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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B. Sc. (General) Degree

Second Year - End Semester Examination (Semester I), April / May 2016

MAP 2301 – Algebra

Answer All Questions **Time allowed: Two hours**

1. (*a*) Prove the followings:

    (*i*) $A \cap (B \cap C) = (A \cap B) \cap C$,where $A, B\, and\, C$ are sets.

    (*ii*) $A \Delta B = (A - B) \cup (B - A)$ is called the symmetric difference of two sets $A\, and\, B$. Show that, $A\Delta(B\Delta C) = (A\Delta B)\Delta C$ where $A, B\, and\, C$ are sets.

    (*b*) According to the survey of 85 students are asked them about the subjects they are studying. 35 students study math, 37 student study chemistry, and 26 study physics. 20 study math and chemistry, 14 study math and physics, and 3 study chemistry and physics. Two of them study all three subjects.

    (*i*) How many of these students study math or physics?

    (*ii*) How many of these students didn't study any of the three subjects?

    (*iii*) How many of these students study math and chemistry but not physics?

    (*c*) Construct a truth table for $(p \vee r) \wedge (\sim q \vee \sim r)$, where $p, q\, and\, r$ are propositions.

2. (*a*) Let *R* be a relation on a non-empty set *A*. Define the following properties of *R*.

    (*i*) Reflexive

    (*ii*) Symmetric

    (*iii*) Transitive

    Let *R* be the set of real numbers, define a relation *R* by $R = \{ (a,b) \in R \times R : |a - b|$ is an integer$\}$. Is *R* an equivalence relation? Justify your answer.

    (*b*) Define an *injective* (one to one) and a *surjective* (onto) function.

    Let $f : N \to Z$ be defined by $f(x) = \begin{cases} -\dfrac{x}{2}, & if\ x\, is\, even \\ \dfrac{x+1}{2}, & if\ x\, is\, odd \end{cases}$.

    Then prove that *f* is one to one function. Is *f* onto?

    (*c*) Consider the following permutations in S_9:

    $\alpha = (1527)(34)(839),\ \beta = (19765248)\, and\, \gamma = (1234)(61879)$

    Compute the disjoint cycles of the followings:

    $\beta^8, \gamma^{20}, \alpha\beta^8\gamma\ and\ \alpha\gamma^{-20}\beta$.

3. (*a*) Write the axioms of a group G with usual notations.
Prove that the following statement, "A non-empty subset H of a group G is a subgroup of G if and only if $\forall\, a,b \in H \Rightarrow ab^{-1} \in H$".

(*b*) Define an operation * on Q^{+} by $x * y = \frac{xy}{5}$ *where* $x, y \in Q^{+}$.

(*i*) Show that * is a binary operation on Q^{+}.

(*ii*) Is $(Q^{+}, *)$ a group? Justify your answer.

(*c*) Let G be a group. If $(ab)^2 = a^2b^2$, $\forall\, a,b \in G$ then show that G is *abelian* group.

4. (*a*) Prove that, the linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$, where $d = \gcd(a,b)$. If x_0, y_0 is any particular solution of this equation, then show that all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t,\ y = y_0 - \left(\frac{a}{d}\right)t$ where t is an arbitrary integer.

(*b*) Using part (*a*) find the general solution for the linear Diophantine equation $11x + 30y = 21$

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Prove that the following statement, "A non-empty subset H of a group G is a subgroup of G if and only if $\forall\, a,b \in H \Rightarrow ab^{-1} \in H$".

(b) Define an operation $*$ on Q^{+} by $x * y = \dfrac{xy}{5}$ $where\, x, y \in Q^{+}$.

   (i) Show that $*$ is a binary operation on Q^{+}.
   (ii) Is $(Q^{+}, *)$ a group? Justify your answer.

(c) Let G be a group. If $(ab)^2 = a^2b^2$, $\forall\, a,b \in G$ then show that G is *abelian* group.

4. (a) Prove that, the linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$, where $d = \gcd(a,b)$. If x_0, y_0 is any particular solution of this equation, then show that all other solutions are given by $x = x_0 + \left(\dfrac{b}{d}\right)t,\ y = y_0 - \left(\dfrac{a}{d}\right)t$ where t is an arbitrary integer.

(b) Using part (a) find the general solution for the linear Diophantine equation $11x + 30y = 21$