RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year - Semester II Examination – July 2020

MAT 4307 – DESIGN OF EXPERIMENTS

Time allowed: Three (03) hours

Instructions,

Answer four (**04**) questions only.
Each question carries equal marks of **25**.
Total marks allocation for this paper is **100**.
Statistical tables and calculators will be provided.

1. An experiment was conducted to evaluate the effect of four (4) medicines (M_1, M_2, M_3, M_4). The selected people were grouped by age and weight class. The observed responses (serum antibodies) of each person are given below.

Age	Weight class	Medicine	Response
1	1	1	105
1	2	2	150
1	3	3	80
1	4	4	90
2	1	2	155
2	2	3	90
2	3	4	98
2	4	1	108
3	1	3	78
3	2	4	82
3	3	1	110
3	4	2	160
4	1	4	85
4	2	1	100
4	3	2	160
4	4	3	85

a. Conduct an appropriate statistical test to conclude about the effect of medicines on serum antibodies. Clearly, state Null and Alternative hypotheses.   (10 marks)

b. If M_4 is the standard medicine, compare rest of the medicines with M_4. (07 marks)

c. Find out the best medicine/s for increasing serum antibodies.   (08 marks)

2. Five different breakfast cereals (A to E) were tested for their digestible energy. A randomized complete block design with four replicates were used to test the significances. Measured energy values are summarized in the following table. **(25 marks)**

	A	B	C	D	E
Block 1	164	168	121	75	169
Block 2	166	64	96	74	91
Block 3	168	160	110	78	145
Block 4	159	161	108	72	130

a. Construct the ANOVA and make your conclusions. Clearly, state Null and Alternative hypotheses.
    (hint: higher mean is the best)   (10 marks)

b. Write an orthogonal contrast set for the given experiment.   (05 marks)

c. Show that "Treatment SS = Sum of contrast SS".   (05 marks)

d. Test the given contrasts: (05 marks)
    i.   AB vs CDE
    ii.   BE vs CD

3. An experiment was conducted to evaluate the effect of 3 different growth hormones (T_1, T_2 & T_3) on the growth of birds using randomized complete block design (RCBD). The measurements on initial weight (X) and the weight after 10 days (Y) were taken. The results are given in following table:

Block	Treatment					
	T_1		T_2		T_3	
	X	Y	X	Y	X	Y
B1	32	48	30	45	23	48
B2	26	31	28	39	25	50
B3	25	30	25	32	26	52

a. Conduct an appropriate statistical test to conclude about the effects of the hormone. Clearly, state Null and Alternative hypotheses.

   Note: Use following formulas to calculate sum of square (SS) and cross products. **(10 marks)**

b. List the adjusted treatment means for T_1, T_2 and T_3. **(06 marks)**

c. If lower means are preferable, find the best treatment for above experiment. **(06 marks)**

d. State the usefulness of a covariate in this experiment. **(03 marks)**

$$\text{Treatment} = T_{yy} - \frac{(T_{XY} + E_{XY})^2}{(T_{xx} + E_{xx})} + \frac{E_{xy}^2}{E_{xx}} \qquad \text{Covariate} = \frac{E_{xy}^2}{E_{xx}}$$

$$\text{Block} = B_{yy} - \frac{(B_{XY} + E_{XY})^2}{(B_{xx} + E_{xx})} + \frac{E_{xy}^2}{E_{xx}} \qquad \text{Residual} = E_{yy} - \frac{E_{xy}^2}{E_{xx}}$$

4. A researcher plans to conduct an experiment with four factors A, B, C and D with three (03), three (03), three (03) and four (04) levels in each, respectively. The experimental design intended to be used is RCBD with four blocks.

   a. Give the layout of one block, if all factors need small plots of the same size. **(01 marks)**

   b. Give the breakdown of ANOVA for part (a) (sources of variation and degrees of freedom (df) only). **(04 marks)**

   c. Suppose factor A needs large plots, what is the most appropriate design? **(01 marks)**

   d. Give the layout if a single block for part (c). **(01 marks)**

   e. Give the breakdown of ANOVA for part (c) (sources of variation and df only). **(04 marks)**

   f. Suppose that factor A and B need large plots of equal size, what is the most appropriate design? **(01 marks)**

   g. Give the layout if a single block for part (f). **(01 marks)**

   h. Give the breakdown of ANOVA for part (f) (sources of variation and df only). **(04 marks)**

   i. Suppose A and B factors need large plots and factor C requires a medium plot, what is the most appropriate design? **(01 marks)**

j. Give the layout of a single block for part (i).

(01 marks)

k. Give the breakdown of ANOVA for part (i) (sources of variation and df only).

(04 marks)

l. Explain the difference of ANOVA in part (b), part (e) and part (k).

(02 marks)

5. A researcher wants to study performances of six (06) food rations on body weight of rabbits. Six (06) farms are selected for this experiment in a same location and (06) rabbits of different weight classes are available at each farm. Weight classes across farms are same and a total of thirty-six (36) rabbits will be used in the experiment.

   a. What is the most appropriate experimental design for this study?

   (01 marks)

   b. Give the layout of the design and the statistical model.

   (02 marks)

   c. Give the breakdown of the ANOVA (sources of variability and df only).

   (05 marks)

   d. Give the appropriate test to determine the effect of food rations.

   (01 marks)

   e. If the above study is repeated in four (04) different locations (at the same time), what is the most appropriate experimental design? **(01 marks)**

   f. Give the statistical model for experimental design of part (e).

   (01 marks)

   g. Give the breakdown of the ANOVA (sources of variability and df only) for the design specified in part (e). **(05 marks)**

   h. If the weight classes are different at each location, what is the most appropriate experimental design? **(01 marks)**

   i. Give the statistical model for experimental design of part (h). **(01 marks)**

   j. Give the breakdown of the ANOVA (sources of variability and df) for the design specified in part (h). **(05 marks)**

   k. Explain the reasons for the changes of ANOVA in part (c), (g), & (j).

   (02 marks)

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