Rank-76100 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_0^2X_3 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108374

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	4
Number of double points	6
Number of single points	129
Number of points off lines	182
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^4, 2^6, 1^{129}, 0^{182}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \delta^5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{48048} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{48048} = \mathbf{Pl}(10,0,0,1,0,0)_{43}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & \delta^{10} \\ 0 & 1 & 0 & 0 \end{array}\right]_{43680} = \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \end{array}\right]_{43680} = \mathbf{Pl}(11,0,0,1,0,0)_{44}$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & \delta^{5} \\ 0 & 0 & 1 & 0 \end{array}\right]_{48304} = \left[\begin{array}{cccc} 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{array}\right]_{48304} = \mathbf{Pl}(0,11,1,0,0,0)_{28}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{array}\right]_{43936} = \left[\begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{array}\right]_{43936} = \mathbf{Pl}(0,10,1,0,0,0)_{27}$$

$$\ell_6 = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69921} = \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{69921} = \mathbf{Pl}(0,1,0,1,0,0)_{49}$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 0 & 1 & \delta^{5} \\ 0 & 1 & 1 & 0 \end{array}\right]_{48322} = \left[\begin{array}{cccc} 1 & 0 & 1 & 11 \\ 0 & 1 & 1 & 0 \end{array}\right]_{48322} = \mathbf{Pl}(10,11,1,11,0,1)_{7845}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & 0 & 1 & \delta^{10} \\ 0 & 1 & 1 & 0 \end{array}\right]_{43954} = \left[\begin{array}{cccc} 1 & 0 & 1 & 10 \\ 0 & 1 & 1 & 0 \end{array}\right]_{43954} = \mathbf{Pl}(11,10,1,10,0,1)_{7621}$$

Rank of lines: (69904, 48048, 43680, 70160, 48304, 43936, 69921, 48322, 43954)
Rank of points on Klein quadric: (33, 43, 44, 1, 28, 27, 49, 7845, 7621)

Eckardt Points

The surface has 4 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$,
$2 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$,
$3 : P_{35} = \mathbf{P}(0,1,1,0) = \mathbf{P}(0,1,1,0)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{284} = (10,0,0,1) = \ell_1 \cap \ell_4$
$P_{444} = (10,10,0,1) = \ell_1 \cap \ell_7$
$P_{285} = (11,0,0,1) = \ell_2 \cap \ell_5$
$P_{461} = (11,11,0,1) = \ell_2 \cap \ell_8$
$P_{2843} = (10,0,10,1) = \ell_4 \cap \ell_7$
$P_{3100} = (11,0,11,1) = \ell_5 \cap \ell_8$

Single Points

The surface has 129 single points:
The single points on the surface are:

$0 :\ P_{290} = (0,1,0,1)$ lies on line ℓ_0
$1 :\ P_{300} = (10,1,0,1)$ lies on line ℓ_1
$2 :\ P_{301} = (11,1,0,1)$ lies on line ℓ_2
$3 :\ P_{306} = (0,2,0,1)$ lies on line ℓ_0
$4 :\ P_{316} = (10,2,0,1)$ lies on line ℓ_1
$5 :\ P_{317} = (11,2,0,1)$ lies on line ℓ_2
$6 :\ P_{322} = (0,3,0,1)$ lies on line ℓ_0
$7 :\ P_{332} = (10,3,0,1)$ lies on line ℓ_1
$8 :\ P_{333} = (11,3,0,1)$ lies on line ℓ_2
$9 :\ P_{338} = (0,4,0,1)$ lies on line ℓ_0
$10 :\ P_{348} = (10,4,0,1)$ lies on line ℓ_1
$11 :\ P_{349} = (11,4,0,1)$ lies on line ℓ_2
$12 :\ P_{354} = (0,5,0,1)$ lies on line ℓ_0
$13 :\ P_{364} = (10,5,0,1)$ lies on line ℓ_1

