RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Fourth Year - Semester I Examination – September / October 2019

PHY 4312 – STATISTICAL THERMODYNAMICS

Time: Three (03) hours

- **Answer all questions**
- A non-programmable calculator is permitted.

Values of constants

speed of light in a vacuum	$c = 3.00 \times 10^{8}$ m s^{-1}
electron charge	$e = 1.60 \times 10^{-19}\ C$
Plank constant	$h = 6.63 \times 10^{-34}$ J s
mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Boltzmann constant	$k_B = 1.381 \times 10^{-23}$ J K^{-1}
electron volt	$1\text{ eV} = 1.60 \times 10^{-19}$ J
Atomic mass unit	$1\text{ U} = 931.5$ M eV

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1. a) Using *Stirling's approximation*

$$\ln n! = n \ln n - n + \frac{1}{2} \ln 2\pi n$$

Derive the formula for $n!$

(02 marks)

b) The binomial expansion is given as;

$$P_N(n) = \frac{N!}{n!\,(N-n)!} p^n q^{N-n}$$

Using Stirling's formula, calculate the probability of getting exactly 500 heads and 500 tails when flipping 1000 coins.

(04 marks)

Continued…

c) The Gaussian form of the probability distribution, $P_N(n)$ can be derived as,

$$\ln P(n) = \ln \frac{N!}{n!\,(N-n)!} p^n q^{N-n} = \ln N! - \ln n! - \ln(N-n)! + n \ln p + (N-n) \ln q$$

i. Rewrite this expression, expanding all the factorials on the right-hand side using Stirling's approximation. **(04 marks)**

ii. Show that,

$$\ln P(n = \bar{n}) = \frac{1}{2} \ln \frac{1}{2\pi N p q} = \frac{1}{2} \ln \frac{1}{2\pi \sigma^2},$$

using $\bar{n} = Np$, $q = 1 - p$, $\ln ab = \ln a + \ln b$, and $\sigma = \sqrt{Npq}$. **(05 marks)**

2. Consider a small system that is interacting with a reservoir at temperature 400 K, pressure 10^8 Pa, and chemical potential -0.3 eV.

a) The small system must take an additional 0.03 eV of energy and 10^{-29} m^3 of volume from the reservoir to go from state 1 to state 2. How many times more probable is it for the small system to be in state 1 than in state 2? **(05 marks)**

b) The small system must take 0.4 eV of energy and one particle (but no extra volume) from the reservoir to go from state 1 to state 3. How many times more probable is it for the small system to be in state 1 than in state 3? **(05 marks)**

c) The small system must take no energy but one particle and $10^{-27} m^3$ of volume from the reservoir to go from state 1 to state 4. How many times more probable is it that the small system is in state 1 than in state 4? **(05 marks)**

3. Consider the photons inside an oven at 500 K. Photons are bosons, so any number of photons may occupy one state. If the chemical potential of a photon is zero and the energy of a certain state is 0.2 eV, find

a) The factor $\beta(\epsilon - \mu)$ for this state, **(03 marks)**

b) The constant C in the formula $P_n = Ce^{-n\beta(\epsilon-\mu)}$, accurate to three decimal places, **(04 marks)**

Continued...

c) The probability of there being no photons in this state at any particular moment,
(04 marks)

d) The probability of there being two photons in this state at any particular moment.
(04 marks)

4. For a certain molecule in a system at 500 K, the energies of the various quantum states, measured relative to the ground state, are given by $\varepsilon = n(0.1 \text{ eV}), n = 0, 1, 2, \ldots$.

a) To three significant figures, what is the value of the constant C in the formula $P_s = Ce^{-\beta\varepsilon_s}$? **(05 marks)**

b) What is the probability that the molecule is in the level $n = 1$? **(05 marks)**

c) What is the probability that it is in the level $n = 2$? **(05 marks)**

$(1 + x + x^2 + x^3 + \ldots = 1/(1 - x))$

5. a) Define the *partition function* for a small system interacting with a large reservoir.
(03 marks)

b) Show that $\overline{E^2} = (1/Z)(\partial^2 Z/\partial\beta^2)_{V,N}$. **(04 marks)**

c) Show that the square of the standard deviation for the energy of a system interacting with a reservoir, $\sigma^2 = \overline{E^2} - \bar{E}^2$, is given by $\sigma^2 = (\partial^2 \ln Z/\partial\beta^2)_{V,N}$
(04 marks)

d) Using $\bar{E} = -\partial \ln Z/\partial\beta$, prove that $\bar{F} = -kT \ln Z$ is a solution to the differential equation $\bar{F} - T(\partial\bar{F}/\partial T)_{V,N} = \bar{E}$. **(04 marks)**

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