

# Screening with damages and ordeals

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## Example: affordable housing

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- Wait-times differ substantially between developments...
- ...and are a key factor in applicants' choice of development

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- Affordable housing programs offer units that vary in location and size
- Wait-times differ substantially between developments...
- ...and are a key factor in applicants' choice of development
- Thus, wait-times largely assume the role of prices:
  - They screen out low-value agents to balance supply and demand...
  - ...and "sort" participants into different types of units

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  - Each period of waiting deprives the household of the apartment's flow value
  - Thus, the cost of delaying receipt is **multiplicative with value**
- Other screening devices impose costs that are **separable** from values
  - E.g. differences in rent subsidies, application hassles...

## Two kinds of screening devices

- I draw the distinction between **damages** and **ordeals**
- The cost of **damages** increases with the value for the good:
  - Waitlists and delays (through discounting or lost periods of use)
  - Damaged goods and usage restrictions (Deneckere and McAfee, 1996)
  - Network restrictions and changing claims rules in healthcare

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  - Damaged goods and usage restrictions (Deneckere and McAfee, 1996)
  - Network restrictions and changing claims rules in healthcare
- The cost of **ordeals** is separable from the value for the good:
  - Queues (Nichols et al., 1971)
  - Travelling to a distant office (Dupas et al., 2016)
  - Application hassles, bureaucracy (Deshpande and Li, 2019)



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- **Result 1:** with one good, **never optimal** to use damages
- **Result 2:** with two goods, using **damages can be optimal**
- **Result 3:** under regularity conditions, damages **suboptimal** even with two goods!

## Also in the paper

- Heterogeneous costs of ordeals
- Monetary payments as *partially* wasteful screening
- Steady-state microfoundation for waitlist example
- Implications for affordable housing allocation

# Model

# Goods

- The designer distributes two kinds of goods,  $A$  and  $B$ ,
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  - There is  $\mu_A > 0$  of good  $A$  and  $\mu_B > 0$  of good  $B$
- Agents' values for  $A$  and  $B$  are given by two-dimensional types  $(a, b)$ 
  - Values  $(a, b)$  distributed according to  $F$  defined on  $[0, 1]^2$



# Allocations

- The designer chooses a menu of **damage and ordeal options** for each of the goods
- That is, she chooses **allocations** of:
  1. Ordeals  $t : [0, 1]^2 \rightarrow \mathbb{R}_+$
  2. Qualities  $x : [0, 1]^2 \rightarrow [0, 1]$
  3. Goods:  $y : [0, 1]^2 \rightarrow \{A, B, \emptyset\}$
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- When  $x < 1$ , we say the good is **damaged**
- Type  $(a, b)$  who gets a good of quality  $x$  and completes an ordeal  $t$  gets utility:

$$\begin{array}{ll} x \cdot a - t & \text{if she gets } A, \\ x \cdot b - t & \text{if she gets } B. \end{array}$$

## Designer's problem

- The designer maximizes total welfare:

$$W = \int U[a, b, (t, x, y)(a, b)] dF(a, b)$$

- She faces **IC**, **IR** and **supply** constraints:

$$\text{for every } (a, b), (a', b') \in [0, 1]^2, \quad U[a, b, (t, x, y)(a, b)] \geq U[a, b, (t, x, y)(a', b')] \quad (\text{IC})$$

$$\text{for every } (a, b) \in [0, 1]^2, \quad U[a, b, (t, x, y)(a, b)] \geq 0 \quad (\text{IR})$$

$$\int \mathbb{1}_{\text{gets } A} dF(a, b) \leq \mu_A, \quad \int \mathbb{1}_{\text{gets } B} dF(a, b) \leq \mu_B \quad (\text{S})$$

One good case

## One good case

- Suppose only good  $A$  is scarce,  $\mu_A < 1$
- Good  $B$  is an **unlimited outside option**,  $\mu_B = \infty$ , with a **common value**  $b$

### Proposition 1

Any mechanism that uses **damages**, so features  $x(a, b) < 1$ , is **Pareto dominated** by a mechanism that uses **only ordeals**.

## Proposition 1: intuition

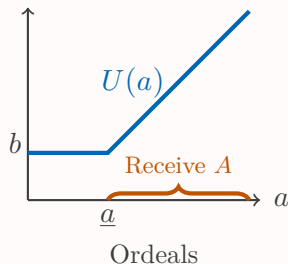
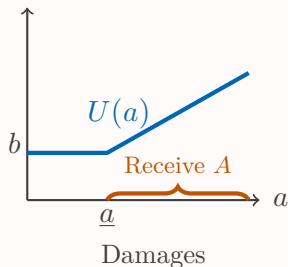
- Every feasible (deterministic) mechanism allocates  $A$  to types above some  $a$
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- ...so ordeals **leave more rents to inframarginal takers of  $A$ !**



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- ...but damages more costly to inframarginal types...
- ...while ordeals "equally costly" to everyone
  
- However, here, the  $A$ -good always goes to **an upper interval of types**
- With **2D** heterogeneity in values, there is **no fixed order!**
- Damages and ordeals **sort agents into goods** in different ways!

