MATH 1: INTRODUCTION TO CALCULUS

OPERATIONS ON FUNCTIONS, EVEN AND ODD FUNCTIONS

COMBINE FUNCTIONS

- With mathematical operators

$$(f+g)(x) = f(x) + g(x) \qquad \textit{Sum}$$

$$(f-g)(x) = f(x) - g(x) \qquad \textit{Difference}$$

$$(f \cdot g)(x) = f(x)\,g(x) \qquad \textit{Product}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0 \qquad \textit{Quotient}$$

COMBINE FUNCTIONS: WITH MATHEMATICAL OPERATIONS

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$

OPERATION	NEW FUNCTION
sum	
difference	
product	
quotient	

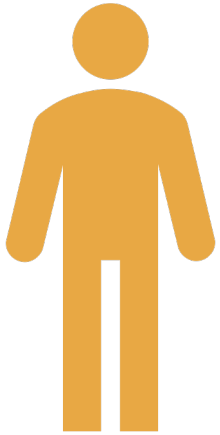
COMBINE FUNCTIONS: WITH MATHEMATICAL OPERATIONS

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$

OPERATION	NEW FUNCTION
sum	$(x-1)^2+\frac{1}{x+2}+2$
difference	$(x-1)^2-\frac{1}{x+2}+8$
product	$[(x-1)^2+5](\frac{1}{x+2}-3)$
quotient	$-\frac{[(x-1)^2+5](x+2)}{3x+5}$

FUNCTION COMPOSITION

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$
- $(f \circ g)(x) = ?$
- $(g \circ f)(x) = ?$
- Recall what we have encountered last class (evaluate functions)



PUT ON CLOTHES

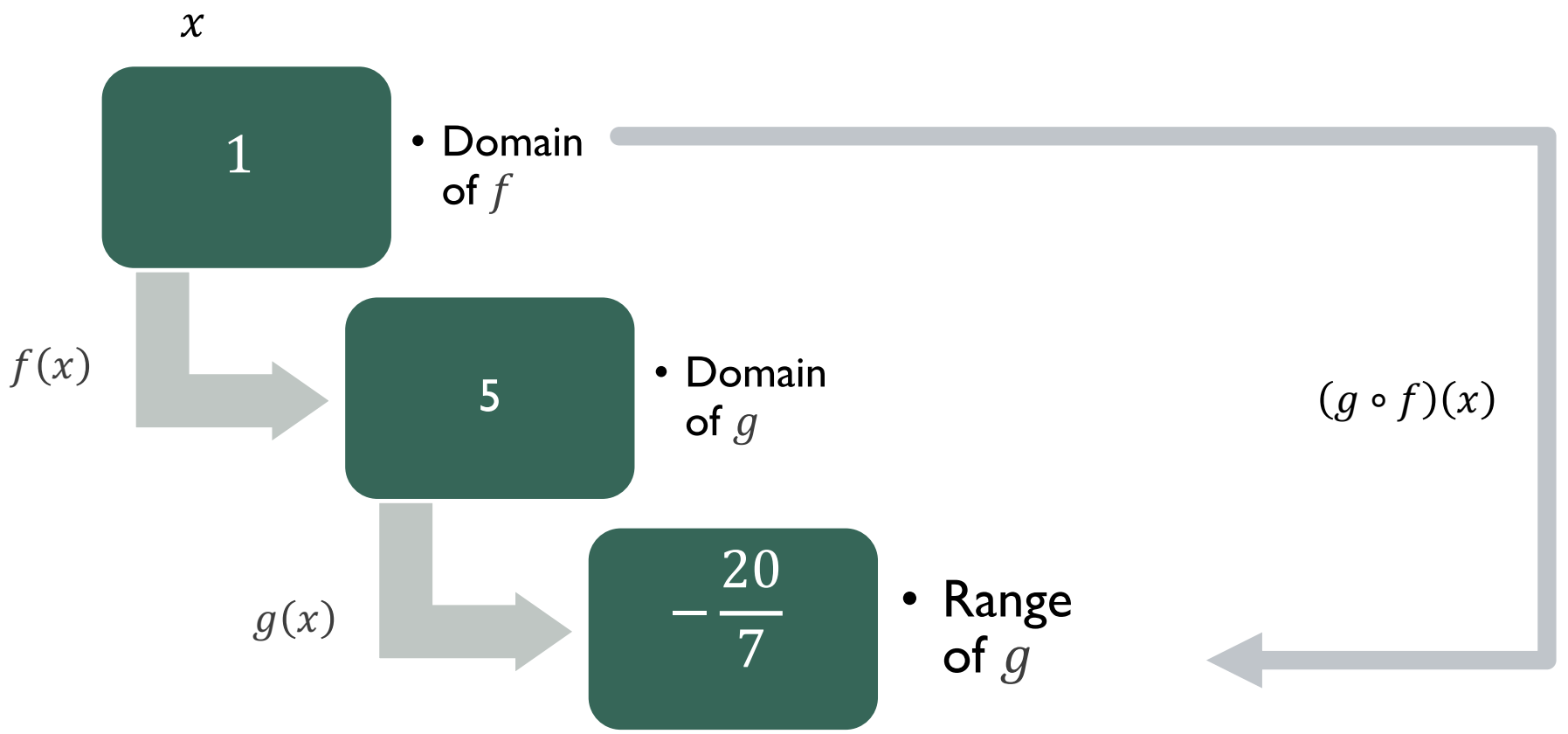
$$f(x) = (x-1)^2+5$$

$$g(x) = \frac{1}{x+2} - 3$$

- First put on the T-shirt, then the jacket.
- Body (x) inside, T-shirt (first function) in between, jacket (second function) outside.
- $(f \circ g)(x) = f\left(\frac{1}{x+2} - 3\right) = (\frac{1}{x+2} - 4)^2+5$
- $(g \circ f)(x) = g\big((x-1)^2 + 5\big) = \frac{1}{(x-1)^2+7} - 3$

