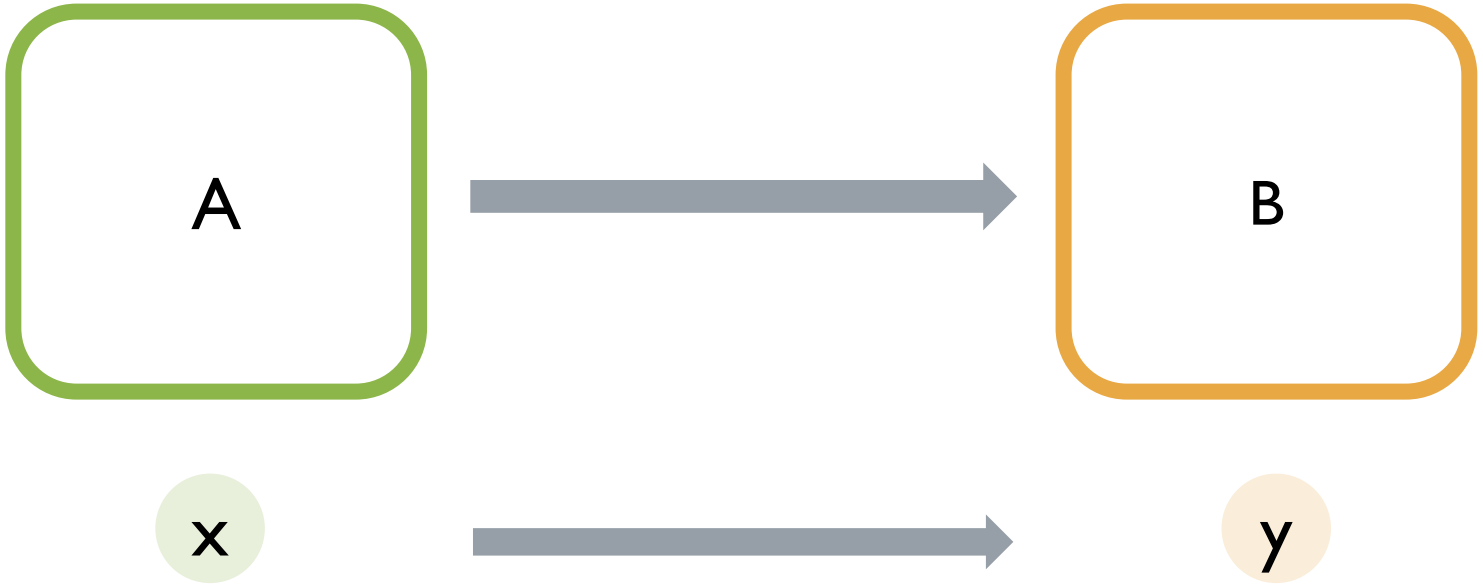
MATH 1: INTRODUCTION TO CALCULUS

FUNCTIONS AND GRAPHS (09/16)

WHAT IS A FUNCTION?

- Two sets A and B
  - x is an element of A
  - y is an element of B
- A relation from A to B
- Ordered pairs (x, y)
- …?



IS IT A FUNCTION?

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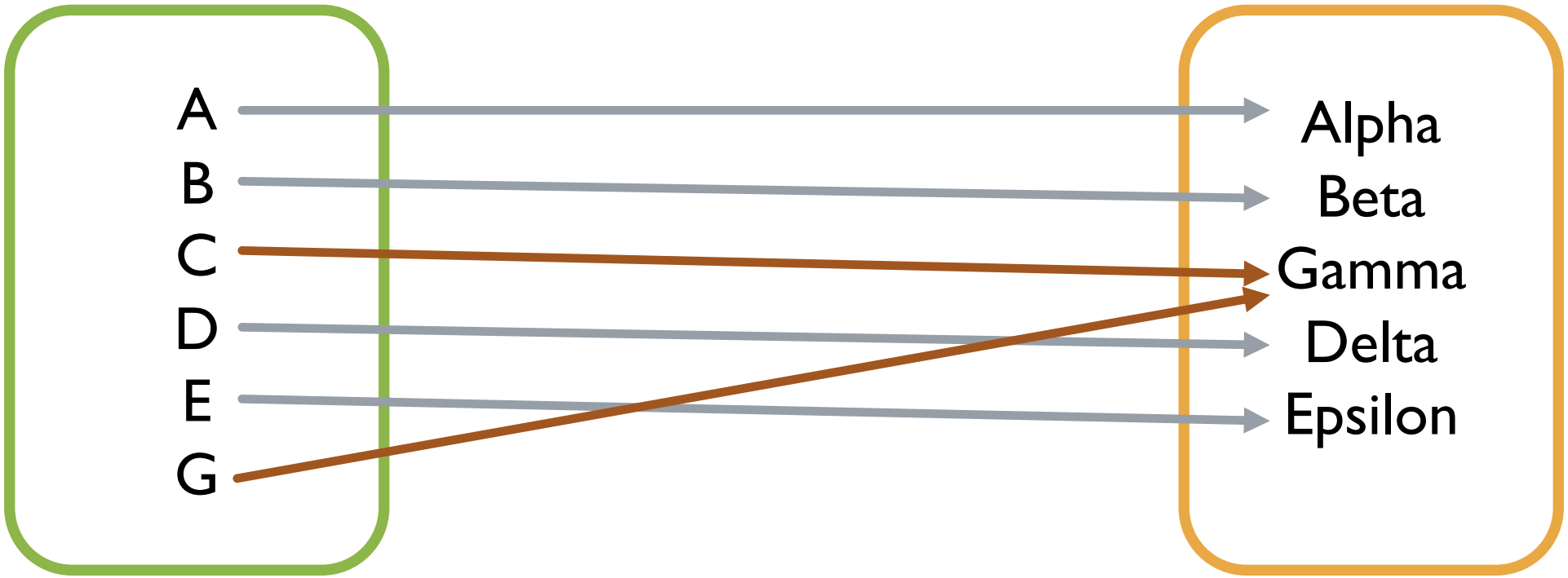
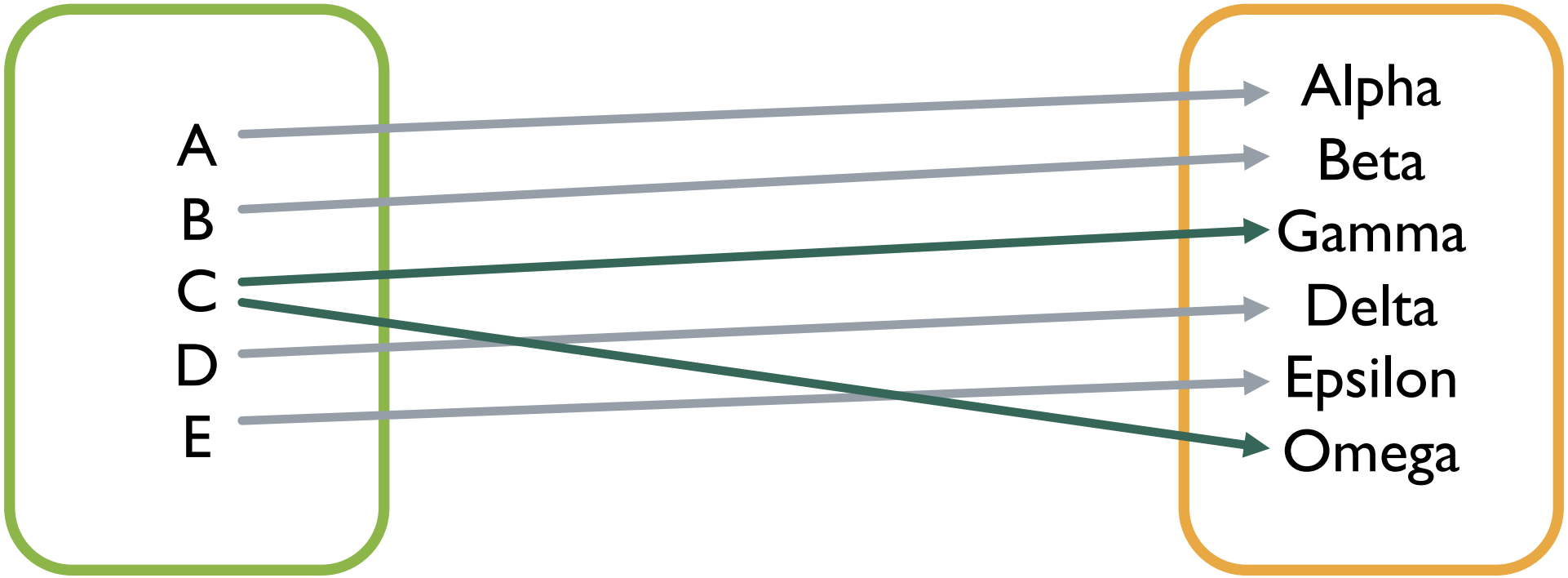
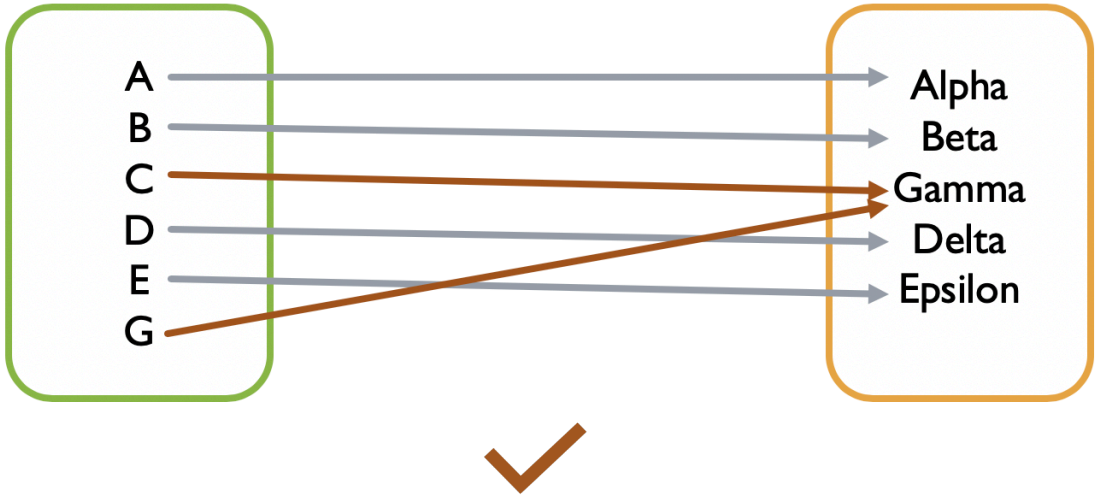


