

Rank-65687 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_1 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1107566661

General information

Number of lines	64
Number of points	4097
Number of singular points	65
Number of Eckardt points	0
Number of double points	0
Number of single points	4096
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{64}
Type of lines on points	$64, 1^{4096}$

Singular Points

The surface has 65 singular points:

0 : $P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$	8 : $P_{33288} = \mathbf{P}(\epsilon^{39}, \epsilon^{39}, \epsilon^{39}, 1) = \mathbf{P}(7, 7, 7, 1)$
1 : $P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$	9 : $P_{37449} = \mathbf{P}(\epsilon^3, \epsilon^3, \epsilon^3, 1) = \mathbf{P}(8, 8, 8, 1)$
2 : $P_{132} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$	10 : $P_{41610} = \mathbf{P}(\epsilon^{34}, \epsilon^{34}, \epsilon^{34}, 1) = \mathbf{P}(9, 9, 9, 1)$
3 : $P_{12483} = \mathbf{P}(\epsilon, \epsilon, \epsilon, 1) = \mathbf{P}(2, 2, 2, 1)$	11 : $P_{45771} = \mathbf{P}(\epsilon^{54}, \epsilon^{54}, \epsilon^{54}, 1) = \mathbf{P}(10, 10, 10, 1)$
4 : $P_{16644} = \mathbf{P}(\epsilon^{58}, \epsilon^{58}, \epsilon^{58}, 1) = \mathbf{P}(3, 3, 3, 1)$	12 : $P_{49932} = \mathbf{P}(\epsilon^{18}, \epsilon^{18}, \epsilon^{18}, 1) = \mathbf{P}(11, 11, 11, 1)$
5 : $P_{20805} = \mathbf{P}(\epsilon^2, \epsilon^2, \epsilon^2, 1) = \mathbf{P}(4, 4, 4, 1)$	13 : $P_{54093} = \mathbf{P}(\epsilon^{60}, \epsilon^{60}, \epsilon^{60}, 1) = \mathbf{P}(12, 12, 12, 1)$
6 : $P_{24966} = \mathbf{P}(\epsilon^{53}, \epsilon^{53}, \epsilon^{53}, 1) = \mathbf{P}(5, 5, 5, 1)$	14 : $P_{58254} = \mathbf{P}(\epsilon^{31}, \epsilon^{31}, \epsilon^{31}, 1) = \mathbf{P}(13, 13, 13, 1)$
7 : $P_{29127} = \mathbf{P}(\epsilon^{59}, \epsilon^{59}, \epsilon^{59}, 1) = \mathbf{P}(6, 6, 6, 1)$	15 : $P_{62415} = \mathbf{P}(\epsilon^{40}, \epsilon^{40}, \epsilon^{40}, 1) = \mathbf{P}(14, 14, 14, 1)$

