Sparse recovery problem in a hierarchical Bayesian framework

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Abstract

A common task in inverse problems and imaging is finding a solution that is sparse, in the sense that most of its components vanish. In the framework of compressed sensing, general results guaranteeing exact recovery have been proven [5, 6]. In practice, sparse solutions are often computed combining ℓ_1-penalized least squares optimization with an appropriate numerical scheme to accomplish the task. A computationally efficient alternative for finding sparse solutions to linear inverse problems is provided by Bayesian hierarchical models, in which the sparsity is encoded by defining a conditionally Gaussian prior model with the prior parameter obeying a generalized gamma distribution. An iterative alternating sequential (IAS) algorithm has been demonstrated to lead to a computationally efficient scheme, and combined with Krylov subspace iterations with an early termination condition, the approach is particularly well suited for large scale problems [1, 4, 3]. In this talk, we will discuss a hybrid version of the original IAS that first exploits the global convergence associated with gamma hyperpriors to arrive in a neighborhood of the unique minimizer, then adopts a generalized gamma hyperprior that promotes sparsity more strongly [2]. The hybrid IAS, that has been originally designed for linear inverse problems, is also extended to the case of non-linear forward model operators [8]. The algorithm will be tested on synthetic data for the linear image restoration inverse problem, and on real data for the non-linear electrical impedance tomography reconstruction [7].

References

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