RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year – Semester I Examination – October/ November 2015

MAT 4301 Operational Research I

Answer ALL Questions **Time: Three (03) hours.**

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01. Two players A and B play a zero sum game in which A has strategies A1, A2, A3 and B has strategies B1, B2 , B3 , B4. The payoff to A when A plays strategy A*i* and B plays strategy B*j* is given by the entry in row *i* and column *j* of the following matrix D:

$$\text{Player A} \begin{bmatrix} 3 & 0 & 3 & 4 \\ 1 & 5 & 2 & 0 \\ 5 & 6 & 6 & 1 \end{bmatrix} \quad \text{Player B}$$

(i) Show that this game is not stable.

(ii) Explain briefly how the matrix D can be reduced by dominance to a 2x2 matrix.

(iii) Develop a Linear programming model with respect to Player A to find optimal strategies.

(iv) Develop a Linear programming model with respect to Player B to find optimal strategies.

The final tableau and the corresponding feasible dictionary for the solution of the linear programming formulation of player B's problem is given below. Here S1and S2 denote the slack variables in the first and second constraints respectively.

Basic Variable	X_1	X_2	X_3	S_1	S_2	Value
z	1/3	0	0	1/6	1/6	1/3
X_3	3/5	0	1	1/5	0	1/5
X_2	11/15	1	0	1/30	1/6	2/15

(a) Using the above table find the optimal mixed strategy for each player in the reduced game.

(b) Hence find the optimal mixed strategy for each player in the original game and state the value of the game.

02. On an average 96 patients per day (24 hours) require the service of an emergency clinic. Also on an average a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes and thus each minute of decrease in this average time would cost Rs.10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patients?

03. Customers arrive at a one- window drive in a bank according to a Poisson distribution with mean 10per hour. Service time per customer is exponential with mean 5 minutes. The car space in front of the window, including that for the serviced can accommodate a maximum of 3 cars. Other cars can wait outside this space.

(a) What is the probability that an arriving customer can drive directly to the space in front of the window?

(b) What is the probability that an arriving customer will have to wait outside the indicated space?

(c) How long is an arriving customer expected to wait before starting service?

04. A tax consulting firm has 3 counters in its office to receive people who have problems concerning their income, wealth and sales taxes. On the average 48 persons arrive in an 8-hour day. Each tax adviser spends 15 minutes on an average on an arrival. If the arrivals follow a Poisson distribution and service times are according to exponential distribution , find

(a) The average number of customers in the system.

(b) Average number of customers waiting to be served.

(c) Average time a customer spends in the system.

(d) Average waiting time for a customer

(e) The number of hours each week a tax driver spends performing his job

(f) The probability that a customer has to wait before he gets service.

05. State the steps of Johnson's method for n jobsthrough 3 machines problem.

A ready-made garment manufacturer has to process 7 items through three stages of production: cutting , sewing , pressing and packing namely. The time taken for each of these items at the different stages is given below in hours:

Item	1	2	3	4	5	6	7
Cutting time	5	7	3	4	6	7	12
Sewing time	2	6	7	5	9	5	8
Pressing and packing	10	12	11	13	12	10	11

(a) Find an order in which these items are to be processed through these stages so as to minimize the total processing time.

(b) What is minimum elapsed time?

(c) Find the idle time for each stage.