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A hierarchical design framework for distributed control of multi-agent systems☆

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ABSTRACT

In this paper, a hierarchical design framework is proposed for distributed control of multi-agent systems. Different from the traditional distributed design philosophy that directly couples the agents' cooperation and individual regulations, the proposed framework decouples these regulations and separates the distributed control design into two layers, i.e., the layer of reference signal generator design and the layer of tracking controllers design. Specifically, the nodes of the generator play the roles of some virtual agents that achieve the distributed control goal and the agents' tracking controllers are used to track the reference signals produced by the generator. The novel design framework makes the agents reach the distributed control goal well and it has better design flexibility/practicality and stronger versatility/scalability. To demonstrate these advantages, two types of representative examples on hierarchical design are exhibited for multi-agent systems with mismatched disturbances and those with communication/input delays, respectively. All examples confirm that the proposed design framework is a natural choice in the presence of some complex factors, which provides not only simpler design but also more autonomy to the agents.

1. Introduction

Over the past years, distributed control of multi-agent systems has been an active research issue, mainly because of higher efficiency compared with traditional centralized control and wide applications in practice. Typical distributed control problems include but not limited to consensus, containment, formation, flocking, coverage control and so on. With the research development, more and more factors are concerned from both the agent models and communication networks, such as disturbances acting on the agents (Ménard, Ali Ajwad, Moulay, Coirault, & Defoort, 2020) and time delays occurring in control or communication process (Tian & Liu, 2008). Research on distributed control with these factors has attracted considerable attention.

On anti-disturbance distributed control, most of the results have been concentrated on the agents with matched disturbances that are in the same channels as the control inputs. Differently, the mismatched disturbances are in different channels from the control inputs. Mismatched disturbances also widely exist in the agents' dynamics, such as the skidding/slipping disturbances in mobile robots (Chwa, 2016), the capacitance parametric perturbations and load variations in converters/inverters of the smart grids (Kumar, Mohanty, & Kumar, 2020; Nguyen, Kim, Kim, Choi, & Jung, 2018), and the friction acting on the hydraulic cylinder piston rods of the multi-hydraulic manipulator systems (Zeng & Sepehri, 2005). In contrast with the matched disturbances, the mismatched disturbances of a certain agent not only affect itself but also interfere with its neighbors through cooperation error dynamics. To cope with the cooperative control problems with mismatched disturbances, several kinds of control methods have been proposed including feedback control methods via feedback suppression (Chen, Chen, & Astolfi, 2021, 2022; Monshizadeh & De Persis, 2017; Shen & Shi, 2015; Wang, Huang, Wen, & Fan, 2014) and feedforward-feedback composite control methods via disturbance estimation/distributed compensation (Hua, Dong, Han, Li, & Ren, 2021; Wang, Li, & Lam, 2016; Wang, Li, Yu, & Yang, 2017; Wang, Zheng, & Wang, 2023; Xiao, Ren, Qi, Li, & Lu, 2022; Yu, Long, & Guo, 2016). Both the above feedback and feedforward–feedback composite control methods directly use the cooperation errors to conduct control design on the agents' models. Since the cooperation error dynamics for a certain agent contain the mismatched disturbances of its neighbors,

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the feedback controller (or composite controller) needs to suppress (or compensate) the mismatched disturbances from both itself and its neighbors, which requires extra information on the bounds of its neighbors' mismatched disturbances (which is difficult to obtain in practice) or communication cost on distributed compensation of mismatched disturbances from its neighbors. In addition, both kinds of control methods are "ex-post-facto" remedies, which work after the effects of mismatched disturbances on the cooperation process. Accordingly, a more concise and "pre-prevention" distributed control scheme is desired in the presence of mismatched disturbances.

Due to retransmission, congestion, limited bandwidth, slow processing time and other impacts, communication delays and input delays generally exist in multi-agent systems. Communication delays caused by interaction from one agent to another are common delays in networked systems. Input delays are inducing delays while exchanging data among devices (such as sensor-to-controller delays and controller-to-actuator delays) (Tian & Liu, 2008). These two types of time delays may deteriorate the system performances and even lead to instability (Gu, Niculescu, & Chen, 2005). Consequently, distributed control of multi-agent systems with time delays has been widely studied in recent years. Designing distributed controllers for multi-agent systems with a single type of delays is not trivial as the delays make the systems infinite dimensional, let alone solutions of the cases with both types of delays. Many research efforts have been spent on distributed control with a single type of delays (Hatanaka, Chopra, Ishizaki, & Li, 2018; Li, Hua, You, & Guan, 2022; Loizou, Lui, Petrillo, & Santini, 2021; Ni, Liu, Liu, & Liu, 2017; Nuño, Loría, Hernández, Maghenem, & Panteley, 2020; Olfati-Saber & Murray, 2004; Yu, Chen, & Cao, 2010). The more challenging distributed control problems with both types of delays are studied in Cepeda-Gomez and Olgac (2013), Dai, Liu, and Liu (2013), Ma and Xu (2022), Meng, Ren, Cao, and You (2011), Ponomarev, Chen, and Zhang (2018), Tian and Liu (2008) and Xu, Liu, and Feng (2018). It is noticed that in the literature considering both types of delays, communication and input delays are usually coupled together in the process of controller design and stability analysis, which makes the design and analysis complicated. As two different and independent delays, a natural idea is to decouple communication and input delays and cope with them separately, so that the existing mature methods can be flexibly adopted to handle them, respectively.

It is worth pointing out that either the aforementioned traditional distributed control methods for multi-agent systems with mismatched disturbances or those with communication/input time delays directly couple the agents' cooperation and individual regulations together. This philosophy makes the design heavily rely on the agents' cooperation errors and models, which causes some problems. Firstly, since the distributed control design is directly made on the agents' models by using cooperation errors, the design flexibility is restricted. Secondly, the design complexity increases significantly as the agent dynamics complexity/homogeneity or the cooperation task complexity grows, which constrains the design practicality. Thirdly, the controllers need to be redesigned frequently for various tasks and this leads to extra cost and is even infeasible with controller encapsulation, which limits the design versatility and scalability. To this end, a design philosophy that decouples the agents' cooperation regulations and individual regulations is yearned for. Actually, the intelligent hierarchical control approach can provide some inspiration (Antsaklis, 1999; Passino, 2001), which consists of three layers (i.e., organization layer, coordination layer and execution layer from the highest to the lowest). Specifically, the coordination layer offers commands to the execution layer and control is executed in the execution layer, which can decouple the agents' cooperation and individual regulations to some content. Similarly, the embedded control technique also has a hierarchical structure (Hristu-Varsakelis & Levine, 2005) and it is composed of the reference signal design and tracking control design, which corresponds to the coordination and execution layers, respectively. Based on the hierarchical design philosophy, some results have been reported on distributed control of smart grid (Bidram & Davoudi, 2012; Guerrero, Vasquez, de Vicuna, & Castilla, 2011) and distributed optimization (Tang, Deng, & Hong, 2019; Wang, Wang, et al., 2020), but there are rarely related results on general distributed control problems.

In this paper, a hierarchical design framework is built for general distributed control of multi-agent systems. The design framework constitutes two layers, i.e., Layer I-Reference signal generator design and Layer II-Agents' tracking controllers design. The generator is set to provide reference signals for the agents (of which the nodes play the roles of some "virtual agents" that "achieve" the distributed control goal) and the tracking controllers are designed for the agents to track the reference signals from the generator. With both layers, the distributed task can be completed well. The proposed design framework decouples the agents' cooperation and individual regulations, which brings some nice features: (1) The generator design and tracking controllers design are "separative" in the proposed framework, which allows more design flexibility with specific demands and provides more autonomy to the agents. (2) The proposed framework simplifies the distributed control design and especially weakens the design difficulties caused by complex agents' models or tasks, which strengthens the design practicality. (3) The proposed framework provides a possible manner where the agents can realize the distributed control tasks without changing their original controllers, which enhances the design versatility and scalability. To demonstrate these features, two kinds of examples are given on distributed control of multi-agent systems with mismatched disturbances and those with communication/input delays. Besides, in the example with mismatched disturbances, the proposed framework makes "pre-prevention" on effects of the mismatched disturbances from the cooperation process. In the example with communication/input delays, the proposed framework decouples the two types of delays and handles them separately in Layers I and II, respectively.

The remainder of the paper is organized as follows. In Section 2, some preliminaries and the problem formulation are given. In Section 3, two hierarchical design examples are given on distributed control of multi-agent systems with mismatched disturbances. In Section 4, another hierarchical design example is given on distributed control of multi-agent systems with communication and input disturbances. In Section 5, some numerical simulations are performed to validate the proposed framework. Finally, some conclusions are drawn in Section 6.

2. Preliminaries and problem formulation

2.1. Notations

Denote $\mathbf{0}_{n\times m}$ as the $n \times m$ null matrix and I_m as the $m \times m$ identity matrix, and $\mathbf{1}_m = [1, \ldots, 1]^T \in \mathbb{R}^m$, $\mathbf{0}_m = [0, \ldots, 0]^T \in \mathbb{R}^m$. For a vector $x = [x_1, \ldots, x_m]^T \in \mathbb{R}^m$ and $\alpha \in \mathbb{R}$, denote $x^\alpha = [x_1^\alpha, \ldots, x_m^\alpha]^T$, $\mathrm{sig}^\alpha(x) = [\mathrm{sig}^\alpha(x_1), \ldots, \mathrm{sig}^\alpha(x_m)]^T$, where $\mathrm{sig}^\alpha(z) = |z|^\alpha \mathrm{sgn}(z), \forall z, \alpha \in \mathbb{R}$ and $\mathrm{sgn}(\cdot)$ is the standard sign function. Especially, $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_m)]^T$. Denote $\|x\|_1 = \sum_{i=1}^m |x_i|$, $\|x\|_2 = \sqrt{x^T x}$, $\|x\|_\infty = \max_{i=1,\ldots,m}\{|x_i|\}$ as the 1-norm, Euclidean norm and infinity norm of vector x, respectively. A basic property is that $\|x\|_2 \le \|x\|_1 \le \sqrt{m}\|x\|_2, \forall x \in \mathbb{R}^m$. For a matrix $B \in \mathbb{R}^{n\times m}$, denote $[B]_{i,j}$ as its (i,j)-th element and denote $\|B\|_2 = \sqrt{\lambda_{\max}(B^T B)}$ and $\|B\|_\infty = \max_{i=1,\ldots,n}\left\{\sum_{j=1}^m |[B]_{i,j}|\right\}$

as its Euclidean norm and infinity norm, respectively. For a symmetric matrix $P \in \mathbb{R}^{m\times m}$, its eigenvalues are denoted as $\lambda_{\min}(P) = \lambda_1(P) \le \lambda_2(P) \le \cdots \le \lambda_m(P) = \lambda_{\max}(P)$ in a non-decreasing order. For two matrices $A \in \mathbb{R}^{m\times m}, B \in \mathbb{R}^{m\times m}$, $A > (\ge) B$ means that $A - B$ is positive (semi-)definite and $A < (\le) B$ means that $B - A$ is positive (semi-)definite. For any two matrices $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{p\times q}$, $C \otimes D \in \mathbb{R}^{mp\times nq}$ denotes their Kronecker product, where m, n, p, q are positive integers.

2.2. Lemmas and definitions

Consider the system $\dot{x} = f(t, x, u)$, where $f : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u. $u(t)$ is piecewise continuous and bounded in t, $\forall t \ge 0$.

Lemma 1 (Khalil, 2002)**.** *If the above system is input-to-state stable (ISS) and* $\lim_{t\to\infty} u(t) = 0$, *then* $\lim_{t\to\infty} x(t) = 0$.

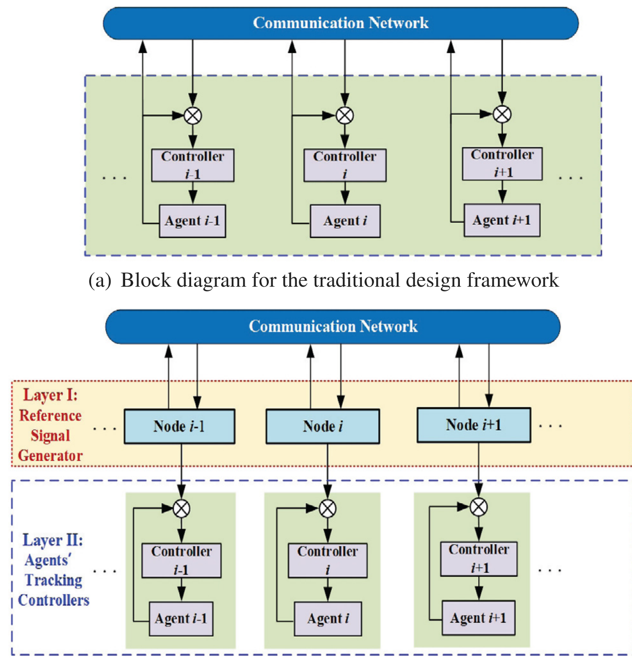
Lemma 2 (Khalil, 2002)**.** *Suppose* $f(t, x, u)$ *is continuously differentiable and globally Lipschitz in* (x, u), *uniformly in* t. *If the origin* $x = \mathbf{0}_n$ *is the globally uniformly exponentially stable equilibrium of the unforced system* $\dot{x} = f(t, x, 0)$, *then system* $\dot{x} = f(t, x, u)$ *is ISS.*

2.3. Graph theory notions

For the leaderless case, the communication topology of the agents is denoted by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, \ldots, N\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix. $a_{ij} = a_{ji} > 0$ if $(j, i) \in \mathcal{E}$ while $a_{ij} = 0$ otherwise, and $a_{ii} = 0, \forall i \in \mathcal{V}$. The neighbor set of node i is $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. The Laplacian matrix of $\mathcal{G}$ is $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$, where $l_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. In the graph $\mathcal{G}$, a path from node i to node j is a sequence of adjacent edges as $(i, k_1), (k_1, k_2), \ldots, (k_l, j)$ with distinct nodes $k_m, m = 1, \ldots, l$. An undirected graph is connected if there is a path between each node pair. For the leader–follower case, the leader is denoted by node 0 and the followers are denoted by nodes $1, \ldots, N$ and the topology of the followers is still denoted by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the Laplacian matrix $\mathcal{L}$ and the graph of all the leader–follower agents is denoted by a directed graph $\bar{G} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ with $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$. The communication between the leader and follower i is unidirectional from the leader to the follower and the edge weight is b_i. $b_i > 0$ if follower i is connected to the leader, while $b_i = 0$ otherwise. $\mathcal{B} = \text{diag}\{b_1, \ldots, b_N\}$ is the leader adjacency matrix and $\bar{\mathcal{L}} = \mathcal{L} + \mathcal{B} \in \mathbb{R}^{N\times N}$ is the graph matrix of $\bar{G}$. If there is a directed edge from node i to node j, then node i is the parent and node j is the child. A directed tree is a graph, of which each node has only one parent except for the root node having no parent, and the root node has a directed path to each node. For a directed graph, a directed spanning tree is a directed tree formed by graph edges which connect all the graph nodes. A directed graph has a directed spanning tree if at least one node has paths to all the other nodes.

Lemma 3 (Olfati-Saber & Murray, 2004)**.** *For a connected undirected graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, *its Laplacian matrix* $\mathcal{L}$ *is positive semi-definite.* 0 *is a simple eigenvalue of* $\mathcal{L}$ *and* $\mathbf{1}_N$ *is the associated eigenvector. The eigenvalues of* $\mathcal{L}$ *satisfy* $0 < \lambda_2(\mathcal{L}) \le \cdots \le \lambda_N(\mathcal{L}) = \lambda_{\max}(\mathcal{L})$. *Moreover, if* $\mathbf{1}_N^T x = 0$ *for* $x \in \mathbb{R}^N$, *then* $x^T\mathcal{L}x \ge \lambda_2(\mathcal{L})x^T x$.

Lemma 4 (Hong, Hu, & Gao, 2006)**.** *For a leader–follower multi-agent system, if the communication topology graph* $\bar{G} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ *contains at least one directed spanning tree (i.e.,* $\mathcal{B} \neq \mathbf{0}$*), then* $\bar{\mathcal{L}}$ *is positive definite.*



(a) Block diagram for the traditional design framework

(b) Block diagram for the hierarchical design framework

Fig. 1. Block diagrams for the traditional design framework and the hierarchical design framework of distributed control.

2.4. Problem formulation

Shown by Fig. 1(a), the traditional design framework of distributed control (such as consensus, containment, formation control, and so on) relies on the cooperation errors between the neighboring agents. This is natural but improvable: (1) Since the traditional design framework makes the agents' cooperation and individual regulations directly coupled, the design flexibility is restricted. (2) The design complexity increases significantly when facing complicated agent models or tasks, of which the practicality needs to be strengthened. (3) For different tasks, the controllers need to be frequently changed, which limits the design versatility and scalability since the agents' basic (encapsulated) controllers may not be changed at will.

Shown by Fig. 1(b), different from the traditional design framework, the hierarchical design framework does not directly rely on the cooperation errors and it decouples the distributed control design task into two layers, i.e., Layer I-Reference signal generator design and Layer II-Agents' tracking controllers design. The reference signal generator is used to produce reference signals for the agents, of which the nodes play the roles of "virtual agents" that "achieve" the desired cooperation task. The tracking controllers are used to track the references for the agents. The "virtual" signals used in Layer I are produced by the agents' CPUs and the signals used in Layer II come from both the agents' sensors and Layer I. Since both layers have access to the agents' sensed and produced signals, the relationship between the two layers is not real "communication", which is more like a kind of signal utilization. Compared with the traditional design framework, the hierarchical design framework has some important advantages: (1) Since both the generator and the tracking controllers can be designed according to specific demands, this framework allows more flexibility in design and provides more autonomy to the agents. (2) Since this framework "separates" the agents' cooperation and individual regulations into two layers, the cooperation is not made directly on agents' models and the agents' individual regulations are not directly restrained by cooperation, the design complexity is reduced and practicality is strengthened. (3) Since the cooperation regulation is moved to Layer I, this framework provides a strong plug-play function to the agents (i.e., they can join into the cooperative tasks without changing their basic encapsulated controllers), which enhances versatility and scalability of the design.

