RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences

Third Year -Semester II Examination – July 2020

MAT 3310 – INTEGER PROGRAMMING

Time: Three (3) hours

Answer All Questions.

Calculators will be provided.

1. i. What are the main components of integer programming? **(10 marks)**

   ii. What are the three types of integer programming models? **(10 marks)**

   iii. Select the correct answer to the following multiple-choice questions from A to D.

   A. If the optimal solution to the linear programming relaxation problem is integer, then it is ..... to the integer programming problem.

      a) a real solution
      b) a degenerate solution
      c) an infeasible solution
      d) the optimal solution
      e) a feasible solution

B. The 0-1 integer programming problem

a) requires the decision variables to have values of 0 or 1.

b) requires that all the constraints have RHS values of 0 or 1.

c) requires that the decision variable have coefficients of 0 or 1.

d) includes the transportation type of problems.

C. An integer solution can never produce a greater profit than the linear programming solution to the same problem.

a) True

b) False

D. In a mixed integer programming problem,

a) some integers must be even and others must be odd.

b) some decision variables must only require integer results and some variables must allow for continuous results.

c) different objectives are mixed together even though they sometimes have relative priorities established.

(20 marks)

iv. Why we use cut plan (cut) in an integer programming problem? **(10 marks)**

v. What are the main four types of cutting plan algorithms? **(10 marks)**

vi. What are the main two conditions of the canonical form? **(10 marks)**

vii. What is the best branch and bound method to get an integer solution? **(05 marks)**

viii. Write down two uses of LINGO software. **(05 marks)**

ix. What are the limitations of lingo software? **(10 marks)**

x. Integer variables in LINGO can be either 0/1 or general. What are the specifications you have to enter to the LINGO window to identify these variables separately? **(10 marks)**

2. World renowned ice cream entrepreneur Sydney and Eden produce two types of premium dairy ice-cream products: Sydney and Eden's Chocolate Concussion and Vanilla Brain Freeze. Their chocolate ice-cream requires 6 oz milk and 8 oz of peanuts per pint size container while the vanilla option requires 9 oz milk and 5 oz peanuts. Sydney and Eden currently enjoys a surplus of all other ingredients required for their ice-cream but only have 360 oz of milk and 400 oz of peanuts for this limited production to run. The entrepreneur charges $5 for each container of Chocolate Concussion and $7 for each Vanilla Brain Freeze. Sydney and Eden require to maximize their profits.

(Note: oz is an abbreviation of ounces)

a) Formulate an integer linear programming model for the above problem. **(20 marks)**

b) Determine the Integer solution to the above problem using **Dual Fractional Cutting Plan Algorithm.** **(50 marks)**

c) Verify the results of **part (b)**, using **graphical** method. **(30 marks)**

3. Consider the following integer programming problem.

Minimize $x_0 = 10x_1 + 14x_2 + 21x_3$

Subject to,

$$4x_1 + \frac{11}{2}x_2 + \frac{9}{2}x_3 \geq 6$$

$$x_1 + x_2 + \frac{7}{2}x_3 \geq 7$$

$$3x_1 + 2x_2 + x_3 \geq \frac{10}{3}$$

$x_1, x_2, x_3 \geq 0$ and integer.

a) Solve the problem using **Dual All Integer – Integer Programming Algorithm** to find the x_1, x_2 and x_3. **(70 marks)**

b) Express each additional constraints in terms of original variables. **(30 marks)**

4. i. Branch and bound is a.....

   a) Problem solving technique
   b) Data structure
   c) Sorting algorithm
   d) Type of tree **(15 marks)**

   ii. A company produces two types of cookies. Chocolate Chip and Oatmeal Raisin require cocoa and oatmeal. Each piece of Chocolate Chip requires 12 grams of cocoa and 4 grams of oatmeal while each piece of Oatmeal Raisin requires 4 gram of cocoa and 8 gram of oatmeal. The company can purchase 36 grams of coca and 24 grams of oatmeal. Each piece of Chocolate Chip brings a profit of 160 dollars and each piece of Oatmeal Raisin brings a profit of 200 dollars.

   a. Formulate an integer programming model to determine the number of pieces of each type of cookie that should be produced to maximize the profit of the company. **(25 marks)**

   b. Use an appropriate **Branch and Bound method** to solve the above model. **(60 marks)**

5. Solve the following integer programming problem using **Dual Fractional Mixed Integer Programming Algorithm.**

   maximize $z = -5x_1 - x_2 - x_3 - 2x_4 - 3x_5$

   subject to,

   $$-x_2 + 5x_3 - x_4 - 2x_5 \leq 2$$
   $$-5x_1 + x_2 - x_5 \leq -7$$
   $$-x_1 - x_2 - 6x_3 - x_4 \leq -4$$

   $x_1, x_2, x_3, x_4, x_5 \geq 0$ and x_1, x_2 are integers.

   (100 marks)

6. A manufacturer of animal feed who is producing feed mix for dairy cattle. The feed mix contains two active ingredients and a filler to provide bulk. One kg of feed mix must contain a minimum number of teaspoons of each of four nutrients as below:

Nutrient	A	B	C	D
Gram	90	50	20	2

The ingredients have the following nutrient values and cost:

	A	B	C	D	Cost/kg
Ingredient 1 (gram/kg)	100	80	40	10	40
Ingredient 2 (gram/kg)	200	150	20	-	60

The additional conditions are:

- If you use any of the two ingredients, we incur a fixed cost of 15.
- You need not satisfy all four nutrient constraints, but only need to satisfy three of them.

a) Formulate an integer programming model to find the amount of active ingredients and filler in one kg of feed mix. **(50 marks)**

b) Explain how you would solve the problem in part (a), using LINGO (Write down steps with expressions). **(50 marks)**

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