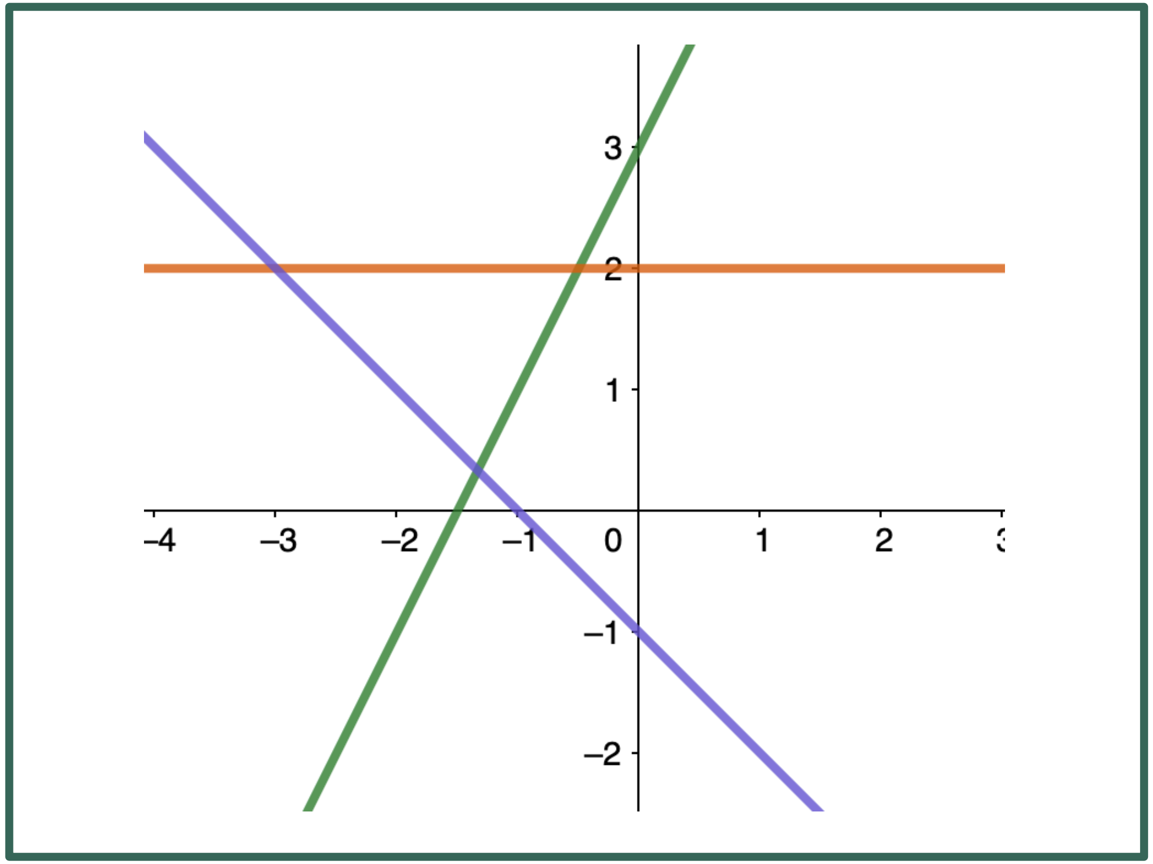
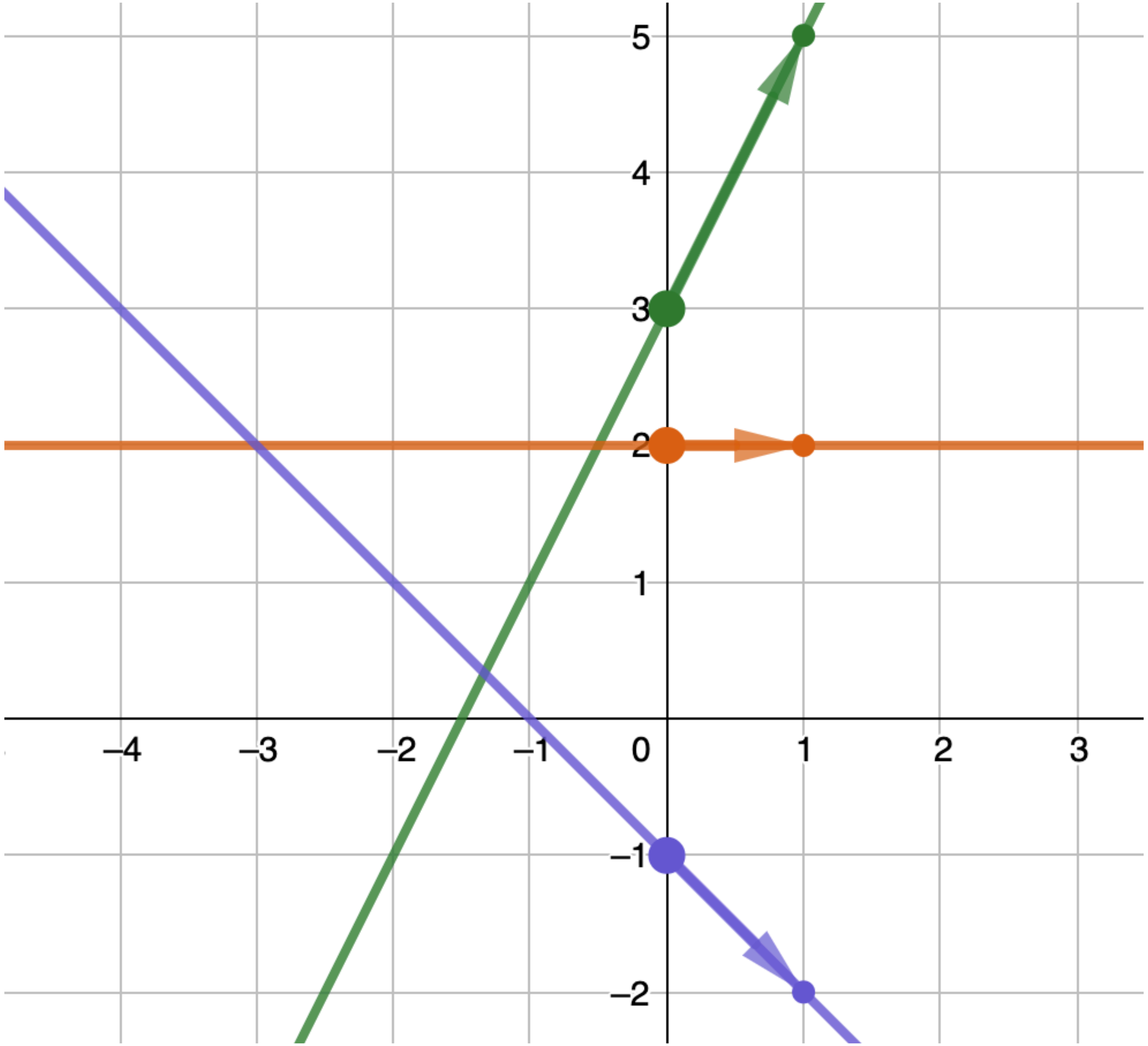
LIBRARY OF FUNCTIONS

MATH 1: INTRODUCTION TO CALCULUS

LINEAR FUNCTIONS



- $f(x) = ax + b$
  - $f(x) = 2x + 3$
  - $f(x) = 2$
  - $f(x) = -x - 1$



LINEAR FUNCTION: SLOPE

THE SLOPE IS THE CHANGE OF y FOR EACH UNIT CHANGE IN x.

