

# Equitable screening\*

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## Abstract

In many mechanism design problems the designer is concerned about the perceived fairness of the allocation. I provide a notion of equity capturing such concerns in a variety of settings. According to it, an allocation is equitable if it does not systematically differ between agents who are equally deserving but belong to different protected groups. I apply this notion to the problem of a government distributing in-kind benefits and ask what forms of efficiency-enhancing screening are compatible with equity. While the government cannot equitably screen with a single instrument (e.g. payments or queueing), combining multiple instruments, which on their own favor different groups, allows it to screen while still producing an allocation that is seen as fair.

## 1 Introduction

In 1999, the UK government aimed to universalize the list of treatments covered by the National Health Service. To that end, it established the National Institute for Clinical Excellence (NICE)—an organization tasked with ranking treatments according to their efficacy and cost-effectiveness; these assessments then informed coverage decisions. NICE’s cost-benefit analyses recommended certain cheap but noncritical procedures while excluding other extremely costly but life-saving ones. The fact that some critically ill patients were covered while others (e.g. those suffering from extremely rare cancers) were not, provoked backlash which forced the government to compromise ([Einav and Finkelstein \(2023\)](#)). NICE responded to public outcry by i.a. dramatically increasing expenditure limits for drugs and treatments for rare life-threatening diseases. Attempts at institutionalizing cost-benefit analyses in healthcare fared similarly in many other rich countries. Some, including Norway, Sweden and the Netherlands, responded to public pressure by explicitly requiring that treatments to those in greatest need be covered regardless of cost.

More recently, many US colleges have shifted to test-optional, or even test-blind admissions policies. Test-based admissions have long been criticized for providing an unfair advantage to wealthy applicants who, unlike their poorer peers, could invest in tutoring and preparation. Despite tests being highly informative selection tools, the reputational cost from being seen as inequitable led many colleges to abandon them ([Dessein et al. \(2023\)](#)).

In both of these cases, the backlash resulted from policies treating equally deserving subjects differently based on some characteristic that, from an ethical standpoint, should not matter.

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In the case of NICE, patients with grave diseases whose treatment happened to be more costly were denied coverage, while those with equally severe but more common conditions got it. In the case of standardized tests, similarly able applicants from poorer backgrounds lacked the recourses for preparation that their wealthier peers possessed. Other policies motivated by similar equity concerns include non-discrimination clauses, affirmative action, and disability accommodations.

This paper considers mechanism design problems where the designer is bound by equity concerns. My notion of equity builds on context-specific *merit functions* specifying how entitled each agent is to the allocated good. I introduce an equity constraint requiring that, among agents with equal merit, the allocation should not depend on *protected characteristics*, such as race or gender. This definition permits systematic differences in allocations based on merit (e.g. allocating vaccines to doctors and nurses before everyone else), as well as differences in allocations among agents with equal merit that are ‘due to noise’ (e.g. a university can reject talented applicants who were unlucky on the exam). It does not, however, allow such errors to systematically bias allocations between protected groups. Importantly, allocating through uniform lotteries will always be equitable.<sup>1</sup>

My notion is also much stronger than the requirement that the procedure not explicitly depend on protected characteristics. Consider, for instance, an insurer who is banned from discriminating on applicants’ race but prices policies based on proxies for it, essentially bypassing regulation. While the insurer is not *explicitly* discriminating based on race, his policy is still inequitable in the sense of this paper. This makes my equity notion akin to disparate impact laws—these prohibit practices in i.a. employment and housing that adversely affect some protected group where there is no ‘business necessity’ to do so; these laws apply even if said practice is formally neutral or unintentional (*EEOC v. Sambo’s of Georgia* (1981)).

My primary application studies the problem of providing in-kind benefits, such as vaccines or public housing, when the government is concerned about efficiency and redistribution. The literature studying the optimal design of such programs (e.g. [Condorelli \(2013\)](#), [Akbarpour et al. \(2020\)](#)) finds that the optimal mechanism often involves screening recipients, e.g. through payments. For instance, [Akbarpour et al. \(2020\)](#) find that, under fairly general conditions, the government should sell the good (at possibly non-market prices) and then redistribute the revenue to poorer agents. However, such mechanisms might be infeasible when the government faces equity constraints. Consider for instance the problem of distributing Covid-19 vaccines studied by [Akbarpour et al. \(2023\)](#). They show that the optimal mechanism combines priorities to vulnerable groups with a market mechanism under which one can pay to be vaccinated early. The authors themselves acknowledge that such mechanisms might provoke backlash on fairness grounds. Indeed, similar objections have been raised by both academic philosophers ([Kass \(1997\)](#), [Walzer \(1983\)](#)) and the general public.<sup>2</sup>

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<sup>1</sup>In some contexts uniform randomization within a particular group would not be considered fair. [Kamada and Kojima \(2023\)](#) provide examples of Japanese disaster relief efforts where organizers, motivated by perceived equity concerns, preferred not allocating a resource to anyone in a given group to running a lottery.

To capture such notions of equity, one could amend my definition with a requirement of *individual fairness*—a criterion insisting that similar individuals be treated similarly ([Dwork et al. \(2012\)](#)).

<sup>2</sup><https://www.statnews.com/2020/12/03/how-rich-and-privileged-can-skip-the-line-for-covid-19-vaccines>

This paper relates to work examining how moral sentiments constrain market designers and policymakers. Roth (2007) discusses how the repugnance of certain transactions precludes the use of markets in settings where they would be efficient. The literature following Abdulkadiroğlu and Sönmez (2003) models fairness concerns in matching markets through assigning *priorities* to agents. It then studies matching mechanisms that eliminate *justified envy*—a notion capturing perceived injustice. Finally, Dessein et al. (2023) argue that many US colleges switched to test-optional admissions to reduce public scrutiny of their admission decisions. However, to my knowledge, no existing work provides a fairness criterion for mechanism design problems of the sort this paper considers. In attempting to do so, my paper also comes close to the literatures on algorithmic fairness in Computer Science, and on discrimination in Economics, which attempt to conceptualize bias, unfairness and discrimination (see Alves et al. (2023) and Onuchic (2022) for respective surveys). However, in both cases researchers focus on problems of classification or statistical inference, and hence do not account for the strategic behavior of agents. By contrast, the purpose of this paper is to study mechanisms that are fair after accounting for strategic responses.

The rest of the paper is structured as follows. First, I formalize my notion of equity and contrast it with notions in the literature. Then, in Section 3, I employ this notion to ask whether governments can equitably screen when providing in-kind benefits.

