



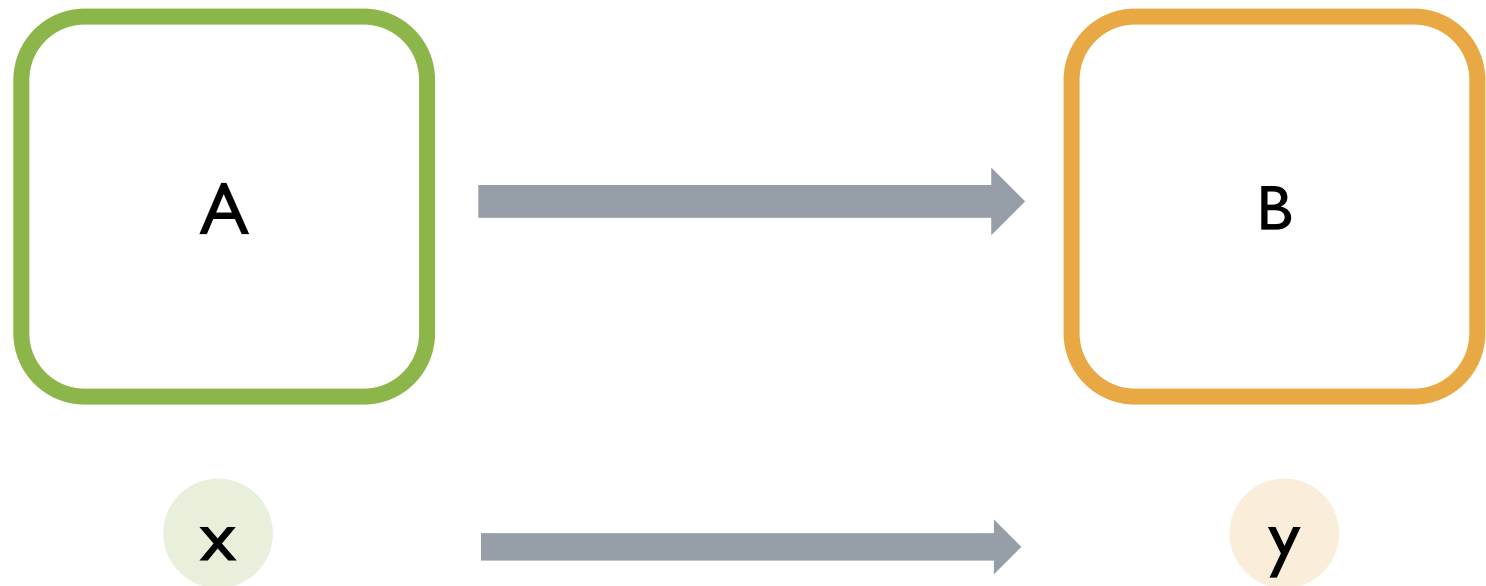
MATH I: INTRODUCTION TO CALCULUS

FUNCTIONS AND GRAPHS (09/16)



WHAT IS A FUNCTION?

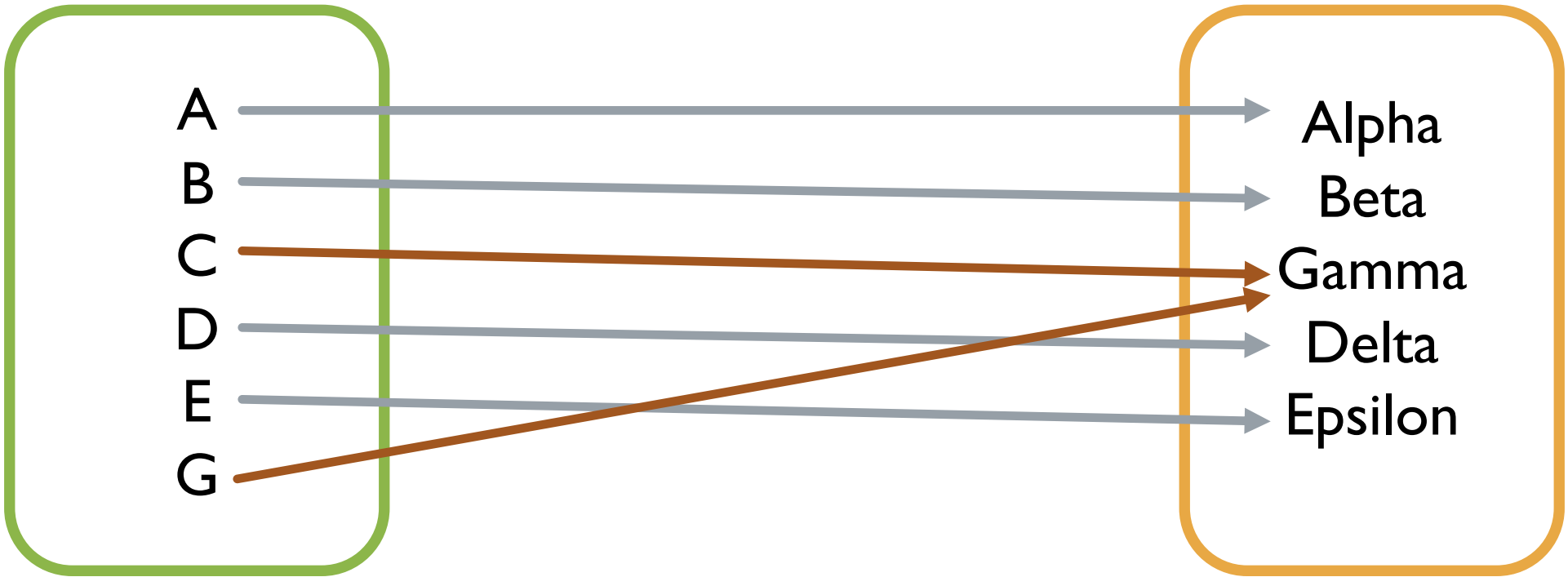
- Two sets A and B
 - x is an element of A
 - y is an element of B
- A relation from A to B
- Ordered pairs (x, y)
- ...?



