

Rank-65542 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_0X_1X_2 = 0$$

(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(64) is 1090785414

General information

Number of lines	64
Number of points	4097
Number of singular points	65
Number of Eckardt points	0
Number of double points	0
Number of single points	4096
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^{64}
Type of lines on points	$64, 1^{4096}$

Singular Points

The surface has 65 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

$$1 : P_3 = \mathbf{P}(0, 0, 0, 1) = \mathbf{P}(0, 0, 0, 1)$$

$$2 : P_{8258} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1)$$

$$3 : P_{12353} = \mathbf{P}(0, 0, \epsilon, 1) = \mathbf{P}(0, 0, 2, 1)$$

$$4 : P_{16449} = \mathbf{P}(0, 0, \epsilon^{58}, 1) = \mathbf{P}(0, 0, 3, 1)$$

$$5 : P_{20545} = \mathbf{P}(0, 0, \epsilon^2, 1) = \mathbf{P}(0, 0, 4, 1)$$

$$6 : P_{24641} = \mathbf{P}(0, 0, \epsilon^{53}, 1) = \mathbf{P}(0, 0, 5, 1)$$

$$7 : P_{28737} = \mathbf{P}(0, 0, \epsilon^{59}, 1) = \mathbf{P}(0, 0, 6, 1)$$

$$8 : P_{32833} = \mathbf{P}(0, 0, \epsilon^{39}, 1) = \mathbf{P}(0, 0, 7, 1)$$

$$9 : P_{36929} = \mathbf{P}(0, 0, \epsilon^3, 1) = \mathbf{P}(0, 0, 8, 1)$$

$$10 : P_{41025} = \mathbf{P}(0, 0, \epsilon^{34}, 1) = \mathbf{P}(0, 0, 9, 1)$$

$$11 : P_{45121} = \mathbf{P}(0, 0, \epsilon^{54}, 1) = \mathbf{P}(0, 0, 10, 1)$$

$$12 : P_{49217} = \mathbf{P}(0, 0, \epsilon^{18}, 1) = \mathbf{P}(0, 0, 11, 1)$$

$$13 : P_{53313} = \mathbf{P}(0, 0, \epsilon^{60}, 1) = \mathbf{P}(0, 0, 12, 1)$$

$$14 : P_{57409} = \mathbf{P}(0, 0, \epsilon^{31}, 1) = \mathbf{P}(0, 0, 13, 1)$$

$$15 : P_{61505} = \mathbf{P}(0, 0, \epsilon^{40}, 1) = \mathbf{P}(0, 0, 14, 1)$$

$$16 : P_{65601} = \mathbf{P}(0, 0, \epsilon^{48}, 1) = \mathbf{P}(0, 0, 15, 1)$$

