Rank-346 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 = 0$$

(1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 303112750

General information

Number of lines	9
Number of points	321
Number of singular points	0
Number of Eckardt points	6
Number of double points	0
Number of single points	135
Number of points off lines	180
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^9
Type of lines on points	3^6, 1^{135}, 0^{180}

Singular Points

The surface has 0 singular points:

The 9 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0,0,1,1,1,1)_{9426}$$

$$\ell_1 = \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3269} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3269} = \mathbf{Pl}(0,0,11,10,11,1)_{50536}$$

$$\ell_2 = \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{2997} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{2997} = \mathbf{Pl}(0,0,10,11,10,1)_{46425}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(1,1,1,0,1,1)_{8976}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & \delta^5 & 0 \\ 0 & 1 & 1 & \delta^{10} \end{bmatrix}_{3164} = \begin{bmatrix} 1 & 0 & 11 & 0 \\ 0 & 1 & 1 & 10 \end{bmatrix}_{3164} = \mathbf{Pl}(11,10,11,0,11,1)_{49936}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & \delta^{10} & 0 \\ 0 & 1 & 1 & \delta^5 \end{bmatrix}_{2907} = \begin{bmatrix} 1 & 0 & 10 & 0 \\ 0 & 1 & 1 & 11 \end{bmatrix}_{2907} = \mathbf{Pl}(10,11,10,0,10,1)_{45840}$$

$$\ell_6 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \mathbf{Pl}(1,1,0,1,1,1)_{9201}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & \delta^5 & 1 \\ 0 & 1 & 0 & \delta^{10} \end{bmatrix}_{7531} = \begin{bmatrix} 1 & 0 & 11 & 1 \\ 0 & 1 & 0 & 10 \end{bmatrix}_{7531} = \mathbf{Pl}(11,10,0,10,11,1)_{50146}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & \delta^{10} & 1 \\ 0 & 1 & 0 & \delta^5 \end{bmatrix}_{7274} = \begin{bmatrix} 1 & 0 & 10 & 1 \\ 0 & 1 & 0 & 11 \end{bmatrix}_{7274} = \mathbf{Pl}(10,11,0,11,10,1)_{46080}$$

Rank of lines: (530, 3269, 2997, 290, 3164, 2907, 4657, 7531, 7274)
Rank of points on Klein quadric: (9426, 50536, 46425, 8976, 49936, 45840, 9201, 50146, 46080)

Eckardt Points

The surface has 6 Eckardt points:
$0 : P_{699} = \mathbf{P}(\delta^{10}, \delta^{10}, 1, 1) = \mathbf{P}(10, 10, 1, 1)$,
$1 : P_{716} = \mathbf{P}(\delta^5, \delta^5, 1, 1) = \mathbf{P}(11, 11, 1, 1)$,
$2 : P_{2860} = \mathbf{P}(\delta^5, 1, \delta^{10}, 1) = \mathbf{P}(11, 1, 10, 1)$,
$3 : P_{3019} = \mathbf{P}(\delta^{10}, \delta^5, \delta^{10}, 1) = \mathbf{P}(10, 11, 10, 1)$,
$4 : P_{3115} = \mathbf{P}(\delta^{10}, 1, \delta^5, 1) = \mathbf{P}(10, 1, 11, 1)$,
$5 : P_{3260} = \mathbf{P}(\delta^5, \delta^{10}, \delta^5, 1) = \mathbf{P}(11, 10, 11, 1)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 135 single points:
The single points on the surface are:

$0 :\ P_4 = (1,1,1,1)$ lies on line ℓ_0
$1 :\ P_5 = (1,1,0,0)$ lies on line ℓ_0
$2 :\ P_{14} = (10,1,0,0)$ lies on line ℓ_1
$3 :\ P_{15} = (11,1,0,0)$ lies on line ℓ_2
$4 :\ P_{20} = (1,0,1,0)$ lies on line ℓ_3
$5 :\ P_{29} = (10,0,1,0)$ lies on line ℓ_4
$6 :\ P_{30} = (11,0,1,0)$ lies on line ℓ_5
$7 :\ P_{36} = (1,1,1,0)$ lies on line ℓ_6
$8 :\ P_{45} = (10,1,1,0)$ lies on line ℓ_7
$9 :\ P_{46} = (11,1,1,0)$ lies on line ℓ_8
$10 :\ P_{290} = (0,1,0,1)$ lies on line ℓ_6
$11 :\ P_{291} = (1,1,0,1)$ lies on line ℓ_3
$12 :\ P_{434} = (0,10,0,1)$ lies on line ℓ_8
$13 :\ P_{435} = (1,10,0,1)$ lies on line ℓ_5
$14 :\ P_{450} = (0,11,0,1)$ lies on line ℓ_7
$15 :\ P_{451} = (1,11,0,1)$ lies on line ℓ_4

