Equitable screening*

Filip Tokarski

December 2, 2023

Abstract

In many mechanism design problems the designer is concerned about the perceived fairness of the allocation. I provide a notion of equity capturing such concerns in a variety of settings. According to it, an allocation is equitable if it does not systematically differ between agents who are equally deserving but belong to different protected groups. I apply this notion to the problem of a government distributing in-kind benefits and ask what forms of efficiency-enhancing screening are compatible with equity. While the government cannot equitably screen with a single instrument (e.g. payments or queueing), combining multiple instruments, which on their own favor different groups, allows it to screen while still producing an allocation that is seen as fair.

1 Introduction

In 1999, the UK government aimed to universalize the list of treatments covered by the National Health Service. To that end, it established the National Institute for Clinical Excellence (NICE)—an organization tasked with ranking treatments according to their efficacy and cost-effectiveness; these assessments then informed coverage decisions. NICE's cost-benefit analyses recommended certain cheap but noncritical procedures while excluding other extremely costly but life-saving ones. The fact that some critically ill patients were covered while others (e.g. those suffering from extremely rare cancers) were not, provoked backlash which forced the government to compromise (Einav and Finkelstein (2023)). NICE responded to public outcry by i.a. dramatically increasing expenditure limits for drugs and treatments for rare life-threatening diseases. Attempts at institutionalizing cost-benefit analyses in healthcare faired similarly in many other rich countries. Some, including Norway, Sweden and the Netherlands, responded to public pressure by explicitly requiring that treatments to those in greatest need be covered regardless of cost.

More recently, many US colleges have shifted to test-optional, or even test-blind admissions policies. Test-based admissions have long been criticized for providing an unfair advantage to wealthy applicants who, unlike their poorer peers, could invest in tutoring and preparation. Despite tests being highly informative selection tools, the reputational cost from being seen as inequitable led many colleges to abandon them (Dessein et al. (2023)).

In both of these cases, the backlash resulted from policies treating equally deserving subjects differently based on some characteristic that, from an ethical standpoint, should not matter.

*I am grateful to Paula Onuchic, Piotr Dworczak and Joey Feffer for their helpful comments.

In the case of NICE, patients with grave diseases whose treatment happened to be more costly were denied coverage, while those with equally severe but more common conditions got it. In the case of standardized tests, similarly able applicants from poorer backgrounds lacked the recourses for preparation that their wealthier peers possessed. Other policies motivated by similar equity concerns include non-discrimination clauses, affirmative action, and disability accommodations.

This paper considers mechanism design problems where the designer is bound by equity concerns. My notion of equity builds on context-specific *merit functions* specifying how entitled each agent is to the allocated good. I introduce an equity constraint requiring that, among agents with equal merit, the allocation should not depend on *protected characteristics*, such as race or gender. This definition permits systematic differences in allocations based on merit (e.g. allocating vaccines to doctors and nurses before everyone else), as well as differences in allocations among agents with equal merit that are 'due to noise' (e.g. a university can reject talented applicants who were unlucky on the exam). It does not, however, allow such errors to systematically bias allocations between protected groups. Importantly, allocating through uniform lotteries will always be equitable.[1]

My notion is also much stronger than the requirement that the procedure not explicitly depend on protected characteristics. Consider, for instance, an insurer who is banned from discriminating on applicants' race but prices policies based on proxies for it, essentially bypassing regulation. While the insurer is not *explicitly* discriminating based on race, his policy is still inequitable in the sense of this paper. This makes my equity notion akin to disparate impact laws—these prohibit practices in i.a. employment and housing that adversely affect some protected group where there is no 'business necessity' to do so; these laws applies even if said practice is formally neutral or unintentional (*EEOC v. Sambo's of Georgia* (1981)).

My primary application studies the problem of providing in-kind benefits, such as vaccines or public housing, when the government is concerned about efficiency and redistribution. The literature studying the optimal design of such programs (e.g. Condorelli (2013), Akbarpour et al. (2020)) finds that the optimal mechanism often involves screening recipients, e.g. through payments. For instance, Akbarpour et al. (2020) find that, under fairly general conditions, the government should sell the good (at possibly non-market prices) and then redistribute the revenue to poorer agents. However, such mechanisms might be infeasible when the government faces equity constraints. Consider for instance the problem of distributing Covid-19 vaccines studied by Akbarpour et al. (2023). They show that the optimal mechanism combines priorities to vulnerable groups with a market mechanism under which one can pay to be vaccinated early. The authors themselves acknowledge that such mechanisms might provoke backlash on fairness grounds. Indeed, similar objections have been raised by both academic philosophers (Kass (1997), Walzer (1983)) and the general public.[2]

[1] In some contexts uniform randomization within a particular group would not be considered fair. Kamada and Kojima (2023) provide examples of Japanese disaster relief efforts where organizers, motivated by perceived equity concerns, preferred not allocating a resource to anyone in a given group to running a lottery. To capture such notions of equity, one could amend my definition with a requirement of *individual fairness*—a criterion insisting that similar individuals be treated similarly (Dwork et al. (2012)).

[2] https://www.statnews.com/2020/12/03/how-rich-and-privileged-can-skip-the-line-for-covid19-vaccines

This paper relates to work examining how moral sentiments constrain market designers and policymakers. Roth (2007) discusses how the repugnance of certain transactions precludes the use of markets in settings where they would be efficient. The literature following Abdulkadiroğlu and Sönmez (2003) models fairness concerns in matching markets through assigning *priorities* to agents. It then studies matching mechanisms that eliminate *justified envy*—a notion capturing perceived injustice. Finally, Dessein et al. (2023) argue that many US colleges switched to test-optional admissions to reduce public scrutiny of their admission decisions. However, to my knowledge, no existing work provides a fairness criterion for mechanism design problems of the sort this paper considers. In attempting to do so, my paper also comes close to the literatures on algorithmic fairness in Computer Science, and on discrimination in Economics, which attempt to conceptualize bias, unfairness and discrimination (see Alves et al. (2023) and Onuchic (2022) for respective surveys). However, in both cases researchers focus on problems of classification or statistical inference, and hence do not account for the strategic behavior of agents. By contrast, the purpose of this paper is to study mechanisms that are fair after accounting for strategic responses.

