RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

First Year - Semester I Examination - July/August 2023

MAP 1302 - DIFFERENTIAL EQUATIONS I

Time allowed: **Three (3) hours**

Answer any five (5) questions only.

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1. a) Solve the following differential equation:

$$y\left(x^2+1\right)\frac{dy}{dx}=x\left(y^2+1\right).$$

(30 marks)

b) Using the substitution $v=\frac{y}{x}$, solve the differential equation given below:

$$y+(x-2y)\frac{dy}{dx}=0.$$

(35 marks)

c) Using suitable substitutions for x and y, reduce the following differential equation to a homogeneous form:

$$\frac{dy}{dx}=-\frac{3y-7x+7}{7y-3x+3}.$$

Hence, obtain the solution for the given differential equation.

(35 marks)

2. a) A moving object is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x and v are the displacement and velocity of the object at that instant and where $b(\neq 0)$ and c are constants. Find the velocity of the object in terms of x, if it starts at rest.

(30 marks)

b) The rate at which a body cools is proportional to the difference between the temperature of the body and that of the surrounding air. If a body in air at 25^0C will cool from 100^0C to 75^0C in one minute, find its temperature at the end of three minutes.

(30 marks)

c) For steady heat flow through the wall of a hollow sphere of inner and outer radii r_1 and r_2, respectively, the temperature u at a distance $r(r_1 < r < r_2)$ from of the center of the sphere is given by

$$r\frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0.$$

If u_1 and u_2 are the temperatures at inner and outer surfaces of the sphere, find u in terms of r.

(40 marks)

3. a) Show that the following differential equation is exact, and hence solve it:

$$(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0.$$

(30 marks)

b) Solve the following linear differential equation using an integrating factor:

$$\frac{dy}{dx} + 2xy = 2e^{-x^2}.$$

(35 marks)

Continued.

c) Using a suitable substitution, convert the following Bernoulli equation to a linear differential equation:

$$(1-x^2)\frac{dy}{dx}+xy=xy^2.$$

Solving the obtained linear differential equation, find the solution of the given Bernoulli equation.

(35 marks)

4. a) Solve the following homogeneous differential equations with constant coefficients:

   i. $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$

   ii. $\frac{d^3y}{dx^3}-4\frac{d^2y}{dx^2}-3\frac{dy}{dx}+18y=0$

   iii. $\left(\frac{dy}{dx}-y\right)^2\left(\frac{d^2y}{dx^2}+y\right)^2=0$

(60 marks)

b) Solve the following differential equations:

   i. $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=2e^{-3x}$

   ii. $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=2e^{x}\cos 2x$

(40 marks)

5. a) Solve the homogeneous linear differential equation given below:

$$x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2\ln x.$$

(30 marks)

b) Solve the following Riccati equations:

   i. $\frac{dy}{dx}=-2-5y-2y^2$, and

   ii. $\frac{dy}{dx}+y^2=1+x^2$.

(50 marks)

Continued

c) Solve: $p = \log(px - y)$, where $p = \frac{dy}{dx}$.

(20 marks)

6. Solve the following simultaneous linear differential equations:

a)

$$\begin{cases} \frac{dx}{dt} = 3x + 8y \\ \frac{dy}{dt} = -x - 3y, \end{cases}$$

with $x(0) = 6$ and $y(0) = -2$.

(50 marks)

b)

$$\begin{cases} \frac{d^2x}{dt^2} + y = \sin t \\ \frac{d^2y}{dt^2} + x = \cos t \end{cases}$$

(50 marks)

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