115

RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. in Information Technology
First Year - Semester I Examination – March 2021

ICT 1404 – Mathematics and Statistics for Computing

Time: Three (03) hours

- Answer any **FIVE (05)** questions.
- Calculators are allowed.
- Each question carries equal marks.
- Statistical tables and formulas are provided as attachments.

1. a) Find the number of four-character passwords that can be made using 11 characters of which 4 are S's , 3 are M's, 2 are L's and 2 each are X and Y,
   (i) If they are not case sensitive
   (ii) If they are case sensitive

   (10 Marks)

   b) Out of seven (07) engineers and five (05) mathematicians, a committee consisting of **five (05) members** is to be formed. In how many ways can this be done if
   (a) any three (03) engineers and any two (02) mathematicians can be included
   (b) a particular engineer and a particular mathematician must be included

   (10 Marks)

2. a) If $\begin{vmatrix} \lambda & 3 & 5 \\ -2 & \lambda & 4 \\ 1 & 4 & 9 \end{vmatrix} = 20$ find λ.

   (10 Marks)

   b) Using matrices or Cramer's rule solve the following linear equations.
   $9x+4y+3z= -1$, $2x+y+2z=1$, $7x+3y+4z=1$

   (10 Marks)

3. a) Three (03) university students decide to enter a marathon race. The respective probabilities that they will complete the marathon are 0.9, 0.7 and 0.6. Assuming that their performances are independent, find the probabilities that,
   i. they all complete the marathon
   ii. at least two (02) complete the marathon

   (08 Marks)

   b) A doctor is consulted by patients for a certain disease. Out of them 60% are more than 65 years in age, 30% between 65 years and 18 years and 10% are less than 18 years. The probabilities that the patients of those age groups suffering from the disease are 0.04, 0.03 and 0.01 respectively. If a patient is selected randomly
      i. Find the probability that he is suffering from the disease.
      ii. Using the Bayes' theorem, find the probability that he is less than 18 years of age given that he is suffering from the disease.

   (12 Marks)

4. a) In a test there are ten (10) multiple choice questions. For each question there is a choice of five (05) answers, only one (01) of which is correct. A student guesses each of the answers.
   i. Find the probability that he gets more than seven (07) correct answers.
   ii. How much correct answers can be expected in average?

   (10 Marks)

   b) Suppose that the number of errors per page in a printed document has randomly distributed according to a Poisson distribution with a mean error of 1. If a page is randomly selected find the probabilities that page has
      i. No errors ii. at most two (02) errors

   (10 Marks)

5. a) Packets of sugar filled by a particular machine have masses which are normally distributed with a mean 1kg and the standard deviation of 25g.
   i. Find the probability that the mass of a randomly selected packets of sugar is between 975g and 1010g.
   ii. 1% of the packets are rejected for being under-mass and 2% of the packets are rejected for being over-mass. Between what ranges of values should the mass of a packet of sugar lie if it is to be accepted?

   (12 Marks)

   b) Assuming that the life expectancy of a person is exponentially distributed with mean, $\beta=50$ years. Find the probability that he lives for more than 77 years, if he is already 57 years old. ($e^{-0.4}=0.67032$)

   (08 Marks)

6. The number of covid-19 patients and the number of deaths among them reported weekly for 8 consecutive weeks are given as follows.

No of patients ('000) Y	No of deaths X
15.02	86
13.37	78
12.30	64
11.90	49
10.04	50
9.70	48
7.80	40
5.30	36

a) Calculate the equation of line of regression of Y on X using least square method. (10 Marks)

b) Estimate the number of deaths when the number of patients is 400,000. (03 Marks)

c) What is the death rate per thousand patients? (02 Marks)

d) Compute the coefficient of Determination and interpret it. (05 Marks)

--END--

Statistical Formulae

Probability Distributions

Normal distribution $Z = \frac{X - \mu}{\sigma}$, here μ = mean and σ = standard deviation

Binonmial distribution $P(x) = {}^{n}C_{x}p^{x}q^{n-x}$; here, q=1-p , x=0,1,2,...n

Poisson distribution $P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$; here λ =mean=variance

Exponential distribution $P(x < T) = 1 - e^{-\frac{T}{\beta}}$ or $P(x \geq T) = e^{-\frac{T}{\beta}}$ here β = mean

"no-memory property". $P(x > a + b \mid x > a) = P(x > b)$

Uniform distribution

$$P(x) = \frac{1}{b-a} \quad for \quad a < x < b$$
$$= 0 \quad elsewhere$$

Correlation and Regression Analysis

Karl Pearson's coefficient of correlaton, $r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$

Line of regression, $y = a + bx$ where $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$ $a = \bar{y} - b\bar{x}$

Coefficient of Determination, R $R^2 = b^2\left(\frac{\sum x^2 - n\bar{x}^2}{\sum y^2 - n\bar{y}^2}\right)$ or $R^2 = b^2\left(\frac{n\sum x^2 - (\sum x)^2}{n\sum y^2 - (\sum y)^2}\right)$

$R^2 = r^2$

Standard Normal Probabilities

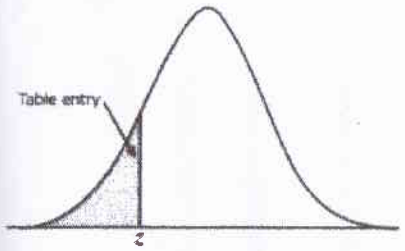

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

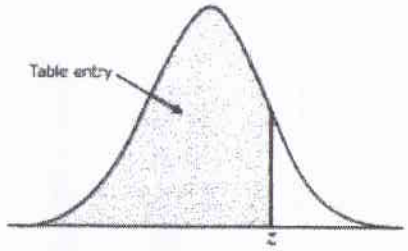

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$e^{-\lambda}$ Table

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	1.00000	0.90484	0.81873	0.74082	0.67032	0.60653	0.54881	0.49659	0.44933	0.40657
1	0.36788	0.33287	0.30119	0.27253	0.24660	0.22313	0.20190	0.18268	0.16530	0.14957
2	0.13534	0.12246	0.11080	0.10026	0.09072	0.08208	0.07427	0.06721	0.06081	0.05502
3	0.04979	0.04505	0.04076	0.03688	0.03337	0.03020	0.02732	0.02472	0.02237	0.02024
4	0.01832	0.01657	0.01500	0.01357	0.01228	0.01111	0.01005	0.00910	0.00823	0.00745
5	0.00674	0.00610	0.00552	0.00499	0.00452	0.00409	0.00370	0.00335	0.00303	0.00274
6	0.00248	0.00224	0.00203	0.00184	0.00166	0.00150	0.00136	0.00123	0.00111	0.00101
7	0.00091	0.00083	0.00075	0.00068	0.00061	0.00055	0.00050	0.00045	0.00041	0.00037
8	0.00034	0.00030	0.00027	0.00025	0.00022	0.00020	0.00018	0.00017	0.00015	0.00014
9	0.00012	0.00011	0.00010	0.00009	0.00008	0.00007	0.00007	0.00006	0.00006	0.00005
10	0.00005	0.00004	0.00004	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002
11	0.00002	0.00002	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001

Example : $e^{-2.4} = 0.09072$