

# Rank-73753 over GF(4)

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## The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(4) is 1498764909

## General information

Number of lines	5
Number of points	29
Number of singular points	0
Number of Eckardt points	1
Number of double points	2
Number of single points	18
Number of points off lines	8
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$5^5$
Type of lines on points	$3, 2^2, 1^{18}, 0^8$

## Singular Points

The surface has 0 singular points:

## The 5 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{345} = \left[ \begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{345} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_{13} \\ \ell_1 &= \left[ \begin{array}{cccc} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{355} = \left[ \begin{array}{cccc} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]_{355} = \mathbf{Pl}(0, 3, 0, 1, 0, 0)_{15}\end{aligned}$$

$$\begin{aligned}\ell_2 &= \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & 1 & \omega^2 \end{bmatrix}_{160} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}_{160} = \mathbf{Pl}(1, 1, 3, 2, 1, 1)_{234} \\ \ell_3 &= \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0, 2, 0, 1, 0, 0)_{14} \\ \ell_4 &= \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & 1 & \omega \end{bmatrix}_{135} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}_{135} = \mathbf{Pl}(1, 1, 2, 3, 1, 1)_{225}\end{aligned}$$

Rank of lines: ( 345, 355, 160, 350, 135 )

Rank of points on Klein quadric: ( 13, 15, 234, 14, 225 )

### Eckardt Points

The surface has 1 Eckardt points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1).$$

### Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_{61} = (0, 2, 2, 1) = \ell_0 \cap \ell_2$$

$$P_{81} = (0, 3, 3, 1) = \ell_0 \cap \ell_4$$

### Single Points

The surface has 18 single points:

The single points on the surface are:

$$0 : P_{11} = (0, 1, 1, 0) \text{ lies on line } \ell_0$$

$$1 : P_{15} = (0, 2, 1, 0) \text{ lies on line } \ell_1$$

$$2 : P_{16} = (1, 2, 1, 0) \text{ lies on line } \ell_2$$

$$3 : P_{19} = (0, 3, 1, 0) \text{ lies on line } \ell_3$$

$$4 : P_{20} = (1, 3, 1, 0) \text{ lies on line } \ell_4$$

$$5 : P_{28} = (2, 1, 0, 1) \text{ lies on line } \ell_2$$

$$6 : P_{29} = (3, 1, 0, 1) \text{ lies on line } \ell_4$$

$$7 : P_{42} = (0, 1, 1, 1) \text{ lies on line } \ell_0$$

$$8 : P_{45} = (0, 2, 1, 1) \text{ lies on line } \ell_1$$

$$9 : P_{47} = (2, 2, 1, 1) \text{ lies on line } \ell_4$$

$$10 : P_{49} = (0, 3, 1, 1) \text{ lies on line } \ell_3$$

$$11 : P_{52} = (3, 3, 1, 1) \text{ lies on line } \ell_2$$

$$12 : P_{54} = (1, 0, 2, 1) \text{ lies on line } \ell_4$$

$$13 : P_{57} = (0, 1, 2, 1) \text{ lies on line } \ell_3$$

$$14 : P_{65} = (0, 3, 2, 1) \text{ lies on line } \ell_1$$

$$15 : P_{70} = (1, 0, 3, 1) \text{ lies on line } \ell_2$$

$$16 : P_{73} = (0, 1, 3, 1) \text{ lies on line } \ell_1$$

$$17 : P_{77} = (0, 2, 3, 1) \text{ lies on line } \ell_3$$

The single points on the surface are:

### Points on surface but on no line

The surface has 8 points not on any line:

The points on the surface but not on lines are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_5 = (1, 1, 0, 0)$   
 2 :  $P_{12} = (1, 1, 1, 0)$   
 3 :  $P_{39} = (1, 0, 1, 1)$   
 4 :  $P_{60} = (3, 1, 2, 1)$

5 :  $P_{62} = (1, 2, 2, 1)$   
 6 :  $P_{75} = (2, 1, 3, 1)$   
 7 :  $P_{82} = (1, 3, 3, 1)$

## Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	0	1	0
2	1	0	0	0	0
3	1	1	0	0	0
4	1	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$
in point	$P_3$	$P_{61}$	$P_3$	$P_{81}$

Line 1 intersects

Line	$\ell_0$	$\ell_3$
in point	$P_3$	$P_3$

Line 2 intersects

Line	$\ell_0$
in point	$P_{61}$

Line 3 intersects

Line	$\ell_0$	$\ell_1$
in point	$P_3$	$P_3$

Line 4 intersects

Line	$\ell_0$
in point	$P_{81}$

The surface has 29 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_3 = (0, 0, 0, 1)$   
 2 :  $P_5 = (1, 1, 0, 0)$   
 3 :  $P_{11} = (0, 1, 1, 0)$   
 4 :  $P_{12} = (1, 1, 1, 0)$   
 5 :  $P_{15} = (0, 2, 1, 0)$   
 6 :  $P_{16} = (1, 2, 1, 0)$   
 7 :  $P_{19} = (0, 3, 1, 0)$   
 8 :  $P_{20} = (1, 3, 1, 0)$   
 9 :  $P_{28} = (2, 1, 0, 1)$

10 :  $P_{29} = (3, 1, 0, 1)$   
 11 :  $P_{39} = (1, 0, 1, 1)$   
 12 :  $P_{42} = (0, 1, 1, 1)$   
 13 :  $P_{45} = (0, 2, 1, 1)$   
 14 :  $P_{47} = (2, 2, 1, 1)$   
 15 :  $P_{49} = (0, 3, 1, 1)$   
 16 :  $P_{52} = (3, 3, 1, 1)$   
 17 :  $P_{54} = (1, 0, 2, 1)$   
 18 :  $P_{57} = (0, 1, 2, 1)$   
 19 :  $P_{60} = (3, 1, 2, 1)$

20 :  $P_{61} = (0, 2, 2, 1)$   
 21 :  $P_{62} = (1, 2, 2, 1)$   
 22 :  $P_{65} = (0, 3, 2, 1)$   
 23 :  $P_{70} = (1, 0, 3, 1)$   
 24 :  $P_{73} = (0, 1, 3, 1)$   
 25 :  $P_{75} = (2, 1, 3, 1)$   
 26 :  $P_{77} = (0, 2, 3, 1)$   
 27 :  $P_{81} = (0, 3, 3, 1)$   
 28 :  $P_{82} = (1, 3, 3, 1)$