Low-rank approximations for large-scale nonlinear feedback control

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Abstract

Computation of the optimal feedback law for general (nonlinear/unstable/stochastic) dynamical systems requires solving the Hamilton-Jacobi-Bellman Partial Differential Equation (PDE), which suffers from the curse of dimensionality. We develop a unified framework for computing a fast surrogate model of the feedback control function based on low-rank decompositions of matrices and tensors.

Firstly, we propose a Statistical Proper Orthogonal Decomposition (SPOD) for Model Order Reduction of very high-dimensional systems, such as the discretized Navier-Stokes equation or other PDEs. SPOD finds a reduced state subspace by a low-rank factorization of the matrix of snapshots of stabilized trajectories of the original dynamics; however, unlike the traditional POD or Balanced Truncation, SPOD collects snapshots corresponding to random samples of all parameters in the system, initial condition and time [2]. This makes the reduced model asymptotically accurate in expectation over all possible controlled system outcomes.

Secondly, we compute a low-rank Functional Tensor Train (TT) approximation of the feedback control function for the reduced model. The low dimensionality of the reduced model enables fast on-demand closed-loop synthesis of control- and value function samples via the State-Dependent Riccati Equations or Pontryagin's Maximum Principle. These samples are used to actively learn the TT decomposition by the cross interpolation algorithm [1].

Finally, thus pre-trained TT representation of the control function of the reduced state can be used for real-time online generation of the control signal. Using the proposed combination of SPOD and TT approximations, we demonstrate a controller computable in milliseconds that achieves lower vorticity of the Navier-Stokes flow with random inflow compared to using the mean inflow to produce either reduced bases or controllers (both full and reduced).

References

[1] S. Dolgov, D. Kalise, and L. Saluzzi. Data-driven Tensor Train gradient cross approximation for Hamilton–Jacobi–Bellman equations. *SIAM Journal on Scientific Computing*, 45(5):A2153–A2184, 2023. `https://doi.org/10.1137/22M1498401`

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