RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

First Year-Semester I Examination-October 2014

MAP 1301 – LINEAR ALGEBRA

Answer **SIX** Questions with **including Question no 1** Time allowed: **Three hours**

1.

i. Which of the following matrices is in **row echelon** form? Justify your answer.

$$\underset{\mathbf{A}}{\begin{pmatrix}0&1\\1&0\\0&0\end{pmatrix}}\quad\underset{\mathbf{B}}{\begin{pmatrix}1&2\\0&1\\0&0\end{pmatrix}}\quad\underset{\mathbf{C}}{\begin{pmatrix}1&2\\0&1\\0&1\end{pmatrix}}\quad\underset{\mathbf{D}}{\begin{pmatrix}1&0\\0&0\\0&1\end{pmatrix}}$$

(A) Matrix **A** (B) Matrix **B** (C) Matrix **C**

(D) Matrix **D** (E) None of the above

ii. Which of the following matrices are in **reduced row echelon** form? Justify your answer.

$$\underset{\mathbf{A}}{\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}}\quad\underset{\mathbf{B}}{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}}\quad\underset{\mathbf{C}}{\begin{pmatrix}0&1&0&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}}$$

(A) Only matrix **A** (B) Only matrix **B** (C) Only matrix **C**

(D) All of the above (E) None of the above

iii. Consider the matrix **X**, shown below.

$$X = \begin{pmatrix}0&1\\1&2\\0&5\end{pmatrix}$$

54

Which of the following matrices is the reduced row echelon form of matrix **X** ? Justify your answer.

$$\begin{pmatrix}0 & 1\\ 1 & 0\\ 0 & 0\end{pmatrix} \quad \begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix} \quad \begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 1\end{pmatrix} \quad \begin{pmatrix}1 & 0\\ 0 & 0\\ 0 & 1\end{pmatrix}$$

A **B** **C** **D**

(A) Matrix **A** (B) Matrix **B** (C) Matrix **C**
(D) Matrix **D** (E) None of the above

iv. Consider the row vectors shown below.

(0 1 2) (3 2 1) (3 3 3) (3 4 5)
a **b** **c** **d**

Which of the following statements are true? Justify your answer.

I. Vectors **a**, **b**, and **c** are linearly dependent.
II. Vectors **a**, **b**, and **d** are linearly dependent.
III. Vectors **b**, **c**, and **d** are linearly dependent.
IV. All of the above. (E) None of the above.

v. Consider the matrix **X**, shown below.

$$X = \begin{pmatrix}1 & 2 & 4 & 4\\ 3 & 4 & 8 & 0\end{pmatrix}$$

What is its rank? Justify your answer.
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2.

a) Define a symmetric matrix and a skew symmetric matrix. Show that the elements on the main diagonal of a skew symmetric matrix are all zero.

b) If A is any $n \times n$ matrix, show that

i. $A + A^T$ is symmetric

ii. $A - A^T$ is skew symmetric

c) Find the nine cofactors C_{ij}, where $i, j = 1,2,3$, of the matrix $A = \begin{bmatrix}2 & 1 & 3\\ 1 & -1 & 1\\ 1 & 4 & -2\end{bmatrix}$ and

hence obtain the matrix defined as $adj\ A$, and show that $A\,adjA = 14I$.

Find the inverse of the matrix A and the value of the determinant of the matrix $adj\ A$.

3.

i. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

Check your answer by multiplication.

ii. Using the result of part (i), solve the system of linear equations:

$$\begin{aligned} x + y + z + t &= 0 \\ y + z + t &= 1 \\ x + y + 2z + t &= 1 \end{aligned}$$

4.

Let $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 2 & 5 \end{pmatrix}$:

(a) Find the eigenvalues of A. (Hint: $\lambda^2 - 9\lambda + 18 = (\lambda - 3)(\lambda - 6)$)

(b) Find bases for the eigenspaces of A.

(c) Write down an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Briefly explain yourself.

5.

i. Find bases for the **Column Space** of the matrix $A = \begin{bmatrix} 1 & 4 & 11 & -4 & -9 \\ -1 & -2 & -5 & 6 & 16 \\ 0 & 4 & 12 & 5 & 18 \\ -1 & 2 & 7 & 4 & 6 \end{bmatrix}$ and its **Null Space.**

ii. Express each of the matrices A and A^T in **row echelon form,** where $A = \begin{pmatrix} 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \\ -3 & 6 & -1 & 1 & -7 \end{pmatrix}$, and hence find the rank of the matrix A.

Obtain the solution set of the homogeneous system $AX = O$, in parametric vector form, and hence find a basis for the null space of the matrix A. What is the dimension of the null space?

Verify the formula: Rank A + Nullity A = 5, the number of columns of A.

6.

a. Determine the value of k such that the system in unknowns x,y,z has,

1. no solution
2. a unique solution
3. infinitely many solutions;

$$kx + y + z = 1$$

$$x + ky + z = 1$$

$$x + y + kz = 1$$

b. Show that $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ is non-singular, find A^{-1} and express A as a product of elementary row matrices.

7.

i. Solve the linear system by reducing the augmented matrix to row echelon form indicating the elementary row operations used at each step:

$$\begin{aligned} X_1 + 2X_2 - 3X_3 &= 9 \\ 2X_1 - X_2 + X_3 &= 0 \\ 4X_1 - X_2 + X_3 &= 4 \end{aligned}$$

ii. Show that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$?

8.

a. Let $S = \{v_1; :::; v_k\}$ be a subset of a vector space V over a field K. Define what we mean by the following.

(i) V is spanned by S.
(ii) S is linearly independent.
(iii) S is a basis for V.
(iv) V is fnite dimensional.

b. State the Basis Theorem and define what is meant by the dimension of a finite dimensional vector space.

c. Show that the set $\{1 + x + x^2; (1 + x)^2; 1 - x^2\}$ spans the vector space $P2$ of polynomials of degree less than or equal to two over **IR**.

d. Determine whether or not the set $S = \{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}\}$ is linearly independent in the **IR**-space M_2(**IR**). Find the dimension of *span(S)*