MATH 1: INTRODUCTION TO CALCULUS
MIDTERM EXAM #1

Name: ______________________________

Section (please circle): 10 Winkeler 11 Tripp 12 Chen 2 Xiao

(1) Write your name *legibly* and circle your section above.
(2) This exam is closed book. You may not use notes, calculators, or any other external resource. It is a violation of the honor code to give or receive help on this exam.
(3) You must explain what you are doing, justify your answer, and show your work. You will be *graded on your work*, not just on your answer. Please write clearly.
(4) It is fine to leave your answer in a form such as $14\sqrt{239}$. However, if an expression can be easily simplified (such as $\cos(\pi/2)$ or $2+3$), please simplify it.
(5) If you use the backside of a page or the scratch paper at the end of the exam, and you want us to look at it, please write on a problem "Continued on back/page…".

Problem	**Score**	**Problem**	**Score**
1	/12	6	/8
2	/9	7	/9
3	/8	8	/6
4	/7		
5	/12		
		Total	/71

Date: Thursday, 10 October 2019.

Problem 1. [12 points]

(a) Simplify $2\ln(e^3)$.

(b) Let $f(x) = \cos(x)$ and $g(x) = e^{x^2}$. Find $(g \circ f)(x)$ and $(f \circ g)(x)$.

(c) Solve the equation $\ln(\sqrt{x-5}) = 1$ for x.

(d) Solve the equation $9^{(3^x)} = 3^{(27^x)}$ for x.

Problem 2. [9 points]

(a) Find all solutions of $2\cos^2(\theta)\tan(\theta) = \sin(\theta)$ for θ in the interval $[0, 2\pi]$.

(b) Let $f(x) = (x-3)^2$ for $x \leq 3$. Find $f^{-1}(x)$.

(c) Solve the equation $\log_2(\sqrt{x}) + \log_2(\sqrt[3]{x}) = 2$ for x.

Problem 3. [8 points] Mark the following statements as true or false.

_____ $\ln(e^{-2}) = e^{\ln(-2)}$

_____ If k is positive, 10^{-k} is negative.

_____ $\log_5(xy) = \log_5(x)\log_5(y)$

_____ If a is a positive, constant, then $\ln(a^r) = r\ln(a)$.

_____ $\log_3 0 = 1$

_____ The domain of $f(x) = 2^x + 1$ is $(2, \infty)$.

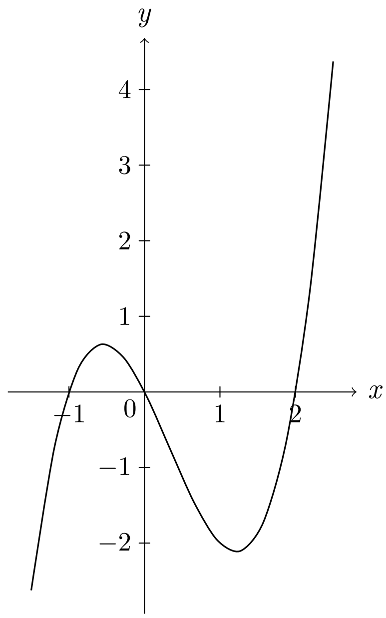
_____ The graph of $y = e^{\ln x}$ is a parabola.

_____ The slope of a secant line passing through the point $(a, f(a))$ and a nearby point on the graph of $f(x)$ approximates the instantaneous rate of change of $f(x)$ at a.

