RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Third Year - Semester I Examination – July / August 2023

PHY 3209 – SOLID STATE PHYSICS

Time: 2 hours

Answer four (04) questions only.

Symbols have their usual meaning.
Use of a calculator is permitted.

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1. a) Explain how the Miller indices of planes and directions in a crystal are determined. (15 marks)

   b) Calculate the "packing fraction" for simple cubic (SC), body centered cubic (BCC) and face centered cubic (FCC) unit cells. (Assume that the atoms are rigid spheres touching each other) (15 marks)

   c) Explain the concept of planar density in crystals. Show that the planar density in (110) plane in BCC lattice is $1.414/a^2$, where a is the length of the edges of the cubic unit cell. (10 marks)

   d) Explain the concept of linear density in crystals. Show that the linear density in [110] direction in FCC lattice is $1.414/a$, where a is the length of the edges of the cubic unit cell. (10 marks)

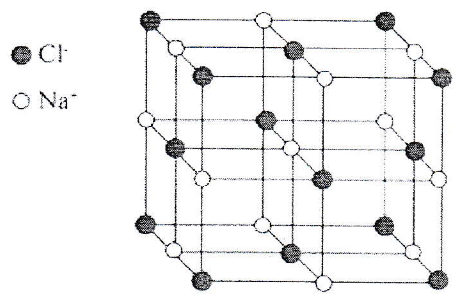
2. a) Define the terms space lattice, basis and crystal structure and explain the relationship among them. (10 marks)

   b) Discuss the five 2D space lattices (Bravais lattices). (15 marks)

   c) Show that the "Centered square space lattice" (which does not exist in reality) is same as "Square space lattice". (15 marks)

   d) "*Hexagonal structure is a space lattice whereas honeycomb structure is not*" Substantiate the above statement. (10 marks)

3. a) $NaCl$ crystallizes in face centered cubic (FCC) lattice with two – atom basis (one Na atom and one Cl atom) with the lattice constant of 5.64×10^{-10} m (see the figure below)



FCC Structure of NaCl

i. Show that the above unit cell consists of eight atoms (four Na atoms and four Cl atoms). (10 marks)

ii. Calculate the theoretical density of $NaCl$. (Atomic mass of Na, atomic mass of Cl and Avogadro's Number are 22.99 g mol^{-1}, 35.45 g mol^{-1} and 6.023×10^{23} mol^{-1} respectively) (15 marks)

b) Define the following

i. Wigner – Seitz cell (05 marks)
ii. Bragg planes (05 marks)
iii. Brillouin zone (05 marks)

c) Draw the first and the second Brillouin zones of the square lattice. (10 marks)

4. The electron theory of solids aims to explain the electrical, thermal and magnetic properties of solids and has developed in three stages; classical free electron theory, quantum free electron theory and the band theory of solids.

a) i. State three drawbacks of classical free electron theory which lead to the development of the band theory of solids. (06 marks)

ii. Describe using a rough sketch the main assumptions given by Sommerfield in quantum free electron theory. (12 marks)

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b) Krönig and Penney suggested a simple model to obtain mathematical solution that confirms energy band formation in crystals.

i. Show that the permitted energies for the Delta-function potential with $P \to \infty$ in the Krönig-Penney model equal the energies of the electron in a box with impenetrable walls. Note: For the Delta-function potential, Krönig-Penney equation is,

$$\frac{P\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka, \text{ where } \alpha^2 = \frac{2m_e E}{\hbar^2}.$$ (10 marks)

ii. Describe " P " in the above Krönig-Penney equation. (08 marks)

iii. Explain the concept of forbidden energy bands with the use of Krönig- Penny model. (14 marks)

5. In Solid State Physics, the dynamics of the electron in the crystal lattice are fully contained in the E-k diagram. The electron's wave function group velocity and the effective mass can be extracted from the slope and concavity of the plot of E vs k.

a) i. Show that the curvature of the energy band in which the electron moves is inversely proportional to the mass of the electron. (10 marks)

ii. Draw band energy $E(k)$ for electrons in a crystalline solid in the first Brillouin zone (i.e. E vs k graph). (08 marks)

iii. Show graphically the dependence of electron wave function group velocity v_g and the effective mass m^* on the band energy $E(k)$ within the first Brillouin zone. (14 marks)

b) i. The electron energies in a system can be fitted to the expression $E(k) = ak^2 - bk^4$, where a and b are constants for that system. Find the group and phase velocities in the system. (10 marks)

ii. What are the values of k in terms of a and b at which the two types of velocities are equal? (08 marks)

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