RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science Honours in Industrial Mathematics
Fourth Year - Semester I Examination – August/September 2023

MAT 4306 – Optimization Modelling

Answer **ALL** questions **Time: Three (3) hours**

1. The DSI Company manufactures two products: slippers and shoes. To manufacture a pair of slippers, 1.5 machine hours and 2.5 labour hours are required. To manufacture a pair of shoes, 2.5 machine hours and 1.5 labour hours are required. In a month, 300 machine hours and 240 labour hours are available. The profit per pair of slippers and pair of shoes is Rs. 50 and Rs. 40 respectively.

   a) Formulate an LP model to maximize the total monthly profit. **(40 marks)**
   b) Solve it by using Excel Solver. **(60 marks)**

2. A firm has 3 factories: A, E, and K. There are four major warehouses situated at B, C, D, and M. Average daily product at A, E, and K is 30, 40, and 50 units respectively. The average daily requirement of this product at B, C, D, and M is 35, 28, 32, and 25 units respectively. The transportation cost (in Rs.) per unit of product from each factory to each warehouse is given below:

Factory	Warehouse			
	B	C	D	M
A	6	8	8	5
E	5	11	9	7
K	8	9	7	13

   Determine a routing plan that minimizes the total transportation costs using Excel Solver. **(100 marks)**

3. Maximize $Z = 2x_1 + x_2$,
   subject to the constraints
   $2x_1 + 5x_2 \leq 17$
   $3x_1 + 2x_2 \leq 10$
   $x_1, x_2 \geq 0$

   a) Solve the above problem as an LP problem. **(50 marks)**
   b) If the optimal solutions are not integer-valued, use the Fractional Cutting Plane Algorithm to derive the optimal integer solutions. **(50 marks)**

4. Wheat is harvested in the Midwest and stored in grain elevators in three different cities: Kansas City, Omaha, and Des Moines. These elevators supply grain to three flour mills, located in Chicago, St. Louis, and Cincinnati. Grain is shipped to the mills in railroad cars, each of which is capable of holding one ton of wheat. Each grain elevator can supply the following number of tons (i.e., railroad cars) of wheat to the mills on a monthly basis:

Grain Elevator	Supply
Kanas City	150
Omaha	175
Des Moines	275
Total	600 tons

Each mill demands the following number of tons of wheat per month.

Mill	Demand
Chicago	200
St. Louis	100
Cincinnati	300
Total	600 tons

The cost of transporting one ton of wheat from each grain elevator (source) to each mill (destination) differs according to the distance and rail system. These costs are shown in the following table. For example, the cost of shipping one ton of wheat from the grain elevator at Omaha to the mill in Chicago is $7.

Grain Elevator	Mill		
	Chicago	St. Louis	Cincinnati
Kanas City	$6	$8	$10
Omaha	$7	$11	$11
Des Moines	$4	$5	$12

The problem is to determine how many tons of wheat to transport from each grain elevator to each mill on a monthly basis to minimize the total cost of transportation.

5. Solve the following NLP problem using Lagrange Multiplier Method:

Optimize the function $Z = {x_1}^2 + {x_2}^2 + {x_3}^2$
subject to the following constraints
$x_1 + x_2 + 3x_3 = 2$
$5x_1 + 2x_2 + x_3 = 5$
$x_1, x_2, x_3 > 0$

(100 marks)

6. Four precision components are to be shaped using four machine tools, one tool is assigned to each component. The machining times in minutes, are given in the table below. Find the best solution assignment using Excel Solver to minimize the total time.

Machine Tool	Component			
	1	2	3	4
A	21	20	39	36
B	25	22	24	25
C	36	22	36	26
D	34	21	25	39

(100 marks)

END