Real-Time Collision-Free Motion Planning of a Mobile Robot Using a Neural Dynamics-Based Approach

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***Abstract*—A neural dynamics based approach is proposed for real-time motion planning with obstacle avoidance of a mobile robot in a nonstationary environment. The dynamics of each neuron in the topologically organized neural network is characterized by a shunting equation or an additive equation. The real-time collision-free robot motion is planned through the dynamic neural activity landscape of the neural network without any learning procedures and without any local collision-checking procedures at each step of the robot movement. Therefore the model algorithm is computationally simple. There are only local connections among neurons. The computational complexity linearly depends on the neural network size. The stability of the proposed neural network system is proved by qualitative analysis and a Lyapunov stability theory. The effectiveness and efficiency of the proposed approach are demonstrated through simulation studies.**

***Index Terms*—Collision avoidance, dynamic environment, mobile robot, motion planning, neural dynamics, neural networks, real-time navigation.**

I. Introduction

REAL-TIME collision-free motion planning of mobile robots is one of the most important issues in robotics. A small and maneuverable mobile robot can be treated as a point robot, with the comparison of the size of the robot and its maneuvering possibilities with the size of free workspace. For example, in practice, cars in the traffic planning in large cities or tanks in field military operations can be treated as point robots. However, in many situations, e.g., when the size of the robot is comparable to the free space, or the length of the robot is obviously larger than its width (e.g., a rectangular robot), the robot should be considered with its shape and size. A holonomic mobile robot is a freely movable object with three degree-of-freedom (DOF), i.e., the translation along X and Y axes in the two-dimensional (2-D) Cartesian workspace, the orientation with respect to the base point of the robot. In many realistic robotic applications, since the wheels cannot slide, the mobile robot is nonholonomically constrained and is not freely movable objects. Because of the kinematic constraint, the d.o.f. of a nonholonomic mobile robot (also called "car-like" robot) becomes two. Due to the mechanical bound on the steering angle, the turning radius of a nonholonomic mobile robot is lower-bounded. In order to plan the motion path of a mobile robot (in this paper, a mobile robot is referred as a nonholonomic mobile robot, not a point robot), the control variables of the robot, the robot velocity and the curvature of the robot moving curve, should be discretized [1]–[3]. For example, for a given robot configuration, in Podsedkowski's model [2] there are at most six possible next configurations; in Kreczmer's model[3] there are at most ten possible next configurations.

There are many studies on motion planning of mobile robots using various approaches (e.g., [1]–[36]). Many previous models deal with static environments only, and need a local collision checking procedure at each step of the robot movement. For example, to detect local collisions, Barraquand and Lotombe's [5] model uses a divide-and-conquer technique; Zelinsky's [13] model uses a hierarchical collision-testing procedure based on "distance space bubbles."

Most previous models use two-step approaches that consist of first computing a collision-free holonomic path, and then transforming this path by a sequence of feasible ones. The quality of the solution and the computational cost of the second step depend on the shape of the holonomic path. For example, Moutarlier *et al.* [19] proposed a model for planning the shortest path in configuration space of a mobile robot, which is based on a Lagrange method for optimizing a function. Paromtchik and Laugier [20] proposed an iterative algorithm for motion generation for parking a mobile robot. Khatib *et al.*'s [24] motion planning model for mobile robots uses a bubble method to find the locally reachable space and a parameterization method to satisfy the kinematic constraint. Jiang *et al.* [23] proposed a time-optimal motion planning method for a robot with kinematic constraints, which consists of three stages: first planning for a point robot; then planning for a robot with size and shape; and finally optimizing cost functions for a time-optimal solution. Podsedkowski [2] proposed a path planner for a nonholonomic mobile robot using a search based algorithm, which requires a local collision-checking procedure and the minimization of cost functions. Sekhavat *et al.* [34] proposed a multi-level approach to motion planning of a nonholonomic mobile robot, where at the first level, a path is found that disrespects the nonholonomic constraints; at each of the next levels, a new path is generated by transformation of the path generated at the previous level; at the final level, all nonholonomic constraints are respected. Ong and Gilbert [31] proposed a new search based model for path

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planning with penetration growth distance, which searches over the collision paths instead of searching over the free workspace as most other models.

There are some learning based models for motion planning of mobile robots. For example, Gambardella and Versino [9] proposed a learning method for path planning of a robot in a cluttered workspace where the dynamic local minima can be detected. Through learning it can avoid the local minima, such as in deadlock situations. Svestka and Overmars [27] proposed a probabilistic learning approach to motion planning of a mobile robot, which involves a learning phase and a query phase and uses a local method to compute feasible paths for the robot. Fujii *et al.* [37] proposed a multilayer reinforcement learning model for path planning of multiple mobile robots. However, the planned robot motion using learning based approaches is not efficient and is computational expensive, especially in its initial learning phase.

In this paper, a novel neural dynamics based approach is proposed for real-time collision-free motion planning of a *nonholonomic mobile robot* in a nonstationary environment, which is based on the basic concept of the neural network models in [38]–[41] for path planning of a *point robot*, but with a significantly different neural network design and a path-planning algorithm. The proposed neural network is topologically organized. Unlike the models in [38]–[41] where the state space is the 2-D Cartesian workspace, the state space of the proposed neural network is the 3-D configuration space of the mobile robot. The dynamics of each neuron is characterized by a shunting equation or a simple additive equation, which is derived from Hodgkin and Huxley's [42] membrane equation. In the proposed model, with a proper design of external inputs and internal neural connections in the neural network, the target *globally* attracts the robot in the entire state space through neural activity propagation, while the obstacles *locally* push the robot away to avoid possible collisions. In contrast to the models in [38]–[41] where neural connections are a function of distance only and the neural activity propagation is *onmi-directional*, in the proposed model the neural connections are a function of both distance *and* direction that is subject to the nonholonomic constraint, and the neural activity propagation is *directionally selective*. Each neuron has only local and excitatory lateral connections to its neighboring neurons in the neural network. Therefore, the computational complexity linearly depends on the neural network size, the number of neurons that depends on the discretization of the workspace. In addition, unlike some previous models such as [4], the proposed model is not sensitive to any irrelevant obstacles nor sensor noise (for details, see [43]).

In the proposed model, the dynamically varying environment is represented by the dynamic neural activity landscape, which is formed by the neural activities of all the neurons in the topically organized neural network. The real-time robot motion is directly planned through the dynamic activity landscape of the neural network without any *prior* knowledge of the dynamic environment. For this purpose the proposed method needs the complete knowledge of the current environment. In particular, it is assumed that the locations of the robot, obstacles, and target are all completely known through various sensor measurements (multisensory fusion to obtain the environment knowledge is beyond the scope of this paper. A related study with limited sensory information in a completely unknown environment can be found in [44]). Knowing the locations of obstacles and the robot in the environment at every time instant involves the research and application of map building and localization methods, respectively.

