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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
Second year – Semester II Examination-September /October 2013
Algebra– MAP 2301

Answer six questions. **Time allowed: 3 hours only.**

1). a). Define the following terms.
i. Reflexive relation
ii. Symmetric relation
iii. Transitive relation

b). Let R be a relation on x={1,2,3,4,5} such that $(x, y) \in R$. If x + y < 7. Check whether R is an equivalence relation. Find the equivalence class of each element.

c). An equivalence relation S defined on A={1,2,3,4} contains the pairs (1,1),(1,2),(2,3). Find S, given that S is not the whole $A \times A$.

2). Consider the group $\mathbb{Z}_{12} = \{0,1,2, \ldots ,11\}$
i).List all subgroups of $\mathbb{Z}_{12}$.
ii).Find the order of the elements 1,3,4,7,9
iii).Is $\mathbb{Z}_{12}$ cyclic? If so, find all generators of $\mathbb{Z}_{12}$.

3). a). Let $\mathbb{Z}$ be the set of integers. Define an operation $*$ on $\mathbb{Z}$ by $a* b= a + b - 7$ where $a, b \in \mathbb{Z}$.
i).Show that $(\mathbb{Z},*)$ is a group.
ii).Is $(\mathbb{Z},*)$ abelian? Justify your answer.

b). Let $\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 5 & 4 & 1 & 2 & 7 \end{pmatrix}$ and $\upsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 3 & 7 & 2 & 1 & 4 \end{pmatrix}$ be permutations in S_7 .
i).Write γ and υ as product of disjoint cycles and calculate their orders.
ii).Find γ^{-1} , υ^{-1} and $\gamma\upsilon\gamma^{-1}$
iii).Find α in S_7 such that $\alpha\gamma = \gamma\upsilon$

4). a). Refer to the mappings f: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ and g: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by f(x, y)= $(x^2+1, x+y)$ and g(x,y)=(2x+y). Find the followings:

i).f(1,4) iii). (g∘f)(2,3)

ii).g(1,4) iv). $f^3(1,4)$

b). Solve the congruence, 296 X ≡ 176 (mod 114)

5). i). Show that the equation $ax + by = c$ has integer solutions if and only if $(a,b)|c$. If (x_0, y_0) is solution, then show that all integer solutions are given by

$$x = x_0 + \frac{b}{(a,b)} n \quad \text{and} \quad y = y_0 + \frac{a}{(a,b)} n \quad \text{where } n \in Z$$

ii). Solve the equation 247 x + 91 y =39

iii). Solve the equation 6x + 10 y +15 z =5

6).

i). If a ≡ b (mod m) and c ≡ d (mod m) , then show that ac ≡ bd (mod m)

ii). If a≡ b (mod m) then show that $a^k \equiv b^k$ (mod m) for all non-negative integers k.

iii). Using the Chinese reminder theorem, solve the following system of congruences.

2x≡ 3(mod 5)
4x≡ 1(mod 7)
2x≡ 5(mod 9)

7). a). Define the following terms

i) Abelian group
ii) Normal subgroup
iii) Homomorphism
iv). Sub group

b). Let $\varphi: G \rightarrow H$ be a homomorphism from a group G to a group H.Then show that

i). The kernel of φ is a normal subgroup of G
ii). The image of φ is a subgroup of H.