$$\begin{aligned}
16 : P_{66576} &= \mathbf{P}(\epsilon^{48}, \epsilon^{48}, \epsilon^{48}, 1) = \mathbf{P}(15, 15, 15, 1) \\
17 : P_{70737} &= \mathbf{P}(\epsilon^4, \epsilon^4, \epsilon^4, 1) = \mathbf{P}(16, 16, 16, 1) \\
18 : P_{74898} &= \mathbf{P}(\epsilon^{43}, \epsilon^{43}, \epsilon^{43}, 1) = \mathbf{P}(17, 17, 17, 1) \\
19 : P_{79059} &= \mathbf{P}(\epsilon^{35}, \epsilon^{35}, \epsilon^{35}, 1) = \mathbf{P}(18, 18, 18, 1) \\
20 : P_{83220} &= \mathbf{P}(\epsilon^{22}, \epsilon^{22}, \epsilon^{22}, 1) = \mathbf{P}(19, 19, 19, 1) \\
21 : P_{87381} &= \mathbf{P}(\epsilon^{55}, \epsilon^{55}, \epsilon^{55}, 1) = \mathbf{P}(20, 20, 20, 1) \\
22 : P_{91542} &= \mathbf{P}(\epsilon^{15}, \epsilon^{15}, \epsilon^{15}, 1) = \mathbf{P}(21, 21, 21, 1) \\
23 : P_{95703} &= \mathbf{P}(\epsilon^{19}, \epsilon^{19}, \epsilon^{19}, 1) = \mathbf{P}(22, 22, 22, 1) \\
24 : P_{99864} &= \mathbf{P}(\epsilon^{26}, \epsilon^{26}, \epsilon^{26}, 1) = \mathbf{P}(23, 23, 23, 1) \\
25 : P_{104025} &= \mathbf{P}(\epsilon^{61}, \epsilon^{61}, \epsilon^{61}, 1) = \mathbf{P}(24, 24, 24, 1) \\
26 : P_{108186} &= \mathbf{P}(\epsilon^{51}, \epsilon^{51}, \epsilon^{51}, 1) = \mathbf{P}(25, 25, 25, 1) \\
27 : P_{112347} &= \mathbf{P}(\epsilon^{32}, \epsilon^{32}, \epsilon^{32}, 1) = \mathbf{P}(26, 26, 26, 1) \\
28 : P_{116508} &= \mathbf{P}(\epsilon^{29}, \epsilon^{29}, \epsilon^{29}, 1) = \mathbf{P}(27, 27, 27, 1) \\
29 : P_{120669} &= \mathbf{P}(\epsilon^{41}, \epsilon^{41}, \epsilon^{41}, 1) = \mathbf{P}(28, 28, 28, 1) \\
30 : P_{124830} &= \mathbf{P}(\epsilon^{13}, \epsilon^{13}, \epsilon^{13}, 1) = \mathbf{P}(29, 29, 29, 1) \\
31 : P_{128991} &= \mathbf{P}(\epsilon^{49}, \epsilon^{49}, \epsilon^{49}, 1) = \mathbf{P}(30, 30, 30, 1) \\
32 : P_{133152} &= \mathbf{P}(\epsilon^{11}, \epsilon^{11}, \epsilon^{11}, 1) = \mathbf{P}(31, 31, 31, 1) \\
33 : P_{137313} &= \mathbf{P}(\epsilon^5, \epsilon^5, \epsilon^5, 1) = \mathbf{P}(32, 32, 32, 1) \\
34 : P_{141474} &= \mathbf{P}(\epsilon^6, \epsilon^6, \epsilon^6, 1) = \mathbf{P}(33, 33, 33, 1) \\
35 : P_{145635} &= \mathbf{P}(\epsilon^{44}, \epsilon^{44}, \epsilon^{44}, 1) = \mathbf{P}(34, 34, 34, 1) \\
36 : P_{149796} &= \mathbf{P}(\epsilon^7, \epsilon^7, \epsilon^7, 1) = \mathbf{P}(35, 35, 35, 1) \\
37 : P_{153957} &= \mathbf{P}(\epsilon^{36}, \epsilon^{36}, \epsilon^{36}, 1) = \mathbf{P}(36, 36, 36, 1) \\
38 : P_{158118} &= \mathbf{P}(\epsilon^{45}, \epsilon^{45}, \epsilon^{45}, 1) = \mathbf{P}(37, 37, 37, 1) \\
39 : P_{162279} &= \mathbf{P}(\epsilon^{23}, \epsilon^{23}, \epsilon^{23}, 1) = \mathbf{P}(38, 38, 38, 1) \\
40 : P_{166440} &= \mathbf{P}(\epsilon^8, \epsilon^8, \epsilon^8, 1) = \mathbf{P}(39, 39, 39, 1) \\
41 : P_{170601} &= \mathbf{P}(\epsilon^{56}, \epsilon^{56}, \epsilon^{56}, 1) = \mathbf{P}(40, 40, 40, 1) \\
42 : P_{174762} &= \mathbf{P}(\epsilon^{37}, \epsilon^{37}, \epsilon^{37}, 1) = \mathbf{P}(41, 41, 41, 1) \\
43 : P_{178923} &= \mathbf{P}(\epsilon^{16}, \epsilon^{16}, \epsilon^{16}, 1) = \mathbf{P}(42, 42, 42, 1) \\
44 : P_{183084} &= \mathbf{P}(\epsilon^{46}, \epsilon^{46}, \epsilon^{46}, 1) = \mathbf{P}(43, 43, 43, 1) \\
45 : P_{187245} &= \mathbf{P}(\epsilon^{20}, \epsilon^{20}, \epsilon^{20}, 1) = \mathbf{P}(44, 44, 44, 1) \\
46 : P_{191406} &= \mathbf{P}(\epsilon^{24}, \epsilon^{24}, \epsilon^{24}, 1) = \mathbf{P}(45, 45, 45, 1) \\
47 : P_{195567} &= \mathbf{P}(\epsilon^{27}, \epsilon^{27}, \epsilon^{27}, 1) = \mathbf{P}(46, 46, 46, 1) \\
48 : P_{199728} &= \mathbf{P}(\epsilon^9, \epsilon^9, \epsilon^9, 1) = \mathbf{P}(47, 47, 47, 1) \\
49 : P_{203889} &= \mathbf{P}(\epsilon^{62}, \epsilon^{62}, \epsilon^{62}, 1) = \mathbf{P}(48, 48, 48, 1) \\
50 : P_{208050} &= \mathbf{P}(\epsilon^{57}, \epsilon^{57}, \epsilon^{57}, 1) = \mathbf{P}(49, 49, 49, 1) \\
51 : P_{212211} &= \mathbf{P}(\epsilon^{52}, \epsilon^{52}, \epsilon^{52}, 1) = \mathbf{P}(50, 50, 50, 1) \\
52 : P_{216372} &= \mathbf{P}(\epsilon^{38}, \epsilon^{38}, \epsilon^{38}, 1) = \mathbf{P}(51, 51, 51, 1) \\
53 : P_{220533} &= \mathbf{P}(\epsilon^{33}, \epsilon^{33}, \epsilon^{33}, 1) = \mathbf{P}(52, 52, 52, 1) \\
54 : P_{224694} &= \mathbf{P}(\epsilon^{17}, \epsilon^{17}, \epsilon^{17}, 1) = \mathbf{P}(53, 53, 53, 1) \\
55 : P_{228855} &= \mathbf{P}(\epsilon^{30}, \epsilon^{30}, \epsilon^{30}, 1) = \mathbf{P}(54, 54, 54, 1) \\
56 : P_{233016} &= \mathbf{P}(\epsilon^{47}, \epsilon^{47}, \epsilon^{47}, 1) = \mathbf{P}(55, 55, 55, 1) \\
57 : P_{237177} &= \mathbf{P}(\epsilon^{42}, \epsilon^{42}, \epsilon^{42}, 1) = \mathbf{P}(56, 56, 56, 1) \\
58 : P_{241338} &= \mathbf{P}(\epsilon^{21}, \epsilon^{21}, \epsilon^{21}, 1) = \mathbf{P}(57, 57, 57, 1) \\
59 : P_{245499} &= \mathbf{P}(\epsilon^{14}, \epsilon^{14}, \epsilon^{14}, 1) = \mathbf{P}(58, 58, 58, 1) \\
60 : P_{249660} &= \mathbf{P}(\epsilon^{25}, \epsilon^{25}, \epsilon^{25}, 1) = \mathbf{P}(59, 59, 59, 1) \\
61 : P_{253821} &= \mathbf{P}(\epsilon^{50}, \epsilon^{50}, \epsilon^{50}, 1) = \mathbf{P}(60, 60, 60, 1) \\
62 : P_{257982} &= \mathbf{P}(\epsilon^{28}, \epsilon^{28}, \epsilon^{28}, 1) = \mathbf{P}(61, 61, 61, 1) \\
63 : P_{262143} &= \mathbf{P}(\epsilon^{12}, \epsilon^{12}, \epsilon^{12}, 1) = \mathbf{P}(62, 62, 62, 1) \\
64 : P_{266304} &= \mathbf{P}(\epsilon^{10}, \epsilon^{10}, \epsilon^{10}, 1) = \mathbf{P}(63, 63, 63, 1)
\end{aligned}$$

