

Joint Research of Optics and Fluid Interface

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1 Setup

Let Δ be the total width the light travels with path not being straight, hence:

$$\forall x \notin [-\frac{\Delta}{2}, \frac{\Delta}{2}] : \frac{d^2}{dx^2}y = 0 \quad (1)$$

Now define the light ray incoming from left-hand side towards right-hand side and (suppose transition layers' width approaches zero) refracts at origin. And let h be the thickness of the transition layers.

Now let's assume the incoming transition layer has the same thickness as outbounding transition layer, hence we get:

$$\Delta = \frac{h}{2}(\tan \theta_{in} + \tan \theta_{out}) \quad (2)$$

Now we can first confirm our path function is a function of $\theta_{in}, \theta_{out}, h$

$$P(\theta_{in}, \theta_{out}, h) = \quad (3)$$

2 Playing with Bezier Curve

First setup the environment

```
h, t=var("h t")
hh=h/2
theta, phi=var("theta phi")
assume(h, "constant")
assume(h>0)
assume(0<theta, theta<pi/2)
assume(0<phi, phi<pi/2)
```

And now we could try to solve functions like $y(x)$ from parametric bezier curve:

```
# x(t)=(t-1)^2*tan(theta)*hh*-1+t^2*tan(phi)*hh
y=(t-1)^2*hh-t^2*hh
```

First we try to find $t(x)$

```
result=[]
[[(abs(res.rhs()(x=tan(theta)*hh*-1)(h=2, theta=0.2, phi=0.4))<1e-12 and
abs(res.rhs()(x=tan(phi)*hh)(h=2, theta=0.2, phi=0.4)-1)<1e-12) and
result.append(res))
```

```
for res in solve((t-1)^2*tan(theta)*hh*-1+t^2*tan(phi)*hh==x, t)]
t=result[0].rhs() if len(result)==1 else None
```

Where we get two solutions and only one will satisfied our assumptions (the first print is 0 and the second yields 1¹), which is:

$$-\frac{h \tan(\theta) - \sqrt{h^2 \tan(\phi) \tan(\theta) + 2(h \tan(\phi) - h \tan(\theta))x}}{h \tan(\phi) - h \tan(\theta)}$$

And substituting back to $y(t)$ we could get

3 Analyzing

Given a full path from v_1 to v_n , whose path will be $y(v_1, v_n, h)(x)$, we could separate it into different sections.

$$\Delta_{i \rightarrow i+1} = \frac{h}{n-1} \frac{1}{2} (\tan \theta_i + \tan \theta_{i+1}) \quad (4)$$

$$\sum_{i=1}^{n-1} \Delta_{i \rightarrow i+1} = \frac{h}{2} (\tan \theta_1 + \tan \theta_n)$$

$$\forall x \in \Delta_{i \rightarrow i+1} : \quad (5)$$

$$(x, y(v_1, v_2, h_{12})(x)) + \left(\Delta_{2 \rightarrow n}, \frac{h}{2 \rightarrow n} \right)$$

¹ Here we use $EPS = 1e - 12$ as we're testing it with arbitrarily chosen values which makes them no longer symbolic.

4 Finding path

4.1 Constructing Layers

Let function $v(\theta_{in}, \theta_{out}, \Delta, \delta)$ be the velocity of layer δ (from the entry interface), whose entry velocity is θ_{in} , exit velocity is θ_{out} , and the whole layer thickness is Δ .²

$$a = v(x, y, \beta - \alpha, \alpha) \tag{6a}$$

$$b = v(x, y, \beta - \alpha, \beta) \tag{6b}$$

$$\alpha \leq \beta \tag{6c}$$

Let $a = v(x, y, \alpha)$ and $b = v(x, y, \beta)$ ($\alpha \leq \beta$), $\forall p \in [\alpha, \beta] : v(a, b, p - \alpha) = v(x, y, p)$

4.2 Fractal Structure

Let $P(\theta_{in}, \theta_{out}, h)$ be the path taken (7)

$$P(\theta_{in \rightarrow \alpha}, \alpha) = P(\theta_{in}, \theta_{\alpha}) \tag{8}$$

² Here we suppose every layer paraelles

5 Regarding Special Relativity

So first let the real velocity v , whom maintains the linear properties of traditional non-relativistic velocity.

