Integration Rules for

$$\int \left(\sin^{j}(z)\right)^{m} \left(A\left(\sin^{k}(z)\right)^{p} + B\left(\sin^{k}(z)\right)^{p+1} + C\left(\sin^{k}(z)\right)^{p+2}\right) dz \text{ when}$$

$$j^{2} = 1 \bigwedge k^{2} = 1$$

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m}\,\left(A\,\left(\sin\left[c+d\,x\right]^{k}\right)^{p}+B\,\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1}\right)\,dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z}) \land p \neq 1 \land p \neq -2$, then

$$\int \left(\sin[c+dx]^{j}\right)^{m} \left(A\left(\sin[c+dx]^{k}\right)^{p} + B\left(\sin[c+dx]^{k}\right)^{p+1}\right) dx \longrightarrow$$

$$\int \left(\sin[c+dx]^{j}\right)^{m+jkp} \left(A+B\sin[c+dx]^{k}\right) dx$$

■ Program code:

```
 \begin{split} & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{j}}_{-} \right) ^{\text{m}}_{-} * \left( \text{A}_{-} * \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{p}}_{-} + \text{B}_{-} * \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} \right) , \text{x\_symbol} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} + \text{B}_{-} * \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} \right) , \text{x\_symbol} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} + \text{B}_{-} * \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} \right) , \text{x\_symbol} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} + \text{A}_{-} * \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{q}}_{-} \right) , \text{x\_symbol} \right] \\ & \text{FreeQ} \left[ \left\{ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right\} ^{\text{k}}_{-} \right\} & \text{\&\& ZeroQ} \left[ \text{j}^{2} - 1 \right] & \text{\&\& ZeroQ} \left[ \text{k}^{2} - 1 \right] & \text{\&\& ZeroQ} \left[ \text{p+1-q} \right] & \text{\&\& ZeroQ} \left[ \text{j}^{2} - 1 \right] & \text{\&\& p+2} \\ & \text{(ZeroQ} \left[ \text{j-k} \right] & \text{| IntegerQ} \left[ \text{p} \right] \right) & \text{\&\& p+2} \end{aligned}
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- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\begin{split} \int \left(\sin\left[c+d\,\mathbf{x}\right]^{-k}\right)^{m} \left(\mathbf{A} \left(\sin\left[c+d\,\mathbf{x}\right]^{k}\right)^{p} + \mathbf{B} \left(\sin\left[c+d\,\mathbf{x}\right]^{k}\right)^{p+1}\right) d\mathbf{x} &\rightarrow \\ \int \left(\sin\left[c+d\,\mathbf{x}\right]^{k}\right)^{p-m} \left(\mathbf{A} + \mathbf{B} \sin\left[c+d\,\mathbf{x}\right]^{k}\right) d\mathbf{x} \end{split}$$

```
 \begin{split} & \operatorname{Int} \left[ \left( \sin \left[ \operatorname{c}_{-} + \operatorname{d}_{-} * \operatorname{x}_{-} \right] ^{-} \right) ^{m}_{-} * \left( \operatorname{A}_{-} * \left( \sin \left[ \operatorname{c}_{-} + \operatorname{d}_{-} * \operatorname{x}_{-} \right] ^{k}_{-} \right) ^{p}_{-} + \operatorname{B}_{-} * \left( \sin \left[ \operatorname{c}_{-} + \operatorname{d}_{-} * \operatorname{x}_{-} \right] ^{k}_{-} \right) ^{q}_{-} \right) , \operatorname{x_{Symbol}} \right] \\ & \operatorname{Int} \left[ \left( \sin \left[ \operatorname{c}_{+} + \operatorname{d}_{+} \operatorname{x}_{-} \right] ^{k} \right) ^{k}_{-} \right) ^{p}_{-} + \operatorname{B}_{-} * \left( \sin \left[ \operatorname{c}_{-} + \operatorname{d}_{-} * \operatorname{x}_{-} \right] ^{k}_{-} \right) ^{q}_{-} \right) , \operatorname{x_{Symbol}} \right] \\ & \operatorname{Int} \left[ \left( \sin \left[ \operatorname{c}_{-} + \operatorname{d}_{-} * \operatorname{x}_{-} \right] ^{k}_{-} \right) ^{k}_{-} \right) ^{q}_{-} \right) , \operatorname{x_{Symbol}} \right] \\ & \operatorname{FreeQ} \left[ \left\{ \operatorname{c}_{+} \operatorname{d}_{+} \operatorname{x}_{-} \right\} ^{k}_{-} \right) ^{q}_{-} \right) , \operatorname{x_{Symbol}} \right] \\ & \operatorname{FreeQ} \left[ \left\{ \operatorname{c}_{+} \operatorname{d}_{+} \operatorname{x}_{-} \right\} ^{k}_{-} \right) ^{q}_{-} \right] , \operatorname{x_{Symbol}} \right] \\ & \operatorname{Not} \left[ \operatorname{IntegerQ} \left[ \operatorname{p} \right] \right] \end{aligned}
```

■ Derivation: Piecewise constant extraction and algebraic normalization

■ Basis:
$$\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$$

■ Rule: If $k^2 = 1 \bigwedge m - \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1}\right) dx \rightarrow \\ \sqrt{\csc\left[c+d\,x\right]} \, \sqrt{\sin\left[c+d\,x\right]} \, \int \left(\sin\left[c+d\,x\right]^{k}\right)^{p-m} \, \left(A+B\sin\left[c+d\,x\right]^{k}\right) dx$$

■ Program code:

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol]
Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k),x]] /;
FreeQ[{c,d,A,B,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && IntegerQ[m-1/2] &&
    Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) dx \ \longrightarrow \\ \sqrt{Csc[c+d\,x]} \ \sqrt{Sin[c+d\,x]} \ \left(\left(\sin[c+d\,x]^{-k}\right)^{m-p} \left(A+B\sin[c+d\,x]^k\right) dx \ \right)$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_),x_Symbol]
Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+B*sin[c+d*x]^k),x]] /;
FreeQ[{c,d,A,B,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] && IntegerQ[p-1/2] &&
Not[IntegerQ[2*m]]
```

