RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four year) Degree in Applied Sciences
Fourth Year – Semester II Examination – June/July 2018

PHY 4203 – CLASSICAL MECHANICS

Time: Two (02) Hours

Answer All Questions

A sheet of paper (A4 size) is allowed with written key notes

Unless otherwise specified, symbols have their usual meaning

1. a) Show that the geodesic on the surface of a right circular cylinder is a helix.

(13 marks)

b) A cylinder of radius R , height h and mass M rolls without slipping down a frictionless plane which is inclined from the horizontal by an angle θ in a uniform gravitational field g in the vertical direction. Find the Lagrangian L and the Lagrange equations of motion. The moment of inertia I of a uniform cylinder of radius R and mass M about an axis through its center is equal to ½ $M R^2$. (12 marks)

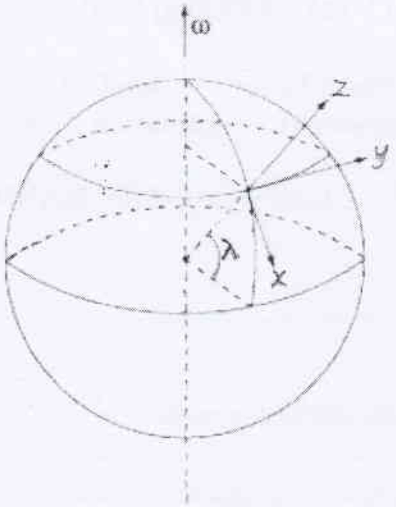
2. Consider a simple, plane pendulum which consists of a mass m attached to a string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate $\frac{dl}{dt} = -\alpha =$ constant. The suspension point remains fixed.

a) Find the Lagrangian and Hamiltonian functions. (15 marks)

b) Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system. (10 marks)

Contd.

3. A particle is projected vertically upwards to a height h above a point on Earth's surface at a northern latitude λ as shown in the figure below.



In Cartesian coordinate system, if the velocity of the particle and the rotational frequency of the Earth are expressed as $v = (0,0,\dot{z})$ and $\omega = (-\omega\cos\lambda\,, 0\,, \omega\sin\lambda)$ respectively,

a) Show that the acceleration due to Coriolis force($-2m(\omega \times \dot{r})$ is given by,
$a = 2\omega(0, -z\cos\lambda\,, 0)$ (07 Marks)

b) If the highest point the particle can reach is h, show that the initial velocity $v_0 = \sqrt{2gh}$ (03 Marks)

c) If the equation of motion along the x-axis is given by $z = v_0 t - \frac{1}{2}gt^2$ and the initial boundary condition $y\ (z = 0) = 0$, show that $\dot{y} = \omega\cos\lambda\,[gt^2 - 2v_0 t^2]$ (07 Marks)

d) If the initial condition $y\ (t = 0) = 0$, and the time the particle takes to strike the ground $t = \frac{2v_0}{g}$, show that the lateral displacement of the particle is given by,

$y = -\frac{4}{3}\omega\cos\lambda\sqrt{\frac{8h^3}{g}}$ (08 Marks)

Contd.

4. Consider the following inertia tensor.

$$\{I\} = \begin{pmatrix} \frac{1}{2}(A+B) & \frac{1}{2}(A-B) & 0 \\ \frac{1}{2}(A-B) & \frac{1}{2}(A+B) & 0 \\ 0 & 0 & C \end{pmatrix}$$

The above inertia tensor is rotated by angle θ, where the rotation matrix is

$$(\lambda) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Evaluate the transformed tensor elements $I' = (\lambda)(I)(\lambda')$. (Marks 13)

b) Show that the choice of $\theta = \frac{\pi}{4}$ renders the inertia tensor diagonal with elements A, B, and C.

(Marks 12)

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