**RAJARATA UNIVERSITY OF SRI LANKA**
**FACULTY OF APPLIED SCIENCES**

**Bachelor of Science in Applied Sciences**
**Third Year - Semester I Examination – July/August 2023**

**MAT 3213– Graph Theory**

Answer **All** questions. Time allowed: **One (2) hours**

1. Underline the most suitable answer.

i. The degree of any vertex of graph is

A. the number of edges incident with vertex.

B. number of vertex in a graph.

C. number of vertices adjacent to that vertex.

D. number of edges in a graph. **(10 marks)**

ii. A minimal spanning tree of a graph G is

A. a spanning sub graph

B. a tree

C. a connected graph

D. All of the above **(10 marks)**

iii. Which of the following statements is/are TRUE for undirected graphs?
P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.
A. P B. Q C. Both P and Q D. Neither P or Q **(10 marks)**

iv. Let $G$ be a graph with $V(G) = \{1.2.3 \ldots., n\}$, where $n \geq 5$, such that two numbers $i$ and $j$ in $V(G)$ are adjacent if and only if $|i - j| = 5$. How many components does $G$ have?

A. 4 B. 5 C. 6 D. 7 **(10 marks)**

v. A simple graph's degree sequence is the graph's nodes in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

(I) 7, 6, 5, 4, 4, 3, 2, 1
(II) 6, 6, 6, 6, 3, 3, 2, 2
(III) 7, 6, 6, 4, 4, 3, 2, 2
(IV) 8, 7, 7, 6, 4, 2, 1, 1

A. IV only
B. III and IV
C. I and II
D. II and IV **(10 marks)**

vi. Let D be a simple graph on 10 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6, a vertex of degree 7, a vertex of degree 8 and a vertex of degree 9. What can be the degree of the last vertex?

A.4
B.0
C.2
D. 5 **( 10 marks)**

vii. Let, $D = < A, R >$ be a directed graph or digraph, then $D' = < A', R' >$ is a subgraph if

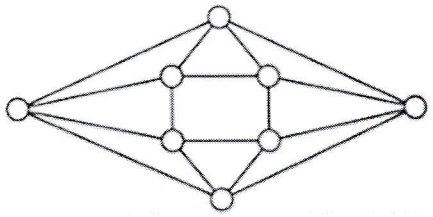
A. $A' \subset A$ and $R' = R \cap (A' \times A')$
B. $A' \subset A$ and $R \subset R' \cap (A' \times A')$
C. $R' = R \cap (A' \times A')$
D. $A' \subseteq A$ and $R \subseteq R' \cap (A' \times A')$ **(10 marks)**

viii. The degree sequence for

$$G=\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \text{ is}$$

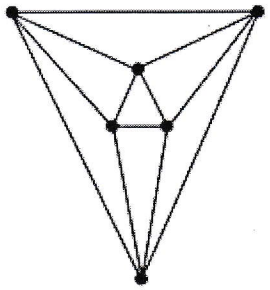
A. (2,2,3,3,4) B. (0,1,1,2,0) C. (0,0,1,1,2) D. None of the above **(10 marks)**

ix. The minimum number of colours required to colour the following graph ,such that no adjacent vertices are assigned the same colour is



A. 2 B. 3 C. 3 D. 4 E. 5 **(10 marks)**

x. Is the following graph



A. a Hamiltonian.

B. an Eulerian.

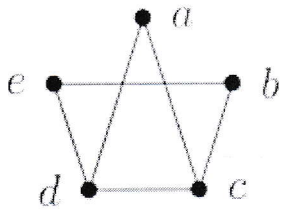
C. Neither an Eulerian nor a Hamiltonian.

D. Satisfies both an Eulerian and a Hamiltonian. **(10 marks)**

2. Are the two graphs below equal? Are they isomorphic? If they are isomorphic, give the isomorphism. If not, justify your answer.

   Graph 1: V={a,b,c,d,e},E={{a,b},{a,c},{a,e},{b,d},{b,e},{c,d}}.

