Screening with damages and ordeals

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Example: affordable housing

- Public housing programs, different units come with different wait-times

- Applicants "sort" themselves into developments based on wait-time

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  - Each period of waiting deprives the household of the apartment's flow value
  - Thus, the cost of delaying receipt is **multiplicative with value**
- Other screening devices impose costs that are **separable** from values
  - E.g. differences in rent subsidies, application hassles...

Two kinds of screening devices

- I draw the distinction between **damages** and **ordeals**
- The cost of **damages** increases with the value for the good:
    - Waitlists and delays (through discounting or lost periods of use)
    - Damaged goods and usage restrictions (Deneckere and McAfee, 1996)
    - Network restrictions and changing claims rules in healthcare

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    - Waitlists and delays (through discounting or lost periods of use)
    - Damaged goods and usage restrictions (Deneckere and McAfee, 1996)
    - Network restrictions and changing claims rules in healthcare
- The cost of **ordeals** is separable from the value for the good:
    - Queues (Nichols et al., 1971)
    - Travelling to a distant office (Dupas et al., 2016)
    - Application hassles, bureaucracy (Deshpande and Li, 2019)

Model

Goods

- The designer distributes two kinds of goods, $\boldsymbol{A}$ and $\boldsymbol{B}$,
    - There is $\boldsymbol{\mu_A} > 0$ of good A and $\boldsymbol{\mu_B} > 0$ of good B

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    - There is $\boldsymbol{\mu_A} > 0$ of good A and $\boldsymbol{\mu_B} > 0$ of good B
- Agents' values for A and B are given by two-dimensional types $\boldsymbol{(a, b)}$
    - Values (a, b) distributed according to $\boldsymbol{F}$ defined on $[0, 1]^2$

Allocations

- The designer chooses a menu of **damage and ordeal options** for each of the goods
- That is, she chooses **allocations** of:
    1. Ordeals $t : [0,1]^2 \to \mathbb{R}_+$
    2. Qualities $x : [0,1]^2 \to [0,1]$
    3. Goods: $y : [0,1]^2 \to \{A, B, \varnothing\}$
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- When $x < 1$, we say the good is **damaged**
- Type (a, b) who gets a good of quality $\boldsymbol{x}$ and completes an ordeal $\boldsymbol{t}$ gets utility:

$$\begin{aligned} x \cdot a - t \quad &\text{if she gets } A, \\ x \cdot b - t \quad &\text{if she gets } B. \end{aligned}$$

Designer's problem

- The designer maximizes total welfare:

$$W = \int U[a, b, \ (t, x, y)(a, b)] \, \mathrm{d}F(a, b)$$

- She faces **IC, IR** and **supply** constraints:

$$\text{for every } (a, b), (a', b') \in [0, 1]^2, \quad U[a, b, (t, x, y)(a, b)] \geq U[a, b, (t, x, y)(a', b')] \tag{IC}$$

$$\text{for every } (a, b) \in [0, 1]^2, \quad U[a, b, (t, x, y)(a, b)] \geq 0 \tag{IR}$$

$$\int \mathbb{1}_{\text{gets } A} \, \mathrm{d}F(a, b) \leq \mu_A, \qquad \int \mathbb{1}_{\text{gets } B} \, \mathrm{d}F(a, b) \leq \mu_B \tag{S}$$

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- **Result 2:** with two goods, using **damages can be optimal**
- **Result 3:** under regularity conditions, damages **suboptimal** even with two goods!

One good case

One good case

- Suppose only good A is scarce, $\mu_A < 1$
- Good B is an **unlimited outside option**, $\mu_B = \infty$, with a **common value** b

Proposition 1

Any mechanism that uses **damages**, so features $x(a, b) < 1$, is **Pareto dominated** by a mechanism that uses **only ordeals**.