The rest of the paper is structured as follows. First, I formalize my notion of equity and contrast it with notions in the literature. Then, in Section 3, I employ this notion to ask whether governments can equitably screen when providing in-kind benefits.

2 Equity constraints

The designer allocates goods from $\mathcal{X} \subseteq \mathbb{R}_+$ to agents. Each agent has a *protected characteristic* $\alpha \in \mathcal{A}$ representing an identity that is salient for equity, and an *unprotected characteristic* $\beta \in \mathcal{B}$ that is irrelevant to equity considerations. The allocation rule is $x : \mathcal{A} \times \mathcal{B} \to \mathcal{X}$. I model equity considerations using an exogenous *merit function* $\eta : \mathcal{A} \times \mathcal{B} \to \mathbb{R}$.

Definition 1 (Equitable allocation)**.** *An allocation rule x is **equitable** if, for all $\alpha_1, \alpha_2 \in \mathcal{A}$ and all $\gamma \in \mathbb{R}$, we have:*

$$\mathbb{E}_\beta[x(\alpha_1, \beta) \mid \eta(\alpha_1, \beta) = \gamma] \;=\; \mathbb{E}_\beta[x(\alpha_2, \beta) \mid \eta(\alpha_2, \beta) = \gamma]. \tag{1}$$

In words, an allocation is equitable if, among agents with equal merit, expected allocations are the same across all protected groups. For intuition, consider the following example:

Example 1. *Consider the problem of allocating university places; then $\mathcal{X} = \{0, 1\}$. Let α be a vector of demographic characteristics (gender, race, etc.) and $\beta = (\beta_1, \beta_2)$ specify academic potential (β_1) and test scores (β_2). Assume α and β_2 are public but academic potential β_1 is private information. Assume also that the only determinant of merit is academic potential:*

$$\eta(\alpha, \beta) \;=\; \beta_1.$$

Equity then requires that demographic characteristics do not systematically affect admission chances among students with the same academic potential. For instance, if some demographic group systematically underperforms on test scores (relative to their potential), equity demands that it face a lower admission threshold to correct for that.

A related criterion, known as the *priority point system*, is commonly used by hospitals in rationing scarce medical resources, such as ICU beds and ventilators (Pathak et al. (2021)).[3] Under this system, the hospital often uses an explicit formula to assign patients to priority tiers, and allocates resources in that order. In line with Definition 1, ties among patients in the same tier are often broken by lottery (Zucker et al. (2015), Emanuel et al. (2020)).[4]

Definition 1 is related to the literature on algorithmic fairness in Computer Science, where various notions of fairness and non-discrimination have been proposed to conceptualize bias in algorithms and machine learning[5] (see Barocas et al. (2019) for an introduction). This literature considers the problem of designing a (usually binary) classifier $\hat{Y}$ to predict an agents' true outcome Y based on observed characteristics X. My approach is close to the family of criteria known as (conditional) independence requiring that some aspect of classification be (conditionally) independent of some protected characteristic. Within this class, Definition 1 is closest to the notion of *conditional statistical parity* requiring that, conditional on certain covariates deemed legitimate, the classification rate be the same across protected groups (Corbett-Davies et al. (2017)). My notion differs from it in that it does not condition directly on agents' characteristics, but on the value of the merit function.

The merit function approach provides more flexibility in capturing context-specific concerns. In particular, it nests many other notions from the independence family—for instance, conditional statistical parity can be recovered by assigning a different value to each combination of legitimate covariates; group fairness (requiring the allocation not depend on protected characteristics) can be recovered by setting the merit function to a constant.

Some further properties of Definition 1 are worth mentioning. First, I model equity as a constraint, not as a term included in the designer's objective that can be traded off against other values. This reflects the rigid nature of ethical concerns, like those leading to banning organ trade or the use of race in pricing insurance. Second, the framework only addresses fairness concerns relating to resulting allocations, and not other features of the process. Third, I require that (conditional on merit) the protected characteristic not affect the allocation only *in expectation*; the algorithmic fairness literature typically requires *independence*.

3 Equitable screening in public provision

I consider a government allocating in-kind benefits such as vaccines, affordable housing or, in many developing countries, basic food items. Standard economic logic suggests that such programs can be made more efficient through costly screening—if agents need to pay or queue to get the benefit, only those who need it will do so. However, different agents with the same need for the good might find queueing or paying burdensome to different extents. For instance, a poor person with severe health conditions might be less willing to pay to get

[3] Pathak et al. (2021) summarize ethical arguments against this criterion, many of which apply to Definition 1. However, I do not claim that my definition is normatively valid, but rather that it captures empirically important moral sentiments in the problems I study.

[4] Let the priority score be the merit function. Then, strictly speaking, Definition 1 does not require higher allocations for agents with higher priorities. But if we assume that every individual is a separate protected group, it does require equal expected allocations for all agents with the same priority.

[5] In this literature, the terms 'fairness', 'equity' and 'non-discrimination' are used synonymously.

a vaccine early than would a rich person of similar health. Consequently, we might worry that efficiency-enhancing screening could lead to inequitable allocations. Indeed, in many cases governments forgo screening in favor of mechanisms based purely on observables, or lotteries.[6] Examples of goods allocated by lottery include primary and secondary school places (Stone (2008)), public housing (Elster (1989), p.63) and US green cards.[7]

I therefore ask how (if at all) the government can screen agents when allocations are subject to equity constraints. I look at screening using only payments (which are less costly to the rich), only queueing (less costly to the poor) and both of these instruments at once. In the former two cases, equitable screening is impossible. However, when the designer uses both payments and queueing, she has significant freedom to screen despite equity constraints.

