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Joint Research of Optics and Fluid Interface

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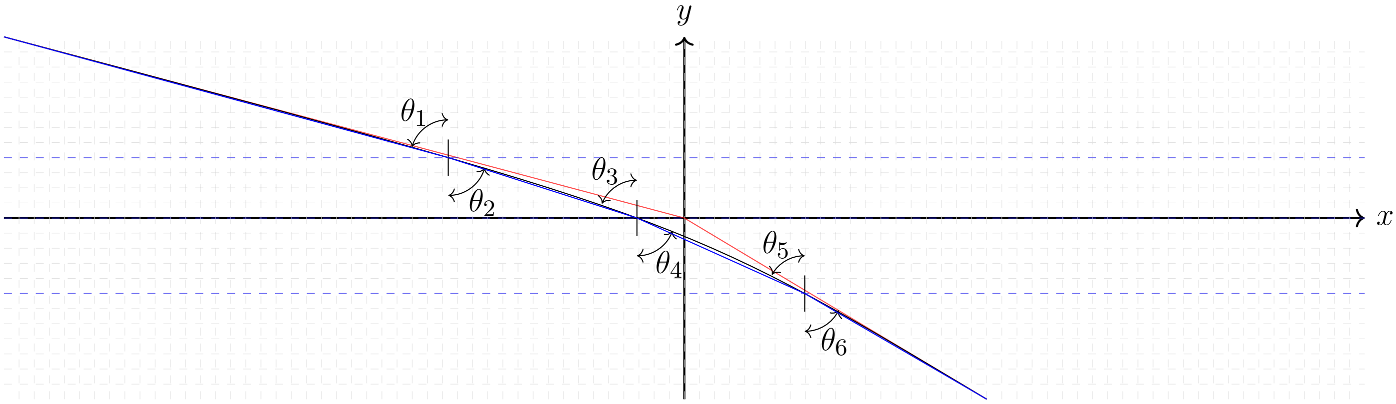


1 Differential Analytic

$$\forall i \in \mathbb{N} : \frac{\sin \theta_{2i-1}}{\sin \theta_{2i}} = \frac{v_{2i-1}}{v_{2i}} \tag{1}$$

Let the ray inbound from upper toward lower and refracs at origin. Let $\theta(y)$ be the acute angle between the velocity vector at that instant and the y-axis.

Let $y = P(x)$ be the path light actually taken, now our goal is to solve $P(x)$ and also the function of speed of light within specific y $v(y) = v(P(x))$.

$$\lim_{\Delta x \to 0} \frac{\sin \theta(P(x))}{\sin(P(x + \Delta x))} = \frac{v(P(x))}{v(P(x + \Delta x))} \tag{2}$$

$$\lim_{\Delta y \to 0} \frac{\sin \theta(y)}{\sin(y + \Delta y)} = \frac{v(y)}{v(y + \Delta y)} \tag{3}$$

$$\lim_{r \to 0} \frac{\sin \theta(y)}{\sin(\theta(y) + r\theta'(y))} = \frac{v(y)}{v(y) + rv'(y)} \tag{4}$$

$$\lim_{r \to 0} \sin \theta(y) v(y) + rv'(y) = v(y) \sin\big(\theta(y) + r\theta'(y)\big) \tag{5}$$

2 Setup

Let Δ be the total width the light travels with path not being straight, hence:

$$\forall x \notin [-\frac{\Delta}{2}, \frac{\Delta}{2}] : \frac{d^2}{dx^2}y = 0 \tag{6}$$

Now define the light ray incoming from left-hand side towards right-hand side and (suppose transition layers' width approaches zero) refracs at origin. And let h be the thickness of the transition layers.

Now let's assume the incoming transition layer has the same thickness as outbounding transition layer, hence we get:

$$\Delta = \frac{h}{2}(\tan\theta_{in} + \tan\theta_{out}) \tag{7}$$

Now we can first confirm our path function is a function of $\theta_{in}, \theta_{out}, h$

$$P(\theta_{in}, \theta_{out}, h) = \tag{8}$$

3 Continuous Geometric Mean

$$v(x) = \begin{cases} a, & x < -\hbar \\ b, & x > \hbar \\ a(\frac{b}{a})^{\frac{x-(-\hbar)}{h}} & \text{otherwise} \end{cases} \tag{9}$$

4 Playing with Bezier Curve (failed)

First setup the environment

```
h, t=var("h t")
hh=h/2
theta=var("theta")
tprime=var("tprime", latex_name=r"\theta^\prime")
assume(h, "constant")
assume(h>0)
assume(0<theta, theta<pi/2)
assume(0<tprime, tprime<pi/2)
```

And now we could try to solve functions like $y(x)$ from parametric bezier curve:

```
# x(t)=(t-1)^2*tan(theta)*hh*-1+t^2*tan(tprime)*hh
y=(t-1)^2*hh-t^2*hh
```

First we try to find $t(x)$

```
result=[]
[((abs(res.rhs()(x=tan(theta)*hh*-1)(h=2, theta=0.2, tprime=0.4))<1e-12 and
 abs(res.rhs()(x=tan(tprime)*hh)(h=2, theta=0.2, tprime=0.4)-1)<1e-12) and
 result.append(res))
 for res in solve((t-1)^2*tan(theta)*hh*-1+t^2*tan(tprime)*hh==x, t)]
t=result[0].rhs() if len(result)==1 else None
```

Where we get two solutions and only one will satisfied our assumptions (the first print is 0 and the second yields 1[1]), which is:

$$t(x) = \frac{h\tan(\theta) - \sqrt{h^2\tan(\theta)\tan(\theta') - 2\,(h\tan(\theta) - h\tan(\theta'))x}}{h\tan(\theta) - h\tan(\theta')}$$

