

RAJARATA UNIVERSITY OF SRI LANKA FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
First Year – Semester I Examination– May/June 2016

MAA 1201 - Mathematical Methods I

Answer Four questions only.

Time allowed: Two Hours

1.

a. Prove $A \cdot (B + C) = A \cdot B + A \cdot C$

b. Prove that A X(B XC) + B X(C XA) + C X(A XB) = 0

c. Find the angle between $\mathbf{A} = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

d. Determine a unite vector perpendicular to the plane of P = 2i - 6j + 3k and Q = 4i + 3j - k

e. Prove the law of sines for plane triangles.

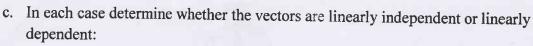
f. Find an equation for the plane determined by the points $P_1(2, -1.1)$, $P_2(3, 2, -1)$ and $P_3(-1, 3, 2)$.

2.

a. Find the equation of a straight line which passes through two given points A and B having position vectors a and b with respect to an origin O.

b. Show that the equation of a plane which passes through three given points A,B,C not in same straight line and having position vectors **a**, **b**, **c** relative to an origin O, can be written $\mathbf{r} = \frac{\alpha \mathbf{a} + \beta \mathbf{b} + \gamma \mathbf{c}}{\alpha + \beta + \gamma}$

Where α , β and γ are scalars. Verify that the equation is independent of the origin.



i.
$$A = 2i + j - 3k$$
, $B = i - 4k$, $C = 4i + 3j - k$

ii.
$$P = i - 3j + 2k$$
, $Q = 2i - 4j - k$, $R = 3i + 2j - k$

3.

- a. Show that $A \cdot (B \times C)$ is an absolute value equal to the volume of a parallelepiped with sides A, B and C.
- b. Prove that a necessary and sufficient condition for the vectors A, B and C to be coplanar is that $A \cdot (B \times C) = 0$.
- c. Prove that p, q and r are non coplanar then xp + yq + zr = 0 implies x = y = z = 0.
- d. Find the constant μ such that vectors 2i j + k, i + 2j 3k and $3i + \mu j + 5k$ are coplanar.

4.

- a. Find the unite tangent vector to any point on the curve $x = t^2 + 1$, y = 4t 3, $z = 2t^2 6t$.
- b. Determine the unite tangent at the point where t = 2.
- c. Find the velocity and acceleration of a particle which moves along the curve $x = 2 \sin 3t$, $y = 2 \cos 3t$, z = 8t at any time t > 0. Find the magnitude of the velocity and acceleration.
- d. Find the curvature and the torsion for the space curve $x = \theta \sin \theta$, $y = 1 \cos \theta$, $z = 4\sin \left(\frac{\theta}{2}\right)$.

5.

- a. Prove that the vector $\mathbf{A} = 3y^4z^2\mathbf{i} + 4x^3z^2\mathbf{j} 3x^2y^2\mathbf{k}$ is solenoidal.
- b. Prove $curl(\varphi \operatorname{grad} \varphi) = 0$
- c. If f(r) is differentiable, prove that f(r) r is irrotational.
- d. Find an equation for the tangent plane to the surface $xz^2 + x^2y = z 1$ at the point (1, -3, 2).