16 : $P_{530} = (0, 0, 1, 1)$ lies on line ℓ_0
17 : $P_{531} = (1, 0, 1, 1)$ lies on line ℓ_6
18 : $P_{546} = (0, 1, 1, 1)$ lies on line ℓ_3
19 : $P_{563} = (2, 2, 1, 1)$ lies on line ℓ_0
20 : $P_{580} = (3, 3, 1, 1)$ lies on line ℓ_0
21 : $P_{597} = (4, 4, 1, 1)$ lies on line ℓ_0
22 : $P_{614} = (5, 5, 1, 1)$ lies on line ℓ_0
23 : $P_{631} = (6, 6, 1, 1)$ lies on line ℓ_0
24 : $P_{648} = (7, 7, 1, 1)$ lies on line ℓ_0
25 : $P_{665} = (8, 8, 1, 1)$ lies on line ℓ_0
26 : $P_{682} = (9, 9, 1, 1)$ lies on line ℓ_0
27 : $P_{733} = (12, 12, 1, 1)$ lies on line ℓ_0
28 : $P_{750} = (13, 13, 1, 1)$ lies on line ℓ_0
29 : $P_{767} = (14, 14, 1, 1)$ lies on line ℓ_0
30 : $P_{784} = (15, 15, 1, 1)$ lies on line ℓ_0
31 : $P_{804} = (3, 1, 2, 1)$ lies on line ℓ_3
32 : $P_{835} = (2, 3, 2, 1)$ lies on line ℓ_6
33 : $P_{928} = (15, 8, 2, 1)$ lies on line ℓ_8
34 : $P_{942} = (13, 9, 2, 1)$ lies on line ℓ_7
35 : $P_{959} = (14, 10, 2, 1)$ lies on line ℓ_5
36 : $P_{973} = (12, 11, 2, 1)$ lies on line ℓ_4
37 : $P_{1059} = (2, 1, 3, 1)$ lies on line ℓ_3
38 : $P_{1076} = (3, 2, 3, 1)$ lies on line ℓ_6
39 : $P_{1176} = (7, 8, 3, 1)$ lies on line ℓ_7
40 : $P_{1189} = (4, 9, 3, 1)$ lies on line ℓ_8
41 : $P_{1206} = (5, 10, 3, 1)$ lies on line ℓ_5
42 : $P_{1223} = (6, 11, 3, 1)$ lies on line ℓ_4
43 : $P_{1318} = (5, 1, 4, 1)$ lies on line ℓ_3
44 : $P_{1381} = (4, 5, 4, 1)$ lies on line ℓ_6
45 : $P_{1463} = (6, 10, 4, 1)$ lies on line ℓ_5
46 : $P_{1475} = (2, 11, 4, 1)$ lies on line ℓ_4
47 : $P_{1528} = (7, 14, 4, 1)$ lies on line ℓ_8
48 : $P_{1540} = (3, 15, 4, 1)$ lies on line ℓ_7
49 : $P_{1573} = (4, 1, 5, 1)$ lies on line ℓ_3
50 : $P_{1622} = (5, 4, 5, 1)$ lies on line ℓ_6
51 : $P_{1726} = (13, 10, 5, 1)$ lies on line ℓ_5
52 : $P_{1737} = (8, 11, 5, 1)$ lies on line ℓ_4
53 : $P_{1786} = (9, 14, 5, 1)$ lies on line ℓ_7
54 : $P_{1805} = (12, 15, 5, 1)$ lies on line ℓ_8
55 : $P_{1832} = (7, 1, 6, 1)$ lies on line ℓ_3
56 : $P_{1927} = (6, 7, 6, 1)$ lies on line ℓ_6
57 : $P_{1978} = (9, 10, 6, 1)$ lies on line ℓ_5
58 : $P_{2000} = (15, 11, 6, 1)$ lies on line ℓ_4
59 : $P_{2009} = (8, 12, 6, 1)$ lies on line ℓ_8
60 : $P_{2031} = (14, 13, 6, 1)$ lies on line ℓ_7
61 : $P_{2087} = (6, 1, 7, 1)$ lies on line ℓ_3
62 : $P_{2168} = (7, 6, 7, 1)$ lies on line ℓ_6
63 : $P_{2227} = (2, 10, 7, 1)$ lies on line ℓ_5
64 : $P_{2246} = (5, 11, 7, 1)$ lies on line ℓ_4
65 : $P_{2261} = (4, 12, 7, 1)$ lies on line ℓ_7
66 : $P_{2276} = (3, 13, 7, 1)$ lies on line ℓ_8
67 : $P_{2346} = (9, 1, 8, 1)$ lies on line ℓ_3
68 : $P_{2367} = (14, 2, 8, 1)$ lies on line ℓ_8
69 : $P_{2375} = (6, 3, 8, 1)$ lies on line ℓ_7
70 : $P_{2473} = (8, 9, 8, 1)$ lies on line ℓ_6
71 : $P_{2496} = (15, 10, 8, 1)$ lies on line ℓ_5
72 : $P_{2504} = (7, 11, 8, 1)$ lies on line ℓ_4
73 : $P_{2601} = (8, 1, 9, 1)$ lies on line ℓ_3
74 : $P_{2621} = (12, 2, 9, 1)$ lies on line ℓ_7
75 : $P_{2630} = (5, 3, 9, 1)$ lies on line ℓ_8
76 : $P_{2714} = (9, 8, 9, 1)$ lies on line ℓ_6
77 : $P_{2741} = (4, 10, 9, 1)$ lies on line ℓ_5
78 : $P_{2766} = (13, 11, 9, 1)$ lies on line ℓ_4
79 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_2
80 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_8
81 : $P_{2880} = (15, 2, 10, 1)$ lies on line ℓ_2
82 : $P_{2885} = (4, 3, 10, 1)$ lies on line ℓ_2
83 : $P_{2904} = (7, 4, 10, 1)$ lies on line ℓ_2
84 : $P_{2925} = (12, 5, 10, 1)$ lies on line ℓ_2
85 : $P_{2937} = (8, 6, 10, 1)$ lies on line ℓ_2
86 : $P_{2948} = (3, 7, 10, 1)$ lies on line ℓ_2
87 : $P_{2975} = (14, 8, 10, 1)$ lies on line ℓ_2
88 : $P_{2982} = (5, 9, 10, 1)$ lies on line ℓ_2
89 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_5
90 : $P_{2994} = (1, 10, 10, 1)$ lies on line ℓ_2
91 : $P_{3034} = (9, 12, 10, 1)$ lies on line ℓ_2
92 : $P_{3043} = (2, 13, 10, 1)$ lies on line ℓ_2
93 : $P_{3063} = (6, 14, 10, 1)$ lies on line ℓ_2
94 : $P_{3086} = (13, 15, 10, 1)$ lies on line ℓ_2
95 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_1
96 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_7
97 : $P_{3134} = (13, 2, 11, 1)$ lies on line ℓ_1
98 : $P_{3144} = (7, 3, 11, 1)$ lies on line ℓ_1
99 : $P_{3156} = (3, 4, 11, 1)$ lies on line ℓ_1
100 : $P_{3178} = (9, 5, 11, 1)$ lies on line ℓ_1
101 : $P_{3199} = (14, 6, 11, 1)$ lies on line ℓ_1
102 : $P_{3205} = (4, 7, 11, 1)$ lies on line ℓ_1
103 : $P_{3223} = (6, 8, 11, 1)$ lies on line ℓ_1
104 : $P_{3245} = (12, 9, 11, 1)$ lies on line ℓ_1
105 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_4
106 : $P_{3266} = (1, 11, 11, 1)$ lies on line ℓ_1
107 : $P_{3286} = (5, 12, 11, 1)$ lies on line ℓ_1
108 : $P_{3312} = (15, 13, 11, 1)$ lies on line ℓ_1
109 : $P_{3321} = (8, 14, 11, 1)$ lies on line ℓ_1
110 : $P_{3331} = (2, 15, 11, 1)$ lies on line ℓ_1
111 : $P_{3374} = (13, 1, 12, 1)$ lies on line ℓ_3
112 : $P_{3450} = (9, 6, 12, 1)$ lies on line ℓ_8
113 : $P_{3462} = (5, 7, 12, 1)$ lies on line ℓ_7
114 : $P_{3513} = (8, 10, 12, 1)$ lies on line ℓ_5
115 : $P_{3525} = (4, 11, 12, 1)$ lies on line ℓ_4
116 : $P_{3565} = (12, 13, 12, 1)$ lies on line ℓ_6
117 : $P_{3629} = (12, 1, 13, 1)$ lies on line ℓ_3
118 : $P_{3712} = (15, 6, 13, 1)$ lies on line ℓ_7
119 : $P_{3715} = (2, 7, 13, 1)$ lies on line ℓ_8
120 : $P_{3764} = (3, 10, 13, 1)$ lies on line ℓ_5
121 : $P_{3791} = (14, 11, 13, 1)$ lies on line ℓ_4
122 : $P_{3806} = (13, 12, 13, 1)$ lies on line ℓ_6
123 : $P_{3888} = (15, 1, 14, 1)$ lies on line ℓ_3

