RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Third Year - Semester II Examination – July 2020

MAT 3302 – NETWORK OPTIMIZATION

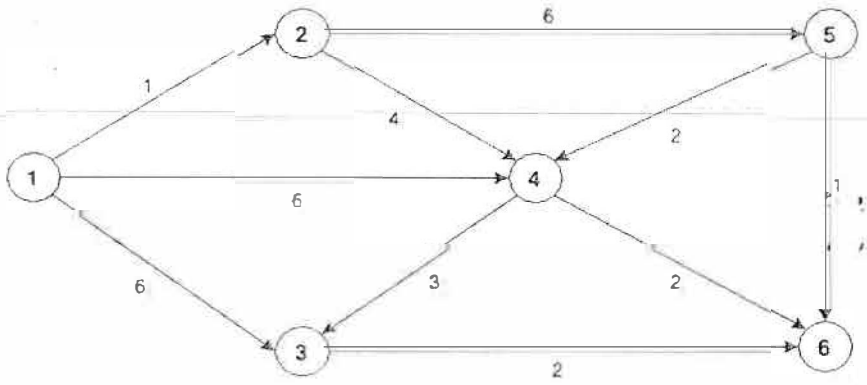
Time: Three (03) hours

Answer only five questions.

Calculators and Statistical tables will be provided.

1. a) Compare and contrast the Floyd's and Dijkstra's Algorithm in finding the shortest path of a weighted network. (10 marks)

b) The network shown below gives the distances in miles among six cities. Use this network to answer the following questions.



i. Formulate a linear programming model to find the shortest path from City 1 to City 6.

ii. Use the Dijkstra's Algorithm to find the shortest paths from City 1 to all the other cities.

iii. Justify your answer for part (ii) using the Floyd's Algorithm.

(90 marks)

2. a) Define each of the following terms:

   i. Connected graph

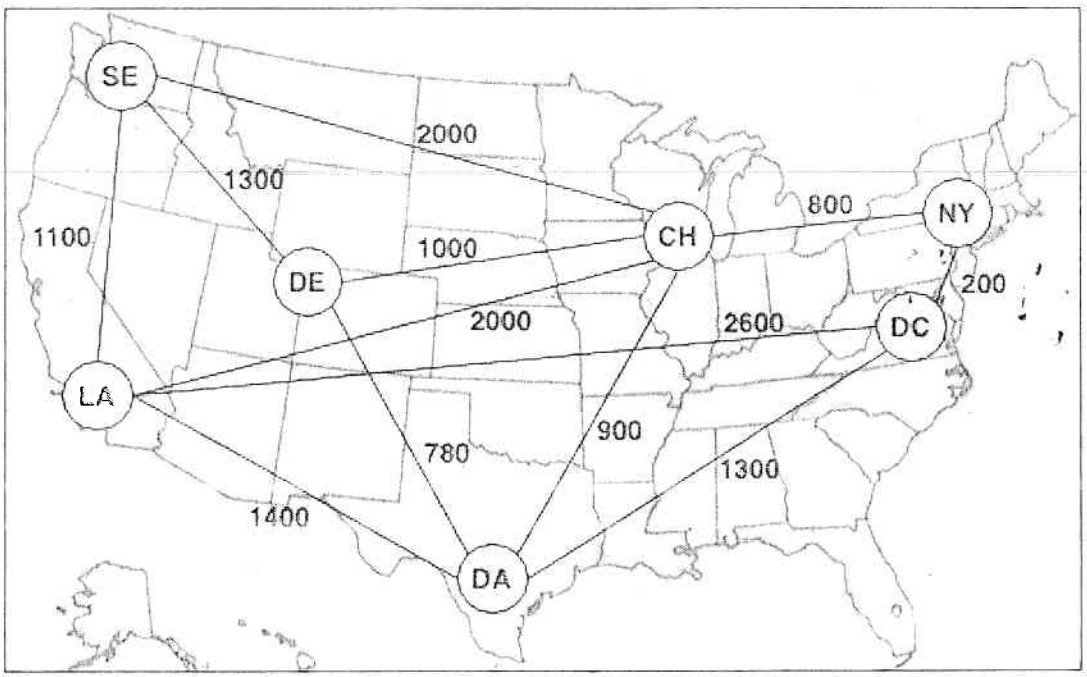
   ii. Spanning tree

   iii. Minimum spanning tree

(15 marks)

