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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B. Sc. (Four Year) Degree in Industrial Mathematics

Fourth Year - Semester I Examination – August 2023

MAT 4301 – OPERATIONAL RESEARCH I

Answer all questions   **Time allowed: Three hours**

Information:

- This paper contains **FIVE** questions from **Page 1** to **Page 4**.

- This is a closed book examination.

- This examination accounts for 100% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in square brackets.

1. The firms ABC Ltd. and XYZ Corp. are competitors in the market of electronic goods. In order to increase market share, each of the firm can opt for any of the following three: high advertising, moderate advertising and low advertising. Corresponding to different possible conditions, the pay-offs in terms of percent market share are given below:

		XYZ Corp.		
		High	*Moderate*	*Low*
ABC Ltd.	*High*	2	3	5
	Moderate	2	0	6
	Low	0	2	-1

Using the dominant property or otherwise, find the optimal strategies for the two firms.

[100 marks]

2. Define the *two-person zero-sum game*. **[10 marks]**

Using the dominant property and mixed strategies solve the zero-sum game of the following payoff matrix:

		Player B		
		1	*2*	*3*
	1	0	-2	2
Player A	*2*	5	4	-3
	3	2	3	-4

[90 marks]

3. Consider the following queuing model:

Poisson arrival, Poisson departure, Single server, Infinite capacity and First come first served discipline.

With the usual notation, prove that $P_n = \left(\dfrac{\lambda}{\mu}\right)^n P_0$ when the system is in steady state.

[25 marks]

Also, prove that $P_n = \rho^n(1-\rho)$ and expected waiting line in the queue is $\dfrac{\rho}{1-\rho}$.

[25 marks]

Consider a switch which has an infinite buffer and an infinite number of users generating messages according to a Poisson process with an average inter-arrival time of 700 milliseconds. The switch serves requests with a service time that is exponentially distributed with an average service time of 400 milliseconds.

(i)   What is the average number of customers in the system and service time?

[25 marks]

(ii)   Suppose that the switch is upgraded to reduce average service time to 300 milliseconds. How would that affect the average time? **[25 marks]**

4. Briefly explain the method in stepwise form to find the optimal sequence of processing n jobs in m machines. **[15 marks]**

Six jobs have to be processed on three machines A, B and C in the order **BAC**. The time taken for each job on each machine is indicated below:

Machine	Job					
	J_1	J_2	J_3	J_4	J_5	J_6
A	3	5	4	2	6	2
B	7	10	9	6	10	8
C	3	4	2	5	1.5	4

(i) Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs. **[15 marks]**

(ii) Construct a table showing the time-in and time-out of each machine and the idle time of each activity. **[40 marks]**

(iii) Find the total minimum elapsed time if no passing of jobs is permitted. **[15 marks]**

(iv) Construct a Gannt chart showing the sequence of processing the jobs. **[15 marks]**

5. (a) The maintenance cost and resale value per year of a machine whose purchase price is Rs. 7 000 are given below:

Year	1	2	3	4
Maintenance cost (Rs.)	900	1 200	1 600	2 100
Resale value (Rs.)	4 000	2 000	1 200	600

When should the machine be replaced? **[50 marks]**

(b) Purchase price of a machine is Rs. 3 000 and its running cost is given in the table below:

Year	1	2	3	4	5	6	7
Running cost (Rs.)	500	600	800	1 000	1 300	1 600	2 000

If the discount rate is 0.90, find at what age the machine should be replaced. **[50 marks]**