RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)

Second year – Semester 1 Examination-March/April 2014

MAP 2301- Algebra

Answer six questions. **Time allowed: 3 hours only.**

1.

a. Define each of the following terms.
Proposition, Tautology and Contradiction

b. Let p, q and r be propositions .Determine whether the following compound statements are a tautology, a contradiction or a contingency.

i. $p \vee (\neg p \vee q)$

ii. $(p \Rightarrow q) \wedge (p \wedge q)$

iii. $(p \wedge q) \wedge (\neg p \vee \neg q)$

c. Show that $p \Rightarrow (\neg q \vee r) \equiv (p \wedge q) \Rightarrow r$

d. If p is true and q, r are false find the truth value of the compound proposition
$[(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)]$

2.

a) A relation R on $\mathbb{Z}^+ \times \mathbb{Z}^+$ is defined by $(a,b)R(c,d)$. If $a^2 + d^2 = c^2 + b^2$. Show that R is an equivalence relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$.

b) Let f, g and h be functions $\mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 3$, $g(x) = \dfrac{1}{x^2 + 4}$ and $h(x) = \sqrt{x^2 - 3}$. Find expressions for each of the followings,

i. $goh(x)$ ii. $fog(x)$ iii. $fo(goh)(x)$

3. Show that the linear Diophantine equation ax + by = c is soluble if and only if $(a, b)|$c. Also, show that if x=x_0 and y=y_0 is a particular solution of ax +by = c, then x= $x_0+\frac{b}{(a,b)}t$, y= $y_0-\frac{a}{(a,b)}t$; where t is integer, is the general solution of ax +by = c?. Find the general form of all the positive integers which divided by 5,7,8, leave remainders 3,4,5 respectively.

4.

i. If $a \equiv b(\text{mod } m)$ and $c \equiv d(\text{mod } m)$, then $ac \equiv bd(\text{mod } m)$

ii. If $a \equiv b(\text{mod } m)$ then $a^k \equiv b^k(\text{mod } m)$ for all non-negative integers k.

iii. Using the Chinese reminder theorem, solve the following system of congruences.

3x = 6 (mod 12)
2x = 5 (mod 7)
3x = 1 (mod 5)

5.

a. Show that the liner equation $a_1x_1+ a_2x_2+ a_3x_3+ \dots\dots +a_nx_n= c$ has integer solutions if and only if $(a_1,a_2,a_3,\dots\dots,a_n)|c$.

b. Solve the equation 6x+10y+15z=5

6.

i. Let $\mathbb{Z}$ be the set of integers. Define an operation $*$ on $\mathbb{Z}$ by $a * b = a + b - 7$ where $a, b \in \mathbb{Z}$

(i) Show that $(\mathbb{Z},*)$ is a group.

(ii) Is $(\mathbb{Z},*)$ abelian? Justify your answer.

ii. Consider the group $G = \mathbb{Z}_{12} = \{0,1,2, \dots ,11\}$ and $H = \{0,4,8\}$.

(i) Find all the left cosets of H in G.

(ii) What is $[G:H]$?

7.

I. Let $a, b \in \mathbb{Z}$. Then prove the following ;

a. If $c|a$ and $c|b$ for any $c \in \mathbb{Z}^+$, then $\left(\frac{a}{c},\frac{b}{c}\right)=\frac{1}{c}(a,b)$

b. If $(a,b)=1$ and $(b,c)=1$ for any $c \in \mathbb{Z}$, then $(ab,c)=1$

c. If $c|ab$ and $(b,c)=1$ for any$c \in \mathbb{Z}$, then $c|a$

II. Let $(a,b)=1$where$a, b \in \mathbb{Z}$. Show that

(i) (a +b ,a- b) = 1or 2

(ii) (a+ b, ab) = 1