Rank-65869 over GF(16)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^3 + X_3^3 + X_0^2X_3 + X_1^2X_2 + X_0X_1X_2 = 0$$

(0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(16) is 303112485

General information

Number of lines	2
Number of points	273
Number of singular points	2
Number of Eckardt points	0
Number of double points	1
Number of single points	32
Number of points off lines	240
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^2
Type of lines on points	2, 1^{32}, 0^{240}

Singular Points

The surface has 2 singular points:

0 : $P_2 = \mathbf{P}(0,0,1,0) = \mathbf{P}(0,0,1,0)$
1 : $P_{275} = \mathbf{P}(1,0,0,1) = \mathbf{P}(1,0,0,1)$

The 2 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{256} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4624} = \mathbf{Pl}(0,1,1,0,0,0)_{18}$$

Rank of lines: (256, 4624)
Rank of points on Klein quadric: (2, 18)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 1 Double points:
The double points on the surface are:

$P_2 = (0,0,1,0) = \ell_0 \cap \ell_1$

Single Points

The surface has 32 single points:
The single points on the surface are:

0 : $P_0 = (1,0,0,0)$ lies on line ℓ_0
1 : $P_{20} = (1,0,1,0)$ lies on line ℓ_0
2 : $P_{21} = (2,0,1,0)$ lies on line ℓ_0
3 : $P_{22} = (3,0,1,0)$ lies on line ℓ_0
4 : $P_{23} = (4,0,1,0)$ lies on line ℓ_0
5 : $P_{24} = (5,0,1,0)$ lies on line ℓ_0
6 : $P_{25} = (6,0,1,0)$ lies on line ℓ_0
7 : $P_{26} = (7,0,1,0)$ lies on line ℓ_0
8 : $P_{27} = (8,0,1,0)$ lies on line ℓ_0
9 : $P_{28} = (9,0,1,0)$ lies on line ℓ_0
10 : $P_{29} = (10,0,1,0)$ lies on line ℓ_0
11 : $P_{30} = (11,0,1,0)$ lies on line ℓ_0
12 : $P_{31} = (12,0,1,0)$ lies on line ℓ_0
13 : $P_{32} = (13,0,1,0)$ lies on line ℓ_0
14 : $P_{33} = (14,0,1,0)$ lies on line ℓ_0
15 : $P_{34} = (15,0,1,0)$ lies on line ℓ_0
16 : $P_{275} = (1,0,0,1)$ lies on line ℓ_1
17 : $P_{531} = (1,0,1,1)$ lies on line ℓ_1
18 : $P_{786} = (1,0,2,1)$ lies on line ℓ_1
19 : $P_{1042} = (1,0,3,1)$ lies on line ℓ_1
20 : $P_{1298} = (1,0,4,1)$ lies on line ℓ_1
21 : $P_{1554} = (1,0,5,1)$ lies on line ℓ_1
22 : $P_{1810} = (1,0,6,1)$ lies on line ℓ_1
23 : $P_{2066} = (1,0,7,1)$ lies on line ℓ_1
24 : $P_{2322} = (1,0,8,1)$ lies on line ℓ_1
25 : $P_{2578} = (1,0,9,1)$ lies on line ℓ_1
26 : $P_{2834} = (1,0,10,1)$ lies on line ℓ_1
27 : $P_{3090} = (1,0,11,1)$ lies on line ℓ_1
28 : $P_{3346} = (1,0,12,1)$ lies on line ℓ_1
29 : $P_{3602} = (1,0,13,1)$ lies on line ℓ_1
30 : $P_{3858} = (1,0,14,1)$ lies on line ℓ_1
31 : $P_{4114} = (1,0,15,1)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 240 points not on any line:
The points on the surface but not on lines are:

