



Diffusion Models and Scientific Generative AI

**—— Lecture 3: Foundations of Generative
Models (2)**

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AI3 institute, Fudan University**

Contents

- **Last Class Review**
- **Generative Model**
 - **Autoregressive Model (PixelRNN/PixelCNN)**
 - **Variational Autoencoder (VAEs)**
 - **Generative Adversarial Networks (GAN)**

Supervised vs Unsupervised Learning

Supervised Learning

Data: (x, y)
 x is data, y is label

Goal: Learn a function to map
 $x \rightarrow y$

Examples: Classification,
regression, object detection,
semantic segmentation,
image captioning, etc.

Unsupervised Learning

Data: x
Just data, no labels!

Goal: Learn some underlying
hidden structure of the data

Examples: Clustering,
dimensionality reduction,
density estimation, etc.

Self-Supervised Learning

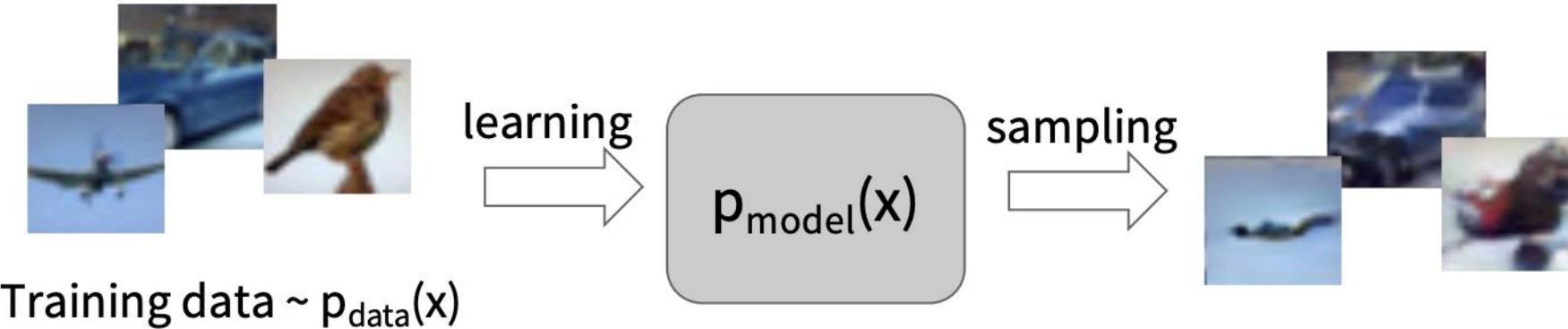
Data: $(x, \text{pseudo generated } y)$
No manual labels!

Goal: Learn to generate good
features (reduce the data to
useful/generic features)

Example: Classification in
downstream applications
where we have limited data

Generative Model

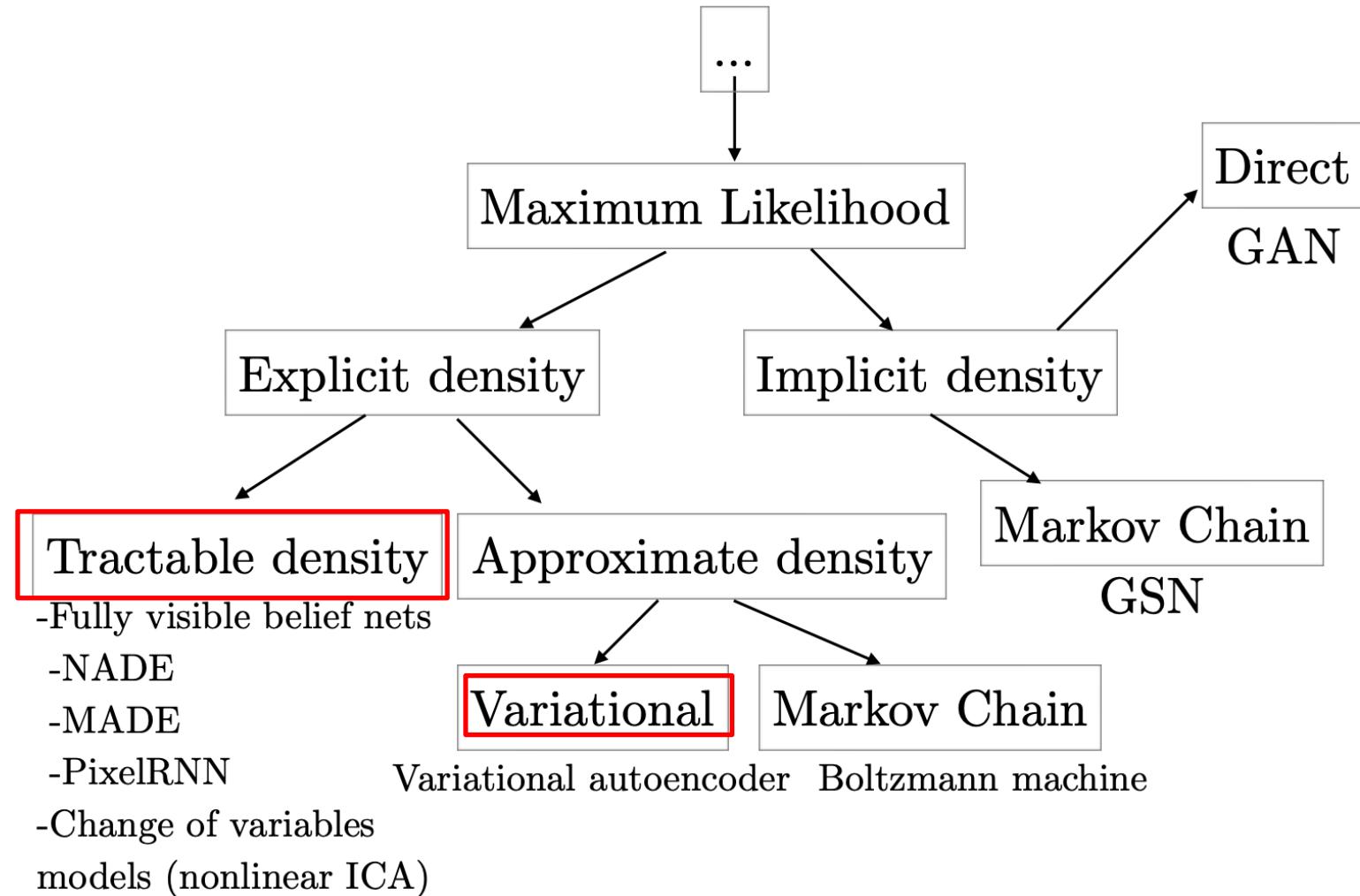
- Given training data, generate new samples from same distribution.



Formulate as density estimation problems:

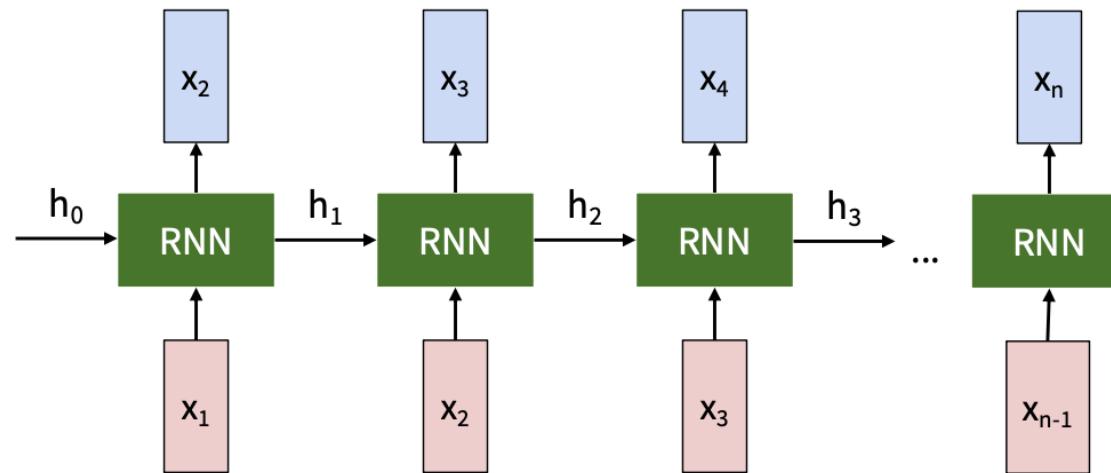
- Explicit density estimation: explicitly define and solve for $p_{\text{model}}(x)$
- Implicit density estimation: learn model that can sample from $p_{\text{model}}(x)$ without explicitly defining it.

Taxonomy of Generative Models



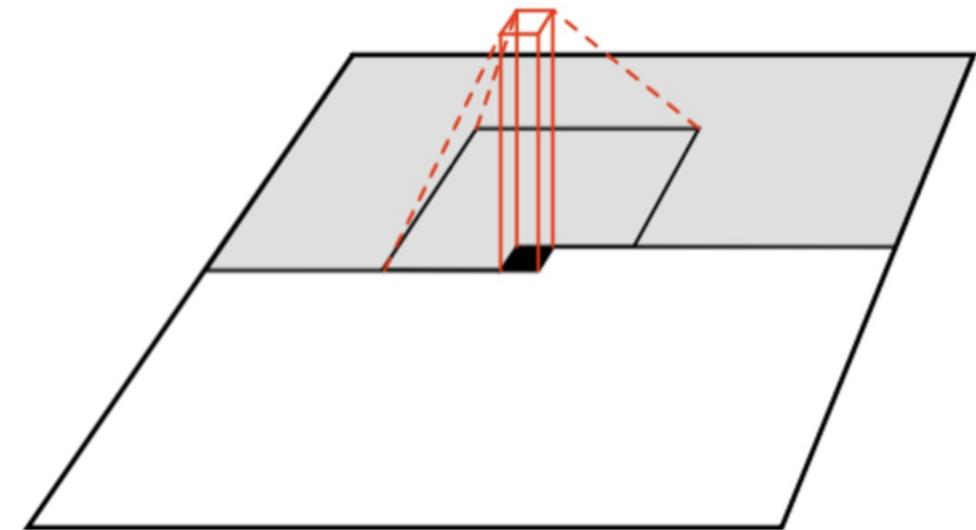
Autoregressive Models

- PixelRNN/ PixelCNN [van der Oord et al. 2016]



