

# Mathematica 7 Test Results

## For Integration Problems Involving Inverse Trig Functions

### Problems involving inverse sines

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcSin}[a + b x]^2}{x^2}, x, 10, 0 \right\}$$

$$- \frac{\text{ArcSin}[a + b x]^2}{x} + \frac{2 i b \text{ArcSin}[a + b x] \text{Log}\left[1 + \frac{i e^{i \text{ArcSin}[a + b x]}}{a - \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} -$$

$$\frac{2 i b \text{ArcSin}[a + b x] \text{Log}\left[1 + \frac{i e^{i \text{ArcSin}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} + \frac{2 b \text{PolyLog}\left[2, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a - \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} - \frac{2 b \text{PolyLog}\left[2, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}}$$

$$\begin{aligned}
& b \left( -\frac{\text{ArcSin}[a + b x]^2}{b x} + \right. \\
& 2 \left( \frac{\pi \text{ArcTan}\left[\frac{1-a \text{Tan}\left[\frac{1}{2} \text{ArcSin}[a + b x]\right]}{\sqrt{-1+a^2}}\right]}{\sqrt{-1+a^2}} + \frac{1}{\sqrt{1-a^2}} \left( -2 \text{ArcCos}[a] \text{ArcTanh}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] - \right. \right. \\
& (\pi - 2 \text{ArcSin}[a + b x]) \text{ArcTanh}\left[\frac{(-1+a) \text{Tan}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] + \left. \left( \text{ArcCos}[a] - \right. \right. \\
& 2 i \left( \text{ArcTanh}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] + \text{ArcTanh}\left[\frac{(-1+a) \text{Tan}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] \right) \left. \right) \\
& \text{Log}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{1-a^2} e^{-\frac{1}{2} i \text{ArcSin}[a + b x]}}{\sqrt{b x}}\right] + \left( \text{ArcCos}[a] + 2 i \text{ArcTanh}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] + \right. \\
& 2 i \text{ArcTanh}\left[\frac{(-1+a) \text{Tan}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] \left. \right) \text{Log}\left[\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sqrt{1-a^2} e^{\frac{1}{2} i \text{ArcSin}[a + b x]}}{\sqrt{b x}}\right] - \\
& \left( \text{ArcCos}[a] - 2 i \text{ArcTanh}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] \right) \\
& \text{Log}\left[-\frac{(-1+a) \left(i + i a + \sqrt{1-a^2}\right) \left(-i + \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]\right)}{1-a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]} \right] - \\
& \left( \text{ArcCos}[a] + 2 i \text{ArcTanh}\left[\frac{(1+a) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]}{\sqrt{1-a^2}}\right] \right) \\
& \text{Log}\left[-\frac{(-1+a) \left(-i - i a + \sqrt{1-a^2}\right) \left(i + \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]\right)}{1-a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]} \right] + \\
& i \left( -\text{PolyLog}\left[2, \frac{\left(a - i \sqrt{1-a^2}\right) \left(-1+a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]\right)}{1-a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]} \right] + \right. \\
& \left. \text{PolyLog}\left[2, \frac{\left(a + i \sqrt{1-a^2}\right) \left(-1+a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]\right)}{1-a + \sqrt{1-a^2} \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcSin}[a + b x])\right]} \right] \right) \left. \right) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcSin}[a + b x]^2}{x^3}, x, 15, 0 \right\}$$

$$\begin{aligned}
& -\frac{b\sqrt{1-(a+bx)^2}\operatorname{ArcSin}[a+bx]}{(1-a^2)x} - \frac{\operatorname{ArcSin}[a+bx]^2}{2x^2} - \frac{iab^2\operatorname{ArcSin}[a+bx]\operatorname{Log}\left[1+\frac{ie^{i\operatorname{ArcSin}[a+bx]}}{a-\sqrt{-1+a^2}}\right]}{(-1+a^2)^{3/2}} + \\
& \frac{iab^2\operatorname{ArcSin}[a+bx]\operatorname{Log}\left[1+\frac{ie^{i\operatorname{ArcSin}[a+bx]}}{a+\sqrt{-1+a^2}}\right]}{(-1+a^2)^{3/2}} + \frac{b^2\operatorname{Log}[-bx]}{1-a^2} - \frac{ab^2\operatorname{PolyLog}\left[2,-\frac{ie^{i\operatorname{ArcSin}[a+bx]}}{a-\sqrt{-1+a^2}}\right]}{(-1+a^2)^{3/2}} + \frac{ab^2\operatorname{PolyLog}\left[2,-\frac{ie^{i\operatorname{ArcSin}[a+bx]}}{a+\sqrt{-1+a^2}}\right]}{(-1+a^2)^{3/2}} \\
& b^2\left(\frac{\sqrt{1-(a+bx)^2}\operatorname{ArcSin}[a+bx]}{(-1+a^2)bx} - \frac{\operatorname{ArcSin}[a+bx]^2}{2b^2x^2} + \frac{\operatorname{Log}\left[-\frac{bx}{a}\right]}{1-a^2} - \right. \\
& \left. \frac{1}{-1+a^2}a\left(\frac{\pi\operatorname{ArcTan}\left[\frac{1-a\tan\left[\frac{1}{2}\operatorname{ArcSin}[a+bx]\right]}{\sqrt{-1+a^2}}\right]}{\sqrt{-1+a^2}} + \frac{1}{\sqrt{1-a^2}}\left(-2\operatorname{ArcCos}[a]\operatorname{ArcTanh}\left[\frac{(1+a)\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right] - \right.\right.\right. \\
& \left.\left.\left(\pi-2\operatorname{ArcSin}[a+bx]\right)\operatorname{ArcTanh}\left[\frac{(-1+a)\tan\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right] + \left(\operatorname{ArcCos}[a] - \right.\right.\right. \\
& \left.\left.2i\left(\operatorname{ArcTanh}\left[\frac{(1+a)\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right] + \operatorname{ArcTanh}\left[\frac{(-1+a)\tan\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right]\right)\right)\right) \\
& \operatorname{Log}\left[\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-a^2}e^{-\frac{1}{2}i\operatorname{ArcSin}[a+bx]}}{\sqrt{bx}}\right] + \left(\operatorname{ArcCos}[a] + 2i\operatorname{ArcTanh}\left[\frac{(1+a)\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right] + \right. \\
& \left.2i\operatorname{ArcTanh}\left[\frac{(-1+a)\tan\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right]\right)\operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1-a^2}e^{\frac{1}{2}i\operatorname{ArcSin}[a+bx]}}{\sqrt{bx}}\right] - \\
& \left(\operatorname{ArcCos}[a] - 2i\operatorname{ArcTanh}\left[\frac{(1+a)\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right]\right) \\
& \operatorname{Log}\left[-\frac{(-1+a)\left(i+ia+\sqrt{1-a^2}\right)\left(-i+\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]\right)}{1-a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}\right] - \\
& \left(\operatorname{ArcCos}[a] + 2i\operatorname{ArcTanh}\left[\frac{(1+a)\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}{\sqrt{1-a^2}}\right]\right) \\
& \operatorname{Log}\left[-\frac{(-1+a)\left(-i-ia+\sqrt{1-a^2}\right)\left(i+\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]\right)}{1-a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}\right] + \\
& i\left(-\operatorname{PolyLog}\left[2,\frac{\left(a-i\sqrt{1-a^2}\right)\left(-1+a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]\right)}{1-a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}\right] + \right. \\
& \left.\operatorname{PolyLog}\left[2,\frac{\left(a+i\sqrt{1-a^2}\right)\left(-1+a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]\right)}{1-a+\sqrt{1-a^2}\operatorname{Cot}\left[\frac{1}{4}(\pi+2\operatorname{ArcSin}[a+bx])\right]}\right]\right)\right)
\end{aligned}$$

