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Glass transition in disordered clusters

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ABSTRACT

Using molecular dynamics simulations, the melting behavior of disordered aluminum clusters with 30 \sim 100 atoms has been systematically studied. We find that, all the disordered clusters undergo a glass transition before completely melting. We identify two characteristic temperatures, *i.e.* T_s and T_g (the glass transition temperature). When the temperature is higher than T_s , the cluster shape starts to change, accordingly the configurational entropy emerges. For temperatures between T_s and T_g , the disordered clusters show a remarkable cooperative diffusion, more importantly, the activation energy of cooperative diffusions below T_g is always lower than that of the liquid state. We have tried to understand such behavior from potential landscape and configurational entropies.

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1. Introduction

Even now, glass and glass transitions are a mystery [1–8]. The mystery is manifold, but the dual characters of solid-like physical properties and liquid-like structures are likely key issues. Mechanically, a glass is rigid, like a crystalline solid; however structurally, a glass has disordered features, like a supercooled liquid. Incredibly, it remains unsolved whether or not from glass to liquid, or vice versa, is a true thermodynamic phase transition [9–15]. Thus, it is hard to give a clear definition for the glass transition within the framework of thermodynamics. It is safe to say that the scientific debate on the glassy state is still lively.

Although the thermodynamic features of a phase transition is ambiguous during glass transitions, some physical properties, such as viscosity [16–19], do change abruptly around the glass transition temperature (T_g). The dramatic increase in viscosity is believed to deeply connect with the atomic dynamics. In fact, most thermodynamic theories on glass transitions hold that the dynamics behavior of atoms is involved. Especially, in Model-Coupling Theory (MCT) developed independently by Leutheusser [20], and Bengtzelius and coworkers [10], the dynamics process plays an essential role. The solution of MCT equations indicates that, there

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exist at least three atomic relaxation time scales, which make a glass different from both liquids and crystals [21].

For a long time, many investigations focused on atomic level dynamics both experimentally and theoretically [22–27]. Among them, computer simulations, such as molecular dynamics (MD) and Monte Carlo simulations, which can give the most direct observation of atomic motions, were widely adopted in previous studies. In fact, recent progress in understanding glasses at the atomic level are largely due to computer simulations [7,17,24,26, 28–33]. Although great efforts had been made in studies of glasses, one of the key issues, namely atomic level diffusion around T_g , is much less understood. The reason is that, for most glasses atomic diffusion is almost frozen except for local vibrations around T_g , which makes computer simulations inefficient due to time scale limitations.

In 1998, two of the present authors (DY and XG) showed that an Al_{43} cluster in its most stable configuration has disordered character, and melts through a typical glass transition process [34]. This glass was identified as an ideal glass [35], which is defined as a glass state with zero configurational entropy [36,37]. Recent study on this nanoscale glass indicates that below T_g , although atomic diffusion definitely slows down, the diffusive motion of atoms can be obtained within affordable simulation times [38]. The natural question is then, is this unusual phenomenon universal in glass transitions, or just in this case? If this phenomenon is common in glassy states, what does it mean for glasses or glass transitions?

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In this paper, we report a MD study on melting of disordered nanoclusters. We find that before melting, all the disordered clusters show typical glass transition behavior, associated with a remarkable cooperative diffusion.

2. Computational methods

In this paper, disordered aluminum clusters in the range of 30 to 100 atoms are investigated. For comparison, two highly symmetric clusters (Al $_{38}$ with D_{6h} symmetry and Al $_{55}$ with I_h symmetry) are also studied. The glue potential [39], which is widely used in MD simulations, is adopted to describe the interatomic interaction of Al. The ground state of the clusters is taken from previous work [40]. The constant-temperature MD method with free boundary conditions is used in all calculations. The simulation spans the sub-microsecond scale in sub-picosecond steps. In whole simulations, the momentum of the center of mass and the total angular momentum of clusters are carefully eliminated from the calculations, thus the temperature refers only to the vibrational modes.

To characterize the shape of a cluster, the principal radii of gyration are calculated [41]. The principal radii of gyration (R_1 , R_2 , R_3) of a cluster are calculated by $R_i = \sqrt{I_i/N}$, where I_i (i=1,2,3) is the principal moment of inertia of clusters. The volume of clusters is calculated based on the Wigner-Seitz primitive cell approximation [41]. Diffusion constants, which are an important thermodynamic quantity for identifying phases, are obtained through calculating the mean-square displacement of each atom.

