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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year – Semester I Examination – Jan/Feb 2021
MAT4306 – Optimization Modeling

Answer ONLY Six Questions **Time Allowed: 3 hours**

1. A company makes four products: A, B, C, D and has available three workstations: W_1, W_2 and W_3. The production time (in minutes) per tonne varies from workstation to workstation as shown below:

Workstations	Products A	B	C	D
W_1	15	18	16	17
W_2	16	10	16	15
W_3	17	12	15	16

It is also given that the profit contribution (in dollars) per tonne varies from workstation to workstation as below:

Workstations	Products A	B	C	D
W_1	6	5	6	7
W_2	4	5	7	5
W_3	6	8	5	6

For a particular week, there are 40 working hours available at each workstation. It is required to produce at least 80 tonnes of product A and 75 tonnes of product B. Also, the company requires that no more than 100 tonnes of product C and 50 tonnes of product D should be produced. The manager of the company needs to decide how much of each product should be produced at each workstation so that the profit earn is maximized. Formulate this problem as *linear programming model.* [100 marks]

2. A company is planning to built two service stations to provide the row materials to the three construction sites A, B and C. The following table provides the information about the three sites.

Sites	Coordinates	Requirement (in tonnes) per month
A	(1,2)	30
B	(-1,3)	24
C	(5,-1)	18

Ctd ...

The company is to determine where the service station should be located and how many tonnes of suppliers for each sites should be shipped to each sites in such a way that the costs of transporting materials from the service stations to the sites is minimized. It is given that the maximum capacity of the service stations is such that no more than 65% of the total tonnage is moved through either sites. It is further given that the cost of moving one tonne of material one unit of distance is \$ 1.
Formulate a *nonlinear programming* model for this problem. [100 marks]

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3. A manager of a company is attempting to devise a shift pattern for his workforce. Each day of every working week is divided into two shits; shift 1 and shift 2. The plant must be manned at all times and the minimum number of workers required for each of these shifts over any working week is as below:

Shifts	Days						
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Shift 1	6	5	4	5	6	4	3
Shift 2	4	8	6	4	6	5	2

In total there are currently 60 workers and each worker is assigned to work only one shift a day and once a worker has been assigned to a shift they must remain on the same shift every day that they work. It is further require that each worker works four consecutive days during any seven day period. Formulate this problems as an *integer linear programming model* to find the number of workers should start working on each shift on each day, so that the total number of assignment is minimum. [100 marks]

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4. (a) Write down the KKT conditions for the following optimization problem;

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \quad i = 1, \ldots, p; \\ & h_j(x) = 0, \quad j = 1, \ldots, q. \end{aligned}$$

where $f, g_i, h_j : \mathbb{R}^n \to \mathbb{R}$ for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, q$. [30 marks]

(b) Consider the following optimization problem:

$$\begin{array}{lrcl} \min & (x-3)^2 + (y-2)^2 & & \\ \text{s.t.} & x^2 + y^2 & \leq & 5 \\ & -x & \leq & 0 \\ & -y & \leq & 0 \\ & x + 2y & = & 4 \end{array}$$

(i) Write the KKT conditions for this problem. [20 marks]

(ii) Verify that these conditions are satisfied at the point $x^* = (2, 1)^T$. [30 marks]

(iii) Is the point x^* is optimal? Justify your answer. [20 marks]

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5. Transform each of the following optimization problems into regular linear or integer liner programming problems:

(a)

$$\begin{aligned} \text{minimize} \quad & (\text{maximize } \{x+y-z, 2x-y+2z, x-y+2z\}) \\ \text{subject to} \quad & 5x-2y-z \geq 2, \\ & -x+y-z \geq 1, \\ & x, y, z \geq 0 \end{aligned}$$

[35 marks]

(b)

$$\begin{array}{llcll} \min & 2x+10y & & & \\ \text{s.t.} & x-y & \leq & 3 & (i) \\ & 2x-3y & \leq & 4 & (ii) \\ & 3x-y & \leq & 5 & (iii) \\ & x,\ y & \geq & 0 & \end{array}$$

two of the above constraints (i), (ii), or (iii) must be satisfied. [35 marks]

(c)

$$\begin{array}{llcl} \min & f_1(x_1)+x_2 & & \\ \text{s.t.} & x_1+x_2 & \leq & 3 \\ & x_1,\ x_2 & \geq & 0, \end{array}$$

$$\text{where } f_1(x_1) = \begin{cases} x_1, & \text{if } 0 \leq x_1 \leq 2; \\ 4-x_1, & \text{if } 2 \leq x_1 \leq 3; \\ 0, & \text{otherwise.} \end{cases}$$

[30 marks]

6. (a) Let $f : \mathbb{R}^3 \mapsto \mathbb{R}$ be given by

$$f(x,y,z) = 4x^2+3y^2+5z^2+6xy+xz-3x-2y+15.$$

(i) Show that f is convex. [30 marks]

(ii) Solve the following unconstrained optimization problem:

$$\text{minimize } f(x,y,z) = 4x^2+3y^2+5z^2+6xy+xz-3x-2y+15.$$

[20 marks]

(b) Consider the following problem:

$$\begin{aligned} \text{minimize} \quad & f(x,y) = x^2-x+y+xy \\ \text{subject to} \quad & x \geq 0, \qquad y \geq 0, \end{aligned}$$

(i) Find the gradient and Hessian of f at the point $(x,y)^T$. [20 marks]

(ii) Is the first-order necessary condition (FONC) for a local minimizer satisfied at $(0,0)^T$? Justify your answer. [15 marks]

(iii) Is the FONC satisfied at $(\frac{1}{2},0)^T$? Justify your answer. [15 marks]

7. Write down the *Nelder and Mead's Simplex Search Algorithm* in step wise form.

[20 marks]

Consider the following unconstrained optimization problem:

$$\text{minimize } f(x_1, x_2) = x_1^2 + x_2^2 + x_1 x_2$$

Using $(1, 1), (2, 2)$ and $(1, 3)$ as the initial simplex, perform two iterations of Nelder and Mead's simplex search algorithm. [80 marks]

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