RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

Second Year - Semester I Examination – September/October 2013

MAA 2302 - Probability and Statistics II

Answer ALL FIVE QUESTIONS. Time allowed: **THREE hours**

[Statistical tables and calculators will be provided.]

1. Let (X, Y) be a two dimensional continuous random variable with joint probability density function,

$$f(x,y) = \begin{cases} k(2x+3y) & ; 0 \le x \le 2, \ 0 \le y \le 1 \\ 0 & ; otherwise \end{cases}$$

(i) Show that $k = \frac{1}{7}$

(ii) Find the marginal probability density functions of the random variables X and Y.

(iii) Determine whether X and Y are independent random variables.

(iv) Find $f_{X|Y}(x|y)$ and hence determine the probability $P\left(\frac{1}{2} < X < 1 \,\middle|\, Y = \frac{2}{3}\right)$.

2.

(a) Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{12}{(x^6+2)} & ; x > 0 \\ 0 & ; otherwise \end{cases}$$

Find the probability density function of the random variable $Y = 8X^3$.

(b) Suppose a fair die with its faces numbered 1, 2, 3, 4, 5, 6 is tossed until number 5 occurs. Let X be the number of failures until the first success occurs.

(i) Determine the probability mass function of X.

(ii) Find the probability mass function of $Y = X + 2$.

(c) Let X and Y be the continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 24x^2y & ; 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ; otherwise \end{cases}$$

(i) Find the joint probability density function of the random variables U and V, where $U = X + Y$ and $V = 2X$.

3. (a)

(i) Let $x_1, x_2, x_3, \dots, x_n$ be a random sample taken from a normal distribution with mean μ and variance σ^2. Let $\bar{X}$ and S^2 be the sample mean and the sample variance.

Show that the random variable $T = \dfrac{\bar{X} - \mu}{S / \sqrt{n}}$ has a t-distribution with $n - 1$degrees of freedom.

(ii) It is known from the past experience that the mean height of a certain plant is $1.6\ m$. A random sample of 16 plants was selected and the standard deviation of the sample was found to be $0.16\ m$. Find the probability that the mean of the sample is greater than $1.67\ m$, stating any assumptions you make.

(b)

(i) State the central limit theorem.

(ii) The diameter of ping-pong balls manufactured by a certain machine has a mean of $3.4\ cm$ and a standard deviation of $0.7\ cm$. If a random sample of 49 ping-pong balls were selected, find the probability that the sample mean is less than $3.24\ cm$.

4.

(a) Let $x_1, x_2, x_3, \dots, x_n$ be a random sample of size n_1 taken from a normal population with mean μ_1 and variance σ_1^2 and let $y_1, y_2, y_3, \dots, y_n$ be a random sample of size n_2 taken from a normal population with mean μ_2 and variance σ_2^2.

(i) Find the mean and the variance for the difference between the two sample means.

(ii) Find 95% confidence interval for the difference between the two sample means.

(b) A company produces two types of light bulbs, type S and type T. A random sample of 35 light bulbs taken from type S indicates a mean life expectancy of 375 hours and a random sample of 45 light bulbs taken from type T indicates a mean life expectancy of 362 hours. The population standard deviation of type S is 110 hours and the population standard deviation of type T is 125 hours. Find a 95% confidence interval for the difference between the two sample means, stating any assumptions you make.

5. Let $x_1, x_2, x_3, \dots, x_n$ be a random sample of size n taken from a Binomial Distribution, $Bin(m, p)$, where m is a known integer. The probability mass function of the distribution is given by

$$f(x;p) = \binom{m}{x} p^x (1-p)^{m-x} \qquad \text{where } x = 0,1,2, \dots m$$

Two estimators for the probability p are defined as $P_1 = \dfrac{\overline{X}}{m}$ and $P_2 = \dfrac{\overline{X}+1}{m+2}$

(i) Stating the conditions satisfied by an unbiased estimator, determine the unbiased estimator/s of the two estimators defined above.

(ii) Determine the best estimator for p, using Cramer-Rao Theory.

END