Rank-76387 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_2 + X_0^2X_3 + X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 304156949

General information

Number of lines	4
Number of points	289
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	60
Number of points off lines	225
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^4
Type of lines on points	2^4, 1^{60}, 0^{225}

Singular Points

The surface has 2 singular points:

$0:\ P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
$1:\ P_5 = \mathbf{P}(1,1,0,0) = \mathbf{P}(1,1,0,0)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,0,1,0,1,0)_{321}$$

$$\ell_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

Rank of lines: (0, 17, 70160, 530)
Rank of points on Klein quadric: (0, 321, 1, 9426)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_3$
$P_4 = (1,1,1,1) = \ell_1 \cap \ell_3$
$P_{530} = (0,0,1,1) = \ell_2 \cap \ell_3$

Single Points

The surface has 60 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_2 = (0,0,1,0)$ lies on line ℓ_2
2 : $P_3 = (0,0,0,1)$ lies on line ℓ_2
3 : $P_6 = (2,1,0,0)$ lies on line ℓ_0
4 : $P_7 = (3,1,0,0)$ lies on line ℓ_0
5 : $P_8 = (4,1,0,0)$ lies on line ℓ_0
6 : $P_9 = (5,1,0,0)$ lies on line ℓ_0
7 : $P_{10} = (6,1,0,0)$ lies on line ℓ_0
8 : $P_{11} = (7,1,0,0)$ lies on line ℓ_0
9 : $P_{12} = (8,1,0,0)$ lies on line ℓ_0
10 : $P_{13} = (9,1,0,0)$ lies on line ℓ_0
11 : $P_{14} = (10,1,0,0)$ lies on line ℓ_0
12 : $P_{15} = (11,1,0,0)$ lies on line ℓ_0
13 : $P_{16} = (12,1,0,0)$ lies on line ℓ_0
14 : $P_{17} = (13,1,0,0)$ lies on line ℓ_0
15 : $P_{18} = (14,1,0,0)$ lies on line ℓ_0
16 : $P_{19} = (15,1,0,0)$ lies on line ℓ_0
17 : $P_{546} = (0,1,1,1)$ lies on line ℓ_1
18 : $P_{547} = (2,1,1,1)$ lies on line ℓ_1
19 : $P_{548} = (3,1,1,1)$ lies on line ℓ_1
20 : $P_{549} = (4,1,1,1)$ lies on line ℓ_1
21 : $P_{550} = (5,1,1,1)$ lies on line ℓ_1
22 : $P_{551} = (6,1,1,1)$ lies on line ℓ_1
23 : $P_{552} = (7,1,1,1)$ lies on line ℓ_1
24 : $P_{553} = (8,1,1,1)$ lies on line ℓ_1
25 : $P_{554} = (9,1,1,1)$ lies on line ℓ_1
26 : $P_{555} = (10,1,1,1)$ lies on line ℓ_1
27 : $P_{556} = (11,1,1,1)$ lies on line ℓ_1
28 : $P_{557} = (12,1,1,1)$ lies on line ℓ_1
29 : $P_{558} = (13,1,1,1)$ lies on line ℓ_1
30 : $P_{559} = (14,1,1,1)$ lies on line ℓ_1
31 : $P_{560} = (15,1,1,1)$ lies on line ℓ_1
32 : $P_{563} = (2,2,1,1)$ lies on line ℓ_3
33 : $P_{580} = (3,3,1,1)$ lies on line ℓ_3
34 : $P_{597} = (4,4,1,1)$ lies on line ℓ_3
35 : $P_{614} = (5,5,1,1)$ lies on line ℓ_3
36 : $P_{631} = (6,6,1,1)$ lies on line ℓ_3
37 : $P_{648} = (7,7,1,1)$ lies on line ℓ_3
38 : $P_{665} = (8,8,1,1)$ lies on line ℓ_3
39 : $P_{682} = (9,9,1,1)$ lies on line ℓ_3
40 : $P_{699} = (10,10,1,1)$ lies on line ℓ_3
41 : $P_{716} = (11,11,1,1)$ lies on line ℓ_3
42 : $P_{733} = (12,12,1,1)$ lies on line ℓ_3
43 : $P_{750} = (13,13,1,1)$ lies on line ℓ_3