3.1 Environment

The government allocates goods $x \in \mathcal{X} = [\underline{x}, \bar{x}]$ to agents with two-dimensional types $(\alpha, \beta) \in \Theta \subseteq \mathbb{R}^2$. Θ is open, connected and bounded. β represents an agent's value for the good and α represents her value for money (higher α means the agent is poorer). I consider two screening instruments—payments and queueing. Payments, $p \in \mathbb{R}$, are costlier for poorer agents (higher α), while queueing, $q \in \mathbb{R}_+$, is costlier for richer agents. Agents have utility:

$$U[\alpha, \beta; x, p, q] = v(\beta, x) - w(\alpha, p) - z(\alpha, q).$$

Assumption 1. *The following conditions hold on the closure of* Θ*:*

1. $v_\beta > 0$, $v_x > 0$, $w_\alpha > 0$, $w_p > 0$, $z_\alpha < 0$, $z_q > 0$, *where the derivatives are continuous.*
2. $v_{\beta x} > 0$; $w_{\alpha p} > 0$; $z_{\alpha q} < 0$.

I assume that both α and β are private information (I discuss this assumption in Subsection 3.5). The government therefore chooses allocations $x : \Theta \to \mathcal{X} \subseteq \mathbb{R}_+$, $p : \Theta \to \mathbb{R}$, $q : \Theta \to \mathbb{R}_+$ subject and IR and IC constraints (I write $y = (x, p, q)$):

$$\text{for all } (\alpha, \beta) \in \Theta, \quad U[\alpha, \beta; y(\alpha, \beta)] \geq 0, \tag{IR}$$

$$\text{for all } (\alpha, \beta) \in \Theta, \quad U[\alpha, \beta; y(\alpha, \beta)] \geq \max_{(\alpha', \beta') \in \Theta} U[\alpha, \beta; y(\alpha', \beta')]. \tag{IC}$$

[6] One might argue, along the lines of Weitzman (1977), that governments eschew screening not due to fairness concerns, but because screening is not optimal when the government has distributional concerns. For instance, if a government allocating affordable housing screened using payments, the allocation would be biased towards wealthier agents. This would be suboptimal for a government redistributive preferences. There are two responses to this argument. First, the government could use a screening instrument other than payments whose cost is *negatively* correlated with wealth; in Subsection 3.3 I study screening with queueing, which plausibly has this feature. Secondly, as observed by Akbarpour et al. (2020), screening with payments is still often optimal even in such settings. This is because the government can generate revenue from selling the good to rich agents and then redistribute it to the poor who value money highly. These observations suggest a deeper reason for the frequent absence of screening in such programs.

[7] My model can capture lotteries when the allocation is binary or the utility for the good is linear: $v(\beta, x) = \beta \cdot x$. In the former case, we can interpret $x \in [0, 1]$ as the probability of being allocated the good.

The government also faces an equity constraint (1) where the merit function $\eta(\alpha,\beta)$ is continuous and strictly increasing in both arguments. That is, agents are more entitled to receiving the good if they value it more or if they are poorer (as richer agents can more easily satisfy their needs without government assistance). I denote the extreme values of $\eta(\alpha,\beta)$ on Θ by:

$$\underline{\eta} = \inf_{(\alpha,\beta)\in\Theta}\{\eta(\alpha,\beta)\}, \quad \overline{\eta} = \sup_{(\alpha,\beta)\in\Theta}\{\eta(\alpha,\beta)\}.$$

Since Θ is connected and $\eta(\alpha,\beta)$ is continuous, for every $\eta' \in (\underline{\eta},\overline{\eta})$ there is some (α',β') such that $\eta(\alpha',\beta') = \eta'$.

I present some useful properties of equitable and implementable allocations. The following lemma shows that, in this environment, the equity constraint (1) simplifies as follows:

Lemma 1. *An allocation rule x is equitable if and only if $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$ for some $\hat{x} : \mathbb{R} \to \mathcal{X}$.*

Proof. If $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, equity holds by definition. For the reverse implication, I show the contrapositive. Let (α_1,β_1) and (α_2,β_2) be distinct types such that $\eta(\alpha_1,\beta_1) = \eta(\alpha_2,\beta_2)$ but $x(\alpha_1,\beta_1) \neq x(\alpha_2,\beta_2)$. If $\alpha_1 \neq \alpha_2$, equity fails, so suppose $\alpha_1 = \alpha_2$. Then, since the types were distinct, $\beta_1 \neq \beta_2$ and, by strict monotonicity, $\eta(\alpha_1,\beta_1) \neq \eta(\alpha_1,\beta_2) = \eta(\alpha_2,\beta_2)$. □

Moreover, any equitable and implementable allocation rule has to be increasing in merit.

Lemma 2. *If $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$ is implementable, then $\hat{x}(\cdot)$ is weakly increasing.*

Proof. Any implementable $x(\alpha,\beta)$ has to be implementable on the subset of agents with $\alpha = \alpha^*$, for any α^*. We can write the utility of such agents as $v(\beta, x(\alpha^*,\beta)) - t(\beta)$, where $t(\beta) := w(\alpha^*, p(\alpha^*,\beta)) + z(\alpha^*, q(\alpha^*,\beta))$. This is a one-dimensional quasi-linear screening problem and so any implementable allocation has to be weakly increasing in β. Hence, $x(\alpha,\beta)$ must be weakly increasing in β for every α. By Lemma 1, an equitable allocation must take the form $\hat{x}(\eta(\alpha,\beta))$. Since η is strictly increasing in β, it follows that $\hat{x}(\cdot)$ has to be increasing. □

3.2 Screening with payments

I first show that the government cannot equitably screen using only payments.

