Rank-74052 over GF(16)

January 15, 2021

The equation

The equation of the surface is:

$$X_0^3 + X_0^2 X_3 + X_1^2 X_2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0) The point rank of the equation over $\mathrm{GF}(16)$ is 303108374

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	141
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	$5, 3^2, 2^9, 1^{141}, 0^{168}$

Singular Points

The surface has 1 singular points:

$$0: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_{33}$$

$$\ell_{1} = \begin{bmatrix} 1 & 0 & 0 & \delta^{5} \\ 0 & 1 & 0 & 0 \end{bmatrix}_{48048} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{48048} = \mathbf{Pl}(10,0,0,1,0,0)_{43}$$

$$\ell_{2} = \begin{bmatrix} 1 & 0 & 0 & \delta^{10} \\ 0 & 1 & 0 & 0 \end{bmatrix}_{43680} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{43680} = \mathbf{Pl}(11,0,0,1,0,0)_{44}$$

$$\ell_{3} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_{4} = \begin{bmatrix} 1 & 0 & 0 & \delta^{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48304} = \mathbf{Pl}(0,11,1,0,0,0)_{28}$$

$$\ell_{5} = \begin{bmatrix} 1 & 0 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{43936} = \mathbf{Pl}(0,10,1,0,0,0)_{27}$$

$$\ell_{6} = \begin{bmatrix} 1 & 1 & 0 & \delta^{5} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \begin{bmatrix} 1 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{48577} = \mathbf{Pl}(0,11,1,0,0,1)_{4682}$$

$$\ell_{7} = \begin{bmatrix} 1 & 1 & 0 & \delta^{10} \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \begin{bmatrix} 1 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{44209} = \mathbf{Pl}(0,10,1,0,0,1)_{4681}$$

$$\ell_{8} = \begin{bmatrix} 1 & 0 & 1 & \delta^{10} \\ 0 & 1 & 0 & 1 \end{bmatrix}_{43969} = \begin{bmatrix} 1 & 0 & 1 & 10 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{43969} = \mathbf{Pl}(1,1,0,10,1,1)_{9336}$$

$$\ell_{9} = \begin{bmatrix} 1 & 0 & 1 & \delta^{5} \\ 0 & 1 & 0 & 1 \end{bmatrix}_{48337} = \begin{bmatrix} 1 & 0 & 1 & 11 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{48337} = \mathbf{Pl}(1,1,0,11,1,1)_{9351}$$

Rank of lines: (69904, 48048, 43680, 70160, 48304, 43936, 48577, 44209, 43969, 48337) Rank of points on Klein quadric: (33, 43, 44, 1, 28, 27, 4682, 4681, 9336, 9351)

Eckardt Points

The surface has 2 Eckardt points:

$$0: P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0),$$

1:
$$P_{290} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1).$$

Double Points

The surface has 9 Double points:

The double points on the surface are:

$$\begin{array}{ll} P_3 = (0,0,0,1) = \ell_0 \cap \ell_3 & P_{2843} = (10,0,10,1) = \ell_4 \cap \ell_9 \\ P_{284} = (10,0,0,1) = \ell_1 \cap \ell_4 & P_{3100} = (11,0,11,1) = \ell_5 \cap \ell_8 \\ P_{444} = (10,10,0,1) = \ell_1 \cap \ell_6 & P_{3003} = (10,10,10,1) = \ell_6 \cap \ell_8 \\ P_{285} = (11,0,0,1) = \ell_2 \cap \ell_5 & P_{3276} = (11,11,11,1) = \ell_7 \cap \ell_9 \\ P_{461} = (11,11,0,1) = \ell_2 \cap \ell_7 & P_{3276} = (11,11,11,1) = \ell_7 \cap \ell_9 \end{array}$$

