Rank-65743 over GF(8)

January 15, 2021

The equation

The equation of the surface is:

$$X_2^3 + X_3^3 + X_0^2 X_3 + X_0 X_1^2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(8)$ is 1229493389

General information

Number of lines	1
Number of points	65
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	9
Number of points off lines	56
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9
Type of lines on points	$1^9, 0^{56}$

Singular Points

The surface has 1 singular points:

$$0: P_{75} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$$

The 1 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{769}$$

Rank of lines: (4673)

Rank of points on Klein quadric: (769)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points: The double points on the surface are:

Single Points

The surface has 9 single points: The single points on the surface are:

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\begin{array}{lll} 0: \ P_1 = (0,1,0,0) \ \mbox{lies on line} \ \ell_0 \\ 1: \ P_{138} = (0,0,1,1) \ \mbox{lies on line} \ \ell_0 \\ 2: \ P_{146} = (0,1,1,1) \ \mbox{lies on line} \ \ell_0 \\ 3: \ P_{153} = (0,2,1,1) \ \mbox{lies on line} \ \ell_0 \\ 4: \ P_{161} = (0,3,1,1) \ \mbox{lies on line} \ \ell_0 \end{array}
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The single points on the surface are:

Points on surface but on no line

The surface has 56 points not on any line: The points on the surface but not on lines are:

$0: P_0 = (1, 0, 0, 0)$	18: $P_{190} = (5, 6, 1, 1)$
$1: P_{29} = (2, 2, 1, 0)$	19: $P_{198} = (5, 7, 1, 1)$
$2: P_{37} = (2,3,1,0)$	$20: P_{203} = (2,0,2,1)$
$3: P_{47} = (4, 4, 1, 0)$	$21: P_{214} = (5, 1, 2, 1)$
$4: P_{55} = (4, 5, 1, 0)$	$22: P_{215} = (6, 1, 2, 1)$
$5: P_{66} = (7, 6, 1, 0)$	$23: P_{219} = (2, 2, 2, 1)$
$6: P_{74} = (7, 7, 1, 0)$	$24: P_{230} = (5, 3, 2, 1)$
$7: P_{75} = (1,0,0,1)$	$25: P_{231} = (6, 3, 2, 1)$
$8: P_{92} = (2, 2, 0, 1)$	$26: P_{271} = (6, 0, 3, 1)$
$9: P_{96} = (6, 2, 0, 1)$	$27: P_{274} = (1, 1, 3, 1)$
$10: P_{109} = (3, 4, 0, 1)$	$28: P_{276} = (3, 1, 3, 1)$
$11: P_{110} = (4, 4, 0, 1)$	$29: P_{282} = (1, 2, 3, 1)$
$12: P_{135} = (5, 7, 0, 1)$	$30: P_{284} = (3, 2, 3, 1)$
13: $P_{137} = (7, 7, 0, 1)$	$31: P_{295} = (6, 3, 3, 1)$
$14: P_{159} = (6, 2, 1, 1)$	$32: P_{333} = (4,0,4,1)$
$15: P_{167} = (6, 3, 1, 1)$	$33: P_{340} = (3, 1, 4, 1)$
$16: P_{172} = (3,4,1,1)$	$34: P_{343} = (6, 1, 4, 1)$
$17: P_{180} = (3, 5, 1, 1)$	$35: P_{365} = (4, 4, 4, 1)$

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36: P_{372} = (3, 5, 4, 1)
                                                                  47: P_{510} = (5, 6, 6, 1)
37: P_{375} = (6, 5, 4, 1)
                                                                  48: P_{514} = (1, 7, 6, 1)
38: P_{396} = (3, 0, 5, 1)
                                                                  49: P_{519} = (6,7,6,1)
39: P_{402} = (1, 1, 5, 1)
                                                                  50: P_{528} = (7, 0, 7, 1)
40: P_{406} = (5, 1, 5, 1)
                                                                  51: P_{532} = (3, 1, 7, 1)
41: P_{426} = (1, 4, 5, 1)
                                                                  52: P_{534} = (5, 1, 7, 1)
42: P_{430} = (5, 4, 5, 1)
                                                                  53: P_{572} = (3, 6, 7, 1)
43: P_{436} = (3, 5, 5, 1)
                                                                  54: P_{574} = (5, 6, 7, 1)
44: P_{462} = (5, 0, 6, 1)
                                                                  55: P_{584} = (7,7,7,1)
45: P_{466} = (1, 1, 6, 1)
46: P_{471} = (6, 1, 6, 1)
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Line Intersection Graph

