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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

Third Year - Semester II Examination – Jan/Feb 2023

MAT 3208 – Time Series

Time: Two (02) hours

Answer **All** questions.

1\. a) i. Distinguish between strictly stationary time series and weakly stationary time series.

(05 marks)

ii. Hence, write down the Autocovariance and Autocorrelation function (ACF) of a stationary time series.

(05 marks)

b) i. Discuss the stationarity of the white noise series (w_t).

(05 marks)

ii. Let's consider a three-point moving average process x_t(where it is the average of immediate neighbours in the future and past of w_t) and discuss the stationarity of the process.

(10 marks)

2\. a) i. Explain the conditions that should satisfy any series to be considered as "*Trend Stationarity*".

(05 marks)

ii. Consider the time series $x_t = \beta_1 + \beta_2 t + w_t$, where β_1 and β_2 are known constants and w_t is a white noise process with variance σ_w^2.

a. Determine whether x_t is stationary or not.

b. Show that the process $y_t = x_t - x_{t-1}$ is stationary.

(10 marks)

b) For a moving average process of the form $x_t = w_{t-1} + 2w_t + w_{t+1}$, where w_i are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and plot the ACF as a function of h.

(10 marks)

3. a) Consider two series, x_t and y_t, formed from the sum and difference of two successive values of a white noise process. Show that the series are jointly stationary.

(05 marks)

b) For MA(1) model, $x_t = w_t + \theta w_{t-1}$, show that $|\rho_x(1)| \leq {}^1/_2$ for any number θ. For which values of θ does $\rho_x(1)$ attain its maximum and minimum?

(10 marks)

c) Identify the following as ARMA(p, q) models, and determine whether they are causal and/or invertible:

i. $x_t = 0.80\, x_{t-1} - 0.15\, x_{t-2} + w_t - 0.30\, w_{t-1}$

ii. $x_t = x_{t-1} - 0.50 x_{t-2} + w_t - w_{t-1}$

(10 marks)

4. a) Show that the autocovariance function of a stationary series is symmetric around the origin, i.e., $\gamma(h) = \gamma(-h)$.

(2.5 marks)

b) Let consider the AR(1) model given by $x_t = \phi\, x_{t-1} + w_t$

i. Show that, $x_t = \phi^k\, x_{t-k} + \sum_{j=0}^{k-1} \phi^j w_{t-j}$

(2.5 marks)

ii. If provided that $|\phi| < 1$ and $sup_t\; var(x_t) < \infty$, show that AR(1) model as a linear process given by $x_t = \sum_{j=0}^{\infty} \phi^j w_{t-j}$.

(2.5 marks)

iii. Hence, show that above AR(1) process is a stationary process.

(10 marks)

iv. Using the results of part a), show that $\rho(h) = \phi^h$, where $h > 0$.

(05 marks)

v. Hence, show that autocorrelation function of AR(1) satisfies the recursion $\rho(h) = \phi\, \rho(h-1)\,, h = 1,2, \dots$.

(2.5 marks)

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