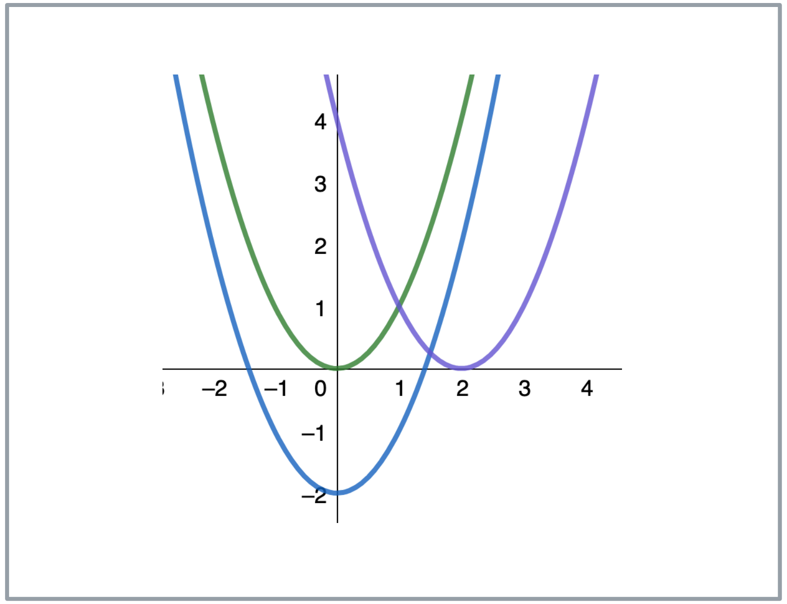
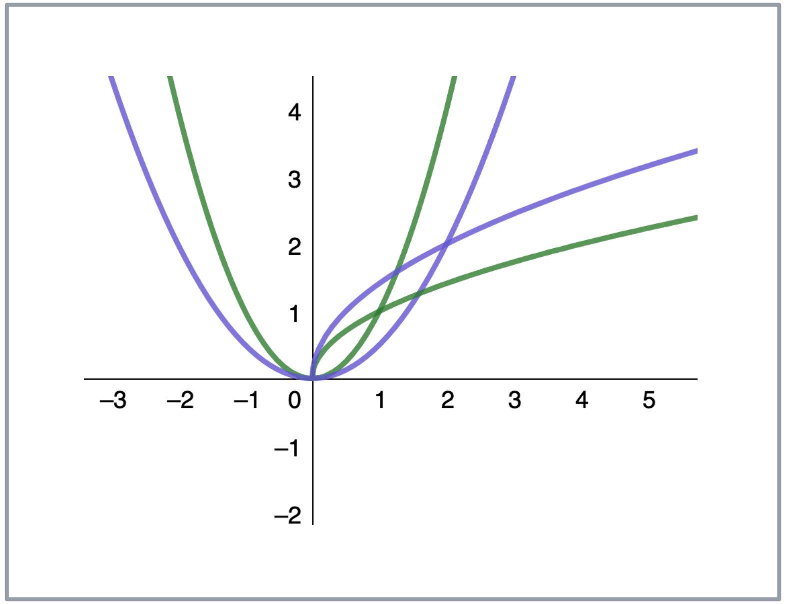
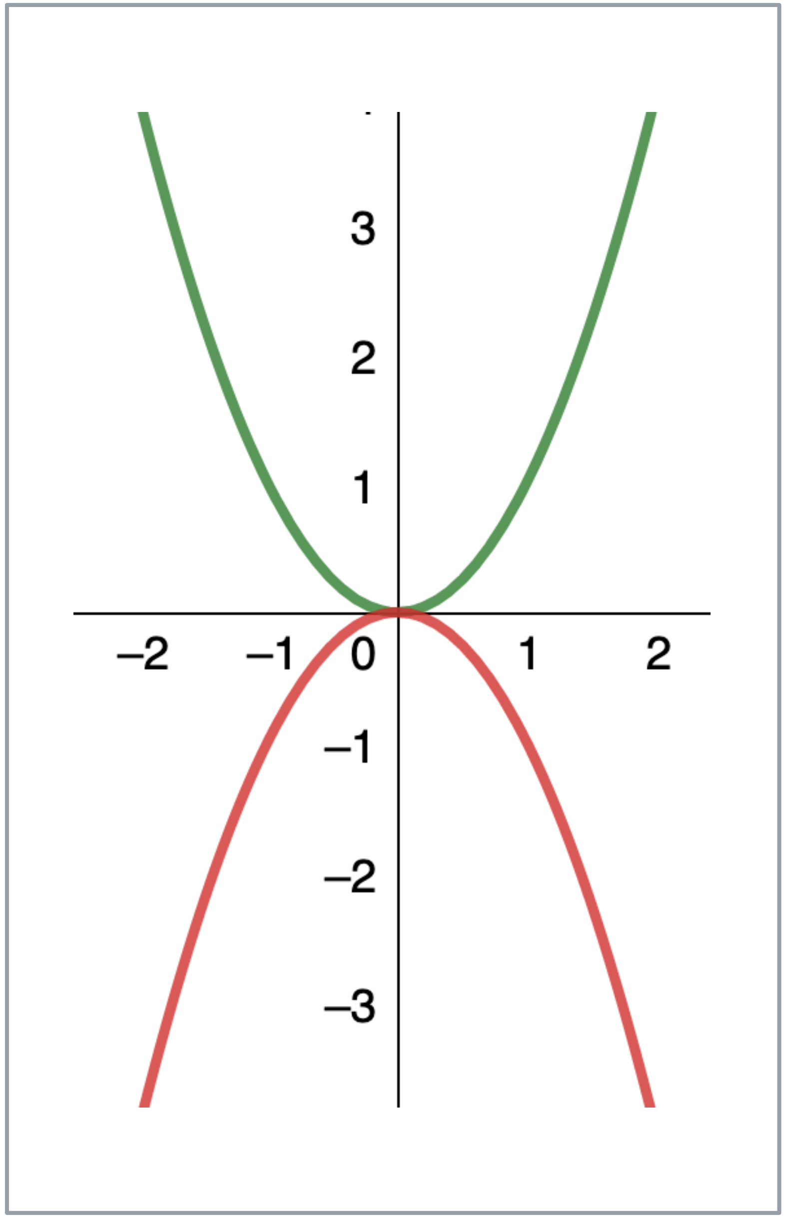
COMPOSITE FUNCTIONS

INTRODUCTION TO CALCULUS



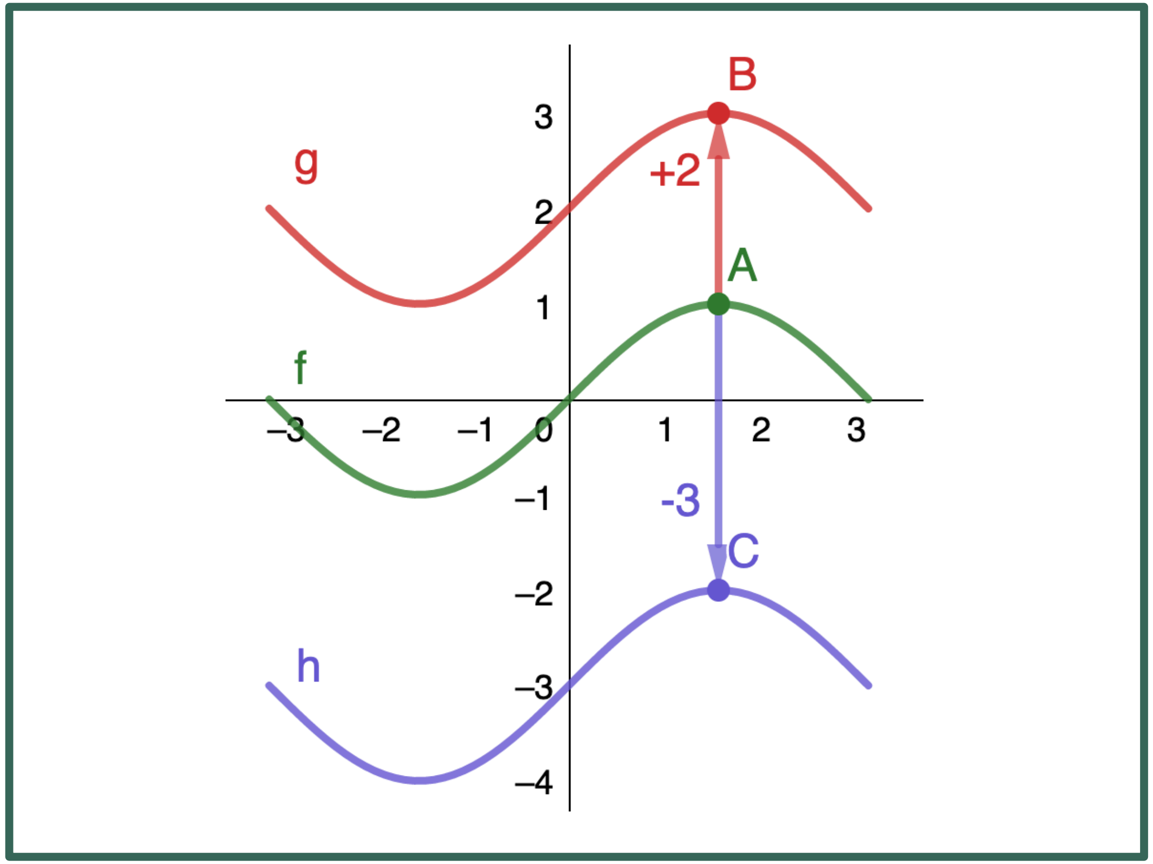




TRANSFORMATION OF A FUNCTION

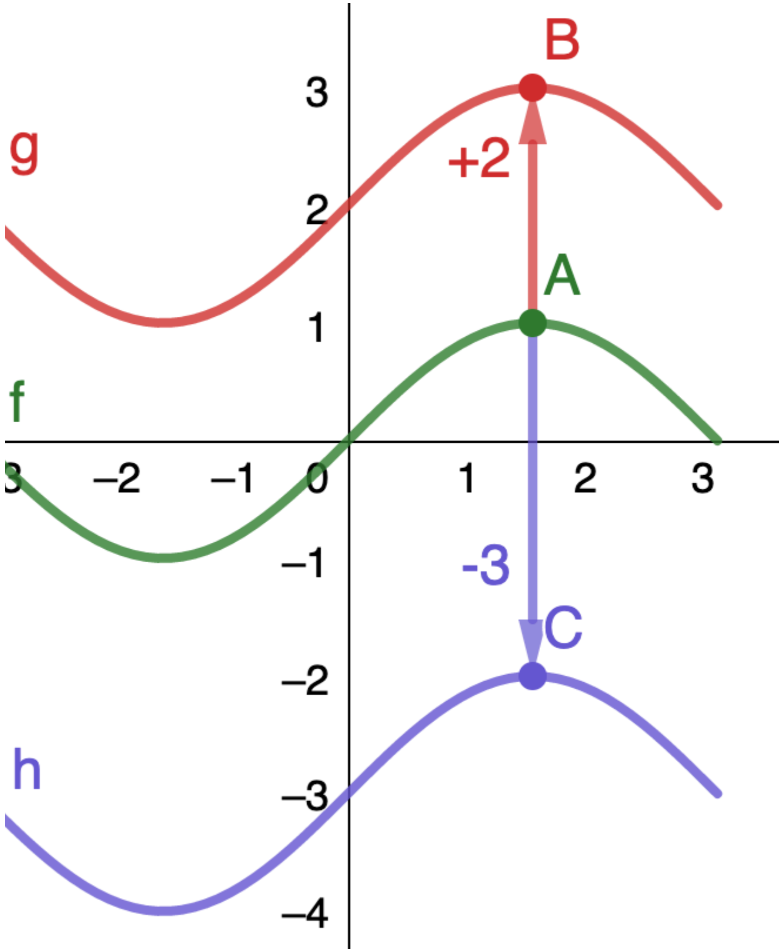
- Shift
  - Horizontal
  - Vertical
- Scaling
  - Horizontal
  - Vertical
- Reflection about the axis

TRANSFORMATION: VERTICAL SHIFT



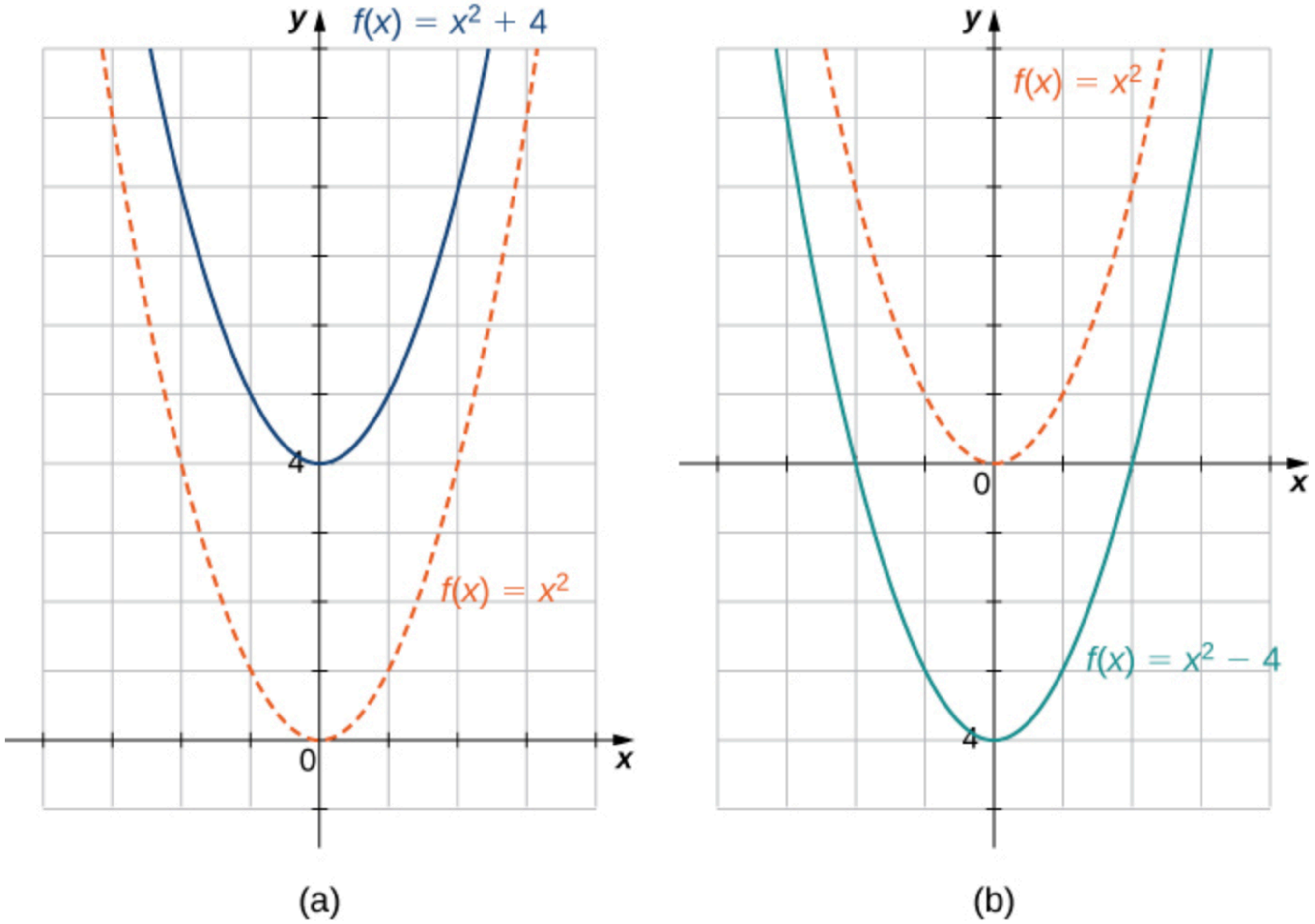
- Original
  - $f(x) = \sin(x)$
- Vertical shift up 2 units
  - ?
- Vertical shift down 3 units
  - ?

