RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. Degree in Applied Sciences
FourthYear- Semester I Examination – June/July 2018

MAT 4310 – COMPUTATIONAL MATHEMATICS

Time:Three (3) hours

Answer five questions only

1.

a) Find the Taylor series forf(x) = cos x at $x = 0$. **(15 marks)**

b) Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$ **(20 marks)**

c) Evaluate $\lim_{x \to 0} \frac{\sin x - x}{x^3}$ **(15 marks)**

d)

i. Find the 3rd –degree Taylor polynomial for the function $f(x) = \ln x$ centered at x=1.

ii. Use your answer from part(i) to approximate $ln(1.15)$ **(25 marks)**

e) Find the second order Taylor expansion of $f(x, y) = \sqrt{1 + x^2 + y^2}$ about$(x_0, y_0) = (1,2)$ and use it to compute approximately $f(1.1\,, 2.05)$. **(25 marks)**

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2. Consider the following **Runge-Kutta** method for the differential equation $y' = f(x,y)$

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{K_1}{2})$$

$$K_3 = hf(x_n + h, y_n - K_1 + 2K_2)$$

a) Compute $y(0.4)$ when $y' = \frac{y+x}{y-x}$, $y(0) = 1$ and $h = 0.2$. Round to fivedecimal Places. **(60 marks)**

b) What is the result after one step of length h when$y' = -y, y(0) = 1$ **(40 marks)**

3.

a) Let $y = f(x,y), y(x_0) = y_0$, step size h and$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$.
Use Euler's method with step size 0.3 to compute the approximate value y(0.9) of thesolution of initial value problem. Given that $y' = x^2, y(0) = 1$. **(60 marks)**

b) Solve the Initial Value Problem $\ddot{Y} + 6\dot{Y} + 9Y = 4\,e^{-3t}$, $Y(0) = 1$ and$\dot{Y}(0) = 0$. **(40 marks)**

4.

a) Solve the differential equation$\Delta^2 y_{n+1} + \frac{1}{2}\Delta^2 y_n = 0$, $n = 0,1,2,..$,when $y_0 = 0$,$y_1 = \frac{1}{2} y_2 = \frac{1}{4}$. Is this method numerically stable? **(60 marks)**

b) Find the inverse of the matrix $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$ by the LU decomposition method. **(40 marks)**

5.

a) The matrix $A = \begin{pmatrix} 1+s & -s \\ s & 1-s \end{pmatrix}$ is given. Calculate p and q such that $A^n = pA + qI$ and determine e^A. **(40 marks)**

b) Calculate $C^T A^{-1} B$ when $A = LL^T$ with

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}, C = \begin{pmatrix} 1 \\ 5 \\ 14 \\ 30 \end{pmatrix}$$

(60 marks)

6.

a) Solve the equations

$$10x_1 - x_2 + 2x_3 = 4$$

$$x_1 + 10x_2 - x_3 = 3$$

$$2x_1 + 3x_2 + 20x_3 = 7$$

Using the Gauss elimination method. **(50 marks)**

b) Consider the equations

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4$$

Use the decomposition method to solve the system. **(50 marks)**

END