Since the cooperation regulation is assigned to the reference signal generator in Layer I of the hierarchical design framework, the main challenge is how to design such a generator that its nodes can "achieve" the desired cooperation task and its outputs are suitable for the agents to track. In Sections 3 and 4, two kinds of typical examples are exhibited to validate the nice features of the hierarchical design framework, i.e., distributed control of multi-agent systems with mismatched disturbances, and those with both communication and input delays.

3. Hierarchical design for distributed control of multi-agent systems with mismatched disturbances

Without loss of generality, the following multi-agent system with mismatched disturbances is considered

$$\begin{aligned} \dot{x}_{i,k} &= x_{i,k+1} + d_{i,k},\ \ k = 1,\dots,M-1, \\ \dot{x}_{i,M} &= u_i + d_{i,M},\ \ i \in \mathcal{V}, \end{aligned} \tag{1}$$

where M is the order of agent i model and $M \geq 2$, $x_{i,k} = [x_{i,k}^1,\dots,x_{i,k}^m]^T \in \mathbb{R}^m, k = 1,\dots,M$ are the state elements, $u_i = [u_i^1,\dots,u_i^m]^T \in \mathbb{R}^m$ is the control input, $d_{i,k} = [d_{i,k}^1,\dots,d_{i,k}^m]^T \in \mathbb{R}^m, k = 1,\dots,M$ are the disturbances, $y_i = [y_i^1,\dots,y_i^m]^T = x_{i,1}$ is the output, and m is the system dimension. The communication topology of leaderless system (1) is depicted by an undirected graph $\mathcal{G} = (\mathcal{V},\mathcal{E},\mathcal{A})$. In the case with a leader, the leader dynamics are described by

$$\dot{x}_{0,k} = x_{0,k+1},\ \ k = 1,\dots,M-1,\ \ \dot{x}_{0,M} = u_0, \tag{2}$$

where $x_{0,k} = [x_{0,k}^1,\dots,x_{0,k}^m] \in \mathbb{R}^m, k = 1,\dots,M$ are the state elements, $x_0 = [x_{0,1}^T,\dots,x_{0,M}^T]^T \in \mathbb{R}^{Mm}$, $y_0 = [y_0^1,\dots,y_0^m]^T = x_{0,1}$ is the output, and $u_0 = [u_i^1,\dots,u_i^m]^T \in \mathbb{R}^m$ is the preset input. $u_0(t)$ is bounded by $\|u_0(t)\|_\infty \leq \bar{u}_0, \forall t \in [0,+\infty)$ with $\bar{u}_0$ being a bounded positive constant. The communication topology of leader–follower multi-agent system (1) and (2) is depicted by a directed graph $\bar{G} = (\bar{\mathcal{V}},\bar{\mathcal{E}},\bar{\mathcal{A}})$ with $\overline{\mathcal{V}} = \mathcal{V} \cup \{0\}$.

3.1. Traditional consensus design

In the presence of mismatched disturbances, output consensus problems are considered. For the leaderless cases, the final consensus states can be regarded as virtual leaders. Turning to the leader–follower cases, there are real leaders. In most cases, not all the agents are accessible to information of the virtual/real leaders. Consensus design under the traditional design framework relies on state exchanges between the neighboring agents. The output consensus errors are commonly defined as ($i \in \mathcal{V}$)

$$e_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) \text{ or } \varepsilon_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) + b_i(y_i - y_0), \tag{3}$$

where $e_{i,1}$ are defined for leaderless cases and $\varepsilon_{i,1}$ are defined for leader–follower cases. Then it follows that $e_{i,1}^{(k)} = \sum_{j\in\mathcal{N}_i} a_{ij}\Big[x_{i,k+1} + \sum_{l=1}^k d_{i,l}^{(k-l)} - \Big(x_{j,k+1} + \underline{\sum_{l=1}^k d_{j,l}^{(k-l)}}\Big)\Big]$, $\varepsilon_{i,1}^{(k)} = \sum_{j\in\mathcal{N}_i} a_{ij}\Big[x_{i,k+1} + \sum_{l=1}^k d_{i,l}^{(k-l)} - \Big(x_{j,k+1} + \underline{\sum_{l=1}^k d_{j,l}^{(k-l)}}\Big)\Big] + b_i\Big[\Big(x_{i,k+1} + \sum_{l=1}^k d_{i,l}^{(k-l)}\Big) - x_{0,k+1}\Big]$, $k = 1,\dots,M-1$, where $d_{j,l}^{(k-l)}$ are the time derivatives of mismatched disturbances (if there are any) from the neighboring agent j of agent i. Based on the above-designed consensus errors, the consensus problems are converted into stabilization tasks of the error systems. However, the agents' mismatched disturbances directly affect the cooperation process. Under this situation, controller $u_{i,k}$ of agent i is required to deal with the mismatched disturbances from its neighbors, which is a tough mission since even its neighbors probably have no exact information of their mismatched disturbances, not to mention agent i. Feedback suppression and feedforward compensation are two commonly used methods to deal with these mismatched disturbances under the traditional design framework. However, both the methods belong to "ex-post-facto" remedies. Then a challenging issue arises: whether can "pre-prevention" be made for the adverse effects of mismatched disturbances on the cooperation process and how to do it if there are any possible ways?

3.2. Hierarchical consensus design

The answer is "yes" and the hierarchical design framework provides an effective solution. Under this new framework, the cooperation regulation is moved to the reference signal generator in Layer I, and the agents' controllers in Layer II do not directly take charge of cooperation regulation while only need to track the reference signals from Layer I. The reference signal generator is not designed based on cooperation errors like (3), while it is composed of some "virtual agents" that "achieve" the expected consensus goal instead. The dynamics of the "virtual agents" are independent of the real agents' dynamics, which can be flexibly designed according to the desired performances.

Through the two-layer design, the direct state exchanges between the neighboring real agents are not required and the agents' mismatched disturbances do not directly affect the cooperation process (i.e., the mismatched disturbances of a certain agent do not appear in the closed-loop dynamics of its neighbors) anymore. Therefore, under the hierarchical design framework, pre-prevention is successfully made for the adverse effects of mismatched disturbances on the cooperation process.

In Layer I, the reference signal generator is described as

$$\dot{z}_i = \phi_i(z_i, a_{ij}z_j, b_i x_0),\ \ i \in \mathcal{V}, \tag{4}$$

where $\phi_i(\cdot)$ is a vector function, $z_i = [z_{i,1}^T,\dots,z_{i,n}^T]^T \in \mathbb{R}^{nm}$ is the state of node i, $z_{i,l} \in \mathbb{R}^m, l = 1,\dots,n, n \geq 1$, and $y_i^{\text{rsg}} = \big[(y_i^{\text{rsg}})^1,\dots,(y_i^{\text{rsg}})^m\big]^T = z_{i,1}$ is the output. Reference signal generator (4) satisfies that $y_i^{\text{rsg}}(t) - y_j^{\text{rsg}}(t) \to \mathbf{0}_{nm}, \forall i \neq j \in \mathcal{V}$ as $t \to +\infty$. If there is a leader, $y_i^{\text{rsg}}(t) - y_0(t) \to \mathbf{0}_m, \forall i \in \mathcal{V}$ as $t \to +\infty$. By this design, the "virtual agents" (i.e., the nodes of generator (4)) reach the expected consensus goal. Correspondingly, in Layer II, the agents' tracking controllers are depicted by

$$u_i = g_i(x_i, z_i),\ \ i \in \mathcal{V}, \tag{5}$$

where $g_i(\cdot,\cdot)$ is a vector function. Under tracking controllers (5), $y_i(t) - y_i^{\text{rsg}}(t) \to \mathbf{0}_m$ as $t \to +\infty$, i.e., the output y_i of real agent i tracks the output y_i^{rsg} of generator (4) asymptotically. Reference signal generator (4) of Layer I and tracking controllers (5) of Layer II hold the hierarchical design scheme.

3.3. Examples

To show more details, a leaderless consensus example and a leader–follower consensus example are given.

3.3.1. A leaderless consensus example

Consider an "average-rate" consensus problem of a leaderless second-order multi-agent system with mismatched disturbances as (1) (i.e., $M = 2$). "Average-rate" means that all the agents reach consensus with the same rate equal to the average of their (estimated) initial values. To focus on the operations on mismatched disturbances, the matched disturbances $d_{i,2}$ are not considered. The consensus design schemes under the traditional design framework and the hierarchical design framework are explicitly presented.

Since $d_{i,1}, i \in \mathcal{V}$ are unknown mismatched disturbances, $\dot{e}_{i,1}$ cannot be directly used. Pure feedback control is hard to suppress the mismatched disturbances if the disturbances are not slowly time varying or even have high-order forms (such as polynomial disturbances with orders higher than one). The feedforward–feedback composite control method is a feasible way to deal with such disturbances, which uses disturbance estimators to estimate the disturbances and compensate them (Hua et al., 2021; Wang et al., 2016, 2023; Xiao et al., 2022; Yu et al., 2016).

Assumption 1. There are some proper disturbance estimators that produce asymptotic estimates $\widehat{d_{i,1}} = \left[\widehat{d_{i,1}^1}, \dots, \widehat{d_{i,1}^m}\right]^T$ for $d_{i,1}, i \in \mathcal{V}$, i.e., the estimation errors $e_{d_{i,1}} = \left[e_{d_{i,1}^1}, \dots, e_{d_{i,1}^m}\right]^T = d_{i,1} - \widehat{d_{i,1}}, i \in \mathcal{V}$ converge to zero asymptotically.

Remark 1. Assumption 1 is reasonable. For different types of disturbances (e.g., constant/slowly time-varying/ramp/higher-order disturbances or disturbances generated from some exogeneous systems), some specific disturbance estimators can provide accurate disturbance estimates (Wang, Li, et al., 2020). Consider the following general disturbed system

$$\dot{x} = f(x) + bu + d, \tag{6}$$

where $x \in \mathbb{R}^m$ is the state, $u \in \mathbb{R}^m$ is the control input, $d \in \mathbb{R}^m$ is the disturbance, $f(x)$ is a smooth function in x, and $b \in \mathbb{R}^{m\times m}$ is a coefficient matrix. For instance, in a three-phase DC–AC inverter system, there are usually LC filter parametric perturbations and load variations, which lead to step and harmonic disturbances in the inverter dynamics (Guo, Bacha, Alamir, Hably, & Boudinet, 2021). As for an unmanned vehicle or a surface vessel, there are usually periodic disturbances or hybrid disturbances caused by winds, waves, and other factors (Liu, Chen, & Andrews, 2012; Xiao, Yang, & Huo, 2017). Here, three typical kinds of disturbance estimators are given.

(a) If d is constant or slowly time-varying and tends to be constant: a disturbance observer (DO) can be designed as (Kim, Rew, & Kim, 2010)

$$\dot{\psi} = f(x) + bu + \hat{d}, \ \ \hat{d} = \varrho(x - \psi), \tag{7}$$

where $\varrho \in \mathbb{R}^{m\times m}$ is a positive-definite diagonal gain matrix, $\hat{d}$ is the estimate of d, $\psi \in \mathbb{R}^m$ is an auxiliary variable, and $\dot{\hat{d}} = \varrho e_d$ with the estimation error $e_d = d - \hat{d}$. The estimation error system is $\dot{e}_d = -\varrho e_d + \dot{d}$. By Lemma 2, this systems is ISS by treating $\dot{d}$ as the input. Since $\dot{d}(t) \equiv \mathbf{0}_m$ or $\lim_{t\to\infty} \dot{d}(t) = \mathbf{0}_m$, by Lemma 1, $\lim_{t\to\infty} e_d(t) = \mathbf{0}_m$.

(b) If d is in a ramp or higher-order form: assume that d is pth differentiable and $\lim_{t\to\infty} d^{(p)}(t) = \mathbf{0}_m, p \geq 2$, and denote $\zeta_1 = d, \zeta_{k+1} = d^{(k)}, k = 1, \dots, p-1$. A generalized proportional-integral observer (GPIO) is designed as (Sira-Ramírez & Oliver-Salazar, 2013)

$$\begin{aligned}
\dot{\hat{x}} &= f(x) + bu + \hat{\zeta}_1 + \varrho_1(x - \hat{x}),\\
\dot{\hat{\zeta}}_k &= \hat{\zeta}_{k+1} + \varrho_{k+1}(x - \hat{x}), \ \ k = 1, \dots, p-1,\\
\dot{\hat{\zeta}}_p &= \varrho_{p+1}(x - \hat{x}),
\end{aligned} \tag{8}$$

where $\hat{x}$ is the estimate of x, $\hat{\zeta}_k$ is the estimate of $\zeta_k, k = 1, \dots, p$, and $\varrho_1, \dots, \varrho_{p+1} \in \mathbb{R}^{m\times m}$ are proper gain matrices. By denoting $e_x = x - \hat{x}, e_{\zeta_k} = \zeta_k - \hat{\zeta}_k$, the estimation error system is

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_{\zeta_1} \\ \vdots \\ \dot{e}_{\zeta_{p-1}} \\ \dot{e}_{\zeta_p} \end{bmatrix} = \begin{bmatrix} -\varrho_1 I_m & I_m & \mathbf{0}_m & \dots & \mathbf{0}_m \\ -\varrho_2 I_m & \mathbf{0}_m & I_m & \dots & \mathbf{0}_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\varrho_p I_m & \mathbf{0}_m & \mathbf{0}_m & \dots & I_m \\ -\varrho_{p+1} I_m & \mathbf{0}_m & \mathbf{0}_m & \dots & \mathbf{0}_m \end{bmatrix} \begin{bmatrix} e_x \\ e_{\zeta_1} \\ \vdots \\ e_{\zeta_{p-1}} \\ e_{\zeta_p} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_m \\ \mathbf{0}_m \\ \vdots \\ \mathbf{0}_m \\ d^{(p)} \end{bmatrix}. \tag{9}$$

The gain matrices $\varrho_1, \dots, \varrho_{p+1}$ can be selected such that the system matrix of (9) is Hurwitz. Then by Lemma 2, the estimation error system (9) is ISS if regarding $d^{(p)}$ as the input. Since $\lim_{t\to\infty} d^{(p)}(t) = \mathbf{0}_m$, by Lemma 1, asymptotic convergence of GPIO (8) is obtained. Otherwise, the observation errors asymptotically converge to a neighborhood of the origin with adjustable bounds. The order p of the disturbance d is the characteristic parameter for GPIO design and it can be got via mechanism analysis or data identification. GPIO (8) is a useful tool to estimate the disturbances, since many disturbances can be described or approximated by Taylor polynomials.

(c) If d is generated from an exogeneous system:

$$\dot{\sigma} = W\sigma, \ \ d = C\sigma, \tag{10}$$

where $W \in \mathbb{R}^{q\times q}$, $C \in \mathbb{R}^{m\times q}$, and (W, C) is an observable pair, a nonlinear DO can be designed as (Chen, Yang, Guo, & Li, 2016)

$$\begin{aligned}
\dot{\phi} &= (W - \gamma C)\phi + W\gamma x - \gamma(C\gamma x + f(x) + bu),\\
\hat{\sigma} &= \phi + \gamma x, \ \ \hat{d} = C\hat{\sigma},
\end{aligned} \tag{11}$$

where $\hat{\sigma}, \hat{d}$ are the estimates of σ, d, and $\gamma \in \mathbb{R}^{q\times m}$ is a gain matrix. By denoting $e_\sigma = \sigma - \hat{\sigma}$, $e_d = d - \hat{d} = Ce_\sigma$, the estimation error dynamics are $\dot{e}_\sigma = (W - \gamma C)e_\sigma$. Via choosing γ to make $W - \gamma C$ Hurwitz, e_σ and e_d asymptotically converge to zero. System (10) describes a sinusoidal disturbance if it is neutrally stable. For instance, if $d(t) = A\sin(\omega t + \varphi)$, it can be written as (10) with $W = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$, $C = [1, 0]^T$, $\sigma(0) = [A\sin(\varphi), A\cos(\varphi)]^T$. For d, the frequency ω is its characteristic parameter for DO design. The amplitude A and phase φ are unnecessary to be known. In practice, the frequency ω can be identified based on mechanism or data analysis, such as fast Fourier transform on the system output and other methods.

Other commonly used disturbance estimators can be found in Back and Shim (2008), Chen et al. (2016), Ginoya, Shendge, and Phadke (2014), Han (2009), Li, Yang, Chen, and Chen (2014) and the references therein.

