RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Joint Major) Degree in Chemistry & Physics

Fourth Year - Semester I Examination - October/November 2015

PHY 4210 - ADVANCED QUANTUM MECHANICS

Answer all four questions. Time: Two hours

Use of a non programmable calculator is permitted.
Unless otherwise specified, all the symbols have their usual meaning.

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(1) (a) Describe the five basic postulates of quantum mechanics. [10 marks]

(b) What can you say about the eigenvalues of an operator that is both Hermitian and unitary? Justify your answer. [06 marks]

(c) Show that the eigenfunctions of Hermitian operators are orthogonal. [09 marks]

(2) (a) Prove the following operator identities;

(i) $\left[\hat{A}, \hat{B}\hat{C}\right] = \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right]$ [05 marks]

(ii) $\left[\hat{A}, \hat{B}^{-1}\right] = -\hat{B}^{-1}\left[\hat{A}, \hat{B}\right]\hat{B}^{-1}$ [05 marks]

(b) If $\hat{L}$ is a non-Hermitian operator, then, show that $\left(\hat{L} + \hat{L}^{\dagger}\right)$ is Hermitian. [09 marks]

(c) If $\hat{A}$ and $\hat{B}$ are integrals of motion, then, show that $i\left[\hat{A}, \hat{B}\right]$ is also an integral of motion.

Hint: If $\hat{A}$ and $\hat{B}$ are integrals of motion, then they commute with $\hat{H}$. [06 marks]

(3) (a) A particle is confined to a cubical box of edge length L with one corner at the origin and edges lined up with the coordinate axes. Using, the technique of separation of variables, solve the energy eigenvalue equation for this system in the region inside the box to obtain appropriately normalized energy eigenfunctions and corresponding eigenvalues. Assume that the wave function satisfies periodic boundary conditions at the edges of the box, i.e., $\phi(L, y, z) = \phi(0, y, z)$, $\phi(x, L, z) = \phi(x, 0, z)$ and $\phi(x, y, L) = \phi(x, y, 0)$.

[16 marks]

(b) Are the energy eigenfunctions also the eigenstates of momentum $\hat{\vec{p}} = -i\hbar\vec{\nabla}$?

[09 marks]

(4) Consider a one-dimensional harmonic oscillator described by the Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2$ and a wave function given by $\psi(x) = \alpha^2 - x^2$ for $|x| < \alpha$, and $\psi(x) = 0$ elsewhere.

(a) Calculate the expectation value for the energy using $\psi(x)$. Note that the wave function has to be normalized.

[10 marks]

(b) Use the variational method with $\psi(x)$ as a trail wave function (i.e. α as a variational parameter) to obtain an upper bound for the ground state energy. Compare your result with the known exact result.

[15 marks]

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