RAJARATA UNIVERSITY OF SRILANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
Second Year-Semester II Examination-April 2015

MAP 2202 – REAL ANALYSIS II

Answer **FOUR** Questions Only — Time Allowed: **Two hours**

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1. i Give an example of a convergent series $\sum_{n=1}^{\infty} a_n$ and a divergent series $\sum_{n=1}^{\infty} b_n$ with the property that $b_n \le a_n$ for all n. **(20 Marks)**

 ii Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ and a convergent series $\sum_{n=1}^{\infty} b_n$ with the property that $a_n \le b_n$ for all n. **(20 Marks)**

 iii Determine if the following series converge or diverge. If they converge give the value of the series.

 a) $$\sum_{n=1}^{\infty} 9^{-n+2}4^{n+1}$$ **(30 Marks)**

 b) $$\sum_{n=0}^{\infty} \frac{(-4)^{3n}}{5^{n-1}}$$ **(30 Marks)**

2. i Let $f(x) = \frac{x^2-2x-15}{x+3}$ $x \neq 3$
 How should $f(-3)$ be defined so that f is continuous at -3? **(20 Marks)**

 ii Without detailed proofs evaluate the following limits.

 a) $\lim_{x\to +1} \frac{x^2-x-2}{x+1}$ **(10 Marks)**

 b) $\lim_{x\to 1}\left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$ **(10Marks)**

 c) $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}}$ **(20 Marks)**

 d) $\lim_{x\to -1} \frac{x^2-x-2}{x+1}$ **(10Marks)**

 iii Define $f:\mathbb{R}\to\mathbb{R}$ by $f(x) = \frac{x^3}{1+x^2}$
 Show that f is continuous on $\mathbb{R}$. Is f uniformly continuous on $\mathbb{R}$? **(30 Marks)**

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3. i Compute the radius and interval of convergence of $f(x)=\sum_{k=1}^{\infty} k\,x^k$. **(15 Marks)**

ii Use the fact that $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ for all $|x|<1$ to show that $\sum_{k=0}^{\infty} k\,x^k = \frac{x}{(1-x)^2}$ for all $|x|<1$. **(20 Marks)**

iii Use the part ii) and compute the series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{3^k}$. **(15 Marks)**

iv Show that $\log(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots, \quad -1 \le x \le 1$ and deduce that $\log 2 = 1 - 1/2 + 1/3 - 1/4 + \ldots$ **(20 Marks)**

v Compute lower Riemann sum $L(p,f)$ and upper Riemann sum $U(p,f)$ if $f(x)=x^2$ on [0,1] and $P=\{0,\ 1/4,\ 2/4,\ 3/4, 1\}$ be a partition of [0,1]. **(15 Marks)**

vi Show that $(3x+1)$ is Riemann integrable on [1,2] and $\int_1^2 (3x+1)\,dx = \frac{11}{2}$. **(15 Marks)**

4 i Determine if the following limits exist or not. If they do exist give the value of the limit.

a) $$\lim_{(x,y)\to(0,0)} \left(\frac{x^2-y^2}{x^2+y^2}\right)^2$$ **(15 Marks)**

b) $$\lim_{(x,y)\to(1,1)} \frac{xy-1}{1+xy}$$ **(15 Marks)**

c) $$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$$ **(20 Marks)**

ii Discuss the continuity of the functions:

a) $$f(x,y)=\begin{cases} \dfrac{x^2+2xy^2+y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$ **(25 Marks)**

b) $$g(x,y)=\begin{cases} \dfrac{4x^2y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 2 & (x,y)=(0,0) \end{cases}$$ **(25 Marks)**

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5 i Find the location and nature (maxima , minima and saddle) of the stationery value of the function $f(x,y) = x^2y - y^2x + 4xy - 4x^2 - 4y^2$. **(30 Marks)**

ii Assuming the validity of differentiation under the integral sign, show that

$$\int_0^{\pi/2} \log\left(1 - x^2\cos^2\theta\right)d\theta = \pi\left\{\log(1+\sqrt{1-x^2}) - \log 2\right\},\ x^2 \le 1$$

(30 Marks)

iii A jewel box is to be constructed of material that costs \$1 per square inch for the bottom, \$2 per square inch for the sides, and \$5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction? **(40 Marks)**

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