Filip Todorić, 0036506267

Izvještaj

NENR, 6. zadaća

Zadatak 1.

IZVODI PRAVILA UČENJA PARAMETARA SUSTAVA ANFIS

PARAMETRI: $a_i\ b_i\ c_i\ d_i$ u premisi pravila i

$p_i\ q_i\ r_i$ u konzekvensu pravila i

$$\mu_{A_i}(x) = \frac{1}{1+e^{b_i(x-a_i)}} \qquad \mu_{B_i}(x) = \frac{1}{1+e^{d_i(x-c_i)}} \qquad \alpha_i = \mu_{A_i}\cdot\mu_{B_i}$$

$z_i(x,y) = p_i x + q_i y + r_i$

y_k - stvarni izlaz sustava $\qquad o_k$ - izlaz ANFIS-a

$$o = \frac{\sum_{i=1}^{m}\alpha_i z_i}{\sum_{i=1}^{m}\alpha_i} \qquad E_k = \frac{1}{2}(y_k - o_k)^2$$

Ažuriranje pojedinih parametara:

STOHASTIČKA VARIJANTA: $\psi(t+1) = \psi(t) - \eta\cdot\frac{\partial E_k}{\partial\psi}$

GRUPNA VARIJANTA: $\psi(t+1) = \psi(t) - \eta\cdot\sum_{i=1}^{N}\frac{\partial E_k}{\partial\psi}$

$\boxed{\text{PARAMETAR } a_i}$ PRINCIP ULANČAVANJA PARCIJALNIH DERIVACIJA:

$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial\alpha_i}\cdot\frac{\partial\alpha_i}{\partial a_i} \qquad \boxed{\frac{\partial E_k}{\partial o_k} = -(y_k - o_k)}$$

$$\frac{\partial o_k}{\partial\alpha_i} = \frac{\left(\sum_{j=1}^{m}\alpha_j z_j\right)'_{\alpha_i}\left(\sum_{j=1}^{m}\alpha_j\right) - \left(\sum_{j=1}^{m}\alpha_j z_j\right)\cdot\left(\sum_{j=1}^{m}\alpha_j\right)'_{\alpha_i}}{\left(\sum_{j=1}^{m}\alpha_j\right)^2}$$

$$= \frac{z_i\cdot\left(\sum_{j=1}^{m}\alpha_j\right) - \left(\sum_{j=1}^{m}\alpha_j z_j\right)\cdot 1}{\left(\sum_{j=1}^{m}\alpha_j\right)^2} = \frac{\sum_{j=1}^{m}\alpha_j z_i - \sum_{j=1}^{m}\alpha_j z_j}{\left(\sum_{j=1}^{m}\alpha_j\right)^2}$$

$$\boxed{\frac{\partial o_k}{\partial\alpha_i} = \frac{\sum_{j=1,\, j\neq i}^{m}\alpha_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\alpha_j\right)^2}}$$

$$\frac{\partial\alpha_i}{\partial a_i} = \frac{\partial(\mu_{A_i}\cdot\mu_{B_i})}{\partial a_i} = \frac{\partial\mu_{A_i}}{\partial a_i}\cdot\mu_{B_i}$$

$$\frac{\partial\mu_{A_i}}{\partial a_i} = \mu_{A_i}(1-\mu_{A_i})\cdot\frac{\partial(-b_i(x-a_i))}{\partial a_i} = \boxed{\mu_{A_i}(1-\mu_{A_i})\cdot b_i}$$

$$\frac{\partial E_k}{\partial a_i} = -(y_k - o_k)\cdot\frac{\sum_{j=1,\, j\neq i}^{m}\alpha_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\alpha_j\right)^2}\cdot\mu_{A_i}(1-\mu_{A_i})\cdot b_i\cdot\mu_{B_i}$$

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STOHASTIČKO AŽURIRANJE:

$$a_i(t+1) = a_i(t) + \eta \cdot (y_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2} \cdot \mu_{A_i}(1-\mu_{A_i}) \cdot b_i \cdot \mu_{B_i}$$

