

# ***Mathematica 7* Test Results**

## **For Rational Function Integration Problems**

Rational function problems involving linear polynomials

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + b x)^3}{x^5}, x, 1, 0 \right\}$$

$$- \frac{(a + b x)^4}{4 a x^4}$$

$$- \frac{a^3 + 4 a^2 b x + 6 a b^2 x^2 + 4 b^3 x^3}{4 x^4}$$

Valid but unnecessarily complicated antiderivative:

$$\{x (a + b x)^7, x, 2, 0\}$$

$$- \frac{a (a + b x)^8}{72 b^2} + \frac{x (a + b x)^8}{9 b}$$

$$\frac{a^7 x^2}{2} + \frac{7}{3} a^6 b x^3 + \frac{21}{4} a^5 b^2 x^4 + 7 a^4 b^3 x^5 + \frac{35}{6} a^3 b^4 x^6 + 3 a^2 b^5 x^7 + \frac{7}{8} a b^6 x^8 + \frac{b^7 x^9}{9}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + b x)^7}{x^9}, x, 1, 0 \right\}$$

$$- \frac{(a + b x)^8}{8 a x^8}$$

$$- \frac{a^7 + 8 a^6 b x + 28 a^5 b^2 x^2 + 56 a^4 b^3 x^3 + 70 a^3 b^4 x^4 + 56 a^2 b^5 x^5 + 28 a b^6 x^6 + 8 b^7 x^7}{8 x^8}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + b x)^7}{x^{10}}, x, 2, 0 \right\}$$

$$- \frac{(a + b x)^8}{9 a x^9} + \frac{b (a + b x)^8}{72 a^2 x^8}$$

$$- \frac{1}{72 x^9} (8 a^7 + 63 a^6 b x + 216 a^5 b^2 x^2 + 420 a^4 b^3 x^3 + 504 a^3 b^4 x^4 + 378 a^2 b^5 x^5 + 168 a b^6 x^6 + 36 b^7 x^7)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^5}{(a + b x)^7}, x, 1, 0 \right\}$$

$$\frac{x^6}{6 a (a + b x)^6}$$

$$-\frac{a^5 + 6 a^4 b x + 15 a^3 b^2 x^2 + 20 a^2 b^3 x^3 + 15 a b^4 x^4 + 6 b^5 x^5}{6 b^6 (a + b x)^6}$$

Valid but unnecessarily complicated antiderivative:

$$\{(a + b x)^4 (c + d x), x, 2, 0\}$$

$$\frac{(6 b c - a d) (a + b x)^5}{30 b^2} + \frac{d x (a + b x)^5}{6 b}$$

$$\frac{1}{30} x \left( 15 a^4 (2 c + d x) + 20 a^3 b x (3 c + 2 d x) + 15 a^2 b^2 x^2 (4 c + 3 d x) + 6 a b^3 x^3 (5 c + 4 d x) + b^4 x^4 (6 c + 5 d x) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\{(a + b x)^5 (c + d x), x, 2, 0\}$$

$$\frac{(7 b c - a d) (a + b x)^6}{42 b^2} + \frac{d x (a + b x)^6}{7 b}$$

$$a^5 c x + \frac{1}{2} a^4 (5 b c + a d) x^2 + \frac{5}{3} a^3 b (2 b c + a d) x^3 + \frac{5}{2} a^2 b^2 (b c + a d) x^4 + a b^3 (b c + 2 a d) x^5 + \frac{1}{6} b^4 (b c + 5 a d) x^6 + \frac{1}{7} b^5 d x^7$$

Valid but unnecessarily complicated antiderivative:

$$\{(a + b x)^4 (c + d x)^2, x, 3, 0\}$$

$$\frac{(b c - a d)^2 (a + b x)^5}{105 b^3} + \frac{(b c - a d) (a + b x)^5 (c + d x)}{21 b^2} + \frac{(a + b x)^5 (c + d x)^2}{7 b}$$

$$a^4 c^2 x + a^3 c (2 b c + a d) x^2 + \frac{1}{3} a^2 (6 b^2 c^2 + 8 a b c d + a^2 d^2) x^3 +$$

$$a b (b^2 c^2 + 3 a b c d + a^2 d^2) x^4 + \frac{1}{5} b^2 (b^2 c^2 + 8 a b c d + 6 a^2 d^2) x^5 + \frac{1}{3} b^3 d (b c + 2 a d) x^6 + \frac{1}{7} b^4 d^2 x^7$$

Valid but unnecessarily complicated antiderivative:

$$\{(a + b x)^5 (c + d x)^2, x, 3, 0\}$$

$$\frac{(b c - a d)^2 (a + b x)^6}{168 b^3} + \frac{(b c - a d) (a + b x)^6 (c + d x)}{28 b^2} + \frac{(a + b x)^6 (c + d x)^2}{8 b}$$

$$a^5 c^2 x + \frac{1}{2} a^4 c (5 b c + 2 a d) x^2 + \frac{1}{3} a^3 (10 b^2 c^2 + 10 a b c d + a^2 d^2) x^3 + \frac{5}{4} a^2 b (2 b^2 c^2 + 4 a b c d + a^2 d^2) x^4 +$$

$$a b^2 (b^2 c^2 + 4 a b c d + 2 a^2 d^2) x^5 + \frac{1}{6} b^3 (b^2 c^2 + 10 a b c d + 10 a^2 d^2) x^6 + \frac{1}{7} b^4 d (2 b c + 5 a d) x^7 + \frac{1}{8} b^5 d^2 x^8$$

Valid but unnecessarily complicated antiderivative:

$$\{(a + b x)^4 (c + d x)^3, x, 4, 0\}$$

$$\frac{(b c - a d)^3 (a + b x)^5}{280 b^4} + \frac{(b c - a d)^2 (a + b x)^5 (c + d x)}{56 b^3} + \frac{3 (b c - a d) (a + b x)^5 (c + d x)^2}{56 b^2} + \frac{(a + b x)^5 (c + d x)^3}{8 b}$$

$$a^4 c^3 x + \frac{1}{2} a^3 c^2 (4 b c + 3 a d) x^2 + a^2 c (2 b^2 c^2 + 4 a b c d + a^2 d^2) x^3 + \frac{1}{4} a (4 b^3 c^3 + 18 a b^2 c^2 d + 12 a^2 b c d^2 + a^3 d^3) x^4 +$$

$$\frac{1}{5} b (b^3 c^3 + 12 a b^2 c^2 d + 18 a^2 b c d^2 + 4 a^3 d^3) x^5 + \frac{1}{2} b^2 d (b^2 c^2 + 4 a b c d + 2 a^2 d^2) x^6 + \frac{1}{7} b^3 d^2 (3 b c + 4 a d) x^7 + \frac{1}{8} b^4 d^3 x^8$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a+bx)^5 (c+dx)^3, x, 4, 0 \right\}$$

$$\frac{(bc-ad)^3 (a+bx)^6}{504 b^4} + \frac{(bc-ad)^2 (a+bx)^6 (c+dx)}{84 b^3} + \frac{(bc-ad) (a+bx)^6 (c+dx)^2}{24 b^2} + \frac{(a+bx)^6 (c+dx)^3}{9 b}$$

$$\frac{1}{504} x \left( 126 a^5 \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) + 126 a^4 b x \left( 10 c^3 + 20 c^2 d x + 15 c d^2 x^2 + 4 d^3 x^3 \right) + \right.$$

$$84 a^3 b^2 x^2 \left( 20 c^3 + 45 c^2 d x + 36 c d^2 x^2 + 10 d^3 x^3 \right) + 36 a^2 b^3 x^3 \left( 35 c^3 + 84 c^2 d x + 70 c d^2 x^2 + 20 d^3 x^3 \right) +$$

$$9 a b^4 x^4 \left( 56 c^3 + 140 c^2 d x + 120 c d^2 x^2 + 35 d^3 x^3 \right) + b^5 x^5 \left( 84 c^3 + 216 c^2 d x + 189 c d^2 x^2 + 56 d^3 x^3 \right) \Big)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^6}{(c+dx)^2}, x, 8, 0 \right\}$$

$$\frac{5 b^2 (bc-ad)^4 x}{d^6} - \frac{2 b (bc-ad)^3 (a+bx)^2}{d^5} + \frac{b (bc-ad)^2 (a+bx)^3}{d^4} -$$

$$\frac{b (bc-ad) (a+bx)^4}{2 d^3} + \frac{b (a+bx)^5}{5 d^2} - \frac{(bc-ad)^6}{d^7 (c+dx)} - \frac{6 b (bc-ad)^5 \text{Log}[c+dx]}{d^7}$$

$$\frac{1}{10 d^7 (c+dx)} \left( 60 a^5 b c d^5 - 10 a^6 d^6 + 150 a^4 b^2 d^4 \left( -c^2 + c d x + d^2 x^2 \right) + 100 a^3 b^3 d^3 \left( 2 c^3 - 4 c^2 d x - 3 c d^2 x^2 + d^3 x^3 \right) + \right.$$

$$50 a^2 b^4 d^2 \left( -3 c^4 + 9 c^3 d x + 6 c^2 d^2 x^2 - 2 c d^3 x^3 + d^4 x^4 \right) + 5 a b^5 d \left( 12 c^5 - 48 c^4 d x - 30 c^3 d^2 x^2 + 10 c^2 d^3 x^3 - 5 c d^4 x^4 + 3 d^5 x^5 \right) +$$

$$b^6 \left( -10 c^6 + 50 c^5 d x + 30 c^4 d^2 x^2 - 10 c^3 d^3 x^3 + 5 c^2 d^4 x^4 - 3 c d^5 x^5 + 2 d^6 x^6 \right) - 60 b (bc-ad)^5 (c+dx) \text{Log}[c+dx] \Big)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^5}{(c+dx)^8}, x, 2, 0 \right\}$$

$$\frac{(a+bx)^6}{7 (bc-ad) (c+dx)^7} + \frac{b (a+bx)^6}{42 (bc-ad)^2 (c+dx)^6}$$

$$- \frac{1}{42 d^6 (c+dx)^7} \left( 6 a^5 d^5 + 5 a^4 b d^4 (c+7 d x) + 4 a^3 b^2 d^3 (c^2 + 7 c d x + 21 d^2 x^2) + 3 a^2 b^3 d^2 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + \right.$$

$$2 a b^4 d (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + b^5 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) \Big)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^6}{(c+dx)^8}, x, 1, 0 \right\}$$

$$\frac{(a+bx)^7}{7 (bc-ad) (c+dx)^7}$$

$$- \frac{1}{7 d^7 (c+dx)^7} \left( a^6 d^6 + a^5 b d^5 (c+7 d x) + a^4 b^2 d^4 (c^2 + 7 c d x + 21 d^2 x^2) + a^3 b^3 d^3 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + \right.$$

$$a^2 b^4 d^2 (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + a b^5 d (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) +$$

$$b^6 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) \Big)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^8}{(c+dx)^8}, x, 10, 0 \right\}$$

