

Rank-346 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)

The point rank of the equation over GF(8) is 2659486

General information

Number of lines	15
Number of points	105
Number of singular points	0
Number of Eckardt points	15
Number of double points	0
Number of single points	90
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{15}
Type of lines on points	$3^{15}, 1^{90}$

Singular Points

The surface has 0 singular points:

The 15 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{138} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{138} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{1322} \\ \ell_1 &= \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right]_{82} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right]_{82} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{1224}\end{aligned}$$

$$\begin{aligned}
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{665} = \mathbf{Pl}(1, 1, 0, 1, 1, 1)_{1273} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{3510} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{3510} = \mathbf{Pl}(2, 6, 1, 1, 0, 0)_{33} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{1758} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{1758} = \mathbf{Pl}(2, 6, 2, 1, 0, 0)_{40} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & \gamma^6 & 0 \end{bmatrix}_{2926} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 6 & 0 \end{bmatrix}_{2926} = \mathbf{Pl}(2, 6, 5, 1, 0, 0)_{61} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \gamma & \gamma^4 \\ 0 & 1 & \gamma^4 & \gamma \end{bmatrix}_{4257} = \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 7 & 2 \end{bmatrix}_{4257} = \mathbf{Pl}(4, 3, 7, 5, 1, 1)_{1710} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{3507} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{3507} = \mathbf{Pl}(4, 3, 6, 1, 0, 0)_{70} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{1755} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{1755} = \mathbf{Pl}(4, 3, 1, 1, 0, 0)_{35} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & \gamma^5 & 0 \end{bmatrix}_{2923} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{2923} = \mathbf{Pl}(4, 3, 4, 1, 0, 0)_{56} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & \gamma^2 & \gamma \\ 0 & 1 & \gamma & \gamma^2 \end{bmatrix}_{1494} = \begin{bmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 2 & 4 \end{bmatrix}_{1494} = \mathbf{Pl}(7, 5, 2, 6, 1, 1)_{1482} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & \gamma^4 & \gamma^2 \\ 0 & 1 & \gamma^2 & \gamma^4 \end{bmatrix}_{2907} = \begin{bmatrix} 1 & 0 & 7 & 4 \\ 0 & 1 & 4 & 7 \end{bmatrix}_{2907} = \mathbf{Pl}(2, 6, 4, 3, 1, 1)_{1575} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{3509} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{3509} = \mathbf{Pl}(7, 5, 7, 1, 0, 0)_{80} \\
\ell_{13} &= \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{1757} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{1757} = \mathbf{Pl}(7, 5, 3, 1, 0, 0)_{52} \\
\ell_{14} &= \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & \gamma^3 & 0 \end{bmatrix}_{2925} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 5 & 0 \end{bmatrix}_{2925} = \mathbf{Pl}(7, 5, 1, 1, 0, 0)_{38}
\end{aligned}$$

Rank of lines: (138, 82, 665, 3510, 1758, 2926, 4257, 3507, 1755, 2923, 1494, 2907, 3509, 1757, 2925)

Rank of points on Klein quadric: (1322, 1224, 1273, 33, 40, 61, 1710, 70, 35, 56, 1482, 1575, 80, 52, 38)

Eckardt Points

The surface has 15 Eckardt points:

- 0 : $P_{27} = \mathbf{P}(0, \gamma, 1, 0) = \mathbf{P}(0, 2, 1, 0)$,
- 1 : $P_{43} = \mathbf{P}(0, \gamma^2, 1, 0) = \mathbf{P}(0, 4, 1, 0)$,
- 2 : $P_{67} = \mathbf{P}(0, \gamma^4, 1, 0) = \mathbf{P}(0, 7, 1, 0)$,
- 3 : $P_{76} = \mathbf{P}(\gamma, 0, 0, 1) = \mathbf{P}(2, 0, 0, 1)$,
- 4 : $P_{78} = \mathbf{P}(\gamma^2, 0, 0, 1) = \mathbf{P}(4, 0, 0, 1)$,
- 5 : $P_{81} = \mathbf{P}(\gamma^4, 0, 0, 1) = \mathbf{P}(7, 0, 0, 1)$,
- 6 : $P_{155} = \mathbf{P}(\gamma, \gamma, 1, 1) = \mathbf{P}(2, 2, 1, 1)$,
- 7 : $P_{173} = \mathbf{P}(\gamma^2, \gamma^2, 1, 1) = \mathbf{P}(4, 4, 1, 1)$,
- 8 : $P_{200} = \mathbf{P}(\gamma^4, \gamma^4, 1, 1) = \mathbf{P}(7, 7, 1, 1)$,
- 9 : $P_{227} = \mathbf{P}(\gamma, \gamma^5, \gamma, 1) = \mathbf{P}(2, 3, 2, 1)$,
- 10 : $P_{275} = \mathbf{P}(\gamma, 1, \gamma^5, 1) = \mathbf{P}(2, 1, 3, 1)$,
- 11 : $P_{373} = \mathbf{P}(\gamma^2, \gamma^3, \gamma^2, 1) = \mathbf{P}(4, 5, 4, 1)$,
- 12 : $P_{405} = \mathbf{P}(\gamma^2, 1, \gamma^3, 1) = \mathbf{P}(4, 1, 5, 1)$,
- 13 : $P_{472} = \mathbf{P}(\gamma^4, 1, \gamma^6, 1) = \mathbf{P}(7, 1, 6, 1)$,
- 14 : $P_{576} = \mathbf{P}(\gamma^4, \gamma^6, \gamma^4, 1) = \mathbf{P}(7, 6, 7, 1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 90 single points:
The single points on the surface are:

- | | |
|--|--|
| 0 : $P_4 = (1, 1, 1, 1)$ lies on line ℓ_0 | 41 : $P_{305} = (0, 5, 3, 1)$ lies on line ℓ_{11} |
| 1 : $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 | 42 : $P_{306} = (1, 5, 3, 1)$ lies on line ℓ_{10} |
| 2 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_1 | 43 : $P_{315} = (2, 6, 3, 1)$ lies on line ℓ_3 |
| 3 : $P_{20} = (1, 1, 1, 0)$ lies on line ℓ_2 | 44 : $P_{317} = (4, 6, 3, 1)$ lies on line ℓ_4 |
| 4 : $P_{39} = (4, 3, 1, 0)$ lies on line ℓ_6 | 45 : $P_{320} = (7, 6, 3, 1)$ lies on line ℓ_5 |
| 5 : $P_{58} = (7, 5, 1, 0)$ lies on line ℓ_{10} | 46 : $P_{332} = (3, 0, 4, 1)$ lies on line ℓ_{11} |
| 6 : $P_{61} = (2, 6, 1, 0)$ lies on line ℓ_{11} | 47 : $P_{342} = (5, 1, 4, 1)$ lies on line ℓ_1 |
| 7 : $P_{82} = (0, 1, 0, 1)$ lies on line ℓ_2 | 48 : $P_{364} = (3, 4, 4, 1)$ lies on line ℓ_{10} |
| 8 : $P_{83} = (1, 1, 0, 1)$ lies on line ℓ_1 | 49 : $P_{371} = (2, 5, 4, 1)$ lies on line ℓ_3 |
| 9 : $P_{96} = (6, 2, 0, 1)$ lies on line ℓ_{11} | 50 : $P_{376} = (7, 5, 4, 1)$ lies on line ℓ_5 |
| 10 : $P_{109} = (3, 4, 0, 1)$ lies on line ℓ_6 | 51 : $P_{379} = (2, 6, 4, 1)$ lies on line ℓ_{12} |
| 11 : $P_{135} = (5, 7, 0, 1)$ lies on line ℓ_{10} | 52 : $P_{381} = (4, 6, 4, 1)$ lies on line ℓ_{13} |
| 12 : $P_{138} = (0, 0, 1, 1)$ lies on line ℓ_0 | 53 : $P_{384} = (7, 6, 4, 1)$ lies on line ℓ_{14} |
| 13 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_2 | 54 : $P_{387} = (2, 7, 4, 1)$ lies on line ℓ_7 |
| 14 : $P_{146} = (0, 1, 1, 1)$ lies on line ℓ_1 | 55 : $P_{389} = (4, 7, 4, 1)$ lies on line ℓ_8 |
| 15 : $P_{157} = (4, 2, 1, 1)$ lies on line ℓ_4 | 56 : $P_{392} = (7, 7, 4, 1)$ lies on line ℓ_9 |
| 16 : $P_{160} = (7, 2, 1, 1)$ lies on line ℓ_5 | 57 : $P_{403} = (2, 1, 5, 1)$ lies on line ℓ_{12} |
| 17 : $P_{164} = (3, 3, 1, 1)$ lies on line ℓ_0 | 58 : $P_{408} = (7, 1, 5, 1)$ lies on line ℓ_{14} |
| 18 : $P_{171} = (2, 4, 1, 1)$ lies on line ℓ_7 | 59 : $P_{419} = (2, 3, 5, 1)$ lies on line ℓ_7 |
| 19 : $P_{176} = (7, 4, 1, 1)$ lies on line ℓ_9 | 60 : $P_{421} = (4, 3, 5, 1)$ lies on line ℓ_8 |
| 20 : $P_{182} = (5, 5, 1, 1)$ lies on line ℓ_0 | 61 : $P_{424} = (7, 3, 5, 1)$ lies on line ℓ_9 |
| 21 : $P_{191} = (6, 6, 1, 1)$ lies on line ℓ_0 | 62 : $P_{430} = (5, 4, 5, 1)$ lies on line ℓ_2 |
| 22 : $P_{195} = (2, 7, 1, 1)$ lies on line ℓ_{12} | 63 : $P_{441} = (0, 6, 5, 1)$ lies on line ℓ_6 |
| 23 : $P_{197} = (4, 7, 1, 1)$ lies on line ℓ_{13} | 64 : $P_{442} = (1, 6, 5, 1)$ lies on line ℓ_{11} |
| 24 : $P_{207} = (6, 0, 2, 1)$ lies on line ℓ_{10} | 65 : $P_{451} = (2, 7, 5, 1)$ lies on line ℓ_3 |
| 25 : $P_{212} = (3, 1, 2, 1)$ lies on line ℓ_1 | 66 : $P_{453} = (4, 7, 5, 1)$ lies on line ℓ_4 |
| 26 : $P_{223} = (6, 2, 2, 1)$ lies on line ℓ_6 | 67 : $P_{456} = (7, 7, 5, 1)$ lies on line ℓ_5 |
| 27 : $P_{229} = (4, 3, 2, 1)$ lies on line ℓ_{13} | 68 : $P_{467} = (2, 1, 6, 1)$ lies on line ℓ_3 |
| 28 : $P_{232} = (7, 3, 2, 1)$ lies on line ℓ_{14} | 69 : $P_{469} = (4, 1, 6, 1)$ lies on line ℓ_4 |
| 29 : $P_{235} = (2, 4, 2, 1)$ lies on line ℓ_3 | 70 : $P_{475} = (2, 2, 6, 1)$ lies on line ℓ_7 |
| 30 : $P_{237} = (4, 4, 2, 1)$ lies on line ℓ_4 | 71 : $P_{477} = (4, 2, 6, 1)$ lies on line ℓ_8 |
| 31 : $P_{240} = (7, 4, 2, 1)$ lies on line ℓ_5 | 72 : $P_{480} = (7, 2, 6, 1)$ lies on line ℓ_9 |
| 32 : $P_{243} = (2, 5, 2, 1)$ lies on line ℓ_7 | 73 : $P_{481} = (0, 3, 6, 1)$ lies on line ℓ_{10} |
| 33 : $P_{245} = (4, 5, 2, 1)$ lies on line ℓ_8 | 74 : $P_{482} = (1, 3, 6, 1)$ lies on line ℓ_6 |
| 34 : $P_{248} = (7, 5, 2, 1)$ lies on line ℓ_9 | 75 : $P_{499} = (2, 5, 6, 1)$ lies on line ℓ_{12} |
| 35 : $P_{277} = (4, 1, 3, 1)$ lies on line ℓ_8 | 76 : $P_{501} = (4, 5, 6, 1)$ lies on line ℓ_{13} |
| 36 : $P_{280} = (7, 1, 3, 1)$ lies on line ℓ_9 | 77 : $P_{504} = (7, 5, 6, 1)$ lies on line ℓ_{14} |
| 37 : $P_{284} = (3, 2, 3, 1)$ lies on line ℓ_2 | 78 : $P_{519} = (6, 7, 6, 1)$ lies on line ℓ_2 |
| 38 : $P_{299} = (2, 4, 3, 1)$ lies on line ℓ_{12} | 79 : $P_{526} = (5, 0, 7, 1)$ lies on line ℓ_6 |
| 39 : $P_{301} = (4, 4, 3, 1)$ lies on line ℓ_{13} | 80 : $P_{535} = (6, 1, 7, 1)$ lies on line ℓ_1 |
| 40 : $P_{304} = (7, 4, 3, 1)$ lies on line ℓ_{14} | 81 : $P_{539} = (2, 2, 7, 1)$ lies on line ℓ_{12} |