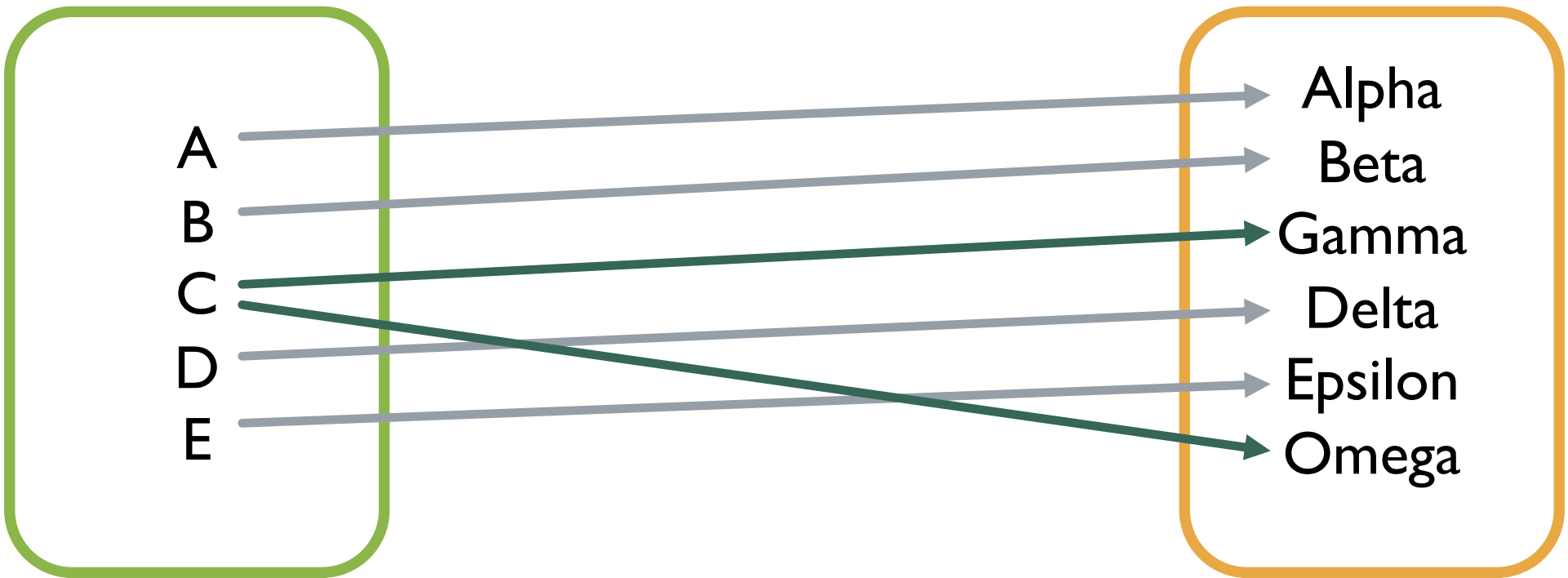
IS IT A FUNCTION?

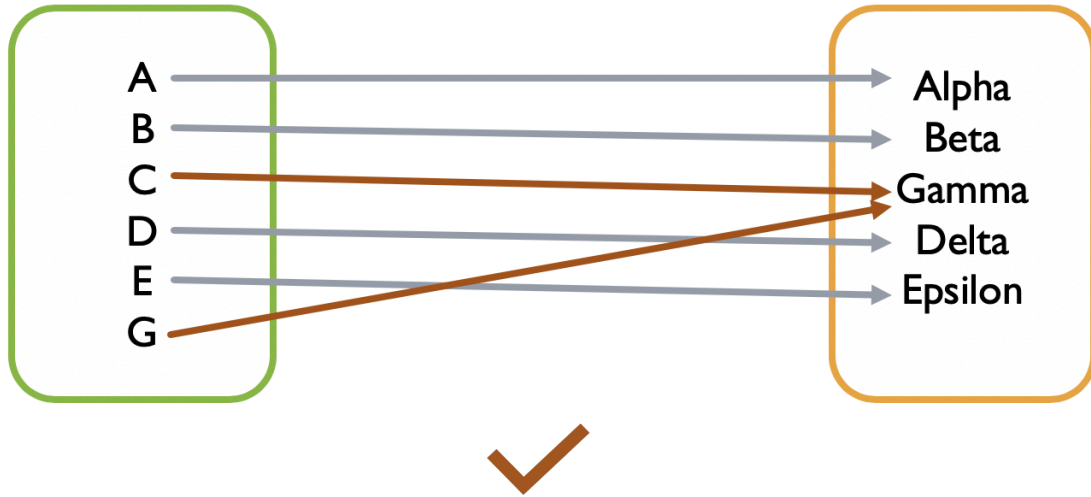


IS IT A FUNCTION?



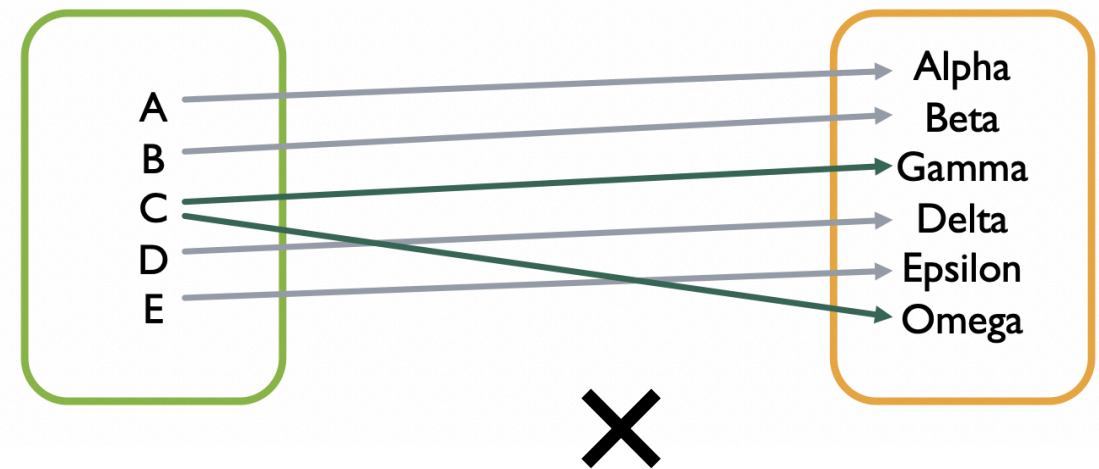
IS IT A FUNCTION?





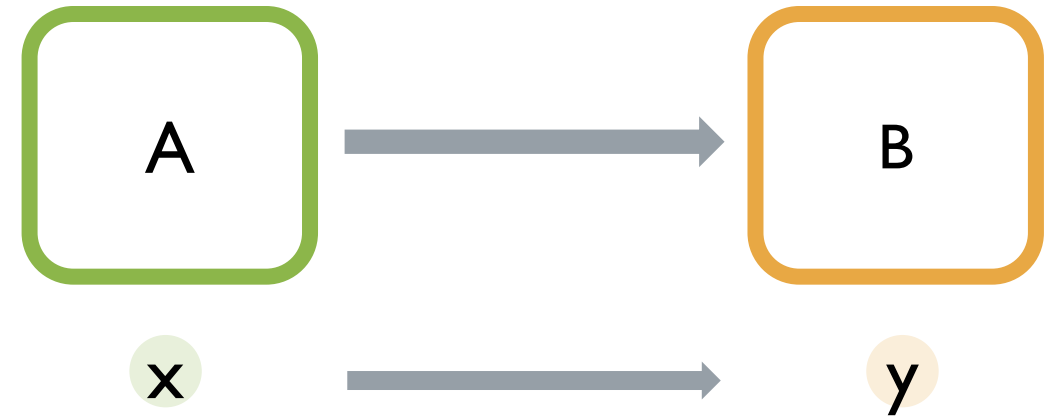
ONE MORE RESTRICTION

- Every element of the first set is related to **exactly one** element of the second set.



