

Problem 1. Section 2.4 #132

Determine the point(s), if any, at which the function $f(x) = \frac{2}{x^2+1}$ is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

Since $x^2+1 = 0$ has no (real) solutions, we conclude that the function has no discontinuities.

Problem 2. Section 2.4 #134

Determine the point(s), if any, at which the function $g(t) = t^{-1} + 1$ is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

Since we can write $g(t) = \frac{1}{t} + 1$, we see that g has an infinite discontinuity at $t = 0$.

Problem 3. Section 2.4 #136

Determine the point(s), if any, at which the function $f(x) = \frac{|x-2|}{x-2}$ is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

We can rewrite f as a piecewise function:

$$f(x) = \begin{cases} -1 & x < 2 \\ 1 & x > 2 \end{cases}$$

We see that f has a jump discontinuity at $x = 2$.

Problem 4. Section 2.4 #138

Determine the point(s), if any, at which the function $f(t) = \frac{t+3}{t^2+5t+6}$ is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

We can factor the denominator $t^2 + 5t + 6 = (t+2)(t+3)$, so $f(t) = \frac{1}{t+2}$, keeping in mind that $t = -3$ is still not in the domain of f . Since the denominator of f can still be zero at $t = -2$, we see that we have a removable discontinuity at $t = -3$ and an infinite discontinuity at $t = -2$.

Problem 5. Section 2.4 #142

Decide if the function $f(y) = \frac{\sin(\pi y)}{\tan(\pi y)}$ is continuous at $y = 1$. If it is discontinuous, what type of discontinuity is it?

$$f(y) = \frac{\sin(\pi y)}{\tan(\pi y)} = \frac{\sin(\pi y)}{\frac{\sin(\pi y)}{\cos(\pi y)}} = \cos(\pi y)$$

This last step involved cancelling a $\sin(\pi y)$, so our function has removable discontinuities at $\sin(\pi y) = 0$. $y = 1$ is a solution to this equation, so $f(y)$ has a removable discontinuity at $y = 1$.

Problem 6. Section 2.4 #146

Find the value(s) of k that make the function continuous over the given interval.

$$f(\theta) = \begin{cases} \sin(\theta) & 0 \leq \theta < \frac{\pi}{2} \\ \cos(\theta + k) & \frac{\pi}{2} \leq \theta \leq \pi \end{cases}$$

$$\sin\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} + k\right)$$

$$1 = \cos\left(\frac{\pi}{2} + k\right)$$

$$\arccos(1) + 2\pi n = \frac{\pi}{2} + k$$

$$0 + 2\pi n = \frac{\pi}{2} + k$$

$$\boxed{k = -\frac{\pi}{2} + 2\pi n}$$

Problem 7. Section 2.4 #148

Find the value(s) of k that make the function continuous over the given interval.

$$f(x) = \begin{cases} e^{kx} & 0 \leq x < 4 \\ x + 3 & 4 \leq x \leq 8 \end{cases}$$

$$e^k(4) = (4) + 3$$

$$e^{4k} = 7$$

$$4k = \ln 7$$

$$\boxed{k = \frac{1}{4} \ln 7}$$

Problem 8. Section 2.4 #154

Consider the graph of the function $y = f(x)$ shown in the graph.

- a. Find all values for which the function is discontinuous.

The function is discontinuous at $\boxed{x = -1, 0}$.

- b. For each value in part a., state why the formal definition of continuity does not apply.

For the discontinuity at $x = -1$, the left-sided limit is 3, which does not agree with the right-sided limit of 1. For the discontinuity at $x = 0$, the left-sided limit is ∞ .

- c. Classify each discontinuity as either jump, removable, or infinite.

The discontinuity at $x = -1$ is a jump discontinuity. The discontinuity at $x = 0$ is an infinite discontinuity.