44 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_3
45 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_3
46 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_2
47 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_2
48 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_2
49 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_2
50 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_2
51 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_2
52 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_2
53 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_2
54 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_2
55 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_2
56 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_2
57 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_2
58 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_2
59 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 225 points not on any line:
The points on the surface but not on lines are:

0 : $P_{45} = (10, 1, 1, 0)$
1 : $P_{46} = (11, 1, 1, 0)$
2 : $P_{56} = (5, 2, 1, 0)$
3 : $P_{58} = (7, 2, 1, 0)$
4 : $P_{91} = (8, 4, 1, 0)$
5 : $P_{95} = (12, 4, 1, 0)$
6 : $P_{169} = (6, 9, 1, 0)$
7 : $P_{178} = (15, 9, 1, 0)$
8 : $P_{183} = (4, 10, 1, 0)$
9 : $P_{193} = (14, 10, 1, 0)$
10 : $P_{197} = (2, 11, 1, 0)$
11 : $P_{204} = (9, 11, 1, 0)$
12 : $P_{246} = (3, 14, 1, 0)$
13 : $P_{256} = (13, 14, 1, 0)$
14 : $P_{275} = (1, 0, 0, 1)$
15 : $P_{300} = (10, 1, 0, 1)$
16 : $P_{301} = (11, 1, 0, 1)$
17 : $P_{374} = (4, 6, 0, 1)$
18 : $P_{375} = (5, 6, 0, 1)$
19 : $P_{400} = (14, 7, 0, 1)$
20 : $P_{401} = (15, 7, 0, 1)$
21 : $P_{440} = (6, 10, 0, 1)$
22 : $P_{441} = (7, 10, 0, 1)$
23 : $P_{462} = (12, 11, 0, 1)$
24 : $P_{463} = (13, 11, 0, 1)$
25 : $P_{468} = (2, 12, 0, 1)$
26 : $P_{469} = (3, 12, 0, 1)$
27 : $P_{490} = (8, 13, 0, 1)$
28 : $P_{491} = (9, 13, 0, 1)$
29 : $P_{793} = (8, 0, 2, 1)$
30 : $P_{849} = (0, 4, 2, 1)$
31 : $P_{856} = (7, 4, 2, 1)$
32 : $P_{869} = (4, 5, 2, 1)$
33 : $P_{875} = (10, 5, 2, 1)$
34 : $P_{931} = (2, 9, 2, 1)$
35 : $P_{933} = (4, 9, 2, 1)$
36 : $P_{962} = (1, 11, 2, 1)$
37 : $P_{973} = (12, 11, 2, 1)$
38 : $P_{987} = (10, 12, 2, 1)$
39 : $P_{994} = (1, 13, 2, 1)$
40 : $P_{1001} = (8, 13, 2, 1)$
41 : $P_{1016} = (7, 14, 2, 1)$
42 : $P_{1021} = (12, 14, 2, 1)$
43 : $P_{1053} = (12, 0, 3, 1)$
44 : $P_{1077} = (4, 2, 3, 1)$
45 : $P_{1084} = (11, 2, 3, 1)$
46 : $P_{1098} = (9, 3, 3, 1)$
47 : $P_{1100} = (11, 3, 3, 1)$
48 : $P_{1108} = (3, 4, 3, 1)$
49 : $P_{1114} = (9, 4, 3, 1)$
50 : $P_{1121} = (0, 5, 3, 1)$
51 : $P_{1128} = (7, 5, 3, 1)$
52 : $P_{1181} = (12, 8, 3, 1)$
53 : $P_{1192} = (7, 9, 3, 1)$
54 : $P_{1195} = (10, 9, 3, 1)$
55 : $P_{1221} = (4, 11, 3, 1)$
56 : $P_{1227} = (10, 11, 3, 1)$
57 : $P_{1312} = (15, 0, 4, 1)$
58 : $P_{1335} = (6, 2, 4, 1)$
59 : $P_{1341} = (12, 2, 4, 1)$
60 : $P_{1404} = (11, 6, 4, 1)$
61 : $P_{1410} = (1, 7, 4, 1)$
62 : $P_{1424} = (15, 7, 4, 1)$
63 : $P_{1434} = (9, 8, 4, 1)$
64 : $P_{1436} = (11, 8, 4, 1)$
65 : $P_{1441} = (0, 9, 4, 1)$
66 : $P_{1453} = (12, 9, 4, 1)$
67 : $P_{1458} = (1, 10, 4, 1)$
68 : $P_{1463} = (6, 10, 4, 1)$
69 : $P_{1525} = (4, 14, 4, 1)$

