

# Rank-65633 over GF(64)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2 X_1 + X_0^2 X_3 + X_0 X_1 X_2 = 0$$

( 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(64) is 1107828869

## General information

Number of lines	2
Number of points	4225
Number of singular points	0
Number of Eckardt points	0
Number of double points	0
Number of single points	130
Number of points off lines	4095
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$65^2$
Type of lines on points	$1^{130}, 0^{4095}$

## Singular Points

The surface has 0 singular points:

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right]_{64} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right]_{64} = \mathbf{Pl}(1, 0, 0, 0, 1, 0)_{4226} \\ \ell_1 &= \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{8258} = \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]_{8258} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{544578}\end{aligned}$$

Rank of lines: ( 64, 8258 )  
Rank of points on Klein quadric: ( 4226, 544578 )

### Eckardt Points

The surface has 0 Eckardt points:

### Double Points

The surface has 0 Double points:  
The double points on the surface are:

### Single Points

The surface has 130 single points:  
The single points on the surface are:

- |   |   |
|---|---|
| 0 : $P_0 = (1, 0, 0, 0)$ lies on line $\ell_0$        | 33 : $P_{4256} = (30, 1, 0, 1)$ lies on line $\ell_0$ |
| 1 : $P_4 = (1, 1, 1, 1)$ lies on line $\ell_1$        | 34 : $P_{4257} = (31, 1, 0, 1)$ lies on line $\ell_0$ |
| 2 : $P_5 = (1, 1, 0, 0)$ lies on line $\ell_1$        | 35 : $P_{4258} = (32, 1, 0, 1)$ lies on line $\ell_0$ |
| 3 : $P_{4226} = (0, 1, 0, 1)$ lies on line $\ell_0$   | 36 : $P_{4259} = (33, 1, 0, 1)$ lies on line $\ell_0$ |
| 4 : $P_{4227} = (1, 1, 0, 1)$ lies on line $\ell_0$   | 37 : $P_{4260} = (34, 1, 0, 1)$ lies on line $\ell_0$ |
| 5 : $P_{4228} = (2, 1, 0, 1)$ lies on line $\ell_0$   | 38 : $P_{4261} = (35, 1, 0, 1)$ lies on line $\ell_0$ |
| 6 : $P_{4229} = (3, 1, 0, 1)$ lies on line $\ell_0$   | 39 : $P_{4262} = (36, 1, 0, 1)$ lies on line $\ell_0$ |
| 7 : $P_{4230} = (4, 1, 0, 1)$ lies on line $\ell_0$   | 40 : $P_{4263} = (37, 1, 0, 1)$ lies on line $\ell_0$ |
| 8 : $P_{4231} = (5, 1, 0, 1)$ lies on line $\ell_0$   | 41 : $P_{4264} = (38, 1, 0, 1)$ lies on line $\ell_0$ |
| 9 : $P_{4232} = (6, 1, 0, 1)$ lies on line $\ell_0$   | 42 : $P_{4265} = (39, 1, 0, 1)$ lies on line $\ell_0$ |
| 10 : $P_{4233} = (7, 1, 0, 1)$ lies on line $\ell_0$  | 43 : $P_{4266} = (40, 1, 0, 1)$ lies on line $\ell_0$ |
| 11 : $P_{4234} = (8, 1, 0, 1)$ lies on line $\ell_0$  | 44 : $P_{4267} = (41, 1, 0, 1)$ lies on line $\ell_0$ |
| 12 : $P_{4235} = (9, 1, 0, 1)$ lies on line $\ell_0$  | 45 : $P_{4268} = (42, 1, 0, 1)$ lies on line $\ell_0$ |
| 13 : $P_{4236} = (10, 1, 0, 1)$ lies on line $\ell_0$ | 46 : $P_{4269} = (43, 1, 0, 1)$ lies on line $\ell_0$ |
| 14 : $P_{4237} = (11, 1, 0, 1)$ lies on line $\ell_0$ | 47 : $P_{4270} = (44, 1, 0, 1)$ lies on line $\ell_0$ |
| 15 : $P_{4238} = (12, 1, 0, 1)$ lies on line $\ell_0$ | 48 : $P_{4271} = (45, 1, 0, 1)$ lies on line $\ell_0$ |
| 16 : $P_{4239} = (13, 1, 0, 1)$ lies on line $\ell_0$ | 49 : $P_{4272} = (46, 1, 0, 1)$ lies on line $\ell_0$ |
| 17 : $P_{4240} = (14, 1, 0, 1)$ lies on line $\ell_0$ | 50 : $P_{4273} = (47, 1, 0, 1)$ lies on line $\ell_0$ |
| 18 : $P_{4241} = (15, 1, 0, 1)$ lies on line $\ell_0$ | 51 : $P_{4274} = (48, 1, 0, 1)$ lies on line $\ell_0$ |
| 19 : $P_{4242} = (16, 1, 0, 1)$ lies on line $\ell_0$ | 52 : $P_{4275} = (49, 1, 0, 1)$ lies on line $\ell_0$ |
| 20 : $P_{4243} = (17, 1, 0, 1)$ lies on line $\ell_0$ | 53 : $P_{4276} = (50, 1, 0, 1)$ lies on line $\ell_0$ |
| 21 : $P_{4244} = (18, 1, 0, 1)$ lies on line $\ell_0$ | 54 : $P_{4277} = (51, 1, 0, 1)$ lies on line $\ell_0$ |
| 22 : $P_{4245} = (19, 1, 0, 1)$ lies on line $\ell_0$ | 55 : $P_{4278} = (52, 1, 0, 1)$ lies on line $\ell_0$ |
| 23 : $P_{4246} = (20, 1, 0, 1)$ lies on line $\ell_0$ | 56 : $P_{4279} = (53, 1, 0, 1)$ lies on line $\ell_0$ |
| 24 : $P_{4247} = (21, 1, 0, 1)$ lies on line $\ell_0$ | 57 : $P_{4280} = (54, 1, 0, 1)$ lies on line $\ell_0$ |
| 25 : $P_{4248} = (22, 1, 0, 1)$ lies on line $\ell_0$ | 58 : $P_{4281} = (55, 1, 0, 1)$ lies on line $\ell_0$ |
| 26 : $P_{4249} = (23, 1, 0, 1)$ lies on line $\ell_0$ | 59 : $P_{4282} = (56, 1, 0, 1)$ lies on line $\ell_0$ |
| 27 : $P_{4250} = (24, 1, 0, 1)$ lies on line $\ell_0$ | 60 : $P_{4283} = (57, 1, 0, 1)$ lies on line $\ell_0$ |
| 28 : $P_{4251} = (25, 1, 0, 1)$ lies on line $\ell_0$ | 61 : $P_{4284} = (58, 1, 0, 1)$ lies on line $\ell_0$ |
| 29 : $P_{4252} = (26, 1, 0, 1)$ lies on line $\ell_0$ | 62 : $P_{4285} = (59, 1, 0, 1)$ lies on line $\ell_0$ |
| 30 : $P_{4253} = (27, 1, 0, 1)$ lies on line $\ell_0$ | 63 : $P_{4286} = (60, 1, 0, 1)$ lies on line $\ell_0$ |
| 31 : $P_{4254} = (28, 1, 0, 1)$ lies on line $\ell_0$ | 64 : $P_{4287} = (61, 1, 0, 1)$ lies on line $\ell_0$ |
| 32 : $P_{4255} = (29, 1, 0, 1)$ lies on line $\ell_0$ | 65 : $P_{4288} = (62, 1, 0, 1)$ lies on line $\ell_0$ |

