RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year - Semester I Examination – September/October 2019

MAT 4306 – OPTIMIZATION MODELLING

Time allowed: Three (03) hours

- Show all work, simplify your answers and write out your work neatly for full credit.
- Answer **SIX** questions only.
- Calculators will be provided.

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1. An individual investor has \$ 70,000 to divide among several investments. The alternative investments are municipal bonds with an 8.5% annual return, certificates of deposit with a 5% return, treasury bills with a 6.5% return, and a growth stock fund with a 13% annual return. The investments are all evaluated after one year. However, each investment alternative has a different perceived risk to the investor; thus, it is advisable to diversify. The investor wants to know how much to invest in each alternative in order to maximize return. The following guidelines have been established for diversifying the investments and lessening the risk perceived by the investor.

   (i) No more than 20%of the total investment should be in municipal bonds.

   (ii) The amount invested in certificates of deposit should not exceed the amount invested in the other three alternatives.

   (iii) At least 30% of the investment shout be in treasury bills and certificates of deposit.

   (iv) To be safe, more should be invested in CDs and treasury bills than in municipal bonds and the growth stock fund by ratio of at least 1.2 to 1.

   The investor wants to invest the entire \$70, 000. Formulate a linear programming model for this problem. (Do not solve).

2. A community council must decide which recreation facilities to conduct in its community. Four recreation facilities have been proposed; a swimming pool, a tennis center, an athletic field, and a gymnasium. The council wants to construct facilities that will maximize the expected daily usage by the residents of the community subject to land and cost limitations. The expected daily and cost and land requirements for each facility are given below.

Recreation Facility	**Expected Usage (people/day)**	**Cost ($)**	**Land Requirements (acres)**
Swimming pool	300	35,000	4
Tennis center	90	10,000	2
Athletic field	400	25,000	7
Gymnasium	150	90,000	3

The community has a $ 120,000 construction budget and 12 acres of land. Because the swimming pool and tennis center must be built on the same part of the land parcel, however, only one of these two facilities can be constructed. The council wants to know which of the recreation facilities to construct to maximize the expected daily usage.

Formulate an integer linear programming model for this problem.

3. The XYZ Company produces two products. The total profit achieved from these products is described by the following equation:

$$\text{Total profit} = -0.2x_1^2 - 0.4x_2^2 + 8x_1 + 12x_2 + 1500,$$

where x_1 = thousands of units of product 1

x_2 = thousands of units of product 2.

Every 1,000 units of x_1 requires one hour of time in the shipping department, and every 1,000 units of x_2 requires 30 minutes in the shipping department. Each unit of each product requires two pounds of a special ingredient, of which 64,000 pounds are available. Additionally, 80 hours of shipping labor are available. Demand for x_1 and x_2 is unlimited.

Formulate a nonlinear programming model for this problem.

4. Transform each of the following optimization problems into regular linear or integer liner programming problems:

(a) Max $x-3|y|-4|z|$

subject to $5x+3y+|z|\geq 10$

$x-4|y|+z\geq 2$

and $x\geq 0$, y, z free.

(b) Min $\dfrac{2x+5y+3z+4}{x+y+z+2}$

subject to $x+y-z\geq 1$

$2x-y+5z\geq 4$

$2x-4y-z\leq 5$

and $x, y, z\geq 0$.

(c) Min $\sum_{j\in J} c_j x_j$

subject to if $\sum_{j\in J} a_{1j}x_j \leq b_1$ holds, then $\sum_{j\in J} a_{2j}x_j \leq b_2$ must hold.

and $x_j\geq 0$ for all $j\in J$.

(d) Min $(\max\{2x+y, 4x-y, -x+10y\})$

subject to $5x+2y\geq 2$

$x-y\leq 4$

and $x, y\geq 0$.

5. Write down the *Nelder and Mead's Simplex Search Algorithm* in step wise form.

Consider the following unconstrained optimization problem:

$$\text{Min } f(x,y)=x-y+2x^2+2xy+y^2.$$

Using $x^{(1)}=(4, 4)^T$, $x^{(2)}=(5, 4)^T$, and $x^{(3)}=(4, 5)^T$ as the initial simplex of three points and assuming $\beta=0.5$ and $\gamma=2$, perform **two** iterations of Nelder and Mead's simplex search algorithm.

6. Write down the KKT conditions for the following optimization problem;

$$\text{Min } f(x)$$
$$\text{Subject to } g_i(x) \geq 0 \text{ for } i = 1, 2, ..., p;$$
$$h_j(x) = 0 \text{ for } j = 1, 2, ..., q,$$

where $f, g_i, h_j : \Re^n \to \Re$ for $i = 1, 2, ..., p$ and $j = 1, 2, ..., q$.

Consider the following optimization problem:

$$\text{Min } f(x, y) = \left(x - \frac{9}{4}\right)^2 + (y-2)^2$$
$$\text{Subject to } g_1(x, y) = y - x^2 \geq 0$$
$$g_2(x, y) = x + y \leq 6$$
$$\text{and } x, y \geq 0.$$

write the Kuhn-Tucker necessary optimality conditions and verify that these conditions are satisfied at the point $x = \left(\frac{3}{2}, \frac{9}{4}\right)^T$

7. (a) Consider the function $f : \Re^2 \to \Re$ given below;

$$f(x, y) = x^2 + 3x + 6xy + 5y + 7y^2 + 6$$

(i) Find the gradient and Hessian of f at the point $(1, 1)^T$.

(ii) Find the directional derivative of f at the point $(1, 1)^T$ with respect to a unit vector in the direction of maximal rate of increase.

(b) Consider the following problem:

$$\text{minimize } f(x, y) = 2x^2 + y^2 + 6y + 9$$

$$\text{subject to } x, y \geq 0.$$

(i) Is the first-order necessary condition (FONC) for a local minimizer satisfied at $x^{(1)} = (1, 3)^T$? Justify your answer.

(ii) Is the FONC for a local minimizer satisfied at $x^{(1)} = (1, 0)^T$? Justify your answer.

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