82 : $P_{541} = (4, 2, 7, 1)$ lies on line ℓ_{13}
83 : $P_{544} = (7, 2, 7, 1)$ lies on line ℓ_{14}
84 : $P_{547} = (2, 3, 7, 1)$ lies on line ℓ_3
85 : $P_{549} = (4, 3, 7, 1)$ lies on line ℓ_4
86 : $P_{552} = (7, 3, 7, 1)$ lies on line ℓ_5

87 : $P_{571} = (2, 6, 7, 1)$ lies on line ℓ_7
88 : $P_{573} = (4, 6, 7, 1)$ lies on line ℓ_8
89 : $P_{582} = (5, 7, 7, 1)$ lies on line ℓ_{11}

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	1	0	0	1	0	1	0	1	1	0	0	1
1	0	0	0	0	0	1	1	1	0	0	1	1	0	1	0
2	0	0	0	0	1	0	1	0	0	1	1	1	1	0	0
3	1	0	0	0	1	1	0	1	0	0	1	0	1	0	0
4	0	0	1	1	0	1	1	0	1	0	0	0	0	1	0
5	0	1	0	1	1	0	0	0	0	1	0	1	0	0	1
6	1	1	1	0	1	0	0	1	0	0	0	0	0	0	1
7	0	1	0	1	0	0	1	0	1	1	0	0	1	0	0
8	1	0	0	0	1	0	0	1	0	1	0	1	0	1	0
9	0	0	1	0	0	1	0	1	1	0	1	0	0	0	1
10	1	1	1	1	0	0	0	0	0	1	0	0	0	1	0
11	1	1	1	0	0	1	0	0	1	0	0	0	1	0	0
12	0	0	1	1	0	0	0	1	0	0	0	1	0	1	1
13	0	1	0	0	1	0	0	0	1	0	1	0	1	0	1
14	1	0	0	0	0	1	1	0	0	1	0	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_3	ℓ_6	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}
in point	P_{155}	P_{200}	P_{173}	P_{155}	P_{173}	P_{200}

Line 1 intersects

Line	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{13}
in point	P_{472}	P_{275}	P_{275}	P_{405}	P_{472}	P_{405}

Line 2 intersects

Line	ℓ_4	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_{373}	P_{373}	P_{576}	P_{576}	P_{227}	P_{227}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{12}
in point	P_{155}	P_{27}	P_{27}	P_{76}	P_{155}	P_{76}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_8	ℓ_{13}
in point	P_{373}	P_{27}	P_{27}	P_{373}	P_{78}	P_{78}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_9	ℓ_{11}	ℓ_{14}
in point	P_{472}	P_{27}	P_{27}	P_{81}	P_{472}	P_{81}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_7	ℓ_{14}
in point	P_{200}	P_{275}	P_{373}	P_{373}	P_{275}	P_{200}