124 : $P_{3927} = (6, 4, 14, 1)$ lies on line ℓ_8
125 : $P_{3945} = (8, 5, 14, 1)$ lies on line ℓ_7
126 : $P_{4024} = (7, 10, 14, 1)$ lies on line ℓ_5
127 : $P_{4042} = (9, 11, 14, 1)$ lies on line ℓ_4
128 : $P_{4111} = (14, 15, 14, 1)$ lies on line ℓ_6
129 : $P_{4143} = (14, 1, 15, 1)$ lies on line ℓ_3
130 : $P_{4179} = (2, 4, 15, 1)$ lies on line ℓ_7
131 : $P_{4206} = (13, 5, 15, 1)$ lies on line ℓ_8
132 : $P_{4285} = (12, 10, 15, 1)$ lies on line ℓ_5
133 : $P_{4292} = (3, 11, 15, 1)$ lies on line ℓ_4
134 : $P_{4352} = (15, 14, 15, 1)$ lies on line ℓ_6

The single points on the surface are:

Points on surface but on no line

The surface has 180 points not on any line:
The points on the surface but not on lines are:

0 : $P_{585} = (8, 3, 1, 1)$
1 : $P_{587} = (10, 3, 1, 1)$
2 : $P_{620} = (11, 5, 1, 1)$
3 : $P_{624} = (15, 5, 1, 1)$
4 : $P_{660} = (3, 8, 1, 1)$
5 : $P_{667} = (10, 8, 1, 1)$
6 : $P_{692} = (3, 10, 1, 1)$
7 : $P_{697} = (8, 10, 1, 1)$
8 : $P_{710} = (5, 11, 1, 1)$
9 : $P_{720} = (15, 11, 1, 1)$
10 : $P_{774} = (5, 15, 1, 1)$
11 : $P_{780} = (11, 15, 1, 1)$
12 : $P_{809} = (8, 1, 2, 1)$
13 : $P_{811} = (10, 1, 2, 1)$
14 : $P_{858} = (9, 4, 2, 1)$
15 : $P_{872} = (7, 5, 2, 1)$
16 : $P_{894} = (13, 6, 2, 1)$
17 : $P_{911} = (14, 7, 2, 1)$
18 : $P_{918} = (5, 8, 2, 1)$
19 : $P_{924} = (11, 8, 2, 1)$
20 : $P_{981} = (4, 12, 2, 1)$
21 : $P_{1006} = (13, 13, 2, 1)$
22 : $P_{1015} = (6, 14, 2, 1)$
23 : $P_{1039} = (14, 15, 2, 1)$
24 : $P_{1081} = (8, 2, 3, 1)$
25 : $P_{1083} = (10, 2, 3, 1)$
26 : $P_{1112} = (7, 4, 3, 1)$
27 : $P_{1123} = (2, 5, 3, 1)$
28 : $P_{1149} = (12, 6, 3, 1)$
29 : $P_{1155} = (2, 7, 3, 1)$
30 : $P_{1212} = (11, 10, 3, 1)$
31 : $P_{1216} = (15, 10, 3, 1)$
32 : $P_{1247} = (14, 12, 3, 1)$
33 : $P_{1262} = (13, 13, 3, 1)$
34 : $P_{1274} = (9, 14, 3, 1)$
35 : $P_{1288} = (7, 15, 3, 1)$
36 : $P_{1324} = (11, 1, 4, 1)$
37 : $P_{1328} = (15, 1, 4, 1)$
38 : $P_{1342} = (13, 2, 4, 1)$
39 : $P_{1347} = (2, 3, 4, 1)$
40 : $P_{1402} = (9, 6, 4, 1)$
41 : $P_{1416} = (7, 7, 4, 1)$
42 : $P_{1437} = (12, 8, 4, 1)$
43 : $P_{1455} = (14, 9, 4, 1)$
44 : $P_{1491} = (2, 12, 4, 1)$
45 : $P_{1512} = (7, 13, 4, 1)$
46 : $P_{1545} = (8, 15, 4, 1)$
47 : $P_{1547} = (10, 15, 4, 1)$
48 : $P_{1599} = (14, 2, 5, 1)$
49 : $P_{1613} = (12, 3, 5, 1)$
50 : $P_{1628} = (11, 4, 5, 1)$
51 : $P_{1632} = (15, 4, 5, 1)$
52 : $P_{1651} = (2, 6, 5, 1)$
53 : $P_{1672} = (7, 7, 5, 1)$
54 : $P_{1685} = (4, 8, 5, 1)$
55 : $P_{1709} = (12, 9, 5, 1)$
56 : $P_{1732} = (3, 11, 5, 1)$
57 : $P_{1739} = (10, 11, 5, 1)$
58 : $P_{1749} = (4, 12, 5, 1)$
59 : $P_{1767} = (6, 13, 5, 1)$
60 : $P_{1850} = (9, 2, 6, 1)$
61 : $P_{1859} = (2, 3, 6, 1)$
62 : $P_{1886} = (13, 4, 6, 1)$
63 : $P_{1901} = (12, 5, 6, 1)$
64 : $P_{1943} = (6, 8, 6, 1)$
65 : $P_{1959} = (6, 9, 6, 1)$
66 : $P_{1990} = (5, 11, 6, 1)$
67 : $P_{1996} = (11, 11, 6, 1)$
68 : $P_{2004} = (3, 12, 6, 1)$
69 : $P_{2011} = (10, 12, 6, 1)$
70 : $P_{2042} = (9, 14, 6, 1)$
71 : $P_{2053} = (4, 15, 6, 1)$
72 : $P_{2104} = (7, 2, 7, 1)$
73 : $P_{2120} = (7, 3, 7, 1)$
74 : $P_{2131} = (2, 4, 7, 1)$
75 : $P_{2159} = (14, 5, 7, 1)$

