Rank-74248 over GF(64)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_2^3 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(64)$ is 1090789446

General information

Number of lines	21
Number of points	4481
Number of singular points	1
Number of Eckardt points	0
Number of double points	75
Number of single points	1209
Number of points off lines	3196
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{21}
Type of lines on points	$6, 2^{75}, 1^{1209}, 0^{3196}$

Singular Points

The surface has 1 singular points:

0:
$$P_{4163} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$$

The 21 Lines

The lines and their Pluecker coordinates are:

$$\ell_1 = \begin{bmatrix} 1 & 0 & \epsilon^{54} & \epsilon^{54} \\ 0 & 1 & \epsilon^{5} & \epsilon^{27} \\ -\epsilon^{27} \end{bmatrix}_{2707641} = \begin{bmatrix} 1 & 0 & 10 & 10 \\ 0 & 1 & 47 & 46 \end{bmatrix}_{2707641} = \mathbf{PI}(1,1,1,36,1)_{9727584}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \epsilon^{55} & \epsilon^{45} \\ 0 & 1 & \epsilon^{18} & \epsilon^{54} \end{bmatrix}_{10007566} = \begin{bmatrix} 1 & 0 & 37 & 37 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{10007856} = \mathbf{PI}(1,1,1,47,1)_{12611031}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{62} & \epsilon^{60} \end{bmatrix}_{267120} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 48 & 12 \end{bmatrix}_{267120} = \mathbf{PI}(24,4,4,8,1,0)_{48349}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{57} & \epsilon^{57} \end{bmatrix}_{269464} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 24 & 49 \end{bmatrix}_{269464} = \mathbf{PI}(6,16,16,33,1,0)_{148312}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \epsilon^{28} & \epsilon^{21} \\ 0 & 1 & \epsilon^{55} & \epsilon^{42} \end{bmatrix}_{15436751} = \begin{bmatrix} 1 & 0 & 61 & 57 \\ 0 & 1 & 18 & 56 \end{bmatrix}_{15436751} = \mathbf{PI}(63,5,35,40,58,1)_{15625769}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \epsilon^{7} & \epsilon^{21} \\ 0 & 1 & \epsilon^{56} & \epsilon^{42} \end{bmatrix}_{15325857} = \begin{bmatrix} 1 & 0 & 35 & 57 \\ 0 & 1 & 40 & 56 \end{bmatrix}_{15235877} = \mathbf{PI}(27,30,58,18,1)_{5124938}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{56} & \epsilon^{45} \end{bmatrix}_{269710} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 6 & 25 \end{bmatrix}_{267910} = \mathbf{PI}(20,39,39,62,1,0)_{264876}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \epsilon^{6} & \epsilon^{27} \\ 0 & 1 & \epsilon^{57} & \epsilon^{45} \end{bmatrix}_{13238714} = \begin{bmatrix} 1 & 0 & 33 & 46 \\ 0 & 1 & 49 & 37 \end{bmatrix}_{12389714} = \mathbf{PI}(24,8,56,57,7,1)_{2343040}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \epsilon^{56} & \epsilon^{45} \end{bmatrix}_{9887188} = \begin{bmatrix} 1 & 0 & 8 & 37 \\ 0 & 1 & 12 & 10 \end{bmatrix}_{9887188} = \mathbf{PI}(26,13,57,56,25,1)_{7062079}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \epsilon^{56} & \epsilon^{27} \\ 0 & 1 & \epsilon^{56} & \epsilon^{27} \end{bmatrix}_{29923991} = \begin{bmatrix} 1 & 0 & 40 & 56 \\ 0 & 1 & 35 & 57 \end{bmatrix}_{15083147} = \mathbf{PI}(44,17,58,30,61,1)_{16501072}$$

