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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
First Year – Semester 1 Examination – Oct. / Nov. 2014

MAA 1201 - MATHEMATICAL METHODS I

Answer FOUR questions only **Time Allowed: TWO HOURS**

1.

i. Find the angle between the two vectors $\underline{A} = 2i + 2j - k$ and $\underline{B} = 6i - 3j + 2k$, where i, j, k denote unit vectors along the positive directions of the rectangular coordinate axes Ox, Oy, Oz, respectively.

ii. Determine the value of β so that the two vectors $\underline{A} = 2i + \beta j + k$ and $\underline{B} = 4i - 2j - 2k$ are perpendicular to each other.

iii. Find the cross product of the two vectors $\underline{A} = 2i - 6j - 3k$ and $\underline{B} = 4i + 3j - k$. **Hence** find (a) the area of a parallelogram, two adjacent edges of which represent these two vectors, and (b) a unit vector perpendicular to the plane determined by these two vectors.

iv. Find the work done in moving a particle along a vector $\underline{r} = 3i + 2j - 5k$ if the applied force is $\underline{F} = 2i - j - k$

2. (a) **Assuming** properties of triple scalar products and a formula for the expansion of a triple vector product of three vectors $\underline{a}$, $\underline{b}$, $\underline{c}$, show that

(i) $(\underline{A}\times\underline{B}) \cdot (\underline{C}\times\underline{D}) = (\underline{A} \,.\, \underline{C})(\underline{B} \,.\underline{D}) - (\underline{A} \,.\, \underline{D})\,(\underline{B} \,.\, \underline{D})$,

(ii) $(\underline{A}\times\underline{B}) \cdot (\underline{C}\times\underline{D}) + (\underline{B}\times\underline{C}) \cdot (\underline{A}\times\underline{D}) + (\underline{C}\times\underline{A}) \cdot (\underline{B}\times\underline{D}) = 0$,
for any four vectors $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$.

(b) Find the gradient of the scalar function $\phi(x,y,z) \equiv x^2y^2z + z^2 + xy$, and hence a unit normal to the surface $\phi(x,y,z) = 10$ at the point A (-1, 2, 2). Also find the Cartesian equations of the tangent plane and the normal line to the surface $\phi(x,y,z) = 10$ at the point (-1, 2, 2).

3.

a) Given $\mathbf{A} = x^2z^2\,\mathbf{i} - 2y^2z^2\,\mathbf{j} - xy^2z\,\mathbf{k}$, find div **A** at the point P(1,-1,1).

b) Show that $\nabla.\nabla\,\psi = \nabla^2\,\psi$, where $\nabla = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$ and find div grad ψ, where $\psi = 6x^3y^2z$.

(c) If ∇^2 denotes the Laplacian operator,show further that $\psi = \frac{1}{r}$ is a solution of Laplace's equation $\nabla^2\psi = 0$, where $r^2 = x^2 + y^2 + z^2$..

4.

i. Given $\underline{A} = yz^2\,\underline{i} - 3xz^2\,\underline{j} - 2\,xyz\,\underline{k}$, $\underline{B} = 3x\,\underline{i} - +4z\,\underline{j} - xy\,\underline{k}$, $\emptyset = xyz$, find (a) $A \times (\nabla\emptyset)$, (b) $(A \times \nabla)\emptyset$, (c) $(\nabla \times A) \times B$ and (d) $B \cdot \nabla \times A$

ii. Show the vector field $\underline{F} = (y+z)\,\underline{i} + (z+x)\,\underline{j} + (x+y)\,\underline{k}$, is irrotational, and find a scalar field Φ such $\underline{F} = \text{grad}\,\Phi$.

Determine whether $\underline{F}$ is solenoidal as well.

5.

(i) If $\varphi = e^x\sin(yz)$ and $\bar{F} = x^2y\,\boldsymbol{i} + (z^2 - y^2)\,\boldsymbol{j} + xy\,\boldsymbol{k}$,
Find $div\,(\varphi\bar{F})$, using an **identity to be established**.

(ii) Verify the identity $div\,(\bar{F} \times \bar{G}) = \bar{G}.curl\,\bar{F} - \bar{F}.curl\,\bar{G}$. for the two vectors
$\bar{F} = e^x\,\boldsymbol{i} + \sin(y)\,\boldsymbol{j} + y^2z\,\boldsymbol{k}$ and $\bar{G} = xy\,\boldsymbol{i} + \cos(z)\,\boldsymbol{j} + xyz\,\boldsymbol{k}$,

(iii)Establish the identity $curl\,(\varphi\bar{F}) = (\,grad\,\varphi\,) \times \bar{F} + \varphi\;curl\,\bar{F}$.
If $\varphi = xyz$ and $\bar{F} = \sin(y)\,\boldsymbol{i} + e^x\,\boldsymbol{j} + \tan(z)\,\boldsymbol{k}$, find $curl\,(\varphi\bar{F})$.

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