RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. General Degree in Applied Sciences
First Year - Semester I Examination – June/July 2018
MAP 1301 – LINEAR ALGEBRA

Time: Three (03) hours

Answer All (06) questions

1. *a*) Let V be the set of all ordered pairs (a,b) of real numbers. Determine whether the set V is a vector space under the following operations:

$$(a_1,b_1)\oplus(a_2,b_2)=\left(a_1^{\,2}+a_2^{\,2},b_1^{\,2}+b_2^{\,2}\right)$$
$$\alpha\otimes(a,b)=\left(\alpha^2 a,\alpha^2 b\right), \text{where } \alpha\in R$$

b) Prove that W is a subspace of a vector space V over the field F if and only if for all

$\alpha,\beta\in F$ and $x,y\in W, \alpha x+\beta y\in W$.

c) Which of the following sets are subspaces of R^3? Justify your answer:

i. $\{(x,y,z)\in R^3 : x-y=z\}$
ii. $\{(x,y,z)\in R^3 : x+y^2=0\}$.

2. *a*) Let $S=\{v_1,v_2,v_3,\ldots,v_k\}$ be a subset of a vector space V over a field F.
Explain the following briefly:
i. V is spanned by S.
ii. S is linearly independent.
iii. S is a basis for V.
iv. V is finite dimensional.

b) In the vector space of polynomials of degree less than or equal to 3, P_3, determine whether the set S is linearly independent or linear dependent.

$$S=\left\{2+x-3x^2-8x^3, 1+x+x^2+5x^3, 3-4x^2-7x^3\right\}$$

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c) Show that the set S forms a basis for the vector space, $M_{2\times 2}(R)$,

where $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \right\}$.

d) Find the dimension of the subspace W of R^4,

where $W = \left\{ \begin{bmatrix} a+b \\ a+c \\ a+d \\ d \end{bmatrix} a,b,c,d \in R \right\}$.

3. a)Find the inverse of the following matrix.

$$\begin{bmatrix} 2 & 1 & 2 \\ 3 & -1 & 4 \\ 4 & 3 & 6 \end{bmatrix}$$

b) Solve the following system of linear equations using row reduced form.

$$2x + y + 2z = 3$$
$$3x - y + 4z = 7$$
$$4x + 3y + 6z = 5$$

4. a) Define a linear transformation.

Which of the following mappings $T : R^2 \to R^2$ are linear transformations?

i. $T((a,b)) = (b, 3a - 2b + 1)$.

ii. $T((a,b)) = (a,b) + (1,1)$.

b) Define a Kernal and Image of a linear transformation.

Let $T : V \to W$ be a linear transformation, where V and W are vector spaces over the field F.

Show that, the Kernal of T is a sub space of V and the image of T is a sub space of W.

c) Define an isomorphism of a linear transformation.

Let $T : C \to R^2$ be defined by $T(a+ib) = (a,b)$. Show that T is an isomorphism.

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5. a) State the Rank-Nullity theorem.

b) Define an inner product space.

Show that $\langle u,v\rangle = x_1y_1 + x_2y_2 - x_3y_3$ is not an inner product on R^3, where $u=(x_1,x_2,x_3),\ v=(y_1,y_2,y_3)\,where\,u,v \in R^3$.

c) Prove that $\langle x_1+x_2, y_1+y_2\rangle = \langle x_1,y_1\rangle + \langle x_1,y_2\rangle + \langle x_2,y_1\rangle + \langle x_2,y_2\rangle$ on any inner product space.

d) Let $p(x)$ and $q(x)$ be arbitrary polynomials. Inner product $\langle p,q\rangle$ is defined by

$\langle p,q\rangle = \int_0^1 p(x)q(x)dx$. Find $\langle p,q\rangle$ where $p(x)= 2x^2-1$ and $q(x)=x^2+x-1$.

6. *a*) Define an eigenvalue of amatrix of order n.

Let $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ be 3×3 matrix. Find eigenvalues and eigenvectors of A .

b) Solve the following system of linear equations using Cramer's rule:

$$\begin{aligned} 2x + z &= 5 \\ -x+2y+3z &= 3 \\ x+2z &= 4. \end{aligned}$$

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