Rank-43 over GF(16)

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The equation

The equation of the surface is :

$$X_2^3 + X_3^3 + X_0^2 X_1 = 0$$

(0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
The point rank of the equation over GF(16) is 8737

General information

Number of lines	6
Number of points	305
Number of singular points	1
Number of Eckardt points	2
Number of double points	3
Number of single points	90
Number of points off lines	210
Number of Hesse planes	0
Number of axes	0
Type of points on lines	17^6
Type of lines on points	3^2, 2^3, 1^{90}, 0^{210}

Singular Points

The surface has 1 singular points:

$0: \ P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$

The 6 Lines

The lines and their Pluecker coordinates are:

$$\ell_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{257} = \mathbf{Pl}(0,0,1,0,1,0)_{320}$$

$$\ell_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \delta^5 \end{bmatrix}_{267} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix}_{267} = \mathbf{Pl}(0,0,10,0,1,0)_{599}$$

$$\ell_2 = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{array}\right]_{266} = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{array}\right]_{266} = \mathbf{Pl}(0,0,11,0,1,0)_{630}$$

$$\ell_3 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{69889} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]_{69889} = \mathbf{Pl}(0,0,0,1,0,1)_{5121}$$

$$\ell_4 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^{5} \end{array}\right]_{69899} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 11 \end{array}\right]_{69899} = \mathbf{Pl}(0,0,0,11,0,1)_{5431}$$

$$\ell_5 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta^{10} \end{array}\right]_{69898} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \end{array}\right]_{69898} = \mathbf{Pl}(0,0,0,10,0,1)_{5400}$$

Rank of lines: (257, 267, 266, 69889, 69899, 69898)
Rank of points on Klein quadric: (320, 599, 630, 5121, 5431, 5400)

Eckardt Points

The surface has 2 Eckardt points:
$0 : P_0 = \mathbf{P}(1,0,0,0) = \mathbf{P}(1,0,0,0)$,
$1 : P_1 = \mathbf{P}(0,1,0,0) = \mathbf{P}(0,1,0,0)$.

Double Points

The surface has 3 Double points:
The double points on the surface are:

$P_{530} = (0,0,1,1) = \ell_0 \cap \ell_3$
$P_{2833} = (0,0,10,1) = \ell_1 \cap \ell_4$
$P_{3089} = (0,0,11,1) = \ell_2 \cap \ell_5$

Single Points

The surface has 90 single points:
The single points on the surface are:

$0 : P_{531} = (1,0,1,1)$ lies on line ℓ_0
$1 : P_{532} = (2,0,1,1)$ lies on line ℓ_0
$2 : P_{533} = (3,0,1,1)$ lies on line ℓ_0
$3 : P_{534} = (4,0,1,1)$ lies on line ℓ_0
$4 : P_{535} = (5,0,1,1)$ lies on line ℓ_0
$5 : P_{536} = (6,0,1,1)$ lies on line ℓ_0
$6 : P_{537} = (7,0,1,1)$ lies on line ℓ_0
$7 : P_{538} = (8,0,1,1)$ lies on line ℓ_0
$8 : P_{539} = (9,0,1,1)$ lies on line ℓ_0
$9 : P_{540} = (10,0,1,1)$ lies on line ℓ_0
$10 : P_{541} = (11,0,1,1)$ lies on line ℓ_0
$11 : P_{542} = (12,0,1,1)$ lies on line ℓ_0
$12 : P_{543} = (13,0,1,1)$ lies on line ℓ_0
$13 : P_{544} = (14,0,1,1)$ lies on line ℓ_0
$14 : P_{545} = (15,0,1,1)$ lies on line ℓ_0
$15 : P_{546} = (0,1,1,1)$ lies on line ℓ_3
$16 : P_{561} = (0,2,1,1)$ lies on line ℓ_3
$17 : P_{577} = (0,3,1,1)$ lies on line ℓ_3
$18 : P_{593} = (0,4,1,1)$ lies on line ℓ_3
$19 : P_{609} = (0,5,1,1)$ lies on line ℓ_3
$20 : P_{625} = (0,6,1,1)$ lies on line ℓ_3
$21 : P_{641} = (0,7,1,1)$ lies on line ℓ_3
$22 : P_{657} = (0,8,1,1)$ lies on line ℓ_3
$23 : P_{673} = (0,9,1,1)$ lies on line ℓ_3
$24 : P_{689} = (0,10,1,1)$ lies on line ℓ_3
$25 : P_{705} = (0,11,1,1)$ lies on line ℓ_3
$26 : P_{721} = (0,12,1,1)$ lies on line ℓ_3
$27 : P_{737} = (0,13,1,1)$ lies on line ℓ_3
$28 : P_{753} = (0,14,1,1)$ lies on line ℓ_3
$29 : P_{769} = (0,15,1,1)$ lies on line ℓ_3
$30 : P_{2834} = (1,0,10,1)$ lies on line ℓ_1
$31 : P_{2835} = (2,0,10,1)$ lies on line ℓ_1
$32 : P_{2836} = (3,0,10,1)$ lies on line ℓ_1
$33 : P_{2837} = (4,0,10,1)$ lies on line ℓ_1
$34 : P_{2838} = (5,0,10,1)$ lies on line ℓ_1
$35 : P_{2839} = (6,0,10,1)$ lies on line ℓ_1

