49

**RAJARATA UNIVERSITY OF SRI LANKA**

**FACULTY OF APPLIED SCIENCES, MIHINTALE**

**B.SC (General) Degree**

**Second Year – Semester I Examination – October/ November 2014**

**MAA 2204 - Linear Programming**

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Time allowed: **Two** hours. Number of pages: 03.

Answer **FOUR** Questions Only. Calculators will be provided.

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01. (a) (i) What are the basic components of a Linear Programming Problem? **[10 Marks]**

(ii) Distinguish between feasible solution and optimal solution. **[10 Marks]**

(b) Consider the following linear programming problem:

Minimize $Z = x_1 + 2x_2 - 3x_3 + x_4$

subject to the constraints

$x_1 + 2x_2 - 3x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + 2x_4 = 4$

$x_j \geq 0$ for $j = 1,2,3,4.$

(i) In addition to the variable $-Z$, how many basic variables are there in this initial basic solution? **[05 Marks]**

(ii) What is the maximum number of basic solutions (either feasible or infeasible) which might exist? **[05 Marks]**

(iii) Find and list all the basic solutions of the constraint equations. **[30 Marks]**

(iv) Is the number of basic solutions in part (iii) equal to the maximum possible number which you specified in part (ii)? **[05 Marks]**

(v) How many of basic solutions in part (iii) are feasible (non-negative)? **[05 Marks]**

(vi) By evaluating the objective function in each basic solution, find the optimal solution. **[30 Marks]**

*Turn Over*

50

02. (a) Define the terms **slack, surplus** and **artificial variables.** **[15 Marks]**

(b) State the purpose of minimum ratio test in the Simplex method. **[10 Marks]**

(c) Solve the following problem using Big-M-Method:

Minimize $Z = 4x_1 + x_2$

subject to

$2x_1 + x_2 \geq 10$

$-3x_1 + 2x_2 \leq 6$

$x_1 + x_2 \geq 6$

$x_1 \geq 0, x_2 \geq 0.$

**[75 Marks]**

03. (a) Outline the Two-Phase Method in Linear Programming. **[20 Marks]**

(b) While solving the linear programming problem given below, using two–phase simplex method, suppose we reach the following table as the optimal solution for phase-I-problem:

Maximize $Z = 3x_1 - 6x_2 + x_3$

Subject to the following constraints

$x_1 + x_2 + x_3 \geq 8$

$2x_1 - x_2 = 5$

$-x_1 + 3x_2 + 2x_3 \leq 7$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

| Basic variable | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $y_1$ | $y_2$ | $s_2$ | constants |
|---|---|---|---|---|---|---|---|---|
| ...... | 0 | 1 | $2/3$ | $-2/3$ | $2/3$ | $-1/3$ | 0 | $11/3$ |
| ...... | 1 | 0 | $1/3$ | $-1/3$ | $1/3$ | $1/3$ | 0 | $13/3$ |
| ...... | 0 | 0 | $1/3$ | $5/3$ | $-5/3$ | $4/3$ | 1 | $1/3$ |
| -w | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

where $y_1, y_2$ are artificial variables, $s_1$ and $s_2$ are slack and surplus variables and $w$ is the objective function for phase $I$.

(i). Identify the basic variables in the optimal solution for phase-I. **[05 Marks]**

(ii). Continue the solution procedure with the phase-II ( if required), and find the optimal solution for the original problem. **[75 Marks]**

04. (a) Obtain the general Linear Programming Problem in matrix form. **[10 Marks]**

(b) ATK – Sugar Company manufactures three types of candy bar. Each candy bar consists of totally sugar and chocolate. Type-1 of candy bar is composed of one unit of sugar and two units of chocolate, Type-2 is composed of one unit of sugar and three units of chocolate and Type-3 is composed of one unit of sugar and one unit of chocolate. Profit per one candy bar of types 1, 2 and 3 are \$3 , \$6 and \$4, respectively. In order to manufacture all types of candy bars, 50 units of sugar and 80 units of chocolate are available.

(i). Formulate the problem as a Linear Programming model so as to maximize the total profit. **[30 Marks]**

(ii) Use the Revised Simplex method to determine the quantity of each of the products company should produce in order to maximize the total profit. **[60 Marks]**

05. Consider the following Linear Programming Problem [Primal Problem]:

Maximize $Z = 2x_1 + 6x_2 + 9x_3$

subject to the constraints

$x_1 + \quad x_3 \le 3$

$x_2 + 2x_3 \le 5$

$x_j \ge 0$ for $j = 1,2,3.$

(i). Write the Dual Linear Programming Problem formulation. **[20 Marks]**

(ii). Plot the feasible region of the dual problem. **[20 Marks]**

(iii).Determine the optimal solution of the dual problem by comparing its objective value at the corner point solutions. **[20 Marks]**

(iv)Which variables are basic in the optimal dual solution? **[10 Marks]**

(v) From your results above, determine the optimal solution of the primal problem. **[30 Marks]**