Unlike some previous search based models (e.g., [2], [3], [5], [13], [31], and [45]–[53]) whose robot motion is planned by searching over either the free workspace or the collision paths, there are no explicit *global* searching procedures to find a possible path in the proposed neural model. Note there is a *local* searching described in (13) to find the next robot position among its at most six possible positions. In contrast to some previous models where the optimal robot motion is generated by optimizing some global cost functions, there is no explicit optimization of any global cost functions in the proposed model, although there is a local selection of the maximum neural activity to find the next robot location. The optimal robot motion results from the nature of the neural network design. Distinct from some previous neural network based motion-planning models, no learning procedures are needed in the proposed model. The neural connection weights are constants predefined at the stage of neural network design, which can be any monotonically decreasing function of the distance. Thus it is very easy to choose the neural connection weights that are good enough for the robot path planning. It is not necessary to add a learning procedure to obtain the best or optimal neural connection weights, which will result in an increase in algorithm complexity and computational cost. The geometric shape and the kinematic constraint of the mobile robot are also included at the stage of neural network design. Unlike some previous motion-planning models for mobile robots with size and shape, no local collision-checking procedures are needed at each step of the robot movement. Therefore, the model algorithm is computationally simple. The planned robot path in a static environment is globally optimal in the sense of the least steps from the starting location to the target if it exists (if there are more than one shortest paths due to the discretization and symmetry, the model gives one of them). The optimality in the real-time motion planning in a nonstationary environment is in the sense of a continuous, smooth path toward the target. The term "real-time" is in the sense that the robot motion planner responds immediately to the dynamic environment, including the robot, target, obstacles and sensor noise. To the best of our knowledge, it is the first time that the real-time motion planning of a nonholonomic mobile robot is studied using a nonlearning based neural dynamics based approach.

This paper is organized as follows. In Section II, the mathematical model of a nonholonomic mobile robot is first briefly introduced, and the biological inspiration, model algorithm and stability analysis of the proposed neural dynamics based approach to real-time collision-free motion planning of the mobile robot are presented. Then, simulation studies in both static and dynamic environments are presented in Section III, including motion planning for parallel parking, motion planning in a complex house-like environment, and motion planning with sudden environmental changes. In Section IV, the parameter sensitivity and a variation of the proposed model are discussed. A conclusion noticing several feature properties of the proposed neural dynamics based model is addressed in Section V.

II. Model

In this section, the mathematical model of a nonholonomic mobile robot is briefly introduced. Then, the proposed neural dynamics based model for real-time collision-free motion planning of a mobile robot is presented, including the originality, motion planning algorithm and stability analysis.

A. Mobile Robot Model

For a mobile robot with size and shape, its location in the 2-D Cartesian workspace $\mathcal{W}$ can be *uniquely* determined by the spatial position (x, y) of the base point and the orientation angle θ with respect to the base point [see Fig. 1(a)]. Thus, a robot location in $\mathcal{W}$, also called a robot configuration, uniquely corresponds to a point (x, y ,θ) in the robot configuration space $\mathcal{C}$. The kinematic constraint of a nonholonomic mobile robot is described as [1], [2]

$$-\dot{x}\sin\theta + \dot{y}\cos\theta = 0. \tag{1}$$

Equation (1) can be parameterized by time t. Given the robot velocity v and the curvature K of the curve followed by the robot, the so called *control variables*, the velocity parameters of the robot are given by

$$\begin{aligned}\dot{x} &= v\cos\theta\\ \dot{y} &= v\sin\theta\\ \dot{\theta} &= vK\end{aligned} \tag{2}$$

where the control variables v and K are limited to $v_{\max}$ and $K_{\max}$, respectively, i.e.,

$$|K| \le K_{\max} \quad \text{and} \quad |v| \le v_{\max}. \tag{3}$$

Thus, the control space may be denoted as the Cartesian product of two intervals, $[-v_{\max}, v_{\max}] \times [-K_{\max}, K_{\max}]$. The minimum turning radius R is given by $R = 1/K_{\max}$. When planning the robot motion in a discretized workspace, the control variables, v and K, should be discretized as well [1]–[3]. For simplicity but without losing generality, a given present robot configuration has at most six possible next robot configurations by setting the v and K according to one of the six elements of the following Cartesian product:

$$\{-v_0, v_0\} \times \{-K_{\max}, 0, K_{\max}\} \tag{4}$$

where v_0 is the robot moving velocity [1], [2]. Such a discretization is used to generate the robot movement (one step). After the integration over the time interval of one step, the next robot position is given by

$$\begin{aligned}\theta(t+\Delta t) &= \theta(t) + vK\Delta t\\ x(t+\Delta t) &= x(t) + \frac{1}{K}\left(\sin\theta(t+\Delta t) - \sin\theta(t)\right)\\ y(t+\Delta t) &= y(t) - \frac{1}{K}\left(\cos\theta(t+\Delta t) - \cos\theta(t)\right)\end{aligned} \tag{5}$$

where Δt is the time interval of one step. Fig. 1(b) shows an example of the possible next robot configurations of a given robot configuration. Please note that in realistic robot motion planning, the length of one step of robot movement is significantly smaller, so the next robot configuration partially overlaps themselves. Similar to other methods that require discretization of the workspace, a smaller discretization step results in a smoother robot path and more computational cost.

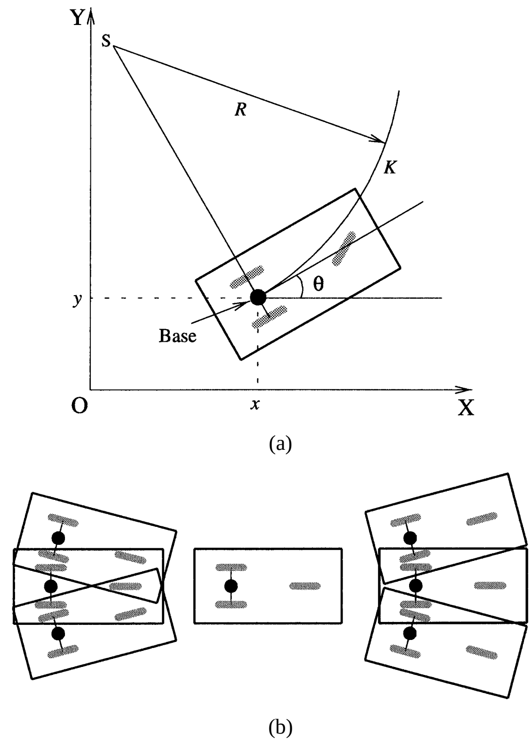

Fig. 1. Schematic diagram for a nonholonomic mobile robot. (a) Kinematics of a mobile robot, where (x, y) is the base point, θ is the orientation angle, R is the turning radius, and K is the curvature of curve followed by the robot. (b) Six possible next robot configurations of a given robot configuration.

