Rank-76389 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2X_2 + X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 304156965

General information

Number of lines	4
Number of points	289
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	60
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^4
Type of lines on points	2^4, 1^{60}, 0^{225}

Singular Points

The surface has 2 singular points:

0 : $P_{3020} = \mathbf{P}(\delta^5, \delta^5, \delta^{10}, 1) = \mathbf{P}(11, 11, 10, 1)$
1 : $P_{3259} = \mathbf{P}(\delta^{10}, \delta^{10}, \delta^5, 1) = \mathbf{P}(10, 10, 11, 1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{4898} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{4898} = \mathbf{Pl}(0,1,1,1,1,1)_{9442}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^5 & \delta^5 \end{array}\right]_{4555} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 11 & 11 \end{array}\right]_{4555} = \mathbf{Pl}(1,1,1,10,1,0)_{3275}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & \delta^{10} & \delta^{10} \end{array}\right]_{4538} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 10 & 10 \end{array}\right]_{4538} = \mathbf{Pl}(1,1,1,11,1,0)_{3500}$$

Rank of lines: (70160, 4898, 4555, 4538)
Rank of points on Klein quadric: (1, 9442, 3275, 3500)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_{530} = (0,0,1,1) = \ell_0 \cap \ell_1$
$P_{3020} = (11,11,10,1) = \ell_1 \cap \ell_2$
$P_{3259} = (10,10,11,1) = \ell_1 \cap \ell_3$
$P_{275} = (1,0,0,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 60 single points:
The single points on the surface are:

0 : $P_2 = (0,0,1,0)$ lies on line ℓ_0
1 : $P_3 = (0,0,0,1)$ lies on line ℓ_0
2 : $P_{36} = (1,1,1,0)$ lies on line ℓ_1
3 : $P_{180} = (1,10,1,0)$ lies on line ℓ_2
4 : $P_{196} = (1,11,1,0)$ lies on line ℓ_3
5 : $P_{291} = (1,1,0,1)$ lies on line ℓ_1
6 : $P_{689} = (0,10,1,1)$ lies on line ℓ_2
7 : $P_{705} = (0,11,1,1)$ lies on line ℓ_3
8 : $P_{785} = (0,0,2,1)$ lies on line ℓ_0
9 : $P_{836} = (3,3,2,1)$ lies on line ℓ_1
10 : $P_{996} = (3,13,2,1)$ lies on line ℓ_2
11 : $P_{1028} = (3,15,2,1)$ lies on line ℓ_3
12 : $P_{1041} = (0,0,3,1)$ lies on line ℓ_0
13 : $P_{1075} = (2,2,3,1)$ lies on line ℓ_1
14 : $P_{1107} = (2,4,3,1)$ lies on line ℓ_3
15 : $P_{1155} = (2,7,3,1)$ lies on line ℓ_2
16 : $P_{1297} = (0,0,4,1)$ lies on line ℓ_0
17 : $P_{1350} = (5,3,4,1)$ lies on line ℓ_2
18 : $P_{1382} = (5,5,4,1)$ lies on line ℓ_1
19 : $P_{1414} = (5,7,4,1)$ lies on line ℓ_3
20 : $P_{1553} = (0,0,5,1)$ lies on line ℓ_0
21 : $P_{1621} = (4,4,5,1)$ lies on line ℓ_1
22 : $P_{1701} = (4,9,5,1)$ lies on line ℓ_2
23 : $P_{1749} = (4,12,5,1)$ lies on line ℓ_3
24 : $P_{1809} = (0,0,6,1)$ lies on line ℓ_0
25 : $P_{1928} = (7,7,6,1)$ lies on line ℓ_1
26 : $P_{1944} = (7,8,6,1)$ lies on line ℓ_3
27 : $P_{2040} = (7,14,6,1)$ lies on line ℓ_2
28 : $P_{2065} = (0,0,7,1)$ lies on line ℓ_0
29 : $P_{2119} = (6,3,7,1)$ lies on line ℓ_3
30 : $P_{2135} = (6,4,7,1)$ lies on line ℓ_2
31 : $P_{2167} = (6,6,7,1)$ lies on line ℓ_1
32 : $P_{2321} = (0,0,8,1)$ lies on line ℓ_0
33 : $P_{2426} = (9,6,8,1)$ lies on line ℓ_2
34 : $P_{2474} = (9,9,8,1)$ lies on line ℓ_1
35 : $P_{2554} = (9,14,8,1)$ lies on line ℓ_3
36 : $P_{2577} = (0,0,9,1)$ lies on line ℓ_0
37 : $P_{2665} = (8,5,9,1)$ lies on line ℓ_3
38 : $P_{2713} = (8,8,9,1)$ lies on line ℓ_1
39 : $P_{2777} = (8,12,9,1)$ lies on line ℓ_2
40 : $P_{2833} = (0,0,10,1)$ lies on line ℓ_0
41 : $P_{2860} = (11,1,10,1)$ lies on line ℓ_3
42 : $P_{3089} = (0,0,11,1)$ lies on line ℓ_0
43 : $P_{3115} = (10,1,11,1)$ lies on line ℓ_2

