

# Rank-76291 over GF(16)

January 15, 2021

## The equation

The equation of the surface is :

$$X_1^2 X_3 + X_1 X_2^2 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

( 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0 )

The point rank of the equation over GF(16) is 286331157

## General information

Number of lines	16
Number of points	337
Number of singular points	2
Number of Eckardt points	3
Number of double points	29
Number of single points	195
Number of points off lines	108
Number of Hesse planes	0
Number of axes	0
Type of points on lines	$17^{16}$
Type of lines on points	$5^2, 3^3, 2^{29}, 1^{195}, 0^{108}$

## Singular Points

The surface has 2 singular points:

$$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0)$$

$$1 : P_{20} = \mathbf{P}(1, 0, 1, 0) = \mathbf{P}(1, 0, 1, 0)$$

## The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0, 0, 1, 0, 0, 0)_2 \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{273} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{273} = \mathbf{Pl}(1, 0, 0, 0, 0, 1)_{4626} \\
\ell_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(1, 0, 1, 0, 1, 0)_{321} \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^5 & \delta^{10} \end{bmatrix}_{171} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{171} = \mathbf{Pl}(11, 0, 10, 0, 1, 0)_{610} \\
\ell_5 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \delta^{10} & \delta^5 \end{bmatrix}_{186} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 10 & 11 \end{bmatrix}_{186} = \mathbf{Pl}(10, 0, 11, 0, 1, 0)_{640} \\
\ell_6 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{70160} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\
\ell_7 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{530} = \mathbf{Pl}(0, 0, 1, 1, 1, 1)_{9426} \\
\ell_8 &= \begin{bmatrix} 1 & \delta^5 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{3269} = \begin{bmatrix} 1 & 11 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{3269} = \mathbf{Pl}(0, 0, 11, 10, 11, 1)_{50536} \\
\ell_9 &= \begin{bmatrix} 1 & \delta^{10} & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{2997} = \begin{bmatrix} 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{2997} = \mathbf{Pl}(0, 0, 10, 11, 10, 1)_{46425} \\
\ell_{10} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{290} = \mathbf{Pl}(1, 1, 1, 0, 1, 1)_{8976} \\
\ell_{11} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^{10} & \delta^5 \end{bmatrix}_{459} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & 11 \end{bmatrix}_{459} = \mathbf{Pl}(10, 11, 11, 0, 11, 1)_{49935} \\
\ell_{12} &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \delta^5 & \delta^{10} \end{bmatrix}_{444} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 11 & 10 \end{bmatrix}_{444} = \mathbf{Pl}(11, 10, 10, 0, 10, 1)_{45841} \\
\ell_{13} &= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4898} = \mathbf{Pl}(0, 1, 1, 1, 1, 1)_{9442} \\
\ell_{14} &= \begin{bmatrix} 1 & \delta^{10} & 0 & \delta^5 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{51045} = \begin{bmatrix} 1 & 10 & 0 & 11 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{51045} = \mathbf{Pl}(0, 10, 10, 11, 10, 1)_{46450} \\
\ell_{15} &= \begin{bmatrix} 1 & \delta^5 & 0 & \delta^{10} \\ 0 & 0 & 1 & \delta^{10} \end{bmatrix}_{46949} = \begin{bmatrix} 1 & 11 & 0 & 10 \\ 0 & 0 & 1 & 10 \end{bmatrix}_{46949} = \mathbf{Pl}(0, 11, 11, 10, 11, 1)_{50562}
\end{aligned}$$

Rank of lines: ( 0, 256, 273, 17, 171, 186, 70160, 530, 3269, 2997, 290, 459, 444, 4898, 51045, 46949 )

Rank of points on Klein quadric: ( 0, 2, 4626, 321, 610, 640, 1, 9426, 50536, 46425, 8976, 49935, 45841, 9442, 46450, 50562 )

### Eckardt Points

The surface has 3 Eckardt points:

$$0 : P_{530} = \mathbf{P}(0, 0, 1, 1) = \mathbf{P}(0, 0, 1, 1),$$

$$1 : P_{2833} = \mathbf{P}(0, 0, \delta^{10}, 1) = \mathbf{P}(0, 0, 10, 1),$$

$$2 : P_{3089} = \mathbf{P}(0, 0, \delta^5, 1) = \mathbf{P}(0, 0, 11, 1).$$

### Double Points

The surface has 29 Double points:

The double points on the surface are:

$$\begin{aligned}
P_1 &= (0, 1, 0, 0) = \ell_0 \cap \ell_2 \\
P_5 &= (1, 1, 0, 0) = \ell_0 \cap \ell_7 \\
P_{14} &= (10, 1, 0, 0) = \ell_0 \cap \ell_8 \\
P_{15} &= (11, 1, 0, 0) = \ell_0 \cap \ell_9 \\
P_2 &= (0, 0, 1, 0) = \ell_1 \cap \ell_6 \\
P_{36} &= (1, 1, 1, 0) = \ell_2 \cap \ell_{13} \\
P_{180} &= (1, 10, 1, 0) = \ell_2 \cap \ell_{14} \\
P_{196} &= (1, 11, 1, 0) = \ell_2 \cap \ell_{15} \\
P_4 &= (1, 1, 1, 1) = \ell_3 \cap \ell_7 \\
P_{546} &= (0, 1, 1, 1) = \ell_3 \cap \ell_{10} \\
P_{556} &= (11, 1, 1, 1) = \ell_3 \cap \ell_{14} \\
P_{555} &= (10, 1, 1, 1) = \ell_3 \cap \ell_{15} \\
P_{3019} &= (10, 11, 10, 1) = \ell_4 \cap \ell_9 \\
P_{3009} &= (0, 11, 10, 1) = \ell_4 \cap \ell_{12} \\
P_{3020} &= (11, 11, 10, 1) = \ell_4 \cap \ell_{13}
\end{aligned}$$

$$\begin{aligned}
P_{3010} &= (1, 11, 10, 1) = \ell_4 \cap \ell_{15} \\
P_{3260} &= (11, 10, 11, 1) = \ell_5 \cap \ell_8 \\
P_{3249} &= (0, 10, 11, 1) = \ell_5 \cap \ell_{11} \\
P_{3259} &= (10, 10, 11, 1) = \ell_5 \cap \ell_{13} \\
P_{3250} &= (1, 10, 11, 1) = \ell_5 \cap \ell_{14} \\
P_{699} &= (10, 10, 1, 1) = \ell_7 \cap \ell_{11} \\
P_{716} &= (11, 11, 1, 1) = \ell_7 \cap \ell_{12} \\
P_{3115} &= (10, 1, 11, 1) = \ell_8 \cap \ell_{10} \\
P_{3266} &= (1, 11, 11, 1) = \ell_8 \cap \ell_{12} \\
P_{2860} &= (11, 1, 10, 1) = \ell_9 \cap \ell_{10} \\
P_{2994} &= (1, 10, 10, 1) = \ell_9 \cap \ell_{11} \\
P_{291} &= (1, 1, 0, 1) = \ell_{10} \cap \ell_{13} \\
P_{445} &= (11, 10, 0, 1) = \ell_{11} \cap \ell_{15} \\
P_{460} &= (10, 11, 0, 1) = \ell_{12} \cap \ell_{14}
\end{aligned}$$

