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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
First year - Semester II Examination – February/ March 2013

MAA 1203 – NUMERICAL ANALYSIS

Answer four questions only **Time:2 hours**

The Newton's Forward formula , $P_k = \sum_{i=0}^{k} \binom{k}{i} \Delta^i y_0$

The Newton's Backward formula

$$P_k = y_0 + k\,\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \cdots + \frac{k\ldots(k+n-1)}{n!}\nabla^n y_0\,; \quad \text{where k=0,-1,…,-n}$$

The Gauss forward formula, $P_k = y_0 + \sum_{i=1}^{n}\left[\binom{k+i-1}{2i-1}\delta^{2i-1}y_{1/2} + \binom{k+i-1}{2i}\delta^{2i}y_0\right]$

The Gauss backward formula, $P_k = y_0 + \sum_{i=1}^{n}\left[\binom{k+i-1}{2i-1}\delta^{2i-1}y_{-1/2} + \binom{k+i}{2i}\delta^{2i}y_0\right]$

The stirling's formula , $P_k = y_0 + \binom{k}{1}\delta\mu y_0 + \frac{(k)}{2}\binom{k}{1}\delta^2 y_0 + \binom{k+1}{3}\delta^3\mu y_0 + \frac{(k)}{4}\binom{k+1}{3}\delta^4 y_0 + \ldots + \binom{k+n-1}{2n-1}\delta^{2n-1}\mu y_0 + \frac{(k)}{2n}\binom{k+n-1}{2n-1}\delta^{2n} y_0$

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1). Solve the following difference equations:

I. $y_{k+2} - 7y_{k+1} + 12y_k = \cos k$ with $y_0 = 0$, $y_1 = 0$
II. $y_{k+2} + 6y_{k+1} + 25y_k = 2^{kK}$ with $y_0 = 0$, $y_1 = 0$
III. $2y_{k+2} - 5y_{k+1} + 2y_k = 0$ with $y_0 = 0$, $y_1 = 1$

2).(a) Prove that any positive integer k,

$\Delta^k y_0 = \sum_{i=0}^{k}(-1)^i\binom{k}{i}y_{k-i}$, Where the familiar symbol for binomial coefficients, $\binom{k}{i} = \frac{k!}{i!(k-i)!}$, has been used.

(b) Use **Lagrange formula** to produce a fourth degree polynomial which includes the following x_k , $f(x_i)$ number pairs. Thus evaluate this polynomial for x_k=9 .

x_k	5	7	11	13	17
$f(x_i)$	150	392	1452	2366	5202

3.(a) Apply **Stirling's formula** with n=2 to find a polynomial of degree four or less which takes the y_k values given in the table below.

k	-2	-1	0	1	2
x_k	2	4	6	8	10
y_k	-2	1	3	8	20

(b) The equation $f(x) = 0$, where

$f(x) = 0.1 - \frac{x}{(1!)^2} + \frac{x^2}{(2!)^2} - \frac{x^2}{(3!)^2} + \frac{x^2}{(4!)^2} - \cdots$,with $x_0 = 0.2$
has one root in the interval (0,1). Calculate this root correct to 5 decimals , using the **Newton-Raphson** method.

4.(a) **Factorial polynomial are defined by**

$$\boldsymbol{k^{(n)} = k(k-1)(k-2) \ldots\ldots (k-n+1)}$$

Prove that

$$\Delta k^{(n)} = nk^{(n-1)} \text{for all integers.}$$

(b) Show that $\binom{k}{n} = \frac{k^{(n)}}{n!}$ and prove the recursion formula, $\binom{k+1}{n+1} = \binom{k}{n+1} + \binom{k}{n}$

(c) Show that $k^{(n)} = \frac{k^{(n+1)}}{k-n}$ and Find $\Delta k^{(-1)}$

5). (a) **Apply Gauss's forward** central formula to find polynomial of degree four or less which takes the values given below.

x_k	2	4	6	8	10
y_k	0	0	1	0	0

(b) Approximate f(2.0) using the following data and the **Newton backward** divided-difference formula

x	1.0	1.3	1.6	1.9	2.2
$f(x)$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

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6). (a) Apply the **Gauss backward** formula to the following data.

x_k	3	4	5	6
y_k	6	24	60	120

with the argument k shifted so that $k=0$ at $x=6$

(b) Find the fourth order **Taylor polynomial** for $f(x) = \ln x$ about $x = 1$.

(c) Compute the 7th degree **Maclaurin polynomial** for the function $f(x) = \log(\cos x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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