RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

First Year Semester I Examination – May/June 2016

MAP 1301 – Linear Algebra

Answer **all** Questions. Time allowed: **Three Hours**

1.

(i). Show that

$$\left\{\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}-1\\2\\0\end{pmatrix}\right\}$$ is a linearly independent subset of $\mathbb{R}^3$.

(ii). Show that $\begin{pmatrix}3\\2\\0\end{pmatrix}$ is in the span of by finding c_1 and c_2 giving a linear relationship.

$$c_1\begin{pmatrix}1\\1\\0\end{pmatrix}+c_1\begin{pmatrix}-1\\2\\0\end{pmatrix}=\begin{pmatrix}3\\2\\0\end{pmatrix}.$$

Show that the pair c_1, c_2 is unique.

(iii). Show that this subset of $\mathbb{R}^3$

$\left\{\begin{pmatrix}a\\d\\g\end{pmatrix},\begin{pmatrix}b\\e\\h\end{pmatrix},\begin{pmatrix}c\\f\\i\end{pmatrix}\right\}$ is linearly independent iff

$$aei + bfg + cdh - hfa - idb - gec \neq 0.$$

(iv). Let $\boldsymbol{A} = \begin{pmatrix}0 & 3\\-1 & 2\end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix}-1 & 2 & 0\\3 & 1 & -2\end{pmatrix}$. For each of the products A^2, AB, BA, B^2 state whether or not it exists; if it exists then evaluate it.

(v). Prove or disprove that is a vector space under these operation.

$$\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}+\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \text{ and } r\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}rx\\ry\\rz\end{pmatrix}$$

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(vi). For each, decide if it is a vector space; the intended operations are the natural ones.This set of 2×2 matrices

$$\begin{pmatrix} x & x+y \\ x+y & y \end{pmatrix} \mid x, y \in \mathbb{R}.$$

(vii). Show that $\{a_0 + a_1x + a_2x^2 | a_0 + a_1 + a_2 = 0\}$ is a subspace of the vector space of degree two polynomials.

2.

(i). Let A be a column matrix and B a row matrix,

$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ and $\boldsymbol{B} = [\boldsymbol{b_1}\ \boldsymbol{b_2}\ \boldsymbol{b_3}\ \boldsymbol{b_4}]$ then their product $\boldsymbol{C} = \boldsymbol{AB}$ is a 4×4 matrix.

a) Can the matrix $\boldsymbol{C}$ be invertible?. Justify your answer; no marks will be given for answer "yes" or "no" if it is not supported by a clear explanation.

b) What are the possible values of the rank of the matrix $\boldsymbol{C}$? Justify your answer; no marks will be given for an answer not supported by clear explanation.

(ii). State the cayley-Hamilton Theorem.

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 \\ 5 & -1 & -2 \\ 3 & 1 & 1 \end{pmatrix}$,Find the charateristic polynomial of $\boldsymbol{A}$ and verify the cayleyHamilton Theorem.

3.

(i). Find $a \in \mathbb{R}$ such that the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & a \\ 0 & a & -a \end{pmatrix}$ is **Orthogonal**.

(ii). Orthogonally diagonalise the matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

that is find an orthogonal matrix $\boldsymbol{P}$ and a diagonal matrix $\boldsymbol{D}$. Such that $\boldsymbol{P^TAP = D}$.

(iii). Is the matrix $\boldsymbol{B} = \begin{pmatrix} 2 & 0 & 0 & 5 \\ 0 & -2 & 3 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ diagonalizable? Justify your answer.

4.

(i). Which of the following matrices (if any) are in row echelon form?

a) $\begin{pmatrix} 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

(ii). Consider the linear system.

$$3x_1 - 6x_2 + 3x_3 + 9x_4 = 3$$

$$2x_1 - 3x_2 + 3x_3 + 4x_4 = 4$$

$$-3x_1 + 7x_2 - 2x_3 - 10x_4 = -1$$

a) Write down the augmented matrix of the system.

b) Transform the augmented matrix to row echelon form. Indicate which elementary row operation you use at each step.

c) Identify the leading and the free variables, and write down the solution set of the system.

5.

(i). Consider the following subspace of $\mathbb{R}^4$:

$$S = Span\left(\begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \\ 3 \\ 9 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -2 \\ 5 \end{pmatrix} \right)$$

Find a basis for S.

a) Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1,1)$ and $(1,2)$respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = (2, -1)$.

b) Let V be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation

$$x + 2y + z = 0$$

Let W be the subspace of $\mathbb{R}^3$ spanned by $(1,1,1)$. Find vectors $v \in V$ and $w \in W$ so $v + w = (1,1,0)$.

c) Determine whether each of the following quadratic forms Q is positve definite.

i. $Q_1(x) = {x_1}^2 + 2{x_2}^2 - 4x_1x_2 - 4x_2x_3 + 7{x_3}^2$

ii. $Q_2(x) = {x_1}^2 + {x_2}^2 + 2x_1x_2 + 4x_2x_3 + 3{x_3}^2$