RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Third Year - Semester II Examination – Jan/Feb 2023

MAT 3310 – INTEGER PROGRAMMING

Time: Three (03) hours

Answer **all** questions.
Calculators will be provided.

1. Consider the following integer programming problem:

$$Max\ Z = -4x_1 - 5x_2$$
$$S.t.$$
$$x_1 + 4x_2 \geq 5$$
$$3x_1 + 2x_2 \geq 7$$
$$x_1, x_2 \geq 0\ \&\ integer.$$

a) Rewrite the above integer programming problem in Canonical Form. **(10 marks)**

b) Convert the part a) problem into the Standard Form by introducing slack or surplus (s_i) variables. **(10 marks)**

c) The LP-Relaxation of the Part b) problem has been solved using the Dual Simplex Algorithm and the resulting final simplex table is given below:

#	x_1	x_2	s_1	s_2	RHS
x_2	0	1	$-3/10$	$1/10$	$8/10$
x_1	1	0	$2/10$	$-4/10$	$18/10$
$-Z$	0	0	$-7/10$	$-11/10$	$112/10$

Obtain the optimal solution of the LP-Relaxation from the given simplex table. **(10 marks)**

d) Determine the solution to the given integer programming problem using the Factional Cutting Plane Algorithm. **(70 marks)**

2. a) Define variables and then formulate constraints for each of the following conditions in an employee selection model. The employees are numbered as 1, 2, ... and 10.

   i. Exactly one employee from the set (1, 3, 5) must be selected.

   ii. Employee number 2 can be selected only if employee number 9 is selected. However, employee number 9 can be selected without employee number 2 being selected.

   iii. No more than two employees from the set (2, 4, 6, 8, 10) can be selected.

   iv. If employee number 3 is selected, employee number 9 cannot be selected.

   v. Employee numbers 5 and 10 must be selected or both be deselected.

(25 marks)

b) Solve the following integer programming problem using the Branch and Bound Algorithm. Given that the solution of the LP-Relaxation is $x_1^* = 7.4, x_2^* = 6.7, x_3^* = 2.4$ and $Z^* = 17.6$.

$$Max\ Z = -x_1 + 3x_2 + 2x_3$$
$$S.t.$$
$$3x_1 - 3x_2 + 2x_3 \leq 7$$
$$-x_1 + 2x_2 \leq 6$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1 \in \{0,1\}$$
$$x_2 \in Z^+$$
$$x_3 \geq 0$$

(75 marks)

3. a) Solve the following binary integer programming problem using the Branch and Bound Algorithm. Given that the solution of the LP-Relaxation is $x_1^* = 3.75, x_2^* = 3.33, x_3^* = 0$ and $Z^* = 53.75$.

$$Max\ Z = 9x_1 + 6x_2 + 5x_3$$
$$S.t.$$
$$2x_1 + 3x_2 + 7x_3 \leq \frac{35}{2}$$
$$4x_1 + 9x_3 \leq 15$$
$$x_1, x_2, x_3 \in \{0,1\}$$

(75 marks)

b) Verify the solution of Part a) by the Implicit Enumeration Method.

(25 marks)

4. a) Compare and contrast the Branch and Bound Algorithm and the Cutting Plane Algorithm. **(20 marks)**

   b) Solve the following problem using the Fractional Mixed Integer Cutting Plane Algorithm.

$$Min\ Z = -5x_1 - 6x_2 - 4x_3$$
$$S.t.$$
$$5x_1 + 3x_2 + 6x_3 \leq 20$$
$$x_1 + 3x_3 \leq 12$$
$$x_1, x_3 \geq 0$$
$$x_2 \geq 0\ and\ an\ integer.$$

(80 marks)

5. a) Briefly explain the steps of the Primal All Integer – Integer Programming Algorithm. **(20 marks)**

   b) A company produces two types of medals A and B that require gold and silver. Each unit of type A requires 2 grams of silver and 3 grams of gold while each unit of type B requires 5 grams of silver and 2 grams of gold. The company can buy 17 grams of silver and 10 grams of gold. Each unit of type A brings a profit of 20 dollars and each unit of type B brings a profit of 10 dollars.

   i. Formulate an integer programming model to determine the number of units of each type of medal that should be produced to maximize the profit of the company.

   ii. Use the Primal All Integer – Integer Programming Algorithm to solve the model formulated in Part i.

(80 marks)

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