RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
Second year – Semester 11 Examination-September /October 2013

Complex Calculus– MAP 2204

Answer four questions. **Time allowed: 2 hours only.**

1).

a).Using the definition, find the derivative of each function at the indicated points.

i). $f(z) = \frac{2Z-1}{3Z+2}$ at $z = z_0$; $z_0 \neq {}^{-2}/_3$

ii). $f(z) = 3z^{-2}$ at $z = 1 + i$

b).Find the limits

i).

$$\lim_{Z\to 1+i} \frac{Z^2 - Z + 1 - i}{Z^2 - 2Z + 2}$$

ii).

$$\lim_{Z\to e^{i\pi/3}} \left(z - e^{i\pi/3}\right)\left(\frac{Z}{Z^3+1}\right)$$

2). State and prove Cauchy Riemann equations.
Show that function $u(x,y) = 4xy - x^3 + 3xy^2$, satisfies the Laplace equation and find the conjugate harmonic function $v(x,y)$ of $u(x,y)$ such that the sum $u(x,y) + iv(x,y)$ is analytic. Express this sum as a function of z.

3). i).Find the Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in $1< |z| < 2$

ii).Expand the function $f(z) = \frac{1}{z+1}$ in a Taylor series about the point $z = 1$.

iii).Find the roots of equation $(1+z)^5 = (1-z)^5$

4). State and prove Cauchy integral formula.
In each of the following cases, use Cauchy integral formula to evaluate the integral

i). $\oint_C \frac{e^{2z}}{(z+1)^4} dz$; where C is the circle $|z| = 3$

ii). $\oint_C \frac{e^z}{z(z+1)} dz$; where C is the circle $|z-1| = 3$, each oriented counter clockwise.

5). Show, using contour integrals , that

i). $\int_0^{2\pi} \frac{\cos 3\varphi}{5-4\cos\varphi} d\varphi = \frac{\pi}{12}$

ii). $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $(|a| > |b|)$

iii). $\int_0^{2\pi} \frac{d\emptyset}{3-2\cos\emptyset+\sin\emptyset} = \pi$

6). State Cauchy Residue formula, and find

(i) $\oint_C \frac{z^2 - e^{-z^2}}{z(z^2 - 1)(z+3)} dz$, where C is the Jordan curve $|z| = 2$, mapped counterclockwise;

(ii) the residue of $f(z) = \frac{\log(1+z)}{(z^2+1)^2}$ at each of its poles.