Rank-76291 over GF(4)

January 15, 2021

The equation

The equation of the surface is :

$$X_1^2X_3 + X_1X_2^2 + X_0X_3^2 + X_0X_1X_2 = 0$$

(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(4) is 1503221081

General information

Number of lines	16
Number of points	37
Number of singular points	2
Number of Eckardt points	3
Number of double points	29
Number of single points	3
Number of points off lines	0
Number of Hesse planes	0
Number of axes	0
Type of points on lines	5^{16}
Type of lines on points	5^2, 3^3, 2^{29}, 1^3

Singular Points

The surface has 2 singular points:

$0: \ P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$
$1: \ P_8 = \mathbf{P}(1,0,1,0) = \mathbf{P}(1,0,1,0)$

The 16 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_0 = \mathbf{Pl}(1,0,0,0,0,0)_0$$

$$\ell_1 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{16} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]_{16} = \mathbf{Pl}(0,0,1,0,0,0)_2$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_{21} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]_{21} = \mathbf{Pl}(1,0,0,0,0,1)_{102}$$

$$\ell_3 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{5} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{5} = \mathbf{Pl}(1,0,1,0,1,0)_{33}$$

$$\ell_4 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega^2 & \omega \end{array}\right]_{11} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 \end{array}\right]_{11} = \mathbf{Pl}(3,0,2,0,1,0)_{42}$$

$$\ell_5 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & \omega & \omega^2 \end{array}\right]_{14} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{array}\right]_{14} = \mathbf{Pl}(2,0,3,0,1,0)_{48}$$

$$\ell_6 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]_{356} = \mathbf{Pl}(0,1,0,0,0,0)_1$$

$$\ell_7 = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{38} = \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{38} = \mathbf{Pl}(0,0,1,1,1,1)_{198}$$

$$\ell_8 = \left[\begin{array}{cccc} 1 & \omega^2 & 0 & 0 \\ 0 & 0 & 1 & \omega \end{array}\right]_{81} = \left[\begin{array}{cccc} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]_{81} = \mathbf{Pl}(0,0,3,2,3,1)_{332}$$

$$\ell_9 = \left[\begin{array}{cccc} 1 & \omega & 0 & 0 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{61} = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]_{61} = \mathbf{Pl}(0,0,2,3,2,1)_{265}$$

$$\ell_{10} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{26} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]_{26} = \mathbf{Pl}(1,1,1,0,1,1)_{180}$$

$$\ell_{11} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega & \omega^2 \end{array}\right]_{35} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{array}\right]_{35} = \mathbf{Pl}(2,3,3,0,3,1)_{307}$$

$$\ell_{12} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & \omega^2 & \omega \end{array}\right]_{32} = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 2 \end{array}\right]_{32} = \mathbf{Pl}(3,2,2,0,2,1)_{245}$$

$$\ell_{13} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{122} = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]_{122} = \mathbf{Pl}(0,1,1,1,1,1)_{202}$$

$$\ell_{14} = \left[\begin{array}{cccc} 1 & \omega & 0 & \omega^2 \\ 0 & 0 & 1 & \omega^2 \end{array}\right]_{313} = \left[\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]_{313} = \mathbf{Pl}(0,2,2,3,2,1)_{270}$$

$$\ell_{15} = \left[\begin{array}{cccc} 1 & \omega^2 & 0 & \omega \\ 0 & 0 & 1 & \omega \end{array}\right]_{249} = \left[\begin{array}{cccc} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]_{249} = \mathbf{Pl}(0,3,3,2,3,1)_{338}$$

Rank of lines: (0, 16, 21, 5, 11, 14, 356, 38, 81, 61, 26, 35, 32, 122, 313, 249)
Rank of points on Klein quadric: (0, 2, 102, 33, 42, 48, 1, 198, 332, 265, 180, 307, 245, 202, 270, 338)

Eckardt Points

The surface has 3 Eckardt points:
$0 : P_{38} = \mathbf{P}(0,0,1,1) = \mathbf{P}(0,0,1,1)$,
$1 : P_{53} = \mathbf{P}(0,0,\omega,1) = \mathbf{P}(0,0,2,1)$,
$2 : P_{69} = \mathbf{P}(0,0,\omega^2,1) = \mathbf{P}(0,0,3,1)$.

