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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
Fourth Year – Semester II Examination – Sep./Oct. 2013

Computational Mathematics–MAT 4310

Time allowed: 3 hours only.

Answer six questions. ***Calculators will be provided***

1).

i. Each term in the sequence $0, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \ldots$..is equal to the arithmetic mean of the two proceeding terms. Find the general term.

ii. Find the general solution of the recurrence relation
$Y_{n+2} + 2bY_{n+1} + cY_n = 0$.

Where b and c are real constants.

Show that solutions tend to zero as $n \to \infty$ if and only if the point (b,c) lies in the interior of a certain region in the b-c plane and determine this region.

2). A sequences of functions $f_n(x);\ n = 0,1,2,\ldots$.. defines a recursion formula,

$$f_{n+1}(x) = 2x f_n(x) - f_{n-1}(x), |x| < 1$$
$$f_0(x) = 0,\ f_1(x) = 1$$

a) Show that $f_n(x)$ is a polynomial and give its degree and the leading coefficient.

b) Show that

$$\begin{pmatrix} f_{n+1}(x) \\ T_{n+1}(x) \end{pmatrix} = \begin{pmatrix} x & 1 \\ x^2 - 1 & 1 \end{pmatrix} \begin{pmatrix} f_n(x) \\ T_n(x) \end{pmatrix}$$

Where $T_n(x) = \cos(n \cos^{-1} x)$

3). Consider the following **Runge-Kutta** method for the differential equation $y' = f(x,y)$

$$Y_{n+1} = Y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$

$$K_1 = h f(x_n, y_n)$$

$$K_2 = h f(x_n + \frac{h}{2}, y_n + \frac{K_1}{2})$$

$$K_3 = h f(x_n + h, y_n - K_1 + 2K_2)$$

a). Compute y(0.4) when $y' = \frac{y+x}{y-x}$, $y(0) = 1$ and h = 0.2, Round off to five decimal places.

c) What is the result after one step of length h when $y' = -y$, $y(0) = 1$?

4).

a). $y = F(x,y), Y(x_0) = Y_0$

step size h

$$y_n = y_{n-1} + h F(x_{n-1}, y_{n-1})$$

Use Euler's method with step size 0.3 to compute the approximate value y(0.9) of the solution of initial value problem

$$y' = x^2 , y(0) = 1$$

b). Solve the Initial Value Problem

$$\ddot{Y} + 6\dot{Y} + 9Y = 4e^{-3t} \qquad Y(0) = 1, \ \dot{Y}(0) = 0$$

5). Compute an Approximation to y(1), y′(1), y″(1) with Taylor's algorithm of order two and step length h=1 when y(x) is the solution to the initial value problem

$$Y''' + 2y'' + y' - y = \cos x \ , 0 \leq x \leq 1 ,$$

$$y(0) = 0, \ y'(0) = 1, \ y''(0) = 2$$

6).

a) Use Gaussian elimination with backward substitution and two digit rounding arithmetic to solve the following system:

$$4x_1+x_2+x_3=4$$
$$x_1+4x_2-2x_3=4$$
$$3x_1+2x_2-4x_3=6$$

b) Given the liner system

$$2x_1 \ -6\alpha x_2 = 3$$
$$3\alpha x_1 - x_2 = \frac{3}{2}$$

i. Find value(s) of α for which the system has no solutions.
ii. Find value(s) of α for which the system has an infinite number of solutions.
iii. Assuming that a unique solution exists for a given α, find the solution.

7).

i. The matrix **A** is rectangular with m rows and n columns , n < m. The matrix $\mathbf{A}^T\mathbf{A}$ is regular. Let $X = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$. Show that $\mathbf{A}\ X\ \mathbf{A} = \mathbf{A}$ and $X\mathbf{A}X = X$. Show that in the sense of the method of least squares, the solution of the system $\mathbf{A}\,\underline{x} = \underline{b}$ can be written as $\underline{x} = X\mathrm{b}$. Calculate x when

$$A=\begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$$

ii).Compute $\mathbf{A}^{10}$

$$\text{where}\quad \mathbf{A} = \frac{1}{9}\begin{bmatrix} 4 & 1 & -8 \\ 7 & 4 & 4 \\ 4 & -8 & 1 \end{bmatrix}$$

8).

i). Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. by the Gauss-Jordan method .

ii).The following system of equations is given

$$\begin{aligned} 4x + y + 2z &= 4 \\ 3x + 5y + z &= 7 \\ X + y + 3z &= 3, \end{aligned}$$

Set up the Jacobi and Gauss-Seidel iterative schemes for the solution and iterate three times starting with the initial vector $x^{(0)}=0$.