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_8	ℓ_9	ℓ_{12}
in point	P_{275}	P_{76}	P_{275}	P_{43}	P_{43}	P_{76}

Line 8 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_9	ℓ_{11}	ℓ_{13}
in point	P_{173}	P_{78}	P_{43}	P_{43}	P_{173}	P_{78}

Line 9 intersects

Line	ℓ_2	ℓ_5	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{14}
in point	P_{576}	P_{81}	P_{43}	P_{43}	P_{576}	P_{81}

Line 10 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_9	ℓ_{13}
in point	P_{155}	P_{405}	P_{576}	P_{155}	P_{576}	P_{405}

Line 11 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5	ℓ_8	ℓ_{12}
in point	P_{173}	P_{472}	P_{227}	P_{472}	P_{173}	P_{227}

Line 12 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_{11}	ℓ_{13}	ℓ_{14}
in point	P_{227}	P_{76}	P_{76}	P_{227}	P_{67}	P_{67}

Line 13 intersects

Line	ℓ_1	ℓ_4	ℓ_8	ℓ_{10}	ℓ_{12}	ℓ_{14}
in point	P_{405}	P_{78}	P_{78}	P_{405}	P_{67}	P_{67}

Line 14 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_9	ℓ_{12}	ℓ_{13}
in point	P_{200}	P_{81}	P_{200}	P_{81}	P_{67}	P_{67}

The surface has 105 points:

The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$	20 : $P_{146} = (0, 1, 1, 1)$	40 : $P_{237} = (4, 4, 2, 1)$
1 : $P_5 = (1, 1, 0, 0)$	21 : $P_{155} = (2, 2, 1, 1)$	41 : $P_{240} = (7, 4, 2, 1)$
2 : $P_{12} = (1, 0, 1, 0)$	22 : $P_{157} = (4, 2, 1, 1)$	42 : $P_{243} = (2, 5, 2, 1)$
3 : $P_{20} = (1, 1, 1, 0)$	23 : $P_{160} = (7, 2, 1, 1)$	43 : $P_{245} = (4, 5, 2, 1)$
4 : $P_{27} = (0, 2, 1, 0)$	24 : $P_{164} = (3, 3, 1, 1)$	44 : $P_{248} = (7, 5, 2, 1)$
5 : $P_{39} = (4, 3, 1, 0)$	25 : $P_{171} = (2, 4, 1, 1)$	45 : $P_{275} = (2, 1, 3, 1)$
6 : $P_{43} = (0, 4, 1, 0)$	26 : $P_{173} = (4, 4, 1, 1)$	46 : $P_{277} = (4, 1, 3, 1)$
7 : $P_{58} = (7, 5, 1, 0)$	27 : $P_{176} = (7, 4, 1, 1)$	47 : $P_{280} = (7, 1, 3, 1)$
8 : $P_{61} = (2, 6, 1, 0)$	28 : $P_{182} = (5, 5, 1, 1)$	48 : $P_{284} = (3, 2, 3, 1)$
9 : $P_{67} = (0, 7, 1, 0)$	29 : $P_{191} = (6, 6, 1, 1)$	49 : $P_{299} = (2, 4, 3, 1)$
10 : $P_{76} = (2, 0, 0, 1)$	30 : $P_{195} = (2, 7, 1, 1)$	50 : $P_{301} = (4, 4, 3, 1)$
11 : $P_{78} = (4, 0, 0, 1)$	31 : $P_{197} = (4, 7, 1, 1)$	51 : $P_{304} = (7, 4, 3, 1)$
12 : $P_{81} = (7, 0, 0, 1)$	32 : $P_{200} = (7, 7, 1, 1)$	52 : $P_{305} = (0, 5, 3, 1)$
13 : $P_{82} = (0, 1, 0, 1)$	33 : $P_{207} = (6, 0, 2, 1)$	53 : $P_{306} = (1, 5, 3, 1)$
14 : $P_{83} = (1, 1, 0, 1)$	34 : $P_{212} = (3, 1, 2, 1)$	54 : $P_{315} = (2, 6, 3, 1)$
15 : $P_{96} = (6, 2, 0, 1)$	35 : $P_{223} = (6, 2, 2, 1)$	55 : $P_{317} = (4, 6, 3, 1)$
16 : $P_{109} = (3, 4, 0, 1)$	36 : $P_{227} = (2, 3, 2, 1)$	56 : $P_{320} = (7, 6, 3, 1)$
17 : $P_{135} = (5, 7, 0, 1)$	37 : $P_{229} = (4, 3, 2, 1)$	57 : $P_{332} = (3, 0, 4, 1)$
18 : $P_{138} = (0, 0, 1, 1)$	38 : $P_{232} = (7, 3, 2, 1)$	58 : $P_{342} = (5, 1, 4, 1)$
19 : $P_{139} = (1, 0, 1, 1)$	39 : $P_{235} = (2, 4, 2, 1)$	59 : $P_{364} = (3, 4, 4, 1)$

