RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester I Examination – September/October 2019

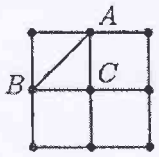
MAT 3213 – GRAPH THEORY

Time: Two (02) hours

- **Answer all questions.**
- **Answer Question number 1 on this paper itself and attache to the answer booklet.**

1. Select the **most suitable** answer for the following questions:

a) In the given graph, the number of walks of length 3 through vertices A, B and C is?



(a) 3 (c) 12
(b) 6 (d) 1

b) Let G be a graph and A be the adjacency matrix of G.

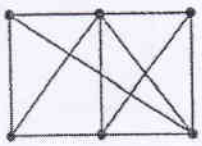
$$A = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The degree sequence of G is,

(a) (2,2,3,3,4) (c) (0,1,1,2,0)
(b) (0,0,1,1,2 (d) None of the above

c) Let H be the plane drawing of the G. Then the graph H has,

G:



(a) 10 faces
(b) 11 faces
(c) 5 faces
(d) 7 faces.

d) The chromatic number of the complete graph K_{15} is,

(a) 3
(b) 6
(c) 2
(d) 15

e) If G is a connected planar graph of order v, size e and with f faces, then,

(a) $v - e + f = 2$
(b) $v + e - f = 2$
(c) $e - v + f = 2$
(d) None of the above.

f) A graph with all vertices having equal degree is known as,

(a) Multi graph
(b) Regular graph
(c) Simple graph
(d) Complete graph

g) The chromatic index of cyclic graph C_{15} is?

(a) 5
(b) 2
(c) 3
(d) None of the above

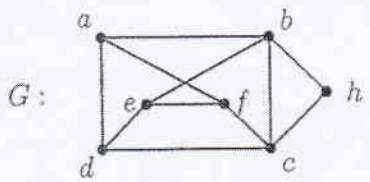
h) Which of the following combinations of the degrees of vertices a of connected graph be **Eulerian**?

(a) 1,2,3
(b) 2,3,4
(c) 2,4,6
(d) 1,3,5

i) Which of the following statement is **not true**?

(a) The cube graph Q_5 is planar
(b) The complete graph K_7 is non-planar
(c) The complete bipartite graph $K_{4,3}$ is non-planar
(d) None of the above

For the questions **j** to **m**, refer to the graph G drawn below:



j) The *dacbeb* is,

(a) a walk in G
(b) a trail but not a path in G
(c) a walk but not atrial in G
(d) not a walk in G

k) The graph G is,

(a) Semi-Hamiltonian but not Eulerian
(b) Hamiltonian but not Eulerian
(c) Hamiltonian and semi-Eulerian
(d) Not Hamiltonian nor Eulerian

l) If $x = ef$ is the line of G, then the degree of x in $L(G)$ is,

(a) 4
(b) 3
(c) 5
(d) 1

m) The graph G is best described as,

(a) A multi graph
(b) A pseudograph
(c) A simple graph
(d) A complete graph

n) Let $H = (V, E)$ is a graph, where $V = \{a, b, c, d, e, f\}$ and $E = \{ab, ad, ac, bc, be, cd, cf, de, df\}$. The edge set of the complement of H is?

(a) $\{ab, be, de, bf, cf, cd\}$
(b) $\{af, fb, bd, dc, ce, ea\}$
(c) $\{af, ad, ac, bc, be, cd, cf, ae, df\}$
(d) None of the above.

o) A tree has five vertices of degree 2, three vertices of degree 3 and four vertices of degree 4. The number of vertices of degree 1 is,

(a) 13
(b) 11
(c) 10
(d) None of the above

(100 marks)

2. a) Suppose G is a regular graph of degree k, where k is odd. Prove that the number of edges in G is a multiple of k.

(20 marks)

b) Show that if a tree has two vertices of maximum degree 3, then it must have at least 4 vertices of degree 1.

(30 marks)

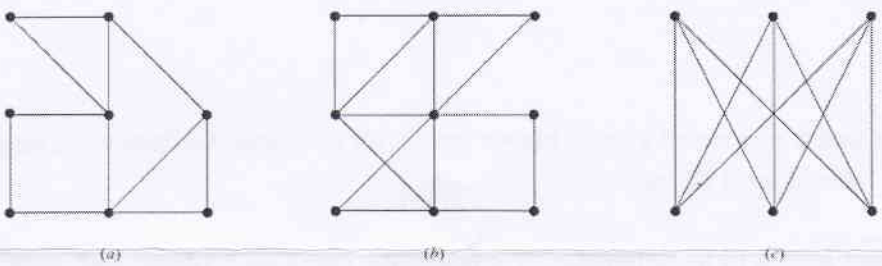
c) A company is considering to build a gas pipeline to connect 7 wells (A, B, …, G) to process plant H. The possible pipelines that they can construct and their costs are listed below.

Pipe line	Cost (×1000 $)	Pipe line	Cost (×1000 $)
AB	23	CG	10
AE	17	DE	14
AD	19	DF	20
BC	15	EH	28
BE	30	FG	11
BF	27	FH	35

Determine a pipeline network to be built in order to reduce the total cost.

(50 marks)

3. a) Determine whether each of the following graphs are traversable (has a Euler path), Eulerian (has Euler circuit), Hamiltonian or neither. Justify your answer.



(40 marks)

b) A salesman wishes to visit four cities starting from the city Q and return home. The distances between each city is given below.

	P	Q	R	S	T
P		18	22	15	20
Q	18		11	12	22
R	22	11		16	10
S	15	12	16		13
T	20	22	10	13	

By using nearest neighbor algorithm, find the minimum distance the salesman should travel.

(60 marks)

4. a) A lecture time table is to be drawn. Since some students wish to attend several lectures, certain lectures must not be coincide. The asterisks (*) in the following table show which pair of lectures cannot coincide. Find the minimum number of time slots needed to timetable all seven lectures.

	a	***b***	***c***	*d*	*e*	***f***	***g***
a	–	*	*	*	–	–	*
b	*	–	*	*	*	–	*
c	*	*	–	*	–	*	–
d	*	*	*	–	–	*	–
e	–	*	–	–	–	–	–
f	–	–	*	*	–	–	*
g	*	*	–	–	–	*	–

(50 marks)

b) Show that a connected simple planar graph, all of whose vertices have degree at least 5, must have at least 12 vertices.

(25 marks)

c) A graph is said to be polyhedral if it is simple, connected, planar and every vertex has at least degree 3. Prove that no polyhedral graph has 30 edges and 11 regions.

(25 marks)

END