Formation Control: A Review and A New Consideration

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***Abstract*— In this paper, we presented a review on the current control issues and strategies on a group of unmanned autonomous vehicles/robots formation. Formation control has broad applications and becomes an active research topic in the recent years. In this paper, we attempt to review the key issues in formation control with a focus on the main control strategies for formation control under different kinds of scenarios. Then, we point out some important open questions and the possible future research directions on formation control. This paper contributes with a new and interesting consideration on formation control and its application in distributed parameter systems. We pointed out that formation control should be classified as formation regulation control and formation tracking control, similar to regulator and tracker in conventional control.**

***Index Terms*— Formation control, stability analysis, graph theory, Lyapunov analysis, distributed parameter system, pattern formation, formation regulation control, formation tracking control, morphological pattern formation tracking control, adaptive mesh.**

I. Introduction

Formation control is an important issues in coordinated control for a group of unmanned autonomous vehicles/robots. In many applications, a group of autonomous vehicles are required to follow a predefined trajectory while maintaining a desired spatial pattern. Moving in formation has many advantages over conventional systems, for example, it can reduce the system cost, increase the robustness and efficiency of the system while providing redundancy, reconfiguration ability and structure flexibility for the system [1], [2], [3], [4].

Formation control has broad applications, for example, security patrols, search and rescue in hazardous environments. In military missions, a group of autonomous vehicles are required to keep in a specified formation for area coverage and reconnaissance; in small satellite clustering, formation helps to reduce the fuel consumption for propulsion and expand their sensing capabilities [5]; In automated highway system (AHS), the throughput of the transportation network can be greatly increased if vehicles can form to platoons at a desired velocity while keeping a specified distance between vehicles [6]. Research on formation control also helps people to better understand some biological social behaviors, such as swarm of insects and flocking of birds.

In formation control for a group of coordinated robots, different control topologies can be adopted depending on the specific scenarios. There may exists one or more leaders in the group while other robots follow one or more leaders in a specified way. Each robot has onboard sensing and computation ability. In some application scenarios, robots can have limited communication ability. But generally speaking, not all the global information is available for each robot. A centralized controller usually is not assumed to exist. The design of the controller for each robot has to be based on the local information. If no leader is designated, then all robots will have to coordinate with each other by relying on some global consensus for a common goal achievement.

There are many issues need to be considered when designing a distributed controller for mobile robot formation, such as the stability of the formation, controllability of different formation patterns, safety and uncertainties in formations. Many control approaches have been put forward to solve the problems in formation control, for example, leader-follower strategy [7], virtual structure approach [8] and behavior-based method [2]. In this paper, we will cover the main issues in formation control and give a review on current technologies in formation control. In addition, this paper contributes with a new consideration on formation control and its application in distributed parameter systems. We pointed out that formation control should be classified as formation regulation control and formation tracking control, similar to regulating and tracking tasks in conventional control.

The remaining part of this paper is organized as follows. In Sec. II, the stabilities in formation control are presented. In Sec. III, we will introduce different tools in formation characterization and stability analysis. Section IV is devoted to review all the main design methods for distributed formation control under different scenarios. We will discuss the challenging problem in formation control from the networked control system point of view in Sec. V. In Sec. VI, we present a new consideration on the application of mobile robot formation in distributed parameter system (DPS) measurement and control. Finally, conclusions are made in Sec. VII.

II. Stability notions of formation control

In this section, we will introduce three different but related concepts of stability: string stability, mesh stability and leader-to-formation stability.

String stability concerns about the disturbance propagation in platoon formation. A stable platoon formation requires that the effects of disturbance propagation should dampen away when travelling from the source to the followers. The definition of string stability in the sense of ℓ_∞ is defined in [9] as follows:

Definition 2.1: Consider the following interconnected system:

$$\dot{x}_i = f(x_i, x_{i-1} \cdots, x_{i-r+1}) \tag{1}$$

where $x_i \in \mathcal{N}$, $x_{i-j} \equiv 0 \ \forall i \leq j$, $x \in \mathcal{R}^n$, f :

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$\underbrace{\mathcal{R}^n \times \cdots \times \mathcal{R}^n}_{r \text{ times}} \rightarrow \mathcal{R}^n$ and $f(0, \cdots, 0) = 0$. The origin of (1) is called string-stable, if given any $\epsilon > 0$, there exists a $\delta > 0$ such that $\|x_i(0)\|_\infty < \delta \Rightarrow \sup_i \|x_i(\cdot)\|_\infty < \epsilon$.

The string stabilities in the sense of asymptotical (exponential) stability and ℓ_p are defined accordingly in [9]. It is demonstrated that exponential string stability is preserved under small structural perturbations and small singular perturbations.

