Lecture 4

Better Explained Blog on LinAlg:

Linear algebra gives you mini-spreadsheets for your math equations.

(most elementary view of linear algebra)

GE Demo

Vocabulary: Pivot, Multiplier.   The multipliers go in L. With ones on the diagonal, LU = A.

Transpose (aka adjoint); Flip along main diagonal. Rows & Columns Interchange

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

Main Diagonal

If A is m x n, A^T is n x m

$A_{ij} = (A^T)_{ji}$

Julia: Uses apostrophe, and uses “lazy evaluation.” (Does not store the transpose, works with it when needed.)

```
A = [1 2;3 4;5 6]
B = A'
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2×3 LinearAlgebra.Adjoint{Int64,Array{Int64,2}}:
 1  3  5
 2  4  6
```
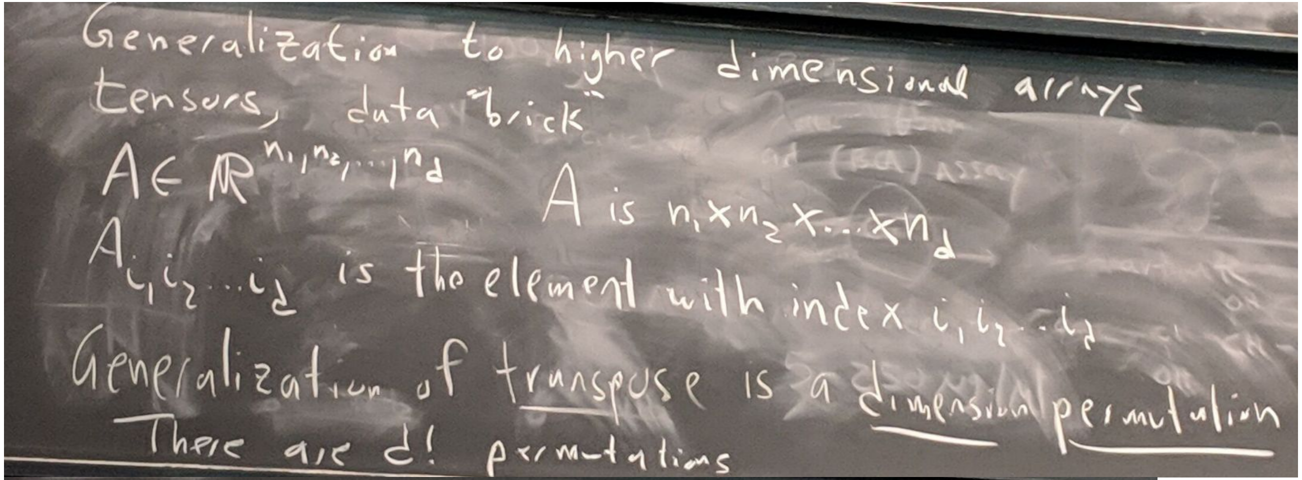
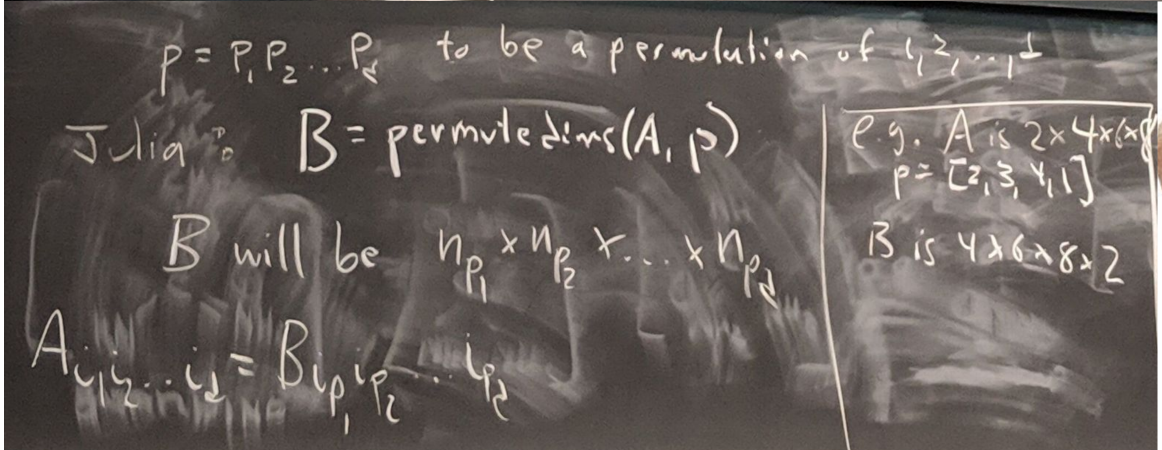

