

RAJARATA UNIVERSITY OF SRI LANKA FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

Second Year - Semester I Examination - September/October 2013

MAA 2302 - Probability and Statistics II

Answer ALL FIVE QUESTIONS.

Time allowed: THREE hours

[Statistical tables and calculators will be provided.]

1. Let (X, Y) be a two dimensional continuous random variable with joint probability density function,

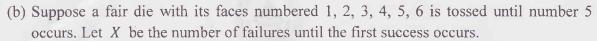
$$f(x,y) = \begin{cases} k(2x+3y) & ; 0 \le x \le 2, \ 0 \le y \le 1 \\ 0 & ; otherwise \end{cases}$$

- (i) Show that $k = \frac{1}{7}$
- (ii) Find the marginal probability density functions of the random variables X and Y.
- (iii) Determine whether X and Y are independent random variables.
- (iv) Find $f_{X|Y}(x|y)$ and hence determine the probability $P\left(\frac{1}{2} < X < 1 \mid Y = \frac{2}{3}\right)$.

(a) Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{12}{(x^6 + 2)} & ; x > 0 \\ 0 & ; otherwise \end{cases}$$

Find the probability density function of the random variable $Y = 8X^3$.



- (i) Determine the probability mass function of X.
- (ii) Find the probability mass function of Y = X + 2.
- (c) Let X and Y be the continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 24x^2y & ; 0 \le x \le 1, \ 0 \le y \le 1 \\ 0 & ; otherwise \end{cases}$$

(i) Find the joint probability density function of the random variables U and V, where U = X + Y and V = 2X.

3. (a)

(i) Let x_1 , x_2 , x_3 , ..., x_n be a random sample taken from a normal distribution with mean μ and variance σ^2 . Let \bar{X} and S^2 be the sample mean and the sample variance.

Show that the random variable $T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$ has a t-distribution with n - 1 degrees of freedom.

(ii) It is known from the past experience that the mean height of a certain plant is $1.6\,m$. A random sample of 16 plants was selected and the standard deviation of the sample was found to be $0.16\,m$. Find the probability that the mean of the sample is greater than $1.67\,m$, stating any assumptions you make.

(b) (i) State the central limit theorem.

(ii) The diameter of ping-pong balls manufactured by a certain machine has a mean of 3.4 cm and a standard deviation of 0.7 cm. If a random sample of 49 pingpong balls were selected, find the probability that the sample mean is less than 3.24 cm.

4.

- (a) Let $x_1, x_2, x_3, ..., x_n$ be a random sample of size n_1 taken from a normal population with mean μ_1 and variance σ_1^2 and let $y_1, y_2, y_3, ..., y_n$ be a random sample of size n_2 taken from a normal population with mean μ_2 and variance σ_2^2 .
 - (i) Find the mean and the variance for the difference between the two sample means.
 - (ii) Find 95% confidence interval for the difference between the two sample means.
- (b) A company produces two types of light bulbs, type S and type T. A random sample of 35 light bulbs taken from type S indicates a mean life expectancy of 375 hours and a random sample of 45 light bulbs taken from type T indicates a mean life expectancy of 362 hours. The population standard deviation of type S is 110 hours and the population standard deviation of type T is 125 hours. Find a 95% confidence interval for the difference between the two sample means, stating any assumptions you make.
- 5. Let $x_1, x_2, x_3, ..., x_n$ be a random sample of size n taken from a Binomial Distribution, Bin(m, p), where m is a known integer. The probability mass function of the distribution is given by

$$f(x;p) = {m \choose x} p^x (1-p)^{m-x}$$
 where $x = 0,1,2,...m$

Two estimators for the probability p are defined as $P_1 = \frac{\overline{X}}{m}$ and $P_2 = \frac{\overline{X} + 1}{m + 2}$

- (i) Stating the conditions satisfied by an unbiased estimator, determine the unbiased estimator/s of the two estimators defined above.
- (ii) Determine the best estimator for p, using Cramer-Rao Theory.

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