Assignment 5

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Question 3.6

To show that H is PAC learnable, let $h \in H$ be target function, and let D be any distribution over X. Examine the distribution D on $X \times \{1,1\}$ obtained by drawing $x \in X$ according to D, and taking the pair $(x, h(x))$.note that with realizability we have $inf_{h \in H} L_D(h) = 0$. set $\varepsilon, \delta \in (0,1)$ and S consisting of i.i.d instances which are labeled by f, so because H is agnostic PAC learnable ,for every $m > m(\varepsilon, \delta)$, with probability at least $1-\delta$ we have:

$$L_D(h) \leq inf_{h' \in H} L_D(h') + \varepsilon \Rightarrow L_D(h) \leq 0 + \varepsilon$$

Therefore H is PAC learnable too.

Question 3.7

First we want determine $L_D(f_D)$ in one arbitrary point x that its true labeling is y.

$$L_D(f_D(x)) = P[f_D(x) \neq y]$$

define $a_x = P[y = 1|x]$, so if $a_x < \frac{1}{2}$ we have $f_D(x) = 0$, otherwise $f_D(x) = 1$, now with this, we can write:

$$P[f_D(x) \neq y] = P[y \neq 0] I(a_x < \frac{1}{2}) + P[y \neq 1] I(a_x \geq \frac{1}{2})$$

$y = 0$ or 1 so:

$$P[f_D(x) \neq y] = P[y = 1] I(a_x < \frac{1}{2}) + P[y = 0] I(a_x \geq \frac{1}{2}) = a_x I(a_x < \frac{1}{2}) + (1 - a_x) I(a_x \geq \frac{1}{2})$$

Note that if $a_x < 1 - a_x$ or $a_x < \frac{1}{2}$ therefor above equation is equal to a_x, otherwise that is equal to $1 - a_x$ so:

$$P[f_D(x) \neq y] = min\{a_x, 1 - a_x\}$$

Now let g arbitrary classifier from X to $\{0,1\}$:

$$\begin{aligned} P[g(x) \neq y] &= P[y = 1] P[g(x) = 0] + P[y = 0] P[g(x) = 1] \\ &= a_x P[g(x) = 0] + (1 - a_x) P[g(x) = 1] \\ &\geq min\{a_x, 1 - a_x\} P[g(x) = 0] + min\{a_x, 1 - a_x\} P[g(x) = 1] \\ &= min\{a_x, 1 - a_x\} \end{aligned}$$

The statement follows now due to the fact that the above is true for every $x \in X$