Equitable screening*

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Abstract

I study the problem of a government providing benefits while considering the perceived equity of the resulting allocation. Such concerns are modeled through an equity constraint requiring that equally deserving agents receive equal allocations. I ask what forms of screening are compatible with equity and show that while the government cannot equitably screen with a single instrument (e.g. payments or wait times), combining multiple instruments, which on their own favor different groups, allows it to screen while still producing an equitable allocation.

1 Introduction

In 2017, the French president Emmanuel Macron unveiled a set of environmental policies aimed at reducing carbon emissions. The plan involved a gradual increase in fuel taxes, including a significant hike in diesel and petrol prices the following year. However, the proposed tax increase sparked protests and riots which later expanded into the 'Yellow Vest' movement. The protesters, largely from rural and less affluent areas, objected to the policy on equity grounds. They claimed that households in poor financial standing were disproportionately bearing the cost of decarbonization, and that the tax unfairly burdened those who could not cut down on driving or switch to greener alternatives.[1] Ultimately, mounting unrest led the French government to suspend the planned fuel tax hike.

This paper considers a government allocating goods such as vaccines, affordable housing, basic food items, or emission rights while being concerned about the perceived equity of the allocation. I model equity concerns using a *merit function* specifying societal perceptions of how entitled each agent is to the allocated good. For instance, in the case of emissions, rural households reliant on cars for transportation would be more deserving of a right to emit than urban ones. I then introduce an equity constraint requiring that agents with equal merit receive the same allocation.

I ask what policy designs are compatible with equity. Standard economic logic suggests that public programs can be made more efficient through costly screening—if agents need to pay or go through ordeals to get the benefit, only those who need it will do so (Nichols and Zeckhauser, 1982). Indeed, the literature studying the optimal design of such programs

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[1] https://www.nytimes.com/2018/12/06/world/europe/france-fuel-carbon-tax.html

(e.g. Condorelli (2013), Akbarpour et al. (2020)) finds that the optimal mechanism often involves screening recipients, e.g. through payments. For instance, Akbarpour et al. (2020) find that, under fairly general conditions, the government should sell the good (at possibly non-market prices) and redistribute the revenue to poorer agents.

However, we might worry that screening could produce inequitable allocations, as people with the same need for the good may find waiting or paying burdensome to different extents. Consider for instance the problem of distributing Covid-19 vaccines studied by Akbarpour et al. (2023); the authors show that the optimal policy combines priorities to vulnerable groups with a market mechanism under which one can pay to be vaccinated early. While proxying for need with willingness to pay may improve targeting, such inference is cofounded by differences in wealth—a poor person with severe health conditions might still be less willing to pay for early vaccination than someone who is healthy but rich. Related objections to selling vaccines have been raised by both academic philosophers (Kass, 1997; Walzer, 1983) and the public.[2] Indeed, the authors themselves acknowledge that their proposed policy might provoke backlash on fairness grounds. Such equity concerns may explain why governments often forgo screening in favor of mechanisms based purely on observables, or lotteries.[3] Examples of goods allocated by lottery include school places (Stone, 2008), public housing (Elster (1989), p.63) and US green cards.

I therefore ask how (if at all) the government can screen agents when allocations are subject to equity constraints. I look at screening using only payments (which are less costly to the rich), only waiting (less costly to the poor) and both of these instruments at once. In the former two cases equitable screening is impossible. However, when the government uses both payments and waiting, it has significant freedom to screen despite equity constraints.

This paper relates to work examining how moral sentiments constrain market designers and policymakers. Roth (2007) discusses how the repugnance of certain transactions precludes the use of markets in settings where they would be efficient. The literature following Abdulkadiroğlu and Sönmez (2003) models fairness concerns in matching markets through assigning *priorities* to agents. It then studies matching mechanisms that eliminate *justified envy*—a notion capturing perceived injustice. Finally, Dessein et al. (2023) argue that many US colleges switched to test-optional admissions to reduce public scrutiny of their admission decisions. However, no existing work studies equity constraints in mechanism design problems of the sort this paper considers. In doing so, my paper also relates to the literatures on algorithmic fairness in Computer Science and on discrimination in Economics, which attempt to conceptualize bias, unfairness and discrimination (see Alves et al. (2023)

[2] `https://www.statnews.com/2020/12/03/how-rich-and-privileged-can-skip-the-line-for-covid19-vaccines`

[3] One might argue, along the lines of Weitzman (1977), that governments eschew screening not due to fairness concerns, but because screening is not optimal when the government has distributional concerns. For instance, if a government allocating affordable housing screened using payments, the allocation would be biased towards wealthier people. This would be suboptimal for a government with redistributive preferences. There are two responses to this argument. First, the government could use a screening instrument other than payments whose cost is *negatively* correlated with wealth; in Subsection 3.2 I study screening with waiting which plausibly has this feature. Secondly, as observed by Akbarpour et al. (2020), screening with payments is still often optimal even in such settings. This is because the government can raise revenue from selling the good to richer agents and then redistribute it to the poor who value money highly. These observations suggest a deeper reason for the frequent absence of screening in public programs.

and Onuchic (2022) for respective surveys). However, both of these literatures focus on problems of classification or statistical inference and typically do not account for the strategic behavior of agents. By contrast, the purpose of this paper is to study mechanisms that are fair after accounting for strategic responses.

The rest of the paper is structured as follows. I first introduce the model of public provision with an equity constraint. Section 3 then discusses the forms of screening that are feasible in this environment. In Section 4 I ask what allocation rules can be equitably implemented if the government also observes agents' wealth. Section 5 discusses a relaxed version of the equity constraint; Section 6 concludes.

2 Model

The government allocates goods $x \in [0,1]$ to agents with types $(\alpha,\beta) \in \Theta \subseteq \mathbb{R}^2$. Θ is open, convex and bounded. β represents an agent's value for the good and α represents her value for money (higher α means the agent is poorer). I consider two screening instruments—payments and wait times. Payments, $p \in \mathbb{R}$, are more burdensome for poorer agents (higher α), while waiting, $q \in \mathbb{R}_+$, is costlier for richer agents. Utility is given by:

$$U[\alpha,\beta;x,p,q] = v(\beta,x) - w(\alpha,p) - z(\alpha,q).$$

Assumption 1. *The following conditions hold for all $(\alpha,\beta) \in \Theta$ and all $x \in [0,1], p \in \mathbb{R}, q \in \mathbb{R}_+$:*

1. *v, w, z are twice continuously differentiable.*
2. $v_\beta > 0$, $v_x > 0$, $w_\alpha > 0$, $w_p > 0$, $z_\alpha < 0$, $z_q > 0$.
3. $v_{\beta x} > 0$, $w_{\alpha p} > 0$, $z_{\alpha q} < 0$.
4. $v(\beta,0) = w(\alpha,0) = z(\alpha,0) = 0$.

I assume that need for money α and need for the good β are private information (I discuss this assumption in Section 4). The government therefore chooses an allocation rule $x : \Theta \to [0,1]$, payment rule $p : \Theta \to \mathbb{R}$ and waiting rule $q : \Theta \to \mathbb{R}_+$ subject and (IC) and (IR) constraints:

$$\text{for all } (\alpha,\beta) \in \Theta, \quad U[\alpha,\beta;(x,p,q)(\alpha,\beta)] \geq \sup_{(\alpha',\beta')\in\Theta} U[\alpha,\beta;(x,p,q)(\alpha',\beta')], \tag{IC}$$

$$\text{for all } (\alpha,\beta) \in \Theta, \quad U[\alpha,\beta;(x,p,q)(\alpha,\beta)] \geq 0. \tag{IR}$$

An allocation rule $x^*(\alpha,\beta)$ is *implementable* if there exist payment and waiting rules $p^*(\alpha,\beta)$ and $q^*(\alpha,\beta)$ such that (x^*,p^*,q^*) satisfies (IC) and (IR).

The government also faces an *equity constraint*, which I model using an exogenous *merit function* $\eta : \mathbb{R}^2 \to \mathbb{R}$ specifying how entitled each agent is to the good. I assume that $\eta(\alpha,\beta)$ is twice continuously differentiable with $\eta_\alpha, \eta_\beta > 0$. That is, agents are more entitled to the good if they value it more or if they are poorer (as richer agents can more easily satisfy their needs without government assistance). Since Θ is open, bounded and connected, and the merit function is strictly increasing and continuous, the set of values attained by $\eta(\alpha,\beta)$ on Θ is an open interval, which I denote by $(\underline{\eta},\overline{\eta})$.

The equity constraint requires that all agents with the same merit receive equal amounts of the good:[4]

$$\eta(\alpha^a,\beta^a)=\eta(\alpha^b,\beta^b) \quad\implies\quad x(\alpha^a,\beta^a)=x(\alpha^b,\beta^b). \tag{E}$$

I call an allocation rule satisfying (E) *equitable*. Note that an allocation rule $x(\alpha,\beta)$ is equitable if and only if it can be written in the form $x(\alpha,\beta)\equiv\hat{x}(\eta(\alpha,\beta))$ for some $\hat{x}:(\underline{\eta},\overline{\eta})\to[0,1]$.