Single Points

The surface has 141 single points: The single points on the surface are:

```
54: P_{955} = (10, 10, 2, 1) lies on line \ell_6
0: P_{180} = (1, 10, 1, 0) lies on line \ell_8
1: P_{196} = (1, 11, 1, 0) lies on line \ell_9
                                                                      55: P_{972} = (11, 11, 2, 1) lies on line \ell_7
2: P_{300} = (10, 1, 0, 1) lies on line \ell_1
                                                                      56: P_{979} = (2, 12, 2, 1) lies on line \ell_8
3: P_{301} = (11, 1, 0, 1) lies on line \ell_2
                                                                      57: P_{1011} = (2, 14, 2, 1) lies on line \ell_9
4: P_{306} = (0, 2, 0, 1) lies on line \ell_0
                                                                      58: P_{1041} = (0,0,3,1) lies on line \ell_3
                                                                      59: P_{1051} = (10, 0, 3, 1) lies on line \ell_4
5: P_{316} = (10, 2, 0, 1) lies on line \ell_1
6: P_{317} = (11, 2, 0, 1) lies on line \ell_2
                                                                      60: P_{1052} = (11, 0, 3, 1) lies on line \ell_5
7: P_{322} = (0, 3, 0, 1) lies on line \ell_0
                                                                      61: P_{1124} = (3, 5, 3, 1) lies on line \ell_9
8: P_{332} = (10, 3, 0, 1) lies on line \ell_1
                                                                      62: P_{1140} = (3, 6, 3, 1) lies on line \ell_8
9: P_{333} = (11, 3, 0, 1) lies on line \ell_2
                                                                      63: P_{1211} = (10, 10, 3, 1) lies on line \ell_6
10: P_{338} = (0, 4, 0, 1) lies on line \ell_0
                                                                      64: P_{1228} = (11, 11, 3, 1) lies on line \ell_7
                                                                      65: P_{1297} = (0, 0, 4, 1) lies on line \ell_3
11: P_{348} = (10, 4, 0, 1) lies on line \ell_1
12: P_{349} = (11, 4, 0, 1) lies on line \ell_2
                                                                      66: P_{1307} = (10, 0, 4, 1) lies on line \ell_4
13: P_{354} = (0, 5, 0, 1) lies on line \ell_0
                                                                      67: P_{1308} = (11, 0, 4, 1) lies on line \ell_5
                                                                      68: P_{1333} = (4, 2, 4, 1) lies on line \ell_8
14: P_{364} = (10, 5, 0, 1) lies on line \ell_1
15: P_{365} = (11, 5, 0, 1) lies on line \ell_2
                                                                      69: P_{1397} = (4, 6, 4, 1) lies on line \ell_9
16: P_{370} = (0, 6, 0, 1) lies on line \ell_0
                                                                      70: P_{1467} = (10, 10, 4, 1) lies on line \ell_6
17: P_{380} = (10, 6, 0, 1) lies on line \ell_1
                                                                      71: P_{1484} = (11, 11, 4, 1) lies on line \ell_7
18: P_{381} = (11, 6, 0, 1) lies on line \ell_2
                                                                      72: P_{1553} = (0, 0, 5, 1) lies on line \ell_3
19: P_{386} = (0,7,0,1) lies on line \ell_0
                                                                      73: P_{1563} = (10, 0, 5, 1) lies on line \ell_4
20: P_{396} = (10, 7, 0, 1) lies on line \ell_1
                                                                      74: P_{1564} = (11, 0, 5, 1) lies on line \ell_5
21: P_{397} = (11, 7, 0, 1) lies on line \ell_2
                                                                      75: P_{1686} = (5, 8, 5, 1) lies on line \ell_8
22: P_{402} = (0, 8, 0, 1) lies on line \ell_0
                                                                      76: P_{1723} = (10, 10, 5, 1) lies on line \ell_6
                                                                      77 : P_{1740} = (11, 11, 5, 1) lies on line \ell_7
23: P_{412} = (10, 8, 0, 1) lies on line \ell_1
24: P_{413} = (11, 8, 0, 1) lies on line \ell_2
                                                                      78: P_{1766} = (5, 13, 5, 1) lies on line \ell_9
25: P_{418} = (0, 9, 0, 1) lies on line \ell_0
                                                                      79: P_{1809} = (0, 0, 6, 1) lies on line \ell_3
26: P_{428} = (10, 9, 0, 1) lies on line \ell_1
                                                                      80: P_{1819} = (10, 0, 6, 1) lies on line \ell_4
27: P_{429} = (11, 9, 0, 1) lies on line \ell_2
                                                                      81: P_{1820} = (11, 0, 6, 1) lies on line \ell_5
28: P_{434} = (0, 10, 0, 1) lies on line \ell_0
                                                                      82: P_{1959} = (6, 9, 6, 1) lies on line \ell_9
29: P_{445} = (11, 10, 0, 1) lies on line \ell_2
                                                                      83: P_{1979} = (10, 10, 6, 1) lies on line \ell_6
30: P_{450} = (0, 11, 0, 1) lies on line \ell_0
                                                                      84: P_{1996} = (11, 11, 6, 1) lies on line \ell_7
31: P_{460} = (10, 11, 0, 1) lies on line \ell_1
                                                                      85: P_{2055} = (6, 15, 6, 1) lies on line \ell_8
32: P_{466} = (0, 12, 0, 1) lies on line \ell_0
                                                                      86: P_{2065} = (0,0,7,1) lies on line \ell_3
                                                                      87: P_{2075} = (10, 0, 7, 1) lies on line \ell_4
33: P_{476} = (10, 12, 0, 1) lies on line \ell_1
34: P_{477} = (11, 12, 0, 1) lies on line \ell_2
                                                                      88: P_{2076} = (11, 0, 7, 1) lies on line \ell_5
