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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
First Year – Semester I Examination– May/June 2016

MAA 1201 – Mathematical Methods I

Answer **Four** questions only. Time allowed: **Two** Hours

1.

a. Prove $\boldsymbol{A}\bullet(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{A}\bullet\boldsymbol{B}+\boldsymbol{A}\bullet\boldsymbol{C}$

b. Prove that $\boldsymbol{A}\,X(\boldsymbol{B}\,X\boldsymbol{C})+\boldsymbol{B}\,X(\boldsymbol{C}\,X\boldsymbol{A})+\boldsymbol{C}\,X(\boldsymbol{A}\,X\boldsymbol{B})=\boldsymbol{0}$

c. Find the angle between $\boldsymbol{A}=3\boldsymbol{i}+2\boldsymbol{j}-6\boldsymbol{k}$ *and* $\boldsymbol{B}=4\boldsymbol{i}-3\boldsymbol{j}+\boldsymbol{k}$

d. Determine a unite vector perpendicular to the plane of $\boldsymbol{P}=2\boldsymbol{i}-6\boldsymbol{j}+3\boldsymbol{k}$ and $\boldsymbol{Q}=4\boldsymbol{i}+3\boldsymbol{j}-\boldsymbol{k}$

e. Prove the **law of sines** for plane triangles.

f. Find an equation for the plane determined by the points $P_1(2,-1.1), P_2(3,2,-1)$ and $P_3(-1,3,2)$.

2.

a. Find the equation of a straight line which passes through two given points A and B having position vectors **a** and **b** with respect to an origin O.

b. Show that the equation of a plane which passes through three given points A,B,C not in same straight line and having position vectors **a**, **b**, **c** relative to an origin O, can be written $\boldsymbol{r}=\frac{\alpha\,\boldsymbol{a}+\beta\,\boldsymbol{b}+\gamma\,\boldsymbol{c}}{\alpha+\beta+\gamma}$

Where α, β *and* γ are scalars. Verify that the equation is independent of the origin.

c. In each case determine whether the vectors are linearly independent or linearly dependent:

i. $\boldsymbol{A} = 2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k},\ \boldsymbol{B} = \boldsymbol{i} - 4\boldsymbol{k},\ \boldsymbol{C} = 4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$

ii. $\boldsymbol{P} = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},\ \boldsymbol{Q} = 2\boldsymbol{i} - 4\boldsymbol{j} - \boldsymbol{k},\ \boldsymbol{R} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$

3.

a. Show that $\boldsymbol{A} \bullet (\boldsymbol{B}\, X \boldsymbol{C})$ is an absolute value equal to the volume of a parallelepiped with sides $A,\ B$ and C.

b. Prove that a necessary and sufficient condition for the vectors $A,\ B$ and C to be coplanar is that $\boldsymbol{A} \bullet (\boldsymbol{B}\, X \boldsymbol{C}) = 0$.

c. Prove that p, q and r are non – coplanar then $xp + yq + zr = 0$ implies $x = y = z = 0$.

d. Find the constant μ such that vectors $2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},$, $\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$ and $3\boldsymbol{i} + \mu\boldsymbol{j} + 5\boldsymbol{k}$ are coplanar.

4.

a. Find the unite tangent vector to any point on the curve $x = t^2 + 1, y = 4t - 3,$ $z = 2t^2 - 6t.$

b. Determine the unite tangent at the point where $t = 2$.

c. Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t, y = 2\cos 3t,\ z = 8t$ at any time $t > 0$. Find the magnitude of the velocity and acceleration.

d. Find the curvature and the torsion for the space curve $x = \theta - \sin\theta,$ $y = 1 - \cos\theta, z = 4\sin\left(\frac{\theta}{2}\right).$

5.

a. Prove that the vector $\boldsymbol{A} = 3y^4z^2\boldsymbol{i} + 4x^3z^2\boldsymbol{j} - 3x^2y^2\boldsymbol{k}$ is **solenoidal.**

b. Prove $curl(\varphi\ grad\ \varphi) = 0$

c. If $f(r)$ is differentiable, prove that $f(r)\,\boldsymbol{r}$ is **irrotational.**

d. Find an equation for the tangent plane to the surface $xz^2 + x^2y = z - 1$ at the point $(1, -3,\ 2)$.