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A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements

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Most existing flocking algorithms rely on information about both relative position and relative velocity among neighbouring agents. In this article, we investigate the flocking problem with only position measurements. We propose a provably-stable flocking algorithm, in which an output vector is produced by distributed filters based on position information alone but not velocity information. Under the assumption that the initial interactive network is connected, the flocking algorithm not only can steer a group of agents to a stable flocking motion, but also can preserve the connectivity of the interactive network during the dynamical evolution. Moreover, we investigate the flocking algorithm with a virtual leader and show that all agents can asymptotically attain a desired velocity even if only one agent in the team has access to the information of the virtual leader. We finally show some numerical simulations to illustrate the theoretical results.

Keywords: distributed control; non-linear system; flocking; network connectivity; multi-agent system

1. Introduction

In recent years, flocking problems have attracted much attention among researchers from diverse fields (Okubo 1986; Reynolds 1987; Vicsek, Cziro'ok, Ben-Jacob, Cohen, and Shochet 1995). Flocking behaviours are characterised by distributed control, local interactions and self-organisation, which have many advantages such as avoiding predators and increasing chances of finding food. Understanding the mechanisms of flocking behaviours can help develop many artificial autonomous systems such as formation control of mobile robots, unmanned air vehicles and automated highway systems.

In the 1980s, Reynolds proposed a flocking algorithm to generate a computer animation model of bird flocks. He introduced three heuristic rules (Reynolds 1987): separation, alignment and cohesion. Stimulated by Reynolds' model, other flocking algorithms have been proposed by a combining a local artificial potential field with a velocity consensus component (Olfati-Saber 2006; Tanner, Jadbabaie, and Pappas 2007). The local artificial potential field produces attractive forces for larger distances and repulsive forces for small distances between agents, which has been widely used to regulate the distances between agents (Gazi and Passino 2003, 2004; Liu, Passino, and Polycarpou 2003a, b). On the other hand, velocity consensus component regulates the relative velocity of one agent to a weighted average of velocities of its neighbouring agents (Jadbabaie, Lin, and Morse 2003; Olfati-Saber and Murray 2004; Ren and Beard 2005; Xiao and Wang 2006). In addition, many other rules have been added to the flocking algorithms, such as obstacle avoidance (Khatib 1986; Chang, Shadden, Marsden, and Olfati-Saber 2003; Olfati-Saber and Murray 2003; Tanner 2004) and goal seeking (Leonard and Friorelli 2003; Shi, Wang, and Chu 2006; Su and Wang 2008; Su, Wang and Lin 2009). Since the assumption that the underlying topology of the network remaining connected at all times is very difficult to verify in practice, a new potential function was recently proposed for a flocking algorithm to achieve flocking behaviour while preserving the network connectivity (Zavlanos, Jadbabaie, and Pappas 2007).

Existing flocking algorithms rely on information about both the relative position and the relative velocity information among neighbouring agents. However, not all the agents (robots, air vehicles, manipulators and so on) in practice are equipped with speed sensors. Furthermore, an algorithm using only position measurements has the advantage of decreasing equipment cost and network traffic. In this work, therefore, we study the flocking problem based only on position measurements. We modify the passivity approach in Lizarralde and Wen (1996); Lawton and Beard (2002); Ren (2007) and use it for the

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proposed flocking algorithm to produce an output vector, which can replace the roles of the velocity. Our proposed control input has the same function as that in Ren (2007) while a non-linear gradient-based term is used to replace the linear one in Ren (2007). Moreover, we extend the results in Ren (2007) to the case of switching topology. In addition, we propose two rules that a potential function should obey in order to preserve the connectivity of the network as long as the initial network is connected. We prove the global asymptotic stability by applying the LaSalle Invariance Principle. Furthermore, we investigate the flocking algorithm with a virtual leader and show that all agents can asymptotically attain the desired velocity even if only one agent in the team has access to the information of the virtual leader.

The remainder of the article is organised as follows. Section 2 motivates the problem to be investigated. The flocking algorithm and its stability analysis are established in §3. Section 4 discusses the case with a virtual leader. Section 5 presents the simulation results. Finally, §6 draws the conclusions.

2. Problem statement

We consider N agents moving in an n-dimensional Euclidean space. The motion of each agent is described by a double-integrator system of the form

$$\begin{aligned} \dot{q}_i &= p_i, \\ \dot{p}_i &= u_i, \quad i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $q_i \in \mathrm{R}^n$ is the position vector of agent i, $p_i \in \mathrm{R}^n$ is its velocity vector and $u_i \in \mathrm{R}^n$ is the (force) control input acting on agent i. Our objective is to design control inputs $u_i \in \mathrm{R}^n$ so that all agents move with a common velocity while avoiding collisions among agents based only on position measurements, under the assumption that the initial network is connected in the usual sense of a connected graph. In the situation that there is a virtual leader in the group of agents, the flocking algorithm should be designed to enable all agents to asymptotic track the virtual leader, in the sense that all agents asymptotically move with the same velocity as the virtual leader.

Suppose that each agent has the same influencing/sensing radius r. Let $\varepsilon \in (0, r)$ be a given constant. We call $G(t) = (V, E(t))$ a dynamic undirected graph consisting of a set of vertices $V = \{1, 2, \ldots, N\}$ numbered by the indexes of the agents and a time-varying set of links $E(t) = \{(i,j) | i, j \in V\}$ such that

(i) initial links are generated by $E(0) = \{(i,j) | \|q_i(0) - q_j(0)\| < r, i, j \in V\}$;

(ii) if $(i,j) \notin E(t^-)$ and $\|q_i(t) - q_j(t)\| < r - \varepsilon$, then (i,j) is a new link to be added to $E(t)$;

(iii) if $\|q_i(t) - q_j(t)\| \geq r$, then $(i,j) \notin E(t)$.

