Rank-68 over GF(2)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

(0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 68

General information

Number of lines	9
Number of points	11
Number of singular points	3
Number of Eckardt points	6
Number of double points	0
Number of single points	4
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^9
Type of lines on points	5, 3^6, 1^4

Singular Points

The surface has 3 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1)$
$2 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0)$

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{Pl}(1,0,1,0,0,0)_3$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0,0,0,0,1,0)_9$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_3 = \mathbf{Pl}(1,0,1,0,1,0)_{13}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0,1,0,0,1,0)_{11}$$

$$\ell_5 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0,1,0,1,0,0)_7$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{15} = \mathbf{Pl}(1,1,1,1,0,0)_8$$

$$\ell_7 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0,1,0,1,1,0)_{17}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,1,1,1,1,0)_{18}$$

Rank of lines: (1, 6, 3, 13, 20, 33, 15, 27, 17)
Rank of points on Klein quadric: (3, 9, 13, 15, 11, 7, 8, 17, 18)

Eckardt Points

The surface has 6 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0),\ T = 5$
$1 : P_4 = \mathbf{P}(1,1,1,1) = \mathbf{P}(1,1,1,1),\ T = -1$
$2 : P_7 = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0),\ T = 5$
$3 : P_8 = \mathbf{P}(1,1,1,0) = \mathbf{P}(1,1,1,0),\ T = -1$
$4 : P_9 = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1),\ T = 5$
$5 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 5$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 4 single points:
The single points on the surface are:

$0 :\ P_5 = (1,1,0,0)$ lies on line ℓ_3
$1 :\ P_6 = (1,0,1,0)$ lies on line ℓ_4
$2 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_3
$3 :\ P_{13} = (1,0,1,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	0	0	1	1	1	1
1	1	0	1	1	1	1	1	1	1
2	1	1	0	0	0	1	1	1	1
3	0	1	0	0	1	1	0	1	0
4	0	1	0	1	0	1	0	1	0
5	1	1	1	1	1	0	1	1	1
6	1	1	1	0	0	1	0	1	1
7	1	1	1	1	1	1	1	0	1
8	1	1	1	0	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_0	P_7	P_7	P_8	P_8

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_0	P_3	P_3	P_3	P_9	P_3	P_9

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_0	P_{14}	P_4	P_4	P_{14}

Line 3 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7
in point	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_7	P_3	P_{14}	P_3	P_3	P_7	P_3	P_{14}

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5	ℓ_7	ℓ_8
in point	P_7	P_9	P_4	P_7	P_4	P_9

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_8
in point	P_8	P_3	P_4	P_3	P_3	P_3	P_4	P_8

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_5	ℓ_6	ℓ_7
in point	P_8	P_9	P_{14}	P_{14}	P_9	P_8

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_3 = (0,0,0,1)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (1,0,1,0)$
$5: P_7 = (0,1,1,0)$
$6: P_8 = (1,1,1,0)$
$7: P_9 = (1,0,0,1)$
$8: P_{11} = (1,1,0,1)$
$9: P_{13} = (1,0,1,1)$
$10: P_{14} = (0,1,1,1)$