$76: P_{2202} = (9, 8, 7, 1)$
$77: P_{2211} = (2, 9, 7, 1)$
$78: P_{2252} = (11, 11, 7, 1)$
$79: P_{2256} = (15, 11, 7, 1)$
$80: P_{2281} = (8, 13, 7, 1)$
$81: P_{2283} = (10, 13, 7, 1)$
$82: P_{2301} = (12, 14, 7, 1)$
$83: P_{2318} = (13, 15, 7, 1)$
$84: P_{2387} = (2, 4, 8, 1)$
$85: P_{2407} = (6, 5, 8, 1)$
$86: P_{2426} = (9, 6, 8, 1)$
$87: P_{2446} = (13, 7, 8, 1)$
$88: P_{2468} = (3, 9, 8, 1)$
$89: P_{2475} = (10, 9, 8, 1)$
$90: P_{2486} = (5, 10, 8, 1)$
$91: P_{2492} = (11, 10, 8, 1)$
$92: P_{2525} = (12, 12, 8, 1)$
$93: P_{2533} = (4, 13, 8, 1)$
$94: P_{2551} = (6, 14, 8, 1)$
$95: P_{2570} = (9, 15, 8, 1)$
$96: P_{2596} = (3, 1, 9, 1)$
$97: P_{2603} = (10, 1, 9, 1)$
$98: P_{2636} = (11, 3, 9, 1)$
$99: P_{2640} = (15, 3, 9, 1)$
$100: P_{2648} = (7, 4, 9, 1)$
$101: P_{2661} = (4, 5, 9, 1)$
$102: P_{2677} = (4, 6, 9, 1)$
$103: P_{2701} = (12, 7, 9, 1)$
$104: P_{2781} = (12, 12, 9, 1)$
$105: P_{2799} = (14, 13, 9, 1)$
$106: P_{2803} = (2, 14, 9, 1)$
$107: P_{2823} = (6, 15, 9, 1)$
$108: P_{2854} = (5, 1, 10, 1)$
$109: P_{2864} = (15, 1, 10, 1)$
$110: P_{2870} = (5, 2, 10, 1)$
$111: P_{2876} = (11, 2, 10, 1)$
$112: P_{2932} = (3, 6, 10, 1)$
$113: P_{2939} = (10, 6, 10, 1)$
$114: P_{2953} = (8, 7, 10, 1)$
$115: P_{2955} = (10, 7, 10, 1)$
$116: P_{2988} = (11, 9, 10, 1)$
$117: P_{2992} = (15, 9, 10, 1)$
$118: P_{3012} = (3, 11, 10, 1)$
$119: P_{3017} = (8, 11, 10, 1)$
$120: P_{3108} = (3, 1, 11, 1)$
$121: P_{3113} = (8, 1, 11, 1)$
$122: P_{3161} = (8, 4, 11, 1)$
$123: P_{3163} = (10, 4, 11, 1)$
$124: P_{3254} = (5, 10, 11, 1)$
$125: P_{3264} = (15, 10, 11, 1)$
$126: P_{3292} = (11, 12, 11, 1)$
$127: P_{3296} = (15, 12, 11, 1)$
$128: P_{3302} = (5, 13, 11, 1)$
$129: P_{3308} = (11, 13, 11, 1)$
$130: P_{3316} = (3, 14, 11, 1)$
$131: P_{3323} = (10, 14, 11, 1)$
$132: P_{3383} = (6, 2, 12, 1)$
$133: P_{3400} = (7, 3, 12, 1)$
$134: P_{3421} = (12, 4, 12, 1)$
$135: P_{3437} = (12, 5, 12, 1)$
$136: P_{3468} = (11, 7, 12, 1)$
$137: P_{3472} = (15, 7, 12, 1)$
$138: P_{3475} = (2, 8, 12, 1)$
$139: P_{3493} = (4, 9, 12, 1)$
$140: P_{3508} = (3, 10, 12, 1)$
$141: P_{3515} = (10, 10, 12, 1)$
$142: P_{3573} = (4, 14, 12, 1)$
$143: P_{3599} = (14, 15, 12, 1)$
$144: P_{3647} = (14, 2, 13, 1)$
$145: P_{3658} = (9, 3, 13, 1)$
$146: P_{3679} = (14, 4, 13, 1)$
$147: P_{3685} = (4, 5, 13, 1)$
$148: P_{3702} = (5, 6, 13, 1)$
$149: P_{3708} = (11, 6, 13, 1)$
$150: P_{3735} = (6, 8, 13, 1)$
$151: P_{3752} = (7, 9, 13, 1)$
$152: P_{3769} = (8, 10, 13, 1)$
$153: P_{3771} = (10, 10, 13, 1)$
$154: P_{3838} = (13, 14, 13, 1)$
$155: P_{3854} = (13, 15, 13, 1)$
$156: P_{3878} = (5, 1, 14, 1)$
$157: P_{3884} = (11, 1, 14, 1)$
$158: P_{3893} = (4, 2, 14, 1)$
$159: P_{3918} = (13, 3, 14, 1)$
$160: P_{3940} = (3, 5, 14, 1)$
$161: P_{3947} = (10, 5, 14, 1)$
$162: P_{3959} = (6, 6, 14, 1)$
$163: P_{3971} = (2, 7, 14, 1)$
$164: P_{3994} = (9, 8, 14, 1)$
$165: P_{4013} = (12, 9, 14, 1)$
$166: P_{4055} = (6, 12, 14, 1)$
$167: P_{4074} = (9, 13, 14, 1)$
$168: P_{4158} = (13, 2, 15, 1)$
$169: P_{4175} = (14, 3, 15, 1)$
$170: P_{4215} = (6, 6, 15, 1)$
$171: P_{4234} = (9, 7, 15, 1)$
$172: P_{4254} = (13, 8, 15, 1)$
$173: P_{4261} = (4, 9, 15, 1)$
$174: P_{4297} = (8, 11, 15, 1)$
$175: P_{4299} = (10, 11, 15, 1)$
$176: P_{4312} = (7, 12, 15, 1)$
$177: P_{4335} = (14, 13, 15, 1)$
$178: P_{4342} = (5, 14, 15, 1)$
$179: P_{4348} = (11, 14, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	1	1	0	1	1
1	0	0	0	1	0	1	1	0	1
2	0	0	0	1	1	0	1	1	0
3	0	1	1	0	0	0	0	1	1
4	1	0	1	0	0	0	1	0	1
5	1	1	0	0	0	0	1	1	0
6	0	1	1	0	1	1	0	0	0
7	1	0	1	1	0	1	0	0	0
8	1	1	0	1	1	0	0	0	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{716}	P_{699}	P_{699}	P_{716}

