Rank-74264 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_2^3 + X_0^2X_1 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 286396950

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	0
Number of double points	18
Number of single points	117
Number of points off lines	186
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	2^{18}, 1^{117}, 0^{186}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{273} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{273} = \mathbf{Pl}(1,0,0,0,0,1)_{4626}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \delta^8 & \delta \\ 0 & 1 & \delta^5 & 1 \end{bmatrix}_{12585} = \begin{bmatrix} 1 & 0 & 14 & 2 \\ 0 & 1 & 11 & 1 \end{bmatrix}_{12585} = \mathbf{Pl}(14,7,7,14,7,1)_{35704}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \delta & \delta^2 \\ 0 & 1 & \delta^{10} & 1 \end{bmatrix}_{18044} = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 10 & 1 \end{bmatrix}_{18044} = \mathbf{Pl}(2,12,12,2,12,1)_{57247}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & \delta^2 & \delta^4 \\ 0 & 1 & \delta^5 & 1 \end{bmatrix}_{40431} = \begin{bmatrix} 1 & 0 & 4 & 9 \\ 0 & 1 & 11 & 1 \end{bmatrix}_{40431} = \mathbf{Pl}(4,6,6,4,6,1)_{31419}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^{12} & \delta^4 \\ 0 & 1 & \delta^7 & \delta^6 \end{bmatrix}_{40378} = \begin{bmatrix} 1 & 0 & 3 & 9 \\ 0 & 1 & 7 & 15 \end{bmatrix}_{40378} = \mathbf{Pl}(8,3,14,7,5,1)_{29128}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \delta^3 & \delta \\ 0 & 1 & \delta^{13} & \delta^9 \end{bmatrix}_{11006} = \begin{bmatrix} 1 & 0 & 8 & 2 \\ 0 & 1 & 6 & 5 \end{bmatrix}_{11006} = \mathbf{Pl}(3,8,4,6,15,1)_{67673}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & \delta^9 & \delta^8 \\ 0 & 1 & \delta^{14} & \delta^{12} \end{bmatrix}_{62577} = \begin{bmatrix} 1 & 0 & 5 & 14 \\ 0 & 1 & 12 & 3 \end{bmatrix}_{62577} = \mathbf{Pl}(15,5,2,12,8,1)_{38675}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^6 & \delta^2 \\ 0 & 1 & \delta^{11} & \delta^3 \end{bmatrix}_{21708} = \begin{bmatrix} 1 & 0 & 15 & 4 \\ 0 & 1 & 13 & 8 \end{bmatrix}_{21708} = \mathbf{Pl}(5,15,9,13,3,1)_{19840}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^4 & \delta^8 \\ 0 & 1 & \delta^{10} & 1 \end{bmatrix}_{63635} = \begin{bmatrix} 1 & 0 & 9 & 14 \\ 0 & 1 & 10 & 1 \end{bmatrix}_{63635} = \mathbf{Pl}(9,13,13,9,13,1)_{61529}$$

Rank of lines: (273, 12585, 18044, 40431, 40378, 11006, 62577, 21708, 63635)
Rank of points on Klein quadric: (4626, 35704, 57247, 31419, 29128, 67673, 38675, 19840, 61529)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 18 Double points:
The double points on the surface are:

$P_{52} = (1,2,1,0) = \ell_0 \cap \ell_1$
$P_{84} = (1,4,1,0) = \ell_0 \cap \ell_2$
$P_{164} = (1,9,1,0) = \ell_0 \cap \ell_3$
$P_{244} = (1,14,1,0) = \ell_0 \cap \ell_8$
$P_{3105} = (0,1,11,1) = \ell_1 \cap \ell_3$
$P_{2882} = (1,3,10,1) = \ell_1 \cap \ell_4$
$P_{3815} = (6,13,13,1) = \ell_1 \cap \ell_6$
$P_{2190} = (13,7,7,1) = \ell_2 \cap \ell_5$
$P_{3170} = (1,5,11,1) = \ell_2 \cap \ell_6$
$P_{2849} = (0,1,10,1) = \ell_2 \cap \ell_8$
$P_{2962} = (1,8,10,1) = \ell_3 \cap \ell_5$
$P_{3544} = (7,12,12,1) = \ell_3 \cap \ell_7$
$P_{2272} = (15,12,7,1) = \ell_4 \cap \ell_6$
$P_{3721} = (8,7,13,1) = \ell_4 \cap \ell_7$
$P_{1917} = (12,6,6,1) = \ell_4 \cap \ell_8$
$P_{3444} = (3,6,12,1) = \ell_5 \cap \ell_6$
$P_{2022} = (5,13,6,1) = \ell_5 \cap \ell_7$
$P_{3330} = (1,15,11,1) = \ell_7 \cap \ell_8$

Single Points

The surface has 117 single points:
The single points on the surface are:

0 : $P_1 = (0,1,0,0)$ lies on line ℓ_0
1 : $P_{20} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{36} = (1,1,1,0)$ lies on line ℓ_0
3 : $P_{68} = (1,3,1,0)$ lies on line ℓ_0
4 : $P_{100} = (1,5,1,0)$ lies on line ℓ_0
5 : $P_{116} = (1,6,1,0)$ lies on line ℓ_0
6 : $P_{132} = (1,7,1,0)$ lies on line ℓ_0
7 : $P_{148} = (1,8,1,0)$ lies on line ℓ_0
8 : $P_{180} = (1,10,1,0)$ lies on line ℓ_0
9 : $P_{182} = (3,10,1,0)$ lies on line ℓ_4