$70: P_{1530} = (9, 14, 4, 1)$
$71: P_{1559} = (6, 0, 5, 1)$
$72: P_{1626} = (9, 4, 5, 1)$
$73: P_{1627} = (10, 4, 5, 1)$
$74: P_{1643} = (10, 5, 5, 1)$
$75: P_{1647} = (14, 5, 5, 1)$
$76: P_{1681} = (0, 8, 5, 1)$
$77: P_{1693} = (12, 8, 5, 1)$
$78: P_{1702} = (5, 9, 5, 1)$
$79: P_{1711} = (14, 9, 5, 1)$
$80: P_{1722} = (9, 10, 5, 1)$
$81: P_{1724} = (11, 10, 5, 1)$
$82: P_{1788} = (11, 14, 5, 1)$
$83: P_{1789} = (12, 14, 5, 1)$
$84: P_{1799} = (6, 15, 5, 1)$
$85: P_{1823} = (14, 0, 6, 1)$
$86: P_{1833} = (8, 1, 6, 1)$
$87: P_{1834} = (9, 1, 6, 1)$
$88: P_{1848} = (7, 2, 6, 1)$
$89: P_{1855} = (14, 2, 6, 1)$
$90: P_{1880} = (7, 4, 6, 1)$
$91: P_{1939} = (2, 8, 6, 1)$
$92: P_{1946} = (9, 8, 6, 1)$
$93: P_{1955} = (2, 9, 6, 1)$
$94: P_{1959} = (6, 9, 6, 1)$
$95: P_{2017} = (0, 13, 6, 1)$
$96: P_{2027} = (10, 13, 6, 1)$
$97: P_{2041} = (8, 14, 6, 1)$
$98: P_{2043} = (10, 14, 6, 1)$
$99: P_{2069} = (4, 0, 7, 1)$
$100: P_{2083} = (2, 1, 7, 1)$
$101: P_{2084} = (3, 1, 7, 1)$
$102: P_{2104} = (7, 2, 7, 1)$
$103: P_{2106} = (9, 2, 7, 1)$
$104: P_{2115} = (2, 3, 7, 1)$
$105: P_{2122} = (9, 3, 7, 1)$
$106: P_{2132} = (3, 4, 7, 1)$
$107: P_{2139} = (10, 4, 7, 1)$
$108: P_{2213} = (4, 9, 7, 1)$
$109: P_{2215} = (6, 9, 7, 1)$
$110: P_{2257} = (0, 12, 7, 1)$
$111: P_{2267} = (10, 12, 7, 1)$
$112: P_{2295} = (6, 14, 7, 1)$
$113: P_{2334} = (13, 0, 8, 1)$
$114: P_{2359} = (6, 2, 8, 1)$
$115: P_{2363} = (10, 2, 8, 1)$
$116: P_{2382} = (13, 3, 8, 1)$
$117: P_{2451} = (2, 8, 8, 1)$
$118: P_{2460} = (11, 8, 8, 1)$
$119: P_{2476} = (11, 9, 8, 1)$
$120: P_{2479} = (14, 9, 8, 1)$
$121: P_{2507} = (10, 11, 8, 1)$
$122: P_{2511} = (14, 11, 8, 1)$
$123: P_{2547} = (2, 14, 8, 1)$
$124: P_{2553} = (8, 14, 8, 1)$
$125: P_{2561} = (0, 15, 8, 1)$
$126: P_{2567} = (6, 15, 8, 1)$
$127: P_{2580} = (3, 0, 9, 1)$
$128: P_{2618} = (9, 2, 9, 1)$
$129: P_{2623} = (14, 2, 9, 1)$
$130: P_{2647} = (6, 4, 9, 1)$
$131: P_{2654} = (13, 4, 9, 1)$
$132: P_{2754} = (1, 11, 9, 1)$
$133: P_{2766} = (13, 11, 9, 1)$
$134: P_{2770} = (1, 12, 9, 1)$
$135: P_{2772} = (3, 12, 9, 1)$
$136: P_{2795} = (10, 13, 9, 1)$
$137: P_{2801} = (0, 14, 9, 1)$
$138: P_{2807} = (6, 14, 9, 1)$
$139: P_{2827} = (10, 15, 9, 1)$
$140: P_{2831} = (14, 15, 9, 1)$
$141: P_{2843} = (10, 0, 10, 1)$
$142: P_{2855} = (6, 1, 10, 1)$
$143: P_{2856} = (7, 1, 10, 1)$
$144: P_{2903} = (6, 4, 10, 1)$
$145: P_{2908} = (11, 4, 10, 1)$
$146: P_{2995} = (2, 10, 10, 1)$
$147: P_{3002} = (9, 10, 10, 1)$
$148: P_{3009} = (0, 11, 10, 1)$
$149: P_{3026} = (1, 12, 10, 1)$
$150: P_{3027} = (2, 12, 10, 1)$
$151: P_{3042} = (1, 13, 10, 1)$
$152: P_{3050} = (9, 13, 10, 1)$
$153: P_{3064} = (7, 14, 10, 1)$
$154: P_{3068} = (11, 14, 10, 1)$
$155: P_{3100} = (11, 0, 11, 1)$
$156: P_{3117} = (12, 1, 11, 1)$
$157: P_{3118} = (13, 1, 11, 1)$
$158: P_{3131} = (10, 2, 11, 1)$
$159: P_{3133} = (12, 2, 11, 1)$
$160: P_{3186} = (1, 6, 11, 1)$
$161: P_{3189} = (4, 6, 11, 1)$
$162: P_{3202} = (1, 7, 11, 1)$
$163: P_{3215} = (14, 7, 11, 1)$
$164: P_{3243} = (10, 9, 11, 1)$
$165: P_{3246} = (13, 9, 11, 1)$
$166: P_{3249} = (0, 10, 11, 1)$
$167: P_{3269} = (4, 11, 11, 1)$
$168: P_{3279} = (14, 11, 11, 1)$
$169: P_{3354} = (9, 0, 12, 1)$
$170: P_{3365} = (4, 1, 12, 1)$
$171: P_{3366} = (5, 1, 12, 1)$
$172: P_{3390} = (13, 2, 12, 1)$
$173: P_{3421} = (12, 4, 12, 1)$
$174: P_{3423} = (14, 4, 12, 1)$
$175: P_{3429} = (4, 5, 12, 1)$
$176: P_{3439} = (14, 5, 12, 1)$
$177: P_{3441} = (0, 6, 12, 1)$

