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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
Third Year – Semester II Examination – March / April 2014

MAT 3319- FLUID MECHANICS I

Answer **five** questions only   Time allowed: **03** hours

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1. Derive the continuity equation, $\frac{\partial \rho}{\partial t} + div(\rho \underline{\mathbf{q}}) = 0$, where ρ is the density and $\underline{\mathbf{q}}$ is the velocity of a moving fluid, and deduce that div $\underline{\mathbf{q}} = 0$, for incompressible fluid.

   The velocity vector $\underline{\mathbf{q}}$, has components $\underline{\mathbf{q}} = \left(\frac{m}{r^2}, 0, 0\right)$, $r > 0$, in spherical polar coordinates (r, θ, φ), where m is a positive constant. Show that motion of an incompressible fluid with this velocity is possible, and that this motion is irrotational. Find the corresponding velocity potential of the motion.

   Show that the outward volume flux across a spherical surface S, center origin, is $4\pi m$

2. Velocity $\underline{\mathbf{q}}$ at a point (x, y, z) has components in two different motions as follows:

   (a) $(-\omega y, \omega x, 0)$, (b) $\left(-\frac{\omega y}{r^2}, \frac{\omega x}{r^2}, 0\right)$, where ω is a constant, $r = \sqrt{x^2 + y^2} > 0$. Show that, in either case, motion of an incompressible fluid is possible and that the streamlines are the circles $x^2 + y^2 = \text{const}$, $z = \text{const}$. Determine the vorticity vector $\underline{\zeta} = curl \underline{\mathbf{q}}$, in each case.

   Find the circulation of velocity round the circle C: $r = a$, z = 0, and the flux of vorticity across the area bounded by C, in the case when the motional is rotational.

   Find the velocity potential in the case of irrotational motion.

3. Suppose an incompressible liquid of constant density ρ is moving radially outwards from a point fixed O, with spherical symmetry so that the velocity at a point P at distance r from O is $\mathbf{q} = q\underline{\mathbf{e}}_r$, where the speed q is a function of distance r and time t only.

If the motion is unsteady, then show that equation of the continuity equation takes the form $r^2 q = F(t)$, where $F(t)$ is a function of time t only.

A pulsating sphere having fixed centre O and radius R(t) at time t is the inner boundary of an infinite mass of homogeneous liquid of density ρ the pressure at infinity being p_∞. Assuming Euler's equation of motion foe a perfect liquid, show that the pressure p_1 at the surface of the sphere at time t is $p_1 = p_\infty + \dfrac{\rho}{2}\left\{\dfrac{d^2}{dt^2}(R^2) + \left(\dfrac{dR}{dt}\right)^2\right\}$.

Deduce the least value of p_1 when $R(t) = a + b\cos\omega t$, where a, b, ω are positive constants and $a > b$.

4. Show that Euler's equation of motion (which may be assumed without proof) for a homogeneous non-viscous liquid moving under an external force derivable from a potential Ω, may be written in the form $\dfrac{\partial \mathbf{q}}{\partial t} + grad\left(\dfrac{p}{\rho} + \dfrac{1}{2}q^2 + \Omega\right) = \underline{\mathbf{q}} \times \underline{\zeta}$, where $\underline{\zeta} = curl\underline{\mathbf{q}}$ is the vorticity vector. Hence derive Bernoulli's equation $\dfrac{p}{\rho} + \dfrac{q^2}{2} + \Omega = Const.$ for steady motion for a perfect fluid, stating other conditions for its validity

A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure Π at a place, where the sectional area B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth below the pipe, s being the delivery per second.

$$\frac{s^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$$

5. Obtain Cauchy-Riemann equations in polar form given by

$$\frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \text{ and } \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r},$$

where ϕ and $\psi$are the velocity potential and the stream function, respectively

Moreover, if the strength of a source in two-dimensions is given by m, then show that $\psi = -m\theta$ and $\phi = -m\log r$.

A source and a sink of equal strength are placed at the points $\left(\pm\frac{a}{2}, 0\right)$, respectively, within a fixed circular boundary $x^2 + y^2 = a^2$. Show that the streamlines are given by

$$\left(r^2 - \frac{1}{4}a^2\right)(r^2 - 4a^2) - 4a^2y^2 = Cy(r^2 - a^2).$$

6. Show that the kinetic energy T of a fluid moving irrotationally in a simply-connected region is given by the surface integral $T = -\frac{\rho}{2}\iint_S \phi\left(\frac{\partial \phi}{\partial n}\right) dS$, in the usual notation.

A rigid sphere of radius a moves with constant velocity U in a liquid which is otherwise at rest. Assuming the form $\Phi = \frac{A}{r^2}\cos\theta$, for the velocity potential, in terms of suitably defined spherical polar coordinates, show that the constant $A = \frac{1}{2}Ua^3$.

Using the above formula, or otherwise, show that the kinetic energy of the fluid is $\frac{1}{3}\rho\pi a^3 U^2$.