$$p(x_i|x_1, \dots, x_{i-1})$$

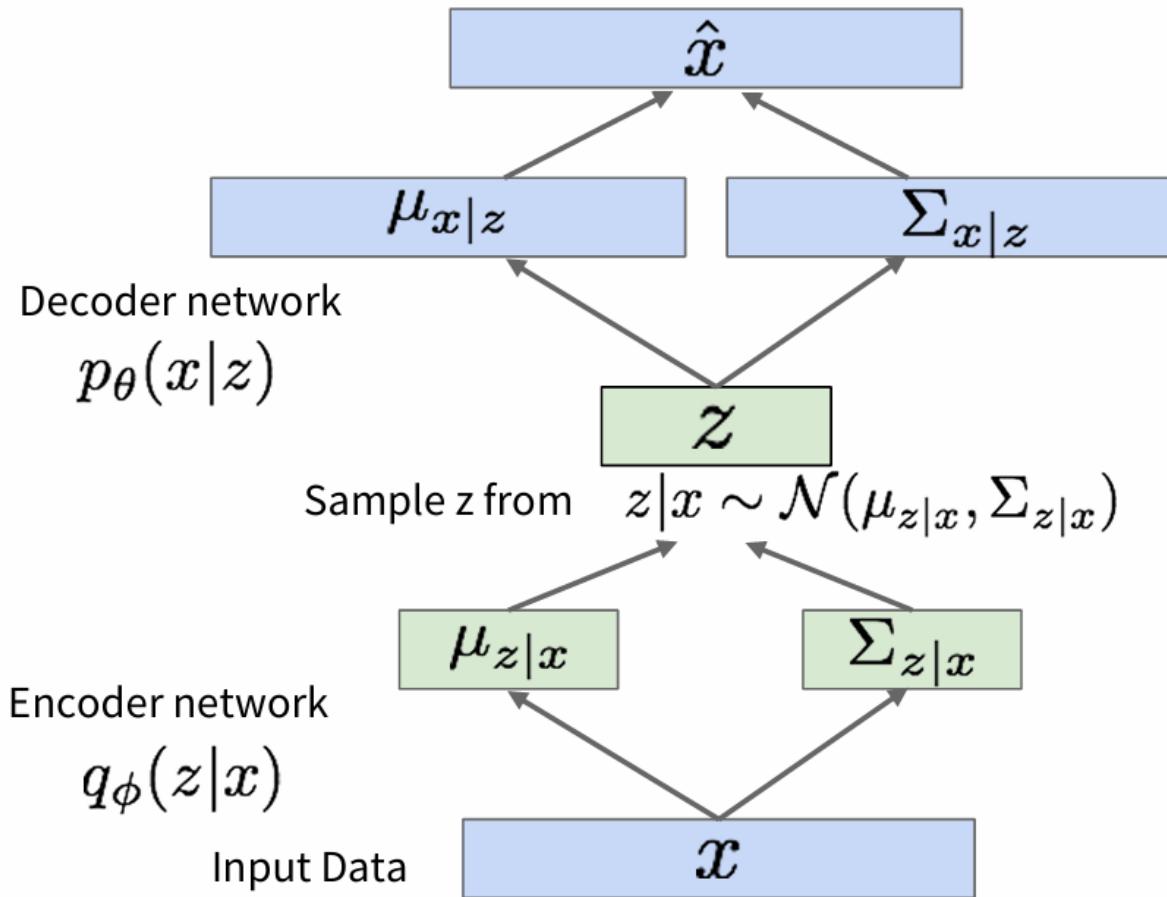
PixelRNN



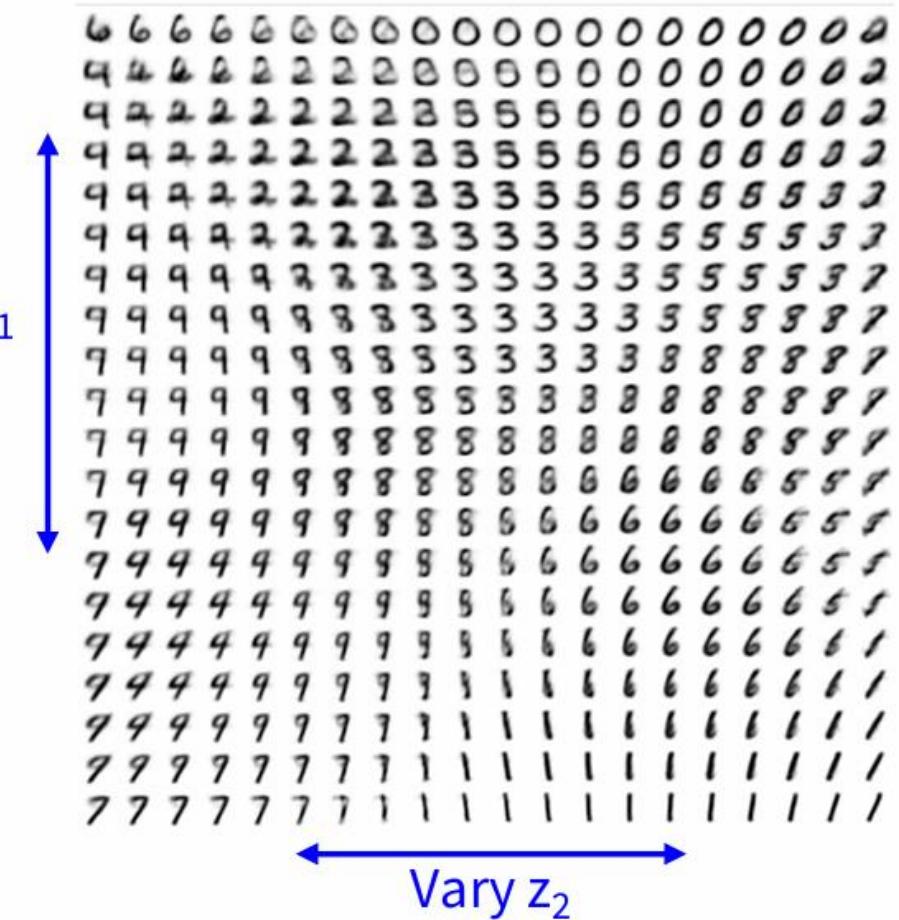
PixelCNN

Variational Autoencoder (VAE)

- VAE [Kingma et al. 2014]



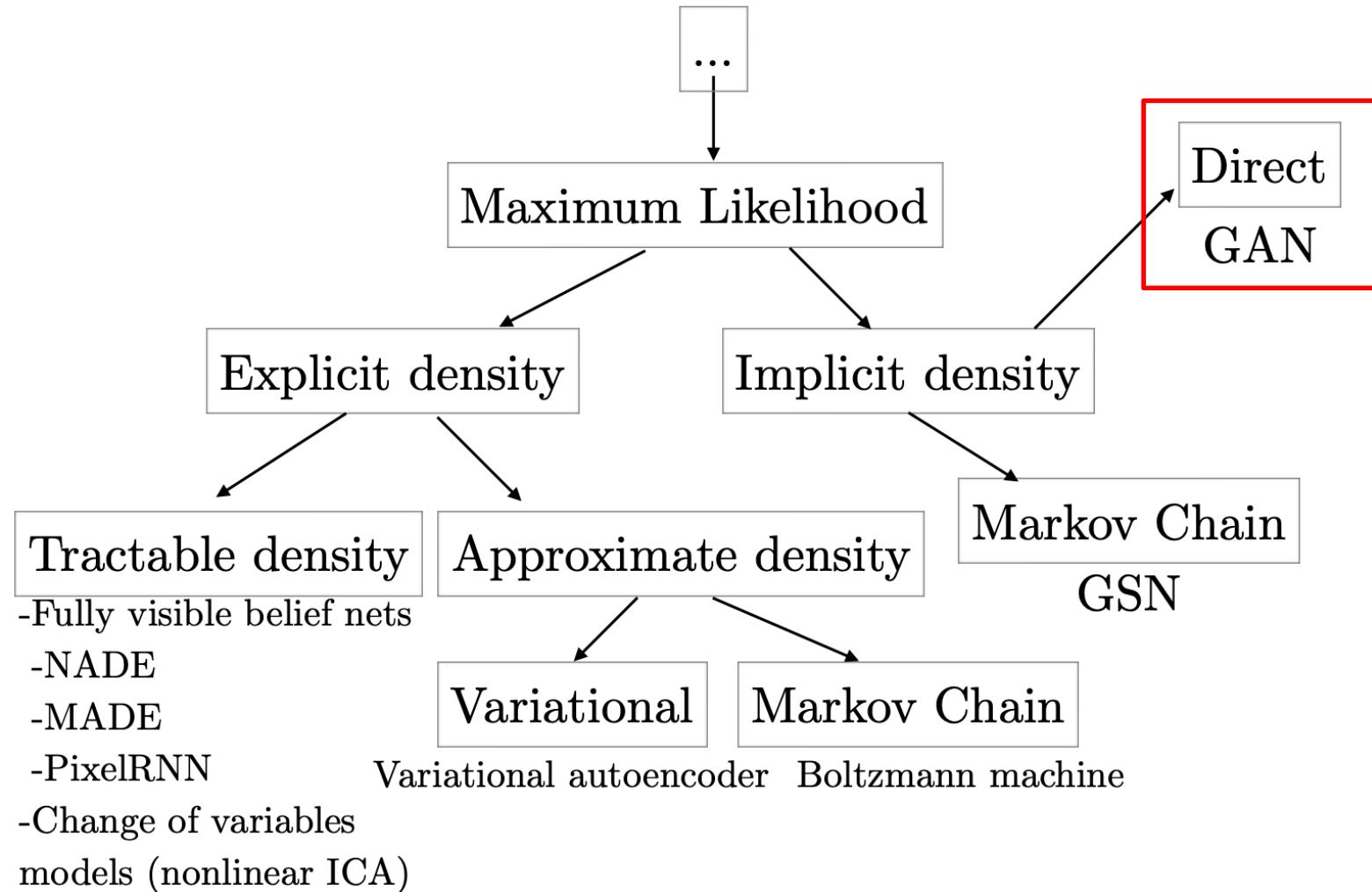
Data manifold for 2-d z



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Taxonomy of Generative Models



Generative Adversarial Networks

PixelRNN/CNNs define tractable density function, optimize likelihood of training data:

$$p_{\theta}(x) = \prod_{i=1}^n p_{\theta}(x_i|x_1, \dots, x_{i-1})$$

VAEs define intractable density function with latent z:

$$p_{\theta}(x) = \int p_{\theta}(z)p_{\theta}(x|z)dz$$

Cannot optimize directly, derive and optimize lower bound on likelihood instead

Generative Adversarial Networks

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Cannot optimize directly, derive and optimize lower bound on likelihood instead

What if we give up on explicitly modeling density, and just want ability to sample?

