# Equitable screening*

Filip Tokarski

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**Abstract**

I study the problem of a government providing benefits while considering the perceived equity of the resulting allocation. Such concerns are modeled through an equity constraint requiring that equally deserving agents receive equal allocations. I ask what forms of efficiency-enhancing screening are compatible with equity and show that while the government cannot equitably screen with a single instrument (e.g. payments or wait times), combining multiple instruments, which on their own favor different groups, allows it to screen while still producing an equitable allocation.

## 1 Introduction

In 2017, the French president Emmanuel Macron unveiled a set of environmental policies aimed at reducing carbon emissions. The plan involved a gradual increase in fuel taxes, including a significant hike in diesel and petrol prices the following year. However, the proposed tax increase sparked protests and riots which later expanded into the 'Yellow Vest' movement. The protesters, largely from rural and less affluent areas, objected to the policy on equity grounds. They claimed that households in poor financial standing were disproportionately bearing the cost of decarbonization, and that the tax unfairly burdened those who couldn't cut down on driving or switch to greener alternatives.[1] Ultimately, mounting unrest led the French government to suspend the planned fuel tax hike.

This paper considers a government allocating goods such as vaccines, affordable housing, basic food items, or emission rights while being concerned about the perceived equity of the allocation. I model equity concerns using a *merit function* specifying societal perceptions of how entitled each agent is to the allocated good. For instance, in the case of emissions, rural households reliant on cars for transportation would be more deserving of a right to emit than urban ones. I then introduce an equity constraint requiring that agents with equal merit receive the same allocation.

I ask what policy designs are compatible with equity. Standard economic logic suggests that public programs can be made more efficient through costly screening—if agents need to pay or go through ordeals to get the benefit, only those who need it will do so (Nichols and Zeckhauser, 1982). Indeed, the literature studying the optimal design of such programs

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[1] https://www.nytimes.com/2018/12/06/world/europe/france-fuel-carbon-tax.html

(e.g. Condorelli (2013), Akbarpour et al. (2020)) finds that the optimal mechanism often involves screening recipients, e.g. through payments. For instance, Akbarpour et al. (2020) find that, under fairly general conditions, the government should sell the good (at possibly non-market prices) and redistribute the revenue to poorer agents.

However, we might worry that screening mechanisms could produce inequitable allocations as people with the same need for the good may find waiting or paying burdensome to different extents. Consider for instance the problem of distributing Covid-19 vaccines studied by Akbarpour et al. (2023); the authors show that the optimal policy combines priorities to vulnerable groups with a market mechanism under which one can pay to be vaccinated early. While proxying for need with willingness-to-pay may improve targeting, such inference is cofounded by differences in wealth—a poor person with severe health conditions might still be less willing to pay for early vaccination than someone who is healthy but rich. Related objections to selling vaccines have been raised by both academic philosophers (Kass, 1997; Walzer, 1983) and the public.[2] Indeed, the authors themselves acknowledge that their proposed policy might provoke backlash on fairness grounds. Such equity concerns may explain why governments often forgo screening in favor of mechanisms based purely on observables, or lotteries.[3] Examples of goods allocated by lottery include primary and school places (Stone, 2008), public housing (Elster (1989), p.63) and US green cards.

I therefore ask how (if at all) the government can screen agents when allocations are subject to equity constraints. I look at screening using only payments (which are less costly to the rich), only waiting (less costly to the poor) and both of these instruments at once. In the former two cases equitable screening is impossible. However, when the designer uses both payments and waiting, she has significant freedom to screen despite equity constraints.

This paper relates to work examining how moral sentiments constrain market designers and policymakers. Roth (2007) discusses how the repugnance of certain transactions precludes the use of markets in settings where they would be efficient. The literature following Abdulkadiroğlu and Sönmez (2003) models fairness concerns in matching markets through assigning *priorities* to agents. It then studies matching mechanisms that eliminate *justified envy*—a notion capturing perceived injustice. Finally, Dessein et al. (2023) argue that many US colleges switched to test-optional admissions to reduce public scrutiny of their admission decisions. However, no existing work studies equity constraints in mechanism design problems of the sort this paper considers. In doing so, my paper also relates to the literatures on algorithmic fairness in Computer Science and on discrimination in Economics, which attempt to conceptualize bias, unfairness and discrimination (see Alves et al. (2023)

[2]https://www.statnews.com/2020/12/03/how-rich-and-privileged-can-skip-the-line-for-covid19-vaccines

[3]One might argue, along the lines of Weitzman (1977), that governments eschew screening not due to fairness concerns, but because screening is not optimal when the government has distributional concerns. For instance, if a government allocating affordable housing screened using payments, the allocation would be biased towards wealthier agents. This would be suboptimal for a government with redistributive preferences. There are two responses to this argument. First, the government could use a screening instrument other than payments whose cost is *negatively* correlated with wealth; in Subsection 3.2 I study screening with waiting which plausibly has this feature. Secondly, as observed by Akbarpour et al. (2020), screening with payments is still often optimal even in such settings. This is because the government can generate revenue from selling the good to rich agents and then redistribute it to the poor who value money highly. These observations suggest a deeper reason for the frequent absence of screening in such programs.

and Onuchic (2022) for respective surveys). However, in both cases, researchers focus on problems of classification or statistical inference and hence do not account for the strategic behavior of agents. By contrast, the purpose of this paper is to study mechanisms that are fair after accounting for strategic responses.

The rest of the paper is structured as follows. I first introduce the model of public provision with an equity constraint. Section 3 then discusses the forms of screening that are feasible in this environment. In Section 4 I ask what allocation rules can be equitably implemented if the government also observes agents' wealth. Section 5 discusses a relaxed version of the equity constraint; Section 6 concludes.

## 2 Model

The government allocates goods $x \in [0,1]$ to agents with types $(\alpha,\beta) \in \Theta \subseteq \mathbb{R}^2$. $\Theta$ is open, connected and bounded. $\beta$ represents an agent's value for the good and $\alpha$ represents her value for money (higher $\alpha$ means the agent is poorer). I consider two screening instruments—payments and wait times. Payments, $p \in \mathbb{R}$, are more burdensome for poorer agents (higher $\alpha$), while waiting, $q \in \mathbb{R}_+$, is costlier for richer agents. Utility is given by:

$$U[\alpha,\beta;x,p,q] = v(\beta,x) - w(\alpha,p) - z(\alpha,q).$$

**Assumption 1.** *The following conditions hold on the closure of* $\Theta$*:*

1. *$v, w, z$ are twice continuously differentiable.*
2. *$v_\beta > 0$, $v_x > 0$, $w_\alpha > 0$, $w_p > 0$, $z_\alpha < 0$, $z_q > 0$.*
3. *$v_{\beta x} > 0$; $\quad w_{\alpha p} > 0$; $\quad z_{\alpha q} < 0$.*

I assume that both $\alpha$ and $\beta$ are private information (I discuss this assumption in Section 4). The government therefore chooses $x : \Theta \to [0,1]$, $p : \Theta \to \mathbb{R}$, $q : \Theta \to \mathbb{R}_+$ subject and (IC) and (IR) constraints (I write $y = (x,p,q)$):

$$\text{for all } (\alpha,\beta) \in \Theta, \quad U[\alpha,\beta; y(\alpha,\beta)] \geq \max_{(\alpha^a,\beta^a)\in\Theta} U[\alpha,\beta; y(\alpha^a,\beta^a)], \tag{IC}$$

$$\text{for all } (\alpha,\beta) \in \Theta, \quad U[\alpha,\beta; y(\alpha,\beta)] \geq v(\beta,0) - w(\alpha,0) - z(\alpha,0). \tag{IR}$$

I call allocations rules $x(\alpha,\beta)$ satisfying (IC) and (IR) for some $p(\alpha,\beta), q(\alpha,\beta)$ *implementable*.

