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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.Sc. (General Degree)
First Year Semester II Examination – March/ April 2014

MAA 1302 – PROBABILTY & STATISTICS I

*Answer **Five (05)** questions only.* Time: **2 ½ Hours**

1. The table below gives the durations of 60 journeys of a lorry on the same route:

Time of journey (in hours)	Number of journeys
5.6 – 5.8	2
5.8 – 6.0	7
6.0 – 6.2	16
6.2 – 6.4	21
6.4 – 6.6	12
6.6 – 6.8	2

(*i*) Draw a histogram to represent the data.
(*ii*) Calculating the values of appropriate measurements comment on the shape of the distribution of the times of the journeys of the lorry.
Draw the graph of the distribution.

2. (*a*) State the Axiomatic Definition of Probability.
Let A and B be any two events.
Using the laws of set Algebra and the above definition of probability prove that $P(A \cap B') = P(A) - P(A \cap B)$, where B' is the complementary event of B.
Hence, show that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Deduce the formula for the probability of the union of any three events.

(*b*) State and prove Bayes' Theorem.

(*c*) It is suspected that 90% of the people in a certain population are honest and the other 10% are dishonest. A testing agency claims that its honesty test is 75% accurate. That is, if a person is deceitful, there is a 75% chance that this person will fail the test and be considered dishonest. Likewise, if a person is honest, there is a 75% chance that this person will pass the test and be correctly identified. A person is randomly selected from this population.
(i) Find the probability that he is correctly identified by the honesty test.
(ii) Given that he passes the honesty test, find the probability that he is dishonest.

Contd... 1

3. A cell of type A, when subjected to a certain treatment, dies, survives or divides into two or three cells of type A with probabilities $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{1}{15}$ respectively. A cell of type B, when subjected to the same treatment in an unrelated experiment, dies, survives or divides into two cells of type B with probabilities 0.3, 0.3 and 0.4 respectively.
   (i) Determine the probability distribution of the random variable X which denotes the total number of cells after the two experiments.
   (ii) Find $F(x)$, the cumulative probability distribution of X.
   (iii) Draw the graph of $F(x)$ and **hence** verify the properties of the cumulative probability distribution function.
   (iv) **Using the cumulative probability distribution** of X find
      (*a*) the probability that there will be at least 3 cells after the two experiments,
      (*b*) the conditional probability that there will be 4 cells given that there are at least 3 cells after the two experiments.

4. A random variable X has a Poisson distribution with probability distribution

$$P[X = x] = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ for } x = 0, 1, 2, \cdots.$$

   (i) Obtain the moment generating function of X and **hence** show that the mean and the variance of the distribution are the same and equal to λ.
   (ii) If $P[X = k] = P[X = k+1]$ for some positive integer k, show that λ must also be an integer.
   (iii) If λ is not an integer, show that m, the mode of the distribution is such that $\lambda - 1 < m < \lambda$.

   In the manufacture of commercial carpets, small faults occur at random in the carpet at an average of 0.50 per 20 m^2.
   Find the probability that in a randomly selected 20 m^2 area of this carpet contain
   (i) no faults,
   (ii) at most 2 faults.

   The ground floor of a new office building has 10 rooms each with 80 m^2 floor area. These rooms have been carpeted using the commercial carpet described above.
   Find the probability that the carpet in any one of these rooms contains
   (i) at least 2 faults,
   (ii) exactly 3 faults,
   (iii) at most 4 faults.

5. The time T, in hours, take to perform a particular task is considered to be a continuous random variable with probability density function $f(t)$ of the form

$$f(t) = \begin{cases} 10ct^2, & \text{if } 0 \le t < 0.6 \\ 9c(1-t), & \text{if } 0.6 \le t < 1, \\ 0, & \text{otherwise} \end{cases}$$

Contd... 2

(i) Show that $c = \frac{25}{36}$.

(ii) Draw the graph of $y = f(x)$ and **hence** find the most likely time.

(iii) Obtain the cumulative probability distribution function of T and **hence** find the probability that the time will be

(*a*) more than 48 minutes,

(*b*) between 24 and 48 minutes.