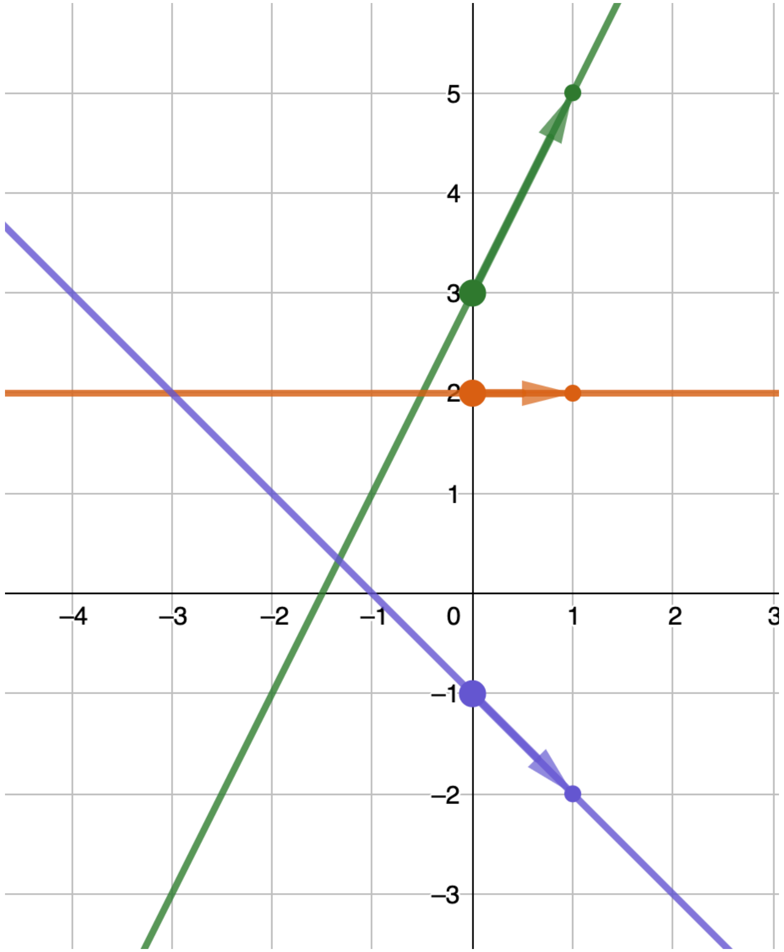
LINEAR FUNCTION: SLOPE

DEFINITION

Consider line L passing through points (x_1, y_1) and (x_2, y_2). Let $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$ denote the changes in y and x, respectively. The **slope** of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}. \qquad \boxed{1.3}$$

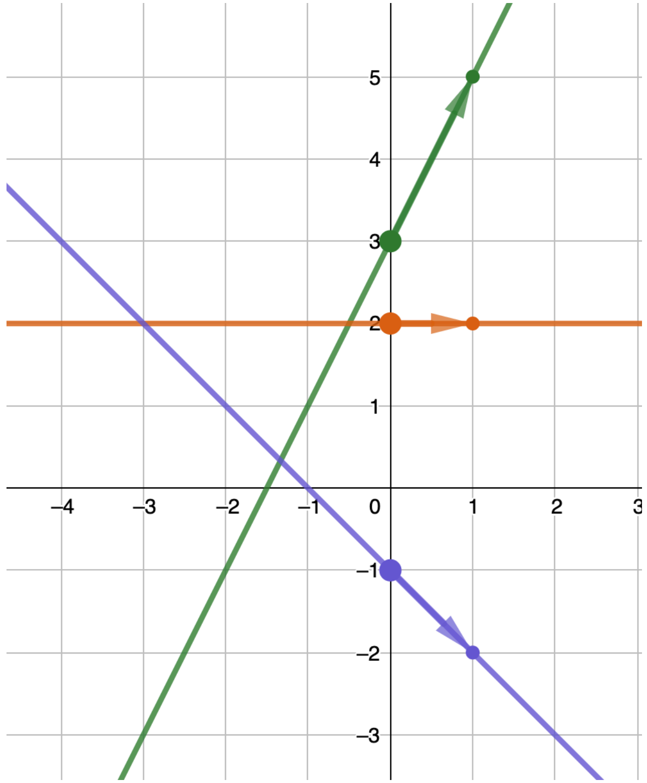
LINEAR FUNCTION: SLOPE-INTERCEPT FORM

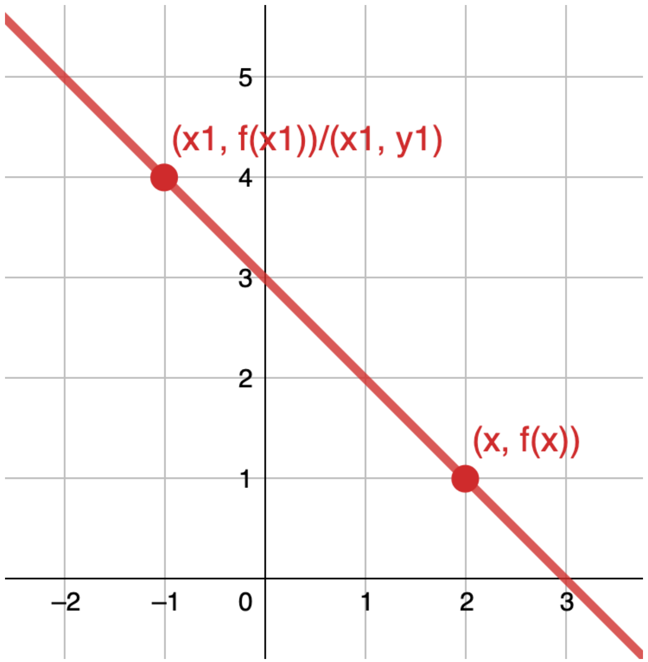


- $\boldsymbol{f(x) = mx + b}$
  - $\begin{cases} m > 0 \\ m = 0 \\ m < 0 \end{cases}$
- How to prove that m is the slope?
- We now know that m is the slope, what can we say about b? $(0, b)$?

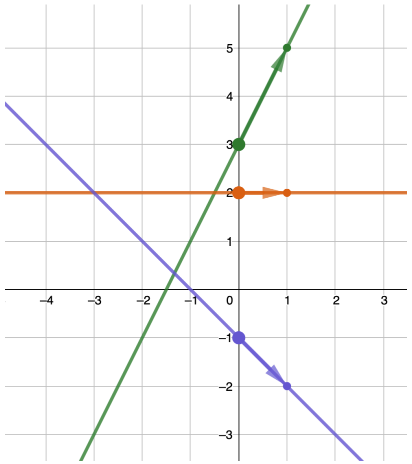
LINEAR FUNCTION: POINT-SLOPE EQUATION

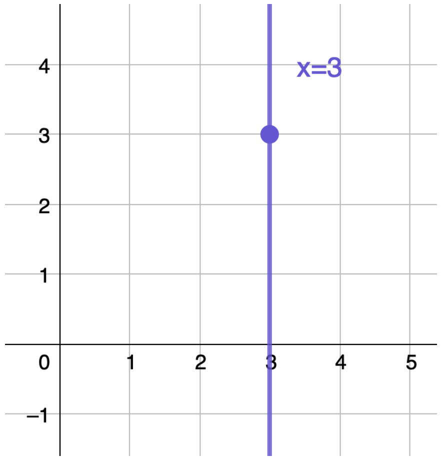
- Point
  - (x_1, y_1)
- Slope
  - m
- Equation?
  - $f(x) - y_1 = m(x - x_1)$





LINEAR FUNCTION: STANDARD FORM OF A LINE





- A vertical line
  - $x = k$
- Generalization
  - $\boldsymbol{ax + by = c}$

LINEAR FUNCTION: THREE FORMS

DEFINITION

Consider a line passing through the point (x_1, y_1) with slope m. The equation

$$y - y_1 = m(x - x_1) \tag{1.4}$$

is the **point-slope equation** for that line.