$$17 : P_{69697} = \mathbf{P}(0, 0, \epsilon^4, 1) = \mathbf{P}(0, 0, 16, 1)$$

$$\begin{aligned}
18 : P_{73793} &= \mathbf{P}(0, 0, \epsilon^{43}, 1) = \mathbf{P}(0, 0, 17, 1) & 42 : P_{172097} &= \mathbf{P}(0, 0, \epsilon^{37}, 1) = \mathbf{P}(0, 0, 41, 1) \\
19 : P_{77889} &= \mathbf{P}(0, 0, \epsilon^{35}, 1) = \mathbf{P}(0, 0, 18, 1) & 43 : P_{176193} &= \mathbf{P}(0, 0, \epsilon^{16}, 1) = \mathbf{P}(0, 0, 42, 1) \\
20 : P_{81985} &= \mathbf{P}(0, 0, \epsilon^{22}, 1) = \mathbf{P}(0, 0, 19, 1) & 44 : P_{180289} &= \mathbf{P}(0, 0, \epsilon^{46}, 1) = \mathbf{P}(0, 0, 43, 1) \\
21 : P_{86081} &= \mathbf{P}(0, 0, \epsilon^{55}, 1) = \mathbf{P}(0, 0, 20, 1) & 45 : P_{184385} &= \mathbf{P}(0, 0, \epsilon^{20}, 1) = \mathbf{P}(0, 0, 44, 1) \\
22 : P_{90177} &= \mathbf{P}(0, 0, \epsilon^{15}, 1) = \mathbf{P}(0, 0, 21, 1) & 46 : P_{188481} &= \mathbf{P}(0, 0, \epsilon^{24}, 1) = \mathbf{P}(0, 0, 45, 1) \\
23 : P_{94273} &= \mathbf{P}(0, 0, \epsilon^{19}, 1) = \mathbf{P}(0, 0, 22, 1) & 47 : P_{192577} &= \mathbf{P}(0, 0, \epsilon^{27}, 1) = \mathbf{P}(0, 0, 46, 1) \\
24 : P_{98369} &= \mathbf{P}(0, 0, \epsilon^{26}, 1) = \mathbf{P}(0, 0, 23, 1) & 48 : P_{196673} &= \mathbf{P}(0, 0, \epsilon^9, 1) = \mathbf{P}(0, 0, 47, 1) \\
25 : P_{102465} &= \mathbf{P}(0, 0, \epsilon^{61}, 1) = \mathbf{P}(0, 0, 24, 1) & 49 : P_{200769} &= \mathbf{P}(0, 0, \epsilon^{62}, 1) = \mathbf{P}(0, 0, 48, 1) \\
26 : P_{106561} &= \mathbf{P}(0, 0, \epsilon^{51}, 1) = \mathbf{P}(0, 0, 25, 1) & 50 : P_{204865} &= \mathbf{P}(0, 0, \epsilon^{57}, 1) = \mathbf{P}(0, 0, 49, 1) \\
27 : P_{110657} &= \mathbf{P}(0, 0, \epsilon^{32}, 1) = \mathbf{P}(0, 0, 26, 1) & 51 : P_{208961} &= \mathbf{P}(0, 0, \epsilon^{52}, 1) = \mathbf{P}(0, 0, 50, 1) \\
28 : P_{114753} &= \mathbf{P}(0, 0, \epsilon^{29}, 1) = \mathbf{P}(0, 0, 27, 1) & 52 : P_{213057} &= \mathbf{P}(0, 0, \epsilon^{38}, 1) = \mathbf{P}(0, 0, 51, 1) \\
29 : P_{118849} &= \mathbf{P}(0, 0, \epsilon^{41}, 1) = \mathbf{P}(0, 0, 28, 1) & 53 : P_{217153} &= \mathbf{P}(0, 0, \epsilon^{33}, 1) = \mathbf{P}(0, 0, 52, 1) \\
30 : P_{122945} &= \mathbf{P}(0, 0, \epsilon^{13}, 1) = \mathbf{P}(0, 0, 29, 1) & 54 : P_{221249} &= \mathbf{P}(0, 0, \epsilon^{17}, 1) = \mathbf{P}(0, 0, 53, 1) \\
31 : P_{127041} &= \mathbf{P}(0, 0, \epsilon^{49}, 1) = \mathbf{P}(0, 0, 30, 1) & 55 : P_{225345} &= \mathbf{P}(0, 0, \epsilon^{30}, 1) = \mathbf{P}(0, 0, 54, 1) \\
32 : P_{131137} &= \mathbf{P}(0, 0, \epsilon^{11}, 1) = \mathbf{P}(0, 0, 31, 1) & 56 : P_{229441} &= \mathbf{P}(0, 0, \epsilon^{47}, 1) = \mathbf{P}(0, 0, 55, 1) \\
33 : P_{135233} &= \mathbf{P}(0, 0, \epsilon^5, 1) = \mathbf{P}(0, 0, 32, 1) & 57 : P_{233537} &= \mathbf{P}(0, 0, \epsilon^{42}, 1) = \mathbf{P}(0, 0, 56, 1) \\
34 : P_{139329} &= \mathbf{P}(0, 0, \epsilon^6, 1) = \mathbf{P}(0, 0, 33, 1) & 58 : P_{237633} &= \mathbf{P}(0, 0, \epsilon^{21}, 1) = \mathbf{P}(0, 0, 57, 1) \\
35 : P_{143425} &= \mathbf{P}(0, 0, \epsilon^{44}, 1) = \mathbf{P}(0, 0, 34, 1) & 59 : P_{241729} &= \mathbf{P}(0, 0, \epsilon^{14}, 1) = \mathbf{P}(0, 0, 58, 1) \\
36 : P_{147521} &= \mathbf{P}(0, 0, \epsilon^7, 1) = \mathbf{P}(0, 0, 35, 1) & 60 : P_{245825} &= \mathbf{P}(0, 0, \epsilon^{25}, 1) = \mathbf{P}(0, 0, 59, 1) \\
37 : P_{151617} &= \mathbf{P}(0, 0, \epsilon^{36}, 1) = \mathbf{P}(0, 0, 36, 1) & 61 : P_{249921} &= \mathbf{P}(0, 0, \epsilon^{50}, 1) = \mathbf{P}(0, 0, 60, 1) \\
38 : P_{155713} &= \mathbf{P}(0, 0, \epsilon^{45}, 1) = \mathbf{P}(0, 0, 37, 1) & 62 : P_{254017} &= \mathbf{P}(0, 0, \epsilon^{28}, 1) = \mathbf{P}(0, 0, 61, 1) \\
39 : P_{159809} &= \mathbf{P}(0, 0, \epsilon^{23}, 1) = \mathbf{P}(0, 0, 38, 1) & 63 : P_{258113} &= \mathbf{P}(0, 0, \epsilon^{12}, 1) = \mathbf{P}(0, 0, 62, 1) \\
40 : P_{163905} &= \mathbf{P}(0, 0, \epsilon^8, 1) = \mathbf{P}(0, 0, 39, 1) & 64 : P_{262209} &= \mathbf{P}(0, 0, \epsilon^{10}, 1) = \mathbf{P}(0, 0, 63, 1) \\
41 : P_{168001} &= \mathbf{P}(0, 0, \epsilon^{56}, 1) = \mathbf{P}(0, 0, 40, 1)
\end{aligned}$$

