38

RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

First Year – Semester II Examination – March/ April 2014

MAP 1302DIFFRENTIAL EQUATIONS I

Answer ALL Questions. Time allowed: Three hours.

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1. (a) Suppose that a, b are real numbers and $a > 0$. Show that, every solution of the differential equation $\dfrac{dy}{dx} + ay = be^{-3t}$ goes to 0 as $t \to \infty$. **[20 Marks]**

   (b) Determine whether the given equation, $dx + \left(\dfrac{x}{y} - \sin y\right)dy = 0$ is exact. If so, find the general solution. **[30 Marks]**

   (c) Show that, the substitution $v = \dfrac{y^2}{x}$ converts any differential equation of the form $\dfrac{dy}{dx} = \dfrac{1}{y} f\left(\dfrac{y^2}{x}\right)$ into a separable differential equation. **[20 Marks]**

   Using the above method, find all solutions of the equation $\dfrac{dy}{dx} = \dfrac{1}{y}\left(\dfrac{y^4}{x^2} - \dfrac{y^2}{2x}\right)$. **[30 Marks]**

2. (a) Define the following terms:

   (i). Ordinary differential equation , **[10 Marks]**

   (ii). Order of a differential equation , **[10 Marks]**

   (iii). Degree of a differential equation. **[10 Marks]**

Turn Over

(b) An object having an initial temperature 25^0C is placed in a medium which has the same initial ambient temperature. The ambient temperature of the medium is raised linearly from 25^0C to 30^0C in 5 minutes. According to Newton's law of cooling, the rate of change of the temperature of the object is proportional to the difference between its temperature and the ambient temperature with proportionality constant $-k$, where $k > 0$.

(i) Write a first order differential equation that describes the situation. **[10 Marks]**

(ii) Find the temperature $T(t)$, for $0 \le t \le 5$ in terms of k, by solving the equation in part (a). **[20 Marks]**

(iii) Find, $k \xrightarrow{\lim} 0 \quad T(5)$. **[10 Marks]**

(c) Suppose that you took out collage loans totaling \$60,000 with the interest of 7.5%. You have an online payment plan which continuously deducts money from your bank account at a rate which comes out to \$15,000 per year. Use the differential equation method to estimate the time duration to pay off the loan. **[30 Marks]**

3. (a) Let $F(D) = p_0 D^n + p_1 D^{n-1} + \ldots + p_n$ where $p_0, p_1, \ldots, p_n$ are constants and n is a positive integer. Prove that,

(i) $F(D)e^{ax} = e^{ax}F(a)$, **[15 Marks]**

(ii) $F(D^2)\cos ax = F(-a^2)\cos ax$, **[15 Marks]**

(iii) $\dfrac{1}{D+a} f(x) = e^{-ax} \dfrac{1}{D} e^{ax} f(x)$. **[20 Marks]**

(b) Find the general solutions of the following differential equations:

(i) $(D^3 + D^2 - D - 1)y = \cos 2x$, **[30 Marks]**

(ii) $(D^2 - 3D + 2)y = e^x$. **[20 Marks]**

40

4. (a) Discuss the method of finding the general solution of the Clairaut's equation $y = px + f(p)$, where $f(p)$ is a function of $p = \frac{dy}{dx}$. **[20 Marks]**

Use this method to find the general solution of the equation $y^2 + x^2p^2 - 2xyp = 4p^2$.

[20 Marks]

(b) In the following differential equations , $p = \frac{dy}{dx}$.

(i) Solve the differential equation, $p^2 + 2py\cot x - y^2 = 0$, by first expressing p in terms of x and y. **[20 Marks]**

(ii). Solve the differential equation $6y^2p^2 + 3xp - y = 0$, for x. **[20 Marks]**

(iii).Solve the differential equation $xp^2 + 2xp - y = 0$, for y. **[20 Marks]**

5. Equations of the form, $\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x)$ are called Riccati's equations. If $y_1(x)$ is a known particular solution to a Riccati's equation, then the substitution $v = y - y_1$ will transform the Riccati equation into a Bernoulli equation.

(a) If $v(x) = y(x) - y_1(x)$, express $y(x)$ and $\frac{dy}{dx}$ in terms of v and y_1. **[20 Marks]**

(b) Suppose that $y_1(x)$ is a solution to the Riccati's equation $\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x)$. Make the change of variable $v = y - y_1$ to transform this equation into a Bernoulli equation.

[25 Marks]

Consider the differential equation:

$$\frac{dy}{dx} = y^2 - \frac{y}{x} - \frac{1}{x^2} \ ;x > 0.$$

(i) Show that , $y_1 = \frac{1}{x}$ is a solution of above equation. **[20 Marks]**

(ii) Using the method in part (b), solve the given differential equation. **[35 Marks]**