Rank-67115 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_1^2X_3 + X_0X_2^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1432967577

General information

Number of lines	12
Number of points	33
Number of singular points	3
Number of Eckardt points	0
Number of double points	18
Number of single points	12
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{12}
Type of lines on points	4^3, 2^{18}, 1^{12}

Singular Points

The surface has 3 singular points:

$0:\ P_{26} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$
$1:\ P_{27} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
$2:\ P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 12 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4} = \mathbf{Pl}(1,0,0,0,1,0)_{26}$$

$$\ell_4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{341} = \mathbf{Pl}(0,1,0,0,0,1)_{105}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{320} = \mathbf{Pl}(1,0,1,1,2,1)_{259}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{215} = \mathbf{Pl}(1,0,1,1,3,1)_{319}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \omega^2 & 1 \\ 0 & 1 & \omega^2 & 0 \end{bmatrix}_{150} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix}_{150} = \mathbf{Pl}(1,1,1,2,0,1)_{159}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{298} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{298} = \mathbf{Pl}(1,1,0,2,3,1)_{312}$$

$$\ell_{10} = \begin{bmatrix} 1 & 0 & \omega & 1 \\ 0 & 1 & \omega & 0 \end{bmatrix}_{128} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{128} = \mathbf{Pl}(1,1,1,3,0,1)_{168}$$

$$\ell_{11} = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & 0 & 1 \end{bmatrix}_{235} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{235} = \mathbf{Pl}(1,1,0,3,2,1)_{255}$$

Rank of lines: (0, 22, 336, 4, 341, 320, 215, 26, 150, 298, 128, 235)
Rank of points on Klein quadric: (0, 109, 101, 26, 105, 259, 319, 180, 159, 312, 168, 255)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 18 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_0 = (1,0,0,0) = \ell_0 \cap \ell_3$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_5$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_6$
$P_{11} = (0,1,1,0) = \ell_1 \cap \ell_2$
$P_8 = (1,0,1,0) = \ell_1 \cap \ell_7$
$P_{18} = (3,2,1,0) = \ell_1 \cap \ell_9$
$P_{21} = (2,3,1,0) = \ell_1 \cap \ell_{11}$
$P_2 = (0,0,1,0) = \ell_2 \cap \ell_4$
$P_{15} = (0,2,1,0) = \ell_2 \cap \ell_8$
$P_{19} = (0,3,1,0) = \ell_2 \cap \ell_{10}$
$P_{52} = (3,3,1,1) = \ell_5 \cap \ell_9$
$P_{46} = (1,2,1,1) = \ell_5 \cap \ell_{10}$
$P_{50} = (1,3,1,1) = \ell_6 \cap \ell_8$
$P_{47} = (2,2,1,1) = \ell_6 \cap \ell_{11}$
$P_{62} = (1,2,2,1) = \ell_8 \cap \ell_9$
$P_{82} = (1,3,3,1) = \ell_{10} \cap \ell_{11}$

Single Points

The surface has 12 single points:
The single points on the surface are:

0 : $P_{28} = (2, 1, 0, 1)$ lies on line ℓ_3
1 : $P_{29} = (3, 1, 0, 1)$ lies on line ℓ_3
2 : $P_{40} = (2, 0, 1, 1)$ lies on line ℓ_5
3 : $P_{41} = (3, 0, 1, 1)$ lies on line ℓ_6
4 : $P_{54} = (1, 0, 2, 1)$ lies on line ℓ_{10}
5 : $P_{56} = (3, 0, 2, 1)$ lies on line ℓ_{11}
6 : $P_{57} = (0, 1, 2, 1)$ lies on line ℓ_4
7 : $P_{60} = (3, 1, 2, 1)$ lies on line ℓ_7
8 : $P_{70} = (1, 0, 3, 1)$ lies on line ℓ_8
9 : $P_{71} = (2, 0, 3, 1)$ lies on line ℓ_9
10 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_4
11 : $P_{75} = (2, 1, 3, 1)$ lies on line ℓ_7

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	1	1	0	1	1	0	0	0	0	0
1	1	0	1	0	0	0	0	1	0	1	0	1
2	1	1	0	0	1	0	0	0	1	0	1	0
3	1	0	0	0	1	0	0	1	1	1	1	1
4	0	0	1	1	0	1	1	1	0	1	0	1
5	1	0	0	0	1	0	1	1	0	1	1	0
6	1	0	0	0	1	1	0	1	1	0	0	1
7	0	1	0	1	1	1	1	0	1	0	1	0
8	0	0	1	1	0	0	1	1	0	1	1	0
9	0	1	0	1	1	1	0	0	1	0	0	1
10	0	0	1	1	0	1	0	1	1	0	0	1
11	0	1	0	1	1	0	1	0	0	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_5	P_1	P_0	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_7	ℓ_9	ℓ_{11}
in point	P_5	P_{11}	P_8	P_{18}	P_{21}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_8	ℓ_{10}
in point	P_1	P_{11}	P_2	P_{15}	P_{19}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}
in point	P_0	P_{26}	P_{27}	P_{27}	P_{26}	P_{27}	P_{26}

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9	ℓ_{11}
in point	P_2	P_{26}	P_{42}	P_{42}	P_{42}	P_{26}	P_{26}

Line 5 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}
in point	P_6	P_{42}	P_{42}	P_{42}	P_{52}	P_{46}

Line 6 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_{11}
in point	P_7	P_{42}	P_{42}	P_{42}	P_{50}	P_{47}

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}
in point	P_8	P_{27}	P_{42}	P_{42}	P_{42}	P_{27}	P_{27}

Line 8 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7	ℓ_9	ℓ_{10}
in point	P_{15}	P_{27}	P_{50}	P_{27}	P_{62}	P_{27}

Line 9 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_8	ℓ_{11}
in point	P_{18}	P_{26}	P_{26}	P_{52}	P_{62}	P_{26}

Line 10 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_7	ℓ_8	ℓ_{11}
in point	P_{19}	P_{27}	P_{46}	P_{27}	P_{27}	P_{82}

Line 11 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_6	ℓ_9	ℓ_{10}
in point	P_{21}	P_{26}	P_{26}	P_{47}	P_{26}	P_{82}

The surface has 33 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (2,1,0,0)$
$5: P_7 = (3,1,0,0)$
$6: P_8 = (1,0,1,0)$
$7: P_{11} = (0,1,1,0)$
$8: P_{15} = (0,2,1,0)$
$9: P_{18} = (3,2,1,0)$
$10: P_{19} = (0,3,1,0)$
$11: P_{21} = (2,3,1,0)$
$12: P_{26} = (0,1,0,1)$
$13: P_{27} = (1,1,0,1)$
$14: P_{28} = (2,1,0,1)$
$15: P_{29} = (3,1,0,1)$
$16: P_{40} = (2,0,1,1)$
$17: P_{41} = (3,0,1,1)$
$18: P_{42} = (0,1,1,1)$
$19: P_{46} = (1,2,1,1)$
$20: P_{47} = (2,2,1,1)$
$21: P_{50} = (1,3,1,1)$
$22: P_{52} = (3,3,1,1)$
$23: P_{54} = (1,0,2,1)$
$24: P_{56} = (3,0,2,1)$
$25: P_{57} = (0,1,2,1)$
$26: P_{60} = (3,1,2,1)$
$27: P_{62} = (1,2,2,1)$
$28: P_{70} = (1,0,3,1)$
$29: P_{71} = (2,0,3,1)$
$30: P_{73} = (0,1,3,1)$
$31: P_{75} = (2,1,3,1)$
$32: P_{82} = (1,3,3,1)$