The 64 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4160} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4160} = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_{4225} \\
\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17043520} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{129} \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8321} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8321} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{12353} \\
\ell_3 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{274625} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{274625} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{12417} \\
\ell_4 &= \begin{bmatrix} 1 & \epsilon^{51} & \epsilon^{50} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16086425} = \begin{bmatrix} 1 & 25 & 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16086425} = \mathbf{Pl}(0, 60, 0, 25, 1, 0)_{15524} \\
\ell_5 &= \begin{bmatrix} 1 & \epsilon^{15} & \epsilon^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15537173} = \begin{bmatrix} 1 & 21 & 58 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15537173} = \mathbf{Pl}(0, 58, 0, 21, 1, 0)_{15014} \\
\ell_6 &= \begin{bmatrix} 1 & \epsilon^{30} & \epsilon^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16473398} = \begin{bmatrix} 1 & 54 & 61 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16473398} = \mathbf{Pl}(0, 61, 0, 54, 1, 0)_{19208} \\
\ell_7 &= \begin{bmatrix} 1 & \epsilon^{39} & \epsilon^{37} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10951751} = \begin{bmatrix} 1 & 7 & 41 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10951751} = \mathbf{Pl}(0, 41, 0, 7, 1, 0)_{13219} \\
\ell_8 &= \begin{bmatrix} 1 & \epsilon^{30} & \epsilon^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12478838} = \begin{bmatrix} 1 & 54 & 46 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12478838} = \mathbf{Pl}(0, 46, 0, 54, 1, 0)_{19193} \\
\ell_9 &= \begin{bmatrix} 1 & \epsilon^{42} & \epsilon^{39} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2101304} = \begin{bmatrix} 1 & 56 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2101304} = \mathbf{Pl}(0, 7, 0, 56, 1, 0)_{19408}
\end{aligned}$$

