Rank-74051 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303108373

General information

Number of lines	10
Number of points	321
Number of singular points	1
Number of Eckardt points	2
Number of double points	9
Number of single points	141
Number of points off lines	168
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^{10}
Type of lines on points	5, 3^2, 2^9, 1^{141}, 0^{168}

Singular Points

The surface has 1 singular points:

$0 : \ P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{529} = \mathbf{Pl}(0,0,1,0,0,1)_{4656}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{69904} = \mathbf{Pl}(0,0,0,1,0,0)_{33}$$

$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{4368} = \mathbf{Pl}(1,0,0,1,0,0)_{34}$$

$$\ell_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_6 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4897} = \mathbf{Pl}(0,1,1,0,0,1)_{4672}$$

$$\ell_7 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0,1,1,0,0,0)_{18}$$

$$\ell_8 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{289} = \mathbf{Pl}(1,1,0,0,1,1)_{8961}$$

$$\ell_9 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{4657} = \mathbf{Pl}(1,1,0,1,1,1)_{9201}$$

Rank of lines: (0, 256, 529, 69904, 4368, 70160, 4897, 4624, 289, 4657)
Rank of points on Klein quadric: (0, 2, 4656, 33, 34, 1, 4672, 18, 8961, 9201)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$,
$1 : P_{290} = \mathbf{P}(0,1,0,1) = \mathbf{P}(0,1,0,1)$.

Double Points

The surface has 9 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_2$
$P_{20} = (1,0,1,0) = \ell_1 \cap \ell_8$
$P_{36} = (1,1,1,0) = \ell_2 \cap \ell_9$
$P_3 = (0,0,0,1) = \ell_3 \cap \ell_5$
$P_{291} = (1,1,0,1) = \ell_4 \cap \ell_6$
$P_{275} = (1,0,0,1) = \ell_4 \cap \ell_7$
$P_4 = (1,1,1,1) = \ell_6 \cap \ell_8$
$P_{531} = (1,0,1,1) = \ell_7 \cap \ell_9$

Single Points

The surface has 141 single points:
The single points on the surface are:

$0:\ P_{6} = (2,1,0,0)$ lies on line ℓ_0
$1:\ P_{7} = (3,1,0,0)$ lies on line ℓ_0
$2:\ P_{8} = (4,1,0,0)$ lies on line ℓ_0
$3:\ P_{9} = (5,1,0,0)$ lies on line ℓ_0
$4:\ P_{10} = (6,1,0,0)$ lies on line ℓ_0
$5:\ P_{11} = (7,1,0,0)$ lies on line ℓ_0
$6:\ P_{12} = (8,1,0,0)$ lies on line ℓ_0
$7:\ P_{13} = (9,1,0,0)$ lies on line ℓ_0
$8:\ P_{14} = (10,1,0,0)$ lies on line ℓ_0
$9:\ P_{15} = (11,1,0,0)$ lies on line ℓ_0
$10:\ P_{16} = (12,1,0,0)$ lies on line ℓ_0
$11:\ P_{17} = (13,1,0,0)$ lies on line ℓ_0
$12:\ P_{18} = (14,1,0,0)$ lies on line ℓ_0
$13:\ P_{19} = (15,1,0,0)$ lies on line ℓ_0
$14:\ P_{21} = (2,0,1,0)$ lies on line ℓ_1
$15:\ P_{22} = (3,0,1,0)$ lies on line ℓ_1
$16:\ P_{23} = (4,0,1,0)$ lies on line ℓ_1
$17:\ P_{24} = (5,0,1,0)$ lies on line ℓ_1
$18:\ P_{25} = (6,0,1,0)$ lies on line ℓ_1
$19:\ P_{26} = (7,0,1,0)$ lies on line ℓ_1
$20:\ P_{27} = (8,0,1,0)$ lies on line ℓ_1
$21:\ P_{28} = (9,0,1,0)$ lies on line ℓ_1
$22:\ P_{29} = (10,0,1,0)$ lies on line ℓ_1
$23:\ P_{30} = (11,0,1,0)$ lies on line ℓ_1
$24:\ P_{31} = (12,0,1,0)$ lies on line ℓ_1
$25:\ P_{32} = (13,0,1,0)$ lies on line ℓ_1
$26:\ P_{33} = (14,0,1,0)$ lies on line ℓ_1
$27:\ P_{34} = (15,0,1,0)$ lies on line ℓ_1
$28:\ P_{53} = (2,2,1,0)$ lies on line ℓ_2
$29:\ P_{70} = (3,3,1,0)$ lies on line ℓ_2
$30:\ P_{87} = (4,4,1,0)$ lies on line ℓ_2
$31:\ P_{104} = (5,5,1,0)$ lies on line ℓ_2
$32:\ P_{121} = (6,6,1,0)$ lies on line ℓ_2
$33:\ P_{138} = (7,7,1,0)$ lies on line ℓ_2
$34:\ P_{155} = (8,8,1,0)$ lies on line ℓ_2
$35:\ P_{172} = (9,9,1,0)$ lies on line ℓ_2
$36:\ P_{189} = (10,10,1,0)$ lies on line ℓ_2
$37:\ P_{206} = (11,11,1,0)$ lies on line ℓ_2
$38:\ P_{223} = (12,12,1,0)$ lies on line ℓ_2
$39:\ P_{240} = (13,13,1,0)$ lies on line ℓ_2
$40:\ P_{257} = (14,14,1,0)$ lies on line ℓ_2
$41:\ P_{274} = (15,15,1,0)$ lies on line ℓ_2
$42:\ P_{306} = (0,2,0,1)$ lies on line ℓ_3
$43:\ P_{307} = (1,2,0,1)$ lies on line ℓ_4
$44:\ P_{322} = (0,3,0,1)$ lies on line ℓ_3
$45:\ P_{323} = (1,3,0,1)$ lies on line ℓ_4
$46:\ P_{338} = (0,4,0,1)$ lies on line ℓ_3
$47:\ P_{339} = (1,4,0,1)$ lies on line ℓ_4
$48:\ P_{354} = (0,5,0,1)$ lies on line ℓ_3
$49:\ P_{355} = (1,5,0,1)$ lies on line ℓ_4
$50:\ P_{370} = (0,6,0,1)$ lies on line ℓ_3
$51:\ P_{371} = (1,6,0,1)$ lies on line ℓ_4
$52:\ P_{386} = (0,7,0,1)$ lies on line ℓ_3
$53:\ P_{387} = (1,7,0,1)$ lies on line ℓ_4
$54:\ P_{402} = (0,8,0,1)$ lies on line ℓ_3
$55:\ P_{403} = (1,8,0,1)$ lies on line ℓ_4
$56:\ P_{418} = (0,9,0,1)$ lies on line ℓ_3
$57:\ P_{419} = (1,9,0,1)$ lies on line ℓ_4
$58:\ P_{434} = (0,10,0,1)$ lies on line ℓ_3
$59:\ P_{435} = (1,10,0,1)$ lies on line ℓ_4
$60:\ P_{450} = (0,11,0,1)$ lies on line ℓ_3
$61:\ P_{451} = (1,11,0,1)$ lies on line ℓ_4
$62:\ P_{466} = (0,12,0,1)$ lies on line ℓ_3
$63:\ P_{467} = (1,12,0,1)$ lies on line ℓ_4
$64:\ P_{482} = (0,13,0,1)$ lies on line ℓ_3
$65:\ P_{483} = (1,13,0,1)$ lies on line ℓ_4
$66:\ P_{498} = (0,14,0,1)$ lies on line ℓ_3
$67:\ P_{499} = (1,14,0,1)$ lies on line ℓ_4
$68:\ P_{514} = (0,15,0,1)$ lies on line ℓ_3
$69:\ P_{515} = (1,15,0,1)$ lies on line ℓ_4
$70:\ P_{530} = (0,0,1,1)$ lies on line ℓ_5
$71:\ P_{785} = (0,0,2,1)$ lies on line ℓ_5
$72:\ P_{786} = (1,0,2,1)$ lies on line ℓ_7
$73:\ P_{802} = (1,1,2,1)$ lies on line ℓ_6
$74:\ P_{803} = (2,1,2,1)$ lies on line ℓ_8
$75:\ P_{835} = (2,3,2,1)$ lies on line ℓ_9
$76:\ P_{1041} = (0,0,3,1)$ lies on line ℓ_5
$77:\ P_{1042} = (1,0,3,1)$ lies on line ℓ_7
$78:\ P_{1058} = (1,1,3,1)$ lies on line ℓ_6
$79:\ P_{1060} = (3,1,3,1)$ lies on line ℓ_8
$80:\ P_{1076} = (3,2,3,1)$ lies on line ℓ_9
$81:\ P_{1297} = (0,0,4,1)$ lies on line ℓ_5
$82:\ P_{1298} = (1,0,4,1)$ lies on line ℓ_7
$83:\ P_{1314} = (1,1,4,1)$ lies on line ℓ_6
$84:\ P_{1317} = (4,1,4,1)$ lies on line ℓ_8
$85:\ P_{1381} = (4,5,4,1)$ lies on line ℓ_9
$86:\ P_{1553} = (0,0,5,1)$ lies on line ℓ_5
$87:\ P_{1554} = (1,0,5,1)$ lies on line ℓ_7
$88:\ P_{1570} = (1,1,5,1)$ lies on line ℓ_6
$89:\ P_{1574} = (5,1,5,1)$ lies on line ℓ_8
$90:\ P_{1622} = (5,4,5,1)$ lies on line ℓ_9
$91:\ P_{1809} = (0,0,6,1)$ lies on line ℓ_5
$92:\ P_{1810} = (1,0,6,1)$ lies on line ℓ_7
$93:\ P_{1826} = (1,1,6,1)$ lies on line ℓ_6
$94:\ P_{1831} = (6,1,6,1)$ lies on line ℓ_8
$95:\ P_{1927} = (6,7,6,1)$ lies on line ℓ_9
$96:\ P_{2065} = (0,0,7,1)$ lies on line ℓ_5
$97:\ P_{2066} = (1,0,7,1)$ lies on line ℓ_7
$98:\ P_{2082} = (1,1,7,1)$ lies on line ℓ_6
$99:\ P_{2088} = (7,1,7,1)$ lies on line ℓ_8
$100:\ P_{2168} = (7,6,7,1)$ lies on line ℓ_9
$101:\ P_{2321} = (0,0,8,1)$ lies on line ℓ_5
$102:\ P_{2322} = (1,0,8,1)$ lies on line ℓ_7
$103:\ P_{2338} = (1,1,8,1)$ lies on line ℓ_6
$104:\ P_{2345} = (8,1,8,1)$ lies on line ℓ_8
$105:\ P_{2473} = (8,9,8,1)$ lies on line ℓ_9
$106:\ P_{2577} = (0,0,9,1)$ lies on line ℓ_5
$107:\ P_{2578} = (1,0,9,1)$ lies on line ℓ_7