GRUPNO AŽURIRANJE:

$$a_i(t+1) = a_i(t) + \eta \cdot \sum_{k=1}^{N} (y_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2} \cdot \mu_{A_i}(1-\mu_{A_i}) \cdot b_i \cdot \mu_{B_i}$$

PARAMETAR b_i

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial b_i}$$

$$\boxed{\frac{\partial E_k}{\partial o_k} = -(y_k - o_k)}$$

$$\boxed{\frac{\partial o_k}{\partial \alpha_i} = \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2}}$$

$$\frac{\partial \alpha_i}{\partial b_i} = \frac{\partial(\mu_{A_i}\mu_{B_i})}{\partial b_i} = \frac{\partial \mu_{A_i}}{\partial b_i} \cdot \mu_{B_i}$$

$$\frac{\partial \mu_{A_i}}{\partial b_i} = \mu_{A_i}(1-\mu_{A_i}) \cdot \frac{\partial(-b_i(x-a_i))}{\partial b_i} = \mu_{A_i}(1-\mu_{A_i}) \cdot (a_i - x)$$

$$\boxed{\frac{\partial \alpha_i}{\partial b_i} = \mu_{A_i}(1-\mu_{A_i}) \cdot (a_i - x) \cdot \mu_{B_i}}$$

$$\frac{\partial E_k}{\partial b_i} = -(y_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2} \cdot \mu_{A_i}(1-\mu_{A_i})(a_i - x) \cdot \mu_{B_i}$$

STOHASTIČKO AŽURIRANJE:

$$b_i(t+1) = b_i(t) + \eta \cdot (y_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2} \cdot \mu_{A_i}(1-\mu_{A_i})(a_i - x)\mu_{B_i}$$

GRUPNO AŽURIRANJE:

$$b_i(t+1) = b_i(t) + \eta \cdot \sum_{k=1}^{N} (y_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} \alpha_j (z_i - z_j)}{\left(\sum_{j=1}^{m} \alpha_j\right)^2} \cdot \mu_{A_i}(1-\mu_{A_i})(a_i - x)\mu_{B_i}$$

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$\boxed{\text{PARAMETAR } c_i}$

$$\mu_{B_i} = \frac{1}{1+e^{d_i(x-c_i)}}$$

$$\frac{\partial E_k}{\partial c_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial c_i}$$

isto kao i prije

$$\frac{\partial \alpha_i}{\partial c_i} = \frac{\partial \mu_{A_i}\mu_{B_i}}{\partial c_i} = \frac{\partial \mu_{B_i}}{\partial c_i}\mu_{A_i}$$

$$\frac{\partial \mu_{B_i}}{\partial c_i} = \mu_{B_i}(1-\mu_{B_i}) \cdot \frac{\partial(-d_i(x-c_i))}{\partial c_i} = \mu_{B_i}(1-\mu_{B_i}) \cdot d_i$$

$$\boxed{\frac{\partial \alpha_i}{\partial c_i} = \mu_{B_i}(1-\mu_{B_i}) \cdot d_i \cdot \mu_{A_i}}$$

$$\frac{\partial E_k}{\partial c_i} = -(y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i}) \cdot d_i \cdot \mu_{A_i}$$

STOHASTIČKO:

$$c_i(t+1) = c_i(t) + \eta \cdot (y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i}) \cdot d_i \cdot \mu_{A_i}$$

GRUPNO:

$$c_i(t+1) = c_i(t) + \eta \cdot \sum_{k=1}^{N}(y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i}) \cdot d_i \cdot \mu_{A_i}$$

