RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year, Semester II Examination – April / May 2015

MAA 2203 - Numerical Analysis II

Answer **FOUR** Questions Only Time allowed: **Two hours**

Calculators are allowed

1. i. **Define** the divided difference $f[x_i, x_{i+1}, \ldots, x_{i+k}]$ for a function $f(x)$.

   ii. Construct the quadratic polynomial

   $P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$.

   Show that the polynomial interpolates $f(x)$ at the point $(x_i, f(x_i)), i = 0,1,2.$

   iii. Consider the points $x_0 = 0$, $x_1 = 0.4$, $x_2 = 0.7$ and for a function f(x), the divided differences are $f[x_2] = 6$, $f[x_1, x_2] = 10$, $f[x_0, x_1, x_2] = \frac{50}{7}$. Use these information, construct the complete divided differences table for the given points.

2. i. A trunnion of diameter 12.363 *inches*has to be cooled from a room temperature of 80 ^0F before it is shrinking fit into a steel hub. The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is-108 ^0F) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6})\, dT$$

   Use Simpson's Rule with $n = 4$ to find the diametric contraction.

   ii. Use (a) the Trapezoidal Rule and (b) the Midpoint Rule with n= 10 to approximate the integral $\int_0^1 \sqrt{1 + x^3}\,dx$

3. A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table where the time is in seconds, the distance is in meter, and the speed is in ms^{-1}.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

(a) Find the degree of the polynomial.

(b) Use a **Hermitepolynomial** to Find the **position of the car** and **speed** when $t = 10s$.

(c) Use the **Derivative of the Hermite polynomial** to determine whether the car ever exceeds a $55ms^{-1}$ speed limit on the road. What is the first time the car exceed this speed?

(d) What is the **predicted maximum speed** for the car?

4. Determine all the values of a, b, c, d, e for whichthe following function is a cubic spline,

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \in (-\infty, 1] \\ c(x-2)^2 & x \in [1,3] \\ d(x-2)^2 + e(x-3)^3 & x \in [3, \infty) \end{cases}$$

The above cubic spline interpolates the values of this table.

x	0	1	4
y	26	7	25

5. Consider the following system of linear equations

$$\begin{aligned} 3x_1 - 7x_2 - 2x_3 + 2x_4 &= -9 \\ -3x_1 + 5x_2 + x_3 &= 5 \\ 6x_1 - 4x_2 - 5x_4 &= 7 \\ -9x_1 + 5x_2 - 5x_3 + 12x_4 &= 11 \end{aligned}$$

Solve the linear system using **LU Factorization**

6.

a) The Boundary value problem

$$y'' = 4(y - x); \qquad 0 \le x \le 1, y(0) = 1, y(1) = 2$$

has the solution $y(x) = \frac{e^2}{e^4 - 1}\left(e^{2x} - e^{-2x}\right) + x$

b) Consider the data (0, 0),(0.5, α),(1, 3) and (2, 2). Then use the **Lagrange polynomial** to find α if the coefficient of x^3 in the polynomial is 6.

4(