Rank-331 over GF(2)

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The equation

The equation of the surface is :
$$X_0^2 X_3 + X_1^2 X_2 = 0$$
(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(2)$ is 331

General information

Number of lines	5
Number of points	9
Number of singular points	3
Number of Eckardt points	0
Number of double points	6
Number of single points	3
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	$2^6, 1^3$

Singular Points

The surface has 3 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$ $2: P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
$1: P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

Rank of lines: (0, 4, 30, 34, 12)
Rank of points on Klein quadric: (0, 2, 5, 1, 32)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_3 = (0,0,0,1) = \ell_2 \cap \ell_3$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$ $P_{12} = (0,0,1,1) = \ell_3 \cap \ell_4$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_4$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_3$

Single Points

The surface has 3 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_4 $2 : P_{10} = (0,1,0,1)$ lies on line ℓ_2
$1 : P_6 = (1,0,1,0)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

$$
\begin{array}{c|ccccc}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 \\
2 & 1 & 0 & 0 & 1 & 0 \\
3 & 0 & 1 & 1 & 0 & 1 \\
4 & 1 & 0 & 0 & 1 & 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_0	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4
in point	P_2	P_3	P_{12}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{12}

The surface has 9 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$

$1: P_1 = (0,1,0,0)$

$2: P_2 = (0,0,1,0)$

$3: P_3 = (0,0,0,1)$

$4: P_4 = (1,1,1,1)$

$5: P_5 = (1,1,0,0)$

$6: P_6 = (1,0,1,0)$

$7: P_{10} = (0,1,0,1)$

$8: P_{12} = (0,0,1,1)$