

Rank-65611 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1431659929

General information

Number of lines	5
Number of points	25
Number of singular points	3
Number of Eckardt points	2
Number of double points	2
Number of single points	15
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$3^2, 2^2, 1^{15}, 0^6$

Singular Points

The surface has 3 singular points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$$

$$2 : P_{23} = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1)$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{101} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_{10} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_6
\end{aligned}$$

Rank of lines: (0, 16, 336, 84, 100)

Rank of points on Klein quadric: (0, 2, 101, 10, 6)

Eckardt Points

The surface has 2 Eckardt points:

$$0 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$$

$$1 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0).$$

Double Points

The surface has 2 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_{23} = (1, 0, 0, 1) = \ell_3 \cap \ell_4$$

Single Points

The surface has 15 single points:

The single points on the surface are:

$$0 : P_5 = (1, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$2 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$3 : P_8 = (1, 0, 1, 0) \text{ lies on line } \ell_1$$

$$4 : P_9 = (2, 0, 1, 0) \text{ lies on line } \ell_1$$

$$5 : P_{10} = (3, 0, 1, 0) \text{ lies on line } \ell_1$$

$$6 : P_{11} = (0, 1, 1, 0) \text{ lies on line } \ell_2$$

$$7 : P_{15} = (0, 2, 1, 0) \text{ lies on line } \ell_2$$

$$8 : P_{19} = (0, 3, 1, 0) \text{ lies on line } \ell_2$$

$$9 : P_{27} = (1, 1, 0, 1) \text{ lies on line } \ell_3$$

$$10 : P_{31} = (1, 2, 0, 1) \text{ lies on line } \ell_3$$

$$11 : P_{35} = (1, 3, 0, 1) \text{ lies on line } \ell_3$$

$$12 : P_{39} = (1, 0, 1, 1) \text{ lies on line } \ell_4$$

$$13 : P_{54} = (1, 0, 2, 1) \text{ lies on line } \ell_4$$

$$14 : P_{70} = (1, 0, 3, 1) \text{ lies on line } \ell_4$$

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:

The points on the surface but not on lines are:

$0 : P_{43} = (2, 1, 1, 1)$
 $1 : P_{44} = (3, 1, 1, 1)$
 $2 : P_{67} = (2, 3, 2, 1)$
 $3 : P_{68} = (3, 3, 2, 1)$

$4 : P_{79} = (2, 2, 3, 1)$
 $5 : P_{80} = (3, 2, 3, 1)$

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	0
1	1	0	1	0	1
2	1	1	0	1	1
3	1	0	1	0	1
4	0	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_1	P_2	P_1	P_2

Line 3 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_1	P_1	P_{23}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_2	P_2	P_{23}

The surface has 25 points:

The points on the surface are:

$0 : P_0 = (1, 0, 0, 0)$
 $1 : P_1 = (0, 1, 0, 0)$
 $2 : P_2 = (0, 0, 1, 0)$
 $3 : P_5 = (1, 1, 0, 0)$
 $4 : P_6 = (2, 1, 0, 0)$
 $5 : P_7 = (3, 1, 0, 0)$
 $6 : P_8 = (1, 0, 1, 0)$
 $7 : P_9 = (2, 0, 1, 0)$
 $8 : P_{10} = (3, 0, 1, 0)$

$9 : P_{11} = (0, 1, 1, 0)$
 $10 : P_{15} = (0, 2, 1, 0)$
 $11 : P_{19} = (0, 3, 1, 0)$
 $12 : P_{23} = (1, 0, 0, 1)$
 $13 : P_{27} = (1, 1, 0, 1)$
 $14 : P_{31} = (1, 2, 0, 1)$
 $15 : P_{35} = (1, 3, 0, 1)$
 $16 : P_{39} = (1, 0, 1, 1)$
 $17 : P_{43} = (2, 1, 1, 1)$

$18 : P_{44} = (3, 1, 1, 1)$
 $19 : P_{54} = (1, 0, 2, 1)$
 $20 : P_{67} = (2, 3, 2, 1)$
 $21 : P_{68} = (3, 3, 2, 1)$
 $22 : P_{70} = (1, 0, 3, 1)$
 $23 : P_{79} = (2, 2, 3, 1)$
 $24 : P_{80} = (3, 2, 3, 1)$