And substituting back to $y(t)$ we could get

5 Analyzing

Given a full path from v_1 to v_n, whose path will be $y(v_1, v_n, h)(x)$, we could seperate it into different sections.

$$\underset{i\to i+1}{\Delta} = \frac{h}{n-1}\frac{1}{2}[-\tan\theta_i, \tan\theta_{i+1}] \tag{10}$$

$$\sum_{i=1}^{n-1} \underset{i\to i+1}{\Delta} = \frac{h}{2}(\tan\theta_1 + \tan\theta_n)$$

$$\forall x \in \underset{i\to i+1}{\Delta} : \tag{11}$$

$(x, y(v_1, v_2, h_{12})(x)) + (\underset{2\to n}{\Delta}, \underset{2\to n}{h})$

[1] Here we use $EPS = 1e - 12$ as we're testing it with arbritarily chosen values which makes them no longer symbolic.

6 Finding path

6.1 Constructing Layers

Let function $v(\theta_{in}, \theta_{out}, \Delta, \delta)$ be the velocity of layer δ (from the entry interface), whose entry velocity is θ_{in}, exit velocity is be θ_{out}, and the whole layer thickness is Δ.[2]

$$a = v(x, y, \beta - \alpha, \alpha) \tag{12a}$$

$$b = v(x, y, \beta - \alpha, \beta) \tag{12b}$$

$$\alpha \leq \beta \tag{12c}$$

Let $a = v(x, y, \alpha)$ and $b = v(x, y, \beta)$ $(\alpha \leq \beta)$, $\forall p \in [\alpha, \beta] : v(a, b, p - \alpha) = v(x, y, p)$

6.2 Fractal Structure

$$\text{Let } P(\theta_{in}, \theta_{out}, h) \text{ be the path taken} \tag{13}$$

$$P(\theta_{in \to \alpha}, \alpha) = P(\theta_{in}, \theta_{\alpha} \tag{14}$$

[2] Here we suppose every layer paraelles

7 Regarding Special Relativity

So first let the real velocity v, whom maintains the linear properties of traditional non-relativistic velocity.

Linear to non-linear (relativistic) velocity:

$$v_S = \frac{v_{\text{リ}} \text{エ}_S \text{リ}}{\text{エ}_S S} = v_{\text{リ}} \text{エ}_S \text{リ} \tag{15}$$

7.1 Constant force/acceleration

Let a point located at origin, with initial velocity $v(0) = 0$.

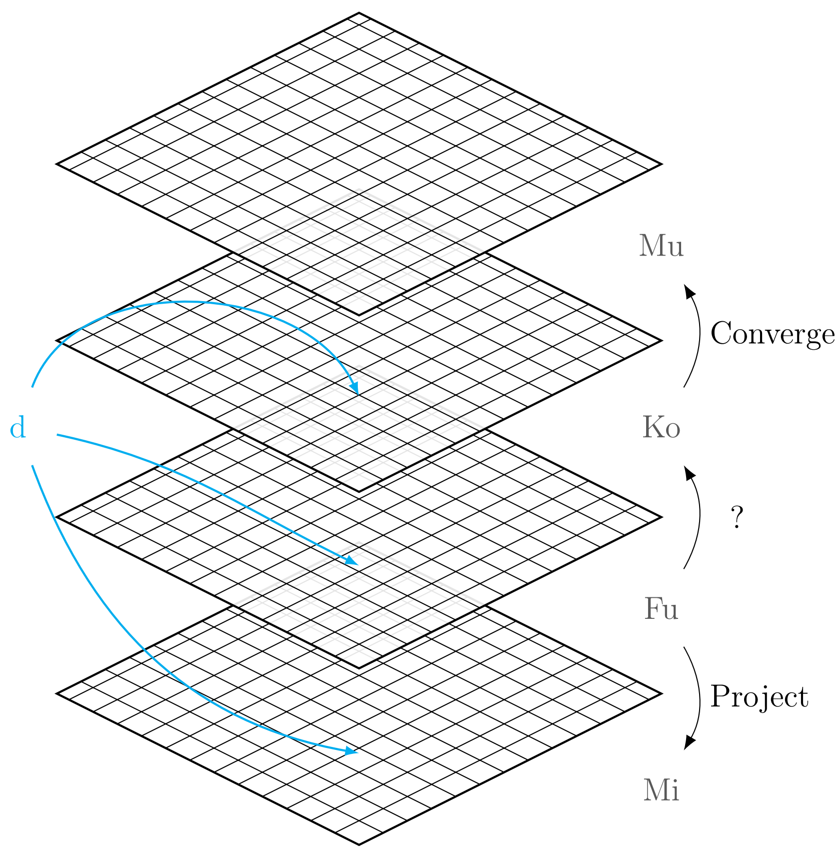
$$a_{\text{ヒ}}(v, F, m_{rest}) = \frac{F}{m_{rest}} \sqrt{1 - \frac{v^2}{c^2}}^3 \tag{16a}$$

$$a_{\text{リ}}(t) = k \tag{16b}$$

$$v_{\text{ヒ}}(t) = v_{\text{リ}} \text{エ}_S \text{リ} = kt \text{エ}_S \text{リ} \tag{16c}$$

$$v_{\text{ヒ}}(t) = \int \frac{F}{m_{rest}} \sqrt{1 - \frac{v_{\text{ヒ}}(t)^2}{c^2}}^3 \tag{16d}$$

8 Model of Particles



9 Conclusion

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Acknowledgment

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References

[1] https://www.mail-archive.com/dou-geometry@googlegroups.com/msg00004/____________-_____________
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