44 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_0
45 : $P_{3438} = (13, 5, 12, 1)$ lies on line ℓ_2
46 : $P_{3502} = (13, 9, 12, 1)$ lies on line ℓ_3
47 : $P_{3566} = (13, 13, 12, 1)$ lies on line ℓ_1
48 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_0
49 : $P_{3645} = (12, 2, 13, 1)$ lies on line ℓ_3
50 : $P_{3805} = (12, 12, 13, 1)$ lies on line ℓ_1
51 : $P_{3853} = (12, 15, 13, 1)$ lies on line ℓ_2
52 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_0
53 : $P_{3968} = (15, 6, 14, 1)$ lies on line ℓ_3
54 : $P_{4000} = (15, 8, 14, 1)$ lies on line ℓ_2
55 : $P_{4112} = (15, 15, 14, 1)$ lies on line ℓ_1
56 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_0
57 : $P_{4159} = (14, 2, 15, 1)$ lies on line ℓ_2
58 : $P_{4335} = (14, 13, 15, 1)$ lies on line ℓ_3
59 : $P_{4351} = (14, 14, 15, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_{35} = (0, 1, 1, 0)$
2 : $P_{76} = (9, 3, 1, 0)$
3 : $P_{77} = (10, 3, 1, 0)$
4 : $P_{110} = (11, 5, 1, 0)$
5 : $P_{113} = (14, 5, 1, 0)$
6 : $P_{149} = (2, 8, 1, 0)$
7 : $P_{157} = (10, 8, 1, 0)$
8 : $P_{190} = (11, 10, 1, 0)$
9 : $P_{205} = (10, 11, 1, 0)$
10 : $P_{263} = (4, 15, 1, 0)$
11 : $P_{270} = (11, 15, 1, 0)$
12 : $P_{290} = (0, 1, 0, 1)$
13 : $P_{320} = (14, 2, 0, 1)$
14 : $P_{321} = (15, 2, 0, 1)$
15 : $P_{334} = (12, 3, 0, 1)$
16 : $P_{335} = (13, 3, 0, 1)$
17 : $P_{340} = (2, 4, 0, 1)$
18 : $P_{341} = (3, 4, 0, 1)$
19 : $P_{360} = (6, 5, 0, 1)$
20 : $P_{361} = (7, 5, 0, 1)$
21 : $P_{414} = (12, 8, 0, 1)$
22 : $P_{415} = (13, 8, 0, 1)$
23 : $P_{422} = (4, 9, 0, 1)$
24 : $P_{423} = (5, 9, 0, 1)$
25 : $P_{446} = (12, 10, 0, 1)$
26 : $P_{447} = (13, 10, 0, 1)$
27 : $P_{456} = (6, 11, 0, 1)$
28 : $P_{457} = (7, 11, 0, 1)$
29 : $P_{506} = (8, 14, 0, 1)$
30 : $P_{507} = (9, 14, 0, 1)$
31 : $P_{520} = (6, 15, 0, 1)$
32 : $P_{521} = (7, 15, 0, 1)$
33 : $P_{574} = (13, 2, 1, 1)$
34 : $P_{585} = (8, 3, 1, 1)$
35 : $P_{600} = (7, 4, 1, 1)$
36 : $P_{624} = (15, 5, 1, 1)$
37 : $P_{627} = (2, 6, 1, 1)$
38 : $P_{650} = (9, 7, 1, 1)$
39 : $P_{660} = (3, 8, 1, 1)$
40 : $P_{685} = (12, 9, 1, 1)$
41 : $P_{735} = (14, 12, 1, 1)$
42 : $P_{741} = (4, 13, 1, 1)$
43 : $P_{759} = (6, 14, 1, 1)$
44 : $P_{774} = (5, 15, 1, 1)$
45 : $P_{793} = (8, 0, 2, 1)$
46 : $P_{807} = (6, 1, 2, 1)$
47 : $P_{808} = (7, 1, 2, 1)$
48 : $P_{829} = (12, 2, 2, 1)$
49 : $P_{832} = (15, 2, 2, 1)$
50 : $P_{842} = (9, 3, 2, 1)$
51 : $P_{857} = (8, 4, 2, 1)$
52 : $P_{864} = (15, 4, 2, 1)$
53 : $P_{871} = (6, 5, 2, 1)$
54 : $P_{873} = (8, 5, 2, 1)$
55 : $P_{907} = (10, 7, 2, 1)$
56 : $P_{912} = (15, 7, 2, 1)$
57 : $P_{968} = (7, 11, 2, 1)$
58 : $P_{971} = (10, 11, 2, 1)$
59 : $P_{988} = (11, 12, 2, 1)$
60 : $P_{1003} = (10, 13, 2, 1)$
61 : $P_{1026} = (1, 15, 2, 1)$
62 : $P_{1053} = (12, 0, 3, 1)$
63 : $P_{1086} = (13, 2, 3, 1)$
64 : $P_{1113} = (8, 4, 3, 1)$
65 : $P_{1159} = (6, 7, 3, 1)$
66 : $P_{1174} = (5, 8, 3, 1)$
67 : $P_{1218} = (1, 11, 3, 1)$
68 : $P_{1232} = (15, 11, 3, 1)$
69 : $P_{1312} = (15, 0, 4, 1)$