FUNCTION COMPOSITION

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$



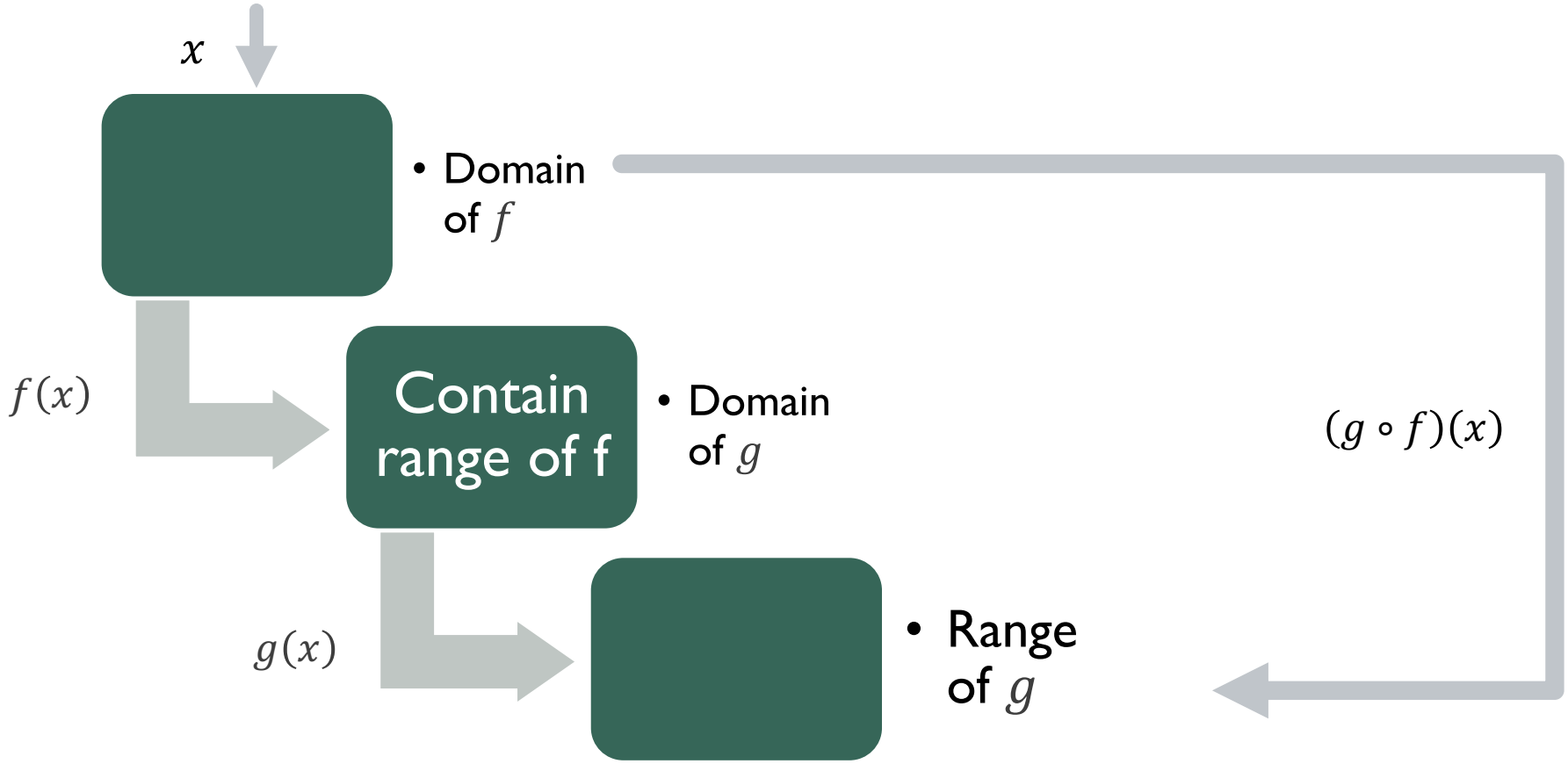
DEFINITION

Consider the function f with domain A and range B, and the function g with domain D and range E. If B is a subset of D, then the **composite function** $(g \circ f)(x)$ is the function with domain A such that