$\boxed{\text{PARAMETAR } d_i}$

$$\frac{\partial E_k}{\partial d_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d_i}$$

$$\frac{\partial \alpha_i}{\partial d_i} = \frac{\partial \mu_{A_i}\mu_{B_i}}{\partial d_i} = \frac{\partial \mu_{B_i}}{\partial d_i} \cdot \mu_{A_i}$$

$$\frac{\partial \mu_{B_i}}{\partial d_i} = \mu_{B_i}(1-\mu_{B_i}) \cdot \frac{\partial(-d_i(x-c_i))}{\partial d_i} = \mu_{B_i}(1-\mu_{B_i}) \cdot (c_i - x)$$

$$\frac{\partial E_k}{\partial d_i} = -(y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i})(c_i - x) \cdot \mu_{A_i}$$

STOHASTIČKO:

$$d_i(t+1) = d_i(t) + \eta \cdot (y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i})(c_i - x)\mu_{A_i}$$

GRUPNO:

$$d_i(t+1) = d_i(t) + \eta \cdot \sum_{k=1}^{N}(y_k - o_k) \cdot \frac{\sum_{j=1\, j\neq i}^{m} \alpha_j (z_i - z_j)}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \mu_{B_i}(1-\mu_{B_i})(c_i - x)\mu_{A_i}$$

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PARAMETAR p_i

$$z_i = p_i x + q_i y + r_i$$

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial p_i}$$

$$\frac{\partial E_k}{\partial o_k} = -(y_k - o_k)$$

$$\frac{\partial o_k}{\partial z_i} = \left(\frac{\sum_{j=1}^{m} \alpha_j z_j}{\sum_{j=1}^{m} \alpha_j} \right)'_{z_i} = \boxed{\frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j}}$$

$$\frac{\partial z_i}{\partial p_i} = x$$

$$\frac{\partial E_k}{\partial p_i} = -(y_k - o_k) \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot x$$

STOHASTIČKO:

$$p_i(t+1) = p_i(t) + \eta (y_k - o_k) \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot x$$

GRUPNO:

$$p_i(t+1) = p_i(t) + \eta \cdot \sum_{k=1}^{N} (y_k - o_k) \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot x$$

PARAMETAR q_i

$$\frac{\partial z_i}{\partial q_i} = y \qquad \frac{\partial E_k}{\partial p_i} = -(y_k - o_k) \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot y$$

STOHASTIČKO:

$$q_i(t+1) = q_i(t) + \eta (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot y$$

GRUPNO:

$$q_i(t+1) = q_i(t) + \eta \cdot \sum_{k=1}^{N} (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j} \cdot y$$

PARAMETAR r_i

$$\frac{\partial z_i}{\partial r_i} = 1 \qquad \frac{\partial E_k}{\partial r_i} = -(y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j}$$

STOHASTIČKO:

$$r_i(t+1) = r_i(t) + \eta \cdot (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j}$$

GRUPNO:

$$r_i(t+1) = r_i(t) + \eta \cdot \sum_{k=1}^{N} (y_k - o_k) \cdot \frac{\alpha_i}{\sum_{j=1}^{m} \alpha_j}$$

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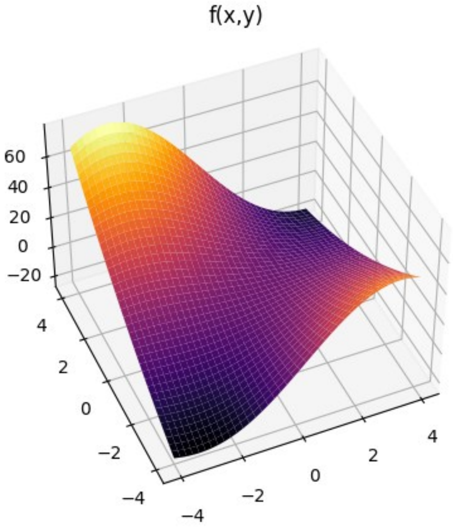
Zadatak 2.

Implementacija ANFIS sustava.

