# Rank-76308 over GF(8)

January 15, 2021

## The equation

The equation of the surface is :

$$X_0^3 + X_0^2 X_1 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over $GF(8)$ is $1361355342$

## General information

| | |
|---|---:|
| Number of lines | 4 |
| Number of points | 89 |
| Number of singular points | 0 |
| Number of Eckardt points | 1 |
| Number of double points | 1 |
| Number of single points | 31 |
| Number of points off lines | 56 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $9^4$ |
| Type of lines on points | $3, 2, 1^{31}, 0^{56}$ |

**Singular Points**

The surface has 0 singular points:

## The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0,1,1,0,0,0)_{10}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{138} = \mathbf{Pl}(0,0,1,1,1,1)_{1322}$$

$$\ell_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{722} = \mathbf{Pl}(0,1,1,1,1,1)_{1330}$$

Rank of lines: ( 4744, 648, 138, 722 )
Rank of points on Klein quadric: ( 1, 10, 1322, 1330 )

**Eckardt Points**

The surface has 1 Eckardt points:
$0 : P_{138} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$.

**Double Points**

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

**Single Points**

The surface has 31 single points:
The single points on the surface are:

$0 : P_3 = (0,0,0,1)$ lies on line $\ell_0$
$1 : P_4 = (1,1,1,1)$ lies on line $\ell_2$
$2 : P_5 = (1,1,0,0)$ lies on line $\ell_2$
$3 : P_{20} = (1,1,1,0)$ lies on line $\ell_3$
$4 : P_{75} = (1,0,0,1)$ lies on line $\ell_1$
$5 : P_{83} = (1,1,0,1)$ lies on line $\ell_3$
$6 : P_{139} = (1,0,1,1)$ lies on line $\ell_1$
$7 : P_{155} = (2,2,1,1)$ lies on line $\ell_2$
$8 : P_{164} = (3,3,1,1)$ lies on line $\ell_2$
$9 : P_{173} = (4,4,1,1)$ lies on line $\ell_2$
$10 : P_{182} = (5,5,1,1)$ lies on line $\ell_2$
$11 : P_{191} = (6,6,1,1)$ lies on line $\ell_2$
$12 : P_{200} = (7,7,1,1)$ lies on line $\ell_2$
$13 : P_{201} = (0,0,2,1)$ lies on line $\ell_0$
$14 : P_{202} = (1,0,2,1)$ lies on line $\ell_1$
$15 : P_{228} = (3,3,2,1)$ lies on line $\ell_3$

$16 : P_{265} = (0,0,3,1)$ lies on line $\ell_0$
$17 : P_{266} = (1,0,3,1)$ lies on line $\ell_1$
$18 : P_{283} = (2,2,3,1)$ lies on line $\ell_3$
$19 : P_{329} = (0,0,4,1)$ lies on line $\ell_0$
$20 : P_{330} = (1,0,4,1)$ lies on line $\ell_1$
$21 : P_{374} = (5,5,4,1)$ lies on line $\ell_3$
$22 : P_{393} = (0,0,5,1)$ lies on line $\ell_0$
$23 : P_{394} = (1,0,5,1)$ lies on line $\ell_1$
$24 : P_{429} = (4,4,5,1)$ lies on line $\ell_3$
$25 : P_{457} = (0,0,6,1)$ lies on line $\ell_0$
$26 : P_{458} = (1,0,6,1)$ lies on line $\ell_1$
$27 : P_{520} = (7,7,6,1)$ lies on line $\ell_3$
$28 : P_{521} = (0,0,7,1)$ lies on line $\ell_0$
$29 : P_{522} = (1,0,7,1)$ lies on line $\ell_1$
$30 : P_{575} = (6,6,7,1)$ lies on line $\ell_3$

The single points on the surface are:

**Points on surface but on no line**

The surface has 56 points not on any line:
The points on the surface but not on lines are:

$0: P_1 = (0,1,0,0)$
$1: P_{32} = (5,2,1,0)$
$2: P_{39} = (4,3,1,0)$
$3: P_{49} = (6,4,1,0)$
$4: P_{58} = (7,5,1,0)$
$5: P_{61} = (2,6,1,0)$
$6: P_{70} = (3,7,1,0)$
$7: P_{146} = (0,1,1,1)$
$8: P_{159} = (6,2,1,1)$
$9: P_{168} = (7,3,1,1)$
$10: P_{172} = (3,4,1,1)$
$11: P_{179} = (2,5,1,1)$
$12: P_{189} = (4,6,1,1)$
$13: P_{198} = (5,7,1,1)$
$14: P_{216} = (7,1,2,1)$
$15: P_{229} = (4,3,2,1)$
$16: P_{233} = (0,4,2,1)$
$17: P_{236} = (3,4,2,1)$
$18: P_{240} = (7,4,2,1)$
$19: P_{245} = (4,5,2,1)$
$20: P_{258} = (1,7,2,1)$
$21: P_{277} = (4,1,3,1)$
$22: P_{285} = (4,2,3,1)$
$23: P_{305} = (0,5,3,1)$
$24: P_{307} = (2,5,3,1)$
$25: P_{312} = (7,5,3,1)$
$26: P_{320} = (7,6,3,1)$
$27: P_{322} = (1,7,3,1)$
$28: P_{339} = (2,1,4,1)$

$29: P_{346} = (1,2,4,1)$
$30: P_{376} = (7,5,4,1)$
$31: P_{384} = (7,6,4,1)$
$32: P_{385} = (0,7,4,1)$
$33: P_{387} = (2,7,4,1)$
$34: P_{390} = (5,7,4,1)$
$35: P_{408} = (7,1,5,1)$
$36: P_{410} = (1,2,5,1)$
$37: P_{419} = (2,3,5,1)$
$38: P_{432} = (7,4,5,1)$
$39: P_{441} = (0,6,5,1)$
$40: P_{443} = (2,6,5,1)$
$41: P_{445} = (4,6,5,1)$
$42: P_{467} = (2,1,6,1)$
$43: P_{481} = (0,3,6,1)$
$44: P_{485} = (4,3,6,1)$
$45: P_{488} = (7,3,6,1)$
$46: P_{490} = (1,4,6,1)$
$47: P_{501} = (4,5,6,1)$
$48: P_{515} = (2,7,6,1)$
$49: P_{533} = (4,1,7,1)$
$50: P_{537} = (0,2,7,1)$
$51: P_{541} = (4,2,7,1)$
$52: P_{543} = (6,2,7,1)$
$53: P_{547} = (2,3,7,1)$
$54: P_{554} = (1,4,7,1)$
$55: P_{571} = (2,6,7,1)$

## Line Intersection Graph

|   | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 1 |
| 3 | 1 | 0 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ | $\ell_2$ | $\ell_3$ |
|---|---|---|---|
| in point | $P_2$ | $P_{138}$ | $P_{138}$ |

Line 1 intersects

| Line | $\ell_0$ |
|---|---|
| in point | $P_2$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_3$ |
|---|---|---|
| in point | $P_{138}$ | $P_{138}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_2$ |
|---|---|---|
| in point | $P_{138}$ | $P_{138}$ |

The surface has 89 points:
The points on the surface are:

3

$0 : P_1 = (0, 1, 0, 0)$

$1 : P_2 = (0, 0, 1, 0)$

$2 : P_3 = (0, 0, 0, 1)$

$3 : P_4 = (1, 1, 1, 1)$

$4 : P_5 = (1, 1, 0, 0)$

$5 : P_{20} = (1, 1, 1, 0)$

$6 : P_{32} = (5, 2, 1, 0)$

$7 : P_{39} = (4, 3, 1, 0)$

$8 : P_{49} = (6, 4, 1, 0)$

$9 : P_{58} = (7, 5, 1, 0)$

$10 : P_{61} = (2, 6, 1, 0)$

$11 : P_{70} = (3, 7, 1, 0)$

$12 : P_{75} = (1, 0, 0, 1)$

$13 : P_{83} = (1, 1, 0, 1)$

$14 : P_{138} = (0, 0, 1, 1)$

$15 : P_{139} = (1, 0, 1, 1)$

$16 : P_{146} = (0, 1, 1, 1)$

$17 : P_{155} = (2, 2, 1, 1)$

$18 : P_{159} = (6, 2, 1, 1)$

$19 : P_{164} = (3, 3, 1, 1)$

$20 : P_{168} = (7, 3, 1, 1)$

$21 : P_{172} = (3, 4, 1, 1)$

$22 : P_{173} = (4, 4, 1, 1)$

$23 : P_{179} = (2, 5, 1, 1)$

$24 : P_{182} = (5, 5, 1, 1)$

$25 : P_{189} = (4, 6, 1, 1)$

$26 : P_{191} = (6, 6, 1, 1)$

$27 : P_{198} = (5, 7, 1, 1)$

$28 : P_{200} = (7, 7, 1, 1)$

$29 : P_{201} = (0, 0, 2, 1)$

$30 : P_{202} = (1, 0, 2, 1)$

$31 : P_{216} = (7, 1, 2, 1)$

$32 : P_{228} = (3, 3, 2, 1)$

$33 : P_{229} = (4, 3, 2, 1)$

$34 : P_{233} = (0, 4, 2, 1)$

$35 : P_{236} = (3, 4, 2, 1)$

$36 : P_{240} = (7, 4, 2, 1)$

$37 : P_{245} = (4, 5, 2, 1)$

$38 : P_{258} = (1, 7, 2, 1)$

$39 : P_{265} = (0, 0, 3, 1)$

$40 : P_{266} = (1, 0, 3, 1)$

$41 : P_{277} = (4, 1, 3, 1)$

$42 : P_{283} = (2, 2, 3, 1)$

$43 : P_{285} = (4, 2, 3, 1)$

$44 : P_{305} = (0, 5, 3, 1)$

$45 : P_{307} = (2, 5, 3, 1)$

$46 : P_{312} = (7, 5, 3, 1)$

$47 : P_{320} = (7, 6, 3, 1)$

$48 : P_{322} = (1, 7, 3, 1)$

$49 : P_{329} = (0, 0, 4, 1)$

$50 : P_{330} = (1, 0, 4, 1)$

$51 : P_{339} = (2, 1, 4, 1)$

$52 : P_{346} = (1, 2, 4, 1)$

$53 : P_{374} = (5, 5, 4, 1)$

$54 : P_{376} = (7, 5, 4, 1)$

$55 : P_{384} = (7, 6, 4, 1)$

$56 : P_{385} = (0, 7, 4, 1)$

$57 : P_{387} = (2, 7, 4, 1)$

$58 : P_{390} = (5, 7, 4, 1)$

$59 : P_{393} = (0, 0, 5, 1)$

$60 : P_{394} = (1, 0, 5, 1)$

$61 : P_{408} = (7, 1, 5, 1)$

$62 : P_{410} = (1, 2, 5, 1)$

$63 : P_{419} = (2, 3, 5, 1)$

$64 : P_{429} = (4, 4, 5, 1)$

$65 : P_{432} = (7, 4, 5, 1)$

$66 : P_{441} = (0, 6, 5, 1)$

$67 : P_{443} = (2, 6, 5, 1)$

$68 : P_{445} = (4, 6, 5, 1)$

$69 : P_{457} = (0, 0, 6, 1)$

$70 : P_{458} = (1, 0, 6, 1)$

$71 : P_{467} = (2, 1, 6, 1)$

$72 : P_{481} = (0, 3, 6, 1)$

$73 : P_{485} = (4, 3, 6, 1)$

$74 : P_{488} = (7, 3, 6, 1)$

$75 : P_{490} = (1, 4, 6, 1)$

$76 : P_{501} = (4, 5, 6, 1)$

$77 : P_{515} = (2, 7, 6, 1)$

$78 : P_{520} = (7, 7, 6, 1)$

$79 : P_{521} = (0, 0, 7, 1)$

$80 : P_{522} = (1, 0, 7, 1)$

$81 : P_{533} = (4, 1, 7, 1)$

$82 : P_{537} = (0, 2, 7, 1)$

$83 : P_{541} = (4, 2, 7, 1)$

$84 : P_{543} = (6, 2, 7, 1)$

$85 : P_{547} = (2, 3, 7, 1)$

$86 : P_{554} = (1, 4, 7, 1)$

$87 : P_{571} = (2, 6, 7, 1)$

$88 : P_{575} = (6, 6, 7, 1)$