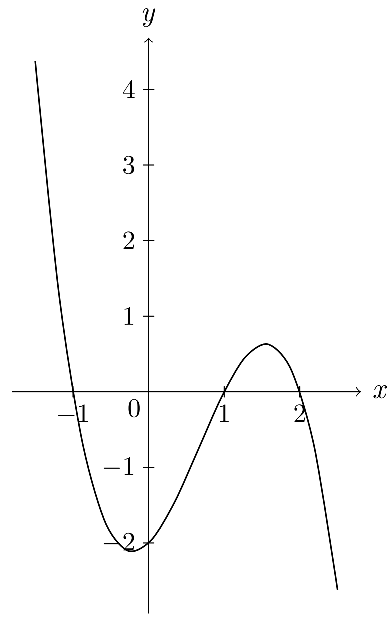
Problem 4. [7 points]

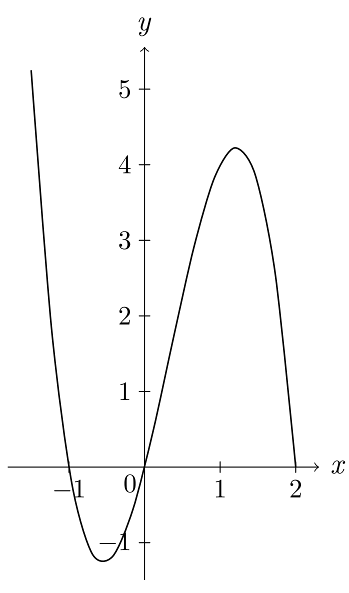
(1) Match the transformations of $f(x)$ with their graphs.

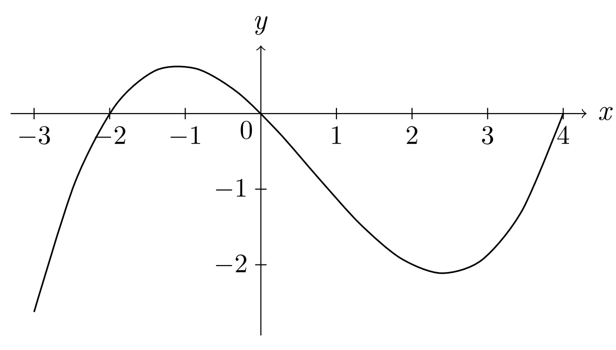
Options: $-2f(x)$, $f(x/2)$, $f(1-x)$



$f(x)$







(2) Is $f(x)$ even, odd, or neither?

Problem 5. [12 points]

(a) Plot the graph of $f(x) = e^x$ and its inverse function $f^{-1}(x) = \ln(x)$.

(b) Let $g(x) = e^{2x}$ and $h(x) = e^{x+1}$. What are $g^{-1}(x)$ and $h^{-1}(x)$?

(c) Describe which **basic** transformation we need to

- transform the graph of $f(x) = e^x$ into the graph of $g(x) = e^{2x}$.
- transform the graph of $f^{-1}(x) = \ln(x)$ into the graph of $g^{-1}(x)$, as found above.

(d) Describe which **basic** transformation we need to

- transform the graph of $f(x) = e^x$ into the graph of $h(x) = e^{x+1}$.
- transform the graph of $f^{-1}(x) = \ln(x)$ into the graph of $h^{-1}(x)$ as found above.

(e) Fill in the blanks below with options from the bank of terms.

If we __________ the original function by a factor of a, its corresponding inverse function will be __________ by a factor of a ($a > 1$).

Term Bank: horizontally compress, horizontally stretch, vertically compress, vertically stretch

If we __________ the original function a ($a > 0$) units, its corresponding inverse function will be __________ a units ($a > 0$).

Term Bank: horizontally shift left, horizontally shift right, vertically shift up, vertically shift down

Problem 6. [8 points]

The number of hours of daylight in a northeastern city is modeled by the function

$$N(t) = 12 + 3\sin[\frac{2\pi}{365}(t - 79)]$$

where t is the number of days after January 1.

