Rank-67150 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_3^3 + X_0^2 X_3 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is 1091047558

General information

Number of lines	3
Number of points	4033
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	3840
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	$3, 1^{192}, 0^{3840}$

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & \epsilon^9 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17046511} = \begin{bmatrix} 0 & 1 & 0 & 47 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17046511} = \mathbf{Pl}(0, 47, 0, 0, 0, 1)_{270511}$$

$$\ell_1 = \begin{bmatrix} 0 & 1 & 0 & \epsilon^{18} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17044171} = \begin{bmatrix} 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{17044171} = \mathbf{Pl}(0, 11, 0, 0, 0, 1)_{270475}$$

$$\ell_2 = \left[\begin{array}{cccc} 0 & 1 & 0 & \epsilon^{36} \\ 0 & 0 & 1 & 0 \end{array}\right]_{17045796} = \left[\begin{array}{cccc} 0 & 1 & 0 & 36 \\ 0 & 0 & 1 & 0 \end{array}\right]_{17045796} = \mathbf{Pl}(0,36,0,0,0,1)_{270500}$$

Rank of lines: (17046511, 17044171, 17045796)
Rank of points on Klein quadric: (270511, 270475, 270500)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_{4802} = (0,10,0,1)$ lies on line ℓ_0
$1 : P_{6530} = (0,37,0,1)$ lies on line ℓ_1
$2 : P_{7106} = (0,46,0,1)$ lies on line ℓ_2
$3 : P_{8897} = (0,10,1,1)$ lies on line ℓ_0
$4 : P_{10625} = (0,37,1,1)$ lies on line ℓ_1
$5 : P_{11201} = (0,46,1,1)$ lies on line ℓ_2
$6 : P_{12993} = (0,10,2,1)$ lies on line ℓ_0
$7 : P_{14721} = (0,37,2,1)$ lies on line ℓ_1
$8 : P_{15297} = (0,46,2,1)$ lies on line ℓ_2
$9 : P_{17089} = (0,10,3,1)$ lies on line ℓ_0
$10 : P_{18817} = (0,37,3,1)$ lies on line ℓ_1
$11 : P_{19393} = (0,46,3,1)$ lies on line ℓ_2
$12 : P_{21185} = (0,10,4,1)$ lies on line ℓ_0
$13 : P_{22913} = (0,37,4,1)$ lies on line ℓ_1
$14 : P_{23489} = (0,46,4,1)$ lies on line ℓ_2
$15 : P_{25281} = (0,10,5,1)$ lies on line ℓ_0
$16 : P_{27009} = (0,37,5,1)$ lies on line ℓ_1
$17 : P_{27585} = (0,46,5,1)$ lies on line ℓ_2
$18 : P_{29377} = (0,10,6,1)$ lies on line ℓ_0
$19 : P_{31105} = (0,37,6,1)$ lies on line ℓ_1
$20 : P_{31681} = (0,46,6,1)$ lies on line ℓ_2
$21 : P_{33473} = (0,10,7,1)$ lies on line ℓ_0
$22 : P_{35201} = (0,37,7,1)$ lies on line ℓ_1
$23 : P_{35777} = (0,46,7,1)$ lies on line ℓ_2
$24 : P_{37569} = (0,10,8,1)$ lies on line ℓ_0
$25 : P_{39297} = (0,37,8,1)$ lies on line ℓ_1
$26 : P_{39873} = (0,46,8,1)$ lies on line ℓ_2
$27 : P_{41665} = (0,10,9,1)$ lies on line ℓ_0
$28 : P_{43393} = (0,37,9,1)$ lies on line ℓ_1

