RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree in Applied Sciences
First Year – Semester II Examination – April / May 2015

MAP 1302 Differential Equations I

Answer ALL Questions **Time: Three (03) hours.**

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1. (a) Define the following terms:

(i). Ordinary differential equation , **[05 Marks]**

(ii). Degree of a differential equation. **[05 Marks]**

(b) Let $F(D)=p_0D^n+p_1D^{n-1}+...+p_n$ where $p_0,p_1,...,p_n$ are constants and n is a positive integer. Prove that,

(i) $F(D)e^{ax}=e^{ax}F(a)$, **[15 Marks]**

(ii) $F(D^2)\cos ax=F(-a^2)\cos ax$, **[15 Marks]**

(iii) $\dfrac{1}{D+a}f(x)=e^{-ax}\dfrac{1}{D}e^{ax}f(x)$. **[20 Marks]**

(c) Find the general solutions of the following differential equations:

(i) $(D^3+D^2-D-1)y=\cos 2x$, **[20 Marks]**

(ii). $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{e^x}$ **[20 Marks]**

Turn Over

2. (a) Discuss the method of finding the general solution of the Clairaut's equation $y = px + f(p)$, where $f(p)$ is a function of $p = \frac{dy}{dx}$. **[20 Marks]**

Use this method to find the general solution of the equation $y^2 + x^2p^2 - 2xyp = 4p^2$.

[20 Marks]

(b) In the following differential equations, $p = \frac{dy}{dx}$.

(i) Solve the differential equation, $p^2 + 2py\cot x - y^2 = 0$, by first expressing p in terms of x and y. **[20 Marks]**

(ii). Solve the differential equation $6y^2p^2 + 3xp - y = 0$, for x. **[20 Marks]**

(iii). Solve the differential equation $xp^2 + 2xp - y = 0$, for y. **[20 Marks]**

3. If M and N are functions of x and y, then the equation $M(x,y)dx + N(x,y)dy = 0$ is called exact when there exists a function $f(x,y) = C$, where C is an arbitrary constant, such that $df(x,y) = M(x,y)dx + N(x,y)dy$.

Show that the necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. **[40 Marks]**

Verify that the differential equation, $x(1 - \sin y)\,dy = (\cos x - \cos y - y)\,dx$ is exact and solve it.

[60 Marks]

4. (a). Show that, if $n \neq 0, 1$, the change of the dependent variable **given** by the substitution $z = y^{1-n}$, transforms the equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ into a linear differential equation in z. **[25 Marks]**

Hence, solve the differential equation $\frac{dy}{dx} - y = e^{-x}y^2$. **[25 Marks]**

(b) Show that, the substitution $v = \frac{y^2}{x}$ converts any differential equation of the form $\frac{dy}{dx} = \frac{1}{y} f\left(\frac{y^2}{x}\right)$ into a separable differential equation. **[20 Marks]**

Using the above method, find all solutions of the equation $\frac{dy}{dx} = \frac{1}{y}\left(\frac{y^4}{x^2} - \frac{y^2}{2x}\right)$. **[30 Marks]**

5. (a). Obtain the method of finding the general solution of the Riccati's equation,

$$\frac{dy}{dx} = P(x) + Q(x)y + R(x)y^2.$$

[30 Marks]

(b). Outline the method of finding the general solution of Riccati's equation when one particular integral is known. **[20 Marks]**

Show that $y = x + \frac{1}{x}$ is a particular integral of the differential equation $y_1 = 2 + \frac{1}{2}\left(x - \frac{1}{x}\right)y - \frac{1}{2}y^2$, and hence obtain its general solution. **[50 Marks]**

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