Proposition 1: intuition

- Every feasible (deterministic) mechanism allocates A to types above some $\underline{\boldsymbol{a}}$
- We can enforce this cutoff by imposing **ordeals** on recipients of A or by **damaging** it

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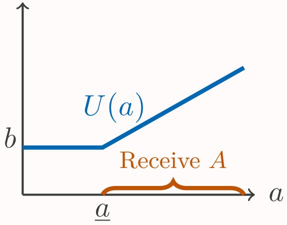
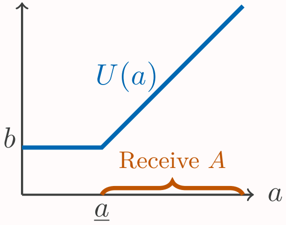


Damages



Ordeals

- $\ldots$ so ordeals **leave more rents to inframarginal** takers of A!

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- Ordeal and damage mechanisms equally good for the cutoff type...
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- However, here, the A-good always goes to **an upper interval of types**
- With **2D** heterogeneity in values, there is **no fixed order**!
- Damages and ordeals **sort agents into goods** in different ways!

Two good case

Two good case

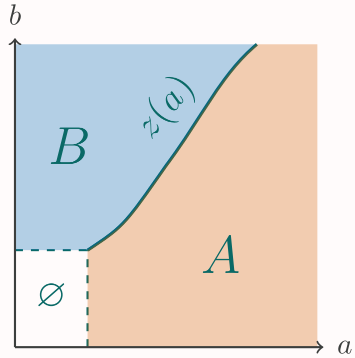
- Consider the case where both goods are scarce: $\mu_A + \mu_B \leq 1$
- F, the distribution of values (a, b), has full support on $[0, 1]^2$

Feasible mechanisms

Who gets which good?

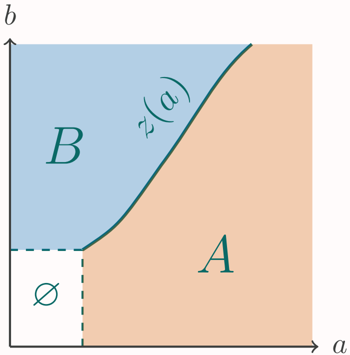
- When both goods come with ordeals, some types do not participate ($\varnothing$)

- The rest pick their favourite (ordeal, damage) option for one of the goods



Who gets which good?

- When both goods come with ordeals, some types do not participate ($\varnothing$)
- The rest pick their favourite (ordeal, damage) option for one of the goods
- Types on the **boundary** $\boldsymbol{z}$ indifferent between their best options for both goods
- Types below z pick some option for A, types above z pick some option for B



Damages can be optimal

Ordeals and damages sort agents differently

- Consider a mechanism with an **ordeal for each good**: c_A, c_B
- Then type (a, b) selects good A if $a - c_A \geq b - c_B$

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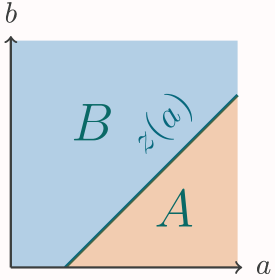
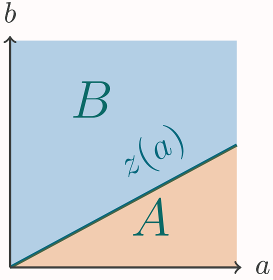


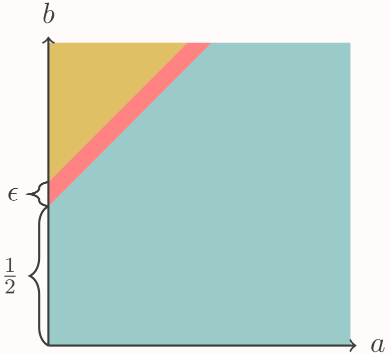
Only ordeals with $\mu_A + \mu_B = 1$



Only damages with $\mu_A + \mu_B = 1$

Example: damages can be optimal

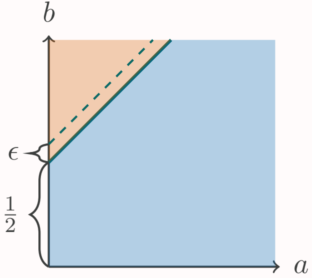
- Put mass $\boldsymbol{\epsilon}$ on the **mustard** region...
- ...mass $\boldsymbol{k}$ on the **red** region...
- ...and mass $\mathbf{1} - \boldsymbol{k} - \boldsymbol{\epsilon}$ on the **green** region
- Set supplies $\boldsymbol{\mu_A} = \mathbf{1} - \boldsymbol{k} - \boldsymbol{\epsilon}, \;\; \boldsymbol{\mu_B} = \boldsymbol{k} + \boldsymbol{\epsilon}$