$$\ell_{12} = \begin{bmatrix} 1 & 0 & \epsilon^{13} & \epsilon^{25} \\ 0 & 1 & \epsilon^{55} & \epsilon^{25} \end{bmatrix}_{2923991} = \begin{bmatrix} 1 & 0 & 51 \\ 0 & 1 & 25 & 46 \end{bmatrix}_{2923991} = \mathbf{PI}(42,45,75,6,21,1)_{6013296}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \epsilon^{13} & \epsilon^{24} \\ 0 & 1 & \epsilon^{55} & \epsilon^{25} \end{bmatrix}_{267703} = \begin{bmatrix} 1 & 0 & 62 & 10 \\ 0 & 1 & 25 & 46 \end{bmatrix}_{2923991} = \mathbf{PI}(44,47,58,30,61,1)_{16501072}$$

$$\ell_{14} = \begin{bmatrix} 1 & 0 & \epsilon^{13} & \epsilon^{24} \\ 0 & 1 & \epsilon^{55} & \epsilon^{25} \end{bmatrix}_{14991631} = \begin{bmatrix} 1 & 0 & 51 \\ 0 & 1 & 55 & 21 \end{bmatrix}_{267703} = \mathbf{PI}(44,47,58,30,61,1)_{13041999}$$

$$\ell_{15} = \begin{bmatrix} 1 & 0 & \epsilon^{14$$

Rank of lines: (12443794, 2707641, 10007856, 267120, 269464, 15436751, 15328587, 267910, 12389714, 269773, 9887188, 15083147, 2923991, 266772, 2882410, 267703, 14991631, 15158040, 10041140, 12314788, 15307800)

Rank of points on Klein quadric: (3173883, 9727584, 12611031, 48349, 148312, 15625769, 5124938, 264876, 2343040, 222883, 7062079, 16501072, 6013296, 197627, 13349999, 77507, 8308813, 9532126, 14659617, 3658013, 11010151)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 75 Double points: The double points on the surface are:

 $P_{195595} = (10, 47, 46, 1) = \ell_0 \cap \ell_1$ $P_{158063} = (46, 36, 37, 1) = \ell_0 \cap \ell_2$ $P_{137354} = (9, 33, 32, 1) = \ell_0 \cap \ell_3$ $P_{162356} = (51, 39, 38, 1) = \ell_0 \cap \ell_8$ $P_{62509} = (44, 15, 14, 1) = \ell_0 \cap \ell_{13}$ $P_{212237} = (12, 51, 50, 1) = \ell_0 \cap \ell_{17}$ $P_{16627} = (50, 2, 3, 1) = \ell_0 \cap \ell_{19}$ $P_{216264} = (7, 50, 51, 1) = \ell_0 \cap \ell_{20}$ $P_{45862} = (37, 11, 10, 1) = \ell_1 \cap \ell_2$ $P_{266209} = (32, 62, 63, 1) = \ell_1 \cap \ell_4$ $P_{120754} = (49, 29, 28, 1) = \ell_1 \cap \ell_5$ $P_{183006} = (29, 42, 43, 1) = \ell_1 \cap \ell_{12}$ $P_{24925} = (28, 4, 5, 1) = \ell_1 \cap \ell_{14}$ $P_{224591} = (14, 52, 53, 1) = \ell_1 \cap \ell_{15}$ $P_{124758} = (21, 28, 29, 1) = \ell_1 \cap \ell_{16}$ $P_{99831} = (54, 22, 23, 1) = \ell_2 \cap \ell_6$ $P_{187328} = (63, 45, 44, 1) = \ell_2 \cap \ell_7$ $P_{41590} = (53, 8, 9, 1) = \ell_2 \cap \ell_9$ $P_{74839} = (22, 16, 17, 1) = \ell_2 \cap \ell_{10}$ $P_{95770} = (25, 23, 22, 1) = \ell_2 \cap \ell_{11}$ $P_{116440} = (23, 26, 27, 1) = \ell_2 \cap \ell_{18}$ $P_{37956} = (3, 16, 8, 1) = \ell_3 \cap \ell_6$ $P_{217741} = (12, 9, 52, 1) = \ell_3 \cap \ell_{14}$ $P_{166945} = (32, 47, 39, 1) = \ell_3 \cap \ell_{16}$ $P_{183844} = (35, 55, 43, 1) = \ell_3 \cap \ell_{18}$ $P_{176960} = (63, 11, 42, 1) = \ell_4 \cap \ell_6$ $P_{39026} = (49, 32, 8, 1) = \ell_4 \cap \ell_{10}$ $P_{141830} = (5, 39, 33, 1) = \ell_4 \cap \ell_{17}$ $P_{115643} = (58, 13, 27, 1) = \ell_4 \cap \ell_{19}$ $P_{46049} = (32, 14, 10, 1) = \ell_5 \cap \ell_6$ $P_{260818} = (17, 42, 62, 1) = \ell_5 \cap \ell_7$ $P_{15375} = (14, 47, 2, 1) = \ell_5 \cap \ell_{13}$ $P_{196738} = (1, 1, 47, 1) = \ell_5 \cap \ell_{16}$ $P_{42487} = (54, 22, 9, 1) = \ell_5 \cap \ell_{18}$ $P_{257224} = (7, 50, 61, 1) = \ell_5 \cap \ell_{19}$ $P_{193197} = (44, 9, 46, 1) = \ell_5 \cap \ell_{20}$ $P_{64712} = (7, 50, 14, 1) = \ell_6 \cap \ell_8$ $P_{49282} = (1, 1, 11, 1) = \ell_6 \cap \ell_{11}$