178 : $P_{3452} = (11, 6, 12, 1)$
179 : $P_{3494} = (5, 9, 12, 1)$
180 : $P_{3500} = (11, 9, 12, 1)$
181 : $P_{3578} = (9, 14, 12, 1)$
182 : $P_{3582} = (13, 14, 12, 1)$
183 : $P_{3603} = (2, 0, 13, 1)$
184 : $P_{3631} = (14, 1, 13, 1)$
185 : $P_{3632} = (15, 1, 13, 1)$
186 : $P_{3644} = (11, 2, 13, 1)$
187 : $P_{3648} = (15, 2, 13, 1)$
188 : $P_{3667} = (2, 4, 13, 1)$
189 : $P_{3677} = (12, 4, 13, 1)$
190 : $P_{3713} = (0, 7, 13, 1)$
191 : $P_{3724} = (11, 7, 13, 1)$
192 : $P_{3757} = (12, 9, 13, 1)$
193 : $P_{3829} = (4, 14, 13, 1)$
194 : $P_{3838} = (13, 14, 13, 1)$
195 : $P_{3845} = (4, 15, 13, 1)$
196 : $P_{3855} = (14, 15, 13, 1)$
197 : $P_{3862} = (5, 0, 14, 1)$
198 : $P_{3889} = (0, 2, 14, 1)$
199 : $P_{3902} = (13, 2, 14, 1)$
200 : $P_{3907} = (2, 3, 14, 1)$
201 : $P_{3916} = (11, 3, 14, 1)$
202 : $P_{3923} = (2, 4, 14, 1)$
203 : $P_{3935} = (14, 4, 14, 1)$
204 : $P_{3954} = (1, 6, 14, 1)$
205 : $P_{3958} = (5, 6, 14, 1)$
206 : $P_{3980} = (11, 7, 14, 1)$
207 : $P_{4008} = (7, 9, 14, 1)$
208 : $P_{4014} = (13, 9, 14, 1)$
209 : $P_{4018} = (1, 10, 14, 1)$
210 : $P_{4024} = (7, 10, 14, 1)$
211 : $P_{4120} = (7, 0, 15, 1)$
212 : $P_{4149} = (4, 2, 15, 1)$
213 : $P_{4160} = (15, 2, 15, 1)$
214 : $P_{4161} = (0, 3, 15, 1)$
215 : $P_{4174} = (13, 3, 15, 1)$
216 : $P_{4188} = (11, 4, 15, 1)$
217 : $P_{4190} = (13, 4, 15, 1)$
218 : $P_{4200} = (7, 5, 15, 1)$
219 : $P_{4275} = (2, 10, 15, 1)$
220 : $P_{4284} = (11, 10, 15, 1)$
221 : $P_{4339} = (2, 14, 15, 1)$
222 : $P_{4347} = (10, 14, 15, 1)$
223 : $P_{4357} = (4, 15, 15, 1)$
224 : $P_{4363} = (10, 15, 15, 1)$

