



RAJARATA UNIVERSITY OF SRI LANKA, MIHINTALE

FACULTY OF APPLIED SCIENCES, DEPARTMENT OF PHYSICAL SCIENCES

B.SC (General) Degree Third Year – Semester II Examination – September 2013

MAT 3310 Integer Programming

Time allowed: **Two hours.**

Answer **FOUR** questions selecting **question No.01** and **three** of the remaining questions.

01. (a) Using the matrix notation, write the model for a Mixed Integer Programming Problem. **[25 Marks]**

(b) A cement distribution company has six warehouses supplying eight customers. Each warehouse has a supply of cement bags that cannot be exceeded, and each customer has a demand for cement bags that must be satisfied. The unit transportation costs, demands and capacities of each warehouse and each customer are given in the following table:

	Customer								Capacity
	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	
A	6	2	6	7	4	2	5	9	60
B	4	9	5	3	8	5	8	2	55
C	5	2	1	9	7	4	3	3	51
D	7	6	7	3	9	2	7	1	43
E	2	3	9	5	7	2	6	5	41
F	5	5	2	2	8	1	4	3	52
Demand	35	37	22	32	41	32	43	38	

Distribution company wants to determine the number of cement bags to ship from each warehouse to each customer so as to minimize the total transportation cost.

- (i) Formulate an integer programming model to find the minimum cost for the distribution company. **[50Marks]**
- (ii) Write a complete computer program using LINGO programming language to solve the problem. **[50Marks]**

Turn Over

02. (a) Briefly explain the steps of “*Dual - Fractional Integer Programming Algorithm*”.

[20 Marks]

(b) The following table specifies the solution to the linear programme associated with the integer programme presented below:

	Constant	$-x_3$	$-x_4$
x_0	$133/5$	$11/5$	$2/5$
x_1	$16/5$	$2/5$	$-1/5$
x_2	$23/5$	$1/5$	$2/5$
x_3	0	-1	0
x_4	0	0	-1

Maximize $x_0 = 4x_1 + 3x_2$,

subject to the constraints

$$2x_1 + x_2 \leq 11,$$

$$-x_1 + 2x_2 \leq 6,$$

where integers $x_1, x_2 \geq 0$.

Here the variables x_3 and x_4 are slack variables for the two constraints.

(i) Derive cuts from the rows in the optimal linear programming table, including the objective function to obtain the optimal integer solution. [50 Marks]

(ii) Express the cuts in terms of the variables x_1 and x_2 . [30 Marks]

(iii) Graph the feasible region for x_1 and x_2 and illustrate the cuts on the graph.

[25 Marks]

03. Solve the following problem using *Dual All Integer-Integer Programming Algorithm* and compare the true optimum integer solution with that obtained by rounding-off the continuous optimum solution:

Minimize $x_0 = -3x_1 - 4x_2$,

subject to the constraints

$$2x_1 + 5x_2 \leq 15,$$

$$2x_1 - 2x_2 \leq 5,$$

where integers $x_1, x_2 \geq 0$.

[125 Marks]

04. Outline the *Land and Doig Algorithm* of the Branch and Bound method, and explain how this algorithm can be applied to a “0 - 1 Integer Programming Problem”.

[25 Marks]

The owner of the readymade garments store sells two types of shirts: Zee-shirts and Button-down shirts. He makes a profit of Rs.3 and Rs. 12 per shirt on Zee-shirts and Button-down shirts, respectively. He has two tailors, *A* and *B* at his disposal to stitch the shirts. Tailors *A* and *B* can devote at the most 7 hours and 15 hours per day, respectively. Both these shirts are to be stitched by both the tailors. Tailors *A* and *B* spend 2 hours and 5 hours, respectively in stitching one Zee-shirt, and 4 hours and 3 hours, respectively in stitching one Button-down shirt.

- (i) Formulate an integer programming model to determine the number of shirts of both types should be stitched in order to maximize daily profit.

[25 Marks]

- (ii) Find the integer solution to the above model using the *Daking Variation* of the branch and bound method.

Demonstrate the solution partitioning graphically.

[50 Marks]

05. A company produces two types of medals *A* and *B* that require gold and silver. Each unit of type *A* requires 3 grams of silver and 1 gram of gold while each unit of type *B* requires 1 gram of silver and 2 grams of gold. The company can buy 9 grams of silver and 8 grams of gold. Each unit of type *A* brings a profit of 40 dollars and each unit of type *B* brings a profit of 50 dollars.

- (i) Formulate an integer programming model to determine the number of units of each type that should be produced to maximize the profit of the company.

[25 Marks]

- (ii) Use *Primal All Integer- Integer Programming Algorithm* to solve the above model.

[100 Marks]