$$\begin{aligned}
& \frac{b^8 x}{d^8} - \frac{(b c - a d)^8}{7 d^9 (c + d x)^7} + \frac{4 b (b c - a d)^7}{3 d^9 (c + d x)^6} - \frac{28 b^2 (b c - a d)^6}{5 d^9 (c + d x)^5} + \frac{14 b^3 (b c - a d)^5}{d^9 (c + d x)^4} - \\
& \frac{70 b^4 (b c - a d)^4}{3 d^9 (c + d x)^3} + \frac{28 b^5 (b c - a d)^3}{d^9 (c + d x)^2} - \frac{28 b^6 (b c - a d)^2}{d^9 (c + d x)} - \frac{8 b^7 (b c - a d) \operatorname{Log}[c + d x]}{d^9} \\
& - \frac{1}{105 d^9 (c + d x)^7} \\
& \left( 15 a^8 d^8 + 20 a^7 b d^7 (c + 7 d x) + 28 a^6 b^2 d^6 (c^2 + 7 c d x + 21 d^2 x^2) + 42 a^5 b^3 d^5 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + 70 a^4 b^4 d^4 \right. \\
& \quad \left( c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4 \right) + 140 a^3 b^5 d^3 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \\
& \quad 420 a^2 b^6 d^2 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \\
& \quad 2 a b^7 c d (1089 c^6 + 7203 c^5 d x + 20139 c^4 d^2 x^2 + 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + 13230 c d^5 x^5 + 2940 d^6 x^6) + b^8 \\
& \quad (1443 c^8 + 9261 c^7 d x + 24843 c^6 d^2 x^2 + 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) + \\
& \quad \left. 840 b^7 (b c - a d) (c + d x)^7 \operatorname{Log}[c + d x] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{(a + b x)^9}{(c + d x)^8}, x, 11, 0 \right\} \\
& - \frac{8 b^8 (b c - a d) x}{d^9} + \frac{b^7 (a + b x)^2}{2 d^8} + \frac{(b c - a d)^9}{7 d^{10} (c + d x)^7} - \frac{3 b (b c - a d)^8}{2 d^{10} (c + d x)^6} + \frac{36 b^2 (b c - a d)^7}{5 d^{10} (c + d x)^5} - \\
& \frac{21 b^3 (b c - a d)^6}{d^{10} (c + d x)^4} + \frac{42 b^4 (b c - a d)^5}{d^{10} (c + d x)^3} - \frac{63 b^5 (b c - a d)^4}{d^{10} (c + d x)^2} + \frac{84 b^6 (b c - a d)^3}{d^{10} (c + d x)} + \frac{36 b^7 (b c - a d)^2 \operatorname{Log}[c + d x]}{d^{10}} \\
& - \frac{1}{70 d^{10} (c + d x)^7} \\
& \left( 10 a^9 d^9 + 15 a^8 b d^8 (c + 7 d x) + 24 a^7 b^2 d^7 (c^2 + 7 c d x + 21 d^2 x^2) + 42 a^6 b^3 d^6 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + 84 a^5 b^4 d^5 \right. \\
& \quad \left( c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4 \right) + 210 a^4 b^5 d^4 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \\
& \quad 840 a^3 b^6 d^3 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \\
& \quad 6 a^2 b^7 c d^2 (1089 c^6 + 7203 c^5 d x + 20139 c^4 d^2 x^2 + 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + 13230 c d^5 x^5 + 2940 d^6 x^6) + 6 a b^8 d \\
& \quad (1443 c^8 + 9261 c^7 d x + 24843 c^6 d^2 x^2 + 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) - \\
& \quad b^9 (3349 c^9 + 20923 c^8 d x + 53949 c^7 d^2 x^2 + 72275 c^6 d^3 x^3 + 50225 c^5 d^4 x^4 + 12495 c^4 d^5 x^5 - \\
& \quad 4655 c^3 d^6 x^6 - 3185 c^2 d^7 x^7 - 315 c d^8 x^8 + 35 d^9 x^9) - 2520 b^7 (b c - a d)^2 (c + d x)^7 \operatorname{Log}[c + d x] \Big)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{(a + b x)^7}{(c + d x)^2}, x, 9, 0 \right\} \\
& - \frac{6 b^2 (b c - a d)^5 x}{d^7} + \frac{5 b (b c - a d)^4 (a + b x)^2}{2 d^6} - \frac{4 b (b c - a d)^3 (a + b x)^3}{3 d^5} + \\
& \frac{3 b (b c - a d)^2 (a + b x)^4}{4 d^4} - \frac{2 b (b c - a d) (a + b x)^5}{5 d^3} + \frac{b (a + b x)^6}{6 d^2} + \frac{(b c - a d)^7}{d^8 (c + d x)} + \frac{7 b (b c - a d)^6 \operatorname{Log}[c + d x]}{d^8} \\
& - \frac{1}{60 d^8 (c + d x)} \\
& \left( 420 a^6 b c d^6 - 60 a^7 d^7 + 1260 a^5 b^2 d^5 (-c^2 + c d x + d^2 x^2) + 1050 a^4 b^3 d^4 (2 c^3 - 4 c^2 d x - 3 c d^2 x^2 + d^3 x^3) + 700 a^3 b^4 d^3 \right. \\
& \quad \left( -3 c^4 + 9 c^3 d x + 6 c^2 d^2 x^2 - 2 c d^3 x^3 + d^4 x^4 \right) + 105 a^2 b^5 d^2 (12 c^5 - 48 c^4 d x - 30 c^3 d^2 x^2 + 10 c^2 d^3 x^3 - 5 c d^4 x^4 + 3 d^5 x^5) + \\
& \quad 42 a b^6 d (-10 c^6 + 50 c^5 d x + 30 c^4 d^2 x^2 - 10 c^3 d^3 x^3 + 5 c^2 d^4 x^4 - 3 c d^5 x^5 + 2 d^6 x^6) + \\
& \quad b^7 (60 c^7 - 360 c^6 d x - 210 c^5 d^2 x^2 + 70 c^4 d^3 x^3 - 35 c^3 d^4 x^4 + 21 c^2 d^5 x^5 - 14 c d^6 x^6 + 10 d^7 x^7) + \\
& \quad \left. 420 b (b c - a d)^6 (c + d x) \operatorname{Log}[c + d x] \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^7}{(c+dx)^3}, x, 9, 0 \right\}$$

$$\frac{15b^3(bc-ad)^4x}{d^7} - \frac{5b^2(bc-ad)^3(a+bx)^2}{d^6} + \frac{2b^2(bc-ad)^2(a+bx)^3}{d^5} -$$

$$\frac{3b^2(bc-ad)(a+bx)^4}{4d^4} + \frac{b^2(a+bx)^5}{5d^3} + \frac{(bc-ad)^7}{2d^8(c+dx)^2} - \frac{7b(bc-ad)^6}{d^8(c+dx)} - \frac{21b^2(bc-ad)^5 \text{Log}[c+dx]}{d^8}$$

$$1$$

$$20d^8(c+dx)^2$$

$$\begin{aligned} & (-10a^7d^7 - 70a^6bd^6(c+2dx) + 210a^5b^2cd^5(3c+4dx) + 350a^4b^3d^4(-5c^3 - 4c^2dx + 4cd^2x^2 + 2d^3x^3) + 350a^3b^4d^3 \\ & (7c^4 + 2c^3dx - 11c^2d^2x^2 - 4cd^3x^3 + d^4x^4) + 70a^2b^5d^2(-27c^5 + 6c^4dx + 63c^3d^2x^2 + 20c^2d^3x^3 - 5cd^4x^4 + 2d^5x^5) + \\ & 35ab^6d(22c^6 - 16c^5dx - 68c^4d^2x^2 - 20c^3d^3x^3 + 5c^2d^4x^4 - 2cd^5x^5 + d^6x^6) + \\ & b^7(-130c^7 + 160c^6dx + 500c^5d^2x^2 + 140c^4d^3x^3 - 35c^3d^4x^4 + 14c^2d^5x^5 - 7cd^6x^6 + 4d^7x^7) - \\ & 420b^2(bc-ad)^5(c+dx)^2 \text{Log}[c+dx]) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^7}{(c+dx)^6}, x, 9, 0 \right\}$$

$$-\frac{6b^6(bc-ad)x}{d^7} + \frac{b^5(a+bx)^2}{2d^6} + \frac{(bc-ad)^7}{5d^8(c+dx)^5} - \frac{7b(bc-ad)^6}{4d^8(c+dx)^4} +$$

$$\frac{7b^2(bc-ad)^5}{d^8(c+dx)^3} - \frac{35b^3(bc-ad)^4}{2d^8(c+dx)^2} + \frac{35b^4(bc-ad)^3}{d^8(c+dx)} + \frac{21b^5(bc-ad)^2 \text{Log}[c+dx]}{d^8}$$

$$1$$

$$20d^8(c+dx)^5$$

$$\begin{aligned} & (4a^7d^7 + 7a^6bd^6(c+5dx) + 14a^5b^2d^5(c^2 + 5cdx + 10d^2x^2) + 35a^4b^3d^4(c^3 + 5c^2dx + 10cd^2x^2 + 10d^3x^3) + 140a^3b^4d^3 \\ & (c^4 + 5c^3dx + 10c^2d^2x^2 + 10cd^3x^3 + 5d^4x^4) - 7a^2b^5cd^2(137c^4 + 625c^3dx + 1100c^2d^2x^2 + 900cd^3x^3 + 300d^4x^4) + \\ & 14ab^6d(87c^6 + 375c^5dx + 600c^4d^2x^2 + 400c^3d^3x^3 + 50c^2d^4x^4 - 50cd^5x^5 - 10d^6x^6) - \\ & b^7(459c^7 + 1875c^6dx + 2700c^5d^2x^2 + 1300c^4d^3x^3 - 400c^3d^4x^4 - 500c^2d^5x^5 - 70cd^6x^6 + 10d^7x^7) - \\ & 420b^5(bc-ad)^2(c+dx)^5 \text{Log}[c+dx]) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^7}{(c+dx)^7}, x, 9, 0 \right\}$$

$$\frac{b^7x}{d^7} + \frac{(bc-ad)^7}{6d^8(c+dx)^6} - \frac{7b(bc-ad)^6}{5d^8(c+dx)^5} + \frac{21b^2(bc-ad)^5}{4d^8(c+dx)^4} -$$

$$\frac{35b^3(bc-ad)^4}{3d^8(c+dx)^3} + \frac{35b^4(bc-ad)^3}{2d^8(c+dx)^2} - \frac{21b^5(bc-ad)^2}{d^8(c+dx)} - \frac{7b^6(bc-ad) \text{Log}[c+dx]}{d^8}$$

$$1$$

$$60d^8(c+dx)^6$$

$$\begin{aligned} & (10a^7d^7 + 14a^6bd^6(c+6dx) + 21a^5b^2d^5(c^2 + 6cdx + 15d^2x^2) + 35a^4b^3d^4(c^3 + 6c^2dx + 15cd^2x^2 + 20d^3x^3) + 70a^3b^4d^3 \\ & (c^4 + 6c^3dx + 15c^2d^2x^2 + 20cd^3x^3 + 15d^4x^4) + 210a^2b^5d^2(c^5 + 6c^4dx + 15c^3d^2x^2 + 20c^2d^3x^3 + 15cd^4x^4 + 6d^5x^5) - \\ & 7ab^6cd(147c^5 + 822c^4dx + 1875c^3d^2x^2 + 2200c^2d^3x^3 + 1350cd^4x^4 + 360d^5x^5) + \\ & b^7(669c^7 + 3594c^6dx + 7725c^5d^2x^2 + 8200c^4d^3x^3 + 4050c^3d^4x^4 + 360c^2d^5x^5 - 360cd^6x^6 - 60d^7x^7) + \\ & 420b^6(bc-ad)(c+dx)^6 \text{Log}[c+dx]) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a+bx)^7}{(c+dx)^9}, x, 1, 0 \right\}$$

$$\frac{(a + b x)^8}{8 (b c - a d) (c + d x)^8} - \frac{1}{8 d^8 (c + d x)^8} \left( a^7 d^7 + a^6 b d^6 (c + 8 d x) + a^5 b^2 d^5 (c^2 + 8 c d x + 28 d^2 x^2) + a^4 b^3 d^4 (c^3 + 8 c^2 d x + 28 c d^2 x^2 + 56 d^3 x^3) + a^3 b^4 d^3 (c^4 + 8 c^3 d x + 28 c^2 d^2 x^2 + 56 c d^3 x^3 + 70 d^4 x^4) + a^2 b^5 d^2 (c^5 + 8 c^4 d x + 28 c^3 d^2 x^2 + 56 c^2 d^3 x^3 + 70 c d^4 x^4 + 56 d^5 x^5) + a b^6 d (c^6 + 8 c^5 d x + 28 c^4 d^2 x^2 + 56 c^3 d^3 x^3 + 70 c^2 d^4 x^4 + 56 c d^5 x^5 + 28 d^6 x^6) + b^7 (c^7 + 8 c^6 d x + 28 c^5 d^2 x^2 + 56 c^4 d^3 x^3 + 70 c^3 d^4 x^4 + 56 c^2 d^5 x^5 + 28 c d^6 x^6 + 8 d^7 x^7) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + b x)^7}{(c + d x)^{10}}, x, 2, 0 \right\} \frac{(a + b x)^8}{9 (b c - a d) (c + d x)^9} + \frac{b (a + b x)^8}{72 (b c - a d)^2 (c + d x)^8} - \frac{1}{72 d^8 (c + d x)^9} \left( 8 a^7 d^7 + 7 a^6 b d^6 (c + 9 d x) + 6 a^5 b^2 d^5 (c^2 + 9 c d x + 36 d^2 x^2) + 5 a^4 b^3 d^4 (c^3 + 9 c^2 d x + 36 c d^2 x^2 + 84 d^3 x^3) + 4 a^3 b^4 d^3 (c^4 + 9 c^3 d x + 36 c^2 d^2 x^2 + 84 c d^3 x^3 + 126 d^4 x^4) + 3 a^2 b^5 d^2 (c^5 + 9 c^4 d x + 36 c^3 d^2 x^2 + 84 c^2 d^3 x^3 + 126 c d^4 x^4 + 126 d^5 x^5) + 2 a b^6 d (c^6 + 9 c^5 d x + 36 c^4 d^2 x^2 + 84 c^3 d^3 x^3 + 126 c^2 d^4 x^4 + 126 c d^5 x^5 + 84 d^6 x^6) + b^7 (c^7 + 9 c^6 d x + 36 c^5 d^2 x^2 + 84 c^4 d^3 x^3 + 126 c^3 d^4 x^4 + 126 c^2 d^5 x^5 + 84 c d^6 x^6 + 36 d^7 x^7) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{2}{-1 + 4 x^2}, x, 2, 0 \right\} - \text{ArcTanh}[2 x] \frac{1}{2} (\text{Log}[1 - 2 x] - \text{Log}[1 + 2 x])$$