14 : $P_{365} = (11, 5, 0, 1)$ lies on line ℓ_2
15 : $P_{370} = (0, 6, 0, 1)$ lies on line ℓ_0
16 : $P_{380} = (10, 6, 0, 1)$ lies on line ℓ_1
17 : $P_{381} = (11, 6, 0, 1)$ lies on line ℓ_2
18 : $P_{386} = (0, 7, 0, 1)$ lies on line ℓ_0
19 : $P_{396} = (10, 7, 0, 1)$ lies on line ℓ_1
20 : $P_{397} = (11, 7, 0, 1)$ lies on line ℓ_2
21 : $P_{402} = (0, 8, 0, 1)$ lies on line ℓ_0
22 : $P_{412} = (10, 8, 0, 1)$ lies on line ℓ_1
23 : $P_{413} = (11, 8, 0, 1)$ lies on line ℓ_2
24 : $P_{418} = (0, 9, 0, 1)$ lies on line ℓ_0
25 : $P_{428} = (10, 9, 0, 1)$ lies on line ℓ_1
26 : $P_{429} = (11, 9, 0, 1)$ lies on line ℓ_2
27 : $P_{434} = (0, 10, 0, 1)$ lies on line ℓ_0
28 : $P_{445} = (11, 10, 0, 1)$ lies on line ℓ_2
29 : $P_{450} = (0, 11, 0, 1)$ lies on line ℓ_0
30 : $P_{460} = (10, 11, 0, 1)$ lies on line ℓ_1
31 : $P_{466} = (0, 12, 0, 1)$ lies on line ℓ_0
32 : $P_{476} = (10, 12, 0, 1)$ lies on line ℓ_1
33 : $P_{477} = (11, 12, 0, 1)$ lies on line ℓ_2
34 : $P_{482} = (0, 13, 0, 1)$ lies on line ℓ_0
35 : $P_{492} = (10, 13, 0, 1)$ lies on line ℓ_1
36 : $P_{493} = (11, 13, 0, 1)$ lies on line ℓ_2
37 : $P_{498} = (0, 14, 0, 1)$ lies on line ℓ_0
38 : $P_{508} = (10, 14, 0, 1)$ lies on line ℓ_1
39 : $P_{509} = (11, 14, 0, 1)$ lies on line ℓ_2
40 : $P_{514} = (0, 15, 0, 1)$ lies on line ℓ_0
41 : $P_{524} = (10, 15, 0, 1)$ lies on line ℓ_1
42 : $P_{525} = (11, 15, 0, 1)$ lies on line ℓ_2
43 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_3
44 : $P_{540} = (10, 0, 1, 1)$ lies on line ℓ_4
45 : $P_{541} = (11, 0, 1, 1)$ lies on line ℓ_5
46 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_6
47 : $P_{700} = (11, 10, 1, 1)$ lies on line ℓ_8
48 : $P_{715} = (10, 11, 1, 1)$ lies on line ℓ_7
49 : $P_{785} = (0, 0, 2, 1)$ lies on line ℓ_3
50 : $P_{795} = (10, 0, 2, 1)$ lies on line ℓ_4
51 : $P_{796} = (11, 0, 2, 1)$ lies on line ℓ_5
52 : $P_{817} = (0, 2, 2, 1)$ lies on line ℓ_6
53 : $P_{923} = (10, 8, 2, 1)$ lies on line ℓ_7
54 : $P_{940} = (11, 9, 2, 1)$ lies on line ℓ_8
55 : $P_{1041} = (0, 0, 3, 1)$ lies on line ℓ_3
56 : $P_{1051} = (10, 0, 3, 1)$ lies on line ℓ_4
57 : $P_{1052} = (11, 0, 3, 1)$ lies on line ℓ_5
58 : $P_{1089} = (0, 3, 3, 1)$ lies on line ℓ_6
59 : $P_{1180} = (11, 8, 3, 1)$ lies on line ℓ_8
60 : $P_{1195} = (10, 9, 3, 1)$ lies on line ℓ_7
61 : $P_{1297} = (0, 0, 4, 1)$ lies on line ℓ_3
62 : $P_{1307} = (10, 0, 4, 1)$ lies on line ℓ_4
63 : $P_{1308} = (11, 0, 4, 1)$ lies on line ℓ_5
64 : $P_{1361} = (0, 4, 4, 1)$ lies on line ℓ_6
65 : $P_{1531} = (10, 14, 4, 1)$ lies on line ℓ_7
66 : $P_{1548} = (11, 15, 4, 1)$ lies on line ℓ_8
67 : $P_{1553} = (0, 0, 5, 1)$ lies on line ℓ_3
68 : $P_{1563} = (10, 0, 5, 1)$ lies on line ℓ_4
69 : $P_{1564} = (11, 0, 5, 1)$ lies on line ℓ_5
70 : $P_{1633} = (0, 5, 5, 1)$ lies on line ℓ_6
71 : $P_{1788} = (11, 14, 5, 1)$ lies on line ℓ_8
72 : $P_{1803} = (10, 15, 5, 1)$ lies on line ℓ_7
73 : $P_{1809} = (0, 0, 6, 1)$ lies on line ℓ_3
74 : $P_{1819} = (10, 0, 6, 1)$ lies on line ℓ_4
75 : $P_{1820} = (11, 0, 6, 1)$ lies on line ℓ_5
76 : $P_{1905} = (0, 6, 6, 1)$ lies on line ℓ_6
77 : $P_{2011} = (10, 12, 6, 1)$ lies on line ℓ_7
78 : $P_{2028} = (11, 13, 6, 1)$ lies on line ℓ_8
79 : $P_{2065} = (0, 0, 7, 1)$ lies on line ℓ_3
80 : $P_{2075} = (10, 0, 7, 1)$ lies on line ℓ_4
81 : $P_{2076} = (11, 0, 7, 1)$ lies on line ℓ_5
82 : $P_{2177} = (0, 7, 7, 1)$ lies on line ℓ_6
83 : $P_{2268} = (11, 12, 7, 1)$ lies on line ℓ_8
84 : $P_{2283} = (10, 13, 7, 1)$ lies on line ℓ_7
85 : $P_{2321} = (0, 0, 8, 1)$ lies on line ℓ_3
86 : $P_{2331} = (10, 0, 8, 1)$ lies on line ℓ_4
87 : $P_{2332} = (11, 0, 8, 1)$ lies on line ℓ_5
88 : $P_{2363} = (10, 2, 8, 1)$ lies on line ℓ_7
89 : $P_{2380} = (11, 3, 8, 1)$ lies on line ℓ_8
90 : $P_{2449} = (0, 8, 8, 1)$ lies on line ℓ_6
91 : $P_{2577} = (0, 0, 9, 1)$ lies on line ℓ_3
92 : $P_{2587} = (10, 0, 9, 1)$ lies on line ℓ_4
93 : $P_{2588} = (11, 0, 9, 1)$ lies on line ℓ_5
94 : $P_{2620} = (11, 2, 9, 1)$ lies on line ℓ_8
95 : $P_{2635} = (10, 3, 9, 1)$ lies on line ℓ_7
96 : $P_{2721} = (0, 9, 9, 1)$ lies on line ℓ_6
97 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_3
98 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_5
99 : $P_{2860} = (11, 1, 10, 1)$ lies on line ℓ_8
100 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_6
101 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_3
102 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_4
103 : $P_{3115} = (10, 1, 11, 1)$ lies on line ℓ_7
104 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_6
105 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_3
106 : $P_{3355} = (10, 0, 12, 1)$ lies on line ℓ_4
107 : $P_{3356} = (11, 0, 12, 1)$ lies on line ℓ_5
108 : $P_{3451} = (10, 6, 12, 1)$ lies on line ℓ_7
109 : $P_{3468} = (11, 7, 12, 1)$ lies on line ℓ_8
110 : $P_{3537} = (0, 12, 12, 1)$ lies on line ℓ_6
111 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_3
112 : $P_{3611} = (10, 0, 13, 1)$ lies on line ℓ_4
113 : $P_{3612} = (11, 0, 13, 1)$ lies on line ℓ_5
114 : $P_{3708} = (11, 6, 13, 1)$ lies on line ℓ_8
115 : $P_{3723} = (10, 7, 13, 1)$ lies on line ℓ_7
116 : $P_{3809} = (0, 13, 13, 1)$ lies on line ℓ_6
117 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_3
118 : $P_{3867} = (10, 0, 14, 1)$ lies on line ℓ_4
119 : $P_{3868} = (11, 0, 14, 1)$ lies on line ℓ_5
120 : $P_{3931} = (10, 4, 14, 1)$ lies on line ℓ_7
121 : $P_{3948} = (11, 5, 14, 1)$ lies on line ℓ_8