Consider a line with slope m and y-intercept $(0, b)$. The equation

$$y = mx + b \tag{1.5}$$

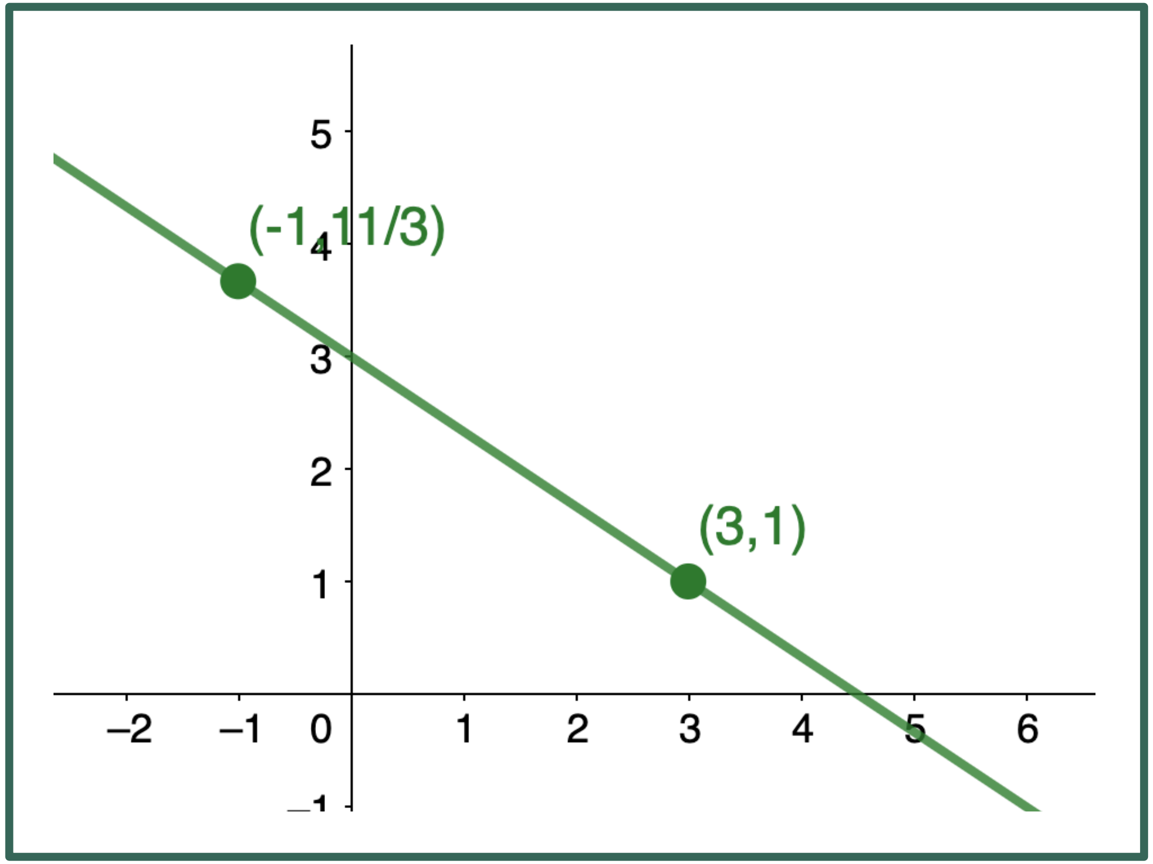
is an equation for that line in **slope-intercept form**.

The **standard form of a line** is given by the equation

$$ax + by = c, \tag{1.6}$$

where a and b are both not zero. This form is more general because it allows for a vertical line, $x = k$.

LINEAR FUNCTION: EXAMPLES



- Consider the line passing through points $(3,1)$ and $(-1,\frac{11}{3})$.
- Find an equation of that line in
  - point-slope form.
  - slope-intercept form.
  - standard form.

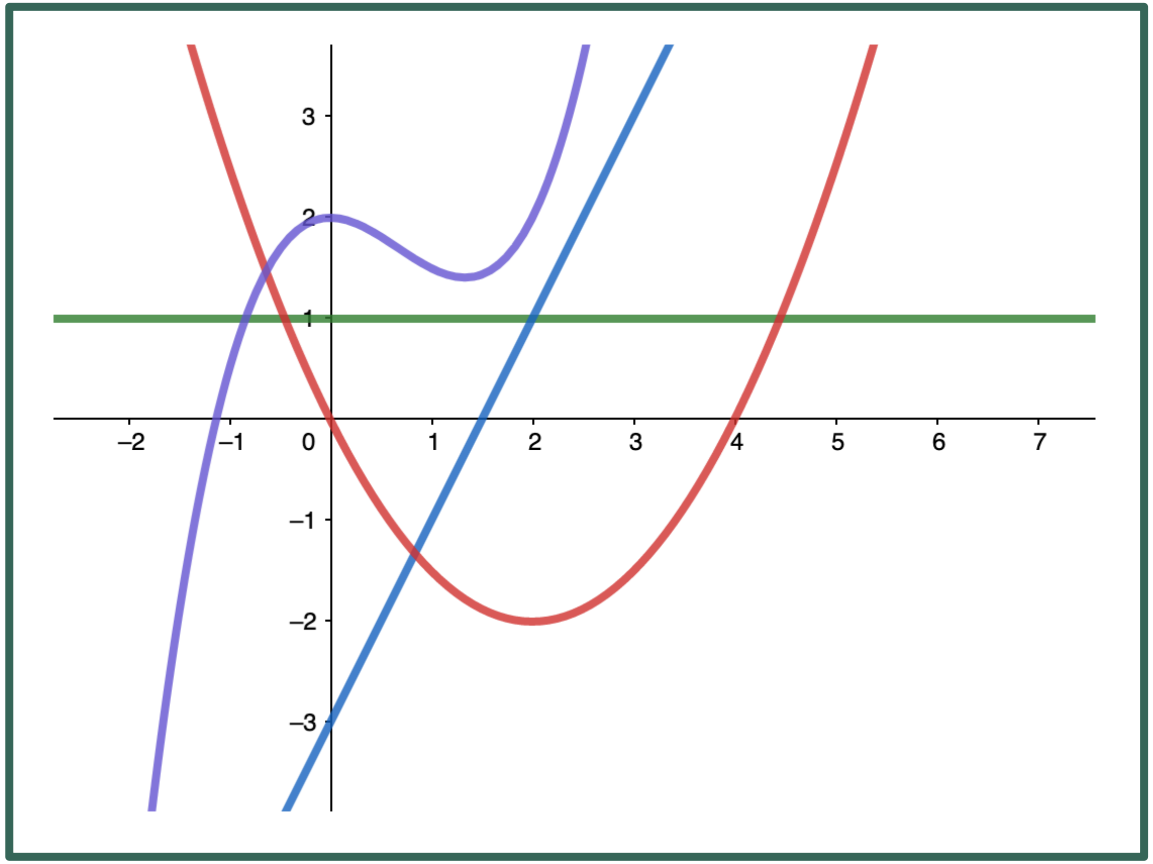
POLYNOMIALS

- A linear function is a special type of a more general class of functions: polynomials.
- A **polynomial function** is any function that can be written in the form

$$\boldsymbol{f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}$$

- for some integer $n \geq 0$ and constants a_n (the **leading coefficient**), $a_{n-1}, \cdots, a_0$, where $a_n \neq 0$.
- In the case when $n = 0$, we allow for $a_0 = 0$. (If $a_0 = 0$, the function $f(x) = 0$ is called the **zero function**.)