Unable to integrate:

$$\left\{ \frac{\text{ArcSin}[a + b x]^3}{x^2}, x, 12, 0 \right\}$$

$$-\frac{\text{ArcSin}[a + b x]^3}{x} + \frac{3 i b \text{ArcSin}[a + b x]^2 \text{Log}\left[1 + \frac{i e^{i \text{ArcSin}[a + b x]}}{a - \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} -$$

$$\frac{3 i b \text{ArcSin}[a + b x]^2 \text{Log}\left[1 + \frac{i e^{i \text{ArcSin}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} + \frac{6 b \text{ArcSin}[a + b x] \text{PolyLog}\left[2, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a - \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} -$$

$$\frac{6 b \text{ArcSin}[a + b x] \text{PolyLog}\left[2, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} + \frac{6 i b \text{PolyLog}\left[3, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a - \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}} - \frac{6 i b \text{PolyLog}\left[3, -\frac{i e^{i \text{ArcSin}[a + b x]}}{a + \sqrt{-1 + a^2}}\right]}{\sqrt{-1 + a^2}}$$

$$\int \frac{\text{ArcSin}[a + b x]^3}{x^2} dx$$

Unable to integrate:

$$\left\{ x^2 \text{ArcSin}[a + b x]^n, x, 19, 0 \right\}$$

$$-\frac{i (1 + 4 a^2) (-i \text{ArcSin}[a + b x])^{-n} \text{ArcSin}[a + b x]^n \text{Gamma}[1 + n, -i \text{ArcSin}[a + b x]]}{8 b^3} +$$

$$\frac{i (1 + 4 a^2) (i \text{ArcSin}[a + b x])^{-n} \text{ArcSin}[a + b x]^n \text{Gamma}[1 + n, i \text{ArcSin}[a + b x]]}{8 b^3} -$$

$$\frac{i 2^{-2-n} a (-i \text{ArcSin}[a + b x])^{-1-n} \text{ArcSin}[a + b x]^{1+n} \text{Gamma}[1 + n, -2 i \text{ArcSin}[a + b x]]}{b^3} +$$

$$\frac{i 2^{-2-n} a (i \text{ArcSin}[a + b x])^{-1-n} \text{ArcSin}[a + b x]^{1+n} \text{Gamma}[1 + n, 2 i \text{ArcSin}[a + b x]]}{b^3} +$$

$$\frac{3^{-1-n} (-i \text{ArcSin}[a + b x])^{-1-n} \text{ArcSin}[a + b x]^{1+n} \text{Gamma}[1 + n, -3 i \text{ArcSin}[a + b x]]}{8 b^3} +$$

$$\frac{3^{-1-n} (i \text{ArcSin}[a + b x])^{-1-n} \text{ArcSin}[a + b x]^{1+n} \text{Gamma}[1 + n, 3 i \text{ArcSin}[a + b x]]}{8 b^3}$$

$$\int x^2 \text{ArcSin}[a + b x]^n dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \text{ArcSin}\left[\frac{a}{x}\right], x, 4, 0 \right\}$$

$$x \text{ArcCsc}\left[\frac{x}{a}\right] + a \text{ArcTanh}\left[\sqrt{1 - \frac{a^2}{x^2}}\right]$$

$$x \text{ArcSin}\left[\frac{a}{x}\right] - \frac{a \sqrt{1 - \frac{a^2}{x^2}} x \text{ArcTan}\left[\frac{x}{\sqrt{a^2 - x^2}}\right]}{\sqrt{a^2 - x^2}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcSin}[a x^n]}{x}, x, 5, 0 \right\}$$

# Mathematica 7 Test Results for Integration Problems Involving Inverse Trig Functions

$$-\frac{i \operatorname{ArcSin}[a x^n]^2}{2 n} + \frac{\operatorname{ArcSin}[a x^n] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcSin}[a x^n]}\right]}{n} - \frac{i \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcSin}[a x^n]}\right]}{2 n}$$

$$\operatorname{ArcSin}[a x^n] \operatorname{Log}[x] + \frac{1}{2 \sqrt{-a^2} n} a \left( \operatorname{ArcSinh}\left[\sqrt{-a^2} x^n\right]^2 + \right. \\ \left. 2 \operatorname{ArcSinh}\left[\sqrt{-a^2} x^n\right] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcSinh}\left[\sqrt{-a^2} x^n\right]}\right] - 2 n \operatorname{Log}[x] \operatorname{Log}\left[\sqrt{-a^2} x^n + \sqrt{1 - a^2 x^{2 n}}\right] - \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}\left[\sqrt{-a^2} x^n\right]}\right] \right)$$

Unable to integrate:

$$\{\operatorname{ArcSin}[c e^{a+b x}], x, 6, 0\}$$

$$-\frac{i \operatorname{ArcSin}[c e^{a+b x}]^2}{2 b} + \frac{\operatorname{ArcSin}[c e^{a+b x}] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcSin}[c e^{a+b x}]}\right]}{b} - \frac{i \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcSin}[c e^{a+b x}]}\right]}{2 b}$$

$$\int \operatorname{ArcSin}[c e^{a+b x}] dx$$

Valid but unnecessarily complicated antiderivative:

$$\{\operatorname{ArcSin}\left[\frac{c}{a+b x}\right], x, 4, 0\}$$

$$\frac{(a+b x) \operatorname{ArcCsc}\left[\frac{a}{c} + \frac{b x}{c}\right]}{b} + \frac{c \operatorname{ArcTanh}\left[\sqrt{1 - \frac{c^2}{(a+b x)^2}}\right]}{b}$$

$$x \operatorname{ArcSin}\left[\frac{c}{a+b x}\right] + \frac{(a+b x) \sqrt{\frac{a^2 - c^2 + 2 a b x + b^2 x^2}{(a+b x)^2}} \left( i a \operatorname{Log}\left[-\frac{2 b^2 \left(-i c + \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}\right)}{a (a+b x)}\right] + c \operatorname{Log}\left[2 \left(a + b x + \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}\right)\right] \right)}{b \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}}$$

## Problems involving inverse cosines

Valid but unnecessarily complicated antiderivative:

$$\left\{ \text{ArcCos}\left[\frac{a}{x}\right], x, 4, 0 \right\}$$

$$x \text{ArcSec}\left[\frac{x}{a}\right] - a \text{ArcTanh}\left[\sqrt{1 - \frac{a^2}{x^2}}\right]$$

$$x \text{ArcCos}\left[\frac{a}{x}\right] + \frac{a \sqrt{1 - \frac{a^2}{x^2}} x \text{ArcTan}\left[\frac{x}{\sqrt{a^2 - x^2}}\right]}{\sqrt{a^2 - x^2}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCos}[a x^n]}{x}, x, 5, 0 \right\}$$

$$-\frac{i \text{ArcCos}[a x^n]^2}{2 n} + \frac{\text{ArcCos}[a x^n] \text{Log}\left[1 + e^{2 i \text{ArcCos}[a x^n]}\right]}{n} - \frac{i \text{PolyLog}\left[2, -e^{2 i \text{ArcCos}[a x^n]}\right]}{2 n}$$

$$\text{ArcCos}[a x^n] \text{Log}[x] + \frac{1}{2 \sqrt{-a^2} n} a \left( -\text{ArcSinh}\left[\sqrt{-a^2} x^n\right]^2 - 2 \text{ArcSinh}\left[\sqrt{-a^2} x^n\right] \text{Log}\left[1 - e^{-2 \text{ArcSinh}\left[\sqrt{-a^2} x^n\right]}\right] + 2 n \text{Log}[x] \text{Log}\left[\sqrt{-a^2} x^n + \sqrt{1 - a^2 x^{2 n}}\right] + \text{PolyLog}\left[2, e^{-2 \text{ArcSinh}\left[\sqrt{-a^2} x^n\right]}\right] \right)$$

Unable to integrate:

$$\left\{ \text{ArcCos}\left[c e^{a+b x}\right], x, 6, 0 \right\}$$

$$-\frac{i \text{ArcCos}\left[c e^{a+b x}\right]^2}{2 b} + \frac{\text{ArcCos}\left[c e^{a+b x}\right] \text{Log}\left[1 + e^{2 i \text{ArcCos}\left[c e^{a+b x}\right]}\right]}{b} - \frac{i \text{PolyLog}\left[2, -e^{2 i \text{ArcCos}\left[c e^{a+b x}\right]}\right]}{2 b}$$

$$\int \text{ArcCos}\left[c e^{a+b x}\right] dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \text{ArcCos}\left[\frac{c}{a+b x}\right], x, 4, 0 \right\}$$

$$\frac{(a+b x) \text{ArcSec}\left[\frac{a}{c} + \frac{b x}{c}\right] - c \text{ArcTanh}\left[\sqrt{1 - \frac{c^2}{(a+b x)^2}}\right]}{b}$$

$$x \text{ArcCos}\left[\frac{c}{a+b x}\right] - \frac{(a+b x) \sqrt{\frac{a^2 - c^2 + 2 a b x + b^2 x^2}{(a+b x)^2}} \left( i a \text{Log}\left[-\frac{2 b^2 \left(-i c + \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}\right)}{a (a+b x)}\right] + c \text{Log}\left[2 \left(a + b x + \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}\right)\right] \right)}{b \sqrt{a^2 - c^2 + 2 a b x + b^2 x^2}}$$

