86

RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year Semester II Examination – February /March 2019

MAT 3310 – INTEGER PROGRAMMING

Time: 03 hours

Answer only FIVE Questions
Calculator is permitted

1. (a) Define an Integer Linear Programming Model. (10 marks)

(b) Explain why Simplex Algorithm cannot be used alone to solve Integer Linear Programming models.

(20 marks)

(c) Spencer Enterprises is attempting to choose new investment alternatives. The potential investment alternatives, the net present value of the future stream of returns, the capital requirements, and the available capital funds over the next three years are summarized as follows:

Alternative	Net Present Value ($)	Capital Requirements ($)		
		Year 1	Year 2	Year 3
Limited warehouse expansion	4,000	3,000	1,000	4,000
Extensive warehouse expansion	6,000	2,500	3,500	3,500
Test market new product	10,500	6,000	4,000	5,000
Advertising campaign	4,000	2,000	1,500	1,800
Basic research	8,000	5,000	1,000	4,000
Purchase new equipment	3,000	1,000	500	900
Capital funds available		10,500	7,000	8,750

(i) Develop an integer programming model to maximize the net present value.

(50 marks)

(ii) Assuming that only one of the warehouse expansion projects can be implemented, modify your model of part (i) to incorporate this assumption.

(10 marks)

(iii) Suppose that, if test marketing of the new product is carried out, the advertising campaign also must be considered. Modify your formulation of part (i) to reflect this new situation.

(10 marks)

2. (a) Briefly explain ***Branch and Bound Algorithm.***

   (b) Solve the following integer programming problem using ***Branch and Bound Algorithm***.

$$\text{Maximize } Z = 100x_1 + 50x_2$$

Subject to

$$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$$

$$15x_1 + 30x_2 \leq 200$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

(80 marks)

3. (a) Briefly explain the cutting plane algorithm used in solving Integer Programming models.

(20 marks)

   (b) Solve the following problem using the ***Dual Fractional Integer Programming Algorithm***:

$$\text{Maximize } Z = -4x_1 - 5x_2$$

Subject to

$$-3x_1 - 2x_2 \leq -7$$

$$-x_1 - 4x_2 \leq -5$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

(80 marks)

4. (a) Briefly explain the steps of ***Dual All Integer - Integer Programming Algorithm***.

(20 marks)

(b) Consider the following Integer Programming Problem (IPP):

$$\text{Maximize } Z = 3x_1 + 8x_2$$

Subject to

$$10x_1 + 10x_2 \leq 9$$

$$10x_1 + 5x_2 \geq 1$$

$$-x_1 + 2x_2 \leq 1$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

(i) Using the algorithm in part (a), show that the above IPP has no feasible solution.

(ii) Verify the result in part (i), using the graphical method.

(80 marks)

5. (a) Briefly explain the steps of ***Primal All Integer- Integer Programming Algorithm***.

(20 marks)

(b) A company produces two types of medal A and B that require gold and silver. Each unit of type A requires 3 grams of silver and 1 gram of gold while each unit of type B requires 1 gram of silver and 2 grams of gold. The company can buy 9 grams of silver and 8 grams of gold. Each unit of type A brings a profit of 40 dollars and each unit of type B brings a profit of 50 dollars.

(i) Formulate an integer programming model to determine the number of units of each type of medal that should be produced to maximize the profit of the company.

(20 marks)

(ii) Use ***Primal All Integer- Integer Programming Algorithm*** to solve the above model.

(60 marks)

6. (a) Write the stepwise form of ***Search Enumeration Algorithm***. (20 marks)

(b) Solve the following problem using the search enumeration algorithm:

$$\text{Maximize } Z = 5x_1 + 6x_2 + 4x_3$$

subject to

$$5x_1 + 3x_2 + 6x_3 \leq 20$$

$$x_1 + 3x_2 \leq 12$$

$$x_1, x_2, x_3 = 0 \text{ or } 1$$

(40 marks)

(c) Consider the following Binary Integer Programming problem (P_1), where the decision variables are $X=(x_1, x_2, x_3, x_4)$.

$$\text{Maximize } Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

Subject to

$$6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$$

$$x_3 + x_4 \leq 1$$

$$-x_1 + x_3 \leq 0$$

$$-x_2 + x_4 \leq 0$$

$$x_i = 0 \text{ or } 1, i = 1, \ldots, 4$$

The following information is given:

(i) The optimal solution to the relaxed linear problem is $X= (5/6, 1, 0, 1)$ with $z=16.5$.

(ii) The optimum solution to P_1 when $x_1 =1$ is $X= (1, 0.8, 0, 0.8)$ with $z=16.2$.

Using above information, find the optimum solution to the integer problem using ***Branch and Bound algorithm.***

(40 marks)

END