36 : $P_{2840} = (7, 0, 10, 1)$ lies on line ℓ_1
37 : $P_{2841} = (8, 0, 10, 1)$ lies on line ℓ_1
38 : $P_{2842} = (9, 0, 10, 1)$ lies on line ℓ_1
39 : $P_{2843} = (10, 0, 10, 1)$ lies on line ℓ_1
40 : $P_{2844} = (11, 0, 10, 1)$ lies on line ℓ_1
41 : $P_{2845} = (12, 0, 10, 1)$ lies on line ℓ_1
42 : $P_{2846} = (13, 0, 10, 1)$ lies on line ℓ_1
43 : $P_{2847} = (14, 0, 10, 1)$ lies on line ℓ_1
44 : $P_{2848} = (15, 0, 10, 1)$ lies on line ℓ_1
45 : $P_{2849} = (0, 1, 10, 1)$ lies on line ℓ_4
46 : $P_{2865} = (0, 2, 10, 1)$ lies on line ℓ_4
47 : $P_{2881} = (0, 3, 10, 1)$ lies on line ℓ_4
48 : $P_{2897} = (0, 4, 10, 1)$ lies on line ℓ_4
49 : $P_{2913} = (0, 5, 10, 1)$ lies on line ℓ_4
50 : $P_{2929} = (0, 6, 10, 1)$ lies on line ℓ_4
51 : $P_{2945} = (0, 7, 10, 1)$ lies on line ℓ_4
52 : $P_{2961} = (0, 8, 10, 1)$ lies on line ℓ_4
53 : $P_{2977} = (0, 9, 10, 1)$ lies on line ℓ_4
54 : $P_{2993} = (0, 10, 10, 1)$ lies on line ℓ_4
55 : $P_{3009} = (0, 11, 10, 1)$ lies on line ℓ_4
56 : $P_{3025} = (0, 12, 10, 1)$ lies on line ℓ_4
57 : $P_{3041} = (0, 13, 10, 1)$ lies on line ℓ_4
58 : $P_{3057} = (0, 14, 10, 1)$ lies on line ℓ_4
59 : $P_{3073} = (0, 15, 10, 1)$ lies on line ℓ_4
60 : $P_{3090} = (1, 0, 11, 1)$ lies on line ℓ_2
61 : $P_{3091} = (2, 0, 11, 1)$ lies on line ℓ_2
62 : $P_{3092} = (3, 0, 11, 1)$ lies on line ℓ_2
63 : $P_{3093} = (4, 0, 11, 1)$ lies on line ℓ_2
64 : $P_{3094} = (5, 0, 11, 1)$ lies on line ℓ_2
65 : $P_{3095} = (6, 0, 11, 1)$ lies on line ℓ_2
66 : $P_{3096} = (7, 0, 11, 1)$ lies on line ℓ_2
67 : $P_{3097} = (8, 0, 11, 1)$ lies on line ℓ_2
68 : $P_{3098} = (9, 0, 11, 1)$ lies on line ℓ_2
69 : $P_{3099} = (10, 0, 11, 1)$ lies on line ℓ_2
70 : $P_{3100} = (11, 0, 11, 1)$ lies on line ℓ_2
71 : $P_{3101} = (12, 0, 11, 1)$ lies on line ℓ_2
72 : $P_{3102} = (13, 0, 11, 1)$ lies on line ℓ_2
73 : $P_{3103} = (14, 0, 11, 1)$ lies on line ℓ_2
74 : $P_{3104} = (15, 0, 11, 1)$ lies on line ℓ_2
75 : $P_{3105} = (0, 1, 11, 1)$ lies on line ℓ_5
76 : $P_{3121} = (0, 2, 11, 1)$ lies on line ℓ_5
77 : $P_{3137} = (0, 3, 11, 1)$ lies on line ℓ_5
78 : $P_{3153} = (0, 4, 11, 1)$ lies on line ℓ_5
79 : $P_{3169} = (0, 5, 11, 1)$ lies on line ℓ_5
80 : $P_{3185} = (0, 6, 11, 1)$ lies on line ℓ_5
81 : $P_{3201} = (0, 7, 11, 1)$ lies on line ℓ_5
82 : $P_{3217} = (0, 8, 11, 1)$ lies on line ℓ_5
83 : $P_{3233} = (0, 9, 11, 1)$ lies on line ℓ_5
84 : $P_{3249} = (0, 10, 11, 1)$ lies on line ℓ_5
85 : $P_{3265} = (0, 11, 11, 1)$ lies on line ℓ_5
86 : $P_{3281} = (0, 12, 11, 1)$ lies on line ℓ_5
87 : $P_{3297} = (0, 13, 11, 1)$ lies on line ℓ_5
88 : $P_{3313} = (0, 14, 11, 1)$ lies on line ℓ_5
89 : $P_{3329} = (0, 15, 11, 1)$ lies on line ℓ_5

The single points on the surface are:

Points on surface but on no line

The surface has 210 points not on any line:
The points on the surface but not on lines are:

0 : $P_{36} = (1, 1, 1, 0)$
1 : $P_{58} = (7, 2, 1, 0)$
2 : $P_{72} = (5, 3, 1, 0)$
3 : $P_{95} = (12, 4, 1, 0)$
4 : $P_{107} = (8, 5, 1, 0)$
5 : $P_{117} = (2, 6, 1, 0)$
6 : $P_{140} = (9, 7, 1, 0)$
7 : $P_{162} = (15, 8, 1, 0)$
8 : $P_{169} = (6, 9, 1, 0)$
9 : $P_{189} = (10, 10, 1, 0)$
10 : $P_{206} = (11, 11, 1, 0)$
11 : $P_{225} = (14, 12, 1, 0)$
12 : $P_{231} = (4, 13, 1, 0)$
13 : $P_{256} = (13, 14, 1, 0)$
14 : $P_{262} = (3, 15, 1, 0)$
15 : $P_{291} = (1, 1, 0, 1)$
16 : $P_{313} = (7, 2, 0, 1)$
17 : $P_{327} = (5, 3, 0, 1)$
18 : $P_{350} = (12, 4, 0, 1)$
19 : $P_{362} = (8, 5, 0, 1)$
20 : $P_{372} = (2, 6, 0, 1)$
21 : $P_{395} = (9, 7, 0, 1)$
22 : $P_{417} = (15, 8, 0, 1)$
23 : $P_{424} = (6, 9, 0, 1)$
24 : $P_{444} = (10, 10, 0, 1)$
25 : $P_{461} = (11, 11, 0, 1)$
26 : $P_{480} = (14, 12, 0, 1)$
27 : $P_{486} = (4, 13, 0, 1)$
28 : $P_{511} = (13, 14, 0, 1)$
29 : $P_{517} = (3, 15, 0, 1)$
30 : $P_{805} = (4, 1, 2, 1)$
31 : $P_{822} = (5, 2, 2, 1)$

$32: P_{846} = (13, 3, 2, 1)$
$33: P_{851} = (2, 4, 2, 1)$
$34: P_{876} = (11, 5, 2, 1)$
$35: P_{889} = (8, 6, 2, 1)$
$36: P_{912} = (15, 7, 2, 1)$
$37: P_{927} = (14, 8, 2, 1)$
$38: P_{930} = (1, 9, 2, 1)$
$39: P_{948} = (3, 10, 2, 1)$
$40: P_{968} = (7, 11, 2, 1)$
$41: P_{987} = (10, 12, 2, 1)$
$42: P_{1002} = (9, 13, 2, 1)$
$43: P_{1015} = (6, 14, 2, 1)$
$44: P_{1037} = (12, 15, 2, 1)$
$45: P_{1066} = (9, 1, 3, 1)$
$46: P_{1086} = (13, 2, 3, 1)$
$47: P_{1095} = (6, 3, 3, 1)$
$48: P_{1113} = (8, 4, 3, 1)$
$49: P_{1128} = (7, 5, 3, 1)$
$50: P_{1148} = (11, 6, 3, 1)$
$51: P_{1167} = (14, 7, 3, 1)$
$52: P_{1179} = (10, 8, 3, 1)$
$53: P_{1189} = (4, 9, 3, 1)$
$54: P_{1213} = (12, 10, 3, 1)$
$55: P_{1222} = (5, 11, 3, 1)$
$56: P_{1236} = (3, 12, 3, 1)$
$57: P_{1264} = (15, 13, 3, 1)$
$58: P_{1266} = (1, 14, 3, 1)$
$59: P_{1283} = (2, 15, 3, 1)$
$60: P_{1322} = (9, 1, 4, 1)$
$61: P_{1342} = (13, 2, 4, 1)$
$62: P_{1351} = (6, 3, 4, 1)$
$63: P_{1369} = (8, 4, 4, 1)$
$64: P_{1384} = (7, 5, 4, 1)$
$65: P_{1404} = (11, 6, 4, 1)$
$66: P_{1423} = (14, 7, 4, 1)$
$67: P_{1435} = (10, 8, 4, 1)$
$68: P_{1445} = (4, 9, 4, 1)$
$69: P_{1469} = (12, 10, 4, 1)$
$70: P_{1478} = (5, 11, 4, 1)$
$71: P_{1492} = (3, 12, 4, 1)$
$72: P_{1520} = (15, 13, 4, 1)$
$73: P_{1522} = (1, 14, 4, 1)$
$74: P_{1539} = (2, 15, 4, 1)$
$75: P_{1583} = (14, 1, 5, 1)$
$76: P_{1586} = (1, 2, 5, 1)$
$77: P_{1605} = (4, 3, 5, 1)$
$78: P_{1624} = (7, 4, 5, 1)$
$79: P_{1646} = (13, 5, 5, 1)$
$80: P_{1654} = (5, 6, 5, 1)$
$81: P_{1668} = (3, 7, 5, 1)$
$82: P_{1693} = (12, 8, 5, 1)$
$83: P_{1712} = (15, 9, 5, 1)$
$84: P_{1721} = (8, 10, 5, 1)$
$85: P_{1735} = (6, 11, 5, 1)$
$86: P_{1747} = (2, 12, 5, 1)$
$87: P_{1771} = (10, 13, 5, 1)$
$88: P_{1786} = (9, 14, 5, 1)$
$89: P_{1804} = (11, 15, 5, 1)$
$90: P_{1827} = (2, 1, 6, 1)$
$91: P_{1855} = (14, 2, 6, 1)$
$92: P_{1867} = (10, 3, 6, 1)$
$93: P_{1874} = (1, 4, 6, 1)$
$94: P_{1898} = (9, 5, 6, 1)$
$95: P_{1909} = (4, 6, 6, 1)$
$96: P_{1932} = (11, 7, 6, 1)$
$97: P_{1944} = (7, 8, 6, 1)$
$98: P_{1965} = (12, 9, 6, 1)$
$99: P_{1982} = (13, 10, 6, 1)$
$100: P_{2000} = (15, 11, 6, 1)$
$101: P_{2006} = (5, 12, 6, 1)$
$102: P_{2025} = (8, 13, 6, 1)$
$103: P_{2036} = (3, 14, 6, 1)$
$104: P_{2055} = (6, 15, 6, 1)$
$105: P_{2090} = (9, 1, 7, 1)$
$106: P_{2110} = (13, 2, 7, 1)$
$107: P_{2119} = (6, 3, 7, 1)$
$108: P_{2137} = (8, 4, 7, 1)$
$109: P_{2152} = (7, 5, 7, 1)$
$110: P_{2172} = (11, 6, 7, 1)$
$111: P_{2191} = (14, 7, 7, 1)$
$112: P_{2203} = (10, 8, 7, 1)$
$113: P_{2213} = (4, 9, 7, 1)$
$114: P_{2237} = (12, 10, 7, 1)$
$115: P_{2246} = (5, 11, 7, 1)$
$116: P_{2260} = (3, 12, 7, 1)$
$117: P_{2288} = (15, 13, 7, 1)$
$118: P_{2290} = (1, 14, 7, 1)$
$119: P_{2307} = (2, 15, 7, 1)$
$120: P_{2339} = (2, 1, 8, 1)$
$121: P_{2367} = (14, 2, 8, 1)$
$122: P_{2379} = (10, 3, 8, 1)$
$123: P_{2386} = (1, 4, 8, 1)$
$124: P_{2410} = (9, 5, 8, 1)$
$125: P_{2421} = (4, 6, 8, 1)$
$126: P_{2444} = (11, 7, 8, 1)$
$127: P_{2456} = (7, 8, 8, 1)$
$128: P_{2477} = (12, 9, 8, 1)$
$129: P_{2494} = (13, 10, 8, 1)$
$130: P_{2512} = (15, 11, 8, 1)$
$131: P_{2518} = (5, 12, 8, 1)$
$132: P_{2537} = (8, 13, 8, 1)$
$133: P_{2548} = (3, 14, 8, 1)$
$134: P_{2567} = (6, 15, 8, 1)$
$135: P_{2607} = (14, 1, 9, 1)$
$136: P_{2610} = (1, 2, 9, 1)$
$137: P_{2629} = (4, 3, 9, 1)$
$138: P_{2648} = (7, 4, 9, 1)$
$139: P_{2670} = (13, 5, 9, 1)$

