Rank-65871 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is 1091051589

General information

Number of lines	4
Number of points	4161
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	252
Number of points off lines	3905
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{4}
Type of lines on points	$2^4, 1^{252}, 0^{3905}$

Singular Points

The surface has 2 singular points:

$$\begin{array}{l} 0: \ P_{4163} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1) \\ 1: \ P_{4227} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1) \end{array}$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{266304} = \mathbf{Pl}(1,0,0,1,0,0)_{130}$$

$$\ell_2 = \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{241274} = \mathbf{Pl}(0,0,1,1,56,1)_{14958978}$$

$$\ell_3 = \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{237113} = \mathbf{Pl}(0,0,1,1,57,1)_{15221058}$$

Rank of lines: (0, 266304, 241274, 237113)

Rank of points on Klein quadric: (0, 130, 14958978, 15221058)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points: The double points on the surface are:

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{60} = (56, 1, 0, 0) = \ell_0 \cap \ell_2$$

$$P_{61} = (57, 1, 0, 0) = \ell_0 \cap \ell_3$$

$P_{8258} = (0,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 252 single points: The single points on the surface are:

 $0: P_0 = (1, 0, 0, 0)$ lies on line ℓ_0 $22: P_{26} = (22, 1, 0, 0)$ lies on line ℓ_0 1: $P_5 = (1, 1, 0, 0)$ lies on line ℓ_0 23: $P_{27} = (23, 1, 0, 0)$ lies on line ℓ_0 2: $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0 24: $P_{28} = (24, 1, 0, 0)$ lies on line ℓ_0 $3: P_7 = (3,1,0,0)$ lies on line ℓ_0 25: $P_{29} = (25, 1, 0, 0)$ lies on line ℓ_0 4: $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0 26: $P_{30} = (26, 1, 0, 0)$ lies on line ℓ_0 5: $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0 27: $P_{31} = (27, 1, 0, 0)$ lies on line ℓ_0 6: $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0 28: $P_{32} = (28, 1, 0, 0)$ lies on line ℓ_0 7: $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0 29: $P_{33} = (29, 1, 0, 0)$ lies on line ℓ_0 8: $P_{12} = (8, 1, 0, 0)$ lies on line ℓ_0 $30: P_{34} = (30, 1, 0, 0)$ lies on line ℓ_0 9: $P_{13} = (9, 1, 0, 0)$ lies on line ℓ_0 $31: P_{35} = (31, 1, 0, 0)$ lies on line ℓ_0 10: $P_{14} = (10, 1, 0, 0)$ lies on line ℓ_0 $32: P_{36} = (32, 1, 0, 0)$ lies on line ℓ_0 11: $P_{15} = (11, 1, 0, 0)$ lies on line ℓ_0 33: $P_{37} = (33, 1, 0, 0)$ lies on line ℓ_0 12 : $P_{16} = (12, 1, 0, 0)$ lies on line ℓ_0 $34: P_{38} = (34, 1, 0, 0)$ lies on line ℓ_0 13: $P_{17} = (13, 1, 0, 0)$ lies on line ℓ_0 $35: P_{39} = (35, 1, 0, 0)$ lies on line ℓ_0 14: $P_{18} = (14, 1, 0, 0)$ lies on line ℓ_0 $36: P_{40} = (36, 1, 0, 0)$ lies on line ℓ_0 15: $P_{19} = (15, 1, 0, 0)$ lies on line ℓ_0 37 : $P_{41} = (37, 1, 0, 0)$ lies on line ℓ_0 16: $P_{20} = (16, 1, 0, 0)$ lies on line ℓ_0 $38: P_{42} = (38, 1, 0, 0)$ lies on line ℓ_0 17: $P_{21} = (17, 1, 0, 0)$ lies on line ℓ_0 $39: P_{43} = (39, 1, 0, 0)$ lies on line ℓ_0 18 : $P_{22} = (18, 1, 0, 0)$ lies on line ℓ_0 40: $P_{44} = (40, 1, 0, 0)$ lies on line ℓ_0 19 : $P_{23} = (19,1,0,0)$ lies on line ℓ_0 41: $P_{45} = (41, 1, 0, 0)$ lies on line ℓ_0 20: $P_{24} = (20, 1, 0, 0)$ lies on line ℓ_0 42: $P_{46} = (42, 1, 0, 0)$ lies on line ℓ_0 21: $P_{25} = (21, 1, 0, 0)$ lies on line ℓ_0 43: $P_{47} = (43, 1, 0, 0)$ lies on line ℓ_0