Generative Adversarial Networks

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$$p_{\theta}(x) = \int p_{\theta}(z)p_{\theta}(x|z)dz$$

Cannot optimize directly, derive and optimize lower bound on likelihood instead

What if we give up on explicitly modeling density, and just want ability to sample?

GANs: not modeling any explicit density function!

Generative Adversarial Networks

Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

Solution: Sample from a simple distribution we can easily sample from, e.g. random noise. Learn transformation to training distribution.

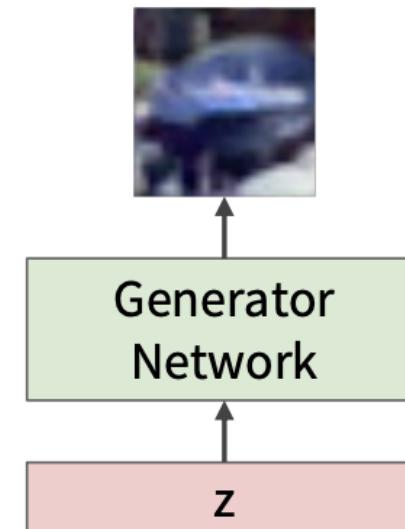
Generative Adversarial Networks

Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

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Output: Sample from training distribution

Input: Random noise



Generative Adversarial Networks

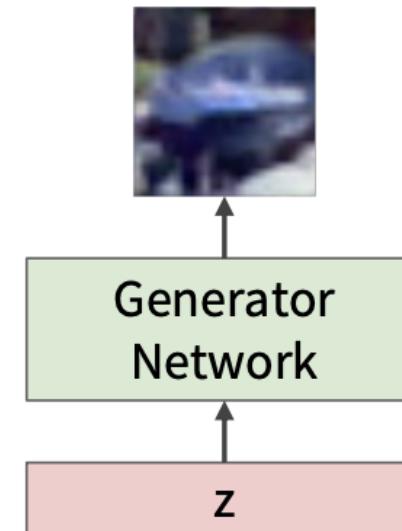
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But we don't know which sample z maps to which training image -> can't learn by reconstructing training images

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Generative Adversarial Networks

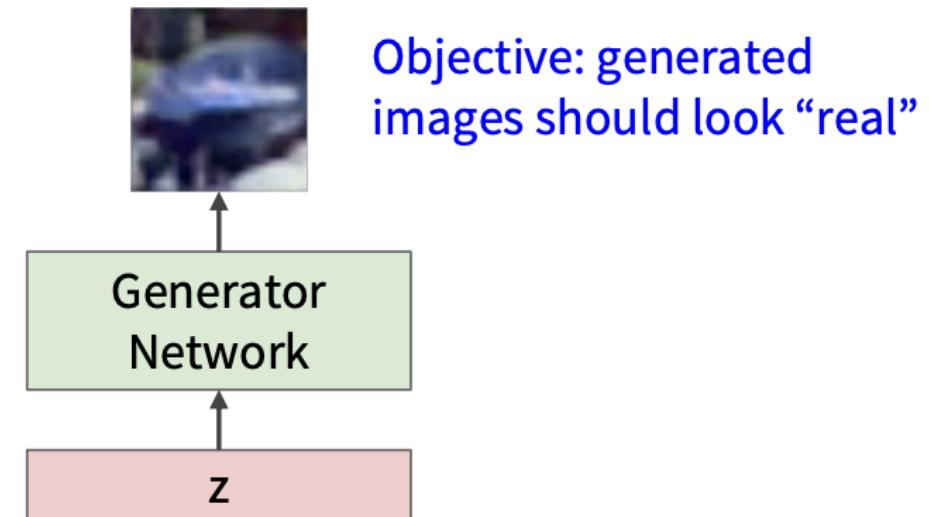
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Output: Sample from training distribution

Input: Random noise



Objective: generated images should look “real”

Generative Adversarial Networks

Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

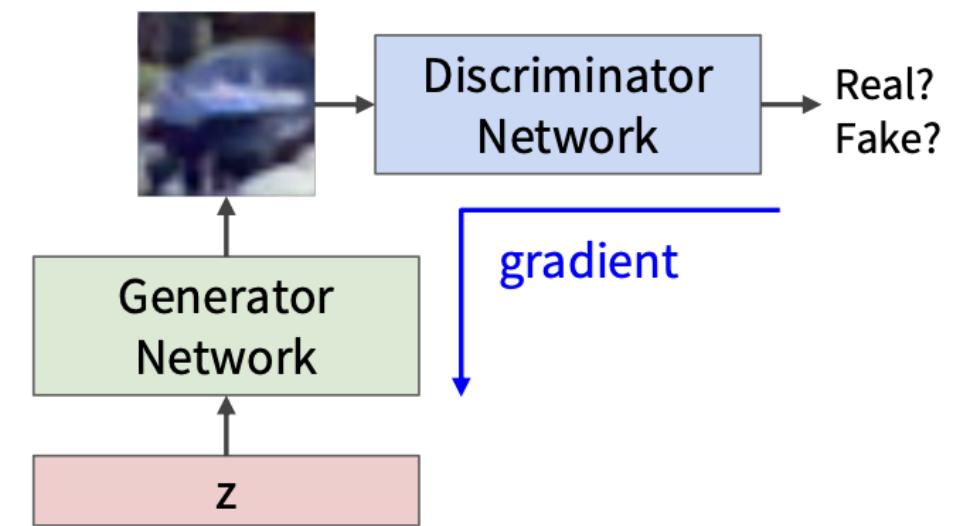
Solution: Sample from a simple distribution we can easily sample from, e.g. random noise. Learn transformation to training distribution.

But we don't know which sample z maps to which training image -> can't learn by reconstructing training images

Solution: Use a discriminator network to tell whether the generate image is within data distribution ("real") or not

Output: Sample from training distribution

Input: Random noise



Training GANs: Two-player game

- Optimization Goal

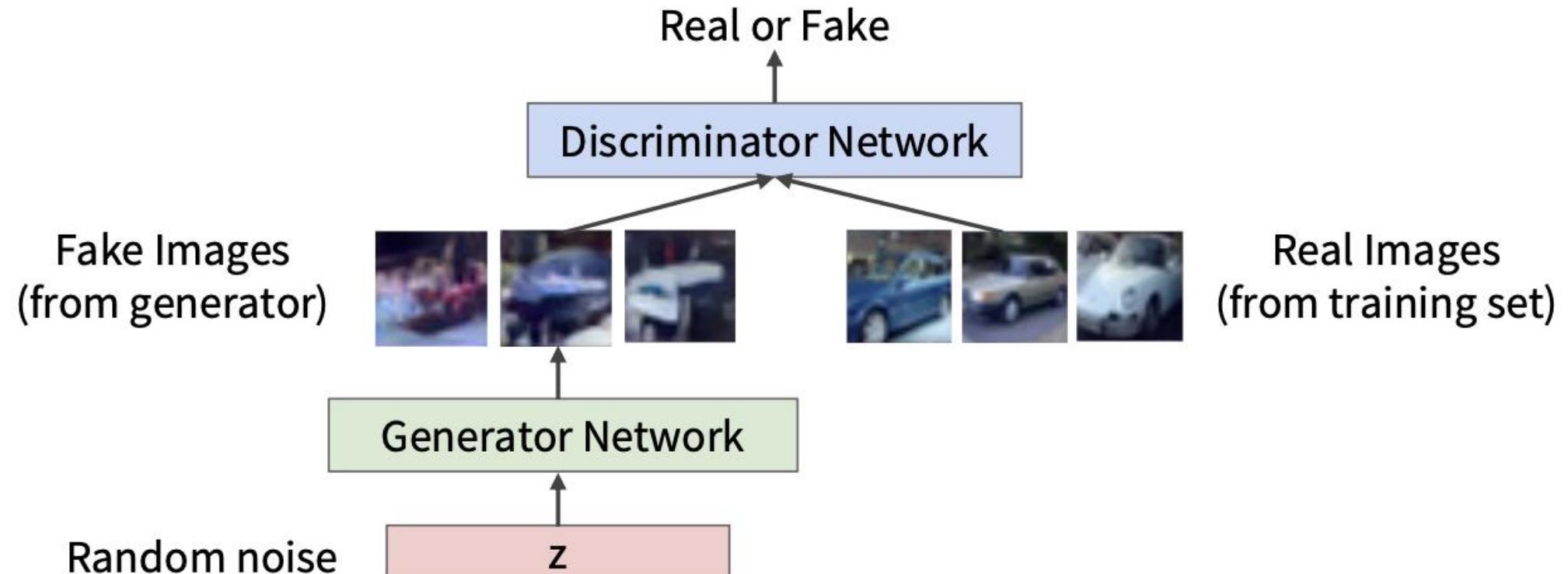
Discriminator network: try to distinguish between real and fake images

Generator network: try to fool the discriminator by generating real-looking images

Training GANs: Two-player game

Discriminator network: try to distinguish between real and fake images

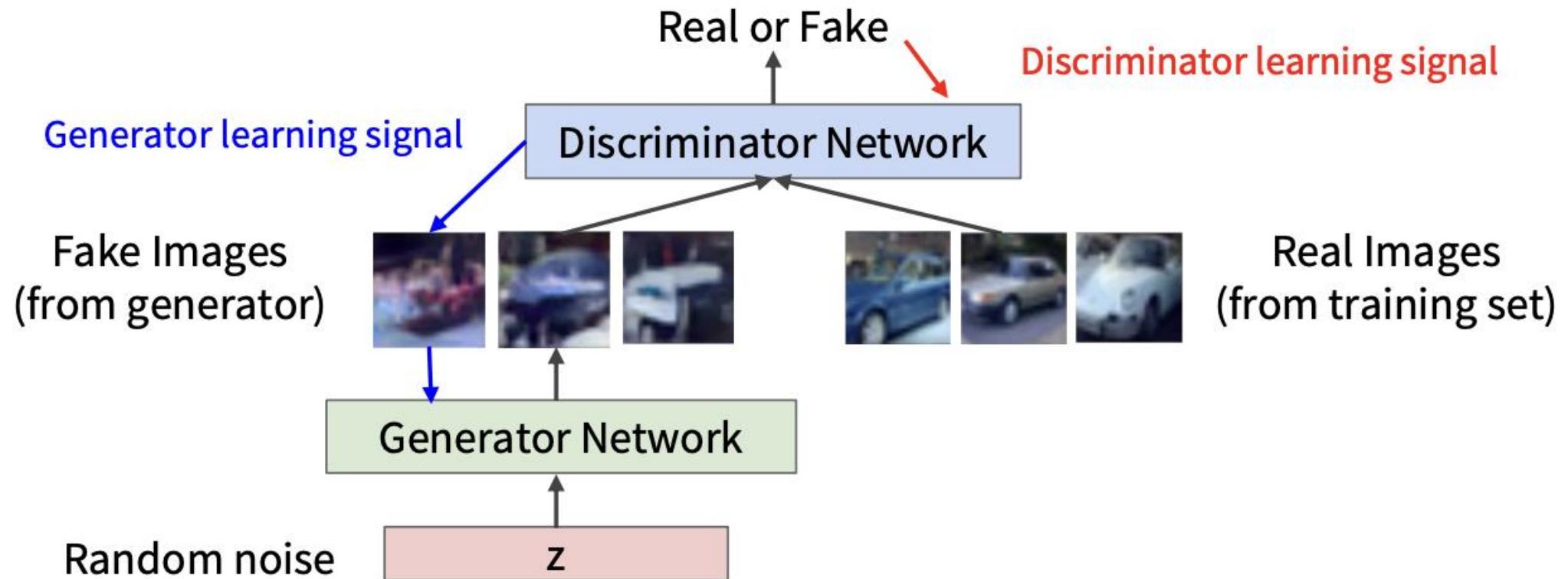
Generator network: try to fool the discriminator by generating real-looking images