140 : $P_{2678} = (5, 6, 9, 1)$
141 : $P_{2692} = (3, 7, 9, 1)$
142 : $P_{2717} = (12, 8, 9, 1)$
143 : $P_{2736} = (15, 9, 9, 1)$
144 : $P_{2745} = (8, 10, 9, 1)$
145 : $P_{2759} = (6, 11, 9, 1)$
146 : $P_{2771} = (2, 12, 9, 1)$
147 : $P_{2795} = (10, 13, 9, 1)$
148 : $P_{2810} = (9, 14, 9, 1)$
149 : $P_{2828} = (11, 15, 9, 1)$
150 : $P_{3375} = (14, 1, 12, 1)$
151 : $P_{3378} = (1, 2, 12, 1)$
152 : $P_{3397} = (4, 3, 12, 1)$
153 : $P_{3416} = (7, 4, 12, 1)$
154 : $P_{3438} = (13, 5, 12, 1)$
155 : $P_{3446} = (5, 6, 12, 1)$
156 : $P_{3460} = (3, 7, 12, 1)$
157 : $P_{3485} = (12, 8, 12, 1)$
158 : $P_{3504} = (15, 9, 12, 1)$
159 : $P_{3513} = (8, 10, 12, 1)$
160 : $P_{3527} = (6, 11, 12, 1)$
161 : $P_{3539} = (2, 12, 12, 1)$
162 : $P_{3563} = (10, 13, 12, 1)$
163 : $P_{3578} = (9, 14, 12, 1)$
164 : $P_{3596} = (11, 15, 12, 1)$
165 : $P_{3621} = (4, 1, 13, 1)$
166 : $P_{3638} = (5, 2, 13, 1)$
167 : $P_{3662} = (13, 3, 13, 1)$
168 : $P_{3667} = (2, 4, 13, 1)$
169 : $P_{3692} = (11, 5, 13, 1)$
170 : $P_{3705} = (8, 6, 13, 1)$
171 : $P_{3728} = (15, 7, 13, 1)$
172 : $P_{3743} = (14, 8, 13, 1)$
173 : $P_{3746} = (1, 9, 13, 1)$
174 : $P_{3764} = (3, 10, 13, 1)$
175 : $P_{3784} = (7, 11, 13, 1)$
176 : $P_{3803} = (10, 12, 13, 1)$
177 : $P_{3818} = (9, 13, 13, 1)$
178 : $P_{3831} = (6, 14, 13, 1)$
179 : $P_{3853} = (12, 15, 13, 1)$
180 : $P_{3875} = (2, 1, 14, 1)$
181 : $P_{3903} = (14, 2, 14, 1)$
182 : $P_{3915} = (10, 3, 14, 1)$
183 : $P_{3922} = (1, 4, 14, 1)$
184 : $P_{3946} = (9, 5, 14, 1)$
185 : $P_{3957} = (4, 6, 14, 1)$
186 : $P_{3980} = (11, 7, 14, 1)$
187 : $P_{3992} = (7, 8, 14, 1)$
188 : $P_{4013} = (12, 9, 14, 1)$
189 : $P_{4030} = (13, 10, 14, 1)$
190 : $P_{4048} = (15, 11, 14, 1)$
191 : $P_{4054} = (5, 12, 14, 1)$
192 : $P_{4073} = (8, 13, 14, 1)$
193 : $P_{4084} = (3, 14, 14, 1)$
194 : $P_{4103} = (6, 15, 14, 1)$
195 : $P_{4133} = (4, 1, 15, 1)$
196 : $P_{4150} = (5, 2, 15, 1)$
197 : $P_{4174} = (13, 3, 15, 1)$
198 : $P_{4179} = (2, 4, 15, 1)$
199 : $P_{4204} = (11, 5, 15, 1)$
200 : $P_{4217} = (8, 6, 15, 1)$
201 : $P_{4240} = (15, 7, 15, 1)$
202 : $P_{4255} = (14, 8, 15, 1)$
203 : $P_{4258} = (1, 9, 15, 1)$
204 : $P_{4276} = (3, 10, 15, 1)$
205 : $P_{4296} = (7, 11, 15, 1)$
206 : $P_{4315} = (10, 12, 15, 1)$
207 : $P_{4330} = (9, 13, 15, 1)$
208 : $P_{4343} = (6, 14, 15, 1)$
209 : $P_{4365} = (12, 15, 15, 1)$