                                                                      89 : P_{2104} = (7, 2, 7, 1) lies on line \ell_9
35: P_{482} = (0, 13, 0, 1) lies on line \ell_0
36: P_{492} = (10, 13, 0, 1) lies on line \ell_1
                                                                      90: P_{2152} = (7, 5, 7, 1) lies on line \ell_8
37: P_{493} = (11, 13, 0, 1) lies on line \ell_2
                                                                      91: P_{2235} = (10, 10, 7, 1) lies on line \ell_6
38: P_{498} = (0, 14, 0, 1) lies on line \ell_0
                                                                      92: P_{2252} = (11, 11, 7, 1) lies on line \ell_7
                                                                      93: P_{2321} = (0,0,8,1) lies on line \ell_3
39: P_{508} = (10, 14, 0, 1) lies on line \ell_1
40: P_{509} = (11, 14, 0, 1) lies on line \ell_2
                                                                      94: P_{2331} = (10, 0, 8, 1) lies on line \ell_4
41: P_{514} = (0, 15, 0, 1) lies on line \ell_0
                                                                      95: P_{2332} = (11, 0, 8, 1) lies on line \ell_5
42: P_{524} = (10, 15, 0, 1) lies on line \ell_1
                                                                      96: P_{2441} = (8,7,8,1) lies on line \ell_8
43: P_{525} = (11, 15, 0, 1) lies on line \ell_2
                                                                      97: P_{2491} = (10, 10, 8, 1) lies on line \ell_6
44: P_{530} = (0, 0, 1, 1) lies on line \ell_3
                                                                      98: P_{2508} = (11, 11, 8, 1) lies on line \ell_7
45: P_{540} = (10, 0, 1, 1) lies on line \ell_4
                                                                      99: P_{2569} = (8, 15, 8, 1) lies on line \ell_9
46: P_{541} = (11, 0, 1, 1) lies on line \ell_5
                                                                      100: P_{2577} = (0, 0, 9, 1) lies on line \ell_3
47: P_{690} = (1, 10, 1, 1) lies on line \ell_9
                                                                      101: P_{2587} = (10, 0, 9, 1) lies on line \ell_4
48: P_{699} = (10, 10, 1, 1) lies on line \ell_6
                                                                      102: P_{2588} = (11, 0, 9, 1) lies on line \ell_5
49: P_{706} = (1, 11, 1, 1) lies on line \ell_8
                                                                      103: P_{2650} = (9, 4, 9, 1) lies on line \ell_9
50: P_{716} = (11, 11, 1, 1) lies on line \ell_7
                                                                      104: P_{2747} = (10, 10, 9, 1) lies on line \ell_6
51: P_{785} = (0,0,2,1) lies on line \ell_3
                                                                      105: P_{2764} = (11, 11, 9, 1) lies on line \ell_7
                                                                      106 : P_{2794} = (9, 13, 9, 1) lies on line \ell_8
52: P_{795} = (10, 0, 2, 1) lies on line \ell_4
53: P_{796} = (11, 0, 2, 1) lies on line \ell_5
                                                                      107: P_{2833} = (0, 0, 10, 1) lies on line \ell_3
```

```
125 : P_{3788} = (11, 11, 13, 1) lies on line \ell_7
108: P_{2844} = (11, 0, 10, 1) lies on line \ell_5
109: P_{3020} = (11, 11, 10, 1) lies on line \ell_7
                                                                    126: P_{3838} = (13, 14, 13, 1) lies on line \ell_8
110: P_{3089} = (0, 0, 11, 1) lies on line \ell_3
                                                                    127: P_{3857} = (0, 0, 14, 1) lies on line \ell_3
111: P_{3099} = (10, 0, 11, 1) lies on line \ell_4
                                                                    128: P_{3867} = (10, 0, 14, 1) lies on line \ell_4
112: P_{3259} = (10, 10, 11, 1) lies on line \ell_6
                                                                    129: P_{3868} = (11, 0, 14, 1) lies on line \ell_5
113: P_{3345} = (0, 0, 12, 1) lies on line \ell_3
                                                                    130: P_{3983} = (14, 7, 14, 1) lies on line \ell_9
114: P_{3355} = (10, 0, 12, 1) lies on line \ell_4
                                                                    131: P_{4015} = (14, 9, 14, 1) lies on line \ell_8
115: P_{3356} = (11, 0, 12, 1) lies on line \ell_5
                                                                     132: P_{4027} = (10, 10, 14, 1) lies on line \ell_6
116: P_{3421} = (12, 4, 12, 1) lies on line \ell_8
                                                                    133: P_{4044} = (11, 11, 14, 1) lies on line \ell_7
117: P_{3485} = (12, 8, 12, 1) lies on line \ell_9
                                                                    134: P_{4113} = (0, 0, 15, 1) lies on line \ell_3
118: P_{3515} = (10, 10, 12, 1) lies on line \ell_6
                                                                    135: P_{4123} = (10, 0, 15, 1) lies on line \ell_4
119: P_{3532} = (11, 11, 12, 1) lies on line \ell_7
                                                                    136: P_{4124} = (11, 0, 15, 1) lies on line \ell_5
120: P_{3601} = (0, 0, 13, 1) lies on line \ell_3
                                                                    137: P_{4176} = (15, 3, 15, 1) lies on line \ell_8
121: P_{3611} = (10, 0, 13, 1) lies on line \ell_4
                                                                    138: P_{4283} = (10, 10, 15, 1) lies on line \ell_6
122: P_{3612} = (11, 0, 13, 1) lies on line \ell_5
                                                                    139: P_{4300} = (11, 11, 15, 1) lies on line \ell_7
123: P_{3662} = (13, 3, 13, 1) lies on line \ell_9
                                                                    140: P_{4320} = (15, 12, 15, 1) lies on line \ell_9
124: P_{3771} = (10, 10, 13, 1) lies on line \ell_6
```

