# RAJARATA UNIVERSITY OF SRILANKA
## FACULTY OF APPLIED SCIENCES

B.Sc. Four Year Degree in Industrial Mathematics
Fourth Year-Semester I Examination- April / May 2015

**MAT 4306 – OPTIMIZATION MODELING**

**Answer all questions** Time Allowed: **Three hours**

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1. Health care Hospital is encountering a problem in supplying for the demand in its fluid analysis lab. This lab has three machines that analyze various fluid samples. Recently, the demand for analyzing blood samples has increased so much that the lab director is having difficulty in getting all the samples analyzed quickly enough and also to complete the other fluid work that comes into the lab. The lab works with five types of blood specimens. Any machine can be used to process any of the specimens. However, the amount of time required by each machine varies depending on the type of specimen being analyzed. These times are summarized in the following table.

**Required Specimen Processing Time in Minutes**

| | Specimen Type | | | | |
|---|---|---|---|---|---|
| **Machine** | **1** | **2** | **3** | **4** | **5** |
| **A** | 3 | 4 | 4 | 5 | 3 |
| **B** | 5 | 3 | 5 | 4 | 5 |
| **C** | 2 | 5 | 3 | 3 | 4 |

Each machine can be used a total of 8 hours a day. Blood samples collected on a given day arrive at the lab and are stored overnight and processed the next day. So, at the beginning of each day, the lab director must determine how to allocate the various samples to the machines for analysis. This morning, the lab has **80**, **type – 1** specimens, **75**, **type – 2** specimens, **80**, **type – 3** specimens, **120**, **type – 4** specimens, and **60**, **type – 5** specimens to be processed. The lab director wants to know how many of each type of specimen should be analyzed on each machine in order to minimize the total time the machines are devoted in analyzing blood samples.

i. Formulate an LP model for this problem. *[ 20 ]*

ii. Create a spreadsheet model for this problem and solve it using Solver. *[ 20 ]*

iii. Obtain sensitivity analysis report and briefly discuss it. *[ 30 ]*

iv. How much processing time will be available on each machine if this solution is implemented? [ 10 ]

v. How would the model and solution change if the lab director wanted to balance the each machine so that each machine will be used approximately the same amount of time? [ 20]

2. Paper Ceylon Company (PCC) imports the stocks of hard boards with the standard length of 25 – foot. These boards can be cut to fulfill individual customer orders. An order has just come in for 5,000 of 7 – foot boards, 1,200 of 9 – foot boards, and 300 of 11 – foot boards. The PCC manager has identified six ways to cut the 25 – foot boards to fill this order. The six cutting patterns are summarized in the following table.

**Number of Boards Produced**

| Cutting Pattern | 7 ft | 9 ft | 11 ft |
|---|---|---|---|
| 1 | 3 | 0 | 0 |
| 2 | 2 | 1 | 0 |
| 3 | 2 | 0 | 1 |
| 4 | 1 | 2 | 0 |
| 5 | 0 | 1 | 1 |
| 6 | 0 | 0 | 2 |

Above table describes, Cutting Pattern 1: cut a 25 – foot board into three 7 – foot boards, and not to cut any 9 or 11 – foot boards. This pattern uses a 21 feet of board and leaves a 4 – foot piece of scrap. Another possibility cutting pattern 4: cut a 25- foot board in to one 7-footboard and two 9- foot boards using all 25 foot board. The remaining cutting patterns have similar interpretation. The PCC manager wants to fill this order using the fewest number of 25 – feet boards as possible. To do this manger needs to determine how many 25 – foot boards to run through each cutting pattern.

i. Write down LP model to find the requirement of manger. [ 30 ]

ii. Implement the above model in a spreadsheet and solve it using Solver. [ 25 ]

iii. Discuss the optimal solution and limitations of the optimal answer using sensitivity analysis. [ 25 ]

iv. Suppose the manger wants to minimize waste. Would the solution change? [ 20 ]