POLYNOMIALS: DEGREE



$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Degree of the polynomial: n

- Constant function: $n = 0$
- Linear function: $n = 1$
- Quadratic function: $n = 2$
  - $f(x) = ax^2 + bx + c$
- Cubic function: $n = 3$
  - $f(x) = ax^3 + bx^2 + cx + d$

POLYNOMIALS VERSUS POWER FUNCTIONS

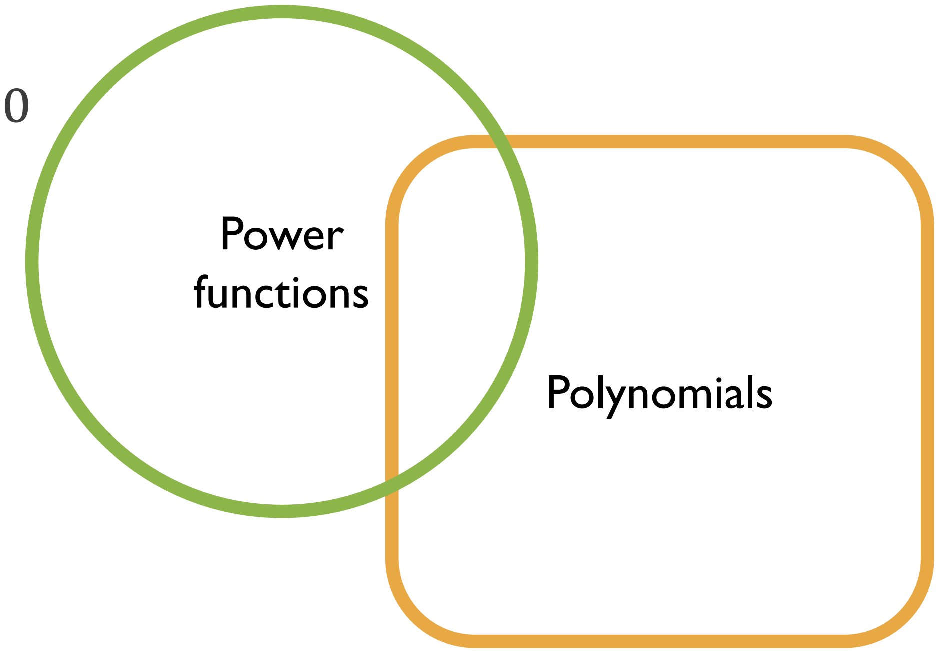
- Polynomials

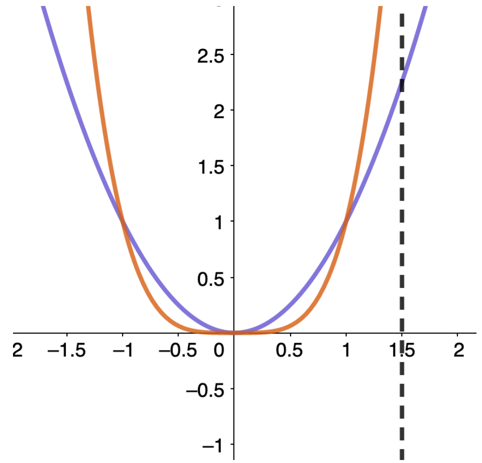
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

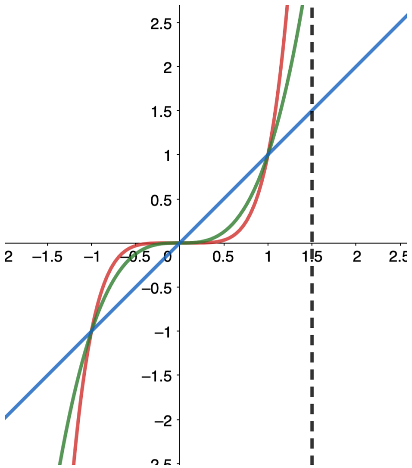
- Power functions

$$f(x) = a x^b$$

- where a and b are any real numbers.
- Intersection?
  - where the exponent is a nonnegative integer.





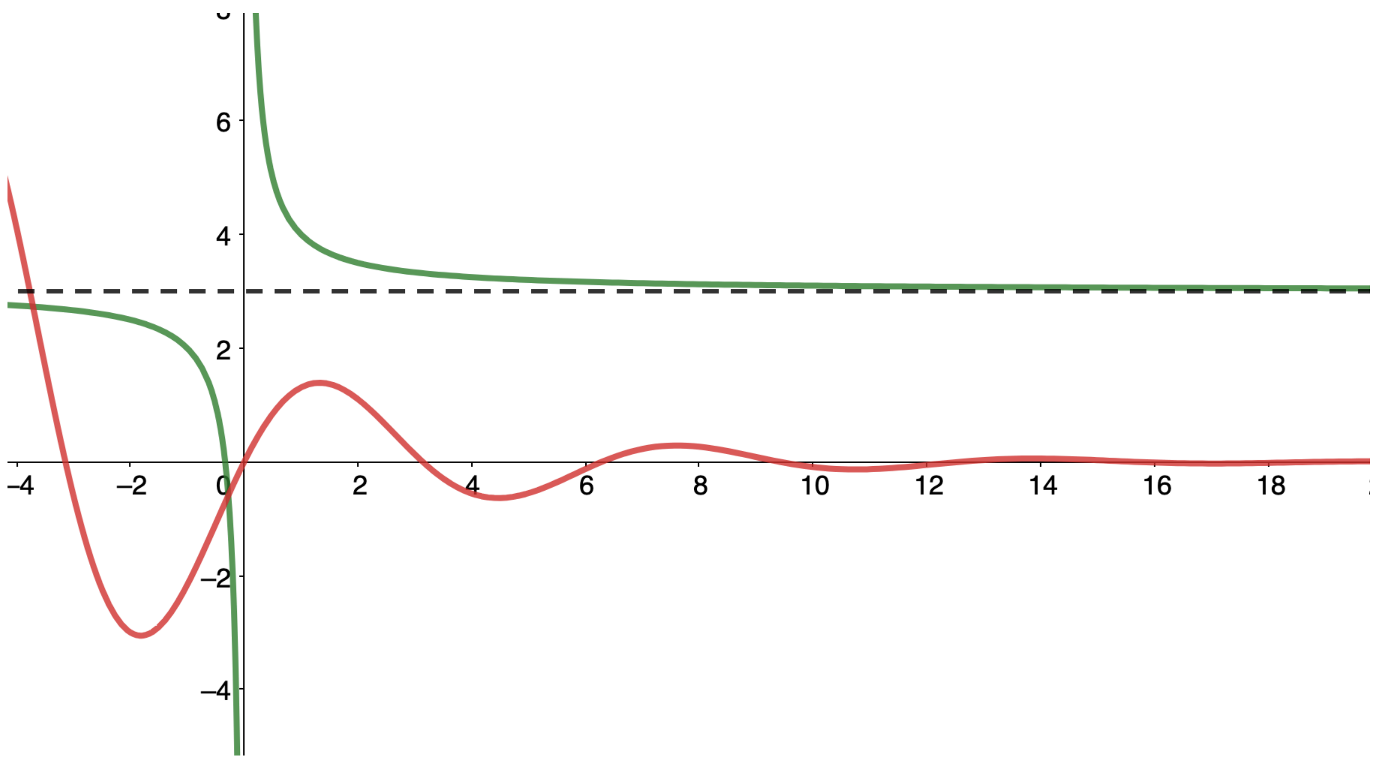


REVISIT ODD AND EVEN FUNCTIONS

- $f(x) = ax^n$, where n is a positive integer.
- x^2, x^4
- x, x^3, x^5
- If n is ___, then $f(x)$ is an ___ function (magic!).