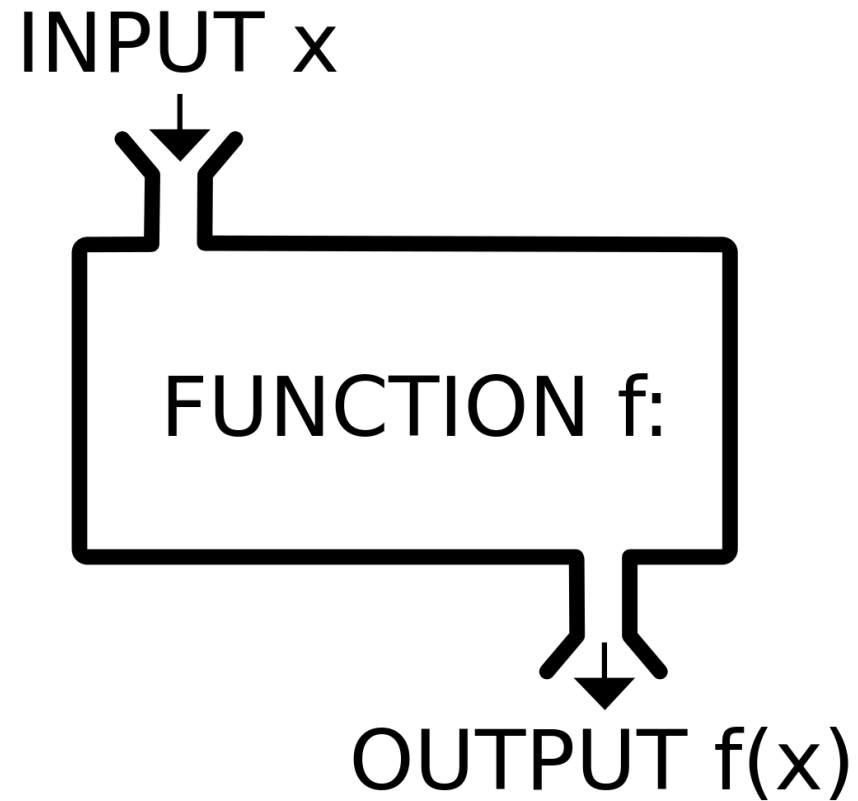
MORE THINGS TO SAY...

- Every element of the first set is related to **exactly one** element of the second set.
- The element of the first set is called the *input*.
- The element of the second set is called the *output*.



EXAMPLES OF A FUNCTION

- Remember to specify
 - the inputs
 - the outputs
 - the relation



EXAMPLES OF A FUNCTION

- $f(x) = 38x$
- What is the input?
- What is the output?

A screenshot of a flight search interface. At the top, there are two tabs: "ONE WAY" (highlighted in red) and "ROUND TRIP". Below the tabs, there are two input fields for location: "Boston / Logan Airport, MA" (with a location pin icon) and "Hanover, NH" (with a location pin icon). Below the first location field is a "Leave on..." field with a calendar icon. At the bottom left is a green "Search" button. On the right side, there are dropdown menus for "Adult" (showing a checkmark next to 1) and "Children" (showing a list from 1 to 7).

EXAMPLES OF A FUNCTION

Temperature

0

Celsius

Fahrenheit

Formula $(0^{\circ}\text{C} \times 9/5) + 32 = 32^{\circ}\text{F}$

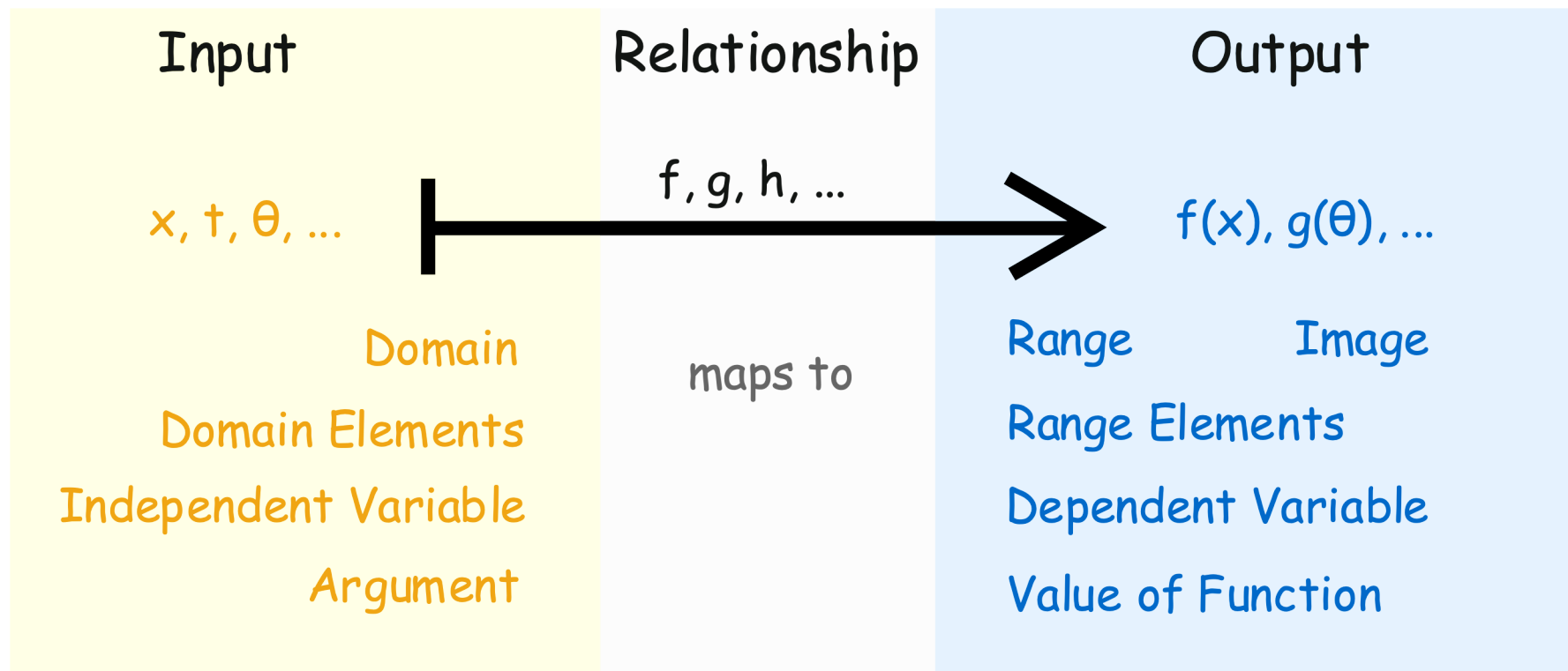
More info

- $f(x) = \frac{9}{5}x + 32$
- What is input?
- What is the output?

