RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year – Semester II Examination– November/December 2016

MAA 1104 – MATHEMATICAL MODELLING

Answer **TWO QUESTIONS** only. Time allowed: **One (1) Hour.**

01. (i) A tank with a capacity of 500 litres originally contains 200 litres of saline - water with 5 kg of salt in the solution. Saline water containing 1 gramme of salt per litre is entering at a rate of 3 lit./min. and the mixture is allowed to flow out at 2 lit./min., both processes starting at time $t = 0$.

a) What is the volume $V(t)$ of the solution in tank at time t, before it overflows? (10 marks)

b) What is the rate of the amount of salt entering the tank? (10 marks)

c) Concentration of salt is decreasing in two ways. Besides the fact that salt is leaving the tank, extra water is entering the tank. (one litre extra per minute, to be exact). Find the rate of the amount of salt leaving the tank? (10 marks)

d) Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. (10 marks)

e) What is the concentration of the solution in the tank just before it overflows? (10 marks)

f) What is the theoretical limiting concentration, if the tank has infinite capacity? (10 marks)

Turn Over

ii.) The number of bacteria cells in a liquid culture is observed to grow at a rate proportional to the number of cells present. A culture starts with 10 000 cells and the number of cells doubles every 40 minutes.

a) Find a function that models the number of bacteria at time t. (10 marks)

b) Find the number of bacteria after one hour. (10 marks)

c) After how many minutes will there be 50 000 bacteria? (10 marks)

d) Sketch a graph of the number of bacteria at time t. (10 marks)

02. i.) You are the manager of *TRENDY-CREATIONS*, a new trend-setting clothing manufacturer. The cost function for the very exclusive T-shirts of the Company is $F(x)=0.02x^2+7.5x+600$ dollars, where x = the number of T- shirts produced. The selling price of this particular T-shirt is $ 20 each.

Find

a) the variable cost and the fixed cost. (10 marks)

b) the revenue function $R(x)$ and the profit function $P(x)$. (10 marks)

c) the minimum number of T- shirts you should make and sell for break-even of profit. (neither profit nor loss) (10 marks)

d) the profit from making and selling 100 T- shirts. (10 marks)

e) the approximate cost of making the 101st T- shirt. (10 marks)

f) the approximate profit on the next T- shirt after selling 100 T- shirts. (10 marks)

g) average cost per T- shirt of 100 T- shirts. (10 marks)

ii.) Let $Y(t)$ be the total national income and $D(t)$ be the total national debt at time t. Domar's second debt model assumes that

- rate at which national debt changes is proportional to national income.
- the rate of increase of national income is proportional to the national income.

a) Express the above two assumptions as differential equations. (10 marks)

b) Solving these differential equations, show that $\lim_{t\to\infty} \frac{D(t)}{Y(t)}$ is a constant.

(20 marks)

03. i.) The general demand and supply functions for the cobweb model, are given as

$Q_t^d = a + bP_t;\ b < 0$

$Q_t^s = c + dP_{t-1};\ d > 0$

According to the cobweb model, price should be adjusted so that the quantity demanded is equal to the quantity supplied.

a) Show that, at the equilibrium point,

$$P_t = \left(\frac{c-a}{b}\right) + \left(\frac{d}{b}\right)P_{t-1}$$ (15 marks)

b) Find the general solution of the above first order difference equation.

(35 marks)

ii.) The respective demand and supply functions for a certain cobweb model are as follows:

$Q_t^d = 400 - 20P_t;$

$Q_t^s = -50 + 10P_{t-1};$ where P_t is the time path price function.

Given further that $P_0 = 12$, find the general solution for P_t and interpret the result graphically.

(50 marks)

.....*End*.....