### Single Points

The surface has 195 single points:  
The single points on the surface are:

$$\begin{aligned}
0 : P_3 &= (0, 0, 0, 1) \text{ lies on line } \ell_6 \\
1 : P_6 &= (2, 1, 0, 0) \text{ lies on line } \ell_0 \\
2 : P_7 &= (3, 1, 0, 0) \text{ lies on line } \ell_0 \\
3 : P_8 &= (4, 1, 0, 0) \text{ lies on line } \ell_0 \\
4 : P_9 &= (5, 1, 0, 0) \text{ lies on line } \ell_0 \\
5 : P_{10} &= (6, 1, 0, 0) \text{ lies on line } \ell_0 \\
6 : P_{11} &= (7, 1, 0, 0) \text{ lies on line } \ell_0 \\
7 : P_{12} &= (8, 1, 0, 0) \text{ lies on line } \ell_0 \\
8 : P_{13} &= (9, 1, 0, 0) \text{ lies on line } \ell_0 \\
9 : P_{16} &= (12, 1, 0, 0) \text{ lies on line } \ell_0 \\
10 : P_{17} &= (13, 1, 0, 0) \text{ lies on line } \ell_0 \\
11 : P_{18} &= (14, 1, 0, 0) \text{ lies on line } \ell_0 \\
12 : P_{19} &= (15, 1, 0, 0) \text{ lies on line } \ell_0 \\
13 : P_{21} &= (2, 0, 1, 0) \text{ lies on line } \ell_1 \\
14 : P_{22} &= (3, 0, 1, 0) \text{ lies on line } \ell_1 \\
15 : P_{23} &= (4, 0, 1, 0) \text{ lies on line } \ell_1 \\
16 : P_{24} &= (5, 0, 1, 0) \text{ lies on line } \ell_1 \\
17 : P_{25} &= (6, 0, 1, 0) \text{ lies on line } \ell_1 \\
18 : P_{26} &= (7, 0, 1, 0) \text{ lies on line } \ell_1 \\
19 : P_{27} &= (8, 0, 1, 0) \text{ lies on line } \ell_1 \\
20 : P_{28} &= (9, 0, 1, 0) \text{ lies on line } \ell_1 \\
21 : P_{29} &= (10, 0, 1, 0) \text{ lies on line } \ell_1 \\
22 : P_{30} &= (11, 0, 1, 0) \text{ lies on line } \ell_1 \\
23 : P_{31} &= (12, 0, 1, 0) \text{ lies on line } \ell_1 \\
24 : P_{32} &= (13, 0, 1, 0) \text{ lies on line } \ell_1 \\
25 : P_{33} &= (14, 0, 1, 0) \text{ lies on line } \ell_1 \\
26 : P_{34} &= (15, 0, 1, 0) \text{ lies on line } \ell_1 \\
27 : P_{52} &= (1, 2, 1, 0) \text{ lies on line } \ell_2 \\
28 : P_{68} &= (1, 3, 1, 0) \text{ lies on line } \ell_2 \\
29 : P_{84} &= (1, 4, 1, 0) \text{ lies on line } \ell_2 \\
30 : P_{100} &= (1, 5, 1, 0) \text{ lies on line } \ell_2 \\
31 : P_{116} &= (1, 6, 1, 0) \text{ lies on line } \ell_2
\end{aligned}$$

$$\begin{aligned}
32 : P_{132} &= (1, 7, 1, 0) \text{ lies on line } \ell_2 \\
33 : P_{148} &= (1, 8, 1, 0) \text{ lies on line } \ell_2 \\
34 : P_{164} &= (1, 9, 1, 0) \text{ lies on line } \ell_2 \\
35 : P_{212} &= (1, 12, 1, 0) \text{ lies on line } \ell_2 \\
36 : P_{228} &= (1, 13, 1, 0) \text{ lies on line } \ell_2 \\
37 : P_{244} &= (1, 14, 1, 0) \text{ lies on line } \ell_2 \\
38 : P_{260} &= (1, 15, 1, 0) \text{ lies on line } \ell_2 \\
39 : P_{547} &= (2, 1, 1, 1) \text{ lies on line } \ell_3 \\
40 : P_{548} &= (3, 1, 1, 1) \text{ lies on line } \ell_3 \\
41 : P_{549} &= (4, 1, 1, 1) \text{ lies on line } \ell_3 \\
42 : P_{550} &= (5, 1, 1, 1) \text{ lies on line } \ell_3 \\
43 : P_{551} &= (6, 1, 1, 1) \text{ lies on line } \ell_3 \\
44 : P_{552} &= (7, 1, 1, 1) \text{ lies on line } \ell_3 \\
45 : P_{553} &= (8, 1, 1, 1) \text{ lies on line } \ell_3 \\
46 : P_{554} &= (9, 1, 1, 1) \text{ lies on line } \ell_3 \\
47 : P_{557} &= (12, 1, 1, 1) \text{ lies on line } \ell_3 \\
48 : P_{558} &= (13, 1, 1, 1) \text{ lies on line } \ell_3 \\
49 : P_{559} &= (14, 1, 1, 1) \text{ lies on line } \ell_3 \\
50 : P_{560} &= (15, 1, 1, 1) \text{ lies on line } \ell_3 \\
51 : P_{563} &= (2, 2, 1, 1) \text{ lies on line } \ell_7 \\
52 : P_{580} &= (3, 3, 1, 1) \text{ lies on line } \ell_7 \\
53 : P_{597} &= (4, 4, 1, 1) \text{ lies on line } \ell_7 \\
54 : P_{614} &= (5, 5, 1, 1) \text{ lies on line } \ell_7 \\
55 : P_{631} &= (6, 6, 1, 1) \text{ lies on line } \ell_7 \\
56 : P_{648} &= (7, 7, 1, 1) \text{ lies on line } \ell_7 \\
57 : P_{665} &= (8, 8, 1, 1) \text{ lies on line } \ell_7 \\
58 : P_{682} &= (9, 9, 1, 1) \text{ lies on line } \ell_7 \\
59 : P_{733} &= (12, 12, 1, 1) \text{ lies on line } \ell_7 \\
60 : P_{750} &= (13, 13, 1, 1) \text{ lies on line } \ell_7 \\
61 : P_{767} &= (14, 14, 1, 1) \text{ lies on line } \ell_7 \\
62 : P_{784} &= (15, 15, 1, 1) \text{ lies on line } \ell_7 \\
63 : P_{785} &= (0, 0, 2, 1) \text{ lies on line } \ell_6
\end{aligned}$$

