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RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

First Year-Semester I Examination-April/May 2015

MAP 1301 – LINEAR ALGEBRA

Proper Candidates (with Mid – Semester Marks): Time Allowed: **Two Hours**
Answer **Three** questions **including Qu.No.1**.

Other Candidates (without Mid-Semester Marks): Time Allowed: **Three Hours**
Answer **Five** questions **including Qu.No.1**.

1.

i. Let A be the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{pmatrix}$$

a. Compute the determinant of A. (write the answer in terms of t). [20 marks]

b. For what values of t is A invertible? [20 marks]

ii. Consider the following basis for $\mathbb{R}^2$.

$$E = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right\}$$

a. Find the coordinates for the vector $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$, in terms of the basis E. [25 marks]

b. Let $L: \mathbb{R}^2 \to \mathbb{R}^2$ be the following linear transformation:

$$L(x, y) = (2x - y, 3x - 2y)$$

Find the matrix representing L with respect to the basis E . [25 marks]

c. Prove that for any three linearly independent vectors u, v and w , the vectors u-v , u-w , w + u will also be a linearly independent of vectors. [25 marks]

d. Let V and W be the subspaces of $\mathbb{R}^2$spanned by (1,1) and (1,2), respectively. Find vectors v ∈ V and w ∈ W so v + w = (2, −1). [25 marks]

2. Let **A** be the matrix given below:

$$\begin{pmatrix} 1 & 0 & 6 & 0 & 0 \\ 0 & 1 & 0 & -9 & 0 \\ 0 & -3 & 0 & 27 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

a) Find the reduced row **echelon matrix** associated with **A** . [20 marks]

b) Put **A** into its normal form, **N**. [20 marks]

c) What is the column rank of **A**?. [20 marks]

d) Find matrices **P** and **Q** such that **N=PAQ**. [20 marks]

3.

a. Consider the system of linear equations $A\underline{x} = \underline{b}$, where λ and μ are constants:

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 5 & 1 & \lambda \\ 1 & -1 & 1 \end{pmatrix} \quad \underline{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \underline{b} = \begin{pmatrix} 2 \\ 7 \\ \mu \end{pmatrix}$$

Compute the **determinant** of A. Determine for which values of λ and μ this system has:

i. a unique solution [15 marks]

ii. no solutions [15 marks]

iii. infinitely many solutions. [15 marks]

b. Consider the system of equations

$$x + y - z = a$$
$$x - y + 2z = b$$

i. Find the general solution of the **homogeneous** equation. [15 marks]

ii. Find a particular solution of the **inhomogeneous** equation when $a = 3$ and $b = 6$. [20 marks]

4.

a. Suppose a matrix **A** has eigenvalues 2 and 3. What is the characteristic polynomial of $\mathbf{A}^{-1}$? [10 marks]

b. Suppose A^2= I. What are the possible characteristic polynomials of A? [10 marks]

c. Suppose A has eigenvalues 2 and 3. What are the eigenvalues of A^T? [20 marks]

d. The matrix, $M = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -3 & 1 \\ 3 & -5 & 3 \end{bmatrix}$

i. Given that $\underline{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\underline{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ are eigenvectors of M, find the eigenvalues corresponding to $\underline{u}$ and $\underline{v}$. [20 marks]

ii. Given also that the third eigenvalue of M is 1, find a corresponding eigenvector. [20 marks]

02

5.

a. Let $A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$. and define $T:\mathbb{R}^2\to\mathbb{R}^2$ by $T(\underline{x})=A\underline{x}$. Find the $T(u)$ where $\underline{u}=\begin{bmatrix}1\\ -3\end{bmatrix}$.

[20 marks]

b. Let $\underline{b}=\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}$, and let A be the matrix $\begin{bmatrix}1 & -3 & 5 & -5\\ 0 & 1 & -3 & 5\\ 2 & -4 & 4 & -4\end{bmatrix}$. Is $\underline{b}$ in the range of the linear transformation $\underline{x}\mapsto A\underline{x}$?

[20 marks]

c. Let $T:\mathbb{R}^2\to\mathbb{R}^2$ be a linear transformation that maps $\underline{u}=\begin{bmatrix}3\\ 4\end{bmatrix}$ into $\begin{bmatrix}4\\ 1\end{bmatrix}$ and maps $\underline{v}=\begin{bmatrix}3\\ 3\end{bmatrix}$ into $\begin{bmatrix}-1\\ 3\end{bmatrix}$. Use the fact that T is **linear** to find the images under T of $2\underline{u}, 3\underline{v}$ and $2\underline{u}+3\underline{v}$.

[20 marks]

d. Given $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(\underline{e}_1)=\begin{bmatrix}1\\ 4\end{bmatrix}$, $T(\underline{e}_2)=\begin{bmatrix}-2\\ 9\end{bmatrix}$ and $T(\underline{e}_3)=\begin{bmatrix}3\\ -8\end{bmatrix}$, where $\underline{e}_1,\underline{e}_2$ and $\underline{e}_3$ are the columns of the 3×3 identity matrix. Assuming that T is a linear transformation, find the *standard matrix A* of T.

[20 marks]

6.

i. Let $\underline{v}_1=\begin{bmatrix}2\\ 3\\ -5\end{bmatrix}$ and $\underline{v}_2=\begin{bmatrix}-4\\ -5\\ 8\end{bmatrix}$. Is the vector $\underline{v}_3=\begin{bmatrix}8\\ 2\\ -9\end{bmatrix}$ in $Span\{\underline{v}_1,\underline{v}_2\}$?

[30 marks]

ii. Find a basis for the null space and the column space of the matrix

$$B=\begin{pmatrix}1 & 2 & 1 & 3 & 0\\ 0 & 1 & 1 & 1 & -1\\ 1 & 3 & 2 & 0 & 1\end{pmatrix}.$$

[25 marks]

iii. Find the rank of B and Verify that $rank(B)+nullity(B)=\dim(B)$.

[25 marks]