RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Information Technology
First Year - Pre Semester Examination - February 2023

FDN 1306 - BASIC MATHEMATICS FOR NON-MATHEMATICS STUDENTS

Time allowed: **Two and half ($2\frac{1}{2}$) hours**

Answer ALL (05) questions

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1. a) A survey of faculty and graduate students at the University of Florida's film school revealed the following information: 51 admire Moe 49 admire Larry 60 admire Curly 34 admire Moe and Larry 32 admire Larry and Curly 36 admire Moe and Curly 24 admire all three of the Stooges 1 admires none of the Three Stooges.
   i. How many people were surveyed?
   ii. How many admire Curly, but not Larry nor Moe?
   iii. How many admire Larry or Curly?
   iv. How many admire exactly one of the Stooges?
   v. How many admire exactly two of the Stooges?

(25 marks)

b) i. Solve the quadratic equation $8x^2 + 10x - 3 = 0$.
   ii. The length of a rectangle is 4 meters more than twice its width. If the area of the rectangle is 126 square meters, find its length and width.

(25 marks)

c) i. Solve the inequality $(x - 1)(6x^2 + 25x + 14) \leq 0$.
   ii. Sahan had scores of 91, 80, 76, and 92 on her first four mathematics exams of the semester. What score must she obtain on the fifth exam to have an average of 83 or better for the five exams?

(30 marks)

d) The shadow of a tower standing on a level ground is found to be $40m$ longer when the Sun's altitude is $30°$ than when it is $60°$. Find the height of the tower.

(20 marks)

2. a) Find the following limits:

i. $\lim_{x\to 1} \dfrac{x-1}{x^3-x^2+x-1}$

02

ii. $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

(20 marks)

b) Differentiate the following functions with respect to x:

i. $f(x) = x^2 + \ln(x^3+1)$.

ii. $f(x) = e^{2x} - \sin^2 x$.

iii. $f(x) = \frac{\sec^2 x + x^{-2}}{x^2+\cos 2x}$.

(45 marks)

c) A rocket R is launched vertically and its tracked from a radar station S which is 4 miles away from the launch site at the same height above sea level. At a certain instant after launch, R is 5 miles away from S and the distance from R to S is increasing at a rate of 3600 miles per hour. Compute the vertical speed v of the rocket at this instant.

(15 marks)

d) A water tank is in the shape of a cone with vertical axis and vertex downward. The tank has radius $3m$ and is $5m$ high. At first the tank is full of water,but at time $t = 0$ (in seconds), a small hole at the vertex is opened and the water begins to drain. When the height of water in the tank has dropped to $3m$, the water is owing out at $2m^3/s$. At what rate, in meters per second,is the water level dropping then?

(20 marks)

3. a) Consider the function $f(x) = \frac{x}{x^2-1}$.

Find the following:

i. The domain of $f(x)$.

ii. $f'(x)$.

iii. The turning points, if any.

iv. The x and y coordinates of all intercepts of f, if any.

v. Horizontal and vertical asymptotes, if any.

vi. The open intervals on which f is increasing as well those on which f is decreasing.

vii. The inflection point, if any.

viii. Sketch the graph of $f(x)$ using all of the above information. All relevant points must be labeled.

(60 marks)

b) Evaluate the following:

i. $\displaystyle\int_0^1 \frac{2x}{x^2+1}\,dx$.

ii. $\int_0^{\frac{\pi}{2}} \frac{2\sin x}{3\tan x}\,dx.$

03

iii. $\int_0^{2} \frac{e^{2x+3}+e^{2x-3}}{e^{-x}}\,dx.$

(40 marks)

4. a) Let $A = \begin{pmatrix} \alpha+1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \alpha & 2 \end{pmatrix}$ and $C = \begin{pmatrix} \alpha & 1 \\ \alpha & 2 \end{pmatrix}$, where $\alpha \in \mathbb{R}$.

i. Show that $A^T B - I = C$; where I is the identity matrix of order 2.

ii. Show that C^{-1} exists if and only if $\alpha \neq 0$.

iii. Let $\alpha = 1$, using this value for α write down C^{-1}.

iv. Find the matrix D such that $CDC = 2I + C$.

(50 marks)

b) $ABCDEF$ is a regular hexagon and O is its centre. If $\overrightarrow{OA} = \underset{\sim}{a}, \overrightarrow{OB} = \underset{\sim}{b}$, find $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}$ and $\overrightarrow{FA}$ in terms of $\underset{\sim}{a}, \underset{\sim}{b}$.

(20 marks)

c) Let $\underset{\sim}{a} = p\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 6\underset{\sim}{j}$, where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors with their usual meanings.

i. Find the value of p.

ii. Find $|\underset{\sim}{a}|$ and $|3\underset{\sim}{b} - \underset{\sim}{a}|$.

iii. Find $\underset{\sim}{a}.(3\underset{\sim}{b} - \underset{\sim}{a})$.

iv. Find the angle between $\underset{\sim}{a}$ and $(3\underset{\sim}{b} - \underset{\sim}{a})$.

(30 marks)

5. a) Explain the distinction between a population and a sample.

b) Explain the following terms.

i. Sample space.

ii. Events.

iii. Random variables.

c) On the first day of classes, 20 students were asked for their one-way travel time from home to university (to the nearest 5 minutes). The resulting data were as follows.

15	20	30	35	10
50	20	15	10	10
20	30	25	35	40
50	45	25	30	45

Construct

i. A frequency distribution.

ii. A relative frequency distribution.

iii. A histogram.

iv. A frequency polygon. 04

d) Calculate the sample mean and variance using the following class mid values and frequencies.

x	25	35	45	55	65	75	85	95
f	1	2	2	3	12	14	12	4

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