TRANSFORMATION: VERTICAL SHIFT

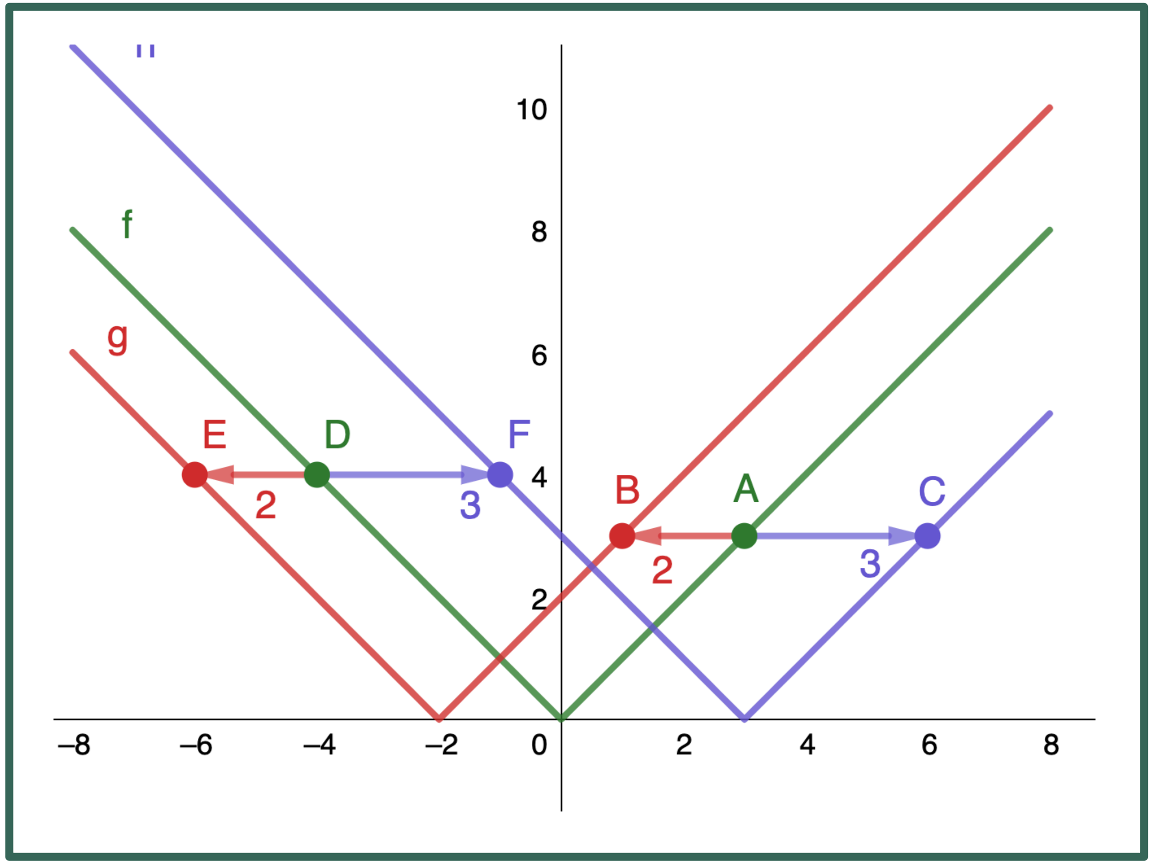


- Original
  - $f(x)$
- Vertical shift up c units $(c > 0)$
  - $f(x) + c$
- Vertical shift down c units $(c > 0)$
  - $f(x) - c$

A vertical shift of a function occurs if we add or subtract the same constant to each output y. For $c > 0$, the graph of $f\,(x) + c$ is a shift of the graph of $f(x)$ up c units, whereas the graph of $f\,(x) - c$ is a shift of the graph of $f\,(x)$ down c units. For example, the graph of the function $f(x) = x^3 + 4$ is the graph of $y = x^3$ shifted up 4 units; the graph of the function $f(x) = x^3 - 4$ is the graph of $y = x^3$ shifted down 4 units (Figure 1.23).

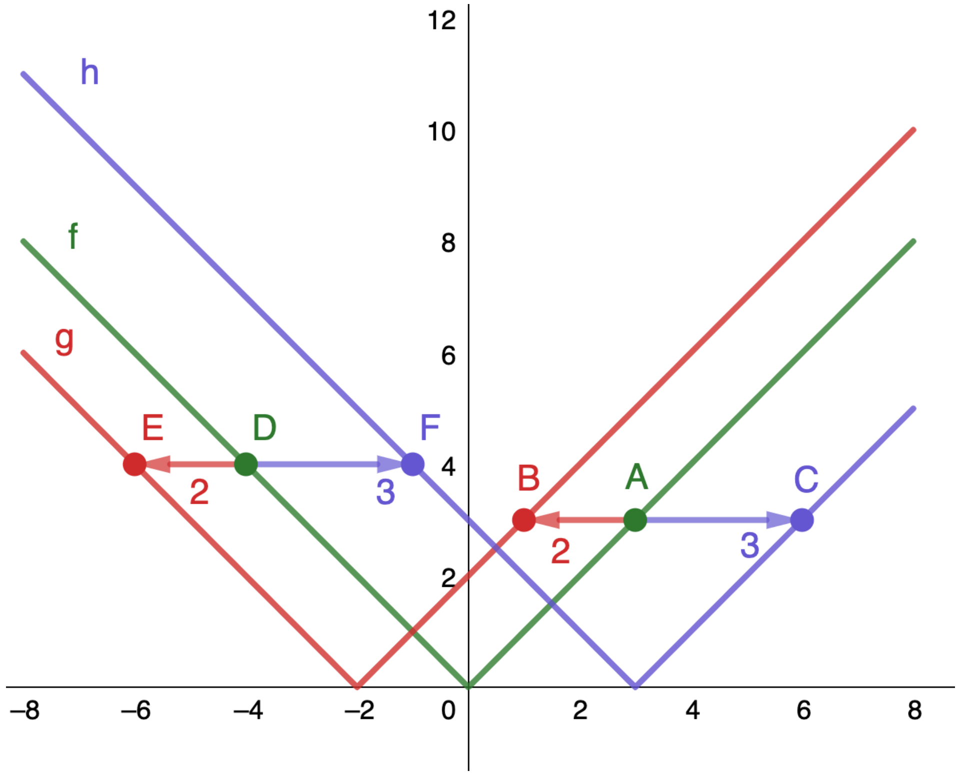


TRANSFORMATION: HORIZONTAL SHIFT



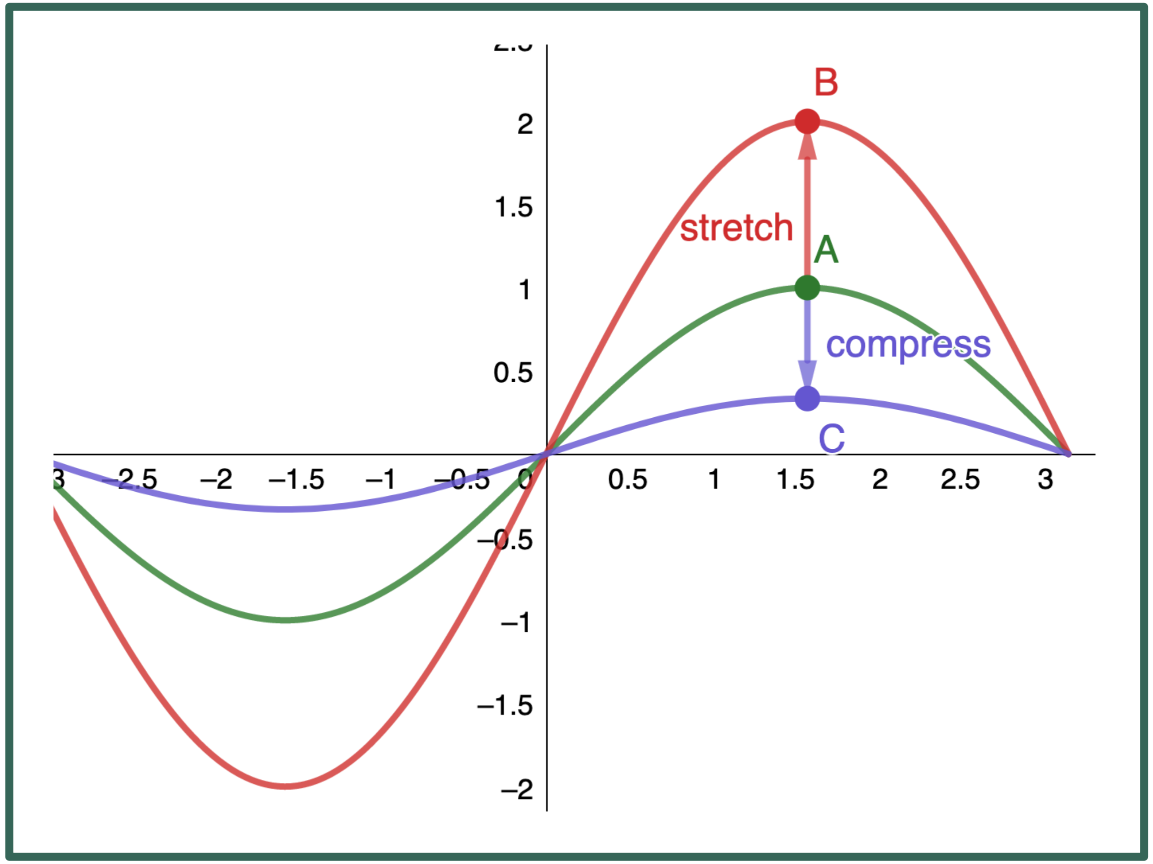
- Original
  - $f(x) = |x|$
- Horizontal shift left 2 units
  - ?
- Horizontal shift right 3 units
  - ?

TRANSFORMATION: HORIZONTAL SHIFT



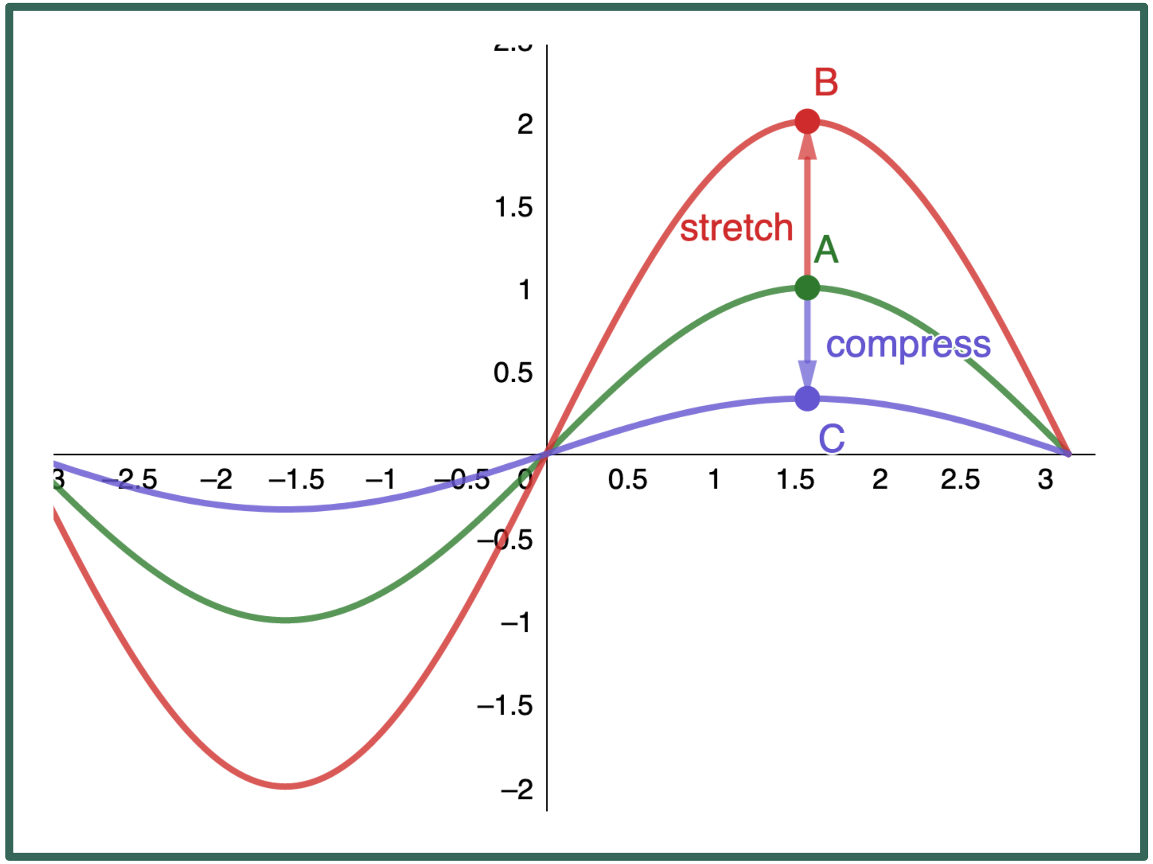
- Original
  - $f(x)$
- Horizontal shift left c units $(c > 0)$
  - $f(x + c)$
- Horizontal shift right c units $(c > 0)$
  - $f(x - c)$

TRANSFORMATION: VERTICAL SCALING



- Original
  - $f(x) = \sin(x)$
- Vertical scaling by a factor of 2
  - ?
- Vertical scaling by a factor of $\frac{1}{3}$
  - ?
- What is the difference between vertical scaling and vertical shift?

