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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester I Examination – September/October 2019

MAP 2203– DIFFERENTIAL EQUATIONS II

Time: Two (02) hours

Answer all questions.

1. a) Consider the differential equation

$$(x^2+1)y'' + y' - x^2y = 0.$$

(i) Show that $x = 0$ is an ordinary point.

(ii) Find a recurrence relation for the coefficients of the power series solution about $x = 0$.

(iii) Find the general solution using the above result.

(60 marks)

b) Find the general solution of the differential equation,

$$3x^2y'' - xy' + y = 0.$$

(i) Show that the point $x = 0$ is a regular singular point.

(ii) Find the general solution about $x = 0$, using the **Frobenius method**.

(40 marks)

2. a) Using Picard's iterative method, solve the following initial value problem.

$$x'(t) = x(t)t;\ \ x(0) = 1$$

(40 marks)

b) State **Picard's existence and uniqueness theorem.**
Consider the initial value problem (IVP)

$$y' = e^{-x^2} + y^2; y(0) = 1.$$

Show that the solution of the IVP exists for all x in which

$$0 \leq x \leq \frac{\sqrt{2}}{1+(1+\sqrt{2})^2}.$$

(60 marks)

3. a) Let $\underline{X}_1 = \varphi_1(t)$ and $\underline{X}_2 = \varphi_2(t)$ be the two solutions of the system

$$\frac{d\underline{X}}{dt} = A\underline{X}.$$

Show that $\underline{X} = c_1\varphi_1(t) + c_2\varphi_2(t)$ is also a solution of the system. (20 marks)

b) Find the eigen values and eigen vectors of the following matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix},$$

and hence, solve the system

$$\frac{d\underline{X}}{dt} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\underline{X}.$$

(80 marks)

4. a) Solve the following first order partial differential equations, given with the usual notations:

(i) $xyp + y^2q = zxy - 2x^2$,

(ii) $y^2p - xyq = x(z - 2y)$,

(iii) $z(x+y)p + z(x-y)q = x^2 + y^2$.

(60 marks)

b) Solve the following second order partial differential equations, given with the usual notations.

(i) $xr = p$

(ii) $xr + 2p = 0$

(40 marks)

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