Line 1 intersects

Line	ℓ_3	ℓ_5	ℓ_6	ℓ_8
in point	P_{3115}	P_{3260}	P_{3260}	P_{3115}

Line 2 intersects

Line	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_{2860}	P_{3019}	P_{3019}	P_{2860}

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_7	ℓ_8
in point	P_{3115}	P_{2860}	P_{2860}	P_{3115}

Line 4 intersects

Line	ℓ_0	ℓ_2	ℓ_6	ℓ_8
in point	P_{716}	P_{3019}	P_{3019}	P_{716}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_6	ℓ_7
in point	P_{699}	P_{3260}	P_{3260}	P_{699}

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{3260}	P_{3019}	P_{3019}	P_{3260}

Line 7 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_{699}	P_{2860}	P_{2860}	P_{699}

Line 8 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{716}	P_{3115}	P_{3115}	P_{716}

The surface has 321 points:
The points on the surface are:

0 : $P_4 = (1, 1, 1, 1)$
1 : $P_5 = (1, 1, 0, 0)$
2 : $P_{14} = (10, 1, 0, 0)$
3 : $P_{15} = (11, 1, 0, 0)$
4 : $P_{20} = (1, 0, 1, 0)$
5 : $P_{29} = (10, 0, 1, 0)$
6 : $P_{30} = (11, 0, 1, 0)$
7 : $P_{36} = (1, 1, 1, 0)$
8 : $P_{45} = (10, 1, 1, 0)$
9 : $P_{46} = (11, 1, 1, 0)$
10 : $P_{290} = (0, 1, 0, 1)$
11 : $P_{291} = (1, 1, 0, 1)$
12 : $P_{434} = (0, 10, 0, 1)$
13 : $P_{435} = (1, 10, 0, 1)$
14 : $P_{450} = (0, 11, 0, 1)$
15 : $P_{451} = (1, 11, 0, 1)$
16 : $P_{530} = (0, 0, 1, 1)$
17 : $P_{531} = (1, 0, 1, 1)$
18 : $P_{546} = (0, 1, 1, 1)$
19 : $P_{563} = (2, 2, 1, 1)$
20 : $P_{580} = (3, 3, 1, 1)$