$0:\ P_{35} = (0,1,1,0)$
$1:\ P_{57} = (6,2,1,0)$
$2:\ P_{73} = (6,3,1,0)$
$3:\ P_{96} = (13,4,1,0)$
$4:\ P_{112} = (13,5,1,0)$
$5:\ P_{126} = (11,6,1,0)$
$6:\ P_{142} = (11,7,1,0)$
$7:\ P_{154} = (7,8,1,0)$
$8:\ P_{170} = (7,9,1,0)$
$9:\ P_{180} = (1,10,1,0)$
$10:\ P_{196} = (1,11,1,0)$
$11:\ P_{221} = (10,12,1,0)$
$12:\ P_{237} = (10,13,1,0)$
$13:\ P_{255} = (12,14,1,0)$
$14:\ P_{271} = (12,15,1,0)$
$15:\ P_{290} = (0,1,0,1)$
$16:\ P_{310} = (4,2,0,1)$
$17:\ P_{331} = (9,3,0,1)$
$18:\ P_{347} = (9,4,0,1)$
$19:\ P_{368} = (14,5,0,1)$
$20:\ P_{372} = (2,6,0,1)$
$21:\ P_{395} = (9,7,0,1)$
$22:\ P_{404} = (2,8,0,1)$
$23:\ P_{432} = (14,9,0,1)$
$24:\ P_{434} = (0,10,0,1)$
$25:\ P_{450} = (0,11,0,1)$
$26:\ P_{480} = (14,12,0,1)$
$27:\ P_{486} = (4,13,0,1)$
$28:\ P_{500} = (2,14,0,1)$
$29:\ P_{518} = (4,15,0,1)$
$30:\ P_{555} = (10,1,1,1)$
$31:\ P_{556} = (11,1,1,1)$
$32:\ P_{589} = (12,3,1,1)$
$33:\ P_{592} = (15,3,1,1)$
$34:\ P_{612} = (3,5,1,1)$
$35:\ P_{615} = (6,5,1,1)$
$36:\ P_{662} = (5,8,1,1)$
$37:\ P_{670} = (13,8,1,1)$
$38:\ P_{690} = (1,10,1,1)$
$39:\ P_{700} = (11,10,1,1)$
$40:\ P_{706} = (1,11,1,1)$
$41:\ P_{715} = (10,11,1,1)$
$42:\ P_{776} = (7,15,1,1)$
$43:\ P_{777} = (8,15,1,1)$
$44:\ P_{806} = (5,1,2,1)$
$45:\ P_{808} = (7,1,2,1)$
$46:\ P_{826} = (9,2,2,1)$
$47:\ P_{830} = (13,2,2,1)$
$48:\ P_{866} = (1,5,2,1)$
$49:\ P_{876} = (11,5,2,1)$
$50:\ P_{898} = (1,7,2,1)$
$51:\ P_{912} = (15,7,2,1)$
$52:\ P_{949} = (4,10,2,1)$
$53:\ P_{954} = (9,10,2,1)$
$54:\ P_{964} = (3,11,2,1)$
$55:\ P_{973} = (12,11,2,1)$
$56:\ P_{998} = (5,13,2,1)$
$57:\ P_{999} = (6,13,2,1)$
$58:\ P_{1009} = (0,14,2,1)$
$59:\ P_{1014} = (5,14,2,1)$
$60:\ P_{1035} = (10,15,2,1)$
$61:\ P_{1038} = (13,15,2,1)$
$62:\ P_{1083} = (10,2,3,1)$
$63:\ P_{1085} = (12,2,3,1)$
$64:\ P_{1123} = (2,5,3,1)$
$65:\ P_{1134} = (13,5,3,1)$
$66:\ P_{1141} = (4,6,3,1)$
$67:\ P_{1151} = (14,6,3,1)$
$68:\ P_{1183} = (14,8,3,1)$
$69:\ P_{1184} = (15,8,3,1)$
$70:\ P_{1239} = (6,12,3,1)$
$71:\ P_{1244} = (11,12,3,1)$
$72:\ P_{1249} = (0,13,3,1)$
$73:\ P_{1263} = (14,13,3,1)$
$74:\ P_{1321} = (8,1,4,1)$
$75:\ P_{1325} = (12,1,4,1)$
$76:\ P_{1329} = (0,2,4,1)$
$77:\ P_{1337} = (8,2,4,1)$
$78:\ P_{1352} = (7,3,4,1)$
$79:\ P_{1356} = (11,3,4,1)$
$80:\ P_{1368} = (7,4,4,1)$
$81:\ P_{1375} = (14,4,4,1)$
$82:\ P_{1417} = (8,7,4,1)$
$83:\ P_{1422} = (13,7,4,1)$
$84:\ P_{1426} = (1,8,4,1)$
$85:\ P_{1435} = (10,8,4,1)$
$86:\ P_{1462} = (5,10,4,1)$
$87:\ P_{1463} = (6,10,4,1)$
$88:\ P_{1482} = (9,11,4,1)$
$89:\ P_{1487} = (14,11,4,1)$
$90:\ P_{1490} = (1,12,4,1)$
$91:\ P_{1492} = (3,12,4,1)$
$92:\ P_{1623} = (6,4,5,1)$
$93:\ P_{1628} = (11,4,5,1)$
$94:\ P_{1659} = (10,6,5,1)$
$95:\ P_{1662} = (13,6,5,1)$
$96:\ P_{1665} = (0,7,5,1)$
$97:\ P_{1667} = (2,7,5,1)$
$98:\ P_{1685} = (4,8,5,1)$
$99:\ P_{1688} = (7,8,5,1)$
$100:\ P_{1763} = (2,13,5,1)$
$101:\ P_{1770} = (9,13,5,1)$
$102:\ P_{1795} = (2,15,5,1)$
$103:\ P_{1796} = (3,15,5,1)$
$104:\ P_{1844} = (3,2,6,1)$
$105:\ P_{1856} = (15,2,6,1)$
$106:\ P_{1859} = (2,3,6,1)$
$107:\ P_{1865} = (8,3,6,1)$

