RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES
DEPARTMENT OF PHYSICAL SCIENCES
B.Sc. (General) Degree

Second Year Semester I Examination – April/May 2016

MAP 2203 – Differential Equations II

Answer **Four** Questions only

Time allowed: **Two** hours

1.

i. Discuss the **Forbenius method** for solving a second order linear differential equation,

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

ii. Find the series solution of the differential equation,

$2x^2y'' - xy' + (1 - x^2)y = 0$ by using **Forbenius method.**

2.

i. Consider the initial value problem of the form , $\frac{dy}{dx} = F(x, y)$; $y(x_0) = y_0$, discuss the **Picard's iteration method** for n^{th} approximation $y_n(x)$.

ii. Apply **Picard's method** to find the solution of the problem

$$\frac{dy}{dx} = y - x\,,\, y(0) = 2.$$

3.

i. Discuss **Picard's Existence** and **Uniqueness Theorem.**

ii. Show that $\frac{dy}{dx} = (y + 1)\cos(x^2y)$ has a unique solution with the initial condition $y(2) = -1$ and find it.

iii. Consider the initial value problem $\frac{dy}{dx} = y^2 + \cos(x^2)$; $y(0) = 0$, show that the initial value problem has an unique solution $y(x)$ on the interval $|x| \leq \frac{1}{2}$ and $|y| \leq 1$.

[P.T.O.]

4.

i. Find the general solution of the system,

$$X'(t) = \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix} X(t).$$

ii. Find e^{At} of above system.

5.

i. Form partial differential equations for each of the following, by eliminating arbitrary constants.

a. $Z = (x-a)^2 + (y-b)^2$

b. $Z = axe^y + \frac{1}{2}a^2e^{2y} + b$

ii. Solve the following partial differential equations, given with the usual notations.

a. $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \sin x$

b. $p - q = \frac{z}{x+y}$

c. $\left(\frac{b-c}{a}\right)yzp + \left(\frac{c-a}{b}\right)zxq = (\frac{a-b}{c})xy$

d. $t - xq = x^2$

e. $t + s + q = 0$