RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

BSc in Applied Sciences

Second Year-Semester I Examination – June/ July 2022

MAP 2301- ALGEBRA

Time: Three (03) hours.

Answer all (06) questions.

1.(a) Let A and B be subsets of a set S. Using the laws of algebra of sets, prove that $(A \cap B')' \cup B = A' \cup B$.

(b) A survey among 475 customers at an ice-cream parlor has shown that, out of the 3 ice-cream flavors - chocolate, strawberry and vanilla, 165 like chocolate, 175 like strawberry, 190 like vanilla, 100 like chocolate but not strawberry, 120 like strawberry but not vanilla, 130 like vanilla but not chocolate, and 75 do not like any of these flavors.
Find how many like

(i) all flavors
(ii) exactly 2 flavors
(iii) exactly 1 flavor
(iv) only chocolate

(100 marks)

2. Define the greatest common divisor (a, b) of two integers a and b and the least common multiple $[a, b]$ of two integers a and b.

(a) Find each of the following:

(i) $(d, 3d)$ (ii) $(d-1, d+1)$ (iii) $(d, d+1)$

(iv) $(24, 0, 36, 48)$ (v) $[d, 3d]$ (vi) $[36, 48]$

(b) Using Euclidean algorithm, find the greatest common divisor of 7002 and 875.

(100 marks)

3. (a) Define the terms reflexive relation, symmetric relation, and transitive relation.

Let $\Re$ denote the set of real numbers. Determine which of the following relations are reflexive, symmetric, and transitive. Justify your answers.

(i) $R_1 = \{(a,b) \mid a \leq b;\ a,b \in \Re\}$

(ii) $R_2 = \{(a,b) \mid a < b;\ a,b \in \Re\}$

(b) Let $A = \{1, 2, 3, 4\}$. Determine which of the following relations are reflexive, symmetric and transitive.

Justify your answers.

(i) $R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (3, 2)\}$

(ii) $R_2 = \{(3, 2)\}$

(iii) $R_3 = \{(3, 2), (2, 4), (3, 4), (4, 1)\}$

(100 marks)

4. (a) Determine which of the following functions are injective and which are surjective.

Justify your answers.

(i) $f : \Re \to \Re$ is defined by $f(x) = x^3 + 2$.

(ii) $g : \Re \to \Re$ is defined by $g(x) = |x - 3|$.

(iii) $h : \Re \to \Re$ is defined by $h(x) = 10$.

(b) Let $f : \Re \to \Re$ be defined by $f(x) = \log_{10}\left(\frac{5-x}{x+2}\right)$. Find the domain of the function.

(100 marks)

5. (a) Let $\mathbb{Z}$ be the set of integers. An operation * on $\mathbb{Z}$ is defined by $a * b = a + b - 10$.

(i) Show that * is a binary operation on $\mathbb{Z}$.

(ii) Prove that $(\mathbb{Z}, *)$ is a group.

(iii) Is $(\mathbb{Z}, *)$ an abelian group? Justify your answer.

(iv) Solve the equation $3 * x * 7 = 21$.

(b) Let H be a non-empty subset of a group G. If $ab^{-1} \in H$ for all $a, b \in H$, then prove that H is a subgroup of G.

(100 marks)

6. (a) Solve each of the following linear Diophantine equations.

(i) $3x + 4y = 22$.
(ii) $12x + 20y = 54$.
(iii) $11x + 13y = 37$.

(b) In how many ways can Rs. 2.60 be made using 25 cents and 10 cents coins (at least one of each kind).

(100 marks)

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