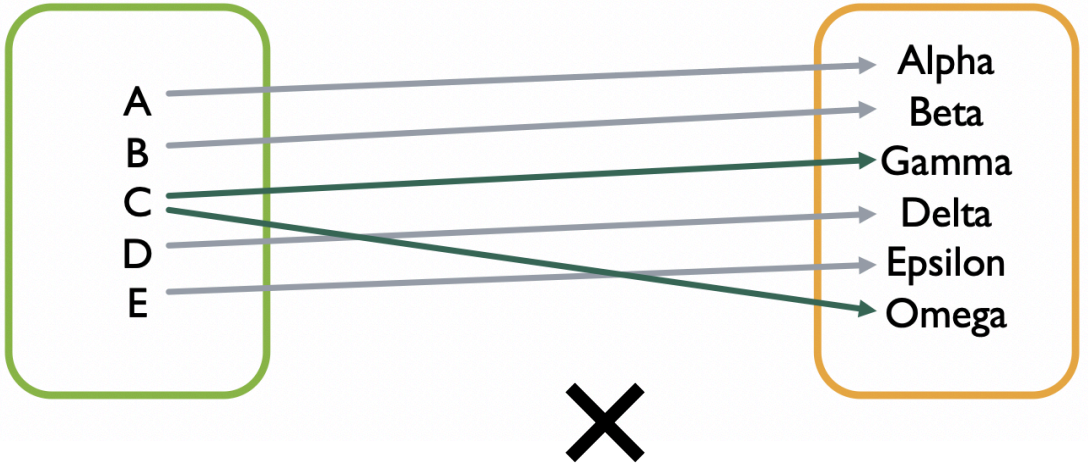
IS IT A FUNCTION?





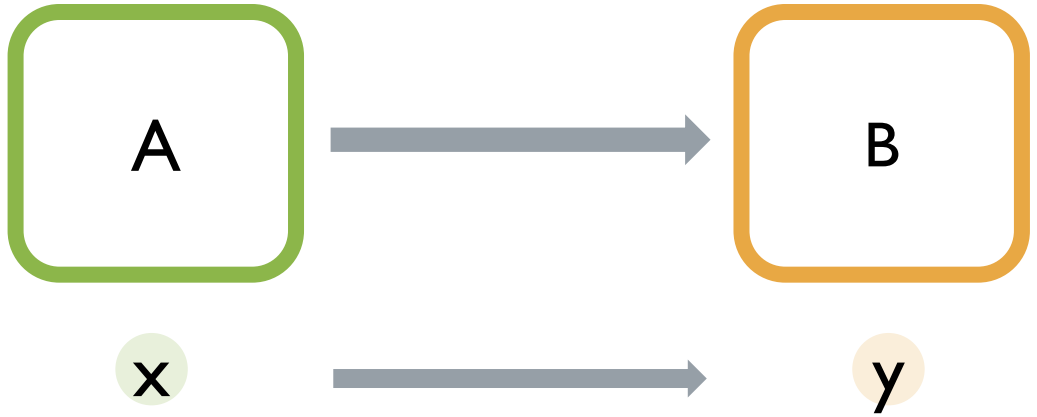
ONE MORE RESTRICTION

- Every element of the first set is related to **exactly one** element of the second set.



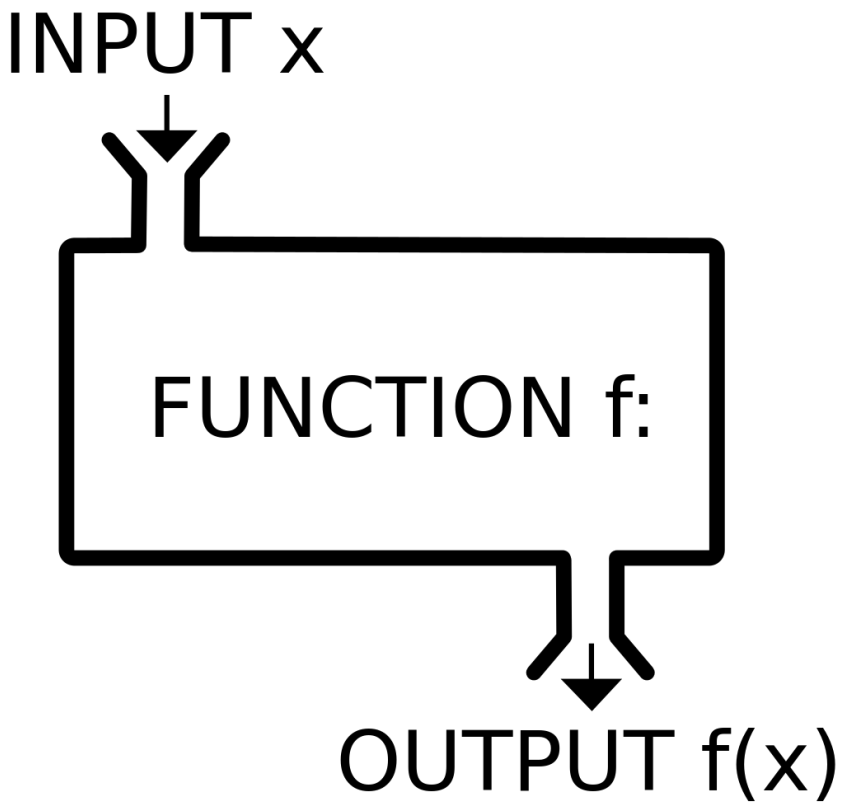
MORE THINGS TO SAY…

- Every element of the first set is related to **exactly one** element of the second set.
- The element of the first set is called the *input*.
- The element of the second set is called the *output*.