66 :  $P_{4289} = (63, 1, 0, 1)$  lies on line  $\ell_0$   
 67 :  $P_{8258} = (0, 0, 1, 1)$  lies on line  $\ell_1$   
 68 :  $P_{8387} = (2, 2, 1, 1)$  lies on line  $\ell_1$   
 69 :  $P_{8452} = (3, 3, 1, 1)$  lies on line  $\ell_1$   
 70 :  $P_{8517} = (4, 4, 1, 1)$  lies on line  $\ell_1$   
 71 :  $P_{8582} = (5, 5, 1, 1)$  lies on line  $\ell_1$   
 72 :  $P_{8647} = (6, 6, 1, 1)$  lies on line  $\ell_1$   
 73 :  $P_{8712} = (7, 7, 1, 1)$  lies on line  $\ell_1$   
 74 :  $P_{8777} = (8, 8, 1, 1)$  lies on line  $\ell_1$   
 75 :  $P_{8842} = (9, 9, 1, 1)$  lies on line  $\ell_1$   
 76 :  $P_{8907} = (10, 10, 1, 1)$  lies on line  $\ell_1$   
 77 :  $P_{8972} = (11, 11, 1, 1)$  lies on line  $\ell_1$   
 78 :  $P_{9037} = (12, 12, 1, 1)$  lies on line  $\ell_1$   
 79 :  $P_{9102} = (13, 13, 1, 1)$  lies on line  $\ell_1$   
 80 :  $P_{9167} = (14, 14, 1, 1)$  lies on line  $\ell_1$   
 81 :  $P_{9232} = (15, 15, 1, 1)$  lies on line  $\ell_1$   
 82 :  $P_{9297} = (16, 16, 1, 1)$  lies on line  $\ell_1$   
 83 :  $P_{9362} = (17, 17, 1, 1)$  lies on line  $\ell_1$   
 84 :  $P_{9427} = (18, 18, 1, 1)$  lies on line  $\ell_1$   
 85 :  $P_{9492} = (19, 19, 1, 1)$  lies on line  $\ell_1$   
 86 :  $P_{9557} = (20, 20, 1, 1)$  lies on line  $\ell_1$   
 87 :  $P_{9622} = (21, 21, 1, 1)$  lies on line  $\ell_1$   
 88 :  $P_{9687} = (22, 22, 1, 1)$  lies on line  $\ell_1$   
 89 :  $P_{9752} = (23, 23, 1, 1)$  lies on line  $\ell_1$   
 90 :  $P_{9817} = (24, 24, 1, 1)$  lies on line  $\ell_1$   
 91 :  $P_{9882} = (25, 25, 1, 1)$  lies on line  $\ell_1$   
 92 :  $P_{9947} = (26, 26, 1, 1)$  lies on line  $\ell_1$   
 93 :  $P_{10012} = (27, 27, 1, 1)$  lies on line  $\ell_1$   
 94 :  $P_{10077} = (28, 28, 1, 1)$  lies on line  $\ell_1$   
 95 :  $P_{10142} = (29, 29, 1, 1)$  lies on line  $\ell_1$   
 96 :  $P_{10207} = (30, 30, 1, 1)$  lies on line  $\ell_1$   
 97 :  $P_{10272} = (31, 31, 1, 1)$  lies on line  $\ell_1$   
 98 :  $P_{10337} = (32, 32, 1, 1)$  lies on line  $\ell_1$

