Writeup: How the this works

We can pair off all the people, and for each pair of people p_i and p_j we can create a boolean $x_{i,j}$ that is true if p_i is older than p_j and false otherwise. Then for each constraint of people p_i, p_j, and p_k, we are essentially given that either p_k is older than both p_i and p_j or younger than both.

This is equivalent to saying we must have $x_{i,k} \odot x_{j,k}$ (XNOR p_k is between p_i and p_j). We can rewrite this relationship as $(x_{i,k} \vee \bar{x}_{j,k}) \wedge (\bar{x}_{i,k} \vee x_{j,k})$.We then AND all these constraints together to get the total set of constraints. However, this does not account for the fact that the people ages must form a DAG.

To do this we add another set of constraints: for every triple of people p_i, p_j, p_k with $i < j < k$ we add the constraints $(\bar{x}_{i,j} \vee \bar{x}_{j,k} \vee \mathrm{x}_{i,k}) \wedge (\mathrm{x}_{i,j} \vee \mathrm{x}_{j,k} \vee \bar{\mathrm{x}}_{i,k})$. We now have a 3-SAT problem comprised of the DAG constraints and the constraints given in the problem instance. After solving the 3-SAT problem using the Glucose SAT solver, we can form a DAG whose edges consist of the order relationships between pairs of people (these edges will have been found by solving for all the $\mathrm{x}_{i,j}$ in the 3-SAT problem). To output a valid ordering of the people we can just run a topological sort on the DAG.

**DAG - directed acyclic graph