Adaptive Pole Placement Control

Contents

APPC = a pole placement control law + parameter adaptive law

- The desired properties of the plant are expressed in terms of desired pole locations to be placed by the controller
- applicable to both minimum- and nonminimum-phase systems
- includes *direct* and *indirect* two categories.
- the use of adaptive laws with normalization and without normalization does not extend to APPC.

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Scalar Example: Adaptive Regulation

Consider the scalar plant

$$\dot{y} = ay + bu$$

where a and $b \neq 0$ are *unknown* constants, and the sign of b is known.

The control objective: choose u so that the closed-loop pole is placed at $-a_m < 0$, and $y \to 0$ as $t \to \infty$.

The idea control law

$$u = -k^* y, \quad k^* = \frac{a + a_m}{b}$$

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The closed-loop plant

$$\dot{y} = -a_m y$$

achieves the control objective. Consider a, b unknown, use the certainty equivalence approach to form the APPC as

$$u = -ky$$

with the estimate k is generated in two different ways:

- Direct approach : $\dot{\hat{k}} = \phi(y)$
- Indirect approach: $k = \frac{\hat{a}+a_m}{\hat{b}}, \hat{b} \neq 0$

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Direct Adaptive Regulation

Add and subtract the term $-a_m y$ in the plant equation to obtain

$$\begin{aligned}\dot{y} &= -a_m y + (a + a_m)\, y + bu \\ &= -a_m y + b\,(k^* y + u) \\ &= -a_m y - b\tilde{k}y\end{aligned}$$

where $\tilde{k} \triangleq k - k^*$ is the parameter error. Equation relates the parameter error term $b\tilde{k}y$ with the regulation error y and motivates the Lyapunov function

$$V = \frac{y^2}{2} + \frac{\tilde{k}^2|b|}{2\gamma}$$

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time derivative of V is

$$\dot{V} = -a_m y^2 - b\tilde{k}y^2 + \frac{|b|}{\gamma}\tilde{k}\dot{\tilde{k}} = -a_m y^2 - b\tilde{k}y^2 + b\tilde{k}\frac{\dot{\tilde{k}}}{\gamma}\operatorname{sgn}(b)$$

The adaptive law

$$\dot{k} = \gamma y^2 \operatorname{sgn}(b)$$

leads to $\dot{V} = -a_m y^2 \leq 0$ which implies

- $y, \tilde{k}, k \in \mathcal{L}_\infty$ and $y \in \mathcal{L}_2$.
- $\dot{y}, u \in \mathcal{L}_\infty$; (Barbalat's lemma) and $y(t) \to 0$ as $t \to \infty$

However, this does not guarantee that the closed-loop pole of the plant is placed at $-a_m$ even asymptotically with time.

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where we have using the algebraic equation

$$k = \frac{\hat{a} + a_m}{\hat{b}}, u = ky$$

and $\tilde{a} = \hat{a} - a, \quad \tilde{b} = \hat{b} - b$. This equation relates the regulation or estimation error y with the parameter errors $\tilde{a}, \tilde{b}$.

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Indirect Adaptive Regulation

Motivates the Lyapunov candidate function

$$V = \frac{y^2}{2} + \frac{\tilde{a}^2}{2\gamma_1} + \frac{\tilde{b}^2}{2\gamma_2}$$

for some $\gamma_1, \gamma_2 > 0$. The time derivative $\dot{V}$ is given by

$$\dot{V} = -a_m y^2 - \tilde{a} y^2 - \tilde{b} u y + \frac{\tilde{a}\dot{\tilde{a}}}{\gamma_1} + \frac{\tilde{b}\dot{\tilde{b}}}{\gamma_2}$$

Choosing

$$\dot{\tilde{a}} = \dot{\hat{a}} = \gamma_1 y^2, \quad \dot{\tilde{b}} = \dot{\hat{b}} = \gamma_2 y u$$

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Indirect Adaptive Regulation