21 : $P_{585} = (8, 3, 1, 1)$
22 : $P_{587} = (10, 3, 1, 1)$
23 : $P_{597} = (4, 4, 1, 1)$
24 : $P_{614} = (5, 5, 1, 1)$
25 : $P_{620} = (11, 5, 1, 1)$
26 : $P_{624} = (15, 5, 1, 1)$
27 : $P_{631} = (6, 6, 1, 1)$
28 : $P_{648} = (7, 7, 1, 1)$
29 : $P_{660} = (3, 8, 1, 1)$
30 : $P_{665} = (8, 8, 1, 1)$
31 : $P_{667} = (10, 8, 1, 1)$
32 : $P_{682} = (9, 9, 1, 1)$
33 : $P_{692} = (3, 10, 1, 1)$
34 : $P_{697} = (8, 10, 1, 1)$
35 : $P_{699} = (10, 10, 1, 1)$
36 : $P_{710} = (5, 11, 1, 1)$
37 : $P_{716} = (11, 11, 1, 1)$
38 : $P_{720} = (15, 11, 1, 1)$
39 : $P_{733} = (12, 12, 1, 1)$
40 : $P_{750} = (13, 13, 1, 1)$
41 : $P_{767} = (14, 14, 1, 1)$
42 : $P_{774} = (5, 15, 1, 1)$
43 : $P_{780} = (11, 15, 1, 1)$
44 : $P_{784} = (15, 15, 1, 1)$
45 : $P_{804} = (3, 1, 2, 1)$
46 : $P_{809} = (8, 1, 2, 1)$
47 : $P_{811} = (10, 1, 2, 1)$
48 : $P_{835} = (2, 3, 2, 1)$
49 : $P_{858} = (9, 4, 2, 1)$
50 : $P_{872} = (7, 5, 2, 1)$
51 : $P_{894} = (13, 6, 2, 1)$
52 : $P_{911} = (14, 7, 2, 1)$
53 : $P_{918} = (5, 8, 2, 1)$
54 : $P_{924} = (11, 8, 2, 1)$
55 : $P_{928} = (15, 8, 2, 1)$
56 : $P_{942} = (13, 9, 2, 1)$
57 : $P_{959} = (14, 10, 2, 1)$
58 : $P_{973} = (12, 11, 2, 1)$
59 : $P_{981} = (4, 12, 2, 1)$
60 : $P_{1006} = (13, 13, 2, 1)$
61 : $P_{1015} = (6, 14, 2, 1)$
62 : $P_{1039} = (14, 15, 2, 1)$
63 : $P_{1059} = (2, 1, 3, 1)$
64 : $P_{1076} = (3, 2, 3, 1)$
65 : $P_{1081} = (8, 2, 3, 1)$
66 : $P_{1083} = (10, 2, 3, 1)$
67 : $P_{1112} = (7, 4, 3, 1)$
68 : $P_{1123} = (2, 5, 3, 1)$
69 : $P_{1149} = (12, 6, 3, 1)$
70 : $P_{1155} = (2, 7, 3, 1)$
71 : $P_{1176} = (7, 8, 3, 1)$
72 : $P_{1189} = (4, 9, 3, 1)$
73 : $P_{1206} = (5, 10, 3, 1)$
74 : $P_{1212} = (11, 10, 3, 1)$
75 : $P_{1216} = (15, 10, 3, 1)$
76 : $P_{1223} = (6, 11, 3, 1)$
77 : $P_{1247} = (14, 12, 3, 1)$
78 : $P_{1262} = (13, 13, 3, 1)$
79 : $P_{1274} = (9, 14, 3, 1)$
80 : $P_{1288} = (7, 15, 3, 1)$
81 : $P_{1318} = (5, 1, 4, 1)$
82 : $P_{1324} = (11, 1, 4, 1)$
83 : $P_{1328} = (15, 1, 4, 1)$
84 : $P_{1342} = (13, 2, 4, 1)$
85 : $P_{1347} = (2, 3, 4, 1)$
86 : $P_{1381} = (4, 5, 4, 1)$
87 : $P_{1402} = (9, 6, 4, 1)$
88 : $P_{1416} = (7, 7, 4, 1)$
89 : $P_{1437} = (12, 8, 4, 1)$
90 : $P_{1455} = (14, 9, 4, 1)$
91 : $P_{1463} = (6, 10, 4, 1)$
92 : $P_{1475} = (2, 11, 4, 1)$
93 : $P_{1491} = (2, 12, 4, 1)$
94 : $P_{1512} = (7, 13, 4, 1)$
95 : $P_{1528} = (7, 14, 4, 1)$
96 : $P_{1540} = (3, 15, 4, 1)$
97 : $P_{1545} = (8, 15, 4, 1)$
98 : $P_{1547} = (10, 15, 4, 1)$
99 : $P_{1573} = (4, 1, 5, 1)$
100 : $P_{1599} = (14, 2, 5, 1)$
101 : $P_{1613} = (12, 3, 5, 1)$
102 : $P_{1622} = (5, 4, 5, 1)$
103 : $P_{1628} = (11, 4, 5, 1)$