To differentiate a cooperative diffusion from an individual diffusion, we have calculated the time-dependent displacement [38]

$$R_{c}(t) = \left(\frac{1}{N} \sum_{i=1}^{N} (r_{i}(t) - r_{i}(t + \Delta t))^{2}\right)^{\frac{1}{2}},$$

where N is the number of atoms, $r_i(t)$ denotes the position of the ith atom at time t, and Δt is a short duration, 5 ps in these studies. For a well-defined solid, the atoms only vibrate around their equilibrium positions, thus $R_c(t)$ has a very small amplitude and a small variation with time. If diffusion is fast and less cooperative among atoms, $R_c(t)$ will be very large and fluctuate uniformly with time, which corresponds to liquid sates. On the contrary, if diffusion is intermittently triggered by strong cooperative motion among atoms, $R_c(t)$ should show a remarkable variation with time.

3. Results and discussion

Fig. 1 depicts the average energy and volume per atom as a function of temperature for the ordered cluster Al₃₈ (top panel). A jump occurs at around 580 K (melting temperature) in both energy and volume, indicting a typical first-order solid-liquid phase transition. Associated with the solid-liquid phase transition, the radii of gyration also have a jump as shown in the lower panel of Fig. 1. Before melting, Al₃₈ keeps an almost ellipsoid shape due to its original symmetry, indicated by the two equal short and one long radii of gyration. After melting, the radii of gyration are equal, since a free-standing liquid droplet always has a spherical shape. A similar melting behavior is also found for an Al₅₅ cluster, which accords with previous studies [42].

The disordered clusters behave differently than the first-order transition of the ordered clusters. Fig. 2 presents the average energy and volume per atom as a function of temperature for disordered Al₄₆, Al₆₂ and Al₉₀ clusters. From Fig. 2, one can see that the melting behavior is totally different from that for ordered clusters (see Fig. 1). There is no abrupt change in either energy or volume, rather they change continuously over the entire temperature range. This is a typical glass transition behavior. T_g is estimated by

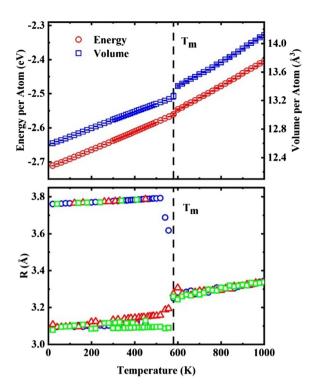


Fig. 1. (Color online) Top panel: the energy (circle) and volume (square) as a function of temperature for an ordered cluster Al_{38} . Bottom panel: three principal radii of gyration ($R_{1,2,3}$ indicated by different colors) for Al_{38} . A remarkable first order solid-liquid transition occurs at around 580 K.

linear extrapolation of the low temperature and high temperature dependence of energy or volume.

The radii of gyration of these disordered clusters also show a different trend from high symmetry clusters. Fig. 3 shows the corresponding radii of gyration for disordered clusters mentioned above. For these three disordered clusters, one can identity two characteristic temperatures (T_s and T_g) as shown in Fig. 3. For temperature lower than T_s (290 K, 80 K and 380 K for Al₄₆, Al₆₂ and Al₉₀, respectively), the three radii of gyration can be distinguished from one another, and are much closer to 0 K value, with only slight thermal expansion. This reflects the fact that these clusters keep their initial structures or shapes before T_s . For temperatures above T_g (527 K, 627 K and 497 K for Al_{46} , Al_{62} and Al₉₀, respectively), the cluster is completely melted. Accordingly, the three radii of gyration become equal. It is remarkable that, at temperatures between T_s and T_g , the shape of the disordered clusters changes, consequently, the three radii of gyration fluctuate dramatically. If the change in cluster shape is frequent, or simulations run long enough, the three radii of gyration should become much closer. In fact, the radii of gyration of Al₆₂ are almost equal, since this cluster has much lower diffusion activation energies (see below). Thus, T_s can be considered the starting temperature of the glass transition, since at temperature higher than T_s , the shape or structure of these clusters begins to change remarkably. Although Fig. 3 only presents the data for the three disordered clusters, a similar glass transition behavior has been observed in all studied disordered clusters. Namely, two characteristic temperatures (T_s and T_g) are universal in the melting process of disordered clus-

Fig. 4 shows the diffusion constants versus temperatures for the disordered clusters, where the vertical axis is a logarithmic scale and the horizontal axis is the reciprocal temperature. For disordered clusters, there exists a remarkable non-negligible atomic diffusivity below $T_{\rm g}$. The diffusion-temperature curve can be divided into two ranges higher or lower than $T_{\rm g}$. Diffusions in both

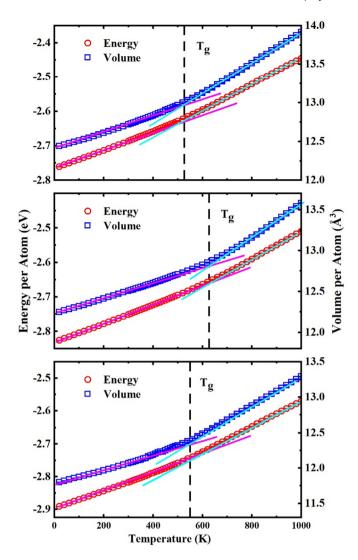


Fig. 2. (Color online) The energy (circle) and volume (square) per atom as a function of temperature for disordered clusters Al_{46} , Al_{62} and Al_{90} . Both energy and volume continuously change with temperature, indicating a glass transition behavior. The glass transition temperature (T_g) is identified by extrapolation of the low temperature and high temperature dependence of energy or volume (solid lines).

ranges can be well fitted with the Arrhenius relationship, but with different diffusion activation energies.