While the equity constraint does not require that agents with higher merit receive more of the good, this will still be the case for any implementable equitable allocation:

Lemma 1. *If the allocation rule $x(\alpha,\beta)\equiv\hat{x}(\eta(\alpha,\beta))$ is implementable, then $\hat{x}$ is weakly increasing.*

Proof. Any implementable $x(\alpha,\beta)$ has to be implementable for the subset of agents with need for money $\alpha=\alpha^a$, for any α^a. We can write the utility of such agents as $v(\beta,x(\alpha^a,\beta))-t(\beta)$, where $t(\beta):=w(\alpha^a,p(\alpha^a,\beta))+z(\alpha^a,q(\alpha^a,\beta))$. This is a one-dimensional quasi-linear screening problem so any implementable allocation has to be weakly increasing in the value for the good β. Hence, $x(\alpha,\beta)$ must be weakly increasing in β for every α. Since the allocation rule takes the form $\hat{x}(\eta(\alpha,\beta))$ and the merit function $\eta(\alpha,\beta)$ is strictly increasing in β, it follows that $\hat{x}$ has to be increasing. □

3 Implementable equitable allocations

I now ask what forms of screening are compatible with equity. I study the sets of implementable equitable allocations in three cases: when the government uses only payments to screen, when it uses only waiting, and when it uses both screening instruments.

3.1 Screening with payments

I first show that the government cannot equitably screen using only payments.

Proposition 1. *Suppose we do not use waiting to screen, so $q\equiv 0$. Then every equitable and implementable allocation rule $x(\alpha,\beta)$ is constant.*

Proof. By Lemma 1, any implementable and equitable allocation rule $x(\alpha,\beta)$ can be written as $\hat{x}(\eta(\alpha,\beta))$, where $\hat{x}$ is weakly increasing. Note also that the payment rule has to take the form $p(\alpha,\beta)\equiv\hat{p}(\eta(\alpha,\beta))$ because identical allocations of x must require identical payments. An argument analogous to the proof of Lemma 1 tells us that any implementable payment rule $p(\alpha,\beta)$ has to be weakly decreasing in α. Since the merit function $\eta(\alpha,\beta)$ is strictly increasing in need for money α, it follows that $\hat{p}$ must be weakly decreasing. However, $\hat{p}$ also has to be weakly increasing—otherwise one could deviate and receive a weakly greater allocation of x for a strictly smaller payment. Therefore, $\hat{p}$ has to be constant. Such a payment rule can only support a constant allocation. □

[4] My model can capture lotteries when the allocation is binary or the utility for the good is linear: $v(\beta,x)=\beta\cdot x$. In the former case, we can interpret x as the probability of being allocated the good.

Intuitively, equity requires that poorer and richer agents of equal merit receive the same allocation, even though the richer agents have higher need for the good. However, these richer agents with higher need for the good have greater willingness to pay for it (Figure 1). Therefore, any mechanism that sells the good will allocate more of it to the richer agents, and hence violate equity.

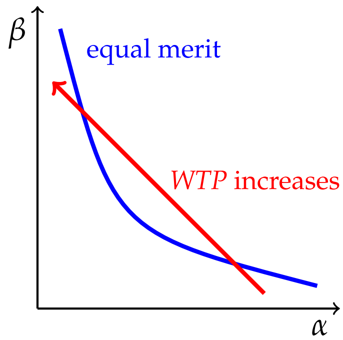

Figure 1: While merit increases in the north-east direction, willingness to pay increases in the north-west direction.

3.2 Screening with waiting

I now show that the government cannot screen equitably using only waiting unless certain knife-edge conditions on primitives hold. Intuitively, screening with waiting differs from screening with payments in that the allocation it produces is biased *towards the poor*. That is, between two people with the same value for the good β, the poorer one will be more eager to wait to get the good, and hence will receive a higher allocation. Unlike in the case of screening with payments, this direction of bias is consistent with what the merit function requires. Notice, however, that equity constraints impose requirements not only on the *direction* of the allocation's bias, but also on its exact form—the merit function $\eta(\alpha, \beta)$ specifies exactly the sets of types that have to be treated identically. However, with one screening device only, the government will generically have 'too few degrees of freedom' to pool agents in this exact way. Proposition 2 formalizes this reasoning.

Proposition 2. *Suppose we do not use payments to screen, so $p \equiv 0$. Then, if a non-constant allocation rule $x(\alpha, \beta)$ is equitable and implementable, one of the following conditions has to hold for some $\eta^* \in (\underline{\eta}, \overline{\eta})$:*

1. *There exist $x \in [0,1]$, $q \in \mathbb{R}_+$ and $k \in \mathbb{R}$ such that for all types (α, β) with merit $\eta(\alpha, \beta) = \eta^*$ we have:*
$$\frac{v_x(\beta, x)}{z_q(\alpha, q)} = k.$$

2. *There exist $x^a \neq x^b \in [0,1]$, $q^a, q^b \in \mathbb{R}_+$ such that for all types (α, β) with merit $\eta(\alpha, \beta) = \eta^*$ we have:*
$$v(\beta, x^a) - z(\alpha, q^a) = v(\beta, x^b) - z(\alpha, q^b).$$

While the proof is relegated to the appendix, I provide the key intuition behind this result. Notice that the waiting rule has to take the form $q(\alpha, \beta) \equiv \hat{q}(\eta(\alpha, \beta))$ since identical allocations of x have to come with equal wait times. Moreover, note that $\hat{q}$ has to be increasing—otherwise one could deviate and receive weakly more x for waiting strictly less. Now, consider the case where $\hat{x}$ and $\hat{q}$ are smoothly increasing around some merit level η^*. Then the first-order conditions for all agents with merit η^* must hold there:

$$\text{for all } (\alpha, \beta) \text{ such that } \eta(\alpha, \beta) = \eta^*, \quad \frac{v_x(\beta, \hat{x}(\eta^*))}{z_q(\alpha, \hat{q}(\eta^*))} = \frac{\hat{q}'(\eta^*)}{\hat{x}'(\eta^*)}. \tag{1}$$

Satisfying this condition would require equating the MRSs of all agents with merit η^*, of whom there are uncountably many. However, the government can only attempt doing so by choosing three values: the allocation and waiting time at η^*, and the value all the MRSs would take. Therefore, except for knife-edge cases, the government will 'lack degrees of freedom' to ensure this condition (Figure 2). Condition 2. in Proposition 2 relates to a similar problem for merit levels where the allocation rule $\hat{x}$ and waiting rule $\hat{q}$ jump discontinuously.

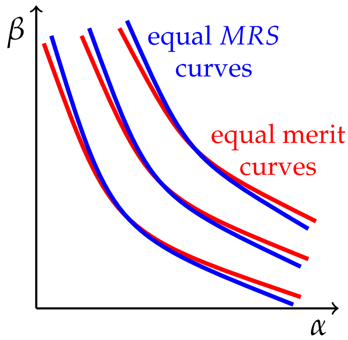

Figure 2: Except in knife-edge cases, no choice of $\hat{x}(\eta^*)$ and $\hat{q}(\eta^*)$ can equate the MRSs of all agents with merit η^*.

3.3 Screening with payments and waiting

I now let the government use both screening instruments at once. To keep the model tractable, I assume $\Theta \subset \mathbb{R}_+ \times \mathbb{R}$ and impose more structure on the utility function:

$$U[\alpha, \beta; x, p, q] = \beta x - w(\alpha)p - z(\alpha)q,$$

where the following properties hold for all $\alpha \in \mathbb{R}$:

1. $w(\alpha), z(\alpha)$ are twice continuously differentiable.
2. $w(\alpha), z(\alpha) > 0$.
3. $w'(\alpha) > 0$ and $z'(\alpha) < 0$.

I also impose the following technical assumption:

Assumption 2. *The following properties hold for all* $(\alpha, \beta) \in \mathbb{R}^2$*:*

1. $\frac{d}{d\alpha}\frac{z(\alpha)}{w(\alpha)} \neq 0$.

2. $\lim_{\alpha\to\infty}\frac{z(\alpha)}{w(\alpha)} = 0$, $\lim_{\alpha\to-\infty}\frac{z(\alpha)}{w(\alpha)} = \infty$.

3. *$\eta_\beta(\alpha,\beta)$ is uniformly bounded away from zero.*

As it turns out, using both screening devices allows for rich screening without violating the equity constraint. Intuitively, every amount of the good can now come with a menu of payment options composed of different fee and wait time pairs. Since the government has one screening device preferred by the poor and another one preferred by the rich, it can fine-tune such 'payment menus' to produce precisely the bias in allocation that equity requires. Loosely speaking, being able to compose such menus fixes the problem of 'too few degrees of freedom' we encountered when only wait times were used.

Theorem 1. *An allocation rule $x(\alpha,\beta)$ is equitable and implementable if and only if $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, where $\hat{x}$ is increasing.*

The proof proceeds in three steps. First, I reparametrize the type space so that the new types (κ,λ) correspond to agents' relative values for the good and time (κ), and money and time (λ). I enlarge the reparametrized type space to ensure convexity.

I then consider *threshold rules*, i.e. allocation rules that give $x = 1$ to agents with merit above a certain level and $x = 0$ to those with merit below it. I show that any threshold rule is implementable. The proof uses the characterization by Rochet (1987), which states that a (multidimensional) allocation rule is implementable in a linear multidimensional setting if and only if it is a subgradient of some convex indirect utility function. I prove that such indirect utility functions can be constructed for every threshold rule by making the slope of the waiting rule increase sufficiently quickly in λ.

Lastly, I use the fact that any allocation rule of the form $x(\alpha,\beta) = \hat{x}(\eta(\alpha,\beta))$ that is increasing in merit can be written as a combination of threshold rules under some probability measure. Since (IC) and (IR) are linear, the fact that threshold rules are implementable means that any such $x(\alpha,\beta)$ is implementable too.