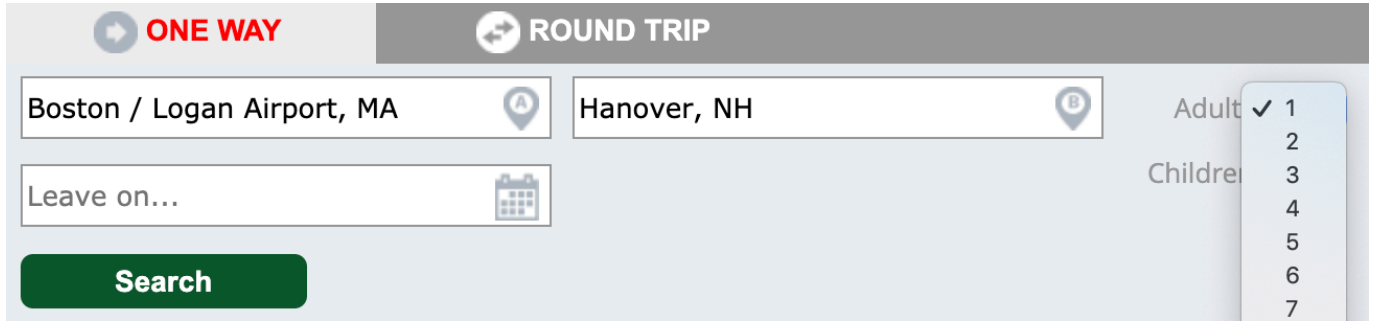
EXAMPLES OF A FUNCTION

- Remember to specify
  - the inputs
  - the outputs
  - the relation

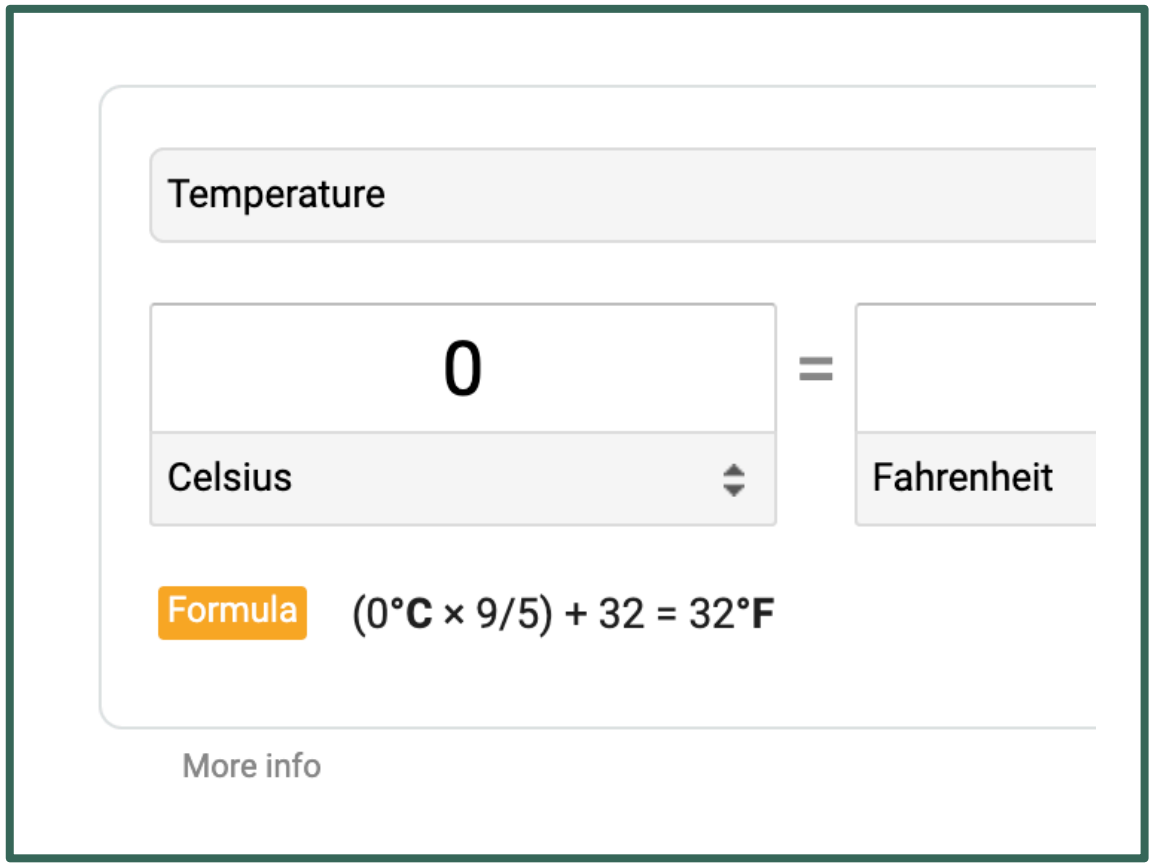


EXAMPLES OF A FUNCTION

- $f(x) = 38x$
- What is the input?
- What is the output?



EXAMPLES OF A FUNCTION

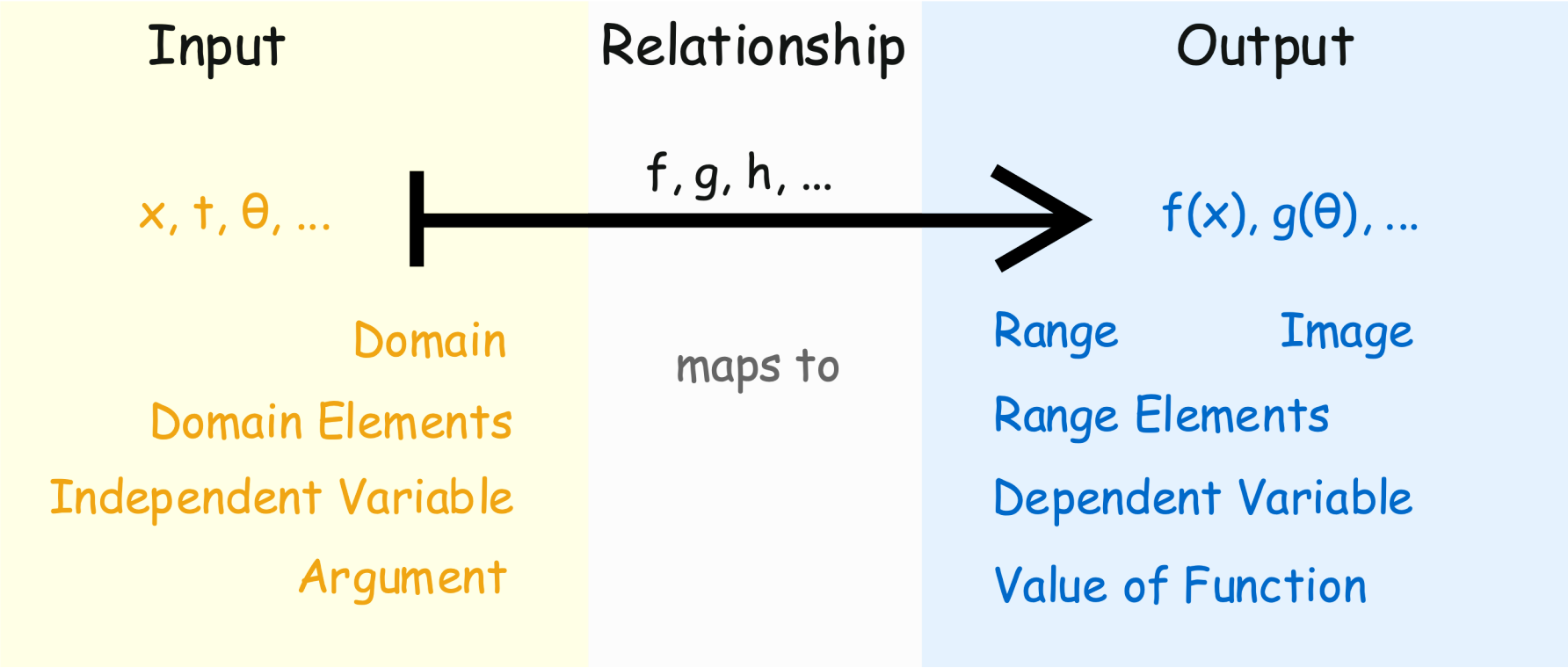


- $f(x) = \frac{9}{5}x + 32$
- What is input?
- What is the output?