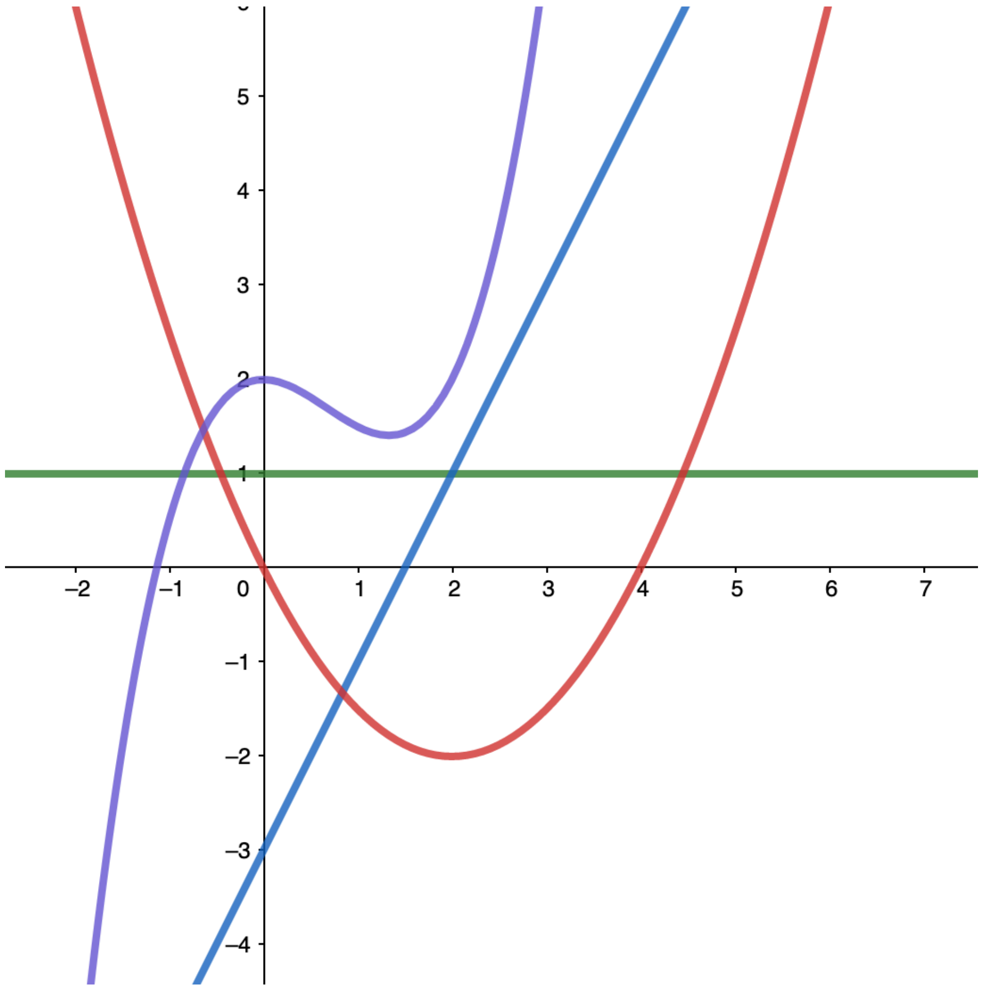
BEHAVIOR AT INFINITY

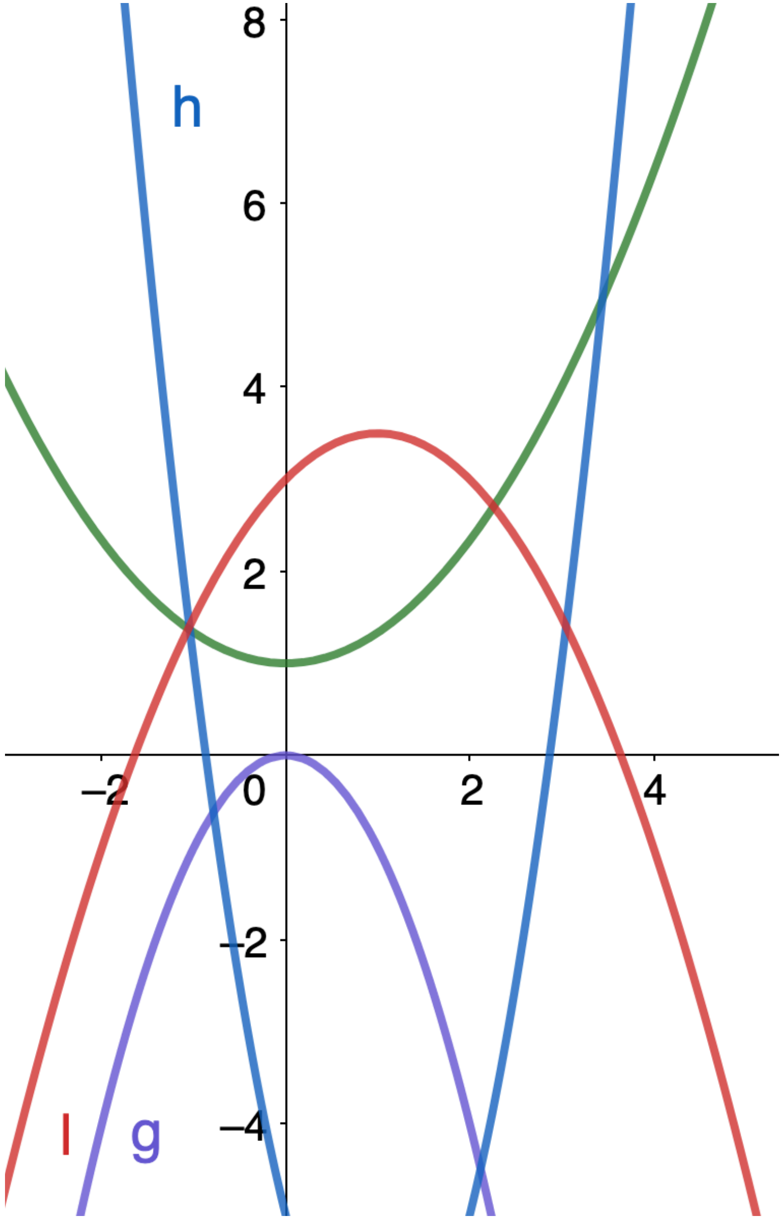
- When x goes to infinity
  - $x \to \infty$
- $f(x)$ approaches ___ as x goes to infinity.
- $f(x) = \frac{1}{x} + 3$
  - $f(x) \to 3$ as $x \to +\infty$
- $f(x) = 2e^{-\frac{x}{4}}\sin(x)$
  - $f(x) \to 0$ as $x \to +\infty$



POLYNOMIALS: BEHAVIOR AT INFINITY (THE END BEHAVIOR)

- $f(x)$ approaches infinity as x goes to infinity?





