


MATH 1: INTRODUCTION TO CALCULUS MIDTERM EXAM #2

Name: _____

Section (please circle):  10 Winkeler  11 Tripp  12 Chen  2 Xiao

- (1) Write your name *legibly* and circle your section above.
- (2) This exam is closed book. You may not use notes, calculators, or any other external resource. It is a violation of the honor code to give or receive help on this exam.
- (3) You must explain what you are doing, justify your answer, and show your work. You will be *graded on your work*, not just on your answer. Please write clearly.
- (4) It is fine to leave your answer in a form such as $14\sqrt{239}$. However, if an expression can be easily simplified (such as $\cos(\pi/2)$ or $2 + 3$), please simplify it.
- (5) If you use the backside of a page or the scratch paper at the end of the exam, and you want us to look at it, please write on a problem “Continued on back/page...”.

Problem	Score	Problem	Score
1	/10	6	/6
2	/8	7	/12
3	/8	8	/8
4	/10		
5	/12		
		Total	/74

 **Problem 1.**[10 points] Mark the following statements as true or false.

_____ $f'(a)$ is the slope of the tangent line to $f(x)$ at a .

_____ $g(x) = \frac{1}{x}$ is continuous on its domain.

_____ $2, 4, 6, 8, 10, 12, \dots$ is a geometric sequence.

_____ The sequence $a_n = \sin(n)$ is bounded.

_____ $\lim_{x \rightarrow 0} \ln x = -\infty$.

_____ If $h(x) = 2x$, then $h'(x) = x^2$.

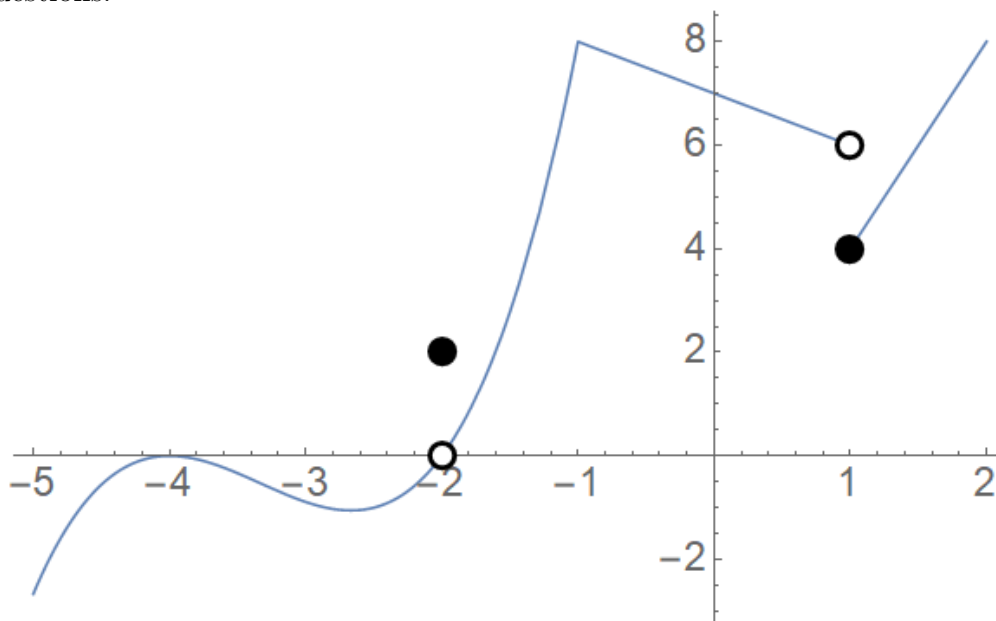
_____ If $j(x)$ is a continuous function such that $j(0) = 0$ and $j(1) = 1$, then $j(\frac{1}{2}) = \frac{1}{2}$.

_____ A monotonic sequence is either bounded above or bounded below.

_____ The value of a continuous function cannot approach ∞ as $x \rightarrow \infty$.

_____ The derivative of $g(x) = |x|$ at $x = 0$ is 0.

🔗 **Problem 2.** [8 points] Use the graph of $y = f(x)$ below to answer the following questions.