Linear to non-linear (relativistic) velocity:

$$v_S = \frac{v_{\mathfrak{I}} \mathfrak{I}_S^{\mathfrak{I}}}{\mathfrak{I}_S S} = v_{\mathfrak{I}} \mathfrak{I}_S^{\mathfrak{I}} \quad (9)$$

5.1 Constant force/acceleration

Let a point located at origin, with initial velocity $v(0) = 0$.

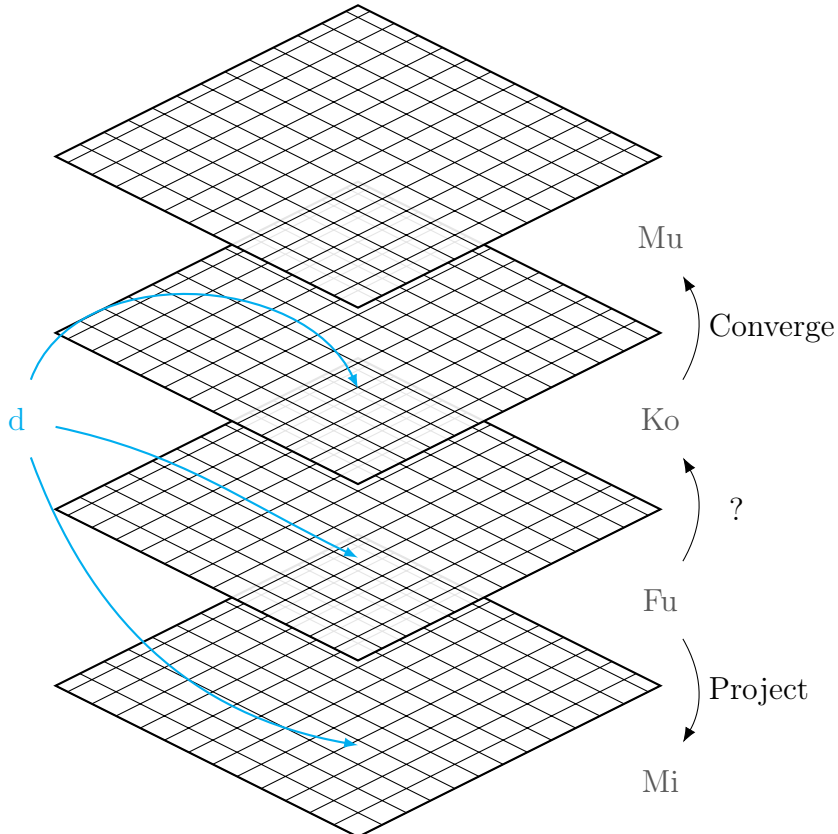
$$a_{\mathfrak{I}}(v, F, m_{rest}) = \frac{F}{m_{rest}} \sqrt{1 - \frac{v^2}{c^2}}^3 \quad (10a)$$

$$a_{\mathfrak{I}}(t) = k \quad (10b)$$

$$v_{\mathfrak{I}}(t) = v_{\mathfrak{I}} \mathfrak{I}_S^{\mathfrak{I}} = kt \mathfrak{I}_S^{\mathfrak{I}} \quad (10c)$$

$$v_{\mathfrak{I}}(t) = \int \frac{F}{m_{rest}} \sqrt{1 - \frac{v_{\mathfrak{I}}(t)^2}{c^2}}^3 \quad (10d)$$

6 Model of Particles



7 Conclusion

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Acknowledgment

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References

- [1] <https://www.mail-archive.com/dou-geometry@googlegroups.com/msg00004/------.docx>

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