RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree) Examination

Third Year – Semester I Examination, February 2013

MAT 3208 – TIME SERIES

Answer **ALL** questions. Time allowed: **TWO HOURS**

Scientific calculators will be provided.

1.

a) State the conditions for a given time series $\{ Y_t \}$ to be stationary.

b) Let $\{ X_t \}$ be a stationary process with zero mean and auto covariance $\gamma_X(h)$. Show that the time series $\{ Y_t \}$, defined by the following relationship, for all $t > 0$, $Y_t = \frac{1}{3} (X_t + 2 X_{t+1})$ is also stationary.

c) Let $\{ X_t \}$ be a $MA(1)$ process with $cov (X_{t+1}, X_t) = 0.7 \sigma_w^2$ where σ_w^2 is the variance of the associated white noise process.

(i) Derive the auto covariance function of $MA(1)$ process.

(ii) Find the auto correlation function for $\{ X_t \}$.

(iii) Show that the auto covariance function of $\{ Y_t \}$ defined in part b) is

$$\gamma_Y (h) = \begin{cases} 1.1389\, \sigma_w^2 & ; \ h = 0 \\ 0.7200\, \sigma_w^2 & ; \ |h| = 1 \\ 0.1556\, \sigma_w^2 & ; \ |h| = 2 \\ 0 & ; \ |h| \geq 3 \end{cases}$$

2.

a) Define the following terms :

(i) **Moving average process** of order q , denoted by $MA\ (q)$,

(ii) **Auto regressive process** of order p , denoted by $AR\ (p)$,

(iii) **Auto regressive moving average process**, denoted by $ARMA\ (p,q)$.

b) Define the two terms **Stationary** and **Invertible** , in relation to an $ARMA\ (p,q)$ process.

c) Express each of the following models in B notation, and determine whether the models are stationary **and / or** invertible:

(i) $X_t = 0.5\, X_{t-1} - 0.1\, X_{t-2} + Z_t$,

(ii) $X_t = Z_t - 0.5\, Z_{t-1}$,

(iii) $X_t - 0.5\, X_{t-1} + 1.2\, X_{t-2} = Z_t + 0.7\, Z_{t-1}$.

3.

a) State four methods which can be used to measure the trend.

b) Discuss the determination of trend line by the method of Least Squares.

State the required conditions and equations with notations.

c) The following table gives the data related to tourist arrivals in Sri Lanka, from the year 2003 to 2011, **except** for the year 2004.

Year	2003	2005	2006	2007	2008	2009	2010	2011
Tourist Arrivals	500,640	549,310	559,600	494,010	438,470	447,890	654,477	855,975

Using the above data, do the following calculations, giving the answers to the nearest fourth decimal place.

(i) Using the method of least squares, fit the above data to a straight line trend.

(ii) Estimate the trend values for each of the given years.

(iii) Find the trend value for the missing year 2004.

(iv) Calculate the expected tourist arrivals for next two years.

4.

a) Briefly explain the exponential smoothing method.

b) Consider the following set of data consisting of 12 observations taken over one month – intervals, in the year 2012.

Month	Observation Y_t
January	10
February	15
March	11
April	10
May	12
June	11
July	12
August	10
September	13
October	14
November	10
December	15

(i) By taking the smoothing factor as **0.1**, perform the simple exponential smoothing procedure to find the estimated smoothed statistics, separately for the above periods of time.

(ii) Find the forecast values for first three months of next year.