99 :  $P_{10402} = (33, 33, 1, 1)$  lies on line  $\ell_1$   
 100 :  $P_{10467} = (34, 34, 1, 1)$  lies on line  $\ell_1$   
 101 :  $P_{10532} = (35, 35, 1, 1)$  lies on line  $\ell_1$   
 102 :  $P_{10597} = (36, 36, 1, 1)$  lies on line  $\ell_1$   
 103 :  $P_{10662} = (37, 37, 1, 1)$  lies on line  $\ell_1$   
 104 :  $P_{10727} = (38, 38, 1, 1)$  lies on line  $\ell_1$   
 105 :  $P_{10792} = (39, 39, 1, 1)$  lies on line  $\ell_1$   
 106 :  $P_{10857} = (40, 40, 1, 1)$  lies on line  $\ell_1$   
 107 :  $P_{10922} = (41, 41, 1, 1)$  lies on line  $\ell_1$   
 108 :  $P_{10987} = (42, 42, 1, 1)$  lies on line  $\ell_1$   
 109 :  $P_{11052} = (43, 43, 1, 1)$  lies on line  $\ell_1$   
 110 :  $P_{11117} = (44, 44, 1, 1)$  lies on line  $\ell_1$   
 111 :  $P_{11182} = (45, 45, 1, 1)$  lies on line  $\ell_1$   
 112 :  $P_{11247} = (46, 46, 1, 1)$  lies on line  $\ell_1$   
 113 :  $P_{11312} = (47, 47, 1, 1)$  lies on line  $\ell_1$   
 114 :  $P_{11377} = (48, 48, 1, 1)$  lies on line  $\ell_1$   
 115 :  $P_{11442} = (49, 49, 1, 1)$  lies on line  $\ell_1$   
 116 :  $P_{11507} = (50, 50, 1, 1)$  lies on line  $\ell_1$   
 117 :  $P_{11572} = (51, 51, 1, 1)$  lies on line  $\ell_1$   
 118 :  $P_{11637} = (52, 52, 1, 1)$  lies on line  $\ell_1$   
 119 :  $P_{11702} = (53, 53, 1, 1)$  lies on line  $\ell_1$   
 120 :  $P_{11767} = (54, 54, 1, 1)$  lies on line  $\ell_1$   
 121 :  $P_{11832} = (55, 55, 1, 1)$  lies on line  $\ell_1$   
 122 :  $P_{11897} = (56, 56, 1, 1)$  lies on line  $\ell_1$   
 123 :  $P_{11962} = (57, 57, 1, 1)$  lies on line  $\ell_1$   
 124 :  $P_{12027} = (58, 58, 1, 1)$  lies on line  $\ell_1$   
 125 :  $P_{12092} = (59, 59, 1, 1)$  lies on line  $\ell_1$   
 126 :  $P_{12157} = (60, 60, 1, 1)$  lies on line  $\ell_1$   
 127 :  $P_{12222} = (61, 61, 1, 1)$  lies on line  $\ell_1$   
 128 :  $P_{12287} = (62, 62, 1, 1)$  lies on line  $\ell_1$   
 129 :  $P_{12352} = (63, 63, 1, 1)$  lies on line  $\ell_1$

The single points on the surface are:

### Points on surface but on no line

The surface has 4095 points not on any line:  
Too many to print.

### Line Intersection Graph

	0	1
0	0	0
1	0	0

Neighbor sets in the line intersection graph:  
Line 0 intersects

Line
in point

Line 1 intersects

Line
in point

The surface has 4225 points:  
Too many to print.