15

RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
SecondYear- Semester II Examination – Jan/Feb2023

MAA 2201 – MATHEMATICAL METHODS II

Answer **FOUR (04)** questions — Time: **Two (2) hours**

1.

a) Convert the point $(-2,-2,1)$ from Cartesian coordinates to (i) cylindrical and (b) spherical coordinates. (40 marks)

b) Plot the point with cylindrical coordinates $(4, \frac{2\pi}{3}, -2)$ and express its location in rectangular coordinates. (20 marks)

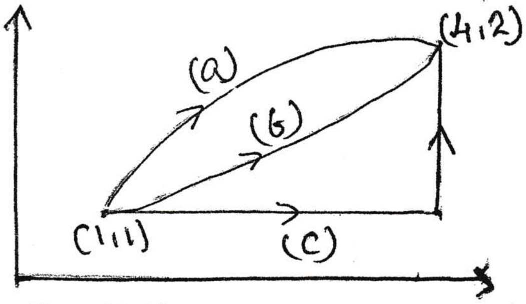
c) Convert each of the following into an equation in the given coordinate system.

i) Convert $2x - 5x^3 = 1 + xy$ into polar coordinates.

ii) Convert $r = -8\cos\theta$ into Cartesian coordinates.

(40 marks)

2. Evaluate the line integral $I = \int_C \underline{a}.d\underline{r}$ where $\underline{a} = (x+y)\mathbf{i} + (y-x)\mathbf{j}$, along each of the paths in the xy-plane shown in the figure below, namely,



a) the parabola $y^2 = x$, from $(1,1)$ to $(4,2)$ (30 marks)

b) the curve $x = 2u^2 + u + 1, y = 1 + u^2$ from $(1,1)$ to $(4,2)$ (35 marks)

c) the line $y = 1$ from $(1,1)$ to $(4,1)$, followed by the line ~~$y = x$~~ from $(4,1)$ to $(4,2)$.
$x=4$ (35 marks)

3. Consider the parabolic cylindrical coordinate system(u, v, z) defined by;
$x = \frac{1}{2}(u^2 - v^2), y = uv$, and $z = z$.
i.Show that, $ds^2 = (u^2 + v^2)(du)^2 + (u^2 + v^2)(dv)^2 + (dz)^2$ (30 marks)
ii.Show that scale factors as $h_u = h_v = \sqrt{(u^2 + v^2)}$ and $h_z = 1$ (30 marks)
iii. Prove that (u, v, w) orthogonal (40 marks)
z ?

4.

a) Verify the Divergence Theorem for the field $F = \langle x, y, z \rangle$ over the sphere $x^2 + y^2 + z^2 = R^2$. (50 marks)
b) Verify Stokes' Theorem for the field $F = \langle x^2, 2x, z^2 \rangle$on the ellipse$S = \{(x, y, z): 4x^2 + y^2 \leq 4, z = 0\}$. (50 marks)

5.

a) Show, for a given constant a,that
   i. $L[\cos at] = \frac{s}{s^2+a^2}$,
   ii. $L[\sinh at] = \frac{a}{s^2-a^2}$. (40 marks)
b) If $L[f(t)] = F(s)$, then show that $L[t^n f(t)] = (-1)^n \frac{d^nF}{ds^n}$,and find;
   i. $L[t \sin 6t]$
   ii. $L[t^2 \cos 4t]$ (60 marks)

6.

a) If ~~the~~$\mathcal{L}[f(t)] = F(s)$,then show that $\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$. (30 marks)
b) Solve the initial value problem by Laplace transform,

$$Y'' - Y' - 2Y = e^{2t}, \text{ where } Y(0) = 0 \text{ and } Y'(0) = 1.$$

[Hint: $\mathcal{L}[Y''] = s^2Y(s) - sY(\underset{0}{\cancel{s}}) - Y'(0)$] (70 marks)

END