The government also faces an *equity constraint* which I model using an exogenous *merit function* $\eta : \mathbb{R}^2 \to \mathbb{R}$. I assume $\eta(\alpha,\beta)$ is twice continuously differentiable and strictly increasing in both arguments. That is, agents are more entitled to receiving the good if they value it more or if they are poorer (as richer agents can more easily satisfy their needs without government assistance). Since $\Theta$ is open and connected, and the merit function is strictly increasing and continuous, the set of values attained by $\eta(\alpha,\beta)$ on $\Theta$ is an open interval, which I denote by $(\underline{\eta}, \overline{\eta})$. The equity constraint requires that all agents with the same merit receive equal amounts of the good:[4]

$$\eta(\alpha^a, \beta^a) = \eta(\alpha^b, \beta^b) \quad \Longrightarrow \quad x(\alpha^a, \beta^a) = x(\alpha^b, \beta^b). \tag{E}$$

I will call any allocation satisfying (E) *equitable*. Note also that an allocation $x(\alpha, \beta)$ is equitable if and only if it can be written in the form $x(\alpha, \beta) = \hat{x}(\eta(\alpha, \beta))$ for some $\hat{x} : (\underline{\eta}, \overline{\eta}) \to [0, 1]$.

While the equity constraint does not require that agents with higher merit receive more of the good, this will still be the case for any implementable equitable allocation:

**Lemma 1.** *If $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$ is implementable, then $\hat{x}$ is weakly increasing.*

*Proof.* Any implementable $x(\alpha, \beta)$ has to be implementable for the subset of agents with $\alpha = \alpha^a$, for any $\alpha^a$. We can write the utility of such agents as $v(\beta, x(\alpha^a, \beta)) - t(\beta)$, where $t(\beta) := w(\alpha^a, p(\alpha^a, \beta)) + z(\alpha^a, q(\alpha^a, \beta))$. This is a one-dimensional quasi-linear screening problem so any implementable allocation has to be weakly increasing in $\beta$. Hence, $x(\alpha, \beta)$ must be weakly increasing in $\beta$ for every $\alpha$. Recall that an equitable allocation must take the form $\hat{x}(\eta(\alpha, \beta))$. Since $\eta$ is strictly increasing in $\beta$, it follows that $\hat{x}$ has to be increasing. □

# 3 Implementable equitable allocations

I now ask what forms of screening are compatible with equity. I study the sets of implementable equitable allocations in three cases: when the government uses only payments to screen, when it uses only waiting, and when it combines both screening instruments.

## 3.1 Screening with payments

I first show that the government cannot equitably screen using only payments.

**Proposition 1.** *Suppose $q \equiv 0$. Then every equitable and implementable $x(\alpha, \beta)$ is constant.*

*Proof.* By Lemma 1, any implementable and equitable allocation $x(\alpha, \beta)$ can be written as $\hat{x}(\eta(\alpha, \beta))$, where $\hat{x}$ is weakly increasing. Note also that $p(\alpha, \beta) \equiv \hat{p}(\eta(\alpha, \beta))$ because identical allocations of $x$ must require identical payments. An argument analogous to the proof of Lemma 1 tells us that any implementable $p(\alpha, \beta)$ has to be weakly decreasing in $\alpha$. Since $\eta$ is strictly increasing in $\alpha$, it follows that $\hat{p}$ must be weakly decreasing. However, $\hat{p}$ also has to be weakly increasing—otherwise one could deviate and receive a weakly greater allocation of $x$ for a strictly smaller $p$. Therefore, $\hat{p}$ has to be constant. Such a payment scheme can only support a constant allocation. □

Intuitively, equity requires that poorer and richer agents of equal merit receive the same allocation, even though the richer agents have higher need. However, these richer agents with higher need have greater willingness to pay for the good (Figure 1). Therefore, any mechanism that sells the good will allocate more to richer agents, and hence violate equity.

[4] My model can capture lotteries when the allocation is binary or the utility for the good is linear: $v(\beta, x) = \beta \cdot x$. In the former case, we can interpret $x \in [0, 1]$ as the probability of being allocated the good.

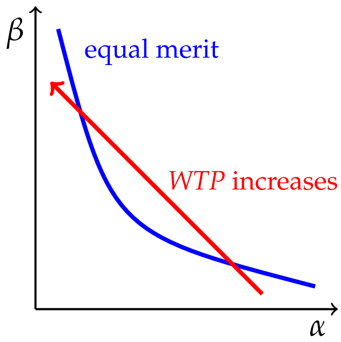


Figure 1

### 3.2 Screening with waiting

I now show that, generically, the government cannot screen equitably using only waiting. Intuitively, screening with waiting differs from screening with payments in that the allocation it produces is skewed *towards the poor*. That is, between two people with the same $\beta$, the poorer one will be more eager to wait to get the good, and hence will receive a higher allocation. Unlike in the case of screening with payments, this direction of bias is consistent with what the merit function requires. It is therefore less clear why screening with waiting is also incompatible with equity. Notice, however, that equity constraints impose requirements not only on the direction of the bias, but also on its exact form—the merit function $\eta(\alpha, \beta)$ specifies exactly the sets of types that have to be treated identically. However, with one screening device only, the designer will generically have 'too few degrees of freedom' to pool agents in this exact way. Proposition 2 formalizes this reasoning.

**Proposition 2.** *Suppose $p \equiv 0$. Then a non-constant $x(\alpha, \beta)$ can be equitable and implementable only for non-generic merit functions.*

While the proof and a discussion of the genericity notion used here are relegated to the appendix, I provide key intuition behind this result. Notice $q(\alpha, \beta) \equiv \hat{q}(\eta(\alpha, \beta))$ since identical allocations of $x$ have to come with equal wait times. Moreover, note that $\hat{q}$ has to be increasing—otherwise one could deviate and receive weakly more $x$ for a strictly smaller $q$. Now, consider the case where $\hat{x}$ and $\hat{q}$ are smoothly increasing around some $\eta^a$. Then the FOCs of all agents with merit $\eta^a$ must hold there:

$$\text{for all } (\alpha, \beta) \text{ such that } \eta(\alpha, \beta) = \eta^a, \quad \frac{v_x(\beta, \hat{x}(\eta^a))}{z_q(\alpha, \hat{q}(\eta^a))} = \frac{\hat{q}'(\eta^a)}{\hat{x}'(\eta^a)}. \tag{1}$$

However, a pair of $\hat{x}(\eta^a)$ and $\hat{q}(\eta^a)$ at which the *MRS*s of all agents with merit $\eta^a$ are equal will only exist for non-generic merit functions. That is, even if $\hat{x}(\eta^a)$ and $\hat{q}(\eta^a)$ satisfying (1) exist for some merit function, they no longer will after the merit function is perturbed slightly. As a result, equal *MRS* curves will not align with equal merit curves (Figure 2) and some agents with merit $\eta^a$ will want to deviate to allocations slightly above or below $\hat{x}(\alpha^a)$.

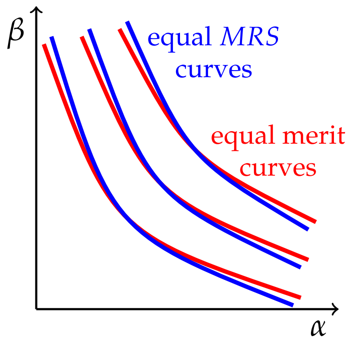


Figure 2

### 3.3 Screening with payments and waiting

I now let the government use both screening instruments at once. To keep the model tractable, I impose more structure on the utility function:

$$U[\alpha,\beta;x,p,q] = \beta x - w(\alpha)p - z(\alpha)q.$$

As it turns out, using both screening devices allows for rich screening without violating the equity constraint. Intuitively, every amount of $x$ can now come with a menu of payment options composed of different amounts of $p$ and $q$. Since the designer has one screening device preferred by the poor and another one preferred by the rich, she can fine-tune such 'payment menus' to produce precisely the bias in allocation that equity requires. Loosely speaking, being able to compose such menus fixes the problem of 'too few degrees of freedom' we encountered when only wait times were used.

**Theorem 1.** *An allocation rule $x(\alpha,\beta)$ is equitable and implementable if and only if $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, where $\hat{x}$ is increasing.*

*Proof.* To simplify the proof, I first reparametrize types:

$$\kappa = \frac{\beta}{z(\alpha)}, \qquad \lambda = \frac{w(\alpha)}{z(\alpha)}, \qquad \tilde{\Theta} = \left\{\left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)}\right) : (\alpha,\beta) \in \Theta\right\}.$$

Then every $(\alpha,\beta)$ corresponds to a unique $(\kappa,\lambda)$. To see that, take $(\alpha^a,\beta^a)$, $(\alpha^b,\beta^b)$ such that:

$$\frac{\beta^a}{z(\alpha^a)} = \frac{\beta^b}{z(\alpha^b)}; \qquad \frac{w(\alpha^a)}{z(\alpha^a)} = \frac{w(\alpha^b)}{z(\alpha^b)}. \tag{2}$$

Suppose that $\alpha^a > \alpha^b$. Since $z_\alpha < 0$ and $w_\alpha > 0$, this imlies:

$$\frac{w(\alpha^a)}{z(\alpha^a)} > \frac{w(\alpha^b)}{z(\alpha^b)},$$

which contradicts (2), so $\alpha^a = \alpha^b$. The first equation in (2) then gives $\beta^a = \beta^b$ (note that the argument would no longer work if both screening devices were less costly to the rich or to the poor).

