RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – November/December 2016

MAA 1203 – Numerical Analysis I

Time: Two (2) hours

Answer **Four** Questions **only**.

Calculators will be **provided.**

Newton's Forward formula

$$y_k = \sum_{i=0}^{n} \binom{k}{i} \Delta^i y_0$$

Newton's Backward formula

$$y_k = y_0 + k\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \frac{k(k+1)(k+2}{3!}\nabla^3 y_0 + \ldots + \frac{k(k+1)\ldots(k+n-1)}{n!}\nabla^n y_0$$

Gauss's Backward formula

$$y_k = y_0 + \sum_{i=1}^{n}\left[\binom{k+i-1}{2i-1}\Delta^{2i-1} y_{-i} + \binom{k+i}{2i}\Delta^{2i} y_{-i}\right]$$

Stirling's formula

$$y_k = y_0 + \frac{k(\Delta y_{-1} + \Delta y_0)}{2} + \frac{k^2}{2!}\Delta^2 y_{-1} + \frac{k(k^2-1)}{3!} \times \frac{(\Delta^3 y_{-1} + \Delta^3 y_{-2})}{2} + \ldots$$

$$+ \frac{k(k^2-1)\ldots(k^2-(n-1)^2)}{(2n-1)!} \times \frac{(\Delta^{2n-1} y_{-(n-1)} + \Delta^{2n-1} y_{-n})}{2} + \frac{k^2(k^2-1)\ldots(k^2-(n-1)^2)}{(2n)!}\Delta^{2n} y_{-n}$$

01

a) Prove that for any positive integer k, **(50 marks)**

$$\Delta^k y_0 = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{k-i}$$

Where the familiar symbol,

$$\binom{k}{i} = \frac{k!}{i!(k-i)!}$$

b) Find a real root of the non linear equation $x^3 - 9x + 1 = 0$ on the interval [2, 4], correct to two significant figures using Bisection method.

(50 marks)

02

a) Briefly explain the steps of Newton Raphson method to solve system of m non-linear equations with n unknowns. **(20 marks)**

b) Solve the following system of non-linear equations using Newton's Raphson Method by assuming the initial estimation as $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (Proceed only two iterations)

(80 marks)

$$4x^2 - y^3 + 28 = 0$$
$$3x^3 + 4y^2 - 145 = 0$$

03

a) Briefly explain the applicability of Newton's Backward formula in interpolation.

(10 marks)

b) The following table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

Height	100	150	200	250	300	350	400
Distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

i. Formulate a cubic polynomial in terms of k to represent the above data set bv using suitable interpolation formula. **(40 marks)**

ii. Rewrite the above polynomial in terms of x by using the relationship between x and k. **(40 marks)**

iii. Use the cubic polynomial of part ii, to extrapolate the distances in nautical miles of the visible horizon when height equals to 410 feet. **(10 marks)**

04

a) Briefly explain the formulation of the Ramanujan's Method for solving non-linear equation.
(20 marks)

b) Obtain a root, correct to two decimal places for the non-linear equation

$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - .... = 0$ using Ramanujan's Method. **(40 marks)**

c) Use second order Runge-kutta method with $h = 0.2$ to find the value of y at $x = 0.2$ and $x = 0.4$, given that $\frac{dy}{dx} = 1 + y^2, y(0) = 0$. **(40 marks)**

05

a) Prove the Gauss's Backward formula by using Newton's Forward formula.
(40 marks)

b) Consider the following tabula data,

x	0	5	10	15	20
$y(x)$	7	11	14	18	24

i. Use Gauss's Backward formula to find $y(8)$ for the above data. **(50 marks)**

ii. What will be the solution, using Stirling's formula for $y(8)$. (Justify your answer) **(10 marks)**

06

a) Write down an algorithm for solving first order ordinary differential equations by using Taylor's series method.
(20 marks)

b) Find $y(1.1)$ for $\frac{dy}{dx} = x^2 + y^2, y(1) = 2.3$ using Taylor's series method of order three.
(40 marks)

c) The following table summarizes marks of 190 students for mathematics of a particular class. Use suitable interpolation formula to estimate the number of students who obtained marks between 40 and 45. (Consider up to 3rd difference operator)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of Students	31	42	51	35	31

(40 marks)

END