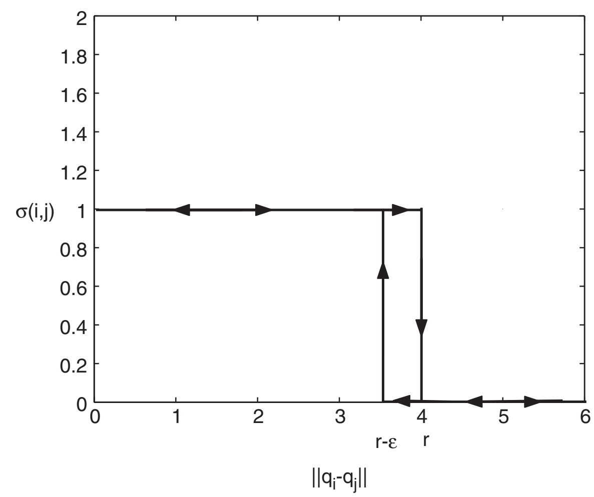

Figure 1. Indicator function $\sigma(i,j)$.

We can use a symmetric indicator function $\sigma(i,j) \in \{0, 1\}$ to describe whether or not there is a link between agent i and agent j at time t, which is defined as

$$\sigma(i,j)[t] = \begin{cases} 0, & \text{if } ((\sigma(i,j)[t^-] = 0) \cap (\|q_i(t) - q_j(t)\| \geq r - \varepsilon)) \\ & \quad \cup ((\sigma(i,j)[t^-] = 1) \cap (\|q_i(t) - q_j(t)\| \geq r)), \\ 1, & \text{if } ((\sigma(i,j)[t^-] = 1) \cap (\|q_i(t) - q_j(t)\| < r)) \\ & \quad \cup ((\sigma(i,j)[t^-] = 0) \cap (\|q_i(t) - q_j(t)\| < r - \varepsilon)). \end{cases}$$

We can see from Figure 1 that there is a hysteresis in the process of adding new links in the graph. This hysteresis is crucial in preserving connectivity of a dynamically interactive network (Ji and Egerstedt 2007; Zavlanos et al. 2007; Su and Wang 2008).

3. Fundamental flocking algorithm

3.1 *Main results*

Motivated by the flocking algorithms developed in Olfati-Saber (2006), Tanner et al. (2007), Zavlanos et al. (2007) and the passivity approach used in Lizarralde and Wen (1996), Lawton and Beard (2002), and Ren (2007), we present a new flocking algorithm using only position measurements, which is described as follows:

$$\begin{cases} u_i = -\sum_{j \in N_i(t)} (\nabla_{q_i} \psi(\|q_{ij}\|) - w_{ij}(y_i - y_j)), \\ y_i = PT\hat{x}_i + P \sum_{j \in N_i(t)} w_{ij} q_{ij}, \\ \dot{\hat{x}}_i = T\hat{x}_i + \sum_{j \in N_i(t)} w_{ij} q_{ij}, \end{cases} \tag{2}$$

where $\hat{x}_i, y_i \in \mathrm{R}^n$, $y = [y_1^T, \ldots, y_N^T]^T$, $q_{ij} = q_i - q_j$, $T \in \mathrm{R}^{n\times n}$ is a Hurwitz matrix, $P \in \mathrm{R}^{n\times n}$ is a symmetric positive-definite matrix such that $T^TP + PT = -Q$ is a symmetric positive-definite matrix, the constant $w_{ij} = w_{ji} > 0$ for all $i, j \in V$, ∇ is the gradient operator, $\psi(\cdot)$ is a potential function to be specified below in (4), $N_i(t)$ is the neighbourhood of agent i at time t which is defined as

$$N_i(t) = \{j | \sigma(i,j)[t] = 1, j \neq i, j = 1, \ldots, N\}. \quad (3)$$

In the proposed flocking algorithm (2), the output vector y is produced by distributed filters based only on relative position measurements. The gradient-based term of control input u_i is responsible for collision avoidance and for maintaining the connectivity of the network. Every output vector, which regulates the velocity of each agent, tends to zero if all agents attain a common velocity. Note that the form of the proposed control input is not the same as that in Ren (2007). The control input in Ren (2007) cannot perform the function in this article, even if the linear gradient-based term in Ren (2007) is replaced by the proposed non-linear one in this article. This is because the negative semi-definiteness of the derivative of the Lyapunov function relies on the linear property of gradient-based term. However, our proposed control input has the same function as that in Ren (2007) when the proposed non-linear gradient-based term is reduced to the linear one in Ren (2007).

The non-negative potential $\psi(\|q_{ij}\|)$ is a function of the distance $\|q_{ij}\|$ between agent i and agent j, which is differentiable for $\|q_{ij}\| \in (0, r)$, satisfying

(i) $\psi(\|q_{ij}\|) \to \infty$ as $\|q_{ij}\| \to 0$ or $\|q_{ij}\| \to r$;
(ii) $\psi(\|q_{ij}\|)$ attains its unique minimum when $\|q_{ij}\|$ equals a desired distance.

