QCQI Chapter 4 Exercises

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October 20, 2018

4.2: Let x be a real number and A a matrix such that $A^2 = I$. Show that

$$\exp(iAx) = \cos(x)I + i\sin(x)A$$

Answer:

$$\exp(iAx) = \sum_{n=1}^{\infty} \frac{1}{n!}(iAx)^n \tag{1}$$

$$= \sum_{n=1}^{\infty} \frac{1}{(2n)!}(iAx)^{2n} + \frac{1}{(2n+1)!}(iAx)^{2n+1} \tag{2}$$

$$= I\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} + \frac{1}{(2n+1)!}(iAx)^{2n+1} \tag{3}$$

$$= I\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} + iA\frac{(-1)^n}{(2n+1)!}x^n \tag{4}$$

$$= \cos(x)I + i\sin(x)A \tag{5}$$