

Rank-76389 over GF(32)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(32) is -2079292347

General information

Number of lines	2
Number of points	1089
Number of singular points	0
Number of Eckardt points	0
Number of double points	1
Number of single points	64
Number of points off lines	1024
Number of Hesse planes	0
Number of axes	0
Type of points on lines	33^2
Type of lines on points	$2, 1^{64}, 0^{1024}$

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{1083424} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{1083424} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_1 &= \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]_{35906} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]_{35906} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{70594}\end{aligned}$$

Rank of lines: (1083424, 35906)
Rank of points on Klein quadric: (1, 70594)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$$P_{2082} = (0, 0, 1, 1) = \ell_0 \cap \ell_1$$

Single Points

The surface has 64 single points:
The single points on the surface are:

- | | |
|--|--|
| 0 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_0 | 33 : $P_{18002} = (17, 17, 16, 1)$ lies on line ℓ_1 |
| 1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_0 | 34 : $P_{18465} = (0, 0, 17, 1)$ lies on line ℓ_0 |
| 2 : $P_{68} = (1, 1, 1, 0)$ lies on line ℓ_1 | 35 : $P_{18993} = (16, 16, 17, 1)$ lies on line ℓ_1 |
| 3 : $P_{1091} = (1, 1, 0, 1)$ lies on line ℓ_1 | 36 : $P_{19489} = (0, 0, 18, 1)$ lies on line ℓ_0 |
| 4 : $P_{3105} = (0, 0, 2, 1)$ lies on line ℓ_0 | 37 : $P_{20116} = (19, 19, 18, 1)$ lies on line ℓ_1 |
| 5 : $P_{3204} = (3, 3, 2, 1)$ lies on line ℓ_1 | 38 : $P_{20513} = (0, 0, 19, 1)$ lies on line ℓ_0 |
| 6 : $P_{4129} = (0, 0, 3, 1)$ lies on line ℓ_0 | 39 : $P_{21107} = (18, 18, 19, 1)$ lies on line ℓ_1 |
| 7 : $P_{4195} = (2, 2, 3, 1)$ lies on line ℓ_1 | 40 : $P_{21537} = (0, 0, 20, 1)$ lies on line ℓ_0 |
| 8 : $P_{5153} = (0, 0, 4, 1)$ lies on line ℓ_0 | 41 : $P_{22230} = (21, 21, 20, 1)$ lies on line ℓ_1 |
| 9 : $P_{5318} = (5, 5, 4, 1)$ lies on line ℓ_1 | 42 : $P_{22561} = (0, 0, 21, 1)$ lies on line ℓ_0 |
| 10 : $P_{6177} = (0, 0, 5, 1)$ lies on line ℓ_0 | 43 : $P_{23221} = (20, 20, 21, 1)$ lies on line ℓ_1 |
| 11 : $P_{6309} = (4, 4, 5, 1)$ lies on line ℓ_1 | 44 : $P_{23585} = (0, 0, 22, 1)$ lies on line ℓ_0 |
| 12 : $P_{7201} = (0, 0, 6, 1)$ lies on line ℓ_0 | 45 : $P_{24344} = (23, 23, 22, 1)$ lies on line ℓ_1 |
| 13 : $P_{7432} = (7, 7, 6, 1)$ lies on line ℓ_1 | 46 : $P_{24609} = (0, 0, 23, 1)$ lies on line ℓ_0 |
| 14 : $P_{8225} = (0, 0, 7, 1)$ lies on line ℓ_0 | 47 : $P_{25335} = (22, 22, 23, 1)$ lies on line ℓ_1 |
| 15 : $P_{8423} = (6, 6, 7, 1)$ lies on line ℓ_1 | 48 : $P_{25633} = (0, 0, 24, 1)$ lies on line ℓ_0 |
| 16 : $P_{9249} = (0, 0, 8, 1)$ lies on line ℓ_0 | 49 : $P_{26458} = (25, 25, 24, 1)$ lies on line ℓ_1 |
| 17 : $P_{9546} = (9, 9, 8, 1)$ lies on line ℓ_1 | 50 : $P_{26657} = (0, 0, 25, 1)$ lies on line ℓ_0 |
| 18 : $P_{10273} = (0, 0, 9, 1)$ lies on line ℓ_0 | 51 : $P_{27449} = (24, 24, 25, 1)$ lies on line ℓ_1 |
| 19 : $P_{10537} = (8, 8, 9, 1)$ lies on line ℓ_1 | 52 : $P_{27681} = (0, 0, 26, 1)$ lies on line ℓ_0 |
| 20 : $P_{11297} = (0, 0, 10, 1)$ lies on line ℓ_0 | 53 : $P_{28572} = (27, 27, 26, 1)$ lies on line ℓ_1 |
| 21 : $P_{11660} = (11, 11, 10, 1)$ lies on line ℓ_1 | 54 : $P_{28705} = (0, 0, 27, 1)$ lies on line ℓ_0 |
| 22 : $P_{12321} = (0, 0, 11, 1)$ lies on line ℓ_0 | 55 : $P_{29563} = (26, 26, 27, 1)$ lies on line ℓ_1 |
| 23 : $P_{12651} = (10, 10, 11, 1)$ lies on line ℓ_1 | 56 : $P_{29729} = (0, 0, 28, 1)$ lies on line ℓ_0 |
| 24 : $P_{13345} = (0, 0, 12, 1)$ lies on line ℓ_0 | 57 : $P_{30686} = (29, 29, 28, 1)$ lies on line ℓ_1 |
| 25 : $P_{13774} = (13, 13, 12, 1)$ lies on line ℓ_1 | 58 : $P_{30753} = (0, 0, 29, 1)$ lies on line ℓ_0 |
| 26 : $P_{14369} = (0, 0, 13, 1)$ lies on line ℓ_0 | 59 : $P_{31677} = (28, 28, 29, 1)$ lies on line ℓ_1 |
| 27 : $P_{14765} = (12, 12, 13, 1)$ lies on line ℓ_1 | 60 : $P_{31777} = (0, 0, 30, 1)$ lies on line ℓ_0 |
| 28 : $P_{15393} = (0, 0, 14, 1)$ lies on line ℓ_0 | 61 : $P_{32800} = (31, 31, 30, 1)$ lies on line ℓ_1 |
| 29 : $P_{15888} = (15, 15, 14, 1)$ lies on line ℓ_1 | 62 : $P_{32801} = (0, 0, 31, 1)$ lies on line ℓ_0 |
| 30 : $P_{16417} = (0, 0, 15, 1)$ lies on line ℓ_0 | 63 : $P_{33791} = (30, 30, 31, 1)$ lies on line ℓ_1 |
| 31 : $P_{16879} = (14, 14, 15, 1)$ lies on line ℓ_1 | |
| 32 : $P_{17441} = (0, 0, 16, 1)$ lies on line ℓ_0 | |

The single points on the surface are:

Points on surface but on no line

The surface has 1024 points not on any line:
Too many to print.

Line Intersection Graph

	0 1
0	0 1
1	1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_{2082}

Line 1 intersects

Line	ℓ_0
in point	P_{2082}

The surface has 1089 points:
Too many to print.