## Problems involving inverse tangents

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcTan}\left[\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right]}{1-a^2x^2}, x, -5, 0 \right\}$$

$$-\frac{i \operatorname{PolyLog}\left[2, -\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right]}{2a} + \frac{i \operatorname{PolyLog}\left[2, \frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right]}{2a}$$

$$\int \frac{\operatorname{ArcTan}\left[\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right]}{1-a^2x^2} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcTan}[1+x]}{2+2x}, x, 5, 0 \right\}$$

$$\frac{1}{4} i \operatorname{PolyLog}[2, -i(1+x)] - \frac{1}{4} i \operatorname{PolyLog}[2, i(1+x)]$$

$$-\frac{1}{16} i$$

$$\left( \pi^2 - 4\pi \operatorname{ArcTan}[1+x] + 8 \operatorname{ArcTan}[1+x]^2 - i\pi \operatorname{Log}[16] + 4i\pi \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}[1+x]}\right] - 8i \operatorname{ArcTan}[1+x] \operatorname{Log}\left[1 + e^{-2i \operatorname{ArcTan}[1+x]}\right] + 8i \operatorname{ArcTan}[1+x] \operatorname{Log}\left[1 - e^{2i \operatorname{ArcTan}[1+x]}\right] + 2i\pi \operatorname{Log}\left[2 + 2x + x^2\right] + 4 \operatorname{PolyLog}\left[2, -e^{-2i \operatorname{ArcTan}[1+x]}\right] + 4 \operatorname{PolyLog}\left[2, e^{2i \operatorname{ArcTan}[1+x]}\right] \right)$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcTan}[x]}{a+bx+cx^2}, x, 7, 0 \right\}$$

$$-\frac{i \operatorname{Log}[1+ix] \operatorname{Log}\left[-\frac{i(b-\sqrt{b^2-4ac}+2cx)}{2c-i(b-\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} + \frac{i \operatorname{Log}[1-ix] \operatorname{Log}\left[\frac{i(b-\sqrt{b^2-4ac}+2cx)}{2c+i(b-\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} +$$

$$\frac{i \operatorname{Log}[1+ix] \operatorname{Log}\left[-\frac{i(b+\sqrt{b^2-4ac}+2cx)}{2c-i(b+\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} - \frac{i \operatorname{Log}[1-ix] \operatorname{Log}\left[\frac{i(b+\sqrt{b^2-4ac}+2cx)}{2c+i(b+\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} + \frac{i \operatorname{PolyLog}\left[2, \frac{2c(1-ix)}{2c+i(b-\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} -$$

$$\frac{i \operatorname{PolyLog}\left[2, \frac{2c(1-ix)}{2c+i(b+\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} - \frac{i \operatorname{PolyLog}\left[2, \frac{2c(1+ix)}{2c-i(b-\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}} + \frac{i \operatorname{PolyLog}\left[2, \frac{2c(1+ix)}{2c-i(b+\sqrt{b^2-4ac})}\right]}{2\sqrt{b^2-4ac}}$$

$$\int \frac{\operatorname{ArcTan}[x]}{a+bx+cx^2} dx$$

Unable to integrate:

$$\left\{ \frac{\operatorname{ArcTan}[d+ex]}{a+bx^2}, x, 8, 0 \right\}$$

$$\begin{aligned}
& \frac{i \operatorname{Log}\left[\frac{e^{\left(\sqrt{-a}-\sqrt{b} x\right)}}{\sqrt{b}(i+d)+\sqrt{-a} e}\right] \operatorname{Log}\left[-i(i+d)-i e x\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{i \operatorname{Log}\left[-\frac{e^{\left(\sqrt{-a}+\sqrt{b} x\right)}}{\sqrt{b}(i+d)-\sqrt{-a} e}\right] \operatorname{Log}\left[-i(i+d)-i e x\right]}{4 \sqrt{-a} \sqrt{b}} - \\
& \frac{i \operatorname{Log}\left[\frac{i e^{\left(\sqrt{-a}-\sqrt{b} x\right)}}{\sqrt{b}(1+i d)+i \sqrt{-a} e}\right] \operatorname{Log}\left[1+i d+i e x\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{Log}\left[-\frac{i e^{\left(\sqrt{-a}+\sqrt{b} x\right)}}{\sqrt{b}(1+i d)-i \sqrt{-a} e}\right] \operatorname{Log}\left[1+i d+i e x\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(1+i d+i e x)}{\sqrt{b}(1+i d)-i \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} - \\
& \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(1+i d+i e x)}{\sqrt{b}(1+i d)+i \sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(i+d+e x)}{\sqrt{b}(i+d)-\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(i+d+e x)}{\sqrt{b}(i+d)+\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} \\
& \int \frac{\operatorname{ArcTan}[d+e x]}{a+b x^2} d x
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{\frac{\operatorname{ArcTan}[d+e x]}{a+b x+c x^2}, x, 7, 0\right\} \\
& \frac{i \operatorname{Log}\left[\frac{i e^{\left(b-\sqrt{b^2-4 a c}+2 c x\right)}}{2 c(1-i d)+i\left(b-\sqrt{b^2-4 a c}\right) e}\right] \operatorname{Log}\left[1-i d-i e x\right]}{2 \sqrt{b^2-4 a c}} - \frac{i \operatorname{Log}\left[\frac{i e^{\left(b+\sqrt{b^2-4 a c}+2 c x\right)}}{2 c(1-i d)+i\left(b+\sqrt{b^2-4 a c}\right) e}\right] \operatorname{Log}\left[1-i d-i e x\right]}{2 \sqrt{b^2-4 a c}} - \\
& \frac{i \operatorname{Log}\left[-\frac{i e^{\left(b-\sqrt{b^2-4 a c}+2 c x\right)}}{2 c(1+i d)-i\left(b-\sqrt{b^2-4 a c}\right) e}\right] \operatorname{Log}\left[1+i d+i e x\right]}{2 \sqrt{b^2-4 a c}} + \frac{i \operatorname{Log}\left[-\frac{i e^{\left(b+\sqrt{b^2-4 a c}+2 c x\right)}}{2 c(1+i d)-i\left(b+\sqrt{b^2-4 a c}\right) e}\right] \operatorname{Log}\left[1+i d+i e x\right]}{2 \sqrt{b^2-4 a c}} + \\
& \frac{i \operatorname{PolyLog}\left[2, \frac{2 c(1-i d-i e x)}{2 c(1-i d)+i\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2-4 a c}} - \frac{i \operatorname{PolyLog}\left[2, \frac{2 c(1-i d-i e x)}{2 c(1-i d)+i\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2-4 a c}} - \\
& \frac{i \operatorname{PolyLog}\left[2, \frac{2 c(1+i d+i e x)}{2 c(1+i d)-i\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2-4 a c}} + \frac{i \operatorname{PolyLog}\left[2, \frac{2 c(1+i d+i e x)}{2 c(1+i d)-i\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2-4 a c}} \\
& \int \frac{\operatorname{ArcTan}[d+e x]}{a+b x+c x^2} d x
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{\frac{\operatorname{ArcTan}[b x]}{1+x^2}, x, 9, 0\right\} \\
& \frac{1}{4} \operatorname{Log}\left[-\frac{i b(i-x)}{1+b}\right] \operatorname{Log}\left[1-i b x\right] - \frac{1}{4} \operatorname{Log}\left[\frac{i b(i+x)}{1-b}\right] \operatorname{Log}\left[1-i b x\right] - \\
& \frac{1}{4} \operatorname{Log}\left[\frac{i b(i-x)}{1-b}\right] \operatorname{Log}\left[1+i b x\right] + \frac{1}{4} \operatorname{Log}\left[-\frac{i b(i+x)}{1+b}\right] \operatorname{Log}\left[1+i b x\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1-i b x}{1-b}\right] + \\
& \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1-i b x}{1+b}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+i b x}{1-b}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+i b x}{1+b}\right]
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{4\sqrt{-b^2}} b \left( -4 \operatorname{ArcTan}[b x] \operatorname{ArcTanh}\left[\frac{\sqrt{-b^2}}{b x}\right] + 2 \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTanh}\left[\frac{b x}{\sqrt{-b^2}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{b x}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b(-i+\sqrt{-b^2})(-i+b x)}{(-1+b^2)(b+\sqrt{-b^2} x)}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{b x}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b(i+\sqrt{-b^2})(i+b x)}{(-1+b^2)(b+\sqrt{-b^2} x)}\right] \right) + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( \operatorname{ArcTanh}\left[\frac{\sqrt{-b^2}}{b x}\right] + \operatorname{ArcTanh}\left[\frac{\sqrt{-b^2} x}{b}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-b^2} e^{-i \operatorname{ArcTan}[b x]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\cos[2 \operatorname{ArcTan}[b x]]}}\right] + \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( -\operatorname{ArcTanh}\left[\frac{\sqrt{-b^2}}{b x}\right] + \operatorname{ArcTanh}\left[\frac{b x}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-b^2} e^{i \operatorname{ArcTan}[b x]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\cos[2 \operatorname{ArcTan}[b x]]}}\right] + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(1+b^2-2i\sqrt{-b^2})(b-\sqrt{-b^2} x)}{(-1+b^2)(b+\sqrt{-b^2} x)}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2i\sqrt{-b^2})(b-\sqrt{-b^2} x)}{(-1+b^2)(b+\sqrt{-b^2} x)}\right] \right) \right)
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTan}[a+b x]}{1+x^2}, x, 9, 0 \right\} \\
& \frac{1}{4} \operatorname{Log}\left[\frac{i b(i-x)}{i(i+a)-b}\right] \operatorname{Log}[-i(i+a)-i b x] - \frac{1}{4} \operatorname{Log}\left[-\frac{i b(i+x)}{i(i+a)+b}\right] \operatorname{Log}[-i(i+a)-i b x] - \\
& \frac{1}{4} \operatorname{Log}\left[\frac{i b(i-x)}{1+i a-b}\right] \operatorname{Log}[1+i a+i b x] + \frac{1}{4} \operatorname{Log}\left[-\frac{i b(i+x)}{1+i a+b}\right] \operatorname{Log}[1+i a+i b x] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+i a+i b x}{1+i a-b}\right] + \\
& \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+i a+i b x}{1+i a+b}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{i(i+a+b x)}{i(i+a)-b}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{i(i+a+b x)}{i(i+a)+b}\right] \\
& \int \frac{\operatorname{ArcTan}[a+b x]}{1+x^2} dx
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTan}[x]}{(a+b x^2)^{3/2}}, x, 3, 0 \right\} \\
& \frac{x \operatorname{ArcTan}[x]}{a \sqrt{a+b x^2}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a-b}}\right]}{a \sqrt{a-b}} \\
& \frac{2 x \operatorname{ArcTan}[x]}{\sqrt{a+b x^2}} + \frac{\operatorname{Log}\left[-\frac{4 a(a-i b x+\sqrt{a-b}\sqrt{a+b x^2})}{\sqrt{a-b}(i+x)}\right] + \operatorname{Log}\left[-\frac{4 a(a+i b x+\sqrt{a-b}\sqrt{a+b x^2})}{\sqrt{a-b}(-i+x)}\right]}{\sqrt{a-b}} \\
& 2 a
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcTan}[x]}{(a + b x^2)^{5/2}}, x, 8, 0 \right\}$$