Data “Bricks” aka Tensors or Multidimensional Arrays





Transpose generalizes to a “dimension permutation”

Inverses

Only applies to n x n square

A^{-1} is pronounced “A Inverse”

$A^{-1}A = I$

If $A^{-1}A = I$ then $AA^{-1} = I$ and vice versa.

If A^{-1} does not exist, we say that A is “singular”.

If A^{-1} does exist we say that A is “nonsingular” or “invertible”

$(AB)^{-1} = B^{-1}A^{-1}$

$Ax = b$ (n x n system of linear equations)

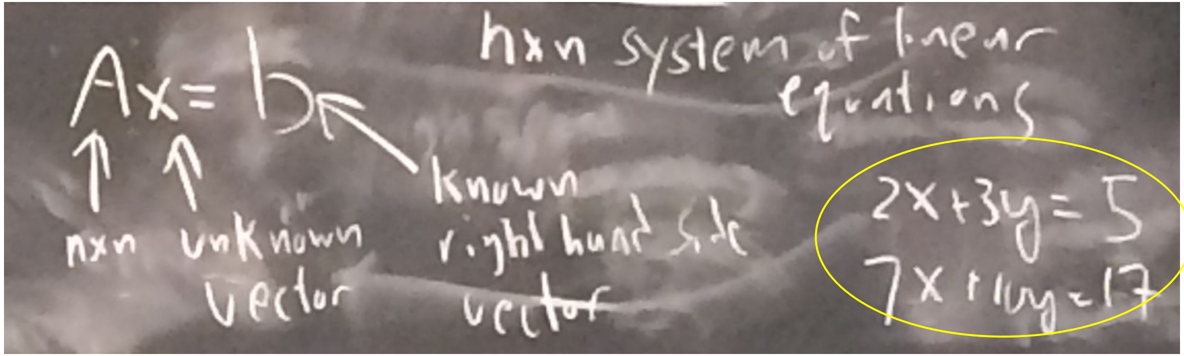
Given n x n A, and right hand side b, if A is invertible, the unique solution is

$$x=A^{-1}b$$

In software it is often considered disadvantageous for complexity and numerical reasons to ever compute A^{-1} explicitly. A common way to write the solution is

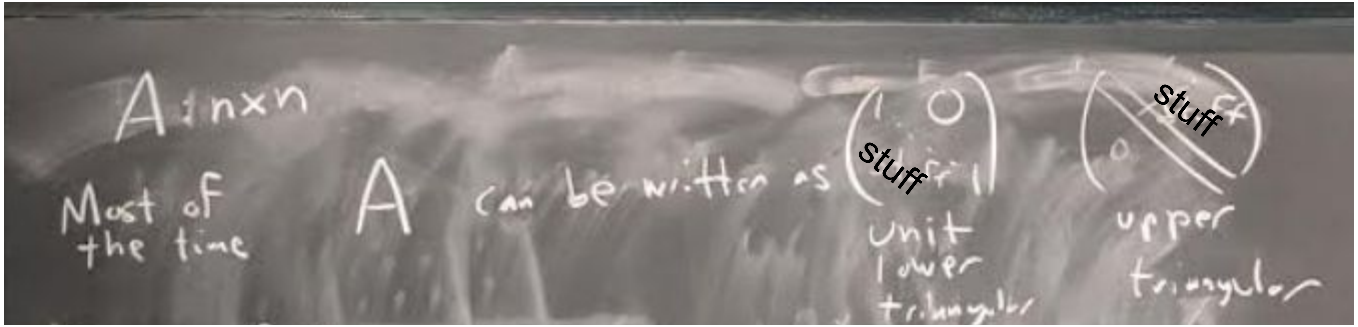
$$x=A\backslash b$$

emphasizing the leftward looking division.



