RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (Four Year) Degree
In Industrial Mathematics
Fourth Year Semester II Examination – September/October 2014

MAT 4301 – OPERATIONS RESEARCH I

Answer ALL questions **Time Allowed: 3 hours**

---

1. (i) Using dominance property, reduce the following payoff matrix of a game and hence solve it using graphical method:

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 18 & 4 & 6 & 4 \\ 6 & 2 & 13 & 7 \\ 11 & 5 & 17 & 3 \\ 7 & 6 & 12 & 2 \end{bmatrix} \end{array}$$

(ii) Sole the following game using Linear Programming method:

$$\begin{array}{c} \\ \text{Player I} \end{array} \begin{array}{c} \text{Player II} \\ \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \end{array}$$

2. Four jobs A, B, C and D are to be processed on each of the five machines M_1, M_2, M_3 and M_4 in the order $M_1M_2M_3M_4$. Find the total elapsed time if no passing of jobs is permitted. Also find the idle time for each machine.

Machines

Jobs	M_1	M_2	M_3	M_4
A	24	7	7	29
B	16	9	5	15
C	22	8	6	14
D	21	6	8	32

[Turn over

3. Consider the following queuing model:
Poisson arrival, Poisson departure, Single server, Infinite capacity and First come first served discipline.

With the usual notation prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ when the system is in steady state.

Also, prove that $P_n = \rho^n(1-\rho)$.

At a one-man barber shop, customers arrive according to Poisson distribution with a mean arrival rate of 5 per hour and the haircutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate the following:

(i) Average number of customers in the shop and the average numbers waiting for a haircut.

(ii) The percent of time, arrival can walk in right without having to wait.

(iii) The percentage of customers who have to wait before getting into the barber's chair.

4. Consider the following queuing model:
Poisson arrival, Poisson departure, Single server, First comes first served discipline and the maximum number of customers is limited to N.

With the usual notation prove that $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ when the system is in steady state and

$P_n = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n$, for $n = 0,1,2,\ldots,N$.

In a railway marshalling yard, goods trains arrive at the rate of 30 trains per day. Assume that the inter arrival time follows an exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate

(i) The probability that the yard is empty.

(ii) The average queue length assuming that the line capacity of the yard is 9 trains.

5. A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that each barber serves customers according to an exponential distribution with mean 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. What is the expected number of customers in the shop?