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RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (4 year) Degree in Applied Sciences
B.Sc. (Joint Major) Degree in Chemistry and Physics

Fourth Year - Semester II Examination – February /March 2019

PHY 4312 - STATISTICAL THERMODYNAMICS

Time: Two (3) hours

Answer all **five** questions
The use of a non-programmable electronic calculator is permitted.

1.

a) Obtain the binomial probability distribution function, $W_N(n_1) = [N!/(n_1!n_2!)]p^{n1}q^{n2}$ using approximations in one dimensional random walk, where p and q are probabilities of stepping to the right and left respectively. n_1 is number of steps taken to the right and n_2 is number of steps to the left out of total N steps.

(20 marks)

b) Prove that (symbols have their usual meanings);
   i) the mean value of n_1 is given by $\overline{n_1} = Np$,
   ii) $\overline{n_1 + n_2} = N$
   iii) the mean value of n_2 is given by $\overline{n_2} = Nq$ and
   iv) the mean displacement is given by $\overline{m} = N(p - q)$ and
   v) mean dispersion is given by $\overline{\Delta n_1^2} = Npq$

(60 marks)

c) Assume that a person is taking steps randomly in a one-dimensional space. The probability of taking a step to the right is 0.75 and to the left is 0.25. Find the values of $\overline{n_1}$, $\overline{n_1 + n_2}$, $\overline{m}$ and $\overline{\Delta n_1^2}$, if the total number of steps is 10.

(20 marks)

2. Prove that the binomial probability distribution can be simplified to gaussian distribution, $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, for large N values ($N \gg 1$), where μ and σ^2 are the most probable value and dispersion of x. (hint; Use the Taylor series expansion. For large n values $\frac{d}{dn}\ln(n!) = \ln(n)$).

(60 marks)

a) Show that the mean value is equal to the most probable value for the gaussian distribution.

(20 marks)

b) A box contains 100 Ω resistors which are known to have a standard deviation of 2 Ω. What is the probability of selecting a resistor with a value of 95 Ω?

(20 marks)

3.

a) A particle of mass m is free to move in one dimension. It´s position and momentum coordinates are denoted by x and p respectively. Suppose, this particle is confined to a box so as to be located between $x = 0$ and $x = L$, and its energy is known to lie between E and $E+dE$. Draw the classical phase space of this particle, indicating the regions of this space which are accessible to the particle.

(25 marks)

b) Write down the energy of the single particle system mentioned above as a function of L, m and number of accessible states (solutions of Schrödinger equation can be assumed)

(25 marks)

c) Prove that the number of states accessible to the system mentioned above is $\frac{L\sqrt{2m}}{2\pi\hbar\sqrt{E}}\delta E$

(symbols have their usual meaning).

(50 marks)

4.

a) Probability $W_N(n)$ that an event characterized a probability p occurs n times in N trails is given by the binomial distribution, $W_N(n) = [N!/(n!(N-n)!)]p^n(1-p)^{N-n}$. Consider a situation where n is very small compared to N ($n<<N$). Using the result $\ln(1-p) \approx -p$ for very small p, show that $(1-p)^{N-n} \approx e^{-Np}$.

(20 marks)

b) Prove that $\frac{N!}{(N-n)!} \approx N^n$

(20 marks)

c) Obtain the poisons probability distribution function, $W_N(n) = \frac{\lambda^n}{n!}e^{-\lambda}$ where λ is the mean number of events.

(20 marks)

d) Prove that $W_N(n) = \frac{\lambda^n}{n!} e^{-\lambda}$ is normalized

(20 marks)

e) Prove that $\bar{n} = \overline{\Delta n_1^2} = \lambda$.

(20 marks)

5.

a) Show that $\beta(\tilde{E}) =$ Constant for a purely thermal quasi static macroscopic interaction of two systems, where $\Omega(E)$ is number of accessible states to the system, $\tilde{E}$ is most probable energy of the system and $\beta(E) = \frac{\partial}{\partial E} \ln \Omega(E)$.

(50 marks)

b) Prove that when the system is at the equilibrium,

i) the entropy change is given by $\frac{dE}{T}$,

ii) the entropy change in the combined system is maximum and

iii) the temperature difference between two systems is zero.

(30 marks)

c) Assume that total number of accessible states to a system with N particles is proportional to the V^N and $E^{\frac{3N}{2}}$ (symbols have their usual meaning). Prove that;

i) mean pressure of the system is given by nKT and

ii) mean energy is given by $\frac{3}{2}NKT$.

(20 marks)

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