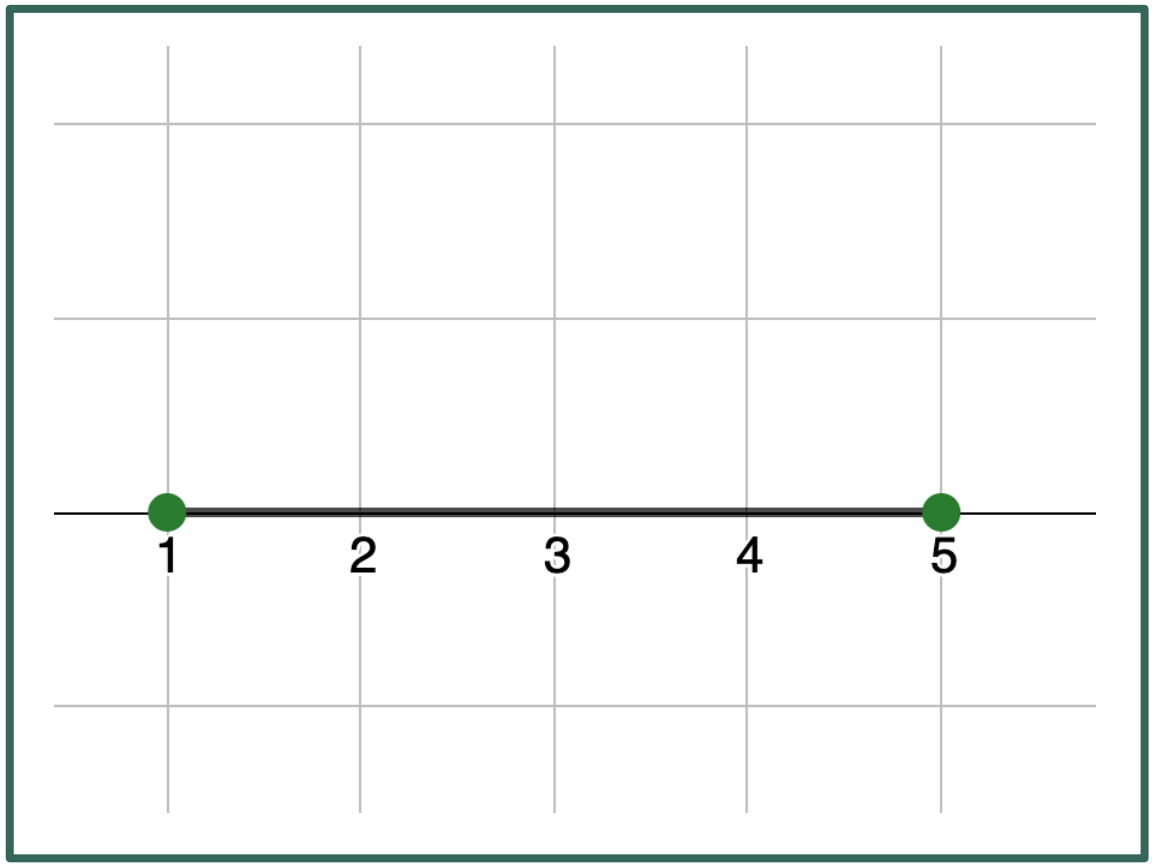
FORMAL DEFINITION

- A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output.
- The set of inputs is called the **domain** of the function.
- The set of outputs is called the **range** of the function.

IN GENERAL


Input
Relationship
Output
x, t, θ, ...
f, g, h, ...
f(x), g(θ), ...
Domain
Domain Elements
Independent Variable
Argument
maps to
Range
Image
Range Elements
Dependent Variable
Value of Function

HOW TO REPRESENT THE DOMAIN OR THE RANGE?



- Mathematically!
  - $\{x|1 < x < 5\}$
  - $(1, 5)$
  - $(1, 5) = \{x|1 < x < 5\}$

SET-BUILDER AND INTERVAL NOTATIONS

- Set builder
  - $\{x | x \text{ has some properties}\}$
- Interval
  - Open: (a, b)
  - Close: $[a, b]$
  - Left open, right close: $(a, b]$
  - Left close, right open: $[a, b)$

SET-BUILDER AND INTERVAL NOTATIONS (IS THE RELATION A FUNCTION?)

- All real numbers
- All positive real numbers
- All negative real numbers
- All non-negative real numbers

$[0, +\infty)$

$\{x|x \text{ is a real number}\}$

$(0, +\infty)$

$\mathbb{R}$

$\{x|x < 0\}$

$\{x|x \geq 0\}$

$(-\infty, +\infty)$

$(-\infty, 0)$

SET-BUILDER AND INTERVAL NOTATIONS (IS THE RELATION A FUNCTION?)

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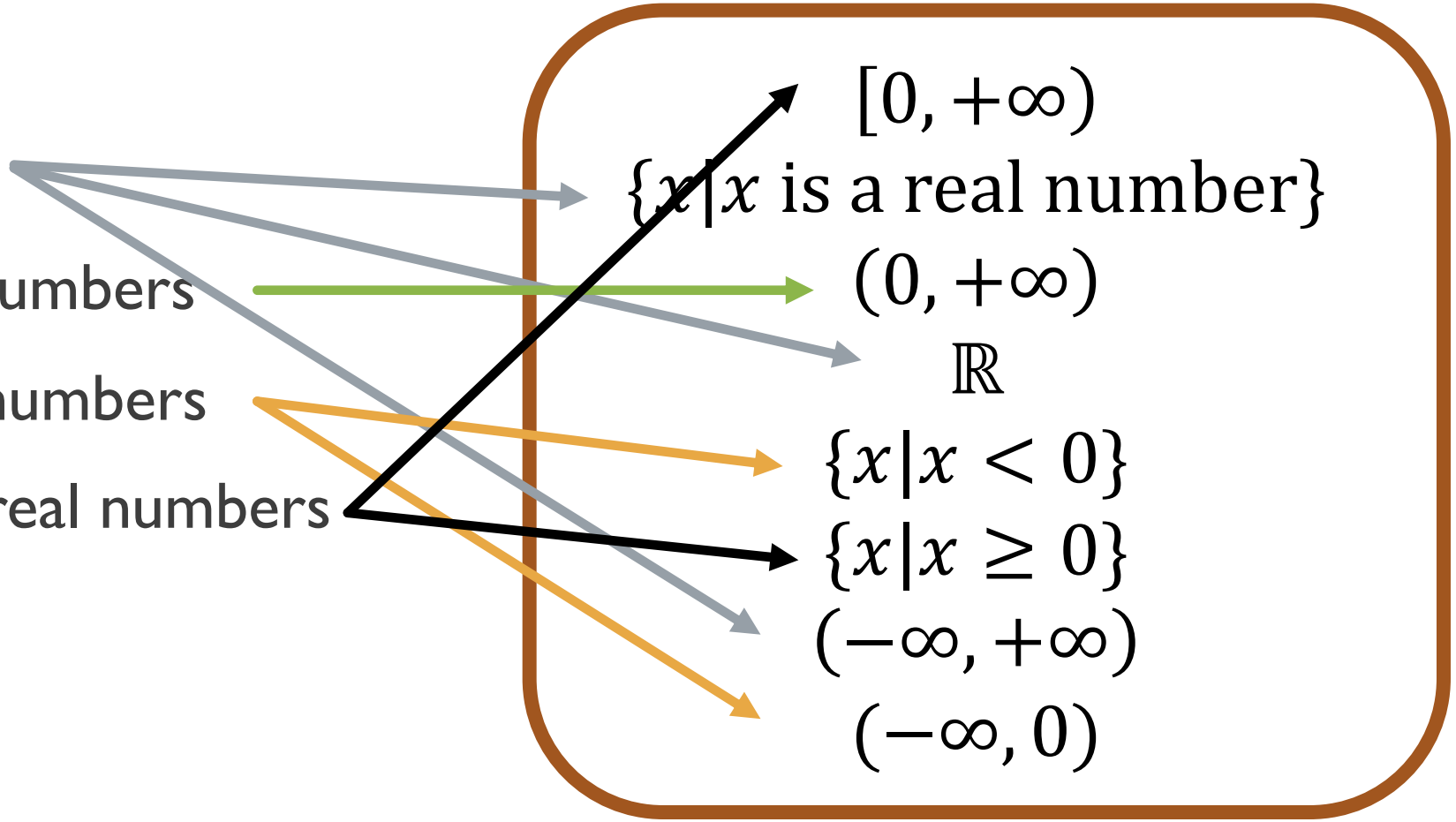