## 2 Equity constraints

The designer allocates goods from  $\mathcal{X} \subseteq \mathbb{R}_+$  to agents. Each agent has a *protected characteristic*  $\alpha \in \mathcal{A}$  representing an identity that is salient for equity, and an *unprotected characteristic*  $\beta \in \mathcal{B}$  that is irrelevant to equity considerations. The allocation rule is  $x : \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{X}$ . I model equity considerations using an exogenous *merit function*  $\eta : \mathcal{A} \times \mathcal{B} \rightarrow \mathbb{R}$ .

**Definition 1** (Equitable allocation). *An allocation rule  $x$  is **equitable** if, for all  $\alpha_1, \alpha_2 \in \mathcal{A}$  and all  $\gamma \in \mathbb{R}$ , we have:*

$$\mathbb{E}_\beta[x(\alpha_1, \beta) \mid \eta(\alpha_1, \beta) = \gamma] = \mathbb{E}_\beta[x(\alpha_2, \beta) \mid \eta(\alpha_2, \beta) = \gamma]. \quad (1)$$

In words, an allocation is equitable if, among agents with equal merit, expected allocations are the same across all protected groups. For intuition, consider the following example:

**Example 1.** *Consider the problem of allocating university places; then  $\mathcal{X} = \{0, 1\}$ . Let  $\alpha$  be a vector of demographic characteristics (gender, race, etc.) and  $\beta = (\beta_1, \beta_2)$  specify academic potential ( $\beta_1$ ) and test scores ( $\beta_2$ ). Assume  $\alpha$  and  $\beta_2$  are public but academic potential  $\beta_1$  is private information. Assume also that the only determinant of merit is academic potential:*

$$\eta(\alpha, \beta) = \beta_1.$$

*Equity then requires that demographic characteristics do not systematically affect admission chances among students with the same academic potential. For instance, if some demographic group systematically underperforms on test scores (relative to their potential), equity demands that it face a lower admission threshold to correct for that.*

A related criterion, known as the *priority point system*, is commonly used by hospitals in rationing scarce medical resources, such as ICU beds and ventilators (Pathak et al. (2021)).<sup>3</sup> Under this system, the hospital often uses an explicit formula to assign patients to priority tiers, and allocates resources in that order. In line with Definition 1, ties among patients in the same tier are often broken by lottery (Zucker et al. (2015), Emanuel et al. (2020)).<sup>4</sup>

Definition 1 is related to the literature on algorithmic fairness in Computer Science, where various notions of fairness and non-discrimination have been proposed to conceptualize bias in algorithms and machine learning<sup>5</sup> (see Barocas et al. (2019) for an introduction). This literature considers the problem of designing a (usually binary) classifier  $\hat{Y}$  to predict an agents' true outcome  $Y$  based on observed characteristics  $X$ . My approach is close to the family of criteria known as (conditional) independence requiring that some aspect of classification be (conditionally) independent of some protected characteristic. Within this class, Definition 1 is closest to the notion of *conditional statistical parity* requiring that, conditional on certain covariates deemed legitimate, the classification rate be the same across protected groups (Corbett-Davies et al. (2017)). My notion differs from it in that it does not condition directly on agents' characteristics, but on the value of the merit function.

The merit function approach provides more flexibility in capturing context-specific concerns. In particular, it nests many other notions from the independence family—for instance, conditional statistical parity can be recovered by assigning a different value to each combination of legitimate covariates; group fairness (requiring the allocation not depend on protected characteristics) can be recovered by setting the merit function to a constant.

Some further properties of Definition 1 are worth mentioning. First, I model equity as a constraint, not as a term included in the designer's objective that can be traded off against other values. This reflects the rigid nature of ethical concerns, like those leading to banning organ trade or the use of race in pricing insurance. Second, the framework only addresses fairness concerns relating to resulting allocations, and not other features of the process. Third, I require that (conditional on merit) the protected characteristic not affect the allocation only *in expectation*; the algorithmic fairness literature typically requires *independence*.

### 3 Equitable screening in public provision

I consider the problem of a government choosing a mechanism for allocating in-kind benefits such as vaccines, affordable housing or, in many developing countries, basic food items. Standard economic logic suggests that such programs can be made more efficient through costly screening—if agents need to pay or queue to get the benefit, only those who need it will choose to do so. However, different agents with the same need for the good might find queueing or paying burdensome to different extents. For instance, a poor person with severe

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<sup>3</sup>Pathak et al. (2021) summarize ethical arguments against this criterion, many of which apply to Definition 1. However, I do not claim that my definition is normatively valid, but rather that it captures empirically important moral sentiments in the problems I study.

<sup>4</sup>Let the priority score be the merit function. Then, strictly speaking, Definition 1 does not require higher allocations for agents with higher priorities. But if we assume that every individual is a separate protected group, it does require equal expected allocations for all agents with the same priority.

<sup>5</sup>In this literature, the terms 'fairness', 'equity' and 'non-discrimination' are used synonymously.

health conditions might be less willing to pay to get a vaccine early than would a rich person of similar health. Consequently, we might worry that efficiency-enhancing screening could lead to inequitable allocations. Indeed, in many cases governments forgo screening in favor of mechanisms based purely on observables, or lotteries.<sup>6</sup> Examples of goods allocated by lottery include primary and secondary school places (Stone (2008)), public housing (Elster (1989), p.63) and US green cards.<sup>7</sup>

I therefore ask how (if at all) the government can screen agents when allocations are subject to equity constraints. I look at screening using only payments (which are less costly to the rich), only queueing (less costly to the poor) and both of these instruments at once. In the former two cases, equitable screening is impossible—the only equitable allocation is the same for all agents. However, when the designer can use both payments and queueing, she has significant freedom to screen agents without violating equity constraints.