$$(g \circ f)(x) = g(f(x)). \tag{1.1}$$

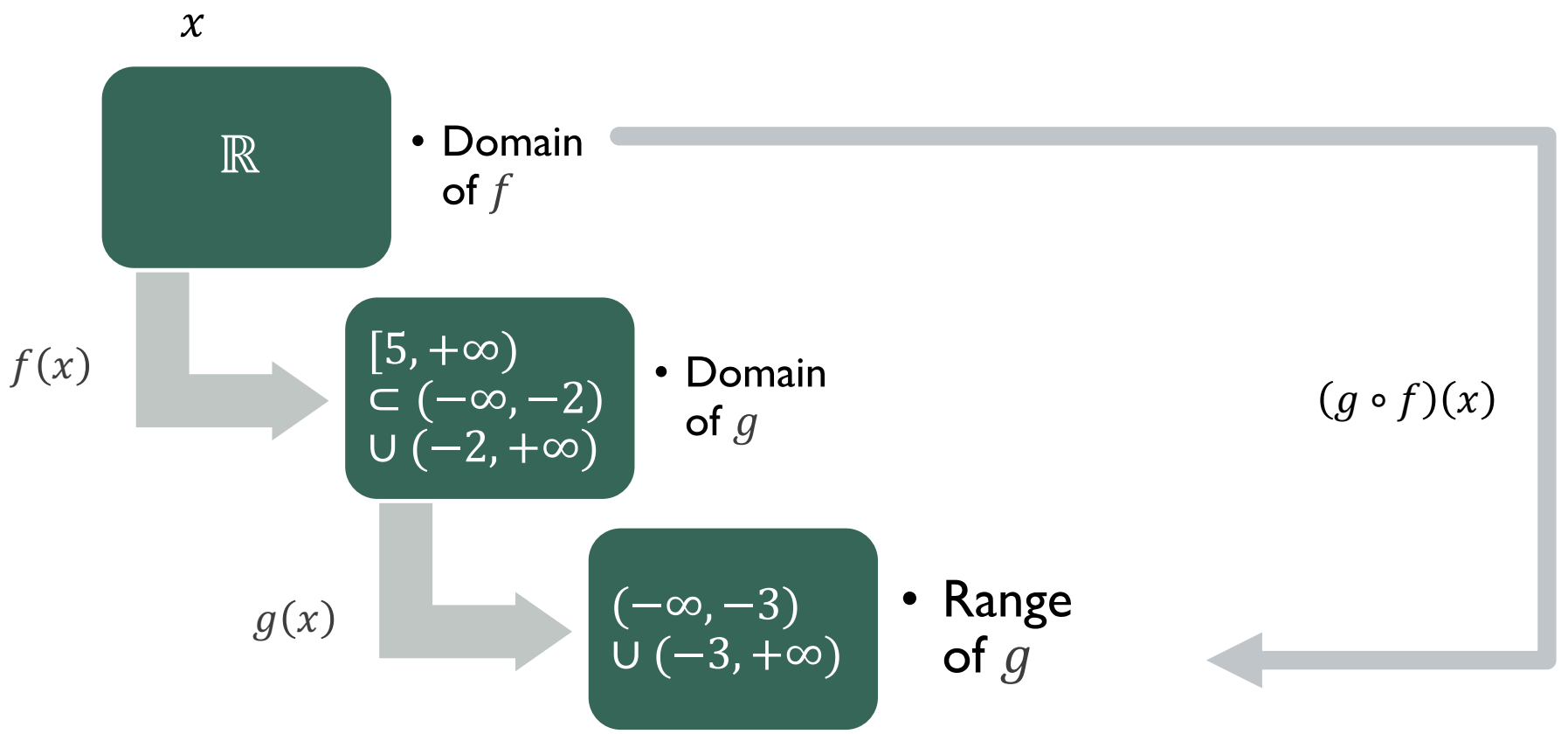
FUNCTION COMPOSITION

FUNCTION COMPOSITION



FUNCTION COMPOSITION

- $f(x) = (x-1)^2+5$
- $g(x) = \frac{1}{x+2} - 3$



Lumos fact file

TYPE

Charm

INCANTATION

Lumos

PURPOSE

To light up dark places at the flick of a wand

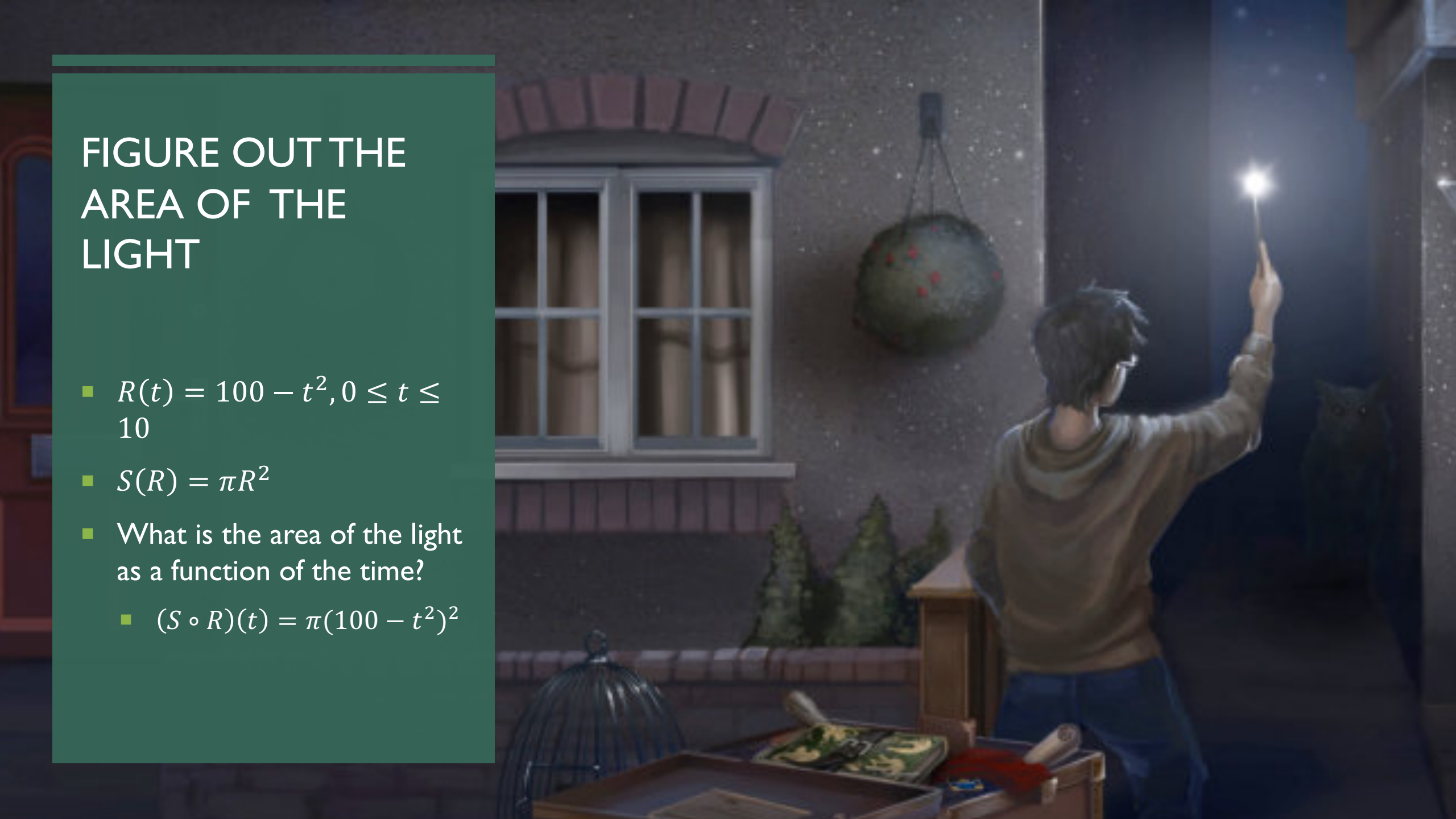
AN APPLICATION OF FUNCTION COMPOSITION

