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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree) Examination

First Year – Semester 11 Examination- March/ April 2014

MAA 1203 – Numerical Analysis I

Answer Four questions only **Time:2 hours**

Calculators will be provided.

The Newton's Forward formula $P_k = \sum_{i=0}^{k} \binom{k}{i} \Delta^i y_0$

Stirling's formula $P_k = y_0 + \sum_{i=1}^{n} \left[\binom{k+i-1}{2i-1} \delta^{2i-1} \mu\, y_0 + \frac{k}{2i} \binom{k+i-1}{2i-1} \delta^{2i} y_0 \right]$

Bessel's formula $P_k = \sum_{i=0}^{n} \left[\binom{k+i-1}{2i} \mu\, \delta^{2i} y_{1/2} + \frac{1}{2i+1} \left(k - \frac{1}{2}\right) \binom{k+i-1}{2i} \delta^{2i+1} y_{1/2} \right]$

1. Which of the following numbers has the greatest precision.

a) 4.3201 b) 4. 32 c) 4 .320106

i. Sum the numbers 0. 1532 , 15. 45, 0. 000354, 305. 1, 8 .12 , 143 .3, 0. 0212 , 0 .643 and 0.1734 · where in each of which all the given digits are correct.

ii. How many digits are to be taken in computing $\sqrt{20}$ so that the error does not exceed 0.01% ·

iii. If $U = \frac{5xy^2}{z^3}$ then find relative maximum error in u , given that $\Delta x = \Delta y = \Delta z = 0.001$ and $x = y = z = 1$.

2.

I. Write the **Bisection method** algorithm.

II. Find a positive root of the equation $xe^x = 1$, which lies between 0 and 1 by bisection method.

III. Find the **Lagrange interpolation** polynomial that takes the values prescribed below

x_k	0	1	2	4
$f(x_k)$	1	1	2	5

3.

a. Prove that

i. $U_3 = U_2 + \Delta U_1 + \Delta^2 U_0 + \Delta^3 U_0$

ii. $U_4 = U_3 + \Delta U_2 + \Delta^2 U_1 + \Delta^3 U_1^2$

iii. $(1+\Delta)(1-\nabla) = 1$

iv. Given sin 45°= 0.7071, sin 50°= 0.7660, sin 55°=0.8192, sin 60°= 0.8660. Find sin 48° by using the **Newton's Forward formula.**

4.

I. The values of tan x are given for values of x in the following table. Estimate tan(0.26)

x	0.10	0.15	0,20	0.25	0.30
y	0.1003	0.1511	0.2027	0.2553	0.3093

(Hint: Newton's Backward formula

$$P_k = y_0 + k\,\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \cdots + \frac{k-(k+n-1)}{n!}\nabla^n y_0\ ; \qquad \text{where } k=0,-1,\ldots,-n)$$

II. The equation $2\,e^x = \frac{1}{x+2} + \frac{1}{x+1}$ has two roots greater than -1. Calculate these roots correct to five decimal places. using the initial estimate $x_0 = -0.6$.

5.

i. A student use **Newton Raphson method** to solve the equation $x^{100} = 0$; using the initial estimate $x_0 = 0.1$. Calculate the next four Newton Method estimates.

ii. Using **Stirling's formula** compute **f(12.2)** from the data

x	10	11	12	13	14
f(x)	0.23967	0.28060	0.31788	0.35209	0.38368

6.

A. Using Bessel's formula find 3rd degree polynomial that approximates the following data: f(0) = 2, f(1) = 3, f(2) = 8, f(3)=23.

B. Find the general solution of $y_{k+2} - 2Ay_{k+1} + y_k = 0$(A is a constant)

C. Find the general solution of $y_{k+2} - y_{k+1} - y_k = c\alpha^{k}$,where c and α are constant.