Proposition 1. *If $q \equiv 0$, any equitable and implementable $x(\alpha,\beta)$ is the same for all $(\alpha,\beta) \in \Theta$.*

Proof. By Lemmas 1 and 2, any equitable and implementable allocation $x(\alpha,\beta)$ can be written as $\hat{x}(\eta(\alpha,\beta))$, where $\hat{x}(\cdot)$ is weakly increasing. Note also that $p(\alpha,\beta) \equiv \hat{p}(\eta(\alpha,\beta))$ because identical allocations of x have to require identical payments. Moreover, $\hat{p}(\cdot)$ has to be increasing—otherwise one could deviate and receive weakly more x for a strictly smaller p. However, an argument analogous to the proof of Lemma 2 tells us that any implementable $p(\alpha,\beta)$ has to be weakly decreasing in α. Since η is strictly increasing in α, it follows that $\hat{p}(\cdot)$ must be weakly decreasing, and hence constant. Such a payment scheme can only support a constant allocation. □

Intuitively, equity requires that poor agents get the same allocation as richer agents, even when their need is higher. However, these richer agents will have a higher willingness to pay for the good, as they value it more and value money less (Figure 1). Therefore, any mechanism that sells the good will allocate more to the rich agents, and hence violate equity.

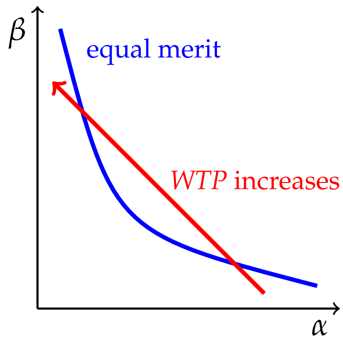

Figure 1

A higher-level intuition is also useful here—notice we can only screen agents based on their willingness to pay. While some agents are willing to pay more because they value the good more (higher β), others are willing to do so because they are wealthier (i.e. because of the value of their protected characteristic α). Screening for the latter would violate equity, but screening for the former would not. However, agents' characteristics are not observed, so the government cannot isolate the former effect. As a result, any form of screening inevitably features both effects, and hence violates equity.

3.3 Screening with queueing

I now show that, generically, the government cannot screen equitably using only queueing. Intuitively, screening with queueing differs from screening with payments in that the allocation it produces is skewed *towards the poor*. That is, between two people with the same β, the poorer one will be more eager to queue for the good, and hence will receive a higher allocation. In contrast to the case of screening with payments, this direction of bias is consistent with what the merit function requires. It is therefore less clear why screening with queueing is also incompatible with equity. Notice, however, that equity constraints impose requirements not only on the direction of the bias, but also on its exact form—the merit function $\eta(\alpha, \beta)$ specifies exactly the sets of types that have to be treated identically. However, with one screening device only, the designer will generically have 'too few degrees of freedom' to pool agents in this exact way. Proposition 2 formalizes this reasoning.

Definition 2. *A merit function is **generic** if the following conditions fail for every $\eta^* \in (\underline{\eta}, \overline{\eta})$:*

1. *There exist $x \in \mathcal{X}$ and $q, k \in \mathbb{R}$ such that for all (α, β) with $\eta(\alpha, \beta) = \eta^*$:*

$$\frac{v_x(\beta, x)}{z_q(\alpha, q)} = k. \tag{2}$$

2. *There exist $x_1 \neq x_2 \in \mathcal{X}, q_1, q_2 \in \mathbb{R}$ such that for all (α, β) with $\eta(\alpha, \beta) = \eta^*$:*

$$v(\beta, x_1) - z(\alpha, q_1) = v(\beta, x_2) - z(\alpha, q_2). \tag{3}$$

Proposition 2. *Suppose $p \equiv 0$. Then, if the merit function η is generic, any equitable and implementable $x(\alpha, \beta)$ is the same for all $(\alpha, \beta) \in \Theta$.*

While the proof is relegated to the appendix, I provide its key intuition. Notice $q(\alpha, \beta) \equiv \hat{q}(\eta(\alpha, \beta))$ since identical allocations of x have to come with equal queueing requirements. Moreover, note that $\hat{q}(\cdot)$ has to be increasing—otherwise one could deviate and receive weakly more x for a strictly smaller q. Now, consider the case where $\hat{x}(\cdot)$ and $\hat{q}(\cdot)$ are smoothly increasing around some η^*. Then the FOCs of all agents with merit η^* must hold:

$$\text{for all } (\alpha, \beta) : \ \eta(\alpha, \beta) = \eta^*, \quad \frac{v_x(\beta, \hat{x}(\eta^*))}{z_q(\alpha, \hat{q}(\eta^*))} = \frac{\hat{q}'(\eta^*)}{\hat{x}'(\eta^*)}. \tag{4}$$

However, Condition 2 of Definition 2 says that (4) cannot hold for all such agents when $\eta(\alpha, \beta)$ is generic (Figure 2). Hence, some of them will have a larger (smaller) *MRS* at that allocation, and will want to mimic agents with a slightly higher (lower) η. Condition 3 relates to a similar requirement for values of $\eta(\alpha, \beta)$ where $\hat{x}(\cdot)$ and $\hat{q}(\cdot)$ jump discontinuously.

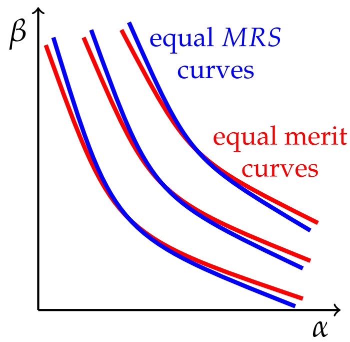

Figure 2

3.4 Screening with payments and queueing

I now let the government use both screening devices at once. To keep the model tractable, I impose more structure:

$$U[\alpha, \beta; x, p, q] = \beta x - w(\alpha) p - z(\alpha) q.$$

Assumption 2. *The following conditions hold on the closure of Θ:*

1. *β, $w(\alpha)$, $z(\alpha) > 0$.*

2. *$w(\alpha)$ and $z(\alpha)$ are twice continuously differentiable.*

3. *The first, second and cross partials of $\eta(\alpha, \beta)$ exist and are continuous.*

As it turns out, using both screening devices allows for rich screening without violating the equity constraint. Intuitively, every amount of x can now come with a menu of payment options composed of different amounts of p and q. Since the designer has one screening device preferred by the poor and another one preferred by the rich, she can fine-tune such 'payment menus' to produce precisely the bias in allocation that equity requires. Loosely speaking, being able to compose such menus fixes the problem of 'too few degrees of freedom' we encountered when only queueing was available.

