RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES, MIHINTALE

B.SC (General) Degree

Third year – Semester I Examination – February 2013

MAT 3301 ADVANCED LINEAR PROGRAMMING

Answer FIVE Questions Only.

Calculators will be provided. **Time allowed: Three hours.**

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1. A multi- plant company has three manufacturing plants , A , B and C and two markets X and Y . Production cost at A , B and C is Rs. 1500 , Rs.1600 and Rs.1700 per piece, respectively. Selling price in X and Y are Rs. 4400 and Rs. 4700 , respectively. Demands in X and Y are 3500 and 3600 pieces , respectively. Production capacities at A , B and C are 2000 , 3000 and 4000 pieces, respectively. Transportation costs are as shown in the following matrix:

		Market	
		X	Y
Plant	A	1000	1500
	B	2000	3000
	C	1500	2500

(i). Formulate the problem as a transportation problem so as to maximize the total profit. **[25 Marks]**

(ii). Is this a balanced problem? Justify your answer. **[10 Marks]**

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(iii). Construct the profit matrix for the above problem. **[10 Marks]**

(iv). Find the Initial Basic Feasible solution using Vogel's Approximation Method. **[15 Marks]**

(v). Test for optimality and find the optimal solution. **[45 Marks]**

2. (i). What is an assignment problem? Give two applications. **[10 Marks]**

(ii). Give an algorithm to solve an assignment problem. **[10 Marks]**

(iii). How do you deal with the assignment problems, where

(a). the objective function is to be maximized? **[05 Marks]**

(b). some assignments are prohibited? **[05 Marks]**

(iv). A city cooperation has decided to carry out road repairs on four main roads of the city. The government has agreed to make a special grant of Rs.5,000,000 towards the cost with a condition that the repairs be done at the lowest cost and quickest time. If the conditions warrant, a supplementary token grant will also be considered favourably. The cooperation has floated tenders and five contractors have sent their bids. In order to expedite work , one road will be awarded to only one contactor.

Cost of Repairs (Rupees in lakh)

Contactor	R_1	R_2	R_3	R_4
C_1	9	14	19	15
C_2	7	17	20	19
C_3	9	18	21	18
C_4	10	12	18	19
C_5	10	15	21	16

(i). Find the best way of assigning the repair work to the contractors and the costs. **[60 Marks]**

(ii). If it is necessary to seek supplementary grants, what should be the amount sought? **[05 Marks]**

(iii). Which of the five contractors will be unsuccessful in his bid? **[05 Marks]**

3. A firm is designing the introduction of three products 1 , 2 and 3 , in its three plants A , B and C . Only a single product is decided to be introduced in each of the three plants. The unit cost of producing a product in a plant , is given in the following matrix:

Product	Plant A	Plant B	Plant C
1	8	12	-
2	10	6	4
3	7	6	6

(i). Formulate a Linear Programming model so as to minimize the total unit cost . **[25 Marks]**

(ii). How should be the product to be assigned so that the total unit cost is minimized? **[35 Marks]**

(iii). If quantities of the different products are to be produced as follows, then what assignment shall minimize the aggregate production cost? **[15 Marks]**

Product	Quantity (in units)
1	2000
2	2000
3	10,000

(iv). What would your answer be if the three products were to be produced in equal quantities? **[05 Marks]**

Turn Over

(v). It is expected that the selling price of the products produced by three different plants would be different , as shown in the following table:

		Plant		
		A	*B*	*C*
Product	1	15	18	-
	2	18	16	10
	3	12	10	8

Assuming the quantities mentioned in part (iii) above would be produced and sold, how should the products be assigned to the plants to obtain maximum profits?

[20 Marks]

4. Explain the following terms:

(i). Deviational Variable. **[05 Marks]**

(ii). Preemptive Goal Programming . **[05 Marks]**

A company produces two kinds of products *A* and *B*. Production of either *A* or *B* requires 3 hours of production capacity in the plant. The plant has a maximum production capacity of 30 hours per week to manufacture these two products. The overtime should not exceed 5 hours per week. The plant manager has set the following goals arranged in order of importance:

Goal 1: To avoid any underutilization of production capacity.

Goal 2: To limit the overtime hours to 5 hours.

Goal 3: To minimize the overtime operation of the plant as much as possible.

(i). Define the decision variables to reach the above objectives. **[10 Marks]**

(ii). Write each goal using the decision variables. **[15 Marks]**

(iii). Add deviational variables to the goal constraints. **[05 Marks]**

(iv). Formulate the linear goal programming model to solve the above problem.

[15 Marks]

(v). Solve the model using Goal Programming Simplex Method. **[30 Marks]**

(vi). Using the solution model, determine the deviation from the goal. **[15 Marks]**

5. State the steps of Dantzig-Wolfe Decomposition Algorithm . **[30 Marks]**

Consider the following Linear Programming Problem:

Minimize $Z = -x_1 - 2x_2 - 4x_3 - 3x_4$

Subject to

$x_1 + x_2 + 2x_3 \leq 4$

$x_2 + x_3 + x_4 \leq 3$

$2x_1 + x_2 \leq 4$

$x_1 + x_2 \leq 2$

$x_3 + x_4 \leq 2$

$3x_3 + 2x_4 \leq 5$

$x_i \geq 0$ for $i = 1, 2, 3, 4$.

Solve the above problem using Dantzig- Wolfe Decomposition Algorithm. **[70 Marks]**

6. A small paint company manufactures two types of paint, latex and enamel. In production, the company uses 10 hours of labor to produce 100 gallons of latex and 15 hours of labour to produce 100 gallons of enamel. The company has 40 hours of daily labour and 30 hours of overtime labour available each week. Furthermore, if enamel paint is produced, latex paint must also be produced. Each paint generates a profit at the rate of $1.00 per gallon. The company has the following objectives listed in decreasing priority:

- avoid the use of overtime
- achieve a weekly profit of $1000
- produce at least 700 gallons of enamel paint each week

(i). Decide the decision variables to reach above objectives. **[05 Marks]**

Turn Over

(ii). State the constraints using the decision variables. **[10 Marks]**

(iii). Write each goal using the decision variables. **[15 Marks]**

(iv). Add deviational variables to the goal constraints. **[10 Marks]**

(v). Determine the variables need to be minimized in the objective function. **[15 Marks]**

(vi). Formulate the linear goal programming model to solve the above problem. **[05 Marks]**

(vii). Solve the model using goal programming simplex method. **[40 Marks]**

7. Explain, briefly, the steps of the dual simplex method. **[15 Marks]**

Consider the following primal problem:

Minimize $z = -3x_1 - 5x_2$

Subject to

$2x_1 \leq 4$

$3x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0\ ,\ x_2 \geq 0$

(i). Give the corresponding dual linear problem. **[10 Marks]**

(ii). Solve the dual using the dual simplex method. **[50 Marks]**

(iii). Obtain the optimal solution to the primal problem using the optimal solution to the dual problem. **[25 Marks]**