70 : $P_{1325} = (12, 1, 4, 1)$
71 : $P_{1326} = (13, 1, 4, 1)$
72 : $P_{1346} = (1, 3, 4, 1)$
73 : $P_{1364} = (3, 4, 4, 1)$
74 : $P_{1367} = (6, 4, 4, 1)$
75 : $P_{1391} = (14, 5, 4, 1)$
76 : $P_{1403} = (10, 6, 4, 1)$
77 : $P_{1420} = (11, 7, 4, 1)$
78 : $P_{1438} = (13, 8, 4, 1)$
79 : $P_{1440} = (15, 8, 4, 1)$
80 : $P_{1444} = (3, 9, 4, 1)$
81 : $P_{1456} = (15, 9, 4, 1)$
82 : $P_{1468} = (11, 10, 4, 1)$
83 : $P_{1469} = (12, 10, 4, 1)$
84 : $P_{1492} = (3, 12, 4, 1)$
85 : $P_{1500} = (11, 12, 4, 1)$
86 : $P_{1559} = (6, 0, 5, 1)$
87 : $P_{1624} = (7, 4, 5, 1)$
88 : $P_{1712} = (15, 9, 5, 1)$
89 : $P_{1714} = (1, 10, 5, 1)$
90 : $P_{1716} = (3, 10, 5, 1)$
91 : $P_{1758} = (13, 12, 5, 1)$
92 : $P_{1801} = (8, 15, 5, 1)$
93 : $P_{1823} = (14, 0, 6, 1)$
94 : $P_{1866} = (9, 3, 6, 1)$
95 : $P_{1872} = (15, 3, 6, 1)$
96 : $P_{1873} = (0, 4, 6, 1)$
97 : $P_{1889} = (0, 5, 6, 1)$
98 : $P_{1904} = (15, 5, 6, 1)$
99 : $P_{1916} = (11, 6, 6, 1)$
100 : $P_{1917} = (12, 6, 6, 1)$
101 : $P_{1936} = (15, 7, 6, 1)$
102 : $P_{1949} = (12, 8, 6, 1)$
103 : $P_{2009} = (8, 12, 6, 1)$
104 : $P_{2014} = (13, 12, 6, 1)$
105 : $P_{2038} = (5, 14, 6, 1)$
106 : $P_{2050} = (1, 15, 6, 1)$
107 : $P_{2061} = (12, 15, 6, 1)$
108 : $P_{2069} = (4, 0, 7, 1)$
109 : $P_{2126} = (13, 3, 7, 1)$
110 : $P_{2144} = (15, 4, 7, 1)$
111 : $P_{2146} = (1, 5, 7, 1)$
112 : $P_{2158} = (13, 5, 7, 1)$
113 : $P_{2166} = (5, 6, 7, 1)$
114 : $P_{2188} = (11, 7, 7, 1)$
115 : $P_{2190} = (13, 7, 7, 1)$
116 : $P_{2195} = (2, 8, 7, 1)$
117 : $P_{2198} = (5, 8, 7, 1)$
118 : $P_{2276} = (3, 13, 7, 1)$
119 : $P_{2285} = (12, 13, 7, 1)$
120 : $P_{2289} = (0, 14, 7, 1)$
121 : $P_{2305} = (0, 15, 7, 1)$
122 : $P_{2310} = (5, 15, 7, 1)$
123 : $P_{2334} = (13, 0, 8, 1)$
124 : $P_{2384} = (15, 3, 8, 1)$
125 : $P_{2424} = (7, 6, 8, 1)$
126 : $P_{2477} = (12, 9, 8, 1)$
127 : $P_{2498} = (1, 11, 8, 1)$
128 : $P_{2502} = (5, 11, 8, 1)$
129 : $P_{2548} = (3, 14, 8, 1)$
130 : $P_{2580} = (3, 0, 9, 1)$
131 : $P_{2599} = (6, 1, 9, 1)$
132 : $P_{2600} = (7, 1, 9, 1)$
133 : $P_{2658} = (1, 5, 9, 1)$
134 : $P_{2678} = (5, 6, 9, 1)$
135 : $P_{2683} = (10, 6, 9, 1)$
136 : $P_{2707} = (2, 8, 9, 1)$
137 : $P_{2726} = (5, 9, 9, 1)$
138 : $P_{2734} = (13, 9, 9, 1)$
139 : $P_{2759} = (6, 11, 9, 1)$
140 : $P_{2763} = (10, 11, 9, 1)$
141 : $P_{2779} = (10, 12, 9, 1)$
142 : $P_{2796} = (11, 13, 9, 1)$
143 : $P_{2804} = (3, 14, 9, 1)$
144 : $P_{2806} = (5, 14, 9, 1)$
145 : $P_{2820} = (3, 15, 9, 1)$
146 : $P_{2824} = (7, 15, 9, 1)$
147 : $P_{2843} = (10, 0, 10, 1)$
148 : $P_{2859} = (10, 1, 10, 1)$
149 : $P_{2929} = (0, 6, 10, 1)$
150 : $P_{2931} = (2, 6, 10, 1)$
151 : $P_{2945} = (0, 7, 10, 1)$
152 : $P_{2954} = (9, 7, 10, 1)$
153 : $P_{2996} = (3, 10, 10, 1)$
154 : $P_{3001} = (8, 10, 10, 1)$
155 : $P_{3029} = (4, 12, 10, 1)$
156 : $P_{3032} = (7, 12, 10, 1)$
157 : $P_{3047} = (6, 13, 10, 1)$
158 : $P_{3055} = (14, 13, 10, 1)$
159 : $P_{3100} = (11, 0, 11, 1)$
160 : $P_{3116} = (11, 1, 11, 1)$
161 : $P_{3194} = (9, 6, 11, 1)$
162 : $P_{3197} = (12, 6, 11, 1)$
163 : $P_{3203} = (2, 7, 11, 1)$
164 : $P_{3214} = (13, 7, 11, 1)$
165 : $P_{3270} = (5, 11, 11, 1)$
166 : $P_{3280} = (15, 11, 11, 1)$
167 : $P_{3281} = (0, 12, 11, 1)$
168 : $P_{3295} = (14, 12, 11, 1)$
169 : $P_{3297} = (0, 13, 11, 1)$
170 : $P_{3301} = (4, 13, 11, 1)$
171 : $P_{3354} = (9, 0, 12, 1)$
172 : $P_{3377} = (0, 2, 12, 1)$
173 : $P_{3393} = (0, 3, 12, 1)$
174 : $P_{3401} = (8, 3, 12, 1)$
175 : $P_{3432} = (7, 5, 12, 1)$
176 : $P_{3462} = (5, 7, 12, 1)$
177 : $P_{3463} = (6, 7, 12, 1)$

