Rank-65666 over GF(64)

January 15, 2021

The equation

The equation of the surface is :

$$X_0^3 + X_1^3 + X_2^3 + X_3^3 + X_0^2X_1 + X_0^2X_2 + X_0^2X_3 + X_0X_1X_2 = 0$$

(1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0)
The point rank of the equation over GF(64) is -2113396602

General information

Number of lines	3
Number of points	4033
Number of singular points	0
Number of Eckardt points	1
Number of double points	0
Number of single points	192
Number of points off lines	3840
Number of Hesse planes	0
Number of axes	0
Type of points on lines	65^3
Type of lines on points	3, 1^{192}, 0^{3840}

Singular Points

The surface has 0 singular points:

The 3 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 1 & \epsilon^9 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{12520450} = \begin{bmatrix} 1 & 0 & 1 & 47 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{12520450} = \mathbf{Pl}(10, 47, 1, 47, 0, 1)_{469113}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 1 & \epsilon^{18} \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2933506} = \begin{bmatrix} 1 & 0 & 1 & 11 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{2933506} = \mathbf{Pl}(37, 11, 1, 11, 0, 1)_{326256}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 1 & \epsilon^{36} \\ 0 & 1 & 1 & 0 \end{array}\right]_{9591106} = \left[\begin{array}{cccc} 1 & 0 & 1 & 36 \\ 0 & 1 & 1 & 0 \end{array}\right]_{9591106} = \mathbf{Pl}(46, 36, 1, 36, 0, 1)_{425490}$$

Rank of lines: (12520450, 2933506, 9591106)
Rank of points on Klein quadric: (469113, 326256, 425490)

Eckardt Points

The surface has 1 Eckardt points:
$0 : P_{131} = \mathbf{P}(0, 1, 1, 0) = \mathbf{P}(0, 1, 1, 0)$.

Double Points

The surface has 0 Double points:
The double points on the surface are:

Single Points

The surface has 192 single points:
The single points on the surface are:

$0 : P_{4812} = (10, 10, 0, 1)$ lies on line ℓ_0
$1 : P_{6567} = (37, 37, 0, 1)$ lies on line ℓ_1
$2 : P_{7152} = (46, 46, 0, 1)$ lies on line ℓ_2
$3 : P_{8971} = (10, 11, 1, 1)$ lies on line ℓ_0
$4 : P_{10598} = (37, 36, 1, 1)$ lies on line ℓ_1
$5 : P_{11311} = (46, 47, 1, 1)$ lies on line ℓ_2
$6 : P_{12875} = (10, 8, 2, 1)$ lies on line ℓ_0
$7 : P_{14886} = (37, 39, 2, 1)$ lies on line ℓ_1
$8 : P_{15215} = (46, 44, 2, 1)$ lies on line ℓ_2
$9 : P_{17035} = (10, 9, 3, 1)$ lies on line ℓ_0
$10 : P_{18918} = (37, 38, 3, 1)$ lies on line ℓ_1
$11 : P_{19375} = (46, 45, 3, 1)$ lies on line ℓ_2
$12 : P_{21451} = (10, 14, 4, 1)$ lies on line ℓ_0
$13 : P_{22694} = (37, 33, 4, 1)$ lies on line ℓ_1
$14 : P_{23279} = (46, 42, 4, 1)$ lies on line ℓ_2
$15 : P_{25611} = (10, 15, 5, 1)$ lies on line ℓ_0
$16 : P_{26726} = (37, 32, 5, 1)$ lies on line ℓ_1
$17 : P_{27439} = (46, 43, 5, 1)$ lies on line ℓ_2
$18 : P_{29515} = (10, 12, 6, 1)$ lies on line ℓ_0
$19 : P_{31014} = (37, 35, 6, 1)$ lies on line ℓ_1
$20 : P_{31343} = (46, 40, 6, 1)$ lies on line ℓ_2
$21 : P_{33675} = (10, 13, 7, 1)$ lies on line ℓ_0
$22 : P_{35046} = (37, 34, 7, 1)$ lies on line ℓ_1
$23 : P_{35503} = (46, 41, 7, 1)$ lies on line ℓ_2
$24 : P_{37067} = (10, 2, 8, 1)$ lies on line ℓ_0
$25 : P_{39407} = (46, 38, 8, 1)$ lies on line ℓ_2
$26 : P_{39846} = (37, 45, 8, 1)$ lies on line ℓ_1
$27 : P_{41227} = (10, 3, 9, 1)$ lies on line ℓ_0
$28 : P_{43567} = (46, 39, 9, 1)$ lies on line ℓ_2
$29 : P_{43878} = (37, 44, 9, 1)$ lies on line ℓ_1
$30 : P_{45131} = (10, 0, 10, 1)$ lies on line ℓ_0
$31 : P_{47471} = (46, 36, 10, 1)$ lies on line ℓ_2
$32 : P_{48166} = (37, 47, 10, 1)$ lies on line ℓ_1
$33 : P_{49291} = (10, 1, 11, 1)$ lies on line ℓ_0
$34 : P_{51631} = (46, 37, 11, 1)$ lies on line ℓ_2
$35 : P_{52198} = (37, 46, 11, 1)$ lies on line ℓ_1
$36 : P_{53707} = (10, 6, 12, 1)$ lies on line ℓ_0
$37 : P_{55535} = (46, 34, 12, 1)$ lies on line ℓ_2
$38 : P_{55974} = (37, 41, 12, 1)$ lies on line ℓ_1
$39 : P_{57867} = (10, 7, 13, 1)$ lies on line ℓ_0
$40 : P_{59695} = (46, 35, 13, 1)$ lies on line ℓ_2
$41 : P_{60006} = (37, 40, 13, 1)$ lies on line ℓ_1
$42 : P_{61771} = (10, 4, 14, 1)$ lies on line ℓ_0
$43 : P_{63599} = (46, 32, 14, 1)$ lies on line ℓ_2
$44 : P_{64294} = (37, 43, 14, 1)$ lies on line ℓ_1
$45 : P_{65931} = (10, 5, 15, 1)$ lies on line ℓ_0
$46 : P_{67759} = (46, 33, 15, 1)$ lies on line ℓ_2
$47 : P_{68326} = (37, 42, 15, 1)$ lies on line ℓ_1
$48 : P_{71371} = (10, 26, 16, 1)$ lies on line ℓ_0
$49 : P_{73126} = (37, 53, 16, 1)$ lies on line ℓ_1
$50 : P_{73711} = (46, 62, 16, 1)$ lies on line ℓ_2
$51 : P_{75531} = (10, 27, 17, 1)$ lies on line ℓ_0
$52 : P_{77158} = (37, 52, 17, 1)$ lies on line ℓ_1
$53 : P_{77871} = (46, 63, 17, 1)$ lies on line ℓ_2
$54 : P_{79435} = (10, 24, 18, 1)$ lies on line ℓ_0
$55 : P_{81446} = (37, 55, 18, 1)$ lies on line ℓ_1
$56 : P_{81775} = (46, 60, 18, 1)$ lies on line ℓ_2
$57 : P_{83595} = (10, 25, 19, 1)$ lies on line ℓ_0

