Numerical preservation of stochastic dissipativity

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Abstract

Standard numerical analysis for stochastic differential equations (SDEs) has a clear understanding of stability in the linear case or when the drift coefficient satisfies a one-sided Lipschitz condition and the diffusion term is globally Lipschitz [3], [5], [7]. By looking at many applications, it is obvious that we need a deeper mathematical and numerical insight into stability of problems with non-global Lipschitz coefficients [6].

This talk is aimed to analyze nonlinear stability properties of θ-methods for stochastic differential equations under non-global Lipschitz conditions on the coefficients, motivated by [8] and of which a more detailed exposition can be found in [1]. In particular, the concept of exponential mean-square contractivity is introduced for the exact dynamics; additionally, stepsize restrictions in order to inherit the contractive behaviour over the discretized dynamics are also given. A selection of numerical tests confirming the theoretical expectations is also presented.

Moreover, we will apply a similar approach to stochastic partial differential equations (SPDEs) [2]. We consider long-time dynamics of dissipative SPDEs when using finite difference method in space, that is transforming it into a system of SDEs and hence, giving us an intuition of applying a similar theory as before. As a time discretization, we compare θ-Maruyama method with IMEX method, that is considered motivated by the properties of the problem under consideration. We establish monotonicity under very light hypotheses. All the results are presented both theoretically and experimentally supported by the examples. This is a current work in progress and it is motivated by the results carried out in [4].

References

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