### 3.1 Environment

The government allocates goods  $x \in \mathcal{X} = [\underline{x}, \bar{x}]$  to agents with a two-dimensional type  $(\alpha, \beta) \in \Theta \subseteq \mathbb{R}^2$ .  $\Theta$  is open, connected and bounded.  $\beta$  represents an agent's value for the good and  $\alpha$  represents her value for money (higher  $\alpha$  means the agent is poorer). I consider two screening instruments—payments and queueing. Payments,  $p \in \mathbb{R}$ , are costlier for poorer agents (higher  $\alpha$ ), while queueing,  $q \in \mathbb{R}_+$ , is costlier for richer agents. Agents have utility:

$$U[\alpha, \beta; x, p, q] = v(\beta, x) - w(\alpha, p) - z(\alpha, q).$$

#### Assumption 1.

1.  $v_\beta > 0, v_x > 0; \quad w_\alpha > 0, w_p > 0; \quad z_\alpha < 0, z_q > 0.$
2.  $v_{\beta x} > 0; \quad w_{\alpha p} > 0; \quad z_{\alpha q} < 0.$
3. *The above derivatives are continuous and uniformly bounded.*

I assume that both  $\alpha$  and  $\beta$  are private information (I discuss this assumption in Subsection 3.5). The government therefore chooses allocations  $x : \Theta \rightarrow \mathcal{X} \subseteq \mathbb{R}_+, p : \Theta \rightarrow \mathbb{R}, q : \Theta \rightarrow \mathbb{R}_+$  subject and IR and IC constraints (I write  $y = (x, p, q)$ ):

$$\forall (\alpha, \beta) \in \Theta, \quad U[\alpha, \beta; y(\alpha, \beta)] \geq 0, \tag{IR}$$

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<sup>6</sup>One might argue, along the lines of Weitzman (1977), that governments eschew screening not due to fairness concerns as such, but because screening is not optimal when the government has distributional concerns. For instance, if a government allocating affordable housing screened using payments, the allocation would be biased towards wealthier agents. This would be suboptimal for a government redistributive preferences. There are two responses to this argument. First, the government could use a screening instrument other than payments whose cost is *negatively* correlated with wealth; in Subsection 3.3 I study screening with queueing, which plausibly has this feature. Secondly, as observed by Akbarpour et al. (2020), screening with payments is still often optimal even in such settings. This is because the government can generate revenue from selling the good to rich agents and then redistribute it to the poor who value money highly. These observations suggest a deeper reason for the frequent absence of screening in such programs.

<sup>7</sup>My model can capture lotteries when the allocation is binary or the utility for the good is linear:  $v(\beta, x) = \beta \cdot x$ . In the former case, we can interpret  $x \in [0, 1]$  as the probability of being allocated the good.

$$\forall (\alpha, \beta) \in \Theta, \quad U[\alpha, \beta; y(\alpha, \beta)] \geq \max_{(\alpha', \beta') \in \Theta} U[\alpha, \beta; y(\alpha', \beta')]. \quad (\text{IC})$$

The government also faces an equity constraint (1) where the merit function  $\eta(\alpha, \beta)$  is continuous and strictly increasing in both arguments. That is, agents are more entitled to receiving the good if they value it more or if they are poorer (as richer agents can more easily satisfy their needs without government assistance). Under this assumption, the equity constraint (1) simplifies as follows:

**Lemma 1.** *Let  $\alpha, \beta$  be one-dimensional and  $\eta(\alpha, \beta)$  be strictly monotonic in both arguments. Then the allocation rule  $x$  is equitable if and only if*

$$x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta)), \quad (2)$$

for some  $\hat{x} : \mathbb{R} \rightarrow \mathcal{X}$ .

*Proof.* ( $\rightarrow$ ) follows because under (2) everyone with the same merit receives the same allocation. For ( $\leftarrow$ ), I will show that if (2) fails, equity fails too. Suppose there are distinct  $(\alpha_1, \beta_1)$  and  $(\alpha_2, \beta_2)$  such that  $\eta(\alpha_1, \beta_1) = \eta(\alpha_2, \beta_2)$  (else, (2) is trivially satisfied). If  $\alpha_1 \neq \alpha_2$  and  $x(\alpha_1, \beta_1) \neq x(\alpha_2, \beta_2)$ , equity fails by definition. So suppose  $x(\alpha_1, \beta_1) = x(\alpha_2, \beta_2)$  and  $\alpha_1 = \alpha_2$ . But since the types were distinct, it has to be that  $\beta_1 \neq \beta_2$ . Then, by strict monotonicity,  $\eta(\alpha_1, \beta_1) \neq \eta(\alpha_1, \beta_2) = \eta(\alpha_2, \beta_2)$ ; contradiction.  $\square$

Finally, define:

$$\underline{\eta} = \inf_{(\alpha, \beta) \in \Theta} \{\eta(\alpha, \beta)\}, \quad \bar{\eta} = \sup_{(\alpha, \beta) \in \Theta} \{\eta(\alpha, \beta)\}.$$

Notice that since  $\Theta$  is connected and  $\eta(\alpha, \beta)$  is continuous, for every  $\eta' \in (\underline{\eta}, \bar{\eta})$  there is some  $(\alpha', \beta')$  such that  $\eta(\alpha', \beta') = \eta'$ .

### 3.2 Screening with payments

I first consider a setting where the government screens only using payments, and not queueing. I show that if it cannot observe agents' wealth and need, it cannot screen equitably, and hence must give the same allocation to everyone.

**Proposition 1.** *Any equitable and implementable  $x(\alpha, \beta)$  is the same for all  $(\alpha, \beta) \in \Theta$ .*

The proof builds on the following lemma:

**Lemma 2.** *We say type  $(\alpha_1, \beta_1)$   $p$ -dominates  $(\alpha_2, \beta_2)$  if*

$$\alpha_1 \leq \alpha_2, \quad \beta_1 \geq \beta_2, \quad (3)$$

*with at least one inequality holding strictly. Then, under any mechanism,  $x(\alpha_1, \beta_1) \geq x(\alpha_2, \beta_2)$ .*



*Proof.* Denote  $x(\alpha_1, \beta_1) = x_1$ ,  $x(\alpha_2, \beta_2) = x_2$  and  $p(\alpha_1, \beta_1) = p_1$ ,  $p(\alpha_2, \beta_2) = p_2$ . Suppose that  $x_1 < x_2$ ; then  $p_1 < p_2$ , or else both types would strictly prefer  $(x_2, p_2)$ . By revealed preference:

$$\begin{aligned} v(\beta_1, x_1) - w(\alpha_1, p_1) &\geq v(\beta_1, x_2) - w(\alpha_1, p_2), \\ v(\beta_2, x_2) - w(\alpha_2, p_2) &\geq v(\beta_2, x_1) - w(\alpha_2, p_1). \end{aligned}$$

The latter inequality implies that:

$$v(\beta_2, x_2) - v(\beta_2, x_1) \geq w(\alpha_2, p_2) - w(\alpha_2, p_1).$$

However, by  $p$ -dominance and strictly increasing differences, this gives:

$$\begin{aligned} v(\beta_1, x_2) - v(\beta_1, x_1) &> w(\alpha_1, p_2) - w(\alpha_1, p_1), \\ v(\beta_1, x_2) - w(\alpha_1, p_2) &> v(\beta_1, x_1) - w(\alpha_1, p_1). \end{aligned}$$

That is,  $(\alpha_1, \beta_1)$  prefers  $(x_2, p_2)$  over  $(x_1, p_2)$ ; contradiction.  $\square$

In general, agents in my setting cannot be ordered according to single-crossing. However, we can recover such a ranking for subsets of the type space—an agent who values money strictly less (lower  $\alpha$ ) and the good strictly more (higher  $\beta$ ) will always consume more of the good.  $p$ -dominance captures this partial order which can be applied to all agents along any downwards-sloping line (Figure 1a). I now proceed to showing Proposition 1.

