RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester II Examination – July 2020

PHY 3302 – MATHEMATICAL METHODS FOR PHYSICISTS

Time: Three (03) hours

Answer Any 5 Questions.

Unless otherwise specified, symbols have their usual meaning.
A non-programmable calculator is permitted.

---

1. a) Find the magnitude and unit vector of the following vectors.

i. $A = 3i + 4j$
ii. $B = 4i - 5j + 6k$
iii. $C = ki - kj$

(03 Marks)

b) Find the angle between the vectors $\boldsymbol{A} = 3\boldsymbol{i} + 6\boldsymbol{j} + 9\boldsymbol{k}$ and $\boldsymbol{B} = -2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

(03 Marks)

c) Find the cross product of the vectors $\boldsymbol{A} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{B} = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$

(04 Marks)

d) Using the vector triple cross product $\{A \times (B \times C) = (A.C)B - (A.B)C\}$, find the value of Jacobi's identity for vector products $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) + \boldsymbol{B} \times (\boldsymbol{C} \times \boldsymbol{A}) + \boldsymbol{C} \times (\boldsymbol{A} \times \boldsymbol{B})$

(04 Marks)

e) Find the volume of the parallelogram (A,B,C) where A=(5,-1,1), B=(-2,3,4), C=(3,4,5)

(06 Marks)

2. a) Draw three diagrams to illustrate the difference between point P in Cartesian P(x, y, z), Cylindrical P(ρ, θ, z) and Spherical P(r, θ, ϕ)coordinates.

(02 Marks)

Contd.

b) Write down the relation between Cartesian / cylindrical coordinates, Cartesian/ spherical coordinates and calculate
i. (3, $\pi/3$, -4) from cylindrical to Cartesian
ii. (-2, 2, 3) from Cartesian to cylindrical
iii. (8, $\pi/4$, $\pi/6$) from spherical to Cartesian
iv. ($2\sqrt{3}$, 6, -4) from Cartesian to spherical

(08 Marks)

c) Convert vectors
i. $x^2 + y^2 = 25$ from Cartesian to cylindrical coordinates
ii. $x^2 + y^2 - z^2 = 1$ from Cartesian to spherical coordinates
iii. $r = 2cos\emptyset$ from spherical to cylindrical coordinates
iv. $\rho = 2sin\theta$ from cylindrical to Cartesian coordinates
v. $rsin\theta = 1$ from spherical to Cartesian coordinates

(10 Marks)

3. a) Show that

$$det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -det\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

(02 Marks)

b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{vmatrix} 2 & 7 \\ -1 & -6 \end{vmatrix}$$

(04 Marks)

c) Let the matrix **A** and **B** are

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 5 & 6 \end{pmatrix}$$

Find the matrix multiplications of **AB** and **BA**

(04 Marks)

d) Find the values of x_1, x_2 *and* x_3 using the knowledge of inverse matrix which satisfies the equations

$$-x_1 + 3x_2 - 2x_3 = 7$$
$$3x_1 + 3x_3 = -3$$
$$2x_1 + x_2 + 2x_3 = 1$$

(10 Marks)

4. a) Write down two different formats representing complex value **Z** using polar coordinates (r, θ) and Euler's formula.

(02 Marks)

b) Make the denominator a real number

$$\frac{2+5i}{1-3i}$$

(02 Marks)

Contd.

c) Solve $\sqrt[3]{27}$ (cubic root of 27) using De Moivre's theorem of roots of a complex number and write d●wn values for k = 1, 2 and 3.

(06 Marks)

d) i. Find $(1+i)^8$

ii. Show that $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(04 Marks)

e) Verify that the following complex number is holomorphic using the Cauchy – Riemann relationship

$$z^2 = (x^2 - y^2) - 2xyi$$

(06 Marks)

5. a) Show that $y(x) = x^{-2/3}$ is a solution for

$$4x^2y'' + 12xy' + 3y = 0 \; for \; x > 0$$

(02 Marks)

b) Solve the following equation

$$y' = \frac{3x^2 + 4x - 4}{2y - 4} \quad where \; y(1) = 3$$

(02 Marks)

c) Show that $(4x - 3y)dx + (-3x + 5)dy = 0$ is an exact differential equation and solve the equation.

(04 Marks)

d) A copper ball with temperature 100 ^{o}C is dropped into a large water tank with temperature 30 ^{o}C. After 3 minutes the temperature of the ball has decreases to 70 ^{o}C. The Newton's law of cooling is stated below, where T is the temperature of the ball, t is the time expends, K is an arbitrary constant and T_w is the temperature of water. (Assume that temperature of the water will not increase)

$$\frac{dT}{dt} = K(T - T_w)$$

i. Obtain an expression for the temperature of the ball (T) after t time.
ii. Calculate the value of constant K.
iii. How long will it take for the ball to reach the temperature of 31 ^{o}C?

(12 Marks)

Contd.

6. a) The Fourier series of a periodic function $f(x)$ of period T is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nx}{T} + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi nx}{T}$$

Where the coefficients are given by,

$$a_n = \frac{2}{T}\int_0^T f(x)\cos\frac{2\pi nx}{T}\,dx$$

$$b_n = \frac{2}{T}\int_0^T f(x)\sin\frac{2\pi nx}{T}\,dx$$

Calculate the Fourier series for the following periodic signal

$$f(x) = \begin{cases} A, & 0 < x \le \pi \\ -A, & \pi < x \le 2\pi \end{cases}$$

(10 Marks)

b) Suppose that $f(x)$ has a power series expansion at $x = a$ with radius of convergence $R > 0$. Then the Tayler series expansion of $f(x)$ takes the form

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

i. Calculate the Tayler series and Maclaurin series of the function $f(x) = cos(x)$

ii. Calculate the Tayler series and Maclaurin series of the function $f(x) = sin(x)$

(10 Marks)

End.