

# Mathematica 7 Test Results

## For Integration Problems Involving Inverse Hyperbolic Functions

### Problems involving inverse hyperbolic sines

Unable to integrate:

$$\{\text{ArcSinh}[c e^{a+bx}], x, 6, 0\}$$

$$-\frac{\text{ArcSinh}[c e^{a+bx}]^2}{2b} + \frac{\text{ArcSinh}[c e^{a+bx}] \text{Log}[1 - e^{2 \text{ArcSinh}[c e^{a+bx}]}]}{b} + \frac{\text{PolyLog}[2, e^{2 \text{ArcSinh}[c e^{a+bx}]}]}{2b}$$

$$\int \text{ArcSinh}[c e^{a+bx}] dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcSinh}[a x^n]}{x}, x, 5, 0 \right\}$$

$$-\frac{\text{ArcSinh}[a x^n]^2}{2n} + \frac{\text{ArcSinh}[a x^n] \text{Log}[1 - e^{2 \text{ArcSinh}[a x^n]}]}{n} + \frac{\text{PolyLog}[2, e^{2 \text{ArcSinh}[a x^n]}]}{2n}$$

$$\text{ArcSinh}[a x^n] \text{Log}[x] + \frac{1}{2\sqrt{a^2} n} a \left( \text{ArcSinh}[\sqrt{a^2} x^n]^2 + 2 \text{ArcSinh}[\sqrt{a^2} x^n] \text{Log}[1 - e^{-2 \text{ArcSinh}[\sqrt{a^2} x^n]}] - 2n \text{Log}[x] \text{Log}[\sqrt{a^2} x^n + \sqrt{1 + a^2 x^{2n}}] - \text{PolyLog}[2, e^{-2 \text{ArcSinh}[\sqrt{a^2} x^n]}] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \text{ArcSinh}\left[\frac{c}{a+bx}\right], x, 2, 0 \right\}$$

$$\frac{(a+bx) \text{ArcCsch}\left[\frac{a}{c} + \frac{bx}{c}\right]}{b} + \frac{c \text{ArcTanh}\left[\sqrt{1 + \frac{c^2}{(a+bx)^2}}\right]}{b}$$

$$x \text{ArcSinh}\left[\frac{c}{a+bx}\right] + \left( (a+bx) \sqrt{\frac{a^2 + c^2 + 2abx + b^2 x^2}{(a+bx)^2}} \right)$$

$$\left( -a \text{Log}[a c (a+bx)] + a \text{Log}[-2b^2 c \left( c + \sqrt{a^2 + c^2 + 2abx + b^2 x^2} \right)] + c \text{Log}\left[2 \left( a+bx + \sqrt{a^2 + c^2 + 2abx + b^2 x^2} \right) \right] \right) \Bigg/ \left( b \sqrt{a^2 + c^2 + 2abx + b^2 x^2} \right)$$

## Problems involving inverse hyperbolic cosines

Unable to integrate:

$$\{\text{ArcCosh}[c e^{a+bx}], x, 6, 0\}$$

$$-\frac{\text{ArcCosh}[c e^{a+bx}]^2}{2b} + \frac{\text{ArcCosh}[c e^{a+bx}] \text{Log}[1 + e^{2 \text{ArcCosh}[c e^{a+bx}]}]}{b} + \frac{\text{PolyLog}[2, -e^{2 \text{ArcCosh}[c e^{a+bx}]}]}{2b}$$

$$\int \text{ArcCosh}[c e^{a+bx}] dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCosh}[a x^n]}{x}, x, 5, 0 \right\}$$

$$-\frac{\text{ArcCosh}[a x^n]^2}{2n} + \frac{\text{ArcCosh}[a x^n] \text{Log}[1 + e^{2 \text{ArcCosh}[a x^n]}]}{n} + \frac{\text{PolyLog}[2, -e^{2 \text{ArcCosh}[a x^n]}]}{2n}$$

$$\frac{1}{2} \left( 2 \text{ArcCosh}[a x^n] \text{Log}[x] + \frac{1}{\sqrt{-a^2} n \sqrt{1 - a^2 x^{2n}}} a \sqrt{-1 + a x^n} \sqrt{1 + a x^n} \left( -\text{ArcSinh}[\sqrt{-a^2} x^n]^2 - 2 \text{ArcSinh}[\sqrt{-a^2} x^n] \text{Log}[1 - e^{-2 \text{ArcSinh}[\sqrt{-a^2} x^n]}] \right) + 2n \text{Log}[x] \text{Log}[\sqrt{-a^2} x^n + \sqrt{1 - a^2 x^{2n}}] + \text{PolyLog}[2, e^{-2 \text{ArcSinh}[\sqrt{-a^2} x^n]}] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\{\text{ArcCosh}[\frac{c}{a+bx}], x, 2, 0\}$$

$$\frac{(a+bx) \text{ArcSech}[\frac{a}{c} + \frac{bx}{c}]}{b} - \frac{2c \text{ArcTan}[\sqrt{\frac{1 - \frac{a}{c} \frac{bx}{c}}{1 + \frac{a}{c} \frac{bx}{c}}}] }{b}$$

$$x \text{ArcCosh}[\frac{c}{a+bx}] + \frac{\sqrt{a-c+bx} \left( i a \text{Log}\left[-\frac{2b^2 \left( -i c + \sqrt{a-c+bx} \sqrt{a+c+bx} \right)}{a(a+bx)} \right] + c \text{Log}\left[2 \left( a+bx + \sqrt{a-c+bx} \sqrt{a+c+bx} \right) \right] \right)}{b \sqrt{-\frac{a-c+bx}{a+c+bx}} \sqrt{a+c+bx}}$$

## Problems involving inverse hyperbolic tangents

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcTanh}[a x]^3}{\sqrt{1 - a^2 x^2}}, x, 11, 0 \right\}$$

$$\frac{2 \text{ArcTan}\left[e^{\text{ArcTanh}[a x]}\right] \text{ArcTanh}[a x]^3}{a} - \frac{3 i \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcTanh}[a x]}\right]}{a} +$$

$$\frac{3 i \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right]}{a} + \frac{6 i \text{ArcTanh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcTanh}[a x]}\right]}{a} -$$

$$\frac{6 i \text{ArcTanh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcTanh}[a x]}\right]}{a} - \frac{6 i \text{PolyLog}\left[4, -i e^{\text{ArcTanh}[a x]}\right]}{a} + \frac{6 i \text{PolyLog}\left[4, i e^{\text{ArcTanh}[a x]}\right]}{a}$$

$$- \frac{1}{64 a} i \left( 7 \pi^4 + 8 i \pi^3 \text{ArcTanh}[a x] + 24 \pi^2 \text{ArcTanh}[a x]^2 - 32 i \pi \text{ArcTanh}[a x]^3 - \right.$$

$$16 \text{ArcTanh}[a x]^4 + 8 i \pi^3 \text{Log}\left[1 + i e^{-\text{ArcTanh}[a x]}\right] + 48 \pi^2 \text{ArcTanh}[a x] \text{Log}\left[1 + i e^{-\text{ArcTanh}[a x]}\right] -$$

$$96 i \pi \text{ArcTanh}[a x]^2 \text{Log}\left[1 + i e^{-\text{ArcTanh}[a x]}\right] - 64 \text{ArcTanh}[a x]^3 \text{Log}\left[1 + i e^{-\text{ArcTanh}[a x]}\right] -$$

$$48 \pi^2 \text{ArcTanh}[a x] \text{Log}\left[1 - i e^{\text{ArcTanh}[a x]}\right] + 96 i \pi \text{ArcTanh}[a x]^2 \text{Log}\left[1 - i e^{\text{ArcTanh}[a x]}\right] - 8 i \pi^3 \text{Log}\left[1 + i e^{\text{ArcTanh}[a x]}\right] +$$

$$64 \text{ArcTanh}[a x]^3 \text{Log}\left[1 + i e^{\text{ArcTanh}[a x]}\right] + 8 i \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (\pi + 2 i \text{ArcTanh}[a x])\right]\right] -$$

$$48 (\pi - 2 i \text{ArcTanh}[a x])^2 \text{PolyLog}\left[2, -i e^{-\text{ArcTanh}[a x]}\right] + 192 \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcTanh}[a x]}\right] -$$

$$48 \pi^2 \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right] + 192 i \pi \text{ArcTanh}[a x] \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right] + 192 i \pi \text{PolyLog}\left[3, -i e^{-\text{ArcTanh}[a x]}\right] +$$

$$384 \text{ArcTanh}[a x] \text{PolyLog}\left[3, -i e^{-\text{ArcTanh}[a x]}\right] - 384 \text{ArcTanh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcTanh}[a x]}\right] -$$

$$192 i \pi \text{PolyLog}\left[3, i e^{\text{ArcTanh}[a x]}\right] + 384 \text{PolyLog}\left[4, -i e^{-\text{ArcTanh}[a x]}\right] + 384 \text{PolyLog}\left[4, -i e^{\text{ArcTanh}[a x]}\right] \Big)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2 \text{ArcTanh}[a x]^3}{(1 - a^2 x^2)^{3/2}}, x, 14, 0 \right\}$$

$$- \frac{6}{a^3 \sqrt{1 - a^2 x^2}} + \frac{6 x \text{ArcTanh}[a x]}{a^2 \sqrt{1 - a^2 x^2}} - \frac{3 \text{ArcTanh}[a x]^2}{a^3 \sqrt{1 - a^2 x^2}} + \frac{x \text{ArcTanh}[a x]^3}{a^2 \sqrt{1 - a^2 x^2}} -$$

$$\frac{2 \text{ArcTan}\left[e^{\text{ArcTanh}[a x]}\right] \text{ArcTanh}[a x]^3}{a^3} + \frac{3 i \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcTanh}[a x]}\right]}{a^3} -$$

$$\frac{3 i \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right]}{a^3} - \frac{6 i \text{ArcTanh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcTanh}[a x]}\right]}{a^3} +$$

$$\frac{6 i \text{ArcTanh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcTanh}[a x]}\right]}{a^3} + \frac{6 i \text{PolyLog}\left[4, -i e^{\text{ArcTanh}[a x]}\right]}{a^3} - \frac{6 i \text{PolyLog}\left[4, i e^{\text{ArcTanh}[a x]}\right]}{a^3}$$

