RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree

Third Year Semester I Examination - March/April 2014
MAT 3213 - GRAPH THEORY

Answer **four** Questions including the 1st question — Time allowed: 2 hours

1.

a) Identify each statement as true or false. If the statement is false, **give an example that demonstrates it is false.**

Let G and G' be graphs, if G and G' are connected, then they must be isomorphic to each other.

I. If a graph has a bridge, it must be connected.

II. If G and G' have the same number of vertices and equal numbers of vertices with the same degree, then $G \cong G'$.

III. Every connected graph must have a Hamilton circuit.

IV. If G and G' both do not have a Hamilton circuit, then they must be isomorphic to each other.

b) Fill in the blanks with most **suitable answer from the four options** given,

i. All vertices are of equal degree is a .....

a) null graph b) regular graph c) planar graph d) euler graph

ii. A graph G= (V, E), it is possible for the edge set E to be empty, such graph is .....

a) planar graph b) euler graph c) null graph d) none of these

iii. A vertex having no incident edge is .....

a) end vertex b) isolated vertex c) pendent vertex d) none of these

iv. A graph G is said to be ..... if there is at least one path between every pair of vertices in G.

a) disconnected graph b) connected graph c) euler graph d) hamiltonian graph

v. The maximum degree of any vertex in simple graph(with n vertices) is .....

a) n b) n+1 c) n-1 d) 2n

vi. A graph G with n-vertices is called a tree if .....

a) G is circuit less and has n-1 edges b) G is minimum connected graph c) G is connected and is circuit less d) All of the above

c) Fill in the blanks.

i. The total number of edges in the complete graph K_{12} is _____ .

ii. The total number of Hamilton circuits in K_8 is _____ .

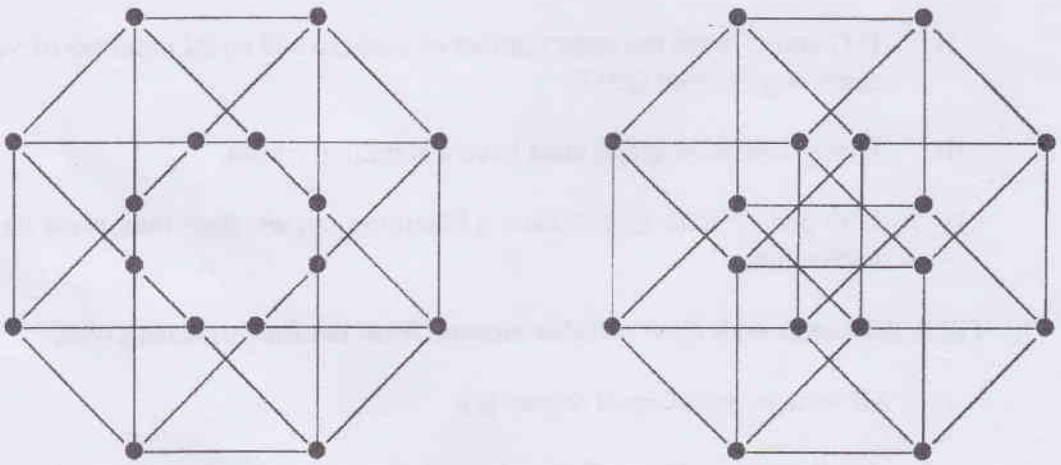
iii. The degree sum of a graph with 150 edges is _____ .

iv. The number of edges contained in a graph having 500 vertices each of degree 8 is _____.

2.

a) Draw all regular graphs of order n, where $2 \leq n \leq 6$.

b) Determine whether the following graphs are isomorphic.



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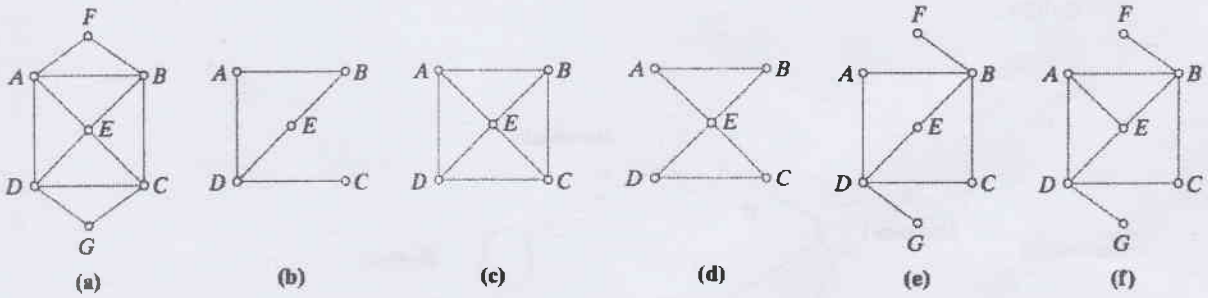
3.

i. A salesman based in Seattle needs to visit Salt Lake City, Houston, Dallas, and Memphis. What sequence of trips would consist of the least total distance traveled? In other words, find the optimal solution to the ***Traveling Salesman Problem*** for these five cities. The distances between the given cities are,