178 : $P_{3474} = (1, 8, 12, 1)$
179 : $P_{3480} = (7, 8, 12, 1)$
180 : $P_{3492} = (3, 9, 12, 1)$
181 : $P_{3544} = (7, 12, 12, 1)$
182 : $P_{3547} = (10, 12, 12, 1)$
183 : $P_{3561} = (8, 13, 12, 1)$
184 : $P_{3589} = (4, 15, 12, 1)$
185 : $P_{3593} = (8, 15, 12, 1)$
186 : $P_{3603} = (2, 0, 13, 1)$
187 : $P_{3641} = (8, 2, 13, 1)$
188 : $P_{3650} = (1, 3, 13, 1)$
189 : $P_{3655} = (6, 3, 13, 1)$
190 : $P_{3684} = (3, 5, 13, 1)$
191 : $P_{3695} = (14, 5, 13, 1)$
192 : $P_{3704} = (7, 6, 13, 1)$
193 : $P_{3712} = (15, 6, 13, 1)$
194 : $P_{3729} = (0, 8, 13, 1)$
195 : $P_{3732} = (3, 8, 13, 1)$
196 : $P_{3745} = (0, 9, 13, 1)$
197 : $P_{3796} = (3, 12, 13, 1)$
198 : $P_{3815} = (6, 13, 13, 1)$
199 : $P_{3819} = (10, 13, 13, 1)$
200 : $P_{3847} = (6, 15, 13, 1)$
201 : $P_{3862} = (5, 0, 14, 1)$
202 : $P_{3885} = (12, 1, 14, 1)$
203 : $P_{3886} = (13, 1, 14, 1)$
204 : $P_{3894} = (5, 2, 14, 1)$
205 : $P_{3897} = (8, 2, 14, 1)$
206 : $P_{3910} = (5, 3, 14, 1)$
207 : $P_{3917} = (12, 3, 14, 1)$
208 : $P_{3964} = (11, 6, 14, 1)$
209 : $P_{3979} = (10, 7, 14, 1)$
210 : $P_{3986} = (1, 8, 14, 1)$
211 : $P_{4028} = (11, 10, 14, 1)$
212 : $P_{4030} = (13, 10, 14, 1)$
213 : $P_{4073} = (8, 13, 14, 1)$
214 : $P_{4076} = (11, 13, 14, 1)$
215 : $P_{4088} = (7, 14, 14, 1)$
216 : $P_{4089} = (8, 14, 14, 1)$
217 : $P_{4101} = (4, 15, 14, 1)$
218 : $P_{4120} = (7, 0, 15, 1)$
219 : $P_{4150} = (5, 2, 15, 1)$
220 : $P_{4196} = (3, 5, 15, 1)$
221 : $P_{4274} = (1, 10, 15, 1)$
222 : $P_{4281} = (8, 10, 15, 1)$
223 : $P_{4333} = (12, 13, 15, 1)$
224 : $P_{4343} = (6, 14, 15, 1)$

