# **Trig Integration Recurrence Equations**

## **Sine Integration Recurrence Equations**

 $\int \sin[c+dx]^{m} (A+B\sin[c+dx]) (a+b\sin[c+dx])^{n} dx \text{ when } a^{2}-b^{2}=0$ 

• Symmetric sine recurrence 1a: If  $a^2 - b^2 = 0$ , then

 $\int Sin[c+dx]^{m+1} ((Ab+aB) (m+1) - Ab (n-1) + (aAn + (aA+bB) (m+1)) Sin[c+dx]) (a+bSin[c+dx])^{n-1} dx$ 

• Symmetric sine recurrence 1b: If  $a^2 - b^2 = 0$ , then

$$d (m+n+1) \int \sin[c+dx]^{m} (A+B\sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ -bB\cos[c+dx] \sin[c+dx]^{m+1} (a+b\sin[c+dx])^{n-1} + \\ d \int \sin[c+dx]^{m} (aAn+(aA+bB) (m+1) + (Ab+aBn+(Ab+aB) (m+n)) \sin[c+dx]) (a+b\sin[c+dx])^{n-1} dx$$

• Symmetric sine recurrence 2a: If  $a^2 - b^2 = 0$ , then

$$a^{2} d (2n+1) \int \sin[c+dx]^{m} (A+B \sin[c+dx]) (a+b \sin[c+dx])^{n} dx = \\ a (Ab-aB) \cos[c+dx] \sin[c+dx]^{m} (a+b \sin[c+dx])^{n} + \\ d \int \sin[c+dx]^{m-1} (-m (Ab-aB) + (bBn+aA (n+1) + m (aA-bB)) \sin[c+dx]) (a+b \sin[c+dx])^{n+1} dx$$

• Symmetric sine recurrence 2b: If  $a^2 - b^2 = 0$ , then

$$b^{2} d (2n+1) \int \sin[c+dx]^{m} (A+B\sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ -b (Ab-aB) \cos[c+dx] \sin[c+dx]^{m+1} (a+b\sin[c+dx])^{n} + \\ d \int \sin[c+dx]^{m} (aA (2n+1) + (aA-bB) (m+1) - (Ab-aB) (m+n+2) \sin[c+dx]) (a+b\sin[c+dx])^{n+1} dx$$

■ Symmetric sine recurrence 3a: If  $a^2 - b^2 = 0$ , then

$$ad (m+n+1) \int \sin[c+dx]^{m} (A+B\sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ -aB\cos[c+dx] \sin[c+dx]^{m} (a+b\sin[c+dx])^{n} + \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]) (a+b\sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx])^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+1)) \sin[c+dx]^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+(bBn+aA(m+n+1)) \sin[c+dx]^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+1)) \sin[c+dx]^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+1)) \sin[c+dx]^{n} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+1)) \sin[c+dx]^{m-1} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+1)) \sin[c+dx]^{m-1} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+1)) \sin[c+dx]^{m-1} dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+n+1)) dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+n+1) dx = \\ d \int \sin[c+dx]^{m-1} (aBm+aA(m+n+n+n+n+$$

• Symmetric sine recurrence 3b: If  $a^2 - b^2 = 0$ , then

$$ad (m+1) \int Sin[c+dx]^{m} (A+BSin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ aACos[c+dx] Sin[c+dx]^{m+1} (a+bSin[c+dx])^{n} + \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+1) - bAn + aA (m+n+2) Sin[c+dx]) (a+bSin[c+dx])^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+n+2) Sin[c+dx]^{n} dx = \\ d\int Sin[c+dx]^{m+1} (aB (m+n+2) Sin[c+dx]^{n} dx = \\ dx =$$

$$\int \sin[c + dx]^{m} (A + B\sin[c + dx] + C\sin[c + dx]^{2})$$

$$(a + b\sin[c + dx])^{n} dx \text{ when } a^{2} - b^{2} \neq 0$$

■ Sine recurrence 1a:

$$d (m+1) \int \sin[c+dx]^{m} (A+B\sin[c+dx] + C\sin[c+dx]^{2}) (a+b\sin[c+dx])^{n} dx = \\ A \cos[c+dx] \sin[c+dx]^{m+1} (a+b\sin[c+dx])^{n} + \\ d \int \sin[c+dx]^{m+1} (aB (m+1) - bAn + (aA+bB+aC) (m+1)) \sin[c+dx] + \\ b (A (n+1) + (A+C) (m+1)) \sin[c+dx]^{2}) (a+b\sin[c+dx])^{n-1} dx$$

■ Sine recurrence 1b:

$$d (m+n+2) \int Sin[c+dx]^{m} (A+BSin[c+dx]+CSin[c+dx]^{2}) (a+bSin[c+dx])^{n} dx = \\ -CCos[c+dx] Sin[c+dx]^{m+1} (a+bSin[c+dx])^{n} + \\ d \int Sin[c+dx]^{m} (a (A (n+1)+(A+C) (m+1)) + (Ab+aB+(Ab+aB+bC) (m+n+1)) Sin[c+dx] + \\ (aCn+bB (m+n+2)) Sin[c+dx]^{2}) (a+bSin[c+dx])^{n-1} dx$$

■ Sine recurrence 2a:

$$bd (n+1) (a^{2}-b^{2}) \int Sin[c+dx]^{m} (A+BSin[c+dx]+CSin[c+dx]^{2}) (a+bSin[c+dx])^{n} dx = \\ - (Ab^{2}-abB+a^{2}C) Cos[c+dx] Sin[c+dx]^{m} (a+bSin[c+dx])^{n+1} + \\ d \int Sin[c+dx]^{m-1} (m (Ab^{2}-abB+a^{2}C)+b (aA-bB+aC) (n+1) Sin[c+dx] - \\ (b (Ab-aB+bC) (n+1)+(Ab^{2}-abB+a^{2}C) (m+1)) Sin[c+dx]^{2}) (a+bSin[c+dx])^{n+1} dx$$

■ Sine recurrence 2b:

$$\begin{array}{l} a\,d\,\left(n+1\right)\,\left(a^{2}-b^{2}\right)\,\int\!\sin\left[c+d\,x\right]^{m}\,\left(A+B\,\sin\left[c+d\,x\right]+C\,\sin\left[c+d\,x\right]^{2}\right)\,\left(a+b\,\sin\left[c+d\,x\right]\right)^{n}\,dx \; = \\ & \left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,\cos\left[c+d\,x\right]\,\sin\left[c+d\,x\right]^{m+1}\,\left(a+b\,\sin\left[c+d\,x\right]\right)^{n+1}+\\ & d\,\int\!\sin\left[c+d\,x\right]^{m}\,\left(A\,\left(a^{2}-b^{2}\right)\,\left(n+1\right)-\left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,\left(m+1\right)-\\ a\,\left(A\,b-a\,B+b\,C\right)\,\left(n+1\right)\,\sin\left[c+d\,x\right]+\left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,\left(m+n+3\right)\,\sin\left[c+d\,x\right]^{2}\right)\,\left(a+b\,\sin\left[c+d\,x\right]\right)^{n+1}\,dx \end{array}$$

■ Sine recurrence 3a:

$$bd (m+n+2) \int Sin[c+dx]^{m} (A+B Sin[c+dx] + C Sin[c+dx]^{2}) (a+b Sin[c+dx])^{n} dx = \\ -C Cos[c+dx] Sin[c+dx]^{m} (a+b Sin[c+dx])^{n+1} + \\ d \int Sin[c+dx]^{m-1} (aCm+b (A+(A+C) (m+n+1)) Sin[c+dx] + (bB (n+1) + (bB-aC) (m+1)) Sin[c+dx]^{2}) \\ (a+b Sin[c+dx])^{n} dx$$