 $\begin{array}{c|c} 0 \\ \hline 0 & 0 \end{array}$

Neighbor sets in the line intersection graph: Line 0 intersects

Line in point

The surface has 65 points:

The points on the surface are:

$0: P_0 = (1,0,0,0)$	$22: P_{172} = (3, 4, 1, 1)$	$44: P_{365} = (4, 4, 4, 1)$
$1: P_1 = (0, 1, 0, 0)$	$23: P_{177} = (0, 5, 1, 1)$	$45: P_{372} = (3, 5, 4, 1)$
$2: P_{29} = (2, 2, 1, 0)$	$24: P_{180} = (3, 5, 1, 1)$	$46: P_{375} = (6, 5, 4, 1)$
$3: P_{37} = (2, 3, 1, 0)$	$25: P_{185} = (0, 6, 1, 1)$	$47: P_{396} = (3, 0, 5, 1)$
$4: P_{47} = (4, 4, 1, 0)$		
	$26: P_{190} = (5, 6, 1, 1)$	$48: P_{402} = (1, 1, 5, 1)$
$5: P_{55} = (4, 5, 1, 0)$	$27: P_{193} = (0, 7, 1, 1)$	$49: P_{406} = (5, 1, 5, 1)$
$6: P_{66} = (7, 6, 1, 0)$	$28: P_{198} = (5, 7, 1, 1)$	$50: P_{426} = (1,4,5,1)$
$7: P_{74} = (7,7,1,0)$	$29: P_{203} = (2,0,2,1)$	$51: P_{430} = (5, 4, 5, 1)$
$8: P_{75} = (1,0,0,1)$	$30: P_{214} = (5, 1, 2, 1)$	$52: P_{436} = (3, 5, 5, 1)$
$9: P_{92} = (2, 2, 0, 1)$	$31: P_{215} = (6, 1, 2, 1)$	$53: P_{462} = (5, 0, 6, 1)$
$10: P_{96} = (6, 2, 0, 1)$	$32: P_{219} = (2, 2, 2, 1)$	$54: P_{466} = (1, 1, 6, 1)$
11: $P_{109} = (3, 4, 0, 1)$	$33: P_{230} = (5, 3, 2, 1)$	$55: P_{471} = (6, 1, 6, 1)$
$12: P_{110} = (4, 4, 0, 1)$	$34: P_{231} = (6, 3, 2, 1)$	$56: P_{510} = (5, 6, 6, 1)$
13: $P_{135} = (5, 7, 0, 1)$	$35: P_{271} = (6,0,3,1)$	$57: P_{514} = (1, 7, 6, 1)$
$14: P_{137} = (7, 7, 0, 1)$	$36: P_{274} = (1, 1, 3, 1)$	$58: P_{519} = (6, 7, 6, 1)$
15: $P_{138} = (0, 0, 1, 1)$	$37: P_{276} = (3, 1, 3, 1)$	$59: P_{528} = (7, 0, 7, 1)$
16: $P_{146} = (0, 1, 1, 1)$	$38: P_{282} = (1, 2, 3, 1)$	$60: P_{532} = (3, 1, 7, 1)$
17: $P_{153} = (0, 2, 1, 1)$	$39: P_{284} = (3, 2, 3, 1)$	$61: P_{534} = (5, 1, 7, 1)$
$18: P_{159} = (6, 2, 1, 1)$	$40: P_{295} = (6, 3, 3, 1)$	$62: P_{572} = (3, 6, 7, 1)$
19: $P_{161} = (0, 3, 1, 1)$	$41: P_{333} = (4,0,4,1)$	$63: P_{574} = (5, 6, 7, 1)$
$20: P_{167} = (6, 3, 1, 1)$	$42: P_{340} = (3, 1, 4, 1)$	$64: P_{584} = (7, 7, 7, 1)$
,	: : : : : : : : : : : : : : : : : : :	01.1084 - (1,1,1,1)
$21: P_{169} = (0, 4, 1, 1)$	$43: P_{343} = (6, 1, 4, 1)$	