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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES
B.Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – November/December 2016

MAP 1203 – Real Analysis I

Answer All Questions **Time allowed: Two hours**

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1. (*a*) Find a constant $M\,(>0)$ such that $\left|\dfrac{3x^4+2x^2-1}{5x+1}\right| \le M$ whenever $1 \le x \le 2$.

   (*b*) Define the terms *Infimum* and *Suprimum*.

   Which of the following sets are bounded above, bounded below or otherwise? Also find the Infimum and Suprimum, if they exists.

   (*i*) $\left\{\left(\dfrac{3n+4}{n}\right); n \in \mathrm{N}\right\}$,

   (*ii*) $\left\{\left(1-\dfrac{1}{n}\right)\sin\dfrac{n\pi}{2} : n \in N\right\}$

   (*c*) Let A, B and C be non empty subsets of $\Re$ and let

   $A+B+C=\{a+b+c \mid a\in A, b\in B, c\in C\}$. Prove that

   $Sup(A+B+C) = SupA + SupB + SupC$.

   (*d*) Let $x, y \in \Re$. Then show that $|x+y| \le |x| + |y|$.

2. Prove that "*If a sequence converges, then its limit is unique*".

   (*a*) Using $\varepsilon-\delta$ definition, prove that,

   $(i)\ \lim\limits_{x\to a}\dfrac{x^2-a^2}{x-a} = 2a$. $\quad (ii)\ \lim\limits_{x\to 0} x\sin\dfrac{1}{x} = 0$

   (*b*) Find $\lim\limits_{x\to a} f(x)$ where

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$$f(x)=\begin{cases}\left(\dfrac{x^2}{a}\right)-a : for\ 0<x<a\\ 0 \quad : for\ x=a\\ a-\left(\dfrac{a^3}{x^2}\right) : for\ x>a\end{cases}.$$

3. (*a*) Let $f, g\ and\ h$ are three functions defined from $R \to R$ as follows:

$f(x)=4x-5, g(x)=11\sin 3x\ and\ h(x)=10x^2, where\ x\in R$. Determine the following compositions,

(*i*) gof (*ii*) goh (*iii*) $gofoh$

(*b*) Define the continuity of a function.

Prove that,

$$f(x)=\begin{cases} x\sin\dfrac{1}{x}, & for\ x\neq 0\\ 0, & for\ x=0\end{cases}$$, is continuous at $x=0$.

(*c*) Let f be a function defined as

$$f(x)=\begin{cases}2x+1, & for\ x\le 1\\ ax^2+b, & for\ 1<x<3\\ 5x+2a, & for\ x\ge 3\end{cases}$$ where a and b are constants. Function $f(x)$ is continuous everywhere then find the constants a and b.

4. (*a*) Examine the following function for differentiability at $x=0$ and $x=1$.

$$f(x)=\begin{cases}x^2, & for\ x\le 0\\ 1, & for\ 0<x\le 1.\\ \dfrac{1}{x}, & for\ x>1\end{cases}$$

(*b*) State the following theorems:.

(*i*) Rolle's theorem,

(*ii*) Lagrange's Mean Value Theorem,

(*iii*) Cauchy's Mean Value Theorem.

(*c*) State the *L'Hospital Rule.*

Using *L'Hospital Rule* find the following limits:

(*i*) $\lim\limits_{x\to 0}\dfrac{e^{-x^2}\log(1+x)}{x}$, (*ii*) $\lim\limits_{x\to 0}\dfrac{(4-4\cos x-2\sin^2 x)}{x^4}$.