$$\begin{aligned}
\ell_{10} &= \begin{bmatrix} 1 & \epsilon^9 & \epsilon^{18} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3129071} = \begin{bmatrix} 1 & 47 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3129071} = \mathbf{Pl}(0, 11, 0, 47, 1, 0)_{18269} \\
\ell_{11} &= \begin{bmatrix} 1 & \epsilon^{27} & \epsilon^{36} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9782510} = \begin{bmatrix} 1 & 46 & 36 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9782510} = \mathbf{Pl}(0, 36, 0, 46, 1, 0)_{18167} \\
\ell_{12} &= \begin{bmatrix} 1 & \epsilon^{48} & \epsilon^{30} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14446991} = \begin{bmatrix} 1 & 15 & 54 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14446991} = \mathbf{Pl}(0, 54, 0, 15, 1, 0)_{14248} \\
\ell_{13} &= \begin{bmatrix} 1 & \epsilon^6 & \epsilon^{51} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6799073} = \begin{bmatrix} 1 & 33 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6799073} = \mathbf{Pl}(0, 25, 0, 33, 1, 0)_{16505} \\
\ell_{14} &= \begin{bmatrix} 1 & \epsilon^{42} & \epsilon^{57} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13286072} = \begin{bmatrix} 1 & 56 & 49 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13286072} = \mathbf{Pl}(0, 49, 0, 56, 1, 0)_{19450} \\
\ell_{15} &= \begin{bmatrix} 1 & \epsilon^{39} & \epsilon^{54} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2696327} = \begin{bmatrix} 1 & 7 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2696327} = \mathbf{Pl}(0, 10, 0, 7, 1, 0)_{13188} \\
\ell_{16} &= \begin{bmatrix} 1 & \epsilon^{60} & \epsilon^{56} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10706252} = \begin{bmatrix} 1 & 12 & 40 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10706252} = \mathbf{Pl}(0, 40, 0, 12, 1, 0)_{13853} \\
\ell_{17} &= \begin{bmatrix} 1 & \epsilon^{15} & \epsilon^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8346965} = \begin{bmatrix} 1 & 21 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8346965} = \mathbf{Pl}(0, 31, 0, 21, 1, 0)_{14987} \\
\ell_{18} &= \begin{bmatrix} 1 & \epsilon^{33} & \epsilon^{61} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6611828} = \begin{bmatrix} 1 & 52 & 24 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6611828} = \mathbf{Pl}(0, 24, 0, 52, 1, 0)_{18917} \\
\ell_{19} &= \begin{bmatrix} 1 & \epsilon^9 & \epsilon^{37} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11118191} = \begin{bmatrix} 1 & 47 & 41 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11118191} = \mathbf{Pl}(0, 41, 0, 47, 1, 0)_{18299} \\
\ell_{20} &= \begin{bmatrix} 1 & \epsilon^3 & \epsilon^{47} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14684168} = \begin{bmatrix} 1 & 8 & 55 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14684168} = \mathbf{Pl}(0, 55, 0, 8, 1, 0)_{13360} \\
\ell_{21} &= \begin{bmatrix} 1 & \epsilon^{54} & \epsilon^{35} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4839242} = \begin{bmatrix} 1 & 10 & 18 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4839242} = \mathbf{Pl}(0, 18, 0, 10, 1, 0)_{13577} \\
\ell_{22} &= \begin{bmatrix} 1 & \epsilon^{24} & \epsilon^{61} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6582701} = \begin{bmatrix} 1 & 45 & 24 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6582701} = \mathbf{Pl}(0, 24, 0, 45, 1, 0)_{18028} \\
\ell_{23} &= \begin{bmatrix} 1 & \epsilon^{54} & \epsilon^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16290314} = \begin{bmatrix} 1 & 10 & 61 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16290314} = \mathbf{Pl}(0, 61, 0, 10, 1, 0)_{13620} \\
\ell_{24} &= \begin{bmatrix} 1 & \epsilon^{39} & \epsilon^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9353927} = \begin{bmatrix} 1 & 7 & 35 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9353927} = \mathbf{Pl}(0, 35, 0, 7, 1, 0)_{13213} \\
\ell_{25} &= \begin{bmatrix} 1 & \epsilon^{57} & \epsilon^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15919985} = \begin{bmatrix} 1 & 49 & 59 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15919985} = \mathbf{Pl}(0, 59, 0, 49, 1, 0)_{18571} \\
\ell_{26} &= \begin{bmatrix} 1 & \epsilon^{33} & \epsilon^{55} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5546612} = \begin{bmatrix} 1 & 52 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5546612} = \mathbf{Pl}(0, 20, 0, 52, 1, 0)_{18913} \\
\ell_{27} &= \begin{bmatrix} 1 & \epsilon^{27} & \epsilon^{49} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8184686} = \begin{bmatrix} 1 & 46 & 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8184686} = \mathbf{Pl}(0, 30, 0, 46, 1, 0)_{18161} \\
\ell_{28} &= \begin{bmatrix} 1 & \epsilon^{12} & \epsilon^{62} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13044734} = \begin{bmatrix} 1 & 62 & 48 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13044734} = \mathbf{Pl}(0, 48, 0, 62, 1, 0)_{20211} \\
\ell_{29} &= \begin{bmatrix} 1 & \epsilon^{27} & \epsilon^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15641198} = \begin{bmatrix} 1 & 46 & 58 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15641198} = \mathbf{Pl}(0, 58, 0, 46, 1, 0)_{18189} \\
\ell_{30} &= \begin{bmatrix} 1 & \epsilon^{48} & \epsilon^{62} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12849167} = \begin{bmatrix} 1 & 15 & 48 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12849167} = \mathbf{Pl}(0, 48, 0, 15, 1, 0)_{14242}
\end{aligned}$$