64 :  $P_{804} = (3, 1, 2, 1)$  lies on line  $\ell_{10}$   
 65 :  $P_{836} = (3, 3, 2, 1)$  lies on line  $\ell_{13}$   
 66 :  $P_{874} = (9, 5, 2, 1)$  lies on line  $\ell_{15}$   
 67 :  $P_{889} = (8, 6, 2, 1)$  lies on line  $\ell_{14}$   
 68 :  $P_{954} = (9, 10, 2, 1)$  lies on line  $\ell_{11}$   
 69 :  $P_{969} = (8, 11, 2, 1)$  lies on line  $\ell_{12}$   
 70 :  $P_{1041} = (0, 0, 3, 1)$  lies on line  $\ell_6$   
 71 :  $P_{1059} = (2, 1, 3, 1)$  lies on line  $\ell_{10}$   
 72 :  $P_{1075} = (2, 2, 3, 1)$  lies on line  $\ell_{13}$   
 73 :  $P_{1209} = (8, 10, 3, 1)$  lies on line  $\ell_{11}$   
 74 :  $P_{1226} = (9, 11, 3, 1)$  lies on line  $\ell_{12}$   
 75 :  $P_{1242} = (9, 12, 3, 1)$  lies on line  $\ell_{14}$   
 76 :  $P_{1273} = (8, 14, 3, 1)$  lies on line  $\ell_{15}$   
 77 :  $P_{1297} = (0, 0, 4, 1)$  lies on line  $\ell_6$   
 78 :  $P_{1318} = (5, 1, 4, 1)$  lies on line  $\ell_{10}$   
 79 :  $P_{1382} = (5, 5, 4, 1)$  lies on line  $\ell_{13}$   
 80 :  $P_{1439} = (14, 8, 4, 1)$  lies on line  $\ell_{14}$   
 81 :  $P_{1472} = (15, 10, 4, 1)$  lies on line  $\ell_{11}$   
 82 :  $P_{1487} = (14, 11, 4, 1)$  lies on line  $\ell_{12}$   
 83 :  $P_{1520} = (15, 13, 4, 1)$  lies on line  $\ell_{15}$   
 84 :  $P_{1553} = (0, 0, 5, 1)$  lies on line  $\ell_6$   
 85 :  $P_{1573} = (4, 1, 5, 1)$  lies on line  $\ell_{10}$   
 86 :  $P_{1600} = (15, 2, 5, 1)$  lies on line  $\ell_{14}$   
 87 :  $P_{1621} = (4, 4, 5, 1)$  lies on line  $\ell_{13}$   
 88 :  $P_{1663} = (14, 6, 5, 1)$  lies on line  $\ell_{15}$   
 89 :  $P_{1727} = (14, 10, 5, 1)$  lies on line  $\ell_{11}$   
 90 :  $P_{1744} = (15, 11, 5, 1)$  lies on line  $\ell_{12}$   
 91 :  $P_{1809} = (0, 0, 6, 1)$  lies on line  $\ell_6$   
 92 :  $P_{1832} = (7, 1, 6, 1)$  lies on line  $\ell_{10}$   
 93 :  $P_{1854} = (13, 2, 6, 1)$  lies on line  $\ell_{15}$   
 94 :  $P_{1901} = (12, 5, 6, 1)$  lies on line  $\ell_{14}$   
 95 :  $P_{1928} = (7, 7, 6, 1)$  lies on line  $\ell_{13}$   
 96 :  $P_{1982} = (13, 10, 6, 1)$  lies on line  $\ell_{11}$   
 97 :  $P_{1997} = (12, 11, 6, 1)$  lies on line  $\ell_{12}$   
 98 :  $P_{2065} = (0, 0, 7, 1)$  lies on line  $\ell_6$   
 99 :  $P_{2087} = (6, 1, 7, 1)$  lies on line  $\ell_{10}$   
 100 :  $P_{2167} = (6, 6, 7, 1)$  lies on line  $\ell_{13}$   
 101 :  $P_{2221} = (12, 9, 7, 1)$  lies on line  $\ell_{15}$   
 102 :  $P_{2237} = (12, 10, 7, 1)$  lies on line  $\ell_{11}$   
 103 :  $P_{2254} = (13, 11, 7, 1)$  lies on line  $\ell_{12}$   
 104 :  $P_{2318} = (13, 15, 7, 1)$  lies on line  $\ell_{14}$   
 105 :  $P_{2321} = (0, 0, 8, 1)$  lies on line  $\ell_6$   
 106 :  $P_{2346} = (9, 1, 8, 1)$  lies on line  $\ell_{10}$   
 107 :  $P_{2388} = (3, 4, 8, 1)$  lies on line  $\ell_{15}$   
 108 :  $P_{2474} = (9, 9, 8, 1)$  lies on line  $\ell_{13}$   
 109 :  $P_{2484} = (3, 10, 8, 1)$  lies on line  $\ell_{11}$   
 110 :  $P_{2499} = (2, 11, 8, 1)$  lies on line  $\ell_{12}$   
 111 :  $P_{2531} = (2, 13, 8, 1)$  lies on line  $\ell_{14}$   
 112 :  $P_{2577} = (0, 0, 9, 1)$  lies on line  $\ell_6$   
 113 :  $P_{2601} = (8, 1, 9, 1)$  lies on line  $\ell_{10}$   
 114 :  $P_{2692} = (3, 7, 9, 1)$  lies on line  $\ell_{14}$   
 115 :  $P_{2713} = (8, 8, 9, 1)$  lies on line  $\ell_{13}$   
 116 :  $P_{2739} = (2, 10, 9, 1)$  lies on line  $\ell_{11}$   
 117 :  $P_{2756} = (3, 11, 9, 1)$  lies on line  $\ell_{12}$

118 :  $P_{2819} = (2, 15, 9, 1)$  lies on line  $\ell_{15}$   
 119 :  $P_{2880} = (15, 2, 10, 1)$  lies on line  $\ell_9$   
 120 :  $P_{2885} = (4, 3, 10, 1)$  lies on line  $\ell_9$   
 121 :  $P_{2904} = (7, 4, 10, 1)$  lies on line  $\ell_9$   
 122 :  $P_{2925} = (12, 5, 10, 1)$  lies on line  $\ell_9$   
 123 :  $P_{2937} = (8, 6, 10, 1)$  lies on line  $\ell_9$   
 124 :  $P_{2948} = (3, 7, 10, 1)$  lies on line  $\ell_9$   
 125 :  $P_{2975} = (14, 8, 10, 1)$  lies on line  $\ell_9$   
 126 :  $P_{2982} = (5, 9, 10, 1)$  lies on line  $\ell_9$   
 127 :  $P_{3011} = (2, 11, 10, 1)$  lies on line  $\ell_4$   
 128 :  $P_{3012} = (3, 11, 10, 1)$  lies on line  $\ell_4$   
 129 :  $P_{3013} = (4, 11, 10, 1)$  lies on line  $\ell_4$   
 130 :  $P_{3014} = (5, 11, 10, 1)$  lies on line  $\ell_4$   
 131 :  $P_{3015} = (6, 11, 10, 1)$  lies on line  $\ell_4$   
 132 :  $P_{3016} = (7, 11, 10, 1)$  lies on line  $\ell_4$   
 133 :  $P_{3017} = (8, 11, 10, 1)$  lies on line  $\ell_4$   
 134 :  $P_{3018} = (9, 11, 10, 1)$  lies on line  $\ell_4$   
 135 :  $P_{3021} = (12, 11, 10, 1)$  lies on line  $\ell_4$   
 136 :  $P_{3022} = (13, 11, 10, 1)$  lies on line  $\ell_4$   
 137 :  $P_{3023} = (14, 11, 10, 1)$  lies on line  $\ell_4$   
 138 :  $P_{3024} = (15, 11, 10, 1)$  lies on line  $\ell_4$   
 139 :  $P_{3034} = (9, 12, 10, 1)$  lies on line  $\ell_9$   
 140 :  $P_{3043} = (2, 13, 10, 1)$  lies on line  $\ell_9$   
 141 :  $P_{3063} = (6, 14, 10, 1)$  lies on line  $\ell_9$   
 142 :  $P_{3086} = (13, 15, 10, 1)$  lies on line  $\ell_9$   
 143 :  $P_{3134} = (13, 2, 11, 1)$  lies on line  $\ell_8$   
 144 :  $P_{3144} = (7, 3, 11, 1)$  lies on line  $\ell_8$   
 145 :  $P_{3156} = (3, 4, 11, 1)$  lies on line  $\ell_8$   
 146 :  $P_{3178} = (9, 5, 11, 1)$  lies on line  $\ell_8$   
 147 :  $P_{3199} = (14, 6, 11, 1)$  lies on line  $\ell_8$   
 148 :  $P_{3205} = (4, 7, 11, 1)$  lies on line  $\ell_8$   
 149 :  $P_{3223} = (6, 8, 11, 1)$  lies on line  $\ell_8$   
 150 :  $P_{3245} = (12, 9, 11, 1)$  lies on line  $\ell_8$   
 151 :  $P_{3251} = (2, 10, 11, 1)$  lies on line  $\ell_5$   
 152 :  $P_{3252} = (3, 10, 11, 1)$  lies on line  $\ell_5$   
 153 :  $P_{3253} = (4, 10, 11, 1)$  lies on line  $\ell_5$   
 154 :  $P_{3254} = (5, 10, 11, 1)$  lies on line  $\ell_5$   
 155 :  $P_{3255} = (6, 10, 11, 1)$  lies on line  $\ell_5$   
 156 :  $P_{3256} = (7, 10, 11, 1)$  lies on line  $\ell_5$   
 157 :  $P_{3257} = (8, 10, 11, 1)$  lies on line  $\ell_5$   
 158 :  $P_{3258} = (9, 10, 11, 1)$  lies on line  $\ell_5$   
 159 :  $P_{3261} = (12, 10, 11, 1)$  lies on line  $\ell_5$   
 160 :  $P_{3262} = (13, 10, 11, 1)$  lies on line  $\ell_5$   
 161 :  $P_{3263} = (14, 10, 11, 1)$  lies on line  $\ell_5$   
 162 :  $P_{3264} = (15, 10, 11, 1)$  lies on line  $\ell_5$   
 163 :  $P_{3286} = (5, 12, 11, 1)$  lies on line  $\ell_8$   
 164 :  $P_{3312} = (15, 13, 11, 1)$  lies on line  $\ell_8$   
 165 :  $P_{3321} = (8, 14, 11, 1)$  lies on line  $\ell_8$   
 166 :  $P_{3331} = (2, 15, 11, 1)$  lies on line  $\ell_8$   
 167 :  $P_{3345} = (0, 0, 12, 1)$  lies on line  $\ell_6$   
 168 :  $P_{3374} = (13, 1, 12, 1)$  lies on line  $\ell_{10}$   
 169 :  $P_{3400} = (7, 3, 12, 1)$  lies on line  $\ell_{15}$   
 170 :  $P_{3512} = (7, 10, 12, 1)$  lies on line  $\ell_{11}$   
 171 :  $P_{3527} = (6, 11, 12, 1)$  lies on line  $\ell_{12}$

