RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
Second year – Semester I Examination – February 2013
MAP 2203 - DIFFERENTIAL EQUATIONS II

Answer all questions. **Duration: 2 hours**

01.

a) Discuss the Frobenius method for solving a second order linear differential equation,

$$a_2(x)\,y'' + a_1(x)\,y' + a_0(x)\,y = 0$$

b) Show that the differential equation $2x^2\,y'' - x\,y' + (1+x)\,y = 0$ has a regular singular point at the origin.

c) Find the general solution of the above differential equation by using Frobenius method.

02.

a) Consider the initial value problem of the form, $\frac{dy}{dx} = F(x,y)\ ;\ y(x_0) = y_0$

Discuss the Picard's iteration method for n^{th} approximation $y_n(x)$.

b)

i) Apply Picard's iteration method to solve the following initial value problem up to 3rd approximation.

$$\frac{dy}{dx} = 3e^x + 2y \quad ; \quad y(0) = 0$$

ii) Use $y_3(x)$ to estimate the value $y\,(0.7)$.

iii) Find the exact solution of the above differential equation and obtain the actual value $y\,(0.7)$.

Compare the result with the Picard's approximated value.

03.

a) Find the general solution of the differential equation,

$$\frac{d\underline{X}}{dt} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix} \underline{X} \quad \text{, subject to the initial condition} \quad X(0) = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

b) Find e^{At} where $A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

04.

a) Form a partial differential equation by eliminating arbitrary constants a and b in each of the following equations of surfaces :

i) $(x-a)^2 + (y-b)^2 + z^2 = c^2$, where c is the radius of the sphere.

ii) $z = (x^2 + a)(y^2 + b)$

b) Solve the following partial differential equations, given with the usual notations:

i) $py + qx = xyz^2 (x^2 - y^2)$

ii) $z(x+y)p + z(x-y)q = x^2 + y^2$

iii) $xys - qy = x^2$

iv) $s - t = {}^{x}/_{y^2}$