



RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year – Semester II Examination – October/November 2017

MAP 2202 – REAL ANALYSIS II

Time : Two (02) hours

Answer all questions
Calculators will be provided

1.

- a) Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. (25 marks)
- b) Using comparison test to find $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ is convergent or divergent. (25 marks)
- c) Use alternating series test to find $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is convergent. (25 marks)
- d) Show that $\int_0^x \left\{ \int_0^t F(u) du \right\} dt = \int_0^x (x - u) F(u) du$ (25 marks)

2.

- a) State the Riemann Integral Theorem and Show that
 - i. $\lim_{n \rightarrow \infty} \left[\frac{1 + 2^{p-1} + \dots + n^{p-1}}{n^p} \right] = \frac{1}{p}$ (20 marks)
 - ii. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[1 + \cos \frac{t}{n} + \cos \frac{2t}{n} + \dots + \cos \frac{(n-1)t}{n} \right] = \frac{1}{x} \sin x$ (20 marks)
 - iii. $\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{n}{n^2 + x^2 k^2} = \frac{\tan^{-1} x}{x}$ (20 marks)

b) State the Mean value theorem for integral and Show that

i. $\int_0^1 \frac{\sin \pi x}{x^2+1} = \frac{2}{\pi(c^2+1)} = \frac{\pi}{4} \sin \pi d$. Where c and $d \in [0,1]$. (20marks)

ii. a and $b > 0$, $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log \frac{b}{a}$. (20marks)

3.

a) Prove the following ,

i. $\Gamma(x+1) = x\Gamma(x)$ (10 marks)

ii. $B(x,y) = B(y,x)$ (10 marks)

iii. $B(x,y) = \int_0^{\frac{\pi}{2}} \sin^{2x-1} \theta \cos^{2y-1} \theta d\theta$, Here $x > 0$ and $y > 0$ (20 marks)

b) Evaluate the following improper integrals:

i. $\int_1^\infty e^{-x^2} dx$ (15 marks)

ii. $\int_{-\infty}^\infty \frac{1}{1+x^2} dx$ (15 marks)

iii. $\int_0^{\frac{\pi}{2}} \sec x dx$ (15 marks)

iv. $\int_0^\pi \sec^2 x dx$ (15 marks)

4.

a) Find the double Riemann sum $\sum \sum_D (3x + 4y) \Delta x \Delta y$, where D is a square

$0 \leq x, y \leq 1$, and $\Delta x = \frac{1}{4}$, $\Delta y = \frac{1}{5}$ (30marks)

b) Let D be the region bounded by the curves $x = y^2$, $x = y + 2$. Evaluate the double integral $\iint_D xy dA$. (35marks)

c) A square plate bounded by the lines $\frac{-1}{2} \leq x \leq \frac{1}{2}$, $\frac{-1}{2} \leq y \leq \frac{1}{2}$ has density $\rho = 1$.

Find the moment of inertia about the origin. (35marks)

END