Training GANs: Two-player game

Discriminator network: try to distinguish between real and fake images

Generator network: try to fool the discriminator by generating real-looking images



Training GANs: Two-player game

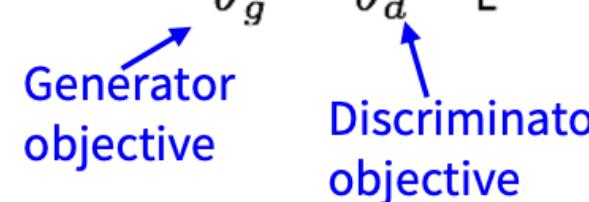
Discriminator network: try to distinguish between real and fake images

Generator network: try to fool the discriminator by generating real-looking images

Train jointly in **minimax game**

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$


Generator objective Discriminator objective

Training GANs: Two-player game

Discriminator network: try to distinguish between real and fake images

Generator network: try to fool the discriminator by generating real-looking images

Train jointly in minimax game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log \underbrace{D_{\theta_d}(x)}_{\text{Discriminator output for real data } x} + \mathbb{E}_{z \sim p(z)} \log(1 - \underbrace{D_{\theta_d}(G_{\theta_g}(z))}_{\text{Discriminator output for generated fake data } G(z)}) \right]$$

- Discriminator (θ_d) wants to maximize objective such that $D(x)$ is close to 1 (real) and $D(G(z))$ is close to 0 (fake)
- Generator (θ_g) wants to minimize objective such that $D(G(z))$ is close to 1 (discriminator is fooled into thinking generated $G(z)$ is real)

Training GANs: Two-player game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Alternate between:

1. Gradient ascent on discriminator

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. Gradient descent on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

Training GANs: Two-player game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Alternate between:

1. Gradient ascent on discriminator

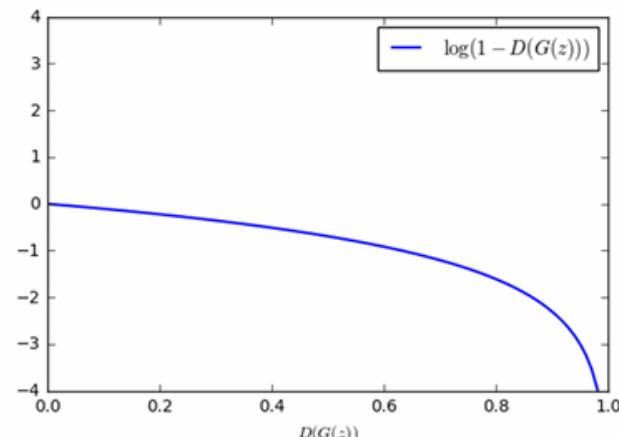
$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. Gradient descent on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

In practice, optimizing this generator objective does not work well!

When sample is likely fake, want to learn from it to improve generator (move to the right on X axis).



Training GANs: Two-player game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Alternate between:

1. Gradient ascent on discriminator

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Gradient signal dominated by region where sample is already good

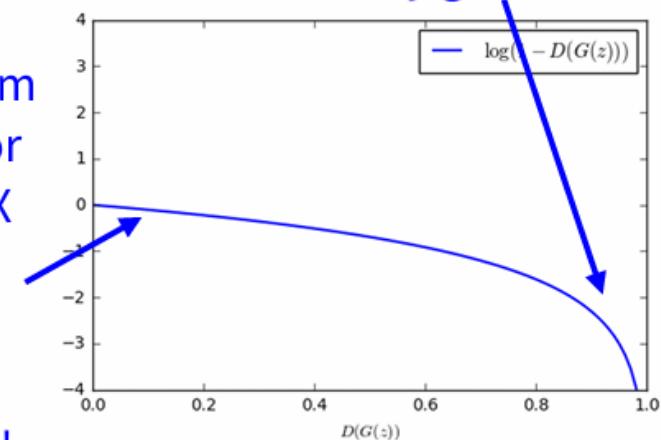
2. Gradient descent on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

In practice, optimizing this generator objective does not work well!

When sample is likely fake, want to learn from it to improve generator (move to the right on X axis).

But gradient in this region is relatively flat!



Training GANs: Two-player game

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

Alternate between:

1. Gradient ascent on discriminator

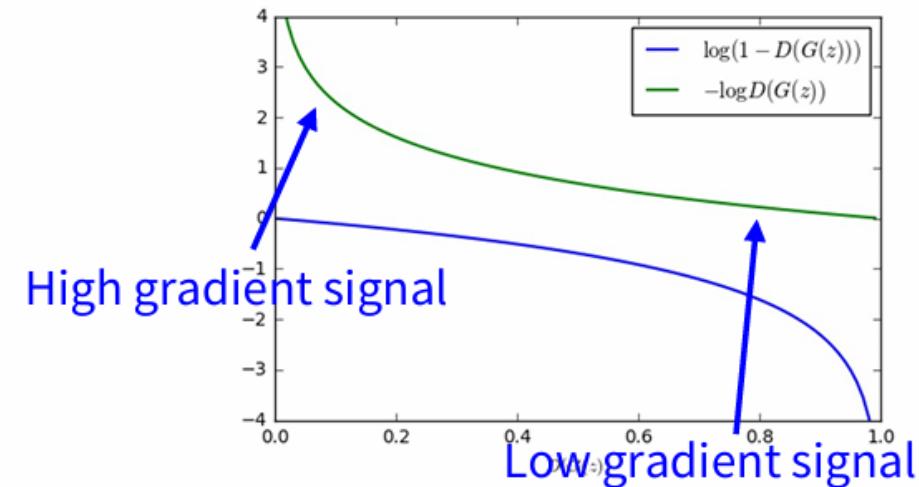
$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. Instead: Gradient **ascent** on generator, different objective

$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(D_{\theta_d}(G_{\theta_g}(z)))$$

Instead of minimizing likelihood of discriminator being correct, now maximize likelihood of discriminator being wrong.

Same objective of fooling discriminator, but now higher gradient signal for bad samples => works much better! Standard in practice.



Training GANs: Two-player game

Putting it together: GAN training algorithm

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
- Sample minibatch of m examples $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\mathbf{x})$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D_{\theta_d}(\mathbf{x}^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)}))) \right]$$

end for

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.

- Update the generator by ascending its stochastic gradient (improved objective):

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)})))$$

end for

Training GANs: Two-player game

Putting it together: GAN training algorithm

```

for number of training iterations do
  for  $k$  steps do
    • Sample minibatch of  $m$  noise samples  $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$  from noise prior  $p_g(\mathbf{z})$ .
    • Sample minibatch of  $m$  examples  $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)}\}$  from data generating distribution  $p_{\text{data}}(\mathbf{x})$ .
    • Update the discriminator by ascending its stochastic gradient:
  
