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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
Second Year – Semester I Examination – October/ Nov. 2015

MAP 2301 – Algebra

Answer **Five** Questions **only**. Time allowed: **Three Hours**

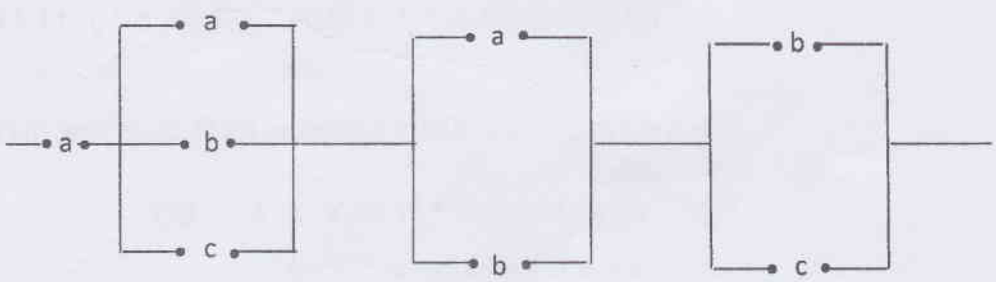
1.

a) 65 elderly men failed a medical test because of defects in at least one of these organs, the heart, lung or kidneys. 29 had heart disease, 28 lung disease and 31 kidney disease. 8 of them had both lung and heart disease. 11 had lung and kidney diseases. While 12 had kidney and heart diseases.

Draw a Venn diagram to show this information.
Find the number of men who

i. Suffer from all three diseases
ii. Suffer from at least two diseases
iii. Suffer from lung disease and exactly one other disease
iv. Suffer from heart disease and lung disease, but not kidney disease

b)



i. Write down a Boolean expression for above circuit.
ii. Simplify the expression and draw the corresponding circuit.

[P.T.O.]

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2.

a) Prove that $[ca, cb] = c[a, b]$ if c is a nonnegative number.

b) State a necessary and sufficient condition for the solubility of the Diophantine equation $ax + by = c$.

c) Solve the equation $247x + 91y = 39$

d) Solve the equation of $6x + 10y + 15z = 5$

3.

a) Show that the congruence $ax \equiv b(mod\ m)$ is soluble if and only if $(a, m)|b$.

b) Solve the congruence $296\ x \equiv 176(mod\ 114)$.

c) State the Chinese Reminder Theorem and Solve the following system of congruence:

$$2x \equiv 3(\mathrm{mod}\, 5)$$
$$4x \equiv 1(\mathrm{mod}\, 7)$$
$$2x \equiv 5(\mathrm{mod}\, 9)$$

4.

a) Define each of the following terms.

i. Reflexive relation.

ii. Symmetric relation.

iii. Transitive relation.

b ~~a~~) A relation $\mathbb{R}$ on $\mathbb{Z}^+ X\ \mathbb{Z}^+$ is defined by $(a, b)\mathcal{R}(c, d)$. If $a^2 + d^2 = c^2 + b^2$ then show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}^+ X\ \mathbb{Z}^+$.

c ~~b~~) Refer to the mapping $f: \mathbb{R}^2 \to \mathbb{R}^2$ and $g: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x^2\ + 1, x + y)$ and $g(x, y) = 2x + y$.
Find the followings:

$$(\mathrm{i}).\ f(1,4)(\mathrm{ii}).\ g\ \circ f(2,3)(\mathrm{iii}).\ g(1,4)(\ \mathrm{iv}).\ f^3(1,4)$$

5.

a) G is a set and * is a binary operation on G. Such that the following conditions are satisfied.

i. $a * (b * c) = (a * b) * c, \forall\ a, b, c \in G$

ii. $\exists$ e $\in$ G such that $e * e = e$

iii. For each $a \in$ G , $\forall$ b $\in$ G such that b * a = e.

iv. For $a, b, c \in$ G, $a * b = e$ and $a * c = e \Rightarrow b = c$
Prove that G is a group.

b) Let $S = \{\begin{pmatrix} x & y \\ x & y \end{pmatrix} \mid x, y \in \mathbb{R} \text{ with } x + y \neq 0\}$

Show that

i. S is associative under matrix multiplication.

ii. S has left identity.

iii. Each element in S has a right inverse.

Is S a group under matrix multiplication?

6. $\propto = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$ $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 6 & 1 & 5 \end{pmatrix}$ and

$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix}$.

Compute $\propto^3$,$(\propto \beta)^{-1}$ and $\gamma^{-1}\beta\alpha$ and find which of these are even. Express α , β , γ as a product of disjoint cycles and then as a product of transpositions. Also find the orders of α , β and γ.

7.

a) Let H be a subgroup of a group G. Show that the following conditions are equivalent:

i. $g\,h\,g^{-1} \in H$, for all g in G and h in H

ii. $g\,H\,g^{-1} = H$, for all g in G

iii. $g\,H = H\,g$, for all g in G

b) Let G be a group and H a non-empty subset of G.

i. State the conditions H must satisfy in order for it to be a subgroup of G.

ii. Prove that H is a subgroup of G if and only if $xy^{-1} \in H$ whenever $x, y \in H$.

iii. Let g be a given element of G, and let $S = \{x \in G \mid xg = gx\}$. Show that S is non-empty and that S is a subgroup of G.