Proposition 3. *Let $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, where $\hat{x}(\cdot)$ is increasing and twice differentiable. Then $x(\alpha,\beta)$ is equitable and implementable.*

Proof. To simplify the proof, I first reparametrize types:

$$\kappa = \frac{\beta}{z(\alpha)}, \quad \lambda = \frac{w(\alpha)}{z(\alpha)}, \quad \tilde{\Theta} = \left\{ \left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)} \right) : (\alpha,\beta) \in \Theta \right\}.$$

Then every (α,β) corresponds to a unique (κ,λ). To see that, take (α_1,β_1), (α_2,β_2) such that:

$$\frac{\beta_1}{z(\alpha_1)} = \frac{\beta_2}{z(\alpha_2)}; \quad \frac{w(\alpha_1)}{z(\alpha_1)} = \frac{w(\alpha_2)}{z(\alpha_2)} \tag{5}$$

Suppose that $\alpha_1 > \alpha_2$. Then, since $z_\alpha < 0$ and $w_\alpha > 0$,

$$\frac{w(\alpha_1)}{z(\alpha_1)} > \frac{w(\alpha_2)}{z(\alpha_2)},$$

which contradicts (5), so $\alpha_1 = \alpha_2$. Then, the first equation in (5) gives $\beta_1 = \beta_2$.[8]

Now, define:

$$\underline{\kappa} = \inf_{(\alpha,\beta)\in\Theta} \frac{\beta}{z(\alpha)}, \quad \bar{\kappa} = \sup_{(\alpha,\beta)\in\Theta} \frac{\beta}{z(\alpha)}, \quad \underline{\lambda} = \inf_{(\alpha,\beta)\in\Theta} \frac{w(\alpha)}{z(\alpha)}, \quad \bar{\lambda} = \sup_{(\alpha,\beta)\in\Theta} \frac{w(\alpha)}{z(\alpha)}.$$

Since Θ is bounded, Assumption 2 guarantees that $\infty > \underline{\lambda}, \underline{\kappa}, \bar{\lambda}, \bar{\kappa} > -\infty$, so $\tilde{\Theta}$ is bounded. Agents' utilities (up to scaling) in the reparametrized model are given by:

$$\tilde{U}[\kappa,\lambda; x, p, q] = \kappa x - \lambda p - q.$$

I will also write the payment rule and the merit function in the (κ,λ) space:

$$\tilde{p}\left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)} \right) \equiv p(\alpha,\beta), \quad \tilde{\eta}\left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)} \right) \equiv \eta(\alpha,\beta). \tag{6}$$

Since $\eta(\alpha,\beta)$ was strictly increasing in β and α, $\tilde{\eta}(\kappa,\lambda)$ is strictly increasing in κ and λ. Moreover, Assumption 2 ensures that $\tilde{\eta}(\kappa,\lambda)$ has continuous first and second partials on the closure of $\tilde{\Theta}$.

By Lemma 1, any allocation of the form $\hat{x}(\tilde{\eta}(\kappa,\lambda))$ is equitable. By Proposition 2 in Rochet

[8] Note this would no longer hold if both screening devices were less costly to the rich (or to the poor).

(1987), $\hat{x}(\tilde{\eta}(\kappa,\lambda))$ is implementable if there is a convex function $V(\kappa,\lambda):\tilde{\Theta}\to\mathbb{R}$ such that:[9]

$$\frac{d}{d\kappa}V(\kappa,\lambda)=\hat{x}(\tilde{\eta}(\kappa,\lambda)). \tag{R}$$

Notice that (R) holds for the following $V(\kappa,\lambda)$:

$$V(\kappa,\lambda)=\int_{\underline{\kappa}}^{\kappa}\hat{x}(\tilde{\eta}(\tau,\lambda))\,d\tau+\int_{\underline{\lambda}}^{\lambda}\zeta(\tau)\,d\tau, \tag{7}$$

where $\zeta:[\underline{\lambda},\bar{\lambda}]\to\mathbb{R}$ is some differentiable function. I now show that we can choose $\zeta(\cdot)$ so that V defined by (7) is convex. The Hessian of $V(\kappa,\lambda)$ is:

$$H(\kappa,\lambda)=\begin{bmatrix}\hat{x}'(\tilde{\eta}(\kappa,\lambda))\cdot\tilde{\eta}_{\kappa}(\kappa,\lambda) & \hat{x}'(\tilde{\eta}(\kappa,\lambda))\cdot\tilde{\eta}_{\lambda}(\kappa,\lambda)\\ \hat{x}'(\tilde{\eta}(\kappa,\lambda))\cdot\tilde{\eta}_{\lambda}(\kappa,\lambda) & r(\kappa,\lambda)\end{bmatrix},$$

where

$$r(\kappa,\lambda)=\int_{\underline{\kappa}}^{\kappa}\underbrace{\hat{x}''(\tilde{\eta}(\tau,\lambda))\cdot\tilde{\eta}_{\lambda}^{2}(\tau,\lambda)+\hat{x}'(\tilde{\eta}(\tau,\lambda))\cdot\tilde{\eta}_{\lambda\lambda}(\tau,\lambda)}_{:=D(\kappa,\lambda)}\,d\tau+\zeta'(\lambda).$$