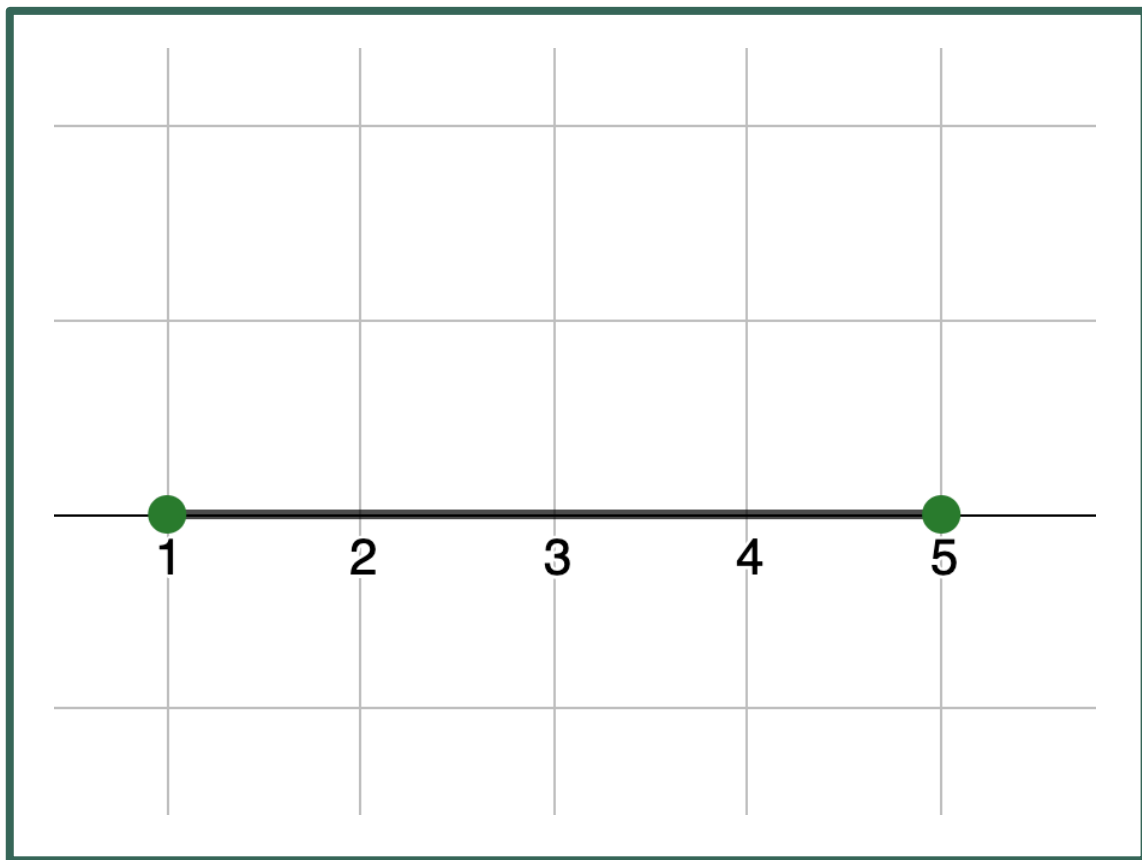
FORMAL DEFINITION

- A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output.
- The set of inputs is called the **domain** of the function.
- The set of outputs is called the **range** of the function.

IN GENERAL



HOW TO REPRESENT THE DOMAIN OR THE RANGE?



- Mathematically!
 - $\{x|1 < x < 5\}$
 - $(1, 5)$
 - $(1, 5) = \{x|1 < x < 5\}$

SET-BUILDER AND INTERVAL NOTATIONS

- Set builder
 - $\{x|x \text{ has some properties}\}$
- Interval
 - Open: (a, b)
 - Close: $[a, b]$
 - Left open, right close: $(a, b]$
 - Left close, right open: $[a, b)$

SET-BUILDER AND INTERVAL NOTATIONS (IS THE RELATION A FUNCTION?)

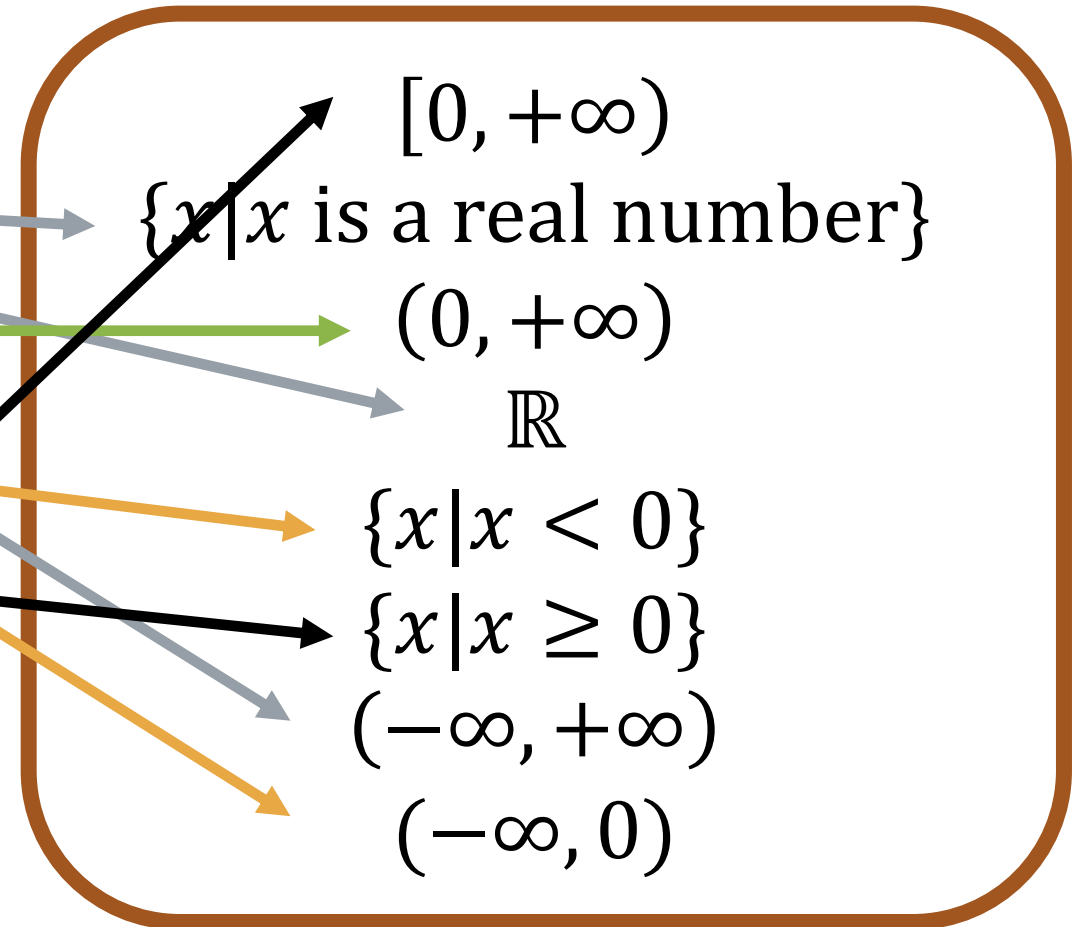
- All real numbers
- All positive real numbers
- All negative real numbers
- All non-negative real numbers

The diagram shows four arrows originating from the list items on the left and pointing to specific interval notations on the right. The first arrow points from 'All real numbers' to $\{x | x \text{ is a real number}\}$. The second arrow points from 'All positive real numbers' to $(0, +\infty)$. The third arrow points from 'All negative real numbers' to $(-\infty, 0)$. The fourth arrow points from 'All non-negative real numbers' to $\{x | x \geq 0\}$.

