

**RAJARATA UNIVERSITY OF SRILANKA, MIHINTALE****FACULTY OF APPLIED SCIENCES****B.Sc. (General) Degree****Third Year-Semester II Examination- October/ November 2014****MAT 3205-INTRODUCTION TO STATISTICAL DECISION THEORY****Answer All Questions****Time: Two Hours**

1. A florist orders and receives batches of a dozen roses early in the morning each day. He then sells them in batches of a dozen roses only and does not sell them in individual pieces. The cost per dozen is \$10.00 and he sells it for \$18.00. Based on the previous year's sales statistics, the florist estimates his daily sale of roses (in dozens) as follows:

Demand	Probability of demand
15	0.4
16	0.3
17	0.2
18	0.1

Any unsold roses have to be thrown away at the end of the day.

- Construct a payoff table.(hint: take demand level be the states of nature and sales order level be the alternatives)
- How many dozens of roses should the florist stock each day so as to maximize his profits in the long run?(hint: use EMV method or EOL method)
- Find the expected value of perfect information.

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2. a) Briefly describe:

- i. Prior probability.
- ii. Posterior probability.

b) A businessman wants to construct a hotel in a city. He generally builds 3 types of hotels. These are 50 rooms, 100 rooms and 150 rooms hotels, depending upon the demand for the rooms which is a function of the area in which the hotel is located and traffic flow. The demand can be categorized as low, medium or high. Depending upon these various demands, the businessman has made some preliminary assessment of his net profits and possible losses (in 1000 rupees) for these various types of hotels. These payoffs are shown in the following table.

	State of nature Demand for rooms		
	Low (A1)	Medium (A2)	High (A3)
R1 = 50	25	35	50
R2 = 100	-10	40	70
R3 = 150	-30	20	100

The businessman has also assigned "prior probabilities" to the demand structure or rooms as follows.

Demand for rooms	Probability of demand
Low (A1)	0.2
Medium (A2)	0.5
High (A3)	0.3

Now the hotelier must decide whether to gather additional information regarding the state of nature, so that these states can be predicted more accurately than the preliminary assessment.

Suppose that the businessman asks a consultant to study the market and predict the state of nature more accurately. The consultant made some studies and came up

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with the estimates of low demand (X1), medium demand (X2) and high demand (X3) with the degree of reliability in these estimates. This degree of reliability is expressed as conditional probability which is the probability that the consultant's estimate of low demand will be correct and the demand will be actually low.

Similarly, there will be a conditional probability of the consultant's estimate of medium demand, when actually the demand is low and so on. These conditional probabilities are expressed in the following table.

State of nature	Conditional probabilities		
	X1	X2	X3
A1	0.5	0.3	0.2
A2	0.2	0.6	0.2
A3	0.1	0.3	0.6

For example probability that the consultant's prediction will be for low demand (X1) when the demand is actually low is 0.5. Similarly, probability that the consultant's prediction will be for medium demand (X2) when the demand is actually low is 0.3 and so on. For example, $P(X_1 | A_1) = 0.5$.

- i. Find the best decision using EMV.
 - ii. Find the posterior probabilities and using Bayesian analysis determine the optimal hotel type.
3. Hero Motorcycle Company is interested in diversifying into bicycle manufacturing. The president is not sure as to whether to open a small plant or large plant. The market demand is uncertain and will become known only after the plant has been built. If the demand is indeed high and a small plant is built initially, then it can be expanded to accommodate high demand. The marketing department has estimated that probability for high demand is 0.7 and for low demand such probability is 0.3. A cost-benefit analyst has provided the following information:

Cost of building a large plant:	\$8million
Cost of building a small plant:	\$4 million
Cost of expanding small plant:	\$3 million
Revenue from high demand for large plant or small expanded plant:	\$18 million
Revenue from high demand without expansion for small plant:	\$11 million
Revenue from low demand:	\$8 million

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- i. Construct a decision tree for Hero Motorcycle Company.
 - ii. Find the expected value for each node.
 - iii. What should be the optimal policy?
4. a) Briefly describe the three types of decision makers in utility.
- b) Company A is planning to build a condominium development in a city. The company is trying to decide between building a small, medium or large development. The payoffs received for each size of development will depend on the market demand for condominiums in the area, which could be low, medium or high. The payoff table of this problem is:

<i>Size of development</i>	<i>Market demand</i>		
	<i>low</i>	<i>medium</i>	<i>high</i>
<i>Small</i>	400	400	400
<i>Medium</i>	200	500	500
<i>large</i>	-400	300	800

The owner of the company estimates a 20% chance that market demand will be low, a 30% chance that it will be medium, and a 50% chance that it will be high. Suppose that the utility function for the owner of company A can be approximated by exponential utility function:

$$U(x) = 1 - e^{-x/R}, \text{ where risk tolerance value } R \text{ is } 100.$$

- i. Convert this payoff matrix to utility values.
- ii. What decision provides the owner of the company with the largest expected utility?