(a) Traditional design: Under Assumption 1, $\dot{e}_{i,1}, i \in \mathcal{V}$ can be replaced by $e_{i,2} = \sum_{j\in\mathcal{N}_i} a_{ij}\left[\left(x_{i,2} + \widehat{d_{i,1}}\right) - \left(x_{j,2} + \widehat{d_{j,1}}\right)\right]$, where $x_{i,2} + \widehat{d_{i,1}}$ replaces $\dot{y}_i$. Denote some vectors $Y = \left[y_1^T, \dots, y_N^T\right]^T$, $X_k = \left[x_{1,k}^T, \dots, x_{N,k}^T\right]^T$, $k = 1, 2$, $E_1 = \left[e_{1,1}^T, \dots, e_{N,1}^T\right]^T$, $E_2 = \left[e_{1,2}^T, \dots, e_{N,2}^T\right]^T$, $U = \left[u_1^T, \dots, u_N^T\right]^T$, $D_1 = \left[d_{1,1}^T, \dots, d_{N,1}^T\right]^T$, $\widehat{D_1} = \left[\widehat{d_{1,1}}^T, \dots, \widehat{d_{N,1}}^T\right]^T$ and $E_{D_1} = \left[e_{d_{1,1}}^T, \dots, e_{d_{N,1}}^T\right]^T = D_1 - \widehat{D_1}$. Based on (1), the consensus error dynamics are written as

$$\dot{E}_1 = E_2 + (\mathcal{L} \otimes I_m)E_{D_1}, \ \ \dot{E}_2 = (\mathcal{L} \otimes I_m)\left(U + \dot{\widehat{D_1}}\right). \tag{12}$$

The consensus protocols are designed as

$$u_i = -k_1 e_{i,1} - k_2 e_{i,2} - \dot{\widehat{d_{i,1}}}, \ \ i \in \mathcal{V} \tag{13}$$

with positive gains k_1, k_2.

Proposition 1. *If the graph $\mathcal{G}$ is connected and Assumption 1 holds, under protocols (13), $y_i(t)-y_j(t) \to \mathbf{0}_m, \dot{y}_i(t) \to \frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}$, $\forall i,j \in \mathcal{V}$ as $t \to +\infty$, namely, the "average-rate" consensus goal is asymptotically achieved for multi-agent system (1).*

Proof. Substituting protocols (13) into (1) yields

$$\dot{E}_1 = E_2 + (\mathcal{L} \otimes I_m)E_{D_1}, \ \dot{E}_2 = (\mathcal{L} \otimes I_m)(-k_1E_1 - k_2E_2). \tag{14}$$

Consider the reduced system without E_{D_1}

$$\dot{E}_1 = E_2, \ \dot{E}_2 = (\mathcal{L} \otimes I_m)(-k_1E_1 - k_2E_2). \tag{15}$$

Set the Lyapunov function $V_{\text{II}} = \frac{k_1}{2}E_1^T(\mathcal{L} \otimes I_m)E_1 + \frac{1}{2}E_2^TE_2$. Since $\mathbf{1}_{Nm}^TE_1 = 0$, by Lemma 3, $E_1^T(\mathcal{L} \otimes I_m)E_1 \geq \lambda_2(\mathcal{L})E_1^TE_1$. Hence, V_{II} is positive definite about $[E_1^T, E_2^T]^T$. $\dot{V}_{\text{II}}$ along (15) is

$$\dot{V}_{\text{II}} = -k_2E_2^T(\mathcal{L} \otimes I_m)E_2. \tag{16}$$

Since $\mathbf{1}_{Nm}^TE_2 = 0$, by Lemma 3, $E_2^T(\mathcal{L} \otimes I_m)E_2 \geq \lambda_2(\mathcal{L})E_2^TE_2$. Then it follows from (16) that $\dot{V}_{\text{II}} \leq -k_2\lambda_2(\mathcal{L})E_2^TE_2$. Denote the invariant set $\Upsilon = \left\{[E_1^T, E_2^T]^T | \dot{V}_{\text{II}} \equiv 0\right\}$. From $\dot{V}_{\text{II}} \equiv 0$, it is derived that $E_2 \equiv \mathbf{0}_{Nm}$. Then it is obtained from (15) that $(\mathcal{L} \otimes I_m)E_1 \equiv \mathbf{0}_{Nm}$. By the fact $E_1^T(\mathcal{L} \otimes I_m)E_1 \geq \lambda_2(\mathcal{L})E_1^TE_1$, it follows that $E_1 \equiv \mathbf{0}_{Nm}$. Thus, by LaSalle's invariant principle (Khalil, 2002), system (15) is globally asymptotically stable, i.e., $\lim_{t\to+\infty} E_1(t) = \mathbf{0}_{Nm}, \lim_{t\to+\infty} E_2(t) = \mathbf{0}_{Nm}$. Since system (15) is a linear system, it is also globally exponentially stable.

By Lemma 2, system (14) is ISS by treating E_{D_1} as the input. Moreover, since $\lim_{t\to+\infty} E_{D_1}(t) = \mathbf{0}_{Nm}$ under Assumption 1, by Lemma 1, $\lim_{t\to+\infty} E_1(t) = \mathbf{0}_{Nm}, \lim_{t\to+\infty} E_2(t) = \mathbf{0}_{Nm}$. By noticing that $E_1 = (\mathcal{L} \otimes I_m)Y, E_2 = (\mathcal{L} \otimes I_m)\big(X_2(t) + \widehat{D_1}(t)\big)$, it is verified that $y_i(t) - y_j(t) \to \mathbf{0}_m, \big(x_{i,2}(t) + \widehat{d_{i,1}}(t)\big) - \big(x_{j,2}(t) + \widehat{d_{j,1}}(t)\big) \to \mathbf{0}_m, \forall i \neq j \in \mathcal{V}$ as $t \to +\infty$. In others words, under protocols (13), the asymptotic output consensus of multi-agent system (1) is achieved. In addition, it follows from (1) and (13) that $\dot{X}_2 + \dot{\widehat{D}}_1 = -k_1E_1 - k_2E_2$. Notice that $\mathbf{1}_{Nm}^TE_1(t) \equiv 0$, $\mathbf{1}_{Nm}^TE_2(t) \equiv 0, \forall t \geq 0$. Then $\mathbf{1}_{Nm}^T\big(\dot{X}_2(t) + \dot{\widehat{D}}_1(t)\big) \equiv 0$, i.e., $\sum_{i=1}^{N}\big(\dot{x}_{i,2}(t) + \dot{\widehat{d_{i,1}}}(t)\big) \equiv \mathbf{0}_m, \forall t \geq 0$. This means that $x_{i,2} + \widehat{d_{i,1}}, i \in \mathcal{V}$ as well as $\dot{y}_i, i \in \mathcal{V}$ asymptotically converge to the same constant value $\frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}$. This completes the proof. ■

Remark 2. For one thing, although under protocols (13), the agents achieve asymptotic consensus, the price is distributed compensations of mismatched disturbances between the neighboring agents, which results in additional communication cost. For another, this design scheme is an "ex-post-facto" remedy, which cannot prevent the adverse effects of mismatched disturbances from the cooperation process.

(b) Hierarchical design: According to the introduction in Section 2.4, the hierarchical design has two layers, i.e., Layer I-Reference signal generator design and Layer II-Agents' tracking controllers design.

(i) Layer I-Reference signal generator design: Since the "virtual agents" are required to "achieve" "average-consensus", the following reference signal generator is designed ($i \in \mathcal{V}$)

$$\begin{aligned} \dot{z}_{i,1} &= z_{i,2}, \\ \dot{z}_{i,2} &= -l_1 \sum_{j\in\mathcal{N}_i} a_{ij}(z_{i,1} - z_{j,1}) - l_2 \sum_{j\in\mathcal{N}_i} a_{ij}(z_{i,2} - z_{j,2}), \end{aligned} \tag{17}$$

where $z_{i,1} \in \mathbb{R}^m, z_{i,2} \in \mathbb{R}^m$ are state elements for node i (i.e., virtual agent i) of the generator, $y_i^{\text{rsg}} = z_{i,1}$ is the output of node i, and l_1, l_2 are positive gains. In addition, $z_{i,2}(0) = x_{i,2}(0) + \widehat{d_{i,1}}(0), i \in \mathcal{V}$. Similar to the proof of Proposition 1, it can be verified that if the graph $\mathcal{G}$ is connected, then $y_i^{\text{rsg}}(t) - y_j^{\text{rsg}}(t) \to \mathbf{0}_m, \dot{y}_i^{\text{rsg}}(t) \to \frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}, \forall i,j \in \mathcal{V}$ as $t \to +\infty$, that is, the "average-rate" consensus for the nodes of generator (17) is asymptotically reached.

(ii) Layer II-Agents' tracking controllers design: Denote the tracking errors as $\xi_{i,1} = [\xi_{i,1}^1, \ldots, \xi_{i,1}^m]^T = y_i - y_i^{\text{rsg}}, \xi_{i,2} = [\xi_{i,2}^1, \ldots, \xi_{i,2}^m]^T = x_{i,2} + \widehat{d_{i,1}} - z_{i,2}, i \in \mathcal{V}$. From (1) and (17), the tracking error dynamics are

$$\dot{\xi}_{i,1} = \xi_{i,2} + e_{d_{i,1}}, \ \dot{\xi}_{i,2} = u_i + \dot{\widehat{d_{i,1}}} - \dot{z}_{i,2}, \ i \in \mathcal{V}. \tag{18}$$

To make the agents asymptotically track the generator's outputs, the following tracking controllers are designed

$$u_i = -\rho_{i,1}\xi_{i,1} - \rho_{i,2}\xi_{i,2} - \dot{\widehat{d_{i,1}}}, \ i \in \mathcal{V}, \tag{19}$$

where $\rho_{i,1}, \rho_{i,2}, i \in \mathcal{V}$ are positive gains.

Proposition 2. *If the graph $\mathcal{G}$ is connected and Assumption 1 holds, under reference signal generator (17) and tracking controllers (19), $y_i(t) - y_j(t) \to \mathbf{0}_m, \dot{y}_i(t) \to \frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}, \forall i,j \in \mathcal{V}$ as $t \to +\infty$, i.e., the "average-rate" consensus goal is asymptotically achieved for multi-agent system (1).*

Proof. By substituting controllers (19) into (18) yields

$$\dot{\xi}_{i,1} = \xi_{i,2} + e_{d_{i,1}}, \ \dot{\xi}_{i,2} = -\rho_{i,1}\xi_{i,1} - \rho_{i,2}\xi_{i,2} - \dot{z}_{i,2}, \ i \in \mathcal{V}. \tag{20}$$

Since the reduced system of system (20) without $e_{d_{i,1}}, \dot{z}_{i,2}, i \in \mathcal{V}$ is globally exponentially stable, system (20) is ISS by treating $e_{d_{i,1}}, \dot{z}_{i,2}, i \in \mathcal{V}$ as control inputs. In addition, $\lim_{t\to+\infty} \dot{z}_{i,2}(t) = \mathbf{0}_m$ and $\lim_{t\to+\infty} e_{d_{i,1}}(t) = \mathbf{0}_m, i \in \mathcal{V}$ under Assumption 1. By Lemma 1, it is obtained that $\lim_{t\to+\infty} \xi_{i,1}(t) = \mathbf{0}_m, \lim_{t\to+\infty} \xi_{i,2}(t) = \mathbf{0}_m, i \in \mathcal{V}$. Then it follows that $y_i(t) - y_j(t) \to \mathbf{0}_m, \dot{y}_i(t) \to \frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}, \forall i,j \in \mathcal{V}$ as $t \to +\infty$. This completes the proof. ■

Remark 3. It seems that there are no signals feedback from Layer II in reference signal generator (17) for normal cases, but it does not mean that Layer I is totally separated from Layer II. If some agents work not well in the case of actuator failures or other reasons, Layer I can get the necessary information of Layer II and adjust the design and settings of the reference signal generator to reduce adverse effects. Specific operations may be different in accordance with different types of faults. Since the cooperation regulations are moved to Layer I of the framework, the real agents do not need to exchange their states directly. In this manner, the effects of the agents' mismatched disturbances are prevented from the cooperation process, which makes pre-prevention. Additionally, transmissions of the estimates for mismatched disturbances and corresponding distributed compensations between the neighboring agents (which are required in traditional design given in the above subsection) are also not needed. Thus, this scheme saves communication and computation cost, and has more conciseness and flexibility. For agent $i (i \in \mathcal{V})$, its matched disturbance $d_{i,M}$ in model (1) is in the same channel as the control input u_i, so it can deal with $d_{i,2}$ independently and the proposed framework can also adapt to the cases with matched disturbances well, which will be shown in the following example.

3.3.2. A leader–follower consensus example

Consider the consensus problem of a leader–follower second-order multi-agent system as (1)–(2) (i.e., $M = 2$). The consensus design schemes under the two design frameworks are given as follows.

(a) Traditional design: By denoting $x_0 = \left[x_{0,1}^T, x_{0,2}^T\right]^T, x_i = \left[x_{i,1}^T, x_{i,2}^T\right]^T, i \in \mathcal{V}, A = \begin{bmatrix} \mathbf{0}_{m\times m} & I_m \\ \mathbf{0}_{m\times m} & \mathbf{0}_{m\times m} \end{bmatrix}, B = \begin{bmatrix} \mathbf{0}_{m\times m} & I_m \end{bmatrix}^T$, system (1)–(2) is rewritten as

$$\dot{x}_0 = Ax_0 + Bu_0,\ \dot{x}_i = Ax_i + Bu_i + \left[d_{i,1}^T, d_{i,2}^T\right]^T,\ i \in \mathcal{V}. \tag{21}$$

Assumption 2. There are some proper disturbance estimators that produce asymptotic estimates $\widehat{d_{i,1}} = \left[\widehat{d_{i,1}^1}, \dots, \widehat{d_{i,1}^m}\right]^T, \widehat{d_{i,2}} = \left[\widehat{d_{i,2}^1}, \dots, \widehat{d_{i,2}^m}\right]^T$ for $d_{i,1}, d_{i,2}, i \in \mathcal{V}$, i.e., the estimation errors $e_{d_{i,1}} = \left[e_{d_{i,1}^1}, \dots, e_{d_{i,1}^m}\right]^T = d_{i,1} - \widehat{d_{i,1}}, e_{d_{i,2}} = \left[e_{d_{i,2}^1}, \dots, e_{d_{i,2}^m}\right]^T = d_{i,2} - \widehat{d_{i,2}}, i \in \mathcal{V}$ converge to zero asymptotically.

Denote consensus errors $\varepsilon_{i,2} = \sum_{j\in\mathcal{N}_i} a_{ij}\left[\left(x_{i,2} + \widehat{d_{i,1}}\right) - \left(x_{j,2} + \widehat{d_{j,1}}\right)\right] + b_i\left[\left(x_{i,2} + \widehat{d_{i,1}}\right) - x_{0,2}\right], \varepsilon_i = \left[\varepsilon_{i,1}^T, \varepsilon_{i,2}^T\right]^T, i \in \mathcal{V}$, where $\varepsilon_{i,1}$ $(i \in \mathcal{V})$ are defined as (3). Inspired by Li, Liu, Ren, and Xie (2013), the following consensus protocols are designed

$$u_i = -c_1 K\varepsilon_i - c_2 \mathrm{sgn}(K\varepsilon_i) - \widehat{\dot{d}_{i,1}} - \widehat{d_{i,2}},\ i \in \mathcal{V} \tag{22}$$

where c_1, c_2 are positive gains and $K \in \mathbb{R}^{m\times 2m}$ are the gain matrices to be determined. Denote $\vartheta_i = [\vartheta_{i,1}, \vartheta_{i,2}]^T = \left[x_{i,1}^T - x_{0,1}^T, (x_{i,2} + \widehat{d_{i,1}})^T - x_{0,2}^T\right]^T, i \in \mathcal{V}, \vartheta = \left[\vartheta_1^T, \dots, \vartheta_N^T\right]^T, E_{d_i} = \left[e_{d_{i,1}}^T, e_{d_{i,2}}^T\right]^T, E_d = \left[E_{d_1}^T, \dots, E_{d_N}^T\right]^T$. By (22) and (1), it yields

$$\begin{aligned} \dot{\vartheta}_i &= A\vartheta_i - c_1 BK\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_i - \vartheta_j) + b_i\vartheta_i\right] + E_{d_i} - Bu_0 \\ &\quad - c_2 B\mathrm{sgn}\left(K\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_i - \vartheta_j) + b_i\vartheta_i\right]\right), i \in \mathcal{V}, \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{\vartheta} &= (I_N \otimes A - c_1\bar{\mathcal{L}} \otimes BK)\vartheta + E_d \\ &\quad - c_2(I_N \otimes B)\mathrm{sgn}((\bar{\mathcal{L}} \otimes K)\vartheta) - (\mathbf{1}_N \otimes B)u_0. \end{aligned} \tag{24}$$

Proposition 3. *For leader–follower multi-agent system (1)–(2), if the graph $\bar{G}$ has at least one direct spanning tree and Assumption 2 holds, and $c_1 \geq \lambda_{\min}^{-1}(\bar{\mathcal{L}}), c_2 \geq \bar{u}_0$ and $K = B^T P^{-1}$ for protocols (22) where $P \in \mathbb{R}^{2m\times 2m}$ is a positive-definite matrix that makes $AP + PA^T - 2BB^T < \mathbf{0}_{2m\times 2m}$, then $y_i(t) - y_0(t) \to \mathbf{0}_m, \forall i \in \mathcal{V}$ as $t \to +\infty$, i.e., the consensus goal is asymptotically reached.*

Proof. The proof is based on that of Theorem 1 in Li et al. (2013). Define an energy function $V_{\mathrm{lf}} = \vartheta^T(\bar{\mathcal{L}} \otimes P^{-1})\vartheta$. V_{lf} is positive definite in ϑ and $\dot{V}_{\mathrm{lf}}$ along system (24) satisfies $\dot{V}_{\mathrm{lf}} \leq \sum_{i=1}^N \lambda_i(\bar{\mathcal{L}})\bar{\vartheta}_i^T(AP + PA^T - 2c_1\lambda_i(\bar{\mathcal{L}})BB^T)\bar{\vartheta}_i + 2\vartheta^T(\bar{\mathcal{L}} \otimes P^{-1})E_d$, where $\bar{\vartheta} = \left[\bar{\vartheta}_1^T, \dots, \bar{\vartheta}_N^T\right]^T = (U^T \otimes P^{-1})\vartheta$ with $U \in \mathbb{R}^{N\times N}$ being a unitary matrix that $U^T\bar{\mathcal{L}}U = \Lambda = \mathrm{diag}\{\lambda_1(\bar{\mathcal{L}}), \dots, \lambda_N(\bar{\mathcal{L}})\}$. Since $AP + PA^T - 2c_1\lambda_i(\bar{\mathcal{L}})BB^T < \mathbf{0}_{2m\times 2m}, \forall i \in \mathcal{V}$, there exists $r > 0$ (that is adjustable with the protocol parameters) so that

$$\dot{V}_{\mathrm{lf}} \leq -r\|\vartheta\|_2^2 + 2\vartheta^T(\bar{\mathcal{L}} \otimes P^{-1})E_d. \tag{25}$$

By the basic inequality, $2\vartheta^T(\bar{\mathcal{L}} \otimes P^{-1})E_d \leq \epsilon\|\vartheta\|_2^2 + \frac{\lambda_{\max}^2(\bar{\mathcal{L}}\otimes P^{-1})}{\epsilon} \times \|E_d\|_2^2$, where $\epsilon < r$ is a small enough positive constant. In addition, $V_{\mathrm{lf}} \leq \lambda_{\max}(\bar{\mathcal{L}} \otimes P^{-1})\|\vartheta\|_2^2$. Then it follows from (25) that

$$\dot{V}_{\mathrm{lf}} \leq -\frac{r - \epsilon}{\lambda_{\max}(\bar{\mathcal{L}} \otimes P^{-1})}V_{\mathrm{lf}} + \frac{\lambda_{\max}^2(\bar{\mathcal{L}} \otimes P^{-1})}{\epsilon}\|E_d\|_2^2. \tag{26}$$

Since $\lim_{t\to+\infty} E_d(t) = \mathbf{0}_{2Nm}$ under Assumption 2, it follows from (26) that $\lim_{t\to+\infty} V_{\mathrm{lf}}(t) = 0$. Then $y_i(t) - y_0(t) \to \mathbf{0}_m, \forall i \in \mathcal{V}$ as $t \to +\infty$. This completes the proof. ■

Remark 4. Different from leaderless consensus protocols (13), there are some discontinuous terms used to suppress the unknown bounded leader input u_0 in leader–follower consensus protocols (22). This is already one of the most concise schemes for leader–follower consensus design since it only requires one-hop neighbor information exchanges. Many other leader–follower consensus design schemes require two-hop neighbors (i.e., the neighbors' neighbors) information exchanges or even $\bar{\mathcal{L}}^{-1}$ that is hard to get in practice. Without exception, all the methods are "ex-post-facto" remedies with distributed disturbance compensations.