Proof. I first reparametrize types:

$$\kappa = \frac{\beta}{w(\alpha)}, \quad \lambda = -\frac{z(\alpha)}{w(\alpha)}, \quad \tilde{\Theta} = \left\{\left(\frac{\beta}{w(\alpha)}, -\frac{z(\alpha)}{w(\alpha)}\right) : (\alpha,\beta)\in\Theta\right\}.$$

Notice that the mapping between (α,β) and (κ,λ) is one-to-one (this would not necessarily be the case if both screening devices were less costly to the rich or to the poor). Moreover, Assumption 1 guarantees that the reparametrized type space $\tilde{\Theta}$ is bounded, so there exist $\underline{\kappa},\bar{\kappa}\in\mathbb{R}$, $\underline{\lambda},\bar{\lambda}\in\mathbb{R}_{--}$ such that $\tilde{\Theta}\subset[\underline{\kappa},\bar{\kappa}]\times[\underline{\lambda},\bar{\lambda}]$. Agents' utilities (up to scaling) in the reparametrized model are given by:

$$\tilde{U}[\kappa,\lambda;x,p,q] = \kappa x + \lambda q - p.$$

I also rewrite the merit function in terms of (κ, λ); let $\tilde{\eta} : \mathbb{R} \times \mathbb{R}_{--} \to \mathbb{R}$ be defined by:

$$\tilde{\eta}\left(\frac{\beta}{w(\alpha)}, -\frac{z(\alpha)}{w(\alpha)}\right) \equiv \eta(\alpha, \beta).$$

Fact 1 (shown in the appendix) establishes some useful properties of $\tilde{\eta}(\kappa, \lambda)$:

Fact 1. *$\tilde{\eta}(\kappa, \lambda)$ is defined everywhere on $\mathbb{R} \times \mathbb{R}_{--}$. Moreover, it satisfies the following properties:*

1. *$\tilde{\eta}(\kappa, \lambda)$ is twice continuously differentiable on $\mathbb{R} \times \mathbb{R}_{--}$.*
2. *$\tilde{\eta}_\kappa(\kappa, \lambda)$ is positive and uniformly bounded away from zero on $\mathbb{R} \times [\underline{\lambda}, \overline{\lambda}]$.*

We are now ready to prove Theorem 1. By Lemma 1, any equitable and implementable allocation rule has to take the form $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$, where $\hat{x}$ is increasing. It therefore suffices to show that any such allocation rule is implementable on Θ. In the reparametrized type space this amounts to showing that we can implement any $\tilde{x}(\kappa, \lambda) : \tilde{\Theta} \to [0, 1]$ such that $\tilde{x}(\kappa, \lambda) \equiv \hat{x}(\tilde{\eta}(\kappa, \lambda))$, where $\hat{x}$ is increasing. In fact, I prove a stronger statement: consider an extension of the reparametrized type space to $[\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$. Let $\underline{\tilde{\eta}}$ and $\overline{\tilde{\eta}}$ be the minimum and maximum of $\tilde{\eta}(\kappa, \lambda)$ on $[\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$ and notice that $(\underline{\eta}, \overline{\eta}) \subseteq [\underline{\tilde{\eta}}, \overline{\tilde{\eta}}]$. I then show that for any increasing $\hat{x} : [\underline{\tilde{\eta}}, \overline{\tilde{\eta}}] \to [0, 1]$ there exists an allocation rule $\tilde{x} : [\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}] \to [0, 1]$ that is implementable on the extended type space and satisfies $\tilde{x}(\kappa, \lambda) = \hat{x}(\tilde{\eta}(\kappa, \lambda))$.

To that end, define a *threshold rule* as a function $\hat{x} : [\underline{\tilde{\eta}}, \overline{\tilde{\eta}}] \to [0, 1]$ satisfying:

$$\hat{x}(\eta) = \begin{cases} 1 & \text{if } \eta > \eta^* \\ 1 \text{ or } 0 & \text{if } \eta = \eta^* \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

for some $\eta^* \in [\underline{\tilde{\eta}}, \overline{\tilde{\eta}}]$ (Figure 3). Let $\mathcal{T}$ be the set of threshold rules.

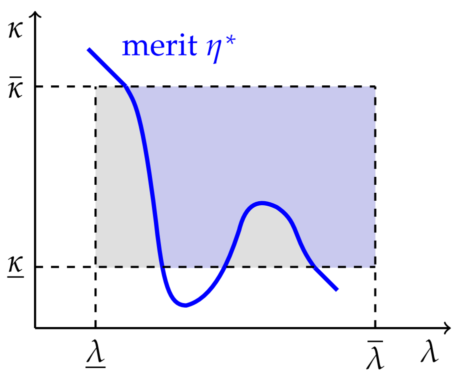

Figure 3: A threshold rule allocates $x = 1$ to agents with merit above η^* (blue region) and $x = 0$ to those with merit below η^* (grey region).

Fix any threshold rule $\hat{x} \in \mathcal{T}$. I will show there exists an allocation rule $\tilde{x}$ that is implementable on $[\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$ and satisfies $\tilde{x}(\kappa, \lambda) \equiv \hat{x}(\tilde{\eta}(\kappa, \lambda))$. Following Rochet (1987), the allocation rule $\tilde{x}(\kappa, \lambda)$ is implementable on $[\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$ if there exists a bounded, non-negative waiting rule $\tilde{q}(\kappa,\lambda)$ and a bounded, convex indirect utility function $V(\kappa,\lambda)$ such that for every (κ,λ), $[\hat{x}(\tilde{\eta}(\kappa,\lambda)),\tilde{q}(\kappa,\lambda)]$ belongs to the subdifferential of $V(\kappa,\lambda)$ at that point. The following lemma (shown in the appendix) asserts that such a waiting rule and indirect utility function indeed exist.

Lemma 2. *There exists a bounded, non-negative waiting rule $\tilde{q}(\kappa,\lambda)$ and a bounded, convex indirect utility function $V(\kappa,\lambda)$ such that $[\hat{x}(\tilde{\eta}(\kappa,\lambda)),\tilde{q}(\kappa,\lambda)]$ belongs to the subdifferential of $V(\kappa,\lambda)$ for every $(\kappa,\lambda) \in [\underline{\kappa},\overline{\kappa}] \times [\underline{\lambda},\overline{\lambda}]$.*

Now, let $\mathcal{A}$ be the set of weakly increasing functions $\hat{x} : [\underline{\tilde{\eta}},\overline{\tilde{\eta}}] \to [0,1]$. $\mathcal{A}$ is convex and compact in the product topology and $\mathcal{T}$ is the set of its extreme points. Hence, by Choquet's theorem, for every $\hat{x}^* \in \mathcal{A}$ there exists a probability measure μ on $\mathcal{T}$ such that $\hat{x}^* = \int_{\mathcal{T}} \hat{x}\mu(d\hat{x})$ (Phelps, 2001). However, we have already shown that for every $\hat{x} \in \mathcal{T}$ there exists an allocation rule $\tilde{x}[\hat{x}]$ satisfying $\tilde{x}[\hat{x}](\kappa,\lambda) \equiv \hat{x}(\tilde{\eta}(\kappa,\lambda))$ that is implementable on $[\underline{\kappa},\overline{\kappa}] \times [\underline{\lambda},\overline{\lambda}]$. Define:

$$\tilde{x}^* := \int_{\mathcal{T}} \tilde{x}[\hat{x}]\mu(d\hat{x}) = \int_{\mathcal{T}} \hat{x}\mu(d\hat{x}) = \hat{x}^*.$$

Since all $\tilde{x}[\hat{x}]$ are implementable and (IC) and (IR) are linear in x, p and q, it follows that $\tilde{x}^*$ is implementable too. □

4 Observable wealth

While I assumed that neither need nor wealth are observable, the government usually has some information about them. For instance, in the problem of vaccine allocation, age and medical history are good indicators of need. Similarly, tax data proxies for one's wealth, even if some income sources or assets remain unobserved. In such cases, agents' private information can be thought of as *residual uncertainty* after accounting for observables. Such uncertainty can still be substantial even in means-tested programs.[5]

Still, what can the government do if it perfectly observes agents' need for money α? As it turns out, the set of equitable allocation rules that are implementable with one instrument when α is observed is identical to the set of equitable allocation rules that are implementable with two instruments when α is private. Intuitively, when the government observes α, it can 'control for' the fact that some agents prefer the good because they are wealthy and screen purely based on need.

Proposition 3. *Suppose need for money α is observable and the government uses either only payments or only wait times to screen. Then the allocation rule $x(\alpha,\beta)$ is equitable and implementable if and only if $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$, where $\hat{x}$ is increasing.*

Proof. Since α is observable, the allocation rule can be implemented separately for every value of need for money α. Thus, the allocation rule $x(\alpha,\beta)$ is implementable if and only if it is weakly increasing in the value for the good β. Recall the equity constraint is satisfied if and only if $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$. Since $\eta(\alpha,\beta)$ is strictly increasing in β, $x(\alpha,\beta) \equiv \hat{x}(\eta(\alpha,\beta))$ is equitable and implementable if and only if $\hat{x}$ is weakly increasing. □

[5] https://thehill.com/regulation/administration/268409-outrage-builds-over-wealthy-families-in-public-housing/

5 Relaxing the equity constraint

My model of societal perceptions of equity was built around a single merit function. In reality, however, people often share general principles concerning equity and desert but hold different opinions about finer trade-offs or ways in which these principles should be applied. For instance, Craxì et al. (2021) find that while healthcare workers tended to agree which groups should get priority in the Covid-19 vaccine rollout, their opinions differed on how these groups should be ranked.