108 : $P_{2594} = (1, 1, 9, 1)$ lies on line ℓ_6
109 : $P_{2602} = (9, 1, 9, 1)$ lies on line ℓ_8
110 : $P_{2714} = (9, 8, 9, 1)$ lies on line ℓ_9
111 : $P_{2833} = (0, 0, 10, 1)$ lies on line ℓ_5
112 : $P_{2834} = (1, 0, 10, 1)$ lies on line ℓ_7
113 : $P_{2850} = (1, 1, 10, 1)$ lies on line ℓ_6
114 : $P_{2859} = (10, 1, 10, 1)$ lies on line ℓ_8
115 : $P_{3019} = (10, 11, 10, 1)$ lies on line ℓ_9
116 : $P_{3089} = (0, 0, 11, 1)$ lies on line ℓ_5
117 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_7
118 : $P_{3106} = (1, 1, 11, 1)$ lies on line ℓ_6
119 : $P_{3116} = (11, 1, 11, 1)$ lies on line ℓ_8
120 : $P_{3260} = (11, 10, 11, 1)$ lies on line ℓ_9
121 : $P_{3345} = (0, 0, 12, 1)$ lies on line ℓ_5
122 : $P_{3346} = (1, 0, 12, 1)$ lies on line ℓ_7
123 : $P_{3362} = (1, 1, 12, 1)$ lies on line ℓ_6
124 : $P_{3373} = (12, 1, 12, 1)$ lies on line ℓ_8
125 : $P_{3565} = (12, 13, 12, 1)$ lies on line ℓ_9
126 : $P_{3601} = (0, 0, 13, 1)$ lies on line ℓ_5
127 : $P_{3602} = (1, 0, 13, 1)$ lies on line ℓ_7
128 : $P_{3618} = (1, 1, 13, 1)$ lies on line ℓ_6
129 : $P_{3630} = (13, 1, 13, 1)$ lies on line ℓ_8
130 : $P_{3806} = (13, 12, 13, 1)$ lies on line ℓ_9
131 : $P_{3857} = (0, 0, 14, 1)$ lies on line ℓ_5
132 : $P_{3858} = (1, 0, 14, 1)$ lies on line ℓ_7
133 : $P_{3874} = (1, 1, 14, 1)$ lies on line ℓ_6
134 : $P_{3887} = (14, 1, 14, 1)$ lies on line ℓ_8
135 : $P_{4111} = (14, 15, 14, 1)$ lies on line ℓ_9
136 : $P_{4113} = (0, 0, 15, 1)$ lies on line ℓ_5
137 : $P_{4114} = (1, 0, 15, 1)$ lies on line ℓ_7
138 : $P_{4130} = (1, 1, 15, 1)$ lies on line ℓ_6
139 : $P_{4144} = (15, 1, 15, 1)$ lies on line ℓ_8
140 : $P_{4352} = (15, 14, 15, 1)$ lies on line ℓ_9

The single points on the surface are:

Points on surface but on no line

The surface has 168 points not on any line:
The points on the surface but not on lines are:

0 : $P_{586} = (9, 3, 1, 1)$
1 : $P_{588} = (11, 3, 1, 1)$
2 : $P_{619} = (10, 5, 1, 1)$
3 : $P_{623} = (14, 5, 1, 1)$
4 : $P_{659} = (2, 8, 1, 1)$
5 : $P_{668} = (11, 8, 1, 1)$
6 : $P_{691} = (2, 10, 1, 1)$
7 : $P_{698} = (9, 10, 1, 1)$
8 : $P_{709} = (4, 11, 1, 1)$
9 : $P_{719} = (14, 11, 1, 1)$
10 : $P_{773} = (4, 15, 1, 1)$
11 : $P_{779} = (10, 15, 1, 1)$
12 : $P_{826} = (9, 2, 2, 1)$
13 : $P_{829} = (12, 2, 2, 1)$
14 : $P_{838} = (5, 3, 2, 1)$
15 : $P_{886} = (5, 6, 2, 1)$
16 : $P_{889} = (8, 6, 2, 1)$
17 : $P_{968} = (7, 11, 2, 1)$
18 : $P_{970} = (9, 11, 2, 1)$
19 : $P_{984} = (7, 12, 2, 1)$
20 : $P_{1017} = (8, 14, 2, 1)$
21 : $P_{1021} = (12, 14, 2, 1)$
22 : $P_{1077} = (4, 2, 3, 1)$
23 : $P_{1123} = (2, 5, 3, 1)$
24 : $P_{1133} = (12, 5, 3, 1)$
25 : $P_{1141} = (4, 6, 3, 1)$
26 : $P_{1152} = (15, 6, 3, 1)$
27 : $P_{1155} = (2, 7, 3, 1)$
28 : $P_{1163} = (10, 7, 3, 1)$
29 : $P_{1174} = (5, 8, 3, 1)$
30 : $P_{1197} = (12, 9, 3, 1)$
31 : $P_{1200} = (15, 9, 3, 1)$
32 : $P_{1254} = (5, 13, 3, 1)$
33 : $P_{1259} = (10, 13, 3, 1)$
34 : $P_{1335} = (6, 2, 4, 1)$
35 : $P_{1344} = (15, 2, 4, 1)$
36 : $P_{1367} = (6, 4, 4, 1)$
37 : $P_{1375} = (14, 4, 4, 1)$
38 : $P_{1385} = (8, 5, 4, 1)$
39 : $P_{1405} = (12, 6, 4, 1)$
40 : $P_{1469} = (12, 10, 4, 1)$
41 : $P_{1471} = (14, 10, 4, 1)$
42 : $P_{1513} = (8, 13, 4, 1)$
43 : $P_{1520} = (15, 13, 4, 1)$
44 : $P_{1626} = (9, 4, 5, 1)$
45 : $P_{1673} = (8, 7, 5, 1)$
46 : $P_{1676} = (11, 7, 5, 1)$
47 : $P_{1685} = (4, 8, 5, 1)$
48 : $P_{1687} = (6, 8, 5, 1)$
49 : $P_{1749} = (4, 12, 5, 1)$
50 : $P_{1756} = (11, 12, 5, 1)$
51 : $P_{1764} = (3, 13, 5, 1)$
52 : $P_{1770} = (9, 13, 5, 1)$
53 : $P_{1780} = (3, 14, 5, 1)$