172 :  $P_{3566} = (13, 13, 12, 1)$  lies on line  $\ell_{13}$   
 173 :  $P_{3575} = (6, 14, 12, 1)$  lies on line  $\ell_{14}$   
 174 :  $P_{3601} = (0, 0, 13, 1)$  lies on line  $\ell_6$   
 175 :  $P_{3629} = (12, 1, 13, 1)$  lies on line  $\ell_{10}$   
 176 :  $P_{3672} = (7, 4, 13, 1)$  lies on line  $\ell_{14}$   
 177 :  $P_{3735} = (6, 8, 13, 1)$  lies on line  $\ell_{15}$   
 178 :  $P_{3767} = (6, 10, 13, 1)$  lies on line  $\ell_{11}$   
 179 :  $P_{3784} = (7, 11, 13, 1)$  lies on line  $\ell_{12}$   
 180 :  $P_{3805} = (12, 12, 13, 1)$  lies on line  $\ell_{13}$   
 181 :  $P_{3857} = (0, 0, 14, 1)$  lies on line  $\ell_6$   
 182 :  $P_{3888} = (15, 1, 14, 1)$  lies on line  $\ell_{10}$   
 183 :  $P_{3909} = (4, 3, 14, 1)$  lies on line  $\ell_{14}$

184 :  $P_{4022} = (5, 10, 14, 1)$  lies on line  $\ell_{11}$   
 185 :  $P_{4037} = (4, 11, 14, 1)$  lies on line  $\ell_{12}$   
 186 :  $P_{4054} = (5, 12, 14, 1)$  lies on line  $\ell_{15}$   
 187 :  $P_{4112} = (15, 15, 14, 1)$  lies on line  $\ell_{13}$   
 188 :  $P_{4113} = (0, 0, 15, 1)$  lies on line  $\ell_6$   
 189 :  $P_{4143} = (14, 1, 15, 1)$  lies on line  $\ell_{10}$   
 190 :  $P_{4229} = (4, 7, 15, 1)$  lies on line  $\ell_{15}$   
 191 :  $P_{4262} = (5, 9, 15, 1)$  lies on line  $\ell_{14}$   
 192 :  $P_{4277} = (4, 10, 15, 1)$  lies on line  $\ell_{11}$   
 193 :  $P_{4294} = (5, 11, 15, 1)$  lies on line  $\ell_{12}$   
 194 :  $P_{4351} = (14, 14, 15, 1)$  lies on line  $\ell_{13}$

The single points on the surface are:

### Points on surface but on no line

The surface has 108 points not on any line:

The points on the surface but not on lines are:

0 :  $P_{310} = (4, 2, 0, 1)$   
 1 :  $P_{327} = (5, 3, 0, 1)$   
 2 :  $P_{347} = (9, 4, 0, 1)$   
 3 :  $P_{362} = (8, 5, 0, 1)$   
 4 :  $P_{383} = (13, 6, 0, 1)$   
 5 :  $P_{398} = (12, 7, 0, 1)$   
 6 :  $P_{417} = (15, 8, 0, 1)$   
 7 :  $P_{432} = (14, 9, 0, 1)$   
 8 :  $P_{472} = (6, 12, 0, 1)$   
 9 :  $P_{489} = (7, 13, 0, 1)$   
 10 :  $P_{500} = (2, 14, 0, 1)$   
 11 :  $P_{517} = (3, 15, 0, 1)$   
 12 :  $P_{828} = (11, 2, 2, 1)$   
 13 :  $P_{849} = (0, 4, 2, 1)$   
 14 :  $P_{903} = (6, 7, 2, 1)$   
 15 :  $P_{925} = (12, 8, 2, 1)$   
 16 :  $P_{940} = (11, 9, 2, 1)$   
 17 :  $P_{1005} = (12, 13, 2, 1)$   
 18 :  $P_{1011} = (2, 14, 2, 1)$   
 19 :  $P_{1031} = (6, 15, 2, 1)$   
 20 :  $P_{1103} = (14, 3, 3, 1)$   
 21 :  $P_{1120} = (15, 4, 3, 1)$   
 22 :  $P_{1121} = (0, 5, 3, 1)$   
 23 :  $P_{1148} = (11, 6, 3, 1)$   
 24 :  $P_{1164} = (11, 7, 3, 1)$   
 25 :  $P_{1200} = (15, 9, 3, 1)$   
 26 :  $P_{1263} = (14, 13, 3, 1)$   
 27 :  $P_{1284} = (3, 15, 3, 1)$   
 28 :  $P_{1333} = (4, 2, 4, 1)$   
 29 :  $P_{1358} = (13, 3, 4, 1)$   
 30 :  $P_{1371} = (10, 4, 4, 1)$   
 31 :  $P_{1415} = (6, 7, 4, 1)$

