Discovering compact latent structures in PDE solution manifolds

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Abstract

Mathematical models based on differential equations, such as Partial Differential Equations (PDEs) and Stochastic Differential Equations (SDEs), provide a principled framework for quantitative predictions of physical phenomena. However, their effective use is often challenged by the complexity of the underlying physics and the high computational cost of numerical solvers, particularly in many-query scenarios such as sensitivity analysis, parameter estimation, and uncertainty quantification. In this talk, we present Scientific Machine Learning (SciML) methods that integrate physics-based modeling with data-driven methods to accelerate the evaluation of differential problems by discovering low-dimensional latent structures in their solution manifolds [1]. Specifically, we introduce Universal Solution Manifold Networks [2], a class of mesh-free emulators that efficiently capture the input-output mapping of differential models while automatically encoding geometric variability. By avoiding the constraints of traditional discretization schemes, this approach learns a compact, structured representation of the solution space, enabling fast and accurate predictions across varying domains. Furthermore, we present the Latent Dynamics Network [3], a space-time operator-learning method that extracts intrinsic structures governing the system's evolution. This model achieves state-of-the-art accuracy (normalized error reduced by a factor of 5) with an order-of-magnitude reduction in trainable parameters compared to conventional approaches. Numerical results demonstrate how these methods enhance efficiency and accuracy in solving high-dimensional PDE problems, enabling fast approximations of quantities of interest and robust inference under uncertainty.

References

[1] Alfio Quarteroni, Paola Gervasio, and Francesco Regazzoni. Combining physics–based and data–driven models: advancing the frontiers of research with scientific machine learning. *Mathematical Models and Methods in Applied Sciences*, 2025.

[2] Francesco Regazzoni, Stefano Pagani, and Alfio Quarteroni. Universal Solution Manifold Networks (USM-Nets): Non-Intrusive Mesh-Free Surrogate Models for Problems in Variable Domains. *Journal of Biomechanical Engineering*, 144(12):121004, 09 2022. 121004.

[3] Francesco Regazzoni, Stefano Pagani, Matteo Salvador, Luca Dede', and Alfio Quarteroni. Learning the intrinsic dynamics of spatio-temporal processes through Latent Dynamics Networks. *Nature Communications*, 15(1):1834, 2024.