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Markov chain modeling for very-short-term wind power forecasting

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ABSTRACT

A Wind power forecasting method based on the use of discrete time Markov chain models is developed starting from real wind power time series data. It allows to directly obtain in an easy way an estimate of the wind power distributions on a very short-term horizon, without requiring restrictive assumptions on wind power probability distribution. First and Second Order Markov Chain Model are analytically described. Finally, the application of the proposed method is illustrated with reference to a set of real data.

1. Introduction

Wind power forecasting methods can be divided into two main groups: physical and statistical methods [1,2]. The former [3–6] are based on physical considerations to provide estimates of future wind power output starting from meteorological predictions. The latter [7–10] consist of emulating the relationship between historical values of wind power, historical and forecasted values of meteorological variables and future wind power output, whose parameters have to be estimated from data, without making any assumption on the physics of the phenomenon under study.

Both approaches are used to provide wind power forecasts on very short-term (up to 30 min ahead), short-term (from 30 min up to 6 h ahead), medium-term (from 6 h to 1 day ahead) and long-term (from 1 day up to 1 week ahead) [11–13]. Very short-term forecasting models are usually statistically-based [1].

In the case of short or longer term forecasts, statistical methods need Numerical Weather Predictions to provide an acceptable forecast accuracy. On the contrary, for a very short-term, pure statistical methods, including the sole autoregressive part, exhibit good performances [2]. Combinations of physical and statistical approaches and combinations of different time-scale models (short-term and medium-term) are referred to as hybrid approach [12–17].

The main limitation of many of the abovementioned models consists in the fact that their use only enables to perform point forecast of the random variable of interest (i.e. the wind power generated in a future time), whereas they do not allow to formulate its probability distribution. In fact, decision making processes in electrical power systems management [18] and electricity market trading strategies [19,20], generally, require more information than a point forecast.

Models which allow formulating the probability distribution functions of the wind power are proposed in [21–28]. Unfortunately, also these models presents some limitations. Indeed, they adopt generalist methods, that are either too complex to be applied in practice or based on assumptions that are usually far to be verified in the application (e.g. residuals are independent and identically distributed Gaussian random variables). In addition, all these models are difficult to calibrate on the basis of the kind of data that are commonly available in practical settings.

The models proposed in this paper fall in the category of pure statistical methods. They have been formulated, starting from an initial idea presented in [29], on the basis of the Markov Chain (MC) theory, a kind of approach that have been already used in relevant literature for the generation of synthetic wind speed and wind power time series [30–33].

These Markov models are based on few non restrictive hypotheses and can be calibrated and applied on the basis of set of data that are usually available in practice. Indeed, only past values of wind power are required for their use. With respect to the models presented in [29], here, the First Order Markov Chain model (FOMC) is strongly reformulated while the Second Order Markov Chain Model (SOMC) is comprehensively formulated by introducing the concepts of auxiliary transition matrices and auxiliary state vec-

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tor probabilities which allow treating SOMC (or even higher order models) as an ordinary FOMC from a mathematical point of view. Moreover, interval prediction, which was not considered in [29], is addressed also discussing in major details point prediction both for SOMC and FOMC. Finally, a more extensive literature review is presented.

From the applicative point of view, the main characteristic of the proposed models is that they allow to estimate the probability distribution of wind power over future time horizon, deriving from it point forecasts and other figures. This gives to the analysts all the information they need to perform risk analyses and economic performances evaluation, which are required in electrical power systems management. Of course the proposed statistical model can be included in a hybrid model that uses numerical weather predictions.

In what follows, two models are presented, one based on the use of First Order Markov Chain, and the other on the use of Second Order Markov Chain. Then, the probability estimation procedure is described. In the last section, the application of the proposed method is briefly illustrated with reference to a case-study.

2. Proposed method

In order to formulate the proposed models, the time axis is divided into contiguous and equispaced intervals of length $\Delta t = 10$ min. Moreover, the state variable is discretized defining a finite set of (representative) values $\{s_1, s_2, \ldots, s_N\}$, where N is a calibration parameter. Finally, the average power generated by the wind farm over the time interval $[t_{h-1}, t_h]$, where $t_h = h \cdot \Delta t$, is considered as state variable of the process, $S_P(t_h)$. So stated, let $\left\{S_P(t_h), h = 0, 1, 2, \ldots\right\}$ denote a discrete time Markov Chain that describes the evolution of the state variable over the time.

In order to define the set $\{s_1, s_2, \ldots, s_N\}$ it is to consider that, very often wind farm output equals zero, because the individual turbines deliver no output outside the so-called cut-in and cut-out wind speed interval. Moreover, very frequently, the output equals the nominal wind farm power, P_n, because the turbines deliver their nominal power when the nominal wind speed is reached, and cut-out conditions do not apply.

