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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (General) Degree in Applied Sciences
First Year - Semester I Examination – September/October 2019
MAA1201 – MATHEMATICAL METHODS I

Time allowed: Two (02) hours

- Answer any Four (04) questions only
- This paper contains **FIVE** questions from **Page 1** to **Page 4**.
- This is a closed book examination.
- This examination accounts for 60% of the module assessment. A total maximum mark obtainable is 100. The marks assigned for each question and parts thereof are indicated in square brackets.

1.

a) Let $OACB$ be a rectangular and let E and F be two points lie on AC and BC, respectively, such that $AE:EC = 1:2$ and $BF:FC = 3:1$. The position vectors of points A and B with respect to point O are $\lambda\mathbf{a}$ ($\lambda > 0$, $\mathbf{a} \neq \mathbf{0}$) and $\mathbf{b}$ ($\neq \mathbf{0}$), respectively.

i. Find $\overrightarrow{OE}$ and $\overrightarrow{OF}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ. **(10 marks)**

ii. If OF is perpendicular to FE, show that

$$\lambda = \frac{4\sqrt{2}|\mathbf{b}|}{3|\mathbf{a}|}.$$ **(20 marks)**

b) Let $\mathbf{u}$ ($\neq \mathbf{0}$) and $\mathbf{v}$ be perpendicular vectors and let $\mathbf{w}$ be the vector defined by

$$\mathbf{w} = \frac{1}{|\mathbf{u}|^2}(\mathbf{u} \times \mathbf{v}) + \lambda\mathbf{u},$$

where λ is a scalar parameter. Show that $\mathbf{w} \times \mathbf{u} = \mathbf{v}$. **(20 marks)**

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c) Determine a vector $\mathbf{x}$ that satisfies the vector equation

$$\mathbf{x} \times \mathbf{b} = \mathbf{c} \times \mathbf{b},$$

provided that $\mathbf{x} \cdot \mathbf{a} = 0$, where $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are fixed vectors and where $\mathbf{a}$ and $\mathbf{b}$ are non-perpendicular vectors. **(25 marks)**

d) Let $\mathbf{p} = 2\mathbf{i} - \mathbf{j}, \mathbf{q} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r} = 7\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.
Show that the set $\{\mathbf{p}, \mathbf{q}, \mathbf{r}\}$ is linearly independent. **(25 marks)**

2.

a) Let A, B and C be three points in space and let their position vectors, in that order, with respect to a fixed point O in space, be $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, respectively.
Prove that A, B and C are collinear if and only if there exist three scalars c_1, c_2 and c_2 not all zero such that $c_1\mathbf{a} + c_2\mathbf{b} + c_3\mathbf{c} = \mathbf{0}$ and $c_1 + c_2 + c_3 = 0$. **(25 marks)**

Let $\overrightarrow{OA} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 7\mathbf{i} - \mathbf{k}$.
Determine if or not A, B and C are collinear. **(15 marks)**

b) A line L passes through the point P with position vector, $\overrightarrow{OP} = 6\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
If L is in the direction of $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, write down its equation in vector form. **(10 marks)**

The Cartesian equations of a line L' are:

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}.$$

Find each of the following:

i. The equation of L' in vector form, **(15 marks)**
ii. The point of intersection of L and L', **(20 marks)**
iii. The angle between L and L'. **(15 marks)**

3.

a) Show that the vector equation of the plane which is at a distance d from the origin O and which is perpendicular to the unit vector $\mathbf{n}$, being directed away from O, is

$$\mathbf{r}.\mathbf{n} = d.$$ **(10 marks)**

Let Π_1 be the plane on which the points $A(5,0,0), B(0,1,0), C(0,0,5)$ are.
Find the equation of Π_1 in the form

$$\mathbf{r} \cdot \mathbf{n} = d,$$

where $\mathbf{r}, \mathbf{n}$ and d are to be determined. **(30 marks)**

Contd.

Let L be the line whose vector equation is

$$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k}),$$

where λ is a scalar parameter.

i. Show that L is parallel to Π_1. **(10 marks)**

ii. Find the equation of the plane Π_2 which is parallel to Π_1 and which contains L. **(10 mark)**

iii. What is the distance of Π_2 from the origin? **(10 marks)**

iv. Deduce that the distance between Π_1 and Π_2 is $\frac{10}{\sqrt{27}}$. **(10 marks)**

b) Find the vector equation of the line of intersection of the following two planes:

$$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i} - \mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k}),$$
$$\mathbf{r}_2 = 3\mathbf{i} - \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where λ, μ, s and t are scalar parameters. **(20 marks)**

4.

a) Let $\mathbf{a} = t\mathbf{c} + \frac{\mathbf{d}}{t}$ and $\mathbf{b} = \frac{\mathbf{c}}{t} + t\mathbf{d}$ be vector functions of t, where $\mathbf{c}$ and $\mathbf{d}$ are constant vectors and where $t \neq 0$. Find $\frac{d}{dt}(\mathbf{a} \times \mathbf{b})$ and $\frac{d}{dt}(\mathbf{a} \,.\, \mathbf{b})$. **(25 marks)**

b) A space curve $\mathbf{r} = \mathbf{r}(t)$ is defined by the vector equation

$$\mathbf{r} = 8\cos^3 t\mathbf{i} + 8\sin^3 t\mathbf{j} + 3\cos 2t\mathbf{k}, \qquad 0 \leq t \leq 2\pi.$$

Find each of the following at t:

i. The principal unit normal vector, the unit tangent vector and the unit binormal vector, **(40 marks)**

ii. The curvature and the radius of curvature, **(20 marks)**

iii. The torsion of the curve. **(15 marks)**

5.

a) The temperature $T = T(x, y, z)$, at point (x, y, z), in a rectangular box is given by

$$T(x, y, z) = xyz(1 - x)(2 - y)(3 - z),$$

where $0 \leq x \leq 1, 0 \leq y \leq 2,\ 0 \leq z \leq 3$.

i. Find the gradient of T at a point (x, y, z) in the box. **(25marks)**

ii. Find the directional derivative of T at $(1/2, 1, 1)$ in the direction of $\mathbf{i} + \mathbf{j}$. **(25 marks)**

iii. A mosquito is located at $(1/2, 1, 1)$.
In which direction should it fly to cool off as rapidly as possible?
Justify your response. **(10 marks)**

Contd.

b) If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, find

$$\operatorname{div}\left\{\frac{\mathbf{r}}{r^2} + \operatorname{curl}\left(\frac{\mathbf{r}}{r^2}\right)\right\},$$

where $r = |\mathbf{r}| > 0$. **(40 marks)**

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