## Rational function problems involving binomials

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + b x^2)^5}{x^{13}}, x, 1, 0 \right\}$$

$$- \frac{(a + b x^2)^6}{12 a x^{12}}$$

$$- \frac{a^5 + 6 a^4 b x^2 + 15 a^3 b^2 x^4 + 20 a^2 b^3 x^6 + 15 a b^4 x^8 + 6 b^5 x^{10}}{12 x^{12}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{a + b x^2}{1 - x^2}, x, 3, 0 \right\}$$

$$- b x + (a + b) \operatorname{ArcTanh}[x]$$

$$\frac{1}{2} (-2 b x - (a + b) \operatorname{Log}[-1 + x] + (a + b) \operatorname{Log}[1 + x])$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^2 (a + b x^3)^3, x, 2, 0 \right\}$$

$$\frac{(a + b x^3)^4}{12 b}$$

$$\frac{1}{12} x^3 (4 a^3 + 6 a^2 b x^3 + 4 a b^2 x^6 + b^3 x^9)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(1 + x)^2}{c + d x^3}, x, 12, 0 \right\}$$

$$- \frac{2 \operatorname{ArcTan}\left[\frac{c^{1/3} - 2 d^{1/3} x}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{1/3} d^{2/3}} - \frac{\operatorname{ArcTan}\left[\frac{c^{1/3} - 2 d^{1/3} x}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} d^{1/3}} - \frac{2 \operatorname{Log}[c^{1/3} + d^{1/3} x]}{3 c^{1/3} d^{2/3}} +$$

$$\frac{\operatorname{Log}[c^{1/3} + d^{1/3} x]}{3 c^{2/3} d^{1/3}} + \frac{\operatorname{Log}[c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2]}{3 c^{1/3} d^{2/3}} - \frac{\operatorname{Log}[c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2]}{6 c^{2/3} d^{1/3}} + \frac{\operatorname{Log}[c + d x^3]}{3 d}$$

$$\frac{\operatorname{RootSum}\left[c - d + 3 d \sqrt[3]{1} - 3 d \sqrt[3]{1}^2 + d \sqrt[3]{1}^3 \&, \frac{\operatorname{Log}[1 + x - \sqrt[3]{1}] \sqrt[3]{1}^2}{1 - 2 \sqrt[3]{1} + \sqrt[3]{1}^2} \&\right]}{3 d}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(1 + x)^3}{c + d x^3}, x, 12, 0 \right\}$$

$$\frac{x}{d} + \frac{(c - d) \operatorname{ArcTan}\left[\frac{c^{1/3} - 2 d^{1/3} x}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} d^{4/3}} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{c^{1/3} - 2 d^{1/3} x}{\sqrt{3} c^{1/3}}\right]}{c^{1/3} d^{2/3}} - \frac{(c - d) \operatorname{Log}[c^{1/3} + d^{1/3} x]}{3 c^{2/3} d^{4/3}} -$$

$$\frac{\operatorname{Log}[c^{1/3} + d^{1/3} x]}{c^{1/3} d^{2/3}} + \frac{(c - d) \operatorname{Log}[c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2]}{6 c^{2/3} d^{4/3}} + \frac{\operatorname{Log}[c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2]}{2 c^{1/3} d^{2/3}} + \frac{\operatorname{Log}[c + d x^3]}{d}$$

$$\frac{1+x}{d} + \frac{\text{RootSum}\left[c-d+3d\sqrt[3]{1-3d\sqrt[3]{1}^2+d\sqrt[3]{1}^3} \&, \frac{-c\text{Log}[1+x-\sqrt[3]{1}]+d\text{Log}[1+x-\sqrt[3]{1}]-3d\text{Log}[1+x-\sqrt[3]{1}]\sqrt[3]{1}+3d\text{Log}[1+x-\sqrt[3]{1}]\sqrt[3]{1}^2}{1-2\sqrt[3]{1}+\sqrt[3]{1}^2}\right]}{3d^2}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{(1+x)^4}{c+dx^3}, x, 13, 0\right\}$$

$$\frac{4x}{d} + \frac{x^2}{2d} + \frac{(c-4d)\text{ArcTan}\left[\frac{c^{1/3}-2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{1/3}d^{5/3}} + \frac{(4c-d)\text{ArcTan}\left[\frac{c^{1/3}-2d^{1/3}x}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{(c-4d)\text{Log}\left[c^{1/3}+d^{1/3}x\right]}{3c^{1/3}d^{5/3}} - \frac{(4c-d)\text{Log}\left[c^{1/3}+d^{1/3}x\right]}{3c^{2/3}d^{4/3}} -$$

$$\frac{(c-4d)\text{Log}\left[c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2\right]}{6c^{1/3}d^{5/3}} + \frac{(4c-d)\text{Log}\left[c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2\right]}{6c^{2/3}d^{4/3}} + \frac{2\text{Log}\left[c+dx^3\right]}{d}$$

$$\frac{3d(7+8x+x^2)+2\text{RootSum}\left[c-d+3d\sqrt[3]{1-3d\sqrt[3]{1}^2+d\sqrt[3]{1}^3} \&, \frac{-3c\text{Log}[1+x-\sqrt[3]{1}]+3d\text{Log}[1+x-\sqrt[3]{1}]-c\text{Log}[1+x-\sqrt[3]{1}]\sqrt[3]{1}-8d\text{Log}[1+x-\sqrt[3]{1}]\sqrt[3]{1}+6d\text{Log}[1+x-\sqrt[3]{1}]\sqrt[3]{1}^2}{1-2\sqrt[3]{1}+\sqrt[3]{1}^2}\right] \&}{6d^2}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{x^3}{a+b(c+dx)^3}, x, 13, 0\right\}$$

$$\frac{c+dx}{bd^4} - \frac{\sqrt{3}c^2\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{a^{1/3}b^{2/3}d^4} + \frac{(a+b c^3)\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{c^2\text{Log}\left[a^{1/3}+b^{1/3}(c+dx)\right]}{a^{1/3}b^{2/3}d^4} -$$

$$\frac{(a+b c^3)\text{Log}\left[a^{1/3}+b^{1/3}(c+dx)\right]}{3a^{2/3}b^{4/3}d^4} + \frac{c^2\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2\right]}{2a^{1/3}b^{2/3}d^4} +$$

$$\frac{(a+b c^3)\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2\right]}{6a^{2/3}b^{4/3}d^4} - \frac{c\text{Log}\left[a+b(c+dx)^3\right]}{bd^4}$$

$$- \frac{-3bdx + \text{RootSum}\left[a+b c^3+3b c^2 d\sqrt[3]{1}+3b c d^2\sqrt[3]{1}^2+b d^3\sqrt[3]{1}^3 \&, \frac{a\text{Log}[x-\sqrt[3]{1}]+b c^3\text{Log}[x-\sqrt[3]{1}]+3b c^2 d\text{Log}[x-\sqrt[3]{1}]\sqrt[3]{1}+3b c d^2\text{Log}[x-\sqrt[3]{1}]\sqrt[3]{1}^2}{c^2+2c d\sqrt[3]{1}+d^2\sqrt[3]{1}^2}\right] \&}{3b^2d^4}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{x^2}{a+b(c+dx)^3}, x, 13, 0\right\}$$

$$\frac{2c\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{1/3}b^{2/3}d^3} - \frac{c^2\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{2/3}b^{1/3}d^3} + \frac{2c\text{Log}\left[a^{1/3}+b^{1/3}(c+dx)\right]}{3a^{1/3}b^{2/3}d^3} + \frac{c^2\text{Log}\left[a^{1/3}+b^{1/3}(c+dx)\right]}{3a^{2/3}b^{1/3}d^3} -$$

$$\frac{c\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2\right]}{3a^{1/3}b^{2/3}d^3} - \frac{c^2\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2\right]}{6a^{2/3}b^{1/3}d^3} + \frac{\text{Log}\left[a+b(c+dx)^3\right]}{3bd^3}$$

$$\frac{\text{RootSum}\left[a+b c^3+3b c^2 d\sqrt[3]{1}+3b c d^2\sqrt[3]{1}^2+b d^3\sqrt[3]{1}^3 \&, \frac{\text{Log}[x-\sqrt[3]{1}]\sqrt[3]{1}^2}{c^2+2c d\sqrt[3]{1}+d^2\sqrt[3]{1}^2}\right] \&}{3bd}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{x^2(a+b(c+dx)^3)}, x, 15, 0\right\}$$



$$\begin{aligned}
& -\frac{1}{(a+bc^3)x} + \frac{b^{1/3}(a-2bc^3) \operatorname{d} \operatorname{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{1/3}(a+bc^3)^2} + \\
& \frac{b^{2/3}c(2a-bc^3) \operatorname{d} \operatorname{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+dx)}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{2/3}(a+bc^3)^2} - \frac{3bc^2 \operatorname{d} \operatorname{Log}[-dx]}{(a+bc^3)^2} + \frac{b^{1/3}(a-2bc^3) \operatorname{d} \operatorname{Log}[a^{1/3}+b^{1/3}(c+dx)]}{3a^{1/3}(a+bc^3)^2} - \\
& \frac{b^{2/3}c(2a-bc^3) \operatorname{d} \operatorname{Log}[a^{1/3}+b^{1/3}(c+dx)]}{3a^{2/3}(a+bc^3)^2} - \frac{b^{1/3}(a-2bc^3) \operatorname{d} \operatorname{Log}[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2]}{6a^{1/3}(a+bc^3)^2} + \\
& \frac{b^{2/3}c(2a-bc^3) \operatorname{d} \operatorname{Log}[a^{2/3}-a^{1/3}b^{1/3}(c+dx)+b^{2/3}(c+dx)^2]}{6a^{2/3}(a+bc^3)^2} + \frac{bc^2 \operatorname{d} \operatorname{Log}[a+b(c+dx)^3]}{(a+bc^3)^2} \\
& \frac{1}{3(a+bc^3)^2x} \left( -3(a+bc^3+3bc^2dx \operatorname{Log}[x]) + dx \operatorname{RootSum}\left[a+bc^3+3bc^2d\#1+3bc^2d^2\#1^2+b^3d^3\#1^3 \&, \right. \right. \\
& \left. \left. \frac{-3ac \operatorname{Log}[x-\#1]+6bc^4 \operatorname{Log}[x-\#1]-ad \operatorname{Log}[x-\#1]\#1+8bc^3d \operatorname{Log}[x-\#1]\#1+3bc^2d^2 \operatorname{Log}[x-\#1]\#1^2}{c^2+2cd\#1+d^2\#1^2} \& \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \{(c+dx)^3(a+b(c+dx)^3), x, 3, 0\} \\
& \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^7}{7d} \\
& c^3(a+bc^3)x + \frac{3}{2}c^2(a+2bc^3)dx^2 + c(a+5bc^3)d^2x^3 + \frac{1}{4}(a+20bc^3)d^3x^4 + 3bc^2d^4x^5 + bcd^5x^6 + \frac{1}{7}bd^6x^7
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \{(c+dx)^3(a+b(c+dx)^3)^2, x, 3, 0\} \\
& \frac{a^2(c+dx)^4}{4d} + \frac{2ab(c+dx)^7}{7d} + \frac{b^2(c+dx)^{10}}{10d} \\
& c^3(a+bc^3)^2x + \frac{3}{2}c^2(a^2+4abc^3+3b^2c^6)dx^2 + c(a^2+10abc^3+12b^2c^6)d^2x^3 + \frac{1}{4}(a^2+40abc^3+84b^2c^6)d^3x^4 + \\
& \frac{6}{5}bc^2(5a+21bc^3)d^4x^5 + bcd(2a+21bc^3)d^5x^6 + \frac{2}{7}b(a+42bc^3)d^6x^7 + \frac{9}{2}b^2c^2d^7x^8 + b^2cd^8x^9 + \frac{1}{10}b^2d^9x^{10}
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \{(c+dx)^3(a+b(c+dx)^3)^3, x, 3, 0\} \\
& \frac{a^3(c+dx)^4}{4d} + \frac{3a^2b(c+dx)^7}{7d} + \frac{3ab^2(c+dx)^{10}}{10d} + \frac{b^3(c+dx)^{13}}{13d} \\
& c^3(a+bc^3)^3x + \frac{3}{2}c^2(a+bc^3)^2(a+4bc^3)dx^2 + c(a^3+15a^2bc^3+36ab^2c^6+22b^3c^9)d^2x^3 + \\
& \frac{1}{4}(a^3+60a^2bc^3+252ab^2c^6+220b^3c^9)d^3x^4 + \frac{9}{5}bc^2(5a^2+42abc^3+55b^2c^6)d^4x^5 + \\
& 3bcd(a^2+21abc^3+44b^2c^6)d^5x^6 + \frac{3}{7}b(a^2+84abc^3+308b^2c^6)d^6x^7 + \frac{9}{2}b^2c^2(3a+22bc^3)d^7x^8 + \\
& b^2c(3a+55bc^3)d^8x^9 + \frac{1}{10}b^2(3a+220bc^3)d^9x^{10} + 6b^3c^2d^{10}x^{11} + b^3cd^{11}x^{12} + \frac{1}{13}b^3d^{12}x^{13}
\end{aligned}$$

