Rank-38 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 122

General information

Number of lines	9
Number of points	37
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	36
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	9, 1^{36}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \mathbf{Pl}(0,3,0,0,1,0)_{31}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \mathbf{Pl}(0,2,0,0,1,0)_{30}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{125} = \mathbf{Pl}(0,1,0,1,1,0)_{57}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{293} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{293} = \mathbf{Pl}(0,3,0,1,1,0)_{59}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209} = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{209} = \mathbf{Pl}(0,2,0,1,1,0)_{58}$$

Rank of lines: (104, 272, 188, 345, 125, 355, 293, 350, 209)
Rank of points on Klein quadric: (29, 31, 30, 13, 57, 15, 59, 14, 58)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 36 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_4
1 : $P_8 = (1,0,1,0)$ lies on line ℓ_0
2 : $P_9 = (2,0,1,0)$ lies on line ℓ_1
3 : $P_{10} = (3,0,1,0)$ lies on line ℓ_2
4 : $P_{11} = (0,1,1,0)$ lies on line ℓ_3
5 : $P_{12} = (1,1,1,0)$ lies on line ℓ_4
6 : $P_{15} = (0,2,1,0)$ lies on line ℓ_5
7 : $P_{17} = (2,2,1,0)$ lies on line ℓ_6
8 : $P_{19} = (0,3,1,0)$ lies on line ℓ_7
9 : $P_{22} = (3,3,1,0)$ lies on line ℓ_8
10 : $P_{39} = (1,0,1,1)$ lies on line ℓ_0
11 : $P_{40} = (2,0,1,1)$ lies on line ℓ_1
12 : $P_{41} = (3,0,1,1)$ lies on line ℓ_2
13 : $P_{42} = (0,1,1,1)$ lies on line ℓ_3
14 : $P_{45} = (0,2,1,1)$ lies on line ℓ_5
15 : $P_{47} = (2,2,1,1)$ lies on line ℓ_6
16 : $P_{49} = (0,3,1,1)$ lies on line ℓ_7
17 : $P_{52} = (3,3,1,1)$ lies on line ℓ_8
18 : $P_{54} = (1,0,2,1)$ lies on line ℓ_2
19 : $P_{55} = (2,0,2,1)$ lies on line ℓ_0
20 : $P_{56} = (3,0,2,1)$ lies on line ℓ_1
21 : $P_{57} = (0,1,2,1)$ lies on line ℓ_7
22 : $P_{58} = (1,1,2,1)$ lies on line ℓ_8
23 : $P_{61} = (0,2,2,1)$ lies on line ℓ_3
24 : $P_{63} = (2,2,2,1)$ lies on line ℓ_4
25 : $P_{65} = (0,3,2,1)$ lies on line ℓ_5
26 : $P_{68} = (3,3,2,1)$ lies on line ℓ_6
27 : $P_{70} = (1,0,3,1)$ lies on line ℓ_1

28 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_2
29 : $P_{72} = (3, 0, 3, 1)$ lies on line ℓ_0
30 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_5
31 : $P_{74} = (1, 1, 3, 1)$ lies on line ℓ_6
32 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_7
33 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_8
34 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_3
35 : $P_{84} = (3, 3, 3, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3 4 5 6 7 8
0	0 1 1 1 1 1 1 1 1
1	1 0 1 1 1 1 1 1 1
2	1 1 0 1 1 1 1 1 1
3	1 1 1 0 1 1 1 1 1
4	1 1 1 1 0 1 1 1 1
5	1 1 1 1 1 0 1 1 1
6	1 1 1 1 1 1 0 1 1
7	1 1 1 1 1 1 1 0 1
8	1 1 1 1 1 1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 37 points:
The points on the surface are:

0 : $P_3 = (0,0,0,1)$
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