$29 : P_{43969} = (0,46,9,1)$ lies on line ℓ_2
$30 : P_{45761} = (0,10,10,1)$ lies on line ℓ_0
$31 : P_{47489} = (0,37,10,1)$ lies on line ℓ_1
$32 : P_{48065} = (0,46,10,1)$ lies on line ℓ_2
$33 : P_{49857} = (0,10,11,1)$ lies on line ℓ_0
$34 : P_{51585} = (0,37,11,1)$ lies on line ℓ_1
$35 : P_{52161} = (0,46,11,1)$ lies on line ℓ_2
$36 : P_{53953} = (0,10,12,1)$ lies on line ℓ_0
$37 : P_{55681} = (0,37,12,1)$ lies on line ℓ_1
$38 : P_{56257} = (0,46,12,1)$ lies on line ℓ_2
$39 : P_{58049} = (0,10,13,1)$ lies on line ℓ_0
$40 : P_{59777} = (0,37,13,1)$ lies on line ℓ_1
$41 : P_{60353} = (0,46,13,1)$ lies on line ℓ_2
$42 : P_{62145} = (0,10,14,1)$ lies on line ℓ_0
$43 : P_{63873} = (0,37,14,1)$ lies on line ℓ_1
$44 : P_{64449} = (0,46,14,1)$ lies on line ℓ_2
$45 : P_{66241} = (0,10,15,1)$ lies on line ℓ_0
$46 : P_{67969} = (0,37,15,1)$ lies on line ℓ_1
$47 : P_{68545} = (0,46,15,1)$ lies on line ℓ_2
$48 : P_{70337} = (0,10,16,1)$ lies on line ℓ_0
$49 : P_{72065} = (0,37,16,1)$ lies on line ℓ_1
$50 : P_{72641} = (0,46,16,1)$ lies on line ℓ_2
$51 : P_{74433} = (0,10,17,1)$ lies on line ℓ_0
$52 : P_{76161} = (0,37,17,1)$ lies on line ℓ_1
$53 : P_{76737} = (0,46,17,1)$ lies on line ℓ_2
$54 : P_{78529} = (0,10,18,1)$ lies on line ℓ_0
$55 : P_{80257} = (0,37,18,1)$ lies on line ℓ_1
$56 : P_{80833} = (0,46,18,1)$ lies on line ℓ_2
$57 : P_{82625} = (0,10,19,1)$ lies on line ℓ_0

$58: P_{84353} = (0, 37, 19, 1)$ lies on line ℓ_1
$59: P_{84929} = (0, 46, 19, 1)$ lies on line ℓ_2
$60: P_{86721} = (0, 10, 20, 1)$ lies on line ℓ_0
$61: P_{88449} = (0, 37, 20, 1)$ lies on line ℓ_1
$62: P_{89025} = (0, 46, 20, 1)$ lies on line ℓ_2
$63: P_{90817} = (0, 10, 21, 1)$ lies on line ℓ_0
$64: P_{92545} = (0, 37, 21, 1)$ lies on line ℓ_1
$65: P_{93121} = (0, 46, 21, 1)$ lies on line ℓ_2
$66: P_{94913} = (0, 10, 22, 1)$ lies on line ℓ_0
$67: P_{96641} = (0, 37, 22, 1)$ lies on line ℓ_1
$68: P_{97217} = (0, 46, 22, 1)$ lies on line ℓ_2
$69: P_{99009} = (0, 10, 23, 1)$ lies on line ℓ_0
$70: P_{100737} = (0, 37, 23, 1)$ lies on line ℓ_1
$71: P_{101313} = (0, 46, 23, 1)$ lies on line ℓ_2
$72: P_{103105} = (0, 10, 24, 1)$ lies on line ℓ_0
$73: P_{104833} = (0, 37, 24, 1)$ lies on line ℓ_1
$74: P_{105409} = (0, 46, 24, 1)$ lies on line ℓ_2
$75: P_{107201} = (0, 10, 25, 1)$ lies on line ℓ_0
$76: P_{108929} = (0, 37, 25, 1)$ lies on line ℓ_1
$77: P_{109505} = (0, 46, 25, 1)$ lies on line ℓ_2
$78: P_{111297} = (0, 10, 26, 1)$ lies on line ℓ_0
$79: P_{113025} = (0, 37, 26, 1)$ lies on line ℓ_1
$80: P_{113601} = (0, 46, 26, 1)$ lies on line ℓ_2
$81: P_{115393} = (0, 10, 27, 1)$ lies on line ℓ_0
$82: P_{117121} = (0, 37, 27, 1)$ lies on line ℓ_1
$83: P_{117697} = (0, 46, 27, 1)$ lies on line ℓ_2
$84: P_{119489} = (0, 10, 28, 1)$ lies on line ℓ_0
$85: P_{121217} = (0, 37, 28, 1)$ lies on line ℓ_1
$86: P_{121793} = (0, 46, 28, 1)$ lies on line ℓ_2
$87: P_{123585} = (0, 10, 29, 1)$ lies on line ℓ_0
$88: P_{125313} = (0, 37, 29, 1)$ lies on line ℓ_1
$89: P_{125889} = (0, 46, 29, 1)$ lies on line ℓ_2
$90: P_{127681} = (0, 10, 30, 1)$ lies on line ℓ_0
$91: P_{129409} = (0, 37, 30, 1)$ lies on line ℓ_1
$92: P_{129985} = (0, 46, 30, 1)$ lies on line ℓ_2
$93: P_{131777} = (0, 10, 31, 1)$ lies on line ℓ_0
$94: P_{133505} = (0, 37, 31, 1)$ lies on line ℓ_1
$95: P_{134081} = (0, 46, 31, 1)$ lies on line ℓ_2
$96: P_{135873} = (0, 10, 32, 1)$ lies on line ℓ_0
$97: P_{137601} = (0, 37, 32, 1)$ lies on line ℓ_1
$98: P_{138177} = (0, 46, 32, 1)$ lies on line ℓ_2
$99: P_{139969} = (0, 10, 33, 1)$ lies on line ℓ_0
$100: P_{141697} = (0, 37, 33, 1)$ lies on line ℓ_1
$101: P_{142273} = (0, 46, 33, 1)$ lies on line ℓ_2
$102: P_{144065} = (0, 10, 34, 1)$ lies on line ℓ_0
$103: P_{145793} = (0, 37, 34, 1)$ lies on line ℓ_1
$104: P_{146369} = (0, 46, 34, 1)$ lies on line ℓ_2
$105: P_{148161} = (0, 10, 35, 1)$ lies on line ℓ_0
$106: P_{149889} = (0, 37, 35, 1)$ lies on line ℓ_1
$107: P_{150465} = (0, 46, 35, 1)$ lies on line ℓ_2
$108: P_{152257} = (0, 10, 36, 1)$ lies on line ℓ_0
$109: P_{153985} = (0, 37, 36, 1)$ lies on line ℓ_1
$110: P_{154561} = (0, 46, 36, 1)$ lies on line ℓ_2
$111: P_{156353} = (0, 10, 37, 1)$ lies on line ℓ_0

