RAJARATA UNIVERSITY OF SRILANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year Semester I Examination – September/October 2019

MAT 4305 – STOCHASTIC PROCESSES

Time allowed: **Three Hours**

Answer **ALL** Questions

1. (a) Clearly define following terms:
   (i) Associate State
   (ii) Communicate State
   (iii) Irreducible State
   (iv) Recurrent State
   (v) Transient State

   (b) Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and the transition probability matrix $P = \begin{pmatrix} 1/4 & 3/4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$.

   (i) Draw a directed graph for the chain.

   (ii) Identify associated communicating classes of the above probability matrix.

   (iii) Classify the above states $S = \{1, 2, 3, 4\}$ as periodic or aperiodic, recurrent or transient, positive-recurrent, ergodic.

2 (a) Define what is *Markov Chain.*

   (b) Clearly state what are *Chapman-Kolmogorov Equations.*

   (c) Suppose that coin 1 has probability 0.7 of coming up heads, and coin 2 has probability 0.6 of coming up heads. If the coin flipped today comes up heads, then we select coin 1 to flip tomorrow, and if it comes up tails, then we select coin 2 to flip tomorrow.

Contd.

(i) Find $E(X_3)$.

(ii) If the coin initially flipped is equally likely to be coin 1 or coin 2, then what is the probability that the coin flipped on the third day after the initial flip is coin 1?

(iii) Suppose that the coin flipped on Monday comes up heads. What is the probability that the coin flipped on Friday of the same week also comes up heads?

3. (a) Clearly state what are the limiting probabilities in terms of *Markov-Chains.*

(b) State the theorem related to deriving limiting probabilities.

(c) College offers 4-year degree program. Each student repeats year, or progresses to next year, or drops out / graduates with different probabilities, depending on which year he is currently in. The probabilities are described by transition matrix P

$$P = \begin{array}{c|ccccc} & Y1 & Y2 & Y3 & Y4 & D/G \\ \hline Y1 & 0.2 & 0.7 & 0 & 0 & 0.1 \\ Y2 & 0 & 0.15 & 0.8 & 0 & 0.05 \\ Y3 & 0 & 0 & 0.1 & 0.85 & 0.05 \\ Y4 & 0 & 0 & 0 & 0.05 & 0.95 \\ D/G & 0 & 0 & 0 & 0 & 1 \end{array}$$

D/G - Drop Out/Graduation State

(i) If annual tuition is Rs. 1 million for years 1 & 2, and Rs. 1.2 million for years 3 & 4, what is average student expected to pay from start to end, i.e. from Y1 until drop-out or graduation?

(ii) What proportion of freshmen make it to Y4?

4. (a) Clearly define what is *Hidden Markov Chains.*

(b) Consider a production process that in each period is either in a good state (state 1) or in a poor state (state 2). If the process is in state 1 during a period then, independent of the past, with probability 0.9 it will be in state 1 during the next period and with probability 0.1 it will be in state 2. Once in state 2, it remains in that state forever. Suppose that a single item is produced each period and that each item produced when the process is in state 1 is of acceptable quality with probability 0.99, while each item produced when the process is in state 2 is of acceptable quality with probability 0.96.

(i) State the transition probability matrix for the above scenario.

(ii) Clearly explain how the above scenario will become a Hidden Markov Chain.

Contd.

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(c) Suppose in previous Example, that P $\{X_1 = 1\} = 0.8$. It is given that the successive conditions of the first three items produced are acceptable (a), un-acceptable (u) and acceptable (a) respectively.

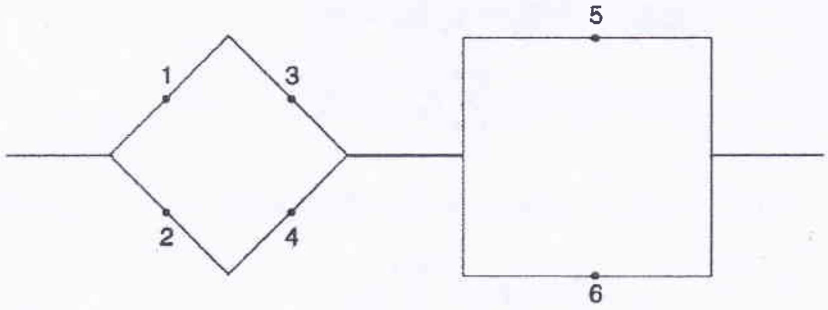
(i) What is the probability that the process was in its good state when the third item was produced?

(ii) What is the probability that X_4 is 1?

(iii) What is the probability that the next item produced is acceptable?

5. (i) Consider a job shop that consists of M machines and one serviceman. Suppose that the amount of time each machine runs before breaking down is exponentially distributed with mean $1/\lambda$ and suppose that the amount of time that it takes for the serviceman to fix a machine is exponentially distributed with mean $1/\mu$. What is the average number of machines not in use?

(ii) What are the minimal path sets for the following structure.



(iii) What are the minimal cut sets for the above model?

(iv) Write the structure function for the structure given above (part ii).

(v) Derive reliability function of the structure in part ii.

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List of Equations

$$s_{ij} = \delta_{i,j} + \sum_{k=1}^{t} P_{ik} s_{kj}$$

$$S = (I - P_T)^{-1}$$

$$f_{ij} = \frac{s_{ij} - \delta_{i,j}}{s_{jj}}$$

$$\mu = \sum_{j=0}^{\infty} j P_j$$

$$\sigma^2 = \sum_{j=0}^{\infty} (j - \mu)^2 P_j$$

$$F_n(j) = P\{S_n = s_n, X_n = j\}$$

$$F_n(j) = p(s_n|j) \sum_i F_{n-1}(i) P_{i,j}$$

$$P\{X_n = j, S_n = s_n | X_{n-1} = i\} = P_{i,j} p(s_n|j)$$

$$v_i = \sum_j v_i P_{ij} = \sum_j q_{ij}$$

$$v_j P_j = \sum_{k \neq j} q_{kj} P_k \quad , all\ states\ j$$