

Rank-73733 over GF(2)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 73733

General information

Number of lines	2
Number of points	7
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	4
Number of points off lines	2
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^2
Type of lines on points	$2, 1^4, 0^2$

Singular Points

The surface has 2 singular points:

$$\begin{aligned} 0 : P_0 &= \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0) \\ 1 : P_2 &= \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0) \end{aligned}$$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_4 = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2$$

$$\ell_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{34} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1$$

Rank of lines: (4, 34)

Rank of points on Klein quadric: (2, 1)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:

The double points on the surface are:

$$P_2 = (0, 0, 1, 0) = \ell_0 \cap \ell_1$$

Single Points

The surface has 4 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_1

2 : $P_6 = (1, 0, 1, 0)$ lies on line ℓ_0

3 : $P_{12} = (0, 0, 1, 1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 2 points not on any line:

The points on the surface but not on lines are:

0 : $P_8 = (1, 1, 1, 0)$

1 : $P_{11} = (1, 1, 0, 1)$

Line Intersection Graph

$$\begin{array}{c|c} 0 & 1 \\ \hline 0 & 0 \\ 1 & 1 \end{array}$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 7 points:

The points on the surface are:

$$\begin{aligned} 0 : P_0 &= (1, 0, 0, 0) \\ 1 : P_2 &= (0, 0, 1, 0) \\ 2 : P_3 &= (0, 0, 0, 1) \end{aligned}$$

$$\begin{aligned} 3 : P_6 &= (1, 0, 1, 0) \\ 4 : P_8 &= (1, 1, 1, 0) \\ 5 : P_{11} &= (1, 1, 0, 1) \end{aligned}$$

$$6 : P_{12} = (0, 0, 1, 1)$$