Data-driven approximation of linear operators: convergence, error bounds, and applications to dynamical systems

Liam Llamazares-Elias*[1], Samir Llamazares-Elias[2], Jonas Latz[3], and Stefan Klus[4]

[1]School of Mathematics, University of Edinburgh, UK, ✉ l.s.llamazares-elias@sms.ed.ac.uk
[2]Department of Mathematics, University of Salamanca, Spain, ✉ samirllamazares@usal.es
[3]Department of Mathematics, University of Manchester, UK, ✉ jonas.latz@manchester.ac.uk
[4]School of Mathematical & Computer Sciences, Heriot–Watt University, UK, ✉ s.klus@hw.ac.uk

Abstract

Global information about dynamical systems can be extracted by analysing associated infinite-dimensional transfer operators, such as Perron–Frobenius and Koopman operators as well as their infinitesimal generators. In practice, these operators typically need to be approximated from data. Popular approximation methods are *extended dynamic mode decomposition* (EDMD) and *generator extended mode decomposition* (gEDMD). We propose a unified framework that leverages Monte Carlo sampling to approximate the operator of interest on a finite-dimensional space spanned by a set of basis functions. Our framework contains EDMD and gEDMD as special cases, but can also be used to approximate more general operators. Our key contributions are proofs of the convergence of the approximating operator and its spectrum under non-restrictive conditions. Moreover, we derive explicit convergence rates and account for the presence of noise in the observations. Whilst all these results are broadly applicable, they also refine previous analyses of EDMD and gEDMD. We verify the analytical results with the aid of several numerical experiments.

*Presenting author