To guarantee $\hat{b} \neq 0$, introduce a projection

$$\dot{\hat{a}} = \gamma_1 y^2,$$

$$\dot{\hat{b}} = \begin{cases} \gamma_2 yu & \text{if } |\hat{b}| > b_0 \text{ or if } |\hat{b}| = b_0 \text{ and } \operatorname{sgn}(b) yu \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\hat{b}(0)$ is chosen so that $\hat{b}(0)\operatorname{sgn}(b) \geq b_0$. Furthermore, the time derivative $\dot{V}$ satisfies

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The projection introduces the extra term $-\tilde{b}yu$

$$\tilde{b}yu = \hat{b}yu - byu = (|\hat{b}| - |b|)\,\mathrm{sgn}(b)yu = (b_0 - |b|)\,\mathrm{sgn}(b)yu \geq 0$$

Hence the projection can only make $\dot{V}$ more negative and

$$\dot{V} \leq -a_m y^2 \quad \forall t \geq 0$$

always hold.

- $y, \tilde{a}, \tilde{b} \in \mathcal{L}_\infty, y \in \mathcal{L}_2$, and $|\hat{b}(t)| \geq b_0 \forall t \geq 0$, which implies that $\tilde{k} \in \mathcal{L}_\infty$.
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Scalar example : Adaptive Tracking

Consider the plant

$$\dot{y} = ay + bu$$

with a, b are unknown constants, but the sign of $b \neq 0$ is known.

Control objective:

i) closed-loop pole is at $-a_m$ and $u, y \in \mathcal{L}_\infty$

ii) $y(t)$ tracks the reference signal $y_m(t) = c$, $\forall t \geq 0$, where $c \neq 0$ is a known constant.

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Define the tracking error $e = y - c$ satisfies

$$\dot{e} = ae + ac + bu$$

Then, the tracking objective becomes $e(t) = y(t) - y_m \to 0$ as $t \to \infty$ exponentially fast. If a, b, c are known, we can choose an idea control law

$$u = -k_1^* e - k_2^*, \quad k_1^* = \frac{a + a_m}{b}, \quad k_2^* = \frac{ac}{b}$$

leading to

$$\dot{e} = -a_m e$$

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Direct Adaptive Tracking

The CE control law

$$u = -k_1 e - k_2$$

As before, we rewrite the error equation as

$$\begin{aligned}\dot{e} &= -a_m e + b\left(u + k_1^* e + k_2^*\right) \\ &= -a_m e - b(\tilde{k}_1 e + \tilde{k}_2)\end{aligned}$$

where $\tilde{k}_1 = k_1 - k_1^*, \tilde{k}_2 = k_2 - k_2^*,$which relates the tracking error e with the parameter errors $\tilde{k}_1, \tilde{k}_2$ and motivates the Lyapunov function

$$V = \frac{e^2}{2} + \frac{\tilde{k}_1^2 |b|}{2\gamma_1} + \frac{\tilde{k}_2^2 |b|}{2\gamma_2}$$

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The time derivative $\dot{V}$ is forced to satisfy

$$\dot{V} = -a_m e^2$$

by choosing

$$\dot{k}_1 = \gamma_1 e^2 \operatorname{sgn}(b), \quad \dot{k}_2 = \gamma_2 e \operatorname{sgn}(b)$$

The APPC scheme may be written as

$$u = -k_1 e - \gamma_2 \operatorname{sgn}(b) \int_0^t e(\tau) d\tau$$

$$\dot{k}_1 = \gamma_1 e^2 \operatorname{sgn}(b)$$

which is referred to as the direct adaptive proportional plus integral (API) controller.

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provided $\hat{b} \neq 0$. Again we rewrite the error equation as

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Leading to an adaptive law

$$\dot{\hat{a}} = \gamma_1 e y,$$
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where $\hat{b}(0)$ satisfies $\hat{b}(0)\operatorname{sgn}(b) \geq b_0$. One can verify the boundedness of and convergence of the tracking error by considering the Lyapunov function

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PPC

Consider the SISO LTI plant

$$y_p = G_p(s) u_p, \quad G_p(s) = \frac{Z_p(s)}{R_p(s)}$$

where $G_p(s)$ is proper and $R_p(s)$ is a monic polynomial.

