RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
Second Year- Semester I Examination- October/November 2015
MAP 2203 - Differential Equations II

Answer **FOUR** Questions only Time: **Two hours**

1.

a) Define ordinary point and singular point of the differential equation,
$P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$. Where the P's are polynomial in x.

b) Following statements are true or false, justify your answers?

i. For the equation $x^2y'' + (x^2 - x)y' + 2y = 0,\ x = 0$ is singular point.

ii. For the equation $(1 + x)y'' + 2xy' - 3y = 0,\ x = -1$ is singular point.

iii. For the equation $x^3y'' + x^2y' + y = 0,\ x = 0$ is singular point.

c) Solve $y'' + (x-1)y' + y = 0$ in powers of $(x - 2)$.

2.

a) Discuss the Frobenius method for solving a second order linear differential equation given , with the usual notations, as
$$a_2(x)y'' + a_1(x)y' + a_0(x)\,y = 0$$

b) Show that the differential equation $2x^2y'' - x\,y' + (1 + x)\,y = 0$ has a regular singular point at the origin.

c) Find the general solution of the above differential equation by using Frobenius method.

3.

a) Show that $\lambda = 2$ is a triple root of the characteristic equation for the system $\dot{x} = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix} x$ and find three linearly independent solutions.

b) Show that the fundamental matrix solution of the system $\dot{x} = Ax$, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{pmatrix}$ is $X(t) = \begin{pmatrix} e^{t} & e^{3t} & e^{5t} \\ 0 & 2e^{3t} & 2e^{5t} \\ 0 & 0 & 2e^{5t} \end{pmatrix}$.

Hence find e^{At}.

4.

a) Solve the first order ordinary differential equation,

$\frac{dy}{dx} + 2xy^2 = 0$, given that $y = 1$ when $x = 0$

Hence find the value of y when $x = 0.1$

b) Use Picard's iteration method to find the approximate solutions $y_1(x)$, $y_2(x)$ and $y_3(x)$ of the above differential equation in part a).

Use $y_3(x)$ to estimate the value of y when $x = 0.1$.

c) Compare the two results obtained in the above parts for y, when $x = 0.1$.

5.

a. Form a partial differential equation by eliminating arbitrary constants a and b from the following equations :

i. $z = a(x+y)+b$

ii. $z = axe^{y} + (1/2)a^{2}e^{y} + b$

b. Form a partial differential equation by eliminating the arbitrary functions f and F from the following equations:

i. $z = f(x+iy)+F(x-iy)$ where $i^{2} = -1$

ii. $y = f(x-at)+F(x+at)$

c. Solve the following partial differential equations, given with the usual notations:

i. $p\tan x + q\tan y = \tan z$

ii. $xyp + y^{2}q = zxy - 2x^{2}$