Rank-65665 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431661229

General information

Number of lines	4
Number of points	25
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	12
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	$2^4, 1^{12}, 0^9$

Singular Points

The surface has 2 singular points:

$0 : P_{27} = \mathbf{P}(1,1,0,1) = \mathbf{P}(1,1,0,1)$
$1 : P_{39} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{22} = \mathbf{Pl}(1,0,1,0,0,1)_{109}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \mathbf{Pl}(1,0,0,0,1,0)_{26}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{106} = \mathbf{Pl}(1,1,1,1,0,1)_{150}$$

Rank of lines: (22, 4, 17, 106)
Rank of points on Klein quadric: (109, 26, 32, 150)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_{11} = (0,1,1,0) = \ell_0 \cap \ell_3$ $P_{39} = (1,0,1,1) = \ell_2 \cap \ell_3$
$P_0 = (1,0,0,0) = \ell_1 \cap \ell_2$
$P_{27} = (1,1,0,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 12 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_0 $7 : P_{38} = (0,0,1,1)$ lies on line ℓ_2
$1 : P_8 = (1,0,1,0)$ lies on line ℓ_0 $8 : P_{40} = (2,0,1,1)$ lies on line ℓ_2
$2 : P_{18} = (3,2,1,0)$ lies on line ℓ_0 $9 : P_{41} = (3,0,1,1)$ lies on line ℓ_2
$3 : P_{21} = (2,3,1,0)$ lies on line ℓ_0 $10 : P_{66} = (1,3,2,1)$ lies on line ℓ_3
$4 : P_{26} = (0,1,0,1)$ lies on line ℓ_1 $11 : P_{78} = (1,2,3,1)$ lies on line ℓ_3
$5 : P_{28} = (2,1,0,1)$ lies on line ℓ_1
$6 : P_{29} = (3,1,0,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

$0 : P_{15} = (0,2,1,0)$ $5 : P_{43} = (2,1,1,1)$
$1 : P_{19} = (0,3,1,0)$ $6 : P_{44} = (3,1,1,1)$
$2 : P_{23} = (1,0,0,1)$ $7 : P_{53} = (0,0,2,1)$
$3 : P_{30} = (0,2,0,1)$ $8 : P_{69} = (0,0,3,1)$
$4 : P_{34} = (0,3,0,1)$

Line Intersection Graph

$$
\begin{array}{c|cccc}
 & 0\ 1\ 2\ 3 \\
\hline
0 & 0\ 0\ 0\ 1 \\
1 & 0\ 0\ 1\ 1 \\
2 & 0\ 1\ 0\ 1 \\
3 & 1\ 1\ 1\ 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_3
in point	P_{11}

Line 1 intersects

Line	ℓ_2	ℓ_3
in point	P_0	P_{27}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_0	P_{39}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_{11}	P_{27}	P_{39}

The surface has 25 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$

$1: P_5 = (1,1,0,0)$

$2: P_8 = (1,0,1,0)$

$3: P_{11} = (0,1,1,0)$

$4: P_{15} = (0,2,1,0)$

$5: P_{18} = (3,2,1,0)$

$6: P_{19} = (0,3,1,0)$

$7: P_{21} = (2,3,1,0)$

$8: P_{23} = (1,0,0,1)$

$9: P_{26} = (0,1,0,1)$

$10: P_{27} = (1,1,0,1)$

$11: P_{28} = (2,1,0,1)$

$12: P_{29} = (3,1,0,1)$

$13: P_{30} = (0,2,0,1)$

$14: P_{34} = (0,3,0,1)$

$15: P_{38} = (0,0,1,1)$

$16: P_{39} = (1,0,1,1)$

$17: P_{40} = (2,0,1,1)$

$18: P_{41} = (3,0,1,1)$

$19: P_{43} = (2,1,1,1)$

$20: P_{44} = (3,1,1,1)$

$21: P_{53} = (0,0,2,1)$

$22: P_{66} = (1,3,2,1)$

$23: P_{69} = (0,0,3,1)$

$24: P_{78} = (1,2,3,1)$