RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester I Examination – June/July 2018

MAP 2203– DIFFERENTIAL EQUATIONS II

Time: Two (02) hours

Answer all questions.

1.

a) Show that $x=0$ is an ordinary point of the differential equation,
$(x^2+1)y''-4xy'+6y=0$.
Find the general solution of the above differential equation, about $x=0$.

(50 marks)

b) By using the **Method of Frobenius**, find the general solution of the differential equation, $2x^2y''+7x(x+1)y'-3y=0$, about $x=0$.

(50 marks)

2.

a) Consider the initial value problem $y'=x\ln y; y(1)=1$ and determine the existence of a unique solution of it.

(20 marks)

b) Consider the initial value problem $y'=1+y^2; y(0)=0$.

Find the largest interval on which the Existence and Uniqueness theorem guarantees the existence of a unique solution.

(40 marks)

c) Find the first three successive approximations of the initial value problem,
$y'=t^2+y^2; y(0)=1$. (**Hint**: Use Picard's iterative method)

(40 marks)

3.

a) Show that, if $\underline{X}(t) = \underline{Y}(t) + i\underline{Z}(t)$ is a complex-valued solution of the first order linear homogeneous system of ordinary differential equations $\frac{d\underline{X}}{dt} = A\underline{X}$, then both the $\underline{Y}(t)$ and $\underline{Z}(t)$ are real-valued solutions of the above system.

(20 marks)

b) Consider the following First order Linear Homogeneous system of Ordinary Differential Equations, $\frac{d\underline{X}}{dt} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \underline{X}$.

(i) Find the eigenvalues and corresponding eigenvectors of the above system.

(ii) Hence find the general solution of the system.

(80 marks)

4. Solve the following partial differential equations, given in the usual notations.

(i) $\left(\frac{(b-c)yz}{a}\right)p + \left(\frac{(c-a)zx}{b}\right)q = \left(\frac{(a-b)xy}{c}\right)$

(ii) $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$

(iii) $t + s + q = 0$

(iv) $xys = 1$

(v) $t = 6x^3y$

(100 marks)

END