Preprint number: XXXX-XXXX

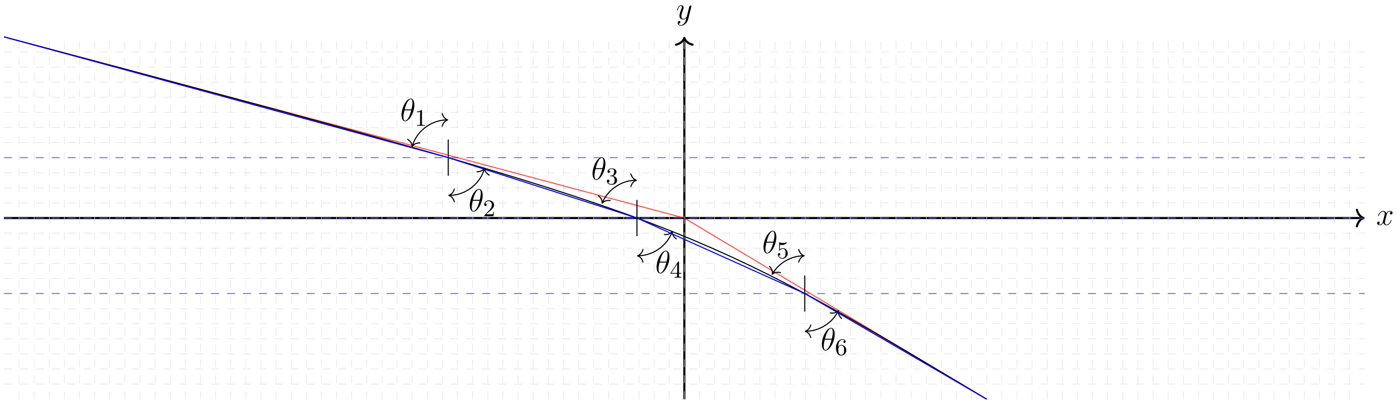
Joint Research of Optics and Fluid Interface

Doumu/Fudepia

E-mail: fudepia@outlook.jp

The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here. The abstract text goes here.

Subject Index xxxx, xxx



1 Differential Analytic

$$\forall i \in \mathbb{N}: \frac{\sin\theta_{2i-1}}{\sin\theta_{2i}} = \frac{v_{2i-1}}{v_{2i}} \tag{1}$$

Let the ray inbound from upper toward lower and refracs at origin. Let $\theta(y)$ be the acute angle between the velocity vector at that instant and the y-axis.

Let $y = P(x)$ be the path light actually taken, now our goal is to solve $P(x)$ and also the function of speed of light within specific y $v(y) = v(P(x))$.

$$\lim_{\Delta x \to 0} \frac{\sin\theta(P(x))}{\sin(P(x+\Delta x))} = \frac{v(P(x))}{v(P(x+\Delta x))} \tag{2}$$

$$\lim_{\Delta y \to 0} \frac{\sin\theta(y)}{\sin(y+\Delta y)} = \frac{v(y)}{v(y+\Delta y)} \tag{3}$$

$$\lim_{r \to 0} \frac{\sin\theta(y)}{\sin\left(\theta(y) + r\theta'(y)\right)} = \frac{v(y)}{v(y) + rv'(y)} \tag{4}$$

$$\lim_{r \to 0} \sin\theta(y)v(y) + rv'(y) = v(y)\sin\big(\theta(y) + r\theta'(y)\big) \tag{5}$$

2 Setup

Let Δ be the total width the light travels with path not being straight, hence:

$$\forall x \notin [-\frac{\Delta}{2}, \frac{\Delta}{2}] : \frac{d^2}{dx^2}y = 0 \tag{6}$$

Now define the light ray incoming from left-hand side towards right-hand side and (suppose transition layers' width approaches zero) refracs at origin. And let h be the thickness of the transition layers.

Now let's assume the incoming transition layer has the same thickness as outbounding transition layer, hence we get:

$$\Delta = \frac{h}{2}(\tan\theta_{in} + \tan\theta_{out}) \tag{7}$$

Now we can first confirm our path function is a function of $\theta_{in}, \theta_{out}, h$

$$P(\theta_{in}, \theta_{out}, h) = \tag{8}$$

3 Playing with Bezier Curve

First setup the environment

```
h, t=var("h t")
hh=h/2
theta=var("theta")
tprime=var("tprime", latex_name=r"\theta^\prime")
assume(h, "constant")
assume(h>0)
assume(0<theta, theta<pi/2)
assume(0<tprime, tprime<pi/2)
```

And now we could try to solve functions like $y(x)$ from parametric bezier curve:

```
# x(t)=(t-1)^2*tan(theta)*hh*-1+t^2*tan(tprime)*hh
y=(t-1)^2*hh-t^2*hh
```

First we try to find $t(x)$

```
result=[]
[((abs(res.rhs()(x=tan(theta)*hh*-1)(h=2, theta=0.2, tprime=0.4))<1e-12 and
 abs(res.rhs()(x=tan(tprime)*hh)(h=2, theta=0.2, tprime=0.4)-1)<1e-12) and
```

```
    result.append(res))
    for res in solve((t-1)^2*tan(theta)*hh*-1+t^2*tan(tprime)*hh==x, t)]
  t=result[0].rhs() if len(result)==1 else None
```

Where we get two solutions and only one will satisfied our assumptions (the first print is 0 and the second yields 1[1]), which is:

$$t(x) = \frac{h\tan(\theta) - \sqrt{h^2\tan(\theta)\tan(\theta') - 2(h\tan(\theta) - h\tan(\theta'))x}}{h\tan(\theta) - h\tan(\theta')}$$