*Proof.* We can partition the type space into iso- $\eta$  curves (Figure 1b). Since  $\eta(\alpha, \beta)$  is strictly increasing in both arguments, iso- $\eta$  curves never cross. Recall that equity constraints require that agents on the same iso- $\eta$  curve must receive the same allocation.

I first show that the allocation must also be the same within some neighborhood around any point on the iso- $\eta$  curve. Fix any  $(\alpha, \beta) \in \Theta$ ; since  $\Theta$  is open, there exist  $\underline{\epsilon}, \underline{\delta}, \bar{\epsilon}, \bar{\delta} > 0$  such that

$$\eta(\alpha - \bar{\epsilon}, \beta + \bar{\delta}) = \eta(\alpha, \beta) = \eta(\alpha + \underline{\epsilon}, \beta - \underline{\delta}). \quad (4)$$

Now, take any  $(\alpha', \beta')$  such that  $\alpha' \in (\alpha - \bar{\epsilon}, \alpha + \underline{\epsilon})$  and  $\beta' \in (\beta - \underline{\delta}, \beta + \bar{\delta})$ . Then:

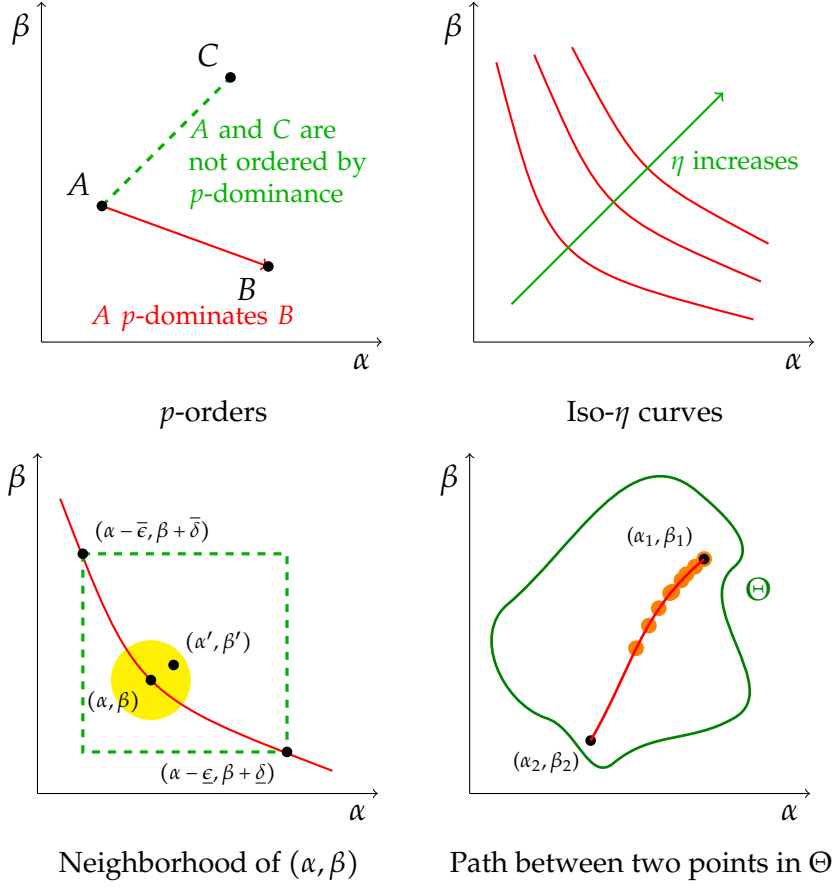
$$(\alpha - \bar{\epsilon}, \beta + \bar{\delta}) \succ_p (\alpha', \beta') \succ_p (\alpha + \underline{\epsilon}, \beta - \underline{\delta}). \quad (5)$$

That is, we found a neighborhood around  $(\alpha, \beta)$  that is ‘sandwiched’ by points on its iso- $\eta$  in the sense of  $p$ -dominance (Figure 1c). Now, since  $(\alpha - \bar{\epsilon}, \beta + \bar{\delta})$  and  $(\alpha + \underline{\epsilon}, \beta - \underline{\delta})$  have the same  $\eta$  as  $(\alpha, \beta)$ :

$$x(\alpha - \bar{\epsilon}, \beta + \bar{\delta}) = x(\alpha, \beta) = x(\alpha + \underline{\epsilon}, \beta - \underline{\delta}).$$

Therefore, (5) and Lemma 2 imply that  $x(\alpha', \beta') = x(\alpha, \beta)$ .

Hence, every  $(\alpha, \beta)$  is surrounded by some neighborhood within which the allocation  $x(\alpha, \beta)$  is constant. Now, take any two  $(\alpha_1, \beta_1), (\alpha_2, \beta_2) \in \Theta$ . Since  $\Theta$  is connected, there exists a continuous path between these points (Figure 1d). Every point along this path has the same allocation as the points within its neighborhood. Therefore, the allocation  $x(\alpha, \beta)$  has to be constant along the whole path, including the end-points  $(\alpha_1, \beta_1)$  and  $(\alpha_2, \beta_2)$ .  $\square$



A higher-level intuition is useful here—notice we can only screen agents based on their willingness-to-pay for the good. While some agents are willing to pay more for it because it gives them greater utility (higher  $\beta$ ), others are willing to do so because they are wealthier (i.e. because of the value of their protected characteristic  $\alpha$ ). Screening through the latter channel would violate equity, but screening using the former would not. However, agents' characteristics are not observed, so the government cannot isolate the former channel. As a result if it screens, it inevitably uses both, so all forms of screening will violate equity.

### 3.3 Screening with queueing

I now consider a setting where the government screens only using queueing. This time I show that the government cannot screen for *generic* merit functions. Intuitively, screening with queueing differs from screening with payments in that the allocation it produces is biased *towards the poor*. That is, among two people with the same  $\beta$  the poorer person will be more eager to queue for the good and hence receive the higher allocation. In contrast to the case of screening in payments, this direction of bias is consistent with what the merit function requires. It is therefore less clear why, generically, screening with queueing is also incompatible with equity. Notice, however, that equity constraints impose requirements not only on the direction of the bias, but also on its exact form—the merit function  $\eta(\alpha, \beta)$  specifies exactly which types have to be bunched together. However, with one screening device only, the designer will typically have 'too few degrees of freedom' to pool agents in this exact way. Therefore, she can equitably screen only in knife-edge cases where the



pooling required by  $\eta$  is actually achievable.