(b) Hierarchical design: A two-layer design scheme is given.

(i) Layer I-Reference signal generator design: Inspired by the distributed observer designed in Wang, Li, and Chen (2018), the following reference signal generator is designed ($i \in \mathcal{V}$)

$$\begin{aligned} \dot{\phi}_{i,1} &= \phi_{i,2} - \mu_1\left[\sum_{j\in\mathcal{V}} a_{ij}(\phi_{i,1} - \phi_{j,1}) + b_i(\phi_{i,1} - x_{0,1})\right], \\ \dot{\phi}_{i,2} &= \phi_{i,3} - \mu_2\left[\sum_{j\in\mathcal{V}} a_{ij}(\phi_{i,2} - \phi_{j,2}) + b_i(\phi_{i,2} - x_{0,2})\right], \\ \dot{\phi}_{i,3} &= -\mu_3\mathrm{sgn}\left(\sum_{j\in\mathcal{V}} a_{ij}(\phi_{i,3} - \phi_{j,3}) + b_i(\phi_{i,3} - u_0)\right), \end{aligned} \tag{27}$$

where $\phi_{i,1}, \phi_{i,2}, \phi_{i,3} \in \mathbb{R}^m$ are state elements for node i (i.e., virtual agent i) of the generator, $y_i^{\mathrm{rsg}} = \phi_{i,1}$ is the output of node i, and $\mu_1, \mu_2 > 0$, $u_3 > \bar{u}_0$ are the gains. By proof like that of Proposition 2 in Wang et al. (2018), if the graph $\bar{G}$ has at least one directed spanning tree, then $\lim_{t\to+\infty}(\phi_{i,1}(t) - x_{0,1}(t)) = \mathbf{0}_m, \lim_{t\to+\infty}(\phi_{i,2}(t) - x_{0,2}(t)) = \mathbf{0}_m, \lim_{t\to+\infty}(\phi_{i,3}(t) - u_0(t)) = \mathbf{0}_m, \forall i \in \mathcal{V}$, i.e., the virtual agents asymptotically reach consensus with the real leader.

(ii) Layer II-Agents' tracking controllers design: Denote the tracking errors as $\xi_{i,1} = y_i - y_i^{\mathrm{rsg}}, \xi_{i,2} = x_{i,2} + \widehat{d_{i,1}} - \phi_{i,2}, i \in \mathcal{V}$. From (1) and (27), the tracking error dynamics are

$$\begin{aligned} \dot{\xi}_{i,1} &= \xi_{i,2} + e_{d_{i,1}} + (\phi_{i,2} - \dot{\phi}_{i,1}), \\ \dot{\xi}_{i,2} &= u_i + \widehat{\dot{d}_{i,1}} + d_{i,2} - \dot{\phi}_{i,2}, i \in \mathcal{V}. \end{aligned} \tag{28}$$

Then the tracking controllers are designed as

$$u_i = -\rho_{i,1}\xi_{i,1} - \rho_{i,2}\xi_{i,2} - \widehat{\dot{d}_{i,1}} - \widehat{d_{i,2}} + \phi_{i,3},\ i \in \mathcal{V}, \tag{29}$$

where $\rho_{i,1}, \rho_{i,2}, i \in \mathcal{V}$ are positive gains.

Proposition 4. *If the graph $\bar{G}$ has at least one directed spanning tree and Assumption 2 holds, under reference signal generator (27) and tracking controllers (29), $y_i(t) - y_0(t) \to \mathbf{0}_m, \forall i \in \mathcal{V}$ as $t \to +\infty$, i.e., the consensus goal is asymptotically reached for leader–follower multi-agent system (1)–(2).*

Proof. By substituting controllers (29) into (28) yields

$$\begin{aligned} \dot{\xi}_{i,1} &= \xi_{i,2} + e_{d_{i,1}} + (\phi_{i,2} - \dot{\phi}_{i,1}), \\ \dot{\xi}_{i,2} &= -\rho_{i,1}\xi_{i,1} - \rho_{i,2}\xi_{i,2} + e_{d_{i,2}} + (\phi_{i,3} - \dot{\phi}_{i,2}),\ i \in \mathcal{V}. \end{aligned} \tag{30}$$

By noticing $\lim_{t\to+\infty} e_{d_{i,1}}(t) = \mathbf{0}_m, \lim_{t\to+\infty} e_{d_{i,2}}(t) = \mathbf{0}_m, i \in \mathcal{V}$ under Assumption 2 and $\lim_{t\to+\infty}(\phi_{i,2}(t) - \dot{\phi}_{i,1}(t)) = \mathbf{0}_m, \lim_{t\to+\infty}(\phi_{i,3}(t) - \dot{\phi}_{i,2}(t)) = \mathbf{0}_m, i \in \mathcal{V}$, the proof is similar to that of Proposition 2 and is omitted. ■

Remark 5. To guarantee the continuity of $\dot{\phi}_{i,2}, i \in \mathcal{V}$, reference signal generator (27) is third-order. Actually, the generator do not have to be in a distributed observer form like (27), and it can be designed as (23) and other possible forms that guarantee convergence of its nodes with respect to the leader and its trackability by the real agents. Once again, it has been shown that the hierarchical design framework realizes pre-prevention of mismatched disturbances from the cooperation process. Moreover, either the hierarchical design scheme proposed in Section 3.3.1 or that proposed in Section 3.3.2 is also effective for heterogeneous cases, e.g., the agents have different orders or different nonlinearities. While it is not easy for traditional design schemes to work in such cases, since definitions of cooperation errors and handling of heterogeneous nonlinearities in distributed cooperative control design are both hard problems. Thus, the hierarchical design framework has better flexibility and versatility.

4. Hierarchical design for distributed control of multi-agent systems with communication and input delays

A leaderless consensus design example is given in the presence of both communication and input delays. Both the tradition design scheme and the hierarchical design scheme are exhibited to illustrate the advantages of the hierarchical design framework in dealing with the distributed control problems with communication and input delays. The following multi-agent system is considered

$$\dot{x}_{i,1}(t) = x_{i,2}(t),\ \ \dot{x}_{i,2}(t) = u_i(t-h_i),\ \ i \in \mathcal{V}, \tag{31}$$

where $x_{i,k} = [x_{i,k}^1, \ldots, x_{i,k}^m]^T \in \mathbb{R}^m$, $k = 1, 2$ and $u_i = [u_i^1, \ldots, u_i^m]^T \in \mathbb{R}^m$ are the state elements and the control input of agent i, h_i is the known constant input delay, and $y_i = x_{i,1} = [y_i^1, \ldots, y_i^m]^T$ is the output. The communication topology is described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Each agent possesses state sensors and transmits information to its neighbors through the communication network. Usually the communications are affected by time-varying delays.

Assumption 3. The communication from agent j to agent i is subject to a variable time delay $\tau_{ij}(t)$, which has a known upper bound $\bar{\tau}_{ij}$ and a bounded time derivative, and $\bar{\tau}_{ij}(t) = \bar{\tau}_{ji}(t)$, $\forall i \neq j \in \mathcal{V}$.

The consensus problem of multi-agent system (31) is considered, i.e., designing distributed controllers $u_i, i \in \mathcal{V}$ such that $\lim_{t\to+\infty}(y_i(t) - y_j(t)) = \mathbf{0}_m, \forall i \neq j \in \mathcal{V}$ in the presence of time-varying communication delays and constant input delays.

4.1. Traditional consensus design

Under the traditional design framework, inspired by Cepeda-Gomez and Olgac (2013), Dai et al. (2013), Ma and Xu (2022) and Meng et al. (2011), consensus protocols for system (31) can be designed as ($i \in \mathcal{V}$)

$$u_i(t) = -\frac{k_3}{\Delta_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,1}(t) - x_{j,1}(t-\tau_{ij}(t))\right) - k_4 x_{i,2}(t), \tag{32}$$

$$\text{or } u_i(t) = -\frac{k_3}{\Delta_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,1}(t) - x_{j,1}(t-\tau_{ij}(t))\right) - \frac{k_4}{\Delta_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,2}(t) - x_{j,2}(t-\tau_{ij}(t))\right), \tag{33}$$

where k_3, k_4 are positive gains and $\Delta_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. Take protocols (32) for example, the closed-loop system is

$$\begin{aligned}\dot{x}_{i,1}(t) &= x_{i,2}(t),\\ \dot{x}_{i,2}(t) &= -\frac{k_3}{\Delta_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,1}(t-h_i) - x_{j,1}\big(t-h_i-\tau_{ij}(t)\big)\right)\\ &\quad - k_4 x_{i,2}(t-h_i),\ i \in \mathcal{V}.\end{aligned} \tag{34}$$

Define $\bar{x}_{i,1} = x_{i,1} - x_{1,1}$, $\bar{x}_{i,2} = x_{i,2} - x_{1,2}$, $i \in \mathcal{V}$. Then from (34),

$$\begin{aligned}\dot{Y}(t) &= (H_1 \otimes I_m)\,Y(t) + (H_2 \otimes I_m)\,Y(t-h_i)\\ &\quad + (H_3 \otimes I_m)\,Y\left(t-h_i-\tau_{ij}(t)\right),\end{aligned} \tag{35}$$

where $\bar{x}_1 = \left[\bar{x}_{2,1}^T, \ldots, \bar{x}_{N,1}^T\right]^T$, $\bar{x}_2 = \left[\bar{x}_{2,2}^T, \ldots, \bar{x}_{N,2}^T\right]^T$, $Y = \left[\bar{x}_1^T, \bar{x}_2^T\right]^T$, $E = \left[-\mathbf{1}_{N-1}\quad I_{N-1}\right]$, $\mathbf{0}^* = \mathbf{0}_{(N-1)\times(N-1)}$, $H_2 = \begin{bmatrix}\mathbf{0}^* & \mathbf{0}^*\\ -k_3 I_{N-1} & -k_4 I_{N-1}\end{bmatrix}$, $H_3 = \begin{bmatrix}\mathbf{0}^* & \mathbf{0}^*\\ -k_3 E\Delta^{-1}\mathcal{A} & \mathbf{0}^*\end{bmatrix}$ and $H_1 = \begin{bmatrix}\mathbf{0}^* & I_{N-1}\\ \mathbf{0}^* & \mathbf{0}^*\end{bmatrix}$, $\Delta = \mathrm{diag}\{\Delta_1, \ldots, \Delta_N\}$.

The consensus analysis of multi-agent system (34) is equivalent to the stability analysis of error system (35). It can be seen from (32) that for each agent, the given protocol directly considers the communication delays of its neighbors' states and its own input delays, which causes some problems. Since this protocol considers both the communication and input delays, the error system (35) includes these two delay-related states $Y(t-h_i)$ and $Y(t-h_i-\tau_{ij}(t))$. Therefore, in the consensus analysis, in addition to designing $\int_{-\bar{\tau}_{ij}}^{0}\int_{t+s}^{t} Y^T(\theta)Q_1 Y(\theta)\mathrm{d}\theta\mathrm{d}s$ and $\int_{-h_i}^{0}\int_{t+s}^{t} Y^T(\theta)Q_2 Y(\theta)\mathrm{d}\theta\mathrm{d}s$ for dealing with the terms of communication delays $Y(t-\tau_{ij}(t))$ and input delays $Y(t-h_i)$, it is also necessary to employ extra terms $\int_{-h_i-\bar{\tau}_{ij}}^{-\bar{\tau}_{ij}}\int_{t+s}^{t} Y^T(\theta)Q_3 Y(\theta)\mathrm{d}\theta\mathrm{d}s$ and $\int_{-h_i-\bar{\tau}_{ij}}^{-h_i}\int_{t+s}^{t} Y^T(\theta)Q_4 Y(\theta)\mathrm{d}\theta\mathrm{d}s$ for dealing with the total delay state $Y(t-h_i-\tau_{ij}(t))$, where Q_1, Q_2, Q_3, Q_4 are positive-definite matrices. This makes the analysis process complicated. Moreover, although there are some mature and effective methods to cope with communication delays and input delays, the results with both types of delays are limited. Based on the above facts, a challenging question is: whether the two types of delays can be decoupled and the existing methods can be directly used to handle the impacts of each type of delays separately? If "yes", how to work it out? The hierarchical design framework provides an effective solution.

4.2. Hierarchical consensus design

A hierarchical consensus design scheme is given for system (31). It has two layers, i.e., Layer I-Reference signal generator design with communication delays and Layer II-Agents' tracking controllers design with input delays.

4.2.1. Layer I-Reference signal generator design

Considering time-varying communication delays, the following generator is designed

$$\dot{\eta}_i = v_i,\ \ \dot{v}_i = -l_3\sum_{j\in\mathcal{N}_i} a_{ij}\big(\eta_i - \eta_j(t-\tau_{ij}(t))\big) - l_4 v_i,\ \ i \in \mathcal{V}, \tag{36}$$

where $\eta_i = [\eta_i^1, \ldots, \eta_i^m]^T \in \mathbb{R}^m$, $v_i = [v_i^1, \ldots, v_i^m]^T \in \mathbb{R}^m$ are the states of node i, and l_3, l_4 are positive gains to be determined.