122 : $P_{4081} = (0, 14, 14, 1)$ lies on line ℓ_6
123 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_3
124 : $P_{4123} = (10, 0, 15, 1)$ lies on line ℓ_4
125 : $P_{4124} = (11, 0, 15, 1)$ lies on line ℓ_5
126 : $P_{4188} = (11, 4, 15, 1)$ lies on line ℓ_8
127 : $P_{4203} = (10, 5, 15, 1)$ lies on line ℓ_7
128 : $P_{4353} = (0, 15, 15, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 182 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_{36} = (1, 1, 1, 0)$
2 : $P_{74} = (7, 3, 1, 0)$
3 : $P_{77} = (10, 3, 1, 0)$
4 : $P_{80} = (13, 3, 1, 0)$
5 : $P_{106} = (7, 5, 1, 0)$
6 : $P_{110} = (11, 5, 1, 0)$
7 : $P_{111} = (12, 5, 1, 0)$
8 : $P_{153} = (6, 8, 1, 0)$
9 : $P_{157} = (10, 8, 1, 0)$
10 : $P_{159} = (12, 8, 1, 0)$
11 : $P_{265} = (6, 15, 1, 0)$
12 : $P_{270} = (11, 15, 1, 0)$
13 : $P_{272} = (13, 15, 1, 0)$
14 : $P_{586} = (9, 3, 1, 1)$
15 : $P_{623} = (14, 5, 1, 1)$
16 : $P_{659} = (2, 8, 1, 1)$
17 : $P_{693} = (4, 10, 1, 1)$
18 : $P_{703} = (14, 10, 1, 1)$
19 : $P_{707} = (2, 11, 1, 1)$
20 : $P_{714} = (9, 11, 1, 1)$
21 : $P_{773} = (4, 15, 1, 1)$
22 : $P_{847} = (14, 3, 2, 1)$
23 : $P_{873} = (8, 5, 2, 1)$
24 : $P_{930} = (1, 9, 2, 1)$
25 : $P_{946} = (1, 10, 2, 1)$
26 : $P_{952} = (7, 10, 2, 1)$
27 : $P_{995} = (2, 13, 2, 1)$
28 : $P_{1032} = (7, 15, 2, 1)$
29 : $P_{1033} = (8, 15, 2, 1)$
30 : $P_{1039} = (14, 15, 2, 1)$
31 : $P_{1066} = (9, 1, 3, 1)$
32 : $P_{1087} = (14, 2, 3, 1)$
33 : $P_{1107} = (2, 4, 3, 1)$
34 : $P_{1117} = (12, 4, 3, 1)$
35 : $P_{1120} = (15, 4, 3, 1)$
36 : $P_{1123} = (2, 5, 3, 1)$
37 : $P_{1174} = (5, 8, 3, 1)$
38 : $P_{1184} = (15, 8, 3, 1)$
39 : $P_{1221} = (4, 11, 3, 1)$
40 : $P_{1226} = (9, 11, 3, 1)$
41 : $P_{1229} = (12, 11, 3, 1)$
42 : $P_{1237} = (4, 12, 3, 1)$
43 : $P_{1252} = (3, 13, 3, 1)$
44 : $P_{1270} = (5, 14, 3, 1)$
45 : $P_{1295} = (14, 15, 3, 1)$
46 : $P_{1347} = (2, 3, 4, 1)$
47 : $P_{1357} = (12, 3, 4, 1)$
48 : $P_{1360} = (15, 3, 4, 1)$
49 : $P_{1379} = (2, 5, 4, 1)$
50 : $P_{1413} = (4, 7, 4, 1)$
51 : $P_{1440} = (15, 8, 4, 1)$
52 : $P_{1474} = (1, 11, 4, 1)$
53 : $P_{1485} = (12, 11, 4, 1)$
54 : $P_{1522} = (1, 14, 4, 1)$
55 : $P_{1583} = (14, 1, 5, 1)$
56 : $P_{1593} = (8, 2, 5, 1)$
57 : $P_{1603} = (2, 3, 5, 1)$
58 : $P_{1619} = (2, 4, 5, 1)$
59 : $P_{1658} = (9, 6, 5, 1)$
60 : $P_{1670} = (5, 7, 5, 1)$
61 : $P_{1685} = (4, 8, 5, 1)$
62 : $P_{1700} = (3, 9, 5, 1)$
63 : $P_{1701} = (4, 9, 5, 1)$
64 : $P_{1703} = (6, 9, 5, 1)$
65 : $P_{1719} = (6, 10, 5, 1)$
66 : $P_{1722} = (9, 10, 5, 1)$
67 : $P_{1727} = (14, 10, 5, 1)$
68 : $P_{1796} = (3, 15, 5, 1)$
69 : $P_{1801} = (8, 15, 5, 1)$
70 : $P_{1898} = (9, 5, 6, 1)$
71 : $P_{1919} = (14, 6, 6, 1)$
72 : $P_{1920} = (15, 6, 6, 1)$
73 : $P_{1978} = (9, 10, 6, 1)$
74 : $P_{1991} = (6, 11, 6, 1)$
75 : $P_{2047} = (14, 14, 6, 1)$
76 : $P_{2064} = (15, 15, 6, 1)$
77 : $P_{2133} = (4, 4, 7, 1)$
78 : $P_{2150} = (5, 5, 7, 1)$
79 : $P_{2181} = (4, 7, 7, 1)$