(a) Find the following limits. Write DNE if the limit does not exist.

(i) $\lim_{x \rightarrow -4^+} f(x) = \underline{\hspace{2cm}}$

(ii) $\lim_{x \rightarrow -2} f(x) = \underline{\hspace{2cm}}$

(iii) $\lim_{x \rightarrow -1} f(x) = \underline{\hspace{2cm}}$

(iv) $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$

(v) $\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$

(b) At what values of x is $f(x)$ not differentiable?

🐧 **Problem 3.** [8 points]

(a) Use the Squeeze Theorem to find $\lim_{x \rightarrow 0} x^2 \sin(\ln|x|)$

(b) Use the Intermediate Value Theorem to show that $f(x) = x + 2^x$ has a zero.

~~3.~~ **Problem 4.** [10 points]

(a) State the three requirements for $f(x)$ to be continuous at a point a .

(b) List the three types of discontinuities a function can have.

(c) Find and classify all discontinuities of $f(x) = \frac{x+1}{x^2-1}$.

 **Problem 5.** [12 points] Consider the piecewise function

$$f(x) = \begin{cases} x + 2, & x < 2 \\ a, & x = 2 \\ \frac{c(x^2 - b^2)}{x - 2}, & x > 2 \end{cases}.$$

where a, b and c are some constants.

(a) Let $c = 1$. Solve for a and b so that this function is continuous everywhere.

(b) Let $a = 4, b = 3, c = 1$, so we get

$$f(x) = \begin{cases} x + 2, & x < 2 \\ 4, & x = 2 \\ \frac{(x^2 - 9)}{x - 2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

(c) Let $a = 3$, $b = 2$, $c = 1$, so we get


$$f(x) = \begin{cases} x + 2, & x < 2 \\ 3, & x = 2 \\ \frac{(x^2-4)}{x-2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

(d) Let $a = 1$, $b = 2$, $c = 3$, so we get

$$f(x) = \begin{cases} x + 2, & x < 2 \\ 1, & x = 2 \\ \frac{3(x^2-4)}{x-2}, & x > 2 \end{cases}.$$

Is $f(x)$ continuous at $x = 2$? If not, what type of discontinuity does it have?

 **Problem 6.** [6 points]

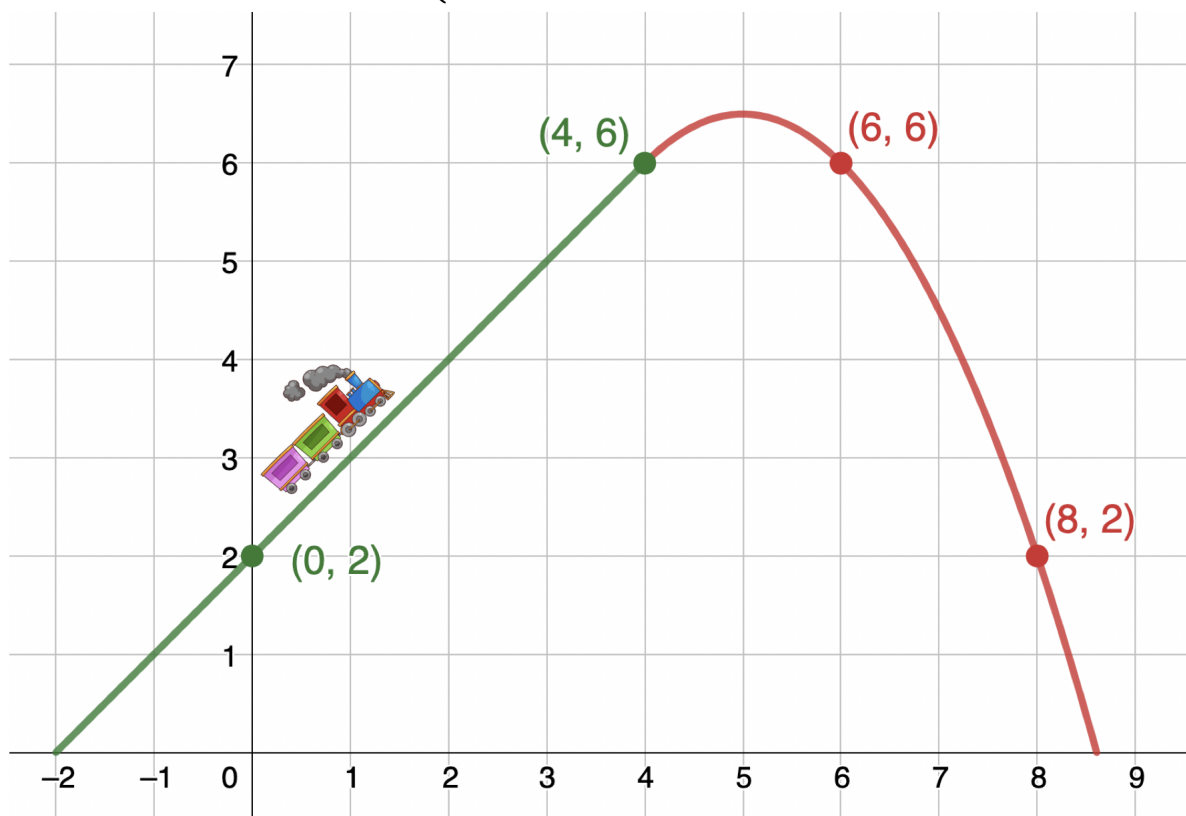
(a) Suppose $\lim_{x \rightarrow 2} f(x) = 3$. Use the algebraic limit laws to compute $\lim_{x \rightarrow 2} \sqrt{10 - (f(x))^2}$.