It is widely known that an infinite string of autonomous vehicles can not be stabilized using only the preceding vehicle's information by any linear controller [10]. This string instability can be corrected by using a variable spacing policy [11]. But in situations where the constant spacing policy is required, both the information of the leader and the predecessor are needed for local controllers for string stability. This requires a communication network for the platoon. In [12], a non-identical linear approach is presented to remove the requirement on communication networks. The controller parameters for each vehicles is different and follows an updating procedure to make sure that the error propagation gain is less than 1.

The concept of mesh stability notion is introduced in [13], [14], [15] and finds its application in [16]. Mesh stability guarantees error attenuation and the stability are preserved when the look-ahead system is augmenting. In [13], an interconnected system is called look-ahead, if the (i, j)-th subsystem is connected only to the subsystems (k, l) such that $k \leq i$ and $l \leq j$. The exponential mesh stability is defined as follows:

Definition 2.2: Consider a system of the form:

$$\dot{x}_{i,j} = f_{i,j}(x_{i,j}, \cdots, x_{1,j}, x_{i,j-1}, \cdots, x_{i,1}, \cdots, x_{1,1}, t). \quad (2)$$

where $i \in \{1, \cdots, N\}$, $x_i \in \mathcal{R}^n$, $f : \mathcal{R}^n \times \cdots \times \mathcal{R} \rightarrow \mathcal{R}^n$ and $f(0, \cdots, 0) = 0$. $x = [x_1^T, \cdots x_n^T]^T \in \mathcal{R}^{N \times n}$. The origin $x = 0$ of the dynamical system (2) is globally exponentially mesh stable if

1) given any $\epsilon > 0$, $\exists \delta > 0$ such that, $\|x(0)\|_\infty < \delta \Rightarrow \|x(t)\|_\infty < \epsilon$
2) $x \rightarrow 0$, exponentially $\forall x \in \mathcal{R}^{N \times n}$
3) $\|x_{ij}(t)\| \leq \|x(k, l)(t)\|_\infty$, for $k \leq i$ and $l \leq j$.

If the system is globally Lipschitz and all the subsystems are globally exponentially stable, then the exponential stability can be achieved if additional conditions on Lipschitz constants are satisfied. The conditions are presented by using Lyapunov theory of stability and the comparison theorem for vector Lyapunov functions. The sufficient conditions under which the unperturbed mesh stable system can still be mesh stable under structural perturbations and singular perturbations are also investigated in [13].

In [17], the concept of leader-to-formation stability (LFS) based on leader-follower approach is put forward. LFS quantifies error amplification during signal propagation in leader-following formations. Different from mesh stability, leader-to-formation puts more emphasis on how the leader behavior can affect the interconnection errors resulted in the formation. This is an extension to their previous work on the concept of input-to-state stability in formation control [18].

Definition 2.3: A formation is called LFS if there is a class-KL function β and a class-K function γ such that for any initial formation error $\widetilde{x}(0)$ and for any bounded inputs of the formation leaders, ω_l, the formation error satisfies

$$\|\widetilde{x}(t)\| \leq \beta(\|\widetilde{x}(0)\|, t) + \sum_{l \in L_F} \gamma_l(\sup_{[0,t]} \|\omega_l\|). \quad (3)$$

The functions $\beta(r, t)$ and $\gamma_l(r)$ are called transient and asymptotic LFS gains for the formation. LFS relates interconnection topology to stability and performance.

III. Controllability analysis of formation control

A. Controllability analysis via graph theory

Graph theory is an important tool in the stability analysis of the formations. A graph is a natural presentation of the interconnection of coordinated robots for information exchange. The characterization of the topology of a graph can be used in the analysis of the stability and controllability of robot formations. It can also be used to choose an appropriate controller for a specific formation pattern or even decide if such a controller can exist. Because of its importance, we will give some preliminary introduction on it. There are rich literatures on graph theory, what we present here is based on [19], [20], [21].

An undirected graph $\mathcal{G}$ consists of a vertex set $\mathcal{V}(\mathcal{G})$ and an edge set $\mathcal{E}(\mathcal{G})$, where an edge is denoted as a pair of distinct vertices of $\mathcal{G}$. If (x,y) is a edge, then x and y are said to be adjacent or y is a neighbor of x and is denoted by $x \sim y$. The valency of a vertex x is the number of neighbor of neighbors of x. A path of length r from x to y in a graph is a sequence of $r + 1$ distinct vertices starting with x and ending with y such that consecutive vertices are adjacent. The valency matrix $\Delta(\mathcal{G})$ of a graph $\mathcal{G}$ is a diagonal matrix in which the (i, i) entry is the valency of vertex i. The adjacency matrix of a graph $\mathcal{G}$ is defined by

$$A_{ij} = \begin{cases} 1 & , \quad i, j \ \text{adjacent;} \\ 0 & , \quad \text{otherwise.} \end{cases}$$

The Laplacian of a graph $\mathcal{G}$ is defined as

$$L(\mathcal{G}) = \triangle(\mathcal{G}) - A(\mathcal{G}).$$

The adjacency matrix $A(\mathcal{G})$ is symmetric while the Laplacian is symmetric and positive semidefinite.