```

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D_{\theta_d}(\mathbf{x}^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)}))) \right]$$

```

    end for
    • Sample minibatch of  $m$  noise samples  $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$  from noise prior  $p_g(\mathbf{z})$ .
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```

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)})))$$

end for

Arjovsky et al. "Wasserstein gan." arXiv preprint arXiv:1701.07875 (2017)

Berthelot, et al. "Began: Boundary equilibrium generative adversarial networks." arXiv preprint arXiv:1703.10717 (2017)

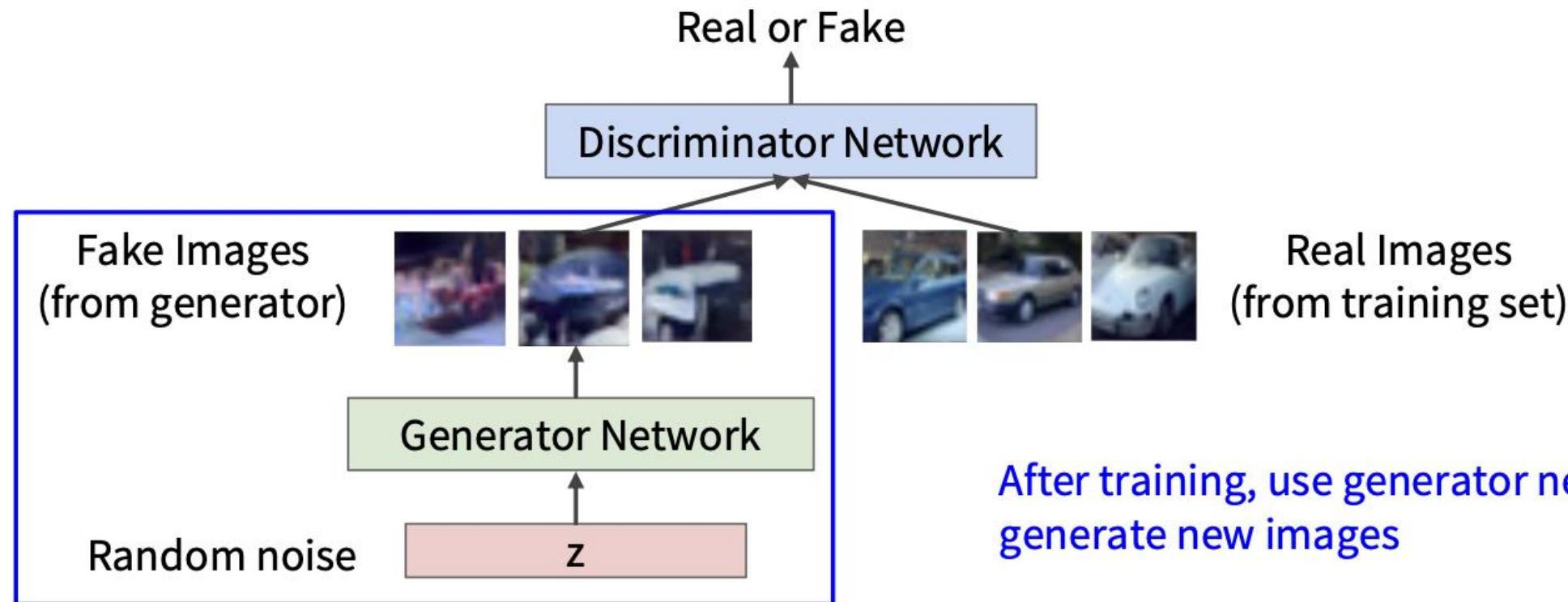
Some find $k=1$ more stable, others use $k > 1$, no best rule.

Followup work (e.g. Wasserstein GAN, BEGAN) alleviates this problem, better stability!

Training GANs: Two-player game

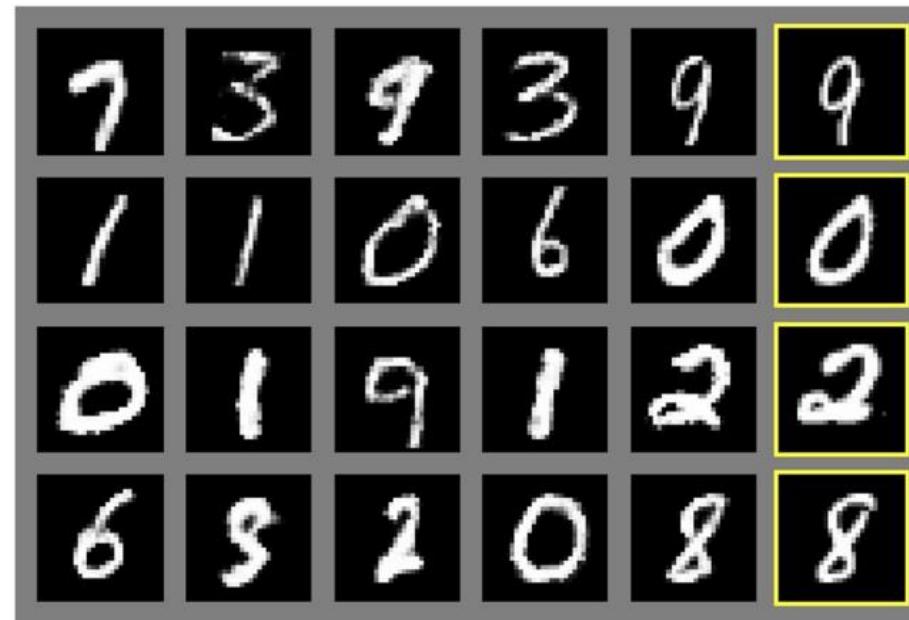
Generator network: try to fool the discriminator by generating real-looking images

Discriminator network: try to distinguish between real and fake images



Generative Adversarial Nets

Generated samples



Nearest neighbor from training set

Figures copyright Ian Goodfellow et al., 2014. Reproduced with permission.

Generative Adversarial Nets

Generated samples (CIFAR-10)



Nearest neighbor from training set

