Rank-65899 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431726489

General information

Number of lines	8
Number of points	29
Number of singular points	3
Number of Eckardt points	2
Number of double points	6
Number of single points	18
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^8
Type of lines on points	4, 3^2, 2^6, 1^{18}, 0^2

Singular Points

The surface has 3 singular points:

$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1: P_{31} = \mathbf{P}(1,\omega,0,1) = \mathbf{P}(1,2,0,1)$
$2: P_{35} = \mathbf{P}(1,\omega^2,0,1) = \mathbf{P}(1,3,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{58} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{58} = \mathbf{Pl}(0,0,3,0,0,1)_{122}$$

$$\ell_2 = \begin{bmatrix} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{79} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{79} = \mathbf{Pl}(0,0,2,0,0,1)_{115}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84} = \mathbf{Pl}(1,0,0,1,0,0)_{10}$$

$$\ell_4 = \begin{bmatrix} 1 & \omega & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{142} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{142} = \mathbf{Pl}(0,3,3,0,0,1)_{128}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega^2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{163} = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{163} = \mathbf{Pl}(0,2,2,0,0,1)_{120}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{236} = \mathbf{Pl}(3,2,2,3,2,1)_{290}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \omega & \omega^2 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{299} = \mathbf{Pl}(2,3,3,2,3,1)_{352}$$

Rank of lines: (0, 58, 79, 84, 142, 163, 236, 299)
Rank of points on Klein quadric: (0, 122, 115, 10, 128, 120, 290, 352)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{31} = \mathbf{P}(1,\omega,0,1) = \mathbf{P}(1,2,0,1)$,
$1 : P_{35} = \mathbf{P}(1,\omega^2,0,1) = \mathbf{P}(1,3,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_7 = (3,1,0,0) = \ell_0 \cap \ell_1$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_3$
$P_{16} = (1,2,1,0) = \ell_1 \cap \ell_6$
$P_{20} = (1,3,1,0) = \ell_2 \cap \ell_7$
$P_{42} = (0,1,1,1) = \ell_6 \cap \ell_7$

Single Points

The surface has 18 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 :\ P_{13} = (2,1,1,0)$ lies on line ℓ_2
$3 :\ P_{14} = (3,1,1,0)$ lies on line ℓ_1
$4 :\ P_{18} = (3,2,1,0)$ lies on line ℓ_2
$5 :\ P_{21} = (2,3,1,0)$ lies on line ℓ_1
$6 :\ P_{23} = (1,0,0,1)$ lies on line ℓ_3
$7 :\ P_{27} = (1,1,0,1)$ lies on line ℓ_3
$8 :\ P_{46} = (1,2,1,1)$ lies on line ℓ_4
$9 :\ P_{50} = (1,3,1,1)$ lies on line ℓ_5
$10 :\ P_{56} = (3,0,2,1)$ lies on line ℓ_6
$11 :\ P_{62} = (1,2,2,1)$ lies on line ℓ_4
$12 :\ P_{66} = (1,3,2,1)$ lies on line ℓ_5
$13 :\ P_{68} = (3,3,2,1)$ lies on line ℓ_7
$14 :\ P_{71} = (2,0,3,1)$ lies on line ℓ_7
$15 :\ P_{78} = (1,2,3,1)$ lies on line ℓ_4
$16 :\ P_{79} = (2,2,3,1)$ lies on line ℓ_6
$17 :\ P_{82} = (1,3,3,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:
The points on the surface but not on lines are:

$0: P_{61} = (0, 2, 2, 1)$
$1: P_{81} = (0, 3, 3, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7
0	0 1 1 1 0 0 0 0
1	1 0 1 0 1 1 1 0
2	1 1 0 0 1 1 0 1
3	1 0 0 0 1 1 1 1
4	0 1 1 1 0 1 0 1
5	0 1 1 1 1 0 1 0
6	0 1 0 1 0 1 0 1
7	0 0 1 1 1 0 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_7	P_6	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_5	ℓ_6
in point	P_7	P_2	P_2	P_2	P_{16}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7
in point	P_6	P_2	P_2	P_2	P_{20}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_1	P_{31}	P_{35}	P_{35}	P_{31}

Line 4 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_7
in point	P_2	P_2	P_{31}	P_2	P_{31}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_2	P_2	P_{35}	P_2	P_{35}

Line 6 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7
in point	P_{16}	P_{35}	P_{35}	P_{42}

Line 7 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6
in point	P_{20}	P_{31}	P_{31}	P_{42}

The surface has 29 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (2,1,0,0)$
$5: P_7 = (3,1,0,0)$
$6: P_{13} = (2,1,1,0)$
$7: P_{14} = (3,1,1,0)$
$8: P_{16} = (1,2,1,0)$
$9: P_{18} = (3,2,1,0)$
$10: P_{20} = (1,3,1,0)$
$11: P_{21} = (2,3,1,0)$
$12: P_{23} = (1,0,0,1)$
$13: P_{27} = (1,1,0,1)$
$14: P_{31} = (1,2,0,1)$
$15: P_{35} = (1,3,0,1)$
$16: P_{42} = (0,1,1,1)$
$17: P_{46} = (1,2,1,1)$
$18: P_{50} = (1,3,1,1)$
$19: P_{56} = (3,0,2,1)$
$20: P_{61} = (0,2,2,1)$
$21: P_{62} = (1,2,2,1)$
$22: P_{66} = (1,3,2,1)$
$23: P_{68} = (3,3,2,1)$
$24: P_{71} = (2,0,3,1)$
$25: P_{78} = (1,2,3,1)$
$26: P_{79} = (2,2,3,1)$
$27: P_{81} = (0,3,3,1)$
$28: P_{82} = (1,3,3,1)$