In [21], the dynamic model of agent i are considered as first order dynamics

$$\dot{x}_i = u_i, i = 1, \cdots, N$$

and the control law applied is

$$u_i = -\frac{1}{\Delta_{ii}} \sum_{j \sim i} (x_i - x_j).$$

Then, it is shown that topology of the interconnection graph formed by agent formation completely determines the controllability of the agent group. The system can be presented as:

$$\dot{x} = -\Delta^{-1/2} L \Delta^{-1/2} x.$$

In [22], the directed graph is used to represent a formation of agents while the dynamics of the agents are represented by the linear time-invariant system. By analyzing the eigenvalues of the graph Laplacian matrix, a Nyquist criterion is developed to determine the effect of the communication topology on formation stability.

In [23], the connection between the spectral graph theory and the control problem in vehicle formations is further investigated. The vehicles exchange information according to a pre-specified undirected communication graph. A state-space approach was developed to stabilize the formation. It is proved that a linear stabilizing feedback law always exists provided that the communication graph is connected. The rate of convergence to formation is governed by the size of the smallest positive eigenvalue of the Laplacian of the communication graph.

Some research has been focused on how the characteristics of the interconnection graph will change when the formation is changed from one pattern to another. In [24], under the framework of leader-following approach, the number of possible control graphs is derived, depending how the following pattern for each local controller is chosen. This result is used to search for the possible transient control graph when the formation is changing.

In [25], the geometric formations of multiple vehicles are studied under cyclic pursuit control law. The stability of the equilibrium formations of unicycle robots is related to the graph characteristic of the patterns.

In [26], based on the analysis of the directed graph from the interconnection of individual robots, the feasibility of achieving a desired pattern is investigated.

B. Controllability analysis via Lyapunov function

In [27], the multi-agent coordination problem is studied under the framework of control Lyapunov functions. The main assumption was that individual robot has a control Lyapunov function. Then, sufficient conditions are derived so that there exists a control Lyapunov function for the formation of robots. This function is a weighted sum of individual control Lyapunov function of each robot. Further investigation on the properties of the control Lyapunov function to maintain formation stability is applied by parameterized formation approach.

IV. Control strategies for formation control

A. Formation control via behavior-based approach and potential field approach

Behavior-based approach and potential field approach are always combined in the application of formation control. In behavior-based approach [28], [29], each robot has basic motor schemas. Each schema generates a vector representing the desired behavior response to sensory input. Possible motor schemas include collision avoidance, obstacle avoidance, goal seeking, and formation keeping. The control action of each robot is a vector weighted average of the control for each motor schema behavior. In [30], the Genetic Algorithm is used to decide the control weights and choose the appropriate behavior for formation maintains and obstacle avoidance.

In [31], the robot's behavior is based on a subsumption architecture. The primary behavior explored in this work is a group formation behaviors based on social potential fields [32]. In this paper, the social potential fields method is extended and evaluated in the presence of agent failure and imperfect sensory input.

In [33], the behavior-based formation control is modelled into a non-linear dynamic systems for trajectory generation and obstacle avoidance.

In [34], the desired formation pattern and trajectory for the robot group is represented by artificial potential trenches. Each robot will be attracted and move along the bottom of the potential field and automatically distribute with respect to each other.

B. Formation control via leader-follower approach

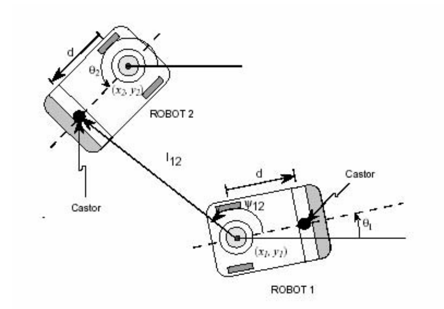

Fig. 1. Notion for the $l-\psi$ controller [35]

Another approach uses leader-follower patterns [35] in formation control. It is assumed that only local sensor-based information is available for each robots. There are two types of feedback controllers for maintaining formations of multiple robots. The first one is the $l-\psi$ controller and the second one is the $l-l$ controller. In the $l-\psi$ controller, the objective is to maintain a desired length l_{12}^d and a desired relative angle ψ_{12}^d between the leader and the follower as shown in Fig. 1 for two-wheeled ground mobile robots. Using input/output feedback linearizaiton, a controller can be designed so that l_{12}^d and ψ_{12}^d can exponentially convergence to the desired value.