Figures copyright Ian Goodfellow et al., 2014. Reproduced with permission.

Generative Adversarial Nets: Convolutional Architectures

Generator is an upsampling network with fractionally-strided convolutions
Discriminator is a convolutional network

Architecture guidelines for stable Deep Convolutional GANs

- Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator).
- Use batchnorm in both the generator and the discriminator.
- Remove fully connected hidden layers for deeper architectures.
- Use ReLU activation in generator for all layers except for the output, which uses Tanh.
- Use LeakyReLU activation in the discriminator for all layers.

Radford et al, “Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks”, ICLR 2016

Generative Adversarial Nets: Convolutional Architectures

Samples from the model look much better!

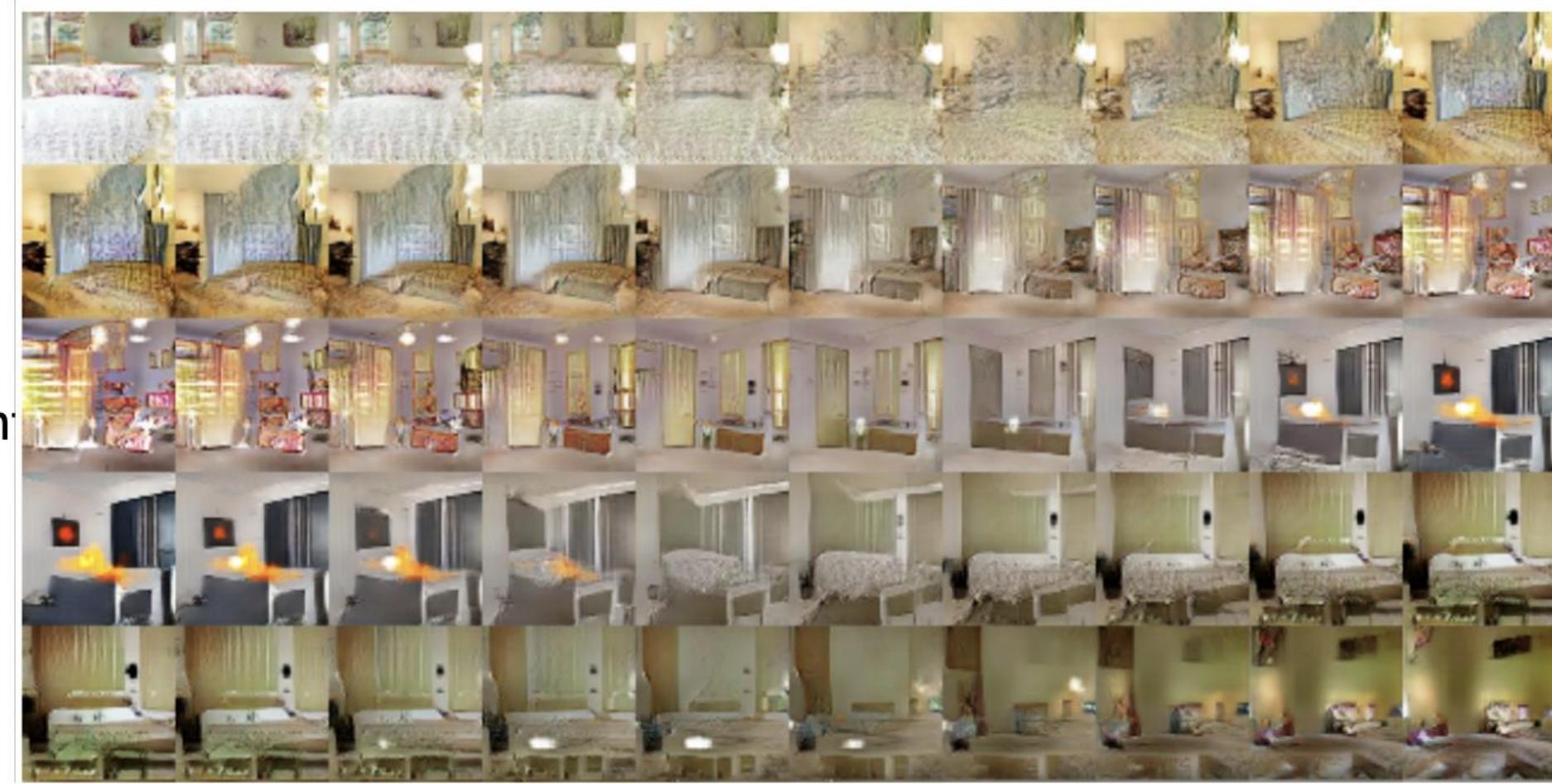
Radford et al,
ICLR 2016



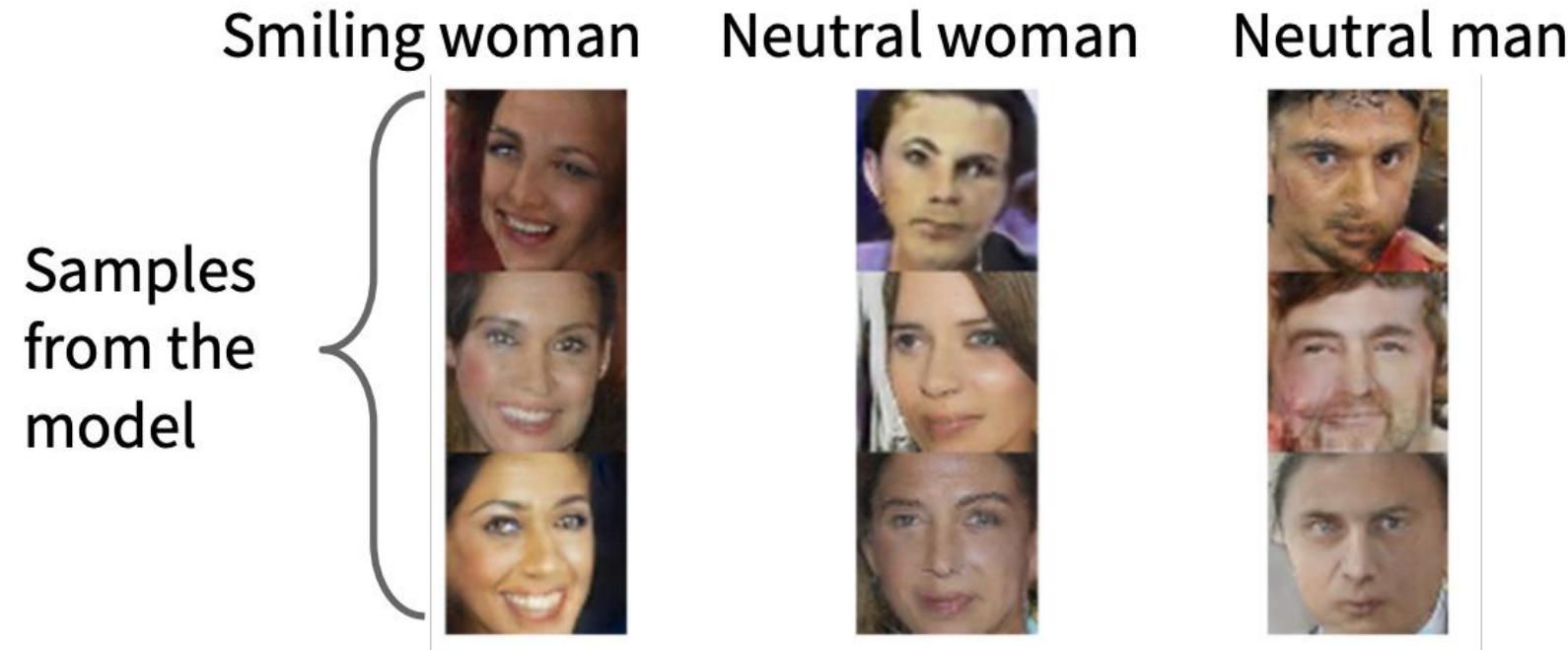
Generative Adversarial Nets: Convolutional Architectures

Interpolating
between
random
points in latent
space

Radford et al,
ICLR 2016

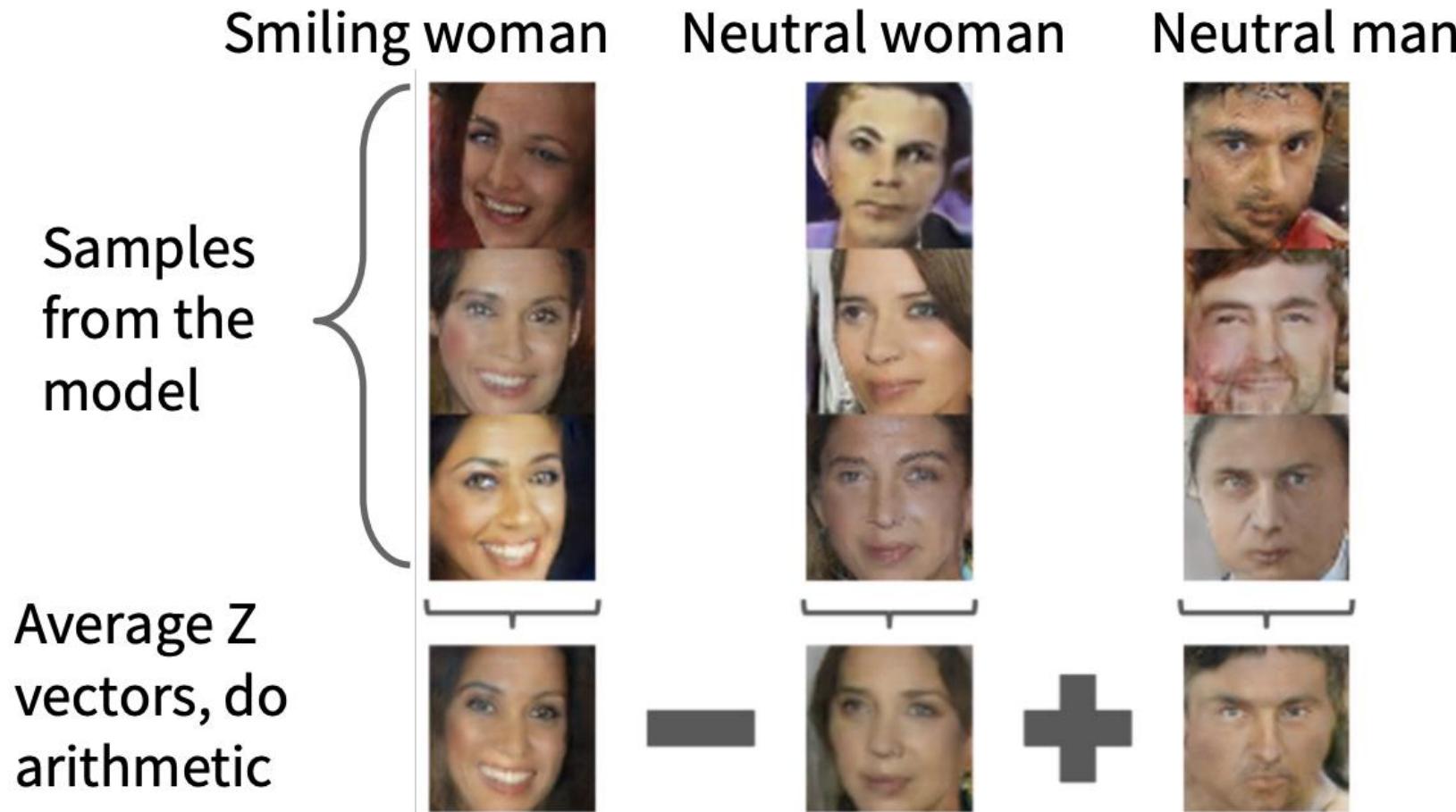


Generative Adversarial Nets: Interpretable Vector Math



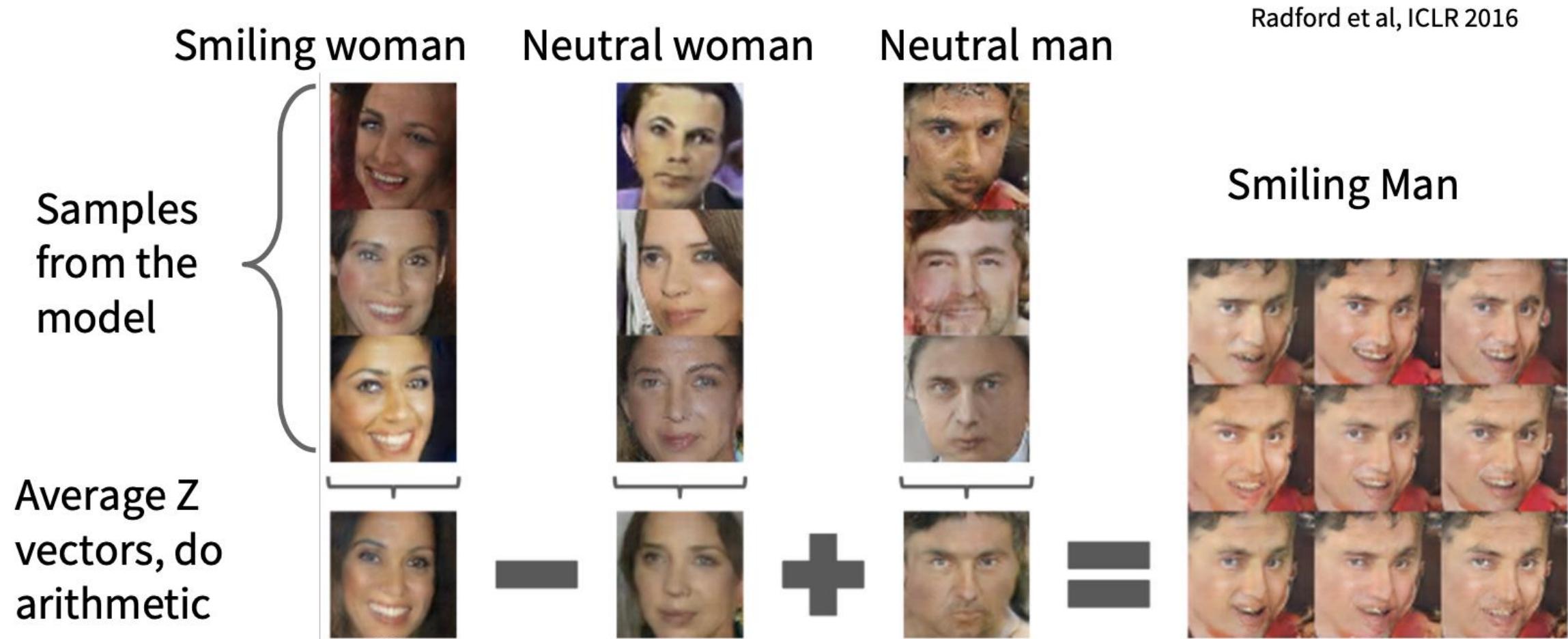
Radford et al, ICLR 2016

Generative Adversarial Nets: Interpretable Vector Math



Radford et al, ICLR 2016

Generative Adversarial Nets: Interpretable Vector Math