The main difference between the potential function $\psi(\|q_{ij}\|)$ here and the potential function in Tanner et al. (2007) is that $\psi(\|q_{ij}\|)$ tends to infinite when the distance between agent i and agent j tends to r. This property can guarantee that no initial connections will be lost. Note that the potential function in Zavlanos et al. (2007) is an example which satisfies the above definition of the potential function $\psi(\|q_{ij}\|)$. Another example is as follows (see Figure 2):

$$\psi(\|q_{ij}\|) = \begin{cases} +\infty, & \|q_{ij}\| = 0, \\ \dfrac{r}{\|q_{ij}\|(r - \|q_{ij}\|)}, & \|q_{ij}\| \in (0, r), \\ +\infty, & \|q_{ij}\| = r. \end{cases} \quad (4)$$

The adjacent matrix $A(t)$ of a graph $G(t)$ is defined as

$$a_{ij}(t) = \begin{cases} w_{ij}, & (i,j) \in E(t), \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

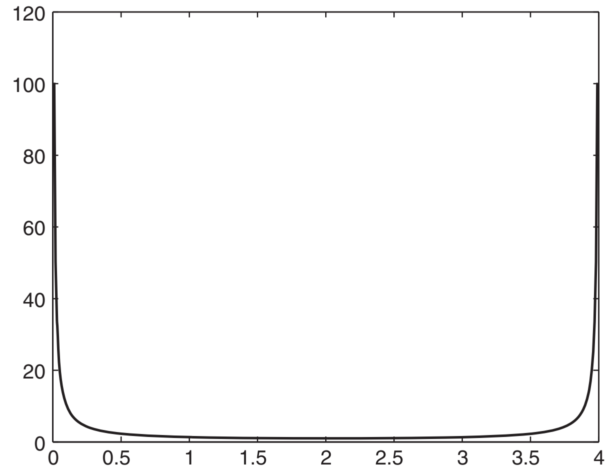

Figure 2. An example of the potential function $\psi(\|q_{ij}\|)$.

The corresponding Laplacian is $L(t) = \Delta(A(t)) - A(t)$, where the degree matrix $\Delta(A(t))$ is a diagonal matrix with the i-th diagonal element being $\sum_{j=1, j\neq i}^N a_{ij}(t)$. Denote the eigenvalues of $L(t)$ as $\lambda_1(L(t)) \leq \cdots \leq \lambda_N(L(t))$. Then it is well known that $\lambda_1(L(t)) = 0$ and $1 = [1, 1, \ldots, 1]^T \in \mathrm{R}^N$ is the corresponding eigenvector; moreover, if $G(t)$ is a connected graph, then $\lambda_2(L(t)) > 0$ (Godsil and Royle 2001).

The energy function for the system is defined as follows:

$$W = \frac{1}{2}\sum_{i=1}^N \left(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + p_i^T p_i \right) + \frac{1}{2}\dot{\hat{x}}^T (I_N \otimes P)\dot{\hat{x}}, \quad (6)$$

Clearly, $W = W(\widehat{q}(t), p(t), \hat{x}(t)) = W(t)$ is a positive semi-definite function (Horn and Johnson 1987), where $\widehat{q} = [q_{11}^T, \ldots, q_{1N}^T, \ldots, q_{N1}^T, \ldots, q_{NN}^T]^T$ and $\hat{x} = [\hat{x}_1^T, \ldots, x_N^T]^T$. We state the main result in the following.

Theorem 1: *Consider a system of N mobile agents with dynamics* (1), *each being steered by protocol* (2). *Suppose that the initial network* $G(0)$ *is connected and the initial energy* $W_0 = W(\widehat{q}(0), p(0), \hat{x}(0))$ *is finite. Then, the following hold*:

(i) $G(t)$ will remain to be connected for all $t \geq 0$;
(ii) all agents asymptotically move with the same velocity;
(iii) almost every final configuration locally minimises each agent's global potential $\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|)$;
(iv) collisions between the agents are avoided.

3.2 *Proof of the theorem*

(a) *Proof of part* (*i*)
Assume that $G(t)$ switches at time t_k ($k = 1, 2, \ldots$), which implies that $G(t)$ is a fixed graph in each

time-interval $[t_{k-1}, t_k)$. Note that W_0 is finite and the time derivative of $W(t)$ in $[t_0, t_1)$ is

$$\begin{aligned}
\dot{W}(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}\dot{\psi}(\|q_{ij}\|) + \sum_{i=1}^{N} p_i^T u_i \\
&\quad + \frac{1}{2}\ddot{\hat{x}}^T(I_N \otimes P)\dot{\hat{x}} + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)\ddot{\hat{x}} \\
&= \sum_{i=1}^{N} p_i^T \sum_{j\in N_i}\nabla_{q_i}\psi(\|q_{ij}\|) \\
&\quad - \sum_{i=1}^{N} p_i^T\left(\sum_{j\in N_i(t)}\nabla_{q_i}\psi(\|q_{ij}\|) + \sum_{j\in N_i(t)} w_{ij}(y_i - y_j)\right) \\
&\quad + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes T^T)(I_N \otimes P)\dot{\hat{x}} \\
&\quad + \frac{1}{2}p^T(L^T \otimes I_n)(I_N \otimes P)\dot{\hat{x}} \\
&\quad + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)(I_N \otimes T)\dot{\hat{x}} \\
&\quad + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)(L \otimes I_n)p \\
&= -p^T(L \otimes I_n)y + p^T(L \otimes I_n)(I_N \otimes P)\dot{\hat{x}} \\
&\quad - \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes Q)\dot{\hat{x}} \\
&= -\frac{1}{2}\dot{\hat{x}}^T(I_N \otimes Q)\dot{\hat{x}} \le 0, \qquad (7)
\end{aligned}$$

where $p = [p_1^T, \ldots, p_N^T]^T$ and $\otimes$ stands for the Kronecker product, which implies that

$$W(t) \le W_0 < \infty \quad \text{for } t \in [t_0, t_1).$$

From the definition of the potential function, we have $\lim_{\|q_{ij}(t)\|\to r} \psi(\|q_{ij}(t)\|) = \infty$. Therefore, no distance of existing edges will tend to r for $t \in [t_0, t_1)$, which also implies that no existing edges will be lost before time t_1. Hence, new edges must be added in the interaction network at switching time t_1. Note that the hysteresis ensures that if a finite number of links are added to $G(t)$, then the associated potentials remain finite. Thus, $W(t_1)$ is finite.