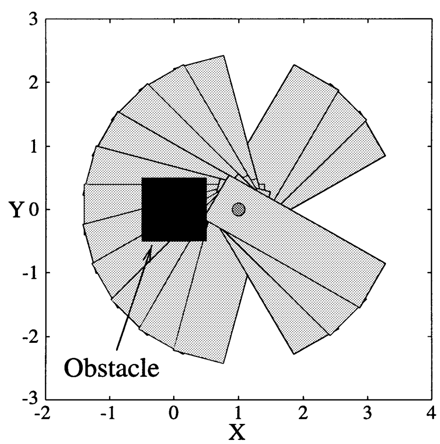

Fig. 2. Forbidden robot locations in a discretized workspace when the obstacle and the robot base are at (0,0) and (1,0), respectively.

For a mobile robot, if there is an obstacle in the discretized 2-D Cartesian workspace $\mathcal{W}$, several robot configurations in $\mathcal{C}$ are not allowed (“forbidden”, called the obstacle configurations in $\mathcal{C}$). For example, when the obstacle and the robot base point are at Positions (0,0) and (1,0) in $\mathcal{W}$, respectively, the forbidden robot configurations are shown in Fig. 2, where the obstacle is shown by a filled square, while each robot configuration is shown by light solid rectangular with its base point represented by a filled circle. The map from an obstacle in $\mathcal{W}$ to the forbidden configurations in $\mathcal{C}$ is exactly the same for all obstacles in the discretized workspace. Thus, it can be done *automatically* once an obstacle appears in the workspace.

B. Neural Dynamics-Based Model

The proposed neural dynamics based approach to real-time collision-free motion planning is motivated by the neural dynamics behavior and computation in biological neural systems. The fundamental concepts of the proposed model is to develop a neural network architecture, whose dynamic neural activity landscape represents the dynamically varying environment. By properly defining the external inputs from the varying environment and internal neural connections, the target and obstacles are guaranteed to stay at the peak and valley of the activity landscape of the neural network, respectively. The target *globally* attracts the robot in the whole state space through neural activity propagation, while the obstacles have only *local* effect to avoid collisions. The neural activity propagation is subject to the kinematic constraint for a nonholonomic mobile robot. The real-time collision-free robot motion is planned through the dynamic activity landscape of the neural network, also subject to the robot kinematic constraint.

1) Originality: In 1952, Hodgkin and Huxley [42] proposed a computational model for a patch of membrane in a biological neural system using electrical circuit elements. This modeling work together with other experimental work led them to a Nobel prize in 1963, for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane. In their membrane model, the dynamics of the voltage across the membrane V_m is described using a state equation technique as

$$C_m \frac{dV_m}{dt} = -(E_p + V_m)g_p + (E_{Na} - V_m)g_{Na} - (E_K + V_m)g_K \tag{6}$$

where C_m is the membrane capacitance, E_K, E_{Na} and E_p are the Nernst potentials (saturation potentials) for potassium ions, sodium ions and passive leak current in the membrane, respectively. Parameters g_K, g_{Na} and g_p represent the conductances of the potassium, sodium, and passive channels, respectively. This model provided the foundation of the shunting model and led to a lot of model variations and applications [43], [54], [55].

By setting $C_m = 1$ and substituting $\xi_i = E_p + V_m$, $A = g_p$, $B = E_{Na} + E_p$, $D = E_k - E_p$, $S_i^e = g_{Na}$, and $S_i^i = g_K$ in (6), a shunting equation is obtained [56]

$$\frac{d\xi_i}{dt} = -A\xi_i + (B - \xi_i)S_i^e(t) - (D + \xi_i)S_i^i(t) \tag{7}$$

where ξ_i is the neural activity (membrane potential) of the ith neuron. Parameters A, B, and D are nonnegative constants representing the passive decay rate, the upper and lower bounds of the neural activity, respectively. Variables S_i^e and S_i^i are the excitatory and inhibitory inputs to the neuron. This shunting model was first proposed by Grossberg to understand the real-time adaptive behavior of individuals to complex and dynamic environmental contingencies [55], [57]–[59], and has many applications in visual perception, sensory motor control, and many other areas (e.g., [55], [56], [58], and [60]–[63]).

2) Model Algorithm: The neural network architecture of the proposed model is a discrete topographically organized map. The state space of the neural network is the configuration space of the mobile robot. Thus the state space $\mathcal{S}$ of the neural network is in 3-D, where two of the coordinates represent the spatial position in the 2-D Cartesian workspace and the third coordinate represents the orientation of the robot with respect to the base point, i.e., the state space $\mathcal{S}$ is the configuration space $\mathcal{C}$ of the mobile robot. The location of the ith neuron at the grid in $\mathcal{S}$, denoted by a vector $p_i \in R^3$, uniquely represents a configuration in $\mathcal{C}$, or a location in $\mathcal{W}$. The dynamics of each neuron in the network is characterized by a shunting equation derived from (7). Each neuron has a local lateral connections to its neighboring neurons that constitute a subset $\mathcal{R}_i$ in $\mathcal{S}$. The subset $\mathcal{R}_i$ is called the receptive field of the ith neuron in neurophysiology. The neuron responds only to the stimulus within its receptive field.

In the proposed neural network model, the excitatory input, S_i^e in (7), results from the target and its neighboring neurons, where the connections among the neighboring neurons are directionally selective due to the nonholonomic robot kinematics, distinct from the omni-directional connections in the neural network models for a point robot in [38]–[41], [44]. The inhibitory input S_i^i results from the obstacles only. Thus the dynamics of the ith neuron in the neural network is characterized by a shunting equation as

$$\frac{d\xi_i}{dt} = -A\xi_i + (B - \xi_i)\left([I_i]^+ + \sum_{j=1}^{N} w_{ij}[\xi_j]^+\right) - (D + \xi_i)[I_i]^-. \tag{8}$$

where N is the total number of neurons in the neural network. The terms $[I_i]^+ + \sum_{j=1}^{N} w_{ij}[\xi_j]^+$ and $[I_i]^-$ are the excitatory and inhibitory inputs, S_i^e and S_i^i in (7), respectively. The external input I_i to the ith neuron is defined as

$$I_i = \begin{cases} E, & \text{if there is a target} \\ -E, & \text{if there is an obstacle} \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $E \gg B$ is a very large positive constant. From (8), it shows that the proposed model requires the *current* complete knowledge, $I_i(t)$, $i = 1, 2, \ldots, N$, of the dynamic environment, which can be obtained from the various sensors [44]. Function $[a]^+$ is a linear-above-threshold function defined as $[a]^+ = \max\{a, 0\}$, and the nonlinear function $[a]^-$ is defined as $[a]^- = \max\{-a, 0\}$.

