_{555} = (2, 4, 7, 1)$

Line Intersection Graph

$$\begin{array}{c|c} & 0 \\ \hline 0 & 0 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line
in point

The surface has 57 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	20 : $P_{200} = (7, 7, 1, 1)$	40 : $P_{401} = (0, 1, 5, 1)$
1 : $P_1 = (0, 1, 0, 0)$	21 : $P_{203} = (2, 0, 2, 1)$	41 : $P_{446} = (5, 6, 5, 1)$
2 : $P_4 = (1, 1, 1, 1)$	22 : $P_{215} = (6, 1, 2, 1)$	42 : $P_{448} = (7, 6, 5, 1)$
3 : $P_5 = (1, 1, 0, 0)$	23 : $P_{237} = (4, 4, 2, 1)$	43 : $P_{452} = (3, 7, 5, 1)$
4 : $P_{19} = (0, 1, 1, 0)$	24 : $P_{239} = (6, 4, 2, 1)$	44 : $P_{456} = (7, 7, 5, 1)$
5 : $P_{20} = (1, 1, 1, 0)$	25 : $P_{257} = (0, 7, 2, 1)$	45 : $P_{462} = (5, 0, 6, 1)$
6 : $P_{75} = (1, 0, 0, 1)$	26 : $P_{261} = (4, 7, 2, 1)$	46 : $P_{465} = (0, 1, 6, 1)$
7 : $P_{83} = (1, 1, 0, 1)$	27 : $P_{271} = (6, 0, 3, 1)$	47 : $P_{475} = (2, 2, 6, 1)$
8 : $P_{138} = (0, 0, 1, 1)$	28 : $P_{273} = (0, 1, 3, 1)$	48 : $P_{478} = (5, 2, 6, 1)$
9 : $P_{155} = (2, 2, 1, 1)$	29 : $P_{301} = (4, 4, 3, 1)$	49 : $P_{483} = (2, 3, 6, 1)$
10 : $P_{158} = (5, 2, 1, 1)$	30 : $P_{303} = (6, 4, 3, 1)$	50 : $P_{487} = (6, 3, 6, 1)$
11 : $P_{164} = (3, 3, 1, 1)$	31 : $P_{308} = (3, 5, 3, 1)$	51 : $P_{528} = (7, 0, 7, 1)$
12 : $P_{168} = (7, 3, 1, 1)$	32 : $P_{309} = (4, 5, 3, 1)$	52 : $P_{534} = (5, 1, 7, 1)$
13 : $P_{173} = (4, 4, 1, 1)$	33 : $P_{333} = (4, 0, 4, 1)$	53 : $P_{539} = (2, 2, 7, 1)$
14 : $P_{175} = (6, 4, 1, 1)$	34 : $P_{340} = (3, 1, 4, 1)$	54 : $P_{542} = (5, 2, 7, 1)$
15 : $P_{179} = (2, 5, 1, 1)$	35 : $P_{345} = (0, 2, 4, 1)$	55 : $P_{553} = (0, 4, 7, 1)$
16 : $P_{182} = (5, 5, 1, 1)$	36 : $P_{352} = (7, 2, 4, 1)$	56 : $P_{555} = (2, 4, 7, 1)$
17 : $P_{189} = (4, 6, 1, 1)$	37 : $P_{388} = (3, 7, 4, 1)$	
18 : $P_{191} = (6, 6, 1, 1)$	38 : $P_{392} = (7, 7, 4, 1)$	
19 : $P_{196} = (3, 7, 1, 1)$	39 : $P_{396} = (3, 0, 5, 1)$	