Line Intersection Graph

	0 1 2 3
0	0 1 0 0
1	1 0 1 1
2	0 1 0 1
3	0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{530}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_{530}	P_{3020}	P_{3259}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{3020}	P_{275}

Line 3 intersects

Line	ℓ_1	ℓ_2
in point	P_{3259}	P_{275}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_{35} = (0, 1, 1, 0)$
4 : $P_{36} = (1, 1, 1, 0)$
5 : $P_{76} = (9, 3, 1, 0)$
6 : $P_{77} = (10, 3, 1, 0)$
7 : $P_{110} = (11, 5, 1, 0)$
8 : $P_{113} = (14, 5, 1, 0)$

$9: P_{149} = (2, 8, 1, 0)$
$10: P_{157} = (10, 8, 1, 0)$
$11: P_{180} = (1, 10, 1, 0)$
$12: P_{190} = (11, 10, 1, 0)$
$13: P_{196} = (1, 11, 1, 0)$
$14: P_{205} = (10, 11, 1, 0)$
$15: P_{263} = (4, 15, 1, 0)$
$16: P_{270} = (11, 15, 1, 0)$
$17: P_{275} = (1, 0, 0, 1)$
$18: P_{290} = (0, 1, 0, 1)$
$19: P_{291} = (1, 1, 0, 1)$
$20: P_{320} = (14, 2, 0, 1)$
$21: P_{321} = (15, 2, 0, 1)$
$22: P_{334} = (12, 3, 0, 1)$
$23: P_{335} = (13, 3, 0, 1)$
$24: P_{340} = (2, 4, 0, 1)$
$25: P_{341} = (3, 4, 0, 1)$
$26: P_{360} = (6, 5, 0, 1)$
$27: P_{361} = (7, 5, 0, 1)$
$28: P_{414} = (12, 8, 0, 1)$
$29: P_{415} = (13, 8, 0, 1)$
$30: P_{422} = (4, 9, 0, 1)$
$31: P_{423} = (5, 9, 0, 1)$
$32: P_{446} = (12, 10, 0, 1)$
$33: P_{447} = (13, 10, 0, 1)$
$34: P_{456} = (6, 11, 0, 1)$
$35: P_{457} = (7, 11, 0, 1)$
$36: P_{506} = (8, 14, 0, 1)$
$37: P_{507} = (9, 14, 0, 1)$
$38: P_{520} = (6, 15, 0, 1)$
$39: P_{521} = (7, 15, 0, 1)$
$40: P_{530} = (0, 0, 1, 1)$
$41: P_{574} = (13, 2, 1, 1)$
$42: P_{585} = (8, 3, 1, 1)$
$43: P_{600} = (7, 4, 1, 1)$
$44: P_{624} = (15, 5, 1, 1)$
$45: P_{627} = (2, 6, 1, 1)$
$46: P_{650} = (9, 7, 1, 1)$
$47: P_{660} = (3, 8, 1, 1)$
$48: P_{685} = (12, 9, 1, 1)$
$49: P_{689} = (0, 10, 1, 1)$
$50: P_{705} = (0, 11, 1, 1)$
$51: P_{735} = (14, 12, 1, 1)$
$52: P_{741} = (4, 13, 1, 1)$
$53: P_{759} = (6, 14, 1, 1)$
$54: P_{774} = (5, 15, 1, 1)$
$55: P_{785} = (0, 0, 2, 1)$
$56: P_{793} = (8, 0, 2, 1)$
$57: P_{807} = (6, 1, 2, 1)$
$58: P_{808} = (7, 1, 2, 1)$
$59: P_{829} = (12, 2, 2, 1)$
$60: P_{832} = (15, 2, 2, 1)$
$61: P_{836} = (3, 3, 2, 1)$
$62: P_{842} = (9, 3, 2, 1)$
$63: P_{857} = (8, 4, 2, 1)$
$64: P_{864} = (15, 4, 2, 1)$
$65: P_{871} = (6, 5, 2, 1)$
$66: P_{873} = (8, 5, 2, 1)$
$67: P_{907} = (10, 7, 2, 1)$
$68: P_{912} = (15, 7, 2, 1)$
$69: P_{968} = (7, 11, 2, 1)$
$70: P_{971} = (10, 11, 2, 1)$
$71: P_{988} = (11, 12, 2, 1)$
$72: P_{996} = (3, 13, 2, 1)$
$73: P_{1003} = (10, 13, 2, 1)$
$74: P_{1026} = (1, 15, 2, 1)$
$75: P_{1028} = (3, 15, 2, 1)$
$76: P_{1041} = (0, 0, 3, 1)$
$77: P_{1053} = (12, 0, 3, 1)$
$78: P_{1075} = (2, 2, 3, 1)$
$79: P_{1086} = (13, 2, 3, 1)$
$80: P_{1107} = (2, 4, 3, 1)$
$81: P_{1113} = (8, 4, 3, 1)$
$82: P_{1155} = (2, 7, 3, 1)$
