57

RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (Four Year) Degree in Applied Sciences /

B. Sc. (Joint Major) Degree in Chemistry and Physics

Fourth Year - Semester I Examination – January / February 2021

PHY4210 - ADVANCED QUANTUM MECHANICS

Time: Two (02) hours

Answer ALL four questions.

Unless otherwise specified all symbols have their usual meaning.

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1. a) Describe the five basic postulates of quantum mechanics. (10 marks)

   b) An operator $\hat{A}$ is said to be Hermitian if it satisfiesthe condition $\langle\psi_1|\hat{A}\psi_2\rangle = \langle\hat{A}\psi_1|\psi_2\rangle$ for any two functions ψ_1 and ψ_2 of the function space in which the operator $\hat{A}$ acts on. Prove that the momentum operator $-i\hbar\overline{\nabla}$ is Hermitian. (08 marks)

   c) Show that the eigenvalues of a Hermitian operator are real. (07 marks)

2. a) Prove the following operator identities:

   (i) $[\hat{A}, \hat{B} + \hat{C}] = [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}]$ (05 marks)

   (ii) $[\hat{A}, \hat{B}^{-1}] = -\hat{B}^{-1}[\hat{A}, \hat{B}]\hat{B}^{-1}$ (05 marks)

   b) If $\hat{L}$ is a non-Hermitian operator, then show that $(\hat{L} + \hat{L}^{\dagger})$ is Hermitian. (07marks)

Contd.

58

c) If $\hat{A}$ and $\hat{B}$ are integrals of motion, then show that $i[\hat{A},\hat{B}]$ is also an integral of motion. **Hint:** If $\hat{A}$ and $\hat{B}$ are integrals of motion, then they commute with $\hat{H}$.

(08 marks)

3. In one dimension, the parity operator $\hat{\pi}$ for a single particle acts in the position representation to replace the wave function $\psi(x)$ by the wave function $\hat{\pi}\psi(x) = \psi(-x)$ and acts in the wave vector representation in a similar fashion, i.e. $\hat{\pi}\psi(k) = \psi(-k)$.

a) Determine the commutators: $[\hat{\pi},\hat{x}]$, $[\hat{\pi},\hat{k}]$, $[\hat{\pi},\hat{x}^2]$ and $[\pi,\hat{k}^2]$.

(16 marks)

b) Determine the eigenvalues of the parity operator. Show that, any state $|\psi\rangle$ which is an eigenfunction of the parity operator can be represented by a wave function which is either even $[\psi(x) = \psi(-x)]$ or odd $[\psi(x) = -\psi(-x)]$ about the origin.

(09 marks)

4. a) A particle is confined to a cubic box of edge length L with one corner at the origin and edges lined up with the coordinate axes. Using, e.g., separation of variables, solve the energy eigenvalue equation for this system in the region inside the box to obtain appropriately normalized energy eigenfunctions and eigenvalues, assuming that the wave function satisfies periodic boundary conditions at the edges of the box, i.e., $\phi(L,y,z) = \phi(0,y,z)$, $\phi(x,L,z) = \phi(x,0,z)$, and $\phi(x,y,L) = \phi(x,y,0)$.

(16 marks)

b) Are the energy eigenfunctions also eigenstates of momentum $\vec{P} = -i\hbar\vec{\nabla}$? Explain.

(09 marks)

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