# Mathematica 7 Test Results for Rational Function Integration Problems

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x^3 (a - b x^4)}, x, 5, 0 \right\}$$

$$-\frac{1}{2 a x^2} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right]}{2 a^{3/2}}$$

$$\frac{-2 \sqrt{a} - \sqrt{b} x^2 \operatorname{Log}\left[-a^{1/4} + b^{1/4} x\right] - \sqrt{b} x^2 \operatorname{Log}\left[a^{1/4} + b^{1/4} x\right] + \sqrt{b} x^2 \operatorname{Log}\left[\sqrt{a} + \sqrt{b} x^2\right]}{4 a^{3/2} x^2}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{1 + a + (-1 + a) x^4}, x, 3, 0 \right\}$$

$$\frac{\operatorname{ArcTan}\left[\frac{(1-a)^{1/4} x}{(1+a)^{1/4}}\right]}{2 (1-a)^{1/4} (1+a)^{3/4}} + \frac{\operatorname{ArcTanh}\left[\frac{(1-a)^{1/4} x}{(1+a)^{1/4}}\right]}{2 (1-a)^{1/4} (1+a)^{3/4}}$$

$$\frac{1}{4 \sqrt{2} (-1+a)^{1/4} (1+a)^{3/4}} \left( -2 \operatorname{ArcTan}\left[1 - \sqrt{2} \left(\frac{-1+a}{1+a}\right)^{1/4} x\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \left(\frac{-1+a}{1+a}\right)^{1/4} x\right] - \right.$$

$$\left. \operatorname{Log}\left[\sqrt{1+a} - \sqrt{2} (-1+a)^{1/4} (1+a)^{1/4} x + \sqrt{-1+a} x^2\right] + \operatorname{Log}\left[\sqrt{1+a} + \sqrt{2} (-1+a)^{1/4} (1+a)^{1/4} x + \sqrt{-1+a} x^2\right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^3 (a + b x^4)^3, x, 2, 0 \right\}$$

$$\frac{(a + b x^4)^4}{16 b}$$

$$\frac{1}{16} x^4 (4 a^3 + 6 a^2 b x^4 + 4 a b^2 x^8 + b^3 x^{12})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1 + x^2}{1 - x^4}, x, 2, 0 \right\}$$

$$\operatorname{ArcTanh}[x]$$

$$\frac{1}{2} (\operatorname{Log}[-1 - x] - \operatorname{Log}[-1 + x])$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^3}{a + b (c + d x)^4}, x, 12, 0 \right\}$$

$$\begin{aligned}
& \frac{3 c^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{2 \sqrt{a} \sqrt{b} d^4} + \frac{c \left(3 \sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[\frac{a^{1/4} - \sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \\
& \frac{c \left(3 \sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[\frac{a^{1/4} + \sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \frac{c \left(3 \sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \\
& \frac{c \left(3 \sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{\operatorname{Log}\left[a + b (c+dx)^4\right]}{4 b d^4} \\
& \frac{\operatorname{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\operatorname{Log}[x-\#1] \#1^3}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \&\right]}{4 b d}
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{x^2}{a + b (c+dx)^4}, x, 10, 0 \right\} \\
& - \frac{c \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b} d^3} - \frac{\left(\sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[\frac{a^{1/4} - \sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3} + \\
& \frac{\left(\sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[\frac{a^{1/4} + \sqrt{2} b^{1/4} (c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3} + \frac{\left(\sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3} - \\
& \frac{\left(\sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3} \\
& \frac{\operatorname{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\operatorname{Log}[x-\#1] \#1^2}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \&\right]}{4 b d}
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{x}{a + b (c+dx)^4}, x, 8, 0 \right\} \\
& \frac{c \operatorname{ArcTan}\left[\frac{b^{1/4} (c+dx)}{(-a)^{1/4}}\right]}{2 (-a)^{3/4} b^{1/4} d^2} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right]}{2 \sqrt{a} \sqrt{b} d^2} + \frac{c \operatorname{ArcTanh}\left[\frac{b^{1/4} (c+dx)}{(-a)^{1/4}}\right]}{2 (-a)^{3/4} b^{1/4} d^2} \\
& \frac{1}{8 a^{3/4} \sqrt{b} d^2} \\
& \left( \left( 4 a^{1/4} - 2 \sqrt{2} b^{1/4} c \right) \operatorname{ArcTan}\left[\frac{-\sqrt{2} a^{1/4} + 2 b^{1/4} (c+dx)}{\sqrt{2} a^{1/4}}\right] - 2 \left( 2 a^{1/4} + \sqrt{2} b^{1/4} c \right) \operatorname{ArcTan}\left[\frac{\sqrt{2} a^{1/4} + 2 b^{1/4} (c+dx)}{\sqrt{2} a^{1/4}}\right] + \right. \\
& \left. \sqrt{2} b^{1/4} c \left( \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right] - \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+dx) + \sqrt{b} (c+dx)^2\right] \right) \right)
\end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\begin{aligned}
& \left\{ \frac{1}{a + b (c+dx)^4}, x, 4, 0 \right\} \\
& - \frac{\operatorname{ArcTan}\left[\frac{b^{1/4} (c+dx)}{(-a)^{1/4}}\right]}{2 (-a)^{3/4} b^{1/4} d} - \frac{\operatorname{ArcTanh}\left[\frac{b^{1/4} (c+dx)}{(-a)^{1/4}}\right]}{2 (-a)^{3/4} b^{1/4} d}
\end{aligned}$$

$$\frac{1}{4 \sqrt{2} a^{3/4} b^{1/4} d} \left( 2 \operatorname{ArcTan} \left[ \frac{-\sqrt{2} a^{1/4} + 2 b^{1/4} (c + d x)}{\sqrt{2} a^{1/4}} \right] + 2 \operatorname{ArcTan} \left[ \frac{\sqrt{2} a^{1/4} + 2 b^{1/4} (c + d x)}{\sqrt{2} a^{1/4}} \right] - \right. \\ \left. \operatorname{Log} \left[ \sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2 \right] + \operatorname{Log} \left[ \sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2 \right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x^2 (a + b (c + d x)^4)}, x, 15, 0 \right\} \\ - \frac{1}{(a + b c^4) x} - \frac{b^{1/4} (a - 3 b c^4) d \operatorname{ArcTan} \left[ \frac{b^{1/4} (c + d x)}{(-a)^{1/4}} \right]}{2 (-a)^{1/4} (a + b c^4)^2} + \frac{b^{3/4} c^2 (3 a - b c^4) d \operatorname{ArcTan} \left[ \frac{b^{1/4} (c + d x)}{(-a)^{1/4}} \right]}{2 (-a)^{3/4} (a + b c^4)^2} - \\ \frac{\sqrt{b} c (a - b c^4) d \operatorname{ArcTan} \left[ \frac{\sqrt{b} (c + d x)^2}{\sqrt{a}} \right]}{\sqrt{a} (a + b c^4)^2} + \frac{b^{1/4} (a - 3 b c^4) d \operatorname{ArcTanh} \left[ \frac{b^{1/4} (c + d x)}{(-a)^{1/4}} \right]}{2 (-a)^{1/4} (a + b c^4)^2} + \\ \frac{b^{3/4} c^2 (3 a - b c^4) d \operatorname{ArcTanh} \left[ \frac{b^{1/4} (c + d x)}{(-a)^{1/4}} \right]}{2 (-a)^{3/4} (a + b c^4)^2} - \frac{4 b c^3 d \operatorname{Log} [-d x]}{(a + b c^4)^2} + \frac{b c^3 d \operatorname{Log} [a + b (c + d x)^4]}{(a + b c^4)^2} \\ \frac{1}{4 (a + b c^4)^2 x} \left( -4 (a + b c^4 + 4 b c^3 d x \operatorname{Log} [x]) + d x \operatorname{RootSum} [a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \right. \\ \left. (-6 a c^2 \operatorname{Log} [x - \#1] + 10 b c^6 \operatorname{Log} [x - \#1] - 4 a c d \operatorname{Log} [x - \#1] \#1 + 20 b c^5 d \operatorname{Log} [x - \#1] \#1 - a d^2 \operatorname{Log} [x - \#1] \#1^2 + \right. \\ \left. 15 b c^4 d^2 \operatorname{Log} [x - \#1] \#1^2 + 4 b c^3 d^3 \operatorname{Log} [x - \#1] \#1^3) / (c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3) \& \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (c + d x)^3 (a + b (c + d x)^4)^3, x, 2, 0 \right\} \\ \frac{(a + b (c + d x)^4)^4}{16 b d} \\ \frac{1}{16} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left( 4 a^3 + 6 a^2 b \left( 2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) + \right. \\ \left. 4 a b^2 \left( 3 c^8 + 12 c^7 d x + 34 c^6 d^2 x^2 + 60 c^5 d^3 x^3 + 71 c^4 d^4 x^4 + 56 c^3 d^5 x^5 + 28 c^2 d^6 x^6 + 8 c d^7 x^7 + d^8 x^8 \right) + \right. \\ \left. b^3 \left( 4 c^{12} + 24 c^{11} d x + 100 c^{10} d^2 x^2 + 280 c^9 d^3 x^3 + 566 c^8 d^4 x^4 + 848 c^7 d^5 x^5 + \right. \right. \\ \left. \left. 952 c^6 d^6 x^6 + 800 c^5 d^7 x^7 + 496 c^4 d^8 x^8 + 220 c^3 d^9 x^9 + 66 c^2 d^{10} x^{10} + 12 c d^{11} x^{11} + d^{12} x^{12} \right) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (c + d x)^3 (a + b (c + d x)^4)^2, x, 2, 0 \right\} \\ \frac{(a + b (c + d x)^4)^3}{12 b d} \\ \frac{1}{12} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left( 3 a^2 + 3 a b \left( 2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) + \right. \\ \left. b^2 \left( 3 c^8 + 12 c^7 d x + 34 c^6 d^2 x^2 + 60 c^5 d^3 x^3 + 71 c^4 d^4 x^4 + 56 c^3 d^5 x^5 + 28 c^2 d^6 x^6 + 8 c d^7 x^7 + d^8 x^8 \right) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (c + d x)^3 (a + b (c + d x)^4), x, 2, 0 \right\}$$

$$\frac{a (c + d x)^4}{4 d} + \frac{b (c + d x)^8}{8 d}$$

$$\frac{1}{8} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \left( 2 a + b \left( 2 c^4 + 4 c^3 d x + 6 c^2 d^2 x^2 + 4 c d^3 x^3 + d^4 x^4 \right) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2}{1 - x^6}, x, 2, 0 \right\}$$

$$\frac{\text{ArcTanh}[x^3]}{3}$$

$$\frac{1}{6} \left( \text{Log}[-1 - x^3] - \text{Log}[-1 + x^3] \right)$$

Incorrect antiderivative:

$$\left\{ \frac{1}{1 - x^7}, x, 9, 0 \right\}$$

$$\begin{aligned} & \frac{2}{7} \text{ArcTan}\left[\text{Sec}\left[\frac{\pi}{14}\right] \left(x + \sin\left[\frac{\pi}{14}\right]\right)\right] \cos\left[\frac{\pi}{14}\right] + \frac{2}{7} \text{ArcTan}\left[\text{Sec}\left[\frac{3\pi}{14}\right] \left(x - \sin\left[\frac{3\pi}{14}\right]\right)\right] \cos\left[\frac{3\pi}{14}\right] - \\ & \frac{1}{7} \text{Log}[1 - x] + \frac{1}{7} \cos\left[\frac{\pi}{7}\right] \text{Log}\left[1 + x^2 + 2x \cos\left[\frac{\pi}{7}\right]\right] + \frac{1}{7} \text{Log}\left[1 + x^2 + 2x \sin\left[\frac{\pi}{14}\right]\right] \sin\left[\frac{\pi}{14}\right] + \\ & \frac{2}{7} \text{ArcTan}\left[\left(x + \cos\left[\frac{\pi}{7}\right]\right) \csc\left[\frac{\pi}{7}\right]\right] \sin\left[\frac{\pi}{7}\right] - \frac{1}{7} \text{Log}\left[1 + x^2 - 2x \sin\left[\frac{3\pi}{14}\right]\right] \sin\left[\frac{3\pi}{14}\right] \\ & \frac{1}{7} \left( 2 \text{ArcTan}\left[x \text{Sec}\left[\frac{\pi}{14}\right] + \tan\left[\frac{\pi}{14}\right]\right] \cos\left[\frac{\pi}{14}\right] + \right. \\ & \quad \left. 2 \text{ArcTan}\left[\text{Sec}\left[\frac{3\pi}{14}\right] \left(x - \sin\left[\frac{3\pi}{14}\right]\right)\right] \cos\left[\frac{3\pi}{14}\right] - \text{Log}[-1 + x] + \cos\left[\frac{\pi}{7}\right] \text{Log}\left[1 + x^2 + 2x \cos\left[\frac{\pi}{7}\right]\right] + \right. \\ & \quad \left. \text{Log}\left[1 + x^2 + 2x \sin\left[\frac{\pi}{14}\right]\right] \sin\left[\frac{\pi}{14}\right] + 2 \text{ArcTan}\left[\cot\left[\frac{\pi}{7}\right] + x \csc\left[\frac{\pi}{7}\right]\right] \sin\left[\frac{\pi}{7}\right] - \text{Log}\left[1 + x^2 - 2x \sin\left[\frac{3\pi}{14}\right]\right] \sin\left[\frac{3\pi}{14}\right] \right) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{a + b x^8}, x, 7, 0 \right\}$$

$$\begin{aligned} & \frac{\left(-\sqrt{-a}\right)^{1/4} \text{ArcTan}\left[\frac{b^{1/8} x}{\left(-\sqrt{-a}\right)^{1/4}}\right]}{4 a b^{1/8}} - \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} b^{1/8}} + \frac{\left(-\sqrt{-a}\right)^{1/4} \text{ArcTanh}\left[\frac{b^{1/8} x}{\left(-\sqrt{-a}\right)^{1/4}}\right]}{4 a b^{1/8}} - \frac{\text{ArcTanh}\left[\frac{b^{1/8} x}{(-a)^{1/8}}\right]}{4 (-a)^{7/8} b^{1/8}} \\ & \frac{1}{8 a^{7/8} b^{1/8}} \left( 2 \text{ArcTan}\left[\frac{b^{1/8} x \text{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}} - \tan\left[\frac{\pi}{8}\right]\right] \cos\left[\frac{\pi}{8}\right] + 2 \text{ArcTan}\left[\frac{b^{1/8} x \text{Sec}\left[\frac{\pi}{8}\right]}{a^{1/8}} + \tan\left[\frac{\pi}{8}\right]\right] \cos\left[\frac{\pi}{8}\right] - \right. \\ & \quad \cos\left[\frac{\pi}{8}\right] \text{Log}\left[a^{1/4} + b^{1/4} x^2 - 2 a^{1/8} b^{1/8} x \cos\left[\frac{\pi}{8}\right]\right] + \cos\left[\frac{\pi}{8}\right] \text{Log}\left[a^{1/4} + b^{1/4} x^2 + 2 a^{1/8} b^{1/8} x \cos\left[\frac{\pi}{8}\right]\right] - \\ & \quad 2 \text{ArcTan}\left[\cot\left[\frac{\pi}{8}\right] - \frac{b^{1/8} x \csc\left[\frac{\pi}{8}\right]}{a^{1/8}}\right] \sin\left[\frac{\pi}{8}\right] + 2 \text{ArcTan}\left[\cot\left[\frac{\pi}{8}\right] + \frac{b^{1/8} x \csc\left[\frac{\pi}{8}\right]}{a^{1/8}}\right] \sin\left[\frac{\pi}{8}\right] - \\ & \quad \left. \text{Log}\left[a^{1/4} + b^{1/4} x^2 - 2 a^{1/8} b^{1/8} x \sin\left[\frac{\pi}{8}\right]\right] \sin\left[\frac{\pi}{8}\right] + \text{Log}\left[a^{1/4} + b^{1/4} x^2 + 2 a^{1/8} b^{1/8} x \sin\left[\frac{\pi}{8}\right]\right] \sin\left[\frac{\pi}{8}\right] \right) \end{aligned}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{\left(a + \frac{b}{x^2}\right)^3}{x^3}, x, 2, 0 \right\}$$

