Recommender System

Spring 2024

Hongchang Gao

Review

- Step 1: Mean subtraction

$$\tilde{X} = X - \frac{1}{n} X \mathbf{1}\mathbf{1}^T$$

- Step 2: Compute the covariance matrix

$$A = \tilde{X}\tilde{X}^T$$

- Step 3: Eigen-decomposition

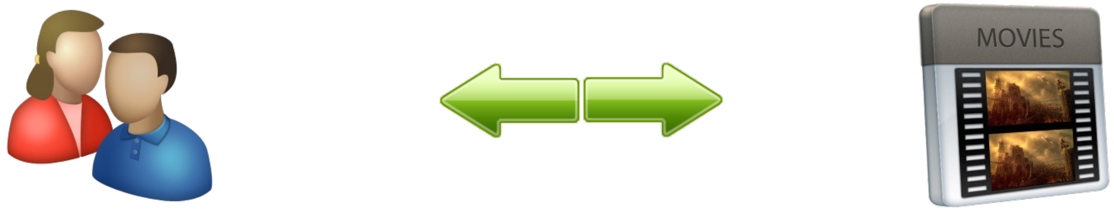
$$A = U \Sigma U^T$$

- Step 4: Keep the largest k eigenvectors

$$W = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_k] \in \mathbb{R}^{d \times k}$$

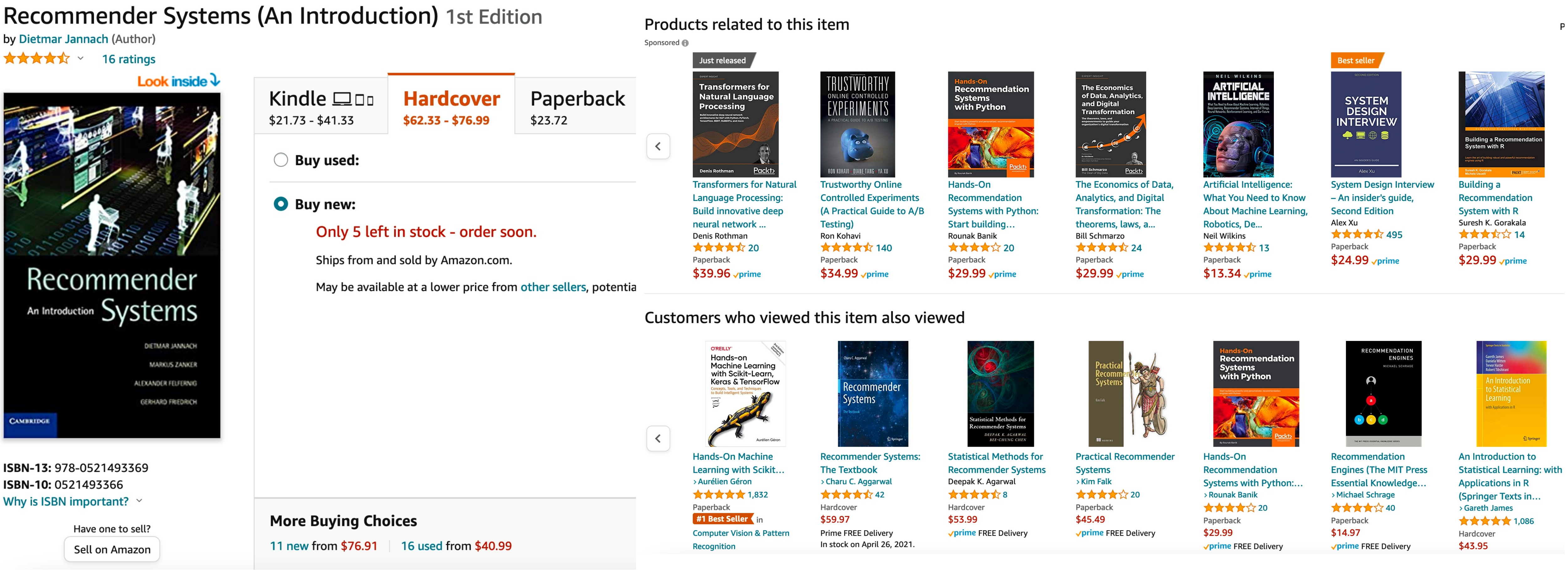
Introduction

- Recommender Systems
  - A particular type of personalized Web-based applications
  - Provide users personalized recommendations about content they may be interested
- Example:
  - Amazon: product recommendation
  - Netflix: movies recommendation
  - Google: news recommendation



