Rank-65921 over GF(4)

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The equation

The equation of the surface is :

$$X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 1, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431726765

General information

Number of lines	6
Number of points	29
Number of singular points	1
Number of Eckardt points	1
Number of double points	6
Number of single points	15
Number of points off lines	7
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^6
Type of lines on points	3, 2^6, 1^{15}, 0^7

Singular Points

The surface has 1 singular points:

$0: \ P_{39} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_4 = \mathbf{Pl}(1,0,0,0,1,0)_{26}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{17} = \mathbf{Pl}(0,0,1,0,1,0)_{32}$$
$$\ell_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110} = \mathbf{Pl}(1,0,1,1,1,1)_{199}$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{89} = \mathbf{Pl}(1,1,1,1,1,0)_{74}$$
$$\ell_5 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109} = \mathbf{Pl}(1,1,0,1,1,1)_{189}$$

Rank of lines: (4, 17, 38, 110, 89, 109)
Rank of points on Klein quadric: (26, 32, 198, 199, 74, 189)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{39} = \mathbf{P}(1,0,1,1) = \mathbf{P}(1,0,1,1).$

Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_0 = (1,0,0,0) = \ell_0 \cap \ell_1$
$P_{26} = (0,1,0,1) = \ell_0 \cap \ell_5$
$P_{38} = (0,0,1,1) = \ell_1 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_2 \cap \ell_3$
$P_{42} = (0,1,1,1) = \ell_3 \cap \ell_4$
$P_{12} = (1,1,1,0) = \ell_4 \cap \ell_5$

Single Points

The surface has 15 single points:
The single points on the surface are:

$0 : P_4 = (1,1,1,1)$ lies on line ℓ_2
$1 : P_{23} = (1,0,0,1)$ lies on line ℓ_4
$2 : P_{27} = (1,1,0,1)$ lies on line ℓ_0
$3 : P_{28} = (2,1,0,1)$ lies on line ℓ_0
$4 : P_{29} = (3,1,0,1)$ lies on line ℓ_0
$5 : P_{40} = (2,0,1,1)$ lies on line ℓ_1
$6 : P_{41} = (3,0,1,1)$ lies on line ℓ_1
$7 : P_{47} = (2,2,1,1)$ lies on line ℓ_2
$8 : P_{48} = (3,2,1,1)$ lies on line ℓ_3
$9 : P_{51} = (2,3,1,1)$ lies on line ℓ_3
$10 : P_{52} = (3,3,1,1)$ lies on line ℓ_2
$11 : P_{64} = (3,2,2,1)$ lies on line ℓ_4
$12 : P_{67} = (2,3,2,1)$ lies on line ℓ_5
$13 : P_{80} = (3,2,3,1)$ lies on line ℓ_5
$14 : P_{83} = (2,3,3,1)$ lies on line ℓ_4

The single points on the surface are:

Points on surface but on no line

The surface has 7 points not on any line:
The points on the surface but not on lines are:

0 : $P_8 = (1, 0, 1, 0)$
1 : $P_{30} = (0, 2, 0, 1)$
2 : $P_{34} = (0, 3, 0, 1)$
3 : $P_{53} = (0, 0, 2, 1)$
4 : $P_{61} = (0, 2, 2, 1)$
5 : $P_{69} = (0, 0, 3, 1)$
6 : $P_{81} = (0, 3, 3, 1)$

Line Intersection Graph

	0 1 2 3 4 5
0	0 1 0 0 0 1
1	1 0 1 1 0 1
2	0 1 0 1 0 0
3	0 1 1 0 1 1
4	0 0 0 1 0 1
5	1 1 0 1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_5
in point	P_0	P_{26}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_5
in point	P_0	P_{38}	P_{39}	P_{39}

Line 2 intersects

Line	ℓ_1	ℓ_3
in point	P_{38}	P_5

Line 3 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_5
in point	P_{39}	P_5	P_{42}	P_{39}

Line 4 intersects

Line	ℓ_3	ℓ_5
in point	P_{42}	P_{12}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4
in point	P_{26}	P_{39}	P_{39}	P_{12}

The surface has 29 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_4 = (1, 1, 1, 1)$
2 : $P_5 = (1, 1, 0, 0)$
3 : $P_8 = (1, 0, 1, 0)$
4 : $P_{12} = (1, 1, 1, 0)$
5 : $P_{23} = (1, 0, 0, 1)$
6 : $P_{26} = (0, 1, 0, 1)$
7 : $P_{27} = (1, 1, 0, 1)$
8 : $P_{28} = (2, 1, 0, 1)$
9 : $P_{29} = (3, 1, 0, 1)$
10 : $P_{30} = (0, 2, 0, 1)$
11 : $P_{34} = (0, 3, 0, 1)$
12 : $P_{38} = (0, 0, 1, 1)$
13 : $P_{39} = (1, 0, 1, 1)$
14 : $P_{40} = (2, 0, 1, 1)$
15 : $P_{41} = (3, 0, 1, 1)$
16 : $P_{42} = (0, 1, 1, 1)$
17 : $P_{47} = (2, 2, 1, 1)$
18 : $P_{48} = (3, 2, 1, 1)$
19 : $P_{51} = (2, 3, 1, 1)$
20 : $P_{52} = (3, 3, 1, 1)$
21 : $P_{53} = (0, 0, 2, 1)$
22 : $P_{61} = (0, 2, 2, 1)$
23 : $P_{64} = (3, 2, 2, 1)$
24 : $P_{67} = (2, 3, 2, 1)$
25 : $P_{69} = (0, 0, 3, 1)$
26 : $P_{80} = (3, 2, 3, 1)$
27 : $P_{81} = (0, 3, 3, 1)$
28 : $P_{83} = (2, 3, 3, 1)$