MORE EXAMPLES: FIND DOMAIN AND RANGE

function	Domain	Range
$f(x)$	$\mathbb{R}$	$[0, +\infty)$
$g(x)$		
$h(x)$		
$c(x)$		
$s(x)$		
$exp(x)$		

- $f(x) = x^2$
- $g(x) = \sqrt{x}$
- $h(x) = \sqrt{x}, x > 4$
- $c(x) = \cos x$
- $s(x) = \sin x$
- $exp(x) = e^x$

MORE EXAMPLES: FIND DOMAIN AND RANGE

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function	Domain	Range
$f(x)$	$\mathbb{R}$	$[0, +\infty)$
$g(x)$	$[0, +\infty)$	$[0, +\infty)$
$h(x)$	$(4, +\infty)$	$(2, +\infty)$
$c(x)$	$\mathbb{R}$	$[-1,1]$
$s(x)$	$\mathbb{R}$	$[-1,1]$
$exp(x)$	$\mathbb{R}$	$(0, +\infty)$

EVALUATE FUNCTIONS

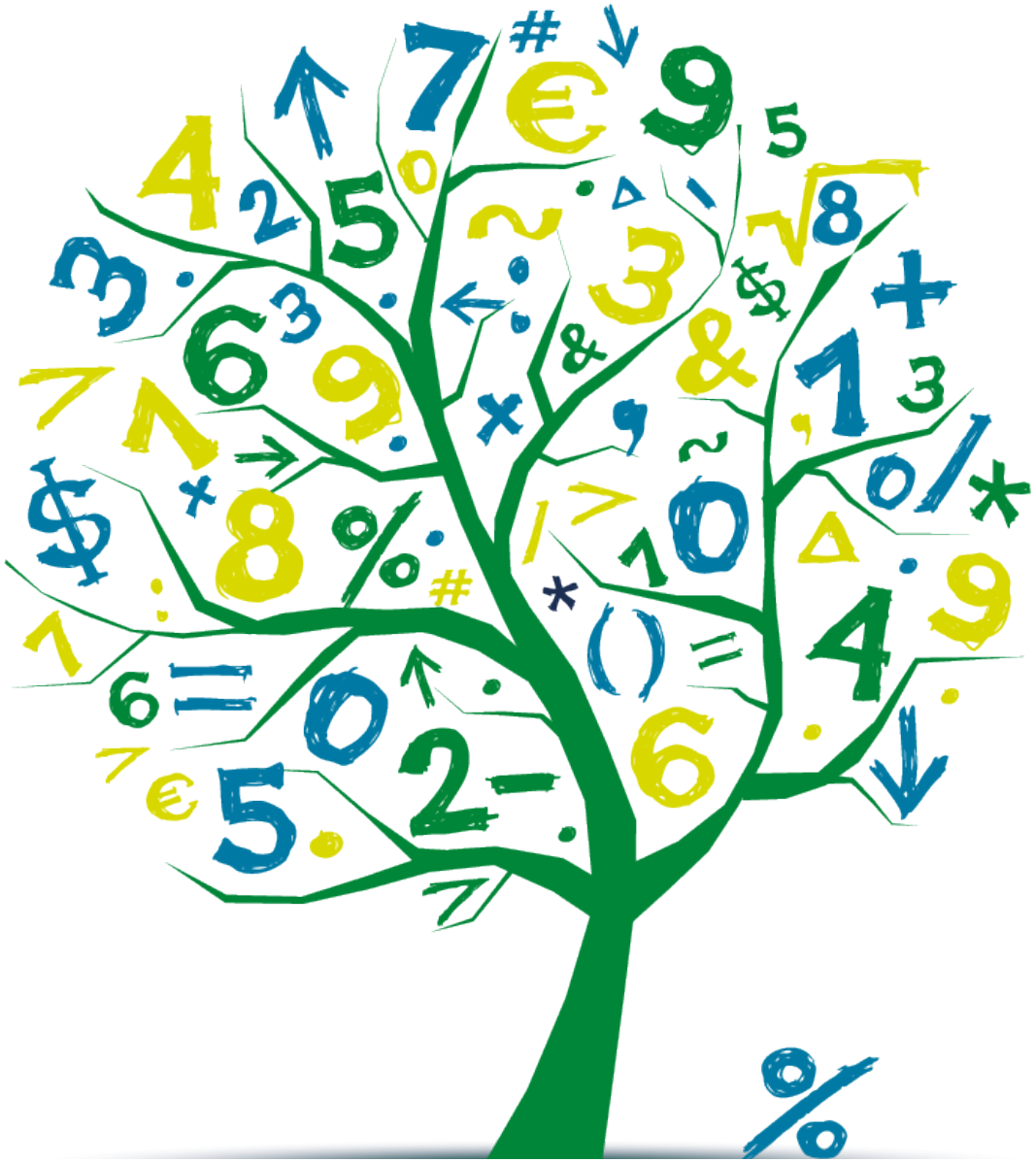
function	domain	range	$f(-1)$	$f(6)$
$f(x) = (x-1)^2+5$				
$f(x) = \frac{1}{x+2} - 3$				
$f(x) = x^3 + \sqrt{x-2}$				

EVALUATE FUNCTIONS

function	domain	range	$f(-1)$	$f(6)$
$f(x) = (x-1)^2+5$	$\mathbb{R}$	$[5, +\infty)$	9	30
$f(x) = \frac{1}{x+2} - 3$	$(-\infty, -2)$ $\cup (-2, +\infty)$	$(-\infty, -3)$ $\cup (-3, +\infty)$	-2	$-2\frac{7}{8}$ $(-\frac{23}{8})$
$f(x) = x^3 + \sqrt{x-2}$	$[2, +\infty)$	$[8, +\infty)$		218

EVALUATE FUNCTIONS

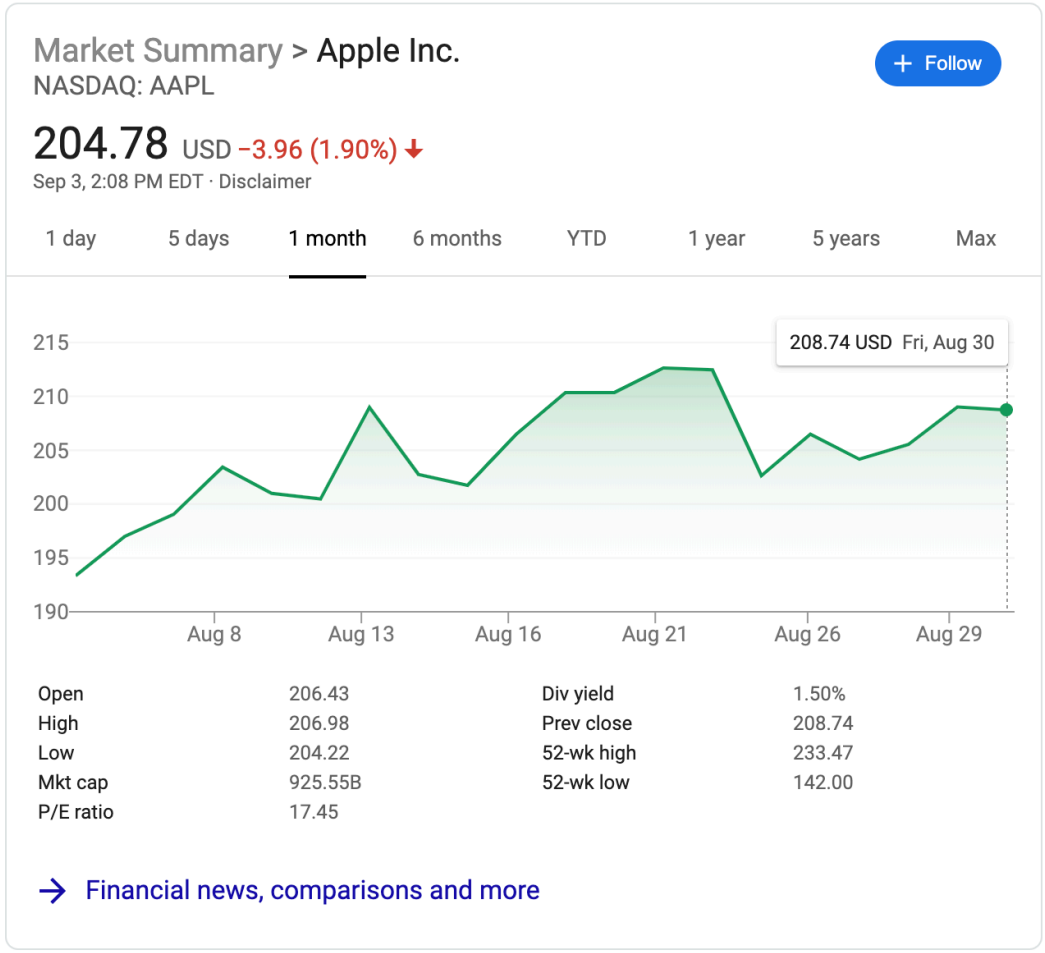
- $f(x) = (x-1)^2+5$
  - $f(2) = (2-1)^2+5 = 6$
- $g(x) = \frac{1}{x+2} - 3$
  - $g(6) = \frac{1}{6+2} - 3 = -\frac{23}{8}$
- $(g \circ f)(x) = ?$
  - $(g \circ f)(2) = ?$
- Will talk more about it next class (Operations on Functions)!



