Derivation of time-discrete Physics-Informed Neural Networks for PDEs models

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Abstract

The efficient numerical solution of advection-reaction-diffusion partial differential equations (PDEs) is a topic of great interest nowadays. In fact, several real-world applications and processes are modeled through PDEs, with the aim of describing and possibly predicting the dynamics of the considered phenomenon. Among the current applications of great interest, we mention: models for the formation of patterns in the context of the evolution of vegetation in arid and semi-arid environments [1]; models for the formation of patterns in the charging/discharging processes of batteries [2]; phase-field models for the corrosion of metallic materials [3]; sustainability models for renewable energy production [4].

For the efficient solution of PDEs, in addition to classical methods, new methods based on the use of artificial neural networks, called Physics-Informed Neural Networks (PINNs), have recently been proposed [5]. PINNs are artificial neural networks in which the loss function is defined based on the residual due to the problem to be solved, the initial conditions and the boundary conditions. Classical PINNs provide a continuous approximation in time and space of the solution of PDEs, and research on them has significantly intensified in recent years, with increasingly refined techniques aimed at improving their efficiency and reliability, see e.g. [6]. To have more control over the obtained outputs, the so-called time-discrete PINNs have also been introduced, which establish connections between the loss function and the approximations provided by the stages of a classical Runge-Kutta (RK) method for Initial Value Problems (IVPs) [5]. Time-discrete PINNs provide an approximation of the solution of PDEs that is continuous in space but discrete in time. This talk focuses on the derivation of novel time-discrete PINNs based on implicit one-step one-stage methods for IVPs, with the aim of reducing the operations required by existing PINNs based on RK methods for the computation of the solution [7]. Several numerical tests show the potential of the new PINNs and their advantages over existing PINNs from the literature [7,8].

This talk is mainly based on reference [7].

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