80 : $P_{2182} = (5, 7, 7, 1)$
81 : $P_{2227} = (2, 10, 7, 1)$
82 : $P_{2248} = (7, 11, 7, 1)$
83 : $P_{2307} = (2, 15, 7, 1)$
84 : $P_{2339} = (2, 1, 8, 1)$
85 : $P_{2374} = (5, 3, 8, 1)$
86 : $P_{2384} = (15, 3, 8, 1)$
87 : $P_{2400} = (15, 4, 8, 1)$
88 : $P_{2405} = (4, 5, 8, 1)$
89 : $P_{2469} = (4, 9, 8, 1)$
90 : $P_{2499} = (2, 11, 8, 1)$
91 : $P_{2510} = (13, 11, 8, 1)$
92 : $P_{2511} = (14, 11, 8, 1)$
93 : $P_{2521} = (8, 12, 8, 1)$
94 : $P_{2543} = (14, 13, 8, 1)$
95 : $P_{2550} = (5, 14, 8, 1)$
96 : $P_{2554} = (9, 14, 8, 1)$
97 : $P_{2558} = (13, 14, 8, 1)$
98 : $P_{2570} = (9, 15, 8, 1)$
99 : $P_{2610} = (1, 2, 9, 1)$
100 : $P_{2660} = (3, 5, 9, 1)$
101 : $P_{2661} = (4, 5, 9, 1)$
102 : $P_{2663} = (6, 5, 9, 1)$
103 : $P_{2709} = (4, 8, 9, 1)$
104 : $P_{2738} = (1, 10, 9, 1)$
105 : $P_{2743} = (6, 10, 9, 1)$
106 : $P_{2778} = (9, 12, 9, 1)$
107 : $P_{2820} = (3, 15, 9, 1)$
108 : $P_{2853} = (4, 1, 10, 1)$
109 : $P_{2863} = (14, 1, 10, 1)$
110 : $P_{2866} = (1, 2, 10, 1)$
111 : $P_{2872} = (7, 2, 10, 1)$
112 : $P_{2919} = (6, 5, 10, 1)$
113 : $P_{2922} = (9, 5, 10, 1)$
114 : $P_{2927} = (14, 5, 10, 1)$
115 : $P_{2938} = (9, 6, 10, 1)$
116 : $P_{2947} = (2, 7, 10, 1)$
117 : $P_{2978} = (1, 9, 10, 1)$
118 : $P_{2983} = (6, 9, 10, 1)$
119 : $P_{3005} = (12, 10, 10, 1)$
120 : $P_{3006} = (13, 10, 10, 1)$
121 : $P_{3037} = (12, 12, 10, 1)$
122 : $P_{3054} = (13, 13, 10, 1)$
123 : $P_{3075} = (2, 15, 10, 1)$
124 : $P_{3077} = (4, 15, 10, 1)$
125 : $P_{3080} = (7, 15, 10, 1)$
126 : $P_{3107} = (2, 1, 11, 1)$
127 : $P_{3114} = (9, 1, 11, 1)$
128 : $P_{3141} = (4, 3, 11, 1)$
129 : $P_{3146} = (9, 3, 11, 1)$
130 : $P_{3149} = (12, 3, 11, 1)$
131 : $P_{3154} = (1, 4, 11, 1)$
132 : $P_{3165} = (12, 4, 11, 1)$
133 : $P_{3191} = (6, 6, 11, 1)$
134 : $P_{3208} = (7, 7, 11, 1)$
135 : $P_{3219} = (2, 8, 11, 1)$
136 : $P_{3230} = (13, 8, 11, 1)$
137 : $P_{3231} = (14, 8, 11, 1)$
138 : $P_{3271} = (6, 11, 11, 1)$
139 : $P_{3272} = (7, 11, 11, 1)$
140 : $P_{3285} = (4, 12, 11, 1)$
141 : $P_{3311} = (14, 13, 11, 1)$
142 : $P_{3314} = (1, 14, 11, 1)$
143 : $P_{3326} = (13, 14, 11, 1)$
144 : $P_{3397} = (4, 3, 12, 1)$
145 : $P_{3481} = (8, 8, 12, 1)$
146 : $P_{3498} = (9, 9, 12, 1)$
147 : $P_{3517} = (12, 10, 12, 1)$
148 : $P_{3525} = (4, 11, 12, 1)$
149 : $P_{3545} = (8, 12, 12, 1)$
150 : $P_{3546} = (9, 12, 12, 1)$
151 : $P_{3635} = (2, 2, 13, 1)$
152 : $P_{3652} = (3, 3, 13, 1)$
153 : $P_{3743} = (14, 8, 13, 1)$
154 : $P_{3774} = (13, 10, 13, 1)$
155 : $P_{3791} = (14, 11, 13, 1)$
156 : $P_{3811} = (2, 13, 13, 1)$
157 : $P_{3812} = (3, 13, 13, 1)$
158 : $P_{3910} = (5, 3, 14, 1)$
159 : $P_{3922} = (1, 4, 14, 1)$
160 : $P_{3967} = (14, 6, 14, 1)$
161 : $P_{3990} = (5, 8, 14, 1)$
162 : $P_{3994} = (9, 8, 14, 1)$
163 : $P_{3998} = (13, 8, 14, 1)$
164 : $P_{4034} = (1, 11, 14, 1)$
165 : $P_{4046} = (13, 11, 14, 1)$
166 : $P_{4106} = (9, 15, 14, 1)$
167 : $P_{4133} = (4, 1, 15, 1)$
168 : $P_{4152} = (7, 2, 15, 1)$
169 : $P_{4153} = (8, 2, 15, 1)$
170 : $P_{4159} = (14, 2, 15, 1)$
171 : $P_{4175} = (14, 3, 15, 1)$
172 : $P_{4196} = (3, 5, 15, 1)$
173 : $P_{4201} = (8, 5, 15, 1)$
174 : $P_{4224} = (15, 6, 15, 1)$
175 : $P_{4227} = (2, 7, 15, 1)$
176 : $P_{4250} = (9, 8, 15, 1)$
177 : $P_{4260} = (3, 9, 15, 1)$
178 : $P_{4275} = (2, 10, 15, 1)$
179 : $P_{4277} = (4, 10, 15, 1)$
180 : $P_{4280} = (7, 10, 15, 1)$
181 : $P_{4346} = (9, 14, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	1	0	0
1	1	0	1	0	1	0	0	1	0
2	1	1	0	0	0	1	0	0	1
3	1	0	0	0	1	1	1	0	0
4	0	1	0	1	0	1	0	1	0
5	0	0	1	1	1	0	0	0	1
6	1	0	0	1	0	0	0	1	1
7	0	1	0	0	1	0	1	0	1
8	0	0	1	0	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6
in point	P_1	P_1	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7
in point	P_1	P_1	P_{284}	P_{444}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_8
in point	P_1	P_1	P_{285}	P_{461}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6
in point	P_3	P_2	P_2	P_3