Generative Adversarial Nets: Interpretable Vector Math

Glasses man



No glasses man



No glasses woman



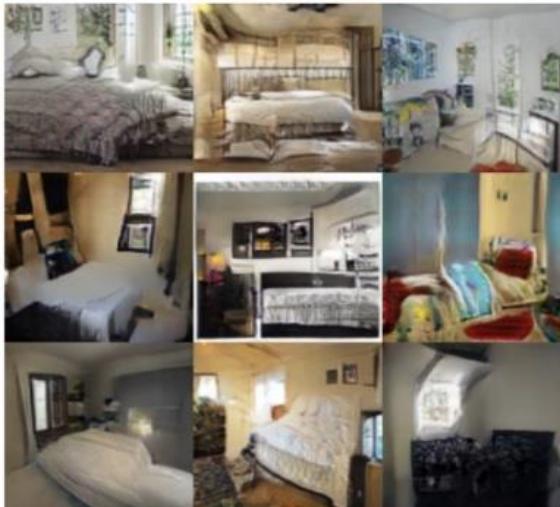
Radford et al,
ICLR 2016

Woman with glasses



Generative Adversarial Nets: Interpretable Vector Math

Better training and generation



LSGAN, Zhu 2017.



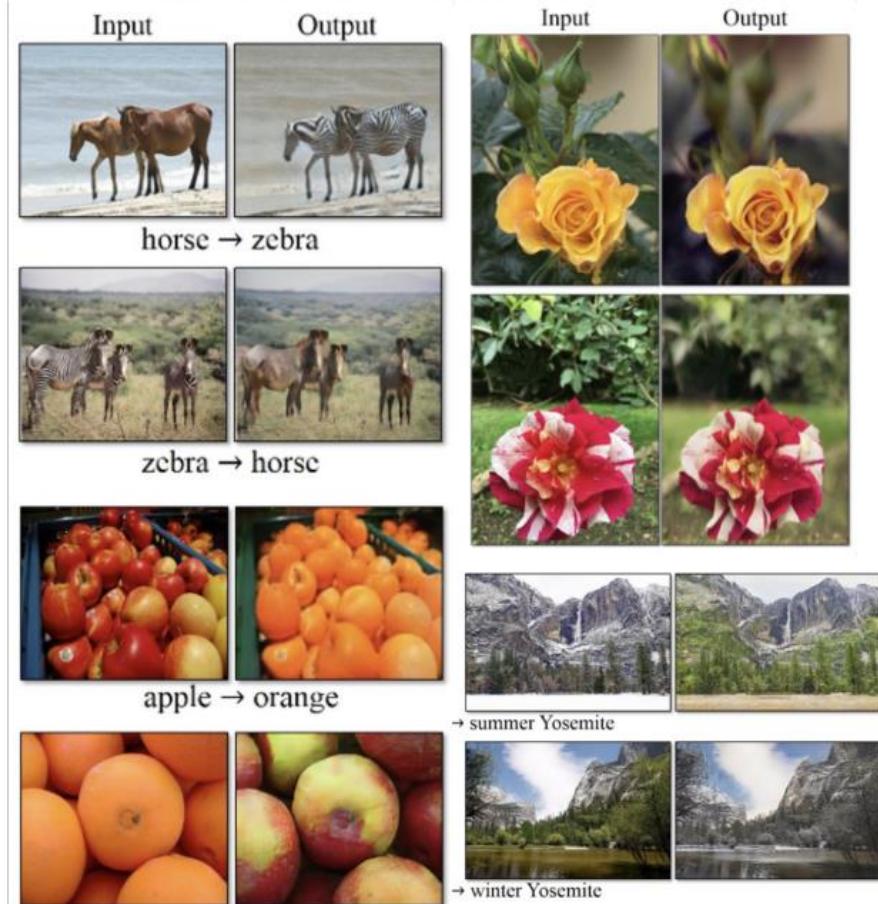
Wasserstein GAN, Arjovsky 2017.
Improved Wasserstein GAN, Gulrajani 2017.



Progressive GAN, Karras 2018.

2017: Explosion of GANs

Source->Target domain transfer



CycleGAN. Zhu et al. 2017.

Text -> Image Synthesis

this small bird has a pink breast and crown, and black primaries and secondaries.



this magnificent fellow is almost all black with a red crest, and white cheek patch.



Reed et al. 2017.

Many GAN applications



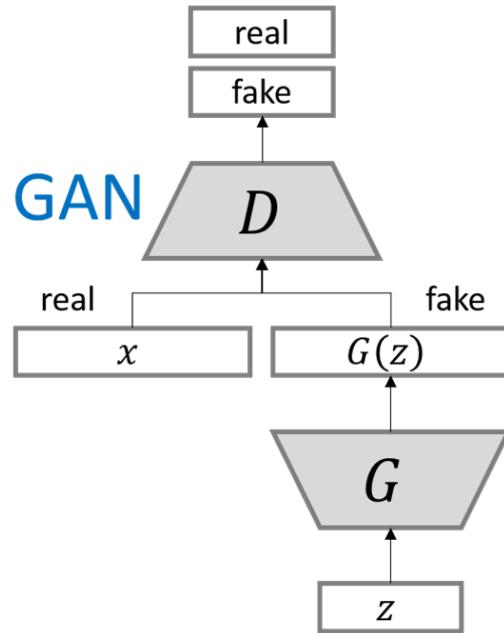
Pix2pix. Isola 2017. Many examples at <https://phillipi.github.io/pix2pix/>

2019: BigGAN

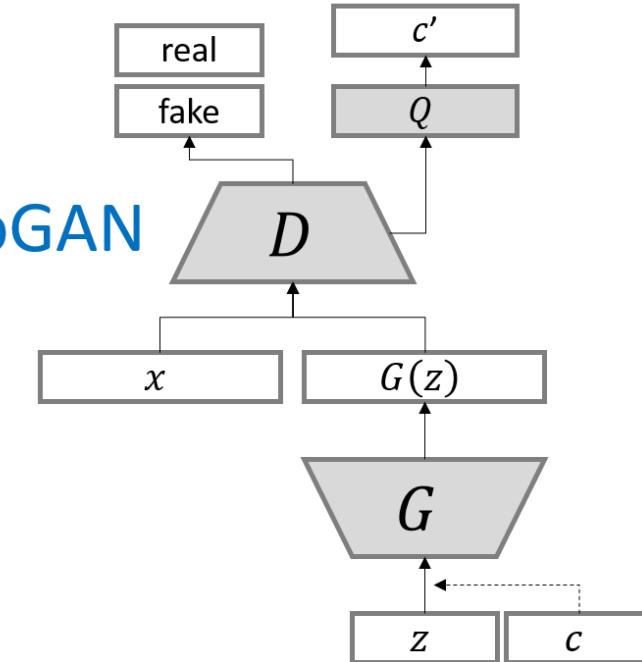
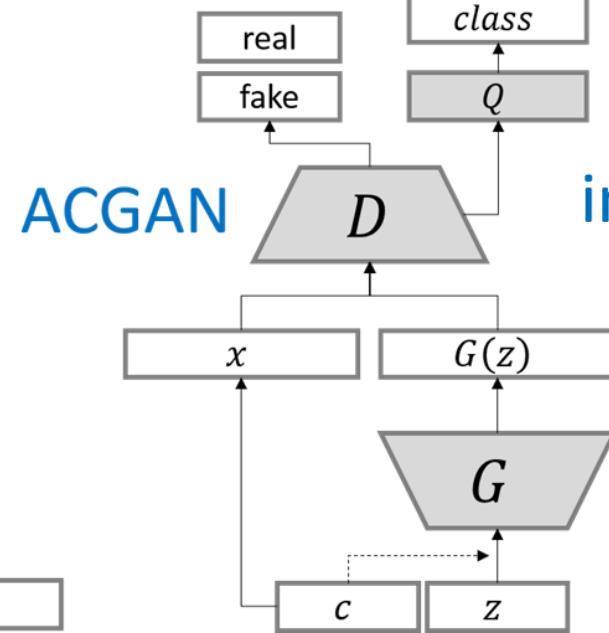
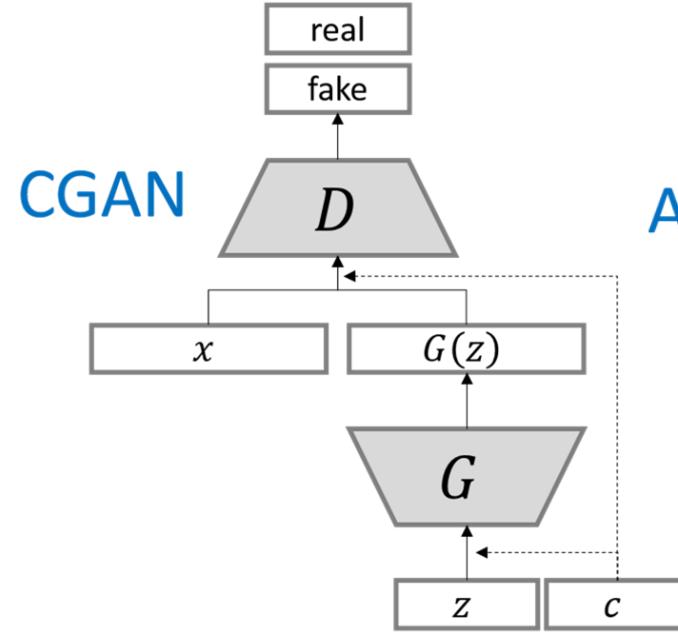


Brock et al., 2019

Generative Adversarial Nets



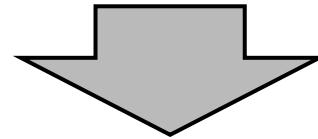
GAN
 Goodfellow et al.
 NIPS 2014



VQ-GAN

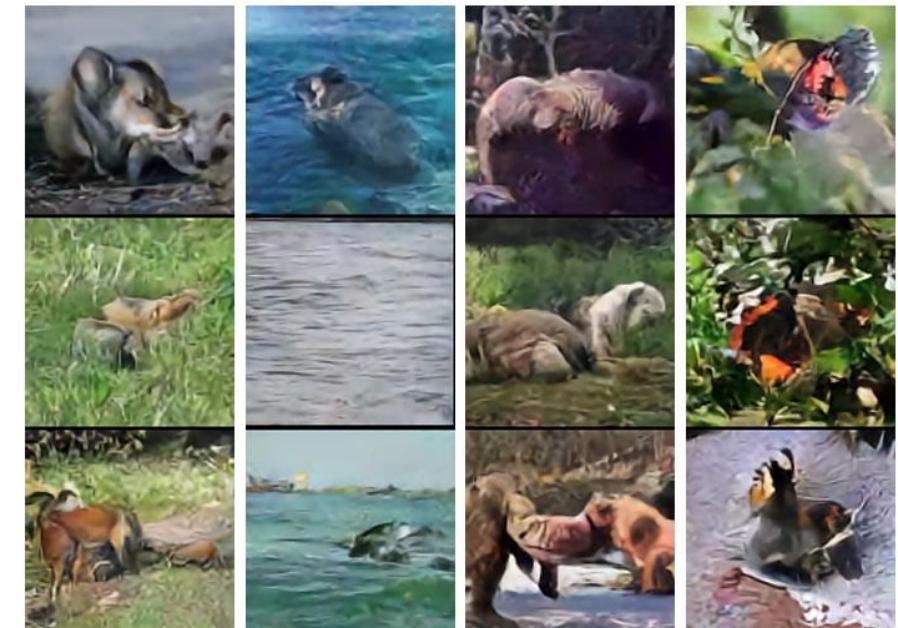
Problems:

- Insufficient fine details in reconstructions & low resolution
- Restricted long-term dependencies