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98: P_{6467} = (1, 36, 0, 1) lies on line \ell_1
44: P_{48} = (44, 1, 0, 0) lies on line \ell_0
                                                                      99: P_{6531} = (1, 37, 0, 1) lies on line \ell_1
45: P_{49} = (45, 1, 0, 0) lies on line \ell_0
46: P_{50} = (46, 1, 0, 0) lies on line \ell_0
                                                                      100: P_{6595} = (1, 38, 0, 1) lies on line \ell_1
47: P_{51} = (47, 1, 0, 0) lies on line \ell_0
                                                                      101: P_{6659} = (1, 39, 0, 1) lies on line \ell_1
48: P_{52} = (48, 1, 0, 0) lies on line \ell_0
                                                                      102: P_{6723} = (1, 40, 0, 1) lies on line \ell_1
49: P_{53} = (49, 1, 0, 0) lies on line \ell_0
                                                                      103: P_{6787} = (1, 41, 0, 1) lies on line \ell_1
50: P_{54} = (50, 1, 0, 0) lies on line \ell_0
                                                                      104: P_{6851} = (1, 42, 0, 1) lies on line \ell_1
51: P_{55} = (51, 1, 0, 0) lies on line \ell_0
                                                                      105: P_{6915} = (1, 43, 0, 1) lies on line \ell_1
52: P_{56} = (52, 1, 0, 0) lies on line \ell_0
                                                                      106: P_{6979} = (1, 44, 0, 1) lies on line \ell_1
53: P_{57} = (53, 1, 0, 0) lies on line \ell_0
                                                                      107: P_{7043} = (1, 45, 0, 1) lies on line \ell_1
54: P_{58} = (54, 1, 0, 0) lies on line \ell_0
                                                                      108: P_{7107} = (1, 46, 0, 1) lies on line \ell_1
55: P_{59} = (55, 1, 0, 0) lies on line \ell_0
                                                                      109: P_{7171} = (1, 47, 0, 1) lies on line \ell_1
56: P_{62} = (58, 1, 0, 0) lies on line \ell_0
                                                                      110: P_{7235} = (1, 48, 0, 1) lies on line \ell_1
57: P_{63} = (59, 1, 0, 0) lies on line \ell_0
                                                                      111: P_{7299} = (1, 49, 0, 1) lies on line \ell_1
58: P_{64} = (60, 1, 0, 0) lies on line \ell_0
                                                                      112: P_{7363} = (1, 50, 0, 1) lies on line \ell_1
59: P_{65} = (61, 1, 0, 0) lies on line \ell_0
                                                                      113: P_{7427} = (1, 51, 0, 1) lies on line \ell_1
60: P_{66} = (62, 1, 0, 0) lies on line \ell_0
                                                                      114: P_{7491} = (1, 52, 0, 1) lies on line \ell_1
61: P_{67} = (63, 1, 0, 0) lies on line \ell_0
                                                                      115: P_{7555} = (1, 53, 0, 1) lies on line \ell_1
                                                                      116: P_{7619} = (1, 54, 0, 1) lies on line \ell_1
62: P_{4163} = (1,0,0,1) lies on line \ell_1
63: P_{4227} = (1, 1, 0, 1) lies on line \ell_1
                                                                      117: P_{7683} = (1, 55, 0, 1) lies on line \ell_1
64: P_{4291} = (1, 2, 0, 1) lies on line \ell_1
                                                                      118: P_{7747} = (1, 56, 0, 1) lies on line \ell_1
65: P_{4355} = (1, 3, 0, 1) lies on line \ell_1
                                                                      119: P_{7811} = (1, 57, 0, 1) lies on line \ell_1
66: P_{4419} = (1, 4, 0, 1) lies on line \ell_1
                                                                      120: P_{7875} = (1, 58, 0, 1) lies on line \ell_1
67: P_{4483} = (1, 5, 0, 1) lies on line \ell_1
                                                                      121: P_{7939} = (1, 59, 0, 1) lies on line \ell_1
68: P_{4547} = (1, 6, 0, 1) lies on line \ell_1
                                                                      122: P_{8003} = (1, 60, 0, 1) lies on line \ell_1
69: P_{4611} = (1, 7, 0, 1) lies on line \ell_1
                                                                      123: P_{8067} = (1, 61, 0, 1) lies on line \ell_1
70: P_{4675} = (1, 8, 0, 1) lies on line \ell_1
                                                                      124: P_{8131} = (1, 62, 0, 1) lies on line \ell_1
71: P_{4739} = (1, 9, 0, 1) lies on line \ell_1