Line Intersection Graph

	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	1	0
2	1	1	0	0	0	1
3	1	0	0	0	1	1
4	0	1	0	1	0	1
5	0	0	1	1	1	0

Neighbor sets in the line intersection graph:
Line 0 intersects

Line	ℓ_1	ℓ_2	ℓ_3
in point	P_0	P_0	P_{530}

Line 1 intersects

Line	ℓ_0	ℓ_2	ℓ_4
in point	P_0	P_0	P_{2833}

Line 2 intersects

Line	ℓ_0	ℓ_1	ℓ_5
in point	P_0	P_0	P_{3089}

Line 3 intersects

Line	ℓ_0	ℓ_4	ℓ_5
in point	P_{530}	P_1	P_1

Line 4 intersects

Line	ℓ_1	ℓ_3	ℓ_5
in point	P_{2833}	P_1	P_1

Line 5 intersects

Line	ℓ_2	ℓ_3	ℓ_4
in point	P_{3089}	P_1	P_1

The surface has 305 points:
The points on the surface are:

0 : $P_0 = (1, 0, 0, 0)$
1 : $P_1 = (0, 1, 0, 0)$
2 : $P_{36} = (1, 1, 1, 0)$
3 : $P_{58} = (7, 2, 1, 0)$
4 : $P_{72} = (5, 3, 1, 0)$
5 : $P_{95} = (12, 4, 1, 0)$
6 : $P_{107} = (8, 5, 1, 0)$
7 : $P_{117} = (2, 6, 1, 0)$
8 : $P_{140} = (9, 7, 1, 0)$
9 : $P_{162} = (15, 8, 1, 0)$
10 : $P_{169} = (6, 9, 1, 0)$
11 : $P_{189} = (10, 10, 1, 0)$
12 : $P_{206} = (11, 11, 1, 0)$
13 : $P_{225} = (14, 12, 1, 0)$
14 : $P_{231} = (4, 13, 1, 0)$
15 : $P_{256} = (13, 14, 1, 0)$
16 : $P_{262} = (3, 15, 1, 0)$
17 : $P_{291} = (1, 1, 0, 1)$
18 : $P_{313} = (7, 2, 0, 1)$
19 : $P_{327} = (5, 3, 0, 1)$
20 : $P_{350} = (12, 4, 0, 1)$
21 : $P_{362} = (8, 5, 0, 1)$
22 : $P_{372} = (2, 6, 0, 1)$
23 : $P_{395} = (9, 7, 0, 1)$
24 : $P_{417} = (15, 8, 0, 1)$
25 : $P_{424} = (6, 9, 0, 1)$
26 : $P_{444} = (10, 10, 0, 1)$
27 : $P_{461} = (11, 11, 0, 1)$
28 : $P_{480} = (14, 12, 0, 1)$
29 : $P_{486} = (4, 13, 0, 1)$
30 : $P_{511} = (13, 14, 0, 1)$
31 : $P_{517} = (3, 15, 0, 1)$
32 : $P_{530} = (0, 0, 1, 1)$
33 : $P_{531} = (1, 0, 1, 1)$
34 : $P_{532} = (2, 0, 1, 1)$
35 : $P_{533} = (3, 0, 1, 1)$
36 : $P_{534} = (4, 0, 1, 1)$
37 : $P_{535} = (5, 0, 1, 1)$
38 : $P_{536} = (6, 0, 1, 1)$
39 : $P_{537} = (7, 0, 1, 1)$
40 : $P_{538} = (8, 0, 1, 1)$
41 : $P_{539} = (9, 0, 1, 1)$
42 : $P_{540} = (10, 0, 1, 1)$
43 : $P_{541} = (11, 0, 1, 1)$
44 : $P_{542} = (12, 0, 1, 1)$
45 : $P_{543} = (13, 0, 1, 1)$
46 : $P_{544} = (14, 0, 1, 1)$
47 : $P_{545} = (15, 0, 1, 1)$
48 : $P_{546} = (0, 1, 1, 1)$
49 : $P_{561} = (0, 2, 1, 1)$
50 : $P_{577} = (0, 3, 1, 1)$
51 : $P_{593} = (0, 4, 1, 1)$
52 : $P_{609} = (0, 5, 1, 1)$
53 : $P_{625} = (0, 6, 1, 1)$
54 : $P_{641} = (0, 7, 1, 1)$
55 : $P_{657} = (0, 8, 1, 1)$
56 : $P_{673} = (0, 9, 1, 1)$
57 : $P_{689} = (0, 10, 1, 1)$
58 : $P_{705} = (0, 11, 1, 1)$
59 : $P_{721} = (0, 12, 1, 1)$
60 : $P_{737} = (0, 13, 1, 1)$
61 : $P_{753} = (0, 14, 1, 1)$
62 : $P_{769} = (0, 15, 1, 1)$
63 : $P_{805} = (4, 1, 2, 1)$
64 : $P_{822} = (5, 2, 2, 1)$
65 : $P_{846} = (13, 3, 2, 1)$
66 : $P_{851} = (2, 4, 2, 1)$
67 : $P_{876} = (11, 5, 2, 1)$
68 : $P_{889} = (8, 6, 2, 1)$
69 : $P_{912} = (15, 7, 2, 1)$
70 : $P_{927} = (14, 8, 2, 1)$
71 : $P_{930} = (1, 9, 2, 1)$
72 : $P_{948} = (3, 10, 2, 1)$
73 : $P_{968} = (7, 11, 2, 1)$
74 : $P_{987} = (10, 12, 2, 1)$
75 : $P_{1002} = (9, 13, 2, 1)$
76 : $P_{1015} = (6, 14, 2, 1)$
77 : $P_{1037} = (12, 15, 2, 1)$
78 : $P_{1066} = (9, 1, 3, 1)$
79 : $P_{1086} = (13, 2, 3, 1)$
80 : $P_{1095} = (6, 3, 3, 1)$
81 : $P_{1113} = (8, 4, 3, 1)$
82 : $P_{1128} = (7, 5, 3, 1)$
83 : $P_{1148} = (11, 6, 3, 1)$
84 : $P_{1167} = (14, 7, 3, 1)$
85 : $P_{1179} = (10, 8, 3, 1)$
86 : $P_{1189} = (4, 9, 3, 1)$
87 : $P_{1213} = (12, 10, 3, 1)$
88 : $P_{1222} = (5, 11, 3, 1)$
89 : $P_{1236} = (3, 12, 3, 1)$
90 : $P_{1264} = (15, 13, 3, 1)$
91 : $P_{1266} = (1, 14, 3, 1)$
92 : $P_{1283} = (2, 15, 3, 1)$
93 : $P_{1322} = (9, 1, 4, 1)$
94 : $P_{1342} = (13, 2, 4, 1)$
95 : $P_{1351} = (6, 3, 4, 1)$
96 : $P_{1369} = (8, 4, 4, 1)$
97 : $P_{1384} = (7, 5, 4, 1)$
98 : $P_{1404} = (11, 6, 4, 1)$
99 : $P_{1423} = (14, 7, 4, 1)$
100 : $P_{1435} = (10, 8, 4, 1)$
101 : $P_{1445} = (4, 9, 4, 1)$
102 : $P_{1469} = (12, 10, 4, 1)$
103 : $P_{1478} = (5, 11, 4, 1)$
104 : $P_{1492} = (3, 12, 4, 1)$
105 : $P_{1520} = (15, 13, 4, 1)$
106 : $P_{1522} = (1, 14, 4, 1)$
107 : $P_{1539} = (2, 15, 4, 1)$
108 : $P_{1583} = (14, 1, 5, 1)$
109 : $P_{1586} = (1, 2, 5, 1)$
110 : $P_{1605} = (4, 3, 5, 1)$

