Assignment 5

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Question 3.5

First we set $h \in H$,if we have:

$$L_{(\tilde{D}_m,f)}(h) > \varepsilon$$

so:

$$P_{x\sim\tilde{D_m}}[h(x) \neq f(x)] > \varepsilon \Rightarrow P_{x\sim\tilde{D_m}}[h(x) = f(x)] < 1-\varepsilon$$

$$\Rightarrow \frac{P_{x\sim D_1}[h(x) = f(x)] + \ldots + P_{x\sim D_m}[h(x) = f(x)]}{m} < 1-\varepsilon$$

and:

$$P_{s\sim\prod D_i}[L_{(S,f)} = 0] = \prod_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)]$$

We can rewrite this as below:

$$\prod_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)] = \left(\left(\prod_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)]\right)^{1/m}\right)^{m}$$

Now if we use geometric-arithmetic mean inequality:

$$\left(\left(\prod_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)]\right)^{1/m}\right)^{m} \leq \left(\frac{\sum_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)]}{m}\right)^{1/m}$$

but we know $\frac{\sum_{i=1}^{m} P_{x\sim D_i}[f(x)=h(x)]}{m}$ is less than $1-\varepsilon$, so:

$$\left(\frac{\sum_{i=1}^{m} P_{x\sim D_i}[f(x) = h(x)]}{m}\right)^{1/m} \leq (1-\varepsilon)^{1/m} \leq e^{-m\varepsilon}$$

we conclude that for fixed $h \in H$, and H is finite, therefore if we use union bound, we have:

$$P[\exists h \in H \text{ s.t } L_{(\tilde{D}_m,f)}(h) > \varepsilon \text{ and } L_{(S,f)}(h) = 0] \leq |H|e^{-m\varepsilon}$$