RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

BSc in Applied Sciences
First Year - Semester I Examination – May 2022

MAA 1201 – MATHEMATICAL METHODS I

Time: Two (02) hours.

Answer all (04) questions.

1. a) If the position vectors of the points A and B are $\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and $2\mathbf{i} + 3\,\mathbf{j} + 5\,\mathbf{k}$, respectively, find the position vector of the point C that divides internally AB in the ratio $AC{:}\,CB = 2{:}\,3$ and its direction cosines. (25 marks)

b) Show that the vectors $\mathbf{v}_1 = 2\mathbf{i} - \mathbf{j}$, $\mathbf{v}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, and $\mathbf{v}_3 = 7\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ are linearly independent. (20 marks)

c) Let $\mathbf{a} = -3\,\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} + 7\,\mathbf{j} - 3\mathbf{k}$, and $\mathbf{c} = 7\,\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$.

i. Determine a unit vector perpendicular to the plane spanned by the vectors $\mathbf{a}$ and $\mathbf{b}$. Also, find the angle between $\mathbf{a}$ and $\mathbf{b}$. (20 marks)

ii. Find the volume of the parallelepiped made by $\boldsymbol{a}$, $\mathbf{b}$, and $\mathbf{c}$. (15 marks)

d) Given constant vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ such that $\mathbf{a} \cdot \mathbf{b} \neq 0$, solve the following vector equation for $\mathbf{x}$:

$$\mathbf{x} \times \mathbf{b} = \mathbf{c} \times \mathbf{b},$$

provided $\mathbf{x} \cdot \mathbf{a} = 0$. (20 marks)

2. a) The Cartesian(symmetric) equations of a straight line (l) are:

$$\frac{x-2}{1}=\frac{y-9}{2}=\frac{z-13}{3}.$$

i. Find the vector equation of the straight line l.

(10 marks)

ii. If the line (m) $\mathbf{r}=a\,\mathbf{i}+7\,\mathbf{j}-2\,\mathbf{k}+\mu\,(-\mathbf{i}+2\mathbf{j}-3\,\mathbf{k})$, μ being a scalar parameter, cuts the line l given in part (i), find the value of a and the position vector of the point of intersection.
Also, find the acute angle between l and m.

(30 marks)

b) Find the Cartesian equation of the plane (α) containing the points:
$A(1,5,0), B(3,0,-1)$ and $C(0,3,-1)$. (25 marks)

i. Find the vector equation of the line of intersection of the plane (β) given by the equation $2x-y+z=4$ and α. (20 marks)

ii. Determine the angle between the planes α and β. (15 marks)

3. Let $\mathbf{r}=\mathbf{r}(s)$ be a vector function defining a smooth curve C, where s denotes the arc - length of C. Let $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ denote the unit tangent vector, the principle unit normal vector, and the unit bi-normal vector to C, respectively.

a) State Frenet-Serret formulas for this curve in terms of $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$. (20 marks)

b) Let C be the curve defined by $\mathbf{r}=\ a\cos(\frac{\omega s}{c})\,\mathbf{i}+a\sin(\frac{\omega s}{c})\,\mathbf{j}+\frac{bs}{c}\,\mathbf{k}$, where $a(>0), b, c\neq 0$, and ω are constants.

i. Find the vectors $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$. (50 marks)

ii. Show that the curvature(κ) and the torsion(τ) of C are given by

$$\kappa=\frac{a\omega^2}{a\omega^2+b^2}$$

and

$$\tau=\frac{b\omega}{a\omega^2+b^2},$$

respectively. (30 marks)

4. a) Let $\mathrm{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ with $r = |\mathbf{r}| > 0$.

Find curl $\left(\frac{\mathrm{i} \times \mathrm{r}}{r^2}\right)$ and div $\left(\frac{\mathrm{i} \times \mathrm{r}}{r^2}\right)$. (35 marks)

b) Let $\varphi = 2xy^2 + z^2xy + x^2$.

i. Find the directional derivative of φ in the direction of the vector $v = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ at $P(1, 1, 2)$.

(35 marks)

ii. Determine the maximum and minimum values of directional derivative at P. (15 marks)

iii. Find the Laplacian of φ. (15 marks)

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