RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree) -Industrial Mathematics
Fourth Year– Semester I Examination – April./May. 2015

MAT 4310 - Computational Mathematics

Time allowed: 3 hours only.

Answer **six** questions with **Qu. 1** — ***Calculators will be provided***

1.

i. Find the **Lagrange interpolation** polynomial that takes the values prescribed below

x_k	0	1	3	5
$f(x_k)$	1	2	6	7

ii. Use the fourth degree Taylor polynomial of $cos(2x)$ to find the **exact** value of

$$lim_{x\to 0}\frac{1-cos(2x)}{3x^2}$$

iii. Find the **exact** value of the following series:

$$\frac{4}{5}-\frac{16}{25}+\frac{64}{125}-\frac{256}{625}+\cdots$$

iv. Find the Maclaurin series for $\tan^{-1}(x^2)$

v. Find the inverse of the following $n\times n$ matrix.

$$\begin{pmatrix} 1 & 0 & 0 & & 0 & 0 & 0 \\ x & 1 & 0 & \cdots & 0 & 0 & 0 \\ x^2 & x & 1 & & 0 & 0 & 0 \\ & \vdots & & \ddots & & \vdots & \\ x^{n-1} & x^{n-2} & x^{n-3} & \cdots & x^2 & x & 1 \end{pmatrix}$$

2. Evaluate the following system via **Gaussian elimination**

$$\begin{aligned} x_1 + x_2 \qquad + 3x_4 &= 4 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 3x_1 - x_2 - x_3 + 2x_4 &= -3 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= 4 \end{aligned}$$

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3.

a. Find the inverse matrix $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$, by the Gauss-Jordan method.

b. The following system of equations is given

$$\begin{aligned} 4x &+ y + 2z = 4 \\ 3x &+ 5y + z = 7 \\ X &+ y + 3z = 3, \end{aligned}$$

Set up the **Jacobi and Gauss-Seidel iterative schemes** for the solution and iterate three times starting with the initial vector

$x^{(0)} = 0$.

4.

i. Solve the differential equation
$y_{n+1} - 2\sin x\, y_n + y_{n-1} = 0$, when $y_0 = 0$ and $y_{n-1} = \cos x$.

ii. Find y_n ,from the difference equation
$\Delta^2 y_{n+1} + \frac{1}{2}\Delta^2 y_n = 0$,n=0,1,2,….,when $y_0 = 0, y_1 = \frac{1}{2}$ and $y_2 = \frac{1}{4}$

5.

i. Each term in the sequence $0, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \ldots$ ..is equal to the arithmetic mean of the two proceeding terms. Find the general term.

ii. Find the general solution of the recurrence relation
$y_{n+2} + 2by_{n+1} + cy_n = 0$, Where b and c are real constants.

Show that solutions tend to zero as $n \to \infty$,if and only if ,the point (b,c) lies in the interior of a certain region in the b-c plane , and determine this region.

6. Consider the following **Runge-Kutta** method for the differential equation $y' = f(x,y)$

$$Y_{n+1} = Y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$

$$K_1 = h f(x_n, y_n)$$

$$K_2 = h f(x_n + \frac{h}{2}, y_n + \frac{K_1}{2})$$

$$K_3 = h f(x_n + h, y_n - K_1 + 2K_2)$$

a). Compute y(0.4) when $y' = \frac{y+x}{y-x}$,$y(0) = 1$ and h = 0.2, Round off to five decimal places.

a) What is the result after one step of length h when $y' = -y$,$y(0) = 1$?

7. Compute an Approximation to $y(1)$, $y'(1)$, $y''(1)$ with Taylor's algorithm of order two and step length h=1 when y(x) is the solution to the initial value problem
$Y''' + 2y'' + y' - y = \cos x \quad , 0 \le x \le 1 ,$
$y(0) = 0, \; y'(0) = 1, \; y''(0) = 2$

8. Consider the initial value problem

$$y' = x(y + x) - 2, y(0) = 2$$

   i. Use Euler's method with step sizes h=0.3,h=0.2 and h=0.15 to compute approximation to y(0.6) (5 decimals).
   ii. Improve the approximations in (a) to $O(h^3)$ by Richardson extrapolation.