THE SHAPE OF THE LIGHT IS A CIRCLE, WITH RADIUS R.

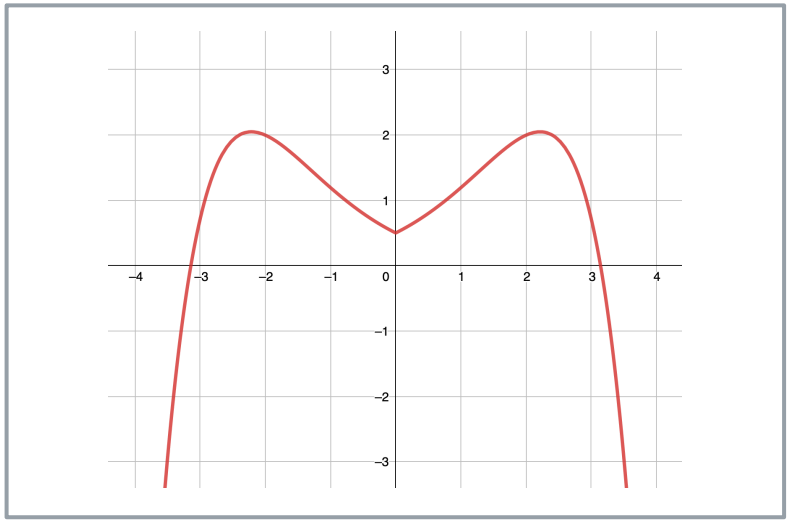
THE RADIUS R IS A FUNCTION OF TIME (THE CHARM WILL BE INVALID GRADUALLY).

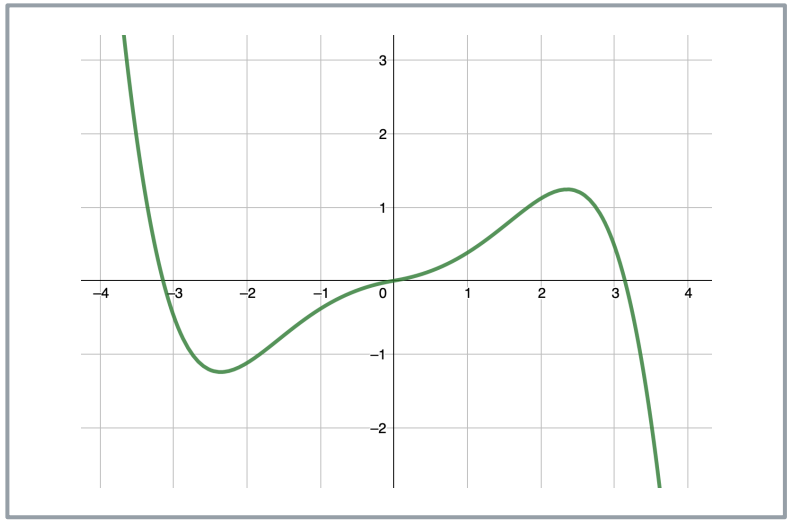
FIGURE OUT THE AREA OF THE LIGHT

- $R(t) = 100 - t^2, 0 \leq t \leq 10$
- $S(R) = \pi R^2$
- What is the area of the light as a function of the time?
  - $(S \circ R)(t) = \pi(100 - t^2)^2$