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line: The points on the surface but not on lines are:

```
0: P_{78} = (11, 3, 1, 0)
                                                                  27: P_{1037} = (12, 15, 2, 1)
                                                                  28: P_{1065} = (8, 1, 3, 1)
1: P_{109} = (10, 5, 1, 0)
2: P_{128} = (13, 6, 1, 0)
                                                                  29: P_{1093} = (4, 3, 3, 1)
                                                                  30: P_{1157} = (4,7,3,1)
3: P_{143} = (12,7,1,0)
4: P_{158} = (11, 8, 1, 0)
                                                                  31: P_{1175} = (6, 8, 3, 1)
                                                                  32: P_{1193} = (8, 9, 3, 1)
5: P_{185} = (6, 10, 1, 0)
6: P_{186} = (7, 10, 1, 0)
                                                                  33: P_{1271} = (6, 14, 3, 1)
7: P_{207} = (12, 11, 1, 0)
                                                                  34: P_{1326} = (13, 1, 4, 1)
                                                                  35: P_{1330} = (1, 2, 4, 1)
8: P_{208} = (13, 11, 1, 0)
9: P_{217} = (6, 12, 1, 0)
                                                                  36: P_{1346} = (1, 3, 4, 1)
10: P_{234} = (7, 13, 1, 0)
                                                                  37: P_{1351} = (6, 3, 4, 1)
11: P_{269} = (10, 15, 1, 0)
                                                                  38: P_{1383} = (6, 5, 4, 1)
12: P_{566} = (5, 2, 1, 1)
                                                                  39: P_{1418} = (9,7,4,1)
13: P_{601} = (8, 4, 1, 1)
                                                                  40: P_{1502} = (13, 12, 4, 1)
14: P_{640} = (15, 6, 1, 1)
                                                                  41: P_{1530} = (9, 14, 4, 1)
                                                                  42: P_{1584} = (15, 1, 5, 1)
15: P_{646} = (5, 7, 1, 1)
16: P_{688} = (15, 9, 1, 1)
                                                                  43: P_{1598} = (13, 2, 5, 1)
17: P_{729} = (8, 12, 1, 1)
                                                                  44: P_{1642} = (9, 5, 5, 1)
                                                                  45: P_{1754} = (9, 12, 5, 1)
18: P_{740} = (3, 13, 1, 1)
19: P_{756} = (3, 14, 1, 1)
                                                                  46: P_{1792} = (15, 14, 5, 1)
20: P_{807} = (6, 1, 2, 1)
                                                                  47: P_{1806} = (13, 15, 5, 1)
21: P_{845} = (12, 3, 2, 1)
                                                                  48: P_{1839} = (14, 1, 6, 1)
                                                                  49: P_{1862} = (5, 3, 6, 1)
22: P_{903} = (6,7,2,1)
23: P_{933} = (4, 9, 2, 1)
                                                                  50: P_{1904} = (15, 5, 6, 1)
24: P_{997} = (4, 13, 2, 1)
                                                                  51: P_{1910} = (5, 6, 6, 1)
25: P_{1010} = (1, 14, 2, 1)
                                                                  52: P_{1914} = (9, 6, 6, 1)
26: P_{1026} = (1, 15, 2, 1)
                                                                  53: P_{1918} = (13, 6, 6, 1)
```

```
54: P_{1973} = (4, 10, 6, 1)
                                                                 108: P_{3127} = (6, 2, 11, 1)
                                                                 109: P_{3129} = (8, 2, 11, 1)
55: P_{1984} = (15, 10, 6, 1)
                                                                 110: P_{3136} = (15, 2, 11, 1)
56: P_{1992} = (7, 11, 6, 1)
                                                                 111: P_{3159} = (6, 4, 11, 1)
57: P_{1998} = (13, 11, 6, 1)
58: P_{2008} = (7, 12, 6, 1)
                                                                 112: P_{3183} = (14, 5, 11, 1)
59: P_{2037} = (4, 14, 6, 1)
                                                                 113: P_{3236} = (3, 9, 11, 1)
60: P_{2058} = (9, 15, 6, 1)
                                                                 114: P_{3238} = (5, 9, 11, 1)
61: P_{2063} = (14, 15, 6, 1)
                                                                 115: P_{3240} = (7, 9, 11, 1)
62: P_{2085} = (4, 1, 7, 1)
                                                                 116: P_{3252} = (3, 10, 11, 1)
63: P_{2143} = (14, 4, 7, 1)
                                                                 117: P_{3257} = (8, 10, 11, 1)
64: P_{2147} = (2, 5, 7, 1)
                                                                 118: P_{3269} = (4, 11, 11, 1)
65: P_{2149} = (4, 5, 7, 1)
                                                                 119: P_{3279} = (14, 11, 11, 1)
66: P_{2179} = (2,7,7,1)
                                                                 120: P_{3282} = (1, 12, 11, 1)
67: P_{2189} = (12, 7, 7, 1)
                                                                 121: P_{3286} = (5, 12, 11, 1)