Double Points

The surface has 29 Double points:
The double points on the surface are:

$P_1 = (0,1,0,0) = \ell_0 \cap \ell_2$
$P_5 = (1,1,0,0) = \ell_0 \cap \ell_7$
$P_6 = (2,1,0,0) = \ell_0 \cap \ell_8$
$P_7 = (3,1,0,0) = \ell_0 \cap \ell_9$
$P_2 = (0,0,1,0) = \ell_1 \cap \ell_6$
$P_{12} = (1,1,1,0) = \ell_2 \cap \ell_{13}$
$P_{16} = (1,2,1,0) = \ell_2 \cap \ell_{14}$
$P_{20} = (1,3,1,0) = \ell_2 \cap \ell_{15}$
$P_4 = (1,1,1,1) = \ell_3 \cap \ell_7$
$P_{42} = (0,1,1,1) = \ell_3 \cap \ell_{10}$
$P_{44} = (3,1,1,1) = \ell_3 \cap \ell_{14}$
$P_{43} = (2,1,1,1) = \ell_3 \cap \ell_{15}$
$P_{67} = (2,3,2,1) = \ell_4 \cap \ell_9$
$P_{65} = (0,3,2,1) = \ell_4 \cap \ell_{12}$
$P_{68} = (3,3,2,1) = \ell_4 \cap \ell_{13}$
$P_{66} = (1,3,2,1) = \ell_4 \cap \ell_{15}$
$P_{80} = (3,2,3,1) = \ell_5 \cap \ell_8$
$P_{77} = (0,2,3,1) = \ell_5 \cap \ell_{11}$
$P_{79} = (2,2,3,1) = \ell_5 \cap \ell_{13}$
$P_{78} = (1,2,3,1) = \ell_5 \cap \ell_{14}$
$P_{47} = (2,2,1,1) = \ell_7 \cap \ell_{11}$
$P_{52} = (3,3,1,1) = \ell_7 \cap \ell_{12}$
$P_{75} = (2,1,3,1) = \ell_8 \cap \ell_{10}$
$P_{82} = (1,3,3,1) = \ell_8 \cap \ell_{12}$
$P_{60} = (3,1,2,1) = \ell_9 \cap \ell_{10}$
$P_{62} = (1,2,2,1) = \ell_9 \cap \ell_{11}$
$P_{27} = (1,1,0,1) = \ell_{10} \cap \ell_{13}$
$P_{33} = (3,2,0,1) = \ell_{11} \cap \ell_{15}$
$P_{36} = (2,3,0,1) = \ell_{12} \cap \ell_{14}$

Single Points

The surface has 3 single points:
The single points on the surface are:

$0:\ P_3 = (0,0,0,1)$ lies on line ℓ_6
$1:\ P_9 = (2,0,1,0)$ lies on line ℓ_1
$2:\ P_{10} = (3,0,1,0)$ lies on line ℓ_1

The single points on the surface are:

Points on surface but on no line

The surface has 0 points not on any line:
The points on the surface but not on lines are:

Line Intersection Graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0
1	1	0	1	1	1	1	1	0	0	0	1	1	1	0	0	0
2	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1
3	1	1	0	0	1	1	0	1	0	0	1	0	0	0	1	1
4	1	1	0	1	0	1	0	0	0	1	0	0	1	1	0	1
5	1	1	0	1	1	0	0	0	1	0	0	1	0	1	1	0
6	0	1	0	0	0	0	0	1	1	1	0	0	0	1	1	1
7	1	0	0	1	0	0	1	0	0	0	0	1	1	1	0	0
8	1	0	0	0	0	1	1	0	0	0	1	0	1	0	0	1
9	1	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0
10	0	1	1	1	0	0	0	0	1	1	0	1	1	1	0	0
11	0	1	1	0	0	1	0	1	0	1	1	0	1	0	0	1
12	0	1	1	0	1	0	0	1	1	0	1	1	0	0	1	0
13	0	0	1	0	1	1	1	1	0	0	1	0	0	0	0	0
14	0	0	1	1	0	1	1	0	0	1	0	0	1	0	0	0
15	0	0	1	1	1	0	1	0	1	0	0	1	0	0	0	0

Neighbor sets in the line intersection graph:

Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_7	ℓ_8	ℓ_9
in point	P_0	P_1	P_0	P_0	P_0	P_5	P_6	P_7

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{12}
in point	P_0	P_8	P_0	P_0	P_0	P_2	P_8	P_8	P_8

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_{10}	ℓ_{11}	ℓ_{12}	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_1	P_8	P_8	P_8	P_8	P_{12}	P_{16}	P_{20}

Line 3 intersects

Line	ℓ_0	ℓ_1	ℓ_4	ℓ_5	ℓ_7	ℓ_{10}	ℓ_{14}	ℓ_{15}
in point	P_0	P_0	P_0	P_0	P_4	P_{42}	P_{44}	P_{43}

Line 4 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_5	ℓ_9	ℓ_{12}	ℓ_{13}	ℓ_{15}
in point	P_0	P_0	P_0	P_0	P_{67}	P_{65}	P_{68}	P_{66}

