RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

First Year - Semester I Examination - July/August 2023

MAT 1201 - MATHEMATICAL METHODS I

Time allowed: **Two (2) hours**

Answer ALL (04) questions

1. a) If the position vectors of points A and B are $2\mathbf{i} - 9\mathbf{j} - 4\mathbf{k}$ and $6\mathbf{i} - 3\mathbf{j} + 8\mathbf{k}$, respectively, find $\overrightarrow{AB}$ and a unit vector in the same direction as $\overrightarrow{AB}$.

    (20 marks)

    b) Let $ABCD$ be a parallelogram and P and Q be the mid-points of BC and CD, respectively. If $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$, find, in terms of $\mathbf{a}$ and $\mathbf{b}$, $\overrightarrow{BD}$, $\overrightarrow{AP}$, $\overrightarrow{AQ}$, $\overrightarrow{PQ}$, and $\overrightarrow{PD}$.

    (35 marks)

    c) Determine whether the following vectors are linearly independent or not:

    i. $4\mathbf{i} - 8\mathbf{j}$, $-6\mathbf{i} + 12\mathbf{j}$ in $\mathbb{R}^2$

    ii. $\mathbf{i} + \mathbf{j}$, $\mathbf{j}$, $2\mathbf{i} + \mathbf{j}$ in $\mathbb{R}^2$

    iii. $\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{i} + \mathbf{j}$ in $\mathbb{R}^3$

    Determine which of the above vector sets form a basis for the given space.

    (45 marks)

2. a) If $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, find $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$.

    (20 marks)

    b) Determine the scalar m such that the vector $2\mathbf{i} + \mathbf{j} - m\mathbf{k}$ is perpendicular to the sum of the vectors $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

    (20 marks)

    c) If $\mathbf{u_1}$ and $\mathbf{u_2}$ are unit vectors making an angle $\frac{\pi}{3}$ with each other, find $|\mathbf{u_1} - \mathbf{u_2}|$.

    (20 marks)

    d) Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Determine the vector $\mathbf{r}$ and the scalar λ such that $\mathbf{a} \times \mathbf{r} = \mathbf{b} + \lambda \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{r} = 3$.

    (40 marks)

3. a) Consider the line (L) through the points $(1, 1, -3)$ and $(4, 7, 1)$.

    i. Find a vector equation, parametric equations, and symmetric equations for L.

    ii. Show that the line L is parallel to the line (M) which is given by $\mathbf{r} = \mathbf{i} - \mathbf{k} + \lambda \left(\frac{3}{2}\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \right)$.

    iii. Find the coordinates of the point where M crosses the xy-plane.

    iv. Find the distance between L and M.

    (60 marks)

    b) Find the point of intersection of the given lines $\mathbf{r} = \mathbf{a} - 2\mathbf{b} + t(\mathbf{b} + 2\mathbf{a})$ and $\mathbf{r} = 2\mathbf{a} - \mathbf{b} + u(\mathbf{a} + 2\mathbf{b})$, where $\mathbf{a}$ and $\mathbf{b}$ are linearly independent vectors and t and u are scalar parameters. Obtain an expression for the angle between the lines.

    (40 marks)

4. a) Find the Cartesian equation of the plane containing the point $(4, -1, 3)$ with the normal vector $\mathbf{n} = 2\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$.

(20 marks)

b) Find the equation of the plane passing through the pont $(1, 3, -6)$ that is perpendicular to the line given by:

$$\frac{x-1}{4} = \frac{y}{5} = \frac{z+5}{6}.$$

(25 marks)

c) Show that the plane with vector equation $\mathbf{r} \cdot (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3$ contains the line given by $\mathbf{r} = \mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$, where λ is a scalar parameter.

(30 marks)

d) Find the vector equation of the line passing through the point $(3, 1, 2)$ and perpendicular to the plane $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 4$.

(25 marks)

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