Proposition 5. *For system (36), if the graph $\mathcal{G}$ is connected and Assumption 3 holds, and $l_4 > \frac{l_3}{2}\max_{i\in\mathcal{V}}\left\{\sum_{j\in\mathcal{N}_i} a_{ij}\left(p_i + \frac{\bar{\tau}_{ij}^2}{p_j}\right)\right\}$, $p_i > 0, i \in \mathcal{V}$, then the consensus goal is asymptotically realized in the presence of time-varying communication delays, i.e., $\lim_{t\to+\infty}(\eta_i(t) - \eta_j(t)) = \mathbf{0}_m$, $\lim_{t\to+\infty} v_i(t) = \mathbf{0}_m$, $\forall i \neq j \in \mathcal{V}$.*

Proof. Consider the Lyapunov Krasovskii-functional

$$V_s = \sum_{i=1}^{N}\Bigg[\frac{1}{2l_3}v_i^T v_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_i-\eta_j)^T(\eta_i-\eta_j) + \frac{1}{2p_i}\sum_{j\in\mathcal{N}_i} a_{ij}\bar{\tau}_{ij}\int_{-\bar{\tau}_{ij}}^{0}\int_{t+s}^{t} v_j^T(\sigma)v_j(\sigma)\mathrm{d}\sigma\,\mathrm{d}s\Bigg]. \tag{37}$$

V_s is positive definite and radially unbounded with respect to v_i and $\eta_i-\eta_j$. Differentiating V_s along (36) yields

$$\begin{aligned}\dot{V}_s = -\sum_{i=1}^{N}\Bigg[&\frac{l_4}{l_3}v_i^T v_i + \dot{\eta}_i^T\sum_{j\in\mathcal{N}_i} a_{ij}\big(\eta_i-\eta_j(t-\tau_{ij}(t))\big)\\ &-\frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\eta}_i-\dot{\eta}_j)^T(\eta_i-\eta_j)\\ &-\frac{1}{2p_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\bar{\tau}_{ij}^2 v_j^T v_j - \bar{\tau}_{ij}\int_{t-\bar{\tau}_{ij}}^{t} v_j^T(s)v_j(s)\mathrm{d}s\right)\Bigg].\end{aligned} \tag{38}$$

By Assumption 3, it holds that $\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\eta}_i-\dot{\eta}_j)^T(\eta_i-\eta_j) = \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a_{ij}\dot{\eta}_i^T(\eta_i-\eta_j)$. Based on the Newton–Leibniz formula, it follows that $\eta_j-\eta_j\big(t-\tau_{ij}(t)\big) = \int_{t-\tau_{ij}(t)}^{t}\dot{\eta}_j(s)\mathrm{d}s$. Then it is verified from (38) that

$$\begin{aligned}\dot{V}_s = -\sum_{i=1}^{N}\Bigg[&\frac{l_4}{l_3}v_i^T v_i + \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\eta}_i^T\int_{t-\tau_{ij}}^{t}\dot{\eta}_j(s)\mathrm{d}s\\ &-\frac{1}{2p_i}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\bar{\tau}_{ij}^2 v_j^T v_j - \bar{\tau}_{ij}\int_{t-\bar{\tau}_{ij}}^{t} v_j^T(s)v_j(s)\mathrm{d}s\right)\Bigg].\end{aligned} \tag{39}$$

By combining Young's and Cauchy–Schwarz' inequalities (Hardy, Littlewood, & Polya, 1952), the second righthand term of (39) is transformed into $-\dot{\eta}_i^T\int_{t-\tau_{ij}}^{t}\dot{\eta}_j(s)\mathrm{d}s \le \frac{p_i}{2}\|\dot{\eta}_i\|_2^2+\frac{1}{2p_i}\left\|\int_{t-\tau_{ij}(t)}^{t}\dot{\eta}_j(s)\mathrm{d}s\right\|_2^2 \le \frac{p_i}{2}\dot{\eta}_i^T\dot{\eta}_i + \frac{\bar{\tau}_{ij}}{2p_i}\int_{t-\bar{\tau}_{ij}}^{t}\dot{\eta}_j^T(s)\dot{\eta}_j(s)\mathrm{d}s$. Based on this inequality, it follows from (39) that

$$\dot{V}_s \le -\sum_{i=1}^{N}\left[\left(\frac{l_4}{l_3}-\sum_{j\in\mathcal{N}_i} a_{ij}\frac{p_i}{2}\right)v_i^T v_i - \sum_{j\in\mathcal{N}_i} a_{ij}\frac{\bar{\tau}_{ij}^2}{2p_i}v_j^T v_j\right]. \tag{40}$$

Define $e_{\eta_i} = \sum_{j\in\mathcal{N}_i} a_{ij}\big(\eta_i-\eta_j(t-\tau_{ij}(t))\big), i\in\mathcal{V}, \upsilon = [v_1^T v_1,\ldots,v_N^T v_N]^T$ and

$$\Phi = \begin{bmatrix} \frac{l_4}{l_3}-\Delta_1\frac{p_1}{2} & -\frac{\bar{\tau}_{21}^2}{2p_1}a_{12} & \cdots & -\frac{\bar{\tau}_{N1}^2}{2p_1}a_{1N}\\ -\frac{\bar{\tau}_{12}^2}{2p_2}a_{21} & \frac{l_4}{l_3}-\Delta_2\frac{p_2}{2} & \cdots & -\frac{\bar{\tau}_{N2}^2}{2p_2}a_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ -\frac{\bar{\tau}_{1N}^2}{2p_N}a_{N1} & -\frac{\bar{\tau}_{2N}^2}{2p_N}a_{N2} & \cdots & \frac{l_4}{l_3}-\Delta_N\frac{p_N}{2}\end{bmatrix}.$$

Then, (40) is written as $\dot{V}_s \le -\mathbf{1}_N^T\Phi\upsilon$, or equivalently,

$$\dot{V}_s \le -\sum_{i=1}^{N}\left[\frac{l_4}{l_3}-\sum_{j\in\mathcal{N}_i} a_{ij}\left(\frac{p_i}{2}+\frac{\bar{\tau}_{ij}^2}{2p_j}\right)\right]v_i^T v_i. \tag{41}$$

If $l_4 > \frac{l_3}{2}\max_{i\in\mathcal{V}}\left\{\sum_{j\in\mathcal{N}_i} a_{ij}\left(p_i+\frac{\bar{\tau}_{ij}^2}{p_j}\right)\right\}$, then there exists $\lambda > 0$ such that $\dot{V}_s \le -\sum_{i=1}^{N}\lambda v_i^T v_i$. This implies that $V_s\in\mathcal{L}_\infty$ and $v_i\in\mathcal{L}_2$. Then v_i and $\eta_i-\eta_j$ belong to $\mathcal{L}_\infty$, consequently, $e_{\eta_i}\in\mathcal{L}_\infty$. It follows from (36) that $\dot{v}_i\in\mathcal{L}_\infty$. Then, by Barbalat's lemma (Slotine & Li, 1991), $\lim_{t\to+\infty} v_i(t) = \mathbf{0}_m$. Note that $\lim_{t\to+\infty}\int_0^t \dot{v}_i(s)\mathrm{d}s = \lim_{t\to+\infty}\big(v_i(t)-v_i(0)\big) = -v_i(0)$. Differentiating $\dot{v}_i$ yields $\ddot{v}_i = -l_3\dot{e}_{\eta_i}-l_4\dot{v}_i,\ i\in\mathcal{V}$. In view of Assumption 3 and the fact of $\dot{e}_{\eta_i}$ and $\dot{v}_i\in\mathcal{L}_\infty$, $\ddot{v}_i\in\mathcal{L}_\infty$. Then, a direct use of Barbalat's lemma (Slotine & Li, 1991) gives that $\lim_{t\to+\infty}\dot{v}_i(t) = \mathbf{0}_m$. According to $\lim_{t\to+\infty}\dot{v}_i(t) = \mathbf{0}_m$, $\lim_{t\to+\infty} v_i(t) = \mathbf{0}_m$ and (36), it is verified that $\lim_{t\to+\infty} e_{\eta_i}(t) = \lim_{t\to+\infty}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\eta_i(t)-\eta_j(t)+\int_{t-\tau_{ij}(t)}^{t} v_j(s)\mathrm{d}s\right) = \mathbf{0}_m$. Then it is obtained that $\lim_{t\to+\infty}\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_i(t)-\eta_j(t)) = \mathbf{0}_m, \forall i\in\mathcal{V}$, which implies that $\lim_{t\to+\infty}(\mathcal{L}\otimes I_m)\eta(t) = \mathbf{0}_{Nm}$ with $\eta = \big[\eta_1^T,\ldots,\eta_N^T\big]^T$. By Lemma 3, $\lim_{t\to+\infty}\big(\eta_i(t)-\eta_j(t)\big) = \mathbf{0}_m, \forall i\ne j\in\mathcal{V}$. This completes the proof. ■

4.2.2. Layer II-Agents' tracking controllers design

By embedding the outputs of generator (36) into the feedback loop and setting them as the references of the real agents, for system (31), the controllers u_i are designed to make the agents' outputs y_i track the generated references η_i in the presence of inherent input delays. Specifically, a concise design strategy is used and a predictor-based state transformation is employed to deal with the input delays. Denote the tracking errors $\mathfrak{e}_{i,1} = y_i-\eta_i, \mathfrak{e}_{i,2} = x_{i,2}, i\in\mathcal{V}$. By (31), the tracking error system is

$$\dot{\mathfrak{e}}_{i,1} = \mathfrak{e}_{i,2}-v_i,\ \dot{\mathfrak{e}}_{i,2} = u_i(t-h_i),\ i\in\mathcal{V}, \tag{42}$$

In order to transform (42) into a delay-free system, a transformation is introduced as ($i\in\mathcal{V}$)

$$w_{i,1} = \mathfrak{e}_{i,1}+\int_{-h_i}^{0}\int_{t}^{t+s} u_i(\sigma)\mathrm{d}\sigma\,\mathrm{d}s,\ w_{i,2} = \mathfrak{e}_{i,2}+\int_{t-h_i}^{t} u_i(s)\mathrm{d}s. \tag{43}$$

From (43), $\dot{w}_{i,1} = w_{i,2}-h_iu_i-v_i, \dot{w}_{i,2} = u_i,\ i\in\mathcal{V}$. Define

$$\varsigma_{i,1} = w_{i,1}+h_iw_{i,2}, \varsigma_{i,2} = w_{i,2},\ i\in\mathcal{V}. \tag{44}$$

Then it is obtained that

$$\dot{\varsigma}_{i,1} = \varsigma_{i,2}-v_i, \dot{\varsigma}_{i,2} = u_i,\ i\in\mathcal{V}. \tag{45}$$

The following controllers are designed based on system (45)

$$u_i = -\rho_{i,3}\varsigma_{i,1}-\rho_{i,4}\varsigma_{i,2},\ i\in\mathcal{V}, \tag{46}$$

where $\rho_{i,3}, \rho_{i,4}$ are positive gains.

Proposition 6. *If Assumption 3 holds, under reference signal generator (36) and tracking controllers (46), the tracking errors $\mathfrak{e}_{i,1}(t)\to\mathbf{0}_m, i\in\mathcal{V}$ as $t\to+\infty$, i.e., the consensus goal is achieved asymptotically for multi-agent systems (31) in the presence of both communication and input delays.*

Proof. From (45) and (46), the closed-loop error system is

$$\dot{E}_{\varsigma,i} = \Psi_i(E_{\varsigma,i},\varpi_i),\ i\in\mathcal{V}, \tag{47}$$

where $E_{\varsigma,i} = \big[\varsigma_{i,1}^T,\varsigma_{i,2}^T\big]^T, \varpi_i = -v_i$, and $\Psi_i(E_{\varsigma,i},\varpi_i) = \big[\varsigma_{i,2}^T+\varpi_i^T, -\rho_{i,3}\varsigma_{i,1}^T-\rho_{i,4}\varsigma_{i,2}^T\big]^T$. For i ($i\in\mathcal{V}$), consider the system

$$\dot{E}_{\varsigma,i} = \Psi_i(E_{\varsigma,i},\mathbf{0}_m). \tag{48}$$

Since system (48) is globally exponentially stable, by Lemma 2, system (47) is ISS. Moreover, by Proposition 5, $\lim_{t\to+\infty}\varpi_i(t) = \mathbf{0}_m, \forall i\in\mathcal{V}$. From Lemma 1, $\lim_{t\to+\infty}E_{\varsigma,i}(t) = \mathbf{0}_{2m}, \forall i\in\mathcal{V}$. Hence, $\varsigma_{i,1}(t)\to\mathbf{0}_m, \varsigma_{i,2}(t)\to\mathbf{0}_m, \forall i\in\mathcal{V}$ as $t\to+\infty$. According to (44) and (46), $w_{i,1}(t)\to\mathbf{0}_m, w_{i,2}(t)\to\mathbf{0}_m, u_i(t)\to\mathbf{0}_m, \forall i\in\mathcal{V}$ as $t\to+\infty$. Hence, by (43), $\mathfrak{e}_{i,1}(t)\to\mathbf{0}_m, \mathfrak{e}_{i,2}(t)\to\mathbf{0}_m, \forall i\in\mathcal{V}$ as $t\to+\infty$. This completes the proof. ■

Remark 6. The gains $\rho_{i,3}, \rho_{i,4}$ in controllers (46) can be any positive values theoretically (where the larger the gains, the faster the convergence rate), but their selections are not arbitrary in practice. Usually, there are some critical values for the maximum allowable time delays, which may be obtained through analysis and calculation (Ma & Xu, 2022; Yu et al., 2010). It is necessary to select appropriate control gains with considerations of actual object constraints such as actuator saturation and different requirements for system performances. Thus, to fix the values of $\rho_{i,3}, \rho_{i,4}$, the balance needs to be carefully considered between the desired performances and practical constraints. In implementations, the values of $\rho_{i,3}, \rho_{i,4}$ can be determined by trial and error to make the agents asymptotically track the reference signals from generator (36) such that consensus is achieved.

Remark 7. This subsection shows that the hierarchical design framework realizes decoupling treatment of communication delays and input delays in the cooperation process. Therefore, the existing control methods on handling of communication delays and input delays can be flexibly used to solve the corresponding time-delay distributed control problems. The reference signal generator only needs to deal with communication delays and the agents' tracking controllers only need to cope with input delays. The proposed scheme is also effective for heterogeneous multi-agent systems. Moreover, this framework can also better solve distributed control problems for more complex systems or more complex delays, such as nonlinear systems and multi-type delays caused by multi-sensor fusion. It is worth mentioning that both the generator design and the tracking controllers design are not unique within the framework. For example, as given in De, Sahoo, and Wahi (2023) and Jiang, Liu, and Charalambous (2022), the generator can be designed as the only-communication delays-related observers and the tracking controllers can also be designed as the only-input delays-related controllers.

Remark 8. The future researches may be conducted from the following directions: Firstly, not limited to the leader–follower case considered in Section 3.3.2, if the agents' communication topology is directed, the proposed hierarchical design framework is also valuable, where the reference signal generator design is required to adapt to the topology. The generator design needs to use the distributed control design methods suitable for directed networks, which deserves deeper research. Secondly, how to make the framework more robust to communication topology switching (e.g., there may be some agents leaving or entering the group) is also worth careful study, since this will enhance the versatility and scalability of the framework further. Thirdly, making the cooperation goals achieved fast in finite time is also meaningful, where the generator is required to have finite-time convergence and the tracking controllers should make the agents track the references from the generator in finite time as well. To achieve this, some useful nonsmooth control technologies need to be employed.

5. Numerical simulations

Some simulations are given to illustrate the feasibility and effectiveness of the proposed hierarchical design framework. For convenience, $0 - 1$ weights are set.

5.1. Simulations for Section 3: Hierarchical design for distributed control with mismatched disturbances

5.1.1. A leaderless consensus example

A leaderless second-order multi-agent system as (1) with 6 agents is considered. The system dimension is $m = 2$. The communication topology is shown in Fig. 2(a). The generator's and agents' initial states are listed in Table 1. There are no disturbances when $t < 20$ s and the disturbances are suddenly imposed on the agents at $t = 20$ s as shown in Table 5. The parameters of reference signal generator (17) are set as $l_1 = 12, l_2 = 14$ and those of tracking controllers (19) are listed in Table 1. To estimate the agents' disturbances, some disturbance estimators are employed (i.e. disturbance observers (DOs) (Chen et al., 2016) and GPIOs (Sira-Ramírez & Oliver-Salazar, 2013)), whose structures and parameters are given in Table 5. The simulation lasts 50s and the step length is 0.001s.

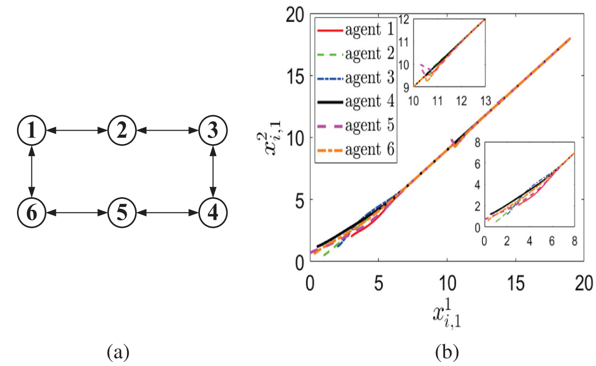


Fig. 2. (a) The communication topology ($\mathcal{V} = \{1, 2, 3, 4, 5, 6\}$). (b) Agents' trajectories in the 2-D space.

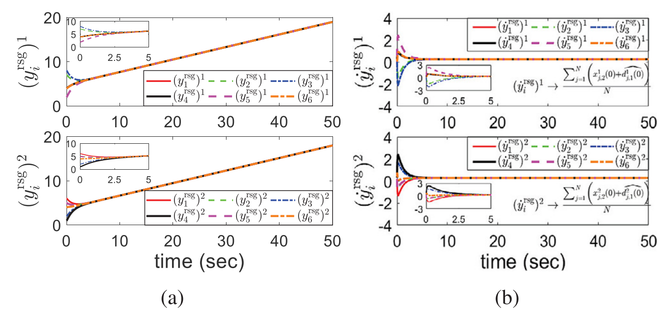


Fig. 3. Response curves of reference signal generator (17). (a) The outputs. (b) The derivatives of the outputs.

The simulation results are shown in Figs. 2(b)–5. From Figs. 2(b)–3, the "average-rate" consensus for the nodes of generator (17) is fast reached, i.e., $y_i^{\rm rsg}(t) - y_j^{\rm rsg}(t) \to \mathbf{0}_m, \dot{y}_i^{\rm rsg}(t) \to \frac{\sum_{j=1}^{N}(x_{j,2}(0)+\widehat{d_{j,1}}(0))}{N}$, $\forall i, j \in \mathcal{V}$. From Fig. 5(a), the disturbances are estimated accurately and quickly under the disturbance estimators. With the help of disturbance compensations in controllers (19), as shown in Figs. 2(b) and 4, the agents track the reference signals from generator (17) and reach "average-rate" consensus asymptotically. Time histories of the agents' inputs are shown in Fig. 5(b). Usually, dynamics of the reference signal generator is set faster than the tracking control part so that the agents converge more precisely and promptly.

5.1.2. A leader–follower consensus example

A leader–follower second-order multi-agent system as (1)–(2) with a leader agent and 5 follower agents is considered. The system dimension is $m = 2$. The communication topology is shown in Fig. 6. The generator's initial states and the agents' ones are listed in Table 2. The disturbances are imposed on the agents initially as shown in Table 6. The input of the leader agent is set as $u_0(t) = [0.01, 0.01]^T, t \geq 0$. In this subsection, comparisons between the proposed hierarchical design framework and the

Table 1
Initial states and controller parameters.