Agents' utilities (up to scaling) in the reparametrized model are given by:

$$\tilde{U}[\kappa,\lambda;x,p,q] = \kappa x - \lambda p - q.$$

I also write the merit function in $(\kappa,\lambda)$ space:

$$\tilde{\eta}\left(\frac{\beta}{z(\alpha)},\frac{w(\alpha)}{z(\alpha)}\right) \equiv \eta(\alpha,\beta). \tag{3}$$

Since $\eta(\alpha,\beta)$ was twice continuously differentiable and strictly increasing in $\alpha$ and $\beta$, $\tilde{\eta}(\kappa,\lambda)$ is twice continuously differentiable and strictly increasing in $\kappa$ and $\lambda$. Finally, Assumption 1 guarantees that the following bounds are finite:

$$\underline{\kappa} = \inf_{(\alpha,\beta)\in\Theta}\frac{\beta}{z(\alpha)},\quad \bar{\kappa} = \sup_{(\alpha,\beta)\in\Theta}\frac{\beta}{z(\alpha)},\quad \underline{\lambda} = \inf_{(\alpha,\beta)\in\Theta}\frac{w(\alpha)}{z(\alpha)},\quad \bar{\lambda} = \sup_{(\alpha,\beta)\in\Theta}\frac{w(\alpha)}{z(\alpha)}.$$

By Lemma 1, any equitable and implementable allocation rule has to take the form $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, where $\hat{x}$ is increasing. It therefore suffices to show that any such allocation rule is implementable on $\Theta$. In the reparametrized type space this amounts to showing that any $\tilde{x}(\kappa,\lambda) : \tilde{\Theta} \to [0,1]$ such that $\tilde{x}(\kappa,\lambda) \equiv \hat{x}(\tilde{\eta}(\kappa,\lambda))$, where $\hat{x}$ is increasing, is implementable on $\tilde{\Theta}$. In fact, I prove a stronger statement: consider a extension of the reparametrized type space to $[\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}]$. Define $\underline{\tilde{\eta}} := \tilde{\eta}(\underline{\kappa},\underline{\lambda})$ and $\bar{\tilde{\eta}} := \tilde{\eta}(\bar{\kappa},\bar{\lambda})$ and notice that $(\underline{\eta},\bar{\eta}) \subseteq [\underline{\tilde{\eta}},\bar{\tilde{\eta}}]$. I then show that for any increasing $\hat{x} : [\underline{\tilde{\eta}},\bar{\tilde{\eta}}] \to [0,1]$ there exists an allocation rule $\tilde{x} : [\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}] \to [0,1]$ that is implementable on the extended type space and satisfies $\tilde{x}(\kappa,\lambda) = \hat{x}(\tilde{\eta}(\kappa,\lambda))$. To that end, define a *threshold rule* as a function $\hat{x} : [\underline{\tilde{\eta}},\bar{\tilde{\eta}}] \to [0,1]$ satisfying:

$$\hat{x}(\eta) = \begin{cases} 1 & \text{if } \eta \geq \eta^* \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

for some $\eta^* \in [\underline{\tilde{\eta}},\bar{\tilde{\eta}}]$. Let $\mathcal{T}$ be the set of threshold rules.

**Lemma 2.** *For every threshold rule $\hat{x} \in \mathcal{T}$ there exists an allocation rule $\tilde{x}$ that is implementable on $[\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}]$ and satisfies $\tilde{x}(\kappa,\lambda) \equiv \hat{x}(\tilde{\eta}(\kappa,\lambda))$.*

*Proof.* Fix some $\hat{x} \in \mathcal{T}$ and define $\kappa^*(\lambda)$ by $\eta^* = \tilde{\eta}(\kappa^*(\lambda),\lambda)$. Since $\tilde{\eta}(\kappa,\lambda)$ was twice continuously differentiable and strictly increasing, $\kappa^*(\lambda)$ is twice continuously differentiable and strictly decreasing. Let $M := \max_{\lambda\in[\underline{\lambda},\bar{\lambda}]} \kappa^{*\prime\prime}(\lambda)$ be the (finite) upper bound on its second derivative.

I now prove there exists $\tilde{x}$ that is implementable on $[\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}]$ and satisfies $\tilde{x}(\kappa,\lambda) \equiv \hat{x}(\tilde{\eta}(\kappa,\lambda))$. Following Rochet (1987), I do so by constructing an indirect utility function $V(\kappa,\lambda)$ such that *a*) there exists a payment rule $\tilde{p}(\kappa,\lambda)$ for which$(\hat{x}(\tilde{\eta}(\kappa,\lambda)),\tilde{p}(\kappa,\lambda))$ belongs to the subgradient of $V(\kappa,\lambda)$ for every $(\kappa,\lambda)\in[\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}]$, and *b*) $V(\kappa,\lambda)$ is convex. Consider the function:

$$V^*(\kappa,\lambda) = \max[0,\kappa-\kappa^*(\lambda)] + \zeta\cdot\frac{\lambda^2}{2}, \tag{5}$$

where $\zeta \in \mathbb{R}_+$. Notice that $a)$ holds for $V^*(\kappa, \lambda)$ for the following payment rule:

$$\tilde{p}(\kappa, \lambda) = \begin{cases} \zeta \cdot \lambda - \kappa^{*\prime}(\lambda) & \text{if } \eta \geq \eta^* \\ \zeta \cdot \lambda & \text{otherwise.} \end{cases} \tag{6}$$

I now show that $V^*(\kappa, \lambda)$ is convex for $\zeta$ sufficiently large. Recall that $V^*(\kappa, \lambda)$ is convex it its gradient is monotone, that is, for every $(\kappa_1, \lambda_1), (\kappa_2, \lambda_2) \in [\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$ we have:

$$\Delta\kappa \cdot \Delta\tilde{x} + \Delta\lambda \cdot \Delta\tilde{p} \geq 0, \tag{7}$$

where $\Delta\kappa := \kappa_2 - \kappa_1$, $\Delta\lambda := \lambda_2 - \lambda_1$, $\Delta\tilde{x} := \tilde{x}(\kappa_2, \lambda_2) - \tilde{x}(\kappa_1, \lambda_1)$ and $\Delta\tilde{p} := \tilde{p}(\kappa_2, \lambda_2) - \tilde{p}(\kappa_1, \lambda_1)$. First, notice that setting $\zeta > M$ ensures that $\tilde{p}(\kappa, \lambda)$ is increasing in $\lambda$. Since $-\kappa^{*\prime}(\lambda) > 0$, $\tilde{p}(\kappa, \lambda)$ is increasing in both variables. Moreover, $\tilde{x}(\kappa, \lambda) \equiv \hat{x}(\tilde{\eta}(\kappa, \lambda))$ is weakly increasing in both $\kappa$ and $\lambda$, and therefore (7) holds trivially whenever $\Delta\kappa$ and $\Delta\lambda$ are of the same sign. Now, consider the case where $\Delta\lambda > 0$ and $\Delta\kappa < 0$ (the opposite case is analogous). This case is trivial if $\Delta\tilde{x} = -1$ or $0$, so suppose $\Delta\tilde{x} = 1$. Inequality (7) then becomes:

$$\Delta\kappa + \Delta\lambda[\Delta\lambda \cdot \zeta - \kappa^{*\prime}(\lambda_2)] \geq 0.$$

Equivalently:

$$\zeta \geq \frac{\kappa^{*\prime}(\lambda_2) - \frac{\Delta\kappa}{\Delta\lambda}}{\Delta\lambda}. \tag{8}$$

