



**RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES**

**B. Sc. (Four Year) Degree in Industrial Mathematics
Fourth Year - Semester II Examination – July 2020**

MAT 4309 – COMBINATORICS

Time allowed: Three (03) hours

Answer all questions

1. a) Define a *difference set*. (05 marks)
 Construct a $(11, 5, 2)$ - difference set using quadratic residues. (35 marks)
 Hence, obtain a Hadamard matrix of order 12. (15 marks)
- b) Construct a $3 - (8, 4, 1)$ design. (30 marks)
 Hence, obtain $2 - (7, 3, 1)$ - design. (15 marks)
2. Define a *Steiner Triple System* STS (n) of order n . (05 marks)
 Construct STS (7). (30 marks)
 Using STS (7), construct STS (21) and STS (49). (40 marks)
 Hence, construct STS (63). (25 marks)
3. a) Determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ given by $a_n = n \cdot 4^n$ is a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$, $n \geq 3$. (40 marks)
- b) Solve each of the following recurrence relations:
 - (i) $a_n = 7a_{n-1} - 12a_{n-2}$ where $a_0 = 5$, $a_1 = 8$ and $n \geq 2$. (30 marks)
 - (ii) $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ where $a_0 = 7$, $a_1 = -4$, $a_2 = 8$ and $n \geq 3$. (30 marks)

4. a) Solve the recurrence relation $a_n = 3a_{n-1} + 4^{n-1}$ with the initial condition $a_0 = 1$, using the generating function. (65 marks)
- b) Construct a tournament schedule for *seven* teams. Hence, obtain a schedule for *eight* teams. (35 marks)
5. a) Find the number of bit strings of length *six* that starts with the digit 1 and ends with 00s. (25 marks)
- b) What is the minimum number of students required in a discrete mathematics class to be sure that at least *ten* will receive the same grade, if there are *five* possible grades, *A, B, C, D* and *E*? (20 marks)
- c) How many different license plates can be made using *three* English Alphabets followed by *three* digits? (25 marks)
- d) How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16? (30 marks)
6. a) Each user on a computer system has a password which is **seven** to **eight** characters long where each character is English Alphabet letter or a digit. How many possible passwords are there if each password must contain at least one digit? (50 marks)
- b) Define Bell numbers. Find the first four Bell numbers. (25 marks)
- c) Define Catalan numbers. Find the first four Catalan numbers. (25 marks)

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