Problem 1. Section 3.8 #304

Differentiating, applying the product rule, and the chain rule, we get that $y + x\frac{\mathrm{dy}}{\mathrm{dx}} + sin(xy)(y + x\frac{\mathrm{dy}}{\mathrm{dx}}) = 0$. We can do algebra to get $(x + \sin(xy)x)\frac{\mathrm{dy}}{\mathrm{dx}} = -y - y\sin(xy)$, so $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-y(1+\sin(xy))}{x(1+\sin(xy))} = \frac{-y}{x}$.

Problem 2. Section 3.8 #308

Differentiating, applying the power rule and chain rule, we get $2(xy)(y + x\frac{\mathrm{dy}}{\mathrm{dx}}) + 3 = 2y\frac{\mathrm{dy}}{\mathrm{dx}}$. Solving for $\frac{\mathrm{dy}}{\mathrm{dx}}$, we get $(2y - 2x^2y)\frac{\mathrm{dy}}{\mathrm{dx}} = 2xy^2 + 3$, so $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{2xy^2+3}{2y-2x^2y}$.

Problem 3. Section 3.8 #312

We can use the chain rule and product rule to implicitly differentiate and get $\sec^2(xy)(x\frac{\mathrm{dy}}{\mathrm{dx}} + y) = \frac{\mathrm{dy}}{\mathrm{dx}}$, or $\sec^2(xy)y = \frac{\mathrm{dy}}{\mathrm{dx}}(1 - \sec^2(xy)x)$, so $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sec^2(xy)y}{1-\sec^2(xy)x}$. Plugging in the point $(\pi/4, 1)$, we get $\frac{\mathrm{dy}}{\mathrm{dx}}|_{(\pi/4,1)} = \frac{\sec^2(\pi/4)}{1-\pi/4\sec^2(\pi/4)} = \frac{2}{1-\pi/2}$. With the slope of the line and a point on the line $(\pi/4, 1)$, we can compute the equation of the tangent line is $y - 1 = \frac{2}{1-\pi/2}(x - \pi/4)$.

Problem 4. Section 3.8 #314

We can use quotient rule to get $\frac{y - x\frac{\mathrm{dy}}{\mathrm{dx}}}{y^2} + 5 = -3/4\frac{\mathrm{dy}}{\mathrm{dx}}$, or $\frac{1}{y} + 5 = \frac{\mathrm{dy}}{\mathrm{dx}}(\frac{-3}{4} + \frac{x}{y^2})$, so $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1/y+5}{-3/4+x/y^2} = \frac{1+5y}{-3y/4+x/y} = \frac{4y(1+5y)}{-3y^2+4x}$. Plugging in the point $(1, 2)$, we get $\frac{\mathrm{dy}}{\mathrm{dx}}|_{(1,2)} = \frac{88}{-8} = -11$, so with out point on the line we get the equation of the tangent line is $y - 2 = (-11)(x - 1)$.

Problem 5. Section 3.8 #316

only part (a), find the equation of the tangent line We can implicitly differentiate to get $6x^2 + 6y^2\frac{\mathrm{dy}}{\mathrm{dx}} - 9(x\frac{\mathrm{dy}}{\mathrm{dx}} + y) = 0$, and solve for $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{2x^2-3y}{3x-2y^2}$. At the point $(2, 1)$, we get $\frac{\mathrm{dy}}{\mathrm{dx}}|_{(2,1)} = \frac{5}{4}$, so the tangent line has equation $y - 1 = 5/4(x - 2)$.

Problem 6. Section 3.8 #318

First implicitly define to get $3y^2\frac{\mathrm{dy}}{\mathrm{dx}} - 27\frac{\mathrm{dy}}{\mathrm{dx}} = 2x$, so $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{2x}{3y^2-27}$. This derivative is a vertical line when $\frac{\mathrm{dy}}{\mathrm{dx}}$ is infinite, so $3y^2 - 27 = 0$, or $y^2 - 9 = 0$. This is at $y = \pm 3$. What are the corresponding x points? If $y = 3$, then $x^2 - 90 = -54$, so $x^2 = 36$ and $x = \pm 6$. If $y = -3$, then $x^2 - 90 = 54$, or $x^2 = 144$, so $x = \pm 12$. Thus the four points are $(3, 6)$, $(3, -6)$, $(-3, 12)$, and $(-3, -12)$.