RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

BSc in Applied Sciences
Second Year - Semester I Examination – June/July 2022

MAA 2204 – Linear Programming

Time: Two (02) hours

Answer **all** questions.

1. a) Suppose the ABC company produces three different products, namely P1, P2 and P3. Each product requires processing on three machines M1, M2 and M3. The product P1 requires 8 hours on M1, 5 hours on M2 and 2 hours on M3. The product P2 requires 6, 5 and 4 hours on M1, M2 and M3 respectively. The product P3 requires 9, 6 and 4 hours on M1, M2 and M3 respectively. For the next planning period, 3200, 2600 and 1800 machine hours are available on M1, M2 and M3 respectively. The ABC company must produce at least 100 P1 products and 150 P2 products due to government regulations. The profit contribution of P1, P2 and P3 are $12, $9 and $10 per unit respectively. Formulate a linear programming model to maximize the total profit.

   (40 marks)

   b) Check the convexity of the following functions:

   i. $f_1(x) = x_1^2 - x_1 x_2 + 3x_2^2 + 2x_1$

   ii. $f_2(x) = 3x_1^2 + 3x_1 x_2 + 2x_2^2 - x_1 + 2x_2 + 6$

   iii. $f_3(x) = x_1^2 - 3x_1 x_2 + x_2^2 + 3x_1 - 2x_2 + 1$

   (30 marks)

c) Determine the convexity of the following sets:

   i. $\{(x_1, x_2): x_1^2 + x_2^2 \geq 3\}$

   ii. $\{(x_1, x_2): |x_1| < 3, x_2 = 2\}$

   iii. $\{(x_1, x_2): x_2 - x_1^2 = 0\}$ **(30 marks)**

2. Consider the following linear programming problem (LPP):

$$Min\ Z = 4x_1 + 3x_2$$
$$s.t.$$
$$3x_1 + x_2 \geq 50,$$
$$x_1 + 2x_2 \geq 60,$$
$$x_1, x_2 \geq 0.$$

a) Solve the above LPP by applying the Two-Phase Method.   **(75 marks)**

b) Formulate the corresponding dual problem of the given LLP.   **(10 marks)**

c) Obtain the optimal solution of the dual problem in Part b) using the Strong Duality Theorem.   **(15 marks)**

3. a) Define Slack, Surplus and Artificial variables and explain their importance in linear programming.   **(15 marks)**

   b) Distinguish the Standard and Revised Simplex methods.   **(15 marks)**

   c) Solve the following LPP using the Revised Simplex method:

$$Max\ Z = x_1 - x_2 + 3x_3$$
$$s.t.$$
$$x_1 - x_2 \geq -20,$$
$$x_1 + x_3 = 5,$$
$$x_2 + x_3 \leq 10,$$
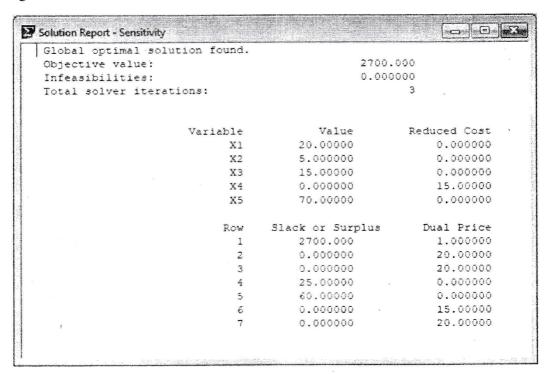$$x_1, x_2, x_3 \geq 0.$$ **(70 marks)**

4. A small-scale furniture manufacturing company produces tables, chairs, sofas, beds and cupboards for the next quarter of 2022. The unit price and estimated maximum demands of each furniture are given in the below table.