$108: P_{1885} = (12, 4, 6, 1)$
$109: P_{1886} = (13, 4, 6, 1)$
$110: P_{1889} = (0, 5, 6, 1)$
$111: P_{1896} = (7, 5, 6, 1)$
$112: P_{1964} = (11, 9, 6, 1)$
$113: P_{1968} = (15, 9, 6, 1)$
$114: P_{2027} = (10, 13, 6, 1)$
$115: P_{2032} = (15, 13, 6, 1)$
$116: P_{2102} = (5, 2, 7, 1)$
$117: P_{2108} = (11, 2, 7, 1)$
$118: P_{2196} = (3, 8, 7, 1)$
$119: P_{2202} = (9, 8, 7, 1)$
$120: P_{2214} = (5, 9, 7, 1)$
$121: P_{2217} = (8, 9, 7, 1)$
$122: P_{2262} = (5, 12, 7, 1)$
$123: P_{2267} = (10, 12, 7, 1)$
$124: P_{2301} = (12, 14, 7, 1)$
$125: P_{2302} = (13, 14, 7, 1)$
$126: P_{2305} = (0, 15, 7, 1)$
$127: P_{2311} = (6, 15, 7, 1)$
$128: P_{2373} = (4, 3, 8, 1)$
$129: P_{2374} = (5, 3, 8, 1)$
$130: P_{2437} = (4, 7, 8, 1)$
$131: P_{2447} = (14, 7, 8, 1)$
$132: P_{2475} = (10, 9, 8, 1)$
$133: P_{2478} = (13, 9, 8, 1)$
$134: P_{2513} = (0, 12, 8, 1)$
$135: P_{2517} = (4, 12, 8, 1)$
$136: P_{2536} = (7, 13, 8, 1)$
$137: P_{2540} = (11, 13, 8, 1)$
$138: P_{2570} = (9, 15, 8, 1)$
$139: P_{2573} = (12, 15, 8, 1)$
$140: P_{2599} = (6, 1, 9, 1)$
$141: P_{2608} = (15, 1, 9, 1)$
$142: P_{2641} = (0, 4, 9, 1)$
$143: P_{2656} = (15, 4, 9, 1)$
$144: P_{2667} = (10, 5, 9, 1)$
$145: P_{2669} = (12, 5, 9, 1)$
$146: P_{2674} = (1, 6, 9, 1)$
$147: P_{2678} = (5, 6, 9, 1)$
$148: P_{2723} = (2, 9, 9, 1)$
$149: P_{2733} = (12, 9, 9, 1)$
$150: P_{2739} = (2, 10, 9, 1)$
$151: P_{2751} = (14, 10, 9, 1)$
$152: P_{2761} = (8, 11, 9, 1)$
$153: P_{2766} = (13, 11, 9, 1)$
$154: P_{2776} = (7, 12, 9, 1)$
$155: P_{2784} = (15, 12, 9, 1)$
$156: P_{2818} = (1, 15, 9, 1)$
$157: P_{2828} = (11, 15, 9, 1)$
$158: P_{2853} = (4, 1, 10, 1)$
$159: P_{2863} = (14, 1, 10, 1)$
$160: P_{2898} = (1, 4, 10, 1)$
$161: P_{2899} = (2, 4, 10, 1)$
$162: P_{2996} = (3, 10, 10, 1)$
$163: P_{3001} = (8, 10, 10, 1)$
$164: P_{3015} = (6, 11, 10, 1)$
$165: P_{3016} = (7, 11, 10, 1)$
$166: P_{3033} = (8, 12, 10, 1)$
$167: P_{3038} = (13, 12, 10, 1)$
$168: P_{3044} = (3, 13, 10, 1)$
$169: P_{3053} = (12, 13, 10, 1)$
$170: P_{3058} = (1, 14, 10, 1)$
$171: P_{3066} = (9, 14, 10, 1)$
$172: P_{3107} = (2, 1, 11, 1)$
$173: P_{3114} = (9, 1, 11, 1)$
$174: P_{3122} = (1, 2, 11, 1)$
$175: P_{3135} = (14, 2, 11, 1)$
$176: P_{3192} = (7, 6, 11, 1)$
$177: P_{3200} = (15, 6, 11, 1)$
$178: P_{3206} = (5, 7, 11, 1)$
$179: P_{3207} = (6, 7, 11, 1)$
$180: P_{3234} = (1, 9, 11, 1)$
$181: P_{3237} = (4, 9, 11, 1)$
$182: P_{3261} = (12, 10, 11, 1)$
$183: P_{3262} = (13, 10, 11, 1)$
$184: P_{3270} = (5, 11, 11, 1)$
$185: P_{3280} = (15, 11, 11, 1)$
$186: P_{3383} = (6, 2, 12, 1)$
$187: P_{3384} = (7, 2, 12, 1)$
$188: P_{3393} = (0, 3, 12, 1)$
$189: P_{3406} = (13, 3, 12, 1)$
$190: P_{3417} = (8, 4, 12, 1)$
$191: P_{3419} = (10, 4, 12, 1)$
$192: P_{3449} = (8, 6, 12, 1)$
$193: P_{3452} = (11, 6, 12, 1)$
$194: P_{3577} = (8, 14, 12, 1)$
$195: P_{3584} = (15, 14, 12, 1)$
$196: P_{3590} = (5, 15, 12, 1)$
$197: P_{3599} = (14, 15, 12, 1)$
$198: P_{3668} = (3, 4, 13, 1)$
$199: P_{3670} = (5, 4, 13, 1)$
$200: P_{3685} = (4, 5, 13, 1)$
$201: P_{3696} = (15, 5, 13, 1)$
$202: P_{3716} = (3, 7, 13, 1)$
$203: P_{3724} = (11, 7, 13, 1)$
$204: P_{3729} = (0, 8, 13, 1)$
$205: P_{3741} = (12, 8, 13, 1)$
$206: P_{3751} = (6, 9, 13, 1)$
$207: P_{3752} = (7, 9, 13, 1)$
$208: P_{3828} = (3, 14, 13, 1)$
$209: P_{3835} = (10, 14, 13, 1)$
$210: P_{3876} = (3, 1, 14, 1)$
$211: P_{3886} = (13, 1, 14, 1)$
$212: P_{3906} = (1, 3, 14, 1)$
$213: P_{3915} = (10, 3, 14, 1)$
$214: P_{3956} = (3, 6, 14, 1)$
$215: P_{3965} = (12, 6, 14, 1)$

