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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

Third Year-Semester I Examination-September/October 2014
MAT 3213-Graph Theory

Answer **four Questions including Qu.No.1** **Time allowed: 2 hours**

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1. For question i to xv , Find the most suitable answer that you think correct.

i. A Hamiltonian cycle in a Hamiltonian graph of order 24 has

a) 12 edges b) 24 edges c) 23 edges d) none of the above

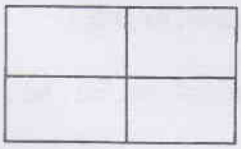
ii. A simple graph G with 13cvertices has 4 vertices of degree 3, 3 vertices of degree 4 and 6 vertices of degree 1.The graph G must be a tree:

a) True b) False

iii. A spanning tree for a simple graph of order 24 has

a) 12 edges b) 6 edges c) 23 edges d) none of the above

iv. The graph given below is bipartite.



a) True b) false

v. If G is a simple connected 3-regular planar graph where every face is bounded by exactly 3 edges, then the size of G is

a) 3 b) 4 c) 5 d) none of the above

vi. Consider a simple connected graph G with n vertices and n edges (n>2). Then which of the following statements are TRUE ?

a) G has atleast one cycle
b) G has no cycles
c) The graph obtained by removing any edge from G is not connected.
d) G has atleast one cycle and The graph obtained by removing any edge from G is not connected.

vii. The number of distinct simple graphs with up to three nodes is.....

a) 9 b) 7 c) 10 d) 15

viii. Consider the graph G where V(G)={A,B,C,D} and E(G)=[{A,B},{B,C},{C,D}]. The degree of each vertices A,B,C,D respectively in G are.....

a) 1,2,3,2 b) 1,3,2,2 c) 1,1,1,1 d) 1,2,2,3

ix. Let G be a graph with 100 vertices numbered 1 to 100. Two vertices i and j are adjacent if |i-j|=8 or |i-j|=12. The number of connected components in G is.....

a) 8 b)12 c) 25 d) 4

x. A graph in which all nodes are of equal degree is known as.....

a) complete graph b) multi graph c) regular graph d) non regular graph

xi. The minimum number of spanning trees in a connected graph with "n" nodes is.....

a) n-1 b) n/2 c) 2 d) 1

xii. The minimum number of edges in a connected cyclic graph on 'n' vertices is.....

a) n-1 b) n c) n+1 d) none of these

xiii. The order of the 4-cube graph Q_4 is :

a) 16 b) 4 c) 8 d) 32

xiv. H= (V , E) is a graph, where V = {a, b, c ,d ,e ,f} and E = {ab , ad, ac, bc, be , cd, cf, de , df}. The edge set for the complement of H is :

a) {ab , be , de , be, cf , cd} b) {af ,ad , ac , bc , be , cd ,cf, ae , df }

c){ af , fb , bd , dc , ce , ea) d) {ac, fb , ba , dc , fe , ea}

xv. If G is a connected plane graph of oder v , size e and with f faces, then

a) $v - e + f = 2$ b) $e - v + f = 2$ c) $e + v - f = 2$ d) none of the above

2.

a. Let G be a graph with V (G) = {1, 2, · · · , n}, where n ≥ 5, such that two numbers i and j in V (G) are adjacent if and only if |i − j| = 5. How many components does G have?

b. The graph G be a disconnected graph with n vertices where n is even. If G has 2 components each of which is complete, prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.

c. Find the incidence matrix of the graph G where Adjacency matrix of a graph G is given below.

$$\begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

d. What is the largest number of vertices in a graph with;

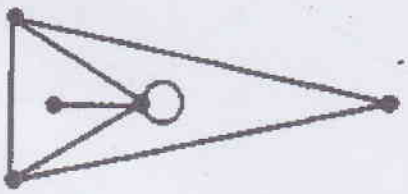
i. 35 edges if all vertices are of degree at least 3

ii. 24 edges and all vertices of the same degree

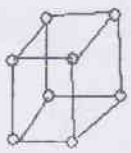
3.

i. Let G = (V;E) and H = (W;B) be two graphs. Prove that G and H are isomorphic if and only if, G^c and H^c are isomorphic.

ii. Define isomorphic graph. Draw three isomorphic graph of the following graph.

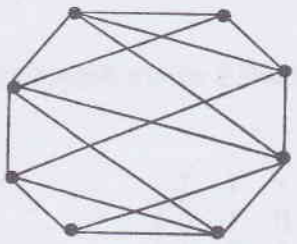


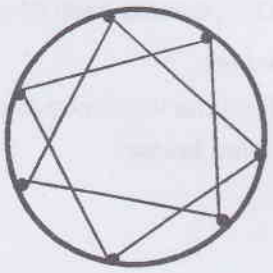
iii. Determine whether the following two graphs are isomorphic. Justify your answer.



4.

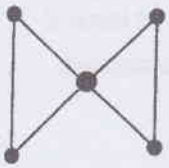
i. State Euler's formula for the planar graph.
ii. Using above formula show that k_5 and $k_{3,3}$ are non-planar.
iii. Determine which of the following graphs are planar.if so, find the number of regions of each planar graph.



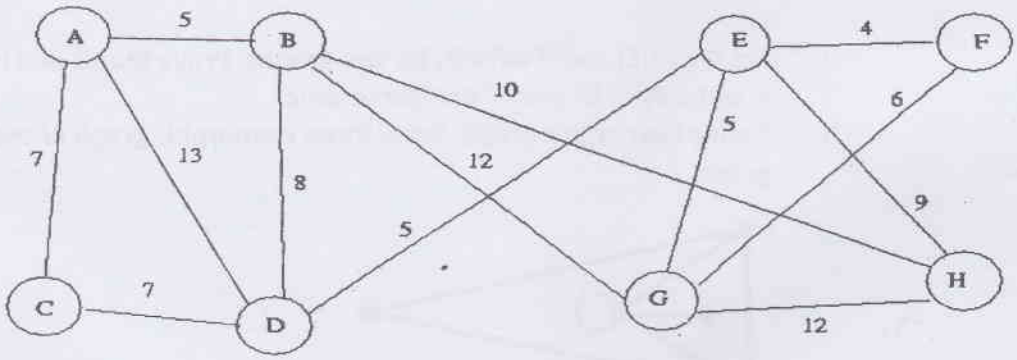


5.

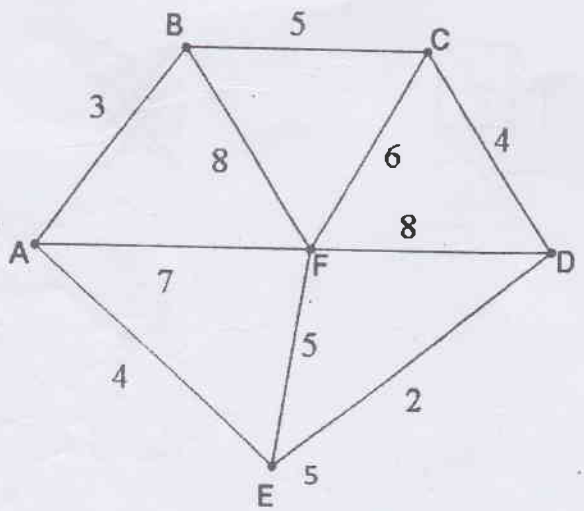
i. Draw all the spanning trees of this graph:



ii. Solve the following by applying Kruskal's Algorithm.



iii. Solve the following by applying **Prim's algorithm**.



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