Rank-74055 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(2) is 74055

General information

Number of lines	4
Number of points	9
Number of singular points	1
Number of Eckardt points	1
Number of double points	1
Number of single points	7
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^4
Type of lines on points	3, 2, 1^7

Singular Points

The surface has 1 singular points:

$0:\ P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1)$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{30} = \mathbf{Pl}(0,0,0,1,0,0)_5$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right]_{14} = \mathbf{Pl}(1,0,0,1,0,0)_6$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

Rank of lines: (0, 30, 14, 33)
Rank of points on Klein quadric: (0, 5, 6, 7)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0).\ T = 2$

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_1 \cap \ell_3$

Single Points

The surface has 7 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 :\ P_7 = (0,1,1,0)$ lies on line ℓ_3
$3 :\ P_9 = (1,0,0,1)$ lies on line ℓ_2
$4 :\ P_{10} = (0,1,0,1)$ lies on line ℓ_1
$5 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_2
$6 :\ P_{14} = (0,1,1,1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0 1 2 3
0	0 1 1 0
1	1 0 1 1
2	1 1 0 0
3	0 1 0 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_1	P_1	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_1

Line 3 intersects

Line	ℓ_1
in point	P_3

The surface has 9 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_7 = (0, 1, 1, 0)$
5 : $P_9 = (1, 0, 0, 1)$
6 : $P_{10} = (0, 1, 0, 1)$
7 : $P_{11} = (1, 1, 0, 1)$
8 : $P_{14} = (0, 1, 1, 1)$