RAJARATA UNIVERSITY OF SRI LANKA

FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences

First Year Semester I Examination– May/ June 2016

FDN 1204 – Basic Mathematics

Time: 02hours

Answer <u>four (04)</u> questions, including the question number <u>one (01)</u>.

1. Given that, $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 1\\ 2 & 0 & 1\end{pmatrix}$ $B=\begin{pmatrix}0 & -2 & -3\\ 0 & 0 & -1\\ -2 & 0 & 0\end{pmatrix}$ $C=\begin{pmatrix}3\\ 1\\ 0\end{pmatrix}$ $D=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$

   (a) Find A+B and what is the uniqueness of the resulted matrix?

   (b) Find, (i) A^T (ii) D^{-1} (iii) $A+B^T$ (iv) AC
   (v) CA (vi) $D^{-1}D$ (vii) DD^{-1}

   (c) Explain the property of D and D^{-1} matrices. (25 Marks)

2. (a) Simplify and express in the form of $(a+bi)$, $\frac{3i^8-i^{11}}{2i-\sqrt{3}}$

   (b) Find the real numbers x and y such that $i^3+6y+3ix-iy+5x=-1+3i$

   (c) (i) What is the complex conjugate of $e^{i\theta}=\cos\theta+i\sin\theta$?

   Given that, $e^{i\theta}=\cos\theta+i\sin\theta$
   $e^{-i\theta}=\cos\theta-i\sin\theta$

   Show that, (ii) $\cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}$ (iii) $\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}$
   (iv) $\sin^2\theta+\cos^2\theta=1$ (v) $\cos2\theta+i\sin2\theta=e^{2i\theta}$

   (d) Given that $\log_5(x^3+x^2-9)=3\log_5 x$, find the value x.

   (25 Marks)

3. (a) In a combustion reaction, the mass (in gram) of fuel decreased according to the formula $M = 4 - 0.05t^2$ (t in hours).
   (i) Find the average rate of reaction in the interval from $t = 2$ to $t = 3$.
   (ii) Find the rate of reaction at $t = 4$.
   (iii) State whether the curve of combustion reaction is parabola or a straight line.
   (iv) Find the turning point of this curve and determine whether it is a maximum, minimum or a point of inflexion.

   (b) Given that Arrhenius equation as $k = Ae^{-\frac{E_a}{RT}}$ where, k is rate of the reaction, T is the temperature, R is the Universal gas constant, A is pre-exponential factor and E_a is activation energy.
   (i) Express the equation in linear form using logarithms.
   (ii) If $\frac{1}{T}$ is the independent variable and $\ln k$ is the dependant variable convert the Arrhenius equation into the form of $y = mx + c$.
   (iii) Write the values of gradient (m) and intercept (c).
   (iv) Write down the 1st partial derivative of Arrhenius equation with respect to T.

(25 marks)

4. (a) Solve the equation $x^4 - 7x^2 + 12 = 0$

   (b) Find the area of the region bound by the x-axis $\sqrt{3}$ and 2 of the curve $y = x^4 - 7x^2 + 12$, explain the sign of the area obtained.

   (c) In chemical thermodynamics the G = H – TS where, G = Gibbs free energy, H = enthalpy, T = temperature and S = entropy are state variables. The infinitesimal change can be denoted as (G+∂G), (H+∂H), (S+∂S) and (T+∂T).
   (i) Show that $\partial G = \partial H - T\partial S - S\partial T$.
   (ii) Find all partial derivatives of G = H – TS.
   (iii) Write the total differential of the function G = H – TS. (25 Marks)

5. (a) Factorise the equation $3x^3 + 4x^2 - 4x$

   (b) Integrate $\int \frac{x^2+4}{3x^3+4x^2-4x} dx$

   (c) Determine $\frac{dy}{dx}$ from 1st principle of the function $y = 3x^2 + 5$. (25 Marks)