$$-\frac{\left(a + \frac{b}{x^2}\right)^4}{8b}$$

$$-\frac{b^3 + 4ab^2x^2 + 6a^2bx^4 + 4a^3x^6}{8x^8}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{a + \frac{b}{x^4}}, x, 5, 0 \right\}$$

$$\frac{x}{a} - \frac{(-b)^{1/4} \operatorname{ArcTan}\left[\frac{a^{1/4}x}{(-b)^{1/4}}\right]}{2a^{5/4}} - \frac{(-b)^{1/4} \operatorname{ArcTanh}\left[\frac{a^{1/4}x}{(-b)^{1/4}}\right]}{2a^{5/4}}$$

$$\frac{1}{8a^{5/4}} \left( 8a^{1/4}x + 2\sqrt{2}b^{1/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}a^{1/4}x}{b^{1/4}}\right] - 2\sqrt{2}b^{1/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}a^{1/4}x}{b^{1/4}}\right] + \right.$$

$$\left. \sqrt{2}b^{1/4} \operatorname{Log}\left[\sqrt{b} - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a}x^2\right] - \sqrt{2}b^{1/4} \operatorname{Log}\left[\sqrt{b} + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a}x^2\right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{-1-9n} (a + bx^n)^8, x, 1, 0 \right\}$$

$$-\frac{x^{-9n} (a + bx^n)^9}{9an}$$

$$-\frac{x^{-9n} (a^8 + 9a^7bx^n + 36a^6b^2x^{2n} + 84a^5b^3x^{3n} + 126a^4b^4x^{4n} + 126a^3b^5x^{5n} + 84a^2b^6x^{6n} + 36ab^7x^{7n} + 9b^8x^{8n})}{9n}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{(a + bx^3)^8}{x^{28}}, x, 1, 0 \right\}$$

$$-\frac{(a + bx^3)^9}{27ax^{27}}$$

$$-\frac{a^8 + 9a^7bx^3 + 36a^6b^2x^6 + 84a^5b^3x^9 + 126a^4b^4x^{12} + 126a^3b^5x^{15} + 84a^2b^6x^{18} + 36ab^7x^{21} + 9b^8x^{24}}{27x^{27}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{1 - (c + dx)^2}, x, 2, 0 \right\}$$

$$\frac{\operatorname{ArcTanh}[c + dx]}{d}$$

$$\frac{\operatorname{Log}[-1 - c - dx] - \operatorname{Log}[-1 + c + dx]}{2d}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{1 - (1 + x)^2}, x, 4, 0 \right\}$$

$$\text{ArcTanh}[1 + x]$$

$$\frac{1}{2} (-\text{Log}[x] + \text{Log}[2 + x])$$

## Rational function problems involving trinomials

Valid but unnecessarily complicated antiderivative:

$$\left\{x^{-1+n} (a + b x^n)^{16}, x, 2, 0\right\}$$

$$\frac{(a + b x^n)^{17}}{17 b n}$$

$$\frac{1}{17 n} x^n \left(17 a^{16} + 136 a^{15} b x^n + 680 a^{14} b^2 x^{2n} + 2380 a^{13} b^3 x^{3n} + 6188 a^{12} b^4 x^{4n} + 12376 a^{11} b^5 x^{5n} + 19448 a^{10} b^6 x^{6n} + 24310 a^9 b^7 x^{7n} + 24310 a^8 b^8 x^{8n} + 19448 a^7 b^9 x^{9n} + 12376 a^6 b^{10} x^{10n} + 6188 a^5 b^{11} x^{11n} + 2380 a^4 b^{12} x^{12n} + 680 a^3 b^{13} x^{13n} + 136 a^2 b^{14} x^{14n} + 17 a b^{15} x^{15n} + b^{16} x^{16n}\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{x^2 (a + b x^3)^{16}, x, 2, 0\right\}$$

$$\frac{(a + b x^3)^{17}}{51 b}$$

$$\frac{1}{51} x^3 \left(17 a^{16} + 136 a^{15} b x^3 + 680 a^{14} b^2 x^6 + 2380 a^{13} b^3 x^9 + 6188 a^{12} b^4 x^{12} + 12376 a^{11} b^5 x^{15} + 19448 a^{10} b^6 x^{18} + 24310 a^9 b^7 x^{21} + 24310 a^8 b^8 x^{24} + 19448 a^7 b^9 x^{27} + 12376 a^6 b^{10} x^{30} + 6188 a^5 b^{11} x^{33} + 2380 a^4 b^{12} x^{36} + 680 a^3 b^{13} x^{39} + 136 a^2 b^{14} x^{42} + 17 a b^{15} x^{45} + b^{16} x^{48}\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(b + 2 c x) (a + b x + c x^2)^{12}, x, 1, 0\right\}$$

$$\frac{1}{13} (a + b x + c x^2)^{13}$$

$$a^{12} x (b + c x) + 6 a^{11} x^2 (b + c x)^2 + 22 a^{10} x^3 (b + c x)^3 + 55 a^9 x^4 (b + c x)^4 + 99 a^8 x^5 (b + c x)^5 + 132 a^7 x^6 (b + c x)^6 + 132 a^6 x^7 (b + c x)^7 + 99 a^5 x^8 (b + c x)^8 + 55 a^4 x^9 (b + c x)^9 + 22 a^3 x^{10} (b + c x)^{10} + 6 a^2 x^{11} (b + c x)^{11} + a x^{12} (b + c x)^{12} + \frac{1}{13} x^{13} (b + c x)^{13}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(b + 2 c x + 3 d x^2) (a + b x + c x^2 + d x^3)^7, x, 2, 0\right\}$$

$$\frac{1}{8} (a + b x + c x^2 + d x^3)^8$$

$$\frac{1}{8} x (b + x (c + d x)) \left(8 a^7 + 28 a^6 x (b + x (c + d x)) + 56 a^5 x^2 (b + x (c + d x))^2 + 70 a^4 x^3 (b + x (c + d x))^3 + 56 a^3 x^4 (b + x (c + d x))^4 + 28 a^2 x^5 (b + x (c + d x))^5 + 8 a x^6 (b + x (c + d x))^6 + x^7 (b + x (c + d x))^7\right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(b + 3 d x^2) (a + b x + d x^3)^7, x, 2, 0\right\}$$

$$\frac{1}{8} (a + b x + d x^3)^8$$

$$\frac{1}{8} x (b + d x^2) \left(8 a^7 + 28 a^6 x (b + d x^2) + 56 a^5 x^2 (b + d x^2)^2 + 70 a^4 x^3 (b + d x^2)^3 + 56 a^3 x^4 (b + d x^2)^4 + 28 a^2 x^5 (b + d x^2)^5 + 8 a x^6 (b + d x^2)^6 + x^7 (b + d x^2)^7\right)$$



Valid but unnecessarily complicated antiderivative:

$$\left\{ \left( 2 c x + 3 d x^2 \right) \left( a + c x^2 + d x^3 \right)^7, x, 2, 0 \right\}$$

$$\frac{1}{8} \left( a + c x^2 + d x^3 \right)^8$$

$$\frac{1}{8} x^2 (c + d x) \left( 8 a^7 + 28 a^6 x^2 (c + d x) + 56 a^5 x^4 (c + d x)^2 + \right.$$

$$\left. 70 a^4 x^6 (c + d x)^3 + 56 a^3 x^8 (c + d x)^4 + 28 a^2 x^{10} (c + d x)^5 + 8 a x^{12} (c + d x)^6 + x^{14} (c + d x)^7 \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x \left( 2 c + 3 d x \right) \left( a + c x^2 + d x^3 \right)^7, x, 2, 0 \right\}$$

$$\frac{1}{8} \left( a + c x^2 + d x^3 \right)^8$$

$$\frac{1}{8} x^2 (c + d x) \left( 8 a^7 + 28 a^6 x^2 (c + d x) + 56 a^5 x^4 (c + d x)^2 + \right.$$

$$\left. 70 a^4 x^6 (c + d x)^3 + 56 a^3 x^8 (c + d x)^4 + 28 a^2 x^{10} (c + d x)^5 + 8 a x^{12} (c + d x)^6 + x^{14} (c + d x)^7 \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{12} \left( a + b x^{13} \right)^{12}, x, 2, 0 \right\}$$

$$\frac{\left( a + b x^{13} \right)^{13}}{169 b}$$

$$\frac{1}{169} x^{13} \left( 13 a^{12} + 78 a^{11} b x^{13} + 286 a^{10} b^2 x^{26} + 715 a^9 b^3 x^{39} + 1287 a^8 b^4 x^{52} + 1716 a^7 b^5 x^{65} + \right.$$

$$\left. 1716 a^6 b^6 x^{78} + 1287 a^5 b^7 x^{91} + 715 a^4 b^8 x^{104} + 286 a^3 b^9 x^{117} + 78 a^2 b^{10} x^{130} + 13 a b^{11} x^{143} + b^{12} x^{156} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{24} \left( a + b x^{25} \right)^{12}, x, 2, 0 \right\}$$

$$\frac{\left( a + b x^{25} \right)^{13}}{325 b}$$

$$\frac{1}{325} x^{25} \left( 13 a^{12} + 78 a^{11} b x^{25} + 286 a^{10} b^2 x^{50} + 715 a^9 b^3 x^{75} + 1287 a^8 b^4 x^{100} + 1716 a^7 b^5 x^{125} + \right.$$

$$\left. 1716 a^6 b^6 x^{150} + 1287 a^5 b^7 x^{175} + 715 a^4 b^8 x^{200} + 286 a^3 b^9 x^{225} + 78 a^2 b^{10} x^{250} + 13 a b^{11} x^{275} + b^{12} x^{300} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{36} \left( a + b x^{37} \right)^{12}, x, 2, 0 \right\}$$

$$\frac{\left( a + b x^{37} \right)^{13}}{481 b}$$

$$\frac{1}{481} x^{37} \left( 13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + \right.$$

$$\left. 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444} \right)$$

Valid but unnecessarily complicated antiderivative:

# Mathematica 7 Test Results for Rational Function Integration Problems

$$\left\{x^{12m} (a + b x^{1+12m})^{12}, x, 2, 0\right\}$$

$$\frac{(a + b x^{1+12m})^{13}}{13 b (1 + 12 m)}$$

$$\frac{1}{13 + 156 m} x^{1+12m} (13 a^{12} + 78 a^{11} b x^{1+12m} + 286 a^{10} b^2 x^{2+24m} + 715 a^9 b^3 x^{3+36m} + 1287 a^8 b^4 x^{4+48m} + 1716 a^7 b^5 x^{5+60m} + 1716 a^6 b^6 x^{6+72m} + 1287 a^5 b^7 x^{7+84m} + 715 a^4 b^8 x^{8+96m} + 286 a^3 b^9 x^{9+108m} + 78 a^2 b^{10} x^{10+120m} + 13 a b^{11} x^{11+132m} + b^{12} x^{12+144m})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(a x + b x^{14})^{12}, x, 3, 0\right\}$$

$$\frac{(a + b x^{13})^{13}}{169 b}$$

$$\frac{1}{169} x^{13} (13 a^{12} + 78 a^{11} b x^{13} + 286 a^{10} b^2 x^{26} + 715 a^9 b^3 x^{39} + 1287 a^8 b^4 x^{52} + 1716 a^7 b^5 x^{65} + 1716 a^6 b^6 x^{78} + 1287 a^5 b^7 x^{91} + 715 a^4 b^8 x^{104} + 286 a^3 b^9 x^{117} + 78 a^2 b^{10} x^{130} + 13 a b^{11} x^{143} + b^{12} x^{156})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{x^{12} (a x + b x^{26})^{12}, x, 3, 0\right\}$$

$$\frac{(a + b x^{25})^{13}}{325 b}$$

$$\frac{1}{325} x^{25} (13 a^{12} + 78 a^{11} b x^{25} + 286 a^{10} b^2 x^{50} + 715 a^9 b^3 x^{75} + 1287 a^8 b^4 x^{100} + 1716 a^7 b^5 x^{125} + 1716 a^6 b^6 x^{150} + 1287 a^5 b^7 x^{175} + 715 a^4 b^8 x^{200} + 286 a^3 b^9 x^{225} + 78 a^2 b^{10} x^{250} + 13 a b^{11} x^{275} + b^{12} x^{300})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{x^{24} (a x + b x^{38})^{12}, x, 3, 0\right\}$$

$$\frac{(a + b x^{37})^{13}}{481 b}$$

$$\frac{1}{481} x^{37} (13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{x^{12 (-1+m)} (a x + b x^{2+12m})^{12}, x, 3, 0\right\}$$

$$\frac{(a + b x^{1+12m})^{13}}{13 b (1 + 12 m)}$$

$$\frac{1}{13 + 156 m} x^{1+12m} (13 a^{12} + 78 a^{11} b x^{1+12m} + 286 a^{10} b^2 x^{2+24m} + 715 a^9 b^3 x^{3+36m} + 1287 a^8 b^4 x^{4+48m} + 1716 a^7 b^5 x^{5+60m} + 1716 a^6 b^6 x^{6+72m} + 1287 a^5 b^7 x^{7+84m} + 715 a^4 b^8 x^{8+96m} + 286 a^3 b^9 x^{9+108m} + 78 a^2 b^{10} x^{10+120m} + 13 a b^{11} x^{11+132m} + b^{12} x^{12+144m})$$

