Assignment # 4

1) a. $f(x,y,z) = \frac{(x-c_x)^2}{r_x^2} - \frac{(y-c_y)^2}{r_y^2} - 1$

$$Q = \begin{bmatrix} \frac{1}{r_x^2} & 0 & 0 & -\frac{c_x}{r_x^2} \\ 0 & \frac{1}{r_y^2} & 0 & -\frac{c_y}{r_y^2} \\ 0 & 0 & 0 & 0 \\ -\frac{c_x}{r_x^2} & -\frac{c_y}{r_y^2} & 0 & \frac{c_x^2}{r_x^2} + \frac{c_y^2}{r_y^2} - 1 \end{bmatrix}$$

b. $n(x,y,z) = \begin{bmatrix} \frac{\partial}{\partial x} f(x,y,z) \\ \frac{\partial}{\partial y} f(x,y,z) \\ \frac{\partial}{\partial z} f(x,y,z) \end{bmatrix} = \begin{bmatrix} \frac{2}{r_x^2}(x-c_x) \\ \frac{2}{r_y^2}(y-c_y) \\ 0 \end{bmatrix}$

c. $s(\theta,z) = \begin{bmatrix} r_x \cos\theta + c_x \\ r_y \sin\theta + c_y \\ z \\ 1 \end{bmatrix}, \quad -\pi < \theta < \pi$

d. $n(\theta,z) = \frac{\partial s}{\partial \theta} \times \frac{\partial s}{\partial z}$

$\frac{\partial s}{\partial \theta} = \begin{bmatrix} -r_x \sin\theta \\ r_y \cos\theta \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$n(\theta,z) = \begin{bmatrix} r_y \cos\theta \\ r_x \sin\theta \\ 0 \end{bmatrix}$

$\hat{n}(\theta,z) = \frac{n(\theta,z)}{|n(\theta,z)|}$

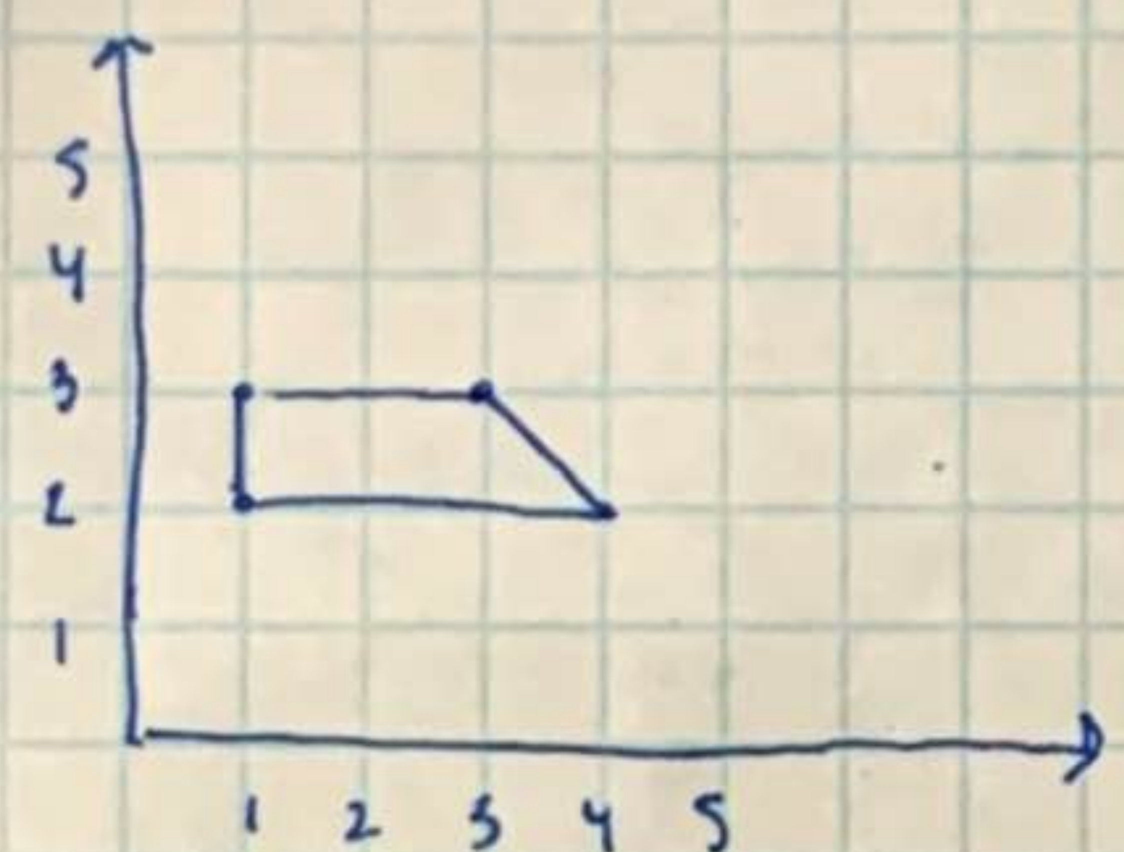
2) a. $L(u) = P_0 + u(P_1 - P_0) = \begin{bmatrix} 2-2u \\ 0 \\ 8u \end{bmatrix}$