10 : $P_{187} = (8, 10, 1, 0)$ lies on line ℓ_5
11 : $P_{196} = (1, 11, 1, 0)$ lies on line ℓ_0
12 : $P_{200} = (5, 11, 1, 0)$ lies on line ℓ_6
13 : $P_{210} = (15, 11, 1, 0)$ lies on line ℓ_7
14 : $P_{212} = (1, 12, 1, 0)$ lies on line ℓ_0
15 : $P_{228} = (1, 13, 1, 0)$ lies on line ℓ_0
16 : $P_{260} = (1, 15, 1, 0)$ lies on line ℓ_0
17 : $P_{316} = (10, 2, 0, 1)$ lies on line ℓ_2
18 : $P_{329} = (7, 3, 0, 1)$ lies on line ℓ_5
19 : $P_{349} = (11, 4, 0, 1)$ lies on line ℓ_3
20 : $P_{366} = (12, 5, 0, 1)$ lies on line ℓ_7
21 : $P_{408} = (6, 8, 0, 1)$ lies on line ℓ_4
22 : $P_{428} = (10, 9, 0, 1)$ lies on line ℓ_8
23 : $P_{509} = (11, 14, 0, 1)$ lies on line ℓ_1
24 : $P_{527} = (13, 15, 0, 1)$ lies on line ℓ_6
25 : $P_{566} = (5, 2, 1, 1)$ lies on line ℓ_4
26 : $P_{601} = (8, 4, 1, 1)$ lies on line ℓ_6
27 : $P_{636} = (11, 6, 1, 1)$ lies on line ℓ_2
28 : $P_{652} = (11, 7, 1, 1)$ lies on line ℓ_8
29 : $P_{688} = (15, 9, 1, 1)$ lies on line ℓ_5
30 : $P_{731} = (10, 12, 1, 1)$ lies on line ℓ_1
31 : $P_{747} = (10, 13, 1, 1)$ lies on line ℓ_3
32 : $P_{756} = (3, 14, 1, 1)$ lies on line ℓ_7
33 : $P_{792} = (7, 0, 2, 1)$ lies on line ℓ_6
34 : $P_{865} = (0, 5, 2, 1)$ lies on line ℓ_4
35 : $P_{953} = (8, 10, 2, 1)$ lies on line ℓ_2
36 : $P_{954} = (9, 10, 2, 1)$ lies on line ℓ_1
37 : $P_{956} = (11, 10, 2, 1)$ lies on line ℓ_7
38 : $P_{985} = (8, 12, 2, 1)$ lies on line ℓ_8
39 : $P_{1023} = (14, 14, 2, 1)$ lies on line ℓ_5
40 : $P_{1034} = (9, 15, 2, 1)$ lies on line ℓ_3
41 : $P_{1061} = (4, 1, 3, 1)$ lies on line ℓ_7
42 : $P_{1082} = (9, 2, 3, 1)$ lies on line ℓ_8
43 : $P_{1111} = (6, 4, 3, 1)$ lies on line ℓ_5
44 : $P_{1145} = (8, 6, 3, 1)$ lies on line ℓ_3
45 : $P_{1177} = (8, 8, 3, 1)$ lies on line ℓ_1
46 : $P_{1219} = (2, 11, 3, 1)$ lies on line ℓ_6
47 : $P_{1274} = (9, 14, 3, 1)$ lies on line ℓ_2
48 : $P_{1284} = (3, 15, 3, 1)$ lies on line ℓ_4
49 : $P_{1309} = (12, 0, 4, 1)$ lies on line ℓ_5
50 : $P_{1331} = (2, 2, 4, 1)$ lies on line ℓ_7
51 : $P_{1359} = (14, 3, 4, 1)$ lies on line ℓ_8
52 : $P_{1408} = (15, 6, 4, 1)$ lies on line ℓ_1
53 : $P_{1425} = (0, 8, 4, 1)$ lies on line ℓ_6
54 : $P_{1483} = (10, 11, 4, 1)$ lies on line ℓ_4
55 : $P_{1487} = (14, 11, 4, 1)$ lies on line ℓ_2
56 : $P_{1488} = (15, 11, 4, 1)$ lies on line ℓ_3
57 : $P_{1578} = (9, 1, 5, 1)$ lies on line ℓ_4
58 : $P_{1599} = (14, 2, 5, 1)$ lies on line ℓ_3
59 : $P_{1606} = (5, 3, 5, 1)$ lies on line ℓ_6
60 : $P_{1631} = (14, 4, 5, 1)$ lies on line ℓ_1
61 : $P_{1710} = (13, 9, 5, 1)$ lies on line ℓ_7
62 : $P_{1717} = (4, 10, 5, 1)$ lies on line ℓ_5
63 : $P_{1776} = (15, 13, 5, 1)$ lies on line ℓ_8
64 : $P_{1808} = (15, 15, 5, 1)$ lies on line ℓ_2
65 : $P_{1822} = (13, 0, 6, 1)$ lies on line ℓ_3
66 : $P_{1854} = (13, 2, 6, 1)$ lies on line ℓ_1
67 : $P_{1869} = (12, 3, 6, 1)$ lies on line ℓ_2
68 : $P_{1931} = (10, 7, 6, 1)$ lies on line ℓ_6
69 : $P_{2077} = (12, 0, 7, 1)$ lies on line ℓ_1
70 : $P_{2171} = (10, 6, 7, 1)$ lies on line ℓ_7
71 : $P_{2206} = (13, 8, 7, 1)$ lies on line ℓ_8
72 : $P_{2221} = (12, 9, 7, 1)$ lies on line ℓ_3
73 : $P_{2351} = (14, 1, 8, 1)$ lies on line ℓ_6
74 : $P_{2372} = (3, 3, 8, 1)$ lies on line ℓ_3
75 : $P_{2387} = (2, 4, 8, 1)$ lies on line ℓ_8
76 : $P_{2409} = (8, 5, 8, 1)$ lies on line ℓ_5
77 : $P_{2436} = (3, 7, 8, 1)$ lies on line ℓ_1
78 : $P_{2467} = (2, 9, 8, 1)$ lies on line ℓ_2
79 : $P_{2506} = (9, 11, 8, 1)$ lies on line ℓ_7
80 : $P_{2552} = (7, 14, 8, 1)$ lies on line ℓ_4
81 : $P_{2583} = (6, 0, 9, 1)$ lies on line ℓ_7
82 : $P_{2645} = (4, 4, 9, 1)$ lies on line ℓ_4
83 : $P_{2659} = (2, 5, 9, 1)$ lies on line ℓ_1
84 : $P_{2739} = (2, 10, 9, 1)$ lies on line ℓ_3
85 : $P_{2740} = (3, 10, 9, 1)$ lies on line ℓ_8
86 : $P_{2748} = (11, 10, 9, 1)$ lies on line ℓ_6
87 : $P_{2788} = (3, 13, 9, 1)$ lies on line ℓ_2
88 : $P_{2817} = (0, 15, 9, 1)$ lies on line ℓ_5
89 : $P_{2911} = (14, 4, 10, 1)$ lies on line ℓ_7
90 : $P_{3061} = (4, 14, 10, 1)$ lies on line ℓ_6
91 : $P_{3130} = (9, 2, 11, 1)$ lies on line ℓ_5
92 : $P_{3235} = (2, 9, 11, 1)$ lies on line ℓ_4
93 : $P_{3351} = (6, 0, 12, 1)$ lies on line ℓ_2
94 : $P_{3564} = (11, 13, 12, 1)$ lies on line ℓ_4
95 : $P_{3575} = (6, 14, 12, 1)$ lies on line ℓ_8
96 : $P_{3592} = (7, 15, 12, 1)$ lies on line ℓ_1
97 : $P_{3608} = (7, 0, 13, 1)$ lies on line ℓ_8
98 : $P_{3672} = (7, 4, 13, 1)$ lies on line ℓ_2
99 : $P_{3687} = (6, 5, 13, 1)$ lies on line ℓ_3
100 : $P_{3804} = (11, 12, 13, 1)$ lies on line ℓ_5
101 : $P_{3870} = (13, 0, 14, 1)$ lies on line ℓ_4
102 : $P_{3905} = (0, 3, 14, 1)$ lies on line ℓ_7
103 : $P_{3974} = (5, 7, 14, 1)$ lies on line ℓ_3
104 : $P_{3989} = (4, 8, 14, 1)$ lies on line ℓ_2
105 : $P_{4010} = (9, 9, 14, 1)$ lies on line ℓ_6
106 : $P_{4037} = (4, 11, 14, 1)$ lies on line ℓ_8
107 : $P_{4038} = (5, 11, 14, 1)$ lies on line ℓ_1
108 : $P_{4043} = (10, 11, 14, 1)$ lies on line ℓ_5
109 : $P_{4131} = (2, 1, 15, 1)$ lies on line ℓ_5
110 : $P_{4157} = (12, 2, 15, 1)$ lies on line ℓ_6
111 : $P_{4198} = (5, 5, 15, 1)$ lies on line ℓ_8
112 : $P_{4256} = (15, 8, 15, 1)$ lies on line ℓ_7
113 : $P_{4261} = (4, 9, 15, 1)$ lies on line ℓ_1
114 : $P_{4287} = (14, 10, 15, 1)$ lies on line ℓ_4
115 : $P_{4310} = (5, 12, 15, 1)$ lies on line ℓ_2
116 : $P_{4341} = (4, 14, 15, 1)$ lies on line ℓ_3