$$\int \left(\sin\left[c+dx\right]^{j}\right)^{m} \left(A\left(\sin\left[c+dx\right]^{k}\right)^{p} + C\left(\sin\left[c+dx\right]^{k}\right)^{p+2}\right) dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z})$, then

$$\int \left(\sin[c+d\,x]^{j}\right)^{m} \left(A\left(\sin[c+d\,x]^{k}\right)^{p} + C\left(\sin[c+d\,x]^{k}\right)^{p+2}\right) dx \rightarrow$$

$$\int \left(\sin[c+d\,x]^{j}\right)^{m+j\,k\,p} \left(A + C\sin[c+d\,x]^{2\,k}\right) dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol]
Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+2-r] &&
    (ZeroQ[j-k] || IntegerQ[p])
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\begin{split} & \int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \, \left(A \, \left(\sin\left[c+d\,x\right]^{k}\right)^{p} + C \, \left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) \, \mathrm{d}x \, \, \rightarrow \\ & \quad \int \left(\sin\left[c+d\,x\right]^{k}\right)^{p-m} \, \left(A + C \sin\left[c+d\,x\right]^{2\,k}\right) \, \mathrm{d}x \end{split}$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_symbol]
Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[m] &&
Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + C\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) dx \ \longrightarrow \\ \sqrt{\operatorname{Csc}\left[c+d\,x\right]} \ \sqrt{\sin\left[c+d\,x\right]} \ \int \left(\sin\left[c+d\,x\right]^{k}\right)^{p-m} \left(A+C\sin\left[c+d\,x\right]^{2\,k}\right) dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol]
Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[m-1/2] &&
    Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\begin{split} &\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m}\,\left(A\,\left(\sin\left[c+d\,x\right]^{k}\right)^{p}+C\,\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right)\,\mathrm{d}x\,\to\\ &\sqrt{Csc\left[c+d\,x\right]}\,\,\sqrt{\sin\left[c+d\,x\right]}\,\,\left[\left(\sin\left[c+d\,x\right]^{-k}\right)^{m-p}\,\left(A+C\sin\left[c+d\,x\right]^{2\,k}\right)\,\mathrm{d}x \end{split}$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symbol]
Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
    Int[(sin[c+d*x]^j)^(m-p)*(A+C*sin[c+d*x]^(2*k)),x]] /;
FreeQ[{c,d,A,C,m,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] && IntegerQ[p-1/2] &&
Not[IntegerQ[2*m]]
```

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m}\,\left(A+B\sin\left[c+d\,x\right]+C\sin\left[c+d\,x\right]^{-1}\right)\,dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = 1$, then

$$\begin{split} & \int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \, \left(A+B\sin\left[c+d\,x\right]+C\sin\left[c+d\,x\right]^{-1}\right) \, \mathrm{d}x \, \to \\ & \int \left(\sin\left[c+d\,x\right]^{j}\right)^{m-j} \, \left(C+A\sin\left[c+d\,x\right]+B\sin\left[c+d\,x\right]^{2}\right) \, \mathrm{d}x \end{split}$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+B_.*sin[c_.+d_.*x_]+C_.*sin[c_.+d_.*x_]^(-1)),x_Symbol] :=
   Int[(sin[c+d*x]^j)^(m-j)*(C+A*sin[c+d*x]+B*sin[c+d*x]^2),x] /;
   FreeQ[{c,d,A,B,C,m},x] && ZeroQ[j^2-1]
```

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z})$, then

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) dx \rightarrow \int \left(\sin\left[c+d\,x\right]^{j}\right)^{m+j\,k\,p} \left(A+B\sin\left[c+d\,x\right]^{k} + C\sin\left[c+d\,x\right]^{2\,k}\right) dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_.+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_),x_Symb
Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k)),x] /;
FreeQ[{c,d,A,B,C,m,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+1-q] && ZeroQ[p+2-r] &&
    (ZeroQ[j-k] || IntegerQ[p])
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) dx \ \longrightarrow \\ \left(\left(\sin\left[c+d\,x\right]^{k}\right)^{p-m} \left(A+B\sin\left[c+d\,x\right]^{k} + C\sin\left[c+d\,x\right]^{2\,k}\right) dx$$

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1} + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) dx \rightarrow \\ \sqrt{\operatorname{Csc}[c+d\,x]} \ \sqrt{\sin[c+d\,x]} \ \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A + B\sin[c+d\,x]^k + C\sin[c+d\,x]^{2\,k}\right) dx$$

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) dx \ \rightarrow \\ \sqrt{\csc\left[c+d\,x\right]} \ \sqrt{\sin\left[c+d\,x\right]} \ \int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m-p} \left(A+B\sin\left[c+d\,x\right]^{k} + C\sin\left[c+d\,x\right]^{2\,k}\right) dx$$

Integration Rules for

$$\int \left(A\left(\sin^{i}(z)\right)^{p} + B\left(\sin^{i}(z)\right)^{p+1} + C\left(\sin^{i}(z)\right)^{p+2}\right) \left(a + b\sin^{k}(z)\right)^{n} dz \text{ when}$$

$$i^{2} = 1 \bigwedge k^{2} = 1$$

$$\int \left(A \left(\sin[c + dx]^{k} \right)^{p} + B \left(\sin[c + dx]^{k} \right)^{p+1} \right) \left(a + b \sin[c + dx]^{k} \right)^{n} dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land p \neq 1$, then

