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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Second Year - Semester I Examination – July/August 2023

MAA 2204 – LINEAR PROGRAMMING

Time: Two (02) hours

Answer **all** questions.

1. a) The Nippon Metal Corporation is planning to produce a new alloy that is by weight 35% of metal M1 and 40% of metal M2. For the rest of the new alloy, the metal M3 and M4 can be used accordingly. Five alloys are available at various costs as stated in the following table.

Metal	Alloy				
	A1	A2	A3	A4	A5
M1 (%)	5	20	40	60	80
M2 (%)	70	40	35	30	15
M3 (%)	20	30	15	6	5
M4 (%)	5	10	10	4	0
Cost/lb ($)	3	4	2	3.5	2.5

The new alloy will be produced by mixing the above alloys. The amount of metal M3 in the new alloy must be at least twice as the metal M4. The total cost for alloys A3 and A5, should not be less than the cost for alloy A2. The corporation wants to determine the amounts of the various alloys, needed to produce the desired new alloy at a minimum total cost. Formulate this problem as a linear programming model. **(40 marks)**

b) Determine the interval of 'a' ($a \in \mathbb{R}^+$) in which the function $f(x_1, x_2) = x_1^2 - 4ax_1x_2 + 3x_2^2$ is convex and concave. **(30 marks)**

c) Let $X_1 = (0,0)$, $X_2 = (2,0)$ and $X_3 = (1,1)$ be the extreme points of a triangular shaped feasible region. Express the point $X_0 = (1, 0.5)$ as a convex combination of X_1, X_2 and X_3. **(30 marks)**

2. Consider the following linear programming problem (LPP):

$$\begin{aligned}
&\text{Max } Z = 3x_1 + 6x_2 - 3x_3\\
&\text{S.t.}\\
&\quad 3x_1 - 3x_2 + 2x_3 \le 8,\\
&\quad 2x_1 - 3x_2 \ge -9,\\
&\quad -4x_1 + 3x_2 + 3x_3 \le 10,\\
&\quad x_1, x_2, x_3 \ge 0.
\end{aligned}$$

a) Solve the above LPP by applying the Simplex Method. **(75 marks)**

b) Formulate the corresponding dual problem of the given LLP. **(10 marks)**

c) Obtain the optimal solution of the dual problem in Part b) using the Strong Duality Theorem.

$$\left[\text{Help: inverse of the matrix} \begin{pmatrix} 3 & -3 & 0 \\ -2 & 3 & 0 \\ -4 & 3 & 1 \end{pmatrix} \text{ is } \begin{pmatrix} 1 & 1 & 0 \\ 2/3 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}.\right]$$

(15 marks)

3. a) In the context of solving an LPP using the Two-Phase Method, under what conditions do infeasible and unbounded solutions occur? **(20 marks)**

b) Consider the following linear programming problem:

$$\begin{aligned}
&\text{Min } Z = 4x_1 + x_2\\
&\text{S.t.}\\
&\quad 3x_1 + x_2 = 3,\\
&\quad 4x_1 + 3x_2 \ge 6,\\
&\quad x_1 + 2x_2 \le 3,\\
&\quad x_1, x_2 \ge 0.
\end{aligned}$$

The following tableau shows the optimal solution for Phase I of the above linear programming problem:

B.V.	x_1	x_2	s_1	s_2	y_1	y_2	Constants
x_1	1	0	1/5	0	3/5	-1/5	3/5
x_2	0	1	-3/5	0	-4/5	3/5	6/5
s_2	0	0	1	1	1	-1	0
$-w$	0	0	0	0	1	1	0

, where y_1 and y_2 are artificial variables, s_1 and s_2 are surplus and slack variables, and w is the objective function for phase I.

Continue the solution procedure with phase II and find the optimal solution. During the execution of phase II, if you encounter any special cases of linear programming, discuss them in detail. **(80 marks)**

4. a) Solve the following linear programming problem using the Graphical Method.

$$\text{Max } Z = 4x_1 + 2x_2$$
$$\text{S.t.}$$
$$2x_1 + x_2 \leq 5,$$
$$x_1 + x_2 \leq 4,$$
$$4x_1 + 5x_2 \leq 20,$$
$$x_1, x_2 \geq 0.$$

If exist, identify the binding, non-binding, and redundant constraints of the above problem based on the solution that you have obtained. **(50 marks)**

b) Solve the following LPP using the Revised Simplex Method:

$$\text{Max } Z = x_1 - x_2 + 3x_3$$
$$\text{S.t.}$$
$$x_1 - x_2 \geq -20,$$
$$x_1 + x_3 = 5,$$
$$x_2 + x_3 \leq 10,$$
$$x_1, x_2, x_3 \geq 0.$$

(50 marks)

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