For this reason, the minimum and maximum values, s_1 and s_N, of the state variable are set to 0 and P_n, respectively. The remaining values $s_2, s_3, \ldots, s_{N-1}$ are set to the centers of the N-2 classes of equal length defined on the interval$]0, P_n[$.

2.1. First Order Markov Chain

A FOMC satisfies the following equality:

$$\begin{aligned} & Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_{i_h}, S_P(t_{h-1}) = s_{i_{h-1}}, \ldots, S_P(t_1) = s_{i_1}\right\} \\ & = Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_{i_h}\right\}, \\ & \text{for each} \quad j, i_1, i_2, \ldots, i_h \in \left\{1, \ldots, N\right\} \end{aligned} \tag{1}$$

Eq. (1) states that, in a FOMC, the probability that $S_P(t_{h+1})$ at t_{h+1} is s_j, given the state of the process at t_h, does not depend on the previous history of the process.

Hence, in order to completely define the process it is necessary to formulate the one-step transition matrix $\mathbf{P}(t_h)$, whose generic element, $p_{ij}(t_h)$, represents the probability that the state of process at t_{h+1} is s_j, given that the state at t_h is s_i:

$$p_{ij}(t_h) = Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_i\right\}. \tag{2}$$

Since, in general, the evolution over the time of power generated from a wind farm cannot be modeled via an homogeneous Markov process (i.e. a process with a stationary transition matrix, $\mathbf{P}(t_h)$), it is necessary to define the one step transition matrix for each h.

	1	**2**	**…**	**N-1**	**N**
1	$\hat{p}_{1,1}$	$\hat{p}_{1,2}$	…	$\hat{p}_{1,N-1}$	$\hat{p}_{1,N}$
2	$\hat{p}_{2,1}$	$\hat{p}_{2,2}$	…	$\hat{p}_{2,N-1}$	$\hat{p}_{2,N}$
⋮	⋮	⋮	⋮	⋮	⋮
N-1	$\hat{p}_{N-1,1}$	$\hat{p}_{N-1,1}$		$\hat{p}_{N-1,N-1}$	$\hat{p}_{N-1,N}$
N	$\hat{p}_{N,1}$	$\hat{p}_{N,2}$	…	$\hat{p}_{N,N-1}$	$\hat{p}_{N,N}$

Fig. 1. First order one-step transition matrix.

In order to obtain an estimate, $\hat{\mathbf{P}}(t_h)$, of the transition matrix at time step t_h, the (most recent) data, collected in the time window, $[t_{h-\text{ws}}, t_h]$, that slides with t_h, can be used, where the sliding window size, ws, is a calibration parameter.

In particular, an estimate for $p_{ij}(t_h)$ can be (easily) obtained as:

$$\hat{p}_{ij}(t_h) = \frac{n_{ij}(t_h)}{\sum_j n_{ij}(t_h)} \quad \forall i, j, \quad \text{with} \quad \sum_{j=1}^{N} \hat{p}_{ij}(t_h) = 1 \quad \forall i, \tag{3}$$

where $n_{ij}(t_h)$ indicates the number of transitions from state s_i to state s_j observed in the sequence of wind power data contained in the sliding window $[t_{h-ws}, t_h]$. Estimates (3) are the maximum likelihood estimates of the transition probabilities [30].

If for a given i it is $n_{ij}(t_h) = 0\, \forall j = 1, 2, \ldots, N$, then it is assumed:

$$\hat{p}_{ij}(t_h) = \begin{cases} 1 & j = i, \\ 0 & \forall j \neq i. \end{cases} \tag{4}$$

Estimates of the transition probabilities at time t_{h+1} can be easily obtained updating those performed at time t_h, by means of recursive algorithms.

For N states, the first order transition matrix is an $N \times N$ matrix. According to the representation reported in Fig. 1, each row of the matrix corresponds to the current state of the process, while each column corresponds to one of the N possible states at next time step. The elements of each row of the matrix sum up to 1, since this sum corresponds to the probability of a transition from a current state to any possible state (i.e. $\mathbf{P}(t_h)$ is a stochastic matrix).

2.2. Second Order Markov Chain

For a SOMC it results:

$$\begin{aligned} & Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_{i_h}, S_P(t_{h-1}) = s_{i_{h-1}}, \ldots, S_P(t_1) = s_{i_1}\right\} \\ & = Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_{i_h}, S_P(t_{h-1}) = s_{i_{h-1}}\right\} \\ & \text{for each} \quad j, i_1, i_2, \ldots, i_h \in \left\{1, \ldots, N\right\} \end{aligned} \tag{5}$$

Eq. (5) states that, in a SOMC, the probability that the process is in the state s_j, at t_{h+1}, given the state of the process at t_h and t_{h-1} does not depend on the previous history.

