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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B. Sc. (General) Degree

Second Year - End Semester Examination (Semester II), April / May 2016

MAP 2202 – Real Analysis II

Answer All Questions **Time allowed: Two hours**

1. (*a*) Prove that, if the series $\sum_{n=1}^{\infty} U_n$ converges, then $\lim_{n\to\infty} U_n = 0$.
Is converse of the above statement true? Justify your answer.

(*b*) The positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$.

(*c*) State the *Raabe's* test.
Using the *Raabe's* test examines the following test for convergence:
$$\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{2.4.6...2n} . \frac{x^{2n}}{2n}$$

2. (*a*) Define a *partition P* of a closed interval $[a,b]$ and define *Upper Riemann* sum $U(P,f)$ and *Lower Riemann* sum $L(P,f)$, where *f* is a function defined on $[a,b]$.

(*b*) Let *f* be a bounded function defined on $[a,b]$ and let *m*, *M* be the infimum and supremum of *f* on $[a,b]$. Then prove that any partition *P* of $[a,b]$, we have $m(b-a) \le L(P,f) \le U(P,f) \le M(b-a)$.

(*c*) Define *Upper Riemann integral* and *Lower Riemann integral* with usual notations. If $f(x) = x^3$ is defined on $[0,a]$, then show that $f \in R[0,a]$ (*i.e. f is Riemann integral on*$[0,a]$)nd $\int_0^a f(x)dx = \frac{a^4}{4}$.

(*d*) Let $f(x)$ be defined on $[a,b]$ as follows,
$$f(x) = \begin{cases} 0, & when\ x\ is\ rational \\ 1, & when\ x\ is\ irrational \end{cases}$$
Then show that $f(x)$ is not Riemann integral on $[a,b]$.

[P.T.O.

3. (*a*) Using practical comparison test for improper integrals, examine the convergence of the following improper integrals

(*i*) $\int_0^1 \frac{dx}{x^{\frac{1}{3}}(1+x^2)}$ (*ii*) $\int_0^1 \frac{dx}{x^2(1+x)^2}$

(*b*) Show that the *Beta function* $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ converges if and only if $m > 0$ *and* $n > 0$..

4. (*a*) Define a *Metric Space* with usual notations.

(*i*) Let X be a non-empty set and define a mapping $d : X \times X \to R$ by

$$d(x,y) = \begin{cases} 0 & when\ x = y \\ 1 & when\ x \neq y \end{cases}, \ \forall\ x, y \in X.$$

Show that d is a metric on X.

(*ii*) Let R be the set of real numbers. Show that the function $d : R \times R \to R$ defined by $d(x,y) = |x - y|,\ \forall x, y \in R$ is a metric on R.

(*b*) Define a *Pseudometric Space* with usual notations.
Let R be the set of real numbers. Show that the function $d : R \times R \to R$ defined by $d(x,y) = |x^2 - y^2|,\ \forall x, y \in R$ is a pseudometric on R ,which is not a metric on R .