It suffices to show that the determinant of H, and $H_{(1,1)},H_{(2,2)}$ are positive everywhere. $H_{(1,1)}$ is positive since $\tilde{\eta}_{\kappa}(\kappa,\lambda)>0$ and $\hat{x}(\cdot)$ is increasing by assumption. For the determinant, we want to show that for every $\kappa\in(\underline{\kappa},\bar{\kappa})$, $\lambda\in(\underline{\lambda},\bar{\lambda})$:

$$\hat{x}'(\tilde{\eta}(\kappa,\lambda))\cdot\tilde{\eta}_{\kappa}(\kappa,\lambda)\cdot\ r(\kappa,\lambda)\ -\ [\hat{x}'(\tilde{\eta}(\kappa,\lambda))\cdot\tilde{\eta}_{\lambda}(\kappa,\lambda)]^{2}\geq 0. \tag{8}$$

If for some (κ,λ) we have $\hat{x}'(\tilde{\eta}(\kappa,\lambda))=0$, the inequality holds, so consider the opposite case. Then $\hat{x}'(\tilde{\eta}(\kappa,\lambda)),\eta_{\kappa}(\kappa,\lambda),\eta_{\lambda}(\kappa,\lambda)>0$, so (8) is equivalent to:

$$r(\kappa,\lambda)\geq x'(\tilde{\eta}(\kappa,\lambda))\cdot\frac{\tilde{\eta}_{\lambda}(\kappa,\lambda)^{2}}{\tilde{\eta}_{\kappa}(\kappa,\lambda)}.$$

$$\zeta'(\lambda)\geq x'(\tilde{\eta}(\kappa,\lambda))\cdot\frac{\tilde{\eta}_{\lambda}(\kappa,\lambda)^{2}}{\tilde{\eta}_{\kappa}(\kappa,\lambda)}-\int_{\underline{\kappa}}^{\kappa}D(\kappa,\lambda)d\kappa.$$

Notice that it suffices to show that the RHS is uniformly bounded across κ.

Recall that the first and second partials of $\tilde{\eta}$ and the first and second derivatives of $\hat{x}$ are continuous on the closure of $\tilde{\Theta}$. Since $\tilde{\Theta}$ is bounded, the first term on the RHS and $D(\kappa,\lambda)$ are uniformly bounded. Since κ is bounded by $\underline{\kappa},\bar{\kappa}\in\mathbb{R}$, the RHS is indeed uniformly bounded. Therefore, we can ensure (8) by choosing high-enough values for all $\zeta'(\lambda)$. Moreover, we can choose these values high enough to make $r(\kappa,\lambda)\geq 0$ everywhere. This also ensures $H_{(2,2)}\geq 0$ everywhere. □

[9] Rochet (1987) assumes $\tilde{\Theta}$ is convex. However, I can add 'fictitious types' enlarging the type space to $(\underline{\kappa},\bar{\kappa})\times(\underline{\lambda},\bar{\lambda})\supseteq\tilde{\Theta}$ and implement an extended allocation on it that coincides with $\hat{x}(\tilde{\eta}(\kappa,\lambda))$ on $\tilde{\Theta}$.

3.5 Observable characteristics

While I assumed that neither need nor wealth are observable, the government usually has some information about them. For instance, in the problem of vaccine allocation, age and medical history are good indicators of need. Similarly, tax data gives the government an idea of one's wealth (even if some income sources or assets remain unobserved). In such cases, agents' private information can be thought of as *residual uncertainty* after accounting for these observables. Indeed, even public programs conditioned on earnings face substantial uncertainty over one's wealth.[10]

Still, what can the government do if it perfectly observes the protected characteristic? As it turns out, this gives it significant freedom to screen as long as it suitably adjusts the mechanism. Intuitively, the designer can now 'control for' the fact that some agents prefer the good (relative to money or queueing) because of their protected characteristic and screen purely based on need.

Proposition 4. *Suppose α is observable and the government uses either only money or only queueing to screen. Then $x(\alpha, \beta)$ is equitable and implementable if and only if $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$, where $\hat{x}$ is increasing.*

Proof. Since α is observable, the allocation can be implemented separately for every value of α. Then, by the argument in the proof of Lemma 2, any implementable $x(\alpha, \beta)$ must be weakly increasing in β. By Lemma 1 the equity constraint is satisfied if and only if $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$. Since $\eta(\alpha, \beta)$ is strictly increasing in β, it follows that $\hat{x}$ is implementable if and only if it is weakly increasing. □

3.6 Discussion

While my approach to modeling perceived fairness is highly stylized, it offers more general qualitative conclusions. First, every screening instrument will bias the allocation towards the group for whom this instrument is less costly; this makes screening with payments especially problematic from an equity standpoint. Using a different instrument (like queueing) could reverse this bias, but the designer's control over the allocation would still be limited. Consequently, the resulting bias might still not satisfy the public. I show this problem can be solved by combining multiple screening instruments which on their own favor different social groups. Doing so gives the designer freedom to temper with various groups' differential cost of the allocated good, and therefore to improve efficiency through screening while still producing an allocation that is seen as fair.

[10] https://thehill.com/regulation/administration/268409-outrage-builds-over-wealthy-families-in-public-housing/

4 Appendix: omitted proofs

4.1 Proof of Proposition 2

Since $\hat{x}(\cdot)$ and $\hat{q}(\cdot)$ are weakly increasing on $(\underline{\eta}, \overline{\eta})$, they have left and right limits everywhere. I denote them by $\hat{x}_+(\cdot)$, $\hat{x}_-(\cdot)$ and $\hat{q}_+(\cdot)$, $\hat{q}_-(\cdot)$. I first show that $\hat{q}(\cdot)$ is discontinuous at η^* if and only if $\hat{x}(\cdot)$ is. Consider any type (α_1, β_1) with merit η^* and define[11]

$$\Phi(\tau) := U[\alpha_1 + \tau, \beta_1 + \tau;\ \eta(\alpha_1 + \tau, \beta_1 + \tau)].$$

