



**RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES**

**B.Sc. (General) Degree in Applied Sciences
Second Year - Semester II Examination – September/ October 2020**

MAA 2201 – MATHEMATICAL METHODS II

Time: Two (02) hours

Answer all (04) questions

1. A coordinate system (u, v, w) is related to the cartesian coordinates by

$$x = uvw, y = uv(1 - w^2)^{1/2}, z = \frac{(u^2 - v^2)}{2}.$$

- a) Find the scale factors h_u, h_v , and h_w . **(15 points)**
 - b) Show that (u, v, w) system is orthogonal. **(05 points)**
 - c) Find the volume element in the (u, v, w) coordinate system. **(05 points)**
2. a) Use Green's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where
 $\vec{F}(x, y) = \langle y \cos x - xy \sin x, xy + x \cos(x) \rangle$ and C is the triangle with vertices
 $(0,0), (0,4)$ and $(2,0)$ in counterclockwise direction. **(15 points)**
- b) Use the Divergence Theorem to calculate the surface integral $\iint_S \vec{F} \cdot d\vec{s}$ where
 $\vec{F}(x, y) = \langle xye^z, xy^2z^3, -ye^z \rangle$ and S is the surface of the box bounded by
the coordinate planes and the planes $x = 5, y = 2$, and $z = 1$. **(10 points)**

3. a) Using the Laplace transforms, find the solution of the initial value problem

$$\frac{d^2 y}{dt^2} + 25y = 10 \cos 5t, \quad y(0) = 2, \quad y'(0) = 0.$$

(15 points)

- b) Applying convolution, solve the following initial value problem

$$\frac{d^2 y}{dt^2} + y = \sin 3t, \quad y(0) = 0, \quad y'(0) = 0.$$

(10 points)

4. a) Find the Fourier Transform of the function

$$f(x) = \begin{cases} 1 + \frac{x}{a}; & -a < x < 0 \\ 1 - \frac{x}{a}; & 0 < x < a \\ 0 & ; \text{otherwise} \end{cases}$$

(15 points)

- b) Let $F(x)$ be the Complex Fourier Transform of $f(x)$.

Show that:

$$(i) \quad F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right), \quad \text{where } a \neq 0.$$

(05 points)

$$(ii) \quad F\{f(x-a)\} = e^{-isa} F(s).$$

(05 points)