Problem 1. Section 1.1 #8

For the following exercises, find the values for $f(x) = 4x^2 - 3x + 1$; if they exist, then simplify.

a.
$$f(0) = 4(0)^2 - 3(0) + 1 = \boxed{1}$$

b.
$$f(1) = 4(1)^2 - 3(1) + 1 = 4 - 3 + 1 = \boxed{2}$$

c.
$$f(3) = 4(3)^2 - 3(3) + 1 = 36 - 9 + 1 = 28$$

d.
$$f(-x) = 4(-x)^2 - 3(-x) + 1 = 4x^2 + 3x + 1$$

e.
$$f(a) = 4(a)^2 - 3(a) + 1 = 4a^2 - 3a + 1$$

f.
$$f(a+h) = 4(a+h)^2 - 3(a+h) + 1 = 4(a^2 + 2ah + h^2) - 3(a+h) + 1 = 4(a^2 + 8ah + 4h^2 - 3a - 3h + 1)$$

Problem 2. Section 1.1 #10

For the following exercises, find the values for f(x) = |x-7| + 8; if they exist, then simplify.

a.
$$f(0) = |0 - 7| + 8 = |-7| + 8 = 7 + 8 = \boxed{15}$$

b.
$$f(1) = |1 - 7| + 8 = |-6| + 8 = 6 + 8 = \boxed{14}$$

c.
$$f(3) = |3 - 7| + 8 = |-4| + 8 = 4 + 8 = \boxed{12}$$

d.
$$f(-x) == [-x-7|+8]$$

e.
$$f(a) = |a-7| + 8$$

f.
$$f(a+h) = |a+h-7| + 8$$

Problem 3. Section 1.1 #36

For f(x) = 3x + 4 and g(x) = x - 2, find f + g, f - g, $f \cdot g$, and f/g. Determine the domain of each of these new functions.

a.
$$(f+g)(x) = f(x) + g(x) = (3x+4) + (x-2) = 4x+2$$

The domain is \mathbb{R} .

b.
$$(f-g)(x) = f(x) - g(x) = (3x+4) - (x-2) = 2x+6$$

The domain is \mathbb{R} .

c.
$$(f \cdot g)(x) = f(x) \cdot g(x) = (3x+4)(x-2) = 3x^2 - 2x - 8$$

The domain is \mathbb{R}

d.
$$(f/g)(x) = \frac{f(x)}{g(x)} = \left[\frac{3x+4}{x-2}\right]$$
.

The domain is $[x \mid x \neq 2]$

Problem 4. Section 1.1 #40

For $f(x) = \sqrt{x}$ and g(x) = x - 2, find f + g, f - g, $f \cdot g$, and f/g. Determine the domain of each of these new functions.

a. $(f+g)(x) = (\sqrt{x}) + (x-2) = \sqrt{x} + x - 2$.

The domain is $[0, \infty)$.

b. $(f-g)(x) = (\sqrt{x}) - (x-2) = \sqrt{x-x+2}$

The domain is $[0, \infty)$.

c. $(f \cdot g)(x) = (\sqrt{x}) \cdot (x-2) = \left| x^{\frac{3}{2}} - 2\sqrt{x} \right|$

The domain is $[0, \infty)$.

d. $(f/g)(x) = \sqrt{\frac{x}{x-2}}$

The domain is $\{x \mid x \ge 0, x \ne 2\}$

Problem 5. Section 1.1 #42

For f(x) = 3x and g(x) = x + 5, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Simplify. Find the domain of each of the results.

a. $(f \circ g)(x) = f(g(x)) = f(x+5) = 3(x+5) = 3x+15$

The domain is \mathbb{R} .

b. $(q \circ f)(x) = q(f(x)) = q(3x) = 3x + 5$.

The domain is \mathbb{R} .

Problem 6. Section 1.1 #46

For $f(x) = \sqrt{x}$ and g(x) = x + 9, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Simplify. Find the domain of each of the results.

a. $(f \circ g)(x) = f(g(x)) = f(x+9) = \sqrt{x+9}$

The domain is $[-9, \infty)$.

b. $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x+9}$

The domain is $[0, \infty)$.

Problem 7. Section 1.1 #56

An American tourist visits Paris and must convert U.S. dollars to Euros, which can be done using the function E(x) = 0.79x, where x is the number of U.S. dollars and E(x) is the equivalent number of Euros. Since conversion rates fluctuate, when the tourist returns to the United States 2 weeks later, the conversion from Euros to U.S. dollars is D(x) = 1.245x, where x is the number of Euros and D(x) is the equivalent number of U.S. dollars.

a. Find the composite function that converts directly from U.S. dollars to U.S. dollars via Euros. Did this tourist lose value in the conversion process?

Since we want to first convert U.S. dollars to Euros, we start with the function E(x). Then, since we want to convert those Euros back to U.S. dollars, we apply the function D to E(x), to get D(E(x)), also written $(D \circ E)(x)$. We calculate this function to be:

$$D(E(x)) = D(0.79x) = 1.245(0.79x) = 0.98355x$$

Since this function makes positive numbers smaller, we conclude that the tourist lost value in the conversion process.

b. Use (a) to determine how many U.S. dollars the tourist would get back at the end of her trip if she converted an extra \$200 when she arrived in Paris.

We want to calculate $(D \circ E)(200)$, which is $0.98355 \cdot 200 = 196.71$, so she would get back $\boxed{\$196.71}$.