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RAJARATA UNIVERSITY OF SRI LANKA, MIHINTHALE
FACULTY OF APPLIED SCIENCES

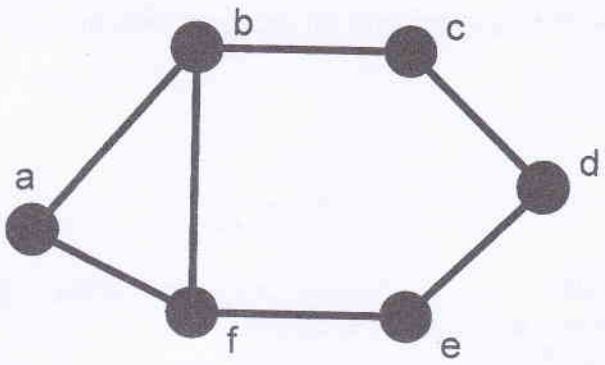
B.Sc. General Degree in Applied Sciences
Third Year-Semester 1 Examination- October/Nov. 2015
MAT-3213 GRAPH THEORY

Answer all Questions **Time: Two hours**

1.

a. Define a graph G such that V(G)={2,3,4,5,1,12,13,14} and two vertices 's' and 't' are adjacent if and only if gcd{s,t}=1. Draw a diagram of G and find its size e (G).

b. Find the adjacent matrix of the following graph G.



c. The adjacency matrix of a multigraph G is shown below:

$$\begin{pmatrix} 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 1 & 2 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 1 & 0 \end{pmatrix}$$

Draw a diagram of G.

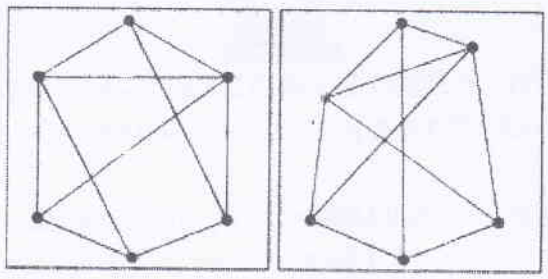
d. Construct a multigraph of order 6 and 7 in which every vertex is odd.

e. Draw all regular graphs of order n, where $2 \leq n \leq 6$.

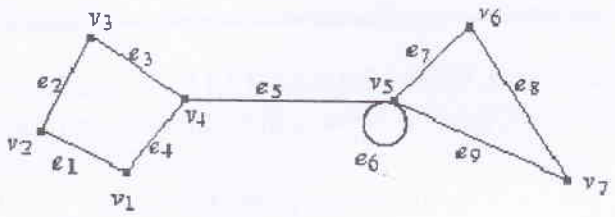
f. Let G be a graph with V(G)={1,2,3,4,.....,n}, where $n \geq 5$, such that two numbers i and j in V(G) are adjacent if and only if $|i - j| = 5$. How many components does G have?

2.

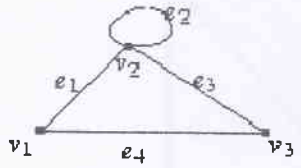
i. Show that the two graphs below are isomorphic:



ii. Find an Eulerian path for the graph G below



iii. Consider the following graph and fill the blank with most suitable answers.



a. Walk ……, ……$v_2e_2v_2$ and vertex …… are all circuits, but vertex is…… a trivial circuit.

b. ……is an Eulerian circuit but not a ……circuit.

c. $v_1e_1v_2e_3v_3e_4v_1$ is a ……circuit, but not an ……circuit.

3.

i. Show that, $A(p,q)$ graph is a tree if and only if it is acyclic and $p=q+1$.

ii. A tree has five vertex of degree 2, three vertices of degree 3 and four vertices of degree 4. How many vertices of degree 1 does it have?

iii. Construct two-non isomorphic trees having exactly 4 pendent vertices on 6 vertices.

iv. Let G be a graph with k components where each component is a tree. If n is the number of vertices and m is the number of edges in G, prove that $n=m+k$.

v. Suppose that a tree T has N_1 vertices of degree 1, N_2 vertices of degree 2, N_3 vertices of degree 3, ……, N_k vertices of degree k. Prove that
$N_1 = 2 + N_3 + 2N_4 + 3N_5 + \cdots\ldots\ldots + (k-2)N_k$.

4.

a) The chief green keeper of a nine hole golf cause plans to install an automatic sprinkler system using water from the mains at the club house to water the greens. The distances in meters between the greens are as shown in the below table below. Use Prim's algorithm to decide where the pipes should be installed to make their total length a minimum.

CH	1	2	3	4	5	6	7	8
1	250							
2	600	350						
3	400	50	300					
4	100	200	600	300				
5	500	350	200	150	400			
6	800	550	200	500	750	350		
7	600	400	150	300	550	250	150	
8	350	200	350	100	350	50	400	250
9	50	300	600	400	100	500	850	600	350

b) A Sri Lankan tourist arrives at Katunayaka and wants to visit A, B, C, D, E, F, G and her ancestral home in H before returning to Katunayaka (K) to catch a flight back to USA. The travel times in hours between places are as shown in the below. If she wants to spend 6 hours in each place, can she complete such a journey in 75 walking hours before her flight leaves?

	A	B	C	D	E	F	G	H	K
A	-	3/2	3/2	3	5	9/2	2	3	1
B	3/2	-	3	3/2	6	6	2	9/2	3/2
C	3/2	3	-	4	6	11/2	7/2	2	5/2
D	3	3/2	4	-	5	13/2	7/2	11/2	3
E	5	6	6	5	-	4	7	8	6
F	9/2	6	11/2	13/2	4	-	13/2	7	11/2
G	2	2	7/2	7/2	7	13/2	-	5	2
H	3	9/2	2	11/2	8	7	5	-	4
K	1	3/2	5/2	3	6	11/2	2	4	-