$$\begin{aligned}
& \frac{1}{64 a^3} \left( 7 i \pi^4 - \frac{384}{\sqrt{1-a^2 x^2}} - 8 \pi^3 \operatorname{ArcTanh}[a x] + \frac{384 a x \operatorname{ArcTanh}[a x]}{\sqrt{1-a^2 x^2}} + 24 i \pi^2 \operatorname{ArcTanh}[a x]^2 - \frac{192 \operatorname{ArcTanh}[a x]^2}{\sqrt{1-a^2 x^2}} + 32 \pi \operatorname{ArcTanh}[a x]^3 + \right. \\
& \frac{64 a x \operatorname{ArcTanh}[a x]^3}{\sqrt{1-a^2 x^2}} - 16 i \operatorname{ArcTanh}[a x]^4 - 8 \pi^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] + 48 i \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] + \\
& 96 \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - 64 i \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - \\
& 48 i \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] - 96 \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] + \\
& 8 \pi^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] + 64 i \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] - 8 \pi^3 \operatorname{Log}\left[\tan\left[\frac{1}{4}(\pi + 2 i \operatorname{ArcTanh}[a x])\right]\right] - \\
& 48 i (\pi - 2 i \operatorname{ArcTanh}[a x])^2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 192 i \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right] - \\
& 48 i \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] - 192 \pi \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] - 192 \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] + \\
& 384 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] - 384 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right] + \\
& \left. 192 \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right] + 384 i \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 384 i \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcTanh}[a x]}\right] \right)
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^2, x, 11, 0 \right\} \\
& - \frac{\operatorname{ArcTan}\left[\frac{a x}{\sqrt{1-a^2 x^2}}\right]}{a} + \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{a} + \frac{1}{2} x \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^2 + \\
& \frac{\operatorname{ArcTan}\left[e^{\operatorname{ArcTanh}[a x]}\right] \operatorname{ArcTanh}[a x]^2}{a} - \frac{i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a} + \\
& \frac{i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right]}{a} + \frac{i \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a} - \frac{i \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right]}{a} \\
& \int \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^2 dx
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^3, x, 16, 0 \right\} \\
& - \frac{6 \operatorname{ArcTan}\left[e^{\operatorname{ArcTanh}[a x]}\right] \operatorname{ArcTanh}[a x]}{a} + \frac{3 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^2}{2 a} + \frac{1}{2} x \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]^3 + \\
& \frac{\operatorname{ArcTan}\left[e^{\operatorname{ArcTanh}[a x]}\right] \operatorname{ArcTanh}[a x]^3}{a} + \frac{3 i (2 - \operatorname{ArcTanh}[a x]^2) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right]}{2 a} - \\
& \frac{3 i (2 - \operatorname{ArcTanh}[a x]^2) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right]}{2 a} + \frac{3 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a} - \\
& \frac{3 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right]}{a} - \frac{3 i \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a} + \frac{3 i \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcTanh}[a x]}\right]}{a}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{128a} i \left( 7\pi^4 + 8i\pi^3 \operatorname{ArcTanh}[ax] + 24\pi^2 \operatorname{ArcTanh}[ax]^2 + \frac{192i \operatorname{ArcTanh}[ax]^2}{\sqrt{1-a^2x^2}} - \frac{192ia^2x^2 \operatorname{ArcTanh}[ax]^2}{\sqrt{1-a^2x^2}} - \right. \\
& 32i\pi \operatorname{ArcTanh}[ax]^3 + \frac{64iax \operatorname{ArcTanh}[ax]^3}{\sqrt{1-a^2x^2}} - \frac{64ia^3x^3 \operatorname{ArcTanh}[ax]^3}{\sqrt{1-a^2x^2}} - 16 \operatorname{ArcTanh}[ax]^4 - \\
& 384 \operatorname{ArcTanh}[ax] \operatorname{Log}\left[1 - ie^{-\operatorname{ArcTanh}[ax]}\right] + 8i\pi^3 \operatorname{Log}\left[1 + ie^{-\operatorname{ArcTanh}[ax]}\right] + 384 \operatorname{ArcTanh}[ax] \operatorname{Log}\left[1 + ie^{-\operatorname{ArcTanh}[ax]}\right] + \\
& 48\pi^2 \operatorname{ArcTanh}[ax] \operatorname{Log}\left[1 + ie^{-\operatorname{ArcTanh}[ax]}\right] - 96i\pi \operatorname{ArcTanh}[ax]^2 \operatorname{Log}\left[1 + ie^{-\operatorname{ArcTanh}[ax]}\right] - \\
& 64 \operatorname{ArcTanh}[ax]^3 \operatorname{Log}\left[1 + ie^{-\operatorname{ArcTanh}[ax]}\right] - 48\pi^2 \operatorname{ArcTanh}[ax] \operatorname{Log}\left[1 - ie^{\operatorname{ArcTanh}[ax]}\right] + \\
& 96i\pi \operatorname{ArcTanh}[ax]^2 \operatorname{Log}\left[1 - ie^{\operatorname{ArcTanh}[ax]}\right] - 8i\pi^3 \operatorname{Log}\left[1 + ie^{\operatorname{ArcTanh}[ax]}\right] + 64 \operatorname{ArcTanh}[ax]^3 \operatorname{Log}\left[1 + ie^{\operatorname{ArcTanh}[ax]}\right] + \\
& 8i\pi^3 \operatorname{Log}\left[\tan\left[\frac{1}{4}(\pi + 2i \operatorname{ArcTanh}[ax])\right]\right] - 48(8 + \pi^2 - 4i\pi \operatorname{ArcTanh}[ax] - 4 \operatorname{ArcTanh}[ax]^2) \operatorname{PolyLog}\left[2, -ie^{-\operatorname{ArcTanh}[ax]}\right] + \\
& 384 \operatorname{PolyLog}\left[2, ie^{-\operatorname{ArcTanh}[ax]}\right] + 192 \operatorname{ArcTanh}[ax]^2 \operatorname{PolyLog}\left[2, -ie^{\operatorname{ArcTanh}[ax]}\right] - 48\pi^2 \operatorname{PolyLog}\left[2, ie^{\operatorname{ArcTanh}[ax]}\right] + \\
& 192i\pi \operatorname{ArcTanh}[ax] \operatorname{PolyLog}\left[2, ie^{\operatorname{ArcTanh}[ax]}\right] + 192i\pi \operatorname{PolyLog}\left[3, -ie^{-\operatorname{ArcTanh}[ax]}\right] + \\
& 384 \operatorname{ArcTanh}[ax] \operatorname{PolyLog}\left[3, -ie^{-\operatorname{ArcTanh}[ax]}\right] - 384 \operatorname{ArcTanh}[ax] \operatorname{PolyLog}\left[3, -ie^{\operatorname{ArcTanh}[ax]}\right] - \\
& \left. 192i\pi \operatorname{PolyLog}\left[3, ie^{\operatorname{ArcTanh}[ax]}\right] + 384 \operatorname{PolyLog}\left[4, -ie^{-\operatorname{ArcTanh}[ax]}\right] + 384 \operatorname{PolyLog}\left[4, -ie^{\operatorname{ArcTanh}[ax]}\right] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[a+bx]}{x}, x, 5, 0 \right\} \\
& -\frac{1}{2} \operatorname{Log}\left[\frac{bx}{1-a}\right] \operatorname{Log}[1-a-bx] + \frac{1}{2} \operatorname{Log}\left[-\frac{bx}{1+a}\right] \operatorname{Log}[1+a+bx] - \frac{1}{2} \operatorname{PolyLog}\left[2, 1 - \frac{bx}{1-a}\right] + \frac{1}{2} \operatorname{PolyLog}\left[2, 1 + \frac{bx}{1+a}\right] \\
& -\frac{1}{2} i \left( i (\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx])^2 - \right. \\
& \frac{1}{4} i (\pi - 2i \operatorname{ArcTanh}[a+bx])^2 - 2i (\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a] - 2 \operatorname{ArcTanh}[a+bx]}\right] + \\
& (\pi - 2i \operatorname{ArcTanh}[a+bx]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] - (\pi - 2i \operatorname{ArcTanh}[a+bx]) \operatorname{Log}\left[\frac{2}{\sqrt{1-(a+bx)^2}}\right] + \\
& 2i \operatorname{ArcTanh}[a+bx] \left( -\operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] + \operatorname{Log}[-i \operatorname{Sinh}[\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]]] \right) + \\
& 2i (\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]) \operatorname{Log}[-2i \operatorname{Sinh}[\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]]] - \\
& \left. i \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a] - 2 \operatorname{ArcTanh}[a+bx]}\right] - i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[a+bx]}\right] \right)
\end{aligned}$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcTanh}[a+bx]^2}{x}, x, -3, 0 \right\}$$

$$\begin{aligned}
& -\frac{2}{3} \operatorname{ArcTanh}[a + b x]^3 - \operatorname{ArcTanh}[a + b x]^2 \operatorname{Log}\left[\frac{2}{1 + a + b x}\right] + \operatorname{ArcTanh}[a + b x]^2 \operatorname{Log}\left[1 - \frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] + \\
& \operatorname{ArcTanh}[a + b x]^2 \operatorname{Log}\left[1 + \frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] + 2 \operatorname{ArcTanh}[a + b x] \operatorname{PolyLog}\left[2, -\frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] + \\
& 2 \operatorname{ArcTanh}[a + b x] \operatorname{PolyLog}\left[2, \frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] + \operatorname{ArcTanh}[a + b x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + a + b x}\right] - \\
& 2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] - 2 \operatorname{PolyLog}\left[3, \frac{\sqrt{\frac{1-a}{b}} (1 + a + b x)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a + b x)^2}}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + a + b x}\right] \\
& \int \frac{\operatorname{ArcTanh}[a + b x]^2}{x} dx
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[1 + x]}{2 + 2 x}, x, 5, 0 \right\} \\
& -\frac{1}{4} \operatorname{PolyLog}[2, -1 - x] + \frac{1}{4} \operatorname{PolyLog}[2, 1 + x] \\
& \frac{1}{16} \left( -\pi^2 + 4 i \pi \operatorname{ArcTanh}[1 + x] + 8 \operatorname{ArcTanh}[1 + x]^2 + 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[1+x]}\right] - \right. \\
& 4 i \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[1+x]}\right] - 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[1+x]}\right] - 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[\frac{1}{\sqrt{-x} (2 + x)}\right] + \\
& 4 i \pi \operatorname{Log}\left[\frac{2}{\sqrt{-x} (2 + x)}\right] + 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[\frac{2}{\sqrt{-x} (2 + x)}\right] + 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[\frac{i (1 + x)}{\sqrt{-x} (2 + x)}\right] - \\
& \left. 8 \operatorname{ArcTanh}[1 + x] \operatorname{Log}\left[\frac{2 i (1 + x)}{\sqrt{-x} (2 + x)}\right] - 4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[1+x]}\right] - 4 \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[1+x]}\right] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[x]}{a + b x}, x, 3, 0 \right\} \\
& \frac{\operatorname{Log}[1 + x] \operatorname{Log}\left[\frac{a+bx}{a-b}\right]}{2 b} - \frac{\operatorname{Log}[1 - x] \operatorname{Log}\left[\frac{a+bx}{a+b}\right]}{2 b} - \frac{\operatorname{PolyLog}\left[2, \frac{b(1-x)}{a+b}\right]}{2 b} + \frac{\operatorname{PolyLog}\left[2, -\frac{b(1+x)}{a-b}\right]}{2 b}
\end{aligned}$$

$$\frac{1}{8b} \left( -\pi^2 + 4 \operatorname{ArcTanh}\left[\frac{a}{b}\right]^2 + 4i\pi \operatorname{ArcTanh}[x] + 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{ArcTanh}[x] + 8 \operatorname{ArcTanh}[x]^2 - \right. \\
4i\pi \operatorname{Log}\left[1 + e^{2\operatorname{ArcTanh}[x]}\right] - 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 + e^{2\operatorname{ArcTanh}[x]}\right] + 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + \\
8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + 4i\pi \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] + 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] + 4 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1-x^2\right] + \\
8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{Log}\left[2i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - \\
8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[2i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - 4 \operatorname{PolyLog}\left[2, -e^{2\operatorname{ArcTanh}[x]}\right] - 4 \operatorname{PolyLog}\left[2, e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] \left. \right)$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcTanh}[x]}{a+bx+cx^2}, x, 7, 0 \right\}$$

