RAJARATA UNIVERSITY

FACULTY OF APPLIED SCIENCES, MIHINTALE

B. Sc. (General) Degree

Third Year - Semester I Examination - October/November 2014

MAT 3319 FLUID MECHANICS

Answer ALL questions **Time allowed: 3 hours**

1)

a) At a point P having spherical polar coordinates (r, θ, ω), fluid velocity is given by $\underline{q} = \left(\frac{\operatorname{cosec}\theta}{r}\right)\underline{e_\theta}$. Verify that motion is incompressible and irrotational with velocity $\underline{q}$. Hence show that velocity potential ϕ exists such that $\phi = -\ln|\tan(\theta/2)|$.

Sketch the streamlines.

b) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the streamlines at time t are the curves

$$(x-t)^2 - (y-t)^2 = \text{constant}.$$

2)

The pressure P of an inviscid incompressible fluid of density ρ with no body force, undergoing irrotational motion, is given by $\frac{P}{\rho} - \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 = F(t)$, where ϕ is the velocity potential. The fluid occupies an infinite domain and contains a spherical gas bubble of radius $R(t)$ in which the pressure is P_g. The pressure in the fluid at infinity is P_∞.

(i) Show that

$$\ddot{R}R + \frac{3}{2}\dot{R}^2 = \frac{P_g - P_\infty}{\rho}.$$

(ii) The bubble contains a fixed mass M of gas in which $P_g = C(M/R^3)^2$ for a constant C. At time $t = 0$, $R = R_0$, $\dot{R} = 0$ and $P_g = P_\infty/2$.

Show that

$$\dot{R}^2 R^3 = \frac{P_\infty}{\rho}\left[{R_0}^3 - \frac{{R_0}^6}{3R^3} - \frac{2}{3}R^3\right].$$

3)

a) Consider a uniform flow of speed U in the negative Oz-direction, superposed with a three-dimensional source of strength m placed at the origin. Find an equation for the dividing stream tube that outlines the Rankine half-body created by the source in this flow field.

b) An infinite ocean of an incompressible perfect liquid of density ρ is streaming past a fixed spherical obstacle of radius a. The velocity is uniform and speed equals to U except in so far it is distributed by the sphere. The pressure in the liquid at a great distance from the obstacle is given as Π.

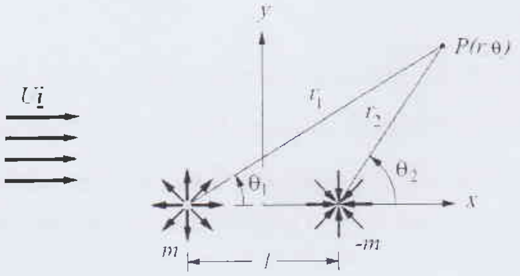
i) Write down the velocity potential of the liquid streaming past the fixed sphere.

ii) Find velocity components at any point of the boundary of the sphere $r = a$.

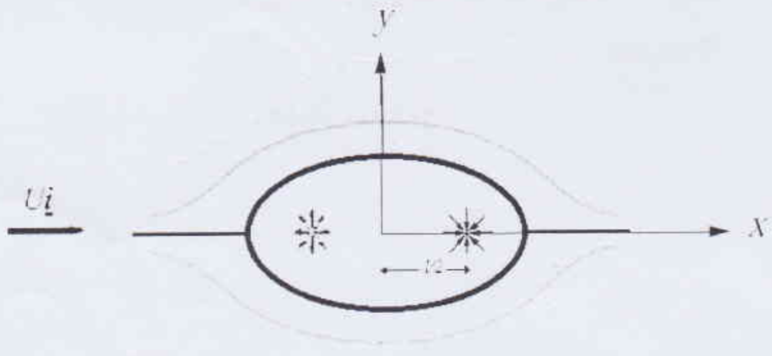
iii) Using the Bernoulli's equation for steady motion in absence of external forces, show that the pressure $P_{(r=a)}$ on the sphere is given by, $P_{(r=a)} = \Pi + \rho U^2/2 - (9/8)\rho U^2 \sin^2\theta$.

4)

a) Write down the velocity potential of the flow field due to a 2D source and a sink placed with distance l apart in a uniform flow of speed U directing $\overrightarrow{Ox}$ as shown in the following figure.



b) Two dimensional flow around the following sketched body (rankine oval) shall be described with the potential theory by superimposing a parallel flow, a source , and a sink with a distance $l/2$ from the center, respectively. Derive the positions of the stagnation points. Also find the equation to determine the contour of the body.



5)

a) A two dimensional doublet of strength $\mu \underline{i}$ is at the point $z = ia$, where a is a real constant, in a stream of velocity $-U\underline{\mathbf{i}}$ in a semi-infinite liquid of constant density occupying the haif-plane $y > 0$ and having x −axis as a rigid boundary. Show that the complex potential of the motion is

$$w = Uz + \frac{2\mu z}{(z^2 + a^2)}.$$

Show also that, for $0 < \mu < 4Ua^2$, there are no stagnation points on this boundary.

b) State The Circle Theorem of Milne – Thomson.

A rectilinear vortex of strength κ is situated at point $A(z=c)$; where c is a complex constant, in an infinite fluid surrounding a fixed circular cylinder of radius a. Here $|c| > |a|$ and there is no circulation in any circuit which does not enclose the vortex. Write down the complex potential due to the vortex and identify the image system. Hence find the velocity of the vortex.