■ Sine recurrence 3b:

$$ad \ (m+1) \ \int Sin[c+dx]^m \left(A+BSin[c+dx]+CSin[c+dx]^2\right) \ (a+bSin[c+dx])^n \ dx = \\ ACos[c+dx] \ Sin[c+dx]^{m+1} \ (a+bSin[c+dx])^{n+1} + \\ d \\ \int Sin[c+dx]^{m+1} \left((aB-bA) \ (m+1) - bA \ (n+1) + a \ (A+(A+C) \ (m+1)) \ Sin[c+dx] + bA \ (m+n+3) \ Sin[c+dx]^2\right)$$

 $(a + b Sin[c + dx])^n dx$ 

# **Cosecant Integration Recurrence Equations**

 $\int Csc[c+dx]^m (A+BCsc[c+dx]) (a+bCsc[c+dx])^n dx \text{ When } a^2-b^2=0$ 

• Symmetric cosecant recurrence 1a: If  $a^2 - b^2 = 0$ , then

• Symmetric cosecant recurrence 1b: If  $a^2 - b^2 = 0$ , then

$$d \ (m+n) \ \int Csc[c+dx]^m \ (A+BCsc[c+dx]) \ (a+bCsc[c+dx])^n \ dx = \\ -bBCos[c+dx] \ Csc[c+dx]^{m+1} \ (a+bCsc[c+dx])^{n-1} + \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]) \ (a+bCsc[c+dx])^{n-1} \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]) \ (a+bCsc[c+dx])^{n-1} \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]) \ (a+bCsc[c+dx])^{n-1} \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx])^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ (aAn+(aA+bB)m+(Ab+aBn+(Ab+aBn+(Ab+aB)(m+n-1)) \ Csc[c+dx]^n \ dx = \\ d \ \int Csc[c+dx]^m \ dx = \\ d \ \int Csc[c+dx]^n \ dx = \\ dx = \\ dx = \\ dx$$

• Symmetric cosecant recurrence 2a: If  $a^2 - b^2 = 0$ , then

$$a^{2} d (2n+1) \int Csc[c+dx]^{m} (A+BCsc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ a (Ab-aB) Cos[c+dx] Csc[c+dx]^{m} (a+bCsc[c+dx])^{n} + \\ d \int Csc[c+dx]^{m-1} \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (bBn+aA (n+1) + (aA-bB) (m-1)) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1)) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (aA-bB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (Ab-aB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (Ab-aB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (Ab-aB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (Ab-aB) (m-1) Csc[c+dx])^{n+1} dx = \\ (-(Ab-aB) (m-1) + (Ab-aB) (m-1) Csc[c+dx]$$

■ Symmetric cosecant recurrence 2b: If  $a^2 - b^2 = 0$ , then

$$b^{2} d (2n+1) \int Csc[c+dx]^{m} (A+BCsc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ -b (Ab-aB) Cos[c+dx] Csc[c+dx]^{m+1} (a+bCsc[c+dx])^{n} + \\ d \int Csc[c+dx]^{m} (aA (2n+1) + (aA-bB) m - (Ab-aB) (m+n+1) Csc[c+dx]) (a+bCsc[c+dx])^{n+1} dx$$

■ Symmetric cosecant recurrence 3a: If  $a^2 - b^2 = 0$ , then

$$ad (m+n) \int Csc[c+dx]^{m} (A+BCsc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ -aBCos[c+dx] Csc[c+dx]^{m} (a+bCsc[c+dx])^{n} + \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]) (a+bCsc[c+dx])^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int Csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[c+dx]^{n} dx = \\ d \int csc[c+dx]^{m-1} (aB (m-1) + (bBn+aA (m+n)) Csc[$$

