RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year - Semester I Examination – Jan/Feb 2021

MAT 4302 – FINANCIAL MATHEMATICS

Time: Three (03) hours.

- Answer **all (05)** questions.
- Students may use calculators and Normal Probability Tables.

1.

(a) Shama makes deposits of \$100 at time 0, and \$$X$ at time 3. The fund grows at a force of interest

$$\delta_t = \frac{t^2}{100}, \quad t > 0.$$

The amount of interest earned from time 3 to time 6 is also X. Calculate X.

(b) Nimal deposits \$100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semiannually. At the same time, Kamala deposits \$100 into a separate account. Kamala's account is credited interest at a force of interest of δ. After 7.25 years, the value of each account is the same. Calculate δ.

(c) Jagath can receive one of the following two payment streams:

(i) \$100 at time 0, \$200 at time n, and \$300 at time $2n$
(ii) \$600 at time 10

At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.76$, determine i.

(100 marks)

2.

(a) Define clearly what is meant by a European put option on a stock.

(b) What are the six factors affecting the price of a stock option?

(c) Write down clearly two portfolios you consider when you derive the put-call parity formula.

(d) Hence derive the put-call parity formula with the usual notations.

(e) Write down lower and upper bounds for the price difference between American call and put options, $C - P$, defined on the same no dividend-paying stock.

(f) Modify the above two formulas in part (d) and Part (e) for the dividend-paying stock case.

(g) A one-month European put option on a non-dividend-paying paying stock is currently selling for $ 2.50. The stock price is $ 47, the strike price is $ 50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur?

(h) The price of a European call that expires in six months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rate being 10%. What is the price of a European put option that expires in six months and has a strike price of $30?

(100 marks)

3.

(a) Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European options using a one-step binomial tree.

(b) What is meant by the delta of a stock option?

(c) A stock price is currently $100. Over each of the next two six -month periods it is expected go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding.

(i) Compute the risk-neutral probability p.

(ii) What is the value of a one-year European call option with a strike price of $100.

(100 marks)

4.

(a) Explain two ways in which a bear spread can be created.

(b) When is it appropriate for an investor to purchase a butterfly spread?

(c) What is the difference between a strangle and a straddle?

(d) Suppose that put options on a stock with strike prices \$30, and \$35 cost \$4 and \$7, respectively. How can these options be used to create

(i) A bull spread?
(ii) A bear spread?
(iii)Construct a table that shows the profit and payoff for both spreads.

(e) A call option with a strike price of \$50 costs \$2. A put option with a strike price of \$45 costs \$3. Explain how a strangle can be created from these two options. Construct a table that shows the profit from this strangle.

(f) Use put-call parity to relate the initial investment for a bull spread created using calls to the initial investment for a bull spread created using puts.

(100 marks)

5.

(a) Write down the Black-Scholes-Merton differential equation for a value of derivative f.

(b) Write down five main assumptions you made to derive above formula in part (a).

(c) Write down the complete Black-Scholes-Merton pricing formulas for European call and put options.

(d) Using Black-Scholes-Merton pricing formulas, find the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5% per annum, the volatility is 30% per annum, and the time to maturity is six months.

(100 marks)

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