 $P_{149426} = (49, 29, 35, 1) = \ell_6 \cap \ell_{12}$ $P_{159798} = (53, 63, 37, 1) = \ell_6 \cap \ell_{20}$ $P_{143386} = (25, 63, 33, 1) = \ell_7 \cap \ell_8$ $P_{19582} = (61, 48, 3, 1) = \ell_7 \cap \ell_{14}$ $P_{113005} = (44, 36, 26, 1) = \ell_7 \cap \ell_{17}$ $P_{14703} = (46, 36, 2, 1) = \ell_8 \cap \ell_{10}$ $P_{23563} = (10, 47, 4, 1) = \ell_8 \cap \ell_{12}$ $P_{74207} = (30, 6, 17, 1) = \ell_8 \cap \ell_{15}$ $P_{81165} = (12, 51, 18, 1) = \ell_8 \cap \ell_{16}$ $P_{151728} = (47, 1, 36, 1) = \ell_8 \cap \ell_{19}$ $P_{161107} = (18, 20, 38, 1) = \ell_9 \cap \ell_{12}$ $P_{217436} = (27, 4, 52, 1) = \ell_9 \cap \ell_{16}$ $P_{69047} = (54, 53, 15, 1) = \ell_9 \cap \ell_{19}$ $P_{72010} = (9, 36, 16, 1) = \ell_9 \cap \ell_{20}$ $P_{26217} = (40, 24, 5, 1) = \ell_{10} \cap \ell_{13}$ $P_{111398} = (37, 11, 26, 1) = \ell_{10} \cap \ell_{14}$ $P_{185882} = (25, 23, 44, 1) = \ell_{10} \cap \ell_{16}$ $P_{49317} = (36, 1, 11, 1) = \ell_{10} \cap \ell_{18}$ $P_{128503} = (54, 22, 30, 1) = \ell_{10} \cap \ell_{20}$ $P_{190183} = (38, 26, 45, 1) = \ell_{11} \cap \ell_{13}$ $P_{169814} = (21, 28, 40, 1) = \ell_{11} \cap \ell_{14}$ $P_{21302} = (53, 11, 4, 1) = \ell_{11} \cap \ell_{15}$ $P_{47183} = (14, 32, 10, 1) = \ell_{11} \cap \ell_{16}$ $P_{159168} = (63, 53, 37, 1) = \ell_{11} \cap \ell_{17}$ $P_{138509} = (12, 51, 32, 1) = \ell_{11} \cap \ell_{19}$ $P_{260936} = (7, 44, 62, 1) = \ell_{12} \cap \ell_{13}$ $P_{196748} = (11, 1, 47, 1) = \ell_{12} \cap \ell_{14}$ $P_{223062} = (21, 28, 53, 1) = \ell_{12} \cap \ell_{17}$ $P_{70438} = (37, 11, 16, 1) = \ell_{12} \cap \ell_{18}$ $P_{179211} = (10, 47, 42, 1) = \ell_{14} \cap \ell_{19}$ $P_{264114} = (49, 29, 63, 1) = \ell_{14} \cap \ell_{20}$ $P_{189398} = (21, 14, 45, 1) = \ell_{15} \cap \ell_{18}$ $P_{65772} = (43, 2, 15, 1) = \ell_{15} \cap \ell_{20}$ $P_{195402} = (9, 44, 46, 1) = \ell_{16} \cap \ell_{17}$ $P_{243226} = (25, 23, 58, 1) = \ell_{17} \cap \ell_{18}$ $P_{151682} = (1, 1, 36, 1) = \ell_{17} \cap \ell_{20}$ $P_{166255} = (46, 36, 39, 1) = \ell_{18} \cap \ell_{19}$