$$\begin{split} & \int \left(A \left(\sin[c+d\,x]^k \right)^p + B \left(\sin[c+d\,x]^k \right)^{p+1} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \, \to \\ & \int \left(\sin[c+d\,x]^k \right)^p \left(A + B \sin[c+d\,x]^k \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \end{split}$$

```
 Int \left[ \left( A_{**} \left( \sin[c_{**}d_{**}x_{-}]^k A_{**} \right)^p + B_{**} \left( \sin[c_{**}d_{**}x_{-}]^k A_{**} \right)^q \right) * \left( a_{**}b_{**}\sin[c_{**}d_{**}x_{-}]^k A_{**} \right)^n A_{**} \right] \\ Int \left[ \left( \sin[c_{*}d_{*}x_{-}]^k A_{**} \right)^p + \left( A_{**}B_{*}\sin[c_{*}d_{*}x_{-}]^k A_{**} \right)^n A_{**} \right] / ; \\ FreeQ \left[ \left\{ a_{*}b_{*}c_{*}d_{*}A_{*}B_{*}n_{*}p \right\}_{*} \right] & & ZeroQ \left[ k^2 - 1 \right] & & ZeroQ \left[ p + 1 - q \right] & & Not \left[ a_{**}B_{*}a_{*}b_{*} \right] - 2 & & B_{**}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_{*}B_{*}a_
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$$\int \left(A \left(\sin[c+dx]^{-k} \right)^{p} + B \left(\sin[c+dx]^{-k} \right)^{p+1} \right) \left(a + b \sin[c+dx]^{k} \right)^{n} dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\begin{split} & \int \left(A + B \sin \left[c + d \, x \right]^{-k} \right) \, \left(a + b \sin \left[c + d \, x \right]^{k} \right)^{n} \, dx \, \rightarrow \\ & \int \! \sin \left[c + d \, x \right]^{-k} \, \left(B + A \sin \left[c + d \, x \right]^{k} \right) \, \left(a + b \sin \left[c + d \, x \right]^{k} \right)^{n} \, dx \end{split}$$

```
 Int [ (A_{+B_{-}}*sin[c_{-}+d_{-}*x_{-}]^i_{-}) * (a_{-}+b_{-}*sin[c_{-}+d_{-}*x_{-}]^k_{-})^n_{-}, x_Symbol ] := \\ Int[sin[c+d*x]^(-k) * (B+A*sin[c+d*x]^k) * (a+b*sin[c+d*x]^k)^n, x] /; \\ FreeQ[\{a,b,c,d,A,B,n\},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && Not[a===0 && b===1]
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land p \neq 1 \land p \neq -2$, then

$$\int \left(A \left(\sin[c+dx]^{-k} \right)^{p} + B \left(\sin[c+dx]^{-k} \right)^{p+1} \right) \left(a + b \sin[c+dx]^{k} \right)^{n} dx \longrightarrow$$

$$\int \left(\sin[c+dx]^{-k} \right)^{p+1} \left(B + A \sin[c+dx]^{k} \right) \left(a + b \sin[c+dx]^{k} \right)^{n} dx$$

```
Int[(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x__
Int[(sin[c+d*x]^(-k))^(p+1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
Not[a===0 && b===1] && p≠-2
```

$$\int \left(A \left(\sin[c+dx]^k \right)^p + C \left(\sin[c+dx]^k \right)^{p+2} \right) \left(a + b \sin[c+dx]^k \right)^n dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\begin{split} & \int \left(A \left(\sin[c+d\,x]^k \right)^p + C \left(\sin[c+d\,x]^k \right)^{p+2} \right) \, \left(a + b \sin[c+d\,x]^k \right)^n \, dx \, \rightarrow \\ & \int \left(\sin[c+d\,x]^k \right)^p \, \left(A + C \sin[c+d\,x]^{2\,k} \right) \, \left(a + b \sin[c+d\,x]^k \right)^n \, dx \end{split}$$

```
Int[(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
   Int[(sin[c+d*x]^k)^p*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[p+2-r]
```

$$\int \left(A \left(\sin[c+dx]^{-k} \right)^p + C \left(\sin[c+dx]^{-k} \right)^{p+2} \right) \left(a + b \sin[c+dx]^k \right)^n dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\int \left(\mathbf{A} + \mathbf{C} \sin[\mathbf{c} + \mathbf{d} \, \mathbf{x}]^{-2 \, k}\right) \, \left(\mathbf{a} + \mathbf{b} \sin[\mathbf{c} + \mathbf{d} \, \mathbf{x}]^{k}\right)^{n} \, d\mathbf{x} \, \rightarrow$$

$$\int \sin[\mathbf{c} + \mathbf{d} \, \mathbf{x}]^{-2 \, k} \, \left(\mathbf{C} + \mathbf{A} \sin[\mathbf{c} + \mathbf{d} \, \mathbf{x}]^{2 \, k}\right) \, \left(\mathbf{a} + \mathbf{b} \sin[\mathbf{c} + \mathbf{d} \, \mathbf{x}]^{k}\right)^{n} \, d\mathbf{x}$$

```
 Int [ (A_+C_.*sin[c_.+d_.*x_]^i2_) * (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol ] := \\ Int[sin[c+d*x]^(-2*k) * (C+A*sin[c+d*x]^(2*k)) * (a+b*sin[c+d*x]^k)^n,x] /; \\ FreeQ[\{a,b,c,d,A,C,n\},x] && ZeroQ[k^2-1] && ZeroQ[k+i2/2] \\ \end{cases}
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\begin{split} & \int \left(A \left(\sin[c+d\,x]^{-k} \right)^p + C \left(\sin[c+d\,x]^{-k} \right)^{p+2} \right) \left(a+b \sin[c+d\,x]^k \right)^n dx \, \longrightarrow \\ & \int \left(\sin[c+d\,x]^{-k} \right)^{p+2} \left(C+A \sin[c+d\,x]^{2\,k} \right) \left(a+b \sin[c+d\,x]^k \right)^n dx \end{split}$$

```
Int[(A_.*(sin[c_.+d_.*x_]^i_.)^p_.+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*(a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x
Int[(sin[c+d*x]^(-k))^(p+2)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+2-r]
```

$$\int (A + B \sin[c + dx]^k + C \sin[c + dx]^{-k}) (a + b \sin[c + dx]^k)^n dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\begin{split} \int \left(A + B \sin[c + dx]^k + C \sin[c + dx]^{-k} \right) & \left(a + b \sin[c + dx]^k \right)^n dx & \rightarrow \\ & \int \sin[c + dx]^{-k} & \left(C + A \sin[c + dx]^k + B \sin[c + dx]^{2k} \right) & \left(a + b \sin[c + dx]^k \right)^n dx \end{split}$$

