RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B. Sc. (General) Degree

First Year – Semester II Examination – April/May 2015

MAP 1203 – Real Analysis I

Answer All Questions **Time allowed: Two hours**

1. (*a*) Show that for all $x, y \in R$ we have $|x+y| \le |x| + |y|$.

   (*b*) Prove that the set of all complex numbers "C"|is not an ordered field.

   (*c*) Define the terms *Infimum* and *Suprimum*.

   Which of the following sets are bounded above, bounded below or otherwise? Also find the Infimum and Suprimum, if they exist.

   (*i*) $\left\{\frac{(-1)^n}{n}; n \in \aleph\right\}$,

   (*ii*) $\left\{1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{2^2}, \ldots, 1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^{n-1}}, \ldots\right\}$

   (*c*) Let A, B and C be non empty subsets of $\Re$ and let

   $A+B+C = \{a+b+c \mid a \in A, b \in B, c \in C\}$. Prove that

   $Inf(A+B+C) = InfA + InfB + InfC$.

2. (*a*) Find the following limit:

$$\lim_{x \to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}.$$

   (*b*) State $\varepsilon - \delta$ definition for continuity of a function.

   Show that the following function is discontinuous at every point of R

$$f(x) = \begin{cases} 1 & if \quad x \text{ is rational} \\ -1 & if \quad x \text{ is irrational} \end{cases}$$

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(*c*) Determined the constants a and b so that the function f defined below is continuous

every where $f(x)=\begin{cases} 2x+1, & for\ x\le 2 \\ ax^3+b, & for\ 2<x<5 \\ 5\sqrt{x}+2a, & for\ x\ge 5 \end{cases}$.

3. (*a*) If f and g are two differentiable functions at $x=c$, then show that fg also

   differentiable function $x=c$ and $(fg)'(c)=f(c)g'(c)+g(c)f'(c)$

   (*a*) Examine the following function for differentiability at $x=0$ and $x=1$.

$$f(x)=\begin{cases} x^2, & for\ x\le 0 \\ 1, & for\ 0<x\le 1. \\ \frac{1}{x}, & for\ x>1 \end{cases}$$

   (*c*) Show that the function $f(x)$ is continues at $x=0$ but not differentiable at $x=0$ where

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & if\ \ x\ne 0 \\ 0 & if\ \ x=0 \end{cases}$$

4. (*a*) State the following theorems:

   (*i*) Rolle's theorem,

   (*ii*) Lagrange's Mean Value Theorem,

   (*iii*) Cauchy's Mean Value Theorem.

   (*b*) Verify whether the function $f(x)=\sin x$ in $[0,\pi]$ satisfies the conditions of Rolle's theorem and hence find c as prescribed by the theorem.

   (*c*) Show that $\frac{v-u}{1+v^2}<\tan^{-1}v-\tan^{-1}u<\frac{v-u}{1+u^2}$ where $0<u<v$. Deduce that

$$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}.$$

   (*d*) Using *L'Hospital Rule* find the following limit:

$$\lim_{x\to 0}\frac{\tan x-x}{x^2\tan x},$$

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