216 : $P_{3991} = (6, 8, 14, 1)$
217 : $P_{3996} = (11, 8, 14, 1)$
218 : $P_{4001} = (0, 9, 14, 1)$
219 : $P_{4004} = (3, 9, 14, 1)$
220 : $P_{4024} = (7, 10, 14, 1)$
221 : $P_{4032} = (15, 10, 14, 1)$
222 : $P_{4035} = (2, 11, 14, 1)$
223 : $P_{4037} = (4, 11, 14, 1)$
224 : $P_{4066} = (1, 13, 14, 1)$
225 : $P_{4073} = (8, 13, 14, 1)$
226 : $P_{4085} = (4, 14, 14, 1)$
227 : $P_{4087} = (6, 14, 14, 1)$
228 : $P_{4167} = (6, 3, 15, 1)$
229 : $P_{4175} = (14, 3, 15, 1)$
230 : $P_{4201} = (8, 5, 15, 1)$
231 : $P_{4202} = (9, 5, 15, 1)$
232 : $P_{4209} = (0, 6, 15, 1)$
233 : $P_{4218} = (9, 6, 15, 1)$
234 : $P_{4235} = (10, 7, 15, 1)$
235 : $P_{4237} = (12, 7, 15, 1)$
236 : $P_{4307} = (2, 12, 15, 1)$
237 : $P_{4314} = (9, 12, 15, 1)$
238 : $P_{4344} = (7, 14, 15, 1)$
239 : $P_{4348} = (11, 14, 15, 1)$

