Rank-65562 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_0^2X_1 + X_0X_1X_2 = 0$$

(1, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1431656046

General information

Number of lines	8
Number of points	33
Number of singular points	1
Number of Eckardt points	0
Number of double points	0
Number of single points	32
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^8
Type of lines on points	8, 1^{32}

Singular Points

The surface has 1 singular points:

$0: \ P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$

The 8 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{104} = \mathbf{Pl}(0,1,0,0,1,0)_{29}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{272} = \mathbf{Pl}(0,3,0,0,1,0)_{31}$$

$$\ell_2 = \begin{bmatrix} 1 & 0 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{188} = \mathbf{Pl}(0,2,0,0,1,0)_{30}$$

$$\ell_3 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345} = \mathbf{Pl}(0,1,0,1,0,0)_{13}$$

$$\ell_4 = \begin{bmatrix} 1 & \omega^2 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{335} = \begin{bmatrix} 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{335} = \mathbf{Pl}(0,3,0,3,1,0)_{73}$$

$$\ell_5 = \begin{bmatrix} 1 & \omega & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{230} = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{230} = \mathbf{Pl}(0,2,0,2,1,0)_{65}$$

$$\ell_6 = \begin{bmatrix} 0 & 1 & \omega^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{355} = \mathbf{Pl}(0,3,0,1,0,0)_{15}$$

$$\ell_7 = \begin{bmatrix} 0 & 1 & \omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{350} = \mathbf{Pl}(0,2,0,1,0,0)_{14}$$

Rank of lines: (104, 272, 188, 345, 335, 230, 355, 350)
Rank of points on Klein quadric: (29, 31, 30, 13, 73, 65, 15, 14)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 32 single points:
The single points on the surface are:

0 : $P_8 = (1,0,1,0)$ lies on line ℓ_0
1 : $P_9 = (2,0,1,0)$ lies on line ℓ_1
2 : $P_{10} = (3,0,1,0)$ lies on line ℓ_2
3 : $P_{11} = (0,1,1,0)$ lies on line ℓ_3
4 : $P_{13} = (2,1,1,0)$ lies on line ℓ_4
5 : $P_{14} = (3,1,1,0)$ lies on line ℓ_5
6 : $P_{15} = (0,2,1,0)$ lies on line ℓ_6
7 : $P_{19} = (0,3,1,0)$ lies on line ℓ_7
8 : $P_{39} = (1,0,1,1)$ lies on line ℓ_0
9 : $P_{40} = (2,0,1,1)$ lies on line ℓ_1
10 : $P_{41} = (3,0,1,1)$ lies on line ℓ_2
11 : $P_{42} = (0,1,1,1)$ lies on line ℓ_3
12 : $P_{43} = (2,1,1,1)$ lies on line ℓ_4
13 : $P_{44} = (3,1,1,1)$ lies on line ℓ_5
14 : $P_{45} = (0,2,1,1)$ lies on line ℓ_6
15 : $P_{49} = (0,3,1,1)$ lies on line ℓ_7
16 : $P_{54} = (1,0,2,1)$ lies on line ℓ_2
17 : $P_{55} = (2,0,2,1)$ lies on line ℓ_0
18 : $P_{56} = (3,0,2,1)$ lies on line ℓ_1
19 : $P_{57} = (0,1,2,1)$ lies on line ℓ_7
20 : $P_{61} = (0,2,2,1)$ lies on line ℓ_3
21 : $P_{62} = (1,2,2,1)$ lies on line ℓ_5
22 : $P_{64} = (3,2,2,1)$ lies on line ℓ_4
23 : $P_{65} = (0,3,2,1)$ lies on line ℓ_6
24 : $P_{70} = (1,0,3,1)$ lies on line ℓ_1
25 : $P_{71} = (2,0,3,1)$ lies on line ℓ_2
26 : $P_{72} = (3,0,3,1)$ lies on line ℓ_0
27 : $P_{73} = (0,1,3,1)$ lies on line ℓ_6

28 : $P_{77} = (0, 2, 3, 1)$ lies on line ℓ_7
29 : $P_{81} = (0, 3, 3, 1)$ lies on line ℓ_3
30 : $P_{82} = (1, 3, 3, 1)$ lies on line ℓ_4
31 : $P_{83} = (2, 3, 3, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7
0	0	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1
3	1	1	1	0	1	1	1	1
4	1	1	1	1	0	1	1	1
5	1	1	1	1	1	0	1	1
6	1	1	1	1	1	1	0	1
7	1	1	1	1	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

Line 7 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_3	P_3	P_3	P_3	P_3	P_3	P_3

The surface has 33 points:
The points on the surface are:

$0:\ P_3 = (0,0,0,1)$
$1:\ P_8 = (1,0,1,0)$
$2:\ P_9 = (2,0,1,0)$
$3:\ P_{10} = (3,0,1,0)$
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