RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year Semester II Examination – February /March 2019

MAT 3205 – INTRODUCTION TO STATISTICAL DECISION THEORY

Answer All Questions
Calculator is permitted   **Time: 02 hours**

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1. a)   Write down the basic elements of decision theory.   (15 marks)

   b)   A garment factory receives its monthly supply of shirts every 3rd week of each month. This shipment must last until the following 3rd week after shipment is received. Shirts are sold for \$14 per hundred and the production cost is \$8 per hundred. The following table gives the record of the last 10 months of sales of shirts:

History of the sales (in hundred)	Month of given sales
2000	2
3000	4
4000	3
5000	1

   Construct the opportunity loss table and determine the action alternative associated with the minimization of expected losses.   (35 marks)

   c)   A company has to decide whether to develop and launch a new product. Research and development cost is expected to be \$ 400,000 and it is expected that there is a 70% chance that the product launch will be successful and 30% chance that it will fail. If it is successful, then the levels of expected profits and the probability of each occurring are estimated as follows:

Products popularity	Probability	Profit per annum for 2 years
High	0.2	$ 500,000
Medium	0.5	$ 400,000
Low	0.3	$ 300,000

If it is a failure, then there is a 0.6 probability that the research and development work can be sold for $ 50,000 and 0.4 probability that it will be worth nothing at all. Using a decision tree, find the optimal solution.   (50 marks)

2. Management of the Amaya Hotel wants to construct a hotel in a city. Management has three different hotel designs (R_1, R_2, and R_3) with 80, 100 and 120 rooms. The demand can be categorized as high, medium and low. Depending upon these various demands, the group of management made some preliminary assessment of its net profit and possible losses (in 1000 rupees) for these various types of hotels. Payoffs and prior probabilities are shown in the following table:

	Low	Medium	High
$R_1 = 80$	40	60	80
$R_2 = 100$	-10	6	8
$R_3 = 120$	-25	70	100
Probability of demand	0.4	0.2	0.4

Hotel management conducted a market survey and the results are categorized as favorable or unfavorable. The conditional probabilities of the survey are:

$P(favorable/80\ rooms) = 0.8$
$P(favorable/100\ rooms) = 0.5$
$P(favorable/120\ rooms) = 0.3$

a) Find the best decision using Expected Monitory Value.   (25 marks)

b) Using the Bayesian analysis, find the posterior probabilities and hence, determine the optimal hotel type.   (75 marks)

3. a) Name the three types of decision curve in utility. (15 marks)

b) A construction company is planning to build a house apartment in Colombo city. The company has to decide whether to build small, medium or large apartments. The payoffs received for each size of apartment will depend on the market demand for apartments in the area, which could be low, medium or high. The payoff table for this problem is given below:

Apartment Size	Market Demand		
	Low	Medium	High
Small	200	500	700
Medium	300	700	1000
Large	- 400	800	1200

The owner of the company estimates a 10% chance that market demand will be low, a 30% chance that will be medium and a 60% chance that will be high. Suppose that the utility function for the owner of the company can be estimated by $U(x) = 10 + 0.5x + 0.0003x^2$,

i. Convert this pay off matrix to utility values.
ii. What decision provides the owner of the company with the largest expected utility?

(40 marks)

c) Brenda Kelley runs a specialty ski clothing shop outside of North Carolina. She must place her order for ski parkas well in advance of ski season because her manufacturer produces them in the summer. Brenda needs to determine whether to place a large, medium or small order for parkas. The number sold will depend largely on whether the area receives a heavy, normal or light amount of snow during the ski season. The following table summarizes the pay- offs Brenda expects to receive under each scenario:

Size of order	Amount of Snow		
	Heavy	Normal	Light
Large	10 000	7 000	3 000
Medium	8 000	8 000	6 000
Small	4 000	4 000	4 000

i. What decision should be made according to the maxi-max decision rule?

ii. What decision should be made according to the maxi-min decision rule?

iii. What decision should be made according to the mini-max regret decision rule?

(45 marks)

4. a) Stating the assumptions, briefly explain the Economic Order Quantity (EOQ) model with uniform rate of demand. (15 marks)

b) Consider the following usual notations which are related to EOQ model:

q - order quantity

R – demand rate

C_s – ordering cost per production run

C_h – holding cost for one unit per unit time

t_0 – cycle length

Prove that the optimum order quantity, $q^* = \sqrt{\dfrac{2C_sR}{C_h}}$ units and optimum cycle time,

$t_0^* = \sqrt{\dfrac{2C_s}{RC_h}}$ time units.

Further prove that the average cost per unit is $\sqrt{2C_hC_sR}$. (45 marks)

c) A particular product has a demand of 9000 units per year. The cost of one order is Rs.250 and holding cost is Rs. 350 per unit per year. The replenishment is instantaneous and no shortages are allowed.

i. Determine the optimum order quantity and cycle length.

ii. Find the number of production runs per year.

iii. Calculate the total inventory cost per unit time. (40 marks)