58 : $P_{85478} = (37, 54, 19, 1)$ lies on line ℓ_1
59 : $P_{85935} = (46, 61, 19, 1)$ lies on line ℓ_2
60 : $P_{88011} = (10, 30, 20, 1)$ lies on line ℓ_0
61 : $P_{89254} = (37, 49, 20, 1)$ lies on line ℓ_1
62 : $P_{89839} = (46, 58, 20, 1)$ lies on line ℓ_2
63 : $P_{92171} = (10, 31, 21, 1)$ lies on line ℓ_0
64 : $P_{93286} = (37, 48, 21, 1)$ lies on line ℓ_1
65 : $P_{93999} = (46, 59, 21, 1)$ lies on line ℓ_2
66 : $P_{96075} = (10, 28, 22, 1)$ lies on line ℓ_0
67 : $P_{97574} = (37, 51, 22, 1)$ lies on line ℓ_1
68 : $P_{97903} = (46, 56, 22, 1)$ lies on line ℓ_2
69 : $P_{100235} = (10, 29, 23, 1)$ lies on line ℓ_0
70 : $P_{101606} = (37, 50, 23, 1)$ lies on line ℓ_1
71 : $P_{102063} = (46, 57, 23, 1)$ lies on line ℓ_2
72 : $P_{103627} = (10, 18, 24, 1)$ lies on line ℓ_0
73 : $P_{105967} = (46, 54, 24, 1)$ lies on line ℓ_2
74 : $P_{106406} = (37, 61, 24, 1)$ lies on line ℓ_1
75 : $P_{107787} = (10, 19, 25, 1)$ lies on line ℓ_0
76 : $P_{110127} = (46, 55, 25, 1)$ lies on line ℓ_2
77 : $P_{110438} = (37, 60, 25, 1)$ lies on line ℓ_1
78 : $P_{111691} = (10, 16, 26, 1)$ lies on line ℓ_0
79 : $P_{114031} = (46, 52, 26, 1)$ lies on line ℓ_2
80 : $P_{114726} = (37, 63, 26, 1)$ lies on line ℓ_1
81 : $P_{115851} = (10, 17, 27, 1)$ lies on line ℓ_0
82 : $P_{118191} = (46, 53, 27, 1)$ lies on line ℓ_2
83 : $P_{118758} = (37, 62, 27, 1)$ lies on line ℓ_1
84 : $P_{120267} = (10, 22, 28, 1)$ lies on line ℓ_0
85 : $P_{122095} = (46, 50, 28, 1)$ lies on line ℓ_2
86 : $P_{122534} = (37, 57, 28, 1)$ lies on line ℓ_1
87 : $P_{124427} = (10, 23, 29, 1)$ lies on line ℓ_0
88 : $P_{126255} = (46, 51, 29, 1)$ lies on line ℓ_2
89 : $P_{126566} = (37, 56, 29, 1)$ lies on line ℓ_1
90 : $P_{128331} = (10, 20, 30, 1)$ lies on line ℓ_0
91 : $P_{130159} = (46, 48, 30, 1)$ lies on line ℓ_2
92 : $P_{130854} = (37, 59, 30, 1)$ lies on line ℓ_1
93 : $P_{132491} = (10, 21, 31, 1)$ lies on line ℓ_0
94 : $P_{134319} = (46, 49, 31, 1)$ lies on line ℓ_2
95 : $P_{134886} = (37, 58, 31, 1)$ lies on line ℓ_1
96 : $P_{135590} = (37, 5, 32, 1)$ lies on line ℓ_1
97 : $P_{136175} = (46, 14, 32, 1)$ lies on line ℓ_2
98 : $P_{137931} = (10, 42, 32, 1)$ lies on line ℓ_0
99 : $P_{139622} = (37, 4, 33, 1)$ lies on line ℓ_1
100 : $P_{140335} = (46, 15, 33, 1)$ lies on line ℓ_2
101 : $P_{142091} = (10, 43, 33, 1)$ lies on line ℓ_0
102 : $P_{143910} = (37, 7, 34, 1)$ lies on line ℓ_1
103 : $P_{144239} = (46, 12, 34, 1)$ lies on line ℓ_2
104 : $P_{145995} = (10, 40, 34, 1)$ lies on line ℓ_0
105 : $P_{147942} = (37, 6, 35, 1)$ lies on line ℓ_1
106 : $P_{148399} = (46, 13, 35, 1)$ lies on line ℓ_2
107 : $P_{150155} = (10, 41, 35, 1)$ lies on line ℓ_0
108 : $P_{151718} = (37, 1, 36, 1)$ lies on line ℓ_1
109 : $P_{152303} = (46, 10, 36, 1)$ lies on line ℓ_2
110 : $P_{154571} = (10, 46, 36, 1)$ lies on line ℓ_0
111 : $P_{155750} = (37, 0, 37, 1)$ lies on line ℓ_1