It suffices to uniformly bound the RHS across all $\Delta\kappa, \Delta\lambda$. Notice that $\Delta\tilde{x} = 1$ implies $\tilde{\eta}(\kappa_1, \lambda_1) < \eta^* \leq \tilde{\eta}(\kappa_2, \lambda_2)$. Since $\kappa^{*\prime}(\lambda) < 0$, we get the following bounds (Figure 3):

$$-\frac{\Delta\kappa}{\Delta\lambda} \leq \frac{\kappa^*(\lambda_1) - \kappa^*(\lambda_2)}{\Delta\lambda} \leq \frac{\int_{\lambda_1}^{\lambda_1+\Delta\lambda} -\kappa^{*\prime}(\tau) d\tau}{\Delta\lambda} = \max_{\lambda \in [\lambda_1, \lambda_2]} -\kappa^{*\prime}(\lambda).$$

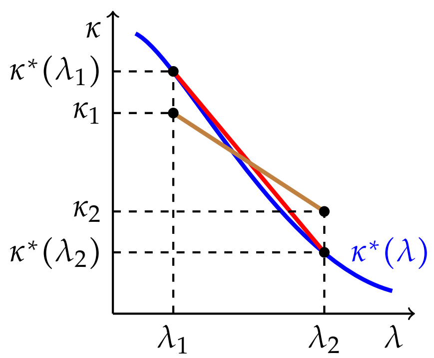

Figure 3

We can then bound the RHS of (8) as follows:

$$\frac{\kappa^{*\prime}(\lambda_2) - \frac{\Delta\kappa}{\Delta\lambda}}{\Delta\lambda} \leq \frac{\kappa^{*\prime}(\lambda_2) + \max_{\lambda \in [\lambda_1, \lambda_2]} -\kappa^{*\prime}(\lambda)}{\Delta\lambda} \leq \frac{\int_{\lambda_1}^{\lambda_1+\Delta\lambda} M d\tau}{\Delta\lambda} = M.$$

Therefore, $V^*(\kappa, \lambda)$ is convex whenever $\zeta > M$. $\square$

Now, let $\mathcal{A}$ be the set of increasing functions $\hat{x} : [\underline{\tilde{\eta}}, \overline{\tilde{\eta}}] \to [0,1]$. $\mathcal{A}$ is convex and compact in the product topology and $\mathcal{T}$ is the set of its extreme points. Hence, by Choquet's theorem, for every $\hat{x}^* \in \mathcal{A}$ there exists a probability measure $\mu$ on $\mathcal{T}$ such that $\hat{x}^* = \int_{\mathcal{T}} \hat{x}\mu(d\hat{x})$ (Phelps, 2001). By Lemma 2, for every $\hat{x} \in \mathcal{T}$ there exists an allocation rule $\tilde{x}[\hat{x}]$ s.t. $\tilde{x}[\hat{x}](\kappa, \lambda) \equiv \hat{x}(\tilde{\eta}(\kappa, \lambda))$ that is implementable on $[\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$. Then:

$$\tilde{x}^* := \int_{\mathcal{T}} \tilde{x}[\hat{x}]\mu(d\hat{x}) = \int_{\mathcal{T}} \hat{x}\mu(d\hat{x}) = \hat{x}^*.$$

Finally, since all $\tilde{x}[\hat{x}]$ are implementable and (IC) and (IR) are linear, it follows that $\tilde{x}^*$ is implementable too. □

## 4 Observable wealth

While I assumed that neither need nor wealth are observable, the government usually has some information about them. For instance, in the problem of vaccine allocation, age and medical history are good indicators of need. Similarly, tax data proxies for one's wealth (even if some income sources or assets remain unobserved). In such cases, agents' private information can be thought of as *residual uncertainty* after accounting for observables. Indeed, even public programs conditioned on earnings face substantial uncertainty over one's wealth.[5]

Still, what can the government do if it perfectly observes agents' $\alpha$? As it turns out, the set of equitable allocation rules that are implementable with one instrument when $\alpha$ is observed is identical to the set of equitable allocation rules that are implementable with two instruments when $\alpha$ is private. Intuitively, when the designer observes $\alpha$, she can 'control for' the fact that some agents prefer the good because they are wealthy and screen purely based on need.

**Proposition 3.** *Suppose $\alpha$ is observable and the government uses either only money or only wait times to screen. Then $x(\alpha, \beta)$ is equitable and implementable if and only if $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$, where $\hat{x}$ is increasing.*

*Proof.* Since $\alpha$ is observable, the allocation can be implemented separately for every value of $\alpha$ so $x(\alpha, \beta)$ is implementable if and only if it is weakly increasing in $\beta$. Recall the equity constraint is satisfied if and only if $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$. Since $\eta(\alpha, \beta)$ is strictly increasing, $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$ is equitable and implementable if and only if $\hat{x}$ is weakly increasing. □

## 5 Relaxing the equity constraint

My model of societal perceptions of equity was built around a single merit function. In reality, however, people often share general principles concerning equity and desert, but hold different opinions about finer trade-offs or ways in which these principles should be applied. For instance, Craxì et al. (2021) found that while healthcare workers tend to agree

[5] https://thehill.com/regulation/administration/268409-outrage-builds-over-wealthy-families-in-public-housing/

which groups should get priority in the Covid-19 vaccine rollout, their opinions differ on how these groups should be ranked. As a result, real-world equity constraints on policymakers would be less demanding than my analysis suggests. While Theorem 1 says that screening with multiple instruments is possible even under such strong restrictions, it is also interesting to compare mechanisms screening with only payments and only waiting when the equity constraint is relaxed. To that end, I develop a measure of *equity violation* capturing how far away a particular allocation is from satisfying the equity constraint (here, I interpret the merit function as a 'rough consensus' among the public). My measure assumes that every agent assesses the allocation's equity by looking at agents similar to herself, and comparing those with the same allocation as her to those with the same merit as her. In other words, she compares agents similar to herself who *are* treated the same as her with those who *should* be treated the same. The further apart these two sets are, the more inequitable the allocation seems to her. Then, the degree to which the whole allocation violates equity is the size of the largest such 'local equity violation'.

**Definition 1** (Equity violation)**.** *For every type $(\alpha, \beta) \in \Theta$, let $D(\alpha, \beta)$ be the set of directions in which the allocation is locally constant:*

$$D(\alpha, \beta) = \{d \in \mathbb{R}^2 : \nabla_d x(\alpha, \beta) = 0\}.$$

*Let the* ***local equity violation*** *for type $(\alpha, \beta)$ be:*

$$l(\alpha, \beta) = \begin{cases} \infty & \text{if } D(\alpha, \beta) = \varnothing, \\ \inf_{d \in D(\alpha,\beta)} \left| \arctan(d) - \arctan\left(-\frac{\eta_\alpha(\alpha,\beta)}{\eta_\beta(\alpha,\beta)}\right) \right| & \text{otherwise.}^{6} \end{cases} \tag{9}$$

*The* ***equity violation*** *of the allocation rule $x(\alpha, \beta)$, denoted $L(x)$, is its largest local equity violation:*

$$L(x) := \sup_{(\alpha,\beta) \in \Theta} l(\alpha, \beta).$$

To build intuition, consider a smooth allocation that is strictly increasing in $\beta$ and fix some type $(\alpha^a, \beta^a)$. Then all the types with allocations equal to that of $(\alpha^a, \beta^a)$ will lie on a smooth curve passing through $(\alpha^a, \beta^a)$. Figure 4a illustrates such a curve together with this type's *iso-merit curve*, that is, the set of types with the same merit as $(\alpha^a, \beta^a)$. Since both of these curves are smooth, we can compare 'how far apart' they are in the neighborhood of $(\alpha^a, \beta^a)$ by comparing their slopes there. The distance between these slopes, as measured by the angle between them, is the *local equity violation* at $(\alpha^a, \beta^a)$ (Figure 4b).

We can ask which of the two screening instruments, when used on its own, will produce larger equity violations. A naïve (but incorrect) intuition suggests that screening with waiting will produce an allocation closer to the equitable one as it biases the allocation 'in the right direction'. However, as discussed in Subsection 3.2, a mechanism screening with waiting will generically fail to pool together agents in a way that 'matches the shape' of the

[6] My results would be unaffected if arctan was replaced with another bounded and strictly increasing function. However, using arctan will let us visualize the size the local equity violation as an angle.