The 64 Lines

The lines and their Pluecker coordinates are:

$$\begin{aligned}
\ell_0 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{17047616} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_1 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8321} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8321} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{12353} \\
\ell_2 &= \begin{bmatrix} 1 & \epsilon^{21} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{241337} = \begin{bmatrix} 1 & 57 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{241337} = \mathbf{Pl}(0, 0, 0, 57, 1, 0)_{19465} \\
\ell_3 &= \begin{bmatrix} 1 & \epsilon^{42} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{237176} = \begin{bmatrix} 1 & 56 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{237176} = \mathbf{Pl}(0, 0, 0, 56, 1, 0)_{19338} \\
\ell_4 &= \begin{bmatrix} 1 & \epsilon^{45} & \epsilon^{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10011365} = \begin{bmatrix} 1 & 37 & 37 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10011365} = \mathbf{Pl}(0, 37, 0, 37, 1, 0)_{17025} \\
\ell_5 &= \begin{bmatrix} 1 & \epsilon^{27} & \epsilon^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12445550} = \begin{bmatrix} 1 & 46 & 46 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12445550} = \mathbf{Pl}(0, 46, 0, 46, 1, 0)_{18177} \\
\ell_6 &= \begin{bmatrix} 1 & \epsilon^{54} & \epsilon^{54} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2708810} = \begin{bmatrix} 1 & 10 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2708810} = \mathbf{Pl}(0, 10, 0, 10, 1, 0)_{13569} \\
\ell_7 &= \begin{bmatrix} 1 & \epsilon^8 & \epsilon^{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16677287} = \begin{bmatrix} 1 & 39 & 62 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16677287} = \mathbf{Pl}(0, 62, 0, 39, 1, 0)_{17304} \\
\ell_8 &= \begin{bmatrix} 1 & \epsilon^{47} & \epsilon^8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10618871} = \begin{bmatrix} 1 & 55 & 39 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10618871} = \mathbf{Pl}(0, 39, 0, 55, 1, 0)_{19313} \\
\ell_9 &= \begin{bmatrix} 1 & \epsilon^{19} & \epsilon^{43} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4622870} = \begin{bmatrix} 1 & 22 & 17 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4622870} = \mathbf{Pl}(0, 17, 0, 22, 1, 0)_{15100}
\end{aligned}$$