```
 \begin{split} & \text{Int} \left[ \left( \texttt{A}_{-} + \texttt{B}_{-} * \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{k}_{-} + \texttt{C}_{-} * \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{l}_{-} \right) * \left( \texttt{a}_{-} + \texttt{b}_{-} * \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{k}_{-} \right) \wedge \texttt{n}_{-}, \texttt{x}_{-} \text{Symbol} \\ & \text{Int} \left[ \sin[\texttt{c}_{-} + \texttt{d}_{+} \texttt{x}_{-}] \wedge (-\texttt{k}) * (\texttt{c}_{-} + \texttt{d}_{+} * \texttt{x}_{-}] \wedge \texttt{k}_{-} + \texttt{b}_{-} * \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{k}_{-} \right) \wedge \texttt{n}_{-}, \texttt{x}_{-} \text{Symbol} \\ & \text{Int} \left[ \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge (-\texttt{k}) * (\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{k}_{-} + \texttt{b}_{-} * \sin[\texttt{c}_{-} + \texttt{d}_{-} * \texttt{x}_{-}] \wedge \texttt{k}_{-} \right) \wedge \texttt{n}_{-}, \texttt{x}_{-} \text{Symbol} \\ & \text{FreeQ} \left[ \{ \texttt{a}_{-} + \texttt{b}_{-} + \texttt{d}_{-} * \texttt{x}_{-} + \texttt{d}_{-} * \texttt{x}_{-} + \texttt{d}_{-} * \texttt{x}_{-} \right] \wedge \texttt{k}_{-} + \texttt{d}_{-} * \texttt{k}_{-} + \texttt{d}_{-} * \texttt{k}_{-} \right] \wedge \texttt{k}_{-} + \texttt{d}_{-} \times \texttt{k}_{-} + \texttt{d}_{-} \times \texttt{k}_{-} + \texttt{d}_{-} \times \texttt{k}_{-} \right] \wedge \texttt{k}_{-} + \texttt{d}_{-} \times \texttt{k}_{-} +
```

$$\int (A (\sin[c+dx]^k)^p + B (\sin[c+dx]^k)^{p+1} + C (\sin[c+dx]^k)^{p+2})$$

$$(a+b\sin[c+dx]^k)^n dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\int \left(A \left(\sin[c+d\,x]^k \right)^p + B \left(\sin[c+d\,x]^k \right)^{p+1} + C \left(\sin[c+d\,x]^k \right)^{p+2} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \rightarrow \\ \int \left(\sin[c+d\,x]^k \right)^p \left(A + B \sin[c+d\,x]^k + C \sin[c+d\,x]^{2k} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx$$

```
 \begin{split} & \text{Int} \left[ \left( \text{A.*} \left( \sin[\text{c.*d.*x.}] \wedge \text{k..} \right) \wedge \text{p.*+B..*} \left( \sin[\text{c.*+d..*x.}] \wedge \text{k..} \right) \wedge \text{q.+C..*} \left( \sin[\text{c.*+d..*x.}] \wedge \text{k..} \right) \wedge \text{r..} \right) \times \\ & \left( \text{a..+b..*sin} \left[ \text{c..+d..*x.}] \wedge \text{k..} \right) \wedge \text{n..,x.} \text{Symbol} \right] := \\ & \text{Int} \left[ \left( \sin[\text{c+d*x}] \wedge \text{k} \right) \wedge \text{p*} \left( \text{A+B*sin} \left[ \text{c+d*x} \right] \wedge \text{k+C*sin} \left[ \text{c+d*x} \right] \wedge \left( 2 \times \text{k} \right) \right) \times \left( \text{a+b*sin} \left[ \text{c+d*x} \right] \wedge \text{k} \right) \wedge \text{n.x.} \right] \right. / ; \\ & \text{FreeQ} \left[ \left\{ \text{a,b,c,d,A,B,C,n,p} \right\}, \text{x} \right] \right. \& \& \text{ZeroQ} \left[ \text{k} \wedge 2 - 1 \right] \right. \& \& \text{ZeroQ} \left[ \text{p+1-q} \right] \right. \& \& \text{ZeroQ} \left[ \text{p+2-r} \right] \end{aligned}
```

$$\int (A (\sin[c+dx]^{-k})^{p} + B (\sin[c+dx]^{-k})^{p+1} + C (\sin[c+dx]^{-k})^{p+2})$$

$$(a+b\sin[c+dx]^{k})^{n} dx$$

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\int \left(A + B \sin[c + dx]^{-k} + C \sin[c + dx]^{-2k}\right) \left(a + b \sin[c + dx]^{k}\right)^{n} dx \rightarrow$$

$$\int \sin[c + dx]^{-2k} \left(C + B \sin[c + dx]^{k} + A \sin[c + dx]^{2k}\right) \left(a + b \sin[c + dx]^{k}\right)^{n} dx$$

```
 \begin{split} & \text{Int} \left[ \left( \textbf{A}_{-} + \textbf{B}_{-} * \sin \left[ \textbf{c}_{-} + \textbf{d}_{-} * \textbf{x}_{-} \right] ^{i} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{x}_{-} \right] ^{i} \textbf{2}_{-} \right) * \left( \textbf{a}_{-} + \textbf{b}_{-} * \sin \left[ \textbf{c}_{-} + \textbf{d}_{-} * \textbf{x}_{-} \right] ^{k} \textbf{c}_{-} \right) ^{n} \textbf{c}_{-} \textbf{x}_{-} \\ & \text{Int} \left[ \sin \left[ \textbf{c}_{-} + \textbf{d}_{+} \textbf{x}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{x}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} \right] ^{k} \textbf{c}_{-} + \textbf{d}_{-} * \textbf{c}_{-} + \textbf{d}_{-} + \textbf
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1$, then

$$\begin{split} & \int \left(A \left(\sin[c+d\,x]^{-k} \right)^p + B \left(\sin[c+d\,x]^{-k} \right)^{p+1} + C \left(\sin[c+d\,x]^{-k} \right)^{p+2} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \\ & \qquad \qquad \int \left(\sin[c+d\,x]^{-k} \right)^{p+2} \left(C + B \sin[c+d\,x]^k + A \sin[c+d\,x]^{2\,k} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \end{split}$$