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7
in point	P_{284}	P_2	P_2	P_{2843}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_8
in point	P_{285}	P_2	P_2	P_{3100}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_7	ℓ_8
in point	P_3	P_3	P_{35}	P_{35}

Line 7 intersects

Line	ℓ_1	ℓ_4	ℓ_6	ℓ_8
in point	P_{444}	P_{2843}	P_{35}	P_{35}

Line 8 intersects

Line	ℓ_2	ℓ_5	ℓ_6	ℓ_7
in point	P_{461}	P_{3100}	P_{35}	P_{35}

The surface has 321 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_{35} = (0, 1, 1, 0)$
5 : $P_{36} = (1, 1, 1, 0)$
6 : $P_{74} = (7, 3, 1, 0)$
7 : $P_{77} = (10, 3, 1, 0)$
8 : $P_{80} = (13, 3, 1, 0)$
9 : $P_{106} = (7, 5, 1, 0)$
10 : $P_{110} = (11, 5, 1, 0)$
11 : $P_{111} = (12, 5, 1, 0)$
12 : $P_{153} = (6, 8, 1, 0)$
13 : $P_{157} = (10, 8, 1, 0)$
14 : $P_{159} = (12, 8, 1, 0)$
15 : $P_{265} = (6, 15, 1, 0)$
16 : $P_{270} = (11, 15, 1, 0)$
17 : $P_{272} = (13, 15, 1, 0)$
18 : $P_{284} = (10, 0, 0, 1)$
19 : $P_{285} = (11, 0, 0, 1)$
20 : $P_{290} = (0, 1, 0, 1)$