This reasoning will be formalized in Proposition 2. Before discussing it, I make a few helpful observations. The following is an analog of Lemma 2 from the previous subsection:

**Lemma 3.** *We say type  $(\alpha_1, \beta_1)$   $q$ -dominates  $(\alpha_2, \beta_2)$  if*

$$\alpha_1 \geq \alpha_2, \quad \beta_1 \geq \beta_2, \quad (6)$$

*with at least one inequality holding strictly. Then, under any mechanism,  $x(\alpha_1, \beta_1) \geq x(\alpha_2, \beta_2)$ .*

We know from Lemma 1 that any equitable allocation rule has to be of the form  $\hat{x}(\eta(\alpha, \beta))$ . Since  $\eta(\alpha, \beta)$  is increasing in both arguments, Lemma 3 tells us that any implementable  $\hat{x}(\cdot)$  has to be increasing. Notice also that  $q(\alpha, \beta) \equiv \hat{q}(\eta(\alpha, \beta))$  because identical allocations of  $x$  have to come with identical queueing requirements. I assume (without loss) that  $\hat{q}(\cdot)$  is left-continuous.

**Definition 2.** *A merit function is **non-generic** if either of the following conditions hold at any  $\eta^*$ :*

1. *There exist  $x_1 \neq x_2 \in \mathcal{X}, q_1, q_2 \in \mathbb{R}$  such that:*

$$\forall (\alpha, \beta) : \eta(\alpha, \beta) = \eta^*, \quad v(\beta, x_1) - z(\alpha, q_1) = v(\beta, x_2) - z(\alpha, q_2). \quad (7)$$

2. *There exist  $x \in \mathcal{X}$  and  $q, k \in \mathbb{R}$  such that:*

$$\forall (\alpha, \beta) : \eta(\alpha, \beta) = \eta^*, \quad \frac{v_x(\beta, x)}{z_q(\alpha, q)} = k. \quad (8)$$

**Proposition 2.** *If the merit function  $\eta$  is generic, any equitable and implementable  $x(\alpha, \beta)$  is the same for all  $(\alpha, \beta) \in \Theta$ .*

As mentioned, equity constraints precisely specify which sets of types have to be treated identically. The above knife-edge conditions describe when the designer can pick a mechanism that makes agents self-select into different allocations in exactly that way.

For further intuition suppose  $\hat{x}(\cdot)$  and  $\hat{q}(\cdot)$  are smoothly increasing around some  $\eta^*$ . Then the FOCs of all agents with merit  $\eta^*$  must bind exactly at  $\eta^*$  (n.b. Condition 8):

$$\forall (\alpha, \beta) : \eta(\alpha, \beta) = \eta^*, \quad \frac{v_x(\beta, \hat{x}(\eta^*))}{z_q(\alpha, \hat{q}(\eta^*))} = \frac{\hat{q}'(\eta^*)}{\hat{x}'(\eta^*)}.$$

If some agent with merit  $\eta^*$  had a larger (smaller) MRS at that allocation, she would be tempted to mimic an agent with a slightly higher (lower)  $\eta$ . Condition 7 relates to a similar requirement for values of  $\eta$  where  $\hat{x}(\cdot)$  jumps discontinuously.

### 3.4 Screening with payments and queueing

I will now let the government use both screening devices at once. To keep the model tractable, I will impose more structure on agents' utilities:

$$U[\alpha, \beta; x, p, q] = \beta x - w(\alpha)p - z(\alpha)q,$$

where  $\beta, w(\alpha), z(\alpha) > 0$  for all  $(\alpha, \beta) \in \Theta$ . I also impose the following technical assumption:

**Assumption 2.**  *$w, z$  and  $\eta$  are twice differentiable, with first, second and cross partials uniformly bounded.*

As it turns out, using both screening devices allows for rich screening without violating the equity constraint. Intuitively, every amount of  $x$  can now come with a menu of payment options composed of different amounts of  $p$  and  $q$ . Since the designer has one screening device preferred by the poor and another one preferred by the rich, she can fine-tune such 'payment menus' to produce precisely the bias in allocation that equity requires. Loosely speaking, being able to compose such menus fixes the problem of 'too few degrees of freedom' we encountered when only queueing was available.

**Proposition 3.** *Let  $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$ , where  $\hat{x}(\cdot)$  is increasing and twice differentiable with uniformly bounded derivatives. Then  $x(\alpha, \beta)$  is equitable and implementable.*

While Proposition 3 provides only sufficient conditions, it is worth noting that any equitable and implementable allocation has to be increasing in merit. Intuitively, IC requires that among agents with the same  $\alpha$ , those with higher  $\beta$  receive weakly higher allocations; hence  $x(\alpha, \beta)$  is increasing in  $\beta$ . However, by Lemma 1 an equitable allocation must take the form  $\hat{x}(\eta(\alpha, \beta))$ . Since  $\eta$  is strictly increasing in  $\beta$ , it follows that  $\hat{x}(\eta)$  has to be increasing.

*Proof.* To simplify the proof, I first reparametrize types:

$$\kappa = \frac{\beta}{z(\alpha)}, \quad \lambda = \frac{w(\alpha)}{z(\alpha)}, \quad \tilde{\Theta} = \left\{ \left( \frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)} \right) : (\alpha, \beta) \in \Theta \right\}.$$

**Lemma 4.** *Every  $(\alpha, \beta)$  corresponds to a unique  $(\kappa, \lambda)$ .*

*Proof.* Take  $(\alpha_1, \beta_1), (\alpha_2, \beta_2)$  such that:

$$\frac{\beta_1}{z(\alpha_1)} = \frac{\beta_2}{z(\alpha_2)}, \quad \frac{w(\alpha_1)}{z(\alpha_1)} = \frac{w(\alpha_2)}{z(\alpha_2)} \tag{9}$$

Suppose that  $\alpha_1 > \alpha_2$ . Then, since  $z_\alpha < 0$  and  $w_\alpha > 0$ ,

$$\frac{w(\alpha_1)}{z(\alpha_1)} > \frac{w(\alpha_2)}{z(\alpha_2)},$$

which contradicts (9), so  $\alpha_1 = \alpha_2$ . Then, the first equation in (9) gives  $\beta_1 = \beta_2$ . □