$83: P_{1159} = (6, 7, 3, 1)$
$84: P_{1174} = (5, 8, 3, 1)$
$85: P_{1218} = (1, 11, 3, 1)$
$86: P_{1232} = (15, 11, 3, 1)$
$87: P_{1297} = (0, 0, 4, 1)$
$88: P_{1312} = (15, 0, 4, 1)$
$89: P_{1325} = (12, 1, 4, 1)$
$90: P_{1326} = (13, 1, 4, 1)$
$91: P_{1346} = (1, 3, 4, 1)$
$92: P_{1350} = (5, 3, 4, 1)$
$93: P_{1364} = (3, 4, 4, 1)$
$94: P_{1367} = (6, 4, 4, 1)$
$95: P_{1382} = (5, 5, 4, 1)$
$96: P_{1391} = (14, 5, 4, 1)$
$97: P_{1403} = (10, 6, 4, 1)$
$98: P_{1414} = (5, 7, 4, 1)$
$99: P_{1420} = (11, 7, 4, 1)$
$100: P_{1438} = (13, 8, 4, 1)$
$101: P_{1440} = (15, 8, 4, 1)$
$102: P_{1444} = (3, 9, 4, 1)$
$103: P_{1456} = (15, 9, 4, 1)$
$104: P_{1468} = (11, 10, 4, 1)$
$105: P_{1469} = (12, 10, 4, 1)$
$106: P_{1492} = (3, 12, 4, 1)$
$107: P_{1500} = (11, 12, 4, 1)$
$108: P_{1553} = (0, 0, 5, 1)$
$109: P_{1559} = (6, 0, 5, 1)$
$110: P_{1621} = (4, 4, 5, 1)$
$111: P_{1624} = (7, 4, 5, 1)$
$112: P_{1701} = (4, 9, 5, 1)$
$113: P_{1712} = (15, 9, 5, 1)$
$114: P_{1714} = (1, 10, 5, 1)$
$115: P_{1716} = (3, 10, 5, 1)$
$116: P_{1749} = (4, 12, 5, 1)$
$117: P_{1758} = (13, 12, 5, 1)$
$118: P_{1801} = (8, 15, 5, 1)$
$119: P_{1809} = (0, 0, 6, 1)$
$120: P_{1823} = (14, 0, 6, 1)$
$121: P_{1866} = (9, 3, 6, 1)$
$122: P_{1872} = (15, 3, 6, 1)$
$123: P_{1873} = (0, 4, 6, 1)$
$124: P_{1889} = (0, 5, 6, 1)$
$125: P_{1904} = (15, 5, 6, 1)$
$126: P_{1916} = (11, 6, 6, 1)$
$127: P_{1917} = (12, 6, 6, 1)$
$128: P_{1928} = (7, 7, 6, 1)$
$129: P_{1936} = (15, 7, 6, 1)$
$130: P_{1944} = (7, 8, 6, 1)$
$131: P_{1949} = (12, 8, 6, 1)$
$132: P_{2009} = (8, 12, 6, 1)$
$133: P_{2014} = (13, 12, 6, 1)$
$134: P_{2038} = (5, 14, 6, 1)$
$135: P_{2040} = (7, 14, 6, 1)$
$136: P_{2050} = (1, 15, 6, 1)$
$137: P_{2061} = (12, 15, 6, 1)$
$138: P_{2065} = (0, 0, 7, 1)$
$139: P_{2069} = (4, 0, 7, 1)$
$140: P_{2119} = (6, 3, 7, 1)$
$141: P_{2126} = (13, 3, 7, 1)$
$142: P_{2135} = (6, 4, 7, 1)$
$143: P_{2144} = (15, 4, 7, 1)$
$144: P_{2146} = (1, 5, 7, 1)$
$145: P_{2158} = (13, 5, 7, 1)$
$146: P_{2166} = (5, 6, 7, 1)$
$147: P_{2167} = (6, 6, 7, 1)$
$148: P_{2188} = (11, 7, 7, 1)$
$149: P_{2190} = (13, 7, 7, 1)$
$150: P_{2195} = (2, 8, 7, 1)$
$151: P_{2198} = (5, 8, 7, 1)$
$152: P_{2276} = (3, 13, 7, 1)$
$153: P_{2285} = (12, 13, 7, 1)$
$154: P_{2289} = (0, 14, 7, 1)$
$155: P_{2305} = (0, 15, 7, 1)$
$156: P_{2310} = (5, 15, 7, 1)$
$157: P_{2321} = (0, 0, 8, 1)$
$158: P_{2334} = (13, 0, 8, 1)$
$159: P_{2384} = (15, 3, 8, 1)$
$160: P_{2424} = (7, 6, 8, 1)$
$161: P_{2426} = (9, 6, 8, 1)$
$162: P_{2474} = (9, 9, 8, 1)$
$163: P_{2477} = (12, 9, 8, 1)$
$164: P_{2498} = (1, 11, 8, 1)$
$165: P_{2502} = (5, 11, 8, 1)$
$166: P_{2548} = (3, 14, 8, 1)$
$167: P_{2554} = (9, 14, 8, 1)$
$168: P_{2577} = (0, 0, 9, 1)$
$169: P_{2580} = (3, 0, 9, 1)$
$170: P_{2599} = (6, 1, 9, 1)$