Valid but unnecessarily complicated antiderivative:

$$\left\{(a x + b x^{14})^{12}, x, 3, 0\right\}$$

# Mathematica 7 Test Results for Rational Function Integration Problems

$$\frac{(a + b x^{13})^{13}}{169 b}$$

$$\frac{1}{169} x^{13} \left( 13 a^{12} + 78 a^{11} b x^{13} + 286 a^{10} b^2 x^{26} + 715 a^9 b^3 x^{39} + 1287 a^8 b^4 x^{52} + 1716 a^7 b^5 x^{65} + \right. \\ \left. 1716 a^6 b^6 x^{78} + 1287 a^5 b^7 x^{91} + 715 a^4 b^8 x^{104} + 286 a^3 b^9 x^{117} + 78 a^2 b^{10} x^{130} + 13 a b^{11} x^{143} + b^{12} x^{156} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a x^2 + b x^{27})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{25})^{13}}{325 b}$$

$$\frac{1}{325} x^{25} \left( 13 a^{12} + 78 a^{11} b x^{25} + 286 a^{10} b^2 x^{50} + 715 a^9 b^3 x^{75} + 1287 a^8 b^4 x^{100} + 1716 a^7 b^5 x^{125} + \right. \\ \left. 1716 a^6 b^6 x^{150} + 1287 a^5 b^7 x^{175} + 715 a^4 b^8 x^{200} + 286 a^3 b^9 x^{225} + 78 a^2 b^{10} x^{250} + 13 a b^{11} x^{275} + b^{12} x^{300} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a x^3 + b x^{40})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{37})^{13}}{481 b}$$

$$\frac{1}{481} x^{37} \left( 13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + \right. \\ \left. 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a x^m + b x^{1+13 m})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{1+12 m})^{13}}{13 b (1 + 12 m)}$$

$$\frac{1}{13 + 156 m} x^{1+12 m} \left( 13 a^{12} + 78 a^{11} b x^{1+12 m} + 286 a^{10} b^2 x^{2+24 m} + 715 a^9 b^3 x^{3+36 m} + 1287 a^8 b^4 x^{4+48 m} + 1716 a^7 b^5 x^{5+60 m} + 1716 a^6 b^6 x^{6+72 m} + \right. \\ \left. 1287 a^5 b^7 x^{7+84 m} + 715 a^4 b^8 x^{8+96 m} + 286 a^3 b^9 x^{9+108 m} + 78 a^2 b^{10} x^{10+120 m} + 13 a b^{11} x^{11+132 m} + b^{12} x^{12+144 m} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^p (a x^n + b x^{1+13 n+p})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{1+12 n+p})^{13}}{13 b (1 + 12 n + p)}$$

$$\frac{1}{13 (1 + 12 n + p)} x^{1+12 n+p} \\ \left( 13 a^{12} + 78 a^{11} b x^{1+12 n+p} + 286 a^3 b^9 x^{9 (1+12 n+p)} + 78 a^2 b^{10} x^{10 (1+12 n+p)} + 13 a b^{11} x^{11 (1+12 n+p)} + b^{12} x^{12 (1+12 n+p)} + 286 a^{10} b^2 x^{2+24 n+2 p} + \right. \\ \left. 715 a^9 b^3 x^{3+36 n+3 p} + 1287 a^8 b^4 x^{4+48 n+4 p} + 1716 a^7 b^5 x^{5+60 n+5 p} + 1716 a^6 b^6 x^{6+72 n+6 p} + 1287 a^5 b^7 x^{7+84 n+7 p} + 715 a^4 b^8 x^{8+96 n+8 p} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{12} (a + b x^{13})^{12}, x, 2, 0 \right\}$$

# Mathematica 7 Test Results for Rational Function Integration Problems

$$\frac{(a + b x^{13})^{13}}{169 b}$$

$$\frac{1}{169} x^{13} \left( 13 a^{12} + 78 a^{11} b x^{13} + 286 a^{10} b^2 x^{26} + 715 a^9 b^3 x^{39} + 1287 a^8 b^4 x^{52} + 1716 a^7 b^5 x^{65} + \right. \\ \left. 1716 a^6 b^6 x^{78} + 1287 a^5 b^7 x^{91} + 715 a^4 b^8 x^{104} + 286 a^3 b^9 x^{117} + 78 a^2 b^{10} x^{130} + 13 a b^{11} x^{143} + b^{12} x^{156} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{12} (a x + b x^{26})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{25})^{13}}{325 b}$$

$$\frac{1}{325} x^{25} \left( 13 a^{12} + 78 a^{11} b x^{25} + 286 a^{10} b^2 x^{50} + 715 a^9 b^3 x^{75} + 1287 a^8 b^4 x^{100} + 1716 a^7 b^5 x^{125} + \right. \\ \left. 1716 a^6 b^6 x^{150} + 1287 a^5 b^7 x^{175} + 715 a^4 b^8 x^{200} + 286 a^3 b^9 x^{225} + 78 a^2 b^{10} x^{250} + 13 a b^{11} x^{275} + b^{12} x^{300} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{12} (a x^2 + b x^{39})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{37})^{13}}{481 b}$$

$$\frac{1}{481} x^{37} \left( 13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + \right. \\ \left. 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{24} (a + b x^{25})^{12}, x, 2, 0 \right\}$$

$$\frac{(a + b x^{25})^{13}}{325 b}$$

$$\frac{1}{325} x^{25} \left( 13 a^{12} + 78 a^{11} b x^{25} + 286 a^{10} b^2 x^{50} + 715 a^9 b^3 x^{75} + 1287 a^8 b^4 x^{100} + 1716 a^7 b^5 x^{125} + \right. \\ \left. 1716 a^6 b^6 x^{150} + 1287 a^5 b^7 x^{175} + 715 a^4 b^8 x^{200} + 286 a^3 b^9 x^{225} + 78 a^2 b^{10} x^{250} + 13 a b^{11} x^{275} + b^{12} x^{300} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{24} (a x + b x^{38})^{12}, x, 3, 0 \right\}$$

$$\frac{(a + b x^{37})^{13}}{481 b}$$

$$\frac{1}{481} x^{37} \left( 13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + \right. \\ \left. 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ x^{36} (a + b x^{37})^{12}, x, 2, 0 \right\}$$

$$\frac{(a + b x^{37})^{13}}{481 b}$$

$$\frac{1}{481} x^{37} \left( 13 a^{12} + 78 a^{11} b x^{37} + 286 a^{10} b^2 x^{74} + 715 a^9 b^3 x^{111} + 1287 a^8 b^4 x^{148} + 1716 a^7 b^5 x^{185} + \right. \\ \left. 1716 a^6 b^6 x^{222} + 1287 a^5 b^7 x^{259} + 715 a^4 b^8 x^{296} + 286 a^3 b^9 x^{333} + 78 a^2 b^{10} x^{370} + 13 a b^{11} x^{407} + b^{12} x^{444} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\{ (5 - 2x)^6 (2 + 3x)^3 (-16 + 33x), x, 1, 0 \}$$

$$-\frac{1}{2} (5 - 2x)^7 (2 + 3x)^4$$

$$\frac{1}{2} x \left( -4\,000\,000 - 75\,000 x + 7\,975\,000 x^2 - 98\,125 x^3 - \right. \\ \left. 7\,632\,450 x^4 + 2\,994\,460 x^5 + 2\,470\,808 x^6 - 2\,512\,752 x^7 + 904\,608 x^8 - 153\,792 x^9 + 10\,368 x^{10} \right)$$

Valid but unnecessarily complicated antiderivative:

$$\{ x (a + bx) (c + dx)^{16}, x, 3, 0 \}$$

$$\frac{(bc - 18ad) (2bc + 17ad) (c + dx)^{17}}{5814bd^3} - \frac{(2bc + 17ad) x (c + dx)^{17}}{342d^2} + \frac{(a + bx)^2 (c + dx)^{17}}{19bd}$$

$$\frac{1}{2} a c^{16} x^2 + \frac{1}{3} c^{15} (bc + 16ad) x^3 + 2 c^{14} d (2bc + 15ad) x^4 + 8 c^{13} d^2 (3bc + 14ad) x^5 + \frac{70}{3} c^{12} d^3 (4bc + 13ad) x^6 +$$

$$52 c^{11} d^4 (5bc + 12ad) x^7 + 91 c^{10} d^5 (6bc + 11ad) x^8 + \frac{1144}{9} c^9 d^6 (7bc + 10ad) x^9 + 143 c^8 d^7 (8bc + 9ad) x^{10} +$$

$$130 c^7 d^8 (9bc + 8ad) x^{11} + \frac{286}{3} c^6 d^9 (10bc + 7ad) x^{12} + 56 c^5 d^{10} (11bc + 6ad) x^{13} + 26 c^4 d^{11} (12bc + 5ad) x^{14} +$$

$$\frac{28}{3} c^3 d^{12} (13bc + 4ad) x^{15} + \frac{5}{2} c^2 d^{13} (14bc + 3ad) x^{16} + \frac{8}{17} c d^{14} (15bc + 2ad) x^{17} + \frac{1}{18} d^{15} (16bc + ad) x^{18} + \frac{1}{19} b d^{16} x^{19}$$

Valid but unnecessarily complicated antiderivative:

$$\{ x (a + bx)^2 (c + dx)^{16}, x, 4, 0 \}$$

$$-\frac{(bc - ad)^2 (3bc + 17ad) (c + dx)^{17}}{58140bd^4} + \frac{(bc - ad) (3bc + 17ad) (a + bx) (c + dx)^{17}}{3420bd^3} -$$

$$\frac{(3bc + 17ad) (a + bx)^2 (c + dx)^{17}}{380bd^2} + \frac{(a + bx)^3 (c + dx)^{17}}{20bd}$$

$$\frac{1}{2} a^2 c^{16} x^2 + \frac{2}{3} a c^{15} (bc + 8ad) x^3 + \frac{1}{4} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^4 + \frac{16}{5} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^5 +$$

$$\frac{10}{3} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^6 + 8 c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^7 + \frac{91}{2} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^8 +$$

$$\frac{208}{9} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^9 + \frac{143}{5} c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{10} +$$

$$260 c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{11} + \frac{143}{6} c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{12} +$$

$$16 c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{13} + 26 c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{14} +$$

$$\frac{56}{15} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{15} + \frac{5}{4} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{16} +$$

$$\frac{16}{17} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{17} + \frac{1}{18} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{18} + \frac{2}{19} b d^{15} (8 b c + a d) x^{19} + \frac{1}{20} b^2 d^{16} x^{20}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a + b x) \left( 1 + \left( a x + \frac{b x^2}{2} \right)^4 \right), x, 2, 0 \right\}$$

$$a x + \frac{b x^2}{2} + \frac{1}{160} x^5 (2 a + b x)^5$$

$$\frac{1}{160} x \left( 32 a^5 x^4 + 80 a^4 b x^5 + 80 a^3 b^2 x^6 + 40 a^2 b^3 x^7 + 10 a (16 + b^4 x^8) + b x (80 + b^4 x^8) \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a + b x) \left( 1 + \left( c + a x + \frac{b x^2}{2} \right)^4 \right), x, 2, 0 \right\}$$

$$c + a x + \frac{b x^2}{2} + \frac{1}{160} (2 c + 2 a x + b x^2)^5$$

$$\frac{1}{160} x (2 a + b x) \left( 80 + 80 c^4 + 16 a^4 x^4 + 32 a^3 b x^5 + 24 a^2 b^2 x^6 + 8 a b^3 x^7 + b^4 x^8 + 80 c^3 x (2 a + b x) + 40 c^2 x^2 (2 a + b x)^2 + 10 c x^3 (2 a + b x)^3 \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a + c x^2) \left( 1 + \left( a x + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$a x + \frac{c x^3}{3} + \frac{x^6 (3 a + c x^2)^6}{4374}$$

$$\frac{x \left( 729 a^6 x^5 + 1458 a^5 c x^7 + 1215 a^4 c^2 x^9 + 540 a^3 c^3 x^{11} + 135 a^2 c^4 x^{13} + 18 a (243 + c^5 x^{15}) + c x^2 (1458 + c^5 x^{15}) \right)}{4374}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (a + c x^2) \left( 1 + \left( d + a x + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$d + a x + \frac{c x^3}{3} + \frac{(3 d + 3 a x + c x^3)^6}{4374}$$

$$\frac{1}{4374} x (3 a + c x^2) \left( 1458 + 1458 d^5 + 243 a^5 x^5 + 405 a^4 c x^7 + 270 a^3 c^2 x^9 + 90 a^2 c^3 x^{11} + 15 a c^4 x^{13} + c^5 x^{15} + 1215 d^4 (3 a x + c x^3) + 540 d^3 (3 a x + c x^3)^2 + 135 d^2 (3 a x + c x^3)^3 + 18 d (3 a x + c x^3)^4 \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (b x + c x^2) \left( 1 + \left( \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$\frac{b x^2}{2} + \frac{c x^3}{3} + \frac{x^{12} (3 b + 2 c x)^6}{279936}$$

$$\frac{x^2 \left( 729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b (243 + c^5 x^{15}) + 64 c x (1458 + c^5 x^{15}) \right)}{279936}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \left( b x + c x^2 \right) \left( 1 + \left( d + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$d + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{(6 d + 3 b x^2 + 2 c x^3)^6}{279936}$$

