33

**RAJARATA UNIVERSITY OF SRI LANKA**
**FACULTY OF APPLIED SCIENCES**

**B.Sc. (General) Degree in Applied Sciences**
**Second Year Semester I Examination – September/October 2019**

# MAA2302 – PROBABILITY AND STATISTICS II

**Time: 03 hours**

---

Answer all question.

Your exam paper should have 04 pages with 06 problems including this page.

Calculators, Statistical distribution tables will be provided.

Following information can be use if needed.

| Distribution | Probability Function | Mean | Variance | Moment Generating Function |
|---|---|---|---|---|
| Binomial | $p(y) = \binom{n}{y} p^y (1-p)^{n-y}$ | $np$ | $np(1-p)$ | $[pe^t + (1-p)]^n$ |
| Normal | $p(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ | $\mu$ | $\sigma^2$ | $exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$ |

---

01. a) Suppose $A$ and $B$ are two events such that $P(A)$=0.8 and $P(B)$=0.7 **(10 Marks)**

i. Is it possible that $P(A \cap B) = 0.1$? Give reasons?

ii. Determine the smallest possible value $P(A \cap B)$ can take.

iii. Is it possible that $P(A \cap B) = 0.77$? Give reasons?

iv. Determine the largest possible value $P(A \cap B)$ can take.

b) Two events $A$ and $B$ are such that $P(A)$=0.2, $P(B)$=0.3, and $P(A \cup B)$=0.4. Find the following; **(10 Marks)**

i. $P(A \cap B)$ ii. $P(\bar{A} \cup \bar{B})$

iii. $P(\bar{A} \cap \bar{B})$ iv. $P(\bar{A} | B)$

*Continued…*

c) If $A$ and $B$ are independent events with $P(A)$ =0.5 and $P(B)$ =0.2, find the following; **(05 Marks)**

i. $P(A \cup B)$ ii. $P(\bar{A} \cap \bar{B})$ iii. $P(\bar{A} \cup \bar{B})$

d) If $A$ and $B$ are mutually exclusive events and $P(B) > 0$, show that $P(A|A \cup B) = \frac{P(A)}{P(A)+P(B)}$. **(10 Marks)**

02. a) Suppose the *probability density function (pdf)* of the magnitude measurement scalar $X$ is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x & ; 0 \leq x \leq 2, \\ 0 & ; \text{otherwise.} \end{cases}$$

Find the *continuous distribution function (cdf)* of $X$ for $0 \leq x \leq 2$. **(10 Marks)**

b) Sketch the *pdf* and *cdf* of $X$. **(05 Marks)**

c) Find the probability that the magnitude of scalar $X$ is in between 1 and 1.5. Also find the probability that the magnitude of the scalar $X$ exceeds 1. **(10 Marks)**

d) Find the mode and mean of the magnitude of scalar $X$. **(10 Marks)**

03. a) Suppose the probability density function of a random variable $Y$ is given by,

$$P(Y = k) = p(k) = \begin{cases} 0.3 & ; \text{if } k = 8 \\ 0.2 & ; \text{if } k = 10 \\ 0.5 & ; \text{if } k = 6 \end{cases}$$

Determine the *moment generating function (mgf)* for $Y$. **(10 Marks)**

b) Using the *mgf* find the mean and variance of $Y$. **(10 Marks)**

c) Suppose that the moment generating function of the random variable $X$ is given by, $M_X(t) = [(1 + 3e^t)/4]^{10}$. Identify the distribution followed by the $X$ and denote it in the standard notation. **(10 Marks)**

d) Hence find the mean and variance of $X$. **(05 Marks)**

04. Fast food outlet operates both; a drive-in facility and a walk-in window. Let $X$ be the proportion of time that the drive-in facility is in (at least one customer is being served or waiting to be served), and $Y$ be the proportion of time that the walk-in window is in use (on a randomly selected day). Then the set of possible values for $(X, Y)$ is the rectangle or the support $D \in \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$. Suppose the *joint probability density function* of $(X, Y)$ is given by

$$f_{X,Y}(x,y) = \begin{cases} k\,(x + y^2) & ; 0 \le x \le 1, 0 \le y \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

a) Find the value of $k$. **(05 Marks)**

b) What is the probability that neither facility is busy more than one-quarter of the time? **(05 Marks)**

c) Find the *marginal probability density functions* of $X$ and $Y$. **(10 Marks)**

d) Find the probability that the busy time is greater than one quarter of time, and less than three quarter of time, for the drive-in facility without reference to the walk-in window **(05 Marks)**

e) Suppose the life-times of two components (drive-in and walk-in) are independent of each other, and that the lifetime of the first, $X_1$, has an exponential distribution with parameter $\lambda_1$, whereas the second, $X_2$, has an exponential distribution with parameter $\lambda_2$. Find the joint probability density function of $X_1$ and $X_2$.

Let the $\lambda_1$ be $1/1000$ and $\lambda_2$ be $1/1200$, so the expected lifetimes are 1000 hours and 12000 hours respectively. Find the probability that the lifetimes of both components are at least 1500 hours.

Note: If $X$ is an exponentially distributed random variable with parameter $\lambda$ *pdf* of $X$ is denoted by,

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & ; \text{for } x > 0 \\ 0 & ; \text{for } x \le 0 \end{cases} \quad \text{and} \quad E(X) = {}^{1}/_{\lambda}$$

**(10 Marks)**

05. a) Describe the difference between the "point estimate" and "point estimator". **(05 Marks)**

b) Explain what is an unbiassed estimator. Hence define the term ***bias***. **(05 Marks)**

c) Let $X_1, X_2, X_3, \dots, X_n$ be a random sample obtained from a distribution with mean $\mu$ and variance $\sigma^2$. Show that the estimator

$$\hat{\sigma}^2 = S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

is unbiased for estimating $\sigma^2$. **(10 Marks)**

***Continued...***

d) If you have unbiassed estimators $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ to estimate parameter $\theta$. Justify how you select the best unbiassed estimator for $\theta$. **(05 Marks)**

06. Suppose $X_1, X_2, X_3, \dots, X_n$ is a random sample from Poisson distribution with parameter denoted $\lambda$, so that
$f_X(x;\theta) \equiv f_X(x;\lambda) = \frac{\lambda^x}{x!} e^{-\lambda}$, where $x = 0,1,2,\dots$, for some $\lambda > 0$ .

a) Use 1st moments of the distribution to find the estimator of $\lambda$. **(05 Marks)**

b) Calculate the likelihood function $L(\lambda)$. **(05 Marks)**

c) Calculate the log-likelihood $\log L(\lambda)$. **(05 Marks)**

d) Differentiate $\log L(\lambda)$ with respect to $\lambda$ and find the **MLE** of $\lambda$. **(10 Marks)**

e) The following data are the observed frequencies of occurrence of domestic accidents:

| No of accidents | Frequency |
|---|---|
| 0 | 447 |
| 1 | 132 |
| 2 | 42 |
| 3 | 21 |
| 4 | 3 |
| 5 | 2 |

Assume that the above data follows a Poison distribution and estimate its parameter $\lambda$, using maximum likelihood estimator. **(10 Marks)**

**... End ...**