RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year Semester II Examination – February /March 2019

PHY 3302 – MATHEMATICAL METHODS FOR PHYSICISTS

Time: Three (03) hours

Answer 5 Questions.

Unless otherwise specified, symbols have their usual meaning.
A non-programmable calculator is permitted.

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1. (a) What are the conditions for vectors $\vec{A}$ and $\vec{B}$ to be parallel and orthogonal?

(02 Marks)

(b) Given three vectors, $\vec{P}$, $\vec{Q}$, and $\vec{R}$, such that:

$$\vec{P} = 3\hat{e}_x + 2\hat{e}_y - \hat{e}_z\ ,$$

$$\vec{Q} = -6\hat{e}_x - 4\hat{e}_y + 2\hat{e}_z\ , \text{and}$$

$$\vec{R} = \hat{e}_x - 2\hat{e}_y - \hat{e}_z\ .$$

Identify from the above set, which two vectors are orthogonal and which two are parallel (or antiparallel).

(06 Marks)

(c) Prove Jacobi's identity for vector products:

$$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + \vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) + \vec{\mathbf{c}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = \mathbf{0}$$

(06 Marks)

(d) The magnetic induction $\vec{B}$ is defined by the Lorentz force equation,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{B})$$

Carrying out three experiments, we find that:

$$\text{if } \vec{\mathbf{v}} = \hat{e}_x, \quad then \quad \frac{\vec{\mathbf{F}}}{q} = 2\hat{e}_z - 4\hat{e}_y$$

Contd.

$$\text{if } \vec{\mathbf{v}} = \hat{e}_y, \quad then \quad \frac{\vec{\mathbf{F}}}{q} = 4\hat{e}_x - \hat{e}_z, \boldsymbol{and}$$

$$\text{if } \vec{\mathbf{v}} = \hat{e}_z, \quad then \quad \frac{\vec{\mathbf{F}}}{q} = \hat{e}_y - 2\hat{e}_x.$$

Use these results to determine the magnetic induction $\vec{\boldsymbol{B}}$.

(06 Marks)

2. (a) Express the vector $\vec{\mathbf{B}}$ (given below in spherical coordinates) in **Cartesian** and **Cylindrical** coordinates.

$$\vec{\mathbf{B}} = \frac{10}{r}\hat{\mathbf{a}}_r + r\,\cos\theta\,\hat{\mathbf{a}}_\theta + \hat{\mathbf{a}}_\phi,$$

Evaluate $\vec{\mathbf{B}}(-3,4,0)$ and $\vec{\mathbf{B}}\,(5,\pi/2,-2)$. (08 Marks)

(b) Let $\vec{\mathbf{A}} = \rho\cos\theta\,\hat{\mathbf{a}}_\rho + \rho z^2 \sin\phi\,\hat{\mathbf{a}}_z$

(i) Transform $\vec{\mathbf{A}}$ into rectangular (Cartesian) coordinates and evaluate its magnitude at point $(3,-4,0)$.

(06 Marks)

(ii) Transform $\vec{\mathbf{A}}$ into spherical system and evaluate its magnitude at point$(3,-4,0)$.

(06 Marks)

3. (a) Evaluate the following determinants

(i) $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$, (ii) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$ (04 Marks)

(b) Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$ (04 Marks)

(c) Show, by computing a matrix inverse, that the solution to the following system of equations is $x_1 = 4, \;\; x_2 = 1$.

$$x_1 - x_2 = 3$$

$$x_1 + x_2 = 5$$ (04Marks)

(d) The three Pauli spin matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that,

(i) They are Hermitian (04 Marks)

(ii) $\sigma_i^2 = \boldsymbol{I}, \;\; i = 1,2,3$ (04 Marks)

4. (a) Show that the complementary function of the second order differential equation $m\frac{d^2y}{dt^2} + k\frac{dy}{dt} + \lambda y = F\cos(nt)$ is given by $e^{-ct}(Ae^{\sqrt{c^2-\mu^2}t} + Be^{-\sqrt{c^2-\mu^2}t})$, where $c = \frac{k}{2m}$, $\mu^2 = \frac{\lambda}{m}$. A, B, m, k, and λ are constants. $F\cos(nt)$ is the magnitude of a periodic force.

(10 Marks)

(b) A condenser of capacity C discharged through an inductance L and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$. Given that $L = 0.25$ henry, $R = 250$ ohms, $C = 2 \times 10^{-6}$ farad, and when $t = 0$, charge q is 0.002 Coulomb and the current $\frac{dq}{dt} = 0$, obtain q(t), the value of q as a function of t.

(10 Marks)

5. (a) (i) Show that under the transformation $y = \frac{u}{\sqrt{x}}$, the Bessel's equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ becomes $\frac{d^2u}{dx^2} + \left(1 + \frac{1-4n^2}{4x^2}\right)u = 0$. (07 Marks)

(ii) Hence deduce that for $x \to \infty$ the solution of Bessel's equation has the form, $\frac{1}{\sqrt{x}}[C_1 \sin x + C_2 \cos x]$. C_1, C_2 are constants. (05 Marks)

(b) Prove that $J_n''(x) = \frac{1}{4}[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)]$. You may use the following recurrence relation for the Bessel functions; $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$.

(08 Marks)

6. (a) (i) Using Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$, evaluate $P_1(x), P_2(x)$, and $P_3(x)$. (06 Marks)

(ii) Plot the graphs of $P_1(x), P_2(x)$, and $P_3(x)$ for $-1 \le x \le 1$. (04 Marks)

(ii) Express $f(x) = 5x^3 - x + 2$ in terms of Legendre's polynomials. (04 Marks)

(b) Prove that $P_{n+1}'(x) - P_{n-1}'(x) = (2n+1)P_n(x)$.
Hint: $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ and
$P_{n+1}'(x) + P_{n-1}'(x) = 2xP_n'(x) + P_n(x)$. (06 Marks)

End.