$$\frac{1}{279936} x^2 (3 b + 2 c x) (46656 + 46656 d^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + 240 b c^4 x^{14} + 32 c^5 x^{15} + 19440 d^4 x^2 (3 b + 2 c x) + 4320 d^3 x^4 (3 b + 2 c x)^2 + 540 d^2 x^6 (3 b + 2 c x)^3 + 36 d x^8 (3 b + 2 c x)^4)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \left( a + b x + c x^2 \right) \left( 1 + \left( a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{x^6 (6 a + 3 b x + 2 c x^2)^6}{279936}$$

$$\frac{a^6 x^6}{6} + \frac{1}{6} a^5 x^7 (3 b + 2 c x) + \frac{5}{72} a^4 x^8 (3 b + 2 c x)^2 + \frac{5}{324} a^3 x^9 (3 b + 2 c x)^3 +$$

$$\frac{5 a^2 x^{10} (3 b + 2 c x)^4}{2592} + a \left( x + \frac{b^5 x^{11}}{32} + \frac{5}{48} b^4 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^3 x^{14} + \frac{5}{162} b c^4 x^{15} + \frac{c^5 x^{16}}{243} \right) +$$

$$\frac{x^2 (729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b (243 + c^5 x^{15}) + 64 c x (1458 + c^5 x^{15}))}{279936}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \left( a + b x + c x^2 \right) \left( 1 + \left( d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right), x, 2, 0 \right\}$$

$$d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{(6 d + 6 a x + 3 b x^2 + 2 c x^3)^6}{279936}$$

$$\frac{1}{279936} x (6 a + x (3 b + 2 c x))$$

$$(46656 + 46656 d^5 + 7776 a^5 x^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + 240 b c^4 x^{14} + 32 c^5 x^{15} + 6480 a^4 x^6 (3 b + 2 c x) + 2160 a^3 x^7 (3 b + 2 c x)^2 + 360 a^2 x^8 (3 b + 2 c x)^3 + 30 a x^9 (3 b + 2 c x)^4 + 19440 d^4 x (6 a + x (3 b + 2 c x)) + 4320 d^3 x^2 (6 a + x (3 b + 2 c x))^2 + 540 d^2 x^3 (6 a + x (3 b + 2 c x))^3 + 36 d x^4 (6 a + x (3 b + 2 c x))^4)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (2 + 3 x)^6 (1 + (2 + 3 x)^7 + (2 + 3 x)^{14}), x, 2, 0 \right\}$$

$$\frac{1}{21} (2 + 3 x)^7 + \frac{1}{42} (2 + 3 x)^{14} + \frac{1}{63} (2 + 3 x)^{21}$$

$$1056832 x + 15808800 x^2 + 149902032 x^3 + 1010576952 x^4 + 5149786572 x^5 + 20588764518 x^6 + 66158154783 x^7 + 173635132896 x^8 + 376174427616 x^9 + 677082445416 x^{10} + 1015602174288 x^{11} + 1269491970942 x^{12} + 1318314865122 x^{13} + \frac{15819767221203 x^{14}}{14} + 790988281344 x^{15} + 444930908256 x^{16} + 196293047760 x^{17} + 65431015920 x^{18} + 15496819560 x^{19} + 2324522934 x^{20} + \frac{1162261467 x^{21}}{7}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ (2 + 3 x)^6 (1 + (2 + 3 x)^7 + (2 + 3 x)^{14})^2, x, 3, 0 \right\}$$

# Mathematica 7 Test Results for Rational Function Integration Problems

$$\frac{1}{21} (2 + 3 x)^7 + \frac{1}{21} (2 + 3 x)^{14} + \frac{1}{21} (2 + 3 x)^{21} + \frac{1}{42} (2 + 3 x)^{28} + \frac{1}{105} (2 + 3 x)^{35}$$

$$17451466816 x + 443569828128 x^2 + 7299544818384 x^3 + 87406679578680 x^4 + \frac{4057390785756924 x^5}{5} + 6077684727888102 x^6 +$$

$$37727143432895007 x^7 + 197897276851452864 x^8 + 889942562270387136 x^9 + \frac{17344958593049772048 x^{10}}{5} +$$

$$11821487501620716192 x^{11} + 35454069480572048124 x^{12} + 94069263918929616324 x^{13} + 221699757548270194389 x^{14} +$$

$$465517091041681015296 x^{15} + 872775774067455498528 x^{16} + 1463104032160519033200 x^{17} + 2194577166014752240080 x^{18} +$$

$$2945285062308448290360 x^{19} + 3534290697929473864098 x^{20} + \frac{26506949038858918036881 x^{21}}{7} +$$

$$3614565944605222108800 x^{22} + 3064515076512846852480 x^{23} + 2298383223254096766840 x^{24} +$$

$$\frac{7584660010542711771792 x^{25}}{5} + 875152864622814086340 x^{26} + 437576396725285446564 x^{27} +$$

$$\frac{2625458326972530284475 x^{28}}{14} + 67899784121041365504 x^{29} + \frac{101849676181562048256 x^{30}}{5} + 4928210137817518464 x^{31} +$$

$$924039400840784712 x^{32} + 126005372841925188 x^{33} + 11118121133111046 x^{34} + \frac{16677181699666569 x^{35}}{35}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{9 + 5 x^2 + x^4}, x, 3, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{11} x}{3 - x^2}\right]}{6 \sqrt{11}} + \frac{1}{6} \text{ArcTanh}\left[\frac{x}{3 + x^2}\right]$$

$$\frac{1}{66} i \left( -\sqrt{22 (5 + i \sqrt{11})} \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2} (5 - i \sqrt{11})}}\right] + \sqrt{22 (5 - i \sqrt{11})} \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2} (5 + i \sqrt{11})}}\right] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2}{1 - x^2 + x^4}, x, 3, 0 \right\}$$

$$\frac{1}{2} \text{ArcTan}\left[\frac{x}{1 - x^2}\right] - \frac{\text{ArcTanh}\left[\frac{\sqrt{3} x}{1 + x^2}\right]}{2 \sqrt{3}}$$

$$\frac{(i + \sqrt{3}) \text{ArcTan}\left[\frac{1}{2} (1 - i \sqrt{3}) x\right]}{\sqrt{-6 + 6 i \sqrt{3}}} + \frac{(-i + \sqrt{3}) \text{ArcTan}\left[\frac{1}{2} (1 + i \sqrt{3}) x\right]}{\sqrt{-6 - 6 i \sqrt{3}}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1 + x^2}{1 + b x^2 + x^4}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2+b} x}{1 - x^2}\right]}{\sqrt{2 + b}}$$



# Mathematica 7 Test Results for Rational Function Integration Problems

$$\frac{\frac{(2-b+\sqrt{-4+b^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{b-\sqrt{-4+b^2}}}\right]}{\sqrt{b-\sqrt{-4+b^2}}} + \frac{(-2+b+\sqrt{-4+b^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2} \sqrt{-4+b^2}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1-x^2}{1+bx^2+x^4}, x, 1, 0 \right\}$$

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{2-b} x}{1+x^2}\right]}{\sqrt{2-b}} - \frac{\frac{(2+b-\sqrt{-4+b^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{b-\sqrt{-4+b^2}}}\right]}{\sqrt{b-\sqrt{-4+b^2}}} - \frac{(2+b+\sqrt{-4+b^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2} x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2} \sqrt{-4+b^2}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}, x, 1, 0 \right\}$$

$$\frac{e \operatorname{ArcTan}\left[\frac{\sqrt{b+\frac{2cd}{e}} ex}{\sqrt{c} (d-ex^2)}\right]}{\sqrt{c} \sqrt{b+\frac{2cd}{e}}} - \frac{e^{3/2} \left( \frac{(2cd-be+\sqrt{-4c^2d^2+b^2e^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{e} x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right]}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}} + \frac{(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}) \operatorname{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}\sqrt{e} x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right]}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{-4c^2d^2+b^2e^2}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{2+x^3+x^6}, x, 11, 0 \right\}$$

$$\frac{i \operatorname{ArcTan}\left[\frac{2^{2/3} (1-i\sqrt{7})^{1/3} -4x}{2^{2/3} \sqrt{3} (1-i\sqrt{7})^{1/3}}\right]}{\sqrt{21} \left(\frac{1}{2} (1-i\sqrt{7})\right)^{2/3}} - \frac{i \operatorname{ArcTan}\left[\frac{(1-i\sqrt{7})^{1/3} (2^{2/3} (1+i\sqrt{7})^{1/3} -4x)}{2 \times 2^{2/3} \sqrt{3}}\right]}{\sqrt{21} \left(\frac{1}{2} (1+i\sqrt{7})\right)^{2/3}} -$$

$$\frac{i \operatorname{Log}\left[2^{2/3} (1-i\sqrt{7})^{1/3} +2x\right]}{3 \sqrt{7} \left(\frac{1}{2} (1-i\sqrt{7})\right)^{2/3}} + \frac{i \operatorname{Log}\left[2^{2/3} (1+i\sqrt{7})^{1/3} +2x\right]}{3 \sqrt{7} \left(\frac{1}{2} (1+i\sqrt{7})\right)^{2/3}} +$$

$$\frac{i \operatorname{Log}\left[2^{1/3} (1-i\sqrt{7})^{2/3} -2^{2/3} (1-i\sqrt{7})^{1/3} x +2x^2\right]}{3 \times 2^{1/3} \sqrt{7} (1-i\sqrt{7})^{2/3}} - \frac{i \operatorname{Log}\left[(\sqrt{2}+i\sqrt{14})^{2/3} -2^{2/3} (1+i\sqrt{7})^{1/3} x +2x^2\right]}{3 \sqrt{7} (\sqrt{2}+i\sqrt{14})^{2/3}}$$

# Mathematica 7 Test Results for Rational Function Integration Problems

$$\frac{1}{3} \text{RootSum}\left[2 + \#1^3 + \#1^6 \&, \frac{\text{Log}[x - \#1]}{\#1^2 + 2 \#1^5} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^3}{2 + x^3 + x^6}, x, 11, 0 \right\}$$

$$-\frac{(7 + i\sqrt{7}) \text{ArcTan}\left[\frac{2^{2/3}(1-i\sqrt{7})^{1/3} - 4x}{2^{2/3}\sqrt{3}(1-i\sqrt{7})^{1/3}}\right]}{7 \times 2^{1/3}\sqrt{3}(1-i\sqrt{7})^{2/3}} - \frac{(7 - i\sqrt{7}) \text{ArcTan}\left[\frac{(1-i\sqrt{7})^{1/3}(2^{2/3}(1+i\sqrt{7})^{1/3} - 4x)}{2 \times 2^{2/3}\sqrt{3}}\right]}{7\sqrt{3}(\sqrt{2} + i\sqrt{14})^{2/3}} +$$

$$\frac{i\left(\frac{1}{2}(1-i\sqrt{7})\right)^{1/3} \text{Log}\left[2^{2/3}(1-i\sqrt{7})^{1/3} + 2x\right]}{3\sqrt{7}} + \frac{(7 - i\sqrt{7}) \text{Log}\left[2^{2/3}(1+i\sqrt{7})^{1/3} + 2x\right]}{21(\sqrt{2} + i\sqrt{14})^{2/3}} -$$

$$\frac{(7 + i\sqrt{7}) \text{Log}\left[2^{1/3}(1-i\sqrt{7})^{2/3} - 2^{2/3}(1-i\sqrt{7})^{1/3}x + 2x^2\right]}{42 \times 2^{1/3}(1-i\sqrt{7})^{2/3}} - \frac{(7 - i\sqrt{7}) \text{Log}\left[(\sqrt{2} + i\sqrt{14})^{2/3} - 2^{2/3}(1+i\sqrt{7})^{1/3}x + 2x^2\right]}{42(\sqrt{2} + i\sqrt{14})^{2/3}}$$

$$\frac{1}{3} \text{RootSum}\left[2 + \#1^3 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1}{1 + 2 \#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x(1 + x^3 + x^6)}, x, 3, 0 \right\}$$

$$-\frac{\text{ArcTan}\left[\frac{1+2x^3}{\sqrt{3}}\right]}{3\sqrt{3}} + \text{Log}[x] - \frac{1}{6} \text{Log}[1 + x^3 + x^6]$$

$$\text{Log}[x] - \frac{1}{3} \text{RootSum}\left[1 + \#1^3 + \#1^6 \&, \frac{\text{Log}[x - \#1] + \text{Log}[x - \#1] \#1^3}{1 + 2 \#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{2 + x^4 + x^8}, x, 7, 0 \right\}$$

$$-\frac{\left(\sqrt{2} - \sqrt{4 - \sqrt{2}}\right) \text{ArcTan}\left[\frac{\sqrt{2\sqrt{2} - \sqrt{4 - \sqrt{2}}}}{2^{1/8}(2^{1/4} - x^2)}x\right]}{4 \times 2^{7/8} \sqrt{(-1 + 2\sqrt{2})} \left(2\sqrt{2} - \sqrt{4 - \sqrt{2}}\right)} + \frac{\left(\sqrt{2} + \sqrt{4 - \sqrt{2}}\right) \text{ArcTan}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1 + 2\sqrt{2}}}}{2^{1/4} - x^2}x\right]}{8 \sqrt{(-1 + 2\sqrt{2})} \left(2 \times 2^{1/4} + \sqrt{-1 + 2\sqrt{2}}\right)} -$$