111 : $P_{1624} = (7, 4, 5, 1)$
112 : $P_{1646} = (13, 5, 5, 1)$
113 : $P_{1654} = (5, 6, 5, 1)$
114 : $P_{1668} = (3, 7, 5, 1)$
115 : $P_{1693} = (12, 8, 5, 1)$
116 : $P_{1712} = (15, 9, 5, 1)$
117 : $P_{1721} = (8, 10, 5, 1)$
118 : $P_{1735} = (6, 11, 5, 1)$
119 : $P_{1747} = (2, 12, 5, 1)$
120 : $P_{1771} = (10, 13, 5, 1)$
121 : $P_{1786} = (9, 14, 5, 1)$
122 : $P_{1804} = (11, 15, 5, 1)$
123 : $P_{1827} = (2, 1, 6, 1)$
124 : $P_{1855} = (14, 2, 6, 1)$
125 : $P_{1867} = (10, 3, 6, 1)$
126 : $P_{1874} = (1, 4, 6, 1)$
127 : $P_{1898} = (9, 5, 6, 1)$
128 : $P_{1909} = (4, 6, 6, 1)$
129 : $P_{1932} = (11, 7, 6, 1)$
130 : $P_{1944} = (7, 8, 6, 1)$
131 : $P_{1965} = (12, 9, 6, 1)$
132 : $P_{1982} = (13, 10, 6, 1)$
133 : $P_{2000} = (15, 11, 6, 1)$
134 : $P_{2006} = (5, 12, 6, 1)$
135 : $P_{2025} = (8, 13, 6, 1)$
136 : $P_{2036} = (3, 14, 6, 1)$
137 : $P_{2055} = (6, 15, 6, 1)$
138 : $P_{2090} = (9, 1, 7, 1)$
139 : $P_{2110} = (13, 2, 7, 1)$
140 : $P_{2119} = (6, 3, 7, 1)$
141 : $P_{2137} = (8, 4, 7, 1)$
142 : $P_{2152} = (7, 5, 7, 1)$
143 : $P_{2172} = (11, 6, 7, 1)$
144 : $P_{2191} = (14, 7, 7, 1)$
145 : $P_{2203} = (10, 8, 7, 1)$
146 : $P_{2213} = (4, 9, 7, 1)$
147 : $P_{2237} = (12, 10, 7, 1)$
148 : $P_{2246} = (5, 11, 7, 1)$
149 : $P_{2260} = (3, 12, 7, 1)$
150 : $P_{2288} = (15, 13, 7, 1)$
151 : $P_{2290} = (1, 14, 7, 1)$
152 : $P_{2307} = (2, 15, 7, 1)$
153 : $P_{2339} = (2, 1, 8, 1)$
154 : $P_{2367} = (14, 2, 8, 1)$
155 : $P_{2379} = (10, 3, 8, 1)$
156 : $P_{2386} = (1, 4, 8, 1)$
157 : $P_{2410} = (9, 5, 8, 1)$
158 : $P_{2421} = (4, 6, 8, 1)$
159 : $P_{2444} = (11, 7, 8, 1)$
160 : $P_{2456} = (7, 8, 8, 1)$
161 : $P_{2477} = (12, 9, 8, 1)$
162 : $P_{2494} = (13, 10, 8, 1)$
163 : $P_{2512} = (15, 11, 8, 1)$
164 : $P_{2518} = (5, 12, 8, 1)$
165 : $P_{2537} = (8, 13, 8, 1)$
166 : $P_{2548} = (3, 14, 8, 1)$
167 : $P_{2567} = (6, 15, 8, 1)$
168 : $P_{2607} = (14, 1, 9, 1)$
169 : $P_{2610} = (1, 2, 9, 1)$
170 : $P_{2629} = (4, 3, 9, 1)$
171 : $P_{2648} = (7, 4, 9, 1)$
172 : $P_{2670} = (13, 5, 9, 1)$
173 : $P_{2678} = (5, 6, 9, 1)$
174 : $P_{2692} = (3, 7, 9, 1)$
175 : $P_{2717} = (12, 8, 9, 1)$
176 : $P_{2736} = (15, 9, 9, 1)$
177 : $P_{2745} = (8, 10, 9, 1)$
178 : $P_{2759} = (6, 11, 9, 1)$
179 : $P_{2771} = (2, 12, 9, 1)$
180 : $P_{2795} = (10, 13, 9, 1)$
181 : $P_{2810} = (9, 14, 9, 1)$
182 : $P_{2828} = (11, 15, 9, 1)$
183 : $P_{2833} = (0, 0, 10, 1)$
184 : $P_{2834} = (1, 0, 10, 1)$
185 : $P_{2835} = (2, 0, 10, 1)$
186 : $P_{2836} = (3, 0, 10, 1)$
187 : $P_{2837} = (4, 0, 10, 1)$
188 : $P_{2838} = (5, 0, 10, 1)$
189 : $P_{2839} = (6, 0, 10, 1)$
190 : $P_{2840} = (7, 0, 10, 1)$
191 : $P_{2841} = (8, 0, 10, 1)$
192 : $P_{2842} = (9, 0, 10, 1)$