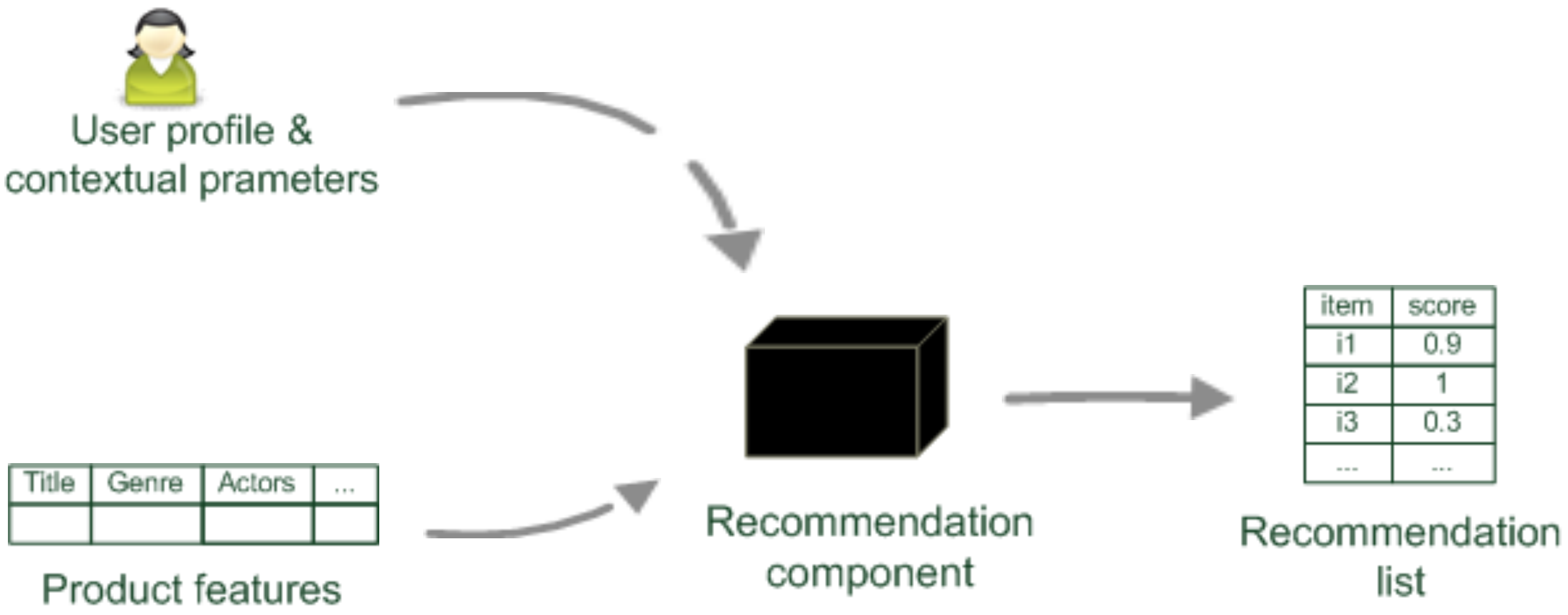
Example

- Book Recommendation: a lot of sales from recommendations



Recommender System

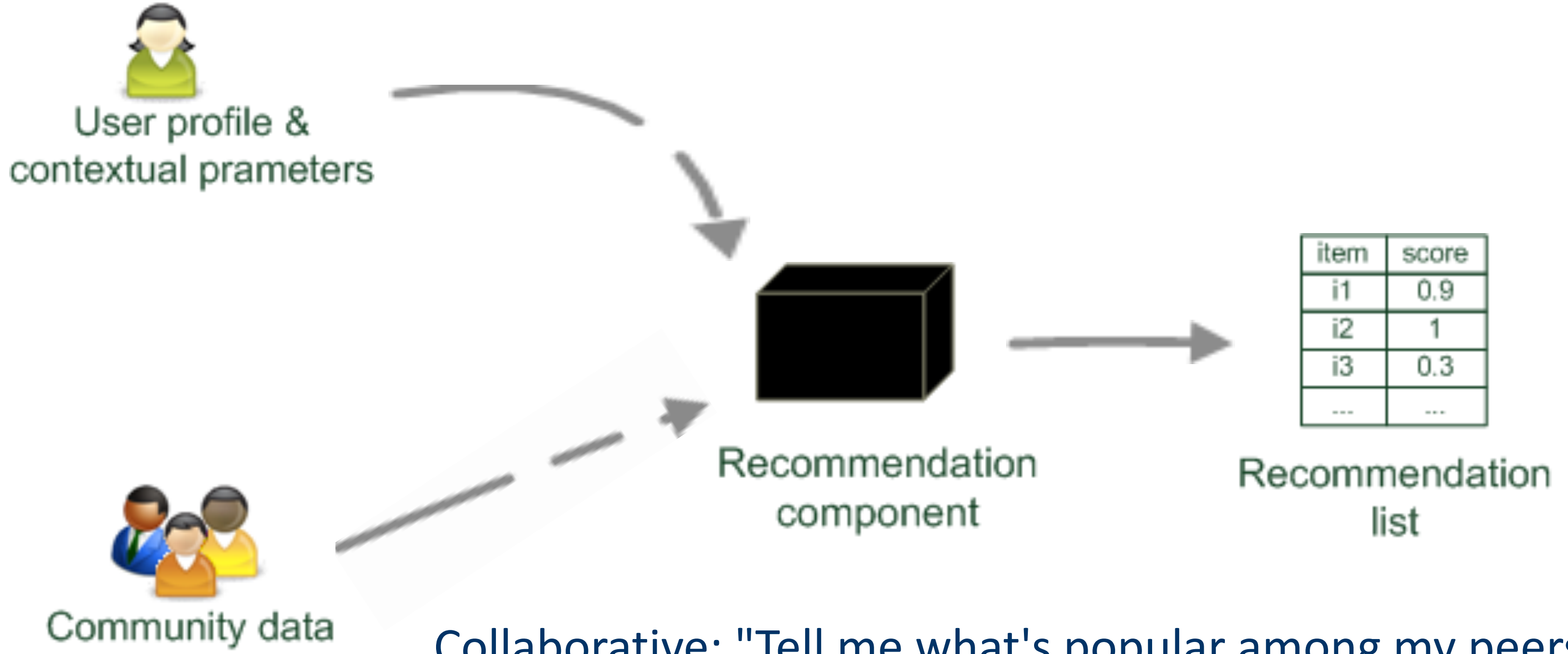
- 1. Content-based filtering
  - recommendations based on item descriptions/features



Content-based: "Show me more of the same what I've liked"

