RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

Second Year - Semester I Examination – July/Augest 2023

MAA 2201 – Mathematical Method II

Time: Two (02) hour

Answer **all** questions

1.

a. Convert the Cartesian coordinates $(-4,-1,8)$ for the point into Cylindrical coordinates. **(25 marks)**

b. Convert the equation written in cylindrical coordinates, $4\,Sin\theta - 2\,\text{Cos}\,\theta = \frac{r}{2}$ into an equation in Cartesian coordinates. **(25 marks)**

c. Mark the point with spherical coordinates $(8, \frac{\pi}{3}, \frac{\pi}{6})$ and express its location in both rectangular and cylindrical coordinates. **(30 marks)**

d. Convert $x^2 + y^2 = 4x + z - 2$ written in Cartesian coordinates into an equation in Spherical coordinates. **(20 marks)**

2.

a. Show that the inverse Laplace transform of $\frac{a}{s^2+a^2}$ is $Sin\ at$. **(10 marks)**

b. Explain why $\frac{2}{(s^2+16)} + \frac{1}{(s^2+4)}$ is the inverse Laplace transform of $Sin\ 3t\ Cos\ t$.

(20 marks)

c. If $L(f(t)) = F(s)$ then show that $L(e^{-at}f(t) = F(s+a)$, where a is a constant.

Diduse that $L(e^{-t}\cosh 4t) = \frac{a}{(s+1)^2-16}$ **(30 marks)**

d. Using the Laplace transform find the solution for the following equation

$$y'' - 10\,y' + 9y = 5t, \text{ with initial values, } y(0) = -1, y'(0) = 2$$

(40 marks)

3.

a. Evaluate the integral $\oint_C z^2 dx + y\, dy + 2y\, dz$, where C is the intersection of the cylinder $x^2 + y^2 = 16$ and the plane $z = 3$ from $(0, 4, 3)$ to $(-4, 0, 3)$.
(25 marks)

b. Evaluate the line integral $\oint_C y^2\, dx + x\, dy$, where C is the arc of the parabola $x = 4 - y^2$ from $(-5, -3)$ to $(0, 2)$. **(25 marks)**

c. Evaluate $\oint_C (3x - 5y)dx + (x - 6y)dy$, where C is the ellipse $\frac{x^2}{4} + y^2 = 1$ in the anticlockwise direction. Evaluate the integral by;

(i) Green's Theorem,

(ii) directly. **(25 marks)**

d. Using the divergence theorem, evaluate $\iint_S \boldsymbol{F}.\boldsymbol{dS}$, where $\boldsymbol{F} = (xy^2, yz^2, x^2y)$ and S is the sphere of radius 3 centered at origin. **(25 marks)**

4. The scale factors h_i, are defind by

$$h_i = \sqrt{\sum_{k=1}^{n} \left(\frac{\partial x_k}{\partial q_i}\right)^2},$$

where x_i is the Cartesian coordinates and q_k is the spherical coordinates .

a. Find h_1, h_2, and h_3. **(40 marks)**

b. Find the expression for $\nabla\varphi$ in spherical coordinates using the general form given below: **(30 marks)**

$$\nabla\varphi = \frac{1}{h_1}\frac{\partial\varphi}{\partial u_1}\bar{u}_1 + \frac{1}{h_2}\frac{\partial\varphi}{\partial u_2}\bar{u}_2 + \frac{1}{h_3}\frac{\partial\varphi}{\partial u_3}\bar{u}_3.$$

c. Find the $Curl\ F = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\bar{u}_1 & h_2\bar{u}_2 & h_3\bar{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1F_1 & h_2F_2 & h_3F_3 \end{vmatrix}$. **(30 marks)**