Rank-331 over GF(4)

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The equation

The equation of the surface is :

$$X_0^2 X_3 + X_1^2 X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over $GF(4)$ is 25953

General information

Number of lines	7
Number of points	25
Number of singular points	5
Number of Eckardt points	0
Number of double points	10
Number of single points	15
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^7
Type of lines on points	$2^{10}, 1^{15}$

Singular Points

The surface has 5 singular points:

$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$
$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$
$2 : P_{38} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$

$3 : P_{53} = \mathbf{P}(0, 0, \omega, 1) = \mathbf{P}(0, 0, 2, 1)$
$4 : P_{69} = \mathbf{P}(0, 0, \omega^2, 1) = \mathbf{P}(0, 0, 3, 1)$

The 7 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1, 0, 0, 0, 0, 0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340} = \mathbf{Pl}(0,0,0,1,0,0)_9$$

$$\ell_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{bmatrix}_{81} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{81} = \mathbf{Pl}(0,0,3,2,3,1)_{332}$$

$$\ell_6 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{bmatrix}_{61} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{61} = \mathbf{Pl}(0,0,2,3,2,1)_{265}$$

Rank of lines: (0, 16, 340, 356, 38, 81, 61)
Rank of points on Klein quadric: (0, 2, 9, 1, 198, 332, 265)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 10 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $P_3 = (0,0,0,1) = \ell_2 \cap \ell_3$

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$ $P_{38} = (0,0,1,1) = \ell_3 \cap \ell_4$

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_4$ $P_{69} = (0,0,3,1) = \ell_3 \cap \ell_5$

$P_6 = (2,1,0,0) = \ell_0 \cap \ell_5$ $P_{53} = (0,0,2,1) = \ell_3 \cap \ell_6$

$P_7 = (3,1,0,0) = \ell_0 \cap \ell_6$

$P_2 = (0,0,1,0) = \ell_1 \cap \ell_3$

Single Points

The surface has 15 single points:
The single points on the surface are:

0 : $P_4 = (1,1,1,1)$ lies on line ℓ_4 8 : $P_{52} = (3,3,1,1)$ lies on line ℓ_4

1 : $P_8 = (1,0,1,0)$ lies on line ℓ_1 9 : $P_{60} = (3,1,2,1)$ lies on line ℓ_6

2 : $P_9 = (2,0,1,0)$ lies on line ℓ_1 10 : $P_{62} = (1,2,2,1)$ lies on line ℓ_6

3 : $P_{10} = (3,0,1,0)$ lies on line ℓ_1 11 : $P_{67} = (2,3,2,1)$ lies on line ℓ_6

4 : $P_{26} = (0,1,0,1)$ lies on line ℓ_2 12 : $P_{75} = (2,1,3,1)$ lies on line ℓ_5

5 : $P_{30} = (0,2,0,1)$ lies on line ℓ_2 13 : $P_{80} = (3,2,3,1)$ lies on line ℓ_5

6 : $P_{34} = (0,3,0,1)$ lies on line ℓ_2 14 : $P_{82} = (1,3,3,1)$ lies on line ℓ_5

7 : $P_{47} = (2,2,1,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

```
  |0 1 2 3 4 5 6
--+-------------
0 |0 1 1 0 1 1 1
1 |1 0 0 1 0 0 0
2 |1 0 0 1 0 0 0
3 |0 1 1 0 1 1 1
4 |1 0 0 1 0 0 0
5 |1 0 0 1 0 0 0
6 |1 0 0 1 0 0 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_1	P_5	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_3
in point	P_1	P_3

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_2	P_3	P_{38}	P_{69}	P_{53}

Line 4 intersects

Line	ℓ_0	ℓ_3
in point	P_5	P_{38}

Line 5 intersects

Line	ℓ_0	ℓ_3
in point	P_6	P_{69}

Line 6 intersects

Line	ℓ_0	ℓ_3
in point	P_7	P_{53}

The surface has 25 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_4 = (1,1,1,1)$
5 : $P_5 = (1,1,0,0)$
6 : $P_6 = (2,1,0,0)$
7 : $P_7 = (3,1,0,0)$
8 : $P_8 = (1,0,1,0)$

9 : $P_9 = (2,0,1,0)$
10 : $P_{10} = (3,0,1,0)$
11 : $P_{26} = (0,1,0,1)$
12 : $P_{30} = (0,2,0,1)$
13 : $P_{34} = (0,3,0,1)$
14 : $P_{38} = (0,0,1,1)$
15 : $P_{47} = (2,2,1,1)$
16 : $P_{52} = (3,3,1,1)$
17 : $P_{53} = (0,0,2,1)$

18 : $P_{60} = (3,1,2,1)$
19 : $P_{62} = (1,2,2,1)$
20 : $P_{67} = (2,3,2,1)$
21 : $P_{69} = (0,0,3,1)$
22 : $P_{75} = (2,1,3,1)$
23 : $P_{80} = (3,2,3,1)$
24 : $P_{82} = (1,3,3,1)$