Problem Set -6.
Book 1.

9.2. $\sigma = 40$, $\bar{X} = 780$, $n = 30$. Find a 96% confidence interval for the population mean μ.

1) $\alpha = 1 - 0.96 = 0.04$

2) $1 - \frac{\alpha}{2} = 1 - \frac{0.04}{2} = 0.98$

3) $Z_{1-\alpha/2} = Z_{0.98} = 2.05$

4) Confidence Interval:

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 780 \pm (2.05)\left(\frac{40}{\sqrt{30}}\right) = 780 \pm 14.971$$

$(780 - 14.971, 780 + 14.971) = (765.03, 794.971)$

9.6. From Theorem 9.2:

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2 = \left(\frac{(2.05)(40)}{10}\right)^2 = 67.24 \approx 68$$

n - required sample size

$z_{\alpha/2}$ - critical value of the z-distribution

e - acceptable error (the maximum acceptable difference between the sample mean $\bar{X}$ and the true mean μ)

σ - standard deviation (of population)

α - significance level.

9.9. $n = \cancel{30}\ 20$, $\bar{X} = 11.3$, $S = 2.45$. Construct a 95% confidence interval for the mean μ.

1) $\alpha = 1 - 0.95 = 0.05$

2) $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

3) $t_{0.025} = 2.093$ for $v = \cancel{30}\ 20 - 1 = 19$ degrees of freedom.

4) Confidence Interval:

$$\bar{X} \pm t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} = 11.3 \pm (2.093)\left(\frac{2.45}{\sqrt{20}}\right) = 11.3 \pm 1.1470$$

$(11.3 - 1.1470, 11.3 + 1.1470) = (10.153, 12.447)$

9.11. $n = 9$. Find a 99% confidence interval for the mean μ.

1) $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{1.01 + 0.97 + 1.03 + 1.04 + 0.99 + 0.98 + 0.99 + 1.01 + 1.0...}{9}$

$= 1.005$

2) $s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \frac{(1.01 - 1.005)^2 + (0.97 - 1.005)^2 + \ldots + (1.03 - 1.005)^2}{8}$

$= 0.0006031$

$s = \sqrt{0.000603} = 0.025$

3) $\alpha = 1 - 0.99 = 0.01$

4) $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

5) $t_{0.005} = 3.355$ and $v = n-1 = 9-1 = 8$ degrees of freedom.

6) Confidence Interval:

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 1.005 \pm (3.355)\left(\frac{0.025}{\sqrt{9}}\right) = 1.005 \pm 0.028$

$(1.005 - 0.028,\ 1.005 + 0.028) = (0.977,\ 1.033)$

9.14. $n = 15$. Find a 95% prediction interval.

$x_i = \{3.4, 2.5, 4.8, 2.9, 3.6, 2.8, 3.3, 5.6, 3.7, 2.8, 4.4, 4.0, 5.2, 3.0, 4.8\}$

1) $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{2.5 + 2.8 + 2.8 + 2.9 + 3.0 + 3.3 + 3.4 + 3.7 + 3.6 + 4.0 + 4.4 + 4.8 + 4.8 + 5.2 + 5.6}{15} = \frac{56.8}{15} = 3.787$

2) $s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \frac{(2.5 - 3.787)^2 + (2.8 - 3.787)^2 + (2.8 - 3.787)^2 + (2.9 - 3.787)^2 + (3.0 - 3.787)^2 + (3.4 - 3.787)^2 + (3.6 - 3.787)^2 + (3.7 - 3.787)^2 + (\overset{4.0}{\cancel{3.6}} - 3.787)^2 + (4.4 - 3.787)^2 + (4.8 - 3.787)^2 + (4.8 - 3.787)^2 + (5.2 - 3.787)^2 + (5.6 - 3.787)^2}{14} = \frac{\cancel{296.4}}{\cancel{225}}$

$\approx 13.1973 \approx 0.9\cancel{2}5726 \approx 0.9427$

$s = \sqrt{0.9427} = 0.97$

3) $\alpha = 1 - 0.95 = 0.05$

4) $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

5) $t_{0.025} = 2.145$ and $v = n - 1 = 15 - 1 = 14$ degrees of freedom.