$[0, +\infty)$
 $\{x | x \text{ is a real number}\}$
 $(0, +\infty)$
 \mathbb{R}
 $\{x | x < 0\}$
 $\{x | x \geq 0\}$
 $(-\infty, +\infty)$
 $(-\infty, 0)$

SET-BUILDER AND INTERVAL NOTATIONS (IS THE RELATION A FUNCTION?)

- All real numbers
- All positive real numbers
- All negative real numbers
- All non-negative real numbers



MORE EXAMPLES: FIND DOMAIN AND RANGE

function	Domain	Range
$f(x)$	\mathbb{R}	$[0, +\infty)$
$g(x)$		
$h(x)$		
$c(x)$		
$s(x)$		
$\exp(x)$		

- $f(x) = x^2$
- $g(x) = \sqrt{x}$
- $h(x) = \sqrt{x}, x > 4$
- $c(x) = \cos x$
- $s(x) = \sin x$
- $\exp(x) = e^x$

MORE EXAMPLES: FIND DOMAIN AND RANGE

- $f(x) = x^2$
- $g(x) = \sqrt{x}$
- $h(x) = \sqrt{x}, x > 4$
- $c(x) = \cos x$
- $s(x) = \sin x$
- $\exp(x) = e^x$

function	Domain	Range
$f(x)$	\mathbb{R}	$[0, +\infty)$
$g(x)$	$[0, +\infty)$	$[0, +\infty)$
$h(x)$	$(4, +\infty)$	$(2, +\infty)$
$c(x)$	\mathbb{R}	$[-1, 1]$
$s(x)$	\mathbb{R}	$[-1, 1]$
$\exp(x)$	\mathbb{R}	$(0, +\infty)$

EVALUATE FUNCTIONS

function	domain	range	$f(-1)$	$f(6)$
$f(x) = (x - 1)^2 + 5$				
$f(x) = \frac{1}{x + 2} - 3$				
$f(x) = x^3 + \sqrt{x - 2}$				

EVALUATE FUNCTIONS

function	domain	range	$f(-1)$	$f(6)$
$f(x) = (x - 1)^2 + 5$	\mathbb{R}	$[5, +\infty)$	9	30
$f(x) = \frac{1}{x + 2} - 3$	$(-\infty, -2) \cup (-2, +\infty)$	$(-\infty, -3) \cup (-3, +\infty)$	-2	$-2\frac{7}{8}$ $(-\frac{23}{8})$
$f(x) = x^3 + \sqrt{x - 2}$	$[2, +\infty)$	$[8, +\infty)$		218

EVALUATE FUNCTIONS

- $f(x) = (x - 1)^2 + 5$
 - $f(2) = (2 - 1)^2 + 5 = 6$
- $g(x) = \frac{1}{x+2} - 3$
 - $g(6) = \frac{1}{6+2} - 3 = -\frac{23}{8}$
- $(g \circ f)(x) = ?$
 - $(g \circ f)(2) = ?$
- Will talk more about it next class
(Operations on Functions)!



HOW TO REPRESENT A FUNCTION?

RANK	PEAK	TITLE	WORLDWIDE GROSS	YEAR
1	1	<u>Avengers: Endgame</u>	\$2,796,255,402	2019
2	1	<u>Avatar</u>	\$2,789,679,794	2009
3	1	<u>Titanic</u>	\$2,187,463,944	1997
4	3	<u>Star Wars: The Force Awakens</u>	\$2,068,223,624	2015
5	4	<u>Avengers: Infinity War</u>	\$2,048,359,754	2018
6	3	<u>Jurassic World</u>	\$1,671,713,208	2015
7	7	<u>The Lion King</u>	\$1,569,877,040	2019
8	3	<u>The Avengers</u>	\$1,518,812,988	2012
9	4	<u>Furious 7</u>	\$1,516,045,911	2015
10	5	<u>Avengers: Age of Ultron</u>	\$1,405,403,694	2015

REPRESENT FUNCTIONS: THREE METHODS

Market Summary > Apple Inc.
NASDAQ: AAPL

+ Follow

204.78 USD **-3.96 (1.90%)** ↓

Sep 3, 2:08 PM EDT · Disclaimer

1 day 5 days **1 month** 6 months YTD 1 year 5 years Max



Open	206.43	Div yield	1.50%
High	206.98	Prev close	208.74
Low	204.22	52-wk high	233.47
Mkt cap	925.55B	52-wk low	142.00
P/E ratio	17.45		

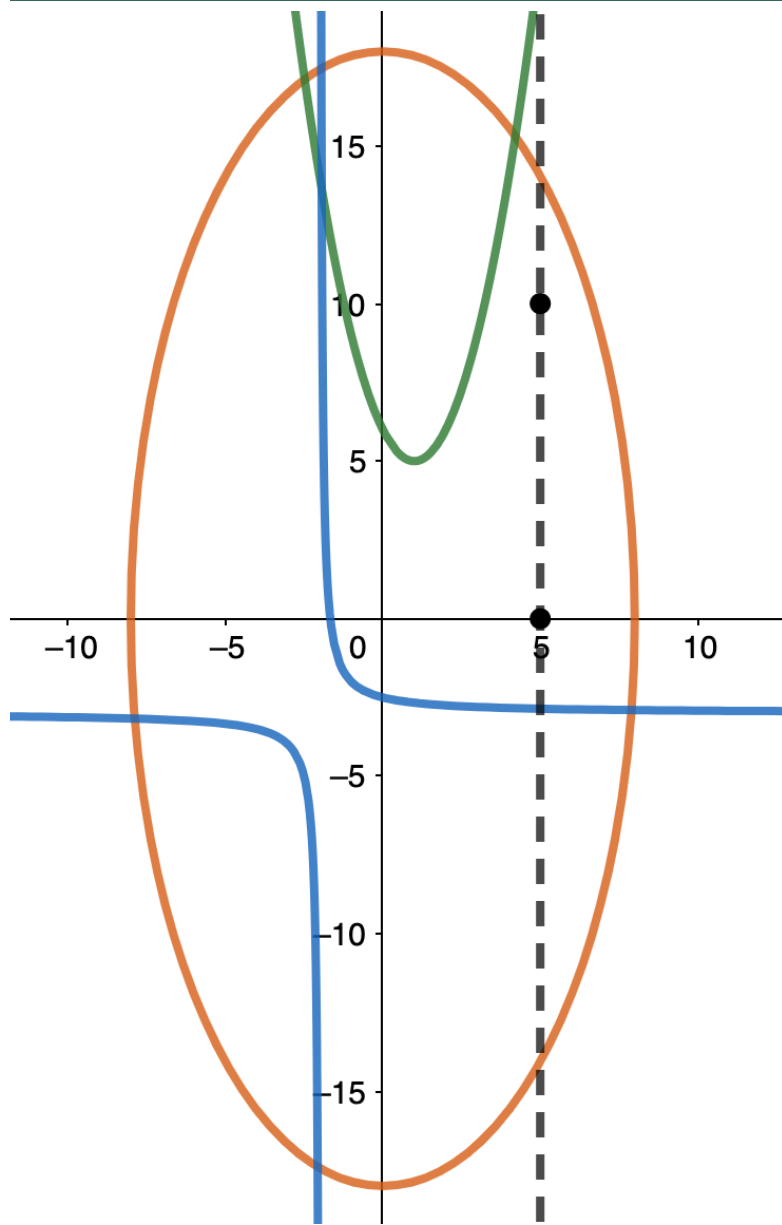
→ [Financial news, comparisons and more](#)

