

Rank-65869 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is 1244173397

General information

Number of lines	11
Number of points	97
Number of singular points	2
Number of Eckardt points	0
Number of double points	15
Number of single points	60
Number of points off lines	20
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^{11}
Type of lines on points	$5, 4, 2^{15}, 1^{60}, 0^{20}$

Singular Points

The surface has 2 singular points:

$$\begin{aligned} 0 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0) \\ 1 : P_{75} &= \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1) \end{aligned}$$

The 11 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{648} = \mathbf{Pl}(0, 1, 1, 0, 0, 0)_{10} \\
\ell_2 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3641} = \begin{bmatrix} 1 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3641} = \mathbf{Pl}(0, 6, 1, 0, 0, 1)_{677} \\
\ell_3 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1889} = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{1889} = \mathbf{Pl}(0, 3, 1, 0, 0, 1)_{674} \\
\ell_4 &= \begin{bmatrix} 1 & 1 & 0 & \gamma^3 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3057} = \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3057} = \mathbf{Pl}(0, 5, 1, 0, 0, 1)_{676} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & \gamma^6 & 0 \\ 0 & 1 & \gamma^6 & \gamma^6 \end{bmatrix}_{492} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 6 & 6 \end{bmatrix}_{492} = \mathbf{Pl}(2, 6, 1, 0, 1, 1)_{1225} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & \gamma^5 & 0 \\ 0 & 1 & \gamma^5 & \gamma^5 \end{bmatrix}_{246} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 3 \end{bmatrix}_{246} = \mathbf{Pl}(4, 3, 1, 0, 1, 1)_{1227} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & \gamma^3 & 0 \\ 0 & 1 & \gamma^3 & \gamma^3 \end{bmatrix}_{410} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 5 & 5 \end{bmatrix}_{410} = \mathbf{Pl}(7, 5, 1, 0, 1, 1)_{1230} \\
\ell_8 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \gamma^2 & \gamma \end{bmatrix}_{604} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 4 & 2 \end{bmatrix}_{604} = \mathbf{Pl}(6, 2, 2, 6, 1, 0)_{563} \\
\ell_9 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \gamma^4 & \gamma^2 \end{bmatrix}_{623} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 7 & 4 \end{bmatrix}_{623} = \mathbf{Pl}(3, 4, 4, 3, 1, 0)_{427} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \gamma & \gamma^4 \end{bmatrix}_{642} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 7 \end{bmatrix}_{642} = \mathbf{Pl}(5, 7, 7, 5, 1, 0)_{548}
\end{aligned}$$

Rank of lines: (64, 648, 3641, 1889, 3057, 492, 246, 410, 604, 623, 642)

Rank of points on Klein quadric: (2, 10, 677, 674, 676, 1225, 1227, 1230, 563, 427, 548)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 15 Double points:

The double points on the surface are:

$$\begin{aligned}
P_{13} &= (2, 0, 1, 0) = \ell_0 \cap \ell_5 \\
P_{15} &= (4, 0, 1, 0) = \ell_0 \cap \ell_6 \\
P_{18} &= (7, 0, 1, 0) = \ell_0 \cap \ell_7 \\
P_{92} &= (2, 2, 0, 1) = \ell_2 \cap \ell_5 \\
P_{347} &= (2, 2, 4, 1) = \ell_2 \cap \ell_{10} \\
P_{110} &= (4, 4, 0, 1) = \ell_3 \cap \ell_6 \\
P_{557} &= (4, 4, 7, 1) = \ell_3 \cap \ell_8 \\
P_{137} &= (7, 7, 0, 1) = \ell_4 \cap \ell_7
\end{aligned}$$

$$\begin{aligned}
P_{264} &= (7, 7, 2, 1) = \ell_4 \cap \ell_9 \\
P_{414} &= (5, 2, 5, 1) = \ell_5 \cap \ell_8 \\
P_{285} &= (4, 2, 3, 1) = \ell_5 \cap \ell_9 \\
P_{495} &= (6, 4, 6, 1) = \ell_6 \cap \ell_9 \\
P_{432} &= (7, 4, 5, 1) = \ell_6 \cap \ell_{10} \\
P_{515} &= (2, 7, 6, 1) = \ell_7 \cap \ell_8 \\
P_{324} &= (3, 7, 3, 1) = \ell_7 \cap \ell_{10}
\end{aligned}$$

Single Points

The surface has 60 single points:

The single points on the surface are:

