RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Joint Major) Degree in Chemistry & Physics

Fourth Year - Semester I Examination - March/April 2014

# PHY 4210 – ADVANCED QUANTUM MECHANICS

Answer all four questions — Time: Two hours

**Unless otherwise specified, all the symbols have their usual meaning.**

**(1)** (a) Describe the five basic postulates of quantum mechanics. [10 marks]

(b) Show that every unitary operator is normal. [06 marks]

(c) What can you say about the eigenvalues of an operator that is both Hermitian and unitary? Justify your answer. [09 marks]

**(2)** (a) Prove the following operator identities;

(i) $[\mathbf{AB}, \mathbf{C}] = \mathbf{A}[\mathbf{B}, \mathbf{C}] + [\mathbf{A}, \mathbf{C}]\mathbf{B}$ [05 marks]

(ii) $(\mathbf{AB})^{\dagger} = \mathbf{B}^{\dagger}\mathbf{A}^{\dagger}$ [05 marks]

(b) If **A** and **B** are Hermitian operators, then show that the product **C** = **AB** is Hermitian only if [**A**, **B**] = 0. [07 marks]

(c) If **A** and **B** are integrals of motion, then show that i[**A**,**B**] is also an integral of motion.
Hint: If **A** & **B** are integrals of motion, then they commute with **H**. [08 marks]

**Contd.**

(3) A particle moves in 1-D in the presence of an attractive potential V(x) which is infinite for $x < 0$, is equal to the constant value $-V_0$ in the region $a > x > 0$, and is equal to 0 for $x > a$.

(a) Obtain the functional form of positive energy solutions ($E > 0$) to the energy eigenvalue equation in the three regions of interest. [10 marks]

(b) What are appropriate boundary conditions for this system at $x = 0$ and $x = a$? [06 marks]

(c) Applying the boundary conditions, determine up to a single normalization constant A, the eigenstates of this system for positive energy solutions. For what energies, if any, are the solutions with $E > 0$ square normalizable? [09 marks]

(4) Consider a one-dimensional harmonic oscillator $\mathbf{H} = \dfrac{\mathbf{P}^2}{2m} + \dfrac{1}{2} m\omega_0^2 x^2$ and a trial wave function given by $\psi(x) = \cos(\alpha x)$ for $|\alpha x| < \pi/2$, zero elsewhere.

(a) Calculate the expectation value for the energy using $\psi(x)$. Note that the wave function has to be normalized. [10 marks]

(b) Use the variation method ( i.e. $\alpha$ as a parameter) to obtain an upper bound for the ground state energy. Compare your result with the known exact result. [15 marks]

You may use the results:

$$\int_{-\pi/2\alpha}^{\pi/2\alpha} \cos^2(\alpha x)dx = \pi/2\alpha \quad \text{and} \quad \int_{-\pi/2\alpha}^{\pi/2\alpha} x^2 \cos^2(\alpha x)dx = \pi(\pi^2 - 6)/24\alpha^3$$

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