Mathematica 7 Test Results

For Integration Problems of the Form $\text{Csc}[x]^m \left(A + B\,\text{Csc}[x] + C\,\text{Csc}[x]^2\right) (a + b\,\text{Csc}[x])^n$

Problems of the form $\text{Csc}[x]^m \left(A + B\,\text{Csc}[x] + C\,\text{Csc}[x]^2\right) (a + b\,\text{Csc}[x])^n$ when $a^2 = b^2$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{\text{Csc}[x]^3}{a + a\,\text{Csc}[x]}, x, 3, 0\right\}$$

$$\frac{\text{ArcTanh}[\text{Cos}[x]]}{a} - \frac{2\,\text{Cot}[x]}{a} + \frac{\text{Cot}[x]\,\text{Csc}[x]}{a + a\,\text{Csc}[x]}$$

$$\frac{-\text{Cot}\left[\frac{x}{2}\right] + 2\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] - 2\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] + \frac{4\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]} + \text{Tan}\left[\frac{x}{2}\right]}{2\,a}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{\text{Csc}[x]^2}{a + a\,\text{Csc}[x]}, x, 3, 0\right\}$$

$$-\frac{\text{ArcTanh}[\text{Cos}[x]]}{a} + \frac{\text{Cot}[x]}{a + a\,\text{Csc}[x]}$$

$$\frac{-\text{Log}\left[2\,\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[2\,\text{Sin}\left[\frac{x}{2}\right]\right] - \frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}}{a}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{\text{Csc}[x]}{a + a\,\text{Csc}[x]}, x, 1, 0\right\}$$

$$-\frac{\text{Cot}[x]}{a + a\,\text{Csc}[x]}$$

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{a\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{(a + a\,\text{Csc}[x])^{3/2}}, x, 4, 0\right\}$$

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{a+a\,\text{Csc}[x]}}\right]}{a^{3/2}} + \frac{5\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{2}\,\sqrt{a+a\,\text{Csc}[x]}}\right]}{2\,\sqrt{2}\,a^{3/2}} + \frac{\text{Cot}[x]}{2\,(a + a\,\text{Csc}[x])^{3/2}}$$

$$-\left(\left(\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right)\left(5\sqrt{2}\ \mathrm{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]\sqrt{-1+\mathrm{Csc}[x]}\ \left(\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right)^2+\right.\right.$$
$$2\left(-1+\mathrm{Sin}[x]+2\,\mathrm{ArcTan}\left[\frac{-2+\sqrt{1+\mathrm{Csc}[x]}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]\sqrt{-1+\mathrm{Csc}[x]}\ (1+\mathrm{Sin}[x])-\right.$$
$$\left.\left.\left.2\,\mathrm{ArcTan}\left[\frac{2+\sqrt{1+\mathrm{Csc}[x]}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]\sqrt{-1+\mathrm{Csc}[x]}\ (1+\mathrm{Sin}[x])\right)\right)\right)\Big/\left(4\,a\sqrt{a\,(1+\mathrm{Csc}[x])}\ \left(\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right)(1+\mathrm{Sin}[x])\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{(a+a\,\mathrm{Csc}[x])^{5/2}},\ x,\ 5,\ 0\right\}$$

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\ \mathrm{Cot}[x]}{\sqrt{a+a\,\mathrm{Csc}[x]}}\right]}{a^{5/2}}+\frac{43\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\ \mathrm{Cot}[x]}{\sqrt{2}\ \sqrt{a+a\,\mathrm{Csc}[x]}}\right]}{16\sqrt{2}\ a^{5/2}}+\frac{\mathrm{Cot}[x]}{4\,(a+a\,\mathrm{Csc}[x])^{5/2}}+\frac{11\,\mathrm{Cot}[x]}{16\,a\,(a+a\,\mathrm{Csc}[x])^{3/2}}$$

$$\frac{1}{64\,(a\,(1+\mathrm{Csc}[x]))^{5/2}}(1+\mathrm{Csc}[x])^3\left(\frac{22\sqrt{2}\ \mathrm{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]\mathrm{Cos}[x]\sqrt{-1+\mathrm{Csc}[x]}\ (1+\mathrm{Csc}[x])\,\mathrm{Sin}[x]^2}{(-1+\mathrm{Sin}[x])\,(1+\mathrm{Sin}[x])^2}+\right.$$
$$\frac{64\left(\sqrt{2}\ \mathrm{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]+\mathrm{ArcTan}\left[\frac{-2+\sqrt{1+\mathrm{Csc}[x]}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]-\mathrm{ArcTan}\left[\frac{2+\sqrt{1+\mathrm{Csc}[x]}}{\sqrt{-1+\mathrm{Csc}[x]}}\right]\right)\mathrm{Cos}[x]\sqrt{-1+\mathrm{Csc}[x]}\ (1+\mathrm{Csc}[x])\,\mathrm{Sin}[x]^2}{(-1+\mathrm{Sin}[x])\,(1+\mathrm{Sin}[x])^2}+$$
$$\left.\frac{8\,\mathrm{Cos}\left[\frac{x}{2}\right]+7\,\mathrm{Cos}\left[\frac{3x}{2}\right]-15\,\mathrm{Cos}\left[\frac{5x}{2}\right]-8\,\mathrm{Sin}\left[\frac{x}{2}\right]+7\,\mathrm{Sin}\left[\frac{3x}{2}\right]+15\,\mathrm{Sin}\left[\frac{5x}{2}\right]}{\left(\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right)^5}\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(a-a\,\mathrm{Csc}[x])^{3/2},\ x,\ 3,\ 0\right\}$$