Similar to the above analysis, the time derivative of $W(t)$ in every $[t_{k-1}, t_k)$ is

$$\dot{W}(t) = -\frac{1}{2}\dot{\hat{x}}^T(I_N \otimes Q)\dot{\hat{x}} \le 0, \qquad (8)$$

which implies that

$$W(t) \le W(t_{k-1}) < \infty \quad \text{for } t \in [t_{k-1}, t_k), \quad k = 1, 2, \ldots. \qquad (9)$$

Therefore, no distance of existing edges will tend to r for $t \in [t_{k-1}, t_k)$, which also implies that no edges will be lost before time t_k and $W(t_k)$ is finite.

Since $G(0)$ is connected and no edges in $E(0)$ were lost, $G(t)$ will remain connected for all $t \ge 0$.

(b) *Proof of parts* (*ii*) *and* (*iii*)
Assume that m_k new edges are added to the network at time t_k. Clearly, we have

$$0 < m_k \le \frac{N(N-1)}{2} - (N-1) \triangleq M.$$

From (6) and (9), we have $W(t_k) \le W_0 + (m_1 + \cdots + m_k)\psi(\|r - \varepsilon\|)$. Since there are at most M new edges that can be added to $G(t)$, we have $k \le M, m_1 + \cdots + m_k \le M$ and $W(t) \le W_0 + M\psi(\|r - \varepsilon\|) \triangleq W_{\max}$ for all $t \ge 0$. Therefore, the number of switching times k of the system (1) is finite, which implies the interaction network $G(t)$ eventually becomes fixed. Thus, the remaining discussions can be restricted on the time interval (t_k, ∞). Note that the distance of edge is not longer than $\max\{\psi^{-1}(W_{\max})\}$, or not shorter than $\min\{\psi^{-1}(W_{\max})\}$. Hence, the set

$$\Omega = \left\{\widehat{q} \in D_g, p, \hat{x} \in \mathrm{R}^{Nn} \,\middle|\, W(\widehat{q}, p, \hat{x}) \le W_{\max}\right\}, \qquad (10)$$

is positively invariant, where

$$D_g = \Big\{\widehat{q} \in \mathrm{R}^{N^2 n} \,\big|\, \|q_{ij}\| \in [\min\{\psi^{-1}(W_{\max})\}, \max\{\psi^{-1}(W_{\max})\}], \quad \forall (i,j) \in E(t), t \in (t_k, \infty)\Big\}.$$

Since $G(t)$ is connected for all $t \ge 0$, $\|q_{ij}\| \le (N-1)r$ for any i and j. Because $W(t) \le W_{\max}$, we have $p_i^T p_i \le 2W_{\max}$ and $\lambda_{\max}(P)\dot{\hat{x}}_i^T\dot{\hat{x}}_i \le 2W_{\max}$, where $\lambda_{\max}(P)$ is the largest eigenvalue of matrix P, which implies that $\hat{x}_i$ from (2) is also bounded. Therefore, the set Ω such that $W(t) \le W_{\max}$ is closed and bounded, hence is compact. Note that system (1) with control input (2) is an autonomous system, at least on the concerned time interval (t_k, ∞). It follows from LaSalle's invariance principle (Khalil 2002) that if the initial conditions of the system (1) lie in Ω, its trajectories will converge to the largest invariant set inside the region $S = \{\widehat{q} \in D_g, p, \hat{x} \in \mathrm{R}^{Nn} \mid \dot{W} = 0\}$. From (8), $\dot{W} = 0$ if and only if $\dot{\hat{x}} = 0$, which implies that $\ddot{\hat{x}} = 0$. From (2), we have

$$\ddot{\hat{x}}_i = T\dot{\hat{x}}_i + \sum_{j\in N_i(t)} w_{ij}(p_i - p_j), \quad i = 1, \ldots, N, \qquad (11)$$

which implies that $(L \otimes I_n)p = 0$. Since the dynamic network is connected all the time, from the property of the Laplacian matrix $L \otimes I_n$ (Godsil and Royle 2001), p converges asymptotically to $1_N \otimes \beta$, for some constant vector $\beta \in R^n$, which implies that $\|p_i - p_j\| \to 0$ for all i, j as $t \to \infty$.

Since $(L \otimes I_n)p = 0$, we have $(L \otimes I_n)u = 0$, which implies that $u = 1_N \otimes \alpha$, for some constant vector

$\alpha \in R^n$. From (2) and $\dot{\hat{x}} = 0$, we have $y = 0$, which implies that

$$u = -\begin{bmatrix} \sum_{j\in N_1(t)} \nabla_{q_1}\psi(\|q_{1j}\|) \\ \vdots \\ \sum_{j\in N_N(t)} \nabla_{q_N}\psi(\|q_{Nj}\|) \end{bmatrix}. \quad (12)$$

It follows from (12) and $\psi(\|q_{ij}\|) = \psi(\|q_{ji}\|)$ that

$$\nabla_{q_i}\psi(\|q_{ij}\|) = -\nabla_{q_j}\psi(\|q_{ij}\|) = -\nabla_{q_j}\psi(\|q_{ji}\|),$$

$$\sum_{j\in N_1(t)} \nabla_{q_1}\psi(\|q_{1j}\|) + \cdots + \sum_{j\in N_N(t)} \nabla_{q_N}\psi(\|q_{Nj}\|) = 0,$$

$$(1_N \otimes \alpha)^T u = 0, \quad (13)$$

which implies that u is orthogonal to $1_N \otimes \alpha$. Since $u = \{1_N \otimes \alpha\} \cap \{1_N \otimes \alpha\}^{\perp}$, we have $u = 0$. From (12), we have

$$\begin{bmatrix} -\sum_{j\in N_1(t)} \nabla_{q_1}\psi(\|q_{1j}\|) \\ \vdots \\ -\sum_{j\in N_N(t)} \nabla_{q_N}\psi(\|q_{Nj}\|) \end{bmatrix} = 0. \quad (14)$$