REPRESENT FUNCTIONS

THREE METHODS

REPRESENT FUNCTIONS: THREE METHODS

- A table
- A graph
- A formula
 - $f(x) = \dots$

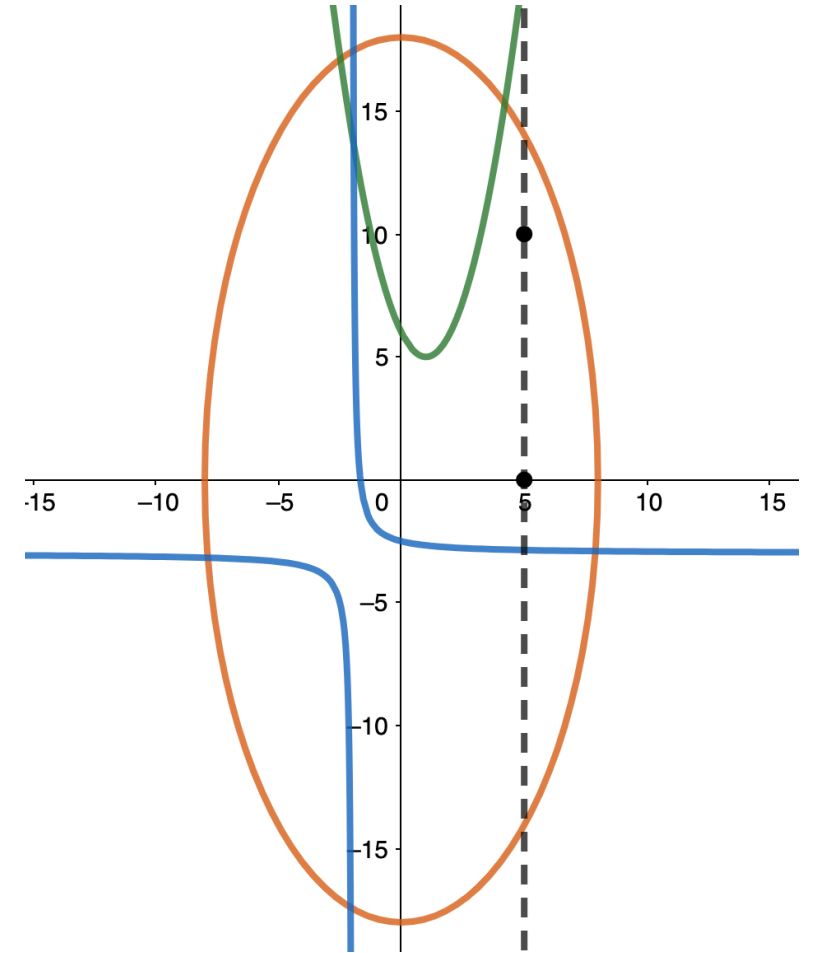


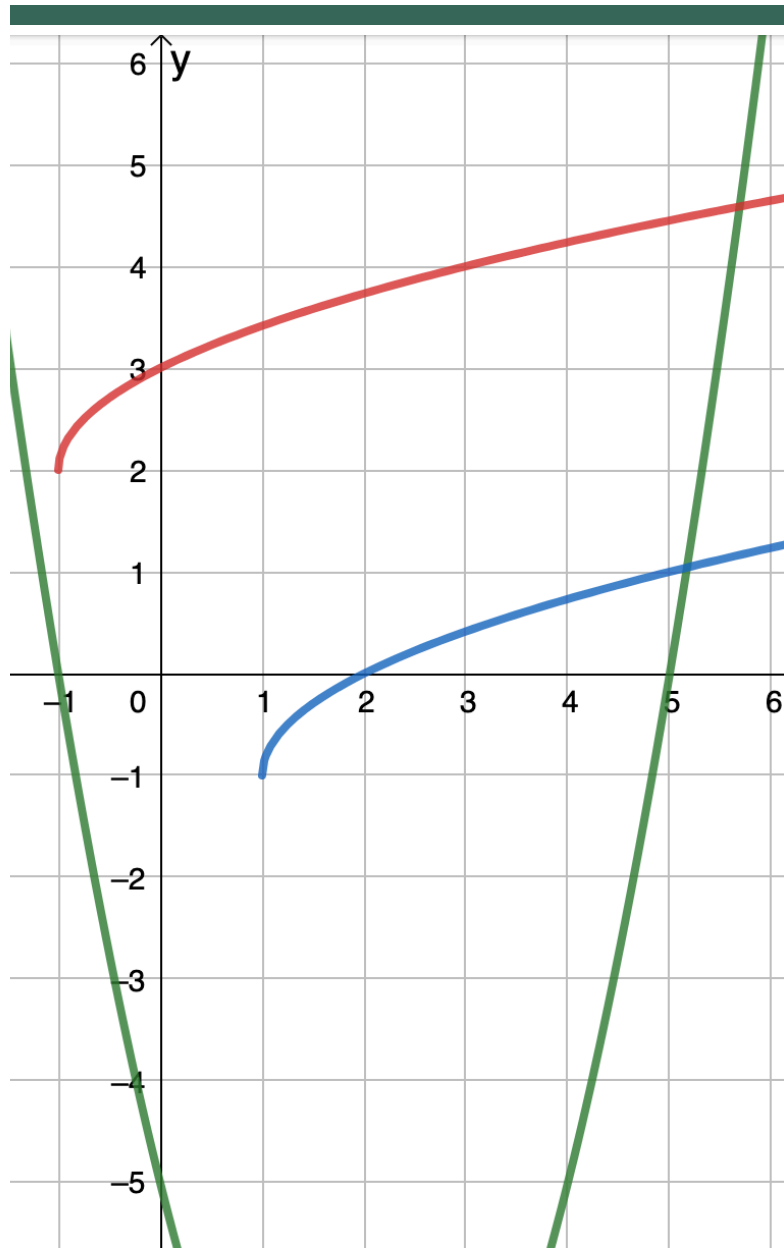
GRAPH: VERTICAL LINE TEST

- Among the three curves, which are functions and which are not?
- Given a function, every vertical line that may be drawn intersects the graph **no more than once!**

