

Rank-65831 over GF(2)

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The equation

The equation of the surface is :

$$X_2^3 + X_0^2 X_2 + X_1^2 X_2 + X_0 X_1 X_2 = 0$$

(0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(2) is 65831

General information

Number of lines	10
Number of points	13
Number of singular points	1
Number of Eckardt points	6
Number of double points	0
Number of single points	6
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	3^{10}
Type of lines on points	$6, 3^6, 1^6$

Singular Points

The surface has 1 singular points:

$$0 : P_3 = \mathbf{P}(0,0,0,1) = \mathbf{P}(0,0,0,1)$$

The 10 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\begin{aligned}
\ell_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_6 = \mathbf{Pl}(0, 0, 0, 0, 1, 0)_9 \\
\ell_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_2 = \mathbf{Pl}(1, 0, 0, 0, 1, 0)_{10} \\
\ell_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{30} = \mathbf{Pl}(0, 0, 0, 1, 0, 0)_5 \\
\ell_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{14} = \mathbf{Pl}(1, 0, 0, 1, 0, 0)_6 \\
\ell_5 &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{13} = \mathbf{Pl}(0, 0, 0, 1, 1, 0)_{15} \\
\ell_6 &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{16} = \mathbf{Pl}(1, 0, 0, 1, 1, 0)_{16} \\
\ell_7 &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{20} = \mathbf{Pl}(0, 1, 0, 0, 1, 0)_{11} \\
\ell_8 &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{33} = \mathbf{Pl}(0, 1, 0, 1, 0, 0)_7 \\
\ell_9 &= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(0, 1, 0, 1, 1, 0)_{17}
\end{aligned}$$

Rank of lines: (0, 6, 2, 30, 14, 13, 16, 20, 33, 27)

Rank of points on Klein quadric: (0, 9, 10, 5, 6, 15, 16, 11, 7, 17)

Eckardt Points

The surface has 6 Eckardt points:

$$0 : P_0 = \mathbf{P}(1, 0, 0, 0) = \mathbf{P}(1, 0, 0, 0), \quad T = 2$$

$$1 : P_1 = \mathbf{P}(0, 1, 0, 0) = \mathbf{P}(0, 1, 0, 0), \quad T = 2$$

$$2 : P_5 = \mathbf{P}(1, 1, 0, 0) = \mathbf{P}(1, 1, 0, 0), \quad T = 2$$

$$3 : P_9 = \mathbf{P}(1, 0, 0, 1) = \mathbf{P}(1, 0, 0, 1), \quad T = 2$$

$$4 : P_{10} = \mathbf{P}(0, 1, 0, 1) = \mathbf{P}(0, 1, 0, 1), \quad T = 2$$

$$5 : P_{11} = \mathbf{P}(1, 1, 0, 1) = \mathbf{P}(1, 1, 0, 1), \quad T = 2$$

Double Points

The surface has 0 Double points:

The double points on the surface are:

Single Points

The surface has 6 single points:

The single points on the surface are:

$$0 : P_4 = (1, 1, 1, 1) \text{ lies on line } \ell_9$$

$$1 : P_6 = (1, 0, 1, 0) \text{ lies on line } \ell_7$$

$$2 : P_7 = (0, 1, 1, 0) \text{ lies on line } \ell_8$$

$$3 : P_8 = (1, 1, 1, 0) \text{ lies on line } \ell_9$$

$$4 : P_{13} = (1, 0, 1, 1) \text{ lies on line } \ell_7$$

$$5 : P_{14} = (0, 1, 1, 1) \text{ lies on line } \ell_8$$

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:

The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9
0	0	1	1	1	1	1	1	0	0	0
1	1	0	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	0	0	0
3	1	1	1	0	1	1	1	1	1	1
4	1	1	1	1	0	1	1	0	0	0
5	1	1	1	1	1	0	1	1	1	1
6	1	1	1	1	1	1	0	0	0	0
7	0	1	0	1	0	1	0	0	1	1
8	0	1	0	1	0	1	0	1	0	1
9	0	1	0	1	0	1	0	1	1	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_0	P_1	P_1	P_5	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_0	P_0	P_3	P_9	P_3	P_9	P_3	P_3	P_3

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_5	ℓ_6
in point	P_0	P_0	P_{10}	P_{11}	P_{11}	P_{10}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_1	P_3	P_{10}	P_1	P_3	P_{10}	P_3	P_3	P_3

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_5	ℓ_6
in point	P_1	P_9	P_{11}	P_1	P_{11}	P_9

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_7	ℓ_8	ℓ_9
in point	P_5	P_3	P_{11}	P_3	P_{11}	P_5	P_3	P_3	P_3

Line 6 intersects

Line	ℓ_0	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5
in point	P_5	P_9	P_{10}	P_{10}	P_9	P_5

Line 7 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_8	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3

Line 8 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9
in point	P_3	P_3	P_3	P_3	P_3

Line 9 intersects

Line	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_8
in point	P_3	P_3	P_3	P_3	P_3

The surface has 13 points:

The points on the surface are:

$$0 : P_0 = (1, 0, 0, 0)$$

$$1 : P_1 = (0, 1, 0, 0)$$

$$2 : P_3 = (0, 0, 0, 1)$$

$$3 : P_4 = (1, 1, 1, 1)$$

$$4 : P_5 = (1, 1, 0, 0)$$

$$5 : P_6 = (1, 0, 1, 0)$$

$$6 : P_7 = (0, 1, 1, 0)$$

$$7 : P_8 = (1, 1, 1, 0)$$

$$8 : P_9 = (1, 0, 0, 1)$$

$$9 : P_{10} = (0, 1, 0, 1)$$

$$10 : P_{11} = (1, 1, 0, 1)$$

$$11 : P_{13} = (1, 0, 1, 1)$$

$$12 : P_{14} = (0, 1, 1, 1)$$