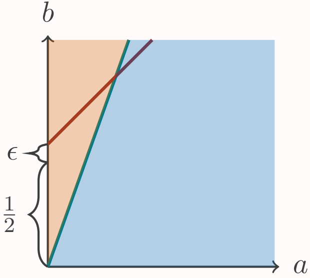
Example: damages can be optimal

- An "ordeal only" mechanism has $\boldsymbol{c_B = 1/2,\ c_A = 0}$

- But this **eats away** almost all the surplus from getting B over A!



Good B given with an ordeal



Good B damaged

- But a mechanism that damages B **leaves surplus** to agents close to the b-axis!

When are damages suboptimal?

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1. Consider piece-wise continuously differentiable $x : [0,1]^2 \to [0,1]$

2. The following are strictly increasing in one of a and b and non-decreasing in the other:

$$\frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}, \quad \frac{F_{B|A}(b|a)}{f_{B|A}(b|a)},$$

Theorem 1

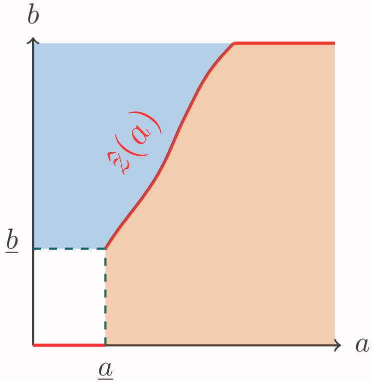
The optimal mechanism implements the efficient allocation of goods, and allocates both of them **without damages**. It posts a **single ordeal** for each good.

Proof strategy

Rewriting the objective

- Let $\boldsymbol{U_A} : [0,1] \to \mathbb{R}_+$ be the indirect utility conditional on getting A
- Write total welfare as a function of $\boldsymbol{U_A}$ and the **extended boundary** $\hat{\boldsymbol{z}}$

$$U_A(1) - \int_0^1 U_A'(a) \cdot F(a, \hat{z}(a))da$$



Extended boundary $\hat{z}$.

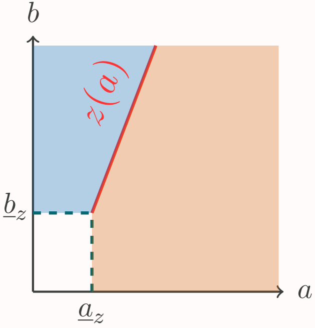
Proof strategy

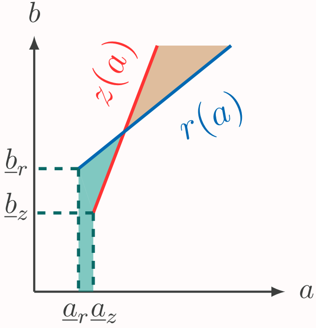
$$U_A(1) - \int_0^1 U_A'(a) \cdot F(a, \hat{z}(a)) da$$

1. Characterize implementable pairs (U_A, z)
2. Pick the optimal U_A for every fixed boundary z
3. Optimize over the space of optimally implemented boundaries z
4. Show the optimal boundary has a slope of $1 \rightarrow$ implementable without damages!