Line Intersection Graph

	0 1 2 3
0	0 1 0 1
1	1 0 0 1
2	0 0 0 1
3	1 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_3
in point	P_0	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_4

Line 2 intersects

Line	ℓ_3
in point	P_{530}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_5	P_4	P_{530}

The surface has 289 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_2 = (0, 0, 1, 0)$
3 : $P_3 = (0, 0, 0, 1)$
4 : $P_4 = (1, 1, 1, 1)$
5 : $P_5 = (1, 1, 0, 0)$
6 : $P_6 = (2, 1, 0, 0)$
7 : $P_7 = (3, 1, 0, 0)$
8 : $P_8 = (4, 1, 0, 0)$

$9: P_9 = (5,1,0,0)$
$10: P_{10} = (6,1,0,0)$
$11: P_{11} = (7,1,0,0)$
$12: P_{12} = (8,1,0,0)$
$13: P_{13} = (9,1,0,0)$
$14: P_{14} = (10,1,0,0)$
$15: P_{15} = (11,1,0,0)$
$16: P_{16} = (12,1,0,0)$
$17: P_{17} = (13,1,0,0)$
$18: P_{18} = (14,1,0,0)$
$19: P_{19} = (15,1,0,0)$
$20: P_{45} = (10,1,1,0)$
$21: P_{46} = (11,1,1,0)$
$22: P_{56} = (5,2,1,0)$
$23: P_{58} = (7,2,1,0)$
$24: P_{91} = (8,4,1,0)$
$25: P_{95} = (12,4,1,0)$
$26: P_{169} = (6,9,1,0)$
$27: P_{178} = (15,9,1,0)$
$28: P_{183} = (4,10,1,0)$
$29: P_{193} = (14,10,1,0)$
$30: P_{197} = (2,11,1,0)$
$31: P_{204} = (9,11,1,0)$
$32: P_{246} = (3,14,1,0)$
$33: P_{256} = (13,14,1,0)$
$34: P_{275} = (1,0,0,1)$
$35: P_{300} = (10,1,0,1)$
$36: P_{301} = (11,1,0,1)$
$37: P_{374} = (4,6,0,1)$
$38: P_{375} = (5,6,0,1)$
$39: P_{400} = (14,7,0,1)$
$40: P_{401} = (15,7,0,1)$
$41: P_{440} = (6,10,0,1)$
$42: P_{441} = (7,10,0,1)$
$43: P_{462} = (12,11,0,1)$
$44: P_{463} = (13,11,0,1)$
$45: P_{468} = (2,12,0,1)$
$46: P_{469} = (3,12,0,1)$
$47: P_{490} = (8,13,0,1)$
$48: P_{491} = (9,13,0,1)$
$49: P_{530} = (0,0,1,1)$
$50: P_{546} = (0,1,1,1)$
$51: P_{547} = (2,1,1,1)$
$52: P_{548} = (3,1,1,1)$
$53: P_{549} = (4,1,1,1)$
$54: P_{550} = (5,1,1,1)$
$55: P_{551} = (6,1,1,1)$
$56: P_{552} = (7,1,1,1)$
$57: P_{553} = (8,1,1,1)$
$58: P_{554} = (9,1,1,1)$
$59: P_{555} = (10,1,1,1)$
$60: P_{556} = (11,1,1,1)$
$61: P_{557} = (12,1,1,1)$
$62: P_{558} = (13,1,1,1)$
$63: P_{559} = (14,1,1,1)$
$64: P_{560} = (15,1,1,1)$
$65: P_{563} = (2,2,1,1)$
$66: P_{580} = (3,3,1,1)$
$67: P_{597} = (4,4,1,1)$
$68: P_{614} = (5,5,1,1)$
$69: P_{631} = (6,6,1,1)$
$70: P_{648} = (7,7,1,1)$
$71: P_{665} = (8,8,1,1)$
$72: P_{682} = (9,9,1,1)$
$73: P_{699} = (10,10,1,1)$
$74: P_{716} = (11,11,1,1)$
$75: P_{733} = (12,12,1,1)$
$76: P_{750} = (13,13,1,1)$
$77: P_{767} = (14,14,1,1)$
$78: P_{784} = (15,15,1,1)$
$79: P_{785} = (0,0,2,1)$
$80: P_{793} = (8,0,2,1)$
$81: P_{849} = (0,4,2,1)$
$82: P_{856} = (7,4,2,1)$