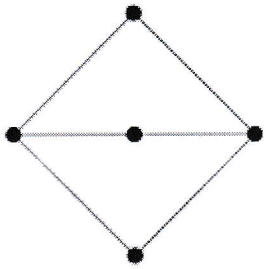
   Graph 2:



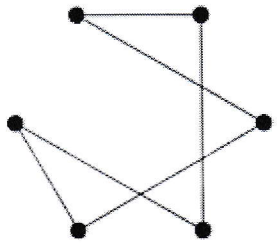

**(50 marks)**

3. Which of the graphs below are bipartite? Justify your answers.

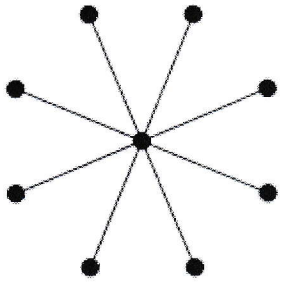
A.



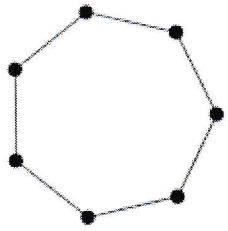
B.



C.



D.



**(50 marks)**

4. Weights are given for edges between 7 vertices, labelled A – G.

| | | | | | | |
|---|---|---|---|---|---|---|
| A | | | | | | |
| 11 | B | | | | | |
| 17 | 9 | C | | | | |
| 17 | 12 | 14 | D | | | |
| 11 | 17 | 15 | 10 | E | | |
| 16 | 9 | 9 | 10 | 8 | F | |
| 20 | 10 | 21 | 19 | 8 | 12 | G |

Find a minimal weight spanning tree. What is the total weight of this spanning tree?

**(100 marks)**

5. Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other.

- A and C
- A and D
- B and C
- C and D
- C and E

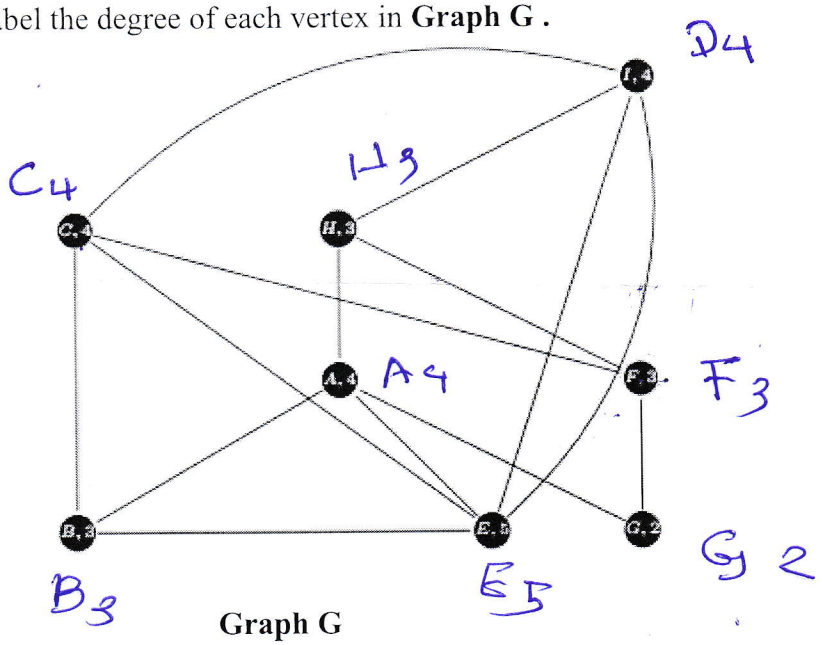
a. Draw a graph G to represent this situation. **(30 marks)**

b. List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, respectively, in G = {V, E}. **(30 marks)**

**c.** Draw an adjacency matrix for G. **(40 marks)**

42 (C, 4)

6. (a) Label the degree of each vertex in **Graph G .** **(40 marks)**




**Graph G**

(b) Determine if **Graph G** has an Euler path or an Euler cycle. If it does, then find one. If not, then explain why not. **(60 marks)**