Solving parabolic PDEs with Extreme Learning Machines

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Abstract

We investigate different ways to use Neural Networks trained with the Extreme Learning Machine (ELM) algorithm in the numerical resolution of parabolic PDEs. ELMs are neural networks with a single hidden layer where only the weights between hidden and output layer are optimized: all the remaining parameters are fixed randomly at the beginning. This results in a linear problem that can be solved with much faster and more accurate algorithms compared to the non-convex optimization that is usually used for Deep Learning Neural Networks. This remarkable efficiency and accuracy allows the design of numerical solvers that mix the approximation capabilities of neural networks with the stability properties of classical methods [1].
We also investigate how ELMs can handle the direct solution of parabolic PDEs both as a system of ODEs [2] and with Space-Time methods.

References

[1] Francesco Calabrò et al. "Time discretization in the solution of parabolic PDEs with ANNs". In: *Applied Mathematics and Computation* 458 (2023), p. 128230. ISSN: 0096-3003. DOI: https://doi.org/10.1016/j.amc.2023.128230. URL: https://www.sciencedirect.com/science/article/pii/S0096300323003995.

[2] Enrico Schiassi, Francesco Calabrò, and Davide Elia De Falco. "Pontryagin Neural Networks for the Class of Optimal Control Problems With Integral Quadratic Cost". In: *Aerospace Research Communications* 2 (2024). ISSN: 2813-6209. DOI: 10.3389/arc.2024.13151. URL: https://www.frontierspartnerships.org/journals/aerospace-research-communications/articles/10.3389/arc.2024.13151.