HOW TO REPRESENT A FUNCTION?

RANK	PEAK	TITLE	WORLDWIDE GROSS	YEAR
1	1	Avengers: Endgame	$2,796,255,402	2019
2	1	Avatar	$2,789,679,794	2009
3	1	Titanic	$2,187,463,944	1997
4	3	Star Wars: The Force Awakens	$2,068,223,624	2015
5	4	Avengers: Infinity War	$2,048,359,754	2018
6	3	Jurassic World	$1,671,713,208	2015
7	7	The Lion King	$1,569,877,040	2019
8	3	The Avengers	$1,518,812,988	2012
9	4	Furious 7	$1,516,045,911	2015
10	5	Avengers: Age of Ultron	$1,405,403,694	2015

REPRESENT FUNCTIONS: THREE METHODS

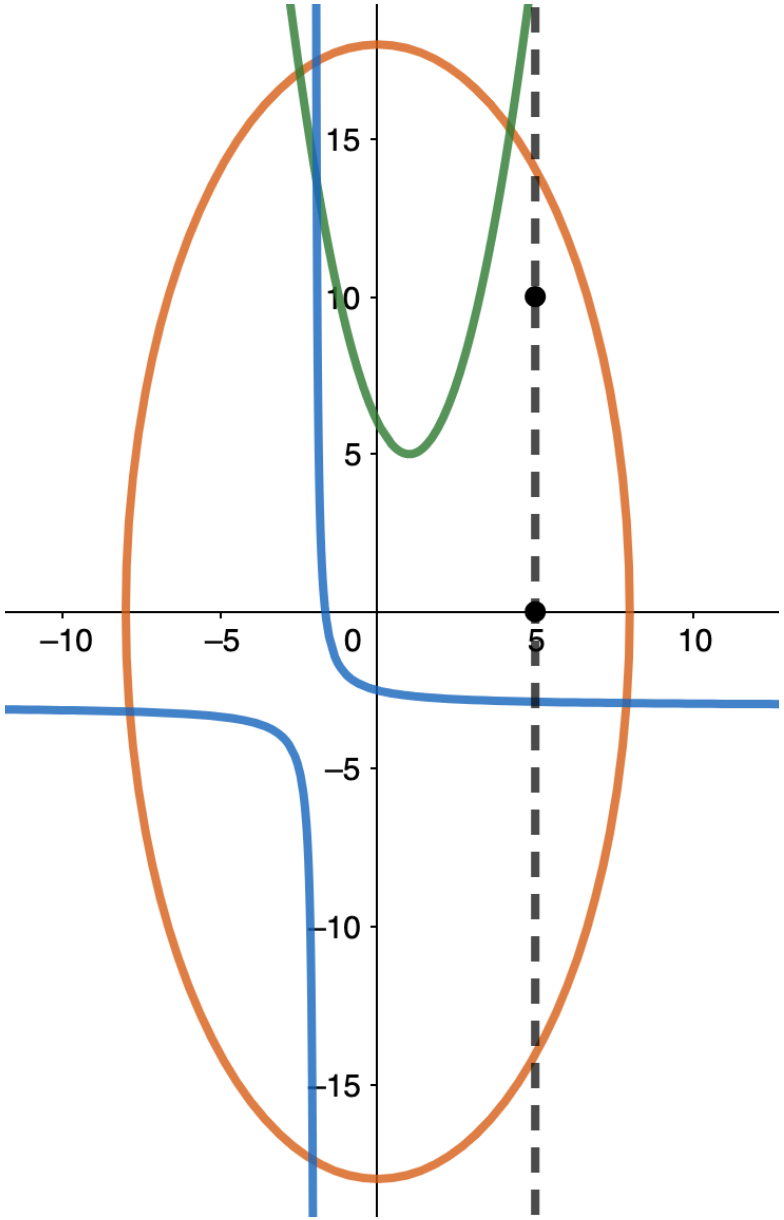


REPRESENT FUNCTIONS