193 : $P_{2843} = (10, 0, 10, 1)$
194 : $P_{2844} = (11, 0, 10, 1)$
195 : $P_{2845} = (12, 0, 10, 1)$
196 : $P_{2846} = (13, 0, 10, 1)$
197 : $P_{2847} = (14, 0, 10, 1)$
198 : $P_{2848} = (15, 0, 10, 1)$
199 : $P_{2849} = (0, 1, 10, 1)$
200 : $P_{2865} = (0, 2, 10, 1)$
201 : $P_{2881} = (0, 3, 10, 1)$
202 : $P_{2897} = (0, 4, 10, 1)$
203 : $P_{2913} = (0, 5, 10, 1)$
204 : $P_{2929} = (0, 6, 10, 1)$
205 : $P_{2945} = (0, 7, 10, 1)$
206 : $P_{2961} = (0, 8, 10, 1)$
207 : $P_{2977} = (0, 9, 10, 1)$
208 : $P_{2993} = (0, 10, 10, 1)$
209 : $P_{3009} = (0, 11, 10, 1)$
210 : $P_{3025} = (0, 12, 10, 1)$
211 : $P_{3041} = (0, 13, 10, 1)$
212 : $P_{3057} = (0, 14, 10, 1)$
213 : $P_{3073} = (0, 15, 10, 1)$
214 : $P_{3089} = (0, 0, 11, 1)$
215 : $P_{3090} = (1, 0, 11, 1)$
216 : $P_{3091} = (2, 0, 11, 1)$
217 : $P_{3092} = (3, 0, 11, 1)$
218 : $P_{3093} = (4, 0, 11, 1)$
219 : $P_{3094} = (5, 0, 11, 1)$
220 : $P_{3095} = (6, 0, 11, 1)$
221 : $P_{3096} = (7, 0, 11, 1)$
222 : $P_{3097} = (8, 0, 11, 1)$
223 : $P_{3098} = (9, 0, 11, 1)$
224 : $P_{3099} = (10, 0, 11, 1)$
225 : $P_{3100} = (11, 0, 11, 1)$
226 : $P_{3101} = (12, 0, 11, 1)$
227 : $P_{3102} = (13, 0, 11, 1)$
228 : $P_{3103} = (14, 0, 11, 1)$
229 : $P_{3104} = (15, 0, 11, 1)$
230 : $P_{3105} = (0, 1, 11, 1)$
231 : $P_{3121} = (0, 2, 11, 1)$
232 : $P_{3137} = (0, 3, 11, 1)$
233 : $P_{3153} = (0, 4, 11, 1)$
234 : $P_{3169} = (0, 5, 11, 1)$
235 : $P_{3185} = (0, 6, 11, 1)$
236 : $P_{3201} = (0, 7, 11, 1)$
237 : $P_{3217} = (0, 8, 11, 1)$
238 : $P_{3233} = (0, 9, 11, 1)$
239 : $P_{3249} = (0, 10, 11, 1)$
240 : $P_{3265} = (0, 11, 11, 1)$
241 : $P_{3281} = (0, 12, 11, 1)$
242 : $P_{3297} = (0, 13, 11, 1)$
243 : $P_{3313} = (0, 14, 11, 1)$
244 : $P_{3329} = (0, 15, 11, 1)$
245 : $P_{3375} = (14, 1, 12, 1)$
246 : $P_{3378} = (1, 2, 12, 1)$
247 : $P_{3397} = (4, 3, 12, 1)$
248 : $P_{3416} = (7, 4, 12, 1)$
249 : $P_{3438} = (13, 5, 12, 1)$
250 : $P_{3446} = (5, 6, 12, 1)$
251 : $P_{3460} = (3, 7, 12, 1)$
252 : $P_{3485} = (12, 8, 12, 1)$
253 : $P_{3504} = (15, 9, 12, 1)$
254 : $P_{3513} = (8, 10, 12, 1)$
255 : $P_{3527} = (6, 11, 12, 1)$
256 : $P_{3539} = (2, 12, 12, 1)$
257 : $P_{3563} = (10, 13, 12, 1)$
258 : $P_{3578} = (9, 14, 12, 1)$
259 : $P_{3596} = (11, 15, 12, 1)$
260 : $P_{3621} = (4, 1, 13, 1)$
261 : $P_{3638} = (5, 2, 13, 1)$
262 : $P_{3662} = (13, 3, 13, 1)$
263 : $P_{3667} = (2, 4, 13, 1)$
264 : $P_{3692} = (11, 5, 13, 1)$
265 : $P_{3705} = (8, 6, 13, 1)$
266 : $P_{3728} = (15, 7, 13, 1)$
267 : $P_{3743} = (14, 8, 13, 1)$
268 : $P_{3746} = (1, 9, 13, 1)$
269 : $P_{3764} = (3, 10, 13, 1)$
270 : $P_{3784} = (7, 11, 13, 1)$
271 : $P_{3803} = (10, 12, 13, 1)$
272 : $P_{3818} = (9, 13, 13, 1)$