SYMMETRY OF FUNCTION

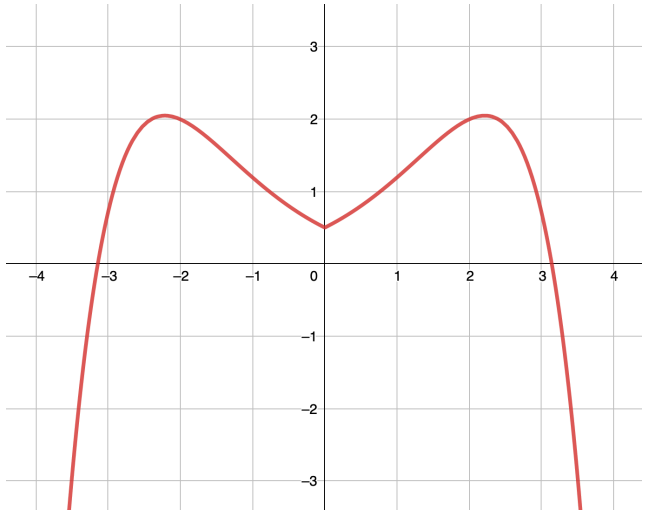


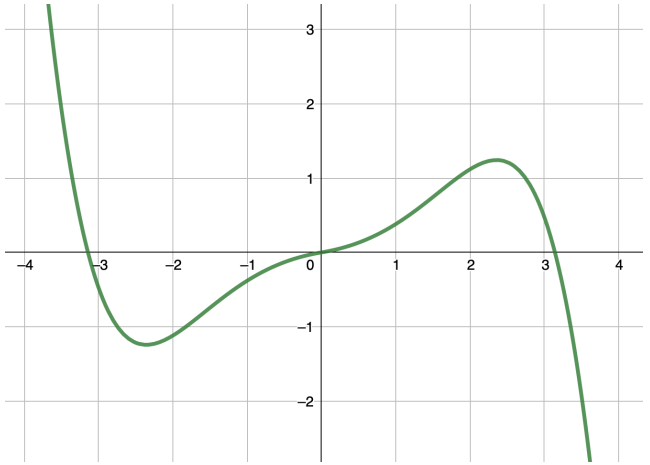


- Symmetry about the y-axis
- Symmetry about the origin

EVEN AND ODD FUNCTIONS

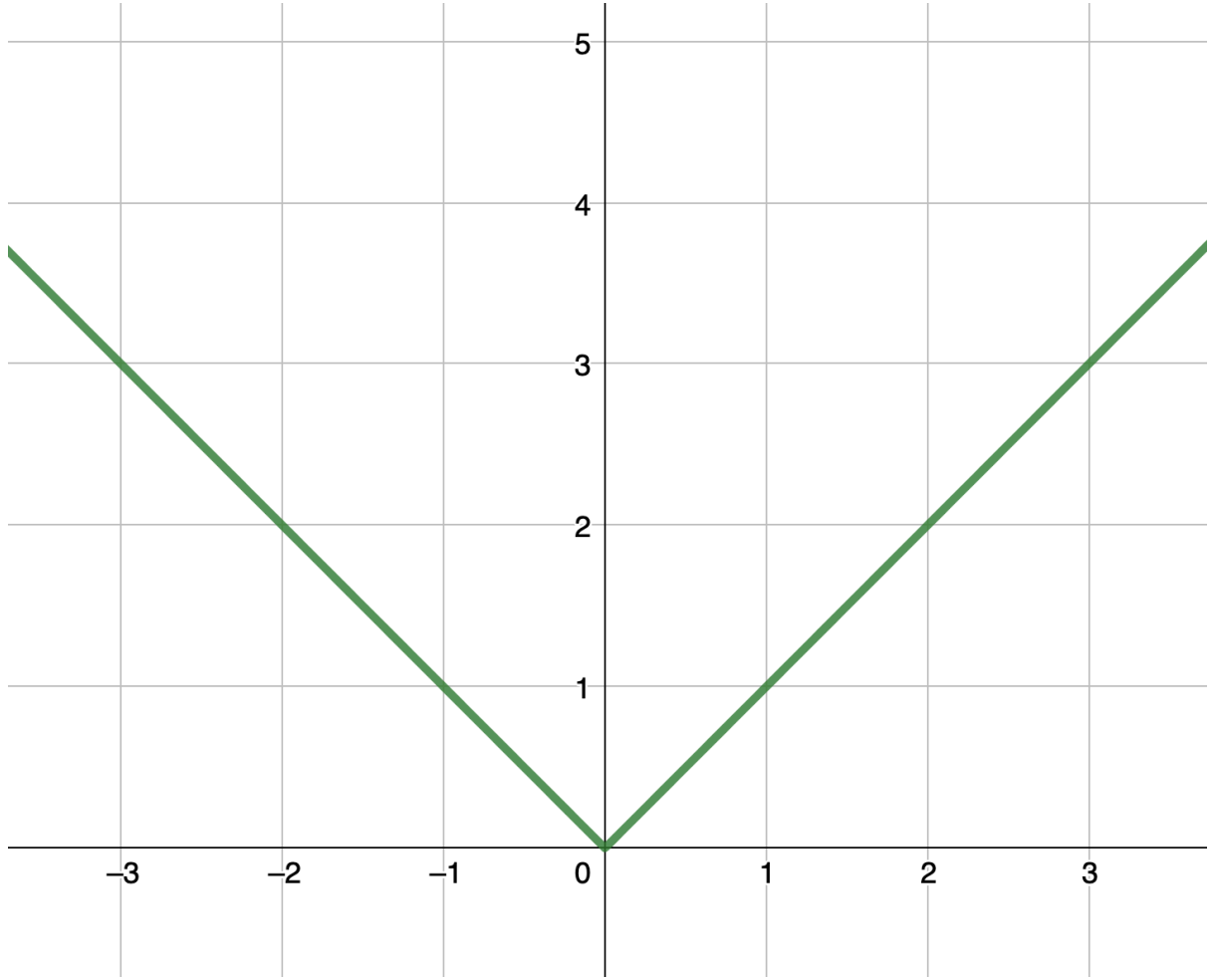
- Even functions:
  - $f(x) = f(-x)$
  - Symmetry about the y-axis
- Odd functions:
  - $-f(x) = f(-x)$
  - Symmetry about the origin





AN EXAMPLES OF EVEN AND ODD FUNCTIONS

- Absolute value function
  $f(x) = |x| =$
  $\begin{cases} -x, x \leq 0 \\ x, x > 0 \end{cases}$
  - even or odd?



FIND THE EVEN AND THE ODD FUNCTIONS!

