RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES
B.Sc. (General) Degree

Third Year - Semester I Examination - February/March 2013

GRAPH THEORY – MAT 3213

Answer four questions including question No.01. **Time: Two (02) hours**

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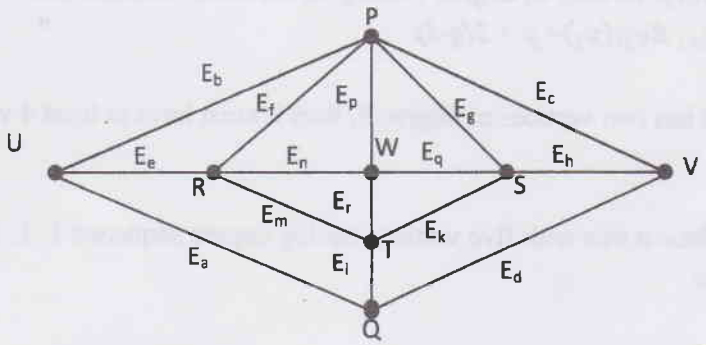
1. a) Find each of the following:



i) A U-V walk that is not a trail.
ii) A U-V trail that is not a path.
iii) A U-U circuit that is not a cycle.
iv) A U-V path of length 5.
v) A U-U cycle of length 8.

b). How many vertices and edges do each of the following graphs have?

i). K_n ii). $K_{m,n}$ iii). C_n iv). Q_n v). *Peterson graph*

c). Give an example for each of the following graphs:

i) A **complete graph** that is a **wheel**

ii) A **bipartite graph** that is a **regular graph** of degree 5

d). A graph G has the **adjacency** matrix

$$A=\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

i) Is G a simple graph?
ii) What is the degree sequence of G?
iii) How many edges does G have?

2). i) For every tree $T = (V, E)$, show that $|V| = |E| + 1$

ii) Let T be a tree with p vertices of degree 1 and q be the other vertices. Show that
$\sum_{deg(v_i)>1} deg(v_i) = p + 2(q-1)$

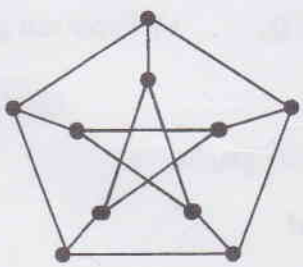
iii) Show that, if a tree has two vertices of degree 3, then it must have at least 4 vertices of degree 1.

iv) Is it possible to a draw a tree with five vertices having degree sequence 1, 1, 2, 2, 4 ? Justify your answer.

3). a) State and prove **Euler's theorem**.

b) If a **planar graph** G of order n and size m has r regions and k components , then show that $n - m + r = k + l$

c) Show that the graph G is non planar.

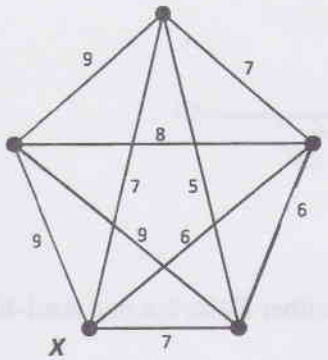


G

d) Draw all planar graphs with 5 vertices which are not isomorphic to each other.

4).

i) How many **Hamiltonian cycles** are there in the following graph?

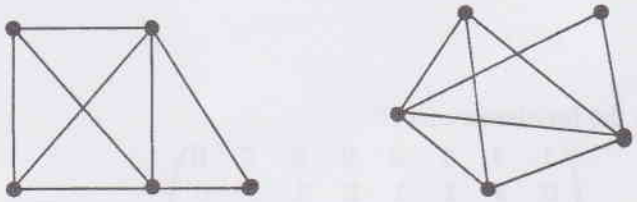


ii) Delete the vertex labeled X. How many **spanning trees** are there on the remaining sub graph? How many of these **spanning trees** are part of a **Hamiltonian cycle**?