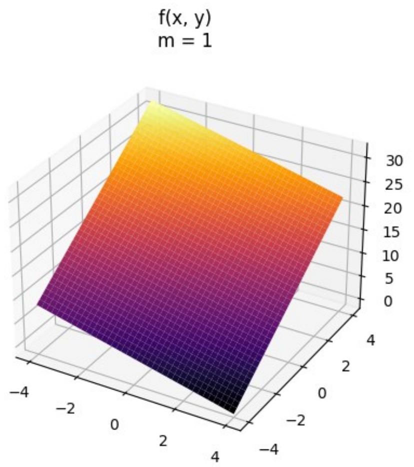
Zadatak 3.

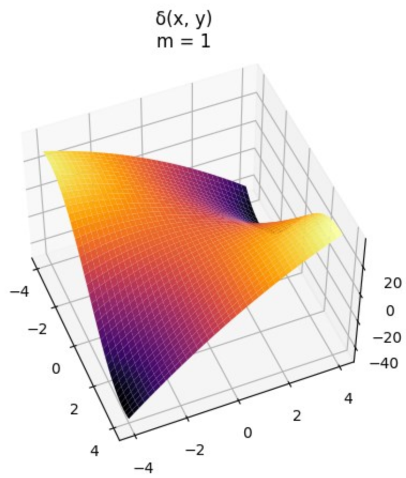


Zadatak 4.

Grupni algoritam češće pronalazi sustav parametara s manjom ukupnom pogreškom s prikladnim skupom parametara **η**. Prikladni parametri **η** veći su za grupni algoritam nego za stohastički. Veći **η** ubrzava postupak nalaženja boljih parametara, ali se riskira postizanje divergencije.

U sljedećim primjerima broj **m** označava broj pravila koje trenirani sustav ANFIS koristi.





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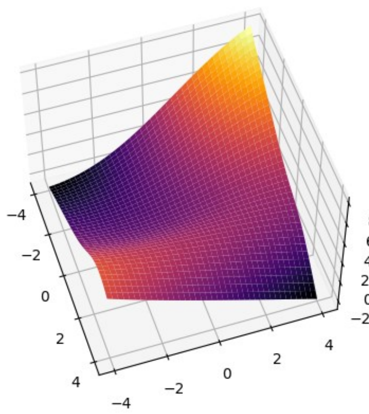
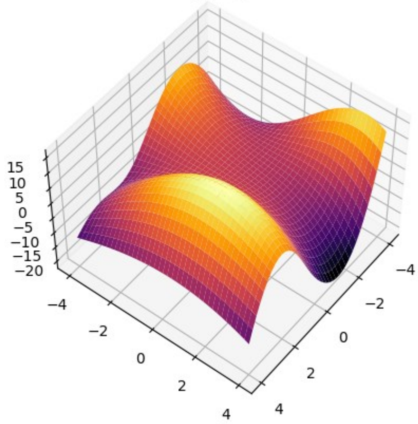
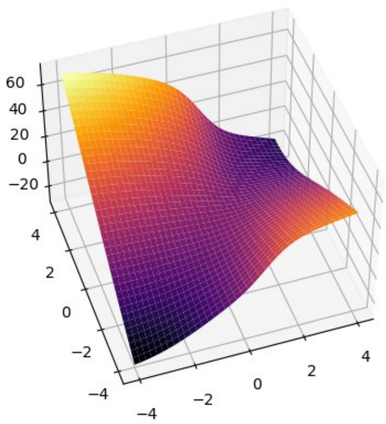
f(x, y)
m = 2