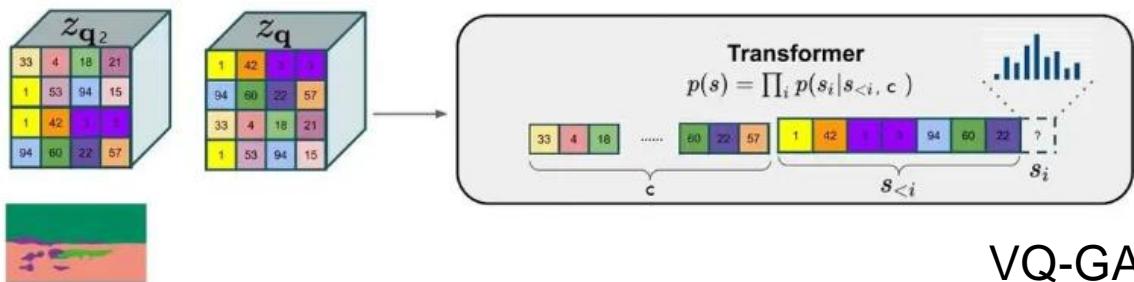
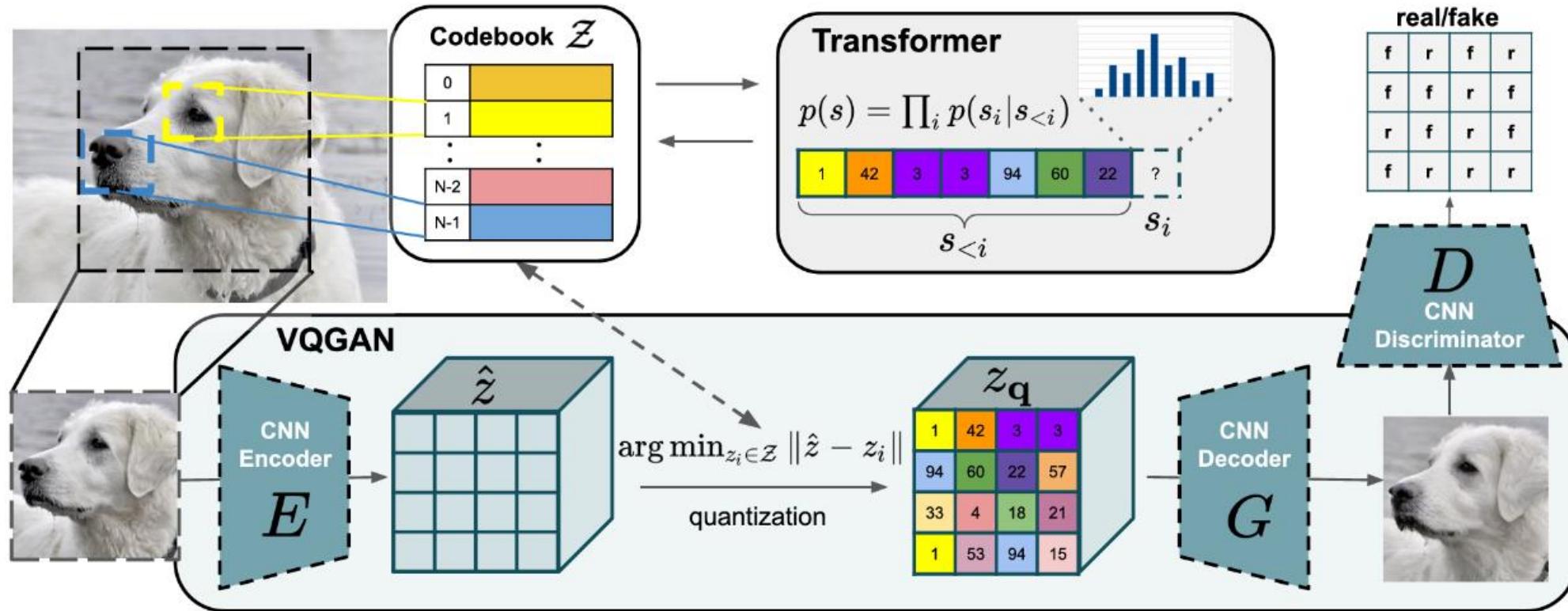
Solution:

- Perceptual loss & generative adversarial nets
- Transformer in discrete latent space



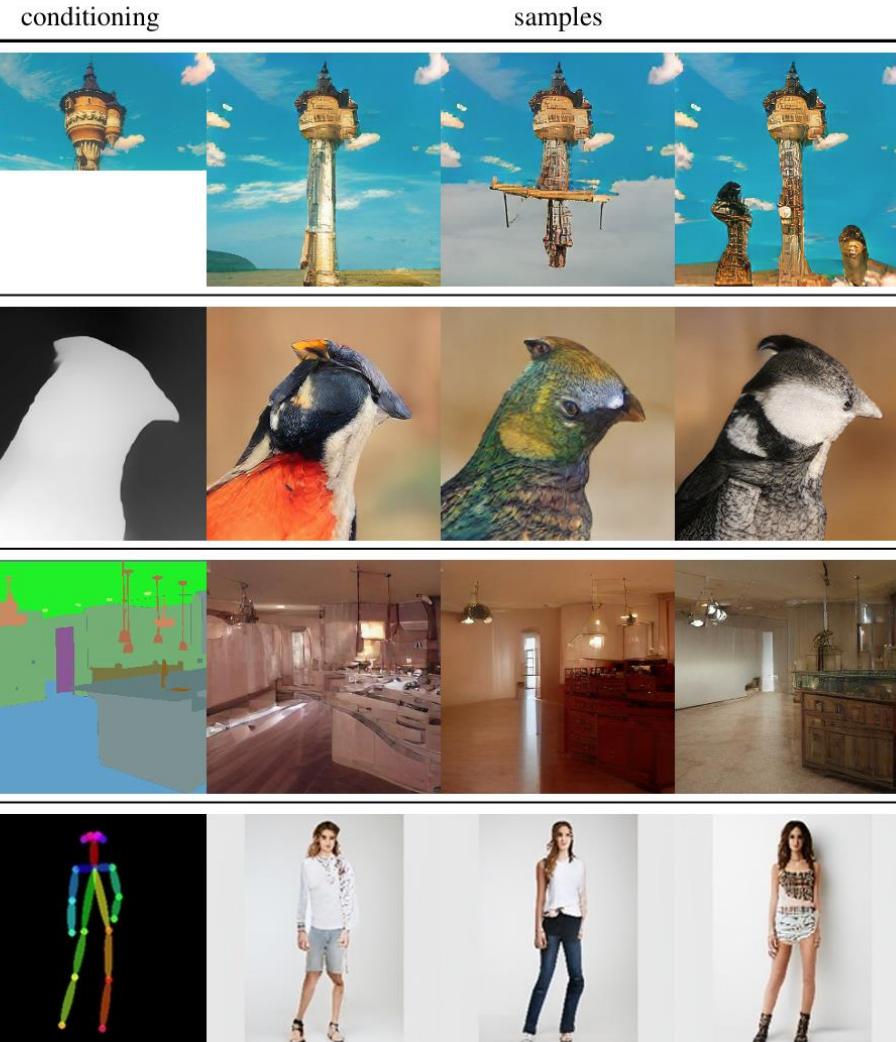
Generations from VQ-VAE

VQ-GAN



- Generative adversarial nets
- Perceptual loss
- Conditional inference

VQ-GAN



Generations with conditioning inputs



Generations with different resolution

Summary: GANs

Don't work with an explicit density function

Take game-theoretic approach: learn to generate from training distribution through 2-player game

Pros:

- Beautiful samples!

Cons:

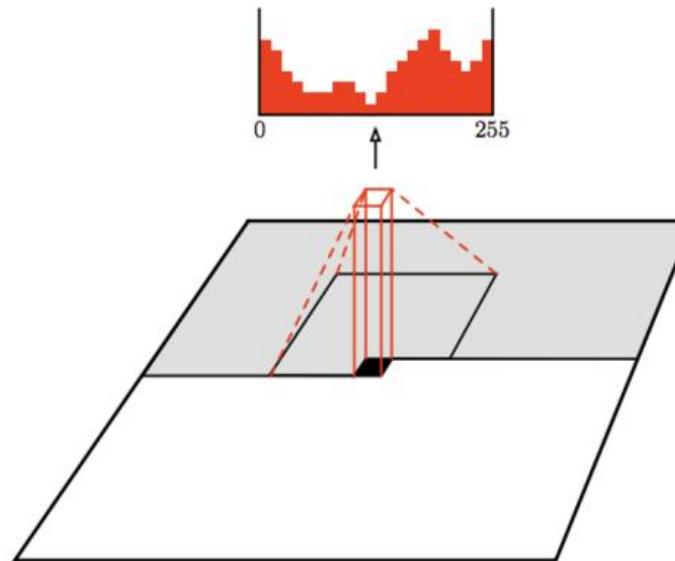
- Trickier / more unstable to train
- Can't solve inference queries such as $p(x)$, $p(z|x)$

Active areas of research:

- Better loss functions, more stable training (Wasserstein GAN, LSGAN, many others)
- Conditional GANs, GANs for all kinds of applications

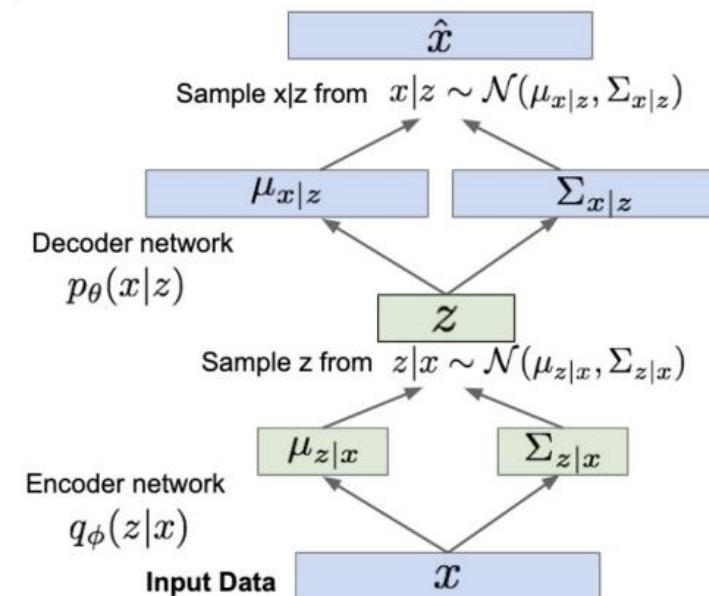
Summary

Autoregressive models:
PixelRNN, PixelCNN



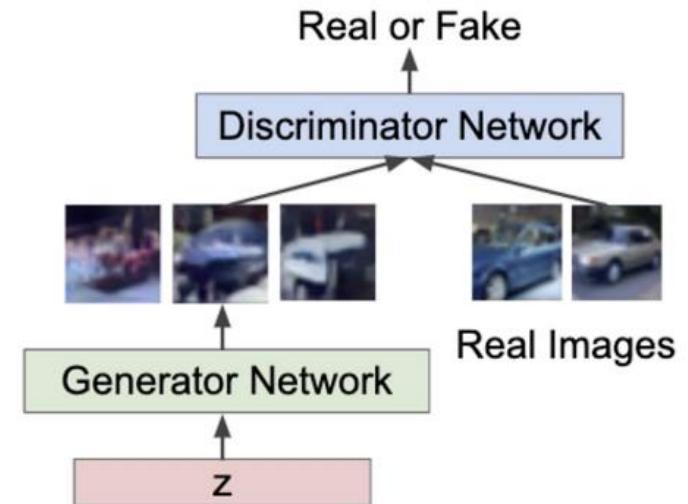
Van der Oord et al, “Conditional image generation with pixelCNN decoders”, NIPS 2016

Variational Autoencoders



Kingma and Welling, “Auto-encoding variational bayes”, ICLR 2013

Generative Adversarial Networks (GANs)



Goodfellow et al, “Generative Adversarial Nets”, NIPS 2014



Thanks!