POLYNOMIALS: BEHAVIOR AT INFINITY (THE END BEHAVIOR)

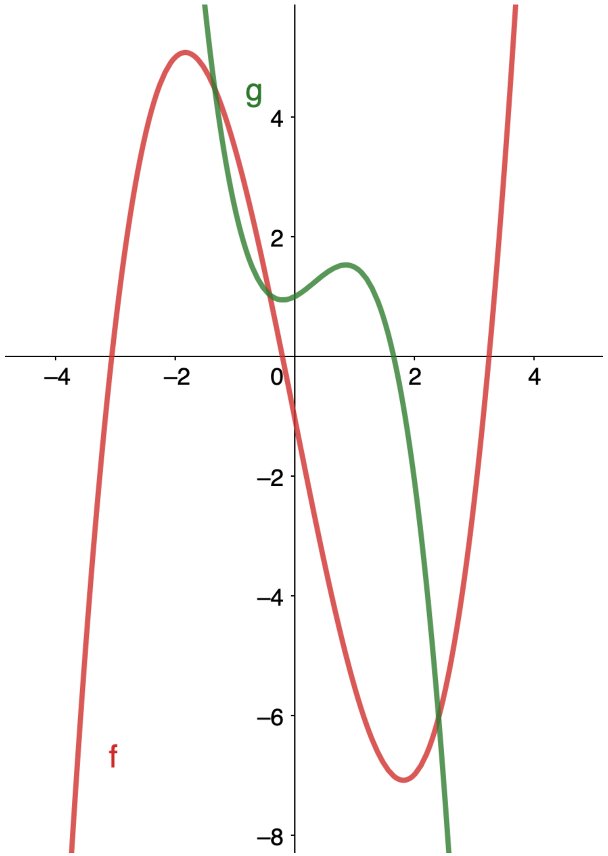
- Quadratic function $ax^2 + bx + c$

$$f(x) = \frac{1}{3}x^2 + 1, g(x) = -x^2$$

$$h(x) = 2x^2 - 4x - 5, l(x) = -\frac{1}{2}x^2 + x + 3$$

- If $a > 0$
  - the parabola opens upward
  - $f(x) \to \infty$ as $x \to \pm\infty$
- if $a < 0$
  - the parabola opens downward
  - $f(x) \to -\infty$ as $x \to \pm\infty$
- **The leading term of the polynomial determines the end behavior!**

POLYNOMIALS: BEHAVIOR AT INFINITY (THE END BEHAVIOR)



- Cubic function $ax^3 + bx^2 + cx + d$

$f(x) = \frac{1}{2}x^3 - 5x - 1, g(x) = -x^3 + x^2 + \frac{1}{2}x + 1$

- If $a > 0$
  - $f(x) \to +\infty$ as $x \to +\infty$
  - $f(x) \to -\infty$ as $x \to -\infty$
- If $a < 0$
  - $f(x) \to -\infty$ as $x \to +\infty$
  - $f(x) \to +\infty$ as $x \to -\infty$
- **The leading term of the polynomial determines the end behavior!**

POLYNOMIALS: BEHAVIOR AT INFINITY (THE END BEHAVIOR)

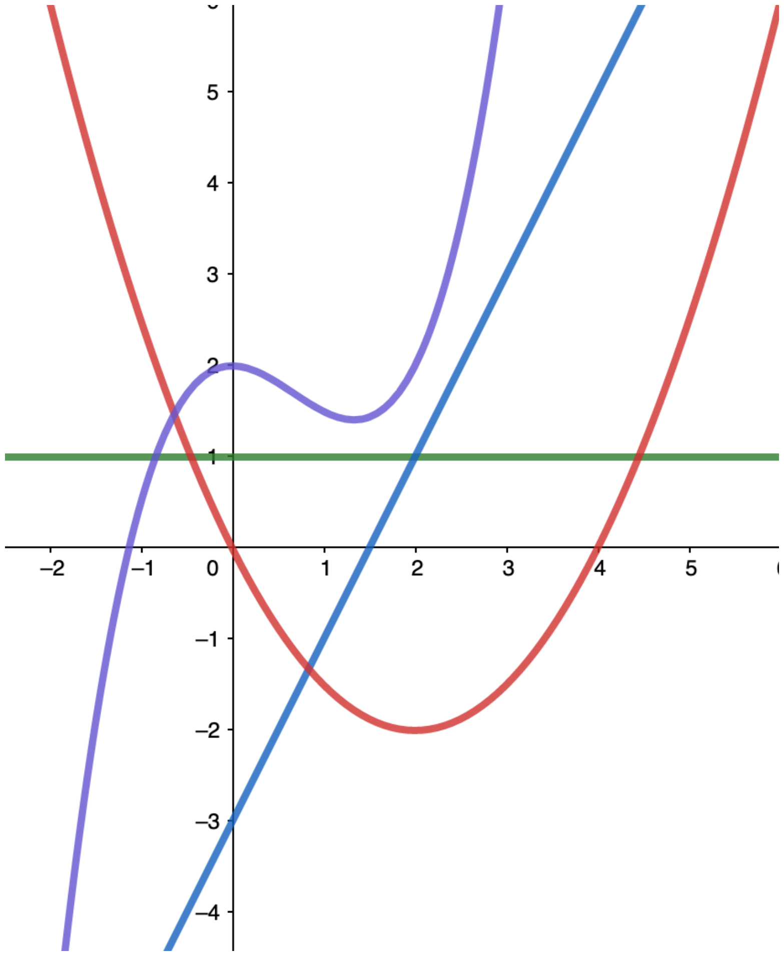
The leading term of the polynomial determines the end behavior!

- $f(x) = \frac{1}{2019}x^4 - 2019x - 2019$
  - $f(x) \to +\infty$ as $x \to +\infty$
  - $f(x) \to +\infty$ as $x \to -\infty$

- $f(x) = -\frac{1}{1984}x^{2019} + 1984x + 1984$
  - $f(x) \to -\infty$ as $x \to +\infty$
  - $f(x) \to +\infty$ as $x \to -\infty$

POLYNOMIALS: ZEROS

- $f(x) = k$
- $f(x) = mx + b$
- $f(x) = ax^2 + bx + c$
- $f(x) = ax^3 + bx^2 + cx + d$
- $\ldots$



Consider the quadratic equation

$$ax^2 + bx + c = 0,$$

where $a \neq 0$. The solutions of this equation are given by the quadratic formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.} \tag{1.8}$$

If the discriminant $b^2 - 4ac > 0$, this formula tells us there are two real numbers that satisfy the quadratic equation. If $b^2 - 4ac = 0$, this formula tells us there is only one solution, and it is a real number. If $b^2 - 4ac < 0$, no real numbers satisfy the quadratic equation.

