**RAJARATA UNIVERSITY OF SRI LANKA**
**FACULTY OF APPLIED SCIENCES**

**Bachelor of Sciences Honours in Industrial Mathematics**
**Fourth Year - Semester II Examination – February 2023**

**MAT 4307 – DESIGN OF EXPERIMENTS**

**Time allowed: Three (03) hours**

**Instructions:**

**Answer four (04) questions only.**
**Each question carries equal marks.**
**Statistical tables are provided on request.**
**Calculators are allowed.**

1. Five (05) rocket propellant formulations i.e., A, B, C, D, & E were tested in an experiment for the energy output. The formulations were different from batch to batch, while the operation of the rocket engine was done using different operators. Energy values of each tested propellent formulations are given in the table.

| | Operators | | | | |
|---|---|---|---|---|---|
| Batches of Raw Material | 1 | 2 | 3 | 4 | 5 |
| 1 | A = 24 | B = 20 | C = 19 | D = 24 | E = 24 |
| 2 | B = 17 | C = 24 | D = 30 | E = 27 | A = 36 |
| 3 | C = 18 | D = 38 | E = 26 | A = 27 | B = 21 |
| 4 | D = 26 | E = 31 | A = 26 | B = 23 | C = 22 |
| 5 | E = 22 | A = 30 | B = 20 | C = 29 | D = 31 |

a) Perform an appropriate statistical test to evaluate the rocket propellant formulations (Clearly indicate your Null hypothesis and Alternative hypothesis, use 0.05 $\alpha$ level). **(10 marks)**

b) Assuming formulation B is the standard, make conclusions upon the rest. **(07 marks)**

c) Rank the rocket propellant formulations and state similarities and dissimilarities based on the energy output. **(08 marks)**

2. An experiment on citrus is laid out in a completely randomised design with three treatments and five plots per treatment. The variable of interest was the crop yield ($Y$ kg/plot) and the covariate used was the yield, $X$, from same plots on previous year when no treatments were applied. The observations are given in the following table.

| **Treatment 1** | | **Treatment 2** | | **Treatment 3** | |
|---|---|---|---|---|---|
| $X_1$ | $Y_1$ | $X_2$ | $Y_2$ | $X_3$ | $Y_3$ |
| 29.5 | 16.5 | 24 | 17.2 | 22.4 | 18.3 |
| 30.4 | 20.3 | 26.4 | 21.4 | 29.7 | 25.6 |
| 29.8 | 22.2 | 30.7 | 22.9 | 30.5 | 30 |
| 36 | 29 | 31.1 | 22 | 28.1 | 24 |
| 32.5 | 26 | 23.9 | 12.9 | 27.1 | 24.5 |

Formular for constructing **Sum of Squares** are as follows.

$$Treatment = Tyy - \frac{(Sxy)^2}{Sxx} + \frac{(Exy)^2}{Exx}$$

$$Covariate = \frac{(Exy)^2}{Exx}$$

$$Error = Eyy - \frac{(Exy)^2}{Exx}$$

a) Make your conclusions after conducting an appropriate analysis (use 0.05 α level).

**(15 marks)**

b) Calculate adjusted treatment means. **(05 marks)**

c) Decide the best treatment in this experiment assuming highest yield is the best.

**(05 marks)**

3. Five different breakfast cereals (A to E) were tested for their digestible energy. A randomized complete block design with four replicates were used to test significances. Measured energy values are summarized in the given table.

| | ***A*** | ***B*** | ***C*** | ***D*** | ***E*** |
|---|---|---|---|---|---|
| ***Block 1*** | 164 | 168 | 121 | 75 | 169 |
| ***Block 2*** | 166 | 64 | 96 | 74 | 91 |
| ***Block 3*** | 168 | 160 | 110 | 78 | 145 |
| ***Block 4*** | 159 | 161 | 108 | 72 | 130 |

a) Construct the ANOVA and make your conclusions (hint: higher mean is the best). Clearly indicate your Null hypothesis and Alternative hypothesis. **(08 marks)**

b) Write an orthogonal contrast set for the given experiment. **(02 marks)**

c) Test the given contrasts:

i. A vs BCDE **(05 marks)**

ii. AB vs CDE **(05 marks)**

iii. BE vs CD **(05 marks)**

4. An experiment is expected to conduct with three factors A, B and C, where each factor containing three (03) four (04) and three (03) levels, respectively. Design the experiment considering the following circumstances.