Thus, the configuration converges asymptotically to a fixed configuration that is an extremum of all agent global potentials. If the solution of (14) starts at an equilibrium, which is a local maximum or saddle point, the solution will be invariant at all times. Then, not all solutions of (14) converge to local minima. However, every point but local minima is an unstable equilibrium (Olfati-Saber 2006). Thus, almost every final configuration locally minimises each agent's global potential $\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|)$.

(c) *Proof of part* (*iv*)

In view of (10), $W(\widehat{q}, p, \hat{x}) \le W_{\max}$ for all $t \ge 0$. However, from the definition of potential function, we have $\lim_{\|q_{ij}\|\to 0}\psi(\|q_{ij}\|) = \infty$. Therefore, collisions among agents are avoided.

4. Flocking with a virtual leader

4.1 *Main results*

In some situations, the regulation of agents has certain purpose, such as achieving a desired common velocity, or arriving at a desired destination. In this section, we investigate the flocking algorithm with a virtual leader. We assume that the virtual leader moves along a fixed direction with a constant velocity p_γ, which is the same as the investigation in Shi et al. (2006).

In some nature examples, few individuals have the pertinent information, such as knowledge of the location of a food source, or of a migration route (Couzin, Krause, Franks, and Levin 2005). We assume that only one agent is being informed about the virtual leader. This assumption is of practical value since it requires only one agent to know the global moving target. Certainly, the result can be generalised to the case with multiple informed agents. The control input for agent i is designed as

$$\begin{cases} u_i = -\sum_{j\in N_i(t)} (\nabla_{q_i}\psi(\|q_{ij}\|) - w_{ij}(y_i - y_j)) - h_i y_i, \\ y_i = PT\hat{x}_i + P\sum_{j\in N_i(t)} w_{ij}q_{ij} + Ph_i(q_i - q_\gamma), \\ \dot{\hat{x}}_i = T\hat{x}_i + \sum_{j\in N_i(t)} w_{ij}q_{ij} + h_i(q_i - q_\gamma), \end{cases} \quad (15)$$

where q_γ is the position of the virtual leader. If agent i is the informed agent, then $h_i = 1$; otherwise, $h_i = 0$. Without loss of generality, we assume that the first agent is the informed agent, that is, $h_i = 1$ for $i = 1$ and $h_i = 0$ for all the others.

The energy function for the system is defined as follows:

$$U = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + (p_i - p_\gamma)^T(p_i - p_\gamma)\right) + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)\dot{\hat{x}}, \quad (16)$$

Clearly, $U = U(\widehat{q}(t), p(t), \hat{x}(t)) = U(t)$ is a positive semi-definite function. Our second main result on tracking the virtual leader is stated in the following theorem.

Theorem 2: *Consider a system of N mobile agents with dynamics* (1), *each being steered by protocol* (15). *Suppose that the initial network* $G(0)$ *is connected and the initial energy* $U_0 = U(\widehat{q}(0), p(0), \hat{x}(0))$ *is finite. Then, the following hold*:

(i) $G(t)$ will remain to be connected for all $t \ge 0$;
(ii) all agents asymptotically move with the desired velocity p_γ;
(iii) almost every final configuration locally minimises each agent's global potential $\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|)$;
(iv) collisions between the agents are avoided.

4.2 *Proof of Theorem 2*

We first prove part (i) of Theorem 2.

Denote the position difference vector and the velocity difference vector between agent i and the virtual leader as $\tilde{q}_i = q_i - q_\gamma$ and $\tilde{p}_i = p_i - p_\gamma$, respectively. Then

$$\begin{aligned}\dot{\tilde{q}}_i &= \tilde{p}_i,\\ \dot{\tilde{p}}_i &= u_i, \quad i = 1, \ldots, N.\end{aligned} \tag{17}$$

By the definition of $\psi(\|q_{ij}\|)$, we have

$$\psi(\|q_{ij}\|) = \psi(\|\tilde{q}_{ij}\|), \tag{18}$$

where $\tilde{q}_{ij} = \tilde{q}_i - \tilde{q}_j$. Thus, the control input (15) for agent i can be rewritten as

$$\begin{cases} u_i = -\sum_{j\in N_i(t)} (\nabla_{\tilde{q}_i}\psi(\|\tilde{q}_{ij}\|) - w_{ij}(y_i - y_j)) - h_i y_i,\\ y_i = PT\hat{x}_i + P\sum_{j\in N_i(t)} w_{ij}\tilde{q}_{ij} + Ph_i\tilde{q}_i,\\ \dot{\hat{x}}_i = T\hat{x}_i + \sum_{j\in N_i(t)} w_{ij}\tilde{q}_{ij} + h_i\tilde{q}_i, \end{cases} \tag{19}$$

and the positive semi-definite energy function (16) can be rewritten as

$$U = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} \psi(\|\tilde{q}_{ij}\|) + \tilde{p}_i^T\tilde{p}_i\right) + \frac{1}{2}\dot{\hat{x}}^T(I_N \otimes P)\dot{\hat{x}}, \tag{20}$$