$$\begin{aligned}
\ell_{31} &= \begin{bmatrix} 1 & \epsilon^{36} & \epsilon^{50} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16132196} = \begin{bmatrix} 1 & 36 & 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16132196} = \mathbf{Pl}(0, 60, 0, 36, 1, 0)_{16921} \\
\ell_{32} &= \begin{bmatrix} 1 & \epsilon^{21} & \epsilon^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5833721} = \begin{bmatrix} 1 & 57 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5833721} = \mathbf{Pl}(0, 21, 0, 57, 1, 0)_{19549} \\
\ell_{33} &= \begin{bmatrix} 1 & \epsilon^{60} & \epsilon^{54} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2717132} = \begin{bmatrix} 1 & 12 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2717132} = \mathbf{Pl}(0, 10, 0, 12, 1, 0)_{13823} \\
\ell_{34} &= \begin{bmatrix} 1 & \epsilon^3 & \epsilon^{59} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1635272} = \begin{bmatrix} 1 & 8 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1635272} = \mathbf{Pl}(0, 6, 0, 8, 1, 0)_{13311} \\
\ell_{35} &= \begin{bmatrix} 1 & \epsilon^{18} & \epsilon^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8305355} = \begin{bmatrix} 1 & 11 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8305355} = \mathbf{Pl}(0, 31, 0, 11, 1, 0)_{13717} \\
\ell_{36} &= \begin{bmatrix} 1 & \epsilon^{33} & \epsilon^{60} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3416180} = \begin{bmatrix} 1 & 52 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3416180} = \mathbf{Pl}(0, 12, 0, 52, 1, 0)_{18905} \\
\ell_{37} &= \begin{bmatrix} 1 & \epsilon^{12} & \epsilon^{39} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2126270} = \begin{bmatrix} 1 & 62 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2126270} = \mathbf{Pl}(0, 7, 0, 62, 1, 0)_{20170} \\
\ell_{38} &= \begin{bmatrix} 1 & \epsilon^{54} & \epsilon^9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12562058} = \begin{bmatrix} 1 & 10 & 47 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12562058} = \mathbf{Pl}(0, 47, 0, 10, 1, 0)_{13606} \\
\ell_{39} &= \begin{bmatrix} 1 & \epsilon^{18} & \epsilon^{36} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9636875} = \begin{bmatrix} 1 & 11 & 36 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9636875} = \mathbf{Pl}(0, 36, 0, 11, 1, 0)_{13722} \\
\ell_{40} &= \begin{bmatrix} 1 & \epsilon^{30} & \epsilon^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5288630} = \begin{bmatrix} 1 & 54 & 19 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5288630} = \mathbf{Pl}(0, 19, 0, 54, 1, 0)_{19166} \\
\ell_{41} &= \begin{bmatrix} 1 & \epsilon^{57} & \epsilon^{49} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8197169} = \begin{bmatrix} 1 & 49 & 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8197169} = \mathbf{Pl}(0, 30, 0, 49, 1, 0)_{18542} \\
\ell_{42} &= \begin{bmatrix} 1 & \epsilon^{24} & \epsilon^{31} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3653357} = \begin{bmatrix} 1 & 45 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3653357} = \mathbf{Pl}(0, 13, 0, 45, 1, 0)_{18017} \\
\ell_{43} &= \begin{bmatrix} 1 & \epsilon^{18} & \epsilon^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15761867} = \begin{bmatrix} 1 & 11 & 59 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15761867} = \mathbf{Pl}(0, 59, 0, 11, 1, 0)_{13745} \\
\ell_{44} &= \begin{bmatrix} 1 & \epsilon^{60} & \epsilon^{44} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9108428} = \begin{bmatrix} 1 & 12 & 34 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9108428} = \mathbf{Pl}(0, 34, 0, 12, 1, 0)_{13847} \\
\ell_{45} &= \begin{bmatrix} 1 & \epsilon^{51} & \epsilon^{35} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4901657} = \begin{bmatrix} 1 & 25 & 18 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4901657} = \mathbf{Pl}(0, 18, 0, 25, 1, 0)_{15482} \\
\ell_{46} &= \begin{bmatrix} 1 & \epsilon^{21} & \epsilon^{60} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3436985} = \begin{bmatrix} 1 & 57 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3436985} = \mathbf{Pl}(0, 12, 0, 57, 1, 0)_{19540} \\
\ell_{47} &= \begin{bmatrix} 1 & \epsilon^{51} & \epsilon^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12358169} = \begin{bmatrix} 1 & 25 & 46 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12358169} = \mathbf{Pl}(0, 46, 0, 25, 1, 0)_{15510} \\
\ell_{48} &= \begin{bmatrix} 1 & \epsilon^{36} & \epsilon^9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12670244} = \begin{bmatrix} 1 & 36 & 47 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12670244} = \mathbf{Pl}(0, 47, 0, 36, 1, 0)_{16908} \\
\ell_{49} &= \begin{bmatrix} 1 & \epsilon^{45} & \epsilon^{18} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3087461} = \begin{bmatrix} 1 & 37 & 11 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3087461} = \mathbf{Pl}(0, 11, 0, 37, 1, 0)_{16999} \\
\ell_{50} &= \begin{bmatrix} 1 & \epsilon^{24} & \epsilon^{15} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5783789} = \begin{bmatrix} 1 & 45 & 21 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5783789} = \mathbf{Pl}(0, 21, 0, 45, 1, 0)_{18025} \\
\ell_{51} &= \begin{bmatrix} 1 & \epsilon^3 & \epsilon^{57} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13086344} = \begin{bmatrix} 1 & 8 & 49 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13086344} = \mathbf{Pl}(0, 49, 0, 8, 1, 0)_{13354}
\end{aligned}$$