THREE METHODS

REPRESENT FUNCTIONS: THREE METHODS

- A table
- A graph
- A formula
  - $f(x) = \cdots$

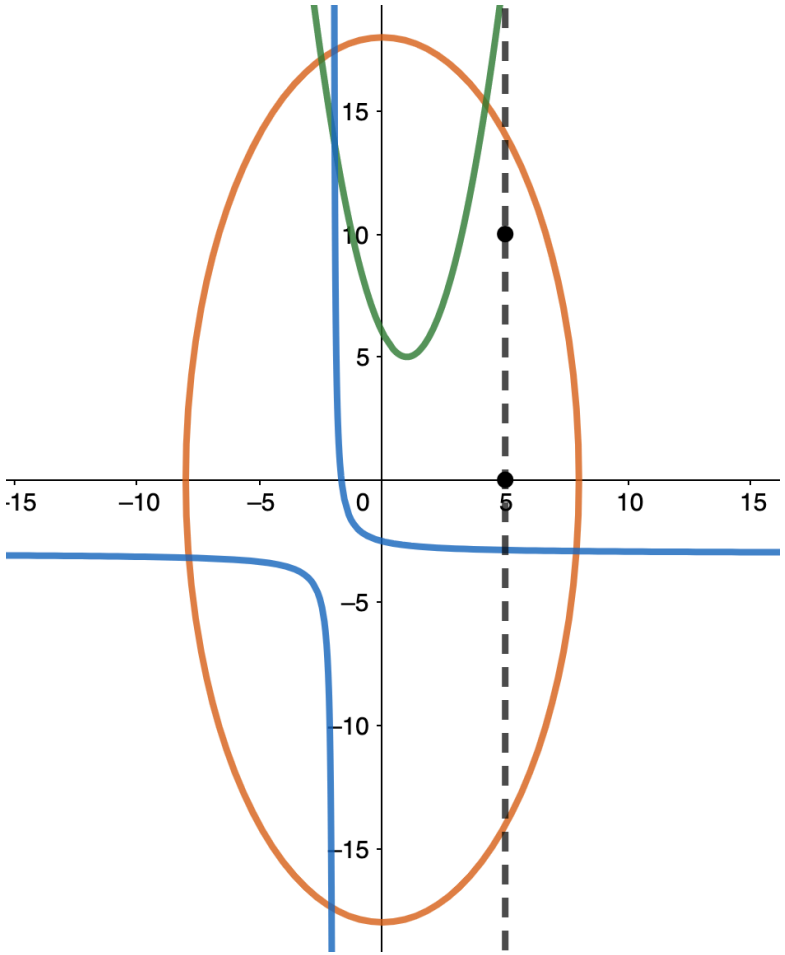


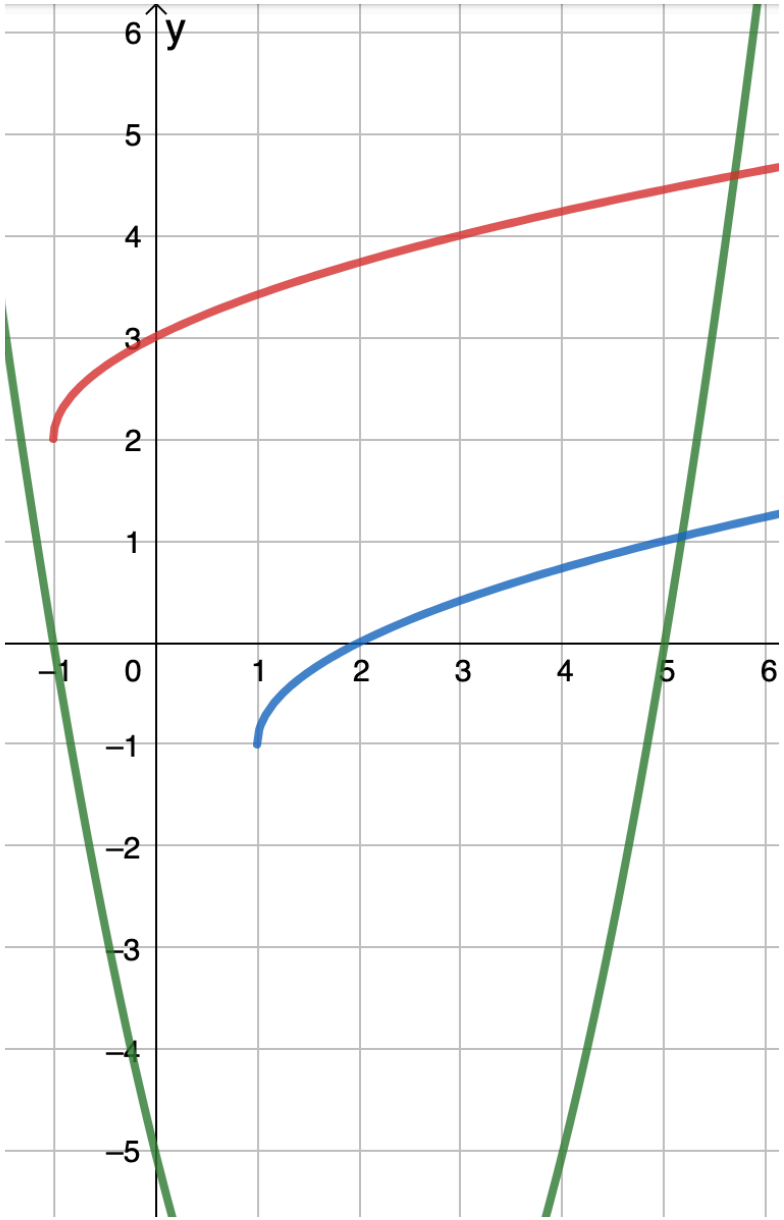
GRAPH: VERTICAL LINE TEST

- Among the three curves, which are functions and which are not?
- Given a function, every vertical line that may be drawn intersects the graph **no more than once**!

GRAPH: SPECIAL POINTS

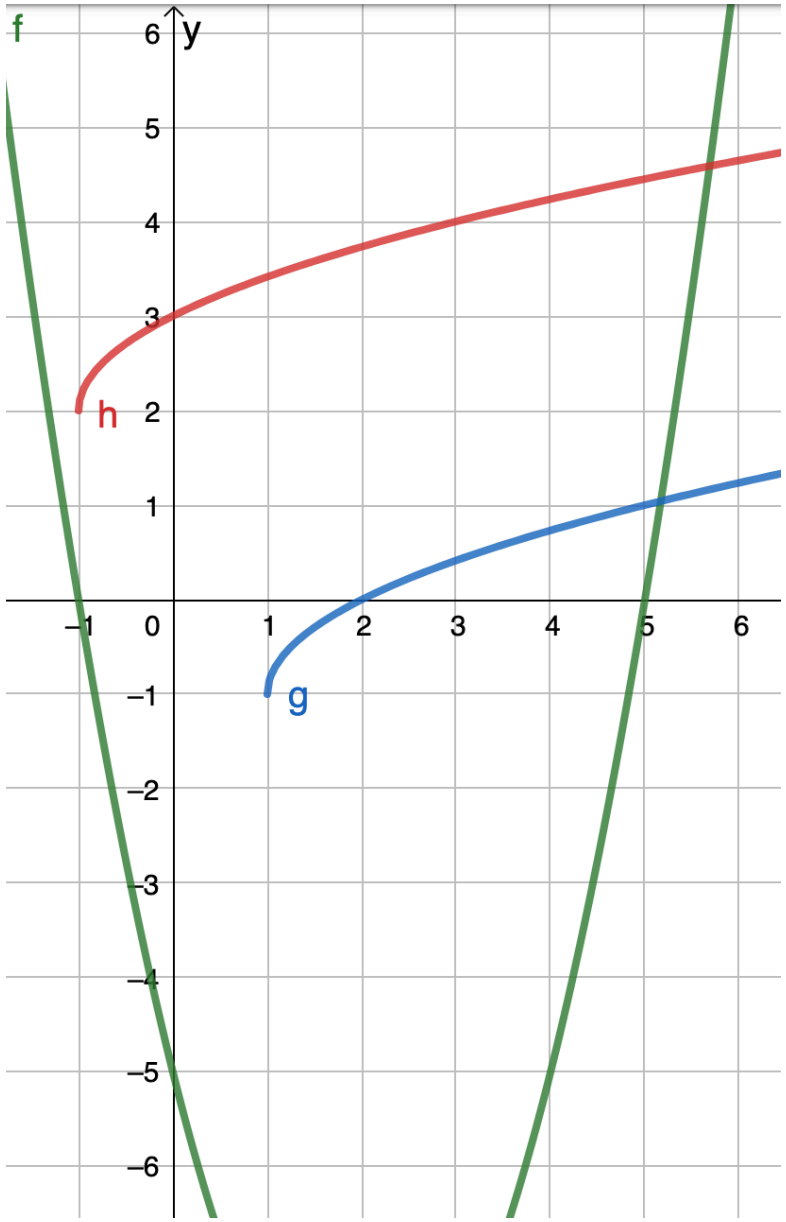
- zeros
  - the **values** of x where f(x)=0.
- x-intercepts
- y-intercepts
  - the y-intercept is given by **(0, f(0))**, if any.