$54: P_{1783} = (6, 14, 5, 1)$
$55: P_{1801} = (8, 15, 5, 1)$
$56: P_{1875} = (2, 4, 6, 1)$
$57: P_{1900} = (11, 5, 6, 1)$
$58: P_{1902} = (13, 5, 6, 1)$
$59: P_{1907} = (2, 6, 6, 1)$
$60: P_{1919} = (14, 6, 6, 1)$
$61: P_{1933} = (12, 7, 6, 1)$
$62: P_{1950} = (13, 8, 6, 1)$
$63: P_{1951} = (14, 8, 6, 1)$
$64: P_{1990} = (5, 11, 6, 1)$
$65: P_{1997} = (12, 11, 6, 1)$
$66: P_{2038} = (5, 14, 6, 1)$
$67: P_{2044} = (11, 14, 6, 1)$
$68: P_{2117} = (4, 3, 7, 1)$
$69: P_{2125} = (12, 3, 7, 1)$
$70: P_{2140} = (11, 4, 7, 1)$
$71: P_{2144} = (15, 4, 7, 1)$
$72: P_{2174} = (13, 6, 7, 1)$
$73: P_{2181} = (4, 7, 7, 1)$
$74: P_{2186} = (9, 7, 7, 1)$
$75: P_{2254} = (13, 11, 7, 1)$
$76: P_{2256} = (15, 11, 7, 1)$
$77: P_{2298} = (9, 14, 7, 1)$
$78: P_{2316} = (11, 15, 7, 1)$
$79: P_{2317} = (12, 15, 7, 1)$
$80: P_{2358} = (5, 2, 8, 1)$
$81: P_{2366} = (13, 2, 8, 1)$
$82: P_{2384} = (15, 3, 8, 1)$
$83: P_{2426} = (9, 6, 8, 1)$
$84: P_{2427} = (10, 6, 8, 1)$
$85: P_{2438} = (5, 7, 8, 1)$
$86: P_{2447} = (14, 7, 8, 1)$
$87: P_{2479} = (14, 9, 8, 1)$
$88: P_{2523} = (10, 12, 8, 1)$
$89: P_{2528} = (15, 12, 8, 1)$
$90: P_{2570} = (9, 15, 8, 1)$
$91: P_{2574} = (13, 15, 8, 1)$
$92: P_{2644} = (3, 4, 9, 1)$
$93: P_{2654} = (13, 4, 9, 1)$
$94: P_{2692} = (3, 7, 9, 1)$
$95: P_{2704} = (15, 7, 9, 1)$
$96: P_{2720} = (15, 8, 9, 1)$
$97: P_{2723} = (2, 9, 9, 1)$
$98: P_{2734} = (13, 9, 9, 1)$
$99: P_{2755} = (2, 11, 9, 1)$
$100: P_{2759} = (6, 11, 9, 1)$
$101: P_{2791} = (6, 13, 9, 1)$
$102: P_{2872} = (7, 2, 10, 1)$
$103: P_{2876} = (11, 2, 10, 1)$
$104: P_{2920} = (7, 5, 10, 1)$
$105: P_{2928} = (15, 5, 10, 1)$
$106: P_{2983} = (6, 9, 10, 1)$
$107: P_{2988} = (11, 9, 10, 1)$
$108: P_{2998} = (5, 10, 10, 1)$
$109: P_{3008} = (15, 10, 10, 1)$
$110: P_{3078} = (5, 15, 10, 1)$
$111: P_{3079} = (6, 15, 10, 1)$
$112: P_{3145} = (8, 3, 11, 1)$
$113: P_{3150} = (13, 3, 11, 1)$
$114: P_{3163} = (10, 4, 11, 1)$
$115: P_{3165} = (12, 4, 11, 1)$
$116: P_{3220} = (3, 8, 11, 1)$
$117: P_{3229} = (12, 8, 11, 1)$
$118: P_{3268} = (3, 11, 11, 1)$
$119: P_{3273} = (8, 11, 11, 1)$
$120: P_{3323} = (10, 14, 11, 1)$
$121: P_{3326} = (13, 14, 11, 1)$
$122: P_{3391} = (14, 2, 12, 1)$
$123: P_{3399} = (6, 3, 12, 1)$
$124: P_{3403} = (10, 3, 12, 1)$
$125: P_{3431} = (6, 5, 12, 1)$
$126: P_{3434} = (9, 5, 12, 1)$
$127: P_{3492} = (3, 9, 12, 1)$
$128: P_{3499} = (10, 9, 12, 1)$
$129: P_{3508} = (3, 10, 12, 1)$
$130: P_{3512} = (7, 10, 12, 1)$
$131: P_{3546} = (9, 12, 12, 1)$
$132: P_{3551} = (14, 12, 12, 1)$
$133: P_{3560} = (7, 13, 12, 1)$
$134: P_{3641} = (8, 2, 13, 1)$
$135: P_{3643} = (10, 2, 13, 1)$
$136: P_{3736} = (7, 8, 13, 1)$
$137: P_{3739} = (10, 8, 13, 1)$
$138: P_{3749} = (4, 9, 13, 1)$
$139: P_{3767} = (6, 10, 13, 1)$
$140: P_{3769} = (8, 10, 13, 1)$
$141: P_{3799} = (6, 12, 13, 1)$
$142: P_{3811} = (2, 13, 13, 1)$
$143: P_{3813} = (4, 13, 13, 1)$
$144: P_{3843} = (2, 15, 13, 1)$
$145: P_{3848} = (7, 15, 13, 1)$
$146: P_{3982} = (13, 7, 14, 1)$
$147: P_{4006} = (5, 9, 14, 1)$
$148: P_{4008} = (7, 9, 14, 1)$
$149: P_{4021} = (4, 10, 14, 1)$
$150: P_{4030} = (13, 10, 14, 1)$
$151: P_{4052} = (3, 12, 14, 1)$
$152: P_{4054} = (5, 12, 14, 1)$
$153: P_{4085} = (4, 14, 14, 1)$
$154: P_{4088} = (7, 14, 14, 1)$
$155: P_{4100} = (3, 15, 14, 1)$
$156: P_{4168} = (7, 3, 15, 1)$
$157: P_{4175} = (14, 3, 15, 1)$
$158: P_{4184} = (7, 4, 15, 1)$
$159: P_{4185} = (8, 4, 15, 1)$
$160: P_{4196} = (3, 5, 15, 1)$
$161: P_{4212} = (3, 6, 15, 1)$