32 :  $P_{1441} = (0, 9, 4, 1)$   
 33 :  $P_{1502} = (13, 12, 4, 1)$   
 34 :  $P_{1531} = (10, 14, 4, 1)$   
 35 :  $P_{1543} = (6, 15, 4, 1)$   
 36 :  $P_{1606} = (5, 3, 5, 1)$   
 37 :  $P_{1635} = (2, 5, 5, 1)$   
 38 :  $P_{1667} = (2, 7, 5, 1)$   
 39 :  $P_{1681} = (0, 8, 5, 1)$   
 40 :  $P_{1700} = (3, 9, 5, 1)$   
 41 :  $P_{1755} = (10, 12, 5, 1)$   
 42 :  $P_{1771} = (10, 13, 5, 1)$   
 43 :  $P_{1780} = (3, 14, 5, 1)$   
 44 :  $P_{1858} = (1, 3, 6, 1)$   
 45 :  $P_{1914} = (9, 6, 6, 1)$   
 46 :  $P_{1938} = (1, 8, 6, 1)$   
 47 :  $P_{1956} = (3, 9, 6, 1)$   
 48 :  $P_{2007} = (6, 12, 6, 1)$   
 49 :  $P_{2017} = (0, 13, 6, 1)$   
 50 :  $P_{2036} = (3, 14, 6, 1)$   
 51 :  $P_{2058} = (9, 15, 6, 1)$   
 52 :  $P_{2105} = (8, 2, 7, 1)$   
 53 :  $P_{2114} = (1, 3, 7, 1)$   
 54 :  $P_{2137} = (8, 4, 7, 1)$   
 55 :  $P_{2147} = (2, 5, 7, 1)$   
 56 :  $P_{2179} = (2, 7, 7, 1)$   
 57 :  $P_{2194} = (1, 8, 7, 1)$   
 58 :  $P_{2257} = (0, 12, 7, 1)$   
 59 :  $P_{2280} = (7, 13, 7, 1)$   
 60 :  $P_{2358} = (5, 2, 8, 1)$   
 61 :  $P_{2409} = (8, 5, 8, 1)$   
 62 :  $P_{2428} = (11, 6, 8, 1)$   
 63 :  $P_{2444} = (11, 7, 8, 1)$

64 : $P_{2453} = (4, 8, 8, 1)$	87 : $P_{3710} = (13, 6, 13, 1)$
65 : $P_{2517} = (4, 12, 8, 1)$	88 : $P_{3713} = (0, 7, 13, 1)$
66 : $P_{2550} = (5, 14, 8, 1)$	89 : $P_{3823} = (14, 13, 13, 1)$
67 : $P_{2561} = (0, 15, 8, 1)$	90 : $P_{3830} = (5, 14, 13, 1)$
68 : $P_{2620} = (11, 2, 9, 1)$	91 : $P_{3842} = (1, 15, 13, 1)$
69 : $P_{2638} = (13, 3, 9, 1)$	92 : $P_{3889} = (0, 2, 14, 1)$
70 : $P_{2650} = (9, 4, 9, 1)$	93 : $P_{3931} = (10, 4, 14, 1)$
71 : $P_{2664} = (7, 5, 9, 1)$	94 : $P_{3944} = (7, 5, 14, 1)$
72 : $P_{2680} = (7, 6, 9, 1)$	95 : $P_{3960} = (7, 6, 14, 1)$
73 : $P_{2732} = (11, 9, 9, 1)$	96 : $P_{3997} = (12, 8, 14, 1)$
74 : $P_{2782} = (13, 12, 9, 1)$	97 : $P_{4015} = (14, 9, 14, 1)$
75 : $P_{2801} = (0, 14, 9, 1)$	98 : $P_{4077} = (12, 13, 14, 1)$
76 : $P_{3424} = (15, 4, 12, 1)$	99 : $P_{4091} = (10, 14, 14, 1)$
77 : $P_{3426} = (1, 5, 12, 1)$	100 : $P_{4153} = (8, 2, 15, 1)$
78 : $P_{3441} = (0, 6, 12, 1)$	101 : $P_{4161} = (0, 3, 15, 1)$
79 : $P_{3469} = (12, 7, 12, 1)$	102 : $P_{4185} = (8, 4, 15, 1)$
80 : $P_{3477} = (4, 8, 12, 1)$	103 : $P_{4218} = (9, 6, 15, 1)$
81 : $P_{3504} = (15, 9, 12, 1)$	104 : $P_{4256} = (15, 8, 15, 1)$
82 : $P_{3541} = (4, 12, 12, 1)$	105 : $P_{4315} = (10, 12, 15, 1)$
83 : $P_{3586} = (1, 15, 12, 1)$	106 : $P_{4331} = (10, 13, 15, 1)$
84 : $P_{3638} = (5, 2, 13, 1)$	107 : $P_{4362} = (9, 15, 15, 1)$
85 : $P_{3663} = (14, 3, 13, 1)$	
86 : $P_{3682} = (1, 5, 13, 1)$	

## Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0
1	1	0	1	1	1	1	1	0	0	0	1	1	1	0	0	0
2	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1
3	1	1	0	0	1	1	0	1	0	0	1	0	0	0	1	1
4	1	1	0	1	0	1	0	0	0	1	0	0	1	1	0	1
5	1	1	0	1	1	0	0	0	1	0	0	1	0	1	1	0
6	0	1	0	0	0	0	0	1	1	1	0	0	0	1	1	1
7	1	0	0	1	0	0	1	0	0	0	0	1	1	1	0	0
8	1	0	0	0	0	1	1	0	0	0	1	0	1	0	0	1
9	1	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0
10	0	1	1	1	0	0	0	0	1	1	0	1	1	1	0	0
11	0	1	1	0	0	1	0	1	0	1	1	0	1	0	0	1
12	0	1	1	0	1	0	0	1	1	0	1	1	0	0	1	0
13	0	0	1	0	1	1	1	1	0	0	1	0	0	0	0	0
14	0	0	1	1	0	1	1	0	0	1	0	0	1	0	0	0
15	0	0	1	1	1	0	1	0	1	0	0	1	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_7$	$\ell_8$	$\ell_9$
in point	$P_0$	$P_1$	$P_0$	$P_0$	$P_0$	$P_5$	$P_{14}$	$P_{15}$

Line 1 intersects

Line	$\ell_0$	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$
in point	$P_0$	$P_{20}$	$P_0$	$P_0$	$P_0$	$P_2$	$P_{20}$	$P_{20}$	$P_{20}$

Line 2 intersects

Line	$\ell_0$	$\ell_1$	$\ell_{10}$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_1$	$P_{20}$	$P_{20}$	$P_{20}$	$P_{20}$	$P_{36}$	$P_{180}$	$P_{196}$

Line 3 intersects

Line	$\ell_0$	$\ell_1$	$\ell_4$	$\ell_5$	$\ell_7$	$\ell_{10}$	$\ell_{14}$	$\ell_{15}$
in point	$P_0$	$P_0$	$P_0$	$P_0$	$P_4$	$P_{546}$	$P_{556}$	$P_{555}$

Line 4 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_5$	$\ell_9$	$\ell_{12}$	$\ell_{13}$	$\ell_{15}$
in point	$P_0$	$P_0$	$P_0$	$P_0$	$P_{3019}$	$P_{3009}$	$P_{3020}$	$P_{3010}$

Line 5 intersects

Line	$\ell_0$	$\ell_1$	$\ell_3$	$\ell_4$	$\ell_8$	$\ell_{11}$	$\ell_{13}$	$\ell_{14}$
in point	$P_0$	$P_0$	$P_0$	$P_0$	$P_{3260}$	$P_{3249}$	$P_{3259}$	$P_{3250}$

Line 6 intersects

Line	$\ell_1$	$\ell_7$	$\ell_8$	$\ell_9$	$\ell_{13}$	$\ell_{14}$	$\ell_{15}$
in point	$P_2$	$P_{530}$	$P_{3089}$	$P_{2833}$	$P_{530}$	$P_{2833}$	$P_{3089}$

Line 7 intersects

Line	$\ell_0$	$\ell_3$	$\ell_6$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$
in point	$P_5$	$P_4$	$P_{530}$	$P_{699}$	$P_{716}$	$P_{530}$

Line 8 intersects

Line	$\ell_0$	$\ell_5$	$\ell_6$	$\ell_{10}$	$\ell_{12}$	$\ell_{15}$
in point	$P_{14}$	$P_{3260}$	$P_{3089}$	$P_{3115}$	$P_{3266}$	$P_{3089}$