Line Intersection Graph

	0	1
0	0	1
1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1
in point	P_2

Line 1 intersects

Line	ℓ_0
in point	P_2

The surface has 273 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_2 = (0, 0, 1, 0)$
2 : $P_{20} = (1, 0, 1, 0)$
3 : $P_{21} = (2, 0, 1, 0)$
4 : $P_{22} = (3, 0, 1, 0)$
5 : $P_{23} = (4, 0, 1, 0)$
6 : $P_{24} = (5, 0, 1, 0)$
7 : $P_{25} = (6, 0, 1, 0)$
8 : $P_{26} = (7, 0, 1, 0)$
9 : $P_{27} = (8, 0, 1, 0)$
10 : $P_{28} = (9, 0, 1, 0)$
11 : $P_{29} = (10, 0, 1, 0)$
12 : $P_{30} = (11, 0, 1, 0)$
13 : $P_{31} = (12, 0, 1, 0)$
14 : $P_{32} = (13, 0, 1, 0)$
15 : $P_{33} = (14, 0, 1, 0)$
16 : $P_{34} = (15, 0, 1, 0)$
17 : $P_{35} = (0, 1, 1, 0)$
18 : $P_{57} = (6, 2, 1, 0)$
19 : $P_{73} = (6, 3, 1, 0)$
20 : $P_{96} = (13, 4, 1, 0)$
21 : $P_{112} = (13, 5, 1, 0)$
22 : $P_{126} = (11, 6, 1, 0)$
23 : $P_{142} = (11, 7, 1, 0)$
24 : $P_{154} = (7, 8, 1, 0)$
25 : $P_{170} = (7, 9, 1, 0)$
26 : $P_{180} = (1, 10, 1, 0)$
27 : $P_{196} = (1, 11, 1, 0)$
28 : $P_{221} = (10, 12, 1, 0)$
29 : $P_{237} = (10, 13, 1, 0)$
30 : $P_{255} = (12, 14, 1, 0)$
31 : $P_{271} = (12, 15, 1, 0)$
32 : $P_{275} = (1, 0, 0, 1)$
33 : $P_{290} = (0, 1, 0, 1)$
34 : $P_{310} = (4, 2, 0, 1)$
35 : $P_{331} = (9, 3, 0, 1)$
36 : $P_{347} = (9, 4, 0, 1)$
37 : $P_{368} = (14, 5, 0, 1)$
38 : $P_{372} = (2, 6, 0, 1)$
39 : $P_{395} = (9, 7, 0, 1)$
40 : $P_{404} = (2, 8, 0, 1)$
41 : $P_{432} = (14, 9, 0, 1)$
42 : $P_{434} = (0, 10, 0, 1)$
43 : $P_{450} = (0, 11, 0, 1)$
44 : $P_{480} = (14, 12, 0, 1)$
45 : $P_{486} = (4, 13, 0, 1)$
46 : $P_{500} = (2, 14, 0, 1)$
47 : $P_{518} = (4, 15, 0, 1)$
48 : $P_{531} = (1, 0, 1, 1)$
49 : $P_{555} = (10, 1, 1, 1)$
50 : $P_{556} = (11, 1, 1, 1)$
51 : $P_{589} = (12, 3, 1, 1)$
52 : $P_{592} = (15, 3, 1, 1)$
53 : $P_{612} = (3, 5, 1, 1)$
54 : $P_{615} = (6, 5, 1, 1)$
55 : $P_{662} = (5, 8, 1, 1)$
56 : $P_{670} = (13, 8, 1, 1)$
57 : $P_{690} = (1, 10, 1, 1)$
58 : $P_{700} = (11, 10, 1, 1)$
59 : $P_{706} = (1, 11, 1, 1)$
60 : $P_{715} = (10, 11, 1, 1)$
61 : $P_{776} = (7, 15, 1, 1)$
62 : $P_{777} = (8, 15, 1, 1)$
63 : $P_{786} = (1, 0, 2, 1)$
64 : $P_{806} = (5, 1, 2, 1)$
65 : $P_{808} = (7, 1, 2, 1)$
66 : $P_{826} = (9, 2, 2, 1)$
67 : $P_{830} = (13, 2, 2, 1)$
68 : $P_{866} = (1, 5, 2, 1)$

