RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B. Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – February/March 2019
MAP 1203 – REAL ANALYSIS I

Time: Two (02) hours

Answer all (04) questions

1. a) Prove that $\sqrt{7}+\sqrt{11}$ is an irrational number.

   b) Define the following terms for a set:
      i. Bounded.
      ii. Supremum (Least Upper Bound).
      iii. Infimum (Greatest Lower Bound).

   c) Find the Supremum, Infimum, Maximum and Minimum for each the following sets if they exist:
      i. $\left\{\frac{4n+3}{n}\middle| n\in N\right\}$.
      ii. $\left\{1,1+\frac{1}{2},1+\frac{1}{2}+\frac{1}{2^2},\dots,1+\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^{n-1}},\dots\middle| n\in N\right\}$.
      iii. $\left\{-\frac{n+1}{n}\middle| n\in N\right\}$.
      iv. $\left\{(-1)^n\frac{1}{n}\middle| n\in N\right\}$.

   d) Let A and B be the sets which are bounded above. Show that $Sup(A+B)=SupA+SupB$.

2. a) Prove that '*Every convergent sequence has a unique limit*'.

   b) Using the definition, show that
      i. $a_n=\frac{3n-1}{4n+5}$ converges to $\frac{3}{4}$, where $n\in N$.
      ii. $a_n=\frac{1}{n}$ converges to 0, where $n\in N$.

   c) Show that the sequence $\{x_n\}$ where $x_1=1$ and $x_n=\sqrt{2+x_{n-1}}$ for all $n\geq 2$ is monotonic, convergent and converges to 2.

3. a) State $\varepsilon-\delta$ definition for limit of a function. Prove that, $\lim_{x\to a}\dfrac{x^2-a^2}{x-a}=2a$.

b) Find $\lim_{x\to c} f(x)$, where

$$f(x)=\begin{cases}\dfrac{x^2}{c}-c & , for\ 0<x<c\\ 0 & , for\ x=0\\ c-\dfrac{c^2}{x^2} & , for\ x>c\end{cases}.$$

c) State $\varepsilon-\delta$ definition for continuity of a function at a point. Using the definition prove that $f(x)$ is continuous at $x=0$, where

$$f(x)=\begin{cases}x\sin\dfrac{1}{x}, & for\ x\neq 0\\ 0 & , for\ x=0\end{cases}.$$

4. a) Define a derivative of a function at a point. Show that the following function $f(x)$ is continuous at $x=1$, for all values of p,

where $f(x)=\begin{cases}px+1 & , for\ x\geq 1\\ x^2+p & , for\ x<1\end{cases}$.

Find the condition for existence of the derivative at $x=1$.

b) State the following theorems

i. Rolle's Theorem.

ii. Lagrange Mean Value Theorem.

iii. Cauchy's Mean Value Theorem.

c) Show that $x-\dfrac{x^2}{2}<\log(x+1)<x-\dfrac{x^2}{2(1+x)}$, where $x>0$.

d) State L'Hospital rule.

Evaluate $\lim_{x\to 0}\dfrac{\tan x-x}{x^2\tan x}$.

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