Recommender System

- 2. Collaborative filtering
  - Look at the ratings of like-minded users to provide recommendations
  - Users who have expressed similar interests in the past will share common interests in the future.



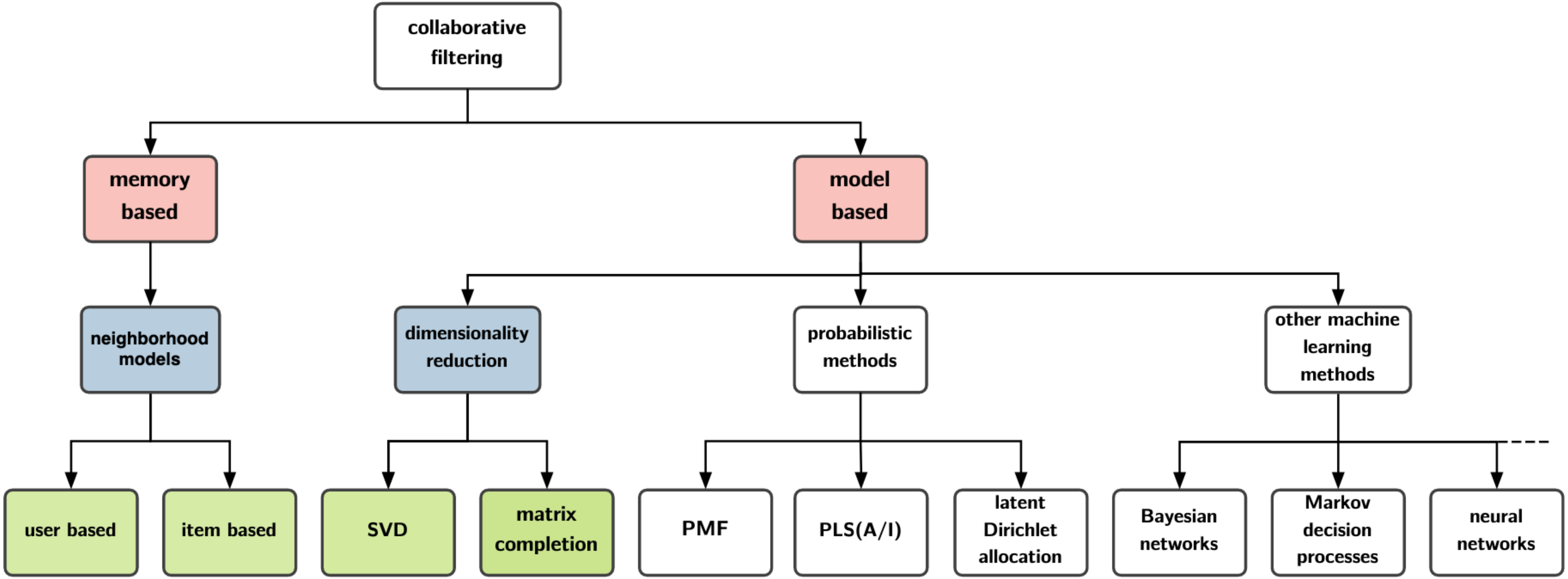
Collaborative: "Tell me what's popular among my peers"

