Rank-20 over GF(4)

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The equation

The equation of the surface is :

$$X_0^3+X_1^3+X_2^3+X_3^3+X_0^2X_1+X_0^2X_2+X_0^2X_3+X_0X_1^2+X_1^2X_2+X_1^2X_3$$
$$+X_0X_2^2+X_1X_2^2+X_2^2X_3+X_0X_3^2+X_1X_3^2+X_2X_3^2+X_0X_1X_2+X_0X_1X_3+X_0X_2X_3+X_1X_2X_3=0$$

(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
The point rank of the equation over GF(4) is 20

General information

Number of lines	9
Number of points	29
Number of singular points	4
Number of Eckardt points	5
Number of double points	6
Number of single points	18
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^9
Type of lines on points	3^5, 2^6, 1^{18}

Singular Points

The surface has 4 singular points:

$0:\ P_{12}=\mathbf{P}(1,1,1,0)=\mathbf{P}(1,1,1,0)$
$1:\ P_{27}=\mathbf{P}(1,1,0,1)=\mathbf{P}(1,1,0,1)$
$2:\ P_{39}=\mathbf{P}(1,0,1,1)=\mathbf{P}(1,0,1,1)$
$3:\ P_{42}=\mathbf{P}(0,1,1,1)=\mathbf{P}(0,1,1,1)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1,0,1,1,1,1)_{199}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \mathbf{Pl}(1,1,1,1,0,0)_{16}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1,1,0,1,1,1)_{189}$$

$$\ell_8 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122} = \mathbf{Pl}(0,1,1,1,1,1)_{202}$$

Rank of lines: (38, 110, 25, 26, 85, 106, 89, 109, 122)
Rank of points on Klein quadric: (198, 199, 177, 180, 16, 150, 74, 189, 202)

Eckardt Points

The surface has 5 Eckardt points:
$0 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$,
$1 : P_{12} = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$,
$2 : P_{27} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$,
$3 : P_{39} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$,
$4 : P_{42} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$
$P_{38} = (0,0,1,1) = \ell_0 \cap \ell_8$
$P_8 = (1,0,1,0) = \ell_2 \cap \ell_3$
$P_{26} = (0,1,0,1) = \ell_2 \cap \ell_7$
$P_{11} = (0,1,1,0) = \ell_4 \cap \ell_5$
$P_{23} = (1,0,0,1) = \ell_4 \cap \ell_6$

Single Points

The surface has 18 single points:
The single points on the surface are:

0 : $P_{47} = (2, 2, 1, 1)$ lies on line ℓ_0
1 : $P_{48} = (3, 2, 1, 1)$ lies on line ℓ_1
2 : $P_{51} = (2, 3, 1, 1)$ lies on line ℓ_1
3 : $P_{52} = (3, 3, 1, 1)$ lies on line ℓ_0
4 : $P_{59} = (2, 1, 2, 1)$ lies on line ℓ_2
5 : $P_{60} = (3, 1, 2, 1)$ lies on line ℓ_3
6 : $P_{62} = (1, 2, 2, 1)$ lies on line ℓ_4
7 : $P_{64} = (3, 2, 2, 1)$ lies on line ℓ_6
8 : $P_{66} = (1, 3, 2, 1)$ lies on line ℓ_5
9 : $P_{67} = (2, 3, 2, 1)$ lies on line ℓ_7
10 : $P_{68} = (3, 3, 2, 1)$ lies on line ℓ_8
11 : $P_{75} = (2, 1, 3, 1)$ lies on line ℓ_3
12 : $P_{76} = (3, 1, 3, 1)$ lies on line ℓ_2
13 : $P_{78} = (1, 2, 3, 1)$ lies on line ℓ_5
14 : $P_{79} = (2, 2, 3, 1)$ lies on line ℓ_8
15 : $P_{80} = (3, 2, 3, 1)$ lies on line ℓ_7
16 : $P_{82} = (1, 3, 3, 1)$ lies on line ℓ_4
17 : $P_{83} = (2, 3, 3, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	0	1	0	0	0	1
1	1	0	0	1	0	1	1	1	0
2	1	0	0	1	1	0	0	1	0
3	0	1	1	0	0	1	1	0	1
4	1	0	1	0	0	1	1	0	0
5	0	1	0	1	1	0	0	1	1
6	0	1	0	1	1	0	0	1	1
7	0	1	1	0	0	1	1	0	1
8	1	0	0	1	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_8
in point	P_5	P_4	P_4	P_{38}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_5	P_{42}	P_{39}	P_{42}	P_{39}

Line 2 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_7
in point	P_4	P_8	P_4	P_{26}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{42}	P_8	P_{27}	P_{42}	P_{27}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6
in point	P_4	P_4	P_{11}	P_{23}

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{39}	P_{27}	P_{11}	P_{39}	P_{27}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_7	ℓ_8
in point	P_{42}	P_{42}	P_{23}	P_{12}	P_{12}

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_8
in point	P_{39}	P_{26}	P_{39}	P_{12}	P_{12}

Line 8 intersects

Line	ℓ_0	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_{38}	P_{27}	P_{27}	P_{12}	P_{12}

The surface has 29 points:
The points on the surface are:

$0: P_4 = (1,1,1,1)$
$1: P_5 = (1,1,0,0)$
$2: P_8 = (1,0,1,0)$
$3: P_{11} = (0,1,1,0)$
$4: P_{12} = (1,1,1,0)$
$5: P_{23} = (1,0,0,1)$
$6: P_{26} = (0,1,0,1)$
$7: P_{27} = (1,1,0,1)$
$8: P_{38} = (0,0,1,1)$
$9: P_{39} = (1,0,1,1)$
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$11: P_{47} = (2,2,1,1)$
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$27: P_{82} = (1,3,3,1)$
$28: P_{83} = (2,3,3,1)$