RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. General Degree in Applied Sciences
Second Year - Semester I Examination – June/July 2018
MAP 2301 – ALGEBRA

Time: Three (03) hours

Answer All (06) questions

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1. *a*) Let A and B be two sets. Show that $A \subseteq B$ if and only if $B = A \cup (B \setminus A)$.

   b) The symmetric difference of the sets A and B is defined by $A \Delta B = (A \setminus B) \cup (B \setminus A)$. Let A, B and C be three sets. Prove the following:

   i. $A \Delta \phi = A$.

   ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

   c) Compute the truth tables for the following propositional formulas:

   i. $(p \wedge \neg q) \vee \neg(p \rightarrow q)$.

   ii. $(p \wedge \neg q) \vee (r \rightarrow \neg s) \vee \neg(p \rightarrow r)$.

2. *a*) Define the following terms:

   i. Reflexive relation
   ii. Symmetric relation
   iii. Anti-symmetric relation
   iv. Transitive relation
   v. Equivalence relation

   b) Let R be a relation defined on Z by xRy if $x^2 - y^2$ divisible by 7 for $x, y \in Z$.

   Is R an equivalence relation? Justify your answer.

   c) Let $S = \Re - \{0\}$ be a set. Define a relation R on the set S by xRy if $\frac{x}{y} \in Q$ for each $x, y \in S$

   Prove that S is an equivalence relation.

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3. *a) Define* each the following terms:
   - i. Well defined function
   - ii. One to one function
   - iii. Onto function
   - iv. Bijective function

   b) Which of the following functions are Well defined , one to one , onto or Bijective ?Justify your answers

   i. $f:\Re \to \Re, where\ f(x) = x^3 - 2x + 1$

   ii. $f:\Re - \{1\} \to \Re, where\ f(x) = \dfrac{2x+3}{x-1}$

   iii. $f:\Re \to \Re, where\ f(x) = \begin{cases} x^2 & ; x \le 0 \\ x+2 & ; x > 0 \end{cases}$

   iv. $f:\Re \to \Re, where\ f(x) = \sin x$

   c) Which of the followings are binary operations? Justify your answer

   i. $a * b = \dfrac{ab}{5}, where\ a,b \in \mathbb{Q}.$

   ii. $a * b = \dfrac{a}{a^2 + b^2}, where\ a,b \in \Re.$

   iii. $a * b = \sqrt{a}, where\ a,b \in \Re^+.$

   iv. $a * b = e^{\frac{a}{b}}, where\ a,b \in \mathbb{Z}.$

4. (a). Determine whether the following permutations are even or odd.

   i. $(2\ \ 3\ \ 7\ \ 5)(1\ \ 3\ \ 4)$

   ii. $(1\ \ 9\ \ 4\ \ 8\ \ 2)(1\ \ 3\ \ 4)(5\ \ 8\ \ 7)$

   (b). Let $\alpha = (1\ \ 9\ \ 2)(3\ \ 5\ \ 8\ \ 7), \beta = (3\ \ 7\ \ 5\ \ 4)\ and\ \gamma = (3\ \ 2\ \ 7)(1\ \ 5\ \ 4).$

   i. Find the orders of $\alpha, \beta\ and\ \gamma$.

   ii. Find $\alpha^{-1}, \beta^{-3}, \alpha^2\beta^3\gamma^{-4}$.

   iii. $\alpha^{2018}\beta^{400}\gamma^{99}$

5. *a)* Write the axioms of the groups .

Let $G = \left\{ M = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| a, b \in R, \det M \neq 0 \right\}$ and Let $\times$ denotes the ordinary matrix multiplication. Show that $(G, \times)$ is a group.

b) Let G be a group. Suppose $(ab)^n = a^n b^n$ *for all* $a, b \in G$ where $n > 1$ is a fixed integer.

Show that:

i. $(ab)^{n-1} = b^{n-1} a^{n-1}$

ii. $a^n b^{n-1} = b^{n-1} a^n$

c) Define a sub group of a group G .

Prove that, a non-empty subset H of a group G is a sub group of G if and only if $ab^{-1} \in H$ for all $a, b \in H$.

6. *a)* Prove that, $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$ for all $n \geq 1$.

b) Given integers a and b , with $b > 0$, show that there exist unique integers q and r satisfying $a = qb + r,\ 0 \leq r < b$.

c) Prove that a linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$, where $d = \gcd(a, b)$. If (x_0, y_0) is a particular solution of this equation, then the General solution is given by

$$x = x_0 + \left(\frac{b}{d}\right)t,\ y = y_0 - \left(\frac{a}{d}\right)t \text{ where } t \text{ an integer}$$

d) Solve the Diophantine linear $172x + 20y = 1000$.

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