The single points on the surface are:

Points on surface but on no line

The surface has 186 points not on any line:
The points on the surface but not on lines are:

$0: P_3 = (0,0,0,1)$
$1: P_4 = (1,1,1,1)$
$2: P_5 = (1,1,0,0)$
$3: P_{29} = (10,0,1,0)$
$4: P_{30} = (11,0,1,0)$
$5: P_{71} = (4,3,1,0)$
$6: P_{73} = (6,3,1,0)$
$7: P_{108} = (9,5,1,0)$
$8: P_{112} = (13,5,1,0)$
$9: P_{154} = (7,8,1,0)$
$10: P_{161} = (14,8,1,0)$
$11: P_{261} = (2,15,1,0)$
$12: P_{271} = (12,15,1,0)$
$13: P_{275} = (1,0,0,1)$
$14: P_{291} = (1,1,0,1)$
$15: P_{330} = (8,3,0,1)$
$16: P_{334} = (12,3,0,1)$
$17: P_{360} = (6,5,0,1)$
$18: P_{369} = (15,5,0,1)$
$19: P_{385} = (15,6,0,1)$
$20: P_{391} = (5,7,0,1)$
$21: P_{405} = (3,8,0,1)$
$22: P_{415} = (13,8,0,1)$
$23: P_{474} = (8,12,0,1)$
$24: P_{485} = (3,13,0,1)$
$25: P_{519} = (5,15,0,1)$
$26: P_{521} = (7,15,0,1)$
$27: P_{546} = (0,1,1,1)$
$28: P_{585} = (8,3,1,1)$
$29: P_{624} = (15,5,1,1)$
$30: P_{660} = (3,8,1,1)$
$31: P_{774} = (5,15,1,1)$
$32: P_{824} = (7,2,2,1)$
$33: P_{887} = (6,6,2,1)$
$34: P_{903} = (6,7,2,1)$
$35: P_{943} = (14,9,2,1)$
$36: P_{974} = (13,11,2,1)$
$37: P_{1011} = (2,14,2,1)$
$38: P_{1036} = (11,15,2,1)$
$39: P_{1038} = (13,15,2,1)$
$40: P_{1053} = (12,0,3,1)$
$41: P_{1095} = (6,3,3,1)$
$42: P_{1109} = (4,4,3,1)$
$43: P_{1169} = (0,8,3,1)$
$44: P_{1187} = (2,9,3,1)$
$45: P_{1222} = (5,11,3,1)$
$46: P_{1229} = (12,11,3,1)$
$47: P_{1238} = (5,12,3,1)$
$48: P_{1333} = (4,2,4,1)$
$49: P_{1352} = (7,3,4,1)$
$50: P_{1355} = (10,3,4,1)$
$51: P_{1373} = (12,4,4,1)$
$52: P_{1464} = (7,10,4,1)$
$53: P_{1502} = (13,12,4,1)$
$54: P_{1518} = (13,13,4,1)$
$55: P_{1523} = (2,14,4,1)$
$56: P_{1559} = (6,0,5,1)$
$57: P_{1646} = (13,5,5,1)$
$58: P_{1657} = (8,6,5,1)$
$59: P_{1706} = (9,9,5,1)$
$60: P_{1719} = (6,10,5,1)$
$61: P_{1721} = (8,10,5,1)$
$62: P_{1781} = (4,14,5,1)$
$63: P_{1793} = (0,15,5,1)$
$64: P_{1846} = (5,2,6,1)$
$65: P_{1851} = (10,2,6,1)$
$66: P_{1857} = (0,3,6,1)$
$67: P_{1872} = (15,3,6,1)$
$68: P_{1892} = (3,5,6,1)$
$69: P_{1907} = (2,6,6,1)$
$70: P_{1913} = (8,6,6,1)$
$71: P_{1938} = (1,8,6,1)$
$72: P_{1968} = (15,9,6,1)$
$73: P_{1972} = (3,10,6,1)$
$74: P_{1993} = (8,11,6,1)$
$75: P_{2007} = (6,12,6,1)$
$76: P_{2035} = (2,14,6,1)$
$77: P_{2050} = (1,15,6,1)$
$78: P_{2102} = (5,2,7,1)$
$79: P_{2114} = (1,3,7,1)$
$80: P_{2138} = (9,4,7,1)$
$81: P_{2146} = (1,5,7,1)$
$82: P_{2180} = (3,7,7,1)$
$83: P_{2186} = (9,7,7,1)$
$84: P_{2193} = (0,8,7,1)$
$85: P_{2198} = (5,8,7,1)$
$86: P_{2219} = (10,9,7,1)$
$87: P_{2224} = (15,9,7,1)$
$88: P_{2233} = (8,10,7,1)$
$89: P_{2244} = (3,11,7,1)$