$$\frac{\operatorname{Log}[1+x] \operatorname{Log}\left[\frac{b-\sqrt{b^2-4ac}+2cx}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{Log}[1-x] \operatorname{Log}\left[\frac{b-\sqrt{b^2-4ac}+2cx}{b+2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} - \\
\frac{\operatorname{Log}[1+x] \operatorname{Log}\left[\frac{b+\sqrt{b^2-4ac}+2cx}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{Log}[1-x] \operatorname{Log}\left[\frac{b+\sqrt{b^2-4ac}+2cx}{b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{PolyLog}\left[2, \frac{2c(1-x)}{b+2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} + \\
\frac{\operatorname{PolyLog}\left[2, \frac{2c(1-x)}{b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left[2, -\frac{2c(1+x)}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{PolyLog}\left[2, -\frac{2c(1+x)}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} \\
\int \frac{\operatorname{ArcTanh}[x]}{a+bx+cx^2} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcTanh}[d+ex]}{a+bx}, x, 3, 0 \right\} \\
-\frac{\operatorname{Log}\left[\frac{e(a+bx)}{b(1-d)+ae}\right] \operatorname{Log}[1-d-ex]}{2b} + \frac{\operatorname{Log}\left[-\frac{e(a+bx)}{b(1+d)-ae}\right] \operatorname{Log}[1+d+ex]}{2b} - \frac{\operatorname{PolyLog}\left[2, \frac{b(1-d-ex)}{b(1-d)+ae}\right]}{2b} + \frac{\operatorname{PolyLog}\left[2, \frac{b(1+d+ex)}{b(1+d)-ae}\right]}{2b}$$

$$\begin{aligned}
& -\frac{1}{2b} i \left( i \left( \operatorname{ArcTanh}\left[d - \frac{ae}{b}\right] - \operatorname{ArcTanh}[d + ex] \right)^2 - \frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[d + ex])^2 + \right. \\
& \quad 2 i \left( -\operatorname{ArcTanh}\left[d - \frac{ae}{b}\right] + \operatorname{ArcTanh}[d + ex] \right) \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcTanh}\left[d - \frac{ae}{b}\right] - \operatorname{ArcTanh}[d + ex]\right)}\right] + \\
& \quad (\pi - 2 i \operatorname{ArcTanh}[d + ex]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[d + ex]}\right] - (\pi - 2 i \operatorname{ArcTanh}[d + ex]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - (d + ex)^2}}\right] + \\
& \quad 2 i \operatorname{ArcTanh}[d + ex] \left( -\operatorname{Log}\left[\frac{1}{\sqrt{1 - (d + ex)^2}}\right] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{-bd + ae}{b}\right] + \operatorname{ArcTanh}[d + ex]\right]\right] \right) \\
& \quad 2 i \left( \operatorname{ArcTanh}\left[d - \frac{ae}{b}\right] - \operatorname{ArcTanh}[d + ex] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{-bd + ae}{b}\right] + \operatorname{ArcTanh}[d + ex]\right]\right] - \\
& \quad i \operatorname{PolyLog}\left[2, e^{2 \left(\operatorname{ArcTanh}\left[d - \frac{ae}{b}\right] - \operatorname{ArcTanh}[d + ex]\right)}\right] - i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[d + ex]}\right] \left. \right)
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[d + ex]}{a + bx^2}, x, 7, 0 \right\} \\
& - \frac{\operatorname{Log}\left[-\frac{e^{\left(\sqrt{-a} - \sqrt{b} x\right)}}{\sqrt{b} (1-d) - \sqrt{-a} e}\right] \operatorname{Log}[1 - d - ex]}{4 \sqrt{-a} \sqrt{b}} + \frac{\operatorname{Log}\left[\frac{e^{\left(\sqrt{-a} + \sqrt{b} x\right)}}{\sqrt{b} (1-d) + \sqrt{-a} e}\right] \operatorname{Log}[1 - d - ex]}{4 \sqrt{-a} \sqrt{b}} + \\
& \frac{\operatorname{Log}\left[\frac{e^{\left(\sqrt{-a} - \sqrt{b} x\right)}}{\sqrt{b} (1+d) + \sqrt{-a} e}\right] \operatorname{Log}[1 + d + ex]}{4 \sqrt{-a} \sqrt{b}} - \frac{\operatorname{Log}\left[-\frac{e^{\left(\sqrt{-a} + \sqrt{b} x\right)}}{\sqrt{b} (1+d) - \sqrt{-a} e}\right] \operatorname{Log}[1 + d + ex]}{4 \sqrt{-a} \sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b} (1-d-ex)}{\sqrt{b} (1-d) - \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \\
& \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b} (1-d-ex)}{\sqrt{b} (1-d) + \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b} (1+d+ex)}{\sqrt{b} (1+d) - \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b} (1+d+ex)}{\sqrt{b} (1+d) + \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} \\
& \int \frac{\operatorname{ArcTanh}[d + ex]}{a + bx^2} dx
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[d + ex]}{a + bx + cx^2}, x, 7, 0 \right\} \\
& - \frac{\operatorname{Log}\left[\frac{e^{\left(b - \sqrt{b^2 - 4ac} + 2cx\right)}}{2c(1-d) + \left(b - \sqrt{b^2 - 4ac}\right)e}\right] \operatorname{Log}[1 - d - ex]}{2 \sqrt{b^2 - 4ac}} + \frac{\operatorname{Log}\left[\frac{e^{\left(b + \sqrt{b^2 - 4ac} + 2cx\right)}}{2c(1-d) + \left(b + \sqrt{b^2 - 4ac}\right)e}\right] \operatorname{Log}[1 - d - ex]}{2 \sqrt{b^2 - 4ac}} + \\
& \frac{\operatorname{Log}\left[-\frac{e^{\left(b - \sqrt{b^2 - 4ac} + 2cx\right)}}{2c(1+d) - \left(b - \sqrt{b^2 - 4ac}\right)e}\right] \operatorname{Log}[1 + d + ex]}{2 \sqrt{b^2 - 4ac}} - \frac{\operatorname{Log}\left[-\frac{e^{\left(b + \sqrt{b^2 - 4ac} + 2cx\right)}}{2c(1+d) - \left(b + \sqrt{b^2 - 4ac}\right)e}\right] \operatorname{Log}[1 + d + ex]}{2 \sqrt{b^2 - 4ac}} - \frac{\operatorname{PolyLog}\left[2, \frac{2c(1-d-ex)}{2c(1-d) + \left(b - \sqrt{b^2 - 4ac}\right)e}\right]}{2 \sqrt{b^2 - 4ac}} + \\
& \frac{\operatorname{PolyLog}\left[2, \frac{2c(1-d-ex)}{2c(1-d) + \left(b + \sqrt{b^2 - 4ac}\right)e}\right]}{2 \sqrt{b^2 - 4ac}} + \frac{\operatorname{PolyLog}\left[2, \frac{2c(1+d+ex)}{2c(1+d) - \left(b - \sqrt{b^2 - 4ac}\right)e}\right]}{2 \sqrt{b^2 - 4ac}} - \frac{\operatorname{PolyLog}\left[2, \frac{2c(1+d+ex)}{2c(1+d) - \left(b + \sqrt{b^2 - 4ac}\right)e}\right]}{2 \sqrt{b^2 - 4ac}}
\end{aligned}$$



$$\int \frac{\text{ArcTanh}[d + e x]}{a + b x + c x^2} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcTanh}[b x]}{1 - x^2}, x, 9, 0 \right\}$$

$$\frac{1}{4} \text{Log}\left[-\frac{b(1-x)}{1-b}\right] \text{Log}[1-bx] - \frac{1}{4} \text{Log}\left[\frac{b(1+x)}{1+b}\right] \text{Log}[1-bx] - \frac{1}{4} \text{Log}\left[\frac{b(1-x)}{1+b}\right] \text{Log}[1+bx] + \frac{1}{4} \text{Log}\left[-\frac{b(1+x)}{1-b}\right] \text{Log}[1+bx] +$$

$$\frac{1}{4} \text{PolyLog}\left[2, \frac{1-bx}{1-b}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1-bx}{1+b}\right] + \frac{1}{4} \text{PolyLog}\left[2, \frac{1+bx}{1-b}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1+bx}{1+b}\right]$$

$$- \frac{1}{4\sqrt{-b^2}} b \left( 2 i \text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \text{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] - 4 \text{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] \text{ArcTanh}[bx] - \left( \text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \text{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \right.$$

$$\left. \left( \text{Log}[2] + \text{Log}\left[\frac{b(-i + \sqrt{-b^2})(-1+bx)}{(-1+b^2)(-ib + \sqrt{-b^2}x)}\right] \right) - \left( \text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \text{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \text{Log}\left[\frac{2b(i + \sqrt{-b^2})(1+bx)}{(-1+b^2)(-ib + \sqrt{-b^2}x)}\right] + \right.$$

$$\left. \left( \text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \left( \text{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] + \text{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{2}\sqrt{-b^2}e^{-\text{ArcTanh}[bx]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)}\text{Cosh}[2\text{ArcTanh}[bx]]}\right] +$$

$$\left( \text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \left( \text{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] + \text{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \right) \text{Log}\left[\frac{\sqrt{2}\sqrt{-b^2}e^{\text{ArcTanh}[bx]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)}\text{Cosh}[2\text{ArcTanh}[bx]]}\right] +$$

$$i \left( \text{PolyLog}\left[2, \frac{(1+b^2-2i\sqrt{-b^2})(b-i\sqrt{-b^2}x)}{(-1+b^2)(b+i\sqrt{-b^2}x)}\right] - \text{PolyLog}\left[2, \frac{(1+b^2+2i\sqrt{-b^2})(b-i\sqrt{-b^2}x)}{(-1+b^2)(b+i\sqrt{-b^2}x)}\right] \right)$$

Unable to integrate:

$$\left\{ \frac{\text{ArcTanh}[a + bx]}{1 - x^2}, x, 9, 0 \right\}$$

$$\frac{1}{4} \text{Log}\left[-\frac{b(1-x)}{1-a-b}\right] \text{Log}[1-a-bx] - \frac{1}{4} \text{Log}\left[\frac{b(1+x)}{1-a+b}\right] \text{Log}[1-a-bx] -$$

$$\frac{1}{4} \text{Log}\left[\frac{b(1-x)}{1+a+b}\right] \text{Log}[1+a+bx] + \frac{1}{4} \text{Log}\left[-\frac{b(1+x)}{1+a-b}\right] \text{Log}[1+a+bx] + \frac{1}{4} \text{PolyLog}\left[2, \frac{1-a-bx}{1-a-b}\right] -$$

$$\frac{1}{4} \text{PolyLog}\left[2, \frac{1-a-bx}{1-a+b}\right] + \frac{1}{4} \text{PolyLog}\left[2, \frac{1+a+bx}{1+a-b}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1+a+bx}{1+a+b}\right]$$

$$\int \frac{\text{ArcTanh}[a + bx]}{1 - x^2} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcTanh}[x]}{(a + bx^2)^{3/2}}, x, 3, 0 \right\}$$

$$\frac{x \text{ArcTanh}[x]}{a\sqrt{a + bx^2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a+b}}\right]}{a\sqrt{a+b}}$$

$$\frac{1}{2 a \sqrt{a+b} \sqrt{a+b x^2}} \left( 2 \sqrt{a+b} x \operatorname{ArcTanh}[x] + \sqrt{a+b x^2} \left( \log \left[ \sqrt{a+b} (-1+x) \right] + \log \left[ \sqrt{a+b} (1+x) \right] - \log \left[ 4 a \left( a - b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right] - \log \left[ 4 a \left( a + b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right] \right) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcTanh}[x]}{(a+b x^2)^{5/2}}, x, 8, 0 \right\}$$

$$\frac{1}{3 a (a+b) \sqrt{a+b x^2}} + \frac{x (3 a+2 b x^2) \operatorname{ArcTanh}[x]}{3 a^2 (a+b x^2)^{3/2}} - \frac{(3 a+2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a+b}}\right]}{3 a^2 (a+b)^{3/2}}$$

