

Rank-65900 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_3^3 + X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1244206158

General information

Number of lines	3
Number of points	73
Number of singular points	1
Number of Eckardt points	1
Number of double points	0
Number of single points	24
Number of points off lines	48
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^3
Type of lines on points	$3, 1^{24}, 0^{48}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & \gamma^6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \left[\begin{array}{cccc} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \end{array} \right]_{3504} = \mathbf{Pl}(2, 0, 0, 1, 0, 0)_{19}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & \gamma^5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{1752} = \mathbf{Pl}(4, 0, 0, 1, 0, 0)_{21}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & 0 & \gamma^3 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{2920} = \mathbf{Pl}(7, 0, 0, 1, 0, 0)_{24}$$

Rank of lines: (3504, 1752, 2920)

Rank of points on Klein quadric: (19, 21, 24)

Eckardt Points

The surface has 1 Eckardt points:

0 : $P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0)$.

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 24 single points:

The single points on the surface are:

0 : $P_{76} = (2, 0, 0, 1)$ lies on line ℓ_0
1 : $P_{78} = (4, 0, 0, 1)$ lies on line ℓ_1
2 : $P_{81} = (7, 0, 0, 1)$ lies on line ℓ_2
3 : $P_{84} = (2, 1, 0, 1)$ lies on line ℓ_0
4 : $P_{86} = (4, 1, 0, 1)$ lies on line ℓ_1
5 : $P_{89} = (7, 1, 0, 1)$ lies on line ℓ_2
6 : $P_{92} = (2, 2, 0, 1)$ lies on line ℓ_0
7 : $P_{94} = (4, 2, 0, 1)$ lies on line ℓ_1
8 : $P_{97} = (7, 2, 0, 1)$ lies on line ℓ_2
9 : $P_{100} = (2, 3, 0, 1)$ lies on line ℓ_0
10 : $P_{102} = (4, 3, 0, 1)$ lies on line ℓ_1
11 : $P_{105} = (7, 3, 0, 1)$ lies on line ℓ_2
12 : $P_{108} = (2, 4, 0, 1)$ lies on line ℓ_0

13 : $P_{110} = (4, 4, 0, 1)$ lies on line ℓ_1
14 : $P_{113} = (7, 4, 0, 1)$ lies on line ℓ_2
15 : $P_{116} = (2, 5, 0, 1)$ lies on line ℓ_0
16 : $P_{118} = (4, 5, 0, 1)$ lies on line ℓ_1
17 : $P_{121} = (7, 5, 0, 1)$ lies on line ℓ_2
18 : $P_{124} = (2, 6, 0, 1)$ lies on line ℓ_0
19 : $P_{126} = (4, 6, 0, 1)$ lies on line ℓ_1
20 : $P_{129} = (7, 6, 0, 1)$ lies on line ℓ_2
21 : $P_{132} = (2, 7, 0, 1)$ lies on line ℓ_0
22 : $P_{134} = (4, 7, 0, 1)$ lies on line ℓ_1
23 : $P_{137} = (7, 7, 0, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 48 points not on any line:

The points on the surface but not on lines are:

0 : $P_2 = (0, 0, 1, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_{12} = (1, 0, 1, 0)$
3 : $P_{20} = (1, 1, 1, 0)$
4 : $P_{31} = (4, 2, 1, 0)$
5 : $P_{42} = (7, 3, 1, 0)$
6 : $P_{50} = (7, 4, 1, 0)$
7 : $P_{53} = (2, 5, 1, 0)$