Intuition behind distributional conditions

- Consider a linear boundary $\boldsymbol{z}$ with slope > 1...
- Our distributional assumptions will guarantee a **less steep boundary is better**
- Pick a less steep $\boldsymbol{r}$ such that z and r allocate the **same amounts** of A and B

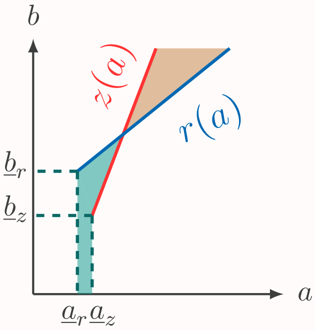




Welfare difference between r and z

$$\Delta \quad = \quad \Delta\, U_A(1) \quad - \quad \Delta\left(\int_0^1 U_A'(a)\, F(a, \hat{z}(a))\, \mathrm{d}a \right)$$

$$= \quad (\underline{a}_z - \underline{a}_r) \quad - \quad \left(\int_{\overline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b) \; - \; \int_{\underline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b) \right)$$



Welfare difference between r and z

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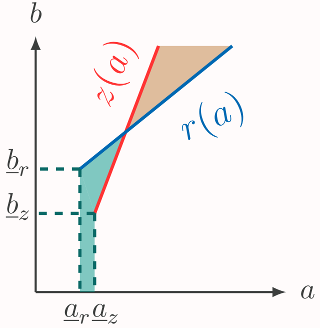
\begin{aligned}
\Delta &= \Delta\, U_A(1) - \Delta\left(\int_0^1 U_A'(a)\, F(a, \hat{z}(a))\, \mathrm{d}a\right) \\
&= (\underline{a}_z - \underline{a}_r) - \left(\int_{\overline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b) - \int_{\underline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b)\right)
\end{aligned}
$$

- But $\underline{\boldsymbol{a}}_z > \underline{\boldsymbol{a}}_r \ldots$



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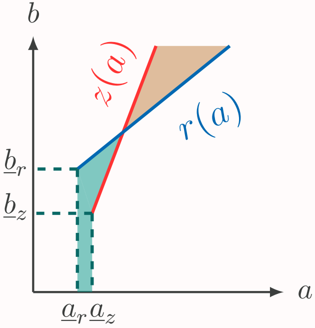
- But $\boldsymbol{\underline{a}_z} > \boldsymbol{\underline{a}_r}$…
- …the masses in the **brown** and **green** regions are equal…

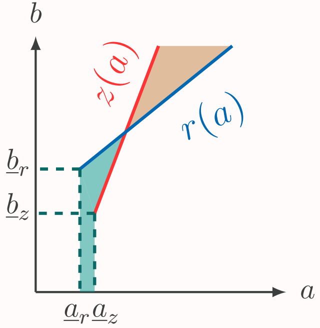


Welfare difference between r and z

$$\Delta \;=\; \Delta\, U_A(1) \;-\; \Delta\left(\int_0^1 U_A'(a)\, F(a, \hat{z}(a))\, \mathrm{d}a\right)$$

$$=\; (\underline{a}_z - \underline{a}_r) \;-\; \left(\int_{\overline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b) \;-\; \int_{\underline{\mathcal{D}}} \frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}\, f(a,b)\, \mathrm{d}(a,b)\right)$$

- But $\boldsymbol{\underline{a}_z} > \boldsymbol{\underline{a}_r}$...
- ...the masses in the **brown** and **green** regions are equal...
- ...and $\frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}$ is increasing in the $\nearrow$ direction by assumption!



Extensions

Also in the paper

- Heterogeneous costs of ordeals
- Monetary payments as *partially* wasteful screening
- Steady-state microfoundation for waitlist example

Conclusions

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  - **Damages** impose costs that increase in one's value for the good
  - **Ordeals** impose costs that are separable from recipients' values

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- Screening devices differ in how they interact with agents' values
    - **Damages** impose costs that increase in one's value for the good
    - **Ordeals** impose costs that are separable from recipients' values
- Using **damages** is never optimal with only one kind of good
- And while they **can** be useful when many kinds of goods are offered...
- ...this is not the case for **"regular"** distributions

Conclusions

- Implications for **public housing** allocation?
  - Such programs often offer **heterogeneous units**, with different **wait-times**
  - Even if some wait-time is often inevitable in reality…
  - …we should be worried about large **imbalances** in wait-times!
  - We should "sort" applicants using other instruments, e.g. by **readjusting subsidies**

Thank you!