Single Points

The surface has 1209 single points: Too many to print.

Points on surface but on no line

The surface has 3196 points not on any line: Too many to print.

Line Intersection Graph

	01234	56	789	10	11	12	13	14	15	16	17	18	19 :	20
0	01110	0.0	010	0	0	0	1	0	0	0	1	0	1	1
1	10101	10	0 0 0	0	0	1	0	1	1	1	0	0	0	0
2	11000	01	101	1	1	0	0	0	0	0	0	1	0	0
3	10001	0.1	101	0	0	0	1	1	1	1	0	1	0	0
4	01010	01	101	1	0	0	1	0	1	0	1	0	1	0
5	01000	01	100	0	0	0	1	0	0	1	0	1	1	1
6	00111	10	010	0	1	1	0	0	0	0	0	0	0	1
7	00111	10	011	0	0	0	1	1	1	0	1	0	0	0
8	10000	01	100	1	0	1	0	0	1	1	0	0	1	0
9	00111	00	100	0	0	1	1	0	1	1	0	0	1	1
10	00101	00	010	0	0	0	1	1	0	1	0	1	0	1
11	00100	01	0 0 0	0	0	0	1	1	1	1	1	0	1	0
12	01000	0.1	011	0	0	0	1	1	0	0	1	1	0	0
13	10011	10	101	1	1	1	0	0	1	0	0	0	0	0
14	01010	00	100	1	1	1	0	0	0	0	0	0	1	1
15	01011	00	111	0	1	0	1	0	0	0	0	1	0	1
16	01010	10	011	1	1	0	0	0	0	0	1	0	0	0
17	10001	00	100	0	1	1	0	0	0	1	0	1	0	1
18	00110	10	0 0 0	1	0	1	0	0	1	0	1	0	1	0
19	10001	10	011	0	1	0	0	1	0	0	0	1	0	0
20	10000	11	001	1	0	0	0	1	1	0	1	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_8	ℓ_{13}	ℓ_{17}	ℓ_{19}	ℓ_{20}
in point	P_{195595}	P_{158063}	P_{137354}	P_{162356}	P_{62509}	P_{212237}	P_{16627}	P_{216264}

${\bf Line~1~intersects}$

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_{12}	ℓ_{14}	ℓ_{15}	ℓ_{16}
in point	P_{195595}	P_{45862}	P_{266209}	P_{120754}	P_{183006}	P_{24925}	P_{224591}	P_{124758}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{18}
in point	P_{158063}	P_{45862}	P_{99831}	P_{187328}	P_{41590}	P_{74839}	P_{95770}	P_{116440}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{18}
in point	P_{137354}	P_{4163}	P_{37956}	P_{4163}	P_{4163}	P_{4163}	P_{217741}	P_{4163}	P_{166945}	P_{183844}

Line 4 intersects

	Line	ℓ_1	ℓ_3	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{13}	ℓ_{15}	ℓ_{17}	ℓ_{19}
ĺ	in point	P_{266209}	P_{4163}	P_{176960}	P_{4163}	P_{4163}	P_{39026}	P_{4163}	P_{4163}	P_{141830}	P_{115643}

${\bf Line~5~intersects}$

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_{13}	ℓ_{16}	ℓ_{18}	ℓ_{19}	ℓ_{20}
in point	P_{120754}	P_{46049}	P_{260818}	P_{15375}	P_{196738}	P_{42487}	P_{257224}	P_{193197}

