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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

First year – Semester I Examination – September / October 2013

MAA 1201 Mathematical Methods I

Time allowed: **Two** hours.

Answer **FOUR** Questions selecting **Question No. 01** and **three** of the remaining questions.

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01. (i). By vector methods, find the perimeter of a triangle whose vertices are the points $(3, 1, 5)$, $(-1, -1, 9)$ and $(0, -5, 1)$. **[25 Marks]**

(ii). Find the value of α such that the vectors $2\underline{i}-\underline{j}+\underline{k}$, $\underline{i}+2\underline{j}-3\underline{k}$ and $3\underline{i}+\alpha\underline{j}+5\underline{k}$ are coplanar. **[25 Marks]**

(iii). Find the vector equation of the plane through the three points $(-1, 1, 2)$, $(1, -2, 1)$ and $(2, 2, 4)$ in terms of $\underline{i}$, $\underline{j}$, $\underline{k}$ and two parameters. **[25 Marks]**

(iv). Show by vector methods that the triangle whose vertices are $A(2, -1, 1)$, $B(1, -3, -5)$ and $C(3, -4, -4)$ is right-angled. **[25 Marks]**

(v). If $\underline{a}\times\underline{b}=\underline{c}\times\underline{d}$ and $\underline{a}\times\underline{c}=\underline{b}\times\underline{d}$, show that $\underline{a}-\underline{d}$ and $\underline{b}-\underline{c}$ are parallel vectors. **[25 Marks]**

(vi). Find the area of the triangle whose vertices are the points with rectangular Cartesian co-ordinates $(1, 2, 3)$, $(-2, 1, -4)$ and $(3, 4, -2)$. **[25 Marks]**

(vii). Show that $\underline{i}\times(\underline{a}\times\underline{i})+\underline{j}\times(\underline{a}\times\underline{j})+\underline{k}\times(\underline{a}\times\underline{k})=2\underline{a}$. **[25 Marks]**

(viii). A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+5$, where t is the time. Using the vector method find the components of its velocity and acceleration at $t=1$. **[25 Marks]**

Turn Over

(ix). Show that the vector $\underline{r} = (\sin y + z)\underline{i} + (x\cos y - z)\underline{j} + (x - y)\underline{k}$ is irrotational.

[25 Marks]

(x). Determine the constant "c" so that the vector $\underline{f} = (x + 3y)\underline{i} + (y - 2z)\underline{j} + (x + cz)\underline{k}$ is solenoidal. **[25 Marks]**

02. (a).(i). Let $\underline{a} = \underline{i}$, $\underline{b} = \underline{i} + \underline{j}$ and $\underline{c} = 2\underline{i} + \underline{j} + 3\underline{k}$. Show that the set of vectors $\{\underline{a}, \underline{b}, \underline{c}\}$ is linearly dependent.

Is the set of vectors $\{\underline{a}, \underline{b}, \underline{c}\}$ a basis? Justify your answer. **[75 Marks]**

(ii). Show that the points A, B, C with position vectors $-2\underline{i} + 3\underline{j} + 5\underline{k}$, $\underline{i} + 2\underline{j} + 3\underline{k}$ and $7\underline{i} - \underline{k}$ are collinear. **[25 Marks]**

(b).(i). OB and OC are two lines and D is a point on BC such that $\dfrac{BD}{DC} = \dfrac{m}{n}$. Show that

$\overrightarrow{OD} = \dfrac{n\,\overrightarrow{OB} + m\,\overrightarrow{OC}}{m+n}$. **[50 Marks]**

(ii). OAB is a given triangle. X is a point on OA such that $\overrightarrow{OX} = 2\overrightarrow{OA}$ and Y is a point on OB, such that $\overrightarrow{OY} = \dfrac{2}{3}\overrightarrow{OB}$. The line XY meets the side AB of the triangle at P. Find the ratio in which P divides AB. Find also the ratio in which P divides YX.

[100 Marks]

03. (a). Define the *gradient* of a function $\phi(x, y, z)$, where ϕ is a continuously differentiable function of x, y, z. **[10 Marks]**

Show that $grad\,\phi = \underline{0}$ if and only if ϕ is a constant. **[50 Marks]**

(b). Define the *divergence* for a differentiable vector valued function. **[10 Marks]**

Show that,

(i) $div(\phi\underline{F}) = \phi\, div\underline{F} + grad\phi \bullet \underline{F}$, **[30 Marks]**

(ii) $div(grad\,\phi) = \nabla^2\phi$, **[30 Marks]**

where $\underline{F}$ is a vector field and ϕ is a scalar field.

(c). Let $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ and $|\underline{r}| = r$.

Show that $\frac{1}{r}$ is a solution of the Laplace equation, $\nabla^2\phi = 0$. **[45 Marks]**

(d). Show that, $curl\ (\phi \underline{F}) = \phi\ curl\, \underline{F}\ +\ grad\phi \times \underline{F}$, where $\underline{F}$ is a vector field and ϕ is a scalar field. **[30 Marks]**

Using the above result, find $curl\ (r^n \underline{r})$, where $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ and $|\underline{r}| = r$.

[45 Marks]

04. (a). Define the *scalar product* of two vectors $\underline{a}$ and $\underline{b}$. **[20 Marks]**

Using the above product, show that the angle subtended at a point on a semi-circle by two ends of a diameter is a right-angle. **[40 Marks]**

(b). Find the reciprocal set of vectors $\{\underline{a}', \underline{b}', \underline{c}'\}$ and verify that

$$[\underline{a}, \underline{b}, \underline{c}][\underline{a}', \underline{b}', \underline{c}'] = 1$$ **[95 Marks]**

(c). Find the vector $\underline{x}$ and the scalar λ which satisfy the equations:

$$\underline{a} \times \underline{x} = \underline{b} + \lambda \underline{a}$$
$$\underline{a} . \underline{x} = 2 \qquad ;\text{where } \underline{a} = \underline{i} + 2\underline{j} - \underline{k}\ ,\ \underline{b} = 2\underline{i} - \underline{j} + \underline{k}$$ **[95 Marks]**

05. Let C be the curve given by, $\underline{r}(\theta) = 2\theta\underline{i} + \cos\theta\,\underline{j} + \sin\theta\,\underline{k}$, where θ is a parameter.

(i). Find the unit tangent vector and the unit principal normal vector at the point with parameter θ. **[120Marks]**

(ii). Let l be the tangent line to C at the point $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Find the equation of l.

[80 Marks]

(iii). Find the curvature of the curve. **[50 Marks]**