RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year Semester II Examination –February /March 2019

MAA 2203 – NUMERICAL ANALYSIS II

Time: Two (02) hours

Answer ALL Questions
Non-Programmable Calculator is permitted

1) a) Determine the constants a, b, c, d and e so that S is a natural cubic spline.

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3 & x \in [0,1] \\ (x-1)^3 + ex^2 - 1 & x \in [1,2] \end{cases}$$

(50 marks)

b) Complete the following table.

x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
$x_0 = 0.0$	$f[x_0] = 1$		
$x_1 = 0.3$	$f[x_1]$	$f[x_0, x_1] = 4$	
$x_2 = 0.8$	$f[x_2]$	$f[x_1, x_2]$	$f[x_0, x_1, x_2] = 5$

(50 marks)

2) a) Using the following data obtain the least square polynomial approximation of degree two.

x	0	1	2	3	4
y	-4	-1	4	11	20

(50 marks)

b) The following table gives the viscosity of sulfuric acid, in millipascal seconds (centipoises), as a function of concentration, in mass percent:

Concentration (C)	0	20	40	60	80	100
Viscosity (V)	0.80	1.40	2.51	5.37	17.4	24.2

i. Using the given data, construct a linear spline L(C) to approximate the viscosity V(C).
(25 marks)

ii. Estimate the viscosity by using L(C) when the concentration is 5%, 63% and 92%.
(25 marks)

03. a) Using Lagrange formula, find the quadratic polynomial $P_2(x)$ that interpolates through the points (0, -1), (1, -1) and (2, 7).

(40 marks)

b) The upward velocity of a rocket is given as a function of time in the following table.

Velocity as a function of time

t(s)	*v(t) (m/s)*
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

i. Calculate the value of the velocity at t = 16 seconds with third order polynomial interpolation using Newton's divided difference polynomial method.

(20 marks)

ii. Using the third order polynomial interpolate for velocity, find the distance covered by the rocket from t =11s to t = 16 s.

(20 marks)

iii. Using the third order polynomial interpolate for velocity, find the exact value of the acceleration of the rocket at t =16 s.

(20 marks)

04. a) Find y_n, from the difference equation $\Delta^2 y_n - 3\Delta y_n + 2y_n = 0$ and show that the solution is unbounded.

(30 marks)

b) Solve the difference equation $\Delta^2 y_k + 3\Delta y_k - 4y_k = k^2$ with the initial conditions $y_0 = 2, y_2 = 2$.

(30 marks)

c) Use Simpson's rule with $n = 6$ to approximate the integral $\int_1^4 \frac{1}{x} dx$, (Round your answers to six decimal places.)

(40 marks)