

Problem 1. Section 3.3 #112

For the following exercises, find $f'(x)$ for each function.

Here $f(x) = (x + 2)(2x^2 - 3)$.

We have $f'(x) = (2x^2 - 3) + (x + 2)(4x) = 6x^2 + 8x - 3$.

Problem 2. Section 3.3 #116

For the following exercises, find $f'(x)$ for each function.

Here $f(x) = \frac{x^2+4}{x^2-4} = 1 + \frac{8}{x^2-4}$.

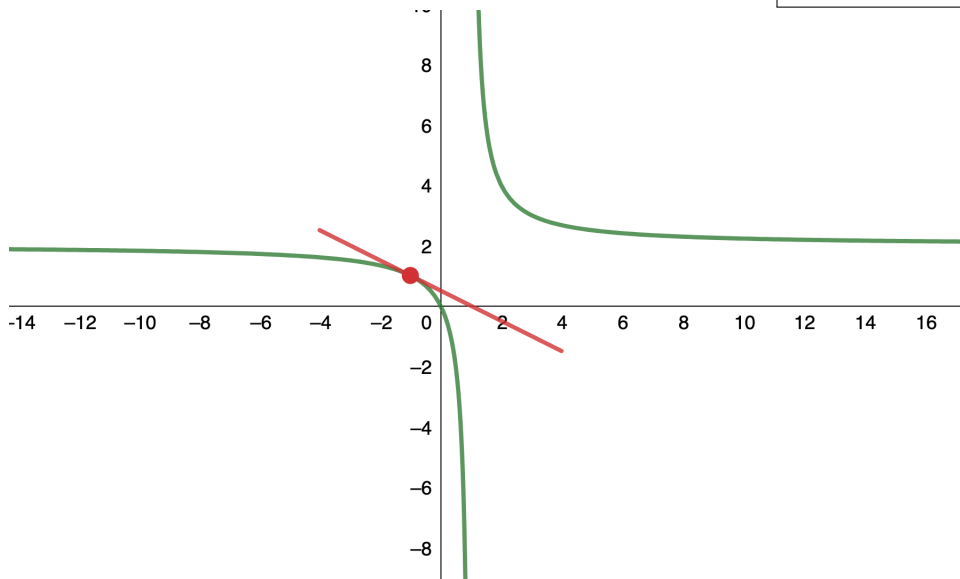
We have $f'(x) = \frac{0-8(2x)}{(x^2-4)^2} = -\frac{16x}{(x^2-4)^2}$.

Problem 3. Section 3.3 #120

For the following exercises, find the equation of the tangent line $T(x)$ to the graph of the given function at the indicated point. Graph the function and the tangent line.

Given that $y = \frac{2x}{x-1}$, we get $\frac{dy}{dx} = \frac{2(x-1)-2x}{(x-1)^2} = -\frac{2}{(x-1)^2}$. Therefore, the slope of the tangent function at $(-1, 1)$ is $\left. \frac{dy}{dx} \right|_{x=-1} = -\frac{1}{2}$.

The slope-point form of the tangent line $T(x)$ is simply $y - 1 = -\frac{1}{2}(x + 1)$.



Problem 4. Section 3.3 #126

For the following exercises, assume that $f(x)$ and $g(x)$ are both differentiable functions with values as given in the following table. Use the following table to calculate the following derivatives.

Since $h(x) = xf(x) + 4g(x)$, we immediately obtain $\boxed{h'(x) = f(x) + xf'(x) + 4g'(x)}$.

When $x = 1$, we have $\boxed{h'(1) = f(1) + f'(1) + 4g'(1) = 3 - 1 + 16 = 18}$.

Problem 5. Section 3.3 #128

For the following exercises, assume that $f(x)$ and $g(x)$ are both differentiable functions with values as given in the following table. Use the following table to calculate the following derivatives.

Since $h(x) = 2x + f(x)g(x)$, we immediately obtain $\boxed{h'(x) = 2 + f'(x)g(x) + f(x)g'(x)}$.

When $x = 3$, we have $\boxed{h'(1) = 2 + f'(3)g(3) + f(3)g'(3) = 2 - 32 - 4 = -34}$.

Problem 6. Section 3.3 #132

For the following exercises, use the following figure to find the indicated derivatives, if they exist.

We know that $h(x) = \frac{f(x)}{g(x)}$, as a result of which, we get $\boxed{h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}}$.

a. $\boxed{h'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{g^2(1)} = \frac{-1-3}{1} = -4}$.

b. $\boxed{h'(3) \text{ does not exist as } f'(3) \text{ does not exist}}$.

c. $\boxed{h'(4) = \frac{f'(4)g(4) - f(4)g'(4)}{g^2(4)} = \frac{\frac{5}{2} - 0}{(\frac{5}{2})^2} = \frac{2}{5}}$.

Problem 7. Section 3.3 #146

The population in millions of arctic flounder in the Atlantic Ocean is modeled by the function $P(t) = \frac{8t+3}{0.2t^2+1}$, where t is measured in years.

- a. We have $P(0) = \frac{3}{1} = 3$. That is to say, there are 3 millions of flounder initially.
- b. We have $P'(t) = \frac{8(0.2t^2+1)-(8t+3)(0.4t)}{(0.2t^2+1)^2}$. Hence we get $P'(10) = \frac{8 \cdot 21 - 83 \cdot 4}{21^2} = -\frac{164}{441}$. That is to say, the instantaneous growth rate of the population of arctic flounder is $-\frac{164}{441}$ at the 10th year and the population is actually reducing at that time.

Problem 8. Section 3.3 #148

A book publisher has a cost function given by $C(x) = \frac{x^3+2x+3}{x^2}$, where x is the number of copies of a book in thousands and C is the cost, per book, measured in dollars. Evaluate $C'(2)$ and explain its meaning.

$$\text{Here we have } C'(x) = \frac{(3x^2+2)x^2 - (x^3+2x+3)(2x)}{x^4} = \frac{x^3-2x-6}{x^3}.$$

Hence, $C'(2) = \frac{8-4-6}{8} = -\frac{1}{4}$. That is to say, when the number of copies is 2000, the instantaneous rate of change of cost per book is $-\frac{1}{4}$ and the cost is actually decreasing at that point.