Inspired by the biological fact that a neuron in a neural system responds only to the stimulus within its receptive field, in the proposed model the ith neuron has only excitatory connections to neurons in a small local region $\mathcal{R}_i$. The lateral connection weight, w_{ij}, from the jth neuron to the ith neuron are defined as

$$w_{ij} = \begin{cases} f(d_{ij}), & \text{if Neurons } j \text{ and } i \text{ are neighbors} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

i.e., the connection weight w_{ij} is not zero if and only if that the jth and ith neurons are neighboring neurons (neighbors), where their locations and orientations, (x, y, θ), in the configuration space $\mathcal{C}$ satisfy the nonholonomic kinematics described in (2) or (5). Variable $d_{ij} = |p_i - p_j|$ is the Euclidean distance between positions p_j and p_i in $\mathcal{S}$. Function $f(d_{ij})$ is a monotonically decreasing function, e.g., a function defined as

$$f(d_{ij}) = \frac{\mu}{d_{ij}} \tag{11}$$

where μ and r_0 are positive constants. Due to the kinematic constraint in (1), the neural connection weight w_{ij} is not only a function of distance d_{ij}, but also a function of the robot orientation θ. Thus, for a mobile robot, each neuron has a maximum of six lateral connections. Unlike the neural network models for a point robot in [38]–[41], [44] where the lateral neural connections are omni-directional, the neural connection for a nonholonomic mobile robot has directional selectivity. Note that the neural connection weight w_{ij} defined in (10) is based on the fundamental concepts of the proposed approach, which guarantees that the robot moves toward the target subject to the nonholonomic kinematics. An example illustrating all the neighboring configurations (next robot locations) of a given robot configuration is shown in Fig. 1(b). All the neurons having lateral connection to the ith neuron are defined as its *neighboring neurons*. Obviously the lateral neural connection weights are symmetric, $w_{ij} = w_{ji}$, i.e., the connection weight from the jth to the ith neuron is equal to the weight from the ith to the jth neuron. Therefore, the dynamics of the ith neuron can be further written as

$$\frac{d\xi_i}{dt} = -A\xi_i + (B-\xi_i)\left([I_i]^+ + \sum_{j=1}^{k} w_{ij}[\xi_j]^+\right) - (D+\xi_i)[I_i]^- \tag{12}$$

where k is the number of neighboring neurons of the ith neuron.

In the proposed neural network model characterized by (12), the design of lateral neural connections guarantees that only the positive neural activity can propagate to the whole state space. The negative neural activity stays locally only. In addition, the design of the external input from the environment guarantees that the target globally influences the whole state space to attract the robot through neural activity propagation, while the obstacles have only local effect to avoid collisions. The activity propagation is subject to the kinematic constraint described in (1). The locations of the target and obstacles may vary with time. The activity landscape of the neural network dynamically changes due to the varying external inputs from the target and obstacles and the internal activity propagation among neurons. The robot motion is planned from the dynamic activity landscape by a steepest gradient ascent rule. For a given present robot location in $\mathcal{S}$ (i.e., a location in $\mathcal{W}$ or a configuration in $\mathcal{C}$), denoted by p_p, the next robot location p_n (also called "command location") is obtained by

$$p_n \quad \Leftarrow \quad \xi_{p_n} = \max\{\xi_j, j = 1, 2, \ldots, k\} \tag{13}$$

where k is the number of *neighboring neurons* of the p_pth neuron, i.e., all the possible next locations of the present location p_p. Because of the definition of neighboring neurons, the found next position p_n satisfies the nonholonomic kinematics in (5). After the present location reaches its next location, the next location becomes a new present location (if the found next location is the same as the present location, the robot stays there without any movement). The current robot location *adaptively* changes according to the varying environment. Note that the shape and size of the robot, and the nonholonomic constraint of the robot kinematics are considered at the very beginning stage of neural network design, thus no local collision-detecting procedures are needed at each step of robot movement during the real-time robot navigation.

The dynamic activity landscape of the topologically organized neural network is used to determine the next robot location. The robot movement is also determined by the robot speed when the next location is available from the current activity landscape. However, when the next location is not immediately available, e.g., in a deadlock situation, the robot has to wait until the next location toward the target is available from the neural activity landscape. Whenever the neural activity at the current robot location is smaller than the largest neural activity of its neighbors, the robot starts to move to its next location. Thus the robot movement is determined by both the robot speed and the neural activity landscape. In a dynamic environment, the neural activity landscape will never reach a steady state as in a static environment. In a fast changing environment, e.g., where obstacles suddenly appear in front of the robot, the neural activities at those locations will immediately reduce to a very large negative value due to their very large inhibitory input. Thus the robot should be able to avoid those suddenly appearing obstacles. In the proposed model, due to the very large external input constant E, the target and obstacles keep staying at the peak and valley of the activity landscape of the neural network, respectively. The robot keeps moving toward an unclean area with obstacle avoidance until the designated objective is achieved or it is forced to stop by a human operator.

3) Stability Analysis: The proposed neural network is a stable system. The neural activity in the shunting equation is bounded in the finite interval $[-D, B]$. In the shunting model in (7) or (12), the neural activity ξ_i increases at a rate of $(B - \xi_i)S_i^e$, which is not only proportional to the excitatory input S_i^e, but also proportional to an *auto gain control* term $(B - \xi_i)$. Thus, with an equal amount of input S_i^e, the closer value of ξ_i and B, the slower ξ_i increases. When the activity ξ_i is below B, the excitatory term is positive causing an increase in the neural activity. If ξ_i is equal to B, the excitatory term becomes zero and ξ_i will no longer increase no matter how strong the excitatory input is. In case the activity ξ_i exceeds B, $(B - \xi_i)$ becomes negative and the shunting term pulls ξ_i back to B. Therefore, ξ_i is forced to stay below B, the upper bound of the neural activity. Similarly, the inhibitory term forces the neural activity to stay above the lower bound $-D$. Therefore, once the neural activity goes into the finite region $[-D, B]$, it is guaranteed that the neural activity will eventually stay in this region for any value of the total excitatory and inhibitory inputs [63].

