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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science Honors in Industrial Mathematics
Fourth Year - Semester I Examination – August/September 2023

MAT 4302 – FINANCIAL MATHEMATICS

Time: Three (03) hours

Answer **All** questions

Calculators will be provided

1. a) Explain the difference between the following concepts in your own words:

     i.   simple interest vs. compound interest

     ii. ordinary annuity vs. annuity due

     iii. call option vs. put option

     iv. long position vs. short position

     v.   spot price vs. strike price

     (50 marks)

    b) Suppose that you need $12,000 in 2 years. How much should you deposit today at a discount rate of 6% compounded monthly?

    (20 marks)

    c) Suppose at the beginning of every year you put $200 in a savings account that pays 6% interest compounded monthly. What is the balance of your account after five years?

    (20 marks)

2. a) A mortgage of \$10,000 is repaid over a year by equal monthly payments at the effective interest rate of 12%. Calculate the amortization table.

    (30 marks)

    b) A certain machine costing \$15,000 has an estimated salvage value of \$2,500 and a probable lifespan of 25,000 operating hours. Calculate the depreciation schedule using the straight-line method.

    (30 marks)

    c) Suppose you invest \$2,000 today, and another \$6,000 after two years. In return, you will receive \$7,000 after one year of the first payment.

        i.   Write net present value (NPV) as a function of the interest rate.
        ii. Compute the internal rate(s) of return (IRR) for this investment.
        iii. Which interest rate would yield the highest net present value?

    (40 marks)

3. a) The market department of ABC company recommends manufacturing and marketing a new school bag. The financial department provides the estimated cost function $C(Q) = 74{,}000 + 120Q$, and the estimated revenue function $R(Q) = 18Q + 0.5Q^2$. Sketch their graphs in the same diagram. Find the following:

        i.   the marginal cost   iii.   the profit function
        ii. the average revenue   iv.   the break-even point

    (40 marks)

    b) Consider the following two-commodity (demand and supply) market model:
    $$Q_d = 12 - 2P + P',\ Q_s = 3P - 3, \qquad Q_d' = 21 + P - 2P',\ Q_s' = P' - 4$$

        i.   Sketch the region for unit prices P and P' such that both demand functions Q_d and Q_d' are positive.
        ii. Find the market equilibrium values.

    (40 marks)

4.   a) The following table shows the daily closing prices of a stock over 10 trading days.

01	02	03	04	05	06	07	08	09	10
100	98	105	112	110	105	108	95	100	115

   i.   Compute the logarithmic return for each day.

   ii.   Using i), compute the drift and the volatility of the stock.

(30 marks)

   b) An investor instructs his broker to buy the call option on 100 shares of a certain stock at a strike price of $20. The price of the contract is $1.65 per share.

   i)   If the price of the stock does not rise above $20 calculate the loss.

   ii) If the price rises to $30 calculate the gain for buying 100 shares.

   iii) Write down the payoff function P at the spot price S_T.

(30 marks)

   c) Suppose to implement a bear spread strategy on a certain stock, you simultaneously execute the following European style options trades:

   • Buy a put option with a strike price of $55 for a premium of $3 per share.

   • Sell a put option with a strike price of $45 for a premium of $1 per share.

   Find the payoff P as a function of spot price S_T.

(20 marks)

END