Data-Driven Algorithms for Online Identification and Control of Partial Differential Equations

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Abstract

We propose online algorithms to identify and control Partial Differential Equations (PDEs) with unknown parameters, addressing key challenges in the control of complex dynamical systems.

We consider two scenarios. In the first, the PDE is assumed observable given a control input and an initial condition. Using an initial parameter estimate, we compute the control via the State-Dependent Riccati Equation (SDRE). The observed trajectory is then used to iteratively update the parameter configuration via Bayesian Linear Regression until a stopping criterion is met.

The second scenario addresses cases with incomplete information, such as unknown initial conditions, sparse boundary data and coefficient in the PDE. We leverage Physics-Informed Neural Networks (PINNs) for open-loop optimal control problems, deriving optimality conditions via Lagrangian multipliers. Neural networks predict the state, adjoint, and control variables, while identifying parameters online based on sparse data obtained from the uncontrolled problem.

Numerical examples demonstrate the effectiveness of the proposed methods, even in highly challenging scenarios characterized by significant uncertainty and incomplete information.

References

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