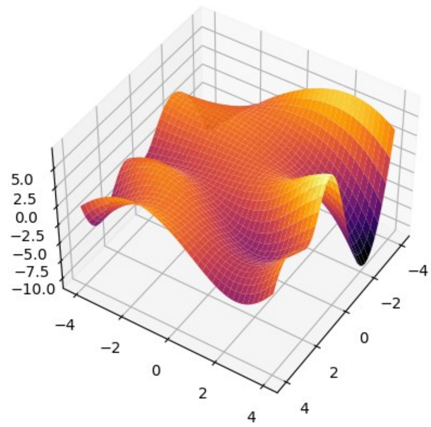
δ(x, y)
m = 2



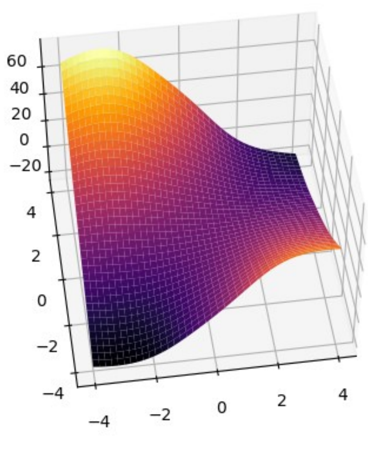
f(x, y)
m = 4



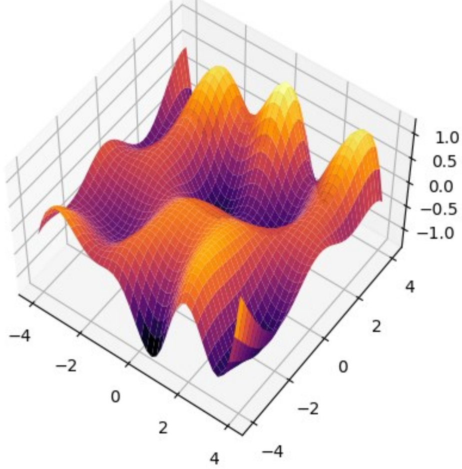
δ(x, y)
m = 4



f(x, y)
m = 7



δ(x, y)
m = 7



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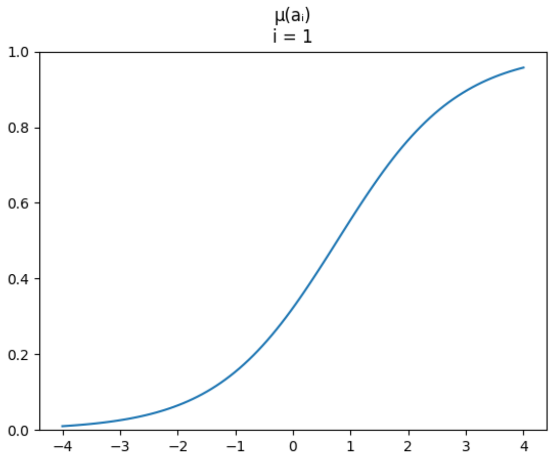
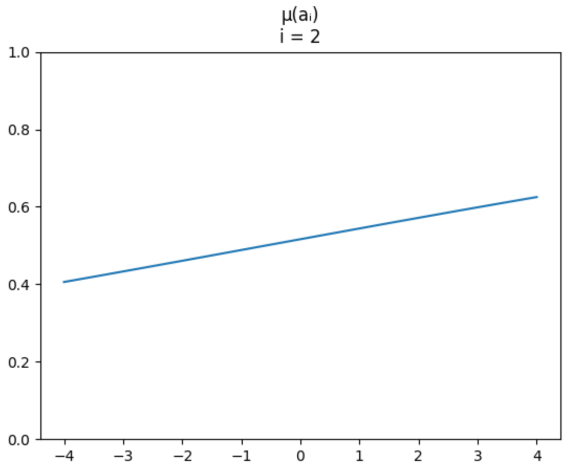
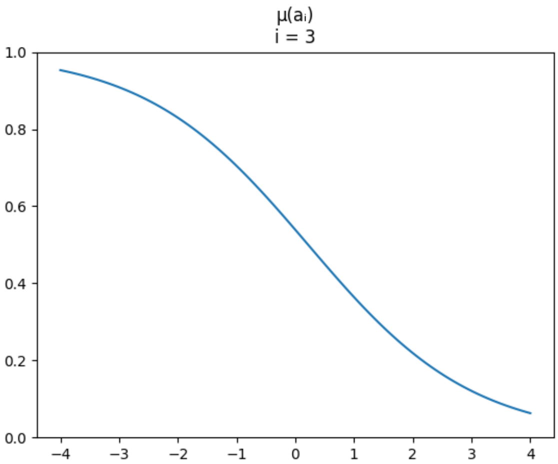
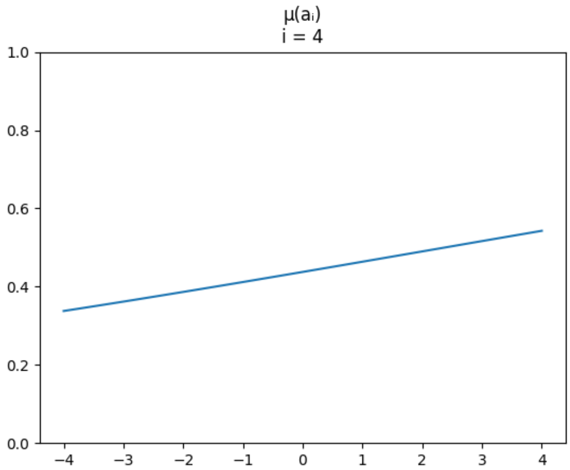
Zadatak 5.