Line 6 intersects

	Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_{11}	ℓ_{12}	ℓ_{20}
ĺ	in point	P_{99831}	P_{37956}	P_{176960}	P_{46049}	P_{64712}	P_{49282}	P_{149426}	P_{159798}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_{187328}	P_{4163}	P_{4163}	P_{260818}	P_{143386}	P_{4163}	P_{4163}	P_{19582}	P_{4163}	P_{113005}

Line 8 intersects

Line	ℓ_0	ℓ_6	ℓ_7	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{16}	ℓ_{19}
in point	P_{162356}	P_{64712}	P_{143386}	P_{14703}	P_{23563}	P_{74207}	P_{81165}	P_{151728}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_7	ℓ_{12}	ℓ_{13}	ℓ_{15}	ℓ_{16}	ℓ_{19}	ℓ_{20}
in point	P_{41590}	P_{4163}	P_{4163}	P_{4163}	P_{161107}	P_{4163}	P_{4163}	P_{217436}	P_{69047}	P_{72010}

Line 10 intersects

Line	ℓ_2	ℓ_4	ℓ_8	ℓ_{13}	ℓ_{14}	ℓ_{16}	ℓ_{18}	ℓ_{20}
in point	P_{74839}	P_{39026}	P_{14703}	P_{26217}	P_{111398}	P_{185882}	P_{49317}	P_{128503}

Line 11 intersects

Line	ℓ_2	ℓ_6	ℓ_{13}	ℓ_{14}	ℓ_{15}	ℓ_{16}	ℓ_{17}	ℓ_{19}
in point	P_{95770}	P_{49282}	P_{190183}	P_{169814}	P_{21302}	P_{47183}	P_{159168}	P_{138509}

Line 12 intersects

Line	ℓ_1	ℓ_6	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{17}	ℓ_{18}
in point	P_{183006}	P_{149426}	P_{23563}	P_{161107}	P_{260936}	P_{196748}	P_{223062}	P_{70438}

${\rm Line}\ 13\ {\rm intersects}$

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{15}
in point	P_{62509}	P_{4163}	P_{4163}	P_{15375}	P_{4163}	P_{4163}	P_{26217}	P_{190183}	P_{260936}	P_{4163}

Line 14 intersects

Line	ℓ_1	ℓ_3	ℓ_7	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{19}	ℓ_{20}
in point	P_{24925}	P_{217741}	P_{19582}	P_{111398}	P_{169814}	P_{196748}	P_{179211}	P_{264114}

Line 15 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{13}	ℓ_{18}	ℓ_{20}
in point	P_{224591}	P_{4163}	P_{4163}	P_{4163}	P_{74207}	P_{4163}	P_{21302}	P_{4163}	P_{189398}	P_{65772}

Line 16 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{17}
in point	P_{124758}	P_{166945}	P_{196738}	P_{81165}	P_{217436}	P_{185882}	P_{47183}	P_{195402}

Line 17 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_{11}	ℓ_{12}	ℓ_{16}	ℓ_{18}	ℓ_{20}
in point	P_{212237}	P_{141830}	P_{113005}	P_{159168}	P_{223062}	P_{195402}	P_{243226}	P_{151682}

${\rm Line}\ 18\ {\rm intersects}$

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_{10}	ℓ_{12}	ℓ_{15}	ℓ_{17}	ℓ_{19}
in point	P_{116440}	P_{183844}	P_{42487}	P_{49317}	P_{70438}	P_{189398}	P_{243226}	P_{166255}

Line 19 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{14}	ℓ_{18}
in point	P_{16627}	P_{115643}	P_{257224}	P_{151728}	P_{69047}	P_{138509}	P_{179211}	P_{166255}

Line 20 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_9	ℓ_{10}	ℓ_{14}	ℓ_{15}	ℓ_{17}
in point	P_{216264}	P_{193197}	P_{159798}	P_{72010}	P_{128503}	P_{264114}	P_{65772}	P_{151682}

The surface has 4481 points:

Too many to print.