$83: P_{869} = (4,5,2,1)$
$84: P_{875} = (10,5,2,1)$
$85: P_{931} = (2,9,2,1)$
$86: P_{933} = (4,9,2,1)$
$87: P_{962} = (1,11,2,1)$
$88: P_{973} = (12,11,2,1)$
$89: P_{987} = (10,12,2,1)$
$90: P_{994} = (1,13,2,1)$
$91: P_{1001} = (8,13,2,1)$
$92: P_{1016} = (7,14,2,1)$
$93: P_{1021} = (12,14,2,1)$
$94: P_{1041} = (0,0,3,1)$
$95: P_{1053} = (12,0,3,1)$
$96: P_{1077} = (4,2,3,1)$
$97: P_{1084} = (11,2,3,1)$
$98: P_{1098} = (9,3,3,1)$
$99: P_{1100} = (11,3,3,1)$
$100: P_{1108} = (3,4,3,1)$
$101: P_{1114} = (9,4,3,1)$
$102: P_{1121} = (0,5,3,1)$
$103: P_{1128} = (7,5,3,1)$
$104: P_{1181} = (12,8,3,1)$
$105: P_{1192} = (7,9,3,1)$
$106: P_{1195} = (10,9,3,1)$
$107: P_{1221} = (4,11,3,1)$
$108: P_{1227} = (10,11,3,1)$
$109: P_{1297} = (0,0,4,1)$
$110: P_{1312} = (15,0,4,1)$
$111: P_{1335} = (6,2,4,1)$
$112: P_{1341} = (12,2,4,1)$
$113: P_{1404} = (11,6,4,1)$
$114: P_{1410} = (1,7,4,1)$
$115: P_{1424} = (15,7,4,1)$
$116: P_{1434} = (9,8,4,1)$
$117: P_{1436} = (11,8,4,1)$
$118: P_{1441} = (0,9,4,1)$
$119: P_{1453} = (12,9,4,1)$
$120: P_{1458} = (1,10,4,1)$
$121: P_{1463} = (6,10,4,1)$
$122: P_{1525} = (4,14,4,1)$
$123: P_{1530} = (9,14,4,1)$
$124: P_{1553} = (0,0,5,1)$
$125: P_{1559} = (6,0,5,1)$
$126: P_{1626} = (9,4,5,1)$
$127: P_{1627} = (10,4,5,1)$
$128: P_{1643} = (10,5,5,1)$
$129: P_{1647} = (14,5,5,1)$
$130: P_{1681} = (0,8,5,1)$
$131: P_{1693} = (12,8,5,1)$
$132: P_{1702} = (5,9,5,1)$
$133: P_{1711} = (14,9,5,1)$
$134: P_{1722} = (9,10,5,1)$
$135: P_{1724} = (11,10,5,1)$
$136: P_{1788} = (11,14,5,1)$
$137: P_{1789} = (12,14,5,1)$
$138: P_{1799} = (6,15,5,1)$
$139: P_{1809} = (0,0,6,1)$
$140: P_{1823} = (14,0,6,1)$
$141: P_{1833} = (8,1,6,1)$
$142: P_{1834} = (9,1,6,1)$
$143: P_{1848} = (7,2,6,1)$
$144: P_{1855} = (14,2,6,1)$
$145: P_{1880} = (7,4,6,1)$
$146: P_{1939} = (2,8,6,1)$
$147: P_{1946} = (9,8,6,1)$
$148: P_{1955} = (2,9,6,1)$
$149: P_{1959} = (6,9,6,1)$
$150: P_{2017} = (0,13,6,1)$
$151: P_{2027} = (10,13,6,1)$
$152: P_{2041} = (8,14,6,1)$
$153: P_{2043} = (10,14,6,1)$
$154: P_{2065} = (0,0,7,1)$
$155: P_{2069} = (4,0,7,1)$
$156: P_{2083} = (2,1,7,1)$
$157: P_{2084} = (3,1,7,1)$
$158: P_{2104} = (7,2,7,1)$
$159: P_{2106} = (9,2,7,1)$
$160: P_{2115} = (2,3,7,1)$
$161: P_{2122} = (9,3,7,1)$
$162: P_{2132} = (3,4,7,1)$
$163: P_{2139} = (10,4,7,1)$
$164: P_{2213} = (4,9,7,1)$
$165: P_{2215} = (6,9,7,1)$
$166: P_{2257} = (0,12,7,1)$
$167: P_{2267} = (10,12,7,1)$
$168: P_{2295} = (6,14,7,1)$
$169: P_{2321} = (0,0,8,1)$
$170: P_{2334} = (13,0,8,1)$