$90: P_{2280} = (7, 13, 7, 1)$
$91: P_{2313} = (8, 15, 7, 1)$
$92: P_{2334} = (13, 0, 8, 1)$
$93: P_{2362} = (9, 2, 8, 1)$
$94: P_{2369} = (0, 3, 8, 1)$
$95: P_{2456} = (7, 8, 8, 1)$
$96: P_{2510} = (13, 11, 8, 1)$
$97: P_{2512} = (15, 11, 8, 1)$
$98: P_{2544} = (15, 13, 8, 1)$
$99: P_{2559} = (14, 14, 8, 1)$
$100: P_{2613} = (4, 2, 9, 1)$
$101: P_{2650} = (9, 4, 9, 1)$
$102: P_{2668} = (11, 5, 9, 1)$
$103: P_{2669} = (12, 5, 9, 1)$
$104: P_{2680} = (7, 6, 9, 1)$
$105: P_{2696} = (7, 7, 9, 1)$
$106: P_{2727} = (6, 9, 9, 1)$
$107: P_{2765} = (12, 11, 9, 1)$
$108: P_{2855} = (6, 1, 10, 1)$
$109: P_{2856} = (7, 1, 10, 1)$
$110: P_{2871} = (6, 2, 10, 1)$
$111: P_{2876} = (11, 2, 10, 1)$
$112: P_{2880} = (15, 2, 10, 1)$
$113: P_{2894} = (13, 3, 10, 1)$
$114: P_{2896} = (15, 3, 10, 1)$
$115: P_{2901} = (4, 4, 10, 1)$
$116: P_{2966} = (5, 8, 10, 1)$
$117: P_{2973} = (12, 8, 10, 1)$
$118: P_{2982} = (5, 9, 10, 1)$
$119: P_{2984} = (7, 9, 10, 1)$
$120: P_{2988} = (11, 9, 10, 1)$
$121: P_{3019} = (10, 11, 10, 1)$
$122: P_{3021} = (12, 11, 10, 1)$
$123: P_{3022} = (13, 11, 10, 1)$
$124: P_{3071} = (14, 14, 10, 1)$
$125: P_{3117} = (12, 1, 11, 1)$
$126: P_{3118} = (13, 1, 11, 1)$
$127: P_{3123} = (2, 2, 11, 1)$
$128: P_{3156} = (3, 4, 11, 1)$
$129: P_{3163} = (10, 4, 11, 1)$
$130: P_{3166} = (13, 4, 11, 1)$
$131: P_{3172} = (3, 5, 11, 1)$
$132: P_{3176} = (7, 5, 11, 1)$
$133: P_{3242} = (9, 9, 11, 1)$
$134: P_{3255} = (6, 10, 11, 1)$
$135: P_{3256} = (7, 10, 11, 1)$
$136: P_{3260} = (11, 10, 11, 1)$
$137: P_{3321} = (8, 14, 11, 1)$
$138: P_{3323} = (10, 14, 11, 1)$
$139: P_{3325} = (12, 14, 11, 1)$
$140: P_{3335} = (6, 15, 11, 1)$
$141: P_{3337} = (8, 15, 11, 1)$
$142: P_{3408} = (15, 3, 12, 1)$
$143: P_{3417} = (8, 4, 12, 1)$
$144: P_{3426} = (1, 5, 12, 1)$
$145: P_{3469} = (12, 7, 12, 1)$
$146: P_{3474} = (1, 8, 12, 1)$
$147: P_{3503} = (14, 9, 12, 1)$
$148: P_{3510} = (5, 10, 12, 1)$
$149: P_{3536} = (15, 11, 12, 1)$
$150: P_{3542} = (5, 12, 12, 1)$
$151: P_{3551} = (14, 12, 12, 1)$
$152: P_{3572} = (3, 14, 12, 1)$
$153: P_{3580} = (11, 14, 12, 1)$
$154: P_{3585} = (0, 15, 12, 1)$
$155: P_{3593} = (8, 15, 12, 1)$
$156: P_{3637} = (4, 2, 13, 1)$
$157: P_{3650} = (1, 3, 13, 1)$
$158: P_{3673} = (8, 4, 13, 1)$
$159: P_{3676} = (11, 4, 13, 1)$
$160: P_{3681} = (0, 5, 13, 1)$
$161: P_{3684} = (3, 5, 13, 1)$
$162: P_{3710} = (13, 6, 13, 1)$
$163: P_{3734} = (5, 8, 13, 1)$
$164: P_{3776} = (15, 10, 13, 1)$
$165: P_{3782} = (5, 11, 13, 1)$
$166: P_{3813} = (4, 13, 13, 1)$
$167: P_{3824} = (15, 13, 13, 1)$
$168: P_{3828} = (3, 14, 13, 1)$
$169: P_{3842} = (1, 15, 13, 1)$
$170: P_{3930} = (9, 4, 14, 1)$
$171: P_{3991} = (6, 8, 14, 1)$
$172: P_{3995} = (10, 8, 14, 1)$
$173: P_{4015} = (14, 9, 14, 1)$
$174: P_{4023} = (6, 10, 14, 1)$
$175: P_{4061} = (12, 12, 14, 1)$
$176: P_{4077} = (12, 13, 14, 1)$
$177: P_{4094} = (13, 14, 14, 1)$
$178: P_{4120} = (7, 0, 15, 1)$
$179: P_{4147} = (2, 2, 15, 1)$
$180: P_{4191} = (14, 4, 15, 1)$
$181: P_{4193} = (0, 5, 15, 1)$
$182: P_{4228} = (3, 7, 15, 1)$
$183: P_{4276} = (3, 10, 15, 1)$
$184: P_{4280} = (7, 10, 15, 1)$
$185: P_{4365} = (12, 15, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	1
1	1	0	0	1	1	0	1	0	0
2	1	0	0	0	0	1	1	0	1
3	1	1	0	0	0	1	0	1	0
4	0	1	0	0	0	0	1	1	1
5	0	0	1	1	0	0	1	1	0
6	0	1	1	0	1	1	0	0	0
7	0	0	0	1	1	1	0	0	1
8	1	0	1	0	1	0	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_8
in point	P_{52}	P_{84}	P_{164}	P_{244}