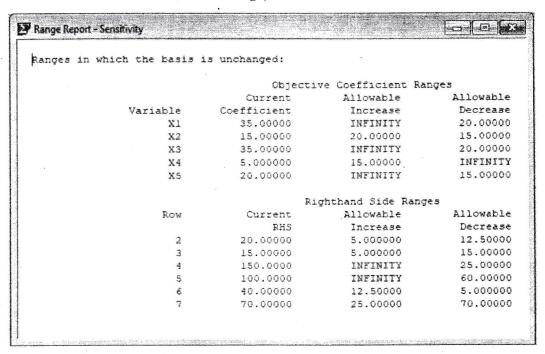| Furniture	Unit profit ($)	Estimated Max demand
Table (x_1)	35	20
Chair (x_2)	15	-
Sofa (x_3)	35	15
Bed (x_4)	5	-
Cupboard (x_5)	20	-

The formulated linear programming model for maximizing the total profit of the company is given below:

$$Max\ Z = 35x_1 + 15x_2 + 35x_3 + 5x_4 + 20x_5$$
$$s.t.$$

$$x_1 \leq 20 \quad \text{(Max demand for tables)},$$
$$x_3 \leq 15 \quad \text{(Max demand for sofas)},$$
$$2x_2 + 3x_3 + 2x_4 + x_5 \leq 150 \quad \text{(Availability of raw material-I)},$$
$$x_1 + 4x_2 + 2x_4 \leq 100 \quad \text{(Availability of raw material-II)},$$
$$x_1 + x_2 + x_3 \leq 40,$$
$$x_4 + x_5 \leq 70,$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

The solution report and the range report obtained by the LINGO for the above LP model are given below.

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Solution Report - Sensitivity                                    [ - ][ □ ][ X ]

    Global optimal solution found.
    Objective value:                          2700.000
    Infeasibilities:                          0.000000
    Total solver iterations:                         3


              Variable           Value        Reduced Cost
                    X1        20.00000            0.000000
                    X2        5.000000            0.000000
                    X3        15.00000            0.000000
                    X4        0.000000           15.00000
                    X5        70.00000            0.000000

              Row        Slack or Surplus      Dual Price
                1             2700.000           1.000000
                2             0.000000           20.00000
                3             0.000000           20.00000
                4             25.00000            0.000000
                5             60.00000            0.000000
                6             0.000000           15.00000
                7             0.000000           20.00000
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Range Report - Sensitivity                                          [ _ ][ □ ][ ✕ ]

Ranges in which the basis is unchanged:

                            Objective Coefficient Ranges
                         Current      Allowable      Allowable
             Variable    Coefficient   Increase       Decrease
               X1        35.00000     INFINITY       20.00000
               X2        15.00000     20.00000       15.00000
               X3        35.00000     INFINITY       20.00000
               X4        5.000000     15.00000       INFINITY
               X5        20.00000     INFINITY       15.00000

                            Righthand Side Ranges
             Row         Current      Allowable      Allowable
                         RHS          Increase       Decrease
                2        20.00000     5.000000       12.50000
                3        15.00000     5.000000       15.00000
                4        150.0000     INFINITY       25.00000
                5        100.0000     INFINITY       60.00000
                6        40.00000     12.50000       5.000000
                7        70.00000     25.00000       70.00000
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Answer the following questions with the justifications. If it is impossible to find the answer to any question, clearly specify the reasons.

a) Determine the best product mix and the maximum total profit for the next quarter.

(10 marks)

b) Briefly explain what are the binding constraints of an LPP. Then identify the binding constraints of the above LPP.

(20 marks)

c) If the unit profit of Tables is reduced to $5, what will be the impact on the total profit of the company? **(15 marks)**

d) What is the amount of penalty that the company has to pay to produce 10 beds?

(15 marks)

e) What would be the impact on the optimal total profit, if the company increases the unit profit of all the furniture by $5? **(20 marks)**

f) Due to the current financial crisis of the country, the expected maximum demand of Sofas will be decreased by 5 units and the availability of raw materials I and II will be limited to 140 and 90 units respectively in the next quarter. Calculate the expected total profit loss of the company. **(20 marks)**

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