TRANSFORMATION: VERTICAL SCALING

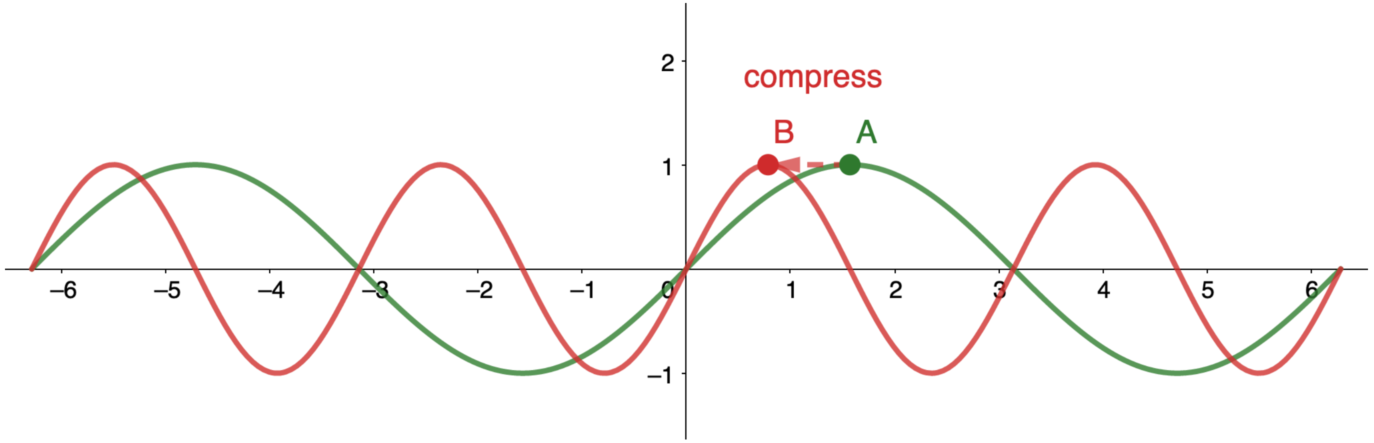


- Original
  - $f(x)$
- New
  - $cf(x)$
- Vertical scaling by a factor of $c > 1$
  - stretching
- Vertical scaling by a factor of $0 < c < 1$
  - compressing

TRANSFORMATION: HORIZONTAL SCALING

- Original
  - $f(x) = \sin(x)$
- Horizontal scaling by a factor of 2
  - ?

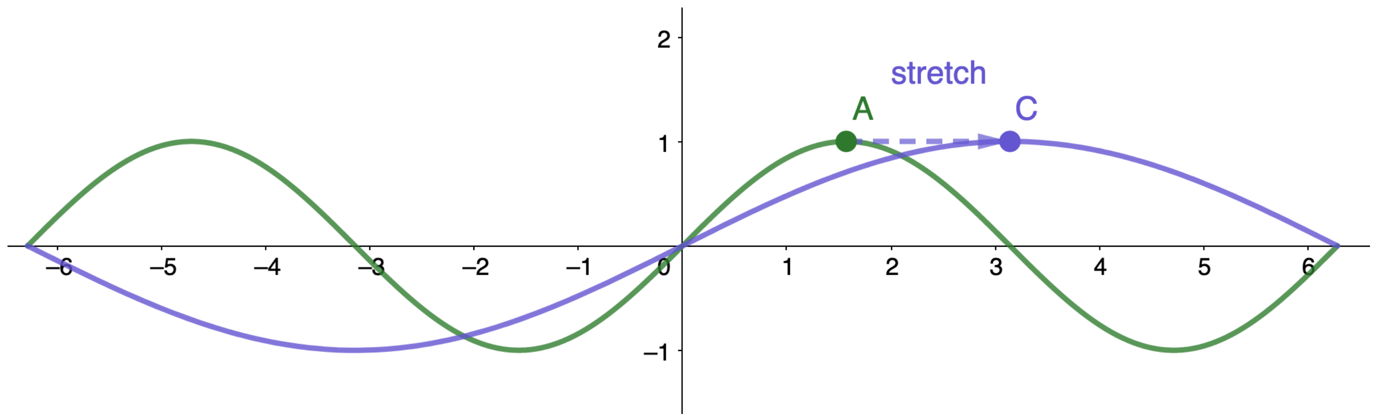
Assume that we know $\sin\left(\frac{\pi}{2}\right) = 1$



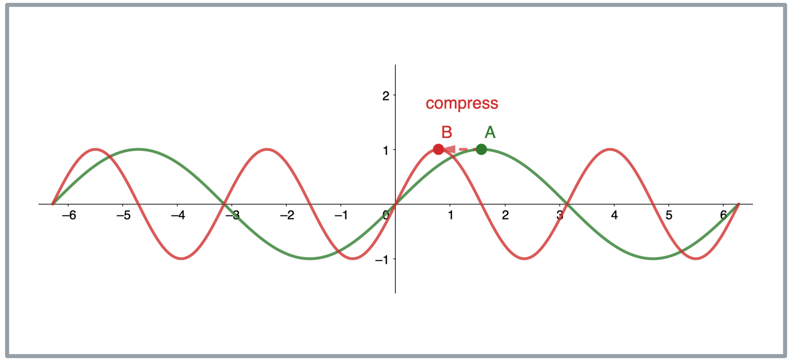
TRANSFORMATION: HORIZONTAL SCALING

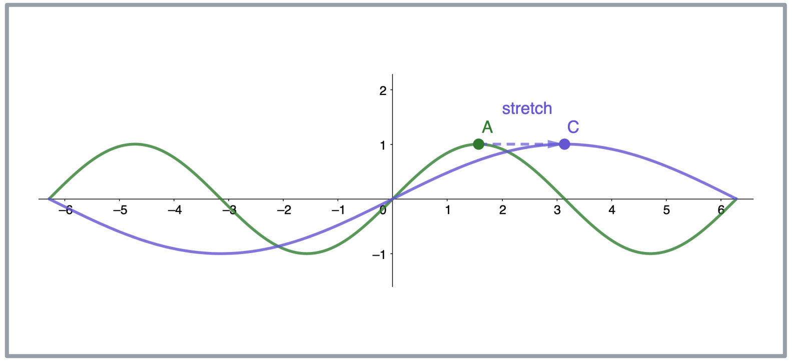
- Original
  - $f(x) = \sin(x)$
- Horizontal scaling by a factor of $\frac{1}{2}$
  - ?
- What is the difference between horizontal scaling and horizontal shift?

Assume that we know $\sin\left(\frac{\pi}{2}\right) = 1$



TRANSFORMATION: HORIZONTAL SCALING

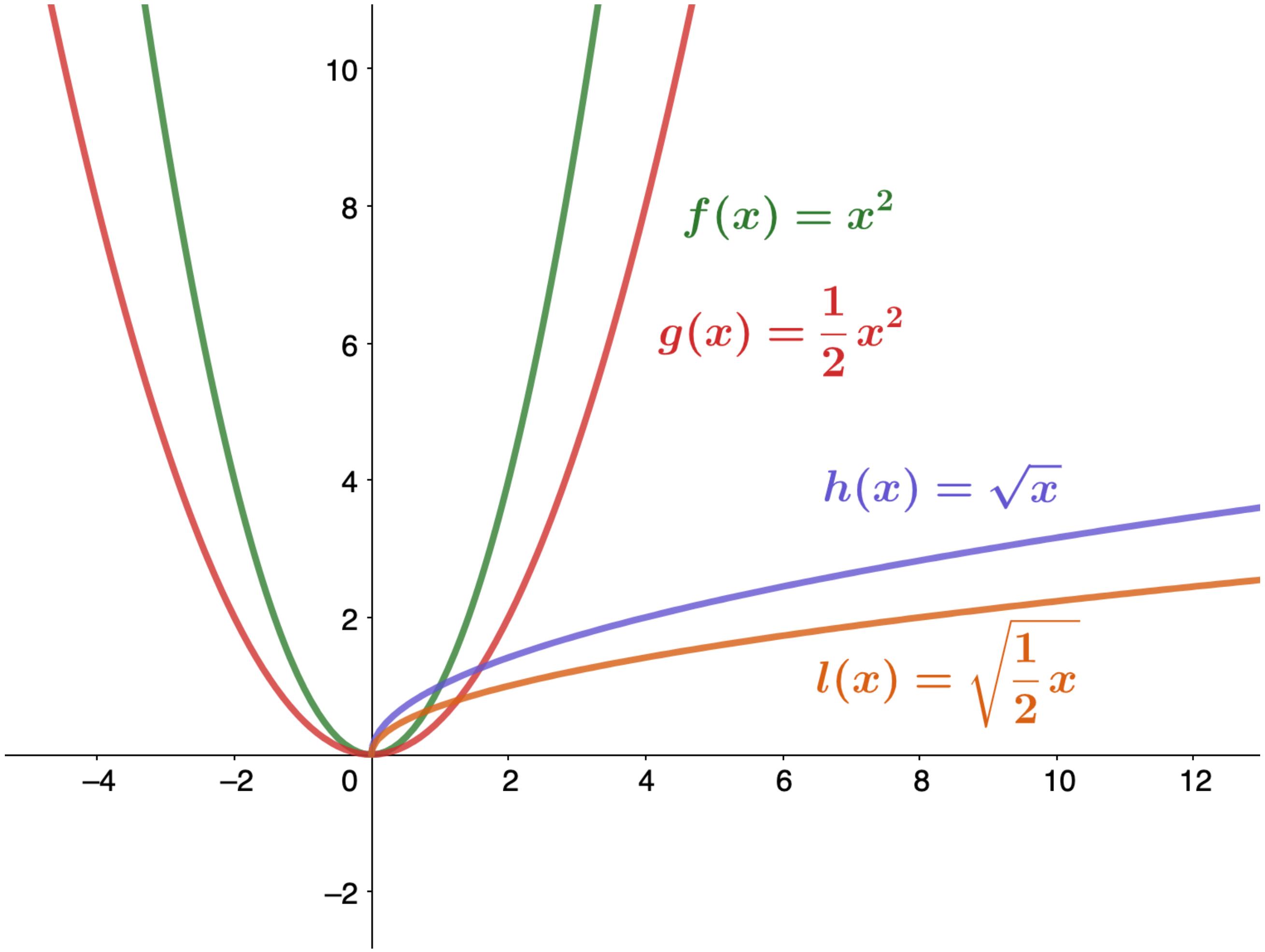


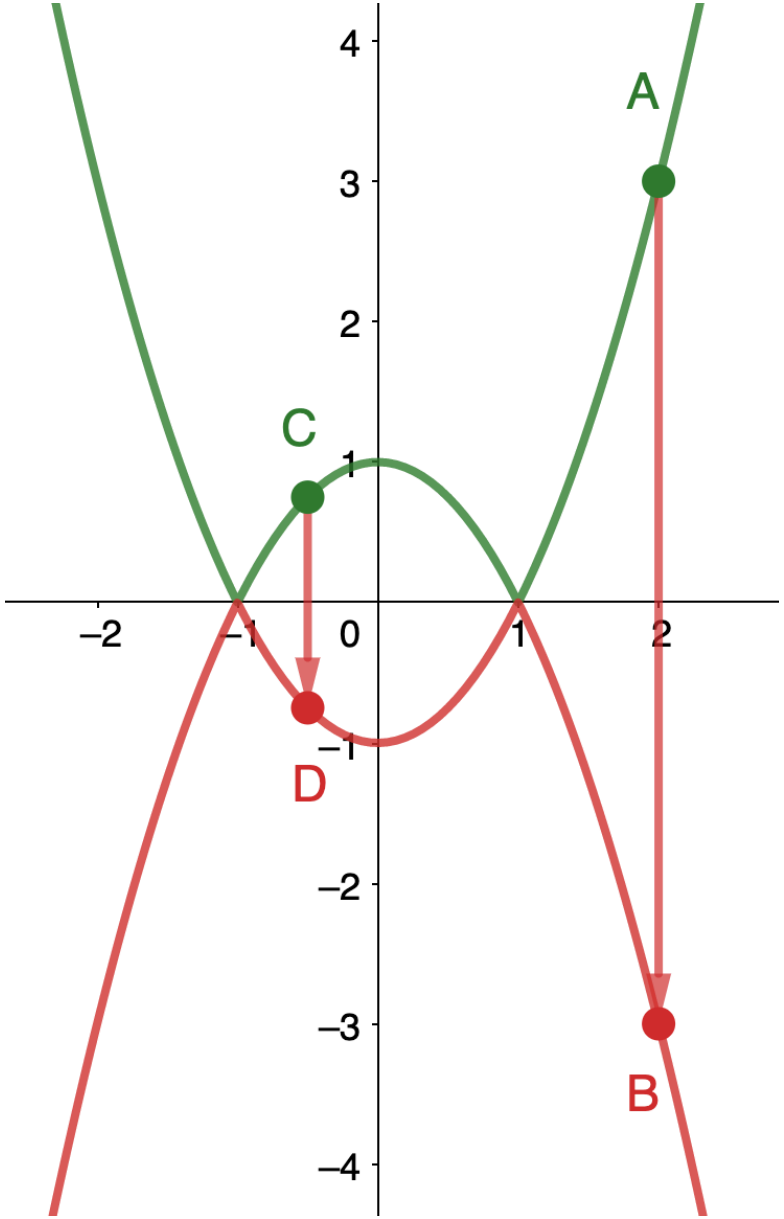


- Original
  - $f(x)$
- New
  - $f(cx)$
- Horizontal scaling by a factor of $c > 1$
  - compressing
- Horizontal scaling by a factor of $0 < c < 1$
  - stretching

TRANSFORMATION: REFLECTION

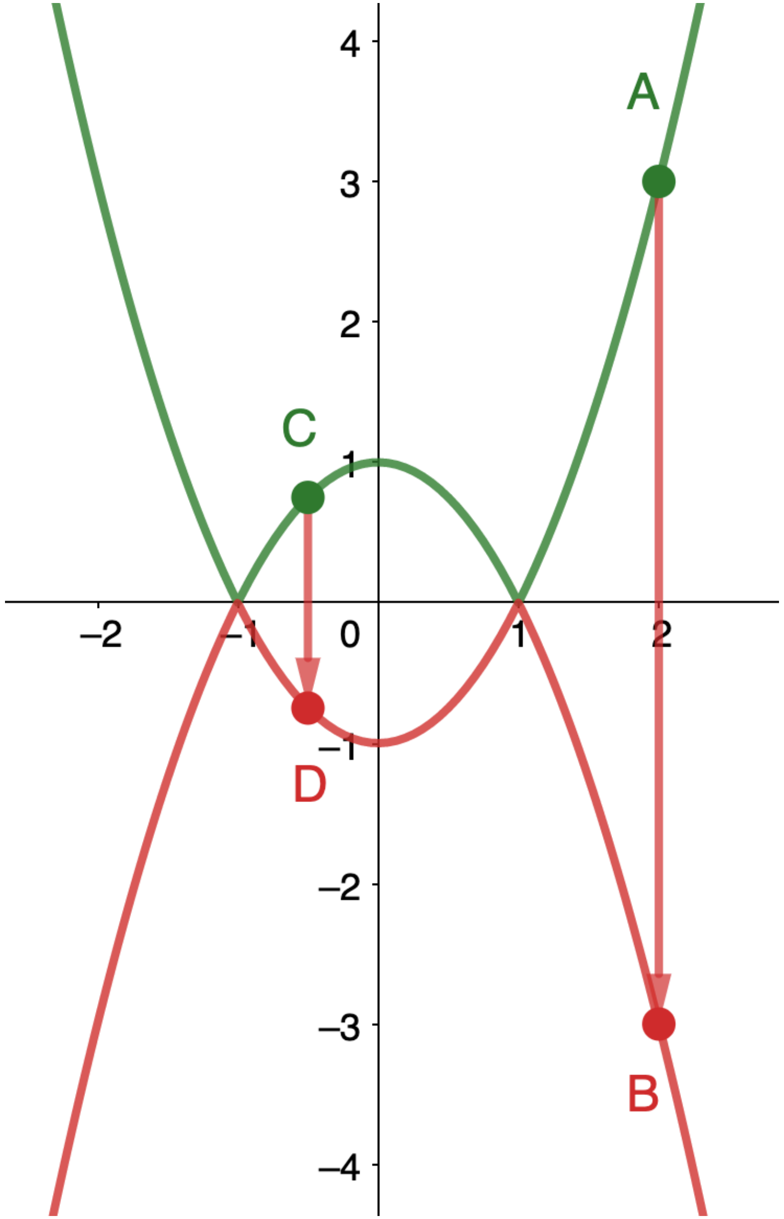
- Vertical scaling
  - $cf(x)$
- Horizontal scaling
  - $f(cx)$
- What if $c < 0$?
- In particular, what if $c = -1$?