Notice Lemma 4 would no longer hold if both screening devices were less costly to the rich (or to the poor). Agents' utilities (up to scaling) in the reparametrized model are given by:

$$\tilde{U}[\kappa, \lambda; x, p, q] = \kappa x - \lambda p - q.$$

I will also write the payment rule and the merit function in the  $(\kappa, \lambda)$  space:

$$\tilde{p}\left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)}\right) \equiv p(\alpha, \beta), \quad \tilde{\eta}\left(\frac{\beta}{z(\alpha)}, \frac{w(\alpha)}{z(\alpha)}\right) \equiv \eta(\alpha, \beta). \quad (10)$$

Since  $\eta$  was strictly increasing in  $\beta$  and  $\alpha$ ,  $\tilde{\eta}$  is strictly increasing in  $\kappa$  and  $\lambda$ . Moreover,  $\eta, w, z$  were twice differentiable with uniformly bounded derivatives, so  $\tilde{\eta}$  is too. Finally, define:

$$\underline{\kappa} = \inf_{(\alpha, \beta) \in \Theta} \frac{\beta}{z(\alpha)}, \quad \bar{\kappa} = \sup_{(\alpha, \beta) \in \Theta} \frac{\beta}{z(\alpha)}, \quad \underline{\lambda} = \inf_{(\alpha, \beta) \in \Theta} \frac{w(\alpha)}{z(\alpha)}, \quad \bar{\lambda} = \sup_{(\alpha, \beta) \in \Theta} \frac{w(\alpha)}{z(\alpha)}.$$

Since  $\Theta$  is bounded and  $z, w$  have uniformly bounded derivatives,  $\infty > \underline{\lambda}, \underline{\kappa}, \bar{\lambda}, \bar{\kappa} > -\infty$ .

By Lemma 2, an allocation of the form  $\hat{x}(\tilde{\eta}(\kappa, \lambda))$  is equitable. By Proposition 2 in [Rochet \(1987\)](#),  $\hat{x}(\tilde{\eta}(\kappa, \lambda))$  is implementable if there is a convex function  $V(\kappa, \lambda) : \tilde{\Theta} \rightarrow \mathbb{R}$  such that:<sup>8</sup>

$$\frac{d}{d\kappa} V(\kappa, \lambda) = \hat{x}(\tilde{\eta}(\kappa, \lambda)). \quad (R)$$

Notice that (R) holds for:

$$V(\kappa, \lambda) = \int_{\underline{\kappa}}^{\kappa} \hat{x}(\tilde{\eta}(\tau, \lambda)) d\tau + \int_{\underline{\lambda}}^{\lambda} \zeta(\tau) d\tau, \quad (11)$$

where  $\zeta : [\underline{\lambda}, \bar{\lambda}] \rightarrow \mathbb{R}$  is some differentiable function. I now show that we can choose  $\zeta(\cdot)$  so that  $V$  defined by (11) is convex. The Hessian of  $V(\kappa, \lambda)$  is:

$$H(\kappa, \lambda) = \begin{bmatrix} \hat{x}'(\tilde{\eta}(\kappa, \lambda)) \cdot \tilde{\eta}_{\kappa}(\kappa, \lambda) & \hat{x}'(\tilde{\eta}(\kappa, \lambda)) \cdot \tilde{\eta}_{\lambda}(\kappa, \lambda) \\ \hat{x}'(\tilde{\eta}(\kappa, \lambda)) \cdot \tilde{\eta}_{\lambda}(\kappa, \lambda) & r(\kappa, \lambda) \end{bmatrix},$$

where

$$r(\kappa, \lambda) = \int_{\underline{\kappa}}^{\kappa} \underbrace{\hat{x}''(\tilde{\eta}(\tau, \lambda)) \cdot \tilde{\eta}_{\lambda}^2(\tau, \lambda) + \hat{x}'(\tilde{\eta}(\tau, \lambda)) \cdot \tilde{\eta}_{\lambda\lambda}(\tau, \lambda)}_{:=D(\kappa, \lambda)} d\tau + \zeta'(\lambda).$$

It suffices to show that the determinant of  $H$ , and  $H_{(1,1)}, H_{(2,2)}$  are positive everywhere.  $H_{(1,1)}$  is positive since  $\tilde{\eta}_{\kappa}(\kappa, \lambda) > 0$  and  $\hat{x}(\cdot)$  is increasing by assumption. For the determinant, we want to show that for every  $\kappa \in (\underline{\kappa}, \bar{\kappa})$ ,  $\lambda \in (\underline{\lambda}, \bar{\lambda})$ :

$$\hat{x}'(\tilde{\eta}(\kappa, \lambda)) \cdot \tilde{\eta}_{\kappa}(\kappa, \lambda) \cdot r(\kappa, \lambda) - [\hat{x}'(\tilde{\eta}(\kappa, \lambda)) \cdot \tilde{\eta}_{\lambda}(\kappa, \lambda)]^2 \geq 0. \quad (12)$$

If for some  $(\kappa, \lambda)$  we have  $\hat{x}'(\tilde{\eta}(\kappa, \lambda)) = 0$ , the inequality holds, so consider the opposite case.

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<sup>8</sup>[Rochet \(1987\)](#) assumes  $\tilde{\Theta}$  is convex. However, I can add 'fictitious types' enlarging the type space to  $(\underline{\kappa}, \bar{\kappa}) \times (\underline{\lambda}, \bar{\lambda}) \supseteq \tilde{\Theta}$  and implement an extended allocation on it that coincides with  $\hat{x}(\tilde{\eta}(\kappa, \lambda))$  on  $\tilde{\Theta}$ .