$$-\frac{1}{3 a (a-b) \sqrt{a+b x^2}} + \frac{x (3 a+2 b x^2) \text{ArcTan}[x]}{3 a^2 (a+b x^2)^{3/2}} + \frac{(3 a-2 b) \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a-b}}\right]}{3 a^2 (a-b)^{3/2}}$$

$$-\frac{2 a}{(a-b) \sqrt{a+b x^2}} + \frac{2 x (3 a+2 b x^2) \text{ArcTan}[x]}{(a+b x^2)^{3/2}} + \frac{(3 a-2 b) \text{Log}\left[-\frac{12 a^2 \sqrt{a-b} \left(a-i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{(3 a-2 b) (i+x)}\right]}{(a-b)^{3/2}} + \frac{(3 a-2 b) \text{Log}\left[-\frac{12 a^2 \sqrt{a-b} \left(a+i b x+\sqrt{a-b} \sqrt{a+b x^2}\right)}{(3 a-2 b) (-i+x)}\right]}{(a-b)^{3/2}}$$


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$$6 a^2$$

Unable to integrate:

$$\left\{ \text{ArcTan}[a + b f^{c+dx}], x, 5, 0 \right\}$$

$$x \text{ArcTan}[a + b f^{c+dx}] + \frac{1}{2} i x \text{Log}\left[1 - \frac{b f^{c+dx}}{i-a}\right] - \frac{1}{2} i x \text{Log}\left[1 + \frac{b f^{c+dx}}{i+a}\right] + \frac{i \text{PolyLog}\left[2, \frac{b f^{c+dx}}{i-a}\right]}{2 d \text{Log}[f]} - \frac{i \text{PolyLog}\left[2, -\frac{b f^{c+dx}}{i+a}\right]}{2 d \text{Log}[f]}$$

$$\int \text{ArcTan}[a + b f^{c+dx}] dx$$

Unable to integrate:

$$\left\{ x \text{ArcTan}[a + b f^{c+dx}], x, 7, 0 \right\}$$

$$\frac{1}{2} x^2 \text{ArcTan}[a + b f^{c+dx}] - \frac{1}{4} i x^2 \text{Log}\left[1 - \frac{i b f^{c+dx}}{1-i a}\right] + \frac{1}{4} i x^2 \text{Log}\left[1 + \frac{i b f^{c+dx}}{1+i a}\right] -$$

$$\frac{i x \text{PolyLog}\left[2, \frac{i b f^{c+dx}}{1-i a}\right]}{2 d \text{Log}[f]} + \frac{i x \text{PolyLog}\left[2, -\frac{i b f^{c+dx}}{1+i a}\right]}{2 d \text{Log}[f]} + \frac{i \text{PolyLog}\left[3, \frac{i b f^{c+dx}}{1-i a}\right]}{2 d^2 \text{Log}[f]^2} - \frac{i \text{PolyLog}\left[3, -\frac{i b f^{c+dx}}{1+i a}\right]}{2 d^2 \text{Log}[f]^2}$$

$$\int x \text{ArcTan}[a + b f^{c+dx}] dx$$

Unable to integrate:

$$\left\{ x^2 \text{ArcTan}[a + b f^{c+dx}], x, 9, 0 \right\}$$

$$\frac{1}{3} x^3 \text{ArcTan}[a + b f^{c+dx}] - \frac{1}{6} i x^3 \text{Log}\left[1 - \frac{i b f^{c+dx}}{1-i a}\right] + \frac{1}{6} i x^3 \text{Log}\left[1 + \frac{i b f^{c+dx}}{1+i a}\right] - \frac{i x^2 \text{PolyLog}\left[2, \frac{i b f^{c+dx}}{1-i a}\right]}{2 d \text{Log}[f]} +$$

$$\frac{i x^2 \text{PolyLog}\left[2, -\frac{i b f^{c+dx}}{1+i a}\right]}{2 d \text{Log}[f]} + \frac{i x \text{PolyLog}\left[3, \frac{i b f^{c+dx}}{1-i a}\right]}{d^2 \text{Log}[f]^2} - \frac{i x \text{PolyLog}\left[3, -\frac{i b f^{c+dx}}{1+i a}\right]}{d^2 \text{Log}[f]^2} - \frac{i \text{PolyLog}\left[4, \frac{i b f^{c+dx}}{1-i a}\right]}{d^3 \text{Log}[f]^3} + \frac{i \text{PolyLog}\left[4, -\frac{i b f^{c+dx}}{1+i a}\right]}{d^3 \text{Log}[f]^3}$$

$$\int x^2 \text{ArcTan}[a + b f^{c+dx}] dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \text{ArcTan}[b \text{Tan}[x]], x, 12, 0 \right\}$$

$$x \text{ArcTan}[b \text{Tan}[x]] + \frac{1}{2} i x \text{Log}\left[1 + \frac{(1-b^2) e^{2 i x}}{1-2 b+b^2}\right] -$$

$$\frac{1}{2} i x \text{Log}\left[1 + \frac{(1-b^2) e^{2 i x}}{1+2 b+b^2}\right] + \frac{1}{4} \text{PolyLog}\left[2, -\frac{(1-b^2) e^{2 i x}}{1-2 b+b^2}\right] - \frac{1}{4} \text{PolyLog}\left[2, -\frac{(1-b^2) e^{2 i x}}{1+2 b+b^2}\right]$$

$$\begin{aligned}
& x \operatorname{ArcTan}[b \operatorname{Tan}[x]] + \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTanh}\left[\frac{\operatorname{Cot}[x]}{\sqrt{-b^2}}\right] + 2 \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] \operatorname{ArcTanh}\left[\sqrt{-b^2} \operatorname{Tan}[x]\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\operatorname{Cot}[x]}{\sqrt{-b^2}}\right] - 2 i \operatorname{ArcTanh}\left[\sqrt{-b^2} \operatorname{Tan}[x]\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-i x}}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2) \cos[2 x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\operatorname{Cot}[x]}{\sqrt{-b^2}}\right] + 2 i \operatorname{ArcTanh}\left[\sqrt{-b^2} \operatorname{Tan}[x]\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{i x}}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2) \cos[2 x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2 i \operatorname{ArcTanh}\left[\sqrt{-b^2} \operatorname{Tan}[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(i b^2 + \sqrt{-b^2}) (-i + \operatorname{Tan}[x])}{(-1+b^2) (-1 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2 i \operatorname{ArcTanh}\left[\sqrt{-b^2} \operatorname{Tan}[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(-i b^2 + \sqrt{-b^2}) (i + \operatorname{Tan}[x])}{(-1+b^2) (-1 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) + \right. \\
& \left. i \left( -\operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (1 + \sqrt{-b^2} \operatorname{Tan}[x])}{(-1+b^2) (-1 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] + \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (1 + \sqrt{-b^2} \operatorname{Tan}[x])}{(-1+b^2) (-1 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) \right)
\end{aligned}$$

