RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester I Examination – November/December 2016

MAT 3213 – Graph Theory

Time: Two (2) hours

Answer **only Four** Questions **including First question.**

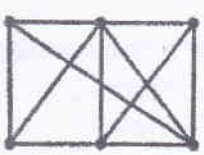
01 Select the most suitable answer. (Justify your answer) **(100 marks)**

a. A spanning tree for a simple graph of order 24 has
   (a) 12 edges
   (b) 6 edges
   (c) 23 edges
   (d) None of the above

b. A Hamiltonian cycle in a Hamiltonian graph of order 24 has
   (a) 12 edges
   (b) 24 edges
   (c) 23 edges
   (d) None of the above

c. The order of a forest, F with 17 vertices and 4 components is
   (a) 17
   (b) 4
   (c) 16
   (d) None of the above

d. The size of a forest, F with 17 vertices and 4 components is
   (a) 14
   (b) 4
   (c) 16
   (d) None of the above

G: 

e. Let H be the plane drawing of the planar graph G, drawn above. The graph H has
   (a) 10 faces
   (b) 5 faces
   (c) 11 faces
   (d) 6 faces
   (e) None of the above

f. A simple graph G with 13 vertices has 4 vertices of degree 3, 3 vertices of degree 4 and 6 vertices of degree 1. Then the graph G must be a tree.

(a) True (b) False

g. The cub graph Q_5 is planar.

(a) True (b) False

h. The complete bipartite graph $K_{4,3}$ is non-planar.

(a) True (b) False

i. If G is a simple connected regular planar graph in degree 3 where every face is bounded by exactly 3 edges, then the size of G is

(a) 3
(b) 4
(c) 6
(d) 5
(e) None of the above

The next two questions refer to A which is an adjacency matrix of an undirected graph.

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

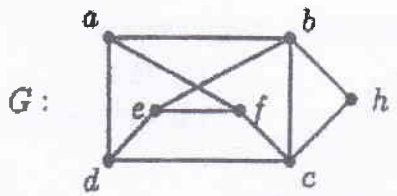
j. The degree sequence of the above graph is

(a) (2, 2, 3, 3, 4)
(b) (0, 1, 1, 2, 0)
(c) (0, 0, 1, 1, 2)
(d) (2, 3, 4)
(e) None of the above.

k. The size of the above graph is

(a) 2
(b) 3
(c) 7
(d) None of the above.

The next four questions refer to the graph G drawn below.



l. The graph G is not a regular graph because:

(a) It has a vertex of degree 3
(b) It is a complete graph
(c) Not all vertices have the same degree
(d) Not all edges are the same length

m. The graph G is a bipartite graph.

(a) True (b) False

n. The order of (G-h) is

(a) 4
(b) 6
(c) 5
(d) 7

o. befcdef is

(a) a trial
(b) a cycle
(c) a walk but not a trail
(d) a path but not a cycle

02 Salvadore is visiting six famous places in Colombo: Zoo (Z), Viharamahadevi Park (V), National Museum (M), Independence Square (I), Galle Face (G) and Taj Samudra Hotel (H). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel. The following table shows the times, in minutes that it will take to travel between the six places.

From \ To	Zoo (Z)	Viharamahadevi Park (V)	National Museum (M)	Independence Square (I)	Galle Face (G)	Hotel (H)
Zoo (Z)	–	35	30	30	37	35
Viharamahadevi Park (V)	25	–	20	21	25	40
National Museum (M)	15	40	–	25	30	29
Independence Square (I)	30	35	25	–	35	20
Galle Face (G)	20	30	17	25	–	25
Hotel (H)	25	35	29	20	30	–

a) Define Hamiltonian cycle and Eulerian circuit and give an example for each. from the context of the above situation (if there is). **(20 marks)**

b) Salvadore intends to travel from one place to another until he has visited all of the places exactly once before returning to his place.

i. Find the total traveling time for Salvadore using nearest neighbour algorithm. staring from Taj Samudra Hotel (H). **(40 marks)**

ii. Explain why your answer to part b) i. is an upper bound for the minimum traveling time for Salvadore. **(15 marks)**

iii. Salvadore starts from Taj Samudra Hotel (H) and then visit Galle Face (G). Given that he visits Viharamahadevi Park (V) before Independence Square (I), find an improved upper bound for the total traveling time for Salvadore. **(25 marks)**

03

a) If a tree is consist with n_1 vertices of degree 1, n_2 vertices of degree 2, n_3 vertices of degree 3, ... and n_k vertices of degree k, prove that **(40 marks)**

$n_1 = 2 + n_3 + 2n_4 + 3n_5 + ... + (k-2)n_k$.

b) Suppose XYZ Drilling Company has four oil wells that must be connected via a pipeline network to a storage tank. The cost of each pipeline (in millions of dollars) is shown in the following table.

From / To	1st oil well	2nd oil well	3rd oil well	4th oil well	Tank
1st oil well	–	1	4	2	3
2nd oil well	1	–	3	1	2
3rd oil well	4	3	–	1	4
4th oil well	2	1	1	–	1
Tank	3	2	4	1	–

i. Represent this information on a weighted graph. **(10 marks)**

ii. Find the minimum cost pipeline network that links together all of the wells and the storage tank using a suitable algorithm. **(40 marks)**

iii. What is the minimum cost of the above network? **(10 marks)**

04

a) In a tree; if degree of every non pendant vertex is 3, show that the number of vertices of that tree is even. **(30 marks)**

b) Suppose that in a group of 5 people: A, B, C, D and E, the following pairs of people are acquainted with each other.

- A and C
- A and D
- B and C
- C and D
- C and E

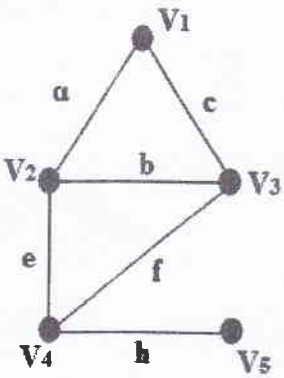
i. Draw a graph G to represent this situation. **(10 marks)**

ii. List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, respectively, in G = {V, E}. **(10 marks)**

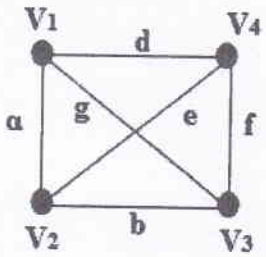
iii. Determine the adjacency matrix and incident matrix of G. **(30 marks)**

iv. Consider the following two graphs and draw $G_1 \cup G_2$, $G_1 \cap G_2$, $G_1 \oplus G_2$ and G_1'.

(20 marks)



G1



G2

05

a) State and prove the Euler's formula for planar graph. **(40 marks)**

b) Show that,

i. If G is a connected simple planar graph with n vertices and e edges and every region is bounded by k edges, $e = \dfrac{k(n-2)}{(k-2)}$ **(20 marks)**

ii. If G is a connected simple planar graph with $n(\geq 3)$ vertices and e edges and no circuits of length 3, $e \leq 2n-4$ **(20 marks)**

c) A connected planar graph has seven vertices having degrees 2, 2, 3, 4, 4, 5 and 6. Determine the number of edges and faces of the graph. **(20 marks)**

END