$$\begin{aligned}
\ell_{10} &= \begin{bmatrix} 1 & \epsilon^{60} & \epsilon^{21} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15233420} = \begin{bmatrix} 1 & 12 & 57 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15233420} = \mathbf{Pl}(0, 57, 0, 12, 1, 0)_{13870} \\
\ell_{11} &= \begin{bmatrix} 1 & \epsilon^{37} & \epsilon^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2305193} = \begin{bmatrix} 1 & 41 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2305193} = \mathbf{Pl}(0, 8, 0, 41, 1, 0)_{17504} \\
\ell_{12} &= \begin{bmatrix} 1 & \epsilon^{18} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{316235} = \begin{bmatrix} 1 & 11 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{316235} = \mathbf{Pl}(0, 1, 0, 11, 1, 0)_{13687} \\
\ell_{13} &= \begin{bmatrix} 1 & \epsilon^{40} & \epsilon^{22} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5122190} = \begin{bmatrix} 1 & 14 & 19 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5122190} = \mathbf{Pl}(0, 19, 0, 14, 1, 0)_{14086} \\
\ell_{14} &= \begin{bmatrix} 1 & \epsilon^5 & \epsilon^{50} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16115552} = \begin{bmatrix} 1 & 32 & 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16115552} = \mathbf{Pl}(0, 60, 0, 32, 1, 0)_{16413} \\
\ell_{15} &= \begin{bmatrix} 1 & \epsilon^{26} & \epsilon^{29} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{7290071} = \begin{bmatrix} 1 & 23 & 27 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{7290071} = \mathbf{Pl}(0, 27, 0, 23, 1, 0)_{15237} \\
\ell_{16} &= \begin{bmatrix} 1 & \epsilon^{61} & \epsilon & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{636632} = \begin{bmatrix} 1 & 24 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{636632} = \mathbf{Pl}(0, 2, 0, 24, 1, 0)_{15339} \\
\ell_{17} &= \begin{bmatrix} 1 & \epsilon^{39} & \epsilon^{42} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14946311} = \begin{bmatrix} 1 & 7 & 56 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14946311} = \mathbf{Pl}(0, 56, 0, 7, 1, 0)_{13234} \\
\ell_{18} &= \begin{bmatrix} 1 & \epsilon & \epsilon^{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13860290} = \begin{bmatrix} 1 & 2 & 52 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13860290} = \mathbf{Pl}(0, 52, 0, 2, 1, 0)_{12595} \\
\ell_{19} &= \begin{bmatrix} 1 & \epsilon^{25} & \epsilon^{48} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4244219} = \begin{bmatrix} 1 & 59 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4244219} = \mathbf{Pl}(0, 15, 0, 59, 1, 0)_{19797} \\
\ell_{20} &= \begin{bmatrix} 1 & \epsilon^7 & \epsilon^{35} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4943267} = \begin{bmatrix} 1 & 35 & 18 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4943267} = \mathbf{Pl}(0, 18, 0, 35, 1, 0)_{16752} \\
\ell_{21} &= \begin{bmatrix} 1 & \epsilon^{16} & \epsilon^{24} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12162602} = \begin{bmatrix} 1 & 42 & 45 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12162602} = \mathbf{Pl}(0, 45, 0, 42, 1, 0)_{17668} \\
\ell_{22} &= \begin{bmatrix} 1 & \epsilon^{38} & \epsilon^{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10335923} = \begin{bmatrix} 1 & 51 & 38 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10335923} = \mathbf{Pl}(0, 38, 0, 51, 1, 0)_{18804} \\
\ell_{23} &= \begin{bmatrix} 1 & \epsilon^{31} & \epsilon^{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11243021} = \begin{bmatrix} 1 & 13 & 42 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11243021} = \mathbf{Pl}(0, 42, 0, 13, 1, 0)_{13982} \\
\ell_{24} &= \begin{bmatrix} 1 & \epsilon^{57} & \epsilon^{42} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15121073} = \begin{bmatrix} 1 & 49 & 56 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15121073} = \mathbf{Pl}(0, 56, 0, 49, 1, 0)_{18568} \\
\ell_{25} &= \begin{bmatrix} 1 & \epsilon^{46} & \epsilon^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12433067} = \begin{bmatrix} 1 & 43 & 46 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12433067} = \mathbf{Pl}(0, 46, 0, 43, 1, 0)_{17796} \\
\ell_{26} &= \begin{bmatrix} 1 & \epsilon^{53} & \epsilon^{27} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12274949} = \begin{bmatrix} 1 & 5 & 46 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12274949} = \mathbf{Pl}(0, 46, 0, 5, 1, 0)_{12970} \\
\ell_{27} &= \begin{bmatrix} 1 & \epsilon^4 & \epsilon^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8858768} = \begin{bmatrix} 1 & 16 & 33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8858768} = \mathbf{Pl}(0, 33, 0, 16, 1, 0)_{14354} \\
\ell_{28} &= \begin{bmatrix} 1 & \epsilon^{55} & \epsilon^4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4348244} = \begin{bmatrix} 1 & 20 & 16 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4348244} = \mathbf{Pl}(0, 16, 0, 20, 1, 0)_{14845} \\
\ell_{29} &= \begin{bmatrix} 1 & \epsilon^{30} & \epsilon^{42} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15141878} = \begin{bmatrix} 1 & 54 & 56 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15141878} = \mathbf{Pl}(0, 56, 0, 54, 1, 0)_{19203} \\
\ell_{30} &= \begin{bmatrix} 1 & \epsilon^{41} & \epsilon^{53} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1452188} = \begin{bmatrix} 1 & 28 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1452188} = \mathbf{Pl}(0, 5, 0, 28, 1, 0)_{15850}
\end{aligned}$$