Then  $\hat{x}'(\tilde{\eta}(\kappa, \lambda)), \eta_\kappa(\kappa, \lambda), \eta_\lambda(\kappa, \lambda) > 0$ , so (12) is equivalent to:

$$r(\kappa, \lambda) \geq x'(\tilde{\eta}(\kappa, \lambda)) \cdot \frac{\tilde{\eta}_\lambda(\kappa, \lambda)^2}{\tilde{\eta}_\kappa(\kappa, \lambda)}.$$

$$\zeta'(\lambda) \geq x'(\tilde{\eta}(\kappa, \lambda)) \cdot \frac{\tilde{\eta}_\lambda(\kappa, \lambda)^2}{\tilde{\eta}_\kappa(\kappa, \lambda)} - \int_{\underline{\kappa}}^{\kappa} D(\kappa, \lambda) d\kappa.$$

Notice that it suffices to show that the RHS is uniformly bounded across  $\kappa$ . Recall that all first, second, and cross partials of  $\tilde{\eta}$ , as well as first and second derivatives of  $\hat{x}$ , are uniformly bounded; therefore the first term on the RHS and  $D(\kappa, \lambda)$  are uniformly bounded. Since  $\kappa$  is bounded by  $\underline{\kappa}, \bar{\kappa} \in \mathbb{R}$ , the RHS is indeed uniformly bounded. Therefore, we can ensure (12) by choosing high-enough values for all  $\zeta'(\lambda)$ . Moreover, we can choose these values high enough to make  $r(\kappa, \lambda) \geq 0$  everywhere. This also ensures  $H_{(2,2)} \geq 0$  everywhere.  $\square$

### 3.5 Observable characteristics

While I assumed that neither need nor wealth are observable, the government usually has some information about them. For instance, in the problem of vaccine allocation, age and medical history are good indicators of need. Similarly, tax data gives the government an idea of one's wealth (even if some income sources or assets remain unobserved). In such cases, agents' private information can be thought of as *residual uncertainty* after accounting for these observables. Indeed, even public programs conditioned on earnings face substantial uncertainty over one's wealth.<sup>9</sup>

Still, what can the government do if it perfectly observes the protected characteristic? As it turns out, this gives it significant freedom to screen as long as it suitably adjusts the mechanism. Intuitively, the designer can now 'control for' the fact that some agents prefer the good (relative to money or queueing) because of their protected characteristic and screen purely based on need.

**Proposition 4.** *Suppose  $\alpha$  is observable and the government uses either only money or only queueing to screen. Then  $x(\alpha, \beta)$  is equitable and implementable if and only if  $x(\alpha, \beta) \equiv \hat{x}(\eta(\alpha, \beta))$ , where  $\hat{x}$  is increasing.*

*Proof.* By Lemma 1, the equity constraint is satisfied if and only if  $x(\alpha, \beta)$  can be written as  $\hat{x}(\eta(\alpha, \beta))$ . Fix any  $\hat{x}(\eta(\alpha, \beta))$ . Since  $\alpha$  is observable, it can be implemented separately for every value of  $\alpha$ . But then, by the standard result, any  $\hat{x}(\eta(\alpha, \beta))$  is implementable if and only if it is increasing in  $\beta$ .<sup>10</sup> Since  $\eta$  is strictly increasing in  $\beta$ , it follows that  $\hat{x}$  is implementable if and only if it is weakly increasing.  $\square$

<sup>9</sup><https://thehill.com/regulation/administration/268409-outrage-builds-over-wealthy-families-in-public-housing/>

<sup>10</sup>For any fixed value of  $\alpha$ , we can rewrite the problem as a quasi-linear one by interpreting  $w(\alpha, p)$  (if we screen with money) or  $z(\alpha, q)$  (if we screen with queueing) as the transfer.

### 3.6 Discussion

While my approach to modeling perceived fairness is highly stylized, it offers more general qualitative conclusions. First, every screening instrument will bias the allocation towards the group for whom this instrument is less costly; this makes screening with payments especially problematic from an equity standpoint. Using a different instrument (like queueing) could reverse this bias, but the designer's control over the allocation would still be limited. Consequently, the resulting bias might still not satisfy the public. I show this problem can be solved by combining multiple screening instruments which on their own favor different social groups. Doing so gives the designer freedom to temper with various groups' differential cost of the allocated good, and therefore to improve efficiency through screening while still producing an allocation that is seen as fair.

## 4 Appendix: omitted proofs

### 4.1 Proof of Proposition 2

Consider some  $\hat{x}(\cdot)$  that is equitable, implementable and not constant. Recall it has to be weakly increasing, which implies  $\hat{q}(\cdot)$  is weakly increasing too.<sup>11</sup>

**Case 1:  $\hat{x}(\cdot)$  has a jump discontinuity at some  $\eta^*$ .** Since  $\hat{x}(\cdot)$  and  $\hat{q}(\cdot)$  are weakly increasing, they have left and right limits everywhere. I denote them by  $\hat{x}_+(\cdot)$ ,  $\hat{x}_-(\cdot)$  and  $\hat{q}_+(\cdot)$ ,  $\hat{q}_-(\cdot)$ . Moreover, since  $\hat{x}$  is discontinuous at  $\eta^*$ ,  $\hat{x}(\eta^*)_- \neq \hat{x}(\eta^*)_+$ .

In what follows I sometimes abuse notation by writing  $U$  as a function of  $\eta$  rather than  $\hat{x}(\eta)$  and  $\hat{q}(\eta)$ . Consider any type  $(\alpha_1, \beta_1)$  with merit  $\eta^*$ . Define

$$\Phi(\tau) := U[\alpha_1 + \tau, \beta_1 + \tau; \eta(\alpha_1 + \tau, \beta_1 + \tau)].$$

Since  $\Theta$  is open, there exists  $\chi > 0$  such that  $(\alpha_1 - \chi, \beta_1 - \chi), (\alpha_1 + \chi, \beta_1 + \chi) \in \Theta$ . Then  $\Phi(\tau)$  is continuous on  $[-\chi, \chi]$  (Milgrom and Segal (2002)). Therefore,  $\lim_{\epsilon \rightarrow 0+} \Phi(\epsilon) = \lim_{\epsilon \rightarrow 0-} \Phi(\epsilon)$ . Furthermore, by the continuity of  $v$  and  $z$  this implies:

$$U[\alpha_1, \beta_1; \hat{x}_+(\eta^*), \hat{q}(\eta^*)_+] = U[\alpha_1, \beta_1; \hat{x}_-(\eta^*), \hat{q}(\eta^*)_-].$$

Hence, every agent with merit  $\eta^*$  is indifferent between getting  $(\hat{x}(\eta^*)_-, \hat{q}(\eta^*)_-)$  and getting  $(\hat{x}(\eta^*)_+, \hat{q}(\eta^*)_+)$ . This implies Condition 7 at  $\eta^*$ .

**Case 2:  $\hat{x}(\cdot)$  is continuous.** I will write  $x_\eta := \hat{x}(\eta)$  and  $q_\eta := \hat{q}(\eta)$ ; I first show the following:

**Fact 1.** For some  $\eta^* \in (\underline{\eta}, \bar{\eta})$ , there exists a sequence  $\{\eta_i\}_i, \eta_i \rightarrow \eta^*$  such that for every  $i > j$ ,  $x_{\eta_i} > x_{\eta_j}$ .

