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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Joint Major) Degree in Chemistry & Physics

Fourth Year - Semester I Examination - March/April 2013

PHY 4210 – Advanced Quantum Mechanics

Answer all four questions — Time: Two hours

Unless otherwise specified, all the symbols have their usual meaning.

(1) (a) Briefly explain the situations where one can use approximation methods; the perturbation theory and the variation method, in quantum mechanics. [9 pts.]

(b) Use the variation method to find the ground state energy of a particle defined by:

$$U = \infty \quad \text{for } x < 0$$
$$U = U_0 x \quad \text{for } x > 0.$$

Hint: The trial wave function which satisfies the boundary conditions [$\psi(x) = 0$ for $x = 0$ and $x = \infty$] can be taken as $\psi(x) = Axe^{-\alpha x}$, where α is a variation parameter and $\int_0^\infty x^n e^{-\mu x} dx = n!\mu^{-n-1}$.

[16 pts.]

(2) (a) Prove the following operator identities;

(i) $[\mathbf{A}, \mathbf{BC}] = [\mathbf{A}, \mathbf{B}]\mathbf{C} + \mathbf{B}[\mathbf{A}, \mathbf{C}]$ [5 pts.]

(ii) $\mathbf{L}^{\dagger\dagger} = \mathbf{L}$. [5 pts.]

Contd.

(b) If **A** and **B** are Hermitian operators, then show that $\frac{1}{2}(\mathbf{AB} + \mathbf{BA})$ is also Hermitian. [7 pts.]

(c) Let **A** and **B** be two operators such that each of them commutes with the commutator [**A, B**]. Show, e.g., by induction, that $\left[\mathbf{A},\ \mathbf{B}^n\right] = n\mathbf{B}^{n-1}\left[\mathbf{A},\ \mathbf{B}\right]$.

[8 pts.]

(3) (a) A particle is confined to a cubic box of edge length L with one corner at the origin and edges lined up with the coordinate axes. Using, e.g., separation of variables, solve the energy eigenvalue equation for this system in the region inside the box to obtain appropriately normalized energy eigenfunctions and eigenvalues, assuming that the wave function satisfies periodic boundary conditions at the edges of the box, i.e., $\phi(L,y,z) = \phi(0,y,z)$, $\phi(x,L,z) = \phi(x,0,z)$, and $\phi(x,y,L) = \phi(x,y,0)$. [16 pts.]

(b) Are the energy eigenfunctions also eigenstates of momentum $\vec{P} = -i\hbar\vec{\nabla}$?

[9 pts.]

(4) (a) The Hamiltonian operator for the harmonic oscillator is $\mathbf{H} = \frac{\mathbf{P}^2}{2m} + \frac{1}{2}m\omega_0^2 x^2$ and the eigenvalue equation is $\mathbf{H}\psi_n = E_n\psi_n$. Show that the two operators **a** and $\mathbf{a}^\dagger$ defined by ;

$$\mathbf{a} = \left(\frac{m\omega_0}{2\hbar}\right)^{\frac{1}{2}}\mathbf{x} + i\left(\frac{1}{2m\hbar\omega_0}\right)^{\frac{1}{2}}\mathbf{p}\ , \qquad \mathbf{a}^\dagger = \left(\frac{m\omega_0}{2\hbar}\right)^{\frac{1}{2}}\mathbf{x} - i\left(\frac{1}{2m\hbar\omega_0}\right)^{\frac{1}{2}}\mathbf{p}$$ satisfy the

following commutation rules.

i. $[\mathbf{a}, \mathbf{a}^\dagger] = 1$ [5 pts.]

ii. $[\mathbf{a}^\dagger\mathbf{a}, \mathbf{a}^\dagger] = \mathbf{a}^\dagger$ [5 pts.]

iii. $[\mathbf{a}^\dagger\mathbf{a}, \mathbf{a}] = -\ \mathbf{a}$ [5 pts.]

Contd.

(b) Let $\mathbf{a}_1, \mathbf{a}_1^\dagger$ and $\mathbf{a}_2, \mathbf{a}_2^\dagger$ be annihilation and creation operators for two independent harmonic oscillators, so that $[\mathbf{a}_k, \mathbf{a}_m] = 0 = [\mathbf{a}_k^\dagger, \mathbf{a}_m^\dagger]$, and $[\mathbf{a}_k, \mathbf{a}_m^\dagger] = \delta_{km}, (k, m = 1,2)$. Let

$$\mathbf{J}_1 = \frac{1}{2}\left(\mathbf{a}_2^\dagger \mathbf{a}_1 + \mathbf{a}_1^\dagger \mathbf{a}_2\right), \quad \mathbf{J}_2 = \frac{i}{2}\left(\mathbf{a}_2^\dagger \mathbf{a}_1 - \mathbf{a}_1^\dagger \mathbf{a}_2\right), \text{ and } \mathbf{J}_3 = \frac{1}{2}\left(\mathbf{a}_1^\dagger \mathbf{a}_1 - \mathbf{a}_2^\dagger \mathbf{a}_2\right).$$

Show that

i. $[\mathbf{J}_1, \mathbf{J}_2] = i\mathbf{J}_3$ [5 pts.]

ii. $[\mathbf{J}_2, \mathbf{J}_3] = i\mathbf{J}_1$ [5 pts.]

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