Timed out after 60 seconds:

{ArcTan[a + b Tan[x]], x, 10, 0}

$$\begin{aligned}
& \frac{1}{4} \operatorname{Log}\left[\frac{i b (i - \operatorname{Tan}[x])}{i (i + a) - b}\right] \operatorname{Log}[-i (i + a) - i b \operatorname{Tan}[x]] - \\
& \frac{1}{4} \operatorname{Log}\left[-\frac{i b (i + \operatorname{Tan}[x])}{i (i + a) + b}\right] \operatorname{Log}[-i (i + a) - i b \operatorname{Tan}[x]] - \frac{1}{4} \operatorname{Log}\left[\frac{i b (i - \operatorname{Tan}[x])}{1 + i a - b}\right] \operatorname{Log}[1 + i a + i b \operatorname{Tan}[x]] + \\
& \frac{1}{4} \operatorname{Log}\left[-\frac{i b (i + \operatorname{Tan}[x])}{1 + i a + b}\right] \operatorname{Log}[1 + i a + i b \operatorname{Tan}[x]] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1 + i a + i b \operatorname{Tan}[x]}{1 + i a - b}\right] + \\
& \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1 + i a + i b \operatorname{Tan}[x]}{1 + i a + b}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{i (i + a + b \operatorname{Tan}[x])}{i (i + a) - b}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{i (i + a + b \operatorname{Tan}[x])}{i (i + a) + b}\right]
\end{aligned}$$

???

Valid but unnecessarily complicated antiderivative:

{ArcTan[b Cot[x]], x, 12, 0}

$$\begin{aligned}
& x \operatorname{ArcTan}[b \operatorname{Cot}[x]] - \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2 i x}}{1-2 b+b^2}\right] + \\
& \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2 i x}}{1+2 b+b^2}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2 i x}}{1-2 b+b^2}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2 i x}}{1+2 b+b^2}\right]
\end{aligned}$$

$$\begin{aligned}
& x \operatorname{ArcTan}[b \cot[x]] + \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTanh}[\sqrt{-b^2} \cot[x]] + 2 \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{-b^2}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( \operatorname{ArcTanh}[\sqrt{-b^2} \cot[x]] - \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-i x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \cos[2 x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( -\operatorname{ArcTanh}[\sqrt{-b^2} \cot[x]] + \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{i x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \cos[2 x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (-i + \sqrt{-b^2}) (-i + \tan[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \tan[x])}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\tan[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (i + \sqrt{-b^2}) (i + \tan[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \tan[x])}\right] \right) + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (b^2 - \sqrt{-b^2} \tan[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \tan[x])}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (b^2 - \sqrt{-b^2} \tan[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \tan[x])}\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \{x^2 \operatorname{ArcTan}[\sinh[x]], x, 9, 0\} \\
& -\frac{2}{3} x^3 \operatorname{ArcTan}[e^x] + \frac{1}{3} x^3 \operatorname{ArcTan}[\sinh[x]] + i x^2 \operatorname{PolyLog}[2, -i e^x] - i x^2 \operatorname{PolyLog}[2, i e^x] - \\
& 2 i x \operatorname{PolyLog}[3, -i e^x] + 2 i x \operatorname{PolyLog}[3, i e^x] + 2 i \operatorname{PolyLog}[4, -i e^x] - 2 i \operatorname{PolyLog}[4, i e^x] \\
& \frac{1}{192} i \left( 7 \pi^4 + 8 i \pi^3 x + 24 \pi^2 x^2 - 32 i \pi x^3 - 16 x^4 - 64 i x^3 \operatorname{ArcTan}[\sinh[x]] + 8 i \pi^3 \operatorname{Log}[1 + i e^{-x}] + 48 \pi^2 x \operatorname{Log}[1 + i e^{-x}] - \right. \\
& 96 i \pi x^2 \operatorname{Log}[1 + i e^{-x}] - 64 x^3 \operatorname{Log}[1 + i e^{-x}] - 48 \pi^2 x \operatorname{Log}[1 - i e^x] + 96 i \pi x^2 \operatorname{Log}[1 - i e^x] - 8 i \pi^3 \operatorname{Log}[1 + i e^x] + \\
& 64 x^3 \operatorname{Log}[1 + i e^x] + 8 i \pi^3 \operatorname{Log}\left[\tan\left[\frac{1}{4} (\pi + 2 i x)\right]\right] - 48 (\pi - 2 i x)^2 \operatorname{PolyLog}[2, -i e^{-x}] + 192 x^2 \operatorname{PolyLog}[2, -i e^x] - \\
& 48 \pi^2 \operatorname{PolyLog}[2, i e^x] + 192 i \pi x \operatorname{PolyLog}[2, i e^x] + 192 i \pi \operatorname{PolyLog}[3, -i e^{-x}] + 384 x \operatorname{PolyLog}[3, -i e^{-x}] - \\
& 384 x \operatorname{PolyLog}[3, -i e^x] - 192 i \pi \operatorname{PolyLog}[3, i e^x] + 384 \operatorname{PolyLog}[4, -i e^{-x}] + 384 \operatorname{PolyLog}[4, -i e^x] \left. \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcTan}[a + b x]}{\frac{a d}{b} + d x}, x, 6, 0 \right\} \\
& \frac{i \operatorname{PolyLog}[2, -i (a + b x)]}{2 d} - \frac{i \operatorname{PolyLog}[2, i (a + b x)]}{2 d} \\
& -\frac{1}{8 d} i \left( \pi^2 - 4 \pi \operatorname{ArcTan}[a + b x] + 8 \operatorname{ArcTan}[a + b x]^2 - i \pi \operatorname{Log}[16] + 4 i \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}\right] - \right. \\
& 8 i \operatorname{ArcTan}[a + b x] \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}\right] + 8 i \operatorname{ArcTan}[a + b x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcTan}[a + b x]}\right] + \\
& 2 i \pi \operatorname{Log}\left[1 + a^2 + 2 a b x + b^2 x^2\right] + 4 \operatorname{PolyLog}\left[2, -e^{-2 i \operatorname{ArcTan}[a + b x]}\right] + 4 \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcTan}[a + b x]}\right] \left. \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{\text{ArcTan}[a x^n]}{x}, x, 3, 0\right\}$$
$$\frac{i \text{PolyLog}[2, -i a x^n]}{2 n}-\frac{i \text{PolyLog}[2, i a x^n]}{2 n}$$
$$\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\},\left\{\frac{3}{2}, \frac{3}{2}\right\},-a^2 x^{2 n}\right]}{n}$$

## Problems involving inverse cotangents

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCot}[1+x]}{2+2x}, x, 5, 0 \right\}$$

$$-\frac{1}{4}i \text{PolyLog}\left[2, -\frac{i}{1+x}\right] + \frac{1}{4}i \text{PolyLog}\left[2, \frac{i}{1+x}\right]$$

$$\frac{1}{16} \left( i\pi^2 - 4i\pi \text{ArcTan}[1+x] + 8i \text{ArcTan}[1+x]^2 + \pi \text{Log}[16] - 4\pi \text{Log}\left[1 + e^{-2i \text{ArcTan}[1+x]}\right] + \right.$$

$$8 \text{ArcTan}[1+x] \text{Log}\left[1 + e^{-2i \text{ArcTan}[1+x]}\right] - 8 \text{ArcTan}[1+x] \text{Log}\left[1 - e^{2i \text{ArcTan}[1+x]}\right] + 8 \text{ArcCot}[1+x] \text{Log}[1+x] +$$

$$\left. 8 \text{ArcTan}[1+x] \text{Log}[1+x] - 2\pi \text{Log}[2+2x+x^2] + 4i \text{PolyLog}\left[2, -e^{-2i \text{ArcTan}[1+x]}\right] + 4i \text{PolyLog}\left[2, e^{2i \text{ArcTan}[1+x]}\right] \right)$$