$$\frac{1}{6 a^2} \left( \frac{2 a}{(a+b) \sqrt{a+b x^2}} + \frac{2 x (3 a+2 b x^2) \operatorname{ArcTanh}[x]}{(a+b x^2)^{3/2}} + \frac{(3 a+2 b) \log \left[ \sqrt{a+b} (3 a+2 b) (-1+x) \right]}{(a+b)^{3/2}} + \frac{(3 a+2 b) \log \left[ \sqrt{a+b} (3 a+2 b) (1+x) \right]}{(a+b)^{3/2}} - \frac{(3 a+2 b) \log \left[ 12 a^2 (a+b) \left( a - b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right]}{(a+b)^{3/2}} - \frac{(3 a+2 b) \log \left[ 12 a^2 (a+b) \left( a + b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right]}{(a+b)^{3/2}} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcTanh}[x]}{(a+b x^2)^{7/2}}, x, 8, 0 \right\}$$

$$\frac{1}{15 a (a+b) (a+b x^2)^{3/2}} + \frac{7 a+4 b}{15 a^2 (a+b)^2 \sqrt{a+b x^2}} + \frac{x \left( 8 (a+b x^2)^2 + a (7 a+4 b x^2) \right) \operatorname{ArcTanh}[x]}{15 a^3 (a+b x^2)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a+b}}\right]}{15 a^3 (a+b)^{5/2}}$$

$$\frac{1}{30 a^3} \left( \frac{2 a \left( 8 a^2+4 b^2 x^2+a b \left( 5+7 x^2 \right) \right)}{(a+b)^2 (a+b x^2)^{3/2}} + \frac{2 x \left( 15 a^2+20 a b x^2+8 b^2 x^4 \right) \operatorname{ArcTanh}[x]}{(a+b x^2)^{5/2}} + \frac{(15 a^2+20 a b+8 b^2) \log \left[ \sqrt{a+b} \left( 15 a^2+20 a b+8 b^2 \right) (-1+x) \right]}{(a+b)^{5/2}} + \frac{(15 a^2+20 a b+8 b^2) \log \left[ \sqrt{a+b} \left( 15 a^2+20 a b+8 b^2 \right) (1+x) \right]}{(a+b)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \log \left[ 60 a^3 (a+b)^2 \left( a - b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right]}{(a+b)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \log \left[ 60 a^3 (a+b)^2 \left( a + b x + \sqrt{a+b} \sqrt{a+b x^2} \right) \right]}{(a+b)^{5/2}} \right)$$

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcTanh}[e^x], x, 3, 0\}$ 

$$-\frac{1}{2} \text{PolyLog}[2, -e^x] + \frac{1}{2} \text{PolyLog}[2, e^x]$$

$$\frac{1}{2} (x (2 \text{ArcTanh}[e^x] + \text{Log}[1 - e^x] - \text{Log}[1 + e^x]) - \text{PolyLog}[2, -e^x] + \text{PolyLog}[2, e^x])$$

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcTanh}[b \tanh[x]], x, 12, 0\}$ 

$$\begin{aligned} & x \text{ArcTanh}[b \tanh[x]] - \frac{1}{2} x \text{Log}\left[1 + \frac{(1 - b^2) e^{2x}}{1 - 2b + b^2}\right] + \\ & \frac{1}{2} x \text{Log}\left[1 + \frac{(1 - b^2) e^{2x}}{1 + 2b + b^2}\right] - \frac{1}{4} \text{PolyLog}\left[2, -\frac{(1 - b^2) e^{2x}}{1 - 2b + b^2}\right] + \frac{1}{4} \text{PolyLog}\left[2, -\frac{(1 - b^2) e^{2x}}{1 + 2b + b^2}\right] \\ & x \text{ArcTanh}[b \tanh[x]] - \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-b^2}}\right] + 2 i \text{ArcCos}\left[\frac{1 + b^2}{-1 + b^2}\right] \text{ArcTan}\left[\sqrt{-b^2} \tanh[x]\right] + \right. \\ & \left. \left( \text{ArcCos}\left[\frac{1 + b^2}{-1 + b^2}\right] - 2 \left( \text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-b^2}}\right] + \text{ArcTan}\left[\sqrt{-b^2} \tanh[x]\right] \right) \right) \text{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-x}}{\sqrt{-1 + b^2} \sqrt{-1 - b^2 + (-1 + b^2) \cosh[2x]}}\right] + \right. \\ & \left. \left( \text{ArcCos}\left[\frac{1 + b^2}{-1 + b^2}\right] + 2 \left( \text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-b^2}}\right] + \text{ArcTan}\left[\sqrt{-b^2} \tanh[x]\right] \right) \right) \text{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^x}{\sqrt{-1 + b^2} \sqrt{-1 - b^2 + (-1 + b^2) \cosh[2x]}}\right] - \right. \\ & \left. \left( \text{ArcCos}\left[\frac{1 + b^2}{-1 + b^2}\right] - 2 \text{ArcTan}\left[\sqrt{-b^2} \tanh[x]\right] \right) \left( \text{Log}[2] + \text{Log}\left[-\frac{(i b^2 + \sqrt{-b^2}) (-1 + \tanh[x])}{(-1 + b^2) (i + \sqrt{-b^2} \tanh[x])}\right] \right) - \right. \\ & \left. \left( \text{ArcCos}\left[\frac{1 + b^2}{-1 + b^2}\right] + 2 \text{ArcTan}\left[\sqrt{-b^2} \tanh[x]\right] \right) \left( \text{Log}[2] + \text{Log}\left[-\frac{(-i b^2 + \sqrt{-b^2}) (1 + \tanh[x])}{(-1 + b^2) (i + \sqrt{-b^2} \tanh[x])}\right] \right) + \right. \\ & \left. i \left( -\text{PolyLog}\left[2, \frac{(1 + b^2 - 2 i \sqrt{-b^2}) (-i + \sqrt{-b^2} \tanh[x])}{(-1 + b^2) (i + \sqrt{-b^2} \tanh[x])}\right] + \text{PolyLog}\left[2, \frac{(1 + b^2 + 2 i \sqrt{-b^2}) (-i + \sqrt{-b^2} \tanh[x])}{(-1 + b^2) (i + \sqrt{-b^2} \tanh[x])}\right] \right) \right) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcTanh}[b \coth[x]], x, 12, 0\}$ 

$$\begin{aligned} & x \text{ArcTanh}[b \coth[x]] - \frac{1}{2} x \text{Log}\left[1 - \frac{(1 - b^2) e^{2x}}{1 - 2b + b^2}\right] + \\ & \frac{1}{2} x \text{Log}\left[1 - \frac{(1 - b^2) e^{2x}}{1 + 2b + b^2}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{(1 - b^2) e^{2x}}{1 - 2b + b^2}\right] + \frac{1}{4} \text{PolyLog}\left[2, \frac{(1 - b^2) e^{2x}}{1 + 2b + b^2}\right] \end{aligned}$$

$$\begin{aligned}
& x \operatorname{ArcTanh}[b \operatorname{Coth}[x]] + \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTan}[\sqrt{-b^2} \operatorname{Coth}[x]] + 2 i \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \left( \operatorname{ArcTan}[\sqrt{-b^2} \operatorname{Coth}[x]] + \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \left( \operatorname{ArcTan}[\sqrt{-b^2} \operatorname{Coth}[x]] + \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^x}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (1+i \sqrt{-b^2}) (-1+\operatorname{Tanh}[x])}{(-1+b^2) (b^2+i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \operatorname{Log}\left[\frac{2 b^2 (i + \sqrt{-b^2}) (1+\operatorname{Tanh}[x])}{(-1+b^2) (-i b^2 + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (b^2-i \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (b^2+i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (b^2-i \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (b^2+i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{2 \operatorname{ArcTanh}[x]}}{x}, x, 5, 0 \right\}$$

$$-2 \operatorname{Log}[1-x] + \operatorname{Log}[x]$$

$$\operatorname{Log}\left[2-2 e^{2 \operatorname{ArcTanh}[x]}\right] + \operatorname{Log}\left[-2\left(1+e^{2 \operatorname{ArcTanh}[x]}\right)\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\frac{\operatorname{ArcTanh}[x]}{2}} x^2, x, 14, 0 \right\}$$

$$\begin{aligned}
& \frac{1}{8} e^{\frac{\operatorname{ArcTanh}[x]}{2}} (1-x) + \frac{1}{28} e^{\frac{\operatorname{ArcTanh}[x]}{2}} (1-x)^2 - \frac{1}{7} e^{\frac{\operatorname{ArcTanh}[x]}{2}} (1-x)^3 - \frac{1}{7} e^{\frac{5 \operatorname{ArcTanh}[x]}{2}} (1-x)^3 - \frac{1}{3} e^{\frac{9 \operatorname{ArcTanh}[x]}{2}} (1-x)^3 - \frac{3 \operatorname{ArcTan}\left[1-\sqrt{2} e^{\frac{\operatorname{ArcTanh}[x]}{2}}\right]}{8 \sqrt{2}} + \\
& \frac{3 \operatorname{ArcTan}\left[1+\sqrt{2} e^{\frac{\operatorname{ArcTanh}[x]}{2}}\right]}{8 \sqrt{2}} - \frac{3 \operatorname{Log}\left[1-\sqrt{2} e^{\frac{\operatorname{ArcTanh}[x]}{2}} + e^{\operatorname{ArcTanh}[x]}\right]}{16 \sqrt{2}} + \frac{3 \operatorname{Log}\left[1+\sqrt{2} e^{\frac{\operatorname{ArcTanh}[x]}{2}} + e^{\operatorname{ArcTanh}[x]}\right]}{16 \sqrt{2}} \\
& - \frac{8 e^{\frac{\operatorname{ArcTanh}[x]}{2}} \left(9+6 e^{2 \operatorname{ArcTanh}[x]}+29 e^{4 \operatorname{ArcTanh}[x]}\right)+9\left(1+e^{2 \operatorname{ArcTanh}[x]}\right)^3 \operatorname{RootSum}\left[1+\#1^4 \&, \frac{\operatorname{ArcTanh}[x]-2 \operatorname{Log}\left[-3 e^{\frac{\operatorname{ArcTanh}[x]}{2}}+3 \#1\right]}{\#1^3} \&\right]}{96\left(1+e^{2 \operatorname{ArcTanh}[x]}\right)^3}
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\frac{\operatorname{ArcTanh}[x]}{2}} x, x, 12, 0 \right\}$$