a) Illustrate the layout of one block, if all factors can be assigned to similar and small plots. **(01 mark)**

b) State the statistical model for part (a). **(01 mark)**

c) Give the partial ANOVA table (sources of variation and *df* only) for the design proposed to use in above (a). **(04 marks)**

**d)** Assuming factor A needs large plots, illustrate arrangement of a single block. **(01 mark)**

e) State the statistical model for part (d). **(01 mark)**

f) Give the partial ANOVA table (sources of variation and *df* only) for the design proposed to use in above (d). **(05 marks)**

g) Assuming factor A and B need large plots of equal size, illustrate arrangement of a single block. **(01 mark)**

h) Give the partial ANOVA table (sources of variation and *df* only) for the design proposed to use in (f) above. **(04 marks)**

**i)** Assuming A and B factors need large plots but different sizes, illustrate arrangement of a single block. **(01 mark)**

j) State the statistical model for part (i). **(01 mark)**

k) Give the partial ANOVA table (sources of variation and *df* only) for the design proposed to use in above (i). **(05 marks)**

5. A factorial experiment was carried out in order to determine the response of diabetic patients to three levels of new medicine denoted by P1, P2, & P3 and five level of pasting time (T1, T2, T3, T4, & T5). The serum sugar levels were tested after laying out the experiment on a RCBD design. Observations made were given below.

| **Block** | **P1** | | | | | **P2** | | | | | **P3** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **T1** | **T2** | **T3** | **T4** | **T5** | **T1** | **T2** | **T3** | **T4** | **T5** | **T1** | **T2** | **T3** | **T4** | **T5** |
| **1** | 0.9 | 1.2 | 1.3 | 1.8 | 1.1 | 0.9 | 1.1 | 1.3 | 1.6 | 1.9 | 0.9 | 1.4 | 1.3 | 1.2 | 1.2 |
| **2** | 0.9 | 1.3 | 1.5 | 1.9 | 1.4 | 0.8 | 0.9 | 1.5 | 1.3 | 1.3 | 1.0 | 1.2 | 1.4 | 1.1 | 1.1 |
| **3** | 0.9 | 1.2 | 1.4 | 2.1 | 1.2 | 0.8 | 0.9 | 1.1 | 1.1 | 1.5 | 0.7 | 1.0 | 1.4 | 1.3 | 1.4 |

a) Construct the ANOVA and make your conclusions (Clearly indicate your Null hypothesis and Alternative hypothesis). **(15 marks)**

b) Assuming the lowest sugar level is the best, conclude using an appropriate mean separation method. **(10 marks)**

6. An experiment is expected to study performances of five (05) studying habits of students to their marks in the examination. Student groups from five (05) income categories and five (05) age groups will be selected from one university.

a) What is the most appropriate experimental design for this study? **(01 mark)**

b) Give the layout of the design and the statistical model. **(02 marks)**

c) Give the breakdown of the ANOVA (sources of variability and *df* only). **(05 marks)**

d) If the above study is repeated in six (06) universities of Sri Lanka (at the same time), what is the most appropriate experimental design? **(01 mark)**

e) Give the statistical model for experimental design of part (d). **(01 mark)**

f) Give the breakdown of the ANOVA (sources of variability and *df* only) for the design specified in part (d). **(05 marks)**

g) If the income categories will be different at each university, what is the most appropriate experimental design? **(01 mark)**

h) Give the statistical model for experimental design of part (g). **(01 mark)**

i) Give the breakdown of the ANOVA (sources of variability and *df*) for the design specified in part (g). **(05 marks)**

j) State differences of the ANOVA constructed for part (c), (f) and (i). **(03 marks)**