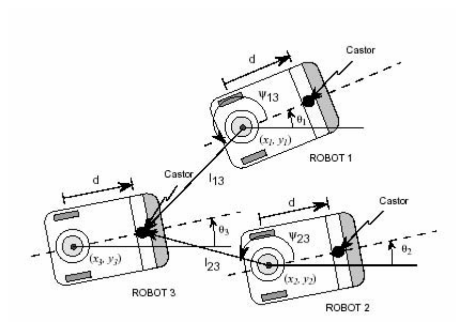

Fig. 2. Notion for the $l-l$ controller [35]

The $l-l$ controller considers the relative position of three mobile robots. Among them, one robot is controlled to follow the other two robots. The objective is to maintain the desired lengths l_{13}^d and l_{23}^d between the follower and its two leaders, as is shown in Fig. 2. A controller is also designed by using input/output feedback linearization in [35]. The application of leader-follower approach can be found in [36].

C. Formation control via generalized coordinates

In [37], a control methodology based on generalized coordinates was presented. The generalized coordinates characterize the vehicle's location (L), orientation (O) and its shape (S) with respect to a formation reference point in the formation. The trajectories of the formation group can be specified in terms of L, O and S coordinates. Formation control laws have been developed for asymptotic tracking of trajectories while maintaining a desired formation geometry. A similar ideal was presented in [38], [39], where the shape of the formation is expressed in shape coordinates.

D. Formation control via virtual structure method

The concept of virtual structure was first introduced in [40]. The virtual structure approach is usually used in spacecraft or small satellite formation flying control [41]. Control methods are developed to force a group of robots to behave in a rigid formation. In virtual structure approach, the controller is derived in three steps. First, the desired dynamics of the virtual structure is defined. Second, the desired motion of the virtual structure is translated into desired motions for each agent. Finally, individual tracking controllers for each agent are derived for agent tracking. In [42], the virtual structure method is combined with leader-following method and behavioral approach to formation control of multiple spacecraft interferometer in deep space. A similar idea in [8] was applied for spacecraft formation flying control.

E. Other control strategies in formation control

There are also many other methods in the application of formation control. For example, in [43], the local controller for formation control is derived from nonlinear servomechanism method. In [44], genetic algorithm and reinforcement learning are used for robot formation control and obstacle avoidance.

Model predictive control (MPC) is used in [45] as a local control law to meet with the overall formation performance under imperfect inter-vehicle communication. The inaccurate inter-vehicle communication is modelled as white noise. The interaction of the quality of information with the formation performance is investigated under this framework.

For a group marine-craft control, the formation maintenance and the trajectory tracking problem are decomposed into a geometric task and dynamic task [46]. The geometric task ensures that the individual ship converges to the position in the formation and the dynamic task will make sure that the ships travel along the trajectory with the desired speed. Similar idea can be found in [47].

In [7], the leader-follower approach is used for formation while the navigation function is used for collision avoidance for the formation. The navigation function method is extended in [48] to deal with partially known environment for mobile robot motion planning. In [49], the fuzzy logic and neural network techniques were used for robot speed control and obstacle avoidance in a formation. In [7], [50], [51], the visual servoing problem is considered for the application of formation control. Moreover, flocking in fixed and switching networks was investigated in [52] and in [53], [54], mobile agents or dynamic agents were considered for coordination control and structural stabilization.

V. Networking issues in formation control

In many situations, a wireless communication network is necessary for the local controller to obtain neighbor's information or even some global information of the formation. The formation control problem can be treated within the framework of networked control system. Until now, most of the research assumes perfect communication with no packet loss and no communication delay. Furthermore, the information the robot can obtain is usually assumed to be accurate. But in the framework of networked control system, the problem intrinsic in the networking can affect the performance of formation control and can even make the control system unstable. Few work has been done on the performance analysis of formation controller under such kind of circumstance.

In [55], the effect of communication delay on string stability was studied. It was assumed that the preceding vehicle's position, velocity and acceleration can be obtained by local sensors via a wireless communication network. Time delay and packet loss, intrinsic characteristics for wireless communication networks, may cause instability of the formation controller and raise the safety issues in platoon formation in AHS. The effect of communication delay on the controller depends on the communication protocols and the way the control law is triggered. In [55], the authors assumed that the network architecture uses time division with token passing for multiple access. The control law for each agent is triggered by the reception of the updated information. The study shows that the string stability is compromised by communication network introduced delay on the controllers. Therefore, a method of synchronizing updating each agent's controller is proposed in [55] to maintain the string stability under small communication delays.