# Mathematica 7 Test Results for Integration Problems Involving Inverse Trig Functions

$$\frac{1}{12} e^{\frac{\text{ArcTan}[x]}{2}} (1-x) - \frac{1}{6} e^{\frac{\text{ArcTan}[x]}{2}} (1-x)^2 - \frac{2}{3} e^{\frac{5 \text{ArcTan}[x]}{2}} (1-x)^2 - \frac{\text{ArcTan}\left[1 - \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}}\right]}{4 \sqrt{2}} +$$

$$\frac{\text{ArcTan}\left[1 + \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}}\right]}{4 \sqrt{2}} - \frac{\text{Log}\left[1 - \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}} + e^{\text{ArcTan}[x]}\right]}{8 \sqrt{2}} + \frac{\text{Log}\left[1 + \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}} + e^{\text{ArcTan}[x]}\right]}{8 \sqrt{2}}$$

$$\frac{2 e^{\frac{\text{ArcTan}[x]}{2}}}{\left(1 + e^{2 \text{ArcTan}[x]}\right)^2} - \frac{5 e^{\frac{\text{ArcTan}[x]}{2}}}{2 \left(1 + e^{2 \text{ArcTan}[x]}\right)} + \frac{1}{16} \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTan}[x] + 2 \text{Log}\left[-e^{\frac{\text{ArcTan}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\frac{\text{ArcTan}[x]}{2}}, x, 8, 0 \right\}$$

$$-(1-x)^{3/4} (1+x)^{1/4} + \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{\sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{2 \sqrt{2}} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{2 \sqrt{2}}$$

$$-\frac{2 e^{\frac{\text{ArcTan}[x]}{2}}}{1 + e^{2 \text{ArcTan}[x]}} + \frac{1}{4} \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTan}[x] + 2 \text{Log}\left[-e^{\frac{\text{ArcTan}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\frac{\text{ArcTan}[x]}{2}}}{x}, x, 12, 0 \right\}$$

$$-2 \text{ArcTan}\left[e^{\frac{\text{ArcTan}[x]}{2}}\right] - \sqrt{2} \text{ArcTan}\left[1 - \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}}\right] + \sqrt{2} \text{ArcTan}\left[1 + \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}}\right] -$$

$$2 \text{ArcTan}\left[e^{\frac{\text{ArcTan}[x]}{2}}\right] - \frac{\text{Log}\left[1 - \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}} + e^{\text{ArcTan}[x]}\right]}{\sqrt{2}} + \frac{\text{Log}\left[1 + \sqrt{2} e^{\frac{\text{ArcTan}[x]}{2}} + e^{\text{ArcTan}[x]}\right]}{\sqrt{2}}$$

$$-2 \text{ArcTan}\left[e^{\frac{\text{ArcTan}[x]}{2}}\right] + \text{Log}\left[-1 + e^{\frac{\text{ArcTan}[x]}{2}}\right] - \text{Log}\left[1 + e^{\frac{\text{ArcTan}[x]}{2}}\right] + \frac{1}{2} \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTan}[x] + 2 \text{Log}\left[-e^{\frac{\text{ArcTan}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\frac{\text{ArcTan}[x]}{3}}, x, 8, 0 \right\}$$

$$-(1-x)^{5/6} (1+x)^{1/6} - \frac{2}{3} \text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \frac{1}{3} \text{ArcTan}\left[\frac{(1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} - (1+x)^{1/3}}\right] + \frac{\text{ArcTan}\left[\frac{\sqrt{3} (1-x)^{1/6} (1+x)^{1/6}}{(1-x)^{1/3} + (1+x)^{1/3}}\right]}{\sqrt{3}}$$

$$-\frac{2 e^{\frac{\text{ArcTan}[x]}{3}}}{1 + e^{2 \text{ArcTan}[x]}} + \frac{2}{3} \text{ArcTan}\left[e^{\frac{\text{ArcTan}[x]}{3}}\right] +$$

$$\frac{1}{9} \text{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{-2 \text{ArcTan}[x] + 6 \text{Log}\left[-e^{\frac{\text{ArcTan}[x]}{3}} + \#1\right] + \text{ArcTan}[x] \#1^2 - 3 \text{Log}\left[-e^{\frac{\text{ArcTan}[x]}{3}} + \#1\right] \#1^2}{-\#1 + 2 \#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\frac{2 \text{ArcTan}[x]}{3}}, x, 7, 0 \right\}$$

$$-(1-x)^{2/3} (1+x)^{1/3} + \frac{2 \text{ArcTan}\left[\frac{1 - \frac{2 (1-x)^{1/3}}{(1+x)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{3} \text{Log}\left[1 + \frac{(1-x)^{2/3}}{(1+x)^{2/3}} - \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right] + \frac{2}{3} \text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

# Mathematica 7 Test Results for Integration Problems Involving Inverse Trig Functions

$$-\frac{2 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{1+e^{2 \operatorname{ArcTanh}[x]}}-\frac{4 \operatorname{ArcTanh}[x]}{9}+\frac{2}{3} \operatorname{Log}\left[-4\left(1+e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right)\right]-\frac{2}{9} \operatorname{RootSum}\left[1-\#1^2+\#1^4 \&, \frac{\operatorname{ArcTanh}[x]-3 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcTanh}[x]}{3}}+\#1\right]+\operatorname{ArcTanh}[x] \#1^2-3 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcTanh}[x]}{3}}+\#1\right] \#1^2}{-2+\#1^2} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{e^{\frac{\operatorname{ArcTanh}[x]}{4}}, x, 22, 0\right\} \\ - (1-x)^{7/8} (1+x)^{1/8} - \frac{1}{2} (-1)^{1/8} \operatorname{ArcTan}\left[\frac{(-1)^{1/8} (1-x)^{1/8}}{(1+x)^{1/8}}\right] - \frac{1}{2} (-1)^{7/8} \operatorname{ArcTan}\left[\frac{(-1)^{7/8} (1-x)^{1/8}}{(1+x)^{1/8}}\right] + \\ \frac{1}{2} (-1)^{1/8} \operatorname{ArcTanh}\left[\frac{(-1)^{1/8} (1-x)^{1/8}}{(1+x)^{1/8}}\right] + \frac{1}{2} (-1)^{7/8} \operatorname{ArcTanh}\left[\frac{(-1)^{7/8} (1-x)^{1/8}}{(1+x)^{1/8}}\right] \\ - \frac{2 e^{\frac{\operatorname{ArcTanh}[x]}{4}}}{1+e^{2 \operatorname{ArcTanh}[x]}} + \frac{1}{16} \operatorname{RootSum}\left[1+\#1^8 \&, \frac{-\operatorname{ArcTanh}[x]+4 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcTanh}[x]}{4}}+\#1\right]}{\#1^7} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{e^{\operatorname{ArcTanh}[a+b x]}}{x\left(1-a^2-2 a b x-b^2 x^2\right)}, x, 7, 0\right\} \\ \frac{e^{\operatorname{ArcTanh}[a+b x]}}{1-a}+\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{-1+a} e^{\operatorname{ArcTanh}[a+b x]}}{\sqrt{1+a}}\right]}{(-1+a)^{3/2} \sqrt{1+a}} \\ -\frac{\sqrt{1-a^2-2 a b x-b^2 x^2}}{-1+a+b x}+\frac{\operatorname{Log}\left[\sqrt{1-a^2} x\right]}{\sqrt{1-a^2}}-\frac{\operatorname{Log}\left[-2(-1+a)\left(-1+a^2+a b x-\sqrt{1-a^2} \sqrt{1-a^2-2 a b x-b^2 x^2}\right)\right]}{\sqrt{1-a^2}} \\ -\frac{}{-1+a}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{e^{\operatorname{ArcTanh}[x]} \sqrt{1-x}, x, 2, 0\right\} \\ \frac{2}{3}(1+x)^{3/2} \\ \frac{2(1+x) \sqrt{1-x^2}}{3 \sqrt{1-x}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{e^{\operatorname{ArcTanh}[x]} x(1+x)^{3/2} \operatorname{Sin}[x], x, 26, 0\right\} \\ \frac{17}{4} \sqrt{1-x} \operatorname{Cos}[x]-5(1-x)^{3/2} \operatorname{Cos}[x]+(1-x)^{5/2} \operatorname{Cos}[x]+\frac{1}{4} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right](2 \operatorname{Cos}[1]-17 \operatorname{Sin}[1])- \\ \frac{1}{4} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right](17 \operatorname{Cos}[1]+2 \operatorname{Sin}[1])-\frac{15}{2} \sqrt{1-x} \operatorname{Sin}[x]+\frac{5}{2}(1-x)^{3/2} \operatorname{Sin}[x]$$

$$\begin{aligned}
& \frac{1}{16 \sqrt{1+x} \sqrt{1-x^2}} \\
& \left( -e^{-i(1+x)} (1+x) \left( (30+16i) e^i + (30-16i) e^{i+2ix} - (18+7i) e^{i(2+x)} \sqrt{\pi} \sqrt{-i(-1+x)} - (18-7i) e^{ix} \sqrt{\pi} \sqrt{i(-1+x)} - \right. \right. \\
& \quad (38-4i) e^i x - (38+4i) e^{i+2ix} x - 20i e^i x^2 + 20i e^{i+2ix} x^2 + 8 e^i x^3 + 8 e^{i+2ix} x^3 + \\
& \quad \left. (18+7i) e^{i(2+x)} \sqrt{\pi} \sqrt{-i(-1+x)} \operatorname{Erf}\left[\sqrt{-i(-1+x)}\right] + (18-7i) e^{ix} \sqrt{\pi} \sqrt{i(-1+x)} \operatorname{Erf}\left[\sqrt{i(-1+x)}\right] \right) - \\
& \quad \frac{1}{\sqrt{-1+x}} 4 e^{-i(1+x)} (-1+x^2) \left( 2 e^i \sqrt{-1+x} ((2-3i) + 2x + e^{2ix} ((2+3i) + 2x)) + \right. \\
& \quad \left. (4+3i) (-1)^{3/4} e^{i(2+x)} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{1/4} \sqrt{-1+x}\right] + (4-3i) (-1)^{1/4} e^{ix} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{3/4} \sqrt{-1+x}\right] \right) + \frac{1}{\sqrt{-1+x}} \\
& \quad (4+4i) (-1+x^2) \left( (-4+4i) \sqrt{-1+x} \cos[x] + (1+4i) \sqrt{2\pi} \operatorname{Erfi}\left[\frac{(1-i)\sqrt{-1+x}}{\sqrt{2}}\right] (\cos[1] - i \sin[1]) - \right. \\
& \quad \left. \left. (4+i) \sqrt{2\pi} \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\cos[1] + i \sin[1]) \right) \right) \Bigg)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[a+bx]}{\frac{ad}{b} + dx}, x, 6, 0 \right\} \\
& - \frac{\operatorname{PolyLog}[2, -a-bx]}{2d} + \frac{\operatorname{PolyLog}[2, a+bx]}{2d} \\
& - \frac{1}{8d} \left( \pi^2 - 4i\pi \operatorname{ArcTanh}[a+bx] - 8 \operatorname{ArcTanh}[a+bx]^2 - 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a+bx]}\right] + \right. \\
& \quad 4i\pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] + 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] + 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] - \\
& \quad 4i\pi \operatorname{Log}\left[\frac{2}{\sqrt{1-(a+bx)^2}}\right] - 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2}{\sqrt{1-(a+bx)^2}}\right] - 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{i(a+bx)}{\sqrt{1-(a+bx)^2}}\right] + \\
& \quad \left. 8 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2i(a+bx)}{\sqrt{1-(a+bx)^2}}\right] + 4 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[a+bx]}\right] + 4 \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcTanh}[a+bx]}\right] \Bigg)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTanh}[ax^n]}{x}, x, 3, 0 \right\} \\
& - \frac{\operatorname{PolyLog}[2, -ax^n]}{2n} + \frac{\operatorname{PolyLog}[2, ax^n]}{2n} \\
& \frac{ax^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, a^2 x^{2n}\right]}{n}
\end{aligned}$$

## Problems involving inverse hyperbolic cotangents

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCoth}[a + b x]}{x}, x, 13, 0 \right\}$$

$$\frac{1}{2} \log[x] \log\left[\frac{1-a-bx}{1-a}\right] - \frac{1}{2} \log[x] \log\left[\frac{1+a+bx}{1+a}\right] - \frac{1}{2} \log[x] \log\left[1 - \frac{1}{a+bx}\right] +$$

$$\frac{1}{2} \log[x] \log\left[1 + \frac{1}{a+bx}\right] + \frac{1}{2} \text{PolyLog}\left[2, \frac{bx}{1-a}\right] - \frac{1}{2} \text{PolyLog}\left[2, -\frac{bx}{1+a}\right]$$

$$(\text{ArcCoth}[a + b x] - \text{ArcTanh}[a + b x]) \log[x] -$$

$$\frac{1}{2} i \left( i (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x])^2 - \frac{1}{4} i (\pi - 2 i \text{ArcTanh}[a + b x])^2 - 2 i (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]) \right.$$

$$\log[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a+bx]}] + (\pi - 2 i \text{ArcTanh}[a + b x]) \log[1 + e^{2 \text{ArcTanh}[a+bx]}] - (\pi - 2 i \text{ArcTanh}[a + b x])$$

$$\log\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] + 2 i \text{ArcTanh}[a + b x] \left( -\log\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + \log[-i \sinh[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]] \right) +$$

$$2 i (\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]) \log[-2 i \sinh[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]]] -$$

$$i \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a+bx]}\right] - i \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[a+bx]}\right] \Bigg)$$