                                                                 122: P_{3298} = (1, 13, 11, 1)
68: P_{2192} = (15, 7, 7, 1)
69: P_{2208} = (15, 8, 7, 1)
                                                                 123: P_{3312} = (15, 13, 11, 1)
                                                                 124: P_{3320} = (7, 14, 11, 1)
70: P_{2230} = (5, 10, 7, 1)
                                                                 125: P_{3333} = (4, 15, 11, 1)
71: P_{2239} = (14, 10, 7, 1)
72: P_{2247} = (6, 11, 7, 1)
                                                                 126: P_{3370} = (9, 1, 12, 1)
73: P_{2253} = (12, 11, 7, 1)
                                                                 127: P_{3401} = (8, 3, 12, 1)
74: P_{2279} = (6, 13, 7, 1)
                                                                 128: P_{3470} = (13, 7, 12, 1)
                                                                 129: P_{3477} = (4, 8, 12, 1)
75: P_{2310} = (5, 15, 7, 1)
                                                                 130: P_{3482} = (9, 8, 12, 1)
76: P_{2340} = (3, 1, 8, 1)
77: P_{2356} = (3, 2, 8, 1)
                                                                 131: P_{3491} = (2, 9, 12, 1)
78: P_{2376} = (7, 3, 8, 1)
                                                                 132: P_{3511} = (6, 10, 12, 1)
79: P_{2392} = (7, 4, 8, 1)
                                                                 133: P_{3518} = (13, 10, 12, 1)
                                                                 134: P_{3523} = (2, 11, 12, 1)
80: P_{2431} = (14, 6, 8, 1)
81: P_{2463} = (14, 8, 8, 1)
                                                                 135: P_{3529} = (8, 11, 12, 1)
82: P_{2600} = (7, 1, 9, 1)
                                                                 136: P_{3540} = (3, 12, 12, 1)
83: P_{2623} = (14, 2, 9, 1)
                                                                 137: P_{3541} = (4, 12, 12, 1)
84: P_{2642} = (1,4,9,1)
                                                                 138: P_{3543} = (6, 12, 12, 1)
85: P_{2658} = (1, 5, 9, 1)
                                                                 139: P_{3588} = (3, 15, 12, 1)
86: P_{2670} = (13, 5, 9, 1)
                                                                 140: P_{3619} = (2, 1, 13, 1)
87: P_{2680} = (7, 6, 9, 1)
                                                                 141: P_{3642} = (9, 2, 13, 1)
                                                                 142: P_{3651} = (2, 3, 13, 1)
88: P_{2718} = (13, 8, 9, 1)
                                                                 143: P_{3663} = (14, 3, 13, 1)
89: P_{2783} = (14, 12, 9, 1)
90: P_{2877} = (12, 2, 10, 1)
                                                                 144: P_{3689} = (8, 5, 13, 1)
                                                                 145: P_{3709} = (12, 6, 13, 1)
91: P_{2890} = (9, 3, 10, 1)
92: P_{2900} = (3, 4, 10, 1)
                                                                 146: P_{3732} = (3, 8, 13, 1)
93: P_{2910} = (13, 4, 10, 1)
                                                                 147: P_{3768} = (7, 10, 13, 1)
94: P_{2912} = (15, 4, 10, 1)
                                                                 148: P_{3773} = (12, 10, 13, 1)
95: P_{2930} = (1, 6, 10, 1)
                                                                 149: P_{3780} = (3, 11, 13, 1)
96: P_{2937} = (8, 6, 10, 1)
                                                                 150: P_{3786} = (9, 11, 13, 1)
                                                                 151: P_{3816} = (7, 13, 13, 1)
97: P_{2946} = (1, 7, 10, 1)
98: P_{2948} = (3, 7, 10, 1)
                                                                 152: P_{3817} = (8, 13, 13, 1)
99: P_{2963} = (2, 8, 10, 1)
                                                                 153: P_{3823} = (14, 13, 13, 1)
100: P_{2990} = (13, 9, 10, 1)
                                                                 154: P_{3885} = (12, 1, 14, 1)
101: P_{2995} = (2, 10, 10, 1)
                                                                 155: P_{3923} = (2, 4, 14, 1)
102: P_{3002} = (9, 10, 10, 1)
                                                                 156: P_{3955} = (2, 6, 14, 1)
103: P_{3014} = (5, 11, 10, 1)
                                                                 157: P_{3986} = (1, 8, 14, 1)
104: P_{3024} = (15, 11, 10, 1)
                                                                 158: P_{3992} = (7, 8, 14, 1)
105: P_{3062} = (5, 14, 10, 1)
                                                                 159: P_{4002} = (1, 9, 14, 1)
106: P_{3065} = (8, 14, 10, 1)
                                                                 160: P_{4077} = (12, 13, 14, 1)
107: P_{3069} = (12, 14, 10, 1)
                                                                 161: P_{4104} = (7, 15, 14, 1)
```

```
\begin{array}{ll} 162: \ P_{4134} = (5,1,15,1) & 166: \ P_{4323} = (2,13,15,1) \\ 163: \ P_{4182} = (5,4,15,1) & 167: \ P_{4355} = (2,15,15,1) \\ 164: \ P_{4205} = (12,5,15,1) & 165: \ P_{4269} = (12,9,15,1) \end{array}
```

