RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year Semester II Examination – February /March 2019

MAA 1203 – NUMERICAL ANALYSIS I

Time: Two (02) hours

Answer Four Questions including Question 01
A non-programmable calculator is permitted

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01. a). Define the absolute error, normalized error and relative error. (05 marks)

b). The given numbers are truncated. Suppose that we are using computer with a fixed ward length of five digits. Find the round off error using chopping and symmetric rounding.

I. $x = 72.32451$
II. $x = 18.63421$ (05 marks)

c). Consider the equation $f(x) = x^2 + 8x + 5$. Find the third order differences using Forward difference method and step size =1. (05 marks)

d). Approximate the derivative of $f(x) = x^2 + 2x$ at $x = 3$ using Central difference method and step size =1. (10 marks)

02. a). I. Write down the digits used in the general base β representation.
II. Let $(a_n a_{n-1} \cdots a_0 . b_1 b_2 b_3 \cdots)_\beta$ be the general form of base β representation. Label the radix point and discuss the use of radix point in the above system.
III. Write down the general form mentioned in part II with summates. (05 marks)

b). Consider the conversion between systems α and β with $\alpha > \beta$. Briefly discuss how the **division algorithm** and **multiplication algorithm** apply in this conversion. (05 marks)

c). Let us consider the operator $fl(\,)$, which is chop any number to 5 digits and $x \oplus y = fl\{fl(x) + fl(y)\}$

I. Given $x = \frac{1}{3}$, $y = \frac{5}{7}$, find $fl(x)$ and $fl(y)$. Hence evaluate $x \oplus y$.
II. Find the exact value of $x + y$.
III. Hence, find the absolute error and the relative error.
IV. At certain situations, finding the relative error is more meaningful compared to calculating other types of errors. Give reasons. (10 marks)

d). Consider the function, $f(x) = e^x$. Find the truncation error at $x = 0.2$ using the first 3 terms. (05 marks)

03. a). Compute $f(x) = \frac{(e^x - 1)}{x}$ for the value x near 0. Explain, why should not you use the given formula for this computation. Suggest a better method for computing $f(x)$ when x is near 0. (05 marks)

b). Let $f(x) = \sqrt{x+1}$. Compute the second order Taylor's polynomial with $x_0 = 0$. Then show that the error in using p_2 to approximate $\sqrt{x+1}$ is $|R_2(x)| = \frac{1}{16}|x|^3|\xi_x + 1\ |^{-5/2}$, where ξ_x is between x and x_0. (05 marks)

c). I. Let $f(x) = x^2 + 8x - 5$. Show that $\Delta^3 f(x) = 0$.
II. Evaluate $\left(\frac{\Delta^2}{E}\right) f(x)$, where $f(x) = x^3$.
III. Prove the relation $E\nabla \equiv \nabla E \equiv \Delta$. (10 marks)

d). Which of the following are polynomial, if not explain why it is not?
I. $x^2 + 2x + 5$
II. $(6x^2 + 3x) \div (3x)$
III. $3x^{1/2} + 2$ (05 marks)

04. a). Briefly explain how the bisection method can be used for solving the nonlinear equation $f(x) = 0$ in the interval $[a, b]$, where f is continuous. (05 marks)

b). I. Sketch the Newton's method and derive the Newton's method iterative formula.
II. Sketch the Secant method and briefly discuss how it proceeds. (10 marks)

c). I. Discuss the cases where Newton's method can fail due to bad starting point.
II. Suggest a method to overcome the cycling issue that can arise when using the Newton's method. (10 marks)

05. a). Distinguish between interpolation and extrapolation. (05 marks)

b). I. Find the polynomial that interpolates $f\left(\frac{1}{3}\right) = 2, f\left(\frac{1}{4}\right) = -1, f(1) = 7$ using system of linear equations.
II. Write down the cardinal functions and interpolate the polynomial in Lagranges's form.
III. Find the Newton's form of the above values and interpolate the polynomial in Newton form. (15 marks)

c). Analyze the efficiency of above three interpolating techniques. (05 marks)

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