RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES MIHINTALE

B.Sc. (General Degree)

Second year – Semester II Examination-September /October 2013

NUMERICAL ANALYSIS II– MAA 2203

Time allowed: 2 hours only.

Answer four questions — ***Calculators will be provided***

1).

a).Define the Chebyshev polynomials and show that $T_n(x) = \cos(n \cos^{-1} x)$ for $x \in [-1,1]$ and $n \geq 0$.

b).Find the hermite interpolation polynomial p(x) that satisfies:
$P(x_0) = y_0$, $P'(x_0) = d_0$, $P(x_1) = y_1$ and $P'(x_1) = d_1$

c).Find the Lagrange form of the polynomial $Q_2(x)$, that interpolates three points: $(x_0, f(x_0))$, $(x_1, f(x_1))$ and $(x_2, f(x_2))$

2). Torque-speed data for an electric motor is given in the first two columns of the table below. Use the ***Newton forward and backward interpolation formula*** to find the torque at 1800 rpm and 2200 rpm.

Speed ω (rpm x 1000)	Torque (Nm)
0.5	42
1.0	38
1.5	33
2.0	19
2.5	3

3).

i). Suppose $x_0, x_1, x_2, \ldots\ldots x_n$ are distinct numbers in the interval [a,b] and $f \in C^{n+1}$ [a,b]. Then show that for each x in [a,b], a number $\zeta(x)$ in (a,b) exists with

$$f(x) = p(x) + \frac{f^{n+1}(\zeta(x))}{(n+1)!}(x - x_0)(x - x_1)\ldots(x - x_n)$$

where $p(x)$ is the interpolating polynomial given by the equation

$$p(x) = \sum_{k=0}^{n} f(x_k)\, L_{n,k}(x) \text{ and } L_{n,k}(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

ii). Suppose a table is to be prepared for the function f(x)= e^x for x in [0,1]. Assume that the number of decimal places to be given per entry is d$\geq$ 8 and that the difference between adjacent x-values, the step size is h. what should be the value of h for linear interpolation (that is the Lagrange polynomial of degree 1) to give an absolute error of at most 10^{-6} ?

4).

i). The vertical distance covered by a rocket from $t = 8$ to $t = 30$ seconds is given by

$$x = \int_{8}^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

Use the two segment trapezoidal rule to find the distance covered from $t = 8$ to $t = 30$ seconds.

ii). In an attempt to understand the mechanism of the depolarization process in a fuel cell, an electro-kinetic model for mixed oxygen-methanol current on platinum was developed in the laboratory. A very simplified model of the reaction developed suggests a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, T(s) is given by

$$T = -\int_{1.22 \times 10^{-6}}^{0.61 \times 10^{-6}} \left(\frac{6.73x + 4.3025 \times 10^{-7}}{2.316 \times 10^{-11}\, x} \right) dx$$

Use Simpson's rule with n=4 to find the time required for 50% of the oxygen to be consumed.

5).

i). A natural cubic ***spline function*** S on [0,2] is defined by

$$S(x) = \begin{cases} S_0(x) = 1+2x-x^3, & \text{if } 0 \le x \le 1 \\ S_1(x) = 2+b(x-1)+c(x-1)^2+d(x-1)^3, & \text{if } 1 \le x \le 2 \end{cases}$$

Find constants b, c and d.

ii). To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at $61°\text{C}$. The specific heat of water is given as a function of time in the following table.

Temperature, T (°C)	22	42	52	82	100
Specific Heat, C_p $(J\,Kg^{-1}\,{}^0C^{-1})$	4181	4179	4186	4199	4217

Determine the value of the specific heat at $T = 61°\text{C}$ using divided difference method of interpolation.