171 : $P_{2600} = (7, 1, 9, 1)$
172 : $P_{2658} = (1, 5, 9, 1)$
173 : $P_{2665} = (8, 5, 9, 1)$
174 : $P_{2678} = (5, 6, 9, 1)$
175 : $P_{2683} = (10, 6, 9, 1)$
176 : $P_{2707} = (2, 8, 9, 1)$
177 : $P_{2713} = (8, 8, 9, 1)$
178 : $P_{2726} = (5, 9, 9, 1)$
179 : $P_{2734} = (13, 9, 9, 1)$
180 : $P_{2759} = (6, 11, 9, 1)$
181 : $P_{2763} = (10, 11, 9, 1)$
182 : $P_{2777} = (8, 12, 9, 1)$
183 : $P_{2779} = (10, 12, 9, 1)$
184 : $P_{2796} = (11, 13, 9, 1)$
185 : $P_{2804} = (3, 14, 9, 1)$
186 : $P_{2806} = (5, 14, 9, 1)$
187 : $P_{2820} = (3, 15, 9, 1)$
188 : $P_{2824} = (7, 15, 9, 1)$
189 : $P_{2833} = (0, 0, 10, 1)$
190 : $P_{2843} = (10, 0, 10, 1)$
191 : $P_{2859} = (10, 1, 10, 1)$
192 : $P_{2860} = (11, 1, 10, 1)$
193 : $P_{2929} = (0, 6, 10, 1)$
194 : $P_{2931} = (2, 6, 10, 1)$
195 : $P_{2945} = (0, 7, 10, 1)$
196 : $P_{2954} = (9, 7, 10, 1)$
197 : $P_{2996} = (3, 10, 10, 1)$
198 : $P_{3001} = (8, 10, 10, 1)$
199 : $P_{3020} = (11, 11, 10, 1)$
200 : $P_{3029} = (4, 12, 10, 1)$
201 : $P_{3032} = (7, 12, 10, 1)$
202 : $P_{3047} = (6, 13, 10, 1)$
203 : $P_{3055} = (14, 13, 10, 1)$
204 : $P_{3089} = (0, 0, 11, 1)$
205 : $P_{3100} = (11, 0, 11, 1)$
206 : $P_{3115} = (10, 1, 11, 1)$
207 : $P_{3116} = (11, 1, 11, 1)$
208 : $P_{3194} = (9, 6, 11, 1)$
209 : $P_{3197} = (12, 6, 11, 1)$
210 : $P_{3203} = (2, 7, 11, 1)$
211 : $P_{3214} = (13, 7, 11, 1)$
212 : $P_{3259} = (10, 10, 11, 1)$
213 : $P_{3270} = (5, 11, 11, 1)$
214 : $P_{3280} = (15, 11, 11, 1)$
215 : $P_{3281} = (0, 12, 11, 1)$
216 : $P_{3295} = (14, 12, 11, 1)$
217 : $P_{3297} = (0, 13, 11, 1)$
218 : $P_{3301} = (4, 13, 11, 1)$
219 : $P_{3345} = (0, 0, 12, 1)$
220 : $P_{3354} = (9, 0, 12, 1)$
221 : $P_{3377} = (0, 2, 12, 1)$
222 : $P_{3393} = (0, 3, 12, 1)$
223 : $P_{3401} = (8, 3, 12, 1)$
224 : $P_{3432} = (7, 5, 12, 1)$
225 : $P_{3438} = (13, 5, 12, 1)$
226 : $P_{3462} = (5, 7, 12, 1)$
227 : $P_{3463} = (6, 7, 12, 1)$
228 : $P_{3474} = (1, 8, 12, 1)$
229 : $P_{3480} = (7, 8, 12, 1)$
