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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

Second Year-Semester I Examination-February/March 2013
PROBABILITY AND STATISTICS II - MAA 2302

Candidates with mid semester marks:
Answer all questions in part A and one question from part B. **Time : 02 hours**

Candidates without mid semester marks:
Answer all questions. **Time: 03 hours**

Statistical tables and calculators will be provided.

Part A

1. Let (X,Y) be a two dimensional continuous random variable with joint probability density function,

$$f(x,y) = \begin{cases} kx(1+3y^2) & ;0<x<2,\ 0<y<1 \\ 0 & ;otherwise \end{cases}$$

(i) Find the value of the constant k.
(ii) Find the conditional probability density function of X for a given $Y=y$, and determine $P(\frac{1}{6}<X<\frac{1}{4}\ |Y=\frac{1}{2})$.
(iii) Find the conditional expectation of X for a given $Y=y$.

2.
a) The random variable Y has a Poisson distribution with parameter λ. Find the probability density functions of

(i) $U=3Y+1$
(ii) $V=Y^{\frac{1}{4}}$

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b) A binary source generates digits 1s and 0s randomly. The probability of generating 1s is 0.7. If a five digit sequence is generated,

(i) determine a formula for the probability distribution X of the number of 1s in the sequence.

(ii) Find the distribution of $Y = 3x - 1$.

3. Let $X_1, X_2, \dots X_n$ be a random sample of size n from the Poisson distribution with parameter λ.

(i) State the inequality of Cramer-Rao lower bound.
Explain how this inequality could be used to identify the best estimator for λ.

(ii) Show that

$$\lambda_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad \lambda_2 = \frac{1}{2}(X_1 + X_2) \quad \text{are both unbiased estimators of } \lambda.$$

(iii) Determine the best estimator for λ, using Cramer-Rao Theory.

Part B

4.

a) Let $\overline{X}$ and $\overline{Y}$ be the sample means of random samples of sizes n_1 and n_2 respectively, and let $\overline{X} \sim N(\mu_1, \sigma_1^2)$ and $\overline{Y} \sim N(\mu_2, \sigma_2^2)$. Find the mean and variance for the difference between the two sample means.

b) Two types of thread are being compared for strength. Type A has a normal distribution with a mean tensile strength of $82.6kg$ and a standard deviation of $6.3kg$. Type B has a normal distribution with a mean tensile strength of $76.4kg$ and a standard deviation of $5.6kg$.

(i) If a sample of 20 pieces of each type of thread are tested, find the probability that the difference between the two sample means is more than $10.8kg$.

(ii) If a sample of size n pieces of each type of thread are tested, find the value of n so that the probability that the difference between the two sample means, is less than $8.7kg$ is 0.95.