$$\begin{aligned}
\ell_{31} &= \begin{bmatrix} 1 & \epsilon^{23} & \epsilon^{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10015526} = \begin{bmatrix} 1 & 38 & 37 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10015526} = \mathbf{Pl}(0, 37, 0, 38, 1, 0)_{17152} \\
\ell_{32} &= \begin{bmatrix} 1 & \epsilon^{58} & \epsilon^{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9869891} = \begin{bmatrix} 1 & 3 & 37 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9869891} = \mathbf{Pl}(0, 37, 0, 3, 1, 0)_{12707} \\
\ell_{33} &= \begin{bmatrix} 1 & \epsilon^{35} & \epsilon^{49} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8068178} = \begin{bmatrix} 1 & 18 & 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8068178} = \mathbf{Pl}(0, 30, 0, 18, 1, 0)_{14605} \\
\ell_{34} &= \begin{bmatrix} 1 & \epsilon^{11} & \epsilon^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8921183} = \begin{bmatrix} 1 & 31 & 33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8921183} = \mathbf{Pl}(0, 33, 0, 31, 1, 0)_{16259} \\
\ell_{35} &= \begin{bmatrix} 1 & \epsilon^{14} & \epsilon^7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9566138} = \begin{bmatrix} 1 & 58 & 35 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9566138} = \mathbf{Pl}(0, 35, 0, 58, 1, 0)_{19690} \\
\ell_{36} &= \begin{bmatrix} 1 & \epsilon^{36} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{420260} = \begin{bmatrix} 1 & 36 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{420260} = \mathbf{Pl}(0, 1, 0, 36, 1, 0)_{16862} \\
\ell_{37} &= \begin{bmatrix} 1 & \epsilon^{17} & \epsilon^{44} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9279029} = \begin{bmatrix} 1 & 53 & 34 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{9279029} = \mathbf{Pl}(0, 34, 0, 53, 1, 0)_{19054} \\
\ell_{38} &= \begin{bmatrix} 1 & \epsilon^{10} & \epsilon^{37} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11184767} = \begin{bmatrix} 1 & 63 & 41 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11184767} = \mathbf{Pl}(0, 41, 0, 63, 1, 0)_{20331} \\
\ell_{39} &= \begin{bmatrix} 1 & \epsilon^{49} & \epsilon^{56} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10781150} = \begin{bmatrix} 1 & 30 & 40 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{10781150} = \mathbf{Pl}(0, 40, 0, 30, 1, 0)_{16139} \\
\ell_{40} &= \begin{bmatrix} 1 & \epsilon^{44} & \epsilon^{24} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12129314} = \begin{bmatrix} 1 & 34 & 45 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12129314} = \mathbf{Pl}(0, 45, 0, 34, 1, 0)_{16652} \\
\ell_{41} &= \begin{bmatrix} 1 & \epsilon^9 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{466031} = \begin{bmatrix} 1 & 47 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{466031} = \mathbf{Pl}(0, 1, 0, 47, 1, 0)_{18259} \\
\ell_{42} &= \begin{bmatrix} 1 & \epsilon^{20} & \epsilon^{11} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8442668} = \begin{bmatrix} 1 & 44 & 31 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8442668} = \mathbf{Pl}(0, 31, 0, 44, 1, 0)_{17908} \\
\ell_{43} &= \begin{bmatrix} 1 & \epsilon^{34} & \epsilon^{25} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15753545} = \begin{bmatrix} 1 & 9 & 59 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15753545} = \mathbf{Pl}(0, 59, 0, 9, 1, 0)_{13491} \\
\ell_{44} &= \begin{bmatrix} 1 & \epsilon^2 & \epsilon^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2151236} = \begin{bmatrix} 1 & 4 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2151236} = \mathbf{Pl}(0, 8, 0, 4, 1, 0)_{12805} \\
\ell_{45} &= \begin{bmatrix} 1 & \epsilon^{59} & \epsilon^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1094342} = \begin{bmatrix} 1 & 6 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1094342} = \mathbf{Pl}(0, 4, 0, 6, 1, 0)_{13055} \\
\ell_{46} &= \begin{bmatrix} 1 & \epsilon^{52} & \epsilon^{58} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1011122} = \begin{bmatrix} 1 & 50 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1011122} = \mathbf{Pl}(0, 3, 0, 50, 1, 0)_{18642} \\
\ell_{47} &= \begin{bmatrix} 1 & \epsilon^{15} & \epsilon^{21} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15270869} = \begin{bmatrix} 1 & 21 & 57 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15270869} = \mathbf{Pl}(0, 57, 0, 21, 1, 0)_{15013} \\
\ell_{48} &= \begin{bmatrix} 1 & \epsilon^{43} & \epsilon^{54} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2737937} = \begin{bmatrix} 1 & 17 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2737937} = \mathbf{Pl}(0, 10, 0, 17, 1, 0)_{14458} \\
\ell_{49} &= \begin{bmatrix} 1 & \epsilon^{29} & \epsilon^{54} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2779547} = \begin{bmatrix} 1 & 27 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2779547} = \mathbf{Pl}(0, 10, 0, 27, 1, 0)_{15728} \\
\ell_{50} &= \begin{bmatrix} 1 & \epsilon^{50} & \epsilon^{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14101628} = \begin{bmatrix} 1 & 60 & 52 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14101628} = \mathbf{Pl}(0, 52, 0, 60, 1, 0)_{19961} \\
\ell_{51} &= \begin{bmatrix} 1 & \epsilon^{62} & \epsilon^{32} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{7127792} = \begin{bmatrix} 1 & 48 & 26 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{7127792} = \mathbf{Pl}(0, 26, 0, 48, 1, 0)_{18411}
\end{aligned}$$