b) State the steps of the Minimum Spanning Tree Algorithm. **(30 marks)**

c) In international transportation, loaded truck trailers are shipped between railroad terminals on special flatbed carts. The following figure shows the locations of the main railroad terminals in the United States and the existing railroad tracks with lengths (in miles). The objective is to decide which tracks should be selected to handle the intermodal traffic such that the total length of the selected tracks is minimized. In particular, the Los Angeles (LA) terminal must be liked directly to Chicago (CH) to accommodate expected heavy traffic. Other than that, all the remaining terminals can be liked directly or indirectly. Determine the railroad tracks that must be included to optimize the traffic and total length of the selected tracks using the Minimum Spanning Tree Algorithm (clearly mention the steps).



(55 marks)

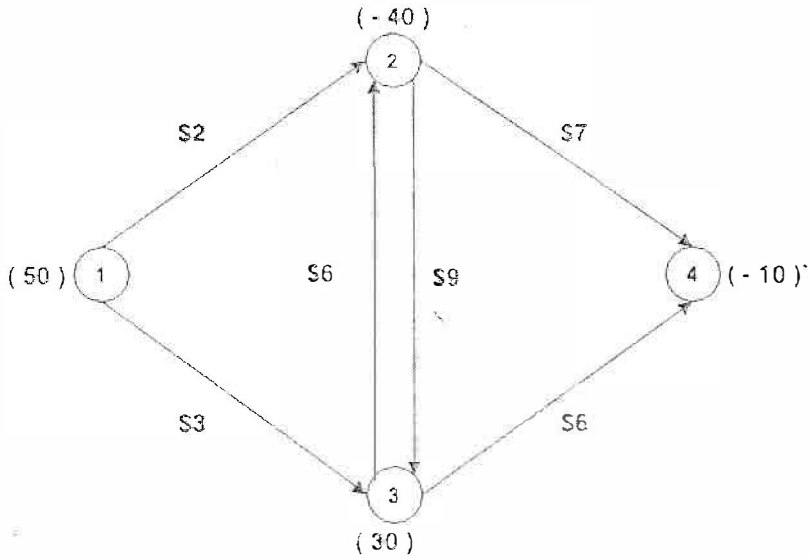
3. a) Write down the general mathematical model of the Minimum Cost Flow Problem.

(15 marks)

b) Briefly explain steps of the Network Simplex Method in solving the Minimum Cost Flow Problem.

(20 marks)

c) Consider the following network diagram which provides the unit costs of arcs and, the supplies and demands of nodes:



Starting with x_{13}, x_{23}, x_{32} and x_{34} as the non-basic variables, find the least cost shipment through the network to satisfy the demands using the Network Simplex Method.

(65 marks)

4. A petroleum company is planning to transport crude oil from an oil well to a refinery through a pipeline network. The following table gives the capacity and direction of each pipeline segment. The oil well and the refinery are denoted by node 1 and node 6 respectively. Intermediate boosters and pumping stations are denoted by the remaining nodes.

Pipeline segment	Flow capacity
1 → 3	45
1 → 4	110
2 → 3	70
4 → 2	40
2 → 5	30
3 → 5	50
3 → 6	45
5 → 6	40

a) Find the maximum capacity and the flow schedule of the pipeline network between the oil well and the refinery using the Maximum Flow Algorithm. (clearly state the steps) **(85 marks)**

b) Verify your result using the Maximum Flow – Minimum Cut Theorem (Enumeration of Cuts Method). **(15 marks)**

5. a) Compare and contrast the Project Evaluation and Review Technique (PERT), and the Critical Path Method (CPM). **(10 marks)**

b) A software developing team of a particular IT company is planning a schedule for a newly assigned project. All the related activities of the project and their information are provided in the table below. Answer the following questions by assuming that the team will work a standard working week (5 working days for a week) and all activities will start as soon as possible.