- 0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0
 1 : $P_{12} = (1, 0, 1, 0)$ lies on line ℓ_0
 2 : $P_{14} = (3, 0, 1, 0)$ lies on line ℓ_0
 3 : $P_{16} = (5, 0, 1, 0)$ lies on line ℓ_0
 4 : $P_{17} = (6, 0, 1, 0)$ lies on line ℓ_0
 5 : $P_{41} = (6, 3, 1, 0)$ lies on line ℓ_8
 6 : $P_{54} = (3, 5, 1, 0)$ lies on line ℓ_9
 7 : $P_{64} = (5, 6, 1, 0)$ lies on line ℓ_{10}
 8 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_1
 9 : $P_{153} = (0, 2, 1, 1)$ lies on line ℓ_5
 10 : $P_{155} = (2, 2, 1, 1)$ lies on line ℓ_2
 11 : $P_{168} = (7, 3, 1, 1)$ lies on line ℓ_8
 12 : $P_{169} = (0, 4, 1, 1)$ lies on line ℓ_6
 13 : $P_{173} = (4, 4, 1, 1)$ lies on line ℓ_3
 14 : $P_{179} = (2, 5, 1, 1)$ lies on line ℓ_9
 15 : $P_{189} = (4, 6, 1, 1)$ lies on line ℓ_{10}
 16 : $P_{193} = (0, 7, 1, 1)$ lies on line ℓ_7
 17 : $P_{200} = (7, 7, 1, 1)$ lies on line ℓ_4
 18 : $P_{202} = (1, 0, 2, 1)$ lies on line ℓ_1
 19 : $P_{215} = (6, 1, 2, 1)$ lies on line ℓ_{10}
 20 : $P_{219} = (2, 2, 2, 1)$ lies on line ℓ_2
 21 : $P_{223} = (6, 2, 2, 1)$ lies on line ℓ_5
 22 : $P_{234} = (1, 4, 2, 1)$ lies on line ℓ_6
 23 : $P_{237} = (4, 4, 2, 1)$ lies on line ℓ_3
 24 : $P_{249} = (0, 6, 2, 1)$ lies on line ℓ_8
 25 : $P_{261} = (4, 7, 2, 1)$ lies on line ℓ_7
 26 : $P_{266} = (1, 0, 3, 1)$ lies on line ℓ_1
 27 : $P_{283} = (2, 2, 3, 1)$ lies on line ℓ_2
 28 : $P_{301} = (4, 4, 3, 1)$ lies on line ℓ_3
 29 : $P_{302} = (5, 4, 3, 1)$ lies on line ℓ_6
 30 : $P_{311} = (6, 5, 3, 1)$ lies on line ℓ_8
 31 : $P_{328} = (7, 7, 3, 1)$ lies on line ℓ_4
 32 : $P_{330} = (1, 0, 4, 1)$ lies on line ℓ_1
 33 : $P_{340} = (3, 1, 4, 1)$ lies on line ℓ_8
 34 : $P_{352} = (7, 2, 4, 1)$ lies on line ℓ_5
 35 : $P_{353} = (0, 3, 4, 1)$ lies on line ℓ_9
 36 : $P_{364} = (3, 4, 4, 1)$ lies on line ℓ_6
 37 : $P_{365} = (4, 4, 4, 1)$ lies on line ℓ_3
 38 : $P_{386} = (1, 7, 4, 1)$ lies on line ℓ_7
 39 : $P_{392} = (7, 7, 4, 1)$ lies on line ℓ_4
 40 : $P_{394} = (1, 0, 5, 1)$ lies on line ℓ_1
 41 : $P_{411} = (2, 2, 5, 1)$ lies on line ℓ_2
 42 : $P_{429} = (4, 4, 5, 1)$ lies on line ℓ_3
 43 : $P_{444} = (3, 6, 5, 1)$ lies on line ℓ_9
 44 : $P_{455} = (6, 7, 5, 1)$ lies on line ℓ_7
 45 : $P_{456} = (7, 7, 5, 1)$ lies on line ℓ_4
 46 : $P_{458} = (1, 0, 6, 1)$ lies on line ℓ_1
 47 : $P_{475} = (2, 2, 6, 1)$ lies on line ℓ_2
 48 : $P_{476} = (3, 2, 6, 1)$ lies on line ℓ_5
 49 : $P_{486} = (5, 3, 6, 1)$ lies on line ℓ_{10}
 50 : $P_{493} = (4, 4, 6, 1)$ lies on line ℓ_3
 51 : $P_{520} = (7, 7, 6, 1)$ lies on line ℓ_4
 52 : $P_{522} = (1, 0, 7, 1)$ lies on line ℓ_1
 53 : $P_{534} = (5, 1, 7, 1)$ lies on line ℓ_9
 54 : $P_{538} = (1, 2, 7, 1)$ lies on line ℓ_5
 55 : $P_{539} = (2, 2, 7, 1)$ lies on line ℓ_2
 56 : $P_{555} = (2, 4, 7, 1)$ lies on line ℓ_6
 57 : $P_{561} = (0, 5, 7, 1)$ lies on line ℓ_{10}
 58 : $P_{582} = (5, 7, 7, 1)$ lies on line ℓ_7
 59 : $P_{584} = (7, 7, 7, 1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 20 points not on any line:

The points on the surface but not on lines are:

- 0 : $P_{19} = (0, 1, 1, 0)$
 1 : $P_{33} = (6, 2, 1, 0)$
 2 : $P_{46} = (3, 4, 1, 0)$
 3 : $P_{72} = (5, 7, 1, 0)$
 4 : $P_{82} = (0, 1, 0, 1)$
 5 : $P_{104} = (6, 3, 0, 1)$
 6 : $P_{117} = (3, 5, 0, 1)$
 7 : $P_{127} = (5, 6, 0, 1)$
 8 : $P_{165} = (4, 3, 1, 1)$
 9 : $P_{184} = (7, 5, 1, 1)$
 10 : $P_{187} = (2, 6, 1, 1)$
 11 : $P_{213} = (4, 1, 2, 1)$
 12 : $P_{250} = (1, 6, 2, 1)$
 13 : $P_{309} = (4, 5, 3, 1)$
 14 : $P_{344} = (7, 1, 4, 1)$
 15 : $P_{354} = (1, 3, 4, 1)$
 16 : $P_{448} = (7, 6, 5, 1)$
 17 : $P_{483} = (2, 3, 6, 1)$
 18 : $P_{531} = (2, 1, 7, 1)$
 19 : $P_{562} = (1, 5, 7, 1)$

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10
0	0	1	1	1	1	1	1	1	0	0	0
1	1	0	1	1	1	0	0	0	1	1	1
2	1	1	0	1	1	1	0	0	0	0	1
3	1	1	1	0	1	0	1	0	1	0	0
4	1	1	1	1	0	0	0	1	0	1	0
5	1	0	1	0	0	0	0	0	1	1	0
6	1	0	0	1	0	0	0	0	0	1	1
7	1	0	0	0	1	0	0	0	1	0	1
8	0	1	0	1	0	1	0	1	0	1	1
9	0	1	0	0	1	1	1	0	1	0	1
10	0	1	1	0	0	0	1	1	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_2	P_2	P_2	P_2	P_{13}	P_{15}	P_{18}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_8	ℓ_9	ℓ_{10}
in point	P_2	P_2	P_2	P_2	P_{75}	P_{75}	P_{75}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_{10}
in point	P_2	P_2	P_2	P_2	P_{92}	P_{347}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_6	ℓ_8
in point	P_2	P_2	P_2	P_2	P_{110}	P_{557}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_7	ℓ_9
in point	P_2	P_2	P_2	P_2	P_{137}	P_{264}

Line 5 intersects

Line	ℓ_0	ℓ_2	ℓ_8	ℓ_9
in point	P_{13}	P_{92}	P_{414}	P_{285}

Line 6 intersects

Line	ℓ_0	ℓ_3	ℓ_9	ℓ_{10}
in point	P_{15}	P_{110}	P_{495}	P_{432}

Line 7 intersects

Line	ℓ_0	ℓ_4	ℓ_8	ℓ_{10}
in point	P_{18}	P_{137}	P_{515}	P_{324}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}
in point	P_{75}	P_{557}	P_{414}	P_{515}	P_{75}	P_{75}

Line 9 intersects

Line	ℓ_1	ℓ_4	ℓ_5	ℓ_6	ℓ_8	ℓ_{10}
in point	P_{75}	P_{264}	P_{285}	P_{495}	P_{75}	P_{75}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_{75}	P_{347}	P_{432}	P_{324}	P_{75}	P_{75}

