RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Third Year Semester II Examination – Jan / Feb 2023

PHY 3302 – MATHEMATICAL METHODS FOR PHYSICISTS

Time: Three (03) hours

Answer Any 5 Questions.

Unless otherwise specified, symbols have their usual meaning.
A non-programmable calculator is permitted.

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1) a) If $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then show that, $(A + B)(A - B) \neq A^2 - B^2$

(02 marks)

b) If $A = \begin{bmatrix} 3 & 6 & 1 \\ 0 & 1 & 3 \\ 2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 4 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^2B.

(04 marks)

c) Find k, if the matrix $P = \begin{bmatrix} k-3 & 1 \\ -5 & k+3 \end{bmatrix}$ is singular. (04 marks)

d) Find the eigenvalue and eigenvector of the matrix,
$C = \begin{bmatrix} 5 & -1 \\ 6 & -2 \end{bmatrix}$ (04 marks)

e) Solve the following three linear equations using determinants.

$$2x + y = 1$$
$$3x + 2y + 2z = 13$$
$$4x - 2y + 3z = 9$$

(06 marks)

2) a) Find the value of x and y in the following equation, given that $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$(x + yi)(2 + 3i) = 3 - 4i$$

(04 marks)

b) Find the magnitude and argument of the following complex umber and write it in polar form.

$$Z = \sqrt{3} - i$$

(04 marks)

Contd.

c) Find the value of $(1+i)^{16}$ using De Moivre's theorem. (06 marks)

d) Verify the complex number, $f(z) = (x^3 - 3xy^2) + i(3x^2y - y^3)$ is holomorphic using the Cauchy-Riemann relationship.

(06 marks)

3) a) Solve the differential equation, $\frac{dy}{dx} - 2xy = x$. (04 marks)

b) Verify that the function $y = e^{-2x}$ is a solution to the differential equation,

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

(04 marks)

c) Show that if n is a constant, then $u(x,t) = Sin\ (nt)\ Cos\ x$ is a solution to, $\frac{d^2u}{dt^2} = n^2\ \frac{\partial^2 u}{\partial x^2}$

(04 marks)

d) Radioactive decay follows the first order differential equation as stated below, where N is the amount of radioactive material present at any time t and k is an arbitrary constant.

$$\frac{dN}{dt} = -kN$$

The half-life of radium is 1600 years and initially the sample contains 50 g of radium.

i. Obtain an expression for the remaining amount of radioactive material after t time.
ii. Calculate the value of constant k.
iii. How long will it take the radium contains to reach 40 g?

(08 marks)

4) Let f(t) be a function of period 2π such that

$$f(t) = \begin{cases} 5, & 0 \leq t \leq \pi \\ 0, & \pi \leq t \leq 2\pi \end{cases}$$

a) Sketch a graph of f(t) in the interval $-2\pi \leq t \leq 2\pi$. (04 marks)

b) Write down expressions for Fourier coefficients a_0, a_n and b_n. (06 marks)

c) Calculate the Fourier series for the above periodic signal. (10 marks)

Contd.

5) Convolution is a mathematical operation on two functions that produce a third function, that express how the shape of one is modified by the other.

   a) Write an expression for convolution of $f(t)$ and $g(t)$ functions. (04 marks)

   b) Find the convolution of $f(t) = sin\ (t)$, and $g(t) = cos\ (t)$. (10 marks)

   c) Sketch graphs of,

      i) $f(t) = sin\ (t)$
      ii) $g(t) = cos\ (t)$
      iii) Convolution of $f(t)$ and $g(t)$, $[(f * g)(t)]$ in the interval of $0 \leq t \leq 2\pi$.
      (06 marks)

6) An infinite series is the sum of infinitely many terms.

   a) The n^{th} partial sum of the infinite series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \frac{3n^3}{(n+1)(n+2)}$.

      i) Determine whether the series converge or diverge. (02 marks)

      ii) Find 7^{th} term (a_7) of the sequence. (02 marks)

   b) If the first term of the sequence is a, and the difference between the terms is d, then derive expression for n^{th} partial sum of the series. (04 marks)

   c) By using the knowledge of series, calculate the summation of first 1000 positive integers. (04 marks)

   d) The 25^{th} term of an arithmetic series is 43, and 59^{th} term is 26. Find the summation of first 100 terms. (04 marks)

   e) By using ratio test, determine whether the following series diverge or converge.
   $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\ldots+\frac{1}{n!}+\ldots$ (04 marks)

End.