RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
Second Year – Semester I Examination – April/ May 2016

MAA 2204 – Linear Programming

Answer **Four** Questions **only.** Time allowed: **Two Hours**

Calculators will be provided

01

(a) "Two Phase method is more efficient than Big-M method." Do you agree with this statement? Discuss your answer.

(b) While finding the solution by Two Phase Method, when does the problem has infeasible and unbounded solutions?

(c) The following tableau show the optimal solution for phase I of the linear programming problem:

Minimize $Z = 4x_1 + x_2$

Subject to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

B.V.	x_1	x_2	s_1	s_2	y_1	y_2	$-w$	Constants
x_1	1	0	$1/5$	0	$3/5$	$-1/5$	0	$3/5$
x_2	0	1	$-3/5$	0	$-4/5$	$3/5$	0	$6/5$
s_2	0	0	1	1	1	−1	0	0
$-w$	0	0	0	0	1	1	1	0

Where y_1, y_2 are artificial variables and s_1, s_2 are surplus and slack variables and w is the objective function for phase I.

Continue the solution procedure with the phase – II (if required) and find the optimal solution.

02

(a) Define a convex set and determine which of the following sets are convex and which are not? (Justify your answers)

I. $\left\{ (x_1, x_2) : x_1^{\,2} + x_2^{\,2} \geq 1 \right\}$

II. $\left\{ (x_1, x_2) : x_1 = 1, |x_2| \leq 4 \right\}$

III. $\left\{ (x_1, x_2) : x_2 - x_1^{\,2} = 0 \right\}$

(b) A company manufactures three products namely *X*, *Y* and *Z*. Each product requires processing on three machines: Turning, Milling and Grinding. Product *X* requires 10 hours of turning, 5 hours of milling and 1 hour of grinding. Product *Y* requires 5 hours of turning, 10 hours of milling and 1 hour of grinding, and product *Z* requires 2 hours of turning, 4 hours of milling and 2 hours of grinding. In the coming planning period, 2700 hours of turning, 2200 hours of milling and 500 hours of grinding are available. The profit contribution of *X*, *Y* and *Z* are Rs. 10, Rs. 15 and Rs. 20 per unit respectively.

I. Formulate the above situation as a linear programming problem in order to maximize the profit.

II. Using Simplex method, find the optimal product mix and maximum profit.

03

(a) Briefly explain the Revised Simplex method in Linear Programming.

(b) Solve the following linear programming problem using Revised Simplex method:

Maximize $Z = -x_1 + 3x_2 - 2x_3$

Subject to

$$3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

04 Consider the following Linear Programming Problem (primal problem):

Maximize $Z = 7x_1 + 5x_2$

Subject to

$$2x_1 + x_2 \leq 100$$
$$4x_1 + 3x_2 \leq 240$$
$$x_1 \geq 0, x_2 \geq 0$$

(a) Formulate the corresponding dual linear programming problem.

(b) Plot the feasible region of the dual problem.

(c) Determine the optimal solution of the dual problem by comparing its objective values at the extreme point solutions.

(d) Determine the optimal solution of the primal problem using above results.

05

(a) Define the terms surplus, slack and artificial variables, and explain their importance in linear programming.

(b) A dietitian wants to design a breakfast menu for certain hospital patients. The menu consists of two items, *A* and *B*. Suppose that each ounce of *A* provides 2 units of vitamin C and 2 units of iron and each ounce of *B* provides 1 unit of vitamin C and 2 units of iron. Suppose that the cost of *A* is Rs. 4 per ounce and the cost for *B* is Rs. 3 per ounce. The breakfast menu must provide at least 8 units of vitamin C and 10 units of iron.

I. Formulate the mathematical model to determine how many ounce of each item should be provide in order to meet the iron and vitamin C for the least cost.

II. Use the Big-M method to solve the above linear programming problem.