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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
First Year Semester II Examination – February/ March 2013

MAA 1201 – MATHEMATICAL METHODS I

Answer four questions only. **Time allowed: Two Hours**

(1) (a) Define the following terms:
(i) a vector,
(ii) collinear vectors,
(iii) coplanar vectors.

(b) Let A, B, C be three points in the space such that $A = (-1, 1, 2)$, $B = (2, 1, 0)$ and $C = (4, 5, 6)$. Let D be the point on the AB line such that $AD : DB = 4 : 1$. Let E be the point on the BC line such that $BE : EC = 2 : 3$. Find the distance between the points D and E by using a vectorial method.

(c) Let ℓ be the straight line passing through the point A with the position vector $\vec{a}$ and parallel to the vector $\vec{b}$. Derive the equation of ℓ in terms of $\vec{a}$ and $\vec{b}$.

(2) (a) Let $\mathcal{P}$ be the plane passing through three non-collinear points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively. Derive the equation of $\mathcal{P}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

(b) (i) Find the equations of the tangent and the normal planes to the surface $x^2 + y^2 + z^2 - 2xyz = 1$ at the point $(-1, -1, 1)$.
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(ii) Let ℓ be the straight line passing through the points $(2, 1, 0)$ and $(-1, 2, 3)$. Find the point where ℓ goes through the tangent plane mentioned in part (i).

(3) Let $\vec{F} = (x^2 + y^2 - yz)\vec{i} + 2x^2y^2z^2\vec{j} + 4xyz\vec{k}$.
Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ along the following paths C :
(i) $x = 1,\ y = t + 1,\ z = t^2 - 1 \quad 0 \leq t \leq 1$.
(ii) the straight line joining $(2, 1, -1)$ and $(-1, 2, 0)$.
(iii) along the path $ABCD$ such that AB, BC, CD are straight lines with $A = (-1, 2, 0),\ B = (2, 2, 0),\ C = (2, 3, 0),\ D = (2, 3, -1)$.

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(4) (a) Find the constants a, b, c such that the field
$\vec{F} = (5x + 3y + az)\vec{i} + (bx + 2y - 3z)\vec{j} + (6x + cy - 7z)\vec{k}$ is irrotational.

(b) What do you mean by a " Conservative Vector Field" ?
Determine whether the field $\vec{F} = e^x \cos z\ \vec{i} + xyz\ \vec{j} + x \sin y\ \vec{k}$ is conservative.

(c) It is given that $div(\vec{F} \times \vec{G}) = (curl\vec{F}) \cdot \vec{G} - \vec{F} \cdot (curl\vec{G})$.
If the vector fields $\vec{F}$ and $\vec{G}$ are irrotational, show that the field $\vec{F} \times \vec{G}$ is solenoidal.

(5) (a) If $\phi = x^2 y + \sin z$ and $\psi = e^{xz} + \cos y$ find $grad(\phi\psi)$.

(b) If $\vec{F} = \cos y\ \vec{i} + ye^x\ \vec{j} + xz^2\ \vec{k}$ and $\vec{G} = e^y\ \vec{i} + (x^2 + z^2)\ \vec{j} + xyz\ \vec{k}$,
find $div(\vec{F} \times \vec{G})$.
(Hint: you may use the information in question (4) (c)).

(c) It is given that $curl(\phi\vec{F}) = grad\ \phi \times \vec{F} + \phi(curl\vec{F})$.
If $\phi = e^{xyz}$ and $\vec{F} = \cos y\ \vec{i} + e^x\ \vec{j} + \sin z\ \vec{k}$, find $curl(\phi\vec{F})$.

(6) Let $\vec{t}, \vec{n}, \vec{b}$ be the fundamental triad defined at a point of a space curve and associated Serret-Frenet Formulae are as follows:

$$\frac{d\vec{t}}{ds} = \kappa\vec{n}, \qquad \frac{d\vec{n}}{ds} = \tau\vec{b} - \kappa\vec{t}, \qquad \frac{d\vec{b}}{ds} = -\tau\vec{n}.$$

Given the space curve

$$\vec{r} = a\cos\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\vec{i} \ + \ a\sin\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\vec{j} \ + \ \frac{bs}{\sqrt{a^2 + b^2}}\vec{k},$$

Find the tangent,the normal and the binormal vectors and curvature(κ), torsion(τ) of the space curve at any point.
(Here a, b are constants and s is the arc legnth measured along the curve.)

End of the test.

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