Na sljedećim primjerima broj **i** nekog neizrazitog skupa predstavlja redni broj pravila sustava ANFIS.

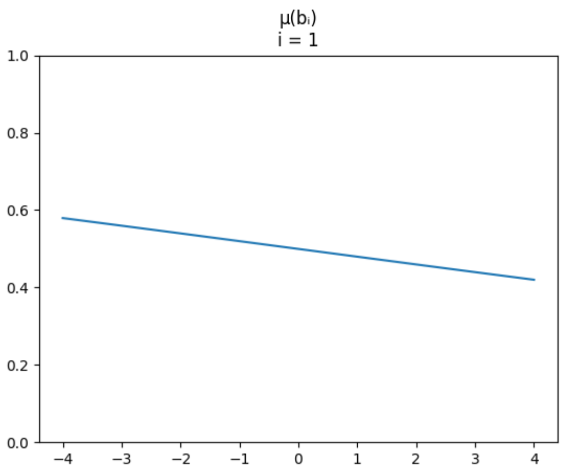


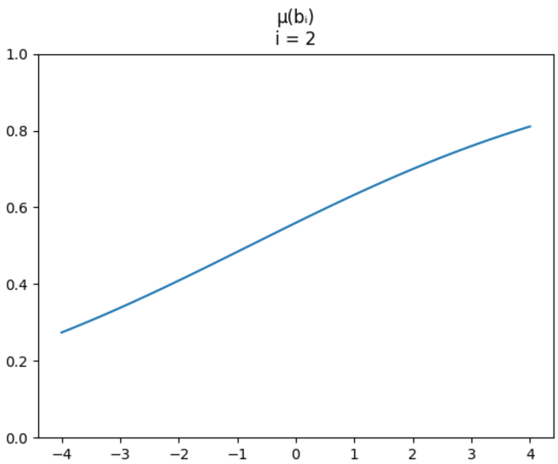


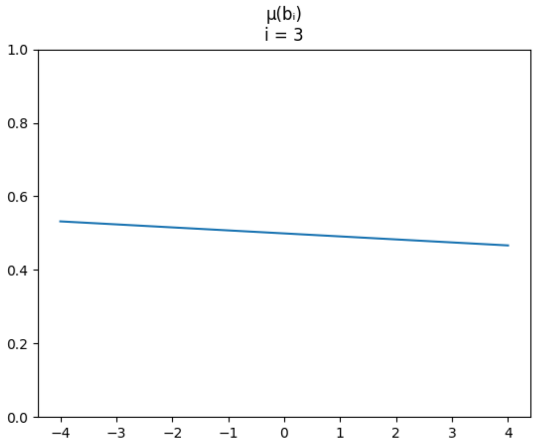


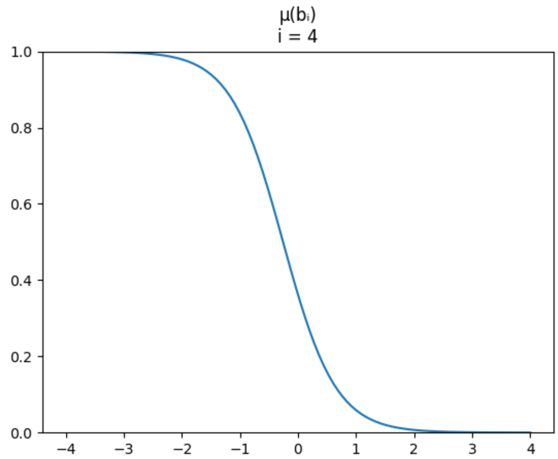


Naprimjer, za funkciju pripadnosti skupa A_1 možemo reći da opisuje „brojeve slične broju 4“ u obliku sigmoidalne funkcije. Takav skup modelira isti koncept kao i skup koji modelira pripadnost skupini visokih ljudi.









