

Rank-76 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_2 = 0$$

(0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(4) is 696

General information

Number of lines	1
Number of points	21
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	5
Number of points off lines	16
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5
Type of lines on points	$1^5, 0^{16}$

Singular Points

The surface has 1 singular points:

$$0 : P_{12} = \mathbf{P}(1, 1, 1, 0) = \mathbf{P}(1, 1, 1, 0)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_1 = \mathbf{PI}(1, 0, 1, 0, 0, 0)_3$$

Rank of lines: (1)
Rank of points on Klein quadric: (3)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 5 single points:
The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0	3 : $P_{13} = (2, 1, 1, 0)$ lies on line ℓ_0
1 : $P_{11} = (0, 1, 1, 0)$ lies on line ℓ_0	4 : $P_{14} = (3, 1, 1, 0)$ lies on line ℓ_0
2 : $P_{12} = (1, 1, 1, 0)$ lies on line ℓ_0	

The single points on the surface are:

Points on surface but on no line

The surface has 16 points not on any line:
The points on the surface but not on lines are:

0 : $P_5 = (1, 1, 0, 0)$	9 : $P_{51} = (2, 3, 1, 1)$
1 : $P_8 = (1, 0, 1, 0)$	10 : $P_{53} = (0, 0, 2, 1)$
2 : $P_{15} = (0, 2, 1, 0)$	11 : $P_{60} = (3, 1, 2, 1)$
3 : $P_{19} = (0, 3, 1, 0)$	12 : $P_{66} = (1, 3, 2, 1)$
4 : $P_{26} = (0, 1, 0, 1)$	13 : $P_{69} = (0, 0, 3, 1)$
5 : $P_{30} = (0, 2, 0, 1)$	14 : $P_{75} = (2, 1, 3, 1)$
6 : $P_{34} = (0, 3, 0, 1)$	15 : $P_{78} = (1, 2, 3, 1)$
7 : $P_{38} = (0, 0, 1, 1)$	
8 : $P_{48} = (3, 2, 1, 1)$	

Line Intersection Graph

0
0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line
in point

The surface has 21 points:
The points on the surface are:

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 1 : $P_5 = (1, 1, 0, 0)$
 2 : $P_8 = (1, 0, 1, 0)$
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