

Rank-69 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

(1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 633

General information

Number of lines	5
Number of points	21
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	20
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^5
Type of lines on points	$5, 1^{20}$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{P}\mathbf{l}(0, 1, 0, 1, 0, 0)_{13}$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0, 3, 0, 1, 0, 0)_{15} \\ \ell_2 &= \begin{bmatrix} 1 & \omega^2 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{251} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{251} = \mathbf{Pl}(0, 2, 0, 3, 1, 0)_{72} \\ \ell_3 &= \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0, 2, 0, 1, 0, 0)_{14} \\ \ell_4 &= \begin{bmatrix} 1 & \omega & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{314} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{314} = \mathbf{Pl}(0, 3, 0, 2, 1, 0)_{66}\end{aligned}$$

Rank of lines: (345, 355, 251, 350, 314)

Rank of points on Klein quadric: (13, 15, 72, 14, 66)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 20 single points:

The single points on the surface are:

- | | |
|--|--|
| 0 : $P_{11} = (0, 1, 1, 0)$ lies on line ℓ_0 | 11 : $P_{60} = (3, 1, 2, 1)$ lies on line ℓ_4 |
| 1 : $P_{15} = (0, 2, 1, 0)$ lies on line ℓ_1 | 12 : $P_{61} = (0, 2, 2, 1)$ lies on line ℓ_0 |
| 2 : $P_{18} = (3, 2, 1, 0)$ lies on line ℓ_2 | 13 : $P_{65} = (0, 3, 2, 1)$ lies on line ℓ_1 |
| 3 : $P_{19} = (0, 3, 1, 0)$ lies on line ℓ_3 | 14 : $P_{66} = (1, 3, 2, 1)$ lies on line ℓ_2 |
| 4 : $P_{21} = (2, 3, 1, 0)$ lies on line ℓ_4 | 15 : $P_{73} = (0, 1, 3, 1)$ lies on line ℓ_1 |
| 5 : $P_{42} = (0, 1, 1, 1)$ lies on line ℓ_0 | 16 : $P_{75} = (2, 1, 3, 1)$ lies on line ℓ_2 |
| 6 : $P_{45} = (0, 2, 1, 1)$ lies on line ℓ_1 | 17 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_3 |
| 7 : $P_{48} = (3, 2, 1, 1)$ lies on line ℓ_2 | 18 : $P_{78} = (1, 2, 3, 1)$ lies on line ℓ_4 |
| 8 : $P_{49} = (0, 3, 1, 1)$ lies on line ℓ_3 | 19 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_0 |
| 9 : $P_{51} = (2, 3, 1, 1)$ lies on line ℓ_4 | |
| 10 : $P_{57} = (0, 1, 2, 1)$ lies on line ℓ_3 | |

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	1	1
1	1	0	1	1	1
2	1	1	0	1	1
3	1	1	1	0	1
4	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4
in point	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3
in point	P_3	P_3	P_3	P_3

The surface has 21 points:

The points on the surface are:

0 : $P_3 = (0, 0, 0, 1)$
 1 : $P_{11} = (0, 1, 1, 0)$
 2 : $P_{15} = (0, 2, 1, 0)$
 3 : $P_{18} = (3, 2, 1, 0)$
 4 : $P_{19} = (0, 3, 1, 0)$
 5 : $P_{21} = (2, 3, 1, 0)$
 6 : $P_{42} = (0, 1, 1, 1)$
 7 : $P_{45} = (0, 2, 1, 1)$

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 9 : $P_{49} = (0, 3, 1, 1)$
 10 : $P_{51} = (2, 3, 1, 1)$
 11 : $P_{57} = (0, 1, 2, 1)$
 12 : $P_{60} = (3, 1, 2, 1)$
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 18 : $P_{77} = (0, 2, 3, 1)$
 19 : $P_{78} = (1, 2, 3, 1)$
 20 : $P_{81} = (0, 3, 3, 1)$