60 : $P_{371} = (2, 5, 4, 1)$
 61 : $P_{373} = (4, 5, 4, 1)$
 62 : $P_{376} = (7, 5, 4, 1)$
 63 : $P_{379} = (2, 6, 4, 1)$
 64 : $P_{381} = (4, 6, 4, 1)$
 65 : $P_{384} = (7, 6, 4, 1)$
 66 : $P_{387} = (2, 7, 4, 1)$
 67 : $P_{389} = (4, 7, 4, 1)$
 68 : $P_{392} = (7, 7, 4, 1)$
 69 : $P_{403} = (2, 1, 5, 1)$
 70 : $P_{405} = (4, 1, 5, 1)$
 71 : $P_{408} = (7, 1, 5, 1)$
 72 : $P_{419} = (2, 3, 5, 1)$
 73 : $P_{421} = (4, 3, 5, 1)$
 74 : $P_{424} = (7, 3, 5, 1)$
 75 : $P_{430} = (5, 4, 5, 1)$

76 : $P_{441} = (0, 6, 5, 1)$
 77 : $P_{442} = (1, 6, 5, 1)$
 78 : $P_{451} = (2, 7, 5, 1)$
 79 : $P_{453} = (4, 7, 5, 1)$
 80 : $P_{456} = (7, 7, 5, 1)$
 81 : $P_{467} = (2, 1, 6, 1)$
 82 : $P_{469} = (4, 1, 6, 1)$
 83 : $P_{472} = (7, 1, 6, 1)$
 84 : $P_{475} = (2, 2, 6, 1)$
 85 : $P_{477} = (4, 2, 6, 1)$
 86 : $P_{480} = (7, 2, 6, 1)$
 87 : $P_{481} = (0, 3, 6, 1)$
 88 : $P_{482} = (1, 3, 6, 1)$
 89 : $P_{499} = (2, 5, 6, 1)$
 90 : $P_{501} = (4, 5, 6, 1)$
 91 : $P_{504} = (7, 5, 6, 1)$

92 : $P_{519} = (6, 7, 6, 1)$
 93 : $P_{526} = (5, 0, 7, 1)$
 94 : $P_{535} = (6, 1, 7, 1)$
 95 : $P_{539} = (2, 2, 7, 1)$
 96 : $P_{541} = (4, 2, 7, 1)$
 97 : $P_{544} = (7, 2, 7, 1)$
 98 : $P_{547} = (2, 3, 7, 1)$
 99 : $P_{549} = (4, 3, 7, 1)$
 100 : $P_{552} = (7, 3, 7, 1)$
 101 : $P_{571} = (2, 6, 7, 1)$
 102 : $P_{573} = (4, 6, 7, 1)$
 103 : $P_{576} = (7, 6, 7, 1)$
 104 : $P_{582} = (5, 7, 7, 1)$