52.

RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

BSc in Applied Sciences

Second Year - Semester I Examination - June/July 2022

MAP 2203 - Differential Equations II

Time allowed: **Two (2) hours**

Answer All Questions

1. a) Consider the following second order differential equation:

$$2xy'' - y' + 2y = 0.$$

i. Show that $x = 0$ is a regular singular point of the above differential equation.

ii. Use the "method of Frobenious" to find the general solution of the above differential equation.

(60 marks)

b) Construct Picard iterations for the following Initial Value Problem:

$$\frac{dy}{dx} = 2x(1 + y), \quad y(0) = 0.$$

Show that the iterations converge to $y(x) = e^{x^2} - 1$.

(40 marks)

2. a) Consider the following system of differential equations:

$$\frac{d\underline{x}(t)}{dt} = A\underline{x}(t); \quad A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

i. Find the general solution of the above system.

ii. Obtain the fundamental matrix solution $\mathbf{\Phi}(t)$ for the above system.

iii. Hence, solve the following Initial Value Problem:

$$\frac{d\underline{x}(t)}{dt} = A\underline{x}(t); \quad \underline{x}(0) = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}.$$

(70 marks)

b) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{bmatrix}$. Using the fact that $A^2 = 0$, find e^{At}.

(30 marks)

3. a) Find the general solution of the homogeneous linear partial differential equation of the form:

$$(D_x - m_1 D_y)(D_x - m_2 D_y)...(D_x - m_n D_y)u = 0,$$

where $D_x = \frac{\partial}{\partial x}$, $D_y = \frac{\partial}{\partial y}$, and $m_1, m_2, ..., m_n$ are distinct constants.

(30 marks)

b) Solve the following partial differential equations:

i. $(D^3 - 4D^2D' + 4DD'^2)z = 2\sin(3x + 2y)$.

ii. $(D^2 - 2DD' + D'^2)u = e^{x+2y} + x + y$.

Here, $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

(70 marks)

4.- a) Consider the following partial differential equation: 54

$$z = px + qy + k\sqrt{1 + p^2 + q^2}.$$

Show that the singular integral of the above partial differential equation is given by

$$x^2 + y^2 + z^2 = k^2,$$

where k is a constant.

(40 marks)

b) Show that the complete integral of the partial differential equation,

$$2xu = px^2 + 2qxy - pq$$

is given by

$$u = ay + b(x^2 - a),$$

where a and b are arbitrary constants.

(30 marks)

c) Find the surface that satisfying the partial differential equation $t = 6x^3y$ and containing the two lines $y = 0 = z$ and $y = 1 = z$. Here, $t = \frac{\partial^2 z}{\partial y^2}$.

(30 marks)

End