RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.SC (General) Degree

Third year – Semester II Examination – October/ November 2014

MAT 3310 Integer Programming

Time allowed: **Two hours.** Number of pages: 04

Answer **FIVE** Questions selecting **Question No.01** and **four** others.

01. (a) State the basic components of an Integer Programming Model. **[15 Marks]**

(b) Using the matrix notation, write the model for a Pure Integer Programming Problem. **[10 Marks]**

(c) You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (1 and 2). The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are given in the table below:

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7	Neither Ballad nor Hit	5
8	Ballad and Hit	4

The assignment of songs to the tape must satisfy the following conditions:

(i) Each side must have exactly two Ballads.

(ii) Side 1 must have at least 3 hit songs.

(iii) Either song 5 or song 6 must be on side 1.

(iv) If both songs 2 and 4 are on side 1, then song 5 must be on side 2.

Formulate an Integer Programming Model to determine whether there is an arrangement of songs satisfying above restrictions. [*Do not solve it* .] **[75 Marks]**

02. (a) What is the cutting plane algorithm in Integer Programming.

[20 Marks]

(b) Solve the following problem using the Dual fractional cutting plane algorithm:

Minimize $Z = 5y_1 + 4y_2 + 3y_3$
subject to the constraints
$y_1 + y_3 \geq 3$
$2y_1 - y_2 \geq -1$
$y_1 + y_2 \geq 2$
$y_1 \geq 0$, $y_2 \geq 0, y_3 \geq 0$ and all are integer .

[80 Marks]

03. In what respects a Mixed Integer Linear Programme differ from Pure Integer Programme?

[20 Marks]

A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 3/4 lb. of clay and each plate uses one lb. of clay. She has 20 hours available for making the cups and plates and has 250 lbs. of clay on hand. She makes a profit of $2 on each cup and $1.50 on each plate. How many cups and how many plates should she make in order to maximize her profit?

04. Briefly explain the steps of ***Dual All Integer– Integer Programming Algorithm.***

Consider the following Integer Programming Problem [IPP]:

$$\begin{aligned}
&\textit{Maximize } z = 3x_1 + 8x_2 \\
&10x_1 + 10x_2 \leq 9 \\
&10x_1 + 5x_2 \geq 1 \\
&-x_1 + 2x_2 \leq 1 \\
&x_1, x_2 \geq 0 \text{ and integer .}
\end{aligned}$$

(i) Using the above algorithm, show that the above IPP has no feasible solution.

(ii) Verify the result in part (i) , using the graphical method.

05. A small manufacturer employs 2 skilled men and 3 semi- skilled men and makes an article in two qualities, a Deluxe model and Ordinary model. The making of Deluxe model requires 2 hours work by a skilled man and 2 hours work by a semi- skilled man. The Ordinary model requires 1 hour work by a skilled man and 3 hours work by a semi-skilled man. By union rules no man can work more than 3 hours per day. The manufacturer's clear profit of the Deluxe model is Rs.3 and of the Ordinary model Rs.4 .

(i) Formulate an **Integer Programming problem** for the above problem so as to maximize the total profit.

(ii) Use ***Dual Fractional Integer Programming Algorithm*** to solve the above problem.

06. Outline the ***Dakin variation*** of the Branch and Bound Algorithm , and explain how this algorithm can be applied to a 0-1 Integer Programming Problem.

A tailor makes wool tweed sport coats and wool slacks. He is able to get a shipment of 150 square yards of wool cloth form Scotland each month to make coats and slacks, and he has 200 hours of his own labor to make them each month. A coat require 3 square yards of wool and 10 hours to make, and a pair of pants require 5 square yards of wool and 4 hours to make. He earns $ 50 in profit from each coat he makes and $40 form each pair of slacks. He wants to know how many coats and slacks to produce to maximize the profit.

(i). Formulate an Integer Linear Programming model for this problem.

(ii). Determine the integer solution to this problem using the Branch and Bound method. Compare this solution with the solution ignoring the integer requirements and indicate if the rounded-off solution would have been optimal.