RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year - Semester I Examination –June / July 2018

MAA 1302 – PROBABILITY AND STATISTICS I

Statistical tables and Calculators will be provided **Time allowed: Three (3) hours**

Answer **Five** questions **only**

1. The ages of the signers of the Declaration of Independence of the United States of America are shown below:

41 54 47 40 39 35 50 37 49 42 70

44 52 39 50 40 30 34 69 39 45 33

44 62 60 27 42 34 50 42 52 38 36

35 43 48 46 31 27 55 63 46 33 60

35 46 45 34 53 50 50 32 52 45 62

a) Using equal class widths 26.5 – 33.5, 33.5 – 40.5 … construct a frequency table.

b) Draw a Histogram, frequency polygon and cumulative frequency curve(less than ogive) to represent the distribution given in the table in part (a).

c) Calculate the mean, median, mode, standard deviation and coefficient of variation using the frequency table constructed in part (a).

2. State the Bayes' Theorem for a random experiment stating all the conditions.

a) A desk lamp produced by The Luminar Company was found to be defective (D). There are three factories (A, B, and C) where such desk lamps are manufactured. A Quality Control Manager (QCM) is responsible for investigating the source of found defects. This is what the QCM knows about the company's desk lamp production and the possible source of defects:

Factory	Percentage of total production	Probability of defective lamps
A	35%	$0.015 = P(D \mid A)$
B	35%	$0.010 = P(D \mid B)$
C	30%	$0.20 = P(D \mid C)$

Answer to the following questions:

(i) if a lamp is picked randomly, what is the probability that it could be a defective?

(ii) If a randomly picked lamp is defective, what is the probability that the lamp was manufactured in factory A?

b) It is given that for the events A and B of a random experiment, $P(A) = 0.3$, $P(A|B) = 0.4$ and $P(B) = 0.5$. Find the following:

(i) $P(A)$.

(ii) $P(A \cup B)$.

(iii) $P(A \cap B)$.

3. The Random Variable X has a Binomial distribution with parameters n and p. Show that,

   (i) $E(X) = np$

   (ii) $Var(X) = np(1-p)$

   a) A manufacturer of metal pistons finds that on the average, 12%of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

      (i) no more than two rejects?

      (ii) at least three rejects?

   b) A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week he will sell,

      (i) at least one policy.

      (ii) two or more policies but less than 5 policies.

4. Let X be a continuous random variable with probability density function:

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ k & 0 \le x < 1 \\ 0 & o.w., \end{cases}$$

   where k is a constant.

   Find

   a) the value of the constant k.

   b) the cumulative distribution function.

   c) $P(X \geq \frac{1}{2})$

5. The discrete random variable X has the probability function:

$$P(X = x) = \begin{cases} kx & x = 2,4,6 \\ k(x-2) & x = 8 \\ 0 & o.w., \end{cases}$$

   where k is a constant

   Show that $k = \frac{1}{18}$, hence find the value of

   a) $E(X)$.

   b) $Var(X)$.

   c) $F(5)$, where $F(x)$is the cumulative distribution function.

6. (a) Let X be a normally distributed random variable with mean 40 and standard deviation 5. Find,

   (i) $P(X < 50)$.

   (ii) $P(X > 21)$.

   (iii) $P(40 < X < 45)$.

   (b) The weight of bags of red gravel is modeled by a normal distribution with mean 25.8 kg and standard deviation 0.5 kg. Determine the percentage that a randomly selected bag of red gravels will weigh:

   (i) less than 25 kg.

   (ii) between 25.5 kg and 26.5 kg.

   (iii) If X represents the random variable of weight of a bag of red gravels, then find w such that $P(X \geq w) = 0.95$.

END