Real-world equity constraints on policymakers would therefore be less demanding than my analysis suggests. While Theorem 1 says that rich screening with multiple instruments is possible even under such overly strong restrictions, it is also interesting to compare mechanisms screening with only payments and only waiting when the equity constraint is relaxed. To that end, I develop a measure of *equity violation* capturing how far away a particular allocation is from satisfying the equity constraint (here, I interpret the merit function as a 'rough consensus' among the public). My measure assumes that every agent assesses the allocation's equity by looking at agents similar to herself and comparing those with the same allocation as her to those with the same merit as her. In other words, she compares agents similar to herself who *are* treated the same as her with those who *should* be treated the same. The further apart these two sets are, the more inequitable the allocation seems to her. Then, the degree to which the whole allocation violates equity is the size of the largest such 'local equity violation'.

Definition 1 (Equity violation). *For every type $(\alpha, \beta) \in \Theta$, let $D(\alpha, \beta)$ be the set of directions in which the allocation is locally constant:*

$$D(\alpha, \beta) = \{d \in \mathbb{R}^2 : \nabla_d x(\alpha, \beta) = 0\}.$$

*Let the **local equity violation** for type (α, β) be:*

$$l(\alpha, \beta) = \begin{cases} \infty & \textit{if } D(\alpha, \beta) = \varnothing, \\ \inf_{d \in D(\alpha,\beta)} \left| \arctan(d) - \arctan\left(-\frac{\eta_\alpha(\alpha,\beta)}{\eta_\beta(\alpha,\beta)}\right) \right| & \textit{otherwise.}^{6} \end{cases} \tag{3}$$

*The **equity violation** of the allocation rule $x(\alpha, \beta)$, denoted $L(x)$, is its largest local equity violation:*

$$L(x) := \sup_{(\alpha,\beta) \in \Theta} l(\alpha, \beta).$$

To build intuition, consider a smooth allocation rule that is strictly increasing in need for the good β and fix some type (α^a, β^a). Then all types with allocations equal to that of (α^a, β^a) will lie on a smooth curve passing through (α^a, β^a). Figure 4a illustrates such a curve together with this type's *iso-merit curve*, that is, the set of types with the same merit as (α^a, β^a). Since both of these curves are smooth, we can compare 'how far apart' they are in the neighbor-

[6] My results would be unaffected if arctan was replaced with another bounded and strictly increasing function. However, using arctan will let us visualize the size the local equity violation as an angle.

hood of (α^a, β^a) by looking at the angle between their tangents there—this angle measures the *local equity violation* at (α^a, β^a) (Figure 4b).

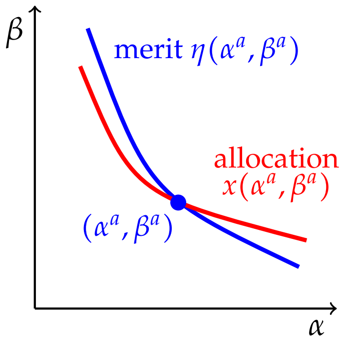

Figure 4a: The iso-merit curve and the iso-allocation curve at type (α^a, β^a).

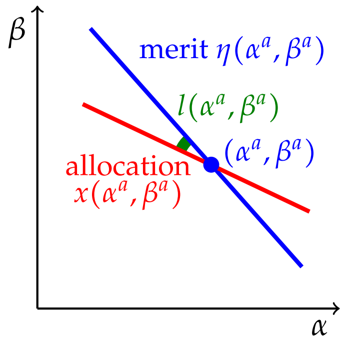

Figure 4b: The slopes of the iso-merit curve and the iso-allocation curve at type (α^a, β^a).

We can ask which of the two screening instruments, when used on its own, produces larger equity violations. A naïve intuition suggests that screening with waiting biases the allocation 'in the right direction' and thus is violates equity less. However, as discussed in Subsection 3.2, a mechanism screening with waiting will generically fail to pool together agents in a way that 'matches the shape' of the merit curve. As it turns out, in some cases this 'shape effect' dominates the aforementioned 'direction effect'.

Screening with waiting will nevertheless produce smaller equity violations when the merit function depends on wealth sufficiently strongly. The following proposition says that, as long as iso-merit curves are sufficiently flat everywhere, there is a non-constant allocation rule implemented with only waiting that violates equity by less than any non-constant allocation rule implemented using only payments.

Proposition 4. *Assume wait times are bounded above by some $\bar{q}$. Then there exists $M < 0$ with the following property: suppose that the slope of the iso-merit curve is flatter than M at every type, so:*

$$\textit{for all } (\alpha, \beta) \in \Theta, \quad -\frac{\eta_\alpha(\alpha, \beta)}{\eta_\beta(\alpha, \beta)} > M.$$

Then there exists a a non-constant allocation rule x_q that is implementable with only waiting that produces a strictly smaller equity violation than any non-constant allocation rule x_p that is implementable with only payments: $L(x_q) < L(x_p)$.

While the proof is relegated to the appendix, I illustrate its key intuition with the case of a smooth allocation that increases in the value for the good β. Fix some type (α^a, β^a) and compare two allocations: x_p implemented using only payments and x_q implemented using only waiting. Like before, the sets of agents with the same allocation as (α^a, β^a) will be smooth curves passing through that point. Moreover, the insights from Subsections 3.1 and 3.2 tell us that the curve for the allocation rule x_p will be upwards-sloping (Figure 5a), while the curve for the allocation rule x_q will be downwards-sloping (Figure 5b).

Let us now compare the local equity violations of these two allocation rules. Figure 6a illustrates that if the iso-merit curve is sufficiently flat, its angle with the iso-allocation curve

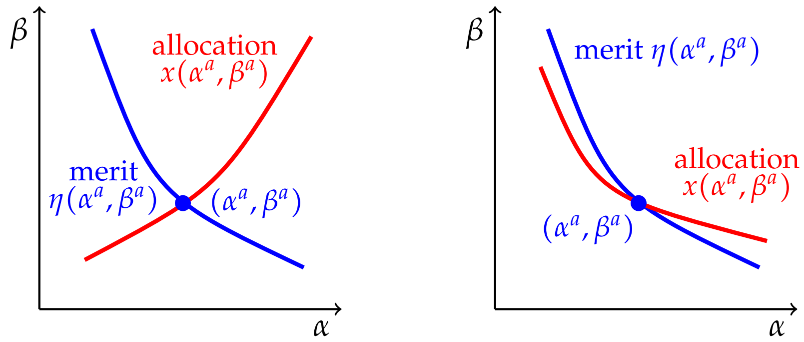

Figure 5a: Allocation rule implemented using only payments

Figure 5b: Allocation rule implemented using only waiting

for x_q will be smaller than that with the iso-allocation curve for x_p. If we can impose a sufficiently low uniform bound on the slopes of iso-merit curves, this will be true for every type and every allocation rule x_p implemented with only payments. Figure 6b, on the other hand, illustrates why the result of Proposition 4 fails when iso-merit curves are not flat enough.

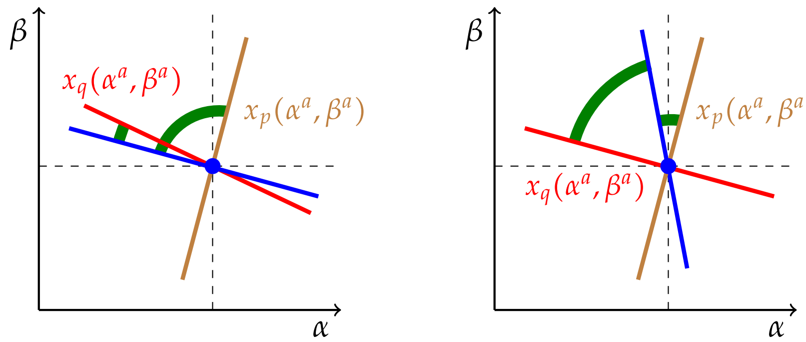

Figure 6a

Figure 6b

Figure 6: If the iso-merit curve is sufficiently steep, screening with money may be more equitable than screening with wait times.

This exercise offers broader qualitative lessons. Roughly speaking, Proposition 4 tells us that screening with waiting violates equity by less than screening with payments if merit depends more strongly on wealth than it does on need. However, the same will be true if agents vary a lot in wealth relative to how much they vary in need. To understand why, consider a mechanism screening with payments. If agents differ greatly in wealth (relative to how much they differ in need), most of the variance in allocations will be explained by preferences for money. This in turn means that the allocation will be heavily skewed towards the rich—this corresponds to a very flat iso-x_p curve in Figure 6a. Consequently, the angle between this curve and the iso-merit curve will be wider.

6 Discussion

While my approach to modeling perceived equity is highly stylized, it offers general qualitative conclusions. First, every screening instrument will bias the allocation towards the group for which this instrument is less costly—this makes screening with payments problematic from an equity standpoint. Using a different instrument (like waiting) could reverse this bias but the government's control over the allocation would still be limited. Consequently, the resulting bias might still not satisfy the public's equity concerns. I show this problem can be solved by combining multiple screening instruments which on their own favor different social groups. Doing so gives the government freedom to tinker with various groups' differential cost of the allocated good, and therefore to improve efficiency through screening while still producing an allocation that is seen as fair. I also highlight that screening with waiting need not always produce more equitable allocations than screening with payments. Waiting is, however, likely to be the more equitable instrument when society is highly averse to handing out benefits to wealthier people, or when the population's wealth heterogeneity is large relative to heterogeneity in terms of need for the good.