In addition, the stability and convergence of the proposed model can be rigorously proved using the Lyapunov stability theory. Introducing the new variables, $\eta_i = \xi_i - B$, the pro-

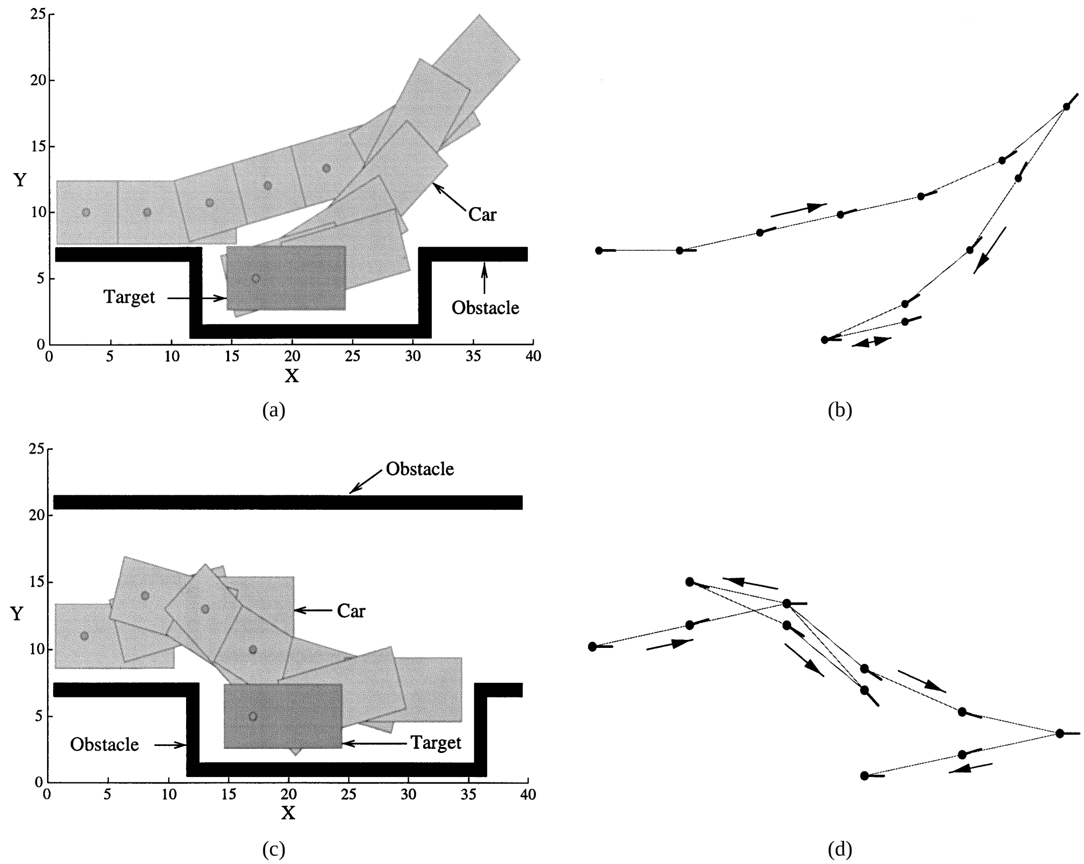

Fig. 3. Motion planning of a nonholonomic car in parallel parking problem. (a) and (b) Parallel parking in a wide area. (c) and (d) Parallel parking in a narrow road.

posed model in (8) or (12) can be written into Grossberg's general form [55], [59],

$$\frac{d\eta_i}{dt} = a_i(\eta_i)\left(b_i(\eta_i) - \sum_{j=1}^{N} c_{ij} d_j(\eta_j)\right) \tag{14}$$

by the following substitutions:

$$a_i(\eta_i) = -\,\eta_i \tag{15}$$

$$b_i(\eta_i) = \frac{1}{\eta_i}\left(AB + \eta_i\left(A + [I_i]^+ + [I_i]^-\right) + (B+D)[I_i]^-\right) \tag{16}$$

$$c_{ij} = -\,w_{ij} \tag{17}$$

$$d_j(\eta_j) = [\eta_j + B]^+. \tag{18}$$

Since the neural connection weight is symmetric, $w_{ij} = w_{ji}$, then $c_{ij} = c_{ji}$ (symmetry). Since η_i varies within the finite interval $[-B-D, 0]$, where B and D are nonnegative constants, then η_i is a nonpositive number. Hence the amplification function $a_i(\eta_i)$ is nonnegative, i.e., $a_i(\eta_i) \geq 0$ (nonnegativity). From the definition of function $[a]^+ = \max\{a, 0\}$, i.e., $[a]^+ = a$ at $a \geq 0$; $[a]^+ = 0$ at $a < 0$, thus $d_j(\eta_j)$ in (18) has $d_j'(\eta_j) = 1$ at $\eta_j \geq -B$ and $d_j'(\eta_j) = 0$ at $\eta_j < -B$. Hence, the signal function $d_j(\eta_j)$ has a nonnegative derivation, i.e., $d_j'(\eta_j) \geq 0$ (monotonicity). Therefore, (12) satisfies all the three stability conditions required by Grossberg's general form [55], [59]. The Lyapunov function candidate for (14) can be chosen as

$$L = -\sum_{i=1}^{N} \int^{\eta_i} b_i(\xi_i) d_i'(\xi_i) d\xi_i + \frac{1}{2} \sum_{j,k=1}^{N} c_{jk} d_j(\eta_j) d_k(\eta_k). \tag{19}$$

The time derivative of L along all the trajectories is given by

$$\frac{dL}{dt} = -\sum_{i=1}^{N} a_i d_i' \left(b_i - \sum_{j=1}^{N} c_{ij} d_j\right)^2. \tag{20}$$

Since $a_i \geq 0$ and $d_i' \geq 0$, then $dL/dt \leq 0$ along all the trajectories. The rigorous proof of the stability and convergence of (14) can be found in [59]. Therefore, the proposed neural network system is stable. The dynamics of the network is guaranteed to converge to an equilibrium state of the system.

III. Simulation Studies

The proposed neural network model is capable of planning real-time motion path with obstacle avoidance for a mobile robot in a nonstationary environment. In this section, several simulations are carried out to demonstrate the effectiveness of the proposed neural network approach. First, the motion planning for parallel parking of a nonholonomic mobile robot is studied. Then the navigation of a mobile robot in a complex house-like environment is investigated. Finally, the proposed model is applied to real-time collision-free motion planning of a mobile robot with sudden environment changes.

