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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (four-year) Degree in Industrial Mathematics

Fourth Year - Semester I Examination – October/November 2017

MAT 4301 – Operational Research I

Time: Three (3) hours

Answer **All** Questions

Calculators will be provided

1. (a) For the following payoff table, determine the optimal strategy for each player by successively eliminating dominated strategies (Indicate the order in which you eliminated strategies):

Strategy		Player 2		
		1	*2*	*3*
Player 1	*1*	1	2	4
	2	1	0	5
	3	0	1	-1

(b) Using dominance property, reduce the following payoff table to two rows and hence, determine the optimal strategy for each player by applying the graphical method:

Strategy		Player 2		
		1	*2*	*3*
Player 1	*1*	0	-2	2
	2	5	4	-3
	3	2	3	-4

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2. Consider the game having the following payoff table:

Strategy		**Player 2**			
		1	*2*	*3*	*4*
Player 1	*1*	5	0	3	1
	2	2	4	3	2
	3	3	2	0	4

(i) Formulate the problem of finding optimal mixed strategies according to the minimax criterion as a linear programming problem.

(ii) Use the simplex method to find these optimal mixed strategies.

3. Briefly explain the method in stepwise form to find the optimal sequence of processing n jobs in m machines.

A company has to process five jobs on three machines A, B, and C. Processing times are given in the following table:

Job	***Machine***		
	A	***B***	***C***
1	4	4	6
2	9	5	9
3	8	3	11
4	6	2	8
5	3	6	7

(i) Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

(ii) Construct a table showing time in and time out of each machine and also, idle time of each activity.

(iii) Find the total minimum elapsed time if no passing of jobs is permitted.

(iv) Construct a Gannt chart showing the sequence of processing the jobs.

4. Consider the following queuing model:

Poisson arrival, Poisson departure, Single server, Infinite capacity and First come first served discipline.

With the usual notation prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ when the system is in steady state.

Also, prove that $P_n = \rho^n(1-\rho)$.

Hence, show the following:

Expected number of customers in the system $= \dfrac{\lambda}{\mu-\lambda}$

Expected waiting time of a customer in the queue $= \dfrac{\lambda}{\mu(\mu-\lambda)}$

The mechanic at a shop is able to install new mufflers (a devise in a vehicle engine) at an average of 3 per hour according to a Poisson distribution. Customers seeking this service arrive at the shop on the average of 2 per hour, following a Poisson distribution. They are served on a first-in, first-out basis and come from very large (almost infinite) population of possible buyers.

Find the following:

(i) Average number of customers in the queue an in the system.

(ii) Average waiting time in the queue and in the system.

(iii) Probability that there are no customers in the system.

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5. Sonali was promoted to Vice President for Customer Services at a reputed bank. One of her responsibilities is to manage how tellers provide services to customers, so she is taking a hard look at this area of the bank's operations. Customers needing teller service arrive randomly at a mean rate of 30 per hour. Customers wait in a single line and are served by the next available teller when they reach the front of the line. Each service takes a variable amount of time (assume an exponential distribution), but on average can be completed in 3 minutes.

(i) If two tellers are used, what will be the average waiting time for a customer before reaching a teller?

(ii) On average, how many customers will be in the bank, including those currently being served?

(iii) Company policy is to have no more than a 10% chance that a customer will need to wait more than 5 minutes before reaching a teller. How many tellers need to be used in order to meet this standard?