# Rank-65542 over GF(2)

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## The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0X_1X_2 = 0$$

( 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(2) is 65542

## General information

| | |
|---|---|
| Number of lines | 2 |
| Number of points | 5 |
| Number of singular points | 3 |
| Number of Eckardt points | 0 |
| Number of double points | 1 |
| Number of single points | 4 |
| Number of points off lines | 0 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $3^2$ |
| Type of lines on points | 2, $1^4$ |

### Singular Points

The surface has 3 singular points:

$0 : P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$ $\quad$ $2 : P_{12} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$
$1 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

## The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0,0,0,1,1,0)_{15}$$

Rank of lines: ( 34, 13 )
Rank of points on Klein quadric: ( 1, 15 )

#### Eckardt Points

The surface has 0 Eckardt points:

#### Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_3 = (0,0,0,1) = \ell_0 \cap \ell_1$

#### Single Points

The surface has 4 single points:
The single points on the surface are:

0 : $P_2 = (0,0,1,0)$ lies on line $\ell_0$
1 : $P_5 = (1,1,0,0)$ lies on line $\ell_1$
2 : $P_{11} = (1,1,0,1)$ lies on line $\ell_1$
3 : $P_{12} = (0,0,1,1)$ lies on line $\ell_0$

The single points on the surface are:

#### Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

## Line Intersection Graph

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_1$ |
|---|---|
| in point | $P_3$ |

Line 1 intersects

| Line | $\ell_0$ |
|---|---|
| in point | $P_3$ |

The surface has 5 points:
The points on the surface are:

$0:\ P_2 = (0,0,1,0)$
$1:\ P_3 = (0,0,0,1)$
$2:\ P_5 = (1,1,0,0)$
$3:\ P_{11} = (1,1,0,1)$
$4:\ P_{12} = (0,0,1,1)$