Line Intersection Graph

 $\begin{array}{c} 0123456789 \\ \hline 0 & 01111000011 \\ 1 & 1010101000 \\ 2 & 1100010100 \\ 3 & 1000111100 \\ 4 & 0101011101 \\ 5 & 0011110110 \\ 6 & 0101111011 \\ 7 & 0011111001 \\ 8 & 1000011001 \\ 9 & 1000100110 \end{array}$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{290}	P_{290}

 ${\bf Line~1~intersects}$

Line	ℓ_0	ℓ_2	ℓ_4	ℓ_6
in point	P_1	P_1	P_{284}	P_{444}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_7
in point	P_1	P_1	P_{285}	P_{461}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_2	P_2	P_2	P_2

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_{284}	P_2	P_2	P_2	P_2	P_{2843}

 ${\bf Line~5~intersects}$

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8
in point	P_{285}	P_2	P_2	P_2	P_2	P_{3100}

 ${\bf Line~6~intersects}$

Line	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_{444}	P_2	P_2	P_2	P_2	P_{3003}

 ${\bf Line}\ 7\ {\bf intersects}$

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_{461}	P_2	P_2	P_2	P_2	P_{3276}

Line 8 intersects

Line ℓ_0		ℓ_5	ℓ_6	ℓ_9
in point	P_{290}	P_{3100}	P_{3003}	P_{290}

Line 9 intersects

Line	ℓ_0	ℓ_4	ℓ_7	ℓ_8	
in point	P_{290}	P_{2843}	P_{3276}	P_{290}	

The surface has 321 points: The points on the surface are:

```
0: P_1 = (0, 1, 0, 0)
                                            54: P_{477} = (11, 12, 0, 1)
                                                                                        108: P_{1307} = (10, 0, 4, 1)
1: P_2 = (0,0,1,0)
                                            55: P_{482} = (0, 13, 0, 1)
                                                                                        109: P_{1308} = (11, 0, 4, 1)
                                                                                        110: P_{1326} = (13, 1, 4, 1)
2: P_3 = (0,0,0,1)
                                            56: P_{492} = (10, 13, 0, 1)
3: P_{78} = (11, 3, 1, 0)
                                                                                        111: P_{1330} = (1, 2, 4, 1)
                                            57: P_{493} = (11, 13, 0, 1)
4: P_{109} = (10, 5, 1, 0)
                                            58: P_{498} = (0, 14, 0, 1)
                                                                                        112: P_{1333} = (4, 2, 4, 1)
                                            59: P_{508} = (10, 14, 0, 1)
                                                                                        113: P_{1346} = (1, 3, 4, 1)
5: P_{128} = (13, 6, 1, 0)
6: P_{143} = (12, 7, 1, 0)
                                            60: P_{509} = (11, 14, 0, 1)
                                                                                        114: P_{1351} = (6, 3, 4, 1)
                                            61: P_{514} = (0, 15, 0, 1)
7: P_{158} = (11, 8, 1, 0)
                                                                                        115: P_{1383} = (6, 5, 4, 1)
8: P_{180} = (1, 10, 1, 0)
                                            62: P_{524} = (10, 15, 0, 1)
                                                                                        116: P_{1397} = (4, 6, 4, 1)
9: P_{185} = (6, 10, 1, 0)
                                            63: P_{525} = (11, 15, 0, 1)
                                                                                        117: P_{1418} = (9,7,4,1)
10: P_{186} = (7, 10, 1, 0)
                                            64: P_{530} = (0,0,1,1)
                                                                                        118: P_{1467} = (10, 10, 4, 1)
11: P_{196} = (1, 11, 1, 0)
                                            65: P_{540} = (10, 0, 1, 1)
                                                                                        119: P_{1484} = (11, 11, 4, 1)
12: P_{207} = (12, 11, 1, 0)
                                            66: P_{541} = (11, 0, 1, 1)
                                                                                        120: P_{1502} = (13, 12, 4, 1)
13: P_{208} = (13, 11, 1, 0)
                                            67: P_{566} = (5, 2, 1, 1)
                                                                                        121: P_{1530} = (9, 14, 4, 1)
                                                                                        122: P_{1553} = (0,0,5,1)
14: P_{217} = (6, 12, 1, 0)
                                            68: P_{601} = (8, 4, 1, 1)
15: P_{234} = (7, 13, 1, 0)
                                            69: P_{640} = (15, 6, 1, 1)
                                                                                        123: P_{1563} = (10, 0, 5, 1)
16: P_{269} = (10, 15, 1, 0)
                                            70: P_{646} = (5, 7, 1, 1)
                                                                                        124: P_{1564} = (11, 0, 5, 1)
17: P_{284} = (10, 0, 0, 1)
                                            71: P_{688} = (15, 9, 1, 1)
                                                                                        125: P_{1584} = (15, 1, 5, 1)
18: P_{285} = (11, 0, 0, 1)
                                                                                        126: P_{1598} = (13, 2, 5, 1)
                                            72: P_{690} = (1, 10, 1, 1)
19: P_{290} = (0, 1, 0, 1)
                                            73: P_{699} = (10, 10, 1, 1)
                                                                                        127: P_{1642} = (9, 5, 5, 1)
20: P_{300} = (10, 1, 0, 1)
                                            74: P_{706} = (1, 11, 1, 1)
                                                                                        128: P_{1686} = (5, 8, 5, 1)
21: P_{301} = (11, 1, 0, 1)
                                                                                        129: P_{1723} = (10, 10, 5, 1)
                                            75: P_{716} = (11, 11, 1, 1)
                                            76: P_{729} = (8, 12, 1, 1)
                                                                                        130: P_{1740} = (11, 11, 5, 1)
22: P_{306} = (0, 2, 0, 1)
23: P_{316} = (10, 2, 0, 1)
                                            77: P_{740} = (3, 13, 1, 1)
                                                                                        131: P_{1754} = (9, 12, 5, 1)
24: P_{317} = (11, 2, 0, 1)
                                            78: P_{756} = (3, 14, 1, 1)
                                                                                        132: P_{1766} = (5, 13, 5, 1)
25: P_{322} = (0, 3, 0, 1)
                                            79: P_{785} = (0, 0, 2, 1)
                                                                                        133: P_{1792} = (15, 14, 5, 1)
26: P_{332} = (10, 3, 0, 1)
                                            80: P_{795} = (10, 0, 2, 1)
                                                                                        134: P_{1806} = (13, 15, 5, 1)
27: P_{333} = (11, 3, 0, 1)
                                            81: P_{796} = (11, 0, 2, 1)
                                                                                        135: P_{1809} = (0, 0, 6, 1)
28: P_{338} = (0,4,0,1)
                                            82: P_{807} = (6, 1, 2, 1)
                                                                                        136: P_{1819} = (10, 0, 6, 1)
29: P_{348} = (10, 4, 0, 1)
                                            83: P_{845} = (12, 3, 2, 1)
                                                                                        137: P_{1820} = (11, 0, 6, 1)
30: P_{349} = (11, 4, 0, 1)
                                            84: P_{903} = (6,7,2,1)
                                                                                        138: P_{1839} = (14, 1, 6, 1)
                                                                                        139: P_{1862} = (5, 3, 6, 1)
31: P_{354} = (0, 5, 0, 1)
                                            85: P_{933} = (4, 9, 2, 1)
