RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
Second Year – Semester I Examination – April/ May 2016

MAA 2201 – Mathematical Methods II

Answer **Four** Questions **only**. Time allowed: **Two Hours**

01.

(a) Spherical polar coordinates (r,θ,ϕ) of a point P are related to its Rectangular Cartesian Coordinates (x,y,z) by the position vector equation $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$, where $r \geq 0, 0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$.

i. Illustrate the relationships between SPC and RCC in a single diagram, when both θ and ϕ are acute angles.

ii. Derive the scale factors (h_r, h_θ, h_ϕ) and base vectors $(\underline{e}_r, \underline{e}_\theta, \underline{e}_\phi)$.

iii. Hence prove that the Spherical Polar Coordinate system is orthogonal.

(b) A vector field $\underline{F}$ has components $\left(\frac{2\cos\theta}{r^n}, \frac{\sin\theta}{r^n}, 0\right)$ respectively, in the directions of unit base vectors $\underline{e}_r, \underline{e}_\theta, \underline{e}_\phi$, in a system of spherical polar coordinates (r,θ,ϕ), where n is a constant. Show each of the followings:

i. $div\underline{F} = \dfrac{2(3-n)\cos\theta}{r^{n+1}}$

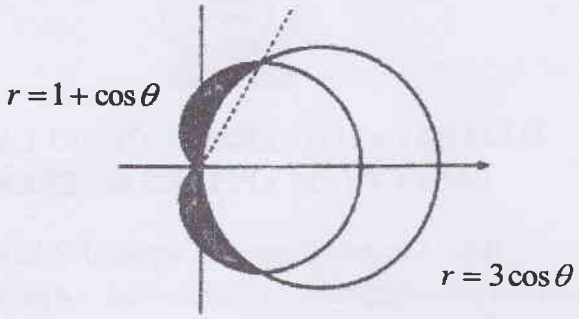
ii. $curl\underline{F} = \dfrac{(3-n)\sin\theta}{r^{n+1}}\underline{e}_\phi$

02.

(a) State (without proof) Stokes' theorem in vector analysis, and prove the Green's theorem in a plane using Stokes' theorem with usual notations.

[P.T.O.]

(b) Find the area of the region inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 3\cos\theta$ by using double integrals and polar coordinates (r, θ).



03.

(a) Find the Laplace Transform of the convolution integral; $F(t) = \int_0^t u^{m-1}(t-u)^{n-1}\,du$.

Hence show that $F(t) = \Gamma(m)\Gamma(n)\dfrac{t^{m+n-1}}{\Gamma(m+n)}$ and deduce the Beta function, $\beta(m,n)$ defined by the integral $\int_0^1 u^{m-1}(t-u)^{n-1}\,du$.

(b) Using Laplace Transform method, solve the following differential equation for $Y(t)$:

$\dfrac{d^2Y}{dt^2} - 6\dfrac{dY}{dt} + 15Y = 2\sin(3t)$, subject to the conditions $Y(0) = -1$ and $Y'(0) = -4$.

04.

(a) V is the volume of the sector of the sphere defined by $0 \le r \le a$ and $0 \le \theta \le \alpha$. Using Spherical Polar Coordinates show that $V = \dfrac{2}{3}a^3\pi(1-\cos\alpha)$.

(b) Evaluate the integral $J = \int_V \underline{r}\,dv$ over the same sector and hence show that the position vector of its centroid is $\dfrac{3}{8}a(\cos\alpha + 1)\underline{k}$.

(c) By using part (a), deduce the volume of the sector of the sphere defined by $0 \le r \le a$ and $\alpha \le \theta \le \pi$.

05.

(a) The Laplace Transform of $F(t)$ is defined by,

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)dt = f(s);\ \text{Where } s \text{ is a parameter.}$$

With the usual notations derive the Laplace Transform of $\sin(at)$ and show that $L\{e^{-at}F(t)\} = f(s+a)$, where a is a constant.

(b) Evaluate the followings:

i. $L\{e^{at}\cos bt\}$

ii. $L^{-1}\left\{\log\left(\frac{s+a}{s-a}\right)\right\}$

iii. $L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}$

Where a and b are constants.

(c) Show that the Laplace Transform of $(te^{-2t}\sin 2t)$ is $\frac{4(s+2)}{\left((s+2)^2+4\right)^2}$. Hence evaluate the integral $\int_0^\infty (te^{-4t}\sin 2t)dt$.