RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree

First Year - Semester II Examination - March /April 2014

MAA 1104 – MATHEMATICAL MODELLING

Answer **all** questions. Time allowed: One hour

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1. (a) Let $x(t)$ be the population size of a certain species at time t and let b be the birth rate and d be the death rate.
   Show that $\frac{dx}{dt} = (b-d)x$

   (b) Suppose that a lake of constant volume V contains at time t an amount $Q(t)$ of pollutant evenly distributed through the lake with a concentration $c(t)$, where $c(t) = \frac{Q(t)}{V}$. Assume that water containing a concentration C of a pollutant enters the lake at a rate R and after mixing thoroughly with the water in the lake the mixture leaves the lake at the same rate. Find an expression for the concentration $c(t)$ at any time t.

2. (a) Let us consider a 95^0 C coffee cup that is in a 30^0 C room. Assume that the proportionality constant is 0.1.
   Find a mathematical equation describing the temperature of the coffee as a function of time.

   (b) Suppose that a savings account pays interest annually at a rate of 5%. An investor deposits an amount $Rs.\ P$ and withdraw $Rs.1000$ from the account at the end of the year.
   Let Y_t be the amount of money in the account after t years.

   (i) Find a mathematical equation for Y_t.

   (ii) Find the general solution of the above equation.

   (iii) If the investor ensures that he can withdraw $Rs.1000$ each year for the next 15 years, maintaining a non-negative balance. Show that

$$P \geq 20000\left(1 - \frac{1}{(1.05)^{15}}\right)$$