■ Symmetric cosecant recurrence 3b: If  $a^2 - b^2 = 0$ , then

$$a \, d \, m \, \int C s c \, [c + d \, x]^m \, \left( A + B \, C s c \, [c + d \, x] \right) \, \left( a + b \, C s c \, [c + d \, x] \right)^n \, dx = \\ a \, A \, C o s \, [c + d \, x] \, C s c \, [c + d \, x]^{m+1} \, \left( a + b \, C s c \, [c + d \, x] \right)^n + \\ d \, \int C s c \, [c + d \, x]^{m+1} \, \left( a \, B \, m - b \, A \, n + a \, A \, \left( m + n + 1 \right) \, C s c \, [c + d \, x] \right) \, \left( a + b \, C s c \, [c + d \, x] \right)^n \, dx$$

$$\int Csc[c+dx]^{m} (A+BCsc[c+dx]+CCsc[c+dx]^{2})$$

$$(a+bCsc[c+dx])^{n} dx \text{ when } a^{2}-b^{2}\neq 0$$

### ■ Cosecant recurrence 1a:

$$d\,m\,\int Csc\,[c+d\,x]^m\,\left(A+B\,Csc\,[c+d\,x]+C\,Csc\,[c+d\,x]^2\right)\,\left(a+b\,Csc\,[c+d\,x]\right)^n\,dx \;=\; \\ A\,Cos\,[c+d\,x]\,\,Csc\,[c+d\,x]^{m+1}\,\left(a+b\,Csc\,[c+d\,x]\right)^n+\\ d\,\int Csc\,[c+d\,x]^{m+1}\,\left(a\,B\,m-b\,A\,n+\,(a\,A+a\,C+b\,B)\,m\right)\,Csc\,[c+d\,x]+b\,\left(A\,(n+1)+\,(A+C)\,m\right)\,Csc\,[c+d\,x]^2\right)\\ \left(a+b\,Csc\,[c+d\,x]\right)^{n-1}\,dx$$

#### **■** Cosecant recurrence 1b:

$$\begin{split} d\ (m+n+1)\ \int & Csc \, [c+d\,x]^m \ \left(A+B\, Csc \, [c+d\,x]+C\, Csc \, [c+d\,x]^2\right) \ (a+b\, Csc \, [c+d\,x])^n \, dx \ = \\ & -C\, Cos \, [c+d\,x]\, \, Csc \, [c+d\,x]^{m+1} \ (a+b\, Csc \, [c+d\,x])^n + \\ d \int & Csc \, [c+d\,x]^m \ \left(a\ (A\ (n+1)+(A+C)\ m)+(b\, A+a\, B+(b\, A+a\, B+b\, C)\ (m+n)\right) \, Csc \, [c+d\,x] + \\ & \left(a\, C\, n+b\, B\, (m+n+1)\right) \, Csc \, [c+d\,x]^2\right) \ (a+b\, Csc \, [c+d\,x])^{n-1} \, dx \end{split}$$

#### ■ Cosecant recurrence 2a:

$$bd (n+1) (a^{2}-b^{2}) \int Csc[c+dx]^{m} (A+BCsc[c+dx]+CCsc[c+dx]^{2}) (a+bCsc[c+dx])^{n} dx = \\ - (Ab^{2}-abB+a^{2}C) Cos[c+dx] Csc[c+dx]^{m} (a+bCsc[c+dx])^{n+1} + \\ d \int Csc[c+dx]^{m-1} ((Ab^{2}-abB+a^{2}C) (m-1)+b (aA-bB+aC) (n+1) Csc[c+dx] - \\ ((Ab^{2}-abB+b^{2}C) (n+1)+(Ab^{2}-abB+a^{2}C) m) Csc[c+dx]^{2}) (a+bCsc[c+dx])^{n+1} dx$$