The control objective is to choose u_p so that

- the closed-loop poles are assigned to those of a monic Hurwitz polynomial $A^*(s)$
- y_p is required to follow a certain class of reference signals y_m

Remark: In general, by assigning the closed-loop poles to those of $A^*(s)$, we can guarantee closed-loop stability and convergence of the plant output y_p to zero, provided that there is no external input.

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Assumptions

P1. $R_p(s)$ is a monic polynomial whose degree n is known.

P2. $Z_p(s), R_p(s)$ are coprime and degree $(Z_p) < n$.

P3. The reference signal $y_m \in \mathcal{L}_\infty$ is assumed to satisfy

$$Q_m(s)y_m = 0$$

where $Q_m(s)$, known as the internal model of y_m, is a known monic polynomial of degree q with all roots in $\Re[s] \leq 0$ and with no repeated roots on the $j\omega$ -axis. The internal model $Q_m(s)$ is assumed to be coprime with $Z_p(s)$.

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$$Q_m(s) y_m = 0$$

and $Q_m(s)$ is assumed to be coprime with $Z_p(s)$.

Example

if $y_m(t) = c + \sin(3t)$, where c is any constant, then

$Q_m(s) = s\left(s^2 + 9\right)$ and, according to $\mathrm{P3}, Z_p(s)$ should not have s

or $s^2 + 9$ as a factor.

Indirect APPC via Polynomial Approach

The design of the APPC scheme is based on three steps:

1. a control law is developed that can meet the control objective in the known parameter case. i.e. PPC
2. an adaptive law is designed to estimate the plant parameters online. The estimated plant parameters are then used to calculate the controller parameters at each time t
3. the APPC is formed by replacing the controller parameters in Step 1 with their online estimates.

APPC via polynomial approach

Step 1. PPC for known parameters. We consider the control law

$$u_p = -\frac{P(s)}{Q_m(s)L(s)}\,(y_p - y_m)$$

where $P(s), L(s)$ are polynomials (with $L(s)$ monic) of degree $q+n-1, n-1$, respectively, chosen to satisfy the polynomial equation

$$L(s)Q_m(s)R_p(s) + P(s)Z_p(s) = A^*(s)$$

Assumptions $\mathrm{P2}$ and $\mathrm{P3}$ guarantee that $Q_m(s)R_p(s)$ and $Z_p(s)$ are coprime, which guarantees hat $L(s), P(s)$ exist and are unique.

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APPC via polynomial approach

The closed-loop plant

$$y_p = \frac{Z_p P}{A^*} y_m$$

and the control law

$$u_p = \frac{R_p P}{A^*} y_m$$

if $y_m \in \mathcal{L}_\infty$ and $\frac{Z_p P}{A^*}, \frac{R_p P}{A^*}$ are proper with stable poles, it follows that $y_p, u_p \in \mathcal{L}_\infty$.

The tracking error $e_1 = y_p - y_m$ can be shown to satisfy

$$e_1 = -\frac{LR_p}{A^*} Q_m y_m = -\frac{LR_p}{A^*}[0]$$

it is easily to see $e_1 \to 0$ as $t \to \infty$.

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APPC via polynomial approach

Since $L(s)$ is not necessarily Hurwitz, the control law

$$u_p = -\frac{P(s)}{Q_m(s)L(s)}\,(y_p - y_m)$$

may have unstable poles. An alternative realization is

$$\Lambda(s)u_p = \Lambda(s)u_p - Q_m(s)L(s)u_p - P(s)\,(y_p - y_m)$$

where $\Lambda(s)$ is any monic Hurwitz polynomial of degree $n+q-1$.

