RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

First Year - Semester II Examination – Jan/Feb 2023

MAA 1203 - NUMERICAL ANALYSIS I

Time: Two (2) hours.

Answer all four (4) questions.

Calculators will be provided.

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1. a) Distinguish the difference between a floating-point number system and the real number system.

   Suppose that $a = \frac{5}{7}$ and $b = \frac{1}{3}$.

   Use five-digit chopping for calculating,

   i. $a + b$,

   ii. $\frac{a}{b}$,

   iii. $a \times b$.

   Calculate the absolute and relative errors in each case above.

(50 marks)

b) Let $x_0, x_1, \dots, x_n$ be n points such that $x_{k+1} - x_k = h$, where h is a constant and $y_k = y(x_k)$ for all $k = 0, 1, \dots, n$. The forward difference operator Δ is defined by

$$\Delta y_k = y_{k+1} - y_k.$$

   i. Find $\Delta^r y_0$ for $r = 2, 3, 4$.

(30 marks)

Contd.

ii. Compute $\Delta^4 y_k$ for the discrete function, $y_k = y(x_k)$, defined in the following table:

x_k	0.1	0.2	0.3	0.4	0.5
y_k	1.40	1.6	1.76	2.00	2.28

(20 marks)

2. a) The equation $x^3 - 5x + 1 = 0$ has a root between 0 and 1.

i. Express x in the form $ax^3 + b$, where a and b should be determined.

ii. Hence, determine an iteration function $\phi(x)$ such that $x_{n+1} = \phi(x_n)$, $n = 0, 1, 2, \ldots$.

iii. Show that the sequence of iterations, (x_n), obtained from the preceding iteration method converges to the root. Starting with $x_0 = 0.5$, perform three iterations of this iteration method to get an approximation for the root.

(40 marks)

b) Let $f(x) = x^3 + x^2 - 3x - 3$ be a function of a real variable x.

i. Show that the equation $f(x) = 0$ has a root between 1 and 2.

ii. Using the method of Regula Falsi for three iterations, determine an approximation for the root.

iii. If the exact root is 1.7321, compute the absolute and relative errors.

(60 marks)

3. a) Differentiate between interpolation and extrapolation.

(10 marks)

b) Write down two advantages of the Lagrange interpolation over direct interpolation.

(10 marks)

c) The following table represents the data for $f(x) = e^x$:

x	0.1	0.2	0.5	0.9
$f(x)$	1.105171	1.221403	1.648721	2.459603

Contd.

i. Using Lagrange interpolation, approximate $f(0.35)$.

(55 marks)

ii. Determine the maximum absolute error at $x = 0.35$, and compare it with the actual error, provided $f(0.35) = 1.419067$.

(25 marks)

4. Consider the initial-value problem:

$$\frac{dy}{dx} = \sqrt{x+y}, \qquad y(0) = 1.$$

a) Write down the numerical schemes of the Euler's and fourth order Runge-Kutta methods for this differential equation. **(20 marks)**

b) Use Euler's method with step size $h = 0.1$ to obtain an approximation for $y(0.2)$.

(40 marks)

c) Use the fourth order Runge-Kutta method with step size $h = 0.1$ to obtain an approximate for $y(0.2)$.

(40 marks)

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