RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Third Year - Semester II Examination – Jan/Feb 2023

MAT 3217 – NONLINEAR PROGRAMMING

Time: Two (02) hours

Answer **all** questions.
Calculators will be provided.

1. a) Briefly explain the initialization, iterative steps and the stopping rule of the following algorithms:
   i. Three-point search
   ii. Fibonacci search

   (20 marks)

   b) Approximate the solution of the following problem using the Three-point search algorithm.

   $$\text{Minimize } f(x) = e^{2x^3+x^2} - 2x - 1.$$

   Take the initial bounds as -1 and 1, and error tolerance as 0.13. **(40 marks)**

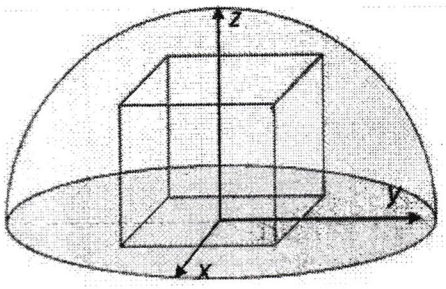
   c) Use the Fibonacci search algorithm to approximate the maximum value of the function f defined below by taking the initial upper bound, initial lower bound and the error tolerance as 2, -2 and 0.75 respectively.

   $$f(x) = \begin{cases} -2x^4 + 3x^2 - x + 2 & ; x \leq 0, \\ -x^2 - 2x + 2 & ; x > 0. \end{cases}$$

   (40 marks)

2. a) Explian the steps of the Lagrange Multiplier Method to find the extreme value/s of the function $f(x,y)$, subject to the m number of constraints: $g_i(x,y) = c_i$, where c_i is a real constant for all $i = 1, 2, .., m$. **(30 marks)**

b) Use the method of Lagrange multipliers to prove that the volume of the largest rectangular parallelepiped that can be inscribed in a hemisphere of radius a is $\frac{4a^3}{3\sqrt{3}}$.



(70 marks)

[Hint: $x^2 + y^2 + z^2 = r^2$ is the equation of a sphere of radius r, centered at the origin.]

3. a) Define the Quadratic Programming (QPP) in optimization theory and state its general mathematical model in matrix form with standard notations.

(15 marks)

b) Consider the following QPP:

$$\text{Maximize } Z = x_1 + 2x_2 - 3x_1^2 - 2x_1x_2 - x_2^2$$

$$S.t. \quad x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

i. Rewrite the above problem in matrix form.

ii. Solve the above QPP using the Wolfe's Modified Simplex Method.

(85 marks)

4. a) Solve the following two variable unconstrained non-linear programming problem, using the Gradient search algorithm:

$$\text{Maximize } f(x,y) = 2xy + 2y - 3x^2 - 2y^2.$$

Take error tolerance as $\varepsilon = 0.34$ and initial trial solution as $X_0 = (0,0)$.

(40 marks)

b) Solve the following Unconstrained Geometric Programming problem using the AM-GM inequality-based method:

$$\text{Minimize } f(x) = 5x_1 + 20x_2 + 10x_1^{-1}x_2^{-1}$$
$$x_1,\ x_2 > 0.$$

(60 marks)

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