RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science Honors in Applied Sciences
B.Sc. (Joint Major) in Chemistry and Physics

Fourth Year-Semester I Examination –July/August 2023

# PHY 4312 - STATISTICAL THERMODYNAMICS

**Time: Three (03) hours**

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**Answer ALL questions.**

Non-programmable calculator is permitted.

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1. Statistical thermodynamics provides a quantitative link between the properties of the microscopic particles and the behavior of the bulk material.

   a) What is meant by macrostate and microstate? Explain using a suitable example.
   (04 Marks)

   b) Write down two (02) advantages of statistical thermodynamics compared to classical thermodynamics.
   (04 Marks)

   c) Four coins are tossed and received 16 outcomes. Calculate the average occupation number of heads.
   (04 Marks)

   d) There is a two-state system with a population of 50.
      i. How many microstates occur in this system?
      ii. What is the percentage of microstates within the 10% of the most probable microstates?

   (08 Marks)

2. a) Stirling's approximation is a method for approximating the value of large factorials.

   i. State Stirling's approximation expression.
   ii. Discuss why this is more reliable for the large factorials.
   iii. Calculate the approximate value for 100! using Stirling's approximation.

(06 Marks)

b) Compare Maxwell – Boltzmann (M-B), Bose- Einstein (B-E) and Fermi- Dirac (F-D) statistics and state their basic differences and/or similarities.

(09 Marks)

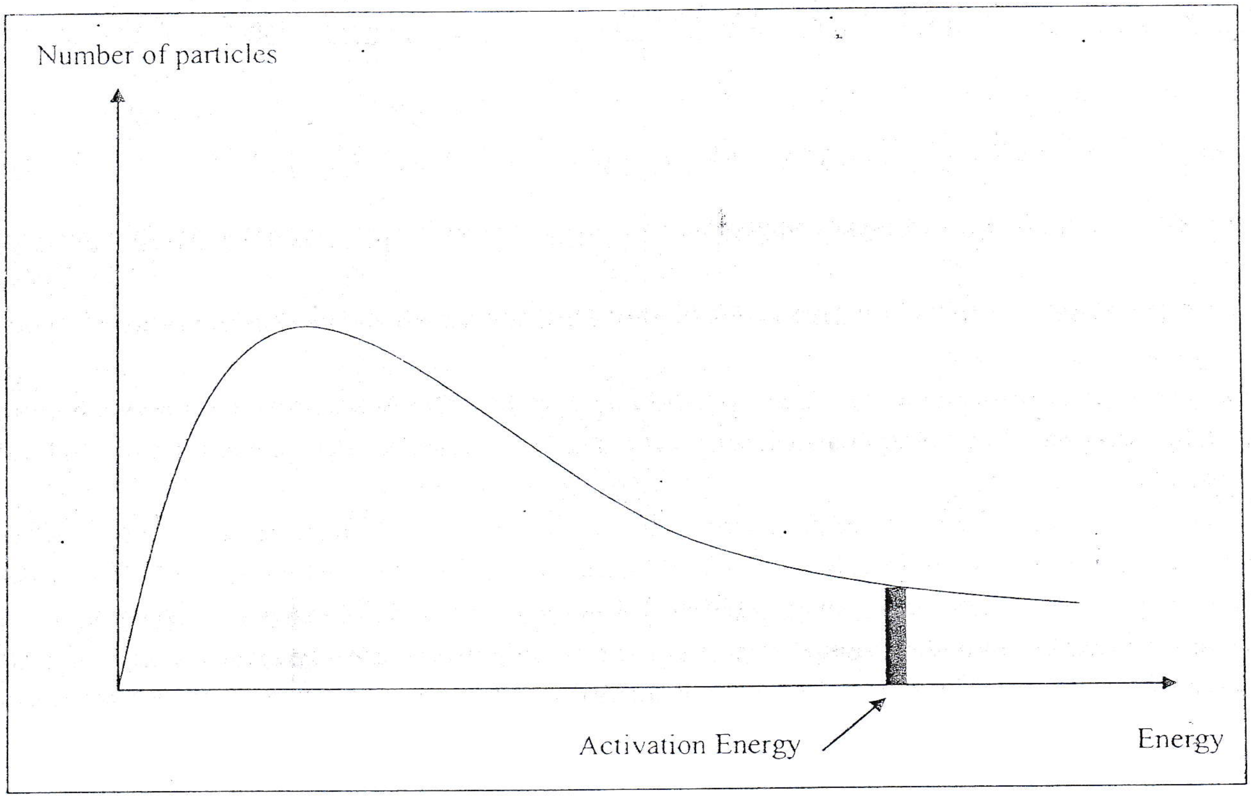
c) Explain what are Bosons and Fermions using the relationship between wave function ($\psi$) and the probability of finding an electron at each point. Give two examples for each.

(06 Marks)

d) State Pauli's Exclusion Principle.

(04 Marks)

3. Following graph shows the distribution of energy of the system with respect to the number of particles at 100 °C temperature.




**Contd.**

a) Explain the following situations using a labeled diagram and compare them with the situation given in the figure above for each case.

i. Use a catalyst for the reaction. (04 Marks)

ii. Temperature of the system increases up to 200 °C. (04 Marks)

iii. Increase the concentration of the system. (04 Marks)

b) Discuss what happens to the area under the graphs in each of the cases in part a). (06 Marks)

4. Consider the system with N identical particles with constant energy E. The total number of particles and the total energy of this system remains constant at a constant temperature.

a) It is stated that the system works under Maxwell-Boltzmann statistics.

i. What are the fundamental assumptions underlying the Maxwell-Boltzmann statistics?

(04 Marks)

ii. Write an expression for the thermodynamic probability ($\omega$) of the distribution in part i.

(03 Marks)

iii. Derive an expression for ln $\omega$ using the Sterling's approximation.

(04 Marks)

iv. State the reason for having only one most probable distribution in the system.

(02 Marks)

v. Derive an expression for the most probable distribution.

(04 Marks)

vi. Derive expressions for two conditions (related to energy and total number of particles) that satisfy the most probable distribution.

(02 Marks)

vii. By using Lagrange's method of undermined multipliers, prove that the generalized form of the Maxwell- Boltzmann distribution is,

$$\frac{n_i}{n} = \frac{e^{-(\frac{E_i}{KT})}}{\sum e^{-(\frac{E_i}{KT})}}.$$

(06 Marks)

b) Suppose that the above system has degenerated energy levels and the amount of degeneracy is equal to $g_i$. The system now performs under the Bose- Einstein statistics.

i. State the meaning of degeneracy of the energy levels.

(02 Marks)

**Contd.**

ii. Distribution of $n_i$ particles among $g_i$ number of states in this system is given by $\prod \frac{(n_i+g_i-1)!}{n_i(g_i-1)!}$. Using this distribution, prove the Bose-Einstein formula $n_i = \frac{g_i}{e^{(\alpha+\beta E_i)}-1}$, where $\alpha$ and $\beta$ are Lagrangian constants.

(10 Marks)

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