This implies that a SOMC can be modeled as a FOMC introducing composite states $\left\{11, 12, \ldots, 1N, 21, \ldots, 2N, \ldots, N1, \ldots, NN\right\}$ [34]. Hence, in order to completely define the SOMC it is necessary to formulate the auxiliary one-step, $N^2 \times N^2$, transition matrix, $\mathbf{P}(t_h, t_{h-1})$, of this “auxiliary” FOMC, where the term “one-step” refers to the number of steps elapsed from the current epoch, t_h, to the subsequent epoch, t_{h+1}.

		yj			
		11	12	21	22
li	11	$P_{11,11}$	$P_{11,12}$	0	0
	12	0	0	$P_{12,21}$	$P_{12,22}$
	21	$P_{21,11}$	$P_{21,12}$	0	0
	22	0	0	$P_{22,21}$	$P_{22,22}$

Fig. 2. Second order auxiliary one-step transition matrix ($N = 2$).

		yj			
		11	12	21	22
li	11	$p_{11,11}^2$	$p_{11,11} \cdot p_{11,12}$	$p_{11,12} \cdot p_{12,21}$	$p_{11,12} \cdot p_{12,22}$
	12	$p_{12,21} \cdot p_{21,11}$	$p_{12,21} \cdot p_{21,12}$	$p_{12,22} \cdot p_{22,21}$	$p_{12,22} \cdot p_{22,22}$
	21	$p_{21,11} \cdot p_{11,11}$	$p_{21,11} \cdot p_{11,12}$	$p_{21,12} \cdot p_{12,21}$	$p_{21,12} \cdot p_{12,22}$
	22	$p_{22,21} \cdot p_{21,11}$	$p_{22,21} \cdot p_{21,12}$	$p_{22,22} \cdot p_{22,21}$	$p_{22,22}^2$

Fig. 3. Second order auxiliary two-step transition matrix ($N = 2$).

For example, in the case $N = 2$ the, $2^2 \times 2^2$, auxiliary one-step transition matrix, $\mathbf{P}(t_h, t_{h-1})$, is reported in Fig. 2:

The generic element, $p_{li,yj}(t_{h-1}, t_h)$, of the auxiliary one-step transition matrix, $\mathbf{P}(t_h, t_{h-1})$, represents the probability that the states of process at t_{h+1} and t_h are s_j and s_y, respectively given that the state at t_h is s_i and the state at t_{h-1} is s_l, indeed it results:

$$p_{li,yj}(t_{h-1}, t_h) = Pr\left\{S_P(t_{h+1}) = s_j, S_P(t_h) = s_y \,\middle|\, S_P(t_h) = s_i\ , S_P(t_{h-1}) = s_l\right\}. \tag{6}$$

It is worthwhile to note that, due to the adopted representation, it results:

$$p_{li,yj}(t_{h-1}, t_h) = \begin{cases} Pr\left\{S_P(t_{h+1}) = s_j \,\middle|\, S_P(t_h) = s_i\ , S_P(t_{h-1}) = s_l\right\} & ,\quad i = y \\ 0 \quad otherwise. \end{cases} \tag{7}$$

It is also useful to note that, the elements of a generic k-step transition matrix, $\mathbf{P}(t_h, t_{h-1})$, obtained as:

$$\mathbf{P}^{(k)}(t_h, t_{h-1}) = [\mathbf{P}(t_h, t_{h-1})]^k, \tag{8}$$

with $k > 1$, could be all different from zero.

For example, in the case $N = 2$ the, $2^2 \times 2^2$, auxiliary two-step transition matrix, $\mathbf{P}^{(2)}(t_h, t_{h-1})$, (reported in Fig. 3) is obtained as:

$$\mathbf{P}^{(2)}(t_h, t_{h-1}) = [\mathbf{P}(t_h, t_{h-1})]^2, \tag{9}$$

where the generic element, $p^{(2)}_{li,yj}(t_{h-1}, t_h)$, represents the probability that the state of process at t_{h+2} is s_j and at t_{h+1} is s_y, given that the state at t_h is s_i and the state at t_{h-1} is s_l:

$$p^{(2)}_{li,yj}(t_{h-1}, t_h) = Pr\left\{S_P(t_{h+2}) = s_j, S_P(t_{h+1}) = s_y \,\middle|\, S_P(t_h) = s_i\ , S_P(t_{h-1}) = s_l\right\}, \tag{10}$$