← Generalizes n=2 like you may have seen in 7th grade.

A = (unit lower triangular) x (upper triangular)

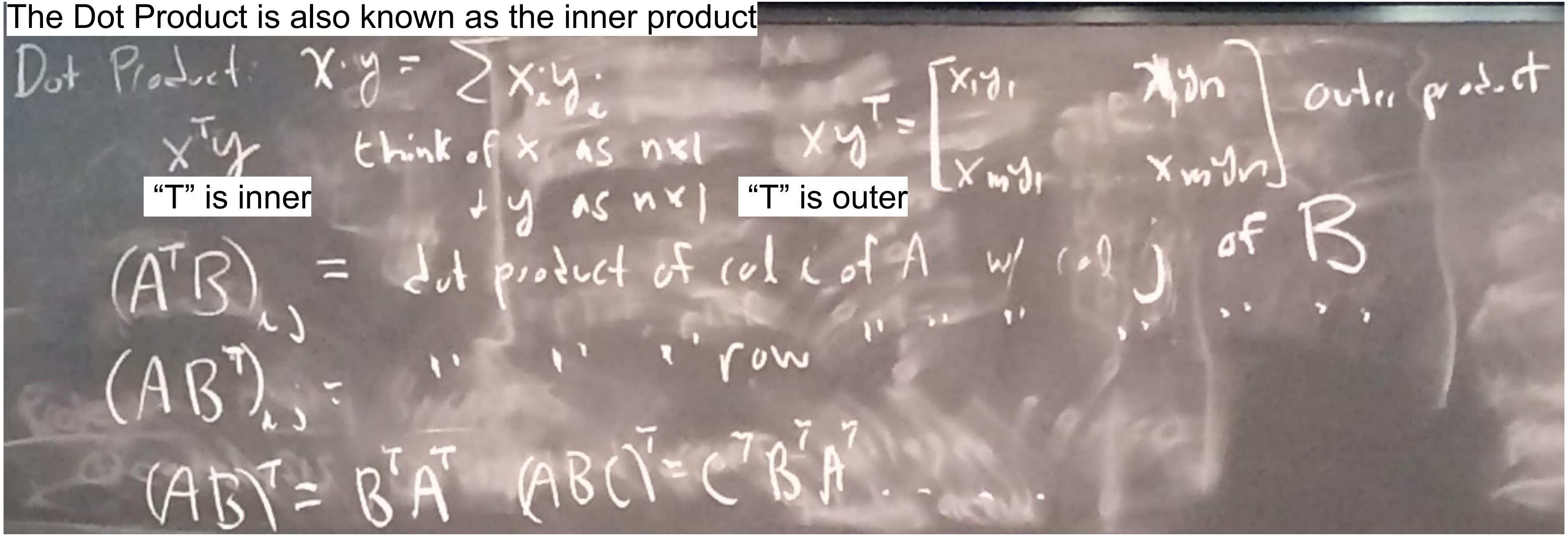


Exists if the top left entry is not 0, and the top left corner 2x2 is invertible, and the top left 3x3 is invertible, …., up to the top left n-1 x n-1.

L has 1 + 2 + .. + n-1 = n(n-1)/2 parameters, U has n(n+1)/2.
Together one gets n^2 parameters, corresponding to that of A.

Inner and Outer Products, and matmul tricks

n vectors are sometimes thought of as n x 1 matrices. This causes little trouble in a linear algebra context and tons of trouble in software with vectors and matrices.



(Matmul is a commonly used abbreviation for Matrix Multiply)