



**RAJARATA UNIVERSITY OF SRI LANKA**  
**FACULTY OF APPLIED SCIENCES, MIHINTALE**

**B.Sc. (General Degree)**  
**Third Year Semester 1 Examination - March/April 2014**

**MAT 3208 – TIME SERIES**

**Answer all questions.**

**Time allowed: 2 hours only.**

1)

- a) State the properties of a Stationary time series.
- b) Consider the following process consisting of a linear trend with an additive noise term  $w_t$ , with zero mean and variance  $\sigma_w^2$ .

$$X_t = \beta_0 + \beta_1 t + w_t ; \text{ Where } \beta_0 \text{ and } \beta_1 \text{ are fixed constants.}$$

- i) Is the process  $X_t$  stationary ?
- ii) Prove that  $X_t - X_{t-1}$  process is stationary.

c)

- i) Find the autocovariance function of  $MA(1)$  process of the form

$$X_t = Z_t + \theta Z_{t-1} \text{ at lag } h \text{ ( } h = 0, \pm 1, \pm 2 \dots \text{ )}$$

$$\text{where } Z_t \sim WN(0, \sigma_w^2) .$$

- ii) Hence find the autocorrelation function of the process

$$X_t = Z_t - 0.5 Z_{t-1}$$

$$\text{Assume that } Z_t \sim WN(0, 0.25).$$

2)

- a) State four methods which can be used to measure the trend.
- b) Briefly explain the "Moving average method".

Discuss the advantages and disadvantages of that method.

- c) Farmer Brown's yield of soya beans per hectare has been monitored over the last ten years. By using modern farming methods he has increased his yield most years. Yields are measured in tonnes per hectare.

| Year | Yield |
|------|-------|
| 2000 | 110   |
| 2001 | 125   |
| 2002 | 115   |
| 2003 | 135   |
| 2004 | 150   |
| 2005 | 165   |
| 2006 | 155   |
| 2007 | 175   |
| 2008 | 180   |
| 2009 | 200   |

- I. Plot the above data in a graph.
- II. Fit a straight line trend for the above data by using the method of least square.
- III. Estimate the trend values for each year.
- IV. Calculate the expected yield for next three years.

3)

- a) Describe two decomposition time series models by using mathematical expressions.
- b) The following table represents the average quarterly price of Coffee ( per 1 kg ) over a period of three years.

| Year | Quarter | Price ( per 1 kg ) |
|------|---------|--------------------|
| 1    | 1       | 10                 |
|      | 2       | 07                 |
|      | 3       | 17                 |
|      | 4       | 34                 |
| 2    | 1       | 09                 |
|      | 2       | 07                 |
|      | 3       | 18                 |
|      | 4       | 40                 |
| 3    | 1       | 27                 |
|      | 2       | 07                 |
|      | 3       | 27                 |
|      | 4       | 100                |

Using the above data, do the following calculations, giving the answers to the *nearest second decimal place*.

- i) Estimate the seasonal factor and trend of the time series by using “Multiplicative decomposition method”.
- ii) Using only the trend and seasonal factors, forecast quarterly price of Coffee for the next year.

4)

- a) Define the followings :
  - i) Moving average process of order  $q$ , denoted by  $MA(q)$ .
  - ii) Auto regressive process of order  $p$ , denoted by  $AR(p)$ .
  - iii) Auto regressive moving average process, denoted by  $ARMA(p, q)$ .
- b) Define the two terms “Stationary model” and “Invertible model”, in relation to an  $ARMA(p, q)$  process.
- c) Express each of the following models in  $B$  notation, and determine whether the models are stationary and/or invertible. Assume that  $Z \sim WN(0, \sigma_w^2)$ .
  - i)  $X_t - Z_t - 0.5 X_{t-1} = 0.7 Z_{t-1} - 1.2 X_{t-2}$
  - ii)  $X_t - 0.5 X_{t-1} = Z_t + 0.3 Z_{t-1} - 0.4 Z_{t-2}$
  - iii)  $X_t + 0.5 Z_{t-1} = Z_t$