$$-x^2$$

$$2x^3 + x$$

$$\frac{1}{x}$$

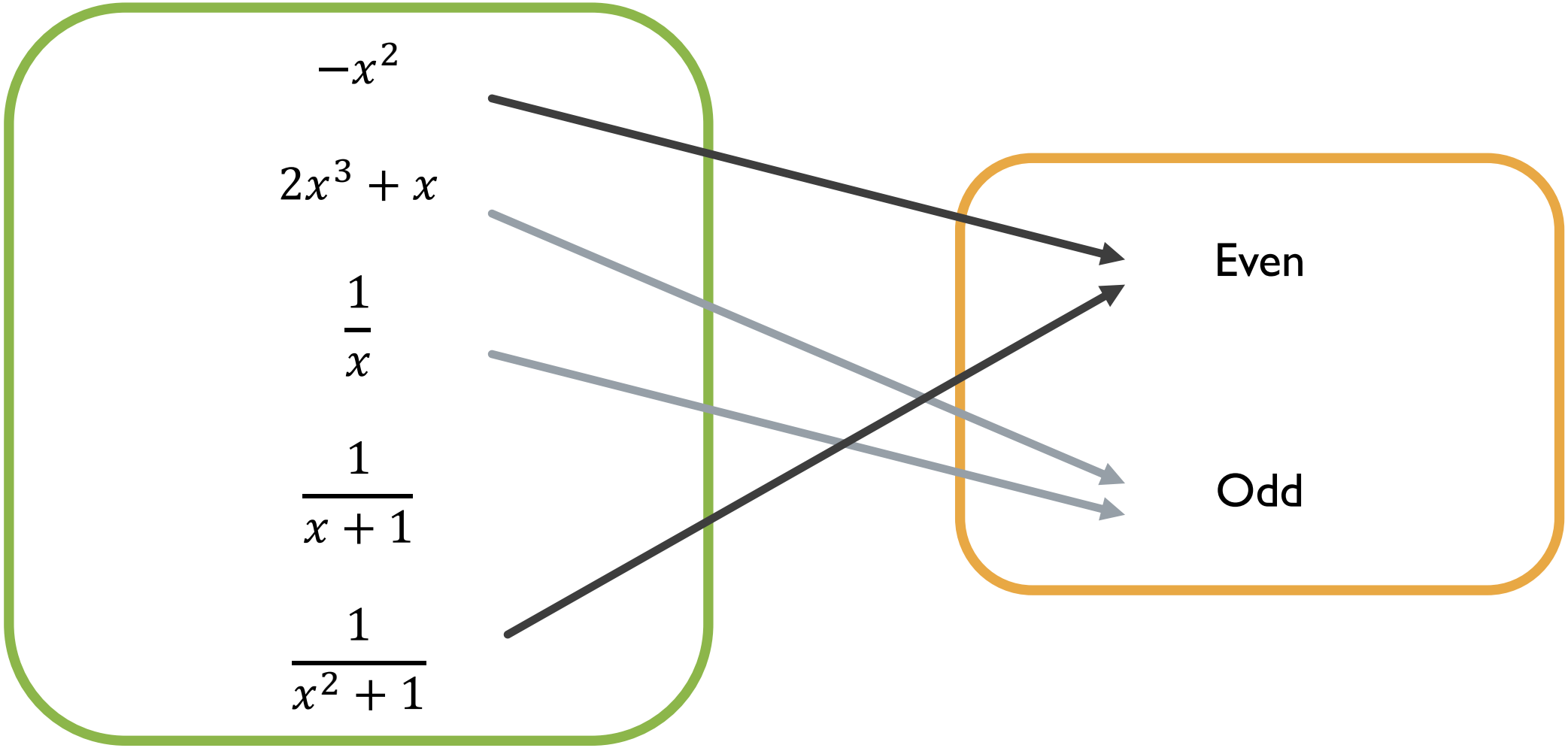
$$\frac{1}{x+1}$$

$$\frac{1}{x^2+1}$$

Even

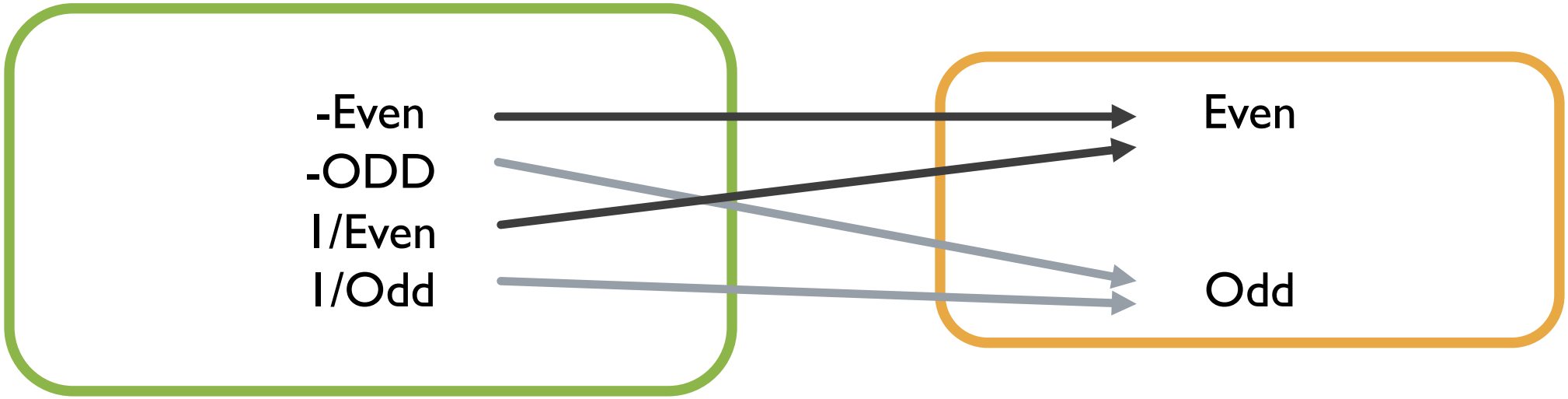
Odd

FIND THE EVEN AND THE ODD FUNCTIONS!



WHAT YOU HAVE FOUND?