162 : $P_{4220} = (11, 6, 15, 1)$
163 : $P_{4307} = (2, 12, 15, 1)$
164 : $P_{4313} = (8, 12, 15, 1)$
165 : $P_{4332} = (11, 13, 15, 1)$
166 : $P_{4335} = (14, 13, 15, 1)$
167 : $P_{4339} = (2, 14, 15, 1)$

Line Intersection Graph

	0 1 2 3 4 5 6 7 8 9
0	0 1 1 1 1 0 0 0 0 0
1	1 0 1 0 0 1 1 1 1 0
2	1 1 0 0 0 1 1 1 0 1
3	1 0 0 0 1 1 0 0 1 1
4	1 0 0 1 0 0 1 1 0 0
5	0 1 1 1 0 0 1 1 0 0
6	0 1 1 0 1 1 0 1 1 0
7	0 1 1 0 1 1 1 0 0 1
8	0 1 0 1 0 0 1 0 0 1
9	0 0 1 1 0 0 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4
in point	P_0	P_5	P_1	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_5	ℓ_6	ℓ_7	ℓ_8
in point	P_0	P_2	P_2	P_2	P_2	P_{20}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5	ℓ_6	ℓ_7	ℓ_9
in point	P_5	P_2	P_2	P_2	P_2	P_{36}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5	ℓ_8	ℓ_9
in point	P_1	P_1	P_3	P_{290}	P_{290}

Line 4 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_7
in point	P_1	P_1	P_{291}	P_{275}

Line 5 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_6	ℓ_7
in point	P_2	P_2	P_3	P_2	P_2

Line 6 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_7	ℓ_8
in point	P_2	P_2	P_{291}	P_2	P_2	P_4

Line 7 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_9
in point	P_2	P_2	P_{275}	P_2	P_2	P_{531}

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_6	ℓ_9
in point	P_{20}	P_{290}	P_4	P_{290}

Line 9 intersects

Line	ℓ_2	ℓ_3	ℓ_7	ℓ_8
in point	P_{36}	P_{290}	P_{531}	P_{290}

The surface has 321 points:
The points on the surface are:

$0: P_0 = (1,0,0,0)$
$1: P_1 = (0,1,0,0)$
$2: P_2 = (0,0,1,0)$
$3: P_3 = (0,0,0,1)$
$4: P_4 = (1,1,1,1)$
$5: P_5 = (1,1,0,0)$
$6: P_6 = (2,1,0,0)$
$7: P_7 = (3,1,0,0)$
$8: P_8 = (4,1,0,0)$
$9: P_9 = (5,1,0,0)$
$10: P_{10} = (6,1,0,0)$
$11: P_{11} = (7,1,0,0)$
$12: P_{12} = (8,1,0,0)$
$13: P_{13} = (9,1,0,0)$
$14: P_{14} = (10,1,0,0)$
$15: P_{15} = (11,1,0,0)$
$16: P_{16} = (12,1,0,0)$
$17: P_{17} = (13,1,0,0)$
$18: P_{18} = (14,1,0,0)$
$19: P_{19} = (15,1,0,0)$
$20: P_{20} = (1,0,1,0)$
$21: P_{21} = (2,0,1,0)$
$22: P_{22} = (3,0,1,0)$
$23: P_{23} = (4,0,1,0)$
$24: P_{24} = (5,0,1,0)$
$25: P_{25} = (6,0,1,0)$
$26: P_{26} = (7,0,1,0)$
$27: P_{27} = (8,0,1,0)$
$28: P_{28} = (9,0,1,0)$
$29: P_{29} = (10,0,1,0)$
$30: P_{30} = (11,0,1,0)$
$31: P_{31} = (12,0,1,0)$
$32: P_{32} = (13,0,1,0)$
$33: P_{33} = (14,0,1,0)$
$34: P_{34} = (15,0,1,0)$
$35: P_{36} = (1,1,1,0)$
$36: P_{53} = (2,2,1,0)$
$37: P_{70} = (3,3,1,0)$
$38: P_{87} = (4,4,1,0)$
$39: P_{104} = (5,5,1,0)$
$40: P_{121} = (6,6,1,0)$
$41: P_{138} = (7,7,1,0)$
$42: P_{155} = (8,8,1,0)$
$43: P_{172} = (9,9,1,0)$
$44: P_{189} = (10,10,1,0)$
$45: P_{206} = (11,11,1,0)$
$46: P_{223} = (12,12,1,0)$
$47: P_{240} = (13,13,1,0)$
$48: P_{257} = (14,14,1,0)$
$49: P_{274} = (15,15,1,0)$
$50: P_{275} = (1,0,0,1)$
$51: P_{290} = (0,1,0,1)$
$52: P_{291} = (1,1,0,1)$
$53: P_{306} = (0,2,0,1)$
$54: P_{307} = (1,2,0,1)$
$55: P_{322} = (0,3,0,1)$
$56: P_{323} = (1,3,0,1)$
$57: P_{338} = (0,4,0,1)$
$58: P_{339} = (1,4,0,1)$
$59: P_{354} = (0,5,0,1)$
$60: P_{355} = (1,5,0,1)$
$61: P_{370} = (0,6,0,1)$
$62: P_{371} = (1,6,0,1)$
$63: P_{386} = (0,7,0,1)$
$64: P_{387} = (1,7,0,1)$
$65: P_{402} = (0,8,0,1)$
$66: P_{403} = (1,8,0,1)$
$67: P_{418} = (0,9,0,1)$
$68: P_{419} = (1,9,0,1)$
$69: P_{434} = (0,10,0,1)$
$70: P_{435} = (1,10,0,1)$
$71: P_{450} = (0,11,0,1)$
$72: P_{451} = (1,11,0,1)$
$73: P_{466} = (0,12,0,1)$
$74: P_{467} = (1,12,0,1)$
$75: P_{482} = (0,13,0,1)$
$76: P_{483} = (1,13,0,1)$
$77: P_{498} = (0,14,0,1)$
$78: P_{499} = (1,14,0,1)$
$79: P_{514} = (0,15,0,1)$
$80: P_{515} = (1,15,0,1)$
$81: P_{530} = (0,0,1,1)$
$82: P_{531} = (1,0,1,1)$
$83: P_{586} = (9,3,1,1)$
$84: P_{588} = (11,3,1,1)$
$85: P_{619} = (10,5,1,1)$
$86: P_{623} = (14,5,1,1)$
$87: P_{659} = (2,8,1,1)$
$88: P_{668} = (11,8,1,1)$
$89: P_{691} = (2,10,1,1)$
$90: P_{698} = (9,10,1,1)$
$91: P_{709} = (4,11,1,1)$
$92: P_{719} = (14,11,1,1)$
$93: P_{773} = (4,15,1,1)$
$94: P_{779} = (10,15,1,1)$
$95: P_{785} = (0,0,2,1)$
$96: P_{786} = (1,0,2,1)$
$97: P_{802} = (1,1,2,1)$
$98: P_{803} = (2,1,2,1)$
$99: P_{826} = (9,2,2,1)$
$100: P_{829} = (12,2,2,1)$
$101: P_{835} = (2,3,2,1)$
$102: P_{838} = (5,3,2,1)$
$103: P_{886} = (5,6,2,1)$
$104: P_{889} = (8,6,2,1)$
$105: P_{968} = (7,11,2,1)$
$106: P_{970} = (9,11,2,1)$
$107: P_{984} = (7,12,2,1)$
$108: P_{1017} = (8,14,2,1)$
$109: P_{1021} = (12,14,2,1)$
$110: P_{1041} = (0,0,3,1)$
$111: P_{1042} = (1,0,3,1)$
$112: P_{1058} = (1,1,3,1)$
$113: P_{1060} = (3,1,3,1)$
$114: P_{1076} = (3,2,3,1)$
$115: P_{1077} = (4,2,3,1)$
$116: P_{1123} = (2,5,3,1)$
$117: P_{1133} = (12,5,3,1)$
$118: P_{1141} = (4,6,3,1)$
$119: P_{1152} = (15,6,3,1)$
$120: P_{1155} = (2,7,3,1)$
$121: P_{1163} = (10,7,3,1)$
$122: P_{1174} = (5,8,3,1)$
$123: P_{1197} = (12,9,3,1)$
$124: P_{1200} = (15,9,3,1)$
$125: P_{1254} = (5,13,3,1)$
$126: P_{1259} = (10,13,3,1)$
$127: P_{1297} = (0,0,4,1)$
$128: P_{1298} = (1,0,4,1)$
$129: P_{1314} = (1,1,4,1)$
$130: P_{1317} = (4,1,4,1)$
$131: P_{1335} = (6,2,4,1)$
$132: P_{1344} = (15,2,4,1)$
$133: P_{1367} = (6,4,4,1)$
$134: P_{1375} = (14,4,4,1)$
$135: P_{1381} = (4,5,4,1)$
$136: P_{1385} = (8,5,4,1)$
$137: P_{1405} = (12,6,4,1)$
$138: P_{1469} = (12,10,4,1)$
$139: P_{1471} = (14,10,4,1)$
$140: P_{1513} = (8,13,4,1)$
$141: P_{1520} = (15,13,4,1)$
$142: P_{1553} = (0,0,5,1)$
$143: P_{1554} = (1,0,5,1)$
$144: P_{1570} = (1,1,5,1)$
$145: P_{1574} = (5,1,5,1)$
$146: P_{1622} = (5,4,5,1)$
$147: P_{1626} = (9,4,5,1)$
$148: P_{1673} = (8,7,5,1)$
$149: P_{1676} = (11,7,5,1)$
$150: P_{1685} = (4,8,5,1)$
$151: P_{1687} = (6,8,5,1)$
$152: P_{1749} = (4,12,5,1)$
$153: P_{1756} = (11,12,5,1)$
$154: P_{1764} = (3,13,5,1)$
$155: P_{1770} = (9,13,5,1)$
$156: P_{1780} = (3,14,5,1)$
$157: P_{1783} = (6,14,5,1)$
$158: P_{1801} = (8,15,5,1)$
$159: P_{1809} = (0,0,6,1)$
$160: P_{1810} = (1,0,6,1)$
$161: P_{1826} = (1,1,6,1)$