Line 5 intersects

Line	ℓ_0	ℓ_1	ℓ_3	ℓ_4	ℓ_8	ℓ_{11}	ℓ_{13}	ℓ_{14}
in point	P_0	P_0	P_0	P_0	P_{80}	P_{77}	P_{79}	P_{78}

Line 6 intersects

Line	ℓ_1	ℓ_7	ℓ_8	ℓ_9	ℓ_{13}	ℓ_{14}	ℓ_{15}
in point	P_2	P_{38}	P_{69}	P_{53}	P_{38}	P_{53}	P_{69}

Line 7 intersects

Line	ℓ_0	ℓ_3	ℓ_6	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_5	P_4	P_{38}	P_{47}	P_{52}	P_{38}

Line 8 intersects

Line	ℓ_0	ℓ_5	ℓ_6	ℓ_{10}	ℓ_{12}	ℓ_{15}
in point	P_6	P_{80}	P_{69}	P_{75}	P_{82}	P_{69}

Line 9 intersects

Line	ℓ_0	ℓ_4	ℓ_6	ℓ_{10}	ℓ_{11}	ℓ_{14}
in point	P_7	P_{67}	P_{53}	P_{60}	P_{62}	P_{53}

Line 10 intersects

Line	ℓ_1	ℓ_2	ℓ_3	ℓ_8	ℓ_9	ℓ_{11}	ℓ_{12}	ℓ_{13}
in point	P_8	P_8	P_{42}	P_{75}	P_{60}	P_8	P_8	P_{27}

Line 11 intersects

Line	ℓ_1	ℓ_2	ℓ_5	ℓ_7	ℓ_9	ℓ_{10}	ℓ_{12}	ℓ_{15}
in point	P_8	P_8	P_{77}	P_{47}	P_{62}	P_8	P_8	P_{33}

Line 12 intersects

Line	ℓ_1	ℓ_2	ℓ_4	ℓ_7	ℓ_8	ℓ_{10}	ℓ_{11}	ℓ_{14}
in point	P_8	P_8	P_{65}	P_{52}	P_{82}	P_8	P_8	P_{36}

Line 13 intersects

Line	ℓ_2	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_{10}
in point	P_{12}	P_{68}	P_{79}	P_{38}	P_{38}	P_{27}

Line 14 intersects

Line	ℓ_2	ℓ_3	ℓ_5	ℓ_6	ℓ_9	ℓ_{12}
in point	P_{16}	P_{44}	P_{78}	P_{53}	P_{53}	P_{36}

Line 15 intersects

Line	ℓ_2	ℓ_3	ℓ_4	ℓ_6	ℓ_8	ℓ_{11}
in point	P_{20}	P_{43}	P_{66}	P_{69}	P_{69}	P_{33}

The surface has 37 points:

The points on the surface are:

$0:\ P_0 = (1,0,0,0)$
$1:\ P_1 = (0,1,0,0)$
$2:\ P_2 = (0,0,1,0)$
$3:\ P_3 = (0,0,0,1)$
$4:\ P_4 = (1,1,1,1)$
$5:\ P_5 = (1,1,0,0)$
$6:\ P_6 = (2,1,0,0)$
$7:\ P_7 = (3,1,0,0)$
$8:\ P_8 = (1,0,1,0)$
$9:\ P_9 = (2,0,1,0)$
$10:\ P_{10} = (3,0,1,0)$
$11:\ P_{12} = (1,1,1,0)$
$12:\ P_{16} = (1,2,1,0)$
$13:\ P_{20} = (1,3,1,0)$
$14:\ P_{27} = (1,1,0,1)$
$15:\ P_{33} = (3,2,0,1)$
$16:\ P_{36} = (2,3,0,1)$
$17:\ P_{38} = (0,0,1,1)$
$18:\ P_{42} = (0,1,1,1)$
$19:\ P_{43} = (2,1,1,1)$
$20:\ P_{44} = (3,1,1,1)$
$21:\ P_{47} = (2,2,1,1)$
$22:\ P_{52} = (3,3,1,1)$
$23:\ P_{53} = (0,0,2,1)$
$24:\ P_{60} = (3,1,2,1)$
$25:\ P_{62} = (1,2,2,1)$
$26:\ P_{65} = (0,3,2,1)$
$27:\ P_{66} = (1,3,2,1)$
$28:\ P_{67} = (2,3,2,1)$
$29:\ P_{68} = (3,3,2,1)$
$30:\ P_{69} = (0,0,3,1)$
$31:\ P_{75} = (2,1,3,1)$
$32:\ P_{77} = (0,2,3,1)$
$33:\ P_{78} = (1,2,3,1)$
$34:\ P_{79} = (2,2,3,1)$
$35:\ P_{80} = (3,2,3,1)$
$36:\ P_{82} = (1,3,3,1)$