RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. General degree in Applied Sciences

Second Year Semester I Examination – April / May 2016
Probability and Statistics II - MAA 2302

Candidates with mid semester marks:
Answer **TWO** questions from **part A** and **TWO** questions from **part B** including **FIRST** question .

Time allowed:**TWO** hours

Candidates without mid semester marks:
Answer **THREE** questions from **part A** including **FIRST** question and **ALL** questions from **part B.**

Time allowed:**THREE** hours

Statistical tables and Calculators will be provided.

Part A

1.

(a) X and Y be jointly continuous random variables with joint PDF

$$f_{X,Y}(x;y) = \begin{cases} 6e^{-(2x+3y)} & ; x, y \geq 0 \\ 0 & ; Otherwise \end{cases}$$

i. Are $X$and Yindependent? Justify your answer.

ii. Find E(Y|X> 2).

iii. Find P(X> Y).

(b) Let X be a continuous random variable with PDF given by f_X(x)=$12e^{-|x|}$, for all $x \in$R.

If $Y = X^2$ find the CDF of Y.

(c) Let $X \sim Uniform(\frac{\pi}{2}, \pi)$ and $Y = Sin(X)$. Find $F_y(y)$.

[Turn Over]

2.

(a) Let X and Y be two discrete random variables.

i. Define joint probability mass function and independence of X and Y.

ii. If X and Y have means $E[X]$ and $E[Y]$ respectively, Show that

$$cov(X,Y) = E[XY] - E[X]E[Y]$$

(b) The number of contracts awarded to firm A (X), and Y, the number of contracts awarded to firm B, is given by the entries in the following table.

Y	**X**			$P_Y(y)$
	0	**1**	**2**	
0	$1/9$		$1/9$	$4/9$
1	$2/9$	$2/9$		
2	$1/9$		0	$1/9$
$P_X(x)$	$4/9$	$4/9$		1

i. Find the missing values in order to fill the table.

ii. Are X and Y independent? Why?

iii. Construct conditional probability distribution of X given $Y =$ ~~$=y$~~ $\forall y$.

iv. Find the covariance between X and Y.

v. Find the variance of $W = 3X + 3Y$.

3. Let X be a continuous random variable and A an event with probability, where $0 < \theta < 1$. Conditional on A, X has cumulative distribution function $F_1(x)$ and expectation μ_1 while, conditional on A' (the complement of A), X has cumulative distribution function $F_2(x)$ and expectation μ_2 . Justify the expression

$F(x) = \theta F_1(x) + (1-\theta)F_2(x)$ for the cumulative distribution function $F(X)$ of X and hence deduce that

$$E(x) = \theta\mu_1 + (1-\theta)\mu_2$$

Small chocolate biscuits of a certain brand are sold in packets of 6 with a nominal weight of 25 g. The weight (g) of an individual biscuit is a $N(4.50.25)$, random variable, and the weights of different biscuits are independent.

i. Find the probability that, in total, 6 of these biscuits weigh less than 25 g.

ii. When 6 biscuits are put together to form a packet, if their total weight is found to be less than 25 g then a seventh biscuit is added to the packet. Find the mean weight of a packet of biscuits.

iii. If it is simply known that the weight of an individual biscuit is a $N(4.5, \sigma^2)$ random variable, for what values of σ is the probability less than 0.01 that 6 biscuits weigh less than 25 g?

4. A process for refining sugar yields up to 1 ton of pure sugar per day, but the actual amount produced, X is a random variable because of machine breakdowns and other slowdowns. Suppose that X has density function given by,

$$f(x) = \begin{cases} 2x & ; 0 \leq x \leq 1 \\ 0 ; & O.w \end{cases}$$

Thus the daily profit, in hundreds of dollars, is $Y = 3X - 1$. Find the probability density function for Y using

i. Method of distribution function
ii. Method of transformation

Part B

5.
(a) Define **TWO** of the following terms:

i. Unbiased Estimator
ii. Mean Square Error (MSE)
iii. Efficient Estimator

(b) Let $X_1, \dots\dots, X_n$ be a random sample from a distribution with mean μ and $var(X) = \sigma^2$. Show that
i. $\bar{X}$ is a unbiased estimator for μ.
ii. $S^2 = 1/(n-1)\sum_{i=1}^{n}(X_i - \overline{X})^2$; S^2 is a unbiased estimator of σ^2.

(c) Let $X_1, \dots\dots, X_n$ be a random sample from a from a poisson distribution with prob. $P(x, \mu) = (e^{-\mu}\mu^x)/x!$. If $T_1 = \bar{X}$ and $T_2 = ½\sum_{i=1}^{n-1}\frac{x_i}{n-1} + \frac{x_n}{2}$ are estimators of μ. Then determine the efficient estimator of μ.

6.
(a) Define the following terms:
i. Rao-Blackwell improvement theorem.
ii. Maximum likelihood estimation.

(b) A random variable $\boldsymbol{x}$ follows a poison distribution with parameter $\boldsymbol{\mu}$ and show that the maximum likelihood estimation of $\boldsymbol{\mu}$ is $\bar{\boldsymbol{x}}$

(c) When $X_i \sim N(\mu, \sigma^2)$: i=1, 2,…,n and μ *and* σ^2 are unknown determine maximum likelihood estimation of μ *and* σ^2.

7.

(a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n taken from a normal population with unknown mean μ and unknown variance σ^2 . Construct a $100(1-\alpha)\%$ confident interval for the ratio of two population variances, σ_2^2/σ_1^2.

(b) A study was conducted to study the effect of an oral anti-plaque rinse on plaque buildup on teeth. For this study, 14 subjects were divided into 2 groups of 7 subjects each. Both groups were assigned to use oral rinses for a 2-week period. Group 1 used rinse that contained an antiplaque agent, while group 2 received a similar rinse except that it contained no antiplaque agent. A plaque index that measures the plaque buildup was recorded after 2 weeks. The sample mean and sample standard deviation for the 2 groups are shown below:

	Group 1	Group 2
Sample size	7	7
Sample mean	0.78	1.26
Sample standard deviation	0.32	0.31

Construct 95% confidence interval for the ratio of two population variances, σ_2^2/σ_1^2 stating any assumptions you make.