RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four year) Degree in Industrial Mathematics
Fourth Year– Semester I Examination – September/October 2019

MAT 4301– OPERATIONAL RESEARCH I

INSTRUCTIONS:

- Answer **ALL** questions
- Time Allowed: **THREE** hours

1. Consider the $(M/M/1):(\infty/FIFO)$ queuing model, where average arrival rate λ is less than the average service rate μ. Assume that the system is in steady state and $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$, where P_n is the probability of n customers in the system.

   With the usual notation derive the formulas for the following:

   (a) Expected number of customers in the system and in the queue.

   (b) Expected waiting time in the system and in the queue.

   Patients arrive at a Government hospital for emergency service at the rate of one every hour. Currently only one emergency case can be handled at a time. Patients spend on average of 20 minutes receiving emergency care. The doctor wishes to have enough seats in the waiting room so that no more than about 1% of arriving patients will have to stand. Find

   (i) the probability that a patient arriving at the hospital will have to wait.

   (ii) the average length of the queue that forms.

   (iii) average time a patient spends in the queue.

   (iv) probability that there will be five or more patients waiting for the service.

2. Consider the $(M/M/1):(N/FIFO)$ queuing model. In the usual notation and steady state, find the probability of n customers in the system. Also, find the average number of customers in the system and in the queue.

Assume a railway marshalling yard is sufficient only for 10 trains. Trains arrive at the rate of 25 trains per day, inter-arrival time and service time follow exponential with an average of 30 minutes. Find

(i) the probability that the yard is empty.

(ii) the average queue length.

(iii) the expected number of trains in the system.

3. (a) Define a zero-sum game in game theory.

Distinguish between ***pure strategies*** and ***mixed strategies***.

Player I and II simultaneously call out one of the numbers one or two. Player I's name is Odd; he wins if the sum of the numbers is odd. Player II's name is even; she wins if the sum of the numbers is even. The amount paid to the winner by the loser is always the sum of the numbers in dollars.

(i) Construct a payoff matrix for the above game.

(ii) Does the above payoff matrix have a saddle point? Justify your answer.

(iii) Determine the mixed strategies of Players I and II, and hence find the value of the game.

(iv) Verify the solution in (ii) graphically.

(b) Consider the payoff matrix: $\begin{pmatrix} 2 & 0 & 4 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{pmatrix}$.

(i) Reduce the given payoff matrix to order two by using the dominance property.

(ii) Solve the reduced payoff matrix in (i) and hence find the value of the game.

4. Consider the following payoff matrix of players A and B:

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{pmatrix} 2 & -1 & 6 \\ 0 & 1 & -1 \\ -2 & 2 & 1 \end{pmatrix}. \end{array}$$

(i) Formulate Linear Programming Models for both players.

(ii) Solve the formulated models in (i) and determine the value of the game, and mixed strategies of each player.

5. Briefly explain the method in stepwise form to find the optimal sequence of processing n jobs in m machines.

The below table gives the processing times (in hours) of seven jobs to be processed on four machines M_1, M_2, M_3, M_4 in that order:

	M_1	M_2	M_3	M_4
A	3	1	4	12
B	8	0	5	15
C	11	3	8	10
D	4	7	3	8
E	5	5	1	10
F	10	2	0	13
G	2	5	6	9

(i) Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

(ii) Construct a table showing time in and time out of each stage and also, idle time of each activity.

(iii) Find the total minimum elapsed time if no passing of jobs is permitted.

(iv) Construct a Gannt chart showing the sequence of processing the jobs.

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