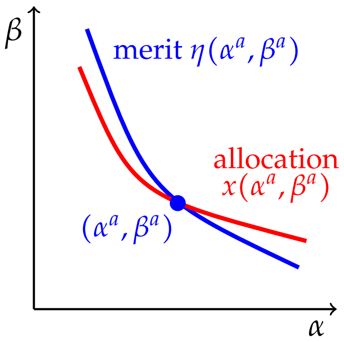


Figure 4a

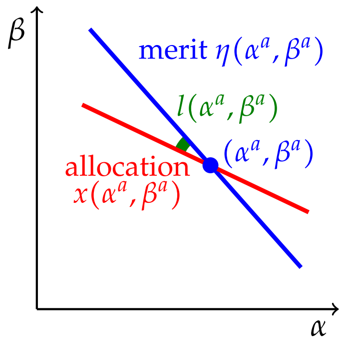


Figure 4b

merit curve. As it turns out, in some environments this 'shape effect' dominates the aforementioned 'direction effect'.

Screening with waiting will nevertheless produce smaller equity violations when the merit function depends on wealth $\alpha$ sufficiently strongly. The following proposition says that, as long as iso-merit curves are sufficiently flat everywhere, every separating allocation implemented using only payments will violate equity by more than some separating allocation implemented with only waiting.

**Proposition 4.** *Suppose wait times are bounded: $q \in [\underline{q}, \bar{q}]$ where $0 \in [\underline{q}, \bar{q}]$. Then there exists $M \in (-\infty, 0)$ with the following property: if for all $(\alpha, \beta) \in \Theta$ we have*

$$-\frac{\eta_\alpha(\alpha, \beta)}{\eta_\beta(\alpha, \beta)} > M,$$

*then for every non-constant allocation rule $x_p$ that is implementable with only payments, there exists a non-constant allocation rule $x_q$ that is implementable with only waiting that produces a strictly smaller equity violation: $L(x_p) < L(x_q)$.*

While the proof is relegated to the appendix, I illustrate its key intuition with the case of a smooth allocation that increases in $\beta$. Fix some type $(\alpha^a, \beta^a)$ and compare two allocations: $x_q$ implemented using only payments and $x_p$ implemented using only waiting. Like before, the sets of agents with the same allocation as $(\alpha^a, \beta^a)$ will be smooth curves passing through that point. Moreover, the insights from Subsections 3.1 and 3.2 tell us that the curve for $x_p$ will be upwards-sloping, while the curve for $x_q$ will be downwards-sloping.

Let us now compare the local equity violations of these two allocations. Figure 6a illustrates that if the iso-merit curve is sufficiently flat, its angle with the iso-$x_q(\alpha^a, \beta^a)$ curve will be smaller than that with the iso-$x_p(\alpha^a, \beta^a)$ one. If we can impose a sufficiently low uniform bound on the slopes of iso-merit curves, this will be true for every type, regardless of $x^q$. Figure 6b, on the other hand, illustrates why the result of Proposition 4 fails when iso-merit curves are not flat enough.

This exercise offers broader qualitative lessons. Roughly speaking, Proposition 4 tells us that screening with waiting violates equity by less than screening with payments if merit depends more strongly on wealth than it does on need. However, the same will be true if agents vary a lot in wealth relative to how much they vary in need. To understand why,

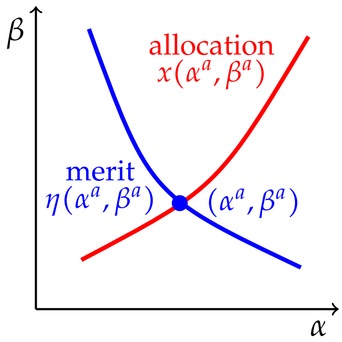


Figure 5a
Screening with only payments

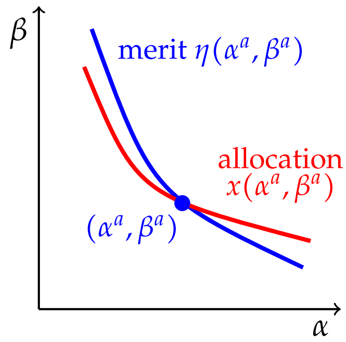


Figure 5b
Screening with only waiting

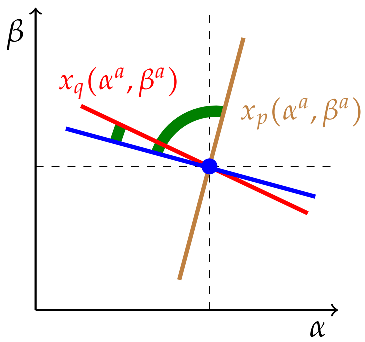


Figure 6a

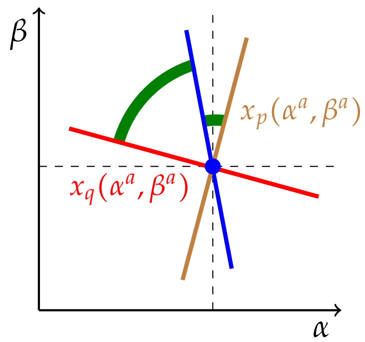


Figure 6b

consider a mechanism screening with payments. If agents differ greatly in wealth (relative to how much they differ in need), most of the variance in allocations will be explained by preferences for money. This in turn means that the allocation will be heavily skewed towards the rich—this corresponds to a very flat iso-$x_p(\alpha^a, \beta^a)$ curve in Figure 6a. Consequently, the angle between this curve and the iso-merit curve will be wider.

## 6 Discussion

While my approach to modeling perceived equity is highly stylized, it offers general qualitative conclusions. First, every screening instrument will bias the allocation towards the group for which this instrument is less costly—this makes screening with payments problematic from an equity standpoint. Using a different instrument (like waiting) could reverse this bias, but the designer's control over the allocation would still be limited. Consequently, the resulting bias might still not satisfy the public. I show this problem can be solved by combining multiple screening instruments which on their own favor different social groups. Doing so gives the designer freedom to tinker with various groups' differential cost of the allocated good, and therefore to improve efficiency through screening while still producing an allocation that is seen as fair. I also highlight that screening with waiting need not always produce more equitable allocations than screening with payments. Waiting is, however, likely to be the more equitable instrument when society is highly averse to handing out benefits to wealthier people, or when the population's wealth heterogeneity is large relative to heterogeneity in terms of need for the good.

# 7 Appendix

In what follows I write $(\alpha^a_\delta, \beta^a_\delta)$ for $(\alpha^a+\delta, \beta^a+\delta)$, $x^a_\delta$ for $x(\alpha^a_\delta, \beta^a_\delta)$, $p^a_\delta$ for $p(\alpha^a_\delta, \beta^a_\delta)$ and $\eta^a_\delta$ for $\eta(\alpha^a_\delta, \beta^a_\delta)$. I omit the subscript when $\delta = 0$.

The following lemma will be useful in the proofs presented here.

**Lemma 3.** *Suppose there exists a decreasing sequence $\{\delta_i\}_i$ such that $\delta_i \to 0$ for which $\{x^a_{\delta_i}\}_i$ is strictly decreasing and $x^a_{\delta_i} \to x^a$. Then, for any $(\alpha^b, \beta^b)$ such that $x^b = x^a$ we have:*

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} \leq \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}.$$

*Proof.* First, note that $\{q^a_{\delta_i}\}_i$ is strictly decreasing since strictly lower allocations must come with strictly lower wait times. I now show that $q^a_{\delta_i} \to q^a$. Indirect utility has to be continuous at every $(\alpha^a, \beta^a)$ (Milgrom and Segal, 2002) so:

$$v(\beta^a, x^a) - z(\alpha^a, q^a) = \lim_{i\to\infty} \left\{ v(\beta^a_{\delta_i}, x^a_{\delta_i}) - z(\alpha^a_{\delta_i}, q^a_{\delta_i}) \right\}.$$

Note that $\lim_{i\to\infty} v(\beta^a_{\delta_i}, x^a_{\delta_i}) = v(\beta^a, x^a)$. Since $\alpha^a_{\delta_i} \to \alpha^a$ and $z$ is continuous and strictly increasing in the latter argument, we have $q^a_{\delta_i} \to q^a$.