Filtering each side with $\frac{1}{\Lambda(s)}$, we obtain

$$u_p = \frac{\Lambda - LQ_m}{\Lambda} u_p - \frac{P}{\Lambda}\,(y_p - y_m)$$

The control law is implemented using $2(n+q-1)$ integrators to realize the proper stable transfer functions $\frac{\Lambda - LQ_m}{\Lambda}, \frac{P}{\Lambda}$.

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APPC via polynomial approach

Step 2. Estimation of plant polynomials The following PM can be derived:

$$z = \theta_p^{*\top} \phi$$

where

$$z = \frac{s^n}{\Lambda_p(s)} y_p, \quad \theta_p^* = \left[\theta_b^{*\top}, \theta_a^{*\top}\right]^\top, \quad \phi = \left[\frac{\alpha_{n-1}^\top(s)}{\Lambda_p(s)} u_p, -\frac{\alpha_{n-1}^\top(s)}{\Lambda_p(s)} y_p\right]^\top$$

$$\alpha_{n-1}(s) = \left[s^{n-1}, \ldots, s, 1\right]^\top, \theta_a^* = [a_{n-1}, \ldots, a_0]^\top, \theta_b^* = [b_{n-1}, \ldots, b_0]^\top$$

and $\Lambda_p(s)$ is a monic Hurwitz polynomial.

Note that, if the degree of $Z_p(s)$ is less than $n-1$, then some of the first elements of θ_b^* will be equal to zero.

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APPC via polynomial approach

A wide class adaptive laws can be designed to estimate θ_p^*. For example, the gradient algorithm

$$\dot{\theta}_p = \Gamma \varepsilon \phi, \quad \varepsilon = \frac{z - \theta_p^T \phi}{m_s^2}, \quad m_s^2 = 1 + \phi^T \phi$$

where $\Gamma = \Gamma^\top > 0$ is the adaptive gain and

$$\theta_p = \left[\hat{b}_{n-1}, \ldots, \hat{b}_0, \hat{a}_{n-1}, \ldots, \hat{a}_0\right]^T$$

are the estimated plant parameters form the plant polynomial

$$\hat{R}_p(s,t) = s^n + \hat{a}_{n-1}(t)s^{n-1} + \cdots + \hat{a}_1(t)s + \hat{a}_0(t),$$
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APPC via polynomial approach

Step 3. Adaptive control law

$$u_p = \Big(\Lambda(s) - \hat{L}(s,t)Q_m(s)\Big)\frac{1}{\Lambda(s)}u_p - \hat{P}(s,t)\frac{1}{\Lambda(s)}(y_p - y_m)$$

with online estimates $\hat{L}(s,t), \hat{P}(s,t)$ calculated at each frozen time t using the polynomial equation

$$\hat{L}(s,t)\cdot Q_m(s)\cdot \hat{R}_p(s,t) + \hat{P}(s,t)\cdot \hat{Z}_p(s,t) = A^*(s)$$

Remark: stabilizability problem in indirect APPC

The existence and uniqueness of $\hat{L}(s,t), \hat{P}(s,t)$ is guaranteed, provided that $\hat{R}_p(s,t)Q_m(s), \hat{Z}_p(s,t)$ are coprime at each frozen time t, which means that at certain points in time the solution $\hat{L}(s,t)$ $\hat{P}(s,t)$ may not exist.

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APPC via polynomial approach

Theorem: Assume that the estimated plant polynomials $\hat{R}_p Q_m, \hat{Z}_p$ are *strongly coprime* at each time t. Then all the signals in the closed loop are uniformly bounded, and the tracking error converges to zero asymptotically with time.

Remark: By strongly coprime we mean that the polynomials do not have roots that are close to becoming common. For example, the polynomials $s+2+\epsilon$, $s+2$, where ϵ is arbitrarily small, are not strongly coprime.

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APPC via polynomial approach

Example: Consider the plant

$$y_p = \frac{b}{s+a} u_p$$

where a and b are constants. The desired roots

$$A^*(s) = (s+1)^2$$

and the reference signal $y_m = 1$.