Rank-65547 over GF(4)

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The equation

The equation of the surface is :

$$X_3^3 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over $GF(4)$ is 1431655833

General information

Number of lines	3
Number of points	21
Number of singular points	3
Number of Eckardt points	0
Number of double points	3
Number of single points	9
Number of points off lines	9
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^3
Type of lines on points	$2^3, 1^9, 0^9$

Singular Points

The surface has 3 singular points:

$0: P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$ $\qquad\qquad 2: P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
$1: P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{336} = \mathbf{Pl}(0,0,0,0,0,1)_{101}$$

Rank of lines: (0, 16, 336)
Rank of points on Klein quadric: (0, 2, 101)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$ $\qquad\qquad\qquad$ $P_2 = (0,0,1,0) = \ell_1 \cap \ell_2$
$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$

Single Points

The surface has 9 single points:
The single points on the surface are:

0 : $P_5 = (1,1,0,0)$ lies on line ℓ_0 $\qquad\qquad$ 5 : $P_{10} = (3,0,1,0)$ lies on line ℓ_1
1 : $P_6 = (2,1,0,0)$ lies on line ℓ_0 $\qquad\qquad$ 6 : $P_{11} = (0,1,1,0)$ lies on line ℓ_2
2 : $P_7 = (3,1,0,0)$ lies on line ℓ_0 $\qquad\qquad$ 7 : $P_{15} = (0,2,1,0)$ lies on line ℓ_2
3 : $P_8 = (1,0,1,0)$ lies on line ℓ_1 $\qquad\qquad$ 8 : $P_{19} = (0,3,1,0)$ lies on line ℓ_2
4 : $P_9 = (2,0,1,0)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 9 points not on any line:
The points on the surface but not on lines are:

0 : $P_4 = (1,1,1,1)$ $\qquad\qquad\qquad\qquad$ 5 : $P_{66} = (1,3,2,1)$
1 : $P_{48} = (3,2,1,1)$ $\qquad\qquad\qquad\quad$ 6 : $P_{75} = (2,1,3,1)$
2 : $P_{51} = (2,3,1,1)$ $\qquad\qquad\qquad\quad$ 7 : $P_{78} = (1,2,3,1)$
3 : $P_{60} = (3,1,2,1)$ $\qquad\qquad\qquad\quad$ 8 : $P_{84} = (3,3,3,1)$
4 : $P_{63} = (2,2,2,1)$

Line Intersection Graph

$$
\begin{array}{c|ccc}
 & 0\ 1\ 2 \\
\hline
0 & 0\ 1\ 1 \\
1 & 1\ 0\ 1 \\
2 & 1\ 1\ 0 \\
\end{array}
$$

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_0	P_1

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_0	P_2

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_1	P_2

The surface has 21 points:
The points on the surface are:

$0 : P_0 = (1,0,0,0)$

$1 : P_1 = (0,1,0,0)$

$2 : P_2 = (0,0,1,0)$

$3 : P_4 = (1,1,1,1)$

$4 : P_5 = (1,1,0,0)$

$5 : P_6 = (2,1,0,0)$

$6 : P_7 = (3,1,0,0)$

$7 : P_8 = (1,0,1,0)$

$8 : P_9 = (2,0,1,0)$

$9 : P_{10} = (3,0,1,0)$

$10 : P_{11} = (0,1,1,0)$

$11 : P_{15} = (0,2,1,0)$

$12 : P_{19} = (0,3,1,0)$

$13 : P_{48} = (3,2,1,1)$

$14 : P_{51} = (2,3,1,1)$

$15 : P_{60} = (3,1,2,1)$

$16 : P_{63} = (2,2,2,1)$

$17 : P_{66} = (1,3,2,1)$

$18 : P_{75} = (2,1,3,1)$

$19 : P_{78} = (1,2,3,1)$

$20 : P_{84} = (3,3,3,1)$