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RAJARATA UNIVERSITY OF SRILANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
Second Year Semester I Examination – September/October 2014

MAA 2201 – MATHEMATICAL METHODS II

Answer **FOUR** Questions Only Time allowed: **Two Hours**

1.

(a) If $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ is the position vector of a variable point, with respect to the origin O show that, the integral $\oint_C \underline{r} \times d\underline{r} = 2\pi ab\underline{k}$, where C is the ellipse: $x = a\cos\theta,\ y = b\sin\theta,\ z = 0,\ \ 0 \le \theta < 2\pi$. Hence find the area enclosed by C.

(b) Evaluate the integral $\int_C \left(y + xy^{-1} + 2yz\right)ds$, taken along the curve C, defined in terms of a parameter t, by $\underline{r} = \left(t^2, t, 1\right),\ 0 < t \le 1$, where s is the arc length.

(c) Given a vector field $\underline{\boldsymbol{F}} = 2xy^3z^4\underline{i} + 3x^2y^2z^4\underline{j} + 4x^2y^3z^3\underline{k}$, show that $curl\underline{\boldsymbol{F}} = \underline{\boldsymbol{0}}$, and find a scalar field $\phi(x, y, z)$ such that $\underline{\boldsymbol{F}} = grad\phi$.

2.

(a) Find, by double integration, the area of the region in the (x, y) –plane lying between the two parabolas $x^2 = 4ay$ and $y^2 = 4ax$.

(b) Evaluate the integral $\underline{\boldsymbol{J}} = \int_S \underline{r}\, dS$ taken over the surface S of the hemisphere $r = a,\ 0 \le \theta \le \pi/2$. Hence find the position vector of the centroid of area of S.

(c) Find by double integration, using polar coordinates (r, θ), the area enclosed by the cardiod $r = a(1 - \cos\theta)$.

3.

(a) Find the integral $I=\int_V dV$, giving the volume of the ***sector*** of the sphere defined by $0\le r\le a, 0\le\theta\le\alpha$. Also evaluate the volume integral $\underline{\mathbf{J}}=\int_V \underline{r}\,dV$, over the same sector, and hence show that, if the sphere is of uniform density ρ, position vector, $\underline{\hat{\mathbf{r}}}$ of its center of mass, defined by $\underline{\hat{\mathbf{r}}}=\underline{\mathbf{J}}/I$,

is $\dfrac{3a\underline{\mathbf{k}}}{8}(1+\cos\alpha)$, where $\underline{\mathbf{k}}$ is a unit vector in the direction of the Oz-axis.

(b) Using the divergence theorem of Gauss, show that the volume of the ***segment*** of the solid sphere: $x^2+y^2+z^2\le a^2$, $\quad -\dfrac{a}{2}\le z\le\dfrac{a}{2}$, is $\dfrac{11}{12}\pi a^3$.

4.

Differentiating $f(s)=\int_0^\infty e^{-st}F(t)dt=L\{F(t)\}$ with respect to parameter s show that

$L\{t\,F(t)\}=-\dfrac{df(s)}{ds}$. Hence find $L\{t\cos at\}$, $L\{\sin at-at\cos at\}$.

Deduce the values of the integrals $\int_0^\infty t\,e^{-2t}\cos t\,dt$, $\int_0^\infty e^{-3t}(\sin 2t-2t\cos 2t)dt$.

Using Laplace transform method solve the differential equation

$\dfrac{d^2y}{dt^2}+y=t\cos 2t$, subject to the initial conditions: $y(0)=0, y'(0)=0$.

5.

Use Laplace transform method to solve the differential equations

(a) $\dfrac{d^2y}{dt^2}-\dfrac{dy}{at}-6y=2$, subject to the initial conditions: $y(0)=1, y'(0)=0$.

(b) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{at}+5y=e^{-t}\sin t$ subject to the initial conditions: $y(0)=0, y'(0)=1$.

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