$112: P_{158081} = (0, 37, 37, 1)$ lies on line ℓ_1
$113: P_{158657} = (0, 46, 37, 1)$ lies on line ℓ_2
$114: P_{160449} = (0, 10, 38, 1)$ lies on line ℓ_0
$115: P_{162177} = (0, 37, 38, 1)$ lies on line ℓ_1
$116: P_{162753} = (0, 46, 38, 1)$ lies on line ℓ_2
$117: P_{164545} = (0, 10, 39, 1)$ lies on line ℓ_0
$118: P_{166273} = (0, 37, 39, 1)$ lies on line ℓ_1
$119: P_{166849} = (0, 46, 39, 1)$ lies on line ℓ_2
$120: P_{168641} = (0, 10, 40, 1)$ lies on line ℓ_0
$121: P_{170369} = (0, 37, 40, 1)$ lies on line ℓ_1
$122: P_{170945} = (0, 46, 40, 1)$ lies on line ℓ_2
$123: P_{172737} = (0, 10, 41, 1)$ lies on line ℓ_0
$124: P_{174465} = (0, 37, 41, 1)$ lies on line ℓ_1
$125: P_{175041} = (0, 46, 41, 1)$ lies on line ℓ_2
$126: P_{176833} = (0, 10, 42, 1)$ lies on line ℓ_0
$127: P_{178561} = (0, 37, 42, 1)$ lies on line ℓ_1
$128: P_{179137} = (0, 46, 42, 1)$ lies on line ℓ_2
$129: P_{180929} = (0, 10, 43, 1)$ lies on line ℓ_0
$130: P_{182657} = (0, 37, 43, 1)$ lies on line ℓ_1
$131: P_{183233} = (0, 46, 43, 1)$ lies on line ℓ_2
$132: P_{185025} = (0, 10, 44, 1)$ lies on line ℓ_0
$133: P_{186753} = (0, 37, 44, 1)$ lies on line ℓ_1
$134: P_{187329} = (0, 46, 44, 1)$ lies on line ℓ_2
$135: P_{189121} = (0, 10, 45, 1)$ lies on line ℓ_0
$136: P_{190849} = (0, 37, 45, 1)$ lies on line ℓ_1
$137: P_{191425} = (0, 46, 45, 1)$ lies on line ℓ_2
$138: P_{193217} = (0, 10, 46, 1)$ lies on line ℓ_0
$139: P_{194945} = (0, 37, 46, 1)$ lies on line ℓ_1
$140: P_{195521} = (0, 46, 46, 1)$ lies on line ℓ_2
$141: P_{197313} = (0, 10, 47, 1)$ lies on line ℓ_0
$142: P_{199041} = (0, 37, 47, 1)$ lies on line ℓ_1
$143: P_{199617} = (0, 46, 47, 1)$ lies on line ℓ_2
$144: P_{201409} = (0, 10, 48, 1)$ lies on line ℓ_0
$145: P_{203137} = (0, 37, 48, 1)$ lies on line ℓ_1
$146: P_{203713} = (0, 46, 48, 1)$ lies on line ℓ_2
$147: P_{205505} = (0, 10, 49, 1)$ lies on line ℓ_0
$148: P_{207233} = (0, 37, 49, 1)$ lies on line ℓ_1
$149: P_{207809} = (0, 46, 49, 1)$ lies on line ℓ_2
$150: P_{209601} = (0, 10, 50, 1)$ lies on line ℓ_0
$151: P_{211329} = (0, 37, 50, 1)$ lies on line ℓ_1
$152: P_{211905} = (0, 46, 50, 1)$ lies on line ℓ_2
$153: P_{213697} = (0, 10, 51, 1)$ lies on line ℓ_0
$154: P_{215425} = (0, 37, 51, 1)$ lies on line ℓ_1
$155: P_{216001} = (0, 46, 51, 1)$ lies on line ℓ_2
$156: P_{217793} = (0, 10, 52, 1)$ lies on line ℓ_0
$157: P_{219521} = (0, 37, 52, 1)$ lies on line ℓ_1
$158: P_{220097} = (0, 46, 52, 1)$ lies on line ℓ_2
$159: P_{221889} = (0, 10, 53, 1)$ lies on line ℓ_0
$160: P_{223617} = (0, 37, 53, 1)$ lies on line ℓ_1
$161: P_{224193} = (0, 46, 53, 1)$ lies on line ℓ_2
$162: P_{225985} = (0, 10, 54, 1)$ lies on line ℓ_0
$163: P_{227713} = (0, 37, 54, 1)$ lies on line ℓ_1
$164: P_{228289} = (0, 46, 54, 1)$ lies on line ℓ_2
$165: P_{230081} = (0, 10, 55, 1)$ lies on line ℓ_0