TRANSFORMATION: REFLECTION $c = -1$

- Original
  - $f(x) = |x^2 - 1|$
- Reflection about the x-axis
  - ?
  - $y \to -y$ ($f(x) \to -f(x)$) or $x \to -x$?

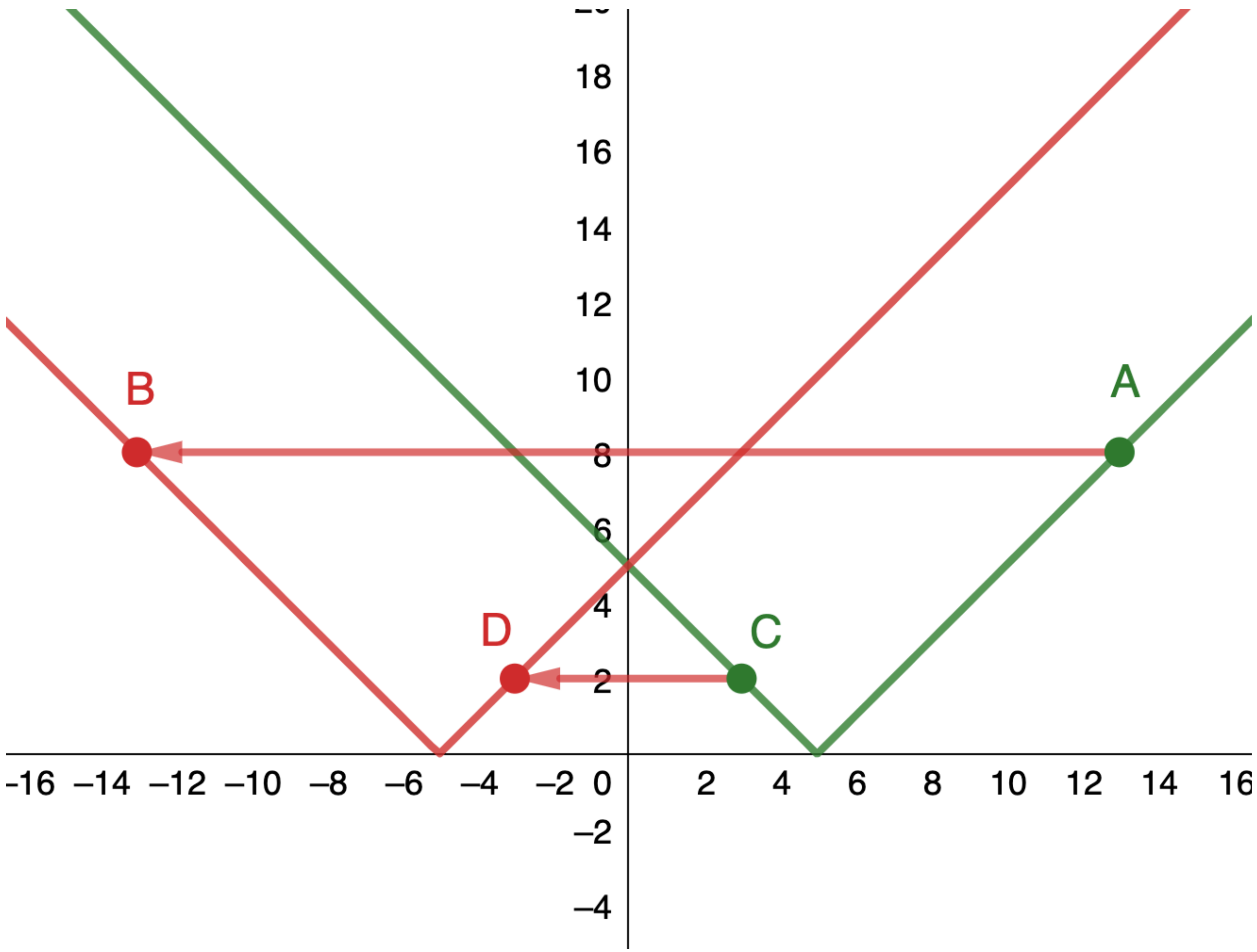


TRANSFORMATION: REFLECTION $c = -1$

- Original
  - $f(x)$
- Reflection about the x-axis
  - $-f(x)$

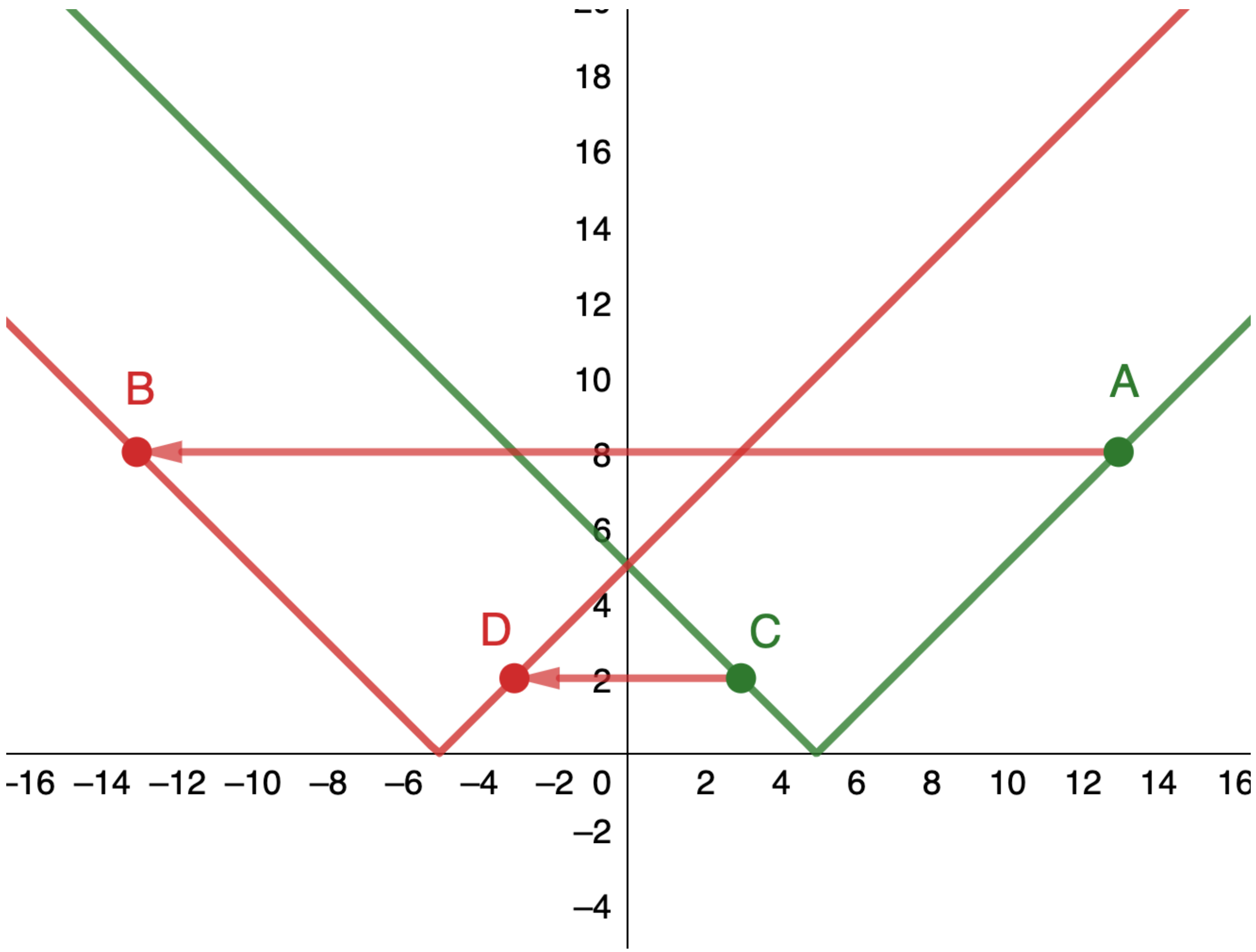
TRANSFORMATION: REFLECTION $c = -1$

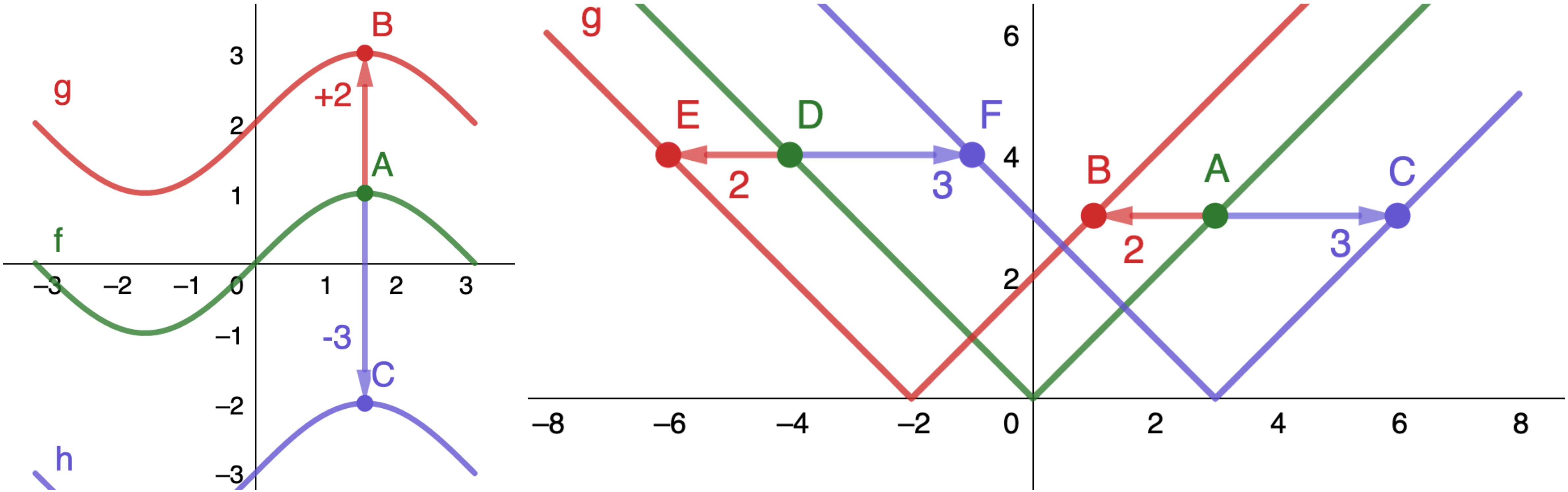
- Original
  - $f(x) = |x - 5|$
- Reflection about the y-axis
  - ?
  - $y \rightarrow -y$ or $x \rightarrow -x$?



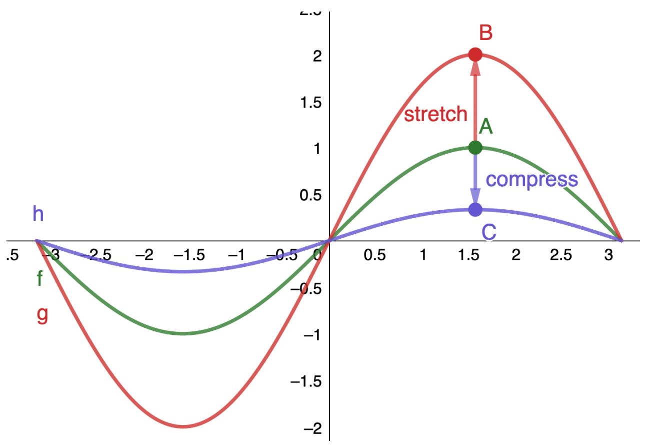
TRANSFORMATION: REFLECTION $c = -1$

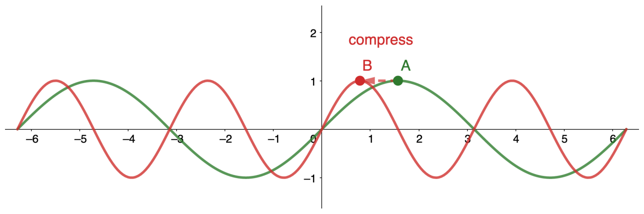
- Original
  - $f(x)$
- Reflection about the y-axis
  - $f(-x)$