3. Describe the use of " ***Balance of Flow Rules***" in solving network flow problems

A construction company wants to determine the optimal replacement policy for the earth mover it owns. The company has a policy of not keeping an earth mover for more than five years, and has estimated the annual operating costs and trade-in values for earth movers during each of the five years they might be kept as :

**Age in Years**

| | 0 – 1 | 1 – 2 | 2 – 3 | 3 – 4 | 4 – 5 |
|---|---|---|---|---|---|
| **Operating Cost (Rs.)** | 80,000 | 91,000 | 107,000 | 92,000 | 110,000 |
| **Trade-in Value (Rs.)** | 140,000 | 90,000 | 60,000 | 35,000 | 20,000 |

Assume that a new earth mover is Rs. 250,000 and the cost is increasing by 4.5 % per year. The company wants to determine when it should plan on replacing its current, 2-year-old earth mover. Use a 5 – year planning horizon.

i. Draw the network representation of this problem. *[ 40 ]*

ii. State the LP model explained by the above network *[ 30 ]*

iii. Solve the problem using Solver and interpret the solution. *[ 30 ]*

4. Sri Lankan Airlines has several non-stop flights between Colombo (Col.) and New Delhi (ND.) every day. The schedules of these flights are shown in the following table.

| Flight | Departs Col. | Arrives ND. | Flight | Departs ND. | Arrives Col. |
|---|---|---|---|---|---|
| 1 | 6 A.M. | 8 A.M. | 1 | 5 A.M. | 9 A.M. |
| 2 | 8 A.M. | 10 A.M. | 2 | 6 A.M. | 10 A.M. |
| 3 | 10 A.M. | Noon | 3 | 9 A.M. | 1 P.M. |
| 4 | Noon | 2 P.M. | 4 | Noon | 4 P.M. |
| 5 | 4 P.M. | 6 P.M. | 5 | 2 P.M. | 6 P.M. |
| 6 | 6 P.M. | 8 P.M. | 6 | 5 P.M. | 9 P.M. |
| 7 | 7 P.M. | 9 P.M. | 7 | 7 P.M. | 11 P.M. |

Sri Lankan Airline wants to determine the optimal way of assigning flight crews to these different flights. The company wants to ensure that the crews always return to the city from which they left each day. According to the SLF regulations at least one hour rest is required for the flight crews between flights. However, flight crews become irritated if they are forced to wait for extremely long period of time between flights, so Sri Lankan Airline wants to find an assignment of flight schedules that minimizes these waiting periods.

i. Draw a network flow model for this problem. [ 30 ]

ii. Implement the problem in a spreadsheet and solve it. [ 25 ]

iii. What is the optimal solution? What is the longest period of time a flight crew has to wait between flights according to your solution? [ 25 ]

iv. Are there alternate optimal solutions to this problem? If so, do any alternate optimal solutions result in a smaller maximum waiting period between flights? [ 20 ]

5. R&D Manufacturing is planning its next production cycle. The company can produce three products, each of which must undergo machining, grinding, and assembly operations. The following table summarizes the hours of machining, grinding, and assembly required by each unit of each product, and the total hours of capacity available for each operation.

| | Hours Required By | | | |
|---|---|---|---|---|
| **Operation** | **Product 1** | **Product 2** | **Product 3** | **Total Hours Available** |
| Machining | 2 | 3 | 6 | 600 |
| Grinding | 6 | 3 | 4 | 300 |
| Assembly | 5 | 6 | 2 | 400 |

The cost accounting department has estimated that each unit of product 1 manufactured and sold will contribute Rs. 48 to profit and each unit of products 2 and 3 contribute Rs. 55 and Rs. 50, respectively. However, manufacturing a unit of product 1 requires a setup operation on the production line that costs Rs. 1000. Similar setups are required for products 2 and 3 at costs of Rs. 800 and Rs. 900, respectively. The marketing department believes it can sell the entire products produced. Therefore management of R&D Manufacturing wants to determine the most profitable mix of products to manufacture. Using appropriate model and spread sheet modeling techniques discuss how to fulfill the manager's requirement. [ 100 ]

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