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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (four - year) Degree in Applied Sciences

Fourth Year - Semester I Examination - September/October 2019

PHY 4210 - ADVANCED QUANTUM MECHANICS

Time: Two (02) hours

Answer **all four** questions

Unless otherwise specified all symbols have their usual meaning.

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1. A particle moves in 1-D in the presence of an attractive potential $V(x)$ which is infinite for $x < 0$, is equal to the constant value $-V_0$ in the region $0 < x < a$, and is equal to 0 for $x > a$.

a) Obtain the functional form of positive energy solutions ($E > 0$) to the energy eigenvalue equation in the three regions of interest. **(10 marks)**

b) What are the appropriate boundary conditions for this system at $x = 0$ and $x = a$? **(06 marks)**

c) Applying the boundary conditions, determine up to a single normalization constant A, the eigenstates of this system for positive energy solutions. For what energies, if any, are the solutions with $E > 0$ square normalizable? **(09 marks)**

2. a) Describe the five basic postulates of quantum mechanics. **(10 marks)**

b) An operator $\hat{A}$ is said to be Hermitian if it satisfies the condition $\langle \psi_1 | \hat{A}\psi_2 \rangle = \langle \hat{A}\psi_1 | \psi_2 \rangle$ for any two functions ψ_1 and ψ_2 of the function space which the operator $\hat{A}$ acts on. Prove that the momentum operator $-i\hbar\overline{\nabla}$ is Hermitian. **(08 marks)**

c) Show that the eigenvalues of a Hermitian operator are real. **(07 marks)**

3. a) Prove the following operator identities:

(i) $[\hat{A}, \hat{B} + \hat{C}] = [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}]$ **(05 marks)**

(ii) $[\hat{A}, \hat{B}^{-1}] = -\hat{B}^{-1}[\hat{A}, \hat{B}]\hat{B}^{-1}$ **(05 marks)**

b) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, then show that the product $\hat{C} = \hat{A}\hat{B}$ is Hermitian, only if $[\hat{A}, \hat{B}] = 0$. **(07 marks)**

c) If $\hat{A}$ and $\hat{B}$ are integrals of motion, then show that $i[\hat{A}, \hat{B}]$ is also an integral of motion.

Hint: If $\hat{A}$ and $\hat{B}$ are integrals of motion, then they commute with $\hat{H}$.

(08 marks)

4. a) Use the variational method to calculate the ground state energy of a harmonic oscillator using a family of trial functions of the form $e^{-\beta x^2}$, where β is a variational parameter. Compare the results with the exact value of the ground state energy. The one-dimensional harmonic oscillator Hamiltonian is $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2 x^2$. **(18 marks)**

b) If we repeat the calculation in (a) using a family of trial functions of the form $xe^{-\beta x^2}$, then the result is $\frac{3}{2}\hbar\omega_0$. Compare and explain the difference between the calculation in (a) with the result of (b) with respect to even and odd trial functions. **(07 marks)**

Helpful integrals:

$$\int_0^\infty e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{4\alpha}} \quad \text{and} \quad \int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\pi/\alpha}^{\pi/\alpha} \sin^2(\alpha x) dx = \pi/\alpha \quad \text{and} \quad \int_{-\pi/\alpha}^{\pi/\alpha} x^2 \sin^2(n\alpha x) dx = \pi(2n^2\pi^2 - 3)/6n^2\alpha^3$$

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