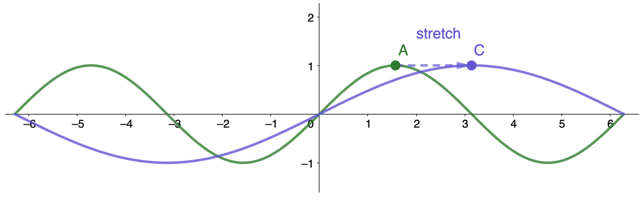




SUMMARY: SHIFT

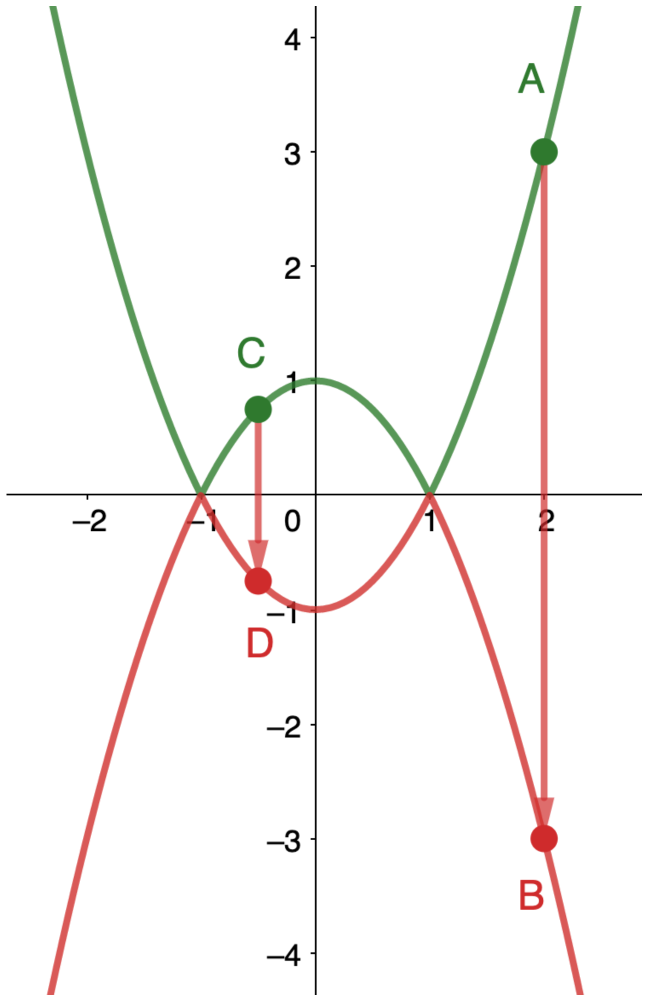


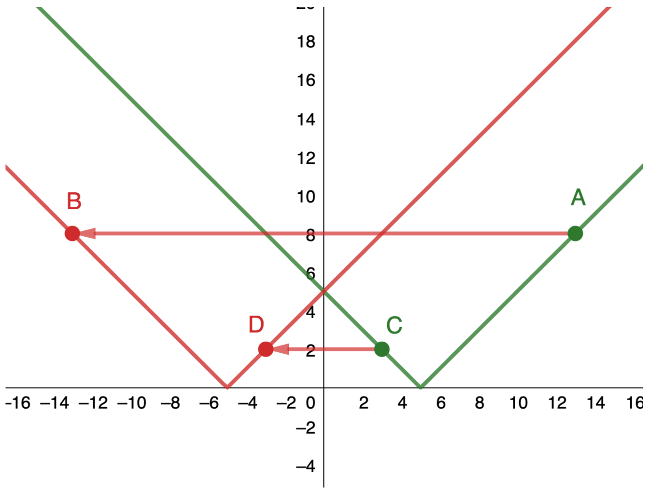




SUMMARY: SCALING (STRETCH AND COMPRESSION)

SUMMARY: REFLECTION

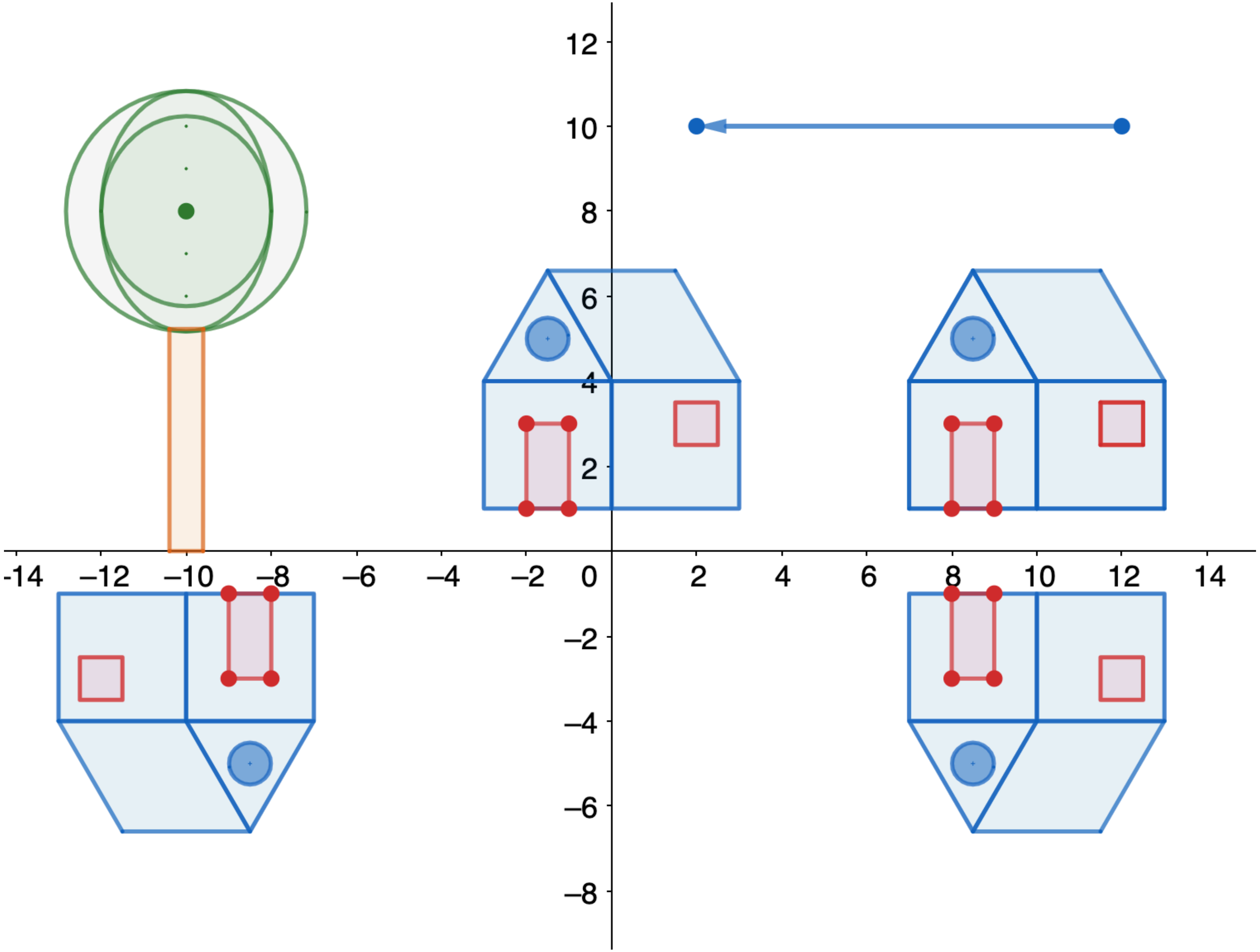




SUMMARY

Transformation of f $(c > 0)$	Effect on the graph of f
$f(x) + c$	Vertical shift up c units
$f(x) - c$	Vertical shift down c units
$f(x + c)$	Shift left by c units
$f(x - c)$	Shift right by c units
$cf(x)$	Vertical stretch if $c > 1$; vertical compression if $0 < c < 1$
$f(cx)$	Horizontal stretch if $0 < c < 1$; horizontal compression if $c > 1$
$-f(x)$	Reflection about the x-axis
$f(-x)$	Reflection about the y-axis

Table 1.7 Transformations of Functions



EXAMPLE

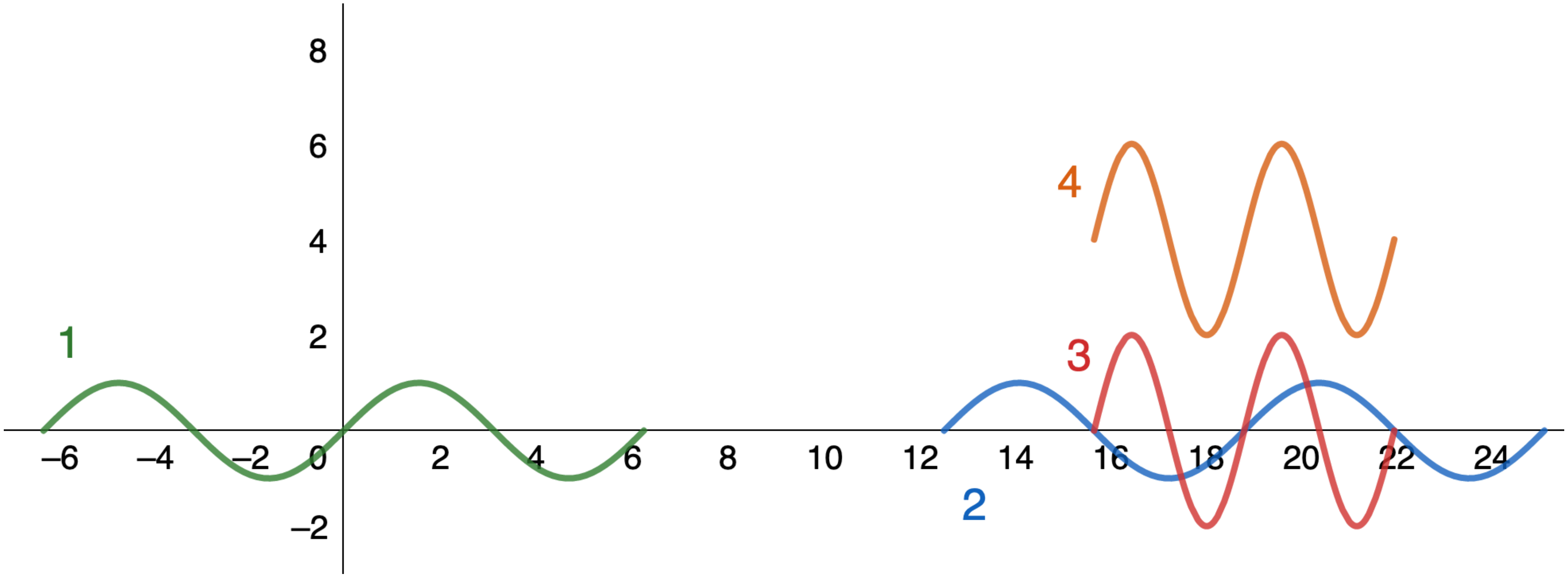
WHICH TRANSFORMATIONS ARE INCLUDED IN THE FIGURE?

- If the graph of a function consists of more than one transformation of another graph, it is important to transform the graph **in the correct order**.

HOW TO DEAL WITH MORE THAN ONE TRANSFORMATION?

FROM 1 TO 2 TO 3 TO 4, WHAT IS THE COMPOUND TRANSFORMATION?

more to cover on trigonometric functions later



ORDER OF TRANSFORMATION

$$f(x)$$

$$cf(a(x+b))+d$$

ORDER OF TRANSFORMATION

$f(x)$

- Original function

$f(x+b)$

ORDER OF TRANSFORMATION

$f(x)$

- Original function

$f(x+b)$

- Horizontal shift. If $b > 0$, shift left, if $b < 0$, shift right.

$f(a(x+b))$

ORDER OF TRANSFORMATION

$f(x)$

- Original function

$f(x+b)$

- Horizontal shift. If $b > 0$, shift left, if $b < 0$, shift right.

$f(a(x+b))$

- Horizontal scaling.

ORDER OF TRANSFORMATION

$f(x)$

- Original function

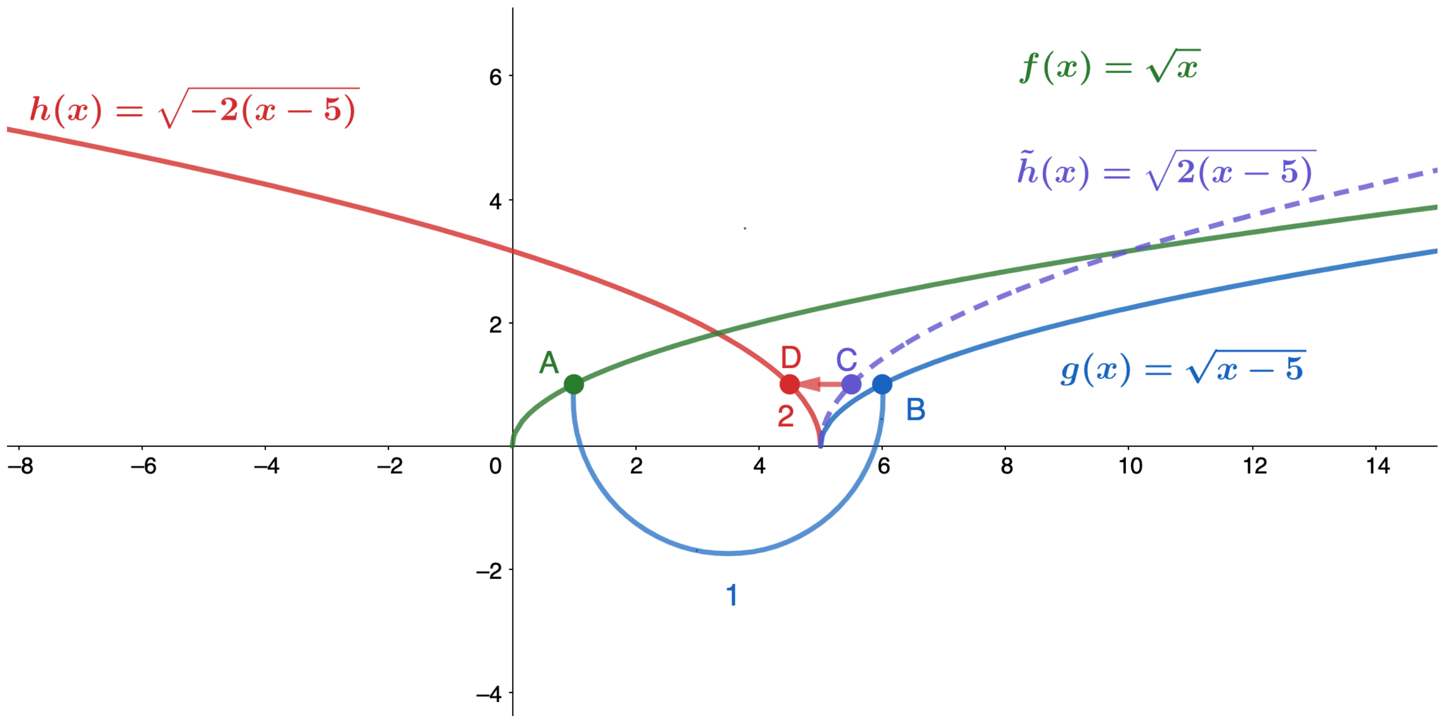
$f(x+b)$

- Horizontal shift. If $b > 0$, shift left, if $b < 0$, shift right.

