RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

Second Year - Semester II Examination – Jan/Feb 2023

MAP 2202 - REAL ANALYSIS II

Time: Two (02) hours

Answer all (4) questions.

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1. a) State the comparison test and its limit form.

Show that the series $\sum_{n=1}^{\infty}\frac{2^n}{7^n(n+1)^5+n^5}$ and $\sum_{n=1}^{\infty}\frac{n}{8n^4-9}$ are convergent.

(35 marks)

b) Show that the following series are divergent:

i. $\sum_{n=1}^{\infty}\frac{1.3.5.\cdots.(2n+1)}{n!}$,

ii. $\sum_{n=2}^{\infty}\frac{1}{n^2\ln(n)}$,

iii. $\sum_{n=1}^{\infty}\left(\frac{3+2(-1)^n}{3+3(-1)^n}\right)^n$.

(40 marks)

c) Determine whether following series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ is convergent or divergent.

(25 marks)

2. a) Let $\sum_{n=0}^{\infty} a_n (x-c)^n$ be a power series with $a_n \neq 0$ for all $n = 0, 1, 2, \cdots$.

Prove that the radius of convergence, R, of the power series is

$$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|,$$

provided the forgoing limit exits.

(20 marks)

b) Determine the radius of convergence and interval of convergence of the following power series:

i. $\sum_{n=0}^{\infty} \frac{x^n}{2^n}$,

ii. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{5^n}(x+1)^n$,

iii. $\sum_{n=1}^{\infty} \frac{1.3.5.\cdots.(2n-1)}{(n+1)!} x^{2n}$.

(60 marks)

c) Let $f: [0,1] \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} 1, & \text{if } x \in \mathrm{Q}, \\ 0, & \text{if } x \in \mathrm{R} - \mathrm{Q}. \end{cases}$$

Determine whether or not f is Riemann integrable on $[0,1]$.

(20 marks)

3. a) Prove that $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2} = 0$.

(25 marks)

b) Find all critical points of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. Discuss the nature of each of the critical points.

(50 marks)

c) Using *the method of Lagrange multipliers,* find the point on the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value, and show that this minimum is given by $\frac{p^2}{a^2+b^2+c^2}$.

(25 marks)

4. a) Reversing the order of integration over the same region of integration, evaluate the integral:

$$\int_0^1 \int_x^1 \exp(y^2)\, dydx.$$

(35 marks)

b) Evaluate $\iint_R (x^2y^2 + 1)\, dA$, where R is the region in the first quadrant bounded by the lines: $xy = 1, xy = 2, y = x, y = 4x$.

(35 marks)

c) Evaluate: $\iiint_E (x^2 + y^2)z\, dx\, dy\, dz$, where E is the region enclosed by the cylinder $x^2 + y^2 = 1$ and the planes: $z = 0$ and $z = 1$. (Hint: Use cylindrical polar coordinates)

(30 marks)

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