GRAPH: SPECIAL POINTS

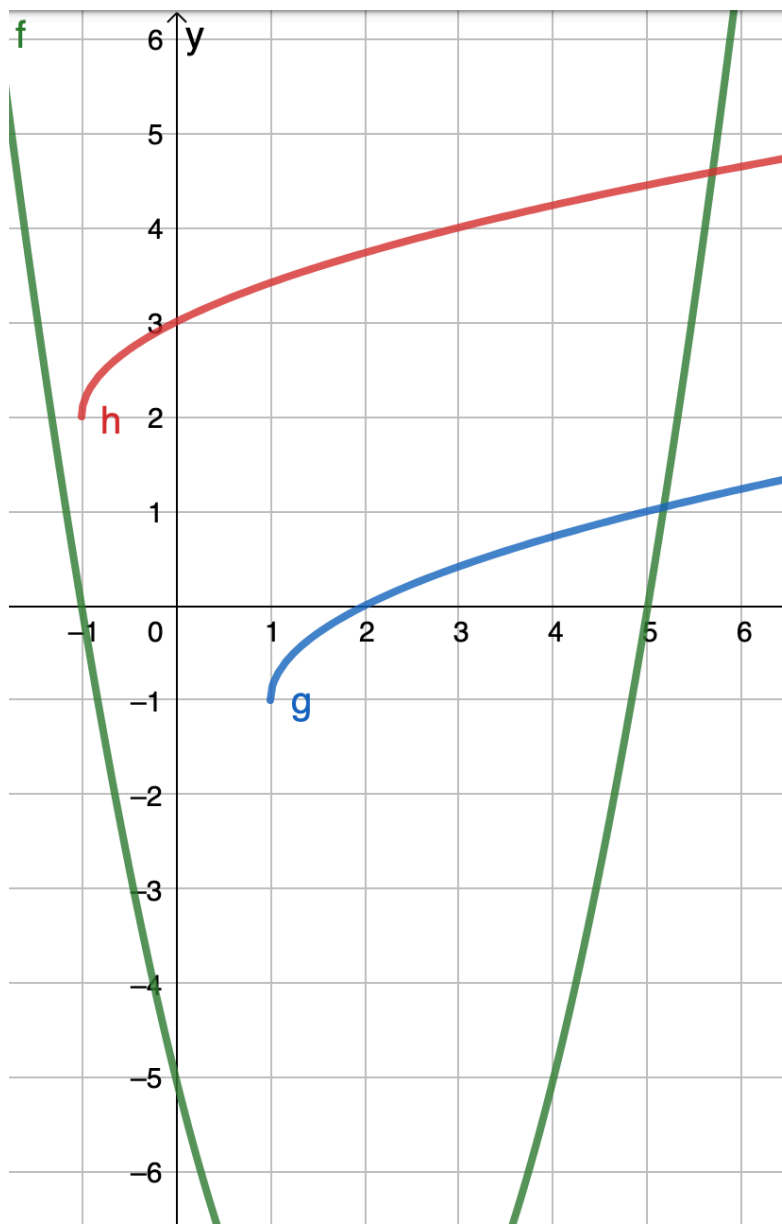
- zeros
 - the **values** of x where $f(x)=0$.
- x-intercepts
- y-intercepts
 - the y-intercept is given by **$(0, f(0))$** , if any.



