**RAJARATA UNIVERSITY OF SRI LANKA**
**FACULTY OF APPLIED SCIENCES**

B. Sc. Four Year Degree in Industrial Mathematics
Fourth Year – Semester I Examination – October/November 2015

**Computational Mathematics–MAT 4310**

Time allowed: **3 hours** only.

Answer **FIVE** questions, including **Qu.1.** *Calculators will be provided*



1.

i. Use Taylor series to compute $\frac{d}{dx}(\sin x)$.

ii. Find the McLaurin series for**cosx.**

iii. Show that the infinite series $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}+\cdots$ is a geometric series by writing it in the form $\sum_{k=0}^{\infty} ar^k$ and calculate the sum.

iv. Express the infinite repeating decimal **0.44444...**as a single fraction by first writing it as a geometric series.

v. Let $T_2(x) = 1 + x + \frac{x^2}{2}$ be the Taylor polynomial of degree two for $f(x) = e^x$, centered at $x = 0$. Verify directly by taking their derivatives that $T_2(x)$ and $f(x)$satisfy the three conditions, $T_2(0) = f(0), T'_2(0) = f'(0)$ and $T''_2(0) = f''(0)$

vi. Suppose $g$ is a function which has continuous derivatives ,and that $g(0) = 3, g'(0) = 2, g''(0) = 1$and$g'''(0) = -3$.

a. What is the Taylor polynomial of degree 2 for $g$, centered at$x = 0$?

b. What is the Taylor polynomial of degree 3 for $g$, centered at $x = 0$?

c. Use $T_2(x)$, and $T_3(x)$ to approximate $g(0.1)$.

vii. Consider the infinite series $\sum_{k=0}^{\infty}(-1)^k$. Write down the values for the first partial sums for the series (i.e.$s_0, s_1, s_2, s_3$). Find the sum of this series (if it exists). Justify your answer.

[P.T.O]

2. 47

i. Solve the differential equation $y_{n+1} - 2\sin x\, y_n + y_{n-1} = 0$, when $y_0 = 0$ and $y_{n-1} = \cos x$. $y_1 = \cos x$

ii. Find $y_n$, from the difference equation $\Delta^2 y_{n+1} + \frac{1}{2}\Delta^2 y_n = 0$, n=0,1,2,…., when $y_0 = 0, y_1 = \frac{1}{2}$ *and* $y_2 = \frac{1}{4}$

3. Compute an Approximation to $y(1)$, $y'(1)$, $y''(1)$ with Taylor's algorithm of order two and step length h=1 when y(x) is the solution to the initial value problem $Y''' + 2y'' + y' - y = \cos x$ $, 0 \leq x \leq 1$,

$$y(0) = 0, \quad y'(0) = 1, \qquad y''(0) = 2$$

4.

i. Perform 4 iterations with the bisection method for the equations $\tan x + \tanh x = 0$

ii. Show that the equation

$$f(x) = \cos\left(\frac{\pi(x+1)}{8}\right) + 0.148x - 0.9062 = 0$$

Has one root in the interval $(-1,0)$ *and one in* $(0,1)$. Calculate the negative root correct to 4 decimals.

iii. Calculate $p$ and $q$, such that $A^n = pA + qI$ and determine $e^A$. Where

$$A = \begin{pmatrix} 1+s & -s \\ s & 1-s \end{pmatrix}$$

5.

i. Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ by the Gauss-Jordan method.

ii. Show that the following matrix is nonsingular but that it cannot be written as the **LU** product of lower and upper triangular matrices.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ -1 & 0 & 2 \end{pmatrix}$$

6. A sequences of functions $f_n(x)$; $n = 0,1,2, \ldots..$ defines a recursion formula,

$$f_{n+1}(x) = 2x\, f_n(x) - f_{n-1}(x), |x| < 1$$

$$f_0(x) = 0, \qquad f_1(x) = 1$$

   i. Show that $f_n(x)$ is a polynomial and give its degree and the leading coefficient.

   ii. Show that

$$\begin{pmatrix} f_{n+1}(x) \\ T_{n+1}(x) \end{pmatrix} = \begin{pmatrix} x & 1 \\ x^2 - 1 & 1 \end{pmatrix} \begin{pmatrix} f_n(x) \\ T_n(x) \end{pmatrix}$$

   Where $T_n(x) = \cos(n \cos^{-1} x)$

7. Consider the following **Runge-Kutta** method for the differential Equation $y' = f(x, y)$

$$Y_{n+1} = Y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$

$$K_1 = h\, f(x_n, y_n)$$

$$K_2 = h\, f(x_n + \frac{h}{2}, y_n + \frac{K_1}{2})$$

$$K_3 = h\, f(x_n + h, y_n - K_1 + 2K_2)$$

   i. Compute y(0.4) when $y' = \frac{y+x}{y-x}$, $y(0) = 1$ and h = 0.2, Round to five decimals.

   ii. What is the result after one step of length h when $y' = -y$, $y(0) = 1$.