21 : $P_{300} = (10, 1, 0, 1)$
22 : $P_{301} = (11, 1, 0, 1)$
23 : $P_{306} = (0, 2, 0, 1)$
24 : $P_{316} = (10, 2, 0, 1)$
25 : $P_{317} = (11, 2, 0, 1)$
26 : $P_{322} = (0, 3, 0, 1)$
27 : $P_{332} = (10, 3, 0, 1)$
28 : $P_{333} = (11, 3, 0, 1)$
29 : $P_{338} = (0, 4, 0, 1)$
30 : $P_{348} = (10, 4, 0, 1)$
31 : $P_{349} = (11, 4, 0, 1)$
32 : $P_{354} = (0, 5, 0, 1)$
33 : $P_{364} = (10, 5, 0, 1)$
34 : $P_{365} = (11, 5, 0, 1)$
35 : $P_{370} = (0, 6, 0, 1)$
36 : $P_{380} = (10, 6, 0, 1)$
37 : $P_{381} = (11, 6, 0, 1)$
38 : $P_{386} = (0, 7, 0, 1)$
39 : $P_{396} = (10, 7, 0, 1)$
40 : $P_{397} = (11, 7, 0, 1)$
41 : $P_{402} = (0, 8, 0, 1)$
42 : $P_{412} = (10, 8, 0, 1)$
43 : $P_{413} = (11, 8, 0, 1)$
44 : $P_{418} = (0, 9, 0, 1)$
45 : $P_{428} = (10, 9, 0, 1)$
46 : $P_{429} = (11, 9, 0, 1)$
47 : $P_{434} = (0, 10, 0, 1)$
48 : $P_{444} = (10, 10, 0, 1)$
49 : $P_{445} = (11, 10, 0, 1)$
50 : $P_{450} = (0, 11, 0, 1)$
51 : $P_{460} = (10, 11, 0, 1)$
52 : $P_{461} = (11, 11, 0, 1)$
53 : $P_{466} = (0, 12, 0, 1)$
54 : $P_{476} = (10, 12, 0, 1)$
55 : $P_{477} = (11, 12, 0, 1)$
56 : $P_{482} = (0, 13, 0, 1)$
57 : $P_{492} = (10, 13, 0, 1)$
58 : $P_{493} = (11, 13, 0, 1)$
59 : $P_{498} = (0, 14, 0, 1)$
60 : $P_{508} = (10, 14, 0, 1)$
61 : $P_{509} = (11, 14, 0, 1)$
62 : $P_{514} = (0, 15, 0, 1)$
63 : $P_{524} = (10, 15, 0, 1)$
64 : $P_{525} = (11, 15, 0, 1)$
65 : $P_{530} = (0, 0, 1, 1)$
66 : $P_{540} = (10, 0, 1, 1)$
67 : $P_{541} = (11, 0, 1, 1)$
68 : $P_{546} = (0, 1, 1, 1)$
69 : $P_{586} = (9, 3, 1, 1)$
70 : $P_{623} = (14, 5, 1, 1)$
71 : $P_{659} = (2, 8, 1, 1)$
72 : $P_{693} = (4, 10, 1, 1)$
73 : $P_{700} = (11, 10, 1, 1)$
74 : $P_{703} = (14, 10, 1, 1)$
75 : $P_{707} = (2, 11, 1, 1)$
76 : $P_{714} = (9, 11, 1, 1)$
77 : $P_{715} = (10, 11, 1, 1)$
78 : $P_{773} = (4, 15, 1, 1)$
79 : $P_{785} = (0, 0, 2, 1)$
80 : $P_{795} = (10, 0, 2, 1)$
81 : $P_{796} = (11, 0, 2, 1)$
82 : $P_{817} = (0, 2, 2, 1)$
83 : $P_{847} = (14, 3, 2, 1)$
84 : $P_{873} = (8, 5, 2, 1)$
85 : $P_{923} = (10, 8, 2, 1)$
86 : $P_{930} = (1, 9, 2, 1)$
87 : $P_{940} = (11, 9, 2, 1)$
88 : $P_{946} = (1, 10, 2, 1)$
89 : $P_{952} = (7, 10, 2, 1)$
90 : $P_{995} = (2, 13, 2, 1)$
91 : $P_{1032} = (7, 15, 2, 1)$
92 : $P_{1033} = (8, 15, 2, 1)$
93 : $P_{1039} = (14, 15, 2, 1)$
94 : $P_{1041} = (0, 0, 3, 1)$
95 : $P_{1051} = (10, 0, 3, 1)$
96 : $P_{1052} = (11, 0, 3, 1)$
97 : $P_{1066} = (9, 1, 3, 1)$
98 : $P_{1087} = (14, 2, 3, 1)$
99 : $P_{1089} = (0, 3, 3, 1)$
100 : $P_{1107} = (2, 4, 3, 1)$
101 : $P_{1117} = (12, 4, 3, 1)$
102 : $P_{1120} = (15, 4, 3, 1)$
103 : $P_{1123} = (2, 5, 3, 1)$