$$\begin{aligned}
\ell_{52} &= \begin{bmatrix} 1 & \epsilon^{48} & \epsilon^{59} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1664399} = \begin{bmatrix} 1 & 15 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1664399} = \mathbf{Pl}(0, 6, 0, 15, 1, 0)_{14200} \\
\ell_{53} &= \begin{bmatrix} 1 & \epsilon^{45} & \epsilon^{56} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10810277} = \begin{bmatrix} 1 & 37 & 40 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10810277} = \mathbf{Pl}(0, 40, 0, 37, 1, 0)_{17028} \\
\ell_{54} &= \begin{bmatrix} 1 & \epsilon^6 & \epsilon^{31} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3603425} = \begin{bmatrix} 1 & 33 & 13 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{3603425} = \mathbf{Pl}(0, 13, 0, 33, 1, 0)_{16493} \\
\ell_{55} &= \begin{bmatrix} 1 & \epsilon^{45} & \epsilon^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9478757} = \begin{bmatrix} 1 & 37 & 35 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9478757} = \mathbf{Pl}(0, 35, 0, 37, 1, 0)_{17023} \\
\ell_{56} &= \begin{bmatrix} 1 & \epsilon^{15} & \epsilon^{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9944789} = \begin{bmatrix} 1 & 21 & 37 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9944789} = \mathbf{Pl}(0, 37, 0, 21, 1, 0)_{14993} \\
\ell_{57} &= \begin{bmatrix} 1 & \epsilon^{21} & \epsilon^{51} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6898937} = \begin{bmatrix} 1 & 57 & 25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{6898937} = \mathbf{Pl}(0, 25, 0, 57, 1, 0)_{19553} \\
\ell_{58} &= \begin{bmatrix} 1 & \epsilon^{36} & \epsilon^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5213732} = \begin{bmatrix} 1 & 36 & 19 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5213732} = \mathbf{Pl}(0, 19, 0, 36, 1, 0)_{16880} \\
\ell_{59} &= \begin{bmatrix} 1 & \epsilon^6 & \epsilon^{55} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5467553} = \begin{bmatrix} 1 & 33 & 20 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5467553} = \mathbf{Pl}(0, 20, 0, 33, 1, 0)_{16500} \\
\ell_{60} &= \begin{bmatrix} 1 & \epsilon^9 & \epsilon^{44} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9254063} = \begin{bmatrix} 1 & 47 & 34 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9254063} = \mathbf{Pl}(0, 34, 0, 47, 1, 0)_{18292} \\
\ell_{61} &= \begin{bmatrix} 1 & \epsilon^{12} & \epsilon^{47} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14908862} = \begin{bmatrix} 1 & 62 & 55 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14908862} = \mathbf{Pl}(0, 55, 0, 62, 1, 0)_{20218} \\
\ell_{62} &= \begin{bmatrix} 1 & \epsilon^{57} & \epsilon^{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10061297} = \begin{bmatrix} 1 & 49 & 37 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10061297} = \mathbf{Pl}(0, 37, 0, 49, 1, 0)_{18549} \\
\ell_{63} &= \begin{bmatrix} 1 & \epsilon^{42} & \epsilon^{30} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14617592} = \begin{bmatrix} 1 & 56 & 54 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14617592} = \mathbf{Pl}(0, 54, 0, 56, 1, 0)_{19455}
\end{aligned}$$