69 : $P_{876} = (11, 5, 2, 1)$
70 : $P_{898} = (1, 7, 2, 1)$
71 : $P_{912} = (15, 7, 2, 1)$
72 : $P_{949} = (4, 10, 2, 1)$
73 : $P_{954} = (9, 10, 2, 1)$
74 : $P_{964} = (3, 11, 2, 1)$
75 : $P_{973} = (12, 11, 2, 1)$
76 : $P_{998} = (5, 13, 2, 1)$
77 : $P_{999} = (6, 13, 2, 1)$
78 : $P_{1009} = (0, 14, 2, 1)$
79 : $P_{1014} = (5, 14, 2, 1)$
80 : $P_{1035} = (10, 15, 2, 1)$
81 : $P_{1038} = (13, 15, 2, 1)$
82 : $P_{1042} = (1, 0, 3, 1)$
83 : $P_{1083} = (10, 2, 3, 1)$
84 : $P_{1085} = (12, 2, 3, 1)$
85 : $P_{1123} = (2, 5, 3, 1)$
86 : $P_{1134} = (13, 5, 3, 1)$
87 : $P_{1141} = (4, 6, 3, 1)$
88 : $P_{1151} = (14, 6, 3, 1)$
89 : $P_{1183} = (14, 8, 3, 1)$
90 : $P_{1184} = (15, 8, 3, 1)$
91 : $P_{1239} = (6, 12, 3, 1)$
92 : $P_{1244} = (11, 12, 3, 1)$
93 : $P_{1249} = (0, 13, 3, 1)$
94 : $P_{1263} = (14, 13, 3, 1)$
95 : $P_{1298} = (1, 0, 4, 1)$
96 : $P_{1321} = (8, 1, 4, 1)$
97 : $P_{1325} = (12, 1, 4, 1)$
98 : $P_{1329} = (0, 2, 4, 1)$
99 : $P_{1337} = (8, 2, 4, 1)$
100 : $P_{1352} = (7, 3, 4, 1)$
101 : $P_{1356} = (11, 3, 4, 1)$
102 : $P_{1368} = (7, 4, 4, 1)$
103 : $P_{1375} = (14, 4, 4, 1)$
104 : $P_{1417} = (8, 7, 4, 1)$
105 : $P_{1422} = (13, 7, 4, 1)$
106 : $P_{1426} = (1, 8, 4, 1)$
107 : $P_{1435} = (10, 8, 4, 1)$
108 : $P_{1462} = (5, 10, 4, 1)$
109 : $P_{1463} = (6, 10, 4, 1)$
110 : $P_{1482} = (9, 11, 4, 1)$
111 : $P_{1487} = (14, 11, 4, 1)$
112 : $P_{1490} = (1, 12, 4, 1)$
113 : $P_{1492} = (3, 12, 4, 1)$
114 : $P_{1554} = (1, 0, 5, 1)$
115 : $P_{1623} = (6, 4, 5, 1)$
116 : $P_{1628} = (11, 4, 5, 1)$
117 : $P_{1659} = (10, 6, 5, 1)$
118 : $P_{1662} = (13, 6, 5, 1)$
119 : $P_{1665} = (0, 7, 5, 1)$
120 : $P_{1667} = (2, 7, 5, 1)$
121 : $P_{1685} = (4, 8, 5, 1)$
122 : $P_{1688} = (7, 8, 5, 1)$
123 : $P_{1763} = (2, 13, 5, 1)$
124 : $P_{1770} = (9, 13, 5, 1)$
125 : $P_{1795} = (2, 15, 5, 1)$
126 : $P_{1796} = (3, 15, 5, 1)$
127 : $P_{1810} = (1, 0, 6, 1)$
128 : $P_{1844} = (3, 2, 6, 1)$
129 : $P_{1856} = (15, 2, 6, 1)$
130 : $P_{1859} = (2, 3, 6, 1)$
131 : $P_{1865} = (8, 3, 6, 1)$
132 : $P_{1885} = (12, 4, 6, 1)$
133 : $P_{1886} = (13, 4, 6, 1)$
134 : $P_{1889} = (0, 5, 6, 1)$
135 : $P_{1896} = (7, 5, 6, 1)$
136 : $P_{1964} = (11, 9, 6, 1)$
137 : $P_{1968} = (15, 9, 6, 1)$
138 : $P_{2027} = (10, 13, 6, 1)$
139 : $P_{2032} = (15, 13, 6, 1)$
140 : $P_{2066} = (1, 0, 7, 1)$
141 : $P_{2102} = (5, 2, 7, 1)$
142 : $P_{2108} = (11, 2, 7, 1)$
143 : $P_{2196} = (3, 8, 7, 1)$
144 : $P_{2202} = (9, 8, 7, 1)$
145 : $P_{2214} = (5, 9, 7, 1)$
146 : $P_{2217} = (8, 9, 7, 1)$
147 : $P_{2262} = (5, 12, 7, 1)$
148 : $P_{2267} = (10, 12, 7, 1)$
149 : $P_{2301} = (12, 14, 7, 1)$
150 : $P_{2302} = (13, 14, 7, 1)$
