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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree
Third Year - Semester II Examination –February / March 2019

MAT 3217 – NON LINEAR PROGRAMMING

Time allowed: **Two (2) hours**

Answer **all questions**.
Calculators will be provided.

1. a) Briefly explain the Lagrangian multiplier method in solving Non Linear Programming Problems. (20 Marks)

   b) An organization wants to decide how to spend a maximum of Rs. 2000.00 for food for a group of people in a village. It decides to allocate money to purchase rice and dhal for Rs.5.00 and Rs.10.00 per sack respectively. The number of people (P) who would be fed if the organization buys x sacks of rice and y sacks of dhal is given by:

   $$P = x + 2y + \frac{x^2y^2}{2 \times 10^8} .$$

   What is the maximum number of people that can be fed, and how should the organization allocate its money? (40 Marks)

   c) Use the Lagrange multiplier method to find the points on the sphere given by the Cartesian equation $x^2 + y^2 + z^2 = 4$, that are closest and farthest from the point $(3, 1, -1)$. (40 Marks)

2. a) Define the Quadratic Programming Problem in optimization theory. (20 Marks)

   b) A firm produces and sells two commodities namely, A and B. When the firm produces $\boldsymbol{x}$ tons of A, its selling price is $(96 - 4\boldsymbol{x})$ dollars per ton, whereas when the firm produces $\boldsymbol{y}$ tons of B, its selling price is $(84 - 2\boldsymbol{y})$ dollars per ton. The total cost of producing $\boldsymbol{x}$ tons of A and $\boldsymbol{y}$ tons of B is:

   $$\boldsymbol{C(x, y) = 2x^2 + 2xy + y^2}$$

Cont...

i. Find the firm's profit function π of selling x and y tons of A and B respectively.

ii. Find all critical points of π, classifying each critical point as a local maximum, a local minimum, or a saddle point.

(20 Marks)

c) Consider the following Non Linear Programming problem:

$$\text{Minimize} z = {x_1}^2 + \frac{3}{2}{x_2}^2 - x_1 - x_2$$
$$\text{subject to}: \quad x_1 + x_2 \geq 6$$
$$x_1, x_2 \geq 0$$

i. Rewrite the above problem in the matrix form. (10 Marks)

ii. Solve the above problem using a suitable algorithm. (50 Marks)

3. a) Explain the following algorithms in stepwise form:

i. One Dimensional Search Procedure.

ii. Gradient Search Procedure. (20 Marks)

b)Use **One Dimensional Search Procedure** to solve the following Non Linear Programming Problem:

$$\text{Minimize } Z = x^2 - 6x$$

Take the initial upper and lower bounds as $\bar{x} = 4.8$, $\underline{x} = 0$ respectively and also error tolerance as $\varepsilon = 0.04$. (40 Marks)

d) Consider the following multi- variable unconstrained optimization problem:

$$\text{Maximize } f(x) = 2x_1x_2 + 2x_2 - {x_1}^2 - 2{x_2}^2$$

Using **Gradient Search Procedure**, find a numerical solution with error tolerance $\varepsilon = 0.01$. (40 Marks)

4. a) Define a posynomial. (10 Marks)

b) Geometric programming deals with problems in which the objective and constraint functions are of the following type:

$$Z = f(x) = \sum_{j=1}^{N} U_j \quad ,$$

$$\text{where } U_j = C_j \prod_{i=1}^{n} {x_i}^{a_{ij}} \qquad j = 1,2, \ldots N$$

Cont...

Let Z^* be the minimum value of Z and $y_j = \frac{U_j^*}{Z^*}$.

Show that $Z^* = \prod_{j=1}^{N} \left(\frac{C_j}{y_j^*}\right)^{y_j^*}$.

(40 Marks)

c) Solve the following Geometric Programming problem:

Minimize $z = \frac{1}{x_1 x_2 x_3} + 2x_2 x_3 + 3x_1 x_3 + 4x_1 x_2$.

subject to $x_1, x_2, x_3 > 0$

(50 Marks)

End