$171: P_{2359} = (6, 2, 8, 1)$
$172: P_{2363} = (10, 2, 8, 1)$
$173: P_{2382} = (13, 3, 8, 1)$
$174: P_{2451} = (2, 8, 8, 1)$
$175: P_{2460} = (11, 8, 8, 1)$
$176: P_{2476} = (11, 9, 8, 1)$
$177: P_{2479} = (14, 9, 8, 1)$
$178: P_{2507} = (10, 11, 8, 1)$
$179: P_{2511} = (14, 11, 8, 1)$
$180: P_{2547} = (2, 14, 8, 1)$
$181: P_{2553} = (8, 14, 8, 1)$
$182: P_{2561} = (0, 15, 8, 1)$
$183: P_{2567} = (6, 15, 8, 1)$
$184: P_{2577} = (0, 0, 9, 1)$
$185: P_{2580} = (3, 0, 9, 1)$
$186: P_{2618} = (9, 2, 9, 1)$
$187: P_{2623} = (14, 2, 9, 1)$
$188: P_{2647} = (6, 4, 9, 1)$
$189: P_{2654} = (13, 4, 9, 1)$
$190: P_{2754} = (1, 11, 9, 1)$
$191: P_{2766} = (13, 11, 9, 1)$
$192: P_{2770} = (1, 12, 9, 1)$
$193: P_{2772} = (3, 12, 9, 1)$
$194: P_{2795} = (10, 13, 9, 1)$
$195: P_{2801} = (0, 14, 9, 1)$
$196: P_{2807} = (6, 14, 9, 1)$
$197: P_{2827} = (10, 15, 9, 1)$
$198: P_{2831} = (14, 15, 9, 1)$
$199: P_{2833} = (0, 0, 10, 1)$
$200: P_{2843} = (10, 0, 10, 1)$
$201: P_{2855} = (6, 1, 10, 1)$
$202: P_{2856} = (7, 1, 10, 1)$
$203: P_{2903} = (6, 4, 10, 1)$
$204: P_{2908} = (11, 4, 10, 1)$
$205: P_{2995} = (2, 10, 10, 1)$
$206: P_{3002} = (9, 10, 10, 1)$
$207: P_{3009} = (0, 11, 10, 1)$
$208: P_{3026} = (1, 12, 10, 1)$
$209: P_{3027} = (2, 12, 10, 1)$
$210: P_{3042} = (1, 13, 10, 1)$
$211: P_{3050} = (9, 13, 10, 1)$
$212: P_{3064} = (7, 14, 10, 1)$
$213: P_{3068} = (11, 14, 10, 1)$
$214: P_{3089} = (0, 0, 11, 1)$
$215: P_{3100} = (11, 0, 11, 1)$
$216: P_{3117} = (12, 1, 11, 1)$
$217: P_{3118} = (13, 1, 11, 1)$
$218: P_{3131} = (10, 2, 11, 1)$
$219: P_{3133} = (12, 2, 11, 1)$
$220: P_{3186} = (1, 6, 11, 1)$
$221: P_{3189} = (4, 6, 11, 1)$
$222: P_{3202} = (1, 7, 11, 1)$
$223: P_{3215} = (14, 7, 11, 1)$
$224: P_{3243} = (10, 9, 11, 1)$
$225: P_{3246} = (13, 9, 11, 1)$
$226: P_{3249} = (0, 10, 11, 1)$
$227: P_{3269} = (4, 11, 11, 1)$
$228: P_{3279} = (14, 11, 11, 1)$
$229: P_{3345} = (0, 0, 12, 1)$
