Rank-73987 over GF(4)

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The equation

The equation of the surface is :

$$X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1498830169

General information

Number of lines	5
Number of points	25
Number of singular points	3
Number of Eckardt points	1
Number of double points	4
Number of single points	14
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	3, 2^4, 1^{14}, 0^6

Singular Points

The surface has 3 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$2: \ P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{16} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{37} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{37} = \mathbf{Pl}(0,0,1,0,0,1)_{108}$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_4 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

Rank of lines: (0, 16, 37, 340, 356)
Rank of points on Klein quadric: (0, 2, 108, 9, 1)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_4$

Single Points

The surface has 14 single points:
The single points on the surface are:

$0 :\ P_6 = (2,1,0,0)$ lies on line ℓ_0
$1 :\ P_7 = (3,1,0,0)$ lies on line ℓ_0
$2 :\ P_8 = (1,0,1,0)$ lies on line ℓ_1
$3 :\ P_9 = (2,0,1,0)$ lies on line ℓ_1
$4 :\ P_{10} = (3,0,1,0)$ lies on line ℓ_1
$5 :\ P_{12} = (1,1,1,0)$ lies on line ℓ_2
$6 :\ P_{17} = (2,2,1,0)$ lies on line ℓ_2
$7 :\ P_{22} = (3,3,1,0)$ lies on line ℓ_2
$8 :\ P_{26} = (0,1,0,1)$ lies on line ℓ_3
$9 :\ P_{30} = (0,2,0,1)$ lies on line ℓ_3
$10 :\ P_{34} = (0,3,0,1)$ lies on line ℓ_3
$11 :\ P_{38} = (0,0,1,1)$ lies on line ℓ_4
$12 :\ P_{53} = (0,0,2,1)$ lies on line ℓ_4
$13 :\ P_{69} = (0,0,3,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:
The points on the surface but not on lines are:

0 : $P_{46} = (1, 2, 1, 1)$
1 : $P_{50} = (1, 3, 1, 1)$
2 : $P_{60} = (3, 1, 2, 1)$
3 : $P_{64} = (3, 2, 2, 1)$
4 : $P_{75} = (2, 1, 3, 1)$
5 : $P_{83} = (2, 3, 3, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	0	1
3	1	0	0	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_5	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_5	P_2	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4
in point	P_1	P_3

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_3

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_5 = (1, 1, 0, 0)$
5 : $P_6 = (2, 1, 0, 0)$
6 : $P_7 = (3, 1, 0, 0)$
7 : $P_8 = (1, 0, 1, 0)$
8 : $P_9 = (2, 0, 1, 0)$
9 : $P_{10} = (3, 0, 1, 0)$
10 : $P_{12} = (1, 1, 1, 0)$
11 : $P_{17} = (2, 2, 1, 0)$
12 : $P_{22} = (3, 3, 1, 0)$
13 : $P_{26} = (0, 1, 0, 1)$
14 : $P_{30} = (0, 2, 0, 1)$
15 : $P_{34} = (0, 3, 0, 1)$
16 : $P_{38} = (0, 0, 1, 1)$
17 : $P_{46} = (1, 2, 1, 1)$
18 : $P_{50} = (1, 3, 1, 1)$
19 : $P_{53} = (0, 0, 2, 1)$
20 : $P_{60} = (3, 1, 2, 1)$
21 : $P_{64} = (3, 2, 2, 1)$
22 : $P_{69} = (0, 0, 3, 1)$
23 : $P_{75} = (2, 1, 3, 1)$
24 : $P_{83} = (2, 3, 3, 1)$