RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree
Second Year Semester I Examination – September/ October 2013

MAP 2203 – Differential Equations II

Answer all questions . **Time: 2 hours**

1.

a) Show that $x = 1$ is an ordinary point of the differential equation , $xy'' + y' + 2y = 0$.

b) Find a power series solution to the above differential equation about $x = 1$.

c) Solve the initial value problem,
$xy'' + y' + 2y = 0$; $y(1) = 1$, $y'(1) = 2$

2.

a) Find the general solution of the first order ordinary differential equation, $\dfrac{dy}{dx} + 2xy^2 = 0$. Find y satisfying the condition $y = 1$ when $x = 0$, and the value of y when $x = 0.1$.

b) Use Picard's iteration method to find the approximate solutions $y_1(x)$, $y_2(x)$ and $y_3(x)$ of the differential equation given in question 2. a) . Use $y_3(x)$ to estimate the value of y when $x = 0.1$.

c) Compare the two results obtained in parts (a) and (b) above for y, when $x = 0.1$.

[P.T.O]

3.

a) Find the general solution of the differential equation,

$$\frac{d\underline{X}}{dt} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 4 & -8 & 2 \end{bmatrix} \underline{X} \text{ , subject to the initial condition } X(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

b) Find the fundamental matrix solution of the differential equation,

$$\frac{d\underline{X}}{dt} = A\underline{X} \text{ where } A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

Hence find e^{At} .

4.

a) Form a partial differential equation by eliminating two arbitrary constants A and p from $z = A\,e^{pt} \sin px$.

b) Find the differential equation of all spheres of constant radius c, having centre $(h, k, 0)$ in the $xy-$ plane where h and k are arbitrary constants.

c) Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$, given with the usual notations.

If the solution of the above equation represents a sphere, what will be the coordinates of its centre?