

RAJARATA UNIVERSITY OF SRI LANKA FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Information and Communication Technology First Year - Semester I Examination – June/July 2018

FDN 1306 - BASIC MATHEMATICS FOR NON-MATHEMATICS STUDENTS

Time: Three (03) hours

Answer ALL Questions

1. a) State the laws of logarithms.

(05 marks)

b) Show that,
$$\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$$
. [Hint: $\frac{1}{\log_b a} = \log_a b$]

(20 marks)

c) Solve each of the following equations for x:

(i)
$$\frac{(3 + \log_7 x)}{(2 - \log_7 x)} = 4$$
 where $x > 0$ (ii) $\log_2(x + 2) + \log_2(3) = \log_2(27)$

(40 marks)

- d) Formulate each of the following problems as an algebraic equation and hence solve:
 - (i) Nuwan has a cord that is 198 cm long. He wishes to cut the cord into two pieces so that one piece will be 26 cm longer than the other. How long should the shorter piece be? (15 marks)
 - (ii) A van can be rented from a company X for Rs. 11,000/- per day plus Rs. 100/- per kilometer. Company Y charges Rs. 5,000/- per day plus Rs.400/- per kilometer to rent the same van. Find the number of kilometers the van should be driven in a day at which the rental costs for both companies are the same.

(20 marks)

[Turn over]

2. a) If $A = \begin{bmatrix} 5 & -3 \\ -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 10 & 7 \\ -2 & -6 \end{bmatrix}$, then find the following:

(i)
$$B(A+C)$$
. (10 marks)

(ii)
$$(AB) + (AC) + (BC)$$
. (20 marks)

(iii)
$$A^{-1}$$
. (10 marks)

b) If
$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1} A^{-1}$.

(10 marks)

c) X, Y and Z are iron balls with different weights. The weights of three bags with different numbers of iron balls are shown below (assume that weight of an empty bag is zero). Formulate a system of linear equations and use Cramer's Rule to solve the system. Hence, find the weight of a bag which has two of X, three of Y and two of Z iron balls:

bag
$$01$$
 – one of X and four of Y = 16 Kg

bag
$$02$$
 – one of **X**, two of **Y** and one of **Z** = 30 Kg

bag
$$03$$
 – two of Y and one of Z = 18 Kg

(50 marks)

3. a) Write in (a + b i) form, where a and b are real numbers: $\frac{(2-5i)}{(1-3i)}$.

(15 marks)

b) If $f(x) = 2x^2 + 3x + 5$, find the corresponding values of f(x) for the domain $\{-2, -1, 0, 1, 2\}$.

(10 marks)

c) Solve each of the following system of linear equations:

$$(i) 3x - y = 7$$

$$2x + 3y = 1 \tag{15 marks}$$

(ii) 5x + 4y = 19

$$3x - 6y = 3 \tag{15 marks}$$

d) Find the roots of each of the following quadratic equations:

(i)
$$5x^2 + 3x = 0$$
. (10 marks)

(ii)
$$x^2 - 5x = -6$$
. (15 marks)

(iii)
$$3x^2 - 5x + 1 = 0$$
. (20 marks)

4. a) Find the following limits:

(i)
$$\lim_{x \to -1} (x^5 + 3x^3 - 2x^2 + 12)$$
. (10 marks)

(ii)
$$\lim_{x \to 4} \frac{(x-4)}{(x^2-16)}$$
. (15 marks)

(iii)
$$\lim_{x \to \infty} \frac{(8x^3 + 5x^2 - 2x + 5)}{(2x^3 - 9x^2 - 4x + 3)}$$
 (20 marks)

b) Check whether the following function is continuous at x = 1:

$$f(x) = \begin{cases} (5 - 2x), & \text{if } x > 1\\ 5, & \text{if } x \le 1 \end{cases}$$
 (10 marks)

c) Find the first derivative of each following functions with respect to x:

(i)
$$f(x) = (5x^2 + 3x + 5)$$
. (10 marks)

(ii)
$$h(x) = \left(2x^{\left(\frac{1}{2}\right)} + \frac{5}{x} + c\right)$$
, where c is a constant. (15 marks)

(iii)
$$l(x) = \left(\frac{x+1}{x}\right)(3x^2 + x\sin x). \tag{20 marks}$$

5. a) Prove each of the following identities:

(i)
$$(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2.$$
 (10 marks)

(ii)
$$\frac{1+\sin y}{\cos y} = \frac{\cos y}{1-\sin y}.$$
 (20 marks)

(iii)
$$\left(\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}\right) = 2 \csc x.$$
 (20 marks)

b) SriLankan Airlines requires that the total outside dimensions (length + width + height) of a checked in baggage should not exceed 72 inches. Suppose you want to check a baggage whose height is equal to its width. Using calculus, find the dimensions of the baggage with the largest volume that can be checked in subject to the given condition.

(50 marks)

END