### ■ Cosecant recurrence 2b:

$$\begin{array}{l} a\,d\,\left(n+1\right)\,\left(a^{2}-b^{2}\right)\,\int\!Csc\left[c+d\,x\right]^{m}\,\left(A+B\,Csc\left[c+d\,x\right]+C\,Csc\left[c+d\,x\right]^{2}\right)\,\left(a+b\,Csc\left[c+d\,x\right]\right)^{n}\,dx \; = \\ & \left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,Cos\left[c+d\,x\right]\,Csc\left[c+d\,x\right]^{m+1}\,\left(a+b\,Csc\left[c+d\,x\right]\right)^{n+1}+\\ & d\,\int\!Csc\left[c+d\,x\right]^{m}\,\left(A\,\left(a^{2}-b^{2}\right)\,\left(n+1\right)-\left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,m-1 \\ & a\,\left(b\,A-a\,B+b\,C\right)\,\left(n+1\right)\,Csc\left[c+d\,x\right]+\left(A\,b^{2}-a\,b\,B+a^{2}\,C\right)\,\left(m+n+2\right)\,Csc\left[c+d\,x\right]^{2}\right)\,\left(a+b\,Csc\left[c+d\,x\right]\right)^{n+1}\,dx \end{array}$$

### ■ Cosecant recurrence 3a:

$$bd (m+n+1) \int Csc[c+dx]^{m} \left(A+BCsc[c+dx]+CCsc[c+dx]^{2}\right) (a+bCsc[c+dx])^{n} dx = \\ -CCos[c+dx] Csc[c+dx]^{m} (a+bCsc[c+dx])^{n+1} + \\ d \int Csc[c+dx]^{m-1} \left(aC (m-1)+b (A+(A+C) (m+n)) Csc[c+dx]+(bB (n+1)+(bB-aC) m) Csc[c+dx]^{2}\right) \\ (a+bCsc[c+dx])^{n} dx$$

#### ■ Cosecant recurrence 3b:

$$a\,d\,m\int Csc\,[c+d\,x]^{m}\,\left(A+B\,Csc\,[c+d\,x]+C\,Csc\,[c+d\,x]^{2}\right)\,\left(a+b\,Csc\,[c+d\,x]\right)^{n}\,d\,x=\\ A\,Cos\,[c+d\,x]\,\,Csc\,[c+d\,x]^{m+1}\,\left(a+b\,Csc\,[c+d\,x]\right)^{n+1}+\\ d\,\int Csc\,[c+d\,x]^{m+1}\\ \left(\left(a\,B-b\,A\right)\,m-b\,A\,\left(n+1\right)+a\,\left(A+\left(A+C\right)\,m\right)\,Csc\,[c+d\,x]+b\,A\,\left(m+n+2\right)\,Csc\,[c+d\,x]^{2}\right)\,\left(a+b\,Csc\,[c+d\,x]\right)^{n}\,d\,x=\\ \left((a\,B-b\,A)\,m-b\,A\,\left(n+1\right)+a\,\left(A+\left(A+C\right)\,m\right)\,Csc\,[c+d\,x]+b\,A\,\left(m+n+2\right)\,Csc\,[c+d\,x]^{2}\right)\,\left(a+b\,Csc\,[c+d\,x]\right)^{n}\,d\,x=\\ \left((a\,B-b\,A)\,m-b\,A\,\left(n+1\right)+a\,\left(A+\left(A+C\right)\,m\right)\,Csc\,[c+d\,x]+b\,A\,\left(m+n+2\right)\,Csc\,[c+d\,x]^{2}\right)$$

# **Tangent Integration Recurrence Equations**

$$\int Tan[c+dx]^m (A+BTan[c+dx]) (a+bTan[c+dx])^n dx \text{ when } a^2+b^2=0$$

• Symmetric tangent recurrence 1a (10): If  $a^2 + b^2 = 0$ , then

$$d (m+1) \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx == \\ A a Tan[c+dx]^{m+1} (a+bTan[c+dx])^{n-1} - \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) (a+bTan[c+dx])^{n-1} dx = \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]) dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx] dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx] dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx] dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx] dx == \\ d \int Tan[c+dx]^{m+1} (Ab (n-1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Ab+Ba) (m+1) + (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Aa (m+n) - Bb (m+1)) Tan[c+dx]^{m+1} dx == \\ d \int Tan[c+dx]^{m+1} (Ab (m+1) - (Aa (m+n) - Bb (m+n)) Tan[c+$$