From	To	Distance(miles)
Seattle	Salt Lake City	700
Seattle	Houston	1891
Seattle	Dallas	1683
Seattle	Memphis	1872
Salt Lake City	Houston	1201
Salt Lake City	Dallas	1003
Salt Lake City	Memphis	1256
Houston	Dallas	224
Houston	Memphis	484
Dallas	Memphis	419

ii. Consider the following graphs,

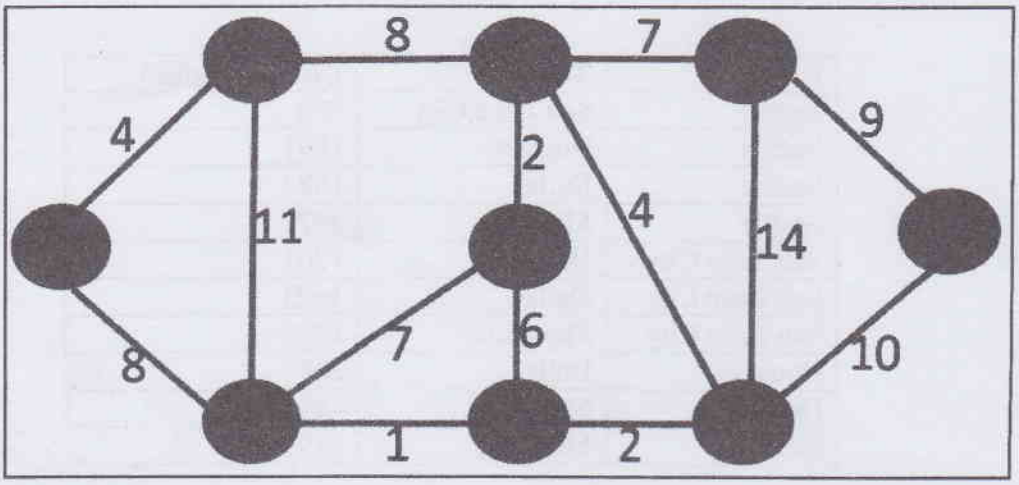


Fill in the blanks with **Yes** or **No** and justify your answer?

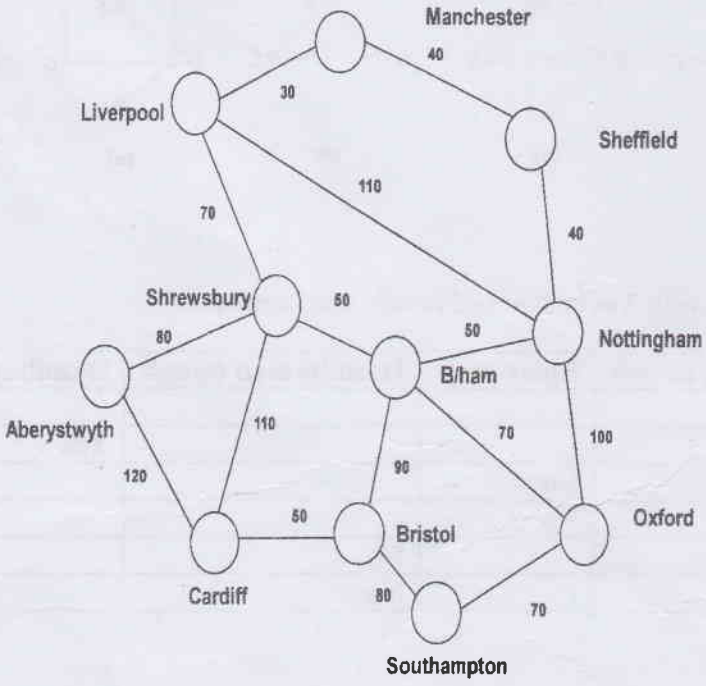
Figure	Euler circuit	Euler path	Hamiltonian circuit	Hamiltonian path
(a)	Yes			
(b)				Yes
(c)		No		
(d)		No		
(e)			No	
(f)			No	

4.

a. Using Kruskal's algorithm, find the minimal spanning tree and its weight for the graph given below



b. Using Prim's algorithm, find the minimal spanning tree and its weight for the graph given below.



5.

a) Construct a multigraph of order 6 and size 7 in which every vertex is odd.
b) Draw all possible trees with 4, 50 or 6 vertices.
c) Suppose G is a k-regular graph of order n and size m ,where $k \geq 0$, $m \geq 0$ and $n \geq 1$. Find a relation linking k, n and m. Justify your answer.
d) Let H be a k -regular graph of order n .If e(H)=10, find all possible values for k and n; and for each case, construct one such graph H.
e) Show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.

6.

I. Let G be a graph with $V(G)=\{1,2,\cdots,10\}$, such that two numbers 'i' and 'j' in V(G) are adjacent if and only if $|i-j|\leq 3$. Draw the graph G and determine e(G).

II. Four teams of three specialist soldiers each (a scout, a sig-naler and a sniper) are to be sent into enemy territory. However, some of the soldiers cannot work well with some others. The following table shows the soldiers, their specializations and who they cannot work with.

Soldier	*Specialization*	*Cannot cooperate with*
1	Scout	5,7,10
2	Scout	-
3	Scout	5,6,8,9,11
4	Scout	8,12
5	Signaler	1,3,9
6	Signaler	3,10,11
7	Signaler	1,9,12
8	Signaler	3,4,9,10
9	Sniper	3,5,7,8
10	Sniper	1,6,8
11	Sniper	3,6
12	Sniper	4,7

a) Draw a multigraph to model the situation so that we may see how to form 3-man teams such that each specialization is represented and every member of the team can work with every other. State clearly what the vertices represent and under what condition(s) two vertices are joined by an edge.

b) Can you form four 3-man teams such that each specialization is represented and all members of the team can work with one another?

c) Construct a simple graph with 5 vertices and 10 edges, or explain why such a graph cannot exist