$273: P_{3831} = (6, 14, 13, 1)$
$274: P_{3853} = (12, 15, 13, 1)$
$275: P_{3875} = (2, 1, 14, 1)$
$276: P_{3903} = (14, 2, 14, 1)$
$277: P_{3915} = (10, 3, 14, 1)$
$278: P_{3922} = (1, 4, 14, 1)$
$279: P_{3946} = (9, 5, 14, 1)$
$280: P_{3957} = (4, 6, 14, 1)$
$281: P_{3980} = (11, 7, 14, 1)$
$282: P_{3992} = (7, 8, 14, 1)$
$283: P_{4013} = (12, 9, 14, 1)$
$284: P_{4030} = (13, 10, 14, 1)$
$285: P_{4048} = (15, 11, 14, 1)$
$286: P_{4054} = (5, 12, 14, 1)$
$287: P_{4073} = (8, 13, 14, 1)$
$288: P_{4084} = (3, 14, 14, 1)$
$289: P_{4103} = (6, 15, 14, 1)$
$290: P_{4133} = (4, 1, 15, 1)$
$291: P_{4150} = (5, 2, 15, 1)$
$292: P_{4174} = (13, 3, 15, 1)$
$293: P_{4179} = (2, 4, 15, 1)$
$294: P_{4204} = (11, 5, 15, 1)$
$295: P_{4217} = (8, 6, 15, 1)$
$296: P_{4240} = (15, 7, 15, 1)$
$297: P_{4255} = (14, 8, 15, 1)$
$298: P_{4258} = (1, 9, 15, 1)$
$299: P_{4276} = (3, 10, 15, 1)$
$300: P_{4296} = (7, 11, 15, 1)$
$301: P_{4315} = (10, 12, 15, 1)$
$302: P_{4330} = (9, 13, 15, 1)$
$303: P_{4343} = (6, 14, 15, 1)$
$304: P_{4365} = (12, 15, 15, 1)$