7 Appendix

In what follows I write $(\alpha^a_\delta, \beta^a_\delta)$ for $(\alpha^a + \delta, \beta^a + \delta)$, x^a_δ for $x(\alpha^a_\delta, \beta^a_\delta)$ and use p^a_δ, q^a_δ and η^a_δ analogously. I omit the subscript when $\delta = 0$.

The following lemma will be useful in the proofs presented here. While it resembles a first-order condition, its proof is more involved because the type space is two-dimensional and the waiting rule need not be piece-wise differentiable.

Lemma 3 (Generalized FOC)**.** *Suppose we do not use payments to screen, so $p \equiv 0$. Fix any type $(\alpha^a, \beta^a) \in \Theta$ and assume there exists a decreasing sequence $\{\delta_i\}_i$ such that $\delta_i \to 0$ for which $\{x^a_{\delta_i}\}_i$ is strictly decreasing and $x^a_{\delta_i} \to x^a$. Then, for any (α^b, β^b) such that $x^b = x^a$ we have:*

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} \leq \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}. \tag{4}$$

Proof. Note that $\{q^a_{\delta_i}\}_i$ is strictly decreasing since strictly lower allocations must come with strictly lower wait times. I first show that $q^a_{\delta_i} \to q^a$. Indirect utility has to be continuous at (α^a, β^a) (Milgrom and Segal, 2002) so:

$$v(\beta^a, x^a) - z(\alpha^a, q^a) = \lim_{i\to\infty} \left\{ v(\beta^a_{\delta_i}, x^a_{\delta_i}) - z(\alpha^a_{\delta_i}, q^a_{\delta_i}) \right\}.$$

Note that $\lim_{i\to\infty} v(\beta^a_{\delta_i}, x^a_{\delta_i}) = v(\beta^a, x^a)$. Since $\alpha^a_{\delta_i} \to \alpha^a$ and z is continuous and strictly increasing in the latter argument, we have $q^a_{\delta_i} \to q^a$.

Now, suppose (4) fails:

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} > \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}.$$

By continuity of v_x and z_q, for i high enough we have:

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} > \frac{v_x(\beta^a_{\delta_i}, x^a)}{z_q(\alpha^a_{\delta_i}, q^a)}. \tag{5}$$

Since $\{x^a_{\delta_i}\}_i$ and $\{q^a_{\delta_i}\}_i$ are strictly decreasing and tend to x^a and q^a, we have that for all α, β:

$$\frac{v(\beta, x^a_{\delta_j}) - v(\beta, x^a)}{x^a_{\delta_j} - x^a} \to v_x(\beta, x^a), \quad \frac{z(\alpha, q^a_{\delta_j}) - z(\alpha, q^a)}{q^a_{\delta_j} - q^a} \to z_q(\alpha, q^a) \quad \text{as } j \to \infty. \tag{6}$$

Fix i high enough that (5) holds. Then, by (6), for j high enough we have:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{\frac{v(\beta^a_{\delta_i}, x^a_{\delta_j}) - v(\beta^a_{\delta_i}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_i}, q^a_{\delta_j}) - z(\alpha^a_{\delta_i}, q^a)}{q^a_{\delta_j} - q^a}}.$$

In particular, take $j > i$ and notice that increasing i to j further relaxes the inequality. Thus:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{\frac{v(\beta^a_{\delta_j}, x^a_{\delta_j}) - v(\beta^a_{\delta_j}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_j}, q^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a)}{q^a_{\delta_j} - q^a}}. \tag{7}$$

Now, by revealed preference we have:

$$v(\beta^a_{\delta_j}, x^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a_{\delta_j}) \geq v(\beta^a_{\delta_j}, x^a) - z(\alpha^a_{\delta_j}, q^a) \implies \frac{\frac{v(\beta^a_{\delta_j}, x^a_{\delta_j}) - v(\beta^a_{\delta_j}, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^a_{\delta_j}, q^a_{\delta_j}) - z(\alpha^a_{\delta_j}, q^a)}{q^a_{\delta_j} - q^a}} \geq \frac{q^a_{\delta_j} - q^a}{x^a_{\delta_j} - x^a}.$$

Combining the latter inequality with (7) gives:

$$\frac{\frac{v(\beta^b, x^a_{\delta_j}) - v(\beta^b, x^a)}{x^a_{\delta_j} - x^a}}{\frac{z(\alpha^b, q^a_{\delta_j}) - z(\alpha^b, q^a)}{q^a_{\delta_j} - q^a}} > \frac{q^a_{\delta_j} - q^a}{x^a_{\delta_j} - x^a} \implies v(\beta^b, x^a_{\delta_j}) - z(\alpha^b, q^a_{\delta_j}) > v(\beta^b, x^a) - z(\alpha^b, q^a).$$

Hence, $x^b \neq x^a$; contradiction. □

7.1 Proof of Proposition 2

Let $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$ be an equitable, implementable and non-constant allocation rule and consider two cases.

Case 1: $\hat{x}$ is discontinuous at some η^a. Indirect utility has to be continuous so, by continuity of η, v and z, the following holds for any type (α, β) with merit η^a:

$$v(\beta, \hat{x}_+(\eta^a)) - z(\alpha, \hat{q}_+(\eta^a)) = v(\beta, \hat{x}_-(\eta^a)) - z(\alpha, \hat{q}_-(\eta^a)).$$

However, the existence of such $\hat{x}_+(\eta^a), \hat{x}_-(\eta^a)$, $q_+(\eta^a), q_-(\eta^a)$ implies condition 2. in Proposition 2.

Case 2: $\hat{x}$ is continuous. The proof of this case relies on the Generalized FOC (Lemma 3). I first show that there exists a sequence that lets us apply it.

Fact 2. *There exists $\eta^a \in (\underline{\eta}, \overline{\eta})$ and a decreasing sequence $\{f_i\}_i$ of $f_i \in (\underline{\eta}, \overline{\eta})$ such that $f_i \to \eta^a$ and $\{\hat{x}(f_i)\}_i$ is strictly decreasing.*

Proof. Recall that $\hat{x}$ is not constant. By Lemma 1 it is also weakly increasing, so there exist $\eta^b, \eta^c \in (\underline{\eta}, \overline{\eta})$ such that $\eta^b < \eta^c$ and $\hat{x}(\eta^b) < \hat{x}(\eta^c)$. Let $\eta^a = \sup\{\eta : \hat{x}(\eta) = \hat{x}(\eta^b)\}$. Since $\hat{x}$ is continuous, $\hat{x}(\eta^a) = \hat{x}(\eta^b)$. Now, take any decreasing sequence $\{e_i\}_i$ such that for every i, $e_i \in (\underline{\eta}, \overline{\eta})$ and $e_i \to \eta^a$. Since $\hat{x}$ is weakly increasing, $\hat{x}(e_i) \leq \hat{x}(e_j)$ whenever $i > j$. Also, by continuity of $\hat{x}$, $\hat{x}(e_i) \to \hat{x}(\eta^a)$ and, by the construction of η^a, $\hat{x}(\eta^a) < \hat{x}(e_i)$ for every i. Since $\hat{x}$ is continuous, there exists a subsequence $\{f_i\}_i$ of $\{e_i\}_i$ for which $\hat{x}(f_i)$ is decreasing strictly. $\square$

Take η^a, $\{f_i\}_i$ from the statement of Fact 2 and consider any (α^b, β^b) with merit η^a. Since $f_i \to \eta^a$ and $\hat{x}$ is continuous, $\hat{x}(f_i) \to \hat{x}(\eta^a)$. Then, by continuity and monotonicity of $\eta(\alpha, \beta)$, for all i high enough there exist δ_i^b such that $f_i = \eta(\alpha^b_{\delta_i}, \beta^b_{\delta_i})$, $\delta_i^b \to 0$ as $f_i \to \eta^a$, and $\{\delta_i^b\}_i$ is decreasing. Since (α^b, β^b) has the same allocation $\hat{x}(\eta^a)$ as (α^a, β^a), Lemma 3 gives:

$$\frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)} \leq \frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)}.$$

However, notice that such a sequence $\{\delta_i^a\}_i$ can also be found for type (α^a, β^a), and thus Lemma 3 gives the reverse inequality too. Hence, all types with merit η^a must the have same value of $v_x(\beta, x^a)/z_q(\alpha, x^a)$, which implies condition 1. in Proposition 2.

7.2 Proof of Fact 1

I first show that $\tilde{\eta}(\kappa, \lambda)$ is defined everywhere on $\mathbb{R} \times \mathbb{R}_{--}$. Fix any $(\kappa^*, \lambda^*) \in \mathbb{R} \times \mathbb{R}_{--}$. Then, by continuity of $-\frac{z(\alpha)}{w(\alpha)}$ and point 2. of Assumption 2 there exists $\alpha^* \in \mathbb{R}$ such that $\lambda^* = -\frac{z(\alpha^*)}{w(\alpha^*)}$. Then $\kappa^* = \frac{\beta^*}{w(\alpha^*)}$ for $\beta^* = \kappa^* \cdot w(\alpha^*)$, so $\tilde{\eta}(\kappa, \lambda)$ is indeed defined everywhere on $\mathbb{R} \times \mathbb{R}_{--}$.

Now, Assumption 1 and point 1. of Assumption 2 ensure that $-\frac{z(\alpha)}{w(\alpha)}$ is strictly increasing and has a twice continuously differentiable inverse $f : \mathbb{R} \to \mathbb{R}$. Thus:

$$\tilde{\eta}(\kappa, \lambda) = \eta\left(f(\lambda),\ \kappa \cdot w(f(\lambda))\right).$$

Calculation confirms that $\tilde{\eta}(\kappa, \lambda)$ is twice continuously differentiable.

