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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.SC (General) Degree

Second Year – Semester I Examination – February 2013

MAA 2204 Linear Programming

Time allowed: Two hours. **Number of pages: 05.**

Answer FOUR Questions Only. **Calculators will be provided.**

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1. (i). Define Linear Programming Problem. **[05 Marks]**

(ii). "Linear programming has no real- life applications". Do you agree with this statement? Discuss your answer. **[05 Marks]**

(iii). Define a convex set. **[05 Marks]**

Prove that the intersection of two convex sets is convex. **[10 Marks]**

(iv). A local travel agent is planning a trip to a major sea resort. The trip is restricted to 200 persons and past experience indicates that there will not be any problem of getting 200 persons. The problem for travel agent is to determine the number of Deluxe, Standard, and Economy tour packages to offer for this trip. These three packages differ according to seating and service for the flight, quality of accommodations, meal plans and tour options. The following table summarizes the estimated price for each of the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of Rs.2,000,000 for the entire trip.

Prices and Costs for Tour Packages per Person

Tour Plan	Price (Rs)	Hotel Costs (Rs)	Meals and Other Expenses (Rs)
Deluxe	10,000	3,000	4,750
Standard	7,000	2,200	2,500
Economy	6,500	1,900	2,200

Turn Over

In planning the trip, the following considerations must be taken into account:

(i). At least 10% of the packages must be of the Deluxe type.

(ii). At least 35% but no more than 70% must be of the standard type.

(iii). At least 30% must be of the Economy type.

(iv). The maximum number of Deluxe packages available in aircraft is restricted to 60.

(v). The hotel desires that at least 120 of tourists should be on the Deluxe and standard packages together.

The travel agent wishes to determine the number of packages to offer in each type so as to maximize the total profit.

(a). Formulate this problem as a linear programming problem. **[35 Marks]**

(b). Re-state the above problem in terms of two decision variables, taking advantage of the fact that 200 packages will be sold. **[10 Marks]**

(c). Find the optimum solution using the graphical method for the restated linear programming problem and interpret your results. **[30 Marks]**

2. (a).Define the following terms:

(i). Initial Basic Feasible Solution **[05 Marks]**

(ii). Slack variable and surplus variable **[10 Marks]**

(b). While finding the solution by simplex method, when does the problem have multiple solutions and infeasible solution? **[10 Marks]**

(c). A furniture manufacturer produces and sells desks, chairs and bookshelves. He has no difficulty in selling the items. However, limited availability of machine time, labour and floor space restrict production. Data on usage of resources, supplies and profits on items are given below:

	Desk	Chair	Bookshelf	Availability
Machine time (Hours/ unit)	8	4	5	1000 Hours
Labour Time (Hours/ Unit)	5	3	3	650 Hours
Floor space(Square feet/ unit)	9	6	9	1250 Square feet
Profit (Rupees/ unit)	270	144	225	

(i). Formulate this problem as a Linear Programming Problem. **[25 Marks]**

(ii). Using x_1 , x_2 , x_3 as respective decision variables, simplex algorithm was initiated for the above problem. At one stage, the following table was reached:

	x_1	x_2	x_3	s_1	s_2	s_3	constants
x_1	1	1/2	5/8	1/8	0	0	125
s_2	0	1/3	-1/8	-5/8	1	0	25
s_3	0	3/2	27/8	-9/8	0	1	135
$-z$	0	9	225/4	-135/4	0	0	.....

