RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

Bachelor of science in Applied Sciences

Second Year - Semester I Examination – July/Aug 2023

MAA 2302– Probability & Statistics II

Time: Three (03) hours

Answer **all** questions.

Calculators and Statistical tables will be provided.

1. a) A company aims to form a Welfare Society by choosing a committee consisting of 4 members from a pool of 10 employees.

   i. Consider a group of 10 people. How many ways can a committee of 4 individuals be formed, disregarding the order of selection?

   ii. Given that the answers obtained in part (i) are equally likely, calculate the probability of a specific individual being selected to be part of the committee.

   iii. Calculate the probability that a particular person is not included in the committee.

   iv. From a group of 10 individuals, determine the number of ways to form a committee of 4 people where one person is designated as the chairperson.

(08 marks)

b) Two events A and B are such that $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$. Find the following probabilities,

   i. $P(A \cap B)$

   ii. $P(\bar{A} \cup \bar{B})$

   iii. $P(\bar{A} \cap \bar{B})$

   iv. $P(\bar{A}|B)$

(08 marks)

2. a) Let X and Y be two discrete random variables.

   i. Define joint probability mass function and independence of X and Y.

   ii. If X and Y have means $E[X]$ and $E[Y]$ respectively, show that

$$cov(X,Y) = E[XY] - E[X]E[Y].$$

(06 marks)

b) Consider the number of contracts awarded to Firm A, represented by the random variable X, and the number of contracts awarded to Firm B, represented by the random variable Y. The joint probability distribution for X and Y is illustrated in the table below.

Y	**X**			$P_Y(y)$
	0	**1**	**2**	
0	1/9	...	1/9	4/9
1	2/9	2/9	...	...
2	1/9	...	0	1/9
$P_X(x)$	4/9	4/9	...	1

   i. Calculate the missing probabilities necessary to fill in the table.

   ii. Investigate the independence between the random variables X and Y. Justify your conclusion with appropriate reasoning.

   iii. Construct the conditional probability distribution of X given Y for all y.

   iv. Find the covariance between X and Y.

(12 marks)

3. Let X and Y be two continuous random variables with the following joint probability density function.

$$f_{(X,Y)}(x,y) = \begin{cases} \dfrac{(x+y)}{3} & ; 0 < x < 1, 0 < y < 2, \\ 0 & ; \text{o/w}. \end{cases}$$

a) Find the marginal density functions of X and Y.

b) Find the marginal distribution functions of X and Y.

c) Check whether random variables X and Y are independent or not.

d) Find the conditional probability density function denoted by $f_{(X|Y)}(x|y)$.

e) Calculate the mean and the variance of the continuous random variable X.

(16 marks)

4. a) In the context of a bakery selling bread, the random variable X represents the amount of bread sold in a day, measured in hundreds of pounds. The probability function for X is defined by the probability density function $f(x)$,

$$f(x) = \begin{cases} Ax & ; 0 \leq x < 10, \\ A(20 - x) & ; 10 \leq x < 20, \\ 0 & ; \text{o/w.} \end{cases}$$

where A is a constant.

i. Find the value of A.

ii. Determine the probabilities corresponding to the given pound values for the amount of bread that will be sold tomorrow is,

I. more than 10 pounds.

II. less than 10 pounds.

III. between 5 and 15 pounds.

(12 marks)

b) Given that the number of customers arriving at a grocery store follows a Poisson distribution with an average arrival rate of 10 customers per hour, let X denote the number of customers arriving between 10.00 am and 11.30 am. Determine the probability, denoted as $P(10 < X \leq 15)$, of having more than 10 but less than or equal to 15 customers.

(05 marks)

5. a) A sample of size $n = 25$ was randomly selected from a population with an unknown mean μ and a known standard deviation $\sigma = 3$. The sample mean $\bar{X}$ was calculated to be 4.5. Using a z-value of 2, compute the *confidence interval estimate for* μ.

Also, determine the followings,

i. The 50% *confidence interval estimate of* μ when $n = 25$

ii. The 50% *confidence interval estimate of* μ when $n = 100$

iii. The 95% *confidence interval estimate of* μ when $n = 100$

(08 marks)

b) Let X be a random variable with mean μ and variance $Var(X)$. Prove the following;

i. $E[aX + b] = aE[X] + b$; where a and b are constants.

ii. $Var[X] = E[X^2] - \mu^2$

(06 marks)

c) For the sample variance S^2, prove that,

$$S^2 \equiv \frac{1}{n-1}\sum_{k=1}^{n}(X_k - \bar{X})^2 \quad = \quad \frac{1}{n}\left[\sum_{k=1}^{n} X_k^2\right] - \bar{X}^2.$$

(05 marks)

6. a) A continuous random variable X has the following p.d.f

$$f(x) = ax \ ; \ 0 \leq x \leq 1.$$

Determine the constant a and also find $P\,[X \leq 1/2]$.

(05 marks)

b) Assuming that $X_n^2 \equiv X_1^2 + X_2^2 + \cdots + X_n^2$ is called the Chi-square random variable with n degrees of freedom, show the following,

i. $E[X_n^2] = n$

ii. $Var(X_n^2) = 2n$

iii. $\sigma(X_n^2) = \sqrt{2n}$

(09 marks)

---END---