■ Symmetric tangent recurrence 1b (9): If  $a^2 + b^2 = 0$ , then

$$d (m+n) \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx = \\ BbTan[c+dx]^{m+1} (a+bTan[c+dx])^{n-1} + \\ d \int Tan[c+dx]^{m} (Aa (n+m) - Bb (m+1) + (Ba (n-1) + (Ab+Ba) (m+n)) Tan[c+dx]) (a+bTan[c+dx])^{n-1} dx$$

■ Symmetric tangent recurrence 2a(7): If  $a^2 + b^2 = 0$ , then

$$2 a^{2} n d \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx = \\ -a (Ab-Ba) Tan[c+dx]^{m} (a+bTan[c+dx])^{n} + \\ d \int Tan[c+dx]^{m-1} ((Ab-Ba) m + (Bb (m-n) + Aa (m+n)) Tan[c+dx]) (a+bTan[c+dx])^{n+1} dx$$

■ Symmetric tangent recurrence 2b (12): If  $a^2 + b^2 = 0$ , then

$$2 a^{2} n d \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx = \\ -a (aA+bB) Tan[c+dx]^{1+m} (a+bTan[c+dx])^{n} + \\ d \int Tan[c+dx]^{m} (bB(m+1)+aA(m+2n+1)-(Ab-aB)(m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n+1} dx$$

■ Symmetric tangent recurrence 3a(8): If  $a^2 + b^2 = 0$ , then

$$ad (m+n) \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx == \\ aBTan[c+dx]^{m} (a+bTan[c+dx])^{n} + \\ d\int Tan[c+dx]^{m-1} (-(Bam) + (Aam + (Aa-Bb) n) Tan[c+dx]) (a+bTan[c+dx])^{n} dx$$

• Symmetric tangent recurrence 3b (11): If  $a^2 + b^2 = 0$ , then

$$ad (m+1) \int Tan[c+dx]^{m} (a+bTan[c+dx])^{n} (A+BTan[c+dx]) dx = \\ aA Tan[c+dx]^{m+1} (a+bTan[c+dx])^{n} - \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]) (a+bTan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx])^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+1)+Aa (m+n+1) Tan[c+dx]^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+n+1)+Aa (m+n+1) Tan[c+dx]^{n} dx = \\ d\int Tan[c+dx]^{m+1} (Abn-Ba (m+n+1)+Aa (m+n+1) Tan[c+dx]^{n} dx = \\ d$$

$$\int Tan[c+dx]^{m} (A+BTan[c+dx]+CTan[c+dx]^{2})$$

$$(a+bTan[c+dx])^{n} dx \text{ when } a^{2}+b^{2} \neq 0$$

■ Tangent recurrence 1a:

$$d (m+1) \int Tan[c+dx]^{m} (A+BTan[c+dx]+CTan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ A Tan[c+dx]^{m+1} (a+bTan[c+dx])^{n} + \\ d \int Tan[c+dx]^{m+1} (aB(m+1)-Abn+(bB-a(A-C)) (m+1) Tan[c+dx] + \\ b (C(m+1)-A(m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n-1} dx$$

■ Tangent recurrence 1b:

$$d \ (m+n+1) \int Tan[c+dx]^m \left(A+B Tan[c+dx]+C Tan[c+dx]^2\right) \ (a+b Tan[c+dx])^n \ dx == \\ C \ Tan[c+dx]^{m+1} \ (a+b Tan[c+dx])^n + \\ d \int Tan[c+dx]^m \left( (Aa \ (m+n+1)-C \ (m+1) \ a) + (aB+b \ (A-C)) \ (m+n+1) \ Tan[c+dx] + (aCn+bB \ (m+n+1)) \ Tan[c+dx]^2 \right) \ (a+b Tan[c+dx])^{n-1} \ dx$$