$f(a(x+b))$

- Horizontal scaling. If $a < 0$, reflect the graph about the y-axis? ✕

ORDER OF TRANSFORMATION



- Original function $f(x) = \sqrt{x}$
- Target function $f(x) = \sqrt{-2(x-5)}$
- Horizontal shift
  - $b = -5 < 0$, shift right
- Horizontal scaling
  - $|a| = 2 > 1$, compress
  - $a = -2 < 0$, reflect about the vertical line $x = -b$.

SECOND TYPO 🚑🚑🚑

If the graph of a function consists of more than one transformation of another graph, it is important to transform the graph in the correct order. Given a function $f(x)$, the graph of the related function $y = cf\left(a\left(x+b\right)\right)+d$ can be obtained from the graph of $y = f(x)$ by performing the transformations in the following order.

1. Horizontal shift of the graph of $y = f(x)$. If $b > 0$, shift left. If $b < 0$, shift right.
2. Horizontal scaling of the graph of $y = f(x+b)$ by a factor of $|a|$. If $a < 0$, reflect the graph about the y-axis.

about $x = -b$

ORDER OF TRANSFORMATION CONTINUED

$f(a(x+b))$

- Horizontal scaling. If $a < 0$, reflect the graph about $x = -b$.

$cf(a(x+b))$

ORDER OF TRANSFORMATION CONTINUED

$f(a(x+b))$

- Horizontal scaling. If $a < 0$, reflect the graph about $x = -b$.

$cf(a(x+b))$

- Vertical scaling. If $\mathrm{c} < 0$, reflect the graph about the x-axis!

$cf\big(a(x+b)\big) + d$

ORDER OF TRANSFORMATION CONTINUED

$f(a(x+b))$

- Horizontal scaling. If $a < 0$, reflect the graph about $x = -b$.

$cf(a(x+b))$

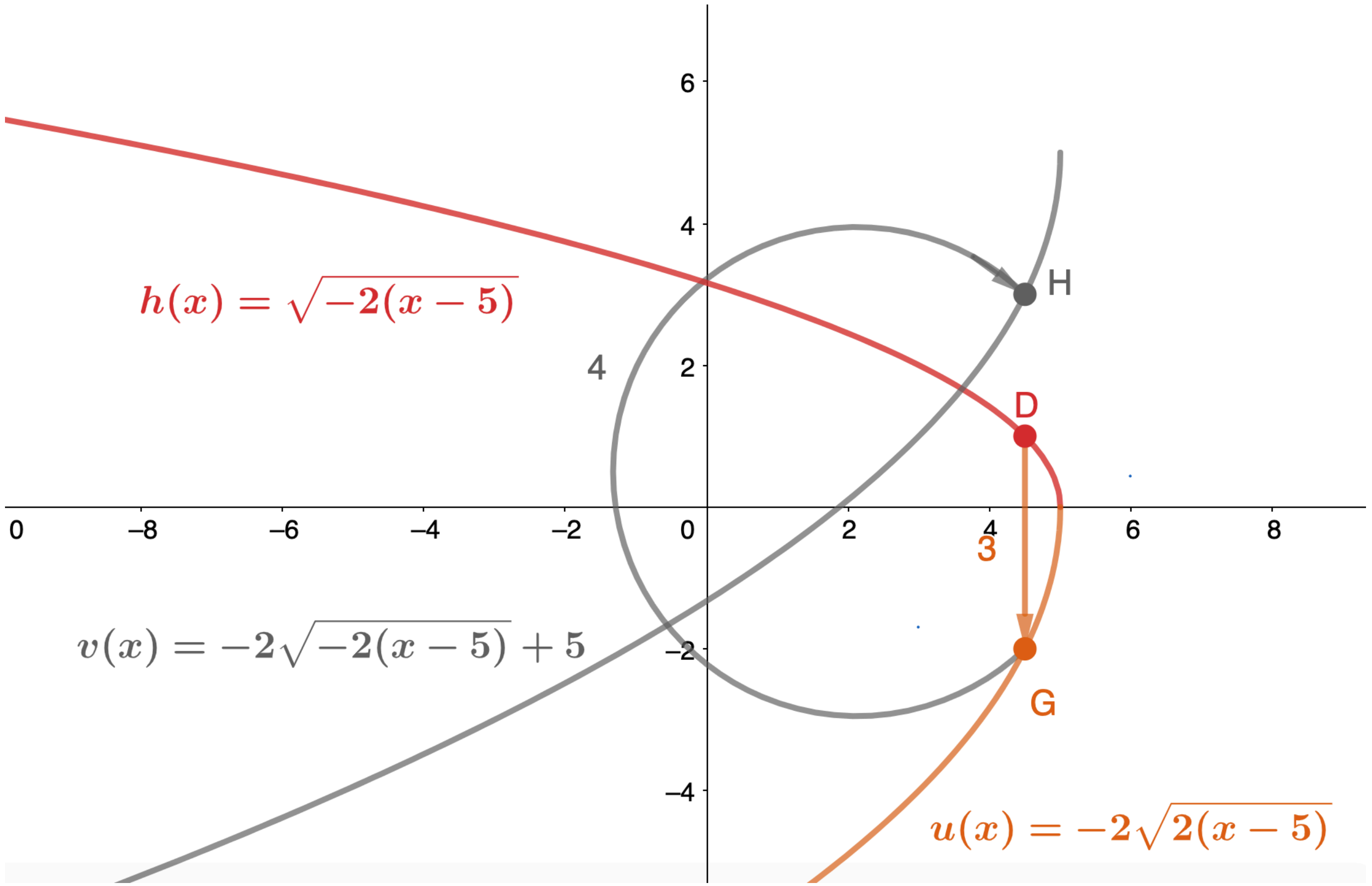
- Vertical scaling. If $c < 0$, reflect the graph about the x-axis!

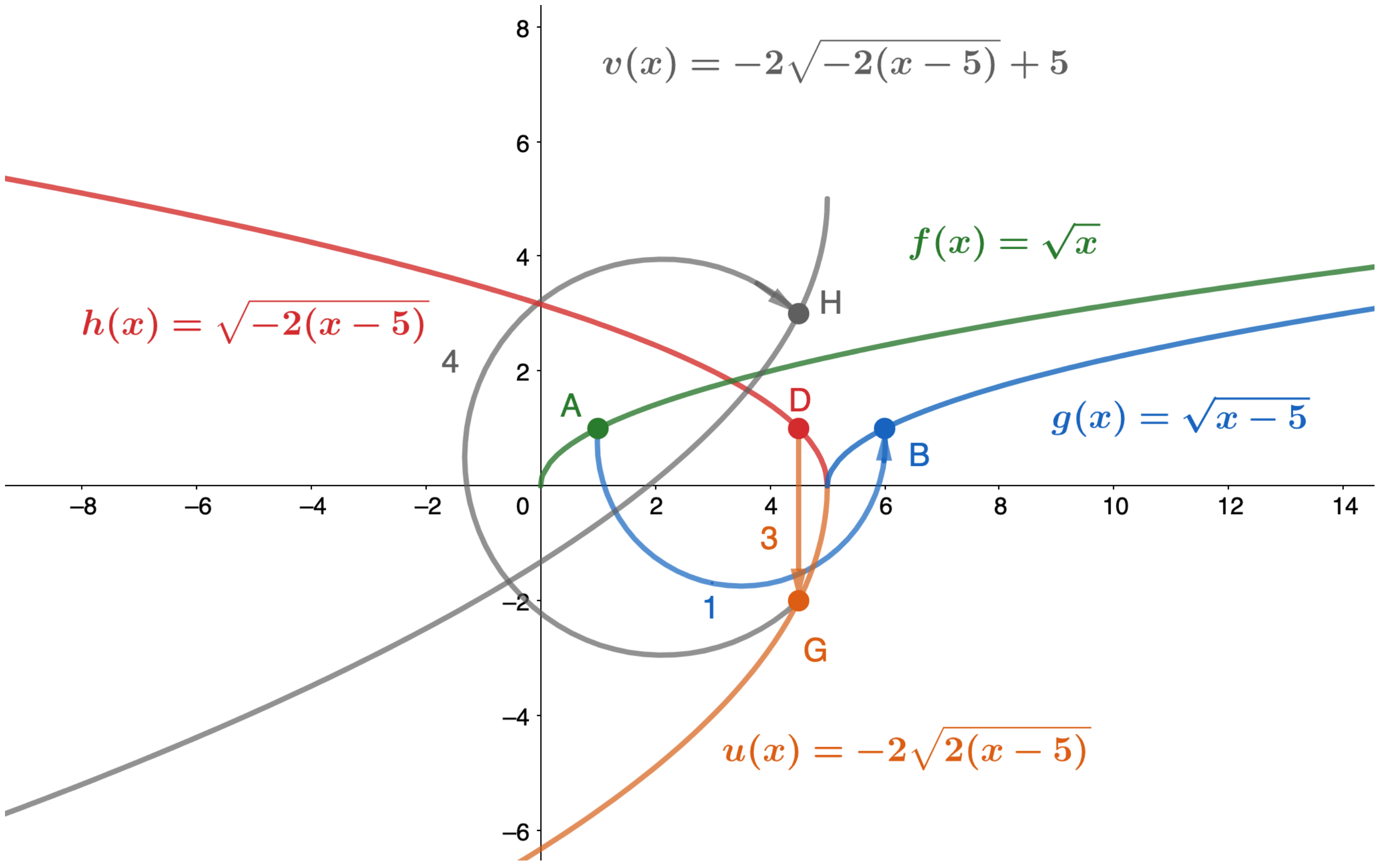
$cf\big(a(x+b)\big) + d$

- Vertical shift. If $d > 0$, shift up. If $d < 0$, shift down.

ORDER OF TRANSFORMATION CONTINUED

- Function from last time $f(x) = \sqrt{-2(x-5)}$
- Target function $f(x) = -2\sqrt{-2(x-5)} + 5$
- Vertical scaling
  - $|c| = 2 > 1$, stretch
  - $c = -2 < 0$, reflect about the x-axis
- Vertical shift
  - $d = 5 > 0$, shift up





ORDER OF TRANSFORMATION CONTINUED

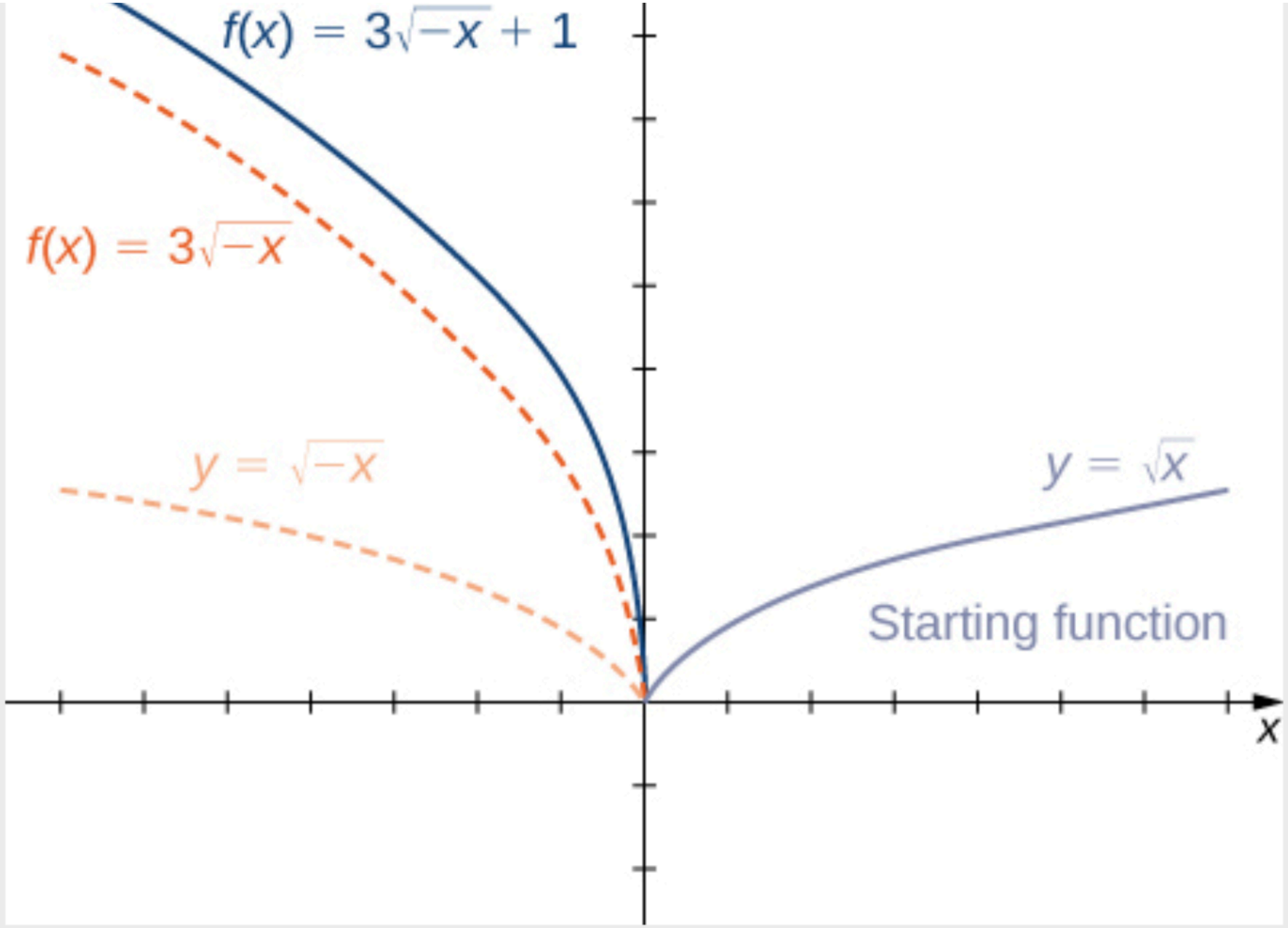


Figure 1.29 The function $f(x) = 3\sqrt{-x} + 1$ can be viewed as a sequence of three transformations of the function $y = \sqrt{x}$.

