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RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

Second Year Semester II Examination – Jan/Feb 2023

MAP 2204 - COMPLEX CALCULUS

Time: Two (02) hours.

Answer all (4) questions.

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1. a) i. By using the triangular inequality, show that if $|z|=4$, then $\left|3z^2+5z+3\right| \leq 71$.

   ii. Write $|z| = 1 - \sqrt{3}\, i$ in Polar form.

   iii. Identify the region given by $E = \{z \in C \colon |z - 5i| \leq 5\}$.

(50 marks)

b) State **Cauchy-Riemann equations.**

Let $f(z) = |z|^2$. Show that the Cauchy-Riemann equations are satisfied at (0,0) and f is differentiable at (0.0).

Give an example of a function which satisfies the Cauchy-Riemann equations at a given point but not differentiable at that point.

(50 marks)

2. a) Define a **Harmonic** function.

Show that $U(x,y) = 4y(x-1)$ is Harmonic in C. Obtain the Harmonic conjugate $V(x,y)$ of $U(x,y)$. Hence, write $f(z) = U + iV$ as a function of z.

(40 marks)

b) State the **Cauchy's Integral formula.**

i. Evaluate $\int_C \frac{\tan z}{z^2 - 3z + 2} dz$, where C is the circle $|z| = 4$.

ii. Using the **Cauchy's Integral formula for derivatives,** evaluate $\int_C \frac{\cos z}{z^3} dz$, where C is the circle $|z| = 1$.

(60 marks)

3. a) Find the Maclaurin series of $f(z) = \sin z$, and hence obtain the Maclaurin series of $f(z) = \cos z$.

(35 marks)

b) Find the Laurent's Series expression of $f(z) = \frac{\cos z}{z}$, $0 < |z| < \infty$.

Hence, determine whether the above function has a removable singularity or not. Justify your answer.

(30 marks)

c) Find the Laurent's Series expression of $f(z) = \frac{\cosh z}{z^5}$, where $\cosh z = \frac{e^z - e^{-z}}{2}$.

Hence, obtain the singularities of $f(z)$ and find their orders.

(35 marks)

4. a) State **Cauchy's Residue Theorem.**

Evaluate $\int_C \frac{z}{z^2+9}\,dz$, where C is the circle $|z| = 4$ oriented counterclockwise.

(40 marks)

b) State Cauchy's residue theorem for a Pole of order m.

Evaluate $\int_C \frac{1}{(z-1)(z-2)^2(z-3)}\,dz$, where C is the circle $|z-2| = 4$ oriented counter clockwise.

(60 marks)

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