RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Second Year - Semester I Examination – Jun/July 2018

MAA 2302 – PROBABILITY & STATISTICS II

Time: Three hours

Answer **Five** Questions Only.
Statistical tables and calculators will be provided.

1. a) Define following terms.

   i. Conditional Expectation of Discrete Radom Variable.
   ii. Conditional Variance
   iii. Covariance
   iv. Correlation

[20marks]

b) Let X and Y be two continuous random variables with the following joint probability density function,

$$f(x,y)=\begin{cases}\dfrac{(x+y)}{3}; & 0<x<1, 0<y<2\\ 0 \quad ; & o.w.\end{cases}$$

i. Find the conditional mean and the conditional variance of X given $Y=1/2$.

[Turn over]

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ii. Find the covariance and the correlation between X and Y.

iii. Find the variance of W= 3X+ 4Y- 3

[80 marks]

2. Three books are selected at random from a bookshelf containing four mathematics, three statistics and two computers. If X and Y are the numbers of mathematics and statistics books respectively. Among these three books drawn from the shelf, assuming that the three are randomly selected from the nine available. The joint probability mass function of X and Y is,

$$P_{(x,y)}(x,y) = \frac{\binom{4}{x}\binom{3}{y}\binom{2}{3-x-y}}{\binom{9}{3}} \text{ where } X,Y > 0$$

i. Find the marginal probability mass function of X, and marginal p.m.f. of Y.

ii. Find the conditional probability mass functions $P_{(x/y)}(x/Y = y)$ and $P_{(y/x)}(y/X = x)$.

iii. Given that two of three books are statistics. Find the probability that the other will be mathematics.

iv. If we let Z denote the number of computer books selected, then $Z = 3 - X - Y$. Find $P(Z = 1/Y = 1)$.

v. Are X and Y independent?.

vi. Find the expected number of statistics books among the three selected.

vii. Find the covariance between X and Y. **[100 marks]**

3. a) Let $x_1, x_2, \ldots, x_n$ denoted the independent and identically distributed random sample of size n with the cumulative distribution $F_x(x)$ and probability density function $f_x(x)$ and let $Y_1, Y_2, \ldots, Y_n$ be the **order statistics**.

i. Using usual notations, derive the formula for the cumulative distributions of $Y_1 =$ min $(x_1, x_2, \ldots, x_n)$ and $Y_n =$ max $(x_1, x_2, \ldots, x_n)$.

[20 marks]

ii. Let $x_1, x_2, \ldots, x_n$ be a random sample from an $Exp(\theta)$ population.

$$f_x(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & ; \quad x > 0 \\ 0 & ; \quad o/w \end{cases}$$

Find the $F_{Y_1}(y), f_{y_1}(y), F_{y_n}(y), f_{y_n}(y)$. **[40 marks]**

b)

i. State the Central Limit Theorem **[10 marks]**

ii. A certain university canteen estimated students are known to prefer noodles 40 percent of times. Consider 2000 randomly chosen students and let X be a number of who order noodles. Find the probability that at most 840 students eat noodles.

[30 marks]

4. a) Explain briefly two methods of deriving point estimators.

[30 marks]

b) Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function,

$$f(y, \theta) = \begin{cases} (\theta + 1) y^{\theta} & ; \quad 0 < y < 1, \theta > -1 \\ 0 & ; \quad o/w \end{cases}$$

i. Find an estimator for θ by the method of moments.

ii. Find also the maximum likelihood estimator for θ.

[70 marks]

5. a) Define the following terms.

i. Unbiased estimators.

ii. Asymptotically unbiased estimator.

[10 marks]

[Turn over]

b) Let $Y_1, Y_2, \ldots, Y_n$ is independent and identical with Exponential with parameter θ. θ is unknown parameter and it has 5 estimators.

$$\hat{\theta}_1 = Y_1, \quad \hat{\theta}_2 = \frac{Y_1 + Y_2}{2}, \quad \hat{\theta}_3 = \frac{Y_1 + 2Y_2}{3}, \qquad \hat{\theta}_4 = \min(Y_1, Y_2, Y_3), \qquad \hat{\theta}_5 = \bar{y}$$

i. Check whether the estimators are unbiased or not and select the unbiased estimator with minimum variance.

ii. Calculate the $eff(\widehat{\theta_1}, \widehat{\theta_4})$, $eff(\widehat{\theta_2}, \widehat{\theta_4})$ and $eff(\widehat{\theta_3}, \widehat{\theta_4})$.

[90 marks]

6. a) State the properties of the pivotal quantity. **[10 marks]**

b) Let consider the two dependent samples, $X_i \sim N(\mu_1, \sigma_1^2)$ for $i = 1,2,..,n$ and $Y_i \sim N(\mu_2, \sigma_2^2)$ for $i = 1,2,..,n$. Then take $\mu_d = \mu_1 - \mu_2$. Use the usual notations and hence derive the $(1-\alpha)100\%$ confidence interval for μ_d is

$$\bar{d} \pm t_{n-1}(\alpha/2)\frac{s_d}{\sqrt{n}}.$$

[30 marks]

c) Here are average weekly losses of man-hours due to accidents in 10 individual plants before and after a certain safety program was put into operation.

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

i. Construct a 95% confidence interval for the mean decrease in weekly man-hours lost due to accidents for all plants after implementing the safety program.

ii. Interpret about the confidence interval and decide if the program seems to be effective or not.

[60 marks]

** END**