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$



RECALL FUNCTION COMBINATION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$

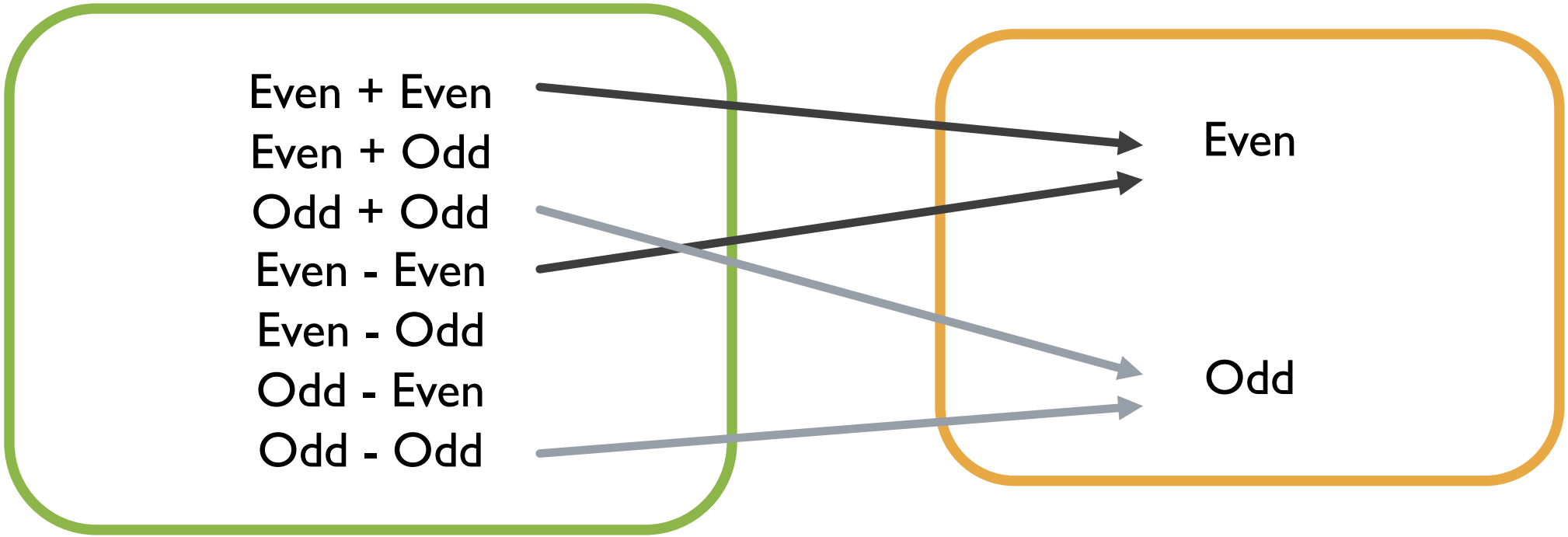
Even + Even
Even + Odd
Odd + Odd
Even - Even
Even - Odd
Odd - Even
Odd - Odd

Even

Odd

RECALL FUNCTION COMBINATION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$



RECALL FUNCTION COMBINATION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$

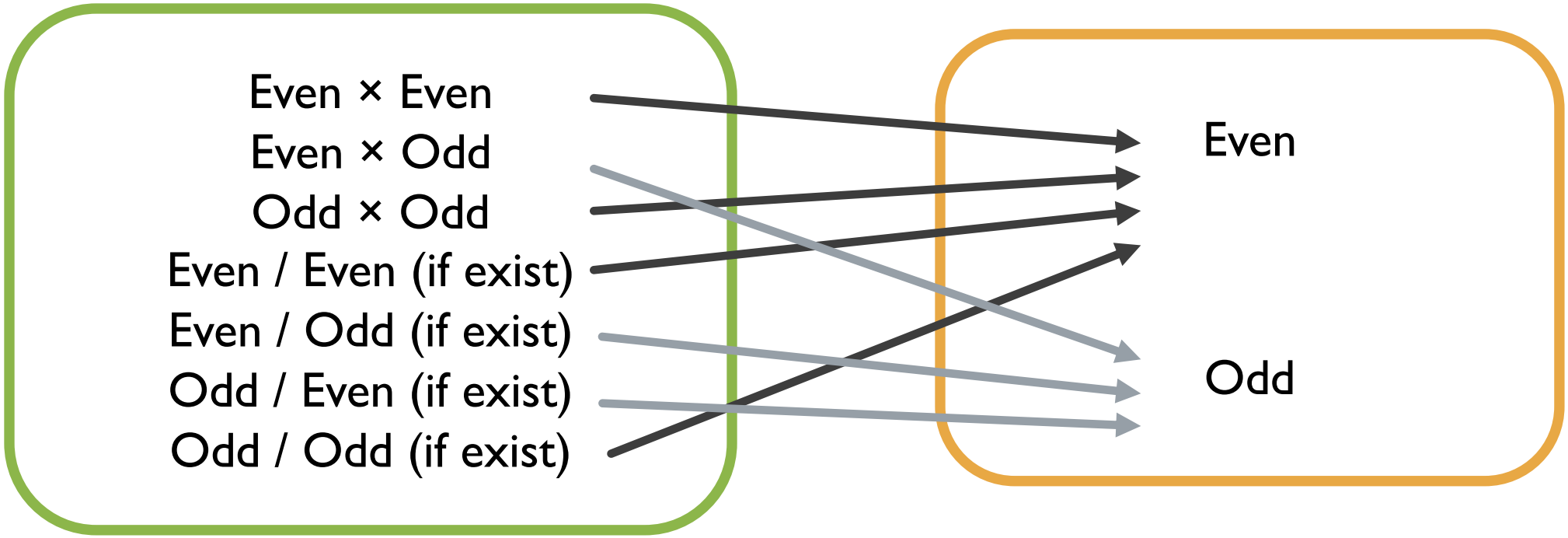
Even × Even
Even × Odd
Odd × Odd
Even / Even (if exist)
Even / Odd (if exist)
Odd / Even (if exist)
Odd / Odd (if exist)

Even

Odd

RECALL FUNCTION COMBINATION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$



RECALL FUNCTION COMPOSITION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$
- $(g \circ f)(x)$

