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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester II Examination – July 2020

MAT 3217 – NONLINEAR PROGRAMMING

Time: Two (02) hours

Answer only **four** questions.
Calculators will be provided.

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1. a) Briefly explain the initialization, iterative steps and the stopping rule of the Gradient search algorithm. **(20 marks)**

   b) Solve the following unconstraint problem, using the above algorithm:

   $$\text{Maximize } f(x,y) = 2xy + 2y - 3x^2 - 2y^2.$$

   Take error tolerance as $\varepsilon = 0.34$ and initial trial solution as $X_0 = (0,0)$. **(40 marks)**

   c) Use the Fibonacci search algorithm to approximate the minimum value of the following function:

   $$f(x) = \begin{cases} -2x^4 + 3x^2 - x + 2 & ; x \leq 0 \\ x^2 + 2x + 2 & ; x > 0. \end{cases}$$

   Take the initial upper bound $(\bar{x})$, initial lower bound $(\underline{x})$ and the error tolerance (ε) as 1, -1 and 0.26 respectively. **(40 marks)**

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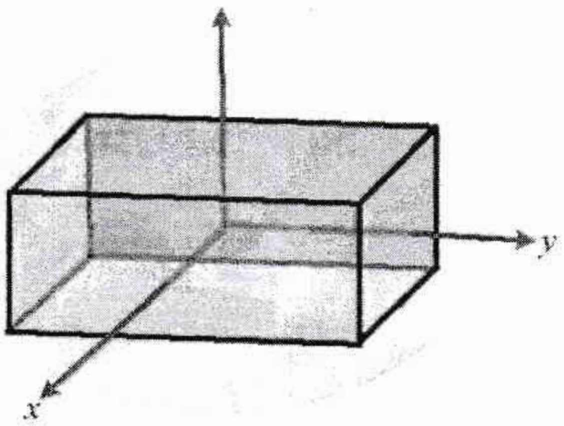
2. a) Explaining each step clearly, obtain the Lagrange multiplier equation $\nabla f = \lambda \nabla g$, where λ ($\neq 0$)is a Lagrange multiplier, to find the extreme value/s of $f(x, y)$, subject to the constraint $g(x, y) = c$, where c is a real constant. **(20 marks)**

   b) Explian the steps of the Lagrange Multiplier Method to find the extreme value/s of the function $f(x, y)$, subject to the m number of constraints: $g_i(x, y) = c_i$; $\forall\, i = 1, 2, .., m$. **(20 marks)**

   c) Use the method of Lagrange multipliers to prove that the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{x^2}{b^2} + \frac{x^2}{c^2} = 1$ is $\frac{8abc}{3\sqrt{3}}$. (*Hint: See the figure below*) **(60 marks)**



3. a) Define the Quadratic Programming Problem (QPP) in optimization theory and state the matrix form of the general mathematical model of QPP using standard notations. **(15 marks)**

   b) Consider the following QPP:

$$\text{Maximize } Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

$$S.t. \quad x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

   i. Rewrite the above problem in matrix form.

   ii. Solve the above QPP using the Wolfe's Modified Simplex Method.

(85 marks)

4. a) Define the term posynomial.

(10 marks)

b) Geometric programming deals with the problems of following types of objective functions:

Minimize
$$Z = \sum_{j=1}^{N} U_j,$$
where, $U_j = C_j \prod_{i=1}^{n} x_i^{a_{ij}}$; $j = 1, 2, .., N$.
$C_j > 0; j = 1, 2, .., N$.

Let Z^* be the optimal value of the objective function, $\delta_i = \frac{U_i}{Z^*}$, and

$$\prod_{j=1}^{N} \left(\prod_{i=1}^{n} (x_i^*)^{a_{ij}} \right)^{\delta_j^*} = 1.$$

Show, by calculus, that $Z^* = \prod_{i=1}^{n} \left(\frac{U_i}{\delta_i} \right)^{\delta_i}$.

(40 marks)

c) Solve the following Geometric programming problem:

$$\text{Minimize } Z = \frac{40}{x_1 x_2 x_3} + 40x_2x_3 + 20x_1x_3 + 10x_1x_2$$

subject to $x_1, x_2, x_3 > 0$.

(50 marks)

5. a) Let $g(x, y) = ax^2 + bxy + cy^2 + dx + ey + f$, where $a, b, c, d, e,$ and f are constants, be the general polynomial of two variables x and y, in degree two. Use principal and leading principal minors to determine, for which values of the constants this function is convex, concave, strictly convex, and strictly concave.

(20 marks)

b) A company produces two types of products. The total profit achieved from these products is described by the following equation:

$$\text{Total profit} = -0.2x_1^2 - 0.4x_2^2 + 8x_1 + 12x_2 + 1500$$

where x_1= thousands of units of the product I

x_2= thousands of units of the product II

Every 1000 units of the product I require one hour, and every 1000 units of product II require 30 minutes in the shipping department. Every thousand units of both products require 2000 kg of a special ingredient, of which 64000 kg are available. Additionally, 80 labour hours are available in the shipping department.

i. Formulate a non-linear programming model to maximize the total profit.

ii. Solve the developed model using Karush-Kuhn-Tucker (KKT) conditions, and find the maximum total profit that can be achieved.

(80 marks)

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