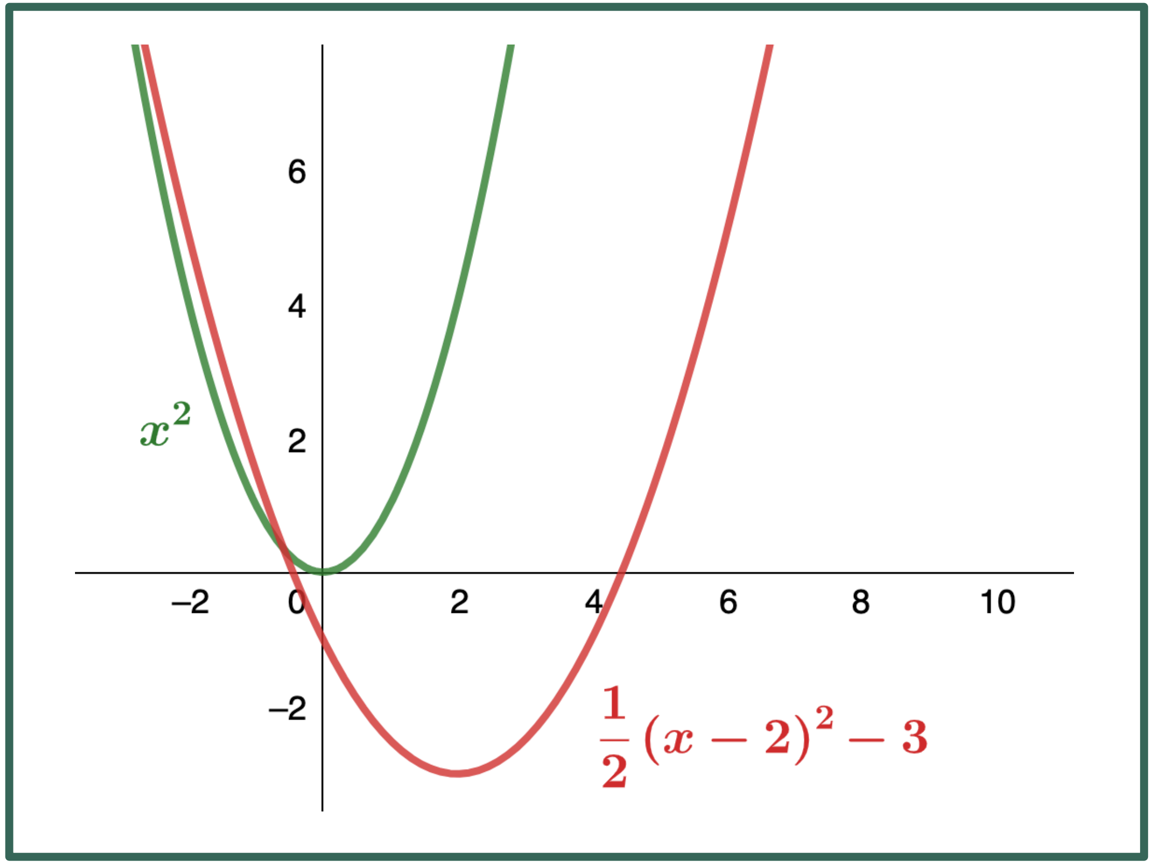
THIRD TYPO

ORDER OF TRANSFORMATION CONTINUED

If the graph of a function consists of more than one transformation of another graph, it is important to transform the graph in the correct order. Given a function $f(x)$, the graph of the related function $y = cf\,(a\,(x + b)) + d$ can be obtained from the graph of $y = f(x)$ by performing the transformations in the following order.

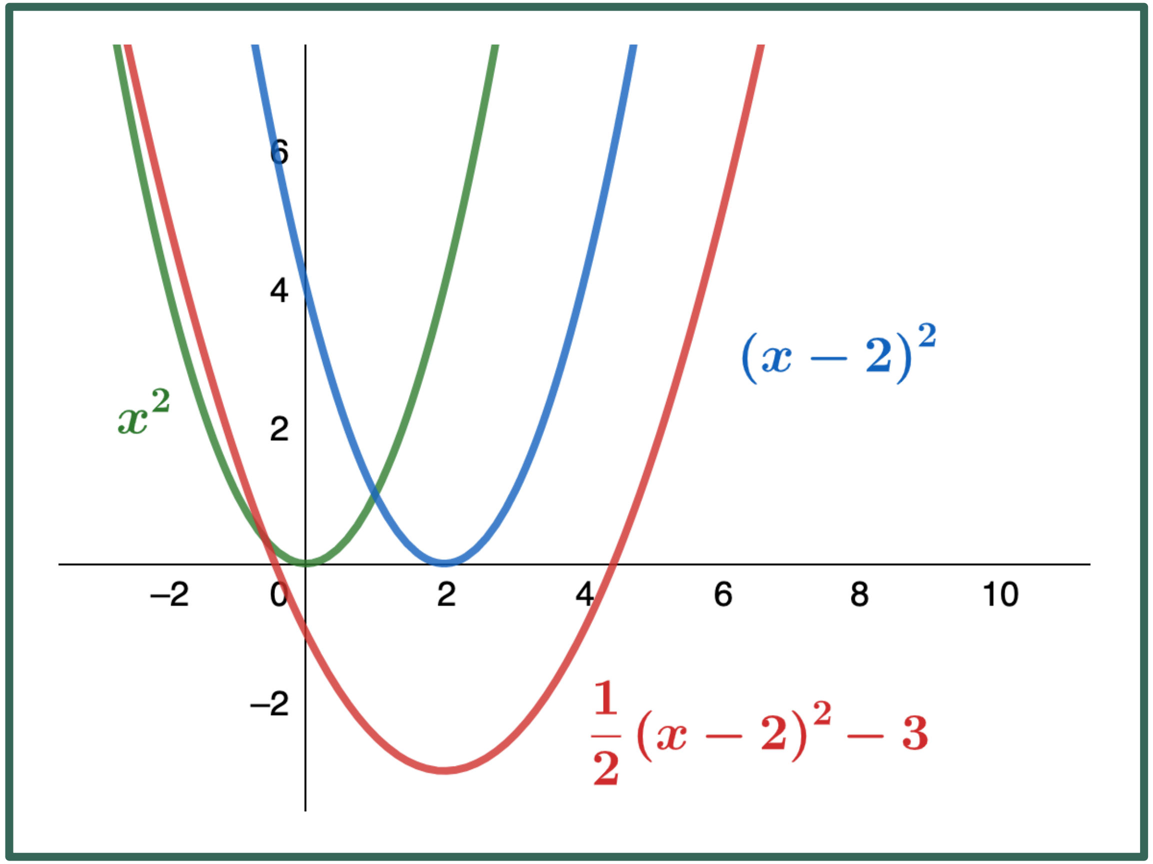
1. Horizontal shift of the graph of $y = f(x)$. If $b > 0$, shift left. If $b < 0$, shift right.
2. Horizontal scaling of the graph of $y = f(x + b)$ by a factor of $|a|$. If $a < 0$, reflect the graph about the y-~~axis.~~ **about $x = -b$**
3. Vertical scaling of the graph of $y = f(a\,(x + b))$ by a factor of $|c|$. If $c < 0$, reflect the graph about the x-axis.
4. Vertical shift of the graph of $y = cf(a\,(x + b))$. If $d > 0$, shift up. If $d < 0$, shift down.

EXERCISE ONE



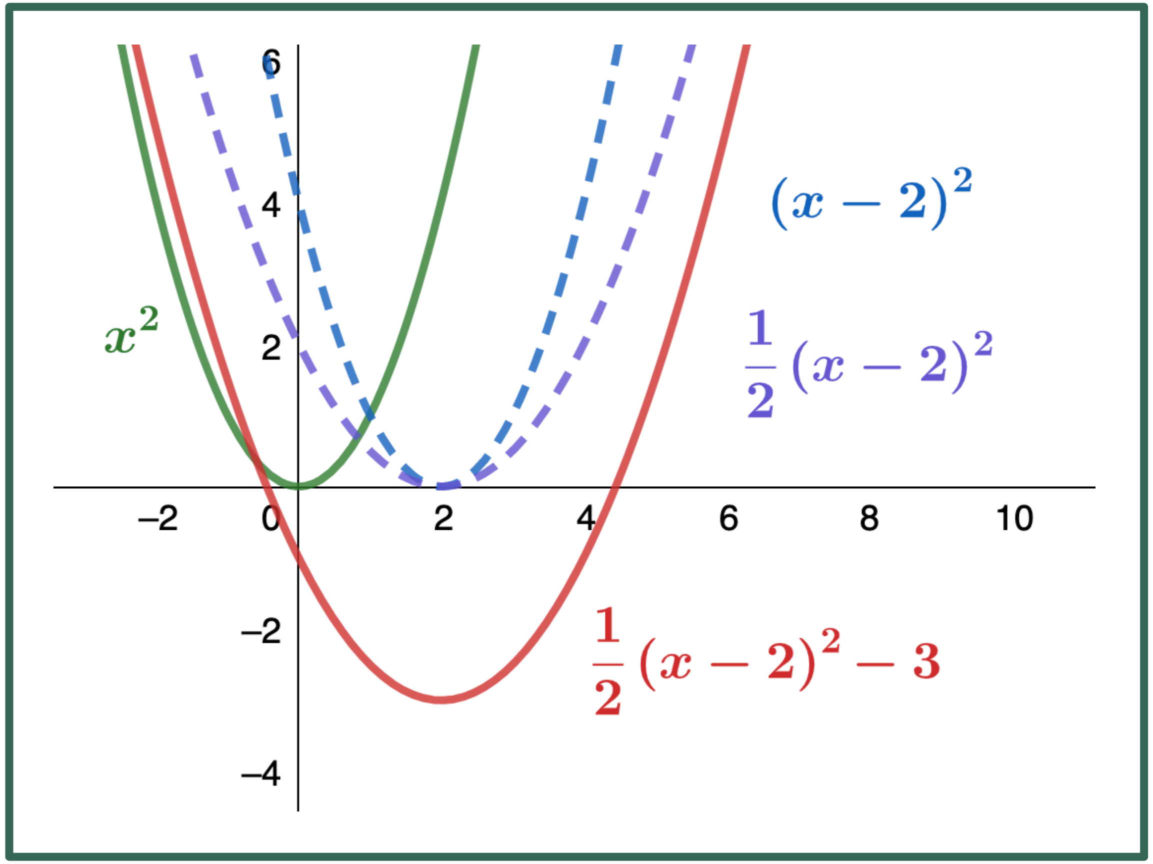
- Describe how the following function can be graphed using **a well-known function** and a sequence of transformations.
- $f(x) = \frac{1}{2}(x-2)^2 - 3$

EXERCISE ONE CONTINUED



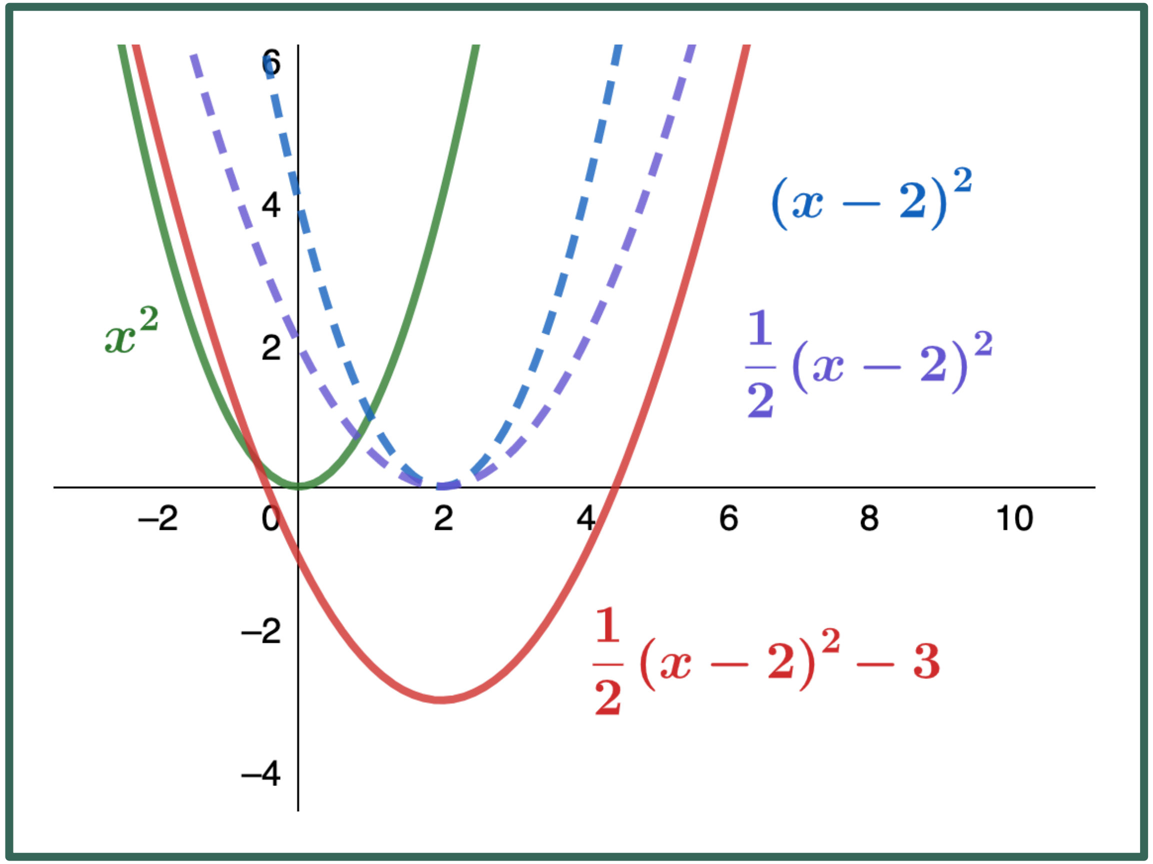
- A well-known function
  - x^2
- Horizontal shift right 2 units
  - $(x-2)^2$
- ...
- The target function
  - $f(x) = \frac{1}{2}(x-2)^2 - 3$

EXERCISE ONE CONTINUED



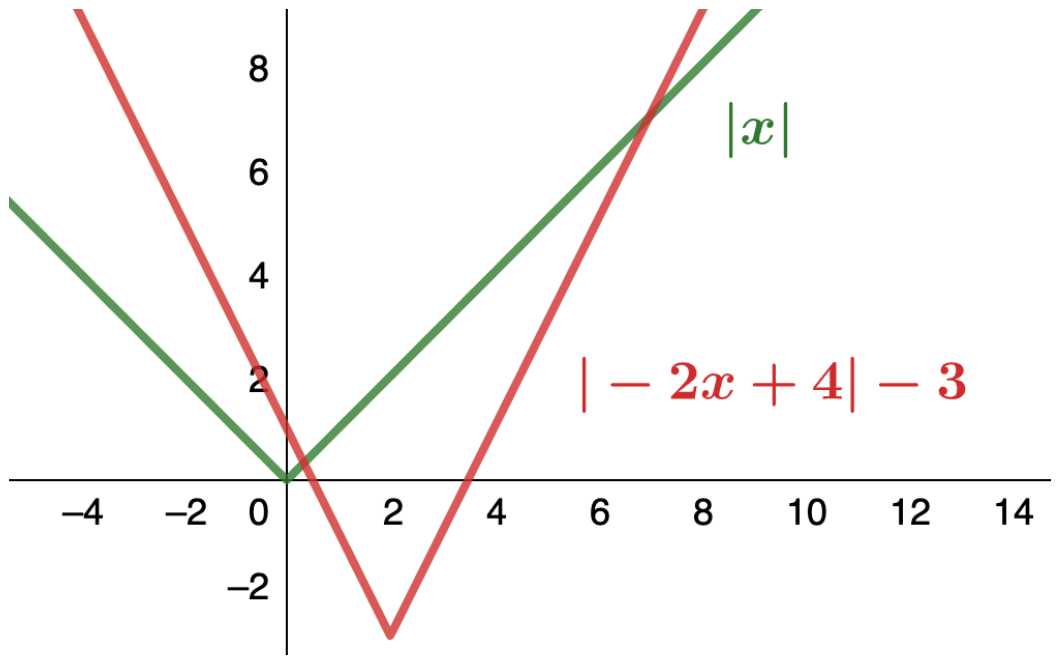
- Horizontal shift right 2 units
  - $(x-2)^2$
- Vertical scaling by a factor of $\frac{1}{2}$
  - $\frac{1}{2}(x-2)^2$
- …
- The target function
  - $f(x) = \frac{1}{2}(x-2)^2 - 3$