I now show that if the desired inequality fails, then $x^b \neq x^a$. Suppose that:

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} > \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}.$$

By continuity of $v_x$ and $z_q$, for $i$ high enough we have:

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} > \frac{v_x(\beta^a_{\delta_i}, x^a)}{z_q(\alpha^a_{\delta_i}, q^a)}. \tag{10}$$

Recall that $\{x^a_{\delta_i}\}_i$ and $\{q^a_{\delta_i}\}_i$ are strictly decreasing and tend to $x^a$ and $q^a$. Hence, for all $\alpha, \beta$:

$$\frac{v(\beta, x^a_{\delta_j}) - v(\beta, x^a)}{x^a_{\delta_j} - x^a} \to v_x(\beta, x^a); \quad \frac{z(\alpha, q^a_{\delta_j}) - z(\alpha, q^a)}{q^a_{\delta_j} - q^a} \to z_q(\alpha, q^a), \tag{11}$$

as $j \to \infty$. Fix $i$ high enough that (10) holds. Then, by (11), for $j$ high enough we have:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{\frac{v(\beta^a_{\delta_i}, x^a_{\delta_j}) - v(\beta^a_{\delta_i}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_i}, q^a_{\delta_j}) - z(\alpha^a_{\delta_i}, q^a)}{q^a_{\delta_j} - q^a}}.$$

In particular, take $j > i$ and notice that increasing $i$ further relaxes the inequality. Hence:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{\frac{v(\beta^a_{\delta_j}, x^a_{\delta_j}) - v(\beta^a_{\delta_j}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_j}, q^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a)}{q^a_{\delta_j} - q^a}}. \tag{12}$$

Now, by revealed preference we have:

$$v(\beta^a_{\delta_j}, x^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a_{\delta_j}) \geq v(\beta^a_{\delta_j}, x^a) - z(\alpha^a_{\delta_j}, q^a) \quad \Longrightarrow \quad \frac{\frac{v(\beta^a_{\delta_j}, x^a_{\delta_j}) - v(\beta^a_{\delta_j}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_j}, q^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a)}{q^a_{\delta_j} - q^a}} \geq \frac{q^a_{\delta_j} - q^a}{x^a_{\delta_j} - x^a}.$$

Combining the latter inequality with (12) gives:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{q^a_{\delta_j} - q^a}{x^a_{\delta_j} - x^a} \quad \Longrightarrow \quad v(\beta^b, x^a_{\delta_j}) - z(\alpha^b, q^a_{\delta_j}) > v(\beta^b, x^a) - z(\alpha^b, q^a).$$

Hence, $x^b(\alpha, \beta) \neq x^a$. □

### 7.1 Notion of genericity in Proposition 2

According to the standard measure-theoretic notion of genericity, a property is generic if it holds for almost all parameter values. However, the merit function cannot be described by a finitely-dimensional parameter and hence the standard notion does not apply. I introduce a novel definition which extends the logic of the standard measure-theoretic one to my environment. My definition is based on a set of parametrized classes of perturbations and says that a property holds only for non-generic merit functions if it fails for almost all perturbations within some such class.

**Definition 2.** *Let any strictly increasing and twice-differentiable function $\eta' : \mathbb{R}^2 \to \mathbb{R}$ be a* ***candidate merit function****. Let a* ***grid*** *be a tuple of types $\{(\alpha_i, \beta_i)\}_{i=1}^N \in \Theta^N$.*

*A property* ***holds only for non-generic merit functions*** *if there exists a grid $\{(\alpha_i, \beta_i)\}_{i=1}^N$ such that for almost all N-sized grids $\{(\alpha'_i, \beta'_i)\}_{i=1}^N$, the property fails for all candidate merit functions $\eta'(\alpha, \beta)$ satisfying $\eta'(\alpha'_i, \beta'_i) = \eta(\alpha_i, \beta_i)$ for every i.*

Intuitively, every choice of the initial grid $\{(\alpha_i, \beta_i)\}_{i=1}^N$ corresponds to a class of candidate merit functions. Every function in this class comes from perturbing the original merit function in the following way: if the original merit function achieved some set of values at $\{(\alpha_i, \beta_i)\}_{i=1}^N$, then the perturbed function attains them at some other grid $\{(\alpha'_i, \beta'_i)\}_{i=1}^N$. Notice that we can think of these 'perturbed grids' as a finite-dimensional parametrization of this class of perturbations and thus we can apply the standard measure-theoretic notion of genericity to it. Then, to show that a certain property holds only for non-generic merit functions, it suffices to find a class of perturbations corresponding to some $\{(\alpha_i, \beta_i)\}_{i=1}^N$ and show the property fails for almost all perturbations in this class.

Finally, note that not every grid $\{(\alpha_i', \beta_i')\}_{i=1}^N$ will have a candidate merit function $\eta'$ satisfying $\eta'(\alpha_i', \beta_i') = \eta(\alpha_i, \beta_i)$ for every $i$. However, for any $N > 0$ such candidate merit functions exist for a positive mass of grids and thus the condition of Definition 2 is never vacuous. In particular, such candidate merit functions exist for any $\{(\alpha_i', \beta_i')\}_{i=1}^N$ for which all $(\alpha_i', \beta_i')$ are sufficiently close to $(\alpha_i, \beta_i)$.

### 7.2 Proof of Proposition 2

**Lemma 4.** *The following properties hold only for non-generic merit functions:*

1. *There exist $\eta^* \in (\underline{\eta}, \overline{\eta})$, $x \in [0,1]$, $q \in \mathbb{R}_+$ and $k \in \mathbb{R}$ such that for all $(\alpha, \beta)$ with $\eta(\alpha, \beta) = \eta^*$ we have:*
$$\frac{v_x(\beta, x)}{z_q(\alpha, q)} = k.$$
2. *There exist $\eta^* \in (\underline{\eta}, \overline{\eta})$, $x^a \neq x^b \in [0,1]$, $q^a, q^b \in \mathbb{R}_+$ such that for all $(\alpha, \beta)$ with $\eta(\alpha, \beta) = \eta^*$ we have:*
$$v(\beta, x^a) - z_q(\alpha, q^a) = v(\beta, x^b) - z_q(\alpha, q^b).$$

*Proof.* I provide only the proof for the former property; the proof for the latter one is analogous. Suppose the property holds for some $\eta^* \in (\underline{\eta}, \overline{\eta})$ and choose a grid $\{(\alpha_i, \beta_i)\}_{i=1}^N$ with $N \geq 4$ such that $\eta(\alpha_i, \beta_i) = \eta^*$ for every $i$. The property implies that the following system of equations has a solution:

$$\begin{aligned} \frac{v_x(\beta_1, x)}{z_q(\alpha_1, q)} - k = 0, \\ \frac{v_x(\beta_2, x)}{z_q(\alpha_2, q)} - k = 0, \\ \vdots \\ \frac{v_x(\beta_N, x)}{z_q(\alpha_N, q)} - k = 0. \end{aligned} \tag{13}$$

It suffices to show that for almost all $N$-sized grids this is not the case for all merit functions consistent with them. Let us express (13) more concisely as $mrs(x, q, k; \alpha, \beta) = 0$. Then the Jacobian $\nabla mrs(x, q, k; \alpha, \beta)$ is an $N \times (3 + 2N)$ matrix where the last $N$ columns are:

$$\begin{bmatrix} \frac{v_{x\beta}(\beta_1, x)}{z_q(\alpha_1, q)} & 0 & \dots & 0 \\ 0 & \frac{v_{x\beta}(\beta_2, x)}{z_q(\alpha_2, q)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{v_{x\beta}(\beta_N, x)}{z_q(\alpha_N, q)} \end{bmatrix}.$$

All diagonal entries are strictly positive by Assumption 1 and thus $\nabla mrs(x, q, k; \alpha, \beta)$ has rank $N$. Now, notice that grids $\{(\alpha_i, \beta_i)\}_{i=1}^N \in \Theta^N$ are vectors of parameter values for a system of equations (13) with $N \geq 4$ equations and 3 variables. The Transversality Theorem then implies that (13) has no solutions for almost all grids in $\Theta^N$. □

Let $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$ be an equitable, implementable and non-constant allocation rule. Indirect utility has to be continuous at $(\alpha^a,\beta^a)$ (Milgrom and Segal, 2002) so, by continuity of $\eta$, $v$ and $z$, the following holds for any $\eta^a \in (\underline{\eta},\overline{\eta})$:

$$v(\beta^a,\hat{x}_+(\eta^a)) - z(\alpha^a,\hat{q}_+(\eta^a)) = v(\beta^a,\hat{x}_-(\eta^a)) - z(\alpha^a,\hat{q}_-(\eta^a)). \tag{14}$$

Thus, if $\hat{x}$ were discontinuous at any $\eta^a$, there would exist $\hat{x}_+(\eta^a) \neq \hat{x}_-(\eta^a)$, $q_+(\eta^a)$ and $q_-(\eta^a)$ satisfying (14) for every $(\alpha,\beta)$ with merit $\eta^a$. However, by Lemma 4 this can only happen for non-generic merit functions and so $\hat{x}$ can only be discontinuous for non-generic merit functions. Let us then consider the case where $\hat{x}$ is continuous.