Line 1 intersects

Line	ℓ_0	ℓ_3	ℓ_4	ℓ_6
in point	P_{52}	P_{3105}	P_{2882}	P_{3815}

Line 2 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_8
in point	P_{84}	P_{2190}	P_{3170}	P_{2849}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_{164}	P_{3105}	P_{2962}	P_{3544}

Line 4 intersects

Line	ℓ_1	ℓ_6	ℓ_7	ℓ_8
in point	P_{2882}	P_{2272}	P_{3721}	P_{1917}

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_{2190}	P_{2962}	P_{3444}	P_{2022}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{3815}	P_{3170}	P_{2272}	P_{3444}

Line 7 intersects

Line	ℓ_3	ℓ_4	ℓ_5	ℓ_8
in point	P_{3544}	P_{3721}	P_{2022}	P_{3330}

Line 8 intersects

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_7
in point	P_{244}	P_{2849}	P_{1917}	P_{3330}

The surface has 321 points:
The points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$
1 : $P_3 = (0, 0, 0, 1)$
2 : $P_4 = (1, 1, 1, 1)$
3 : $P_5 = (1, 1, 0, 0)$
4 : $P_{20} = (1, 0, 1, 0)$
5 : $P_{29} = (10, 0, 1, 0)$
6 : $P_{30} = (11, 0, 1, 0)$
7 : $P_{36} = (1, 1, 1, 0)$
8 : $P_{52} = (1, 2, 1, 0)$
9 : $P_{68} = (1, 3, 1, 0)$
10 : $P_{71} = (4, 3, 1, 0)$
11 : $P_{73} = (6, 3, 1, 0)$
12 : $P_{84} = (1, 4, 1, 0)$
13 : $P_{100} = (1, 5, 1, 0)$
14 : $P_{108} = (9, 5, 1, 0)$
15 : $P_{112} = (13, 5, 1, 0)$
16 : $P_{116} = (1, 6, 1, 0)$
17 : $P_{132} = (1, 7, 1, 0)$
18 : $P_{148} = (1, 8, 1, 0)$
19 : $P_{154} = (7, 8, 1, 0)$
20 : $P_{161} = (14, 8, 1, 0)$