151 : $P_{2305} = (0, 15, 7, 1)$
152 : $P_{2311} = (6, 15, 7, 1)$
153 : $P_{2322} = (1, 0, 8, 1)$
154 : $P_{2373} = (4, 3, 8, 1)$
155 : $P_{2374} = (5, 3, 8, 1)$
156 : $P_{2437} = (4, 7, 8, 1)$
157 : $P_{2447} = (14, 7, 8, 1)$
158 : $P_{2475} = (10, 9, 8, 1)$
159 : $P_{2478} = (13, 9, 8, 1)$
160 : $P_{2513} = (0, 12, 8, 1)$
161 : $P_{2517} = (4, 12, 8, 1)$
162 : $P_{2536} = (7, 13, 8, 1)$
163 : $P_{2540} = (11, 13, 8, 1)$
164 : $P_{2570} = (9, 15, 8, 1)$
165 : $P_{2573} = (12, 15, 8, 1)$
166 : $P_{2578} = (1, 0, 9, 1)$
167 : $P_{2599} = (6, 1, 9, 1)$
168 : $P_{2608} = (15, 1, 9, 1)$
169 : $P_{2641} = (0, 4, 9, 1)$
170 : $P_{2656} = (15, 4, 9, 1)$
171 : $P_{2667} = (10, 5, 9, 1)$
172 : $P_{2669} = (12, 5, 9, 1)$
173 : $P_{2674} = (1, 6, 9, 1)$
174 : $P_{2678} = (5, 6, 9, 1)$
175 : $P_{2723} = (2, 9, 9, 1)$
176 : $P_{2733} = (12, 9, 9, 1)$
177 : $P_{2739} = (2, 10, 9, 1)$
178 : $P_{2751} = (14, 10, 9, 1)$
179 : $P_{2761} = (8, 11, 9, 1)$
180 : $P_{2766} = (13, 11, 9, 1)$
181 : $P_{2776} = (7, 12, 9, 1)$
182 : $P_{2784} = (15, 12, 9, 1)$
183 : $P_{2818} = (1, 15, 9, 1)$
184 : $P_{2828} = (11, 15, 9, 1)$
185 : $P_{2834} = (1, 0, 10, 1)$
186 : $P_{2853} = (4, 1, 10, 1)$
187 : $P_{2863} = (14, 1, 10, 1)$
188 : $P_{2898} = (1, 4, 10, 1)$
189 : $P_{2899} = (2, 4, 10, 1)$
190 : $P_{2996} = (3, 10, 10, 1)$
191 : $P_{3001} = (8, 10, 10, 1)$
192 : $P_{3015} = (6, 11, 10, 1)$
193 : $P_{3016} = (7, 11, 10, 1)$
194 : $P_{3033} = (8, 12, 10, 1)$
195 : $P_{3038} = (13, 12, 10, 1)$
196 : $P_{3044} = (3, 13, 10, 1)$
197 : $P_{3053} = (12, 13, 10, 1)$
198 : $P_{3058} = (1, 14, 10, 1)$
199 : $P_{3066} = (9, 14, 10, 1)$
200 : $P_{3090} = (1, 0, 11, 1)$
201 : $P_{3107} = (2, 1, 11, 1)$
202 : $P_{3114} = (9, 1, 11, 1)$
203 : $P_{3122} = (1, 2, 11, 1)$
204 : $P_{3135} = (14, 2, 11, 1)$
205 : $P_{3192} = (7, 6, 11, 1)$
206 : $P_{3200} = (15, 6, 11, 1)$
207 : $P_{3206} = (5, 7, 11, 1)$
208 : $P_{3207} = (6, 7, 11, 1)$
209 : $P_{3234} = (1, 9, 11, 1)$
210 : $P_{3237} = (4, 9, 11, 1)$
211 : $P_{3261} = (12, 10, 11, 1)$
212 : $P_{3262} = (13, 10, 11, 1)$
213 : $P_{3270} = (5, 11, 11, 1)$
214 : $P_{3280} = (15, 11, 11, 1)$
215 : $P_{3346} = (1, 0, 12, 1)$
216 : $P_{3383} = (6, 2, 12, 1)$
217 : $P_{3384} = (7, 2, 12, 1)$
218 : $P_{3393} = (0, 3, 12, 1)$
219 : $P_{3406} = (13, 3, 12, 1)$
220 : $P_{3417} = (8, 4, 12, 1)$
221 : $P_{3419} = (10, 4, 12, 1)$
222 : $P_{3449} = (8, 6, 12, 1)$
223 : $P_{3452} = (11, 6, 12, 1)$
224 : $P_{3577} = (8, 14, 12, 1)$
225 : $P_{3584} = (15, 14, 12, 1)$
226 : $P_{3590} = (5, 15, 12, 1)$
227 : $P_{3599} = (14, 15, 12, 1)$
228 : $P_{3602} = (1, 0, 13, 1)$
229 : $P_{3668} = (3, 4, 13, 1)$
230 : $P_{3670} = (5, 4, 13, 1)$

