RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (Four Year) Degree in Applied Sciences
Fourth Year - Semester I Examination – October / November 2017

PHY 4312 - STATISTICAL THERMODYNAMICS

Time: Three (3) hours

Answer all **five** questions
The use of a non-programmable calculator is permitted.

1.

a) Derive the binomial distribution function, $W_N(n_1) = [N!/(n_1!n_2!)]p^{n_1}q^{n_2}$ using approximations in one dimensional random walk where, p and q are probabilities of stepping to the right and left respectively, n_1 is number of steps to the right and n_2 is number of steps to the left out of total N steps.

(40 marks)

b) Prove that;
   i) the mean value of n_1 is given by $\overline{n_1} = Np$,
   ii) the mean value of n_2 is given by $\overline{n_2} = Nq$ and
   iii) the mean displacement is given by $\overline{m} = N(p-q)$.
   iv) Mean dispersion $\overline{\Delta n_1^2} = Npq$

(60 marks)

2.

a) A particle of mass m is confined to a one dimensional box with length L. Prove that the number of states accessible to the system is $\dfrac{L\sqrt{2m}}{2\pi\hbar\sqrt{E}}\delta E$ (symbols have their usual meaning).

(50 marks)

b) A particle of mass m is free to move in one dimension and its position and momentum coordinates are x and p respectively. Suppose that this particle is confined within $x = 0$ and $x = L$ and its energy lies between E and $E+\delta E$. Draw classical phase space for this particle, indicating the regions accessible to the particle.

(50 marks)

3.

a) Probability $W_N(n)$ of an event characterized with a probability p occurs n times in N trails is given by the binomial distribution, $W_N(n) = [N!/(n!(N-n)!)]p^n(1-p)^{N-n}$. Consider a situation where n is very small compared to N ($n \ll N$). Using the result $\ln(1-p) \approx -p$ for very small p, show that $(1-p)^{N-n} \approx e^{-Np}$.

(25 marks)

b) Prove that $\dfrac{N!}{(N-n)!} \approx N^n$

(25 marks)

c) Hence, prove that $W_N(n)$ reduces to $W_N(n) = \dfrac{\lambda^n}{n!}e^{-\lambda}$ where λ is the mean number of events.

(25 marks)

d) Pure Na sample prepared in a lab contains K as an impurity. The probability of finding K atom is 0.01 and Na is 99.99. What is the probability of containing exactly five K atoms in a sample of 300 g ?

(25 marks)

4.

a) Define purely thermal interaction and quasi-static process.

(30 marks)

b) An ideal gas has a temperature-independent molar specific heat capacity c_v at constant volume. Let $\gamma = \frac{c_p}{c_v}$ denotes the ratio of its specific heats. The gas is thermally insulated and allowed to expand quasi-statically from an initial volume V_i at temperature T_i to the final volume V_f.

i) Use the relation $pV^{\gamma} = \text{Constant}$, to find the final temperature.

ii) Use the fact that the entropy remains constant in this process to find the final temperature, T_f.

(70 marks)

5.

a) Show that $\beta(\tilde{E})$ is a constant for a purely thermal quasi static macroscopic interaction of two systems where, $\Omega(E)$ is number of accessible states to the system, $\tilde{E}$ is most probable energy of the system and $\beta(E) = \dfrac{\partial}{\partial E}\ln\Omega(E)$.

(60 marks)

b) Prove that, when the system is at the equilibrium,

i) the entropy change is given by $\dfrac{dE}{T}$,

ii) the entropy change in the combined system is maximum and

iii) the temperature difference between two systems is zero.

(40 marks)

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