

Rank-76387 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_0^2 X_3 + X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(4) is 1503226201

General information

Number of lines	4
Number of points	25
Number of singular points	2
Number of Eckardt points	0
Number of double points	4
Number of single points	12
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^4
Type of lines on points	$2^4, 1^{12}, 0^9$

Singular Points

The surface has 2 singular points:

$$0 : P_4 = \mathbf{P}(1, 1, 1, 1) = \mathbf{P}(1, 1, 1, 1)$$

$$1 : P_5 = \mathbf{P}(1, 1, 0, 0) = \mathbf{P}(1, 1, 0, 0)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{33} \\ \ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{356} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_3 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{198}\end{aligned}$$

Rank of lines: (0, 5, 356, 38)

Rank of points on Klein quadric: (0, 33, 1, 198)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_0 = (1, 0, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_4 = (1, 1, 1, 1) = \ell_1 \cap \ell_3$$

$$P_{38} = (0, 0, 1, 1) = \ell_2 \cap \ell_3$$

Single Points

The surface has 12 single points:

The single points on the surface are:

$$0 : P_1 = (0, 1, 0, 0) \text{ lies on line } \ell_0$$

$$1 : P_2 = (0, 0, 1, 0) \text{ lies on line } \ell_2$$

$$2 : P_3 = (0, 0, 0, 1) \text{ lies on line } \ell_2$$

$$3 : P_6 = (2, 1, 0, 0) \text{ lies on line } \ell_0$$

$$4 : P_7 = (3, 1, 0, 0) \text{ lies on line } \ell_0$$

$$5 : P_{42} = (0, 1, 1, 1) \text{ lies on line } \ell_1$$

$$6 : P_{43} = (2, 1, 1, 1) \text{ lies on line } \ell_1$$

$$7 : P_{44} = (3, 1, 1, 1) \text{ lies on line } \ell_1$$

$$8 : P_{47} = (2, 2, 1, 1) \text{ lies on line } \ell_3$$

$$9 : P_{52} = (3, 3, 1, 1) \text{ lies on line } \ell_3$$

$$10 : P_{53} = (0, 0, 2, 1) \text{ lies on line } \ell_2$$

$$11 : P_{69} = (0, 0, 3, 1) \text{ lies on line } \ell_2$$

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:

The points on the surface but not on lines are:

$$0 : P_{13} = (2, 1, 1, 0)$$

$$1 : P_{14} = (3, 1, 1, 0)$$

$$2 : P_{23} = (1, 0, 0, 1)$$

$$3 : P_{28} = (2, 1, 0, 1)$$

$$4 : P_{29} = (3, 1, 0, 1)$$

$$5 : P_{55} = (2, 0, 2, 1)$$

$$6 : P_{65} = (0, 3, 2, 1)$$

$$7 : P_{72} = (3, 0, 3, 1)$$

$$8 : P_{77} = (0, 2, 3, 1)$$

Line Intersection Graph

	0	1	2	3
0	0	1	0	1
1	1	0	0	1
2	0	0	0	1
3	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_3
in point	P_0	P_5

Line 1 intersects

Line	ℓ_0	ℓ_3
in point	P_0	P_4

Line 2 intersects

Line	ℓ_3
in point	P_{38}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2
in point	P_5	P_4	P_{38}

The surface has 25 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
 1 : $P_1 = (0, 1, 0, 0)$
 2 : $P_2 = (0, 0, 1, 0)$
 3 : $P_3 = (0, 0, 0, 1)$
 4 : $P_4 = (1, 1, 1, 1)$
 5 : $P_5 = (1, 1, 0, 0)$
 6 : $P_6 = (2, 1, 0, 0)$
 7 : $P_7 = (3, 1, 0, 0)$
 8 : $P_{13} = (2, 1, 1, 0)$

9 : $P_{14} = (3, 1, 1, 0)$
 10 : $P_{23} = (1, 0, 0, 1)$
 11 : $P_{28} = (2, 1, 0, 1)$
 12 : $P_{29} = (3, 1, 0, 1)$
 13 : $P_{38} = (0, 0, 1, 1)$
 14 : $P_{42} = (0, 1, 1, 1)$
 15 : $P_{43} = (2, 1, 1, 1)$
 16 : $P_{44} = (3, 1, 1, 1)$
 17 : $P_{47} = (2, 2, 1, 1)$

18 : $P_{52} = (3, 3, 1, 1)$
 19 : $P_{53} = (0, 0, 2, 1)$
 20 : $P_{55} = (2, 0, 2, 1)$
 21 : $P_{65} = (0, 3, 2, 1)$
 22 : $P_{69} = (0, 0, 3, 1)$
 23 : $P_{72} = (3, 0, 3, 1)$
 24 : $P_{77} = (0, 2, 3, 1)$