b. $P(u,v) = L(u) + \begin{bmatrix} r_x \cos(2\pi v) \\ r_y \sin(2\pi v) \\ 0 \end{bmatrix} = \begin{bmatrix} 2-2u + r_x \cos(2\pi v) \\ r_y \sin\theta \\ 8u \end{bmatrix}$

c. $N(u,v) = \frac{\partial}{\partial u} P(u,v) \otimes \frac{\partial}{\partial v} P(u,v) =$

$$\begin{bmatrix} -2 \\ 0 \\ 8 \end{bmatrix} \otimes \begin{bmatrix} -2\pi r_x \sin(2\pi v) \\ -2\pi\ r_y \cos(2\pi v) \\ 0 \end{bmatrix} = \begin{bmatrix} -16\pi r_x \cos(2\pi v) \\ -16\pi r_y \sin(2\pi v) \\ -4\pi r_y \cos(2\pi v) \end{bmatrix}$$

3) a.



b. $P'(1) = 3(P_3 - P_2) = 3\left[\binom{3}{3} - \binom{4}{2}\right] = 3\binom{-1}{1} = \binom{-3}{3}$

c. $P_0'(1) = \binom{-3}{3}$

$P_1'(0) = 3(P_1 - P_0) = 3\left[\binom{1}{5} - \binom{3}{3}\right] = 3\binom{-2}{2} = \binom{-6}{6}$

$$P_1'(0) \neq P_0'(1)$$

no continuity since $P_1'(0) \neq P_0'(1)$.

4) a.

$$\begin{bmatrix} P(0) \\ P(1) \\ P'(0) \\ P'(1) \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2}(1-t) & 0 & +\frac{1}{2}(1-t) & 0 \\ 0 & -\frac{1}{2}(1-t) & 0 & +\frac{1}{2}(1-t) \end{bmatrix}}^{\text{"A"}} \begin{bmatrix} P_{k-1} \\ P_k \\ P_{k+1} \\ P_{k+2} \end{bmatrix} =$$

$$\overbrace{\begin{bmatrix} (u^3 & u^2 & u & 1)_{u=0} \\ (u^3 & u^2 & u & 1)_{u=1} \\ (3u^2 & 2u & 1 & 0)_{u=0} \\ (3u^2 & 2u & 1 & 0)_{u=1} \end{bmatrix}}^{\text{"B"}} M_C \begin{bmatrix} P_{k-1} \\ P_k \\ P_{k+1} \\ P_{k+2} \end{bmatrix}.$$

So, because "A" = "B" M_c, we can say this all equals:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} M_c$$

To solve for M_c we can now do:

$$M_c = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2}(1-t) & 0 & +\frac{1}{2}(1-t) & 0 \\ 0 & -\frac{1}{2}(1-t) & 0 & \frac{1}{2}(1-t) \end{bmatrix} =$$

$$\begin{bmatrix} -s & 2-s & s-2 & s \\ 2s & s-3 & 3-2s & -s \\ -s & 0 & s & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ where } s = \tfrac{1}{2}(1-t)$$

b. $$B(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -s & 2-s & s-2 & s \\ 2s & s-3 & 3-2s & -s \\ -s & 0 & s & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad s = \tfrac{1}{2}(1-t)$$

$B_0(u) = -su^3 + 2su^2 - su$

$B_1(u) = (2-s)u^3 + (s-3)u^2 + 1$

$B_2(u) = (s-2)u^3 + (3-2s)u^2 + su$

$B_3(u) = su^3 - su^2$

c. We're looking for $P_{s1}(1) = P_{s2}(0)$ assuming segment 2 immediately follows segment 1 (S1) (S2)

If these satisfy continuity then $P'_{s1}(1) = P'_{s2}(0)$

$$P'_{s1}(1) = [(3u^2 \ 2u \ 1 \ 0)|_{u=1}] M_c \begin{bmatrix} P_{k-1} \\ P \\ P_{k+1} \\ P_{k+2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -s & 0 & s \end{bmatrix} \begin{bmatrix} P_{k-1} \\ P \\ P_{k+1} \\ P_{k+2} \end{bmatrix}$$

$$P'_{s2}(0) = [(3u^2 \ 2u \ 1 \ 0)|_{u=0}] M_c \begin{bmatrix} P_k \\ P_{k+1} \\ P_{k+2} \\ P_{k+3} \end{bmatrix}$$

$$= \begin{bmatrix} -s & 0 & s & 0 \end{bmatrix} \begin{bmatrix} P_k \\ P_{k+1} \\ P_{k+2} \\ P_{k+3} \end{bmatrix}$$

$P'_{s1}(1) = P'_{s2}(0)$, so all adjacent segments satisfy continuity.

d. Changing t only changes the scale factor for P' which only changes magnitude, not the tangent at the endpoints.

$$P'(0) = \tfrac{1}{2}(1-a)(P_{k+1} - P_{k-1}), \quad t=a$$
$$P'(0) = \tfrac{1}{2}(1-b)(P_{k+1} - P_{k-1}), \quad t=b$$
$$P'(1) = \tfrac{1}{2}(1-c)(P_{k+2} - P_k), \quad t=c$$
$$P'(2) = \tfrac{1}{2}(1-d)(P_{k+2} - P_k), \quad t=d$$

So the first two P(0)s are proportional to each other, as are the last two.