RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester II Examination – February/ March 2019

PHY2105 – QUANTUM MECHANICS

Time: One (01) hour

Answer any two questions.

Use of a non-programmable calculator is permitted.

Symbols and notations have their usual meaning.

Some fundamental constants and physical data:

Electron mass $m_e = 9.1 \times 10^{-31}$ kg
Speed of light in vacuum $c = 3.0 \times 10^{8}$ m s^{-1}
Bohr radius $a_0 = 0.529 \times 10^{-10}$ m

Planck constant $h = 6.626 \times 10^{-34}$ J s
Electron volt (1 eV) = 1.6×10^{-19} J
Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}

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1. The total intensity $I(T)$ radiated from a blackbody (at all wavelengths λ) is equal to the integral over all wavelengths, $0 < \lambda < \infty$, of the Planck distribution;

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}.$$

(a) By changing variables to $x = {}^{hc}/_{\lambda k_B T}$, show that $I(T)$ has the form $I(T) = \sigma T^4$, where σ is a constant independent of temperature. (20 marks)

(b) Given that $\int_0^\infty \frac{x^3}{(e^x - 1)} dx = \frac{\pi^4}{15}$, show that the Stephan-Boltzmann constant σ is $\frac{2\pi^5 k_B^4}{15h^3c^2}$. (15 marks)

(c) Evaluate σ numerically and find the total power radiated from a red-hot (T=1000 K) steel ball of 1 cm radius. (15 marks)

Contd.

2. (a) i. State Heisenberg's uncertainty principle and prove that $\Delta E.\Delta T \geq \hbar$. (15 marks)

ii. Use the above relation to explain how one could describe the broadening of spectral lines. (07 marks)

(b) An electron in the n = 2 state of a hydrogen atom remains there on average 10^{-8} s before making a transition to the ground state..

i. Estimate the uncertainty in the energy of the n = 2 state. (06 marks)

ii. What fraction of the transition energy is this? (06 marks)

iii. What is the width of the spectral line emitted in the transition? (10 marks)

(c) "Electron Microscope is more suitable to see objects of atomic size than an optical microscope". Justify this statement. (06 marks)

3. Consider a particle of mass m and energy E approaching from the left, to a one-dimensional potential step given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases}$$

(a) Discuss the motion classically and quantum mechanically for the following cases.

i. $E < V_0$ (12 marks)

ii. $E > V_0$ (12 marks)

(b) Obtain the reflection and transmission coefficients. Note: The probability current density is given by $j = \mathrm{Re}[\psi^* \frac{\hbar}{im} \nabla\psi]$, where Re indicates the "real part of". (18 marks)

(c) Show that the sum of the reflection and transmission coefficients is one for a particle scattered by a potential step. (08 marks)

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