Integration Rules for

$$\int \left(\sin^{j}(z)\right)^{m} \left(A\left(\sin^{i}(z)\right)^{p} + B\left(\sin^{i}(z)\right)^{p+1} + C\left(\sin^{i}(z)\right)^{p+2}\right) \left(a + b\sin^{k}(z)\right)^{n} dz \text{ when}$$

$$j^{2} = 1 \bigwedge j^{2} = 1 \bigwedge k^{2} = 1$$

$$\int \left(\sin\left[c + dx\right]^{j}\right)^{m}$$

$$\left(A\left(\sin\left[c + dx\right]^{k}\right)^{p} + B\left(\sin\left[c + dx\right]^{k}\right)^{p+1}\right) \left(a + b\sin\left[c + dx\right]^{k}\right)^{n} dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z}) \land p \neq 1$, then

$$\begin{split} \int \left(\sin[c+d\,x]^{\,j}\right)^m \left(A\,\left(\sin[c+d\,x]^k\right)^p + B\,\left(\sin[c+d\,x]^k\right)^{p+1}\right) \, \left(a+b\sin[c+d\,x]^k\right)^n \, \mathrm{d}x \, \to \\ \int \left(\sin[c+d\,x]^{\,j}\right)^{m+j\,k\,p} \, \left(A+B\sin[c+d\,x]^k\right) \, \left(a+b\sin[c+d\,x]^k\right)^n \, \mathrm{d}x \end{split}$$

■ Program code:

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+1-q] &&
        (ZeroQ[j-k] || IntegerQ[p])
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \rightarrow \\ \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A+B\sin[c+d\,x]^k\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
        IntegerQ[m] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^{-k}\right)^m \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \rightarrow \\ \sqrt{\csc[c+d\,x]} \sqrt{\sin[c+d\,x]} \int \left(\sin[c+d\,x]^k\right)^{p-m} \left(A+B\sin[c+d\,x]^k\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
        IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^m \left(A\,\left(\sin\left[c+d\,x\right]^k\right)^p + B\,\left(\sin\left[c+d\,x\right]^k\right)^{p+1}\right) \, \left(a+b\sin\left[c+d\,x\right]^k\right)^n \, dx \, \rightarrow \\ \sqrt{\csc\left[c+d\,x\right]} \, \sqrt{\sin\left[c+d\,x\right]} \, \int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m-p} \, \left(A+B\sin\left[c+d\,x\right]^k\right) \, \left(a+b\sin\left[c+d\,x\right]^k\right)^n \, dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1$, then

$$\begin{split} & \int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \left(A+B\sin\left[c+d\,x\right]^{-k}\right) \, \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} \, dx \, \to \\ & \int \left(\sin\left[c+d\,x\right]^{j}\right)^{m-j\,k} \, \left(B+A\sin\left[c+d\,x\right]^{k}\right) \, \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} \, dx \end{split}$$

```
 Int \left[ \left( \sin[c_{-} + d_{-} *x_{-}] ^{j}_{-} \right) ^{m}_{-} * \left( A_{-} + B_{-} * \sin[c_{-} + d_{-} *x_{-}] ^{i}_{-} \right) * \left( a_{-} + b_{-} * \sin[c_{-} + d_{-} *x_{-}] ^{k}_{-} \right) ^{n}_{-} , x_{-} \text{Symbolized} \right] \\ Int \left[ \left( \sin[c_{+} d *x_{-}] ^{j} \right) ^{m}_{-} * \left( B_{+} A * \sin[c_{+} d *x_{-}] ^{k} \right) * \left( a_{+} b * \sin[c_{+} d *x_{-}] ^{k} \right) ^{n}_{-} , x_{-} \text{Symbolized} \right] \\ FreeQ \left[ \left\{ a, b, c, d, A, B, m, n \right\}, x \right] & & ZeroQ \left[ j^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & & ZeroQ \left[ k^{2} - 1 \right] \\ & ZeroQ \left[ k^{2
```

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j + k = 0 \lor p \in \mathbb{Z}) \land p \neq 1 \land p \neq -2$, then

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{-k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+1}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx \rightarrow \\ \left(\sin\left[c+d\,x\right]^{j}\right)^{m-j\,k\,(p+1)} \left(B+A\sin\left[c+d\,x\right]^{k}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^j)^(m-j*k*(p+1))*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i] &&
        ZeroQ[p+1-q] && (ZeroQ[j+k] || IntegerQ[p]) && p≠-2
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{k}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{-k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+1}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx \rightarrow$$

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{p-m+1} \left(B+A\sin\left[c+d\,x\right]^{k}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx$$

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^(-k))^(p-m+1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
        IntegerQ[m] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^k\right)^m \left(A\,\left(\sin\left[c+d\,x\right]^{-k}\right)^p + B\,\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+1}\right) \,\left(a+b\sin\left[c+d\,x\right]^k\right)^n \,dx \, \rightarrow \\ \sqrt{\operatorname{Csc}\left[c+d\,x\right]} \, \sqrt{\sin\left[c+d\,x\right]} \, \int \left(\sin\left[c+d\,x\right]^k\right)^{m-p-1} \,\left(B+A\sin\left[c+d\,x\right]^k\right) \, \left(a+b\sin\left[c+d\,x\right]^k\right)^n \,dx$$

