RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year - Semester I Examination – February/March 2021

MAT 4301 – OPERATIONAL RESEARCH I

Time: Three (03) hours

Answer all questions

Information:

- This paper contains **SIX** questions from **Page 1** to **Page 4**.
- This is a closed book examination.
- This examination accounts for 100% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in parentheses.
- **Save the Excel models and solutions of questions 3 and 6 in pdf format in two separate files labelled by the question numbers and save the files in a folder labelled by your index number.**

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1. a) Briefly explain the method in stepwise form to find the optimal sequence of processing n jobs in m machines. **(10 marks)**

b) A machine operator has to perform three operations; turning, threading and knurling on a number of different jobs. The time required to perform these operations (in hours) for each job is known and given in the table below:

Machine	Job				
	J_1	J_2	J_3	J_4	J_5
M_1	19	43	41	74	36
M_2	22	48	28	42	11
M_3	58	32	61	27	57
M_4	11	50	37	49	22
M_5	69	63	75	79	85

i. Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs. **(25 marks)**

ii. Construct a table showing *time in* and *time out* of each machine and also, idle time of each activity. **(30 marks)**

iii. Find the total minimum elapsed time if no passing of jobs is permitted. **(10 marks)**

iv. Construct a Gannt chart showing the sequence of processing the jobs. **(25 marks)**

2. The following matrix represents the payoff to player P_1 in a rectangular game between two players P_1 and P_2:

Player P_2

Player P_1		*I*	*II*	*III*	*IV*
	I	3	4	10	12
	II	8	4	3	2

Solve the game graphically. **(100 marks)**

3. a) Formulate linear programming models to solve the following game:

Player *B*

Player A		B_1	B_2	B_3
	A_1	3	-4	2
	A_2	1	-3	-7
	A_3	-2	4	7

(50 marks)

b) Interpret the models in Excel Spreadsheet and hence, solve using Excel Solver. Find the optimal strategies. **(50 marks)**

4. a) Consider the following queuing model:
Poisson arrival, Poisson departure, Single server, Infinite capacity and First come first served discipline.

i. With the usual notation prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ when the system is in steady state.

(25 marks)

ii. Also, prove that $P_n = \rho^n(1-\rho)$ and expected waiting line in the queue is $\frac{\rho}{1-\rho}$.

(25 marks)

b) In a public telephone booth, the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find

i. the expected number of callers in the booth at any time **(25 marks)**

ii. the proportion of the time the booth is expected to be idle? **(25 marks)**

5. a) Consider the following queuing model:
Poisson arrival, Poisson departure, Single server, First comes first served discipline and the maximum number of customers is limited to N.

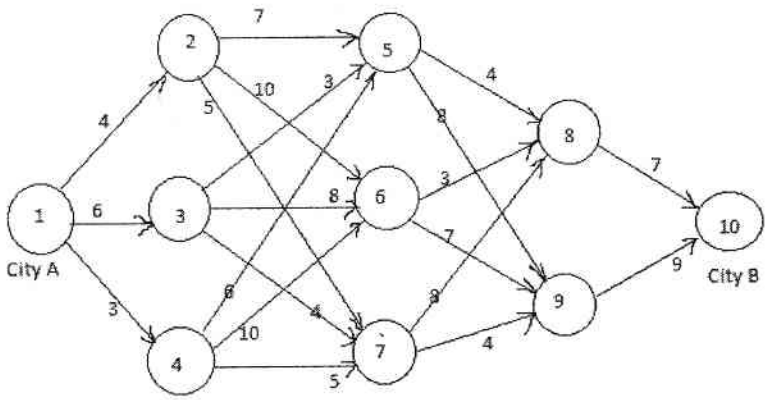
With the usual notation prove that $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ when the system is in steady state and

$P_n = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n$, for $n = 0,1,2,...,N$. **(40 marks)**

b) A barber shop has one barber and three chairs for customers. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that the barber services customers according to an exponential distribution with mean 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. What is the expected number of customers in the shop?

(60 marks)

6. A salesman located in a city A decided to travel to city B. He knows the distances of alternative routes from city A to city B as shown below. The city of origin, A, is city 1. The destination city B, is city 10. Other cities through which the salesman will have to pass through are numbered 2 to 9. The arrow representing routes between cities and distances in kilometers are indicated on each route:



a) Write a mathematical model to determine the shortest route from city A to city B. **(40 marks)**

b) Implement the formulated model in an Excel Spreadsheet. **(30 marks)**

c) Solve the formulated model in (ii) using Excel Solver and hence, determine the shortest distance from city A to city B. **(30 marks)**

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