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RAJARATA UNIVERSITYOF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
Second Year - Semester I Examination-October / Nov. 2015

MAA 2302 – Probability and Statistics II

Answer **five questions** only. **Time: 03 hours**

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1. Let (X,Y) be jointly distributed with density function

$$f(x,y) = \begin{cases} e^{-(x+y)}; & 0 < x < \infty, 0 < y < \infty \\ 0 & ; \ otherwise \end{cases}$$

   I. For find the moment generating function of (X, Y).
   II. Find the covariance between X and Y.
   III. Are X and Y independent? Justify your answer.

2.

   I. The random variable X has possible values 0,1,2,3,………. with probabilities

$$P(X = r) = 2(1-b)b^r \ , \quad r = 0,1,2,3,\ldots\ldots,$$ where b is a constant.

      i. For what values of b is this a valid probability distribution?
      ii. Show that for any positive integers x and y

$$P(X > x + y \mid X > x) = \frac{1}{2}P(X \geq y)$$

   II. The continuous random variable X has the cumulative distribution function,

$$F(x) = \begin{cases} 0 & ,x < 0 \\ 2/\pi \ \sin^{-1}\sqrt{x} & ,0 \leq x \leq 1 \\ 1 & ,x > 1 \end{cases}$$

      a. Obtain the probability density function of X.
      b. Show that this is a valid probability density function.
      c. Find $E(x)$ and $Var(x)$.

3. Let X be a continuous random variable with probability density function f_X given by

$$f_X(x) = \begin{cases} \frac{a}{x^2} & for \quad 1 \le x \le 2 \\ 0 & otherwise, \text{where } a \in \mathbb{R} \end{cases}$$

I. Prove that $a = 2$.
II. Find the mean and variance of X.
III. Find the cumulative distribution function of X.
IV. Hence find the cumulative distribution function of $Y = X^2$.
V. Hence, or otherwise, find the probability density function of Y .

4.

I. A random variable X follows a Poisson distribution with parameter λ and show that the maximum likelihood estimation of λ is $\bar{X}$.

II. Let$X_1, \dots\dots, X_n$be a random sample of size n from the population with the probability density function

$$f_X(x) = \frac{x}{\theta^2} e^{\frac{-x}{\theta}} \quad ,0 < x, \theta < \infty.$$

Find the maximum likelihood estimate for θ.

III. Let $X_1, \dots\dots, X_n$ be a random sample from a distribution with mean μ and $var(X) = \sigma^2$. Show that;

a. $\bar{x}$ is a unbiased estimator for μ.

b. $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$; S^2 is a unbiased estimator of σ^2.

5.

I. Two different plasma etchers in a semiconductor factory have the same mean etch rate. However, machine 1 is newer than machine 2 and consequently has smaller variability in etch rate. We know that the variance of etch rate for machine 1 is σ_1^2 and for machine 2 it is $\sigma_2^2 = \alpha\sigma_1^2$. Suppose that we have $n_1$independent observations on etch rate from machine 1 and n_2 independent observations on etch rate from machine 2.

a. Show that $\hat{\mu} = \alpha\overline{X_1} + (1-\alpha)\overline{X_2}$ is an unbiased estimator of μ for any value of μ between 0 and 1.

b. Find the standard error of the point estimate of μ in part (a). (Hint: The standard error is given by the standard deviation of the estimator.)

II. Briefly explain the difference between point estimators and interval estimators.

III. The administrators for a hospital would like to estimate the mean number of days required for in-patient treatment of patients between the ages of 25 and34 years. A random sample of $n = 500$ hospital patients between these ages. Produced the following sample statistics:
$\overline{X} = 5.4\ days$
$s = 3.1\ days$
Construct a 90% confidence interval for μ, the (population) mean length of stay for this cohort of patients.

6.

I. Briefly explain about 3 properties of point estimators.

II. Snout beetles cause millions of dollars worth of damage each year to cotton crops. Two different chemical treatments are used to control this beetle population using 13 randomly selected plots. Below are the percentages of cotton plants with beetle damage (after treatment) for the plots:
Treatment 1: 22.3, 19.5, 18.6, 24.3, 19.9, 20.4
Treatment 2: 9.8, 12.3, 16.2, 14.1, 15.3, 10.8, 18.3

Under normality, and assuming that these two samples are independent, find a 95% confidence interval for the ratio of the two treatment variances.

7.

I. Suppose there is a random sample of size $2n$ from a population denoted by X, and $E(X) = \mu$ and $Var(X) = \sigma^2$.

Let $\overline{X_1} = \frac{1}{2n}\sum_{i=1}^{2n} X_i$ and $\overline{X_2} = \frac{1}{2}\sum_{i=1}^{n} X_i$ be two estimators of μ.

Which is the better estimator of μ? Explain your choice.

II. If $\overline{x_1}, \overline{x_2}, s_1 \text{and} s_2$ are the values of the means and the standard deviations of independent random samples of size n_1 and n_2 from a normal population with means μ_1 , μ_1 and unknown variances σ_1 , σ_1 respectively. Construct $100(1-\alpha)\%$ confidence interval for the difference between the two population means, with the variances are equal where $(1-\alpha)$ is the confidence coefficient.

III. A study has been made to compare the nicotine content of two brands of cigarettes. 10 cigarettes of brand A had an average nicotine content of 3.1 milligrams with a standard deviation 0.5 milligrams, while 8 cigarettes of brand B had an average nicotine content of 2.7 milligrams with a standard deviation 0.7 milligrams. Assuming that the two sets of data are independent random samples from normal populations with equal variances, construct a 95% confidence interval for the difference between the mean nicotine content of the two brands of cigarettes.