Since Θ is open, there exists $\chi > 0$ such that $(\alpha_1 - \chi, \beta_1 - \chi), (\alpha_1 + \chi, \beta_1 + \chi) \in \Theta$. Then $\Phi(\tau)$ is continuous on $[-\chi, \chi]$ (Milgrom and Segal (2002)). Therefore, $\lim_{\epsilon \to 0+} \Phi(\epsilon) = \lim_{\epsilon \to 0-} \Phi(\epsilon)$. Furthermore, by continuity of η, v and z, this implies:

$$U[\alpha_1, \beta_1;\ \hat{x}_+(\eta^*), \hat{q}_+(\eta^*)] = U[\alpha_1, \beta_1;\ \hat{x}_-(\eta^*), \hat{q}_-(\eta^*)],$$

$$v(\beta_1, \hat{x}_+(\eta^*)) - z(\alpha_1, \hat{q}_+(\eta^*)) = v(\beta_1, \hat{x}_-(\eta^*)) - z(\alpha_1, \hat{q}_-(\eta^*)). \tag{9}$$

Since v and z are weakly increasing in their latter arguments, the left and right limits of $\hat{q}(\cdot)$ at η^* agree if and only if those of $\hat{x}(\cdot)$ do.

Now, fix a generic merit function $\eta(\alpha, \beta)$. If $\hat{x}(\cdot)$ and $\hat{q}(\cdot)$ were discontinuous at some η^*, (9) would hold there. However, $\hat{x}_-(\eta^*) \neq \hat{x}_+(\eta^*)$, which would violate condition 3 in Definition 2. Hence, $\hat{x}(\cdot)$ and $\hat{q}(\cdot)$ have to be continuous on $(\underline{\eta}, \overline{\eta})$.

Suppose towards a contradiction that $\hat{x}$ is not constant. In what follows I write $x_\eta := \hat{x}(\eta)$ and $q_\eta := \hat{q}(\eta)$. I first show the following fact:

Fact 1. *For some $\eta^* \in (\underline{\eta}, \overline{\eta})$, there exists a sequence $\{\eta_i\}_i, \eta_i \to \eta^*$ such that for every $i > j$, $x_{\eta_i} > x_{\eta_j}$.*

Proof. $\hat{x}(\cdot)$ is not constant, so there are $\eta_0, \eta_1' \in (\underline{\eta}, \overline{\eta})$ s.t. $\eta_0 < \eta_1'$ and $x_{\eta_0} < x_{\eta_1'}$. Let $\eta^* = \inf\{\eta : x_\eta = x_{\eta_1'}\}$. Since $\hat{x}(\cdot)$ is continuous, $x_{\eta^*} = x_{\eta_1'}$. Now, take any strictly increasing sequence $\{\eta_i'\}_i$ s.t. for every i, $\eta_i' \in (\underline{\eta}, \overline{\eta})$ and $\eta_i' \to \eta^*$. Since $\hat{x}(\cdot)$ is increasing, $x_{\eta_i} \geq x_{\eta_j}$ for every $i > j$. Also, by continuity of $\hat{x}(\cdot)$, $x_{\eta_i'} \to x_{\eta^*}$. Moreover, by the definition of η^*, $x_{\eta_i'} < x_{\eta^*}$ for every i. Now, we can take a subsequence $\{\eta_i\}_i$ of $\{\eta_i'\}_i$ for which x_{η_i} is increasing strictly. □

Fix η^* and $\{\eta_i\}_i$ as in Fact 1. Then for every $i > j$ we have $x_{\eta_i} > x_{\eta_j}$, and so $q_{\eta_i} > q_{\eta_j}$—otherwise an agent with merit η_j would prefer (x_{η_i}, q_{η_i}) over her assignment. Now, for every (α, β):

$$\frac{z(\alpha, q_{\eta^*}) - z(\alpha, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}} \to z_q(\alpha, q_{\eta^*}); \quad \frac{v(\beta, x_{\eta^*}) - v(\beta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}} \to v_x(\beta, x_{\eta^*}).$$

Since the merit function was generic, Condition 2 guarantees that there exist (α_1, β_1) and (α_2, β_2) with merit η^* such that:

$$\frac{v_x(\beta_2, x_{\eta^*})}{z_q(\alpha_2, q_{\eta^*})} > \frac{v_x(\beta_1, x_{\eta^*})}{z_q(\alpha_1, q_{\eta^*})}.$$

[11] In what follows I sometimes abuse notation by writing U as a function of η rather than $\hat{x}(\eta)$ and $\hat{q}(\eta)$.

Consider $(\alpha_\delta, \beta_\delta) := (\alpha_2 - \delta, \beta_2 - \delta)$ for $\delta > 0$. By continuity of v_x and z_q, there is some $\delta^* > 0$ such that for $\delta \in (0, \delta^*)$ we have:

$$\frac{v_x(\beta_\delta, x_{\eta^*})}{z_q(\alpha_\delta, q_{\eta^*})} > \frac{v_x(\beta_1, x_{\eta^*})}{z_q(\alpha_1, q_{\eta^*})}. \tag{10}$$

Then, for any such δ and sufficiently large i, we have:

$$\frac{\frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} > \frac{\frac{v(\beta_1, x_{\eta^*}) - v(\beta_1, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_1, q_{\eta^*}) - z(\alpha_1, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}}. \tag{11}$$

Now, the following holds by revealed preference:

$$\text{for all } i, \quad U[\alpha_1, \beta_1; \eta^*] \geq U[\alpha_1, \beta_1; \eta_i]. \tag{12}$$

Equivalently:

$$\frac{\frac{v(\beta_1, x_{\eta^*}) - v(\beta_1, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_1, q_{\eta^*}) - z(\alpha_1, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} \geq \frac{q_{\eta^*} - q_{\eta_i}}{x_{\eta^*} - x_{\eta_i}}. \tag{13}$$

But then by (11), for any $\delta \in (0, \delta^*)$ and sufficiently large i:

$$\frac{\frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} > \frac{q_{\eta^*} - q_{\eta_i}}{x_{\eta^*} - x_{\eta_i}} \implies \frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})} > 1. \tag{14}$$

