

Rank-74531 over GF(8)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^2 X_2 + X_1^2 X_2 + X_1^2 X_3 + X_0 X_3^2 + X_0 X_1 X_2 = 0$$

(0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0)

The point rank of the equation over GF(8) is 1378161229

General information

Number of lines	4
Number of points	81
Number of singular points	1
Number of Eckardt points	0
Number of double points	4
Number of single points	28
Number of points off lines	49
Number of Hesse planes	0
Number of axes	0
Type of points on lines	9^4
Type of lines on points	$2^4, 1^{28}, 0^{49}$

Singular Points

The surface has 1 singular points:

$$0 : P_2 = \mathbf{P}(0, 0, 1, 0) = \mathbf{P}(0, 0, 1, 0)$$

The 4 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{PI}(1, 0, 0, 0, 0, 0)_0$$

$$\begin{aligned}\ell_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{4673} = \mathbf{Pl}(0, 0, 0, 1, 0, 1)_{769} \\ \ell_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4744} = \mathbf{Pl}(0, 1, 0, 0, 0, 0)_1 \\ \ell_3 &= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{666} = \mathbf{Pl}(1, 0, 1, 1, 1, 1)_{1323}\end{aligned}$$

Rank of lines: (0, 4673, 4744, 666)

Rank of points on Klein quadric: (0, 769, 1, 1323)

Eckardt Points

The surface has 0 Eckardt points:

Double Points

The surface has 4 Double points:

The double points on the surface are:

$$P_1 = (0, 1, 0, 0) = \ell_0 \cap \ell_1$$

$$P_5 = (1, 1, 0, 0) = \ell_0 \cap \ell_3$$

$$P_{138} = (0, 0, 1, 1) = \ell_1 \cap \ell_2$$

$$P_{146} = (0, 1, 1, 1) = \ell_1 \cap \ell_3$$

Single Points

The surface has 28 single points:

The single points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$ lies on line ℓ_0

1 : $P_2 = (0, 0, 1, 0)$ lies on line ℓ_2

2 : $P_3 = (0, 0, 0, 1)$ lies on line ℓ_2

3 : $P_6 = (2, 1, 0, 0)$ lies on line ℓ_0

4 : $P_7 = (3, 1, 0, 0)$ lies on line ℓ_0

5 : $P_8 = (4, 1, 0, 0)$ lies on line ℓ_0

6 : $P_9 = (5, 1, 0, 0)$ lies on line ℓ_0

7 : $P_{10} = (6, 1, 0, 0)$ lies on line ℓ_0

8 : $P_{11} = (7, 1, 0, 0)$ lies on line ℓ_0

9 : $P_{139} = (1, 0, 1, 1)$ lies on line ℓ_3

10 : $P_{153} = (0, 2, 1, 1)$ lies on line ℓ_1

11 : $P_{156} = (3, 2, 1, 1)$ lies on line ℓ_3

12 : $P_{161} = (0, 3, 1, 1)$ lies on line ℓ_1

13 : $P_{163} = (2, 3, 1, 1)$ lies on line ℓ_3

14 : $P_{169} = (0, 4, 1, 1)$ lies on line ℓ_1

15 : $P_{174} = (5, 4, 1, 1)$ lies on line ℓ_3

16 : $P_{177} = (0, 5, 1, 1)$ lies on line ℓ_1

17 : $P_{181} = (4, 5, 1, 1)$ lies on line ℓ_3

18 : $P_{185} = (0, 6, 1, 1)$ lies on line ℓ_1

19 : $P_{192} = (7, 6, 1, 1)$ lies on line ℓ_3

20 : $P_{193} = (0, 7, 1, 1)$ lies on line ℓ_1

21 : $P_{199} = (6, 7, 1, 1)$ lies on line ℓ_3

22 : $P_{201} = (0, 0, 2, 1)$ lies on line ℓ_2

23 : $P_{265} = (0, 0, 3, 1)$ lies on line ℓ_2

24 : $P_{329} = (0, 0, 4, 1)$ lies on line ℓ_2

25 : $P_{393} = (0, 0, 5, 1)$ lies on line ℓ_2

26 : $P_{457} = (0, 0, 6, 1)$ lies on line ℓ_2

27 : $P_{521} = (0, 0, 7, 1)$ lies on line ℓ_2

The single points on the surface are:

Points on surface but on no line

The surface has 49 points not on any line:

The points on the surface but not on lines are:

0 : $P_{83} = (1, 1, 0, 1)$	25 : $P_{352} = (7, 2, 4, 1)$
1 : $P_{94} = (4, 2, 0, 1)$	26 : $P_{357} = (4, 3, 4, 1)$
2 : $P_{103} = (5, 3, 0, 1)$	27 : $P_{388} = (3, 7, 4, 1)$
3 : $P_{113} = (7, 4, 0, 1)$	28 : $P_{392} = (7, 7, 4, 1)$
4 : $P_{120} = (6, 5, 0, 1)$	29 : $P_{400} = (7, 0, 5, 1)$
5 : $P_{125} = (3, 6, 0, 1)$	30 : $P_{420} = (3, 3, 5, 1)$
6 : $P_{132} = (2, 7, 0, 1)$	31 : $P_{424} = (7, 3, 5, 1)$
7 : $P_{207} = (6, 0, 2, 1)$	32 : $P_{434} = (1, 5, 5, 1)$
8 : $P_{211} = (2, 1, 2, 1)$	33 : $P_{436} = (3, 5, 5, 1)$
9 : $P_{214} = (5, 1, 2, 1)$	34 : $P_{450} = (1, 7, 5, 1)$
10 : $P_{237} = (4, 4, 2, 1)$	35 : $P_{459} = (2, 0, 6, 1)$
11 : $P_{239} = (6, 4, 2, 1)$	36 : $P_{474} = (1, 2, 6, 1)$
12 : $P_{251} = (2, 6, 2, 1)$	37 : $P_{499} = (2, 5, 6, 1)$
13 : $P_{261} = (4, 7, 2, 1)$	38 : $P_{502} = (5, 5, 6, 1)$
14 : $P_{262} = (5, 7, 2, 1)$	39 : $P_{506} = (1, 6, 6, 1)$
15 : $P_{269} = (4, 0, 3, 1)$	40 : $P_{510} = (5, 6, 6, 1)$
16 : $P_{290} = (1, 3, 3, 1)$	41 : $P_{526} = (5, 0, 7, 1)$
17 : $P_{295} = (6, 3, 3, 1)$	42 : $P_{532} = (3, 1, 7, 1)$
18 : $P_{298} = (1, 4, 3, 1)$	43 : $P_{536} = (7, 1, 7, 1)$
19 : $P_{317} = (4, 6, 3, 1)$	44 : $P_{539} = (2, 2, 7, 1)$
20 : $P_{319} = (6, 6, 3, 1)$	45 : $P_{542} = (5, 2, 7, 1)$
21 : $P_{332} = (3, 0, 4, 1)$	46 : $P_{555} = (2, 4, 7, 1)$
22 : $P_{341} = (4, 1, 4, 1)$	47 : $P_{556} = (3, 4, 7, 1)$
23 : $P_{343} = (6, 1, 4, 1)$	48 : $P_{568} = (7, 5, 7, 1)$
24 : $P_{351} = (6, 2, 4, 1)$	

Line Intersection Graph

	0 1 2 3
0	0 1 0 1
1	1 0 1 1
2	0 1 0 0
3	1 1 0 0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_3
in point	P_1	P_5

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3
in point	P_1	P_{138}	P_{146}

Line 2 intersects

Line	ℓ_1
in point	P_{138}

Line 3 intersects

Line	ℓ_0	ℓ_1
in point	P_5	P_{146}

The surface has 81 points:

The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$	28 : $P_{181} = (4, 5, 1, 1)$	56 : $P_{388} = (3, 7, 4, 1)$
1 : $P_1 = (0, 1, 0, 0)$	29 : $P_{185} = (0, 6, 1, 1)$	57 : $P_{392} = (7, 7, 4, 1)$
2 : $P_2 = (0, 0, 1, 0)$	30 : $P_{192} = (7, 6, 1, 1)$	58 : $P_{393} = (0, 0, 5, 1)$
3 : $P_3 = (0, 0, 0, 1)$	31 : $P_{193} = (0, 7, 1, 1)$	59 : $P_{400} = (7, 0, 5, 1)$
4 : $P_5 = (1, 1, 0, 0)$	32 : $P_{199} = (6, 7, 1, 1)$	60 : $P_{420} = (3, 3, 5, 1)$
5 : $P_6 = (2, 1, 0, 0)$	33 : $P_{201} = (0, 0, 2, 1)$	61 : $P_{424} = (7, 3, 5, 1)$
6 : $P_7 = (3, 1, 0, 0)$	34 : $P_{207} = (6, 0, 2, 1)$	62 : $P_{434} = (1, 5, 5, 1)$
7 : $P_8 = (4, 1, 0, 0)$	35 : $P_{211} = (2, 1, 2, 1)$	63 : $P_{436} = (3, 5, 5, 1)$
8 : $P_9 = (5, 1, 0, 0)$	36 : $P_{214} = (5, 1, 2, 1)$	64 : $P_{450} = (1, 7, 5, 1)$
9 : $P_{10} = (6, 1, 0, 0)$	37 : $P_{237} = (4, 4, 2, 1)$	65 : $P_{457} = (0, 0, 6, 1)$
10 : $P_{11} = (7, 1, 0, 0)$	38 : $P_{239} = (6, 4, 2, 1)$	66 : $P_{459} = (2, 0, 6, 1)$
11 : $P_{83} = (1, 1, 0, 1)$	39 : $P_{251} = (2, 6, 2, 1)$	67 : $P_{474} = (1, 2, 6, 1)$
12 : $P_{94} = (4, 2, 0, 1)$	40 : $P_{261} = (4, 7, 2, 1)$	68 : $P_{499} = (2, 5, 6, 1)$
13 : $P_{103} = (5, 3, 0, 1)$	41 : $P_{262} = (5, 7, 2, 1)$	69 : $P_{502} = (5, 5, 6, 1)$
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16 : $P_{125} = (3, 6, 0, 1)$	44 : $P_{290} = (1, 3, 3, 1)$	72 : $P_{521} = (0, 0, 7, 1)$
17 : $P_{132} = (2, 7, 0, 1)$	45 : $P_{295} = (6, 3, 3, 1)$	73 : $P_{526} = (5, 0, 7, 1)$
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21 : $P_{153} = (0, 2, 1, 1)$	49 : $P_{329} = (0, 0, 4, 1)$	77 : $P_{542} = (5, 2, 7, 1)$
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24 : $P_{163} = (2, 3, 1, 1)$	52 : $P_{343} = (6, 1, 4, 1)$	80 : $P_{568} = (7, 5, 7, 1)$
25 : $P_{169} = (0, 4, 1, 1)$	53 : $P_{351} = (6, 2, 4, 1)$	
26 : $P_{174} = (5, 4, 1, 1)$	54 : $P_{352} = (7, 2, 4, 1)$	
27 : $P_{177} = (0, 5, 1, 1)$	55 : $P_{357} = (4, 3, 4, 1)$	