No.	Agents' initial states	Generator's initial states	Controller parameters
1	$x_{1,1}(0) = [3, 2]^T$ $x_{1,2}(0) = [0.3, 0.2]^T$	$z_{1,1}(0) = [4, 6]^T$ $z_{1,2}(0) = [0.3, 0.2]^T$	$\rho_{1,1} = 5,\ \rho_{1,2} = 8$
2	$x_{2,1}(0) = [1, 0.5]^T$ $x_{2,2}(0) = [0.2, 0.2]^T$	$z_{2,1}(0) = [7, 5]^T$ $z_{2,2}(0) = [0.2, 0.2]^T$	$\rho_{2,1} = 5.5,\ \rho_{2,2} = 8.5$
3	$x_{3,1}(0) = [2, 1.2]^T$ $x_{3,2}(0) = [0.1, 0.4]^T$	$z_{3,1}(0) = [8, 2]^T$ $z_{3,2}(0) = [0.1, 0.4]^T$	$\rho_{3,1} = 4.4,\ \rho_{3,2} = 6.8$
4	$x_{4,1}(0) = [0.5, 1.2]^T$ $x_{4,2}(0) = [0.3, 0.3]^T$	$z_{4,1}(0) = [4, 1]^T$ $z_{4,2}(0) = [0.3, 0.3]^T$	$\rho_{4,1} = 6.2,\ \rho_{4,2} = 7.2$
5	$x_{5,1}(0) = [0, 0.7]^T$ $x_{5,2}(0) = [0.5, 0.2]^T$	$z_{5,1}(0) = [2, 5]^T$ $z_{5,2}(0) = [0.5, 0.2]^T$	$\rho_{5,1} = 4.7,\ \rho_{5,2} = 8.8$
6	$x_{6,1}(0) = [0.3, 0.6]^T$ $x_{6,2}(0) = [0.3, 0.4]^T$	$z_{6,1}(0) = [4, 4]^T$ $z_{6,2}(0) = [0.3, 0.4]^T$	$\rho_{6,1} = 3.2,\ \rho_{6,2} = 6.4$

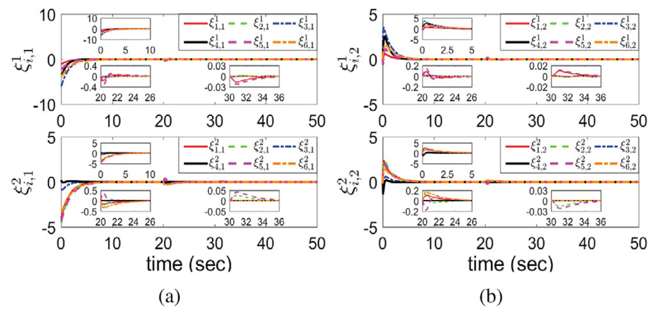


Fig. 4. Response curves for the agents' tracking errors under controllers (19). (a) $\xi_{i,1}$. (b) $\xi_{i,2}$.

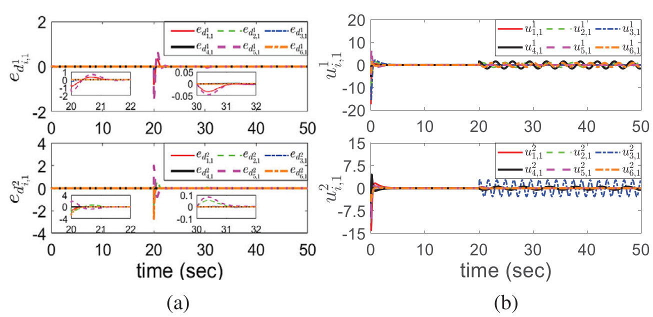


Fig. 5. (a) Response curves of the disturbance estimation errors. (b) Time histories of the agents' inputs.

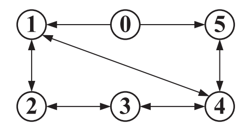


Fig. 6. The communication topology ($\bar{\mathcal{V}} = \{0, 1, 2, 3, 4, 5\}$).

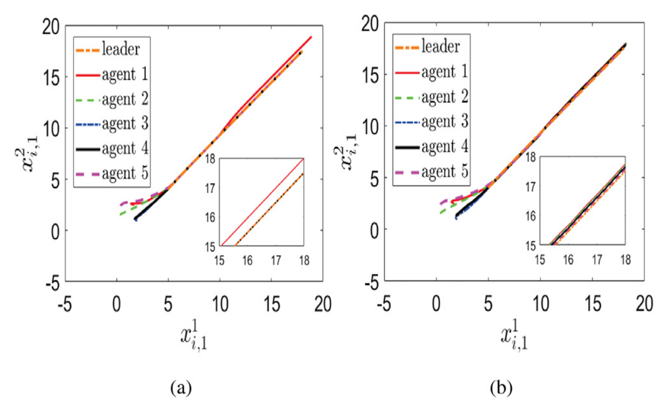


Fig. 7. Agents' trajectories in the 2-D space. (a) Hierarchical framework. (b) Traditional framework.

traditional design framework are presented under the condition that some of agents have faults, e.g., actuator failure. It is set that the actuator output of agent 1 turns to be 0.2 times from $t = 30$ s. Several efforts have been made to guarantee fairness of comparisons, i.e., the amplitudes for both kinds of controllers are in the same levels as much as possible. The parameters of reference signal generator (27) are chosen as $\mu_1 = 1, \mu_2 = 1.2, \mu_3 = 1.8$ and the parameters of tracking controllers (29) are listed in Table 2. For the traditional consensus protocols (22), the parameters are chosen as $c_1 = 15, c_2 = 5, K = [0.7481, 2.2443]^T$ which satisfy Proposition 3. To estimate the agents' disturbances, some disturbance estimators are adopted, whose details are given in Table 6. The simulation lasts 50s and the step length is 0.001 s.

The simulation results are shown in Figs. 7–13. From Fig. 8, the nodes' outputs of generator (27) speedily reach consensus with the leader. From Figs. 9–10, both the mismatched and matched disturbances are estimated preciously and quickly under the disturbance estimators. As shown in Figs. 7, 11 and 12, both schemes make the followers track the leader in the presence of disturbances. However, after actuator failure from $t = 30$ s, only agent 1 fails to track the leader and others can still reach consensus with the leader asymptotically under the tracking controllers (29) of the hierarchical framework. However, all the agents fail to track the leader under the traditional protocols (22). Time histories of agents' inputs are shown in Fig. 13, which indicates the comparison fairness. Hence, the proposed hierarchical design framework is more reliable and has better performances than the traditional design framework, which is in accordance with the aforementioned analysis.

5.2. Simulations for Section 4: Hierarchical design for distributed control with communication and input delays

A leaderless second-order multi-agent system as (31) with 6 agents is given. The system dimension is $m = 3$. The communication topology is shown in Fig. 14. The initial states and input delays of the agents as well as the initial states of reference signal generator (36) and communication delays are listed in Tables 3 and 4, respectively. The parameters of generator (36) are chosen as $l_3 = 60, l_4 = 50$ and the parameters of tracking controllers (46) are listed in Table 3. The simulation lasts 15 s and the step length is variable in [0, 0.003] s.

Table 2
Initial states and controller parameters.

No.	Agents' initial states	Generator's initial states	Controller parameters
0	$x_{0,1}(0) = [5, 4]^T$ $x_{0,2}(0) = [0.01, 0.02]^T$		
1	$x_{1,1}(0) = [1.7, 2.5]^T$ $x_{1,2}(0) = [0.05, 0.01]^T$	$\phi_{1,1}(0) = [2.5, 2]^T$ $\phi_{1,2}(0) = [0.02, 0.01]^T$	$\rho_{1,1} = 10,\ \rho_{1,2} = 18$
2	$x_{2,1}(0) = [0.4, 1.5]^T$ $x_{2,2}(0) = [0.03, 0.01]^T$	$\phi_{2,1}(0) = [0.4, 1.5]^T$ $\phi_{2,2}(0) = [0.03, 0.03]^T$	$\rho_{2,1} = 16,\ \rho_{2,2} = 21$
3	$x_{3,1}(0) = [2, 1]^T$ $x_{3,2}(0) = [0.01, 0.02]^T$	$\phi_{3,1}(0) = [1.8, 1]^T$ $\phi_{3,2}(0) = [0.01, 0.02]^T$	$\rho_{3,1} = 18,\ \rho_{3,2} = 15$
4	$x_{4,1}(0) = [2.2, 1.5]^T$ $x_{4,2}(0) = [0.04, 0.02]^T$	$\phi_{4,1}(0) = [1.2, 0.3]^T$ $\phi_{4,2}(0) = [0.02, 0.03]^T$	$\rho_{4,1} = 12.5,\ \rho_{4,2} = 19$
5	$x_{5,1}(0) = [0.4, 2.4]^T$ $x_{5,2}(0) = [0.01, 0.05]^T$	$\phi_{5,1}(0) = [3.5, 3]^T$ $\phi_{5,2}(0) = [0.01, 0.02]^T$	$\rho_{5,1} = 10.5,\ \rho_{5,2} = 17.8$

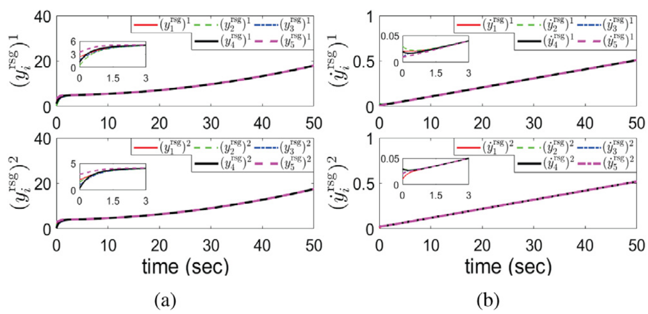


Fig. 8. Response curves of reference signal generator (27). (a) The outputs. (b) The derivatives of the outputs.

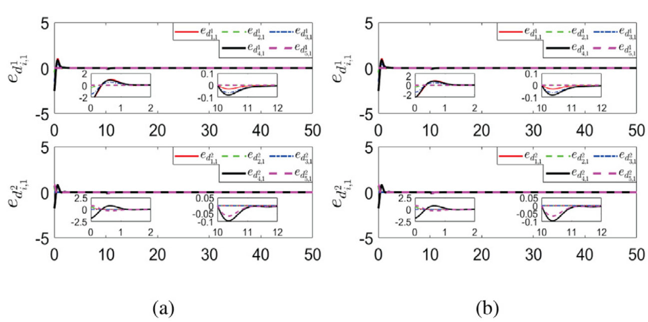


Fig. 9. Response curves of mismatched disturbance estimation errors. (a) Hierarchical framework. (b) Traditional framework.

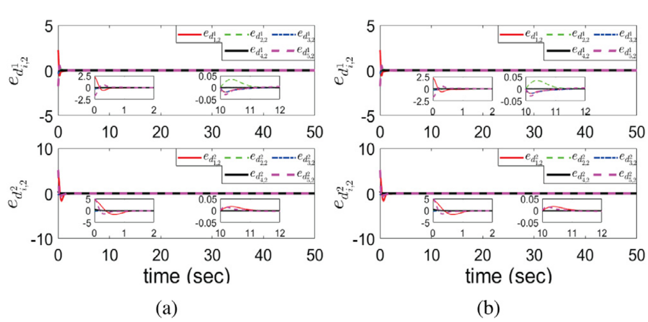


Fig. 10. Response curves of matched disturbance estimation errors. (a) Hierarchical framework. (b) Traditional framework.

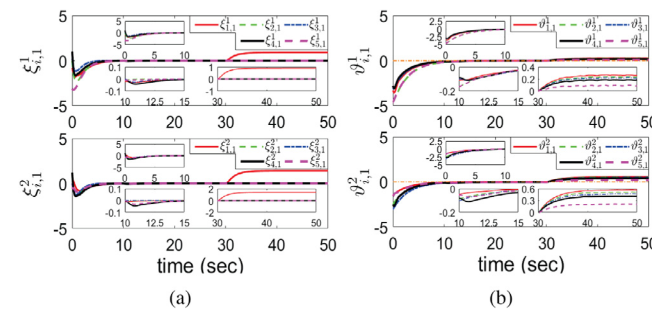


Fig. 11. Response curves for the agents' tracking errors $\xi_{i,1}$ under controllers (29) and $\vartheta_{i,1}$ under protocols (22). (a) Hierarchical framework. (b) Traditional framework.

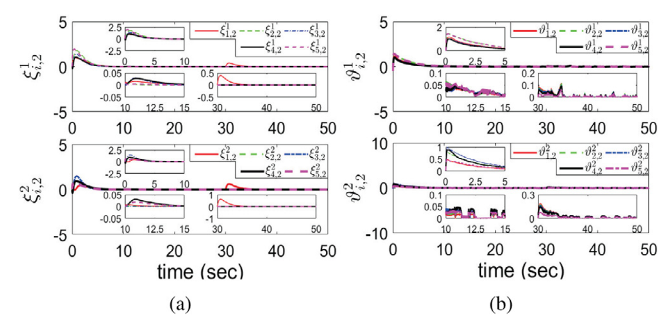


Fig. 12. Response curves for the agents' tracking errors $\xi_{i,2}$ under controllers (29) and $\vartheta_{i,2}$ under protocols (22). (a) Hierarchical framework. (b) Traditional framework.

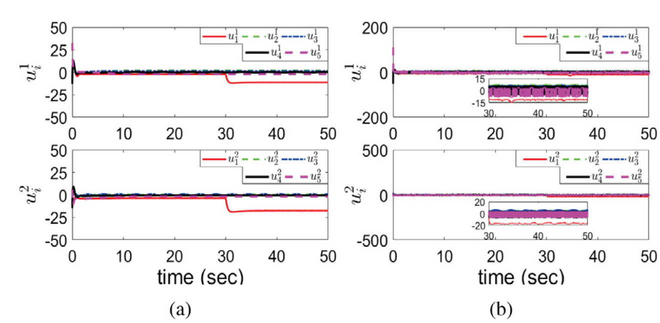


Fig. 13. Time histories of the agents' inputs. (a) Hierarchical framework. (b) Traditional framework.

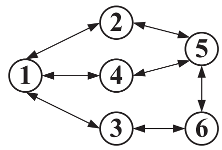


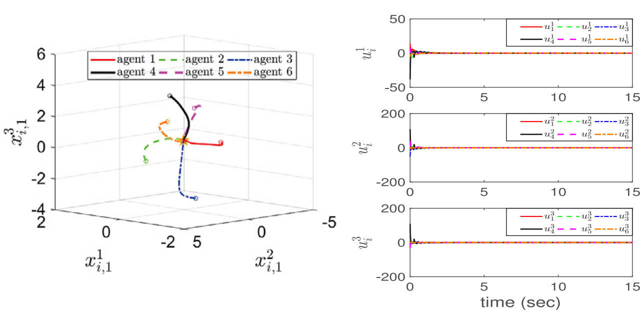
Fig. 14. The communication topology ($\mathcal{V} = \{1, 2, 3, 4, 5, 6\}$).

Table 3
Initial states and input delays of agents.

No.	Agents' initial states	Input delays (s)	Controller parameters
1	$x_{1,1}(0) = [-1.3, 0.4, 0.8]^T$ $x_{1,2}(0) = [-1.2, 1.2, -0.6]^T$	$h_1 = 0.1$	$\rho_{1,3} = 15$, $\rho_{1,4} = 10$
2	$x_{2,1}(0) = [1.3, -0.3, -1.3]^T$ $x_{2,2}(0) = [1.2, -1.4, 1.2]^T$	$h_2 = 0.15$	$\rho_{2,3} = 20$, $\rho_{2,4} = 25$
3	$x_{3,1}(0) = [-1.4, 2.6, -2.5]^T$ $x_{3,2}(0) = [1.2, 3, 0.7]^T$	$h_3 = 0.2$	$\rho_{3,3} = 10$, $\rho_{3,4} = 15$
4	$x_{4,1}(0) = [1.4, -2.4, 2.6]^T$ $x_{4,2}(0) = [-1.2, 0.4, -0.9]^T$	$h_4 = 0.25$	$\rho_{4,3} = 25$, $\rho_{4,4} = 20$
5	$x_{5,1}(0) = [0.4, -1.8, 2.2]^T$ $x_{5,2}(0) = [-0.4, -2, 1.6]^T$	$h_5 = 0.3$	$\rho_{5,3} = 15$, $\rho_{5,4} = 15$
6	$x_{6,1}(0) = [1, -1.2, 1.2]^T$ $x_{6,2}(0) = [0.4, 1.5, -0.3]^T$	$h_6 = 0.35$	$\rho_{6,3} = 10$, $\rho_{6,4} = 10$

Table 4
Generator initial states and communication delays.