I now show that $\tilde{\eta}_\kappa(\kappa, \lambda)$ is positive and bounded away from zero on $\mathbb{R} \times [\underline{\lambda}, \overline{\lambda}]$. Notice:

$$\tilde{\eta}_\kappa(\kappa, \lambda) = \eta_\beta\left(f(\lambda),\ \kappa \cdot w(f(\lambda))\right) \cdot w(f(\lambda)).$$

$\eta_\beta(\alpha, \beta)$ is strictly positive and uniformly bounded away from zero on $\mathbb{R}^2$ by point. 3. of Assumption 2. $w(f(\lambda))$ is strictly positive everywhere. Moreover, w and f are continuous, so $w(f(\lambda))$ attains its (strictly positive) minimum on $[\underline{\lambda}, \overline{\lambda}]$. Thus, $\tilde{\eta}_\kappa(\kappa, \lambda)$ is positive and uniformly bounded away from zero on $\mathbb{R} \times [\underline{\lambda}, \overline{\lambda}]$.

7.3 Proof of Lemma 2

First, define $\kappa^* : \mathbb{R}_{--} \to \mathbb{R}$ such that:

$$\eta^* = \tilde{\eta}(\kappa^*(\lambda), \lambda). \tag{8}$$

That is, $\kappa^*(\lambda)$ gives the value of κ for which type (κ, λ) attains the threshold merit level η^* (see Figure 3). Notice $\kappa^*(\lambda)$ is well-defined everywhere on $\mathbb{R}_{--}$. This is because by, Fact 1, $\tilde{\eta}_\kappa(\kappa, \lambda)$ is positive and uniformly bounded away from zero so, for every $\lambda^* \in \mathbb{R}_{--}$, $\tilde{\eta}(\kappa, \lambda^*)$ attains every value in $\mathbb{R}$ for some κ.

Since Fact 1 guarantees that $\tilde{\eta}(\kappa, \lambda)$ is twice continuously differentiable and $\tilde{\eta}(\kappa, \lambda) > 0$, implicitly differentiating (8) tells us that $\kappa^*(\lambda)$ is twice continuously differentiable too. Thus, the following bounds are finite:

$$M_1 := \max_{\lambda \in [\underline{\lambda}, \overline{\lambda}]} |\kappa^{*\prime}(\lambda)|, \quad M_2 := \max_{\lambda \in [\underline{\lambda}, \overline{\lambda}]} |\kappa^{*\prime\prime}(\lambda)|.$$

Having defined the necessary objects, we can now proceed to proving Lemma 2. Consider first the case where the threshold rule allocates the good to agents with merit at the threshold: $\hat{x}(\eta^*) = 0$ (the case of $\hat{x}(\eta^*) = 1$ will be analogous). Take the following waiting rule, where ψ, ζ are positive constants:

$$\tilde{q}(\kappa, \lambda) = \begin{cases} \psi - \zeta \cdot \overline{\lambda} + \zeta \cdot \lambda - \kappa^{*\prime}(\lambda) & \text{if } \tilde{\eta}(\kappa, \lambda) > \eta^*, \\ \psi - \zeta \cdot \overline{\lambda} + \zeta \cdot \lambda & \text{otherwise.} \end{cases} \tag{9}$$

Let the indirect utility function be:

$$V(\kappa, \lambda) = \max[0,\ \kappa - \kappa^*(\lambda)] + \zeta \cdot \frac{\lambda^2}{2} + \lambda \cdot (\psi - \zeta \cdot \overline{\lambda}). \tag{10}$$

I will show this waiting rule and indirect utility function satisfy the conditions of Lemma 2 for some choice of constants ψ, ζ. First, notice that since $\kappa^*(\lambda)$ is continuous, $V(\kappa, \lambda)$ is continuous too. Thus, $V(\kappa, \lambda)$ is bounded on the extended type space. Notice also that setting $\psi \geq M_1$ guarantees that the waiting rule $\tilde{q}(\kappa, \lambda)$ is positive on the extended type space.

It remains to show we can select constants to make $V(\kappa, \lambda)$ convex and such that the allocation and waiting rule pair belongs to its subdifferential everywhere. To that end, I establish the monotonicity of the gradient $\nabla V(\kappa, \lambda)$, wherever it exists. Notice also that this gradient is equal to $[\hat{x}(\tilde{\eta}(\kappa, \lambda)), \tilde{q}(\kappa, \lambda)]$.

Fact 3. *There exists ζ such that for any $(\kappa_1, \lambda_1), (\kappa_2, \lambda_2) \in [\underline{\kappa}, \overline{\kappa}] \times [\underline{\lambda}, \overline{\lambda}]$ where the gradient ∇V exists, we have:*

$$\Delta\kappa \cdot \Delta\tilde{x} + \Delta\lambda \cdot \Delta\tilde{q} \geq 0, \tag{11}$$

where $\Delta\kappa := \kappa_2 - \kappa_1$, $\Delta\lambda := \lambda_2 - \lambda_1$, $\Delta\tilde{x} := \tilde{x}(\kappa_2, \lambda_2) - \tilde{x}(\kappa_1, \lambda_1)$ and $\Delta\tilde{q} := \tilde{q}(\kappa_2, \lambda_2) - \tilde{q}(\kappa_1, \lambda_1)$.

Proof. Consider three cases.

Case 1: $\Delta\lambda = 0$. This case follows instantly from the fact that $\hat{x}$ is increasing and $\tilde{\eta}(\kappa, \lambda)$ is increasing in κ.

Case 2: $\Delta\lambda, \Delta\tilde{x} \neq 0$. Assume without loss that $\tilde{x}(\kappa_2, \lambda_2) = 1$. Then it has to be that $\tilde{\eta}(\kappa_2, \lambda_2) > \eta^* \geq \tilde{\eta}(\kappa_1, \lambda_1)$, so inequality (11) becomes:

$$\Delta\kappa + \Delta\lambda[\Delta\lambda \cdot \zeta - \kappa^{*\prime}(\lambda_2)] \geq 0.$$

Equivalently:

$$\zeta \geq \frac{1}{(\Delta\lambda)^2}[\Delta\lambda \cdot \kappa^{*\prime}(\lambda_2) - \Delta\kappa]. \tag{12}$$

Notice that to prove (11) holds for all such $\Delta\kappa, \Delta\lambda$, it suffices to uniformly bound the RHS of (12) across them. If such a uniform bound exists, we can then simply choose ζ above it.

Since $\tilde{\eta}(\kappa_2, \lambda_2) > \eta^* \geq \tilde{\eta}(\kappa_1, \lambda_1)$, it has to be that $\kappa^*(\lambda_1) \geq \kappa_1$ and $\kappa^*(\lambda_2) \leq \kappa_2$ (Figure 7). This gives the following inequality:

$$\Delta\kappa = \kappa_2 - \kappa_1 \geq \hat{\kappa}^*(\lambda_2) - \kappa^*(\lambda_1). \tag{13}$$

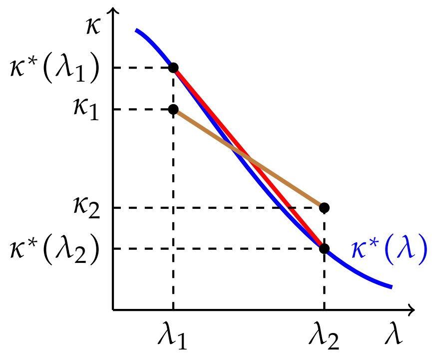

Figure 7

We can now use (13) to bound the RHS of (12) from above:

$$\begin{aligned}
\frac{1}{(\Delta\lambda)^2}[\Delta\lambda \cdot \kappa^{*\prime}(\lambda_2) - \Delta\kappa] &\leq \frac{1}{(\Delta\lambda)^2}[\Delta\lambda \cdot \kappa^{*\prime}(\lambda_2) - (\kappa^*(\lambda_2) - \kappa^*(\lambda_1))] \\
&= \frac{1}{(\Delta\lambda)^2}\left[\Delta\lambda \cdot \kappa^{*\prime}(\lambda_2) - \int_{\lambda_1}^{\lambda_2} \kappa^{*\prime}(\tau)\, d\tau\right] \\
&= \frac{1}{(\Delta\lambda)^2}\left[\Delta\lambda \cdot \kappa^{*\prime}(\lambda_2) - \int_{\lambda_1}^{\lambda_2} \left(\kappa^{*\prime}(\lambda_2) - \int_{\tau}^{\lambda_2} \kappa^{*\prime\prime}(\nu) d\nu\right) d\tau\right] \\
&= \frac{1}{(\Delta\lambda)^2} \int_{\lambda_1}^{\lambda_2} \int_{\tau}^{\lambda_2} \kappa^{*\prime\prime}(\nu) d\nu d\tau \\
&\leq \frac{1}{(\Delta\lambda)^2} \frac{(\Delta\lambda)^2}{2} M_2 = \frac{M_2}{2}.
\end{aligned}$$

Thus, setting $\zeta > M_2/2$ ensures that (11) holds wherever $\nabla V(\kappa, \lambda)$ exists.

