RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General) Degree in Applied Sciences
First Year - Semester II Examination – February/ March 2019

MAA 1104 – MATHEMATICAL MODELING

Time allowed: 01 hour

Answer all questions.
Calculator is permitted.

1. Let $Y(t)$ be the total national income and $D(t)$be the total national debt at time t. ***Domar's first debt model*** assumes that,

   - Rate at national debt changes is proportional to national income.
   - National income increases at a constant rate.

   a) Express the above two assumptions as difference equations. (20 marks)

   b) Solving these difference equations, show that $\lim_{t\to\infty} \frac{D(t)}{Y(t)}$ tends to increase without limit.

   (80 marks)

2. The general demand and supply functions, for the **Cobweb model**, are given by,

   $Q_{dt} = a + bP_t; b < 0$
   $Q_{st} = c + dP_{t-1}; d > 0$

   a) Show that at the equilibrium point,

   $$P_t = \left(\frac{c-a}{b}\right) + \left(\frac{d}{b}\right) P_{t-1}.$$ (10 marks)

   b) Find the general solution for the above first order difference equation.

   (45 marks)

   c) The demand and supply functions, for the **Cobweb model**, are given below:

   $Q_{dt} = 1200 - 6P_t$
   $Q_{st} = 2P_{t-1}$
   P_t is price for time t.

At the equilibrium point, the initial condition for the model is $P_0 = 200$. Find the general solution for P_t.

(45 marks)

3. The weekly demand for the color LED TV is $P = 600 - 0.05Q$ $(0 \leq Q \leq 12000)$ where P denoted the whole sale unit price and Q denoted the quantity demanded. The weekly cost function associated with LED TV production is given by,

$$TC(Q) = 0.000002Q^3 - 0.03Q^2 + 400Q + 80000$$

a) Find the total revenue function and total profit function. (15 marks)

b) Find the marginal cost function, marginal revenue function and the marginal profit function. (45 marks)

c) Find the approximate profit of the 2001st LED TV. (10 marks)

d) Calculate the approximate cost of producing 2001st LED TV. (15 marks)

e) Find the average cost of 2000 LED TVs. (15 marks)