Collaborative Filtering

- Given
  - Set of users
  - Set of items (movies, books, news, …)
  - Feedback (ratings, …)
- Predict the preference of each user for each item
  - Assumption: similar feedback ←→ similar taste

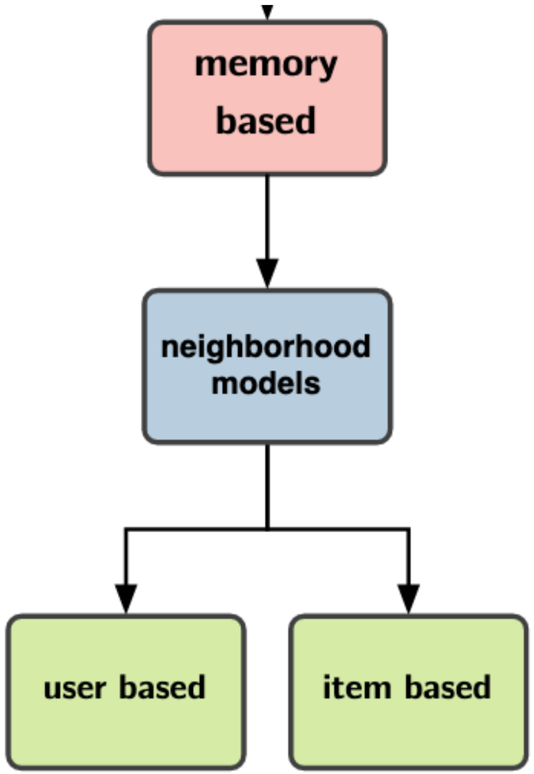
	Avatar	The Matrix	Up
Marco	?	4	2
Luca	3	2	?
Anna	5	?	3

