Problem 1. Section 3.7 #262

We see $f^{-1}(1) = 0$, and $(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$ by the inverse function theorem. It looks approximately as if $f'(f^{-1}(1)) = f'(0) = -1$, so $(f^{-1})'(1) = -1$, approximately.

Problem 2. Section 3.7 #264

(a) We see $\frac{\mathrm{df}}{\mathrm{dx}} = 6$ at $x = a$.

(b) We can find the inverse by letting $y = 6x - 1$, solving $x = (y+1)/6$, and switching the roles of x and y to get $f^{-1}(x) = y = (x+1)/6$. Then $(f^{-1})'(6a-1) = \frac{1}{f'(f^{-1}(a))} = \frac{1}{6}$.

Problem 3. Section 3.7 #270

Let's first note that $f^{-1}(2)$ is the number we need to plug in to $f(x)$ to get 2; visibly, this is 1. Further, $f'(x) = 1 + \frac{1}{2\sqrt{x}}$. Thus we have that $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{3/2} = 2/3$.

Problem 4. Section 3.7 #274

First let's find $f'(x) = \frac{(1+x^2)\frac{d}{dx}4 - 4\frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \frac{-8x}{(1+x^2)^2}$. Then by the inverse function theorem we get that $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{-2} = -\frac{1}{2}$.

Now we know $(f^{-1})(2)$ is the slope of the tangent line to f^{-1} at $x = 2$, and we know a point on this line is $(2,1)$, so we get the equation of the tangent line is $y - 1 = -1/2(x-2)$.

Problem 5. Section 3.7 #286

We need to use chain rule here: $y = f(g(x))$, with $g(x) = -x$ and $f(x) = sec^{-1}(x)$. Then $= f'(g(x))g'(x) = f'(-x)(-1) = \frac{-1}{|-x|\sqrt{(-x)^2-1}} = \frac{-1}{|x|\sqrt{x^2-1}}$.

Problem 6. Section 3.7 #290

By the inverse function theorem, we have that $(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$. We see $f^{-1}(2) = 6$, since $f(6) = 2$, so we have $(f^{-1})'(2) = \frac{1}{f'(6)} = \frac{1}{1/3} = 3$.