Rank-264 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1^2 = 0$$

(0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(2) is 264

General information

Number of lines	5
Number of points	11
Number of singular points	0
Number of Eckardt points	2
Number of double points	0
Number of single points	9
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^5
Type of lines on points	3^2, 1^9

Singular Points

The surface has 0 singular points:

The 5 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{5} = \mathbf{Pl}(0,0,1,0,1,0)_{12}$$

$$\ell_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{29} = \mathbf{Pl}(0,0,0,1,0,1)_{25}$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{12} = \mathbf{Pl}(0,0,1,1,1,1)_{32}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{10} = \mathbf{Pl}(1,1,1,0,1,1)_{30}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1,1,1,1,1,0)_{18}$$

Rank of lines: (5, 29, 12, 10, 17)
Rank of points on Klein quadric: (12, 25, 32, 30, 18)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1),\ T = 1$
$1 : P_{14} = \mathbf{P}(0,1,1,1) = \mathbf{P}(0,1,1,1).\ T = 8$

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 9 single points:
The single points on the surface are:

$0 :\ P_0 = (1,0,0,0)$ lies on line ℓ_0
$1 :\ P_1 = (0,1,0,0)$ lies on line ℓ_1
$2 :\ P_4 = (1,1,1,1)$ lies on line ℓ_2
$3 :\ P_5 = (1,1,0,0)$ lies on line ℓ_2
$4 :\ P_6 = (1,0,1,0)$ lies on line ℓ_3
$5 :\ P_8 = (1,1,1,0)$ lies on line ℓ_4
$6 :\ P_9 = (1,0,0,1)$ lies on line ℓ_4
$7 :\ P_{11} = (1,1,0,1)$ lies on line ℓ_3
$8 :\ P_{13} = (1,0,1,1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	1
2	1	1	0	0	0
3	0	1	0	0	1
4	0	1	0	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{12}	P_{12}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4
in point	P_{12}	P_{12}	P_{14}	P_{14}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{12}	P_{12}

Line 3 intersects

Line	ℓ_1	ℓ_4
in point	P_{14}	P_{14}

Line 4 intersects

Line	ℓ_1	ℓ_3
in point	P_{14}	P_{14}

The surface has 11 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_4 = (1,1,1,1)$
$3: P_5 = (1,1,0,0)$
$4: P_6 = (1,0,1,0)$
$5: P_8 = (1,1,1,0)$
$6: P_9 = (1,0,0,1)$
$7: P_{11} = (1,1,0,1)$
$8: P_{12} = (0,0,1,1)$
$9: P_{13} = (1,0,1,1)$
$10: P_{14} = (0,1,1,1)$