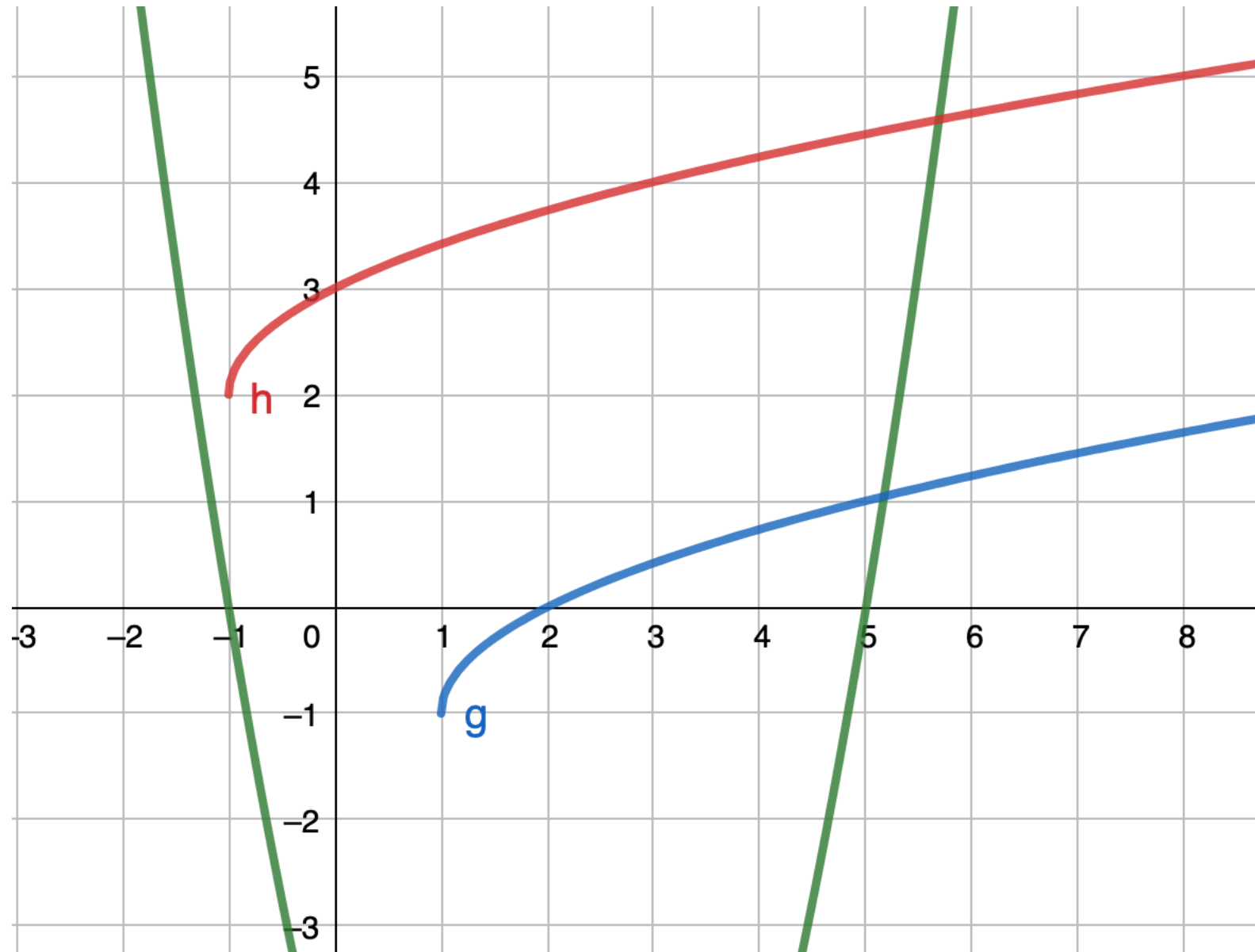


FIND ZEROS AND Y-INTERCETS

- $f(x) = x^2 - 4x - 5$
- $g(x) = \sqrt{x-1} - 1$
- $h(x) = \sqrt{x+1} + 2$



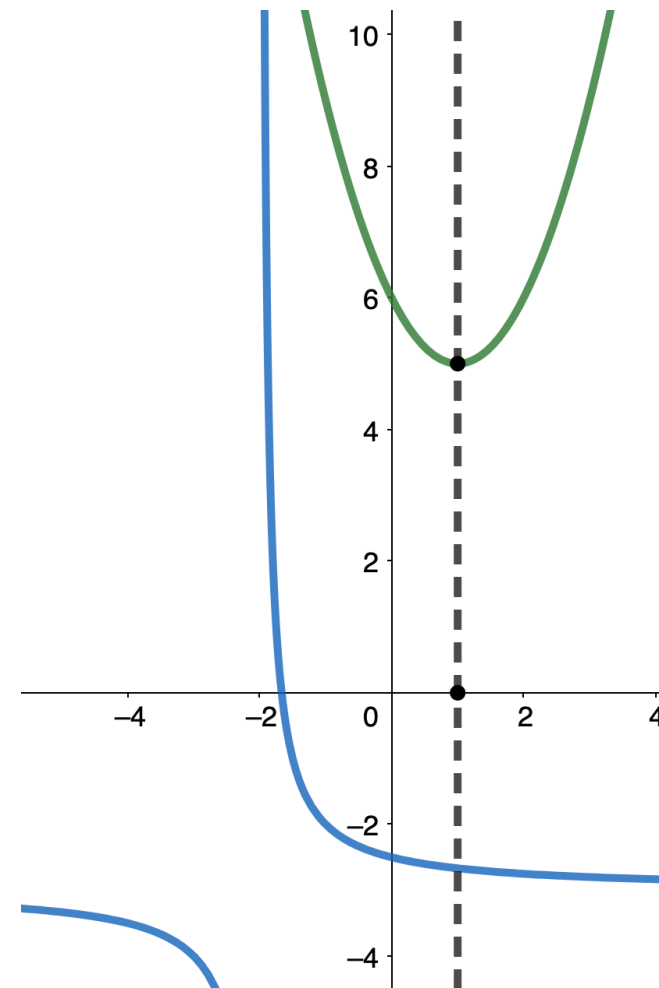
function	zeros	y-intercepts
$f(x) = x^2 - 4x - 5$	-1, 5	(0, -5)
$g(x) = \sqrt{x - 1} - 1$	2	
$h(x) = \sqrt{x + 1} + 2$		(0, 3)



HOW TO
DESCRIBE THE
UPS AND
DOWNS

MONOTONICITY:
INCREASING?
DECREASING?

- $f(x) = (x - 1)^2 + 5$
- $g(x) = \frac{1}{x+2} - 3$



MONOTONICITY

DEFINITION

We say that a function f is **increasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \leq f(x_2) \text{ when } x_1 < x_2.$$

We say f is strictly increasing on the interval I if for all $x_1, x_2 \in I$,

$$f(x_1) < f(x_2) \text{ when } x_1 < x_2.$$

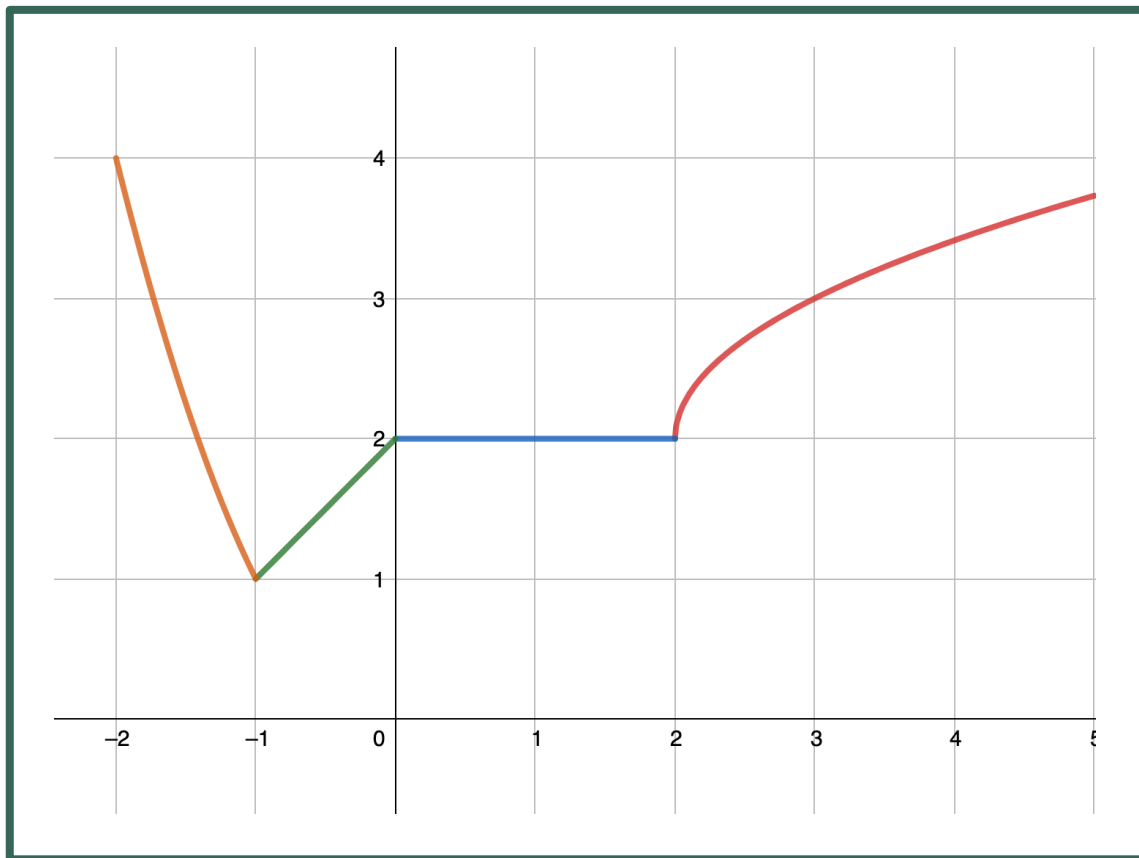
We say that a function f is **decreasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \geq f(x_2) \text{ if } x_1 < x_2.$$

We say that a function f is strictly decreasing on the interval I if for all $x_1, x_2 \in I$,

$$f(x_1) > f(x_2) \text{ if } x_1 < x_2.$$

MONOTONICITY



- A piecewise-defined function

- $$f(x) = \begin{cases} x^2, & -2 \leq x < -1 \\ x + 2, & -1 \leq x < 0 \\ 2, & 0 \leq x < 2 \\ \sqrt{x-2} + 2, & 2 \leq x \leq 5 \end{cases}$$