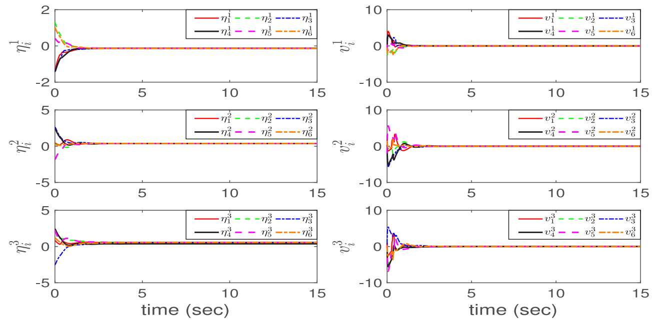
No.	Generator's initial states	Communication delays (s)
1	$\eta_1(0) = [-1.3, 0.4, 0.8]^T$ $\upsilon_1(0) = [-1.2, 2, 0.6]^T$	$\tau_{1,2}(t) = 0.2\sin(0.5t) + 0.3$ $\tau_{1,3}(t) = 0.3\sin(0.3t) + 0.4$ $\tau_{1,4}(t) = 0.2\sin(0.5t) + 0.25$
2	$\eta_2(0) = [1.3, 0.3, 2.3]^T$ $\upsilon_2(0) = [-1.2, 0.4, 1.2]^T$	$\tau_{2,1}(t) = 0.15\sin(0.4t) + 0.35$ $\tau_{2,5}(t) = 0.1\sin(0.1t) + 0.3$
3	$\eta_3(0) = [-1.4, 2.6, -2.5]^T$ $\upsilon_3(0) = [1.2, 3, 0.7]^T$	$\tau_{3,1}(t) = 0.2\sin(0.2t) + 0.5$ $\tau_{3,6}(t) = 0.2\sin(0.1t) + 0.35$
4	$\eta_4(0) = [-1.4, 2.4, 2.6]^T$ $\upsilon_4(0) = [-1.2, 0.4, 0.9]^T$	$\tau_{4,1}(t) = 0.15\sin(0.1t) + 0.3$ $\tau_{4,5}(t) = 0.3\sin(0.1t) + 0.35$
5	$\eta_5(0) = [0.4, -1.8, 2.2]^T$ $\upsilon_5(0) = [0.4, -2, 0.6]^T$	$\tau_{5,2}(t) = 0.2\sin(0.2t) + 0.2$ $\tau_{5,4}(t) = 0.25\sin(0.5t) + 0.4$ $\tau_{5,6}(t) = 0.2\sin(0.3t) + 0.4$
6	$\eta_6(0) = [1, 0.2, 1.2]^T$ $\upsilon_6(0) = [-0.4, 1.5, 0.3]^T$	$\tau_{6,3}(t) = 0.25\sin(0.5t) + 0.3$ $\tau_{6,5}(t) = 0.25\sin(0.1t) + 0.35$



(a) (b)

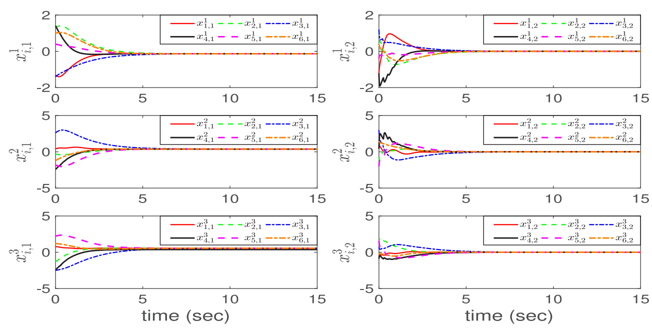
Fig. 15. (a) Agents' trajectories in the 3-D space. (b) Time histories of the agents' inputs.

The consensus goal is achieved in the presence of both communication and input delays by decoupling the distributed control task into Layer I-Reference signal generator design with communication delays and Layer II-Agents' tracking controllers design with input delays. The simulation results are shown in Figs. 15–17. From Fig. 16(a), the outputs of generator (36) rapidly reach consensus in the presence of communication delays. Under tracking controllers (46), as shown in Figs. 15(a) and 17(a), the



(a) (b)

Fig. 16. Response curves of reference signal generator (36). (a) The outputs. (b) The derivatives of the outputs.



(a) (b)

Fig. 17. Response curves of the agents' states under controllers (46). (a) $x_{i,1}$. (b) $x_{i,2}$.

agents track the reference signals from generator (36) in the presence of input delays, and reach consensus asymptotically. Time histories of agents' inputs are shown in Fig. 15(b). As shown in Fig. 15, "∘" denotes the agents' initial positions and "⋆" denotes the consensus position that the agents asymptotically converge to.

6. Conclusions

In this paper, a hierarchical design framework has been constructed for distributed control of multi-agent systems. This framework decouples the agents' cooperation regulations and individual regulations into two layers, i.e., a reference signal generator design layer and a tracking control design layer. Under this framework, the distributed control design is simplified and its flexibility, practicality, versatility and scalability are improved. Two kinds of typical hierarchical design examples have been given on distributed control of multi-agent systems with mismatched disturbances and with communication/input delays, respectively. Future research may be conducted for the cases with more factors, such as directed graphs, time-varying communication topologies, and finite-time convergence.

Acknowledgments

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Appendix

Tables 5 and 6 are listed in this appendix.

Table 5
Disturbances and disturbance estimators.

No.	20 s $\leq t <$ 30 s	$t \geq$ 30 s	Disturbance estimators	Parameters
1	$d^1_{1,1}(t) = 0.15t - 3.8$	$d^1_{1,1}(t) = 0.7$	GPIO $\begin{cases} \dot{\widehat{x^1_{1,1}}} = x^1_{1,2} + \widehat{\zeta^1_{1,1,1}} + \varrho^1_{1,1,1}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \\ \dot{\widehat{\zeta^1_{1,1,1}}} = \widehat{\zeta^1_{1,1,2}} + \varrho^1_{1,1,2}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \quad \left(\widehat{\zeta^1_{1,1,1}} = \widehat{d^1_{1,1}}\right) \\ \dot{\widehat{\zeta^1_{1,1,2}}} = \varrho^1_{1,1,3}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \end{cases}$	$\varrho^1_{1,1,1} = 6$ $\varrho^1_{1,1,2} = 30$ $\varrho^1_{1,1,3} = 37.5$
	$d^2_{1,1}(t) = -1.6$	$d^2_{1,1}(t) = -1.6$	DO $\begin{cases} \dot{\psi}^2_{1,1} = x^2_{1,2} + \widehat{d^2_{1,1}} \\ \widehat{d^2_{1,1}} = \varrho^2_{1,1,1}\left(x^2_{1,1} - \psi^2_{1,1}\right) \end{cases}$	$\varrho^2_{1,1,1} = 8$
2	$d^1_{2,1}(t) = 0.7\sin(1.5(t-20)-4)$	$d^1_{2,1}(t) = 0.7\sin(1.5(t-20)-4)$	DO $\begin{cases} \dot{\phi}^1_{2,1} = \left(W^1_{2,1} - \gamma^1_{2,1}C^1_{2,1}\right)\phi^1_{2,1} + W^1_{2,1}\gamma^1_{2,1}x^1_{2,1} - \gamma^1_{2,1}\left(C^1_{2,1}\gamma^1_{2,1}x^1_{2,1} + x^1_{2,2}\right) \\ \widehat{\sigma^1_{2,1}} = \phi^1_{2,1} + \gamma^1_{2,1}x^1_{2,1} \\ \widehat{d^1_{2,1}} = C^1_{2,1}\widehat{\sigma^1_{2,1}} \end{cases}$	$W^1_{2,1} = \begin{bmatrix} 0 & 1.5 \\ -1.5 & 0 \end{bmatrix}, C^1_{2,1} = [1, 0]$ $\sigma^1_{2,1}(20) = [-0.7\sin(4), 0.7\cos(4)]^T$ $\gamma^1_{2,1} = [6, 4.5]^T$
	$d^2_{2,1}(t) = -0.24t + 3.2$	$d^2_{2,1}(t) = -4$	GPIO $\begin{cases} \dot{\widehat{x^2_{2,1}}} = x^2_{2,2} + \widehat{\zeta^2_{2,1,1}} + \varrho^2_{2,1,1}\left(x^2_{2,1} - \widehat{x^2_{2,1}}\right) \\ \dot{\widehat{\zeta^2_{2,1,1}}} = \widehat{\zeta^2_{2,1,2}} + \varrho^2_{2,1,2}\left(x^2_{2,1} - \widehat{x^2_{2,1}}\right) \quad \left(\widehat{\zeta^2_{2,1,1}} = \widehat{d^2_{2,1}}\right) \\ \dot{\widehat{\zeta^2_{2,1,2}}} = \varrho^2_{2,1,3}\left(x^2_{2,1} - \widehat{x^2_{2,1}}\right) \end{cases}$	$\varrho^2_{2,1,1} = 5.8$ $\varrho^2_{2,1,2} = 29$ $\varrho^2_{2,1,3} = 35$
3	$d^1_{3,1}(t) = -0.3$	$d^1_{3,1}(t) = -0.3$	DO $\begin{cases} \dot{\psi}^1_{3,1} = x^1_{3,2} + \widehat{d^1_{3,1}} \\ \widehat{d^1_{3,1}} = \varrho^1_{3,1,1}\left(x^1_{3,1} - \psi^1_{3,1}\right) \end{cases}$	$\varrho^1_{3,1,1} = 8$
	$d^2_{3,1}(t) = 0.7\sin(4(t-20)+2.5)$	$d^2_{3,1}(t) = 0.7\sin(4(t-20)+2.5)$	DO $\begin{cases} \dot{\phi}^2_{3,1} = \left(W^2_{3,1} - \gamma^2_{3,1}C^2_{3,1}\right)\phi^2_{3,1} + W^2_{3,1}\gamma^2_{3,1}x^2_{3,1} - \gamma^2_{3,1}\left(C^2_{3,1}\gamma^2_{3,1}x^2_{3,1} + x^2_{3,1}\right) \\ \widehat{\sigma^2_{3,1}} = \phi^2_{3,1} + \gamma^2_{3,1}x^2_{3,1} \\ \widehat{d^2_{3,1}} = C^2_{3,1}\widehat{\sigma^2_{3,1}} \end{cases}$	$W^2_{3,1} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}, C^2_{3,1} = [1, 0]$ $\sigma^2_{3,1}(20) = [0.7\sin(2.5), 0.7\cos(2.5)]^T$ $\gamma^2_{3,1} = [10, 8]^T$
4	$d^1_{4,1}(t) = 0.8\sin(2(t-20)-3)$	$d^1_{4,1}(t) = 0.8\sin(2(t-20)-3)$	DO $\begin{cases} \dot{\phi}^1_{4,1} = \left(W^1_{4,1} - \gamma^1_{4,1}C^1_{4,1}\right)\phi^1_{4,1} + W^1_{4,1}\gamma^1_{4,1}x^1_{4,1} - \gamma^1_{4,1}\left(C^1_{4,1}\gamma^1_{4,1}x^1_{4,1} + x^1_{4,1}\right) \\ \widehat{\sigma^1_{4,1}} = \phi^1_{4,1} + \gamma^1_{4,1}x^1_{4,1} \\ \widehat{d^1_{4,1}} = C^1_{4,1}\widehat{\sigma^1_{4,1}} \end{cases}$	$W^1_{4,1} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, C^1_{4,1} = [1, 0]$ $\sigma^1_{4,1}(20) = [-0.8\sin(3), 0.8\cos(3)]^T$ $\gamma^1_{4,1} = [9, 12]^T$
	$d^2_{4,1}(t) = 0.4\sin(1.3(t-20)+5)$	$d^2_{4,1}(t) = 0.4\sin(1.3(t-20)+5)$	DO $\begin{cases} \dot{\phi}^2_{4,1} = \left(W^2_{4,1} - \gamma^2_{4,1}C^2_{4,1}\right)\phi^2_{4,1} + W^2_{4,1}\gamma^2_{4,1}x^2_{4,1} - \gamma^2_{4,1}\left(C^2_{4,1}\gamma^2_{4,1}x^2_{4,1} + x^2_{4,1}\right) \\ \widehat{\sigma^2_{4,1}} = \phi^2_{4,1} + \gamma^2_{4,1}x^2_{4,1} \\ \widehat{d^2_{4,1}} = C^2_{4,1}\widehat{\sigma^2_{4,1}} \end{cases}$	$W^2_{4,1} = \begin{bmatrix} 0 & 1.3 \\ -1.3 & 0 \end{bmatrix}, C^2_{4,1} = [1, 0]$ $\sigma^2_{4,1}(20) = [0.4\sin(5), 0.4\cos(5)]^T$ $\gamma^2_{4,1} = [10, 9]^T$
5	$d^1_{5,1}(t) = 0.2t - 5.5$	$d^1_{5,1}(t) = 0.5$	GPIO $\begin{cases} \dot{\widehat{x^1_{5,1}}} = x^1_{5,2} + \widehat{\zeta^1_{5,1,1}} + \varrho^1_{5,1,1}\left(x^1_{5,1} - \widehat{x^1_{5,1}}\right) \\ \dot{\widehat{\zeta^1_{5,1,1}}} = \widehat{\zeta^1_{5,1,2}} + \varrho^1_{5,1,2}\left(x^1_{5,1} - \widehat{x^1_{5,1}}\right) \quad \left(\widehat{\zeta^1_{5,1,1}} = \widehat{d^1_{5,1}}\right) \\ \dot{\widehat{\zeta^1_{5,1,2}}} = \varrho^1_{5,1,3}\left(x^1_{5,1} - \widehat{x^1_{5,1}}\right) \end{cases}$	$\varrho^1_{5,1,1} = 5.5$ $\varrho^1_{5,1,2} = 27$ $\varrho^1_{5,1,3} = 34$
	$d^2_{5,1}(t) = -0.4t + 10$	$d^2_{5,1}(t) = -2$	GPIO $\begin{cases} \dot{\widehat{x^2_{5,1}}} = x^2_{5,2} + \widehat{\zeta^2_{5,1,1}} + \varrho^2_{5,1,1}\left(x^2_{5,1} - \widehat{x^2_{5,1}}\right) \\ \dot{\widehat{\zeta^2_{5,1,1}}} = \widehat{\zeta^2_{5,1,2}} + \varrho^2_{5,1,2}\left(x^2_{5,1} - \widehat{x^2_{5,1}}\right) \quad \left(\widehat{\zeta^2_{5,1,1}} = \widehat{d^2_{5,1}}\right) \\ \dot{\widehat{\zeta^2_{5,1,2}}} = \varrho^2_{5,1,3}\left(x^2_{5,1} - \widehat{x^2_{5,1}}\right) \end{cases}$	$\varrho^2_{5,1,1} = 7$ $\varrho^2_{5,1,2} = 35$ $\varrho^2_{5,1,3} = 44$
6	$d^1_{6,1}(t) = 0.4\cos(2.4(t-20)-4)$	$d^1_{6,1}(t) = 0.4\cos(2.4(t-20)-4)$	DO $\begin{cases} \dot{\phi}^1_{6,1} = \left(W^1_{6,1} - \gamma^1_{6,1}C^1_{6,1}\right)\phi^1_{6,1} + W^1_{6,1}\gamma^1_{6,1}x^1_{6,1} - \gamma^1_{6,1}\left(C^1_{6,1}\gamma^1_{6,1}x^1_{6,1} + x^1_{6,1}\right) \\ \widehat{\sigma^1_{6,1}} = \phi^1_{6,1} + \gamma^1_{6,1}x^1_{6,1} \\ \widehat{d^1_{6,1}} = C^1_{6,1}\widehat{\sigma^1_{6,1}} \end{cases}$	$W^1_{6,1} = \begin{bmatrix} 0 & 2.4 \\ -2.4 & 0 \end{bmatrix}, C^1_{6,1} = [1, 0]$ $\sigma^1_{6,1}(20) = [0.4\cos(4), 0.4\sin(4)]^T$ $\gamma^1_{6,1} = [9, 6]^T$
	$d^2_{6,1}(t) = -3$	$d^2_{6,1}(t) = -3$	DO $\begin{cases} \dot{\psi}^2_{6,1} = x^2_{6,2} + \widehat{d^2_{6,1}} \\ \widehat{d^2_{6,1}} = \varrho^2_{6,1,1}\left(x^2_{6,1} - \psi^2_{6,1}\right) \end{cases}$	$\varrho^2_{6,1,1} = 8$

Table 6
Disturbances and disturbance estimators.

