

# Rank-67243 over GF(32)

January 15, 2021

## The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_2 + X_0 X_1^2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(32) is 1141965861

## General information

Number of lines	2
Number of points	1089
Number of singular points	0
Number of Eckardt points	0
Number of double points	1
Number of single points	64
Number of points off lines	1024
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$33^2$
Type of lines on points	$2, 1^{64}, 0^{1024}$

## Singular Points

The surface has 0 singular points:

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{1082368} = \left[ \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]_{1082368} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{34849} \\ \ell_1 &= \left[ \begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]_{1082401} = \left[ \begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]_{1082401} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{34881}\end{aligned}$$

Rank of lines: ( 1082368, 1082401 )  
Rank of points on Klein quadric: ( 34849, 34881 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 1 Double points:  
The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

### Single Points

The surface has 64 single points:  
The single points on the surface are:

- |   |  |
|---|--|
| 0 : $P_1 = (0, 1, 0, 0)$ lies on line $\ell_0$        | 33 : $P_{2114} = (0, 1, 1, 1)$ lies on line $\ell_1$   |
| 1 : $P_{67} = (0, 1, 1, 0)$ lies on line $\ell_0$     | 34 : $P_{3137} = (0, 1, 2, 1)$ lies on line $\ell_1$   |
| 2 : $P_{99} = (0, 2, 1, 0)$ lies on line $\ell_0$     | 35 : $P_{4161} = (0, 1, 3, 1)$ lies on line $\ell_1$   |
| 3 : $P_{131} = (0, 3, 1, 0)$ lies on line $\ell_0$    | 36 : $P_{5185} = (0, 1, 4, 1)$ lies on line $\ell_1$   |
| 4 : $P_{163} = (0, 4, 1, 0)$ lies on line $\ell_0$    | 37 : $P_{6209} = (0, 1, 5, 1)$ lies on line $\ell_1$   |
| 5 : $P_{195} = (0, 5, 1, 0)$ lies on line $\ell_0$    | 38 : $P_{7233} = (0, 1, 6, 1)$ lies on line $\ell_1$   |
| 6 : $P_{227} = (0, 6, 1, 0)$ lies on line $\ell_0$    | 39 : $P_{8257} = (0, 1, 7, 1)$ lies on line $\ell_1$   |
| 7 : $P_{259} = (0, 7, 1, 0)$ lies on line $\ell_0$    | 40 : $P_{9281} = (0, 1, 8, 1)$ lies on line $\ell_1$   |
| 8 : $P_{291} = (0, 8, 1, 0)$ lies on line $\ell_0$    | 41 : $P_{10305} = (0, 1, 9, 1)$ lies on line $\ell_1$  |
| 9 : $P_{323} = (0, 9, 1, 0)$ lies on line $\ell_0$    | 42 : $P_{11329} = (0, 1, 10, 1)$ lies on line $\ell_1$ |
| 10 : $P_{355} = (0, 10, 1, 0)$ lies on line $\ell_0$  | 43 : $P_{12353} = (0, 1, 11, 1)$ lies on line $\ell_1$ |
| 11 : $P_{387} = (0, 11, 1, 0)$ lies on line $\ell_0$  | 44 : $P_{13377} = (0, 1, 12, 1)$ lies on line $\ell_1$ |
| 12 : $P_{419} = (0, 12, 1, 0)$ lies on line $\ell_0$  | 45 : $P_{14401} = (0, 1, 13, 1)$ lies on line $\ell_1$ |
| 13 : $P_{451} = (0, 13, 1, 0)$ lies on line $\ell_0$  | 46 : $P_{15425} = (0, 1, 14, 1)$ lies on line $\ell_1$ |
| 14 : $P_{483} = (0, 14, 1, 0)$ lies on line $\ell_0$  | 47 : $P_{16449} = (0, 1, 15, 1)$ lies on line $\ell_1$ |
| 15 : $P_{515} = (0, 15, 1, 0)$ lies on line $\ell_0$  | 48 : $P_{17473} = (0, 1, 16, 1)$ lies on line $\ell_1$ |
| 16 : $P_{547} = (0, 16, 1, 0)$ lies on line $\ell_0$  | 49 : $P_{18497} = (0, 1, 17, 1)$ lies on line $\ell_1$ |
| 17 : $P_{579} = (0, 17, 1, 0)$ lies on line $\ell_0$  | 50 : $P_{19521} = (0, 1, 18, 1)$ lies on line $\ell_1$ |
| 18 : $P_{611} = (0, 18, 1, 0)$ lies on line $\ell_0$  | 51 : $P_{20545} = (0, 1, 19, 1)$ lies on line $\ell_1$ |
| 19 : $P_{643} = (0, 19, 1, 0)$ lies on line $\ell_0$  | 52 : $P_{21569} = (0, 1, 20, 1)$ lies on line $\ell_1$ |
| 20 : $P_{675} = (0, 20, 1, 0)$ lies on line $\ell_0$  | 53 : $P_{22593} = (0, 1, 21, 1)$ lies on line $\ell_1$ |
| 21 : $P_{707} = (0, 21, 1, 0)$ lies on line $\ell_0$  | 54 : $P_{23617} = (0, 1, 22, 1)$ lies on line $\ell_1$ |
| 22 : $P_{739} = (0, 22, 1, 0)$ lies on line $\ell_0$  | 55 : $P_{24641} = (0, 1, 23, 1)$ lies on line $\ell_1$ |
| 23 : $P_{771} = (0, 23, 1, 0)$ lies on line $\ell_0$  | 56 : $P_{25665} = (0, 1, 24, 1)$ lies on line $\ell_1$ |
| 24 : $P_{803} = (0, 24, 1, 0)$ lies on line $\ell_0$  | 57 : $P_{26689} = (0, 1, 25, 1)$ lies on line $\ell_1$ |
| 25 : $P_{835} = (0, 25, 1, 0)$ lies on line $\ell_0$  | 58 : $P_{27713} = (0, 1, 26, 1)$ lies on line $\ell_1$ |
| 26 : $P_{867} = (0, 26, 1, 0)$ lies on line $\ell_0$  | 59 : $P_{28737} = (0, 1, 27, 1)$ lies on line $\ell_1$ |
| 27 : $P_{899} = (0, 27, 1, 0)$ lies on line $\ell_0$  | 60 : $P_{29761} = (0, 1, 28, 1)$ lies on line $\ell_1$ |
| 28 : $P_{931} = (0, 28, 1, 0)$ lies on line $\ell_0$  | 61 : $P_{30785} = (0, 1, 29, 1)$ lies on line $\ell_1$ |
| 29 : $P_{963} = (0, 29, 1, 0)$ lies on line $\ell_0$  | 62 : $P_{31809} = (0, 1, 30, 1)$ lies on line $\ell_1$ |
| 30 : $P_{995} = (0, 30, 1, 0)$ lies on line $\ell_0$  | 63 : $P_{32833} = (0, 1, 31, 1)$ lies on line $\ell_1$ |
| 31 : $P_{1027} = (0, 31, 1, 0)$ lies on line $\ell_0$ |  |
| 32 : $P_{1090} = (0, 1, 0, 1)$ lies on line $\ell_1$  |  |

The single points on the surface are:

**Points on surface but on no line**

The surface has 1024 points not on any line:  
Too many to print.

**Line Intersection Graph**

	0 1
0	0 1
1	1 0

Neighbor sets in the line intersection graph:  
Line 0 intersects

Line	$\ell_1$
in point	$P_2$

Line 1 intersects

Line	$\ell_0$
in point	$P_2$

The surface has 1089 points:  
Too many to print.