RAJARATA UNIVERSITY OF SRILANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

Second Year Semester I Examination - October 2014

MAA 2302 – Probability and statistics II

Answer **Five** Questions Only   Time allowed: **Three Hours**

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1. Identify the choice that best completes the statement or answers the question.

   i. The standard deviation of a standard normal distribution

      a. is always equal to zero
      b. is always equal to one
      c. can be any positive value
      d. can be any value

   ii. X is a normally distributed random variable with a mean of 8 and a standard deviation of 4. The probability that X is between 1.48 and 15.56 is

      a. 0.0222
      b. 0.4190
      c. 0.5222
      d. 0.9190

   iii. A simple random sample of 100 observations was taken from a large population. The sample mean and the standard deviation were determined to be 80 and 12 respectively. The standard error of the mean is

      a. 1.20
      b. 0.12
      c. 8.00
      d. 0.80

   iv. The probability distribution of all possible values of the sample proportion $\bar{p}$ is the

      a. probability density function of $\bar{p}$
      b. sampling distribution of $\bar{x}$
      c. same as $\bar{p}$, since it considers all possible values of the sample proportion
      d. sampling distribution of $\bar{p}$

   v. The sample statistic s is the point estimator of

      a. m
      b. s
      c. $\bar{x}$
      d. $\bar{p}$

2.

a) Define the following terms for two random variables X and Y.

(i) Covariance

(ii) Correlation coefficient

b) Show that, in the usual notation,

$$Cov(X,Y) = E(XY) - E(X)(Y)$$

c) Let X and Y be continuous random variables with joint probability density function,

$$f(x,y) = \begin{cases} kx(x+y) & ;0 < x < 1, 0 < y < 2 \\ 0 & ;otherwise \end{cases}$$

(i) Determine the value of k.

(ii) Find the marginal density functions of X and Y.

(iii) Find $Cov(X,Y)$ and conditional probability function of X given $Y = y$.

d) Let $Z = aX + bY$. Determine the variance of Z in terms of σ_X, σ_Y and σ_{XY}.

3.

a) Let X is a discrete random variable whose probability function is $f(x)$. Suppose that a discrete random variable U is defined in terms of X by $U = \phi(X)$, where to each value of X there corresponding one and only one value of U and conversely, so that $X = \psi(U)$. Prove that the probability function for U is given by $g(u) = f[\psi(U)]$.

b) Let Y be a random variable with density function

$$f_Y(y) = \begin{cases} 2(1-y) & ;0 < y < 1 \\ 0 & ;elsewhere \end{cases}$$

Use the method of transformations to find the density functions of

(i) $U = 2Y - 1$

(ii) $V = Y^2$

c) Let X be a standard normal variable. Find the cumulative distribution function of $Y = X^2$. Hence find the density function of $Y = X^2$.