RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four year) Degree in Industrial Mathematics
Fourth Year– Semester I Examination – June/July 2018

MAT 4301– OPERATIONAL RESEARCH I

INSTRUCTIONS:

- Answer **ALL** questions
- Time Allowed: **THREE** hours

1. Consider the following queuing model:

   Poisson arrival, Poisson departure, Single server, Infinite capacity and First come first served discipline.

   With the usual notation prove that when the system is in steady state, $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$.

   Also, prove that $P_n = \rho^n(1-\rho)$, expected waiting line in the system is $\frac{\rho}{1-\rho}$ and average waiting time of a customer in the queue is $\frac{\rho}{\mu(1-\rho)}$, where $\rho = \frac{\lambda}{\mu}$.

   Customers arrive at one-man barber shop according to a Poisson process with a mean inter-arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as a unit of time, then

   (i) what is the probability that a customer need not wait for a haircut?

   (ii) what is the expected number of customers in the barber shop and in the queue?

   (iii) how much time can a customer expect to spend in the barber shop?

   (iv) find the average time that a customer spends in the queue.

2. (a) Consider a queueing system with the queues which are being served by parallel service channels in which each server has an independently and identically distributed exponential service-time distribution, with the arrival process assumed to be Poisson. With the usual notation, show the probability that an arrival has to wait for service is $\frac{\mu\left(\frac{\lambda}{\mu}\right)^{C}}{(C-1)!(C\mu-\lambda)}P_0$ stating any other assumptions you may use.

A petrol pump station has 2 pumps. The service times follow the exponential distribution with a mean of 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle?

(b) A one-person barber shop has six chairs to accommodate people waiting for a haircut. Assume customers who arrive when all six chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 minutes in the barber chair.

(i) What is the probability that a customer can get directly into the barber chair upon arrival?

(ii) What percentage of time is the barber idle?

(iii) What is the expected number of customers waiting for a haircut?

3. Two oil companies, A and B, operating in a city, are trying to increase their market at the expense of the other. The company A is considering possibilities of decreasing price, giving soft drinks on Rs. 1500 purchases of oil or giving away a drinking glass with each 40 litre purchase. Obviously, company B cannot ignore this and comes out with its own programme to increase its share in the market. The payoff matrix from the viewpoints of increasing or decreasing market shares is given in the table below:

		Company B		
		Decrease Price	***Free Soft Drinks on Rs. 1500 purchase***	***Free Drinking Glass on 40 Litres or More***
Company A	***Decrease Price***	4	1	-3
	Free Soft Drinks on Rs. 1500 purchase	3	1	6
	Free Drinking Glass on 40 Litres or More	-3	4	-2

(i) Formulate the problem of finding optimal mixed strategies according to the minimax and maxmin criterion as a linear programming problem.

(ii) Use the simplex method to find these optimal mixed strategies.

4. Briefly explain the method in stepwise form to find the optimal sequence of processing n jobs in m machines.

A company has to process five jobs on three stages of production, viz, cutting, sewing, and pressing. Processing times are given in the following table:

Job	***Stages***		
	Cutting	***Sewing***	***Pressing***
1	3	3	5
2	8	4	8
3	7	2	10
4	5	1	7
5	2	5	6

(i) Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

(ii) Construct a table showing time in and time out of each stage and also, idle time of each activity.

(iii) Find the total minimum elapsed time if no passing of jobs is permitted.

(iv) Construct a Gannt chart showing the sequence of processing the jobs.

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