Rank-67107 over GF(8)

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The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_3 + X_0 X_2^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(8) is -1859841459

General information

Number of lines	6
Number of points	89
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	39
Number of points off lines	43
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^6
Type of lines on points	$3, 2^6, 1^{39}, 0^{43}$

Singular Points

The surface has 1 singular points:

$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{74} = \mathbf{Pl}(1,0,1,0,0,1)_{665}$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4672} = \mathbf{Pl}(0,0,0,0,0,1)_{649}$$

$$\ell_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{72} = \mathbf{Pl}(0,0,0,0,1,0)_{81}$$

$$\ell_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0,1,0,0,0,0)_{1}$$

$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{656} = \mathbf{Pl}(0,1,0,0,1,0)_{89}$$

Rank of lines: (0, 74, 4672, 72, 4744, 656)
Rank of points on Klein quadric: (0, 665, 649, 81, 1, 89)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$.

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_5 = (1,1,0,0) = \ell_0 \cap \ell_1$ $\qquad\qquad P_{12} = (1,0,1,0) = \ell_1 \cap \ell_5$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$ $\qquad\qquad P_2 = (0,0,1,0) = \ell_2 \cap \ell_4$
$P_0 = (1,0,0,0) = \ell_0 \cap \ell_3$
$P_{19} = (0,1,1,0) = \ell_1 \cap \ell_2$

Single Points

The surface has 39 single points:
The single points on the surface are:

$0 : P_6 = (2,1,0,0)$ lies on line ℓ_0 $\qquad\quad 15 : P_{66} = (7,6,1,0)$ lies on line ℓ_1
$1 : P_7 = (3,1,0,0)$ lies on line ℓ_0 $\qquad\quad 16 : P_{67} = (0,7,1,0)$ lies on line ℓ_2
$2 : P_8 = (4,1,0,0)$ lies on line ℓ_0 $\qquad\quad 17 : P_{73} = (6,7,1,0)$ lies on line ℓ_1
$3 : P_9 = (5,1,0,0)$ lies on line ℓ_0 $\qquad\quad 18 : P_{75} = (1,0,0,1)$ lies on line ℓ_3
$4 : P_{10} = (6,1,0,0)$ lies on line ℓ_0 $\qquad\quad 19 : P_{76} = (2,0,0,1)$ lies on line ℓ_3
$5 : P_{11} = (7,1,0,0)$ lies on line ℓ_0 $\qquad\quad 20 : P_{77} = (3,0,0,1)$ lies on line ℓ_3
$6 : P_{27} = (0,2,1,0)$ lies on line ℓ_2 $\qquad\quad 21 : P_{78} = (4,0,0,1)$ lies on line ℓ_3
$7 : P_{30} = (3,2,1,0)$ lies on line ℓ_1 $\qquad\quad 22 : P_{79} = (5,0,0,1)$ lies on line ℓ_3
$8 : P_{35} = (0,3,1,0)$ lies on line ℓ_2 $\qquad\quad 23 : P_{80} = (6,0,0,1)$ lies on line ℓ_3
$9 : P_{37} = (2,3,1,0)$ lies on line ℓ_1 $\qquad\quad 24 : P_{81} = (7,0,0,1)$ lies on line ℓ_3
$10 : P_{43} = (0,4,1,0)$ lies on line ℓ_2 $\qquad\quad 25 : P_{138} = (0,0,1,1)$ lies on line ℓ_4
$11 : P_{48} = (5,4,1,0)$ lies on line ℓ_1 $\qquad\quad 26 : P_{139} = (1,0,1,1)$ lies on line ℓ_5
$12 : P_{51} = (0,5,1,0)$ lies on line ℓ_2 $\qquad\quad 27 : P_{201} = (0,0,2,1)$ lies on line ℓ_4
$13 : P_{55} = (4,5,1,0)$ lies on line ℓ_1 $\qquad\quad 28 : P_{203} = (2,0,2,1)$ lies on line ℓ_5
$14 : P_{59} = (0,6,1,0)$ lies on line ℓ_2 $\qquad\quad 29 : P_{265} = (0,0,3,1)$ lies on line ℓ_4

30 : $P_{268} = (3,0,3,1)$ lies on line ℓ_5
31 : $P_{329} = (0,0,4,1)$ lies on line ℓ_4
32 : $P_{333} = (4,0,4,1)$ lies on line ℓ_5
33 : $P_{393} = (0,0,5,1)$ lies on line ℓ_4
34 : $P_{398} = (5,0,5,1)$ lies on line ℓ_5

