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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (4 year) Degree in Applied Sciences/ B.Sc. (Joint Major) Degree in Physics and Chemistry
Fourth Year - Semester I Examination – January/February 2021

PHY 4312 – STATISTICAL THERMODYNAMICS

Time: Three (3) hours

Answer All Questions

Instructions:

- This is a **closed book examination**. The examination paper contains 03 pages.
- You are given 3 hours to complete the examination. There are 04 questions to be answered. You may allocate your time wisely. Read the questions carefully before answering them.
- A non-programmable calculator is permitted.
- Plagiarism (copying) is considered as a punishable offence.

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1. A system of two energy levels E_0 and E_1 is populated with N particles at temperature T. The particles populate the energy levels according to Maxwell-Boltzmann (classical) statistics.
   a) Derive an expression for the average energy per particle. (5 marks)
   b) Calculate the average energy per particle when $T \to 0$ and $T \to \infty$. (5 marks)
   c) Derive an expression for the specific heat of the system of N particles. (5 marks)
   d) Calculate the specific heat when $T \to 0$ and $T \to \infty$. (5 marks)

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2. A system of N distinguishable non-interacting particles, each fixed in position and carrying a magnetic moment μ is immersed in a magnetic field H. Hence, each particle exists in either $E = 0$ or $E = 2\mu H$ energy states.

   a) The entropy, S of the system is defined as $S = k \ln \Omega(E)$, where k is the Boltzmann constant and $\Omega(E)$ is a function of the total energy E of the system.
   Provide the definition of $\Omega(E)$. (3 marks)

   b) Write a formula for $S(n)$, where n is the number of distinguishable particles in the upper state. Sketch $S(n)$. (4 marks)

   c) Derive Stirling's approximation for large n:
   $$\ln n! = n \ln n - n$$
   by approximating $\ln n!$ (5 marks)

   d) Using the results in parts (b) and (c), find the value of n for which $S(n)$ is maximum. (5 marks)

   e) Treating E as continuous, show that this system can have negative absolute temperature. (5 marks)

   f) Although negative temperature is possible for this system why is it not possible for a gas in a box? (3 marks)

3. Suppose the energy of a particle is defined by the expression $E(z) = az^2$ where position or momentum, z can assume all values from $-\infty$ to $+\infty$. A system of such particles follows classical (Boltzmann) statistics.

   a) Show that the average energy per particle is $\bar{E} = kT/2$. (6 marks)

   b) State the principle of equipartition of energy, and explain its relation to the above calculation. (4 marks)

4. A gas of N spinless Bose particles of mass m is enclosed in a volume V at a temperature T.

   a) Find an expression for the density of single-particle states $D(\varepsilon)$ as a function of the single-particle energy ε and sketch the result. (5 marks)

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b) What is the mean occupation number of a single particle state $\bar{n}_\varepsilon$ as a function of ε, T, and the chemical potential $\mu(T)$.
Draw this function in the sketch drawn in part (a) for a moderately high temperature (above the Bose-Einstein transition). Show μ $(= \varepsilon)$ on the energy axis.

(4 marks)

c) Derive an expression for N/V that implicitly determines $\mu(T)$. Using the sketch drawn in part (a), determine the movement of direction of $\mu(T)$ when T decreases.

(6 marks)

d) Derive an expression for the Bose-Einstein critical temperature, T_c, below which one must have a macroscopic occupation of certain single particle states.
(Hint: $A = \int_0^\infty \frac{x^{1/2}\,dx}{e^x - 1} = 1.306\sqrt{\pi}$)

(6 marks)

e) What is $\mu(T)$ for $T < T_c$?
Describe $\bar{n}(\varepsilon, T)$ for $T < T_c$.

(5 marks)

f) What is the total energy, $U(T, V)$ of the gas for $T < T_c$.

(4 marks)

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