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+B_.*(sin[c_.+d_.*x_]^i_.)^q_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^k)^(m-p-1)*(B+A*sin[c+d*x]^k)*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,B,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+1-q] &&
        IntegerQ[p-1/2] && Not[IntegerQ[2*m]]
```

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z})$, then

$$\int \left(\sin[c+d\,x]^{j}\right)^{m} \left(A\left(\sin[c+d\,x]^{k}\right)^{p} + C\left(\sin[c+d\,x]^{k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} dx \rightarrow$$

$$\int \left(\sin[c+d\,x]^{j}\right)^{m+j\,k\,p} \left(A+C\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^{k}\right)^{n} dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_.+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^j)^(m+j*k*p)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[p+2-r] &&
        (ZeroQ[j-k] || IntegerQ[p])
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\begin{split} \int \left(\sin\left[c+d\,x\right]^{-k}\right)^m \left(A\,\left(\sin\left[c+d\,x\right]^k\right)^p + C\,\left(\sin\left[c+d\,x\right]^k\right)^{p+2}\right) \,\left(a+b\sin\left[c+d\,x\right]^k\right)^n \,\mathrm{d}x \,\, \to \\ \int \left(\sin\left[c+d\,x\right]^k\right)^{p-m} \,\left(A+C\sin\left[c+d\,x\right]^{2\,k}\right) \,\left(a+b\sin\left[c+d\,x\right]^k\right)^n \,\mathrm{d}x \end{split}$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] &&
IntegerQ[m] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^m \left(A\left(\sin\left[c+d\,x\right]^k\right)^p + C\left(\sin\left[c+d\,x\right]^k\right)^{p+2}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx \ \rightarrow \\ \sqrt{\operatorname{Csc}\left[c+d\,x\right]} \ \sqrt{\sin\left[c+d\,x\right]} \ \int \left(\sin\left[c+d\,x\right]^k\right)^{p-m} \left(A+C\sin\left[c+d\,x\right]^{2k}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^k_.)^p_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^k)^(p-m)*(A+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+2-r] &&
        IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m} \, \left(A\, \left(\sin\left[c+d\,x\right]^{k}\right)^{p} + C\, \left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right) \, \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} \, dx \, \rightarrow \\ \sqrt{\csc\left[c+d\,x\right]} \, \sqrt{\sin\left[c+d\,x\right]} \, \int \left(\sin\left[c+d\,x\right]^{-k}\right)^{m-p} \, \left(A+C\sin\left[c+d\,x\right]^{2\,k}\right) \, \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} \, dx \,$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1$, then

$$\begin{split} &\int \left(\sin[c+d\,x]^{\,j}\right)^m \, \left(A+C\sin[c+d\,x]^{\,-2\,k}\right) \, \left(a+b\sin[c+d\,x]^k\right)^n \, dx \, \to \\ &\int \left(\sin[c+d\,x]^{\,j}\right)^{m-2\,j\,k} \, \left(C+A\sin[c+d\,x]^{\,2\,k}\right) \, \left(a+b\sin[c+d\,x]^k\right)^n \, dx \end{split}$$

```
 \begin{split} & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{j}}_{-} \right) ^{\text{m}}_{-} * \left( \text{A}_{-} + \text{C}_{-} * \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{i}}_{2} \right) * \left( \text{a}_{-} + \text{b}_{-} * \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{n}}_{-} , \text{x\_Symbolic} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{C}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{C}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) ^{\text{n}}_{-} , \text{x\_Symbolic}_{-} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{C}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) ^{\text{n}}_{-} , \text{x\_Symbolic}_{-} \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) ^{\text{k}}_{-} \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \right] \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \text{x}_{-} \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} * \right] ^{\text{k}}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} * \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} + \text{d}_{-} \right] * \left( \text{c}_{-} + \text{d}_{-} \right) * \left( \text{c}_{-} + \text{d}_{-} \right) * \right) * \\ & \text{Int} \left[ \left( \sin \left[ \text{c}_{-} + \text{d}_{-} + \text{d}_{-} \right] * \left( \text{c}_{-} + \text{d}_{-} + \text{d}
```

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j + k = 0 \lor p \in \mathbb{Z})$, then

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{-k}\right)^{p} + C\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+2}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx \rightarrow$$

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m-j\,k\,(p+2)} \left(C+A\sin\left[c+d\,x\right]^{2\,k}\right) \left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_.+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
Int[(sin[c+d*x]^j)^(m-j*k*(p+2))*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i] &&
    ZeroQ[p+2-r] && (ZeroQ[j+k] || IntegerQ[p])
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

$$\begin{split} \int \left(\sin\left[c+d\,x\right]^k\right)^m \left(A\,\left(\sin\left[c+d\,x\right]^{-k}\right)^p + C\,\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+2}\right) \,\left(a+b\sin\left[c+d\,x\right]^k\right)^n \,\mathrm{d}x \,\, \to \\ \int \left(\sin\left[c+d\,x\right]^{-k}\right)^{p-m+2} \,\left(C+A\sin\left[c+d\,x\right]^{2\,k}\right) \,\left(a+b\sin\left[c+d\,x\right]^k\right)^n \,\mathrm{d}x \end{split}$$

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^(-k))^(p-m+2)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+2-r] &&
        IntegerQ[m] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\int \left(\sin[c+d\,x]^k\right)^m \left(A\left(\sin[c+d\,x]^{-k}\right)^p + C\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \ \rightarrow \\ \sqrt{Csc[c+d\,x]} \ \sqrt{Sin[c+d\,x]} \ \int \left(\sin[c+d\,x]^{-k}\right)^{p-m+2} \left(C+A\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^(-k))^(p-m+2)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+2-r] &&
        IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
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$$\begin{split} & \int \left(\sin[c+d\,x]^k\right)^m \left(A\,\left(\sin[c+d\,x]^{-k}\right)^p + C\,\left(\sin[c+d\,x]^{-k}\right)^{p+2}\right) \,\left(a+b\sin[c+d\,x]^k\right)^n dx \, \to \\ & \sqrt{Csc\,[c+d\,x]} \, \sqrt{\sin[c+d\,x]} \, \left[\left(\sin[c+d\,x]^k\right)^{m-p-2} \, \left(C+A\sin[c+d\,x]^{2\,k}\right) \, \left(a+b\sin[c+d\,x]^k\right)^n dx \end{split}$$