                                                                      125: P_{8195} = (1, 63, 0, 1) lies on line \ell_1
72: P_{4803} = (1, 10, 0, 1) lies on line \ell_1
                                                                      126: P_{8377} = (56, 1, 1, 1) lies on line \ell_2
73: P_{4867} = (1, 11, 0, 1) lies on line \ell_1
                                                                      127: P_{8378} = (57, 1, 1, 1) lies on line \ell_3
74: P_{4931} = (1, 12, 0, 1) lies on line \ell_1
                                                                      128: P_{8402} = (17, 2, 1, 1) lies on line \ell_2
75: P_{4995} = (1, 13, 0, 1) lies on line \ell_1
                                                                      129: P_{8404} = (19, 2, 1, 1) lies on line \ell_3
76: P_{5059} = (1, 14, 0, 1) lies on line \ell_1
                                                                      130: P_{8490} = (41, 3, 1, 1) lies on line \ell_2
77: P_{5123} = (1, 15, 0, 1) lies on line \ell_1
                                                                      131: P_{8491} = (42, 3, 1, 1) lies on line \ell_3
78: P_{5187} = (1, 16, 0, 1) lies on line \ell_1
                                                                      132: P_{8547} = (34, 4, 1, 1) lies on line \ell_2
79: P_{5251} = (1, 17, 0, 1) lies on line \ell_1
                                                                      133: P_{8551} = (38, 4, 1, 1) lies on line \ell_3
80: P_{5315} = (1, 18, 0, 1) lies on line \ell_1
                                                                      134: P_{8603} = (26, 5, 1, 1) lies on line \ell_2
81: P_{5379} = (1, 19, 0, 1) lies on line \ell_1
                                                                      135: P_{8608} = (31, 5, 1, 1) lies on line \ell_3
82: P_{5443} = (1, 20, 0, 1) lies on line \ell_1
                                                                      136: P_{8692} = (51, 6, 1, 1) lies on line \ell_2
83: P_{5507} = (1, 21, 0, 1) lies on line \ell_1
                                                                      137: P_{8694} = (53, 6, 1, 1) lies on line \ell_3
84: P_{5571} = (1, 22, 0, 1) lies on line \ell_1
                                                                      138: P_{8716} = (11, 7, 1, 1) lies on line \ell_2
85: P_{5635} = (1, 23, 0, 1) lies on line \ell_1
                                                                      139: P_{8717} = (12, 7, 1, 1) lies on line \ell_3
86 : P_{5699} = (1, 24, 0, 1) lies on line \ell_1
                                                                      140: P_{8806} = (37, 8, 1, 1) lies on line \ell_2
87: P_{5763} = (1, 25, 0, 1) lies on line \ell_1
                                                                      141: P_{8814} = (45, 8, 1, 1) lies on line \ell_3
88: P_{5827} = (1, 26, 0, 1) lies on line \ell_1
                                                                      142: P_{8853} = (20, 9, 1, 1) lies on line \ell_3
89: P_{5891} = (1, 27, 0, 1) lies on line \ell_1
                                                                      143: P_{8862} = (29, 9, 1, 1) lies on line \ell_2
90: P_{5955} = (1, 28, 0, 1) lies on line \ell_1
                                                                      144: P_{8949} = (52, 10, 1, 1) lies on line \ell_2
91: P_{6019} = (1, 29, 0, 1) lies on line \ell_1
                                                                      145: P_{8959} = (62, 10, 1, 1) lies on line \ell_3
92: P_{6083} = (1, 30, 0, 1) lies on line \ell_1
                                                                      146: P_{8968} = (7, 11, 1, 1) lies on line \ell_3
93: P_{6147} = (1, 31, 0, 1) lies on line \ell_1
                                                                      147: P_{8973} = (12, 11, 1, 1) lies on line \ell_2
94: P_{6211} = (1, 32, 0, 1) lies on line \ell_1
                                                                      148: P_{9032} = (7, 12, 1, 1) lies on line \ell_2
95: P_{6275} = (1, 33, 0, 1) lies on line \ell_1
                                                                      149: P_{9036} = (11, 12, 1, 1) lies on line \ell_3
                                                                      150 : P_{9139} = (50, 13, 1, 1) lies on line \ell_3
96: P_{6339} = (1, 34, 0, 1) lies on line \ell_1
97: P_{6403} = (1, 35, 0, 1) lies on line \ell_1
                                                                      151: P_{9152} = (63, 13, 1, 1) lies on line \ell_2
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152: P_{9175} = (22, 14, 1, 1) lies on line \ell_2
