8 3

Rajarata University of Sri Lanka - Sri Lanka

Faculty of Applied Sciences

B. Sc. (General) Degree

First Year – Second Semester Examination (Repeat), March 2013

MAA 1302 – PROBABILITY AND STATISTICS I

Answer **Five (05)** questions only — Time: Two & half (2½) hrs.

1. (*i*) The length of 32 leaves were measured correct to the nearest millimetre. The data obtained are given in the following grouped frequency distribution:

Length (mm)	20 – 22	23 – 25	26 – 28	29 – 31	32 – 34
Number of leaves	3	6	12	9	2

Find, using an appropriate coding method, the mean length correct to the first decimal point and the standard deviation correct to the second decimal point of the distribution.

(*ii*) A group A of 20 students and a group B of 30 students sat for an examination in Statistics. The mean and the variance of group A are 66 and 9 respectively. These values for group B are 51 and 39 respectively. Find the mean mark for all 50 students and show that the standard deviation of all 50 marks is 9

It is suggested that the original marks of students from group A should be linearly scaled so that their scaled marks would have a standard deviation of 9 and a mean mark equal to the mean mark of all 50 students. What effect would this have on an original mark of 60 obtained by a student from group A.

2. The times, to the nearest minute, spent by a group of school children for reading during a particular day are given in the following table:

Time (in minutes)	Number of children
10 – 19	8
20 – 24	15
25 – 29	25
30 – 39	18
40 – 49	12
50 – 64	7
65 – 89	5

(*i*) Draw a histogram to represent the data.

(*ii*) Find the median, mode, mean and the standard deviation of the distribution.

(*iii*) Comment on the shape of the distribution

3. (*a*) Define what is meant by saying that the events A and B are independent of each other.
The probability that a person, who is now 25 years old, will survive till the age of 55 is 0.75 and the probability that a person, who is now 45 years old, will survive till the age of 75 is 0.25. Compute the probability that at least one of them will remain alive for the next 30 years.
Verify your answer by an independent method.

(*b*) An astrologer claims that he can predict before the birth the sex of a baby just to be born. Suppose that the astrologer has no real power, but he tosses a coin just once before every birth and if the head turns up he predicts a boy for that birth and if the tail turns up he predicts a girl. Let α be the probability that at a certain birth a boy is born. Let β be the probability of a head turning up in a single toss with astrologer's coin.
(i) Find the probability of a correct prediction.
(ii) Obtain the probability that at a certain birth a boy is born given that the astrologer made a correct prediction.

4. (*a*) State the Axiomatic Definition of Probability.
Using the above definition prove that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any two events A and B.

(*b*) State and prove Bayes' Theorem.

(c) A machine consists of two components. Each component must function for the machine to operate. The probability that the first component will fail during the time t is p and that for the second component is q. The machine is tested during time t and fails. Find the probability that only the first component failed. If the two components have the same probability of failing during the time t show that the above conditional probability is $\frac{r}{r+1}$, where r is the probability that any component not failing during time t.

5. A and B fire shots each other alternately towards a target. Firing stops when one of them hits the target. A has probability α of missing the target at each shot. B's aim improves with practice in such a way that his probability of missing the target at his n^{th} shot is $\frac{\beta}{n}$. Assume that A fires the first shot.
(i) Show that the probability that A eventually hits the target is $(1-\alpha)e^{\alpha\beta}$.
(ii) Find the conditional probability that A hits the target at his n^{th} attempt, given that A hits the target and obtain the conditional mean and variance of the number of shots fired by A.
(iii) Determine the conditional probability that A hits the target, given that A misses with his first shot.

91

6. Personnel of a certain engineering company use an on-line terminal to make routine engineering calculations. The time each engineer spends in a session at a terminal has an exponential distribution with a mean of 36 minutes.
   (i) Find the probability that an engineer will spend 30 minutes or less in a session at the terminal.
   (ii) Find the probability that an engineer will use the terminal for more than an hour in a session.
   (iii) If an engineer has already been at the terminal for 30 minutes in a session find the conditional probability that he or she will spend more than another hour in that session.
   (iv)If the ninety percent of the sessions end in less than K minutes, find the value of K.
   [You may assume that $\ln 10$ is approximately equal to 2.30259].

7. The scores of a certain examination are normally distributed with a mean of 510 and a standard deviation of 102.
   (*a*) Compute the two scores that lie equidistant form the mean and include 95% of the students who sat the examination.
   (*b*) Five of your friends who sat this examination have obtained the scores 270, 780, 500, 410 and 565.
      (α) Find the z-scores of these five friends.
      (β) Which of the scores obtained at the examination would place your friends,
         (*i*) in the top 10%?
         (*ii*) in the upper quartile?
         (*iii*) above the median?
         (*iv*) in the lowest quartiles?

***