From the above table, what is the current solution? Is it optimum? **[15 Marks]**

(iii). If the current solution above is not optimum, carry out the simplex algorithm an optimum solution is reached. State the optimum solution. **[25 Marks]**

(iv). The Personal Manager of good furnishings claims that by recruiting additional labour force, the profit can be increased. Assuming no other changes, is the claim valid? Justify your answer. **[10 Marks]**

3. (a). Explain the term Artificial variable and its use in Linear Programming. **[10 Marks]**

(b). A small city of 15,000 people requires an average of 300,000 of gallons of water daily. The city is supplied with water purified at a central water works, where water is purified by filtration, chlorination and addition of two chemicals softening chemical *X* and health chemical *Y*. Manager of water works plan to purchase two popular brands of products , product *A* and product *B*, which contain these two elements. One unit of product *A* gives 8 *kg* of *X* and 3*kg* of *Y*. One unit of product *B* gives 4 *kg* of *X* and 9 *kg* of *Y*. To maintain the water at minimum level of softness and meet a minimum in health protection, it is decided that 150 *kg* and 100 *kg* of two chemicals that make up each product must be added daily. At a cost of *Rs.* 8 and *Rs.* 10 per unit respectively for *A* and *B* , the manager wants to decide the quantity of each product that should be used to meet consumer standard.

(i). Formulate the above problem as a linear programming problem to determine the quantity of each product that should be used to meet the consumer standard.

[20 Marks]

(ii). Solve the above model using the Big-M-Method. **[45 Marks]**

(iii). Solve the problem using the graphical method and compare the solution obtained in part (ii). **[25 Marks]**

Turn Over

4. Give outlines of the Two-Phase Method in Linear Programming. **[20 Marks]**

A diet is to contain at least 20 g of proteins and 15 g of carbohydrate. There are three foods A, B and C available in the market, costing Rs.2, Rs.1 and Rs.3 per unit, respectively. Each unit of A contains 2 g of protein and 4 g of carbohydrate; each unit of B contains 3 g of protein and 2 g carbohydrate; each unit of C contains 4 g of protein and 2 g carbohydrate.

(i). Formulate a Linear Programming model so that the cost per unit diet is minimum.

[20 Marks]

(ii).Using the two-phase method, find the number of units of each food the diet should contain. **[60 Marks]**

5. Obtain the general Linear Programming Problem in matrix form. **[20 Marks]**

A pharmaceutical company has 100 kg of A, 180 kg of B and 120 kg of C ingredients available per month. Company can use these materials to make three basic pharmaceutical products namely p_1, p_2 and p_3. The percentage of weight of A, B and C, respectively in each of the products and the cost of these raw materials are as follows:

Product	A	B	C	Inert ingredients
p_1	5%	10%	5%	80%
p_2	5%	5%	10%	80%
p_3	20%	5%	10%	65%
Cost (Rs.)/ kg	80	20	50	20

Ingredient	Cost (Rs)/ kg
A	80
B	20
C	50
Inert ingredients	20

Selling prices of these three products are Rs. 40.50 , Rs.43 and Rs.45 per kg , respectively. There is a capacity restriction of the company for product p_1 , so that company cannot produce more than 30 kg per month.

(i). Formulate a Linear Programming model so as to maximize the monthly profit.

[30 Marks]

(ii). Use the Revised Simplex method to determine the quantity of each of the products company should produce in order to maximize its monthly profit. **[50 Marks]**

6. State the steps involved in the Dual Simplex Algorithm. **[20 Marks]**

A company manufactures two products X and Y on three machines Turning, Milling and Finishing machines. Each unit of X takes, 10 hours of turning machine capacity , 5 hours of milling machine capacity and 1 hour of finishing machine capacity. One unit of Y takes, 6 hours of turning machine capacity , 10 hours of milling machine capacity and 2 hours of finishing machine capacity. The company has 2500 hours of turning machine capacity ,2000 hours of milling machine capacity and 500 hours of finishing machine capacity in the coming planning period. The profit contribution of products X and Y are Rs.23 per unit and Rs. 32 per unit respectively.

(i). Formulate a Linear programming model so as to maximize the total profit. **[15 Marks]**

(ii).Give the corresponding dual linear problem. **[10 Marks]**

(iii). Solve the dual problem using the Dual Simplex method. **[35 Marks]**

(iv). Obtain the optimal solution to the primal problem using the optimal solution to the dual problem. **[20 Marks]**