# Rank-74295 over GF(4)

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## The equation

The equation of the surface is :

$$X_2^3 + X_0^2X_1 + X_0^2X_2 + X_1^2X_3 + X_0X_3^2 + X_0X_1X_2 = 0$$

( 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0 )
The point rank of the equation over GF(4) is 1499028073

## General information

| | |
|---|---|
| Number of lines | 6 |
| Number of points | 29 |
| Number of singular points | 1 |
| Number of Eckardt points | 1 |
| Number of double points | 6 |
| Number of single points | 15 |
| Number of points off lines | 7 |
| Number of Hesse planes | 0 |
| Number of axes | 0 |
| Type of points on lines | $5^6$ |
| Type of lines on points | 3, $2^6$, $1^{15}$, $0^7$ |

### Singular Points

The surface has 1 singular points:

$0: \ P_8 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$

## The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{21} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{21} = \mathbf{Pl}(1,0,0,0,0,1)_{102}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_5 = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$
$$\ell_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega & 1 \end{bmatrix}_{27} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{27} = \mathbf{Pl}(1,1,2,0,1,1)_{183}$$
$$\ell_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{28} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{28} = \mathbf{Pl}(1,1,3,0,1,1)_{186}$$
$$\ell_4 = \begin{bmatrix} 1 & 0 & \omega & \omega \\ 0 & 1 & \omega & 1 \end{bmatrix}_{216} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}_{216} = \mathbf{Pl}(2,3,1,1,3,1)_{340}$$
$$\ell_5 = \begin{bmatrix} 1 & 0 & \omega^2 & \omega^2 \\ 0 & 1 & \omega^2 & 1 \end{bmatrix}_{322} = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 3 & 1 \end{bmatrix}_{322} = \mathbf{Pl}(3,2,1,1,2,1)_{284}$$

Rank of lines: ( 21, 5, 27, 28, 216, 322 )
Rank of points on Klein quadric: ( 102, 33, 183, 186, 340, 284 )

### Eckardt Points

The surface has 1 Eckardt points:
$0 : P_8 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$.

### Double Points

The surface has 6 Double points:
The double points on the surface are:

$P_{16} = (1,2,1,0) = \ell_0 \cap \ell_4$
$P_{20} = (1,3,1,0) = \ell_0 \cap \ell_5$
$P_{44} = (3,1,1,1) = \ell_1 \cap \ell_2$
$P_{43} = (2,1,1,1) = \ell_1 \cap \ell_3$
$P_{57} = (0,1,2,1) = \ell_2 \cap \ell_4$
$P_{73} = (0,1,3,1) = \ell_3 \cap \ell_5$

### Single Points

The surface has 15 single points:
The single points on the surface are:

$0 : P_0 = (1,0,0,0)$ lies on line $\ell_1$
$1 : P_1 = (0,1,0,0)$ lies on line $\ell_0$
$2 : P_4 = (1,1,1,1)$ lies on line $\ell_1$
$3 : P_{12} = (1,1,1,0)$ lies on line $\ell_0$
$4 : P_{28} = (2,1,0,1)$ lies on line $\ell_2$
$5 : P_{29} = (3,1,0,1)$ lies on line $\ell_3$
$6 : P_{32} = (2,2,0,1)$ lies on line $\ell_4$
$7 : P_{37} = (3,3,0,1)$ lies on line $\ell_5$
$8 : P_{40} = (2,0,1,1)$ lies on line $\ell_5$
$9 : P_{41} = (3,0,1,1)$ lies on line $\ell_4$
$10 : P_{42} = (0,1,1,1)$ lies on line $\ell_1$
$11 : P_{58} = (1,1,2,1)$ lies on line $\ell_3$
$12 : P_{62} = (1,2,2,1)$ lies on line $\ell_5$
$13 : P_{74} = (1,1,3,1)$ lies on line $\ell_2$
$14 : P_{82} = (1,3,3,1)$ lies on line $\ell_4$

The single points on the surface are:

### Points on surface but on no line

The surface has 7 points not on any line:
The points on the surface but not on lines are:

0 : $P_3 = (0, 0, 0, 1)$
1 : $P_{17} = (2, 2, 1, 0)$
2 : $P_{22} = (3, 3, 1, 0)$
3 : $P_{31} = (1, 2, 0, 1)$
4 : $P_{35} = (1, 3, 0, 1)$
5 : $P_{67} = (2, 3, 2, 1)$
6 : $P_{80} = (3, 2, 3, 1)$

## Line Intersection Graph

| | 0 1 2 3 4 5 |
|---|---|
| 0 | 0 0 1 1 1 1 |
| 1 | 0 0 1 1 0 0 |
| 2 | 1 1 0 1 1 0 |
| 3 | 1 1 1 0 0 1 |
| 4 | 1 0 1 0 0 0 |
| 5 | 1 0 0 1 0 0 |

Neighbor sets in the line intersection graph:
Line 0 intersects

| Line | $\ell_2$ | $\ell_3$ | $\ell_4$ | $\ell_5$ |
|---|---|---|---|---|
| in point | $P_8$ | $P_8$ | $P_{16}$ | $P_{20}$ |

Line 1 intersects

| Line | $\ell_2$ | $\ell_3$ |
|---|---|---|
| in point | $P_{44}$ | $P_{43}$ |

Line 2 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_3$ | $\ell_4$ |
|---|---|---|---|---|
| in point | $P_8$ | $P_{44}$ | $P_8$ | $P_{57}$ |

Line 3 intersects

| Line | $\ell_0$ | $\ell_1$ | $\ell_2$ | $\ell_5$ |
|---|---|---|---|---|
| in point | $P_8$ | $P_{43}$ | $P_8$ | $P_{73}$ |

Line 4 intersects

| Line | $\ell_0$ | $\ell_2$ |
|---|---|---|
| in point | $P_{16}$ | $P_{57}$ |

Line 5 intersects

| Line | $\ell_0$ | $\ell_3$ |
|---|---|---|
| in point | $P_{20}$ | $P_{73}$ |

The surface has 29 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_3 = (0, 0, 0, 1)$
3 : $P_4 = (1, 1, 1, 1)$
4 : $P_8 = (1, 0, 1, 0)$
5 : $P_{12} = (1, 1, 1, 0)$
6 : $P_{16} = (1, 2, 1, 0)$
7 : $P_{17} = (2, 2, 1, 0)$
8 : $P_{20} = (1, 3, 1, 0)$
9 : $P_{22} = (3, 3, 1, 0)$
10 : $P_{28} = (2, 1, 0, 1)$
11 : $P_{29} = (3, 1, 0, 1)$
12 : $P_{31} = (1, 2, 0, 1)$
13 : $P_{32} = (2, 2, 0, 1)$
14 : $P_{35} = (1, 3, 0, 1)$
15 : $P_{37} = (3, 3, 0, 1)$
16 : $P_{40} = (2, 0, 1, 1)$
17 : $P_{41} = (3, 0, 1, 1)$
18 : $P_{42} = (0, 1, 1, 1)$
19 : $P_{43} = (2, 1, 1, 1)$
20 : $P_{44} = (3, 1, 1, 1)$
21 : $P_{57} = (0, 1, 2, 1)$
22 : $P_{58} = (1, 1, 2, 1)$
23 : $P_{62} = (1, 2, 2, 1)$
24 : $P_{67} = (2, 3, 2, 1)$
25 : $P_{73} = (0, 1, 3, 1)$
26 : $P_{74} = (1, 1, 3, 1)$
27 : $P_{80} = (3, 2, 3, 1)$
28 : $P_{82} = (1, 3, 3, 1)$