Even ∘ Even

Even ∘ Odd

Odd ∘ Even

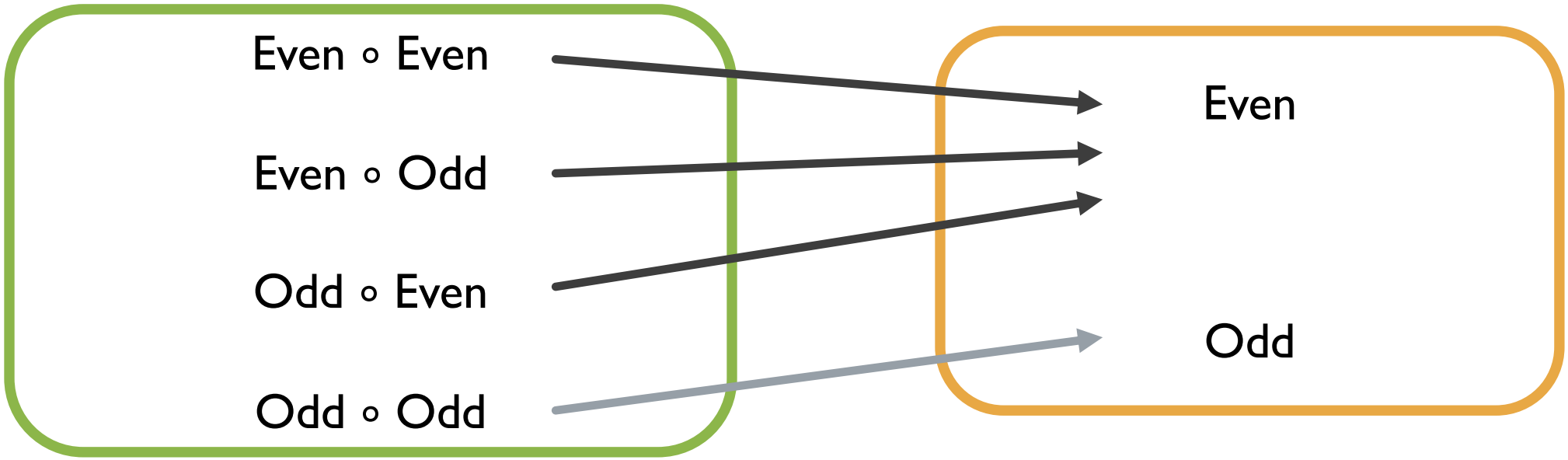
Odd ∘ Odd

Even

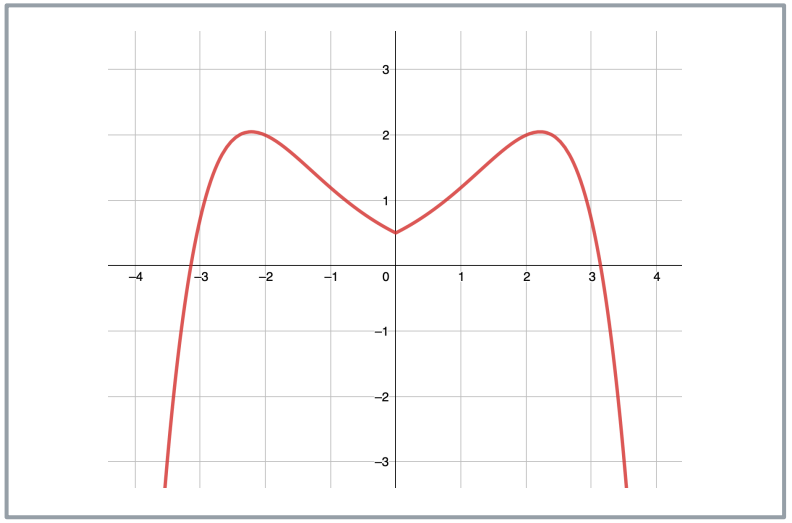
Odd

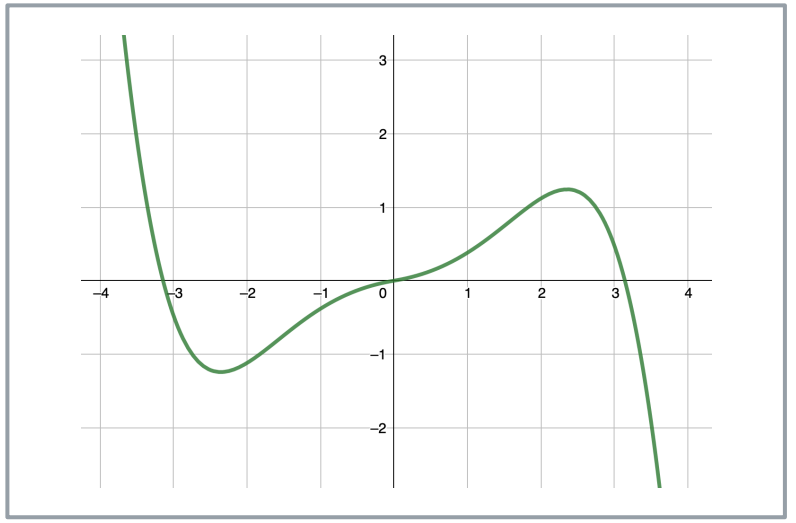
RECALL FUNCTION COMPOSITION

- Even functions: $f(x) = f(-x)$
- Odd functions: $-f(x) = f(-x)$
- $(g \circ f)(x)$



LOOK BACK ON THE TWO EXAMPLES





- Even function: $e^{|x|}$ and $\cos(x)$
- Odd function: $\sin(x)$
- $f(x) = \frac{1}{2}e^{|x|}\cos(\frac{x}{2})$
- $g(x) = \frac{1}{6}e^{|x|}\sin(x)$