The activation energy of diffusion can be obtained by Arrhenius fitting to the diffusion constant-temperature data, namely the slope of the fitted lines. Fig. 5 shows the diffusion activation energy of glasses and liquids for all disordered clusters. Surprisingly, the diffusion activation energy of glasses is always less than that of the liquid state. The lower activation energy of glassy state is also obtained in one previous experiment [43].

The low activation energy of diffusion in glassy states is not due to the well-known pre-melting of surface atoms. To illustrate this, we calculated the diffusion constant of each atom versus its average position, which we defined as the average distance from the mass center of the cluster. Obviously, if only surface atoms melt, the average position of each atom will be almost unchanged. Fig. 6 depicts the data for Al₄₆, Al₆₂ and Al₉₀. It can be seen that, for clusters in liquid state (760 K, 760 K and 800 K for Al₄₆, Al₆₂ and Al₉₀, respectively), the diffusion constant of each atom is almost independent of atom position. Surprisingly, for disordered clusters below T_g (450 K, 200 K and 460 K for Al₄₆, Al₆₂ and Al₉₀, respectively), inner atoms also have non-negligible diffusion, comparable to that of surface atoms.

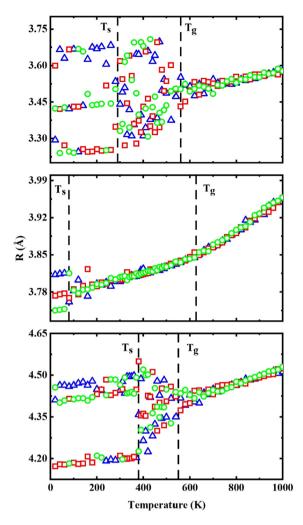


Fig. 3. (Color online) Three principal radii of gyration ($R_{1,2,3}$ indicated by different colors) for Al₄₆, Al₆₂ and Al₉₀ (from top to bottom).

Diffusion below T_g is attributed to cooperative diffusion among atoms, namely solid-solid structure-like transformations. To give evidence for the cooperative diffusion, we calculated time-dependent displacement $(R_c(t))$ at three different temperatures as shown in Fig. 7. It can be seen that, at high temperatures, $R_c(t)$ is large and varies uniformly with time, which is typical behavior for atomic diffusion in liquids with single atom behavior. At low temperatures, the atoms vibrate around their equilibrium positions, thus, $R_c(t)$ has a very small amplitude and a small variation with time, indicating the cluster is in a well-defined solid-like state. However, at intermediate temperatures between T_s and T_g , $R_c(t)$ shows a remarkable variation with time. During some short intervals, $R_c(t)$ is large, but these are interspersed with periods for which $R_c(t)$ exhibits solid behavior. When $R_c(t)$ suddenly becomes very large in a short interval, there are always a few atoms that diffuse quickly simultaneously. Thus, we suggest that, this kind of diffusion is intermittently triggered by strong cooperative motion among atoms. Recall that at this temperature the cluster is in the glassy state, which implies that atom diffusion is induced by a cooperative motion in a short time, i.e., a solid-solid structural transformation. Recent experimental and theoretical results also suggest cooperative motion in other glasses or supercooled liquids [44-51]. Similar behavior has been found for all studied disordered clusters.

Fig. 8 presents T_s and T_g for all the studied disordered clusters ranging from 30 to 100 atoms. With increasing cluster sizes, both T_s and T_g fluctuate. There is no significant correlation between

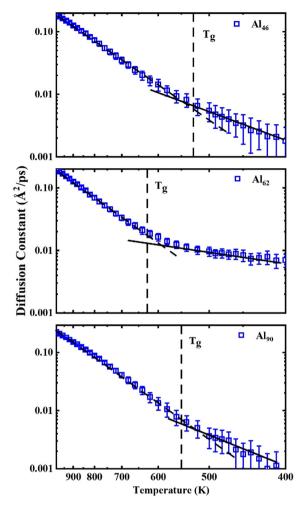


Fig. 4. (Color online) Temperature dependence of the diffusion constant for Al_{46} , Al_{62} and Al_{90} (from top to bottom), where the vertical axis is logarithmic and the horizontal axis is reciprocal temperature. The solid lines are the Arrhenius fits to the data. The slope of the fitted lines indicates the activation energies of diffusions.