Line 9 intersects

Line	$\ell_0$	$\ell_4$	$\ell_6$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$
in point	$P_{15}$	$P_{3019}$	$P_{2833}$	$P_{2860}$	$P_{2994}$	$P_{2833}$

Line 10 intersects

Line	$\ell_1$	$\ell_2$	$\ell_3$	$\ell_8$	$\ell_9$	$\ell_{11}$	$\ell_{12}$	$\ell_{13}$
in point	$P_{20}$	$P_{20}$	$P_{546}$	$P_{3115}$	$P_{2860}$	$P_{20}$	$P_{20}$	$P_{291}$

Line 11 intersects

Line	$\ell_1$	$\ell_2$	$\ell_5$	$\ell_7$	$\ell_9$	$\ell_{10}$	$\ell_{12}$	$\ell_{15}$
in point	$P_{20}$	$P_{20}$	$P_{3249}$	$P_{699}$	$P_{2994}$	$P_{20}$	$P_{20}$	$P_{445}$

Line 12 intersects

Line	$\ell_1$	$\ell_2$	$\ell_4$	$\ell_7$	$\ell_8$	$\ell_{10}$	$\ell_{11}$	$\ell_{14}$
in point	$P_{20}$	$P_{20}$	$P_{3009}$	$P_{716}$	$P_{3266}$	$P_{20}$	$P_{20}$	$P_{460}$

Line 13 intersects

Line	$\ell_2$	$\ell_4$	$\ell_5$	$\ell_6$	$\ell_7$	$\ell_{10}$
in point	$P_{36}$	$P_{3020}$	$P_{3259}$	$P_{530}$	$P_{530}$	$P_{291}$

Line 14 intersects

Line	$\ell_2$	$\ell_3$	$\ell_5$	$\ell_6$	$\ell_9$	$\ell_{12}$
in point	$P_{180}$	$P_{556}$	$P_{3250}$	$P_{2833}$	$P_{2833}$	$P_{460}$

Line 15 intersects

Line	$\ell_2$	$\ell_3$	$\ell_4$	$\ell_6$	$\ell_8$	$\ell_{11}$
in point	$P_{196}$	$P_{555}$	$P_{3010}$	$P_{3089}$	$P_{3089}$	$P_{445}$

The surface has 337 points:

The points on the surface are:

0 :  $P_0 = (1, 0, 0, 0)$   
 1 :  $P_1 = (0, 1, 0, 0)$   
 2 :  $P_2 = (0, 0, 1, 0)$   
 3 :  $P_3 = (0, 0, 0, 1)$   
 4 :  $P_4 = (1, 1, 1, 1)$   
 5 :  $P_5 = (1, 1, 0, 0)$

6 :  $P_6 = (2, 1, 0, 0)$   
 7 :  $P_7 = (3, 1, 0, 0)$   
 8 :  $P_8 = (4, 1, 0, 0)$   
 9 :  $P_9 = (5, 1, 0, 0)$   
 10 :  $P_{10} = (6, 1, 0, 0)$   
 11 :  $P_{11} = (7, 1, 0, 0)$

12 :  $P_{12} = (8, 1, 0, 0)$   
 13 :  $P_{13} = (9, 1, 0, 0)$   
 14 :  $P_{14} = (10, 1, 0, 0)$   
 15 :  $P_{15} = (11, 1, 0, 0)$   
 16 :  $P_{16} = (12, 1, 0, 0)$   
 17 :  $P_{17} = (13, 1, 0, 0)$