The communication loss problem in vehicle following control problem was considered In [56] where the discrete time linear system with stochastic packet loss was modelled as a Markovian jump linear system. A linear matrix inequality (LMI) condition has been established for the existence of a stabilizing dynamic output feedback controller.

In spite of the current research efforts, the performance evaluation of formation control in a real network is hardly touched. The design method for formation control and the approaches in networked control system design have to be combined to give more realistic solutions.

VI. A new consideration - Formation control in distributed parameter systems with mobile actuators and mobile sensors

Here we present a new consideration in formation control. The key question to ask is "*why formation and with what pattern*". For example, when investigating how a group of networked mobile sensors (fog concentration sensors mounted on ground mobile robots) can be coordinated to perform a high level task of diffusion process boundary characterization [57], [58]. Each mobility node has certain sensing ability and limited communication ability [59]. The robots are required to work together to trace the evolving boundary of the diffusion process and furthermore, use the obtained information to control the diffusion process by releasing neutralizing chemicals through "*mobile actuators*". Formation control technology is clearly promising in this application. By forming the robots in a specified geometric pattern, we expect to get more accurate mesh measurement than by using a single robot. So, the open question is how the formation pattern can be decided for a diffusion process. Since there is no predefined trajectory for the formation, we also need to know how to generate the trajectory for the formation based on the knowledge of the diffusion process. Assume that a formation pattern can be found to meet the goal, we still need to solve the problem of how to achieve the formation stability and regional stability at the same time. Since the formation will be maintained under a wireless communication network, the formation control strategy should be robust to network induced uncertainties. The chosen control strategy should also be robust to the sensor noise.

In general, formation control can be considered as a parallel control concept as in a conventional single input-single output (SISO) feedback control system with no spatial variable involved. For example, the static pattern formation can be thought of the ramping up in a step response. Let us assume that, in the group of networked mobile sensors, there is a VIP (very important patrolman) predefined while others are called bodyguards (BGs) which are randomly distributed in a region. Once the VIP (not moving) calls, BGs will join the VIP and form a circle (evenly distributed) around the VIP with a given radius. So, this pattern formation is like a step response and when the localization is not accurate, several iterative control/adjustments among the BGs may be required, as described in [60] with some experimental results. This iterative pattern formation is actually called "*formation regulation control with a given fixed pattern*" which is just like the set-point value in the classical control sense. Now, let us consider that the VIP calls the BGs while it is moving. The task for formation control is the same - form a circle around the moving VIP. This case is simply like a tracking control task in the classical control sense. This case is much more complex than the static case where the VIP does not move since the motion control to achieve this is complicated especially when the global information is not completely available. We may call this case "*formation tracking control with a given fixed pattern*", which has also been implemented in [60] using global information. Here, the stability notion is not only on time domain, such as asymptotic stability, but also in spatial domain, such as the so called "*mesh stability*" [15]. The dominant literature in formation control seems to focus on how to maintain a given (fixed) pattern. However, the question remains "*who decides the pattern/shape to be formed*?" Clearly, to define a pattern for the formation controller to maintain needs a *good* reason.

Let us revisit the MAS-net project [57], [58] where the use of "formation movement" for fog concentration sampling and state reconstruction is definitely appealing. Clearly, there is no reason to do formation control with a fixed pattern, or shape while moving mobile sensors for diffusion process measurements. For example, in the 2×2 square formation movement, what is and who decides the side length? Moreover, why square not triangular? Just similar to solving ODE using Adams variable-step size method, in solving PDE systems, adaptive meshes are usually used. Therefore, the pattern to be formation controlled should be linked to this adaptive mesh of the governing partial differential equation. So, in general, the formation control should be considered in terms of "*morphological pattern formation tracking control.*" Our on-going research efforts are based on this new consideration and based on the simulation platforms we developed [61].

VII. Conclusion

In this paper, we presented a literature review on the current research efforts on formation control for a group of robots/vehicles. Some well-developed control methodologies have been introduced. To make the group of robot maintain formation while following some desired trajectories, the distributed controller needs to communicate with its neighbors. In most situations, we can put the formation control problem under the framework of networked control system. We point out that there are still open questions on how the controllers can be designed and how the formation performance can be evaluated in this way. We also presented a new consideration on formation control which is a promising future research direction on formation control for distributed parameter systems.

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