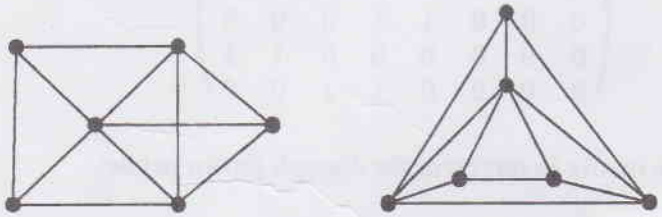
iii) Find a minimum weight **spanning tree** on the remaining sub graph. Hence, show that the weight (or length) of a solution to the **travelling salesman** problem is at least 32.

5). a) Determine whether the following pair of graphs are **isomorphic or not**. Justify your answer.

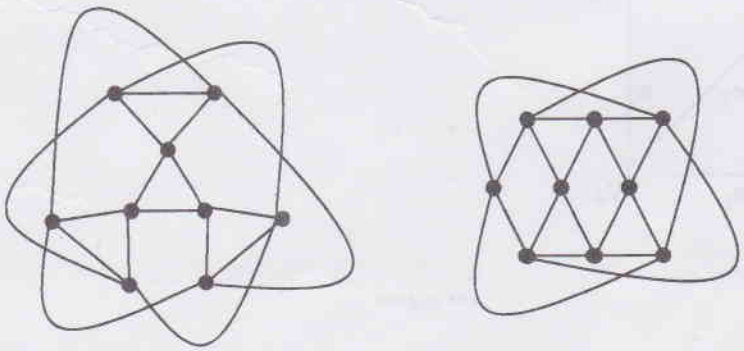
i)



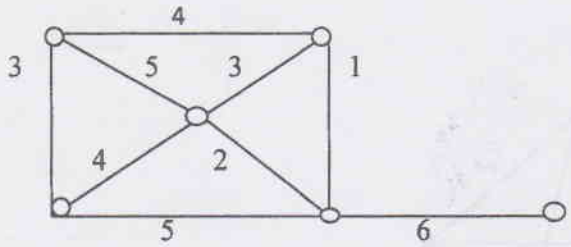
ii)



iii)

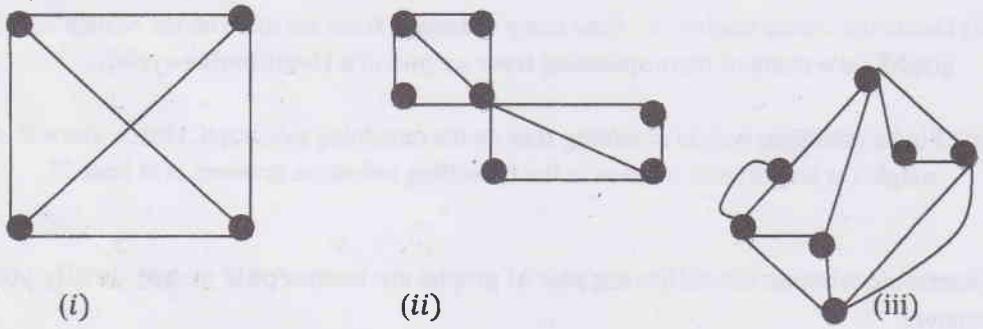


b) Use **Kruskal's algorithm** to find all minimum weight spanning trees for this weighted graph.



6). a) Prove that the **Peterson graph** is neither **Eulerian** nor **semi-Eulerian.**

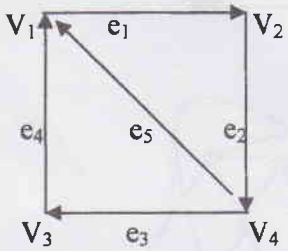
b) Which of the following graphs in figure (*i*) , (*ii*) and (*iii*) have an **Euler circuit, an Euler path** or both.



c) Draw a graph with the **incidence** matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

d) Find the **incidence** matrix to represent the digraph shown below:



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