RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year - Semester I Examination – June/July 2018

MAA 1201 – MATHEMATICAL METHODS I

Time: Two (02) hours

Answer ALL Questions

1. a) Show that the vector expression $4\overrightarrow{AB} - \overrightarrow{CB} - 4\overrightarrow{AC}$ is equal to $3\overrightarrow{CB}$.

[15 marks]

b) Define the linear independence of vectors and determine whether the following vectors in $\mathbf{R}^3$ are linearly independent:

$$x_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, \quad x_2 = \begin{bmatrix}-2\\1\\0\end{bmatrix}, \quad x_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}$$ **[20 marks]**

c) In the usual notation, $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + z\hat{k}$ and $\vec{b} = 4\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$. If $\vec{c} = 3\vec{a} - 2\vec{b}$ is on the *Oxy* plane, find the value of z.

d) OABC is a parallelogram. P is the midpoint of OA and the point D divides PC in the ratio 1:2. Prove that O, D and B are collinear.

[Hint: show that $\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OB}$] **[40 marks]**

2. a) Let A ≡ (-2, 1, 2), B ≡ (-2, 1, 1), C ≡ (2, 4, 4). Find the: .

(i) area of the triangle ABC, **[10 marks]**

(ii) unit vector perpendicular to the plane ABC, **[10 marks]**

(iii) perpendicular distance from the point (1, -3, 6) to the plane ABC.

[15 marks]

[*Turn over*]

b) Find the value of λ such that the vector $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$, $\vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{c} = 4\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$ are coplanar. **[10 marks]**

c) Prove that $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

[Hint: Consider $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$]. **[25 marks]**

d) If $\frac{d\vec{a}}{dt} = \vec{u} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{u} \times \vec{b}$, then prove that $\frac{d}{dt}\left[\vec{a} \times \vec{b}\right] = \vec{u} \times \left(\vec{a} \times \vec{b}\right)$.

[30 marks]

3. a) Sketch the curve given by $\vec{r} = \cos\theta\hat{\imath} + \sin\theta\,\hat{\jmath} + 5\hat{k}$, where θ is the angle to the positive Ox axis. **[20 marks]**

b) A particle moves on the Oxy plane, where its position vector at time t is given by $\vec{r} = \cos\omega t\,\hat{\imath} + \sin\omega t\,\hat{\jmath}$, where ω is a constant.

Show that

(i) the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ **[20 marks]**

(ii) show that $\vec{r} \times \vec{v}$ is a constant vector. **[10 marks]**

c) Consider the space curve given by $\vec{r} = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath} + 4t\,\hat{k}$.

Find

(i) the unit tangent T, **[15 marks]**

(ii) the principal normal N, and **[15 marks]**

(iii) curvature k and radius of curvature ρ. **[20 marks]**

4. a) If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ (or grad ϕ) at the point (1, -2, -1).

[30 marks]

b) If $u = x^2 + y^2 + z^2$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then find $\boldsymbol{div}\ (\boldsymbol{u\vec{r}})$ in terms of u.

[30 marks]

c) Find the scalar potential function f for $\vec{A} = y^2\hat{\imath} + 2xy\hat{\jmath} - z^2\hat{k}$.

[40 marks]

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