Assignment 10

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Question 18.1

part 1

Given h,construct a binary tree, set the root node ($x_1 = 0?$), and other nodes in level i ($x_{i+1} = 0?$).The height of this tree is d and it has 2^d leaves.It is clear that, we can allocate one leaf to any possible combination of values for $x_1, x_2, ..., x_d$, with the leaf's value $h(x) = h((x_1, x_2, ..., x_d))$.

part 2

previous example show that we can shatter the $0, 1^d$, so the $VCdim = 2^d$.

Question 18.2

part 1

We know:

$Gain(S,i) = C(P[y=1]) - (P[x_i=1]C(P[y=1|x_i=1]) + (P[x_i=0]C(P[y=1|x_i=0])$

So if we set C as information gain we have:

$$Gain(S,i) = H(\frac{1}{2}) - (P[x_i=1]H(P[y=1|x_i=1]) + (P[x_i=0]H(P[y=1|x_i=0])$$

For feature 1 we have:

$$Gain(S,1) = H(\frac{1}{2}) - (\frac{3}{4}H(\frac{2}{3}) + (\frac{1}{4}H(0)) =$$

$$-2\frac{1}{2}\log(\frac{1}{2}) - (\frac{3}{4}(-\frac{2}{3})\log(\frac{2}{3}) - \frac{1}{3})\log(\frac{1}{3}) = 0.2$$

For feature 2 , 3 we have:

$$Gain(S,1) = H(\frac{1}{2}) - (\frac{1}{2}H(\frac{1}{2}) + (\frac{1}{2}H(\frac{1}{2})) = 0$$

So we should pick $x_1 = 0?$ as the root. and it means that first three example are in the same subtree. now if we pick $x_2 = 0?$ as next node, $((1,1,1),1)$ and $((1,1,0),0)$ are in the same subtree and we can't classify them. (if we choose $x_3 = 0?$ as the second node we have the similar result.) , so 1 of 4 example has incorrect label, and it follows that the error is $\frac{1}{4}$.

part 2

If we pick $x_2 = 0?$ as the root, and $x_3 = 0?$ as the next node, we can label all example correctly.