18 : $P_{18} = (14, 1, 0, 0)$	72 : $P_{552} = (7, 1, 1, 1)$	126 : $P_{1318} = (5, 1, 4, 1)$
19 : $P_{19} = (15, 1, 0, 0)$	73 : $P_{553} = (8, 1, 1, 1)$	127 : $P_{1333} = (4, 2, 4, 1)$
20 : $P_{20} = (1, 0, 1, 0)$	74 : $P_{554} = (9, 1, 1, 1)$	128 : $P_{1358} = (13, 3, 4, 1)$
21 : $P_{21} = (2, 0, 1, 0)$	75 : $P_{555} = (10, 1, 1, 1)$	129 : $P_{1371} = (10, 4, 4, 1)$
22 : $P_{22} = (3, 0, 1, 0)$	76 : $P_{556} = (11, 1, 1, 1)$	130 : $P_{1382} = (5, 5, 4, 1)$
23 : $P_{23} = (4, 0, 1, 0)$	77 : $P_{557} = (12, 1, 1, 1)$	131 : $P_{1415} = (6, 7, 4, 1)$
24 : $P_{24} = (5, 0, 1, 0)$	78 : $P_{558} = (13, 1, 1, 1)$	132 : $P_{1439} = (14, 8, 4, 1)$
25 : $P_{25} = (6, 0, 1, 0)$	79 : $P_{559} = (14, 1, 1, 1)$	133 : $P_{1441} = (0, 9, 4, 1)$
26 : $P_{26} = (7, 0, 1, 0)$	80 : $P_{560} = (15, 1, 1, 1)$	134 : $P_{1472} = (15, 10, 4, 1)$
27 : $P_{27} = (8, 0, 1, 0)$	81 : $P_{563} = (2, 2, 1, 1)$	135 : $P_{1487} = (14, 11, 4, 1)$
28 : $P_{28} = (9, 0, 1, 0)$	82 : $P_{580} = (3, 3, 1, 1)$	136 : $P_{1502} = (13, 12, 4, 1)$
29 : $P_{29} = (10, 0, 1, 0)$	83 : $P_{597} = (4, 4, 1, 1)$	137 : $P_{1520} = (15, 13, 4, 1)$
30 : $P_{30} = (11, 0, 1, 0)$	84 : $P_{614} = (5, 5, 1, 1)$	138 : $P_{1531} = (10, 14, 4, 1)$
31 : $P_{31} = (12, 0, 1, 0)$	85 : $P_{631} = (6, 6, 1, 1)$	139 : $P_{1543} = (6, 15, 4, 1)$
32 : $P_{32} = (13, 0, 1, 0)$	86 : $P_{648} = (7, 7, 1, 1)$	140 : $P_{1553} = (0, 0, 5, 1)$
33 : $P_{33} = (14, 0, 1, 0)$	87 : $P_{665} = (8, 8, 1, 1)$	141 : $P_{1573} = (4, 1, 5, 1)$
34 : $P_{34} = (15, 0, 1, 0)$	88 : $P_{682} = (9, 9, 1, 1)$	142 : $P_{1600} = (15, 2, 5, 1)$
35 : $P_{36} = (1, 1, 1, 0)$	89 : $P_{699} = (10, 10, 1, 1)$	143 : $P_{1606} = (5, 3, 5, 1)$
36 : $P_{52} = (1, 2, 1, 0)$	90 : $P_{716} = (11, 11, 1, 1)$	144 : $P_{1621} = (4, 4, 5, 1)$
37 : $P_{68} = (1, 3, 1, 0)$	91 : $P_{733} = (12, 12, 1, 1)$	145 : $P_{1635} = (2, 5, 5, 1)$
38 : $P_{84} = (1, 4, 1, 0)$	92 : $P_{750} = (13, 13, 1, 1)$	146 : $P_{1663} = (14, 6, 5, 1)$
39 : $P_{100} = (1, 5, 1, 0)$	93 : $P_{767} = (14, 14, 1, 1)$	147 : $P_{1667} = (2, 7, 5, 1)$
40 : $P_{116} = (1, 6, 1, 0)$	94 : $P_{784} = (15, 15, 1, 1)$	148 : $P_{1681} = (0, 8, 5, 1)$
41 : $P_{132} = (1, 7, 1, 0)$	95 : $P_{785} = (0, 0, 2, 1)$	149 : $P_{1700} = (3, 9, 5, 1)$
42 : $P_{148} = (1, 8, 1, 0)$	96 : $P_{804} = (3, 1, 2, 1)$	150 : $P_{1727} = (14, 10, 5, 1)$
43 : $P_{164} = (1, 9, 1, 0)$	97 : $P_{828} = (11, 2, 2, 1)$	151 : $P_{1744} = (15, 11, 5, 1)$
44 : $P_{180} = (1, 10, 1, 0)$	98 : $P_{836} = (3, 3, 2, 1)$	152 : $P_{1755} = (10, 12, 5, 1)$
45 : $P_{196} = (1, 11, 1, 0)$	99 : $P_{849} = (0, 4, 2, 1)$	153 : $P_{1771} = (10, 13, 5, 1)$
46 : $P_{212} = (1, 12, 1, 0)$	100 : $P_{874} = (9, 5, 2, 1)$	154 : $P_{1780} = (3, 14, 5, 1)$
47 : $P_{228} = (1, 13, 1, 0)$	101 : $P_{889} = (8, 6, 2, 1)$	155 : $P_{1809} = (0, 0, 6, 1)$
48 : $P_{244} = (1, 14, 1, 0)$	102 : $P_{903} = (6, 7, 2, 1)$	156 : $P_{1832} = (7, 1, 6, 1)$
49 : $P_{260} = (1, 15, 1, 0)$	103 : $P_{925} = (12, 8, 2, 1)$	157 : $P_{1854} = (13, 2, 6, 1)$
50 : $P_{291} = (1, 1, 0, 1)$	104 : $P_{940} = (11, 9, 2, 1)$	158 : $P_{1858} = (1, 3, 6, 1)$
51 : $P_{310} = (4, 2, 0, 1)$	105 : $P_{954} = (9, 10, 2, 1)$	159 : $P_{1901} = (12, 5, 6, 1)$
52 : $P_{327} = (5, 3, 0, 1)$	106 : $P_{969} = (8, 11, 2, 1)$	160 : $P_{1914} = (9, 6, 6, 1)$
53 : $P_{347} = (9, 4, 0, 1)$	107 : $P_{1005} = (12, 13, 2, 1)$	161 : $P_{1928} = (7, 7, 6, 1)$
54 : $P_{362} = (8, 5, 0, 1)$	108 : $P_{1011} = (2, 14, 2, 1)$	162 : $P_{1938} = (1, 8, 6, 1)$
55 : $P_{383} = (13, 6, 0, 1)$	109 : $P_{1031} = (6, 15, 2, 1)$	163 : $P_{1956} = (3, 9, 6, 1)$
56 : $P_{398} = (12, 7, 0, 1)$	110 : $P_{1041} = (0, 0, 3, 1)$	164 : $P_{1982} = (13, 10, 6, 1)$
57 : $P_{417} = (15, 8, 0, 1)$	111 : $P_{1059} = (2, 1, 3, 1)$	165 : $P_{1997} = (12, 11, 6, 1)$
58 : $P_{432} = (14, 9, 0, 1)$	112 : $P_{1075} = (2, 2, 3, 1)$	166 : $P_{2007} = (6, 12, 6, 1)$
59 : $P_{445} = (11, 10, 0, 1)$	113 : $P_{1103} = (14, 3, 3, 1)$	167 : $P_{2017} = (0, 13, 6, 1)$
60 : $P_{460} = (10, 11, 0, 1)$	114 : $P_{1120} = (15, 4, 3, 1)$	168 : $P_{2036} = (3, 14, 6, 1)$
61 : $P_{472} = (6, 12, 0, 1)$	115 : $P_{1121} = (0, 5, 3, 1)$	169 : $P_{2058} = (9, 15, 6, 1)$
62 : $P_{489} = (7, 13, 0, 1)$	116 : $P_{1148} = (11, 6, 3, 1)$	170 : $P_{2065} = (0, 0, 7, 1)$
63 : $P_{500} = (2, 14, 0, 1)$	117 : $P_{1164} = (11, 7, 3, 1)$	171 : $P_{2087} = (6, 1, 7, 1)$
64 : $P_{517} = (3, 15, 0, 1)$	118 : $P_{1200} = (15, 9, 3, 1)$	172 : $P_{2105} = (8, 2, 7, 1)$
65 : $P_{530} = (0, 0, 1, 1)$	119 : $P_{1209} = (8, 10, 3, 1)$	173 : $P_{2114} = (1, 3, 7, 1)$
66 : $P_{546} = (0, 1, 1, 1)$	120 : $P_{1226} = (9, 11, 3, 1)$	174 : $P_{2137} = (8, 4, 7, 1)$
67 : $P_{547} = (2, 1, 1, 1)$	121 : $P_{1242} = (9, 12, 3, 1)$	175 : $P_{2147} = (2, 5, 7, 1)$
68 : $P_{548} = (3, 1, 1, 1)$	122 : $P_{1263} = (14, 13, 3, 1)$	176 : $P_{2167} = (6, 6, 7, 1)$
69 : $P_{549} = (4, 1, 1, 1)$	123 : $P_{1273} = (8, 14, 3, 1)$	177 : $P_{2179} = (2, 7, 7, 1)$
70 : $P_{550} = (5, 1, 1, 1)$	124 : $P_{1284} = (3, 15, 3, 1)$	178 : $P_{2194} = (1, 8, 7, 1)$
71 : $P_{551} = (6, 1, 1, 1)$	125 : $P_{1297} = (0, 0, 4, 1)$	179 : $P_{2221} = (12, 9, 7, 1)$