6) Prediction Interval:

$\bar{x} - t_{\alpha/2} s\sqrt{1+1/n} < x_0 < \bar{x} + t_{\alpha/2} s\sqrt{1+1/n}$

$3.8 - (2.145)(0.97)\sqrt{1+\frac{1}{15}} < x_0 < 3.8 + (2.145)(0.97)\sqrt{1+\frac{1}{15}}$

$3.8 - 2.08065\sqrt{1.067} < x_0 < 3.8 + 2.08065\sqrt{1.067}$

$3.8 - 2.150 < x_0 < 3.8 + 2.150$

$1.65 < x_0 < 5.95$

9.52. ~~$\bar{x}$~~ - $n = 100$, $X = 8$. Compute 95% confidence intervals, using 2 methods for the Large-Sample Confidence Intervals for p.

1) $\alpha = 1 - 0.95 = 0.05$

2) $\frac{\alpha}{2} = 0.025 \Rightarrow 1 - \frac{\alpha}{2} = 0.975$

3) $z_{0.025} = 1.96$

4) $\bar{p} = \frac{X}{n} = \frac{8}{100} = 0.08$ - the point estimate of p

5) Confidence Intervals:

$$\frac{\bar{p} + \frac{z_{\alpha/2}^2}{2n}}{1 + \frac{z_{\alpha/2}^2}{n}} - \frac{z_{\alpha/2}}{1 + \frac{z_{\alpha/2}^2}{n}}\sqrt{\frac{\bar{p}\bar{q}}{n} + \frac{z_{\alpha/2}^2}{4n^2}} < p < \frac{\bar{p} + \frac{z_{\alpha/2}^2}{2n}}{1 + \frac{z_{\alpha/2}^2}{n}} + \frac{z_{\alpha/2}}{1 + \frac{z_{\alpha/2}^2}{n}}\sqrt{\frac{\bar{p}\bar{q}}{n} + \frac{z_{\alpha/2}^2}{4n^2}}$$

$$\frac{0.08 + \frac{1.96^2}{(2)(100)}}{1 + \frac{1.96}{100}} \pm \frac{1.96}{1 + \frac{1.96^2}{100}}\sqrt{\frac{(0.08)(0.92)}{100} + \frac{1.96}{(4)(100)^2}} = 0.099 \pm 0.0528$$

$0.0462 < p < 0.1518$

9.54. $n = 500$, $X = 15$. Find a 90% confidence interval for the proportion.

1) $\bar{p} = \frac{X}{n} = \frac{15}{500} = 0.03$

2) $\bar{p}_{pass} = 1 - 0.03 = 0.97$

3) $\alpha = 1 - 0.9 = 0.1$

4) $\cancel{z_{0.1}} = \frac{\alpha}{2} = \frac{0.1}{2} = 0.05$

5) $z_{0.05} = 1.65$

6) Confidence Interval:

$$\bar{p} - z_{\alpha/2}\sqrt{\frac{\bar{p}\bar{q}}{n}} < p < \bar{p} + z_{\alpha/2}\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$0.97 - 1.65\sqrt{\frac{(0.97)(0.03)}{500}} < p < 0.97 + 1.65\sqrt{\frac{(0.97)(0.03)}{500}}$$

$$0.97 - 0.0125 < p < 0.97 + 0.0125$$

$$0.9575 < p < 0.9825$$

Book 2.

Chapter 7.

8. $\bar{x} = ?$, $\sigma = 0.1$, $n = 5$

$x_i = \{3.141, 3.142, 3.150, 3.155, 3.163\}$

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{3.141 + 3.142 + 3.150 + 3.155 + 3.163}{5} = \frac{15.751}{5} =$$

$= 3.1502$

(a) Determine a 95% confidence interval:

1) $\alpha = 1 - 0.95 = 0.05$

2) $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

3) $z_{0.025} = 1.96$

4) Confidence Interval:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 3.1502 \pm (1.96)\frac{(0.1)}{\sqrt{5}} = 3.1502 \pm 0.088$$

$(3.1502 - 0.088, 3.1502 + 0.088) = (3.0622, 3.2382)$

(b) Determine a 99% confidence interval:

1) $\alpha = 1 - 0.99 = 0.01$

2) $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

3) $z_{0.005} = 2.58$

4) Confidence Interval:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 3.1502 \pm (2.58)\frac{(0.1)}{\sqrt{5}} = 3.1502 \pm 0.1154$$

$(3.1502 - 0.1154, 3.1502 + 0.1154) = (3.0348, 3.2656)$

19. $n=9$, $\bar{x} = \$222,000$, $S = \$22,000$. Give a 95% upper confidence interval for the mean μ.

