



RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

Second Year-Semester I Examination-October/November 2014

MAP 2203 – DIFFERENTIAL EQUATIONS II

Answer **ALL** Questions.

Time Allowed: **Two hours.**

1.

- i. Solve the equation $2y'' + xy' + y = 0$ about the ordinary point $x = 0$
- ii. Find a series solution for the differential equation

$$xy''(x) + 4y'(x) - xy(x) = 0, \quad x > 0 \text{ about the point } x = 0$$

2.

- a) Find the general solution of the system of differential equations $\dot{x} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -2 & 0 & -1 \end{pmatrix} x$.

- b) Find all solutions of $\dot{x} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} x$.

3.

- a) Form a partial differential equation by eliminating arbitrary constants a and b from the following equations :

- i. $z = a(x + y) + b$

- ii. $z = axe^y + (1/2)a^2e^y + b$

- b) Form a partial differential equation by eliminating the arbitrary functions f and F from the following equations:

- i. $z = f(x + iy) + F(x - iy)$ where $i^2 = -1$

ii. $y = f(x - at) + F(x + at)$

c) Solve the following partial differential equations,

i. $p \tan x + q \tan y = \tan z$

ii. $xyp + y^2q = zxy - 2x^2$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$,

4.

i. Discuss the Frobenius method for solving a second order linear differential equation given, with the usual notations, as

$$a_2(x) y'' + a_1(x) y' + a_0(x) y = 0$$

ii. Show that the differential equation

$$x^2 y'' - 2(x + x^2) y' + (x^2 + 2x + 2) y = 0 \text{ has a regular singular point at the origin.}$$

iii. Find the first three terms of the Frobenius solution around $x=0$ for

$$x^2 y'' + e^x y = 0$$