EXERCISE ONE CONTINUED



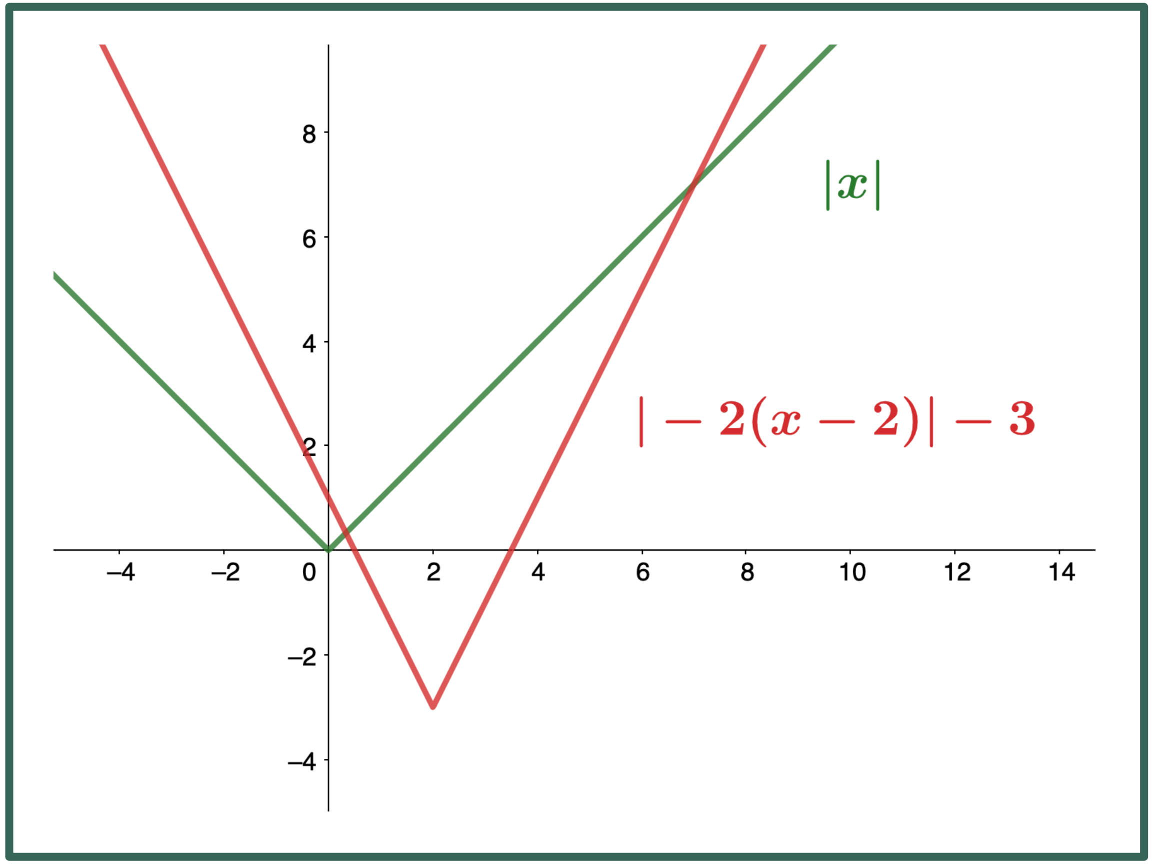
- Vertical scaling by a factor of $\frac{1}{2}$
  - $\frac{1}{2}(x-2)^2$
- Vertical shift down 3 units
  - $f(x) = \frac{1}{2}(x-2)^2 - 3$

EXERCISE TWO



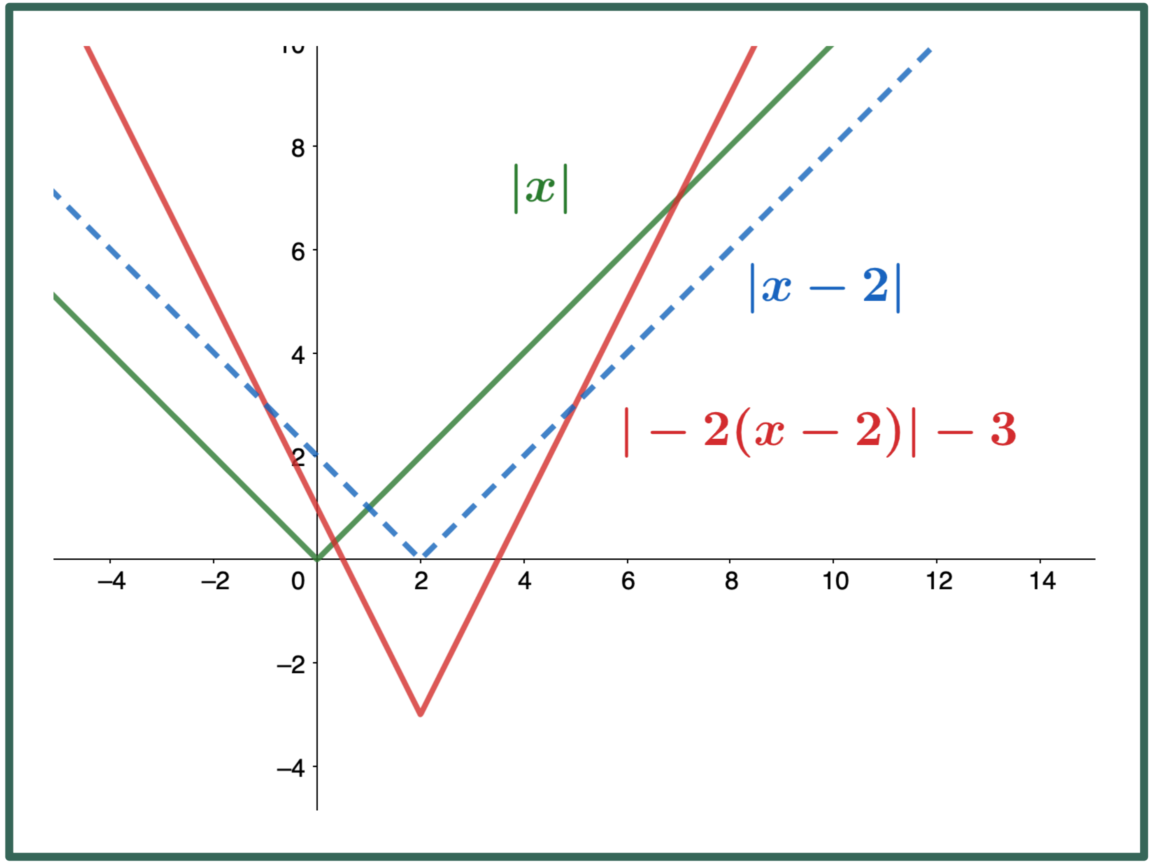
- Describe how the following function can be graphed using a well-known function and a sequence of transformations.
- $f(x) = |-2x + 4| - 3$

EXERCISE TWO CONTINUED



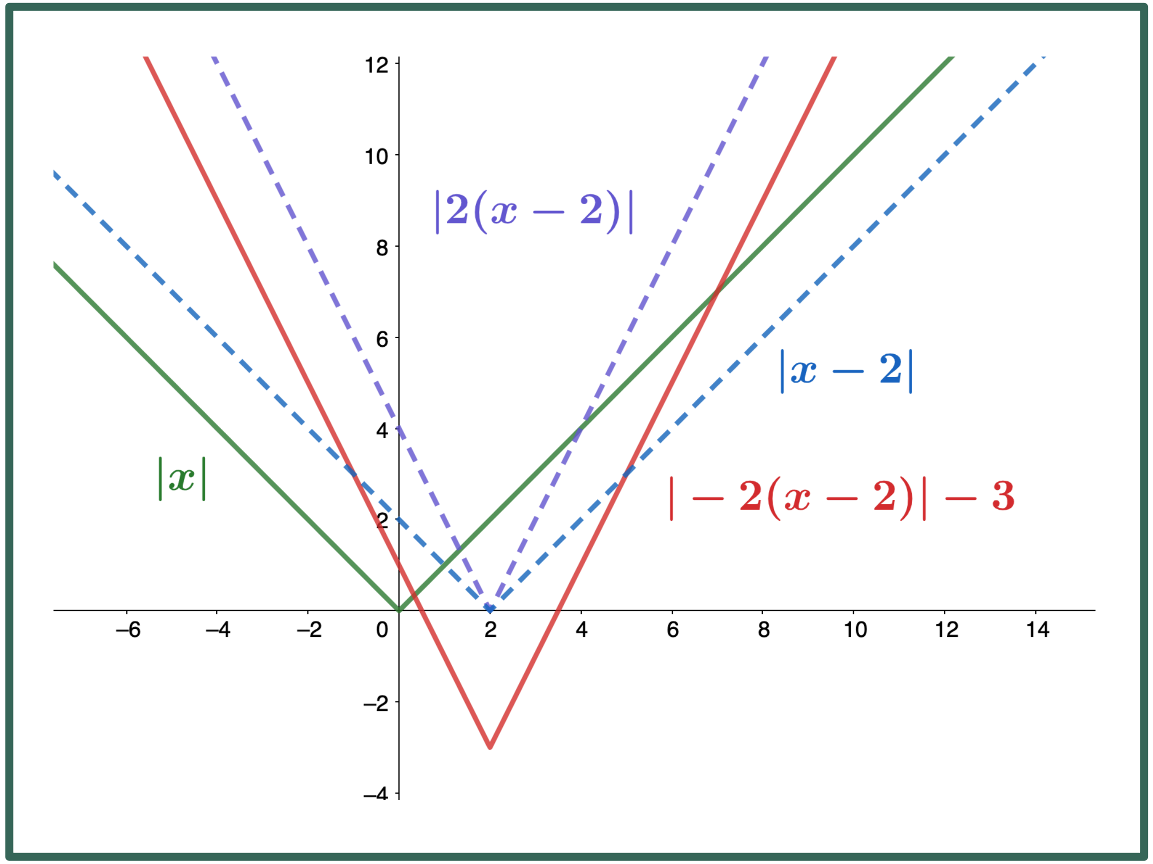
- Step 0
  - write the function in standard form
  - $f(x) = |-2x + 4| - 3 =$ $|2x - 4| - 3 = |2(x - 2)| - 3$

EXERCISE TWO CONTINUED



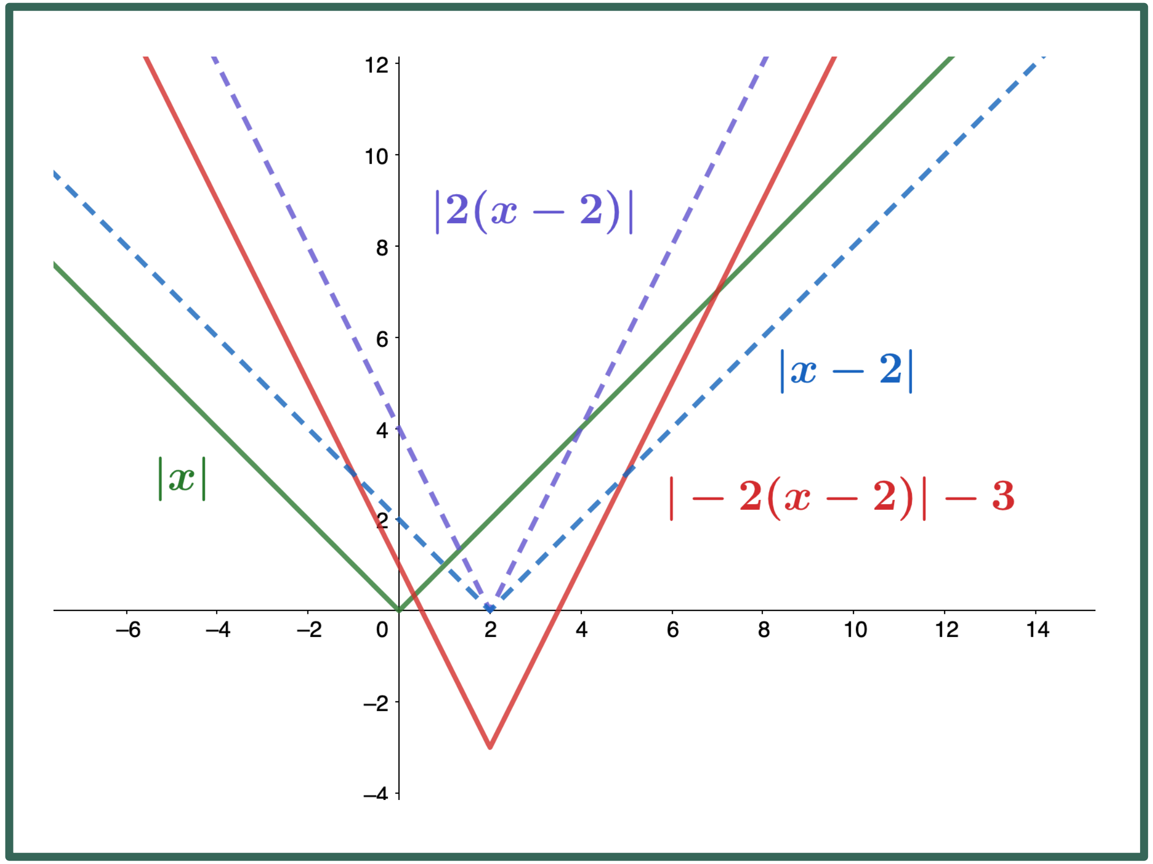
- A well-known function
  - $|x|$
- Horizontal shift right 2 units
  - $|x-2|$
- …
- The target function
  - $f(x) = |2(x-2)| - 3$

EXERCISE TWO CONTINUED



- Horizontal shift right 2 units
  - $|x - 2|$
- Horizontal scaling by a factor of 2
  - $|2(x - 2)|$
- …
- The target function
  - $f(x) = |2(x - 2)| - 3$

EXERCISE TWO CONTINUED



- Horizontal scaling by a factor of 2
  - $|2(x-2)|$
- Vertical shift down 3 units
  - $f(x) = |2(x-2)| - 3$