RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester I Examination – June/July 2018

MAT 3213 – GRAP THEORY

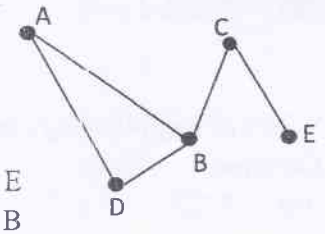
Time: Two (02) hours

Answer **four** questions **including first question.**

1. Select the **most suitable** answer for the following questions:

a) Which one of the following is **true** for any simple graph?
   (a) Every path is a trail.
   (b) Every trail is a path.
   (c) Every trail is a path as well as every path is a trail.
   (d) None of the above.

b) In the given graph cut vertices will be:



   (a) B and E
   (b) C and D
   (c) A and E
   (d) C and B

c) The number of edges in a complete graph having n vertices is,
   (a) $n(n+1)/2$
   (b) $n(n-1)/2$
   (c) n
   (d) None of the above.

d) Maximum number of edges in a bipartite graph having 10 vertices is,
   (a) 24
   (b) 21
   (c) 25
   (d) 16

e) Which of the following is true for a connected graph G having v vertices and e edges and no cycles?

(a) $v = e$ (c) $v + 1 = e$

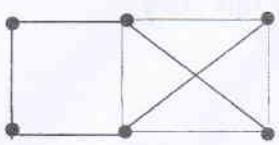
(b) $v = e + 1$ (d) None of the above.

f) A graph with all vertices having equal degree is known as,

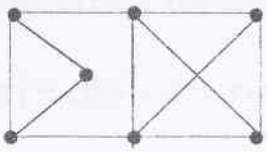
(a) Multi graph (c) Simple graph

(b) Regular graph (d) Complete graph

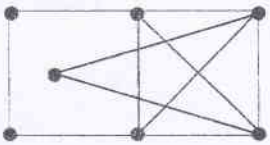
- Use figure below to answer the questions **(g)** and **(h)**:



Graph 1



Graph 2



Graph 3

g) Which of the graphs has an **Euler circuit**?

(a) Graph 2 only (c) Graph 1 and 3

(b) Graphs 3 only (d) Graph 1 only

h) Which of the graphs has **an Euler path but no Euler circuit?**

(a) Graphs 1 only (c) Graph 1and 2

(b) Graph 3 only (d) Graph 2 only

i) Which of the following combinations of the degrees of vertices of connected graph be **Eulerian**?

(a) 1,2,3 (c) 2,4,5

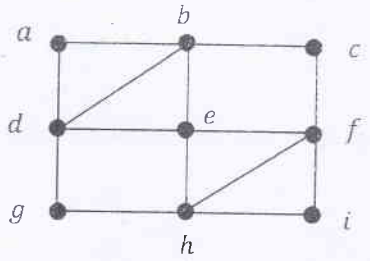
(b) 2,3,4 (d) 1,3,5

j) Number of regions contain in a connected planar graph having 6 vertices and 7 edges is,

(a) 15 (c) 1

(b) 3 (d) 11

k) The chromatic number of the following graph is.



(a) 2
(b) 3
(c) 4
(d) 5

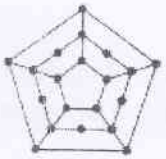
l) G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is,

(a) Regular
(b) Complete
(c) Hamiltonian
(d) Euler

m) Which of the following graphs are **Hamiltonian**?


A


B


C

(a) A only
(b) B only
(c) C only
(d) A and C

n) Let G be the non-planar graph with the minimum possible number of edges. Then G has,

(a) 9 edges and 5 vertices
(b) 9 edges and 6 vertices
(c) 10 edges and 5 vertices
(d) 10 edges and 6 vertices

o) What is the chromatic number of an n vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$.

(a) 2
(b) 3
(c) n
(d) $n-1$

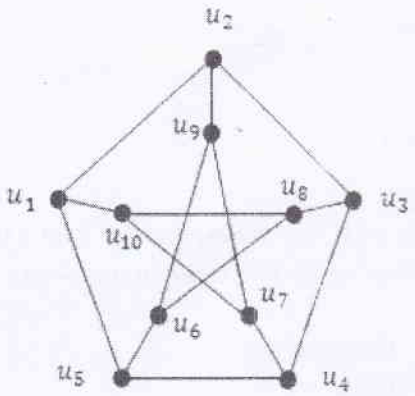
(100 marks)

2.