**Fact 1.** *There exists $\eta^a \in (\underline{\eta},\overline{\eta})$ and a decreasing sequence $\{f_i\}_i$ s.t. $f_i \to \eta^a$, $\hat{x}(f_i) \to \hat{x}(\eta^a)$ and $\{\hat{x}(f_i)\}_i$ is strictly decreasing.*

*Proof.* By Lemma 1 $\hat{x}$ is weakly increasing. It is also not constant, so there exist $\eta^b,\eta^c \in (\underline{\eta},\overline{\eta})$ s.t. $\eta^b < \eta^c$ and $\hat{x}(\eta^b) < \hat{x}(\eta^c)$. Let $\eta^a = \sup\{\eta : \hat{x}(\eta) = \hat{x}(\eta^b)\}$. Since $\hat{x}$ is continuous, $\hat{x}(\eta^a) = \hat{x}(\eta^b)$. Now, take any decreasing sequence $\{e_i\}_i$ s.t. for every $i$, $e_i \in (\underline{\eta},\overline{\eta})$ and $e_i \to \eta^a$. Since $\hat{x}$ is weakly increasing, $\hat{x}(e_i) \leq \hat{x}(e_j)$ whenever $i > j$. Also, by continuity of $\hat{x}$, $\hat{x}(e_i) \to \hat{x}(\eta^a)$ and, by the construction of $\eta^a$, $\hat{x}(\eta^a) < \hat{x}(e_i)$ for every $i$. Since $\hat{x}$ is continuous, there exists a subsequence $\{f_i\}_i$ of $\{e_i\}_i$ for which $\hat{x}(f_i)$ is decreasing strictly. □

Take $\eta^a$, $\{f_i\}_i$ from Fact 1 and any $(\alpha^b,\beta^b)$ with merit $\eta^a$. Since $f_i \to \eta^a$ and $\hat{x}(f_i) \to \hat{x}(\eta^a)$, continuity and monotonicity of $\eta(\alpha,\beta)$ tell us that for all $i$ high enough there are $\delta_i$ such that $f_i = \eta(\alpha^b_{\delta_i},\beta^b_{\delta_i})$, $\delta_i \to 0$ as $f_i \to \eta^a$, and $\{\delta_i\}_i$ is decreasing. Such a sequence $\{\delta_i\}_i$ can be found for any type with merit $\eta^a$ so by Lemma 3 all such types must have same value of $v_x(\beta^b,x^a)/z_q(\alpha^b,x^a)$. However, by Lemma 4 this can only hold for non-generic merit functions.

### 7.3 Proof of Proposition 4

Suppose that we screen only with wait times ($p \equiv 0$) and that the allocation is not constant.

**Fact 2.** *If $\alpha^a \geq \alpha^b$ and $\beta^a \geq \beta^b$ with at least one inequality holding strictly, then $x^a \geq x^b$.*

*Proof.* Suppose that $x^a < x^b$; then $q^a < q^b$ or else both types would strictly prefer $(x^b,q^b)$. By revealed preference:

$$v(\beta^b,x^b) - z(\alpha^b,q^b) \geq v(\alpha^b,x^a) - z(\alpha^b,q^a) \quad \Longrightarrow \quad v(\beta^b,x^b) - v(\beta^b,x^a) \geq z(\alpha^b,q^b) - z(\alpha^b,q^a).$$

By strictly increasing differences, this gives:

$$v(\beta^a,x^b) - v(\beta^a,x^a) > z(\alpha^a,q^b) - z(\alpha^a,q^a) \quad \Longrightarrow \quad v(\beta^a,x^b) - z(\alpha^a,q^b) > v(\beta^a,x^a) - z(\alpha^a,q^a).$$

That is, $(\alpha^a,\beta^a)$ prefers $(x^b,q^b)$ to $(x^a,q^a)$; contradiction. □

In particular, Fact 2 tells us that $x^a_\delta$ is weakly increasing in $\delta$ for every $(\alpha^a,\beta^a)$. Moreover, whenever $x^a > x^b$, it has to be that $q^a > q^b$ or else $(\alpha^b,\beta^b)$ would prefer $(x^a,q^a)$ to her allocation. Therefore, $q^a_\delta$ is also weakly increasing in $\delta$ for every $(\alpha^a,\beta^a)$.

**Fact 3.** *There exists $(\alpha^a,\beta^a)$ such that either a) for every $\delta > 0$, $x^a_\delta > x^a$, or b) for every $\delta < 0$, $x^a_\delta < x^a$.*

*Proof.* Suppose otherwise; then for every $(\alpha^a, \beta^a) \in \Theta$ there exists some $\epsilon > 0$ such that $x^a_\delta = x^a$ for $\delta \in [-\epsilon, \epsilon]$. Take any $(\alpha^b, \beta^b) \in [\alpha^a - \epsilon, \alpha^a + \epsilon] \times [\beta^a - \epsilon, \beta^a + \epsilon]$ and notice that by Fact 2 we have $x(\alpha - \epsilon, \beta - \epsilon) \le x^b \le x(\alpha + \epsilon, \beta + \epsilon)$. However, $x(\alpha - \epsilon, \beta - \epsilon) = x(\alpha + \epsilon, \beta + \epsilon) = x^a$ so $x(\alpha, \beta) = x^a$ for all $(\alpha, \beta) \in [\alpha^a - \epsilon, \alpha^a + \epsilon] \times [\beta^a - \epsilon, \beta^a + \epsilon]$. Consequently, for any $(\alpha, \beta) \in \Theta$ there exists a neighborhood around it in which the allocation is constant. Now, take any $(\alpha^c, \beta^c), (\alpha^d, \beta^d) \in \Theta$. Since $\Theta$ is connected, there exists a continuous path between them. Every point along this path has the same allocation as the points within its neighborhood. Therefore, the allocation has to be constant along the whole path, including the end-points. Since $(\alpha^c, \beta^c)$ and $(\alpha^d, \beta^d)$ were arbitrary, the allocation is constant; contradiction. □

Fix $(\alpha^a, \beta^a)$ from Fact 3 and assume *a*) holds (the argument for *b*) is analogous). Let $x^a_+ = lim_{\delta \to 0^+} x^a_\delta$ and $q^a_+ = lim_{\delta \to 0^+} q^a_\delta$; these exist as $x^a_\delta$ and $q^a_\delta$ are monotonic.

**Fact 4.** *If $x^a_\delta$ is right-continuous at $\delta = 0$, then for every $(\alpha^b, \beta^b)$ such that $x^b = x^a$, we have:*

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} \le \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}.$$

*If $x^a_\delta$ is not right-continuous at $\delta = 0$, then for every $(\alpha^b, \beta^b)$ such that $x^b = x^a$, we have:*

$$\frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z(\alpha^b, q^a_+) - z(\alpha^b, q^a)} \le \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)}.$$

*Proof.* Suppose $x^a_\delta$ is right-continuous at $\delta = 0$ and take a decreasing sequence $\{e_i\}_i$ such that $e_i \to 0$. Then $x^a_{e_i} \to x^a$ and, by case *a*) of Fact 3, $x^a < x^a_{e_i}$ for all $i$. Since $x^a_\delta$ is right-continuous at $\delta = 0$, there exists a subsequence $\{f_i\}_i$ of $\{e_i\}_i$ such that $x^a_{f_i}$ is decreasing strictly. We can therefore apply Lemma 3 which completes the proof of this case.

