

## RAJARATA UNIVERSITY OF SRI LANKA FACULTY OF APPLIED SCIENCES, MIHINTALE

## B. Sc. (Industrial Mathematics) Degree Fourth Year – Semester I Examination – April/May 2015 MAT 4309 – Combinatorics

## **Answer All Questions**

Time allowed: Two hours

1. (a) Determine which of the following are the solutions of the recurrence relation  $a_n = 8a_{n-1} - 16a_{n-2}$ .

(i) 
$$a_n = 2.4^n + 3n4^n$$
 (ii)  $a_n = n^2 4^n$  (iii)  $a_n = 2^n$ .

(b) Find the solution of recurrence relation

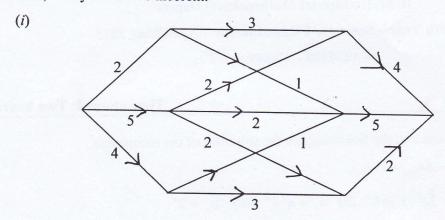
$$a_n = 7a_{n-2} + 6a_{n-3}, \quad a_0 = 9, a_1 = 10, a_2 = 32.$$

- (c) Find a recurrence relation for the number of bit strings of the length n that do not contain three consecutive 0 s. What are the initial conditions?How many bit strings are of length seven that do not contain three consecutive 0 s?
- 2. (a) Using Generating Functions, solve the recurrence relation  $a_n = 3a_{n-1}$  where  $a_0 = 2$ .
  - (b) State the Exclusion and Inclusion principle.
    - (i) How many integers not exceeding 1000 are divisible by 7 or 11?
    - (ii) Find the number of primes not exceeding 100.
  - (c) A survey of households in the United States reveals that 96% have at least one television, 98% have telephones and 95% have telephones and at least one television. What percentage of households in the United States have neither a telephone nor a television?
- 3. Define STS(n), a Steiner Triple System of order n.
  - (a) Construct STS(7).
  - (b) Construct a tournament schedule for eight teams.
  - (c) Using above (a) and (b) construct STS(15).

[P.T.O.

4. Define each of the following terms: *a Walk, a Trial, a Path* and *a Cycle*. State the Maximum Flow and Minimum Cut Theorem.

Find the Maximum Flow and Minimum Cut of the following networks. Hence, verify the above theorem.



(ii)