Case 3: $\Delta\lambda \neq 0, \Delta\tilde{x} = 0$. Since $\zeta \geq 0$, the case where $\tilde{x}(\kappa_2, \lambda_2) = \tilde{x}(\kappa_1, \lambda_1) = 0$ is trivial. If $\tilde{x}(\kappa_2, \lambda_2) = \tilde{x}(\kappa_1, \lambda_1) = 1$, it has to be that $\tilde{\eta}(\kappa_2, \lambda_2), \tilde{\eta}(\kappa_1, \lambda_1) > \eta^*$, and so (11) becomes:

$$\Delta\lambda[\Delta\lambda \cdot \zeta - (\kappa^{*\prime}(\lambda_2) - \kappa^{*\prime}(\lambda_1))] \geq 0.$$

Equivalently:

$$\zeta \geq \frac{1}{(\Delta\lambda)^2}\Delta\lambda[\kappa^{*\prime}(\lambda_2) - \kappa^{*\prime}(\lambda_1)]$$
$$= \frac{1}{(\Delta\lambda)^2}\Delta\lambda \int_{\lambda_1}^{\lambda_2} \kappa^{*\prime\prime}(\tau)d\tau.$$

We can now uniformly bound the RHS from above as follows:

$$\frac{1}{(\Delta\lambda)^2}\Delta\lambda \int_{\lambda_1}^{\lambda_2} \kappa^{*\prime\prime}(\tau)d\tau \leq \frac{1}{(\Delta\lambda)^2}\Delta\lambda \int_{\lambda_1}^{\lambda_2} M_2 d\tau = M_2.$$

Therefore, $V(\kappa,\lambda)$ is convex whenever $\zeta > M_2$. □

Now, notice that $\nabla V(\kappa,\lambda)$ exists almost everywhere on $[\underline{\kappa},\bar{\kappa}] \times [\underline{\lambda},\bar{\lambda}]$, the only exception being the points on the curve $\kappa^*(\lambda)$. Fix a ζ described in Fact 3. I will now show that, whenever it exists, the gradient of $V(\kappa,\lambda)$ is also its subgradient.

Take any (κ_2,λ_2) in the extended type space and any (κ_1,λ_1) in the extended type space such that $\nabla V(\kappa_1,\lambda_1)$ exists. Consider the following parametrization of $V(\kappa,\lambda)$:

$$W(t) := V(\kappa_1 + t(\kappa_2 - \kappa_1), \lambda_1 + t(\lambda_2 - \lambda_1)).$$

Then $W(0) = V(\kappa_1,\lambda_1)$ and $W(1) = V(\kappa_2,\lambda_2)$. Since $V(\kappa,\lambda)$ is absolutely continuous, $W'(t)$ exists almost everywhere and is given by:

$$W'(t) = \nabla V(\kappa_1 + t\Delta\kappa, \lambda_1 + t\Delta\lambda) \cdot [\Delta\kappa, \Delta\lambda],$$

where $\Delta\kappa := \kappa_2 - \kappa_1$ and $\Delta\lambda := \lambda_2 - \lambda_1$. Now, observe that:

$$t(W'(t) - W'(0)) = [\nabla V(\kappa_1 + t\Delta\kappa, \lambda_1 + t\Delta\lambda) - \nabla V(\kappa_1,\lambda_1)] \cdot [(\kappa_1 + t\Delta\kappa) - \kappa_1, (\lambda_1 + t\Delta\lambda) - \lambda_1].$$

Since $\nabla V(\kappa,\lambda) = [\tilde{x}(\kappa,\lambda), \tilde{q}(\kappa,\lambda)]$, Fact 3 implies that $W'(t) - W'(0) \geq 0$.

Now, since $V(\kappa,\lambda)$ is absolutely continuous, we have:

$$\begin{aligned}
V(\kappa_2,\lambda_2) = W(1) &= W(0) + \int_0^1 W'(t)dt \\
&\geq W(0) + \int_0^1 W'(0)dt \\
&\geq W(0) + \nabla V(\kappa_1,\lambda_1) \cdot [\Delta\kappa, \Delta\lambda] \\
&= V(\kappa_1,\lambda_1) + [\tilde{x}(\kappa_1,\lambda_1), \tilde{q}(\kappa_1,\lambda_1)] \cdot [\Delta\kappa, \Delta\lambda].
\end{aligned}$$

Thus, $[\tilde{x}(\kappa_1,\lambda_1), \tilde{q}(\kappa_1,\lambda_1)]$ is indeed the subgradient of $V(\kappa,\lambda)$ almost everywhere on $[\underline{\kappa},\bar{\kappa}] \times [\underline{\lambda},\bar{\lambda}]$, the exception being the curve $\kappa^*(\lambda)$. However, by continuity of $V(\kappa,\lambda)$, $[0, \psi - \zeta \cdot \bar{\lambda} + \zeta \cdot \lambda]$ also belongs to the subdifferential of $V(\kappa,\lambda)$ there, and so $[\tilde{x}(\kappa_1,\lambda_1), \tilde{q}(\kappa_1,\lambda_1)]$ belongs to the subdifferential of $V(\kappa,\lambda)$ everywhere on $[\underline{\kappa},\bar{\kappa}] \times [\underline{\lambda},\bar{\lambda}]$. Finally, since $V(\kappa,\lambda)$ has a subgradient at every point of its domain, it is convex.

The proof for case where $\hat{x}(\eta^*) = 1$ is analogous, except the waiting rule $\tilde{q}(\kappa,\lambda)$ is set to $\psi - \zeta \cdot \bar{\lambda} + \zeta \cdot \lambda - \kappa^{*\prime}(\lambda)$ along the curve $\kappa^*(\lambda)$.

7.4 Proof of Proposition 4

The proof consists of two steps. First, I establish a lower bound on the global equity violations of all non-constant allocation rules implemented with only payments. To that end, I consider a type around which the allocation is not locally constant and show that all types with the same allocation as this one must lie below some differentiable curve. I establish that these curves are downwards-sloping when only waiting is used to screen and upwards-sloping when only payments are used. I then derive expressions for their slopes and use them to bound from below the global equity violations of all allocation rules implemented with only payments.

I then assume that iso-merit curves are sufficiently flat and construct an allocation rule implemented with only waiting whose global equity violation is below the aforementioned lower bound.

I now proceed with the first step. To that end, consider some non-constnat allocation rule x implemented only with waiting (the case where we screen only with payments is analogous). I first prove a few facts about such allocation rules.

Fact 4 (Monotonicity)**.** *If $\alpha^a \geq \alpha^b$ and $\beta^a \geq \beta^b$ with at least one inequality holding strictly, then type (α^a, β^a) gets a weakly higher allocation than type (α^b, β^b): $x^a \geq x^b$.*

Proof. Suppose that $x^a < x^b$. Then $q^a < q^b$ or else both types would strictly prefer (x^b, q^b). By revealed preference:

$$v(\beta^b, x^b) - z(\alpha^b, q^b) \geq v(\alpha^b, x^a) - z(\alpha^b, q^a) \quad \Longrightarrow \quad v(\beta^b, x^b) - v(\beta^b, x^a) \geq z(\alpha^b, q^b) - z(\alpha^b, q^a).$$

By strictly increasing differences we get:

$$v(\beta^a, x^b) - v(\beta^a, x^a) > z(\alpha^a, q^b) - z(\alpha^a, q^a) \quad \Longrightarrow \quad v(\beta^a, x^b) - z(\alpha^a, q^b) > v(\beta^a, x^a) - z(\alpha^a, q^a).$$

That is, (α^a, β^a) prefers (x^b, q^b) to (x^a, q^a); contradiction. □

In particular, Fact 4 tells us that x^a_δ is weakly increasing in δ for every (α^a, β^a). Moreover, whenever $x^a > x^b$ it has to be that $q^a > q^b$ or else (α^b, β^b) would prefer (x^a, q^a) to her allocation. Therefore, q^a_δ is also weakly increasing in δ for every (α^a, β^a).

The following fact proves the existence of a sequence that lets us apply the Generalized FOC (Lemma 3) to some type (α^a, β^a).

Fact 5. *There exists (α^a, β^a) such that either a) for every $\delta > 0$, $x^a_\delta > x^a$, or b) for every $\delta < 0$, $x^a_\delta < x^a$.*

Proof. Suppose otherwise; then for every $(\alpha^a, \beta^a) \in \Theta$ there exists some $\epsilon > 0$ such that $x^a_\delta = x^a$ for $\delta \in [-\epsilon, \epsilon]$. Take any $(\alpha^b, \beta^b) \in [\alpha^a - \epsilon, \alpha^a + \epsilon] \times [\beta^a - \epsilon, \beta^a + \epsilon]$ and notice that by Fact 4 we have $x(\alpha - \epsilon, \beta - \epsilon) \leq x^b \leq x(\alpha + \epsilon, \beta + \epsilon)$. However, $x(\alpha - \epsilon, \beta - \epsilon) = x(\alpha + \epsilon, \beta + \epsilon) = x^a$ so $x(\alpha, \beta) = x^a$ for all $(\alpha, \beta) \in [\alpha^a - \epsilon, \alpha^a + \epsilon] \times [\beta^a - \epsilon, \beta^a + \epsilon]$. Consequently, for any $(\alpha, \beta) \in \Theta$ there exists a neighborhood around it in which the allocation is constant. Now, take any $(\alpha^c, \beta^c), (\alpha^d, \beta^d) \in \Theta$. Since Θ is connected, there exists a continuous path between them and every point along this path has the same allocation as the points within its neighborhood. Therefore, the allocation has to be constant along the whole path, including the end-points. Since (α^c, β^c) and (α^d, β^d) were arbitrary, the allocation is constant; contradiction. □

Fix (α^a, β^a) from Fact 5 and assume $a)$ holds (the argument for $b)$ is analogous). Let $x^a_+ = lim_{\delta\to 0^+} x^a_\delta$ and $q^a_+ = lim_{\delta\to 0^+} q^a_\delta$. These one-sided limits exist as x^a_δ and q^a_δ are increasing in δ.