$$\begin{aligned}
\ell_{52} &= \begin{bmatrix} 1 & \epsilon^{13} & \epsilon^{46} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11575901} = \begin{bmatrix} 1 & 29 & 43 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{11575901} = \mathbf{Pl}(0, 43, 0, 29, 1, 0)_{16015} \\
\ell_{53} &= \begin{bmatrix} 1 & \epsilon^{51} & \epsilon^{21} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15287513} = \begin{bmatrix} 1 & 25 & 57 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15287513} = \mathbf{Pl}(0, 57, 0, 25, 1, 0)_{15521} \\
\ell_{54} &= \begin{bmatrix} 1 & \epsilon^{32} & \epsilon^{48} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4106906} = \begin{bmatrix} 1 & 26 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4106906} = \mathbf{Pl}(0, 15, 0, 26, 1, 0)_{15606} \\
\ell_{55} &= \begin{bmatrix} 1 & \epsilon^3 & \epsilon^{24} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12021128} = \begin{bmatrix} 1 & 8 & 45 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{12021128} = \mathbf{Pl}(0, 45, 0, 8, 1, 0)_{13350} \\
\ell_{56} &= \begin{bmatrix} 1 & \epsilon^{48} & \epsilon^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8854607} = \begin{bmatrix} 1 & 15 & 33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{8854607} = \mathbf{Pl}(0, 33, 0, 15, 1, 0)_{14227} \\
\ell_{57} &= \begin{bmatrix} 1 & \epsilon^{12} & \epsilon^{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14109950} = \begin{bmatrix} 1 & 62 & 52 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{14109950} = \mathbf{Pl}(0, 52, 0, 62, 1, 0)_{20215} \\
\ell_{58} &= \begin{bmatrix} 1 & \epsilon^{24} & \epsilon^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2321837} = \begin{bmatrix} 1 & 45 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2321837} = \mathbf{Pl}(0, 8, 0, 45, 1, 0)_{18012} \\
\ell_{59} &= \begin{bmatrix} 1 & \epsilon^6 & \epsilon^{48} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4136033} = \begin{bmatrix} 1 & 33 & 15 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4136033} = \mathbf{Pl}(0, 15, 0, 33, 1, 0)_{16495} \\
\ell_{60} &= \begin{bmatrix} 1 & \epsilon^{33} & \epsilon^{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16731380} = \begin{bmatrix} 1 & 52 & 62 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16731380} = \mathbf{Pl}(0, 62, 0, 52, 1, 0)_{18955} \\
\ell_{61} &= \begin{bmatrix} 1 & \epsilon^{28} & \epsilon^{14} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15703613} = \begin{bmatrix} 1 & 61 & 58 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{15703613} = \mathbf{Pl}(0, 58, 0, 61, 1, 0)_{20094} \\
\ell_{62} &= \begin{bmatrix} 1 & \epsilon^{56} & \epsilon^{28} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16415144} = \begin{bmatrix} 1 & 40 & 61 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16415144} = \mathbf{Pl}(0, 61, 0, 40, 1, 0)_{17430} \\
\ell_{63} &= \begin{bmatrix} 1 & \epsilon^{22} & \epsilon^{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16594067} = \begin{bmatrix} 1 & 19 & 62 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{16594067} = \mathbf{Pl}(0, 62, 0, 19, 1, 0)_{14764}
\end{aligned}$$