(b) Suppose $\lim_{x \rightarrow 3} g(x) = 4$ and $\lim_{x \rightarrow 3} h(x) = 1$. Use the algebraic limit laws to compute $\lim_{x \rightarrow 3} \frac{4g(x)}{1+h(x)}$.

(c) Suppose $\lim_{x \rightarrow 2^-} s(x) = -3$ and $\lim_{x \rightarrow 2^+} s(x) = 3$. Use the algebraic limit laws to compute $\lim_{x \rightarrow 2} s(x)$ and $\lim_{x \rightarrow 2} (s(x)^2)$.

Problem 7. [12 points] As you may still remember from the First Midterm, the figure below shows a track of the Green-Red Mountain. Suppose the figure depicts the entire mountain, modeled by the function:

$$f(x) = \begin{cases} x + 2, & x \leq 4 \\ -\frac{1}{2}x^2 + 5x - 6 & x > 4 \end{cases}$$



- (a) Let $f(x)$ be a function. State the limit definition of the derivative function $f'(x)$.

- (b) Compute the derivative of $x+2$ at $x=4$ and the derivative of $-\frac{1}{2}x^2+5x-6$ at $x=4$ using the limit definition. Verify that they are the same, and the track smoothly transitions from the green to the red. [Hint: compute the derivative function and plug in, as the derivative function will be used again in the next part.]

- (c) Engineers are working to decide if it is safe for a train to travel on the mountain or if they need to build a tunnel through the mountain. Compute the slope of the tangent line to the mountain at the foot of the mountain, the point $(8,2)$, using the limit definition. If the absolute value of the slope is greater than 2, then it is too dangerous and the engineers need to build a tunnel. Do they need to build a tunnel? [Work must be shown to receive credit on this problem.]

☺ **Problem 8.** [8 points]

(a) For each of the following sequences, state whether they are arithmetic, geometric, neither, or both. Write down an **explicit** formula for each.

(i) $\{a_n\} = \{3, 7, 11, 15, 19, 23, 27, \dots\}$

(ii) $\{b_n\} = \{2, \frac{3}{7}, \frac{4}{49}, \frac{5}{343}, \frac{6}{2401}, \dots\}$ (hint: $7 = 7^1$, $49 = 7^2$, $343 = 7^3$, $2401 = 7^4$)

(b) Suppose $c_n = \frac{3n^3+n-100}{2n^2-23n}$. Evaluate $\lim_{n \rightarrow \infty} c_n$.

(c) Give an example of a sequence that is monotone but is not convergent.

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