6. A cutting machine produces steel rods which must not be more than 100 cm in length. The mean length and the standard deviation of a large batch of rods taken from this cutting machine are found to be 99.80 cm and 0.15 cm respectively.
Assuming the lengths of the rods are normally distributed find, correct to one decimal place, the percentage of rods which are too long.

The position of the cut can be adjusted without altering the standard deviation of the length. Find in cm, correct to 2 decimal places, how small the mean length should be if no more than 2% of the rods are to be rejected for being longer than 100 cm.

If the mean length is maintained at 99.80 cm find to the nearest tenth of a mm, how much the standard deviation must be reduced if no more than 4% of the rods are to be rejected for being longer than 100 cm.

7. (*a*) Let X be a continuous random variable with probability density function $f_X(x)$.
Define $M_X(t)$, the Moment generating function of X.

(*i*) Obtain $E(X)$ and $Var X$ in terms of the Moment generating function of X.

(*ii*) If $Y = aX + b$, where a and b are constants, show that $M_Y(t) = e^{bt} M_X(at)$.
Hence, prove that $E(Y) = a\,E(X) + b$ and $Var Y = a^2\, Var X$.

(*b*) Let X be a continuous random variable with probability density function given by

$$f_X(x) = \begin{cases} \dfrac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$, where λ is a positive constant.

Show that X has moment generating function $M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$, for $t < \lambda$.

Deduce the moment generating function of $Y = 2X + 1$.

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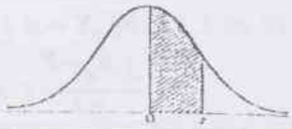
Areas Under the Standard Normal Curve from 0 to z



Z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Poisson Distribution

$$P(x;\lambda) = P\{X = x\} = \frac{\lambda^{x} e^{-\lambda}}{x!};\ x = 0, 1, 2, \ldots$$

	λ									
x	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153
5	-	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031
6	-	-	-	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0005
7	-	-	-	-	-	-	0.0000	0.0000	0.0000	0.0001

	λ									
x	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**6.0**	**7.0**
0	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0025	0.0009
1	0.3347	0.2707	0.2052	0.1494	0.1057	0.0733	0.0500	0.0337	0.0149	0.0064
2	0.2510	0.2707	0.2565	0.2240	0.1850	0.1465	0.1125	0.0842	0.0446	0.0223
3	0.1255	0.1804	0.2138	0.2240	0.2158	0.1954	0.1687	0.1404	0.0892	0.0521
4	0.0471	0.0902	0.1336	0.1680	0.1888	0.1954	0.1898	0.1755	0.1339	0.0912
5	0.0141	0.0361	0.0668	0.1008	0.1322	0.1563	0.1708	0.1755	0.1606	0.1277
6	0.0035	0.0120	0.0278	0.0504	0.0771	0.1042	0.1281	0.1462	0.1606	0.1490
7	0.0008	0.0034	0.0099	0.0216	0.0385	0.0595	0.0824	0.1044	0.1377	0.1490
8	0.0001	0.0009	0.0031	0.0081	0.0169	0.0298	0.0463	0.0653	0.1033	0.1304
9	0.0000	0.0002	0.0009	0.0027	0.0066	0.0132	0.0232	0.0363	0.0688	0.1014
10	-	0.0000	0.0002	0.0008	0.0023	0.0053	0.0104	0.0181	0.0413	0.0710
11	-	-	0.0000	0.0002	0.0007	0.0019	0.0043	0.0082	0.0225	0.0452
12	-	-	-	0.0001	0.0002	0.0006	0.0016	0.0034	0.0113	0.0263
13	-	-	-	0.0000	0.0001	0.0002	0.0006	0.0013	0.0052	0.0142
14	-	-	-	-	0.0000	0.0001	0.0002	0.0005	0.0022	0.0071
15	-	-	-	-	-	0.0000	0.0001	0.0002	0.0009	0.0033
16	-	-	-	-	-	-	0.0000	0.0000	0.0003	0.0014
17	-	-	-	-	-	-	-	-	0.0001	0.0006
18	-	-	-	-	-	-	-	-	0.0000	0.0002
19	-	-	-	-	-	-	-	-	-	0.0001