RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences

First Year-Semester II Examination – September/October 2020

MAA 1203 -NUMERICAL ANALYSIS I

Time: Two (2) hours.

Answer all four (4) questions.

Calculators will be provided.

1. a) A representation of a nonzero number $x \in \mathbb{R}$ in a floating-point number system $F(\beta, k, m, M)$, where β is the base, k is the number of digits in β expansion, and e is the exponent with $m \leq e \leq M$ is given by

$$fl(x) = \pm(0.\beta_1\beta_2\beta_3 \dots \beta_k)_\beta \times \beta^e,$$

where $1 \leq \beta_1 \leq \beta - 1$ and $0 \leq \beta_j \leq \beta - 1$ for $1 < j < \beta - 1$.

i. For chopping, show that

$$\left|\frac{x - fl(x)}{x}\right| \leq \beta^{1-k}.$$

(20 marks)

ii. Let $\beta = 10$, $k = 5$, and $x = 0.1234566 \times 10^7$.

Compute the absolute and relative errors associated with chopping and the machine precision for chopping.

(30 marks)

Contd.

b) Let $x_0, x_1, \dots, x_n$ be n points such that $x_{k+1} - x_k = h$ (constant) and $y_k = y(x_k)$ for all $k = 0, 1, \dots, n$. The forward difference operator Δ is defined by

$$\Delta y_k = y_{k+1} - y_k.$$

i. Find $\Delta^r y_0$ for $r = 2, 3, 4$.

(30 marks)

ii. Compute $\Delta^4 y_k$ for the discrete function, $y_k = y(x_k)$, defined in the following table.

x_k	1.0	1.1	1.2	1.3	1.4	1.0
y_k	7.000	8.093	9.384	10.891	12.632	3.0

(20 marks)

2. a) Write down two advantages of the Newton-Raphson method in seeking numerical solution of nonlinear equations.

(10 marks)

b) Write down the Newton-Raphson iterative formula.

(10 marks)

c) Let $f(x) = x^3 - 2x - 5$.

i. Show that $f(x) = 0$ has a root between 2 and 2.5.

(20 marks)

ii. Performing four iterations of the Newton-Raphson method with initial approximation, $x_0^* = 2$, obtain an approximation for the root.

(60 marks)

3. a) Differentiate between interpolation and extrapolation.

(10 marks)

b) Write down two advantages of the Lagrange interpolation over direct interpolation.

(10 marks)

c) The following table represents the data for $f(x) = e^{-x}$:

x	0.1	0.2	0.4	0.7
$f(x)$	0.904837	0.818731	0.670320	0.496585

Contd.

i. Using an appropriate Lagrange interpolating polynomial with degree 2, approximate $f(0.15)$.

(70 marks)

ii. Compute the absolute error, provided $f(0.15) = 0.8607080$.

(10 marks)

4. Consider the initial-value problem:

$$\frac{dy}{dx} = x + y, \qquad y(1) = 2.$$

a) Write down the numerical schemes of the following methods for solving the above differential equation:

i. The Taylor series method of order 4,

ii. The fourth order Runge-Kutta method.

(30 marks)

b) Write down two advantages of the fourth order Runge-Kutta method over the fourth order Taylor series method.

(10 marks)

c) Using the fourth order Runge-Kutta method, approximate $y(1.2)$ for the initial-value problem with step size $h = 0.1$.

(50 marks)

d) Suppose that the analytic solution, $y(x)$, of the initial-value problem is

$$y(x) = -x - 1 + 4e^{x-1}.$$

Compute the relative error of $y(1.2)$.

(10 marks)

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