230 : $P_{3492} = (3, 9, 12, 1)$
231 : $P_{3502} = (13, 9, 12, 1)$
232 : $P_{3544} = (7, 12, 12, 1)$
233 : $P_{3547} = (10, 12, 12, 1)$
234 : $P_{3561} = (8, 13, 12, 1)$
235 : $P_{3566} = (13, 13, 12, 1)$
236 : $P_{3589} = (4, 15, 12, 1)$
237 : $P_{3593} = (8, 15, 12, 1)$
238 : $P_{3601} = (0, 0, 13, 1)$
239 : $P_{3603} = (2, 0, 13, 1)$
240 : $P_{3641} = (8, 2, 13, 1)$
241 : $P_{3645} = (12, 2, 13, 1)$
242 : $P_{3650} = (1, 3, 13, 1)$
243 : $P_{3655} = (6, 3, 13, 1)$
244 : $P_{3684} = (3, 5, 13, 1)$
245 : $P_{3695} = (14, 5, 13, 1)$
246 : $P_{3704} = (7, 6, 13, 1)$
247 : $P_{3712} = (15, 6, 13, 1)$
248 : $P_{3729} = (0, 8, 13, 1)$
249 : $P_{3732} = (3, 8, 13, 1)$
250 : $P_{3745} = (0, 9, 13, 1)$
251 : $P_{3796} = (3, 12, 13, 1)$
252 : $P_{3805} = (12, 12, 13, 1)$
253 : $P_{3815} = (6, 13, 13, 1)$
254 : $P_{3819} = (10, 13, 13, 1)$
255 : $P_{3847} = (6, 15, 13, 1)$
256 : $P_{3853} = (12, 15, 13, 1)$
257 : $P_{3857} = (0, 0, 14, 1)$
258 : $P_{3862} = (5, 0, 14, 1)$
259 : $P_{3885} = (12, 1, 14, 1)$
260 : $P_{3886} = (13, 1, 14, 1)$
261 : $P_{3894} = (5, 2, 14, 1)$
262 : $P_{3897} = (8, 2, 14, 1)$
263 : $P_{3910} = (5, 3, 14, 1)$
264 : $P_{3917} = (12, 3, 14, 1)$
265 : $P_{3964} = (11, 6, 14, 1)$
266 : $P_{3968} = (15, 6, 14, 1)$
267 : $P_{3979} = (10, 7, 14, 1)$
268 : $P_{3986} = (1, 8, 14, 1)$
269 : $P_{4000} = (15, 8, 14, 1)$
270 : $P_{4028} = (11, 10, 14, 1)$
271 : $P_{4030} = (13, 10, 14, 1)$
272 : $P_{4073} = (8, 13, 14, 1)$
273 : $P_{4076} = (11, 13, 14, 1)$
274 : $P_{4088} = (7, 14, 14, 1)$
275 : $P_{4089} = (8, 14, 14, 1)$
276 : $P_{4101} = (4, 15, 14, 1)$
277 : $P_{4112} = (15, 15, 14, 1)$
278 : $P_{4113} = (0, 0, 15, 1)$
279 : $P_{4120} = (7, 0, 15, 1)$
280 : $P_{4150} = (5, 2, 15, 1)$
281 : $P_{4159} = (14, 2, 15, 1)$
282 : $P_{4196} = (3, 5, 15, 1)$
283 : $P_{4274} = (1, 10, 15, 1)$
284 : $P_{4281} = (8, 10, 15, 1)$
285 : $P_{4333} = (12, 13, 15, 1)$
286 : $P_{4335} = (14, 13, 15, 1)$
287 : $P_{4343} = (6, 14, 15, 1)$
288 : $P_{4351} = (14, 14, 15, 1)$