Now suppose $x^a_\delta$ is not right-continuous at $\delta = 0$ and take a decreasing sequence $\{\delta_i\}_i$ such that $\delta_i \to 0$. Then $x^a_+ > x^a$. Since indirect utility has to be continuous at $(\alpha^a, \beta^a)$ (Milgrom and Segal, 2002), it has to be that $q^a_+ > q_a$ and:

$$v(\beta^a, x^a) - z(\alpha^a, q^a) = v(\beta^a, x^a_+) - z(\alpha^a, q^a_+) \quad \Longrightarrow \quad \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)} = 1. \tag{15}$$

By the fact that $x^b = x^a$ and revealed preference, for every $i$ we have:

$$v(\beta^b, x^a) - z(\alpha^b, q^a) \ge v(\beta^b, x^a_{\delta_i}) - z(\alpha^b, q^a_{\delta_i}) \quad \Longrightarrow \quad \frac{v(\beta^b, x^a_{\delta_i}) - v(\beta^b, x^a)}{z(\alpha^b, q^a_{\delta_i}) - z(\alpha^b, q^a)} \le 1.$$

Taking $i \to \infty$ gives:

$$\frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z(\alpha^b, q^a_+) - z(\alpha^b, q^a)} \le 1. \tag{16}$$

Combining the latter equality in (15) with (16) completes the proof. □

Assumption 1 ensures that, in the respective cases of Fact 4, the sets of $(\alpha^b, \beta^b) \in \Theta$ for which

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} = \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)} \quad \text{and} \quad \frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z_+(\alpha^b, q^a_+) - z(\alpha^b, q^a)} = \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)}, \tag{17}$$

are downwards-sloping and differentiable curves. I will refer to them as *iso-MRS* and *iso-difference* curves at $(\alpha^a, \beta^a)$. The LHSs of both equations in (17) are strictly increasing in $\alpha^b$ and $\beta^b$ so by Fact 4 all types allocated $x^a$ must lie weakly below the iso-$MRS$ curve at $(\alpha^a, \beta^a)$ in the first case, and weakly below the iso-difference curve at $(\alpha^a, \beta^a)$ in the second case. Hence, the only possible element of $D(\alpha^a, \beta^a)$ from Definition 1 is the slope of the iso-$MRS$ curve at $(\alpha^a, \beta^a)$ in the first case, and the slope of the iso-difference curve at $(\alpha^a, \beta^a)$ in the second case:

$$s^q_{MRS}(\alpha^a, \beta^a) := \frac{v_x(\alpha^a, x^a)}{z_q(\alpha^a, q^a)} \frac{z_{\alpha q}(\alpha^a, q^a)}{v_{\beta x}(\alpha^a, x^a)},$$

$$s^q_{diff}(\alpha^a, \beta^a) := \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)} \frac{z_\alpha(\alpha^a, q^a_+) - z_\alpha(\alpha^a, q^a)}{v_\beta(\beta^a, x^a_+) - v_\beta(\beta^a, x^a)}.$$

By Assumption 1, $s^q_{MRS}(\alpha, \beta)$, $s^q_{diff}(\alpha, \beta) < 0$ for every $(\alpha, \beta) \in \Theta$.

An analogous argument pins down the only possible elements of $D(\alpha^a, \beta^a)$ when we screen using only paments. Then, however, $s^p_{MRS}(\alpha, \beta)$, $s^p_{diff}(\alpha, \beta) > 0$ for every $(\alpha, \beta) \in \Theta$. Notice also that the slope of the iso-merit curve equals to $-(\eta_\alpha(\alpha, \beta))/(\eta_\beta(\alpha, \beta))$ and is negative for any $(\alpha, \beta) \in \Theta$ since $\eta_\alpha, \eta_\beta > 0$. Therefore, we have the following lower bound on equity violations that holds across all allocation rules $x_p$ screening with only payments:

$$L(x_p) > \left| \frac{\pi}{2} - \arctan\left( -\frac{\eta_\alpha(\alpha, \beta)}{\eta_\beta(\alpha, \beta)} \right) \right|. \tag{18}$$

Now, the following bounds hold across all $(\alpha^b, \beta^b) \in \Theta$ and all allocation rules $x_q$ implemented with only wait times:

$$\inf_{(\alpha,\beta)\in\Theta,\ x\in[0,1]} v_x(\beta, x) \le v_x(\beta^b, x^b) \le \sup_{(\alpha,\beta)\in\Theta,\ x\in[0,1]} v_x(\beta, x),$$

$$\inf_{(\alpha,\beta)\in\Theta,\ q\in[\underline{q},\bar{q}]} z_q(\alpha, q) \le z_q(\alpha^b, q^b) \le \sup_{(\alpha,\beta)\in\Theta,\ q\in[\underline{q},\bar{q}]} z_q(\alpha, q),$$

$$\inf_{(\alpha,\beta)\in\Theta,\ x\in[0,1]} v_{\beta x}(\beta, x) \le v_{\beta x}(\beta^b, x^b) \le \sup_{(\alpha,\beta)\in\Theta,\ x\in[0,1]} v_{\beta x}(\beta, x),$$

$$\inf_{(\alpha,\beta)\in\Theta,\ q\in[\underline{q},\bar{q}]} z_{\alpha q}(\alpha, q) \le z_{\alpha q}(\alpha^b, q^b) \le \sup_{(\alpha,\beta)\in\Theta,\ q\in[\underline{q},\bar{q}]} z_{\alpha q}(\alpha, q).$$

The same bounds hold for the following terms when $x^b \neq x^b_+$ (as shown, this implies $q^b \neq q^b_+$):

$$\frac{v(\beta^b, x^b_+) - v(\beta^b, x^b)}{x^b_+ - x^b}, \quad \frac{z(\alpha^b, q^b_+) - z(\alpha^b, q^b)}{q^b_+ - q^b}, \quad \frac{v_\beta(\beta^b, x^b_+) - v_\beta(\beta^b, x^b)}{x^b_+ - x^b}, \quad \frac{z_\alpha(\alpha^b, q^b_+) - z_\alpha(\alpha^b, q^b)}{q^b_+ - q^b}.$$

Since $\Theta$ is bounded and $v_x, z_q, v_{\beta x} > 0$, $z_{\alpha q} < 0$ on the closure of $\Theta$, these bounds are finite and there exists $M \in (-\infty, 0)$ such that for any $(\alpha, \beta) \in \Theta$:

$$s^q_{MRS}(\alpha, \beta),\ s^q_{diff}(\alpha, \beta) < M.$$

That is, all iso-$MRS$ and iso-difference curves for all allocation rules $x_q$ screening with only waiting are steeper than $M$. Now, assume that all iso-merit curves are flatter than $M$:

$$\text{for all } (\alpha, \beta) \in \Theta, \quad -\frac{\eta_\alpha(\alpha,\beta)}{\eta_\beta(\alpha,\beta)} > M.$$

I will construct an allocation rule using only waiting whose equity violation is below the bound in (18). Fix $(\alpha^b, \beta^b) \in \Theta$ and $x^b > 0$. Then, for $x^b$ close enough to 0 there exists $q^b \in \mathbb{R}_{++}$ for which:

$$\frac{v(\beta^b, x^b) - v(\beta^b, 0)}{w(\alpha^b, q^b) - w(\alpha^b, 0)} = 1.$$

and which will be sufficiently close to 0 that $q^b \in [\underline{q}, \bar{q}]$. Now, the following allocation satisfies (IR) and (IC): $(\alpha^b, \beta^b)$ and all types weakly above the iso-difference curve at $(\alpha^b, \beta^b)$ take $(x^b, q^b)$; all types strictly below it take $(0,0)$.

Take any $(\alpha^c, \beta^c)$ strictly below or strictly above the iso-difference curve at $(\alpha^b, \beta^b)$. There is a neighborhood around $(\alpha^c, \beta^c)$ in which the allocation is constant, and so it is also constant along the slope of the iso-merit curve at $(\alpha^c, \beta^c)$, giving $l(\alpha^c, \beta^c) = 0$. Now, take any $(\alpha^c, \beta^c)$ on the iso-diffeence curve at $(\alpha^b, \beta^b)$. The allocation is constant along this curve, so $s^q_{diff}(\alpha^c, \beta^c) \in D(\alpha^c, \beta^c)$. Since the iso-merit curve is flatter than $s_{diff}(\alpha^c, \beta^c)$ for any such $(\alpha^c, \beta^c)$, we get:

$$L(x_q) \leq \left| \arctan(s^q_{diff}(\alpha^c, \beta^c)) - \arctan\left(-\frac{\eta_\alpha(\alpha^c,\beta^c)}{\eta_\beta(\alpha^c,\beta^c)}\right)\right| < \left|\frac{\pi}{2} - \arctan\left(-\frac{\eta_\alpha(\alpha^c,\beta^c)}{\eta_\beta(\alpha^c,\beta^c)}\right)\right| < L(x_p).$$

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