Incorrect antiderivative:

$$\{x^2 \text{ArcCoth}[a + b x]^2, x, 16, 0\}$$

$$\frac{a + b x}{3 b^3} - \frac{\text{ArcCoth}[a + b x]}{3 b^3} - \frac{2 a (a + b x) \text{ArcCoth}[a + b x]}{b^3} + \frac{(a + b x)^2 \text{ArcCoth}[a + b x]}{3 b^3} + \frac{\text{ArcCoth}[a + b x]^2}{3 b^3} + \frac{a \text{ArcCoth}[a + b x]^2}{b^3} +$$

$$\frac{a^2 \text{ArcCoth}[a + b x]^2}{b^3} + \frac{a^2 (a + b x) \text{ArcCoth}[a + b x]^2}{b^3} - \frac{a (a + b x)^2 \text{ArcCoth}[a + b x]^2}{b^3} + \frac{(a + b x)^3 \text{ArcCoth}[a + b x]^2}{3 b^3} -$$

$$\frac{2 \text{ArcCoth}[a + b x] \log\left[\frac{2}{1-a-bx}\right]}{3 b^3} - \frac{2 a^2 \text{ArcCoth}[a + b x] \log\left[\frac{2}{1-a-bx}\right]}{b^3} - \frac{a \log[1 - (a + b x)^2]}{b^3} - \frac{(1 + 3 a^2) \text{PolyLog}\left[2, 1 - \frac{2}{1-a-bx}\right]}{3 b^3}$$



$$\begin{aligned}
& -\frac{1}{12b^3} (a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}} (1 - (a+bx)^2) \\
& \left( \frac{4 \operatorname{ArcCoth}[a+bx]}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} + \frac{3 \operatorname{ArcCoth}[a+bx]^2}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} - \frac{12 a \operatorname{ArcCoth}[a+bx]^2}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} + \frac{9 a^2 \operatorname{ArcCoth}[a+bx]^2}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} + \right. \\
& \frac{-1 + 6 a \operatorname{ArcCoth}[a+bx] - 3 (-1 + a^2) \operatorname{ArcCoth}[a+bx]^2}{\sqrt{1 - \frac{1}{(a+bx)^2}}} + \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+bx]] - 6 a \operatorname{ArcCoth}[a+bx] \\
& \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+bx]] + \operatorname{ArcCoth}[a+bx]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+bx]] + 3 a^2 \operatorname{ArcCoth}[a+bx]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a+bx]] + \\
& \frac{6 \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx]}\right]}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} + \frac{18 a^2 \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx]}\right]}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} - \\
& \frac{18 a \operatorname{Log}\left[\frac{1}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}}\right]}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}} + \frac{4 (1 + 3 a^2) \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCoth}[a+bx]}\right]}{(a+bx)^3 \left(1 - \frac{1}{(a+bx)^2}\right)^{3/2}} - \operatorname{ArcCoth}[a+bx]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+bx]] - \\
& \frac{3 a^2 \operatorname{ArcCoth}[a+bx]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+bx]] - 2 \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+bx]] - 6 a^2 \operatorname{ArcCoth}[a+bx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCoth}[a+bx]}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+bx]] +}{6 a \operatorname{Log}\left[\frac{1}{(a+bx) \sqrt{1 - \frac{1}{(a+bx)^2}}}\right] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a+bx]]} \left. \right)
\end{aligned}$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcCoth}[a+bx]^2}{x}, x, -2, 0 \right\}$$

$$\begin{aligned}
& -\frac{2}{3} \operatorname{ArcCoth}[a+bx]^3 - \operatorname{ArcCoth}[a+bx]^2 \operatorname{Log}\left[\frac{2}{1+a+bx}\right] + \operatorname{ArcCoth}[a+bx]^2 \operatorname{Log}\left[1 - \frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] + \\
& \operatorname{ArcCoth}[a+bx]^2 \operatorname{Log}\left[1 + \frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] + 2 \operatorname{ArcCoth}[a+bx] \operatorname{PolyLog}\left[2, -\frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] + \\
& 2 \operatorname{ArcCoth}[a+bx] \operatorname{PolyLog}\left[2, \frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] + \operatorname{ArcCoth}[a+bx] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right] - \\
& 2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] - 2 \operatorname{PolyLog}\left[3, \frac{\sqrt{\frac{1-a}{b}} (1+a+bx)}{\sqrt{\frac{1+a}{b}} \sqrt{1 - (a+bx)^2}}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+a+bx}\right]
\end{aligned}$$

$$\int \frac{\text{ArcCoth}[a + b x]^2}{x} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCoth}[1 + x]}{2 + 2 x}, x, 5, 0 \right\}$$

$$\frac{1}{4} \text{PolyLog}\left[2, -\frac{1}{1 + x}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1}{1 + x}\right]$$

$$\frac{1}{16} \left( -\pi^2 + 4 i \pi \text{ArcTanh}[1 + x] + 8 \text{ArcTanh}[1 + x]^2 + \right.$$

$$8 \text{ArcTanh}[1 + x] \text{Log}\left[1 - e^{-2 \text{ArcTanh}[1 + x]}\right] - 4 i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[1 + x]}\right] - 8 \text{ArcTanh}[1 + x] \text{Log}\left[1 + e^{2 \text{ArcTanh}[1 + x]}\right] +$$

$$8 \text{ArcCoth}[1 + x] \text{Log}[1 + x] - 8 \text{ArcTanh}[1 + x] \text{Log}[1 + x] - 8 \text{ArcTanh}[1 + x] \text{Log}\left[\frac{1}{\sqrt{-x(2 + x)}}\right] +$$

$$4 i \pi \text{Log}\left[\frac{2}{\sqrt{-x(2 + x)}}\right] + 8 \text{ArcTanh}[1 + x] \text{Log}\left[\frac{2}{\sqrt{-x(2 + x)}}\right] + 8 \text{ArcTanh}[1 + x] \text{Log}\left[\frac{i(1 + x)}{\sqrt{-x(2 + x)}}\right] -$$

$$\left. 8 \text{ArcTanh}[1 + x] \text{Log}\left[\frac{2 i(1 + x)}{\sqrt{-x(2 + x)}}\right] - 4 \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[1 + x]}\right] - 4 \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[1 + x]}\right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCoth}[x]}{a + b x}, x, 14, 0 \right\}$$

$$-\frac{\text{Log}\left[1 - \frac{1}{x}\right] \text{Log}[a + b x]}{2 b} + \frac{\text{Log}\left[1 + \frac{1}{x}\right] \text{Log}[a + b x]}{2 b} +$$

$$\frac{\text{Log}\left[\frac{b(1-x)}{a+b}\right] \text{Log}[a + b x]}{2 b} - \frac{\text{Log}\left[-\frac{b(1+x)}{a-b}\right] \text{Log}[a + b x]}{2 b} - \frac{\text{PolyLog}\left[2, \frac{a+bx}{a-b}\right]}{2 b} + \frac{\text{PolyLog}\left[2, \frac{a+bx}{a+b}\right]}{2 b}$$

$$\frac{1}{8 b} \left( -\pi^2 + 4 \text{ArcTanh}\left[\frac{a}{b}\right]^2 + 4 i \pi \text{ArcTanh}[x] + 8 \text{ArcTanh}\left[\frac{a}{b}\right] \text{ArcTanh}[x] + 8 \text{ArcTanh}[x]^2 - \right.$$

$$4 i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[x]}\right] - 8 \text{ArcTanh}[x] \text{Log}\left[1 + e^{2 \text{ArcTanh}[x]}\right] + 8 \text{ArcTanh}\left[\frac{a}{b}\right] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right)}\right] +$$

$$8 \text{ArcTanh}[x] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right)}\right] + 8 \text{ArcCoth}[x] \text{Log}[a + b x] - 8 \text{ArcTanh}[x] \text{Log}[a + b x] +$$

$$4 i \pi \text{Log}\left[\frac{2}{\sqrt{1 - x^2}}\right] + 8 \text{ArcTanh}[x] \text{Log}\left[\frac{2}{\sqrt{1 - x^2}}\right] + 4 \text{ArcTanh}[x] \text{Log}[1 - x^2] +$$

$$8 \text{ArcTanh}[x] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right]\right] - 8 \text{ArcTanh}\left[\frac{a}{b}\right] \text{Log}\left[2 i \text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right]\right] -$$

$$\left. 8 \text{ArcTanh}[x] \text{Log}\left[2 i \text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right]\right] - 4 \text{PolyLog}\left[2, -e^{2 \text{ArcTanh}[x]}\right] - 4 \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{a}{b}\right] + \text{ArcTanh}[x]\right)}\right] \right)$$

Unable to integrate:

$$\left\{ \frac{\text{ArcCoth}[x]}{a + b x + c x^2}, x, 29, 0 \right\}$$

$$\begin{aligned}
& -\frac{\operatorname{Log}\left[1-\frac{1}{x}\right] \operatorname{Log}\left[b-\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{Log}\left[1+\frac{1}{x}\right] \operatorname{Log}\left[b-\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} + \\
& \frac{\operatorname{Log}\left[\frac{2c(1-x)}{b+2c-\sqrt{b^2-4ac}}\right] \operatorname{Log}\left[b-\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{Log}\left[-\frac{2c(1+x)}{b-2c-\sqrt{b^2-4ac}}\right] \operatorname{Log}\left[b-\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} + \\
& \frac{\operatorname{Log}\left[1-\frac{1}{x}\right] \operatorname{Log}\left[b+\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{Log}\left[1+\frac{1}{x}\right] \operatorname{Log}\left[b+\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} - \\
& \frac{\operatorname{Log}\left[\frac{2c(1-x)}{b+2c+\sqrt{b^2-4ac}}\right] \operatorname{Log}\left[b+\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{Log}\left[-\frac{2c(1+x)}{b-2c+\sqrt{b^2-4ac}}\right] \operatorname{Log}\left[b+\sqrt{b^2-4ac}+2cx\right]}{2\sqrt{b^2-4ac}} - \\
& \frac{\operatorname{PolyLog}\left[2, \frac{b-\sqrt{b^2-4ac}+2cx}{b-2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left[2, \frac{b-\sqrt{b^2-4ac}+2cx}{b+2c-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left[2, \frac{b+\sqrt{b^2-4ac}+2cx}{b-2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} - \frac{\operatorname{PolyLog}\left[2, \frac{b+\sqrt{b^2-4ac}+2cx}{b+2c+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2-4ac}} \\
& \int \frac{\operatorname{ArcCoth}[x]}{a+bx+cx^2} dx
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcCoth}[d+ex]}{a+bx^2}, x, 27, 0 \right\} \\
& \frac{\operatorname{Log}\left[a+\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[\frac{\sqrt{b}(1-d-ex)}{\sqrt{b}(1-d)-\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{Log}\left[a-\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[\frac{\sqrt{b}(1-d-ex)}{\sqrt{b}(1-d)+\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} + \\
& \frac{\operatorname{Log}\left[a-\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[\frac{\sqrt{b}(1+d+ex)}{\sqrt{b}(1+d)-\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{Log}\left[a+\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[\frac{\sqrt{b}(1+d+ex)}{\sqrt{b}(1+d)+\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{Log}\left[a-\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[1-\frac{1}{d+ex}\right]}{4\sqrt{-a}\sqrt{b}} - \\
& \frac{\operatorname{Log}\left[a+\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[1-\frac{1}{d+ex}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{Log}\left[a-\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[1+\frac{1}{d+ex}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{Log}\left[a+\sqrt{-a}\sqrt{b}x\right] \operatorname{Log}\left[1+\frac{1}{d+ex}\right]}{4\sqrt{-a}\sqrt{b}} + \\
& \frac{\operatorname{PolyLog}\left[2, -\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(1-d)-\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, \frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(1+d)+\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{PolyLog}\left[2, -\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(1+d)-\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, \frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}(1-d)+\sqrt{-a}e}\right]}{4\sqrt{-a}\sqrt{b}} \\
& \int \frac{\operatorname{ArcCoth}[d+ex]}{a+bx^2} dx
\end{aligned}$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcCoth}[d+ex]}{a+bx+cx^2}, x, 27, 0 \right\}$$

