Navigating Limits in Learning Complex Systems

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Abstract

Addressing today's challenge of understanding complex, high-dimensional systems demands reliable methods for extracting dynamics directly from data. However, existing data-driven techniques often fail due to convergence issues and limited generalizability, raising concerns about their reliability in critical applications. We confront these challenges by establishing fundamental impossibility results, demonstrating that certain adversarial system dynamics inherently resist algorithmic learning with high probability. By analyzing these inherent limitations, we identify clear conditions necessary for successful data-driven learning and propose optimal algorithms designed for robust convergence and rigorous verification. This leads us to a rigorous classification theory for the capabilities and constraints of data-driven methods. We demonstrate this framework through Koopman operators—operators acting on infinite-dimensional spaces that linearize nonlinear dynamics. Applications range from classical oscillators and chaotic fluid flows to Arctic sea ice forecasting. In the Arctic sea ice case, our methods uncover previously hidden dynamics, achieve significantly extended forecast horizons, provide precise error bounds for geographically crucial regions, and substantially outperform state-of-the-art deep learning and traditional dynamical models—all with reduced computational cost, enabling real-time analysis. This talk is based on joint work with Igor Mezić and Alexei Stepanenko.