RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester I Examination – June/July 2018

MAA 2201 – MATHEMATICAL METHODS II

Time: Two (2) hours

Answer FOUR Questions only

---

1. a) Let the Spherical Polar Coordinates be (r, θ, φ) and Rectangular Cartesian Coordinates be (x, y, z) on the 3-dimensional Space.

   i. Express x, y *and* z in terms of r, θ *and* φ by drawing a suitable diagram.

   ii. In the usual notation, derive scale factors $(h_r, h_\theta, h_\varphi)$ and base vectors $(\underline{e}_r\ ,\ \underline{e}_\theta\ ,\ \underline{e}_\varphi)$.

   iii. Using part (ii), prove that system of Spherical Polar Coordinates is orthogonal. **(60 marks)**

   b) Determine the constant 'a' so that the vector
   $\underline{F} = (x + 3y)\ \underline{i} + (y - 2z)\ \underline{j} + (x + az)\ \underline{k}$ is solenoidal. **(20 marks)**

   c) Determine the constants a, b *and* c so that the vector
   $\underline{F} = (x + 2y + az)\ \underline{i} + (bx - 3y - z)\ \underline{j} + (4x + cy + 2z)\ \underline{k}$ is irrotational. **(20 marks)**

2. a) What is a 'Conservative vector field'? **(10 marks)**

b) For the vector field, $\underline{F} = (e^x z - 2xy)\underline{i} + (1 - x^2)\underline{j} + (e^x + z)\underline{k}$,

i. find a scalar function $\emptyset(x, y, z)$ such that $\underline{F} = grad\ \emptyset$.

ii. evaluate the line integral $\int_c \underline{F} \, . \, d\underline{r}$, where c is any path from point $A(0, 1, -1)$ to point $B(2, 3, 0)$.

(25 marks)

c) State Stokes' theorem and use it to prove the Green's theorem. **(25 marks)**

d) Verify Green's theorem for $\oint_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where c is the region bounded by the parabolas $y^2 = x$ and $y = x^2$.

(40 marks)

3. a) State Gauss's divergence theorem. **(10 marks)**

b) Let S be the closed surface enclosing the volume v, which is the upper surface of the sphere $x^2 + y^2 + z^2 = a^2$, cut off by the plane $z = \frac{a}{2}$, where a is a constant.

i. Use Cylindrical Polar Coordinates to show that $v = \frac{5}{24}\pi a^3$.

ii. Using part (i), verify Gauss's divergence theorem for the position vector, $\underline{r} = x\,\underline{i} + y\,\underline{j} + z\,\underline{k}$ over the boundary of the volume v.

(60 marks)

c) Let $\underline{F} = 2xz\,\underline{i} - x\underline{j} + y^2\underline{k}$. Evaluate $\iiint \underline{F}\, dv$, where v is the region bounded by the surfaces $x = 0$, $y = 0$, $y = 6$, $z = x^2$ and $z = 4$. **(30 marks)**

4. a) State the linearity property and first shifting property in Laplace transform.

(20 marks)

b) Let F be a function of t and let $L\{F(t)\} = f(s)$. Using Laplace transform, evaluate the following:

i. $\int_0^\infty \frac{\sin(t)}{t}\, dt$.

ii. $L\{t^2 \sin(at)\}$, where a is a constant. **(20 marks)**

c) State the Convolution Theorem in the Inverse Laplace transform. **(10 marks)**

d) Find the Laplace transform of the convolution integral;

$F(t) = \int_0^t u^{m-1}(t-u)^{n-1}\,du$, where $m > 0$ and $n > 0$. Hence, show that $F(t) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}\, t^{m+n-1}$ and deduce the Beta function, $\beta(m,n)$ defined by the integral $\int_0^1 u^{m-1}(1-u)^{n-1}\,du$.

[**Hint:** $\Gamma(r) = \int_0^\infty u^{r-1}\, e^{-u}\, du$, $r > 0$] **(50 marks)**

5. a) Define Fourier transform and inverse Fourier transform. **(10 marks)**

b) Find the Fourier transform and inverse Fourier transform of $F(x)$ defined by,

$$F(x) = \begin{cases} 1\,, |x| < a \\ 0\,, |x| > a \end{cases},$$

where a is a constant

Hence, evaluate $\int_{-\infty}^{\infty} \frac{\sin(pa)\cos(px)}{p}\, dx$. **(70 marks)**

c) Find the cosine Fourier transform of a function of x , which is unity for $0 < x < a$ and zero for $x \geq a$, where a is a constant. Hence, find the function whose cosine transform is $\frac{\sin(ap)}{p}$. **(20 marks)**

END