Unable to integrate:

$$\left\{ \frac{\text{ArcCot}[x]}{a+bx+cx^2}, x, 31, 0 \right\}$$

$$\frac{i \text{Log}\left[1 - \frac{i}{x}\right] \text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} - \frac{i \text{Log}\left[1 + \frac{i}{x}\right] \text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} -$$

$$\frac{i \text{Log}\left[\frac{2c(i-x)}{b+2ic-\sqrt{b^2-4ac}}\right] \text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} + \frac{i \text{Log}\left[-\frac{2c(i+x)}{b-2ic-\sqrt{b^2-4ac}}\right] \text{Log}\left[b - \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} -$$

$$\frac{i \text{Log}\left[1 - \frac{i}{x}\right] \text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} + \frac{i \text{Log}\left[1 + \frac{i}{x}\right] \text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} +$$

$$\frac{i \text{Log}\left[\frac{2c(i-x)}{b+2ic+\sqrt{b^2-4ac}}\right] \text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} - \frac{i \text{Log}\left[-\frac{2c(i+x)}{b-2ic+\sqrt{b^2-4ac}}\right] \text{Log}\left[b + \sqrt{b^2 - 4ac} + 2cx\right]}{2\sqrt{b^2 - 4ac}} +$$

$$\frac{i \text{PolyLog}\left[2, \frac{b-\sqrt{b^2-4ac}+2cx}{b-2ic-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2 - 4ac}} - \frac{i \text{PolyLog}\left[2, \frac{b-\sqrt{b^2-4ac}+2cx}{b+2ic-\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2 - 4ac}} - \frac{i \text{PolyLog}\left[2, \frac{b+\sqrt{b^2-4ac}+2cx}{b-2ic+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2 - 4ac}} + \frac{i \text{PolyLog}\left[2, \frac{b+\sqrt{b^2-4ac}+2cx}{b+2ic+\sqrt{b^2-4ac}}\right]}{2\sqrt{b^2 - 4ac}}$$

$$\int \frac{\text{ArcCot}[x]}{a+bx+cx^2} dx$$

Unable to integrate:

$$\left\{ \frac{\text{ArcCot}[d+ex]}{a+bx^2}, x, 27, 0 \right\}$$

$$\begin{aligned}
& - \frac{i \operatorname{Log}\left[a + \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[\frac{\sqrt{b} (i-d-e x)}{\sqrt{b} (i-d)-\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{Log}\left[a - \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[\frac{\sqrt{b} (i-d-e x)}{\sqrt{b} (i-d)+\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} \\
& + \frac{i \operatorname{Log}\left[a - \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[\frac{\sqrt{b} (i+d+e x)}{\sqrt{b} (i+d)-\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{Log}\left[a + \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[\frac{\sqrt{b} (i+d+e x)}{\sqrt{b} (i+d)+\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{i \operatorname{Log}\left[a - \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[1 - \frac{i}{d+e x}\right]}{4 \sqrt{-a} \sqrt{b}} + \\
& \frac{i \operatorname{Log}\left[a + \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[1 - \frac{i}{d+e x}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{Log}\left[a - \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[1 + \frac{i}{d+e x}\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{i \operatorname{Log}\left[a + \sqrt{-a} \sqrt{b} x\right] \operatorname{Log}\left[1 + \frac{i}{d+e x}\right]}{4 \sqrt{-a} \sqrt{b}} - \\
& \frac{i \operatorname{PolyLog}\left[2, -\frac{e\left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{b} (i-d)-\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{PolyLog}\left[2, \frac{e\left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{b} (i+d)+\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} - \frac{i \operatorname{PolyLog}\left[2, -\frac{e\left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} (i+d)-\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} + \frac{i \operatorname{PolyLog}\left[2, \frac{e\left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} (i-d)+\sqrt{-a} e}\right]}{4 \sqrt{-a} \sqrt{b}} \\
& \int \frac{\operatorname{ArcCot}[d+e x]}{a+b x^2} d x
\end{aligned}$$

Unable to integrate:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcCot}[d+e x]}{a+b x+c x^2}, x, 27, 0 \right\} \\
& - \frac{i \operatorname{Log}\left[b - \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[\frac{2 c (i-d-e x)}{2 c (i-d)+\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{i \operatorname{Log}\left[b + \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[\frac{2 c (i-d-e x)}{2 c (i-d)+\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} + \\
& \frac{i \operatorname{Log}\left[b - \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[\frac{2 c (i+d+e x)}{2 c (i+d)-\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} - \frac{i \operatorname{Log}\left[b + \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[\frac{2 c (i+d+e x)}{2 c (i+d)-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} + \\
& \frac{i \operatorname{Log}\left[b - \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[1 - \frac{i}{d+e x}\right]}{2 \sqrt{b^2 - 4 a c}} - \frac{i \operatorname{Log}\left[b + \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[1 - \frac{i}{d+e x}\right]}{2 \sqrt{b^2 - 4 a c}} - \\
& \frac{i \operatorname{Log}\left[b - \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[1 + \frac{i}{d+e x}\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{i \operatorname{Log}\left[b + \sqrt{b^2 - 4 a c} + 2 c x\right] \operatorname{Log}\left[1 + \frac{i}{d+e x}\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{i \operatorname{PolyLog}\left[2, -\frac{e\left(b-\sqrt{b^2-4 a c}+2 c x\right)}{2 c (i+d)-\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} - \\
& \frac{i \operatorname{PolyLog}\left[2, \frac{e\left(b-\sqrt{b^2-4 a c}+2 c x\right)}{2 c (i-d)+\left(b-\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} - \frac{i \operatorname{PolyLog}\left[2, -\frac{e\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{2 c (i+d)-\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{i \operatorname{PolyLog}\left[2, \frac{e\left(b+\sqrt{b^2-4 a c}+2 c x\right)}{2 c (i-d)+\left(b+\sqrt{b^2-4 a c}\right) e}\right]}{2 \sqrt{b^2 - 4 a c}} \\
& \int \frac{\operatorname{ArcCot}[d+e x]}{a+b x+c x^2} d x
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcCot}[x]}{(a+b x^2)^{3/2}}, x, 3, 0 \right\}$$

$$\frac{x \operatorname{ArcCot}[x]}{a \sqrt{a + b x^2}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a + b x^2}}{\sqrt{a - b}}\right]}{a \sqrt{a - b}}$$

$$\frac{2 x \operatorname{ArcCot}[x]}{\sqrt{a + b x^2}} + \frac{-\operatorname{Log}\left[\frac{4 a \left(a - i b x + \sqrt{a - b} \sqrt{a + b x^2}\right)}{\sqrt{a - b} (i + x)}\right] - \operatorname{Log}\left[\frac{4 a \left(a + i b x + \sqrt{a - b} \sqrt{a + b x^2}\right)}{\sqrt{a - b} (-i + x)}\right]}{2 a \sqrt{a - b}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\operatorname{ArcCot}[x]}{(a + b x^2)^{5/2}}, x, 8, 0 \right\}$$

$$\frac{1}{3 a (a - b) \sqrt{a + b x^2}} + \frac{x (3 a + 2 b x^2) \operatorname{ArcCot}[x]}{3 a^2 (a + b x^2)^{3/2}} - \frac{(3 a - 2 b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b x^2}}{\sqrt{a - b}}\right]}{3 a^2 (a - b)^{3/2}}$$

$$- \frac{2 a}{(a - b) \sqrt{a + b x^2}} - \frac{2 x (3 a + 2 b x^2) \operatorname{ArcCot}[x]}{(a + b x^2)^{3/2}} + \frac{(3 a - 2 b) \operatorname{Log}\left[\frac{12 a^2 \sqrt{a - b} \left(a - i b x + \sqrt{a - b} \sqrt{a + b x^2}\right)}{(3 a - 2 b) (i + x)}\right]}{(a - b)^{3/2}} + \frac{(3 a - 2 b) \operatorname{Log}\left[\frac{12 a^2 \sqrt{a - b} \left(a + i b x + \sqrt{a - b} \sqrt{a + b x^2}\right)}{(3 a - 2 b) (-i + x)}\right]}{(a - b)^{3/2}}$$

$$- \frac{6 a^2}{6 a^2}$$

Unable to integrate:

$$\left\{ \operatorname{ArcCot}[a + b f^{c + d x}], x, -15, 0 \right\}$$

$$x \operatorname{ArcCot}[a + b f^{c + d x}] - \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{b f^{c + d x}}{i - a}\right] + \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{b f^{c + d x}}{i + a}\right] - \frac{i \operatorname{PolyLog}\left[2, \frac{b f^{c + d x}}{i - a}\right]}{2 d \operatorname{Log}[f]} + \frac{i \operatorname{PolyLog}\left[2, -\frac{b f^{c + d x}}{i + a}\right]}{2 d \operatorname{Log}[f]}$$

$$\int \operatorname{ArcCot}[a + b f^{c + d x}] dx$$

Unable to integrate:

$$\left\{ x \operatorname{ArcCot}[a + b f^{c + d x}], x, 25, 0 \right\}$$

$$-\frac{1}{4} i x^2 \operatorname{Log}\left[1 - \frac{b f^{c + d x}}{i - a}\right] + \frac{1}{4} i x^2 \operatorname{Log}\left[1 + \frac{b f^{c + d x}}{i + a}\right] + \frac{1}{4} i x^2 \operatorname{Log}\left[1 - \frac{i}{a + b f^{c + d x}}\right] - \frac{1}{4} i x^2 \operatorname{Log}\left[1 + \frac{i}{a + b f^{c + d x}}\right] -$$

$$\frac{i x \operatorname{PolyLog}\left[2, \frac{b f^{c + d x}}{i - a}\right]}{2 d \operatorname{Log}[f]} + \frac{i x \operatorname{PolyLog}\left[2, -\frac{b f^{c + d x}}{i + a}\right]}{2 d \operatorname{Log}[f]} + \frac{i \operatorname{PolyLog}\left[3, \frac{b f^{c + d x}}{i - a}\right]}{2 d^2 \operatorname{Log}[f]^2} - \frac{i \operatorname{PolyLog}\left[3, -\frac{b f^{c + d x}}{i + a}\right]}{2 d^2 \operatorname{Log}[f]^2}$$

$$\int x \operatorname{ArcCot}[a + b f^{c + d x}] dx$$

Unable to integrate:

$$\left\{ x^2 \operatorname{ArcCot}[a + b f^{c + d x}], x, 29, 0 \right\}$$

$$-\frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{b f^{c + d x}}{i - a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{b f^{c + d x}}{i + a}\right] + \frac{1}{6} i x^3 \operatorname{Log}\left[1 - \frac{i}{a + b f^{c + d x}}\right] -$$

$$\frac{1}{6} i x^3 \operatorname{Log}\left[1 + \frac{i}{a + b f^{c + d x}}\right] - \frac{i x^2 \operatorname{PolyLog}\left[2, \frac{b f^{c + d x}}{i - a}\right]}{2 d \operatorname{Log}[f]} + \frac{i x^2 \operatorname{PolyLog}\left[2, -\frac{b f^{c + d x}}{i + a}\right]}{2 d \operatorname{Log}[f]} +$$

$$\frac{i x \operatorname{PolyLog}\left[3, \frac{b f^{c + d x}}{i - a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i x \operatorname{PolyLog}\left[3, -\frac{b f^{c + d x}}{i + a}\right]}{d^2 \operatorname{Log}[f]^2} - \frac{i \operatorname{PolyLog}\left[4, \frac{b f^{c + d x}}{i - a}\right]}{d^3 \operatorname{Log}[f]^3} + \frac{i \operatorname{PolyLog}\left[4, -\frac{b f^{c + d x}}{i + a}\right]}{d^3 \operatorname{Log}[f]^3}$$



$$\int x^2 \operatorname{ArcCot}[a + b f^{c+dx}] dx$$

Valid but unnecessarily complicated antiderivative:

$$\{\operatorname{ArcCot}[b \tan[x]], x, 12, 0\}$$

$$\begin{aligned} & x \operatorname{ArcCot}[b \tan[x]] - \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(1-b^2) e^{2ix}}{1-2b+b^2}\right] + \\ & \frac{1}{2} i x \operatorname{Log}\left[1 + \frac{(1-b^2) e^{2ix}}{1+2b+b^2}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, -\frac{(1-b^2) e^{2ix}}{1-2b+b^2}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, -\frac{(1-b^2) e^{2ix}}{1+2b+b^2}\right] \\ & x \operatorname{ArcCot}[b \tan[x]] - \frac{1}{4\sqrt{-b^2}} b \left( -4x \operatorname{ArcTanh}\left[\frac{\cot[x]}{\sqrt{-b^2}}\right] + 2 \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] \operatorname{ArcTanh}\left[\sqrt{-b^2} \tan[x]\right] + \right. \\ & \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2i \operatorname{ArcTanh}\left[\frac{\cot[x]}{\sqrt{-b^2}}\right] - 2i \operatorname{ArcTanh}\left[\sqrt{-b^2} \tan[x]\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-ix}}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2)\cos[2x]}}\right] + \\ & \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2i \operatorname{ArcTanh}\left[\frac{\cot[x]}{\sqrt{-b^2}}\right] + 2i \operatorname{ArcTanh}\left[\sqrt{-b^2} \tan[x]\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{ix}}{\sqrt{-1+b^2} \sqrt{-1-b^2+(-1+b^2)\cos[2x]}}\right] - \\ & \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] + 2i \operatorname{ArcTanh}\left[\sqrt{-b^2} \tan[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(ib^2 + \sqrt{-b^2})(-i + \tan[x])}{(-1+b^2)(-1 + \sqrt{-b^2} \tan[x])}\right] \right) - \\ & \left( \operatorname{ArcCos}\left[\frac{1+b^2}{-1+b^2}\right] - 2i \operatorname{ArcTanh}\left[\sqrt{-b^2} \tan[x]\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[-\frac{(-ib^2 + \sqrt{-b^2})(i + \tan[x])}{(-1+b^2)(-1 + \sqrt{-b^2} \tan[x])}\right] \right) + \\ & i \left( -\operatorname{PolyLog}\left[2, \frac{(1+b^2-2i\sqrt{-b^2})(1+\sqrt{-b^2} \tan[x])}{(-1+b^2)(-1+\sqrt{-b^2} \tan[x])}\right] + \operatorname{PolyLog}\left[2, \frac{(1+b^2+2i\sqrt{-b^2})(1+\sqrt{-b^2} \tan[x])}{(-1+b^2)(-1+\sqrt{-b^2} \tan[x])}\right] \right) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\{\operatorname{ArcCot}[b \cot[x]], x, 12, 0\}$$

$$\begin{aligned} & x \operatorname{ArcCot}[b \cot[x]] + \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2ix}}{1-2b+b^2}\right] - \\ & \frac{1}{2} i x \operatorname{Log}\left[1 - \frac{(1-b^2) e^{2ix}}{1+2b+b^2}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2ix}}{1-2b+b^2}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{(1-b^2) e^{2ix}}{1+2b+b^2}\right] \end{aligned}$$