104 : $P_{1174} = (5, 8, 3, 1)$
105 : $P_{1180} = (11, 8, 3, 1)$
106 : $P_{1184} = (15, 8, 3, 1)$
107 : $P_{1195} = (10, 9, 3, 1)$
108 : $P_{1221} = (4, 11, 3, 1)$
109 : $P_{1226} = (9, 11, 3, 1)$
110 : $P_{1229} = (12, 11, 3, 1)$
111 : $P_{1237} = (4, 12, 3, 1)$
112 : $P_{1252} = (3, 13, 3, 1)$
113 : $P_{1270} = (5, 14, 3, 1)$
114 : $P_{1295} = (14, 15, 3, 1)$
115 : $P_{1297} = (0, 0, 4, 1)$
116 : $P_{1307} = (10, 0, 4, 1)$
117 : $P_{1308} = (11, 0, 4, 1)$
118 : $P_{1347} = (2, 3, 4, 1)$
119 : $P_{1357} = (12, 3, 4, 1)$
120 : $P_{1360} = (15, 3, 4, 1)$
121 : $P_{1361} = (0, 4, 4, 1)$
122 : $P_{1379} = (2, 5, 4, 1)$
123 : $P_{1413} = (4, 7, 4, 1)$
124 : $P_{1440} = (15, 8, 4, 1)$
125 : $P_{1474} = (1, 11, 4, 1)$
126 : $P_{1485} = (12, 11, 4, 1)$
127 : $P_{1522} = (1, 14, 4, 1)$
128 : $P_{1531} = (10, 14, 4, 1)$
129 : $P_{1548} = (11, 15, 4, 1)$
130 : $P_{1553} = (0, 0, 5, 1)$
131 : $P_{1563} = (10, 0, 5, 1)$
132 : $P_{1564} = (11, 0, 5, 1)$
133 : $P_{1583} = (14, 1, 5, 1)$
134 : $P_{1593} = (8, 2, 5, 1)$
135 : $P_{1603} = (2, 3, 5, 1)$
136 : $P_{1619} = (2, 4, 5, 1)$
137 : $P_{1633} = (0, 5, 5, 1)$
138 : $P_{1658} = (9, 6, 5, 1)$
139 : $P_{1670} = (5, 7, 5, 1)$
140 : $P_{1685} = (4, 8, 5, 1)$
141 : $P_{1700} = (3, 9, 5, 1)$
142 : $P_{1701} = (4, 9, 5, 1)$
143 : $P_{1703} = (6, 9, 5, 1)$
144 : $P_{1719} = (6, 10, 5, 1)$
145 : $P_{1722} = (9, 10, 5, 1)$
146 : $P_{1727} = (14, 10, 5, 1)$
147 : $P_{1788} = (11, 14, 5, 1)$
148 : $P_{1796} = (3, 15, 5, 1)$
149 : $P_{1801} = (8, 15, 5, 1)$
150 : $P_{1803} = (10, 15, 5, 1)$
151 : $P_{1809} = (0, 0, 6, 1)$
152 : $P_{1819} = (10, 0, 6, 1)$
153 : $P_{1820} = (11, 0, 6, 1)$
154 : $P_{1898} = (9, 5, 6, 1)$
155 : $P_{1905} = (0, 6, 6, 1)$
156 : $P_{1919} = (14, 6, 6, 1)$
157 : $P_{1920} = (15, 6, 6, 1)$
158 : $P_{1978} = (9, 10, 6, 1)$
159 : $P_{1991} = (6, 11, 6, 1)$
160 : $P_{2011} = (10, 12, 6, 1)$
161 : $P_{2028} = (11, 13, 6, 1)$
162 : $P_{2047} = (14, 14, 6, 1)$
163 : $P_{2064} = (15, 15, 6, 1)$
164 : $P_{2065} = (0, 0, 7, 1)$
165 : $P_{2075} = (10, 0, 7, 1)$
166 : $P_{2076} = (11, 0, 7, 1)$
167 : $P_{2133} = (4, 4, 7, 1)$
168 : $P_{2150} = (5, 5, 7, 1)$
169 : $P_{2177} = (0, 7, 7, 1)$
170 : $P_{2181} = (4, 7, 7, 1)$
171 : $P_{2182} = (5, 7, 7, 1)$
172 : $P_{2227} = (2, 10, 7, 1)$
173 : $P_{2248} = (7, 11, 7, 1)$
174 : $P_{2268} = (11, 12, 7, 1)$
175 : $P_{2283} = (10, 13, 7, 1)$
176 : $P_{2307} = (2, 15, 7, 1)$
177 : $P_{2321} = (0, 0, 8, 1)$
178 : $P_{2331} = (10, 0, 8, 1)$
179 : $P_{2332} = (11, 0, 8, 1)$
180 : $P_{2339} = (2, 1, 8, 1)$
181 : $P_{2363} = (10, 2, 8, 1)$
182 : $P_{2374} = (5, 3, 8, 1)$