Estimates (i.e. Maximum likelihood estimates) of the (non-zero) element of the auxiliary one-step transition matrix probabilities can be obtained as follows:

$$\hat{p}_{li,ij}(t_{h-1}, t_h) = \frac{n_{li,ij}(t_{h-1}, t_h)}{\sum_j n_{li,ij}(t_{h-1}, t_h)} \quad \forall \quad l, \quad i, \quad j,$$

$$\sum_{j=1}^{N} \hat{p}_{li,ij}(t_{h-1}, t_h) = 1, \quad \forall \quad l, \quad i, \tag{11}$$

where $n_{li,ij}(t_{h-1}, t_h)$ is the number of times that this sequence of wind power states is observed in the sliding window used to calibrate the model.

If it results $n_{li,ij}(t_{h-1}, t_h) = 0\ \forall\ l, i, j$, then it is assumed:

$$\hat{p}_{li,ij}(t_{h-1}, t_h) = \begin{cases} 1 & i = j, \\ 0 & elsewhere. \end{cases} \tag{12}$$

Estimates of the transition probabilities at time t_{h+1} can be easily obtained updating those performed at time t_h, by means of recursive algorithms.

2.3. Predictor modeling

In this section, it is described how models results are obtained in the case of FOMC and SOMC.

FOMC

Indicating with $\pi(t_h)$ the state probabilities vector at time t_h:

$$\pi(t_h) = [\pi_1(t_h), \pi_2(t_h), \ldots, \pi_N(t_h)], \tag{13}$$

whose generic i-th element, $\pi_i(t_h)$, represents the probability that the wind power $S_P(t_h)$ at time t_h equals s_i:

$$\pi_i(t_h) = Pr\left\{S_P(t_h) = s_i\right\}, \tag{14}$$

it is possible to obtain the state probabilities vector at time t_{h+1}, as follows:

$$\pi(t_{h+1}) = \pi(t_h) \cdot \mathbf{P}(t_h). \tag{15}$$

Given the measured wind power data necessary to evaluate the transition matrix, $\mathbf{P}(t_h)$, at time t_h, it is possible to obtain an estimate $\hat{\pi}(t_{h+1})$ of the state probability vector, $\pi(t_{h+1})$, as follows:

$$\hat{\pi}(t_{h+1}) = \pi(t_h) \cdot \hat{\mathbf{P}}(t_h), \tag{16}$$

where $\hat{\mathbf{P}}(t_h)$ is the transition matrix whose elements are given by equation (3) and $\pi_0(t_h)$ is the observed state probability vector, whose elements are all zero but the element corresponding to the state s_{i_h} the process is at time t_h, which is set equal to 1.

An estimate, $\hat{\pi}(t_{h+k})$, of the state probability vector $\pi(t_{h+k})$ for every $k = 1,2,\ldots$ can be obtained (on the basis of the same data) via the following formula:

$$\hat{\pi}(t_{h+k}) = \pi_0(t_h) \cdot \left[\hat{\mathbf{P}}(t_h)\right]^k. \tag{17}$$

SOMC

In the case of a SOMC the elements of the state probabilities vector $\pi(t_h)$ at time t_h, can be computed as:

$$\pi_i(t_h) = \sum_{l=1}^{N} \pi_{l,i}(t_{h-1}, t_h), \quad i = 1, 2, \ldots, N, \tag{18}$$

where:

$$\pi_{li}(t_{h-1}, t_h) = Pr\left\{S_P(t_h) = s_i, S_P(t_{h-1}) = s_l\right\}, \quad \forall l, \quad i \tag{19}$$

is the generic element of the auxiliary state probability vector, $\pi(t_{h-1}, t_h)$.

On this basis, analysis of a SOMC can be easily performed adopting the following recursive equation:

$$\pi(t_h, t_{h+1}) = \pi_0(t_{h-1}, t_h) \cdot \mathbf{P}(t_{h-1}, t_h). \tag{20}$$

Hence, given the measured wind power data necessary to evaluate the auxiliary transition matrix, $\mathbf{P}(t_{h-1}, t_h)$, it is possible to obtain an estimate $\hat{\pi}(t_h, t_{h+1})$ of the auxiliary state probability vector, $\pi(t_h, t_{h+1})$, via the following formula:

$$\hat{\pi}(t_h, t_{h+1}) = \pi_0(t_{h-1}, t_h) \cdot \hat{\mathbf{P}}(t_{h-1}, t_h), \tag{21}$$

where $\pi_0(t_{h-1}, t_h)$ is a vector whose elements are all zero but the element corresponding to the values $S_P(t_{h-1})$ and $S_P(t_h)$ assumes at t_{h-1} and t_h, respectively, which is set equal to 1.

