Rank-139 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 + X_0^2 X_3 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 2743

General information

Number of lines	6
Number of points	29
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	18
Number of points off lines	6
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^6
Type of lines on points	$3^2, 2^3, 1^{18}, 0^6$

Singular Points

The surface has 1 singular points:

$0: P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \mathbf{Pl}(1,0,0,0,1,0)_{26}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{25} = \mathbf{Pl}(1,1,0,0,1,1)_{177}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85} = \mathbf{Pl}(1,1,1,1,0,0)_{16}$$

Rank of lines: (1, 4, 17, 38, 25, 85)
Rank of points on Klein quadric: (3, 26, 32, 198, 177, 16)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{11} = (0,1,1,0) = \ell_0 \cap \ell_5$ $P_{38} = (0,0,1,1) = \ell_2 \cap \ell_3$
$P_{26} = (0,1,0,1) = \ell_1 \cap \ell_4$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 : P_5 = (1,1,0,0)$ lies on line ℓ_3 $10 : P_{40} = (2,0,1,1)$ lies on line ℓ_2
$1 : P_8 = (1,0,1,0)$ lies on line ℓ_4 $11 : P_{41} = (3,0,1,1)$ lies on line ℓ_2
$2 : P_{12} = (1,1,1,0)$ lies on line ℓ_0 $12 : P_{47} = (2,2,1,1)$ lies on line ℓ_3
$3 : P_{13} = (2,1,1,0)$ lies on line ℓ_0 $13 : P_{52} = (3,3,1,1)$ lies on line ℓ_3
$4 : P_{14} = (3,1,1,0)$ lies on line ℓ_0 $14 : P_{59} = (2,1,2,1)$ lies on line ℓ_4
$5 : P_{23} = (1,0,0,1)$ lies on line ℓ_5 $15 : P_{62} = (1,2,2,1)$ lies on line ℓ_5
$6 : P_{27} = (1,1,0,1)$ lies on line ℓ_1 $16 : P_{76} = (3,1,3,1)$ lies on line ℓ_4
$7 : P_{28} = (2,1,0,1)$ lies on line ℓ_1 $17 : P_{82} = (1,3,3,1)$ lies on line ℓ_5
$8 : P_{29} = (3,1,0,1)$ lies on line ℓ_1
$9 : P_{39} = (1,0,1,1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 6 points not on any line:
The points on the surface but not on lines are:

0 : $P_{15} = (0,2,1,0)$
1 : $P_{19} = (0,3,1,0)$
2 : $P_{30} = (0,2,0,1)$
3 : $P_{34} = (0,3,0,1)$

4 : $P_{53} = (0,0,2,1)$
5 : $P_{69} = (0,0,3,1)$

Line Intersection Graph

```
 |0 1 2 3 4 5
-+-----------
0|0 1 1 0 0 1
1|1 0 1 0 1 0
2|1 1 0 1 0 0
3|0 0 1 0 1 1
4|0 1 0 1 0 1
5|1 0 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5
in point	P_0	P_0	P_{11}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_0	P_{26}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3
in point	P_0	P_0	P_{38}

Line 3 intersects

Line	ℓ_2	ℓ_4	ℓ_5
in point	P_{38}	P_4	P_4

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{26}	P_4	P_4

Line 5 intersects

Line	ℓ_0	ℓ_3	ℓ_4
in point	P_{11}	P_4	P_4

The surface has 29 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_4 = (1,1,1,1)$
2 : $P_5 = (1,1,0,0)$
3 : $P_8 = (1,0,1,0)$
4 : $P_{11} = (0,1,1,0)$
5 : $P_{12} = (1,1,1,0)$
6 : $P_{13} = (2,1,1,0)$
7 : $P_{14} = (3,1,1,0)$
8 : $P_{15} = (0,2,1,0)$
9 : $P_{19} = (0,3,1,0)$

10 : $P_{23} = (1,0,0,1)$
11 : $P_{26} = (0,1,0,1)$
12 : $P_{27} = (1,1,0,1)$
13 : $P_{28} = (2,1,0,1)$
14 : $P_{29} = (3,1,0,1)$
15 : $P_{30} = (0,2,0,1)$
16 : $P_{34} = (0,3,0,1)$
17 : $P_{38} = (0,0,1,1)$
18 : $P_{39} = (1,0,1,1)$
19 : $P_{40} = (2,0,1,1)$

20 : $P_{41} = (3,0,1,1)$
21 : $P_{47} = (2,2,1,1)$
22 : $P_{52} = (3,3,1,1)$
23 : $P_{53} = (0,0,2,1)$
24 : $P_{59} = (2,1,2,1)$
25 : $P_{62} = (1,2,2,1)$
26 : $P_{69} = (0,0,3,1)$
27 : $P_{76} = (3,1,3,1)$
28 : $P_{82} = (1,3,3,1)$