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RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

Second Year-Semester II Examination-March/April 2014

MAP 2202 - REAL ANALYSIS II

Answer **FOUR** Questions Only TimeAllowed: **Two hours**

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1. (a) Determine whether the following series are convergent or divergent. Specify the test you use in each part.

   i. $\sum_{n=0}^{\infty}\left(\frac{n^2+1}{2n^2-1}\right)^n$

   ii. $\sum_{n=0}^{\infty}\frac{2^{3n}}{3^{2n-1}}$

   iii. $\sum_{n=0}^{\infty} ne^{-n^2}$

   iv. $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$

   v. $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$

(b) Find the radius of convergence and interval of convergence of the series:

i. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n^3}$

ii. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n^3}$

If k is a positive integer, find the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(n!)^k}{(kn)!}x^n$.

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2. (a) Show that a constant function is Riemann integrable.

   (b) If $f(x)=x^3$ is defined on $[0,a]$, show that $\int_0^a f(x)dx=\frac{a^4}{4}$.

3. (a) Determine if the following limits exist or not. If they do exist give the value of the limit.

   i. $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$

   ii. $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

   iii. $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2}$

   iv. $\lim_{(x,y)\to(1,2)} \frac{5x^2y}{x^2+y^2}$

   (b) Determine whether the function is continuous at the stated point.

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+xy+y^2} & ;\ if\ (x,y)\neq(0,0)\\ 0 & ;\ if\ (x,y)=(0,0)\end{cases}$$

4. (a) Find the maxima and minima of the function $z=x^2+xy+y^2-y$.

   (b) Find the minimum and maximum of the function $f(x,y,z)=x^2-y^2+2z^2$ on the surface of the sphere defined by the equation $x^2+y^2+z^2=1$.

   (c) A jewel box is to be constructed of material that costs \$1 per square inch for the bottom, \$2 per square inch for the sides, and \$5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction?

5. Assuming the validity of differentiation under the integral sign,

   i. Show that $\int_0^{\pi/2} \frac{\log(1+\cos\alpha \sin x)}{\cos x} dx = \frac{1}{2}\left(\frac{\pi^2}{4} - \alpha\right)$

   ii. Given $\int_0^x \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$. Show that

   $$\int_0^x \frac{dx}{\left(x^2+a^2\right)^2} = \frac{1}{2a^3}\tan^{-1}\frac{x}{a} + \frac{x}{2a^2(x^2+a^2)}$$

6. (a) Evaluate the integral $\iint_R (x-y^2)dxdy$ over the region $R = \{(x,y) | 2 \le x \le 3, 1 \le y \le 2\}$.

   (b) Evaluate the integral $\iint_R (x^2+y^2)dxdy$, where R is bounded by the lines

   $y = x,\ y = x+a,\ y = a,\ y = 2a\ \ (a > 0)$.