$$-2\,a^{3/2}\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\ \mathrm{Cot}[x]}{\sqrt{a-a\,\mathrm{Csc}[x]}}\right]-\frac{2\,a^2\,\mathrm{Cot}[x]}{\sqrt{a-a\,\mathrm{Csc}[x]}}$$

$$-\frac{2\,a^2\,\mathrm{Csc}[x]\left(\sqrt{a}\ \mathrm{ArcTan}\left[\frac{\sqrt{-a\,(1+\mathrm{Csc}[x])}}{\sqrt{a}}\right]+\sqrt{-a\,(1+\mathrm{Csc}[x])}\right)\left(\mathrm{Cos}\left[\frac{x}{2}\right]-\mathrm{Sin}\left[\frac{x}{2}\right]\right)\left(\mathrm{Cos}\left[\frac{x}{2}\right]+\mathrm{Sin}\left[\frac{x}{2}\right]\right)}{\sqrt{-a\,(1+\mathrm{Csc}[x])}\ \sqrt{a-a\,\mathrm{Csc}[x]}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{\sqrt{a-a\,\mathrm{Csc}[x]}},\ x,\ 3,\ 0\right\}$$

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\ \mathrm{Cot}[x]}{\sqrt{a-a\,\mathrm{Csc}[x]}}\right]}{\sqrt{a}}+\frac{\sqrt{2}\ \mathrm{ArcTan}\left[\frac{\sqrt{a}\ \mathrm{Cot}[x]}{\sqrt{2}\ \sqrt{a-a\,\mathrm{Csc}[x]}}\right]}{\sqrt{a}}$$

$$\frac{1}{\sqrt{a}\,\sqrt{a-a\,\text{Csc}[x]}}(-1+\text{Csc}[x])\,\sqrt{-a\,(1+\text{Csc}[x])}\,\left(\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-a\,(1+\text{Csc}[x])}}\right]-i\left(\text{Log}\left[-\frac{2\,a\left(-2\,i\,\sqrt{a}+\sqrt{-a\,(1+\text{Csc}[x])}+i\,\sqrt{a-a\,\text{Csc}[x]}\right)}{-\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]+\text{Log}\left[\frac{2\,i\,a\left(2\,\sqrt{a}+i\,\sqrt{-a\,(1+\text{Csc}[x])}+\sqrt{a-a\,\text{Csc}[x]}\right)}{\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]\right)\right)\text{Tan}[x]$$

Incorrect antiderivative:

$$\left\{\frac{1}{(a-a\,\text{Csc}[x])^{3/2}},\ x,\ 4,\ 0\right\}$$

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{a-a\,\text{Csc}[x]}}\right]}{a^{3/2}}+\frac{5\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{2}\,\sqrt{a-a\,\text{Csc}[x]}}\right]}{2\,\sqrt{2}\,a^{3/2}}+\frac{\text{Cot}[x]}{2\,(a-a\,\text{Csc}[x])^{3/2}}$$

$$\frac{1}{4\,a^2\,\sqrt{a-a\,\text{Csc}[x]}\,\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^3}\left(\frac{1}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}\sqrt{a}\,\sqrt{-a\,(1+\text{Csc}[x])}\,\left(4\,\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-a\,(1+\text{Csc}[x])}}\right]+i\left(\sqrt{2}\,\text{Log}\left[\frac{2\,i\,\sqrt{2}\,\sqrt{a}-2\,\sqrt{-a\,(1+\text{Csc}[x])}}{\sqrt{a-a\,\text{Csc}[x]}}\right]-4\left(\text{Log}\left[-\frac{2\,a\left(-2\,i\,\sqrt{a}+\sqrt{-a\,(1+\text{Csc}[x])}+i\,\sqrt{a-a\,\text{Csc}[x]}\right)}{-\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]+\text{Log}\left[\frac{2\,i\,a\left(2\,\sqrt{a}+i\,\sqrt{-a\,(1+\text{Csc}[x])}+\sqrt{a-a\,\text{Csc}[x]}\right)}{\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]\right)\right)\right)\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^4+2\,a\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)(-1+\text{Sin}[x])\right)$$