A. Motion Planning for Parallel Parking

The proposed neural network model is first applied to the famous parallel parking problem. Motion planning for parallel parking of a nonholonomic car-like robot under two situations are studied. The neural network has $40 \times 30 \times 24$ topographically ordered neurons with initial neural activities at a value of zero, where 40 $\times$ 30 represents the discretized 2-D workspace at a size of 40 $\times$ 30 (arbitrary units), and 24 represents the number of the orientation angles from 0° to 345° with a step of 15°. In practice, the discretization of state space could be much smaller, depending on the application requirements and computational resource available. Because of geometrically $360° = 0°$, Neuron (x_i, y_i, 0) in the topologically organized neural network is identical to Neuron (x_i, y_i, 24) and is an immediate maneuverable neuron of Neuron (x_i, y_i, 23), and likewise. The model parameters are chosen as: $A = 10$ and $B = D = 1$ for the shunting equation, $\mu = 1$ and $r_0 = 6$ for the lateral connections, and $E = 100$ for the external input. The minimum turning radius of the car is $R = 20$, i.e., the maximum curvature of the curve followed by the car is $K_{\max} = 1/R = 0.05$. On a wide road, the planned motion to park the car is shown in Fig. 3(a), and the dynamic trace of the base point and orientation of the car is shown in Fig. 3(b). It shows that the car first moves forward to the left side, then turns backward to park the car at the target location without any collisions. Note that at the end after the robot base point reaches the target base point but with a different orientation from the target orientation, the car has to move left-forward and move right-backward in order to reach the correct target orientation.

In case of a narrow road as shown in Fig. 3(c), the car has to turn back and forth several times, and eventually the car is also able to properly park at the target location with obstacle avoidance. The real-time robot motion is shown in Fig. 3(c), and the dynamic trace of base point and orientation of the car is shown in Fig. 3(d). It shows that at the third step the car moves left-backward, which is in the direction away from the target direction and toward the upper obstacles. Because of the nonholonomic constraint, the existing robot path by moving left-backward at the third step is shorter than the paths with a forward movement at the third step. The robot motion planning is generated automatically by the path planner without any guidance from operators and without any learning procedures.

B. Motion Planning in a House-Like Environment

Motion planning of a nonholonomic mobile robot in a complex house-like environment is then studied, where there are several deadlock situations that the robot may be trapped in. Fig. 4 shows a house-like environment, where the doors can be opened or closed. The neural network has $90 \times 90 \times 24$ topologically organized neurons, and the model parameters are chosen as the same as in previous case, i.e., $A = 10$, $B = D = 1$, $\mu = 1$, $r_0 = 6$ and $E = 100$. In the case that Door L is opened, the planned robot motion is shown in Fig. 4(a), where the robot moves to the target along the shortest path. When Door L is closed, the planned robot motion is shown in Fig. 4(b). The robot has to travel a much longer path to reach the target. Note there are no learning procedures. The robot is capable of reaching the target along the shortest path without any collisions, without violating the kinematic constraint, and without being trapped in any deadlock situations. The planned path in a static environment is capable of generating the shortest path from starting location to the target, which results from the fact that the positive neural activity propagation from the target to the current robot location is alway along the shortest path available from the target to the current robot location.

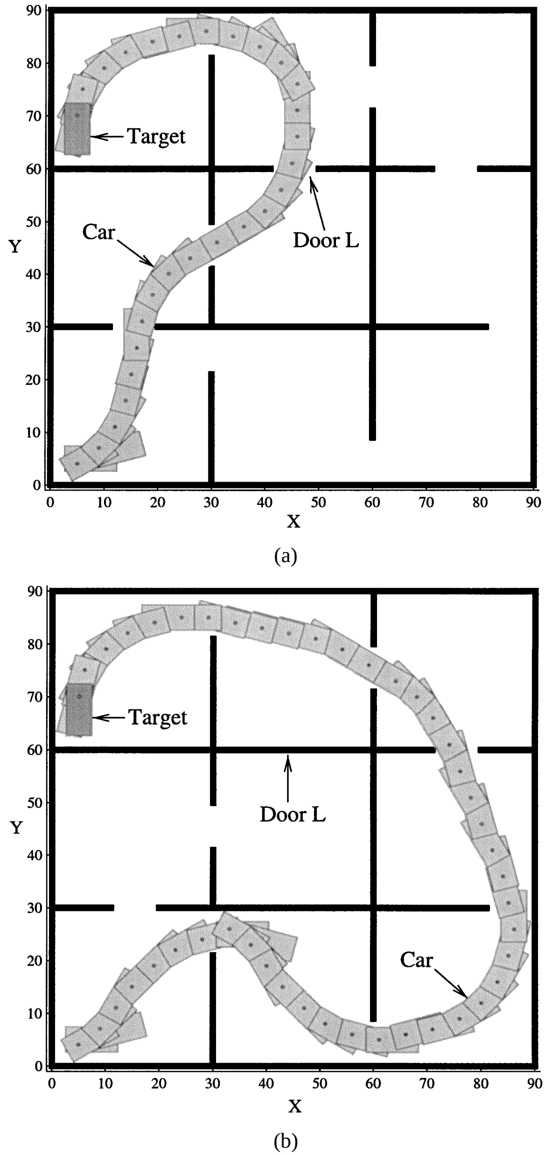

Fig. 4. Motion planning of a nonholonomic mobile robot in a house-like environment with several deadlock situations. (a) Robot motion when door L is opened. (b) When door L is closed.

C. Motion Planning With Sudden Environmental Changes

The proposed neural network model can perform properly in a dynamic environment, even with sudden environmental changes, such as suddenly adding or removing obstacles. A case with sudden placement of obstacles in front of a nonholonomic car is studied. The neural network has $50 \times 30 \times 24$ neurons with the same model parameters as in previous cases.