$230: P_{3354} = (9, 0, 12, 1)$
$231: P_{3365} = (4, 1, 12, 1)$
$232: P_{3366} = (5, 1, 12, 1)$
$233: P_{3390} = (13, 2, 12, 1)$
$234: P_{3421} = (12, 4, 12, 1)$
$235: P_{3423} = (14, 4, 12, 1)$
$236: P_{3429} = (4, 5, 12, 1)$
$237: P_{3439} = (14, 5, 12, 1)$
$238: P_{3441} = (0, 6, 12, 1)$
$239: P_{3452} = (11, 6, 12, 1)$
$240: P_{3494} = (5, 9, 12, 1)$
$241: P_{3500} = (11, 9, 12, 1)$
$242: P_{3578} = (9, 14, 12, 1)$
$243: P_{3582} = (13, 14, 12, 1)$
$244: P_{3601} = (0, 0, 13, 1)$
$245: P_{3603} = (2, 0, 13, 1)$
$246: P_{3631} = (14, 1, 13, 1)$
$247: P_{3632} = (15, 1, 13, 1)$
$248: P_{3644} = (11, 2, 13, 1)$
$249: P_{3648} = (15, 2, 13, 1)$
$250: P_{3667} = (2, 4, 13, 1)$
$251: P_{3677} = (12, 4, 13, 1)$
$252: P_{3713} = (0, 7, 13, 1)$
$253: P_{3724} = (11, 7, 13, 1)$
$254: P_{3757} = (12, 9, 13, 1)$
$255: P_{3829} = (4, 14, 13, 1)$
$256: P_{3838} = (13, 14, 13, 1)$
$257: P_{3845} = (4, 15, 13, 1)$
$258: P_{3855} = (14, 15, 13, 1)$
$259: P_{3857} = (0, 0, 14, 1)$
$260: P_{3862} = (5, 0, 14, 1)$
$261: P_{3889} = (0, 2, 14, 1)$
$262: P_{3902} = (13, 2, 14, 1)$
$263: P_{3907} = (2, 3, 14, 1)$
$264: P_{3916} = (11, 3, 14, 1)$
$265: P_{3923} = (2, 4, 14, 1)$
$266: P_{3935} = (14, 4, 14, 1)$
$267: P_{3954} = (1, 6, 14, 1)$
$268: P_{3958} = (5, 6, 14, 1)$
$269: P_{3980} = (11, 7, 14, 1)$
$270: P_{4008} = (7, 9, 14, 1)$
$271: P_{4014} = (13, 9, 14, 1)$
$272: P_{4018} = (1, 10, 14, 1)$
$273: P_{4024} = (7, 10, 14, 1)$
$274: P_{4113} = (0, 0, 15, 1)$
$275: P_{4120} = (7, 0, 15, 1)$
$276: P_{4149} = (4, 2, 15, 1)$
$277: P_{4160} = (15, 2, 15, 1)$
$278: P_{4161} = (0, 3, 15, 1)$
$279: P_{4174} = (13, 3, 15, 1)$
$280: P_{4188} = (11, 4, 15, 1)$
$281: P_{4190} = (13, 4, 15, 1)$
$282: P_{4200} = (7, 5, 15, 1)$
$283: P_{4275} = (2, 10, 15, 1)$
$284: P_{4284} = (11, 10, 15, 1)$
$285: P_{4339} = (2, 14, 15, 1)$
$286: P_{4347} = (10, 14, 15, 1)$
$287: P_{4357} = (4, 15, 15, 1)$
$288: P_{4363} = (10, 15, 15, 1)$