The state probability vector can be estimated via the following relationship:

$$\hat{\pi}_j(t_{h+1}) = \sum_{i=1}^{N} \hat{\pi}_{ij}(t_h, t_{h+1}),\ j = 1,\ 2,\ \ldots,\ N. \tag{22}$$

Finally, an estimate, $\hat{\pi}(t_{h+k})$, of the state probability vector $\pi(t_{h+k})$, for every k = 1,2,... can be obtained (on the basis of the same data) via the following formulas:

$$\hat{\pi}_j(t_{h+k}) = \sum_{i=1}^{N} \hat{\pi}_{ij}(t_{h+k-1}, t_{h+k}),\ j = 1, 2, \ldots, N, \tag{23}$$

where:

$$\hat{\pi}(t_{h+k-1}, t_{h+k}) = \pi_0(t_{h-1}, t_h) \cdot \left[\hat{\mathbf{P}}(t_{h-1}, t_h)\right]^k. \tag{24}$$

2.4. Predictor calibration

The term calibration in this section refers to the setting of the number of wind power classes, N, and the size of the sliding window, s_w, to use to implement the proposed FOMC and SOMC models.

Setting of N involves a tradeoff between model accuracy and model complexity. Indeed, for a rth-order Markov chain, the number of independent parameters to estimate for evaluating the transition matrix is $N^r(N^r - 1)$.

The parameter s_w corresponds to the amount of data it can be used to estimate the transition matrix. Nonetheless, as s_w increases, averaging over the sliding windows can produce poor estimates of the (non stationary) transition probabilities.

In this paper, calibration is performed setting N and s_w to the values that minimizes the k-step-ahead Normalized Root Mean Square Error, NRMSE [35]. Minimization is accomplished on the basis of predictions performed over an appropriately chosen time interval called training period.

For a given time horizon k, and for a given N, the prediction error is defined as:

$$e_{t_{h+k}|t_h} := S_P(t_{h+k}) - \hat{S}_P(t_{h+k}\,|t_h). \tag{25}$$

being $\hat{S}_P(t_{h+k}|t_h)$ the predictor used that will be defined in the following section.

The normalized prediction error, $\varepsilon_{t_{h+k}|t_h}$, is then computed as:

$$\varepsilon_{t_{h+k}|t_h} := \frac{1}{P_n} e_{t_{h+k}|t_h}, \tag{26}$$

where P_n is the Wind Farm Nominal Power.

Finally, the NRMSE (Normalized Root Mean Square Error) is defined as:

$$NMRSE(s_w, N) = \left[\frac{1}{L}\sum_{h=\max(s_w)}^{sw_M+L} \varepsilon^2_{t_{h+k}|t_h}(s_w, N)\right]^{1/2}, \tag{27}$$

where max(s_w) is the largest value considered for the sliding window length and $L = T_P - \max(s_w) - k + 1$ is the number of predictions performed over the training period (those used to compute the NMRSE).

In [29] a sensitivity analysis has been conducted to show how the optimal parameters of FOMC and SOMC have to be selected. The NRMSE obtained adopting FOMC and SOMC predictors versus the window size, for a time horizon of 2 h and different numbersof power classes, N, is reported and compared in Figs. 4 and 8 of [29]. The main outcomes of the analysis were the following:

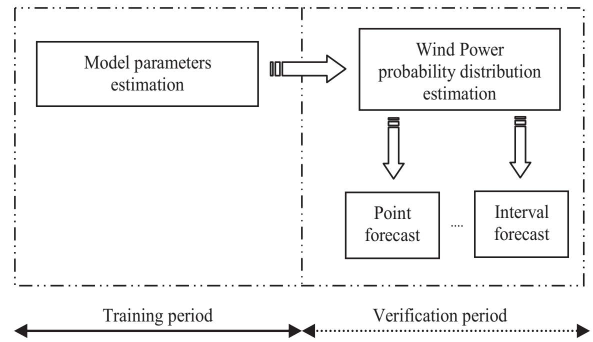

Fig. 4. Application of methods to determine point forecasts and interval forecasts.

- the number of power classes that allows to obtain a satisfying accuracy for both FOMC and SOMC was resulted to be equal to 102;
- the optimal size of the sliding window was resulted to be almost 30 days and 90 days for FOMC and SOMC, respectively.

3. Probability distribution estimation

The proposed method allows to directly obtain an estimate, $\hat{\pi}(t_{h+k})$, of the entire probability distribution $\pi(t_{h+k})$ (i.e., the state probability vector), of the wind power generated over the time interval $[t_{h+k-1}, t_{h+k}]$.