Activity	Description	Duration (in days)	Predecessors
A	Requirement Analysis	10	-
B	Systems design	15	A
C	Programming	25	B
D	Telecoms	20	B
E	Hardware installation	35	B
F	Integration	5	C and D
'G	System testing	15	E and F
H	Handover and go-live	5	G

i. Develop a network to represent the above project by denoting activities on arcs.

ii. Determine the critical path and the project completion time in weeks by using CPM.

iii. Calculate the total float and free float in weeks for all non-critical activities of the project. **(55 marks)**

c) A project consists of 7 activities. Optimistic time estimate (O), pessimistic time estimate (P), and most likely estimate (M) of the activities are given in the table below. Assuming that the A_1, A_3, A_5 and A_7 as critical activities answer the following questions.

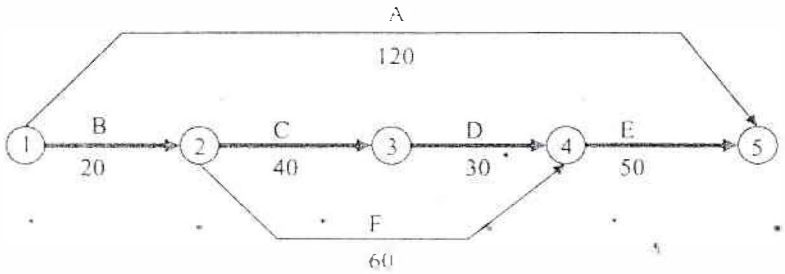
Activity	O (days)	M (days)	P (days)
A_1	7	9	10
A_2	5	8	11
A_3	4	6	7
A_4	5	7	9
A_5	3	4	6
A_6	4	6	7
A_7	8	11	12

i. Calculate the expected project completion time and expected variance of the project completion time.

ii. Evaluate the probabilities that the project will be completed within 28 days and going over 30 days.

(35 marks)

6. a) The network given below shows the normal schedule and the critical path of a construction project. You can crash the durations at an additional expense.



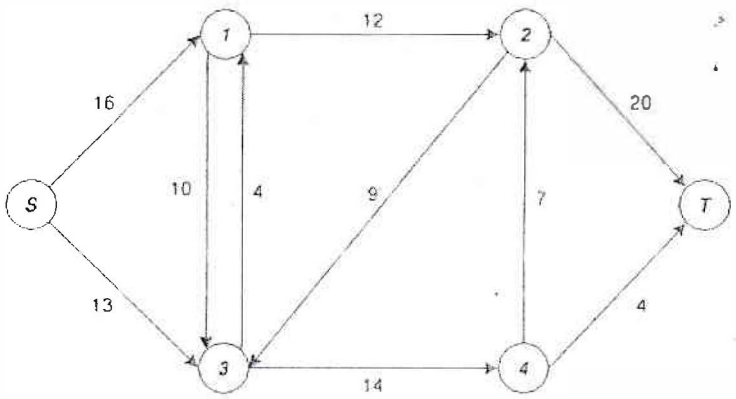
The following table summarizes the time-cost information of the activities in the project. The owner wants you to finish the project as soon as possible while minimizing the crashing cost. Find the duration, schedule, and total cost of the crashed project.

Activity	Normal Time (days)	Crash Time (days)	Normal Cost ($)	Crashed Cost ($)
A	120	100	12000	14000
B	20	15	1800	2800
C	40	30	16000	22000
D	30	20	1400	2000
E	50	40	3600	4800
F	60	45	13500	18000

(50 marks)

b) i. Briefly explain the mechanism of the Ford-Fulkerson Algorithm for solving the Maximum Flow Problem.

ii. Consider the following network:



Using the Ford-Fulkerson Algorithm together with the initial flow along $S \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow T$, find the maximum flow, from node S to node T.

(50 marks)