And substituting back to $y(t)$ we could get

4 Analyzing

Given a full path from v_1 to v_n, whose path will be $y(v_1, v_n, h)(x)$, we could seperate it into different sections.

$$\underset{i\to i+1}{\Delta} = \frac{h}{n-1}\frac{1}{2}[-\tan\theta_i, \tan\theta_{i+1}] \tag{9}$$

$$\sum_{i=1}^{n-1} \underset{i\to i+1}{\Delta} = \frac{h}{2}(\tan\theta_1 + \tan\theta_n)$$

$$\forall x \in \underset{i\to i+1}{\Delta} : \tag{10}$$

$(x, y(v_1, v_2, h_{12})(x)) + (\underset{2\to n}{\Delta}, \underset{2\to n}{h})$

[1] Here we use $EPS = 1e-12$ as we're testing it with arbritarily chosen values which makes them no longer symbolic.

5 Finding path

5.1 Constructing Layers

Let function $v(\theta_{in}, \theta_{out}, \Delta, \delta)$ be the velocity of layer δ (from the entry interface), whose entry velocity is θ_{in}, exit velocity is be θ_{out}, and the whole layer thickness is Δ.[2]

$$a = v(x, y, \beta - \alpha, \alpha) \tag{11a}$$

$$b = v(x, y, \beta - \alpha, \beta) \tag{11b}$$

$$\alpha \leq \beta \tag{11c}$$

Let $a = v(x, y, \alpha)$ and $b = v(x, y, \beta)$ $(\alpha \leq \beta)$, $\forall p \in [\alpha, \beta] : v(a, b, p - \alpha) = v(x, y, p)$

5.2 Fractal Structure

$$\text{Let } P(\theta_{in}, \theta_{out}, h) \text{ be the path taken} \tag{12}$$

$$P(\theta_{in \to \alpha}, \alpha) = P(\theta_{in}, \theta_\alpha \tag{13}$$

[2] Here we suppose every layer paraelles

6 Regarding Special Relativity

So first let the real velocity v, whom maintains the linear properties of traditional non-relativistic velocity.

Linear to non-linear (relativistic) velocity:

$$v_S = \frac{v_{\text{リ}} \text{エ}_S \text{リ}}{\text{エ}_S S} = v_{\text{リ}} \text{エ}_S \text{リ} \tag{14}$$

6.1 Constant force/acceleration

Let a point located at origin, with initial velocity $v(0) = 0$.

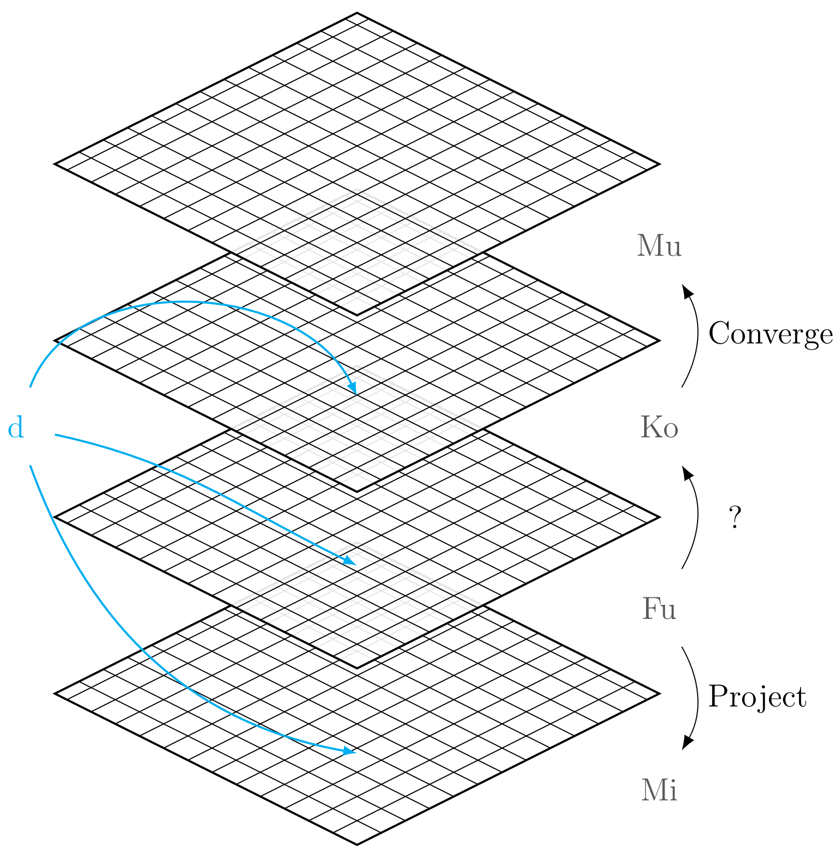
$$a_{\text{ヒ}}(v, F, m_{rest}) = \frac{F}{m_{rest}} \sqrt{1 - \frac{v^2}{c^2}}^3 \tag{15a}$$

$$a_{\text{リ}}(t) = k \tag{15b}$$

$$v_{\text{ヒ}}(t) = v_{\text{リ}} \text{エ}_S \text{リ} = kt\text{エ}_S \text{リ} \tag{15c}$$

$$v_{\text{ヒ}}(t) = \int \frac{F}{m_{rest}} \sqrt{1 - \frac{v_{\text{ヒ}}(t)^2}{c^2}}^3 \tag{15d}$$

7 Model of Particles



8 Conclusion

The conclusion text goes here.

Acknowledgment

Insert the Acknowledgment text here.

References

[1] https://www.mail-archive.com/dou-geometry@googlegroups.com/msg00004/____________-_____________ ______________.docx

A Appendix head