$\hat{\pi}(t_{h+k})$ is given by equation (17) in the case of FOMC and equation (23) in the case of SOMC.

The estimated probability distribution may then be used as such or to formulate point or interval conditional (i.e. given $S_P(t_h)$ in the case of FOMC and given $S_P(t_h)$ and $S_P(t_{h-1})$ in the case of SOMC) predictors.

In particular, the following conditional point predictors can be obtained:

Mean

$$\hat{S}_P(t_{h+k}|t_h)_{Mean} = \sum\nolimits_{i=1}^{N} s_i\hat{\pi}_i(t_{h+k}), \tag{28}$$

Mode

$$\hat{S}_P(t_{h+k}|t_h)_{Mode} = s_j : \max_{i=1,\ldots,N}\left[\hat{\pi}_i(t_{h+k})\right] = \hat{\pi}_j(t_{h+k}), \tag{29}$$

Quantile

$$\hat{S}_P(t_{h+k}|t_h)_\gamma = s_j : \begin{cases} \sum\nolimits_{i=1}^{j}\hat{\pi}_i(t_{h+k}) \geq \gamma \\ \sum\nolimits_{i=1}^{j-1}\hat{\pi}_i(t_{h+k}) < \gamma \end{cases} \quad 0 < \gamma < 1 \tag{30}$$

The conditional median prediction can be obtained from (30) setting γ = 0.5. The “k-step” point forecast, $\hat{S}_P(t_{h+k}\,|t_h)$, is generally set by equation (28). It can be shown that this estimator is optimal in the sense that it minimizes the root mean square prediction error over the power data set used to calibrate the model.

Starting from the estimated probability distribution it is also possible to formulate conditional (i.e. given $S_P(t_h)$ in the case of FOMC and given $S_P(t_h)$ and $S_P(t_{h-1})$ in the case of SOMC) interval predictors. An interval forecast $\hat{I}^{(\alpha)}_{t_{h+k}|t_h}$, estimated at time t_h for a time horizon t_{h+k} is the range of values within which the actual

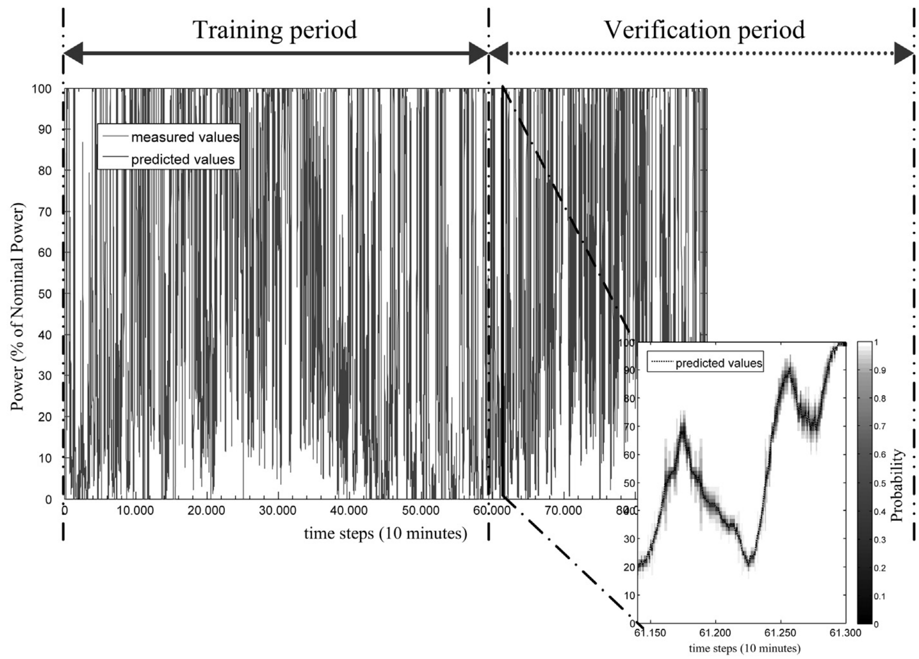

Fig. 5. Results obtained in terms of 1-step-ahead Wind Power point forecasts and interval forecasts centered on the point forecasts, in the case of FOMC with $N = 72$ and $ws = 4 \times 10^3$.

value $S_P(t_{h+k})$ is expected to lie with a certain probability $(1-\alpha)$, denoted as nominal coverage rate [36–38]:

$$Pr\left\{S_P(t_{h+k}) \in \hat{I}^{(\alpha)}_{t_{h+k}|t_h}\right\} = Pr\left\{S_P(t_{h+k}) \in \left[\hat{L}^{(\alpha)}_{t_{h+k}|t_h}, \hat{U}^{(\alpha)}_{t_{h+k}|t_h}\right]\right\} = 1-\alpha, \quad (31)$$

where $\hat{L}^{(\alpha)}_{t_{h+k}|t_h}$ and $\hat{U}^{(\alpha)}_{t_{h+k}|t_h}$ indicate, respectively, the lower and the upper bounds.

