RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year – Semester II Examination – October/November 2017

MAA 2203 – NUMERICAL ANALYSIS II

Time : Two(02) hours

Answer all questions
Calculators will be provided

1.

a) Show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
f(x)	1	4	11	16	13	-4

(40 marks)

b) A **natural cubic spline** S on $[0,2]$ is defined by;

$$S(x) = \begin{cases} S_0(x)=1+2x-x^3 & \text{if } 0\leq x\leq 1 \\ S_1(x)=2+b(x-1)+c(x-1)^2+d(x-1)^3 & \text{if } 1\leq x\leq 2 \end{cases}$$

Find b, c and d.

(60 marks)

2.

a) For a function f, the **forward divide difference** are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_3] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.6$	$f[x_2] = 6$		

Determine the missing entries in the table.

(40 marks)

b) A **clamped cubic spline** S of a function f defined on $[1,3]$ by

$$s(x) = \begin{cases} s_0(x)=3(x-1)+2(x-1)^2-(x-1)^3 & \text{if } 1\leq x\leq 2 \\ s_1(x)=a+b(x-2)+c(x-2)^2+d(x-2)^3 & \text{if } 2\leq x\leq 3 \end{cases}$$

If $f'(1) = f'(3)$, find a, b, c and d.

(60 marks)

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3.

a) The Chebyshev polynomials $[T_{(n)}(x)]$ are orthogonal on $[-1,1]$ with respect to the weight function $w(x) = (1-x^2)^{-\frac{1}{2}}$. Consider $T_{(n)}(x) = \cos(ncos^{-1}x)$ and proof $T_{(n+1)}(x) = 2x\, T_{(n)}(x) - T_{(n-1)}(x)$ for $n \geq 1$.

(50 marks)

b) Use the **Midpoint Rule** and the given data to estimate the value of the integral $\int_1^5 \mathrm{f(x)}\ \mathrm{dx}$.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	2.4	2.9	3.3	3.6	3.8	4.0	4.1	3.9	3.5

(50 marks)

4.

a) Determine the general solution for the following difference equations,

i. $\Delta^2 y_n - 3\Delta y_n + 2y_n = 0$ and the solution is unbounded.

(30 marks)

ii. $\Delta^2 y_k + 3\Delta y_k - 4y_k = k^2$ with the initial conditions $y_0 = 2, y_2 = 2$.

(30 marks)

b) Use Simpson's rule with **n = 10** to approximate the integral $\int_0^1 e^{x^2}\ dx$.

(40 marks)

END