RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree
Third Year - Semester II Examination – September/October 2014

MAT3217 – Nonlinear Programming

Answer only four questions ***Time: 02 hours***

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01. a. Explain the followings which cause difficulties inoptimizing nonlinear models inherently.

I. "It is difficult to distinguish a local optimum from a global optimum"

II. "Optima is not restricted to extreme points"

III. " There may be multiple disconnected feasible regions"

Hint: Use graphical interpretation for explanations

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b. If K units of capital and L units of labor are used, a company can produce KL units of a manufactured good. Capital can be purchase at Rs.4/unit and labor can be purchase at Rs.1/unit. A total of Rs.8 is available to purchase capital and labor. How can the firm maximize the quantity of good that can be manufactured?

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c. Use Golden Section Search to find

$$\begin{aligned} Max \quad & -x^2 - 1 \\ s.t. \quad & -1 \leq x \leq 0.75 \end{aligned}$$

with the final interval of uncertainty having a length less than 0.25

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02. a. Check whether the following functions are either *convex, concave or not.*

I. $f(x_1, x_2) = -x_1^2 - x_1x_2 - 2x_2^2$
II. $f(x_1, x_2) = x_1^2 - 3x_1x_2 + 2x_2^2$; where $S = \mathbb{R}^2$

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Show that following function is a convex function ,

$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 2x_3^2 - x_1x_2 - x_2x_3 - x_1x_3$; where $S = \mathbb{R}^3$

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Cont...

b. Find the value of x which minimizes $f(x) = x^2 - 3x + 5$

Then discuss what will happen to the optimal solution if we add a constraint

I. $x \geq 0$
II. $x \geq 2$

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c. A biomedical instrumentation company sells its main product at the rate of 5 units per day. The instrument is manufactured in lots run every few days. It costs the company \$2000 to set up for production of a lot and \$40 per unit per day to hold finished instruments in inventory between runs. The company would like to choose a lot size that minimize average inventory and setup cost per day assuming that demand occurs smoothly at the given rate.

I. Formulate a 1-variable unconstrained NLP to choose an optimum lot size. *15*

II. Plot the objective function of your model and compute an optimum lot size graphically. *25*

03. a. Local maximum point x^* of an unconstrained function has followed the two properties given below.

$$\frac{df(x^*)}{dx} = 0 \quad , \quad \frac{d^2f(x)}{dx^2} < 0$$

Explain what is meant by each property. *10*

b. Explain what is meant by saddle point
Find all local maxima, local minima, and saddle points for $f(x_1, x_2) = x_1^2 x_2 + x_2^3 x_1 - x_1 x_2$

Hint : there are six stationary points ***40***

c. Note down the main technique of steepest Descent / Ascent method used in solving NLP *10*

d. Briefly discuss the function of steepest Ascent Method *10*

Hence solve the problem

$Max\ z = -(x_1 - 3)^2 - (x_2 - 2)^2 = f(x_1, x_2)\,, s.t. (x_1, x_2)\ \epsilon\ \mathbb{R}^2$ *30*

04. a. State the condition that Lagrangian multipliers can be used to solve NLP *30*

b. Solve the following Non-Linear Programming problem using the Lagrangian multipliers.

$optimize\ \ Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1 x_2$

$$S.t. \quad x_1 + x_2 + x_3 = 15$$
$$2x_1 - x_2 + 2x_3 = 20$$
$$x_1, x_2, x_3 \geq 0$$

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05. a. Solve the NLP given below using the KTK conditions

$Max\ z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

$S.t.\ 2x_1 + 5x_2 \leq 98$

$x_1\ ,\ x_2\ \geq 0$

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b. State necessary conditions to solve a problem as a quadratic programming problem.

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06. a. What is separable programming?
State two distinct examples for separable and non-separable programs.

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b. Consider a separable nonlinear program with objective function and constraint component function for non-negative decision variable w_1 given by,

$f_1(w_1) \triangleq (w_1)^2 - 4w_1 + 22$

$g_{1,1}(w_1) \triangleq \sqrt{w_1 + 9}$

$g_{2,1}(w_1) \triangleq 14w_1$

Form corresponding piece-wise linear approximations using breakpoint $u_{1,1} = 2\ \ and\ u_{1,2} = 5$

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