RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

Bachelor of Science in Applied Sciences
Third Year - Semester I Examination – July/Aug 2023

MAT 3301 – ADVANCED LINEAR PROGRAMMING

Time: Three (03) hours

Answer **all** questions.

1. a) Briefly explain the structural differences between the Simplex and the Dual Simplex algorithms. **(20 marks)**

   b) The Dual Simplex algorithm cannot be applied to solve every Linear Programming Problem (LPP). State the conditions that a given LPP must satisfy in order to be solvable using the Dual Simplex algorithm. **(10 marks)**

   c) Solve the following LPP using the Dual Simplex algorithm:

$$\text{Max } Z = -3x_1 - 2x_2$$
$$\text{S.t.}$$
$$x_1 + x_2 \geq 1,$$
$$x_1 + x_2 \leq 7,$$
$$x_1 + 2x_2 \geq 10,$$
$$x_2 \leq 3,$$
$$x_1, x_2 \geq 0.$$

(70 marks)

2. a) Write short notes for the following special cases of the assignment problem:

   i. Unbalanced assignment problems

   ii. Assignment problems with restrictions

   iii. Maximization type assignment problems

   iv. Multiple solutions in Assignment problems **(30 marks)**

b) A city corporation has decided to carry out road repairs on four main roads of the city. The government has agreed to provide a special grant of Rs. 5,000,000 towards the cost, with the condition that the repairs be done at the lowest cost and in the shortest time possible. If the conditions warrant, a supplementary token grant will also be considered favorably. The corporation has invited tenders, and five contractors have submitted their bids. The bids of all the contractors are summarized in the table below. To expedite the work, each road will be awarded to only one contractor.

Contractors	Bids for the roads (Rupees in lakhs)			
	R1	R2	R3	R4
C1	9	14	19	15
C2	7	17	20	19
C3	9	18	21	18
C4	10	12	18	19
C5	10	15	21	16

i. Determine the best way of assigning the repair work to the contractors and the associated costs.
ii. Is a supplementary grant required? If yes, what should be the sought amount?
iii. Which of the five contractors will not be successful in their bid?

(70 marks)

3. a) Explain the concept of degeneracy in linear programming. How do you identify degeneracy in transportation problems? **(10 marks)**

b) Explain what alternative solutions are in linear programming. How do you determine the existence of alternative solutions for a transportation problem? **(10 marks)**

c) How do you solve a transportation problem with prohibited routes? **(05 marks)**

d) Consider the following transportation model:

$$\text{Min } Z = 8x_{11} + 9x_{12} + 7x_{13} + 4x_{21} + 3x_{22} + 5x_{23} + 8x_{31} + 5x_{32} + 7x_{33}$$

S.t.

$$x_{11} + x_{12} + x_{13} \leq 40,$$
$$x_{21} + x_{22} + x_{23} \leq 25,$$
$$x_{31} + x_{32} + x_{33} \leq 35,$$
$$x_{11} + x_{21} + x_{31} = 30,$$
$$x_{12} + x_{22} + x_{32} = 30,$$
$$x_{13} + x_{23} + x_{33} = 40,$$
$$x_{ij} \geq 0.$$

where x_{ij} = Number of units shipped from i^{th} factory to j^{th} warehouse.

i. Construct the transportation matrix of the above model.

ii. Find the initial basic feasible solution using the Least Cost method.

iii. Determined the optimal solution using the Stepping Stone method.

(75 marks)

4. a) Solve the following LPP using the Bounded Variable Simplex method:

$$\text{Max } Z = 4x_1 + 5x_2 + 3x_3$$

S.t.

$$x_1 + 2x_2 + 2x_3 \leq 12,$$
$$2x_1 + 5x_2 - 3x_3 \leq 30,$$
$$0 \leq x_1 \leq 3,$$
$$1 \leq x_2 \leq 5,$$
$$0 \leq x_3 \leq 2.$$

(80 marks)

b) Briefly explain why the Bounded Variable Simplex method is more appropriate for solving the above LPP and the advantages of using this method.

(20 marks)

5. a) A manufacturing plant faces certain production and marketing constraints for its products, **A** and **B**. The plant has a monthly production capacity of 300 hours, with each product requiring an average of one hour for production. The marketing department has determined that the maximum demand for product **A** is 140 units per month, while for product **B**, it is 200 units per month. It is important to note that these maximum demand levels cannot be exceeded. The net profit obtained from selling one unit of product **A** is Rs. 600, whereas for product **B**, it is Rs. 200. The manager has established the following objectives:

Priority 1: The primary goal is to utilize the normal production capacity fully, without any underutilization.

Priority 2: The manager aims to achieve the maximum possible sales for both products **A** and **B**. Considering that the net profit from product **A** is three times that of product **B**, the manager is three times more motivated to attain sales targets for product **A** compared to product **B**.

Priority 3: Minimizing overtime operations at the plant is another objective. The manager aims to reduce the need for operating the plant beyond normal working hours as much as possible.

Formulate a goal programming model that addresses the objectives and constraints described in the above situation.

(40 marks)

b) Consider the following Linear Programming Problem (LPP):

$$\text{Max } Z = 6x_1 + 5x_2 + 3x_3 + 4x_4$$

S.t.

$$x_1 + x_2 \leq 5,$$
$$3x_1 + 2x_2 \leq 12,$$
$$x_3 + 2x_4 \leq 8,$$
$$2x_3 + x_4 \leq 10,$$
$$x_1 + x_2 + x_3 + x_4 \leq 7,$$
$$2x_1 + x_2 + x_3 + 3x_4 \leq 17,$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

The Dantzig-Wolfe decomposition algorithm can be applied to solve the above LPP.

i. Identify the linking constraints and independent subsets of constraints in the above LPP in order to apply the Dantzig-Wolfe decomposition algorithm.

ii. Solve the identified subproblems using the Graphical method.

iii. Rewrite the given LPP in matrix form.

iv. Obtain the Dantzig-Wolfe formulation (DW–formulation) for the given LPP.

(60 marks)

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