Similar to the proof of part (i) of Theorem 1, the time derivative of $U(t)$ in $[t_{k-1}, t_k)$ is obtained as

$$\begin{aligned}\dot{U}(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i}\dot{\psi}(\|\tilde{q}_{ij}\|) + \sum_{i=1}^{N}\tilde{p}_i^T u_i\\ &\quad + \frac{1}{2}\ddot{\hat{x}}^T(I_N\otimes P)\dot{\hat{x}} + \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes P)\ddot{\hat{x}}\\ &= \sum_{i=1}^{N}\tilde{p}_i^T\sum_{j\in N_i}\nabla_{\tilde{q}_i}\psi(\|\tilde{q}_{ij}\|)\\ &\quad - \sum_{i=1}^{N}\tilde{p}_i^T\left(\sum_{j\in N_i(t)}\nabla_{\tilde{q}_i}\psi(\|\tilde{q}_{ij}\|) + \sum_{j\in N_i(t)} w_{ij}(y_i - y_j) + h_i y_i\right)\\ &\quad + \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes T^T)(I_N\otimes P)\dot{\hat{x}} + \frac{1}{2}\tilde{p}^T(L^T\otimes I_n)(I_N\otimes P)\dot{\hat{x}}\\ &\quad + \frac{1}{2}\tilde{p}^T(H_N^T\otimes I_n)(I_N\otimes P)\dot{\hat{x}} + \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes P)(I_N\otimes T)\dot{\hat{x}}\\ &\quad + \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes P)(L\otimes I_n)\tilde{p} + \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes P)(H_N\otimes I_n)\tilde{p}\\ &= -\tilde{p}^T((L + H_N)\otimes I_n)y + \tilde{p}^T((L + H_N)\otimes I_n)(I_N\otimes P)\dot{\hat{x}}\\ &\quad - \frac{1}{2}\dot{\hat{x}}^T(I_N\otimes Q)\dot{\hat{x}}\\ &= -\frac{1}{2}\dot{\hat{x}}^T(I_N\otimes Q)\dot{\hat{x}} \le 0,\end{aligned} \tag{21}$$

where $H_N = \mathrm{diag}(h_1, \ldots, h_N)$, which implies that

$$U(t) \le U(t_{k-1}) < \infty \quad \text{for var } t \in [t_{k-1}, t_k), k = 1, 2, \ldots. \tag{22}$$

Therefore, no distance of existing edges will tend to r for $t\in[t_{k-1}, t_k)$, which also implies that no existing edges will be lost before time t_k, and $U(t_k)$ is finite. Since $G(0)$ is connected and no edges in $E(0)$ will be lost, $G(t)$ will remain to be connected for all $t \ge 0$.

We now prove part (ii) and (iii) of Theorem 2.

Similar to the proof of part (ii) of Theorem 1, the set

$$\Omega = \left\{\widehat{\tilde{q}} \in D_g, \tilde{p}, \hat{x} \in \mathrm{R}^{Nn} \,\middle|\, U(\widehat{\tilde{q}}, \tilde{p}, \hat{x}) \le U_{\max}\right\}, \tag{23}$$

is compact, where $\widehat{\tilde{q}} = [\tilde{q}_{11}^T, \ldots, \tilde{q}_{1N}^T, \ldots, \tilde{q}_{N1}^T, \ldots, \tilde{q}_{NN}^T]^T$, $\tilde{p} = [\tilde{p}_1^T, \tilde{p}_2^T, \ldots, \tilde{p}_N^T]^T$, $D_g = \{\widehat{\tilde{q}} \in \mathrm{R}^{N^2 n} | \|\tilde{q}_{ij}\| \in [\min\{\psi^{-1}(U_{\max})\}, \max\{\psi^{-1}(U_{\max})\}], \forall(i,j) \in E(t), t \in (t_k, \infty)\}$ and $U_{\max} \triangleq U_0 + g(\frac{N(N-1)}{2} - (N-1)g)\psi(\|r - \varepsilon\|)$. From LaSalle's invariance principle (Khalil 2002); we have

$$\dot{U} = -\frac{1}{2}\dot{\hat{x}}^T(I_N\otimes Q)\dot{\hat{x}} = 0, \tag{24}$$

which implies that $\dot{\hat{x}} = 0$ and $\ddot{\hat{x}} = 0$. From (15), we have

$$\ddot{\hat{x}}_i = T\dot{\hat{x}}_i + \sum_{j\in N_i(t)} w_{ij}(\tilde{p}_i - \tilde{p}_j) + h_i\tilde{p}_i, \quad i = 1, \ldots, N, \tag{25}$$

which implies that $((L + H_N)\otimes I_n)\tilde{p} = 0$. Since the dynamic network is connected all the time and one agent is the informed agent, from Lemma 3 in Hong, Hu, and Gao (2006), the symmetric matrix $(L + H_N)\otimes I_n$ is positive definite, which implies $\|\tilde{p}_i\| \to 0$ for all i as $t\to\infty$, hence $\|p_i - p_\gamma\| \to 0$ for all i as $t\to\infty$.

From the previous analysis, we can see that, in steady state,

$$\dot{p}_i = \dot{p}_\gamma = 0, \quad i = 1, \ldots, N, \tag{26}$$

which implies that $u = 0$. Similar to the proof of part (iii) of Theorem 1, we have

$$u = -\begin{bmatrix}\sum_{j\in N_1(t)}\nabla_{q_1}\psi(\|q_{1j}\|)\\ \vdots\\ \sum_{j\in N_N(t)}\nabla_{q_N}\psi(\|q_{Nj}\|)\end{bmatrix} = 0. \tag{27}$$

Thus, almost every final configuration locally minimises each agent's global potential $\sum_{j\in N_i(t)}\nabla_{q_i}\psi(\|q_{ij}\|)$.