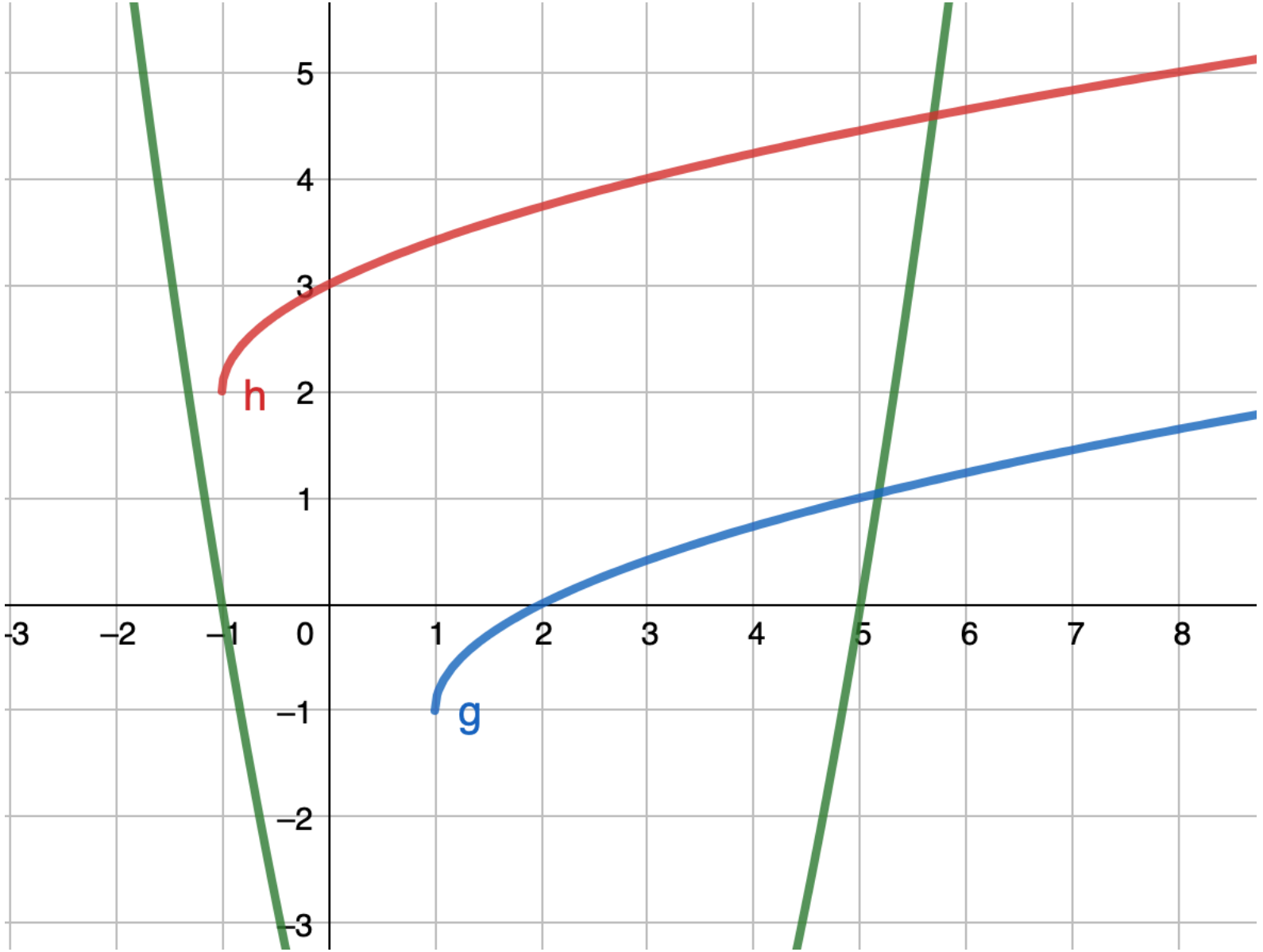


FIND ZEROS AND Y-INTERCETS

- $f(x) = x^2 - 4x - 5$
- $g(x) = \sqrt{x - 1} - 1$
- $h(x) = \sqrt{x + 1} + 2$



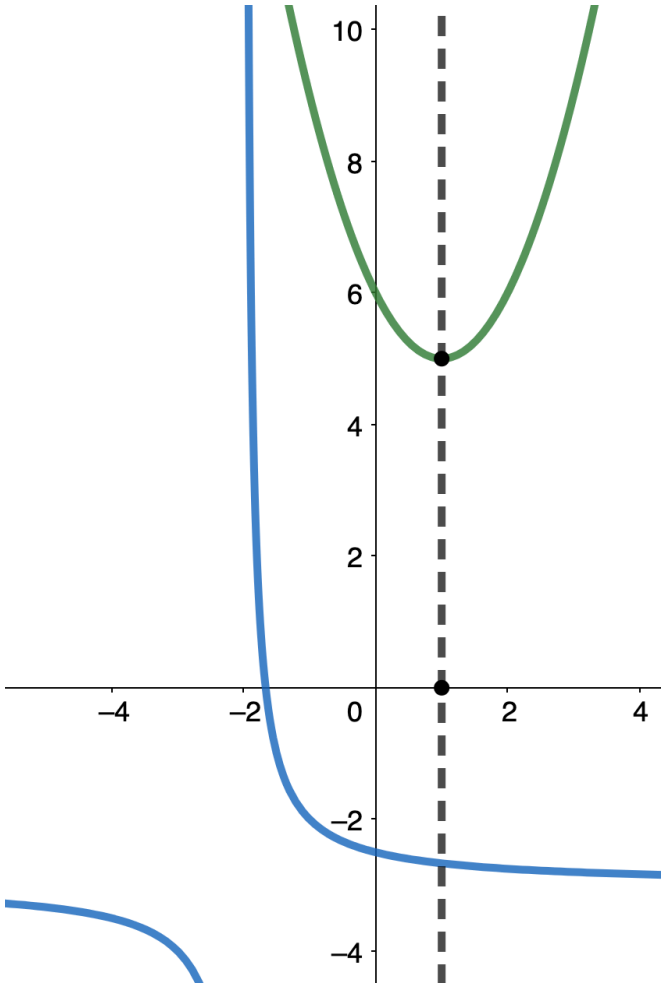
function	zeros	y-intercepts
$f(x) = x^2 - 4x - 5$	$-1, 5$	$(0, -5)$
$g(x) = \sqrt{x - 1} - 1$	2	
$h(x) = \sqrt{x + 1} + 2$		$(0,3)$



HOW TO DESCRIBE THE UPS AND DOWNS

MONOTONICITY: INCREASING? DECREASING?

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$



MONOTONICITY

DEFINITION

We say that a function f is **increasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \leq f(x_2) \text{ when } x_1 < x_2.$$

We say f is strictly increasing on the interval I if for all $x_1, x_2 \in I$,

$$f(x_1) < f(x_2) \text{ when } x_1 < x_2.$$

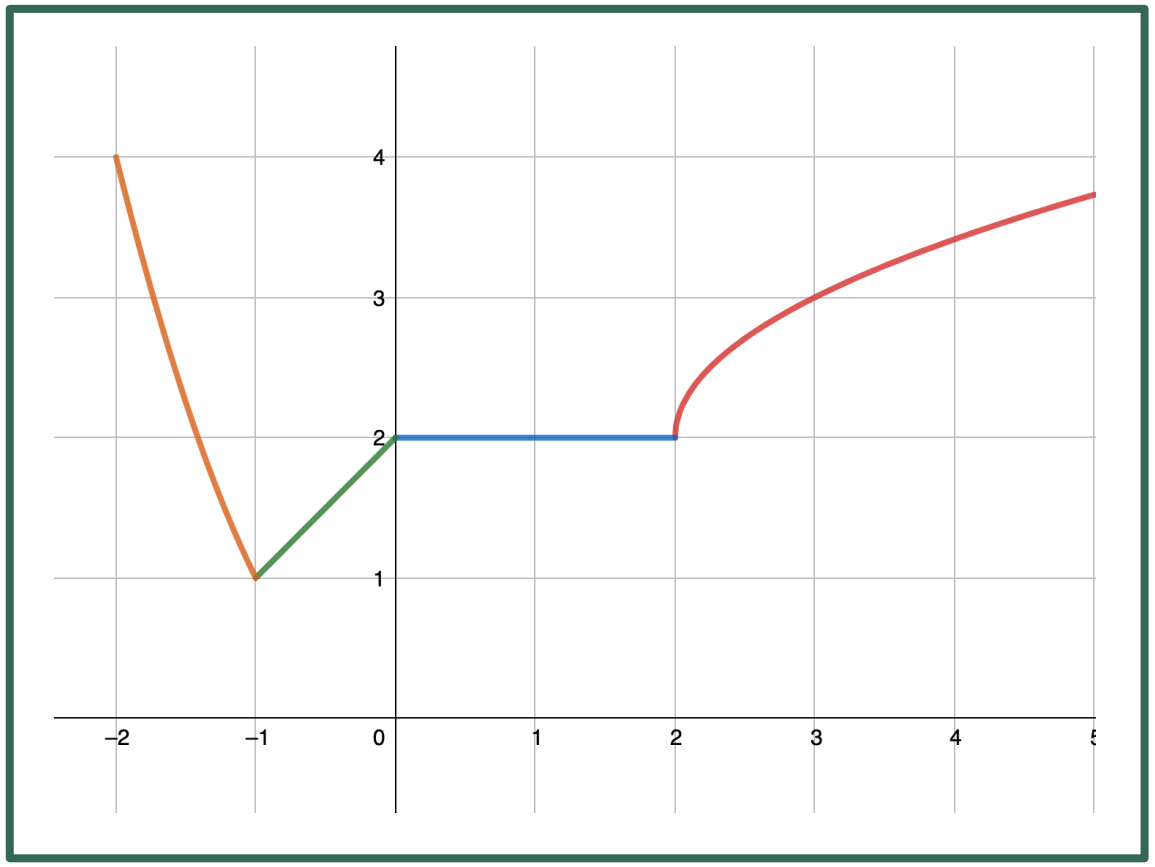
We say that a function f is **decreasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \geq f(x_2) \text{ if } x_1 < x_2.$$

We say that a function f is strictly decreasing on the interval I if for all $x_1, x_2 \in I$,

$$f(x_1) > f(x_2) \text{ if } x_1 < x_2.$$

MONOTONICITY



- A piecewise-defined function
- $$f(x) = \begin{cases} x^2,\ -2 \le x < -1 \\ x + 2,\ -1 \le x < 0 \\ 2, 0 \le x < 2 \\ \sqrt{x-2} + 2, 2 \le x \le 5 \end{cases}$$