Fact 6. *If x^a_δ is right-continuous at $\delta = 0$, then for every (α^b, β^b) such that $x^b = x^a$ we have:*

$$\frac{v_x(\beta^b, x^a)}{z_q(\alpha^b, q^a)} \leq \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}. \tag{14}$$

If x^a_δ is not right-continuous at $\delta = 0$, then for every (α^b, β^b) such that $x^b = x^a$ we have:

$$\frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z(\alpha^b, q^a_+) - z(\alpha^b, q^a)} \leq \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)}. \tag{15}$$

Proof. Consider two cases.

Case 1: x^a_δ is right-continuous at $\delta = 0$. Take a decreasing sequence $\{e_i\}_i$ such that $e_i \to 0$. Then $x^a_{e_i} \to x^a$ by right-continuity and, by case $a)$ of Fact 5, $x^a < x^a_{e_i}$ for all i. Since x^a_δ is right-continuous at $\delta = 0$, there exists a subsequence $\{f_i\}_i$ of $\{e_i\}_i$ such that $x^a_{f_i}$ is decreasing strictly. We can therefore apply the Generalized FOC (Lemma 3), which completes the proof.

Case 2: x^a_δ is not right-continuous at $\delta = 0$. Then $x^a_+ > x^a$. Since indirect utility has to be continuous at (α^a, β^a), it has to be that $q^a_+ > q_a$ and:

$$v(\beta^a, x^a) - z(\alpha^a, q^a) = v(\beta^a, x^a_+) - z(\alpha^a, q^a_+) \implies \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)} = 1. \tag{16}$$

Take a decreasing sequence $\{\delta_i\}_i$ such that $\delta_i \to 0$. By the fact that $x^b = x^a$ and revealed preference, for every i we have:

$$v(\beta^b, x^a) - z(\alpha^b, q^a) \geq v(\beta^b, x^a_{\delta_i}) - z(\alpha^b, q^a_{\delta_i}) \implies \frac{v(\beta^b, x^a_{\delta_i}) - v(\beta^b, x^a)}{z(\alpha^b, q^a_{\delta_i}) - z(\alpha^b, q^a)} \leq 1.$$

Taking $i \to \infty$ gives:

$$\frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z(\alpha^b, q^a_+) - z(\alpha^b, q^a)} \leq 1. \tag{17}$$

Combining the latter equality in (16) with (17) completes the proof. □

By Assumption 1, the LHSs of (14) or (15) are strictly increasing in α^b and β^b. Thus, to satisfy conditions (14) or (15) of Fact 6, type (α^b, β^b) must lie weakly below some differentiable curve. If x^a_δ is right-continuous at $\delta = 0$, this curve is traced out by (α, β) for which:

$$\frac{v_x(\beta, x^a)}{z_q(\alpha, q^a)} = \frac{v_x(\beta^a, x^a)}{z_q(\alpha^a, q^a)}. \tag{18}$$

I will call this curve the *iso-MRS curve at* (α^a, β^a). If x^a_δ is not right-continuous at $\delta = 0$, this curve is given by:

$$\frac{v(\beta^b, x^a_+) - v(\beta^b, x^a)}{z(\alpha^b, q^a_+) - z(\alpha^b, q^a)} = \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)}. \tag{19}$$

I will call it the *iso-difference curve at* (α^a, β^a).

Recall that $D(\alpha^a, \beta^a)$ from Definition 1 is the set of directions in which the allocation is locally constant at type (α^a, β^a). The above observation then tells us that for (α^a, β^a) with merit η^a there is only one candidate element of $D(\alpha^a, \beta^a)$: depending on the right-continuity of x_δ^a at 0, it is either the slope of the iso-*MRS* curve at (α^a, β^a) or the slope of the iso-difference curve at (α^a, β^a). These slopes are given by:

$$s^q_{MRS}(\alpha^a, \beta^a) := \frac{v_x(\alpha^a, x^a)}{z_q(\alpha^a, q^a)} \frac{z_{\alpha q}(\alpha^a, q^a)}{v_{\beta x}(\alpha^a, x^a)}, \tag{20}$$

$$s^q_{diff}(\alpha^a, \beta^a) := \frac{v(\beta^a, x^a_+) - v(\beta^a, x^a)}{z(\alpha^a, q^a_+) - z(\alpha^a, q^a)} \frac{z_\alpha(\alpha^a, q^a_+) - z_\alpha(\alpha^a, q^a)}{v_\beta(\beta^a, x^a_+) - v_\beta(\beta^a, x^a)}. \tag{21}$$

By Assumption 1, $s^q_{MRS}(\alpha, \beta),\ s^q_{diff}(\alpha, \beta) < 0$ for every $(\alpha, \beta) \in \Theta$.

Now, consider the case where we screen only with payments. Analogous reasoning pins down the only candidate elements of $D(\alpha^a, \beta^a)$ for (α^a, β^a) such that x^a_δ is locally non-constant at $\delta = 0$. Then, however, $s^p_{MRS}(\alpha, \beta),\ s^p_{diff}(\alpha, \beta) > 0$ for every $(\alpha, \beta) \in \Theta$.

Notice also that the slope of the iso-merit curve at (α, β) equals to $-\eta_\alpha(\alpha, \beta)/\eta_\beta(\alpha, \beta)$ and is negative for any $(\alpha, \beta) \in \Theta$. We therefore have the following lower bound on global equity violations for any allocation rule x_p implemented with payments only:

$$L(x_p) > \left| \frac{\pi}{2} - \arctan\left(-\frac{\eta_\alpha(\alpha, \beta)}{\eta_\beta(\alpha, \beta)} \right) \right|. \tag{22}$$

Having established this uniform lower bound, I proceed to the second step of the proof. I show that if iso-merit curves are sufficiently flat everywhere, I can construct an allocation rule implemented with waiting only whose global equity violation is strictly below this lower bound.

To that end, I first establish a uniform bound on the slopes $s^q_{MRS}(\alpha^a, \beta^a)$ and $s^q_{diff}(\alpha^a, \beta^a)$ that holds across types and all allocation rules x_q implemented with only wait times. Recall that v, z as well as their partials and cross-partials are continuous. Moreover, these partials and cross-partials are non-zero everywhere. Thus, the absolute values of $v, v_{\beta x}$ and $z_q, z_{\alpha q}$ can be uniformly bounded away from zero and infinity on the whole type space Θ and for all $x \in [0, 1]$, $q \in [0, \bar{q}]$. Analogous bounds hold for the following terms whenever $x^b \neq x^b_+$ (which implies $q^b \neq q^b_+$):

$$\frac{v(\beta^b, x^b_+) - v(\beta^b, x^b)}{x^b_+ - x^b}, \quad \frac{z(\alpha^b, q^b_+) - z(\alpha^b, q^b)}{q^b_+ - q^b}, \quad \frac{v_\beta(\beta^b, x^b_+) - v_\beta(\beta^b, x^b)}{x^b_+ - x^b}, \quad \frac{z_\alpha(\alpha^b, q^b_+) - z_\alpha(\alpha^b, q^b)}{q^b_+ - q^b}.$$

Therefore, by (20) and (21), there exists $M < 0$ such that for any $(\alpha, \beta) \in \Theta$ and x_q:

$$s^q_{MRS}(\alpha, \beta),\ s^q_{diff}(\alpha, \beta) < M.$$

That is, all iso-*MRS* and iso-difference curves for all allocation rules x_q implemented with only waiting are steeper than M. Now, assume that all iso-merit curves are flatter than M:

$$\text{for all } (\alpha, \beta) \in \Theta, \quad -\frac{\eta_\alpha(\alpha, \beta)}{\eta_\beta(\alpha, \beta)} > M.$$

I will construct an allocation rule implemented with only waiting whose equity violation is below the bound in (22). Fix any $(\alpha^b, \beta^b) \in \Theta$. Then, for $x^b > 0$ small enough, q^b defined by

$$v(\beta^b, x^b) = z(\alpha^b, q^b),$$

is sufficiently close to 0 that $q^b \in [0, \bar{q}]$. The following allocation then satisfies (IR) and (IC): (α^b, β^b) and all types weakly above the iso-difference curve at (α^b, β^b) take (x^b, q^b); all types strictly below it take $(0, 0)$.

Take any (α^c, β^c) strictly below or strictly above the iso-difference curve at (α^b, β^b). There is a neighborhood around (α^c, β^c) in which the allocation is constant, and so it is also constant along the slope of the iso-merit curve at (α^c, β^c), giving a local equity violation $l(\alpha^c, \beta^c) = 0$ there. Now, take any (α^c, β^c) on the iso-diffeence curve at (α^b, β^b). The allocation is constant along this curve, so $s^q_{diff}(\alpha^c, \beta^c) \in D(\alpha^c, \beta^c)$. Since the iso-merit curve is flatter than $s_{diff}(\alpha^c, \beta^c)$ for any such (α^c, β^c), we get:

$$L(x_q) \leq \left| \arctan(s^q_{diff}(\alpha^c, \beta^c)) - \arctan\left(-\frac{\eta_\alpha(\alpha^c, \beta^c)}{\eta_\beta(\alpha^c, \beta^c)} \right) \right| < \left| \frac{\pi}{2} - \arctan\left(-\frac{\eta_\alpha(\alpha^c, \beta^c)}{\eta_\beta(\alpha^c, \beta^c)} \right) \right| < L(x_p).$$

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