Finally, we prove part (iv) of Theorem 2. From the earlier analysis, we recall that $U(\widehat{\tilde{q}}, \tilde{p}, \hat{x}) \le U_{\max}$. The results then follow by the same arguments as that used in the proof of part (iv) of Theorem 1. This completes the proof of Theorem 2.

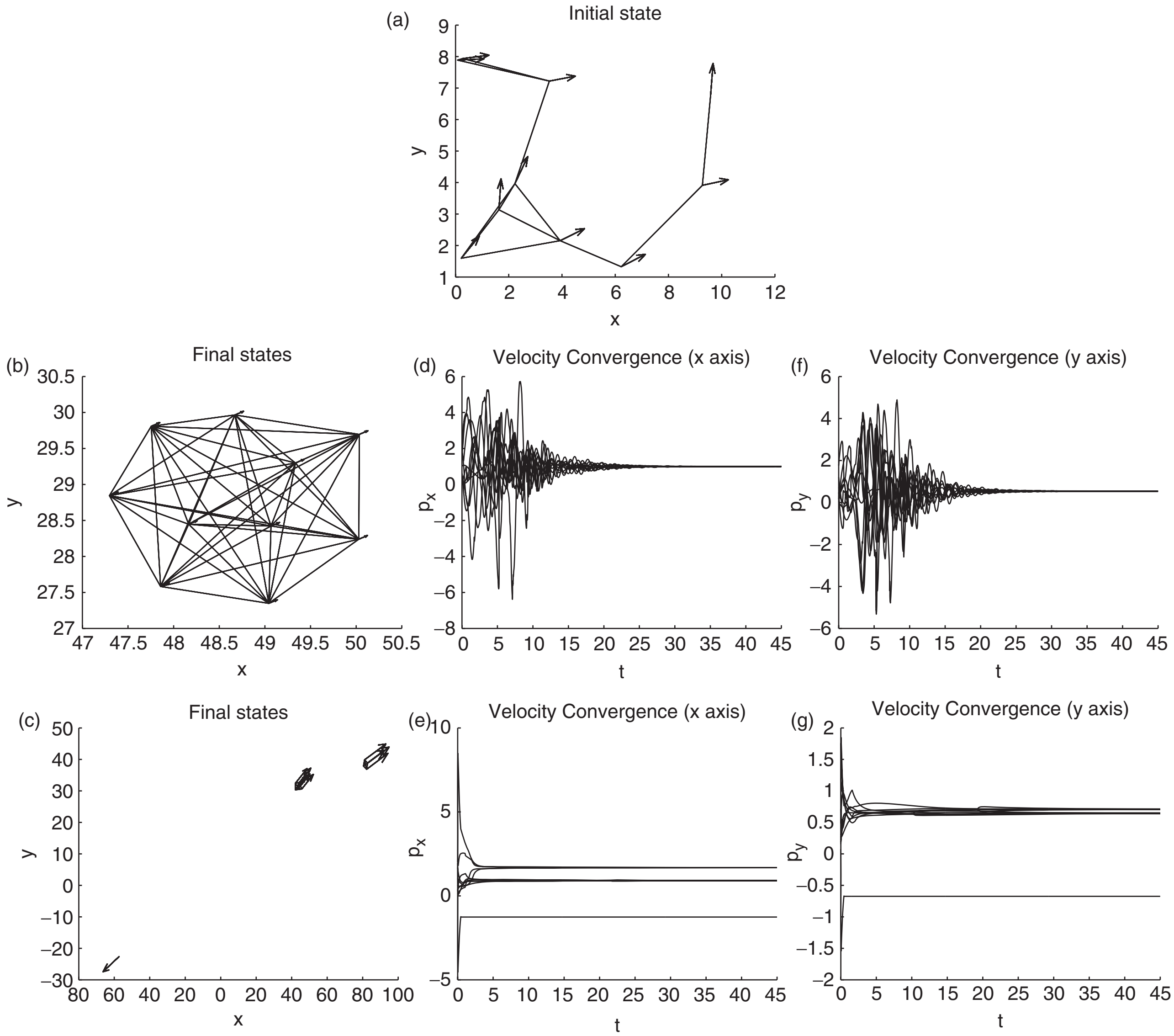

Figure 3. Flocking of 10 agents applying the algorithm (2) and the algorithm in Tanner et al. (2007).

5. Simulation studies

All the simulations are performed with 10 agents moving in the plane whose initial positions and velocities are randomly chosen within $[0, 7] \times [0, 7]$ and $[0, 1] \times [0, 1]$, respectively. We restrict the initial interaction network be connected. In simulations, we choose the influencing/sensing radius $r = 4$, the parameter $\varepsilon = 0.5$, the weights $w_{ij} = 1$ for all $i, j \in V$, the matrices

$$T = -0.5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and the potential function (4) for flocking algorithm (2).

In Figure 3, we compare the flocking algorithm (2) proposed in this work with the flocking algorithm in Tanner et al. (2007). Figure 3 (a) shows the initial states of the agents; Figures 3 (b) and (c) depict the configuration and velocities of the group of agents at $t = 45$ obtained by the algorithm (2) and the algorithm in Tanner et al. (2007), respectively; Figures 3 (d) and (e) show the curves of velocities for the x-axis by the algorithm (2) here and the algorithm in Tanner et al. (2007), respectively; Figures 3 (f) and (g) show the curves of velocities for the y-axis by the algorithm (2) here and the algorithm in Tanner et al. (2007), respectively. We can see that algorithm (2) here produces flocking behaviour, while the algorithm in Tanner et al. (2007) leads to fragmentation. This is due to the fact that the