180 : $P_{2237} = (12, 10, 7, 1)$	233 : $P_{3016} = (7, 11, 10, 1)$	286 : $P_{3512} = (7, 10, 12, 1)$
181 : $P_{2254} = (13, 11, 7, 1)$	234 : $P_{3017} = (8, 11, 10, 1)$	287 : $P_{3527} = (6, 11, 12, 1)$
182 : $P_{2257} = (0, 12, 7, 1)$	235 : $P_{3018} = (9, 11, 10, 1)$	288 : $P_{3541} = (4, 12, 12, 1)$
183 : $P_{2280} = (7, 13, 7, 1)$	236 : $P_{3019} = (10, 11, 10, 1)$	289 : $P_{3566} = (13, 13, 12, 1)$
184 : $P_{2318} = (13, 15, 7, 1)$	237 : $P_{3020} = (11, 11, 10, 1)$	290 : $P_{3575} = (6, 14, 12, 1)$
185 : $P_{2321} = (0, 0, 8, 1)$	238 : $P_{3021} = (12, 11, 10, 1)$	291 : $P_{3586} = (1, 15, 12, 1)$
186 : $P_{2346} = (9, 1, 8, 1)$	239 : $P_{3022} = (13, 11, 10, 1)$	292 : $P_{3601} = (0, 0, 13, 1)$
187 : $P_{2358} = (5, 2, 8, 1)$	240 : $P_{3023} = (14, 11, 10, 1)$	293 : $P_{3629} = (12, 1, 13, 1)$
188 : $P_{2388} = (3, 4, 8, 1)$	241 : $P_{3024} = (15, 11, 10, 1)$	294 : $P_{3638} = (5, 2, 13, 1)$
189 : $P_{2409} = (8, 5, 8, 1)$	242 : $P_{3034} = (9, 12, 10, 1)$	295 : $P_{3663} = (14, 3, 13, 1)$
190 : $P_{2428} = (11, 6, 8, 1)$	243 : $P_{3043} = (2, 13, 10, 1)$	296 : $P_{3672} = (7, 4, 13, 1)$
191 : $P_{2444} = (11, 7, 8, 1)$	244 : $P_{3063} = (6, 14, 10, 1)$	297 : $P_{3682} = (1, 5, 13, 1)$
192 : $P_{2453} = (4, 8, 8, 1)$	245 : $P_{3086} = (13, 15, 10, 1)$	298 : $P_{3710} = (13, 6, 13, 1)$
193 : $P_{2474} = (9, 9, 8, 1)$	246 : $P_{3089} = (0, 0, 11, 1)$	299 : $P_{3713} = (0, 7, 13, 1)$
194 : $P_{2484} = (3, 10, 8, 1)$	247 : $P_{3115} = (10, 1, 11, 1)$	300 : $P_{3735} = (6, 8, 13, 1)$
195 : $P_{2499} = (2, 11, 8, 1)$	248 : $P_{3134} = (13, 2, 11, 1)$	301 : $P_{3767} = (6, 10, 13, 1)$
196 : $P_{2517} = (4, 12, 8, 1)$	249 : $P_{3144} = (7, 3, 11, 1)$	302 : $P_{3784} = (7, 11, 13, 1)$
197 : $P_{2531} = (2, 13, 8, 1)$	250 : $P_{3156} = (3, 4, 11, 1)$	303 : $P_{3805} = (12, 12, 13, 1)$
198 : $P_{2550} = (5, 14, 8, 1)$	251 : $P_{3178} = (9, 5, 11, 1)$	304 : $P_{3823} = (14, 13, 13, 1)$
199 : $P_{2561} = (0, 15, 8, 1)$	252 : $P_{3199} = (14, 6, 11, 1)$	305 : $P_{3830} = (5, 14, 13, 1)$
200 : $P_{2577} = (0, 0, 9, 1)$	253 : $P_{3205} = (4, 7, 11, 1)$	306 : $P_{3842} = (1, 15, 13, 1)$
201 : $P_{2601} = (8, 1, 9, 1)$	254 : $P_{3223} = (6, 8, 11, 1)$	307 : $P_{3857} = (0, 0, 14, 1)$
202 : $P_{2620} = (11, 2, 9, 1)$	255 : $P_{3245} = (12, 9, 11, 1)$	308 : $P_{3888} = (15, 1, 14, 1)$
203 : $P_{2638} = (13, 3, 9, 1)$	256 : $P_{3249} = (0, 10, 11, 1)$	309 : $P_{3889} = (0, 2, 14, 1)$
204 : $P_{2650} = (9, 4, 9, 1)$	257 : $P_{3250} = (1, 10, 11, 1)$	310 : $P_{3909} = (4, 3, 14, 1)$
205 : $P_{2664} = (7, 5, 9, 1)$	258 : $P_{3251} = (2, 10, 11, 1)$	311 : $P_{3931} = (10, 4, 14, 1)$
206 : $P_{2680} = (7, 6, 9, 1)$	259 : $P_{3252} = (3, 10, 11, 1)$	312 : $P_{3944} = (7, 5, 14, 1)$
207 : $P_{2692} = (3, 7, 9, 1)$	260 : $P_{3253} = (4, 10, 11, 1)$	313 : $P_{3960} = (7, 6, 14, 1)$
208 : $P_{2713} = (8, 8, 9, 1)$	261 : $P_{3254} = (5, 10, 11, 1)$	314 : $P_{3997} = (12, 8, 14, 1)$
209 : $P_{2732} = (11, 9, 9, 1)$	262 : $P_{3255} = (6, 10, 11, 1)$	315 : $P_{4015} = (14, 9, 14, 1)$
210 : $P_{2739} = (2, 10, 9, 1)$	263 : $P_{3256} = (7, 10, 11, 1)$	316 : $P_{4022} = (5, 10, 14, 1)$
211 : $P_{2756} = (3, 11, 9, 1)$	264 : $P_{3257} = (8, 10, 11, 1)$	317 : $P_{4037} = (4, 11, 14, 1)$
212 : $P_{2782} = (13, 12, 9, 1)$	265 : $P_{3258} = (9, 10, 11, 1)$	318 : $P_{4054} = (5, 12, 14, 1)$
213 : $P_{2801} = (0, 14, 9, 1)$	266 : $P_{3259} = (10, 10, 11, 1)$	319 : $P_{4077} = (12, 13, 14, 1)$
214 : $P_{2819} = (2, 15, 9, 1)$	267 : $P_{3260} = (11, 10, 11, 1)$	320 : $P_{4091} = (10, 14, 14, 1)$
215 : $P_{2833} = (0, 0, 10, 1)$	268 : $P_{3261} = (12, 10, 11, 1)$	321 : $P_{4112} = (15, 15, 14, 1)$
216 : $P_{2860} = (11, 1, 10, 1)$	269 : $P_{3262} = (13, 10, 11, 1)$	322 : $P_{4113} = (0, 0, 15, 1)$
217 : $P_{2880} = (15, 2, 10, 1)$	270 : $P_{3263} = (14, 10, 11, 1)$	323 : $P_{4143} = (14, 1, 15, 1)$
218 : $P_{2885} = (4, 3, 10, 1)$	271 : $P_{3264} = (15, 10, 11, 1)$	324 : $P_{4153} = (8, 2, 15, 1)$
219 : $P_{2904} = (7, 4, 10, 1)$	272 : $P_{3266} = (1, 11, 11, 1)$	325 : $P_{4161} = (0, 3, 15, 1)$
220 : $P_{2925} = (12, 5, 10, 1)$	273 : $P_{3286} = (5, 12, 11, 1)$	326 : $P_{4185} = (8, 4, 15, 1)$
221 : $P_{2937} = (8, 6, 10, 1)$	274 : $P_{3312} = (15, 13, 11, 1)$	327 : $P_{4218} = (9, 6, 15, 1)$
222 : $P_{2948} = (3, 7, 10, 1)$	275 : $P_{3321} = (8, 14, 11, 1)$	328 : $P_{4229} = (4, 7, 15, 1)$
223 : $P_{2975} = (14, 8, 10, 1)$	276 : $P_{3331} = (2, 15, 11, 1)$	329 : $P_{4256} = (15, 8, 15, 1)$
224 : $P_{2982} = (5, 9, 10, 1)$	277 : $P_{3345} = (0, 0, 12, 1)$	330 : $P_{4262} = (5, 9, 15, 1)$
225 : $P_{2994} = (1, 10, 10, 1)$	278 : $P_{3374} = (13, 1, 12, 1)$	331 : $P_{4277} = (4, 10, 15, 1)$
226 : $P_{3009} = (0, 11, 10, 1)$	279 : $P_{3400} = (7, 3, 12, 1)$	332 : $P_{4294} = (5, 11, 15, 1)$
227 : $P_{3010} = (1, 11, 10, 1)$	280 : $P_{3424} = (15, 4, 12, 1)$	333 : $P_{4315} = (10, 12, 15, 1)$
228 : $P_{3011} = (2, 11, 10, 1)$	281 : $P_{3426} = (1, 5, 12, 1)$	334 : $P_{4331} = (10, 13, 15, 1)$
229 : $P_{3012} = (3, 11, 10, 1)$	282 : $P_{3441} = (0, 6, 12, 1)$	335 : $P_{4351} = (14, 14, 15, 1)$
230 : $P_{3013} = (4, 11, 10, 1)$	283 : $P_{3469} = (12, 7, 12, 1)$	336 : $P_{4362} = (9, 15, 15, 1)$
231 : $P_{3014} = (5, 11, 10, 1)$	284 : $P_{3477} = (4, 8, 12, 1)$	
232 : $P_{3015} = (6, 11, 10, 1)$	285 : $P_{3504} = (15, 9, 12, 1)$	