Collaborative Filtering



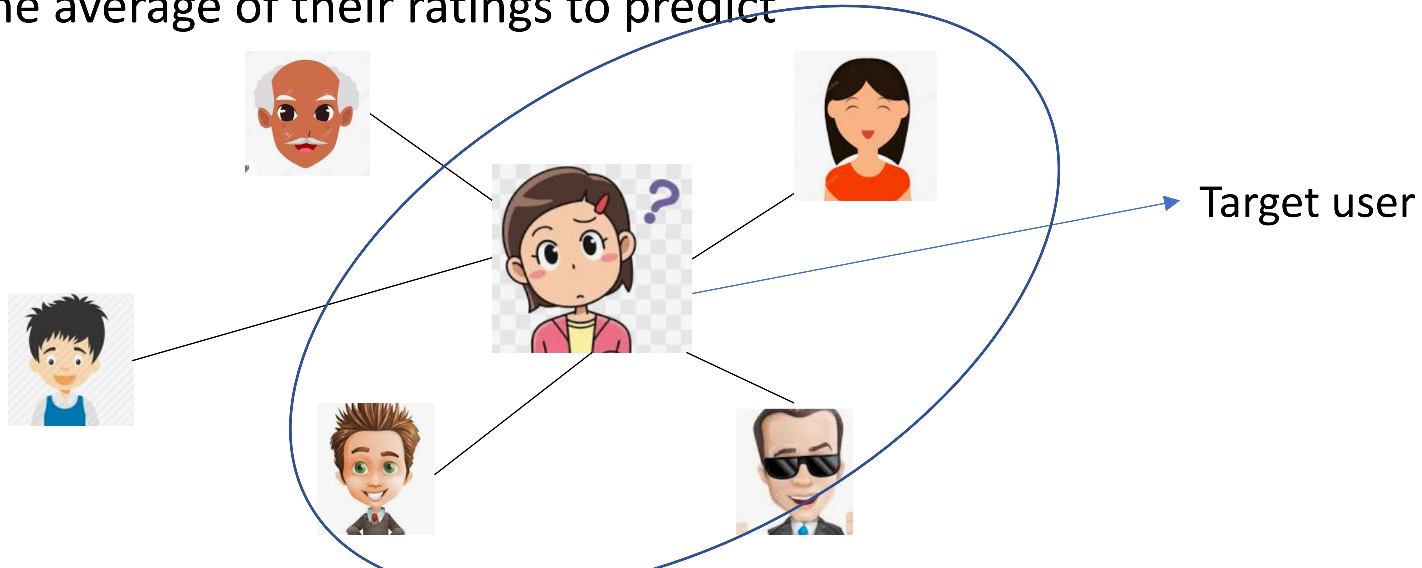
Memory based CF

- Memory based collaborative filtering
  - User based CF
  - Item based CF



Memory based CF: User-CF

- User-CF:
  - Idea
    - If users have similar tastes in the past, they will have similar tastes in the future
  - Recommend item i to Alice?
    - Find a set of users (peers/nearest neighbors) who liked the same items as the target user (Alice) in the past and who have rated item i
    - Use the average of their ratings to predict



Memory based CF: User-CF

- Illustration:
  - A database of ratings of the current user, Alice, and some other users is given
  - Determine whether Alice will like or dislike *Item5*, which Alice has not yet rated or seen

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Memory based CF: User-CF

- Questions:
  - How do we measure similarity?
  - How many neighbors should we consider?
  - How do we generate a prediction from the neighbors' ratings?

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Memory based CF: User-CF

- How to measure the user similarity?
  - Pearson correlation

    a, b : users

    $r_{a,p}$: rating of user a for item p

    P : set of items, rated by both a and b
  - Possible similarity values between -1 and 1

$$\boldsymbol{sim(a,b)} = \frac{\sum_{p \in P}(\boldsymbol{r_{a,p}} - \boldsymbol{\bar{r}_a})(\boldsymbol{r_{b,p}} - \boldsymbol{\bar{r}_b})}{\sqrt{\sum_{p \in P}(\boldsymbol{r_{a,p}} - \boldsymbol{\bar{r}_a})^2}\sqrt{\sum_{p \in P}(\boldsymbol{r_{b,p}} - \boldsymbol{\bar{r}_b})^2}}$$

Memory based CF: User-CF

- Pearson correlation

$$sim(a,b) = \frac{\sum_{p \in P}(r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P}(r_{a,p} - \bar{r}_a)^2}\sqrt{\sum_{p \in P}(r_{b,p} - \bar{r}_b)^2}}$$

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

sim = 0,85

sim = 0,00

sim = 0,70

sim = -0,79

Memory based CF: User-CF

- How to make predictions?
  - Use similarity threshold or fixed number of neighbors

$$\boldsymbol{pred(a,p)} = \overline{\boldsymbol{r_a}} + \frac{\sum_{\boldsymbol{b} \in \boldsymbol{N}} \boldsymbol{sim(a,b)} * (\boldsymbol{r_{b,p}} - \overline{\boldsymbol{r_b}})}{\sum_{\boldsymbol{b} \in \boldsymbol{N}} \boldsymbol{sim(a,b)}}$$

  - Calculate, whether the neighbors' ratings for the unseen item are higher or lower than their average
  - Combine the rating differences – use the similarity with a as a weight
  - Add/subtract the neighbors' bias from the active user's average and use this as a prediction

Memory based CF: User-CF

- Prediction

$$\boldsymbol{pred(a,p) = \overline{r_a} + \frac{\sum_{b \in N} sim(a,b) * (r_{b,p} - \overline{r_b})}{\sum_{b \in N} sim(a,b)}}$$

