RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester II Examination – November/December 2016

MAA 2203 – Numerical Analysis II

Time: Two (2) hours

Answer **ALL** questions. **Calculators will be provided**

1. **Select the most appropriate answer.** **(120 marks)**

a) The following x-y data is given

x	15	18	22
y	24	37	25

The Newton's divided difference second order polynomial for the above data is given by

$$f_2(x) = b_0 + b_1(x-15) + b_2(x-15)(x-22)$$

The value of b_1 is

$i.\ -1.048$ $ii.\ 0.1433$ $iii.\ 4.333$ $iv.\ 24.00$

b) The polynomial that passes through the following x-y data

x	18	22	24
y	?	25	123

Is given by $8.125x^2 - 324.75x + 3237, 18 \le x \le 24$

The corresponding polynomial using Newton's divided difference polynomial is given by

$$f_2(x) = b_0 + b_1(x-18) + b_2(x-18)(x-22)$$

The value of b_2 is

i. 0.2500 *ii.* 8.125 *iii.* 24.00 *iv.* Not obtainable with the information given

c) Velocity vs. time data for a body is approximated by a second order Newton's divided difference polynomial as

$$v(t) = b_0 + 39.622(t-20) + 0.5540(t-20)(t-15), 10 \le t \le 20$$

The acceleration in m/s^2 at is

i. 0.5540 m/s^2 *ii.* 39.622 m/s^2 *iii.* 36.852 m/ s^2 *iv* Not obtainable with the information given

d) The path that a robot is following on a *x*-*y* plane is found by interpolating the following four data points

x	2	45	5.5	7
y	7.5	7.5	6	5

$y(x) = 0.1524x^3 - 2.257x^2 + 9.605x - 3.900$
The length of the path from x=2 to x=7 is

i. $\sqrt{(7.5-7.5)^2+(4.5-2)^2} + \sqrt{(6-7.5)^2+(5.5-4.5)^2} + \sqrt{(5-6)^2+(7-5.5)^2}$

ii. $\int_2^7 \sqrt{1+(0.1524x^3 - 2.257x^2 + 9.605x - 3.900)^2}\, dx$

iii. $\int_2^7 \sqrt{1+(0.4572x^2 - 4.514x + 9.605)^2}\, dx$

iv. $\int_2^7 (0.1524x^3 - 2.257x^2 + 9.605x - 3.900)\, dx$

e) The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at seconds, the three data points of time you would choose for interpolation are

i. 0,15,18 *ii.* 15,18,22 *iii.* 0,15,22 *iv.* 0, 18, 24

f) The value of $\int_{0.2}^{2.2} e^x dx$ by using n=2 **Simpson's rule** is most nearly

i. 7.8306 *ii.* 7.8423 *iii.* 8.4433 *iv.* 10.246

g) The value of $\int_{0.2}^{2.2} xe^x dx$ by using the three-segment **trapezoidal rule** is most nearly

i. 11.672 *ii.* 11.807 *iii.* 12.811 *iv.* 14.633

h) The value of integral $\int_0^1 x^2 dx$ by using the two sub interval **midpoint rule** is most nearly,

i. $\frac{5}{16}$ *ii.* $\frac{1}{4}$ *iii.* $\frac{4}{9}$ *iv.* $\frac{2}{5}$

i) The power series form of the quadratic interpolating polynomial $P_2(x) = a_0 + a_1(x) + a_2x^2$ Interpolating the data (−1, 0), (0, 1) and (1, 3).What is the value of a_0, a_1 *and* a_3?

i. $a_0 = 1, a_1 = \frac{3}{2}$ *and* $a_2 = \frac{1}{2}$. *ii.* $a_0 = 1, a_1 = \frac{1}{2}$ *and* $a_2 = \frac{1}{2}$

iii. $a_0 = 1, a_1 = \frac{3}{2}$ *and* $a_2 = \frac{3}{2}$ *iv.* $a_0 = 1, a_1 = \frac{1}{2}$ *and* $a_2 = \frac{3}{2}$

j) Given the two points $[a, f(a)], [b, f(b)]$,the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by

i. $f_1(x) = \frac{x-b}{a-b}f(a) + \frac{x-a}{a-b}f(b)$

ii. $f_1(x) = \frac{x}{b-a}f(a) + \frac{x}{b-a}f(b)$

iii. $f_1(x) = f(a) + \frac{f(b)-f(a)}{b-a}(b-a)$

iv. $f_1(x) = \frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b)$

k) The following data of the velocity of a body is given as a function of time.

Time (s)	10	15	18	22	24
Velocity (m/s)	22	24	37	25	123

A quadratic Lagrange interpolant is found using three data points, t=15, 18 and 22. From this information, at what of the times given in seconds is the velocity of the body 26 m/s during the time interval of t=15 to 22 seconds.

i. 20.173 *ii.* 21.858 *iii.* 21.667 *iv.* 22.020

l) The Lagrange polynomial that passes through three data points is given by

x	15	18	22
y	24	37	25

$$f_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

The value of $L_1(x)$ at $x = 16$ is

$i. -0.071430$ $ii. 0.50000$ $iii. 0.57143$ $iv. 4.3333$

2. A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table where the time is in seconds, the distance is in meter, and the speed is in ms^{-1}. **(90 marks)**

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

a) Find the degree of the polynomial.

b) Use a **Hermite polynomial** to Find the **position of the car** and **speed** when $t = 10s$.

c) Use the **Derivative of the Hermite polynomial** to determine whether the car ever exceeds a $55ms^{-1}$ speed limit on the road. What is the first time the car exceed this speed?

d) What is the **predicted maximum speed** for the car?

3. Construct the natural cubic spline using the data for the function $y = f(x)$ and with its help find the approximate values for $f(0.5)$and $f(1.7)$. **(90 marks)**

x_i	0.1	1	1.5	2	2.5
y_i	-0.7	0	1	0.8	1.2

END