If prediction intervals are central interval forecasts, there is the same probability $(\alpha/2)$ to observe the future outcome lying below or above the interval bounds:

$$Pr\left(S_P(t_{h+k}) < \hat{L}^{(\alpha)}_{t_{h+k}|t_h}\right) = Pr\left(S_P(t_{h+k}) > \hat{U}^{(\alpha)}_{t_{h+k}|t_h}\right) = \alpha/2. \quad (32)$$

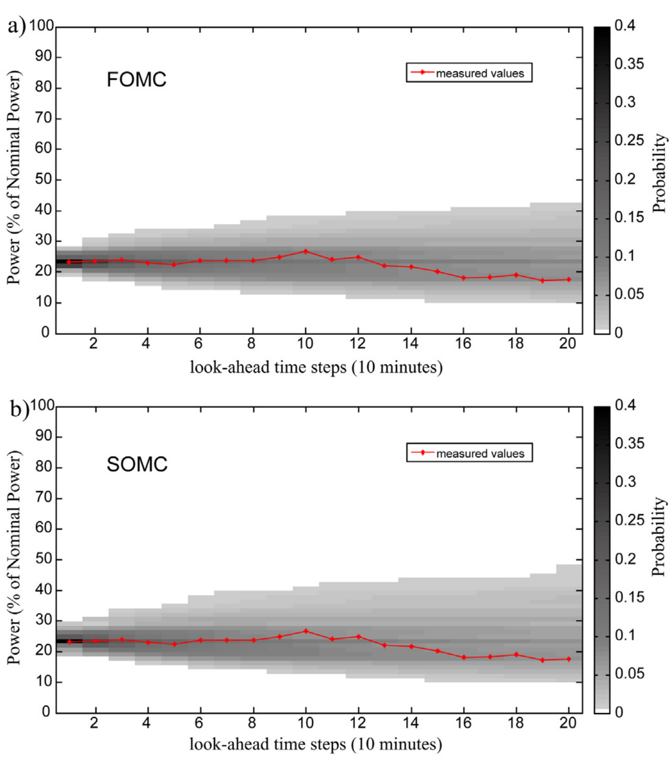

Fig. 6. Wind speed far from extreme values: Wind power measured values and $\hat{\pi}(t_{h+k})$ ($N = 72$) for $k = 1, \ldots, 20$ consecutive look-ahead time steps: (a) FOMC, (b) SOMC.

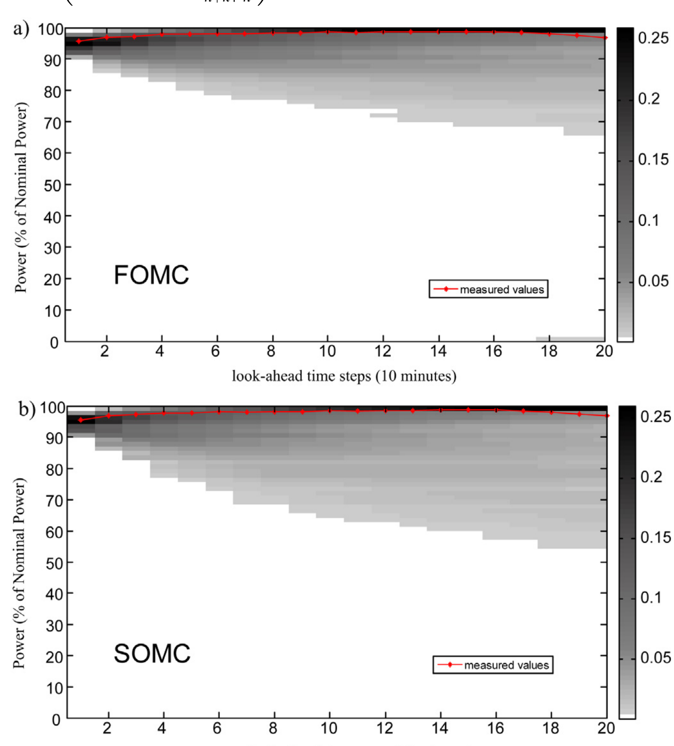

Fig. 7. Wind speed near to cut-out condition: Wind Power measured values and $\hat{\pi}(t_{h+k})$ ($N = 72$) for 20 consecutive look-ahead time steps: (a) FOMC, (b) SOMC.