Zadatak 6.

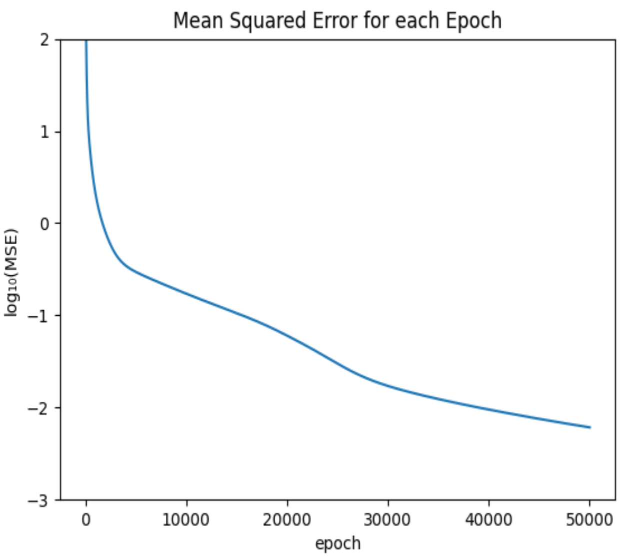
$\delta(x, y) = o_k - y_k$

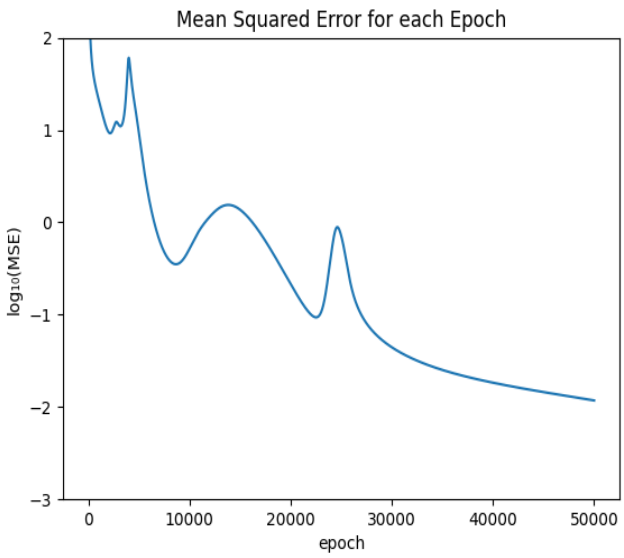
o_k - vrijednost izlaza ANFIS mreže
y_k - prava vrijednost izlaza funkcije

Ovu vrstu pogreške smo koristili za iscrtavanje pogreški svakog uzorka u zadatku 4.

Zadatak 7.

Broj pravila sustava ANFISA je 7 za ovaj primjer. Graf lijevo pokazuje srednju kvadratnu pogrešku svake epohe upotrebom grupnog algoritma, dok desni upotrebom stohastičkog algoritma.





Zadatak 8.

