**RAJARATA UNIVERSITY OF SRI LANKA**
**FACULTY OF APPLIED SCIENCES**
**B. Sc. (General) Degree**
**Second Year – Semester II Examination – September/October 2014**
**MAP 2301 – Algebra**

**Answer All Questions** **Time allowed: Three hours**

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1. *a*) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 6 & 1 & 5 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{pmatrix}$ and $\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 5 & 6 & 1 \end{pmatrix}$ are three permutations, Compute the following,

   i) $\alpha\beta^2\gamma$ ii) $\alpha^6\beta^5\gamma^4$ iii) $\alpha\beta^{-3}\gamma^{-1}$

   *b*) Construct truth tables for the following:

   i) $p \wedge \bar{q} \vee r$ ii) $\overline{p \vee \bar{q}} \vee \bar{r}$

   *c*) Let $A$ be the set of all lines a plane. Let $R \subseteq A \times A$ where $R = \{(l,m) | l, m \in A, l \parallel m\}$.

2. *a*) Let $G = \{(a,b) | \, a, b \, rationals, a \neq 0\}$. Define * on $G$ by $(a,b) * (c,d) = (ac, ad + b)$. Prove that G is a group.

   *b*) Define a sub group of a group.

   Prove that a non empty subset $H$ of a group $G$ is a subgroup of G if and only if the following condition holds for any $a, b \in H$ then $ab^{-1} \in H$.

   *c*) If $H$ and $K$ are subgroups of a group $G$. Then show that $H \cap K$ is also subgroup of $G$.

   *d*) Find the elements of symmetric group $S_3$.

3. *a*) Let G be a cyclic group with $G = < a >$. If $G$ is finite then prove that $G$ has exactly two generators $a$ and $a^{-1}$.

   *b*) State and prove the Lagrange theorem. Using above theorem find all subgroups of $C_{24}$, where is a cyclic group of order 24 generated by *a*. ($C_{24} = \langle a \rangle$)

   *c*) A subgroup $H$ of a group $G$ is normal in $G$ if and only if $g^{-1}hg \in G$ for all $h \in H, g \in G$.

   *d*) Define a center of a group $G$. Prove that the center of the group $G$ is a normal subgroup of the group $G$.

4. *a*) Prove that $\{a+\sqrt{3}b \mid a,b \in Z\}$ is a ring under usual addition and multiplication.
   *b*) Define an Integral Domain. Prove that any finite integral domain is a field.
   *c*) Let $R$ be a commutative ring with identity. Prove that $R$ is a field if and only if ideals of $R$ are $\langle 0 \rangle$ and $R$ itself.

5. *a*) Let $a, b, c, m$ and $n$ be integers. If $c \mid a$ and $c \mid b$, then prove that $c \mid ma+nb$
   *b*) Show that, if $a$ and $b$ are integers such that $b > 0$, then there exists unique integers $q$ and $r$ such that $a = bq + r$ where $0 \le r < b$.
   *c*) Find **gcd** of 1051 and 5072.
   *d*) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$. Show that $a + c \equiv b + d \pmod{m}$.

6. *a*) If $a$ and $b$ be integers and $a = bq + r$ where $q$ and $r$ are integers. Prove that $(a,b) = (b,r)$.
   *b*) Define Euler's phi function $\phi(n)$ where $n$ is a positive integer. Find $\phi(24)$ *and* $\phi(30)$.
   *c*) State the Chinese reminder theorem.
   *d*) Solve the following linear congruence $3x \equiv 2 \pmod{5}$