21 : $P_{164} = (1, 9, 1, 0)$
22 : $P_{180} = (1, 10, 1, 0)$
23 : $P_{182} = (3, 10, 1, 0)$
24 : $P_{187} = (8, 10, 1, 0)$
25 : $P_{196} = (1, 11, 1, 0)$
26 : $P_{200} = (5, 11, 1, 0)$
27 : $P_{210} = (15, 11, 1, 0)$
28 : $P_{212} = (1, 12, 1, 0)$
29 : $P_{228} = (1, 13, 1, 0)$
30 : $P_{244} = (1, 14, 1, 0)$
31 : $P_{260} = (1, 15, 1, 0)$
32 : $P_{261} = (2, 15, 1, 0)$
33 : $P_{271} = (12, 15, 1, 0)$
34 : $P_{275} = (1, 0, 0, 1)$
35 : $P_{291} = (1, 1, 0, 1)$
36 : $P_{316} = (10, 2, 0, 1)$
37 : $P_{329} = (7, 3, 0, 1)$
38 : $P_{330} = (8, 3, 0, 1)$
39 : $P_{334} = (12, 3, 0, 1)$
40 : $P_{349} = (11, 4, 0, 1)$
41 : $P_{360} = (6, 5, 0, 1)$
42 : $P_{366} = (12, 5, 0, 1)$
43 : $P_{369} = (15, 5, 0, 1)$
44 : $P_{385} = (15, 6, 0, 1)$
45 : $P_{391} = (5, 7, 0, 1)$
46 : $P_{405} = (3, 8, 0, 1)$
47 : $P_{408} = (6, 8, 0, 1)$
48 : $P_{415} = (13, 8, 0, 1)$
49 : $P_{428} = (10, 9, 0, 1)$
50 : $P_{474} = (8, 12, 0, 1)$
51 : $P_{485} = (3, 13, 0, 1)$
52 : $P_{509} = (11, 14, 0, 1)$
53 : $P_{519} = (5, 15, 0, 1)$
54 : $P_{521} = (7, 15, 0, 1)$
55 : $P_{527} = (13, 15, 0, 1)$
56 : $P_{546} = (0, 1, 1, 1)$
57 : $P_{566} = (5, 2, 1, 1)$
58 : $P_{585} = (8, 3, 1, 1)$
59 : $P_{601} = (8, 4, 1, 1)$
60 : $P_{624} = (15, 5, 1, 1)$
61 : $P_{636} = (11, 6, 1, 1)$
62 : $P_{652} = (11, 7, 1, 1)$
63 : $P_{660} = (3, 8, 1, 1)$
64 : $P_{688} = (15, 9, 1, 1)$
65 : $P_{731} = (10, 12, 1, 1)$
66 : $P_{747} = (10, 13, 1, 1)$
67 : $P_{756} = (3, 14, 1, 1)$
68 : $P_{774} = (5, 15, 1, 1)$
69 : $P_{792} = (7, 0, 2, 1)$
70 : $P_{824} = (7, 2, 2, 1)$
71 : $P_{865} = (0, 5, 2, 1)$
72 : $P_{887} = (6, 6, 2, 1)$
73 : $P_{903} = (6, 7, 2, 1)$
74 : $P_{943} = (14, 9, 2, 1)$
75 : $P_{953} = (8, 10, 2, 1)$
76 : $P_{954} = (9, 10, 2, 1)$
77 : $P_{956} = (11, 10, 2, 1)$
78 : $P_{974} = (13, 11, 2, 1)$
79 : $P_{985} = (8, 12, 2, 1)$
80 : $P_{1011} = (2, 14, 2, 1)$
81 : $P_{1023} = (14, 14, 2, 1)$
82 : $P_{1034} = (9, 15, 2, 1)$
83 : $P_{1036} = (11, 15, 2, 1)$
84 : $P_{1038} = (13, 15, 2, 1)$
85 : $P_{1053} = (12, 0, 3, 1)$
86 : $P_{1061} = (4, 1, 3, 1)$
87 : $P_{1082} = (9, 2, 3, 1)$
88 : $P_{1095} = (6, 3, 3, 1)$
89 : $P_{1109} = (4, 4, 3, 1)$
90 : $P_{1111} = (6, 4, 3, 1)$
91 : $P_{1145} = (8, 6, 3, 1)$
92 : $P_{1169} = (0, 8, 3, 1)$
93 : $P_{1177} = (8, 8, 3, 1)$
94 : $P_{1187} = (2, 9, 3, 1)$
95 : $P_{1219} = (2, 11, 3, 1)$
96 : $P_{1222} = (5, 11, 3, 1)$
97 : $P_{1229} = (12, 11, 3, 1)$
98 : $P_{1238} = (5, 12, 3, 1)$
99 : $P_{1274} = (9, 14, 3, 1)$
100 : $P_{1284} = (3, 15, 3, 1)$
101 : $P_{1309} = (12, 0, 4, 1)$
102 : $P_{1331} = (2, 2, 4, 1)$
103 : $P_{1333} = (4, 2, 4, 1)$