104 : $P_{1632} = (15, 4, 5, 1)$
105 : $P_{1651} = (2, 6, 5, 1)$
106 : $P_{1672} = (7, 7, 5, 1)$
107 : $P_{1685} = (4, 8, 5, 1)$
108 : $P_{1709} = (12, 9, 5, 1)$
109 : $P_{1726} = (13, 10, 5, 1)$
110 : $P_{1732} = (3, 11, 5, 1)$
111 : $P_{1737} = (8, 11, 5, 1)$
112 : $P_{1739} = (10, 11, 5, 1)$
113 : $P_{1749} = (4, 12, 5, 1)$
114 : $P_{1767} = (6, 13, 5, 1)$
115 : $P_{1786} = (9, 14, 5, 1)$
116 : $P_{1805} = (12, 15, 5, 1)$
117 : $P_{1832} = (7, 1, 6, 1)$
118 : $P_{1850} = (9, 2, 6, 1)$
119 : $P_{1859} = (2, 3, 6, 1)$
120 : $P_{1886} = (13, 4, 6, 1)$
121 : $P_{1901} = (12, 5, 6, 1)$
122 : $P_{1927} = (6, 7, 6, 1)$
123 : $P_{1943} = (6, 8, 6, 1)$
124 : $P_{1959} = (6, 9, 6, 1)$
125 : $P_{1978} = (9, 10, 6, 1)$
126 : $P_{1990} = (5, 11, 6, 1)$
127 : $P_{1996} = (11, 11, 6, 1)$
128 : $P_{2000} = (15, 11, 6, 1)$
129 : $P_{2004} = (3, 12, 6, 1)$
130 : $P_{2009} = (8, 12, 6, 1)$
131 : $P_{2011} = (10, 12, 6, 1)$
132 : $P_{2031} = (14, 13, 6, 1)$
133 : $P_{2042} = (9, 14, 6, 1)$
134 : $P_{2053} = (4, 15, 6, 1)$
135 : $P_{2087} = (6, 1, 7, 1)$
136 : $P_{2104} = (7, 2, 7, 1)$
137 : $P_{2120} = (7, 3, 7, 1)$
138 : $P_{2131} = (2, 4, 7, 1)$
139 : $P_{2159} = (14, 5, 7, 1)$
140 : $P_{2168} = (7, 6, 7, 1)$
141 : $P_{2202} = (9, 8, 7, 1)$
142 : $P_{2211} = (2, 9, 7, 1)$
143 : $P_{2227} = (2, 10, 7, 1)$
144 : $P_{2246} = (5, 11, 7, 1)$
145 : $P_{2252} = (11, 11, 7, 1)$
146 : $P_{2256} = (15, 11, 7, 1)$
147 : $P_{2261} = (4, 12, 7, 1)$
148 : $P_{2276} = (3, 13, 7, 1)$
149 : $P_{2281} = (8, 13, 7, 1)$
150 : $P_{2283} = (10, 13, 7, 1)$
151 : $P_{2301} = (12, 14, 7, 1)$
152 : $P_{2318} = (13, 15, 7, 1)$
153 : $P_{2346} = (9, 1, 8, 1)$
154 : $P_{2367} = (14, 2, 8, 1)$
155 : $P_{2375} = (6, 3, 8, 1)$
156 : $P_{2387} = (2, 4, 8, 1)$
157 : $P_{2407} = (6, 5, 8, 1)$
158 : $P_{2426} = (9, 6, 8, 1)$
159 : $P_{2446} = (13, 7, 8, 1)$
160 : $P_{2468} = (3, 9, 8, 1)$
161 : $P_{2473} = (8, 9, 8, 1)$
162 : $P_{2475} = (10, 9, 8, 1)$
163 : $P_{2486} = (5, 10, 8, 1)$
164 : $P_{2492} = (11, 10, 8, 1)$
165 : $P_{2496} = (15, 10, 8, 1)$
166 : $P_{2504} = (7, 11, 8, 1)$
167 : $P_{2525} = (12, 12, 8, 1)$
168 : $P_{2533} = (4, 13, 8, 1)$
169 : $P_{2551} = (6, 14, 8, 1)$
170 : $P_{2570} = (9, 15, 8, 1)$
171 : $P_{2596} = (3, 1, 9, 1)$
172 : $P_{2601} = (8, 1, 9, 1)$
173 : $P_{2603} = (10, 1, 9, 1)$
174 : $P_{2621} = (12, 2, 9, 1)$
175 : $P_{2630} = (5, 3, 9, 1)$
176 : $P_{2636} = (11, 3, 9, 1)$
177 : $P_{2640} = (15, 3, 9, 1)$
178 : $P_{2648} = (7, 4, 9, 1)$
179 : $P_{2661} = (4, 5, 9, 1)$
180 : $P_{2677} = (4, 6, 9, 1)$
181 : $P_{2701} = (12, 7, 9, 1)$
182 : $P_{2714} = (9, 8, 9, 1)$

