Rank-20 over GF(2)

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The equation

The equation of the surface is :

$$X_0^3+X_1^3+X_2^3+X_3^3+X_0^2X_1+X_0^2X_2+X_0^2X_3+X_0X_1^2+X_1^2X_2+X_1^2X_3+X_0X_2^2+X_1X_2^2+X_2^2X_3+X_0X_3^2+X_1X_3^2+X_2X_3^2+X_0X_1X_2+$$

(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
The point rank of the equation over GF(2) is 20

General information

Number of lines	9
Number of points	11
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	0
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^9
Type of lines on points	$3^5, 2^6$

Singular Points

The surface has 4 singular points:

$0 :\ P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$
$1 :\ P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
$2 :\ P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$
$3 :\ P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{24} = \mathbf{Pl}(1,0,1,1,1,1)_{33}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{9} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{9} = \mathbf{Pl}(1,1,0,0,1,1)_{29}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1,1,1,0,1,1)_{30}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \mathbf{Pl}(1,1,1,1,0,0)_{8}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,1,1,1,0,1)_{28}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,1,1,1,1,0)_{18}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{23} = \mathbf{Pl}(1,1,0,1,1,1)_{31}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(0,1,1,1,1,1)_{34}$$

Rank of lines: (12, 24, 9, 10, 15, 22, 17, 23, 26)
Rank of points on Klein quadric: (32, 33, 29, 30, 8, 28, 18, 31, 34)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1),\ T = 11$
$1 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0),\ T = -1$
$2 : P_{11} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1),\ T = -1$
$3 : P_{13} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1),\ T = -1$
$4 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = -1$

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$ $P_7 = (0,1,1,0) = \ell_4 \cap \ell_5$
$P_{12} = (0,0,1,1) = \ell_0 \cap \ell_8$ $P_9 = (1,0,0,1) = \ell_4 \cap \ell_6$
$P_6 = (1,0,1,0) = \ell_2 \cap \ell_3$
$P_{10} = (0,1,0,1) = \ell_2 \cap \ell_7$

Single Points

The surface has 0 single points:
The single points on the surface are:

The single points on the surface are:

2

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
  |0 1 2 3 4 5 6 7 8
--+-----------------
0 |0 1 1 0 1 0 0 0 1
1 |1 0 0 1 0 1 1 1 0
2 |1 0 0 1 1 0 0 1 0
3 |0 1 1 0 0 1 1 0 1
4 |1 0 1 0 0 1 1 0 0
5 |0 1 0 1 1 0 0 1 1
6 |0 1 0 1 1 0 0 1 1
7 |0 1 1 0 0 1 1 0 1
8 |1 0 0 1 0 1 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_8
in point	P_5	P_4	P_4	P_{12}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_5	P_{14}	P_{13}	P_{14}	P_{13}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_7
in point	P_4	P_6	P_4	P_{10}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{14}	P_6	P_{11}	P_{14}	P_{11}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_4	P_4	P_7	P_9

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{13}	P_{11}	P_7	P_{13}	P_{11}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{14}	P_{14}	P_9	P_8	P_8

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{13}	P_{10}	P_{13}	P_8	P_8

Line 8 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_{12}	P_{11}	P_{11}	P_8	P_8

The surface has 11 points:
The points on the surface are:

$0: P_4 = (1,1,1,1)$ $\quad$ $4: P_8 = (1,1,1,0)$ $\quad$ $8: P_{12} = (0,0,1,1)$

$1: P_5 = (1,1,0,0)$ $\quad$ $5: P_9 = (1,0,0,1)$ $\quad$ $9: P_{13} = (1,0,1,1)$

$2: P_6 = (1,0,1,0)$ $\quad$ $6: P_{10} = (0,1,0,1)$ $\quad$ $10: P_{14} = (0,1,1,1)$

$3: P_7 = (0,1,1,0)$ $\quad$ $7: P_{11} = (1,1,0,1)$