We can now fix some $\delta_1 \in (0, \delta^*)$ and select i so that $\eta_i > \eta(\alpha_{\delta_1}, \beta_{\delta_1})$. Then, since $\eta(\alpha_\delta, \beta_\delta)$ is continuous and strictly decreasing in δ, there exists some $\delta_2 \in (0, \delta_1)$ such that $\eta_i = \eta(\alpha_{\delta_2}, \beta_{\delta_2})$. Moreover, the LHS of the latter inequality in (14) is strictly decreasing in δ, so:

$$v(\beta_{\delta_2}, x_{\eta^*}) - v(\beta_{\delta_2}, x_{\eta(\alpha_{\delta_2}, \beta_{\delta_2})}) > z(\alpha_{\delta_2}, q_{\eta^*}) - z(\alpha_{\delta_2}, q_{\eta(\alpha_{\delta_2}, \beta_{\delta_2})}).$$

Equivalently, $U[\alpha_{\delta_2}, \beta_{\delta_2}; \eta^*] > U[\alpha_{\delta_2}, \beta_{\delta_2}; \eta(\alpha_{\delta_2}, \beta_{\delta_2})]$, which contradicts revealed preference.

References

ABDULKADIROĞLU, A. AND T. SÖNMEZ (2003): "School Choice: A Mechanism Design Approach," *American Economic Review*, 93, 729–747.

AKBARPOUR, M., E. BUDISH, P. DWORCZAK, AND S. D. KOMINERS (2023): "An Economic Framework for Vaccine Prioritization*," *The Quarterly Journal of Economics*, qjad022.

AKBARPOUR, M., P. DWORCZAK, AND S. D. KOMINERS (2020): "Redistributive allocation mechanisms," *Available at SSRN 3609182*.

ALVES, G., F. BERNIER, M. COUCEIRO, K. MAKHLOUF, C. PALAMIDESSI, AND S. ZHIOUA (2023): "Survey on Fairness Notions and Related Tensions," .

BAROCAS, S., M. HARDT, AND A. NARAYANAN (2019): *Fairness and Machine Learning: Limitations and Opportunities*, fairmlbook.org, http://www.fairmlbook.org.

CONDORELLI, D. (2013): "Market and non-market mechanisms for the optimal allocation of scarce resources," *Games and Economic Behavior*, 82, 582–591.

CORBETT-DAVIES, S., E. PIERSON, A. FELLER, S. GOEL, AND A. HUQ (2017): "Algorithmic Decision Making and the Cost of Fairness," in *Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, New York, NY, USA: Association for Computing Machinery, KDD '17, 797–806.

DESSEIN, W., A. FRANKEL, AND N. KARTIK (2023): "Test-Optional Admissions," *arXiv preprint arXiv:2304.07551*.

DWORK, C., M. HARDT, T. PITASSI, O. REINGOLD, AND R. ZEMEL (2012): "Fairness through Awareness," in *Proceedings of the 3rd Innovations in Theoretical Computer Science Conference*, New York, NY, USA: Association for Computing Machinery, ITCS '12, 214–226.

EINAV, L. AND A. FINKELSTEIN (2023): *We've got you covered: Rebooting American Health Care*, Portfolio/Penguin, an imprint of Penguin Random House LLC.

ELSTER, J. (1989): *Solomonic judgements: Studies in the limitation of rationality*, Cambridge University Press.

EMANUEL, E. J., G. PERSAD, R. UPSHUR, B. THOME, M. PARKER, A. GLICKMAN, C. ZHANG, C. BOYLE, M. SMITH, AND J. P. PHILLIPS (2020): "Fair Allocation of Scarce Medical Resources in the Time of Covid-19," *New England Journal of Medicine*, 382, 2049–2055, pMID: 32202722.

KAMADA, Y. AND F. KOJIMA (2023): "Fair Matching under Constraints: Theory and Applications," *The Review of Economic Studies*, rdad046.

KASS, L. R. (1997): "The wisdom of repugnance: Why we should ban the cloning of humans," *Val. UL Rev.*, 32, 679.

MILGROM, P. AND I. SEGAL (2002): "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, 70, 583–601.

ONUCHIC, P. (2022): "Recent contributions to theories of discrimination," *arXiv preprint arXiv:2205.05994*.

PATHAK, P. A., T. SÖNMEZ, M. U. ÜNVER, AND M. B. YENMEZ (2021): "Fair Allocation of Vaccines, Ventilators and Antiviral Treatments: Leaving No Ethical Value Behind in Health Care Rationing," in *Proceedings of the 22nd ACM Conference on Economics and Computation*, New York, NY, USA: Association for Computing Machinery, EC '21, 785–786.

ROCHET, J.-C. (1987): "A necessary and sufficient condition for rationalizability in a quasi-linear context," *Journal of Mathematical Economics*, 16, 191–200.

ROTH, A. E. (2007): "Repugnance as a Constraint on Markets," *Journal of Economic Perspectives*, 21, 37–58.

STONE, P. (2008): "What can lotteries do for education?" *Theory and Research in Education*, 6, 267–282.

EEOC v. Sambo's of Georgia (1981): "Equal Employment Opportunity Commission v. Sambo's of Georgia, Inc., 530 F. Supp. 86, 92 (N.D. Ga. 1981)," .

WALZER, M. (1983): "Spheres of Justice. A defense of Pluralism and Equality. New York: Basis Book," *Inc., Publishers*.

WEITZMAN, M. L. (1977): "Is the Price System or Rationing More Effective in Getting a Commodity to Those Who Need it Most?" *The Bell Journal of Economics*, 8, 517–524.

ZUCKER, H., K. ADLER, D. BERENS, R. BLEICH, R. BRYNNER, K. BUTLER, ET AL. (2015): "Ventilator allocation guidelines. New York State task force on life and the law; 2015," .