RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Fourth Year Semester II Examination – Jan / Feb 2023

PHY 4203 – CLASSICAL MECHANICS

Time: Two (02) hours

Answer All Questions.

Unless otherwise specified, symbols have their usual meaning.

Useful information:

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta},\ \frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r},\ \frac{d(r^2\dot{\theta})}{dt} = 2r\dot{r}\dot{\theta} + r^2\ddot{\theta}$$

Potential energy of an object moving in a circular path $= -\frac{GMm}{r}$

Angular momentum, $\vec{L} = I\vec{\omega}$, $\tau = \frac{dL}{dt}$

Generalized force, $Q_j = \sum_{i=1}^{N} \vec{F}_i \cdot \frac{\partial r_i}{\partial q_j}$

Lagrangian, $L = T - V$

Lagrangian equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

Generalized momenta, $P_i = \frac{\partial L}{\partial \dot{q}_i}$

Hamiltonian, $H = \sum_{i=1}^{N} P_i\dot{q}_i - L$

Hamiltonian equation of motion, $\frac{\partial H}{\partial P_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{P}_i$

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1) Suppose that the position of a particle of mass m moving on a plane is given by the polar coordinates (r, θ).

   a) Write down the equations for radial and transverse velocity vectors of the particle. (02 marks)

Contd.

b) Show that the magnitudes of radial and transverse accelerations of the particle are given by $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = \frac{1}{r}\frac{d}{dt}r^2\dot{\theta}$

(04 marks)

c) Assume that the earth is a uniform sphere of radius R and mass M. Suppose a satellite of mass m is traveling in an elliptical orbit around the earth. The speed of the satellite is v_o when it is at the closest distance to the earth, which is $4R$ from the centre of the earth. The gravitation constant is G.

i. What is the angular momentum of the satellite?
ii. Obtain an expression for the total energy of the satellite.
iii. If the maximum distance to the satellite from the center of the earth is $5R$, find v_o in terms of given constants.

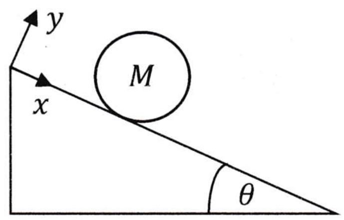
(09 marks)

2) a) Identifying the symbols used, write down the following theorems related to moment of inertia of a system.

i. Parallel axis theorem.
ii. Perpendicular axis theorem.

(04 marks)

b) A solid cylinder of radius R and mass M rolls down without slipping on a rough surface of a fixed wedge inclined to the horizontal at an angle θ.



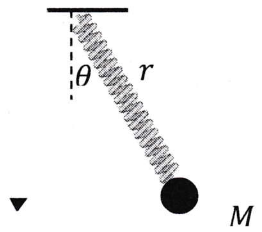
i. Using vector analysis, show that the acceleration is constant and equals to $\frac{2}{3}g\ Sin\ \theta$. ($g$ is the gravitational constant, moment of inertia of the cylinder at center, $I = \frac{1}{2}MR^2$)

(08 marks)

ii. Prove that the coefficient of static friction, μ must be at least $\frac{1}{3}\tan\theta$ for not slipping.

(03 marks)

3) Consider a particle of mass M is connected to a massless spring with length r, suspended freely as shown in the figure. The spring constant is k and the natural length of the spring is l.



Contd.

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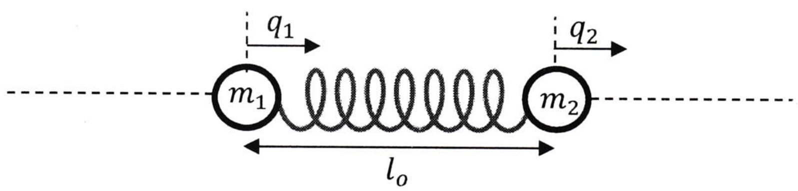
a) Determine the generalized force component Q_r and Q_θ of the forces acting on the system by considering plane polar coordinates.

(06 marks)

b) Write the Lagrangian, L of the system. (03 marks)

c) Hence, determine the equations of motion. (06 marks)

4) Consider the system of two particles connected by a spring of spring constant k and natural length l_o as shown in the figure.



The motion of the system is described by the two generalized coordinates (q_1, q_2) measured along the line joining two masses. Consider the extension as $(q_2 - q_1)$.

a) Write the Lagrangian, L of the system (03 marks)

b) Hence, determine the Hamiltonian, H of the system (06 marks)

c) Determine expressions for $\ddot{q}_1$ and $\ddot{q}_2$ (06 marks)

End.