$162: P_{1831} = (6, 1, 6, 1)$
$163: P_{1875} = (2, 4, 6, 1)$
$164: P_{1900} = (11, 5, 6, 1)$
$165: P_{1902} = (13, 5, 6, 1)$
$166: P_{1907} = (2, 6, 6, 1)$
$167: P_{1919} = (14, 6, 6, 1)$
$168: P_{1927} = (6, 7, 6, 1)$
$169: P_{1933} = (12, 7, 6, 1)$
$170: P_{1950} = (13, 8, 6, 1)$
$171: P_{1951} = (14, 8, 6, 1)$
$172: P_{1990} = (5, 11, 6, 1)$
$173: P_{1997} = (12, 11, 6, 1)$
$174: P_{2038} = (5, 14, 6, 1)$
$175: P_{2044} = (11, 14, 6, 1)$
$176: P_{2065} = (0, 0, 7, 1)$
$177: P_{2066} = (1, 0, 7, 1)$
$178: P_{2082} = (1, 1, 7, 1)$
$179: P_{2088} = (7, 1, 7, 1)$
$180: P_{2117} = (4, 3, 7, 1)$
$181: P_{2125} = (12, 3, 7, 1)$
$182: P_{2140} = (11, 4, 7, 1)$
$183: P_{2144} = (15, 4, 7, 1)$
$184: P_{2168} = (7, 6, 7, 1)$
$185: P_{2174} = (13, 6, 7, 1)$
$186: P_{2181} = (4, 7, 7, 1)$
$187: P_{2186} = (9, 7, 7, 1)$
$188: P_{2254} = (13, 11, 7, 1)$
$189: P_{2256} = (15, 11, 7, 1)$
$190: P_{2298} = (9, 14, 7, 1)$
$191: P_{2316} = (11, 15, 7, 1)$
$192: P_{2317} = (12, 15, 7, 1)$
$193: P_{2321} = (0, 0, 8, 1)$
$194: P_{2322} = (1, 0, 8, 1)$
$195: P_{2338} = (1, 1, 8, 1)$
$196: P_{2345} = (8, 1, 8, 1)$
$197: P_{2358} = (5, 2, 8, 1)$
$198: P_{2366} = (13, 2, 8, 1)$
$199: P_{2384} = (15, 3, 8, 1)$
$200: P_{2426} = (9, 6, 8, 1)$
$201: P_{2427} = (10, 6, 8, 1)$
$202: P_{2438} = (5, 7, 8, 1)$
$203: P_{2447} = (14, 7, 8, 1)$
$204: P_{2473} = (8, 9, 8, 1)$
$205: P_{2479} = (14, 9, 8, 1)$
$206: P_{2523} = (10, 12, 8, 1)$
$207: P_{2528} = (15, 12, 8, 1)$
$208: P_{2570} = (9, 15, 8, 1)$
$209: P_{2574} = (13, 15, 8, 1)$
$210: P_{2577} = (0, 0, 9, 1)$
$211: P_{2578} = (1, 0, 9, 1)$
$212: P_{2594} = (1, 1, 9, 1)$
$213: P_{2602} = (9, 1, 9, 1)$
$214: P_{2644} = (3, 4, 9, 1)$
$215: P_{2654} = (13, 4, 9, 1)$
$216: P_{2692} = (3, 7, 9, 1)$
$217: P_{2704} = (15, 7, 9, 1)$
$218: P_{2714} = (9, 8, 9, 1)$
$219: P_{2720} = (15, 8, 9, 1)$
$220: P_{2723} = (2, 9, 9, 1)$
$221: P_{2734} = (13, 9, 9, 1)$
$222: P_{2755} = (2, 11, 9, 1)$
$223: P_{2759} = (6, 11, 9, 1)$
$224: P_{2791} = (6, 13, 9, 1)$
$225: P_{2833} = (0, 0, 10, 1)$
$226: P_{2834} = (1, 0, 10, 1)$
$227: P_{2850} = (1, 1, 10, 1)$
$228: P_{2859} = (10, 1, 10, 1)$
$229: P_{2872} = (7, 2, 10, 1)$
$230: P_{2876} = (11, 2, 10, 1)$
$231: P_{2920} = (7, 5, 10, 1)$
$232: P_{2928} = (15, 5, 10, 1)$
$233: P_{2983} = (6, 9, 10, 1)$
$234: P_{2988} = (11, 9, 10, 1)$
$235: P_{2998} = (5, 10, 10, 1)$
$236: P_{3008} = (15, 10, 10, 1)$
$237: P_{3019} = (10, 11, 10, 1)$
$238: P_{3078} = (5, 15, 10, 1)$
$239: P_{3079} = (6, 15, 10, 1)$
$240: P_{3089} = (0, 0, 11, 1)$
$241: P_{3090} = (1, 0, 11, 1)$
$242: P_{3106} = (1, 1, 11, 1)$
