RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. General Degree in Applied Sciences
First Year - Semester I Examination – March 2021

MAA 1201 – MATHEMATICAL METHODS I

Time: Two (02) hours.

Answer all (04) questions.

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1. (a) If $\mathbf{u} = 2\,\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} + 2\,\mathbf{k}$, find the modulus and direction cosines of $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$. **(30 marks)**

(b) Let $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ be three vectors with their magnitudes 3, 5, and 7, respectively. Suppose that $\mathbf{v} = \mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$.

i. Considering the dot product $\mathbf{a} \cdot \mathbf{v}$, show that $\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} = -9$. Also, find $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c}$ and $\mathbf{a} \cdot \mathbf{c} + \mathbf{b} \cdot \mathbf{c}$.

ii. Determine the angle between $\mathbf{a}$ and $\mathbf{b}$.

(35 marks)

(c) Consider the vectors: $\mathbf{p} = 2\,\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{q} = -\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$, and $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.

i. Find $\mathbf{p} \times \mathbf{r}$.

ii. If $\mathbf{p} \times \mathbf{r} = \mathbf{q} + \mu\,\mathbf{p}$ and $\mathbf{p} \cdot \mathbf{r} = 3$, where μ is a scalar, find μ and $\mathbf{r}$.

(35 marks)

2. (a) Let the line through the points $(1, -1, 3)$ and $(2, -2, 4)$ be L.
Find each of the following:

i. a vector equation for L,

ii. the coordinates of the point where L crosses the xy-plane,

iii. the point of intersection of the lines L and M, where the line M is given by $\mathbf{r} = 2\,\mathbf{i} + 4\,\mathbf{j} + 6\,\mathbf{k} + \mu\,(2\,\mathbf{i} + \mathbf{j} + 3\,\mathbf{k})$, μ being a scalar parameter,

iv. the angle between L and M.

(60 marks)

(b) Obtain an equation of the plane Π that contains the points: $(1, 0, -1)$, $(3,1, 4)$, and $(2, -2,0)$.

Also, find an equation of the plane passing through the point $(1,2,3)$ that is parallel to Π.

(40 marks)

3. The position vector $\mathbf{r} = \mathbf{r}(t)$ of a moving particle at time t is given by the space curve $r = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 3t\,\mathbf{k}$.

(a) Find the components of the velocity and acceleration of the particle at time $t = \pi/4$ in the direction of $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

(40 marks)

(b) Find each of the following at t:

i. the unit tangent vector, **T**,

ii. the principal unit normal vector, **N**,

iii. the unit bi-normal vector, **B**,

iv. the curvature and torsion of the curve.

(60 marks)

4. (a) Consider the function: $\varphi = xy^2 - 4x^2y + z^2$.

i. Find $\nabla\varphi$ at $A(1, -1, 2)$.

ii. Find the directional derivative of φ at A in the direction of $6\mathbf{i} + 2\,\mathbf{j} + 3\mathbf{k}$.

iii. Determine the maximum and minimum values of the directional derivative at A.

iv. What is the direction in which φ increases most rapidly at A?

(55 marks)

(b) If $\mathbf{F} = (z^2 + 2x + 3y)\mathbf{i} + (3x + 2y + z)\mathbf{j} + (y + 2xz)\mathbf{k}$, find curl $\mathbf{F}$, div $\mathbf{F}$, and div(curl $\mathbf{F}$). **(25 marks)**

(c) Given $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \neq \mathbf{0}$ with $r = |\mathbf{r}|$, show that

$$\nabla \times \left(\mathbf{k} \times \nabla\frac{1}{r}\right) = \frac{-3xz\,\mathbf{i} - 3yz\,\mathbf{j} + (x^2 + y^2 - 2z^2)\,\mathbf{k}}{r^5}.$$

(20 marks)

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