Rank of lines: (17047616, 8321, 241337, 237176, 10011365, 12445550, 2708810, 16677287, 10618871, 4622870, 15233420, 2305193, 316235, 5122190, 16115552, 7290071, 636632, 14946311, 13860290, 4244219, 4943267, 12162602, 10335923, 11243021, 15121073, 12433067, 12274949, 8858768, 4348244, 15141878, 1452188, 10015526, 9869891, 8068178, 8921183, 9566138, 420260, 9279029, 11184767, 10781150, 12129314, 466031, 8442668, 15753545, 2151236, 1094342, 1011122, 15270869, 2737937, 2779547, ...15703613, 16415144, 16594067)

Rank of points on Klein quadric: (1, 12353, 19465, 19338, 17025, 18177, 13569, 17304, 19313, 15100, 13870, 17504, 13687, 14086, 16413, 15237, 15339, 13234, 12595, 19797, 16752, 17668, 18804, 13982, 18568, 17796, 12970, 14354, 14845, 19203, 15850, 17152, 12707, 14605, 16259, 19690, 16862, 19054, 20331, 16139, 16652, 18259, 17908, 13491, 12805, 13055, 18642, 15013, 14458, 15728, ...20094, 17430, 14764)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 4096 single points:
Too many to print.

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

[illegible]

Line 0 intersects

Line 1 intersects

Line 2 intersects

Line 3 intersects

Line 4 intersects

Line 5 intersects

Line 6 intersects

Line 7 intersects

Line 8 intersects

Line 9 intersects

Line 10 intersects

Line 11 intersects

8

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

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Too many to print.