$$\begin{aligned}
& \frac{\text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[\frac{2c(1-d-ex)}{2c(1-d) + (b - \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} - \frac{\text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[\frac{2c(1-d-ex)}{2c(1-d) + (b + \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} \\
& + \frac{\text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[\frac{2c(1+d+ex)}{2c(1+d) - (b - \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} + \frac{\text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[\frac{2c(1+d+ex)}{2c(1+d) - (b + \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} \\
& + \frac{\text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[1 - \frac{1}{d+ex}\right]}{2\sqrt{b^2 - 4ac}} + \frac{\text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[1 - \frac{1}{d+ex}\right]}{2\sqrt{b^2 - 4ac}} + \\
& - \frac{\text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[1 + \frac{1}{d+ex}\right]}{2\sqrt{b^2 - 4ac}} - \frac{\text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right] \text{Log}\left[1 + \frac{1}{d+ex}\right]}{2\sqrt{b^2 - 4ac}} - \frac{\text{PolyLog}\left[2, -\frac{e(b - \sqrt{b^2 - 4ac} + 2cx)}{2c(1+d) - (b - \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} + \\
& + \frac{\text{PolyLog}\left[2, \frac{e(b - \sqrt{b^2 - 4ac} + 2cx)}{2c(1-d) + (b - \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} + \frac{\text{PolyLog}\left[2, -\frac{e(b + \sqrt{b^2 - 4ac} + 2cx)}{2c(1+d) - (b + \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} - \frac{\text{PolyLog}\left[2, \frac{e(b + \sqrt{b^2 - 4ac} + 2cx)}{2c(1-d) + (b + \sqrt{b^2 - 4ac})e}\right]}{2\sqrt{b^2 - 4ac}} \\
& \int \frac{\text{ArcCoth}[d+ex]}{a+bx+cx^2} dx
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\text{ArcCoth}[x]}{(a+bx^2)^{3/2}}, x, 3, 0 \right\} \\
& \frac{x \text{ArcCoth}[x]}{a\sqrt{a+bx^2}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a+b}}\right]}{a\sqrt{a+b}} \\
& \frac{1}{2a\sqrt{a+b}\sqrt{a+bx^2}} \left( 2\sqrt{a+b} x \text{ArcCoth}[x] + \sqrt{a+bx^2} \right. \\
& \quad \left. \left( \text{Log}\left[\sqrt{a+b}(-1+x)\right] + \text{Log}\left[\sqrt{a+b}(1+x)\right] - \text{Log}\left[4a\left(a-bx+\sqrt{a+b}\sqrt{a+bx^2}\right)\right] - \text{Log}\left[4a\left(a+bx+\sqrt{a+b}\sqrt{a+bx^2}\right)\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\text{ArcCoth}[x]}{(a+bx^2)^{5/2}}, x, 8, 0 \right\} \\
& \frac{1}{3a(a+b)\sqrt{a+bx^2}} + \frac{x(3a+2bx^2)\text{ArcCoth}[x]}{3a^2(a+bx^2)^{3/2}} - \frac{(3a+2b)\text{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a+b}}\right]}{3a^2(a+b)^{3/2}}
\end{aligned}$$

$$\frac{1}{6 a^2} \left( \frac{2 a}{(a+b) \sqrt{a+b x^2}} + \frac{2 x (3 a+2 b x^2) \operatorname{ArcCoth}[x]}{(a+b x^2)^{3/2}} + \frac{(3 a+2 b) \operatorname{Log}\left[\sqrt{a+b} (3 a+2 b) (-1+x)\right]}{(a+b)^{3/2}} + \frac{(3 a+2 b) \operatorname{Log}\left[\sqrt{a+b} (3 a+2 b) (1+x)\right]}{(a+b)^{3/2}} - \frac{(3 a+2 b) \operatorname{Log}\left[12 a^2 (a+b) \left(a-b x+\sqrt{a+b} \sqrt{a+b x^2}\right)\right]}{(a+b)^{3/2}} - \frac{(3 a+2 b) \operatorname{Log}\left[12 a^2 (a+b) \left(a+b x+\sqrt{a+b} \sqrt{a+b x^2}\right)\right]}{(a+b)^{3/2}} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcCoth}[x]}{(a+b x^2)^{7/2}}, x, 8, 0 \right\}$$

$$\frac{1}{15 a (a+b) (a+b x^2)^{3/2}} + \frac{7 a+4 b}{15 a^2 (a+b)^2 \sqrt{a+b x^2}} +$$

$$\frac{x \left( 8 (a+b x^2)^2 + a (7 a+4 b x^2) \right) \operatorname{ArcCoth}[x]}{15 a^3 (a+b x^2)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a+b}}\right]}{15 a^3 (a+b)^{5/2}}$$

$$\frac{1}{30 a^3} \left( \frac{2 a (8 a^2+4 b^2 x^2+a b (5+7 x^2))}{(a+b)^2 (a+b x^2)^{3/2}} + \frac{2 x (15 a^2+20 a b x^2+8 b^2 x^4) \operatorname{ArcCoth}[x]}{(a+b x^2)^{5/2}} + \frac{(15 a^2+20 a b+8 b^2) \operatorname{Log}\left[\sqrt{a+b} (15 a^2+20 a b+8 b^2) (-1+x)\right]}{(a+b)^{5/2}} + \frac{(15 a^2+20 a b+8 b^2) \operatorname{Log}\left[\sqrt{a+b} (15 a^2+20 a b+8 b^2) (1+x)\right]}{(a+b)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \operatorname{Log}\left[60 a^3 (a+b)^2 \left(a-b x+\sqrt{a+b} \sqrt{a+b x^2}\right)\right]}{(a+b)^{5/2}} - \frac{(15 a^2+20 a b+8 b^2) \operatorname{Log}\left[60 a^3 (a+b)^2 \left(a+b x+\sqrt{a+b} \sqrt{a+b x^2}\right)\right]}{(a+b)^{5/2}} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\{\operatorname{ArcCoth}[b \operatorname{Tanh}[x]], x, 12, 0\}$$

$$x \operatorname{ArcCoth}[b \operatorname{Tanh}[x]] - \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1-b^2) e^{2x}}{1-2 b+b^2}\right] +$$

$$\frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1-b^2) e^{2x}}{1+2 b+b^2}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, -\frac{(1-b^2) e^{2x}}{1-2 b+b^2}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, -\frac{(1-b^2) e^{2x}}{1+2 b+b^2}\right]$$

$$\begin{aligned}
& x \operatorname{ArcCoth}[b \operatorname{Tanh}[x]] - \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-b^2}}\right] + 2 i \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Tanh}[x]\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2 \left( \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-b^2}}\right] + \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Tanh}[x]\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-x}}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2 \left( \operatorname{ArcTan}\left[\frac{\operatorname{Coth}[x]}{\sqrt{-b^2}}\right] + \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Tanh}[x]\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^x}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2 \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Tanh}[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(i b^2 + \sqrt{-b^2}) (-1 + \operatorname{Tanh}[x])}{(-1+b^2) (i + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2 \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Tanh}[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(-i b^2 + \sqrt{-b^2}) (1 + \operatorname{Tanh}[x])}{(-1+b^2) (i + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) + \right. \\
& \left. i \left( -\operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (-i + \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (i + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] + \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (-i + \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (i + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

{ArcCoth[b Coth[x]], x, 12, 0}

$$\begin{aligned}
& x \operatorname{ArcCoth}[b \operatorname{Coth}[x]] - \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2x}}{1-2b+b^2}\right] + \\
& \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2x}}{1+2b+b^2}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2x}}{1-2b+b^2}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2x}}{1+2b+b^2}\right] \\
& x \operatorname{ArcCoth}[b \operatorname{Coth}[x]] + \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Coth}[x]\right] + 2 i \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] + \right. \\
& \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \left( \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Coth}[x]\right] + \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] + \\
& \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \left( \operatorname{ArcTan}\left[\sqrt{-b^2} \operatorname{Coth}[x]\right] + \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^x}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cosh}[2x]}}\right] - \\
& \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (1+i \sqrt{-b^2}) (-1 + \operatorname{Tanh}[x])}{(-1+b^2) (b^2 + i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right) - \\
& \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \operatorname{ArcTan}\left[\frac{\operatorname{Tanh}[x]}{\sqrt{-b^2}}\right] \right) \operatorname{Log}\left[\frac{2 b^2 (i + \sqrt{-b^2}) (1 + \operatorname{Tanh}[x])}{(-1+b^2) (-i b^2 + \sqrt{-b^2} \operatorname{Tanh}[x])}\right] + \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (b^2 - i \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (b^2 + i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (b^2 - i \sqrt{-b^2} \operatorname{Tanh}[x])}{(-1+b^2) (b^2 + i \sqrt{-b^2} \operatorname{Tanh}[x])}\right] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{2 \operatorname{ArcCoth}[x]}}{x}, x, 5, 0 \right\}$$

$$2 \operatorname{Log}[1 - x] - \operatorname{Log}[x]$$

$$-\operatorname{Log}\left[2 - 2 e^{2 \operatorname{ArcCoth}[x]}\right] - \operatorname{Log}\left[-2 \left(1 + e^{2 \operatorname{ArcCoth}[x]}\right)\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{x}, x, 13, 0 \right\}$$

$$2 \operatorname{ArcTan}\left[e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] +$$

$$2 \operatorname{ArcTanh}\left[e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] + \frac{\operatorname{Log}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{\sqrt{2}}$$

$$2 \operatorname{ArcTan}\left[e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] - \operatorname{Log}\left[-1 + e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] + \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right] - \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcCoth}[x] + 2 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcCoth}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{x^2}, x, 10, 0 \right\}$$

$$-\frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}(1-x)}{x} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right]}{\sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right]}{\sqrt{2}} +$$

$$\frac{\operatorname{Log}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{2 \sqrt{2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{2 \sqrt{2}}$$

$$\frac{2 e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{1 + e^{2 \operatorname{ArcCoth}[x]}} - \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcCoth}[x] + 2 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcCoth}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{x^3}, x, 13, 0 \right\}$$

$$\frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}(1-x)^2}{6 x^2} + \frac{2 e^{\frac{5 \operatorname{ArcCoth}[x]}{2}}(1-x)^2}{3 x^2} + \frac{e^{\frac{\operatorname{ArcCoth}[x]}{2}}(1-x)}{12 x} + \frac{\operatorname{ArcTan}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right]}{4 \sqrt{2}} -$$

$$\frac{\operatorname{ArcTan}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}}\right]}{4 \sqrt{2}} + \frac{\operatorname{Log}\left[1 - \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{8 \sqrt{2}} - \frac{\operatorname{Log}\left[1 + \sqrt{2} e^{\frac{\operatorname{ArcCoth}[x]}{2}} + e^{\operatorname{ArcCoth}[x]}\right]}{8 \sqrt{2}}$$