35 : $P_{457} = (0,0,6,1)$ lies on line ℓ_4
36 : $P_{463} = (6,0,6,1)$ lies on line ℓ_5
37 : $P_{521} = (0,0,7,1)$ lies on line ℓ_4
38 : $P_{528} = (7,0,7,1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 43 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$
1 : $P_{158} = (5,2,1,1)$
2 : $P_{159} = (6,2,1,1)$
3 : $P_{172} = (3,4,1,1)$
4 : $P_{175} = (6,4,1,1)$
5 : $P_{196} = (3,7,1,1)$
6 : $P_{198} = (5,7,1,1)$
7 : $P_{213} = (4,1,2,1)$
8 : $P_{216} = (7,1,2,1)$
9 : $P_{224} = (7,2,2,1)$
10 : $P_{235} = (2,4,2,1)$
11 : $P_{237} = (4,4,2,1)$
12 : $P_{295} = (6,3,3,1)$
13 : $P_{298} = (1,4,3,1)$
14 : $P_{303} = (6,4,3,1)$
15 : $P_{308} = (3,5,3,1)$
16 : $P_{310} = (5,5,3,1)$
17 : $P_{322} = (1,7,3,1)$
18 : $P_{326} = (5,7,3,1)$
19 : $P_{339} = (2,1,4,1)$
20 : $P_{344} = (7,1,4,1)$
21 : $P_{363} = (2,4,4,1)$

22 : $P_{389} = (4,7,4,1)$
23 : $P_{392} = (7,7,4,1)$
24 : $P_{410} = (1,2,5,1)$
25 : $P_{415} = (6,2,5,1)$
26 : $P_{436} = (3,5,5,1)$
27 : $P_{446} = (5,6,5,1)$
28 : $P_{447} = (6,6,5,1)$
29 : $P_{450} = (1,7,5,1)$
30 : $P_{452} = (3,7,5,1)$
31 : $P_{474} = (1,2,6,1)$
32 : $P_{478} = (5,2,6,1)$
33 : $P_{484} = (3,3,6,1)$
34 : $P_{487} = (6,3,6,1)$
35 : $P_{490} = (1,4,6,1)$
36 : $P_{492} = (3,4,6,1)$
37 : $P_{510} = (5,6,6,1)$
38 : $P_{531} = (2,1,7,1)$
39 : $P_{533} = (4,1,7,1)$
40 : $P_{539} = (2,2,7,1)$
41 : $P_{544} = (7,2,7,1)$
42 : $P_{581} = (4,7,7,1)$

Line Intersection Graph

```
 |0 1 2 3 4 5
0|0 1 1 1 0 0
1|1 0 1 0 0 1
2|1 1 0 0 1 0
3|1 0 0 0 1 1
4|0 0 1 1 0 1
5|0 1 0 1 1 0
```

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_5	P_1	P_0

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5
in point	P_5	P_{19}	P_{12}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_4
in point	P_1	P_{19}	P_2

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_0	P_3	P_3

Line 4 intersects

Line	ℓ_2	ℓ_3	ℓ_5
in point	P_2	P_3	P_3

Line 5 intersects

Line	ℓ_1	ℓ_3	ℓ_4
in point	P_{12}	P_3	P_3

The surface has 89 points:
The points on the surface are:

0 : $P_0 = (1,0,0,0)$
1 : $P_1 = (0,1,0,0)$
2 : $P_2 = (0,0,1,0)$
3 : $P_3 = (0,0,0,1)$
4 : $P_4 = (1,1,1,1)$
5 : $P_5 = (1,1,0,0)$
6 : $P_6 = (2,1,0,0)$
7 : $P_7 = (3,1,0,0)$
8 : $P_8 = (4,1,0,0)$
9 : $P_9 = (5,1,0,0)$
10 : $P_{10} = (6,1,0,0)$
11 : $P_{11} = (7,1,0,0)$
12 : $P_{12} = (1,0,1,0)$
13 : $P_{19} = (0,1,1,0)$
14 : $P_{27} = (0,2,1,0)$
15 : $P_{30} = (3,2,1,0)$
16 : $P_{35} = (0,3,1,0)$
17 : $P_{37} = (2,3,1,0)$
18 : $P_{43} = (0,4,1,0)$
19 : $P_{48} = (5,4,1,0)$
20 : $P_{51} = (0,5,1,0)$
21 : $P_{55} = (4,5,1,0)$
22 : $P_{59} = (0,6,1,0)$
23 : $P_{66} = (7,6,1,0)$
24 : $P_{67} = (0,7,1,0)$
25 : $P_{73} = (6,7,1,0)$
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