RAJARATA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

B.Sc. (General) Degree in Applied Sciences
Fourth Year – Semester I Examination –October/November 2017

MAT 4310 – COMPUTATIONAL MATHEMATICS

Time: Three (03) hours

Answer **Five** Questions only
Calculators are allowed

1.

a) Find the Taylor polynomial of degree three for $\mathbf{f(x) = \sin x}$, centered at $\mathbf{x = \frac{5\pi}{6}}$.

(15 marks)

b) Find the Maclaurin series for $\mathbf{\cos x}$.

(15 marks)

c) Evaluate $\int e^{-x^2} dx$.

(15 marks)

d) Let $f(t, y) = 1 + \sin(ty)$. Find the first and second Taylor polynomial of $f(t, y)$centered at $(\pi, 1)$.

(15 marks)

e) The matrix $A = \begin{pmatrix} 1+s & -s \\ s & 1-s \end{pmatrix}$ is given. Calculate p and q such that $A^n = pA + qI$ and determine e^A.

(20 marks)

f) Given the following five data points,

x	1	1.3	1.6	1.9	2.2
$f(x)$	0.1411	-0.6878	-0.9962	-0.5507	0.3115

where $f(x) = sin(3x)$, and estimate $f(1.5)$ by using Lagrange polynomials.

(20 marks)

2. A sequences of functions $f_n(x)$; n = 0,1,2, ... defines a recursion formula,

$$f_{n+1}(x) = 2xf_n(x) - f_{n-1}(x), |x| < 1$$
$$f_0(x) = 0, f_1(x) = 1$$

a) Show that $f_n(x)$ is a polynomial and give the degree and leading coefficient.

(60 marks)

b) Show that

$$\begin{pmatrix} f_{n+1}(x) \\ T_{n+1}(x) \end{pmatrix} = \begin{pmatrix} x & 1 \\ x^2 - 1 & 1 \end{pmatrix} \begin{pmatrix} f_n(x) \\ T_n(x) \end{pmatrix}$$

Where $T_n(x) = \cos(n\cos^{-1}(x))$.

(40 marks)

3. Compute the Approximation to y(1), y′(1), y″ (1) with Taylor's algorithm of order two and step length h=1 when y(x) is the solution to the initial value problem

$$y''' + 2y'' + y' - y = \cos x \quad , 0 \leq x \leq 1 \ , \ y(0) = 0, \ y'(0) = 1, \ y''(0) = 2$$

(100 marks)

4. Consider the following **Runge-Kutta** method for the differential equation y′ = f(x, y)

$$Y_{n+1} = Y_n + \frac{1}{6}(K_1 + 4K_2 + K_3)$$
$$K_1 = h\, f(x_n, y_n)$$
$$K_2 = h\, f\left(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right)$$
$$K_3 = h\, f(x_n + h, y_n - K_1 + 2K_2)$$

a) Compute y(0.4) when $y' = \frac{y+x}{y-x}$, y(0) =1 and h = 0.2. Round to five decimals.

(60 marks)

b) what is the result after one step of length h when y′ = -y ,y (0)=1

(40 marks)

5.

a) Solve the differential equation $y_{n+1} - 2Sin\, x\, y_n + y_{n-1} = 0$, when $y_0 = 0$ and $y_1 = Cos\, x$.

(30 marks)

b) Find the general solution of the difference equation $y_{n+1} - 2y_n = \frac{n}{2^n}$

(40 marks)

c) Find the y_n from the difference equation $\Delta^2 y_{n+1} + \frac{1}{2}\Delta^2 y_n = 0, n = 1,2,3 \ldots$ when $y_0 = 0, y_1 = \frac{1}{2}$ and $y_2 = \frac{1}{4}$.

(30 marks)

6. Given that $\mathbf{A} = \begin{bmatrix} 5.5 & 0 & 0 & 0 & 0 & 3.5 \\ 0 & 5.5 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 6.25 & 0 & 3.75 & 0 \\ 0 & 0 & 0 & 5.5 & 0 & 0.5 \\ 0 & 0 & 3.75 & 0 & 6.25 & 0 \\ 3.5 & 1.5 & 0 & 0.5 & 0 & 5.5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

a) Find the lower triangular matrix **L** of the **Cholesky factorization.**

(70 marks)

b) Solve the system **Ax=b**.

(30 marks)

END