The surface has 97 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	33 : $P_{187} = (2, 6, 1, 1)$	66 : $P_{386} = (1, 7, 4, 1)$
1 : $P_2 = (0, 0, 1, 0)$	34 : $P_{189} = (4, 6, 1, 1)$	67 : $P_{392} = (7, 7, 4, 1)$
2 : $P_{12} = (1, 0, 1, 0)$	35 : $P_{193} = (0, 7, 1, 1)$	68 : $P_{394} = (1, 0, 5, 1)$
3 : $P_{13} = (2, 0, 1, 0)$	36 : $P_{200} = (7, 7, 1, 1)$	69 : $P_{411} = (2, 2, 5, 1)$
4 : $P_{14} = (3, 0, 1, 0)$	37 : $P_{202} = (1, 0, 2, 1)$	70 : $P_{414} = (5, 2, 5, 1)$
5 : $P_{15} = (4, 0, 1, 0)$	38 : $P_{213} = (4, 1, 2, 1)$	71 : $P_{429} = (4, 4, 5, 1)$
6 : $P_{16} = (5, 0, 1, 0)$	39 : $P_{215} = (6, 1, 2, 1)$	72 : $P_{432} = (7, 4, 5, 1)$
7 : $P_{17} = (6, 0, 1, 0)$	40 : $P_{219} = (2, 2, 2, 1)$	73 : $P_{444} = (3, 6, 5, 1)$
8 : $P_{18} = (7, 0, 1, 0)$	41 : $P_{223} = (6, 2, 2, 1)$	74 : $P_{448} = (7, 6, 5, 1)$
9 : $P_{19} = (0, 1, 1, 0)$	42 : $P_{234} = (1, 4, 2, 1)$	75 : $P_{455} = (6, 7, 5, 1)$
10 : $P_{33} = (6, 2, 1, 0)$	43 : $P_{237} = (4, 4, 2, 1)$	76 : $P_{456} = (7, 7, 5, 1)$
11 : $P_{41} = (6, 3, 1, 0)$	44 : $P_{249} = (0, 6, 2, 1)$	77 : $P_{458} = (1, 0, 6, 1)$
12 : $P_{46} = (3, 4, 1, 0)$	45 : $P_{250} = (1, 6, 2, 1)$	78 : $P_{475} = (2, 2, 6, 1)$
13 : $P_{54} = (3, 5, 1, 0)$	46 : $P_{261} = (4, 7, 2, 1)$	79 : $P_{476} = (3, 2, 6, 1)$
14 : $P_{64} = (5, 6, 1, 0)$	47 : $P_{264} = (7, 7, 2, 1)$	80 : $P_{483} = (2, 3, 6, 1)$
15 : $P_{72} = (5, 7, 1, 0)$	48 : $P_{266} = (1, 0, 3, 1)$	81 : $P_{486} = (5, 3, 6, 1)$
16 : $P_{75} = (1, 0, 0, 1)$	49 : $P_{283} = (2, 2, 3, 1)$	82 : $P_{493} = (4, 4, 6, 1)$
17 : $P_{82} = (0, 1, 0, 1)$	50 : $P_{285} = (4, 2, 3, 1)$	83 : $P_{495} = (6, 4, 6, 1)$
18 : $P_{92} = (2, 2, 0, 1)$	51 : $P_{301} = (4, 4, 3, 1)$	84 : $P_{515} = (2, 7, 6, 1)$
19 : $P_{104} = (6, 3, 0, 1)$	52 : $P_{302} = (5, 4, 3, 1)$	85 : $P_{520} = (7, 7, 6, 1)$
20 : $P_{110} = (4, 4, 0, 1)$	53 : $P_{309} = (4, 5, 3, 1)$	86 : $P_{522} = (1, 0, 7, 1)$
21 : $P_{117} = (3, 5, 0, 1)$	54 : $P_{311} = (6, 5, 3, 1)$	87 : $P_{531} = (2, 1, 7, 1)$
22 : $P_{127} = (5, 6, 0, 1)$	55 : $P_{324} = (3, 7, 3, 1)$	88 : $P_{534} = (5, 1, 7, 1)$
23 : $P_{137} = (7, 7, 0, 1)$	56 : $P_{328} = (7, 7, 3, 1)$	89 : $P_{538} = (1, 2, 7, 1)$
24 : $P_{139} = (1, 0, 1, 1)$	57 : $P_{330} = (1, 0, 4, 1)$	90 : $P_{539} = (2, 2, 7, 1)$
25 : $P_{153} = (0, 2, 1, 1)$	58 : $P_{340} = (3, 1, 4, 1)$	91 : $P_{555} = (2, 4, 7, 1)$
26 : $P_{155} = (2, 2, 1, 1)$	59 : $P_{344} = (7, 1, 4, 1)$	92 : $P_{557} = (4, 4, 7, 1)$
27 : $P_{165} = (4, 3, 1, 1)$	60 : $P_{347} = (2, 2, 4, 1)$	93 : $P_{561} = (0, 5, 7, 1)$
28 : $P_{168} = (7, 3, 1, 1)$	61 : $P_{352} = (7, 2, 4, 1)$	94 : $P_{562} = (1, 5, 7, 1)$
29 : $P_{169} = (0, 4, 1, 1)$	62 : $P_{353} = (0, 3, 4, 1)$	95 : $P_{582} = (5, 7, 7, 1)$
30 : $P_{173} = (4, 4, 1, 1)$	63 : $P_{354} = (1, 3, 4, 1)$	96 : $P_{584} = (7, 7, 7, 1)$
31 : $P_{179} = (2, 5, 1, 1)$	64 : $P_{364} = (3, 4, 4, 1)$	
32 : $P_{184} = (7, 5, 1, 1)$	65 : $P_{365} = (4, 4, 4, 1)$	