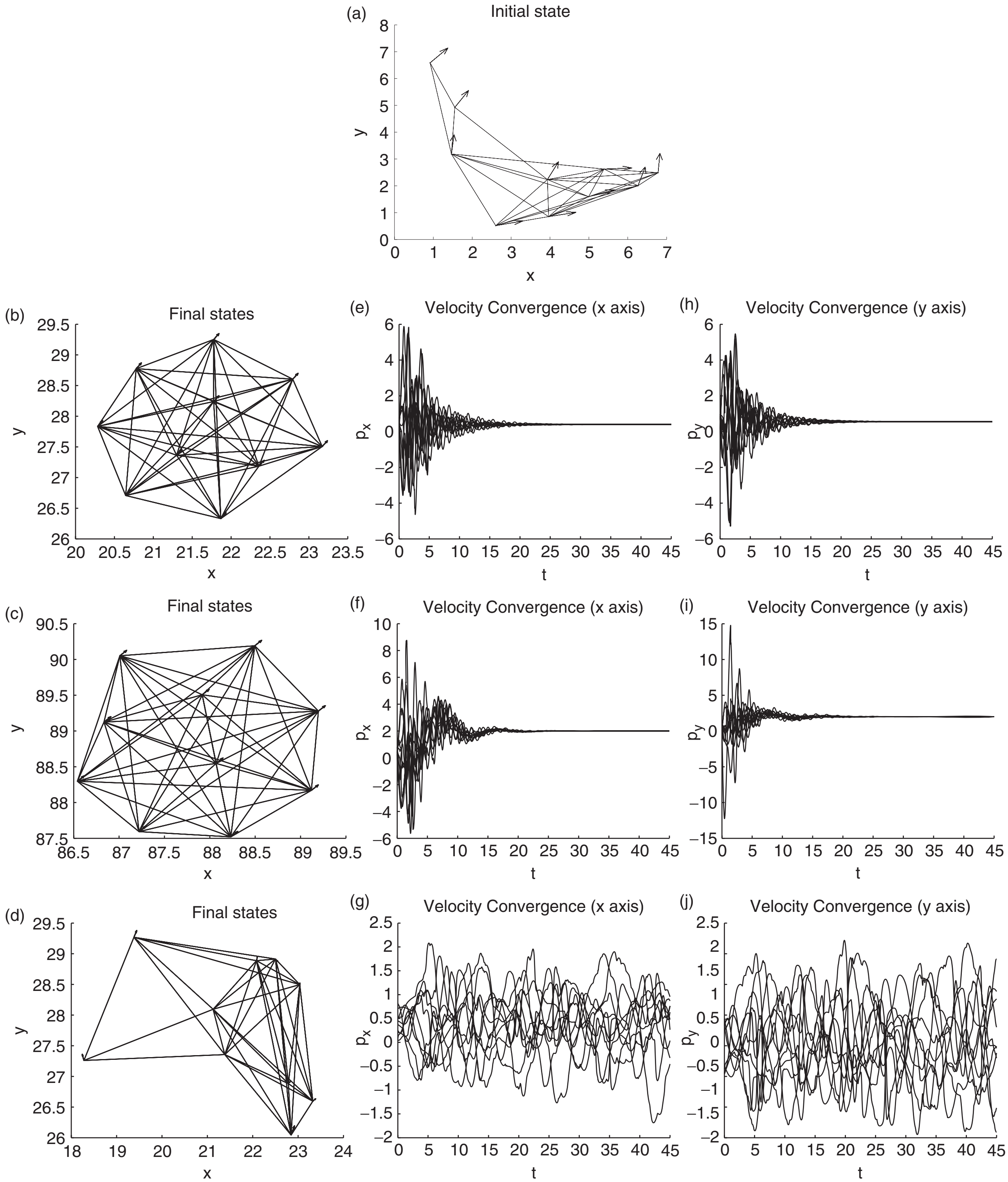

Figure 4. Flocking of 10 agents applying the algorithm (2), the algorithm (15) and the control inputs (28).

connectivity of the initial network cannot guarantee the connectivity of the network during the evolution by using the algorithm in Tanner et al. (2007).

If we directly use the algorithm (2) about without the estimate vector y to control a group of mobile autonomous agents, the control inputs can be simply written as follows:

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|), \quad i = 1, \ldots, N. \tag{28}$$

Under the same initial states of the 10 agents, simulation results for the flocking algorithm (2), the flocking algorithm (15) and the control inputs (28) are shown in Figure 4. The initial position and velocity of the virtual leader are set at $q_\gamma(0)=[0,0]^T$ and $p_\gamma(0)=[2,2]^T$. Figure 4 (a) shows the initial states of the agents; Figures 4(b)–(d) depict the configuration and velocities of the group of agents at $t=45$ obtained by the algorithm (2), the algorithm (15) and the control inputs (28), respectively; Figures 4(e)–(g) show the curves of velocities for the x-axis by the algorithm (2), the algorithm (28) and the control inputs (28), respectively; Figures 4(h)–(j) show the curves of velocities for the y-axis by the algorithm (2), the algorithm (15) and the control inputs (28), respectively. We can see that all agents reach a common velocity by using the flocking algorithm (2), and attain a desired velocity by using the flocking algorithm (15). However, without the estimate vector y, the agents may not achieve velocity consensus.

6. Conclusions

In this article, we have presented a connectivity-preserving flocking algorithm using only position measurements. Under the assumption that the initial network is connected, we have theoretically proved that the network remains connected all the time, all the agents asymptotically move with the same velocity, collision can be avoided between agents, and the final formation minimises all agents' global potentials. Moreover, we have investigated the flocking algorithm with a virtual leader and shown that all agents can asymptotically attain a desired velocity even if only one agent in the team has access to the information about the virtual leader. Future work will be to further investigate the proposed algorithms with bounded control inputs, and to analyse the robustness of the proposed algorithms against noise and time delays.

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