Incorrect antiderivative:

$$\left\{\frac{1}{(a-a\,\text{Csc}[x])^{5/2}},\ x,\ 5,\ 0\right\}$$

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{a-a\,\text{Csc}[x]}}\right]}{a^{5/2}}+\frac{43\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{2}\,\sqrt{a-a\,\text{Csc}[x]}}\right]}{16\,\sqrt{2}\,a^{5/2}}+\frac{\text{Cot}[x]}{4\,(a-a\,\text{Csc}[x])^{5/2}}+\frac{11\,\text{Cot}[x]}{16\,a\,(a-a\,\text{Csc}[x])^{3/2}}$$

$$\frac{1}{32}\left(\left(\sqrt{-a\,(1+\text{Csc}[x])}\left(32\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-a\,(1+\text{Csc}[x])}}\right]+\right.\right.\right.$$

$$i\left(11\sqrt{2}\,\text{Log}\left[\frac{2\,i\,\sqrt{2}\,\sqrt{a}-2\sqrt{-a\,(1+\text{Csc}[x])}}{\sqrt{a-a\,\text{Csc}[x]}}\right]-32\left(\text{Log}\left[-\frac{2\,a\left(-2\,i\,\sqrt{a}+\sqrt{-a\,(1+\text{Csc}[x])}+i\,\sqrt{a-a\,\text{Csc}[x]}\right)}{-\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]+\right.\right.$$

$$\left.\left.\left.\text{Log}\left[\frac{2\,i\,a\left(2\sqrt{a}+i\,\sqrt{-a\,(1+\text{Csc}[x])}+\sqrt{a-a\,\text{Csc}[x]}\right)}{\sqrt{a}+\sqrt{a-a\,\text{Csc}[x]}}\right]\right)\right)\right)$$

$$\left.\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)\right)\Big/\left(a^{5/2}\sqrt{a-a\,\text{Csc}[x]}\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)\right)+$$

$$\left.\frac{\sqrt{a-a\,\text{Csc}[x]}\left(-8\,\text{Cos}\left[\frac{x}{2}\right]-7\,\text{Cos}\left[\frac{3x}{2}\right]+15\,\text{Cos}\left[\frac{5x}{2}\right]-8\,\text{Sin}\left[\frac{x}{2}\right]+7\,\text{Sin}\left[\frac{3x}{2}\right]+15\,\text{Sin}\left[\frac{5x}{2}\right]\right)}{2\,a^3\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^5}\right)$$

Problems of the form $\mathrm{Csc}[x]^m \left(A + B\,\mathrm{Csc}[x] + C\,\mathrm{Csc}[x]^2\right) (a + b\,\mathrm{Csc}[x])^n$ when $a^2 \neq b^2$

Valid but unnecessarily complicated antiderivative:

$$\left\{(a+b\,\mathrm{Csc}[c+d\,x])^3,\ x,\ 4,\ 0\right\}$$

$$a^3\,x-\frac{b\left(6\,a^2+b^2\right)\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{2\,d}-\frac{5\,a\,b^2\,\mathrm{Cot}[c+d\,x]}{2\,d}-\frac{b^2\,\mathrm{Cot}[c+d\,x]\,(a+b\,\mathrm{Csc}[c+d\,x])}{2\,d}$$

$$\frac{1}{8\,d}\left(8\,a^3\,c+8\,a^3\,d\,x-12\,a\,b^2\,\mathrm{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]-b^3\,\mathrm{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2-24\,a^2\,b\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]-4\,b^3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+\right.$$
$$\left.24\,a^2\,b\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+4\,b^3\,\mathrm{Log}\left[\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+b^3\,\mathrm{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2+12\,a\,b^2\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(a+b\,\mathrm{Csc}[c+d\,x])^2,\ x,\ 2,\ 0\right\}$$

$$a^2\,x-\frac{2\,a\,b\,\mathrm{ArcTanh}[\mathrm{Cos}[c+d\,x]]}{d}-\frac{b^2\,\mathrm{Cot}[c+d\,x]}{d}$$

$$\frac{-b^2\,\mathrm{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]+2\,a\left(a\,c+a\,d\,x-2\,b\,\mathrm{Log}\left[2\,\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]+2\,b\,\mathrm{Log}\left[2\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\right)+b^2\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]}{2\,d}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{3+5\,\mathrm{Csc}[c+d\,x]},\ x,\ 2,\ 0\right\}$$

$$-\frac{x}{12}-\frac{5\,\mathrm{ArcTan}\left[\frac{\mathrm{Cos}[c+d\,x]}{3+\mathrm{Sin}[c+d\,x]}\right]}{6\,d}$$

$$\frac{2\,(c+d\,x)-5\,\mathrm{ArcTan}\left[\frac{2\left(\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)}{\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]}\right]}{6\,d}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{5+3\,\mathrm{Csc}[c+d\,x]},\ x,\ 2,\ 0\right\}$$

$$\frac{x}{5}+\frac{3\,\mathrm{ArcTanh}\left[\frac{1}{4}\left(5+3\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right]}{10\,d}$$

$$\frac{4\,(c+d\,x)+3\,\mathrm{Log}\left[3\,\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]-3\,\mathrm{Log}\left[\mathrm{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+3\,\mathrm{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{20\,d}$$