■ Tangent recurrence 2a:

$$bd (n+1) (a^{2}+b^{2}) \int Tan[c+dx]^{m} (A+BTan[c+dx]+CTan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (Ab^{2}-abB+a^{2}C) Tan[c+dx]^{m} (a+bTan[c+dx])^{n+1} + \\ d \int Tan[c+dx]^{m-1} (-(Ab^{2}-abB+a^{2}C)m+b(bB+a(A-C))(n+1) Tan[c+dx] - \\ (b(Ab-aB)(m+n+1)-C(-a^{2}m+b^{2}(n+1))) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n+1} dx$$

■ Tangent recurrence 2b:

$$a\,d\,\left(n+1\right)\,\left(a^2+b^2\right)\,\int\! Tan[c+d\,x]^m\,\left(A+B\,Tan[c+d\,x]+C\,Tan[c+d\,x]^2\right)\,\left(a+b\,Tan[c+d\,x]\right)^n\,dx = \\ -\left(A\,b^2-a\,b\,B+a^2\,C\right)\,Tan[c+d\,x]^{m+1}\,\left(a+b\,Tan[c+d\,x]\right)^{n+1}+ \\ d\,\int\! Tan[c+d\,x]^m\,\left(-a\,\left(b\,B-a\,C\right)\,\left(m+1\right)+A\,\left(a^2\,\left(n+1\right)+b^2\,\left(m+n+2\right)\right)+ \\ a\,\left(a\,B-b\,\left(A-C\right)\right)\,\left(n+1\right)\,Tan[c+d\,x]+\left(A\,b^2-a\,b\,B+a^2\,C\right)\,\left(m+n+2\right)\,Tan[c+d\,x]^2\right)\,\left(a+b\,Tan[c+d\,x]\right)^{n+1}\,dx$$

■ Tangent recurrence 3a:

$$bd (m+n+1) \int Tan[c+dx]^{m} (A+BTan[c+dx]+CTan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ CTan[c+dx]^{m} (a+bTan[c+dx])^{n+1} - \\ d \int Tan[c+dx]^{m-1} (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx])^{n} dx = \\ (aCm-b(A-C) (m+n+1) Tan[c+dx] + (aCm-bB (m+n+1)) Tan[c+dx]^{2}) (a+bTan[c+dx]^{2}) (a+bTan[c+dx]^{$$

■ Tangent recurrence 3b:

$$a\,d\,\left(m+1\right)\,\int\! Tan\big[c+d\,x\big]^{m}\,\left(A+B\,Tan\big[c+d\,x\big]+C\,Tan\big[c+d\,x\big]^{2}\right)\,\left(a+b\,Tan\big[c+d\,x\big]\right)^{n}\,dx = \\ A\,Tan\big[c+d\,x\big]^{m+1}\,\left(a+b\,Tan\big[c+d\,x\big]\right)^{n+1}+\\ d\,\int\! Tan\big[c+d\,x\big]^{m+1} \\ \left(a\,B\,\left(m+1\right)-A\,b\,\left(m+n+2\right)-a\,\left(A-C\right)\,\left(m+1\right)\,Tan\big[c+d\,x\big]-A\,b\,\left(m+n+2\right)\,Tan\big[c+d\,x\big]^{2}\,\right)\,\left(a+b\,Tan\big[c+d\,x\big]\right)^{n}\,dx = \\ \left(a\,B\,\left(m+1\right)-A\,b\,\left(m+n+2\right)-a\,\left(A-C\right)\,\left(m+1\right)\,Tan\big[c+d\,x\big]-A\,b\,\left(m+n+2\right)\,Tan\big[c+d\,x\big]^{2}\,\right)\,\left(a+b\,Tan\big[c+d\,x\big]\right)^{n}\,dx = \\ \left(a\,B\,\left(m+1\right)-A\,b\,\left(m+n+2\right)-a\,\left(A-C\right)\,\left(m+1\right)\,Tan\big[c+d\,x\big]-A\,b\,\left(m+n+2\right)\,Tan\big[c+d\,x\big]^{2}\,\right)$$