RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – February/March 2019

MAP 1302 – DIFFERENTIAL EQUATIONS - I

Time: Three (03) hours

Answer all questions.

1. a) Define the following terms of a differential equation:

   i. Order
   ii. Degree
   iii. Characteristics.

(30 marks)

b) Consider the differential equation $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$.

Using a suitable substitution, reduce the above equation into separable form and solve.

(40 marks)

c) By using a suitable method, solve the differential equation:

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0.$

(30 marks)

2. a) Let $\frac{dy}{dx} = \frac{(x+2y-3)}{(2x+y-3)}$.

   i. Show that the above equation can be reducible to homogeneous form.
   ii. Find the general solution.

(50 marks)

40

b) Consider the differential equation $(2xe^y + 3y^2)(dy/dx) + (3x^2 + \lambda e^y) = 0$.

i. Find λ such that the above equation is exact.

ii. Using the value of λ found in (i), solve the equation.

(50 marks)

3. a) Solve the following linear differential equation:

$$(x + 2y^3)(dy/dx) = y.$$

(30 marks)

b) Let $F(D)y = (a_nD^n + a_{n-1}D^{n-1} + \cdots + a_1D + a_0)y$, where $a_0, a_1, \dots, a_n$ are constant coefficients and the operator $D \equiv \frac{d}{dx}$. Show that $F(D)\{e^{bx}\} = F(b)e^{bx}$, where b is a constant.

(20 marks)

c) Find the general solution of each of the following differential equations:

i. $(4D^2 + 12D + 9)y = 144e^{-3x}$

ii. $(D^2 - 3D + 2)y = \sin 3x$, where $D = \frac{d}{dx}$.

(50 marks)

4. a) Show that $\frac{1}{(D+a)}f(x) = e^{-ax}\frac{1}{D}e^{ax}f(x)$, where $D \equiv \frac{d}{dx}$, $f(x)$ is any function of x and a is a constant.

Hence, find the general solution of $(D^2 - 3D + 2)y = e^x + e^{2x}$.

(50 marks)

b) Consider each of the following differential equations, where $p = \frac{dy}{dx}$.

Solve each equation using the method indicated in each part:

i. $p^2 - 7p + 12 = 0$ (solve for p)

ii. $y = 2px + y^2p^3$ (solve for x).

(50 marks)

5. a) Define Clairaut's equation.

Show that the differential equation $(y - px)(p - 1) = p$, where $p = \frac{dy}{dx}$ is in the Clairaut's form and hence, find the general solution.

(40 marks)

b) Define Riccati's equation.

Consider the equation $x^2(y_1 + y^2) = 2$, where $y_1 = \frac{dy}{dx}$.

i. Show that the above equation is in the form of Riccati's equation.

ii. Show that, there exist two values for the constant k such that $\frac{k}{x}$ is an integral of the above equation.

iii. Hence, find the general solution of the equation.

(60 marks)

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