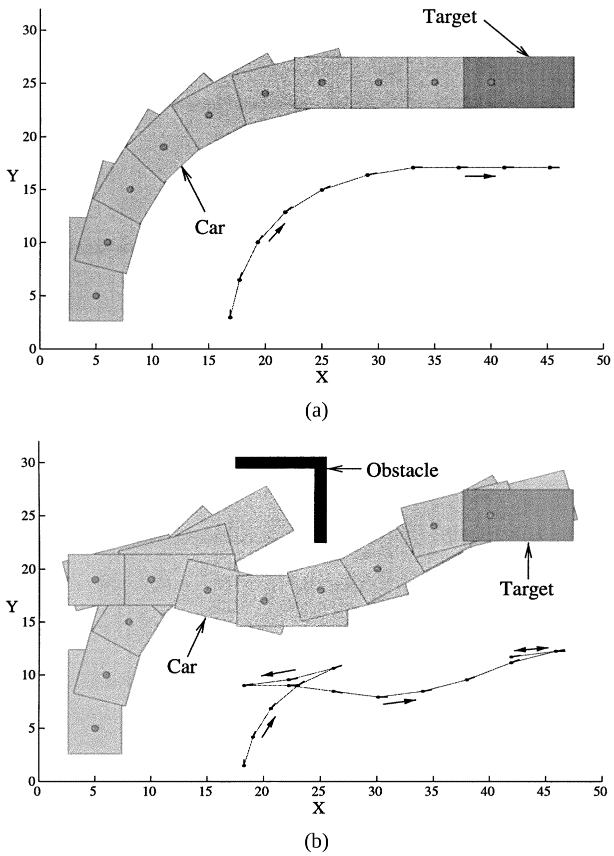

Fig. 5. Real-time planning of a nonholonomic mobile robot with sudden placement of obstacles. (a) Planned robot motion in case of no obstacles. (b) Real-time motion in case that V-shaped obstacles suddenly are placed in front of the car.

The initial and target locations of the car are at Locations (5,5,6) and (40,25,0) in $\mathcal{W}$, respectively. In the first case with no obstacles in the workspace, the robot motion path is shown in Fig. 5(a), where the inset shows the dynamic trace of the robot base point and its orientation. In the second case with the same initial condition, but when the car reaches (15,22,2) on the way toward the target, the obstacles (V-shaped, shown in Fig. 5(b) with filled rectangles) are *suddenly* placed in front of the car. The real-time robot motion is shown in Fig. 5(b), where the inset shows the dynamic trace of the robot base point and orientation. It shows that the car first has to move *away* from the target after the appearance of the obstacles, then passes around these obstacles, and finally reaches the target without any collisions, satisfying the kinematic constraint.

IV. Discussion

In this section, the parameter sensitivity of the proposed neural network model is briefly discussed. Then a simple additive model is proposed by lumping together the excitatory and inhibitory inputs and removing the auto gain control terms in the proposed shunting model.

A. Parameter Sensitivity

Parameter sensitivity is a very important factor to be considered when a model is proposed or evaluated. An acceptable model should be robust, i.e., not very sensitive to changes in its parameter values. There are only few parameters in our proposed model in (12). The upper and lower activity bounds B and D, the receptive field parameter r_0, and external input constant E are not important factors, because the real-time robot motion is generated by the relative values of the neural activities as described in (13) (for details, see [43]). The passive decay rate A determines the transient response of the neurons, which is very important for the model dynamics, particularly when the locations of target and obstacles are varying fast. The lateral connection weight parameter μ is also an important factor, which determines the propagation of the neural activity in the neural network. A detailed discussion of parameters A and μ through descriptive analysis and simulation study can be found in [41], [43].

The proposed models are not very sensitive to the variations of model parameters and the connection weight function. The parameters can be chosen in a very wide range, e.g., all the case studies in this paper use the same model parameters. The weight function can be any monotonically decreasing function. Thus in the proposed model there is no need to use any learning algorithms to find the neural connection weights. A detailed study of the parameter sensitivity of a shunting equation can be found in [43] and [63].

B. Model Variation

If the excitatory and inhibitory terms in the shunting model in (12) are lumped together and the auto gain control terms are removed, then (12) can be written into a simpler form as

$$\frac{d\xi_i}{dt} = -A\xi_i + I_i + \sum_{j=1}^{k} w_{ij}[\xi_j]^+. \tag{21}$$

This is an additive equation [58], which is widely applied to a lot of areas such as vision, associative pattern learning and pattern recognition [55], [58]. The term $I_i + \sum_{j=1}^{k} w_{ij}[\xi_j]^+$ represents the total input to the ith neuron from the external input and lateral connections. The definitions of I_i, $[a]^+$, w_{ij} and k are defined the same as those in (12). The nonlinear function $[a]^+$ guarantees that only the positive neural activity can propagate to the other neurons. In addition, because of the very large external input constant $E \gg B$, the target and obstacles are guaranteed to stay at the peak and valley of the dynamic neural activity landscape, respectively. There are only local lateral neural connections as well, because of the same definition of neighboring neurons. The kinematic constraint is also respected by the definition of w_{ij}. Therefore, this simple additive model also follows the fundamental concept of the proposed approach to real-time collision-free motion planning of a nonholonomic mobile robot. The procedure to plan the real-time robot motion is the same as in (13). This additive model is also capable of planning real-time motion with obstacle avoidance for a mobile robot.

Unlike the shunting model whose neural activity is bounded, the neural activity characterized by the additive model in (21) does not have any bounds. However, it is easy to prove that this additive neural network model is a stable system using the Lyapunov stability theory. Equation (21) can be rewritten into the Grossberg's general form in (14) by substituting

$$\begin{aligned} a_i(\xi_i) &= 1 \\ b_i(\xi_i) &= -A\xi_i + I_i \\ c_{ij} &= -w_{ij} \\ d_j(\xi_j) &= [\xi_j]^+. \end{aligned} \tag{22}$$

Obviously, have $c_{ij} = c_{ji}$ (symmetry), $a_i(\xi_i) \geq 0$ (nonnegativity), and $d'_j(\xi_j) \geq 0$ (monotonicity). Thus (21) satisfies all the three stability conditions required by the Grossberg's general form in (14) [55], [59]. Therefore, this additive neural network system is stable.

There are some important differences between the shunting model in (12) and the additive model in (21), although the additive model is computationally simpler and can also plan real-time robot motion with obstacle avoidance. First, by rewriting the shunting and additive models into the Grossberg's general form in (14), unlike the additive model with a constant $a_i(\xi_i) = 1$ and a linear function $b_i(\xi_i)$, for the shunting model the amplification function $a_i(\eta_i)$ in (15) is not a constant, and the self-signal function $b_i(\eta_i)$ in (16) is nonlinear. Second, the shunting model has two *auto gain control* terms, $(B - \xi_i)$ and $(D + \xi_i)$, which result in that the dynamics of (12) remains sensitive to input fluctuations [55]. Such a property is important for the real-time robot motion planning when the target and obstacles are varying fast. In contrast, the dynamics of the additive equation may saturate in some situations [55]. Third, the activity of the shunting model is bounded in the finite interval $[-D, B]$, while the activity in the additive model does not have any bounds. A detailed analysis of the shunting model and the additive model can be found in [43], [55], [63].