112 : $P_{156463} = (46, 11, 37, 1)$ lies on line ℓ_2
113 : $P_{158731} = (10, 47, 37, 1)$ lies on line ℓ_0
114 : $P_{160038} = (37, 3, 38, 1)$ lies on line ℓ_1
115 : $P_{160367} = (46, 8, 38, 1)$ lies on line ℓ_2
116 : $P_{162635} = (10, 44, 38, 1)$ lies on line ℓ_0
117 : $P_{164070} = (37, 2, 39, 1)$ lies on line ℓ_1
118 : $P_{164527} = (46, 9, 39, 1)$ lies on line ℓ_2
119 : $P_{166795} = (10, 45, 39, 1)$ lies on line ℓ_0
120 : $P_{168431} = (46, 6, 40, 1)$ lies on line ℓ_2
121 : $P_{168870} = (37, 13, 40, 1)$ lies on line ℓ_1
122 : $P_{170187} = (10, 34, 40, 1)$ lies on line ℓ_0
123 : $P_{172591} = (46, 7, 41, 1)$ lies on line ℓ_2
124 : $P_{172902} = (37, 12, 41, 1)$ lies on line ℓ_1
125 : $P_{174347} = (10, 35, 41, 1)$ lies on line ℓ_0
126 : $P_{176495} = (46, 4, 42, 1)$ lies on line ℓ_2
127 : $P_{177190} = (37, 15, 42, 1)$ lies on line ℓ_1
128 : $P_{178251} = (10, 32, 42, 1)$ lies on line ℓ_0
129 : $P_{180655} = (46, 5, 43, 1)$ lies on line ℓ_2
130 : $P_{181222} = (37, 14, 43, 1)$ lies on line ℓ_1
131 : $P_{182411} = (10, 33, 43, 1)$ lies on line ℓ_0
132 : $P_{184559} = (46, 2, 44, 1)$ lies on line ℓ_2
133 : $P_{184998} = (37, 9, 44, 1)$ lies on line ℓ_1
134 : $P_{186827} = (10, 38, 44, 1)$ lies on line ℓ_0
135 : $P_{188719} = (46, 3, 45, 1)$ lies on line ℓ_2
136 : $P_{189030} = (37, 8, 45, 1)$ lies on line ℓ_1
137 : $P_{190987} = (10, 39, 45, 1)$ lies on line ℓ_0
138 : $P_{192623} = (46, 0, 46, 1)$ lies on line ℓ_2
139 : $P_{193318} = (37, 11, 46, 1)$ lies on line ℓ_1
140 : $P_{194891} = (10, 36, 46, 1)$ lies on line ℓ_0
141 : $P_{196783} = (46, 1, 47, 1)$ lies on line ℓ_2
142 : $P_{197350} = (37, 10, 47, 1)$ lies on line ℓ_1
143 : $P_{199051} = (10, 37, 47, 1)$ lies on line ℓ_0
144 : $P_{202150} = (37, 21, 48, 1)$ lies on line ℓ_1
145 : $P_{202735} = (46, 30, 48, 1)$ lies on line ℓ_2
146 : $P_{204491} = (10, 58, 48, 1)$ lies on line ℓ_0
147 : $P_{206182} = (37, 20, 49, 1)$ lies on line ℓ_1
148 : $P_{206895} = (46, 31, 49, 1)$ lies on line ℓ_2
149 : $P_{208651} = (10, 59, 49, 1)$ lies on line ℓ_0
150 : $P_{210470} = (37, 23, 50, 1)$ lies on line ℓ_1
151 : $P_{210799} = (46, 28, 50, 1)$ lies on line ℓ_2
152 : $P_{212555} = (10, 56, 50, 1)$ lies on line ℓ_0
153 : $P_{214502} = (37, 22, 51, 1)$ lies on line ℓ_1
154 : $P_{214959} = (46, 29, 51, 1)$ lies on line ℓ_2
155 : $P_{216715} = (10, 57, 51, 1)$ lies on line ℓ_0
156 : $P_{218278} = (37, 17, 52, 1)$ lies on line ℓ_1
157 : $P_{218863} = (46, 26, 52, 1)$ lies on line ℓ_2
158 : $P_{221131} = (10, 62, 52, 1)$ lies on line ℓ_0
159 : $P_{222310} = (37, 16, 53, 1)$ lies on line ℓ_1
160 : $P_{223023} = (46, 27, 53, 1)$ lies on line ℓ_2
161 : $P_{225291} = (10, 63, 53, 1)$ lies on line ℓ_0
162 : $P_{226598} = (37, 19, 54, 1)$ lies on line ℓ_1
163 : $P_{226927} = (46, 24, 54, 1)$ lies on line ℓ_2
164 : $P_{229195} = (10, 60, 54, 1)$ lies on line ℓ_0
165 : $P_{230630} = (37, 18, 55, 1)$ lies on line ℓ_1