183 : $P_{2380} = (11, 3, 8, 1)$
184 : $P_{2384} = (15, 3, 8, 1)$
185 : $P_{2400} = (15, 4, 8, 1)$
186 : $P_{2405} = (4, 5, 8, 1)$
187 : $P_{2449} = (0, 8, 8, 1)$
188 : $P_{2469} = (4, 9, 8, 1)$
189 : $P_{2499} = (2, 11, 8, 1)$
190 : $P_{2510} = (13, 11, 8, 1)$
191 : $P_{2511} = (14, 11, 8, 1)$
192 : $P_{2521} = (8, 12, 8, 1)$
193 : $P_{2543} = (14, 13, 8, 1)$
194 : $P_{2550} = (5, 14, 8, 1)$
195 : $P_{2554} = (9, 14, 8, 1)$
196 : $P_{2558} = (13, 14, 8, 1)$
197 : $P_{2570} = (9, 15, 8, 1)$
198 : $P_{2577} = (0, 0, 9, 1)$
199 : $P_{2587} = (10, 0, 9, 1)$
200 : $P_{2588} = (11, 0, 9, 1)$
201 : $P_{2610} = (1, 2, 9, 1)$
202 : $P_{2620} = (11, 2, 9, 1)$
203 : $P_{2635} = (10, 3, 9, 1)$
204 : $P_{2660} = (3, 5, 9, 1)$
205 : $P_{2661} = (4, 5, 9, 1)$
206 : $P_{2663} = (6, 5, 9, 1)$
207 : $P_{2709} = (4, 8, 9, 1)$
208 : $P_{2721} = (0, 9, 9, 1)$
209 : $P_{2738} = (1, 10, 9, 1)$
210 : $P_{2743} = (6, 10, 9, 1)$
211 : $P_{2778} = (9, 12, 9, 1)$
212 : $P_{2820} = (3, 15, 9, 1)$
213 : $P_{2833} = (0, 0, 10, 1)$
214 : $P_{2843} = (10, 0, 10, 1)$
215 : $P_{2844} = (11, 0, 10, 1)$
216 : $P_{2853} = (4, 1, 10, 1)$
217 : $P_{2860} = (11, 1, 10, 1)$
218 : $P_{2863} = (14, 1, 10, 1)$
219 : $P_{2866} = (1, 2, 10, 1)$
220 : $P_{2872} = (7, 2, 10, 1)$
221 : $P_{2919} = (6, 5, 10, 1)$
222 : $P_{2922} = (9, 5, 10, 1)$
223 : $P_{2927} = (14, 5, 10, 1)$
224 : $P_{2938} = (9, 6, 10, 1)$
225 : $P_{2947} = (2, 7, 10, 1)$
226 : $P_{2978} = (1, 9, 10, 1)$
227 : $P_{2983} = (6, 9, 10, 1)$
228 : $P_{2993} = (0, 10, 10, 1)$
229 : $P_{3005} = (12, 10, 10, 1)$
230 : $P_{3006} = (13, 10, 10, 1)$
231 : $P_{3037} = (12, 12, 10, 1)$
232 : $P_{3054} = (13, 13, 10, 1)$
233 : $P_{3075} = (2, 15, 10, 1)$
234 : $P_{3077} = (4, 15, 10, 1)$
235 : $P_{3080} = (7, 15, 10, 1)$
236 : $P_{3089} = (0, 0, 11, 1)$
237 : $P_{3099} = (10, 0, 11, 1)$
238 : $P_{3100} = (11, 0, 11, 1)$
239 : $P_{3107} = (2, 1, 11, 1)$
240 : $P_{3114} = (9, 1, 11, 1)$
241 : $P_{3115} = (10, 1, 11, 1)$
242 : $P_{3141} = (4, 3, 11, 1)$
243 : $P_{3146} = (9, 3, 11, 1)$
244 : $P_{3149} = (12, 3, 11, 1)$
245 : $P_{3154} = (1, 4, 11, 1)$
246 : $P_{3165} = (12, 4, 11, 1)$
247 : $P_{3191} = (6, 6, 11, 1)$
248 : $P_{3208} = (7, 7, 11, 1)$
249 : $P_{3219} = (2, 8, 11, 1)$
250 : $P_{3230} = (13, 8, 11, 1)$
251 : $P_{3231} = (14, 8, 11, 1)$
252 : $P_{3265} = (0, 11, 11, 1)$
253 : $P_{3271} = (6, 11, 11, 1)$
254 : $P_{3272} = (7, 11, 11, 1)$
255 : $P_{3285} = (4, 12, 11, 1)$
256 : $P_{3311} = (14, 13, 11, 1)$
257 : $P_{3314} = (1, 14, 11, 1)$
258 : $P_{3326} = (13, 14, 11, 1)$
259 : $P_{3345} = (0, 0, 12, 1)$
260 : $P_{3355} = (10, 0, 12, 1)$
261 : $P_{3356} = (11, 0, 12, 1)$
262 : $P_{3397} = (4, 3, 12, 1)$
263 : $P_{3451} = (10, 6, 12, 1)$
264 : $P_{3468} = (11, 7, 12, 1)$
265 : $P_{3481} = (8, 8, 12, 1)$
266 : $P_{3498} = (9, 9, 12, 1)$
267 : $P_{3517} = (12, 10, 12, 1)$
268 : $P_{3525} = (4, 11, 12, 1)$
269 : $P_{3537} = (0, 12, 12, 1)$
270 : $P_{3545} = (8, 12, 12, 1)$
271 : $P_{3546} = (9, 12, 12, 1)$
272 : $P_{3601} = (0, 0, 13, 1)$
273 : $P_{3611} = (10, 0, 13, 1)$
274 : $P_{3612} = (11, 0, 13, 1)$
275 : $P_{3635} = (2, 2, 13, 1)$
276 : $P_{3652} = (3, 3, 13, 1)$
277 : $P_{3708} = (11, 6, 13, 1)$
278 : $P_{3723} = (10, 7, 13, 1)$
279 : $P_{3743} = (14, 8, 13, 1)$
280 : $P_{3774} = (13, 10, 13, 1)$
281 : $P_{3791} = (14, 11, 13, 1)$
282 : $P_{3809} = (0, 13, 13, 1)$
283 : $P_{3811} = (2, 13, 13, 1)$
284 : $P_{3812} = (3, 13, 13, 1)$
285 : $P_{3857} = (0, 0, 14, 1)$
286 : $P_{3867} = (10, 0, 14, 1)$
287 : $P_{3868} = (11, 0, 14, 1)$
288 : $P_{3910} = (5, 3, 14, 1)$
289 : $P_{3922} = (1, 4, 14, 1)$
290 : $P_{3931} = (10, 4, 14, 1)$
291 : $P_{3948} = (11, 5, 14, 1)$
292 : $P_{3967} = (14, 6, 14, 1)$
293 : $P_{3990} = (5, 8, 14, 1)$
294 : $P_{3994} = (9, 8, 14, 1)$
295 : $P_{3998} = (13, 8, 14, 1)$
296 : $P_{4034} = (1, 11, 14, 1)$
297 : $P_{4046} = (13, 11, 14, 1)$
298 : $P_{4081} = (0, 14, 14, 1)$
299 : $P_{4106} = (9, 15, 14, 1)$
300 : $P_{4113} = (0, 0, 15, 1)$
301 : $P_{4123} = (10, 0, 15, 1)$
302 : $P_{4124} = (11, 0, 15, 1)$
303 : $P_{4133} = (4, 1, 15, 1)$
304 : $P_{4152} = (7, 2, 15, 1)$
305 : $P_{4153} = (8, 2, 15, 1)$
306 : $P_{4159} = (14, 2, 15, 1)$
307 : $P_{4175} = (14, 3, 15, 1)$
308 : $P_{4188} = (11, 4, 15, 1)$
309 : $P_{4196} = (3, 5, 15, 1)$
310 : $P_{4201} = (8, 5, 15, 1)$
311 : $P_{4203} = (10, 5, 15, 1)$
312 : $P_{4224} = (15, 6, 15, 1)$
313 : $P_{4227} = (2, 7, 15, 1)$
314 : $P_{4250} = (9, 8, 15, 1)$
315 : $P_{4260} = (3, 9, 15, 1)$
316 : $P_{4275} = (2, 10, 15, 1)$
317 : $P_{4277} = (4, 10, 15, 1)$
318 : $P_{4280} = (7, 10, 15, 1)$
319 : $P_{4346} = (9, 14, 15, 1)$
320 : $P_{4353} = (0, 15, 15, 1)$