32: P_{364} = (10, 5, 0, 1)
                                            86: P_{955} = (10, 10, 2, 1)
                                                                                        140: P_{1904} = (15, 5, 6, 1)
                                            87: P_{972} = (11, 11, 2, 1)
33: P_{365} = (11, 5, 0, 1)
                                                                                        141: P_{1910} = (5, 6, 6, 1)
34: P_{370} = (0,6,0,1)
                                            88: P_{979} = (2, 12, 2, 1)
                                                                                        142: P_{1914} = (9, 6, 6, 1)
                                                                                        143: P_{1918} = (13, 6, 6, 1)
35: P_{380} = (10, 6, 0, 1)
                                            89: P_{997} = (4, 13, 2, 1)
36: P_{381} = (11, 6, 0, 1)
                                            90: P_{1010} = (1, 14, 2, 1)
                                                                                        144: P_{1959} = (6, 9, 6, 1)
37: P_{386} = (0,7,0,1)
                                            91: P_{1011} = (2, 14, 2, 1)
                                                                                        145: P_{1973} = (4, 10, 6, 1)
38: P_{396} = (10, 7, 0, 1)
                                                                                        146: P_{1979} = (10, 10, 6, 1)
                                            92: P_{1026} = (1, 15, 2, 1)
39: P_{397} = (11,7,0,1)
                                            93: P_{1037} = (12, 15, 2, 1)
                                                                                        147: P_{1984} = (15, 10, 6, 1)
40: P_{402} = (0, 8, 0, 1)
                                            94: P_{1041} = (0, 0, 3, 1)
                                                                                        148: P_{1992} = (7, 11, 6, 1)
41: P_{412} = (10, 8, 0, 1)
                                            95: P_{1051} = (10, 0, 3, 1)
                                                                                        149: P_{1996} = (11, 11, 6, 1)
42: P_{413} = (11, 8, 0, 1)
                                            96: P_{1052} = (11, 0, 3, 1)
                                                                                        150: P_{1998} = (13, 11, 6, 1)
43: P_{418} = (0, 9, 0, 1)
                                            97: P_{1065} = (8, 1, 3, 1)
                                                                                        151: P_{2008} = (7, 12, 6, 1)
44: P_{428} = (10, 9, 0, 1)
                                            98: P_{1093} = (4, 3, 3, 1)
                                                                                        152: P_{2037} = (4, 14, 6, 1)
                                            99: P_{1124} = (3, 5, 3, 1)
                                                                                        153: P_{2055} = (6, 15, 6, 1)
45: P_{429} = (11, 9, 0, 1)
46: P_{434} = (0, 10, 0, 1)
                                            100: P_{1140} = (3, 6, 3, 1)
                                                                                        154: P_{2058} = (9, 15, 6, 1)
47: P_{444} = (10, 10, 0, 1)
                                            101: P_{1157} = (4,7,3,1)
                                                                                        155: P_{2063} = (14, 15, 6, 1)
48: P_{445} = (11, 10, 0, 1)
                                                                                        156: P_{2065} = (0, 0, 7, 1)
                                            102: P_{1175} = (6, 8, 3, 1)
49: P_{450} = (0, 11, 0, 1)
                                            103: P_{1193} = (8, 9, 3, 1)
                                                                                        157: P_{2075} = (10, 0, 7, 1)
50: P_{460} = (10, 11, 0, 1)
                                            104: P_{1211} = (10, 10, 3, 1)
                                                                                        158: P_{2076} = (11, 0, 7, 1)
51: P_{461} = (11, 11, 0, 1)
                                            105: P_{1228} = (11, 11, 3, 1)
                                                                                        159: P_{2085} = (4, 1, 7, 1)
52: P_{466} = (0, 12, 0, 1)
                                            106: P_{1271} = (6, 14, 3, 1)
                                                                                        160: P_{2104} = (7, 2, 7, 1)
53: P_{476} = (10, 12, 0, 1)
                                            107: P_{1297} = (0,0,4,1)
                                                                                        161: P_{2143} = (14, 4, 7, 1)
```

```
162: P_{2147} = (2, 5, 7, 1)
                                           216: P_{2948} = (3, 7, 10, 1)
                                                                                      270: P_{3543} = (6, 12, 12, 1)
163: P_{2149} = (4, 5, 7, 1)
                                           217: P_{2963} = (2, 8, 10, 1)
                                                                                      271: P_{3588} = (3, 15, 12, 1)
                                                                                      272: P_{3601} = (0, 0, 13, 1)
164: P_{2152} = (7, 5, 7, 1)
                                           218: P_{2990} = (13, 9, 10, 1)
165: P_{2179} = (2,7,7,1)
                                           219: P_{2995} = (2, 10, 10, 1)
                                                                                      273: P_{3611} = (10, 0, 13, 1)
166: P_{2189} = (12, 7, 7, 1)
                                           220: P_{3002} = (9, 10, 10, 1)
                                                                                      274: P_{3612} = (11, 0, 13, 1)
167: P_{2192} = (15, 7, 7, 1)
                                           221: P_{3003} = (10, 10, 10, 1)
                                                                                      275: P_{3619} = (2, 1, 13, 1)
168: P_{2208} = (15, 8, 7, 1)
                                           222: P_{3014} = (5, 11, 10, 1)
                                                                                      276: P_{3642} = (9, 2, 13, 1)
169: P_{2230} = (5, 10, 7, 1)
                                           223: P_{3020} = (11, 11, 10, 1)
                                                                                      277: P_{3651} = (2, 3, 13, 1)
170: P_{2235} = (10, 10, 7, 1)
                                           224: P_{3024} = (15, 11, 10, 1)
                                                                                      278: P_{3662} = (13, 3, 13, 1)
171: P_{2239} = (14, 10, 7, 1)
                                           225: P_{3062} = (5, 14, 10, 1)
                                                                                      279: P_{3663} = (14, 3, 13, 1)
172: P_{2247} = (6, 11, 7, 1)
                                           226: P_{3065} = (8, 14, 10, 1)
                                                                                      280: P_{3689} = (8, 5, 13, 1)
173: P_{2252} = (11, 11, 7, 1)
                                           227: P_{3069} = (12, 14, 10, 1)
                                                                                      281: P_{3709} = (12, 6, 13, 1)
174: P_{2253} = (12, 11, 7, 1)
                                           228: P_{3089} = (0, 0, 11, 1)
                                                                                      282: P_{3732} = (3, 8, 13, 1)
175: P_{2279} = (6, 13, 7, 1)
                                           229: P_{3099} = (10, 0, 11, 1)
                                                                                      283: P_{3768} = (7, 10, 13, 1)
                                           230: P_{3100} = (11, 0, 11, 1)
                                                                                      284: P_{3771} = (10, 10, 13, 1)
176: P_{2310} = (5, 15, 7, 1)
177: P_{2321} = (0, 0, 8, 1)
                                           231: P_{3127} = (6, 2, 11, 1)
