10

RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.SC (General) Degree

First Year – Semester II Examination – February 2013

MAA 1104 Mathematical Modeling

Time allowed: **One hour.** **Number of pages: 03.**

Answer **TWO** Questions Only. **Calculators will be provided.**

---

1. Six years ago your parents purchased a house by financing Rs. 8,000,000 , paying monthly payment of Rs.88,000 with a monthly interest of 2%. They have made 72 payments and wish to know how much they owe on the mortgage , which they are considering paying off with an inheritance they received.

   (i). Formulate a dynamical system. **[20 Marks]**

   (ii). What is the equilibrium value of the system. **[10 Marks]**

   (iii). Build a numerical solution for the first five payments the parents would have made. **[10 Marks]**

   (iv). Find the General solution. **[40 Marks]**

   (v). After they have made 72 payments how much more money will they have to pay? **[10 marks]**

   (vi). Instead of paying with the inheritance they received , if they wish to follow the previous method, when will the account be paid off? **[10 Marks]**

Turn Over

2. (a). Predict the long – term behavior of the model $a_{n+1} = ra_n$, when

(i). $r = 1$, **[05 Marks]**

(ii). $r < 0$, **[05 Marks]**

(iii). $|r| < 1$. **[05 Marks]**

(b). State and prove the Cobweb Theorem. **[25 Marks]**

The respective Demand and Supply functions for the cobweb model are below:

$$Q_{dt} = 6000 - 2000p_t$$
$$Q_{st} = -1500 + 1000p_{t-1}$$

where p_t is the time path price function and p_0 is the initial price .

Find the general solution for p_t and discuss the stability of the above dynamical system. **[40 Marks]**

Interpret the results graphically. **[20 Marks]**

3. (a). Wildlife management has become an increasingly important issue as modern civilization puts greater demands on wildlife habitat. As an example, consider a fishing pond that is stocked from a nearby hatchery. Suppose you are in charge of managing the fish population in the pond.

(i). What are the factors to consider in managing the fish population in the pond? List as many factors as you can. **[15 Marks]**

(ii). How could you figure out the current size of the fish population? **[15 Marks]**

(iii). Why would it be useful to be able to predict the year-to-year changes in the fish population? **[10 Marks]**

106

(b). Consider a pond with Gold fish. The fish population model is given by , $N(t) = \frac{M}{1 + Ae^{-kt}}$

where , M= 100,

$N(t)$ = fish population at time t ,

A and k are arbitrary constants.

(i). Suppose you put 10 gold fish in the pond at time zero. Find the value of constant A in the model. **[05 Marks]**

(ii). Suppose there are 20 gold fish in the pond a year later. Find the value of constant k in the model.

[05 Marks]

(iii). Use a calculator to graph the model from time 0 to time 15. **[15 Marks]**

(iv). In which years is the population growing the fastest? **[05 Marks]**

(v). Find the population in years 20, 30, 40. **[15 Marks]**

(vi). What is happening to the population? **[10 Marks]**

(vii).Why might this happen to a population of goldfish in a pond? **[05 Marks]**