$$\frac{\left(\sqrt{2} - \sqrt{4 - \sqrt{2}}\right) \text{ArcTanh}\left[\frac{\sqrt{2\sqrt{2} - \sqrt{4 - \sqrt{2}}}}{2^{1/8}(2^{1/4} + x^2)}x\right]}{4 \times 2^{7/8} \sqrt{(-1 + 2\sqrt{2})} \left(2\sqrt{2} - \sqrt{4 - \sqrt{2}}\right)} + \frac{\left(\sqrt{2} + \sqrt{4 - \sqrt{2}}\right) \text{ArcTanh}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1 + 2\sqrt{2}}}}{2^{1/4} + x^2}x\right]}{8 \sqrt{(-1 + 2\sqrt{2})} \left(2 \times 2^{1/4} + \sqrt{-1 + 2\sqrt{2}}\right)}$$

$$\frac{1}{4} \text{RootSum}\left[2 + \#1^4 + \#1^8 \&, \frac{\text{Log}[x - \#1]}{\#1^3 + 2 \#1^7} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2}{2 + x^4 + x^8}, x, 7, 0 \right\}$$

$$\frac{\frac{\text{ArcTan}\left[\frac{\sqrt{2\sqrt{2}-\sqrt{4-\sqrt{2}}}\,x}{2^{1/8}(2^{1/4}-x^2)}\right]}{4 \times 2^{1/8} \sqrt{(-1+2\sqrt{2})\left(2\sqrt{2}-\sqrt{4-\sqrt{2}}\right)}} - \frac{\frac{\text{ArcTan}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}}\,x}{2^{1/4}-x^2}\right]}{4 \times 2^{1/4} \sqrt{(-1+2\sqrt{2})\left(2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}\right)}}}{\frac{\frac{\text{ArcTanh}\left[\frac{\sqrt{2\sqrt{2}-\sqrt{4-\sqrt{2}}}\,x}{2^{1/8}(2^{1/4}+x^2)}\right]}{4 \times 2^{1/8} \sqrt{(-1+2\sqrt{2})\left(2\sqrt{2}-\sqrt{4-\sqrt{2}}\right)}} + \frac{\frac{\text{ArcTanh}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}}\,x}{2^{1/4}+x^2}\right]}{4 \times 2^{1/4} \sqrt{(-1+2\sqrt{2})\left(2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}\right)}}}{\frac{1}{4} \text{RootSum}\left[2 + \#1^4 + \#1^8 \&, \frac{\text{Log}[x - \#1]}{\#1 + 2 \#1^5} \&\right]}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^4}{2 + x^4 + x^8}, x, 7, 0 \right\}$$

$$\frac{\frac{\text{ArcTan}\left[\frac{\sqrt{2\sqrt{2}-\sqrt{4-\sqrt{2}}}\,x}{2^{1/8}(2^{1/4}-x^2)}\right]}{2 \times 2^{7/8} \sqrt{(-1+2\sqrt{2})\left(2\sqrt{2}-\sqrt{4-\sqrt{2}}\right)}} - \frac{\frac{\text{ArcTan}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}}\,x}{2^{1/4}-x^2}\right]}{4 \sqrt{(-1+2\sqrt{2})\left(2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}\right)}}}{\frac{\frac{\text{ArcTanh}\left[\frac{\sqrt{2\sqrt{2}-\sqrt{4-\sqrt{2}}}\,x}{2^{1/8}(2^{1/4}+x^2)}\right]}{2 \times 2^{7/8} \sqrt{(-1+2\sqrt{2})\left(2\sqrt{2}-\sqrt{4-\sqrt{2}}\right)}} - \frac{\frac{\text{ArcTanh}\left[\frac{\sqrt{2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}}\,x}{2^{1/4}+x^2}\right]}{4 \sqrt{(-1+2\sqrt{2})\left(2 \times 2^{1/4} + \sqrt{-1+2\sqrt{2}}\right)}}}{\frac{1}{4} \text{RootSum}\left[2 + \#1^4 + \#1^8 \&, \frac{\text{Log}[x - \#1] \#1}{1 + 2 \#1^4} \&\right]}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x(1 + x^5 + x^{10})}, x, 3, 0 \right\}$$

$$-\frac{\frac{\text{ArcTan}\left[\frac{1+2x^5}{\sqrt{3}}\right]}{5\sqrt{3}} + \text{Log}[x] - \frac{1}{10} \text{Log}[1 + x^5 + x^{10}]}$$

$$\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{5\sqrt{3}} + \text{Log}[x] - \frac{1}{10} \text{Log}[1+x+x^2] - \frac{1}{5} \text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\&, \right. \\ \left. \frac{-\text{Log}[x-\#1]\#1+2\text{Log}[x-\#1]\#1^2-\text{Log}[x-\#1]\#1^3+3\text{Log}[x-\#1]\#1^4-\text{Log}[x-\#1]\#1^5-3\text{Log}[x-\#1]\#1^6+4\text{Log}[x-\#1]\#1^7}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}\& \right. \\ \left. \right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{1}{x+x^6+x^{11}}, x, 4, 0\right\}$$

$$-\frac{\text{ArcTan}\left[\frac{1+2x^5}{\sqrt{3}}\right]}{5\sqrt{3}} + \text{Log}[x] - \frac{1}{10} \text{Log}[1+x^5+x^{10}]$$

$$\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{5\sqrt{3}} + \text{Log}[x] - \frac{1}{10} \text{Log}[1+x+x^2] - \frac{1}{5} \text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\&, \right. \\ \left. \frac{-\text{Log}[x-\#1]\#1+2\text{Log}[x-\#1]\#1^2-\text{Log}[x-\#1]\#1^3+3\text{Log}[x-\#1]\#1^4-\text{Log}[x-\#1]\#1^5-3\text{Log}[x-\#1]\#1^6+4\text{Log}[x-\#1]\#1^7}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}\& \right. \\ \left. \right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}, x, 1, 0\right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

$$-\frac{\left(-d-2e+\sqrt{d}\sqrt{d+2e}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}}\right]}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}} - \frac{\left(d+2e+\sqrt{d}\sqrt{d+2e}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}\right]}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}$$

$$\frac{2\sqrt{2}\sqrt{d}\sqrt{d+2e}\sqrt{f}}{2\sqrt{2}\sqrt{d}\sqrt{d+2e}\sqrt{f}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{\frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}, x, 1, 0\right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

$$-\frac{\left(-id+2ie+\sqrt{d}\sqrt{-d+2e}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}}\right]}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}} - \frac{\left(id-2ie+\sqrt{d}\sqrt{-d+2e}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}\right]}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}$$

$$\frac{2\sqrt{2}\sqrt{d}\sqrt{-d+2e}\sqrt{f}}{2\sqrt{2}\sqrt{d}\sqrt{-d+2e}\sqrt{f}}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e - 4 f x^3}{e^2 + 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\frac{\text{RootSum}\left[e^2 + 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x-\#1] + 4 f \text{Log}[x-\#1] \#1^3}{2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \&\right]}{4 f}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{e - 4 f x^3}{e^2 - 4 d f x^2 + 4 e f x^3 + 4 f^2 x^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\frac{\text{RootSum}\left[e^2 - 4 d f \#1^2 + 4 e f \#1^3 + 4 f^2 \#1^6 \&, \frac{-e \text{Log}[x-\#1] + 4 f \text{Log}[x-\#1] \#1^3}{-2 d \#1 + 3 e \#1^2 + 6 f \#1^5} \&\right]}{4 f}$$

Unable to integrate:

$$\left\{ \frac{e - 2 f (-1 + n) x^n}{e^2 + 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}(-1+n)x}{e(-1+n)+2f(-1+n)x^n}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 + 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Unable to integrate:

$$\left\{ \frac{e - 2 f (-1 + n) x^n}{e^2 - 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}(-1+n)x}{e(-1+n)+2f(-1+n)x^n}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\int \frac{e - 2 f (-1 + n) x^n}{e^2 - 4 d f x^2 + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2 (3 e + 2 f x^2)}{e^2 + 4 e f x^2 + 4 f^2 x^4 + 4 d f x^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\frac{\text{RootSum}\left[e^2 + 4 e f \#1^2 + 4 f^2 \#1^4 + 4 d f \#1^6 \&, \frac{3 e \text{Log}[x-\#1] \#1 + 2 f \text{Log}[x-\#1] \#1^3}{e+2f\#1^2+3d\#1^4} \&\right]}{8 f}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\frac{\text{RootSum}\left[e^2 + 4ef\#1^2 + 4f^2\#1^4 - 4df\#1^6 \&, \frac{3e\text{Log}[x-\#1]\#1 + 2f\text{Log}[x-\#1]\#1^3}{e+2f\#1^2-3d\#1^4} \&\right]}{8f}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

$$-\frac{\text{RootSum}\left[e^2 + 4ef\#1^3 + 4df\#1^4 + 4f^2\#1^6 \&, \frac{-e\text{Log}[x-\#1]+f\text{Log}[x-\#1]\#1^3}{3e\#1+4d\#1^2+6f\#1^4} \&\right]}{2f}$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 - 4dfx^4 + 4f^2x^6}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

$$-\frac{\text{RootSum}\left[e^2 + 4ef\#1^3 - 4df\#1^4 + 4f^2\#1^6 \&, \frac{-e\text{Log}[x-\#1]+f\text{Log}[x-\#1]\#1^3}{3e\#1-4d\#1^2+6f\#1^4} \&\right]}{2f}$$

Unable to integrate:

$$\left\{ \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}(1+m)(1+m-n)x^{1+m}}{e(1+m)(1+m-n)+2f(1+m)(1+m-n)x^n}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Unable to integrate:

$$\left\{ \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}(1+m)(1+m-n)x^{1+m}}{e(1+m)(1+m-n)+2f(1+m)(1+m-n)x^n}\right]}{2\sqrt{d}\sqrt{f}}$$

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^5}{x - x^3}, x, 5, 0 \right\}$$

$$-x - \frac{x^3}{3} + \text{ArcTanh}[x]$$

$$\frac{1}{6} \left( -2x(3 + x^2) + 3 \log[-1 - x] - 3 \log[-1 + x] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x^3}{x - x^3}, x, 4, 0 \right\}$$

$$-x + \text{ArcTanh}[x]$$

$$\frac{1}{2} (-2x + \log[-1 - x] - \log[-1 + x])$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{x}{x - x^3}, x, 2, 0 \right\}$$

$$\text{ArcTanh}[x]$$

$$\frac{1}{2} (\log[-1 - x] - \log[-1 + x])$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x(x - x^3)}, x, 5, 0 \right\}$$

$$-\frac{1}{x} + \text{ArcTanh}[x]$$

$$\frac{1}{2} \left( -\frac{2}{x} - \log[1 - x] + \log[1 + x] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1}{x^3(x - x^3)}, x, 5, 0 \right\}$$

$$-\frac{1}{3x^3} - \frac{1}{x} + \text{ArcTanh}[x]$$

$$\frac{1}{6} \left( -\frac{2 + 6x^2}{x^3} - 3 \log[1 - x] + 3 \log[1 + x] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1 - 2x^2}{x^2 - x^4}, x, 4, 0 \right\}$$

$$-\frac{1}{x} - \text{ArcTanh}[x]$$

$$\frac{1}{2} \left( -\frac{2}{x} - \text{Log}[-1-x] + \text{Log}[-1+x] \right)$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1+x^3}{x(1-x^3+x^6)}, x, 3, 0 \right\}$$

$$-\frac{\text{ArcTan}\left[\frac{1-2x^3}{\sqrt{3}}\right]}{\sqrt{3}} + \text{Log}[x] - \frac{1}{6} \text{Log}[1-x^3+x^6]$$

$$\text{Log}[x] - \frac{1}{3} \text{RootSum}\left[1-\#1^3+\#1^6 \&, \frac{-2 \text{Log}[x-\#1] + \text{Log}[x-\#1] \#1^3}{-1+2 \#1^3} \&\right]$$

Valid but unnecessarily complicated antiderivative:

$$\left\{ \frac{1+x^3}{x-x^4+x^7}, x, 6, 0 \right\}$$

$$-\frac{\text{ArcTan}\left[\frac{1-2x^3}{\sqrt{3}}\right]}{\sqrt{3}} + \text{Log}[x] - \frac{1}{6} \text{Log}[1-x^3+x^6]$$

$$\text{Log}[x] - \frac{1}{3} \text{RootSum}\left[1-\#1^3+\#1^6 \&, \frac{-2 \text{Log}[x-\#1] + \text{Log}[x-\#1] \#1^3}{-1+2 \#1^3} \&\right]$$

Unable to integrate:

$$\left\{ \frac{e-2 f(-1+n) x^n}{e^2+4 d f x^2+4 e f x^n+4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f}(-1+n) x}{e(-1+n)+2 f(-1+n) x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

$$\int \frac{e-2 f(-1+n) x^n}{e^2+4 d f x^2+4 e f x^n+4 f^2 x^{2n}} dx$$

Unable to integrate:

$$\left\{ \frac{e-2 f(-1+n) x^n}{e^2-4 d f x^2+4 e f x^n+4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f}(-1+n) x}{e(-1+n)+2 f(-1+n) x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

$$\int \frac{e-2 f(-1+n) x^n}{e^2-4 d f x^2+4 e f x^n+4 f^2 x^{2n}} dx$$

Unable to integrate:

$$\left\{ \frac{x^m(e(1+m)+2 f(1+m-n) x^n)}{e^2+4 d f x^{2+2m}+4 e f x^n+4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f}(1+m)(1+m-n) x^{1+m}}{e(1+m)(1+m-n)+2 f(1+m)(1+m-n) x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$



$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 + 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Unable to integrate:

$$\left\{ \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}}, x, 1, 0 \right\}$$

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} (1+m) (1+m-n) x^{1+m}}{e (1+m) (1+m-n) + 2 f (1+m) (1+m-n) x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$