$$-\frac{2 e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{\left(1 + e^{2 \operatorname{ArcCoth}[x]}\right)^2} + \frac{5 e^{\frac{\operatorname{ArcCoth}[x]}{2}}}{2 \left(1 + e^{2 \operatorname{ArcCoth}[x]}\right)} - \frac{1}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcCoth}[x] + 2 \operatorname{Log}\left[-e^{\frac{\operatorname{ArcCoth}[x]}{2}} + \#1\right]}{\#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\operatorname{ArcCoth}[x]}}{1-x}, x, 7, 0 \right\}$$

$$\frac{2 e^{\text{ArcCoth}[x]} - 2 \text{ArcTanh}\left[e^{\text{ArcCoth}[x]}\right] - 2 \sqrt{1 - \frac{1}{x^2}} x - (-1 + x) \text{Log}\left[\left(1 + \sqrt{1 - \frac{1}{x^2}}\right) x\right]}{-1 + x}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\text{ArcCoth}[x]}}{(1 - x)^2}, x, 5, 0 \right\}$$

$$-\frac{1}{3} e^{3 \text{ArcCoth}[x]}$$

$$-\frac{\sqrt{1 - \frac{1}{x^2}} x (1 + x)}{3 (-1 + x)^2}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\text{ArcCoth}[x]}}{1 + x}, x, 5, 0 \right\}$$

$$2 \text{ArcTanh}\left[e^{\text{ArcCoth}[x]}\right]$$

$$\text{Log}\left[\left(1 + \sqrt{1 - \frac{1}{x^2}}\right) x\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\text{ArcCoth}[x]}}{(1 + x)^2}, x, 4, 0 \right\}$$

$$e^{-\text{ArcCoth}[x]}$$

$$\frac{\sqrt{1 - \frac{1}{x^2}} x}{1 + x}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\text{ArcCoth}[x]} \sqrt{1 - \frac{1}{x^2}}, x, 3, 0 \right\}$$

$$x + \text{Log}[x]$$

$$\frac{2}{-1 + e^{2 \text{ArcCoth}[x]}} - \text{Log}\left[2 - 2 e^{2 \text{ArcCoth}[x]}\right] + \text{Log}\left[2 \left(1 + e^{2 \text{ArcCoth}[x]}\right)\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\text{ArcCoth}[x]} \left(1 - \frac{1}{x^2}\right)^{3/2}, x, 3, 0 \right\}$$

$$\frac{1}{2 x^2} + \frac{1}{x} + x + \text{Log}[x]$$



# Mathematica 7 Test Results for Integration Problems Involving Inverse Trig Functions

$$\frac{\left(1 - \frac{1}{x^2}\right) x^2 \left( \frac{2}{-1 + e^{2 \operatorname{ArcCoth}[x]}} - \frac{2 (1 + 2 e^{2 \operatorname{ArcCoth}[x]})}{(1 + e^{2 \operatorname{ArcCoth}[x]})^2} - \operatorname{Log}\left[2 - 2 e^{2 \operatorname{ArcCoth}[x]}\right] + \operatorname{Log}\left[2 (1 + e^{2 \operatorname{ArcCoth}[x]})\right] \right)}{-1 + x^2}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ e^{\operatorname{ArcCoth}[x]} \left(1 - \frac{1}{x^2}\right)^{5/2}, x, 3, 0\right\}$$

$$-\frac{1}{4 x^4} - \frac{1}{3 x^3} + \frac{1}{x^2} + \frac{2}{x} + \operatorname{Log}[x]$$

$$\frac{1}{3 (-1 + e^{2 \operatorname{ArcCoth}[x]}) (1 + e^{2 \operatorname{ArcCoth}[x]})^4} \left(16 + 54 e^{2 \operatorname{ArcCoth}[x]} + 50 e^{4 \operatorname{ArcCoth}[x]} - 18 e^{6 \operatorname{ArcCoth}[x]} - 6 e^{8 \operatorname{ArcCoth}[x]} - \right.$$

$$\left. 3 (-1 + e^{2 \operatorname{ArcCoth}[x]}) (1 + e^{2 \operatorname{ArcCoth}[x]})^4 \operatorname{Log}[1 - e^{2 \operatorname{ArcCoth}[x]}] + 3 (-1 + e^{2 \operatorname{ArcCoth}[x]}) (1 + e^{2 \operatorname{ArcCoth}[x]})^4 \operatorname{Log}[1 + e^{2 \operatorname{ArcCoth}[x]}] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e^{\operatorname{ArcCoth}[x]}}{\sqrt{1 - \frac{1}{x^2}}}, x, 4, 0\right\}$$

$$x + \operatorname{Log}[1 - x]$$

$$\frac{2 - (-1 + e^{2 \operatorname{ArcCoth}[x]}) \operatorname{Log}[2 - 2 e^{2 \operatorname{ArcCoth}[x]}]}{-1 + e^{2 \operatorname{ArcCoth}[x]}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcCoth}[a + b x]}{\frac{a d}{b} + d x}, x, 6, 0\right\}$$

$$\frac{\operatorname{PolyLog}\left[2, -\frac{1}{a + b x}\right]}{2 d} - \frac{\operatorname{PolyLog}\left[2, \frac{1}{a + b x}\right]}{2 d}$$

$$-\frac{1}{8 d} \left( \pi^2 - 4 i \pi \operatorname{ArcTanh}[a + b x] - 8 \operatorname{ArcTanh}[a + b x]^2 - 8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[a + b x]}] + \right.$$

$$4 i \pi \operatorname{Log}[1 + e^{2 \operatorname{ArcTanh}[a + b x]}] + 8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}[1 + e^{2 \operatorname{ArcTanh}[a + b x]}] - 8 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[a + b x] +$$

$$8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}[a + b x] + 8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - 4 i \pi \operatorname{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] -$$

$$8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] - 8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{i (a + b x)}{\sqrt{1 - (a + b x)^2}}\right] +$$

$$\left. 8 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{2 i (a + b x)}{\sqrt{1 - (a + b x)^2}}\right] + 4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[a + b x]}\right] + 4 \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[a + b x]}\right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcCoth}[a x^n]}{x}, x, 3, 0\right\}$$

$$\frac{\operatorname{PolyLog}\left[2, -\frac{x^{-n}}{a}\right]}{2 n} - \frac{\operatorname{PolyLog}\left[2, \frac{x^{-n}}{a}\right]}{2 n}$$

$$\frac{a x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\},\left\{\frac{3}{2}, \frac{3}{2}\right\}, a^2 x^{2 n}\right]}{n}+\left(\operatorname{ArcCoth}\left[a x^n\right]-\operatorname{ArcTanh}\left[a x^n\right]\right) \operatorname{Log}[x]$$

## Problems involving inverse hyperbolic secants

Incorrect antiderivative:

 $\{\text{ArcSech}[a x], x, 1, 0\}$ 

$$x \text{ArcSech}[a x] - \frac{2 \text{ArcTan}\left[\sqrt{\frac{1-a x}{1+a x}}\right]}{a}$$

$$x \text{ArcSech}[a x] + \frac{2 \sqrt{\frac{1-a x}{1+a x}} \sqrt{1-a^2 x^2} \text{ArcSin}\left[\frac{\sqrt{1+a x}}{\sqrt{2}}\right]}{a - a^2 x}$$

Unable to integrate:

 $\{\text{ArcSech}[c e^{a+b x}], x, 7, 0\}$ 

$$\frac{\text{ArcSech}[c e^{a+b x}]^2}{2 b} - \frac{\text{ArcSech}[c e^{a+b x}] \text{Log}[1 + e^{2 \text{ArcSech}[c e^{a+b x}]}]}{b} - \frac{\text{PolyLog}[2, -e^{2 \text{ArcSech}[c e^{a+b x}]}]}{2 b}$$

$$\int \text{ArcSech}[c e^{a+b x}] dx$$

Valid but unnecessarily complicated antiderivative:

 $\left\{\frac{\text{ArcSech}[a x^n]}{x}, x, 7, 0\right\}$ 

$$\frac{\text{ArcSech}[a x^n]^2}{2 n} - \frac{\text{ArcSech}[a x^n] \text{Log}[1 + e^{2 \text{ArcSech}[a x^n]}]}{n} - \frac{\text{PolyLog}[2, -e^{2 \text{ArcSech}[a x^n]}]}{2 n}$$

$$\frac{1}{8} \left( 8 \text{ArcSech}[a x^n] \text{Log}[x] - \frac{1}{n (-1 + a x^n)} \sqrt{\frac{1-a x^n}{1+a x^n}} \left( 4 \sqrt{-1 + a^2 x^{2 n}} \text{ArcTan}\left[\sqrt{-1 + a^2 x^{2 n}}\right] (2 n \text{Log}[x] - \text{Log}[a^2 x^{2 n}]) + \right. \right.$$

$$\left. \sqrt{1 - a^2 x^{2 n}} \left( \text{Log}[a^2 x^{2 n}]^2 - 4 \text{Log}[a^2 x^{2 n}] \text{Log}\left[\frac{1}{2} \left( 1 + \sqrt{1 - a^2 x^{2 n}} \right)\right] + 2 \text{Log}\left[\frac{1}{2} \left( 1 + \sqrt{1 - a^2 x^{2 n}} \right)\right]^2 \right) - \right.$$

$$\left. \left. 4 \sqrt{1 - a^2 x^{2 n}} \text{PolyLog}\left[2, \frac{1}{2} - \frac{1}{2} \sqrt{1 - a^2 x^{2 n}}\right] \right) \right)$$

## Problems involving inverse hyperbolic cosecants

Valid but unnecessarily complicated antiderivative:

$$\{\text{ArcCsch}[c e^{a+b x}], x, 7, 0\}$$

$$\frac{\text{ArcCsch}[c e^{a+b x}]^2}{2 b} - \frac{\text{ArcCsch}[c e^{a+b x}] \text{Log}[1 - e^{2 \text{ArcCsch}[c e^{a+b x}]}]}{b} - \frac{\text{PolyLog}[2, e^{2 \text{ArcCsch}[c e^{a+b x}]}]}{2 b}$$

$$x \text{ArcCsch}[c e^{a+b x}] + \frac{1}{8 b c \sqrt{1 + \frac{e^{-2 (a+b x)}}{c^2}}}$$

$$e^{-a-b x} \sqrt{1 + c^2 e^{2 (a+b x)}} \left( \text{Log}[-c^2 e^{2 (a+b x)}]^2 + \text{ArcTanh}[\sqrt{1 + c^2 e^{2 (a+b x)}}] (-8 b x + 4 \text{Log}[-c^2 e^{2 (a+b x)}]) - 4 \text{Log}[-c^2 e^{2 (a+b x)}] \right)$$

$$\text{Log}\left[\frac{1}{2} \left(1 + \sqrt{1 + c^2 e^{2 (a+b x)}}\right)\right] + 2 \text{Log}\left[\frac{1}{2} \left(1 + \sqrt{1 + c^2 e^{2 (a+b x)}}\right)\right]^2 - 4 \text{PolyLog}\left[2, \frac{1}{2} \left(1 - \sqrt{1 + c^2 e^{2 (a+b x)}}\right)\right]\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{\text{ArcCsch}[a x^n]}{x}, x, 7, 0\right\}$$

$$\frac{\text{ArcCsch}[a x^n]^2}{2 n} - \frac{\text{ArcCsch}[a x^n] \text{Log}[1 - e^{2 \text{ArcCsch}[a x^n]}]}{n} - \frac{\text{PolyLog}[2, e^{2 \text{ArcCsch}[a x^n]}]}{2 n}$$

$$- \frac{x^{-n} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{x^{-2 n}}{a^2}\right]}{a n} + \left(\text{ArcCsch}[a x^n] - \text{ArcSinh}\left[\frac{x^{-n}}{a}\right]\right) \text{Log}[x]$$