104 : $P_{1352} = (7, 3, 4, 1)$
105 : $P_{1355} = (10, 3, 4, 1)$
106 : $P_{1359} = (14, 3, 4, 1)$
107 : $P_{1373} = (12, 4, 4, 1)$
108 : $P_{1408} = (15, 6, 4, 1)$
109 : $P_{1425} = (0, 8, 4, 1)$
110 : $P_{1464} = (7, 10, 4, 1)$
111 : $P_{1483} = (10, 11, 4, 1)$
112 : $P_{1487} = (14, 11, 4, 1)$
113 : $P_{1488} = (15, 11, 4, 1)$
114 : $P_{1502} = (13, 12, 4, 1)$
115 : $P_{1518} = (13, 13, 4, 1)$
116 : $P_{1523} = (2, 14, 4, 1)$
117 : $P_{1559} = (6, 0, 5, 1)$
118 : $P_{1578} = (9, 1, 5, 1)$
119 : $P_{1599} = (14, 2, 5, 1)$
120 : $P_{1606} = (5, 3, 5, 1)$
121 : $P_{1631} = (14, 4, 5, 1)$
122 : $P_{1646} = (13, 5, 5, 1)$
123 : $P_{1657} = (8, 6, 5, 1)$
124 : $P_{1706} = (9, 9, 5, 1)$
125 : $P_{1710} = (13, 9, 5, 1)$
126 : $P_{1717} = (4, 10, 5, 1)$
127 : $P_{1719} = (6, 10, 5, 1)$
128 : $P_{1721} = (8, 10, 5, 1)$
129 : $P_{1776} = (15, 13, 5, 1)$
130 : $P_{1781} = (4, 14, 5, 1)$
131 : $P_{1793} = (0, 15, 5, 1)$
132 : $P_{1808} = (15, 15, 5, 1)$
133 : $P_{1822} = (13, 0, 6, 1)$
134 : $P_{1846} = (5, 2, 6, 1)$
135 : $P_{1851} = (10, 2, 6, 1)$
136 : $P_{1854} = (13, 2, 6, 1)$
137 : $P_{1857} = (0, 3, 6, 1)$
138 : $P_{1869} = (12, 3, 6, 1)$
139 : $P_{1872} = (15, 3, 6, 1)$
140 : $P_{1892} = (3, 5, 6, 1)$
141 : $P_{1907} = (2, 6, 6, 1)$
142 : $P_{1913} = (8, 6, 6, 1)$
143 : $P_{1917} = (12, 6, 6, 1)$
144 : $P_{1931} = (10, 7, 6, 1)$
145 : $P_{1938} = (1, 8, 6, 1)$
146 : $P_{1968} = (15, 9, 6, 1)$
147 : $P_{1972} = (3, 10, 6, 1)$
148 : $P_{1993} = (8, 11, 6, 1)$
149 : $P_{2007} = (6, 12, 6, 1)$
150 : $P_{2022} = (5, 13, 6, 1)$
151 : $P_{2035} = (2, 14, 6, 1)$
152 : $P_{2050} = (1, 15, 6, 1)$
153 : $P_{2077} = (12, 0, 7, 1)$
154 : $P_{2102} = (5, 2, 7, 1)$
155 : $P_{2114} = (1, 3, 7, 1)$
156 : $P_{2138} = (9, 4, 7, 1)$
157 : $P_{2146} = (1, 5, 7, 1)$
158 : $P_{2171} = (10, 6, 7, 1)$
159 : $P_{2180} = (3, 7, 7, 1)$
160 : $P_{2186} = (9, 7, 7, 1)$
161 : $P_{2190} = (13, 7, 7, 1)$
162 : $P_{2193} = (0, 8, 7, 1)$
163 : $P_{2198} = (5, 8, 7, 1)$
164 : $P_{2206} = (13, 8, 7, 1)$
165 : $P_{2219} = (10, 9, 7, 1)$
166 : $P_{2221} = (12, 9, 7, 1)$
167 : $P_{2224} = (15, 9, 7, 1)$
168 : $P_{2233} = (8, 10, 7, 1)$
169 : $P_{2244} = (3, 11, 7, 1)$
170 : $P_{2272} = (15, 12, 7, 1)$
171 : $P_{2280} = (7, 13, 7, 1)$
172 : $P_{2313} = (8, 15, 7, 1)$
173 : $P_{2334} = (13, 0, 8, 1)$
174 : $P_{2351} = (14, 1, 8, 1)$
175 : $P_{2362} = (9, 2, 8, 1)$
176 : $P_{2369} = (0, 3, 8, 1)$
177 : $P_{2372} = (3, 3, 8, 1)$
178 : $P_{2387} = (2, 4, 8, 1)$
179 : $P_{2409} = (8, 5, 8, 1)$
180 : $P_{2436} = (3, 7, 8, 1)$
181 : $P_{2456} = (7, 8, 8, 1)$
182 : $P_{2467} = (2, 9, 8, 1)$