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

sim = 0,85

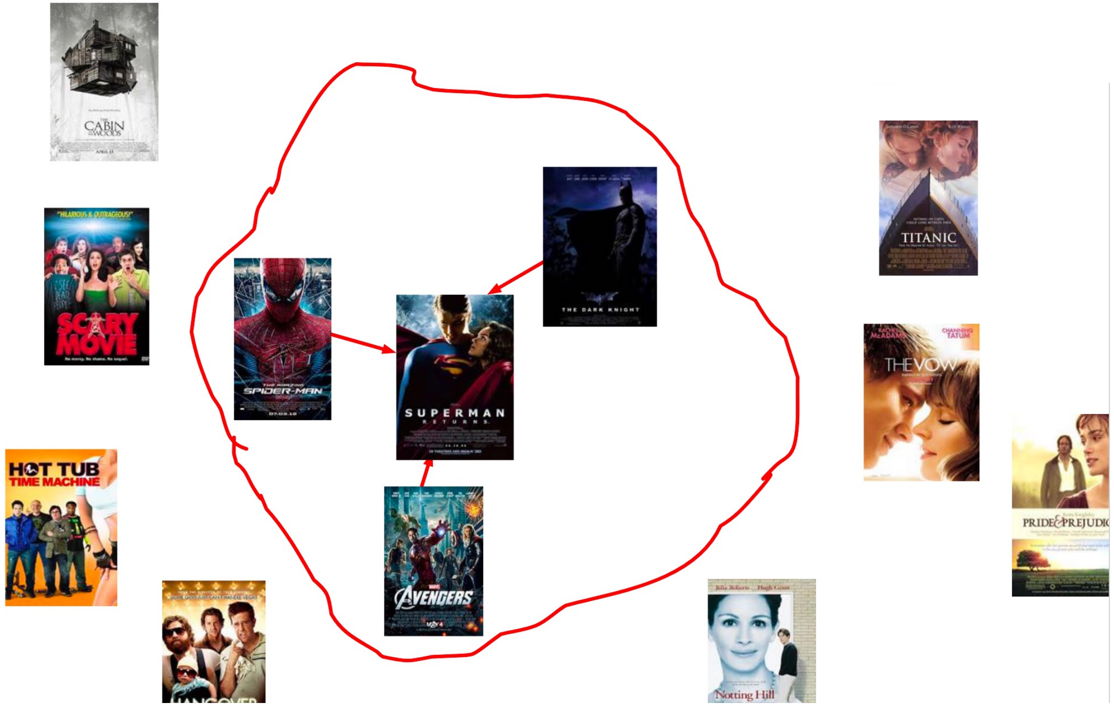
sim = 0,00

sim = 0,70

sim = -0,79

Memory based CF: Item-CF

- Idea:
  - Use the similarity between items (and not users) to make predictions



Memory based CF: Item-CF

- Idea:
  - Use the similarity between items (and not users) to make predictions
- Steps:
  - Look for items that are similar to item5
  - Take Alice's ratings for these items to predict the rating for item5

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Memory based CF: Item-CF

- How to measure the item similarity?
  - Cosine similarity

$$sim(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| * |\vec{b}|}$$

  - Ratings are seen as vector in n-dimensional space

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Memory based CF: Item-CF

- How to make prediction?

$$\boldsymbol{pred(u,p)} = \frac{\sum_{i \in ratedItem(u)} \boldsymbol{sim(i,p)} * \boldsymbol{r_{u,i}}}{\sum_{i \in ratedItem(u)} \boldsymbol{sim(i,p)}}$$

  - Neighborhood size is typically also limited to a specific size
  - Not all neighbors are taken into account for the prediction

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Evaluation

- 1. MAE and RMSE
  - Mean Absolute Error (*MAE*) computes the deviation between predicted ratings and actual ratings

$$MAE = \frac{1}{n}\sum_{i=1}^{n} | p_i - r_i |$$

  - Root Mean Square Error (*RMSE*) is similar to *MAE*, but places more emphasis on larger deviation

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (p_i - r_i)^2}$$

Evaluation

Actually good
Item 237
Item 899

Recommended (predicted as good)
Item 345
Item 237
Item 187

- 2. Precision and Recall
  - Precision: a measure of exactness, determines the fraction of relevant items retrieved out of all items retrieved
    - E.g. the proportion of recommended movies that are actually good

$$Precision = \frac{tp}{tp + fp} = \frac{|good\ movies\ recommended|}{|\text{all recommendations}|}$$

  - Recall: a measure of completeness, determines the fraction of relevant items retrieved out of all relevant items
    - E.g. the proportion of all good movies recommended

$$Recall = \frac{tp}{tp + fn} = \frac{|good\ movies\ recommended|}{|all\ good\ movies|}$$