*Proof.*  $\hat{x}(\cdot)$  is not constant, so there are  $\eta_0, \eta'_1 \in (\underline{\eta}, \bar{\eta})$  s.t.  $\eta_0 < \eta'_1$  and  $x_{\eta_0} < x_{\eta'_1}$ . Let  $\eta^* = \inf\{\eta : x_\eta = x_{\eta'_1}\}$ . Since  $\hat{x}(\cdot)$  is continuous,  $x_{\eta^*} = x_{\eta'_1}$ . Now, take any strictly increasing sequence  $\{\eta'_i\}_i$  s.t. for every  $i$ ,  $\eta'_i \in (\underline{\eta}, \bar{\eta})$  and  $\eta'_i \rightarrow \eta^*$ . Since  $\hat{x}(\cdot)$  is increasing,  $x_{\eta_i} \geq x_{\eta_j}$  for every  $i > j$ .

<sup>11</sup>Suppose there were some  $\eta_1 < \eta_2$  s.t.  $\hat{q}(\eta_1) > \hat{q}(\eta_2)$ . Then, since  $\hat{x}(\eta_1) \leq \hat{x}(\eta_2)$ , an agent with merit  $\eta_1$  would strictly prefer  $(\hat{x}(\eta_2), \hat{q}(\eta_2))$  over her own assignment.

Also, by continuity of  $\hat{x}(\cdot)$ ,  $x_{\eta'_i} \rightarrow x_{\eta^*}$ . Moreover, by the definition of  $\eta^*$ ,  $x_{\eta'_i} < x_{\eta^*}$  for every  $i$ . Now, we can take a subsequence  $\{\eta_i\}_i$  of  $\{\eta'_i\}_i$  for which  $x_{\eta_i}$  is increasing strictly.  $\square$

Fix  $\eta^*$  and  $\{\eta_i\}_i$  as in Fact 1. Then for every  $i > j$  we have  $q_{\eta_i} > q_{\eta_j}$ .<sup>12</sup> We also know  $\hat{q}(\cdot)$  is left-continuous. Since  $\hat{x}(\cdot)$  is continuous too, we have that for every  $(\alpha, \beta)$ :

$$\frac{z(\alpha, q_{\eta^*}) - z(\alpha, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}} \rightarrow z_q(\alpha, q_{\eta^*}).$$

$$\frac{v(\beta, x_{\eta^*}) - v(\beta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}} \rightarrow v_x(\beta, x_{\eta^*}).$$

I will now show Condition 8 has to hold at  $\eta^*$ ; it states that there is  $(x^*, q^*)$  such that all agents on the iso- $\eta^*$  have the same MRS at this  $(x^*, q^*)$ . Suppose Condition 8 failed at  $\eta^*$ ; then there must exist  $(\alpha_1, \beta_1)$  and  $(\alpha_2, \beta_2)$  on the iso- $\eta^*$  such that:

$$\frac{v_x(\beta_2, x_{\eta^*})}{z_q(\alpha_2, q_{\eta^*})} > \frac{v_x(\beta_1, x_{\eta^*})}{z_q(\alpha_1, q_{\eta^*})}.$$

Consider  $(\alpha_\delta, \beta_\delta) := (\alpha_2 - \delta, \beta_2 - \delta)$  for  $\delta > 0$ . By continuity of  $v_x$  and  $z_q$ , there is some  $\delta^* > 0$  such that for  $\delta \in (0, \delta^*)$  we have:

$$\frac{v_x(\beta_\delta, x_{\eta^*})}{z_q(\alpha_\delta, q_{\eta^*})} > \frac{v_x(\beta_1, x_{\eta^*})}{z_q(\alpha_1, q_{\eta^*})}. \quad (13)$$

Then, for any such  $\delta$  and sufficiently large  $i$ , we have:

$$\frac{\frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} > \frac{\frac{v(\beta_1, x_{\eta^*}) - v(\beta_1, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_1, q_{\eta^*}) - z(\alpha_1, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}}. \quad (14)$$

Now, the following holds by revealed preference:

$$\forall i, \quad U[\alpha_1, \beta_1; \eta^*] \geq U[\alpha_1, \beta_1; \eta_i]. \quad (15)$$

Equivalently:

$$\frac{\frac{v(\beta_1, x_{\eta^*}) - v(\beta_1, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_1, q_{\eta^*}) - z(\alpha_1, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} \geq \frac{q_{\eta^*} - q_{\eta_i}}{x_{\eta^*} - x_{\eta_i}}. \quad (16)$$

But then by (14), for any  $\delta \in (0, \delta^*)$  and sufficiently large  $i$ :

$$\frac{\frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{x_{\eta^*} - x_{\eta_i}}}{\frac{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})}{q_{\eta^*} - q_{\eta_i}}} > \frac{q_{\eta^*} - q_{\eta_i}}{x_{\eta^*} - x_{\eta_i}} \implies \frac{v(\beta_\delta, x_{\eta^*}) - v(\beta_\delta, x_{\eta_i})}{z(\alpha_\delta, q_{\eta^*}) - z(\alpha_\delta, q_{\eta_i})} > 1. \quad (17)$$

We can now fix some  $\delta_1 \in (0, \delta^*)$  and select  $i$  so that  $\eta_i > \eta(\alpha_{\delta_1}, \beta_{\delta_1})$ . Then, since  $\eta(\alpha_\delta, \beta_\delta)$  is

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<sup>12</sup>Suppose otherwise; then since  $x_{\eta_i} > x_{\eta_j}$ , an agent with merit  $\eta_j$  would prefer  $(x_{\eta_i}, q_{\eta_i})$  over her assignment.



continuous and strictly decreasing in  $\delta$ , there exists some  $\delta_2 \in (0, \delta_1)$  such that  $\eta_i = \eta(\alpha_{\delta_2}, \beta_{\delta_2})$ . Moreover, the LHS of the latter inequality in (17) is strictly decreasing in  $\delta$ , so:

$$v(\beta_{\delta_2}, x_{\eta^*}) - v(\beta_{\delta_2}, x_{\eta(\alpha_{\delta_2}, \beta_{\delta_2})}) > z(\alpha_{\delta_2}, q_{\eta^*}) - z(\alpha_{\delta_2}, q_{\eta(\alpha_{\delta_2}, \beta_{\delta_2})}).$$

But this is equivalent to  $U[\alpha_{\delta_2}, \beta_{\delta_2}; \eta^*] > U[\alpha_{\delta_2}, \beta_{\delta_2}; \eta(\alpha_{\delta_2}, \beta_{\delta_2})]$  which contradicts revealed preference. Therefore, Condition 8 has to hold in Case 2.

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