```
Int[(sin[c_.+d_.*x_]^k_.)^m_.*(A_.*(sin[c_.+d_.*x_]^i_.)^p_+C_.*(sin[c_.+d_.*x_]^i_.)^r_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^k)^(m-p-2)*(C+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[k+i] && ZeroQ[p+2-r] &&
        IntegerQ[p-1/2] && Not[IntegerQ[2*m]]
```

$$\int \left(\sin\left[c+d\,x\right]^{\,j}\right)^m\,\left(A+B\sin\left[c+d\,x\right]^k+C\sin\left[c+d\,x\right]^{-k}\right)\,\left(a+b\sin\left[c+d\,x\right]^k\right)^n\,dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1$, then

$$\begin{split} &\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m}\,\left(A+B\sin\left[c+d\,x\right]^{k}+C\sin\left[c+d\,x\right]^{-k}\right)\,\left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n}\,\mathrm{d}x\,\,\rightarrow\\ &\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m-j\,k}\,\left(C+A\sin\left[c+d\,x\right]^{k}+B\sin\left[c+d\,x\right]^{2\,k}\right)\,\left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n}\,\mathrm{d}x \end{split}$$

```
 \begin{split} & \text{Int} \left[ \left( \sin[c_{-} + d_{-} *x_{-}] ^{-} j_{-} \right) ^{m}_{-} * \left( A_{-} + B_{-} * \sin[c_{-} + d_{-} *x_{-}] ^{-} k_{-} + C_{-} * \sin[c_{-} + d_{-} *x_{-}] ^{-} 1_{-} \right) * \\ & \left( a_{-} + b_{-} * \sin[c_{-} + d_{-} *x_{-}] ^{-} k_{-} \right) ^{n}_{-} , x_{-} \text{Symbol} \right] := \\ & \text{Int} \left[ \left( \sin[c_{+} d *x_{-}] ^{-} j_{+} \right) * \left( C_{+} A * \sin[c_{+} d *x_{-}] ^{-} k_{+} B * \sin[c_{+} d *x_{-}] ^{-} (2 * k_{-}) \right) * \left( a_{+} b * \sin[c_{+} d *x_{-}] ^{-} k_{+} \right) ^{n}_{-} , x_{-} \right] \right. \\ & \text{FreeQ} \left[ \left\{ a_{+} b_{+} c_{+} d_{-} * x_{-} \right\} ^{n}_{+} \right] & \text{\& ZeroQ} \left[ j_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{+}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_{-}^{2} - 1 \right] & \text{\& ZeroQ} \left[ k_{-}^{2} - 1 \right] \\ & \text{& ZeroQ} \left[ k_
```

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m} \left(A\left(\sin\left[c+d\,x\right]^{k}\right)^{p} + B\left(\sin\left[c+d\,x\right]^{k}\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^{k}\right)^{p+2}\right)$$

$$\left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n} dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j = k \lor p \in \mathbb{Z})$, then

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\begin{split} & \int \left(\sin\left[c+d\,x\right]^{-k}\right)^m \\ & \left(A\left(\sin\left[c+d\,x\right]^k\right)^p + B\left(\sin\left[c+d\,x\right]^k\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^k\right)^{p+2}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx \, \to \\ & \sqrt{Csc\left[c+d\,x\right]} \, \sqrt{Sin\left[c+d\,x\right]} \\ & \int \left(\sin\left[c+d\,x\right]^k\right)^{p-m} \left(A+B\sin\left[c+d\,x\right]^k + C\sin\left[c+d\,x\right]^{2\,k}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx \end{split}$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*
    (A_.*(sin[c_.+d_.*x_]^k_.)^p_+B_.*(sin[c_.+d_.*x_]^k_.)^q_+C_.*(sin[c_.+d_.*x_]^k_.)^r_)*
    (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Dist[Sqrt[Csc[c+d*x]]*Sqrt[Sin[c+d*x]],
        Int[(sin[c+d*x]^k)^(p-m)*(A+B*sin[c+d*x]^k+C*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x]] /;
    FreeQ[{a,b,c,d,A,B,C,m,n,p},x] && ZeroQ[k^2-1] && ZeroQ[j+k] && ZeroQ[p+1-q] &&
        ZeroQ[p+2-r] && IntegerQ[m-1/2] && Not[IntegerQ[p]]
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\begin{split} & \int \left(\sin[c+d\,x]^{-k}\right)^m \\ & \left(A\left(\sin[c+d\,x]^k\right)^p + B\left(\sin[c+d\,x]^k\right)^{p+1} + C\left(\sin[c+d\,x]^k\right)^{p+2}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \, \to \\ & \sqrt{Csc[c+d\,x]} \, \sqrt{\sin[c+d\,x]} \\ & \int \left(\sin[c+d\,x]^{-k}\right)^{m-p} \left(A+B\sin[c+d\,x]^k + C\sin[c+d\,x]^{2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \end{split}$$

$$\int \left(\sin\left[c+d\,x\right]^{j}\right)^{m}$$

$$\left(A\left(\sin\left[c+d\,x\right]^{-k}\right)^{p}+B\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+1}+C\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+2}\right)$$

$$\left(a+b\sin\left[c+d\,x\right]^{k}\right)^{n}dx$$

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1$, then

$$\int \left(\sin[c+d\,x]^{\,j}\right)^m \left(A+B\sin[c+d\,x]^{\,-k}+C\sin[c+d\,x]^{\,-2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx \ \longrightarrow \\ \left(\sin[c+d\,x]^{\,j}\right)^{m-2\,j\,k} \left(C+B\sin[c+d\,x]^k+A\sin[c+d\,x]^{\,2\,k}\right) \left(a+b\sin[c+d\,x]^k\right)^n dx$$

```
Int[(sin[c_.+d_.*x_]^j_.)^m_.*(A_.+B_.*sin[c_.+d_.*x_]^i_.+C_.*sin[c_.+d_.*x_]^i2_)*
        (a_.+b_.*sin[c_.+d_.*x_]^k_.)^n_.,x_Symbol] :=
    Int[(sin[c+d*x]^j)^(m-2*j*k)*(C+B*sin[c+d*x]^k+A*sin[c+d*x]^(2*k))*(a+b*sin[c+d*x]^k)^n,x] /;
    FreeQ[{a,b,c,d,A,B,C,m,n},x] && ZeroQ[j^2-1] && ZeroQ[k^2-1] && ZeroQ[k+i] &&
        ZeroQ[2*i-i2]
```

- Derivation: Algebraic normalization
- Rule: If $j^2 = k^2 = 1 \land (j + k = 0 \lor p \in \mathbb{Z})$, then