                                                                    203: P_{10813} = (60, 39, 1, 1) lies on line \ell_2
                                                                    204 : P_{10835} = (18, 40, 1, 1) lies on line \ell_2
153: P_{9177} = (24, 14, 1, 1) lies on line \ell_3
                                                                    205 : P_{10875} = (58, 40, 1, 1) lies on line \ell_3
154: P_{9250} = (33, 15, 1, 1) lies on line \ell_3
155: P_{9263} = (46, 15, 1, 1) lies on line \ell_2
                                                                    206: P_{10884} = (3, 41, 1, 1) lies on line \ell_3
156: P_{9324} = (43, 16, 1, 1) lies on line \ell_2
                                                                    207: P_{10923} = (42, 41, 1, 1) lies on line \ell_2
157: P_{9340} = (59, 16, 1, 1) lies on line \ell_3
                                                                    208: P_{10948} = (3, 42, 1, 1) lies on line \ell_2
158: P_{9347} = (2, 17, 1, 1) lies on line \ell_3
                                                                    209: P_{10986} = (41, 42, 1, 1) lies on line \ell_3
159: P_{9364} = (19, 17, 1, 1) lies on line \ell_2
                                                                    210: P_{11025} = (16, 43, 1, 1) lies on line \ell_3
160: P_{9449} = (40, 18, 1, 1) lies on line \ell_3
                                                                    211: P_{11068} = (59, 43, 1, 1) lies on line \ell_2
161: P_{9467} = (58, 18, 1, 1) lies on line \ell_2
                                                                    212: P_{11101} = (28, 44, 1, 1) lies on line \ell_3
162: P_{9475} = (2, 19, 1, 1) lies on line \ell_2
                                                                    213: P_{11121} = (48, 44, 1, 1) lies on line \ell_2
163: P_{9490} = (17, 19, 1, 1) lies on line \ell_3
                                                                    214: P_{11145} = (8, 45, 1, 1) lies on line \ell_2
164: P_{9546} = (9, 20, 1, 1) lies on line \ell_2
                                                                    215: P_{11174} = (37, 45, 1, 1) lies on line \ell_3
165: P_{9566} = (29, 20, 1, 1) lies on line \ell_3
                                                                    216: P_{11216} = (15, 46, 1, 1) lies on line \ell_3
166: P_{9637} = (36, 21, 1, 1) lies on line \ell_3
                                                                    217: P_{11234} = (33, 46, 1, 1) lies on line \ell_2
167: P_{9650} = (49, 21, 1, 1) lies on line \ell_2
                                                                    218: P_{11290} = (25, 47, 1, 1) lies on line \ell_2
168: P_{9679} = (14, 22, 1, 1) lies on line \ell_3
                                                                    219: P_{11319} = (54, 47, 1, 1) lies on line \ell_3
169: P_{9689} = (24, 22, 1, 1) lies on line \ell_2
                                                                    220: P_{11357} = (28, 48, 1, 1) lies on line \ell_2
170: P_{9761} = (32, 23, 1, 1) lies on line \ell_2
                                                                    221: P_{11373} = (44, 48, 1, 1) lies on line \ell_3
171: P_{9784} = (55, 23, 1, 1) lies on line \ell_3
                                                                    222: P_{11414} = (21, 49, 1, 1) lies on line \ell_3
172 : P_{9807} = (14, 24, 1, 1) lies on line \ell_2
                                                                    223: P_{11429} = (36, 49, 1, 1) lies on line \ell_2
173 : P_{9815} = (22, 24, 1, 1) lies on line \ell_3
                                                                    224: P_{11470} = (13, 50, 1, 1) lies on line \ell_2
174: P_{9904} = (47, 25, 1, 1) lies on line \ell_3
                                                                     225: P_{11520} = (63, 50, 1, 1) lies on line \ell_3
175: P_{9911} = (54, 25, 1, 1) lies on line \ell_2
                                                                    226: P_{11527} = (6, 51, 1, 1) lies on line \ell_3
176: P_{9926} = (5, 26, 1, 1) lies on line \ell_3
                                                                    227: P_{11574} = (53, 51, 1, 1) lies on line \ell_2
177: P_{9952} = (31, 26, 1, 1) lies on line \ell_2
                                                                    228: P_{11595} = (10, 52, 1, 1) lies on line \ell_3
178: P_{10024} = (39, 27, 1, 1) lies on line \ell_2
                                                                    229: P_{11647} = (62, 52, 1, 1) lies on line \ell_2