To demonstrate the effectiveness, the simple additive model in (21) is applied to the motion planning of a mobile robot in parallel parking under the same conditions as in Fig. 3. The neural network has the same structure with $40 \times 30 \times 24$ neurons and zero initial activities. The model parameters are chosen as $A = 10$, $\mu = 1$, $r_0 = 6$ and $E = 50$. The generated real-time robot motion paths are the same as those in Fig. 3. However, the activity landscape using the additive model has different peak and valley values from those using the shunting model. Because the activity landscape of the neural network is in 4-D, i.e., x, y, θ and ξ, it is not able to display graphically. To illustrate the different activity landscapes using the shunting and additive models, the proposed model is applied to path planning a point robot in a U-shaped environment, with the same model parameters in this paper. The generated path using either the shunting model or the additive model is shown in Fig. 6(a), while the neural activity landscape in 3-D with shunting and additive models are shown in Figs. 6(b) and 6(c), respectively. It shows that the neural activities with a shunting model is bounded in $[-1, 1]$, while those with a additive model is not bounded.

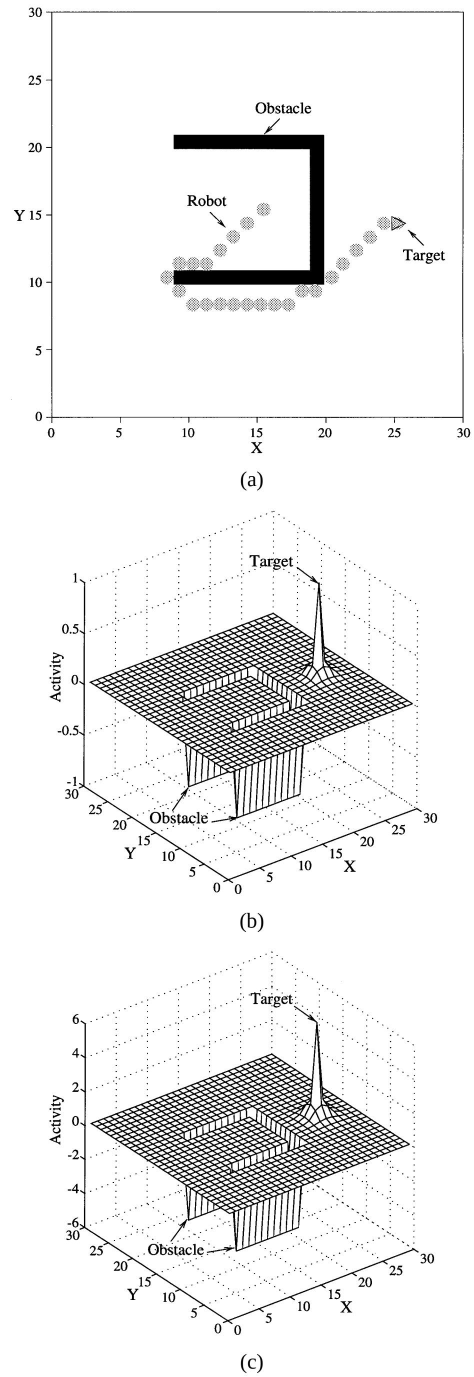

Fig. 6. Real-time planning of a point robot using the shunting and additive models. (a) Planned robot motion. (b) Activity landscape using the shunting model. (c) Activity landscape using the additive model.

V. Conclusion

In this paper, a neural dynamics based approach is proposed for real-time collision-free motion planning of a nonholonomic mobile robot in a nonstationary environment, which is based on the fundamental concept of the motion-planning models for a point robot, but with significant differences in the neural network design and the robot motion planning algorithm. Distinct from the previous path-planning models in for a point robot where the neural connections are a function of distance only [38]–[41], in the proposed model the neural connections depend on both distance *and* direction. In addition, the neural activity propagation is directionally selective, instead of onmi-directional as previous path-planning models in [38]–[41]. Furthermore, the real-time robot motion is generated through the dynamic activity landscape of the neural network, subject to the nonholonomic kinematic constraint of the mobile robot. Several points about the proposed neural dynamics based approach are worth noticing.

- This model is biologically plausible. It is originally derived from Hodgkin and Huxley's [42] biological membrane model. The neural activity is a continuous analog signal and has both upper and lower bounds. In addition, the continuous activity prevents the possible oscillations related to parallel dynamics of discrete neurons [64]–[66]. It has potential VLSI implementation for parallel computation.
- This model will not be trapped in deadlock situations, even in a complex house-like environment. The target globally influences the whole workspace through neural activity propagation. The robot motion is planned through the dynamic activity landscape of the neural network.
- This model can perform properly in an varying environment, even with sudden environmental changes, such as suddenly adding or removing obstacles. This property results from the dynamics behavior of a shunting equation [55].
- This model is not very sensitive to the model parameters and the neural connection weight function. The parameters can be chosen in a very wide range. The weight function can be any monotonically decreasing function.
- This model is not sensitive to any irrelevant obstacles. There are no inhibitory lateral connections in the neural network. The negative neural activity from the obstacle location stays locally only without propagating to any other neurons. Thus the obstacles have only local effect to push the robot away to avoid collisions. Therefore, unlike some previous models (e.g., [4]), the irrelevant obstacles do not influence the robot motion planning (for details, see [43]).
- This model is not very sensitive to sensor noise. The obstacles have only local effect. The false-detected obstacles by the sensors far from the robot will not influence the global robot motion planning. On the other side, because the proposed model is capable of dealing with the sudden environmental changes, the obstacles un-detected by the sensors at a far distance still can be avoided by robot motion planner at a short distance, even when they are very close to the robot.
- The computational complexity linearly depends on the state-space size of the neural network. The number of neurons required is equal to $N = N_x \times N_y \times N_\theta$, where N_x, N_y and N_θ are the discretized size of the Cartesian workspace and robot orientation. Each neuron has at most 6 local connections for a nonholonomic mobile robot. When the state space size N increases, the number of neuron and the total neural connections increases linearly proportional to N. Because the state space of the neural network is the robot configuration space, the proposed algorithm is computationally expensive and thus is not suitable for a very complicated robotic system, e.g., a mobile robot with many trails.
- The model algorithm is computationally simple. The real-time robot motion is planned without explicitly searching over the free workspace or the collision paths, without explicitly optimizing any global cost functions, without any prior knowledge of the dynamic environment, without any learning procedures, and without any local collision-detecting procedures at each step of robot movement during the real-time robot navigation. It has the limitation that the current knowledge of the environment is assumed to be completely known [44].

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