$166 : P_{231809} = (0, 37, 55, 1)$ lies on line ℓ_1
$167 : P_{232385} = (0, 46, 55, 1)$ lies on line ℓ_2
$168 : P_{234177} = (0, 10, 56, 1)$ lies on line ℓ_0
$169 : P_{235905} = (0, 37, 56, 1)$ lies on line ℓ_1
$170 : P_{236481} = (0, 46, 56, 1)$ lies on line ℓ_2
$171 : P_{238273} = (0, 10, 57, 1)$ lies on line ℓ_0
$172 : P_{240001} = (0, 37, 57, 1)$ lies on line ℓ_1
$173 : P_{240577} = (0, 46, 57, 1)$ lies on line ℓ_2
$174 : P_{242369} = (0, 10, 58, 1)$ lies on line ℓ_0
$175 : P_{244097} = (0, 37, 58, 1)$ lies on line ℓ_1
$176 : P_{244673} = (0, 46, 58, 1)$ lies on line ℓ_2
$177 : P_{246465} = (0, 10, 59, 1)$ lies on line ℓ_0
$178 : P_{248193} = (0, 37, 59, 1)$ lies on line ℓ_1
$179 : P_{248769} = (0, 46, 59, 1)$ lies on line ℓ_2

$180 : P_{250561} = (0, 10, 60, 1)$ lies on line ℓ_0
$181 : P_{252289} = (0, 37, 60, 1)$ lies on line ℓ_1
$182 : P_{252865} = (0, 46, 60, 1)$ lies on line ℓ_2
$183 : P_{254657} = (0, 10, 61, 1)$ lies on line ℓ_0
$184 : P_{256385} = (0, 37, 61, 1)$ lies on line ℓ_1
$185 : P_{256961} = (0, 46, 61, 1)$ lies on line ℓ_2
$186 : P_{258753} = (0, 10, 62, 1)$ lies on line ℓ_0
$187 : P_{260481} = (0, 37, 62, 1)$ lies on line ℓ_1
$188 : P_{261057} = (0, 46, 62, 1)$ lies on line ℓ_2
$189 : P_{262849} = (0, 10, 63, 1)$ lies on line ℓ_0
$190 : P_{264577} = (0, 37, 63, 1)$ lies on line ℓ_1
$191 : P_{265153} = (0, 46, 63, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 3840 points not on any line:
Too many to print.

Line Intersection Graph

	0	1	2
0	0	1	1
1	1	0	1
2	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_2	P_2

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_2	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_2	P_2

The surface has 4033 points:
Too many to print.