



RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

First Year - Semester I Examination - July/August 2023

MAT 1204 - DIFFERENTIAL EQUATIONS I

Time allowed: **Two (2) hours**

Answer ALL (4) questions.

1. a) Solve the following differential equation:

$$y(x^2 + 1) \frac{dy}{dx} = x(y^2 + 1).$$

(30 marks)

- b) Using the substitution $v = \frac{y}{x}$, solve the differential equation given below:

$$y + (x - 2y) \frac{dy}{dx} = 0.$$

(35 marks)

- c) Using suitable substitutions for x and y , reduce the following differential equation to a homogeneous form:

$$\frac{dy}{dx} = -\frac{3y - 7x + 7}{7y - 3x + 3}.$$

Hence, obtain the solution for the given differential equation.

(35 marks)

2. a) Show that the following differential equation is exact, and hence solve it:

$$(3x^2 + 4xy) dx + (2x^2 + 2y) dy = 0.$$

(30 marks)

- b) Solve the following linear differential equation using an integrating factor:

$$\frac{dy}{dx} + 2xy = 2e^{-x^2}.$$

(35 marks)

- c) Using a suitable substitution, convert the following Bernoulli equation to a linear differential equation:

$$(1 - x^2) \frac{dy}{dx} + xy = xy^2.$$

Solving the obtained linear differential equation, find the solution of the given Bernoulli equation.

(35 marks)

3. a) Solve the following homogeneous differential equations with constant coefficients:

i. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

ii. $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0$

iii. $\left(\frac{dy}{dx} - y\right)^2 \left(\frac{d^2y}{dx^2} + y\right)^2 = 0$

(60 marks)

- b) Solve the following differential equations:

i. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 2e^{-3x}$

ii. $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 2e^x \cos 2x$

(40 marks)

4. a) Solve the homogeneous linear differential equation given below:

$$x^2 \frac{d^3 y}{dx^3} + 3x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 \ln x.$$

(40 marks)

- b) Solve the following Riccati equations:

i. $\frac{dy}{dx} = -2 - 5y - 2y^2$, and

ii. $\frac{dy}{dx} + y^2 = 1 + x^2$.

[Hint: $y = x$ is a solution of ii.]

(60 marks)

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