```
 \begin{split} & \text{Int} \big[ \big( \sin[c_{-} + d_{-} *x_{-}] ^{-} j_{-} \big) ^{m} - * \\ & \quad \big( A_{-} * \big( \sin[c_{-} + d_{-} *x_{-}] ^{-} i_{-} \big) ^{p} - * B_{-} * \big( \sin[c_{-} + d_{-} *x_{-}] ^{-} i_{-} \big) ^{q} + C_{-} * \big( \sin[c_{-} + d_{-} *x_{-}] ^{-} i_{-} \big) ^{r} \big) * \\ & \quad \big( a_{-} * b_{-} * \sin[c_{-} * d_{-} *x_{-}] ^{k} - \big) ^{n} - i_{-} x_{-} \text{Symbol} \big] := \\ & \quad \text{Int} \big[ \big( \sin[c_{+} d *x_{-}] ^{-} \big) ^{k} \big( (c_{+} B * \sin[c_{+} d *x_{-}] ^{k} + A * \sin[c_{+} d *x_{-}] ^{k} \big) * \big( (a_{+} b * \sin[c_{+} d *x_{-}] ^{k} \big) ^{n} \big) / (a_{+} b * \sin[c_{+} d *x_{-}] ^{k} \big) ^{r} \big] \\ & \quad \text{FreeQ} \big[ \{ a_{+} b_{+} c_{+} d_{+} A_{+} B_{+} C_{+} d_{+} A_{+} B_{+} B_{
```

- Derivation: Algebraic normalization
- Rule: If $k^2 = 1 \land m \in \mathbb{Z} \land p \notin \mathbb{Z}$, then

```
 \begin{split} & \text{Int} \big[ \big( \sin[c_{-} + d_{-} * x_{-}]^{\wedge} k_{-} \big)^{\wedge} m_{-} * \\ & \quad \big( A_{-} * \big( \sin[c_{-} + d_{-} * x_{-}]^{\wedge} i_{-} \big)^{\wedge} p_{-} + B_{-} * \big( \sin[c_{-} + d_{-} * x_{-}]^{\wedge} i_{-} \big)^{\wedge} q_{-} + C_{-} * \big( \sin[c_{-} + d_{-} * x_{-}]^{\wedge} i_{-} \big)^{\wedge} r_{-} \big) * \\ & \quad \big( a_{-} + b_{-} * \sin[c_{-} + d_{-} * x_{-}]^{\wedge} k_{-} \big)^{\wedge} n_{-} , x_{-} \text{Symbol} \big] := \\ & \quad \text{Int} \big[ \big( \sin[c_{+} d * x_{-}]^{\wedge} (-k) \big)^{\wedge} \big( p_{-} m_{+} 2 \big) * \big( C_{+} B * \sin[c_{+} d * x_{-}]^{\wedge} k_{+} A * \sin[c_{+} d * x_{-}]^{\wedge} \big( 2 * k \big) \big) * \big( a_{+} b * \sin[c_{+} d * x_{-}]^{\wedge} k_{+} \big)^{\wedge} n_{+} x_{-} \big] \\ & \quad \text{FreeQ} \big[ \big\{ a_{+} b_{+} c_{+} d_{+} A_{+} n_{+} n_{
```

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge m \frac{1}{2} \in \mathbb{Z} \bigwedge p \notin \mathbb{Z}$, then

$$\begin{split} & \int \left(\sin[c+d\,x]^k\right)^m \\ & \left(A \left(\sin[c+d\,x]^{-k}\right)^p + B \left(\sin[c+d\,x]^{-k}\right)^{p+1} + C \left(\sin[c+d\,x]^{-k}\right)^{p+2} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \to \\ & \sqrt{Csc[c+d\,x]} \sqrt{\sin[c+d\,x]} \\ & \int \left(\sin[c+d\,x]^{-k}\right)^{p-m+2} \left(C + B \sin[c+d\,x]^k + A \sin[c+d\,x]^{2\,k} \right) \left(a + b \sin[c+d\,x]^k \right)^n dx \end{split}$$

- Derivation: Piecewise constant extraction and algebraic normalization
- Basis: $\partial_z \left(\sqrt{\text{Sec}[z]} \sqrt{\text{Cos}[z]} \right) = 0$
- Rule: If $k^2 = 1 \bigwedge p \frac{1}{2} \in \mathbb{Z} \bigwedge 2 m \notin \mathbb{Z}$, then

$$\begin{split} & \int \left(\sin\left[c+d\,x\right]^k\right)^m \\ & \left(A\left(\sin\left[c+d\,x\right]^{-k}\right)^p + B\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+1} + C\left(\sin\left[c+d\,x\right]^{-k}\right)^{p+2}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx \ \longrightarrow \\ & \sqrt{Csc\left[c+d\,x\right]} \, \sqrt{\sin\left[c+d\,x\right]} \\ & \int \left(\sin\left[c+d\,x\right]^k\right)^{m-p-2} \left(C+B\sin\left[c+d\,x\right]^k + A\sin\left[c+d\,x\right]^{2\,k}\right) \left(a+b\sin\left[c+d\,x\right]^k\right)^n dx \end{split}$$

```
Int[u_,x_Symbol] :=
   Int[SubstInertSineForTrigOfLinear[u,x],x] /;
SimplifyFlag && FunctionOfTrigOfLinearQ[u,x],

Int[u_,x_Symbol] :=
   Int[SubstInertSineForTrigOfLinear[u,x],x] /;
FunctionOfTrigOfLinearQ[u,x]]
```

```
Int[u_,x_Symbol] :=
  Defer[Int[u,x]] /;
RecognizedFunctionOfTrigQ[u,x]
```