Rank of lines: (4160, 17043520, 8321, 274625, 16086425, 15537173, 16473398, 10951751, 12478838, 2101304, 3129071, 9782510, 14446991, 6799073, 13286072, 2696327, 10706252, 8346965, 6611828, 11118191, 14684168, 4839242, 6582701, 16290314, 9353927, 15919985, 5546612, 8184686, 13044734, 15641198, 12849167, 16132196, 5833721, 2717132, 1635272, 8305355, 3416180, 2126270, 12562058, 9636875, 5288630, 8197169, 3653357, 15761867, 9108428, 4901657, 3436985, 12358169, 12670244, 3087461, ...14908862, 10061297, 14617592)

Rank of points on Klein quadric: (4225, 129, 12353, 12417, 15524, 15014, 19208, 13219, 19193, 19408, 18269, 18167, 14248, 16505, 19450, 13188, 13853, 14987, 18917, 18299, 13360, 13577, 18028, 13620, 13213, 18571, 18913, 18161, 20211, 18189, 14242, 16921, 19549, 13823, 13311, 13717, 18905, 20170, 13606, 13722, 19166, 18542, 18017, 13745, 13847, 15482, 19540, 15510, 16908, 16999, ...20218, 18549, 19455)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 4096 single points:
Too many to print.

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

[illegible]

Line 0 intersects

Line 1 intersects

Line 2 intersects

Line 3 intersects

Line 4 intersects

Line 5 intersects

Line 6 intersects

Line 7 intersects

Line 8 intersects

Line 9 intersects

Line 10 intersects

Line 11 intersects

8

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