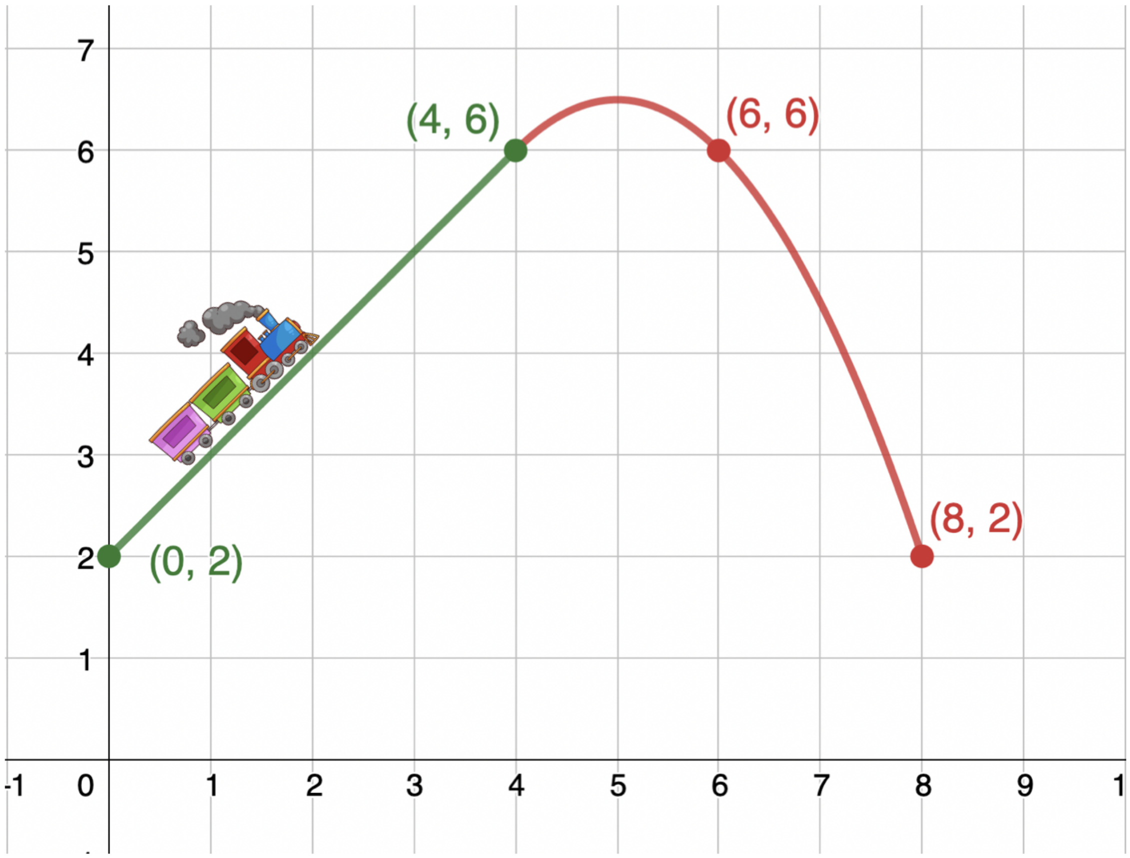
(a) Find the amplitude and period of $N(t)$.

(b) How many hours of sunlight does the model predict on the longest day of the year?

(c) How many hours of sunlight does the model predict 90 days after January 1?

Problem 7. [9 points]

The figure below shows a track of the Green-Red Mountain. The first part of the mountain is linear, while the rest is part of the quadratic curve $-\frac{1}{2}x^2 + 5x - 6$.



(a) Suppose the figure depicts the entire mountain, modeled by a function $f(x)$. Using the given coordinates and the formula of the quadratic curve, write down a piecewise definition of $f(x)$.

(b) Engineers are working to decide if it is safe for a train to travel on the mountain or if they need to build a tunnel through the mountain. If the absolute value of the slope at the foot of the mountain, which is the point $(8, 2)$, is greater than 2, it is too dangerous to run a train, and they need to build a tunnel. Compute the slope of the secant line through $(7, \frac{9}{2})$ and $(8, 2)$, and the slope of the secant line through $(\frac{15}{2}, \frac{27}{8})$ and $(8, 2)$. Use these slopes to estimate the slope of the tangent line at $(8, 2)$ and help the engineers make the decision.

Problem 8. [6 points]

Show algebraically whether the following functions are even, odd, or neither:

(a) $f(x) = e^{|x|}$

(b) $g(x) = \sin x \cos x$

(c) $h(x) = x \sin x$

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