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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences

First Year - Semester I Examination – July/Aug 2023

STA 1202 – Introduction to Probability Theory

Time: Two (02) hours

Answer **all** questions.

Calculators and Statistical tables will be provided.

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1. a) Given that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$ are for events A and B, find the following probabilities,

   (i) $P(A')$ (ii) $P(A \cap B')$

   (iii) $P(B \cap A')$ (iv) $P(A \cup B)$

   (08 marks)

   b) Consider two events, A and B. The probability that either A or B occurred is denoted by p. Additionally, let q represent the probability that exactly one of the events A or B occurs. Prove the following,

   $$P(A') + P(B') = 2 - 2p + q.$$

   (07 marks)

   c) State the Bayes' Theorem.

   A car manufacturing factory consists of two plants, X and Y. Plant X manufactures 70% of cars, while Plant Y manufactures the remaining 30%. Among the cars produced at plant X, 80% are rated as standard quality, while at plant Y, 90% of the cars are rated as standard quality. If a car is randomly chosen and found to be of standard quality, what is the probability that it originated from plant X?

   (10 marks)

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2. a) Let X be a discrete random variable with the following probability mass function.

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2, \\ 0.2 & \text{for } x = 0.4, \\ 0.2 & \text{for } x = 0.5, \\ 0.3 & \text{for } x = 0.8, \\ 0.2 & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Determine the range, denoted by R_X, of the random variable X.

(ii) Find $P(X \leq 0.5)$.

(iii) Find $P(0.25 < X < 0.75)$.

(iv) Find $P(X = 0.2|X < 0.6)$.

(20 marks)

b) In the Prussian army corps, the average number of soldiers who died per year due to horse kicks was 0.61. Calculate the probability of exactly two soldiers dying in the VII Army Corps in 1898, assuming that the number of horse kicking deaths per year follows a Poisson distribution.

(05 marks)

3. a) Distinguish the differences between discrete and continuous random variables.

(05 marks)

b) Let X be a random variable with mean μ and variance $Var(X)$. Prove the following;

(i) $Var(X) = E[X^2] - \mu^2$

(ii) $Var(aX + b) = a^2Var(X)$, where a and b are constants.

(10 marks)

c) The probability density function of a continuous random variable X is given by,

$$f(x) = \begin{cases} x & ; \text{for } 0 \leq x \leq 1, \\ 2 - x & ; \text{for } 1 < x \leq 2, \\ 0 & ; \text{otherwise.} \end{cases}$$

(i) Compute the expected value of X.

(ii) Compute the variance and standard deviation of X.

(10 marks)

4. a) The random variable X has probability function

$$P(X = x) = \begin{cases} kx & ; x = 1,2,3, \\ k(x+1) & ; x = 4,5. \end{cases}$$

where k is a constant.

(i) Find the value of k.

(ii) Find the expected value $E[X]$.

(iii) Compute the variance $Var[X]$.

(15 marks)

b) Suppose the temperature T during May is normally distributed with a mean $\mu = 68°$ and standard deviation $\sigma = 6°$. Find the probability p that the temperature in May is

(i) between $70°$ and $80°$,

(ii) less than $60°$.

(10 marks)

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