                                                                                      285: P_{3773} = (12, 10, 13, 1)
178: P_{2331} = (10, 0, 8, 1)
                                           232: P_{3129} = (8, 2, 11, 1)
                                                                                      286: P_{3780} = (3, 11, 13, 1)
                                                                                      287: P_{3786} = (9, 11, 13, 1)
179: P_{2332} = (11, 0, 8, 1)
                                           233: P_{3136} = (15, 2, 11, 1)
                                                                                      288: P_{3788} = (11, 11, 13, 1)
180: P_{2340} = (3, 1, 8, 1)
                                           234: P_{3159} = (6, 4, 11, 1)
181: P_{2356} = (3, 2, 8, 1)
                                           235: P_{3183} = (14, 5, 11, 1)
                                                                                      289: P_{3816} = (7, 13, 13, 1)
182: P_{2376} = (7, 3, 8, 1)
                                           236: P_{3236} = (3, 9, 11, 1)
                                                                                      290: P_{3817} = (8, 13, 13, 1)
183: P_{2392} = (7, 4, 8, 1)
                                           237: P_{3238} = (5, 9, 11, 1)
                                                                                      291: P_{3823} = (14, 13, 13, 1)
                                           238: P_{3240} = (7, 9, 11, 1)
184: P_{2431} = (14, 6, 8, 1)
                                                                                      292: P_{3838} = (13, 14, 13, 1)
185: P_{2441} = (8,7,8,1)
                                           239: P_{3252} = (3, 10, 11, 1)
                                                                                      293: P_{3857} = (0, 0, 14, 1)
186: P_{2463} = (14, 8, 8, 1)
                                           240: P_{3257} = (8, 10, 11, 1)
                                                                                      294: P_{3867} = (10, 0, 14, 1)
187: P_{2491} = (10, 10, 8, 1)
                                           241: P_{3259} = (10, 10, 11, 1)
                                                                                      295: P_{3868} = (11, 0, 14, 1)
188: P_{2508} = (11, 11, 8, 1)
                                           242: P_{3269} = (4, 11, 11, 1)
                                                                                      296: P_{3885} = (12, 1, 14, 1)
189 : P_{2569} = (8, 15, 8, 1)
                                           243: P_{3276} = (11, 11, 11, 1)
                                                                                      297: P_{3923} = (2, 4, 14, 1)
190: P_{2577} = (0,0,9,1)
                                           244: P_{3279} = (14, 11, 11, 1)
                                                                                      298: P_{3955} = (2, 6, 14, 1)
191: P_{2587} = (10, 0, 9, 1)
                                           245: P_{3282} = (1, 12, 11, 1)
                                                                                      299: P_{3983} = (14, 7, 14, 1)
192: P_{2588} = (11, 0, 9, 1)
                                           246: P_{3286} = (5, 12, 11, 1)
                                                                                      300: P_{3986} = (1, 8, 14, 1)
193: P_{2600} = (7, 1, 9, 1)
                                           247: P_{3298} = (1, 13, 11, 1)
                                                                                      301: P_{3992} = (7, 8, 14, 1)
194: P_{2623} = (14, 2, 9, 1)
                                           248: P_{3312} = (15, 13, 11, 1)
                                                                                      302: P_{4002} = (1, 9, 14, 1)
195: P_{2642} = (1, 4, 9, 1)
                                           249: P_{3320} = (7, 14, 11, 1)
                                                                                      303: P_{4015} = (14, 9, 14, 1)
196: P_{2650} = (9, 4, 9, 1)
                                           250: P_{3333} = (4, 15, 11, 1)
                                                                                      304: P_{4027} = (10, 10, 14, 1)
197: P_{2658} = (1, 5, 9, 1)
                                           251: P_{3345} = (0, 0, 12, 1)
                                                                                      305: P_{4044} = (11, 11, 14, 1)
198: P_{2670} = (13, 5, 9, 1)
                                           252: P_{3355} = (10, 0, 12, 1)
                                                                                      306: P_{4077} = (12, 13, 14, 1)
                                           253: P_{3356} = (11, 0, 12, 1)
199: P_{2680} = (7, 6, 9, 1)
                                                                                      307: P_{4104} = (7, 15, 14, 1)
200: P_{2718} = (13, 8, 9, 1)
                                           254: P_{3370} = (9, 1, 12, 1)
                                                                                      308: P_{4113} = (0, 0, 15, 1)
201: P_{2747} = (10, 10, 9, 1)
                                           255: P_{3401} = (8, 3, 12, 1)
                                                                                      309: P_{4123} = (10, 0, 15, 1)
202: P_{2764} = (11, 11, 9, 1)
                                           256: P_{3421} = (12, 4, 12, 1)
                                                                                      310: P_{4124} = (11, 0, 15, 1)
                                           257: P_{3470} = (13, 7, 12, 1)
                                                                                      311: P_{4134} = (5, 1, 15, 1)
203: P_{2783} = (14, 12, 9, 1)
204: P_{2794} = (9, 13, 9, 1)
                                           258: P_{3477} = (4, 8, 12, 1)
                                                                                      312: P_{4176} = (15, 3, 15, 1)
                                           259: P_{3482} = (9, 8, 12, 1)
                                                                                      313: P_{4182} = (5, 4, 15, 1)
205: P_{2833} = (0, 0, 10, 1)
206: P_{2843} = (10, 0, 10, 1)
                                           260: P_{3485} = (12, 8, 12, 1)
                                                                                      314: P_{4205} = (12, 5, 15, 1)
                                           261: P_{3491} = (2, 9, 12, 1)
207: P_{2844} = (11, 0, 10, 1)
                                                                                      315: P_{4269} = (12, 9, 15, 1)
208: P_{2877} = (12, 2, 10, 1)
                                           262: P_{3511} = (6, 10, 12, 1)
                                                                                      316: P_{4283} = (10, 10, 15, 1)
209: P_{2890} = (9, 3, 10, 1)
                                           263: P_{3515} = (10, 10, 12, 1)
                                                                                      317: P_{4300} = (11, 11, 15, 1)
210: P_{2900} = (3, 4, 10, 1)
                                           264: P_{3518} = (13, 10, 12, 1)
                                                                                      318: P_{4320} = (15, 12, 15, 1)
211: P_{2910} = (13, 4, 10, 1)
                                           265: P_{3523} = (2, 11, 12, 1)
                                                                                      319: P_{4323} = (2, 13, 15, 1)
212: P_{2912} = (15, 4, 10, 1)
                                           266: P_{3529} = (8, 11, 12, 1)
                                                                                      320: P_{4355} = (2, 15, 15, 1)
213: P_{2930} = (1, 6, 10, 1)
                                           267: P_{3532} = (11, 11, 12, 1)
214: P_{2937} = (8, 6, 10, 1)
                                           268: P_{3540} = (3, 12, 12, 1)
215: P_{2946} = (1, 7, 10, 1)
                                           269: P_{3541} = (4, 12, 12, 1)
```