179: P_{10045} = (60, 27, 1, 1) lies on line \ell_3
                                                                    230: P_{11655} = (6, 53, 1, 1) lies on line \ell_2
180: P_{10093} = (44, 28, 1, 1) lies on line \ell_2
                                                                    231: P_{11700} = (51, 53, 1, 1) lies on line \ell_3
181: P_{10097} = (48, 28, 1, 1) lies on line \ell_3
                                                                    232: P_{11738} = (25, 54, 1, 1) lies on line \ell_3
182: P_{10122} = (9, 29, 1, 1) lies on line \ell_3
                                                                    233: P_{11760} = (47, 54, 1, 1) lies on line \ell_2
183: P_{10133} = (20, 29, 1, 1) lies on line \ell_2
                                                                    234: P_{11800} = (23, 55, 1, 1) lies on line \ell_2
184: P_{10212} = (35, 30, 1, 1) lies on line \ell_3
                                                                    235: P_{11809} = (32, 55, 1, 1) lies on line \ell_3
185: P_{10238} = (61, 30, 1, 1) lies on line \ell_2
                                                                    236: P_{11842} = (1, 56, 1, 1) lies on line \ell_3
186: P_{10246} = (5, 31, 1, 1) lies on line \ell_2
                                                                    237: P_{11898} = (57, 56, 1, 1) lies on line \ell_2
187: P_{10267} = (26, 31, 1, 1) lies on line \ell_3
                                                                    238: P_{11906} = (1, 57, 1, 1) lies on line \ell_2
188: P_{10328} = (23, 32, 1, 1) lies on line \ell_3
                                                                    239: P_{11961} = (56, 57, 1, 1) lies on line \ell_3
189 : P_{10360} = (55, 32, 1, 1) lies on line \ell_2
                                                                     240 : P_{11987} = (18, 58, 1, 1) lies on line \ell_3
190 : P_{10384} = (15, 33, 1, 1) lies on line \ell_2
                                                                    241: P_{12009} = (40, 58, 1, 1) lies on line \ell_2
191: P_{10415} = (46, 33, 1, 1) lies on line \ell_3
                                                                    242: P_{12049} = (16, 59, 1, 1) lies on line \ell_2
192: P_{10437} = (4, 34, 1, 1) lies on line \ell_3
                                                                    243: P_{12076} = (43, 59, 1, 1) lies on line \ell_3
193: P_{10471} = (38, 34, 1, 1) lies on line \ell_2
                                                                    244: P_{12124} = (27, 60, 1, 1) lies on line \ell_2
194: P_{10527} = (30, 35, 1, 1) lies on line \ell_2
                                                                    245: P_{12136} = (39, 60, 1, 1) lies on line \ell_3
195: P_{10558} = (61, 35, 1, 1) lies on line \ell_3
                                                                    246: P_{12191} = (30, 61, 1, 1) lies on line \ell_3
196: P_{10582} = (21, 36, 1, 1) lies on line \ell_2
                                                                     247: P_{12196} = (35, 61, 1, 1) lies on line \ell_2
197: P_{10610} = (49, 36, 1, 1) lies on line \ell_3
                                                                     248: P_{12235} = (10, 62, 1, 1) lies on line \ell_2
198: P_{10633} = (8, 37, 1, 1) lies on line \ell_3
                                                                    249: P_{12277} = (52, 62, 1, 1) lies on line \ell_3
199: P_{10670} = (45, 37, 1, 1) lies on line \ell_2
                                                                    250: P_{12302} = (13, 63, 1, 1) lies on line \ell_3
200: P_{10693} = (4, 38, 1, 1) lies on line \ell_2
                                                                    251: P_{12339} = (50, 63, 1, 1) lies on line \ell_2
201: P_{10723} = (34, 38, 1, 1) lies on line \ell_3
202: P_{10780} = (27, 39, 1, 1) lies on line \ell_3
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The single points on the surface are:

Points on surface but on no line

The surface has 3905 points not on any line: Too many to print.

Line Intersection Graph

 $\begin{array}{c|c} & 0 \ 1 \ 2 \ 3 \\ \hline 0 \ 0 \ 1 \ 1 \ 1 \\ 1 \ 1 \ 0 \ 0 \ 0 \\ 2 \ 1 \ 0 \ 0 \ 1 \\ 3 \ 1 \ 0 \ 1 \ 0 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_1	P_{60}	P_{61}

 ${\bf Line~1~intersects}$

Line	ℓ_0
in point	P_1

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_{60}	P_{8258}

Line 3 intersects

Line	ℓ_0	ℓ_2
in point	P_{61}	P_{8258}

The surface has 4161 points:

Too many to print.