QUADRATIC FUNCTIONS: ZEROS

CUBIC FUNCTIONS OR HIGHER DEGREE POLYNOMIALS

- The cubic formula
  - In algebra, the **Abel–Ruffini theorem** (also known as **Abel's impossibility theorem**) states that there is no solution in radicals to general polynomial equations of degree five or higher with arbitrary coefficients.
- Factor theorem
- …

ALGEBRAIC FUNCTIONS

algebraic function

addition, subtraction, multiplication, division, rational powers, and roots

$\sqrt[3]{x-5}$

rational function

$f(x) = p(x)/q(x)$

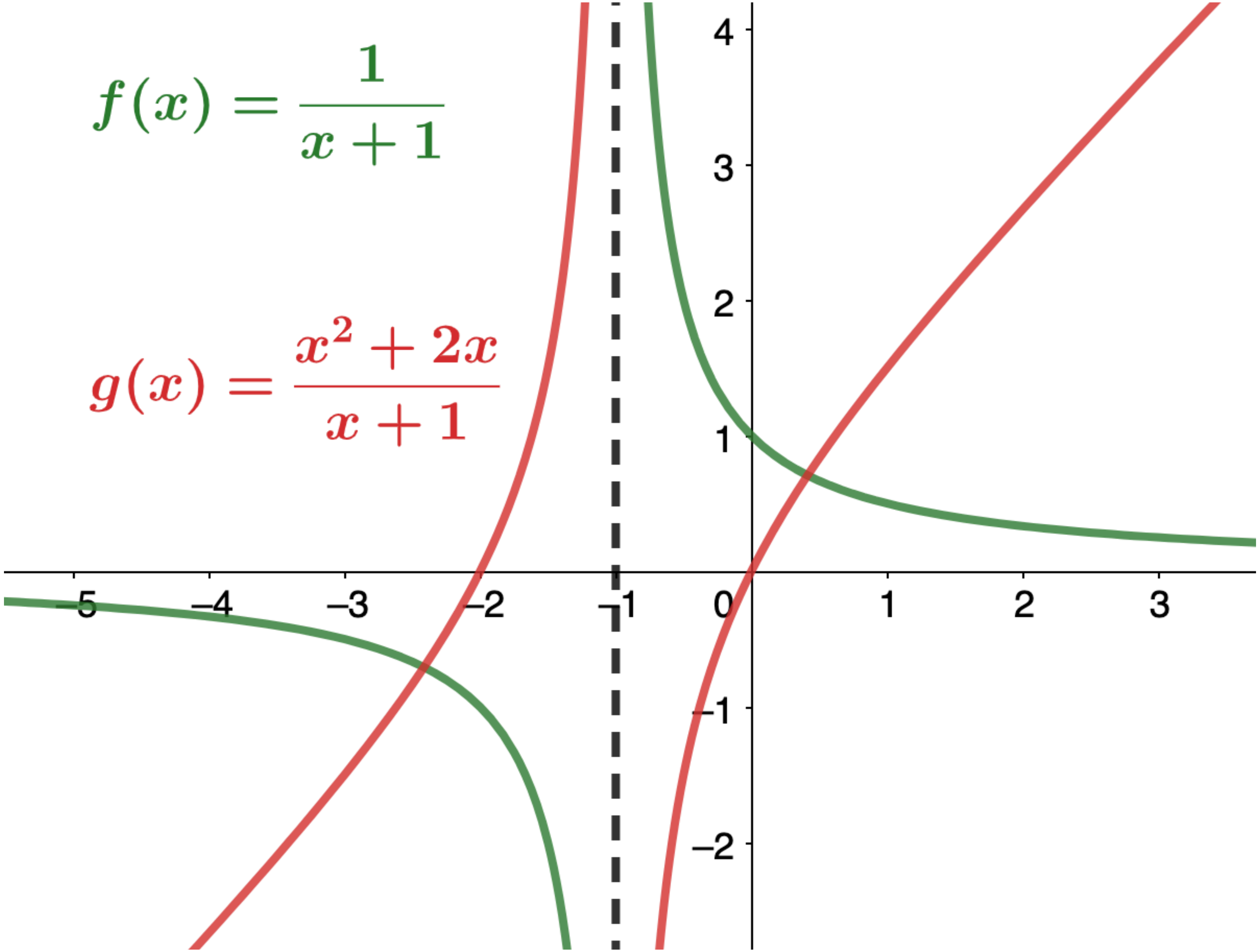
$$\frac{1}{x+1}$$

$$\frac{x^2-2x+3}{6x-5}$$

root function

$f(x) = x^{\frac{1}{n}}$

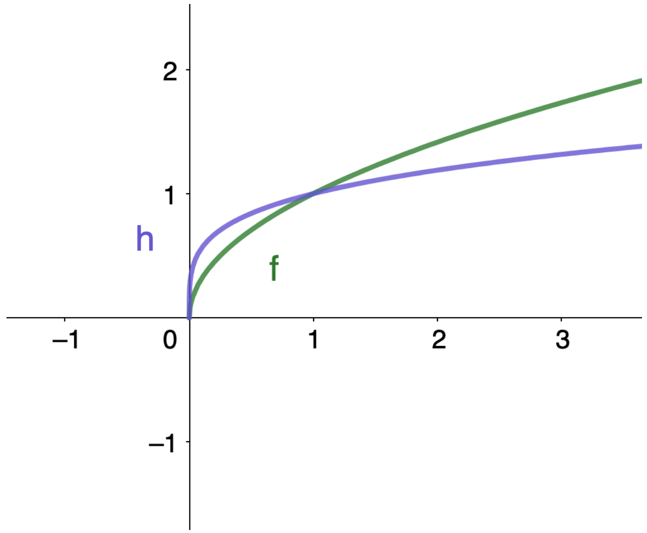
$\sqrt[3]{x}$

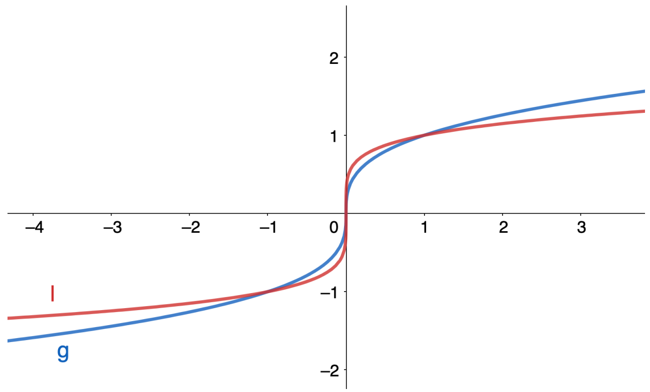


EXAMPLES OF ALGEBRAIC FUNCTIONS

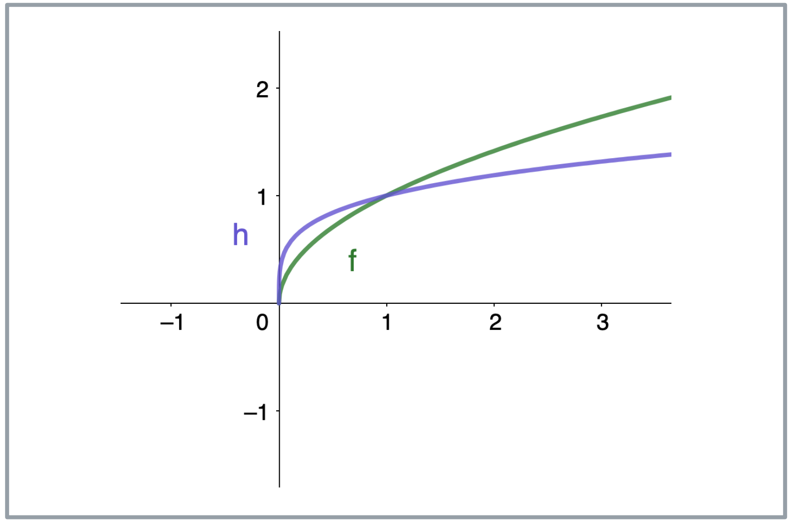
ROOT FUNCTIONS

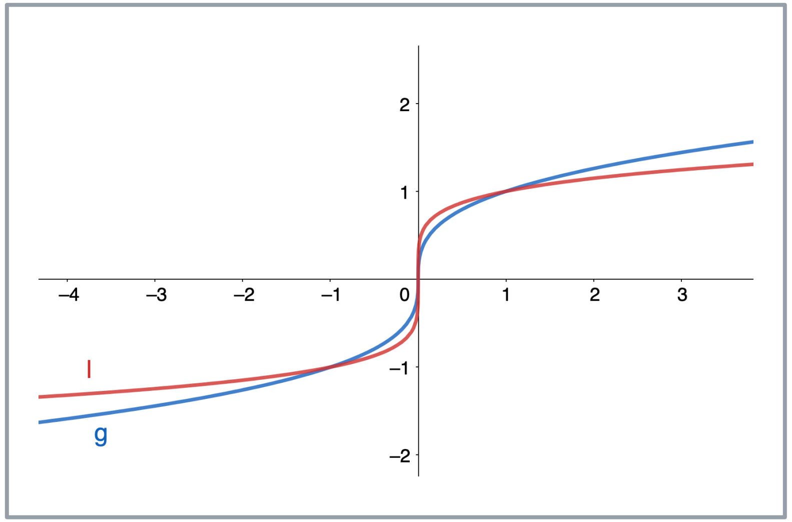
- $f(x) = \sqrt{x}$
- $g(x) = \sqrt[3]{x}$
