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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – October/November 2017

MAA 1203 – Numerical Analysis I

Time: Two (2) hours

Answer **Four** Questions **only**.

Calculators will be provided.

Newton's Forward formula

$$y_k = \sum_{i=0}^{n} \binom{k}{i} \Delta^i y_0$$

Newton's Backward formula

$$y_k = y_0 + k\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \frac{k(k+1)(k+2)}{3!}\nabla^3 y_0 + \ldots + \frac{k(k+1)\ldots(k+n-1)}{n!}\nabla^n y_0$$

Gauss's Forward formula

$$y_k = y_0 + \sum_{i=1}^{n}\left[\binom{k+i-1}{2i-1}\Delta^{2i-1} y_{1-i} + \binom{k+i-1}{2i}\Delta^{2i} y_{-i}\right]$$

Bessel's formula

$$y_k = y_0 + k\Delta y_0 + \frac{k(k-1)}{2!} \times \left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \frac{k(k-1)(k-1/2)}{3!}\Delta^3 y_{-1} + \frac{(k+1)k(k-1)(k-2)}{4!} \times \left(\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}\right) + \ldots$$

01

a) Briefly explain the steps of Newton Raphson method to solve system of m non-linear equations with n unknowns. **(30 marks)**

b) Solve the following system of non-linear equations using Newton's Raphson Method by assuming the initial estimation as $\begin{pmatrix} x_{10} \\ x_{20} \\ x_{30} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. (Proceed only two iterations)

$$3x_1 - Cos(x_2 x_3) - 1.5 = 0$$
$$4x_1^2 - 625x_2^2 + 2x_3 - 1 = 0$$
$$20x_3 + e^{-x_1 x_2} + 9 = 0$$

(70 marks)

02

a) Briefly explain the applicability of Newton's Forward formula in interpolation. **(10 marks)**

b) The population of a town is shown in the following table for several years.

Year	1891	1901	1911	1921	1931
Population in thousands	46	66	81	93	101

i. Formulate a cubic polynomial to represent the above data set by using suitable interpolation formula. **(60 marks)**

ii. Use the above cubic polynomial to estimate the population for the year 1895. **(30 marks)**

03

a) Briefly explain the steps of the Bisection Method for solving non-linear equation. **(10 marks)**

b) Consider the following non-linear equation.

$$f(x) = x^3 - \frac{7}{x} + 2, \quad x > 0$$

i. Show that $f(x) = 0$ has a root between 1.4 and 1.5.

ii. Starting with interval [1.4, 1.5], use the Bisection Method to find a root with two decimal places accuracy.

(50 marks)

c) The following table summarizes marks of 190 students for mathematics of a particular class. Use Bessel's interpolation formula to estimate the number of students who obtained marks between 50 and 55. (Consider up to 2nd difference operator)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of Students	31	42	51	35	31

(40 marks)

04

a) Prove the Gauss's Forward formula by using Newton's Forward formula.

(40 marks)

b) Consider the following tabula data,

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	0.841	0.891	0.932	0.963	0.985

i. Formulate a cubic polynomial in terms of k to represent the above data set by using Gauss's Forward interpolation formula.

ii. Use the above cubic polynomial to estimate $f(1.22)$.

(60 marks)

05

a) Write down an algorithm for solving first order ordinary differential equations by using Taylor's series method. **(30 marks)**

b) Solve the initial value problem $\frac{dy}{dx} = yCos(x)$, $y(0) = 1$ for y at $x = 1$ with step length 0.5 using Taylor's series method of order three. **(70 marks)**

END