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CS480. Problem Set #2

Problem One:

```
//Algorithm to detect CW/CCW order of vertices given
// 2D polygon

Input: v1, ...., vN      // N vertices
Output: True or False    // True for CCW, false otherwise

float total = 0;
for(i=1; i<=N; i++){
   if(i == N)
        j = 1;
   else
        j = i+1;
   total += ( vi.x vj.y - vj.x vi.y); //add area to total
return (0.0 < total); // check sign
```

Problem Two:

(a)

$$M = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta & a \\ 0 & -\sin\theta & \cos\theta & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Assume $\theta = 45°$, so

$\cos(45) = \sin(45) = \frac{1}{\sqrt{2}}$

(b)

Translation, Rotation, Scale

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta & a \\ 0 & -\sin\theta & \cos\theta & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem Three:

For an x-direction shear:

$$\begin{bmatrix} 1 & sh_x & y_{ref} \cdot -Sh_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Say that our y_{REF} is $=4$, and our $sh_x = 0.5$

$$\begin{bmatrix} 1 & 0.5 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem Four:

We know that chaining rotations for quaternions takes the form:

$$p' = qpq^{-1}$$

For an addiitonal rotation the point simply becomes:

$$p' = q_2(q_1 p q_1^{-1})q_2^{-1}$$
$$= (s_2, v_2)(s_1, v_1)\, P\, (s_1, -v_1)(s_2, -v_2)$$

If we switch the order of the rotation we get:

$$p' = q_1(q_2 p q_2^{-1})q_1^{-1}$$
$$= (s_1, v_1)(s_2, v_2)\, P\, (s_2, -v_2)(s_1, -v_1)$$

The only time these two rotations are equal is when

$$q_1 q_2 = q_2 q_1$$

$$(s_1 s_2 - v_1 v_2, s_1 v_2 + s_2 v_1 + v_1 \times v_2) = (s_2 s_1 - v_2 v_1, s_2 v_1 + s_1 v_2 + v_2 \times v_1)$$

using our given q_1 and q_2:

$$(s_1 s_2 - v_1 v_2,\ s_1 v_2 + s_2 v_1 + v_1 \times v_2) = (s_2 s_1 - v_2 v_1,\ s_2 v_1 + s_1 v_2 + v_2 \times v_1)$$

$$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{3}-1}{2\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{3}+1}{2\sqrt{2}}\right) + 0 \stackrel{?}{=}$$

$$\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\frac{1}{\sqrt{2}} - \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{-1}{\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{3}+1}{2\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}-1}{2\sqrt{2}}\right) + 0$$

$$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \stackrel{?}{=} \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \stackrel{\checkmark}{=} \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

The rotation does commute. This is upheld by

$$v_2 \times v_1 = v_1 \times v_2$$

in which all cross products equal 0, bringing any objects back to their original position.

(b) $q_3 = (-1, 0, 0, 0)$

$$q = \left(\cos\frac{\theta}{2},\ u \sin\frac{\theta}{2}\right)$$

$$2\cos^{-1}(-1) = 360^\circ \qquad 2u\sin(0^\circ) = 0^\circ$$

angle of rotation for quaternion is 360°.

Problem Five:

First we need to create a uvn coordinate system (a local coordinate system:

$$v = \frac{V}{|V|}, \quad u = \frac{(V_y, -V_x, V_z)}{|V|}, \quad n = u \times v$$

Define Tin in terms of C:

$$T_{in} = \begin{bmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Define Rin in xyz basis:

$$R_{in} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now scale along u-axis:

$$S = \begin{bmatrix} S_u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate back to uvn:

$$R_{OUT} = \begin{bmatrix} u_x & v_x & n_x & 0 \\ u_y & v_y & n_y & 0 \\ u_z & v_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translate C back to original position

$$T_{out} = \begin{bmatrix} 1 & 0 & 0 & C_x \\ 0 & 1 & 0 & C_y \\ 0 & 0 & 1 & C_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The homogeneous transformation is

$$M = T_{OUT} R_{OUT} S R_{IN} T_{IN}$$

Problem ~~6~~ Six:

First, need to create an origin for this plane coord. system

Need to satisfy $ax_0 + by_0 + cz_0 = d$.

Now map new point (x_0, y_0, z_0) to new canonical origin:

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Need to define an orthogonal basis:

$u = (a, b, c) \rightarrow$ normal

$v = (-b, a, 0) \rightarrow$ orthogonal to normal

$n = v \times u \rightarrow$ orthogonal to u and v

Map canonical origin to orthogonal basis:

$$R_{OUT} = \begin{bmatrix} \frac{n}{|n|} & \frac{v}{|v|} & \frac{u}{|u|} & 0 \\ & 0 & & 1 \end{bmatrix} \qquad R_{in} = R_{OUT}^{-1} = R_{OUT}^{T}$$

New Reflection transformation in canonical plane:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous transformation matrix is

$$M = T^{-1} R_{OUT} Q R_{IN} T$$

(I went based off the book, super sorry if this is completely wrong!)

(Just wanted to give it a shot)