



**RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES**

B.Sc. General Degree in Applied Sciences
Third Year – Semester II Examination – April/ May 2016

MAT 3205 – Introduction to Statistical Decision Theory

Answer **Four** Questions only.

Time allowed: **Two Hours**

Calculators will be provided

- 01 Stereo Industries Ltd. must decide to build either a large or small plant to produce a new mini-disk player. A large plant will cost \$2.8 million to build and put into operation, while a small plant will cost only \$1.4 million. The company's best estimate of a discrete distribution of sales over the relevant planning horizon of 10 years is given in this table:

Customer demand	Probability
High Demand	0.5
Moderate Demand	0.3
Low Demand	0.2

Cost-volume-profit analysis by Stereo Industries suggests the following conditional outcomes under various combinations of plant size and market demand:

- A large plant with high demand would yield \$10 million annually in profits.
- A large plant with moderate demand would yield \$6 million annually in profits.
- A large plant with low demand would lose \$2 million annually, because of production inefficiencies.
- A small plant with high demand would yield only \$2.5 million annually in profits, considering the cost of lost sales because of inability to supply customers.
- A small plant with moderate demand would yield \$4.5 million annually in profits because the cost of lost sales would be somewhat lower.
- A small plant with low demand would yield \$5.5 million annually, because the plant size and the market size would be matched fairly optimally.

- (a) Based on the above given information, which plant would you build? Justify your answer.

- (b) Suppose that Stereo Industries' market research department could conduct a survey to predict the level of future demand, and that survey could have two possible outcomes, either favorable or unfavorable (indicated, respectively, by I_1 and I_2). Suppose further that data can be used to estimate the following conditional probabilities for the results of the research project, given the possible states of nature:

	$I_1 = \text{Favorable}$	$I_2 = \text{Unfavorable}$
High Demand	$P(I_1 s_1) = 0.96$	$P(I_2 s_1) = 0.04$
Moderate Demand	$P(I_1 s_2) = 0.40$	$P(I_2 s_2) = 0.60$
Low Demand	$P(I_1 s_3) = 0.00$	$P(I_2 s_3) = 1.00$

How would market survey affect your decision?

What is the Expected Value of Sample Information?

- 02 Under an employment promotion program it is proposed to allow sale of newspapers on the buses during off peak hours. The vendor can purchase the newspapers at a concessional rate of 25 paise per copy against the selling price of 40 paise. Any unsold copies are, however, a dead loss. A vendor has estimated the following probability distribution for the number of copies demanded.

Number of copies	15	16	17	18	19	20
probability	0.04	0.19	0.33	0.26	0.11	0.07

- Construct a conditional profit table.
 - Determine the action alternative associated with the maximization of expected profit.
 - Compute Expected Profit of Perfect Information (EPPI).
 - The vendor is thinking of spending on a small market survey to obtain additional information regarding the demand levels. How much should he willing to spend on such a survey?
- 03 A product is manufactured by a certain automatic machine in batches of 3 items. The selling price is \$10, and the production cost is \$5 per item. Before each batch is produced, the machine may be adjusted by a skilled mechanic at a cost of \$6 per batch. If the machine is adjusted, there will be no defectives in the batch. If it is not adjusted, some items may be defective. On the basis of past experience, the probability distribution of the number of defectives is estimated as follows:

Number of defectives in batches of 3 items	Probability
0	0.4
1	0.3
2	0.2
3	0.1

Defective items may be sold as scrap for \$5 each, or may be reprocessed at the following cost schedule:

Number of reprocessed items	Total reprocessing cost (\$)
1	6
2	10
3	12

Reprocessed items can be sold as good items. Should the machine be adjusted before each batch is produced? Use a decision tree.

04 (a) Briefly explain any two of the following uncertainty decision criteria.

- i. Maximax criterion
- ii. Maximin criterion
- iii. Minimax regret criterion

(b) Dr. Thomas has been thinking about starting his own independent nursing home. The problem is to decide how large the nursing home should be. The annual returns will depend on both size of nursing home and number of marketing factors. After a careful analysis, Dr. Thomas developed the following table.

Size of nursing home	Good market (Rs.)	Fair market (Rs.)	Poor market (Rs.)
Small	50,000	20,000	-10,000
Medium	70,000	35,000	-25,000
Large	90,000	35,000	-45,000
Very Large	200,000	25,000	-120,000

Find the optimum decision if Dr. Thomas adopts, each of the following strategies:

- i. Maximax criterion.
- ii. Maximin criterion.
- iii. Minimax regret criterion.

05 (a) Consider the following notations in Inventory Control:

- q – order quantity
- R – demand rate
- t – ordering cycle length
- k – setup cost
- h – holding cost

If the demand rate is uniform and replenishment rate is infinite, show that the

optimum inventory policy is $q_0 = \sqrt{\frac{2kR}{h}}$ units and $t_0 = \sqrt{\frac{2k}{hR}}$ time units.

- (b) A manufacturer has to supply 12,000 products per year, to his customers. The demand is fixed, and shortages are not allowed. The inventory holding cost is Rs. 0.2 per unit per month and the setup cost per production run is Rs. 350. Determine the optimum inventory policy.