183 : $P_{2506} = (9, 11, 8, 1)$
184 : $P_{2510} = (13, 11, 8, 1)$
185 : $P_{2512} = (15, 11, 8, 1)$
186 : $P_{2544} = (15, 13, 8, 1)$
187 : $P_{2552} = (7, 14, 8, 1)$
188 : $P_{2559} = (14, 14, 8, 1)$
189 : $P_{2583} = (6, 0, 9, 1)$
190 : $P_{2613} = (4, 2, 9, 1)$
191 : $P_{2645} = (4, 4, 9, 1)$
192 : $P_{2650} = (9, 4, 9, 1)$
193 : $P_{2659} = (2, 5, 9, 1)$
194 : $P_{2668} = (11, 5, 9, 1)$
195 : $P_{2669} = (12, 5, 9, 1)$
196 : $P_{2680} = (7, 6, 9, 1)$
197 : $P_{2696} = (7, 7, 9, 1)$
198 : $P_{2727} = (6, 9, 9, 1)$
199 : $P_{2739} = (2, 10, 9, 1)$
200 : $P_{2740} = (3, 10, 9, 1)$
201 : $P_{2748} = (11, 10, 9, 1)$
202 : $P_{2765} = (12, 11, 9, 1)$
203 : $P_{2788} = (3, 13, 9, 1)$
204 : $P_{2817} = (0, 15, 9, 1)$
205 : $P_{2849} = (0, 1, 10, 1)$
206 : $P_{2855} = (6, 1, 10, 1)$
207 : $P_{2856} = (7, 1, 10, 1)$
208 : $P_{2871} = (6, 2, 10, 1)$
209 : $P_{2876} = (11, 2, 10, 1)$
210 : $P_{2880} = (15, 2, 10, 1)$
211 : $P_{2882} = (1, 3, 10, 1)$
212 : $P_{2894} = (13, 3, 10, 1)$
213 : $P_{2896} = (15, 3, 10, 1)$
214 : $P_{2901} = (4, 4, 10, 1)$
215 : $P_{2911} = (14, 4, 10, 1)$
216 : $P_{2962} = (1, 8, 10, 1)$
217 : $P_{2966} = (5, 8, 10, 1)$
218 : $P_{2973} = (12, 8, 10, 1)$
219 : $P_{2982} = (5, 9, 10, 1)$
220 : $P_{2984} = (7, 9, 10, 1)$
221 : $P_{2988} = (11, 9, 10, 1)$
222 : $P_{3019} = (10, 11, 10, 1)$
223 : $P_{3021} = (12, 11, 10, 1)$
224 : $P_{3022} = (13, 11, 10, 1)$
225 : $P_{3061} = (4, 14, 10, 1)$
226 : $P_{3071} = (14, 14, 10, 1)$
227 : $P_{3105} = (0, 1, 11, 1)$
228 : $P_{3117} = (12, 1, 11, 1)$
229 : $P_{3118} = (13, 1, 11, 1)$
230 : $P_{3123} = (2, 2, 11, 1)$
231 : $P_{3130} = (9, 2, 11, 1)$
232 : $P_{3156} = (3, 4, 11, 1)$
233 : $P_{3163} = (10, 4, 11, 1)$
234 : $P_{3166} = (13, 4, 11, 1)$
235 : $P_{3170} = (1, 5, 11, 1)$
236 : $P_{3172} = (3, 5, 11, 1)$
237 : $P_{3176} = (7, 5, 11, 1)$
238 : $P_{3235} = (2, 9, 11, 1)$
239 : $P_{3242} = (9, 9, 11, 1)$
240 : $P_{3255} = (6, 10, 11, 1)$
241 : $P_{3256} = (7, 10, 11, 1)$
242 : $P_{3260} = (11, 10, 11, 1)$
243 : $P_{3321} = (8, 14, 11, 1)$
244 : $P_{3323} = (10, 14, 11, 1)$
245 : $P_{3325} = (12, 14, 11, 1)$
246 : $P_{3330} = (1, 15, 11, 1)$
247 : $P_{3335} = (6, 15, 11, 1)$
248 : $P_{3337} = (8, 15, 11, 1)$
249 : $P_{3351} = (6, 0, 12, 1)$
250 : $P_{3408} = (15, 3, 12, 1)$
251 : $P_{3417} = (8, 4, 12, 1)$
252 : $P_{3426} = (1, 5, 12, 1)$
253 : $P_{3444} = (3, 6, 12, 1)$
254 : $P_{3469} = (12, 7, 12, 1)$
255 : $P_{3474} = (1, 8, 12, 1)$
256 : $P_{3503} = (14, 9, 12, 1)$
257 : $P_{3510} = (5, 10, 12, 1)$
258 : $P_{3536} = (15, 11, 12, 1)$
259 : $P_{3542} = (5, 12, 12, 1)$
260 : $P_{3544} = (7, 12, 12, 1)$
261 : $P_{3551} = (14, 12, 12, 1)$
262 : $P_{3564} = (11, 13, 12, 1)$
263 : $P_{3572} = (3, 14, 12, 1)$
264 : $P_{3575} = (6, 14, 12, 1)$
265 : $P_{3580} = (11, 14, 12, 1)$
266 : $P_{3585} = (0, 15, 12, 1)$
267 : $P_{3592} = (7, 15, 12, 1)$
268 : $P_{3593} = (8, 15, 12, 1)$
269 : $P_{3608} = (7, 0, 13, 1)$
270 : $P_{3637} = (4, 2, 13, 1)$
271 : $P_{3650} = (1, 3, 13, 1)$
272 : $P_{3672} = (7, 4, 13, 1)$
273 : $P_{3673} = (8, 4, 13, 1)$
274 : $P_{3676} = (11, 4, 13, 1)$
275 : $P_{3681} = (0, 5, 13, 1)$
276 : $P_{3684} = (3, 5, 13, 1)$
277 : $P_{3687} = (6, 5, 13, 1)$
278 : $P_{3710} = (13, 6, 13, 1)$
279 : $P_{3721} = (8, 7, 13, 1)$
280 : $P_{3734} = (5, 8, 13, 1)$
281 : $P_{3776} = (15, 10, 13, 1)$
282 : $P_{3782} = (5, 11, 13, 1)$
283 : $P_{3804} = (11, 12, 13, 1)$
284 : $P_{3813} = (4, 13, 13, 1)$
285 : $P_{3815} = (6, 13, 13, 1)$
286 : $P_{3824} = (15, 13, 13, 1)$
287 : $P_{3828} = (3, 14, 13, 1)$
288 : $P_{3842} = (1, 15, 13, 1)$
289 : $P_{3870} = (13, 0, 14, 1)$
290 : $P_{3905} = (0, 3, 14, 1)$
291 : $P_{3930} = (9, 4, 14, 1)$
292 : $P_{3974} = (5, 7, 14, 1)$
293 : $P_{3989} = (4, 8, 14, 1)$
294 : $P_{3991} = (6, 8, 14, 1)$
295 : $P_{3995} = (10, 8, 14, 1)$
296 : $P_{4010} = (9, 9, 14, 1)$
297 : $P_{4015} = (14, 9, 14, 1)$
298 : $P_{4023} = (6, 10, 14, 1)$
299 : $P_{4037} = (4, 11, 14, 1)$
300 : $P_{4038} = (5, 11, 14, 1)$
301 : $P_{4043} = (10, 11, 14, 1)$
302 : $P_{4061} = (12, 12, 14, 1)$
303 : $P_{4077} = (12, 13, 14, 1)$
304 : $P_{4094} = (13, 14, 14, 1)$
305 : $P_{4120} = (7, 0, 15, 1)$
306 : $P_{4131} = (2, 1, 15, 1)$
307 : $P_{4147} = (2, 2, 15, 1)$
308 : $P_{4157} = (12, 2, 15, 1)$
309 : $P_{4191} = (14, 4, 15, 1)$
310 : $P_{4193} = (0, 5, 15, 1)$
311 : $P_{4198} = (5, 5, 15, 1)$
312 : $P_{4228} = (3, 7, 15, 1)$
313 : $P_{4256} = (15, 8, 15, 1)$
314 : $P_{4261} = (4, 9, 15, 1)$
315 : $P_{4276} = (3, 10, 15, 1)$
316 : $P_{4280} = (7, 10, 15, 1)$
317 : $P_{4287} = (14, 10, 15, 1)$
318 : $P_{4310} = (5, 12, 15, 1)$
319 : $P_{4341} = (4, 14, 15, 1)$
320 : $P_{4365} = (12, 15, 15, 1)$