Two good case

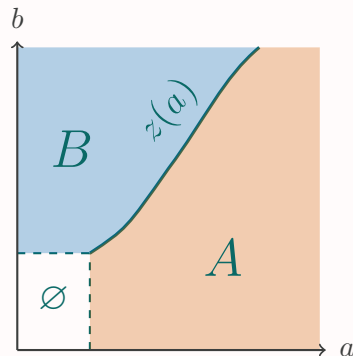
## Two good case

- Consider the case where both goods are scarce:  $\mu_A + \mu_B \leq 1$
- $F$ , the distribution of values  $(a, b)$ , has full support on  $[0, 1]^2$

# Feasible mechanisms

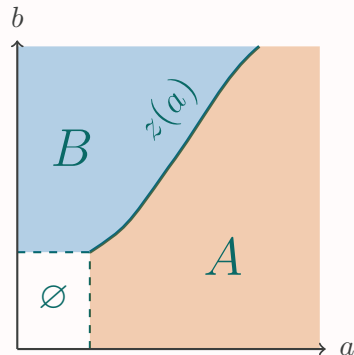
# Who gets which good?

- When both goods come with ordeals, some types do not participate ( $\emptyset$ )
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- The rest pick their favourite (ordeal, damage) option for one of the goods
- Types on the **boundary**  $z$  indifferent between their best options for both goods
- Types below  $z$  pick some option for  $A$ , types above  $z$  pick some option for  $B$



Damages can be optimal



## Ordeals and damages sort agents differently

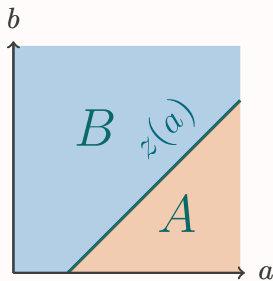
- Consider a mechanism with an **ordeal for each good**:  $c_A, c_B$
- Then type  $(a, b)$  selects good  $A$  if  $a - c_A \geq b - c_B$

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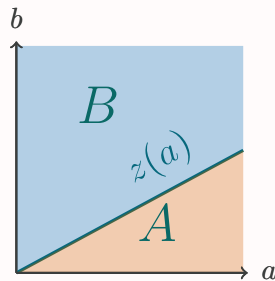
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- Consider a mechanism which uses no ordeals but a **damages** good  $A$  to  $x = q$
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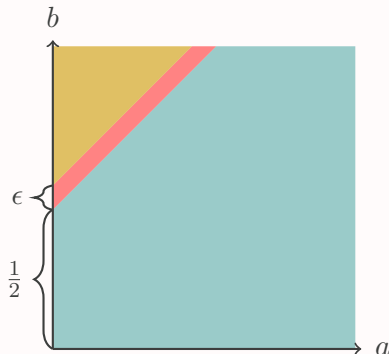
Only ordeals with  $\mu_A + \mu_B = 1$



Only damages with  $\mu_A + \mu_B = 1$

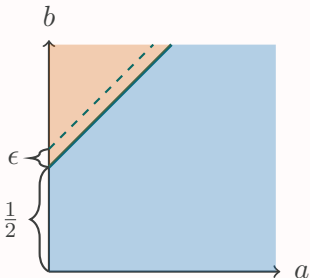
## Example: damages can be optimal

- Put mass  $\epsilon$  on the **mustard** region...
- ...mass  $k$  on the **red** region...
- ...and mass  $1 - k - \epsilon$  on the **green** region
- Set supplies  $\mu_A = 1 - k - \epsilon$ ,  $\mu_B = k + \epsilon$

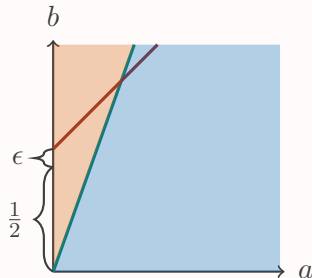


## Example: damages can be optimal

- An "ordeal only" mechanism has  $c_B = 1/2$ ,  $c_A = 0$
- But this **eats away** almost all the surplus from getting  $B$  over  $A$ !



Good  $B$  given with an ordeal



Good  $B$  damaged

- But a mechanism that damages  $B$  **leaves surplus** to agents close to the  $b$ -axis!

When are damages suboptimal?

## When are damages suboptimal?

1. Consider piece-wise continuously differentiable  $x : [0, 1]^2 \rightarrow [0, 1]$
2. The following are strictly increasing in one of  $a$  and  $b$  and non-decreasing in the other:

$$\frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}, \quad \frac{F_{B|A}(b|a)}{f_{B|A}(b|a)},$$

### Theorem 1

The optimal mechanism implements the efficient allocation of goods, and allocates both of them **without damages**. It posts a **single ordeal** for each good.