183 : $P_{2741} = (4, 10, 9, 1)$
184 : $P_{2766} = (13, 11, 9, 1)$
185 : $P_{2781} = (12, 12, 9, 1)$
186 : $P_{2799} = (14, 13, 9, 1)$
187 : $P_{2803} = (2, 14, 9, 1)$
188 : $P_{2823} = (6, 15, 9, 1)$
189 : $P_{2833} = (0, 0, 10, 1)$
190 : $P_{2834} = (1, 0, 10, 1)$
191 : $P_{2854} = (5, 1, 10, 1)$
192 : $P_{2860} = (11, 1, 10, 1)$
193 : $P_{2864} = (15, 1, 10, 1)$
194 : $P_{2870} = (5, 2, 10, 1)$
195 : $P_{2876} = (11, 2, 10, 1)$
196 : $P_{2880} = (15, 2, 10, 1)$
197 : $P_{2885} = (4, 3, 10, 1)$
198 : $P_{2904} = (7, 4, 10, 1)$
199 : $P_{2925} = (12, 5, 10, 1)$
200 : $P_{2932} = (3, 6, 10, 1)$
201 : $P_{2937} = (8, 6, 10, 1)$
202 : $P_{2939} = (10, 6, 10, 1)$
203 : $P_{2948} = (3, 7, 10, 1)$
204 : $P_{2953} = (8, 7, 10, 1)$
205 : $P_{2955} = (10, 7, 10, 1)$
206 : $P_{2975} = (14, 8, 10, 1)$
207 : $P_{2982} = (5, 9, 10, 1)$
208 : $P_{2988} = (11, 9, 10, 1)$
209 : $P_{2992} = (15, 9, 10, 1)$
210 : $P_{2993} = (0, 10, 10, 1)$
211 : $P_{2994} = (1, 10, 10, 1)$
212 : $P_{3012} = (3, 11, 10, 1)$
213 : $P_{3017} = (8, 11, 10, 1)$
214 : $P_{3019} = (10, 11, 10, 1)$
215 : $P_{3034} = (9, 12, 10, 1)$
216 : $P_{3043} = (2, 13, 10, 1)$
217 : $P_{3063} = (6, 14, 10, 1)$
218 : $P_{3086} = (13, 15, 10, 1)$
219 : $P_{3089} = (0, 0, 11, 1)$
220 : $P_{3090} = (1, 0, 11, 1)$
221 : $P_{3108} = (3, 1, 11, 1)$
222 : $P_{3113} = (8, 1, 11, 1)$
223 : $P_{3115} = (10, 1, 11, 1)$
224 : $P_{3134} = (13, 2, 11, 1)$
225 : $P_{3144} = (7, 3, 11, 1)$
226 : $P_{3156} = (3, 4, 11, 1)$
227 : $P_{3161} = (8, 4, 11, 1)$
228 : $P_{3163} = (10, 4, 11, 1)$
229 : $P_{3178} = (9, 5, 11, 1)$
230 : $P_{3199} = (14, 6, 11, 1)$
231 : $P_{3205} = (4, 7, 11, 1)$
232 : $P_{3223} = (6, 8, 11, 1)$
233 : $P_{3245} = (12, 9, 11, 1)$
234 : $P_{3254} = (5, 10, 11, 1)$
235 : $P_{3260} = (11, 10, 11, 1)$
236 : $P_{3264} = (15, 10, 11, 1)$
237 : $P_{3265} = (0, 11, 11, 1)$
238 : $P_{3266} = (1, 11, 11, 1)$
239 : $P_{3286} = (5, 12, 11, 1)$
240 : $P_{3292} = (11, 12, 11, 1)$
241 : $P_{3296} = (15, 12, 11, 1)$
242 : $P_{3302} = (5, 13, 11, 1)$
243 : $P_{3308} = (11, 13, 11, 1)$
244 : $P_{3312} = (15, 13, 11, 1)$
245 : $P_{3316} = (3, 14, 11, 1)$
246 : $P_{3321} = (8, 14, 11, 1)$
247 : $P_{3323} = (10, 14, 11, 1)$
248 : $P_{3331} = (2, 15, 11, 1)$
249 : $P_{3374} = (13, 1, 12, 1)$
250 : $P_{3383} = (6, 2, 12, 1)$
251 : $P_{3400} = (7, 3, 12, 1)$
252 : $P_{3421} = (12, 4, 12, 1)$
253 : $P_{3437} = (12, 5, 12, 1)$
254 : $P_{3450} = (9, 6, 12, 1)$
255 : $P_{3462} = (5, 7, 12, 1)$
256 : $P_{3468} = (11, 7, 12, 1)$
257 : $P_{3472} = (15, 7, 12, 1)$
258 : $P_{3475} = (2, 8, 12, 1)$
259 : $P_{3493} = (4, 9, 12, 1)$
260 : $P_{3508} = (3, 10, 12, 1)$
261 : $P_{3513} = (8, 10, 12, 1)$
262 : $P_{3515} = (10, 10, 12, 1)$
263 : $P_{3525} = (4, 11, 12, 1)$
264 : $P_{3565} = (12, 13, 12, 1)$
265 : $P_{3573} = (4, 14, 12, 1)$
266 : $P_{3599} = (14, 15, 12, 1)$
267 : $P_{3629} = (12, 1, 13, 1)$
268 : $P_{3647} = (14, 2, 13, 1)$
269 : $P_{3658} = (9, 3, 13, 1)$
270 : $P_{3679} = (14, 4, 13, 1)$
271 : $P_{3685} = (4, 5, 13, 1)$
272 : $P_{3702} = (5, 6, 13, 1)$
273 : $P_{3708} = (11, 6, 13, 1)$
274 : $P_{3712} = (15, 6, 13, 1)$
275 : $P_{3715} = (2, 7, 13, 1)$
276 : $P_{3735} = (6, 8, 13, 1)$
277 : $P_{3752} = (7, 9, 13, 1)$
278 : $P_{3764} = (3, 10, 13, 1)$
279 : $P_{3769} = (8, 10, 13, 1)$
280 : $P_{3771} = (10, 10, 13, 1)$
281 : $P_{3791} = (14, 11, 13, 1)$
282 : $P_{3806} = (13, 12, 13, 1)$
283 : $P_{3838} = (13, 14, 13, 1)$
284 : $P_{3854} = (13, 15, 13, 1)$
285 : $P_{3878} = (5, 1, 14, 1)$
286 : $P_{3884} = (11, 1, 14, 1)$
287 : $P_{3888} = (15, 1, 14, 1)$
288 : $P_{3893} = (4, 2, 14, 1)$
289 : $P_{3918} = (13, 3, 14, 1)$
290 : $P_{3927} = (6, 4, 14, 1)$
291 : $P_{3940} = (3, 5, 14, 1)$
292 : $P_{3945} = (8, 5, 14, 1)$
293 : $P_{3947} = (10, 5, 14, 1)$
294 : $P_{3959} = (6, 6, 14, 1)$
295 : $P_{3971} = (2, 7, 14, 1)$
296 : $P_{3994} = (9, 8, 14, 1)$
297 : $P_{4013} = (12, 9, 14, 1)$
298 : $P_{4024} = (7, 10, 14, 1)$
299 : $P_{4042} = (9, 11, 14, 1)$
300 : $P_{4055} = (6, 12, 14, 1)$
301 : $P_{4074} = (9, 13, 14, 1)$
302 : $P_{4111} = (14, 15, 14, 1)$
303 : $P_{4143} = (14, 1, 15, 1)$
304 : $P_{4158} = (13, 2, 15, 1)$
305 : $P_{4175} = (14, 3, 15, 1)$
306 : $P_{4179} = (2, 4, 15, 1)$
307 : $P_{4206} = (13, 5, 15, 1)$
308 : $P_{4215} = (6, 6, 15, 1)$
309 : $P_{4234} = (9, 7, 15, 1)$
310 : $P_{4254} = (13, 8, 15, 1)$
311 : $P_{4261} = (4, 9, 15, 1)$
312 : $P_{4285} = (12, 10, 15, 1)$
313 : $P_{4292} = (3, 11, 15, 1)$
314 : $P_{4297} = (8, 11, 15, 1)$
315 : $P_{4299} = (10, 11, 15, 1)$
316 : $P_{4312} = (7, 12, 15, 1)$
317 : $P_{4335} = (14, 13, 15, 1)$
318 : $P_{4342} = (5, 14, 15, 1)$
319 : $P_{4348} = (11, 14, 15, 1)$
320 : $P_{4352} = (15, 14, 15, 1)$