$$\begin{aligned}
8 : P_{63} &= (4, 6, 1, 0) & 29 : P_{396} &= (3, 0, 5, 1) \\
9 : P_{69} &= (2, 7, 1, 0) & 30 : P_{410} &= (1, 2, 5, 1) \\
10 : P_{139} &= (1, 0, 1, 1) & 31 : P_{418} &= (1, 3, 5, 1) \\
11 : P_{146} &= (0, 1, 1, 1) & 32 : P_{420} &= (3, 3, 5, 1) \\
12 : P_{215} &= (6, 1, 2, 1) & 33 : P_{423} &= (6, 3, 5, 1) \\
13 : P_{236} &= (3, 4, 2, 1) & 34 : P_{425} &= (0, 4, 5, 1) \\
14 : P_{241} &= (0, 5, 2, 1) & 35 : P_{439} &= (6, 5, 5, 1) \\
15 : P_{260} &= (3, 7, 2, 1) & 36 : P_{462} &= (5, 0, 6, 1) \\
16 : P_{263} &= (6, 7, 2, 1) & 37 : P_{490} &= (1, 4, 6, 1) \\
17 : P_{271} &= (6, 0, 3, 1) & 38 : P_{498} &= (1, 5, 6, 1) \\
18 : P_{281} &= (0, 2, 3, 1) & 39 : P_{500} &= (3, 5, 6, 1) \\
19 : P_{294} &= (5, 3, 3, 1) & 40 : P_{502} &= (5, 5, 6, 1) \\
20 : P_{314} &= (1, 6, 3, 1) & 41 : P_{508} &= (3, 6, 6, 1) \\
21 : P_{318} &= (5, 6, 3, 1) & 42 : P_{513} &= (0, 7, 6, 1) \\
22 : P_{319} &= (6, 6, 3, 1) & 43 : P_{534} &= (5, 1, 7, 1) \\
23 : P_{322} &= (1, 7, 3, 1) & 44 : P_{543} &= (6, 2, 7, 1) \\
24 : P_{340} &= (3, 1, 4, 1) & 45 : P_{545} &= (0, 3, 7, 1) \\
25 : P_{348} &= (3, 2, 4, 1) & 46 : P_{558} &= (5, 4, 7, 1) \\
26 : P_{350} &= (5, 2, 4, 1) & 47 : P_{559} &= (6, 4, 7, 1) \\
27 : P_{377} &= (0, 6, 4, 1) \\
28 : P_{390} &= (5, 7, 4, 1)
\end{aligned}$$

Line Intersection Graph

$$\begin{array}{c|ccc}
& 0 & 1 & 2 \\
\hline
0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
2 & 1 & 1 & 0
\end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_1	P_1

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_1

The surface has 73 points:

The points on the surface are:

$$\begin{aligned}
0 : P_1 &= (0, 1, 0, 0) & 11 : P_{76} &= (2, 0, 0, 1) & 22 : P_{105} &= (7, 3, 0, 1) \\
1 : P_2 &= (0, 0, 1, 0) & 12 : P_{78} &= (4, 0, 0, 1) & 23 : P_{108} &= (2, 4, 0, 1) \\
2 : P_4 &= (1, 1, 1, 1) & 13 : P_{81} &= (7, 0, 0, 1) & 24 : P_{110} &= (4, 4, 0, 1) \\
3 : P_{12} &= (1, 0, 1, 0) & 14 : P_{84} &= (2, 1, 0, 1) & 25 : P_{113} &= (7, 4, 0, 1) \\
4 : P_{20} &= (1, 1, 1, 0) & 15 : P_{86} &= (4, 1, 0, 1) & 26 : P_{116} &= (2, 5, 0, 1) \\
5 : P_{31} &= (4, 2, 1, 0) & 16 : P_{89} &= (7, 1, 0, 1) & 27 : P_{118} &= (4, 5, 0, 1) \\
6 : P_{42} &= (7, 3, 1, 0) & 17 : P_{92} &= (2, 2, 0, 1) & 28 : P_{121} &= (7, 5, 0, 1) \\
7 : P_{50} &= (7, 4, 1, 0) & 18 : P_{94} &= (4, 2, 0, 1) & 29 : P_{124} &= (2, 6, 0, 1) \\
8 : P_{53} &= (2, 5, 1, 0) & 19 : P_{97} &= (7, 2, 0, 1) & 30 : P_{126} &= (4, 6, 0, 1) \\
9 : P_{63} &= (4, 6, 1, 0) & 20 : P_{100} &= (2, 3, 0, 1) & 31 : P_{129} &= (7, 6, 0, 1) \\
10 : P_{69} &= (2, 7, 1, 0) & 21 : P_{102} &= (4, 3, 0, 1) & 32 : P_{132} &= (2, 7, 0, 1)
\end{aligned}$$

33 : $P_{134} = (4, 7, 0, 1)$
 34 : $P_{137} = (7, 7, 0, 1)$
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 36 : $P_{146} = (0, 1, 1, 1)$
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 72 : $P_{559} = (6, 4, 7, 1)$