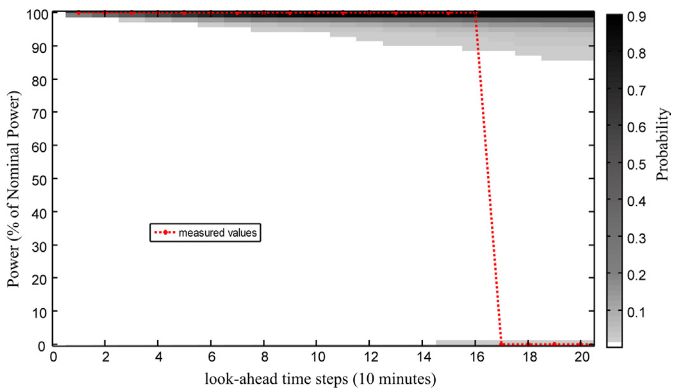

Fig. 8. Cut-out condition: Wind power measured values and $\hat{\pi}(t_{h+k})$ obtained by means of SOMC (N = 72) for 20 consecutive look-ahead time steps.

Interval forecasts can be alternatively constructed in the form of intervals $\hat{I}^{(\alpha)}_{Ct_{h+k}|t_h}$, which is centered (when it is possible) on the point forecast itself:

$$\hat{I}^{(\alpha)}_{Ct_{h+k}|t_h} = \left[\hat{L}^{(\alpha)}_{Ct_{h+k}|t_h}, \hat{U}^{(\alpha)}_{Ct_{h+k}|t_h}\right]. \tag{33}$$

In this case there is the same probability to observe the actual outcome lying in the interval forecasts below or above the point forecast:

$$\begin{aligned} &Pr\left\{S_P(t_{h+k}) \in \left[\hat{L}^{(\alpha)}_{Ct_{h+k}|t_h}, \hat{S}_P(t_{h+k}\,|t_h)\right]\right\} \\ &= Pr\left\{S_P(t_{h+k}) \in \left[\hat{S}_P(t_{h+k}\,|t_h), \hat{U}^{(\alpha)}_{Ct_{h+k}|t_h}\right]\right\} = (1-\alpha)/2. \end{aligned} \tag{34}$$

For a nonlinear and bounded process such as wind power generation, wind power probability distributions can be strongly skewed. For this reason point forecasts can result very close to wind power minimum or maximum values: (i) intervals (32) could not include the point forecast itself; (ii) it could be not possible to formulate intervals like (33) unless setting a very large α with small associated probabilities.

4. Application

The application of both proposed models (FOMC, SOMC) is illustrated in Fig. 4.

The dataset utilized is obtained from data reported in [39]. It refers to average values over 10 min measurements of wind power for 28 months. Datasets containing average values over smaller time intervals, i.e. 5 min or 1 min, will lead to more accurate results but currently the majority of the dataset available from measuring campaigns are taken on 10 min base. The available dataset has been divided in two parts: a training period, going from 01/01/2003 to 08/31/2004, and a verification period, going from 09/01/2004 to 05/01/2005.

In Fig. 5 some results are reported in terms of 1-step-ahead Wind Power point forecasts and interval forecasts centered on the point forecasts, obtained applying the FOMC model in the case of a number of power classes, N, equal to 72. The gray scale reported on the right side of the figure indicates the coverage rates associated to the interval forecasts.

In Fig. 6 the Wind Power measured values (red line) and the estimates of state probabilities vector (N = 72) made at time for 20 consecutive look-ahead time steps are reported for both FOMC (6.a) and SOMC (6.b) models in the case of wind speed far from extreme values. The state probabilities vector is represented indicating the probability associated to each of N values the state can assume, by means of a proper color. The adopted scale is reported on the right side of the figure. It is possible to observe, as expected, that the dispersion of the estimated probability distributions increases as k values increase. This reflects on the uncertainty associated to the obtainable point predictions (see (28) and (29)).

Fig. 7 is the analogous of Fig. 6 in the case of wind speed near to cut-out conditions. FOMC and SOMC results seem to reproduce correctly the abovementioned conditions. Moreover, Fig. 8 reports the results obtained with SOMC in cut-out conditions. It is possible to observe that the proposed model allows to predict the so-called cut-out event.

5. Conclusions

A Wind power forecasting method based on the use of discrete time Markov chain models has been developed starting from wind power time series analysis. It allows to directly obtain in an easy way an estimate of the wind power distributions on a very short-term horizon, without requiring restrictive assumptions on wind power probability distribution. First and Second Order Markov Chain Model have been analytically described. Finally, the application of the proposed method has been illustrated with reference to a set of real wind power data.

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