

Rank-67243 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_3^3 + X_0^2 X_2 + X_0 X_1^2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 67243

General information

| | |
|----------------------------|---------------|
| Number of lines | 2 |
| Number of points | 9 |
| Number of singular points | 0 |
| Number of Eckardt points | 0 |
| Number of double points | 1 |
| Number of single points | 4 |
| Number of points off lines | 4 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | 3^2 |
| Type of lines on points | $2, 1^4, 0^4$ |

Singular Points

The surface has 0 singular points:

The 2 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}\ell_0 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{28} = \mathbf{Pl}(0, 0, 0, 0, 0, 1)_{19} \\ \ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{31} = \mathbf{Pl}(0, 1, 0, 0, 0, 1)_{21}\end{aligned}$$

Rank of lines: (28, 31)

Rank of points on Klein quadric: (19, 21)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

Single Points

The surface has 4 single points:

The single points on the surface are:

0 : $P_1 = (0, 1, 0, 0)$ lies on line ℓ_0

1 : $P_7 = (0, 1, 1, 0)$ lies on line ℓ_0

2 : $P_{10} = (0, 1, 0, 1)$ lies on line ℓ_1

3 : $P_{14} = (0, 1, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 4 points not on any line:

The points on the surface but not on lines are:

0 : $P_0 = (1, 0, 0, 0)$

1 : $P_4 = (1, 1, 1, 1)$

2 : $P_6 = (1, 0, 1, 0)$

3 : $P_8 = (1, 1, 1, 0)$

Line Intersection Graph

$$\begin{array}{c|c} & 01 \\ \hline 0 & 01 \\ 1 & 10 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

| | |
|----------|----------|
| Line | ℓ_1 |
| in point | P_2 |

Line 1 intersects

| | |
|----------|----------|
| Line | ℓ_0 |
| in point | P_2 |

The surface has 9 points:

The points on the surface are:

$$\begin{aligned}
0 : P_0 &= (1, 0, 0, 0) \\
1 : P_1 &= (0, 1, 0, 0) \\
2 : P_2 &= (0, 0, 1, 0) \\
3 : P_4 &= (1, 1, 1, 1)
\end{aligned}$$

$$\begin{aligned}
4 : P_6 &= (1, 0, 1, 0) \\
5 : P_7 &= (0, 1, 1, 0) \\
6 : P_8 &= (1, 1, 1, 0) \\
7 : P_{10} &= (0, 1, 0, 1)
\end{aligned}$$

$$8 : P_{14} = (0, 1, 1, 1)$$