Dovoljno dobri parametri **η** veći su za grupni algoritam nego za stohastički. Veći **η** ubrzava postupak nalaženja boljih parametara, ali se riskira postizanje divergencije. Prikladne vrijednosti su približno:

- grupni algoritam

  η = 10^{-6}

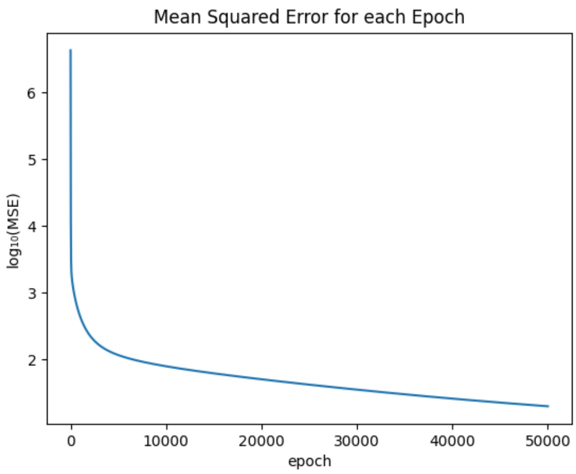
- stohastički algoritam

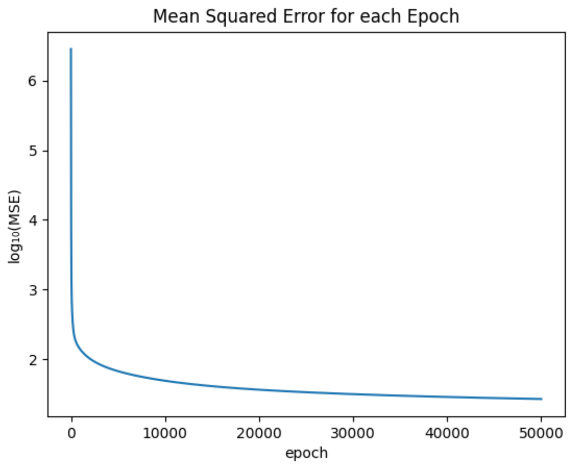
  η = 10^{-7}

Za rubne vrijednosti 10^{-10} i 10^{-1} prikazani su grafovi za grupni i stohastički algoritam.

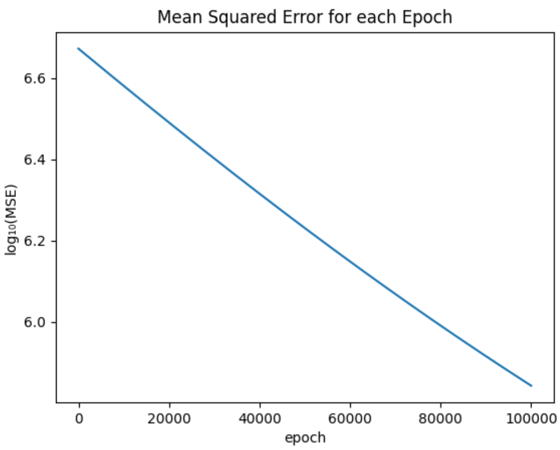
grupni algoritam | **stohastički algoritam**

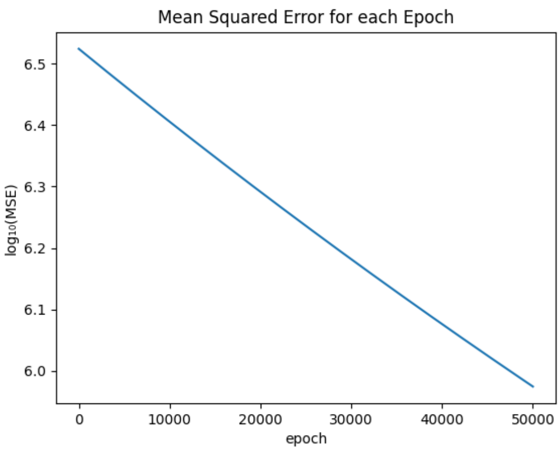
Prikladne vrijednosti parametra **η**:





Premale vrijednosti parametra **η**:





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Prevelike vrijednosti parametra **η:**