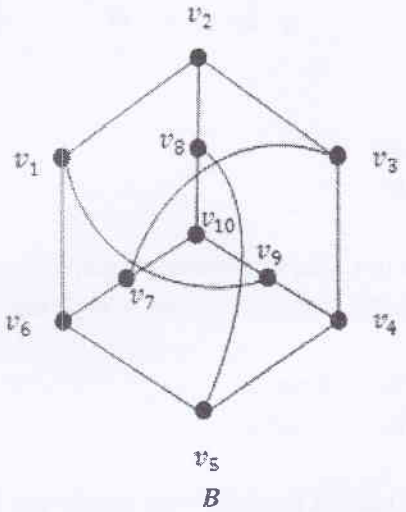
a) Define **graph isomorphism**.

(20 marks)

b) Determine whether the following two graphs are isomorphic or non- isomorphic. Justify your answer.



A



B

(40 marks)

c) For a regular graph G of degree r, where r is odd, show that G has an even number of vertices and the number of edges in G is a multiple of r.

(20 marks)

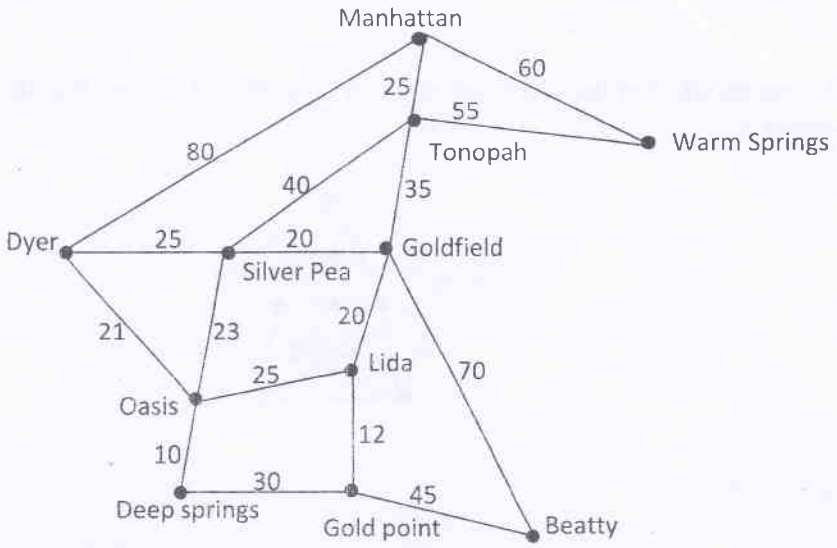
d) Show that the maximum number of edges in a simple graph of n vertices is $\frac{n(n-1)}{2}$.

(20 marks)

3.

a) The unpaved roads between pair of cities in Nevada are represented in the graph, where the distances are represented by edge weight.

Find the roads should be paved so that there exists a path of paved roads between each pair of cities so that a minimum road length is paved.



(50 marks)

b) Draw all possible non-isomorphic trees of 5 vertices.

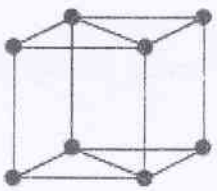
(20 marks)

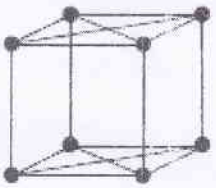
c) If a tree consists of n_1 vertices of degree 1, n_2 vertices of degree 2, n_3 vertices of degree 3, …, and n_k vertices of degree k, prove that $n_1 = 2 + n_3 + 2n_4 + 3n_5 + ... + (k-2)n_k$.

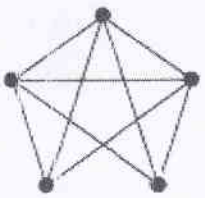
(30 marks)

4.

a) For each of the following graphs, determine whether it has an **Eulerian trail, an Eulerian circuit, both or neither**. Give a brief justification in each case.

i. 

ii. 

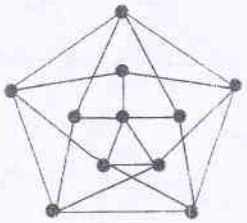
iii. 

(60 marks)

b) Find the values of n (≥ 2) for which K_n is Eulerian and Semi-Eulerian.

(20 marks)

c) Determine whether the following graph is Hamiltonian or not Hamiltonian. Justify your answer.



(20 marks)

5.

a) Six stations located at the distances (miles) shown in the table below:

	1	2	3	4	5	6
1	-	85	175	200	50	100
2	85	-	125	175	100	160
3	175	125	-	100	200	250
4	200	175	100	-	210	220
5	50	100	200	210	-	100
6	100	160	250	220	100	-

If given that two stations cannot use the same channel when they are within 150 miles of each other, find the minimum number of different channe ls needed.

(60 marks)

b) Consider a connected planar simple graph $\boldsymbol{G}$ of $\boldsymbol{n}$ vertices, $\boldsymbol{e}$ edges and $\boldsymbol{r}$ regions.

(i) Prove that $n - e + r = 2$.

(ii) Show that when $v \geq 3$, then $e \leq 3v - 6$ and hence, justify that K_5 is non-planar.

(40 marks)

END