RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (4 Year) Degree in Industrial Mathematics
Fourth Year Semester II Examination – June / July 2018

MAT 4306 – OPTIMIZATION MODELLING

INSTRUCTIONS:

- Show all work, simplify your answers and write out your work neatly for full credit.
- Answer **SIX** questions only.
- Time Allowed: **THREE** hours.
- ***Calculators will be provided***

1. A cargo plane has three compartments for storing cargo: front, center and rear. These compartments have the following limits on both weight and space:

Compartment	Weight capacity (tonnes)	Space capacity (cubic metres)
Front	10	6800
Center	16	8700
Rear	8	5300

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane. The following four cargoes are available for shipment on the next flight:

Cargo	Weight (tonnes)	Volume (cubic metres/tonne)	Profit ($/tonne)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

Any proportion of these cargoes can be accepted. The objective is to determine *how much* (if any) of each cargo C1, C2, C3 and C4 should be accepted and *how to*

P.T.O

2

distribute each among the compartments so that the total profit for the flight is maximized. Formulate this as a linear programming model. Do not solve.

2. Transform each of the following optimization problems into regular linear or integer liner programming problems:

(a) Min $|x_1| + 3|x_2| - 2x_3$

subject to $2x_1 - 3x_2 + 4x_3 \le 5$

$x_1 - 2x_2 + 3x_3 \ge 3$

and x_1, x_2 free and $x_3 \ge 0$.

(b) Min $\dfrac{x_1 - 3x_2 + 5x_3 + 7}{2x_1 - 6x_2 - 5x_3 + 8}$

subject to $2x_1 - 3x_2 + 4x_3 \ge 10$

$-10x_1 + 5x_2 + 3x_3 \ge 5$

and $x_j \ge 0$ for all $j = 1,2,3$.

(c) Min $\left(\mathbf{Max} \sum_{j=1}^{n} c_j x_j \right)$

subject to $\sum_{j=1}^{n} a_{ij} x_j \ge b_i$ for all $i = 1,2,\ldots,m$.

and $x_j \ge 0$ for all $j = 1,2,\ldots,n$.

(d) Max $\sum_{j=1}^{n} c_j x_j$

subject to either $\sum_{j=1}^{n} a_{1j} x_j \le b_1$ or $\sum_{j=1}^{n} a_{2j} x_j \le b_2$

and $x_j \ge 0$ for all $j = 1,2,\ldots,n$.

Cts...

3. The production manager of a chemical plant is attempting to devise a shift pattern for his workforce. Each day of every working week is divided into three eight-hour shift periods (00:01-08:00, 08:01-16:00, 16:01-24:00) denoted by night, day and late respectively. The plant must be manned at all times and the minimum number of workers required for each of these shifts over any working week is as below:

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
Night	5	3	2	4	3	2	2
Day	7	8	9	5	7	2	5
Late	9	10	10	7	11	2	2

The union agreement governing acceptable shifts for workers is as follows:

(i) Each worker is assigned to work *either* a night shift *or* a day shift *or* a late shift and once a worker has been assigned to a shift they must remain on the same shift every day that they work.

(ii) Each worker works four consecutive days during any seven day period.

In total there are currently 60 workers. Formulate the production manager's problem as a integer linear programming model. Do not solve.

4. The Rugger Corporation is a Seattle-based R&D company that recently developed a new type of fiber substrate that is waterproof and resists dirt. Several carpet manufactures in northeast Georgia want to use Rugger as their sole supplier for this new fiber. The locations of the carpet manufactures are summarized in the following table:

Carpet Milll Locations	X-Coordinates	Y-Coordinates
Dalton	9	43
Rome	2	28
Canton	51	36
Kennesaw	19	4

Rugger expects to make 130, 75, 90 and 80 deliveries to the carpet producers in Dalton, Rome, Canton, and Kennesaw respectively. The company wants to build its new plant in the location that would minimize the annual shipping miles. However Rugger also wants to be within 50 miles each of the new customers so that it will be easy to provide on-site technical support for any production problems that may occur. Formulate an nonlinear programming model to this problem. Do not solve.

TA

5. (a) Determine the maximum and minimum values of the function:

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5.$$

(b) Let $f : \Re^2 \to \Re$ be defined by $f(x,y) = (x-y)^4 + x^2 - y^2 - 2x + 2y + 1$. Suppose that we wish to minimize f over $\Re^2$. Find all the points satisfying the FONC. Do these points satisfy the SONC?

(c) Using *Lagrange Multiplier Method or otherwise* solve the following problem:

$$\text{Minimize} \quad f(x,y) = x^2 + 2y^2$$
$$\text{subject to} \quad x^2 + y^2 = 1.$$

6. (a) Consider the following optimization problem:

$$\text{Minimize} \quad f(x,y) = x^2 + \frac{1}{2}y^2 + 3y + \frac{9}{2}$$

$$\text{subject to } (x,y) \in \Omega = \{ (x,y) \in \Re^2 \mid x \geq 0,\ y \geq 0 \}.$$

(i) Find the gradient and Hessian of f at the point $(0,0)^T$.

(ii) Find the directional derivative of f at $(0,0)^T$ in the direction $d = (d_1, d_2)^T$.

(iii) Does the point $(0,0)^T$ satisfy the first-order necessary condition for a minimizer? Justify your answer.

(b) Write the Kuhn-Tucker optimality conditions for the following optimization problem and verify that these conditions are true at the point $\bar{x} = \left(\frac{3}{2}, \frac{9}{4}\right)^T$.

$$\text{Minimize} \quad f(x,y) = \left(x - \frac{9}{4}\right)^2 + (y-2)^2$$

$$\begin{aligned} \text{subject to} \quad & y - x^2 \geq 0 \\ & x + y \leq 6 \\ & x \geq 0 \text{ and } y \geq 0. \end{aligned}$$

7. Write down the algorithm of the Hooke-Jeeves pattern search method.

Perform **THREE** iterations of the Hooke-Jeeves pattern search method to solve the following problem: Minimize $f(x,y) = 3x^2 + y^2 - 12x - 8y$.

Take the initial point $x^{(0)} = (1, 1)^T$ and acceleration factor $\alpha = 2$ with $\Delta = (\frac{1}{2}, \frac{1}{2})^T$.