$243: P_{3116} = (11, 1, 11, 1)$
$244: P_{3145} = (8, 3, 11, 1)$
$245: P_{3150} = (13, 3, 11, 1)$
$246: P_{3163} = (10, 4, 11, 1)$
$247: P_{3165} = (12, 4, 11, 1)$
$248: P_{3220} = (3, 8, 11, 1)$
$249: P_{3229} = (12, 8, 11, 1)$
$250: P_{3260} = (11, 10, 11, 1)$
$251: P_{3268} = (3, 11, 11, 1)$
$252: P_{3273} = (8, 11, 11, 1)$
$253: P_{3323} = (10, 14, 11, 1)$
$254: P_{3326} = (13, 14, 11, 1)$
$255: P_{3345} = (0, 0, 12, 1)$
$256: P_{3346} = (1, 0, 12, 1)$
$257: P_{3362} = (1, 1, 12, 1)$
$258: P_{3373} = (12, 1, 12, 1)$
$259: P_{3391} = (14, 2, 12, 1)$
$260: P_{3399} = (6, 3, 12, 1)$
$261: P_{3403} = (10, 3, 12, 1)$
$262: P_{3431} = (6, 5, 12, 1)$
$263: P_{3434} = (9, 5, 12, 1)$
$264: P_{3492} = (3, 9, 12, 1)$
$265: P_{3499} = (10, 9, 12, 1)$
$266: P_{3508} = (3, 10, 12, 1)$
$267: P_{3512} = (7, 10, 12, 1)$
$268: P_{3546} = (9, 12, 12, 1)$
$269: P_{3551} = (14, 12, 12, 1)$
$270: P_{3560} = (7, 13, 12, 1)$
$271: P_{3565} = (12, 13, 12, 1)$
$272: P_{3601} = (0, 0, 13, 1)$
$273: P_{3602} = (1, 0, 13, 1)$
$274: P_{3618} = (1, 1, 13, 1)$
$275: P_{3630} = (13, 1, 13, 1)$
$276: P_{3641} = (8, 2, 13, 1)$
$277: P_{3643} = (10, 2, 13, 1)$
$278: P_{3736} = (7, 8, 13, 1)$
$279: P_{3739} = (10, 8, 13, 1)$
$280: P_{3749} = (4, 9, 13, 1)$
$281: P_{3767} = (6, 10, 13, 1)$
$282: P_{3769} = (8, 10, 13, 1)$
$283: P_{3799} = (6, 12, 13, 1)$
$284: P_{3806} = (13, 12, 13, 1)$
$285: P_{3811} = (2, 13, 13, 1)$
$286: P_{3813} = (4, 13, 13, 1)$
$287: P_{3843} = (2, 15, 13, 1)$
$288: P_{3848} = (7, 15, 13, 1)$
$289: P_{3857} = (0, 0, 14, 1)$
$290: P_{3858} = (1, 0, 14, 1)$
$291: P_{3874} = (1, 1, 14, 1)$
$292: P_{3887} = (14, 1, 14, 1)$
$293: P_{3982} = (13, 7, 14, 1)$
$294: P_{4006} = (5, 9, 14, 1)$
$295: P_{4008} = (7, 9, 14, 1)$
$296: P_{4021} = (4, 10, 14, 1)$
$297: P_{4030} = (13, 10, 14, 1)$
$298: P_{4052} = (3, 12, 14, 1)$
$299: P_{4054} = (5, 12, 14, 1)$
$300: P_{4085} = (4, 14, 14, 1)$
$301: P_{4088} = (7, 14, 14, 1)$
$302: P_{4100} = (3, 15, 14, 1)$
$303: P_{4111} = (14, 15, 14, 1)$
$304: P_{4113} = (0, 0, 15, 1)$
$305: P_{4114} = (1, 0, 15, 1)$
$306: P_{4130} = (1, 1, 15, 1)$
$307: P_{4144} = (15, 1, 15, 1)$
$308: P_{4168} = (7, 3, 15, 1)$
$309: P_{4175} = (14, 3, 15, 1)$
$310: P_{4184} = (7, 4, 15, 1)$
$311: P_{4185} = (8, 4, 15, 1)$
$312: P_{4196} = (3, 5, 15, 1)$
$313: P_{4212} = (3, 6, 15, 1)$
$314: P_{4220} = (11, 6, 15, 1)$
$315: P_{4307} = (2, 12, 15, 1)$
$316: P_{4313} = (8, 12, 15, 1)$
$317: P_{4332} = (11, 13, 15, 1)$
$318: P_{4335} = (14, 13, 15, 1)$
$319: P_{4339} = (2, 14, 15, 1)$
$320: P_{4352} = (15, 14, 15, 1)$