RAJARATA UNIVERSITY OF SRILANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

First Year-Semester I Examination- May 2015

MAA 1201 – Mathematical Methods I

Answer **three** Questions including **Qu.1**. Time Allowed: **Two hours**

1) *[140 Marks]*

a. Find the angle between $\boldsymbol{A} = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$and $\boldsymbol{B} = 6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

b. If **A** is any vector, prove that **A = (A .i)I + (A .j)j + (A .k)k**

c. Find a unite vector parallel to the resultant vectors,

$$\boldsymbol{r_1} = 2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k} \; and \; \boldsymbol{r_2} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$$

d. Determine the angles α, β, γ which the vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ makes with the positive directions of the coordinates axes and show that$\boldsymbol{cos^2\alpha} + \boldsymbol{cos^2\beta} + \boldsymbol{cos^2\gamma} = \mathbf{1}$

e. Show that the equation of a plane which passes through three given point s A,B,C not in the same straight line and having position vectors **a**, **b**, **c** relative to an origin O, can be written $r = \frac{m\boldsymbol{a}+n\boldsymbol{b}+p\boldsymbol{c}}{m+n+p}$, where m,n,p are the scalars. Verify that the equation is independent of the origin.

f. If $\boldsymbol{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}$and $\boldsymbol{B} = B_1\boldsymbol{i} + B_2\boldsymbol{j} + B_3\boldsymbol{k}$, prove that

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

g. If $\boldsymbol{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}, \boldsymbol{B} = B_1\boldsymbol{i} + B_2\boldsymbol{j} + B_3\boldsymbol{k}$and $\boldsymbol{C} = C_1\boldsymbol{i} + C_2\boldsymbol{j} + C_3\boldsymbol{k}$, show that

$$\boldsymbol{A} \cdot \boldsymbol{B} \times \boldsymbol{C} = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

h. Find an equation for the plane determined by the points $P_1 = (2,-1,1)$, $P_2 = (3,2,-1)$ and $P_3 = (-1,3,2)$.

i. Find the area of the triangle having vertices at $P = (1,3,2), Q = (2,-1,1)$ and $R = (-1,1,2,3)$.

j. Determine a unit vector perpendicular to the plane of $\boldsymbol{A} = 2\boldsymbol{i} - 6\boldsymbol{j} - 3\boldsymbol{k}$and $\boldsymbol{B} = 4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$.

2) *[80 Marks]*

i. In each case determine whether the vectors are linearly independent or linearly dependent.

a. $\boldsymbol{A} = 2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}, \boldsymbol{B} = \boldsymbol{i} - 4\boldsymbol{k}$ and $\boldsymbol{C} = 4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$.

b. $\boldsymbol{A} = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{B} = 2\boldsymbol{i} - 4\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{C} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$.

ii. Given the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Find the,

a. Unit tangential vector

b. Curvature of the curve

c. Principal normal vector

d. Radius of the curvature

3) *[80 Marks]*

i. If $\boldsymbol{v} = \boldsymbol{\alpha} \times \boldsymbol{r}$ then prove that $\boldsymbol{\alpha} = \frac{1}{2} curl\ \boldsymbol{v}$. Where $\propto$ is a constant vector and $\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$

ii. Prove that vector $\boldsymbol{A} = 3y^4z^2\boldsymbol{i} + 4x^3z^2\boldsymbol{j} - 3x^2y^2\boldsymbol{k}$ is **solenoidal**.

iii. Show that $\boldsymbol{A} = (6xy + z^3)\boldsymbol{i} + (3x^2 - z)\boldsymbol{j} + (3xz^2 - y)\boldsymbol{k}$ is **irrotational**. Find φ such that $\boldsymbol{A} = \nabla\varphi$.

4) *[80 Marks]*

i. For what value of the constant σ will the vector

$\boldsymbol{A} = (\sigma xy - z^3)\boldsymbol{i} + (\sigma - 2)x^2\boldsymbol{j} + (1 - \sigma)xz^2\boldsymbol{k}$ have its curl identically equal to zero?

ii. Evaluate $\nabla^2(\ln r)$.

iii. Find and equation for the tangent plane to the surface $(x - 1)^2 + y^2 + (x + 2)^2 = 9$ at the point $(1, -3, 2)$

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