No.	$0\ \mathrm{s} \le t < 10\ \mathrm{s}$	$t \ge 10\ \mathrm{s}$	Disturbance estimators	Parameters
1	$d^1_{1,1}(t) = 0.14t - 2.5$	$d^1_{1,1}(t) = -1.1$	GPIO $\begin{cases} \dot{\widehat{x^1_{1,1}}} = x^1_{1,2} + \widehat{\zeta^1_{1,1,1}} + \varrho^1_{1,1,1}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \\ \dot{\widehat{\zeta^1_{1,1,1}}} = \widehat{\zeta^1_{1,1,2}} + \varrho^1_{1,1,2}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \\ \dot{\widehat{\zeta^1_{1,1,2}}} = \varrho^1_{1,1,3}\left(x^1_{1,1} - \widehat{x^1_{1,1}}\right) \end{cases}$ $\left(\widehat{\zeta^1_{1,1,1}} = \widehat{d^1_{1,1}}\right)$	$\varrho^1_{1,1,1} = 6$ $\varrho^1_{1,1,2} = 31$ $\varrho^1_{1,1,3} = 37$
	$d^2_{1,1}(t) = 0.7$	$d^2_{1,1}(t) = 0.7$	DO $\begin{cases} \dot{\psi}^2_{1,1} = x^2_{1,2} + \widehat{d^2_{1,1}} \\ \widehat{d^2_{1,1}} = \varrho^2_{1,1,1}\left(x^2_{1,1} - \psi^2_{1,1}\right) \end{cases}$	$\varrho^2_{1,1,1} = 8$
	$d^1_{1,2}(t) = 0.02t^2 - 0.2t + 2.25$	$d^1_{1,2}(t) = 2.25$	GPIO $\begin{cases} \dot{\widehat{x^1_{1,2}}} = u^1_1 + \widehat{\zeta^1_{1,2,1}} + \varrho^1_{1,2,1}\left(x^1_{1,2} - \widehat{x^1_{1,2}}\right) \\ \dot{\widehat{\zeta^1_{1,2,1}}} = \widehat{\zeta^1_{1,2,2}} + \varrho^1_{1,2,2}\left(x^1_{1,2} - \widehat{x^1_{1,2}}\right) \\ \dot{\widehat{\zeta^1_{1,2,2}}} = \widehat{\zeta^1_{1,2,3}} + \varrho^1_{1,2,3}\left(x^1_{1,2} - \widehat{x^1_{1,2}}\right) \\ \dot{\widehat{\zeta^1_{1,2,3}}} = \varrho^1_{1,2,4}\left(x^1_{1,2} - \widehat{x^1_{1,2}}\right) \end{cases}$ $\left(\widehat{\zeta^1_{1,2,1}} = \widehat{d^1_{1,2}}\right)$	$\varrho^1_{1,2,1} = 15$ $\varrho^1_{1,2,2} = 174$ $\varrho^1_{1,2,3} = 250$ $\varrho^1_{1,2,4} = 96$
	$d^2_{1,2}(t) = -0.08t + 4.3$	$d^2_{1,2}(t) = 3.5$	GPIO $\begin{cases} \dot{\widehat{x^2_{1,2}}} = u^2_1 + \widehat{\zeta^2_{1,2,1}} + \varrho^2_{1,2,1}\left(x^2_{1,2} - \widehat{x^2_{1,2}}\right) \\ \dot{\widehat{\zeta^2_{1,2,1}}} = \widehat{\zeta^2_{1,2,2}} + \varrho^2_{1,2,2}\left(x^2_{1,2} - \widehat{x^2_{1,2}}\right) \\ \dot{\widehat{\zeta^2_{1,2,2}}} = \varrho^2_{1,2,3}\left(x^2_{1,2} - \widehat{x^2_{1,2}}\right) \end{cases}$ $\left(\widehat{\zeta^2_{1,2,1}} = \widehat{d^2_{1,2}}\right)$	$\varrho^2_{1,2,1} = 6.5$ $\varrho^2_{1,2,2} = 33$ $\varrho^2_{1,2,3} = 40$
2	$d^1_{2,1}(t) = -0.5$	$d^1_{2,1}(t) = -0.5$	DO $\begin{cases} \dot{\psi}^1_{2,1} = x^1_{2,2} + \widehat{d^1_{2,1}} \\ \widehat{d^1_{2,1}} = \varrho^1_{2,1,1}\left(x^1_{2,1} - \psi^1_{2,1}\right) \end{cases}$	$\varrho^1_{2,1,1} = 8$
	$d^2_{2,1}(t) = 0.7$	$d^2_{2,1}(t) = 0.7$	DO $\begin{cases} \dot{\psi}^2_{2,1} = x^2_{2,2} + \widehat{d^2_{2,1}} \\ \widehat{d^2_{2,1}} = \varrho^2_{2,1,1}\left(x^2_{2,1} - \psi^2_{2,1}\right) \end{cases}$	$\varrho^2_{2,1,1} = 7$
	$d^1_{2,2}(t) = -0.16t - 0.26$	$d^1_{2,2}(t) = -1.86$	GPIO $\begin{cases} \dot{\widehat{x^1_{2,2}}} = u^1_2 + \widehat{\zeta^1_{2,2,1}} + \varrho^1_{2,2,1}\left(x^1_{2,2} - \widehat{x^1_{2,2}}\right) \\ \dot{\widehat{\zeta^1_{2,2,1}}} = \widehat{\zeta^1_{2,2,2}} + \varrho^1_{2,2,2}\left(x^1_{2,2} - \widehat{x^1_{2,2}}\right) \\ \dot{\widehat{\zeta^1_{2,2,2}}} = \varrho^1_{2,2,3}\left(x^1_{2,2} - \widehat{x^1_{2,2}}\right) \end{cases}$ $\left(\widehat{\zeta^1_{2,2,1}} = \widehat{d^1_{2,2}}\right)$	$\varrho^1_{2,2,1} = 7$ $\varrho^1_{2,2,2} = 34$ $\varrho^1_{2,2,3} = 45$
	$d^2_{2,2}(t) = 0.7\sin(1.8t + 4)$	$d^2_{2,2}(t) = 0.7\sin(1.8t + 4)$	DO $\begin{cases} \dot{\phi}^2_{2,2} = \left(W^2_{2,2} - \gamma^2_{2,2}C^2_{2,2}\right)\phi^2_{2,2} + W^2_{2,2}\gamma^2_{2,2}x^2_{2,2} - \gamma^2_{2,2}\left(C^2_{2,2}\gamma^2_{2,2}x^2_{2,2} + u^2_2\right) \\ \widehat{\sigma^2_{2,2}} = \phi^2_{2,2} + \gamma^2_{2,2}x^2_{2,2} \\ \widehat{d^2_{2,2}} = C^2_{2,2}\widehat{\sigma^2_{2,2}} \end{cases}$	$W^2_{2,2} = \begin{bmatrix} 0 & 1.8 \\ -1.8 & 0 \end{bmatrix}$, $C^2_{2,2} = [1, 0]$ $\sigma^2_{2,2}(0) = [0.7\sin(4), 0.7\cos(4)]^T$ $\gamma^2_{2,2} = [6, 9]^T$
3	$d^1_{3,1}(t) = 0.3t - 1.5$	$d^1_{3,1}(t) = 1.5$	GPIO $\begin{cases} \dot{\widehat{x^1_{3,1}}} = x^1_{3,2} + \widehat{\zeta^1_{3,1,1}} + \varrho^1_{3,1,1}\left(x^1_{3,1} - \widehat{x^1_{3,1}}\right) \\ \dot{\widehat{\zeta^1_{3,1,1}}} = \widehat{\zeta^1_{3,1,2}} + \varrho^1_{3,1,2}\left(x^1_{3,1} - \widehat{x^1_{3,1}}\right) \\ \dot{\widehat{\zeta^1_{3,1,2}}} = \varrho^1_{3,1,3}\left(x^1_{3,1} - \widehat{x^1_{3,1}}\right) \end{cases}$ $\left(\widehat{\zeta^1_{3,1,1}} = \widehat{d^1_{3,1}}\right)$	$\varrho^1_{3,1,1} = 7.5$ $\varrho^1_{3,1,2} = 38$ $\varrho^1_{3,1,3} = 42$
	$d^2_{3,1}(t) = 0.4\sin(2t + 3.5)$	$d^2_{3,1}(t) = 0.4\sin(2t + 3.5)$	DO $\begin{cases} \dot{\phi}^2_{3,1} = \left(W^2_{3,1} - \gamma^2_{3,1}C^2_{3,1}\right)\phi^2_{3,1} + W^2_{3,1}\gamma^2_{3,1}x^2_{3,1} - \gamma^2_{3,1}\left(C^2_{3,1}\gamma^2_{3,1}x^2_{3,1} + x^2_{3,2}\right) \\ \widehat{\sigma^2_{3,1}} = \phi^2_{3,1} + \gamma^2_{3,1}x^2_{3,1} \\ \widehat{d^2_{3,1}} = C^2_{3,1}\widehat{\sigma^2_{3,1}} \end{cases}$	$W^2_{3,1} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, $C^2_{3,1} = [1, 0]$ $\sigma^2_{3,1}(0) = [0.4\sin(3.5), 0.4\cos(3.5)]^T$ $\gamma^2_{3,1} = [6, 9]^T$
	$d^1_{3,2}(t) = 0.03t^2 - 0.4t - 0.35$	$d^1_{3,2}(t) = -1.35$	GPIO $\begin{cases} \dot{\widehat{x^1_{3,2}}} = u^1_3 + \widehat{\zeta^1_{3,2,1}} + \varrho^1_{3,2,1}\left(x^1_{3,2} - \widehat{x^1_{3,2}}\right) \\ \dot{\widehat{\zeta^1_{3,2,1}}} = \widehat{\zeta^1_{3,2,2}} + \varrho^1_{3,2,2}\left(x^1_{3,2} - \widehat{x^1_{3,2}}\right) \\ \dot{\widehat{\zeta^1_{3,2,2}}} = \widehat{\zeta^1_{3,2,3}} + \varrho^1_{3,2,3}\left(x^1_{3,2} - \widehat{x^1_{3,2}}\right) \\ \dot{\widehat{\zeta^1_{3,2,3}}} = \varrho^1_{3,2,4}\left(x^1_{3,2} - \widehat{x^1_{3,2}}\right) \end{cases}$ $\left(\widehat{\zeta^1_{3,2,1}} = \widehat{d^1_{3,2}}\right)$	$\varrho^1_{3,2,1} = 18$ $\varrho^1_{3,2,2} = 210$ $\varrho^1_{3,2,3} = 300$ $\varrho^1_{3,2,4} = 120$
	$d^2_{3,2}(t) = -0.4$	$d^2_{3,2}(t) = -0.4$	DO $\begin{cases} \dot{\psi}^2_{3,2} = u^2_3 + \widehat{d^2_{3,2}} \\ \widehat{d^2_{3,2}} = \varrho^2_{3,2,1}\left(x^2_{3,2} - \psi^2_{3,2}\right) \end{cases}$	$\varrho^2_{3,2,1} = 8$

(*continued on next page*)

Table 6 (*continued*).

No.	$0\ \text{s} \le t < 10\ \text{s}$	$t \ge 10\ \text{s}$	Disturbance estimators	Parameters
4	$d^1_{4,1}(t) = 0.4t - 2.5$	$d^1_{4,1}(t) = 1.5$	GPIO $\begin{cases} \dot{\widehat{x}}^1_{4,1} = x^1_{4,2} + \widehat{\zeta}^1_{4,1,1} + \varrho^1_{4,1,1}\left(x^1_{4,1} - \widehat{x}^1_{4,1}\right) \\ \dot{\widehat{\zeta}}^1_{4,1,1} = \widehat{\zeta}^1_{4,1,2} + \varrho^1_{4,1,2}\left(x^1_{4,1} - \widehat{x}^1_{4,1}\right) \qquad \left(\widehat{\zeta}^1_{4,1,1} = \widehat{d}^1_{4,1}\right) \\ \dot{\widehat{\zeta}}^1_{4,1,2} = \varrho^1_{4,1,3}\left(x^1_{4,1} - \widehat{x}^1_{4,1}\right) \end{cases}$	$\varrho^1_{4,1,1} = 8$ $\varrho^1_{4,1,2} = 40$ $\varrho^1_{4,1,3} = 50$
	$d^2_{4,1}(t) = 0.45t - 1.75$	$d^2_{4,1}(t) = 2.75$	GPIO $\begin{cases} \dot{\widehat{x}}^2_{4,1} = x^2_{4,2} + \widehat{\zeta}^2_{4,1,1} + \varrho^2_{4,1,1}\left(x^2_{4,1} - \widehat{x}^2_{4,1}\right) \\ \dot{\widehat{\zeta}}^2_{4,1,1} = \widehat{\zeta}^2_{4,1,2} + \varrho^2_{4,1,2}\left(x^2_{4,1} - \widehat{x}^2_{4,1}\right) \qquad \left(\widehat{\zeta}^2_{4,1,1} = \widehat{d}^2_{4,1}\right) \\ \dot{\widehat{\zeta}}^2_{4,1,2} = \varrho^2_{4,1,3}\left(x^2_{4,1} - \widehat{x}^2_{4,1}\right) \end{cases}$	$\varrho^2_{4,1,1} = 6$ $\varrho^2_{4,1,2} = 32$ $\varrho^2_{4,1,3} = 40$
	$d^1_{4,2}(t) = 0.3\sin(1.2t + 2.5)$	$d^1_{4,2}(t) = 0.3\sin(1.2t + 2.5)$	DO $\begin{cases} \dot{\phi}^1_{4,2} = \left(W^1_{4,2} - \gamma^1_{4,2}C^1_{4,2}\right)\phi^1_{4,2} + W^1_{4,2}\gamma^1_{4,2}x^1_{4,2} - \gamma^1_{4,2}\left(C^1_{4,2}\gamma^1_{4,2}x^1_{4,2} + u^1_4\right) \\ \widehat{\sigma}^1_{4,2} = \phi^1_{4,2} + \gamma^1_{4,2}x^1_{4,2} \\ \widehat{d}^1_{4,2} = C^1_{4,2}\widehat{\sigma}^1_{4,2} \end{cases}$	$W^1_{4,2} = \begin{bmatrix} 0 & 1.2 \\ -1.2 & 0 \end{bmatrix}, C^1_{4,2} = [1, 0]$ $\sigma^1_{4,2}(0) = [0.3\sin(2.5), 0.3\cos(2.5)]^T$ $\gamma^1_{4,2} = [15, 20]^T$
	$d^2_{4,2}(t) = 0.5$	$d^2_{4,2}(t) = 0.5$	DO $\begin{cases} \dot{\psi}^2_{4,2} = u^2_4 + \widehat{d}^2_{4,2} \\ \widehat{d}^2_{4,2} = \varrho^2_{4,2,1}\left(x^2_{4,2} - \psi^2_{4,2}\right) \end{cases}$	$\varrho^2_{4,2,1} = 7$
5	$d^1_{5,1}(t) = 0.5\cos(2.4t - 3)$	$d^1_{5,1}(t) = 0.5\cos(2.4t - 3)$	DO $\begin{cases} \dot{\phi}^1_{5,1} = \left(W^1_{5,1} - \gamma^1_{5,1}C^1_{5,1}\right)\phi^1_{5,1} + W^1_{5,1}\gamma^1_{5,1}x^1_{5,1} - \gamma^1_{5,1}\left(C^1_{5,1}\gamma^1_{5,1}x^1_{5,1} + x^1_{5,2}\right) \\ \widehat{\sigma}^1_{5,1} = \phi^1_{5,1} + \gamma^1_{5,1}x^1_{5,1} \\ \widehat{d}^1_{5,1} = C^1_{5,1}\widehat{\sigma}^1_{5,1} \end{cases}$	$W^1_{5,1} = \begin{bmatrix} 0 & 2.4 \\ -2.4 & 0 \end{bmatrix}, C^1_{5,1} = [1, 0]$ $\sigma^1_{5,1}(0) = [0.5\cos(3), 0.5\sin(3)]^T$ $\gamma^1_{5,1} = [8, 12]^T$
	$d^2_{5,1}(t) = 0.32t + 0.8$	$d^2_{5,1}(t) = 4$	GPIO $\begin{cases} \dot{\widehat{x}}^2_{5,1} = x^2_{5,2} + \widehat{\zeta}^2_{5,1,1} + \varrho^2_{5,1,1}\left(x^2_{5,1} - \widehat{x}^2_{5,1}\right) \\ \dot{\widehat{\zeta}}^2_{5,1,1} = \widehat{\zeta}^2_{5,1,2} + \varrho^2_{5,1,2}\left(x^2_{5,1} - \widehat{x}^2_{5,1}\right) \qquad \left(\widehat{\zeta}^2_{5,1,1} = \widehat{d}^2_{5,1}\right) \\ \dot{\widehat{\zeta}}^2_{5,1,2} = \varrho^2_{5,1,3}\left(x^2_{5,1} - \widehat{x}^2_{5,1}\right) \end{cases}$	$\varrho^2_{5,1,1} = 6.5$ $\varrho^2_{5,1,2} = 35$ $\varrho^2_{5,1,3} = 43$
	$d^1_{5,2}(t) = -0.01t^2 + 0.5t - 1.75$	$d^1_{5,2}(t) = 2.25$	GPIO $\begin{cases} \dot{\widehat{x}}^1_{5,2} = u^1_5 + \widehat{\zeta}^1_{5,2,1} + \varrho^1_{5,2,1}\left(x^1_{5,2} - \widehat{x}^1_{5,2}\right) \\ \dot{\widehat{\zeta}}^1_{5,2,1} = \widehat{\zeta}^1_{5,2,2} + \varrho^1_{5,2,2}\left(x^1_{5,2} - \widehat{x}^1_{5,2}\right) \\ \dot{\widehat{\zeta}}^1_{5,2,2} = \widehat{\zeta}^1_{5,2,3} + \varrho^1_{5,2,3}\left(x^1_{5,2} - \widehat{x}^1_{5,2}\right) \qquad \left(\widehat{\zeta}^1_{5,2,1} = \widehat{d}^1_{5,2}\right) \\ \dot{\widehat{\zeta}}^1_{5,2,3} = \varrho^1_{5,2,4}\left(x^1_{5,2} - \widehat{x}^1_{5,2}\right) \end{cases}$	$\varrho^1_{5,2,1} = 14$ $\varrho^1_{5,2,2} = 170$ $\varrho^1_{5,2,3} = 260$ $\varrho^1_{5,2,4} = 88$
	$d^2_{5,2}(t) = 0.02t^2 - 0.55t + 5.25$	$d^2_{5,2}(t) = 1.75$	GPIO $\begin{cases} \dot{\widehat{x}}^2_{5,2} = u^2_5 + \widehat{\zeta}^2_{5,2,1} + \varrho^2_{5,2,1}\left(x^2_{5,2} - \widehat{x}^2_{5,2}\right) \\ \dot{\widehat{\zeta}}^2_{5,2,1} = \widehat{\zeta}^2_{5,2,2} + \varrho^2_{5,2,2}\left(x^2_{5,2} - \widehat{x}^2_{5,2}\right) \\ \dot{\widehat{\zeta}}^2_{5,2,2} = \widehat{\zeta}^2_{5,2,3} + \varrho^2_{5,2,3}\left(x^2_{5,2} - \widehat{x}^2_{5,2}\right) \qquad \left(\widehat{\zeta}^2_{5,2,1} = \widehat{d}^2_{5,2}\right) \\ \dot{\widehat{\zeta}}^2_{5,2,3} = \varrho^2_{5,2,4}\left(x^2_{5,2} - \widehat{x}^2_{5,2}\right) \end{cases}$	$\varrho^2_{5,2,1} = 16$ $\varrho^2_{5,2,2} = 166$ $\varrho^2_{5,2,3} = 266$ $\varrho^2_{5,2,4} = 100$

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