166 : $P_{231087} = (46, 25, 55, 1)$ lies on line ℓ_2
167 : $P_{233355} = (10, 61, 55, 1)$ lies on line ℓ_0
168 : $P_{234991} = (46, 22, 56, 1)$ lies on line ℓ_2
169 : $P_{235430} = (37, 29, 56, 1)$ lies on line ℓ_1
170 : $P_{236747} = (10, 50, 56, 1)$ lies on line ℓ_0
171 : $P_{239151} = (46, 23, 57, 1)$ lies on line ℓ_2
172 : $P_{239462} = (37, 28, 57, 1)$ lies on line ℓ_1
173 : $P_{240907} = (10, 51, 57, 1)$ lies on line ℓ_0
174 : $P_{243055} = (46, 20, 58, 1)$ lies on line ℓ_2
175 : $P_{243750} = (37, 31, 58, 1)$ lies on line ℓ_1
176 : $P_{244811} = (10, 48, 58, 1)$ lies on line ℓ_0
177 : $P_{247215} = (46, 21, 59, 1)$ lies on line ℓ_2
178 : $P_{247782} = (37, 30, 59, 1)$ lies on line ℓ_1
179 : $P_{248971} = (10, 49, 59, 1)$ lies on line ℓ_0
180 : $P_{251119} = (46, 18, 60, 1)$ lies on line ℓ_2
181 : $P_{251558} = (37, 25, 60, 1)$ lies on line ℓ_1
182 : $P_{253387} = (10, 54, 60, 1)$ lies on line ℓ_0
183 : $P_{255279} = (46, 19, 61, 1)$ lies on line ℓ_2
184 : $P_{255590} = (37, 24, 61, 1)$ lies on line ℓ_1
185 : $P_{257547} = (10, 55, 61, 1)$ lies on line ℓ_0
186 : $P_{259183} = (46, 16, 62, 1)$ lies on line ℓ_2
187 : $P_{259878} = (37, 27, 62, 1)$ lies on line ℓ_1
188 : $P_{261451} = (10, 52, 62, 1)$ lies on line ℓ_0
189 : $P_{263343} = (46, 17, 63, 1)$ lies on line ℓ_2
190 : $P_{263910} = (37, 26, 63, 1)$ lies on line ℓ_1
191 : $P_{265611} = (10, 53, 63, 1)$ lies on line ℓ_0

The single points on the surface are:

Points on surface but on no line

The surface has 3840 points not on any line:
Too many to print.

Line Intersection Graph

	0 1 2
0	0 1 1
1	1 0 1
2	1 1 0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2
in point	P_{131}	P_{131}

Line 1 intersects

Line	ℓ_0	ℓ_2
in point	P_{131}	P_{131}

Line 2 intersects

Line	ℓ_0	ℓ_1
in point	P_{131}	P_{131}

The surface has 4033 points:
Too many to print.