RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
Second Year – Semester II Examination – April/May 2015

PHY 2105 – Quantum Mechanics

Answer any **two** questions Time: One hour

Use of a non-programmable calculator is permitted.

Symbols have their usual meaning.

Some fundamental constants and physical data:

Electron mass $m_e = 9.1 \times 10^{-31}$ kg	Planck constant $h = 6.626 \times 10^{-34}$ J s
Speed of light in vacuum $c = 3.0 \times 10^{8}$ m s^{-1}	Electron charge $e = 1.6 \times 10^{-19}$ C
Electron volt $eV = 1.6 \times 10^{-19}$ J	Bohr radius $a_0 = 0.529 \times 10^{-10}$ m
Rydberg constant $R_H = 1.097 \times 10^{7}$ m^{-1}	Proton mass $m_p = 1.672 \times 10^{-27}$ kg

1. (a) Consider a particle of mass m, moving in a one-dimensional infinite square well of width L, such that the left corner of the well is at the origin. Obtain the energy eigenvalues and the corresponding normalized eigenfunctions (wave functions) of the particle. [20 marks]

   (b) Calculate the probability that a particle in a one - dimensional box of length L can be found between 0.4 L to 0.6 L for the (i) ground state, (ii) first excited state (iii) second excited state. [15 marks]

   (c) Show that the de Broglie wavelength of a particle in a one-dimensional box in the first excited state is equal to the length of the box [15 marks]

Contd.

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2. (a) State Heisenberg's uncertainty principle and prove that $\Delta E.\Delta T \geq \hbar$.

[15 marks]

(b) The average lifetime of an excited atomic state is 10^{-8} s. If the wavelength of the spectral line associated with the transition from this state to the ground state is 6000 $\overset{0}{A}$, estimate the width of this line. [15 marks]

(c) Using the uncertainty principle, show that an alpha particle can exist inside a nucleus.

Hint: mass of an alpha particle is four times the mass of a proton and the radius of a typical nucleus is of the order of 10^{-14} m. [20 marks]

3. (a) The mysterious lines observed by Pickering in 1896 in the spectrum of the star ζ - Puppis fit the empirical formula $\frac{1}{\lambda} = R_H\left(\frac{1}{(n_f/2)^2} - \frac{1}{(n_i/2)^2}\right)$. Show that these lines can be explained by the Bohr theory as originating from He^+. In general, to describe a single electron orbiting a fixed nucleus of charge +Ze, Bohr theory gives $r_n = (n^2)a_0/Z$ and $E_n = -\frac{ke^2}{2a_0}\left(\frac{Z^2}{n^2}\right)$, $n = 1,2,3,....$

[12 marks]

(b) Positronium is a hydrogen-like atom consisting of a positron (a positively charged electron) and an electron revolving around each other. Using the Bohr model, find the allowed radii (relative to the center of mass of the two particles) and the allowed energies of the system. [12 marks]

(c) Which state of the triply ionized beryllium has the same orbital radius as that of the ground state of hydrogen? Compare the energies of the two states.

[14 marks]

(d) Suppose that the electron in the hydrogen atom obeyed classical mechanics rather than quantum mechanics. Why should such a "hypothetical" atom emit a continuous spectrum rather than the observed line spectrum?

[12 marks]

End.