$$\begin{aligned}
& x \operatorname{ArcCot}[b \operatorname{Cot}[x]] - \frac{1}{4 \sqrt{-b^2}} b \left( -4 x \operatorname{ArcTanh}[\sqrt{-b^2} \operatorname{Cot}[x]] + 2 \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{-b^2}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( \operatorname{ArcTanh}[\sqrt{-b^2} \operatorname{Cot}[x]] - \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{-i x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cos}[2 x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \left( -\operatorname{ArcTanh}[\sqrt{-b^2} \operatorname{Cot}[x]] + \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-b^2} e^{i x}}{\sqrt{-1+b^2} \sqrt{1+b^2+(-1+b^2) \operatorname{Cos}[2 x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 i \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (-i + \sqrt{-b^2}) (-i + \operatorname{Tan}[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) - \right. \\
& \left. \left( \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 i \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}[x]}{\sqrt{-b^2}}\right] \right) \left( \operatorname{Log}[2] + \operatorname{Log}\left[\frac{b^2 (i + \sqrt{-b^2}) (i + \operatorname{Tan}[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(1+b^2-2 i \sqrt{-b^2}) (b^2 - \sqrt{-b^2} \operatorname{Tan}[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2 i \sqrt{-b^2}) (b^2 - \sqrt{-b^2} \operatorname{Tan}[x])}{(-1+b^2) (b^2 + \sqrt{-b^2} \operatorname{Tan}[x])}\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{e^{\operatorname{ArcCot}[x]}}{(a + a x^2)^{7/2}}, x, 7, 0 \right\} \\
& \frac{e^{\operatorname{ArcCot}[x]} (-9 + 17 x - 14 x^2 + 18 x^3 - 6 x^4 + 6 x^5)}{26 a (a (1 + x^2))^{5/2}} \\
& \frac{1}{416 a^3 \sqrt{a (1 + x^2)}} e^{\operatorname{ArcCot}[x]} \left( -130 + 130 x - 39 \sqrt{1 + \frac{1}{x^2}} x \operatorname{Cos}[3 \operatorname{ArcCot}[x]] + \right. \\
& \left. 5 \sqrt{1 + \frac{1}{x^2}} x \operatorname{Cos}[5 \operatorname{ArcCot}[x]] + 13 \sqrt{1 + \frac{1}{x^2}} x \operatorname{Sin}[3 \operatorname{ArcCot}[x]] - \sqrt{1 + \frac{1}{x^2}} x \operatorname{Sin}[5 \operatorname{ArcCot}[x]] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{\operatorname{ArcCot}[a + b x]}{\frac{a d}{b} + d x}, x, 6, 0 \right\} \\
& - \frac{i \operatorname{PolyLog}\left[2, -\frac{i}{a + b x}\right]}{2 d} + \frac{i \operatorname{PolyLog}\left[2, \frac{i}{a + b x}\right]}{2 d} \\
& \frac{1}{8 d} \left( i \pi^2 - 4 i \pi \operatorname{ArcTan}[a + b x] + 8 i \operatorname{ArcTan}[a + b x]^2 + \right. \\
& \pi \operatorname{Log}[16] - 4 \pi \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}\right] + 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}\left[1 + e^{-2 i \operatorname{ArcTan}[a + b x]}\right] - \\
& 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}\left[1 - e^{2 i \operatorname{ArcTan}[a + b x]}\right] + 8 \operatorname{ArcCot}[a + b x] \operatorname{Log}[a + b x] + 8 \operatorname{ArcTan}[a + b x] \operatorname{Log}[a + b x] - \\
& \left. 2 \pi \operatorname{Log}\left[1 + a^2 + 2 a b x + b^2 x^2\right] + 4 i \operatorname{PolyLog}\left[2, -e^{-2 i \operatorname{ArcTan}[a + b x]}\right] + 4 i \operatorname{PolyLog}\left[2, e^{2 i \operatorname{ArcTan}[a + b x]}\right] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\text{ArcCot}[a x^n]}{x}, x, 3, 0 \right\}$$

$$-\frac{i \text{PolyLog}\left[2, -\frac{i x^n}{a}\right]}{2 n} + \frac{i \text{PolyLog}\left[2, \frac{i x^n}{a}\right]}{2 n}$$

$$-\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -a^2 x^{2 n}\right]}{n} + (\text{ArcCot}[a x^n] + \text{ArcTan}[a x^n]) \text{Log}[x]$$

## Problems involving inverse secants

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcSec}[a + b x], x, 3, 0\}$ 

$$\frac{(a + b x) \text{ArcSec}[a + b x]}{b} - \frac{\text{ArcTanh}\left[\sqrt{1 - \frac{1}{(a + b x)^2}}\right]}{b}$$

$$x \text{ArcSec}[a + b x] - \frac{(a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}} \left( a \text{ArcTan}\left[\frac{1}{\sqrt{-1 + a^2 + 2 a b x + b^2 x^2}}\right] + \text{Log}\left[2 \left( a + b x + \sqrt{-1 + a^2 + 2 a b x + b^2 x^2}\right)\right] \right)}{b \sqrt{-1 + a^2 + 2 a b x + b^2 x^2}}$$

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcSec}[c e^{a + b x}], x, 7, 0\}$ 

$$\frac{i \text{ArcSec}[c e^{a + b x}]^2}{2 b} - \frac{\text{ArcSec}[c e^{a + b x}] \text{Log}\left[1 + e^{2 i \text{ArcSec}[c e^{a + b x}]}\right]}{b} + \frac{i \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[c e^{a + b x}]}\right]}{2 b}$$

$$x \text{ArcSec}[c e^{a + b x}] - \frac{1}{8 b c \sqrt{1 - \frac{e^{-2 (a + b x)}}{c^2}}} e^{-a - b x} \left( 4 \sqrt{-1 + c^2 e^{2 (a + b x)}} \text{ArcTan}\left[\sqrt{-1 + c^2 e^{2 (a + b x)}}\right] (2 b x - \text{Log}[c^2 e^{2 (a + b x)}]) + \right.$$

$$\left. \sqrt{-1 - c^2 e^{2 (a + b x)}} \left( \text{Log}[c^2 e^{2 (a + b x)}]^2 - 4 \text{Log}[c^2 e^{2 (a + b x)}] \text{Log}\left[\frac{1}{2} \left( 1 + \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right] + 2 \text{Log}\left[\frac{1}{2} \left( 1 + \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right]^2 - \right.$$

$$\left. 4 \sqrt{-1 - c^2 e^{2 (a + b x)}} \text{PolyLog}\left[2, \frac{1}{2} \left( 1 - \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right] \right)$$

Valid but unnecessarily complicated antiderivative:

 $\left\{\frac{\text{ArcSec}[a x^n]}{x}, x, 7, 0\right\}$ 

$$\frac{i \text{ArcSec}[a x^n]^2}{2 n} - \frac{\text{ArcSec}[a x^n] \text{Log}\left[1 + e^{2 i \text{ArcSec}[a x^n]}\right]}{n} + \frac{i \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[a x^n]}\right]}{2 n}$$

$$\frac{x^{-n} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{x^{-2 n}}{a^2}\right]}{a n} + \left( \text{ArcSec}[a x^n] + \text{ArcSin}\left[\frac{x^{-n}}{a}\right] \right) \text{Log}[x]$$

## Problems involving inverse cosecants

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcCsc}[a + b x], x, 3, 0\}$ 

$$\frac{(a + b x) \text{ArcCsc}[a + b x]}{b} + \frac{\text{ArcTanh}\left[\sqrt{1 - \frac{1}{(a + b x)^2}}\right]}{b} \\
+ x \text{ArcCsc}[a + b x] + \left( (a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}} \left( a \text{ArcTan}\left[\frac{1}{\sqrt{-1 + a^2 + 2 a b x + b^2 x^2}}\right] + \text{Log}\left[2 \left(a + b x + \sqrt{-1 + a^2 + 2 a b x + b^2 x^2}\right)\right] \right) \right) / \left( b \sqrt{-1 + a^2 + 2 a b x + b^2 x^2} \right)$$

Valid but unnecessarily complicated antiderivative:

 $\{\text{ArcCsc}[c e^{a + b x}], x, 7, 0\}$ 

$$\frac{i \text{ArcCsc}[c e^{a + b x}]^2}{2 b} - \frac{\text{ArcCsc}[c e^{a + b x}] \text{Log}[1 - e^{2 i \text{ArcCsc}[c e^{a + b x}]}]}{b} + \frac{i \text{PolyLog}[2, e^{2 i \text{ArcCsc}[c e^{a + b x}]}]}{2 b} \\
+ x \text{ArcCsc}[c e^{a + b x}] + \frac{1}{8 b c \sqrt{1 - \frac{e^{-2 (a + b x)}}{c^2}}} e^{-a - b x} \left( 4 \sqrt{-1 + c^2 e^{2 (a + b x)}} \text{ArcTan}\left[\sqrt{-1 + c^2 e^{2 (a + b x)}}\right] (2 b x - \text{Log}[c^2 e^{2 (a + b x)}]) + \right. \\
\left. \sqrt{-1 - c^2 e^{2 (a + b x)}} \left( \text{Log}[c^2 e^{2 (a + b x)}]^2 - 4 \text{Log}[c^2 e^{2 (a + b x)}] \text{Log}\left[\frac{1}{2} \left(1 + \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right] + 2 \text{Log}\left[\frac{1}{2} \left(1 + \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right]^2 \right) - \right. \\
\left. 4 \sqrt{-1 - c^2 e^{2 (a + b x)}} \text{PolyLog}\left[2, \frac{1}{2} \left(1 - \sqrt{-1 - c^2 e^{2 (a + b x)}}\right)\right] \right)$$

Valid but unnecessarily complicated antiderivative:

 $\left\{\frac{\text{ArcCsc}[a x^n]}{x}, x, 7, 0\right\}$ 

$$\frac{i \text{ArcCsc}[a x^n]^2}{2 n} - \frac{\text{ArcCsc}[a x^n] \text{Log}[1 - e^{2 i \text{ArcCsc}[a x^n]}]}{n} + \frac{i \text{PolyLog}[2, e^{2 i \text{ArcCsc}[a x^n]}]}{2 n} \\
- \frac{x^{-n} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{x^{-2 n}}{a^2}\right]}{a n} + \left( \text{ArcCsc}[a x^n] - \text{ArcSin}\left[\frac{x^{-n}}{a}\right] \right) \text{Log}[x]$$