231 : $P_{3685} = (4, 5, 13, 1)$
232 : $P_{3696} = (15, 5, 13, 1)$
233 : $P_{3716} = (3, 7, 13, 1)$
234 : $P_{3724} = (11, 7, 13, 1)$
235 : $P_{3729} = (0, 8, 13, 1)$
236 : $P_{3741} = (12, 8, 13, 1)$
237 : $P_{3751} = (6, 9, 13, 1)$
238 : $P_{3752} = (7, 9, 13, 1)$
239 : $P_{3828} = (3, 14, 13, 1)$
240 : $P_{3835} = (10, 14, 13, 1)$
241 : $P_{3858} = (1, 0, 14, 1)$
242 : $P_{3876} = (3, 1, 14, 1)$
243 : $P_{3886} = (13, 1, 14, 1)$
244 : $P_{3906} = (1, 3, 14, 1)$
245 : $P_{3915} = (10, 3, 14, 1)$
246 : $P_{3956} = (3, 6, 14, 1)$
247 : $P_{3965} = (12, 6, 14, 1)$
248 : $P_{3991} = (6, 8, 14, 1)$
249 : $P_{3996} = (11, 8, 14, 1)$
250 : $P_{4001} = (0, 9, 14, 1)$
251 : $P_{4004} = (3, 9, 14, 1)$
252 : $P_{4024} = (7, 10, 14, 1)$
253 : $P_{4032} = (15, 10, 14, 1)$
254 : $P_{4035} = (2, 11, 14, 1)$
255 : $P_{4037} = (4, 11, 14, 1)$
256 : $P_{4066} = (1, 13, 14, 1)$
257 : $P_{4073} = (8, 13, 14, 1)$
258 : $P_{4085} = (4, 14, 14, 1)$
259 : $P_{4087} = (6, 14, 14, 1)$
260 : $P_{4114} = (1, 0, 15, 1)$
261 : $P_{4167} = (6, 3, 15, 1)$
262 : $P_{4175} = (14, 3, 15, 1)$
263 : $P_{4201} = (8, 5, 15, 1)$
264 : $P_{4202} = (9, 5, 15, 1)$
265 : $P_{4209} = (0, 6, 15, 1)$
266 : $P_{4218} = (9, 6, 15, 1)$
267 : $P_{4235} = (10, 7, 15, 1)$
268 : $P_{4237} = (12, 7, 15, 1)$
269 : $P_{4307} = (2, 12, 15, 1)$
270 : $P_{4314} = (9, 12, 15, 1)$
271 : $P_{4344} = (7, 14, 15, 1)$
272 : $P_{4348} = (11, 14, 15, 1)$