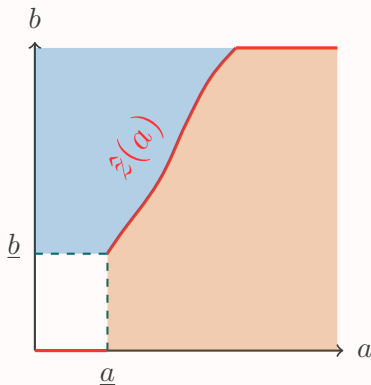
# Proof strategy



## Rewriting the objective

- Let  $\mathbf{u}_A : [0, 1] \rightarrow \mathbb{R}_+$  be the indirect utility conditional on getting  $A$
- Write total welfare as a function of  $\mathbf{u}_A$  and the **extended boundary**  $\hat{z}$

$$U_A(1) - \int_0^1 U'_A(a) \cdot F(a, \hat{z}(a)) da$$



Extended boundary  $\hat{z}$ .

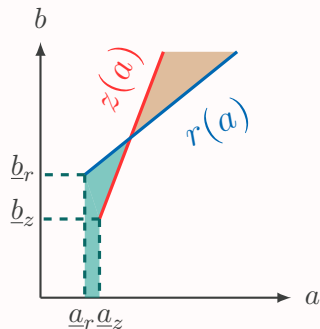
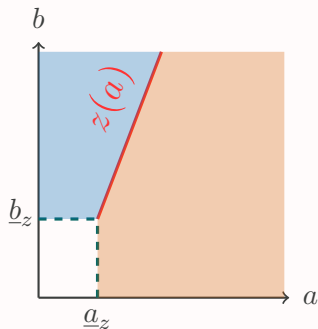
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$$U_A(1) - \int_0^1 U'_A(a) \cdot F(a, \hat{z}(a)) da$$

1. Characterize implementable pairs  $(U_A, z)$
2. Pick the optimal  $U_A$  for every fixed boundary  $z$
3. Optimize over the space of optimally implemented boundaries  $z$
4. Show the optimal boundary has a slope of 1  $\rightarrow$  implementable without damages!

## Intuition behind distributional conditions

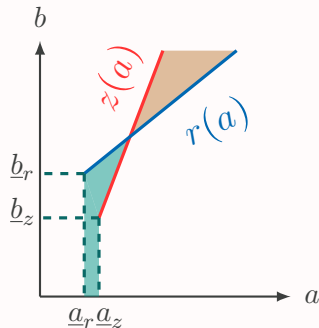
- Consider a linear boundary  $z$  with slope  $> 1$ ...
- Our distributional assumptions will guarantee a **less steep boundary is better**
- Pick a less steep  $r$  such that  $z$  and  $r$  allocate the **same amounts** of  $A$  and  $B$



# Intuition behind distributional conditions

- We can write the **difference in welfare** between  $r$  and  $z$  as:

$$\Delta = (\underline{a}_z - \underline{a}_r) - \left( \int_{\underline{\mathcal{D}}} \frac{F_{A|B}(a | b)}{f_{A|B}(a | b)} f(a, b) \, \mathrm{d}(a, b) - \int_{\underline{\mathcal{D}}} \frac{F_{A|B}(a | b)}{f_{A|B}(a | b)} f(a, b) \, \mathrm{d}(a, b) \right).$$

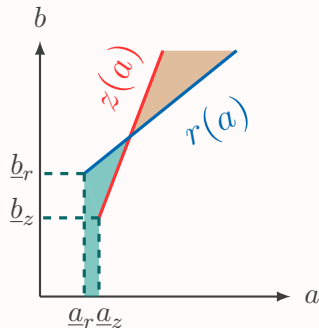


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- But  $\underline{a}_z > \underline{a}_r \dots$

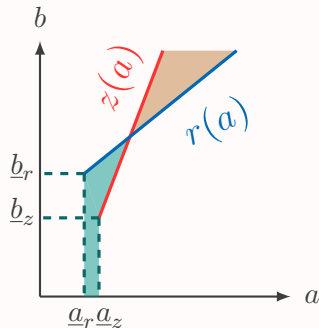


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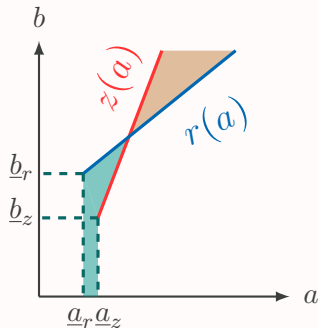


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- But  $\underline{a}_z > \underline{a}_r \dots$
- ...the masses in the **brown** and **green** regions are equal...
- ...and  $\frac{F_{A|B}(a|b)}{f_{A|B}(a|b)}$  is increasing in the ↗ direction by assumption!



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- Screening devices differ in how they interact with agents' values
  - **Damages** impose costs that increase in one's value for the good
  - **Ordeals** impose costs that are separable from recipients' values
- Using **damages** is never optimal with only one kind of good
- And while they **can** be useful when many kinds of goods are offered...
- ...this is not the case for "**regular**" distributions

# Conclusions

- Implications for **public housing** allocation?
  - Such programs often offer **heterogeneous units**, with different **wait-times**
  - Even if some wait-time is often inevitable in reality...
  - ...we should be worried about large **imbalances** in wait-times!
  - We should "sort" applicants using other instruments, e.g. by **readjusting subsidies**

Thank you!