- $h(x) = \sqrt[4]{x}$
- $l(x) = \sqrt[5]{x}$





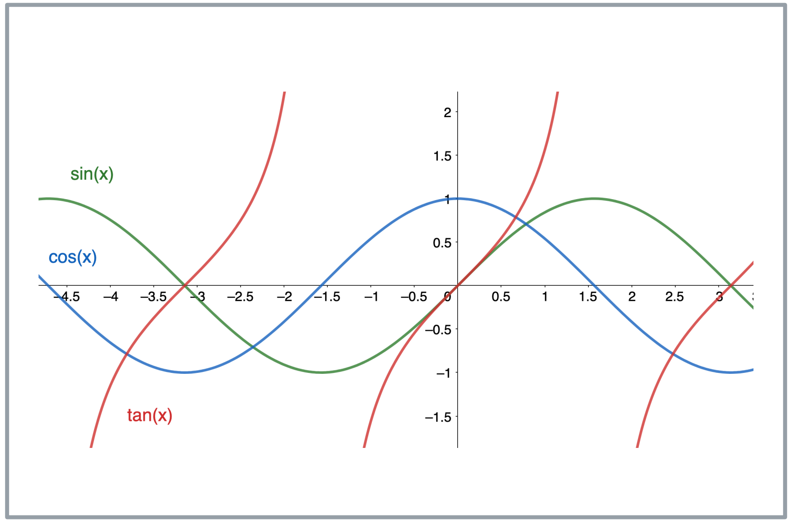
ROOT FUNCTIONS

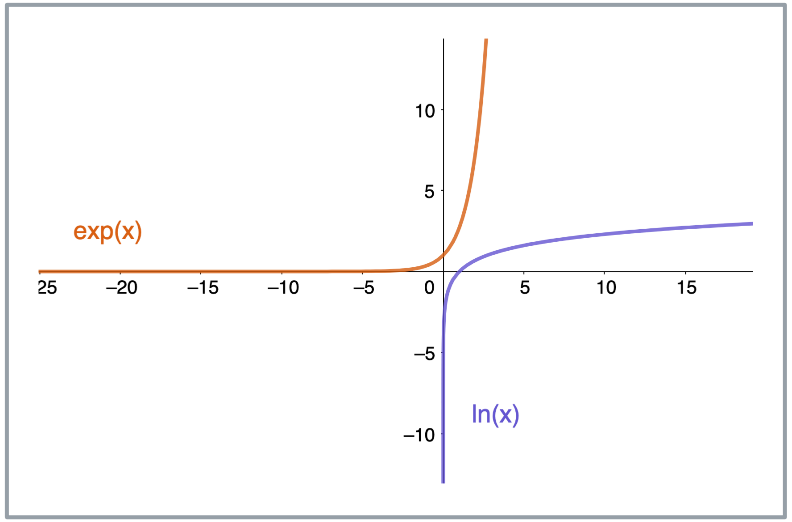




n	Domain	Range	Symmetry
Even	$[0, +\infty)$	$[0, +\infty)$	
Odd	$(-\infty, +\infty)$	$(-\infty, +\infty)$	Odd

TRANSCENDENTAL FUNCTIONS (GO BEYOND ALGEBRA)

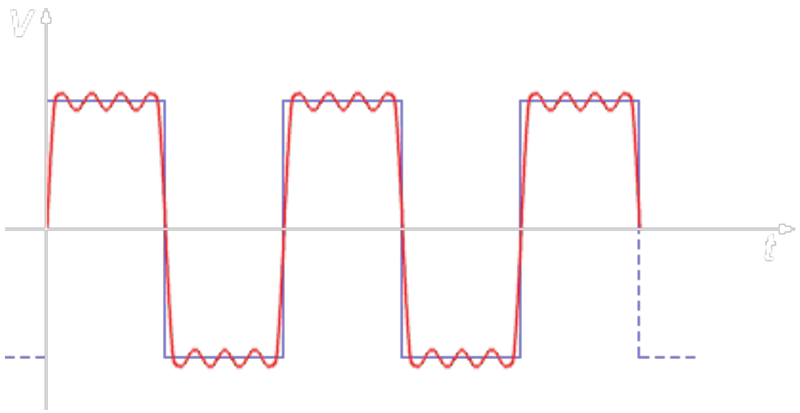


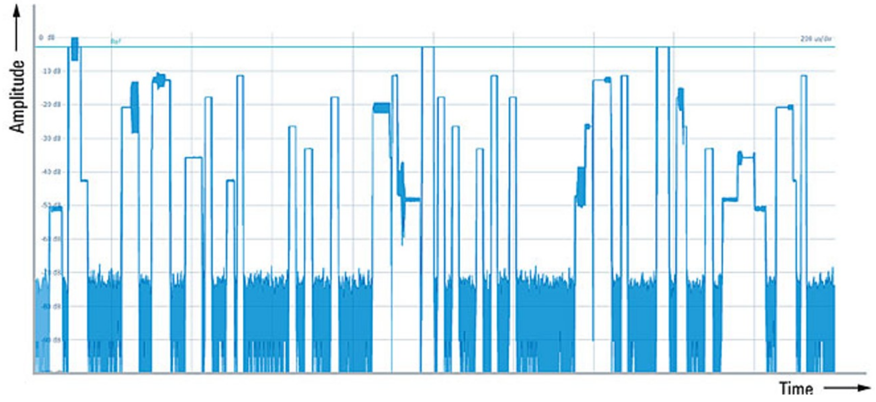


- Most common transcendental functions are
  - Trigonometric: $\sin(x), \cos(x), \tan(x) \ldots$
  - Exponential: b^x
  - Logarithmic: $\log_b(x)$
- We will discuss them later!

PIECEWISE-DEFINED FUNCTIONS

- A function is defined by different formulas on different parts of its domain.
- Recall when we discuss the monotonicity of a function …
- Recall a special example of an even function …





EXAMPLE ONE: RADAR SIGNAL

- The fifth harmonic wave
- The rectangle signal

EXAMPLE TWO: HEART RATE