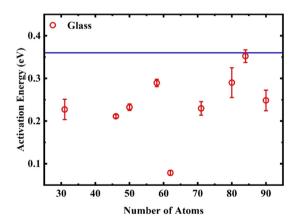


Fig. 5. (Color online) Diffusion activation energy of glass state for different disordered clusters. The value is obtained by Arrhenius fitting to the diffusion constant-temperature data. The activation energy of the liquid state (solid line) is averaged over all clusters. From the figure, it is obvious that the activation energy of glass is always less than that of liquid.

the characteristic temperatures and cluster sizes, which is different from high symmetry clusters, for which T_m usually increases with the increase of cluster size [42]. Interestingly, an approximated anti-correlation between $T_{\rm S}$ and $T_{\rm g}$ can be seen in Fig. 8,

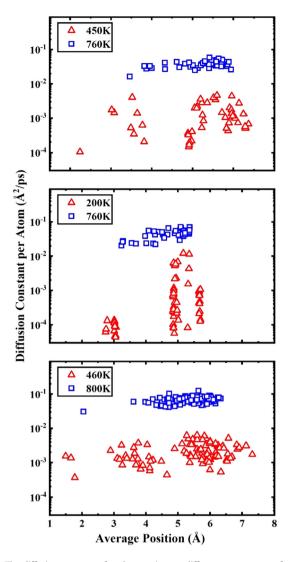


Fig. 6. The diffusion constant of each atom in two different temperatures for Al₄₆, Al₆₂ and Al₉₀ (from top to bottom respectively). Here the average position is defined as the average distance from the mass center of the cluster over the simulation times. In the glass state (below T_g), inner atoms also have non-negligible diffusion compared to surface atoms.

namely the lower T_s is, the higher T_g is. Looking again at Fig. 5, it is seen that there is another positive correlation between T_s and activation energies of diffusion.

As previously asserted [38], the free energy of a glass before melting should involve a contribution from the configuration entropy. As we have pointed out, between T_s and T_g , the configurational transformation dominates the long-range diffusivity of atoms. Considering the contribution of the configuration entropy, a glassy cluster should further lower its free energy, which will affect its glass transition temperature. Although precisely calculating of the configuration entropy is out of our current scope, the correlations between T_s and T_g , as well as T_s and activation energies of diffusion, may support this kind of contribution. The lower T_s reflects the fact that the configurational transformation can be staggered at low temperature. Lower activation energies of diffusion make the system visit more configurations. Both factors should tend to a higher configuration entropy, thus leading to a higher T_g . The data shown in Figs. 5 and 8 seemingly supports this conclusion. It needs to point out that, since T_g is not solely dependent on entropies but also on energies, the exact calculation of T_g is extremely difficult.

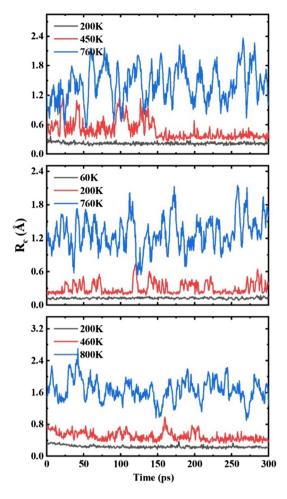


Fig. 7. (Color online) The time-dependent displacement ($R_c(t)$) at three temperatures for Al₄₆, Al₆₂ and Al₉₀ (from top to bottom respectively). At intermediate temperatures between T_s and T_g , $R_c(t)$ shows a remarkable variation with time, indicating a cooperative motion.

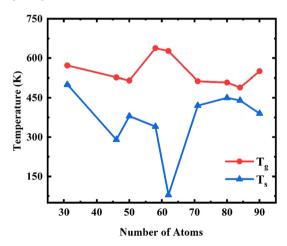


Fig. 8. (Color online) T_g and T_s versus the number of atoms for all the studied disordered clusters. T_g is obtained through extrapolation of temperature dependence of energy or volume. T_s is determined from the change of radii of gyration with temperature.

4. Summary

We have found that the glass transition is universal behavior in disordered clusters. During the transition, energies and volumes of clusters change continuously. For all the disordered clusters, the shape or structure starts to change at a characteristic temperature (T_s) lower than the glass transition temperature (T_g) . Atoms in the disordered clusters exhibit clear cooperative diffusion between T_s and T_g , which has smaller activation energy than that of liquid state. Diffusion below the glass transition temperature indicates the emergence of configuration entropies.

Conflicts of interest

There are no conflicts to declare.

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