



RAJARATA UNIVERSITY OF SRI LANKA  
FACULTY OF APPLIED SCIENCES, MIHINTALE

B.SC (General) Degree

Second year – Semester I Examination – September 2013

MAA 2204 Linear Programming

Time allowed: **Two** hours.

Answer **FOUR** Questions only.

01. Define the general mathematical model of a linear programming problem. [50 Marks]

A company, engaged in producing canned foods, has 500 trained employees on the rolls, each of whom can produce one can of food in one day. Due to the developing taste of the public for this kind of food, the company plans to add to the existing labour force 250 additional employees in a phased manner, over the next five days. The newcomers would have to undergo a two-day training programme before being put to work. The training is to be given by employees from among the existing ones and it is known that one employee can train three trainees. Assume that there would be no contribution to the production from the trainers and the trainees during training period as the training is off-the-job. However, the trainees would be remunerated at the rate of Rs.800 per day, the same rate as for the trainers.

The company has booked the following orders to supply during the next five days:

Day	1	2	3	4	5
No. of cans	280	298	305	360	400

Assume that the production in any day would not be more than the number of cans ordered so that every delivery of the food would be factory fresh.

Formulate this problem as a linear programming model to develop a training schedule that minimizes the labour cost over the five-day period. [Do not solve it]

[100 Marks]

Turn over

02. Explain the graphical method of solving a linear programming problem. [ 20 Marks ]

A local travel agent is planning a trip to a major sea resort. The trip is restricted to 200 persons and past experience indicates that there will not be any problem for getting 200 persons. The problem for the travel agent is to determine the number of Deluxe, Standard, and Economy tour packages to offer for this trip. These three plans differ according to the seating and service on the flight, quality of accommodation, meal plans and tour options. The following table summarizes the estimated prices for the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of Rs.200,000 for the entire trip.

**Prices and Costs for Tour Packages per person**

Tour plan	Price (Rs.)/ Package	Hotel Costs (Rs.)	Meals and Other expenses (Rs.)
Deluxe	10,000	3,000	4,750
Standard	7,000	2,200	2,500
Economy	6,500	1,900	2,200

In planning the trip, the following considerations must be taken into account:

- At least 10% of the packages must be of the Deluxe type.
- At least 35% but not more than 70% must be of the Standard type.
- At least 30% must be of the Economy type.
- The maximum number of Deluxe packages available in any aircraft is restricted to 60.
- The hotel desires that at least 120 of the tourists should be on the Deluxe and Standard packages together.

The travel agent wishes to determine the number of packages to offer in each type so as to maximize the total profit.

(a). Formulate this problem as a linear programming problem. [55 Marks ]

(b). Restate the above linear programming problem in terms of two decision variables, taking the advantage of the fact that 200 packages will be sold. [15 Marks ]

(c). Find the optimum solution using the graphical method for the restated linear programming problem and interpret your results. [60 Marks ]

03. Explain the meaning of basic feasible solution and degenerate solution in a linear programming problem. [ 20 Marks]

A furniture manufacturer produces and sells desks, chairs and bookshelves. They have no difficulty in selling their items. However, limited availability of machine time, labour and floor space restrict production. Data on usage of resources, supplies and profits on items are given in the following table:

	Desk	Chair	Bookshelf	Supply
Machine Time (hrs/unit)	8	4	5	1000 hrs
Labour time (hrs/unit)	5	3	3	650 hrs
Floor Space (square feet/unit)	9	6	9	1260 sqft
Profit (Rs./unit)	270	144	225	

- (a). Formulate this problem as a linear programming problem. [ 35 Marks]  
 (b). Using  $x_1$ ,  $x_2$ ,  $x_3$  as the respective decision variables, simplex algorithm was initiated for the above problem. At one stage, the following table was reached:

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	constant
$x_1$	1	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{8}$	0	0	125
$s_2$	0	$\frac{1}{3}$	$-\frac{1}{8}$	$-\frac{5}{8}$	1	0	25
$s_3$	0	$\frac{3}{2}$	$\frac{27}{8}$	$-\frac{9}{8}$	0	1	135
$-z$	0	9	$\frac{225}{4}$	$-\frac{135}{4}$	0	0	.....

From the above table, find the current solution. Is it optimal?

[30Marks]

- (c). If the current solution above is not optimal, carry out the simplex algorithm until an optimal solution is reached. State the optimal solution. [45 Marks]

*Turn Over*

- (d). The personnel manager of Good furnishing claims that by recruiting additional labour force, the profit can be increased. Assuming no other changes, investigate whether the claim is valid? Explain giving reasons. [20 Marks]

04. Explain the term artificial variables and their use in linear programming. [30 Marks]

Use the Big-M-Method to solve the following linear programming problem:

$$\text{Maximize } z = 3x_1 + 2x_2 + x_3$$

subject to the constraints

$$2x_1 + 5x_2 + x_3 = 12$$

$$3x_1 + 4x_2 = 11$$

$$x_2 \geq 0, x_3 \geq 0 \text{ and } x_1 \text{ unrestricted.}$$

[120 Marks]

05. Describe the two-phase method of solving a linear programming problem. [30 Marks]

A product is manufactured by blending three different raw materials. The finished product should meet certain quality requirements. Using the two-phase method find your recommendation with regard to quantities of raw materials to be blended, which will meet the quality requirements with minimum cost:

Quality characteristics	A	B	C	Minimum quality requirement
1	3	0	1	10
2	5	1	2	15
Cost (Rs. / unit)	2	5	3	

[120 Marks]

06. State the steps of the revised simplex method. [30 Marks]

The annual handmade furniture show and sale is to be held on next month and the school of vocational studies is planning to make furniture for the sale. There are three wood-working classes: I year, II year and III year, at the school and they have decided to make styles of chairs, A, B and C. Each chair must receive work in each class and the time in hours required for each chair in each class is given in the following table:

Chair	I year	II year	III year
A	2	4	3
B	3	3	2
C	2	1	4

During the next month there will be 120 hours available to the I year class, 160 hours to the II year class, and 100 hours to the III year class to produce the chairs. The teachers of the wood-working classes estimate that a maximum of 40 chairs can be sold at the show. They estimate that the profit from each type of chair will be : A, RS.40 ; B, Rs.35 and C, Rs.30.

Use the revised simplex method to find the number of chairs of each type that should be made in order to maximize profits at the show and sale. **[120 Marks]**

07. State various steps involved in the Dual simplex algorithm. **[25 Marks]**

The XYZ company has the option of producing two products during periods of less activity. For the next period, production has been scheduled so that the milling machine is free for 10 hours and skilled labour will have 8 hours of free time available.

Product	Machine time (hrs / unit )	Skilled labour time (hrs/ unit)	Profit ( Rs. / unit )
A	4	2	5
B	2	2	3

- Formulate this problem as a linear programming problem. **[20 Marks]**
- Solve the model using the graphical method. **[25 Marks]**
- Write the dual problem for the model which was obtained in part (a). **[20 Marks]**
- Solve it using the dual simplex method. **[40Marks]**
- Bring out the fact that the optimal solution of the model in part (a) can be obtained from the optimal solution of the dual model. **[20 Marks]**