1) $\alpha = 1 - 0.95 = 0.05$

2) $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

3) $t_{0.025} = \overset{1.86}{\cancel{2.306}}$ and $\nu = n-1 = 9-1 = 8$ degrees of freedom.

4) Upper confidence interval:

$$\left(\bar{x} - t_{\alpha/2, n-1}\frac{S}{\sqrt{n}}, \infty\right) = \left(222,000 - \cancel{(22,000)}(2.306)\frac{(22,000)}{\sqrt{9}}, \infty\right) =$$

$$= (222,000 - 16910.\cancel{7}6\cancel{7}, \infty) = (205089.4, \infty)$$

$$\left(\bar{x} - t_{\alpha, n-1}\frac{S}{\sqrt{n}}, \infty\right) = \left(222,000 - (1.86)\frac{(22,000)}{\sqrt{9}}, \infty\right) =$$

$$= (222,000 - 13639.9\cancel{7}, \infty) = (208360, \infty)$$

18. $n = 18$, $\bar{x} = 133.22$, $S = 10.2128$

1) $$\bar{x} = \sum_{i=1}^{n}\frac{x_i}{n} = \frac{118+119+120+122+127+127+129+130+132+133+136+137+141+142+141+142+150+152}{18} =$$

$$= \frac{2398}{18} = 133.22$$

$$S^2 = \sum_{i=1}^{n}\frac{(x_i - \bar{x})^2}{n-1} = \frac{(118-133.22)^2 + \ldots (152-133.22)^2}{17}$$

2) $S = 10.2128$

((a) 95% confidence interval)

3) $\alpha = 1 - 0.95 = 0.05$

4) $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

5) $t_{0.025} = \overset{2.11}{\cancel{2.306}}$ and $\nu = n-1 = 18-1 = 17$

(a) 95% Confidence interval:

$$\bar{x} \pm t_{\alpha/2, n-1}\frac{S}{\sqrt{n}} = 133.22 \pm (2.11)\frac{(10.2128)}{\sqrt{18}} = 133.22 \pm 5.079$$

$$(133.22 - 5.079,\ 133.22 + 5.079) = (128.141,\ 138.299)$$

(b) 95% lower confidence interval:

1) $t_{0.05} = 1.74$ and $\nu = n-1 = 18-1$ degrees of freedom

2) $\left(-\infty, \bar{X} + t_{\alpha, n-1} \frac{S}{\sqrt{n}}\right)$ ~~$(\bar{X} -$~~

$\left(-\infty, 133.22 + (1.74)\frac{(10.2128)}{\sqrt{18}}\right) = (-\infty, 137.4)$

(c) 95% upper confidence interval:

$\left(\bar{X} - t_{\alpha, n-1}\frac{S}{\sqrt{n}}, \infty\right) =$

$\left(133.22 - (1.74)\frac{(10.2128)}{\sqrt{18}}\right) = (130.8, \infty)$

41. $n = 10$

(a) Two-sided

$\left(\bar{X}_1 - \bar{X}_2 - t_{0.025, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \bar{X}_1 - \bar{X}_2 + t_{0.025, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$

$\left(3358.1 - 3130.4 - 2.101(266.6)\sqrt{\frac{1}{10} + \frac{1}{10}}, 3358.1 - 3130.4 + \right.$

$\left. + 2.101(266.6)\sqrt{\frac{1}{10} + \frac{1}{10}}\right) = (22.8, 478.2)$

(b) One-sided upper:

$\left(\bar{X}_1 - \bar{X}_2 - t_{0.05, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \infty\right)$

$\left(3358.1 - 3130.4 - 1.734(266.6)\sqrt{\frac{1}{10} + \frac{1}{10}}, \infty\right) = (20.96, \infty)$

(c) One-sided lower:

$\left(-\infty, \bar{X}_1 - \bar{X}_2 + t_{0.05, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$

$\left(-\infty, 3358.1 - 3130.4 + 1.734(266.6)\sqrt{\frac{1}{10} + \frac{1}{10}}\right) = (-\infty, 434.44)$